Notices

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Calendar

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting *in person* must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

Meeting Number	Date	Place	Deadline for Abstracts* and News Items
715	May 13-18, 1974	DeKalb, Illinois	March 27, 1974
---	August 1974	No summer meeting; International Congress (see below)	June 15, 1974 (News items only)
716	October 26, 1974	Middletown, Connecticut	Sept. 3, 1974
717	November 8-9, 1974	Nashville, Tennessee	Sept. 25, 1974
718	November 22-23, 1974	Los Angeles, California	Sept. 25, 1974
719	November 23, 1974	Houston, Texas	Sept. 25, 1974
720	January 23-27, 1975 (81st Annual Meeting)	Washington, D.C.	Nov. 6, 1974
	April 18-19, 1975	Monterey, California	
	August 18-22, 1975	Kalamazoo, Michigan	
	January 22-26, 1976 (82nd Annual Meeting)	San Antonio, Texas	

*Deadline for abstracts not presented at a meeting (by title). August 1974 issue: June 8
October 1974 issue: August 29

OTHER EVENTS

August 21-29, 1974	International Congress of Mathematicians Vancouver, B.C., Canada	April 15, 1974

The zip code of the Post Office Box of the Society has been changed from 02904 to 02940. Correspondents are requested to note this change in their records.

Please affix the peel-off label on these *Notices* to correspondence with the Society concerning fiscal matters, changes of address, promotions, or when placing orders for books and journals.

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Notices
OF THE
AMERICAN MATHEMATICAL SOCIETY

Everett Pitcher and Gordon L. Walker, Editors
Wendell H. Fleming, Associate Editor

CONTENTS

The Seven Hundred Thirteenth Meeting
Biltmore Hotel
New York, New York
April 10 – 13, 1974

The seven hundred thirteenth meeting of the American Mathematical Society will be held at the Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York, from Wednesday, April 10, through Saturday, April 13, 1974, in conjunction with the 1974 spring meeting of the Association for Symbolic Logic.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be four one-hour addresses. Professor Sol I. Rubinow of the Graduate School of Medical Sciences, Cornell University, and the Sloan-Kettering Institute will speak on "Some mathematical problems in biology" at 11:00 a.m. on Friday, April 12. Professor George C. Papanicolaou of the Courant Institute of Mathematical Sciences, New York University, will speak on "Asymptotic methods for stochastic equations" at 2:00 p.m. on Friday, April 12. Professor Serge Lang of Yale University will speak on "Higher dimensional diophantine problems" at 11:00 a.m. on Saturday, April 13. Professor William P. Thurston of the Massachusetts Institute of Technology will speak on "The existence of foliations" at 2:00 p.m. on Saturday, April 13.

There will be two special sessions. Professor V. Lakshmikantham of the University of Rhode Island and the University of Texas at Arlington has organized a special session on Nonlinear Problems in Differential and Integral Equations for Friday afternoon and Saturday morning; speakers will be Bernard A. Asner, Jr., Constantin Corduneanu, Paul William Davis, Jerome Eisenfeld, John R. Haddock, Athanassios G. Kartsatos, Gangaram S. Ladde, S. Leela, A. Richard Mitchell. Some of the talks will be based on papers with joint authorship which is indicated in each case in the program of the sessions. Professor Herbert S. Wilf of the University of Pennsylvania and Rockefeller University has organized a special session on Combinatorial Algorithms for Saturday morning and afternoon; speakers will be Jack Edmonds, Daniel J. Kleitman, Albert Nijenhuis, Gian-Carlo Rota, W. T. Tutte, and Herbert S. Wilf.

Sessions for contributed ten-minute papers will be scheduled in the morning and afternoon on Friday and Saturday. No provision will be made for late papers. Each meeting room will be equipped with an overhead projector.

SYMPOSIUM ON MATHEMATICAL ASPECTS OF CHEMICAL AND BIOCHEMICAL PROBLEMS AND QUANTUM CHEMISTRY

With the support of the Office of Naval Research and the Atomic Energy Commission, a symposium on Mathematical Aspects of Chemical and Biochemical Problems and Quantum Chemistry is to be held on Wednesday and Thursday, April 10 and 11. This topic was selected by the AMS-SIAM Committee on Applied Mathematics whose members are Earl A. Coddington, Hirsh G. Cohen (chairman), Lester E. Dubins, Harold Grad, J. Barkley Rosser, and Richard S. Varga. The symposium is a further attempt at encouraging and broadening the inter-disciplinary research in the several wide fields of mutual interest which exist between certain mathematicians, applied mathematicians, and researchers in chemical reaction theory, biochemistry, and quantum chemistry. The Organizing Committee includes Donald S. Cohen (chairman), Hirsh G. Cohen, Julian D. Cole, George R. Gavalas, and Aron Kupperman.

The program will consist of ten lectures. The list of speakers includes Neal R. Amundson (University of Minnesota), Rutherford Aris (University of Minnesota), Joseph Higgins (The Johnson Research Foundation, University of Pennsylvania), Fritz Horn (University of Rochester), Louis N. Howard (Massachusetts Institute of Technology), James C. Keck (Massachusetts Institute of Technology), Herbert B. Keller (California Institute of Technology), Aron Kupperman (California Institute of Technology), Gregoire Nicolis (Université Libre de Bruxelles), and Arthur T. Winfree (Purdue University). The symposium will be held in the Grand Ballroom of the Biltmore Hotel both days.

MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC

Invited addresses and sessions for contributed papers for the spring meeting of the Association for Symbolic Logic are scheduled to be held in the Marlborough Room of the Biltmore Hotel on April 12–13. Programs, listing titles and times for individual papers, will be available at the registration desk located in the Key Room.

There will be two sessions for contributed papers on Friday, April 12, at 9:00 a.m. and 1:30 p.m. An invited expository talk entitled "Recursion on finite type functionals" will be presented by Leo Harrington of SUNY, Buffalo at 3:30 p.m. on Friday. Ashok Chandra of the IBM T.J. Watson Research Center will give an invited talk at 9:30 a.m. on Saturday; the title of his lecture will be "Generality of control structures in uninterpreted computer programs."

The Council of the Association will meet at 8:00 p.m. on Friday, April 12, in the Marlborough Room.

REGISTRATION

The registration desk will be located in the Key Room of the Biltmore Hotel on the nineteenth floor adjacent to the Grand Ballroom. The desk will be open from 8:30 a.m. to 4:30 p.m. on Wednesday, April 10, through Friday, April 12; and from 8:30 a.m. to 3:30 p.m. on Saturday, April 13.

The registration fees for the meeting are as follows:

Member	$3
Student and unemployed	1
Nonmember	5

ACCOMMODATIONS

Persons intending to stay at the Biltmore Hotel should make their own reservations with the hotel. A reservation form and a listing of room rates will be found on page A-362 of the February *Notices*. The deadline for receipt of reservations is April 3, 1974.

TRAVEL

The Biltmore Hotel is located on Madison Avenue at 43rd Street on the east side of New York City. Walkways to Grand Central Station are located under the hotel and signs are posted directing persons to the lobby of the hotel.

Those arriving by bus may take the Independent Subway System from the Port Authority Bus Terminal. There is shuttle bus service from LaGuardia and Kennedy Airports directly to Grand Central Station. Starters can direct participants to the correct bus.

Air passengers arriving at Newark Airport can take a shuttle bus to the East Side Terminal and take either a taxi or a bus to the hotel.

Those arriving by car will find many parking facilities in the neighborhood in addition to those at the hotel. Parking service can be arranged through the hotel doorman at a cost of $8 for a 24-hour period. There will be an additional charge for extra pickup and delivery service if it is required. The parking fee is subject to New York City taxes.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, The Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York 10017.

PROGRAM FOR THE SYMPOSIUM

Wednesday, April 10

First Session, Grand Ballroom (19th floor)
Chairman: Donald S. Cohen, California Institute of Technology and University of Arizona

9:30 a.m. Wave trains, fronts, and transition layers in reaction-diffusion equations. LOUIS N. HOWARD, Massachusetts Institute of Technology

10:45 a.m. Rotating solutions to reaction-diffusion equations in simply-connected media. ARTHUR T. WINFREE, Purdue University

Second Session, Grand Ballroom (19th floor)
Chairman: George R. Gavalas, California Institute of Technology

1:45 p.m. Nonlinear problems in chemical reactor theory. NEAL R. AMUNDSON, University of Minnesota

3:00 p.m. An analysis of the counter-current moving bed reactor. RUTHERFORD ARIS, University of Minnesota

4:15 p.m. Some problems in chemical reactor theory. HERBERT B. KELLER, California Institute of Technology

Thursday, April 11

Third Session, Grand Ballroom (19th floor)
Chairman: Julian D. Cole, University of California at Los Angeles

9:00 a.m. The dynamics of open reaction systems. FRITZ HORN, University of Rochester

10:15 a.m. Phase space theory of atomic and molecular excitation and dissociation. JAMES C. KECK, Massachusetts Institute of Technology

11:30 a.m. Quantum dynamics of reactive molecular collisions. ARON KUPPERMAN, California Institute of Technology

Fourth Session, Grand Ballroom (19th floor)
Chairman: Hirsh G. Cohen, IBM T.J. Watson Research Center

2:00 p.m. Mathematical problems in the analysis of metabolic dynamics. JOSEPH HIGGINS, The Johnson Research Foundation, University of Pennsylvania

3:15 p.m. Patterns of spatio-temporal organization in nonlinear chemical and biochemical kinetics. GREGOIRE NICOLIS, Faculté des Sciences, Université Libre de Bruxelles

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions the time varies. To maintain this schedule, the time limits will be strictly enforced.

FRIDAY, 9:00 A.M.

General Session, Suite J. (1st floor)

9:00- 9:10 (1) How much classical model theory depends on the law of excluded middle? Preliminary report. Professor WILLIAM C. POWELL, State University of New York at Buffalo (713-E1)

9:15- 9:25 (2) Optimal equilibrium harvesting policy of an age-specific population. Preliminary report. Professor CHRIS RORRES*, and Professor WYMAN G. FAIR, Drexel University (713-C3)

9:30- 9:40 (3) Coupled power equations for stochastic systems with propagating and evanescent modes. Preliminary report. Professor WERNER E. KOHLER, Virginia Polytechnic Institute and State University (713-C4)

9:45- 9:55 (4) Equivalence classes of polynomials over finite fields. Preliminary report. Mr. GARY L. MULLEN, Pennsylvania State University (74T-A39)

10:00-10:10 (5) Compact semitopological completely 0-simple semigroups. Preliminary report. Dr. JOHN F. BERGLUND, Virginia Commonwealth University (711-22-26)

FRIDAY, 9:00 A.M.

Session on Analysis I, Vanderbilt Suite (1st floor)

9:00- 9:10 (6) (L_p, L_q) mapping properties of convolution transforms. Preliminary report. Dr. GARY SAMPSON*, Dr. ABRAHAM NAPARSTEK, and Dr. VLADIMIR DROBOT, State University of New York at Buffalo (713-B5)

9:15- 9:25 (7) On Cesaro summability of Fourier series. Preliminary report. Professor DANIEL WATERMAN*, Syracuse University, and Professor BADRI N. SAHNEY, University of Calgary (713-B15)

9:30- 9:40 (8) An analog of the Dirichlet-Jordan theorem for dual Fourier series. Professor ROBERT B. KELMAN, Colorado State University (713-B42)

9:45- 9:55 (9) Fourier transforms of odd and even tempered distributions. Professor E.O. MILTON, University of California, Davis (713-B19) (Introduced by Professor Allan L. Edelson)

10:00-10:10 (10) A characterization of square Banach spaces. Dr. NINA M. ROY, Rosemont College (713-B4)

10:15-10:25 (11) Polytopes in locally convex spaces. Professor M. RAJAGOPALAN*, Memphis State University, and Professor A.K. ROY, Tata Institute of Fundamental Research, Bombay, India (713-B18) (Introduced by Professor Stanley P. Franklin)

10:30-10:40 (12) The second dual of certain Choquet simplexes. Preliminary report. Professor ROBERT E. ATALLA, Ohio University (713-B41)

FRIDAY, 9:00 A.M.

Session on Complex Analysis, French Suite (1st floor)

9:00- 9:10 (13) Entire functions of bounded index, bounded value distribution, and bounded 1-index. Professor BENJAMIN LEPSON*, Naval Research Laboratory, Washington, D.C. and Catholic University, and Professor CHARLES S. DAVIS, Anne Arundel Community College and Catholic University (713-B29)

9:15- 9:25 (14) Entire functions of bounded absolute index. Professor CHARLES S. DAVIS*, Anne Arundel Community College and Catholic University, and Professor BENJAMIN LEPSON, Naval Research Laboratory, Washington, D.C. and Catholic University (713-B30)

9:30- 9:40 (15) On the maximum modulus for meromorphic univalent functions. Professor JAMES E. MILLER, West Virginia University (713-B32)

9:45- 9:55 (16) Conformal deformation of meromorphic functions. Preliminary report. Mr. GARY W. TOWSLEY, University of Rochester (713-B38) (Introduced by Professor Gail S. Young)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

10:00-10:10 (17) Some remarks on analytic continuation. Preliminary report. Professor MARVIN D. TRETKOFF, Stevens Institute of Technology (713-B39)

FRIDAY, 9:00 A.M.

Session on Differential and Integral Equations, Suite G (1st floor)

9:00- 9:10 (18) Continuous paramêter dependence in a class of Volterra integral equations. Preliminary report. Professor KENNETH B. HANNSGEN, Virginia Polytechnic Institute and State University (713-B3)

9:15- 9:25 (19) Asymptotic stability of stochastic differential equations. Professor MARK A. PINSKY, Northwestern University (713-F1)

9:30- 9:40 (20) A remark on time scales. Preliminary report. Professor LAWRENCE E. LEVINE, Stevens Institute of Technology, and Mr. ERIC LUBOT*, Stevens Institute of Technology and Bergen Community College (713-B1)

9:45- 9:55 (21) Convexity arguments for differential equations. I. Professor HEINRICH W. GUGGENHEIMER, Polytechnic Institute of New York (713-B12)

10:00-10:10 (22) Solution in the large of a certain differential equation with an irregular singular point of rank greater than one. Dr. T.K. PUTTASWAMY*, Ball State University and Mr. RAJ PAUL SEEKRI, Texas Instruments, Inc., Dallas (713-B31)

10:15-10:25 (23) Some examples of singularly perturbed boundary value problems with turning points. Professor ROGER Y. LYNN, Villanova University (713-B33)

FRIDAY, 11:00 A.M.

Invited Address, Grand Ballroom (19th floor)

(24) Some mathematical problems in biology. Professor SOL I. RUBINOW, Graduate School of Medical Sciences, Cornell University and Sloan-Kettering Institute (713-C6)

FRIDAY, 2:00 P.M.

Invited Address, Grand Ballroom (19th floor)

(25) Asymptotic methods for stochastic equations. Professor GEORGE C. PAPANICOLAOU, Courant Institute, New York University (713-F4)

FRIDAY, 3:15 P.M.

Special Session on Nonlinear Problems in Differential and Integral Equations I, Windsor Room (18th floor)

3:15- 3:35 (26) Fixed point theorems through cones. Preliminary report. Professor JEROME EISENFELD* and Professor V. LAKSHMIKANTHAM, University of Texas at Arlington (713-B8)

3:45- 4:05 (27) On perturbing Lyapunov functions. Preliminary report. Professor V. LAKSHMIKANTHAM, University of Texas at Arlington, and Professor S. LEELA*, State University of New York, College at Geneseo (713-B10)

4:15- 4:35 (28) Locally invertible operators and existence problems in differential systems. Preliminary report. Professor ATHANASSIOS G. KARTSATOS, University of South Florida (713-B2)

4:45- 5:05 (29) Invariant spaces for a convolution operator and applications to functional equations. Preliminary report. Professor CONSTANTIN CORDUNEANU, University of Rhode Island (713-B40)

FRIDAY, 3:15 P.M.

Session on Algebra I, Vanderbilt Suite (1st floor)

3:15- 3:25 (30) Invertible ideals and theory of grade. Preliminary report. Professor SAMUEL FLOYD BARGER, Youngstown State University (713-A4)

3:30- 3:40 (31) Representations of valued graphs. Dr. VLASTIMIL J. DLAB*, Carleton University, and Dr. CLAUS MICHA RINGEL, Universitat Bonn, Federal Republic of Germany (713-A6)

3:45- 3:55 (32) Prime ideals of Noetherian PI-rings. Dr. LOUIS HALLE ROWEN, University of Chicago (713-A10)

4:00- 4:10 (33) Modules over hereditary noetherian prime rings. Dr. SURJEET SINGH, Ohio University (713-A11) (Introduced by Professor S. K. Jain)

4:15- 4:25	(34)	Semiprimary rings with quasi-projective left ideals. Preliminary report. Professor S. K. JAIN* and Professor SURJEET SINGH, Ohio University (713-A13)

FRIDAY, 3:15 P. M.

Session on Combinatorics, French Suite (1st floor)

3:15- 3:25	(35)	Characterization of projective 3-class association schemes. Dr. ALAN P. SPRAGUE*, and Dr. DWIJENDRA K. RAY-CHAUDHURI, Ohio State University (713-A2)
3:30- 3:40	(36)	Words with prescribed letter and letter-pair frequencies. Preliminary report. Dr. JOAN P. HUTCHINSON*, Dartmouth College and Professor HERBERT S. WILF, Rockefeller University (713-A3)
3:45- 3:55	(37)	Latin k-cubes. Dr. JOSEPH ARKIN*, Spring Valley, New York, and Professor E. G. STRAUS, University of California, Los Angeles (713-A14)
4:00- 4:10	(38)	Two-fold triple systems and graph imbeddings. Professor SETH R. ALPERT, State University of New York, Downstate Medical Center, Brooklyn (713-A17)

FRIDAY, 3:15 P. M.

Session on Probability, Suite G (1st floor)

3:15- 3:25	(39)	Scale and reliability bounds for Gamma under a Bayesian influence. Professor ALEX S. PAPADOPOULOS, Keene State College (713-F3)
3:30- 3:40	(40)	Potential operators in Dirichlet spaces. Preliminary report. Professor DHANDAPANI KANNAN, University of Georgia (713-F5)
3:45- 3:55	(41)	Difference structures in probability and analysis. Preliminary report. Professor JAMES H. ABBOTT, Louisiana State University, New Orleans (713-F6)
4:00- 4:10	(42)	Relaxation times for the evolution of Markov fields. Preliminary report. Dr. WAYNE G. SULLIVAN, Dublin Institute for Advanced Study, Ireland (713-F2)
4:15- 4:25	(43)	Backward-forward duality for certain random evolutions. Preliminary report. Professor GEZA SCHAY, University of Massachusetts, Boston (713-F7)

SATURDAY, 9:00 A.M.

Special Session on Combinatorial Algorithms I, Music Room (1st floor)

9:00- 9:25	(44)	Smith canonical forms of integer matrices. Preliminary report. Professor ALBERT NIJENHUIS, University of Pennsylvania (713-A12)
9:30- 9:55	(45)	Algorithms for generating combinatorial objects uniformly at random. Professor HERBERT S. WILF, Rockefeller University and University of Pennsylvania (713-A19)
10:00-10:25	(46)	Significance arithmetic. Professor GIAN-CARLO ROTA, Massachusetts Institute of Technology (713-C5)

SATURDAY, 9:00 A.M.

Special Session on Nonlinear Problems in Differential and Integral Equations II, Windsor Room (18th floor)

9:00- 9:15	(47)	Asymptotic equilibrium in Banach spaces. Preliminary report. Professor A. RICHARD MITCHELL*, and Professor ROGER W. MITCHELL, University of Texas at Arlington (713-B9)
9:20- 9:35	(48)	Utilization of pointwise degenerate delay-differential equations as comparison functions. Preliminary report. Professor BERNARD A. ASNER, Jr.*, University of Dallas, and Professor V. LAKSHMIKANTHAM, University of Texas at Arlington (713-B13)
9:40- 9:55	(49)	Systems of differential inequalities and stochastic differential equations. I. Preliminary report. Professor GANGARAM S. LADDE, State University of New York, College at Potsdam (713-B14)
10:00-10:15	(50)	Minimum principles and positive solutions for a class of nonlinear diffusion problems. Preliminary report. Dr. JAGDISH CHANDRA, U.S. Army Research Center, Durham, North Carolina, Professor PAUL WILLIAM DAVIS*, Worcester Polytechnic Institute, and Professor BERNARD A. FLEISHMAN, Rensselaer Polytechnic Institute (713-B25)

10:20-10:35	(51)	On the delay-differential equation $x''(t) + a(t)f(x(t-r(t))) = 0$. Professor THEODORE A. BURTON, Southern Illinois University, and Professor JOHN R. HADDOCK*, Memphis State University (713-B26)

SATURDAY 9:00 A.M.

Session on Algebra II, Vanderbilt Suite (1st floor)

9:00- 9:10	(52)	A note on character sums. II. Preliminary report. Professor CLIFTON T. WHYBURN, University of Houston (713-A5)
9:15- 9:25	(53)	The order of $K_2\mathcal{O}/(K_2\mathcal{O})^3$ for quadratic fields. Dr. ALAN CANDIOTTI, University of Missouri-St. Louis (713-A9)
9:30- 9:40	(54)	On a nullity of a product of two linear transformations. Professor JIN BAI KIM, West Virginia University (713-A7)
9:45- 9:55	(55)	Right simple elements in a semigroup. Professor FRANCIS E. MASAT, Glassboro State College (713-A1)
10:00-10:10	(56)	Weaving j-diagrams for torsion free abelian groups. Mr. ERNEST C. ACKERMAN, Pennsylvania State University (713-A18)
10:15-10:25	(57)	The SQ-universality of certain linear groups. Preliminary report. Professor BENJAMIN FINE*, City University of New York, John Jay College, and Professor MARVIN D. TRETKOFF, Stevens Institute of Technology (713-A16)

SATURDAY, 9:00 A.M.

Session on Algebraic Topology, French Suite (1st floor)

9:00- 9:10	(58)	A characterization of fibered knots. Mr. JULIAN R. EISNER, Princeton University (713-G1)
9:15- 9:25	(59)	On 10-crossing knots. Mr. KENNETH A. PERKO, Jr., New York, New York (713-G3)
9:30- 9:40	(60)	Polynomial algebras which are modules over the mod-p Steenrod algebra. Preliminary report. Professor STAVROS G. PAPASTAVRIDIS, Brandeis University (713-G4)
9:45- 9:55	(61)	A geometrical realization of a construction of Bass and Serre. Professor STEVEN C. ALTHOEN, Hofstra University (713-G7)
10:00-10:10	(62)	The shape of a map. Professor DAVID EDWARDS*, and Mrs. PATRICIA McAULEY, State University of New York at Binghamton (713-G15)
10:15-10:25	(63)	An equivariant Serre spectral sequence. Dr. HENRY M. WALKER, Massachusetts Institute of Technology (713-G2)

SATURDAY, 9:00 A.M.

Session on Operator Theory, Suite G (1st floor)

9:00- 9:10	(64)	A note on smooth perturbations. Preliminary report. Professor JOHN B. BUTLER, Jr., Portland State University (713-B6)
9:15- 9:25	(65)	Spectral integrals from the theory of multivariate stationary Payen processes. Preliminary report. Professor MILTON ROSENBERG, City University of New York, Staten Island Community College (713-B16)
9:30- 9:40	(66)	Algebraic relationships that yield subnormality. Professor RAMESH M. KULKARNI, State University of New York, College at Potsdam (713-B36)
9:45- 9:55	(67)	On convergence and evaluation of sums of reciprocal powers of eigenvalues for certain operators on a Hilbert space which are meromorphic functions of the eigenvalue parameter. Preliminary report. Dr. ANTHONY V. LAGINESTRA*, Elmont, New York, and Professor WILLIAM E. BOYCE, Rensselaer Polytechnic Institute (713-B43)
10:00-10:10	(68)	A Hille-Yoshida-Phillips type of theorem for semigroups in a locally convex space. Preliminary report. Professor RATHINDRA N. MUKHERJEE, University of Georgia (713-B28) (Introduced by Professor Richard Bouldin)
10:15-10:25	(69)	Spectral maximal spaces and weak spectral manifolds of unbounded operators. Professor IVAN ERDELYI, Temple University (713-B27)

SATURDAY, 11:00 A.M.

Invited Address, Grand Ballroom (19th floor)

(70) Higher dimensional diophantine problems. Professor SERGE LANG, Yale University (713-A8)

SATURDAY, 2:00 P.M.

Invited Address, Grand Ballroom (19th floor)

(71) The existence of foliations. Professor WILLIAM P. THURSTON, Massachusetts Institute of Technology (713-A15)

SATURDAY, 3:15 P.M.

Special Session on Combinatorial Algorithms II, Music Room (1st floor)

3:15- 3:40 (72) Degree sequences and trees. Professor DANIEL J. KLEITMAN, Massachusetts Institute of Technology (713-A15)

3:45- 4:10 (73) Spanning subgraphs with specified valencies. Professor W.T. TUTTE, University of Waterloo (713-A20)

4:15- 4:40 (74) A linear decomposition theory. Preliminary report. Professor JACK EDMONDS, University of Waterloo (713-A21) (Introduced by Professor Herbert S. Wilf)

SATURDAY, 3:15 P.M.

Session on Analysis II, Vanderbilt Suite (1st floor)

3:15- 3:25 (75) Homoclinic limit cycles. Dr. OKAN GUREL, IBM Scientific Centers, White Plains, New York (713-G9)

3:30- 3:40 (76) The epsilon method, the method of multipliers, and a nonlinear optimal control problem. Preliminary report. Professor RUSSELL D. RUPP, State University of New York at Albany (713-B7)

3:45- 3:55 (77) Some positive integrals of Bessel functions. Preliminary report. Professor GEORGE GASPER, Northwestern University (713-B20)

4:00- 4:10 (78) Leibniz rule for fractional derivatives used to generalize formulas of Walker and Cauchy. Professor THOMAS J. OSLER, Glassboro State College (713-B21)

4:15- 4:25 (79) A spatial analogue of a theorem of Hayman. Preliminary report. Professor M.N.M. TALPUR, University of Illinois (713-B22)

4:30- 4:40 (80) The complete Pade tables of certain series of simple fractions. Professor ALBERT EDREI, Syracuse University (713-B24)

4:45- 4:55 (81) Converting interpolation series into Chebyshev series by recurrence formulas. Dr. HERBERT E. SALZER, Brooklyn, New York (713-C1)

5:00- 5:10 (82) Cubatures of precision 2k and 2k+1 for hyperrectangles. Preliminary report. Dr. DALTON R. HUNKINS, Kutztown State College (713-C2)

SATURDAY, 3:15 P.M.

Session on Partial Differential Equations, French Suite (1st floor)

3:15- 3:25 (83) A note on coercive inequalities for irregular regions. Preliminary report. Professor JAMES M. NEWMAN, City University of New York, Baruch College (713-B11)

3:30- 3:40 (84) Scaling of a system of first order hyperbolic partial differential equations. Professor PAUL GORDON, Drexel University (713-B17)

3:45- 3:55 (85) Wave equation with acoustic boundary conditions. Preliminary report. Dr. STEVEN I. ROSENCRANS*, and Dr. J.T. BEALE, Tulane University (713-B23)

4:00- 4:10 (86) Some functional differential equations associated with a random parabolic equation. Preliminary report. Professor PAO-LIU CHOW, Wayne State University (713-B35)

4:15- 4:25 (87) Uniqueness of solutions of elliptic equations in Lipschitz domains. Preliminary report. Professor JAMES R. DIEDERICH, University of Califcrnia, Davis (713-B37)

4:30- 4:40 (88) Sturm comparison theorems for systems of second order elliptic partial differential equations. Professor COREEN L. METT, Virginia Polytechnic Institute and State University (713-B34)

Session on Topology, Suite G (1st floor)

3:15- 3:25 (89) A generalization of quasi-complete spaces. Preliminary report. Professor HOWARD H. WICKE*, and Professor JOHN M. WORRELL, Jr., Ohio University (713-G11)

3:30- 3:40 (90) Pseudocompactness and invariance of continuity. Dr. JOE A. GUTHRIE and Dr. H. EDWARD STONE*, University of Pittsburgh (713-G12)

3:45- 3:55 (91) The Lorch-Tong property and G_δ diagonals. Preliminary report. Dr. ERIC JOHN BRAUDE, Seton Hall University (713-G13)

4:00- 4:10 (92) Strong 0-dimensionality of some spaces. Preliminary report. Professor STANISLAW G. MROWKA, State University of New York at Buffalo (713-G14)

4:15- 4:25 (93) T^3-actions on simply connected 6-manifolds. Mr. DENNIS McGAVRAN, University of Connecticut, Waterbury Branch (713-G6)

4:30- 4:40 (94) Homeotopy groups of compact 2-manifolds. Preliminary report. Dr. DAVID J. SPROWS, Villanova University (713-G8)

4:45- 4:55 (95) A metrization theorem for 2-manifolds. Preliminary report. Dr. PAUL A. VINCENT, Universite de Moncton (713-G16)

5:00- 5:10 (96) Rational bordism of frame fields, immersions and k-mersions. Professor ULRICH KOSCHORKE, Rutgers University, Busch Campus (713-G5)

5:15- 5:25 (97) Sufficient conditions for connectivity of real-valued functions. Mr. DAVID S. LAWRENCE, Brooklyn, New York (74T-G33)

Walter H. Gottschalk
Associate Secretary

Middletown, Connecticut

PRESENTORS OF PAPERS

Following each name is the number corresponding to the speaker's position on the program

• Invited one-hour lectures

Abbott, James H. #41
Ackermann, Ernest C. #56
Alpert, Seth R. #38
Althoen, Steven C. #61
Arkin, Joseph #37
Asner, Bernard A., Jr. #48
Atalla, Robert E. #12
Barger, Samuel F. #30
Berglund, John F. #5
Braude, Eric J. #91
Butler, John B., Jr. #64
Candiotti, Alan #53
Chow, Pao-Liu #86
Corduneanu, Constantin #29
Davis, Charles S. #14
Davis, Paul W. #50
Diederich, James R. #87
Dlab, Vlastimil J. #31
Edmonds, Jack #74
Edrei, Albert #80
Edwards, David #62
Eisenfeld, Jerome #26
Eisner, Julian R. #58
Erdelyi, Ivan #69
Fine, Benjamin #57
Gasper, George #77
Gordon, Paul #84
Guggenheimer, Heinrich W. #21
Gurel, Okan #75
Haddock, John R. #51
Hannsgen, Kenneth B. #18
Hunkins, Dalton R. #82
Hutchinson, Joan P. #36
Jain, S. K. #34
Kannan, Dhandapani #40
Kartsatos, Athanassios G. #28
Kelman, Robert B. #8
Kim, Jin Bai #54
Kleitman, Daniel J. #72
Kohler, Werner E. #3
Koschorke, Ulrich #96
Kulkarni, Ramesh M. #66
Ladde, Gangaram S. #49
Laginestra, Anthony V. #67
• Lang, Serge #70
Lawrence, David S. #97
Leela, S. #27
Lepson, Benjamin #13
Lubot, Eric #20
Lynn, Roger Y. #23
McGavran, Dennis #93
Masat, Francis E. #55
Mett, Coreen L. #88
Miller, James E. #15
Milton, E. O. #9
Mitchell, A. Richard #47
Mrowka, Stanislaw G. #92
Mukherjee, Rathindra N. #68
Mullen, Gary L. #4
Newman, James M. #83
Nijenhuis, Albert #44
Osler, Thomas J. #78
Papadopoulos, Alex S. #39
• Papanicolaou, George C. #25
Papastavridis, Stavros G. #60
Perko, Kenneth A. #59
Pinsky, Mark A. #19
Powell, William C. #1
Puttaswamy, T. K. #22
Rajagopalan, M. #11
Rorres, Chris #2
Rosenberg, Milton #65
Rosencrans, Steven I. #85
Rota, Gian-Carlo #46
Rowen, Louis H. #32
Roy, Nina M. #10
• Rubinow, Sol I. #24
Rupp, Russell D. #76
Salzer, Herbert E. #81
Sampson, Gary #6
Schay, Geza #43
Singh, Surjeet #33
Sprague, Alan P. #35
Sprows, David J. #94
Stone, H. Edward #90
Sullivan, Wayne G. #42
Talpur, M. N. M. #79
• Thurston, William P. #71
Towsley, Gary W. #16
Tretkoff, Marvin D. #17
Tutte, W. T. #73
Vincent, Paul A. #95
Walker, Henry M. #63
Waterman, Daniel #7
Whyburn, Clifton T. #52
Wicke, Howard H. #89
Wilf, Herbert S. #45

The Seven Hundred Fourteenth Meeting
University of California
Santa Barbara, California
April 27, 1974

The seven hundred fourteenth meeting of the American Mathematical Society will be held at the University of California, Santa Barbara, California, on Saturday, April 27, 1974.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two one-hour addresses. Professor Garth W. Warner of the University of Washington will lecture at 11:00 a.m. The title of his lecture is "L-functions on reductive groups." Professor Hung-Hsi Wu of the University of California, Berkeley, will lecture at 1:30 p.m. on "Applications of some theorems in partial differential equations to geometry." Both hour addresses will be given in room 1920 of Ellison Hall.

Professors Marvin Marcus, Henryk Minc, and Robert C. Thompson of the University of California, Santa Barbara, are organizing two special sessions on Linear Algebra. These sessions will begin at 9:30 a.m. and 2:45 p.m., respectively, and will consist of six thirty-minute talks given by John DePillis, Eugene C. Johnsen, Herbert J. Ryser, Robert C. Thompson, Olga Taussky-Todd, and Stanley G. Williamson.

There will also be sessions for contributed papers. Late papers will be accepted for presentation at the meeting, but will not appear in the printed program of the meeting. Overhead projectors will be available at all of the sessions.

The registration desk will be located in the patio between Ellison Hall and Phelps Hall. Registration will begin at 8:00 a.m. on Saturday.

Santa Barbara has a large number of motels and hotels. Those in Goleta are nearest to the campus, those by the beach are farthest from the campus, while those in the business district on upper State Street are approximately midway. All have easy access to U.S. Highway 101. Reservations should be made directly with the desired motel or hotel and the American Mathematical Society should be mentioned since, in some cases, the rates listed below are special rates.

Goleta

HOLIDAY INN
5650 Calle Real, Goleta 93017
Phone: (805) 964-6241

Single	\$14.50 up
Double	18.50 up

MOTEL 6
5897 Calle Real, Goleta 93017
Phone: (805) 964-1812

Single	\$ 7.00
Double	8.16

Upper State Street

SANDPIPER HYATT LODGE
3525 State Street, Santa Barbara 93105
Phone: (805) 687-5326

Single	\$ 8.50 up
Double	12.00 up

MOTEL 6
3505 State Street, Santa Barbara 93105
Phone: (805) 687-5400

PEPPER TREE MOTOR INN
3850 State Street, Santa Barbara 93110
Phone: (805) 687-5511

Single	\$15.00 up
Double	20.00

Beach Area

LA CASA DEL MAR
28 W. Cabrillo Boulevard, Santa Barbara 93101
Phone: (805) 966-6337

Single	\$10.60 up
Double	14.84 up

MING TREE
930 Orilla del Mar Drive, Santa Barbara 93103
Phone: (805) 966-1641

Single	\$14.00 up
Double	14.00 up

MOTEL 6
443 Corona del Mar, Santa Barbara 93103
Phone: (805) 965-0300

The campus is located on the Pacific Ocean approximately ten miles north of Santa Barbara. When approaching from the south on U.S. 101, drive past the four sets of traffic lights downtown. Continue for about seven miles and take the exit marked "UC Santa Barbara, Airport, Goleta" (California Highway 217). Continue for approximately three miles until you arrive at the entrance to the campus. There is a 25¢ daily parking fee on campus, and the personnel at the kiosk will have information and maps. When aproaching from the north on U.S. 101, take the exit marked "Glen Annie Road, Storke Road" and then follow the signs to the University. It is about two miles from the freeway offramp to the entrance of the campus.

Santa Barbara Airport is served by United Airlines and Air West with service from Los Angeles and San Francisco, and the commuter airline Golden West with service from Oxnard and Los Angeles. There are a Greyhound bus terminal and an Amtrak train station downtown; both offer service from Los Angeles and San

Francisco. There is a bus every thirty minutes between downtown and the campus. Half of these go via Goleta and Isla Vista and the other half via the airport. The distance between the airport and campus on foot is about one and a half miles, and a taxi can be obtained for approximately $2.50.

There is a very slight chance of showers in April and an equally slight chance of a heat wave. The usual weather condition is a temperature of 70° with possibly a slight mist.

Noon meals will be available on campus (grill service only) and there are many restaurants in Isla Vista, within ten-minutes walking distance.

PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the general sessions is ten minutes and in the special session thirty minutes. To maintain this schedule, the time limits will be strictly enforced.

SATURDAY, 9:30 A.M.

Special Session on Linear Algebra I, Room 1440, Ellison Hall

9:30-10:00 (1) Decomposable tensors as sums of dyads. Professor JOHN de PILLIS, University of California, Riverside (714-A2)

10:10-10:40 (2) Essentially doubly stochastic matrices. Professor EUGENE C. JOHNSEN, University of California, Santa Barbara (714-A6)

SATURDAY, 9:30 A.M.

Session on Geometry and Topology, Room 1444, Ellison Hall

9:30- 9:40 (3) Algebraic conics in arbitrary projective planes. Preliminary report. Mrs. CHERYL KOCH*, California State University, Los Angeles, and Professor RAYMOND B. KILLGROVE, San Diego State University (714-D1)

9:45- 9:55 (4) Sylvester-Gallai conjecture does not imply transversal axiom. Professor RAYMOND B. KILLGROVE, San Diego State University (714-D2)

10:00-10:10 (5) A cuspless infinite hyperbolic plane and a remark on series. Mr. K. DEMYS, Santa Barbara, California (714-D4)

10:15-10:25 (6) Monotone decompositions into trees of Hausdorff continua irreducible about a finite subset. Professor ELDON J. VOUGHT, California State University, Chico (714-G1)

10:30-10:40 (7) A covering property of continua having an uncountable set of disjoint non-separating plane embeddings. Dr. CLIFFORD W. ARNQUIST, California State University (714-G2)

SATURDAY, 11:00 A.M.

Invited Address, Room 1920, Ellison Hall

(8) L-functions on reductive groups. Professor GARTH W. WARNER, University of Washington (713-B4)

SATURDAY, 1:30 P.M.

Invited Address, Room 1920, Ellison Hall

(9) Applications of some theorems in partial differential equations to geometry. Professor HUNG-HSI WU, University of California, Berkeley (714-D3)

SATURDAY, 2:45 P.M.

Special Session on Linear Algebra II, Room 1440, Ellison Hall

2:45- 3:15 (10) Indeterminates and incidence matrices. Professor HERBERT J. RYSER, California Institute of Technology (713-A4)

3:25- 3:55 (11) The eigenvalues of a partitioned Hermitian matrix involving a parameter. Professor ROBERT C. THOMPSON, University of California, Santa Barbara (714-A7)

4:05- 4:35 (12) More on the L-property. Professor OLGA TAUSSKY-TODD, California Institute of Technology (713-A3)

4:45- 5:15 (13) On certain linear algebraic techniques in enumeration. Professor STANLEY G. WILLIAMSON, University of California, San Diego (714-A5)

SATURDAY, 2:45 P.M.

Session on Algebra and Analysis, Room 1444, Ellison Hall

2:45- 2:55	(14)	Radical and coradical theory in monoidal categories. Preliminary report. Mr. GORDON H. HUGHES, University of California, Riverside (714-A1) (Introduced by Professor R. E. Block)
3:00 - 3:10	(15)	A classification, decomposition and spectral multiplicity theory for bounded operators on a separable Hilbert space. Professor JOHN A. ERNEST, University of California, Santa Barbara (714-B1)
3:15- 3:25	(16)	On approximate and true solutions of a nonlinear singular perturbation problem. Preliminary report. Mr. FREDRICK A. HOWES, University of Southern California (714-B2)
3:30- 3:40	(17)	On some forms of Whitely. Professor PETER S. BULLEN, University of British Columbia (714-B3)
3:45 -3:55	(18)	On series iteration involving hypergeometric functions. Preliminary report. Professor MOSES E. COHEN, California State University, Fresno (714-B5)
4:00- 4:10	(19)	Quadratic differential equations. Dr. ROY B. LEIPNIK, Naval Weapons Center, China Lake, California (714-B6)

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

Seven Hundred Fifteenth Meeting
Northern Illinois University
DeKalb, Illinois
May 13 – 18, 1974

The seven hundred fifteenth meeting of the American Mathematical Society will be held at Northern Illinois University in DeKalb, Illinois, from Monday, May 13, to Saturday, May 18, 1974. The principal feature of the meeting will be a symposium on Mathematical Developments Arising from the Hilbert Problems. The symposium will be supported by a grant from the National Science Foundation. This topic was chosen by the Committee to Select Hour Speakers for Western Sectional Meetings. The Organizing Committee of the symposium consists of Paul T. Bateman, Felix E. Browder (chairman), R. Creighton Buck, Donald J. Lewis, and Daniel Zelinsky. The tenative list of speakers includes Lipman Bers, Enrico Bombieri, Herbert Busemann, Nicholas M. Katz, Steven L. Kleiman, Georg Kreisel, R. Langlands, George Lorentz, David Mumford, Albrecht Pfister, Julia B. Robinson, Guido Stampacchia, John T. Tate, Robert Tijdeman, and Arthur S. Wightman.

There will be five hour speakers per day in the symposium on Monday through Thursday. In addition there will be four hour speakers for the Society meeting on Friday who in subject matter will be an integral part of the symposium. These will be John Conway, O. Timothy O'Meara, Hugh L. Montgomery, and James B. Serrin.

At the request of the Organizing Committee of the symposium on Mathematical Developments Arising from the Hilbert Problems, Professor Jean Dieudonné has set up a committee of eminent mathematicians to assemble a list of the most outstanding unsolved problems of present day mathematics. It is hoped that such a list might serve to focus the interest of young research mathematicians on worthwhile projects and help them see their way through the considerable amount of mathematical publications. A member of Professor Dieudonné's panel will speak at the symposium.

There will be sessions for the presentation of contributed ten-minute papers on Friday afternoon and Saturday morning, May 17 and 18. The deadline for abstracts was March 27, 1974. There will be a session for late papers if one is needed, but late papers will not be listed in the printed program of the meeting.

All the addresses on the Mathematical Developments Arising from the Hilbert Problems will be presented in the Carl Sandburg Auditorium of the University Center. The sessions of contributed papers will be held in Reavis West, a classroom building four-minutes walk from the University Center.

The Council of the Society will meet at 2:00 p.m. on Sunday, May 12, 1974, at the Holiday Inn, 1212 W. Lincoln Highway in DeKalb.

REGISTRATION

The registration desk will be located in the lobby of the University Center. The desk will be open from 8:30 a.m. to 4:30 p.m. Monday through Friday and from 8:30 a.m. to 12 noon on Saturday. A message center will be maintained at the registration desk. The registration fees for the meeting will be as follows: members \$5.00, nonmembers \$7.50, and students and unemployed mathematicians \$1.00.

ACCOMMODATIONS

The following accommodations will be available on or near the Northern Illinois University campus for this meeting.

A. Dormitory accommodations in Grant South, a student dormitory complex ten-minutes walk from the University Center. These accommodations are not recommended for families with small children. Maid service is not provided. Room rates are \$3.50 per person per night on a double occupancy basis, and \$5.50 per person per night in a single room.

B. Accommodations in the University Plaza, a private dormitory type building five-minutes walk from the University Center. Maid service is provided. Room rates are \$7.50 per person per night on a double occupancy basis, and \$8.50 per person per night in a single room. All rooms have shared baths.

C. A limited number of rooms in the University Center. All of these rooms are for double occupancy at the rate of \$7.50 per person per night, and all have private baths.

Requests for room reservations in Grant South, University Plaza, or the University Center should be sent prior to April 15 to Professor Donald Ostberg, Department of Mathematical Sciences, Northern Illinois University, DeKalb, Illinois 60115. A form for making room reservations is included on the last page of these *Notices*.

Individuals arriving on campus without room reservations should contact the University Center registration desk to arrange accommodations.

There is also a Holiday Inn located within ten-minutes walk of the University Center. Its address and room rates are:

HOLIDAY INN
1212 W. Lincoln Hwy
DeKalb, Illinois 60115
Phone: (815) 758-8661
Single: $13.50
Double: 16.50 (2 persons)
Twin: 18.50 (2 persons)

Reservations should be made directly with the Inn, and mention should be made of this meeting in order to obtain the quoted rates.

FOOD SERVICE

Breakfast, lunch, and dinner will be available in the University Center throughout the conference. A snack bar will be open in Grant South during the evenings, but meals will not be served in the dormitories. There are also several restaurants within walking distance of campus. A list of these restaurants with directions for reaching them will be available at the registration desk.

TRAVEL

DeKalb, Illinois, is located on Illinois Route 38, approximately 70 miles west of Chicago. The campus of Northern Illinois University is adjacent to Route 38 on the west side of DeKalb.

Travelers driving to DeKalb from the east should take one of the routes shown on the map. Travelers driving to DeKalb from the west should arrive on Illinois Route 38.

Travelers arriving by air should fly to O'Hare Airport, Chicago, from which point ground transportation will be available to DeKalb as follows:

A. Continental Air Transport Company and University Bus with a transfer at the Ramada Inn in St. Charles, Illinois. Cost: $3.25 each way.

From O'Hare to DeKalb

	Lv. O'Hare	Ar. DeKalb
Sun. May 12	3:35 p.m. 5:35 p.m. 7:35 p.m.	5:20 p.m. 7:20 p.m. 9:20 p.m.
Mon. May 13	9:35 a.m. 11:35 a.m. 7:35 p.m.	11:20 a.m. 1:20 p.m. 9:20 p.m.
Tues. May 14	7:35 p.m.	9:20 p.m.
Wed. May 15	7:35 p.m.	9:20 p.m.
Thurs. May 16	3:35 p.m. 5:35 p.m.	5:20 p.m. 7:20 p.m.
Fri. May 17	5:35 p.m.	7:20 p.m.

Buses will deliver passengers to their residences for the conference.

From DeKalb to O'Hare

	Lv. DeKalb	Ar. O'Hare
Mon. May 13	4:30 p.m.	6:30 p.m.
Tues. May 14	4:30 p.m.	6:30 p.m.
Wed. May 15	4:30 p.m.	6:30 p.m.
Thurs. May 16	3:30 p.m. 4:30 p.m.	5:30 p.m. 6:30 p.m.
Fri. May 17	4:30 p.m.	6:30 p.m.
Sat. May 18	7:30 a.m. 10:30 a.m. 12:30 p.m.	9:30 a.m. 12:30 p.m. 2:30 p.m.

Buses will pick up passengers at their residences for departure.

B. Direct limousine service is available between O'Hare and DeKalb at a cost of $9.50 per person each way.

Reservations must be made in advance for transportation between O'Hare and DeKalb, using the form on the last page of these *Notices*. This form should be mailed to Professor Donald Ostberg, Department of Mathematical Sciences, Northern Illinois University, DeKalb, Illinois 60115, to arrive prior to May 1, 1974. Do not include payment with the transportation request; payment will be made directly to the carrier involved at the time of the trip.

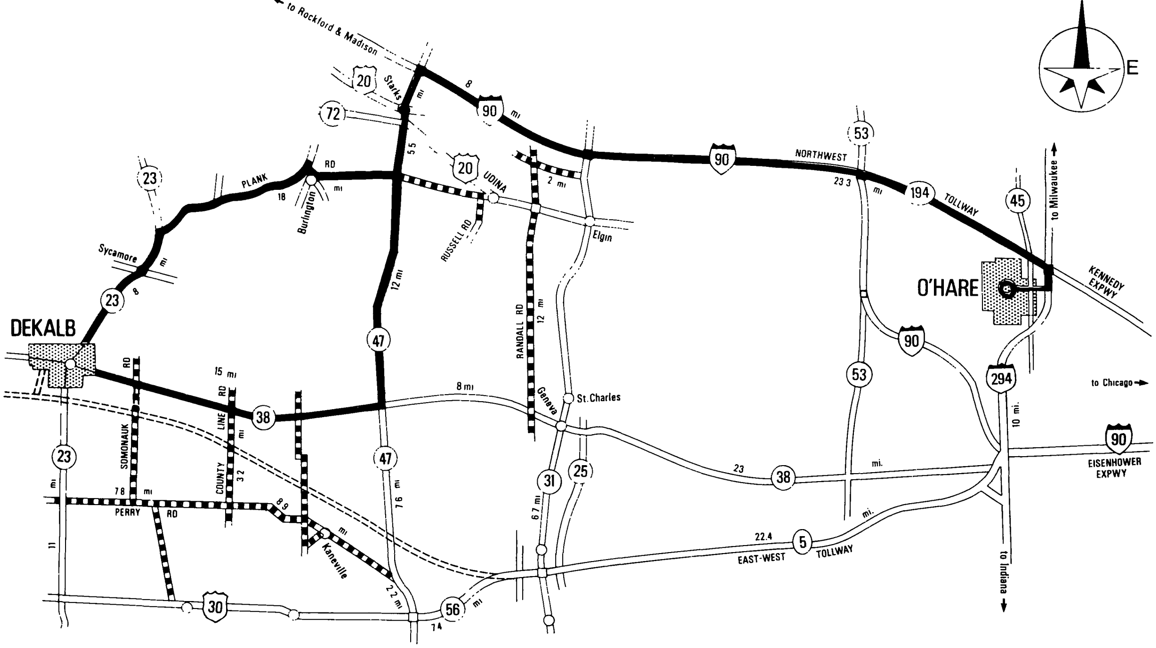
ENTERTAINMENT

A cash bar will be in operation from 5:30 p.m. to 6:30 p.m. on Monday through Friday evenings. In addition, the Department of Mathematical Sciences at Northern Illinois University will host a beer party for all those attending the meeting on Wednesday evening, May 15. Details concerning the location of the cash bar and the time and place of the beer party will be available at the registration desk.

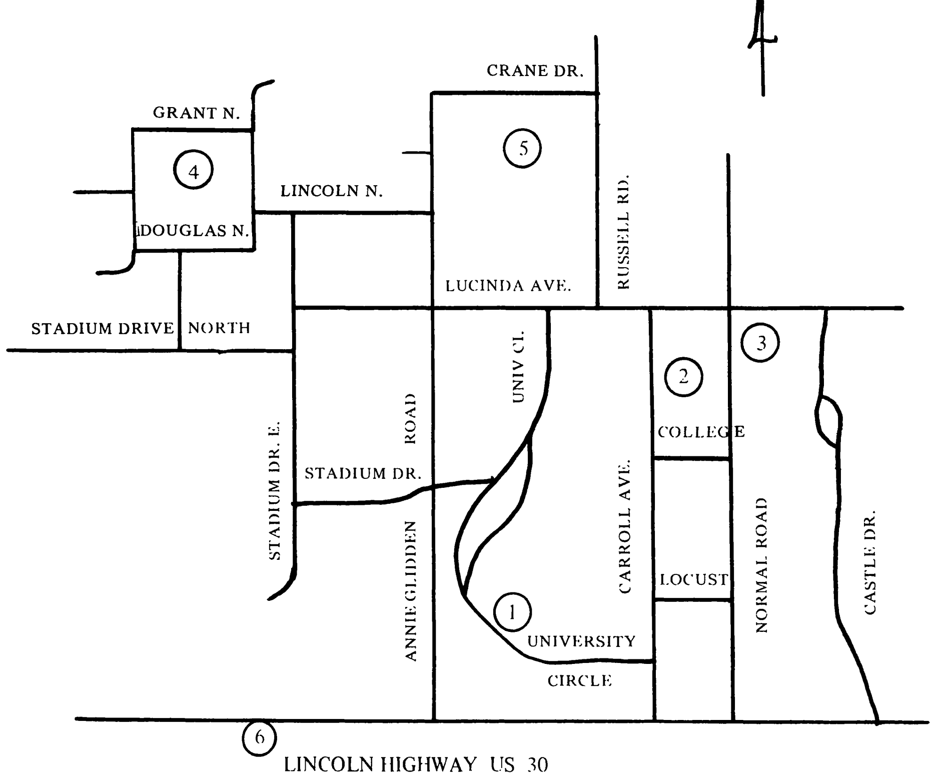
PARKING

Since the meeting will be held after the end of the spring semester at Northern Illinois University ample parking will be available on the campus. Visitors are requested not to park in metered or reserved parking places, or in no parking zones.

SUGGESTED ROUTES O'HARE-DEKALB



NORTHERN ILLINOIS UNIVERSITY



1 -- REAVIS WEST
2 -- UNIVERSITY CENTER & TOWER
3 -- ADAMS HALL (MATHEMATICS DEPARTMENT)
4 -- GRANT TOWERS RESIDENCE HALL
5 -- UNIVERSITY PLAZA
6 -- HOLIDAY INN

TENTATIVE TIME SCHEDULE

SYMPOSIUM ON MATHEMATICAL DEVELOPMENTS
ARISING FROM THE HILBERT PROBLEMS

Monday, May 13

9:00 a.m. - 10:00 a.m.	First problem. Speaker to be announced
10:30 a.m. - 11:30 a.m.	What have we learnt from Hilbert's second problem? GEORGE KREISEL
1:30 p.m. - 2:30 p.m.	Fourth problem. HERBERT BUSEMANN
3:00 p.m. - 4:00 p.m.	Fifth problem. Speaker to be announced
4:30 p.m. - 5:30 p.m.	Sixth problem. ARTHUR S. WIGHTMAN

Tuesday, May 14

9:00 a.m. - 10:00 a.m.	Seventh problem. ROBERT TIJDEMAN
10:30 a.m. - 11:30 a.m.	The Riemann hypothesis for curves. ENRICO BOMBIERI
1:30 p.m. - 2:30 p.m.	The Riemann hypothesis for varieties. NICHOLAS M. KATZ
3:00 p.m. - 4:00 p.m.	Ninth problem. JOHN T. TATE
4:30 p.m. - 5:30 p.m.	Tenth problem. JULIA B. ROBINSON

Wednesday, May 15

9:00 a.m. - 10:00 a.m.	Twelfth problem. R. LANGLANDS
10:30 a.m. - 11:30 a.m.	Thirteenth problem. GEORGE LORENTZ
1:30 p.m. - 2:30 p.m.	Fourteenth problem. DAVID MUMFORD
3:00 p.m. - 4:00 p.m.	Rigorous foundation of Schubert's enumerative calculus, STEVEN L. KLEIMAN
4:30 p.m. - 5:30 p.m.	Seventeenth problem and related problems. ALBRECHT PFISTER

Thursday, May 16

9:00 a.m. - 10:00 a.m.	Nineteenth problem. ENRICO BOMBIERI
10:30 a.m. - 11:30 a.m.	Twenty-first problem. NICHOLAS M. KATZ
1:30 p.m. - 2:30 p.m.	Twenty-second problem. LIPMAN BERS
3:00 p.m. - 4:00 p.m.	Twenty-third problem. GUIDO STAMPACCHIA
4:30 p.m. - 5:30 p.m.	Current problems. Speaker to be announced

SEVEN HUNDRED FIFTEENTH MEETING

Friday, May 17

9:00 a.m. - 10:00 a.m.	Eighth problem. HUGH L. MONTGOMERY
10:30 a.m. - 11:30 a.m.	Eleventh problem. O. TIMOTHY O'MEARA
1:30 p.m. - 2:30 p.m.	Eighteenth problem. JOHN CONWAY
3:00 p.m. - 4:00 p.m.	Twentieth problem. JAMES B. SERRIN
4:15 p.m. - 6:00 p.m.	Sessions for contributed papers

Saturday, May 18

9:00 a.m. - 12:00 noon	Sessions for contributed papers

Paul T. Bateman
Associate Secretary

Urbana, Illinois

INVITED SPEAKERS AT AMS MEETINGS

This section of these *Notices* lists regularly the individuals who have agreed to address the Society at the times and placed noted below. For some future meetings, the lists of speakers are incomplete.

New York, New York, April 1974

Serge Lang — Sol I. Rubinow
George C. Papanicolaou — William P. Thurston

Santa Barbara, California, April 1974

Garth W. Warner — Hung-Hsi Wu

DeKalb, Illinois, May 1974

John Conway — Hugh L. Montgomery
O. Timothy O'Meara — James B. Serrin

Washington, D. C., January 1975

Linda Keen — Wilfried Schmid

CHAIRMEN AND TOPICS OF SPECIAL SESSIONS

New York, New York, April 1974

V. Lakshmikantham, Nonlinear problems in differential and integral equations
Herbert S. Wilf, Combinatorial algorithms

Santa Barbara, California, April 1974

Marvin Marcus, Henryk Minc, and Robert C. Thompson, Linear algebra

NATIONAL SCIENCE FOUNDATION BUDGET

On February 4, 1974, the National Science Foundation distributed its annual budget report, comparing expenditures for the previous fiscal year with those of the current year and with proposed figures for the following year. This year's budget might well be termed the "energy budget" in view of the overwhelming emphasis placed on support for energy directed research, and it is clear that mathematics will not benefit from this emphasis.

The NSF release shows an increase of $141.8 million in its proposed 1975 budget, a rise of 21.9 percent over the 1974 budget. A large proportion of the increase is accounted for by the emphasis on energy concerns, with the result that nearly one-third (32%) of the total NSF budget for 1975 will be devoted to energy. This year's budget was constructed in two stages: the original total budgeted was $672.1 million, including $325 million for Scientific Research Project Support (SRPS). These sums were increased in the so-called "energy amendment" to $788.2 and $363.7 millions, respectively, which are the figures being sent to Congress. In the process, SRPS declined from 48.4 percent to 46.1 percent of the total.

The two major programs receiving the largest increases are Scientific Research Project Support and Research Applied to National Needs (RANN). The entire increase in RANN ($73.8 million) is allocated for energy research, specifically for research directed towards utilization of solar and geothermal resources. The emphasis on energy is also reflected in the funding for Scientific Research Project Support, where $53.6 million of the $72.4 million is allocated to energy-related programs. This will make 36 percent of the total SRPS budget be allocated to energy-related research.

Although SRPS will receive significant increases, mathematics will not benefit as much as other disciplines; the areas receiving the largest increases will be chemistry (42%), engineering (33%), earth sciences (29%), materials research (27.5%), and biological sciences (23%). Support for mathematics, in contrast, will increase only 16 percent, somewhat below the 20 percent total increase for SRPS as a whole. In the original budget, the distribution to each of the various programs was essentially in the same proportion as it had been in the previous year. Mathematics fell behind on a relative basis as a result of the increases included in the energy amendment.

In the past several years the program summary did not even mention mathematics (except for computing activities); this year's summary states, "In mathematics, the emphasis will be given to work on the creation of new mathematical structures and on an array of studies dealing with the application of mathematics to various problems in the biological and social sciences."

The figures in the accompanying tables were extracted from the last three budget reports (January 1972, 1973, and February 1974). The data indicate how mathematics has fared in recent years in comparison with some major groups of items in the NSF budget.

Table I provides a comparison of budgets for the years 1971 through 1975. It gives actual figures for 1971, 1972, and 1973; current estimates for 1974; and budget requests for 1975. The figures in line (6) of Table I demonstrate that the portion of research support funds devoted to mathematics is a generally decreasing function of time. Moreover, the percentage of the total NSF expenditures devoted to mathematical research is monotonically decreasing as indicated by the figures on line (7) of the table.

Table II shows that, of the four sections comprising the Mathematical and Physical Sciences Division of the Foundation, only the Mathematical Sciences Section has received a steadily decreasing share of the funds for the Division. The increases over the past four years in the portions allotted to Physics, Chemistry and Astronomy have clearly been at the expense of the Mathematical Sciences Section. In this interval the fraction allotted to mathematics has fallen from 19.6 percent to 15.5 percent of the total, which represents a reduction on a relative basis of about 20 percent.

Table III shows that the fraction of support for the Mathematical Sciences Section in the total SRPS budget has fallen from 7.2 percent to 4.7 percent, representing a reduction of more than a third, although the fraction of SRPS funds going to the Division as a whole appears to have stabilized at about 30 percent.

TABLE I: NATIONAL SCIENCE FOUNDATION

Millions of Dollars

Fiscal Year	1971 Actual 1/22/72	Change (71–72)	1972 Actual 1/27/73	Change (72–73)	1973 Actual 2/2/74	Change (73–74)	1974 Estimate 2/2/74	Change (74–75)	1975 Budget 2/2/74
(1) Mathematics Research Support	$ 12.9	6.2%	$ 13.7	2.9%	$ 14.1	4.3%	$ 14.7	16.3%	$ 17.1
(2) Other Research Support (Note A)	315.5	37.9%	435.0	11.1%	483.5	2.5%	495.8	30.0%	644.5
(3) Education, Information, Foreign Currency Program (Note B)	145.9	-12.7%	127.4	-40.0%	84.1	+23.9%	104.2	-16.4%	87.1
(4) Program Development and Management ("Overhead") (Note C)	21.8	12.8%	24.6	16.3%	28.6	10.8%	31.7	24.6%	39.5
(5) Totals	$ 496.1	21.1%	$ 600.7	1.6%	$ 610.3	5.9%	$ 646.4	21.9%	$ 788.2
(6) (1) as % of (1) & (2)	3.93%		3.05%		2.78%		2.88%		2.58%
(7) (1) as % of (5)	2.60%		2.28%		2.27%		2.27%		2.17%

NOTE A: Scientific research and facilities (excluding mathematics), national and special research programs (excluding science information activities), national research centers, and research applied to national needs. Support for mathematics has been excluded, cf. items (1) and (3)

NOTE B: The programs in this group are ones in which there is some support for projects in every field, including mathematics. The foreign currency program involves both cooperative scientific research and the dissemination and translation of foreign scientific publications. Foreign currencies in excess of the normal requirements of the U.S. are used.

NOTE C: This heading covers the administrative expenses of operating the Foundation; the funds involved are not considered to constitute direct support for individual projects.

TABLE II: MATHEMATICAL AND PHYSICAL SCIENCES DIVISION

Millions of Dollars

	1971 Actual 1/22/72		1972 Actual 1/27/73		1973 Actual 2/2/74		1974 Estimate 2/2/74		1975 Budget 2/2/74	
Mathematics	$ 12.9	(19.6%)	$ 13.7	(17.2%)	$ 14.1	(17.0%)	$ 14.7	(16.9%)	$ 17.1	(15.5%)
Physics	26.5	(40.3%)	33.3	(41.9%)	34.9	(42.1%)	36.4	(41.8%)	44.8	(40.5%)
Chemistry	19.6	(29.8%)	24.5	(30.8%)	25.1	(30.3%)	26.4	(30.3%)	37.1	(33.6%)
Astronomy	6.7	(10.2%)	8.0	(10.1%)	8.8	(10.6%)	9.6	(11.0%)	11.5	(10.4%)
Total	$ 65.7		$ 79.5		$ 82.9		$ 87.1		$ 110.5	

TABLE III: SCIENTIFIC RESEARCH PROJECT SUPPORT

Mathematical Sciences Section	$ 12.9	(7.2%)	$ 13.7	(5.5%)	$ 14.1	(5.1%)	$ 14.7	(5.0%)	$ 17.1	(4.7%)
Mathematics and Physical Sciences Division	$ 65.7	(36.4%)	$ 79.5	(32.0%)	$ 82.9	(29.9%)	$ 87.1	(29.9%)	$ 110.5	(30.4%)
Total Scientific Research Project Support	$ 180.4		$ 248.6		$ 277.3		$ 291.3		$ 363.7	

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HOW TO TALK MATHEMATICS

By P. R. Halmos

Apology

The purpose of what follows is to suggest to a young mathematician what he might do (and what he had better not do) the first few times that he gives a public lecture on his subject. By a "public lecture" I mean something like a colloquium talk (to more or less the entire mathematics department at a large university), or an invited address (to more or less the entire membership in attendance at a meeting of the American Mathematical Society); I do not mean a classroom lecture (to reluctant beginners) or a seminar talk (to dedicated experts).

That an article on how to talk mathematics might serve a good purpose was suggested by some of the officers of the American Mathematical Society. It seems that there have been more and more complaints about invited addresses ("they are incomprehensible, and therefore useless"), and that, therefore, it might do some good to let a speaker know about such complaints before he adds to the reason for them.

A genius makes his own rules, but a "how to" article is written by one ordinary mortal for the benefit of another. Harpo Marx, one of the greatest harpists of all times, was never taught how to play; everything he did was "wrong" according to standard teaching. Most things that an article such as this one can say have at least one counterexample in the practice of some natural born genius. Authors of articles such as this one know that, but, in the first approximation, they must ignore it, or nothing would ever get done.

Why lecture?

What is the purpose of a public lecture? Answer: to attract and to inform. We like what we do, and we should like for others to like it too; and we believe that the subject's intrinsic qualities are good enough so that anyone who knows what they are cannot help being attracted to them. Hence, better answer: the purpose of a public lecture is to inform, but to do so in a manner that makes it possible for the audience to absorb the information. An attractive presentation with no content is worthless, to be sure, but a lump of indigestible information is worth no more.

The question then becomes this: what is the best way to describe a subject (or that small part of a subject that has recently been the center of the lecturer's attention) to an audience of mathematicians most of whom are interested in something else? The problem is different from describing a subject to students who, willy nilly, must learn it in usable detail, and it is different from sharing a new discovery with fellow experts who have been thinking about the same sort of thing and are wondering what you know that they don't.

Simplicity

Less is more, said the great architect Mies van der Rohe, and if all lecturers remembered that adage, all audiences would be both wiser and happier.

Have you ever disliked a lecture because it was too elementary? I am sure that there are people who would answer yes to that question, but not many. Every time I have asked the question, the person who answered said no, and then looked a little surprised at hearing the answer. A public lecture should be simple and elementary; it should not be complicated and technical. If you believe and can act on this injunction ("be simple"), you can stop reading here; the rest of what I have to say is, in comparison, just a matter of minor detail.

To begin a public lecture to 500 people with "Consider a sheaf of germs of holomorphic functions..." (I have heard it happen) loses people and antagonizes them. If you mention the Künneth formula, it does no harm to say that, at least as far as Betti numbers go, it is just like what happens when you multiply polynomials. If you mention functors, say that a typical example is the formation of the duals of vector spaces and the adjoints of linear transformations.

Be simple by being concrete. Listeners are prepared to accept unstated (but hinted) generalizations much more than they are able, on the spur of the moment, to decode a precisely stated abstraction and to re-invent the special cases that motivated it in the first place. Caution: being concrete should not lead to concentrating on the trees and missing the woods. In many parts of mathematics a generalization is simpler and more incisive than its special parent. (Examples: Artin's solution of Hilbert's 17th problem about definite forms via formally real fields; Gelfand's proof of Wiener's theorem about absolutely convergent Fourier series via Banach algebras.) In such cases there is always a concrete special case that is simpler than the seminal one and that illustrates the generalization with less fuss; the lecturer who knows his subject will explain the complicated special case, and the generalization, by discussing the simple cousin.

Some lecturers defend complications and technicalities by saying that that's what their subject is like, and there is nothing they can do about it. I am skeptical, and I am willing to go so far as to say that such statements indicate incomplete understanding of the subject and of its place in mathematics. Every subject, and even every small part of a subject, if it is identifiable, if it is big enough to give an hour talk on, has its simple aspects, and they, the simple aspects, the roots of the subject, the connections with more widely known and older parts of mathematics, are what a non-specialized audience needs to be told.

Many lecturers, especially those near the foot of the academic ladder, anxious to climb rapidly, feel under pressure to say something brand new - to impress their elders with their brilliance and profundity. Two comments: (1) the best way to do that is to make the talk simple, and (2) it doesn't really have to be done. It may be entirely appropriate to make the lecturer's recent research the focal point of the lecture, but it may also be entirely appropriate not to do so. An audience's evaluation of the merits of a talk is not proportional to the amount of original material included; the explanation of the speaker's latest theorem may fail to improve his chances of creating a good impression.

An oft-quoted compromise between trying to be intelligible and trying to seem deep is this advice: address the first quarter of your talk to your high-school chemistry teacher, the second to a graduate student, the third to an educated mathematician whose interests are different from yours, and the last to the specialists. I have done my duty by reporting the formula, but I'd fail in my duty if I didn't warn that there are many who do not agree with it. A good public lecture should be a work of art. It should be an architectural unit whose parts reinforce each other in conveying the maximum possible amount of information - not a campaign speech that offers something to everybody and, more likely than not, ends by pleasing nobody.

Make it simple, and you won't go wrong.

Details

Some lecturers, with the best of intentions, striving for simplicity, try to achieve it by being overly explicit and overly detailed; that's a mistake.

"Explicit" refers to computations. If a proof can be carried out by multiplying two horrendous expressions, say so and let it go at that; the logical simplicity of the steps doesn't necessarily make the computation attractive or informative to carry out. Landau, legend has it, never omitted a single epsilon from his lectures, and his lectures were inspiring anyway - but that's the exception, not the rule. If, on an exceptional occasion, you think that a brief computation will be decisive and illuminating, put it in, but the rule for ordinary mortals still stands: do not compute in public. It may be an explicit and honest thing to do, but that's not what makes a lecture simple.

"Detailed" refers to definitions. Some lecturers think that the way to reach an audience of non-experts is to tell them everything. ("To get to the theorem I proved last week, I need, starting from the beginning, 14 definitions and 11 theorems that my predecessors have proved. If I talk and write fast, I can present those 25 nuggets in 25 minutes, and in the rest of the time I can state and prove my own thing.") This, too, is honest, and it makes the lecture self-contained, in some sense - but it is impossible to digest, and its effect is dreadful. If someone told you, in half an hour, the meaning of each ideogram on a page of Chinese, could you then read and enjoy the poem on that page in the next half hour?

Proofs

Some lecturers understand the injunction "be simple" to mean "don't prove anything". That isn't quite right. It is true, I think, that it is not the main purpose of a public lecture to prove things, but to prove nothing at all robs the exposition of an essential part of what mathematicians regard as attractive and informative. I would advise every lecturer to be sure to prove something - one little theorem, one usable and elegant lemma, something that is typical of the words and the methods used in the subject. If the proof is short enough, it almost doesn't matter that it may, perhaps, not be understood. It is of value to the listener to hear the lecturer say that Bernoulli numbers enter the theory of stable homotopy groups, even if the listener has only an approximate idea of what Bernoulli numbers or homotopy groups are.

Something that's even better than a sample proof is the idea of a proof, the intuition that suggested it in the first place, the reason why the theorem is true. To find the right words to describe the central idea of a proof is sometimes hard, but it is worth the trouble; when it can be done, it provides the perfect way to communicate mathematics.

Problems

In the same vein, it is a false concept of simplicity that makes a lecturer concentrate only on what is safe and known; I strongly recommend that every public lecture reach the frontiers of knowledge, and at least mention something that is challenging and unknown. It doesn't have to be, it shouldn't be, the most delicate and newest technicality. Don't be afraid of repeating an old one; remember that many in your audience probably haven't heard of your subject since they took a course in it in graduate school, a long time ago. They will learn something just by hearing today that the unsolved problem they learned about years ago is still unsolved. The discussion of unsolved problems is a valuable part of the process of attracting and informing - it is, I think, an indispensable part. A field is not well described if its boundaries are missing from the description; some knowledge of the boundaries is essential for an understanding of where the field is today as well as for enlarging the area of our knowledge tomorrow. A public lecture must be simple, yes, but not at the cost of being empty, or, not quite that bad but bad enough, it must not be incomplete to the point of being dishonest.

Organization

The organization of a talk is like the skeleton of a man: things would fall apart without it, but it's bad if it shows. Organize your public lecture, plan it, prepare it carefully, and then deliver it impromptu, extemporaneously.

To prepare a talk, the first thing to know is the subject, and a very close second is the audience. It's much more important to adjust the level to fit the audience in a public lecture than it is in a book. ("Adjust the level" is not a euphemism for "talk down". Don't insult the audience, but be realistic. Slightly over the mark, very slightly, doesn't do much harm, but too much over is much worse than somewhat under.) A reader can put down a book and come back to it when he has learned more; an annoyed and antagonized listener will, in spirit, leave you, and, as far as this talk is concerned, he'll never come back.

The right level for a talk is a part of what organization is meant to achieve, but, of course, the first and more important thing to organize is the content. Here I have two recommendations (in addition to "prove something" and "ask something", already mentioned): (1) discuss three or four related topics, and the connections between them, rather than relentlessly pursue one central topic, and (2) break each topic into four or five sub-topics, portable, freely addable or subtractable modules, the omission of any one of which would not wreck the continuity.

As for extemporaneous delivery, there are two reasons for that: it sounds good, and it makes possible an interaction between the speaker and the listeners. The faces in the audience can be revealing and helpful: they can indicate the need to slow down, to speed up, to explain something, to omit something.

Preparation

To prepare a lecture means to prepare the subjects it will cover, the order in which those subjects are to come, and the connections between them that you deem worthy of mention; it does not mean to write down all the words with the intention of memorizing them (or, much worse, reading them aloud). Still: to write it all out is not necessarily a bad idea. "All" means all, including, especially, exactly what is to be put on the blackboard (with a clear idea of when it will be put on and whether it will remain for long or be rubbed out right away). To have it all written out will make it easier to run through it once, out loud, by a blackboard, and thus to get an idea of the timing. (Warning: if the dry run takes an hour, then the actual delivery will take an hour and a half.)

Brevity

Most talks are described as "one-hour lectures", but, by a generally shared tradition, most are meant to last for 50 minutes only. Nobody will reproach you for sitting down after 45 minutes, but the majority of the audience will become nervous after 55, and most of them will glare at you, displeased and uncomfortable, after 65.

To take long, to run over time, is rude. Your theorems, or your proofs, are not all that important in other peoples' lives; that hurried, breathless last five minutes is expendable. If you didn't finish, say so, express your regret if you must, but stop; it's better thus than to give the audience cause for regret.

Techniques

A public lecture usually begins with an introduction by the chairman of the session. Rule of etiquette: give him a chance. Before the lecture begins, sit somewhere by the side of the room, or with the audience, near the front; do not stand by or near the blackboard, or hover near the chairman worrying him.

One good trick to overcome initial stage-fright is to memorize one sentence, the opener. After that, the preparation and your knowledge of the subject will take over.

Try very hard to avoid annoying mannerisms. Definition: an annoying mannerism is anything that's repeated more than twice. A mannerism can be verbal ("in other words", pronounced " 'n 'zer w'rs", meaning nothing), it can be visual (surrounding a part of the material on the blackboard by elaborate fences), or it can be dynamic (teeter-tottering at the edge of the platform).

If you are in mechanical trouble, catch the chairman's eye and say, to him only, "I am out of chalk", or "May I have an eraser?". Do not bumble about your awkwardness and do not keep on apologizing. ("Oh, dear, where can I put this - sorry, I seem to have run out of room - well, let's see, perhaps we don't need this anymore...".) Make the appropriate decision and take the appropriate action, but do so silently. Keep your own counsel, and do not distract the audience with irrelevancies.

Silence is a powerful tool at other times too; the best speakers are also the best non-speakers. A long period of silence (five seconds, say, or ten at most) after an important and crisply stated definition or theorem puts the audience on notice ("this is important") and gives them a chance to absorb what was just said. Don't overdo it, but three or four times during the hour, at the three or four high points, you might very well find that the best way to explain something is to say nothing.

Speak slowly and speak loudly; write large and speak as you write; write slowly and do not write much. Intelligently chosen abbreviations, arrows for implications, and just reminder words, not deathless prose, are what a board is for; their purpose is to aid the audience in following you by giving them something to look at as well as something to listen to. (Example: do not write "<u>semisimple is defined as follows:</u>"; write "<u>semisimple:</u>".) Do not, ever, greet an audience with a carefully prepared blackboard (or overhead projector sheets) crammed with formulas, definitions, and theorems. (An occasionally advisable exception to this rule has to do with pictures - if a picture, or two pictures, would help your exposition but would take too long to draw as you talk, at least with the care it deserves, the audience will forgive you for drawing it before the talk begins.) The audience can take pleasure in seeing the visual presentation grow before its eyes - the growth is part of your lecture, or should be.

Flexibility

Because of the unpredictability of the precise timing (you didn't rehearse enough, the audience asks questions during the talk, the lecture room is reserved for another group at 5:00 sharp, or you just plain get mixed up and waste time trying to get unscrambled), flexibility is an important quality to build into a lecture. You must be prepared to omit (or to add!) material, and you must be prepared to do so under pressure, in public, on the spur of the moment, without saying so, and without seeming to do so. There are probably many ways to make a lecture flexible; I'll mention two that I have found useful.

The first is exercises. Prepare two or three statements whose detailed discussion might well be a part of the lecture but whose omission would not destroy continuity, and, at the proper places during your lecture, "assign" them to the audience as exercises. You run the slight risk of losing the attention of some of the more competitive members of the group for the rest of the hour. What you gain is something else that you can gracefully fill out your time with if (unlikely as that may be) you finish everything else too soon, and, at the same time, something that'll never be missed if you do not discuss the solution. (Exercises in this sense may yield another fringe benefit: they'll give the audience something to ask their courtesy questions about.)

A second way to make a lecture flexible is one I mentioned before and I believe is worth emphasizing again: portable modules. My notes for a lecture usually consist of about 20 telegraphically written paragraphs. The detailed presentation of each paragraph may take between 2 and 4 minutes, and at least half the paragraphs (the last 10) are omittable. These omittable modules often contain material dear to my heart: that clever proof, that ingenious generalization, that challenging question - but no one (except me) will miss them if I keep mum. Knowing that those modules are there, I sail through the first half of the period with no worries: I am sure that I won't run out of things to say, and I am sure that everything that I must say will get said. In the second half, or last third, of the period I keep an eye on the time, and, without saying anything about it, make instantaneous decisions about what to throw overboard.

One disadvantage of this method is that at the end of your time you might sound too abrupt, as if you had stopped in the middle of a sentence. To avoid the abrupt ending, prepare your peroration, and do not omit it. The peroration can be a three-sentence summary of the whole lecture, or it can be the statement of the most important unsolved problem of the subject. Make it whatever you think proper for an ending, and then end with it.

Rule of etiquette: when you stop, sit down. Literally sit down. Do not just stop talking and look helpless, and do not ask for questions; that's the chairman's job.

Short talks

Short talks are harder to prepare and to deliver than long ones. The lecturer has less time to lay the groundwork, and the audience has less time to catch on; the lecturer feels under pressure to explain quickly, and the audience is under pressure to understand quickly.

In my experience a 20-minute talk can still be both enjoyable and enlightening; all you need to do is prepare a 10-minute talk and present it leisurely. A 10-minute talk is the hardest to do right; the precepts presented above (simple, organized, and short) must be applied again, but this time there is no room for error. Focus on one idea only, and on its simplest nontrivial special case at that, practice the talk and time it carefully, and under no circumstances allow a 10-minute contributed paper to become a 45-minute uninvited address. It has been done, but the results were neither informative nor attractive.

Some experts are willing to relax the rules for a 10-minute talk: it is all right, they say, to dive into the middle of things immediately, and it is all right, they say, to use prepared projection sheets. Others, having in mind the limited velocity and capacity of the human mind to absorb technicalities, disagree.

Summary

My recommendations amount to this: make it simple, organized, and short. Make your lecture simple (special and concrete); be sure to prove something and ask something; prepare, in detail; organize the content and adjust to the level of the audience; keep it short, and, to be sure of doing so, prepare it so as to make it flexible.

Remember that you are talking in order to attract the listeners to your subject and to inform them about it; and remember that less is more.

LETTERS TO THE EDITOR

Editor, the *Notices*

Many of us have worked to help North Vietnamese mathematicians keep up their active teaching and research under the difficult conditions imposed by the war. Individuals have sent many scientific books, and this year the American Mathematical Society has contributed a large number of its publications.

Such support seems to us just as appropriate now as in the past. Beside sending books, and corresponding with mathematicians in the same field, there is one more thing we can do.

To send a delegation to the International Congress at Vancouver in August could be quite important to the North Vietnamese mathematical community, but would strain available resources considerably. Why doesn't the world mathematical community share the load? Specifically, let's try to collect at least $2,000 toward the air fare to bring North Vietnamese mathematicians to the Congress. (This amount is proposed as one which we can contribute without hardship, yet which might be enough to make a difference to the feasibility of their coming.)

Contributions may be sent to Chandler Davis, Department of Mathematics, University of Toronto, Toronto M5S 1A1, Canada.

Chandler Davis
E. E. Moise
Anatol Rapoport
Rimhak Ree
Steve Smale

QUERIES

Edited by Wendell H. Fleming

The QUERIES column is published in each issue of these *Notices*. This column welcomes questions from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning various conjectures. When appropriate, replies from readers will be edited into a definitive composite answer and published in a subsequent column. All answers received to QUERIES will ultimately be forwarded to the questioner. Consequently, all items submitted for consideration for possible publication in this column should include the name and complete mailing address of the person who is to receive the replies. The queries themselves, and responses to such queries, should be typewritten if at all possible and sent to Professor Wendell H. Fleming, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

QUERIES

35. Morris W. Hirsch (Department of Mathematics, University of California, Berkeley, California 94720). I would like to know who first used triangulations to prove the so-called Gauss-Bonnet theorem: the integral of the Gaussian curvature of a compact closed surface is 2π times the Euler characteristic. (This theorem was first proved, using Kronecker's index of a function, by Walther Dyck, Math. Ann. 32 (1888), pp. 465-512. It should be called the Gauss-Bonnet-Dyck theorem, since Gauss and Bonnet considered only polygons.) The proof by counting simplices appeared in Blaschke's book, Vorlesungen Über Differential Geometrie, in 1924. Is this proof original with Blaschke? Did it appear earlier?

36. R. S. Cunningham (Department of Mathematics, University of Kansas, Lawrence, Kansas 66044). I would like information, especially references, on the uses of algebra in linguistics.

RESPONSES TO QUERIES

Replies have been received to queries published in recent issues of these *Notices*, as follows. The editor wishes to thank those who have responded.

26. (Small, October 1973). The query sought a "simple algebraic proof" or references to such for the following result: Any matrix of the form

$$A = \begin{pmatrix} a & c+di \\ c-di & b \end{pmatrix},$$

where a, b, c, d are integers and a, b are positive and $\det(A) = 1$, can be written in the form $A - BB^*$, where B is a 2×2 matrix with entries $\mathbb{Z}[i]$ and * means conjugate-transpose.

O. Tausky supplied the following proof: "We have $ab = 1 + c^2 + d^2$. This means that the quaternion $d + ic + j$ has its norm equal to ab. By a known theorem (cf.e.g. G. Pall and O. Taussky, Application of quaternions to the representation of a binary quadratic form as a sum of four squares, Proc. Royal Irish Acad. 58A3 (1957), 23–28) there exist two quaternions with integral coordinates $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ respectively $\beta_1, \beta_2, \beta_3, \beta_4$ whose norms are a, b and whose product is $d + ic + j$. Hence by quaternion multiplication:

$$c = \alpha_1\beta_2 + \alpha_2\beta_1 + \alpha_3\beta_4 - \alpha_4\beta_3$$
$$d = \alpha_1\beta_1 - \alpha_2\beta_2 - \alpha_3\beta_3 - \alpha_4\beta_4.$$

Take then the matrix

$$B = \begin{pmatrix} \alpha_1 - i\alpha_2 & \alpha_3 - i\alpha_4 \\ \beta_2 - i\beta_1 & \beta_4 + i\beta_3 \end{pmatrix}.$$

This matrix fulfills the required conditions."

In this proof, the machinery of quadratic forms over $\mathbb{Z}$ enters indirectly, through the Proc. Royal Irish Acad. result cited. The querier would like a proof which avoids this machinery, but now has some doubt whether such a proof can be found.

31. (Shelupsky, February 1974). J. J. Schäffer responds as follows:

"The correct inequalities are $6 \leqq L \leqq 8$. The lower [upper] bound is attained if and only if the unit disk is an affinely regular hexagon [a parallelogram].

"Except for the 'only if' part, the preceding result was obtained by Laugwitz [5]; a complete proof can be found in [6]. The problem, including the case of non-centrally symmetric 'unit disk', goes back to Gołąb [2].

"The following list of references will be found useful in this connection. There is a growing literature on analogous problems in spaces of dimension greater than 2, but I am not listing any references to this development.

1. G.C. Chakerian and W. K. Talley, Some properties of the self-circumference of convex sets, Arch. Math. (Basel) 20 (1969), 431-443.

2. S. Gołąb, Quelques problèmes métriques de la géométrie de Minkowski, Travaux de l'Académie des Mines à Cracovie 6 (1932) (Polish, French summary).

3. B. Grünbaum, Self-circumference of convex sets, Colloq. Math. 13 (1964), 55-57.

4. B. Grünbaum, The perimeter of Minkowski unit disks, Colloq. Math. 15 (1966), 135-139.

5. D. Laugwitz, Konvexe Mittelpunktsbereiche und normierte Räume, Math. Z. 61 (1954), 235-244.

6. J. J. Schäffer, Inner diameter, perimeter, and girth of spheres, Math. Ann. 173 (1967), 59-79.

7. J. J. Schäffer, The self-circumference of polar convex disks, Arch. Math. (Basel) 24 (1973), 87-90."

Dorothy Wolfe pointed out the related article, Inequalities for sums of distances, by G. D. Chakerian and M. S. Klamkin, Amer. Math. Monthly 80 (1973), 1009-1017.

Responses were also received from H. S. Witsenhausen and Ken Davidson.

NEWS ITEMS AND ANNOUNCEMENTS

REQUEST FOR APPLICATIONS OF MATHEMATICS SUITABLE FOR USE IN SECONDARY SCHOOLS

With the financial support of the National Science Foundation, the National Council of Teachers of Mathematics and the Mathematical Association of America through its Committee on the Undergraduate Program in Mathematics are engaged in producing resource materials in all the various applications of mathematics suitable for use by both teachers and students in mathematics instruction for grades 7–12, i.e., the last six years of secondary school. Applications of arithmetic, elementary and advanced algebra, geometry, computing, and other more advanced topics are being worked on. In addition to the uses of mathematics in other disciplines, applications of mathematics in daily life and to skilled trades will be especially emphasized. The readership of this journal is hereby requested to send suggestions regarding this project, sample problems, references, or any other suitable materials ranging from simple exercises to extended model building and mathematical development to CUPM, P.O. Box 1024, Berkeley, California 94701. The readership is reminded that through hobbies or previous employment it may know of special applications that might otherwise escape notice.

EDITORIAL POLICY FOR THE TRANSACTIONS

For economic reasons the Transactions of the American Mathematical Society will have to publish fewer pages this year than last year, and only 80% of the projected number of pages for this year. It is the editors' opinion that it is undesirable to keep accepting papers at the present rate and thereby build up a sizeable backlog and increase the waiting time for publication of papers in the Transactions. We are therefore forced to apply stricter standards for acceptance in the future and a higher percentage of papers will have to be rejected. We hope that the mathematical community will show understanding for this necessity.

SYMPOSIUM TO MARK RETIREMENT

This spring E. J. McShane will retire as Alumni Professor of Mathematics from the faculty of the University of Virginia. Professor McShane was president of the Society from 1959 to 1960. He plans to remain in residence in Charlottesville.

To mark his retirement the Department of Mathematics of the University of Virginia will hold a small, informal symposium April 24 to 26. Invited participants are Monroe D. Donsker, William L. Duren, Jr., Wendell H. Fleming, Kyosi Itô, Victor L. Klee, and B. J. Pettis.

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates, or geographical area become apparent. An announcement will be published in these *Notices* if it contains a call for papers, place, date, subject (when applicable), and speakers; a second announcement will be published only if changes to the original announcement are necessary, or if it appears that additional information should be announced. In general, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry slightly more detailed information. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society. Deadlines for particular issues of the *Notices* are the same as the deadlines for abstracts which appear on the inside front cover of each issue.

April 30–May 2, 1974
SIXTH ANNUAL ACM SYMPOSIUM ON THEORY OF COMPUTING
University Tower Hotel, Seattle, Washington
Program: Thirty-five talks in the areas of analysis of algorithms, computational complexity, formal languages and automata, theory of computation, and theoretical analysis of computer systems.
Information: Professor J. W. Carlyle, Department of System Science, 4531 Boelter Hall, University of California, Los Angeles, Los Angeles, California 90024

May 8–10, 1974
THE JOHN H. BARRETT MEMORIAL LECTURES
University of Tennessee, Knoxville, Tennessee
Program: Professor Zeev Nehari of Carnegie-Mellon University will speak on nonlinear techniques for linear oscillation problems.
Contributed papers: A session of 30-minute contributed papers on ordinary differential equations will be held on May 9.
Abstracts and information: Professor John S. Bradley, Department of Mathematics, University of Tennessee, Knoxville, Tennessee 37916

May 20–24, 1974
REGIONAL CONFERENCE ON IMPROPERLY POSED PROBLEMS
University of New Mexico, Albuquerque, New Mexico
Program: Lawrence E. Payne, principal speaker
Contributed papers: Sessions will be scheduled.
Support: National Science Foundation (pending); travel and subsistence allowances for 25 participants.
Information: Professors A. Carasso or A. Stone, Department of Mathematics, University of New Mexico, Albuquerque, New Mexico 87131

May 20–31, 1974
ADVANCED COURSE ON THE FOUNDATIONS OF COMPUTER SCIENCE
University of Amsterdam, Amsterdam, The Netherlands
Speakers: J. W. de Bakker (Mathematical Centre and Free University), "Fixed points in programming theory"; E. Engeler (Eidgenössische Technische Hochschule), "Algorithmic logic"; A. N. Habermann (Carnegie-Mellon University), "Operating system structures"; R. Kowalski (University of Edinburgh), "Predicate logic as a programming language in artificial intelligence"; E. J. Neuhold (University of Stuttgart), "Formal properties of data bases"; M. S. Paterson (University of Warwick), "Complexity of matrix algorithms."
Organizers: The Mathematical Centre and the European Communities
Sponsor: European Association for Theoretical Computer Science
Support: Netherlands Organization for the Advancement of Pure Research
Information: Professor J. W. de Bakker, Mathematical Centre, EC COURSE, 2e Boerhaavestraat 49, Amsterdam - 1005, The Netherlands

June 3–7, 1974
REGIONAL CONFERENCE ON INTEGRATION IN FUNCTION SPACES WITH APPLICATIONS
University of Connecticut, Storrs, Connecticut
Speakers: M. Kac (Rockefeller University); G. Papanicolaou (Courant Institute); D. Stroock (University of Colorado)
Contributed papers: Abstract deadline April 15, 1974
Information and abstracts: Professor John V. Ryff, Department of Mathematics, University of Connecticut, Storrs, Connecticut 06268

June 10–13, 1974
NUMERICAL SOLUTIONS OF BOUNDARY PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS
University of Maryland Baltimore County, Baltimore, Maryland
Program: Four 90-minute invited survey lectures, 45-minute invited talks, 15-minute short communications
Information: Professor A. K. Aziz, c/o Mrs. M. Hern, Division of Mathematics and Physics, University of Maryland Baltimore County, Baltimore, Maryland 21228

June 10–14, 1974
REGIONAL RESEARCH CONFERENCE ON SPECIAL FUNCTIONS
Virginia Polytechnic Institute and State University, Blacksburg, Virginia
Program: Richard A. Askey, principal speaker
Contributed papers: Sessions will be scheduled.
Support: National Science Foundation (anticipated); travel and subsistence allowances for 25 participants.
Information: Professor James A. Cochran, Department of Mathematics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061

June 17–21, 1974
THEORETICAL BIOLOGY AND BIOMATHEMATICS (GORDON RESEARCH CONFERENCES)
Tilton School, Tilton, New Hampshire
Speakers: George Oster, "Dynamics of interacting populations"; Thomas Nagylaki, "The geographical structure of populations"; Michael Gilpin, "Interference and niche"; Joseph B. Keller, "Population genetics–analysis of stochastic models"; Stuart Kauffman, "The mitotic oscillator and physarun polycephelum"; Robert D. Allen, "Recent advances in the theory of amoeboid movement"; Edward Spiegel, Stephen Childress and Michael Levandowsky, "Pattern formation by swimming microorganisms"; Cyrus Levinthal, "Instructive model for developmental specificity"; Bruce Knight (subject to be announced); Nancy Kopell, "Chemical pattern formation–a mini-survey of mechanisms"; Rutherford Aris, "The mathematical theory of diffusion and reaction in permeable catalytic bodies"; Lee A. Segel, "Theories of bacterial chemotaxis"; Sol Rubinow, "The swimming of microorganisms"; Garrett Odell, "A continuum theory of amoeboid pseudopodium extension"; Julia T. Apter, "A physical model for muscular action."
Program: The Gordon Research Conferences in 1974 will be held in New Hampshire and California. The complete program is published in Science, March 8, 1974. Reprints are available on request from Dr. Alexander M. Cruickshank, Director, Gordon Research Conferences, Pastore Chemical Laboratory, University of Rhode Island, Kingston, Rhode Island 02881.
Registration and reservations: Individuals are requested to send applications in duplicate, no later than two months prior to the conference, to Dr. Alexander M. Cruickshank, Director, Gordon Research Conferences, Colby College-New Hampshire, New London, New Hampshire 03257.

June 24–28, 1974
REGIONAL CONFERENCE ON MODULES OVER COMMUTATIVE RINGS
The University of Nebraska-Lincoln, Lincoln, Nebraska
Principal lecturer: Professor Melvin Hochster
Support: (pending) National Science Foundation. There is support in terms of travel and per diem available for a limited number of participants from the Rocky Mountain and Midwest regions.
Information: Professor Max D. Larsen, 819 Oldfather Hall, University of Nebraska-Lincoln, Lincoln, Nebraska 68508.

July 8–20, 1974
ADVANCED STUDY INSTITUTE ON COMBINATORICS
Nijenrode Castle, The Netherlands
Program: Instructional lectures by leading authorities on theory of designs, graph theory, combinatorial group theory, finite geometry, foundations, partitions and combinatorial geometry, coding theory. There will also be seminars on these topics, and a few sessions for contributed papers.
Sponsors: Mathematisch Centrum of The Netherlands and the American Mathematical Society.
Support: North Atlantic Treaty Organization
Information: Professor Dr. J. H. van Lint, Department of Mathematics, Technological Institute of Eindhoven, Eindhoven, The Netherlands; Professor Marshall Hall, Jr., Department of Mathematics, California Institute of Technology, Pasadena, California 91109

August 5–9, 1974
FOURTH CONFERENCE ON STOCHASTIC PROCESSES
York University, Toronto, Ontario, Canada
Program: Invited and contributed papers
Auspices: Committee on Stochastic Processes and the Institute of Mathematical Statistics
Information: Dr. R. A. Schaufele, Department of Mathematics, York University, Downsview, Ontario M3J 1P3, Canada

September 2–6, 1974
INTERNATIONAL CONFERENCE ON VALUE DISTRIBUTION THEORY
Purdue University, West Lafayette, Indiana
Topic: Value distribution in one and several complex variables
Speakers: L. Ahlfors, A. Edrei, P. Griffiths, P. Lelong, W. Stoll, H. H. Wu, and others to be announced
Information: Professor K. V. R. Rao, Division of Mathematical Sciences, Purdue University, West Lafayette, Indiana 47907

September 4–11, 1974
CONFERENCE ON ANALYTIC FUNCTIONS
Institute of Mathematics, Cracow, Poland
Program: One-hour lectures and ten-minute communications will be delivered on the current problems of the widely conceived theory of analytic functions. Also, there will be three seminars: extremal problems for analytic functions of one complex variable; quasiconformal mappings; functions of several complex variables (including the theory of analytic functions in topological vector spaces).
Organizers: The Institute of Mathematics
Information: Conference on Analytic Functions, UJ, Institute of Mathematics, Reymonta 4, 30-059 Kraków, Poland

October 9–13, 1974
TOPOLOGY CONFERENCE
University of Tennessee, Knoxville, Tennessee
Program: Professor William Browder of Princeton University will give a series of ten lectures on "Surgery Theory and its Applications to Topology."
Contributed papers: There will be opportunity for a few participants to present papers.
Support: Application for support has been filed with the National Science Foundation under the NSF-CBMS regional conferences plan.
Information: Professor L. S. Husch, Department of Mathematics, University of Tennessee, Knoxville, Tennessee 37916

October 21–22, 1974
SECOND LANGLEY CONFERENCE ON SCIENTIFIC COMPUTING: NUMERICAL METHODS FOR PARALLEL AND VECTOR PROCESSORS
Hilton Inn, Virginia Beach, Virginia
Program: A small number of invited and tutorial talks as well as contributed papers dealing with the utilization of parallel and/or vector processors in performing numerical scientific calculations
Sponsors: Institute for Computer Application in Science and Engineering and Society for Industrial and Applied Mathematics
Information: Dr. Robert G. Voigt, ICASE, Mail Stop 132C, NASA-Langley Research Center, Hampton, Virginia 23665

May 20–24, 1975
INTERNATIONAL SYMPOSIUM ON INTERVAL MATHEMATICS
Universität Karlsruhe, Karlsruhe, Germany
Program: Invited lectures describing the state of the art and contributed lectures. Abstracts of all lectures will be available before the start of the symposium. Proceedings will be published.
Information: Professor Karl Nickel, Universität Karlsruhe, Institut für Praktische Mathematik, Postfach 6380, Karlsruhe 1, Germany

NOTE: The Conference on Iterative Algorithms for Numerical Analysis, April 29–May 1, 1974, University of Pittsburgh, has been cancelled.

ERRATUM

Carleton University's Probability and Statistics Day is April 20, 1974, not April 10, 1974, as announced in the February 1974 Notices, p. 121.

NEW AMS PUBLICATIONS

AMS TRANSLATIONS—SERIES 2

NINE PAPERS IN ANALYSIS

Volume 103
208 pages; list price $17.70; member price $13.28; ISBN 0-8218-3053-0
To order, please specify TRANS2/103

This volume of the AMS Translations–Series 2 contains the following nine papers in analysis: "A certain class of operator-valued entire functions," M. S. Brodskiĭ; "Divisors and minorants of operator-functions of bounded form," Ju. P. Ginzburg; "The spectrum of one-dimensional singular integral operators with piecewise continuous coefficients," I. C. Gohberg and N. Ja. Krupnik; "R-functions–analytic functions mapping the upper halfplane into itself" and "On the spectral functions of the string," I. S. Kac and M. G. Kreĭn; "Čebyšev-Markov inequalities in the theory of the spectral functions of the vibrating string," M. G. Kreĭn; "On linear-fractional transformations with operator coefficients," M. G. Kreĭn and Ju. L. Šmul'jan; "The reduction method for operators in Hilbert space" and "A note on the multistage method," A. S. Markus.

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS

HARMONIC ANALYSIS ON HOMOGENEOUS SPACES, edited by Calvin C. Moore

Volume XXVI
480 pages; list price $43.50; member price $32.63; ISBN 0-8218-1426-5
To order, please specify PSPUM/XXVI

This volume constitutes the proceedings of the nineteenth Summer Research Institute of the American Mathematical Society which was held at Williams College on July 31–August 18, 1972, and was supported by a grant from the National Science Foundation. The scientific program of the institute consisted first of all of six major lecture series, each devoted to relatively broad areas within the subject and consisting of from four to six lectures. The lecturers were C. C. Moore, V. S. Varadarajan, Harish-Chandra, H. Furstenberg, S. Helgason, and B. Kostant (jointly with R. Blattner). The first part of this volume contains articles prepared by the lecturers and based on these lecture series. These are intended to be surveys of various aspects of the subject of the institute, and it is hoped that they will provide material not only for advanced graduate students and postdoctoral mathematicians who want to get into the subject, but also for more senior mathematicians who work in related areas and who need a survey of what is known in the subject, and what techniques are available.

The second part of the program consisted of five research seminars, each under the leadership of an invited chairman. The seminars were devoted to the presentation of talks by participants on current research in the various fields: representation theory of solvable groups and harmonic analysis on solvmanifolds; irreducibility and realization of various series of representations of semisimple groups; boundary behavior, special functions, and integral transforms in group representations; representations of p-adic groups; and $L^2(G/\Gamma)$ and automorphic functions. Short summaries of these talks are included in this volume in the format of Research Announcements, and they are arranged under the five seminar headings.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

UNITARY REPRESENTATIONS ON PARTIALLY HOLOMORPHIC COHOMOLOGY SPACES by Joseph A. Wolf

Number 138
156 pages; list price $3.60; member $2.70; ISBN 0-8218-1838-4
To order, please specify MEMO/138

This Memoir is a study of the representation theory for a class of reductive Lie groups that includes all connected semisimple groups. Following the lines of Harish-Chandra's work, a series of unitary representation classes is constructed for each conjugacy class of Cartan subgroups, the characters of those representations are discussed, and a Plancherel theorem is proved. Then the representation classes of the various series are realized in a geometric setting of complex flag manifolds, real group orbits that are partially complex manifolds, hermitian vector bundles over those orbits, and square integrable partially harmonic differential forms with values in the bundles. The entire procedure is first illustrated in elementary form in §1, where it is carried out for the "principal series" by means of a slight extension of the familiar Bott-Borel-Weil Theorem.

THE LEBESGUE-NIKODYM THEOREM FOR VECTOR VALUED RADON MEASURES by Erik Thomas

Number 139
104 pages; list price $3.20; member price $2.40; ISBN 0-8218-1839-2
To order, please specify MEMO/139

This Memoir concerns absolutely continuous Radon measures with values in some locally convex space E. Although the Radon-Nikodym theorem fails in general to produce E-valued densities for such measures, it is shown that if E is appropriately embedded in a larger locally convex space F, the measure will have an F-valued density. The existence and properties of such "densities going out of the space" are studied in detail, and illustrative examples drawn from analysis are provided.

PRODUCT FORMULAS, NONLINEAR SEMIGROUPS, AND ADDITION OF UNBOUNDED OPERATORS by Paul R. Chernoff

Number 140
128 pages; list price $3.30; member price $2.48; ISBN 0-8218-1840-6
To order, please specify MEMO/140

This Memoir is concerned with relations of the form $G(t) = \lim_{n\to\infty} F(t/n)^n$ where $G(t)$ is a semigroup of operators (possibly nonlinear), while $F(t)$ is relatively unrestricted. It deals with aspects of two general questions: first, when can a given semigroup be represented by such a formula, where $F(t)$ has some prescribed form; second, what are the properties of the above limit for general $F(t)$? Of particular interest is the use of the Trotter-Lie product formula $\lim_{n\to\infty} (e^{tA/n} e^{tB/n})^n = e^{tC}$ to define a generalized sum C of two semigroup generators A and B. The properties and pathology of this generalized addition process, and related processes, are discussed.

NEWS ITEMS AND ANNOUNCEMENTS

TOPICS FOR 1975 SUMMER INSTITUTE INVITED

The AMS Committee on Summer Institutes (Louis Auslander, chairman; Richard E. Bellman; S. S. Chern; Richard K. Lashof; Walter Rudin; and John T. Tate) invites proposals for the summer institute in 1975. Such proposals should be concerned with a topic in mathematics of broad, general interest, and should include a list of suggested members of the organizing committee. For your information, the topic of the 1974 summer institute is algebraic geometry; the topics of the last two institutes were differential geometry (1973) and harmonic analysis of homogeneous spaces (1972). All proposals should be submitted to the chairman, Professor Louis Auslander, Department of Mathematics, Graduate Center, City University of New York, 33 West 42nd Street, New York, New York 10036.

NEW MASTER'S LEVEL PROGRAM

The Department of Mathematical Sciences at Florida Technological University, Orlando, is now offering a unique new Master's level Graduate Program in Mathematical Science. The program provides advanced training in the combined areas of mathematics, statistics, and computer science. It is designed to meet the need (in industry, government, business, and education) for professionals who are broadly trained in the techniques of mathematical science. Emphasis is on theory and methods which have broad practical applications. Additional information and application forms may be obtained by writing to the Graduate Committee, Department of Mathematical Sciences, Florida Technological University, Box 25000, Orlando, Florida 32816.

CASE STUDIES

The Society's Committee on Employment and Educational Policy is collecting "case studies" of Ph.D. mathematicians employed in "non-traditional" positions, i.e., neither in academic teaching or research nor in the traditional technical areas such as industrial research and development. The Committee hopes that these case studies will prove helpful to mathematicians seeking non-traditional positions, and is considering the possibility of selecting and editing some of them for possible publication in these *Notices*. Ph.D. mathematicians employed in such positions who feel they might be interested in participating in this project, either by preparing a case study of themselves or by proposing other candidates for such studies, are encouraged to write to Professor Martha K. Smith, Mathematics Department, University of Texas, Austin, Texas 78712.

PERSONAL ITEMS

DUANE C. ABBEY of Iowa State University has been appointed educational coordinator of the Data Processing Division, State Comptroller, Des Moines, Iowa.

JAGDISH C. AGRAWAL, ANDREW J. MACHUSKO, JR., LAWRENCE ROMBOSKI and GEORGE D. NOVAK of California State College, California, Pennsylvania, have formed the Cal Investment Advisory Service.

MARTIN AIGNER of the University of Tubingen, Germany, has been appointed to a professorship at the Freie University of Berlin.

JOHN W. BAKER of Florida State University has been appointed to an assistant professorship at Kent State University.

S. BARON of Clark University has been appointed to an assistant professorship at Virginia State College.

GLORIA B. BARRETT has been appointed a mathematician at the Fleet Combat Direction Systems Support Activity, Dam Neck, Virginia.

JOSEPH J. BASTIAN of Indiana University has been appointed a research associate at Dalhousie University.

RICHARD BEALS of the University of Chicago has been appointed to a visiting professorship at Duke University for the fall 1974 semester.

OTTO B. BEKKEN of the University of Oslo has been appointed to a lectureship at the Agricultural University of Norway.

MICHAEL P. BENSON of Colorado State University has been appointed to an assistant professorship at the University of Cincinnati.

RAMENDRA K. BHATTACHARYA of Southern Illinois University has been appointed to an associate professorship at Pacific University.

JOHN L. BRITTON of the University of Kent at Canterbury has been appointed to a professorship at Queen Elizabeth College, University of London.

PHILIP G. BUCKHIESTER of Clemson University has been appointed to an assistant professorship at Valdosta State College.

DAVID H. BUDENAERS of the Stanford Linear Accelerator Center has been appointed a mathematician and analyst at Systems Control, Inc., Palo Alto, California.

SUNDAY C. CHIKWENDU of the University of California, Los Angeles, has been appointed to a lectureship at the University of Nigeria.

THOMAS R. CHOW of Oregon State University has been appointed a research scientist associate with Lockheed Missile and Space Co., Inc.

ROBERT R. CLOUGH of the University of Notre Dame has been appointed a systems representative with Burroughs Corporation, Chicago, Illinois.

WILLIAM CLOVER of Concordia Teachers College has been appointed an associate scientist with Horrigan Analytics, Chicago, Illinois.

BRIAN W. CONOLLY of NATO, SACLANT ASW Research Center has been appointed to a professorship at Chelsea College, University of London.

WILLIAM H. DAVENPORT of the U.S. Army Missile Command has been appointed to an assistant professorship at the College of Petroleum and Minerals, Dhahran, Saudi Arabia.

JOHN I. DERR of the Rand Corporation has been appointed a senior Technical Staff member at Litton Data Systems, Van Nuys, California.

LEROY J. DICKEY of the University of Waterloo has been appointed managing editor of Aequationes Mathematicae.

ROBERT EASTON of Brown University has been appointed to an associate professorship at the University of Colorado.

HERBERT M. FARKAS has been appointed to a lectureship at the University of Maryland, Princess Ann, Maryland.

LAWRENCE A. FIALKOW of the University of Michigan has been appointed to an assistant professorship at Western Michigan University.

LEIKO H. FINCH of the University of Houston has been appointed an industrial specialist with Brown and Root, Inc., Houston, Texas.

HANS-BJORN FOXBY of the University of Aarhus, Denmark, has been appointed to a visiting lectureship at the University of Illinois.

ROBERT GORDON of the University of Utah has been appointed to an associate professorship at Temple University.

MURLI M. GUPTA of the University of Western Australia has been appointed to a senior lectureship at Papua and New Guinea University of Technology.

AUBREY E. HARVEY III of the University of Arkansas has been appointed to an assistant professorship at Texas A&M University.

ALAN HENNEY of the Naval Ordnance Laboratory has been appointed a division head with the Defense Department.

DOMINGO A. HERRERO of the Universidade Estadual de Campinas has been appointed to a professorship at the Universidad Nacional di Rio N°, Cordoba, Argentina.

LINDA HILL of New Mexico State University has been appointed to an assistant professorship at the University of Colorado.

EINAR HILLE of California State University, San Diego, has been appointed a research mathematician at the University of California, San Diego, La Jolla.

GUY T. HOGAN of the State University of New York, College at Oneonta has been appointed to an associate professorship at the University of Massachusetts at Boston.

RAYMOND W. HONERLAH of Marquette

University has been appointed manager of marketing research at the U.S. National Bank of Oregon.

CARL C. HUGHES of North Carolina State University has been appointed to a visiting assistant professorship at the University of South Carolina.

DAVID A. KLARNER of Stanford University has been appointed to a visiting associate professorship at SUNY at Binghamton.

LARRY E. KNOP of the University of Utah has been appointed to a lectureship at Southern Illinois University, Carbondale.

ATUO KOMATU of Kyoto University has been appointed to a professorship at Science University of Tokyo.

EVERETT LEE LADY of the University of Kansas has been appointed to a visiting lectureship at the University of Illinois.

HOWARD A. LEVINE of the University of Minnesota has been appointed to an assistant professorship at the University of Rhode Island.

J. D. LOGAN of the University of Arizona has been appointed to an assistant professorship at Kansas State University.

DONALD W. LOVELAND of Carnegie-Mellon University has been appointed to a professorship and to the chairmanship of the Computer Science Department at Duke University.

FILOMENA P. LUPO of the American Mathematical Society Editorial Department has been appointed a research analyst at the research and planning unit of the Department of the Attorney General, Providence, Rhode Island.

THOMAS J. MARTINO of the University of Rhode Island has been appointed a scientific analyst at the Center for Naval Analyses.

SYED M. MAZHAR of Aligarh Muslim University has been appointed to a professorship at Kuwait University.

THOMAS A. McINTYRE of the University of Notre Dame has been appointed to an assistant professorship at Tri-State College.

LYNN McLINDEN of the University of Wisconsin, Madison, has been appointed to an assistant professorship at the University of Illinois.

JEFF McLEAN of Ohio State University has been appointed to an assistant professorship at Ohio Northern University.

JAMES R. MILLER of the National Center for Atmospheric Research has been appointed a National Research Council associate at the Goddard Institute for Space Studies, NASA.

JOSEPHINE MITCHELL of SUNY at Buffalo will be visiting at the University of Michigan until August 1974.

K. A. MOHAMMADI of the University of Islamabad has been appointed to an assistant professorship at Baluchistan University, Ouetta, Pakistan.

SIDNEY A. MORRIS of the University of New South Wales has been appointed a senior visiting fellow at the University College of North Wales.

A. NDUKA of the University of Illinois, Urbana, has been appointed to a lectureship at the University of Nigeria.

ROGERS J. NEWMAN of Southern University has been appointed director of the Institute for Higher Educational Opportunity at the Southern Regional Education Board, Atlanta, Georgia.

PAUL OLUM of Cornell University has been appointed dean of the College of Natural Sciences at the University of Texas at Austin.

MARTIN ORR of AVCO Corporation has been appointed a data analyst with the State Law Enforcement Planning Agency, Trenton, New Jersey.

WILLIAM ORR of the University of Wisconsin, Madison, has been appointed to an assistant professorship at Northern Michigan University.

KANTI A. PATEL of North Carolina State University, Raleigh, has been appointed to a professorship at Shaw University.

EMILY M. PECK of Vassar College has been appointed to an assistant professorship at the University of Illinois.

EDWARD A. PEDERSEN of the University of Utah has been appointed an engineer with Mountain Bell, Salt Lake City, Utah.

EDWARD PETTIT of the University of California, Riverside, has been appointed to a visiting assistant professorship at Western Michigan University.

WALTER A. POOR of SUNY at Stony Brook has been appointed to an assistant professorship at Purdue University, Fort Wayne.

DAVID A. POPE of the Aerojet Electrosystems Company has been appointed head of mathematics services at Hughes Aircraft Company.

VENKATA RAO POTLURI of the University of Oregon has been appointed to an assistant professorship at Reed College.

JOHN F. PRICE of the Australian National University has been appointed to a lectureship at the University of New South Wales.

ERNEST PYLE of the University of Texas at Austin has been appointed to an assistant professorship at Houston Baptist University.

WILLIAM C. QUEEN of the University of South Carolina has been appointed to an associate professorship at the University of North Florida.

SAMUEL M. RANKIN III of Vanderbilt University has been appointed to an assistant professorship at the Florida Institute of Technology.

MICHAEL REED of Princeton University has been appointed to a professorship at Duke University.

ALLAN REHM of the Center for Naval Analyses has been appointed to the professional staff at Ketron, Inc.

PAUL ROOS of the Universität Stuttgart has been appointed to a professorship at the Universität Bremen.

ARTHUR D. ROSENBERG of Harcourt Brace has been appointed a mathematics editor with Intext Educational Publishing.

GERALD ROSENFELD of New York University has been appointed to an assistant professorship at the University of Maryland Baltimore County.

JOHN A. ROULIER of Union College has been appointed to an assistant professorship at

North Carolina State University, Raleigh.

BEN RUSSAK of American Elsevier Publishing Company has been appointed president of Crane, Russak and Company, Inc.

PEDRO P. SANCHEZ of the University of Michigan has been appointed to an assistant professorship at Eastern Michigan University.

NIKO SAUER of the National Research Institute for Mathematical Sciences, Pretoria, South Africa, has been appointed to a professorship and to the chairmanship of the Department of Applied Mathematics at the University of Pretoria.

LOWELL SCHOENFELD of SUNY at Buffalo will be visiting at the University of Michigan until August 1974.

WOLFRAM SCHWABHAUSER of the Universität Bonn has been appointed to a professorship at the Universität Stuttgart.

EUGENE R. SEELBACH of the University of Wyoming has been appointed to an assistant professorship at the State University of New York, College at Brockport.

GEORGE E. SINCLAIR of the University of Arizona has been appointed a senior engineer at the Federal Electric Corporation, Vandenberg Air Force Base.

DONALD H. SINGLEY of the University of Minnesota has been appointed to an assistant professorship at Arkansas State University.

MALCOLM SMITH of Scott Research Laboratories has been appointed a senior research associate at Olson Laboratories, Inc., Anaheim, California.

WILLIAM F. SMITH, JR., of Murray State University has been appointed a consultant with Sperry Rand, Space Support Division, Huntsville, Alabama.

RAYMOND E. SMITHSON of the University of Wyoming has been appointed to a visiting associate professorship at the University of Houston.

RUSSELL A. SMUCKER of Indiana University has been appointed to an assistant professorship at Kalamazoo College.

MICHAEL STECHER of Indiana University has been appointed to an assistant professorship at Texas A&M University.

JUAN A. TIRAO of the National University of Cordoba, Argentina, has been appointed a research associate at the University of California, Berkeley.

NEIL S. TRUDINGER of the University of Queensland, Brisbane, Australia, has been appointed to a professorship at the Australian National University, Canberra.

THOMAS W. TUCKER of Princeton University has been appointed to an assistant professorship at Colgate University.

HOMER F. WALKER of Texas Tech University has been appointed to a visiting associate professorship at the University of Denver.

MARTIN WALTER of Queen's University has been appointed to an assistant professorship at the University of Colorado.

NANCY WARREN of Metropolitan State College has been appointed to an assistant professorship at the University of Colorado.

STANLEY G. WAYMENT of Utah State University has been appointed director of the Division of Mathematics and Systems Design at the University of Texas at San Antonio.

DAVID M. WELLS of the University of Pittsburgh has been appointed to an assistant professorship at Ohio Dominican College.

PROMOTIONS

To Chairman and Associate Professor. University of Haifa: JOSEPH ZAKS.

To Coordinator, Division of Science and Mathematics. Utica College of Syracuse University: RONALD W. DeGRAY.

To Senior Vice President. Daniel H. Wagner, Associates: HENRY R. RICHARDSON.

To Senior Specialist Engineer. Boeing Aerospace Company: JOHN N. JOHNSON.

To Professor. Carleton College: ROGER B. KIRCHNER; City University of New York: LINDA KEEN; State University of New York, College at Geneseo: SRINIVASA G. LEELAMMA; University of Colorado: KARL E. GUSTAFSON, WILLIAM B. JONES; University of Illinois: EARL R. BERKSON, WALTER V. PHILIPP; Dept. of Computer Science, University of Maryland: H. P. EDMUNDSON; Western Michigan University: PO FANG HSIEH.

To Associate Professor. Cornell University: LOUIS J. BILLERA; Indiana University: GRAHAME BENNETT; Northern Michigan University: WILLIAM MUTCH, ROBERT H. MYERS; State University of New York, College at Brockport: JOHN G. MICHAELS, SANFORD S. MILLER, KAZUMI NAKANO; University of Colorado: WILLIAM N. REINHARDT, JAY H. WOLKOWISKY; University of Illinois: KENNETH B. STOLARSKY, WILLIAM F. STOUT; University of Oklahoma: JOHN WILLIAM GREEN; University of South Carolina: THOMAS L. MARKHAM; Villanova University: ROBERT E. BECK, FREDERICK W. HARTMANN; Virginia Polytechnic Institute and State University: EZRA BROWN; Western Michigan University: ARTHUR T. WHITE; U.S. Air Force Academy: ARTHUR E. OLSON, JR., WALTER M. PATTERSON.

INSTRUCTORSHIPS

Bainbridge Junior College: W. JERRY HATTAWAY; Carnegie-Mellon University: I. E. LEONARD; Concordia College: D. BRUCE ERICKSON; Fordham University: CHAMOND LIU; Navajo Community College: DON CURLOVIC; Philadelphia Community College and Rutgers University, Camden: LAWRENCE SCHOENFELD; University of Oklahoma: PATRICK D. CASSADY, EVAN G. HOUSTON, JR., ROBERT L. THELE; William Paterson College of New Jersey: JULIUS J. VANDE KOPPLE; Kansas University: CHESTER C. JOHN, JR.

DEATHS

Dr. RICHARD S. BURINGTON of the U. S. Navy and Arlington, Virginia, died on December 24, 1973, at the age of 72. He was a member of the Society for 46 years.

Professor Emeritus HAROLD HOTELLING of the University of North Carolina died on December 26, 1973, at the age of 78. He was a member of the Society for 49 years.

Professor WALTER JAUNZEMIS of the Pennsylvania State University died on August 7, 1973, at the age of 47. He was a member of the Society for 2 1/2 years.

Professor LLOYD J. QUAID of the University of Minnesota died on June 22, 1971, at the age of 73. He was a member of the Society for 25 years.

Professor JOSEPH V. TALACKO of Marquette University died on January 7, 1974, at the age of 64. He was a member of the Society for 24 years.

Professor Emeritus CHARLES F. THOMAS of Case Western Reserve University died on June 10, 1970, at the age of 94. He was a member of the Society for 40 years.

Professor MICHAEL J. WRIGHT of Loyola University of Los Angeles died on November 23, 1973, at the age of 31. He was a member of the Society for 7 years.

NEWS ITEMS AND ANNOUNCEMENTS

A REQUEST FOR COPIES OF PUBLICATIONS ON STATISTICAL DISTRIBUTIONS AND THEIR APPLICATIONS TO VARIOUS FIELDS

A Dictionary and Bibliography of Continuous Distributions is now under preparation for the International Statistical Institute series of Statistical Bibliographies and Teaching Aids. Also, a supplement to the already published Dictionary and Bibliography of Discrete Distributions is under preparation. Further, a display of literature on statistical distributions in scientific work will be organized during the forthcoming NATO Advanced Study Institute to be held in Calgary, Canada, July 29–August 10, 1974. One or two copies of relevant literature in the form of reprints, reports, computer programs, theses, books, etc., would be most welcome. Please send these as soon as possible to Professor G. P. Patil, Distributions Project, Department of Statistics, Pennsylvania State University, University Park, Pennsylvania 16802. Lists of relevant references in one's field of expertise and interest would also be gratefully received and appropriately acknowledged.

ITALIAN NATIONAL RESEARCH COUNCIL FELLOWSHIPS

The Italian National Research Council will award several fellowships for study and research at Italian Universities and Research Institutions, in the field of Mathematics, for the academic year 1974–1975. These fellowships are limited to foreign nationals. The monthly stipend is 180.000 Italian lire in addition to a travel allowance covering round trip transportation to Italy.

Applications should be addressed to Consiglio Nazionale delle Ricerche, piazzale delle Scienze 7; 00100 Roma, Italy. No special application forms are needed, but the application should include the following information: (1) Date and place of birth, (2) country of citizenship, (3) place of residence, (4) curriculum vitae with a list of publications, if any, (5) knowledge of Italian (if any) and of other foreign languages, (6) names of Italian mathematicians under whose direction the applicant would like to work or study. At least two letters of recommendation should accompany the application.

Applications and letters of reference should arrive no later than May 14, 1974.

SUMMER GRADUATE COURSES

Supplementary List

The following is a list of graduate courses being offered in the mathematical sciences during the summer of 1974. Another list appeared in the February issue of these *Notices*, p. 127.

ALABAMA

SAMFORD UNIVERSITY
Birmingham, Alabama 35209
Information: Dr. W. D. Peeples, Jr., Chairman, Department of Mathematics, Engineering and Computer Sciences

June 3 - July 10
Theory of Matrices

July 11 - August 16
Numerical Solution of Differential Equations

ARIZONA

NORTHERN ARIZONA UNIVERSITY
Box 5717, Flagstaff, Arizona 86001
Application deadline: May 30
Information: Dr. Richard D. Meyer, Chairman, Department of Mathematics

June 10 - July 13
Elements of Algebraic Systems
Theory of Functions of a Complex Variable
Foundations of Math for Teachers
Theory of Functions of a Real Variable
Introduction to Higher Algebra

July 15 - August 17
Elements of Analysis
Advanced Calculus
Numerical Analysis
Functional Analysis

MASSACHUSETTS

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge, Massachusetts 02139
Information: Director, Summer Session, Room E19-356

June 17 - 28
Design and Analysis of Scientific Experiments

PENNSYLVANIA

WILKES COLLEGE
Wilkes-Barre, Pennsylvania 18703
Information: Director, Continuing Education and Graduate Studies

June 17 - August 9
Linear Programming
Complex Analysis
Introduction to Geometry
Functional Analysis
Linear Algebra

TEXAS

STEPHEN F. AUSTIN STATE UNIVERSITY
Nacogdoches, Texas 75961
Application deadline: May 1
Information: Dr. W.I. Layton, Chairman, Department of Mathematics

June 3 - July 12
Probability Theory
Statistical Analysis

June 17 - August 9
College Geometry
Advanced Topics in Elementary School

June 18 - August 9
History of Mathematics
Seminar Teaching Secondary Mathematics
Combinatorial Analysis

ASSISTANTSHIPS AND FELLOWSHIPS IN MATHEMATICS 1974—1975

Supplementary List

TYPE of financial assistance (with number anticipated 1974-1975)	STIPEND amount in dollars	STIPEND 9 or 12 months	TUITION if not included in stipend (dollars)	SERVICE REQUIRED hours per week	SERVICE REQUIRED type of service	DEGREES AWARDED Academic year 1972-1973
MASSACHUSETTS						
NORTHEASTERN UNIVERSITY, BOSTON 02115						
DEPARTMENT OF MATHEMATICS David I. Epstein, Chairman						Bachelor's by inst. 2639 Bachelor's by dept. 57 Master's by dept. 30
Teaching Assistantship (28-30)	2,600-3,000	9		4-6	Teaching, Conference	Ph.D. (1970-1973 incl.)
Scholarship-Tuition (6)		9		4-8	Grading papers for instructors	A&NT 1, G&T 3, A&FA 1, AM 2, Other 1. Total: 8

ERRATA
Volume 21

HUMPHREY FONG and LOUIS SUCHESTON. *On a mixing property of operators in L_p spaces,* Abstract 74T-B53, Page A-306.

Line 3, for "$\lim_n \max_i |a_{ni}| = 1$", read "$\lim_n \max_i |a_{ni}| = 0$".

Observe that in all the theorems the condition $\lim_n \Sigma_i a_{ni} = 1$ may be omitted if (B) is weakened to (B′). For each $f \in L_p$, $\exists\ \bar{f} \in L_p \ni \lim_n \Sigma_i a_{ni}(T^i f - \bar{f}) = 0$. The proofs remain essentially the same.

JUDY GREEN. *C/ω compactness for $\mathcal{P}$-admissible sets with urelements,* Abstract 711-02-12, Page A-21.

Line six for "$\forall a \forall b(b = \mathcal{P}(a) \leftrightarrow \forall d(d \in c \to d \in a))$" read "$\forall a \forall b(b = \mathcal{P}(a) \leftrightarrow \forall c(c \in b \leftrightarrow \forall d(d \in c \to d \in a)))$".

ERIC C. NUMMELA. *Projective abelian topological groups need not be free,* Abstract 711-22-6, Page A-107.

Line 2, delete "nonempty".

Lines 4, 5, 6, delete "which is projective ··· subgroup of $Z(U,0)$."

Line 6, for "x_{10}" read "x_{11}".

Line 7, following "is" insert "isomorphic to".

Lines 8, 9, 10, delete "abelian topological group ··· but not free?" and replace with "The kernel of the quotient morphism $Z(U, 0) \to S$ is a free abelian topological group over a noncompact space, and hence has no compact set of generators. This is surprising, since U is compact. An example validating the title is given by the subgroup A of $Z([0, 1], e)$ generated by $]0, 1[$, $0 < e < 1$. Although A is not a closed subgroup of $Z([0, 1], e)$, A is a closed subgroup of $Z(A, e)$ (in fact, a direct summand)."

ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred.

Invited addresses are indicated by •

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the *Notices* but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

*74T-A75. PHILIP A. LEONARD, Arizona State University, Tempe, Arizona 85721 and KENNETH S. WILLIAMS, Carleton University, Ottawa, Ontario, Canada. *The cyclotomic numbers of order eleven.*

Let $e > 1 \in \mathbf{Z}$, p be prime $\equiv 1 \pmod e$, say $p = ef + 1$, and g a primitive root (mod p). The number of solutions (s, t) with $0 \leq s, t \leq f - 1$ of the congruence $g^{es+h} + 1 \equiv g^{et+k} \pmod p$ is denoted by $(h, k)_e$. The numbers $(h, k)_e$ are called cyclotomic numbers and the authors have evaluated them when $e = 11$ in terms of the solutions of a certain system of diophantine equations. (Received August 31, 1973.)

*74T-A76. HOWARD CARY MORRIS, California Institute of Technology, Pasadena, California 91109 and 213 Pennsylvania, Shreveport, Louisiana 71105. *Two pigeon hole principles.*

Let $|X|$ be the number of points in a set X. A set $\mathcal{P}$ of sets is an incomplete partition (i.p.) if the sets of $\mathcal{P}$ are pairwise disjoint. $\|\mathcal{P}\|$ is the number of nonvoid sets in $\mathcal{P}$. The intersection of two i.p.'s $\mathcal{P}$ & $\mathcal{Q}$, $\mathcal{P} \wedge \mathcal{Q} = \{P \cap Q | P \in \mathcal{P}, Q \in \mathcal{Q}\}$. Let $\mathrm{Supp}(\mathcal{P}) = \bigcup P$, $P \in \mathcal{P}$. Let $\mathrm{Res}(\mathcal{P}|Z) = Z \sim \mathrm{Supp}(\mathcal{P})$. If A is a set of i.p.'s s.t. (1) $|\mathrm{Res}(\mathcal{P}|Z)| \leq l \ \forall \mathcal{P} \in A$, and (2) $\|\bigwedge \mathcal{P}\| \leq k$ over every $\mathcal{P} \in B \subset A$, $\exists$ finite functions $\chi a(h, k, l)$ and $\chi e(h, k, l, t)$ s.t. *Theorem* 1. If $|Z| \geq \chi a(h, k, l)$ $\exists\, H \subset Z$, $|H| > h$, s.t. $\forall \mathcal{P} \in A$, $\exists\, P \in \mathcal{P} \ni |H \sim P| \leq l$. *Theorem* 2. If $|Z| \geq \chi e(h, k, l, t)$, $\exists$ an i.p. $\mathcal{G}$ s.t. (1) for every $\mathcal{P} \in A$, $\mathcal{P} \wedge \mathcal{G} = \{\mathrm{Supp}(\mathcal{P})\} \wedge \mathcal{G}$, and (2) $|\mathrm{Supp}(\mathcal{G}) \cap Z| \geq \|\mathcal{G}\| \cdot h + t$. Furthermore if h is sufficiently large (as compared to functions of k and l) then $\chi a(h, k, l) = hk + 1$ and $\chi e(h, k, l, t) = hk + t + (k - 1)l$. When $l = 0$, both theorems reduce to the pigeon hole principle. A special case of Theorem 1 is useful in proving a conjecture concerning the Helly's number for unions of convex sets (proposed by Grünbaum and Motzkin [Proc. Amer. Math. Soc. 12(1961), 607–613]). (Received January 4, 1974.)

*74T-A77. N. S. MENDELSOHN and R. PADMANABHAN, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. *Minimal identities for Boolean groups.*

By a well-known result of Higman and Neumann, Boolean groups (i.e. groups of exponent 2) can be represented as an equational class of groupoids satisfying a single identity. It is easy to see that the only equational class of groupoids definitionally equivalent to Boolean groups is the class of all Boolean groups with the groupoid operation as the group operation. A binary type identity $f(x_1, \cdots, x_n) = g(x_1, \cdots, x_n)$ is called a minimal identity for Boolean groups iff (i) a groupoid $\mathfrak{A} = \langle A, \cdot \rangle$ satisfies '$f = g$' iff $\mathfrak{A}$ is a Boolean group, and (ii) length of '$f = g$' (i.e. length of f + length of g) is six (cf. "A single identity for Boolean groups and Boolean rings", J. Algebra 20(1972), 78–82). *Theorem.* There are just six minimal identities for Boolean groups. They are (i) $(xy)(x(zy)) = z$, (ii) $x((zy)(xy)) = z$, (iii) $x(((xy)z)y) = z$ and their mirror reflections (i.e. by transposing $xy \to yx$). This corrects an error made in Theorem 2 of the above mentioned reference. (Received January 9, 1974.)

*74T-A78. ALLAN B. CRUSE, University of San Francisco, San Francisco, California 94117. *On extending incomplete latin rectangles.* Preliminary report.

An *incomplete latin rectangle* L of type (r, s, t) is a rectangular array of r rows and s columns where a subset of the rs places is occupied by integers from the set $1, \cdots, t$ and no integer occurs more than once in any row or column. An incomplete latin rectangle L' of type (r', s', t') is said to *extend* L if, for any i, j, k satisfying $i \leq r$, $j \leq s$, and $k \leq t$, the place in the ith row and jth column of L' is occupied by the symbol k iff the place in the ith row and jth column of L is occupied by k. *Theorem.* To extend L to a latin square of order n ($n \geq \max(r, s, t)$) it is both necessary and sufficient that L satisfy: (1) each row of L has at least $s + t - n$ occupied places; (2) each column of L has at least $r + t - n$ occupied places; (3) each symbol $1, \cdots, t$ occurs in L at least $r + s - n$ times; (4) the total number of occupied places in $L \not> (rst + (n-r)(n-s)(n-t))/n$. This theorem generalizes a well-known combinatorial criterion of Ryser (Proc. Amer. Math. Soc. 2(1951), 550–552) on extending latin rectangles. Its proof employs a classic theorem of König on sums of permutation matrices, and exploits the symmetries of certain convex polytopes. (Received January 14, 1974.)

*74T-A79. MATTHEW I. GOULD, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada and Vanderbilt University, Nashville, Tennessee 37235. *Automorphism groups of direct squares of universal algebras.*

The characterization of endomorphism monoids of direct squares [Abtract 74T-A7, these *Notices* 21(1974), A-2] is applied in the proof of the following theorem, and is also generalized to yield a characterization of endomorphism monoids of Ith powers (I any fixed nonvoid set). These results and their proofs make plausible the conjecture stated below. *Theorem.* A nontrivial group G is isomorphic to the automorphism group of a direct square iff G contains an element of order two. *Conjecture.* Let I be a nonvoid set and G a nontrivial group. For G to be isomorphic to the automorphism group of the Ith power of a universal algebra it is sufficient that G contain a subgroup anti-isomorphic to the full symmetric group on I. (Necessity of the condition is readily observed.) (Received January 14, 1974.)

74T-A80. JIMMY T. ARNOLD, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. *Krull dimension in stably equivalent rings.* Preliminary report.

Let A and B be commutative rings with identity. A and B are stably equivalent provided $\exists$ an integer n s.t. the polynomial rings $A[X_1, \cdots, X_n]$ and $B[Y_1, \cdots, Y_n]$ are isomorphic. The theorem answers a question of P. Eakin. *Theorem.* Stably equivalent rings have equal (Krull) dimension. (Received January 14, 1974.)

74T-A81. GEORGE HAVAS, Canberra College of Advanced Education, Canberra City, Australia and G. E. WALL, University of Sydney, New South Wales 2006, Australia. *The group $\overline{B}(5, 2)$.* Preliminary report.

$\overline{B}(5, 2)$ is the largest 2-generator finite group of exponent 5. Its Lie ring is shown to be the largest 2-generator Lie algebra over the field of 5 elements satisfying the 4th Engel condition. Hence, $\overline{B}(5, 2)$ has order 5^{34} and class 12. Lie ring methods are used in the proof. John W. Wamsley has reached the same conclusion by a quite different, group-theoretical method. (Received January 14, 1974.) (Authors introduced by Professor G. M. Kelly.)

74T-A82. E. M. WRIGHT, University of Aberdeen, Aberdeen, United Kingdom. *Large cycles in large graphs.* Preliminary report.

Write $P = P(n, q, k)$ for the probability that an (n, q) graph, i.e. a simple graph on n labelled nodes with q edges, contains a cycle of length k and consider the behaviour of P as $n \to \infty$. I gave results for the case of Hamiltonian cycles ($k = n$) in Abstract 73T-A83, these *Notices* 20(1973), A-261. Erdös and Renyi (Magyar Tud. Akad. Mat. Kutató Int. Közl. 5(1960), 17–61) dealt with the case $k = O(1)$. *Results.* (1) If $k > An$ and $qn^{-3/2} \to \infty$, then $P \to 1$; in fact, given any set of k nodes, almost all (n, q) graphs have a cycle through those nodes. (2) If $An^{1/2} \leq k = o(n)$ and $qk^{-1}n^{-1/2} \to \infty$, then $P \to 1$. (3) If $k \to \infty$, $k = o(n^{1/2})$, $\{q - (n/2)\}/k^2 \to \infty$ and $(2qk/n) - k - \log k \to \infty$, then $P \to 1$. Write $P_s = P_s(n, q, k)$ for the probability that an (n, q) graph contains just s cycles of length k. (4) If $k \to \infty$, $k = o\{(n \log n)^{1/3}\}$ and $(2qk/n) - k - \log\{2k\} \to c$, where $-A < c < A$, then $P_s \to e^{cs}\exp(-e^c)/s!$ and $P \to 1 - \exp(-e^c)$. (Received January 17, 1974.)

74T-A83. MICHAEL RICH, Temple University, Philadelphia, Pennsylvania 19121. *Rings with idempotents in their nucleii.*

If R is a prime nonassociative ring and the set of idempotents of R lies in the nucleus or in the alternative nucleus of R (Thedy, Amer. J. Math 119(1971), 42–51) we show that R is an associative or an alternative ring, respectively. Similarly we define an appropriate Jordan nucleus $N_J(R)$ and an appropriate noncommutative Jordan nucleus $N_{NJ}(R)$ s.t. if at least one idempotent is in $N_J(R)$ or in $N_{NJ}(R)$ then R is a Jordan ring or a noncommutative Jordan ring, respectively. (Received January 18, 1974.)

74T-A84. GEORGIA M. BENKART, Yale University, New Haven, Connecticut 06520. *Inner ideals and classical Lie algebras.*

If L is a Lie algebra over a field k, a k-subspace B of L is an *inner ideal* of L if $[B[B\ L]] \subseteq B$. If $[x[x\ L]] = 0 \Rightarrow x = 0$, then L is *not strongly degenerate. Theorem.* Let L be a finite dimensional, not-strongly-degenerate, simple Lie algebra over an algebraically closed field of char $p > 5$. (i) L has proper inner ideals iff L is classical. (ii) $\exists\ y \in L$ with $(ad_y)^n = 0$ for $3 \leq n \leq p-1$ iff L is classical. The proof uses a result of Jacobs (J. Algebra 19(1971), 31–50) and the *Theorem.* If L is an arbitrary Lie algebra, and B is a minimal inner ideal of L, then (1) $B = kx$ and $[x[x\ L]] = 0$; (2) $B = [B[B\ L]]$ and $[BB] = 0$; or (3) B is a simple ideal of L and every inner ideal V of B has $[VV] = 0$. *Theorem.* Let L be a not-strongly-degenerate Lie algebra over a field of char $p \geq 5$ (char 0) satisfying the minimum condition on inner ideals. If $(ad_y)^n = 0$ for $y \in L$, $3 \leq n \leq p-1$ ($n \geq 3$), then L contains a split 3-dimensional simple subalgebra. The adjoint action of the subalgebra decomposes L into six parts: two copies of a Jordan algebra A; two copies of a Jordan bimodule M of A; the multiplication algebra of A; and a Lie algebra of transformations of A and M, with relations between Jordan inner ideals of A and Lie inner ideals of L. (Received January 21, 1974.)

*74T-A85. COLIN W. CRYER, University of Wisconsin, Madison, Wisconsin 53706. *Some properties of totally positive matrices.*

Let A be a real $n \times n$ matrix. A is TP (*totally positive*) if all the minors of A are nonnegative. A has a *LU-factorization* if $A = LU$ where L is a lower triangular matrix and U is an upper triangular matrix. *Theorem* 1. A is TP iff A has an LU-factorization s. t. L and U are TP. *Theorem* 2. If A is TP then $\exists$ a TP matrix S and a tridiagonal TP matrix T s. t.: (i) $TS = SA$; and (ii) the matrices A and T have the same eigenvalues. If A is nonsingular then S is also nonsingular. *Theorem* 3. If A is of rank m then A is TP iff every minor of A formed from any columns $\beta_1, \cdots, \beta_p$ satisfying $\Sigma_{i=2}^{p}|\beta_i - \beta_{i-1}| \leq n-m$ is nonnegative. *Theorem* 4. If A is a nonsingular lower triangular matrix then A is TP iff every minor of A formed from consecutive initial columns is nonnegative. (Received January 21, 1974.)

*74T-A86. MICHAEL DOOB, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. *A generalization of magic graphs.* Preliminary report.

A graph is magic if the edges have real valued labels that are distinct and have the property that the sum of the labels on the edges incident to one vertex is the same for all vertices. The labelings can be considered elements of the real vector space V coordinatized by the edges. *Theorem.* If G is a graph and $r(v)$ is a prescribd real number for each vertex, then the set of labelings of the edges, s. t. the sum of the labels on the edges incident to v is $r(v)$, is a translation of a subspace of V. The dimension is $|E(G)| - |V(G)| + 1$ or the set of labelings $= \emptyset$ if G is bipartite, and the dimension is $|E(G)| - |V(G)|$ otherwise. The construction of all such labelings is given. The concepts of trivially magic, zero magic, and semimagic graphs are defined in Stewart (Canad. J. Math. 18(1966), 1031–1059). *Theorem.* G is trivially magic iff G is a tree whose bipartition sets have different cardinalities. G is zero magic iff G is a bipartite graph whose bipartition sets have different cardinalities and G is not a tree. G is semimagic iff G is not bipartite or G has a bipartition whose sets have equal cardinalities. (Received January 21, 1974.)

74T-A87. SPYROS S. MAGLIVERAS, State University College of New York, Oswego, New York 13126. *On transitive extensions of the Higman-Sims group.* Preliminary report.

The following extends an earlier result by the author on transitive extensions of the Higman-Sims simple group. *Theorem.* There are no simple groups which are rank-3 transitive extensions of the Higman-Sims simple group. (Received January 25, 1974.)

74T-A88. HARRY F. SMITH, Madison College, Harrisonburg, Virginia 22801. *The Wedderburn principal theorem for generalized alternative algebras.* I. Preliminary report.

A generalized alternative ring I is a nonassociative ring R in which the identities $(wx, y, z) + (w, x, (y, z)) - w(x, y, z) - (w, y, z)x$; $((w, x), y, z) + (w, x, yz) - y(w, x, z) - (w, x, y)z$; and (x, x, x) are identically zero. Let A be a finite-dimensional algebra of this type over a field F of characteristic $\neq 2, 3$. Then it is established that (1) A cannot be a nodal algebra, and (2) the standard Wedderburn principal theorem is valid for A. (Received January 28, 1974.)

*74T-A89. ADIL G. NAOUM and R. M. SALLOUM, College of Science, University of Baghdad, Adhamiyah, Baghdad, Iraq. *Annihilators of ideals in commutative Artinian rings.*

Let R be a commutative ring with 1; then we have the following. *Theorems*: 1. If R is Artinian, then the annihilator of every proper ideal of R is not zero. 2. If every ideal in R is the annihilator of a finite number of elements in R, then R is Artinian. 3. If R is semiprime and Noetherian, then R is Artinian iff the annihilator of each proper primary ideal of R is not zero. (Received January 30, 1974.)

*74T-A90. GEORGE E. ANDREWS, Pennsylvania State University, University Park, Pennsylvania 16802. *On the ordered factorization of n and a conjecture of C. Long.*

We prove the following theorem orginally conjectured by C. Long (Canad. Math. Bull. 13(1970), 333–335). *Theorem.* If $1 < n = p_1^{\alpha_1} \cdots p_r^{\alpha_r}$ is the prime factorization of n, then one obtains the number $F(n)$ of ordered factorizations of n by fully expanding the polynomial $2^{\alpha_1 - 1}\Pi_{i=2}^{r}(X_i + Y_i)^{\alpha_i}$ where $X_i = x_1 x_2 \cdots x_{i-1}$, $Y_i = (1 + x_1)(1 + x_2)\cdots(1 + x_{i-1})$ and then replacing each x_i^k by $\binom{\alpha_i}{\alpha_{i+1}+\cdots+\alpha_k - k}$. Long's conjecture follows from *Theorem.* If $1 < n = p_1^{\alpha_1} \cdots p_r^{\alpha_r}$ is the prime factorization of n, then $F(n) = \frac{1}{2}\Sigma\Pi_{j=1}^{r} A_j B_j$, where $A_j = \binom{\alpha_j}{h_j}$, $B_j = \binom{\alpha_j + h_j + 1 + \cdots + h_r}{\alpha_j}$, and where Σ is over all r tuples of nonnegative integers $(h_1, h_2, \cdots, h_r)$. (Received January 30, 1974.)

74T-A91. WILLEM J. BLOK and PHILIP DWINGER, University of Illinois, Chicago, Illinois 60680. *Varieties of closure algebras.* I. Preliminary report.

If K is a class of (similar) algebras, let $V(K)$ be the variety generated by K. B_c and H will denote the varieties of closure algebras and Heyting algebras respectively. If $L \in B_c$, then L° = Heyting algebras of open elements of L, and if $K \subseteq B_c$, then $K^\circ = \{L \in H: L \cong L'^\circ$ for some $L' \in K\}$. *Theorems*: 1. $L \in B_c$ is subdirectly irreducible iff L° is subdirectly irreducible. 2. If $K \subseteq B_c$ then $(V(K))^\circ = V(K^\circ)$. Several results concerning the lattice of subvarieties of B_c are obtained. Particular attention is paid to the variety $Q = \{L \in B_c: L^\circ$ a relative Stone algebra$\}$ and its lattice of subvarieties, using results of Hecht and Katrinak (Notre Dame J. Formal Logic 13(1972), 248–254). Let, for $n \geq 1$, Q_n denote $V(\{L \in B_c: L^\circ$ an n-element chain$\})$. *Theorems*: 3. Q is generated by its finite members. 4. Q is generated by the varieties Q_n, $n \geq 1$. 5. Finitely generated algebras in Q_n, $n \geq 1$, are finite. Equations defining Q and Q_n, $n \geq 1$, are determined, and a description is given of the lattice of those subvarieties of Q which themselves have only finitely many subvarieties. (Received January 31, 1974.)

74T-A92. TEMPLE HAROLD FAY, Hendrix College, Conway, Arkansas 72032. *A natural transformation approach to additivity.* Preliminary report.

Main Theorem. Let C be a category having finite products and a zero object. A necessary and sufficient condition that C be additive is there exists a natural transformation f from $(- \times - \times -)$ to 1_C such that for each object X, (i) $f_X(1_X \times \Delta_X) = \pi_1$ and $f_X(\Delta_X \times 1_X) = \pi_2$ where Δ_X is the diagonal morphism and π_1 and π_2 are the projections of $X \times X$; (ii) $f_X(f_X \times 1_X \times 1_X) = f_X(1_X \times 1_X \times f_X)$. The dual theorem states additivity depends upon a natural transformation g from 1_C to $(-\sqcup-\sqcup-)$ satisfying the dual conditions. It is shown that f and g are unique, determine each other, $f = \langle 1_X, -1_X, 1_X \rangle$, and $g = \{1_X, -1_X, 1_X\}$. If C is finitely complete, an E-M bicategory with E closed under pullbacks, admitting a natural transformation f from $(- \times - \times -)$ to 1_C satisfying condition (i) above, then all equivalence relations on the same object commute. If (C, U) is an algebraic category in the sense of Herrlich and Strecker ("Category theory", Allyn and Bacon, 1973), and if f is a natural transformation from $U(- \times - \times -)$ to U satisfying condition (i) above with X replaced by UX, then all equivalence relations on the same object commute. The former results account for the commuting of congruences in an abelian category and the latter accounts for the situation in the category of all groups. Other equivalent relation theoretic conditions are considered. (Received January 31, 1974.)

*74T-A93. JOHN A. TILLER and TEMPLE HAROLD FAY, Hendrix College, Conway, Arkansas 72032. *Unions in E-M categories and coreflective subcategories.* Preliminary report.

Let C be an E-M category and $d_i: D_i \to X$ a family of M-morphisms and assume each d_i factors through $h: D \to X$. We call (D, h) the *M-union* if whenever $c: B \to X$ is in M and each d_i factors through c, then h factors uniquely through c. (D, h) is called the *strong M-union* if whenever $f: X \to A$ and $c: B \to A$ is in M such that each fd_i factors through c, then fh factors uniquely through c. *Theorem.* If C has weak pullbacks, then strong M-unions and M-unions coincide. *Theorem.* C has strong M-unions if and only if C has M-unions and E-M images distribute over M-unions. *Theorem.* If C is M-locally small and has M-unions, a full replete subcategory K is M-coreflective (coreflection morphisms are M-morphisms) if and only if K is closed under M-unions and M-images. (Received January 31, 1974.)

*74T-A94. KENNETH P. McDOWELL, McMaster University, Hamilton, Ontario K8S 4K1, Canada. *Pseudo-Noetherian rings.*

The rings of Abstract 72T-A268, these *Notices* 19(1972), A-749 will here be called pseudo-Noetherian rings. If R is a local pseudo-Noetherian ring and M is a nonzero finitely presented R-module, then the length of any maximal M-sequence is equal to the grade of M. (If $M = R$, this value coincides with the supremum of the projective dimensions of those finitely presented modules which have finite dimension.) Furthermore, if M has finite Gorenstein dimension, the grade of R is the sum of the grade of M and the Gorenstein dimension of M. Although, in general, coherent local rings need not be pseudo-Noetherian, the observation that a faithfully flat directed colimit of pseudo-Noetherian rings is again pseudo-Noetherian leads to the production of various examples of coherent local rings which are pseudo-Noetherian. E. G. Evans has shown that every coherent Z. D. ring is pseudo-Noetherian (Trans. Amer. Math. Soc. 155(1971), 505–512). However, with the help of the above observation, the converse may be shown to be false even in the local case. (Received February 1, 1974.)

74T-A95. DAVID ZEITLIN, 1650 Vincent Avenue North, Minneapolis, Minnesota 55411. *Identities for integer sequences involving the greatest integer function.* IV. Preliminary report.

M and Q are positive integers; W_n is an integer sequence given by $W_{n+2} = MW_{n+1} + SQW_n$, $S = \pm 1$, where W_0 and W_1 are integers; if $W_0 = 0$, $W_2 = 1$, then $W_n \equiv U_n$; if $W_0 = 2$, $W_1 = M$, then $W_n \equiv V_n$. Set $A(x) = |xW_0 - W_1|$, $D(j) = (V_2 + 1)U_{k+j}$, $F = V_2/(V_2 + 1)$, $T = (M^2 + 4SQ)^{1/2}$, $Z = (W_1^2 - MW_0W_1 - SQW_0^2)/T$, and $[x]$ the greatest integer function. *Case* 1. $S = 1$, with $1 \leq Q < M + 1$ and $M \geq 1$, and $P > 0$ the root of $x^2 - Mx - Q = 0$. *Case* 2. $S = -1$, with $1 \leq Q < M - 1$ and $M \geq 3$, and $P > 1$ the root of $x^2 - Mx + Q = 0$. For both cases, *Theorem* 1. If $A^2(P) < (TP^{2k})/(D(k)Q^{2k})$, then $[P^{2k}W_n^2 + 2ZP^kU_k(-SQ)^n + F] = W_{n+k}^2$ for $n \geq k \geq 0$. *Example* 1. *Fibonacci sequences*, with $S = M = Q = 1$, $U_n \equiv F_n$, $V_n \equiv L_n$, and $P > 0$ the root of $x^2 - x - 1 = 0$. Let $C = 5^{-1/2}$. For

$n \geq 1$, $[P^2F_{2n}^2 + C + 1.75] = F_{2n+1}^2$; $[P^2L_{n+2}^2 + (5 + C^{-1})(-1)^{n+1} + .75] = L_{n+3}^2$; and $[P^2F_{2n-1}^2 - C - .25] = F_{2n}^2$. *Example* 2. $W_{n+2} = 4W_{n+1} - W_n$, $W_0 = 1$, $W_1 = 3$, and $P > 1$ the root of $x^2 - 4x + 1 = 0$. Let $B = 3^{-1/2}$. For $n \geq 1$, $[P^2W_n^2 - 4B - (16/15)] = W_{n+1}^2$. See Abstract 73T-A259, these *Notices* 20(1973), A-567. Additional reports are forthcoming. (Received February 1, 1974.)

*74T-A96. AVIEZRI S. FRAENKEL, Weizmann Institute of Science, Rehovot, Israel. *Combinatorial games with an annihilation rule.*

A two-person game is defined by placing m stones on distinct vertices $(u_1, \cdots, u_m) = u^m$ of a finite directed loopless graph which may contain cycles. A move consists of moving a stone from a vertex u_i to a neighboring vertex u_j along a directed edge. If there was already a stone at u_j, both stones get annihilated. The player making the last move wins. If there is no last move, the game is a tie. Let G denote the generalized Sprague-Grundy function. For an arbitrary game position u^m, suppose that $G(u_i) = \infty\,(1 \leq i \leq n)$, $G(u_i) < \infty$ $(n < i \leq m)$. It is proved that $G(u^m) = G(u^n) +_2 \Sigma_2{}_{i=n+1}^{m} G(u_i)$, where $+_2$ and Σ_2 denote generalized Nim-addition. This completely determines the game's strategy (Proc. Conf. on Influence of Computing on Mathematical Research and Education, Missoula, Montana, 1973, to appear). (Received February 1, 1974.)

*74T-A97. THOMAS S. SHORES, University of Nebraska, Lincoln, Nebraska 68508. *Continuous subfunctors of the identity.* Preliminary report.

For background refer to Bronn [Dissertation, Northwestern University, 1971], Hsü [Math. Ann. 206(1973), 177–186], and Walker and Walker [Rocky Mountain J. Math. 2(1972), 513–555]. Throughout, $\mathfrak{A}$ is a cocomplete abelian category with a generator G. A subobject I of G is *fully invariant* if for every epimorphic image N of a coproduct of copies of G/I and $\alpha: G \to N$, $I \subseteq \operatorname{Ker} \alpha$. *Theorem* 1. Let $\mathfrak{A}$ be Grothendieck, $\mathfrak{B}$ abelian and $F: \mathfrak{A} \to \mathfrak{B}$ an additive functor. Then F preserves all limits iff F preserves inverse limits. *Theorem* 2. Up to a natural equivalence, the only continuous concordant (resp., cocontinuous harmonic) functors on $\mathfrak{A}$ are S_I (resp., Q_I), where for all objects $M \in \mathfrak{A}$, $S_I(M) = \Sigma\{\operatorname{Im} \alpha \mid \alpha: \bigoplus G/I \to M\}$ and $Q_I(M) = M/\Sigma\{\operatorname{Im}(\alpha|I) \mid \alpha: G \to M\}$. S_I and Q_I are adjoint functors. *Remark.* If $\mathfrak{A}$ is a C^2 category in the sense of Mitchell, then every harmonic functor is cocontinuous. (Received February 4, 1974.)

*74T-A98. R. PADMANABHAN, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. *A minimal self-dual equational basis for Boolean algebras.*

The class of all Boolean algebras, considered say, as an equational class of type $\langle 2, 2, 1, 0, 0 \rangle$ is well known to be self-dual. In spite of an abundance of different axiomatic approaches to the subject, however, no independent equational basis which is also self-dual is known to exist (see Problem 29 in G. Grätzer, "Lattice theory: first concepts and distributive lattices", W. H. Freeman & Co., 1971). In this note we give one such basis for Boolean algebras. *Theorem.* The set of six identities $(x \vee y) \wedge y = y$, $x \wedge (y \vee z) = (y \wedge x) \vee (z \wedge x)$, $x \wedge x' = \mathbf{0}$ and their duals form an independent basis for Boolean algebras. The dual $\bar{f}$ of a Boolean polynomial f can be defined in at least two ways: (1) $\bar{f}$ is obtained from f by interchanging the two binary operation symbols $\vee$ and $\wedge$ and the two nullary operation symbols $\mathbf{0}$ and $\mathbf{1}$; (2) if f is a (0,1)-lattice polynomial then f is defined as above and if $'$ symbol occurs in f then $\bar{f}$ is obtained by further interchanging x and x'. Thus, for example, the dual of $x \wedge x' = \mathbf{0}$ is $x \vee x' = 1$ in the first sense while it is $x' \vee x = 1$ in the second sense. The theorem remains valid under both interpretations of duality. (Received February 4, 1974.)

74T-A99. RICHARD A. MOLLIN, Queens's University, Kingston, Ontario K7L 3N6, Canada. *Some results on the group of algebras with uniformly distributed invariants, $U(K)$.* Preliminary report.

Let K be an abelian extension of Q. Let $U(K)$ be the subgroup of the Brauer group $B(K)$ which consists of those classes containing an algebra A satisfying: (1) If index of A is m then $\zeta_m \in K$ where ζ_m is a primitive mth root of unity. (2) Let p be a rational prime. If $\mathfrak{P}$ is a prime of K dividing p, and $\theta \in \mathfrak{Gal}(K/Q)$

with $\zeta_m^\theta = \zeta_m^{b\theta}$ then $\mathrm{inv}_\wp(A) \equiv b_\theta \, \mathrm{inv}_\wp \theta(A) \pmod 1$. (Benard and Schacher (J. Algebra 22(1972), 378–385) have shown that $S(K) \subseteq U(K)$.) Let q be a prime and let $S_q(K)$ (resp. $U_q(K)$) denote the subgroup of elements of q-power order in $S(K)$ (resp. $U(K)$). *Theorem.* If K is a nonreal abelian extension of Q then $|U_2(K): S_2(K)| = \infty$. If q is odd and K is real extension of Q then $U_q(K) = S_q(K) = 1$. *Theorem.* If $n = q^a t$, $(q \nmid t) = 1$ is the order of the group of roots of unity in K, q odd, then (1) if $q \nmid |K: Q(\zeta_q a)|$ then $U_q(K) = S_q(K)$; (2) if $q \mid |K: Q(\zeta_q a)|$ where $a \geq 1$ then $|U_q(K): S_q(K)| = \infty$. These results generalize those obtained by Schacher (Proc. Amer. Math. Soc. 31(1972), 15–17). (Received February 4, 1974.)

*74T-A100. CARL POMERANCE, University of Georgia, Athens, Georgia 30602. *On a problem of Erdős concerning the functions σ and ϕ.*

If σ is the sum of the divisors function and ϕ is Euler's function, then for every $c > 1$, $t > 1$, denote by $S(c, t)$ the set of integers m_1 for which $\sigma(m_1) > cm_1$ and the equation $\sigma(n) - n = m_1$ has at least t solutions; denote by $F(c, t)$ the set of integers m_2 for which $m_2 > c\phi(m_2)$ and the equation $n - \phi(n) = m_2$ has at least t solutions; and denote by $B(c, t)$ that subset of $S(c, t) \times F(c, t)$ where the two indicated equations have at least t simultaneous solutions. P. Erdős (Elem. Math. 28(1973), 83–86) asked if $S(c, t)$, $F(c, t)$, and $B(c, t)$ are always nonempty. We not only prove these sets are nonempty, but we prove *Theorem* 1. For any $c > 1$, $t > 1$, the lower density of $S(c, t) \cap F(c, t)$ is positive; the lower densities of the projections of $B(c, t)$ on its coordinates are both positive. The proof immediately follows from the known partial solution of Goldbach's conjecture: for every t, then (with density 0 exceptions) every positive even integer is a sum of 2 primes in at least t different ways. *Theorem* 2. Schinzel's Conjecture H implies that for every $c > 1$, $t > 1$, $S(c, t)$ and $F(c, t)$ contain even numbers. *Theorem* 3. For every $c > 1$, $t > 1$, if $(m_1, m_2) \in B(c, t)$, then $m_1 \neq m_2$. (Received February 4, 1974.)

74T-A101. WERNER POGUNTKE, Technische Hochschule, 61 Darmstadt FB4 AG1, West Germany. *On the decomposition of S-spaces.* Preliminary report.

Let S be a finite poset; an *S-space* is a vector space V together with a system V_i $(i \in S)$ of subspaces such that $i \leq j$ in S implies $V_i \subseteq V_j$. It is easy to see that if an S-space is directly decomposed into systems $(X, (X_i)_{i\in S})$ and $(Y, (Y_i)_{i\in S})$, the sublattice $\langle V_i \rangle$ of the lattice of all subspaces of V which is generated by the V_i $(i \in S)$ is a subdirect product of $\langle X_i \rangle$ and $\langle Y_i \rangle$. We denote by D_2 the two-element lattice and by M_3 the five-element modular lattice which is not distributive. *Theorem.* If there is a homomorphism from $\langle V_i \rangle$ onto M_3 which is not an isomorphism, then there exists a decomposition with $\langle X_i \rangle \cong M_3$. (A corresponding theorem for the lattice D_2 instead of M_3 is implicitly proved in Brenner, J. Algebra 6(1967), 100–114.) The theorem is the key to apply recent lattice-theoretical results to get the classifications of indecomposable S-spaces for certains posets S: with the help of R. Wille (Math. Z., vol. 131, 241–249), it is easy to show that if S does not contain $1+1+1+1$ nor $1+2+2$ as a subposet, then for every indecomposable S-space it holds either that $\langle V_i \rangle \cong D_2$ or $\langle V_i \rangle \cong M_3$ (cf. also M. M. Kleiner); applying recent results of C. Herrmann, M. Kindermann and R. Wille, the classification of the indecomposable S-spaces for $S = 1+2+2$ can be obtained (cf. also P. Gabriel, Manuscripta Math., vol. 6, 71–103). (Received February 5, 1974.) (Author introduced by Dr. Rudolf Wille.)

*74T-A102. CARL G. WAGNER, University of Tennessee, Knoxville, Tennessee 37916. *Differentiable functions of a p-adic variable.*

Let $\Sigma_{n=0}^\infty a_n \binom{x}{n}$ be the interpolation series for a continuous function $f: Z_p \to Q_p$ [K. Mahler, "Introduction to p-adic numbers and their functions", Cambridge University Press, 1973, Chapter 6, and R. Bojanic, Abstract 711-10-15, these *Notices* 21(1974), A-51]. For each $y \in Z_p$, let $a_j(y) = \Sigma_{k=0}^\infty a_{j+k}\binom{y}{k}$. Mahler showed that $\lim_{j\to\infty} a_j(y)/j = 0$ is necessary and sufficient for the differentiability of f at y, in which case, $f'(y) = \Sigma_{j=1}^\infty (-1)^{j-1} a_j(y)/j$. His proof of necessity is based on a somewhat complicated Tauberian theorem. Using the combinatorial identity $\Sigma_{k=j}^n (-1)^{n-k}\binom{n}{k}\binom{k-1}{j-1} = (-1)^{n-j}$, we show directly from the hypothesis of differentiability that $f'(y) = \Sigma_{j=1}^\infty (-1)^{j-1} a_j(y)/j$, from which it follows a fortiori that $(a_j(y)/j)$ is a p-adic null sequence. (Received February 5, 1974.)

74T-A103. PETER HAGIS, JR., Temple University, Philadelphia, Pennsylvania 19121 and WAYNE L. McDANIEL, University of Missouri, St. Louis, Missouri 63121. *On the largest prime divisor of an odd perfect number.* II. Preliminary report.

Using essentially the same procedure employed in their earlier paper [Math. Comp. 27(1973), 955–957] the authors prove that if n is odd and perfect then n has a prime factor which exceeds 100109. (Received February 6, 1974.)

74T-A104. S. BRENT MORRIS, Duke University, Durham, North Carolina 27706. *Generalized permutation enumerants.*

This note generalizes all enumerants $A(n, \alpha)$ of Z_n, where α is some condition. The process is accomplished by considering one element from each of n disjoint subsets of Z_N, $N \geq n$, and requiring these n elements to have property α. If m_i of the subsets are of size t_i, $i = 1, \cdots, p$, and $B(N, \alpha)$ enumerates these generalized permutations, then $B(N, \alpha) = A(n, \alpha)N!/m_1! \cdots m_p!$. (Received February 7, 1974.)

74T-A105. KARL K. NORTON, 2235 Floral Drive, Boulder, Colorado 80302. *The absolute value of a Gaussian sum.* Preliminary report.

Let μ be the Möbius function, ϕ be Euler's function, n, m be integers $(n > 0)$, and χ be a Dirichlet (residue) character (mod n) with conductor K. Define $B = B(n, m, \chi) = \Sigma_{x=1}^{n} \chi(x)\exp(2\pi imx/n)$. *Theorem.* Write $n_m = n/(n, m)$. If $K \nmid n_m$, then $B = 0$. If $K|n_m$, then $|B| = \mu^2(n_m/K)\chi_K(n_m/K)\phi(n)K^{1/2}/\phi(n_m)$, where χ_K is the principal character (mod K). (This result is well known if χ is primitive, i.e., $K = n$, or if χ is arbitrary and $(m, n) = 1$. Cf. E. Landau ("Handbuch der Lehre von der Verteilung der Primzahlen", Chelsea Publ. Co., New York, 1953, pp. 483–486, 492–494, and "Vorlesungen über Zahlentheorie", Chelsea Publ. Co., New York, 1955, Vol. I, pp. 188–190, Vol. III, pp. 330–334.) (Received February 8, 1974.)

*74T-A106. BJARNI JONSSON, Vanderbilt University, Nashville, Tennessee 37235 and PHILIP OLIN, York University, Downsview, Ontario, Canada. *Elementary equivalence and relatively free products of lattices.* Preliminary report.

If $\mathcal{V}$ is a nontrivial variety of lattices, then there exist $A, B, C \in \mathcal{V}$ such that B and C are elementarily equivalent, but $A * B$ and $A * C$ are not. If $\mathcal{V}$ contains a nondistributive lattice, then A can be taken to consist of just one element. (Received February 12, 1974.)

Analysis

*74T-B75. LEE A. RUBEL, University of Illinois, Urbana, Illinois 61801 and CHUNG-CHUN YANG, Naval Research Laboratory, Washington, D. C. 20390. *On zero-one sets of entire functions.*

A pair $(\{a_n\}, \{b_n\})$, $n = 1, 2, 3, \cdots$, of sequences of complex numbers is called the zero-one set of the entire function f if $\{a_n\}$ is the precise zero sequence of f and $\{b_n\}$ is the precise one sequence of f. Repeated a_n correspond to zeros of the appropriate multiplicity, with a similar convention for the b_n. *Theorem* 1. Given any $\{a_n\}$, there exists a sequence $\{b_n\}$ that is disjoint from $\{a_n\}$ and without finite limit points, such that $(\{a_n\}, \{b_n\})$ is not the zero-one set of any entire function. *Theorem* 2. If the entire function f has the same zero-one set as $\sin z$ then $f(z) = \sin z$. Theorem 2 is much easier to prove than Theorem 1. (Received November 5, 1973.)

*74T-B76. HAROLD EXTON, University of Salford, Salford, United Kingdom. *A type of orthogonal polynomial of the fourth order.* Preliminary report.

A sequence of orthogonal polynomials defined by $X_n(z) = d^{2n}(z^4-1)^n/dz^{2n}$ is studied and shown to be capable of expression in terms of the generalised hypergeometric function ${}_4F_3$ and to be orthogonal over

$[-1, 1]$ w.r.t. the unit weight function. Certain of its properties are shown to be extensions of corresponding properties of the Legendre polynomials. A differential equation and two differential recurrence relations are also given. (Received January 3, 1974.)

*74T-B77. CHANDRA MOHAN JOSHI, University of Jodhpur, Jodhpur, Rajasthan, India 342001 and M. L. PRAJAPAT, Defence Laboratory, P. B. No. 136, Jodhpur, Rajasthan, India 342001. *On some properties of a class of polynomials unifying the generalized Hermite, Laguerre and Bessel polynomials.* II. Preliminary report.

For the polynomial system $\{M_n^{(\alpha)}(x, r, p, b, k, q) \mid n = 0, 1, 2, \cdots\}$ defined in terms of the differential operator $T_{k,q} \equiv x^q(k + xD)$, where k and q are constants, a number of properties, in addition to those discussed earlier [Abstract 73T-B298, these *Notices* 20 (1973), A-582], have been established. (Received January 4, 1974.

74T-B78. DOUGLAS MOREMAN, Emory University, Atlanta, Georgia 30322. *Convex hull-specific transformations in convex topology.* Preliminary report.

See Abstract 701-46-48, these *Notices* 20 (1973), A-151 for background material. Suppose S (S') is the set of all points relative to some meaning of the word "point", B (B') is a topology for S (S'), and C (C') is an intersectional convexity for S (S'). *Definition.* Let T be a transformation from S onto S'; T *is convex hull-specific* if $M \subset S \Rightarrow T(C(M))$ is $C'(T(M))$. T *is a convex hull-specific functional* if S', B', C' is a subspace of the space of all numbers. *Theorems.* T is convex hull-specific only if for each convex subset H of S (K of S'), $T(H)$ ($T^{-1}(K)$) is convex. If T is a convex hull-specific functional then (1) T is continuous only in case T is continuous relative to the convex topology of S, B, C (so that, if S, B, C is a linear topological space and T is a linear functional from S, B, C then T is continuous iff T is continuous relative to the "weak topology" of S, B, C); (2) if the sequence α of points of S converges convexly to the point P, then $T(\alpha)$ converges to $T(P)$; and (3) if S is metric, α is a sequence of points of S that closes convexly, and T is Lipschitz continuous (there is a number k s.t. if $P, Q \in S$ then $T(P)$ differs from $T(Q)$ by no more than $k \cdot d(P, Q)$) then $T(\alpha)$ converges, etc. (Received January 14, 1973.)

*74T-B79. ANDREAS ZACHARIOU, Mathematical Institute, University of Athens, Athens 143, Greece. *A study of inner product spaces.* Preliminary report.

Space means real inner product space. *Theorems*: 1. X, Y are spaces and $f: X \to Y$ is a linear map. f has an adjoint iff $\{x \in X: \langle f(x), y\rangle = 0\}^{\perp} \neq 0$ $\forall y \in Y$. 2. X is a space of uncountable dimension. $X \supset Y$ with $\dim Y = \dim X$, while Y does not admit any orthogonal basis. 3. X is a space. X is finite-dimensional $\Leftrightarrow$ every linear functional f on X is Riesz representable $\Leftrightarrow f$ is continuous $\Leftrightarrow \forall V \subset X$, $X = V \oplus V^{\perp} \Leftrightarrow$ every linear map on X has an adjoint $\Leftrightarrow X$ is Cartesian. 4. X is a space. X is Hilbert $\Leftrightarrow$ every continuous linear functional on X is Riesz representable $\Leftrightarrow \forall V \subset X$, $V^{\perp\perp} = \overline{V} \Leftrightarrow \forall$closed $V \subset X$, $V^{\perp\perp} = V \Leftrightarrow \forall$closed $V \subset X$, $X = V \oplus V^{\perp} \Leftrightarrow$ every continuous linear map on X has an adjoint. 5. Every Hilbert space X does not admit any orthogonal basis. Consequently $\dim X > \aleph_0$. 6. X is a space, and $V \subset X$. $X = V \oplus V^{\perp}$ iff $(x_V^{\perp})_V^{\perp} \neq 0$ $\forall x \in X$. 7. X is a Hilbert space, and V an infinite dimensional subspace of X. If $X = V \oplus V^{\perp}$, then V does not admit any orthogonal basis. Consequently, if $\dim V = \aleph_0$, then $X \neq V \oplus V^{\perp}$. 8. X is a space and V is a Hilbert subspace of X. Then $X = V \oplus V^{\perp}$. 9. X is any infinite-dimensional space. X contains a noncomplete proper subspace V with $\dim V = \dim X$, and $\operatorname{codim}(V) =$ finite. 10. The Witt isometry extension theorem does not hold for infinite-dimensional Hilbert spaces. (Received November 20, 1973.) (Author introduced by Professor Anastasios Mallios.)

*74T-B80. SANFORD S. MILLER, State University College of New York, Brockport, New York 14420 and Babes-Bolyai University, Cluj, Romania, and PETRU T. MOCANU, Babes-Bolyai University, Cluj, Romania. *The H^p classes for alpha-convex functions.* II. Preliminary report.

In a previous paper (Eenigenburg and Miller, Proc. Amer. Math. Soc. 38 (1973), 558–562) the Hardy classes for $f(z)$ and $f'(z)$ were determined for alpha-convex functions when $\alpha \geq 0$. The authors have extended this result to $\alpha < 0$ and investigated the Hardy classes for $f''(z)$, $f'''(z)$ and $(zf'(z)/f(z))'$ for any real α. (Received December 5, 1973.)

74T-B81. DENNIS M. SCHNEIDER, Knox College, Galesburg, Illinois 61401. *Sufficient sets for some spaces of entire functions.*

B. A. Taylor (Trans. Amer. Math. Soc. 163 (1972), 207–214) has shown that the lattice points in the plane form a sufficient set for the space of entire functions of order less than two. We obtain a generalization of this result to functions of several variables and to more general spaces of entire functions. For example, we prove that if $S \subset \mathbf{C}^n$ s.t. $d(z, S) \leq \text{const}\,|z|^{1-\rho/2}\ \forall z \in \mathbf{C}^n$, then S is a sufficient set for the space of entire functions on $\mathbf{C}^n$ of order less than ρ. The proof involves estimating the growth rate of an entire function from its growth rate on S. The concept of a weakly sufficient set is introduced and sufficient conditions are obtained for a set to be weakly sufficient. We prove that sufficient sets and certain types of effective sets (see Iyer, Trans. Amer. Math. Soc. 42 (1937), 358–365; correction, ibid. 43 (1938), 494) are weakly sufficient. (Received January 10, 1974.) (Author introduced by Professor Rothwell Stephens.)

*74T-B82. JON C. HELTON, Arizona State University, Tempe, Arizona 85281. *Product integrals and the solution of integral equations.*

Functions are from $R \times R$ to N, where R = reals and N a normed complete ring. See B. W. Helton [Pacific J. Math. 16(1966), 297–322] for additional background. If $\beta > 0$, H and G are functions from $R \times R$ to N, f and h are functions from R to N, each of H, G and dh is in OB° on $[a, b]$ and $|H| < 1 - \beta$ on $[a, b]$, then the following are equivalent: (1) f is bounded on $[a, b]$, each of $\int_a^b H$, $\int_a^b G$ and (LR) $\int_a^b (fG + fH)$ exists and $f(x) = h(x) + (\text{LR}) \int_a^x (fG + fH)$ for $a \leq x \leq b$, and (2) each of ${}_x\pi^y(1 + \sum_{j=1}^\infty H^j)$, ${}_x\pi^y(1 + G)$ and (R) $\int_x^y dh\,(1 + \sum_{j=1}^\infty H^j)\,{}_s\pi^y(1 + G)(1 + \sum_{j=1}^\infty H^j)$ exists for $a \leq x < y \leq b$ and $f(x) = h(a)\,{}_a\pi^x(1 + G)(1 + \sum_{j=1}^\infty H^j) +$ (R) $\int_a^x dh\,(1 + \sum_{j=1}^\infty H^j)\,{}_s\pi^x(1 + G)(1 + \sum_{j=1}^\infty H^j)$ for $a \leq x \leq b$. This result is obtained without requiring the existence of integrals of the form $\int_a^b |G - \int G| = 0$ and $\int_a^b |1 + G - \pi(1 + G)| = 0$. (Received January 14, 1974.)

*74T-B83. ETHELBERT N. CHUKWU, Cleveland State University, Cleveland, Ohio 44115. *On the boundedness and stability properties of solutions of some differential equations of the fifth order.* Preliminary report.

We investigate $x^{(5)} + f_1(x, x', x'', x''', x^{(iv)})x^{(iv)} + bx''' + f_3(x'') + f_4(x') + f_5(x) = p(t)$ when (i) $p \equiv 0$, (ii) p ($\neq 0$) bounded, assuming b is constant and p, f_i ($i = 1, 3, 4, 5$) are real functions depending only on the arguments explicitly displayed. It is also assumed the functions are such that the existence and uniqueness of solutions, as well as their continuous dependence on initial conditions, are assured. For case (i) the asymptotic stability (in the large) of the trivial solution $x = 0$ is established subject to certain generalizations of the Routh-Hurwitz stability criterion. For case (ii) a boundedness result is deduced with a bound dependent on the initial conditions. The proof of the two main results depends on an explicit Lyapunov function; the results are comparable in generality to known results of third and fourth order equations. (Received January 14, 1974.)

74T-B84. MICHAEL D. RICE, Ohio University, Athens, Ohio 45701. *h-closed σ-algebras.* Preliminary report.

Let (X, Σ) be a measurable space. Then $\{B_s : s \in S\} \subseteq \Sigma$ is a (c.a.) *completely additive* family if $\bigcup\{B_s : s \in S'\} \in \Sigma$ for each $S' \subset S$. Σ is *h-closed* if whenever $\{B_s\}$ is a c.a. disjoint Σ-family and $A_s \subset B_s$ for each s, with $A_s \in \Sigma$, $\{A_s\}$ is a c.a. family. *Theorem.* (CH) Let μ^* be a regular (relative Borel sets) outer measure on a separable metric space and let Σ be the μ^*-measurable sets. Then Σ_H is h-closed for each σ-finite $H \in \Sigma$. Hence the Lebesgue sets on R^n are h-closed. *Notes.* 1. (CH) S has Lebesgue measure zero iff $\exists$ a c.a. disjoint Lebesgue family $\{B_s\}$ with $s \in B_s$. 2. Let μ^* be a σ-finite outer measure on a metric space M with $\mu^*(x) = 0$ for each $x \in X$. Then Σ is h-closed and each set with positive outer measure contains a non-μ^*-measurable set iff whenever $\{B_s\}$ is a c.a. Σ-family of measure zero sets, $\mu^*(\bigcup B_s) = 0$. 3. If the Borel sets of a complete metric space are h-closed, then each Borel measurable map to a metric space is a map of class α, for some $\alpha < \omega_1$. 4. Let X be a topological space of Ulam nonmeasurable power, μ any 0–1 valued countably additive Borel measure with $\mu(x) = 0$ for each $x \in X$, and μ^* the induced outer measure. Then Σ is h-closed. (Such measures exist on any locally compact, nonmetrizable group.) (Received January 14, 1974.) (Author introduced by Professor George Reed.)

74T-B85. G. P. KAPOOR, V. V. Post Graduate College, Shamli (Muzaffarnagar) UP, India. *A note on the growth of functions analytic in a disc.* Preliminary report.

For $f(z) = \sum_{k=0}^{\infty} a_k z^{\lambda_k}$ $(a_k \neq 0 \ \forall k)$ analytic in $D \equiv \{z: |z| < 1\}$, set $\rho_f(\lambda_f) = \limsup_{r\to 1} (\inf)(\log^+\log^+ M(r)/-\log(1-r))$ and $T_f(t_f) = \limsup_{r\to 1} (\inf)(\log M(r)/(1-r)^{-\rho_f})$, $0 < \rho_f < \infty$, where $M(r) = M(r, f) = \max_{|z|=r}|f(z)|$, $0 < r < 1$. For $g(z) = \sum_{k=0}^{\infty} c_k z^{\lambda_k}$ $(c_k \neq 0 \ \forall k)$ analytic in $D_R \equiv \{z: |z| < R\}$, $0 < R < \infty$, set $\rho_g^*(\lambda_g^*) = \limsup_{r\to R} (\inf)\{(\log^+\log^+ M^*(r))/(\log(Rr/(R-r)))\}$ and $T_g^*(t_g^*) = \limsup_{r\to R} (\inf)\{(\log M^*(r))(rR\,(R-r))^{-\rho_g^*}\}$, $0 < \rho_g^* < \infty$, where $M^*(r) = M^*(r, g) = \max_{|z|=r} |g(z)|$, $0 < r < R$. We consider $G(z) = \sum_{k=0}^{\infty} c_k R^{\lambda_k} z^{\lambda_k}$ analytic in the unit disc and show that $\rho_G = \rho_g^*$, $\lambda_G = \lambda_g^*$, $T_G = R^{\rho_G} T_g^*$ and $t_G = R^{\rho_G} t_g^*$. Thus, all the coefficient formulae for ρ_g^*, λ_g^*, T_g^* and t_g^*, and, in particular, the results of Bajpai, Tanne, and Whittier (Abstract 73T–B201, these *Notices* 20 (1973), A–483) immediately follow from the known formulae for ρ_G, λ_G, T_G and t_G (Math. Japon. 17(1972), 49–54; Abstract 73T–B227, these *Notices* 20 (1973), A–490; Kapoor, Ph.D. Thesis, Indian Institute of Technology, Kanpur, 1972). (Received January 15, 1974.) (Author introduced by Dr. Prabha Gaiha.)

*74T-B86. HERBERT E. PEEBLES, St. Bonaventure University, St. Bonaventure, New York 14778. *On convergence of sequences of measurable functions.* Preliminary report.

All possible implications among three modes of convergence (pointwise a.e., measure and mean) of sequences of a.e. finite-valued measurable functions under all possible subsets of a set of twenty auxiliary hypotheses are considered. The first ten hypotheses are concerned with functions in an abstract measure space and the second ten with functions in a locally compact Hausdörff space. Counterexamples establish the nonexistence of the unproved theorems. (Received January 16, 1974.)

74T-B87. LAWRENCE GLUCK, DePaul University, Chicago, Illinois 60614. *On the ideal of equicontinuous multipliers.* Preliminary report.

Notation. G an LCA group and F any Banach space upon which the translations T_g by elements of G is an isometric isomorphism; E a Banach space of type F satisfying (1) $\lim_{g\to 0} \|T_g f - f\|_E = 0 \ \forall f \in E$; a bounded operator S on E is equicontinuous if for each bounded set $B \subset E$, (1) holds for Sf uniformly for $f \in B$; $M(F)$ is the (uniform) Banach algebra of multipliers (i.e. operators on F which commute with all T_g, $g \in G$) and $M_e(F)$ the uniformly closed ideal of those which are equicontinuous; A_E is the projective tensor product of E and E'. Then: $M_e(E)$ (and A_E) are of type E and have bounded approximate identity, and $M(E) = M(M_e(E))$ $(= M(A_E))$. This result uses the McKennon K-topology on $M_e(E)$ and both sharpens and generalizes a result of McKennon. $M(E)$ is the double dual of $M_e(E)$ in its K-topology. Since, for compact G, $M_e(E)$ consists of the compact multipliers of E, this generalizes to noncompact groups the fact that $M(E)$ is the double dual of the compact multipliers. The theorem holds for nonabelian locally compact groups with appropriate modifications. (Received January 17, 1974.)

74T-B88. MEHDI RADJABALIPOUR and HEYDAR RADJAVI, Dalhousie University, Halifax, Nova Scotia, Canada. *On a question of Colojoara and Foias.* Preliminary report.

Let J be a fixed C^2 Jordan curve. A Hilbert space operator T satisfies condition (I) if (a) T is the sum of a normal operator with spectrum on J and an operator of Schatten class C_p $(1 \leq p < \infty)$, (b) $\sigma(T)$ does not fill the interior of J. *Theorem.* Let T be an operator on a Hilbert space. Assume $T|M$ and $T^*|M^\perp$ satisfy condition (I) for all hyperinvariant subspaces M of T. Then T is strongly decomposable. *Corollary* 1. If $T^* - T \in C_p$ then T is strongly decomposable. *Corollary* 2. If $T^*T - I \in C_p$ and $\sigma(T)$ does not fill the unit disc then T is strongly decomposable. A slight generalization of these facts is possible by replacing C_p with the Mačaev's ideal S_ω (Colojoara-Foias, "Theory of generalized spectral operators", pp. 218, 231.) (Received January 21, 1974.)

*74T-B89. ZDZISŁAW LEWANDOWSKI and ELIGIUSZ ZŁOTKIEWICZ, Maria Curie-Skłodowska University, Lublin, Poland, and SANFORD S. MILLER, State University College of New York, Brockport, New York 14420 and Maria Curie-Skłodowska University, Lublin, Poland. *On some classes of starlike functions*. Preliminary report.

Let $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ be regular in the unit disc D, with $f(z)/z$, $f'(z)$, $zf''(z)/f'(z) + 1 \neq 0$ for $z \in D$. Suppose γ is real and (1) $\operatorname{Re}[zf'(z)/f(z)]^{1-\gamma}[zf''(z)/f'(z)+1]^{\gamma} > 0$ for $z \in D$, where the powers appearing in (1) are meant as principal values. The authors show that functions satisfying (1) are necessarily starlike and they call them *gamma-starlike functions*. The class of such functions is denoted by $\mathfrak{L}_\gamma$. In addition, the authors consider *gamma-starlike functions of order* α, $\mathfrak{L}_\gamma(\alpha)$: $\operatorname{Re}[zf'(z)/f(z)]^{1-\gamma}[zf''(z)/f'(z)+1]^{\gamma} > \alpha$, $0 \leq \alpha < 1$, and *strongly gamma-starlike functions of order* α, $\mathfrak{L}^*_\gamma(\alpha)$: $|(1-\gamma)\arg(zf'(z)/f(z)) + \gamma \arg(zf''(z)/f'(z)+1)$ $\leq \alpha\pi/2$, $0 \leq \alpha < 1$. (Received January 28, 1974.)

*74T-B90. HARRY B. COONCE, Mankato State College, Mankato, Minnesota 56001. *Kaplan-Mocanu functions.*

For $\alpha \in \mathbf{R}$ we apply the generalized Kaplan criteria to the Mocanu α-vector, i.e., we appropriately bound the quantity $\int_{\theta_1}^{\theta_2} \operatorname{Re}\{\alpha(1 + zf''(z)/f'(z)) + (1-\alpha)zf'(z)/f(z)\}\,d\theta$, $z = re^{i\theta}$, $r < 1$. Sufficient conditions for univalence are given. For $\alpha > 0$ the classes reduce to classes of Basilevič functions of type $[\lambda, 0]$ $(\alpha\lambda = 1)$. However, for $\alpha < 0$ the classes need not be of Basilevič type $[\lambda, 0]$ for any λ, therefore a new test for univalence is provided. An example of a function satisfying the new criteria is provided as well as an example of a univalent function for which the new test fails. (Received January 28, 1974.)

*74T-B91. CHUNG-LING YU, Florida State University, Tallahassee, Florida 32306. *On the global solvability of the linear elliptic Cauchy problem.*

Consider the elliptic system $w_{\bar{z}} + a(z)w + b(z)\bar{w} = 0$, $z = x + iy$, where the coefficients $a(z)$, $b(z)$ are Hölder continuous in the $z = x + iy$ plane. A necessary and sufficient condition is given for the existence of a unique global solution to the Cauchy problem of the above-mentioned system. Furthermore, if such a solution exists, we shall give a method to construct it. (Received January 28, 1974.)

74T-B92. L. C. HSU, Jilin University, Changchun, People's Republic of China. *Symmetrical inversion formulas for integral transforms*. II.

A real function $y = \phi(x)$ is said to be "self-reciprocal" on the interval $0 \leq x \leq 1$ with $0 \leq y \leq 1$ if the relation $\phi(\phi(x)) = x$ holds on $0 \leq x \leq 1$. Let $\phi(x)$ be a self-reciprocal function defined as above and having derivatives of all orders in (0, 1). Let $H(s, t; m) = L_{m,t}[(\phi(e^{-u}))^s]$, where $L_{m,t}[\cdot]$ is the Post-Widder inversion operator applied to the function of u. *Theorem.* For every real function $f(s) \in L(0, \infty)$ such that the function $G(u) \equiv \int_0^\infty f(s)(\phi(e^{-u}))^s\,ds$ satisfies "Conditions D", there is a real function $g(s) \in L(0, \infty)$ such that $G(u) = \int_0^\infty g(s)e^{-us}\,ds$ and the pair of inversion formulas $g(t) = \lim_{m\to\infty}\int_0^\infty H(s, t; m)f(s)\,ds$ and $f(t) = \lim_{m\to\infty}\int_0^\infty H(s, t; m)g(s)\,ds$ hold for almost all positive t. These formulas are analogous to the author's symmetrical inversion formulas for series transforms. For definitions of $L_{m,t}[\cdot]$ and "Conditions D", see Widder, "The Laplace transform", 1946, Chapter 7, §§6, 17. (Received January 18, 1974.) (Author introduced by Professor Everett Pitcher.)

74T-B93. RONALD F. GARIEPY and JOHN L. LEWIS, University of Kentucky, Lexingon, Kentucky 40506. *A maximum principle with applications to subharmonic functions in n space.*

Denote points in Euclidean n-space $\mathbf{R}^n$, $n \geq 3$, by $x = (x_1, x_2, \cdots, x_n)$ and let $r = |x|$, $x_1 = r\cos\theta$, $0 \leq \theta \leq \pi$. Let H^m denote m-dimensional Hausdorff measure in $\mathbf{R}^n$, let $S = \{x: |x| = 1\}$, and for $0 \leq \theta_0 \leq \pi$, let $C(\theta_0) = S \cap \{x: \theta < \theta_0\}$. Given a function u subharmonic in $\{x: |x| < R\}$, let $\hat{u}(r, \theta) = \sup \int_E u(ry)\,dH^{n-1}y$, where $0 < r < R$, $0 \leq \theta \leq \pi$, and the supremum is taken over all measurable sets $E \subset S$ with $H^{n-1}(E) = H^{n-1}(C(\theta))$.

Let Ω be a bounded region in $\mathbf{R}^n$ of the form $\Omega = \bigcup_{r_1<r<r_2} C(\theta(r))$ where $0<\theta(r)\leq\pi$, $0\leq r_1<r_2<R$. Let h be a function harmonic in Ω that is symmetric w.r.t. the x_1-axis. Then $h(x)$ can be written as $h(r,\theta)$ for $0\leq\theta<\theta(r)$, $r_1<r<r_2$. Suppose that for each r, $r_1<r<r_2$, $h(r,\cdot)$ is a nonincreasing function of θ for $0\leq\theta<\theta(r)$. Then $\hat{h}(r,\theta)=\int_{C(\theta)} h(ry)\,dH^{n-1}y$, if $0\leq\theta<\theta(r)$. It is shown that if $\hat{h}$ has a continuous extension to $\Omega\cup(\partial\Omega-\{0\})$ and $\hat{u}-\hat{h}\leq c$ $(c\geq 0)$ on $\partial\Omega-\{0\}$, then $\hat{u}-\hat{h}\leq c$ in Ω. This maximum principle is then used to obtain "$\cos\pi\rho$" type theorems in $\mathbf{R}^n$. (Received January 31, 1974.)

74T-B94. H. H. PU, Soochow University, Taipei, Republic of China, J. D. CHEN, Institute of Mathematics, Academia Sinica, Taipei, Republic of China and H. W. PU, Institute of Mathematics, Academia Sinica, Taipei, Republic of China and Texas A&M University, College Station, Texas 77843. *A theorem on approximate derivates*. Preliminary report.

Neugebauer (Acta Sci. Math. (Szeged) 23 (1962), 79–81) proved the following interesting theorem: If $f: R\to R$ is continuous, then the set $E=\{x: f^-(x)\neq f^+(x)$ or $f_-(x)\neq f_+(x)\}$ is of first category. We prove that the theorem is also true for approximate derivates. (Received February 4, 1974.)

*74T-B95. CHRISTER BORELL, Institut Mittag-Leffler, Aurav. 17, S-182 62 Djursholm 1, Sweden. *An inequality for a class of harmonic functions in n-space*. Preliminary report.

Let D be a bounded region in R^n for which the Dirichlet problem is solvable. Set $a_i=\min\{x_i: x\in\bar{D}\}$, $b_i=\max\{x_i: x\in\bar{D}\}$, and $\gamma=(\bigcup_1^{n-k}\bar{D}\cap\{x_i=a_i\})\cup(\bigcup_1^{n-k}\bar{D}\cap\{x_i=b_i\})$. Let $f:\partial D\to[0,+\infty)$ be a continuous function, which vanishes on $\partial D\setminus\gamma$, and let $\omega(x,f,D)$ be the solution of Dirichlet's problem with boundary data f. Set $\omega(x,f,D)=0$ when $x\notin\bar{D}$. Let D^0 and f^0 denote certain k-dimensional Steiner symmetrizations of D and f, respectively. It is then proved that the inequality $\int\varphi(\omega(x,f,D))\,dx_{n-k-1}\cdots dx_n\leq\int\varphi(\omega(x,f^0,D^0))\,dx_{n-k+1}\cdots dx_n$ is valid for every nondecreasing convex function on R. This generalizes an inequality due to A. Baernstein. (Received February 4, 1974.) (Author introduced by Matts R. Essén.)

74T-B96. WILLIAM D. L. APPLING, North Texas State University, Denton, Texas 76203. *A representation characterization theorem*.

U, F, p_B, p_{AB}, and the notions of subdivision, refinement and integral are as in previous abstracts of the author. *Theorem*. If S is a subset of p_B and T is a function from S into $\mathbf{R}$, then the following are equivalent: (1) $\exists\ h\in p_{AB}$ s.t. if $A\in S$, then $\int_U A(I)h(I)$ exists and is $T(A)$; (2) $\exists$ a linear subspace S' of p_B, a function P from S' into the nonnegative numbers, a real linear functional T' on S' s.t.: (i) $T\subseteq T'$, (ii) $\{T'(X_V): V\in F\}$ is bounded, where for each $V\in F$, $X_V\in S'$, where $\forall\ I\in F$, $X_V(I)=\{1\}$ if $I\subseteq V$ and is $\{0\}$ otherwise, (iii) $T'(A)\to 0$ as $P(A)\to 0$, $A\in S'$, and (iv) if $A\in S'$ and $0<c$, then there is a subdivision D of U s.t. if E is a refinement of D and for each $V\in E$, $a(V)\in A(V)$, then $P(A-\Sigma_E a(V)X_V)<c$. (Received February 6, 1974.)

74T-B97. TSANG-HAI KUO, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. *On conjugate Banach spaces with the Radon-Nikodým property*.

Let X^* be the conjugate space for a Banach space X. It is shown that if the unit ball $B_{X^{**}}$ of X^{**} is Eberlein compact in the weak* topology or if X^* is isomorphic to a subspace of a weakly compactly generated Banach space then X^* possesses the Radon-Nikodým property (RNP). This result improves the classical theorem of Dunford and Pettis. The possession of RNP by the conjugate spaces of the two specific classes of Banach spaces, the Grothendieck spaces and the Banach spaces X with X^{**}/X separable, is being investigated. For instance, if X is a nonreflexive continuous linear image of $C(S)$ with S a compact F-space, then X^* cannot have RNP; if X is a Banach space with X^{**}/X separable then both X^* and X^{**} (and hence X) have the RNP. It is also shown that if a conjugate space X^* possesses the RNP and X is weak*-sequentially dense in X^{**} then $B_{X^{**}}$ is weak*-sequentially compact. Thus, in particular, if X^{**}/X is separable then $B_{X^{***}}$ is weak*-sequentially compact. (Received February 12, 1974.)

*74T-B98. M. R. PARAMESWARAN, University of Manitoba, Winnipeg, Manitoba R3T 2N2, Canada. *A Tauberian theorem for logarithmic means.*

A sequence $t = \{t_n\}$ is said to be summable by the logarithmic method (L) to λ if $[\log(1-x)]^{-1} \cdot \Sigma_0^{\infty}(n+1)^{-1}t_n x^{n+1} \to \lambda$ as $x \to 1-0$. Let $\{s_n^k\}$ denote the transform of $\{s_n\}$ by the Cesaro matrix C_k. *Theorem.* If $\{s_n^{\alpha}\}$ is (L)-summable to λ (for some α) and $n(s_n^{\beta} - s_{n-1}^{\beta})\log n = O(1)$ for some β, then $\{s_n\}$ is $C_{\beta-1}$-summable to λ. This improves, fully or in part, some results due to Ishiguro [Proc. Japan Acad. 39(1963), 156–159], Rangachari and Sitaraman [Tohoku Math. J. 16(1964), 257–269] and Borwein and Rizvi [Abstract 73T-B163, these Notices 29(1973), A-431]. (Received February 11, 1974.) (Author introduced by Professor H. C. Finlayson.)

74T-B99. S. K. BAJPAI, Indian Institute of Technology, New Delhi, India. *A note on a class of meromorphic univalent functions.*

Let A and B denote the family of functions f which are regular and univalent in the unit disc D and in the punctured disc $E\{0 < |z| < 1\}$ and having Taylor and Laurent expansions $f(z) = z + a_2z^2 + \cdots$ and $f(z) = 1/z + a_0 + a_1z + \cdots$, respectively, in D and E around the origin. We have proved the following analogue of Libera (Proc. Amer. Math. Soc. 16(1965), 755–758) for $f \in B$: *Theorem* I. If $f \in B$ and meromorphically starlike (or convex) then $F(z) = z^{-2}\int_0^z tf(t)dt \in B$ and meromorphically starlike (or convex). Similar results have been found for α-meromorphically starlike functions. (Received February 11, 1974.)

*74T-B100. CARL M. PEARCY and NORBERTO SALINAS, University of Michigan, Ann Arbor, Michigan 48104. *Finite dimensional representations of separable C^*-algebras.*

In our announcement of the same title to appear in Bull. Amer. Math. Soc., we introduce and characterize the sets $R^n(T)$ and $R_e^n(T)$ for a given operator T acting on a Hilbert space $\mathcal{H}$ of dimension $\aleph_0$. Let $C^*(T)$ $[C_e^*(T)]$ denote the C^*-algebra generated by 1 and T [1 and $\pi(T)$, where π is the Calkin map]. Then $R^n(T)$ $[R_e^n(T)]$ is the set of all $n \times n$ matrices L for which there exists an n-dimensional *-representation ϕ of $C^*(T)$ $[C_e^*(T)]$ s.t. $\phi(T) = L[\phi(\pi(T)) = L]$. *Additional results*: *Theorem* 1. Let T be an n-normal operator in $\mathcal{L}(\mathcal{H})$. Then $R_e^n(T)$ (and hence $R^n(T)$) is nonempty, and there exists a diagonable n-normal operator S and a compact operator K s.t. $T = S + K$ and $R^n(S) \subset R_e^n(T)$. Moreover, for every $\epsilon > 0$, there exist in $\mathcal{L}(\mathcal{H})$ a diagonable n-normal operator S_ϵ and a compact operator K_ϵ s.t. $T = S_\epsilon + K_\epsilon$, $\|K_\epsilon\| < \epsilon$, and $R^n(S_\epsilon) \subset R^n(T)$. *Theorem* 2. If S and T are two n-normal operators in $\mathcal{L}(\mathcal{H})$ s.t. $R_e^n(S) = R_e^n(T)$, then S is unitarily equivalent to a compact perturbation of T. Furthermore, if $R^n(S) = R_e^n(S)$ and $R^n(T) = R_e^n(T)$, then the compact perturbation can be taken to have arbitrarily small norm. *Theorem* 3. Let T be an operator $\mathcal{L}(\mathcal{H})$ s.t. $R_e^n(T)$ is nonempty. Then for every n-normal operator S in $\mathcal{L}(\mathcal{H})$ s.t. $R_e^n(S) \subset R_e^n(T)$, the operator $S \oplus T$ is unitarily equivalent to a compact perturbation $T + K$ of T. Moreover, if $R^n(S) \subset R_e^n(T)$, then K can be taken to have arbitrarily small norm. (Received February 12, 1974.)

*74T-B101. RICHARD D. MAULDIN, University of Florida, Gainesville, Florida 32601. *On generalized rectangles and countably generated families.* Preliminary report.

Sierpinski posed the following problem (S): For every family F of real-valued functions defined on a set X with card$(F) \leq 2^{\aleph_0}$, does a countable family G of real valued functions exist on X such that each function in F is the pointwise limit of a sequence from G? Let R be the family of all "rectangles" in the plane E^2: $R = \{A \times B: A, B \subseteq E\}$. Ulam and others have asked (U): Is every subset of the plane in the Borel field generated by R? *Theorem* 1. The answer to (S) is yes iff (*) every subset of E^2 is an $R_{\sigma\delta}$ set. There are a number of generalizations and applications. Some examples: *Theorem* 2. If (*) holds, then there is a rarefied subset of E of cardinality $2^{\aleph_0}$ with Borel order ω_1. *Theorem* 3. If (*) holds and $2^{\aleph_0} = 2^{\aleph_1}$, then there is a normal separable nonmetrizable Moore space. (Received February 12, 1974.)

74T-B102. VASANT A. UBHAYA, Washington University, St. Louis, Missouri 63130. *Infima of integrals involving mollifier functions by a mathematical programming approach.* Preliminary report.

The following results are in addition to those announced by the author in Abstract 73T-B319, these Notices 20(1973), A-638, for the problem defined there. That problem on the underlying infinite dimensional

function space is shown to be equivalent to a nonlinear programming problem involving minimization of a real valued function of $(k-1)$ variables. Let $0 = y_0 < y_1 < \cdots < y_k = 1$ and let $\tau_j(y) = (-1)^j(\pi_{i=0,i\neq j}^{k}|y_i - y_j|) = (\pi_{i=0,i\neq j}^{k}(y_i - y_j))$, $j = 0, 1, \cdots, k$, where $y = (y_1, y_2, \cdots, y_{k-1})$. Thus $\tau_j(y)$ is $(-1)^j$ times the product of the distances of y_j from y_i for $i \neq j$. Define $f_k(y) = \sum_{j=0}^{[(k-1)/2]}(\tau_{k-1-2j}(y))^{-1}$ where $[\alpha]$ denotes the largest integer in $[0, \alpha]$. *Theorem.* There exists a unique minimizer y^* of $f_k(y)$ with $0 = y_0^* < y_1^* < \cdots < y_k^* = 1$. This minimizer satisfies $\partial f_k(y^*)/\partial y_j = 0$, $j = 1, 2, \cdots, k-1$, and $\inf_{\phi\in\Phi}\int_0^1 |\phi^{(k)}(x)|\,dx = 2k!f_k(y^*)$, for all $k = 1, 2, \cdots$. A simple upper bound is given by the following *Theorem.* $\inf_{\phi\in\Phi}\int|\phi^{(k)}(x)|\,dx \leq (2k)^k$, for all $k = 1, 2, \cdots$.
(Received February 14, 1974.)

*74T-B103. DOMINGO A. HERRERO, Universidad Nacional de Río IV, Río IV, Córdoba, Argentina. *Solution to the seventh problem of P. R. Halmos.* Preliminary report.

In "Normal limits of nilpotent operators" (Indiana Univ. Math. J., to appear) the author proved that a normal operator N (on a complex separable Hilbert space $\mathcal{H}$) can be uniformly approximated by nilpotent operators iff its spectrum is a connected set containing the origin. An easy extension of the results and methods of that paper yields the following: Exactly the same result holds for every operator N which can be written as the direct sum of a spectral operator and an operator with totally disconnected spectrum. In particular, this affirmatively answers Problem 7 of Halmos (Bull. Amer. Math. Soc. 76(1970), 915): Every quasinilpotent operator is the norm limit of nilpotent ones. As a byproduct, the following result is obtained: Every (bounded linear) operator on $\mathcal{H}$ is the norm limit of sums of two nilpotent ones. (Received February 13, 1974.)

*74T-B104. ANTHONY G. O'FARRELL, University of California, Los Angeles, California 90024. *A generalised Walsh-Lebesgue theorem.* Preliminary report.

Let X be the boundary of a compact set which does not separate the plane, $\mathbf{C}$. Let Φ and Ψ be homeomorphisms of $\mathbf{C}$ to $\mathbf{C}$ with opposite orientations. Then every continuous complex-valued function on X is the uniform limit on X of sums $p(\Phi) + q(\Psi)$, where p and q are analytic polynomials. (Received February 14, 1974.)

74T-B105. MOSTAFA A. ABDELKADER, 25 Sh. Champollion, Alexandria, Egypt. *Real identities connecting circular and hyperbolic functions.*

The only known identities connecting the circular and hyperbolic functions involve $\sqrt{-1}$ explicitly. We give about thirty identities interrelating these functions, in which only real quantities appear. For instance, a special case of one of these identities (which are given in more general form) yields the following relation, expressing a definite geometrical connection between the graphs of the tangent and the hyperbolic tangent (or the exponential function): For all real x, we have $\tanh 4x/\tanh x = \tan 4y/\tan y$, where $y = \arctan\sqrt{u}$, $0 \leq y < \pi/2$, $u = (5+v^2)/(1+v^2)$, and $v = \tanh x$. Using well-known identities, this identity can be immediately verified.
(Received February 14, 1974.)

Applied Mathematics

*74T-C14. H. MASSAM and SANJO ZLOBEC, McGill University, Montreal, Quebec, Canada. *Various definitions of the derivative in mathematical programming.*

Seven derivatives in mathematical programming in locally convex topological vector spaces are introduced. They have been known in mathematical sciences but never used in mathematical programming. The weakest is the compact derivative of Gil de Lamadrid and Sova. That used by Neustadt in optimization theory is stronger than the compact derivative and is equivalent to that introduced by Michal and Bastiani. *Results.* The optimality conditions of both Lagrange-Kuhn-Tucker and Caratheodory-John types hold for compactly differentiable functions. For finite dimensional spaces, these seven derivatives are equivalent to the Frechet derivative. (Received January 9, 1974.)

74T-C15. WILLIAM C. TROY, State University of New York, Amherst, New York 14226. *Oscillatory phenomena in nerve conduction equations.* Preliminary report.

A widely accepted model of nerve conduction in the squid axon is the system of nonlinear differential equations of Hodgkin and Huxley, which are simplified by the Fitzhugh-Nagumo equations. Under space clamp conditions both systems are reduced to nonlinear ODE of orders four and two, respectively. Under appropriate assumptions on the functions and parameters occurring in the resulting fourth order Hodgkin-Huxley equations there occurs a bifurcation of periodic solutions from a steady state as the 'current' parameter I passes through a critical value. Furthermore, a range of parameters is found in which there exists a large recurrent solution which corresponds to an infinite sequence of action potentials. In the second order Fitzhugh-Nagumo equations we demonstrate the bifurcation, direction and stability of a family of periodic orbits as I passes through a critical value. This family grows to a large periodic solution as I decreases, then shrinks, collapsing onto another bifurcation point as I passes through a second critical value. Using the CDC6400 computer, the bifurcating periodic solutions of both equations are found. It appears that the small bifurcating periodic solutions have not been observed before either numerically or experimentally. (Received January 17, 1974.)

74T-C16. ANDY S. KYDES, State University of New York, Stony Brook, New York 11790. *Pseudo-Chebyschev solution of $n+1$ inconsistent linear equations in n unknowns.*

A solution to the linear inconsistent system $Ax = b$, where A is an $(n+1) \times n$, real-valued matrix of rank n, is defined as any n vector which renders n components of the residual vector equal to zero. An algorithm for computing the solution with minimum nonzero residual component, in the Chebyschev sense, is given and shown to be unique under the assumption that the rows of A satisfy the Haar condition. The cost of the algorithm is shown to be on the order of $n^3 + 2n^2$ operations (Markowitz criterion). (Received January 23, 1974.) (Author introduced by Professor R. P. Tewarson.)

74T-C17. WITHDRAWN

74T-C18. DANIEL S. YEUNG, Case Western Reserve University, Cleveland, Ohio 44106. *Synthesis of time optimal control.* Preliminary report.

We consider the time optimal problem of reaching the origin within the control systems in n-space: $\dot{x}(t) = Ax(t) + Bu(t)$, where A and B are constant $n \times n$ matrices. In a generic case and for small t, the reachable set $R(t)$ consists of at most $2n^{n-1}$ disjoint connected open sets, and of a finite number of analytic manifolds of dimension less than n (the switching loci). Parametric descriptions of these, making it possible to construct an optimal feedback control, will be given in a subsequent paper, and also partial results for more general cases. (Received February 14, 1974.)

Geometry

74T-D11. ERWIN O. KREYSZIG, University of Windsor, Windsor, Ontario, Canada. *Equiareal mapping of surfaces.*

Let Ω be a domain in the xy-plane, and $u, v \in C'(\Omega)$ solutions of grad $u = A$ grad v, $A = (a_{jk}) \in C(\Omega)$. Let S_u and S_v be the surfaces in R_3 represented by $r(x, y) = (x, y, u(x, y))$ and $\bar{r}(\bar{x}, \bar{y}) = (\bar{x}, \bar{y}, v(\bar{x}, \bar{y}))$, $(x, y) \in \Omega$. Then the mapping $S_v \to S_u$ given by $(\bar{x}, \bar{y}) \mapsto (x, y)$ is equiareal iff A is an orthogonal matrix. If $A \in C'(\Omega)$, det $A = +1$, and one expresses the elements of A in terms of $\cos \alpha(x, y)$ and $\sin \alpha(x, y)$; the second order PDE for v, obtained as an integrability condition, includes, as a particular case, an equation which can be applied in connection with an approximation of the Poisson adiabate in compressible fluid flow. (Received January 7, 1974.)

*74T-D12. CHANDAN S. VORA, Jundi Shapur University, Ahwaz, Iran. *Counterexamples on the extension of Lipschitz function with respect to three Hilbert norms and two Lipschitz conditions.* Preliminary report.

The author introduced the concept of comparability of two pairs of norms (Rend. Sem. Mat. Univ. Padova, to appear). For two norms, the author showed in general that if the pairs of norms are not comparable then there is no Lipschitz extension preserving the two Lipschitz conditions. In this paper for three norms, the author shows in general that if the corresponding pairs of norms are not comparable then there is no Lipschitz extension preserving the two Lipschitz conditions. In general, when the corresponding pairs of norms are comparable, the Lipschitz extension preserving the two Lipschitz conditions always exist. The papers in this direction were prompted by the questions arising out of the theory of networks being formulated by Professor Darbo at Genova. The counterexamples for the case of four norms have not yet been obtained by the author for all the subcases arising in it and hence it is still open. (Received February 6, 1974.)

74T-D13. ALCIBIADES RIGAS, University of Chicago, Chicago, Illinois 60637. *Riemannian metrics of nonnegative curvature on stable vector bundles over spheres.* Preliminary report.

Generators are realized for each nonzero stable homotopy group of the infinite orthogonal group and its classifying space as totally geodesic submanifolds of some $O(m)$, respectively $G_{m,2m}$, with their standard Riemannian metric of nonnegative curvature. These generators are isometric to Euclidean spheres of appropriate radius. It follows that all stable classes of vector bundles over a sphere have a representative whose total space admits a Riemannian metric of nonnegative curvature. (Received February 11, 1974.) (Author introduced by Professor Richard K. Lashof.)

Logic and Foundations

74T-E35. KONRAD SUPRUNOWICZ, Utah State University, Logan, Utah 84322. *A connection between facts and theories.* Preliminary report.

This is a generalization of the problem described in Abstract 711-02-34, these *Notices* 21(1974), A-27, to any empirical data. *Thesis.* A short formulation of the principle of RATIONAL INDUCTIVE generalization. On the basis of reliable observations, a rational guess, hypothesis, theory is one that is in agreement with observed events together with all rates of changes of observed characteristics of events. It is not asserted that predictions based on the principle will always be correct; it is asserted that they will be correct more often than those arrived by any other method. Noticing that all familiar objects change in time, all objects are included under the term "event". "Severe tests" of theories, as the concept is used by Karl R. Popper's deductivest school are discussed within new context. (Received October 31, 1973.)

74T-E36. DANIEL SARACINO, Yale University, New Haven, Connecticut 06520. *On existentially complete nilpotent groups.* Preliminary report.

It is known that the theory of abelian groups has a model companion but that for any $n \geq 2$ the theory of groups solvable of length $\leq n$ has no model companion. It is natural to ask what happens for nilpotent groups. *Theorem* 1. For any $n \geq 2$, the theory of groups nilpotent of class $\leq n$ has no model companion, i.e. the class of existentially complete models is not elementary in the wider sense. Our method of proof also applies to some more special classes of nilpotent groups; for example: *Theorem* 2. For any $n \geq 2$ the theory of torsion-free groups nilpotent of class $\leq n$ has no model companion. (Received December 26, 1973.) (Author introduced by Abraham Robinson.)

74T-E37. ANGUS MACINTYRE and DANIEL SARACINO, Yale University, New Haven, Connecticut 06520. *On existentially complete nilpotent Lie algebras.* Preliminary report.

Theorem. For any $n \geq 2$ the theory of rational Lie algebras nilpotent of class $\leq n$ has no model companion. The proof uses the results of the preceding abstract and category equivalences between torsion-free divisible nilpotent groups and nilpotent rational Lie algebras. (Received December 27, 1973.)

74T-E38. ANGUS MACINTYRE, Yale University, New Haven, Connecticut 06520. *On existentially complete Lie algebras.* Preliminary report.

Theorem 1. Let K be an infinite field. The theory of Lie algebras over K has no model companion. *Theorem* 2. Let K be an infinite field. The theory of Lie algebras L over K with $L^3 = 0$ has no model companion. With $K = Q$, this gives an alternative proof of the nonexistence of a model companion for torsion-free nilpotent groups of class 2 (cf. Abstract 74T-E36 above). (Received December 27, 1973.)

*74T-E39. ALBERT A. MULLIN, 9213 Kristin Lane, Fairfax, Virginia 22030. *On the geometry of recursively enumerable sets and degrees.* Preliminary report.

We investigate the interface between Minkowski's geometry and Post's mathematical logic. *Prelemma.* Let $V(K)$ be the n-dimensional Lebesgue measure (not necessarily finite) of a measurable, origin-symmetrical convex set K in E^n. (a) If $V(K) > 2^n$, then K contains at least one nonempty recursive set of lattice points disjoint with the origin of E^n. (b) If $V(K) < 2 \cdot \zeta(n)$ (ζ- Riemann's zeta-function) then K contains no nonempty recursively enumerable set of lattice points disjoint with the origin. *Lemmas.* 1. If $V(K) > 2^n$, $n \geq 2$, then K contains at least one nonempty recursive set of *visible* lattice points (Abstract 73T-A263, these *Notices* 20(1973), A-629). 2. Let B contain an origin-centered n-dimensional ball of radius $r \geq \frac{1}{2}$. $\exists$ a recursive function $f(\cdot)$ s.t. if $V(B) \geq f(n) > 2^n$, $n \geq 2$, then B contains at least one nonempty recursive set of highly visible lattice points. *Scholiums.* 1. Lemma 2 fails for some r, $0 < r < \frac{1}{2}$. 2. Lattice points are basic for relating Minkowski's theory of convex bodies and Post's recursive function theory. (Received January 14, 1974.)

74T-E40. ROBERT I. SOARE, University of Illinois, Chicago, Illinois 60680. *Degrees and structure of speedable sets.*

Let $\{\phi_i: i \in N\}$ be an acceptable numbering of the partial recursive functions and $\{\Phi_i: i \in N\}$ step-counting functions which constitute a complexity measure in the sense of Blum. Let W_i = domain ϕ_i. An r.e. set A is *speedable* if $\forall$ i s.t. $W_i = A$ and $\forall$ recursive functions $r, \exists$ j s.t. $W_j = A$ and $\Phi_i(x) > r(x, \Phi_j(x))$ for infinitely many x. Blum called A *effectively speedable* if j can be found effectively from i and r, and proved that A is effectively speedable iff A is subcreative. A set A is *low* if $\{i: W_i^A \neq \emptyset\} \leq_T \emptyset'$ and *semilow* if $\{i: W_i \cap A \neq \emptyset\} \leq_T \emptyset'$. *Theorem.* An r.e. set A is speedable iff $\bar{A}$ is not semilow. *Corollary* 1. Every r.e. degree contains a nonspeedable r.e. set. *Corollary* 2. An r.e. degree $\underset{\sim}{a}$ contains a speedable set if and only if $\underset{\sim}{a}' > \underset{\sim}{0}'$ (and hence there are nonzero r.e. degrees containing no speedable sets). *Corollary* 3. If an r.e. set A is nonspeedable and $\bar{A}$ is infinite then $\mathcal{E}_{\bar{A}} \simeq_{\mathrm{eff}} \mathcal{E}$, where $\mathcal{E}_{\bar{A}}$ denotes the lattice of sets $\{W_i \cap \bar{A}: i \in N\}$ under inclusion. *Corollary* 4. An r.e. set A must be speedable if A is either atomless, r-maximal, or hyperhypersimple (or even finitely strongly hypersimple),but simple, hypersimple or even dense simple sets may be nonspeedable. (Received January 14, 1974.)

*74T-E41. GIORGIO M. GERMANO, Laboratorio di Cibernetica, 80072 Arco Felice, Italy. *Incompleteness of diophantine arithmetic.*

Let T be a recursively enumerable arithmetical theory (Abstract 70T-E56, these *Notices* 17(1970), 833) in which addition and multiplication are definable. T is diophantine and there is a numbering of T with a polynomial diagonal function d. Therefore there is a formula Θ and a term δ s.t. $\Phi \in T$ iff $\Theta(D_{g(\Phi)}) \in T$ and $\delta(D_n) = D_{d(n)} \in T$. The proof is then concluded as in the above cited abstract. (Received January 15, 1974.)

*74T-E42. WARREN D. GOLDFARB, Department of Philosophy, Harvard University, Cambridge, Massachusetts 02138. *Decision problems for quantificational formulas with few atomic subformulas.*

Theorem. The following classes of prenex formulas are reduction classes: formulas with prefixes $\forall\exists\forall\cdots\forall$ and matrices $(A_1 \vee A_2) \wedge (\neg A_3 \vee \neg A_4)$; formulas with prefixes $\forall\cdots\forall\exists$ and matrices $(A_1 \vee A_2 \vee A_3) \wedge (\neg A_4 \vee \neg A_5)$, where in each case the A_i are atomic formulas all containing the same predicate letter. The theorem settles several decision problems left open by Lewis and Goldfarb [J. Symbolic Logic 38(1973), 478]. It is obtained by improving the two techniques used in that paper: the encoding of two-register machines by quantificational formulas with five atomic subformulas, and the further logical reductions of the class of formulas resulting from this encoding. (Received January 24, 1974.) (Author introduced by Burton Dreben.)

74T-E43. PHILIP OLIN, York University, Downsview, Ontario M3J 1P3, Canada. *Free products and elementary equivalence.* II. Preliminary report.

For earlier work see Abstract 73T-E22, these *Notices* 20(1973), A-284. If C is a variety of algebras having one binary function and $S(C)$, the set of defining equations, consists of the associative and commutative laws then the C-free product preserves $\equiv$ and $\prec$. This applies also to the Abelian free product of Abelian groups. Suppose C is the class of distributive lattices. Then (1) if $L \in C$, L finite, $M_1, M_2 \in C$, $M_1 \equiv (\prec) M_2$, then $L * M_1 \equiv (\prec) L * M_2$. If C is a variety of algebras having two binary functions and $S(C)$ consists of the two associative laws and the two commutative laws then both parts of (1) are false. (Received January 28, 1974.)

74T-E44. WILLIAMS K. FORREST, Simon Fraser University, Burnaby 2, British Columbia, Canada. *An n-cardinal theorem.* Preliminary report.

Suppose that T is a totally transcendental theory and $\psi_0(v_0), \cdots, \psi_n(v_0)$ are elements of $L_1(T)$, $n \geq 1$. If there is a model A of T in which $|\psi_i(A)| \geq \omega$ and $|\psi_i(A)| < |\psi_{i+1}(A)|$ for each $i < \eta$, then for each sequence $\lambda_0 <, \cdots, < \lambda_n$ of infinite cardinals there is a model B of T in which $|\psi_i(B)| = \lambda_i$ for each $i \leq n$. (Received January 30, 1974.) (Author introduced by Dr. Alistar H. Lachlan.)

74T-E45. A. KANAMORI, Cambridge University, Cambridge, England. *A characterization of nonregular ultrafilters.*

If U is an ultrafilter over a cardinal γ and $f: \gamma \to \gamma$, call f a *least function* (mod U) if: (a) if $\alpha < \gamma$, $\{\beta | f(\beta) \geq \alpha\} \in U$, and (b) if $g: \gamma \to \gamma$ and $\{\beta | g(\beta) < f(\beta)\} \in U$, then for some $\alpha < \gamma$, $\{\beta | g(\beta) < \alpha\} \in U$. *Theorem.* If U is a uniform ultrafilter over γ^+, U is not (γ, γ^+)-regular iff there is a least function f (mod U) and $\{\beta | \operatorname{co} f(f(\beta)) = \gamma\} \in U$. *Corollary.* If γ is singular, every uniform ultrafilter over γ^+ is (γ, γ^+)-regular. (Received February 4, 1974.) (Author introduced by A. R. D. Mathias.)

74T-E46. PAUL E. COHEN, Institute for Advanced Study, Princeton, New Jersey 08540. *Preservation of chain conditions in sums.* Preliminary report.

If κ is a weakly compact cardinal and $\lambda < \kappa$ then κ c.c. is preserved under λ-sums of posets (or topological spaces). (Received February 6, 1974.)

74T-E47. ALEXANDER ABIAN, Iowa State University, Ames, Iowa 50010. *Nonexistence of partially ordered sets with denumerably many dense subsets.*

Let $(P, \leq)$ be a poset (i.e., a partially ordered set). A subset D of P is called a dense subset of P iff for every $x \in P$ $\exists$ $y \in D$ s.t. $y \leq x$. *Theorem.* The set of all dense subsets of a poset is finite or of the power greater than or equal to continuum. *Lemma.* A subset W of a nonempty poset P has a nonempty intersection with every dense subset of P iff $I(p) \subseteq W$ for some $p \in P$ where $I(p) = \{x | x \in P$ and $x \leq p\}$. *Corollary.* A subset G of a poset P is generic iff (i) every $x, y \in G$ has a common lower bound in G, (ii) for every $g \in G$, $\{x | x \in P$ and $g \leq x\} \subseteq G$, (iii) for some $p \in P$, $I(p) \subseteq G$. (Received February 14, 1974.)

74T-E48. WITHDRAWN

Statistics and Probability

74T-F6. THOMAS H. SAVITS, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. *Limiting behavior of an age-dependent model.* Preliminary report.

We continue the study of the age-dependent model of Abstract 73T-F21, these *Notices* 20(1973), A-642. Again we suppose that $1 < m < \infty$ and G is continuous. *Theorem* 1. $E_0(Z_t) \sim ce^{\lambda t}$ as $t \to \infty$, where Z_t is the number of particles alive at time t, λ is the Malthusian parameter, and c is some determined positive constant. Under a finite second moment assumption, we prove *Theorem* 2. $Z_t/ce^{\lambda t}$ converges in mean square and w.p. 1 to a nontrivial random variable. Necessary and sufficient conditions for the corresponding martingale $W_t = \check{\phi}(X_t)e^{-\lambda t}$ to converge w.p. 1 to a nontrivial random variable W, $E_0(W) = 1$, are also discussed. In this case we have *Theorem* 3. W has a continuous density except for a jump of size q (the extinction probability) at 0. Let $Z(x, t)$ denote the number of objects alive at time t and of age $\leq x$. *Theorem* 4. If $\phi \geq \alpha > 0$ on $[0, x]$, then $Z(x, t)/ce^{\lambda t} \xrightarrow{\mathfrak{D}} A(x)W$, where $A(x)$ is the limiting age distribution. (Received January 9, 1974.)

74T-F7. RICHARD T. DURRETT, Department of Statistics, Stanford University, Stanford, California 94305 and S. G. GHURYE, University of Alberta, Edmonton, Alberta, Canada. *Waiting times without memory.* Preliminary report.

In probability theory, usually two distributions appear as distributions of waiting times without memory: the exponential on the positive reals and the geometric on a lattice of the positive reals. These are known to be the only distributions of waiting times without memory on their respective domains, which leads to the question: What is the class of all possible domains for waiting times without memory and the corresponding distributions? With the notation, $P\{X > t\} = f(t)$ and A is a subset of the positive reals such that $P\{X \in A\} = 1$, the following two properties are defined: (1) X is a waiting time with partial lack of memory (prospective) iff s, $t \in A$ implies $f(s+t) = f(s)f(t)$; (2) X is a waiting time with partial lack of memory (retrospective) iff s, $t \in A$ and $s < t$ imply $f(t) = f(s)f(t-s)$. The possible forms of f and A under each property are investigated. It is shown that the two usual cases mentioned above are the only solutions having both properties (1) and (2). (Received February 11, 1974.)

*74T-F8. KENNETH S. MILLER, Riverside Research Institute, New York, New York 10023. *Some multivariate complex distributions.*

Let $\{z_n(t+h), z_n(t)\}$, $1 \leq n \leq N$, $h \neq 0$, be vector observations on a mean zero stationary complex Gaussian process with covariance function R, and let $\hat{R}(h) = N^{-1}\Sigma_{n=1}^{N} z_n(t+h)\bar{z}_n(t)$ be an unbiased estimate of $R(h)$. Then the joint four-dimensional density function J of $u = N\operatorname{Re}\hat{R}(h)$, $v = N\operatorname{Im}\hat{R}(h)$, $x = \Sigma_{n=1}^{N}|z_n(t+h)|^2$, $y = \Sigma_{n=1}^{N}|z_n(t)|^2$ is found in closed form. Some practical problems where such a distribution may be exploited are briefly outlined. Other tri-, bi- and uni-variate frequency functions are based on u, v, x, y are calculated. All formulas are expressed in closed form. In particular, $J(u, v, x, y)$ and the joint probability density function of u, v and $\frac{1}{2}(x+y)$ are elementary functions, while the bivariate frequency functions of u, v and x, y each involve modified Bessel functions. (Received February 14, 1974.)

Topology

*74T-G50. R. VASUDEVAN and C. K. GOEL, Institute of Advanced Studies, Meerut University, Meerut 250001, India. *Connectivity properties of bitopological hyperspaces.* Preliminary report.

Given a bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$, the natural bitopological hyperspaces $(2^X, 2^{\mathcal{T}_1}, 2^{\mathcal{T}_2})$ and $(C(X), 2^{\mathcal{T}_1}, 2^{\mathcal{T}_2})$, where 2^X denotes the collection of all nonempty $\mathcal{T}_1$- or $\mathcal{T}_2$-closed sets including singletons, $C(X)$ is the subspace of 2^X consisting of all bicompact sets, $2^{\mathcal{T}_2}$, $i = 1, 2$, is the Vietoris or finite topology [Kuratowski, "Topology". I, Academic Press, New York, 1966, p. 160]. We investigate what connectivity properties of a bitopological space $(X, \mathcal{T}_1, \mathcal{T}_2)$ are carried over to bitopological hyperspaces. The following connectivity properties are studied: pairwise connectedness, pairwise total disconnectedness, pairwise zero-dimensional and pairwise local connectedness. *Result.* Pairwise connectedness of $(X, \mathcal{T}_1, \mathcal{T}_2)$ is equivalent to that of $(2^X, 2^{\mathcal{T}_1}, 2^{\mathcal{T}_2})$, whereas pairwise total disconnectedness, pairwise zero-dimensional, and pairwise local connectedness are equivalent to those of $(C(X), 2^{\mathcal{T}_1}, 2^{\mathcal{T}_2})$. (Received October 23, 1973.) (Authors introduced by Professor J. N. Kapur.)

*74T-G51. RAYMOND F. GITTINGS, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. *Products of generalized metric spaces.* Preliminary report.

For convenience, we assume all spaces are regular T_1-spaces. A space X is called *weakly-θk* if a set $F \subset X$ is closed whenever $F \cap P$ is finite for every closed θ-refinable space P. Every θ-refinable space and every k-space is weakly-θk. *Theorem* 1. The countable product of weakly-θk M (M^*, wM or quasi-complete)-spaces is an M (M^*, wM or quasi-complete)-space. *Corollary* 1. The condition of weakly-θk in Theorem 1 may be replaced by any of the following properties: (a) θ-refinable, (b) k-space, (c) pointwise countable type, (d) every point in the space is a G_δ, (e) sequential. Theorem 1 and Corollary 1 (with the exception of (d)) also hold for Σ and $\Sigma^{\#}$-spaces. *Theorem* 2. If X

is a Q-space and Y is a weakly-θk Q-space, then $X \times Y$ is a Q-space for any of the following properties Q: M, M^*, wM, quasi-complete, Σ or $\Sigma^{\#}$. Results are also obtained for $w\Lambda$-spaces and β-spaces somewhat analogous to the above. An example is provided which (except for the Σ and $\Sigma^{\#}$-cases) shows that weakly-θk is necessary in Theorems 1 and 2. (Received October 31, 1973.) (Author introduced by Professor George M. Rosenstein, Jr.)

*74T-G52. R. VASUDEVAN, Institute of Advanced Studies, Meerut University, Meerut, India. *A note on weaker forms of compactness in hyperspaces.* Preliminary report.

Let 2^X be the space of all nonempty closed subsets of a topological space $(X, \mathfrak{T})$ with the finite topology $2^{\mathfrak{T}}$. It is shown that if $\mathcal{P}$ Hausdorff, Urysohn, regular, almost regular, E_1 then a T_1-space $(X, \mathfrak{T})$ is minimal $\mathcal{P}$ whenever $(2^X, 2^{\mathfrak{T}})$ is minimal $\mathcal{P}$ and $(X, \mathfrak{T})$ is $\mathcal{P}$-closed whenever $(2^X, 2^{\mathfrak{T}})$ is $\mathcal{P}$-closed. (Received December 6, 1973.) (Author introduced by Professor J. N. Kapur.)

*74T-G53. ROBERT A. HERRMANN, U. S. Naval Academy, Annapolis, Maryland 21402. *More nonstandard characteristics for compact-like spaces.* Preliminary report.

We use nonstandard topology. *Definition.* For each $p \in X$, $R(p) = \{$all regular-open sets $X | p \in X\}$. For each $p \in X$, define $r(p) = \bigcap\{\hat{R} | R \in R(p)\}$ and call $r(p)$ an r-monad. *Theorems.* 1. Let F be a filter on X. Then $p \in X$ is a δ-adherent point of F iff Nuc $F \cap r(p) \neq \emptyset$. 2. Let F be an ultrafilter on X. Then F is δ-convergent to $p \in X$ iff Nuc $F \subset r(p)$. 3. A space X is nearly-compact iff $\hat{X} = \bigcup\{r(p) | p \in X\}$. 4. A space X is almost-regular iff (i) for each $p \in X$, $r(p) = \mu(\overline{p})$; (ii) for each $p \in X$ and $q \in \hat{X}$ s.t. $q \notin r(p)$ $\exists$ disjoint regular-open sets G, H s.t. $p \in G$ and $q \in \hat{H}$. Using the c-monad $\mu(\overline{p})$ (Abstract 72T-G152, these *Notices* 19(1972), A-719), $\mu(p) \subset r(p) \subset \mu(\overline{p})$. 5. A space is T_2 iff $r(p) \cap r(q) = \emptyset$ for distinct $p, q \in X$. 6. Let X and Y be semiregular spaces. Then $f: X \to Y$ continuously iff $g[r(p)] \subset r(f(p))$ for each $p \in X$. 7. A space X is semiregular iff $r(p) = \mu(p)$ for each $p \in X$. 8. If $f: X \to Y$ and $g[r(p)] \subset \mu(f(p))$ for each $p \in X$, then f is continuous. 9. Let $f: X \to Y$. Then f is almost-continuous at $p \in X$ iff $g[\mu(p)] \subset r(f(p))$. 10. A space X is nearly-compact T_2 iff X is H-closed, Urysohn iff $\{r(p) | p \in X\}$ partitions $\hat{X}$ iff $\{\mu(\overline{p}) | p \in X\}$ partitions $\hat{X}$. 11. If X is $H(i)$ and F is an open filter with a unique adherent point p, then Nuc $F \subset \mu(\overline{p})$. 12. If X is Urysohn, $p \in X$, F an open filter and Nuc $\subset \mu(\overline{p})$, then p is a unique adherent point for F. (Received December 31, 1973.)

*74T-G54. LOUIE M. MAHONY, State University of New York, Binghamton, New York 13901. *The topology of the finite general linear group.*

If $GL(n, F_q)$ is the finite general linear group with coefficients in the field with q elements, then $GL(n, F_q)$ acts freely on the connected sums $\#_p S^k \times S^k$ of p copies of $S^k \times S^k$ for some p where S^k is the k-dimensional sphere ($k \neq 2^j - 1$, $j \geq 2$, and otherwise arbitrary). See Abstract 73T-G9, these *Notices* 20(1973), A-26. $H^*(\#_p S^k \times S^k, Z)$ is a right $Z[GL(n, F_q)]$ module over the integral group ring $Z[GL(n, F_q)]$. *Theorem* 1. There exists a first quadrant spectral sequence with $E^2_{st} = H^s(\#_p S^k \times S^k, H^t(\#_p S^k \times S^k, Z))$ converging, for $s + t < k$, to a filtration of $H^{s+t}(GL(n, F_q), Z)$ (the filtration is derived from the spectral sequence). *Theorem* 2. (i) There exists a first quadrant spectral sequence $E^2_{st} = H^s(GL(n, F_q), H^t(\#_p S^k \times S^k, Z))$ converging for $s + t < k$ to a filtration of $H^{s+t}(GL(n, F_q), Z)$. (ii) Moreover, by determining the differentials, a long exact *infinite* sequence is constructed involving only $H^s(GL(n, F_q), Z)$ and $H^s(GL(n, F_q), H^t(\#_p S^k \times S^k, Z))$. (iii) The group $H^{2k+i}(GL(n, F_q), Z)$ is isomorphic to the group $H^{k+i-1}(GL(n, F_q), H^k(\#_p S^k \times S^k, Z))$ for $i = 0, 1$. Let S_n be the symmetric group which is naturally contained in $GL(n, F_q)$. By restricting $GL(n, F_q)$ to S_n, using results about the cohomology of S_n, intertwining the above spectral sequences, various specific results are derived about the complex K-theory of the classifying spaces BS_n and $BGL(n, F_q)$. (Received January 2, 1974.)

74T-G55. THOMAS JOHN O'MALLEY, Le Moyne College, Syracuse, New York 13104. *The geometry of S-subgroups of the hyperbolic groups.* Preliminary report.

An S-subgroup H of a locally compact group G is one for which given a neighborhood U of the identity in G and any $g \in G$, $\exists$ an integer $n > 0$ s.t. $g^n \in U \cdot H \cdot U$. A. Selberg observed that if H is a lattice in G (G/H has finite G-invariant measure), then H is an S-subgroup. When G is a solvable Lie group, S. P. Wang proved that if H is a closed S-subgroup, G/H is compact. *Theorem.* A discrete subgroup H of $G_1 = PSL(2, R)$ or $G_2 = PSL(2, C)$ is an S-subgroup iff it is a group of the first kind in the sense of Fuchsian and Kleinian groups. With the exception of one small detail, the proof extends to the identity component of $O(n, 1)$. Dirichlet's theorem in number theory is a key tool. So for discrete subgroups in G_1, lattice and S-subgroup mean the same thing for finitely generated subgroups. But in G_2 this is unknown; there might be finitely generated discrete subgroups of G_2 of the first kind having fundamental domain of infinite volume. (Received January 14, 1974.)

74T-G56. CHARLES E. AULL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. *Notes on accessibility, k-spaces and sequential spaces.* Preliminary report.

Continuing from Abstract 706-54-3, these *Notices* 20(1973), A-533, the following are proved. A space accessible by countably compact (sequential compact) sets is S_6 = sequences convergent uniquely and sequential closures are closed iff every countable compact (sequentially compact) set is closed. As a consequence an E_1-space is S_6 iff it is quasi-k and approximately accessible by countably compact sets. A space is S_5 (a sequential space with unique sequential limits) iff every sequential compact set is closed and it is sequentially k (replace compact by sequentially compact in the definition of k-spaces). In a sequentially k-space no highly divergent sequence has a side point and hence countably compact T_1 sequentially k-spaces are sequentially compact. In an S_5 hereditary normal space C^*-embedded sets are closed. (Received January 16, 1974.)

*74T-G57. D. R. J. CHILLINGWORTH, University of Southampton, Southampton SO9 5NH, United Kingdom and P. STEFAN, University College of North Wales, Bangor LL57 2UW, United Kingdom. *Integrability of singular Banach distributions.* Preliminary report.

Let M be a C^q Banach manifold ($2 \leq q \leq \omega$). A *distribution* on M is a family $B = (B_x : x \in M)$, where B_x is a topological direct summand of T_xM. A vectorfield X *respects* B if B is invariant under the differential of the X-flow; B is *homogeneous* if every vectorfield in B respects B. A regular distribution is one which defines a subbundle of TM. The standard Frobenius theorem asserts that a regular distribution is integrable iff it is involutive. This can now be extended to (differentiable) singular distributions with involutiveness replaced by homogeneity. Necessary and sufficient conditions for homogeneity are given in terms of Lie brackets. Besides containing the Frobenius theorem, these results imply, for example, that a (possibly singular) C^ω distribution is integrable iff it is involutive and 'locally everywhere defined'. This generalizes the well-known theorem of Nagano. (Received January 16, 1974.) (Authors introduced by Professor Ronald Brown.)

74T-G58. MICHAEL C. BIX, University of Chicago, Chicago, Illinois 60637. *Z_2-equivariant embeddings up to cobordism.* Preliminary report.

Let T be the involution on RP^n given in homogeneous coordinates by $T[x_0, x_1, \cdots, x_n] = [-x_0, x_1, \cdots, x_n]$, α be the class of (RP^n, T) in the cobordism ring of manifolds with unrestricted involutions, and let $1^r \oplus (-1)^s$ denote R^{r+s} with the involution $(x_1, \cdots, x_{r+s}) \to (x_1, \cdots, x_r, -x_{r+1}, \cdots, -x_{r+s})$. *Theorem.* Suppose RP^{n-1} can be embedded in R^j. Then (1) there is a representative of α which can be Z_2-equivariantly embedded in $1^j \oplus (-1)^n$; (2) no representative of α can be Z_2-equivariantly immersed in $1^r \oplus (-1)^{n-1}$ or $1^{n-1} \oplus (-1)^r$, for any r; (3) if no nonempty manifold in the unoriented cobordism class of RP^{n-1} can be embedded in R^{j-1}, then no representative of α can be Z_2-equivariantly embedded in $1^{j-1} \oplus (-1)^r$, for any r. (Received January 28, 1974.)

*74T-G59. RONALD BROWN and J. P. L. HARDY, University College of North Wales, Bangor LL57 2UW, United Kingdom. *Subgroups of free topological groups and free products of topological groups.*

The main theorem on free topological groups is *Theorem* 7. Let G be the Graev free topological group on a Hausdorff k_ω-space X, and let H be a closed subgroup of G admitting a continuous Schreier transversal. Then H is a Graev free topological subgroup of G. Here a continuous Schreier transversal for H

is a continuous section $s: G/H \to G$ of the projection $p: G \to G/H$ s.t. $\{s(\bar{a}): \bar{a} \in G/H\}$ is a Schreier transversal in the usual sense. The proof models that given by Higgins, "Categories and groupoids", Van Nostrand, Princeton, N. J., for the abstract case – we use topological groupoids. The theorem implies, for example, that open subgroups of Graev free topological groups on Hausdorff k_ω-spaces are free topological, and there is a similar result for free Abelian topological groups. A topological version of the Kurosch theorem for open subgroups of free topological products follows from similar arguments. (Received January 28, 1974.)

*74T-G60. PHILLIP L. ZENOR, Auburn University, Auburn, Alabama 36830. *Hereditary separability and the hereditary Lindelöf property in product spaces.*

Theorem 1. Suppose that $X \times Y$ is hereditarily $\mathfrak{m}$-compact. Then either X is hereditarily $\mathfrak{m}$-separable or Y is hereditarily $\mathfrak{m}$-Lindelöf. *Theorem* 2 (G.C.H.). Suppose that $X \times Y$ is $\mathfrak{m}$-separable and hereditarily normal. Then either X is hereditarily $\mathfrak{m}$-separable or Y is hereditarily $\mathfrak{m}$-Lindelöf. *Theorem* 3. Suppose that $\{X_a | a \in A\}$ is a collection of spaces such that (i) $|A| \leq \mathfrak{m}$, and (ii) if B is a finite subset of A, then $\pi\{X_a | a \in B\}$ is hereditarily $\mathfrak{m}$-separable (hereditarily $\mathfrak{m}$-Lindelöf). Then $\pi\{X_a | a \in A\}$ is hereditarily $\mathfrak{m}$-separable (hereditarily $\mathfrak{m}$-Lindelöf). There are T_2-spaces S and L such that, for each n, S^n is hereditarily separable and L^n is hereditarily Lindelöf but S is not hereditarily Lindelöf and L is not hereditarily separable. (Received February 7, 1974.)

*74T-G61. GEORGE MICHAEL REED, Ohio University, Athens, Ohio 45101 and PHILLIP L. ZENOR, Auburn University, Auburn, Alabama 36830. *A metrization theorem for normal Moore spaces.*

Theorem. Every normal, locally connected, locally compact Moore space is metrizable. (Received February 7, 1974.)

*74T-G62. V. KANNAN, Madurai University, Madurai 21, India. *Two metric topologies with only two regular open sets in common.*

Here we give two metric topologies on a countable set such that if a set is open in each of them, then it is dense in each of them. We observe that there is an infinite family of topologies on a countable set such that any two of them have the above property. This answers a question of D. Nix (Bull. Amer. Math. Soc. 76(1970), 976). (Received February 14, 1974.) (Author introduced by Professor Stanley P. Franklin.)

Symposium on Some Mathematical Questions in Biology
San Francisco, California, February 25—26, 1974

*Bio 74-6. JOEL B. SWARTZ, Environmental Health Sciences, University of California, Berkeley, California 94720 and HANS BREMERMANN, University of California, Berkeley, California 94720. *Some comments on the parameter estimation and verification problems in biological systems.* Preliminary report.

Some problems crucial to the fitting of biological models to data are discussed. An important part of the task of constructing models in biology is to check their predictions against experimental data, and this requires the identification of the parameters of the model. First the parameter estimation problem is considered. Two new techniques, one based on the optimization technique of Bremermann, are presented, and the results of extensive tests with these are given. The use of the Rosenbrock and Storey technique for the evaluation of parameter estimates is discussed, and some results of its use are presented. Also methods for the verification of models, and some trials with these methods are presented and discussed. (Received February 7, 1974.)

*Bio 74-7. H. L. RESNIKOFF, Rice University, Houston, Texas 77001. *Differential geometry and color perception.*

Newton gave the space X of color perception the geometry of a convex cone equipped with a perceptual metric inherited from its natural embedding in 3-dimensional Euclidean space. In order to improve the correspondence of this theory with experiment, Helmholtz introduced a particular Riemannian metric on X which measures the perceptual distinctions between distinct perceived colors. The present paper introduces the structure of a homogeneous space, $X = G/K$, as a consequence of a natural hypothesis about the subjective invariance of color relations under changes of background illumination. The classification of Lie groups restricts X to one of two essentially distinct alternatives. G-invariance of the perceptual metric determines the possible metrics up to certain characteristic constants. One of the possibilities is Helmholtz's metric; the other is non-Euclidean and appears to be new. It is shown how hue, saturation, and brightness can be defined in terms of the metrical geometry of X, and that the so-called 'Fechner's Law' is compatible with all alternatives. An historical introduction and several illustrative applications are included. (Received February 11, 1974.)

Bio 74-8. V. CHVATAL and DAVID SANKOFF, Université de Montréal, Montréal H3C 3J7, Canada. *Homology of random macromolecules, and inference in molecular evolution.* Preliminary report.

The study of molecular evolution requires the inference of relatedness among two or more macromolecular sequences. This usually involves matching terms $a(i_1) = b(j_1), \cdots, a(i_\lambda) = b(j_\lambda)$ from sequences $a(1), \cdots, a(m)$ and $b(1), \cdots, b(n)$ where $i_k < i_{k+1}$, $j_k < j_{k+1}$, for $k = 1, \cdots, \lambda - 1$ such that λ is as large as possible. However it is not always clear whether such matchings are anything other than accidental, or random, coincidences between the sequences. Accordingly, this paper studies $f(n, k)$, the largest value of λ to be expected for two random k-ary sequences of length n. Exact values of f are found for small k and n, and upper and lower bounds for $c_k = n^{-1} \lim_{n\to\infty} f(n, k)$ are derived. Monte-Carlo estimates of c_k are also given. The implications for molecular evolution studies, of the suprisingly high value of c_k are discussed. (Received February 11, 1974.)

The March Meeting in Gainesville, Florida
March 7—8, 1974

712-E1. J. R. SHOENFIELD, Duke University, Durham, North Carolina 27706. *The decision problem for recursively enumerable degrees.*

• Some recent results on RE degrees are discussed and related to the decision problem for these degrees. Possible ways of solving this problem either positively or negatively are discussed. (Received January 28, 1974.)

The April Meeting in New York, New York
April 10 – 13, 1974
Algebra & Theory of Numbers

*713-A1. FRANCIS E. MASAT, Glassboro State College, Glassboro, New Jersey 08028. *Right simple elements in a semigroup.*

Concepts and properties of right simple (RS) congruences on a semigroup S appeared in Abstract 703-A2, these *Notices* 20(1973), A-356, and Abstract 73T-A111, ibid., A-318. This paper investigates the relationship between the RS part of S and the nontrivial RS homomorphs of S. $x \in S$ is a *right simple element* (RSE) if $xS = S$. (S is RS iff each $x \in S$ is a RSE.) In the following, S has RSE's and is non-RS. Denote the set of RSE's by R and S/R by N. *Theorem* 1. A: N is the unique maximal right ideal of S and R is a subsemigroup of S; B: N is an ideal of S iff R is a RS subsemigroup of S and $rN = N$ for each $r \in R$. *Corollary.* If R is not RS then for each $r \in R$, s.t. $rN = S$, there exists an infinite descending chain of right ideals of S in N. *Theorem* 2. A: For each right 0-simple homomorphism γ on S there corresponds a prime ideal I of S in N; B: $N\gamma = S\gamma$ iff $I = N$. *Theorem* 3. If γ is a RS homomorphism on S and R is RS then $R\gamma \cong R \Rightarrow$

$Sy \cong R$ iff $Ny \cong R$. Conventional semigroups are used to show *Theorem* 4. There exist semigroups with RSE's s.t. the maximum RS homomorphism γ separates none of R and, moreover, there exists a proper subset G of N s.t. $G \cong Sy$. (Received October 12, 1973.)

*713-A2. ALAN P. SPRAGUE and DWIJENDRA K. RAY CHAUDHURI, Ohio State University, Columbus, Ohio 43210. *Characterization of projective 3-class association schemes.*

Let $PG(d-1, q)$ denote the $(d-1)$-dimensional projective space over $GF(q)$. Let Proj$(3, q, d)$ denote the 3-class association scheme with planes of $PG(d-1, q)$ as vertices where two planes are ith associates iff they intersect in a $(2-i)$-dimensional space, $i = 0, 1, 2, 3$. For a graph G, vertices x and y, and integers i, j, let $d(x, y)$ denote the distance between x and y and $p_{ij}(x, y)$ denote the number of vertices z s.t. $d(z, x) = i$ and $d(z, y) = j$. Let $k = (q^d - q^2)/(q^3 - q^2)$. The graph G of the first association class of Proj$(3, q, d)$ has the following properties: (i) G is connected, (ii) $p_{11}(x, x) = (q^2 + q + 1)(k-1)$ $\forall$ vertices, (iii) $p_{11}(x, y) = k - 2 + q^2(q+1)$ $\forall$ vertices x and y with $d(x, y) = 1$, (iv) $p_{11}(x, y) = (q+1)^2$ $\forall$ vertices x and y with $d(x, y) = 2$, and (v) $p_{31}(x, y) = q^2(k - q^2 - q - 1)$ $\forall$ vertices x and y with $d(x, y) = 2$. The following partial converse has been proved. *Theorem.* Let $q, d \in \mathbf{Z}$, $q \geq 2$, $d \geq 9$ and $(q, d) \neq (2,9)$. Let G be a graph satisfying (i) to (v). Then q is a prime power and G is isomorphic to the graph of first associates of Proj$(3, q, d)$. (Received November 30, 1973.)

*713-A3. JOAN P. HUTCHINSON, Dartmouth College, Hanover, New Hampshire 03755 and HERBERT S. WILF, Rockefeller University, New York, New York 10021. *Words with prescribed letter and letter-pair frequencies.* Preliminary report.

It is shown that the number of words on an alphabet of n letters in which letter i occurs ν_i times, and ν_{ij} times letter i is followed by letter j $(i, j = 1, \cdots, n)$ is given by $\{\Pi(\nu_i - 1)!\}\{\Pi\nu_{ij}!\}^{-1}\det(\nu_i\delta_{ij} - \nu_{ij})^n_{i,j=1}$, if certain consistency conditions hold, and zero otherwise. The method is to count the Eulerian paths on a certain directed multigraph. Partial results are obtained in the symmetric case. (Received January 4, 1974.)

713-A4. SAMUEL FLOYD BARGER, Youngstown State University, Youngstown, Ohio 44503. *Invertible ideals and theory of grade.* Preliminary report.

Let $\mathfrak{R}$ be a commutative integral domain with identity, and I a nonzero finitely generated ideal. I has grade 1 if for any $a \neq 0$, $a \in I$, $\exists\ b \notin (a)$ so that $bI \subseteq (a)$. The connection between grade 1 and invertible ideals is established, several characterizations of Dedekind domains are given and connection to π domains are explored. (Received January 14, 1974.)

713-A5. CLIFTON T. WHYBURN, University of Houston, Houston, Texas 77004. *A note on character sums.* II. Preliminary report.

In Abstract 655–58, these *Notices* 15(1968), 482 and subsequently in Duke Math. J. 37(1970), 307, an elementary estimation for $|\Sigma_{a<k\leq b}\chi(k)|$, where $0 \leq a < b$, $b^2 - a^2 \leq m$, χ a nonprincipal character (mod m), is obtained. If $\chi(-1) = 1$, the method used in the earlier paper may be modified to yield an estimate for this sum which is meaningful for a shorter interval of summation. (Received January 28, 1974.)

*713-A6. VLASTIMIL J. DLAB, Carleton University, Ottawa K1S 5B6, Ontario, Canada and CLAUS MICHA RINGEL, Universitat Bonn, Federal Republic of Germany. *Representations of valued graphs.*

A valued graph (Γ, d) is a finite graph (without loops) together with natural numbers (d_{ij}, d_{ji}) for each edge $i \longrightarrow j$. A species $(\Gamma, \mathfrak{F})$ is a set of skew fields F_i together with $F_i - F_j$-bimodules ${}_iM_j, i,j \in \Gamma$, s.t. (i) ${}_jM_i \approx \mathrm{Hom}_{F_i}({}_iM_j, F_i) \approx \mathrm{Hom}_{F_j}({}_iM_j, F_j)$, and (ii) $\dim({}_iM_j)_{F_j} = d_{ij}$. A representation $V = (V_i, {}_j\phi_i)$ of an oriented species is a set of finite-dimensional F_i-spaces V_i and F_j-linear maps ${}_j\phi_i$: $V_i \otimes {}_iM_j \to V_j$ $\forall\ i \longrightarrow j$. The following theorem extends results of Gabriel, Bernstein-Gelfand-Ponomarev, Donovan-Freislich, Nazarova and the authors: Let $(\Gamma, \mathfrak{F})$ be a species and $\mathfrak{O}$ an arbitrary orientation of Γ. (1) Γ is a Dynkin diagram iff the category $\mathfrak{L} = \mathfrak{L}(\Gamma, \mathfrak{F}, \mathfrak{O})$ of all representations of $(F, \mathfrak{F})$ is of finite type. Moreover, $V \mapsto (\dim V_i)$ induces a bijection between the indecomposable objects of $\mathfrak{L}$ and the positive roots of Γ. (2) If Γ is an "extended" Dynkin diagram, then $\mathfrak{L}$ has two types of indecomposable objects. For every positive root (x_i) of Γ, there is precisely one indecomposable object $V \in \mathfrak{L}$ with $\dim V_i = x_i$. The other indecomposable objects are "homogeneous". Moreover, the subcategory of all objects which are direct sums of indecomposable objects of "defect" zero, is a product of a finite number of categories whose objects are serial and the subcategory of all homogeneous objects. (Received January 16, 1974.)

*713-A7. JIN BAI KIM, West Virginia University, Morgantown, West Virginia 26505. *On a nullity of a product of two linear transformations.*

Let V be a vector space over a field F, T_i $(i = 1, 2)$ be linear transformations of V, $N(T_i)$ be the null space of T_i and $\dim N(T_i)$ the dimension of the space $N(T_i)$. We discuss applications of *Theorem.* $\dim N(T_1T_2) = \dim (N(T_1)) + \dim (R(T_1) \cap N(T_2))$, where $R(T_1)$ denotes the range of T_1. (Received February 5, 1974.)

713-A8. SERGE LANG, Yale University, New Haven, Connecticut 06520. *Higher dimensional diophantine problems.*

• 1. *Rational points.* The Mordell conjecture states that a curve of genus ≥ 2 over the rational numbers has only a finite number of rational points. Possible higher dimensional analogue: Let V be a variety, closed in projective space, and quotient of a bounded domain by a discrete group of automorphisms operating freely. Then V should have only a finite number of points rational over a field finitely generated over the rational numbers. Such varieties and subvarieties of abelian varieties not containing translations of abelian subvarieties can be subsumed under the class of hyperbolic projective (nonsingular) varieties for which a similar statement should hold. 2. *Integral points.* An affine open subset of an abelian variety should contain only a finite number of integral points, i.e. points with coordinates in a finitely generated ring over the rationals. One method of proof leads to diophantine inequalities on abelian logarithms (inverse to the abelian exponential map given by theta functions), but in the higher dimensional case, one must deal with one coordinate rather than a set of uniformizing parameters, thus facing serious difficulties due to the higher dimension. 3. *Isogenies.* Serre has proved for elliptic curves with nonintegral invariants over a number field that if their p-adic Galois representations are isomorphic for one prime p, then they are isogenous. The problem is unsolved in general for elliptic curves, let alone abelian varieties. One approach is to try to obtain lower bounds for the degrees of the fields of division points. In the case of two elliptic curves, without complex multiplication, it would suffice to prove that their common field of division points of order p^n has degree at least of the order of magnitude $p^{n(4+\epsilon)}$, unless they are isogenous. The methods of the theory of transcendental numbers can be used in this direction, but so far I have not succeeded in reaching the $4 + \epsilon$. (Received February 8, 1974.)

713-A9. ALAN CANDIOTTI, University of Missouri, St. Louis, Missouri 63121. *The order of $K_2\mathcal{O}/(K_2\mathcal{O})^3$ for quadratic fields.*

Let $k = Q(\sqrt{d}, \sqrt{-3})$ where d is a positive square free integer > 1. Let $k^+ = Q(\sqrt{d})$ and $k^- = Q(\sqrt{-3d})$. Every quadratic number field except $Q(\sqrt{-3})$ is of the form k^+ or k^-. Let A^+ and A^- be the 3-primary subgroups of the ideal class group of k^+ and k^- respectively. Let s and t be the nonnegative integers so that the order of A^+ is 3^s and the order of A^- is 3^t. Let n be the number of primes above 3 in k^+ and m be the number of primes above 3 in k^-. Let $\mathcal{O}^+$ be the ring of integers of k^+ and $\mathcal{O}^-$ be the ring of integers of k^-. Then the order of $K_2\mathcal{O}^+/(K_2\mathcal{O}^+)^3$ is 3^{t-1+m} and the order of $K_2\mathcal{O}^-/(K_2\mathcal{O}^-)^3$ is 3^{s-1+n}. (Received February 11, 1974.)

*713-A10. LOUIS HALLE ROWEN, University of Chicago, Chicago, Illinois 60637. *Prime ideals of Noetherian* PI-*rings.*

Let R be a left and right Noetherian PI-ring with center C. Given an ideal B of C, there exists $t > 0$ and $b_1, \cdots, b_t$ in B such that $BR = b_1R + \cdots + b_tR$; call the smallest such t the *rank* of B. A prime ideal

P of R has *height* h if every chain of prime ideals descending from P has length $\leq h$. Using a principal ideal theorem of Jategaonkar, one obtains *Theorem* 1. If B is an ideal of C of rank t, and if P is a prime ideal of R such that P/BR has height h in R/BR, then P has height $\leq h + t$ in R. Theorem 1 implies that all prime ideals of R have finite height (and gives a bound on the height), establishing a conjecture of L. Small. (Received January 25, 1974.)

*713-A11. SURJEET SINGH, Ohio University, Athens, Ohio 45701. *Modules over hereditary noetherian prime rings.*

Let R be a hereditary noetherian prime ring ((hnp)-ring), which is not right primitive and M_R be a torsion module; $x \in M$ is said to be uniform if xR ($\neq 0$) is a uniform submodule of M. For any uniform $x \in M$, the supremum of composition length $d(T/xR)$, where T is a uniform submodule of M containing x, is called the height of x (denoted by $H(x)$). For any $k \geq 0$, $H_k(M)$ denotes the submodule of M generated by all uniform $x \in M$ with $H(x) \geq k$. M is said to be bounded if $H_k(M) = 0$ for some k. A module N_R is said to be decomposable if it is a direct sum of cyclic modules and finitely generated torsion free uniform modules. Many results on decomposability of R-modules are established which generalize some of the well-known theorems of Kulikov, Szele, Prüfer, Kertesz, Kaplansky and Fuchs for abelian p-groups or modules over Dedekind domains. *Some results.* (I) A torsion R-module M is decomposable iff M is a union of an ascending chain of submodules M_n ($n \geq 1$), such that for each positive integer n, there exists a positive integer k_n satisfying $M_n \cap H_{k_n}(M) = (0)$. (II) Any submodule of a decomposable R-module is decomposable. (III) If N is a bounded submodule of a torsion module M_R such that $H_k(N) = N \cap H_k(M)$ for all k, then N is a direct summand of M. (IV) Every torsion R-module has a uniform direct summand. (Received February 13, 1974.) (Author introduced by Professor S. K. Jain.)

*713-A12. ALBERT NIJENHUIS, University of Pennsylvania, Philadelphia, Pennsylvania 19174. *Smith canonical forms of integer matrices.* Preliminary report.

The S.C.F. of an $n \times m$ matrix A (all entries of all matrices are integers or belong to a P.I.D.) is a diagonal matrix $S(A) = \operatorname{diag}\{d_1, d_2, \cdots\}$ s.t. (1) there are unimodular matrices P and Q for which $A = PS(A)Q$, (2) $d_i | d_{i+1}$, $i = 1, 2, \cdots$. Here is a new method to achieve (1) in two phases; (2) is then easy. Let $m \leq n$ or transpose A. *Phase* I (Reduction to almost-diagonal form). For $i = 1, \cdots, m-1$ perform: I1. Elementary row operations (interchanges, add multiple of one row to another) on rows $i, i+1, \cdots, n$ so the elements in column i below the (i, i) element are zero; I2. Elementary column operations on columns $i+1, \cdots, m$ so the elements in row i right of the $(i, i+1)$ element are zero. *Phase* II (Final reduction). For $i = 1, \cdots, m-1$, repeatedly perform the sequence of operations II1. If matrix entries $(i, i+1)$ and $(i+1, i)$ are both zero, do nothing. (If all off-diagonal elements are zero, exit.) Otherwise take the 2×2 matrix in rows and columns $i, i+1$ and perform elementary column operations or elementary row operations so the previously nonzero off-diagonal element is now zero. Note that I sweeps once through the matrix, while II acts on few ($< 2m$) numbers so very large entries are less likely. (Received February 14, 1974.)

*713-A13. S. K. JAIN and SURJEET SINGH, Ohio University, Athens, Ohio 45701. *Semiprimary rings with quasi-projective left ideals.* Preliminary report.

A ring R is a left (qp)-ring if each of its left ideals is quasi-projective (Abstract 73T-A78, these *Notices* 20(1973), A-260). Harada [Nagoya Math. J. 27(1966), 463–484] showed that semiprimary hereditary rings are generalized triangular matrix rings over semisimple artinian rings. Let R be a semiprimary left (qp)-ring. We show that the basic ring S of R is also a left (qp)-ring (the converse does not hold), and it is a generalized triangular matrix ring over division rings and local left (qp)-rings. In particular, if $S = Se_1 \oplus Se_2$, e_i primitive, S indecomposable (as a ring) and not hereditary, then we can suppose that $e_1Se_2 \neq (0)$, $e_2Se_1 = (0)$; in this case e_1Se_1 is a division ring, e_2Se_2 is a local left (qp)-ring which is not a division ring. Further if $(e_2Ne_2)^2 = (0)$, then $V = e_1Se_2$ is a (e_1Se_1, e_2Se_2) bimodule s.t. either V is faithful as a right e_2Se_2 module, and for any nonzero e_2xe_2, e_2ye_2 in e_2Ne_2, $Ve_2xe_2 \simeq Ve_2ye_2$ as a left e_1Se_1-module, or $Ve_2Ne_2 = (0)$. However if $(e_2Ne_2)^2 \neq (0)$, then for any $e_2xe_2 \in e_2Ne_2$, Ve_2xe_2 is a bisubmodule and all such bisubmodules are totally ordered under inclusion. (Received February 14, 1974.)

*713-A14. JOSEPH ARKIN, 197 Old Nyack Turnpike, Spring Valley, New York 10977 and E. G. STRAUS, University of California, Los Angeles, California 90024. *Latin k-cubes.*

Theorem. If there exist two orthogonal Latin squares of order n then there exist 4 orthogonal cubes of order n and k orthogonal Latin k-cubes for each $k > 3$. *Corollary.* There exist orthogonal k-tuples of Latin k-cubes of order n for every $n > 2$, $n \neq 6$. *Theorem. If* n is a power of a prime and $k < n$, then there exists a system of $n - 1$ orthogonal k-cubes of order n. *Theorem.* If n is a power of 2 then there exists $n - 1$ orthogonal Latin cubes of order n with the property that the corresponding plane sections form systems of $n - 1$ orthogonal Latin squares. *If* n is a power of an odd prime then there exist $n - 1$ orthogonal Latin cubes with the property that the corresponding plane cross-sections in two directions form complete systems of orthogonal Latin squares, while the plane cross-sections in the third direction form a system of $(n - 1)/2$ orthogonal Latin squares, each square occurring twice. (We observe that if we have orthogonal k-cubes of orders m and n then we can form their Kronecker products to obtain orthogonal k-cubes of order mn.) *Corollary. If* $n = p_1^{a_1}p_2^{a_2}\cdots p_s^{a_s}$ and $q = \min_{1\le j\le s} p_j^{a_j}$ then for any $k < q$ THERE EXIST AT LEAST $q - 1$ orthogonal Latin k-cubes of order n. (Received February 18, 1974.)

713-A15. DANIEL J. KLEITMAN, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. *Degree sequences and trees.*

We explore algorithms for constructing a graph containing k disjoint trees having prescribed degree sequences (jointly with D. L. Wang). (Received February 18, 1974.)

713-A16. BENJAMIN FINE, City University of New York, John Jay College, New York, New York 10010 and MARVIN D. TRETKOFF, Stevens Institute of Technology, Hoboken, New Jersey 07030. *The SQ-universality of certain linear groups.* Preliminary report.

We show that the projective special linear groups PSL(2, I) are SQ-universal if I is the ring of integers in the quadratic extension of the rationals obtained by adjoining the square root of $-d$, where $d =$ 1, 2, 3, or 7. Recall that a group G is called SQ-universal if every countable group can be embedded in a quotient of G. In case $d = 2$, 7, or 11, we also show that the groups in question are HNN extensions of SQ-universal groups with free parts of rank 1. Our proofs are obtained from a detailed investigation of presentations of the projective linear groups which yields results to which we can apply various criteria for a group to be SQ-universal. We mention just one of these criteria. Namely, if K is a free product, not $Z_2 * Z_2$, and H is a finitely generated subgroup of infinite index in K, then any HNN extension of K with H tied to any other isomorphic subgroup of K is SQ-universal. (Received February 20, 1974.)

*713-A17. SETH R. ALPERT, Medical Computer Science Program, State University of New York - Downstate Medical Center, Brooklyn, New York 11203. *Two-fold triple systems and graph imbeddings.*

There is a natural bijection between balanced incomplete block designs on v objects with $k = 3$ and $\lambda = 2$, or two-fold triple systems, and a class of topological and combinatorial structures, called triangulation systems on v names, which generalize and include the triangular imbeddings of complete graphs into 2-manifolds. This observation is exploited to use known graph imbeddings to construct several new classes of triple systems. (Received February 21, 1974.)

*713-A18. ERNEST C. ACKERMANN, Pennsylvania State University, University Park, Pennsylvania 16802. *Weaving j-diagrams for torsion free abelian groups.*

Let A be a torsion free abelian group, j an injection of $\mathbf{N}$ into $\mathbf{N}$ such that $j(n) > n \; \forall \; n \geq 0$, and p a prime. An infinite series for A, $A = A_0 > A_1 > \cdots > A_n > \cdots$ is a j-diagram for A if $\bigcap_{n=0} A_n = \{0\}$ and the mapping $a_i + A_{i+1} \mapsto pa_i + A_{j(i)+1}$ defines an isomorphism of A_i/A_{i+1} onto $A_{j(i)}/A_{j(i)+1}$ $\forall$ $i \geq 0$. Let B and C be torsion free abelian groups with j_1 and j_2 diagrams $B = B_0 > B_1 > \cdots > B_n > \cdots$ and $C = C_0 > C_1 > \cdots > C_m > \cdots$, respectively. A j-diagram for A is said to be woven from the j_1 and j_2 diagrams for B and C if $A = B \oplus C$ and if, $\forall\, t \geq 0$, $A_t = B_{t1} \oplus C_{t2}$ with $A_{t+1} = B_{(t1)+1} \oplus C_{t2}$ or $A_{t+1} = B_{t1} + C_{(t2)+1}$. We define weaving of any number of diagrams similarly. Weaving is associative. If A, B, and C are as above with the j-diagram for A woven from the j_1 and j_2 diagrams for B and C, then if $d_i = \max\{j(l) - l;\ l \geq 0\} < \infty$ $(i = 1, 2)$, we have $j(k) - k \leq d_1 + d_2$ for every $k \geq 0$ and $\exists\ t \ni j(s) - s = d_1 + d_2$ for every $s \geq t$. Given diagrams for B and C all diagrams woven from the given diagrams for B and C are determined. (Received February 21, 1974.)

713-A19. HERBERT S. WILF, Rockefeller University, New York, New York 10021 and University of Pennsylvania Philadelphia, Pennsylvania 19104. *Algorithms for generating combinatorial objects uniformly at random.*

Given a family of combinatorial objects, we can often select an object uniformly at random from the family by endowing the recurrence formula which results from logarithmic differentiation of the generating function of the family with a constructive combinatorial interpretation. Examples are given of algorithms for choosing at random a partition of an integer, a partition of an n-set, a labelled forest, and unlabelled rooted tree, etc. This work is due jointly to Professor A. Nijenhuis and the speaker. (Received February 21, 1974.)

713-A20. W. T. TUTTE, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada. *Spanning subgraphs with specified valencies.*

The author has published a necessary and sufficient condition for a finite loopless graph to have a spanning subgraph with a specified positive valency at each vertex. Supplementary theorems are now presented; they facilitate the application of the condition to graph-theoretical problems. (Received February 21, 1974.)

713-A21. JACK EDMONDS, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada. *A linear decomposition theory.* Preliminary report.

For $i = 1$ and 2, let L_i be a system of homogeneous linear equations in the variables $\{X_0, X_i\}$ where X_0, X_1, and X_2 are disjoint sets of variables. Elementary linear algebra tells when and how X_0 can be "eliminated" to obtain a system L in the variables $\{X_1, X_2\}$ s.t. $\{X_1^0, X_2^0\}$ is a solution of L iff, for some value X_0^0 of X_0, $\{X_0^0, X_i^0\}$ is a solution of L_i, $i = 1$ and 2. One main result of a theory developed jointly with W. H. Cunningham is an algorithm for the problem: given a homogeneous linear system L, and an integer k, determine if there is an L_1 and L_2, related to L as described above, s.t. the number of variables in $X_0 < k$ and the number of variables in each X_i is at least k. Another aspect of the theory is certain uniqueness theorems concerning this decomposition. Matroids are a main concept used in the theory. (Received February 21, 1974.) (Author introduced by Professor Herbert S. Wilf.)

Analysis

713-B1. LAWRENCE E. LEVINE, Stevens Institute of Technology, Hoboken, New Jersey 07030 and ERIC LUBOT, Stevens Institute of Technology, Hoboken, New Jersey 07030 and Bergen Community College, Paramus, New Jersey 07652. *A remark on time scales.* Preliminary report.

Consider the question of the form of the minimum number of time variables needed to obtain a uniformly valid approximation $y(t, t_1, t_2, \cdots)$ to the solution of initial value problem (1) $\epsilon^n y^{(m)} + y'' + \epsilon y' + y = 0$, $y^{(i)}(0) = c_i$, $0 \leq i \leq m - 1$, where $m \geq 2$ and n are positive integers. See Reiss [SIAM Rev. 13(1971), 189–196]. It is shown that the minimum number of time variables needed are t, $t_1 = \epsilon t$ and $t_2 = t/\sigma(\epsilon)$, where $\sigma(\epsilon)$ is a function of ϵ to be determined. Using a limit process technique which guarantees that as $\epsilon \to 0$ the unperturbed equation $y'' + y = 0$ is obtained leads to the result that $\sigma = \epsilon^{n/(m-2)}$. Thus $t_2 = t/\epsilon^{n/(m-2)}$. Note that if we take $n = 1$, $m = 3$ in (1), we obtain essentially the equation studied by Matkowsky and Reiss [Arch. Rational Mech. Anal. 42(1971), 194–212]. (Received December 3, 1973.)

*713-B2. ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620. *Locally invertible operators and existence problems in differential systems.* Preliminary report.

Consider the boundary value problem: (*) $x' + A(t)x = F(t, x)$, $Lx = r$, where A is an $n \times n$ matrix, F an n-vector, L a bounded linear operator defined on the space of bounded and continuous n-vector-valued functions on $[0, \infty)$, under the sup-norm, and r a vector in R^n. The existence of a solution to (*) is shown under quite general assumptions on A and F, which include: the homogeneous problem corresponding to (*) has only the zero solution, and the vector $F(t, u)$ is continuously differentiable in u. The method used is based on the fact that a certain operator T associated with (*) is Fréchet differentiable in the neighborhood of a point in its domain. (Received February 4, 1974.)

713-B3. KENNETH B. HANNSGEN, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. *Continuous parameter dependence in a class of Volterra integral equations.* Preliminary report.

For real λ let $a_\lambda \in C(0, \infty) \cap L^1(0, 1)$ be nonnegative, nonincreasing, and convex, with $a_\lambda(t) \not\equiv a_\lambda(\infty)$. It is known that if $x'_\lambda(t) + \int_0^t a_\lambda(t - s)x_\lambda(s)\,dx = k$ $(t \geq 0)$, then $|x_\lambda(t)| \leq M(\lambda)$. We give conditions under which $x_\lambda(t)$ is continuous in λ, uniformly in $0 \leq t < \infty$. We indicate applications to related equations in Hilbert space. (Received February 8, 1974.)

713-B4. NINA M. ROY, Rosemont College, Rosemont, Pennsylvania 19010. *A characterization of square Banach spaces.*

A real Banach space is square if it is linearly isometric to a uniformly closed space X of real functions on a compact Hausdorff space Ω satisfying (i) if $x \in X$ and $f \in C(\Omega)$, then $fx \in X$, and (ii) $|x|$ is upper semicontinuous $\forall\, x \in X$. Square spaces were introduced by F. Cunningham, Jr., who proved that the class of square spaces is properly contained in the class of (Grothendieck) G-spaces. Let X be a real Banach space and E the set of extreme points of the dual ball equipped with the Alfsen-Effros structure topology. Denote by E_σ the quotient space obtained by identifying antipodal points in E. *Theorem.* X is square iff the bounded structurally continuous functions on E_σ separate points in E_σ. *Corollaries.* (1) Every separable G-space is square. (2) The class of (Jerison) C_σ-spaces is properly contained in the class of square spaces. It is also shown that for a G-space, regularity of the structure topology on E_σ is equivalent to complete regularity, and that square spaces exist for which this topology is not regular. The proof of the latter uses the theorem in Abstract 696-46-10, these Notices 19(1972), A-648. Proofs will appear in Israel J. Math. (Received February 7, 1974.)

*713-B5. GARY SAMPSON, ABRAHAM NAPARSTEK and VLADIMIR DROBOT, State University of New York at Buffalo, Amherst, New York 14226. *(L_p, L_q) mapping properties of convolution transforms.* Preliminary report.

Let k and f be two Lebesgue measurable functions on R^n. $k*f(x) = \int_{\mathbf{R}^n} k(x - t)f(t)\,dt$ defines the convolution transforms of k and f. We set $T(f) = k*f$ and give both necessary as well as sufficient conditions for T to map $L_p \to L_q$ continuously. To show that our results are sharp, we work out the exact mapping properties of several examples. The results extend theorems of Hörmander (Acta Math. 104(1960), 93–140) and Hirschman (Duke Math. J. 26(1959), 221–242). Two typical results: (i) Let $0 \leq \lambda \leq 1$, $t \in \mathrm{R}$, $k(t) = \exp(i|t|^a)/|t|^b$, and $1/p - 1/q = 1 - \lambda$. If $a \neq 1$, $b < \lambda$ and $a\lambda/2 + (b - \lambda) > 0$, then $k \in L_p^q$ for $a/(\lambda(a - 1) + b) < q < a/(\lambda - b)$ and $k \in L_p^q$ for $q > a/(\lambda - b)$. (ii) Let $t \in \mathrm{R}^n$, $n \geq 2$, $0 < b < \lambda$, $0 \leq \lambda \leq 1$, and $1/p - 1/q = 1 - \lambda$. Then $k(t) = \exp(i|t|^2)/(|t_1|^{2b} + |t_2|^{2b} + \cdots + |t_n|^{2b}) \in L_p^q(\mathrm{R}^n)$ for $2n/(n\lambda + 2b) < q < 2n/(n\lambda - 2b)$ and $k \notin L_p^q(\mathrm{R}^n)$ for $q > 2n/(n\lambda - 2b)$. (Received February 11, 1974.)

713-B6. JOHN B. BUTLER, JR., Portland State University, Portland Oregon 97207. *A note on smooth perturbations.* Preliminary report.

Let selfadjoint operators H^0, $H^1 = H^0 + AB$ be defined on a rigged Hilbert space $\Phi \subseteq \mathfrak{H} \subseteq \Phi'$ where A, B are bounded and set $R^i = (H^i - \lambda I)^{-1}$, $i = 0, 1$. H^1 is said to be a smooth perturbation of H^0 when for $u \in \mathfrak{H}$, $AR^0(\lambda)u$, $BR^0(\lambda)u$ are in the Hardy class $H_2((-\infty, \infty); \mathfrak{H})$ (T. Kato, Math. Ann. 162(1965/66), 258–279). The question arises as to when $AR^1(\lambda)u$, $BR^1(\lambda)u$ are also in H_2. *Theorem.* Let $R^0 = R_1^0 + R_2^0$ satisfy the following conditions for Im $\lambda \neq 0$: (1) $AR^0(\lambda)u$, $BR^0(\lambda)u$ are in H_2. (2) $\|BR^0(\lambda)A\| < 1$, $|\lambda| \geq 1$, and $\|BR_1^0(\lambda)A\| < 1$, $|\lambda| \leq 1$. (3) $R_2^0(\lambda)$ is a weak limit of degenerate operators $\{R_2^0(\lambda; n)\}$, $R_2^0(\lambda; n)\phi = \Sigma_k u_k(n)(\phi\ v_k(n))$, $v_k(n) \in \mathfrak{H}$, $u_k(n)$, $\phi \in \Phi$. (4) If $c_k(\phi; n)$ are solutions of $\Sigma_k(\delta_{ik} + (v_k(n), Du_k(n)))c_k(n) = (v_i(n), D\phi)$, $D = AB(I + R_1^0(\lambda)AB)^{-1}$ then for some N, $|c_k(n)| \leq \|B\phi\|$, $\phi \in \Phi$, $n \geq N$, $|\lambda| \leq 1$. Then $AR^1(\lambda)u$, $BR^1(\lambda)u$ are in H_2. The theorem also holds for $H_2(\Delta; \mathfrak{H})$, $\Delta \subseteq (-\infty, \infty)$. (Received February 11, 1974.)

713-B7. RUSSELL D. RUPP, State University of New York, Albany, New York 12222. *The epsilon method, the method of multipliers, and a nonlinear optimal control problem.* Preliminary report.

The nonlinear optimal control problem of minimizing $\int_{T_1}^{T_2} L(t, x(t), u(t))\,dt$ subject to the vector equation $\dot{x}(t) = f(t, x(t), u(t))$ is considered. Existence, necessary conditions, and convergence results are given for the augmented functional, $\int_{T_1}^{T_2}[L + p'\phi + (1/2\epsilon)|\phi|^2]\,dt$ where $\phi = \dot{x} - f$, $\epsilon > 0$, and $'$ denotes transpose. Square integrable multiplier terms, deterministic controls, and an unbounded control set are included. Essential use is made of an inequality whose general form is $\|p_n - p_0\|^2 \geq \|p_{n+1} - p_0\|^2 + r_{n+1}^2\|\phi_n\|^2$. The indices refer to the sequence of unconstrained minima associated with the multiplier method. The proper amount of convexity must be assumed, otherwise the epsilon maximum principle is false. (Received February 12, 1974.)

*713-B8. JEROME EISENFELD and V. LAKSHMIKANTHAM, University of Texas, Arlington, Texas 76019. *Fixed point theorems through cones.* Preliminary report.

Employing a cone as a basis for comparison of vectors, a cone-valued metric and a measure of noncompactness of a set are defined. In this framework, two well-known fixed point theorems of Banach and Schander are extended so as to be more useful in applications to differential and integral equations. The results obtained offer sharper conclusions on account of the partial ordering induced by the cone. Applications to initial and boundary value problems as well as nonlinear eigenvalue problems are indicated. (Received February 12, 1974.)

*713-B9. A. RICHARD MITCHELL and ROGER W. MITCHELL, University of Texas, Arlington, Texas 76019. *Asymptotic equilibrium in Banach spaces.* Preliminary report.

The asymptotic equilibrium of differential systems in Euclidean spaces has been considered by several authors. These papers deal with a majorant function, $g(t, u)$, which is either nondecreasing or nonincreasing in u for each t. In extending these results to differential systems in a Banach space additional conditions must be placed on the system. In this paper the Kuratowski measure of noncompactness is used to give conditions yielding asymptotic equilibrium of the system in a Banach space. (Received February 12, 1974.)

*713-B10. V. LAKSHMIKANTHAM, University of Texas, Arlington, Texas 76019 and S. LEELA, State University College of New York, Geneseo, New York 14454. *On perturbing Lyapunov functions.* Preliminary report.

It is known that in proving uniform boundedness of a differential system by means of Lyapunov functions, it is sufficient to impose conditions in the complement of a compact set in R^n, where as in the case of equiboundedness, the proofs demand that the assumptions hold everywhere in R^n. In this paper, a new idea which permits one to discuss nonuniform properties of solutions of differential equations under weaker assumptions is presented. The results obtained also imply that in those situations when the Lyapunov function found does not satisfy all the desired conditions, it is fruitful to perturb it rather than discarding it. (Received February 12, 1974.)

713-B11. JAMES M. NEWMAN, City University of New York, Baruch College, New York, New York 10010. *A note on coercive inequalities for irregular regions.* Preliminary report.

In two previous papers (Comm. Pure Appl. Math. 22(1969), 825–838; Proc. Amer. Math. Soc. 32(1972), 120–126) the author derived sufficient conditions for the coerciveness of formally positive (semidefinite) integrodifferential forms over complex-valued functions of two real variables having zero boundary values, where the boundaries of the planar region are nonsmooth: e.g., may have corners. By using elementary techniques, it is shown coerciveness may also be proved in some cases for indefinite forms over the same types of functions and regions. (Received February 13, 1974.)

713-B12. HEINRICH W. GUGGENHEIMER, Polytechnic Institute of New York, Brooklyn, New York 11201. *Convexity arguments for differential equations.* I.

1. Let $p(\lambda, t, x, x')R \times R \times R^2 \times R^2 \to R^+$ be continuous tending to ∞ with λ uniformly on every compa domain $D \subset R^+ \times R^2 \times R^2$. Also, $p(0, t, x, x') = 0$. The boundary value problem $x'' + px = 0$, $x(t_0) = a$, $x(t) = b$, $\sphericalangle(x(t_0), x'(t_0)) = \mu_0 \det(a, b) \neq 0$, $\mu_0 \not\equiv 0 \pmod{\pi}$ admits positive proper values of all odd orders for $\operatorname{sgn} \mu_0 \neq \operatorname{sgn} \sphericalangle(a, b)$ and proper values of all even positive orders for $\operatorname{sgn} \mu_0 = \operatorname{sgn} \sphericalangle(a, b)$. The order of a proper value is the number of conjugate points to t_0 in $t_0 < t < t_1$. The statement for λ_0 is rather complicated. The proper values can be estimated from below. 2. The problem $x'' + px = 0$, $x(t_0) = a$, $x'(t_0) = b$, $\det(a, b) \neq 0$, $\det(x(t_0), x(t_1)) = 0$ has proper values of all orders. The same holds for the problems $\det(x(t_0), x'(t_1)) = 0$ and $\det(x'(t_0), x(t)) = 0$ or $\det(x'(t_0), x'(t)) = 0$. These problems generalize scalar problems $u'' + pu = 0$, $u(t_0) = u(t_1) = 0$, $u(t_0) = u'(t_1) = 0$, etc. 3. The problem $x'' + px = 0$, $x(t_0) = a$, $x'(t_0) = x_0'$, $\det(a, x_0') \neq 0$, $\arg x'(t_1) = \theta_1$ ($\not\equiv \arg x_0' \pmod{\pi}$) admits proper values of all odd orders for $\operatorname{sgn} \sphericalangle(a, x_0') \neq \operatorname{sgn}(\arg x_1' - \arg x_0')$ and of all even orders otherwise. Here the order denotes the number of coconjugate points in $t_0 < t < t_1$. (Received February 14, 1974.)

713-B13. BERNARD A. ASNER, JR., University of Dallas, Irving, Texas 75060 and V. LAKSHMIKANTHAM, University of Texas, Arlington, Texas 76010. *Utilization of pointwise degenerate delay-differential equations as comparison functions.* Preliminary report.

Liapunov-like vector functions are used to find comparison theorems applicable to the nonlinear delay-differential equation $x'(t) = f(t, x(t), x(t-h))$ where $x \in R^n$ and $f(t, x(t), x(t-h))$ possesses a mixed quasimonotone property. Qualitive information on the solution $x(t)$ is obtained by using pointwise degenerate delay-differential systems as comparison functions. (Received February 18, 1974.)

*713-B14. GANGARAM S. LADDE, State University College of New York, Potsdam, New York 13676. *Systems of differential inequalities and stochastic differential equations.* I. Preliminary report.

By developing a comparison principle, sufficient conditions are given for conditional stability and boundedness in the mean of solutions of stochastic differential systems. Our main tool here is the theory of systems of differential inequalities and vector Lyapunov-like functions. (Received February 18, 1974.)

713-B15. DANIEL WATERMAN, Syracuse University, Syracuse, New York 13210 and BADRI N. SAHNEY, University of Calgary, Calgary, Alberta T2N 1N4, Canada. *On Cesaro summability of Fourier series.* Preliminary report.

Let f be an integrable function on $[0, 2\pi]$ and $S[f]$ its Fourier series. We obtain necessary and sufficient conditions for (C, β) summability, $-1 < \beta < 0$, of $S[f]$ at a point x or uniformly on a set E. This is used to obtain for (C, β) summability of $S[f]$ a test analogous to the Lebesgue convergence test. (Received February 18, 1974.)

713-B16. MILTON ROSENBERG, City University of New York, Staten Island Community College, Staten Island, New York 10301. *Spectral integrals from the theory of multivariate stationary Payen processes.* Preliminary report.

Refer to Mandrekar and Salehi [Indiana Univ. Math. J. 20(1970/71), 545–563; Ann. Inst. H. Poincaré Sect. B 6(1970), 115–130] for definitions, and to Rosenberg [J. Multivariate Anal. (to appear)] for tools for proof. Let $\mathfrak{H}$, $\mathfrak{K}$, $\mathfrak{W}$ be separable Hilbert spaces, E be a projection-valued (spectral) measure on a σ-algebra $\mathfrak{B}$ over a set Ω for $\mathfrak{H}$, and T be a linear operator from H.S. $(\mathfrak{W}, \mathfrak{H})$ to H.S. $(\mathfrak{K}, \mathfrak{H})$ (H.S. = Hilbert-Schmidt; similarly B = bounded, C = closed-densely defined). Let for $X \in$ H.S. $(\mathfrak{W}, \mathfrak{H})$, $\mathfrak{M}_X = \{$closed subspace of $\mathfrak{H}$ generated by the ranges of $E(B)X$, $B \in \mathfrak{B}\}$, P_X = projection onto $\mathfrak{M}_X$. *Theorem* 1. (a) $T(\cdot) = \int (E(dw)(\cdot))\Phi^*(w)$ (cf. M. & S.) where Φ is strongly measurable, $\Phi(w) \in B(\mathfrak{W}, \mathfrak{K})$, and ess sup $|\Phi(w)|_B = K < \infty \iff$ (i) T is bounded with $|T|_B = K$ (H.S. norms), and (ii) $T(X) \in \{P_X Y\colon Y \in \text{H.S.}(\mathfrak{K}, \mathfrak{H})\}$, each X. (b) The Φ in "$\Leftarrow$" is unique a.e. (E). *Theorem* 2. Let T be closed densely-defined (H.S. norms) $\ni P_X T(\cdot) \subseteq T(P_X(\cdot))$, each $X \in$ H.S. $(\mathfrak{W}, \mathfrak{H})$. Then $\exists \Phi$, $\Phi(w) \in C(\mathfrak{W}, \mathfrak{K})$, $\ni T(\cdot) = \int (E(dw)(\cdot))\Phi^*(w)$. (Received February 18, 1974.)

713-B17. PAUL GORDON, Drexel University, Philadelphia, Pennsylvania 19104. *Scaling of a system of first order hyperbolic partial differential equations.*

We derive a means of scaling the dependent variables of a system of first order hyperbolic partial differential equations, in two independent variables, so that the normalized system accurately reflects the dependence of the system on initial data. The scaling is equivalent to row and column scaling commonly used in linear algebra. For the case of positive matrices (that is, for the case where the diagonalizing matrix and its inverse have no zero elements), the theory for the "1" and "∞" norms has been thoroughly developed by Bauer. The assumption of positive matrices is too stringent for some applications. Thus, the theory is generalized to show that for irreducible matrices a best scaling exists (for both the "1" and "∞" norms), while for simply reducible matrices a best scaling exists for at least one of these two norms. Several examples are given. (Received February 18, 1974.)

*713-B18. M. RAJAGOPALAN, Memphis State University, Memphis, Tennessee 38152 and A. K. ROY, Tata Institute of Fundamental Research, Bombay, India. *Polytopes in locally convex spaces.*

A definition of a polytope in infinite dimensional locally convex spaces is given. This generalises the idea of a polyhedran in finite dimensional spaces. Besides sharing most of the properties of finite dimensional polyhedra, these classes enjoy properties similar to that of α-polytopes. It is possible that this could be the right generalisation of the class of polyhedra in Euclidean spaces. (Received February 18, 1974.) (Authors introduced by Professor Stanley P. Franklin.)

*713-B19. E. O. MILTON, University of California, Davis, California 95616. *Fourier transforms of odd and even tempered distributions.*

Certain previous results of the author concerning Abelian theorems for the Fourier transform of a distribution are generalized to two new distribution spaces, those of odd and even tempered distributions. These spaces arise in the consideration of Fourier sine and cosine transforms of distributions. Each of the new spaces is larger than the space of tempered distributions and their intersection is exactly the space of tempered distributions. (Received February 19, 1974.)

*713-B20. GEORGE GASPER, Northwestern University, Evanston, Illinois 60201. *Some positive integrals of Bessel functions.* Preliminary report.

Over the years methods have had to be developed to prove the positivity of various integrals of Bessel functions. We show that many of the known, as well as some new, positivity results for integrals of Bessel functions and for certain generalized hypergeometric functions can be obtained by writing the integrals and functions as a sum of squares of Bessel functions with positive coefficients. In particular, expansions of this type and fractional integrals are used to give a simple proof of the positivity results in Fields and Ismail ["On the positivity of some ${}_1F_2$'s", SIAM J. Math. Anal., to appear] and to give Steinig's result [Trans. Amer. Math. Soc. 163(1972), 123–129] that the Lommel function $s_{\mu,\nu}(x) > 0$ for $x > 0$, if $\mu = ½$ and $-½ < \nu < ½$, or if $\mu > ½$ and $-\mu \leq \nu \leq \mu$. (Received February 19, 1974.)

*713-B21. THOMAS J. OSLER, Glassboro State College, Glassboro, New Jersey 08028. *Leibniz rule for fractional derivatives used to generalize formulas of Walker and Cauchy.*

H. W. Gould [Bul. Inst. Politehn. Iași 18(1972), 47–53] discussed two formulas which are generalizations of Leibniz' rule $D^N uv = \Sigma_{n=0}^N \binom{N}{n} D^{N-n}u\, D^n v$: (1) [due to Walker]. $D^N[f^N uv] = \Sigma_{n=0}^N W(N, n)$, where $W(\alpha, \omega) = \binom{\alpha}{\omega} D^{\alpha-\omega}[f^{\alpha-\omega}v] D^{\omega-1}[f^{\omega}u']$; (2) [due to Cauchy]. $D^{N-1}[f^N D(uv)] = \Sigma_{n=0}^N C(N, n)$, where $C(\alpha, \omega) = \binom{\alpha}{\omega} D^{\alpha-\omega-1}[f^{\alpha-\alpha}v'] D^{\omega-1}[f^{\omega}u']$. We extend the above formulas such that the positive integer "N" is replaced by arbitrary (integer, rational, irrational, or complex) "α". $D^{\alpha}g(z)$ is called the "fractional derivative" of $g(z)$. The extension of (1) is $D^{\alpha}[f^{\alpha}uv] = \Sigma_{n=-\infty}^{\infty} W(\alpha, an + \gamma)a = \int_{-\infty}^{\infty} W(\alpha, \omega + \gamma)\, d\omega$, and the extension of (2) is $D^{\alpha-1}[f^{\alpha}D(uv)] = \Sigma_{n=-\infty}^{\infty} C(\alpha, an + \gamma)a = \int_{-\infty}^{\infty} C(\alpha, \omega + \gamma)\, d\omega$, where γ is an arbitrary complex number, and $0 < a \leq 1$. Examples are also given. (Received February 19, 1974.)

713-B22. M. N. M. TALPUR, University of Illinois, Urbana, Illinois 61801. *A spatial analogue of a theorem of Hayman.* Preliminary report.

Hayman (Acta Math. 112(1964), 181–214) obtained the following subharmonic version of his results on functions meromorphic in a disk: Suppose that $u(z)$ is a negative subharmonic function in a circle $|z| \leq R$ and $f(r, \theta) = \inf_{0 \leq t \leq r} u(te^{i\theta})$. Then $(1/2\pi)\int_0^{2\pi} f(r, \theta)\, d\theta \geq [1 + \psi(r/R)]\, u(0)$, $0 < r < R$, $\psi(t) =$ $(1 - t) \log (1 + 2\pi\sqrt{t}/(1 - t))/\pi\sqrt{t} \log (1/t)$, $\psi(t) \to 0$ as $t \to 0$. The author uses Hayman's method to prove the following analogue in R^3: If $\omega(p)$ (≤ 0) is subharmonic in a sphere with centre at the origin and radius R, and $f(r, \theta, \phi) = \inf_{0 \leq t \leq r} \omega(t, \theta, \phi)$ for fixed θ, ϕ, then $(1/4\pi r^2)\int_{S_r} f(r, \theta, \phi)\, dS_r > [\pi/4 + 1/2 + \phi(r/R)]\omega(0)$, $0 < r < R$, $\phi(t) = \sqrt{t}/(1 - \sqrt{t})$, $S_r =$ surface of sphere with radius r. It is shown that the constant $\pi/4 + 1/2$ is the best possible. An immediate consequence is that a subharmonic function in a space bounded above is also bounded below on almost all straight lines. Extension of the results in R^n is discussed. (Received February 19, 1974.)

713-B23. STEVEN I. ROSENCRANS and J. T. BEALE, Tulane University, New Orleans, Louisiana 70118. *Wave equation with acoustic boundary conditions.* Preliminary report.

We study the propagation of sound waves in the exterior of an obstacle whose surface is assumed to react locally like a resistive harmonic oscillator in response to the excess pressure. (This condition is used in theoretical acoustics, and is a generalization of the Robin boundary condition.) The problem is formulated as an evolution equation in a certain Hilbert space, whose elements are the direct sum of boundary functions and exterior functions. The differential operator is shown to be maximal dissipative. The solution decays locally. It follows from the abstract scattering theory of Lax and Phillips for dissipative operators (J. Functional Analysis 14(1973), 172–235) that wave and scattering operators exist. The extended outgoing resolvent is meromorphic in the complex plane with the two spring frequencies removed. Calculations for a sphere show that these two points are indeed accumulation points of poles. (Received February 19, 1974.)

*713-B24. ALBERT EDREI. Syracuse University, Syracuse, New York 13210. *The complete Padé tables of certain series of simple fractions.*

Let $A(z) = \kappa + \Sigma_{j=1}^{\infty} r_j/(\beta_j - z) = \Sigma_{k=0}^{\infty} a_k z^k$ $(a_0 \neq 0)$, where $r_j > 0$, $\beta_j > 0$, $\kappa \geq 0$, $\Sigma\, r_j/\beta_j < +\infty$, $\Sigma(\beta_j)^{-1} < +\infty$. Then $A(z)$ is necessarily of the form $A(z) = a_0 g(z)/h(z)$ with $g(z) = \Pi_{j=1}^{\infty}(1 - z/\alpha_j)$, $h(z) = \pi_{j=1}^{\infty}(1 - z/\beta_j)$, and $\beta_1 < \alpha_1 < \beta_2 < \alpha_2 < \cdots < \beta_j < \alpha_j < \beta_{j+1} < \cdots$. Let $\{m(\lambda)\}_{\lambda}$, $\{n(\lambda)\}_{\lambda}$ be sequences of positive integers, both tending to ∞ as $\lambda \to \infty$. Consider the Padé approximant P_{λ}/Q_{λ} of the entry $(m(\lambda), n(\lambda))$ of the table of $\Sigma a_k z^k$ (degree $(P_{\lambda}) = m(\lambda)$; degree $(Q_{\lambda}) = n(\lambda)$, $Q_{\lambda}(0) = 1$). Then, $P_{\lambda}(z) \to a_0 g(z)$, $Q_{\lambda}(z) \to h(z)$ $(\lambda \to \infty)$. The convergence is uniform in any bounded region of the complex plane. The substitution $\zeta = z^2$ reduces

$\tan z/z$ to a function of the form $A(\zeta)$. The above result therefore yields complete information about the separate convergence of the Padé polynomials of the table of $\tan z/z$. (Received February 19, 1974.)

713-B25. JAGDISH CHANDRA, U.S. Army Research Center, Durham, North Carolina, PAUL WILLIAM DAVIS, Worcester Polytechnic Institute, Worcester, Massachusetts 01609 and BERNARD A. FLEISHMAN, Rensselaer Polytechnic Institute, Troy, New York 12181. *Minimum principles and positive solutions for a class of nonlinear diffusion problems.* Preliminary report.

Under the assumption that $R(x, u, u', Du')$ obeys a minimum principle, we establish minimum principles and bounds for solutions of the nonlinear diffusion operator $R(x, u, u', Du') - S(x, u, u', Du') - f(x, u)$, subject to a variety of nonlinear boundary conditions. In this operator, D denotes any one of the Dini derivatives, $S(x, u, o, w) = 0$, and $f(x, u)u > 0$, $u = o$, for $0 \le x \le 1$, $-\infty < u, w < \infty$. The complete operator $R - S - f$ need not be elliptic. These results are applied to obtain lower bounds on solutions of nonlinear problems arising in enzyme diffusion-kinetics and gas-film lubrication theory. (Received February 20, 1974.)

*713-B26. THEODORE A. BURTON, Southern Illinois University, Carbondale, Illinois 62901 and JOHN R. HADDOCK, Memphis State University, Memphis, Tennessee 38152. *On the delay-differential equation $x''(t) + a(t)f(x(t - r(t))) = 0$.*

We assume $f: R \to R$ and $a, r: R^+ \to R^+$ are continuous and $xf(x) > 0$ if $x \neq 0$. Without requiring that $r(t)$ be strictly positive, conditions are given which insure that each solution of the equation in the title can be continued to $t = +\infty$. Also, for $a(t) \to +\infty$, conditions are given to guarantee that all solutions tend to zero as $t \to +\infty$. These latter results are obtained by first employing a Liouville transformation and then applying Liapunov techniques. (Received February 20, 1974.)

*713-B27. IVAN ERDELYI, Temple University, Philadelphia, Pennsylvania 19122. *Spectral maximal spaces and weak spectral manifolds of unbounded operators.*

Let C be the unique regular spectral capacity of an unbounded operator $T: D(T)\ (\subset X) \to X$ as defined in [Abstract 711-47-20, these *Notices* 21(1974), A-194], X being a Banach space over the complex field π. Define a spectral maximal space (introduced by C. Foias [Arch. Math. 14(1963), 341–349]) of an unbounded T as a subspace $Y \subset D(T)$ invariant under T such that for any other subspace $Z \subset D(T)$ invariant under T, the inclusion $\mathrm{sp}(T|Z) \subset \mathrm{sp}(T|Y)$ implies $Z \subset Y$. Then, for every compact $K \subset \pi$, $C(K)$ is a spectral maximal space of T. Moreover, if $Y \subset C(K)$ is a spectral maximal space of T then $Y = C(\mathrm{sp}(T|Y))$ and $\mathrm{sp}(T|Y)$ is compact. Next, redefine E. Bishop's concept of weak spectral manifold $N(F, T)$ [Pacific J. Math. 9(1959), 379–397] without the restriction of T being bounded as follows. For every closed set F, $N(F, T)$ is the set of all $x \in X$ which have the property that for each $\epsilon > 0$ there exists an X-valued function $\tilde{x}$ analytic on F^c (^{c} for the complement) such that $\|x - (\lambda - T)\tilde{x}(\lambda)\| < \epsilon$ for all $\lambda \in F^c$. It is shown that for every unbounded operator T with regular spectral capacity C and every closed $F \subset \pi$, $N(F, T) = C(F)$. (Received February 20, 1974.)

*713-B28. RATHINDRA N. MUKHERJEE, University of Georgia, Athens, Georgia 30601. *A Hille-Yoshida-Phillips type of theorem for semigroups in a locally convex space.* Preliminary report.

Let X be a locally convex Hausdorff linear topological space, and $\{T_t;\ t \ge 0\}$ a one-parameter family of continuous linear operators $\in L(X, X)$ s.t. $T_t T_s = T_{t+s}$, $T_0 = I$, $\lim_{t\to t_0} T_t x = T_{t_0} x$, for $t_0 \ge 0$ and $x \in X$. Such a family T_t is called an equicontinuous semigroup of class (C_0) if for any continuous seminorm p on X, there exists a continuous seminorm q on X s.t. $p(T_t x) \le q(x)\ \forall\ t \ge 0$, $\forall\ x \in X$. We consider a discrete semigroup $T(k) = B^k$ $(k = 1, 2, 3, \cdots)$ on a sequentially complete, locally convex, Hausdorff linear topological space X, where $B \in L(X, X)$. Necessary and sufficient conditions for $T(k)$ to be equicontinuous are given. These results essentially are extensions of those obtained for discrete semigroups on a Banach space by Gibson [J. Math. Anal. Appl. 39(1972), 761–770]. (Received February 21, 1974.) (Author introduced by Professor Richard Bouldin.)

*713-B29. BENJAMIN LEPSON, Naval Research Laboratory, Washington, D. C. 20375 and Catholic University, Washington, D. C. 20017 and CHARLES S. DAVIS, Anne Arundel Community College, Arnold, Maryland 21012 and Catholic University, Washington, D. C. 20017. *Entire functions of bounded index, bounded value distribution, and bounded* 1*-index.*

Let B denote the class of entire functions of bounded index, and BVD the class of entire functions of bounded value distribution in the sense of Turán plus the constant functions. The relation between these two classes was found by Hayman (Pacific J. Math. 44(1973), 117–137) and states: $f(z) \in BVD$ iff $f'(z) \in B$. This relation, plus the fact that B is not closed w.r.t. addition of constants, although BVD obviously is, leads to consideration of a subclass T_1 of B, the class of entire functions of bounded 1-index, whose definition is the special case $k = 1$ of Abstract 713-B30. It is shown that an entire function $f(z)$ is of bounded 1-index iff $f'(z)$ is of bounded index. Hayman's result is therefore equivalent to the statement that the classes BVD and T_1 are equal. It is not known if T_1 is the largest subclass of B which is closed w.r.t. addition of constants. (Received February 21, 1974.)

*713-B30. CHARLES S. DAVIS, Anne Arundel Community College, Arnold, Maryland 21012 and Catholic University, Washington, D. C. 20017 and BENJAMIN LEPSON, Naval Research Laboratory, Washington, D. C. and Catholic University, Washington, D. C. 20017. *Entire functions of bounded absolute index.*

An entire function $f(z)$ is of bounded k-index for k a nonnegative integer if there exists an integer $N \geq k$ independent of z s.t. $\max_{k \leq s \leq N}\{|f^{(s)}(z)|/s!\} \geq |f^{(n)}(z)|/n!$ $\forall$ $n \geq k$ and all complex z. Denote by T_k the class of functions of bounded k-index. Let T denote the intersection of all T_k and call T the class of functions of bounded absolute index. It is found that, like the class B (identical to T_0) of entire functions of bounded index, the subclasses T and T_k for each k are closed under indefinite integration, not closed under addition, contain all functions satisfying a linear differential equation with polynomial coefficients, the degree of the leading coefficient maximal, and have a characterization similar to that given by Fricke for the class B. Unlike B, T is shown to be closed under addition of a polynomial and closed under differentiation. A function f is in T iff f and all of its derivatives are in B. (Received February 21, 1974.)

*713-B31. T. K. PUTTASWAMY, Ball State University, Muncie, Indiana 47306 and RAJ PAUL SEEKRI, Texas Instruments, Inc., Dallas, Texas. *Solution in the large of a certain differential equation with an irregular singular point of rank greater than one.*

Our purpose is to solve in the large the linear homogeneous ODE (1) $z\, d^3y/dz^3 + K_1\, d^2y/dz^2 + K_2 z^{\alpha} dy/dz + K_3 z^{\alpha-1} y = 0$, with z complex, the constants K_i $(i = 1, 2, 3)$ real or complex, and α a positive integer. Then (1) will have a regular singular point at $z = 0$ and an irregular singular point at $z = \infty$. In general the rank of $z = \infty$ will be > 1. It is also assumed that $K_1 \not\equiv 1 \pmod{\alpha + 1}$ and $K_1 \not\equiv 2 \pmod{\alpha + 1}$. (Received February 21, 1974.)

*713-B32. JAMES E. MILLER, West Virginia University, Morgantown, West Virginia 26506. *On the maximum modulus for meromorphic univalent functions.*

Let $S(p)$ denote the class of all functions $f(z) = z + a_2 z^2 + \cdots$ that are analytic, except at $z = p$, and univalent in the unit disc. A variational formula is used to determine the form of the extremal function that maximizes the modulus in this class. (Received February 21, 1974.)

713-B33. ROGER Y. LYNN, Villanova University, Villanova, Pennsylvania 19085. *Some examples of singularly perturbed boundary value problems with turning points.*

Quadrature solutions for some boundary value problems of the form $\epsilon y'' + fy' + gy = 0$, $-a \leq x \leq b$ $(a > 0,\ b > 0)$, $y(-a; \epsilon) = A$, $y(b; \epsilon) = B$ with higher order turning points are studied as $\epsilon \to 0^+$. Results are compared with those obtained by O'Malley, Ackerberg & O'Malley, and Dorr. (Received February 21, 1974.)

*713-B34. COREEN L. METT, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24060. *Sturm comparison theorems for systems of second order elliptic partial differential equations.*

The fourth order elliptic partial differential equation $\Delta(\alpha\Delta u) + 2\,\mathrm{div}\,(\beta \text{ grad } u) + \gamma u = 0$ ($\Delta =$ n-dimensional Laplacian) can be expressed as the system of second order equations $\Delta u + bu = ew$, $\Delta w + bw = -cu$. Solutions of this system are compared to solutions of the system $\Delta v + dv = fz$, $\Delta z + dz = -hv$ in a region G in E^n with boundary Γ. A typical theorem asserts that for solutions of the above systems with u satisfying the boundary condition $u = \partial u/\partial n = 0$ on Γ, if $f \geq e \geq 0$, $0 \geq d \geq b$, $c \geq h$, $c \not\equiv h$, $z \leq 0$ in G and $v > 0$ at a point in G, then v must have a zero inside G. Results are obtained from an analog of the fourth order Picone-type identity of Dunninger (Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8) 50(1971), 302–313). (Received February 21, 1974.)

*713-B35. PAO-LIU CHOW, Wayne State University, Detroit, Michigan 48202. *Some functional differential equations associated with a random parabolic equation.* Preliminary report.

A parabolic equation with a generalized stochastic process as coefficient is treated. Under the assumption that the coefficient process is a Gaussian random field whose correlation function has bounded variation, the solutions can be shown to exist in the sense of stochastic integrals introduced by Daleckiĭ and Paramonova [Soviet Math. Dokl. 14(1973), 96–100]. For certain functionals of solution processes, Frechét-Volterra differential equations, analogous to Kolmogorov's backward equations and Hopf's equations in turbulence, are derived. The solution to the Hopf's equation for the characteristic functional is constructed, and the results thus obtained also imply the existence and uniqueness of a solution to the random Cauchy problem for the parabolic equation. (Received February 21, 1974.)

713-B36. RAMESH M. KULKARNI, State University College of New York, Potsdam, New York 13676. *Algebraic relationships that yield subnormality.*

Let A be a bounded linear operator on a Hilbert space. Then *Theorem* 1. If $A = A^*A^2$ then A is subnormal. *Theorem* 2. Let $D = A^*A - AA^* \geq 0$ and let $A^*D = DA$, then A is subnormal. There exists an operator A viz unilateral shift which is subnormal but not normal and satisfies the hypotheses in the two theorems. (The first of these theorems was also obtained independently by Mary Embry but our proof is entirely different.) (Received February 21, 1974.)

713-B37. JAMES R. DIEDERICH, University of California, Davis, California 95616. *Uniqueness of solutions of elliptic equations in Lipschitz domains.* Preliminary report.

Let Ω be an open, bounded region in R^n whose boundary, $\partial\Omega$, satisfies a Lipschitz condition. $Lu = a_{ij}(x)u_{ij} + b_i(x)u_i + cu = 0$, $c \leq 0$, where L is an elliptic operator with Holder-α continuous coefficients in Ω. *Theorem.* If f is a finite valued function on $\partial\Omega$, then there is at most one u satisfying $Lu = 0$ and absolutely integrable on Ω which assumes the boundary values f mean continuously. The main methods involve Serrin's kernel parametrix and Harnack's inequality for elliptic equations. (Received February 21, 1974.)

713-B38. GARY W. TOWSLEY, University of Rochester, Rochester, New York 14627. *Conformal deformation of meromorphic functions.* Preliminary report.

Let M be a compact Riemann surface of genus g, $C(M)$ its meromorphic function field. $C(M)$ is given the compact-open topology as a set of maps into the Riemann sphere. In this topology none of its algebraic operations are continuous. It is shown that the path components of $C(M)$ are the conformal deformation classes of $C(M)$. For certain families of surfaces it is shown that the path components are equal to the degree classes. For degrees greater than the genus of the surface the degree classes equal the path components for all surfaces. An example is given of a surface of genus four for which there are two functions of degree three in different path components. Since these functions are of the same degree they are continuously homotopic but any homotopy must leave $C(M)$ at some point. The proof shows that $C(M)-0$ is a holomorphic C^* fibre bundle over the space of principal divisors and investigates the principal divisors as subspaces of products of symmetric products of M. (Received February 21, 1974.) (Author introduced by Professor Gail S. Young.)

713-B39. MARVIN D. TRETKOFF, Stevens Institute of Technology, Hoboken, New Jersey 07030. *Some remarks on analytic continuation.* Preliminary report.

The present paper provides two applications of abstract group theory to functions of a complex variable. First, the example of a finitely generated group with unsolvable word problem is applied to assert the existence of an interesting multivalued transcendental function. The branches of this function, which has a finite number of isolated singularities on the Riemann sphere, do not arise as the solutions of homogeneous linear ordinary differential equations with single-valued analytic coefficients. Next, suppose that one is given a compact Riemann surface X of genus greater than one and on it a loop c which is not null homotopic. Then it is shown that there is a function element w which can be continued throughout the surface and has the following three properties: (1) w has only a finite number of branches, (2) continuation of w along c does not lead to the original branch, (3) the continuations of w are unramified at every place of X. The proofs depend on covering space interpretations of group theoretical concepts and the notion of the monodromy group of an analytic function. (Received February 21, 1974.)

*713-B40. CONSTANTIN CORDUNEANU, University of Rhode Island, Kingston, Rhode Island 02881. *Invariant spaces for a convolution operator and applications to functional equations.* Preliminary report.

Let $k(t)$ be an n by n matrix kernel defined for t in R_+, s.t. $\|k(t)\| \in L$. Then each function space of the form $\exp(at)L(R_+, R^n)$, $a > 0$, is invariant w.r.t. the convolution operator $(Kx)(t) = \int_0^t k(t-s)x(s)\,ds$. A similar remark is true when the space L is replaced by some other function spaces. These facts are then used to find existence theorems for some classes of functional-integral equations. (Received February 21, 1974.)

*713-B41. ROBERT E. ATALLA, Ohio University, Athens, Ohio 45701. *The second dual of certain Choquet simplexes.* Preliminary report.

Let X be compact and L a closed separating subspace of $C(X)$ with $1 \in L$. Assume (i) the state space of L is a simplex, (ii) each maximal measure assigns full measure to the extreme points, (iii) if $f \in C(X)$, then Pf is Baire, where $Pf(x) = \int f\,dp_x$, p_x the maximal measure for x. Let B_L be the smallest system containing L and closed under bounded pointwise convergence. Using Gordon's representation of $C(X)^{**}$ as $C_b(Y)$, where $Y = Y_\delta$ is a certain extension of X [Amer. J. Math. 88(1966), 827–843], we show that the map $P^{**}: C(X)^{**} \to B^{**}$ (B = bounded Baire functions) induces a projection $Q: C_b(Y) \to C_b(Y)$. *Theorem* 1. The following are equivalent: (a) range $Q = L^{\perp\perp} (= L^{**})$, (b) $f \in C(X) \Rightarrow Pf \in B_L$. *Theorem* 2. Assume $f \in C(X) \Rightarrow Pf \in B_L$. If $g \in C_b(Y)$, then $Qg = g$ iff g is 'locally in B_L'. The B_L concept is (essentially) introduced and used by Lacey and Morris [Proc. Amer. Math. Soc. 23(1969), 151–157]. (Received February 21, 1974.)

*713-B42. ROBERT B. KELMAN, Department of Computer Science, Colorado State University, Ft. Collins, Colorado 80521. *An analog of the Dirichlet-Jordan theorem for dual Fourier series.*

An analog of the Dirichlet-Jordan theorem and a uniqueness theorem are established for dual trigonometric series equations when the right-hand sides of the dual equations are given functions of bounded variation. In the usual fashion there are two series in these equations one of which has coefficients, say, $\{j_n/n\}$ or $\{j_n/n-\frac{1}{2}\}$, and the other coefficients $\{j_n\}$. In the first series we establish ordinary convergence and in the second Abel-Poisson convergence. In general $j_n \neq o(1)$ and the second series does not converge in the ordinary sense on any set of positive measure. Further regularity assumptions are obtained for the given functions of bounded variation needed to insure the square summability of $\{j_n\}$. A best possible estimate on growth conditions for $\{j_n\}$ needed for uniqueness is given. In the proof a mixed boundary value problem of potential theory is associated with the dual series. Conformal mapping replaces this potential problem with one in which Dirichlet boundary conditions can be associated with the dual series. Analysis of this new problem provides the denouement. A typical dual equation to which these results applies is $\Sigma j_n \sin nx = f(x)$ on $(0, c)$ and $\Sigma(j_n/n) \sin nx = g(x)$ on (c, π). (Received February 21, 1974.)

713-B43. ANTHONY V. LAGINESTRA, 67 Diamond Street, Elmont, Long Island, New York 11003 and WILLIAM E. BOYCE, Rensselaer Polytechnic Institute, Troy, New York 12181. *On convergence and evaluation of sums of reciprocal powers of eigenvalues for certain operators on a Hilbert space which are meromorphic functions of the eigenvalue parameter.* Preliminary report.

Notation. $\mathcal{H}$ a complex Hilbert space; $H_0, H_1, \cdots, H_s$ compact linear operators on $\mathcal{H}$, where $H_i \in C_{\alpha(i)}$, $i = 0, \cdots, s$, $\alpha(i) > 0$; $H(\lambda) = \Sigma_{i=0}^{s} \lambda^i H_i$; $k_0 = \max\{(i+1)\,\alpha(i): i = 0, \cdots, s\}$; $x_j(\lambda), y_j(\lambda)$, $j = 1, \cdots, p$, $\mathcal{H}$ valued functions, entire w.r.t. norm on $\mathcal{H}$; $f(\lambda)$ an entire complex-valued function, $f(0) \neq 0$; $P(\lambda)u = (f(\lambda))^{-1} \cdot \Sigma_{j=0}^{p} x_j(\lambda) < u, y_j(\bar{\lambda}) >$; $K(\lambda) = H(\lambda) + P(\lambda)$; k an integer. We wish to study eigenvalues of (1) $\lambda K(\lambda) v = v$. For each $k \geq k_0 - 1$, $\exists\, D(\lambda, k)$ (entire or meromorphic) satisfying $D(0, k) = 1$, $-D'(\lambda, k)/D(\lambda, k) = \operatorname{trace}\{T(\lambda) - \Sigma_{i=0}^{k-1} \lambda^i m_i(T)\}$, $T(\lambda) = [\lambda K(\lambda)]'[I - \lambda K(\lambda)]^{-1}$, $m_i(T) = (i!)^{-1}(d^i/d\lambda^i) T(\lambda)|_{\lambda=0}$. The derivatives are w.r.t. "sup" norm. Eigenvalues of (1) are zeroes of $D(\lambda, k)$; if $D(a, k) = 0$, then either $\lambda = a$ is an eigenvalue of (1) or $f(a) = 0$. Let $\{\nu_i\}$ and $\{\rho_i\}$ be sequences (independent of k) of zeroes and poles of $D(\lambda, k)$, taken according to multiplicity. Suppose $\exists$ a constant $\gamma \geq 0$ and a function $A(\epsilon) > 0 \ni$ for each $\epsilon > 0$, $|f(\lambda)| + \Sigma_{j=0}^{p} \|x_j(\lambda)\| + \|y_j(\lambda)\| \leq A(\epsilon) \exp |\lambda|^{\gamma+\epsilon}$. Then if $\delta = \max\{\gamma, k_0\}$, (2) $\Sigma_i |\nu_i|^{-(\delta+\epsilon)} < +\infty$, and $\Sigma_i |\rho_i|^{-(\delta+\epsilon)} < +\infty$, if $\epsilon > 0$. Also, (3) $\Sigma_i \nu_i^{-(k+1)} - \Sigma_i \rho_i^{-(k+1)} = \operatorname{trace} m_k(T)$ holds for each $k > \delta - 1$. If $x_j(\lambda), y_j(\lambda)$ are polynomials, $f(\lambda) \equiv 1$, we may take $\delta = k_0$ and set $\epsilon = 0$ in (2). Then (3) holds for $k \geq k_0 - 1$. A recursion formula can be used to evaluate the right side of (3). (Received February 21, 1974.)

Applied Mathematics

713-C1. HERBERT E. SALZER, 941 Washington Avenue, Brooklyn, New York 11225. *Converting interpolation series into Chebyshev series by recurrence formulas.*

Interpolation series (divided difference, Gregory-Newton, Gauss, Stirling, Bessel) are converted into Chebyshev (or Jacobi) series by applying a previously derived general five-term recurrence formula (Salzer, *Comm. ACM* 16(1973), 705–707). It employs the coefficients in three-term linear recurrence formulas (same kind as for orthogonal polynomials) which have been found for the mth degree nonorthogonal polynomial coefficients of the differences used in the interpolation series. In the Gauss, Stirling and Bessel series, the coefficients in the recurrence formulas vary with the parity of m. The basic five-term recurrence formula is applicable also to (1) inter- and intraconversion of power series in $ax + b$, divided difference and equal-interval interpolation series (including subtabulation), and Chebyshev series, (2) obtaining Chebyshev series for solutions of difference equations, (3) the derivation of formulas for Chebyshev coefficients in terms of differences, and (4) the conversion of interpolation series into Chebyshev series, for more than one variable. (Received November 21, 1973.)

713-C2. DALTON R. HUNKINS, Kutztown State College, Kutztown, Pennsylvania 19530. *Cubatures of precision $2k$ and $2k + 1$ for hyperrectangles.* Preliminary report.

It is well known that integration formulas of precision $2k$ $(2k + 1)$ for a region in n space which is a Cartesian product of intervals can be obtained from one dimension Radau (Gauss) rules. The number of function evaluations in these product cubatures is $(k + 1)^n$. In this paper, an algorithm is given for obtaining cubatures on hyperrectangles in n space of precision $2k$ (in many instances $2k + 1$) which use $(k + 1)(k)^{n-1}$ number of nodes. The weights and nodes of these new formulas are derived from one dimension generalized Radau rules. (Received January 22, 1974.)

*713-C3. CHRIS RORRES and WYMAN G. FAIR, Drexel University, Philadelphia, Pennsylvania 19104. *Optimal equilibrium harvesting policy of an age-specific population.* Preliminary report.

The optimal equilibrium harvesting policy for an age-specific population of females is investigated. Using a Leslie growth model, admissible harvesting policies are considered in which a certain fraction of each age group is harvested in such a way that the population returns to an identical age configuration after each harvest. The optimal harvesting policy– that policy which maximizes the yield subject to a linear ecological or economic constraint – is found. It is shown to consist of harvesting a certain fraction of a primary age group and all of a secondary age group. The primary age group is younger than, or the same age as, the 'critical' age group – that age group during which the cumulative expected number of daughters born to a female first exceeds one. The secondary age group is older than the critical age group, though in some cases is absent from the problem. The two harvesting ages are dependent on the birth and death rates and on the economic parameters of the problem. (Received February 11, 1974.)

713-C4. WERNER E. KOHLER, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. *Coupled power equations for stochastic systems with propagating and evanescent modes.* Preliminary report.

Let $D_1 \equiv \operatorname{diag}(\beta_1, \cdots, \beta_n) \oplus 0_m$, $D_2 = 0_n \oplus \operatorname{diag}(\kappa_1, \cdots, \kappa_m)$ and let B be an $(n+m) \times (n+m)$ matrix-valued stochastic process. The initial value problem $X' = [jD_1 - D_2 + \epsilon B]X$, $X(0) = X_0$, is studied as a model of a stochastically-coupled system of propagating and evanescent modes. An adaptation of the stochastic perturbation theory of Papanicolau and Keller (SIAM J. Appl. Math. 21(1971), 287–305), is used to derive coupled power equations for the system. As a special case, a result of Matveev (Theor. Probability Appl. 10(1965)) is recovered. (Received February 13, 1974.)

713-C5. GIAN-CARLO ROTA, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. *Significance arithmetic.*

In this work, done jointly with F. Faltin, N. Metropolis and B. Ross, an algorithmic construction is given of the real number field which is based upon the notion of *carrying.* Real numbers are defined as equivalence classes of strings of integers under the equivalence relation of carrying. The structure of the equivalence classes is investigated, and it is shown that the operations of arithmetic can be defined by explicit formulas leading to an analysis of propagation of errors and of complexity. (Received February 18, 1974.)

*713-C6. SOL. I. RUBINOW, Cornell University, Graduate School of Medical Sciences, New York, New York 10021. *Some mathematical problems in biology.*

• A survey will be presented of some mathematical problems encountered in biological studies. A brief description of the biological problems to be discussed are as follows. 1. *Blood flow.* The steady flow of blood through arteries and veins can be represented as flow through elastic tubes. For such flow, the collapsibility of the tube must be taken into account in order to represent it correctly. 2. *Tracer analysis.* The study of physiological systems by means of radioactive tracers is called compartment analysis, in biology. The inferences to be drawn from such studies require the solution to an inverse problem. 3. *Cell populations.* The growth of cell populations can conveniently be described by differential equations which utilize either age and time and maturity and time as independent variables. The different descriptions usually lead to opposing descriptions of the growth process. 4. *Enzymatic reactions.* The Michaelis-Menten equation is the most widely and successfully utilized quantitative representation of enzymatic reactions. Singular perturbation theory underlies its mathematical derivation. (Received February 20, 1974.)

Logic and Foundations

713-E1. WILLIAM C. POWELL, State University of New York at Buffalo, Amherst, New York 14226. *How much classical model theory depends on the law of excluded middle?* Preliminary report.

We show that a generalization of Keisler's theorem for complete embeddings does not depend on the law of excluded middle. The proof is an adaptation of the proof of Corollary 2 in "Variations of Keisler's theorem for complete embeddings", Fund. Math. 81(1973), 41–52, and can be formalized in a natural axiomatization of ZF without the law of excluded middle. We consider only formulas built up from $\neg, \wedge, \vee, \rightarrow, \exists$. Thus, we only require that elementary embeddings preserve formulas in which $\forall$ does not occur. We say a submodel $\mathfrak{A}$ of $\beth$ is $\beth$-definable if the universe A of $\mathfrak{A}$ and all relations in $\mathfrak{A}$ are $\beth$-definable. Furthermore, we say $\mathfrak{A}$ is Skolem in $\beth$ if any $\beth$-definable relations $R \subseteq A^n \times C$ can be uniformized by a $\beth$-definable relation. We say an embedding $j: \mathfrak{A} \rightarrow \mathfrak{B}$ is $\beth$-complete if there are expansions $\mathfrak{A}^\#$ and $\mathfrak{B}^\#$ of $\mathfrak{A}$ and $\mathfrak{B}$ such that $j: \mathfrak{A}^\# \rightarrow \mathfrak{B}^\#$ is elementary and $\mathfrak{A}^\#$ contains all $\beth$-definable relations on A. *Theorem.* Suppose that $\mathfrak{A}$ is a submodel of $\beth$ that is $\beth$-definable and Skolem in $\beth$. Then an elementary embedding $j: \mathfrak{A} \rightarrow \mathfrak{B}$ can be extended to $\beth$ iff $j: \mathfrak{A} \rightarrow \mathfrak{B}$ is $\beth$-complete. (Received February 21, 1974.)

Statistics and Probability

*713-F1. MARK A. PINSKY, Northwestern University, Evanston, Illinois 60201. *Asymptotic stability of stochastic differential equations.*

Let X_t^x be the solution of the system of Itô equations $dX = \sigma(X)dw + b(X)dt$, $X^x = x$; (σ, b) satisfy $|\sigma(x) - \sigma(y)| + |b(x) - b(y)| \leq K|x - y|$, $\sigma(0) = 0 = b(0)$. A sufficient condition that $\lim_{x\to 0} P\{\lim_{t\to\infty} X_t^x - 0\}$ is that there exists a real function f satisfying $Lf \equiv \frac{1}{2}\Sigma_{i,j,s}\sigma_{is}\sigma_{js}f_{x_ix_j} + \Sigma_i b_i f_{x_i} \leq -1$, $\Sigma_{i,s}\sigma_{is}f_{x_i} \leq$ const. for $x \neq 0$, s.t. $\lim_{x\to 0} f(x) = -\infty$. Several sufficient conditions are given in terms of the coefficients (σ, b). This generalizes the work of Hasminskii (Theor. Probability Appl. 12(1967), 144–147). Applications are made to the Dirichlet problem $Lu = 0$ in a bounded domain in R^2, where data is prescribed on the set of boundary points which are limits of X_t^x when $t \rightarrow \infty$. (Received December 21, 1973.)

713-F2. WAYNE G. SULLIVAN, Dublin Institute for Advanced Studies, Dublin 4, Ireland. *Relaxation times for the evolution of Markov fields.* Preliminary report.

Evolution of Markov fields with generators of the form considered by Liggett (Trans. Amer. Math. Soc. 165(1972), 471–481) are studied. Such an evolution is said to have a finite relaxation time if there is a $\tau > 0$ such that each finite cylinder function evolves to a limiting value more rapidly than $\exp(-t/\tau)$. Theorems for comparison of relaxation times are deduced from a certain dissipative property of the generators. A reduction theorem for "attractive" evolutions is proved. These theorems are applied to statistical mechanical models. (Received February 13, 1974.)

*713-F3. ALEX S. PAPADOPOULOS, Keene State College, Keene, New Hampshire 03431. *Scale and reliability bounds for Gamma under a Bayesian influence.*

The Gamma pdf with random scale parameter θ is considered as a failure model. $(1 - a)$ 100% confidence bounds for the random scale parameter θ and the reliability function are obtained when θ is characterized (a) by an Inverted Gamma prior and (b) by a Uniform prior. Let $\underline{x} = (x_1, x_2, \cdots, x_n)$ denote the observed ordered lifetimes of n items that have been placed on a lifetest. The posterior pdf of θ given $\underline{x}$ when the prior of θ is an Inverted Gamma is a Gamma, "closure under sampling." A computer simulation was employed to compare the $(1 - a)$% lower confidence bound of θ and of the reliability function of the MLE approach to the Bayesian approach. As we expected the Bayesian estimates of the $(1 - a)$ 100% lower confidence bound of θ and of the reliability function are better than the MLE. (Received February 15, 1974.)

713-F4. GEORGE C. PAPANICOLAOU, Courant Institute, New York University, New York, New York 10012. *Asymptotic methods for stochastic equations.*

• We will present a survey of asymptotic results for stochastic equations with a small parameter. Each theorem will be accompanied by several examples, some of which are of interest in themselves. We will assess the effectiveness and scope of the presently available results and we will conclude with a few remarks on some unsolved problems. (Received February 19, 1974.)

713-F5. DHANDAPANI KANNAN, University of Georgia, Athens, Georgia 30602. *Potential operators in Dirichlet spaces.* Preliminary report.

Let (D, b) be a regular $L^2(E, m)$-Dirichlet space, where E is a separable locally compact Hausdorff space, m is an everywhere dense Radon measure on E. A positive symmetric linear operator G from a dense linear subset of L^2 to D is called a (symmetric) *potential operator* if, $b(Gf, g) = \langle f, g\rangle_{L^2}$ holds for all $f \in \mathfrak{D}(G)$ and $g \in D$. Let $\{G_\lambda, \lambda > 0\}$ be an L^2-symmetric resolvent family (associated with D) and V be the Yosida potential operator of $\{G_\lambda\}$. It is not hard to see that if V exists, then G exists and $V = G$. We find conditions under which the converse will be true. We can define b in terms of G. We give some basic potential theoretic results in terms of G. (Received February 21, 1974.)

713-F6. JAMES H. ABBCTT, Louisiana State University, New Orleans, Louisiana 70122. *Difference structures in probability and analysis.* Preliminary report.

The following observations on a class $\mathcal{C}$ of subsets of a space Ω extend to some function spaces and abstract difference structures, and have implications in algebratizing developments of integrals and measures: For a proper difference $(\triangleright)$-closed $\mathcal{C}(A \supset B$ each in $\mathcal{C} \Rightarrow A \triangleright B = A\backslash B \in \mathcal{C})$, $\mathcal{C}$ is disjoint union $(\oplus)$-closed $(A, B$ disjoint in $\mathcal{C} \Rightarrow A \oplus B = A \cup B \in \mathcal{C})$ iff $\mathcal{C}$ is disjoint directed $(A, B$ disjoint in $\mathcal{C} \Rightarrow \exists\, C \in \mathcal{C}, A \subset C, B \subset C)$. For $\mathcal{C}$ $(\backslash)$-closed, $\mathcal{C}$ is $(\cup)$-closed iff $\mathcal{C}$ is directed. These difference classes form Moore families (Birkhoff, "Lattice theory") with closure operations $(\triangleright)$ and $(\backslash)$, and here the types of directedness even correspond to closure operations $(\oplus)$ and $(\cup)$. Since $\mathcal{C}$ is directed if $\Omega \in \mathcal{C}$, the joint closure operation $(\Omega, \triangleright) = (\Omega, \triangleright, \oplus)$, and thus disjoint union is a superfluous requirement in a λ-class in Ω (Dynkin, "Markov processes"). Also $\mathcal{C}$ is a field in Ω iff $\mathcal{C}$ is $(\Omega, \backslash)$-closed, i.e. $\Omega \in \mathcal{C}$ and C is $(\backslash)$-closed. Indeed, $(\backslash) = (\triangleright, \cap) = (\triangleright)(\cap)$. (Received February 21, 1974.)

713-F7. GEZA SCHAY, University of Massachusetts, Boston, Massachusetts 02127. *Backward-forward duality for certain random evolutions.* Preliminary report.

Let X be a normed real vector-space with n transformation groups $\{T_i(t); t \geq 0\}$, and $\{v(t); t \geq 0\}$ a Markov chain with state space $N = \{1, \cdots, n\}$. Consider points $(x(t), v(t))$ moving in the phase-space $X \times N$, s.t. if $v(t) = i$ during a time interval $[s, s+u]$, then $(x(t), v(t)) = (T_i(t)x(s), i)$ in this interval, and (x, i) jumps to (x, j) when v jumps from i to j. Let $P_{ij}(t, x, y)$ be the transition probabilities and densities for the $(x(t), v(t))$ process, and f, g be integrable functions from X to R^n with components f_i and g_i. Define operators $S(t)$ and $T(t)$ from R^n to R^n by $(S(t)f)_j(y) = \Sigma_i \int f_i(x) P_{ij}(t, x, y)\,dx$ and $(T(t)g)_i(x) = \Sigma_j \int g_j(y) P_{ij}(t, x, y)\,dy$. If we define a scalar product by $(f, g) = \Sigma_i \int f_i(x) g_i(x)\,dx$, then S and T can be shown to be adjoints, and the functions Sf and Tg to satisfy adjoint forward and backward differential equations respectively. (Received February 21, 1974.)

Topology

*713-G1. JULIAN R. EISNER, Princeton University, Princeton, New Jersey 08540. *A characterization of fibered knots.*

All knots, k, and their spanning surfaces, F, are tamely embedded in S^3, and all spanning surfaces are orientable. A *strong self-equivalence* of F of k is an isotopic deformation of S^3, fixing k (throughout the isotopy) and taking F back onto itself. *Theorem* 1. To any strong self-equivalence H (of F of k) we can associate a winding number $w(H)$; intuitively, $w(H)$ is the number of times F winds around k during the isotopy. *Theorem* 2. If k is a nonfibered knot and H is a strong self-equivalence of F of k, then $w(H) = 0$. *Theorem* 3.

If k is a fibered knot and F is a spanning surface of k, then there is a strong self-equivalence of F with preassigned winding number. *Theorem* 4. A knot is fibered iff it has a spanning surface admitting a strong self-equivalence with nonzero winding number. (Received January 14, 1974.)

*713-G2. HENRY M. WALKER, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. *An equivariant Serre spectral sequence.*

Let G be a compact topological group and let $\mathfrak{F}$ be a fixed family of orbit types. If G and $\mathfrak{F}$ have the property that $(G/K)^L$ is a (nonequivariant) CW complex for (L), $(K) \in \mathfrak{F}$ (e.g., G may be a discrete group or a compact Lie group), then cellular homology and cohomology may be defined (H. M. Walker, Thesis, Massachusetts Institute of Technology, 1973). For $f: E \to B$, a G-map of equivariant CW complexes, suppose f has the G-homotopy lifting property, and B is equivariantly path connected. Then the pullback F_K of E from any G-map $G/K \to B$ is unique up to G-homotopy equivalence, for each $(K) \in \mathfrak{F}$. For a contravariant coefficient system l, suppose the first G-homotopy groups of B act trivially on $H_G^*(F_K; l)$. Then the $H_G^n(F_K; l)$ define a contravariant coefficient system, denoted $H_G^n(F; l)$, and there is a first quadrant spectral sequence with $E_2^{p,q} = H_G^p(B; H_G^q(F; l))$ converging to $H_G^{p+q}(E; l)$. There is a similar result in homology. (Received January 21, 1974.)

*713-G3. KENNETH A. PERKO, JR., 400 Central Park West, New York, New York 10025. *On* 10-*crossing knots.*

We continue our investigation of the 165 10-crossing knot types tabled in the nineteenth century. [See Abstract 73T-G79, these *Notices* 20(1973), A-453. (The citation to "Knotentheorie" should read "Chapter III, § 15.")] They are all, in fact, prime and the only amphicheirals are the 13 identified as such by Little and Tait in 1885. [Cf. Conway's table and beware of typographical errors for δ of 10_9, 10_{91}, $10_{3\,\mathrm{VIII}}$ and $10_{6\,\mathrm{VII}}$.] For most examples Professor Murasugi has kindly advised us of a known method of proof. The rest may be dealt with by linking numbers in, or homology of, appropriate noncyclic coverings. (Received January 28, 1974.)

*713-G4. STAVROS G. PAPASTAVRIDIS, Brandeis University, Waltham, Massachusetts 02139. *Polynomial algebras which are modules over the* mod-p *Steenrod algebra.* Preliminary report.

Let M be a graded truncated polynomial algebra of truncation at least p, which is a module over the mod-p Steenrod algebra (p an odd prime). Let $\overline{M} = \bigoplus_{i \geq 1} M_i$ and $D_i = \overline{M}\overline{M} \cdots \overline{M}$ (i factors). We will call a natural number n regular if M_{2n} does not contain elements of $\bigoplus_{2 \leq i \leq p-1} D_i$. *Main result.* Let $k \geq p - 4$, $1 \leq r \leq p - 1$, all numbers $p(p^i k - a_1 p^{i_1} - a_2 p^{i_2} - \cdots - a_n p^{i_n}) + r + m$, regular $\forall\, 0 \leq m \leq p - 1 - r$ and $a_1, a_2, \cdots, a_n \geq 0$, $a_1 + \cdots + a_n = m$, $i \geq i_1 \geq i_2 \geq \cdots \geq i_n \geq 0$; then, if there is a polynomial generator x of M in dimension $2(pk + r)$, there will be a polynomial generator y in dimension $2p(k - p + r + 1)$ s.t. $x = P^{p-r} y + d$, $d \in Dp$. An analogous, though complex, result exists for polynomial generators appearing in dimension $2p^l(pk + r)$, $l > 0$. The above generalizes results of Thomas (Ann. of Math. 77(1963), 306–317), for $p = 2$. Our proof is patterned after his. (Received February 1, 1974.)

*713-G5. ULRICH KOSCHORKE, Rutgers University, New Brunswick, New Jersey 08903. *Rational bordism of frame fields, immersions and k-mersions.*

Let $\mathfrak{N}_n(m, k)$ (resp. $\Omega_n(m, k)$) denote the group of bordism classes of pairs (M, h) where M is an arbitrary (resp. oriented) closed smooth n-manifold and the bundle morphism $h: TM \to M \times \mathbf{R}^m$ has rank $\geq k$ everywhere. This is bordism of m-frame fields for $n \geq m = k$, of (stably) parallelized manifolds for $n = m = k$, of immersions for $n = k < m$, and of k-mersions for $n < m$. *Theorem.* If $m \equiv 0(4)$, $n \equiv k \equiv 0(2)$, then $\mathfrak{N}_n(m, k) \otimes \mathbf{Q} \cong H_{n+1-(n+1-k)(m+1-k)}(BO(n-k) \times BO(m-k); \mathbf{Q})$, otherwise $\mathfrak{N}_n(m, k) \otimes \mathbf{Q} = 0$. The multiplication in the 3-graded algebra $\Sigma \mathfrak{N}_n(m, k) \otimes \mathbf{Q}$ is described by a formula involving Hankel determinants in Pontrjagin classes. Next let $\Omega_n(m, k) \otimes \mathbf{Q} = \Omega_n^+(m, k) \otimes \mathbf{Q} \oplus \Omega_n^-(m, k) \otimes \mathbf{Q}$ be the decomposition into eigenspaces of the involution obtained by reversing orientations. *Theorem.* Natural forgetful maps give rise to an isomorphism $\Omega_n^+(m, k) \otimes Q \cong \mathfrak{N}_n(m, k) \otimes Q$ and to a monomorphism from $\Omega_n^-(m, k) \otimes Q$ into $\Omega_n \otimes Q$ whose image is described by vanishing conditions on Pontrjagin numbers. *Corollary.* A positive multiple of an oriented n-manifold M is bordant to a manifold with an m-frame field iff all Pontrjagin numbers of M involving some $p_i(M)$, $i > (n - m)/2$, vanish. (Received February 11, 1974.)

713-G6. DENNIS McGAVRAN, University of Connecticut, Waterbury, Connecticut 06710. *T^3-actions on simply connected 6-manifolds.*

We are concerned with T^3-actions on simply connected 6-manifolds M^6. As in the codimension two case, there exists, under certain restrictions, a cross-section to the action. Unlike the codimension two case, the orbit space need not be a disk and there can be finite stability groups. C.T.C. Wall has determined (Invent. Math. 1(1966), 355–374) a complete set of invariants for simply connected 6-manifolds satisfying certain conditions, including $\omega_2(M^6) = 0$. Using equivariant surgery and connected sums, we compute these for certain T^3-manifolds. We then construct a T^3-manifold M^6 with invariants different than any well-known manifold. (Received February 15, 1974.)

*713-G7. STEVEN C. ALTHOEN, Hofstra University, Hempstead, New York 11550. *A geometrical realization of a construction of Bass and Serre.*

Let X be a path connected topological space with a countable open cover with path connected elements, the intersection of any three of which is void. A graph is constructed in X by selecting, for vertices, basepoints in each of the elements of the cover. Points are also selected in each path component of the intersection of elements in the cover. Pairs of paths from these points to the basepoints of the elements of the cover in which they lie provide edges in the obvious way. A graph of groups (for definitions see Bass and Serre, "Groupes discretes", unpublished notes, 1968) is obtained by assigning to each vertex the fundamental group of the cover element at that vertex and by assigning to each edge the fundamental group of the intersection through which the edge runs. The necessary homomorphisms are obtained by using the conjugations induced by the selected paths. A geometric proof of the following theorem (first proved by J. C. Chipman) is given by using the notion of fundamental groupoid to enable a description of the actual isomorphisms involved. *Theorem.* The fundamental group of X is isomorphic to the fundamental group of the graph of groups described above. (Received February 18, 1974.)

713-G8. DAVID J. SPROWS, Villanova University, Villanova, Pennsylvania 19085. *Homeotopy groups of compact 2-manifolds.* Preliminary report.

Let X be a 2-manifold and let $H(X)$ denote the homeotopy group (or mapping class group) of X. Several results have been obtained concerning $H(X)$ in the case X is of the form $M - F_n$ where M is a closed 2-manifold and F_n is a set of n distinct points in M. It is shown that these results give rise immediately to corresponding results for compact 2-manifolds. In particular, it is shown that if Y is the compact 2-manifold obtained by removing the interiors of n disjoint closed discs from some closed manifold M, then $H(Y)$ is isomorphic to $H(M - F_n)$. (Received February 18, 1974.)

713-G9. OKAN GUREL, IBM Scientific Centers, White Plains, New York 10604. *Homoclinic limit cycles.*

In the global analysis of differential systems, (X_m, f) possessing singular solutions, $S[X_m] \in X_m$, bifurcation of $S[X_m]$ at the bifurcation value $m^b \in m$ of the parameter space plays an important role. Although the theory is incomplete many examples have recently been accumulating. The concept of *homoclinic limit cycles* (HLC), coined and introduced in (Abstract 703-G11, these *Notices* 20(1973), A-380), illustrates another possibility of bifurcation. Construction of HLC is based on decomposition of hyperbolic singular points prior to *peeling*. The term peeling is more general than and replaces bifurcation as discussed in the above reference.

Central Proposition. HCL's consist of stable and unstable manifolds, W^s and W^u, respectively, and can only be generated by peeling of $S[X_m]$ with W^s and W^u. Since the simplest example of hyperbolic $S[X_m]$ with W^s and W^u is a saddle point, a singular point with a stability of the second level denoted by $w^1_{m1} \oplus w^1_{m2} \oplus w^2_{m1} \oplus w^2_{m2}$ (Math. Systems Theory 7(1973), 154–163) it is also stated that *Corollary.* The stability of HCL is of the level ≥ 2. (Received February 18, 1974.)

713-G10 WILLIAM P. THURSTON, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. *The existence of foliations.*

• Recent developments have answered the question of existence of foliations in terms of homotopy theory. In particular, *Theorem* 1. A closed manifold has a C^∞ codimension one foliation if its Euler characteristic is zero. *Theorem* 2. Every plane field on a closed manifold is homotopic to a C^0 integrable plane field. On the other hand, Bott has shown that not every plane field is homotopic to a C^2, integrable, plane field; and Reeb showed that the question of finding foliations on manifolds with boundary cannot have quite as simple a solution. The current theory does not give answers for all such questions, but it translates them to a different setting. This theory is the same as the theory developed by Haeflinger in the case of foliations of noncompact manifolds, in which he relied essentially on the Gromov-Phillips theorem. The current method uses, rather, a simplicial technique. (Received February 20, 1974.)

713-G11. HOWARD H. WICKE and JOHN M. WORRELL, JR., Ohio University, Athens, Ohio 45701. *A generalization of quasi-complete spaces.* Preliminary report.

A space X is said to satisfy *condition* (q) iff there exists a monotonically contracting sequence $\mathcal{G}$ of open covers of X such that for all decreasing representatives G of $\mathcal{G}$ for which $\bigcap_n G_n \neq \emptyset$, if K is a decreasing sequence of nonempty closed sets such that $K_n \subseteq G_n$ for all $n \in N$, then $\bigcap_n K_n \neq \emptyset$. The class of spaces satisfying (q) properly includes the classes of quasi-complete spaces of Creede [Pacific J. Math. 32(1970), 47–54] and of β_c-spaces [General Topology and Appl. 1(1971), 85–100]. *Theorem* 1. Every θ-refinable space which satisfies (q) is quasi-complete. (From this and a theorem of Gittings [Abstract 73T-166, these *Notices* 20(1973), A-511] it follows that every regular T_0 θ-refinable space satisfying (q) satisfies Burke's characterization condition [Pacific J. Math. 35(1970), 285–296] for p-space.) *Theorem* 2. A pararegular space has a base of countable order iff it satisfies (q) and has a primitive base [for a definition of *primitive base*, see Abstract 706-54-5, these *Notices* 20(1973), A-533]. *Corollary* 1. A pararegular space is developable iff it satisfies (q), has a primitive base, and is θ-refinable. *Corollary* 2. A T_2 paracompact p-space is metrizable iff it has a primitive base. (Received February 20, 1974.)

*713-G12. JOE A. GUTHRIE and H. EDWARD STONE, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. *Pseudocompactness and invariance of continuity.*

Given a space (X, τ) and a class Σ of spaces, we study the topologies comparable to τ which determine the same continuous functions into all spaces of Σ, which we call the Σ-*invariant* expansions and compressions of τ. *Theorem.* An expansion is Tychonoff-invariant iff it is real-invariant. A class of counterexamples to the first theorem of Abstract 73T-G144, these *Notices* 20(1973), A-599 is presented. Calling an expansion *improper* at x if it does not alter the neighborhood system at x in the positive direction, we have *Theorem.* If σ is an expansion of τ which is improper at a τ-dense set of points and such that every σ-neighborhood of a point is τ-dense in some τ-open set, then σ is a real-invariant expansion. A space is *perfectly Hausdorff* if every point is a zero-set. *Theorem.* Let X be completely regular and perfectly Hausdorff. If X is pseudocompact, it is minimal perfectly Hausdorff. (Received February 20, 1974.)

*713-G13. ERIC JOHN BRAUDE, Seton Hall University, South Orange, New Jersey 07079. *The Lorch-Tong property and G_δ diagonals.* Preliminary report.

The following property was recently formulated by E. R. Lorch and Hing Tong. A space X has the *Lorch-Tong property* if, for each $x \in X$, there is a countable set $\{V(x, n) : n \in N\}$ of open neighborhoods of x such that $\bigcap_{n=1}^{\infty} V(x_n, n) = \emptyset$ or a singleton for every $\{x_n\}_{n=1}^{\infty} \subseteq X$. For every ordinal $\alpha \in [2, \omega]$, the αth-*order diagonal* of X is the subset $\{x^{\alpha}: x \in X\}$ of X^{α}. *Theorem.* The following are equivalent for a topological space X. (1) X has the Lorch-Tong property; (2) there is an ordinal number α in $[2, \omega]$ such that the αth-order diagonal of X is a G_{δ}; (3) X has a G_{δ} diagonal; (4) for every α in $[2, \omega]$, the αth-order diagonal of X is a G_{δ}. The following corollary is the principal result of a recent paper of Lorch and Tong. *Corollary.* A compact Hausdorff space is metrizable iff it has the Lorch-Tong property. In a recent article the author proved that every compact Hausdorff space with a $\mathcal{G}$-*Souslin* diagonal is metrizable. Analogs of the characterizations of G_{δ} diagonality, including that of Lorch and Tong, are given for $\mathcal{G}$-Souslin diagonality, and a corollary follows which is similar to that above. Finally, we have *Theorem.* No compact nonmetrizable Hausdorff space is the countable union of $\mathcal{G}$-Souslin metrizable subspaces. (Received February 20, 1974.)

713-G14. STANISLAW G. MROWKA, State University of New York at Buffalo, Amherst, New York 14226. *Strong 0-dimensionality of some spaces.* Preliminary report.

Let A and B be disjoint subsets of the Cantor set C; for an $x \in C$, G_x stands for an open subset of C with $x \in G_x$. In the space $(A \times B) \cup (B \times B)$ neighborhoods of a point $(x, y) \in A \times B$ are of the form $\{x\} \times G_y$ and those of an $(x, y) \in B \times B$ are of the form $G_x \times G_y$. Teresawa and, independently, Mrowka and Tan have shown that such a space is strongly 0-dimensional. We prove strong 0-dimensionality of "higher dimensional" analogues of such a space; e.g. of the space $(A \times A \times B) \cup (A \times B \times B) \cup (B \times B \times B)$, where neighborhoods of a point (x, y, z) from a given summand are of the form $\{x\} \times \{y\} \times G_z$, $\{x\} \times G_y \times G_z$, $G_x \times G_y \times G_z$, respectively. (Received February 21, 1974.)

*713-G15. DAVID EDWARDS and PATRICIA McAULEY, State University of New York, Binghamton, New York 13901. *The shape of a map.*

Let $\mathcal{S}_0$ ($\mathcal{S}_{0,\mathrm{maps}}$) be the category of pointed simplicial sets (maps). The Vietoris construction defines functors $\mathcal{V}$: $\mathrm{Top}_0 \to \mathrm{Pro} - \mathcal{S}_0$, and $\mathcal{V}$: $\mathrm{Top}_{0,\mathrm{maps}} \to \mathrm{Pro} - \mathcal{S}_{0,\mathrm{maps}}$. *Theorem.* Let $f: (X, x) \to (Y, y)$ be a continuous pointed map. Then we have a long exact sequence $\cdots \to \mathrm{Pro} - \check{\pi}_i(X, x) \to \mathrm{Pro} - \check{\pi}_i(Y, y) \to \mathrm{Pro} - \check{\pi}_i(f) \to \cdots$. If f is movable, then the inverse limit sequence $\to \check{\pi}_i(X, x) \to \check{\pi}_i(Y, y) \to \check{\pi}_i(f) \to \cdots$ is also exact. *Theorem.* $\exists$ canonical maps $\mathcal{V}(f^{-1}) \xrightarrow{\psi} (\mathcal{V}(f))^{-1} \xrightarrow{\phi} (F\mathcal{V}(f))^{-1}$ from the Vietoris type of the geometric fiber of f to the fiber of the Vietoris type of f to the Vietoris theoretic fiber of f (defined as the homotopy theoretic fiber of $\mathcal{V}(f)$). The fibration $[0, 1] \to$ (Warsaw Circle) shows that the composition $\mathcal{V}(f^{-1}) \xrightarrow{\rho} (F\mathcal{V}(f))^{-1}$ need not be a homotopy equivalence even when f is assumed to be a Hurewicz fibration. A map $f: X \to Y$ is a Vietoris quasi-fibration if for every choice of pointings the canonical map $\mathcal{V}(f^{-1}) \xrightarrow{\rho} (F\mathcal{V}(f))^{-1}$ is a weak homotopy equivalence. *Theorem.* If $F \to E \to B$ is a Vietoris quasi-fibration, then $\exists$ an exact sequence $\cdots \to \mathrm{Pro} - \check{\pi}_i(F) \to \mathrm{Pro} - \check{\pi}_i(E) \to \mathrm{Pro} - \check{\pi}_i(B) \to \cdots$. If F, E, and B are movable, then the inverse limit sequence $\cdots \to \check{\pi}_i(F) \to \pi_i(E) \to \check{\pi}_i(B) \to \cdots$ is also exact. (Received February 21, 1974.)

*713-G16. PAUL A. VINCENT, Université de Moncton, Moncton, New Brunswick E1A 3E9, Canada. *A metrization theorem for 2-manifolds.* Preliminary report.

The definition of a net on a 2-manifold M (Abstract 701-54-37, these *Notices* 20(1973), A-179), is broadened. *Definition.* A pair $[\mathfrak{A}, \mathfrak{B}]$ of families of generalized continua in M forms a *net* if (1) $\mathfrak{A}$ and $\mathfrak{B}$ are locally U.S.C., (2) for any $A \in \mathfrak{A}$, $B \in \mathfrak{B}$, $A \cap B$ is discrete; (3) for each point $p \in M$ $\exists$ a disk D about p s.t. (a) each element of $\mathfrak{A}$ and $\mathfrak{B}$ meets $\mathcal{C}D$, and (b) for each point $q \in D$ $\exists$ unique elements $A \in \mathfrak{A}$, $B \in \mathfrak{B}$ s.t. $\{q\} = A_q \cap B_q$ where A_q and B_q are the components of $A \cap D$ and $B \cap D$, respectively, containing q. The theorems in the above mentioned abstract still hold. *Theorem.* If each element of $\mathfrak{A}$ and $\mathfrak{B}$ is separable, M is metrizable. (Received February 21, 1974.)

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Algebra & Theory of Numbers

*714-A1. GORDON H. HUGHES, University of California, Riverside, California 92502. *Radical and coradical theory in monoidal categories.* Preliminary report.

Suppose C is the category of nonunital monoids in a monoidal category $(B, \Box, E)$ where (1) B is abelian and $\Box$ preserves finite colimits. It follows that kernels in C are exactly ideals (suitably defined) and there is a lattice theorem for ideals as well as First and Second Isomorphism Theorems. Call C a *radical (admitting) category* if (2) for any c in C the lub and glb of any collection of ideals of c is an ideal. In this setting an abstract radical theory is defined after Amitsur-Kurosch. Standard results are obtained including theorems on generating upper and lower radical classes. The nilpotent, prime, and Brown-McCoy radicals are defined and structure theorems are obtained. When B is also bicomplete and biclosed, condition (2) is always satisfied. Examples of radical categories are constructed using results from the theory of closed categories. A dual theory is defined in the category C' of noncounital comonoids in a monoidal category $(B', \Box', E')$ satisfying the dual condition $(1)^*$. When C' satisfies $(2)^*$ call C' a *coradical category.* An example is the category of noncounital coalgebras. In this case the dual to the Brown-McCoy radical gives the coradical known from coalgebra theory. (Received January 14, 1974.) (Author introduced by Professor R. E. Block.)

*714-A2. JOHN de PILLIS, University of California, Riverside, California 92502. *Decomposable tensors as sums of dyads.*

A decomposable element $A \otimes B$ of the algebraic tensor product $\mathfrak{A} \otimes \mathfrak{B}$ of matrix (or operator) algebras, $\mathfrak{A}$ and $\mathfrak{B}$, can be identified in two ways as a linear transformation, viz., either (1) $A \otimes_1 B$: $C \to ACB^t$, or the dyadic operator (2) $A \otimes_2 B$: $C \to [C, B]A$ for all "appropriate" C, where B^t is the transpose of B, and the inner product $[C, B] = \text{trace}(B^*C)$. We characterize those dyads $U_i \otimes_2 V_i$ whose sum $\Sigma_i^N U_i \otimes_2 V_i = A \otimes_1 B^t$, for given $A \in \mathfrak{A}$, $B \in \mathfrak{B}$. We show, too, that if $\Sigma U_i \otimes_2 V_i = A \otimes B^t$, then A and B are each idempotent iff $\{U_i\}, \{V_i\}$ are biorthonormal (i.e., $[U_i, V_j] = \delta_{ij}$). Finally, any $A \otimes B^t$ is realized as a sum $\Sigma^n U_i \otimes_2 V_i$, and once the U_i's are arbitrarily chosen (within certain restraints), the choice of the V_i's becomes unique. These results apply in the search for minimizing the number of scalar multiplications, $P(m, k, n)$, required to multiply an $m \times k$ matrix by a $k \times n$ matrix. (The classical definition of matrix multiplication would require mkn scalar multiplications.) (Received February 11, 1974.)

714-A3. OLGA TAUSSKY-TODD, California Institute of Technology, Pasadena, California 91109. *More on the L-property.*

A pair of square matrices A, B with elements in a field F containing their characteristic roots α_i, β_i is said to have the L-property if $\lambda A + \mu B$ has the characteristic roots $\lambda\alpha_i + \mu\beta_i$ for a fixed ordering of α_i, β_i idenpendent of $\lambda, \mu \in F$. The author recently obtained results on 'approximations' to simultaneous triangular similarity for A, B if further conditions are added (J. Algebra 20(1972), 271–283, where references to earlier work can be found). This study is continued. In particular it is shown that a 3×3 L-pair which has the further property that AB has the characteristic roots $\alpha_i\beta_i$ can be transformed to triangular form simultaneously by a similarity – provided $F \neq GF(2)$. In the latter case the theorem does not hold. (Received February 13, 1974.)

714-A4. HERBERT J. RYSER, California Institute of Technology, Pasadena, California 91109. *Indeterminates and incidence matrices.*

We are concerned with a collection of subsets $X_1, \cdots, X_m$ of an n-set $X = \{x_1, \cdots, x_n\}$ and study the structure of such configurations by way of their incidence matrices and certain related matrices involving indeterminates. A detailed discussion of the matrices and the combinatorial theorems derived is available in the following articles by the author: ["A fundamental matrix equation for finite sets", Proc. Amer. Math. Soc. 34(1972), 332–336; "Analogs of a theorem of Schur on matrix transformations", J. Algebra 25(1973), 176–184; "Indeterminates and incidence matrices", Linear and Multilinear Algebra 1(1973), 149–157]. (Received February 15, 1974.)

714-A5. STANLEY G. WILLIAMSON, University of California, La Jolla, California 92037. *On certain linear algebraic techniques in enumeration.*

Pólya's enumeration theorem, the principle of inclusion exclusion, and certain combinatorial problems associated with the exponential generating function have been studied recently from the point of view of linear and multilinear algebra. An indication of some recent results in these areas will be presented. (Received February 18, 1974.)

*714-A6. EUGENE C. JOHNSEN, University of California, Santa Barbara, California 93106. *Essentially doubly stochastic matrices.*

The author has recently been investigating the theory and structure of essentially doubly stochastic (e.d.s.) matrices. This is motivated by questions concerning the algebraic and combinatorial structure of doubly stochastic and related combinatorial matrices. Here will be discussed some of the results obtained on (i) the characterization of all algebra isomorphisms Ω_n between the algebra of e.d.s. matrices of order n over a field F, $\mathfrak{E}_n(F)$, and the total algebra of matrices of order $n-1$ over F, (ii) the determination of the cases when F has an involution and the isomorphism Ω_n preserves adjoints, (iii) the cogredience of certain matrices in $\mathfrak{E}_n(F)$ and the corresponding canonical forms, and (iv) the factorization of e.d.s. matrices into elementary e.d.s. matrices. (Received February 21, 1974.)

Analysis

*714-B1. JOHN A. ERNEST, University of California, Santa Barbara, California 93106. *A classification, decomposition and spectral multiplicity theory for bounded operators on a separable Hilbert space.*

The title seems to refer to standard material in operator theory, except for the omission of the word "normal." Indeed a complete theory for bounded linear operators on a separable Hilbert space is developed, in which operators are classified up to unitary equivalence by a triplet consisting of a Borel space (its quasi-spectrum), a finite measure class on that space, and a spectral multiplicity function defined on the measure classes absolutely continuous with respect to that measure class. The spectral theorem and the spectral multiplicity theory for normal operators is then a (very) special case of the general theory. (Received February 8, 1974.)

714-B2. FREDRICK A. HOWES, University of Southern California, Los Angeles, California 90007. *On approximate and true solutions of a nonlinear singular perturbation problem.* Preliminary report.

The existence and asymptotic behavior of a solution of the nonlinear boundary value problem $\epsilon y'' = f(t, y, y', \epsilon)$, $y(0, \epsilon) = A(\epsilon)$, $y(1, \epsilon) = B(\epsilon)$, are studied under the principal assumption that the approximate problem $\epsilon u'' = f(t, u, u', \epsilon) + O(\eta) + O(\eta\epsilon^{-1}e^{-mkt/\epsilon})$, $u(0, \epsilon) = A(\epsilon) + O(\eta)$, $u(1, \epsilon) = B(\epsilon) + O(\eta)$, has a solution $u = u(t, \epsilon)$. Here ϵ is a small positive parameter, $m > 0$, $\eta = \eta(\epsilon)$ is a gauge function, and k is a positive constant such that $f_{y'}(t, u(t, \epsilon), u'(t, \epsilon), \epsilon) \leq -k$ for $t \in [0, 1]$. Willett (Arch. Rational Mech. Anal. 23(1966), 276–287), Erdelyi (Ibid. 29(1968), 1–17), and Chang (Abstract 710-B4, these *Notices*, 20(1973), A-679) have considered this problem under the restrictions that $f_{y'y'} = O(\epsilon)$ and $\eta = O(\epsilon)$. In this note, results similar to theirs are shown to hold under the milder assumptions that $f_{y'y'} = O(1)$ and $\eta = o(1)$ as $\epsilon \to 0^+$. (Received February 11, 1974.)

*714-B3. PETER S. BULLEN, University of British Columbia, Vancouver 8, British Columbia, Canada. *On some forms of Whiteley.*

In J. London Math. Soc. 37(1962), 459–469, J. N. Whiteley defines forms $S_n^{[k]}(a)$ of degree k. Their importance is that they have simple properties and include many well-known forms as particular cases. Whiteley proves the following important theorem. *Theorem* 1. If (a) is a nonnegative n-tuple and the forms $S_n^{[k-1]}$, $S_n^{[k]}$, $S_n^{[k+1]}$ are of positive type, then (1) $\{S_n^{[k]}(a)\}^2 \geq (k+1)k^{-1}S_n^{[k-1]}(a)S_n^{[k+1]}(a)$. If the forms are of negative type then (1) is reversed. Hence for forms of positive type and $1 \leq r < k$, (2) $\{r!S_n^{[r]}(a)\}^{1/r} \geq \{k!S_n^{[k]}(a)\}^{1/k}$; if the forms are of negative type this inequality is reversed. The values of numerical coefficients imply the following simpler but weaker inequalities. *Corollary* 2. If (a) is a nonnegative n-tuple and the forms are of positive type, then, unless the forms are zero or all the a_i are zero, (3) $\{S_n^{[k]}(a)\}^2 > S_n^{[k-1]}(a)S_n^{[k+1]}(a)$; $\{S_n^{[r]}\}^{1/r} \geq \{S_n^{[k]}\}^{1/k}$. The reverse inequalities to (3) do not follow from Theorem 1; in fact for forms of negative type we just get (4) $\{S_n^{[k]}(a)\}^2 \leq (k+1)k^{-1}S_n^{[k-1]}(a)S_n^{[k+1]}(a)$, $\{S_n^{[r]}(a)\}1/r \leq ((k!)/(r!))^{1/k}\{S_n^{[k]}(a)\}^{1/k}$, $1 \leq r < k$. The reverse inequalities to (3) are stronger than (4) and evidence suggests they are false and (3) hold for forms of negative type as well. With suitable restrictions this is proved to be the case. (Received February 18, 1974.)

714-B4. GARTH W. WARNER, University of Washington, Seattle, Washington 98195. *L-functions on reductive groups.*

• I will discuss recent work of Langlands, Jacquet-Langlands, Jacquet-Godemont dealing with the general problem of attaching L-functions to irreducible representations of reductive algebraic groups over local fields. (Received February 21, 1974.)

714-B5. MOSES E. COHEN, California State University, Fresno, California 93710. *On series iteration involving hypergeometric functions.* Preliminary report.

In Abstract 702-B15, these *Notices* 20(1973), A-352 we presented a generating function for a class of classical polynomials using infinite series transformations. In this paper, the above techniques are modified to prove another class of generating functions for Hermite, Jacobi, Laguerre and related polynomials. (Received February 21, 1974.)

714-B6. ROY B. LEIPNIK, Naval Weapons Center, Code 60706, China Lake, California 93555. *Quadratic differential equations.*

Differential systems of the type $\dot{x}_i = \Sigma_j a_{ij}x_j + \Sigma_{jk} b_{ijk}x_jx_k$, $i+1, \cdots n$ occur in a number of applications. A recursive procedure permits the explicit determination of periodic and aperiodic solution vectors and subspaces, starting from a linear system. General solutions are expressible as multiple periodic and aperiodic series in terms of the eigenvalues of two $n \times n$ matrices associated with the system. Sufficient conditions for absolute convergence of the series are found with the help of solutions of auxiliary functional equations. (Received February 21, 1974.)

Geometry

714-D1. CHERYL KOCH, California State University, Los Angeles, California 90032 and RAYMOND B. KILLGROVE, San Diego State University, San Diego, California 92115. *Algebraic conics in arbitrary projective planes.* Preliminary report.

Four points can be used to define a unique "conic" as follows: let the four points be, in the coordinates of Hall (Trans. Amer. Math. Soc. 54(1943), 229–277), (0, 0), (1, 1), (0), (∞), then the conic will be the point (∞) and the points (x, x^2) for all x in the coordinate set (excluding ∞). The same set of points can also arise as the intersections of corresponding lines of projectively related pencils through (0, 0) and (∞) where the projectivity is defined by the product of the following two perspectivities: the lines through (∞) are perspective to the lines through (0) by the axis $y = x$, the lines through (0) are perspective to the lines through (0, 0) by the axis $x = 1$. In any ordered affine plane the conic is a boundary for a region starshaped with respect to the vertex. In the Moulton plane some such regions are convex while others are not. (Received February 15, 1974.)

*714-D2. RAYMOND B. KILLGROVE, San Diego State University, San Diego, California 92115. *Sylvester-Gallai conjecture does not imply transversal axiom.*

Given a finite set of points in an affine or projective plane with the property that for every pair of distinct points of the set there is a third point of the set collinear with these, then must the entire set be on one line? Sylvester conjectured yes for the usual plane and Gallai proved it (Coxeter, "The real projective plane"). Subsequently it has been shown false for finite planes and the complex coordinatized affine plane but true for ordered planes. Professor Crowe asked if the conjecture being true could imply the transversal axiom (OVI, Forder, "Foundations of geometry"). It can be shown that in a free plane any such set not part of a line is a confined configuration and hence part of the initial partial plane defining the free plane. There is a partial plane in which no such set not part of a line exists and where the transversal axiom fails regardless how order is introduced. (Received February 15, 1974.)

*714-D3. HUNG-HSI WU, University of California, Berkeley, California 94720. *Applications of some theorems in partial differential equations to geometry.*

• This is an elementary expository talk on Riemannian geometry based on the work of R. E. Greene and the author. It deals with the problems of existence and approximation of functions of geometric interest on noncompact Riemannian manifolds, such as convex functions, harmonic functions, and subharmonic functions. Do they always exist, and if so "how many" are there? Given such a continuous function, can it be approximated by C^∞ functions with the same property? Fairly satisfactory answers are known, and they in turn lead to results giving concrete geometric information about the Riemannian manifold itself. The main tools are certain theorems in elliptic and parabolic partial differential equations, including those proved by Aronszajn, Cordes, Lax and Malgrange around 1956. (February 18, 1974.)

*714-D4. K. DEMYS, 844 San Ysidro Lane, Santa Barbara, California 93103. *A cuspless infinite hyperbolic plane and a remark on series.*

Hilbert's objection to Beltrami's realization of the hyperbolic plane on a pseudosphere was that the corresponding hyperbolic plane was made unallowably discontinuous by the cuspal edge of the pseudosphere. One may obtain an infinite hyperbolic plane, freed from Hilbert's objection, on the infinite *parasol surface*, generated by rotating a horizontally asymptotic tractrix about its vertical axis of symmetry (instead of about the horizontal axis, as in the generation of the pseudosphere). The resulting surface of revolution projects as an infinite, cusp-free hyperbolic plane possessing the required Bolyai-Lobachevsky geometry. Related to the hyperbola is the harmonic sequence $1, 1/2, 1/3, 1/4, \cdots, 1/n$. Forming the sum $H_n = \Sigma_1^n k^{-1}$, we have $1 = \Sigma_1^n (kH_n)^{-1}$; whence, as $\underline{n}$ passes beyond assignable limit, $\Sigma_1^\infty (kH_\infty) = 1$, a null series with finite sum: the series analog for the generation of a finite area as the summation of an infinity of differential areas in the integration process. (Received February 21, 1974.)

Topology

*714-G1. ELDON J. VOUGHT, California State University, Chico, California 95926. *Monotone decompositions into trees of Hausdorff continua irreducible about a finite subset.*

Theorem 1. If M is a compact Hausdorff continuum that is irreducible about a finite set of points then M has a decomposition D such that (1) D is upper semicontinuous, (2) the elements of D are continua, (3) each element of D has void interior, (4) the quotient space of D is a tree (a locally connected, hereditarily locally connected continuum) iff M contains no indecomposable subcontinuum with nonvoid interior. Furthermore, if it exists, this decomposition is unique. This generalizes a result for continua irreducible about two points due to Gordh [Pacific J. Math. 36(1971), 647–658] where condition (4) is that the quotient space is a generalized arc (a continuum in which every point except for two is a separating point). Using the notion of closed separators (closed sets which separate the space) necessary and sufficient conditions are given in Theorem 2 such that a compact Hausdorff continuum that is irreducible about a finite set admits a nontrivial decomposition D satisfying conditions (1), (2), (4) above. A final result describes the structure of the elements of D having void interiors (and certain of those having nonvoid interiors) by means of the aposyndetic set function T. (Received February 18, 1974.)

*714-G2. CLIFFORD W. ARNQUIST, California State University, Fullerton, California 92634. *A covering property of continua having an uncountable set of disjoint nonseparating plane embeddings.*

In the following, C will refer to a continuum, that is, a compact, connected metric space. A nonseparating plane embedding of C is a plane set homeomorphic to C whose complement is connected. C is said to be *A-chainable* if for each $p, q \in C$ and $\epsilon > 0$ there exists an open cover $\{O_1, O_2, \cdots, O_n\}$ of C such that $O_i \cap O_j \neq \emptyset$ iff $|i-j| \leq 1$, $p \in O_2$, $q \in O_{n-1}$ and diam $(O_i) < \epsilon$ for $i = 2, \cdots, n-1$. *Theorem.* If C is as described in the title, it is A-chainable. *Theorem.* If C is A-chainable, and $p, q \in C$ and $\epsilon > 0$, then there exists a chainable subcontinuum, C^*, of C s.t. $d(p, C^*) < \epsilon$ and $d(q, C^*) < \epsilon$. C is homogeneous if for any x, $y \in C$ there exists a homeomorphism, f, of C onto C s.t. $f(x) = y$. C is hereditarily equivalent if it is homeomorphic to each of its nondegenerate subcontinua. The above results show that if a plane continuum is indecomposable and either homogeneous or hereditarily equivalent it must have a nondegenerate chainable subcontinuum. This is used in proving the following. *Theorem.* A plane continuum which is hereditarily equivalent is either an arc or a pseudo-arc. *Theorem.* Every nondegenerate proper subcontinuum of a homogeneous indecomposable plane continuum is a pseudo-arc. (Received February 20, 1974.)

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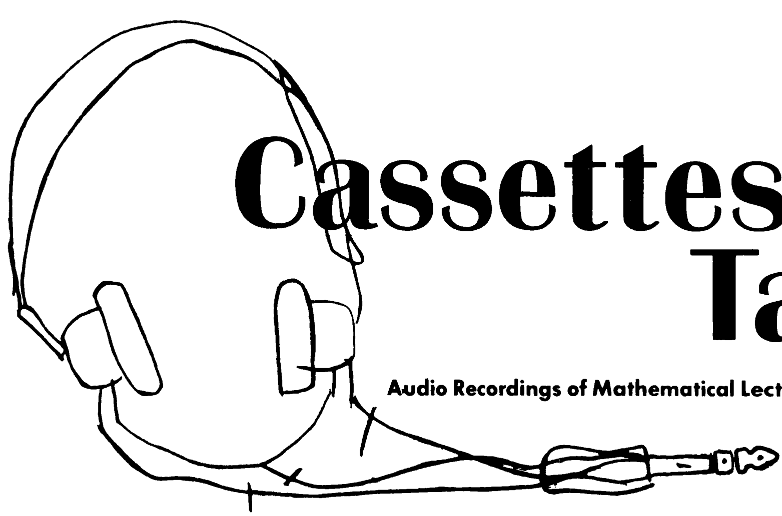

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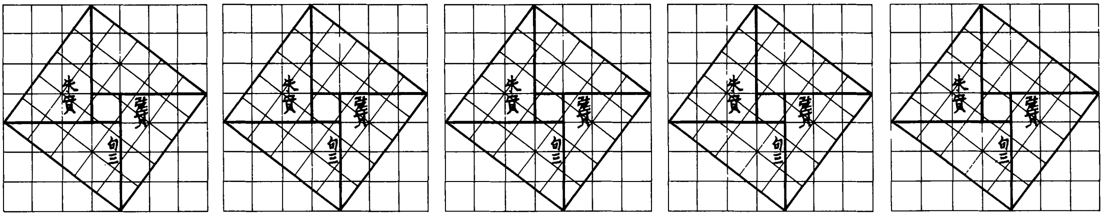

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