Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

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February 1974
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Calendar

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting *in person* must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

Meeting Number	Date	Place	Deadline for Abstracts* and News Items
713	April 10–13, 1974	New York, New York	Feb. 21, 1974
714	April 27, 1974	Santa Barbara, California	Feb. 21, 1974
715	May 13–18, 1974	De Kalb, Illinois	March 27, 1974
---	August 1974	No summer meeting; International Congress (see below)	June 15, 1974 (News Items only)
716	October 26, 1974	Middletown, Connecticut	
717	November 8–9, 1974	Nashville, Tennessee	
718	November 22–23, 1974	Los Angeles, California	
719	November 23, 1974	Houston, Texas	
720	January 23–27, 1975 (81st Annual Meeting)	Washington, D.C.	
	April 18–19, 1975	Monterey, California	
	August 18–22, 1975	Kalamazoo, Michigan	
	January 22–26, 1976 (82nd Annual Meeting)	San Antonio, Texas	

*Deadline for abstracts not presented at a meeting (by title). June 1974 issue: March 20
August 1974 issue: June 8

OTHER EVENTS

August 21–29, 1974	International Congress of Mathematicians Vancouver, B.C., Canada	April 15, 1974

The zip code of the Post Office Box of the Society has been changed from 02904 to 02940. Correspondents are requested to note this change in their records.

Please affix the peel-off label on these *Notices* to correspondence with the Society concerning fiscal matters, changes of address, promotions, or when placing orders for books and journals.

The *Notices* of the American Mathematical Society is published by the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, in January, February, April, June, August, October, November, and December. Price per annual volume is $10. Price per copy $3. Special price for copies sold at registration desks of meetings of the Society, $1 per copy. Subscriptions, orders for back numbers (back issues of the last two years only are available), and inquiries should be addressed to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940. Second class postage paid at Providence, Rhode Island, and additional mailing offices.

Notices OF THE AMERICAN MATHEMATICAL SOCIETY

Everett Pitcher and Gordon L. Walker, Editors
Wendell H. Fleming, Associate Editor

CONTENTS

The Seven Hundred Twelfth Meeting
The University of Florida
Gainesville, Florida
March 7 – 8, 1974

The seven hundred twelfth meeting of the American Mathematical Society will be held at the University of Florida, in Gainesville, Florida, from noon Thursday, March 7, until noon Friday, March 8, 1974. The meeting of the American Mathematical Society will be followed by the regular annual meeting of the Florida Section of the Mathematical Association of America.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, there will be two one-hour addresses presented. Professor A. T. Bharucha-Reid of Georgia Institute of Technology will give an address entitled "Probabilistic operator theory," and an address entitled "The decision problem for recursively enumerable degrees" will be presented by Professor J. R. Shoenfield of Duke University.

There will be two special sessions, one entitled Dynamical Systems, Flows and One-Parameter Semigroups of Transformations, to be organized by Professor John Neuberger of Emory University. The participants will include Professors J. R. Dorroh, J. A. Goldstein, R. Houston, G. G. Johnson, D. L. Lovelady, C. S. Reed, J. A. Reneke, J. T. Sandefur, Jr., and B.J. Wichnoski. The other special session on Ordered Groups, to be organized by Professor Jorge Martinez of the University of Florida, will include Professors R. D. Bleier, P. F. Conrad, A. M. W. Glass, W. C. Holland, S. H. McCleary, J. L. Mott, and J. Scott.

There will also be sessions for contributed papers on Thursday afternoon and Friday morning.

The registration desk will be located in the J. Wayne Reitz Union. Registration hours will be from noon to 5:00 p.m., on Thursday, March 7, and all day Friday, March 8.

The registration fee for the Society meeting will be $1 (noon Thursday, March 7, through noon Friday, March 8). For the regular annual meeting of the Florida Section of MAA (noon Friday, March 8, through Saturday, March 9, the registration fee will be $1 also.

Gainesville is located just east of Interstate 75 and is served by Eastern Air Lines. The most convenient flight into Gainesville from most points in the southeast is through Jacksonville International Airport. Rental cars are also available at Jacksonville International. It is approximately an hour and a half drive from Jacksonville to Gainesville.

Four motels near the campus are holding blocks of rooms with a deadline of February 21. Reservations should be made directly with them, with mention of this meeting included in that correspondence.

FLAGLER INN
P. O. Box 1406
Phone: (904) 376-1661
Single $15
Double 20 ($4.00 each add. person)

GAINESVILLE HILTON
2900 S. W. 13th Street
Phone: (904) 377-4000
Single $18
Double 26 ($5.00 each add. person)

HOLIDAY INN
1900 S. W. 13th Street
Phone: (904) 372-3311
Single $13
Double 18 ($3.00 each add. person over double rate)

UNIVERSITY INN
1901 S. W. 13th Street
Phone: (904) 372-6333
Single $11.50
Double 15.50
17.50 for three persons
18.50 for four persons

There will be a cocktail party (cash bar) on Friday, March 8, at 5:30 p.m., in the Flagler Inn Ballroom.

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, the time limits will be strictly enforced.

THURSDAY, 1:15 P. M.

Invited Address, J. Wayne Reitz Union Auditorium

Probabilistic operator theory. Professor A. T. BHARUCHA-REID, Georgia Institute of Technology (712-F3)

THURSDAY, 2:30 P. M.

Special Session on Dynamical Systems, Flows, and One-Parameter Semigroups of Transformations I, J. Wayne Reitz Union Auditorium

2:30- 2:50 (2) Fréchet type differentiability for nonlinear semigroups. Preliminary report. Professor JAMES R. DORROH, Louisiana State University (712-B4)

3:00- 3:20 (3) Contraction semigroups on Hilbert space. Preliminary report. Professor JEROME A. GOLDSTEIN, Tulane University (712-B6)

3:30- 3:50 (4) On the existence of wave operators for time dependent nonlinear evolution equations. Preliminary report. Mr. BRUNO J. WICHNOSKI, Tulane University (712-B8) (Introduced by Professor Jerome A. Goldstein)

4:00- 4:20 (5) Linear evolution operators and the embedding of time-dependent problems into semigroup theory. Professor DAVID L. LOVELADY, Florida State University (712-B2)

4:30- 4:50 (6) The input-output functions for a class of hereditary systems. Preliminary report. Dr. JAMES A. RENEKE, Clemson University (712-B10)

5:00- 5:20 (7) Higher order abstract Cauchy problems. Preliminary report. Mr. JAMES T. SANDEFUR, Jr., Tulane University (712-B9) (Introduced by Professor Jerome A. Goldstein)

5:30- 5:50 (8) Embedding theorems for dynamical systems. Professor COKE S. REED, Auburn University (712-G1) (Introduced by Professor John W. Neuberger)

THURSDAY, 2:30 P. M.

Special Session on Ordered Groups, Room 361, J. Wayne Reitz Union

2:30- 2:50 (9) Some remarks on the orthocompletion of an ℓ-group. Preliminary report. Dr. ROGER D. BLEIER, University of Texas, Austin (712-A2)

3:00- 3:20 (10) Stabilizer extensions of ordered permutation groups. Professor A. M. W. GLASS and Professor W. CHARLES HOLLAND*, Bowling Green State University (712-A3)

3:30- 3:50 (11) a*-extensions of lattice ordered groups. Professor W. CHARLES HOLLAND and Professor A. M. W. GLASS*, Bowling Green State University (712-A4)

4:00- 4:20 (12) Groups of divisibility and symmetric algebras. Preliminary report. Professor JOE L. MOTT, Florida State University (712-A5)

4:30- 4:50 (13) Monotonic permutations and pairing. Professor JERRY SCOTT, Georgia College (712-A6)

5:00- 5:20 (14) a*-closures of lattice-ordered groups. Preliminary report. Professor PAUL F. CONRAD, University of Kansas (712-A13)

5:30- 5:50 (15) The structure of ordered permutation groups applied to lattice-ordered groups. Professor STEPHEN H. McCLEARY, University of Georgia (712-A14)

THURSDAY, 2:30 P. M.

Session on Topology, Room 349, J. Wayne Reitz Union

2:30- 2:40 (16) A note on C(X). Professor DARRELL C. KENT, Washington State University and Mr. GARY D. RICHARDSON*, East Carolina University (712-G2)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

2:45- 2:55 (17) Embeddings transversal to algebraic sets. Professor J. H. ROBERTS, Duke University (712-G3)

3:00- 3:10 (18) Induced function theorems in topology. Dr. A. R. BEDNAREK, University of Florida and Dr. EUGENE M. NORRIS*, University of South Carolina (712-G4)

3:15- 3:25 (19) Universal cohomology products and the delicate Hopf invariant. Preliminary report. Professor JAMES A. DRAPER, University of Florida (712-G5)

3:30- 3:40 (20) Knots with infinitely many minimal spanning surfaces. Mr. JULIAN R EISNER, Princeton University (712-G6)

3:45- 3:55 (21) On elementary ideals of projective planes in the 4-sphere and θ-curves in the 3-sphere. Preliminary report. Dr. SHIN'ICHI KINOSHITA, Florida State University (712-G9)

4:00- 4:10 (22) A nondensity example in Diff(S^2). Professor DENIS J. BLACKMORE, Newark College of Engineering (712-G7)

4:15- 4:25 (23) Equivalence of connected and Darboux functions under closure. Preliminary report. Mr. MAURICE HUGH MILLER, Jr., University of Alabama (Tuscaloosa) (712-G8)

THURSDAY, 2:30 P. M.

Session on Algebra and Number Theory, Room 346, J. Wayne Reitz Union

2:30- 2:40 (24) Freeness and its generalizations in valued vector spaces. Preliminary report. Mr. DAVID E. THOMAS, Tulane University (712-A1) (Introduced by Professor Laszlo Fuchs)

2:45- 2:55 (25) Ideal transforms of 2-dimensional Noetherian domains. Professor PAUL M. EAKIN, University of Kentucky and Professor WILLIAM J. HEINZER*, Purdue University (712-A7)

3:00- 3:10 (26) Embedding algebras with a solvable word problem in simple algebras--some Boone-Higman type theorems. Professor TREVOR EVANS*, Professor KENNETH I. MANDELBERG and Professor MARY M. NEFF, Emory University (712-A15)

3:15- 3:25 (27) Odd super perfect numbers. Professor JOHN L. HUNSUCKER* and Professor CARL POMERANCE, University of Georgia (712-A8)

3:30- 3:40 (28) Enumerating automorphism orders in groups. Preliminary report. Professor HAROLD S. FINKELSTEIN Emory University (712-A9)

3:45- 3:55 (29) Subgroups of central separable algebras with no zero divisors. Professor EDWARD DAVID ELGETHUN, University of North Florida (712-A10)

4:00- 4:10 (30) A k-measure of irreducibility of doubly stochastic matrices. Preliminary report. Mr. FLOYD L. CHRISTIAN, Jr.*, Roane State Community College and Dr. JAMES J. JOHNSON, University of Mississippi (712-A11)

4:15- 4:25 (31) Minimum seminorm properties of the singular-weighted matrix pseudoinverse. Professor JAMES F. WARD, Jr., University of North Florida (712-A12)

4:30- 4:40 (32) Factorization of Mersenne numbers. Professor ALAN S. COVER and Professor ANDREW SOBCZYK*, Clemson University (712-A16)

4:45- 4:55 (33) Completely positive matrices. Dr. DON M. JORDAN, University of South Carolina (712-A17)

THURSDAY, 2:30 P. M.

Session on Analysis I, Room 347, J. Wayne Reitz Union

2:30- 2:40 (34) Transformations of generalized inverses under changes of projectors. Professor M. ZUHAIR NASHED*, Georgia Institute of Technology and Professor GEORGE F VOTRUBA, University of Montana (712-B1)

2:45- 2:55 (35) Global controllability of perturbed quasi-linear systems. Preliminary report. Professor ATHANASSIOS G. KARTSATOS, University of South Florida (712-B3)

3:00- 3:10 (36) Hadamard miltipliers of infinite matrices. Professor WILLIAM H. RUCKLE, Clemson University (712-B5)

3:15- 3:25 (37) Oscillation theorems for second order nonhomogeneous linear differential equations. Preliminary report. Professor SAMUEL M. RANKIN III, Florida Institute of Technology (712-B11)

3:30- 3:40 (38) On ergodic sequences of measures. Preliminary report. Professor J. R. BLUM* and Professor ROBERT F. COGBURN, University of New Mexico (712-B12)

3:45- 3:55 (39) Asymptotic characterization of solutions of a nonlinear boundary value problem arising in fluid mechanics. Professor JOHN W. HEIDEL*, University of Tennessee and Professor GARY D. JONES, Murray State University (712-B13)

4:00- 4:10 (40) Toeplitz operators on compact symmetric spaces. Preliminary report. Mr. DAVID SHIEN LIANG, Washington University (712-B14)

4:15- 4:25 (41) Stieltjes transform of functions satisfying the Lipschitz condition. Preliminary report. Professor RATHINDRA N. MUKHERJEE*, University of Georgia and Professor DIPENDRA N. BHATTACHARYA, Clarkson College of Technology (712-B19)

4:30- 4:40 (42) The uniform continuity of certain translation semigroups. Mr. JIMMIE LEE JOHNSON, University of Illinois at Chicago Circle (712-B17)

4:45- 4:55 (43) On Radon-Nikodym theorems. Preliminary report. Professor GEORGE YU-HUA CHI, University of Pittsburgh (712-B23)

FRIDAY, 9:00 A. M.

Invited Address, J. Wayne Reitz Union Auditorium

The decision problem for recursively enumerable degrees. Professor J. R. SHOENFIELD, Duke University

FRIDAY, 10:15 A. M.

Special Session on Dynamical Systems, Flows and One-Parameter Semigroups of Transformations II, and Informal Session, Room 349, J. Wayne Reitz Union

10:15-10:35 (45) Quotient groups for semigroups on manifolds and cones. Professor RAYMOND HOUSTON, Auburn University

10:45-11:05 (46) Homeomorphisms, iteration and linear independence. Professor GORDON G. JOHNSON, University of Houston (712-B7)

FRIDAY, 10:15 A. M.

Session on Analysis II, J. Wayne Reitz Union Auditorium

10:15-10:25 (47) On the operator $L = \Delta r^2 + \mu(\partial/\partial r)r + \lambda$. Professor LEONARD J. LIPKIN, University of North Florida (712-B15)

10:30-10:40 (48) Harmonic majorants for plurisubharmonic functions on bounded symmetric domains with applications to the spaces H_φ and N_*. Dr. MANFRED STOLL, University of South Carolina (712-B16)

10:45-10:55 (49) Measures associated with Toeplitz matrices generated by the Laurent expansion of rational functions. Dr. K. MICHAEL DAY, University of Michigan (712-B18)

11:00-11:10 (50) Finitely additive set functions. (I) Order-characterization of a prering of subsets of a set. Professor J. S. Mac NERNEY, University of Houston (712-B20)

11:15-11:25 (51) Convex topology. Preliminary report. Dr. DOUGLAS MOREMAN, Emory University (712-B21)

11:30-11:40 (52) Sums and limits of almost continuous functions. Professor KENNETH R. KELLUM, Miles College (712-B22)

11:45-11:55 (53) Continuous automorphisms on rings of infinite matrices. Preliminary report. Dr. PHILIP C. TONNE, Emory University (712-B24)

12:00-12:10 (54) Continued fraction representation of points of an inner product space. Professor F. A. ROACH, University of Houston (712-B25)

FRIDAY, 10:15 A. M.

Session on Statistics and Probability, Room 346, J. Wayne Reitz Union

10:15-10:25 (55) Uniqueness criteria for Ito's equation. Preliminary report. Mr. THOMAS C. GARD, University of Tennessee (712-F1)

10:30-10:40 (56) Baire category in spaces of probability measures. Professor JACK B. BROWN Auburn University (712-F2)

10:45-10:55 (57) On estimating the expected behavior of a stochastic epidemiological model applicable to small populations. Professor CHRIS P. TSOKOS and Mr. T. L. SMITH*, University of South Florida (712-F4)

FRIDAY, 10:15 A. M.

Session on Applied Mathematics, Room 347, J. Wayne Reitz Union

10:15-10:25 (58) More eigenvalue estimates. Preliminary report. Professor WILLIAM B. DAY, Auburn University (712-C1)

10:30-10:40 (59) Inertial oscillations and miltiple boundary layers in an unsteady rotating flow. Dr. LOKENATH DEBNATH*, East Carolina University and Miss SUKLA MUKHERJEE, University of Calcutta, India (712-C2)

10:45-10:55 (60) Fine topologies in relativity theory. Preliminary report. Dr. GARETH WILLIAMS, Stetson University (712-C3)

11:00-11:10 (61) Orthomodularity and transition probability. Preliminary report. Professor JOHAN G. F. BELINFANTE, Georgia Institute of Technology (712-C4)

11:15-11:25 (62) Properties of a semidiscrete approximation to a two-dimensional partial differential equation. Preliminary report. Dr. LAWRENCE A. KURTZ, Hollins College (712-C5) (Introduced by Professor Claude C. Thompson)

11:30-11:40 (63) An adaptation of Powell's sigma-theta constrained nonlinear optimization method to accomodate inequality constraints. Dr. JAMES V. BLOWERS, Eglin AFB (712-C6)

O. G. Harrold, Jr.
Associate Secretary

Tallahassee, Florida

Symposium on Some Mathematical Questions in Biology
San Francisco, California
February 25 – 26, 1974

The eighth annual symposium on Some Mathematical Questions in Biology will be held on February 25–26, 1974, in the Olympic Suite at the St. Francis Hotel, San Francisco, California. This symposium will be cosponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics in conjunction with the February 1974 meeting of the American Association for the Advancement of Science. It is anticipated that the symposium will be supported by the National Science Foundation. Information on registration and hotel arrangements appeared in the December 21, 1973 issue of SCIENCE.

The program has been arranged by Hans J. Bremermann, Jack D. Cowan, Murray Gerstenhaber, Alston S. Householder, Richard C. Lewontin, and Simon A. Levin (chairman), all of whom are members of the AMS-SIAM Committee on Mathematics in the Life Sciences.

PROGRAM

February 25, 8:30 a. m.

Chairman: Hans J. Bremermann, University of Chicago

8:30 a. m. Some models of mutation in molecular population genetics. MOTOO KIMURA, Institute of Genetics, Mishima, Japan

9:30 a. m. Mathematical and statistical problems arising in the non-Darwinian theory. WARREN J. EWENS, University of Pennsylvania

10:30 a. m. Gradients and catastrophes in developmental biology. E. C. ZEEMAN, University of Warwick, Warwick, England

11:30 a. m. Competition, selection, and evolution in chemical networks. JOHN J. TYSON, Max Planck Institut für Biophysikalische Chemie, Göttingen, Federal Republic of Germany

February 25, 2:30 p. m.

Chairman: Simon A. Levin, Cornell University

Session for Contributed Papers

2:30-2:45 p. m. Some mathematical questions concerning random cellular movements. Preliminary report. LESLIE E. BLUMENSON, Roswell Park Memorial Institute (Bio 74-1)

2:50-3:05 p. m. Catastrophy theory in modelling behavioral stages in Xenopus laevis embryos. Preliminary report. WACLAW G. WOJTKOWSKI, WITA WOJTKOWSKI* and M. MACKLIN, Case Western Reserve University (Bio 74-2)

3:10-3:25 p. m. A mathematical model of the cochlea (inner ear). ALFRED INSELBERG*, IBM Scientific Center, Los Angeles, and RICHARD S. CHADWICK, Israel Institute of Technology, Haifa, Israel (Bio 74-3)

3:30-3:45 p. m. An algebraic analysis of cladistic characters. G. F. ESTABROOK, University of Michigan, CARLOS S. JOHNSON, Jr., and FRED R. McMORRIS*, Bowling Green State University (Bio 74-4)

3:50-4:05 p. m. Mathematical simulation of chemotherapy in cancer. Preliminary report. RICHARD C. BROWN, Mathematics Research Center, University of Wisconsin (Bio 74-5)

4:10-4:25 p. m. Some comments on the parameter estimation and verification problems in biological systems. Preliminary report. JOEL B. SWARTZ, University of California, Berkeley (Introduced by Simon A. Levin)

4:30-4:45 p. m. Differential geometry and color perception. Preliminary report. H. L. RESNIKOFF, Rice University

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

February 26, 8:30 a. m.

Chairman: George Oster, University of California, Berkeley

8:30 a. m. A diffusion model of pattern formation in the insect cuticle. STEPHEN BLOMFIELD, MCR Laboratory of Molecular Biology, Cambridge, England

9:30 a. m. Biological pattern formation on the basis of lateral inhibition. ALFRED GIERER* and H. MEINHARDT, Molikularbiologische Abteilung Max Planck Institut für Virusforschung, Tübingen, Federal Republic of Germany

10:30 a. m. Pattern formation in an oscillating chemical system. NANCY KOPELL, Northeastern University

Simon A. Levin
Chairman

Ithaca, New York

Symposium on Category Theory Applied to Computation and Control San Francisco, California February 25 – 26, 1974

The Society is sponsoring the First International Symposium: Category Theory Applied to Computation and Control which will take place in the Kent Room of the St. Francis Hotel in San Francisco, California, at the annual meeting of the American Association for the Advancement of Science, February 25–26, 1974.

It is the purpose of this symposium to bring together experts in automata, control, systems and programming, who share an interest in algebraic approaches to the theory of their subject, with algebraists and category theorists who believe that their studies can fruitfully be applied in these areas.

The proceedings of the symposium, containing introductory material, abstracts of presented talks and a bibliography, will be published by the mathematics and computer science departments of the University of Massachusetts as a compact bound volume. These proceedings will be distributed to participants of the symposium at registration. After the meeting, the proceedings will be available from the American Mathematical Society for $1 a set.

Information on registration appears in the October 26, 1973, issue of Science. Other information may be obtained by writing to Professor E. G. Manes, Department of Mathematics, University of Massachusetts, Amherst, Massachusetts 01002.

PROGRAM

Monday, February 25

8:30 a.m. -11:30 a.m. Invited hour addresses; E. G. Manes, Chairman

S. Eilenberg, Categories: Use and misuse, Columbia University
J. A. Goguen, Semantics of computation, University of California, Los Angeles
R. E. Kalman, Algebraic-geometric problems in systems theory, University of Florida at Gainesville

2:00 p.m. - 5:00 p.m. J. A. Goguen, Chairman

B. F. Wyman, Linear systems over rings of operators, Ohio State University
J. Rissanen, Duals of input/output maps (with B. F. Wyman), Linköping University (Sweden)
E. Kamen, Control of linear continuous-time systems defined over rings of operators, Georgia Institute of Technology
E. T. Onat, Representation of a class of nonlinear systems (with J. A. Geary), Yale University
E. S. Bainbridge, Addressed machines and duality, University of Ottawa
M. A. Arbib, Time-varying systems (with E. G. Manes), University of Massachusetts
J. L. Baker, Factorization of Scott-style automata, University of British Columbia
R. Brockett, Some structural properties of automata defined on groups (with A. Willsky), Harvard University

Evening

Informal discussion on the role of categorical methods in the mathematical system sciences

Tuesday, February 26

8:30 a.m. -11:30 a.m. M. Wand, Chairman

J. Meseguer, Automata in semimodule categories (with I. Sols), Universidad de Zaragoza (Spain)
L. A. Carlson, Realization is continuously universal, University of Chicago
H. Ehrig, Power and initial automata in pseudoclosed categories (with H.-J. Kreowski), Technische Universität Berlin (Germany)
E. G. Manes, Fuzzy morphisms in automata theory (with M. A. Arbib), University of Massachusetts
L. Budach, Automata in additive categories with applications to stochastic linear automata, Humboldt University (E. Germany)

M. Mesarovic, Application of categorical algebra to classification of systems, Case Western Reserve University

J. Rhodes, Complexity as a general mathematical idea: subadditive Grothendieck functions on categories, University of California, Berkeley

2:00 p. m. - 5:00 p. m. E. S. Bainbridge, Chairman

D. Benson, An abstract machine theory for formal language parsers, Washington State University

M. Wand, An algebraic formulation of the Chomsky hierarchy and the recursive specification of data types, Indiana University

J. Thatcher, Algebraic theory of recursive program schemes (with R. M. Burstall), IBM T. J. Watson Research Center

J. Helton, Scattering theory for computers and other non linear systems (with W. Helton), Dowling College

W. Merzenich, Cellular automata with additive local transition, Universität Dortmund (Germany)

G. Hotz, Strukturelle Verwandtschaften von semi-Thue-Systemen, Universität des Saarlandes (Germany)

A. W. Wymore, The tricotyledon theory of system design, University of Arizona

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Seven Hundred Thirteenth Meeting
Biltmore Hotel
New York, New York
April 10—13, 1974

The seven hundred thirteenth meeting of the American Mathematical Society will be held at the Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York, from Wednesday, April 10, through Saturday, April 13, 1974, in conjunction with the 1974 meeting of the Association for Symbolic Logic.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be four one-hour addresses. The speakers will be Professor Serge Lang of Yale University, Professor George C. Papanicolaou of the Courant Institute of Mathematical Sciences, New York University, Professor Sol I. Rubinow of the Graduate School of Medical Sciences, Cornell University, and the Sloan-Kettering Institute, and Professor William P. Thurston of the Massachusetts Institute of Technology. A tentative title of Professor Lang's talk is "Higher dimensional diophantine problems."

Professor V. Lakshmikantham of the University of Rhode Island and the University of Texas at Arlington is organizing a special session on Nonlinear Problems in Differential and Integral Equations. Professor Herbert S. Wilf of the University of Pennsylvania and Rockefeller University is organizing a special session on Combinatorial Algorithms.

All of the above activities will take place on Friday and Saturday.

Sessions for contributed ten-minute papers will be scheduled in the morning and afternoon on Friday and Saturday. No provision will be made for late papers. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of February 21, 1974. The final program of the meeting will appear in the April issue of these *Notices*.

SYMPOSIUM ON MATHEMATICAL ASPECTS OF CHEMICAL AND BIOCHEMICAL PROBLEMS AND QUANTUM CHEMISTRY

With the support of the Office of Naval Research and the Atomic Energy Commission, a symposium on Mathematical Aspects of Chemical and Biochemical Problems and Quantum Chemistry is scheduled to be held on Wednesday and Thursday, April 10 and 11. This topic was selected by the AMS-SIAM Committee on Applied Mathematics whose members are Earl A. Coddington, Hirsh G. Cohen (chairman), Lester E. Dubins, Harold Grad, J. Barkley Rosser, and Richard S. Varga. The symposium is a further attempt at encouraging and broadening the interdisciplinary research in the several wide fields of mutual interest which exist between certain mathematicians, applied mathematicians, and researchers in chemical reaction theory, biochemistry, and quantum chemistry. The Organizing Committee includes Donald S. Cohen (chairman), Hirsh G. Cohen, Julian D. Cole, George R. Gavalas, and Aron Kupperman.

The program will consist of ten lectures. The list of speakers includes Neal Amundson (University of Minnesota), "Non-linear problems in chemical reaction theory"; Rutherford Aris (University of Minnesota), "An analysis of the counter-current moving bed reactor"; Joseph Higgins (The Johnson Research Foundation, University of Pennsylvania), "Mathematical problems in the analysis of metabolic dynamics"; Fritz Horn (University of Rochester), "The dynamics of open reaction systems"; Louis N. Howard (Massachusetts Institute of Technology), "Wave trains, fronts, and transition layers in reaction-diffusion equations"; James Keck (Massachusetts Institute of Technology), "Phase space theory of atomic ard molecular excitation and disassociation"; Herbert B. Keller (California Institute of Technology), "Some problems in chemical reactor theory"; Aron Kupperman (California Institute of Technology), "Quantum dynamics of reactive molecular collisions"; Gregoire Nicolis (Université Libre de Bruxelles), "Patterns of spatio-temporal organization in nonlinear chemical and biochemical kinetics"; and Arthur Winfree (Purdue University), "Rotating solutions to reaction/diffusion equations in simply-connected media." Further details will be given in the April issue of these *Notices*.

MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

The program for the meeting of the Association for Symbolic Logic on April 12—13 is as follows: Professor Leo Harrington of SUNY at Buffalo will give an invited expository talk at 4:00 p.m., on April 12, on "Recursion on finite type functionals." Dr. Ashok Chandra of the T. J. Watson Research Center at Yorktown Heights, New York, will give an invited talk on the "Generality of control structures in uninterpreted computer programs." There will also be sessions for contributed papers. The chairman of the program committee is Dr. Paul C. Gilmore, IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598.

REGISTRATION

The registration desk will be located in the Key Room of the Biltmore Hotel on the nineteenth floor adjacent to the Grand Ballroom. The desk will be open from 8:30 a.m. to 4:30 p.m. on Wednesday, April 10, through Friday, April 12; and from 8:30 a.m. to 3:30 p.m. on Saturday, April 13.

The registration fees for the meeting are as follows:

Member	$3
Student and unemployed	1
Nonmember	5

ACCOMMODATIONS

Persons intending to stay at the Biltmore Hotel should make their own reservations with the hotel. A reservation form and a listing of room rates will be found on the last page of these *Notices*. The deadline for receipt of reservations is April 3, 1974.

TRAVEL

The Biltmore Hotel is located on Madison Avenue at 43rd Street on the east side of New York City. Walkways to Grand Central Station are located under the hotel and signs are posted directing persons to the lobby of the hotel.

Those arriving by bus may take the Independent Subway System from the Port Authority Bus Terminal. There is shuttle bus service from LaGuardia and Kennedy Airports directly to Grand Central Station. Starters can direct participants to the correct bus.

Air passengers arriving at Newark Airport can take a shuttle bus to the East Side Terminal and take either a subway, taxi, or bus to the hotel.

Those arriving by car will find many parking facilities in the neighborhood in addition to those at the hotel. Parking service can be arranged through the hotel doorman at a cost of $8 for a 24-hour period. There will be an additional charge for extra pickup and delivery service if it is required. The parking fee is subject to New York City taxes.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, The Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York 10017.

Walter H. Gottschalk
Associate Secretary

Middletown, Connecticut

The Seven Hundred Fourteenth Meeting
University of California
Santa Barbara, California
April 27, 1974

The seven hundred fourteenth meeting of the American Mathematical Society will be held at the University of California, Santa Barbara, California, on Saturday, April 27, 1974.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two one-hour addresses. Professor Garth W. Warner of the University of Washington will lecture at 11:00 a.m. The title of his lecture is "L-functions on reductive groups." Professor Hung-Hsi Wu of the University of California, Berkeley, will lecture at 1:30 p.m. on "Applications of some theorems in partial differential equations to geometry." Both hour addresses will be given in room 1920 of Ellison Hall.

Professors Marvin Marcus, Henryk Minc, and Robert C. Thompson of the University of California, Santa Barbara, are organizing two special sessions on Linear Algebra. These sessions will begin at 8:30 a.m. and 2:45 p.m., respectively, and will consist of six thirty-minute talks given by John DePillis, Eugene C. Johnsen, Herbert J. Ryser, Robert C. Thompson, Olga Taussky, and Stanley G. Williamson.

There will also be sessions for contributed papers. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of February 21, 1974. Late papers will be accepted for presentation at the meeting, but will not appear in the printed program of the meeting. Overhead projectors will be available at all of the sessions.

The registration desk will be located in the patio between Ellison Hall and Phelps Hall. Registration will begin at 8:00 a.m. on Saturday.

Santa Barbara has a large number of motels and hotels. Those in Goleta are nearest to the campus, those by the beach are farthest from the campus, while those in the business district on upper State Street are approximately midway. All have easy access to U.S. Highway 101. Reservations should be made directly with the desired motel or hotel and the American Mathematical Society should be mentioned since, in some cases, the rates listed below are special rates.

Goleta

HOLIDAY INN
5650 Calle Real, Goleta 93017
Phone: (805) 964-6241

Single	$14.50 up
Double	18.50 up

MOTEL 6
5897 Calle Real, Goleta 93017
Phone: (805) 964-1812

Single	$ 7.00
Double	8.16

Upper State Street

SANDPIPER HYATT LODGE
3525 State Street, Santa Barbara 93105
Phone: (805) 687-5326
Single $ 8.50 up
Double 12.00 up

MOTEL 6
3505 State Street, Santa Barbara 93105
Phone: (805) 687-5400

PEPPER TREE MOTOR INN
3850 State Street, Santa Barbara 93110
Phone: (805) 687-5511
Single $15.00 up
Double 20.00

Beach Area

LA CASA DEL MAR
28 W. Cabrillo Boulevard, Santa Barbara 93101
Phone: (805) 966-6337
Single $10.60 up
Double 14.84 up

MING TREE
930 Orilla del Mar Drive, Santa Barbara 93103
Phone: (805) 966-1641
Single $14.00 up
Double 14.00 up

MOTEL 6
443 Corona del Mar, Santa Barbara 93103
Phone: (805) 965-0300

The campus is located on the Pacific Ocean approximately ten miles north of Santa Barbara. When approaching from the south on U.S. 101, drive past the four sets of traffic lights downtown. Continue for about seven miles and take the exit marked "UC Santa Barbara, Airport, Goleta" (California Highway 217). Continue for approximately three miles until you arrive at the entrance to the campus. There is a 25¢ daily parking fee on campus, and the personnel at the kiosk will have information and maps. When approaching from the north on U.S. 101, take the exit marked "Glen Annie Road, Storke Road" and then follow the signs to the University. It is about two miles from the freeway offramp to the entrance of the campus.

Santa Barbara Airport is served by United Airlines and Air West with service from Los Angeles and San Francisco, and the commuter airline Golden West with service from Oxnard and Los Angeles. There are a Greyhound bus terminal and an Amtrak train station downtown; both offer service from Los Angeles and San Francisco. There is a bus every thirty minutes between downtown and the campus. Half of these go via Goleta and Isla Vista and the other half via the airport. The distance between the airport and campus on foot is about one and a half miles, and a taxi can be obtained for approximately $2.50.

There is a very slight chance of showers in April and an equally slight chance of a heat wave. The usual weather condition is a temperature of 70° with possibly a slight mist.

Noon meals will be available on campus (grill service only) and there are many restaurants in Isla Vista, within ten-minutes walking distance.

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

Seven Hundred Fifteenth Meeting
Northern Illinois University
DeKalb, Illinois
May 13 – 18, 1974

The seven hundred fifteenth meeting of the American Mathematical Society will be held at Northern Illinois University in DeKalb, Illinois, from Monday, May 13, to Saturday, May 18, 1974. The principal feature of the meeting will be a symposium on Mathematical Developments Arising from the Hilbert Problems; the support of the National Science Foundation is expected under a proposed grant. This topic was chosen by the Committee to Select Hour Speakers for Western Sectional Meetings. The Organizing Committee of the symposium consists of Paul T. Bateman, Felix E. Browder (chairman), R. Creighton Buck, Donald J. Lewis, and Daniel Zelinsky. The tentative list of speakers includes Lipman Bers, Herbert Busemann, John Conway, Georg Kreisel, R. Langlands, George Lorentz, Ju. V. Matijasevič, Hugh L. Montgomery, David Mumford, Albrecht Pfister, James B. Serrin, John T. Tate, and Arthur S. Wightman.

There will be five hour speakers per day in the symposium on Monday through Thursday. In addition the four hour speakers for the Society meeting on Friday and Saturday will be an integral part of the symposium.

There will also be sessions for the presentation of contributed ten-minute papers on Friday and Saturday, May 17 and 18. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of March 27, 1974. Those having time preferences for the presentation of their papers should indicate them on their abstracts. There will be a session for late papers if one is needed, but late papers will not be listed in the printed program of the meeting.

All the addresses on the Mathematical Developments Arising from the Hilbert Problems will be presented in the Carl Sandberg Auditorium of the University Center. The session of contributed papers will be held in Reavis West, a classroom building four minutes walk from the University Center.

The Council of the Society will meet at 2:00 p.m. on Sunday, May 12, 1974, at the Holiday Inn, 1212 W. Lincoln Highway in DeKalb.

REGISTRATION

The registration desk will be located in the lobby of the University Center. The desk will be open from 8:30 a.m. to 4:30 p.m. Monday through Friday and from 8:30 a.m. to 12 noon on Saturday. A message center will be maintained at the registration desk. The registration fees for the meeting will be as follows: members $5.00, nonmembers $7.50, and students and unemployed mathematicians $1.00.

ACCOMMODATIONS

The following accommodations will be available on or near the Northern Illinois University campus for this meeting.

A. Dormitory accommodations in Grant South, a student dormitory complex ten-minutes walk from the University Center. These accommodations are not recommended for families with small children. Maid service is not provided. Room rates are $3.50 per person per night on a double occupancy basis, and $5.50 per person per night in a single room.

B. Accommodations in the University Plaza, a private dormitory type building five-minutes walk from the University Center. Maid service is provided. Room rates are $7.50 per person per night on a double occupancy basis, and $8.50 per person per night in a single room. All rooms have shared baths.

C. A limited number of rooms in the University Center. All of these rooms are for double occupancy at a rate of $7.50 per person per night, and all have private baths.

Requests for room reservations in Grant South, University Plaza, or the University Center should be sent prior to April 15 to Professor Donald Ostberg, Department of Mathematical Sciences, Northern Illinois University, DeKalb, Illinois 60115. Forms for making room reservations are included on page 113 of these *Notices*.

Individuals arriving on campus without room reservations should contact the University Center registration desk to arrange accommodations.

There is also a Holiday Inn located within ten-minutes walk of the University Center. Its address and room rates are:

HOLIDAY INN
1212 W. Lincoln Hwy.
DeKalb, Illinois 60115
Phone: (815) 758-8661

Single:	$13.50
Double:	16.50 (2 persons)
Twin:	18.50 (2 persons)

Reservations should be made directly with the Inn, and mention should be made of this meeting in order to obtain the quoted rates.

FOOD SERVICE

Breakfast, lunch, and dinner will be available in the University Center throughout the conference. A snack bar will be open in Grant South during the evenings, but meals will not be served in the dormitories. There are also several restaurants within walking distance of campus. A list of these restaurants with directions for reaching them will be available at the registration desk.

TRAVEL

DeKalb, Illinois, is located on Illinois Route 38, approximately 70 miles west of Chicago. The campus of Northern Illinois University is adjacent to Route 38 on the west side of DeKalb.

Travelers driving to DeKalb from the east should take one of the routes shown on the map which will be included in the April *Notices*. Travelers driving to DeKalb from the west should arrive on Illinois Route 38.

Travelers arriving by air should fly to O'Hare Airport, Chicago, from which point ground transportation will be available to DeKalb as follows:

A. Continental Air Transport Company and University Bus with a transfer at the Ramada Inn in St. Charles, Illinois. Cost: $3.25 each way.

From O'Hare to DeKalb

	Lv. O'Hare	Ar. DeKalb
Sun. May 12	3:35 p.m. 5:35 p.m. 7:35 p.m.	5:20 p.m. 7:20 p.m. 9:20 p.m.
Mon. May 13	9:35 a.m. 11:35 a.m. 7:35 p.m.	11:20 a.m. 1:20 p.m. 9:20 p.m.
Tues. May 14	7:35 p.m.	9:20 p.m.
Wed. May 15	7:35 p.m.	9:20 p.m.
Thurs. May 16	3:35 p.m. 5:35 p.m.	5:20 p.m. 7:20 p.m.
Fri. May 17	5:35 p.m.	7:20 p.m.

Buses will deliver passengers to their residences for the conference.

From DeKalb to O'Hare

	Lv. DeKalb	Ar. O'Hare
Mon. May 13	4:30 p.m.	6:30 p.m.
Tues. May 14	4:30 p.m.	6:30 p.m.
Wed. May 15	4:30 p.m.	6:30 p.m.
Thurs. May 16	3:30 p.m. 4:30 p.m.	5:30 p.m. 6:30 p.m.
Fri. May 17	4:30 p.m.	6:30 p.m.
Sat. May 18	7:30 a.m. 10:30 a.m. 12:30 p.m.	9:30 a.m. 12:30 p.m. 2:30 p.m.

Buses will pick up passengers at their residences for departure.

B. Direct limousine service is available between O'Hare and DeKalb at a cost of $9.50 per person each way.

Reservations must be made in advance for transportation between O'Hare and DeKalb, using the form on this page of these *Notices*. This form should be mailed to Professor Donald Ostberg, Department of Mathematical Sciences, Northern Illinois University, DeKalb, Illinois 60115, to arrive prior to May 1, 1974. Do not include payment with the transportation request; payment will be made directly to the carrier involved at the time of the trip.

ENTERTAINMENT

A cash bar will be in operation from 5:00 p.m. to 6:30 p.m. on Monday through Friday evenings. In addition, the Department of Mathematical Sciences at Northern Illinois University will host a beer party for all those attending the meeting on Wednesday evening, May 15. Details concerning the location of the cash bar and the time and place of the beer party will be available at the registration desk.

PARKING

Since the meeting will be held after the end of the spring semester at Northern Illinois University ample parking will be available on the campus. Visitors are requested not to park in metered or reserved parking places, or in no parking zones.

ROOM RESERVATIONS

Name: _____

Dates: _____

Accommodations Requested:

First Choice: _____

Second Choice: _____

Traveling to the meeting by (check one)

Car ☐ Plane ☐

TRANSPORTATION: O'HARE - DeKALB

Name: _____

Arrival at O'Hare	Departure from O'Hare
Date: _____	Date: _____
Time: _____	Time: _____
Airline: _____	
Flight No.: _____	

Transportation requested (check one)

Direct to DeKalb ☐

Via St. Charles ☐

Paul T. Bateman
Associate Secretary

Urbana, Illinois

1974 Summer Institute on Algebraic Geometry
Arcata, California
July 29 – August 16, 1974

The twenty-first Summer Research Institute of the American Mathematical Society will be devoted to the topic "Algebriac Geometry," and will be held at Humboldt State University, Arcata, California, for a period of three weeks from July 29 through August 16, 1974. The Organizing Committee consists of Professors Michael Artin, Phillip A. Griffiths, Robin C. Hartshorne, Heisuke Hironaka, Nicholas Katz, and David Mumford (chairman). It is expected that the institute will be supported by a grant from the National Science Foundation.

The scientific program of the institute will consist first of all of a series of expository talks, each surveying an area of current interest and consisting of two-to four-hour talks. The speakers and their topics will be Egbert Brieskorn, "Special singularities–resolution, deformation, and monodromy"; M. Cornalba, "Hodge theory"; Pierre Deligne, "Inputs of etale cohomology"; David Eisenbud, "Projective resolutions, ideals of low codimension and Serre's conjecture"; Robin C. Hartshorne, "Cycles, K-theory, Chow ring and intermediate jacobians"; Joseph Lipman, "Methods of desingularization, old and new"; Barry Mazur, "The zeroes of the zeta-function of a variety in characteristic p"; C. S. Seshadri, "Theory of moduli." Jean-Pierre Serre will also give a talk, the title to be announced.

In addition to these series of lectures, there will be seminars presenting in more detail progress in various areas, each under the supervision of an invited chairman. The seminar program is not yet fixed, but it is anticipated that the following areas will be covered: "Classification of varieties," "Birational geometry," "Equisingularity," "Kähler geometry," "DeRham and crystalline geometry," "Arithmetic and algebraic geometry," "Commutative algebra and algebraic geometry," "Deformation theory and moduli," "Proof of the Weil conjectures." In planning these seminars, the guiding principle will be to communicate rather than merely display mathematics, and also (a) to find out what participants want to learn and (b) when possible, to ask some person x to speak on y's work ($x \neq y$) à la Bourbaki.

Information on travel and accommodations will be available in subsequent announcements.

Funds for participant support will be limited, and it is hoped that a number of participants will find their own sources of support, e.g., travel grants to the International Congress at Vancouver. The institute is open to all mathematicians specializing in algebraic geometry and related topics, and to advanced graduate students in this field. Those wishing to participate should write to Dr. Gordon L. Walker, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940. Recent Ph.D.'s and advanced graduate students who wish to be considered for support should write before March 20, 1974.

1974 Summer Seminar in Applied Mathematics

The eighth AMS Summer Seminar in Applied Mathematics will be held on the campus of the University of California, Los Angeles, from August 5 through August 16, 1974. The seminar will be sponsored jointly by the American Mathematical Society and the Society for Industrial and Applied Mathematics, and will be supported by a grant from the National Science Foundation. The topic, "Inverse problems," was selected by the AMS-SIAM Committee on Applied Mathematics whose members include Earl A. Coddington, Hirsh G. Cohen (chairman), Lester E. Dubins, Harold Grad, J. Barkley Rosser, and Richard S. Varga. The members of the Organizing Committee are Victor Barcilon, University of Chicago (co-chairman); Julian D. Cole, University of California, Los Angeles (co-chairman); Michael Crandall, University of California, Los Angeles; Nathaniel Grossman, University of California, Los Angeles; F. Gilbert, University of California, San Diego; Leon Knopoff, University of California, Los Angeles; R. G. Newton, University of Indiana; and James Ralston, University of California, Los Angeles.

Inverse problems in mathematics are not precisely defined. Rather, the term denotes several classes of interesting problems where unusual or atypical questions are asked. These often arise in a physical or at least nonmathematical context. The mathematical entities involved are most often partial differential questions, integral equations, and systems of ordinary differential equations. Some typical subclasses of inverse problems (not necessarily totally distinct) are inverse eigenvalue problems, inverse boundary value problems, inverse scattering problems, and miscellaneous problems such as finding stiffness and spring matrix from some natural frequencies and mode shapes in large structures, the optimum structural design in (x, t), and similar problems that occur in biological, chemical, and business economic contexts.

The basic plan of the program is to provide a large amount of introductory material especially suited for those participants who have sound mathematical training, but who may not be familiar with the applications. Special topics and

current research reports will, of course, also be presented. The general program will include ten lectures on scattering presented by four lecturers; ten lectures on eigenvalues presented by three lecturers; and ten or eleven lectures on applications presented by three lecturers. The names of these principal lecturers will be announced at a later date.

Dormitory accommodations and food service will be available on campus, and a complete brochure of information will be sent to the participants early in the spring.

Individuals may apply for admission to the seminar. Application blanks for admission and financial assistance can be obtained from the Meeting Arrangements Department, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940. An applicant will be asked to indicate his scientific background and interest; he should have completed at least one year of graduate school. Graduate students should have their faculty advisors write to a member of the Organizing Committee concerning their ability and promise. Those who wish to apply for a grant-in-aid should so indicate. Funds available to the seminar are limited, and those who can obtain support from other sources should do so.

INVITED SPEAKERS AT AMS MEETINGS

This section of these *Notices* lists regularly the individuals who have agreed to address the Society at the times and places noted below. For some future meetings, the lists of speakers are incomplete.

Gainesville, Florida, March 1974

A. T. Bharucha-Reid
J. R. Shoenfield

New York, New York, April 1974

Serge Lang
George C. Papanicolaou
Sol I. Rubinow
William P. Thurston

Santa Barbara, California, April 1974

Garth W. Warner
Hung-Hsi Wu

Washington, D. C., January 1975

Linda Keen
Wilfried Schmid

CHAIRMEN AND TOPICS OF SPECIAL SESSIONS

Gainesville, Florida, March 1974

Jorge Martinez, Ordered groups
John Neuberger, Dynamical systems, flows, and one-parameter semigroups of transformations

New York, New York, April 1974

V. Lakshmikantham, Nonlinear problems in differential and integral equations
Herbert S. Wilf, Combinatorial algorithms

Santa Barbara, California, April 1974

Marvin Marcus, Henryk Minc, and Robert C. Thompson, Linear algebra

AMS Research Fellowship Fund
Request for Contributions

The Council and the Trustees of the Society have established the AMS Research Fellowship Fund to be used for postdoctoral fellowships for research in mathematics during the year 1974–1975. This Fund is to be funded in part by the Society and in part by members and friends of the Society. The Society has agreed to contribute a minimum of $9,000 which guarantees that at least one fellowship can be awarded for next year. In addition, the Society will match one-half the funds, in excess of $18,000, raised from other sources, but will contribute no more than $20,000. It is hoped that individual contributions to the Fund will make it possible to award several fellowships.

At a time when there has been a drastic reduction in funds available for postdoctoral fellowships, it is only fitting that the Society should reaffirm its dedication to mathematical research and establish the Research Fellowship Fund. Although the number of fellowships that will be awarded from the Fund may be small, the Research Fellowship Fund will, nonetheless, serve as a gesture of the weight the Society and its members place on research in mathematics and of their concern that funds for postdoctoral fellowships have been cut.

Members and friends of the Society are asked to send their contributions to the Research Fellowship Fund as soon as possible since the number of fellowships that can be granted will be in direct proportion to the amount in the Fund. (The deadline for applications for the Research Fellowships is March 15, 1974.) It is hoped that every tenured member of the Society will be willing to contribute at least $100 to the Fund; some have already done so. However, any contribution, large or small, will be welcome. All contributions are tax deductible, of course.

Checks should be made payable to the American Mathematical Society, clearly marked AMS Research Fellowship Fund, and sent to the Society at P.O. Box 6248, Providence, Rhode Island 02940.

AMS Research Fellowship

The American Mathematical Society invites applications for the AMS Research Fellowships. These are postdoctoral fellowships to be awarded for research in mathematics during the year 1974–1975, and they are open to individuals who have recently received the Ph.D. The stipend will be in the range of $9,000–$10,000, depending on the amount in the Fund. Of the award, $3,600 plus $500 for travel expenses will be tax deductible.

Completed applications must be received by March 15, 1974. A very small number of fellowships will be awarded, the number depending on the amount contributed to the AMS Research Fellowship Fund. Notification of awards will be made by April 15, 1974.

For further information and application forms write to Dr. Gordon L. Walker, Executive Director, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

LETTERS TO THE EDITOR

Editor, the *Notices*

At the January AMS meetings, the women moderating sessions were called "chair people" while the male moderators were called "chairmen".

I suggest that the three words "head, moderator, convener" are not only inoffensive substitutes for "chairman" but are also far more indicative of actual function and should therefore be uniformly adopted.

Judith Q. Longyear

Editor, the *Notices*

The San Francisco meeting, with its full program, reminded me quite forcibly that the officers of the Society are likely to be so busy with necessary functions at an annual meeting that they have little chance to talk with more than a handful of the members present. Since this was in any event my situation at San Francisco, this brief note is to say that I would be happy this spring to receive letters from members of the Society about questions of Society policy or activities. I will try to answer all such letters as quickly as may be possible. Please note also an erratum to the Annual Report of the President for 1973 in the January 1974 *Notices* (page 79). That Report should have carried at the end my signature.

Saunders Mac Lane

NEWS ITEMS AND ANNOUNCEMENTS

HARVARD UNIVERSITY POSTDOCTORAL FELLOWSHIP

The Department of Mathematics of Harvard University is offering a postdoctoral fellowship with a stipend of $8,000. Candidates should send a brief vita, a statement of research plans, and two letters of recommendation to Professor Raoul H. Bott, Department of Mathematics, Harvard University, 1 Oxford Street, Cambridge, Massachusetts 02138, by March 1, 1974. The award will be announced by April 15, 1974.

RESOLUTION BY THE COMMITTEE ON EMPLOYMENT & EDUCATIONAL POLICY ENDORSED BY THE COUNCIL OF THE AMERICAN MATHEMATICAL SOCIETY

August 21, 1973, Missoula, Montana

Data collected by the AMS in 1973 show that, nationally, the number of teaching assistantships in mathematics departments has changed very little during the past three years. With the virtual certainty that additional academic employment of mathematicians over the next 20 years will be far below the expected number of Ph.D.'s based on the present numbers graduating, this committee urges all graduate departments able to do so to convert some teaching assistantship funds to junior faculty position funds. A reduction of 10% a year in TA's with replacement by junior faculty would both curtail the future number of Ph.D.'s and create an additional 150-200 faculty positions per year, thus significantly alleviating the sad unemployment situation facing many of our young doctorates. This Committee and the Council have made similar suggestions earlier, unfortunately, without much apparent effect.

Since it is steadily becoming clearer that, over the next 20 years, academic job prospects for mathematicians must be expected to get much worse, the community should make great efforts to cut back the numbers of new Ph.D.'s designed purely for academic employment, to convert some new Ph.D.'s toward prospective nonacademic employment, and to actively seek many more opportunities for nonacademic employment and for retraining young Ph.D.'s for such employment.

NSF FACULTY FELLOWSHIPS

The National Science Foundation has announced a program of Faculty Fellowships designed to help teachers broaden their perspective in the applications of science to societal problems. The program is for teachers of science, mathematics, and engineering at universities, colleges, and junior and community colleges. NSF plans to award approximately fifty Faculty Fellowships in Science Applied to Societal Problems on May 31, 1974, with an application deadline of April 1. Awards will be made to faculty members who have had five or more years of full-time teaching experience, who are citizens or nationals of the United States, and who hold at least a baccalaureate degree. Competition will be divided into two categories: applicants with the Ph.D., and applicants without the Ph.D. The fellowships may be used for periods of three to nine months. Stipends are based on salary paid during the preceding year, and travel allowances are also paid. In addition, NSF will provide the fellowship institution with an allowance to assist the institution in meeting tuition and other costs. Application materials may be obtained from the Faculty Fellowship in Science Program, Division of Higher Education in Science, National Science Foundation, Washington, D. C. 20550.

QUERIES

Edited by Wendell H. Fleming

The QUERIES column is published in each issue of these *Notices*. This column welcomes questions from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning various conjectures. When appropriate, replies from readers will be edited into a definitive composite answer and published in a subsequent column. All answers received to QUERIES will ultimately be forwarded to the questioner. Consequently, all items submitted for consideration for possible publication in this column should include the name and complete mailing address of the person who is to receive the replies. The queries themselves, and responses to such queries, should be typewritten if at all possible and sent to Professor Wendell H. Fleming, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

NOTE FROM THE QUERIES COLUMN EDITOR

This column has now been in existence for slightly over a year. We have received a steady flow of queries and responses, from mathematicians in the United States and other countries. In several instances responses have come from leading (even world famous) people in the field.

The queries received up to now can be roughly grouped into three categories, as follows. One category is concerned with a specific technical problem, apparently related to the querier's current research interests. The second is a request for references from experts on some topic about which the querier is interested. The third category might be described as questions intended to enrich the querier's understanding of some part of our mathematical culture and heritage. Examples are queries about the status of famous unsolved conjectures, and attempts to separate fact from folklore. Queries of all three types are welcome. However, the editor wishes to encourage more queries of the last two categories, which tend to interest not only the querier but many other readers of these *Notices*.

The second feature of the column is the Problem Lists. We publicize both information about existing lists, and seek to encourage new ones by routinely contacting organizers of symposia, special sessions at AMS meetings, and so on. Three problem lists generated at the Dallas and Evanston AMS meetings were published in the June 1973 and August 1973 *Notices*. Solutions to two problems on these lists have already been received.

The editor invites suggestions from readers regarding the column.

QUERIES

31. David Shelupsky (Department of Physics, The City College (CUNY), New York, New York 10031). Can anyone tell me if the following statement is true or false or a conjecture (and if true where a proof can be found)? In a two-dimensional Banach space (Minkowski space) the perimeter of the unit circle, L, satisfies the inequalities $4 \leqq L \leqq 8$.

32. William E. Hartnett (Department of Mathematics, SUNY, College at Plattsburgh, Plattsburgh, New York 12901). Let X be a nonempty set and let M be the monoid of all functions from X to X with composition as operation. A submonoid N of M is a continuity monoid iff there exists a topology on X such that N is precisely the set of all functions continuous with respect to that topology. Which are the continuity monoids? Literature references and current status desired.

33. S. Zaidman (Department of Mathematics, Université de Montréal, Montréal, Canada). I would like to have precise information, with detailed proof (if available), of the following result (or conjecture?) by V. V. Zikov:

"Let A be a linear compact operator in a Hilbert or reflexive Banach space; let f(t) be an almost-periodic function with range in the same Banach space, and let u(t) be a strong solution of the equation

$$u'(t) = A\,u(t) + f(t)$$

on the whole real axis, which is bounded on the real line.

"Then u(t) is weakly almost-periodic (which implies that u(t) is almost-periodic)."

A recent paper by B. R. Basit and L. Cend, Differencial'nye Vravnenija 8(1972), 1343-1348 (MR 46 #5783) states more general results. The proof is quite an easy consequence of a proposition by Zikov (3.I of the paper: "Almost-periodic solutions of differential equations in Banach spaces", in the volume "Theory of functions and its applications", Harkov, No. 4(1967), pp. 176-188) not available to the querier and of a paper by Bochner (Proc. Nat. Acad. Sci. 45 (1962), pp. 2039-2043).

34. Cleve Moler (Department of Mathematics, University of New Mexico, Albuquerque, New Mexico 87106). References are sought on applications of mathematics in cryptography.

RESPONSES TO QUERIES

Several responses have been received to queries published in recent issues of these *Notices*. The editor wishes to thank all those who have responded. The following summarizes information given therein, arranged according to query number and name of the questioner.

5. (Shields, Nov. 1972) The following is a new reference regarding the 17th Hilbert problem: Danielle Gondard and Paulo Ribenboim, Sur le 17^{e} problème de Hilbert, C. R. Acad. Sci. Paris 277 (1973), A303-A304. [provided by Ben Silver]

(Previous responses to this query were published in this column in the January, June, and August 1973 *Notices*.)

22. (Gilmer, June 1973) The querier summarized information received from D. L. Johnson, Jürgen Neukirch, Jean-Pierre Serre, and Luis Ribes as follows:

"I would agree with Serre's statement that there is no good survey article on the question of what finite groups can be Galois groups over Q; the best approximations seem to be (in chronological order):

(1) Portions of sections 3 and 4 of chapter V of N. Chebotarev's book Grundzüge der Galois'schen Theorie, P. Noordhoff N. V., Groningen-Djakarta, 1950.

(2) The article Über die absolute Galoisgruppe algebraischer Zahlkörper, by Jürgen Neukirch; Séminaire Delange-Pisot-Poitou, 13^{e} année, 1971/72, no. 18.

(3) Section 3 of Computational problems, methods, and results in algebraic number theory, by Horst G. Zimmer; Lecture Notes in Mathematics No. 262, Springer-Verlag, New York, 1972.

"The one reference I know on this problem that is not covered in the bibliographics of (1), (2), and (3) is the Ph. D. thesis of K. -Y. Shih, Princeton University, 1972.

"On the closely related question of whether the fixed field of a finite permutation group acting on $Q(X_1, \ldots, X_n)$ is purely transcendental over Q, one should consult (1), (2), and the older article On a theorem of E. Noether, by W. Kuyk, in Konink, Nederl. Akad. Wetensch. Ser. A. 67(1964), 32–39."

In addition Walter Feit responded as follows:

"Let G be a Weyl group. By a theorem of Coxeter and Chevalley the ring of invariants of G is a polynomial ring. Thus Hilbert's irreducibility theorem implies that G is a Galois group over the rationals. Thus also every homomorphic image of G is a Galois group over the rationals. The only simple homomorphic image of any Weyl group is $Sp_6(2) \simeq O_7(2)$ which occurs for $G = W(E_7)$. Aside from A_n and some PSL(p) I don't know any other simple group which is known to be a Galois group over the rationals."

(Earlier information regarding this query appeared in the November 1973 *Notices*.)

30. (Edgar, Nov. 1973) This query reads: Let n be an odd perfect number so that we necessarily have $n = p^{\alpha} q_1^{2\beta_1} \ldots q_t^{2\beta_t}$ where p, $q_1, \ldots, q_t$ are distinct odd primes, $p \equiv \alpha \equiv 1 \pmod 4$, $t \geq 6$, etc., etc.

(a) Does it necessarily follow that there exist at least two pairs of subscripts $i < j < k < \ell$ for which $\sigma(q_i^{2\beta_i}) = q_j^{2\beta_j}$ and $\sigma(q_k^{2\beta_k}) = q_\ell^{2\beta_\ell}$?

(b) Does it necessarily follow that there exists exactly one pair of subscripts $i < j$ for which $\sigma(q_i^{2\beta_i}) = q_j^{2\beta_j}$?

Carl Pomerance responded as follows:

"If either (a) or (b) is true, then there are no odd perfect numbers. Indeed it follows from a paper of W. Ljunggren (Noen setninger om ubestente likninger av formen $\frac{x^n - 1}{x - 1} = y^q$, Norsk. Mat. Tidsskr. 1. Hefte 25 (1943), pp. 17–20) that the only solution of the equation $\sigma(q_i^{2\beta_i}) = q_j^{2\beta_j}$ in primes q_i, q_j is $\sigma(3^4) = 11^2$. Hence if there is one pair of subscripts $i \neq j$ with $\sigma(q_i^{2\beta_i}) = q_j^{2\beta_j}$, then $3^4 \cdot 11^2$ is a unitary divisor of the odd perfect number n. Since $\sigma(11^2) = 7 \cdot 19$, we also have $7^2 \cdot 19^2 | n$. But $3^4 \cdot 11^2 \cdot 7^2 \cdot 19^2$ is abundant, contradicting n perfect.

"Note that using methods of a paper of Hunsucker and Pomerance (Odd super perfect numbers, to appear) it is possible to prove the following: there are at least $t - [\log(t+1)/\log 2]$ subscripts i for which there is a subscript $j \neq i$ such that either $q_j | \sigma(q_i^{2\beta_i})$ or $q_i | \sigma(q_j^{2\beta_j})$."

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates, or geographical area become apparent. An announcement will be published in these *Notices* if it contains a call for papers, place, date, subject (when applicable), and speakers; a second announcement will be published only if changes to the original announcement are necessary, or if it appears that additional information should be announced. In general, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry slightly more detailed information. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society. Deadlines for particular issues of the *Notices* are the same as the deadlines for abstracts which appear on the inside front cover of each issue.

ERRATUM: The dates of the Fifth Balkan Mathematical Congress have been changed to June 24–30, 1974, from July 1–7, 1974, as announced in the January 1974 *Notices*

March 5–7, 1974
FACHTAGUNG FÜR PROGRAMMIERSPRACHEN
Universität Kiel, Federal Republic of Germany
Information: Professor Bodo Schlender, Institut für Informatik und Praktische Mathematik der Universität, Olshausenstrasse 40-60, D2300 Kiel 1, Federal Republic of Germany

March 9, 1974
CARLETON AND OTTAWA UNIVERSITY DAY IN TOPOLOGY
Carleton University, Ottawa, Canada
Invited speakers: D. Henderson (Cornell University), J. Stasheff (Temple University), M. Tierney (Rutgers University)
Information: Dr. Clarence Wilkerson, Department of Mathematics, Carleton University, Ottawa, Ontario Canada K1S 5B6

March 18–21, 1974
CONFERENCE ON COMPUTER AIDED GEOMETRIC DESIGN
University of Utah, Salt Lake City, Utah
Contributed papers: Three copies of abstract not later than March 1, 1974, to address below
Information: Professor Robert E. Barnhill, Department of Mathematics, University of Utah, Salt Lake City, Utah 84112

March 23, 1974
THIRTY-FIRST ONTARIO MATHEMATICS MEETING
University of Waterloo, Waterloo, Ontario, Canada
Information: Dr. George Cross, Department of Pure Mathematics, University of Waterloo, Waterloo, Ontario, Canada

March 30, 1974
ELEVENTH ALGEBRA DAY
Carleton University, Ottawa, Canada
Speakers: Michael Barr (McGill University), Israel N. Herstein (University of Chicago), Peter J. Hilton (Case Western Reserve University)
Information: Dr. L. Ribes, Department of Mathematics, Carleton University, Ottawa, Ontario K1S 5B6, Canada

April 1–August 9, 1974
NON-CREDIT SHORT COURSES
Duke University, Durham, North Carolina
April 1–26: Four lectures per week; "Theory of Besov spaces," Jaak Peetre (Tekniska Hogskolan, Lund, Sweden)
July 8–August 9: Four lectures per week; "Homology of commutative algebras," Michel André (Ecole Polytechnique Fédérale, Lausanne, Switzerland); "Pseudo-differential operators, Fourier integral operators, applications," Louis Boutet de Monvel (Université de Paris, Paris, France)
Information: Professor Seth L. Warner, Department of Mathematics, Duke University, Durham, North Carolina 27706

April 10, 1974
PROBABILITY AND STATISTICS DAY
Carleton University, Ottawa, Canada
Speakers: F. J. Ancombe (Yale University), "Regression analysis in the computer age"; W. G. Cochran (Harvard University), "Principles and some current problems in controlled experimentation"; H. A. David (Iowa State University of Science and Technology), "The theory of competing risks"; E. Parzen (State University of New York at Buffalo), "Time series and systems identification"
Information: Dr. E. Saleh, Department of Mathematics, Carleton University, Ottawa, Ontario K1S 5B6, Canada

April 20, 1974
ILLINOIS NUMBER THEORY CONFERENCE
Illinois State University, Normal, Illinois
Abstracts: Fifteen- to twenty-minute talks; two-sentence abstracts to address below. Deadline April 8, 1974
Information: Professor Charles Vanden Eynden, Mathematics Department, Illinois State University, Normal, Illinois 61761

April 29–May 1, 1974
CONFERENCE ON ITERATIVE ALGORITHMS FOR NUMERICAL ANALYSIS
University of Pittsburgh, Pittsburgh, Pennsylvania
Sponsors: University of Pittsburgh, Carnegie-Mellon University, National Science Foundation (pending funds), Society for Industrial and Applied Mathematics
Information: Professor Thomas A. Porsching, Department of Mathematics, University of Pittsburgh, Pittsburgh, Pennsylvania 15260

May 1–3, 1974
CONFERENCE ON BIOMATHEMATICS AND BIOSTATISTICS
Washington State University, Pullman, Washington
Topics: Mathematical genetics, biostochastic processes, control and harvesting with population dynamics
Speakers: D. Chapman (University of Washington), H. L. Lucas (North Carolina State University at Raleigh), E. Pollack (Iowa State University), M. Rosenzweig (University of New Mexico), B. Wallace (Cornell University), C. F. Wehrhahn (University of British Columbia)
Contributed papers: Abstracts to be submitted to Dr. W. Y. Tan, Department of Pure and Applied Mathematics, Washington State University, Pullman, Washington 99163, prior to March 15, 1974
Information: Chairman, Department of Pure and Applied Mathematics, Washington State University, Pullman, Washington 99163

May 14–16, 1974
NINTH NEW ZEALAND MATHEMATICS COLLOQUIUM
University of Auckland, New Zealand
Information: Mr. D. P. Alcorn, Department of Mathematics, University of Auckland, Private Bag, Auckland, New Zealand

May 16–18, 1974
THIRD CONFERENCE ON COMBINATORIAL MATHEMATICS
University of Queensland, St. Lucia, Queensland, Australia
Program: Invited speakers and sessions for contributed papers
Information: A. P. Street, Department of Mathematics, University of Queensland, St. Lucia, Queensland 4067, Australia

May 27–31, 1974
EIGHTEENTH ANNUAL MEETING OF THE AUSTRALIAN MATHEMATICAL SOCIETY
University of Newcastle, New South Wales, Australia
Information: Professor Warren Brisley, Faculty of Mathematics, University of Newcastle, Newcastle, New South Wales 2308, Australia

May 29–31, 1974
CONFERENCE ON MATHEMATICAL SOFTWARE II
Purdue University, West Lafayette, Indiana
Scope: This conference is to focus on real algorithms, the analysis of the behavior of programs, the establishment of standards, comparative testing and evaluation of programs, and the interface between mathematical software and users. Several invited speakers will cover highly important related areas such as software management technology, hardware development, and large scale applications.
Deadlines: Contributed papers with travel support for authors, April 1, 1974; all contributed papers, May 1, 1974; advance registration, May 24, 1974
Information: Professor John R. Rice, Computer Sciences, Mathematical Sciences Building, Lafayette, Indiana 47907

June 1974
CENTRO INTERNAZIONALE MATEMATICO ESTIVO (SUMMER SESSIONS)
June 2–11, Bressanone, Italy
Scientific Director: L. Salvadori, University of Rome
Lecturers: V. Zubov, N. Rouche, V. Laksmikantham, J. Barbata, V. V. Rumiantjez
June 16–25, Bressanone, Italy
Scientific Director: A. Tognoli, University of Pisa
Lecturers: H. Hiromaka, V. Poenaru, F. Lazzeri, A. Tognoli
June 16–26, Varenna, Italy
Scientific Director: G. Prodi, University of Pisa
Lecturers: P. Rabinowitz, P. C. Fife, W. Klingenberg, J. Necas, O. Voivoda, M. Reeken, H. Amman, R. Turner

June 24–26, 1974
SIAM 1974 NATIONAL MEETING
Pasadena, California
Theme: Mathematics applied to societal problems including weather prediction and natural disasters
Program: Invited speakers, sessions for contributed papers
Contributed papers: Abstract deadline, March 2, 1974
Information: Society for Industrial and Applied Mathematics, 33 South 17th Street, Philadelphia, Pennsylvania 19103

June 26–29, 1974
CONFERENCE ON COMPUTING FIXED POINTS AND APPLICATIONS
Clemson University, Clemson, South Carolina
Sponsor: Office of Naval Research
Contributed papers: Deadline for abstracts, April 15, 1974
Information and abstracts: Professor Stepan Karamardian, Graduate School of Administration, University of California, Irvine, California 92664

July 8–19, 1974
SHORT COURSES ON SYSTEMS THEORY AND APPLICATION
Rensselaer Polytechnic Institute, Troy, New York
First week: Digital signal processing fundamentals
Second week: Applications of digital processing
Information: Office of Continuing Studies, Rensselaer Polytechnic Institute, Troy, New York 12181

July 11–22, 1974
SYMPOSIUM ON GLOBAL RIEMANNIAN GEOMETRY
University of Durham, Durham, England
Sponsors: University of Durham, London Mathematical Society, Royal Society, and (tentatively) Science Research Council
Organizers: J. Eells, T. J. Willmore
Application deadline: May 1, 1974
Information and applications: Professor T. J. Willmore, Department of Mathematics, University of Durham, Durham, England

July 15–27, 1974
REUNIÃO DE ANÁLISE FUNCIONAL
São José dos Campos, São Paulo, Brazil
Sponsor: Sociedade Brasileira de Matemática
Speakers: G. S. S. Avila (Brasilia), A. P. Calderón (MIT and Buenos Aires), M. Cotlar (La Plata), P. Hess (Zurich), C. S. Hönig (São Paulo), L. Karlovitz (Maryland), L. Nachbin (Rio de Janeiro), P. Novosad (IMPA), G. Vidossich (Brasilia), and others to be confirmed
Information: Professor D. G. de Figueiredo, Departamento de Matemática, Universidade de Brasilia, 70.000 Brasilia, D. F., Brazil

July 17–August 18, 1974
CONFERENCE ON RECENT DEVELOPMENTS IN THE FOUNDATIONS OF GEOMETRY
University of Toronto, Toronto, Canada
Program: Three series of lectures, seminars, individual reports
Main speakers: F. Bachmann (Kiel), R. Lingenberg (Karlsruhe), J. Tits (Bonn)
Seminar speakers: A. Barlotti (Bologna), F. Buekenbout (Brussels), J. Cofman (Mainz), M. Goetzky (Kiel), J. W. Lorimer (Toronto), H. Lueneburg (Kaiserslautern), E. Mendelsohn (Toronto), T. G. Ostrom (Washington State), H. Salzmann (Tuebingen), J. J. Seidel (Eindhoven), J. Yaqub (Ohio State), and others as yet unannounced
Registration deadline: March 15, 1974
Information: Professor Peter Scherk, Department of Mathematics, University of Toronto, Toronto, Ontario M5S 1A1, Canada

July 22–August 2, 1974

SYMPOSIUM ON FUNCTIONAL ANALYSIS AND STOCHASTIC PROCESSES
University of Durham, Durham, England
Sponsors: University of Durham, London Mathematical Society, Royal Society, and (tentatively) Science Research Council
Organizers: D. J. H. Garling, J. F. C. Kingman
Application deadline: May 1, 1974
Information and applications: Professor T. J. Willmore, Department of Mathematics, University of Durham, Durham, England

July 29–August 2, 1974
THIRD ANNUAL SOUTHERN CALIFORNIA SYMPOSIUM ON ALGEBRAIC AND GEOMETRIC TOPOLOGY
University of California, Los Angeles, California
Information: Professor Robert F. Brown, Department of Mathematics, University of California, Los Angeles, California 90024

August 19–20, 1974
CONFERENCE ON PROBABILISTIC METHODS IN DIFFERENTIAL EQUATIONS
University of Victoria, British Columbia, Canada
Subjects: Stochastic differential equations, random evolutions, diffusions on manifolds, control theory
Speakers: A. Friedman, K. Ito, S. Varadhan
Information: Professor C. R. Miers, Department of Mathematics, University of Victoria, Victoria, British Columbia V8W 2Y2, Canada

August 27–September 6, 1974
NATO ADVANCED STUDY INSTITUTE ON THE THEORY AND APPLICATION OF DIFFERENTIAL GAMES
University of Warwick, Coventry, England
Applicants: Should be mathematicians or mathematical engineers interested in theoretical aspects of control systems and differential equations. Some travel and subsistence available.
Application deadline: May 31, 1974
Information: Professor P. C. Parks, Control Theory Centre, University of Warwick, Coventry CV4 7AL, England

August 30–September 9, 1974
TWELFTH INTERNATIONAL SYMPOSIUM ON FUNCTIONAL EQUATIONS
Waterloo and Victoria Harbour, Ontario, Canada
Application deadline: March 31, 1974 (Those wishing to be invited should state their previous work and interest in the subject in as much detail as possible.)
Abstract deadline: July 31, 1974
Information and applications: Professor J. Aczel, Department of Applied Analysis and Computer Science, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

September 3–7, 1974
INTERNATIONAL CONFERENCE ON DIFFERENTIAL EQUATIONS
University of Southern California, Los Angeles, California
Subjects: Analytic theory, qualitative theory, mathematical control theory
Information: Professor H. A. Antosiewicz, Department of Mathematics, University of Southern California, University Park, Los Angeles, California 90007

September 18–21, 1974
SYMPOSIUM ON QUASIGROUPS AND FUNCTIONAL EQUATIONS
Belgrad, Yugoslavia
Contributed papers: Papers to be submitted (for photocopying) not later than March 15, 1974. Communications may be submitted in any of the Yugoslav languages, English, French, German and Russian.
Information: Matematički Institut, Knez Mihailova 35, 11000 Beograd, Yugoslavia

October 14–16, 1974
FIFTEENTH ANNUAL SYMPOSIUM ON SWITCHING AND AUTOMATA THEORY
New Orleans, Louisiana
Sponsors: Switching and Automata Theory Committee of the IEEE Computer Group, ACM Special Interest Group on Automata and Computability Theory, Louisiana State University at New Orleans
Contributed papers: Extended abstract (seven copies) to Ronald V. Book, Center for Research in Computing Technology, Aiken Computation Laboratory, Harvard University, Cambridge, Massachusetts 02138. Abstract should provide sufficient detail to allow committee to apply uniform criteria for evaluation; should include appropriate references and comparisons with extant work. Helpful to include brief interpretation of major results and explanation of their significance.
Information: Professor Fred Hosch, Department of Computer Science, Louisiana State University at New Orleans, Lake Front, New Orleans, Louisiana 70122

November 5–9, 1974
REGIONAL CONFERENCE ON CURRICULUM AND TEACHER TRAINING FOR MATHEMATICAL EDUCATION
National Institute for Educational Research, Tokyo, Japan
Information: Professor Z. Kobayashi, Japan Society of Mathematical Education, Tokyo University of Education, 29 Outsuka 3-chome, Bujkyo-ku, Tokyo 112, Japan

BACKLOG OF MATHEMATICS RESEARCH JOURNALS

Information on the backlog of papers for research journals is published in the February and August issues of these *Notices* with the cooperation of the respective editorial boards. Since all columns in the table are not self-explanatory, we include further details on their meaning.

Column 3. This is an estimate of the number of printed pages which have been accepted but are not necessary to maintain copy editing and printing schedules.

Column 5. The first (Q_1) and third (Q_3) quartiles are presented to give a measure of normal dispersion. They do not include misleading extremes, the result of unusual circumstances arising in part from the refereeing system.

The observations are made from the latest issue of each journal received at the Headquarters Offices before the deadline for the appropriate issue of these *Notices*. Waiting times are measured in months from receipt of manuscript in final form to receipt of final publication at the Headquarters Offices. When a paper is revised, the waiting time between an editor's receipt of the final revision and its publication may be much shorter than is the case otherwise, so these figures are low to that extent.

	1	2	3		4	5		
JOURNAL	No. issues per year	Approx. no. pages per year	BACKLOG 12/31/73	BACKLOG 6/15/73	Est. time for paper submitted currently to be published (in months)	Observed waiting time in latest published issue (in months) Q_1	Med.	Q_3
Acta Informatica	4	790-860	0	0	7	13	22	24
American J. of Math.	4	1000	1650	1570	24	19	23	27
Annals of Math.	6	1200	NR*	NR*	16-18	10	16	17
Annals of Probability	6	1100	100	0	13	9	10	10
Annals of Statistics	6	NR*	NR*	200	NR*	8	9	10
Arch. Rational Mech.	16	1316	0	0	6-7	6	9	11
Canad. J. of Math.	6	1250**	1300	1500	13	14	17	18
Comm. Math. Physics	22	1828	NR*	0	6-7	8	8	9
Duke Math. J.	4	1000	70	50	8	7	8	9
Illinois J. of Math.	4	700	150	100	12	22	24	25
Indiana Univ. Math. J.	12	1200	300	400	9	9	10	11
Inventiones Math.	16	1337	NR*	0	6-7	9	10	12
J. Amer. Stat. Assoc.	4	1033	192	44	10	8	9	10
J. Assoc. for Comp. Mach.	4	720	460	NR*	10	10	12	13
J. of Comp. and Sys. Scis.	6	800	420	170	12	9	13	23
J. Diff. Geometry	4	620	750	680	12-15	26	31	33
J. Math. Physics	12	2000	0	7	8	7	9	16
J. Symbolic Logic	4	900	0	0	10	13	16	19
Linear Algebra & Appl.	6	576	526	360	12	14	17	32
Math. Biosciences	6	1152	800	900	12		***	
Math. Systems Theory	4	400	0	180	12	19	20	21
Math. of Comp.	4	1200	0	100	8	13	15	17
Math. Annalen	30	2529	172	498	12-13	10	11	17
Math. Zeitschrift	11	1739	0	73	6-7	6	8	9
Michigan Math. J.	4	400	100	100	11	11	13	17
Numerische Math.	9	790-860	0	0	10	9	11	14
Operations Research	6	1350	100	100	18	15	20	30
Pacific J. of Math.	12	3700	NR*	NR*	12	15	18	20
Proceedings of AMS	12	2600	600	150	11	10	10	11
Proc. Nat'l Acad. Sci.	12	NR*	NR*	about 300	NR*	5	5	6
Quarterly of Appl. Math.	4	480	480	500	12-15	15	16	18
Semigroup Forum	8	732	200	0	6	6	6	9
SIAM J. of Appl. Math.	8	1400	0	0	8-10	9	9	10
SIAM J. on Computing	4	700	0	0	4-6	8	8	10
SIAM J. on Control	4	700	0	0	8-10	12	13	17
SIAM J. on Math. Anal.	6	1050	0	0	9-11	16	17	19
SIAM J. on Numer. Anal.	6	1050	0	0	9-11	13	18	21
SIAM Review	4	700	0	0	8-10	15	18	19
Transactions of AMS	14	5600	1200	300	13	13	14	17
Z. Wahrscheinlichkeitstheorie	12	995	NR*	0	9-10	6	8	16

*NR means no response was received to a request for information.

**For 1973. An increase is planned for 1974, which, together with a stricter acceptance policy, should decrease the backlog.

***Date of receipt of manuscript not given in this journal.

DOCTORATES CONFERRED IN 1972-1973

Supplementary List

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during 1972–1973. This is a supplement to the list printed in the October and November 1973 issues of these *Notices*. The numbers appearing in parentheses after each university indicate the following: the first number is the total number of degrees listed for that institution; the next seven numbers are the number of degrees in the categories of 1. Pure Mathematics, 2. Statistics, 3. Operations Research, 4. Computer Science, 5. Applied Mathematics, 6. Mathematics Education, 7. Other. Eight universities are listed with a total of 31 individual names. This total, combined with the other lists, includes doctorates from 150 universities and a total of 1,301 individual names.

CALIFORNIA

STANFORD UNIVERSITY (2;0, 2, 0, 0, 0, 0, 0)

Department of Statistics

Matsubara, Nozomu
Test procedures for general Lehmann alternatives

Smith, Laurel Alison Beckett
Statistical procedures for diagnosis based on binary variables

MISSOURI

SAINT LOUIS UNIVERSITY (1;1, 0, 0, 0, 0, 0, 0)

Becker, David N.
The numerical range in non-unital normed algebras

WASHINGTON UNIVERSITY (2;2, 0, 0, 0, 0, 0, 0)

Liden, Norman Eugene
K-spaces, their antispaces and related maps

Wiener, Jeffrey C.
The extremal length problem applied to a bordered Riemann surface with a finite number of boundary components

NEW JERSEY

RUTGERS UNIVERSITY (10;10, 0, 0, 0, 0, 0, 0)

Arnow, Barry J.
Preregulated lattices and the generation of regulated semilattices

Bredlau, Carl E.
Recursive equivalence types and admissable sets

Fowler, Northrup III
Some theorems on countable spaces with recursive operations

Gaydos, Eugene M.
The theory of L^p-spaces for functions on an abstract semilattice

Gottlieb, Sarah J.
Automorphisms of algebraic groups with stable maximal tori

Iberkleid, Wolf
Splitting the tangent bundle

Miller, Robert C.
A characterization of $SL(4, 4^\gamma)$ by its 3-structure

Rucker, Rudolf V.
Definable sets

Sheng, Hu
Finiteness in dimension two

Slater, David J.
Non-standard languages and their applications

NEW YORK

UNIVERSITY OF ROCHESTER (5;5, 0, 0, 0, 0, 0, 0)

David, John
Some non-Noetherian factorial rings

Hill, Linda
On prescribing Henselizations for valued fields

Lieberman, Gerald
Pullbacks and localization

Stevenson, James
Holomorphy of composition

Vincent, Paul
Families of generalized continua on 2-manifolds

OHIO

UNIVERSITY OF TOLEDO (4;4, 0, 0, 0, 0, 0, 0)

Antoniou, Nicholas
On the convergence of perturbation theories that avoid secular terms in quantum mechanics

McDonald, Dennis
Arithmetical functions on a locally finite, locally modular, local lattice

Movasseghi, Darius
Problems in analytic function theory

Repsys, Rimantas
Spaces of measures on variable sigma algebras

PENNSYLVANIA

PENNSYLVANIA STATE UNIVERSITY (1;0, 0, 0, 1, 0, 0, 0)

Department of Computer Science

Metzner, John R.
Generalized decision table programming

TEXAS

RICE UNIVERSITY (6;0, 1, 1, 1, 3, 0, 0)

Department of Mathematical Sciences

Archer, David Anderson
Some methods of collocation type for differential equations

Chou, Shih-I.
Galerkin approximations on linear elastostatics, elastodynamics, and thermoelastodynamics

Hoard, Robert Earl
A study of estimation problems involving multiple traces

Hsia, Wei-Shen
Decomposition in nonlinear stochastic programming

Mayor de Montricher, Gilbert Franz
Nonparametric Bayesian estimation of probability densities by function space techniques

McCarthy, Mary Anne
Error bounds for the solution of two point boundary value problems based on the Kantorovich theorem

VISITING MATHEMATICIANS
Supplementary List

The list of visiting mathematicians includes both foreign mathematicians visiting in the United States and Canada, and Americans visiting abroad. Note that there are two separate lists.

American and Canadian Mathematicians Visiting Abroad

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Brualdi, Richard A. (U.S.A.)	Hungarian Academy of Sciences	Combinatorics	1/74 - 6/74
Eggleton, Roger (Canada)	The Weizmann Institute of Science	Combinatorics	9/73 - 9/74
Frankowski, K. (U.S.A.)	The Weizmann Institute of Science	Numerical Analysis and Computer Science	1/74 - 8/74
Grossman, Marvin W. (U.S.A.)	University of Negev, Israel	Topological Groups	9/73 - 8/74
Hoppensteadt, Frank (U.S.A.)	University of Oxford, England	Ordinary Differential Equations	1/74 - 6/74
Klerer, M. (U.S.A.)	The Weizmann Institute of Science	Computer Science	9/73 - 8/74
Kruskal, M. D. (U.S.A.)	The Weizmann Institute of Science	Applied Mathematics	9/73 - 8/74
Lakser, H. (Canada)	The Weizmann Institute of Science	Universal Algebra	9/73 - 8/74
Lukacs, Eugene (U.S.A.)	University of Sheffield, England	Probability & Statistics	1/74 - 6/74
Micchelli, C. (U.S.A.)	The Weizmann Institute of Science	Approximation Theory	11/73 - 6/74
Schoenberg, I. J. (U.S.A.)	The Weizmann Institute of Science	Approximation Theory	11/73 - 5/74
Shields, Allen L. (U.S.A.)	Mittag-Leffler Institute, Sweden	Functional Analysis	1/74 - 6/74
Van de Wetering, R. L. (U.S.A.)	University of Groningen, Netherlands	Integral Transforms	9/73 - 9/74
Warga, J. (U.S.A.)	The Weizmann Institute of Science	Optimal Control	7/73 - 2/74

Foreign Mathematicians Visiting in the United States

Bechter, Alfons (Austria)	Michigan State University	General Mathematics	9/73 - 6/74
Benigni, Osvaldo (Argentina)	University of California, Berkeley		9/73 - 4/74
Berthelot, Pierre (France)	Princeton University	Crystalline Cohomology of Schemes	2/74 - 7/74
Chirica, Laurian M. (Romania)	University of California, Los Angeles	Computer Science	9/72 - 6/74
Coman, Gheorghe A. (Romania)	University of Wisconsin, Mathematics Research Center	Numerical Analysis and Computer Programming	8/73 - 6/74
De Melo, Wellington C. (Brazil)	University of California, Berkeley		9/73 - 9/74
Larionescu, Dan (Romania)	University of Florida	Mechanism for Kinematic Analysis & Synthesis by Computers	9/73 - 6/74
Magidor, Menachem (Israel)	University of California, Berkeley	Algebraic Theory for Tree Automata	7/73 - 6/74
Manselli, Paolo (Italy)	University of California, Berkeley		9/73 - 7/74
Nicolaides, Roy A. (United Kingdom)	California Institute of Technology	Applied Mathematics	8/73 - 7/74
Olaofe, Gabriel O. (Nigeria)	University of Southern California	Analysis in Theoretical Physics Problems	6/74 - 9/74
Palis, Jacob (Brazil)	University of California, Berkeley	Ordinary Differential Equations	9/73 - 3/74
Petrov, Valentin (U.S.S.R.)	Tulane University	Probability Theory and Mathematical Statistics	1/74 - 5/74

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Procesi, Abeasis Silvana (Italy)	University of California, Los Angeles	Algebraic Topology	9/73 - 9/74
Rabesaotra, Raymond (Malagasy Republic)	Massachusetts Institute of Technology	Electrically-Charged Fluids	2/74 - 4/74
Ruchti, Rene (Switzerland)	University of California, Berkeley		9/73 - 8/74
Scherer, Karl (Fed. Rep. of Germany)	Kent State University	Approximations and Expansions	1/74 - 3/74
Schwarz, Gideon (Israel)	University of California, Berkeley	Statistics	9/73 - 6/74
Stoy, Gabrielle (United Kingdom)	Massachusetts Institute of Technology	Modern Algebra	8/73 - 8/74
Szafraniec, Franciszek H. (Poland)	University of California, Berkeley	Functional Analysis	9/73 - 9/74
Talpur, Mir Noor M. (Pakistan)	University of Illinois	Subharmonic, Integral & Meromorphic Functions	9/73 - 6/74

SUMMER GRADUATE COURSES

The following is a list of graduate courses being offered in the mathematical sciences during the summer of 1974. Another list will appear in the April issue of these *Notices*.

KANSAS

FORT HAYS KANSAS STATE COLLEGE
Hays, Kansas 67601
Information: E. Beougher, Chairman, Department of Mathematics

June 3 - July 31
Introduction to Computer Programming
Theory of Sets
Applied Mathematics
Advanced Calculus
Abstract Algebra

MICHIGAN

ANDREWS UNIVERSITY
Berrien Springs, Michigan 49104
Application deadline: May 15
Information: Harold T. Jones, Chairman, Department of Mathematical Sciences

June 16 - August 9
Topology

NORTHERN MICHIGAN UNIVERSITY
Marquette, Michigan 49855
Application deadline: May 1
Information: Dean of Graduate Studies

June 17 - August 9
Foundations of Mathematics
Geometry
Probability for Teachers
Seminar

NEW JERSEY

RUTGERS UNIVERSITY
New Brunswick, New Jersey 08903
Application deadline: May 15
Information: Mrs. Annette Roselli, Department of Mathematics

June 24 - August 14
Mathematical Theory of Probability*
General Set Theory
Functional Analysis

*This course carries undergraduate or graduate credit

OKLAHOMA

UNIVERSITY OF OKLAHOMA
Norman, Oklahoma 73069
Application deadline: May 1*
Information: Office of Admissions and Records

June 3 - July 27
Theory of Groups
Analytic Projective Geometry
Theory of Probability
Topics in Topology I

*Preference given to applications received by this date.

ASSISTANTSHIPS AND FELLOWSHIPS IN MATHEMATICS 1974—1975

Supplementary List

FOR GRADUATE STUDY AT UNIVERSITIES

TYPE of financial assistance (with number anticipated 1974-1975)	STIPEND amount in dollars	STIPEND 9 or 12 months	TUITION if not included in stipend (dollars)	SERVICE REQUIRED hours per week	SERVICE REQUIRED type of service	DEGREES AWARDED Academic year 1972-1973

CALIFORNIA

University of California, Berkeley 94720

GROUP IN LOGIC AND THE METHODOLOGY OF SCIENCE
Robert L. Vaught, Chairman

Applications due: 12/1/73*

Ph. D. (1970–1973 incl.)
L&MS 3. Total: 3

Fellowship ( )	2500-3600				
Teaching Assistantship (2)	3996-4104				
Research Assistantship (1)	3177				

*With Graduate Division. Applications for teaching assistantships must be filed by February 1, 1974 with the Department of Mathematics.

DISTRICT OF COLUMBIA

George Washington University, Washington 20006

DEPARTMENT OF STATISTICS
Jerome Cornfield, Chairman

Applications due: 2/15/74

Bachelor's by inst. 1342
Bachelor's by dept. 4
Master's by dept. 4

Ph. D. (1970–1973 incl.)
P&S 23. Total: 23

Teaching Fellowship ( )	2400-2800	9			

FLORIDA

University of Florida, Gainesville 32611

DEPARTMENT OF MATHEMATICS
A. R. Bednarek, Chairman

Applications due: *

Master's by dept. 22

Ph. D. (1970–1973 incl.)
A&NT 4, G&T 8, A&FA 3, AM 1. Total: 16

Fellowship (3)	2400	9			
Teaching Assistantship (20)	2790	9		15	

*Fellowships: 2/15/74; Assistantships: 4/15/74.

PENNSYLVANIA

University of Pennsylvania, Philadelphia 19174

DEPARTMENT OF STATISTICS AND OPERATIONS RESEARCH
John S. de Cani, Chairman

Applications due: 2/1/74

Master's by dept. 8

Ph. D. (1970–1973 incl.)
P&S 6, Other 17.
Total: 23

Fellowship (3)	2400-fees	12	3260		
Teaching Fellowship (6)	2750-3200	9	3260	6 Teaching	

TYPE of financial assistance (with number anticipated 1974-1975)	STIPEND amount in dollars	STIPEND 9 or 12 months	TUITION if not included in stipend (dollars)	SERVICE REQUIRED hours per week	SERVICE REQUIRED type of service	DEGREES AWARDED Academic year 1972-1973

WASHINGTON

University of Washington, Seattle 98195

DEPARTMENT OF BIOSTATISTICS
Donovan J. Thompson, Chairman
Richard A. Kronmal, Chairman, Biomathematics Group program for Ph. D.

Applications due: 7/30/74

Master s by dept. 4
Ph. D. (1970–1973 incl.) P&S 10. Total: 10

Teaching Assistantship (1)	441- 463	9	20		
Research Assistantship (4)	405- 425	9	20		

*Resident: $208; Nonresident: $547

WEST VIRGINIA

Marshall University, Huntington 25701

DEPARTMENT OF MATHEMATICS
John W. Hogan, Chairman

Applications due: 5/1/74*

Bachelor's by inst. 1294
Bachelor's by dept. 10

Teaching Assistantship (3-6)	1800-2400	9	6	Teaching	

*Late applications will be considered.

ERRATA
Volume 20

ROBERT A. HERRMANN. Nonstandard RO(i) strongly semiregular extensions, Abstract 73T-G142, Page A-598.

Delete Theorem 2.

LUDVIK JANOS. A representation of zero-dimensional spaces, Abstract 73T-G117, Page A-593.

Line three should read "...there exists a rigid subset".

ASSISTANTSHIPS AND FELLOWSHIPS. Northwestern University, Page 385.

Tuition for the fellowships and teaching assistantships are waived.

Volume 21

RONALD M. URNER. Topological general position. Preliminary report, Abstract 711-54-5, Page A-210.

Line three for "... $\dim X \leq k, \ldots$" read "$\operatorname{dem} X \leq k$"; line 8 for "... $\dim X \leq k$, $\dim Y \leq \ell \ldots$" read "$\operatorname{dem} X \leq k$, $\operatorname{dem} Y \leq \ell$".

GRAEME C. WAKE and J. M. HEATH. Nonlinear eigenvalue problems with mixed boundary conditions, Abstract 711-35-31, Page A-150.

Line three for "... $B_k(u) \equiv kb(x) \ldots$" read "$B_k(u) \equiv kb(x)u$"; for "... $x \in \partial D_1 = \partial D - \partial D_1 \ldots$" read "$x \in \partial D_1 = \partial D - \partial D_2$".

NEW AMS PUBLICATIONS

PROCEEDINGS OF THE STEKLOV INSTITUTE

BOUNDARY VALUE PROBLEMS OF MATHEMATICAL PHYSICS. VII, edited by O. A. Ladyženskaja

Number 116 (1971)
245 pages; list price $30.60; member price $22.95; ISBN 0-8218-3016-3
To order, please specify STEKLO/116

This volume is dedicated by the authors to Kirill Kapitonovič Golovkin who died in 1969 at the age of 33. One of his papers appears in the collection. The list of papers includes "On the approximate computation of Fourier integrals" by K. K. Golovkin; "Construction of nonsingular isoperimetric films" by M. L. Gromov and Ja. M. Èliašberg; "On the Dirichlet problem for quasilinear nonuniformly elliptic equations of second order" and "Smoothness of generalized solutions of degenerate parabolic equations of second order" by A. V. Ivanov; "Stability of difference schemes for Navier-Stokes equations in cylindrical coordinates and a sweep method for them" by K. P. Ivanov, O. A. Ladyženskaja, and V. Ja. Rivkind; "On a representation of the solution of a linearized stationary problem for the Navier-Stokes equations in the case of two space variables" by N. K. Korenev; "Boundary value problems for mixed equations" by O. A. Ladyženskaja and L. Stupjalis; "On some classes of quasilinear nonuniformly elliptic equations. 2" by A. P. Oskolkov; "A priori estimates for weighted first derivatives for certain classes of quasilinear nonuniformly elliptic equations in an unbounded domain" by A. P. Oskolkov and V. A. Tarasov; "Application of potential theory to the solution of a linearized system of Navier-Stokes equations in the two-dimensional case" by A. N. Popov; "On Green's matrices for elliptic boundary value problems. II" by V. A. Solonnikov; "Nonlinear boundary value problems for equations of minimal-surface type" by N. N. Ural'ceva; "A fractional step scheme for parabolic equations of general form" by A. E. Ščadilov.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

THE ARITHMETICS OF QUADRATIC JORDAN ALGEBRAS by Michel L. Racine

Number 136
132 pages; list price $3.20; member price $2.40; ISBN 0-8218-1836-0
To order, please specify MEMO/136

The first step in establishing an arithmetic theory for finite dimensional quadratic Jordan algebras over the quotient field of a Dedekind ring is the determination of maximal orders. The first results in this direction were obtained by M. Knebusch. In this Memoir, using techniques of associative arithmetic and the arithmetic theory of forms, we determine the maximal orders of special simple quadratic Jordan algebras. It is shown that if the field is local then there are only a finite number of isomorphism classes of maximal orders of such an algebra, and in most cases the actual number is computed Partial results on the maximal orders of exceptional Jordan algebras are also given.

COMPLEX ACTIONS OF LIE GROUPS by Connor Lazarov and Arthur Wasserman

Number 137
84 pages; list price $2.90; member price $2.18; ISBN 0-8218-1837-6
To order, please specify MEMO/137

The objects under study are triples (M, J, ϕ) where M is a smooth manifold, J is a stable almost complex structure on M and $\phi: G \times M \to M$ is an action of the compact Lie group G on M preserving the stable almost complex structure.

The bordism theory of such triples with free action of G is studied by constructing a spectrum and computing the cohomology of the spectrum as a module over the Steenrod algebra and as an $H^*(MU)$ comodule. The bordism groups $\Omega_*^{(G, ad)}$ are determined when G is connected and $H^*(G)$ is torsion free.

The above result is used to study the bordism group of "regular U(n)-manifolds", $\Omega_*^{U(n)}$, where $G = U(n)$ and all isotropy groups are conjugate to subunitary groups. A spectral sequence relating $\Omega_*^{U(n)}$ and the groups $\Omega_*(U(k), ad)(BU(j))$ is constructed and shown to collapse at the E^2 level; $\Omega_*^{U(n)}$ is shown to be a free MU_* module and generators are described.

AMS TRANSLATIONS—SERIES 2

TEN PAPERS IN ANALYSIS

Volume 102
256 pages; list price $19.10; member price $14.32; ISBN 0-8218-3052-X
To order, please specify TRANS2/102

This volume of the AMS Translations–Series 2 contains the following ten papers in analysis: "General theorems on the factorization of matrix-valued functions. I. Fundamental theorem," and "General theorems on the factorization of matrix-valued functions. II. Some tests and their consequences," M. S. Budjanu and I. C. Gohberg; "On a Tauberian theorem," M. V. Keldyš; "On the asymptotic behavior of the spectral function of a selfadjoint differential equation of the second order and on expansion in eigenfunctions" and "On the spectral function of the

equation $y'' = \{\lambda - q(x)\}y = 0$," B. M. Levitan; "Theorems of Tauberian type in the spectral analysis of differential operators," V. A. Marčenko; "D-decomposition of the space of quasi-polynomials (On the stability of linearized distributive systems)," Ju. I. Neĭmark; "Functions with dominant mixed derivative satisfying a multiple Hölder condition" and "Approximation of functions in the mean by trigonometric polynomials," S. M. Nikol'skiĭ; "A limit theorem for difference operators," A. I. Žukov.

SELECTED TRANSLATIONS IN MATHEMATICAL STATISTICS AND PROBABILITY

Volume 13
303 pages; list price $26.40; member price $19.80; ISBN 0-8218-1463-X
To order, please specify STAPRO/13

This volume contains twenty papers by the following authors: M. Arató, A. Ja. Beleckiĭ, A. Bikjalis, E. F. Car'kov, I. D. Čerkasov, Ju. M. Ermol'ev, J. Gergely, B. Grigelionis, Zohel' S. Halil', M. D. Judin, N. P. Kandelaki, Ja. P. Lumel'skiĭ, V. Paulauskas, E. A. Pleszczyńska, A. G. Postnikov, A. Rényi, L. I. Saulis, A. Slušnis, Josef Štěpán, A. D. Tuniev, N. N. Vahanija.

SIAM-AMS PROCEEDINGS

STOCHASTIC DIFFERENTIAL EQUATIONS, edited by J. B. Keller and H. P. McKean

Volume VI
216 pages; list price $17.90; member price $13.43; ISBN 0-8218-1325-0
To order, please specify SIAM/6

This volume constitutes the proceedings of a symposium in applied mathematics, sponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics, held March 29–30, 1972. The papers collected here are extremely varied. The task of the editors, who also served as co-chairmen of the symposium, was to bring together a meeting on stochastic differential equations. Usually a pure mathematician will understand this subject to be part of the integral and differential calculus based upon the Brownian motion, dealing chiefly with problems like $\dot{x} = e(x)b + f(x)$ with known coefficients e and f and a standard "white noise" $\dot{b}$. It was, however, the policy of the editors to interpret the field much more broadly and to admit any kind of problem in which a randomness enters either via the coefficients or the forcing, or both, a simple example being $\ddot{x} + (\epsilon + \alpha^2)x = \cos \beta t + \gamma \dot{b}$, in which ϵ is a (small) stationary noise and α, β, γ are known constants. Clearly, this is a vast subject with connections to the most diverse fields of application and could be treated at the symposium in the most episodic manner only. Correspondingly, the editors' aim is modest, namely to indicate the extraordinary variety of such problems, especially as they arise from practical considerations, and to hope to influence some of the mathematical audience to look into this rich and fascinating field.

1974 MATHEMATICAL SCIENCES ADMINISTRATIVE DIRECTORY

167 pages; list price $5.00; member price $2.50; ISBN 0-8218-0002-7
To order, please specify ADMDIR/74

- Chairmen of Academic Departments in the Mathematical Sciences in the U.S. and Canada coded according to the highest degree offered;
- Heads of Nonacademic Research Groups;
- Heads and key personnel of a selected group of Government Agencies;
- Editors of Mathematics Journals;
- Officers and Committee Members in twenty-three Professional Mathematical organizations;
- An Index for colleges and universities.

PERSONAL ITEMS

NADIM ASSAD has been appointed to a lectureship at the Technion-Israel Institute of Technology.

BARRY J. ARNOW of Rutgers University has been appointed to an assistant professorship at Kean College of New Jersey.

F.S. BECHMAN of IBM has been appointed to a professorship and to the chairmanship of the Computer and Information Sciences Department at Brooklyn College (CUNY).

ROGER BERNARD of the University of Kentucky has been appointed to a lectureship at Texas Tech University.

EVELYN H. BOORMAN of Columbia University has been appointed to an assistant professorship at the University of Michigan.

JOHN E. BROTHERS of Indiana University has been appointed a visiting member at the Institute for Advanced Study.

PAUL-JEAN CAHEN of McGill University has been appointed to an assistant professorship at the University of British Columbia.

JOHN R. CANNON of the University of Texas has been appointed to a visiting professorship at Texas Tech University for the academic year 1973-1974.

SRISADKI CHARMONMAN of the University of Missouri has been appointed to a professorship in the Computer Science Department and to advisor to the vice president for Administration on Computer Usage at the State University of New York, College at Brockport.

MAN-DUEN CHOI of Toronto University has been appointed to a lectureship at the University of California, Berkeley.

ROBERT DELVER of the University of Toronto has been appointed to a visiting assistant professorship at Ryksuniversiteit te Groningen, The Netherlands.

JOHN E. DONER of Purdue University has been appointed to an associate professorship at the University of California, Santa Barbara.

DANIEL M. DRIBIN of the National Security Agency has been appointed an executive of the graduate committee at the University of Maryland.

PATRICK EBERLEIN of the University of California, Berkeley, has been appointed to an assistant professorship at the University of North Carolina at Chapel Hill.

ROBERT D. EDWARDS of the University of California, Los Angeles, has been appointed to a visiting assistant professorship at the University of California, Berkeley.

BRUCE F. GLOBUS of Brandeis University has been appointed to an assistant professorship at Wayne State University.

BRIAN GREENBERG of Rutgers University has been appointed to a visiting assistant professorship at the University of Nebraska, Lincoln.

KENNETH I. GROSS of Darmouth College has been appointed to an associate professorship at the University of North Carolina at Chapel Hill.

JAMES E. HALL of the University of Wisconsin-Extension, Madison, has received the Kiekhofer award for excellence in teaching.

MARSHALL HALL, Jr., of the California Institute of Technology has been appointed to a named professorship at that institute. He has been designated as the first IBM Professor of Mathematics.

MORTON E. HARRIS of the University of Illinois at Chicago Circle has been appointed to a visiting professorship at the University of Minnesota.

MAURICE G. HARTMAN of Universal Oil Products has been appointed an engineer with Offshore Power Systems, Jacksonville, Florida.

ELMER K. HAYASHI of the University of Illinois has been appointed to a visiting assistant professorship at Wake Forest University.

DAVID B. HEISLER of Hofstra University has been appointed to an assistant professorship at John Jay College (CUNY).

J. WILLIAM HELTON of SUNY at Stony Brook has been appointed to a visiting associate professorship at the University of California, Los Angeles.

ANATOLE JOFFE has been appointed as a director of the Centre de Recherches Mathematiques at the University of Montreal.

JOHN W. KENELLY of Clemson University will be on sabbatical leave at the Department of Operations Research, Stanford University, for the academic year 1973-1974.

PATRICIA C. KENSCHAFT has been appointed to an assistant professorship at Montclair State College.

JOSEPH F. KENT III of the University of Florida has been appointed to an assistant professorship at the University of Richmond.

ROBION C. KIRBY of the University of California, Berkeley, has been appointed to a research professorship in the Miller Institute for Basic Research in Science, University of California, Berkeley, for 1973-1974.

MANFRED KOCHEN of the University of Michigan has been appointed honorary visiting research associate at Harvard University until January 31, 1974.

BERTRAM KOSTANT of the Massachusetts Institute of Technology has been appointed to a visiting professorship at the University of California, Berkeley.

ERNESTO A. LACOMBA of the University of Brasilia has been appointed associate researcher at the National University, Mexico.

DOUGLAS A. LIND of Stanford University has been awarded a fellowship by the Miller Institute for Basic Research in Science, Univ-

ersity of California, Berkeley, for 1973-1974.

RICHARD I. LOEBL of the University of California, Santa Cruz, has been appointed to an assistant professorship at Wayne State University.

WILLIAM S. MAHAVIER of Emory University is on sabbatical leave at the University of Houston for the academic year 1973-1974.

KIM E. MICHENER of Purdue University has been appointed to an assistant professorship at Wayne State University.

NORMAN MIRSKY of the University of California, Los Angeles, has been appointed to a lectureship at Texas Tech University.

CALVIN C. MOORE of the University of California, Berkeley, has been elected a fellow of the American Academy of Arts and Sciences.

MICHAEL D. MORLEY of Cornell University has been appointed to a visiting professorship at the University of California, Berkeley.

PAUL NELSON, Jr., of the Oak Ridge National Laboratory has been appointed to a professorship at Texas Tech University.

JOHN W. NEUBERGER of Emory University was on leave for the fall quarter, 1973, at the University of Kentucky.

MARILYN K. OBA of Loyola University of Los Angeles has been appointed to an assistant professorship at California State University, Fullerton.

JOSEPH PLANTE of the University of Wisconsin, Madison, has been appointed to an assistant professorship at the University of North Carolina at Chapel Hill.

ERIC REISSNER of the University of California, San Diego, was the recipient of the Timoshenko Medal at the 94th annual winter meeting of the American Society of Mechanical Engineers. The medal is given in recognition of distinguished contributions to applied mechanics.

LAWRENCE RISMAN has been appointed to a lecturship at the Technion-Israel Institute of Technology.

RONALD S. RIVLIN of the Lehigh University has been elected to a two-year term as a member of the board of governors of the American Institute of Physics, effective March 1974.

RAYMOND G. ROBINSON, Jr., of E.I. DuPont de Nemours and Co., Inc., has been appointed a programmer with Hercules Incorporated, Wilmington, Delaware.

SHOLOM ROSEN of the University of Illinois has been appointed to an assistant professorship at Johns Hopkins University.

RICHARD A. SANERIB, Jr., of the University of Colorado has been appointed to an assistant professorship at Emory University.

ALLEN SHIELDS of the University of Michigan was appointed to a visiting professorship at the University of California, Berkeley, for the fall 1973 quarter.

IAN N. SNEDDON of the University of Glasgow has been awarded the degree of Doctor of Science honoris causa of the University of Warsaw.

JONATHAN WAHL of the Institute for Advanced Study has been appointed to an assistant professorship at the University of North Carolina at Chapel Hill.

BILL WATSON of the University of Washington and the University of Pittsburgh has been appointed to a professorship at the Université de Oriente, Cumaná, Venezuela.

ROTRAUT C. WEISS of SLAC has been appointed a systems analyst at Gould, Inc., Sunnyvale, California.

JEFFREY C. WIENER of Washington University has been appointed to an assistant professorship at Emory University.

ROBERT C. WILLIAMS of the University of Illinois has been appointed to an assistant professorship at Alfred University.

PROMOTIONS

To Assistant Provost. Rochester Institute of Technology: CHARLES W. HAINES.

To Dean, Graduate School of Research. Central Washington State College: DALE R. COMSTOCK.

To Director, Center for Pure and Applied Mathematics. University of California, Berkeley: MURRAY H. PROTTER.

To Chairman, Science Division. Windham College: ROBERT LEWAND.

To Professor and Chairman, Department of Mathematics. College of St. Elizabeth: BARBARA A. MORRISON.

To Chairman, Department of Mathematics. University of California, Berkeley: MAXWELL A. ROSENLICHT; Los Angeles City College: ROBERT B. HERRERA; Washington University: FRANKLIN HAIMO.

To Vice Chairman for Personnel, Department of Mathematics. University of California, Berkeley: SHOSHICHI KOBAYASHI.

To Professor. University of California, Berkeley: MARC A. RIEFFEL, HUNG-SHI WU; University of Maryland: SIDNEY L. GULICK III, RONALD LIPSMAN; University of North Carolina at Chapel Hill: JOSEPH CIMA, WILLIAM GRAVES, ROBERT HEYNEMAN, JON TOLLE.

To Associate Professor. Brock University: SHAO-CHIEN CHANG, VELMER B. HEADLEY; University of California, Berkeley: DAVID M. GOLDSCHMIDT, JOHN B. WAGONER; C.W. Post College: ELLIOTT H. BIRD; University of Illinois: GEORGE K. FRANCIS; Kent State University: ROBERT H. LOHMAN; Kingsborough Community College (CUNY): PHILIP GREENBERG; University of Pittsburgh: JAMES P. FINK; Roanoke College: KENNETH R. GARREN; Texas Tech University: HOMER WALKER; Villanova University: MICHAEL L. LEVITAN; Wayne State University: JOHN C. BRECKENRIDGE, LOWELL J. HANSEN.

To Assistant Professor. Emory University: KENNETH I. MANDELBERG; Tufts University: GERALD McCOLLUM.

INSTRUCTORSHIPS

Emory University: DONALD L. GREENWELL, PETER D. JOHNSON, Jr.; University of North Carolina at Chapel Hill: PAUL KING.

DEATHS

Professor PAUL MASON BATCHELDER of Austin, Texas, died on August 5, 1971, at the age of 85. He was a member of the Society for 51 years.

Dr. WILLIAM R. BURWELL of Wolfeboro, New Hampshire, died in November, 1971, at the age of 77. He was a member of the Society for 49 years.

Professor Emeritus GRIFFITH CONRAD EVANS of the University of California, Berkeley, died on December 8, 1973, at the age of 86. He was a member of the Society for 64 years.

Professor RALPH H. FOX of Princeton University died on December 23, 1973, at the age of 60. He was a member of the Society for 38 years.

Professor Emeritus LAWRENCE M. GRAVES of the University of Chicago died on May 25, 1973, at the age of 76. He was a member of the Society for 49 years.

Professor AUBREY J. KEMPNER of the University of Colorado died on November 18, 1973, at the age of 93. He was a member of the Society for 60 years.

Professor MEIRA LAVIE of the Technion-Israel Institute of Technology died on December 15, 1973, at the age of 38. She was a member of the Society for 6 years.

Professor H. MELVIN LIEBERSTEIN of the University of Newcastle died on August 18, 1973, at the age of 47. He was a member of the Society for 24 years.

Professor Emeritus EUGENE TAYLOR of the University of Idaho died on June 27, 1973, at the age of 93. He was a member of the Society for 52 years.

Professor Emeritus W.J. TRJITZINSKY of the University of Illinois died on December 8, 1973, at the age of 72. He was a member of the Society for 46 years.

Professor JOSEPH L. WALSH of the University of Maryland died on December 10, 1973, at the age of 78. He was a member of the Society for 35 years.

Professor Emeritus ROY M. WINGER of the University of Washington died on April 25, 1973, at the age of 87. He was a member of the Society for 59 years.

ERRATA

In the August, 1973 Notices it was announced that EUGENE LUKACS has been elected a corresponding member of the Australian Academy of Sciences. The item should have read: EUGENE LUKACS of Bowling Green State University has been elected a corresponding member of the Austrian Academy of Sciences.

In the October, 1973 Notices it was reported that Professor LLOYD L. LASSEN of the University of Texas at Arlington died on May 14, 1973, at the age of 90. The item should have read Professor LLOYD L. LASSEN of the University of Texas at Arlington died on May 14, 1973, at the age of 64.

NEWS ITEMS AND ANNOUNCEMENTS

SIGSAM LECTURESHIP PROGRAM

A revised Lectureship Program of ACM's Special Interest Group on Symbolic and Algebraic Manipulation (SIGSAM) has been developed for the academic year 1973–1974. Three rosters of lecturers, one for mathematics, one for computer science, and one for physics, have been drawn up. The SIGSAM Lecturers in Mathematics and titles of their talks are as follows: Elwyn R. Berlekamp (University of California, Berkeley), "Factoring polynomials"; W. Stanley Brown (Bell Laboratories), 'The ALTRAN language for symbolic algebra" and "On Euclid's algorithm and the computation of polynomial greatest common divisors"; George E. Collins (University of Wisconsin), "Computer algebra of polynomials and rational functions" and "Quantifier elimination for elementary algebra by cylindrical algebraic decomposition"; W. Morven Gentleman (University of Waterloo), "The analysis of algorithms for symbolic computation" and "Applications of symbolic computation in numerical analysis"; W. Kahan (University of California, Berkeley), "One numerical analyst's experience with one symbol manipulator"; Joel Moses (Massachusetts Institute of Technology), "Symbolic manipulation and integration." Additional information concerning the SIGSAM Lectureship Program (including biographic material on the speakers and abstracts of their talks) may be obtained by writing to James H. Griesmer, SIGSAM Chairman, IBM Thomas J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598.

ABSTRACTS PRESENTED TO THE SOCIETY

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An individual may present only one abstract by title in any one issue of the *Notices* but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

*74T-A24. PHILIP A. LEONARD, Arizona State University, Tempe, Arizona 85281 and KENNETH S. WILLIAMS, Carleton University, Ottawa, Ontario K1S 5B6, Canada. *The septic character of* 2, 3, 5 *and* 7.

Let p be a prime $\equiv 1 \pmod 7$. The diophantine system $72p = 2x_1^2 + 42(x_2^2 + x_3^2 + x_4^2) + 343(x_5^2 + 3x_6^2)$, $12x_2^2 - 12x_4^2 + 147x_5^2 - 441x_6^2 + 56x_1x_6 + 24x_2x_3 - 24x_2x_4 + 48x_3x_4 + 98x_5x_6 = 0$, $12x_3^2 - 12x_4^2 + 49x_5^2 - 147x_6^2 + 28x_1x_5 + 28x_1x_6 + 48x_2x_3 + 24x_2x_4 + 24x_3x_4 + 490x_5x_6 = 0$, has six solutions $(x_1, x_2, x_3, x_4, x_5, x_6)$ with $x_1 \equiv 1 \pmod 7$, distinct from $(-6t, \pm 2u, \pm 2u, \mp 2u, 0, 0)$, where $p = t^2 + 7u^2$, $t \equiv 1 \pmod 7$. It is shown that for *any* of these six solutions: 2 is a seventh power (mod p) iff $x_1 \equiv 0 \pmod 2$; 3 is a seventh power (mod p) iff $x_5 \equiv x_6 \equiv 0 \pmod 3$; 5 is a seventh power (mod p) iff either $x_2 \equiv x_3 \equiv -x_4 \pmod 5$ and $x_5 \equiv x_6 \equiv 0 \pmod 5$, or $x_1 \equiv 0 \pmod 5$ and $x_2 + x_3 - x_4 \equiv 0 \pmod 5$; 7 is a seventh power (mod p) iff $x_2 - 19x_3 - 18x_4 \equiv 0 \pmod{49}$. (Received December 18, 1973.)

*74T-A25. CHARLES F. WELLS, Case Western Reserve University, Cleveland, Ohio 44106. *Polynomials over finite fields which commute with translations.*

Let k be a finite field with $q = p^n$ elements. Let $a \in k$. Let $f(x)$ be a polynomial with coefficients in k such that $f(x + a) = f(x) + a$. Assume $\deg f \leq q - 1$. Equations are given which determine the coefficients of $f(x)$ uniquely in terms of a and the coefficients of x^{rp} for $r = 0, 1, \cdots, p^{n-1} - 1$. Thus there are exactly qp^{n-1} such polynomials $f(x)$. This result is found by using the following easy generalization of a well-known theorem: Let ϕ be a permutation of a finite set with $n = de$ elements, and suppose that ϕ consists of e disjoint d-cycles. Then the semigroup of transformations of X which commute with ϕ is isomorphic to the wreath product of the cyclic group of order d by the full transformation semigroup on e letters. (Received October 25, 1973.)

*74T-A26. AWAD A. ISKANDER, University of Southwestern Louisiana, Lafayette, Louisiana 70501. *Product of ring varieties and attainability.*

The class of all rings that are Everett extensions of a ring in a variety U by a ring in a variety V is a variety $U \cdot V$. With respect to this product, the set of all ring varieties is a partially ordered groupoid (under inclusion). A variety is idempotent if and only if it is the variety of all rings or generated by a finite number of finite fields. No families of polynomial identities other than those equivalent to $x = x$ or $x = y$ are attainable on the class of all rings or the class of all commutative rings. (Received October 26, 1973.)

74T-A27. C. D. ALIPRANTIS, California Institute of Technology, Pasadena, California 91109 and ERIC S. LANGFORD, University of Maine, Orono, Maine 04473. *Quotient Riesz spaces and quotient seminorms.* Preliminary report.

In what follows, L is a Riesz space with Riesz seminorm ρ and A is an ideal of L. It is known ("Riesz Spaces.I", p. 438) that the quotient seminorm on L/A defined by $[\rho]([f])$ $\inf\{\rho(g);\ g \in [f]\}$, is also a

Riesz seminorm. We investigate the relationship of the A-properties of Luxemburg and Zaanen ("Notes on Banach function spaces, II, III, XI", Nederl. Akad. Wetensch. Proc. Ser. A 66 = Indag. Math 25(1963), 148–153, 239-250; ibid. 67 = Indag. Math. 26(1964), 507-518) in L to those in L/A. We have shown the following: (1) If L has (A, iii), then so does L/A; (2) if L is Archimedean and has (A, ii), then L/A has (A, ii); (3) if L is almost σ-Dedekind complete (see the authors' "Almost σ-Dedekind complete Riesz spaces and the main inclusion theorem", Abstract 711-06-14, these *Notices* 21(1974)) and has (A, i), then L/A has (A, i); (4) it is possible for L to have (A, i) and sufficiently many projections, for A to be a band, but for L/A to fail to have even the σ-Fatou property; (5) if L is σ-Dedekind complete and has the σ-Fatou property, and if A is a σ-ideal, then L/A has the σ-Fatou property; (6) if L is Dedekind complete and has the Fatou property, and if A is a band, then L/A has the Fatou property. (Received October 29, 1973.)

74T-A28. E. J. COCKAYNE, University of Victoria, Victoria, British Columbia V8W 2Y2, Canada. *Some tree-star Ramsey numbers.*

The Ramsey number $r(G_1, G_2)$ is the least integer N such that for every graph G with N vertices, either G has the graph G_1 as a subgraph or $\overline{G}$, the complement of G, has the graph G_2 as a subgraph. In this paper we embed the paths P_m in a much larger class $\mathcal{T}$ of trees and then show how some evaluations by T. D. Parsons of Ramsey numbers $r(P_m, K_{1,n})$ ($K_{1,n}$ is the star of degree n) are also valid for $r(T_m, K_{1,n})$ where $T_m \in \mathcal{T}$. (Received October 29, 1973.) (Author introduced by Dr. G. G. Miller.)

74T-A29. EDWARD SPENCE, University of Glasgow, Glasgow G12 8QW, Scotland. *Hadamard matrices of the Goethals-Seidel type.*

If n is an odd prime power such that $1 + n + n^2$ is either a prime p such that $p \equiv 3$, 5 or 7 (mod 8), or $2n^2 + 2n + 3$ is a prime power, then there exists a skew-Hadamard matrix of the Goethals-Seidel type of order $4(1 + n + n^2)$. This yields skew-Hadamard matrices of orders 52, 124, 1228, 2604, 3028, $\cdots$. In the case $1 + n + n^2 = p \equiv 1$ (mod 8) a Hadamard matrix, not necessarily skew, of order $4(1 + n + n^2)$ is constructed. Partial results are obtained in the case $n = 2^r$. (Received October 29, 1973.)

*74T-A30. GORO AZUMAYA, Indiana University, Bloomington, Indiana 47401. *Characterizations of semiperfect and perfect modules.* Preliminary report.

A projective (say, left) R-module P is called by Mares (Math. Z. 82(1963), 347–360) semiperfect if every homomorphic image of it has a projective cover, and P is called perfect if the direct sum of any (infinite) number of copies of P is semiperfect. *Theorem* 1. The following conditions are equivalent: (1) P is semiperfect; (2) every proper submodule of P is contained in a maximal submodule of P and every simple homomorphic image of P has a projective cover; (3) P is a direct sum of local R-modules and the radical $J(P)$ is small in P (where local R-module means a projective cover of a simple R-module, or equivalently, an R-module isomorphic to a left ideal Re generated by a local idempotent e). *Theorem* 2. P is perfect if and only if every proper submodule of P is contained in a maximal submodule of P and every semisimple R-module which is a direct sum of simple homomorphic images of P has a projective cover. These two theorems can be regarded as natural generalizations of Sandomierski's theorems (Proc. Amer. Math. Soc. 21(1969), 205–207). (Received October 29, 1973.)

74T-A31. ROLANDO B. CHUAQUI, Universidad Católica De Chile, Santiago, Chile and University of California, Berkeley, California 94720. *Some theorems about real multiples in a cardinal algebra.*

A definition of real multiples in a generalized cardinal algebra appeared in the author's "Cardinal algebras and measures", Trans. Amer. Math. Soc. 142(1969), 61–79 (see also Tarski, "Cardinal algebras", Oxford University Press, New York, 1949). Let $\mathfrak{A} = \langle A, +, \Sigma \rangle$ be a G.C.A., $a, b, c \in A$, $A(a)$, $S(a)$, $R(a)$, the sets of b with $a + b = a$, $b \leq a$, and b real multiple of a, respectively. *Theorem* 1. Let $B \subseteq A$ with $0, a \in B$ and B closed under Σ. Then $B \subseteq A(a) \cup R(a)$ iff B satisfies simultaneously: (i) if $b \in B$, there is $c \in {}^{\omega}(B \cap S(a))$ with $b = \Sigma_{i<\omega} c_i$; (ii) if $b, c \in (B \cap S(a)) - A(a)$, then $b \leq c$ or $c \leq b$; (iii) if $c \in A(a)$, $b \in B - A(a)$, then $c \leq b$; (iv) if $b, c \in (B \cap S(a)) - A(a)$, $b \leq c$, then there is $d \in B$ with $b + d = c$. *Theorem* 2. Let B be as in 1 and if $n < \omega$,

there is $b \in B$ with $a = n \cdot b$. Then $B \subseteq A(a) \cup R(a)$ iff (i) and (ii) are satisfied. *Theorem* 3. Let r be a real with $0 \leq r < 1$ and $r \cdot a$ defined. Then $b \quad (1 - r) \cdot a$ iff $r \cdot a + b = a$ and for every $c \in A(a)$, $c \leq b$. *Theorem* 4. Let $a \neq 2 \cdot a$ be completely divisible, $b \neq 0$. Then there is a real $s > 0$ with $b = s \cdot a$ iff for every real $r \geq 0$ with $r \cdot a < b$, there is c with $r \cdot a + c = b$ and $a \leq n \cdot c$ for some integer $n < \infty$. (Received October 31, 1973.)

*74T-A32. ROBERT GILMER and JOSEPH F. HOFFMANN, Florida State University, Tallahassee, Florida 32306. *The integral closure need not be a Prüfer domain.* Preliminary report.

Let K be a field, let L be the quotient field of the formal power series ring $K[[X]]$, and let W be the K-subspace of L consisting of all elements f such that the coefficient of X^{-1} in the Laurent expansion of f is zero. The domain $D = K + WY + Y^2L[[Y]]$ has the following three properties: (1) D has a unique minimal overring, (2) each nonzero finitely generated ideal of D is divisorial, and (3) the integral closure of D is not a Prüfer domain. Observations (1) and (3) show that the domain D provides a negative answer to a question raised by W. Heinzer and the first author in [J. Math. Kyoto Univ. 7(1967), 133–150], while (2) and (3) yield a negative answer to a question closely related to a problem posed by Heinzer in [Mathematika 15(1968), 164–170]. (Received November 14, 1973.)

74T-A33. THOMAS L. MARKHAM, University of South Carolina, Columbia, South Carolina 29208. *Products of inverses of M-matrices.* Preliminary report.

Suppose A is an $n \times n$ real matrix such that $a_{ij} \leq 0$ for $i \neq j$. Then A is a M-matrix ($A \in \mathfrak{M}$) if and only if A^{-1} exists and is nonnegative. If $B = C_1 \cdots C_K$ where $C_i^{-1} \in \mathfrak{M}$, then B is nonnegative and $\det(B) > 0$. However, this condition is not sufficient for B to be a product of inverses of M-matrices. We examine in this paper some sufficient conditions based on triangular factorizations. *Theorem.* Suppose A is nonnegative of order n, and A has nonzero principal minors of all orders. If A can be factored as $L \cdot U$ where $L(U)$ is a lower (upper) triangular matrix which is nonnegative, then A is a product of inverses of M-matrices. Other sufficient conditions are obtained in terms of the Schur complement of certain principal submatrices of A. (Received October 19, 1973.)

*74T-A34. ELLIS D. COOPER, City University of New York, Lehman College, Bronx, New York 10468. *Listing finite partial orders.*

The Hasse diagrams of finite partial orders correspond to certain parenthesized linear expressions called po-strings. A 360 PL/1 computer program exists which allows the construction of all po-strings corresponding to all finite partial orders having n nodes, $n \geq 1$. For instance, when $n = 6$, the computer prints out a list of 318 po-strings. (Received October 31, 1973.) (Author introduced by Professor Myles Tierney.)

74T-A35. SURJEET SINGH, Ohio University, Athens, Ohio 45701 and Aligarh Muslim University, Aligarh, India and QUAZI ZAMEERUDDIN, Kirori Mal College, Delhi - 7, India. *On commutative self-injective rings.*

Let R be a commutative ring with unity $1 \neq 0$. R is said to be a (PMI)-ring if for every prime ideal P, with $P^2 \neq 0$, R/P^2 is a self-injective ring. Noetherian (PMI)-rings were studied by Singh and Wasan [Canad. J. Math. 22(1970), 1101–1108]. A commutative ring R is said to be an n-(PMIE)-ring if for every proper ideal A, A^*, the subring of R generated by $A \cup \{1\}$ is a Noetherian (PMI)-ring. The following structure theorems are established: (I) Let R be a quasi-local ring with M as its maximal ideal, and which is not a domain. Then R is an n-(PMIE)-ring iff for some prime number p, and $n > 1$, $c(R) = p^n$ [$c(X)$ denotes the cardinality of X], and R satisfies one of the following: (1) $M^*/M \cong Z/(p)$ and $M^2 = (0)$; (2) $M^2 \neq (0)$, $n \leq 4$, $R/M \cong Z/(p)$ and R is a special primary ring; (3) $n > 4$ and $R \cong Z/(p^n)$. (II) A commutative ring R is an n-(PMIE)-ring iff it is of one of the following types: (1) R is a (KE)-domain; (2) R is a local n-(PMIE)-ring; (3) R is semisimple artinian of finite characteristic; (4) $R = S \oplus R_1 \oplus \cdots \oplus R_t$, $t \geq 1$, S of type (3) or zero and each R_i is a quasi-local (ME)-ring (not a domain) of order p_i^{ni} for some prime p_i such that (i) p_i are distinct, or (ii) $t = 2$, $p_1 = p_2$,

$n_1 = n_2 = 2$. The structures of (KE)-domains and (ME)-rings were determined by Singh and Kumar [Arch. Math. 23(1972)]. (Received October 26, 1973.) (Authors introduced by Professor S. K. Jain.)

74T-A36. AUREL J. ZAJTA, Kenyatta University College, Nairobi, Kenya. *On the solution of the Diophantine equation* $A^4 + B^4 = C^4 + D^4$.

A new method, using linear transforms, is presented to obtain 8 new solutions $(A^{(k)}, B^{(k)}, C^{(k)}, D^{(k)})$, $k = 1, 2, \cdots, 8$, from a known solution (A, B, C, D) of the equation of the title. The method is illustrated by examples of numerical and parametric solutions. Starting from the well-known parametric solution of Euler, eight new parametric solutions are obtained of which one is trivial and seven are nontrivial. (Received November 14, 1973.)

*74T-A37. HENRIK BRESINSKY, University of Maine, Orono, Maine 04473. *Analytically equivalent planar cubic branches.*

Let α and β be two planar branches with representatives (x_α, y_α), (x_β, y_β) in $K\{u\} \times K\{u\}$, $K\{u\}$ the power series ring over the algebraically closed field K of characteristic 0. α and β are said to be analytically equivalent (a. e.) if there exists a regular transformation f such that $f(K\{x_\alpha, y_\alpha\}) = K\{x_\beta, y_\beta\}$. It follows that any cubic branch is a. e. to a reduced branch with representative $(x_\alpha = u^3, y_\alpha = u^{3q+j} + \sum_{i=0}^{q-1} a_i u^{3(q+1)+2j})$, $j = 1$ or $j = 2$. Let 0_t be the last satellite point of the monomial branch $\bar{\alpha}$ with representative (u^3, u^{3q+j}). *Theorem.* The reduced cubic branch α is a. e. to $\bar{\alpha}$ iff α and $\bar{\alpha}$ have at least $3q + j - 2$ free points following 0_t in common. If α and $\bar{\alpha}$ have only 0_{t+s}, $1 \leq s < 3k + j + 1$, $0 \leq k \leq q - 2$, in common, then α is a. e. to $(u^3, u^{3q+j} + u^{3(q+k)+2j})$. (Received November 16, 1973.)

*74T-A38. JAMES R. WALL, Auburn University, Auburn, Alabama 36830. *Generalized inverses for stochastic matrices.*

Theorem. Let A be an $n \times m$ stochastic matrix with rank k. Then the Moore-Penrose inverse A^+ of A is stochastic if and only if there exist permutation matrices P and Q such that PAQ^T has the block form $PAQ^T = (B_{ij})$, $i = 1, \cdots, k$, $j = 1, \cdots, k$, where each B_{ij} is $n_i \times m_j$, $n_1 + \cdots + n_k = n$, $m_1 + \cdots + m_k = m$; for each i and each j exactly one block B_{ij} is nonzero, and this block has each entry equal to $1/m_j$. The form for A^+ is also given. All stochastic matrices A which have a stochastic semi-inverse (i.e., for which there exists X such that $AXA = A$ and $XAX = X$) are determined. (Received November 16, 1973.)

*74T-A39. GARY L. MULLEN, Pennsylvania State University, University Park, Pennsylvania 16802. *Equivalence classes of polynomials over finite fields.* Preliminary report.

Let $K[x_1, \cdots, x_r] = K[\bar{x}]$ be the ring of polynomials in r indeterminates over the finite field $K = GF(q)$ of order q, and let Ω be a group of permutations of K^r, the product of r copies of K. If $f, g \in K[\bar{x}]$, then f is *right equivalent* to g relative to Ω if $f(\phi(\bar{x})) = g(\bar{x})$ for some $\phi \in \Omega$. The object is to determine the number $\lambda(\Omega)$ of equivalence classes induced by Ω. Algorithms for $\lambda(\Omega)$, the number of equivalence classes of a given order, and $\mu(f, \Omega)$, the order of the class of f, are derived if Ω is cyclic or abelian, and some general results are found for the nonabelian case. Also discussed are the concepts of *automorphisms* of a polynomial f, and *equivalent* decompositions induced by two permutation groups Ω_1 and Ω_2. Several relations to permutation polynomials are also given. (Received November 19, 1973.)

74T-A40. N. S. MENDELSOHN, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. *Commutators in free groups.*

The following theorem is proved. In a free group a word of length $4n + 2$ which lies in the commutator subgroup is the product of at most n commutators. This result is best possible. (Received November 19, 1973.)

74T-A41. HUDSON V. E. KRONK, State University of New York, Binghamton, New York 13790, M. RADLOWSKI, Le Moyne College, Syracuse, New York 13214 and B. TRANEN, Vassar College, Poughkeepsie, New York 12601. *On the line-chromatic number of triangle-free graphs.* Preliminary report.

Vizing [Diskret. Analiz. 5(1965), 9–17] has shown that every planar graph G with maximum degree $\Delta(G) \geq 8$ has line-chromatic number $X'(G) \quad \Delta(G)$. In this paper we obtain similar results for triangle-free graphs. *Theorem* 1. If G is a planar graph with girth $g(G) = 4$ and $\Delta(G) \geq 5$, then $X'(G) = \Delta(G)$. *Theorem* 2. If G is a planar graph with $g(G) \geq 5$ and $\Delta(G) \geq 4$, then $X'(G) = \Delta(G)$. Results analogous to Theorems 1 and 2 are also obtained for triangle-free graphs having positive genus. (Received November 5, 1973.)

* 74T-A42. ALBERT A. MULLIN, 9213 Kristin Lane, Fairfax, Virginia 22030. *More on the geometry of prime numbers.* Preliminary report.

Recall that every (highly) visible lattice point is necessarily distinct from the origin. *Lemma.* Let k be a natural number. Let M be a bounded set in n-dimensional real Euclidean space E^n, $n \geq 2$, with Lebesgue measure $V(M) > k > 0$. Then there exists $z \in E^n$ such that $M + z$ contains at least k *highly visible* lattice points. *Basic lemma.* Let $V(K)$ be the n-dimensional Lebesgue measure of a nonhyperaxial origin-symmetrical convex body K in n-dimensional real Euclidean space E^n, $n \geq 2$. Then there exists a recursive function $f(\cdot)$ such that if $V(K) \geq f(n) > 2^n$ then K contains at least one lattice point all of whose coordinates are prime numbers and no two of which are equal. *Corollary.* There exist infinitely many primes. *Scholium.* These results on specially structured lattice points are germane to the recursive function theory of the class number of quadratic fields. (Received January 2, 1974.)

74T-A43. VINCENT C. HARRIS, San Diego State University, San Diego, California 92115 and M. V. SUBBARAO, University of Alberta, Edmonton, Alberta, Canada. *On the divisor sum function.* Preliminary report.

Let the positive integer n be written as $n = dd_2d_3 \cdots d_k$, the product of $k \geq 1$ factors d_i (with d in place of d_1 for convenience). We define $\sigma_s(n, k) = \Sigma d^s$ summed over all d such that $dd_2 \cdots d_k = n$. Also $\sigma(n, k) \quad \sigma_1(n, k)$. For $k = 2$ and for $s = 0$ this reduces to known functions. It is easily seen that $\sigma_s(n, k)$ is multiplicative and $\sigma(p^a, k) = p^a + \binom{k-1}{1}p^{a-1} + \binom{k}{2}p^{a-2} + \cdots + \binom{k+a-2}{a}$ where p^a is a power of a prime. Some identities and generating functions are obtained as also results concerning K-ply perfect numbers, which are defined to be integers n such that $\sigma(n, k) = kn$. (For example, $\sigma(5487,29) = 29 \cdot 5487$ and $\sigma(105,3) = 3 \cdot 105$.) The density of K-ply perfect numbers is shown to be zero. (Received November 26, 1973.)

74T-A44. WILLIAM A. ALFORD, GEORGE HAVAS and MICHAEL F. NEWMAN, Australian National University, Institute of Advanced Studies, Canberra, A.C.T. 2600, Australia. *Groups of exponent* 4. Preliminary report.

A commutator-power presentation has been obtained (with the aid of a computer) for the free group of rank 4 of the variety of groups of exponent 4 and nilpotency class 8. This has been done using an algorithm developed by I. D. Macdonald (J. Austral. Math. Soc., to appear) and J. W. Wamsley (Proc. Second Internat. Conf. on the Theory of Groups, Lecture Notes in Math., Springer-Verlag, Berlin and New York, to appear) with some refinements. The order of the group is 2^{392}. The order of third term of its derived series is 2^{14}. This confirms (in a different way) the result of Bachmuth, Mochizuki and Weston (Proc. Amer. Math. Soc. 39(1973), 228–234) that there is a group of exponent 4 and soluble length (precisely) 4. (Received November 26, 1973.)

74T-A45. ANDREW MacFARLANE, University of New South Wales, Kensington, New South Wales 2033, Australia. *Topological objects in topoi.* Preliminary report.

Open and closed topological objects can be defined in an elementary topos, $\underline{E}$, in two equivalent ways. Open and closed topologies coincide if $\underline{E}$ is boolean. The categories of open topological objects and closed topological objects both have finite limits and colimits and are $\underline{E}$ categories. The following results are true for both open and closed topological objects; (i) there is a functor Top which takes an object to the object of topologies on it and Top is a strong subfunctor of $P \circ P$ (P the power "set" functor); (ii) $V \subset \mathrm{Top}(X)$ implies $\bigcap(V) \in \mathrm{Top}(X)$; (iii) given a real number object R in $\underline{E}$ one constructs a homotopy category, H, which is an $\underline{E}$

category and for $\underline{E}$ = sheaves on a topological space one has a fundamental group object defined on H. Addition and multiplication on R are continuous for the internal open interval topology. Internal convergence of filters gives the link between open and closed topologies. (Received November 26, 1973.) (Author introduced by Dr. Peter Donovan.)

*74T-A46. S. BRENT MORRIS, Duke University, Durham, North Carolina 27706. *The generalized faro shuffle.*

The standard faro shuffle, an idealized riffle shuffle, divides the deck into two equal portions, and perfectly interlaces them. The simple cut takes one card from the top to the bottom of the deck. It is known that for decks of even size, the faro shuffle and simple cut generate all possible permutations, while if the deck is of odd size, only a small fraction are available. This paper considers a generalized faro shuffle where the deck is divided into n, rather than 2, portions, and these "interlaced" together. The following theorem is obtained. *Theorem.* The number of permutations generated by the generalized out faro shuffle O_n and the simple cut C operating on a deck of size $mn + k$, where $0 \leq k \leq n - 1$, is determined by the following rules: (1) if $m = 0$, k permutations are generated; (2) if $m \geq 1$, $k = n - 1$, and f is the order of O_n, then $f(mn + n - 1)$ permutations are generated; (3) if $m \geq 1$, $k \neq n - 1$, and $mn + k \equiv 0 \pmod 2$ or $mn(n-1)(m+1)/4 + m(n-k)(n-k-1)/2 = 1 \pmod 2$, then $(mn + k)!$ permutations, i.e. the symmetric group S_{mn+k}, are generated, otherwise $(mn + k)!/2$ permutations, i.e. the alternating group A_{mn+k}, are generated. (Received November 29, 1973.)

74T-A47. DANIEL S. KUBERT, Yale University, New Haven, Connecticut 06520. *Universal bounds on the torsion of elliptic curves.* Preliminary report.

Let E below be an elliptic curve defined over Q and E-tor its Q-torsion group. E_l designates the kernel of multiplication by l over $\bar{Q}$. E_l is l-deficient iff the field of definition of the points of E_l has degree over Q not divisible by l. *Result* I. Let l be a prime number for which $x^l + y^l = 1$ has only the trivial Q-solutions. Suppose E_l is l-deficient. Then if p is a prime number dividing the order of E-tor, p belongs to the set $\{2, 3, 5\}$. *Result* II. Let l be a prime number exceeding 3 for which $x^l + y^l = 1$ has only the trivial Q-solutions. Then E-tor does not contain an l^2-point. *Result* III. If E possesses the given isogeny structure, the list of groups following gives all possible structures for E-tor. 4-isogeny and all 2-points rational; $Z/2Z \times Z/2Z$, $Z/2Z \times Z/4Z$ $Z/8Z \times Z/2Z$ - 8-isogeny; $Z/2Z$, $Z/4Z$, $Z/8Z$, $Z/2Z \times Z/2Z$ $Z/4Z \times Z/2Z$, $Z/8Z \times Z/2Z$ - two 3-isogenies; 0, $Z/2Z$, $Z/3Z$, $Z/6Z$ - 9-isogeny; 0, $Z/3Z$, $Z/2Z$, $Z/6Z$, $Z/9Z$ - two 5-isogenies; 0, $Z/5Z$ - 25-isogeny; 0, $Z/5Z$. *Result* IV. Let E-tor (23) be the group generated by the p-parts of E-tor where p is a prime < 23. Then E-tor (23) as an abstract group is one of the following: Z/eZ, $e = 1 \cdots 10$, $Z/12Z$, $Z/2Z \times Z/3Z$ $Z/2Z \times Z/4Z$, $Z/2Z \times Z/8Z$, $Z/2Z \times Z/6Z$. (Received December 3, 1973.) (Author introduced by Professor Serge Lang.)

*74T-A48. ELVIRA RAPAPORT STRASSER, State University of New York, Stony Brook, New York 11790. *Knot-like groups.*

If $P = (x_0, x_1, \cdots, x_n; R_1, \cdots, R_n)$ is a presentation of a group G with factor commutator group, G/G', free cyclic, I call G knot-like. If the Abelianized commutator subgroup, G'/G'', is finitely generated, then it is free and its rank is the degree, d, of the Alexander polynomial of G [E. S. Rapaport, Ann. of Math. 71(1960), 157–162]. If G is actually a knot group and G' is finitely generated, then G' is free of rank d [L. Neuwirth, Ann. of Math., 1963]. The presentation P gives rise to a presentation of a certain group H. Let M be the deficiency of this (the number of defining relations subtracted from the number of generators). The main results are: any two of the following three conditions imply the third: 1. G' is free; 2. G' is finitely generated; 3. $d = M$; for one-relator presentations each of the first two conditions implies the rest; $d \leq M$ and G can have presentations P_1 and P_2 with $d = M_1$ but $d < M_2$; for the case $d < M$ an analogue of the "Haupt-form" of the Freiheitssatz [Magnus, J. Reine Angew. Math., 1930] is found. The proofs are combinatorial. (Received December 3, 1973.)

*74T-A49. ROGER H. HUNTER, Australian National University, Institute of Advanced Studies, Canberra, A.C.T. 2600, Australia. *A characterisation of cotorsion groups and algebraically compact groups.*

Theorem 1. An abelian group G is injective with respect to balanced exact sequences of torsion-free abelian groups if and only if G is cotorsion. *Theorem* 2. G is injective with respect to general balanced exact sequences of abelian groups if and only if G is algebraically compact. (Received December 4, 1973.) (Author introduced by Dr. M. F. Newman.)

*74T-A50. JUDITH Q. LONGYEAR, Dartmouth College, Hanover, New Hampshire 03755. *Patterns: The structure of linear homogeneous sets.*

The structure of totally ordered sets with transitive groups of order preserving bijections (that is, patterns) is investigated, using an extended lexicographic product which is linearly ordered even for infinite index sets. It is determined exactly when simple patterns are order isomorphic with their order duals, and several new questions are raised. (Received December 4, 1973.)

*74T-A51. RICHARD T. BUMBY and DAVID E. DOBBS, Rutgers University, New Brunswick, New Jersey 08903. *Amitsur cohomology of quadratic extensions: formulas and number-theoretic examples.*

Let R be an integral domain with quotient field K; S a flat R-subalgebra of a quadratic separable field extension L of K, such that the Galois group $\langle 1, g\rangle$ of L/K fixes S as a set. Let I be the ideal of S generated by $\langle a - g(a): a \in S\rangle$. *Theorem.* If S is not contained in K, then $H^1(S/R, UK/U) \cong (N(U(L)) \cap A)/N(B)$ and $H^2(S/R, U) \cong (K \cap A)/N(B)$, where $N = N_{L/K}$, $A = \langle x \in U(S): x \equiv 1(I^2)\rangle$ and $B = \langle x \in U(S): x \equiv 1(I)\rangle$. The above formula for $H^2(S/R, U)$ has been obtained by K. I. Mandelberg (to appear) for the special case: R integrally closed, $\mathrm{char}(K) \neq 2$, S free as an R-module with basis $\langle 1, a\rangle$. Since the above isomorphisms are compatible and I^2 may be regarded as the relative discriminant, we apply the Hasse norm theorem and Corollary 1.5 of the second-named author's paper (Canad. J. Math. 24(1972), 239–260) to obtain examples including: (a) *Theorem.* Let R be the integers of a real quadratic algebraic number field. Then there exist infinitely many biquadratic L/K whose integers S satisfy: the homomorphism $H^2(S/R, U) \to B(S/R)$ (see Theorem 7.6 of S. U. Chase and A. Rosenberg, Mem. Amer. Math. Soc. no. 52, (1965), 34–79) is neither injective nor surjective. (b) There are imaginary quadratic R with extensions S for which $H^2(S/R, U) \neq 0$. (Received December 5, 1973.)

*74T-A52. RICHARD J. BONNEAU, Massachusetts Institute of Technology, Project MAC, Cambridge, Massachusetts 02139. *An interactive implementation of the Todd-Coxeter algorithm.* Preliminary report.

The Todd-Coxeter algorithm provides a systematic approach to the enumeration of cosets of a finitely presented group. This memo describes an interactive implementation of this algorithm, including a manual on its use, examples, and methods of accessing the program. Applications of this algorithm are also discussed. (Received December 6, 1973.)

*74T-A53. ROBERTO L. O. CIGNOLI, Princeton University, Princeton, New Jersey 08540 and Universidad Nacional del Sur, Bahia Blanca, Argentina. *Injective De Morgan and Kleene algebras.*

A De Morgan algebra is a system $(A, \wedge, \vee, \sim, 0, 1)$ such that $(A, \wedge, \vee, 0, 1)$ is a distributive lattice with zero 0 and unit 1, and $\sim$ is a unary operation defined on A fulfilling the conditions $\sim\sim x = x$ and $\sim(x \vee y) = \sim x \wedge \sim y$. T will denote the De Morgan algebra obtained by taking as distributive lattice the Boolean algebra with two atoms a and b and $\sim 0 = 1$, $\sim a = a$, $\sim b = b$ and $\sim 1 = 0$. The complement of an element x, if it exists, will be denoted by $-x$, and $K(A) = \{x: -x = \sim x\}$. A De Morgan algebra A is called centered if there is a complemented element c in A such that $c = \sim c$ and any element x admits the representation $x = c \wedge k \vee -c \wedge k'$ with k, k' in $K(A)$. *Theorem.* The following conditions are equivalent for any De Morgan algebra A: (i) A is injective, (ii) there is a nonempty set Λ such that A is a retract of T^{Λ}, and (iii) A is centered and complete. A De Morgan algebra in which $\sim x \wedge x \leq \sim y \vee y$ is called a Kleene algebra. Let L denote the subalgebra of T formed by the elements 0, a and 1. *Theorem.* The following conditions are equivalent for any Kleene algebra A: (i) A is

injective, (ii) there is a nonempty set Λ such that A is a retract of L^{Λ}, and (iii) A is a complete Postalgebra of order three. (Received December 10, 1973.)

*74T-A54. CHARLES C. EDMUNDS, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. *On the endomorphism problem for free groups.*

Given a group G and an element $g \in G$, we say that the endomorphism problem is solvable for g if for any $h \in G$ it can be effectively decided whether or not h is an endomorphic image of g. Henceforth it is assumed that G is absolutely free. For each $g \in G$, we define a unique subset B_g of G which in some sense forms a basis for the set of endomorphic images of g. It is shown that the endomorphism problem is solvable for g if and only if B_g is a recursive set. Next a procedure is given by which B_g can be computed; if B_g is known to be a finite set, the procedure terminates after a finite number of steps. Thus if B_g is finite, B_g is recursive and the endomorphism problem is solvable for g. As an application of our techniques the endomorphism problem is solved for g a quadratic word (i.e. each generator occurring in g appears, with exponent $+1$ or -1, exactly twice). This is accomplished by showing that if g is quadratic, B_g is finite. (Received December 10, 1973.)

74T-A55. KEVIN R. HEBB, University of Alberta, Edmonton, Alberta T6G 2G1, Canada. *Some results on addition chains.* Preliminary report.

Let $l(n)$ be the length of a minimal addition chain for a positive integer n. Brauer (Bull. Amer. Math. Soc. 45(1939), 736–739) proved that $l(n) \leq (r+1)s + 2^r - 2$ for $2^{rs} \leq n < 2^{r(s+1)}$ $(r \geq 1,\ s \geq 0)$. Thurber (Abstract 73T-A112, these Notices 20(1973), A-318) then proved that $l(n) \leq \lambda(n) + 2^{K-1} - (K-1) + [\lambda(n)/K]$ for any integer $K \geq 1$ $(\lambda(n) = [\log n/\log 2])$. We improve this result as follows: *Theorem.* For all integers $r \geq 1$ and $s \geq 0$, $l(n) \leq (r+1)s + 2r - 2$ for $2^{rs} \leq n < 2^{r(s+1)}$. Consequently, it follows that $c(rs + s + r - 2) > 2^{rs}$ for all $r, s \geq 1$ except when $r = s = 1$, where $c(r)$ is the smallest value of n for which $l(n) = r$. Giese (Abstract 72T-A257, these Notices 19(1972), A–688) proved that $l(3n) < l(n)$ for infinitely many n. Thurber then proved that there exist infinitely many infinite classes of integers n for which $l(mn) \leq l(n) - b$ when $m = 2^{2K+1} + 1$ ($K \geq 0$ and b is an arbitrary nonnegative integer). Using Cottrell's result (Abstract 73T–A200, these Notices 20(1973), A–476) we improve Thurber's result by proving. *Theorem* 2. For all integers $m > 1$, $m \neq 2^K$ for any $K \geq 1$ and b an arbitrary nonnegative integer, there exist infinitely many integers n for which $l(mn) \leq l(n) - b$. We also show that each of the diophantine equations $xl(x) = yl(y)$; $yl(x) = xl(y)$ has an infinity of solutions (x, y). (Received November 26, 1973.) (Author introduced by Professor L. H. Erbe.)

74T-A56. BERNHARD GANTER, Technische Hochschule, 61 Darmstadt, FB4, AG1, Federal Republic of Germany. *Partial pairwise balanced designs.*

A *partial* PBD (or partial linear space) (P, B) consists of a set P of *points* together with a collection B of subsets of P (called *blocks*) such that every pairset $\{x, y\}$ of points is contained in at most one block. (P, B) has *block sizes from* K if $|b| \in K$ for all $b \in B$. If every pairset is contained in exactly one block, (P, B) is called a *complete* PBD (or simply a PBD). A complete PBD (P, B) is called a *completion* of a partial PBD (Q, C) if $Q \subseteq P$ and $C \subseteq B$. *Theorem.* Let K be a nonempty set of integers ≥ 2. Every finite partial PBD with block sizes from K has a finite completion with block sizes from K. (Received December 13, 1973.)

*74T-A57. JAMES L. HEIN, Northwestern University, Evanston, Illinois 60201. *The convertibility of* $\mathrm{Ext}_R^n(-, A)$.

Let R be a commutative ring and $\mathrm{Mod}(R)$ the category of R-modules. Call a contravariant functor $F\colon \mathrm{Mod}(R) \to \mathrm{Mod}(R)$ convertible if for every direct system $\{X_\alpha\}$ in $\mathrm{Mod}(R)$ there is a natural isomorphism γ: $F(\text{inj lim } X_\alpha) \to \text{proj lim } F(X_\alpha)$. If A is in $\mathrm{Mod}(R)$ and n is a positive integer then $\mathrm{Ext}_R^n(-, A)$ is not in general convertible. *Purpose.* To study convertibility of Ext and find out more about Ext and the modules A which make

$\mathrm{Ext}_R^n(-, A)$ convertible for all n. *Results.* $\mathrm{Ext}_R^n(-, A)$ is convertible for all A having finite length and all n. If R is Noetherian then A can be Artinian; if R is semilocal Noetherian then A can be linearly compact in the discrete topology. Characterizations are studied and it is shown that if A is a finitely generated module over a semilocal Noetherian ring R, then $\mathrm{Ext}_R^1(-, A)$ is convertible if and only if A is complete in the J-adic topology where J is the Jacobson radical of R. Morita-duality is characterized by the convertibility of $\mathrm{Ext}_R^1(-, R)$ when R is a Noetherian ring, a reflexive ring, or an almost maximal valuation ring. Applications to the vanishing of Ext are studied. (Received December 14, 1973.)

*74T-A58. JONATHAN S. GOLAN, University of Haifa, Haifa, Israel. *A Krull-like dimension for noncommutative rings.*

Let R be an associative ring with 1. For any left R-module M let $\mathrm{ass}(M)$ be the set of all prime torsion theories τ on R-mod for which M has a τ-cocritical submodule. (A module is τ-cocritical iff it is τ-torsionfree and every proper submodule is τ-dense.) Then for any set U of prime torsion theories on R-mod there exists a torsion theory $\delta(U)$ on R-mod such that a module M is $\delta(U)$-torsion iff $\emptyset \neq \mathrm{ass}(M') \subseteq U$ for every homomorphic image M' of M. This construction is used to define a generalization of the Krull dimension to arbitrary associative rings. (Received December 18, 1973.)

*74T-A59. H. PETER GUMM, Mathematisches Institut, Technische Hochschule, D-6100 Darmstadt, Federal Republic of Germany. *Mal'cev conditions in joins of varieties.*

Let E be an equation in the variables $x_1, \cdots, x_n$ and in the function symbols $\wedge$, $\vee$, and $\circ$. We call E *congruence-valid* in a variety K iff for every algebra $A \in K$ the equation holds whenever the variables are interpreted as congruences on A, and $\wedge, \vee$, and $\circ$ are interpreted as the meet, join and the relational product of congruences, respectively. *Theorem.* Let E be an equation in $\wedge$, $\vee$, and $\circ$ which does not hold for the congruences of every algebra. There exist varieties K_1 and K_2 such that E is congruence-valid in K_1 and in K_2 but not in $K_1 \vee K_2$. (Received December 19, 1973.) (Author introduced by Dr. Rudolf Wille.)

*74T-A60. GEORGE A. GRÄTZER and R. PADMANABHAN, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. *Symmetric difference in abelian groups.*

A groupoid $\mathfrak{A} = \langle A; * \rangle$ is called a left (resp. right) difference group if there exists a binary operation $+$ on A such that $\langle A; + \rangle$ is an abelian group and $x * y = -x + y$ (resp. $= x - y$). Let I (resp. J) be the class of all binary identities valid in all left (resp. right) difference groups. A groupoid $\mathfrak{A} = \langle A; * \rangle$ is called a symmetric difference group if it satisfies all the identities in the class $I \cap J$. *Theorem* 1. For a groupoid $\mathfrak{A} = \langle A; * \rangle$ the following statements are equivalent: (i) $\mathfrak{A}$ is a symmetric difference group; (ii) $\mathfrak{A}$ satisfies the identity $(x * y) * (((x * z) * (u * u)) * y) = z$; (iii) there exists a binary operation $+$ in $\mathfrak{A}$ and a map α of A into itself such that $\langle A; + \rangle$ is an abelian group, and α is an endomorphism of $\langle A; + \rangle$ satisfying $\alpha^2 =$ identity map on A and $x * y = x\alpha - y\alpha$. *Theorem* 2. The class of identities $I \cap J$ is one-based, that is, an identity holds for all left and right difference groups iff it follows from the identity given in Theorem 1 (ii). Theorems 1 and 2 can be proved for symmetric difference groups satisfying any fixed "identity" of a given type. This implies that any finitely based theory of symmetric difference groups is one-based. This includes the known results that I and J are one-based. Other known results (R. McKenzie and A. Tarski) on finitely based theories of rings and rings with unit also follow. (Received December 19, 1973.)

*74T-A61. IVO G. ROSENBERG, Université de Montréal, Montréal 101, Québec, Canada. *The set of maximal closed classes of operations on an infinite set A has cardinality* $2^{2^{|A|}}$. Preliminary report.

Let A be an infinite set. A closed class is a set of finitary operations on A which is closed with respect to composition. Let $\mathfrak{L}$ be the set of all closed classes on A. The dual atoms of the lattice $(\mathfrak{L}, \subseteq)$ are maximal or precomplete closed classes. G. P. Gavrilov (Dokl. Akad. Nauk SSSR 158 (1964), 506–508; English transl. Soviet Math. 158(1964), 1239–1242, and Problemy Kibernet.15(1964), 5–64) has shown that the set of maximal classes on a countable A has the same cardinality as $\mathfrak{L}$. This result is proved for any infinite set A. (Received December 21, 1973.)

*74T-A62. KENNETH S. WILLIAMS, Carleton University, Ottawa, Ontario K1S 5B6, Canada. *3 as a ninth power (mod p).*

Let p be a prime $\equiv 1 \pmod 9$ for which 3 is a cube (mod p). A simple necessary and sufficient condition is derived for 3 to be a ninth power (mod p). (Received December 26, 1973.)

74T-A63. PAUL C. DESMARAIS, University of Massachusetts, Amherst, Massachusetts 01002. *Primitive rings with involution and generalized pivotal monomials.* Preliminary report.

Drazin (Proc. Amer. Math. Soc. 8(1958), 352–361) has shown that a primitive ring R satisfies a pivotal monomial (PM) iff $R \cong D_n$, D a division ring. Amitsur (Trans. Amer. Math. Soc. 114(1965), 210–226) has shown that a primitive ring satisfies a generalized pivotal monomial (GPM) iff it has a nonzero socle. In generalizing these theorems to the symmetric elements of a primitive ring with *, an involution, one would at least conjecture that if the symmetric elements of a primitive ring with * satisfy a PM then the ring has a nonzero socle. However, let U be the set of all countably infinite matrices of the form $\left[\begin{smallmatrix} A & 0 \\ 0 & 0 \end{smallmatrix}\right]$, where A is $n \times n$ over F, the reals, and n varies with transpose as *. For a nonprincipal ultrafilter $\mathcal{F}$ on N, the natural numbers, let R denote the ultraproduct $\Pi_{n \in N} U/\mathcal{F}$ and let T be the socle of this primitive ring with *. R/T is primitive with *, has zero socle and the symmetric elements satisfy the PM x since they satisfy the relation $s = s^2 p(s)$, where $p(s)$ is a polynomial in s over $\Pi_{n \in N} F/\mathcal{F}$. We call a GPM *restricted* if all the monomials arising from a substitution have degree ≤ 1 in each variable. *Theorem.* If the symmetric elements of a primitive ring R with * satisfy a restricted GPM then R has nonzero socle. (Received December 26, 1973.) (Author introduced by Professor Wallace S. Martindale III.)

*74T-A64. SABURO TAMURA, Yamaguchi University, Yamaguchi 753, Japan. *Three axioms for commutative rings.*

Theorem. A commutative ring with unity is a set with two nullary operations, 0 and 1, with one unary operation, −, and with two binary operations, + and juxtaposition, such that [I] $a + 0 = a$; [II] $((-b)+b)x + a = a$; and [III] $(ax + b)y + c1 = (yb + c)+a(xy)$ for every a, b, c, x, y. *Proof.* [IV] $((-b)+b)x = ((-b)+b)x + 0 = 0$; [V] $0 + a = ((-b)+b)x + a = a$; [VI] $(-0)x = ((-0)+0)x = 0$; [VII] $c1 = ((-0)x+(-0))(-0) + c1 = ((-0)(-0) + c) + (-0)(x(-0)) = c$; [VIII] $(-b)+b = ((-b)+b)1 = 0$; [IX] $0x = ((-b)+b)x = 0$; [X] $by = (0x + b)y + 01 = (yb + 0)+0(xy) = yb$; [XI] $(ax)y = (ax + 0)y + 01 = (y0 + 0)+a(xy) = a(xy)$; [XII] $a + b = (a1 + b)1 + 01 = (1b + 0)+a(11) = b + a$; [XIII] $(a + b)+c = (a1 + b)1 + c1 = (1b + c)+a(11) = a+(b + c)$; and [XIV] $(a + b)y = (a1 + b)y + 01 = (yb + 0)+a(1y) = ay + by$. □ (Received December 26, 1973.) (Author introduced by Professor G. R. Blakley.)

*74T-A65. ROBERT E. JAMISON II, University of Washington, Seattle, Washington 98195 and FRANK CLARKE, University of British Columbia, Vancouver 8, British Columbia, Canada. *Measures, multicolorings, and games on graphs.*

Suppose w is a weighting of the nodes of a graph G with positive weights so that the sum of all the weights is 1. The selectivity $s(w)$ is the greatest cumulative weight on an independent set of nodes of G. The *selectivity* $\sigma(G)$ of G is the infimum of $s(w)$ taken over all weightings w of G. *Theorem.* For any graph G, $\sigma(G)$ is a rational number. An (r, k)-multicoloring of G is an assignment, from a pool of k colors, of r colors to each node of G so that adjacent nodes have no colors in common. The *multichromatic number* $\chi^*(G)$ of G is the infimum of the ratios k/r taken over all multicolorings of G. *Theorem.* For any graph G, $\chi^*(G)\sigma(G) = 1$. These theorems are proved by a rather novel appeal to the duality results from game theory. (Received December 26, 1973.)

74T-A66. RONALD HIRSHON, Polytechnic Institute of New York, Brooklyn, New York 11201. *On cancelling an infinite cyclic group.* Preliminary report.

We investigate groups G which may be written as a direct product $G = A \times C = B \times D$ where C and D are infinite cyclic. We show: (a) There exists a positive integer n such that the direct product of n copies of A is isomorphic to the direct product of n copies of B. (b) If A is nilpotent with an infinite cyclic center then A and B are isomorphic. (c) If A is hopfian and for each positive integer n the direct product of n copies of A is hopfian, then G is hopfian. (d) There exist finitely generated nilpotent groups A, B, G as above, with A and B not isomorphic but $A \times A$ is isomorphic to $B \times B$. (Received December 27, 1973.)

*74T-A67. JOHN MOSS GROVER, University of California, Irvine, California 92664 and EUGENE MALEK, Faculté des Sciences, Universite de Tours, Parc de Grandmont 37, Tours, France. *Ranks of Whitehead groups.*

E. Malek has proved a variation of the Dirichlet unit theorem (J. Algebra 23(1972), 538–552), giving the rank of the group of units of commutative orders over certain PID's. We apply this to calculating the rank of the Whitehead group K_1B of such a ring B. We consider the property of the special Whitehead group SK_1B being torsion, which happens exactly when K_1B and the units group UB have the same rank. Malek's results give the ranks of units groups of such orders in terms of units groups ranks of suitable residue domains of the order. Results are derived concerning relations between ranks of Whitehead groups of orders and Whitehead group ranks over certain residue domains, paralleling those for units groups. These results are applied to orders over over global rings, to orders for which the residue domains are Euclidean or semilocal, and to finite abelian group rings. (Received November 12, 1973.)

*74T-A68. MARVIN D. TRETKOFF, Stevens Institute of Technology, Hoboken, New Jersey 07030. *Covering space proofs in combinatorial group theory.*

We show how covering spaces can be employed in combinatorial group theory to discover new results and simplify the proofs of existing theorems. As an application of our results we prove that every countable group can be embedded in a quotient of the Picard group, the group of fractional linear transformations $(az + b)/(cz + d)$, a, b, c, and d Gaussian integers, with $ad - bc = 1$. If the groups A and B are suitably restricted, then our results also allow us to verify a conjecture of Karrass and Solitar which states that a finitely generated subgroup H has finite index in the free product $A*B$ if and only if it has nontrivial intersection with every nontrivial normal subgroup of $A*B$. Among our main results are topological proofs of various generalizations to free products of the following theorem of M. Hall, Jr.: If G is a free group and H a finitely generated subgroup which does not contain $g_1, \cdots, g_t$, then there is a subgroup H^* of finite index in G which contains H as a free factor and does not contain $g_1, \cdots, g_t$. (Received January 4, 1974.)

*74T-A69. JITENDRA N. MANOCHA, Kent State University, East Liverpool, Ohio 43920. *σ-noetherian rings.*

Let σ denote an idempotent kernel functor in the sense of Goldman [J. Algebra 13(1969), 10–47] and let $\mathfrak{T}$ ($\mathfrak{F}$) denote the class of torsion (torsion-free) modules. Given σ, there exists an injective module V_R such that M_R is torsion if and only if $\mathrm{Hom}[M_R, V_R] = 0$. A module M_R is σ-finitely generated (σ – FG) if there exists a finitely generated submodule N of M such that the quotient module M/N is torsion. A module M_R is σ-noetherian if each submodule of M is σ – FG. *Theorem* 1. The following statements are equivalent: (a) R_R is σ-noetherian. (b) If V_R is an injective module which determines σ, then V_R is Σ-injective. (c) $\mathfrak{T}$, $\mathfrak{F}$ satisfy conditions of Tepley's theorem [Pacific J. Math. 28(1969), 441–453, Theorem 1.2]. (d) The Grothendieck category $\mathfrak{Q}$ of torsion-free and σ-divisible modules is locally noetherian and the localization functor commutes with direct sums. (e) $Q(R)$, as an object of $\mathfrak{Q}$, is noetherian. *Theorem* 2. Let R be σ-noetherian. Then every torsion-free indecomposable injective is isomorphic to $E(R/I)$ for some critical prime right ideal I of R and, up to relatedness, I is uniquely determined. *Theorem* 3. If R is commutative, R is σ-noetherian if and only if each prime ideal is σ – FG. Results of Lambek-Michler and Cohen follow. Applications to specific torsion theories are discussed. (Received December 6, 1973.)

*74T-A70. KARL K. NORTON, 2235 Floral Drive, Boulder, Colorado 80302. *Applications of the Siegel-Walfisz prime number theorem.* II.

Notation. k, l, n, ν are integers with k, n, ν positive and $(k, l) = 1$. p is prime, ϕ is Euler's function, and $c_1, \cdots, c_5$ are positive absolute constants. $\omega(n)$ is the number of distinct p dividing n, and $\omega(n; k, l)$ is the number of distinct p such that $p|n$ and $p \equiv l \pmod{k}$. $\log_2 x = \log\log x$. *Theorem* 1. Let $\beta > 1$, and let $N(x, \beta)$ be the number of n such that $3 \le n \le x$ and $\omega(n; k, l) > \beta\{\phi(k)\}^{-1}\log_2 n$. Then if x exceeds a certain specific function of β and k, we have $N(x, \beta) \le \exp(c_1\beta)\, x(\log x)^{-\delta}$, where $\delta = \{\phi(k)\}^{-1}(\beta\log\beta - \beta + 1)$.

Theorem 2. Let $\rho_\nu(x; k, l)$ be the number of $n \le x$ for which $\omega(n; k, l) = \omega(n) = \nu$. Then for $x \ge 2$, we have $\rho_\nu(x; k, l) \le c_2 x A B^{-1}$, where $A = \{\log_2 x + c_3\phi(k)\}^{\nu-1}$, $B = \{\phi(k)\}^{\nu-1}(\nu-1)! \log x$. *Theorem* 3. If $k \ge 2$ and $x \ge \exp\{2^\nu \log k\}$, then $\rho_\nu(x; k, l) \le c_4 x A_1 B_1^{-1}$, where $A_1 = \{\log_2 x + c_5\phi^2(k)\}^{\nu-1}$, $B_1 = \{\phi(k)\}^\nu(\nu-1)! \log x$. (Theorems 2 and 3 generalize a result of Hardy and Ramanujan (Quart. J. Math. 48(1917), 76–92) who considered only the case $k = 1$. The inequalities given here are quite precise, as may be seen by comparing them with Delange's asymptotic formula $\rho_\nu(x; k, l) \sim x(\log_2 x)^{\nu-1}D^{-1}$, where $D = \{\varphi(k)\}^\nu(\nu-1)! \cdot \log x$. See C. R. Acad. Sci. Paris 246(1958), 2205–2207. (Received January 7, 1974.)

74T-A71. DAVID ZEITLIN, 1650 Vincent Avenue North, Minneapolis, Minnesota 55411. *A conjecture on the general coefficients of the chromatic polynomial of a complete bipartite graph* $K_{p,q}$.

The polynomial is $F(p, q, t) \equiv \sum_{r=1}^{p} r! \mathfrak{S}_p^r(t)(t-r)^q \equiv \sum_{j=1}^{p+q} A_j t^j$, where (*) $A_j = \sum_{m=j}^{p+q} \mathfrak{S}_p^{m-q} \cdot \sum_{k=0}^{j} \binom{q}{k}(q-m)^{q-k} \cdot S_{m-q}^{j-k}$, with S_n^k and $\mathfrak{S}_n^k$ Stirling numbers of the 1st and 2nd kind. From (*), $A_{p+q-j} = (-1)^j\binom{pq}{j}$, $j = 0, 1, 2$; $4A_{p+q-3} = -4\binom{pq}{3} + 2\binom{pq}{2} + (2 - p - q)\binom{pq}{1}$; $12A_{p+q-4} = 12\binom{pq}{4} - 18\binom{pq}{3} + (-18 + 8(p+q))\binom{pq}{2} + (2 - p^2 - q^2)\binom{pq}{1}$. *Conjecture.* For $j = 2, 3, \cdots, p+q$, we claim that (C) $A_{p+q-j} = (-1)^j\binom{pq}{j} + \sum_{k=1}^{j-1} M_{k,j}(p, q)\binom{pq}{j-k}$, where $M_{k,j}(p, q) = \sum_{i=0}^{k-1} B(i, j)(p^i + q^i)$; $B(i, j)$ are numerical constants, *independent* of p and q. *Remarks.* (C) $\Rightarrow \sum_{k=1}^{j-1} M_{k,j}(1, q)\binom{q}{j-k} \equiv 0$ in q, because $A_{1+q-j} = (-1)^j\binom{q}{j}$. *Results.* 1. (R) $144A_{p+q-5} = -144\binom{pq}{5} + 432\binom{pq}{4} + (336 - 180(p+q))\binom{pq}{3} + (-372 + 84(p+q) + 30(p^2+q^2))\binom{pq}{2} + (-146 + 111(p+q) - 35(p^2+q^2) - 3(p^3+q^3))\binom{pq}{1}$. 2. (H): $F(p, q+1, t) + F(p+1, q, t) \equiv tF(p, q, t) + tF(p, q, t-1)$, which fails to give an effective recursion relation (for A_j) needed to prove (C). Laskar and Hare (Abstract 709-A36, these Notices 20(1973), A-655) gave the chromatic polynomial of a complete r-partite graph $F(p_1, \cdots, p_r, t) \equiv \sum_{j=1}^{d} C_j t^j$, $d = \sum_{i=1}^{r} p_i$. Our results for $r = 2$ generalize as (G) $C_d = 1$, $C_{d-j} = \sum_{k=1}^{r-1}\sum_{i=k+1}^{r}\binom{p_k p_i}{j}(-1)^j$, $j = 1, 2$, and, I presume, similarly for $j \ge 3$ (Abstract 73T-A278, these Notices 20(1973), A-633). (C) can be used to evaluate A_{p+q-j}, $j \ge 6$. (C) gives $\binom{j}{2}$ linear equations in $\binom{j}{2}$ unknowns. (Received December 31, 1973.)

*74T-A72. MARK BLONDEAU HEDRICK, 1205 Eastman, Pasadena, Texas 77506. *The permanent at a minimum on certain classes of doubly stochastic matrices.*

The author proves the following. Let X be a subset of the set of doubly stochastic matrices. Let A be a matrix of X such that for some neighborhood of A, every doubly stochastic matrix in the neighborhood with the same (0, 1)-pattern as A is in X. Under the above assumptions, if the permanent achieves a local minimum (maximum) at A relative to X, then $\operatorname{per} A = \operatorname{per} A(i|j)$ for each positive a_{ij}. (Received December 21, 1973.) (Author introduced by Professor Garret J. Etgen.)

74T-A73. MANFRED EGON SZABO, Sir George Williams University, Montreal 107, Quebec, Canada. *A categorical characterization of Boolean algebras.*

A *Boolean algebra* is a Heyting algebra satisfying the de Morgan laws. Since Heyting algebras, *qua* lattices, are "bicartesian closed categories", it is of interest to determine the class of categories to which Boolean algebras belong. The following theorem solves this problem: *Theorem.* If $\underline{K}$ is a bicartesian closed category in which there exist $\underline{K}$-isomorphisms $A \times B \cong ((A \Rightarrow 0) + (B \Rightarrow 0)) \Rightarrow 0$ and $(A \times B) \Rightarrow 0 \cong (A \Rightarrow 0) + (B \Rightarrow 0)$ for all $\underline{K}$-objects A and B, then any skeleton of $\underline{K}$ is a Boolean algebra. ("$\times$" and "$+$" denote product and sum, "$\Rightarrow$" denotes the internal hom functor, and "0" and "1" stand for an initial and or terminal object of $\underline{K}$, respectively.) (Received January 8, 1974.)

74T-A74. FRANK W. OWENS, Ball State University, Muncie, Indiana 47306. *Sums of consecutive U-numbers.* Preliminary report.

Define the sequence $\{u_n\}$ of positive integers recursively by $u_1 = 1$, $u_2 = 2$, and for $n > 2$, $u_n =$ the least integer which can be uniquely represented as a sum of two distinct preceding terms of the sequence. The terms of this sequence are called U-numbers. Recamán [Amer. Math. Monthly 80(1973), 919–920] asks whether the sum of two consecutive U-numbers apart from $1 + 2 = 3$ can be a U-number. We have obtained $u_{19} = 62$, $u_{20} = 69$ and $u_{31} = 131$. No other occurrences of this phenemenon appear with the sum $\le 99{,}933 = u_{7584}$. (Received January 8, 1974.)

Analysis

*74T-B21. DENNIS J. WILDFOGEL, University of California, Santa Barbara, California 93106. *Double algebras, Segal algebras, and annihilator algebras,* II. *Abstract Tauberian theorems.*

In part I of this paper (Abstract 711–46–6, these Notices 21 (1974)) we have shown that for a D-algebra (B, A) there is an inclusion preserving bijection between the closed right ideals of A and those of B. We call a D-algebra *modular closed* if the maximal modular ideals of A (resp. B) are closed in the A-topology (resp. B-topology) and if B is in addition a right ideal of A. In what follows (B, A) will always denote a modular closed D-algebra. *Theorem* 1. The strong structure spaces (i.e., space of maximal modular ideals with the hull-kernel topology) of A and B are homeomorphic. *Theorem* 2. Every proper closed ideal of B is contained in a maximal modular ideal of B if and only if the analogous assertion is true of A. *Definition.* A topological algebra T is a Silov algebra if any closed ideal I of T contains every element x in ker (hull (I)) such that hull $(x) \cap$ Boundary (hull I) contains no nonempty perfect set. *Theorem* 3. If A is a Silov algebra then B is. Since for a locally compact abelian group G, $L^1(G)$ is a Silov algebra we have *Corollary.* Any Segal algebra is a Silov algebra. *Theorem* 4. A is a Tauberian (respectively N, N^*) algebra if and only if B is. (See Rickart, "Banach algebras", p. 92, for definitions.) (Received October 18, 1973.)

*74T-B22. JAMES D. FABREY, University of North Carolina, Chapel Hill, North Carolina 27514. *A modified Runge-Kutte method.*

The classical fourth-order Runge-Kutte method is viewed in terms of predictors and correctors, rather than fourth-order matching, and then modified for equations of the form $y' = g(y)$ by means of the linear approximation $g(y) = g(y_n) + g'(y_n)(y - y_n)$ on each interval $[x_n, x_{n+1}]$. The resulting method is: $k_1 = hg'(y_n)$, $k_2 = h(e^{k_1} - 1)$, $y_{n+1} = y_n + k_2 g(y_n)/k_1$ $(k_1 \neq 0)$. It requires fewer function evaluations and operations but requires smaller steps, in general, to achieve a given accuracy. These trade-offs are illustrated by two examples of continuation of solutions, in one of which g is "almost" linear. (Received October 19, 1973).

*74T-B23. M. R. PARAMESWARAN, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. *On a theorem of Rhoades on the summability domains of Hausdorff methods.*

B. E. Rhoades [Bull. Amer. Math. Soc. 65(1959), 9–11] raised the question: Given an unbounded sequence x, is there a Hausdorff summability method H such that (*): the summability field (H) of H is precisely $c \oplus x$, the linear space spanned by x and the set of all convergent sequences? This was answered in the negative (for certain sequences x) by the author [Proc. Nat. Inst. Sci. India Part A 27(1961), 175–177] and in the affirmative for a particular sequence x by Rhoades [J. Math. Anal. Appl. 19(1967), 457–468]. Rhoades' proof depends on deep results on Hausdorff methods as well as Zeller's technique for constructing Einfolgenverfahren. In this note an alternative and direct proof of Rhoades' theorem is given. (Received October 24, 1973.) (Author introduced by Professor H. C. Finlayson.)

*74T-B24. JOSE BARRIA-SANCHEZ, Indiana University, Bloomington, Indiana 47401 and Universidad de Concepcion, Concepcion, Chile. *Chains of invariant subspaces.* Preliminary report.

Theorem 1. If H is a separable infinite-dimensional Hilbert space, then there exists a bounded linear operator on H whose lattice of (closed) invariant subspaces is isomorphic to $\omega m + 1 + \omega^* n$. (Here m and n are nonnegative integers such that $m + n \geq 1$, and ω is the ordinal number of the set of positive integers. This theorem is a generalization of the results that appeared in Abstract 72T-B289, these Notices 19(1972), A-705.) *Theorem* 2. Let μ be the sum of Lebesgue measure on $[0, 1]$ and a purely atomic measure with a finite number of atoms in the open interval $(0, 1)$. The lattice of invariant subspaces of the operator V defined on $L^2([0, 1], \mu)$ by $Vf(x) = \int_{[0,x)} f(t)\, d\mu(t)$ (integration over the half open interval $[0, x)$) is isomorphic to $[0, 1] \cup [2, 3] \cup \cdots \cup [2n, 2n + 1]$, where n is the number of atoms of μ. (Received October 26, 1973.)

*74T-B25. ALI A. JAFARIAN, Arya-Mehr University of Technology, Tehran, Iran. *Weak contractions of Sz-Nagy and Foias are decomposable.* Preliminary report.

Let T be a bounded linear operator on a Banach space X. We say that T *has decomposable spectrum* if it is weak decomposable (A. A. Jafarian, Abstract 73T-B179, these Notices 20(1973), A-435) and moreover the spectral maximal subspaces $\mathcal{Y}_i$ can be chosen so that $\sigma(T) = \bigcup_{i=1}^{n} \sigma(T \mid \mathcal{Y}_i)$; and that T *has property* (P) if for every spectral maximal subspace $\mathcal{Y}$ of T we have $\sigma(T^{\mathcal{Y}}) = \overline{\sigma(T) \setminus \sigma(T \mid \mathcal{Y})}$. *Main results.* The first one is a generalization of C. Foias (Rev. Roumaine Math. Pures Appl. 15(1970), 1599–1606). *Theorem* 1. Suppose that T has decomposable spectrum and satisfies the property (P). Then for every closed subset F of $\mathbf{C}$ we have $X_T(F) = N_c(T; F)$, and hence $X_T(F)$ is closed. Using this theorem and a characterization of decomposable operators (A. A. Jafarian, Abstract 73T-B234, these Notices 20(1973), A-492) we can prove *Theorem* 2. Let T be a weak contraction on a Hilbert space. Then T is decomposable. (Received October 26, 1973.)

74T-B26. ROGER D. NUSSBAUM, Rutgers University, New Brunswick, New Jersey 08903. *The structure of the set of periodic solutions of some functional differential equations.* Preliminary report.

We are interested in the structure of the set of periodic solutions of certain parametrized families of nonlinear autonomous functional differential equations. The basic tool is a new global bifurcation theorem with which we obtain the existence of unbounded, closed connected sets of periodic solutions. With the aid of such existence theorems for continua of periodic solutions and with further argument we prove the following *Theorem.* Let $f: \mathbf{R} \to \mathbf{R}$ be a continuous increasing function such that $xf(x) > 0$ for $x \neq 0$ and $f'(0)$ exists and equals one. Assume there exists a positive constant $b < \infty$ such that $f(x) > -b$ for all x and assume there exists a positive constant a such that $f(x) < a$ for all x. We allow the possibility $a = +\infty$. Then for every p such that $4 < p < 2 + a/b + b/a$, there exists $\alpha > 0$ and a periodic solution x of the equation $x'(t) = -\alpha f(x(t-1))$ such that $x(-1) = 0$, x is strictly monotonic increasing on $[-1, 0]$, and x has period precisely p. The above bifurcation theorem can also be applied to Liénard equations with one time lag and to other functional differential equations. (Received October 29, 1973.)

*74T-B27. DOMENICO ROSA, McMaster University, Hamilton, Ontario L8S 4K1, Canada. *Necessary and sufficient conditions for $C(X)$ to be a $B(\underline{L})$ algebra.* Preliminary report.

If $C(X)$, the algebra of complex-valued, continuous functions on a completely regular space X with the compact-open topology, is B-complete, then X is a normal k-space [V. Ptak, Czechoslovak Math. J. 3(78) (1953),301–364]. It is shown here that local compactness and paracompactness of X are not sufficient for $C(X)$ to be B-complete. Let $\underline{L}$ denote the class of all commutative locally m-convex algebras with identity. $A \in \underline{L}$ is said to be a $B(\underline{L})$ algebra if every continuous and almost open algebra homomorphism from A onto an algebra B in $\underline{L}$ is open. The main result is *Theorem* 1. $C(X)$ is a $B(\underline{L})$ algebra iff X is a k-space. The adjoint situation between compact T_2 spaces and commutative Banach algebras with identity is extended to an adjoint situation between completely regular spaces and the category $\underline{CL}$ consisting of all A in $\underline{L}$ whose Gelfand map $A \to CM(A)$ (compact-open topology) is continuous. From this a dual equivalence is obtained between the category of k'-spaces [W. W. Comfort, Trans. Amer. Math. Soc. 131(1968), 107–118] and the category $\underline{Cb}^*$ consisting of all A in $\underline{CL}$ which are complete b^* algebras. Using this duality we obtain *Theorem* 2. $C(X)$ is injective (w.r.t. embeddings) in $\underline{Cb}^*$ iff X is the disjoint topological union of compact, extremally disconnected spaces. (Received October 29, 1973.) (Author introduced by Professor Taqdir Husian.)

*74T-B28. HAROLD EXTON, University of Salford, Salford, Lancashire, England. *On the partial differential system of ${}_n\phi$, the confluent hypergeometric function of several variables.* Preliminary report.

The general solution of the partial differential system associated with the confluent hypergeometric function in n variables ${}_n\phi$ is obtained, and, in addition, further solutions of the system are given in terms of certain multiple hypergeometric functions of generalized Horn type. Solutions in terms of asymptotic expansions are given also. (Received November 2, 1973.)

*74T-B29. LEE A. RUBEL, University of Illinois, Urbana, Illinois 61801 and CHUNG-CHUN YANG, Naval Research Laboratory, Washington, D.C. 20390. *Unavoidable families of meromorphic functions.*

A family F of meromorphic functions in the complex plane is called *unavoidable* if for every meromorphic function g, there is an $f \in F$ and a complex number z such that $g(z) = f(z)$. *Theorem.* The minimum cardinality of an unavoidable family of meromorphic functions is 3. (Received November 5, 1973.)

*74T-B30. HENRY B. COHEN, University of Pittsburgh, Pittsburgh, Pennsylvania 15213. *Isomorphisms of $C(T)$ with small bound.* Preliminary report.

(1) Nonhomeomorphic compact Hausdorff spaces X and Y are constructed admitting an onto isomorphism $L: C(X) \to C(Y)$ such that $\|L\| \|L^{-1}\| = 2$. (2) By using $C(T)^{**}$, a new proof is obtained that X and Y are homeomorphic if the bound of L is less than two (for some isomorphism L), and relationships between X and Y are deduced when $2 \leq \|L\| \|L^{-1}\| < 3$. (Received December 3, 1973.)

74T-B31. PRATIBHA GHATAGE, University of Guelph, Guelph, Ontario, Canada. *An extension of Nagy's theorem.* Preliminary report.

Definition. A polynomially bounded operator T is said to be absolutely continuous if for every x, y in H there exists a measure $\mu_{x,y}$ on the unit circle Γ such that $(p(T)x, y) = \int p(z)\, d\mu_{x,y}(z)$ for all polynomials p and such that μ is absolutely continuous with respect to the Lebesgue measure m. *Theorem.* If T is an absolutely continuous polynomially bounded operator for which $m(\sigma(T) \cap \Gamma) = 0$ then $T^n \to^{st} 0$ and $T^{*n} \to^{st} 0$. This gives an elementary proof of the following well-known theorem of Nagy and Foiaş ("Analyse harmonique des operateurs de l'espace de Hilbert", Chapter II, Proposition 6.7). *Corollary.* If T is a c.n.u. contraction for which $m(\sigma(T) \cap \Gamma) = 0$, then $T \in C_{\infty}$. (Received November 6, 1973.)

74T-B32. GRAHAM D. ALLEN, CHARLES K. CHUI, WOLODYMYR R. MADYCH, F. J. NARCOWICH and PHILIP W. SMITH, Texas A&M University, College Station, Texas 77843. *Pade approximation.* Preliminary report.

The authors have discovered a generalized method of Pade approximation, from which the treatment of Stieltjes series yields the approximants as compact integral formulas. The integral formulas give the exact location of poles and hence the interlacing of poles of successive approximants follows trivially. Furthermore, the method generalizes to L_2 and also classes of analytic functions by using integral representations. (Received November 8, 1973.)

*74T-B33. REKHA PANDA, University of Victoria, Victoria, British Columbia V8W 2Y2, Canada and Ravenshaw College, Cuttack-3, Orissa, India. *A note on some generating functions for the Jacobi polynomials.*

In the present note the author gives a class of bilateral generating functions for a generalization of the classical orthogonal polynomials of Jacobi, Laguerre, and Hermite, the Gould-Hopper polynomials $g_n^m(x, h)$ [Duke Math. J. 29(1962), 58], the Brafman polynomials $B_n^m[a_1, \cdots, a_p; b_1, \cdots, b_q; x]$ [Canad. J. Math. 9(1957), 186], and the Srivastava-Daoust polynomials [Comment. Math. Univ. St. Paul 20(1971), 18]. Several special cases, including generalizations of results due to W. N. Bailey [Proc. London Math. Soc. (2) 28(1928), 247, (3.1)] and H. M. Srivastava [Ann. Polon. Math. 27(1972), 76, (3.1)] are considered. (Received November 8, 1973.)

*74T-B34. SANFORD S. MILLER, State University College of New York, Brockport, New York 14420 and PETRU T. MOCANU, Babes-Bolyai University, Cluj, Romania. *The Hardy class of functions of bounded argument rotation.*

Suppose that $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ is regular in the unit disc D with $f(z)/z \neq 0$ in D. If $\int_0^{2\pi} |\mathrm{Re}\, zf'(z)/f(z)|\, d\theta \leq k\pi$ for $z \in D$ and $k \geq 2$, then $f(z)$ is said to be of bounded argument rotation. These functions were first studied by O. Tammi (Ann. Acad. Sci. Fenn. Ser. A I 114(1952)). The authors determine the Hardy classes to which $f(z)$ and $f'(z)$ belong. In addition a growth condition for a_n is obtained. (Received November 9, 1973.)

*74T-B35. TECK-CHEONG LIM, Dalhousie University, Halifax, Nova Scotia, Canada. *A fixed point theorem for multivalued nonexpansive mappings in a uniformly convex Banach space.*

Another use of Edelstein's notion of asymptotic center is made to prove: *Theorem.* Let X be a uniformly convex Banach space and C be a nonempty closed convex bounded subset of X. Let $T: C \to \mathcal{C}(C)$ be a nonexpansive mapping, where $\mathcal{C}(C)$ denotes the family of nonempty compact subsets of C equipped with the Hausdorff metric. Then there exists $x \in C$ such that $x \in Tx$. In fact, a somewhat stronger result is obtained. This theorem was proved by Browder for spaces having weakly continuous duality mapping, by Markin for Hilbert spaces, by Lami Dozo for spaces satisfying Opial's condition and by Assad and Kirk for a result somewhat stronger then Lami Dozo's The use of Edelstein's asymptotic center was made earlier by the author to improve a result by Belluce and Kirk (Abstract 73T-B224, these Notices 20(1973), A-489). Bruck made another improvement by a different approach (Abstract 73T-B264, these Notices 20(1973), A-573). (Received November 9, 1973.)

*74T-B36. CHARLES W. GROETSCH, University of Cincinnati, Cincinnati, Ohio 45221. *Representations of the generalized inverse.*

Let H_1 and H_2 be Hilbert spaces over the same scalars and suppose $T \in L(H_1, H_2)$ has closed range. Define the operator $A \in L(H, H)$ to be the restriction of $I - T^*T$ to $H = R(T^*)$. *Theorem.* Let Ω be an open set with $\sigma(A) \subset \Omega \subsetneq (-\infty, 1)$ and suppose that $\{S_\beta(x)\}$ is a net of continuous functions on Ω such that $\lim S_\beta(x) = 1/(1-x)$ uniformly on $\sigma(A)$; then $T^\dagger = \lim S_\beta(A)T^*$ in the uniform topology for $L(H_2, H_1)$. By choosing for $S_\beta(x)$ various summability transforms of the geometric series, we obtain as corollaries seven specific representations of $T^\dagger$ including the series and integral representations of Showalter and the limit representation of den Broeder and Charnes. (Received November 12, 1973.)

74T-B37 MICHAEL A. RAINS, University of British Columbia, Vancouver 8, British Columbia, Canada. *On a majorant principle for functions with positive coefficients.* Preliminary report.

Using unpublished results of J. Fournier, the results of H. Shapiro (Abstract 73T-B141, these Notices 20(1973), A-426) are generalized to all infinite compact abelian groups. *Theorem* 1. Let p be an even integer or ∞ and A a symmetric neighbourhood of the identity. Suppose $f \in L'(G)$, $\hat{f} \geq 0$ and $f \in L^p(A)$. Then $f \in L^p(G)$ and there is a constant K, independent of f, such that (*) $\|f\|_{L^p(G)} \leq K \|f\|_{L^p(A)}$. For counterexamples we have *Theorem* 2. Let $p \geq 1$ not be an even integer or ∞ and let A be a closed symmetric neighbourhood of the identity not of full measure. Then no inequality (*) can hold for each $f \in L^p(A)$ with $\hat{f} \geq 0$. More directly: *Theorem* 3. If $p > 2$ is not an even integer or ∞ and A is as in Theorem 2, then there is a $g \in L'(G)$ with $\hat{g} \geq 0$, $g \in L^p(A)$ but g is not in $L^p(G)$. This has the following variant for $1 \leq p < 2$: *Theorem* 4. There is a trigonometric series g with positive coefficients which represents a function in $L^p(A)$ but is not the Fourier series of a function in $L^p(G)$. (Received November 12, 1973.)

*74T-B38. A. R. REDDY, University of Toledo, Toledo, Ohio 43606. *A note on rational approximation on* $[0, \infty)$. Preliminary report.

We announce here the following *Theorem*: Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k \geq 0$ $(k \geq 0)$ be any entire function of order ρ $(0 < \rho < \infty)$. Then we cannot find algebraic polynomials $P(x)$ and $Q(x)$ with nonnegative coefficients and of degree at most n for which for any small ϵ $(0 < \epsilon < \rho)$ $\overline{\lim}_{n\to\infty}\{\|1/f(x) - P(x)/Q(x)\|_{L_\infty[0,\infty)}\}^{(\rho-\epsilon)/n} \leq (2\sqrt{2})^{-1}$ has a solution. *Remark.* If $f(z)$ is of type τ and lower type ω $(0 < \omega \leq \tau < \infty)$, then it is easy to replace $(\rho - \epsilon)$ by $\rho\omega/\tau$ in the above conclusion. (Received November 12, 1973.)

*74T-B39. KOK-KEONG TAN and TECK-CHEONG LIM, Dalhousie University, Halifax, Nova Scotia, Canada. *Hausdorff metric on the family of weakly compact sets.* II. Preliminary report.

Let $(X, \|\ \|)$ be a normed space, $wc(X)$ $(wcc(X))$ be the family of all nonempty weakly compact (and convex) subsets of X and D be the Hausdorff metric on $wc(X)$ (and hence also on $wcc(X)$) induced by the metric

on X which is induced by the norm $\|\ \|$ on X. Then $(X, \|\ \|)$ is a Banach space if and only if $<wc(X), D>$ $(<wcc(X), D>)$ is a complete metric space if and only if cl $(\bigcup_{n=1}^{\infty} A_n)$ is weakly compact for each Cauchy sequence $(A_n)_{n=1}^{\infty}$ in $<wc(X), D>$. (Received November 8, 1973.)

74T-B40. JAMES VINCENT PETERS, Stevens Institute of Technology, Hoboken, New Jersey 07020 and 88-08 235 Street, Queens Village, New York 11427. *Spanning L^p classes of semigroups.* Preliminary report.

It has recently been shown (D. J. Newman, Proc. Amer. Math. Soc. 21(1969), 511–512) that $f \in L^1(R)$ and its translates span $L^1(R^+ \cup \{0\})$ if and only if the Fourier transform of f is not identically zero. This result is generalized to maximal semigroups of Archimedean ordered groups. A counter example for the non-Archimedean case is given to show that this condition is no longer sufficient. Necessary and sufficient conditions for spanning $L^1(S)$ are established where S is a semigroup of the general ordered group. Identical results are obtained for $f \in L^2$ when the transform of f is not zero almost everywhere. A discussion of arbitrary L^p classes is also included. (Received November 14, 1973.)

*74T-B41. ROY A. JOHNSON, Washington State University, Pullman, Washington 99163. *Cross section measure of a Borel set need not be a.e. measurable.*

Let Y be the set of ordinals less than or equal to the first uncountable ordinal Ω with the order topology and let $Y_0 = Y - \{\Omega\}$. Let X be 2^{Y_0} with the product topology, μ be Haar measure on X, and ν be Dieudonné's nonregular Borel measure on Y. If $M = \{(x, y) \in X \times Y: x_\alpha = 1 \text{ if } \alpha \geq y\}$, then M is compact in $X \times Y$, and $\nu(M_x)$ is not μ-almost everywhere measurable. Hence, the Borel measure ρ on $X \times Y$ given by $\rho(M) = \int \nu(M_x)\, d\mu$ does not exist in this case. (Received November 15, 1973.)

74T-B42. DAVID N. BECKER, St. Francis College, Fort Wayne, Indiana 46808 and JAMES M. FRANKLIN, 7700 Stanford Street, St. Louis, Missouri 63130. *Using the numerical range to find a diagonal set containing the spectrum of a matrix.*

Let $X = \mathbb{C}^{n\times 1}$ with $\|x\| = \text{MAX}\{|x_i|: 1 \leq i \leq n\}$. Let $\mathfrak{A} = LC(X, X)$ such that for $A = [a_{ij}] \in \mathfrak{A}$, $\|A\| = \text{MAX}\{\Sigma_{k=1}^{n}|a_{ik}|: 1 \leq i \leq n\}$. Then X is a Banach space and $\mathfrak{A}$ is a Banach algebra. Note that $I =$ the identity $\in \mathfrak{A}$ and $\|I\| = 1$. Let $S = \{f \in \mathfrak{A}^*: 1 = \|f\| = f(I)\}$. *Theorem* 1. The extreme points of S, Ext $(S) = \{f = [\alpha_{ij}] \in \mathfrak{A}^*: |\alpha_{ij}| = |\alpha_{ii}|$ for $1 \leq j \leq n$ and each i; there is p, $1 \leq p \leq n$, with $\alpha_{pp} = 1$ and $\alpha_{ij} = 0$ for $i \neq p\}$. *Theorem* 2. For $A = [a_{pk}] \in \mathfrak{A}$, $V(\mathfrak{A}, A) = \langle \bigcup_{p=1}^{n} \overline{N(a_{pp}, r_p)} \rangle$ where $r_p = \Sigma_{k=1, k\neq p}^{n}|a_{pk}|$. Since $\mathfrak{A}$ is a unital Banach algebra, then $sp_{\mathfrak{A}}(A) \subseteq V(\mathfrak{A}, A)$ for $A \in \mathfrak{A}$. (See Bonsall and Duncan, "Numerical ranges of operators on normed spaces and elements of normed algebras", Cambridge University Press, 1971.) Theorem 2 gives an easy way to calculate a set which contains the spectrum and estimates the location of the spectrum in the complex plane. (Received November 19, 1973.)

74T-B43. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. *A simplest example of a nonmeasurable set.*

Let c be the cardinal of the continuum and let $(C_i)_{i<c}$ denote a well-ordering of all the nondenumerable (and therefore of power c) closed subsets C_i of the real unit interval. Let $(a_i)_{i<c}$ be a sequence of pairwise distinct real numbers a_i such that $a_i \in C_i$. Let $(b_i)_{i<c}$ be a sequence of pairwise distinct real numbers b_i such that $b_i \in C_i$ and $a_i \neq b_j$. Then $(a_i)_{i<c}$ or $(b_i)_{i<c}$ is a Lebesgue nonmeasurable set of real numbers. (Received November 21, 1973.)

*74T-B44. RONALD E. BRUCK, JR., University of Southern California, Los Angeles, California 90007. *Convergence of steepest descent for convex functions in Hilbert space.*

Let H be a real Hilbert space and $f: H \to (-\infty, +\infty]$ be a proper l.s.c. convex function. It is known that for each $x_0 \in D(\partial f)$ there exists a unique absolutely continuous solution $x: R^+ \to D(\partial f)$ of $dx/dt \in -\partial f(x(t))$ for a.e. t, $x(0) = x_0$. *Theorem* 1. If f assumes a minimum in H then the weak $\lim_{t\to\infty} x(t) = x^*$ exists and is a

minimum point of f. *Theorem* 2. If f is also *even*, then the convergence of $x(t)$ to x^* is *strong*. *Theorem* 3. Suppose f is even, $(t_n) \subset R^+$, $\Sigma t_n = +\infty$, and $(x_n) \subset D(\partial f)$ satisfies $x_{n+1} \in x_n - t_n \partial f(x_n)$ for all n. If $\Sigma \|x_n - x_{n+1}\|^2 < +\infty$ then (x_n) converges *strongly* to a minimum point of f. (Received November 23, 1973.)

74T-B45. LOUIS R. BRAGG, Oakland University, Rochester, Michigan 48063. *Singular nonhomogeneous abstract Cauchy and Dirichlet type problems related by a generalized Stieltjes transform.*

Let X be a Banach space, B the infinitesimal generator of an equibounded group in X, and $A = B^2$. Let $F(Y)$ denote a class of strongly continuous functions $\{f\}$ each member of which has domain $[0, \infty)$ and range Y ($Y \subseteq X$) and let $\|f(t)\|$ be the corresponding norm function associated with the function f at the point t. This paper is concerned with the construction of solutions of the nonhomogeneous Dirichlet problem (*) $v_{tt}(t) + t^{\beta} A v(t) = g(t)$; $\beta \geq 0$, $g \in F(D(A))$, $\|v(t)\|_{t \to 0+} \to 0$, in terms of the solutions of the nonhomogeneous Cauchy problem (**) $u_{tt}(t) - t^{\alpha} A u(t) = f(t)$, $\alpha \geq 0$, $f \in F(D(A))$, $\|u(t)\|_{t \to 0+} \to 0$, $\|u_t(t)\|_{t \to 0+} \to 0$. It is shown that $v(t)$ is related to $u(t)$ by means of a generalized Stieltjes transform provided that $g(t)$ is related to $f(t)$ by another such transform. The solution of (**) can be constructed by semigroup methods and the relation between (v,t) and $u(t)$ then defines a solution of (*) provided that $\int_0^{\infty} \|f(t)\| \, dt < \infty$. A discussion is given on the construction of $f(t)$ in (**) for a given $g(t)$ in (*). (Received October 24, 1973.)

* 74T-B46. LUDVIK JANOS, University of Newcastle, New South Wales 2308, Australia. *Linear contractions on Fréchet spaces.* Preliminary report.

Let X be a Fréchet space and $P: X \to X$ a linear operator for which there is a defining family of seminorms $\{\|\cdot\|_n\}$ on X and a constant $q \in (0, 1)$ such that $\|Px\|_n \leq q \|x\|_n$ for all $x \in X$ and all $n = 1, 2, \cdots$ Calling such an operator briefly a "contraction" one may ask a question: Does there exist a metric on X generating the given topology and rendering P a contraction in the ordinary metric sense? We have the following results: *Theorem* 1. If X degenerates to a Banach space the answer is "yes" and there exists a norm on X realizing the desired metric for some Lipschitz constant $q \in (0, 1)$. *Theorem* 2. Let $C(-\infty, \infty)$ be the Fréchet space of all continuous real valued functions on $(-\infty, \infty)$ with the topology of uniform convergence on compacta, and let $P: C(-\infty, \infty) \to C(-\infty, \infty)$ be the operator of integration, i.e., $Pf(x) = \int_0^x f(t)\, dt$ for $f \in C(-\infty, \infty)$. Then the operator P is a contraction in the above generalized sense but it is *not* a contraction relative to any metric on $C(-\infty, \infty)$ which is compatible with the given topology. (Received November 1, 1973.)

* 74T-B47 HECTOR O. FATTORINI, University of California, Los Angeles, California 90024. *Two point boundary value problems for operational differential equations.*

Let n be a positive integer, α_0 and α_1 subsets of the set of integers $\{0, 1, \cdots, n-1\}$, m_0 (resp. m_1) the number of elements of α_0 (resp. α_1), T a positive real number, A a linear operator with domain $D(A)$ dense in the complex Banach space E and range in E. We say that condition (E) (existence) is satisfied if *there exists a dense subspace* D of E *such that the boundary value problem* $u^{(n)}(t) = Au(t)$ $(0 \leq t \leq T)$, $u^{(j)}(0) = u_{0,j}$ $(j \in \alpha_0)$, $u^{(k)}(T) = u_{1,k}$ $(k \in \alpha_1)$ *has a solution for any* f *continuous in* $0 \leq t \leq T$ *and any* $u_{0,j}$, $u_{1,k}$ *in* D. We say condition (CD) (continuous dependence) is satisfied if *for any sequence* u_m *of solutions of* $u_m^{(n)} = Au_m + f_m$ *with* $f_m \to 0$ *uniformly in* $0 \leq t \leq T$ *and* $u_m^{(j)}(0) \to 0$ $(j \in \alpha_0)$, $u_m^{(k)}(0) \to 0$ $(k \in \alpha_1)$ *we have* $u_m \to 0$ *uniformly in* $0 \leq t \leq T$. Assume $\rho(A) \neq \emptyset$. Then we have: *Theorem* 1. Suppose condition (CD) holds. Then $m_0 + m_1 \geq n$. *Theorem* 2. Suppose conditions (E) and (CD) hold. Then $m_0 + m_1 \leq n$. *Theorem* 3. Assume conditions (E) and (CD) hold. Assume, further that (a) n is even and $m_0 < (n-2)/2$ or $m_1 < (n-2)/2$, or that (b) n is odd and $m_0 < (n-1)/2$ or $m_1 < (n-1)/2$. Then A must be bounded. In the cases left out by Theorem 3 the resolvent $R(\lambda; A)$ can be shown to exist in certain regions of the complex plane and can be estimated in these regions. (Received November 29, 1973.)

74T-B48. DEBORAH REBHUHN, Vassar College, Poughkeepsie, New York 12601. *On the set of attainability of nonautonomous nonlinear control systems.* Preliminary report.

Let M be a C^∞ second countable Hausdorff manifold of dimension m. Let Ω be a second countable Hausdorff topological manifold. Let $\sigma: M \times \mathbf{R} \to 2^\Omega$ be a set valued function such that for each $(x', t') \in M \times \mathbf{R}$ there is a neighborhood U of (x', t') and distinct $w_1, w_2 \in \bigcap\{\sigma(x, t): (x, t) \in \Omega\}$. *Definition.* A section $F: M \times \mathbf{R} \times \Omega \to TM$ such that $F(x, t, w) \in T_x M$ is called a C^k controllable vector field if the map $(x, t) \to F(x, t, w)$ is C^k for each fixed $w \in \Omega$ and if the derivatives up to order k of the local representations of F are jointly continuous in (x, t, w). The set of all C^k controllable vector fields in the Whitney C^k topology will be denoted by A^k. Let P be the set of piecewise constant paths from compact subintervals of the reals into Ω. If $F \in A^k$, let $F^+_{(x', t')} = \{(x_u(t), t): u \in P, x_u(t') = x', (d/ds)x_u(s) = F(x_u(s), s, u(s))$ almost everywhere, $t \geq t'$, and $u(s) \in \sigma(x_u(s), s)\}$. *Theorem.* If $k \geq 2m + 2$, then there is an open dense subset O of A^k such that if $F \in O$, then $F^+_{(x', t')}$ is contained in the closure of its own interior for any $(x', t') \in M \times \mathbf{R}$. (Received November 29, 1973.)

74T-B49. MARTIN SCHECHTER, Belfer Graduate School of Science, Yeshiva University, New York, New York 10033. *Completeness of the wave operators.* Preliminary report.

The numbers α, p will be called suitable if $\alpha \geq 0$ and $1 \leq p \leq \infty$ with the additional stipulations that $\alpha > 0$ if $2n/(n + 1) \leq p \leq \frac{1}{2} n$ and $\alpha > 1 - (n/2p)$ if $\frac{1}{2}n < p \leq \infty$. Let $V(x)$ be a real valued function on E^n, and let H, H_1 be the Hamiltonian operators associated with $-\Delta$ and $-\Delta + V(x)$, respectively. *Theorem.* Assume (a) $\sup_x \int_{|x-y|<\delta} |V(y)|\,|x - y|^{2-n}\,dy \to 0$ as $\delta \to 0$, (b) $\int_{|x-y|<1} |V(y)|\,dy \to 0$ as $|x| \to \infty$ and (c) $(1 + |x|)^\alpha \int_{|x-y|<1} |V(y)|\,dy \in L^p$ for some suitable α, p. Then the absolutely continuous part of H_1 is unitarily equivalent to H. The wave operators exist and are complete; the intertwining relations and the invariance principle hold. (Received November 30, 1973.)

* 74T-B50. H. KHARAGHANI, University of Calgary, Calgary, Alberta T2N 1N4, Canada. *Multiplicative invariant means on weakly almost periodic functions.* Preliminary report.

Using de Leeuw-Glicksburg's method, we obtain a necessary and sufficient condition for a topological semigroup S to admit a multiplicative invariant mean on $W(S)$, the weakly almost periodic functions on S. Consequently when G is a group, $W(G)$ admits a multiplicative invariant mean iff $A(G)$, the almost periodic functions on G, consists of constant functions. As an application we give an example to answer negatively R. B. Burckel's third open question in "WAP functions on semigroups", Gordon and Breach, New York, 1970, p. 81 (Burckel has informed us of a different example, yet to be published). Furthermore, we deduce that no nontrivial locally compact abelian group G has a multiplicative invariant mean $W(G)$. We give examples to show the commutativity cannot be dropped entirely. (Received December 3, 1973.) (Author introduced by Dr. James C.S. Wong.)

74T-B51. ARNOLD M. DUNN, Clarkson College of Technology, Potsdam, New York 13676 and State University College of New York, Potsdam, New York 13676. *Schauder decompositions of quasi-reflexive Banach spaces.* II. Preliminary report.

Let (R_n) be a sequence of real Banach spaces and let K be a solid BK-space. Let $(\Sigma R_n)_K = \{(r_n)_{n=1}^\infty : r_n \in R_n$ for each n with $(\|r_n\|)_{n=1}^\infty \in K\}$. It is well known that $(\Sigma R_n)_K$ is Banach with $\|(r_n)_{n=1}^\infty\| = \|(\|r_n\|)_{n=1}^\infty\|_K$. *Definition.* Let (R_n, E_n) be a Schauder decomposition of a Banach space X and let G_n be an operator from R_n to R_n for each n. (R_n, E_n) is said to have the BOC (bounded operator convergence) property if $\sup_n\{\|G_n\|\}$ finite implies $\Sigma G_n(E_n(x))$ converges in X for each $x \in X$. *Results.* 1. The Schauder decomposition (R_n) for $(\Sigma R_n)_K$ has BOC. 2. (R_n, E_n) has BOC implies (R_n, E_n) is an unconditional Schauder decomposition. The converse fails. 3. If X is a Banach space with a Schauder decomposition (R_n) which has BOC then there exists a BK-space K such that X is isomorphic to $(\Sigma R_n)_K$. 4. If $k = 0$, definitions 2 and 3 given in Abstract 73T-B180, these Notices 20(1973), A-435 are equivalent for all spaces of the form $(\Sigma R_n)_K$. 5. Quasi-reflexive

spaces of the form $(\Sigma R_n)_K$ are constructed with the property that none is isomorphic to products of James' space. Similarly, this leads to the construction of spaces with the property that dim $(X^{**}/\hat{X})$ is infinite and $X \equiv X^{**}$. (Received December 3, 1973.)

74T-B52. WITHDRAWN

*74T-B53. HUMPHREY S. FONG and LOUIS SUCHESTON, Ohio State University, Columbus, Ohio 43210. *On a mixing property of operators in L_p spaces.*

A matrix (a_{ni}) is called uniformly regular iff it satisfies (UR): $\sup_n \Sigma_i |a_{ni}| < \infty$, $\lim_n \Sigma_i a_{ni} = 1$, $\lim_n \max_i |a_{ni}| = 1$. Let T be a contraction on L_p of a σ-finite measure space. If $p = 1$ or $p = 2$, then the following conditions are equivalent: (A) T^n converges weakly; (B) $\Sigma_i a_{ni} T^i$ converges strongly for each (UR)-matrix (a_{ni}). If $1 < p < 2$ then (A) and (B) are shown equivalent assuming that T is positive and there exists a strictly positive $h \in L_p$ such that $Th \leq h$. If $2 < p < \infty$ then (A) and (B) are shown equivalent assuming that T is positive and there exists a strictly positive $h \in L_p$ such that $Th \leq h$ and $T^* h^{p-1} \leq h^{p-1}$ (Z. Wahrscheinlichkeitstheorie und Verw. Gebeite, to appear). (Received December 6, 1973.)

74T-B54. RICHARD D. MAULDIN, University of Florida, Gainesville, Florida 32611. *The continuum hypothesis, integration and duals of measure spores.*

Let S be a set, Σ a σ-algebra of subsets of S, X a Banach space having the Radon-Nikodym property. Let $bv(S, \Sigma, X)$ be the space of all countably additive functions from Σ into X which are of bounded variation and let this space be given the variation norm. A bilinear vector integral is defined by considering functions Ψ from Σ into X^* and taking the limit over the net of Σ-subdivisions of S. It is shown that if the cardinality of $ca(S, \Sigma) \leq 2^{N_0}$ (as is the case if S is separable metric and Σ is the family of Borel subsets of S) and $2^{N_0} = N_1$, then for each $T \in bv^*(S, \Sigma, X)$, there is a function Ψ from Σ into X^* such that for every $u \in bv(S, \Sigma, X)$, $(R)T(u) = \int \Psi \, du$ and $|T| = \text{l.u.b.} \{\|\Psi(E)\|: E \in \Sigma\}$. Some necessary and sufficient conditions for this representation are studied and a certain space of functions Ψ appearing in (R) is constructed and shown to be isometrically isomorphic to $bv^*(S, \Sigma, X)$. (Received December 7, 1973.)

74T-B55. DONALD W. HADWIN, Indiana University, Bloomington, Indiana 47401. *Closures of unitary equivalence classes.* Preliminary report.

The main theorem determines the closures of the unitary equivalence classes $\mathcal{U}(T)$ of each operator T in a certain large class of operators on a separable infinite-dimensional Hilbert space $\mathcal{H}$. The class includes all essentially normal operators T (i.e., all T such that $T^*T - TT^*$ is compact). *Special cases.* (1) If T is normal, then $\overline{\mathcal{U}(T)} = \{S: S \text{ is normal, } \sigma(S) = \sigma(T), \text{ the isolated eigenvalues of } S \text{ and } T \text{ have the same multiplicity}\}$. (2) If T is compact and $\ker(T) \cap \ker(T^*) = 0$, then $\overline{\mathcal{U}(T)} = \{S: S \cong T \oplus 0\}$. (3) If T is essentially normal and T has no normal part, then $\overline{\mathcal{U}(T)} = \{S: S \cong T \oplus N, N \text{ is normal, } \sigma(N) \subseteq \sigma_{ess}(T)\}$. The techniques involved make use of two objects: (1) the C^*-algebra $\mathcal{S}(T)$ of all operators S for which $\{U_n^* S U_n\}$ has a convergent subsequence whenever $\{U_n\}$ is a sequence of unitary operators and $\{U_n^* T U_n\}$ converges; (2) the subspace $\mathcal{M}(T)$ of all vectors f for which the projection onto f belongs to $\mathcal{S}(T)$. The main result follows from an auxiliary theorem: if T is essentially normal and $S \in \mathcal{U}(T)$, then the abnormal part of S is unitarily equivalent to the abnormal part of T. This auxiliary result remains true if "normal" is replaced by "hyponormal", or any other property that can be defined by a family of equations involving functions of T and T^*. (Received December 7, 1973.) (Author introduced by Professor Paul R. Halmos.)

*74T-B56. RICHARD S. ELLIS and MARK A. PINSKY, Northwestern University, Evanston, Illinois 60201. *Asymptotic equivalence of the linear Navier-Stokes and heat equations in one dimension.*

For d a positive integer, $t > 0$, x real, and $\{f_k(x)\}_{1 \leq k \leq d}$ nonnegative and smooth, let $n_k = n_k(t, x)$, $1 \leq k \leq d$, solve the system $n_k = \alpha_1^{(k)} n_k' + \Sigma_{1 \leq l \leq d} \alpha_2^{(kl)} n_l''$, $\lim_{t \downarrow 0} n_k(t, x) = f_k(x)$ ($\cdot$ indicates $\partial/\partial t$, $'$, $\partial/\partial x$). This

system is the Navier-Stokes equations for the model Boltzmann equation $p' + vp' = Qp$ discussed in Abstract 703-B2, these Notices 20(1973), A-365. The numbers $\alpha_1^{(k)}$ and $\alpha_2^{(kl)}$ are constants defined in terms of Q and v, and $\alpha_2^{(kk)} > 0$. Let $m_k(t, x)$, $1 \leq k \leq d$, solve the heat equations $m_k' = \alpha_2^{(kk)} m_k''$, $\lim_{t \downarrow 0} m_k(t, x) = f_k(x)$. *Theorem.* For each k, the following hold: (a) $\sup_{-\infty<x<\infty} |n_k(t, x - \alpha_1^{(k)} t) - m_k(t, x)| = O(1/t)$, as $t \to \infty$; (b) $\sup_{-\infty<x<\infty} |n_k(t, x - \alpha_1^{(j)} t)| = O(1/t)$, as $t \to \infty$, for $\alpha_1^{(j)} \neq \alpha_1^{(k)}$; (c) $\sup_{-\infty<x<\infty} |n_k(t, x - \alpha t)| = O(1/t^n)$, as $t \to \infty$, for $\alpha \notin \{\alpha_1^{(1)}, \cdots, \alpha_1^{(d)}\}$ and each positive integer n. The statement and proof of this theorem apply as well to the physical one-dimensional linearized Navier-Stokes equations. We also point out several results dual to these which hold for the solution p of the model Boltzmann equation. (Received December 10, 1973.)

*74T-B57. NORBERTO SALINAS, University of Michigan, Ann Arbor, Michigan 48104. *Subnormal limits of nilpotent operators.*

In this article it is proved that if T is a subnormal operator whose approximate point spectrum is simply connected and contains the origin, then T is the uniform limit of nilpotent operators. In particular, it follows that the direct sum of an isometry and a normal operator whose spectrum coincides with the closed unit disk is the uniform limit of nilpotent operators. Also the class of pseudo-nilpotent operators is introduced and some of its properties are discussed. (Received December 12, 1973.)

74T-B58. VASANT A. UBHAYA, Washington University, St. Louis, Missouri 63130. *Approximation by isotone functions — a* min-max *form of best approximation.* Preliminary report.

Let X be a partially ordered set with partial order $\leq$. Let B be the linear space of bounded real valued functions defined on X. Given $w \in B$, $w(x) \geq \delta > 0$ for all $x \in X$, define a generalized uniform norm $\|\cdot\|_w$ on B by $\|f\|_w = \sup_{x \in X} w(x)|f(x)|$ where $f \in B$. A function $h \in B$ is called isotone if $h(x) \leq h(y)$ whenever $x, y \in X$, $x \leq y$. Let $M \subset B$ be the convex cone of isotone functions. The problem under consideration is: Given $f \in B$ find $k \in M$ such that $\|f - k\|_w = \min_{h \in M} \|f - h\|_w$. Such a k is called a best approximation to f. *Theorem.* Let $f \in B$. For $y, z \in X$, define $K(y, z) = (w(y)f(y) + w(z)f(z))/(w(y) + w(z))$. Then for each fixed $x \in X$, $\inf_{\{z:\, z \geq x\}} \sup_{\{y:\, y \leq x\}} K(y, z) = \sup_{\{y:\, y \leq x\}} \inf_{\{z:\, z \geq x\}} K(y, z) = g(x)$ say. $g \in M$ and is a best approximation to f. (Received December 10, 1973.)

*74T-B59. DONG SIE KIM, University of Florida, Gainesville, Florida 32611. *Complete domains with respect to the Carathéodory distance.*

Let D be a domain of a complex space (X, A). Let $B = B(D)$ be the algebra of bounded holomorphic functions on D. If B separates the points of D then there exists the Carathéodory distance c. If every closed c-ball, $\Delta(p, r) = \{x \in D;\, c(p, x) \leq r\}$, is compact then D is said to be complete. If D is complete it is boundedly holomorphic convex. *Results.* A bounded convex domain in $\mathbf{C}^n$ is complete. Let α be a proper spread map of a hyperbolic complex space (X, A) to a hyperbolic complex space $(\tilde{X}, \tilde{A})$. Then X is complete if and only if $\tilde{X}$ is complete. Let $(X, A; \alpha)$ be a Riemann domain, D a domain in X with $\alpha(D)$ bounded in $\mathbf{C}^n$; $B(D)$ separates the points of D. Suppose there is a compact set K such that for any $x \in D$ there is an analytic automorphism $\sigma \in \mathrm{Aut}\,(D)$ and a point $a \in K$ such that $\sigma(x) = a$. Then D is a domain of bounded holomorphy. (Received December 17, 1973.)

*74T-B60. STYLIANUS K. PICHORIDES, N.R.C. Democritos, Agia Paraskevi Attikis, Athens, Greece. *A lower bound for the L^1-Norm of exponential sums.* Preliminary report.

Let $f(x)$ be a trigonometric polynomial with N (≥ 2) nonzero terms and coefficients of absolute value greater than 1. *Theorem.* The L^1-Norm of f exceeds a fixed positive multiple of $(\log N/\log\log N)^{1/2}$. This result improves a previous one of Cohen and Davenport (the same bound with exponent $\frac{1}{4}$ (H. Davenport, Mathematika 7(1960), 93–97)). The proof is achieved by refining the argument used in the above mentioned paper. (Received December 17, 1973.)

74T-B61. DENNIS D. BERKEY, Miami University, Oxford, Ohio 45056. *Column diagonally dominant linear differential systems.* Preliminary report.

We consider the differential equation (1) $x'(t) = A(t)x(t)$ where $A(t)$ is an $n \times n$ matrix function whose entries are continuous on $(-\infty, \infty)$. Let X denote the fundamental matrix for (1) such that $X(0) = I$. (1) is said to admit an *exponential dichotomy* if there exist constants α_1 and α_2 in $(0, \infty)$, constants K_1 and K_2 in $[1, \infty)$, and a projection P so that $\|X(t)PX^{-1}(s)\| < K_1 e^{\alpha_1(t-s)}$ for all $s < t$ and $\|X(t)(I - P)X^{-1}(s)\| < K_2 e^{\alpha_2(s-t)}$ for all $t < s$. *Theorem.* Let A be bounded on $(-\infty, \infty)$ and let $\delta > 0$ be a constant. Let $\{1, 2, \cdots, n\} = I_1 \cup I_2$ where $I_1 \cap I_2 = \emptyset$, $\mathrm{Re}(a_{ii}(t)) + \Sigma_{j=1; j\neq i}^{n} |a_{ji}(t)| < -\delta < 0$ for all i in I_1 and t in $(-\infty, \infty)$, and $\mathrm{Re}(a_{ii}(t)) - \Sigma_{j=1; j\neq i}^{n} |a_{ji}(t)| > \delta > 0$ for all i in I_2 and t in $(-\infty, \infty)$. Then (1) admits an exponential dichotomy. Our result for column diagonally dominant systems contains a stronger conclusion than that of Lazer (J. Math. Anal. Appl. 35(1971), 215–229) for row diagonally dominant systems. However, we have not succeeded in removing the condition that A be bounded. (Received December 18, 1973.)

74T-B62. GUSTAVO PERLA-MENZALA, Brown University, Providence, Rhode Island 02912 and Federal University of Rio De Janeiro, Rio De Janeiro, Guanabara, Brazil. *A uniqueness theorem for the 3-dimensional inverse scattering problem of the wave equation.* Preliminary report.

We consider a linear perturbation for the wave equation $\Box u = 0$ ($\Box = \partial^2/\partial t^2 - \Delta$, $\Delta = \Sigma_{j=1}^{3} \partial^2/\partial x_j^2$) in $\Omega = \mathbf{R}^3$ by "repulsive" potentials $q(y)$ which are "small at infinity" and suitably small (in magnitude). We prove that the scattering operator $S(q)$ determines *uniquely* the scatterer q (at least in this class). Our method is essentially based on elementary estimates for the Riemann function and standard energy inequalities. *Theorem.* Let $q_1(y)$ and $q_2(y)$ be positive potentials (sufficiently smooth), q_1, q_2 belong to $L^1(\mathbf{R}^3) \cap L^\infty(\mathbf{R}^3)$. Let $S(q_1)$ and $S(q_2)$ be the corresponding scattering operators for the perturbed wave equation by q_1 and q_2 respectively. If q_1 and q_2 are suitably small, then either $S(q_1) \neq S(q_2)$ or $q_1 = q_2$. (Received December 21, 1973.)

74T-B63. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. *A convergence theorem for the dual of a certain function space.*

U, F, p_{AB}, p_A^+ and the notion of integral are as in previous abstracts of the author. *Theorem.* Suppose T is in the dual of p_{AB}, m is in p_A^+, $\{b_n\}_{n=1}^{\infty}$ is a sequence of elements of p_{AB} uniformly absolutely continuous with respect to m with $\{\int_U |b_n(I)| : n = 1, 2, \cdots\}$ bounded, and z is a function from F into $\mathbf{R}$ such that for all V in F, $b_n(V) \to z(V)$, $n \to \infty$. Then z is in p_{AB} and $T(b_n) \to T(z)$, $n \to \infty$. (Received December 26, 1973.)

* 74T-B64. GERASIMOS E. LADAS, University of Rhode Island, Kingston, Rhode Island 02881. *Oscillatory effects of retarded actions.* Preliminary report.

Consider the retarded differential equation (*) $y^{(2n)}(t) - p(t)y(t - \tau) = 0$, $n \geq 1$, where τ is a positive constant and $p(t) > 0$ is a continuous function on $[0, \infty)$. Under certain conditions, which in the case $n = 2$ coincide with Papadakis [Bull. Soc. Math. Grece 13(1972), 133–137], it is proved that (a) every bounded solution of (*) is oscillatory, and (b) for every preassigned initial function (**) $y(t) = \phi(t)$, $-\tau \leq t \leq 0$, the system (*) – (**) has at least one oscillatory solution. Clearly the above oscillations are symptoms of the presence of large delays in (*). (Received December 27, 1973.)

* 74T-B65. JAMES LI-MING WANG, Brown University, Providence, Rhode Island 02912. *Modulus of approximate continuity for $R(X)$.* Preliminary report.

Let X be a compact subset of the plane. We say that ϕ is an admissible function if (a) ϕ is a positive, nondecreasing function on $(0, \infty)$, and (b) the associated function ψ, defined by $\psi(r) = r/\phi(r)$, is also nondecreasing, with $\psi(0^+) = 0$. Let F be a function defined on X, $x \in X$. We say that F admits ϕ as modulus of approximate continuity at x if $|F(y) - F(x)| \leq \phi(|y - x|)$ for all y in a set having full area density at x. Recently("An approximate Taylor's theorem for $R(X)$", Math. Scand., to appear), the author proved *Theorem.* Let ϕ be an admissible function. Suppose there exists a measure μ on X representing x for $R(X)$ such that

$\mu\{x\} = 0$ and $\int d|\mu|(t)\phi(|t - x|) < \infty$. Then the unit ball of $R(X)$ admits $\epsilon\phi$ as modulus of approximate continuity at x for every $\epsilon > 0$. The converse is well known to be true when $\phi = 1$. One might conjecture that the converse is true in general. The main result of this note is to disprove this conjecture. (Received December 27, 1973.)

74T-B66. JOSE GARCIA-CUERVA, Washington University, St. Louis, Missouri 63130. *Characterization of weighted H^p spaces.*

Let $w(x)$ satisfy the A_{p_0} condition (Muckenhoupt, Trans. Amer. Math. Soc.). $H^p(w)$ is defined as the space of analytic functions on $\mathbf{R}^2_+$ such that $N_p(F) = \sup_y \{(\int_{-\infty}^{\infty} |F(x + iy)|^p w(x)\,dx)^{1/p},\ y > 0\} < \infty$. It is shown that $\lim F(x + iy) = f(x) + i\tilde{f}(x)$ as $y \to 0$ exists a.e., and that the following expressions all define equivalent "norms" to $N_p(F)$: $\|f\|_{L^p(w)} + \|\tilde{f}\|_{L^p(w)}$, $\|f^*\|_{L^p(w)}$, $\|R_N^*(f)\|_{L^p(w)}$, where $f^*(x) = \sup\{|f(z)|: |z - x| < \operatorname{Im} z\}$ and $R_N^*(f)(x) = \sup\{\epsilon \in \mathbf{R}: |\int_0^1 [t(1 - t)]^N f(x + \epsilon t)\,dt|\}$ (N sufficiently large). We thus obtain an atomic decomposition of f (Coifman, Studia Math.): For $q > p_0$ a (p, q) atom is a function $a(x)$ supported in an interval I such that (1) $\int_{-\infty}^{\infty} a(x)x^k\,dx = 0$ for k an integer, $0 \le k \le [p/p_0] - 1$; (2) $(\int_I |a(x)|^q w(x)\,dx/w(I))^{1/q} \le 1/w(I)^{1/p}$. f is the real part of the boundary value of an $F \in H^p$ iff $f = \Sigma \alpha_i a_i$ with a_i atoms and $(\Sigma|\alpha_i|^p)^{1/p} \le cN_p(F)$. For $p_0 < 2 \wedge 2p$, $k = 0$ is needed in (1), and L is a bounded linear functional on $H^p(w)$ iff $L(F) = \int fCw$ with C satisfying $\{(\int_I |C - m_I(Cw)/w|^{q'} w\,dx)/w(I)\}^{1/q'} \le c/w(I)^{1 - 1/p}$ for all I and $q' \le p_0' = p_0/(p_0 - 1)$. For any such q' the condition is equivalent to the same condition with $q' = 1$. (Received December 27, 1973.) (Author introduced by Professor Guido L. Weiss.)

74T-B67. GRAHAME BENNETT, Indiana University, Bloomington, Indiana 47401. *Tensor products of l^p spaces.* Preliminary report.

Let p and q denote fixed numbers satisfying $1 < p < q < \infty$. It is shown that, if the infinite matrix $A = (a_{ij})_{i,j=1}^{\infty}$ maps l^p into l^q, then so does the triangular matrix $T = (t_{ij})_{i,j=1}^{\infty}$, where $t_{ij} = a_{ij}$ for $j \le i$ and $t_{ij} = 0$ for $j > i$. This answers a question of Kwapien and Pelczynski [Problem 1, Studia Math. 34(1970), 43–68] It follows from a remark of Lindenstrauss [loc. cit., p. 67] that both the spaces $l^{p*} \hat{\otimes} l^q$ and $l^p \hat{\otimes} l^q$ possess an unconditional basis of finite dimensional subspaces. This complements recent work [unpublished] of Gordon and Lewis, where it is shown that neither space possesses an unconditional basis. (Received December 28, 1973.)

74T-B68. MISCHA COTLAR, Universidad de La Plata, La Plata, Argentina and CORA S. SADOSKY, Uriburu 1252, Buenos Aires, Argentina. *A moment theory approach to the Riesz theorem on the conjugate function with general measures.*

A measure $\mu \ge 0$ belongs to the class $\mathcal{R}_M$ if it satisfies the Riesz inequality $\int_0^{2\pi} |\tilde{f}(t)|^2\,d\mu \le M\int_0^{2\pi} |f(t)|^2\,d\mu$, $\forall f \in L^2(\mu)$, where $\tilde{f}$ is (essentially) the conjugate function of f, with fixed constant M. Applying a moment theory approach, we determine (and give explicit formulae for) the canonical extremal (simple) elements of $\mathcal{R}_M$, which prove to be of the form $R(t)dt$, with $R(t)$ certain rational functions, making up an essential (dense) part of $\mathcal{R}_M$. These particular measures are in $\mathcal{R}_M$ while the Dirac measures are in the class of all measures. Among the possible applications of this parallel construction is the analogue for the $\mathcal{R}_M$-simple measures of Bochner's theorem of decomposition on Dirac measures. (Received December 31, 1973.)

74T-B69. CARL P. McCARTY, LaSalle College, Philadelphia, Pennsylvania 19141. *Analytic functions with initial zero coefficients.* Preliminary report.

Let $P_n(\alpha) = \{P(z) = 1 + b_n z^n + b_{n+1} z^{n+1} + \cdots : b_n \ne 0,\ P(z)$ analytic and $\operatorname{Re}\{P(z)\} > \alpha$ for $|z| < 1\}$ where $\alpha \in [0, 1)$ and n is an integer greater than 1. A sharp lower bound is found for $\operatorname{Re}\{zP'(z)/P(z)\}$. These results extend those found in Tonti and Trahan [Math. Z. 115(1970), 252–258], Singh and Goel [J. Math. Soc. Japan 23(1971)], and the author [Proc. Amer. Math. Soc., Dec. 1973]. Most recently a subclass of these functions was investigated by Shaffer [Proc. Amer. Math. Soc. 39(1973), 281–287]. (Received January 7, 1974.)

*74T-B70. CHANDRA MOHAN JOSHI, University of Jodhpur, Jodhpur, Rajasthan, India 342001 and M. L. PRAJAPAT, Defence Laboratory, P. B. No. 136, Jodhpur, Rajasthan, India 342001. *On a unified presentation of certain classical polynomials.* Preliminary report.

For the polynomial system $\{J_{n,\lambda,\mu}^{(\alpha,\beta)}(x, a, b, c, d, p, q, r) | n = 0, 1, 2, \cdots\}$ defined by $J_{n,\lambda,\mu}^{(\alpha,\beta)}(x, a, b, c, d, p, q, r) = (1/n!q^n)(ax + b)^{-\alpha}(cx + d)^{-\beta}\exp(px^r) D^n[(ax + b)^{\lambda n+\alpha}(cx + d)^{\mu n+\beta}\exp(-px^r)]$, where a, b, c, d, p, q, r are constants and n, λ, μ are essentially nonnegative integers, a theorem, one each, on generating and bilateral generating functions is proved. It is also pointed out that these polynomials provide unification of the various known generalizations of the classical polynomials (cf., Chatterjea [Acta Math. Acad. Sci. Hungar. 17(1966), 377–385], Fujiwara [Math. Japon. 11(1966), 133–148], Krall and Frink [Trans. Amer. Math. Soc. 65(1949), 100–115], Ratan Singh [J. Indian Math. Soc. 36(1972), 127–131], Singh and Srivastava [Ricerca 14(1963), 11–21], and Srivastava and Singhal [Math. Comp. 26(1972), 969–975]). Properties of the various classes of polynomials are obtained from the generalized system, without recourse to limiting processes. (Received January 4, 1973.)

*74T-B71. DENNY GULICK, University of Maryland, College Park, Maryland 20742. *Domination in analysis.*

Let T be a completely regular Hausdorff space, and let $LSC(T)$ and $C(T)$ denote the collections of all lower semicontinuous and of all continuous functions, respectively, in R^T. *Theorem* 1. There exists a measurable cardinal iff there exists a nondiscrete T such that for each $h \in R^T$, there is an $f \in C(T)$ with $f \geq h$. *Theorem* 2. If for each $g \in LSC(T)$, there is an $f \in C(T)$ such that $f \geq g$, then T is necessarily a P-space. Next, let $C_s(T)$ denote $C(T)$ with the topology of simple convergence. *Theorem* 3. The following conditions are equivalent: (a) For each $h \in R^T$ there is a $g \in LSC(T)$ with $g \geq h$. (b) The bidual $C_s''(T) = R^T$. (c) The dual $C_s'(T)$ is ultrabornological. *Theorem* 4. There exists a nondiscrete T such that $C_s''(T)$ is not bornological iff there exists a measurable cardinal. (Received December 27, 1973.)

*74T-B72. NADIM A. ASSAD, Technion, Israel Institute of Technology, Haifa, Israel. *Fixed points of set valued functions.*

Let (M, d) be a complete metrically convex metric space, K a closed nonempty subset of M, and $\mathcal{F}(M)$ = family of nonempty closed bounded subsets of M. For $A, B \in \mathcal{F}(M)$, let $H(A, B)$ denote the Hausdorff metric generated by d; for $x \in M$, $A \in \mathcal{F}(M)$, let $D(x, A) = \inf\{d(x, y): y \in A\}$. The mapping $T:K \to \mathcal{F}(M)$ has property (*) if $\forall x, y \in K$, $H(T(x), T(y)) \leq aD(x, T(x)) + bD(y, T(y)) + cd(x, y)$, where $a, b, c \in [0, 1)$. Put $\beta = \min\{(a + c)/(1 - b), (b + c)/(1 - a)\}$, $\gamma = \min\{(1 + a)/(1 - b), (1 + b)/(1 - a)\}$, and let $\alpha = \gamma\beta$. *Theorem* A. If $T:K \to \mathcal{F}(M)$ has property (*) and $T(x) \subset K$ $\forall x \in \partial K$ (the boundary of K), then $\alpha < 1$ implies that T has a fixed point. *Observations* (In (*)). 1. If $a = b$, $c = 0$, then $\alpha < 1$ is equivalent to the condition $3a < 1$. 2. If $a = b = c$, then $\alpha < 1$ is equivalent to the condition $a < \sqrt{5} - 2$. 3. If $a = b = 0$, then $\alpha < 1$ is equivalent to requiring T to be a contraction $(H(T(x), T(y)) \leq ld(x, y), 0 \leq l < 1)$. Theorem A generalizes an earlier result by Assad and Kirk (Abstract 71T-B33, these Notices 18(1971), 407, Theorem 1) to a wider class of mappings. (Received January 7, 1974.)

*74T-B73. M. ZUHAIR NASHED, Georgia Institute of Technology, Atlanta, Georgia 30332 and ROBERT H. MOORE, University of Wisconsin, Milwaukee, Wisconsin 52301. *Approximations to generalized inverses of linear operators.*

For a linear operator A on a Banach space X, a generalized inverse A^+ is such that $E = AA^+$ and $P = A^+A$ are projectors onto the range of A and a topological complement M of the null space of A, resp. It is shown that, if B approximates A so that $\delta = \|A^+(A - B)(I - EB)\| < 1$ (less generally, if $\|A^+(A - B)\| < 1$) then, while B^+ need not approximate A^+, there is still an operator B^ϕ mapping X onto M such that $F = BB^\phi$ and $Q = B^\phi B$ are projectors onto BM and M, resp., and $\|Q - P\|$, $\|B^\phi x - A^+x\|$, $\|Fx - Ex\|$ are bounded by quantities tending to zero with δ when $\|Bx - Ax\| \to 0$, $x \in X$. This is applied for $A = I - K$, $A_n = I - K_n$ where $\|K_n x - Kx\| \to 0$ and $\{K_n, n = 1, 2, \cdots\}$ is collectively compact (e.g. K is an integral operator, K_n is defined

by numerical quadratures). Results are obtained for approximate least squares solutions of Fredholm integral equations of the second kind. Known results on the generalized inverse of perturbed matrices are also recovered. (Received January 7, 1974.)

74T-B74. L. C. HSU, Jilin University, Changchun, People's Republic of China. *Symmetrical inversion formulas for series transforms.*

An analytic function $\phi(z)$ is said to be "self-reciprocal" on a domain $\mathfrak{D}$ if the relation $\phi(\phi(z)) = z$ holds on $\mathfrak{D}$. *Theorem.* Let $w = \phi(z)$ be an analytic self-reciprocal function on an open disk $\mathfrak{D}(|z| < \rho \leq 1)$ with $|\phi(z)| < 1$ and $|\phi(0)| < \rho$. Define a series transform by $n!g(n) = \Sigma_{k=0}^{\infty} f(k)(d/dz)^n(\phi(z))^k|_{z=0}$ with either $\Sigma f(k)$ or $\Sigma g(k)$ being convergent. Then inversely we have $n!\, f(n) = \Sigma_{k=0}^{\infty} g(k)(d/dz)^n(\phi(z))^k|_{z=0}$. Conversely, if f is given as above, then the first formula is the inversion giving g in terms of f. These symmetrical inversion formulas imply various particular reciprocal pairs and were derived formally, without giving precise conditions, in the author's earlier paper (Acta. Sci. Natur. Univ. Pekinensis No. 1 (1964), 25–33), in which similar theorems replacing the ordinary convergence condition for $\Sigma f(k)$ (or $\Sigma g(k)$) by other appropriate ones have also been established. (Received January 4, 1974.) (Author introduced by Professor Everett Pitcher.)

Applied Mathematics

*74T-C3 GERALD A. HEUER, Mathematisches Institut der Universität zu Köln, 5 Köln 41, Weyertal 86, Federal Republic of Germany and Concordia College, Moorhead, Minnesota 56560. *Uniqueness of equilibrium points in bimatrix games.*

Let F be an ordered field and $(x, y) \in F^m \times F^n$. Under what conditions is there a nonzero-sum game (A, B) over F having (x, y) as its unique equilibrium point? Say that (A, B) has property (P1) if (i) (x, y) is an equilibrium point of (A, B), and (ii) if (x', y') is any equilibrium point of (A, B) with the same pure strategies active that (x, y) has, then $(x', y') = (x, y)$; and property (P2) if (i) as above, and (ii) if (x', y') is any equilibrium point of (A, B) whose active pure strategies are among those of (x, y), then $(x', y') = (x, y)$. Let m_1, n_1, be the number of nonzero components (i.e., number of pure strategies active) in x, y, resp. *Theorem* 1. A game (A, B) satisfying (P1) exists iff (*): $m_1 = n_1$, $n_1 < m_1 < n$, or $m_1 < n_1 < m$. All games satisfying (P1) are constructed. *Theorem* 2. A game (A, B) satisfying (P2) exists iff (*). *Corollary* 1. The condition (*) is necessary for existence of a game having (x, y) as its unique equilibrium point. *Corollary* 2. If x and y are completely mixed (i.e., all strategies active), there is a game with (x, y) as unique equilibrium point iff $m = n$. This extends Theorem 2 of C. B. Millham (Naval Res. Logist. Quart. 19 (1972), 709–714), and complements results of T. E. S. Raghavan (J. London Math. Soc. (2) 2 (1970), 709–712). (Received November 2, 1973.)

*74T-C4 MARCUS DUARTE MAIA, Universidade de Brasilia, 70 000 Brasilia D. F., Brasil. *Spinor geometry in pseudo Euclidean spaces.* Preliminary report.

The spinor representations of the groups of isometries of some pseudo Euclidean spaces are obtained. The spaces considered are the local, isometric embedding of the space-times of general relativity theory. The relation between spinors and geometrical objects in the pseudo Euclidean spaces is given and the reality conditions are examined. A review of spinor theory is given. (Received November 13, 1973.)

*74T-C5 ROBERTO TRIGGIANI, University of Minnesota, Minneapolis, Minnesota 55455. *Controllable systems in Banach space with unbounded operators.* Preliminary report.

Consider the control system $\mathfrak{L}: \dot{x} = Ax + Bu$, where X and U are Banach spaces, $B: U \to X$ is a bounded linear operator, while $A: D(A) (\subset X) \to X$ generates a strongly continuous semigroup $S(t)$, $t \geq 0$. $D_\infty(A) = \bigcap_{n=1}^{\infty} D(A^n)$ is dense in X. Let $U_\infty = \{u : Bu \in D_\infty(A)\}$. It is shown, by using the Hahn-Banach theorem that: if (1) $\overline{sp}\{A^n B U_\infty, n \geq 0\} = X$, then $\mathfrak{L}$ is approximately controllable in any finite interval $[0, T]$. Conversely, if $S(t)$ is analytic for $t > 0$, then approximate controllability on $[0, T]$ implies: (2) $\overline{sp}\{S(\tau)A^n B U_\infty, n \geq 0\} = X$, with τ an arbitrary

positive time. In particular, for the system $\mathfrak{L}_m : \dot{x} = Ax + \Sigma b_i u_i$, $i = 1, \cdots, m$, (1) and (2) become (1′) $\overline{sp}\{A^n b_i,\ 1 \le i \le m;\ n \ge 0\} = X$, and (2′) $\overline{sp}\{S(t)\ A^n b_i,\ 1 \le i \le m;\ n \ge 0\} = X$ respectively, when $b_i \in D_\infty(A)$. *Illustration.* When A is also selfadjoint with compact resolvent and $r = \sup r_j < \infty$, with r_j multiplicity of eigenvalue λ_j of A with eigenvectors x_{jk}, $j = 1, 2, \cdots$; $k = 1, \cdots, r_j$, then (1′), (2′) and the following condition are mutually equivalent: rank $B_j = r_j$, ($\Rightarrow m \ge r$), where B_j is the $r_j \times m$ matrix, whose ith column is $[(b_i, x_{j1}), \cdots, (b_i, x_{jr_j})]^T$. Actually this last condition holds not only for $b_i \in D_\infty(A)$, but for any $b_i \in X$. (Received November 13, 1973.)

*74T-C6 CURTIS LIPKIE and DAVID W. BALLEW, South Dakota School of Mines and Technology, Rapid City, South Dakota 57701. *A new approach to the Lagrange interpolation formula.*

This paper describes a method of constructing the approximating Lagrange interpolation polynomial by the multiplication of a n by n matrix (partial polynomials stored by columns) and a column vector of functional values. The n by n polynomial matrix is constant for a fixed interval but can be converted to any finite interval by an appropriate linear transformation. This property is especially nice for digital computers. The polynomial determining matrix can be determined by another technique which, on the surface, is independent of the Lagrange formula. This is done by the construction of the inverse of the matrix with elements of the form $a_{ij} = (i - 1)^{n-j}$. The matrix exhibits several curious properties. The sum of each row equals zero except for the row corresponding to the zeroth degree polynomial. If the data points are equally spaced on the unit interval all of the columns add to zero except that column corresponding to the extreme right hand data point. This latter property can be extended to arbitrary intervals. (Received November 26, 1973.)

74T-C7 OTOMAR HAJEK, Case Western Reserve University, Cleveland, Ohio 44106. *An unorthodox linearisation.* Preliminary report.

The nonlinear control system $\dot{x} = Ax + f(x) + u$ may be somewhat misinterpreted as the linear game $\dot{x} = Ax + v + u$ with $v(t) = f(x(t))$ representing the unpredictable control variable of a notional opponent. If the game can be won, the resulting strategy still steers to actual (rather than approximate) target, nonoptimally and perhaps suboptimally. *Example.* For the controlled frictionless pendulum $\ddot{x} + \sin x = u$, $|u(t)| \le 1$, the optimal time of steering the position $x = \frac{1}{2}$, $\dot{x} = 0$ to rest is 1.35 ± 0.09 (using a 4 step technique); the bad linearisation $\ddot{x} = u - (\sin x)$ yields 1.96; the linearisation $\ddot{x} + x = u - (\sin x - x)$ yields 1.50 (note: a far longer time results if the feedback control appropriate to $\ddot{y} + y = u$ is used); the moderately clever linearisation $\ddot{x} + \alpha x = u - (\sin x - \alpha x)$, with α determined by least squares in $[-\frac{1}{2}, \frac{1}{2}]$, yields 1.32, a better upper estimate obtained far more easily than by direct integration. (Received December 3, 1973.)

74T-C8. RICHARD E. LADNER, University of Washington, Seattle, Washington 98195, NANCY LYNCH, University of Southern California, Los Angeles, California 90007 and ALAN L. SELMAN, Florida State University, Tallahassee, Florida 32306. *A comparison of polynomial-time bounded reducibilities.* Preliminary report.

$\le_T^P$ and $\le_m^P$ denote polynomial time bounded reducibilities (see Cook, "The complexity of theorem proving procedures", Third Annual ACM Symposium on Theory of Computing, 1971, and Karp, "Complexity of computer computations", ed. R. Miller and J. Thatcher, Plenum Press, New York, 1972, 85–103). *Theorem* 1. $\exists$ an infinite, coinfinite set $A \ni$ (i) A is decidable in time $2^{|s|}$, and (ii) $\overline{A} \not\le_m^P A$. Definitions are given of polynomial time-bounded truth-table reducibility, $\le_{tt}^P$, k-truth-table reducibility, $\le_{k-tt}^P$, $k \ge 1$, bounded truth-table reducibility, $\le_{b-tt}^P$, positive reducibility, $\le_{pos}^P$, and conjunctive and disjunctive reducibilities, $\le_c^P$, and $\le_d^P$. They are analogous to the corresponding reducibilities of recursive function theory, but have the property that if A is reducible to B and A cannot be decided in polynomial time, then neither can B. Define $\le_1$ *transcends* $\le_2$ if $\exists$ recursive sets A and $B \ni A \le_1 B$, $B \le_1 A$, and A and B are $\le_2$-incomparable. *Theorem* 2. (i) $\le_{1-tt}^P$ transcends $\le_m^P$, (ii) $\forall k$, $\le_{k+1-c}^P$ transcends $\le_{k-tt}^P$, (iii) $\forall k$, $\le_{k+1-d}^P$ transcends $\le_{k-tt}^P$, (iv) $\le_{tt}^P$ transcends $\le_{b-tt}^P$, (v) $\le_{2-c}^P$ transcends $\le_d^P$, (vi) $\le_{2-d}^P$ transcends $\le_c^P$, (vii) $\le_{4\text{-}pos}^P$ transcends both $\le_c^P$ and $\le_d^P$, (viii) $\le_{1-tt}^P$ transcends $\le_{pos}^P$. Other similar results are obtained. (Received December 27, 1973.)

74T-C9. PAUL JOSEPH KAISER, Lewis University, Lockport, Illinois 60441. *Closed operators in problems of optimal control with distributed and boundary parameters.*

We consider Lagrange problems with distributed and boundary controls and with state equations in strong or in weak form. In an earlier paper with Cesari the concept of a closed operator was used to obtain an existence theorem; here the concept of a Cauchy operator is also shown to be relevant. By combining these two properties, and convergence properties for nonlinear operators previously used by Cesari and by Cesari and Cowles, we present rather extensive existence theorems. The seminormality properties required here are reduced to a minimum, namely property (Q) or property (Q_ρ) (intermediate between Kuratowski's upper semicontinuity and property (Q)) with respect to the state variable only. (Received December 28, 1973.)

* 74T-C10. HOBART P. YOUNG, City University of New York, Graduate Center, New York, New York 10036. *Social choice scoring functions.* Preliminary report.

Let a committee of voters be considering a finite set $A = \{a_1, a_2, \cdots, a_m\}$ of alternatives for election. Each voter is assumed to rank the alternatives according to his preferences in a strict linear order. A social choice function is a rule which, to every finite committee of voters with specified preference orders, assigns a nonempty subset of A, interpreted as the set of "winners". The function is "consistent" if, whenever two disjoint committees meeting separately choose the same winner(s), then the committees meeting jointly choose precisely these winner(s). The function is "symmetric" if it depends only on the numbers of voters having each preference order, and if it commutes with any permutation of the alternatives. Every symmetric, consistent social choice function is obtained as follows. There is a sequence $s^1, s^2, \cdots, s^k$ of m-tuples from $\mathbf{R}^m$, where $s^j = (s^j_1, s^j_2, \cdots, s^j_m)$ and $k \leq m - 1$, such that each voter assigns score s^1_i to his ith-most preferred alternative, and A^1 is the set of alternatives with highest total score. Similarly, score all alternatives using s^2, and let the highest-scoring alternatives from A^1 (relative to s^2) constitute the set A^2, and so forth. Then A^k is the set of winning alternatives. (Received January 2, 1974.)

74T-C11. S. P. BHATTACHARYYA and S. K. JAIN, Indian Institute of Technology, Powai, Bombay-76, India. *Stability of an infinite horizontal layer of fluid with a spatial heat source in the presence of a magnetic field.* Preliminary report.

The stability of an electrically conducting infinite layer of a viscous fluid which loses heat throughout its volume at a constant rate in the presence of a magnetic field is discussed. The value of the critical Rayleigh number R is found to decrease as the rate of heat loss increases which implies that the layer becomes more unstable. It is observed that the destabilizing effect of the heat source term 'Q' is more prominent when the strength of the magnetic field is low; as the strength of the magnetic field increases the effect of Q decreases rapidly. (Received January 3, 1974.) (Authors introduced by Professor P. C. Jain.)

* 74T-C12. J. THOMAS KING, University of Cincinnati, Cincinnati, Ohio 45221. *New error bounds for the penalty method and extrapolation.*

In [Math. Comp. 27(1973), 221–228] Babuška studied the penalty method for obtaining approximate solutions of the Laplace equation with homogeneous Dirichlet boundary condition. The purpose of this paper is to derive new error bounds for the penalty method which admit the use of extrapolation. It is shown that repeated extrapolation yields "optimal" error estimates. For the particular case of cubic spline approximants the error in the second extrapolate is $O(h^3)$ when measured in the energy norm. (Received January 7, 1974.) (Author introduced by Professor Charles W. Groetsch.)

*74T-C13. K. L. BAGLA, Division of Physical Sciences, Institute of Advanced Studies, Meerut University, Meerut, India 250001. *Instability due to density stratification.*

Here we have investigated the stability of two fluids of different densities flowing one over another between two horizontally placed parallel plates. The viscosities of both the fluids are the same. For nonoscillatory

modes, in the absence of effects due to surface tension, the system is found to be (i) unstable always when the lower fluid is lighter than the upper one ($\rho_2 < \rho_1$), (ii) unstable if the quantity $2(\rho_2^2 - \rho_1^2)/(\rho_1\rho_2)$ just exceeds the Froude number when the lower fluid is heavier than the upper one ($\rho_2 > \rho_1$). (Received January 7, 1974.) (Author introduced by Professor Jagat N. Kapur.)

Geometry

74T-D2. ERWIN O. KREYSZIG, University of Windsor, Windsor, Ontario, Canada. *On a class of space curves in affine differential geometry.*

The definition of a spherical curve used in Euclidean differential geometry becomes meaningless in affine differential geometry. However, to define an analogue of such a curve, one may require that all affine normal planes pass through a common point. It is shown that a C^4-curve with nonzero affine torsion is spherical in the sense of that definition if and only if $\chi'' + \kappa(s)\chi = 0$, where s is the affine arc length, κ the affine curvature, and χ the affine radius of torsion. (Received September 25, 1973.)

*74T-D3. PAUL EWING EHRLICH, State University of New York, Stony Brook, New York 11790. *Complete, nonpositively curved "negatively pointed" metrics on $S^1 \times S^1 \times R^2$.* Preliminary report.

We give an example of an admissible metric g_0 on $S^1 \times S^1 \times R^2$ with everywhere nonpositive sectional curvature and a point of all negative sectional curvatures such that no complete metric for $S^1 \times S^1 \times R^2$ close to g_0 can have everywhere negative sectional curvature. A metric g for $S^1 \times S^1 \times R^2$ is *admissible* iff (1) g is complete, (2) the g-injectivity radius of M is greater than zero, and (3) every nontrivial free homotopy class of curves contains a smooth closed g-geodesic which realizes the minimum of the g-lengths of curves in its free homotopy class. Let g be the metric on $S^1 \times R$ represented by the surface of revolution obtained by revolving $y = x^2 + 1$ around the x-axis. Then g_0 is obtained by warping the product metric $((S^1 \times R) \times (S^1 \times R), g \times g)$ by a suitable convex function. An application of Théorème II.3, "Variétés à courbure négative", Séminaire Berger de Géométrie Riemannienne, 1970-71, p. 44, completes the proof. (Received October 26, 1973.)

*74T-D4. KENNETH B. STOLARSKY, University of Illinois, Urbana, Illinois 61801. *Distances between points on a sphere.* Preliminary report.

If N points $p_1, p_2, \cdots, p_N$ are placed on the unit sphere $x^2 + y^2 + z^2 = 1$ so that the sum of all Euclidean distances which they determine is maximal, then the distance between any two of these points is greater than $2/(N + 5)$. (Received November 9, 1973.)

*74T-D5. ERHARD HEIL, Technische Hochschule, 61 Darmstadt, Federal Republic of Germany. *Lie derivative, torsion, and parallel displacement.* Preliminary report.

The torsion T of an affine connection and the Lie bracket have similar geometric interpretations by "infinitesimal parallelograms". If 2 vector fields X, Y are considered as parallel with respect to an absolute parallelism, then $[X, Y] = -T(X, Y)$ and the interpretations become identical. It is also possible to give a system of axioms which covers Lie and covariant derivatives and also Rund's derivative in Finsler space. (Received November 19, 1973.)

*74T-D6. BILL WATSON, Universidad de Oriente, Cumaná, Venezuela. *Almost semi-Kähler manifolds.* Preliminary report.

Recently, we announced in Abstract 73T-D7, these Notices 20 (1973), A-337, that every almost complex submanifold of a para-Kähler manifold is minimal. A partial generalization exists to the weaker class of almost semi-Kähler manifolds. Recall that an almost Hermitian manifold (M, g, J) with Kähler form Φ, is *para-Kähler* (resp., *almost semi-Kähler*) if $d\Phi^{(1,2)} = d\Phi^{(2,1)} = 0$ (resp., $\delta\Phi = 0$). Para-Kähler implies almost semi-Kähler, but since the complexification of a para-Kähler manifold is Kähler, while that of an almost semi-Kähler

manifold is not, in general, Kähler, the classes are different. *Theorem.* Every codimension 2 almost complexly immersed submanifold of an almost semi-Kähler manifold is minimal. We take this opportunity to also announce that 4-dimensional almost semi-Kähler manifolds are *almost Kähler* ($d\Phi = 0$), thereby generalizing the result of Gray that nearly Kähler manifolds of dimension 4 are Kähler. As a corollary, 4-dimensional complex almost semi-Kähler manifolds are Kähler. (Received November 19,1973.)

*74T-D7. SAMUEL I. GOLDBERG, University of Illinois, Urbana, Illinois 61801 and NICHOLAS C. PETRIDIS, Eastern Illinois University, Charleston, Illinois 61920. *Curvature groups.*

In an effort to prove that a compact orientable Ricci flat manifold is locally flat, a cochain complex associated with the Levi-Civita connection is introduced. Let M be a pseudo-Riemannian manifold with torsion free connection Γ and covariant differential ∇. If (i) λ is an R^n-valued tensorial p-form satisfying $\nabla^2\lambda = 0$, (ii) α is a scalar p-form, and (iii) η is the solder form, consider the set $L^p = \{(\lambda, \alpha)\}$ with operators $D^p(\lambda, \alpha) = (\nabla\lambda - \eta \wedge \alpha, d\alpha)$. The groups $H^p(M, \Gamma) = \operatorname{Ker} D^p/\operatorname{Im} D^{p-1}$, $p = 1, \cdots, n$ of the complex $L = (\bigoplus_{p=0}^{n} L^p, D^p)$ are called curvature groups. The solutions of the differential system (*) $X = c\eta$, $c = \text{const.}$ are called (homothetic) s-fields. *Theorem* 1. The $H^p(M, \Gamma)$ are isomorphic with the cohomology groups of M with coefficients in the sheaf of germs of s-fields. *Corollary* 1.1. If the Ricci tensor is nondegenerate everywhere, the $H^p(M, \Gamma)$ are trivial. *Corollary* 1.2. An Einstein space with at least one nonvanishing curvature group is flat. *Theorem* 2. The $H^p(M, \Gamma)$ of a Riemannian manifold with constant nonzero scalar curvature are isomorphic with the cohomology groups of the manifold with coefficients in the sheaf of germs of its parallel vector fields. *Theorem* 3. If (*) has solutions for some $c \neq 0$, but only the trivial solution for $c = 0$, the $H^p(M, \Gamma)$ are isomorphic with the p-dimensional de Rham groups $p = 1, \cdots, n-1$. (Received October 31, 1973.)

*74T-D8. PETER F. MAH, S. A. NAIMPALLY and JOHN H. M. WHITFIELD, Lakehead University, Thunder Bay, Ontario P7B 5E1, Canada. *Linearization of a convexity structure.* Preliminary report.

Let $(X, \mathcal{C})$ be a convexity space (Kay and Womble, Pacific J. Math 38(1971), 471–485), R the reals with the usual convexity structure and $C(X) = \{f: X \to R: f(C) \text{ is convex for each } C \in \mathcal{C}\}$. It is shown that if there is a subfamily X^* of $C(X)$ such that (i) $\bigcap \{f^{-1}f(x): f \in X^*\} = \{x\}$ for each $x \in X$ and $f(x_0) = 0$ for each $f \in X^*$, (ii) each $f \in X^*$ restricted to any line is either a bijection or a constant map, and (iii) if f and g separate any two points x and y then, restricted to the line determined by x and y, $f = \lambda g + \mu$ where $\lambda, \mu \in R$, then $(X, \mathcal{C})$ can be assigned a real linear structure. If, in addition, $\mathcal{C}$ is join-hull commutative, domain-finite and $\mathcal{C}(x, y) = \mathcal{C}(z, y)$ implies $x = z$, then the family of convex sets from the linear structure is precisely $\mathcal{C}$ and X^* is the set of all linear functionals on the linear space. (Received January 14, 1974.)

* 74T-D9. HOWARD CARY MORRIS, California Institute of Technology, Pasadena, California 91109 and 213 Pennsylvania, Shreveport, Louisiana 71105. *Unions of convexly disjoint sets.*

Let $\mathcal{C}$ be a family of subsets of X that is closed under finite intersection. $\mathcal{D}$ is a collection of convexly disjoint sets if (1) the sets of $\mathcal{D}$ are pairwise disjoint, and if (2) $\forall C \in \mathcal{C}$, with $C \subset \bigcup Y$ (over $Y \in \mathcal{D}$), $\exists Y^* \in \mathcal{D}$ such that $C \subset Y^*$. *Definition.* A unit set is a set that cannot be represented as the union of two nonvoid convexly disjoint sets. Unit sets are generalized polygonally connected sets, and are investigated as such. *Definition.* A family $\mathcal{T}$ of sets has Helly's number b if every subfamily $\mathcal{T}' \subset \mathcal{T}$ with the property (1) that every b members of $\mathcal{T}$ have a point in common also has the property (2) that every finite number of members of $\mathcal{T}'$ have a point in common. *Theorem.* Given: (1) $\mathcal{C}$ has Helly number b, (2) every set of $\mathcal{A}$ can be written as the union of convexly disjoint sets of $\mathcal{C}$, and that (3) the intersection of every m sets of $\mathcal{A}$ can be written as the union of at most k $(k \leq m)$ convexly disjoint sets of $\mathcal{C}$. *Conclusion.* $\mathcal{A}$ has Helly's number $q = \max(m + k, \psi a(b, k, 1) - 1)$ where it is known that for $b \geq 3$, $\psi a(b, k, 1) = bk + 1$ and that $2k + 1 \leq \psi a(2, k, 1) \leq 3k$. Parts of this result were first conjectured by Grünbaüm and Motzkin (Proc. Amer. Math. Soc. 12(1961), 607–613). Except for $b = 2$, the conclusion of the theorem cannot be strengthened. (Received January 4, 1974.)

74T-D10. JEANNE S. HUTCHISON, University of Alabama, Birmingham, Alabama 35294. *Subconvex sets.*

In the Euclidean plane with rectangular coordinates (x, y), the graph of a convex function defined on a closed interval of real numbers satisfies the following properties: (1) it forms with the line segment joining its endpoints the boundary of a convex set; (2) it lies below the line determined by its endpoints; (3) it lies in the strip between two lines, each of which parallels the y-axis and contains an endpoint of the graph; (4) it possesses no line segments parallel to the y-axis. In a two-dimensional topological linear space $\mathfrak{L}_2$, an arc possessing analogues of (1), (2), and (3), but not necessarily (4), is referred to as being stripwise subconvex. A set in $\mathfrak{L}_2$ is stripwise subconvex if each pair of points can be connected by a stripwise subconvex arc lying in the set. It is shown that a closed connected set S in $\mathfrak{L}_2$ is stripwise subconvex iff for each point z of S, there exists a neighborhood N of z such that each pair of points taken from $N \cap S$ can be connected by a stripwise subconvex arc lying in S. This local result extends to an arbitrary topological space $\mathfrak{L}$ in the event that S is somewhat bounded. (Received January 7, 1974.)

Logic and Foundations

74T-E8. H. JEROME KEISLER, University of Wisconsin, Madison, Wisconsin 53706. *The number of types in a first order theory.* Preliminary report.

Given a model A, $S(A)$ denotes the set of all types (maximal consistent sets of formulas in one free variable) in the theory of $(A, a)_{a \in A}$. Given a theory T, S_T is the function on infinite cardinals λ defined by $S_T(\lambda) = \mathrm{Sup}(|S(A)|: A$ is a model of T of power $\lambda)$. Let $\mathrm{ded}(\lambda)$ be $S_{DLO}(\lambda)$ where DLO is the theory of dense linear order. *Theorem.* If T is a theory with infinite models in a countable language, then S_T is one of the following six functions: λ, $\lambda + 2^{\omega}$, λ^{ω}, $\mathrm{ded}(\lambda)$, $(\mathrm{ded}(\lambda))^{\omega}$, 2^{λ}. This partially solves a problem of Shelah (Ann. Math. Logic 3 (1971), 271–362) who proved that either S_T is λ, $\lambda + 2^{\omega}$, λ^{ω}, or $\mathrm{ded}(\lambda) \leq S_T(\lambda) \leq 2^{\lambda}$. If we assume the GCH, then $\mathrm{ded}(\lambda) = 2^{\lambda}$ and the theorem reduces to Shelah's result. (Received October 26, 1973.)

74T-E9. DAN SARACINO, Yale University, New Haven, Connecticut 06520. *A counterexample in the theory of model companions.* Preliminary report.

Let T be a countable first-order theory. We consider the following questions about T. (I) If T is $\aleph_1$-categorical and has no finite models, must T have a model companion? If T is, in addition, finite forcing-complete (i.e. $T = T^f$), must T be model-complete? (II) If the class of infinitely generic structures for T is categorical in some (or all) infinite powers, must T have a model companion? *Theorem.* There exists a countable theory T such that (i) T has no finite models and is $\aleph_1$-categorical; (ii) T is forcing-complete for finite forcing, i.e. $T = T^f$; (iii) T has no model companion (i.e., in light of (ii), T is not model-complete); (iv) the class of infinitely generic structures for T is categorical in every infinite power; (v) every uncountable existentially complete structure for T is infinitely generic; (vi) there is, up to isomorphism, precisely one countable existentially complete model of T^f, and there are no uncountable e.c. models of T^f (in particular, there is just one countable finitely generic structure and there are no uncountable ones); (vii) there are precisely $\aleph_0$ isomorphism types of countable existentially complete structures for T. (Received January 7, 1974.) (Author introduced by Professor Abraham Robinson.)

74T-E10. MICHAEL D. RICE, Ohio University, Athens, Ohio 45701. *Uniformities in the descriptive theory of sets.* Preliminary report.

Let m_* denote the *measurable* operator on uniform spaces defined by Z. Frolik and the author: if uX is a uniform space, let m_*u have a basis of covers of the form $\{A_n \cap C_s^n\}$, where $\{A_n\}$ is a countable disjoint cover of members from the σ-algebra which is generated by the uniform zero sets, and $\{C_s^n\} \in u$, for each n. Define the *bianalytic* operator a_* similarly, where A_n is a bianalytic set relative to the uniform zero sets. In the

following let ρM be a complete metric space. *Theorem* 1. These are equivalent: (i) locally Baire sets are Baire sets; (ii) $m_*\rho M$ is locally fine. Furthermore, each statement implies that $m_*\rho M$ is finest in its proximity class. *Theorem* 2. $a_*\rho M$ is a measurable, locally fine space, which is finest in its proximity class, and the bianalytic sets of M are h-closed (in the sense of Z. Frolik). *Corollary.* Let $B(k) = k^{\aleph_0}$ be the Baire space on a cardinal $k > \aleph_0$; then there exist bianalytic sets in $B(k)$ which are not Borel sets. (*Note.* Certainly this corollary is known, but the author has no reference.) (Received October 29, 1973.) (Author introduced by Professor Robert L. Blair.)

*74T-E11. MICHAEL D. MORLEY, Cornell University, Ithaca, New York 14850 and ROBERT I. SOARE, University of Illinois at Chicago Circle, Chicago, Illinois 60680. *Boolean algebras, splitting theorems, and* Δ_2^0 *sets.*

For $A \subseteq \omega$, define $\mathfrak{E}_A$, the lattice (under inclusion) of r.e. sets restricted to A, to be $\{W \cap A: W \text{ r.e.}\}$. *Theorem* 1. If $A \subseteq \omega$ is infinite and in Δ_2^0 then A is hyperhyperimmune iff $\mathfrak{E}_A$ is a Boolean algebra. This answers a question of Jockusch and Soare and immediately yields (for A co-r.e.) Lachlan's remarkable characterization of bb-simple sets as those coinfinite r.e. sets H whose lattice of supersets $\mathfrak{L}(H) = \{W: W \supseteq H \ \& \ W \text{ r.e.}\}$ forms a Boolean algebra. (Note that H r.e. implies $\mathfrak{L}(H) \cong \mathfrak{E}_{\overline{H}}$.) Theorem 1 is obtained as an immediate corollary of a splitting theorem for noncomplemented elements of $\mathfrak{E}_A$, $A \in \Delta_2^0$, which simultaneously generalizes well-known splitting theorems of Sacks and Owings although its proof is no harder than existing proofs of either. Combining Theorem 1 with results of Cooper and Jockusch, we obtain the following structural characterization of degrees $\underset{\sim}{a} \leq \underset{\sim}{0}'$ which are *high* ($\underset{\sim}{a}' = \underset{\sim}{0}''$). *Theorem* 2. For $\underset{\sim}{a} \leq \underset{\sim}{0}'$, $\underset{\sim}{a}' = \underset{\sim}{0}''$ just if $\mathfrak{E}_A$ is a Boolean algebra for some infinite set $A \in \underset{\sim}{a}$. This contrasts with the case of *low* degrees where the second author has shown that if $\underset{\sim}{a}' = \underset{\sim}{0}'$ then $\mathfrak{E}_A \cong \mathfrak{E}$ for every infinite $A \in \underset{\sim}{a}$. (Received October 31, 1973.)

74T-E12. MELVIN C. FITTING, Lehman College, City University of New York, Bronx, New York 10468. *A recursive function theory for the finite sets.* Preliminary report.

For terminology of elementary formal systems, see R. M. Smullyan, "Theory of formal systems", Princeton University Press, 1961. Let W be the set of words (finite strings) formed from the two symbols, { and }. Certain words in W 'name' sets in a natural way. For example, { } names $\varnothing$, and {{ }} names $\{\varnothing\}$. This naming can be properly defined as a partial function on W; it is not necessary to be more precise in this note however. If w is a 'set-naming' word, let $\overline{w}$ be the set w names. If S is a collection of set-naming words, let $\overline{S}$ be the result of replacing each $w \in S$ by $\overline{w}$. The following has a straightforward but tedious proof. *Theorem.* If S is a subset of W consisting of set-naming words, then S is formally representable in an elementary formal system over W iff $\overline{S}$ is a Σ definable subset of HF, the hereditarily finite sets. (Received October 31, 1973.)

74T-E13. JÖRG FLUM, University of Freiburg, 78 Freiburg, Federal Republic of Germany. *Generalized quantifiers and reduced products.* Preliminary report.

Let L be a first order language, $L(Q)$ the language obtained from L by adding the quantifier "there exist at least $2^{\aleph_0}$ many". *Theorem* 1. Suppose D is an ω-regular filter over a set I, and $\mathfrak{A}_i \equiv \mathfrak{B}_i$ for $i \in I$. Then $\Pi_I \mathfrak{A}_i/D \equiv_{L(Q)} \Pi \mathfrak{B}_i/D$. *Corollary* 1. $L(Q)$-equivalence is preserved by ultraproducts. *Corollary* 2. Reduced products with a countable index set preserve $L(Q)$-equivalence. Call a sentence ϕ of $L(Q)$ product-stable, if whenever $\mathfrak{A} \equiv_{L(Q)} \mathfrak{B}_i$, then $\Pi_I \mathfrak{A}_i \models \phi$ iff $\Pi_I \mathfrak{B}_i \models \phi$. *Theorem* 2. Assume $2^{\aleph_0}$ is singular. There are formulas of the form $Qx \neg Qy \phi_0$ and $Qx \forall y \phi_1$, ϕ_0 and ϕ_1 quantifier-free, which are not product-stable. Every formula of the form $Qx_1 \exists y_1 \cdots Qx_n \exists y_n \phi$, where ϕ is quantifier-free is product-stable. There are generalizations to other cardinals. (Received November 1, 1973.) (Author introduced by Dr. Michael Richter.)

74T-E14. STEPHEN WHITNEY, Université Laval, Québec, Québec G1K 7P4, Canada. *Model classes of linear theories.* Preliminary report.

Consider a first-order language L with equality as the only relation symbol. A term of L is *linear* iff each of its variables occurs only once. An identity $t_1 = t_2$ of L is *semilinear* iff t_1 and t_2 are linear. If,

in addition, the variables of t_1 and t_2 are the same, then the identity is *linear*. In 1957, Gautam (Arch. Math. Logik Grundlagenforsch. 3(1957), 117–124) proved: $t_1 = t_2$ is linear iff $t_1 = t_2$ true in $A \Rightarrow t_1 = t_2$ true in $P(A)$ for all algebras A, where $P(A)$ is the naturally-defined powerset algebra of A. More generally, let us call a class K of algebras *P-closed* if $A \in K \Rightarrow P(A) \in K$, and *$\hat{P}$-closed* if $A \in K \Rightarrow \hat{P}(A) \in K$ where $\hat{P}(A)$ is the set of nonnull subsets of A. We here prove: (1) Any theory T in L is equivalent to a theory consisting of linear identities iff the model class T^* of T is primitive and P-closed. (2) Any theory T of L is equivalent to a theory consisting of semilinear identities iff T^* is primitive and $\hat{P}$-closed. (Received November 2, 1973.) (Author introduced by Professor William S. Hatcher.)

74T-E15. EGON BÖRGER, Universita de Salerno, Istituto di Scienze dell'Informazione 84100 Salerno, Italy and KLAUS HEIDLER, University of Freiburg, 78 Freiburg, Federal Republic of Germany. *The m-degrees of logical decision problems.* Preliminary report.

For a class F of first-order formulae let the decision problem for deducibility (satisfiability, *sf*) of F be the set $A(F)$ of all deducible ($E(F)$ of all satisfiable) formulae in F. *Theorem.* The decision problems for ded (*sf*) of recursive classes of formulae are up to m-equivalence and in an effective way the same as the initialized halting (immortality) problems of 2-register machines; this holds up to recursive isomorphy for classes F with cylindrical decision problem. From this theorem one obtains a solution to a problem formulated by Hao Wang (Proc. Sympos. Math. Theory of Automata, 1962, p. 54): *Corollary* 1. There is an effective procedure which associates to every r.e. m-degree $d \neq \{N\}, \{\varnothing\}$ a very simple closed Krom– and Horn-formula $AxAyf$ with quantifier-free f such that the class $F_f = AxAyf \wedge Pf(K, 0, |)$ satisfies $A(\neg F_f) \in d$ and $E(F_f) \in \overline{d}$ where $Pf(K, 0, |)$ is the set of all prime formulae over the binary predicate symbol K, the individual symbol 0, and the monadic functional symbol $|$. Corollary 1 cannot hold for 1-degrees. *Corollary* 2. $\lambda x(A(W_x) = \varnothing)$ is Π_1-complete, $\lambda x(A(W_x)$ infinite), $\lambda x(W_x \subseteq A(W_x))$ are Π_2-complete, $\lambda x(W_x - A(W_x)$ finite), $\lambda x(A(W_x)$ decidable) are Σ_3-complete. (Received November 12, 1973.)

*74T-E16. MICHAEL MAKKAI, Université de Montréal, Montréal 101, Quebec, Canada. *Applications of a theorem on weak definability in $L_{\omega_1\omega}$.* II.

The following improves Abstract 711–02–11, these Notices, 21(1974), A-21. The theorem of the title is Theorem 1 of the author's "Global definability theory in $L_{\omega_1\omega}$", Bull. Amer. Math. Soc. 79(1973), 916–921, in the full "admissible" version and improved by J. Barwise as described at the end of loc. cit. For any ordinal α and any given structure M, let $t^\alpha(M)$ be the set $\{\phi^\alpha_{a_1,\ldots,a_n} : n < \omega, a_1, \cdots, a_n \in M\}$ where the formulas $\phi^\alpha_{a_1,\ldots,a_n}$ are defined on the basis of M in Chang's proof of Scott's isomorphism theorem as given in Keisler's book. Let A be a countable admissable set, let $\omega \in A$, let T be a theory in $L_A(\vec{R})$ with the language $L(\vec{R}) \in A$, let T be Σ over A, and let K be the class of countable models of $\exists \vec{R} \wedge T$. *Theorem* 1. Suppose that K has $< 2^{\aleph_0}$ isomorphism types of models. Then for all $\alpha \in \mathrm{Ord}_A$ and for all models M in K, $t^\alpha(M)$ is an element of A. *Application.* Any Σ-over-HC set of power $< 2^{\aleph_0}$ is a subset of the constructible universe (result due to R. Mansfield). *Theorem* 2. Suppose, in addition to Theorem 1, that M is in K and there is $\alpha \in \mathrm{Ord}_A$ such that M has a Scott sentence of quantifier rank α. Then the canonical Scott sentence of M belongs to A and has quantifier rank $\leq \alpha + \omega \cdot 2$. Theorem 2 generalizes unpublished results of M. Nadel and G. Sacks. (Received November 12, 1973.) (Author introduced by Dr. Gonzalo Reyes.)

74T-E17. SAHARON SHELAH, Hebrew University, Jerusalem, Israel. *Uniqueness of prime models, with an application to differential fields.* Preliminary report.

For simplicity we restrict ourselves to (complete) countable theory T, and to elementary submodels of a saturated model M_0 of T of large cardinality. *Definition.* A prime model over $A \subseteq |M_0|$ ($|A| < \|M_0\|$) is a model $N \prec M_0, A \subseteq |N|$, such that if $N' \prec M_0$, $A \subseteq N'$ then there is an automorphism of M_0, mapping N' into N which is the identity over A. *Theorem* 1. Suppose (A) there is a prime model over every $A \subseteq |M_0|$; (B) T is stable. *Then* the prime model from (A) is unique up to isomorphism over A. *Theorem* 2. If in Theorem 1 we assume in addition that T is superstable, *then* N is prime over A iff (1) every finite sequence from N realizes over A an isolated

type; (2) in N, there is no uncountable indiscernible set over A. From Theorem 1 we have: *Conclusion.* The prime differentially closed field over any perfect field is unique. (This is new for characteristic $p > 0$.) The satisfaction of (A) was proved by Wood and the author (independently); the satisfaction of (B) by the author; those proofs will soon appear in Israel J. Math. There are generalizations and strengthening of 1,2. (Received November 19, 1973.)

74T-E18. WILLIAMS K. FORREST, Simon Fraser University, Burnaby 2, British Columbia V5A 1S6, Canada. *Omitting open types in universal classes with amalgamation.* Preliminary report.

Let Σ be the class of all models of a countable universal theory and assume that Σ has the amalgamation property. If $C \in \Sigma$ we define $S_A(C)$ to be the set of open types over C. Suppose B, $C \in \Sigma$ with $B \subseteq C$ and $X \subseteq C \sim B$. Then X is L_A-indiscernible over B if for each $n < \omega$ all n-tuples of distinct elements in X realize the same open type over B. Finally, Σ is L_A-$(\aleph_0)$-stable if $\forall C \in \Sigma(|S_A(C)| \leq |C| + \aleph_0)$. *Theorem* 1. Suppose Σ is L_A – $(\aleph_0)$-stable and A, $B \in \Sigma$ with $A \subseteq B$. If $|A| + \aleph_0 < |B|$ then for each regular $\lambda \leq |B|$ there is a subset X of B of cardinality λ such that X is L_A-indiscernible over A. *Theorem* 2. Suppose Σ is L_A–$(\aleph_0)$-stable and A, $B \in \Sigma$ with $A \subseteq B$. If $|A| + \aleph_0 < |B|$ and B omits $\Gamma \in S_A(A)$ then for each $\lambda \geq |A| + \aleph_0$ there is $C \in \Sigma$ of power λ such that $C \geq A$ and C omits Γ. (Received November 23, 1973.) (Author introduced by Professor Alistar H. Lachlan.)

74T-E19. JOHN P. BURGESS, University of California, Berkeley, California 94720. *A new absolute logic.* Preliminary report.

See Barwise, Ann. Math. Logic 4(1972), 309–340, for the notion of an absolute logic. He notes that the $L(\omega)$ of Keisler (Springer Lecture Notes in Math. 72(1968), 96–130) is not absolute. Let K be the countable part of $L(\omega)$, so the formation rules of K are those of $L_{\omega_1\omega}$ with the addition that if Φ is in K so is $\forall x_0 \exists y_0 \forall x_1 \exists y_1 \cdots \Phi$, *provided it has only finitely many free variables.* Let GBD be the proposition $(\forall X)$ (Borel games over X^ω are determined). GBD follows from large cardinals, and possibly from ZFC. *Theorem.* K is absolute relative to ZFC + GBD. Absoluteness implies the Löwenheim-Skolem theorem, which can also be established by a simple direct argument. Using special cases of GBD due to Morton, Davis and Paris, one can show extensive sublanguages of K are absolute relative to ZFC alone. (Received November 26, 1973.) (Author introduced by Professor Robert L. Vaught.)

*74T-E20. DOUGLAS E. MILLER, University of California, Berkeley, California 94720. *Remarks on the transform* $B^* = \{x: \{g: gx \in B\}$ *is comeager*$\}$.

X is a topological space, G a nonmeager topological group with a countable basis. G acts on X according to a map $(g, x) \mapsto gx$ mapping $(G \times X) \to X$ which is continuous in each variable separately. The transform $B \mapsto B^*$ defined in the title was introduced by R. L. Vaught in "A Borel invariantization", Bull. Amer. Math. Soc., to appear. $B^\Delta = \sim(\sim B)^*$. *Theorem* 1. (a) If B is meager, so are B^Δ and B^*. (b) If B has the Baire property, so do B^Δ and B^*. *Theorem* 2. If (B_0, B_1) reduces (A_0, A_1), then (B_0^Δ, B_1^*) reduces (A_0^Δ, A_1^*). Theorem 2 yields a simple proof of Vaught's theorem: If X is Polish, then the class of G-invariant Π_1^1 sets has the reduction property. (Received November 26, 1973.) (Author introduced by Professor Robert L. Vaught.)

*74T-E21. JOHN P. BURGESS and DOUGLAS E. MILLER, University of California, Berkeley, California 94720. *Invariant reduction and uniformization in the projective and analytical hierarchies.*

Let X be a Polish space (resp., let $X = \omega^\omega$). Let $E \subseteq X \times X$ be a $\underset{\sim}{\Sigma}^1_1$ (Σ^1_1) equivalence relation on X. $B \subseteq X$ is E-invariant if $y \in B$ whenever xEy and $x \in B$. *Theorem* 1. (a) The class of E-invariant $\underset{\sim}{\Pi}^1_1$ (Π^1_1) subsets of X has the reduction property; the same holds for $\underset{\sim}{\Sigma}^1_2$ (Σ^1_2). (b) Assuming projective determinacy, the same holds for $\underset{\sim}{\Pi}^1_{2n+1}$ (Π^1_{2n+1}) and $\underset{\sim}{\Sigma}^1_{2n}$ (Σ^1_{2n}), $n \geq 1$; assuming $V = L$ or $V = L^D$, the same holds for $\underset{\sim}{\Sigma}^1_n$ (Σ^1_n), $n \geq 3$. Theorem 1 extends the action-theoretic and game-theoretic results of Vaught and Moschovakis. Even in the previously known cases the proofs are simpler. Form the equivalence E' on $X \times X$ by setting $(x_0, x_1)E'(y_0, y_1)$ if $x_0 E y_0$ and $x_1 E y_1$. A class K of subsets of $X \times X$ satisfies the *invariant uniformization principle* if every E'-

invariant $B \in K$ has an E'-invariant subset $C \in K$ such that (i) $\forall x(\exists y(x, y) \in B \to \exists y(x, y) \in C)$, and (ii) $(x, y_0) \in C$ & $(x, y_1) \in C \to y_0 E y_1$. *Theorem* 2. If $V = L$ and $n \geq 2$, or $V = L^D$ and $n \geq 3$, then the Σ^1_n (Σ^1_n) subsets of X satisfy the invariant uniformization principle. (Received November 26, 1973.) (Authors introduced by Professor Robert L. Vaught.)

*74T-E22. ALAN L. SELMAN, Florida State University, Tallahassee, Florida 32306. *A note on spectra.*

Let Q be the set of Fermat primes. Let $Q' = \{s | 2^s + 1$ belongs to $Q\}$. J. Bennett ["On spectra", Doctoral Dissertation, Princeton University, 1962] showed that Q is a spectrum and that the complement of Q' is a spectrum, but left open the question of whether Q' is a spectrum. *Theorem.* Q' is a spectrum. The proof used V. Pratt's nondeterministic polynomial time algorithm for recognizing prime numbers to show that Q belongs to *NP*. The Theorem follows then from N. Jones and A. Selman ["Turing machines and the spectra of first-order formulas with equality, J. Symbolic Logic, to appear]. (Received November 30, 1973.)

*74T-E23. LESLIE H. THARP, Rockefeller University, New York, New York 10021. *Necessity, apriority and provability.*

Theorems 1–5 are in the framework of Kripke's "Naming and necessity" in Semantics of Natural Language, Harman and Davidson (eds.), D. Reidel, Dordrecht, Holland, 1972. Let Λ be "is a priori" and let $\Box$ be "is necessary." *Theorem* 1. $\forall\phi\exists\psi[\Delta(\phi\leftrightarrow\psi)\wedge(\Box\psi\vee\Box\neg\psi)]$. *Theorem* 2. $\forall\phi\exists\psi[\Box(\phi\leftrightarrow\psi)\wedge(\Delta\psi\vee\Lambda\neg\psi)]$. *Theorem* 3. For certain ϕ there is no ψ such that $[\Box(\phi\leftrightarrow\psi)\wedge(\Delta\psi\vee\Delta\neg\psi)]$ is provable. Let π be a fixed contingent a priori truth, and σ a fixed necessary a posteriori truth; let C be "is a contingent truth" and S be "is an a posteriori truth." Without introducing names one has *Theorem* 4. $\forall\phi\exists\psi[\Delta(\phi\leftrightarrow\psi)\wedge(C\psi\vee C\neg\psi)]$ and *Theorem* 5. $\forall\phi\exists\psi[\Box(\phi\leftrightarrow\psi)\wedge(S\psi\vee S\neg\psi)]$. Interpret '$B$' as provability; let T have as axioms all theorems of the modal logic M (changing $\Box$ to B), and all instances of $\phi \to \neg B\neg B\phi$, with modus ponens as the rule of inference. *Theorem* 6. $T \vdash \phi \to B\phi$. *Theorem* 7. There is an S4 model satisfying $P\wedge\neg BP$ and $B(\alpha \to \neg B\neg B\alpha)$ for all α not containing 'B'. (Received December 6, 1973.)

74T-E24. JOHN STEEL and JOHN P. BURGESS, University of California, Berkeley, California 94720. *On a theorem of Makkai.* Preliminary report.

Notation is as in Makkai, "Vaught sentences and Lindström's regular relations", Proc. 1971 Summer School in Logic, Cambridge, to appear. So p is a sequence of $\exists s$ and $\forall s$; Γ a set of formulas; $G(p, \Gamma)$, $R(G)$, $F_{\infty\omega}(G)$ the associated game, regular relation, class of formulas; p', Γ' Makkai's modification of p, Γ; $G' = G(p', \Gamma')$, $R'(G) = R(G')$, $F'_{\infty\omega}(G) = F_{\infty\omega}(G')$. Let S_κ = (sentences of $L_{\kappa\omega}$)$\cap F_{\infty\omega}(G)$, and S'_κ similarly. The theorem of the title states: For $\kappa = \omega$ or ω_1, a sentence of $L_{\kappa\omega}$ is preserved under $R(G)$ iff equivalent to one in S'_κ. Makkai wonders if S_κ can replace S'_κ, i.e. if every sentence in S'_κ is equivalent to one in S_κ. (Converse is trivial.) *Theorem* 1. If κ inaccessible, e.g. $= \omega$ or ∞, every sentence in S'_κ is equivalent to one in S_κ. (This yields, *via* a theorem of Lindström relating $R(G)$ and S_∞, a new proof of $R'(G) = R(G)$.) *Theorem* 2. If p is any sequence with $P_0 = \exists$, and $\Gamma = \{P_n(v_0)$: $n \in \omega\}\cup\{\sim P_n(v_0)$: $n \in \omega\}$, then $\exists v_0\exists v_1\wedge_{n\in\omega}(P_n(v_0)\leftrightarrow\sim P_n(v_1)) \in S'_{\omega_1}$ is not equivalent to any sentence in S_{ω_1}. (Received December 6, 1973.) (Authors introduced by Stephen G. Simpson.)

*74T-E25. WITHDRAWN

*74T-E26. STEVEN K. THOMASON, Simon Fraser University, Burnaby 2, British Columbia, Canada. *Categories of frames for modal logic.*

Consider the categories $A+$ and $A\#$ whose objects are all complete atomic modal algebras (CAMA's) and whose morphisms are all complete homomorphisms and all homomorphisms respectively. Categories $W+$ and $W\#$ are defined, whose objects are all frames (W, R) $(R \subseteq W \times W \neq \varnothing)$ and which are dual to $A+$ and $A\#$ respectively. Closure of equational classes of CAMA's under complete subalgebras, complete homomorphic images, and direct

products corresponds to closure of axiomatic (in the sense of propositional modal logic) classes of frames under p-morphic images, terminal subframes, and disjoint unions, respectively. The conditions on axiomatic classes of frames corresponding to the closure of equational classes of CAMA's under subalgebras and homorphic images are investigated. *Application.* The class of frames satisfying $(\forall x)(\exists y)(Rxy \,\&\, Ryy)$ (or equivalently $\Diamond(\forall p)(p \to \Diamond p)$) is closed under p-morphic images, terminal subframes, and disjoint unions, but not under the operation dual to subalgebras, and hence is not an axiomatic class. (Received December 17, 1973.)

*74T-E27. FRED. G. ABRAMSON, LEO A. HARRINGTON, EUGENE M. KLEINBERG and WILLIAM S. ZWICKER, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. *Flipping properties: A unified approach to the study of certain large cardinals.*I.

Let κ be a cardinal and $t\colon \delta \to 2^{\kappa}$ be a δ-sequence of subsets of κ. We call t' a flip of t if for every $\alpha < \delta$, $t'(\alpha)$ is either $t(\alpha)$ or its complement relative to κ. A flipping property on κ is a combinatorial assertion to the effect that every sequence of subsets of κ of a particular sort can be flipped so that the resulting sequence has a large "intersection". For example, κ is strongly inaccessible if and only if every shorter-than-κ sequence of subsets of κ has a flip whose intersection has cardinality κ, and *Theorem.* κ is ineffable if and only if every κ-sequence t has a flip t' such that any rearrangement of t has diagonal intersection of cardinality κ. Flipping properties of successively greater uniformity are similarly used to characterize the following successively stronger cardinals: strongly inaccessibles, weakly compacts, weakly ineffables, ineffables, completely ineffables, and measurables. One can obtain weak ultrafilters from flipping properties and use them to build weak ultraproducts which suffice to prove indescribability results. Flipping techniques often simplify proofs of old results and, further, prove new indescribability theorems. Finally various types of forcing inspired by flipping are shown to lead naturally to the existence of various types of M-ultrafilters. (Received December 17, 1973.)

*74T-E28. FRED G. ABRAMSON, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 and GERALD E. SACKS, Harvard University, Cambridge, Massachusetts 02138. *On uncountable α-recursive wellorderings of α.*

A Σ_1-admissible ordinal α is called a *Gandy ordinal* if the supremum of the order types of α-recursive wellorderings of α is α^+ (= least admissible ordinal greater than α). *Theorem* I. Let $\alpha = (\aleph_{\omega}^{+})^{L}$. Then α is a Gandy ordinal. This theorem refutes Gostanian's conjecture that there are no uncountable Gandy ordinals, and adds new examples of countable Gandy ordinals to those previously discovered by H. Friedman. The key to the proof of Theorem I is to replace "countable" in Friedman's technique with "cofinality ω". Theorem I is a special case of Theorem II, which gives many more examples, e.g., the first recursively inaccessible greater than $(\aleph_{\omega})^{L}$, the first nonprojectable greater than $(\aleph_{\aleph_{\omega}})^{L}$. *Theorem* II. Let $\aleph$ be a cardinal of L, α an admissible greater than $\aleph$ such that α is the constructibly countable union of sets of α-cardinality $\aleph$, and such that $L_{\alpha} \models \aleph$ has cofinality ω. Suppose also that there is a sentence S (containing no parameter greater than $\aleph$) such that $\alpha = \text{least } \beta\ [L_{\beta} \models S]$. Then α is a Gandy ordinal. (Received December 21, 1973.)

*74T-E29. WITHDRAWN

74T-E30. JOHANN ANDREAS MAKOWSKY, Department of Philosophy, Stanford University, Stanford, California 94305 and Swiss Federal Institute of Technology, 8006 Zurich, Switzerland. *More about continuous quantifiers.* Preliminary report.

This note is a continuation of Abstract 73T-E84, these Notices 20(1973), A-502. *Definitions.* A formula of $L_{\omega_1\omega}$ is: 1. Σ if it is equivalent to a formula which in negational normal form has only existential quantifiers; 2. $\forall'\Sigma$ if it is equivalent to a countable conjunction of universal closures of Σ-formulas. 3. $\wedge\vee\forall\exists(\vee,\wedge\forall\exists)$ if it is equivalent to a countable conjunction (disjunction) of a countable disjunction (conjunction) of $\forall\exists$-formulas of $L_{\omega\omega}$. *Theorem* 1. An ω-continuous quantifier, Q, is definable in $L_{\omega_1\omega}$ by a sentence ϕ $\ni$ both ϕ and $\neg\phi$ are in $\bigwedge\bigvee\forall\exists\cap\bigvee\bigwedge\forall\exists\cap\forall'\Sigma$. *Theorem* 2. Let Q be an ω- or ω_1-continuous monadic quantifier. Then there is a smallest logic L extending $L_{\omega\omega}[Q]$, which satisfies the interpolation theorem,

and L is of the form L_A where A is a countable admissible set. *Theorem* 3. If L is a k-continuous logic, k regular, and $\{\mathfrak{A}_\alpha\}_{\alpha<k}$ is a L-elementary proper chain, then $\bigcup_{\alpha<k}\mathfrak{A}_\alpha$ is a L-elementary extension of all the $\mathfrak{A}_\alpha$. *Theorem* 4. A quantifier Q is k-continuous, for k regular, iff $L_{\omega\omega}[Q]$ satisfies the Lowenheim-Skolem-Tarski theorem down to $< k$ and both the definable class of Q and its complement are closed under proper submodel chains of length k. *Corollary* 5. A quantifier Q is ω-continuous iff it is definable by a sentence ϕ in $L_{\omega_1\omega}$ ∋ both ϕ and $\neg\phi$ are $\forall'\Sigma$. (Received December 17, 1973.)

74T-E31. JOHN E. HUTCHINSON, Stanford University, Stanford, California 94305 and State University of New York at Buffalo, Amherst, New York 14226. *Extending countable models of set theory.* Preliminary report.

Let $\mathfrak{A} = \langle A, E\rangle$ be a countable model of Z.F.C., c a regular cardinal inside $\mathfrak{A}$. Keisler and Morley (Israel J. Math. 6(1968), 49–65) show that there exists a countable $\mathfrak{B} > \mathfrak{A}$ such that c is enlarged and each $a \in c_E$ remains fixed. This result may be strengthened to show either (i) $\mathfrak{B}$ contains no least new ordinal, or (ii) if $\mathfrak{A} \vDash c \neq \omega$ then $\mathfrak{B}$ does contain a least new ordinal. The proof of (i) is similar to techniques used by Keisler and Morley; the omitting types condition is used to show the existence of a model of $\mathrm{Th}(\mathfrak{A}_A) \cup \{k_m E k_n: n < m < \omega\} \cup \{k_0 E c\}$ omitting $\{vEk_n: n < \omega\} \cup \{v \neq a: a \in c_E\}$. For (ii), a model of $\mathrm{Th}(\mathfrak{A}_A) \cup \{kEs: \mathfrak{A}_A \vDash$ "s is a closed unbounded subset of c"$\}$ omitting $\{vEk\} \cup \{v \neq a: a \in c_E\}$ is shown to exist, using the result that a regressive function on a stationary subset of a regular cardinal $(\neq\omega)$ is constant on a stationary subset. The results apply to some weaker theories than Z.F.C. and, under stronger conditions, to elementary end extensions. *Applications.* Order types of ordinals and to definability problems in infinitary logics, to be discussed in future communications. (Received December 20, 1973.) (Author introduced by Professor John Gregory.)

74T-E32. LEE W. BADGER, University of Colorado, Boulder, Colorado 80302. *Beth's theorem fails in* $L^{<\omega}$. Preliminary report.

Consider the language $L^{<\omega}$ (introduced by Magidor and Malitz, Abstract 73T-E79, these Notices 20(1973), A-501). Let L^1 be the language obtained when only the quantifier Q^1 is added to elementary logic. *Theorem.* There is a predicate implicitly defined by a sentence of L^1 having no explicit definition in $L^{<\omega}$. (Received January 7, 1974.)

74T-E33. THOMAS J. JECH, State University of New York at Buffalo, Amherst, New York 14226. *A descending sequence of models of* HOD *sets.* Preliminary report.

HOD is the class of all hereditarily ordinal definable sets. Let Θ be an ordinal. There is a generic extension M of L, and a descending Θ-sequence of models M_α of ZFC, $\alpha < \Theta$, such that: (i) $M_0 = M$, (ii) $M_{\alpha+1} = (\mathrm{HOD})^{M_\alpha}$, and (iii) $M_\lambda = \bigcap_{\alpha<\lambda} M_\alpha$ if λ is limit. (Received January 7, 1974.)

74T-E34. RONALD FAGIN, IBM T. J. Watson Research Center, Yorktown Heights, New York 10598. *$L_{\kappa,\lambda}$-equivalence of ordinals and a compactness result.*

Throughout $\kappa \geq \lambda \geq \omega$, κ regular; α, β, γ are ordinals. *Definitions.* 1. Write α uniquely as $\gamma\alpha_1 + \alpha_2$, $\alpha_2 < \gamma$. Then $r_\gamma(\alpha)$ and $c_\gamma(\alpha)$ are defined to be α_2 and cofinality $(\gamma\alpha_1)$, respectively. Further, $\alpha \sim \beta \bmod \gamma$, if (1) $r_\gamma(\alpha) = r_\gamma(\beta)$, and (2) $\alpha < \gamma$ iff $\beta < \gamma$. 2. $\mathfrak{A} \equiv \mathfrak{B}$ means that $\mathfrak{A}$ and $\mathfrak{B}$ agree on $L_{\kappa,\lambda}$-sentences. *Theorem.* 1. If λ is a successor or ω, $\kappa = \lambda$, then $\langle\alpha; \epsilon\rangle \equiv_{\kappa,\lambda} \langle\beta; \epsilon\rangle$ iff (1) $\alpha \sim \beta \bmod \kappa^\kappa$, and (2) $c_{\kappa^\kappa}(\alpha)$ and $c_{\kappa^\kappa}(\beta)$ are equal, or both at least λ. 2. If λ is regular, $\kappa > \lambda$, then $\langle\alpha; \epsilon\rangle \equiv_{\kappa,\lambda} \langle\beta; \epsilon\rangle$ iff (1) $\alpha \sim \beta \bmod \kappa$, and (2) $c_\kappa(\alpha)$ and $c_\kappa(\beta)$ are equal, or both at least λ. 3. If λ is weakly inaccessible, $\kappa = \lambda$, then $\langle\alpha; \epsilon\rangle \equiv_{\kappa,\lambda} \langle\beta; \epsilon\rangle$ iff (1) $\alpha \sim \beta \bmod \kappa$, and (2) $c_\kappa(\alpha)$ and $c_\kappa(\beta)$ are equal, or both at least λ. 4. If λ is singular, $\kappa = \lambda^+$, then $\langle\alpha; \epsilon\rangle \equiv_{\kappa,\lambda} \langle\beta; \epsilon\rangle$ iff (1) $\alpha \sim \beta \bmod \kappa^\kappa$, and (2) $c_{\kappa^\kappa}(\alpha)$ and $c_{\kappa^\kappa}(\beta)$ are equal, or both at least λ^+. 5. If λ is singular, $\kappa > \lambda^+$, then $\langle\alpha; \epsilon\rangle \equiv_{\kappa,\lambda} \langle\beta; \epsilon\rangle$ iff (1) $\alpha \sim \beta \bmod \kappa$, and (2) $c_\kappa(\alpha)$ and $c_\kappa(\beta)$ are equal, or both at least λ^+. □ The theorem yields a complete characterization of ordinals in $L_{\kappa\lambda}$ (using only $<$ and $=$), from which follows *Corollary.* Assume $\kappa > \lambda$, or κ is weakly inaccessible; $\kappa = \lambda$. Let Σ be a set of $L_{\kappa\lambda}$ sentences involving only $<$ and $=$, ∋ every subset of Σ having less than κ sentences has a well-ordered model. Then Σ has a well-ordered model. (Received January 7, 1974.)

Statistics and Probability

74T-F4. DAVID LOUIS TANNY, Cornell University, Ithaca, New York 14850. *On multi-type branching processes in random environments* (MBPRE). Preliminary report.

Let $Z_n(i)$ = the number of particles of type i in the nth generation in a MBPRE, $1 \le i \le r$, $n \ge 0$. Each of these particles produces offspring according to the probability distribution $\xi_n(i)$; the expected number of j children of this distribution is $M_n(j, i)$. The "environmental sequence" $\bar{\xi} = (\xi_0(\cdot), \xi_1(\cdot), \cdots)$ is assumed to be a stationary ergodic process. It is known that if $E(\log^+ M(j, i)) < \infty$, then $E = \lim_{n\to\infty} (1/n) \log \Sigma_{1\le i,j\le r} (M_n \cdots M_0) \cdot (i, j)$ exists and $E < \infty$ w.p.1. $\|Z_n\| =_{DEF} \Sigma_{i=1}^r Z_n(i)$. *Theorem* 1. Under regularity conditions, $P(\|Z_n\| \to 0$ or $\|Z_n\| \to \infty$ as $n \to \infty | \bar{\xi}) = 1$ w.p.1. *Theorem* 2. If $E < 1$, then $Z_n(\cdot) = 0$ eventually w.p.1. The same holds for $E = 1$ under regularity conditions. If $E > 1$ then w.p.1. either $\|Z_n\| = 0$ eventually or $\lim_{n\to\infty} (1/n) \log \|Z_n\| > 0$. Under irreducibility conditions, the last limit equals E for every starting vector $Z_0(\cdot)$. This simplifies results of Kaplan [Thesis, Stanford University, 1970]. (Received December 5, 1973.)

*74T-F5. R. V. CHACON, University of Minnesota, Minneapolis, Minnesota 55455. *A 'stopped' proof of convergence.*

A short and elementary proof of the following theorem is given: *Theorem.* If $\{x_n\}$ is a sequence of random variables with either $\lim\sup_{\tau\in\Gamma} \int x_\tau^+ < +\infty$ or $\lim\inf_{\tau\in\Gamma} \int x_\tau^- > -\infty$ then for any two bounded random variables y_1 and y_2 with $x_* \le y_1 \le y_2 \le x^*$ we have $\lim\sup_{\sigma,\tau\in\Gamma} \int (x_\sigma - x_\tau) \ge \int (y_2 - y_1)$, where Γ is the class of bounded stopping times, $x^* = \lim\sup x_n$ and $x_* = \lim\inf x_n$. If $\{x_n\}$ has the property that $\lim\sup_{\tau\in\Gamma} \int |x_\tau| < +\infty$, we obtain as a direct consequence that x_* and x^* are integrable and that $\lim\sup_{\sigma,\tau\in\Gamma} \int (x_\sigma - x_\tau) \ge \int (x^* - x_*)$. The martingale convergence theorem is an immediate consequence and the result also admits of extension to infinite measure spaces. (Received December 27, 1973.) (Author introduced by Professor Naresh C. Jain.)

Topology

*74T-G13. R. VASUDEVAN and C. K. GOEL, Institute of Advanced Studies, Meerut University, Meerut 250001, India. *Some more separation axioms for bitopological hyperspaces.* Preliminary report.

Given a bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$, the natural bitopological hyperspace $(2^X, 2^{\mathcal{T}_1}, 2^{\mathcal{T}_2})$, where 2^X stands for the collection of all nonempty $\mathcal{T}_1$-closed or $\mathcal{T}_2$-closed sets including the singletons, $2^{\mathcal{T}_i}$, $i = 1, 2$, stands for the Vietoris or finite topology as defined by Kuratowski ["Topology. I", Academic Press, New York, 1966, p. 160] is considered. In this paper pairwise normal, pairwise seminormal, pairwise almost normal, pairwise regular, pairwise semiregular and pairwise almost regular separation axioms are investigated. It is found that if $(2^X, 2^{\mathcal{T}_1}, \mathbf{2}^{\mathcal{T}_2})$ is pairwise almost regular or pairwise semiregular then $(X, \mathcal{T}_1, \mathcal{T}_2)$ is pairwise almost normal or pairwise seminormal respectively. Also it is observed that in a bi-T_1 bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ pairwise regularity, pairwise semiregularity and pairwise almost regularity are respectively equivalent to the pairwise normality and pairwise almost normality of $(BC(X), 2^{\mathcal{T}_1}, 2^{\mathcal{T}_2})$ where $BC(X)$ is the space of all biclosed sets (i.e. sets which are closed with respect to $\mathcal{T}_1$ as well as $\mathcal{T}_2$). (Received September 27, 1973.) (Authors introduced by Professor J. N. Kapur.)

74T-G14. JAMES F. CHEW, Michigan State University, East Lansing, Michigan 48823. *Invertibility of function spaces.*

This paper points out a gap in the proof of each of two theorems by Naimpally, Amer. Math. Monthly 73 (1963), 513–515. A topological space X is *homogeneously invertible* if given a nonempty open set U, $a \in U$ and $b \in X - U$, there exists a homeomorphism h of X onto X such that $h(b) = a$ and $h(X - U) \subset U$. Let F denote the collection of all self maps of a topological space X. The compact-open tolopogy will be denoted by $\mathcal{C}$, the

point-open topology by $\mathcal{P}$ and the subset of F consisting of constant maps by M. *Theorem* 1. $(F, \mathcal{C})$ is sub-near-homogeneous if X is a noncompact weakly invertible space. *Theorem* 2. $(F, \mathcal{C})$ is near-homogeneous at points of M if X is a noncompact weakly invertible space. *Theorem* 3. $(F, \mathcal{P})$ is sub-near-homogeneous if X is a near-homogeneous space. *Theorem* 4. $(F, \mathcal{P})$ is near-homogeneous at points of M if X is a weakly invertible infinite space. *Theorem* 5. $(F, \mathcal{P})$ is subinvertible if X is an invertible space. *Theorem* 6. As a closed subspace of $(F, \mathcal{C})$, M is invertible. *Theorem* 7. A homogeneously invertible Hausdorff space is regular. Theorems 1 and 2 are revisions of two theorems in the Naimpally paper whose proofs are incomplete. Theorems 3 and 4 are $\mathcal{P}$-analogues of Theorems 1 and 2. Theorem 5 gives an affirmative answer to a $\mathcal{P}$-analogue of a $\mathcal{C}$-question asked by Naimpally. Theorem 7 suggests a way of connecting invertibility with the separation axioms. (Received October 17, 1973.)

74T-G15. ROBERT D. EDWARDS, University of California, Los Angeles, California 90024 and MARTIN G. SCHARLEMANN, University of California, Berkeley, California 94720. *A remark on suspensions of homology spheres.*

Theorem. (Cf. Glaser in "Topology of manifolds", ed. Cantrell and Edwards, Markham Publ. Co., Chicago, Ill., 1970.) If H_1 and H_2 are homology n-spheres $\ni \Sigma^p H_i \approx S^{n+p}$ for $i = 1, 2$, then $\Sigma^p(H_1 \# H_2) \approx S^{n+p}$ ($\#$ = connected sum). *Proof.* By hypothesis $X \times E^{p-1}$ is a manifold, where $X = Y \cup$ an open exterior collar, $Y = cH_1 \natural cH_2$; c = cone, $\natural$ = boundary connected sum. Clearly $\exists$ a cell $D^n \times I$ in $Y \ni Y/\{w \times I | w \in D^n\} \approx c(H_1 \# H_2)$. A routine adaptation of [Bryant, Fund. Math. 63 (1968), 42–51], keeping in mind [Bryant, "Euclidean n-space modulo an $(n-1)$-cell", Trans. Amer. Math. Soc. (to appear), especially Theorem 4.4], establishes by shrinking that $X/\{w \times I | w \in D^n\} \times E^{p-1} \approx X \times E^{p-1}$. □ Since each $D^n \times I \times z$ is wild in $X \times E^{p-1}$ at only two points, Bryant's Theorem 4.4 is unnecessarily strong. Let, e.g., $\alpha = 0 \times I$ be the arc in $D^n \times I$ containing the two wild points as endpoints. Then $X/\alpha \times E^{p-1} \approx X \times E^{p-1}$ by [Andrews-Curtis, Ann. of Math. 75 (1962), 1–7] trivially generalized. Now one can squeeze each $(D^n \times I)/\alpha \times z$ to $D^n \times z$ with bare hands. Glaser used this latter proof in Proc. First Conf. on Monotone Mappings, 1970, State Univ. of New York, Binghamton, 1971, p. 228, Step 1. (Received October 25, 1973.)

*74T-G16. ROBERT A. HERRMANN, U. S. Naval Academy, Annapolis, Maryland 21402. *A simple set-theoretic characterization for proximity spaces.*

Let $P(X)$ denote the power set of X. *Definition.* Let X be a nonempty set. A map $\rho: P(X) \to P(Y)$, where Y is an arbitrary nonempty set, will be called a generating proximity map on $P(X)$ if it satisfies the conditions (S.1) $\rho(A) = \emptyset$ iff $A = \emptyset$; (S.2) $\rho(A \cup B) = \rho(A) \cup \rho(B)$; if $\rho(A) \cap \rho(B) = \emptyset$, then there exist some C and D such that (i) $A \subset C$ and $B \subset D$, (ii) $C \cup D = X$, (iii) $\rho(A) \cap \rho(D) = \rho(B) \cap \rho(C) = \emptyset$. *Theorem* 1. If ρ is a generating proximity map on $P(X)$, then the binary relation δ on $P(X)$ defined by $A \,\delta\, B$ iff $\rho(A) \cap \rho(B) \neq \emptyset$ is a proximity relation on X. *Theorem* 2. A set X is a proximity space iff there exists a generating proximity map on $P(X)$. (Received October 25, 1973.)

*74T-G17. RICHARD E. HODEL, Duke University, Durham, North Carolina 27706. *On a theorem of Arhangel'skiĭ concerning Lindelöf p-spaces.*

Arhangel'skiĭ (General Topology and Appl. 3 (1973), 39–46) has proved the remarkable theorem that every regular space which is a Lindelöf p-space hereditarily has a countable base. The following analogue of Arhangel'skiĭ s result is obtained. (See Nagami (Fund. Math. 65 (1969), 169–192) for the definition of a strong Σ-space.) *Theorem* 1. Every regular space which is an $\aleph_1$-compact strong Σ-space hereditarily has a countable net. Under the assumption of GCH (generalized continuum hypothesis), Theorem 1 can be extended to higher cardinals. This extension yields the following result, which gives an affirmative answer to Problem 2 in Arhangel'skiĭ s paper. *Theorem* 2. Assume GCH. If X is a paracompact p-space hereditarily, then the weight of X is equal to the cellularity of X. (Received October 29, 1973.)

*74T-G18. JOHN GINSBURG, University of Manitoba, Winnipeg, Manitoba R3T 2N2. Canada. *Countably compact powers and $\mathfrak{D}$-compactness.* Preliminary report.

We characterize those T_1 spaces X such that every power of X is countably compact. Let $\mathfrak{D}$ be a free ultrafilter on N, and let X be a topological space. Let $(x_n : n \in N)$ be a sequence in X. A point $a \in X$ is said to be a $\mathfrak{D}$-limit point of $(x_n : n \in N)$ in X if, for every neighborhood W of a, $\{n : x_n \in W\} \in \mathfrak{D}$ [A. R. Bernstein, Fund. Math. 66 (1970), 185–193]. X is said to be $\mathfrak{D}$-compact if every sequence in X has a $\mathfrak{D}$-limit point in X. *Theorem.* Let X be a T_1 topological space. The following are equivalent: (a) X is $\mathfrak{D}$-compact for some free ultrafilter $\mathfrak{D}$ on N; (b) X^{2^c} is countably compact; (c) every power of X is countably compact. (Received October 30, 1973.)

*74T-G19. RAYMOND F. GITTINGS, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. *On semimetric spaces in which all spheres are open.*

A semimetrizable space X will be called *o-semimetrizable* if there is a compatible semimetric d on X for which all d-spheres are open. *Theorem* 1. A T_0-space X is o-semimetrizable if and only if for each $x \in X$ there is a countable local base $\{g_n(x): n = 1, 2, \ldots\}$ such that if $y \in g_n(x)$ then $x \in g_n(y)$ for all $x, y \in X$. *Theorem* 2. A T_0-space X is o-semimetrizable if and only if there is a semidevelopment $\{G_n : n = 1, 2, \cdots\}$ for X such that for each $x \in X$, $\{St(x, G_n) : n = 1, 2, \cdots\}$ is an open base at x. For the definition of a semidevelopment see C. C. Alexander, Pacific J. Math 37 (1971), 277–293. *Theorem* 3. A T_0-space X is o-semimetrizable if and only if there is a metric space (M, ρ) and a π-map f from M onto X such that $f(S_\rho(f^{-1}(x); 1/n))$ is open in X for every $x \in X$ and $n \in N$. *Theorem* 4. For a semimetric space (X, d), d is separately continuous if and only if d satisfies the following two conditions: (a) all d-spheres are open, (b) for each $x \in X$ and $\epsilon > 0$, $\mathrm{Cl}\, S_d(x, \delta) \subset S_d(x, \epsilon)$ for all $\delta < \epsilon$. (Received May 21, 1973.) (Author introduced by Professor David J. Lutzer.)

74T-G20. RONNIE FRED LEVY, Washington University, St. Louis, Missouri 63130. *Showering spaces.* Preliminary report.

Suppose $\omega \geq 1$ is an ordinal number, $\alpha > 0$ is a cardinal number, and β is an infinite cardinal number. A paracompact Hausdorff space $S^{\omega}_{\alpha,\beta}$, called the showering space of type α, β, ω is constructed. If $\alpha \geq \beta$, $S^{\omega}_{\alpha,\beta}$ is Baire if and only if ω is not a sequential ordinal. If $\omega > 1$, $S^{\omega}_{\alpha,\beta}$ is a P-space if and only if β is not a sequential cardinal. If ω is an infinite limit ordinal and $\alpha \geq \beta$, $S^{\omega}_{\alpha,\beta}$ is dense-in-itself. If $\aleph$ is any uncountable cardinal, $S^{\omega_0}_{\aleph,\aleph_1}$ is a dense-in-itself Lindelöf bihomogeneous P-space of cardinal $\aleph$ which is first category in itself and which has the property that if $f: S^{\omega_0}_{\aleph,\aleph_1} \to \mathbf{R}$ is continuous, $f(S^{\omega_0}_{\aleph,\aleph_1})$ is countable. It is also shown that the space of rationals is a closed subspace of a space Y with the property that every nonempty G_δ of Y has nonempty interior. (Received November 2, 1973.)

74T-G21. ROBERT W. BUTTON, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. *Covers with no finite subcovers.* Preliminary report.

We assume that each enlargement is based upon a structure sufficient to express finiteness. *Theorem.* If $\mathfrak{U}$ is a cover of X, then $\mathfrak{U}$ has a finite subcover iff $\bigcup\{{}^*U: U \in \mathfrak{U}\} = {}^*X$. Define $\mathfrak{U}_z = \{U: U$ is open and $z \notin {}^*U\}$ for remote z in ${}^*X$. *Corollary.* If $z \in {}^*X$ is remote, then $\mathfrak{U}_z$ is an open cover of the space X with no finite subcover. Any open cover of a space X with no finite subcover must be a subcover of some $\mathfrak{U}_z$. The relation $x \sim y$ iff $\mathfrak{U}_x = \mathfrak{U}_y$ partitions the remote points of ${}^*X$ and generates an interesting total monad system [F. Wattenberg, J. Symbolic Logic 36 (1971), 463–476]. Research in this area has led to a short and elegant proof that from the axiom of choice any set may be well-ordered. (Received November 7, 1973.)

*74T-G22. YANG HUA, Dartmouth College, Hanover, New Hampshire 03755, and University of North Carolina, Chapel Hill, North Carolina 27514. *A degenerate principal series of representations of $U(2, 2)$.*

Let $G = U(2, 2)$ be the group of 2×2 matrices over $\mathbf{C}^{2\times 2}$, for which $gpg^* = p$, where $p = \left(\begin{smallmatrix} 0 & I \\ -I & 0 \end{smallmatrix}\right)$, I being the identity in $\mathbf{C}^{2\times 2}$. We are concerned with the irreducibility or reducibility of the representations in the

degenerate principal series corresponding to the maximal parabolic subgroup consisting of $y = \begin{pmatrix} a^{*-1} & 0 \\ * & a \end{pmatrix}$ with $a \in \mathrm{GL}(2, \mathbf{C})$. The representations $T(\rho, m; \cdot)$ are induced from the characters $y \to \chi_{\rho,m}(y) = |\det a|^{i\rho} \cdot (\det a/|\det a|)^m$, where $\rho \in \mathbf{R}$, $m \in \mathbf{Z}$. Our main result is the following *Theorem.* (1) Except for $\rho = 0$, m even, the representations $T(\rho, m; \cdot)$ are all irreducible. (2) If $\rho = 0$, m even, then $T(\rho, m; \cdot)$ is reducible. Our proofs for (1) and (2) are quite different. For (1) we follow Fourier analysis lines. For (2) we compute certain generalized Riesz transforms, the analytic continuation of which yields a nontrivial commuting operator. (Received November 9, 1973.) (Author introduced by Professor Kenneth I. Gross.)

*74T-G23. TOM KNOBLAUCH, 8022 West Dixon Street, Milwaukee, Wisconsin 53214. *Imbedding deleted 3-manifold neighborhoods in* E^3. Preliminary report.

A result of Haken ("Some results on surfaces in 3-manifolds," Studies in Modern Topology, Prentice-Hall, Englewood Cliffs, N. J., 1968, pp. 39–98), is used to prove that a compact set X in an orientable 3-manifold M^3 has a neighborhood N in M^3 such that $N - X$ imbeds in $\mathbf{E}^3$. (Received November 9, 1973.) (Author introduced by Professor Daniel R. McMillan, Jr.)

*74T-G24. DAVID A. EDWARDS, State University of New York, Binghamton, New York 13901. *Etale homotopy theory and shape.*

A survey of recent developments in the algebraic topology of pathological spaces is presented. (Received November 12, 1973.)

74T-G25. HAROLD R. BENNETT, Texas Tech University, Lubbock, Texas 79409. *On a subclass of Arhangel'skiĭ's class* MOBI. Preliminary report.

In (Russian Math. Surveys 21 (1966), 115–162) A. V. Arhangel'skiĭ introduced the class MOBI. Bennett (Proc. Amer. Math. Soc. 26 (1970), 178–180) gave an alternate definition concerning a finite number of open compact maps acting on a finite space. If the class MOBI is restricted to a finite number of open, finite-to-one maps acting on a metric space, then every space in this restricted class is metrizable. *Theorem.* The open, finite-to-one continuous image of a paracompact space is paracompact. (Received November 12, 1973.) (Author introduced by Professor Vadim Komkov.)

*74T-G26. STEPHEN LEON LIPSCOMB, Box 523, Dahlgren, Virginia 22448. *On imbedding finite-dimensional metric spaces.*

Let τ be an infinite cardinal. Let $N(A)$ be Baire's zero-dimensional space with $\aleph(A) = \tau$. Two points $(\alpha_1, \alpha_2, \cdots), (\beta_1, \beta_2, \cdots) \in N(A)$ are R-related if, and only if, $\alpha_i = \beta_i$ for all $i \geq 1$, or, in case there exists a j such that (1) $\alpha_i = \beta_i$ for $i < j$, (2) $\alpha_j \neq \beta_j$, and (3) $\alpha_j = \beta_{j+i}$ for all $i \geq 1$ and $\beta_j = \alpha_{j+i}$ for all $i \geq 1$. Let $N(A)/R = J(\tau)$ where $J(\tau)$ is given the quotient topology induced by the natural map $p: N(A) \to N(A)/R$, i.e., R is an equivalence relation. Then $z \in J(\tau)$ is *rational* if $\aleph(p^{-1}(z)) = 2$, and z is *irrational* if $\aleph(p^{-1}(z)) = 1$. This is a topological generalization of identifying adjacent end points in the Cantor space. Indeed, a copy of the unit interval, together with the subspaces of rational and irrational points, can be obtained by using these definitions. It is only required that one use the Cantor space, considered as the countable product of a two point discrete space, instead of $N(A)$. *Theorem.* A metric space of weight $\tau \geq \aleph_0$ is of dimension $\leq n$ if, and only if, it can be imbedded in the set of points of $J(\tau)^{n+1}$ which have at most n rational coordinates. This answers the long standing problem of obtaining a nonseparable analogue to the classical imbedding theorem in dimension theory. (Received November 15, 1973.)

74T-G27. SHU-CHUNG KOO, City College, City University of New York, New York, New York 10031. *Transitivity of transformation groups in hyperspaces.* Preliminary report.

Let (X, T) be a transformation group with compact Hausdorff phase space X. Let $\mathfrak{U}$ be the unique uniformity on X compatible with the topology for X, and let $2^X = \{E: \emptyset \neq E \subset X \text{ and } E \text{ is closed}\}$. For each $\alpha \in \mathfrak{U}$ and each $E \in 2^X$, we denote $E\alpha = \bigcup\{x\alpha : x \in E\}$, $\tilde{\alpha} = \{(A, B): A, B \in 2^X;\ A \subset B\alpha,\ B \subset A\alpha\}$. Let $\mathfrak{B} = \{\tilde{\alpha}: \alpha \in \mathfrak{U}\}$. Then $\mathfrak{B}$ is a uniformity base for the hyperspace 2^X. Let $\tilde{\mathfrak{U}}$ be the uniformity generated by $\mathfrak{B}$. Then

$(2^X, \tilde{\mathfrak{U}})$ is a uniform space whose topology is also compact Hausdorff. The map $\pi: 2^X \times T \to 2^X$ defined by $(A, t)\pi = At$ $(A \in 2^X, t \in T)$ is continuous so that $(2^X, T)$ is a transformation group. Transitivity of the transformation group $(2^X, T)$ in the sense that there exists a dense orbit in 2^X has been investigated. In particular, it can be shown that if $(X, \mathbf{Z})$ is the symbolic flow, then $(2^X, \mathbf{Z})$ is transitive. If X is the unit interval $[0, 1]$ or the unit circle S^1 and T is $\mathbf{Z}$ or $\mathbf{R}$, then $(2^X, T)$ is not transitive. (Received November 15, 1973.) (Author introduced by Professor Walter H. Gottschalk.)

*74T-G28. STEPHEN WILLARD and S. M. KARNIK, University of Alberta, Edmonton, Alberta T6G 2G1, Canada. *Coherence topologies.*

X is an s_R-space provided every sequentially continuous real-valued function on X is continuous. A structure theorem is provided for s_R-spaces, similar to existing theorems for Frechet and sequential spaces, and various results are obtained on subspaces, products and mappings of s_R-spaces, sequential spaces and Frechet spaces. Most results are cast in a general setting, and thus apply to c-spaces and c_R-spaces, as well as to k'-spaces, k-spaces and k_R-spaces. (Received November 9, 1973.)

74T-G29. BARADA KINKAR RAY, Regional Engineering College, Durgapur 713209, West Bengal, India. *On a fixed point theorem.* Preliminary report.

Let (X, ρ) be a complete metric space, T_i, $i = 1, 2$, be two selfmappings of X such that $\rho(T_1^p x, T_2^q y) \leq \alpha(\rho(x, y))\, \rho(x, T_1^p x) + \beta(\rho(x, y))\, \rho(y, T_2^q y) + \gamma(\rho(x, y))\rho(x, y)$ $\forall\, x, y \in X$, $x \neq y$, $p, q > 0$ integers, $\alpha, \beta, \gamma \in F = \{\lambda: [0, \alpha) \to [0, 1)/\lambda$ monotonically decreasing$\}$ such that $\alpha(r) + \beta(r) + \gamma(r) < \forall\, r \in [0, \infty)$. Then T_1 and T_2 have a unique common fixed point in X. (Received November 12, 1973.)

*74T-G30. FRANKLIN D. TALL, University of Toronto, Toronto, Ontario M5S 1A1 Canada. *On the existence of nonmetrizable hereditarily Lindelöf spaces with point-countable bases.*

Ponomarev's problem concerning the existence of a nonmetrizable hereditarily Lindelöf space with a point-countable base is given a set-theoretic translation. The problem is then related to the normal Moore space problem. (Received November 16, 1973.)

*74T-G31. ANDREW ADLER and R. DOUGLAS WILLIAMS, University of British Columbia, Vancouver 8, British Columbia, Canada. *Rings of continuous E-valued functions.*

Let E be a subring of the real numbers. Recall that a topological space X is called "E-compact" if X is homeomorphic to a closed subset of some power of E. Let $C(X, E)$ denote the partially ordered ring of all continuous functions from X to E. Let X and Y be E-compact. *Theorem* 1. If $\phi: C(X, E) \to C(Y, E)$ is an order preserving isomorphism, then ϕ is induced by a homeomorphism of X and Y. *Theorem* 2. Suppose that E is a subfield of the reals. If $\phi: C(X, E) \to C(Y, E)$ is an isomorphism, then there exists an isomorphism $\tau: C(Y, E) \to C(Y, E)$ such that $\tau \circ \phi$ is induced by a homeomorphism of X and Y. (Received November 28, 1973.)

*74T-G32. KENNETH A. PERKO, JR., 400 Central Park West, New York, New York 10025. *On dihedral linking numbers of knots.*

Let v_{ij}^p (p odd) be the $(p^2 - 1)/8$ linking numbers between branch curves of a p-sheeted dihedral covering space M_p, ordered as in Abstract 73T-G140, these Notices 20(1973), A-598. *Theorem.* For any $i > j > 0$, $2v_{ij}^p = v_{b0}^p + v_{k0}^p$ where $b = i - j$ and k is the smallest value of $|\pm(i + j) \bmod p\,|$. (Proof traces the symmetries of a corresponding $2p$-sheeted dihedral covering through its natural mapping onto M_p. Cf. §4 of the forthcoming paper by J. M. Montesinos, "Representaciones de enlaces en relacion con recubridores dobles ramificados", Collect Math., to appear.) Thus, some $(p^2 - 4p + 3)/8$ linking numbers in M_p are determined by the $(p - 1)/2$ others. Also, if any one linking number is "defined", then they all are, and their sum is just $(p + 1)/4$ times the sum of all v_{i0}^p. We call the latter sum v_p and note that it is a handy invariant of the placement of the branch link

in oriented M_p. (For example, 7_4 is distinguishable from 9_2 either by v_5 or by v_{15} and v_3. Cf. "Knotentheorie", p. 69.) From a result communicated to us by K. Murasugi, it follows that the signature of any 2-bridged knot equals v_p for $p = \Delta(-1)$. (Received December 26, 1973.)

74T-G33. DAVID S. LAWRENCE, 135 Ashland Place 5D, Brooklyn, New York 11201. *Sufficient conditions for connectivity of real-valued functions.*

On an open interval $J \subseteq R$, f is bilaterally exactly semicontinuous (BES) if it is (upper) BES in that for $x \in J$, $f(x) = \limsup f(y)$ as $y \to x^+$ and as $y \to x^-$, or if it is (lower) BES. *Theorem.* If f is BES then its graph is connected. In this case we call f a "weak connectivity function." *Corollary.* If the set $S(f) \subseteq J$, of points x where f is not locally BES, is countable and $f(x)$ lies in the bilateral cluster set of f at x, then f is a weak connectivity function. In the real domain, this is sufficient for f to be a connectivity function. See also E. E. Hargrove, "Connectivity and Darboux functions", University of Alabama, 1972, Dissertation Abstract #73-8040. On R a counterexample will show the intermediate value property is not sufficient for connectivity. Our corollary generalizes to (arcwise connected) Banach manifolds. Some natural questions are raised. (Received November 26, 1973.)

*74T-G34. PETER J. NYIKOS, University of Chicago, Chicago, Illinois 60637. *Proto-metrizable spaces.* Preliminary report.

A *proto-metric* for a set X is a symmetric function q from X^2 to an ordinal τ, satisfying the following conditions: (i) $q(x, x) > q(x, y)$ for all $y \neq x$; (ii) For each $\alpha < \tau$ there exists a (finite) number n such that $\alpha + n \leq \tau$ and such that if $q(x, y) \geq \alpha + n$ and $q(y, z) \geq \alpha + n$, then $q(x, z) \geq \alpha$. A space X is *proto-metrizable* if for each point x of X, the sets $B(x, \alpha) = \{y | q(x, y) > \alpha\}$, where $\alpha < q(x, x)$, form a base for the neighborhoods of x. Every metrizable space is proto-metrizable, as is every non-Archimedean space, i.e. every T_1 space with a rank 1 base. The following theorems extend results by Archangel'skij and Filippov on non-Archimedean spaces [Math. USSR Sb. 16 (1972), 147–158]. *Theorem.* Every proto-metrizable Σ-space, semistratifiable space, separable space, or connected space is metrizable. *Theorem.* A space X is proto-metrizable [non-Archimedean] if, and only if, X is paracompact [ultraparacompact] and there exists a base $\mathcal{B}$ for X such that for every subset $\mathcal{B}'$ of $\mathcal{B}$ either (1) $\bigcap \mathcal{B}'$ is open, or (2) $\bigcap \mathcal{B}'$ is a nonisolated singleton p and $\mathcal{B}'$ is local base at p. *Theorem.* A space X is non-Archimedean if, and only if, it is proto-metrizable via a proto-metric q satisfying (ii) $q(x, z) \geq \min\{q(x, y), q(y, z)\}$ for all points x, y, z of X. (Received November 26, 1973.)

*74T-G35. JAMES W. MAXWELL, Oklahoma State University, Stillwater, Oklahoma 74074. *Regular neighborhoods of immersed manifolds.*

Associated with a PL immersion of a PL manifold M in a PL manifold Q is a regular neighborhood of M. It is shown that in the case the dimension of Q is greater than or equal to twice the dimension of M and the dimension of M is at least three, the regular neighborhoods associated with any homotopic immersions are equivalent. This result follows immediately from the main theorem of the paper which states that any two homotopic immersions in the setting described above are pseudo regularly homotopic. Some examples are mentioned which show that this is the best possible theorem in this generality. (Received October 24, 1973.)

*74T-G36. JOHN M. FRANKS, Northwestern University, Evanston, Illinois 60201. *Time dependent structural stability.* Preliminary report.

A diffeomorphism $f: M \to M$ of a compact manifold is said to be *time dependent structurally stable* if there is a neighborhood N of f in the C^1 topology such that $g_1, g_2, \cdots, g_k \in N$ implies $g_1 \circ g_2 \circ \cdots \circ g_k$ is topologically conjugate to f^k. *Theorem.* f is time dependent structurally stable if and only if it satisfies Axiom A and the strong transversality property. (Received October 29, 1973.)

*74T-G37. MARSHALL M. COHEN, Cornell University, Ithaca, New York 14850. *Dimension estimates in collapsing* $X \times I^q$. Preliminary report.

In the following, X denotes a compact, contractible n-dimensional polyhedron. *Theorem* 1. *If* X is the spine of a q-ball *then* $X \times I^q$ is collapsible. *Corollary* 1. *If* $n = 2$ *then* $X \times I^6$ is collapsible. *If* $n \geq 3$ *then* $X \times I^{2n}$ is collapsible. (*Proof.* Embed X in a large Euclidean space, and take a regular neighborhood.) *Corollary* 2. *If* X is P.L. embedded in a 3-manifold M and $X \times I^3$ is not collapsible then the regular neighborhood of X in M is a counterexample to the Poincaré conjecture. (*Proof.* Use Theorem 1 with $q = 3$.) *Theorem* 2. Let Y and Z be compact simple-homotopy equivalent polyhedra. Let $d = \max\{\dim Y, \dim Z\}$. If $d = 2$ then $X \times I^7 \searrow Y$. If $d \geq 3$ then $X \times I^{2d+2} \searrow Y$. (Received November 29, 1973.)

*74T-G38. MICHAEL A. GRAJEK, Hiram College, Hiram, Ohio 44234. *Obstructions to collapsing* $K \times I$ *to vertical segments*. Preliminary report.

Let K denote a contractible, noncollapsible 2-complex. If $K \times I$ is collapsible, then $K \times I$ collapses to an arc connecting $K \times \{0\}$ to $K \times \{1\}$; however, W.B.R. Lickorish has shown that it is not always possible to make the arc "vertical" ("Topology of manifolds", Cantrell and Edwards, Markham Publishing Co., Chicago, Ill., 1970, pp. 157–160; and Topology 12(1973), 5–8). This suggests the following question: When does $K \times I$ collapse to $P \times I$, where P is a point in K? P. Wright answered this question for all 2-complexes consisting of one 1-cell, a, one vertex, and one 2-cell sewn onto the 1-cell by a word in a and a^{-1} (Proc. Cambridge Philos. Soc. 69(1971), 71–74). In the present paper, we prove four theorems that completely answer the above question for all complexes in $\mathfrak{L}$, the class of all 2-complexes obtained by sewing two disks onto the wedge of two circles, $a \vee b$, by the words $a^p b^q$ and $a^r b^s$, where p, q, r, and s are positive integers satisfying $ps - qr = \pm 1$. The problem of answering a similar question for nonvertical arcs is discussed. Our results do not yield a K in $\mathfrak{L}$ for which $K \times I$ is noncollapsible. (Received November 30, 1973.)

74T-G39. N. N. KAULGUD, Indian Institute of Technology, Powai, Bombay 400076, India. *Fixed point theorems for set-valued mappings*. Preliminary report.

Theorem 1. Let (X, d) be a complete metric space and $(b(X), D)$ be the metric space of nonempty closed-bounded subsets of X equipped with the Hausdorff metric D. Let $F_i: X \to b(X)$, $i = 1, 2$, be a mapping with $D(F_1 x, F_2 y) \leq k\{d(x, F_1 x) + d(y, F_2 y)\}$ for $x, y \in X$ and $0 \leq k < \frac{1}{2}$. Then there exists a point $\bar{x} \in X$ with $\bar{x} \in F_1 \bar{x}$ and $\bar{x} \in F_2 \bar{x}$. *Theorem* 2. Let (X, d) be a compact metric space and $\{F_\lambda\}_{\lambda \in \Lambda}$ be a family of continuous maps from X into $b(X)$ such that $D(F_\lambda x, F_\mu y) \leq k\{d(x, F_\lambda x) + d(y, F_\mu y)\}$ with $0 \leq k < \frac{1}{2}$ for all $x, y \in X$ and $\lambda, \mu \in \Lambda$. Then the family $\{F_\lambda\}$ has a simultaneous fixed point. *Theorem* 3. Let (X, d) be a complete metric space and E be a dense subset of X. Let $F_i: X \to b(X)$, $i = 1, 2$, be continuous mappings with $D(F_1 x, F_2 y) \leq k_1 d(x, F_1 x) + k_2 d(y, F_2 y)$, $k_1 > 0$, $k_2 > 0$, $k_1 + k_2 < 1$, and $x, y \in E$. Then F_1 and F_2 have a common fixed point. (Received December 4, 1973.) (Author introduced by Dr. Padam C. Jain.)

*74T-G40. LOUIE M. MAHONY, State University of New York, Binghamton, New York 13901. *Diffeomorphisms of plumbed four-dimensional manifolds.*

Let $M_i^4, i = 1, \cdots, 5$ be the four-dimensional manifolds obtained by plumbing according to the periodic quadratic forms whose trees were communicated in Abstract 73T-G89, these Notices 20 (1973), A–456. Each of these manifolds has index 8 and they are all nonhomeomorphic. The boundary of say M_1 is the dodecahedron space with finite fundamental group $SL(2, Z_5)$ while the boundary of each of the remaining manifolds has an infinite perfect fundamental group and all these groups are different. *Theorem.* Let D^1 be the unit interval. Then the manifolds $M_i \times D^1$ are *all* diffeomorphic to each other. Moreover, the boundaries $\partial(M_i \times D^1)$ are each diffeomorphic to $\#_8 S^2 \times S^2$, the connected sum of eight copies of $S^2 \times S^2$ where S^2 is the two-dimensional sphere. (Received December 5, 1973.)

*74T-G41. R. VASUDEVAN, Institute of Advanced Studies, Meerut University, Meerut India. *Weaker forms of compactness in hyperspaces.* Preliminary report.

Let 2^X be the space of all nonempty closed sets in a topological space X with the finite topology. It is shown that a T_1 topological space is almost compact if and only if 2^X is almost compact, and is nearly compact if and only if 2^X is nearly compact. It is also shown that a T_1 topological space X is C-compact, functionally compact if 2^X is C-compact, functionally compact respectively. (Received December 6, 1973.) (Author introduced by Professor J. N. Kapur.)

*74T-G42. PAUL J. BANKSTON, University of Wisconsin, Madison, Wisconsin 53701. *Topological ultraproducts and the separation axioms.*

Given an indexed family $\{A_i : i \in I\}$ of spaces and an ultrafilter D on I, the *topological ultraproduct* $\prod_D A_i$ is introduced. A property P of topological spaces is *productive* if P is preserved under the taking of ultraproducts; P is *coproductive* if its complement is productive. Among other things we prove that the separation axioms T_0 through T_3 are both productive and coproductive; that $T_{3.5}$ is productive but not coproductive; and that T_4 is neither productive nor coproductive. All of the negative results involve exotic set-theoretic axioms – namely Martin's axiom and a form of the generalized continuum hypothesis. (Received December 10, 1973.)

74T-G43. ALAN D. WEINSTEIN, University of California, Berkeley, California 94720. *On the spectrum of manifolds, all of whose geodesics are closed.* Preliminary report.

Associated with any compact Riemannian manifold (M, g) is the sequence $0 = \lambda_1(M, g) < \lambda_2(M, g) \leq \lambda_3(M, g) \leq \cdots$ of eigenvalues of its laplacian on real valued functions. Let (M, g_0) be a symmetric space of rank one, and consider a smooth family $\{g_t\}$ of metrics on M. *Theorem.* If, for all t, the geodesics of (M, g_t) are all closed and have the same length (independent of t), then the sequence $|\lambda_n(M, g_t) - \lambda_n(M, g_0)|$ is bounded for each t. The proof uses the theory of Fourier integral operators to construct an approximate conjugacy between the laplacians of (M, g_0) and (M, g_t). The converse theorem, in case the metrics g_t are analytic, has been proven by J. J. Duistermaat and V. Guillemin (unpublished). They have also proven a slightly weaker result than ours for any pair of manifolds whose geodesics are closed and have the same length. (Received December 13, 1973.)

*74T-G44. JAMES R. BOONE, Texas A&M University, College Station, Texas 77843. *On k-quotient mappings.*

Two natural generalizations of Arhangel'skiĭ's compact-covering mappings are introduced, the k-quotient and k'-quotient mappings. They may be used to replace the much stronger perfect mappings in improving some mapping theorems concerning the invariance of topological structures. The defining k-systems and defining k_1-systems of Arhangel'skiĭ are fundamentally related to the k-quotient and k'-quotient mappings. Functional characterizations of various k-spaces as domains and ranges of certain mappings are presented. Examples illustrate the results of this study. (Received December 18, 1973.)

*74T-G45. ROBERT WELLS, Pennsylvania State University, University Park, Pennsylvania 16802 and LUIZ A. FAVARO, Instituto de Ciencias Matematicas de Sao Carlos, Sao Carlos (SP), Brazil. *On a theorem of Kuiper.* Preliminary report.

If Σ is a smooth homotopy n-sphere with $n \geq 5$, it is well known that there is a triangulation $\phi: K \to \Sigma$ where K is a *subdivision* of $\partial\Delta_{n+1}$. Kuiper has shown that if there is a triangulation $\phi: \partial\Delta_{n+1} \to \Sigma$, then Σ is diffeomorphic to S^n. Munkres' definition of triangulation extends to the case where the domain is a polyhedron rather than just a simplicial complex. With this extended definition, this theorem is true: *Theorem.* For each smooth homotopy n-sphere Σ there exists a nonempty class $P(\Sigma)$ of polyhedrons, such that (1) if $Q \in P(\Sigma)$, then there is a smooth triangulation $\phi: Q \to \Sigma$, and (2) if $Q \in P(\Sigma)$ and there is a smooth triangulation $\phi: Q \to \Sigma'$, then Σ and Σ' are smoothly s-cobordant (and so diffeomorphic when $n \geq 5$). *Theorem.* There exists a polyhedron homeomorphic to Δ_8 which cannot be smoothed unless it is first subdivided. (Received January 7, 1974.)

74T-G46. KENNETH O. LELAND, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio 45433 and 619 Omar Circle, Yellow Springs, Ohio 45387. *A direct approach to the theory of the degree of a mapping.*

This theory of mappings from E^n into E^n originates in the following observations. Let P be a simple closed polygonal curve in the plane C and $x \in C - P$. Then if L is a ray terminating in x, containing no vertex of P, then x lies in the interior of P if and only if P cuts L an odd number of times. Generalizing, a direction of traverse is imposed on P, P is no longer required to be simple, L is taken parallel to the x axis and each cut of L by P is assigned the value $+1$ or -1 depending on whether the cut is from below L to above or vice versa. The sum of these "oriented cuts" gives the number of times P winds about x. Generalizing to E^n, let f be a continuous mapping on ∂S into E^n, where S is an n-simplex, and let $z \in E - f(\partial S)$. Let A be a simplicial partition of ∂S and g a mapping of ∂S into E^n, piecewise linear with respect to A, approximating f. Then if A is sufficiently fine, the degree of f at x is given by the degree p of g at x, where p is the number of oriented cuts of elements of $f(A)$ by a suitable ray terminating in x. (Received January 7, 1974.)

*74T-G47. ROBERT D. EDWARDS, University of California, Los Angeles, California 90024 and RICHARD T. MILLER, Michigan State University, East Lansing, Michigan 48823. *Cell-like 0-dimensional decompositions of R^3 are R^4 factors.*

Theorem. The product of a cell-like 0-dimensional upper semicontinuous decomposition of R^3 with a line is R^4. This establishes at once this feature for all the various dogbone inspired decompositions of R^3. The above theorem is the main theorem of L. Rubin [Trans. Amer. Math. Soc. 166(1972), 215–224], whose proof is faulty [Erratum, ibid. 177(1973), 505]. Our proof uses Rubin's basic observation that the universal cover of a cube-with-handles T (in fact any space homotopy equivalent to a countable wedge of circles) 1-1 immerses in $T \times R^1$, such that projection to T is compatible with the covering projection. (Received January 7, 1974.)

*74T-G48. MILTON WANNIER, University of Oregon, Eugene, Oregon 97403. *Free homeomorphisms of the real line.* Preliminary report.

A maximal (abelian) group of fixed-point-free homeomorphisms of the real line, with the compact-open topology, is topologically isomorphic to the additive group of the real line. (Received January 7, 1974.)

74T-G49. MICHAEL HANDEL, University of California, Berkeley, California 94720. *A resolution of two stratification conjectures.* Preliminary report.

Siebemann (Comment. Math. Helv. 47(1972), 123–163) offers two stratification conjectures: (1) that every finite-dimensional locally cone-like metric space is homeomorphic to a CS set, and (2) that any CS set has a unique intrinsic stratification, in the sense that its n-skeleton is the intersection of all possible n-skeleta of CS stratifications of the space. (2) is verified and a counterexample to (1) is produced as follows. Given any topological manifold M without boundary and having one tame end, it is shown that $(M \cup \infty) \times \mathbf{R}$ is locally cone-like. However, granting certain conditions on the end of M, $(M \cup \infty) \times \mathbf{R}$ is not homeomorphic to a CS set. Finally, a purely local definition of a CS set is given. (Received January 7, 1974.)

The November Meeting in Minneapolis, Minnesota
November 3, 1973

708-G18. DEBORAH LOUISE GOLDSMITH, University of Chicago, Chicago, Illinois 60637. *A class of Neuwirth links.* Preliminary report.

A fibered (Neuwirth) link is a link $L \subset S^3$ whose complement is a surface bundle over the circle with fiber, the interior of a compact, orientable surface F with $\partial F = L$. Few examples of fibered links are known. Here is a construction for a large class of them, which exhibit symmetries: Let K be the axis for a closed braid A in S^3, and $\pi: \hat{S}^3 \to S^3$ be a branched covering of S^3 along A such that $\hat{S}^3 = S^3$. Then $\hat{K} = \pi^{-1}(K)$ is a fibered link in S^3, which is left invariant by the covering translations of $\hat{S}^3$. For the theorem, let $B_n = n$-stranded braid

group, let $c \in B_n$ generate Center$[B_n] \simeq \mathbf{Z}$, and if $b \in B_n$ let $\overline{b}$ denote the closed braid obtained from b. *Theorem.* Suppose $\hat{K}$ is a symmetric, fibered knot obtained from the above construction when A is the trivial knot, and $\pi: \hat{S}_3 \to S^3$ is the k-fold cyclic branched covering of S^3 along A; let $A = \overline{b}$ where $b \in B_n$. Then $\hat{K}$ has Property P$\Leftrightarrow$none of the k-fold cyclic branched coverings of S^3 along the knots $\overline{c^{nz} \cdot b}$, $0 \neq z \in Z$, are simply connected. *Corollary.* Let $\hat{K}$ be the fibered knot of the above theorem, where $k = 2$ (i.e., $\hat{K}$ exhibits a symmetry of order 2, for example $\hat{K}$ = figure 8 knot). Then $\hat{K}$ is characterized by its complement $S^3 - \hat{K}$. *Proof.* It is known that S^3 cannot be the double branched cover of S^3 along a knot. (Received November 26, 1973.) (Author introduced by Professor Paul T. Bateman.)

The January Meeting in San Francisco
January 15—19, 1974

* 711-22-32. KENNETH D. JOHNSON, Rutgers University, New Brunswick, New Jersey 08903. *Fourier transforms on Lie groups.* Preliminary report.

Let G be a linear semisimple Lie group with an Iwasawa decomposition $G = KAN$ where K is a maximal compact subgroup of G, A a vector subgroup normalizing N, a simply connected nilpotent group. If M is the centralizer of A in K then $P = MAN$ is a parabolic subgroup of G. Let $\mathcal{P}$ denote the set of equivalence classes of all principal series representations induced from P (not necessarily unitary). For $\pi \in \omega \in \tilde{\mathcal{P}}$ let $\theta_\omega = \theta_\pi$: $C_c^\infty(G) \to \mathbf{C}$ be the character of ω. If $f \in C_c^\infty(G)$ the Fourier transform of f is the function $\hat{f}: \tilde{\mathcal{P}} \to \mathbf{C}$ where $\hat{f}(\omega) = \theta_\omega(f)$. $f \in C_c^\infty(G)$ is K-finite if f is left and right K-finite. We give a characterization of the space of functions from $\tilde{\mathcal{P}}$ to $\mathbf{C}$ which are the Fourier transforms of functions in $C_c^\infty(G)$ which are K-finite. (Received November 28, 1973.)

Symposium on Some Mathematical Questions in Biology
San Francisco, California, February 25—26, 1974

* Bio 74-1. LESLIE E. BLUMENSON, Department of Biostatistics, Roswell Park Memorial Institute, Buffalo, New York 14203 *Some mathematical questions concerning random cellular movements.* Preliminary report.

The use of mathematical models to predict the observed cellular patterns formed during embryonic development has not been very fruitful to date. Sometimes what seem to be organized cellular formations may, on closer examination, actually be due to a complicated chain of random phenomena. A simple example of this was found with the movement of Burkitt lymphoma cells *in vitro*. These cells actually move around on the culture plates, collide with each other, adhere upon collision and so form larger multicellular aggregates which continue to move. After twenty hours the pattern of the number of multicellular aggregates formed seemed at first to depend in a peculiar way on the number of cells originally plated. However the use of Monte Carlo techniques suggest that these patterns were simply the result of random phenomena. The analysis of these data using closed mathematical forms remains an interesting unsolved problem with possible implications both for embryogenesis and cancer research. (Received October 29, 1973.)

* Bio 74-2. WACLAW G. WOJTKOWSKI, WITA WOJTKOWSKI and M. MACKLIN, Case Western Reserve University, Cleveland, Ohio 44106. *Catastrophy theory in modeling behavioral stages in Xenopus laevis embryos.* Preliminary report.

During the early stages of vertebrate development, there is a degree of simplicity which makes it possible to correlate behavior directly with electrical activity from the epithelium of intact animals (J. Comp. Physiol. 84(1973), 41–58). If we implicitly assume that electrical activity can be modelled by generic differential equations, then we can explicitly use catastrophy theory to explain certain developmental phenomena. We abstract the main dynamical qualities of behavioral patterns of developing embryos, and then build mathematical models with these qualities. (Received November 16, 1973.)

* Bio 74-3. ALFRED INSELBERG, IBM Scientific Center, Los Angeles, California 90067 and RICHARD S. CHADWICK, Department of Mechanical Engineering, Technion, Haifa, Israel. *A mathematical model of the cochlea (inner ear).*

A mathematical model of the cochlea is formulated. It consists of two rectangular chambers (the scalae) filled with a viscous incompressible fluid and separated by a thin uniform Euler-Bernoulli beam representing the basilar membrane. The system is driven at one end by the piston-like movement of the stapes. The equations of motion of the system consist of three simultaneous partial differential equations, one of them of fourth order. The solution of the resulting initial and boundary value problem is obtained in closed form. A place principle (a 1-1 correspondence between input frequencies and positions along the basilar membrane) is found. Explicit relations are given for low and high auditory thresholds defined in terms of sensitivity. The low frequency threshold is affected only by the geometry of the basilar membrane, while all model parameters affect the high frequency threshold. For some parameter values, hearing pathologies can be simulated. (Received November 19, 1973.)

* Bio 74-4. G. F. ESTABROOK, University of Michigan, Ann Arbor, Michigan 48104, CARLOS S. JOHNSON, JR. and FRED R. McMORRIS, Bowling Green State University, Bowling Green, Ohio 43403. *An algebraic analysis of cladistic characters.*

We are concerned with obtaining useful algebraic results about cladistic characters. Cladistic characters are those which are potentially useful in estimating the evolutionary history of a given collection of evolutionary units S. Let S' be a tree semilattice containing S. A *cladistic character* is an onto map $K: S' \to P$ where P is a tree semilattice, the *character state tree*. If we now interpret $(S', \leq)$ as the true cladistic history of S, a cladistic character will be ideally related to S' if certain desirable relationships hold between S' and the states of the characters. The following definition is what we feel to be a representation of such a relationship. A cladistic character $K: S' \to P$ is *true* if and only if (i) $\overline{a} \in K^{-1}(K(a))$ where $\overline{a} = \bigwedge K^{-1}(K(a))$, (ii) $a \leq b$ implies $K(a) \leq K(b)$, and (iii) $K(a) \leq K(b)$ implies $\overline{a} \leq \overline{b}$. *Theorem.* A cladistic character is true iff it is a homomorphism. Results concerning the compatibility of characters are also proved and discussed. (Received November 28, 1973.)

Bio 74-5. RICHARD C. BROWN, University of Wisconsin, Madison, Wisconsin 53706. *Mathematical simulation of chemotherapy in cancer.* Preliminary report.

A computer model of tumor growth and treatment with a cycle specific drug is constructed. The program has both deterministic and stochastic features; it allows both for independence of transit times (according to an arbitrary p.d.f.) through G_1, S, and G_2 or for degrees of genetic dependence, as well as for metastasis and effects due to tumor shape. In the simplest case (L1210 Leukemia) the model is a refinement of Shackney's (Cancer Chemotherapy Reports 54 (1970), 399–429). In this and other special cases sufficient conditions for curability, near optimal treatment schedules, and estimates of survival times can be calculated a priori using the theory of irreducible periodic matrices and extreme point theory. Dependence of the process on key parameters (such as variance of cycle time around the mean) is explored. (Received January 24, 1974.)

The March Meeting in Gainesville, Florida
March 7—8, 1974
Algebra & Theory of Numbers

* 712-A1. DAVID E. THOMAS, Tulane University, New Orleans, Louisiana 70118. *Freeness and its generalizations in valued vector spaces.* Preliminary report.

The following criterion for freeness in a countable-valued vector space, V, with values in a totally ordered set with suprema is established. V is free whenever it can be obtained from the set-theoretic union of countably many free subspaces. Since the socle of a primary abelian group may be viewed as a vector space over the field of characteristic p with values in the ordinals, one gets the following criterion for summability. An

Abelian p-group is summable if and only if it is the union of countably many summable, isotype subgroups of countable length. The above criterion is modified to cover smooth chains of closed subspaces of length $\aleph_1$. Using this result one can get a nonfree space of dimension $\aleph_2$ over any field with values in any totally ordered set with suprema which fails to satisfy the maximum condition. This space has the property that every subspace of strictly smaller dimension is free. This generalizes results of R. Nunke and P. Hill. (Received December 27, 1973.) (Author introduced by Professor Laszlo Fuchs.)

712-A2. ROGER D. BLEIER, University of Texas, Austin, Texas 78712. *Some remarks on the orthocompletion of an l-group.* Preliminary report.

A construction of the orthocompletion of a representable l-group is given which generalizes the sheaf-theoretic construction of K. Keimal and the construction given by D. Chambless for the archimedean case. Beginning with any representation of the l-group as an l-subgroup of a product of totally-ordered groups, the construction achieves the orthocompletion in such a way that the group and lattice operations are pointwise almost everywhere. The technique is somewhat similar to that of the author's earlier work on the SP-hull. A variation which begins with the permutation group representation of an l-group may possibly lead to a construction for the lateral completion of any l-group. (Received December 17, 1973.)

712-A3. A. M. W. GLASS and W. CHARLES HOLLAND, Bowling Green State University, Bowling Green, Ohio 43403. *Stabilizer extensions of ordered permutation groups.*

If T is a chain, $A(T)$ the lattice-ordered group of order-preserving permutations of T, let G and H be l-subgroups of $A(T)$ with $G \subseteq H$. Then H is a *stabilizer extension* of G if $X, Y \subseteq \overline{T}$ = the dedekind completion of T, and $G_X = G_Y$ imply $H_X = H_Y$ where these are the stabilizer subgroups. Using wreath products, we prove that every transitive l-permutation group has a maximal transitive stabilizer extension. In certain cases, including most of the o-primitive ones, we show this is unique. (Received January 8, 1974.)

712-A4. W. CHARLES HOLLAND and A. M. W. GLASS, Bowling Green State University, Bowling Green, Ohio 43403. *a^*-extensions of lattice-ordered groups.*

Let G be an l-subgroup of an l-group H. Then H is an a^*-extension of G if the mapping $K \to K \cap G$ provides a one-to-one correspondence between the lattice of all closed convex l-subgroups of H and the lattice of all closed convex l-subgroups of G. If, in addition, G is a complete l-subgroup of H, we say that H is an a^0-extension of G. Using the techniques of the previous abstract, it is shown that any completely distributive l-group has an a^0-closure, and there is a cardinality bound on any a^*-extension. (Received January 8, 1974.)

712-A5. JOE L. MOTT, Florida State University, Tallahassee, Florida 32306. *Groups of divisibility and symmetric algebras.* Preliminary report.

The order exact sequence of (multiplicative) partially ordered groups, $\{1\} \to G \to H \to J \to \{1\}$, splits cardinally if H is 0-isomorphic to the cardinal sum of G and J. An equivalent formulation in terms of splitting maps is immediate, and cardinal splitting is readily translated, via the notion of group of divisibility of an integral domain, to a corresponding formulation for integral domains. See Abstract 699-A27, these Notices 19(1972), A-788. *Theorem* 1. Let A be an integral domain with quotient field K, and X a collection of indeterminates. The following are equivalent: (1) A is a GCD-domain. (2) The group of divisibility of $A[X]$ is the cardinal sum of the groups of divisibility of A and of $K[X]$. *Theorem* 2. Let A be a GCD-domain and M a pure submodule of a free A-module. Then the group of divisibility of the symmetric algebra $S_A(M)$ is the cardinal sum of the groups of divisibility of A and of $S_K(M \otimes_A K)$. If, in addition, A is a UFD, then $S_A(M)$ is a UFD. (Received January 10, 1974.)

712-A6. JERRY SCOTT, Georgia College, Milledgeville, Georgia 31061. *Monotonic permutations and pairing.*

An opp (orp) of a chain C is an order-preserving (reversing) permutation of C. A monotonic permutation of C is either an opp or an orp. The collection $M(C)$ of all monotonic permutations forms a group in which $A(C)$

has index 2. A subgroup K of $M(C)$ is called 1-monotonic if $G = G(K) = K \cap A(C)$ is a transitive 1-subgroup of A and K contains an orp. *Theorem* 1. $K' = K \setminus G$ is a lattice o-isomorphic to G. *Theorem* 2. There exist orps in K' which square to the identity. If $c \in C$ and N is a subgroup of M, $N_c = \{n \in N | cn = c\}$. The orbit of N containing d is $\{dn | n \in N\}$. If D is a G_c-orbit, the paired G_c-orbit of $D = D' = \{cg | dg = c \text{ for some } d \in D\}$. It is well known that pairing provides an o-anti-isomorphism of the G_c-orbits and that $D'' = D$. *Theorem* 3. If K is 1-monotonic, an orbit of K_c is the union of a $(G(K))_c$ G_c-orbit and its paired G_c-orbit. Thus 1-monotonic groups have the pairing-property. There exist c-monotonic groups ($c < d$ implies there exists $g > 1$ in $G(K)$ such that $cg = d$) without the pairing property. (Received December 13, 1973.)

*712-A7. PAUL M. EAKIN, University of Kentucky, Lexington, Kentucky 40506 and WILLIAM J. HEINZER, Purdue University, West Lafayette, Indiana 47907. *Ideal transforms of 2-dimensional noetherian domains.*

Let R be an integral domain with quotient field K. If $\mathfrak{A}$ is an ideal of R, then the ideal transform of R at $\mathfrak{A}$ or $\mathfrak{A}$-transform of R, $T_R(\mathfrak{A}) = \{x \in K | x\mathfrak{A}^n \subseteq R \text{ for some positive integer } N\}$, is said to be finite if $T_R(\mathfrak{A})$ is finitely generated as a ring extension of R. *Theorem* 1. If R is a 2-dimensional noetherian domain such that for each maximal ideal P of R the derived normal ring of R_P is finite, local, and analytically irreducible, then for any nonzero ideal $\mathfrak{A}$ of R the $\mathfrak{A}$-transform of R is finite. *Theorem* 2. If R is a 2-dimensional local Cohen-Macaulay domain such that the integral closure of R is no longer local, then there exist nonzero ideals $\mathfrak{A}$ of R such that the $\mathfrak{A}$-transform of R is not finite. Theorem 2 yields examples of almost noetherian rings (in the sense of Krull) which are not noetherian and for which the integral and complete integral closure do not coincide. *Corollary.* Let A be a 1-dimensional noetherian domain such that the integral closure $\overline{A}$ of A is a finite A-module and let $R = A[X]$ be a polynomial ring in one variable over A. Then all ideal transforms of R (at nonzero ideals) are finite if and only if there is no splitting between A and $\overline{A}$, i.e. lying above any maximal ideal of A there is only one maximal ideal of $\overline{A}$. (Received January 11, 1974.)

*712-A8. JOHN L. HUNSUCKER and CARL POMERANCE, University of Georgia, Athens, Georgia 30602. *Odd super perfect numbers.*

Let σ be the sum of the divisors function. Following Suryanarayana (Elem. Math. 24(1969), 16–17) we call a natural number n *super perfect* if $\sigma(\sigma(n)) = 2n$. Suryanarayana (op.cit.) classified all even super perfect numbers and questioned the existence of odd super perfect numbers. This question appears to be as hard as the corresponding one for odd perfect numbers. Kanold (Elem. Math. 24(1969), 61–62) proved that if an odd super perfect number exists, it must be a square. Let $\omega(n)$ be the number of distinct prime divisors of n. Among the theorems we have proven are: If n is an odd super perfect number, then (i) $\omega(n) \geq 2$, (ii) $\omega(\sigma(n)) \geq 2$, (iii) $\omega(n) + \omega(\sigma(n)) \geq 7$, (iv) $\omega(n\sigma(n)) \geq 5$, (v) $n > 7 \cdot 10^{24}$. We also discuss several connections between perfect numbers and super perfect numbers. In particular, we answer the following outstanding question posed by Suryanarayana (Bull. Amer. Math. Soc. 5(1970), 977) about odd perfect numbers: Let $m = p^a k^2$ be an odd perfect number where p is a prime, $p \nmid k$, and $p = a \equiv 1 \pmod 4$ (every odd perfect number is in such a form); does it necessarily follow that $\sigma(k^2) = p^a$ and $\sigma(p^a) = 2k^2$? However, if these equations hold, k^2 would be an odd super perfect number, contradicting (ii) above. (Received January 11, 1974.)

712-A9. HAROLD S. FINKELSTEIN, Emory University, Atlanta, Georgia 30322. *Enumerating automorphism orders in groups.* Preliminary report.

Let G be a finite group and let $\mathrm{Aut}(G)$ be its automorphism group. *Definition.* If $x \in G$ and $\theta \in \mathrm{Aut}(G)$, the automorphism order of x w.r.t. θ, denoted $\gamma_\theta(x)$, is $\min\{k: x\theta(x)\theta^2(x)\cdots\theta^k(x) = 1\}$, where $\theta(x)$ is the image of x under θ and $\theta^i(x) = \theta(\theta^{i-1}(x))$. $i \geq 2$. Using a method of McKay (Amer. Math. Monthly 66(1959), 119) the following generalization of Cauchy's theorem is valid. *Theorem.* If $|\theta| / p / G$, where p is a prime and $|\theta|$ is the order of θ, then $\exists\, x \in G$ s.t. $\gamma_\theta(x) = p$. *Notation.* Let $b[\theta; s, k](G) = |\{x \in G: k\ \gamma_\theta(x) / sk\}|$ where $\theta \in \mathrm{Aut}(G)$. *Theorem.* If $G = C_{p^n}$, a cyclic group of odd prime power order, then $b[\theta; s, k](C_{p^n}) = (s-1)k + \phi(k)$ for $|\theta| / p^n$ and sk / p^n, where ϕ is the Euler phi function. (Received January 14, 1974.)

712-A10. EDWARD DAVID ELGETHUN, University of North Florida, Jacksonville, Florida 32216. *Subgroups of central separable algebras with no zero divisors.*

A group G is without fixed points if G has a representation $g \to M_g$ by matrices with the property that 1 is a characteristic root of the matrix M_g iff g is the identity of G. *Theorem.* Choose R a commutative ring of characteristic zero, A a central separable R-algebra with no zero divisors, G a group of order n and ϕ an R-algebra homomorphism from RG onto A which is 1-1 on G. Then G is without fixed points. (Received January 14, 1974.)

712-A11. FLOYD L. CHRISTIAN, JR., Roane State Community College, Harriman, Tennessee 37748 and JAMES J. JOHNSON, University of Mississippi, University, Mississippi 38677. *A k-measure of irreducibility of doubly stochastic matrices.* Preliminary report.

We define k-measure of irreducibility of a matrix $A \in R_n$ as $\mu_k(A) = \min_{M \in \mathfrak{M}_k} \sum_{i \in M, j \notin M} a_{ij} = \min_{M \in \mathfrak{M}_k} (e_M, Ae_{M'})$, where $1 \le k \le n-1$ and $\mathfrak{M}_k = \{M: \varnothing \neq M \subsetneq \{1, 2, \cdots, n\}$ and $\overline{\overline{M}} = k\}$. If $A \ge 0$ and $\sum_{i,j=1}^{n} a_{ij} = n$, then $0 \le \mu_k(A) \le k(n-k)/(n-1)$, and we obtain *Theorem.* (1) Suppose $n \ge 2$, n even, $A \in \Omega_n$, the set of $n \times n$ doubly stochastic matrices, and $1 \le k \le n/2$. If $\mu_k(A) > \max\{(k-1)(n-k)/(n-2), k(n-2k)/(n-2)\}$, then A is irreducible. (2) Suppose $n \ge 3$, n odd, $A \in \Omega_n$, and $1 \le k \le (n-1)/2$. If $\mu_k(A) > \max\{(k-1)(n-k)/(n-2), k(n-2k-1)/(n-3)\}$, then A is irreducible. *Theorem.* Suppose $n \ge 4$, S is $n \times n$ stochastic, and for some integer k it is true that $\mu_k(S) = k(n-k)/(n-1)$. Then (1) $\mathrm{Tr}(S) = 0$. (2) $S \in \Omega_n$. (3) If S is symmetric, then $S = (J-I)/(n-1)$, where $J = ee^t$. *Theorem.* Suppose $n \ge 3$, $A \in \Omega_n$ and is symmetric, and g is the second largest eigenvalue of A. Then $1 - g \le (n/k(n-k))\mu_k(A)$. *Theorem.* Suppose $n \ge 3$, $1 \le k \le n-1$, and $A \in \Omega_n$ is symmetric. If g_2 and g_n are the second largest and smallest eigenvalue of A, respectively, then (1) $1 - g_n \ge (n/k(n-k)) \max_{M \in \mathfrak{M}_k} (e_M, Ae_{M'})$; (2) $g_2 - g_n \ge (n/k(n-k))\{\max_{M \in \mathfrak{M}_k} (e_M, Ae_{M'}) - \mu_k(A)\}$. Other results are also obtained. (Received January 14, 1974.)

712-A12. JAMES F WARD, JR., University of North Florida, Jacksonville, Florida 32216. *Minimum seminorm properties of the singular-weighted matrix pseudoinverse.*

Partially order the real n-square matrices by defining $A \le B$ if $B - A$ is positive semidefinite. With respect to this partial ordering, we show that the singular-weighted matrix pseudoinverse X of A has the following seminorm properties: (1) $\|I - AX\|_T = \min \|I - AY\|_T$ and (2) $\|I - A^t X^t\|_S = \min \|I - A^t Y^t\|_S$, where $\|B\|_Q = (\mathrm{tr}\, B^t Q B)^{1/2}$, S and T are given positive semidefinite matrices, and minima are taken over all semi-inverses Y of A. (Received January 14, 1974.)

712-A13. PAUL F. CONRAD, University of Kansas, Lawrence, Kansas 66044. *a^*-closures of lattice-ordered groups.* Preliminary report.

By definition an l-group H is an a^*-extension of G if G is an l-subgroup of H and $C \to C \cap G$ is a $1-1$ map of the closed convex l-subgroups of H onto those of G. In general, $C \cap G$ need not be closed, but it is provided that the map is $1-1$. To prove this one uses the following two results: G is large in H iff for each nonzero closed convex l-subgroup C of H, $C \cap G \neq 0$; if G is large in H then all the joins and intersections in G agree with those in H. If we restrict our attention to abelian groups then in all the known cases where G has a unique a^*-closure H it can be shown that H is a #-extension of the divisible hull G^d of G (i.e., for each $0 < h \in H$ there is a subset $\{g_i \mid i \in I\}$ of G such that h and $\bigvee_H g_i$ are archimedean equivalent). This concept of #-extension simplifies most of the proofs in the theory of a^*-extensions and leads to many new results. (Received January 15, 1974.)

*712-A14. STEPHEN H. McCLEARY, University of Georgia, Athens, Georgia 30602. *The structure of ordered permutation groups applied to lattice-ordered groups.*

Let S be a chain and let G be a (not necessarily transitive) lattice-ordered group of order-preserving permutations of S. The structure theory for such groups, which involves a root system of convex congruences each having an associated "o-primitive component," will be used to discuss problems involving closed convex l-subgroups of G. As is well known, every abstract l-group can be represented as an ordered permutation group. Here the permutation group structure will be used to identify the closed values of G and to discuss their properties (normal, essential, special, etc.). (Received January 15, 1974.)

*712-A15. TREVOR EVANS, KENNETH I. MANDELBERG and MARY M. NEFF, Emory University, Atlanta, Georgia 30322. *Embedding algebras with a solvable word problem in simple algebras — some Boone-Higman type theorems.*

W. W. Boone and G. Higman have proved ("An algebraic characterization of groups with a soluble word problem", J. Austral. Math. Soc., to appear) that a recursively presented group has a solvable word problem if and only if it can be embedded in a simple group which is embeddable in a finitely presented group. They also prove the corresponding theorem for semigroups. In this paper some further examples of this type are given. It is shown that for groupoids, loops, lattices and rings of characteristic p, a recursively presented algebra (of one of these types) has a solvable word problem if and only if it is embeddable in a recursively presented simple algebra. There is obviously no such theorem for rings without the characteristic assumption, commutative rings, commutative semigroups, abelian groups. Nor is it possible to embed the simple algebra in a finitely presented algebra (as in the Boone-Higman theorem) in the cases of groupoids, loops and lattices. We do not know the situation for rings. (Received January 15, 1974.)

*712-A16. ALAN S. COVER and ANDREW SOBCZYK, Clemson University, Clemson, South Carolina 29631. *Factorization of Mersenne numbers.*

Mersenne numbers are integers of the form $M_n = (2^n - 1)$ for n a positive integer. (In the binary notation, the digits of M_n are all 1's, and M_n has n digits.) It is known that for $n \leq 257$, the M_n are primes only for $n = 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127$, and that M_{521} is a prime. In this paper, by adaptation from decimal to binary of the second-named author's proof of a theorem announced in Abstract 709-A40, these Notices 20(1973), A-656, the following is proved: each odd integer m divides a Mersenne number for some n (in case m = a prime $M_{n'}$ m divides $M_{k \cdot n}$ for $k = 1, 2, \cdots$). Using this theorem and a computer, it should easily be possible to extend the list of Mersenne primes and nonprimes. The number M_{91} has factors M_{13}, M_7, and obvious binary "cofactors"; since M_7 divides the cofactor of M_{13}, and M_{13} the cofactor of M_7, complete factorization of M_{91} is quickly achieved. (Received January 15, 1974.)

712-A17. DON M. JORDAN, University of South Carolina, Columbia, South Carolina 29208. *Completely positive matrices.*

We give several necessary and sufficient conditions for a matrix to be completely positive, and as a consequence, illustrate a simple method of determining if a given $n \times n$ matrix is *not* completely positive and demonstrate this method with the classical example given by Hall ("Combinatorial theory", Blaisdell Publ. Co., Mass., 1967, Chapter 16, pp. 252–289). (Received January 15, 1974.)

Analysis

*712-B1. M. ZUHAIR NASHED, Georgia Institute of Technology, Atlanta, Georgia 30332 and GEORGE F. VOTRUBA, University of Montana, Missoula, Montana 59801. *Transformations of generalized inverses under changes of projectors.*

We investigate a unified approach to generalized inverses (g.i.) of linear operators both in the algebraic and topological contexts. This approach makes it possible to relate many of the definitions within the hierarchy of various g.i. Let V and W be (real or complex) vector spaces, and let L be a linear operator from V into W. The range and null spaces of L are denoted by $R(L)$ and $N(L)$. Given $P^2 = P: V \to N(L)$, and $Q^2 = Q: W \to R(L)$, both linear and onto, there is a g.i. $M = L^{\dagger}_{P,Q}$ relative to P and Q, which is the unique solution of $LX = Q$, $XL = I - P$, and $XLX = X$. *Theorem.* (a) If we change P and Q to some other (algebraic) projectors P' and Q', where $R(P') = N(L)$ and $R(Q') = R(L)$, then $L^{\dagger}_{P',Q'} = (2I - ML - P')M(2I - LM + Q')$. (b) Given two g.i. $L^{\dagger}$ and $L^{\#}$, the four projectors $Q = LL^{\dagger}$, $P = I - L^{\dagger}L$, $Q' = LL^{\#}$, and $P' = I - L^{\#}L$ are related as follows: $P' = P + A$, $Q' = Q + B$, where $R(A) \subset N(L) \subset N(A)$, and $R(B) \subset R(L) \subset N(B)$. Equivalently, $P' = P + (L^{\dagger} - L^{\#})L$ and $Q' = Q + L(L^{\#} - L^{\dagger})$. Topologies on V and/or W are induced to study g.i. of arbitrary linear operators on topological linear spaces, together with proximinal properties of g.i. (Received November 1, 1973.)

*712-B2. DAVID L. LOVELADY, Florida State University, Tallahassee, Florida 32306. *Linear evolution operators and the embedding of time-dependent problems into semigroup theory.*

The formula $S(t)[f](r) = U(r, r-t)f(r-t)$ allows us to embed a linear nonexpansive strongly continuous evolution operator U over a Banach space X into a linear nonexpansive strongly continuous semigroup S over the space $\mathcal{C}_{00}$ of all functions f from R to X such that $\|f(-r)\| + \|f(r)\| \to 0$ as $r \to \infty$. Thus with each evolution operator U over X we can associate a generator over $\mathcal{C}_{00}$. The connection between this idea and the more traditional idea of associating evolution operators with time-dependent evolution equations is shown to come through Trotter-Chernoff product limits. Numerous results are given which exploit this technique. (Received November 19, 1973.)

*712-B3. ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620. *Global controllability of perturbed quasi-linear systems.* Preliminary report.

Conditions are given on the matrices $A(t, x, u)$, $B(t, x, u)$ and the vector $Q(t, x, u)$, ensuring the controllability of the system $(*)$ $x' = A(t, x, u)x + B(t, x, u)u + Q(t, x, u)$. The control is continuous and such that $u(0) = 0$. The problem is reduced to solving a system of two integral equations. Schauder's fixed point theorem is applied on a suitable product space. Recent results of Davison and Kunze, Lukes are extended. (Received November 23, 1973.)

712-B4. JAMES R. DORROH, Louisiana State University, Baton Rouge, Louisiana 70803. *Fréchet type differentiability for nonlinear semigroups.* Preliminary report.

Let Y be a Banach space which is densely and continuously contained in the Banach space X, let D denote an open set in Y, and let C denote the closure of D in X. Let $\{T_t\}$ denote a strongly continuous semigroup of nonlinear nonexpansive transformations in D, let A denote the infinitesimal generator of $\{T_t\}$, and suppose that the domain of A contains D. Suppose that A is differentiable in the sense that for each f in D, there is a bounded linear transformation $W(f)$ from Y into X such that $A(f+g) = Af + W(f)g + \|g\|_X \theta(g)$, where $\|\theta(g)\|_X \to 0$ as $\|g\|_Y \to 0$. With a few more restrictions on the semigroup $\{T_t\}$ and the linear transformations $W(f)$, it follows that the transformations T_t possess derivatives in the sense that if f is in D and $t \geq 0$, then there is a bounded linear transformation $V(t, f)$ from X into X such that $T_t(f+g) = T_t f + V(t, f)g + \|g\|_X \theta(g)$, where $\|\theta(g)\|_X \to 0$ as $\|g\|_Y \to 0$. Although this notion of differentiability is weaker than the standard notion of Fréchet differentiability, it still has interesting consequences, and there seem to be interesting cases where this notion obtains and the standard one does not. It is not essential that C be open in Y nor that the semigroup $\{T_t\}$ be nonexpansive. (Received December 6, 1973.)

*712-B5. WILLIAM H. RUCKLE, Clemson University, Clemson, South Carolina 29631. *Hadamard multipliers of infinite matrices.*

For L a space of infinite matrices let $M(L)$ consist of all matrices $G = (G_{ij})$ such that $(G_{ij}F_{ij}) \in L$ for each F in L. Let S and T be sequence spaces in which the coordinate vectors (e_j) form an unconditional basis. If θ is a cross norm satisfying a common homogeneity condition then $M(S \otimes_\theta T)$ contains the space of all matrices of integral mappings from l_1 into m. Here $S \otimes_\theta T$ has its obvious representation as a space of matrices. If L denotes the space of matrices of continuous, compact, nuclear or integral mappings from S into T then $M(L)$ contains $\mathfrak{D}$, the set of all matrices having finite squares of ones along the diagonal and 0's elsewhere. (Received December 7, 1973.)

*712-B6. JEROME A. GOLDSTEIN, Tulane University, New Orleans, Louisiana 70118. *Contraction semigroups on Hilbert space.* Preliminary report.

Let $T = \{T(t): t \geq 0\}$ be a strongly continuous (or (C_0)) semigroup of linear operators on a Banach space $(X, \|\cdot\|)$. Then T satisfies $\|T(t)\| \leq Me^{bt}$ for all $t \geq 0$ with suitable constants b and M, where $M \geq 1$. Many years ago Feller observed that one can replace $\|\cdot\|$ by an equivalent norm $|\cdot|$ so that $|T(t)| \leq e^{bt}$ holds

for all $t \geq 0$; i. e., M can be taken to be 1. If $(X, \|\cdot\|)$ is a Hilbert space, Feller's choice of $|\cdot|$ is not necessarily an inner product norm. (Q) Does the theory of (C_0) semigroups on Hilbert space reduce to the theory of (C_0) contraction semigroups on Hilbert space? The answer to (Q) would help to give a perspective for viewing the theory of nonlinear semigroups on Hilbert space as a complete generalization of the linear theory. The answer to (Q) depends on exactly how the question is phrased. A result of E. Packel [Proc. Amer. Math. Soc. 21(1969), 240–244] suggests that the answer to (Q) is *no*. We have proved a theorem which suggests that the answer to (Q) is *yes*. (Received December 13, 1973.)

712-B7. GORDON G. JOHNSON, University of Houston, Houston, Texas 77004. *Homeomorphisms, iteration and linear independence.*

If h is a homeomorphism of the number interval $[0, 1]$ onto $[0, 1]$ such that $h \circ h$ is not the identity on $[0, 1]$ then $\{h^n: n \text{ is an integer}\}$ is a linearly independent subset of $C_{[0,1]}$ when $h^n = h \circ h \cdots \circ h$, n times and $h^{-n} = h^{-1} \circ \cdots \circ h^{-1}$ n times if $n > 0$. These ideas are extended to flows on $[0, 1]$. (Received January 2, 1974.)

* 712-B8. BRUNO J. WICHNOSKI, Tulane University, New Orleans, Louisiana 70118. *On the existence of wave operators for time dependent nonlinear evolution equations.* Preliminary report.

This paper is concerned with the existence of wave operators for the pair of time dependent nonlinear evolution equations (1) $du/dt = A_i(t)u$, $i = 0, 1$. We postulate the existence of wave operators for the (not necessarily) linear system of time independent evolution equations (2) $du/dt = A_i(\pm\infty)u$, $i = 0, 1$. By imposing decay conditions on $A_i(t) - A_i(\pm\infty)$ on orbits of (2) we get sufficient conditions for the existence of the wave operators governing (1). Also the usual intertwining relations and chain rule are proved for the wave operators of (1). (Received December 28, 1973.) (Author introduced by Professor Jerome A. Goldstein.)

* 712-B9. JAMES T. SANDEFUR, JR., Tulane University, New Orleans, Louisiana 70118. *Higher order abstract Cauchy problems.* Preliminary report.

Let $A_1, \cdots, A_n$ be a set of linear operators on an arbitrary Banach space X. Using semigroup methods, a necessary and sufficient condition for the abstract Cauchy problem (1) $(d/dt - A_n)(d/dt - A_{n-1})\cdots(d/dt - A_1)u(t) = 0$; (2) $u^{(j)}(0) = \phi_j \in X$ for $j = 0, 1, \cdots, n-1$, and consequently, a sufficient condition for (3) $P(d/dt, A)u(t) = \sum_{j=1}^{m}\sum_{k=1}^{n} c_{jk}(d/dt)^j A^k u(t) = 0$, (4) $u^{(j)}(0) = \phi_j \in X$ for $j = 0, 1, \cdots, n-1$, to be "well posed is given. Perturbation and stability theorems are proved, and a unique solution to the inhomogeneous problem is given. Finally these results, which generalize those of Hersh, Mizohata, Mochizuki, et al, are extended to the case in which the operators A_i depend on time. (Received December 26, 1973.) (Author introduced by Professor Jerome A. Goldstein.)

* 712-B10. JAMES A. RENEKE, Clemson University, Clemson, South Carolina 29631. *The input-output functions for a class of hereditary systems.* Preliminary report.

Suppose that G is a linear space of functions from an interval S' of numbers containing $S = [0, \infty)$ into a linear space X, and $\mathcal{H}$ denotes a class of functions from G into G. A function W (alternately, K): $S \times S \to \mathcal{H}$ is a *transition function on* G (an *integral on* G) provided (i) $W(v, x)W(x, u) = W(v, u)$ $(K(u, x) + K(x, v) = K(u, v))$ when $0 \leq u \leq x \leq v$, (ii) $[W(v, u)f](x) = f(x)$ $([K(u, v)f](x) = 0)$ for each f in G, subinterval $[u, v]$ of S, and x in S' which does not exceed u, and (iii) if $[u, v]$ is a subinterval of S, $\{f, g\}$ is an ordered pair in $G \times G$, and $f(x) = g(x)$ for each x in S' which does not exceed v, then $W(v, u)f - f = W(v, u)g - g(K(u, v)f = K(u, v)g)$. A function $A: G \to G$ is an *input-output function for a hereditary system* if $\exists$ a function W (or K): $S \times S \to \mathcal{H}$ which is a transition function on G (an integral on G) $\ni$ an ordered pair $\{f, b\}$ of $G \times G \in A$ only if $b(u) = f(u)$ when $u \leq 0$ and $b(u) = [W(u, 0)f](u)$ $(b(u) = f(u) + [K(0, u)b](u))$ when $0 \leq u$, for each u in S'. Necessary and sufficient conditions are obtained for a function $A: G \to G$ to be an input-output function for a restricted type of hereditary system satisfying a Lipschitz condition. (Received December 26, 1973.)

*712-B11. SAMUEL M. RANKIN III, Florida Institute of Technology, Melbourne, Florida 32901. *Oscillation theorems for second order nonhomogeneous linear differential equations.* Preliminary report.

Let $r(t) > 0$, $p(t)$, and $f(t)$ be continuous real-valued functions on $[a, \infty)$ for some real number a. *Theorem* 1. If (1) $(r(t)u')' + p(t)u = 0$ is oscillatory, and $f(t)$ is oscillatory such that the distance between consecutive zeros of each solution of (1) is less than or equal to the distance between consecutive zeros of $f(t)$, then (2) $(r(t)y')' + p(t)y = f(t)$ is oscillatory. *Theorem* 2. If (1) is oscillatory then all the nonoscillatory solutions of (2) eventually have the same sign. The method of proof of Theorem 1 follows techniques used by M. S. Keener (Applicable Anal. 1(1971), 57–63), while Theorem 2 is an extension of one of his results. (Received January 11, 1974.)

*712-B12. J. R. BLUM and ROBERT F. COGBURN, University of New Mexico, Albuquerque, New Mexico 87131. *On ergodic sequences of measures.* Preliminary report.

Let G be a σ-compact, locally compact abelian group. A sequence $\{\mu_n\}$ of probability measures defined on the Borel sets of G is called ergodic if μ_n converges weakly to m, where m is Haar measure on the Bohr compactification of G. Let μ be Haar measure on G. It can be shown that if $\{A_n\}$ is a sequence of Borel subsets of G such that $\lim_n(\mu(A_n \cap A_n g)/\mu(A_n)) = 1$, for every $g \in G$, where $A_n g$ is A_n translated by g, then the sequence $\{\mu_n\}$ defined by $\mu_n(B) = \mu(A_n \cap B)/\mu(A_n)$ is an ergodic sequence. We construct examples of (random) sequences of sets $\{A_n\}$ such that $\lim_n(\mu(A_n \cap A_n g)/\mu(A_n)) = p$, for every p with $0 \le p \le 1$, and the corresponding sequence $\{\mu_n\}$ is ergodic. (Received January 8, 1974.)

*712-B13. JOHN W. HEIDEL, University of Tennessee, Knoxville, Tennessee 37916 and GARY D. JONES, Murray State University, Murray, Kentucky 42071. *Asymptotic characterization of solutions of a nonlinear boundary value problem arising in fluid mechanics.*

The boundary value problem $y''' + yy'' + \mu y'^2 = 0$, $y(0) = y'(0) = y'(\infty) = 0$ is considered. It is known (Heidel, A. Angew. Math. Mech. 53(1973), 167–170) that there can be a whole family of solutions if $\mu > 0$. It is now shown that uniqueness fails precisely if $0 < \mu \le 1$ or if $\mu = 2$. In these cases there is a one parameter family of solutions where the parameter is related to the asymptotic behavior at infinity. (Received January 10, 1974.)

712-B14. DAVID SHIEN LIANG, Washington University, St. Louis, Missouri 63130. *Toeplitz operators on compact symmetric spaces.* Preliminary report.

Notation. G is a compact group with a normalized Haar measure $m \ni m(G) = 1$; K a closed subgroup of $G \ni G/K$ is a symmetric space; $x \to U_\alpha(x)$, $\alpha \in A$, a complete list of unitary irreducible representations on G of class one relative to K; $d(\alpha)$ the dimension of the space on which $U_\alpha(x)$ acts, and $\phi_\alpha(x)$ the zonal spherical function corresponding to U_α. For each $\epsilon > 0$, let $\infty > \pi(\epsilon) \subseteq A$, $D(\epsilon, x) = \Sigma_{\pi(\epsilon)} d(\alpha)\phi_\alpha(x)$, and $\pi(\epsilon)^\# = D(\epsilon, e) = \Sigma d(\alpha)$. For real $f \in L^1[G/K]$, let $T_f^\epsilon(g)(x) = \int_G[\int_G D(\epsilon, z^{-1}x)f(z)D(\epsilon, y^{-1}z)\,dz]g(y)\,dy$ for $g \in L^2[G/K]$. T_f^ϵ, the Toeplitz operator of index ϵ generated by f, is selfadjoint of rank $\pi(\epsilon)^\#$. $N_f^\epsilon[a, b]$ denotes the number of eigenvalues of T_f^ϵ in $[a, b]$. We show that $N_f^\epsilon[a, b]/\pi(\epsilon)^\# \to m\{x: a \le f(x) \le b\}$ as $\epsilon \to 0$, provided that for each $\alpha \in \hat{A}$, $\lim_{\epsilon \to 0} Q\hat{}(\epsilon, \alpha)/\pi(\epsilon)^\# = 1$, where $Q\hat{}(\epsilon, \alpha) = \int_G |D(\epsilon, x)|^2 \overline{\phi_\alpha(x)}\,dx$. The last condition is necessary and sufficient. Let (U, du) be a measure space and $x \to F(x; u, v)$ a measurable function from G into the selfadjoint Hilbert-Schmidt kernels acting on $L^2(U) \ni \int_G dx \int_U \int_U |F(x; u, v)|^2\,du\,dv < \infty$. Then the theorem extends to $(T_F^\epsilon g)(u, x) = \int_U \int_G [\int_G D(\epsilon, z^{-1}x)F(x; u, v)D_\epsilon(y^{-1}z)\,dz]g(y, v)dy\,dx$ on $L^2[U \times G]$. (Received November 29, 1973.)

712-B15. LEONARD J. LIPKIN, University of North Florida, Jacksonville, Florida 32216. *On the operator* $L = \Delta r^2 + \mu(\partial/\partial r)r + \lambda$.

Let $x = (x_1, \cdots, x_n) \in \mathbf{R}^n$, $r^2 = \Sigma x_i^2$, $(\lambda, \mu) \in \mathbf{C}^2$, $\Lambda = \Sigma \partial^2/\partial x_i^2$, and define the singular elliptic operator L by $Lu - L(\lambda, \mu)u = \Lambda r^2 u + \mu(\partial/\partial r)ru + \lambda u$. We consider the equation (*) $Lu = f$ in a neighborhood of the origin in $\mathbf{R}^n$. In collaboration with M. S. Baouendi and C. Goulaouic we have proved: (1) if $f = 0$, every $\mathcal{C}^\infty$ solution of (*) is analytic; (2) if $(\lambda, \mu) \notin \Sigma(L)$, where $\Sigma(L)$ is a completely determined exceptional set in $\mathbf{C}^2$,

for every f analytic in a neighborhood of the origin there exists a unique solution u of (*) analytic in a neighborhood of the origin; (3) if $(\lambda, \mu) \in \Sigma(L)$ we give a complete description of the kernel of L and compatibility conditions for solvability of (*); (4) L is not hypoelliptic, but if u is C^∞ and Lu is analytic, then u is analytic. (Received January 11, 1974.)

*712-B16. MANFRED STOLL, University of South Carolina, Columbia, South Carolina 29208. *Harmonic majorants for plurisubharmonic functions on bounded symmetric domains with applications to the spaces H_ϕ and N_*.*

Let D be a bounded symmetric domain in $\mathbf{C}^n$ with Bergman-Šilov boundary B and $0 \in D$. Denote by μ the normalized K-invariant measure on B, where K is the isotropy subgroup at 0 of the group of holomorphic automorphisms of D. *Theorem* 1. Let f be plurisubharmonic on D. (a) If $\sup_{0<r<1} \int_B f_r^+ d\mu < \infty$, there exists a minimal regular Borel measure ν_f on B, called the *boundary measure* of f, such that $f(z) \leq \int_B P(z, t)\, d\nu_f(t)$ for all $z \in D$. P denotes the Poisson kernel on $D \times B$. (b) If the family $\{f_f^+: 0 < r < 1\}$ is uniformly integrable on B then $\sigma_f \leq 0$, where σ_f denotes the singular part of ν_f. *Corollary.* Let f be plurisubharmonic on D satisfying $\sup_{0<r<1} \int_B f_r^+ d\mu < \infty$. Then if $d\nu_f = \hat{f} d\mu + d\sigma_f$ is the Lebesgue decomposition of ν_f, $\lim \sup_{r \to 1} \int_B |f_r - \hat{f}|\, d\mu \leq \int_B d|\sigma_f|$. *Theorem* 2. Let f be plurisubharmonic on D, ϕ strongly convex, and assume $\sup_{0<r<1} \int_B \phi(f_r)\, d\mu < \infty$. Then f has a boundary measure ν_f with $\sigma_f \leq 0$ and the boundary measure of $\phi(f)$ is absolutely continuous and equals $\phi(\hat{f}) d\mu$. The results are then applied to the spaces H_ϕ and N_* of holomorphic functions on D. (Received January 11, 1974.)

*712-B17. JIMMIE LEE JOHNSON, University of Illinois, Chicago, Illinois 60680. *The uniform continuity of certain translation semigroups.*

Consider the strongly continuous translation semigroup $\{S_h\}$: $(S_h f)(x) = f(x + h)$, $h \geq 0$, where f, defined on $(0, \infty)$, is square-summable with values in a Hilbert space K of finite dimension. Let L be a closed subspace of $L^2(0, \infty; K)$ such that $S_h L \subset L$ for all $h \geq 0$. The Fourier transform $\hat{L}$ of L decomposes H^2 as $\hat{L} \oplus QH^2$ where Q is a rigid analytic function in the upper half plane. For $\operatorname{im} s > 0$, $Q(s)$ is an operator on K, so it can be represented by a matrix with entries which are analytic in s. *Theorem.* $\{S_h\}$, if restricted to L, is uniformly continuous iff $\det Q(s)$ is analytic at infinity and does not vanish there. The result is obtained using the spectrum of the adjoint of the shift restricted to certain subsets of H^2. (Received January 14, 1974.)

712-B18. K. MICHAEL DAY, University of Michigan, Ann Arbor, Michigan 48104. *Measures associated with Toeplitz matrices generated by the Laurent expansion of rational functions.*

Let $f(z) = \Sigma a_m z^m$ be the Laurent expansion of a rational function, and $T_n(f) = (a_{i-j})_{i,j=0}^n$ be the Toeplitz matrices generated by the function f. Denote by σ_n the set of $n + 1$ eigenvalues of $T_n(f)$, $\sigma_n = \{\lambda_{n0}, \cdots, \lambda_{nn}\}$. Let $B = \{\lambda: \lambda = \lim \lambda_m, \lambda_m \in \sigma_{i_m}\}$, where $i_m \to \infty$. An identity for $\det(T_n(f - \lambda))$ has been derived [Abstract 709-B11, these Notices 20 (1973), A-659] which, using the techniques of Schmidt and Spitzer [Math. Scand. 8(1960), 15–38], allows one to show that B either is a point or consists of analytic arcs. Define a sequence of measures α_n, $\alpha_n(E) = (n + 1)^{-1} \Sigma_{\lambda_{ni} \in E} 1$, where $\lambda_{ni} \in \sigma_n$, and E is an arbitrary set in the λ-plane. If α is any weak limit of the measures α_n, it is shown to be unique. The identity for $\det(T_n(f - \lambda))$ may be used to show that α possesses at most two atoms and does so if the function f is of a certain form, and the weight of the atoms is determined. For all other points the analysis of Hirschman [Illinois J. Math. 11(1967), 145–159] may be applied which shows that the support of α is all of B. These results, along with those announced previously, complete for f a rational generating function, the results of which are obtained by the above mentioned authors when f is a Laurent polynomial, $f(z) = \Sigma_k^h a_n z^n$, $h, k \geq 1$. (Received January 14, 1974.)

*712-B19. RATHINDRA N. MUKHERJEE, University of Georgia, Athens, Georgia 30601, and DIPENDRA N. BHATTACHARYA, Clarkson College of Technology, Potsdam, New York 13676. *Stieltjes transform of functions satisfying the Lipschitz condition.* Preliminary report.

We deal with the Stieltjes transform of a function $\phi(t)$, $S[\phi, x]$ defined by $S[\phi, x] = f(x) =$

$\int_0^\infty (x+t)^{-1}\phi(t)\,dt$, $\phi(t) \in L\ (0, R)\ \forall\ R > 0$, and the integral exists. The jump operator is defined by $J[f, k; t] = 2t\pi^{\frac{1}{2}}k^{-\frac{1}{2}}(-1)^{k-1}/k!(k-2)! \cdot d^{2k-1}\ d^{2k-1}[t^k f(t)]$. The asymptotic behaviour of $J[f, k; t]$ w.r.t. k is discussed under the assumption that $\phi(t)$ satisfies the Lipschitz condition [for the Laplace transform of functions satisfying the Lipschitz condition, see Ditzian, Compositio Math. 22 (1970), 29–38]. For $f(x) = S[\phi, x]$, $\phi(t)$ is called the determining function and $f(x)$ the generating function of the transform. The behaviour of the determining function is studied under certain growth conditions for $J[f, k; t]$. This result is analogous to that of Ditzian for the Laplace transform. (Received January 14, 1974.)

*712-B20. J. S. Mac NERNEY, University of Houston, Houston, Texas 77004. *Finitely additive set functions. (I) Order-characterization of a prering of subsets of a set.*

Suppose that $\{E, \leq\}$ is an upper semilattice D, which is an upper extension of the nondegenerate partially ordered set R without a least element. It is shown that (1) and (2) are equivalent. (1) $\exists$ a function γ from R onto a collection Q of subsets of some set s.t. (a) if $u \in R$, and $\infty > Y \subseteq R$, then $u \leq \sup_D Y$ iff $\gamma(u)$ is covered by the γ-image of Y, and (b) if G is a finite collection of members of Q then there is a collection M of mutually exclusive members of Q s.t. each set in the collection G is filled up by a finite subcollection of M. (2) If $\infty > G \subseteq R$ then $\exists$ $M \subseteq R$ s.t. (i) if $\infty > X \subseteq M$ and $y \in M$ but $y \notin X$ then $\exists$ $t \in R$ s.t. $t \leq \sup_D X$ and $t \leq y$, and (ii) each element of G is the supremum in D of a finite subset of M. Proof that (1) is a consequence of (2) is effected in terms of (A) the set R'' to which P belongs iff $P \subseteq R$ which has, and is maximal w.r.t. having, the property that if $\infty > Y \subseteq P$ then $\exists$ $u \in R$ s.t., for each $w \in Y$, $u \leq w$, and (B) the function γ from R s.t. if $v \in R$ then $\gamma(v)$ is the subset of R'' to which P belongs iff $v \in P$. A *prering* is a collection Q of subsets of a set s.t. (1)(b) is satisfied. (Received January 14, 1974.)

712-B21. DOUGLAS MOREMAN, Emory University, Atlanta, Georgia 30322. *Convex topology.* Preliminary report.

A brief review of some generalizations to metric spaces and to more abstract settings of theorems relating the notions of weak convergence, weak topology, weak Cauchy sequence, centers, near-point properties, normal structure and fixed points, uniformly convex space, and so forth as mentioned in earlier abstracts in these Notices. New material includes properties of convex regularity and convex normality which are analogues of "separation properties" in the context of linear topological space and are also analogues of topological properties. (Received January 15, 1974.)

*712-B22. KENNETH R. KELLUM, Miles College, Birmingham, Alabama 35208. *Sums and limits of almost continuous functions.*

Suppose $f: A \to B$. If each open set in $A \times B$ which contains f also contains a continuous function $g: A \to B$, then f is said to be *almost continuous*. Suppose T is a collection of c-many real functions. We prove that there exists a function g such that $g + t$ is almost continuous for each t in T. We obtain as corollaries that each real function is the sum of two almost continuous functions and the point-wise limit of a sequence of almost continuous functions. This generalizes results of Fast (Colloq. Math. 7(1959), 75–77) and Phillips (Fund. Math. 75(1972), 47–49). We also point out that the uniform limit of a sequence of almost continuous functions need not have the Darboux property. (Received January 14, 1974.)

712-B23. GEORGE YU-HUA CHI, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. *On Radon-Nikodym theorems.* Preliminary report.

Let (Ω, Σ, μ) be a complete probability space, where Σ is a σ-algebra. Let $L_0^1(\Omega, \Sigma, \mu; F)$ be the locally convex space of all strongly integrable Borel measurable functions $f: \Omega \to F$ with the Egoroff property. Let F be a quasi-complete locally convex space with the following properties: (a) Every absolutely convex compact subset of F is metrizable, (b) F has property (C), i.e., for every bounded subset $B \subset l_N^1\{F\}$, the space of absolutely summable sequences, there exists an absolutely convex compact subset $M \subset F$ such that

$\Sigma_{i=1}^{\infty} p_M(x_i) \leq 1$, for every $(x_i)_{i=1}^{\infty} \in B$. Let $m: \Sigma \to F$ be a vector measure. *Theorem* 1. There exists an $f \in L_0^1(\Omega, \Sigma, \mu; F) \ni m = \mu_f$ iff (i) $m << \mu$, (ii) m has bounded variation, and (iii) m has locally relatively compact average range. This holds for the strong duals of metrizable Montel spaces, and hence in particular for the strong duals of Frechet-Montel spaces. It holds also for quasi-complete dual nuclear spaces. In fact, one has *Theorem* 2. Let F be a quasi-complete dual nuclear space. Then there exists $f \in L_0^1(\Omega, \Sigma, \mu; F) \ni m = \mu_f$ iff $m << \mu$. Theorem 2 applies to strong duals of nuclear barreled spaces, nuclear (F)-spaces, complete nuclear (DF)-spaces, and sequential projective limits of the respective latter spaces. (Received January 14, 1974.)

*712-B24. PHILIP C. TONNE, Emory University, Atlanta, Georgia 30322. *Continuous automorphisms on rings of infinite matrices.* Preliminary report.

Suppose that S is a perfect sequence space -- it is its own second dual. Let M denote the class of all infinite matrices which transform S to S. Let T be a sequentially continuous automorphism of M, i.e. T is a reversible (1-1) linear transformation from M onto M which preserves matrix multiplication. *Theorem.* T is an inner automorphism; i.e., $\exists\ U \in M$ which has a two-sided inverse U^{-1} s.t. if $A \in M$ then $T(A) = U^{-1}AU$. Let L be the class of all weakly (sequentially) continuous linear functions from S to S. Let T be a sequentially continuous automorphism of L. *Corollary.* T is an inner automorphism. (Received January 14, 1974.)

* 712-B25. F. A. ROACH, University of Houston, Houston, Texas 77004. *Continued fraction representation of points of an inner product space.*

A continued fraction method for the approximation of points of complete real inner product spaces is given. The continued fractions used are of the type introduced in "Continued fractions over an inner product space", Proc. Amer. Math. Soc. 24(1970), 576–582, and the approximation method is analogous to the one used for the representation of real numbers by regular continued fractions. Error bounds are determined which indicate that the rate of convergence is at least as good as in the real case, and a "uniqueness of representation" result is obtained. An example of this method in the real coordinate space l^2 is given. (Received January 15, 1974.)

Applied Mathematics

*712-C1. WILLIAM B. DAY, Auburn University, Auburn, Alabama 36830. *More eigenvalue estimates.* Preliminary report.

A numerical method for estimating the eigenvalues of the Sturm-Liouville problem has been developed. This method is limited to zero end-point boundary conditions and is based on a linear approximation of the coefficient function rather than the solution. (Received October 23, 1973.)

* 712-C2. LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834 and SUKLA MUKHERJEE, Center of Advanced Study in Applied Mathematics, University of Calcutta, Calcutta, India. *Inertial oscillations and multiple boundary layers in an unsteady rotating flow.*

An asymptotic analysis is made of the unsteady boundary layer flow generated impulsively in an incompressible homogeneous viscous fluid bounded by an infinite porous plate with uniform suction or blowing to describe the manner of the transient approach to the ultimate steady-state. It is shown that the initial motion for small time describes the general features of the unsteady boundary layers on the plate and consists of four boundary layers on the plate which are unaffected by rotation. In subsequent large times, the effect of the rotation manifests itself through inertial oscillations of frequency 2Ω and also the generation of diffused waves propagating with velocity $2(\Omega\nu)^{1/2}$ away from the boundary. It is found that the final steady boundary layers are established in a dimensional time $(4/\Omega)(S^4/4E^2 + 4)^{-1/2}$ which decreases with an increase of the suction parameter. Several limiting cases of interest are recovered. (Received December 31, 1973.)

712-C3. GARETH WILLIAMS, Stetson University, Deland, Florida 32720. *Fine topologies in relativity theory.* Preliminary report.

In the usual differentiable manifold model of space-time it is a pseudo-metric on the tangent bundle,

representing the gravitational field, that determines the "metric" geometry of the space-time. The topology of the manifold determines the allowable coordinate representations and gives a concept of continuity on space-time, but does play a secondary role and a role unrelated to the pseudo-metric. Are there topologies related to the pseudo-metric that can play a significant role in this theory? (Received December 26, 1973.)

712-C4. JOHAN G. F. BELINFANTE, Georgia Institute of Technology, Atlanta, Georgia 30332. *Orthomodularity and transition probability.* Preliminary report.

Transition probability spaces arise in axiomatic quantum mechanics (B. Mielnik, Comm. Math. Phys. 9(1968), 55–80). A subset of a transition probability space is *orthoclosed* if it is its own second orthogonal complement. The orthoclosed subspaces are partially ordered by inclusion. *Theorem.* The union of two orthogonal orthoclosed subspaces is an orthoclosed subspace. *Theorem.* If a subspace T_1 of a transition probability space is contained in a subspace T_2, and if the orthogonal complement of T_1 is disjoint from T_2, then the orthogonal complements of T_1 and T_2 are equal. *Corollary.* The poset of orthoclosed subspaces of a transition probability space is orthomodular. *Remarks.* There is an 8-element transition probability space whose poset of orthoclosed subspaces is not a lattice. This space has two 4-element orthoclosed subspaces whose intersection is an orthoclosed subset which is not a subspace. It also contains a subset whose orthogonal complement is not a subspace. (This settles the conjectures in Abstract 709-C5, these Notices 20(1973), A-668.) (Received January 10, 1974.)

712-C5. LAWRENCE A. KURTZ, Hollins College, Hollins College, Virginia 24020. *Properties of a semidiscrete approximation to a two-dimensional partial differential equation.* Preliminary report.

We study an approximation to the second order two-dimensional PDE: $u_t = [\alpha(x)u_x]_x + [\beta(y)u_y]_y$, $x \in [0, a]$, $y \in [0, b]$, $t \geq 0$, where $\alpha(x)$ and $\beta(y)$ are continuous and positive. Suitable boundary conditions are assumed given along the edges of the region, as well as some initial condition on $u(x, y, t)$ at $t = 0$. Wadsworth and Wragg investigate the solution of the classical two-dimensional heat conduction equation, a special case of the above with $\alpha(x) \equiv 1 \equiv \beta(y)$, by the method of lines, in which only the "space variables" x and y are discretized. This means that there is no trunction error in the t-direction, and the approximation is then equivalent to a linear system of ordinary differential equations in t. We shall investigate the inherent structure of the coefficient matrices involved in a semidiscrete approximation to the above equation, and show how this information leads us to a manageable and meaningful solution of the PDE. (Received January 11, 1974.) (Author introduced by Professor Claude C. Thompson.)

*712-C6. JAMES V. BLOWERS, (AFATL-DLYA), Eglin AFB, Florida 32542. *An adaptation of Powell's sigma-theta constrained nonlinear optimization method to accommodate inequality constraints.*

M. J. D. Powell ("A method for nonlinear constraints in minimization problems", Optimization, ed. Fletcher, pp. 283–298) describes a method for adapting an algorithm for minimizing a nonlinear function of several variables so that it will accommodate nonlinear equality constraints. The only problem in adapting the method to inequality constraints is in deciding whether an active inequality constraint (i.e., one that is satisfied as an equality) should be dropped from the current list of active constraints. He states that the sign of theta for the constraint in question decides whether it should be dropped. This presentation will demonstrate that this observation is correct and will describe a successful computer implementation of this variation of Powell's algorithm. (Received January 11, 1974.)

Statistics and Probability

712-F1. THOMAS C. GARD, University of Tennessee, Knoxville, Tennessee 37916. *Uniqueness criteria for Ito's equation.* Preliminary report.

Let $(\Omega, \mathcal{B}, P)$ be a probability space, and $\{\beta_t\}_{t \in [0,T]}$ a real Wiener process adapted to an increasing family of sub-σ-algebras of $\mathcal{B}$. The Ito equation (I) $x_t - x_0 + \int_0^t f(s, x_s)\,ds + \int_0^t g(s, x_s)\,d\beta_s$ has the pathwise uniqueness

property on $[0, T]$ if given solution processes x_t and y_t with $x_0 = y_0$, then $x_t = y_t$, $t \in [0, T]$ w.p.1. An analogue to a well-known result for ordinary differential equations which contains, as a special case, the classical Ito uniqueness criterion is *Theorem.* Assume there exists a nonnegative function $V(t, x, y)$ satisfying: (i) $V(t, x, y)$ is C^1 in t, and C^2 in (x, y) for $t \in [0, T]$, $x, y \in R$, (ii) $V(t, x, y) = 0$ if and only if $x = y$, (iii) $\partial V/\partial t \leq 0$ and, $(\partial V/\partial x)f(t, x) + (\partial V/\partial y)f(t, y) + \partial V/\partial t + \frac{1}{2}(\partial^2 V/\partial x^2)g^2(t, x) + \frac{1}{2}(\partial^2 V/\partial y^2)g^2(t, y) + (\partial^2 V/\partial x \partial y)g(t, x)g(t, y) \leq 0$. Then (I) has the pathwise uniqueness property for continuous w.p.1. solutions. *Corollary.* If $f(t, x)$ and $g(t, x)$ satisfy generalized one-sided Lipschitz conditions in x, then (I) has the pathwise uniqueness property for continuous w.p.1. solutions. Generalizations encompass a number of results including those of Levy, Yamada and Watanabe, and Skorohod. (Received January 2, 1974.)

*712-F2. JACK B. BROWN, Auburn University, Auburn, Alabama 36830. *Baire category in spaces of probability measures.*

For separable metric spaces X, let $M(X)$ denote the space of probability measures on X, where $M(X)$ is endowed with the weak-* topology (separability of X implies metrizability of this topology). Prohorov proved (Theor. Probability Appl. 1, pp. 157–214) that $M(X)$ is topologically complete iff X is. In general, topological completeness implies pseudo-completeness (i.e. existence of a dense completely metrizable subspace), which implies Baire completeness (i.e. no open sets are 1st category), which implies that the space is 2nd category, and the implications are not reversible. *Theorem* 1. If X is pseudo-complete, then $M(X)$ is pseudo-complete (but the converse fails). *Theorem* 2. If $M(X)$ is Baire complete, then X is Baire complete (but the continuum hypothesis implies that the converse fails). *Theorem* 3. The continuum hypothesis implies the existence of a subspace X of the reals such that $M(X)$ is Baire complete but not pseudo-complete. Parthasarathy, Rao, and Varadhan (Trans. Amer. Math. Soc. 102(1962), 200–217) utilized Prohorov's theorem to establish the abundance of the class of nonatomic measures on a complete, separable, dense in itself space X. This theorem extends to such spaces X which are not complete, but merely such that $M(X)$ is 2nd category. (Received January 7, 1974.)

*712-F3. A. T. BHARUCHA-REID, Georgia Institute of Technology, Atlanta, Georgia 30332. *Probabilistic operator theory.*

A survey of probabilistic operator theory, with particular reference to the following topics: (1) random operators on Banach spaces, (2) operator-valued random functions, (3) measure-theoretic problems, (4) spectral theory of random operators, (5) semigroups of random operators, (6) fixed point theorems, (7) operator-valued solutions of random differential and integral equations, and (8) limit theorems. (Received January 14, 1974.)

*712-F4. CHRIS P. TSOKOS and T. L. SMITH, University of South Florida, Tampa, Florida 33620. *On estimating the expected behavior of a stochastic epidemiological model applicable to small populations.*

Griffith [J. Appl. Probability 10(1973), 15–26] has obtained expressions for the stochastic means of a generalized linear bivariate birth-death process as a model for an epidemic involving a vector. His results are not applicable, however, to the case of a small susceptible population. The aim of the present paper is to give a method for approximating the stochastic means of multivariate birth-death processes. The results are in close agreement with those obtained by Griffith. The stochastic version of Bailey's model is shown to be an extension of Griffith's model which is applicable to small populations. Approximations are obtained for the expected behavior of the extended model. (Received January 15, 1974.)

Topology

*712-G1. COKE S. REED, Auburn University, Auburn, Alabama 36830. *Embedding theorems for dynamical systems.*

Suppose that s is a subspace of a metric space S, and r is a subset of the set of real numbers R such that $0 \in r$, and if $a \in r$ and $b \in r$, then $a + b \in r$. t is a dynamical system on $r \times s$ means that t is a

continuous transformation from $r \times s$ into s with the property that if $a \in r$, $b \in r$ and $p \in s$, then $t(a, t(b, p)) = t(a + b, p)$ and $t(0, p) = p$. A number of properties P, spaces r, s and S, and questions (some settled and some not) of the following form are discussed. Given a dynamical system t on $r \times s$, is there a dynamical system T on $R \times S$ such that T has property P and if $a \in r$ and $p \in s$, then $T(a, p) = t(a, p)$? (Received December 26, 1973.) (Author introduced by Professor J. W. Neuberger.)

* 712-G2. DARRELL C. KENT, Washington State University, Pullman, Washington 99163 and GARY D. RICHARDSON, East Carolina University, Greenville, North Carolina 27834. *A note on $C(X)$.*

Let X denote a convergence space and $C(X)$ the space of all real-valued continuous functions on X equipped with the continuous convergence structure. The *length of the decomposition series* of a convergence space X is the smallest ordinal such that the closure operator becomes idempotent. A space X is *c-embedded* whenever the natural map from X into $C(C(X))$ is an embedding. *Theorem.* For each infinite ordinal σ, there is a locally compact c-embedded space X with length at least σ. *Corollary* 1. There is a completely regular topological space X such that $C(X)$ has length at least σ. *Corollary* 2. There is a space X with length at least σ such that $C(X)$ is a completely regular topological space. (Received January 7, 1974.)

* 712-G3. J. H. ROBERTS, Duke University, Durham, North Carolina 27706. *Embeddings transversal to algebraic sets.*

A 1941 result involving linear spaces is extended to apply to all algebraic sets. An algebraic set $V \subset R^{2n+1}$ is the zero set of some finite set of real polynomials. *Theorem.* Let X be a compact metric space of dimension $\leq n$, and let $F = \{f: X \to R^{2n+1}, f \text{ continuous}\}$. Then there exists a dense G_δ set $G \subset F$ of homeomorphisms, such that for any $f \in G$, any k $(n + 1 \leq k \leq 2n + 1)$, and any V with dim $V = k$, we have dim $(V \cap f(X)) \leq k - n - 1$ (as much less than n as k is less than $2n + 1$). *Application.* A new proof (for the special case of separable metric spaces) of Nagata's theorem that for any metric space X of dimension n, there exists a suitable metric ρ under which all spheres have dimension $< n$. The present result uses the Euclidean metric of R^{2n+1} on $f(M)$, and gives extended results (lower dimensions for intersections of spheres). (Received January 9, 1974.)

* 712-G4. A. R. BEDNAREK, University of Florida, Gainesville, Florida 32601 and EUGENE M. NORRIS, University of South Carolina, Columbia, South Carolina 29208. *Induced function theorems in topology.*

For definitions, see Abstract 699-G19, these Notices 19(1972), A-808. We call a relation $R: X \to Y$ *q-continuous* provided the projection $p: R \to X$ is a quotient map. All l.s.c. relations are q-continuous as are many *u.s.c.* ones, e.g. all point-closed *u.s.c.* relations with compact range. Difunctional q-continuous relations are studied. We prove, under the assumption that all relations discussed are q-continuous, that the following informally stated theorems, in themselves very useful, are logically equivalent: (1) the well-known Sierpinski's lemma, (2) a generalized Bednarek-Wallace induced function theorem [Math. Systems Theory 1(1967), 217–224] which does not hypothesize compactness, and (3) a "noncompact" generalization of the theorem of the above cited abstract. Without using mapping diagrams, the precise statement of these theorems is cumbersome. (Received January 9, 1974.)

712-G5. JAMES A. DRAPER, University of Florida, Gainesville, Florida 32611. *Universal cohomology products and the delicate Hopf invariant.* Preliminary report.

A universal cohomology product defined relative to a cofibration is shown to be related to the delicate Hopf invariant of Ganea (Comment. Math. Helv. 39(1965), 295–322). This generalizes the classical result of Steenrod relating the Hopf invariant to the cup product. This new product yields a criterion for certain maps between co-H-spaces to be co-H-maps which extends a criterion of Hilton (Topology 3(1965), 161–176) concerning induced cofibrations to arbitrary cofibrations. This criterion serves to determine when certain homotopy epimorphisms admit sections. (Received January 14, 1974.)

*712-G6. JULIAN R. EISNER, Princeton University, Princeton, New Jersey 08540. *Knots with infinitely many minimal spanning surfaces.*

In this paper, all knots and their spanning surfaces are tamely embedded in S^3. A knot k has a unique minimal spanning surface in the weak (strong) sense if any two minimal spanning surfaces of k are weakly (strongly) equivalent. (The author has defined weak equivalence and strong equivalence in Abstract 73T-G123, these Notices 20(1973), A-594.) *Theorem* 1. If k_1 and k_2 are nonfibered knots, then $k_1 \# k_2$ has an infinite collection of minimal spanning surfaces, no two of which are strongly equivalent. *Theorem* 2. There are knots k for which the number of strong equivalence classes of (not necessarily orientable) spanning surfaces of maximal characteristic is infinite. (Thus the result announced in Abstract 73T-G125, these Notices 20(1973), A-595 is incorrect.) *Theorem* 3. If the knots k_1 and k_2 both have a unique minimal spanning surface in the weak sense, then so does $k_1 \# k_2$. *Theorem* 4. It is possible to find knots k_1 and k_2 such that k_1 and k_2 both have a unique minimal spanning surface in the strong sense, but $k_1 \# k_2$ does not. *Theorem* 5. There are knots k which have a unique minimal spanning surface in the weak sense but do not have a unique minimal spanning surface in the strong sense. (Received January 14, 1974.)

*712-G7. DENIS J. BLACKMORE, Newark College of Engineering, Newark, New Jersey 07102. *A nondensity example in* Diff (S^2).

Let M be a smooth, compact, finite-dimensional manifold and Diff (M) be the set of self-diffeomorphisms of M. An important part of the research on discrete dynamical systems has been the determination of those manifolds such that the structurally stable and Ω-stable diffeomorphisms are not dense in the uniform C^1 topology on Diff (M) [Proc. Sympos. Pure Math., vol. 14, Amer. Math. Soc., Providence, R. I., 1970]. In this paper, we answer an open question of this type by constructing a self-diffeomorphism of the unit 2-sphere S^2 which has a neighborhood in the C^1 topology on Diff (S^2) containing no Ω-stable diffeomorphisms. (Received January 14, 1974.)

712-G8. MAURICE HUGH MILLER, JR., University of Alabama, University, Alabama 35486. *Equivalence of connected and Darboux functions under closure.* Preliminary report.

Denote by f_i $(i = 1, 2)$ the graph of a real function with domain in the segment (a, b) of the X-axis such that each point of f_i is a limit point of f_i from both the left and the right. A function f_i is Darboux if $f_i(c)$ is connected whenever C is a connected subset of the domain of f_i. A function f_i is connected if the graph of f_i is connected. It is not hard to show that if f_i is Darboux, then $\overline{f}_i$ is connected. *Theorem.* Denote by f_1 a Darboux function. Then there exists a connected function f_2 s.t. $\overline{f}_1$ is $\overline{f}_2$. Hence letting f_1 be equivalent to f_2 iff $\overline{f}_1$ is $\overline{f}_2$, one obtains the same collection of equivalence classes whether f_1 is Darboux or connected. *Corollary.* A sufficient condition that a function f_1 be Darboux (connected) is that K meets f_1 whenever K is a horizontal segment (continuum in the plane) such that $(K \cap \overline{f})_X$ is uncountable. (Received January 14, 1974.)

712-G9. SHIN'ICHI KINOSHITA, Florida State University, Tallahassee, Florida 32306. *On elementary ideals of projective planes in the 4-sphere and θ-curves in the 3-sphere.* Preliminary report.

Let L be a polyhedron in an n-sphere $(n \geq 3)$ that does not separate S^n, and let G_L be the fundamental group of $S^n - L$. Let l be an $(n-2)$-dimensional cycle over integers mod p (p, prime). There is a homomorphism ψ of G_L into the multiplicative cyclic group H_p of order p generated by t such that $g^{\psi} = t^{\text{link}(g,l)}$, where link$(g, l)$ is the linking number mod p between g and l in S^n. Using Fox's free differential calculus, we associate to l the dth elementary ideals $E_d(l)$ of G, evaluated in the group ring JH_p of H_p over the integers. If l and l' are homologous on L, then $E_d(l) = E_d(l')$. We apply these $E_d(l)$ to the study of the position of L in S^n. The following cases are considered: (1) L is a projective plane in S^4 and $p = 2$, and (2) L is a θ-curve in S^3 and $p = 3$. (Received January 11, 1974.)

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MATHEMATICS PROFESSOR. Ph. D., 1959, Yale. Ergodic and Operator Theory. Since 1971 published 8 papers, 1 monograph (U. of Chi. Press), 5 more submitted. Superior teacher, research director. Author of 3 undergraduate texts. Excellent recommendations. P. Shields, Visiting Lecturer, University of Warwick, England.

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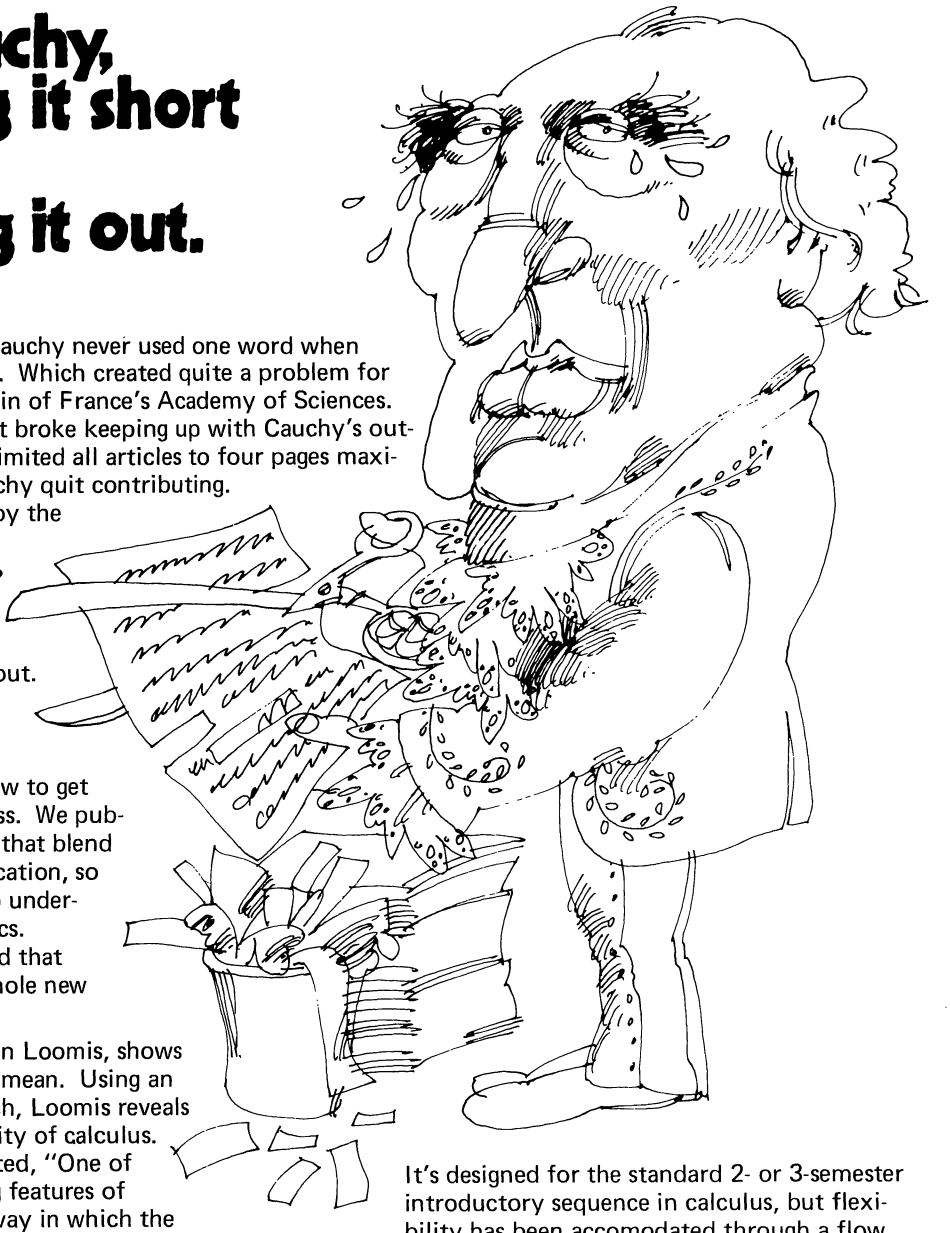

Augustin-Louis Cauchy never used one word when twenty would do. Which created quite a problem for the weekly bulletin of France's Academy of Sciences. They almost went broke keeping up with Cauchy's output. Until they limited all articles to four pages maximum. Then Cauchy quit contributing. That limitation, by the way, still applies.

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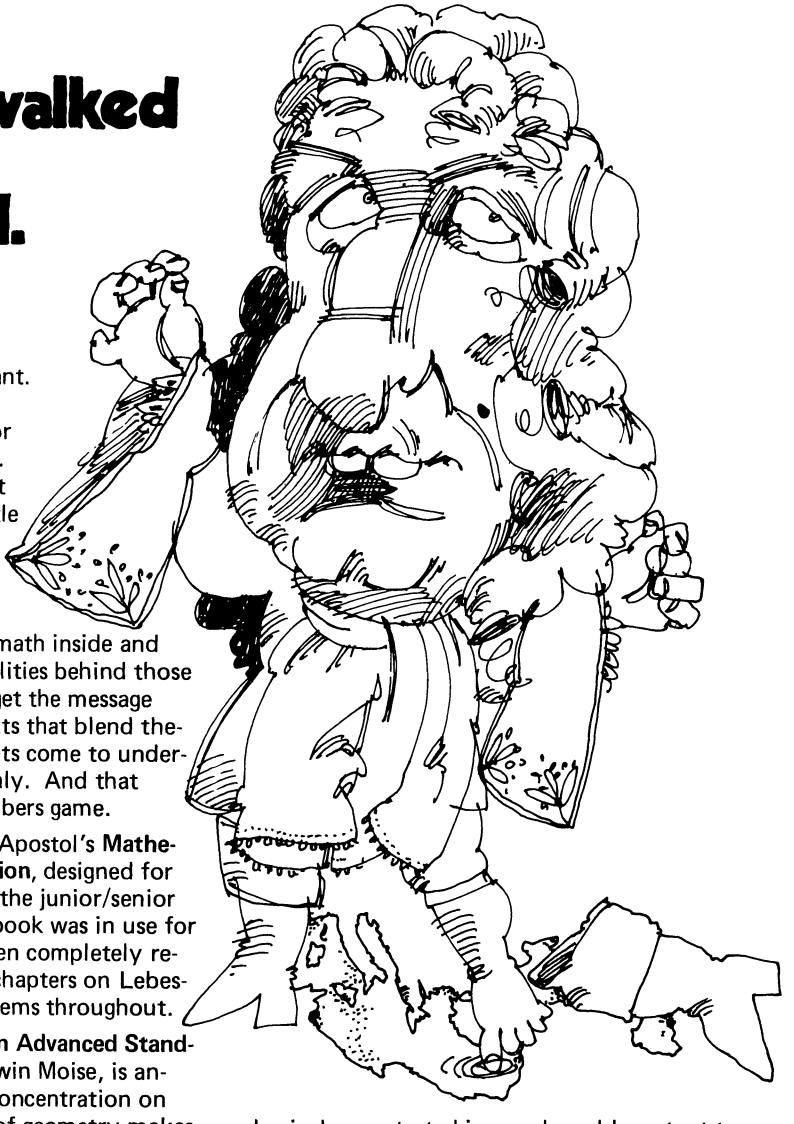

Leibniz was a lot of things. Even something of a clairvoyant. One of his dreams was to wage holy war on Egypt for its conquest and recivilization. Napoleon finally tried it about 100 years later. And was a little upset to find out Leibniz had anticipated him.

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