Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

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January 1975
Issue No. 159

Calendar

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting *in person* must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

Meeting Number	Date	Place	Deadline for Abstracts* and News Items
721	March 20–21, 1975	Mobile, Alabama	Jan. 28, 1975
722	March 23–26, 1975	New York, New York	Jan. 28, 1975
723	April 11–12, 1975	St. Louis, Missouri	Feb. 18, 1975
724	April 19, 1975	Monterey, California	Feb. 18, 1975
725	June 20–21, 1975	Pullman, Washington	Apr. 29, 1975
726	August 18–22, 1975 (79th Summer Meeting)	Kalamazoo, Michigan	June 17, 1975
727	October 25, 1975	Cambridge, Massachusetts	
728	November 1, 1975	Chicago, Illinois	
729	November 7–8, 1975	Blacksburg, Virginia	
730	November 15, 1975	Los Angeles, California	
731	January 22–26, 1976 (82nd Annual Meeting)	San Antonio, Texas	
	April 23–24, 1976	Reno, Nevada	
	June 18–19, 1976	Portland, Oregon	
	November 26–27, 1976	Albuquerque, New Mexico	

*Deadline for abstracts not presented at a meeting (by title).

February 1975 issue: January 21
April 1975 issue: February 11

OTHER EVENTS

January 29–30, 1975	Symposium on Some Mathematical Questions in Biology New York, New York
January 31, 1975	Symposium on Theory vs. Practice in the Finite Element Method New York, New York

Please affix the peel-off label on these *Notices* to correspondence with the Society concerning fiscal matters, changes of address, promotions, or when placing orders for books and journals.

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Notices

OF THE

AMERICAN MATHEMATICAL SOCIETY

Everett Pitcher and Gordon L. Walker, Editors
Wendell H. Fleming, Associate Editor

CONTENTS

The Eighty-First Annual Meeting
Shoreham Hotel
Washington, D. C.
January 21 – 26, 1975

SHORT COURSE ON MATHEMATICS IN OPERATIONS RESEARCH, January 21 and 22

On the recommendation of its Committee on Employment and Educational Policy, the American Mathematical Society will present a one and one-half day Short Course on Mathematics in Operations Research on Tuesday and Wednesday, January 21 and 22, in the Palladian Room of the Shoreham Hotel. The course is designed to identify opportunities for mathematicians in operations research, both in employment and in intellectual content; to give substantial introductions to several of the important areas of operations research; and to introduce some of the mathematically challenging aspects of the subject. This short course, which is open to all who wish to participate, will be similar in format to the short courses given at the 1973 summer meeting in Missoula and the annual meeting in San Francisco last January.

The program is under the direction of Dr. Alan J. Goldman, National Bureau of Standards. The current members of the AMS Committee on Employment and Educational Policy are Michael Artin, Charles W. Curtis, Wendell H. Fleming (chairman), Calvin C. Moore, Martha K. Smith, and Daniel H. Wagner.

The program will consist of lectures by the following six distinguished mathematical scientists: Dr. Ralph E. Gomory, Vice President and Director of Research, IBM Corporation, "Stock cutting and its ramifications: Mathematical operations research in industry"; Dr. Carl M. Harris, Program Director, RMC Research Corporation (Professor on leave from George Washington University), "Queueing theory and applications: Some mathematical frontiers"; Professor Frank Proschan, Florida State University, "Mathematical theory of reliability, with applications"; Dr. Gordon Raisbeck, Vice President (and Head of Physical Systems Research), Arthur D. Little, Inc., "Mathematicians in the practice of operations research"; Professor Arthur F. Veinott, Jr., Stanford University, "Lattice programming and inventory theory"; and Dr. Christoph Witzgall, Senior Mathematician, National Bureau of Standards, "Linear programming and flows in networks".

ANNUAL MEETING, January 23 – 26

The eighty-first annual meeting of the American Mathematical Society will be held at the Shoreham Hotel in Washington, D. C., from Thursday, January 23, through Sunday, January 26, 1975. The meeting will be held in conjunction with the annual meeting of the Association for Symbolic Logic (January 23–24) and the annual meeting of the Mathematical Association of America (January 25–27). The National Council of Teachers of Mathematics will meet jointly with MAA on Saturday and Sunday, January 25 and 26. The Association for Women in Mathematics will hold a session on Friday, January 24 at 10:00 a.m. The Conference Board of the Mathematical Sciences will present a panel discussion on "Wide ranging applications of statistics" at 2:00 p.m. on Saturday, January 25. There will be a panel discussion sponsored by the AMS-MAA Committee on the Training of Graduate Students to Teach, on Sunday evening, January 26, at 8:00 p.m.

Professor Henry L. Alder will retire at the end of January 1975 as Secretary of the Association following fifteen years of service in this position. A luncheon will be held in his honor on Saturday, January 25, at 12:00 noon. Tickets will be on sale in the registration area at the meeting.

The AMS Committee on Employment and Educational Policy is planning a panel discussion on Friday evening, January 24, at 8:30 p.m. Professor Martha K. Smith of the University of Texas, Austin, will serve as moderator; the topic to be discussed is "Seeking employment outside academia: Views from some who have recently succeeded." The members of the panel are: Steve Bravy, John Matherne, and Michael Weiss.

There will be a discussion of public science policy on Saturday, January 25, at 8:30 p.m. This subject and the speakers were selected by a Society committee appointed by the Executive Committee and Board of Trustees. The committee members are R. H. Bing (chairman), Paul Halmos, and A. H. Taub. The Director of the Office of Technology Assessment, Emilio Q. Daddario, and H. Guyford Stever, Director of the National Science Foundation, will be the speakers. The titles of their talks will be "The role of OTA in shaping public science policy" and "Basic research in a problem plagued world," respectively.

There will be one set of Colloquium Lectures which will consist of four one-hour talks. Professor H. Jerome Keisler of the University of Wisconsin, Madison will present these lectures; the title of these talks will be "New directions in model theory."

The Josiah Willard Gibbs Lecture will be presented by Professor Fritz John of the Courant Institute of Mathematical Sciences, New York University, on Thursday, January 23, 1975, at 8:30 p.m. in the Regency Ballroom. He will speak on "A priori estimates, geometric

effects, and asymptotic behavior."

The Retiring Presidential Address will be given at 2:15 p.m. on Friday, January 24, by Professor Saunders Mac Lane of the University of Chicago. The title of his lecture will be "Topology and logic as sources of algebraic ideas."

The Frank Nelson Cole Prize in Algebra and the 1974 Steele Prizes will be awarded at a session at 3:15 p.m. on Friday, January 24.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there will be eight hour addresses. They will be given by Professor Donald W. Anderson, University of California, San Diego; Professor Donald L. Burkholder, University of Illinois, Urbana-Champaign; Professor Sigurdur Helgason, Massachusetts Institute of Technology; Professor Linda Keen, Graduate School and University Center, City University of New York; Professor Haskell P. Rosenthal, University of Illinois, Urbana-Champaign; Professor Rainer Sachs, University of California, Berkeley; Professor Wilfried Schmid, Columbia University; and Professor Nolan R. Wallach, Rutgers University. The titles and times of these lectures can be found in the Timetable which follows this announcement.

There will be sixty-four sessions of contributed ten-minute papers. No provision will be made for late papers.

SPECIAL SESSIONS

Many special sessions of selected papers of varying lengths will be held. The subjects of these special sessions and the names of the mathematicians arranging them are as follows: Operator Theory, Professor William B. Arveson, University of California, Berkeley; Topological Dynamics, Professors Joseph Auslander and Nelson G. Markley, University of Maryland; Probabilistic Analysis, Professor A. T. Bharucha-Reid, Wayne State University; Set-theoretic and Combinatorial Methods in Topology, Professor W. Wistar Comfort, Wesleyan University; Hyperbolic Conservation Laws, Professor Constantine M. Dafermos, Brown University; Expository Session on Some Directions of Current Progress in Commutative Algebra, Professor David Eisenbud, Brandeis University; Singular Cauchy Problems, Professor Bernard A. Fusaro, Salisbury State College; Interpolation of Operators and Applications, Professors John E. Gilbert and George G. Lorentz, University of Texas, Austin; Analytic Number Theory, Professor Emil Grosswald, Temple University; Advances in Graph Theory, Professor Frank Harary, University of Michigan; Interrelations between Computation and Number Theory, Dr. Morris Newman, National Bureau of Standards; Structure and Representations of Lie Algebras over General Fields, Professor George B. Seligman, Yale University; Fourier Integral Operators, Professor François Treves, Rutgers University; and Mathematics and Games, Professor Stanislaw M. Ulam, University of Colorado.

There will be two informal special sessions: Professor Ruth Bari, George Washington University, Color theory; Professor James D. Stasheff, Temple University, Finite H-spaces.

COUNCIL AND BUSINESS MEETING

The Council will meet on Wednesday, January 22, at 2:00 p.m. in the Diplomat Room of the Shoreham Hotel. Most of the meeting is open to members of the Society as observers. The agenda will be posted. The Diplomat Room is located on the lower lobby level of the hotel.

The Business Meeting will be held on Friday, January 24, at 4:30 p.m. in the Regency Ballroom of the Shoreham Hotel which is located one level below the Diplomat Room. The Secretary notes the following resolution of the Council. Each person who attends a Business Meeting of the Society shall be willing and able to identify himself as a member of the Society. In further explanation, it is noted that "each person who is to vote at a meeting is thereby identifying himself as and claiming to be a member of the American Mathematical Society."

MEETING REGISTRATION

Operations Research short course participants may register at the desk in the upper lobby level which is located directly adjacent to the main lobby of the Shoreham. The desk will be open from 11:00 a.m. to 3:00 p.m. on Tuesday, January 21, and from 8:30 a.m. to 1:00 p.m. on Wednesday, January 22.

The registration desk for the joint meeting will be in the same location. The desk will be open from 2:00 p.m. to 8:00 p.m. on Wednesday, January 22; from 8:00 a.m. to 5:00 p.m. on Thursday, January 23; from 8:00 a.m. to 4:00 p.m. on Friday through Sunday, January 24–26; and from 8:30 a.m. to 2:30 p.m. on Monday, January 27.

Please note that separate registration is required for the short course and the joint meetings. Registration fees for the meetings are as follows:

	At meeting
Operations Research Short Course	
All participants	$12
Joint Mathematics Meetings	
Member	12
Student or unemployed member	2
Nonmember	20
One-day fee (Monday, January 27 only)	$4

There will be no extra charge for members of the families of registered participants.

The unemployed status refers to any member currently unemployed and actively seeking employment. It is not intended to include members who have voluntarily resigned or retired from their latest position.

Students are considered to be only those currently working toward a degree who do not receive an annual compensation totaling more than $7,000 from employment, fellowships, and scholarships.

A fifty percent refund of the preregistration fee will be reimbursed for all cancellations received prior to January 21. There will be no refunds granted for cancellations received after

that date or to persons who do not attend the meetings.

EMPLOYMENT REGISTER

The Mathematical Sciences Employment Register will be maintained from 9:00 a.m. to 4:00 p.m. on Friday, January 24, with interviews scheduled from 9:00 a.m. to 5:40 p.m. on Saturday, Sunday, and Monday, January 25–27. The addition of a third day of interviews continues a previously adopted procedure recommended by the Joint Committee on Employment Opportunities in an attempt to expand the interview schedule and to eliminate the necessity for evening interviews. The Register will be located in the Empire Room of the Shoreham Hotel which is adjacent to the lower lobby reception area.

A short meeting of persons planning to participate in the Employment Register has been called by the committee for Thursday, January 23, at 4:30 p.m. in The Forum of the Shoreham Hotel. The purpose of this meeting is to explain Register procedures and the various printed forms which are used. Both applicants and employers are urged to attend the session in order to become familiar with the details in advance. An informal question and answer session will follow the meeting. This orientation session is designed to increase the efficiency of the Register and make it run more smoothly.

EXHIBITS

The book and educational media exhibits will be displayed in the Ambassador Room of the Shoreham from Thursday through Sunday, January 23–26. The exhibits will be displayed from noon to 5:00 p.m. on Thursday; from 9:00 a.m. to 5:00 p.m. on Friday, Saturday, and Sunday. All participants are encouraged to visit the exhibits during the meeting. The Ambassador Room is located adjacent to the Regency Ballroom.

BOOKS AND AUDIO TAPES SALE

Books published by the Society and the Association and audio tapes of invited addresses will be sold for cash prices somewhat below the usual prices when these same books and tapes are sold by mail.

ACCOMMODATIONS

The deadline for making reservations for accommodations was December 20, 1974. Those who have not yet made reservations are advised to try to get reservations by writing directly to a hotel. The Washington Convention & Visitors Bureau may be able to place some applicants. A reservation form will be found on the last page of the November *Notices*. This should be sent to the Washington Convention & Visitors Bureau, 1129 Twentieth Street, N.W., Washington, D.C. 20036, although at this late date no guarantee can be made that requests will be filled.

BARBIZON TERRACE	
Singles	$16
Twins	18
Extra person in room	4.50
BRIGHTON HOTEL	
Singles	$12
Doubles or twins	16
Suites	24
Extra person in room	4
SHERATON-PARK	
Singles	$22, 24, 26, 28, 30
Doubles or twins	26, 28, 30, 32, 34
*Triples	27
Suites	75 and up
Extra person in room	9
SHOREHAM HOTEL	
Singles	$20, 21, 22, 24, 26, 28
Doubles or twins	26, 27, 28, 30, 32
*Triples	27
Suites	60, 70, 80, 90
Extra person in room	6
WASHINGTON HILTON	
Singles	$23, 25, 27
Doubles or twins	28, 30, 32
*Triples	33
YMCA	
Singles or twins	$ 5.25

N.B. Space was available to accommodate only 35 male and 15 female participants; reservations were accepted on a first-come first-served basis and processed by the Housing Bureau in Providence. No further reservations can be accepted at this late date.

YWCA (Strong Residence)
1011 17th Street, N.W.
Washington, D. C. 20036

Strong Residence (for women only) has limited accommodations. YWCA will not quote rates for 1975 until the latter part of November. Singles are currently $8. Reservations will be taken by mail starting in December; write directly.

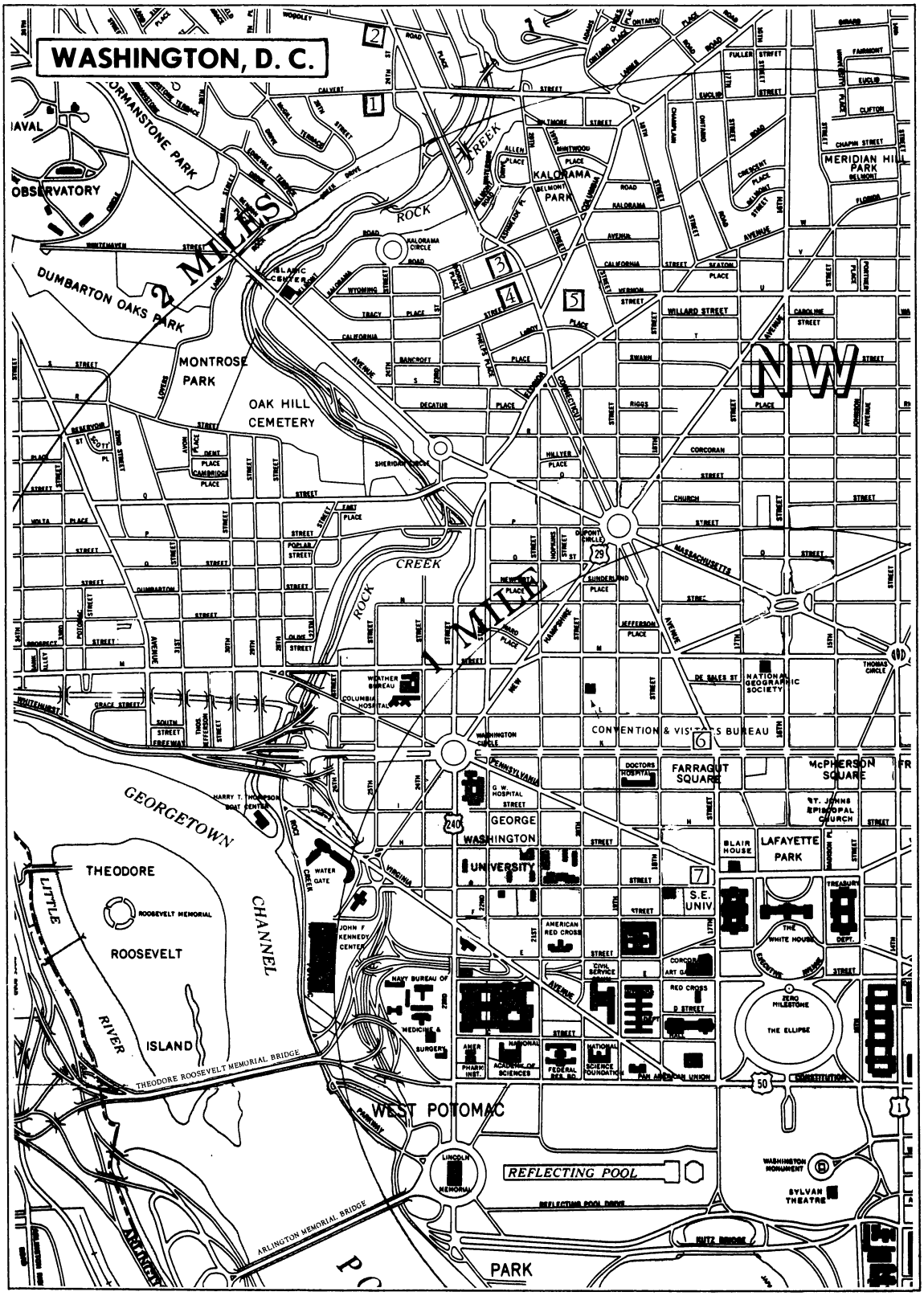
It should be noted that all rooms in the Shoreham Hotel have two beds. Participants, who are able to do so, are urged to share a room whenever possible. This procedure will be economically beneficial. For instance, two participants sharing a double or twin room will each pay only $13, $13.50, $14, $15, or $16 (depending on category of room) daily as compared with the single room rates of $20, $21, $22, $24, $26, and $28.

The Shoreham Hotel has two dining rooms: The International Snack Bar (a coffee shop) which is open daily from 7:00 a.m. to 11:00 p.m.; and The Greenery (a more formal restaurant serving liquor) which is open for breakfast, lunch, and dinner from 7:00 a.m. to 11:00 p.m. every day.

NATIONAL SCIENCE FOUNDATION

NSF staff members will be available to provide counsel and information on NSF programs of interest to mathematicians from 9:00 a.m. to 5:00 p.m. on January 24, 25, and 26, in the Committee Room. This room is located on the main lobby level of the Shoreham Hotel.

*The triples were reserved for students and unemployed members; see special reservation form on penultimate page of November 1974 *Notices* for criteria governing eligibility for these rooms.



1 = Shoreham Hotel
2 = Sheraton-Park Hotel
3 = Barbizon Terrace
4 = Brighton Hotel
5 = Washington Hilton Hotel
6 = Y.W.C.A.
7 = Y.M.C.A.

ENTERTAINMENT

There will be a No-Host cocktail party on Saturday, January 25, from 5:00 p.m. to 6:30 p.m. in the Palladian Room. This will be the only major social function of the meeting, and everyone is invited to attend.

Eighteen million people visit the nation's capitol each year. The "Top Ten Tourist Attractions", according to the Washington Convention & Visitors Bureau, are listed below in no particular order. The eleventh in this list is the most recent addition to the Smithsonian complex.

(1) The U.S. Capitol. Capitol Hill at east end of the Mall. Visitor hours 9:00 a.m. to 4:30 p.m.; guided tours leave from the upstairs rotunda daily every fifteen minutes from 9:00 a.m. to 3:45 p.m. The tour takes 45 minutes. The charge is 25¢ for adults; children are free.

(2) The White House. 1600 Pennsylvania Avenue, N.W. Visitor hours 10:00 a.m. to noon Tuesday through Saturday.

(3) Jefferson Memorial. South side of Tidal Basin. Visitor hours 8:00 a.m. to midnight.

(4) The Washington Monument. For 10¢ you can ride to the top in one minute. Visitor hours daily 9:00 a.m. to 5:00 p.m.

(5) The Lincoln Memorial. Open around the clock. Fifteen minute tours are conducted at random daily by National Park service guides.

(6) The John F. Kennedy Center for the Performing Arts. At Rock Creek Park and New Hampshire Avenue. The Center contains an Opera House, Concert Hall, Eisenhower Theater (for live drama), a Film Theater, an Exhibition Hall, and three restaurants. It is worth a sightseeing trip even if you do not attend one of the performances. The building is open for tours from 10:00 a.m. to 5:00 p.m. daily.

(7) National Archives. 7th and 9th Streets, N.W. Open 9:00 a.m. to 10:00 p.m. weekdays and holidays; 1:00 p.m. to 10:00 p.m. on Sundays.

(8) The National Gallery of Art. Offers general tours which convene in the rotunda at 11:00 a.m., 3:00 p.m., and 5:00 p.m. Monday through Saturday, and at 5:00 p.m. on Sunday. Special tours and concerts are also scheduled. The Gallery is open to visitors from 10:00 a.m. to 5:00 p.m. Monday through Saturday, and from 2:00 p.m. to 10:00 p.m. on Sunday.

(9) The Museum of History and Technology. One of the newest of the Smithsonian buildings, 14th and Constitution on the north side of the Mall. Visitor hours are 9:00 a.m. to 5:00 p.m. daily.

(10) Museum of Arts and Industries. Southside of Mall, Independence at 9th St., S.W. Visitor hours 9:00 a.m. to 5:30 p.m. daily. Apollo 11, Spirit of St. Louis, Wright Brothers plane on display.

(11) The Hirshhorn Museum. The Inaugural Exhibition of this Museum and Sculpture Garden, the newest of the Smithsonian buildings, will continue through the fall of 1975. Located at Independence Avenue at 8th Street, S.W., the building is adjacent to the Museum of Arts and Industries, and is opened daily from 10:00 a.m. to 5:30 p.m. Admission is free. The museum has one of the most comprehensive collections of 20th Century art and has been widely acclaimed since its opening in early October.

Brochures on these and other attractions in the Washington, D.C. area will be available at the Information Desk. It is anticipated that several tours will be organized for registrants at the Mathematics Meetings.

TRAVEL AND LOCAL INFORMATION

The District of Columbia is served by two airports: Washington National (4 miles from city center) and Dulles International (26 miles from city center). Allegheny, American, Braniff, Delta, Eastern, National, Northwest, Piedmont, TWA and United Airlines provide service to Washington National. Several commuter airlines (i.e., Altair, Pennsylvania, etc.) also operate service into National. Transfers are available at variable rates and limousine service at $2.50 is available from National Airport directly to the hotel. A third airport, namely, Baltimore International, located 32 miles (50 minutes) from the District can also be used. Limousine service is available at $3.50.

Dulles International Airport is served by American, Braniff, Delta, Eastern, Northwest, Ozark, Pan American, Piedmont, Southern, TWA, United, and Viasa Airlines, as well as Colgan Airway, a commuter line. Metered taxi service to the hotel will cost approximately $20, plus fifty cents for each additional passenger. Limousine-bus service for $3.75 is available from Dulles to the Washington Hilton, not quite a mile south of the Shoreham. City buses (metrobus L2, L4, and L6) run on Connecticut Avenue past the Washington Hilton and the Shoreham.

Amtrak service is available from many points to Union Station. The high-speed Metroliner runs between New York, Philadelphia and Washington. Metrobus numbers 96 and 98 go from Union Station past the Shoreham. The fare anywhere within the city is 40¢ in exact change.

From Exit 20 on the Capitol Beltway (Interstate 495) south to Calvert Street is approximately 8-1/2 miles. The Shoreham is located at 2500 Calvert Street, N.W., near Connecticut Avenue. There is a parking garage within the hotel. The charge is $2.50 for each 24-hour period; there is no extra charge for in/out service. Parking is available in this garage for commuters as well as hotel residents.

WEATHER

Washington weather in January varies from brisk to balmy (10° to 60°) but rarely prevents travel. The average daily high is 44°; the average daily low is 29°; in January there are approximately eleven days with at least .01 inches of rain. Medium to heavy clothing is recommended.

MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematics Meetings, The Shoreham Hotel & Motor Inn, Calvert Street and Connecticut Avenue, N.W., Washington, D.C. 20008. Mail and telegrams so addressed may be picked up at

the Mail and Information Desk located at the registration area in the upper lobby of the hotel.

A message center will be located in the same area to receive incoming calls for all members in attendance. Messages may be left for registrants during the hours the registration desk is open, cf. the section entitled MEETING REGISTRATION, above. Messages will be taken down, and the name of any member for whom a message has been received will be posted until the message is picked up at the Message Center. Members are advised to leave the following number with anyone who might want to reach them at the meeting (202) 797-8356.

LOCAL ARRANGEMENTS COMMITTEE

H. L. Alder (ex officio), Ruth A. Bari, James A. Donaldson, Irving I. Glick, Walter H. Gottschalk (ex officio), Hewitt Kenyon (chairman), James C. Owings, William Swyter, Choy-tak Taam, Juanita S. Tolson, and Gordon L. Walker (ex officio).

PROGRAM OF THE SESSIONS

SHORT COURSE ON MATHEMATICS IN OPERATIONS RESEARCH

All Sessions in Palladian Room, Shoreham, Lower Lobby Level

TUESDAY, 1:45 P.M. - 5:00 P.M.

1:45	Introductory remarks Dr. ALAN J. GOLDMAN, National Bureau of Standards, Washington, D.C.
2:00- 3:15	Mathematicians in the practice of operations research. Dr. GORDON RAISBECK, Arthur D. Little, Incorporated, Cambridge, Massachusetts
3:45- 5:00 P.M.	Linear programming and flows in networks. Dr. CHRISTOPH WITZGALL, National Bureau of Standards, Washington, D.C.

WEDNESDAY, 9:00 A.M. - 5:00 P.M.

9:00-10:15	Stock cutting and its ramifications: Mathematical operations research in industry. Dr. RALPH E. GOMORY, IBM T.J. Watson Research Center, Yorktown Heights, New York
10:45-12:00	Mathematical theory of reliability, with applications. Professor FRANK PROSCHAN, Florida State University
2:00- 3:15	Lattice programming and inventory theory. Professor ARTHUR F. VEINOTT, Jr., Stanford University
3:45- 5:00	Queueing theory and applications: Some mathematical frontiers. Dr. CARL M. HARRIS, RMC Research Corporation, Bethesda, Maryland

THE EIGHTY-FIRST ANNUAL MEETING

The time limit for each contributed paper in the general sessions is ten minutes. In the special sessions the time varies from session to session and within sessions. To maintain the schedule, the time limits will be strictly enforced.

THURSDAY, 8:30 A.M.

Special Session on Mathematics and Games I, Empire Room, Shoreham

8:30- 8:50	(1)	Introductory remarks. Professor STANISLAW ULAM, University of Colorado, Boulder
8:50-10:00	(2)	Combinatorial game theory. Mr. RICHARD K. GUY, University of Calgary (720-90-6)
10:00-10:20	(3)	Borel determinacy. Professor DONALD A. MARTIN, Rockefeller University (720-02-17) (Introduced by Professor William Mitchell)
10:20-11:00	(4)	Discrete theories of pursuit and evasion. Professor JAN MYCIELSKI, University of Colorado (720-90-4)
11:00-11:30	(5)	Theory of annihilation games. Preliminary report. Dr. AVIEZRI S. FRAENKEL*, and Mr. YAACOV YESHA, Weizmann Institute of Science, Israel (720-05-2)
	(6)	Withdrawn

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

THURSDAY, 8:45 A.M.

Session on Combinatorics, Dover Room, Sheraton-Park (Ground Level)

8:45- 8:55	(7)	Mathematical foundations of interconnected J-K flip-flops. Dr. VERA PLESS, Massachusetts Institute of Technology (720-05-11)
9:00- 9:10	(8)	The combinatorial topology of bordered Riemann surfaces. Professor GEORGE K. FRANCIS, University of Illinois (720-05-12)
9:15- 9:25	(9)	A partial Steiner triple system of order n can be embedded in a Steiner triple system of order 6n + 3. Professor CHARLES C. LINDNER, Auburn University, (720-05-20)
9:30- 9:40	(10)	Doubly incomparable s-systems. Professor MURRAY HOCHBERG, City University of New York, Brooklyn College (720-05-21)
9:45- 9:55	(11)	Ramsey numbers for linear forests. RALPH FAUDREE*, and RICHARD SCHELP, Memphis State University (720-05-28)
10:00-10:10	(12)	An enumeration problem in finite geometries. Preliminary report. Mr. GENE A. BERG, Colorado State University (720-05-29)
10:15-10:25	(13)	A construction of combinatorial geometries. Dr. WILLIAM SPEARS*, and Dr. BARBARA JEFFCOTT, Kansas State University (720-05-38)
10:30-10:40	(14)	Finite embedding theorems for partial pairwise balanced designs. Preliminary report. Mr. WILLIAM B. POUCHER, Auburn University (720-05-31)
10:45-10:55	(15)	Finite nets of dimension d. Preliminary report. Ms. JEAN DUNBAR*, and Professor RENU LASKAR, Clemson University (720-05-40)
11:00-11:10	(16)	Difference sets in 2-groups. Preliminary report. Dr. JOHN F. DILLON, Department of Defense, Fort George G. Meade, Maryland (720-05-42)
11:15-11:25	(17)	Combinatorics and an algebra of divided powers. Preliminary report. Professor LADNOR GEISSINGER, University of North Carolina (720-05-44)

THURSDAY, 8:45 A.M.

Session on Commutative Rings, Wilmington Room, Sheraton-Park (Ground Level)

8:45- 8:55	(18)	Splitting torsion theories over commutative rings. Professor JOHN FUELBERTH, University of Northern Colorado, Professor JAMES KUZMANOVICH, Wake Forest University, and Professor THOMAS SHORES*, University of Nebraska (720-13-3)
9:00- 9:10	(19)	Entropy of ring endomorphisms. Preliminary report. Mr. MICHAEL SCHLOSSER, University of Kentucky (720-13-8)
9:15- 9:25	(20)	Chained rings. Preliminary report. Dr. PAUL A. FROESCHL III, Saint Mary's College (720-13-6)
9:30- 9:40	(21)	The maximal spectrum of a Noetherian ring. Preliminary report. Professor ROGER WIEGAND, and Professor SYLVIA WIEGAND*, University of Nebraska (720-13-9)
9:45- 9:55	(22)	Algebras which resemble symmetric algebras. Preliminary report. DAVID WRIGHT, Columbia University (720-13-11)
10:00-10:10	(23)	Dimension functions on the prime spectrum. Professor ROGER WIEGAND, University of Nebraska (720-13-10)
10:15-10:25	(24)	Concerning filtrations on commutative rings. Professor JOHN W. PETRO, Western Michigan University (720-13-15)
10:30-10:40	(25)	Filtrations with finite power type on Prüfer domains. Preliminary report. Professor JOHN W. PETRO, and Professor M. EDWARD PETTIT, Jr.*, Western Michigan University (720-13-16)
10:45-10:55	(26)	Topologically defined classes of going-down domains. Dr. I.J. PAPICK, Rutgers University (720-13-4)
11:00-11:10	(27)	Integral domains which satisfy Gauss's lemma. Professor JIMMY T. ARNOLD, and Professor PHILIP B. SHELDON*, Virginia Polytechnic Institute and State University (720-13-13)
11:15-11:25	(28)	Irreducible divisors in domains of finite character. Preliminary report. Professor ANNE GRAMS*, University of Tennessee at Nashville, and Professor HOYT WARNER, Vanderbilt University (720-13-17)
11:30-11:40	(29)	Factorization of ideals into semiprime ideals in an almost Dedekind domain. Professor N. H. VAUGHAN*, North Texas State University, and Professor R.W. YEAGY, Stephen F. Austin State University (720-13-18)

11:45-11:55 (30) Completion of local ring which is not Noetherian. Preliminary report. JOONG HO KIM, East Carolina University (720-13-14)

THURSDAY, 8:45 A.M.

Session on Complex Analysis I, Alexandria Room, Sheraton-Park (Ground Level)

8:45- 8:55 (31) A characterisation of convex domains in the plane. Professor F.R. KEOGH, University of Kentucky (720-30-1)

9:00- 9:10 (32) On functions of bounded boundary rotation. Preliminary report. Professor MAXWELL O. READE*, University of Michigan, and Dr. THOMAS ARMSTRONG, Princeton, New Jersey (720-30-7)

9:15- 9:25 (33) On the log of a schlicht function. Preliminary report. Professor JOSEPH CIMA, University of North Carolina at Chapel Hill (720-30-10)

9:30- 9:40 (34) The limiting behavior of $zf''(z)/f'(z)$ and two conjectures on univalent functions. Preliminary report. Dr. DOUGLAS M. CAMPBELL, Brigham Young University (720-30-11)

9:45- 9:55 (35) Powers of p-valent functions. Preliminary report. Professor J.W. NOONAN, Holy Cross College (720-30-13)

10:00-10:10 (36) Rate of growth of spiral functions. Dr. JOSEPH WARREN, Fordham University (720-30-14)

10:15-10:25 (37) A class of differential inequalities implying boundedness. Preliminary report. Professor SANFORD S. MILLER, State University of New York, College at Brockport (720-30-16)

10:30-10:40 (38) Maximal ideals in the algebra N^+. Professor JAMES W. ROBERTS, and Professor MANFRED STOLL*, University of South Carolina (720-30-21)

10:45-10:55 (39) The component of the origin in the Nevanlinna class. Professor JAMES W. ROBERTS, University of South Carolina (720-30-24)

11:00-11:10 (40) Univalence of the integral of $f'(z)^c$. Professor JOHN PFALTZGRAFF, University of North Carolina (720-30-27)

11:15-11:25 (41) Coefficient estimates for univalent polynomials Professor DAVID HOROWITZ, University of South Florida (720-30-28)

THURSDAY, 8:45 A.M.

Session on Fields, Franklin Room, Sheraton-Park (First Floor)

8:45- 8:55 (42) Effective procedures in algebraic number theory. Professor R. MINES*, and Professor F. RICHMAN, New Mexico State University-Las Cruces (720-12-5)

9:00- 9:10 (43) A generalization of Berwick's unit algorithm. Professor R. RUDMAN*, Wayne State University, and Professor R. FINKELSTEIN, Bowling Green State University (720-12-10)

9:15- 9:25 (44) Indecomposable hermitian forms over quadratic number fields. Preliminary report. Professor LARRY J. GERSTEIN, University of California, Santa Barbara (720-12-13)

9:30- 9:40 (45) An enumeration of the orders in cubic number fields. Preliminary report. Mr. ROBERT A. MORRIS, Fort George G. Meade, Maryland (720-12-16)

9:45- 9:55 (46) A local version of the nullstellensatz in rings of integer-valued polynomials. Professor D. BRIZOLIS, University of Southern California (720-12-17)

10:00-10:10 (47) Maximal abelian extensions viewed as Kummer extensions. Professor VICTOR ALBIS-GONZALES, University of Cincinnati (720-12-3)

10:15-10:25 (48) Group, round, and Pfister forms over a linearly compact field. Preliminary report. Professor HOYT D. WARNER, Vanderbilt University (720-12-7)

10:30-10:40 (49) Matrix fields over the integers modulo m. Professor JACOB T.B. BEARD, Jr.*, University of Texas at Arlington and Professor ROBERT M. McCONNEL, University of Tennessee (720-12-9)

10:45-10:55 (50) Local-global principles for formally real fields. Preliminary report. Dr. MURRAY A. MARSHALL, University of Saskatchewan (720-12-11) (Introduced by Dr. Stephan Berman)

11:00-11:10 (51) Rings containing division rings or fields. Professor JACOB T.B. BEARD, Jr., University of Texas at Arlington, and Professor ROBERT M. McCONNEL*, University of Tennessee (720-12-8)

THURSDAY, 8:45 A.M.

Session on Functional Analysis I, Richmond Room, Sheraton-Park (Ground Level)

8:45- 8:55 (52) A noncompact Choquet theorem. Professor G.A. EDGAR, Northwestern University (720-46-5)

9:00- 9:10 (53) Weak parallelogram laws for Banach spaces. Preliminary report. Mr. W.L. BYNUM, College of William & Mary (720-46-8)

9:15- 9:25 (54) Complex Banach lattices with order continuous norm. Professor S.J. BERNAU* and Professor H. ELTON LACY, University of Texas at Austin (720-46-11)

9:30- 9:40 (55) On nonstandard hulls of convex spaces. Professor STEVEN F. BELLENOT, Florida State University (720-46-23)

9:45- 9:55 (56) Equivalent strictly convex increasing norms on C(S) spaces. Preliminary report. JOHN WOLFE, Oklahoma State University (720-46-25)

10:00-10:10 (57) On the other side of nearest points. Professor CLIFFORD KOTTMAN*, Oregon State University, and Professor BOR-LUH LIN, University of Iowa (720-46-27)

10:15-10:25 (58) Compact operators and induced representations. Preliminary report. ROBERT A. FONTENOT*, and IRWIN SCHOCHETMAN, Oakland University (720-46-31)

10:30-10:40 (59) Strict convexity and orthogonality in Banach spaces. Preliminary report. Professor JAMES R. HOLUB, Virginia Polytechnic Institute and State University (720-46-32)

10:45-10:55 (60) Conditionally injective Banach spaces. Professor CHARLES W. NEVILLE, Central Connecticut State College (720-46-40)

11:00-11:10 (61) A cousin of a space of R.C. James. Preliminary report. JAMES N. HAGLER, Catholic University of America (720-46-43)

11:15-11:25 (62) Upstream geometry. Preliminary report. Professor FRANCIS SULLIVAN, Catholic University of America (720-46-44)

THURSDAY, 8:45 A.M.

Session on Graphs I, Continental Room, Sheraton-Park (Lobby Level)

8:45- 8:55 (63) Ramsey theory and chromatic numbers. Dr. GARY CHARTRAND, Western Michigan University, and Dr. ALBERT D. POLIMENI*, State University of New York, College at Fredonia (720-05-1)

9:00- 9:10 (64) Brooks' theorem is true for hypergraphs. Preliminary report. Ms. MARIANNE LEPP GARDNER, Carleton College (720-05-13)

9:15- 9:25 (65) Reducible configurations for the four colour conjecture. Preliminary report. Mr. FRANK ALLAIRE, University of Manitoba (720-05-16) (Introduced by Professor R.C. Mullin)

9:30- 9:40 (66) On the chromatic index of small graphs. Professor L.W. BEINEKE*, Purdue University, and Mr. S. FIORINI, The Open University, Milton Keynes, Bucks, England (720-05-23)

9:45- 9:55 (67) Coloring maps via branched coverings. Preliminary report. Professor PAUL C. KAINEN, Case Western Reserve University (720-05-26)

10:00-10:10 (68) Anticritical graphs. Professor FRANK HARARY*, University of Michigan, Ann Arbor, and Dr. CARSTEN THOMASSEN, Aarhus University, Denmark (720-05-39)

10:15-10:25 (69) Cycles of length four in chromatic graphs. Professor F.D. PARKER, St. Lawrence University (720-05-47)

10:30-10:40 (70) Comparability graphs and a new matroid. MARTIN CHARLES GOLUMBIC, Columbia University (720-05-9)

10:45-10:55 (71) Degree sets and traversability. Professor LINDA LESNIAK, Louisiana State University, Professor ALBERT D. POLIMENI, State University of New York, College at Fredonia, and Professor DONALD W. VANDERJAGT*, Grand Valley State Colleges (720-05-15)

11:00-11:10 (72) On automorphism groups of Cayley graphs. Preliminary report. Professor WILFRIED IMRICH, Montanistische Hochschule Leoben, Austria, and Professor MARK E. WATKINS*, Syracuse University (720-05-32)

11:15-11:25 (73) Unicyclic graphs with cyclic automorphism group. Preliminary report. Professor DONALD J. McCARTHY, St. John's University (720-05-33)

THURSDAY, 8:45 A.M.

Session on General Topology I, Arlington Room, Sheraton-Park (Ground Level)

8:45- 8:55	(74)	Open and uniformly open mapping theorems. Professor PETER F. MAH and Professor S.A. NAIMPALLY*, Lakehead University (720-54-1)
9:00- 9:10	(75)	Functions continuous in some compact metric topology. Preliminary report. Mr. BRUCE LOATMAN, Georgetown University (720-54-16)
9:15- 9:25	(76)	Almost periodic homeomorphisms of E^2 are periodic. Professor BEVERLY L. BRECHNER, University of Florida (720-54-2)
9:30- 9:40	(77)	Generalized almost finite homomorphisms of minimal sets. Preliminary report. Dr. PETER S. SHOENFELD, National Bureau of Standards, Washington, D.C. (720-54-36)
9:45- 9:55	(78)	On the Edelstein contractive mapping theorem. Dr. LUDVIK JANOS, University of Montana (720-54-5)
10:00-10:10	(79)	Weakly smooth continua. Dr. LEWIS LUM, Salem College (720-54-28) (Introduced by G.R. Gordh, Jr.)
10:15-10:25	(80)	The Hahn-Mazurkiewicz theorem for rim-finite continua. Professor L.E. WARD, Jr., University of Oregon (720-54-31)
10:30-10:40	(81)	Characterizing local connectedness in inverse limits. Dr. G.R. GORDH, Jr.*, Guilford College, and Professor SIBE MARDESIĆ, University of Zagreb, Yugoslavia (720-54-63)
10:45-10:55	(82)	Arcwise accessibility in hyperspaces. Professor SAM B. NADLER, Jr., University of Georgia (720-54-35)
11:00-11:10	(83)	Inverse limits and the completeness of quotient groups. Dr. DAVID WIGNER, University of Michigan (720-54-12) (Introduced by Peter Weinberger)
11:15-11:25	(84)	On some subspaces of Helly space. Preliminary report. Professor W.F. PFEFFER, University of California, Davis (720-54-7)

THURSDAY, 8:45 A.M.

Session on Geometry, Wardman Room, Sheraton-Park (Lobby Level)

8:45- 8:55	(85)	Proper lineations on real spaces are collineations. Preliminary report. Dr. LYNN E. GARNER, Brigham Young University (720-50-1)
9:00- 9:10	(86)	Relations between two Baer subplanes of $PG(2,q^2)$. Preliminary report. Dr. J.W. FREEMAN, Virginia Commonwealth University (720-50-6)
9:15- 9:25	(87)	Collineations of order 10 planes. Preliminary report. SUE WHITESIDES, University of Wisconsin (720-50-8) (Introduced by Professor R.H. Bruck)
9:30- 9:40	(88)	Arrangements of colored lines. Professor BRANKO GRÜNBAUM, University of Washington (720-50-5)
9:45- 9:55	(89)	Arrangements generated by polygons. Preliminary report. Mr. THOMAS STROMMER, University of Washington (720-50-2)
10:00-10:10	(90)	A topological measure-theoretic approach to dimension theory. Professor LEOPOLDO V. TORALBALLA, Bronx Community College (720-50-3)
10:15-10:25	(91)	Generalized quadrangles with symmetry. Dr. STANLEY E. PAYNE*, Miami University, and Dr. J.A. THAS, University of Ghent, Belgium (720-50-4)
10:30-10:40	(92)	Disjoint circles in finite miquelian inversive planes. Preliminary report. GARY L. EBERT, University of Wisconsin (720-50-7)
10:45-10:55	(93)	Cusps of simplicial mappings to the plane (with computer graphics analysis) Professor THOMAS F. BANCHOFF*, and Professor CHARLES M. STRAUSS, Brown University (720-50-9)
11:00-11:10	(94)	Linear isotopies. R.H. BING, and MICHAEL STARBIRD*, University of Texas at Austin (720-50-10)
11:15-11:25	(95)	Rest points, tangent circles, and the rational number line. Professor RODNEY T. HOOD, Franklin College (720-50-11)

THURSDAY, 8:45 A.M.

Session on Harmonic Analysis, Assembly Room, Sheraton-Park (Lobby Level)

8:45- 8:55	(96)	A Plancherel theorem for AN. Dr. FREDERICK W. KEENE, University of Miami (720-43-2)
9:00- 9:10	(97)	The spherical Bochner theorem on semisimple Lie groups. WILLIAM H. BARKER, Dartmouth College (720-43-9)

9:15- 9:25 (98) Characteristic principal bundles. Professor HARVEY A. SMITH, Oakland University (720-43-3)

9:30- 9:40 (99) Quadratic forms in harmonic analysis and the Bochner-Eberlein theorem. Professor ROGER A. HORN, Johns Hopkins University (720-43-4)

9:45- 9:55 (100) A Paley-Wiener theorem. Preliminary report. GUNAR E. LIEPINS, Texas Tech University (720-43-6)

10:00-10:10 (101) Generalized multipliers of spaces on locally groups. II. Preliminary report. Professor J. EDWARD KERLIN, University of Kentucky (720-43-7) (Introduced by Professor A. Lambert)

10:15-10:25 (102) Generalizations of P. Levy's inversion formula. Professor P.R. MASANI, University of Pittsburgh (720-43-12)

10:30-10:40 (103) Multipliers on compact groups with unconditionally converging Fourier series. Preliminary report. Dr. ALAN ARMSTRONG, Kansas State University (720-43-8)

10:45-10:55 (104) Multipliers of closed ideals of L^p (D^∞). Preliminary report. Dr. DANIEL M. OBERLIN, Florida State University (720-43-11)

11:00-11:10 (105) Riesz transforms for vector spaces over a local field. Dr. CHARLES P. DOWNEY, University of Nebraska, Omaha (720-43-5)

11:15-11:25 (106) A characterization of H^p - spaces on a local field. Preliminary report. Dr. JIA-ARNG CHAO, University of Texas (720-43-10)

THURSDAY, 8:45 A.M.

Session on Lattices, Adams/Hamilton Room, Sheraton-Park (First Floor)

8:45- 8:55 (107) Lattice orderings on the real field. Mr. ROBERT R. WILSON, California State University, Long Beach (720-06-1)

9:00- 9:10 (108) Means in lattices and semilattices. Dr. SIEMION FAJTLOWICZ, University of Houston (720-06-5)

9:15- 9:25 (109) Some results from the combinatorial approach to quantum logic. RICHARD J. GREECHIE, Kansas State University (720-06-6)

9:30- 9:40 (110) On modular lattices with four generators. II. Preliminary report. Mr. C. HERRMANN*, Vanderbilt University, and Mr. R. WILLE, Technische Hochschule, Darmstadt, West Germany (720-06-10)

9:45- 9:55 (111) Compact convergence lattices. Preliminary report. Dr. ARMANDO R. GINGRAS, IBM Corporation, Boulder, Colorado (720-06-11)

10:00-10:10 (112) Semimodularity in the completion of a poset. Dr. BARBARA JEFFCOTT* and Dr. WILLIAM SPEARS, Kansas State University (720-06-12)

10:15-10:25 (113) Products of symmetric elements in the Noether lattice, $RL(A_i)$. Preliminary report. Professor MICHAEL E. DETLEFSEN, Slippery Rock State College (720-06-13)

10:30-10:40 (114) The hyperoctant property in orthomodular AC-lattices. Dr. RONALD P. MORASH, University of Michigan (720-06-14)

10:45-10:55 (115) A characterization of a type of nearest point set in complete ℓ-groups. Dr. MICHAEL KEISLER, Randolph-Macon College (720-06-3)

11:00-11:10 (116) Doubling chains, singular elements and hyper-$\mathcal{G}\ell$-groups. Dr. JORGE MARTINEZ, University of Florida (720-06-8)

11:15-11:25 (117) Groups of homeomorphisms of the real line with manageable automorphism groups. Professor STEPHEN H. McCLEARY, University of Georgia (720-06-7)

THURSDAY, 8:45 A.M.

Session on Real Analysis, Senate Room, Sheraton-Park (Lobby Level)

8:45- 8:55 (118) Remarks on the Gauss-Lucas theorem in higher dimensional space. Professor A.W. GOODMAN, University of South Florida (720-26-1)

9:00- 9:10 (119) Some analogues of the Lusin-Menchoff theorem and applications. Preliminary report. Mr. STEVE AGRONSKY, University of California, Santa Barbara (720-26-2) (Introduced by Professor A.M. Bruckner)

9:15- 9:25 (120) On the existence of the differential of a function of several real variables. Professor L.V. TORALBALLA, and Professor LOUIS I. ALPERT*, Bronx Community College (720-26-3)

9:30- 9:40 (121) Trjitzinsky theorem in the product space of two abstract measure spaces. Professor MONBILL TONG, California State University (720-28-1) (Introduced by Professor Hugo Sun)

9:45- 9:55 (122) Riesz theorem for H*-algebra valued positive linear mappings. Preliminary report. Professor PARFENY SAWOROTNOW, Catholic University (720-28-2)

10:00-10:10 (123) The Radon-Nikodym theorem in locally convex spaces. Preliminary report. Professor G.Y.H. CHI, University of Florida and University of Pittsburgh (720-28-3)

10:15-10:25 (124) A decomposition of measures. Preliminary report. Professor ROY A. JOHNSON, Washington State University (720-28-4)

10:30-10:40 (125) An example of a universal measure. Preliminary report. Professor WILLIAM GRAVES, University of North Carolina (720-28-5)

10:45-10:55 (126) A relation of measurability with respect to a sigma-ring V to measurability with respect to the smallest sigma-algebra extending V. Professor W.M. BOGDANOWICZ, and Mr. J.P. McCLOSKEY*, Catholic University of America (720-28-6)

11:00-11:10 (127) Mutual existence of product integrals in normed rings. Professor JON C. HELTON, University of Arizona (720-28-7)

11:15-11:25 (128) Absolutely summing and mapping properties of representing measures. Dr. PAUL LEWIS, North Texas State University (720-28-8)

THURSDAY, 8:45 A.M.

Session on Semigroups, Madison Room, Sheraton-Park (First Floor)

8:45- 8:55 (129) Simple inverse semigroups whose $\mathscr{D}$ -classes are ω-semigroups. Professor JANET E. AULT, Madison College (720-20-2)

9:00- 9:10 (130) Homogeneous quotients of an inverse semigroup. Dr. KEVIN E. OSONDU, State University of New York at Buffalo (720-20-8)

9:15- 9:25 (131) Compatible extensions of semigroups. Professor DAVID E. ZITARELLI, Temple University (720-20-10)

9:30- 9:40 (132) Tensor product of commutative semigroups. Dr. JAMES A. ANDERSON, Northern Arizona University (720-20-5)

9:45- 9:55 (133) Regularity and Green's relations for sandwich semigroups. Preliminary report. K.D. MAGILL, Jr.* and S. SUBBIAH, State University of New York at Buffalo (720-20-9)

10:00-10:10 (134) A construction of the idempotent-separating congruences on a bisimple orthodox semigroup. Professor D.R. LATORRE, Clemson University (720-20-26)

10:15-10:25 (135) A generalization of right simple semigroups. Preliminary report. Professor FRANCIS E. MASAT, Glassboro State College (720-20-27)

10:30-10:40 (136) Linear projection operators on standard threads. Dr. GEORGE AKST*, and Dr. THOMAS P. DENCE, New Mexico State University-Las Cruces (720-20-11)

10:45-10:55 (137) Semilattices of topological semigroups. Preliminary report. J.C. HIGGINS, Brigham Young University (720-20-12)

11:00-11:10 (138) Quasidirect products of quasigroups. Professor ROBERT L. WILSON, Jr., University of Wisconsin (720-20-21)

11:15-11:25 (139) A note on some varieties of point algebras. Professor MARSHALL SAADE, University of Georgia (720-20-22)

THURSDAY, 9:00 A.M.

Invited Address, Regency Ballroom, Shoreham

(140) Invariant differential equations on homogeneous manifolds. Professor SIGURDUR HELGASON, Massachusetts Institute of Technology (720-22-11)

THURSDAY, 9:00 A.M.

Special Session on Some Directions of Current Progress in Commutative Algebra I, Palladian Room, Shoreham (Lower Lobby Level)

9:00- 9:45 (141) Invariant theory, Macaulay rings, and Gorenstein rings. Preliminary report. Mr. E. GRAHAM EVANS, Jr., University of Illinois (720-13-7)

10:00-10:45 (142) Involutions in characteristic 2. Professor MICHAEL ARTIN, Massachusetts Institute of Technology

11:00-11:45 (143) Homological questions in commutative rings and Cohen-Macaulay modules. Professor MELVIN HOCHSTER, Purdue University, West Lafayette (720-13-1)

THURSDAY, 9:00 A.M.

Special Session on Probabilistic Analysis I, Heritage Room, Shoreham (Lower Lobby Level)

9:00- 9:20 (144) On the existence of optimal control for systems governed by Ito stochastic differential and functional equations. Professor N. U. AHMED, University of Ottawa (720-93-1) (Introduced by Professor A. T. Bharucha-Reid)

9:30- 9:50 (145) Existence and stability behavior of a class of stochastic functional integral equations. Preliminary report. Professor CHRIS P. TSOKOS* and Professor A.N. RAO, University of South Florida (720-60-11)

10:00-10:20 (146) Limit theorems for solutions to stochastic differential equations. Professor WALTER ROSENKRANTZ, University of Massachusetts (720-60-5)

10:30-10:50 (147) Random solutions of Burger's equation. Preliminary report. Professor MURRAY ROSENBLATT, University of California, San Diego (720-60-8)

11:00-11:20 (148) Asymptotic analysis of the linearized Boltzmann equation. Professor MARK A. PINSKY, Northwestern University (720-60-6)

11:30-12:00 (149) Informal Discussion

THURSDAY, 9:00 A.M.

Special Session on Analytic Number Theory I, Diplomat Room, Shoreham (Lower Lobby Level)

9:00- 9:20 (150) Dedekind sums for a totally real algebraic number field. Preliminary report. Professor LARRY J. GOLDSTEIN*, and Dr. PILOR DE LA TORRE, University of Maryland (720-10-8)

9:30- 9:50 (151) Density functions for prime and relatively prime numbers. Professor PAUL ERDOS, Hungarian National Academy and Professor IAN RICHARDS*, University of Minnesota (720-10-34)

10:00-10:20 (152) Fields with maximal Kronecker constant. Professor THOMAS CALLAHAN, University of Toronto, Dr. MORRIS NEWMAN, National Bureau of Standards, Washington, D.C., and Professor MARK SHEINGORN*, City University of New York, Baruch College (720-12-4)

10:30-10:50 (153) Analytic estimates of discriminants. Professor H.M. STARK, Massachusetts Institute of Technology (720-10-39)

11:00-11:30 (154) Assorted research problems in analytic number theory. Preliminary report. Professor HUGH L. MONTGOMERY, University of Michigan (720-10-40)

THURSDAY, 10:00 A.M.

Special Session on Topological Dynamics I, Executive Room, Shoreham

10:00-12:00 (155) Informal

THURSDAY, 10:15 A.M.

Invited Address, Regency Ballroom, Shoreham

(156) Representations of semisimple Lie groups (a nontechnical survey). Professor Wilfried Schmid, Columbia University

THURSDAY, 12:00 NOON

General Session I, Madison Room, Sheraton-Park (First Floor)

12:00-12:10 (157) Identities. Preliminary report. Professor PRESTON C. HAMMER, Grand Valley State College (720-00-1)

12:15-12:25 (158) The axiom of choice in Church's type theory. Preliminary report. MITSURU YASUHARA, University of Kentucky (720-02-8)

12:30-12:40 (159) Undecidable properties of finite sets of equations. II. GEORGE McNULTY, Dartmouth College (720-02-13)

12:45-12:55 (160) Natural deduction and lattice models of some weak propositional logics. Professor STANLEY E. HAYES, State University of New York, College at New Paltz (720-02-14)

1:00- 1:10 (161) $\mathscr{V}$-filters and $\mathscr{T}$-filters. Mr. ROBERT W. BUTTON, Carnegie-Mellon University (720-02-10)

1:15- 1:25 (162) The indiscrete ultrafilters problem. Preliminary report. Mr. STEVEN GLAZER, University of Kansas (720-04-1)

1:30- 1:40 (163) Locally finite ring variety. Professor AWAD A. ISKANDER, University of Southwestern Louisiana (720-08-1)

1:45- 1:55 (164) Systems of magic Latin k-cubes. Professor JOSEPH ARKIN*, Spring Valley, New York, Professor VERNER E. HOGGATT, Jr., California State University, San Jose, and Professor ERNST G. STRAUS, University of California, Los Angeles (720-05-3)

2:00- 2:10 (165) Row admissible latin squares. Dr. MARK PANKIN, Marshall University (720-05-19)

THURSDAY, 12:00 NOON

Session on Algebra I, Wardman Room, Sheraton-Park (Lobby Level)

12:00-12:10 (166) Generalized cyclotomic polynomials and the irreducible factors of Fibonacci polynomials. Professor CLARK KIMBERLING, University of Evansville (720-12-1)

12:15-12:25 (167) Local permutation polynomials over a finite field. Preliminary report. Dr. GARY L. MULLEN, Pennsylvania State University (720-12-12)

12:30-12:40 (168) The structure of strongly normal difference field extensions. Preliminary report. Professor RONALD P. INFANTE, Seton Hall University (720-12-6)

12:45-12:55 (169) A new proof in the Galois theory of function fields. Dr. DAVID A. GAY, New College and University of Arizona (720-12-15)

1:00- 1:10 (170) On pseudo valuations. Preliminary report. Dr. THOMAS RIGO, Indiana University–Purdue University at Indianapolis (720-12-14)

1:15- 1:25 (171) Prime ideals and pseudo Boolean valuations. Dr. JOSEPH E. CICERO, Clayton Junior College (720-13-5)

1:30- 1:40 (172) Higher derivations and automorphisms of C[[X]]. Preliminary report. Professor HENRY J. SCHULTZ, University of Michigan, Dearborn (720-13-2)

1:45- 1:55 (173) Substitution in Nash functions. Preliminary report. Professor GUSTAVE A. EFROYMSON, University of New Mexico (720-14-1)

2:00- 2:10 (174) On cubic curves over finite fields. Preliminary report. Dr. BASIL GORDON, University of California at Los Angeles, and Dr. LORNE HOUTEN*, Roswell Park Memorial Institute (720-14-2)

2:15- 2:25 (175) Singular orbits for linear algebraic groups. Preliminary report. Dr. FRANK SERVEDIO, Dalhousie University (720-14-3)

THURSDAY, 12:00 NOON

Session on Analysis I, Senate Room, Sheraton-Park (Lobby Level)

12:00-12:10 (176) Subharmonic functions possessing local harmonic majorants. Professor MYRON GOLDSTEIN, Arizona State University (720-31-2)

12:15-12:25 (177) Growth problems for subharmonic functions of finite order in space. Professor N. V. RAO*, University of Toledo, and Professor DAN F. SHEA, University of Wisconsin (720-31-3)

12:30-12:40 (178) Polyharmonic Green's functions and null classes. Preliminary report. Dr. NORMAN MIRSKY, Texas Tech University (720-31-4)

12:45-12:55 (179) Natural limits for harmonic and superharmonic functions. Preliminary report. Professor J. R. DIEDERICH, University of California, Davis (720-31-5)

1:00- 1:10 (180) A converse to Fatou's general theorem. Preliminary report. Mr. EDWIN BRAITHWAITE, University of Illinois (720-31-6)

1:15- 1:25 (181) Infimum and domination principles in vector lattices. Professor PETER A. FOWLER, California State University (720-31-7) (Introduced by Professor George B. Pedrick)

1:30- 1:40 (182) A sum of Jacobi polynomials. Professor JOAQUIN BUSTOZ, University of Cincinnati (720-33-1)

1:45- 1:55 (183) The Bessel polynomials and the student t-distribution. Dr. MOURAD E. H. ISMAIL*, Mathematics Research Center, University of Wisconsin and DOUGLAS KELKER, University of Alberta (720-33-2)

2:00- 2:10 (184) Quadratic transformations of Appell functions. Professor B. C. CARLSON, Ames Laboratory-USAEC, Iowa State University (720-33-3)

THURSDAY, 12:00 NOON

Session on Applied Mathematics I, Richmond Room, Sheraton-Park (Ground Floor)

12:00-12:10 (185) An interactive program for several algorithms for winding numbers. Dr. KENNETH O. LELAND, Navy Personnel Research and Development Center, San Diego (720-68-1)

12:15-12:25 (186) On the propagation of energy in multiphase wavetrains. Preliminary report. Dr. GERALD ROSKES, City University of New York, Queens College (720-70-1)

12:30-12:40 (187) The rotational gradient operator. Preliminary report. Dr. RAY F. SNIPES, Bowling Green State University (720-70-2)

12:45-12:55 (188) On a new analytical method of solution of nonlinear differential equations. I. The nonlinear oscillator equation. Preliminary report. Professor PETER J. MELVIN, University of Illinois (720-70-3) (Introduced by Professor Paul T. Bateman)

1:00- 1:10 (189) On the asymptotic nature of the n-body problem. Professor DONALD G. SAARI, Northwestern University (720-70-4)

1:15- 1:25 (190) Higher order corrections to the boundary layer equations. Professor SHIH-LIANG WEN*, Ohio University, and Dr. L.K. CHI, Sperry Univac Division, Cinnaminson, New Jersey (720-76-1)

1:30- 1:40 (191) On a microcontinuum model of pulsatile blood flow. Dr. LOKENATH DEBNATH, University of Maryland and East Carolina University (720-76-2)

1:45- 1:55 (192) Heat generated by MHD Couette flow with porous walls. Professor M. BALARAM, Grambling State University (720-76-3)

2:00- 2:10 (193) Existence and uniqueness of axisymmetric free boundary flows. DAVID H. HOITSMA, Wesleyan University (720-76-4) (Introduced by Professor J.D. Reid)

2:15- 2:25 (194) Existence of an optimal control for systems with jump Markov disturbances. Preliminary report. Professor ROBERT M. GOOR, University of Kentucky (720-93-2)

2:30- 2:40 (195) Null controllability for parabolic nonlinear equations. Professor WILLIAM C. CHEWNING, University of South Carolina (720-93-3)

THURSDAY, 12:00 NOON

Session on Associative Rings I, Franklin Room, Sheraton-Park (First Floor)

12:00-12:10 (196) Finite groups with semisimple endomorphism rings. Dr. CARTER LYONS, Madison College (720-16-1)

12:15-12:25 (197) A decomposition of rings. Preliminary report. Mr. GARY F. BIRKENMEIER, University of Wisconsin-Milwaukee (720-16-12)

12:30-12:40 (198) Reductivity in rings. Professor DON E. EDMONDSON, University of Texas (720-16-18)

12:45-12:55 (199) Rings whose ideals have projective covers. Dr. ROBERT L. SNIDER, Virginia Polytechnic Institute and State University (720-16-20)

1:00- 1:10 (200) On stable Noetherian rings. Professor ZOLTAN PAPP, George Mason University (720-16-4)

1:15- 1:25 (201) A representation theory for noetherian rings. Professor ROBERT GORDON*, Temple University, and Professor EDWARD L. GREEN, University of Pennsylvania (720-16-7)

1:30- 1:40 (202) σ-noetherian rings and σ-artinian rings. II. Professor JITENDRA N. MANOCHA, Kent State University (720-16-8)

1:45- 1:55 (203) Semiperfect group rings. Preliminary report. Mr. JOHN LAWRENCE, Carleton University (720-16-15)

2:00- 2:10 (204) Hereditary and semihereditary group rings. Preliminary report. Professor KLAUS E. ELDRIDGE, Ohio University, and Mr. SAMSON HSIA*, Ohio University (720-16-23)

2:15- 2:25 (205) Group rings which are domains. Preliminary report. Professor KLAUS E. ELDRIDGE*, and Mr. SAMSON HSIA, Ohio University (720-16-25)

THURSDAY, 12:00 NOON

Session on Finite Groups I, Assembly Room, Sheraton-Park (Lobby Level)

12:00-12:10 (206) Projectively monomial groups. Professor JOHN R. DURBIN, University of Texas and Professor K. BOLLING FARMER*, University of Florida (720-20-1)

12:15-12:25 (207) On solvability of certain groups of central type. Preliminary report. Mr. JAY YELLEN, Colorado State University (720-20-16)

12:30-12:40 (208) Maximal subgroups which determine the properties of a finite group. Preliminary report. MALCOLM OTTAWAY, State University of New York at Binghamton (720-20-30) (Introduced by Ben Brewster)

12:45-12:55 (209) On Fitting classes. Dr. JAMES C. BEIDLEMAN, University of Kentucky (720-20-3)

1:00- 1:10 (210) On the canonical sets of subgroups induced by a Fitting class F. Preliminary report. Mr. MICHAEL A. KLEMBARA, University of Cincinnati (720-20-31)

1:15- 1:25 (211) The formation generated by a Φ - free group. Dr. BEN BREWSTER, State University of New York at Binghamton (720-20-34)

1:30- 1:40 (212) The generalized Artin exponent of a finite group. Preliminary report. JOHN R. RASMUSSEN, Bowdoin College (720-20-14)

1:45- 1:55 (213) On the automorphism group of an integral group ring. Preliminary report. Mr. GARY L. PETERSON, Michigan State University (720-20-15)

2:00- 2:10 (214) The subclass algebra associated with a finite group and subgroup. II. Professor JOHN KARLOF, University of Nebraska (720-20-20)

2:15- 2:25 (215) A characterization of SL(3,8). Preliminary report. Dr. DEAN C. MORROW, State University of New York, College at Oneonta (720-20-24)

THURSDAY, 12:00 NOON

Session on General Topology II, Alexandria Room, Sheraton-Park (Ground Level)

12:00-12:10 (216) Initial topologies, k-spaces and accessibility spaces. CHARLES E. AULL, Virginia Polytechnic Institute and State University (720-54-3)

12:15-12:25 (217) The regular continuous image of a minimal regular space is not necessarily minimal regular. MANUEL P. BERRI*, and CARROLL F. BLAKEMORE, University of New Orleans (720-54-32)

12:30-12:40 (218) Compact basically screenable spaces are metrizable. Preliminary report. GARY GRUENHAGE, Auburn University (720-54-23)

12:45-12:55 (219) Products of symmetrizable spaces. Professor PETER W. HARLEY III and Professor R.M. STEPHENSON, Jr.*, University of South Carolina (720-54-56)

1:00- 1:10 (220) G_δ points in symmetrizable spaces. Professor PETER W. HARLEY III, University of South Carolina (720-54-60)

1:15- 1:25 (221) On E-sequentially regular spaces. Dr. ROMAN FRIČ, University of Transport Engineering, Žilina, Czechoslovakia (720-54-26) (Introduced by Professor Darrell C. Kent)

1:30- 1:40 (222) A note on the foundations of sequential topology. Professor CARLOS A. INFANTOZZI, Universidad de la Republica, Montevideo, Uruguay (720-54-27)

1:45- 1:55 (223) Generalized sequential spaces. Preliminary report. Dr. JOSEPH W. GOLDSTON, North Carolina Central University (720-54-57)

2:00- 2:10 (224) Characterizations of some countable, T_1-, k-spaces having exactly one non-isolated point. Preliminary report. Professor FRANK SIWIEC, City University of New York, John Jay College (720-54-19)

2:15- 2:25 (225) Extending cross-sections onto countable unions. W. HOLSZTYNSKI, Southern Illinois University (720-54-33)

THURSDAY, 12:00 NOON

Session on Linear Algebra, Adams/Hamilton Room, Sheraton-Park (First Floor)

12:00-12:10 (226) Adjoints and the numerical range. Professor MARVIN MARCUS, University of California, Santa Barbara (720-15-1)

12:15-12:25 (227) Polynomial calculus with D-like operators. Professor JAMES W. BURGMEIER* and Professor RONALD E. PRATHER, University of Denver (720-15-2)

12:30-12:40 (228) On the geometry of dual pairs. Professor B.E. CAIN, Iowa State University, Mr. B.D. SAUNDERS*, and Professor H. SCHNEIDER, University of Wisconsin (720-15-6)

12:45-12:55 (229) The role of the group generalized inverse in the theory of finite Markov chains. Dr. CARL D. MEYER, Jr., North Carolina State University (720-15-3)

1:00- 1:10 (230) A minimal permanent-like function. Professor W.C. PYE*, and Professor MELVYN W. JETER, University of Southern Mississippi (720-15-4)

1:15- 1:25	(231)	Singular values and diagonal elements. Professor R. C. THOMPSON, University of California, Santa Barbara (720-15-5)
1:30- 1:40	(232)	On the reduction of functions of matrices to matrix polynomials. Professor EDWIN K. GORA*, and Professor JAMES J. TATTERSALL, Providence College (720-15-7)
1:45- 1:55	(233)	A note on the Jordan canonical form. STEPHEN BANCROFT, Gordon College (720-15-9) (Introduced by Professor Thomas Goulding)
2:00- 2:10	(234)	Induced transformations on exterior product spaces. Professor JIN BAI KIM, West Virginia University (720-15-8)

THURSDAY, 12:00 NOON

Session on Number Theory I, Dover Room, Sheraton-Park (Ground Level)

12:00-12:10	(235)	Isomorphic Pellian quadratic forms and their associated continued fraction expansions. Preliminary report. Professor GREGORY WULCZYN, Bucknell University (720-10-9)
12:15-12:25	(236)	Quasi-genera of quadratic forms. Professor BURTON W. JONES, University of Colorado (720-10-16)
12:30-12:40	(237)	More additive arithmetic functions inversely associated with partition functions. Preliminary report. Professor L.M. CHAWLA, and Dr. ELLEN TORRANCE*, Kansas State University (720-10-5)
12:45-12:55	(238)	On sums analogous to Dedekind's sums. Professor J.M. GANDHI, Western Illinois University (720-10-10)
1:00- 1:10	(239)	The spinor genus of quaternion orders. Professor GORDON L. NIPP, California State University, Los Angeles (720-10-11)
1:15- 1:25	(240)	Asymptotic distribution mod m and the notion of independence. LAWRENCE KUIPERS, Southern Illinois University, Carbondale (720-10-12)
1:30- 1:40	(241)	Finding good rational approximations to power series. Dr. CHARLES F. OSGOOD, Naval Research Laboratory, Washington, D.C. (720-10-13)
1:45- 1:55	(242)	Diophantine equations of the form $x^2 + D = y^p$. II. Professor EZRA BROWN, Virginia Polytechnic Institute and State University (720-10-15)
2:00- 2:10	(243)	Factoring integers whose digits are all ones in base b. Dr. ELMER K. HAYASHI, Wake Forest University (720-10-17)

THURSDAY, 12:00 NOON

Session on Ordinary Differential Equations I, Wilmington Room, Sheraton-Park (Ground Level)

12:00-12:10	(244)	$Y' = G(X,Y)$, $G \in C'$. Professor LLOYD K. WILLIAMS, Atlanta University (720-34-4)
12:15-12:25	(245)	A generalization of the Sturm-Piccone comparison theorem. Professor VADIM KOMKOV, Texas Tech University (720-34-9)
12:30-12:40	(246)	On a partial asymptotic stability theorem of Willett and Wong. Dr. FREDERICK SCOTT, Comcon, Inc., Cinnaminson, New Jersey (720-34-29)
12:45-12:55	(247)	Stability properties of a differential equation. Dr. HERMAN GOLLWITZER, Drexel University (720-34-30)
1:00- 1:10	(248)	On the equation $4x^{n+2}y'' + y^{2n+1} = 0$. MARK FEY, Pennsylvania State University, Capitol Campus (720-34-6) (Introduced by Dr. G. DiAntonio)
1:15- 1:25	(249)	Nonlinear boundary value problems. Dr. RONALD GRIMMER, Southern Illinois University (720-34-19)
1:30- 1:40	(250)	Certain second order boundary value problems. Professor PHILIP W. WALKER, University of Houston (720-34-27)
1:45- 1:55	(251)	Two point connection problem for a certain differential equation with an irregular singular point of rank two. Dr. T.K. PUTTASWAMY, Ball State University (720-34-15)
2:00- 2:10	(252)	The structure of oscillatory solutions of third order linear differential equations. DAVID LOWELL LOVELADY, Florida State University (720-34-37)
2:15- 2:25	(253)	Nonselfadjoint fourth order differential equations with conjugate points. Professor KURT KREITH, University of California, Davis (720-34-1)
2:30- 2:40	(254)	An inverse eigenvalue problem of order four. Dr. JOYCE R. McLAUGHLIN, Rensselaer Polytechnic Institute (720-34-2)

THURSDAY, 12:00 NOON

Session on Partial Differential Equations I, Arlington Room, Sheraton-Park (Ground Level)

12:00-12:10	(255)	Constructive function theoretic methods for fourth order pseudoparabolic and meta-parabolic equations in two space variables. Professor PATRICK M. BROWN, Gonzaga University, Professor ROBERT P. GILBERT, Indiana University, and Professor GEORGE C. HSIAO*, University of Delaware (720-35-7)
12:15-12:25	(256)	On a uniform parabolic equation with mixed boundary condition. Professor C.V. PAO, North Carolina State University (720-35-10)
12:30-12:40	(257)	A representation for distributional solutions of parabolic problems. Preliminary report. Professor HAROLD D. MEYER, Texas Tech University (720-35-11)
12:45-12:55	(258)	On a singular parabolic operator. Preliminary report. Professor LEONARD J. LIPKIN, University of North Florida (720-35-12)
1:00- 1:10	(259)	Saddle point behavior for a nonlinear one-dimensional parabolic equation. Preliminary report. Professor NATHANIEL CHAFEE, Georgia Institute of Technology (720-35-22)
1:15- 1:25	(260)	Parabolic Green functions in open sets. Dr. NEIL EKLUND, Centre College of Kentucky (720-35-25)
1:30- 1:40	(261)	Boundary regularity of weak solutions to a quasilinear parabolic equation. Dr. DANIEL J. DEIGNAN, University of Kentucky (720-35-27)
1:45- 1:55	(262)	Construction of solutions of parabolic partial differential equations. Professor M.J. STECHER, Texas A&M University (720-35-45)
2:00- 2:10	(263)	An algebraic rate of decay of local energy of solutions of hyperbolic systems on exterior regions. Preliminary report. Professor CLIFFORD O. BLOOM, State University of New York at Buffalo (720-35-26)
2:15- 2:25	(264)	Uniqueness in the Cauchy problem for a doubly characteristic operator. Ms. LETITIA SEESE, University of New Mexico (720-35-28)
2:30- 2:40	(265)	Formation of a shock from a compression wave. Preliminary report. Professor BARBARA KEYFITZ, Columbia University (720-35-13)
2:45- 2:55	(266)	Scattering of waves reflected off a moving obstacle. Professor JEFFERY COOPER*, University of Maryland, College Park, and Professor WALTER A. STRASS, Brown University (720-35-17)

THURSDAY, 12:00 NOON

Session on Probability I, Continental Room, Sheraton-Park (Lobby Level)

12:00-12:10	(267)	On admissible translates of infinitely divisible distributions. Professor WILLIAM N. HUDSON*, University of Utah, and Professor HOWARD G. TUCKER, University of California, Irvine (720-60-1)
12:15-12:25	(268)	More on equivalence of infinitely divisible distributions. Professor W.N. HUDSON and Professor J.D. MASON*, University of Utah (720-60-2)
12:30-12:40	(269)	A spherical characterization of the normal distribution. Dr. DAVID H. NASH*, General Motors Technical Center, Warren, Michigan, and Professor MURRAY S. KLAMKIN, University of Waterloo (720-60-19)
12:45-12:55	(270)	On the unimodality of spherically symmetric stable distribution functions. Professor STEPHEN J. WOLFE, University of Delaware (720-60-23)
1:00- 1:10	(271)	Conjugate transforms for τ_T semigroups of probability distribution functions. Mr. RICHARD MOYNIHAN, University of Massachusetts (720-60-28) (Introduced by Professor Berthold Schweizer)
1:15- 1:25	(272)	Existence of Markov processes associated with noncontraction semigroups. T.F. LIN, Louisiana State University (720-60-14)
1:30- 1:40	(273)	How well can one fit two processes together. Professor PAUL C. SHIELDS, University of Toledo (720-60-15)
1:45- 1:55	(274)	Lorentz manifolds and retarded Markov operators. Preliminary report. Dr. S.P. LLOYD, Bell Laboratories, Murray Hill, New Jersey and Professor R.C. SINE*, University of Rhode Island (720-60-18)
2:00- 2:10	(275)	An algebraic model for Wiener measure. Mr. MARK J. CHRISTENSEN*, and Professor A.T. BHARUCHA-REID, Wayne State University (720-60-16)
2:15- 2:25	(276)	Random evolutions and limit theorems for initial value problems. ROBERT P. KERTZ, Georgia Institute of Technology (720-60-24)

THURSDAY, 1:00 P.M.

Colloquium Lectures: Lecture I, Regency Ballroom, Shoreham

(277) New directions in model theory. Professor H. JEROME KEISLER, University of Wisconsin-Madison

THURSDAY, 2:00 P.M.

Special Session on Interpolation of Operators and Applications I, Palladian Room, Shoreham (Lower Lobby Level)

2:00- 3:00 (278) Some recent applications of the theory of interpolation of operators. Professor E.M. STEIN, Princeton University

3:05- 3:25 (279) Interpolation between H^p spaces, $0<p<\infty$. Professor CHARLES FEFFERMAN, University of Chicago, Professor NESTOR M. RIVIÈRE*, University of Minnesota, and Professor Y. SAGHER, University of Minnesota and Weizmann Institute of Sciences, Israel (720-42-13)

3:30- 3:50 (280) Interpolation on H^p spaces and some of their generalizations. Preliminary report. GUIDO WEISS, Washington University (720-47-16)

3:55- 4:15 (281) Interpolation between H^p spaces, the complex method. Preliminary report. Professor A.P. CALDERON, Massachusetts Institute of Technology, and Professor A. TORCHINSKY*, Cornell University (720-42-9)

4:20- 4:40 (282) Measures as convolution operators on $H^1\alpha$ and Lip α. Dr. MISHA ZAFRAN, Stanford University (720-42-10)

4:45- 5:05 (283) Counterexamples in interpolation theory. Professor ROBERT SHARPLEY III, Oakland University (720-46-45)

5:10- 5:30 (284) Interpolation and martingale inequalities. Professor CARL S. HERZ, McGill University

5:35- 5:55 (285) Biorthogonal systems equivalent to Fourier series. Professor HAROLD E. BENZINGER, University of Illinois (720-42-2)

THURSDAY, 2:00 P.M.

Special Session on Probabilistic Analysis II, Heritage Room, Shoreham (Lower Lobby Level)

2:00- 2:20 (286) Recurrence of d-dimensional random walks and the exterior Dirichlet problem. Professor HARRY KESTEN, Cornell University (720-60-12)

2:30- 2:50 (287) Integration of stochastic partial differential equations. Professor PAO-LIU CHOW, Wayne State University (720-60-9)

3:00- 3:20 (288) Existence of densities for degenerate diffusion processes. Professor AVNER FRIEDMAN, Northwestern University (720-60-4)

3:30- 3:50 (289) Fluctuations in the Navier-Stokes equations. Professor GEORGE C. PAPANICOLAOU, Courant Institute, New York University (720-60-10)

4:00- 4:30 (290) Informal Discussion

THURSDAY, 2:00 P.M.

Special Session on Mathematics and Games II, Empire Room, Shoreham

2:00- 2:40 (291) Recent results and open questions for some particular games. Preliminary report. Professor EDWARD THORP, University of California, Irvine (720-90-7)

2:40- 3:00 (292) On some positional games and the use of hypergraphs. Professor CLAUDE BERGE, University of Paris

3:00- 3:40 (293) Infinite games. Professor RALPH McKENZIE, University of California, Berkeley

3:40- 5:10 (294) Roundtable Discussion

5:10- 5:30 (295) Backgammon doubling strategy. EMMETT B. KEELER, Rand Corporation, Santa Monica, California, and JOEL H. SPENCER*, Massachusetts Institute of Technology (720-90-10)

5:30- 5:40 (296) Dynamic metagames of static games. Mr. DAVID S. LAWRENCE, Courant Institute, New York University (74T-C30)

THURSDAY, 2:00 P.M.

Special Session on Topological Dynamics II, Executive Room, Shoreham

2:00- 2:20 (297) The discrete Bebutov dynamical system. Dr. ALLAN JAWORSKI, University of Texas at Austin (720-54-18)

2:30- 2:50 (298) Real prime flows. Professor H.B. KEYNES*, University of Minnesota and Professor D. NEWTON, University of Sussex, England (720-54-9)

3:00- 3:20 (299) Generalized Morse flows on n symbols. Dr. JOHN C. MARTIN, North Dakota State University (720-54-22)

3:30- 3:50 (300) On the connectedness of homomorphisms in topological dynamics. Preliminary report. Dr. DOUGLAS McMAHON*, University of Oregon, and Dr. TA-SUN WU, Case Western Reserve University (720-54-17)

4:00- 4:20 (301) Sofic systems. ETHAN M. COVEN, Wesleyan University and MICHAEL E. PAUL*, University of Maryland, Baltimore County (720-54-10)

4:30- 4:50 (302) Expansive properties of maps. Preliminary report. WILLIAM L. REDDY, Wesleyan University (720-54-39)

5:00- 5:20 (303) Generalized Bohr compactifications. Preliminary report. Professor LEONARD SHAPIRO, University of Minnesota (720-22-12)

THURSDAY, 2:15 P.M.

Invited Address, Regency Ballroom, Shoreham

(304) On real Teichmuller spaces and their modular groups. Professor LINDA KEEN, City University of New York, Lehman College and Graduate School and University Center (720-32-7)

THURSDAY, 2:30 P.M.

General Session II, Madison Room, Sheraton-Park (First Floor)

2:30- 2:40 (305) Isotone functions and diagonal projection operators. Professor TAEN-YU DAI*, City University of New York, York College, and Professor RALPH E. DeMARR, University of New Mexico (720-06-2)

2:45- 2:55 (306) Galois connection algorithms. Preliminary report. Professor EUGENE M. NORRIS, University of South Carolina (720-06-4)

3:00- 3:10 (307) The dimension of planar posets. Preliminary report. Dr. WILLIAM T. TROTTER, Jr.,* and Mr. JOHN I. MOORE, Jr., University of South Carolina (720-05-24)

3:15- 3:25 (308) The dimension of a tree is at most three. Preliminary report. Dr. WILLIAM T. TROTTER, Jr., and Mr. JOHN I. MOORE, Jr.,* University of South Carolina (720-05-25)

3:30- 3:40 (309) A rank equality for matroids. Professor TERRENCE J. BROWN, State University of New York, College at Oswego (720-05-43)

3:45- 3:55 (310) Representation of finite orthomodular posets. Preliminary report. Mr. ERIC G.R. GERELLE, Kansas State University (720-06-9) (Introduced by Dr. R.J. Greechie)

4:00- 4:10 (311) Integration of polynomials without evaluation of antiderivatives. JAMES D. FABREY, University of North Carolina (720-98-1)

4:15- 4:25 (312) The mathematization of analysis. Professor WILLIAM E. HARTNETT, State University of New York, College at Plattsburgh (720-98-2)

4:30- 4:40 (313) Media assisted self study lessons in calculus. Professor THERON ROCKHILL, State University of New York, College at Brockport (720-98-3) (Introduced by Dr. E.O. Stephany)

4:45- 4:55 (314) Method for teaching introduction to computer science. Preliminary report. Dr. LEE D. THOMPSON, Loyola University, Chicago (720-98-4)

5:00- 5:10 (315) Minimal surfaces rediscovered. Sister RITA EHRMANN, Villanova University (720-98-5)

THURSDAY, 2:30 P.M.

Session on Algebra II, Wardman Room, Sheraton-Park (Lobby Level)

2:30- 2:40 (316) Chain conditions on regular near-rings. Professor MARJORY J. JOHNSON, University of South Carolina (720-16-13)

2:45- 2:55 (317) Endomorphism near rings that are rings. Preliminary report. Professor J.J. MALONE, Worcester Polytechnic Institute (720-16-14)

3:00- 3:10 (318) Matrix representation of simple halfrings. Professor H.E. STONE, University of Pittsburgh (720-16-22)

3:15- 3:25 (319) Some forms of certain Hopf algebras. Professor D.E. RADFORD, Lawrence University, Professor E.J. TAFT* and Professor R.L. WILSON, Rutgers University (720-16-9)

3:30- 3:40 (320) The antipode of a Hopf algebra. Preliminary report. Professor DAVID E. RADFORD, Lawrence University (720-16-24)

3:45- 3:55 (321) On the coradical of a Hopf algebra. Preliminary report. Dr. RICHARD K. MOLNAR, Oakland University (720-16-26)

4:00- 4:10 (322) An application of group rings to associator dependent algebras. Professor DENIS R. FLOYD*, and Professor IRVIN ROY HENTZEL, Iowa State University (720-17-6)

4:15- 4:25 (323) Semiprime antiflexible rings. Preliminary report. Professor HASAN A. CELIK, California State Polytechnic University (720-17-11)

4:30- 4:40 (324) A note on idealizers of terms of the ideal closure series of subalgebras of Lie algebras. Preliminary report. Miss ELENA M. MEDINA*, and Dr. BYOUNG-SONG CHWE, University of Alabama (720-17-14)

4:45- 4:55 (325) Radial component of invariant differential operators at principal nilpotent points. Professor MOHSEN PAZIRANDEH, University of Texas (720-22-2) (Introduced by Professor Roger Bleier)

THURSDAY, 2:30 P.M.

Session on Analysis II, Adams/Hamilton Room, Sheraton-Park (First Floor)

2:30- 2:40 (326) Interpolating with discrete natural polynomial spline functions. Professor C.S. DURIS, Drexel University (720-39-1)

2:45- 2:55 (327) On the speed of convergence of iterations of a function. Dr. VLADIMIR DROBOT, University of Santa Clara (720-39-2)

3:00- 3:10 (328) A note on linear hereditary systems in L^p-spaces. Preliminary report. Dr. H.T. BANKS, Brown University, and Dr. J.A. BURNS*, Virginia Polytechnic Institute and State University (720-39-3)

3:15- 3:25 (329) Representation of the solution of a delay-differential equation. Preliminary report. Dr. BERNARD A. ASNER, Jr., University of Dallas (720-39-4)

3:30- 3:40 (330) A Tauberian theorem for stretchings. Professor DAVID F. DAWSON, North Texas State University (720-40-1)

3:45- 3:55 (331) A Tauberian theorem for the generalized logarithmic method of summation. Dr. KUSUM SONI, University of Tennessee (720-40-7)

4:00- 4:10 (332) Summability of rearrangements of sequences Professor J.A. FRIDY, Kent State University (720-40-4)

4:15- 4:25 (333) Matrix summability theorems involving a certain class of sequences. Dr. W. VANCE UNDERHILL, East Texas State University (720-40-6)

4:30- 4:40 (334) Series expansions and linear differential operators. Preliminary report. Dr. J.K. SHAW, Virginia Polytechnic Institute and State University (720-40-3) (Introduced by Professor George W. Crofts)

4:45- 4:55 (335) Generalization of Fibonacci numbers useful in dynamic memory allocation. Dr. HELAMAN FERGUSON, Brigham Young University (720-40-5)

THURSDAY, 2:30 P.M.

Session on Associative Rings II, Franklin Room, Sheraton-Park (First Floor)

2:30- 2:40 (336) Regular modules. Dr. TOM CHEATHAM, Samford University (720-16-2) (Introduced by Dr. W.D. Peeples)

2:45- 2:55 (337) On rings with no nontrivial annihilators. Preliminary report. Professor GEORGE SZETO, Bradley University (720-16-6)

3:00- 3:10 (338) Rings with a class of modules projectives or quasi-injectives. Preliminary report. Mr. S.C. GOEL*, and Dr. S.K. JAIN, Ohio University (720-16-16)

3:15- 3:25 (339) Semiprime splitting rings. Preliminary report. Dr. MARK L. TEPLY, University of Florida (720-16-17)

3:30- 3:40 (340) Primary direct sum decomposition. JOHN FUELBERTH, University of Northern Colorado, and J. KUZMANOVICH*, Wake Forest University (720-16-21)

3:45- 3:55 (341) Finitely generated projective modules. Preliminary report. Dr. ROBERT W. MILLER, College of William and Mary (720-16-27)

4:00- 4:10 (342) Rings generated by their units. Preliminary report. Professor JOE W. FISHER, University of Texas at Austin (720-16-3)

4:15- 4:25 (343) Completions of simple regular rings. Dr. K.R. GOODEARL, University of Utah (720-16-5)

4:30- 4:40 (344) A noncommutative analog of prime ideals. Mr. JAY SHAPIRO, Rutgers University (720-16-10)

4:45- 4:55 (345) Maximal orders. Preliminary report. JOHN H. COZZENS*, Rider College and Kent State University, and FRANCIS L. SANDOMIERSKI, Kent State University (720-16-28)

5:00- 5:10 (346) Presentations of GE(n, R) and computation of $K_2(n, R)$. Preliminary report. Miss SUSAN C. GELLER, Cornell University (720-16-11)

THURSDAY, 2:30 P.M.

Session on Finite Groups II, Assembly Room, Sheraton-Park (Lobby Level)

2:30- 2:40 (347) The subgroup lattice for groups of order p^2q and p^3. Mr. THOMAS KRAFCIK, Miami University (720-20-19) (Introduced by Professor Charles Holmes)

2:45- 2:55 (348) Groups of order p^3q with identical subgroup structures. Dr. CHARLES S. HOLMES, Miami University (720-20-35)

3:00- 3:10 (349) Partitions and representations of the hyperoctahedral groups. Preliminary report. Mr. DENNIS F. KINCH, University of North Carolina (720-20-25)

3:15- 3:25 (350) Automorphism groups of isoclinic p-groups. Professor JOSEPH BUCKLEY, Western Michigan University (720-20-4)

3:30- 3:40 (351) The existence of a normal Sylow p-subgroup or a normal p-complement. Professor JAMES R. WEAVER, University of West Florida (720-20-6)

3:45- 3:55 (352) Aut(G) of a semidirect product of cyclic p-groups. Preliminary report. Dr. LARRY E. KNOP, University of Texas, Austin (720-20-13)

4:00- 4:10 (353) p-groups in which centralizers of noncentral elements have the same order. I. Preliminary report. Dr. DAVID M. ROCKE, Governors State University (720-20-32)

4:15- 4:25 (354) Construction of p-groups of maximal class with group order $p^n \leq p^{2k+6}$. RAY SHEPHERD, Palm Beach Atlantic College (720-20-36)

4:30- 4:40 (355) Connection between the Schur index of a representation and the structure of the group. Professor CHARLES FORD, Washington University (720-20-33)

THURSDAY, 2:30 P.M.

Session on Categories, Arlington Room, Sheraton-Park (Ground Level)

2:30- 2:40 (356) Epireflective subcategories in concrete categories. Preliminary report. Dr. JEAN MARIE McDILL, California Polytechnic State University (720-18-1)

2:45- 2:55 (357) Finite partially ordered sets of cohomological dimension one. Preliminary report. CHARLES CHING-AN CHENG, Rutgers University (720-18-2)

3:00- 3:10 (358) Principal homogeneous objects as representable functors. Preliminary report. Professor D. H. VAN OSDOL, University of New Hampshire (720-18-3)

3:15- 3:25 (359) An application of nonstandard analysis to category theory. Professor HARRY GONSHOR, Rutgers University (720-18-4)

3:30- 3:40 (360) Free triples and the orthogonal subcategory problem. Dr. HARVEY WOLFF, University of Texas (720-18-5)

3:45- 3:55 (361) Adjunctions and comonads in differential algebra. Professor WILLIAM F. KEIGHER, University of Tennessee (720-18-6)

4:00- 4:10 (362) Structured categories and projection functors. Preliminary report. Professor V.S. KRISHNAN, Temple University (720-18-7)

THURSDAY, 2:30 P.M.

Session on General Topology III, Alexandria Room, Sheraton-Park (Ground Level)

2:30- 2:40 (363) Another characterization of semiprime ideals in C(X). Dr. W. T. WHITLEY, Marshall University (720-54-4)

2:45- 2:55 (364) First category function spaces under the topology of pointwise convergence. Professor ROBERT A. McCOY, Virginia Polytechnic Institute and State University (720-54-30)

3:00- 3:10 (365) Maximal ideals in algebras of continuous functions. Dr. EDWARD BECKENSTEIN*, St. John's University, Staten Island, Dr. LAWRENCE NARICI, St. John's University, Jamaica, Dr. CHARLES SUFFEL, Stevens Institute of Technology, and Dr. SETH WARNER, Duke University (720-54-64)

3:15- 3:25 (366) θ-characterizations of generalized paracompact spaces. Preliminary report. Professor J.C. SMITH, Virginia Polytechnic Institute and State University (720-54-11)

3:30- 3:40 (367) Countable paracompactness of F_σ-sets. PHILLIP ZENOR, Auburn University (720-54-34)

3:45- 3:55 (368) On normality and countable paracompactness. Professor GEORGE M. REED, Ohio University (720-54-38)

4:00- 4:10 (369) On extending locally finite collections. Dr. H. L. SHAPIRO, Northern Illinois University, and Dr. F.A. SMITH*, Kent State University (720-54-52)

4:15- 4:25 (370) α-weak normality and related properties. Preliminary report. Dr. EUGENE S. BALL, American University, Cairo, Egypt (720-54-55)

4:30- 4:40 (371) Completeness in semimetric spaces. Preliminary report. Professor S.D. SHORE*, University of New Hampshire, and Professor S.A. KENTON, Eastern Connecticut State College (720-54-45)

THURSDAY, 2:30 P.M.

Session on Graphs II, Continental Room, Sheraton-Park (Lobby Level)

2:30- 2:40 (372) On a method of attacking some combinatorial problems. Preliminary report. E. MAHMOODIAN, University of Pennsylvania (720-05-10)

2:45- 2:55 (373) On pseudosurface imbeddings of graphs. Professor R.D. RINGEISEN*, Purdue University and Professor A.T. WHITE, Western Michigan University (720-05-14)

3:00- 3:10 (374) Self-dual embeddings of graphs. Preliminary report. Mr. SAUL STAHL, Western Michigan University (720-99-1)

3:15- 3:25 (375) The independence ratio of a toroidal graph. Professor MICHAEL O. ALBERTSON*, Smith College, and Dr. JOAN P. HUTCHINSON, Dartmouth College (720-05-27)

3:30- 3:40 (376) A class of panconnected graphs. R.J. FAUDREE, and R.H. SCHELP*, Memphis State University (720-05-17)

3:45- 3:55 (377) A theorem on strongly connected digraphs. Preliminary report. Professor Y. EDMUND LIEN, University of Kansas (720-05-22) (Introduced by Professor Richard Hetherington)

4:00- 4:10 (378) Maximally nonhamiltonian-connected graphs. Preliminary report. Dr. JAMES E. WILLIAMSON, Southern Illinois University (720-05-30)

4:15- 4:25 (379) Nuclei of a point determining graph. Preliminary report. Mr. DENNIS P. GEOFFROY, University of South Carolina (720-05-34)

4:30- 4:40 (380) Nonstandard methods in combinatorics and graph theory. Preliminary report. Professor A.E. HURD, University of Victoria (720-05-45)

4:45- 4:55 (381) Vertex cyclic graphs. Mr. JOHN ROBERTS, Western Michigan University (720-99-3)

THURSDAY, 2:30 P.M.

Session on Ordinary Differential Equations II, Dover Room, Sheraton-Park (Ground Level)

2:30- 2:40 (382) Asymptotic nature of nonoscillatory solutions of nth order retarded equations. Dr. BHAGAT SINGH, University of Wisconsin (720-34-16)

2:45- 2:55 (383) On the sign of Green's functions. Professor ALLAN PETERSON, University of Nebraska (720-34-26)

3:00- 3:10 (384) Oscillations of a forced even order differential equation. Mr. KENT FOSTER, Southern Illinois University (720-34-32)

3:15- 3:25 (385) Linear differential equations subject to integral smallness conditions involving ordinary integrability. Professor WILLIAM F. TRENCH, Drexel University (720-34-31)

3:30- 3:40 (386) A new approach to the method of nonlinear variation of parameters. Preliminary report. Professor M.E. LORD*, and Professor A. RICHARD MITCHELL, University of Texas, Arlington (720-34-21)

3:45- 3:55 (387) On a nonlinear differential equation occurring in astrodynamics. Professor ABOLGHASSEM GHAFFARI, Arya-Mehr University of Technology--Teheran, Iran and NASA, Washington, D.C. (720-34-38)

4:00- 4:10 (388) On singularly perturbed initial value problems. Preliminary report. Professor JOHN V. BAXLEY, Wake Forest University (720-34-35)

4:15- 4:25 (389) Vector Lyapunov functions and perturbation theory. Preliminary report. Professor R.W. MITCHELL, and Professor M.E. MOORE*, University of Texas at Arlington (720-34-18)

4:30- 4:40 (390) Boundedness of solutions to ordinary differential equations using Liapunov functions. Preliminary report. Mr. J.F. WIGGS, North Carolina State University (720-34-24)

4:45- 4:55 (391) Time-delay interconnected systems and extensions of Lyapunov's direct method. Preliminary report. Professor GANGARAM S. LADDE, State University of New York, College at Potsdam (720-34-7)

5:00- 5:10 (392) Singular integro-differential inequalities. Preliminary report. Professor J.S. BRADLEY*, University of Tennessee, and Professor W.N. EVERITT, The University, Dundee, Scotland (720-34-36)

THURSDAY, 2:30 P.M.

Session on Partial Differential Equations II, Senate Room, Sheraton-Park (Lobby Level)

2:30- 2:40 (393) On Holmgren's theorem. Preliminary report. Professor L.R. HUNT and Professor M.J. STRAUSS*, Texas Tech University (720-35-3)

2:45- 2:55 (394) Harmonic Faber polynomials. Preliminary report. Dr. EDWIN T. HOEFER, Rosary Hill College (720-35-14) (Introduced by G. Georgantas)

3:00- 3:10 (395) On the corner problem for symmetric positive systems. Professor GIDEON PEYSER, Newark College of Engineering (720-35-15)

3:15- 3:25 (396) Elementary solutions of P(D). Professor NARENDRA L. MARIA, California State College, Stanislaus (720-35-23)

3:30- 3:40 (397) On a correctness class of the Bessel type differential operator s_μ. Preliminary report. Dr. WILLY LEE, Rutgers University, Camden (720-35-30)

3:45- 3:55 (398) Reflection principles for the iterated Helmholtz equation. Dr. J.B. DIAZ, Rensselaer Polytechnic Institute, and Dr. R.B. RAM*, State University of New York, College at Oneonta (720-35-41)

4:00- 4:10 (399) Analyticity of solutions of nonlinear evolution equations. Dr. FRANK J. MASSEY III, University of Kentucky (720-35-43)

4:15- 4:25 (400) The solution of initial-boundary value problems for equations of Sobolev type. Dr. W. RUNDELL, Texas A&M University (720-35-44) (Introduced by Dr. Michael Stecher)

4:30- 4:40 (401) Extension of two theorems of Payne to some nonlinear Dirichlet problems. Dr. RENE P. SPERB, University of Tennessee (720-35-6) (Introduced by Professor P.W. Schaefer)

4:45- 4:55 (402) The Dirichlet problem for the equation of zero Gauss curvature. Professor J.B. RAUCH*, and Professor B.A. TAYLOR, University of Michigan (720-35-20)

5:00- 5:10 (403) Exterior Dirichlet and Neumann problems in generalized biaxially symmetric potential theory. Preliminary report. Dr. DENNIS W. QUINN, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio (720-35-21)

THURSDAY, 2:45 P.M.

Session on Applied Mathematics II, Richmond Room, Sheraton-Park (Ground Level)

2:45- 2:55 (404) The asymptotic expansion near a focus for almost plane waves. Preliminary report. Dr. JOHN I. BOBBITT, Purdue University, Calumet Campus (720-78-1)

3:00- 3:10 (405) Multi-mode surface wave phenomena. Professor RICHARD C. MORGAN*, St. John's University, and Professor SAMUEL N. KARP, Courant Institute, New York University (720-78-2)

3:15- 3:25 (406) The unitarity equations for matrices and the problem of determining the scattering amplitude from the differential cross-section. Dr. MICHAEL TORTORELLA*, and Dr. JAMES A. LEISE, University of Wisconsin, Milwaukee (720-81-1)

3:30- 3:40 (407) Spectral analysis of the multigroup transport operator. Professor WILLIAM GREENBERG, Virginia Polytechnic Institute and State University (720-82-1)

3:45- 3:55 (408) Interior solutions to plane symmetric Einstein-Maxwell equations. Professor M. HUMI*, and Mr. J.A. LEBRITTON, Worcester Polytechnic Institute (720-83-2)

4:00- 4:10 (409) Transport operators. Dr. R. HECHT-NIELSEN*, and Professor A.P. WANG, Arizona State University (720-99-2)

4:15- 4:25 (410) A mathematical method for approximating the potential and gradient at any point between an insulated cable to plane. Mr. T.J. LANOUE* and Dr. V.L. BOAZ, Westinghouse Electric Corporation, Muncie, Indiana and Dr. MIR M. ALI, Ball State University (720-99-4) (Introduced by Dr. T.K. Puttaswamy)

4:30- 4:40 (411) Limiting and asymptotic behavior of the Bondurant-Hoskold formulas for valuation of coal mine parcels for equitable tax assessment. Preliminary report. Professor H.W. GOULD, West Virginia University (720-99-5)

4:45- 4:55 (412) Complete degrees of finite-state transformability. Professor HENRY G. GORDON, Allentown College of St. Francis DeSales (720-94-1)

5:00- 5:10 (413) Distortion mappings. III: Progress report. PAUL A. WILLIS, Teledyne Geotech Laboratories, Alexandria, Virginia

THURSDAY, 2:45 P.M.

Session on Differential Geometry, Wilmington Room, Sheraton-Park (Ground Level)

2:45- 2:55 (414) On results of Bochner and Nomizu. Professor GEORGE PARKER, Southern Illinois University (720-53-1)

3:00- 3:10 (415) Taut immersions of noncompact surfaces into R^3. Dr. THOMAS E. CECIL, Vassar College (720-53-2)

3:15- 3:25 (416) Submersions from complex projective space. RICHARD H. ESCOBALES, Jr., Canisius College (720-53-4)

3:30- 3:40 (417) Imbeddings of Stiefel manifolds into Grassmannians. Professor GARY R. JENSEN, Washington University (720-53-5)

3:45- 3:55 (418) Kaehler Gauss-Codazzi equations. Preliminary report. HOWARD JACOBOWITZ, Rice University (720-53-9)

4:00- 4:10 (419) On manifolds with nonnegative Ricci curvature. II. Dr. EUGENIO CALABI, University of Pennsylvania (720-53-6)

4:15- 4:25 (420) Pseudo-Riemannian manifolds with totally geodesic bisectors. Professor JOHN K. BEEM, University of Missouri (720-53-8)

4:30- 4:40 (421) The asymptotic expansion of the zeta-function of compact symmetric spaces of rank one. ROBERT S. CAHN*, University of Miami, and JOSEPH A. WOLF, University of California, Berkeley (720-53-3)

4:45- 4:55 (422) On weakly asymptotically simple space-times. Preliminary report. Professor GEORGE DEBNEY, Virginia Polytechnic Institute and State University (720-53-7)

THURSDAY, 3:30 P.M.

Invited Address, Regency Ballroom, Shoreham

(423) Some recent discoveries in the isomorphic theory of Banach spaces. Professor HASKELL P. ROSENTHAL, University of Illinois at Urbana-Champaign (720-46-50)

THURSDAY, 3:30 P.M.

Special Session on Analytic Number Theory II, Diplomat Room, Shoreham (Lower Lobby Level)

3:30- 3:50 (424) 3-class groups of cubic fields. Professor FRANK GERTH III, University of Texas at Austin (720-12-2)

4:00- 4:20 (425) On ideals having only small prime factors. Preliminary report. Dr. DONALD HAZLEWOOD, Southwest Texas State University (720-10-1)

4:30- 4:50 (426) On multiply perfect numbers and Mersenne primes. Dr. CARL POMERANCE, University of Georgia (720-10-2)

5:00- 5:20 (427) A formula relating $\zeta_k(s)$ and $\zeta_k(2-s)$ for k totally real with class number one. Professor A. TERRAS, University of California, San Diego (720-10-7)

5:30-5:50 (428) Quadratic forms over nonformally real fields. Professor CRAIG M. CORDES, Louisiana State University (720-10-3)

6:00- 6:20 (429) A stopping time problem on the positive integers. RIHO TERRAS, Del Mar, California (720-10-6)

THURSDAY, 8:30 P.M.

Gibbs Lecture, Regency Ballroom, Shoreham

(430) A priori estimates, geometric effects, and asymptotic behavior. Professor FRITZ JOHN, Courant Institute, New York University (720-41-16)

FRIDAY, 8:30 A.M.

Special Session on Structure and Representation of Lie Algebras Over General Fields I, Dover Room, Sheraton-Park (Ground Level)

8:30- 9:30 (431) Recent advances in classification of simple Lie algebras. Professor ROBERT LEE WILSON, Rutgers University (720-17-10)

9:45-10:05 (432) Simple modular Lie algebras and Lie algebras over valuation rings. HELMUT STRADE, Universität Hamburg, West Germany (720-17-8)

10:10-10:30 (433) Triple forms and related Lie algebras. Preliminary report. Dr. JOHN B. JACOBS, University of Oregon (720-17-13)

10:35-10:55 (434) On certain classes of Jacobian generated algebras. Preliminary report. Dr. MARGUERITE FRANK, Princeton, New Jersey (720-17-7) (Introduced by Professor George Seligman)

11:00-11:20 (435) Forms of certain generalized Witt algebras. Preliminary report. Dr. DAVID KOPCSO, Babson College (720-17-15)

11:25-11:45 (436) Borel and Borel-like solvable Lie algebras. Professor MICHAEL A. GAUGER, University of Massachusetts (720-17-2)

11:50-12:10 (437) On simple associative algebras arising from solvable Lie algebras. Professor JOHN McCONNELL, University of California, Berkeley (720-16-19) (Introduced by Professor G. Seligman)

12:15-12:35 (438) Representations of solvable p-algebras. Mr. JOHN SCHUE, Macalester College (720-17-4)

FRIDAY, 8:45 A.M.

Session on Analysis III, Wardman Room, Sheraton-Park (Lobby Level)

8:45- 8:55 (439) Contractive perturbations of restricted shifts. Professor JOSEPH BALL, Virginia Polytechnic Institute and State University and Professor ARTHUR LUBIN*, Northwestern University (720-47-2)

9:00- 9:10 (440) Circularly symmetric normal and subnormal operators. Professor RALPH GELLAR, North Carolina State University (720-47-28)

9:15- 9:25 (441) Uniqueness of analytic continuation on a real hypersurface in C^2. Preliminary report. Professor L.R. HUNT, Texas Tech University (720-32-1)

9:30- 9:40 (442) Runge domains and lacunary power series in several variables. Preliminary report. Mr. JOHN PESEK, Jr., University of Michigan (720-32-2)

9:45- 9:55 (443) A second main theorem on Stein manifolds with pseudoconvex exhaustion. Preliminary report. Dr. JOHN J. MURRAY, Texas Tech University (720-32-3) (Introduced by Professor L.R. Hunt)

10:00-10:10 (444) Some analytic uniform algebras. Preliminary report. Dr. WILLIAM R. ZAME, State University of New York at Buffalo (720-32-5)

10:15-10:25 (445) Invariance of holomorphic convexity under proper maps. Dr. ANDREW G. MARKOE, University of Washington (720-32-6)

10:30-10:40 (446) Simultaneous spline approximation and interpolation preserving norm. Professor C.K. CHUI*, Professor E.R. ROZEMA, Professor P.W. SMITH, and Professor J.D. WARD, Texas A&M University (720-41-8)

10:45-10:55 (447) Limits of $H^{k,p}$-splines as $p \to 1$. Professor C.K. CHUI, Professor P.W. SMITH, Professor J.D. WARD*, Texas A&M University (720-41-9)

11:00-11:10 (448) Favards solution is the limit of $H^{k,p}$ splines. Professor C.K. CHUI, Professor P.W. SMITH*, and Professor J.D. WARD, Texas A&M University (720-41-10)

11:15-11:25 (449) Walsh-like expansions and Hadamard matrices. Preliminary report. Professor R.D. LARSEN, and Professor W.R. MADYCH*, Texas A&M University (720-41-12)

11:30-11:40 (450) Extremal polynomials in bounded T-systems. B. GOPINATH, and R.P. KURSHAN*, Bell Laboratories, Murray Hill, New Jersey (720-41-14)

11:45-11:55 (451) Stone-Weierstrass properties in some commutative Banach algebras. Preliminary report. STEPHEN RODI, Mary Baldwin College (720-41-15)(Introduced by John E. Gilbert)

FRIDAY, 8:45 A.M.

Session on Approximation, Adams/Hamilton Room, Sheraton-Park (First Floor)

8:45- 8:55 (452) Error estimates in approximation by extended Hermite-Fejer operators. Dr. SHELDON M. EISENBERG*, University of Hartford and Dr. BRUCE WOOD, University of Arizona (720-41-1)

9:00- 9:10 (453) H-sets, an approach to linear Tchebycheff approximation. Dr. MICHAEL BRANNIGAN, Iowa State University (720-41-2)(Introduced by Richard H. Miller)

9:15- 9:25 (454) Chebychev approximation of completely monotonic functions by sums of exponentials. Professor DAVID W. KAMMLER, Southern Illinois University (720-41-11)

9:30- 9:40 (455) On some polynomial approximation operators. Dr. T.J. RIVLIN, IBM T.J. Watson Research Center, and Professor E.W. CHENEY*, University of Texas at Austin (720-41-6)

9:45- 9:55	(456)	An algorithm for L_1 approximation using near best approximates. Preliminary report. Dr. MICHAEL P. CARROLL, Virginia Polytechnic Institute and State University (720-41-3)
10:00-10:10	(457)	Approximation of random functions. Preliminary report. Professor W. H. LING, Union College, Professor H. W. McLAUGHLIN, and Ms. M. L. SMITH*, Rensselaer Polytechnic Institute (720-41-7)
10:15-10:25	(458)	The complete Pade tables of certain series of simple fractions. II. Professor ALBERT EDREI, Syracuse University (720-41-4)
10:30-10:40	(459)	Rational approximation on certain unbounded domains of the complex plane. Professor P. ERDÖS, Stanford University, and Professor A. R. REDDY*, Michigan State University (720-41-5)
10:45-10:55	(460)	Uniform approximation by rational functions and their derivatives. Preliminary report. Professor URI FIXMAN, Queens University, and Professor LEE A. RUBEL*, University of Illinois (720-41-13)

FRIDAY, 8:45 A.M.

Session on Complex Analysis II, Franklin Room, Sheraton-Park (First Floor)

8:45- 8:55	(461)	On linear combinations of convex functions of order β. Preliminary report. Mr. HERB W. SILVERMAN, University of Delaware, and Ms. EVELYN M. SILVIA*, University of California, Davis (720-30-2)
9:00- 9:10	(462)	Starlike functions as limits of polynomials. Professor TED J. SUFFRIDGE, University of Kentucky (720-30-4)
9:15- 9:25	(463)	On the radius of starlikeness of (zf)' for f univalent. Preliminary report. Dr. ROGER W. BARNARD, Texas Tech University (720-30-8)
9:30- 9:40	(464)	Analytic functions in class $H_{1/2,n}$ and their applications. Dr. DOROTHY BROWNE SHAFFER, Fairfield University (720-30-22)
9:45- 9:55	(465)	Univalent functions with negative coefficients. HERB SILVERMAN, University of Delaware (720-30-5)
10:00-10:10	(466)	Completely convex and positive harmonic functions. Professor DALE H. MUGLER, Syracuse University (720-30-3)
10:15-10:25	(467)	Duality and multipliers in spaces of harmonic functions. Professor A. L. SHIELDS, University of Michigan, and Professor D. L. WILLIAMS*, Syracuse University (720-30-20)
10:30-10:40	(468)	Quadratic differentials. Preliminary report. JEFFREY C. WIENER, Georgia Institute of Technology (720-30-6)
10:45-10:55	(469)	Analytic capacity of open sets. Mr. STEVEN JACOBSON, Yale University (720-30-15)
11:00-11:10	(470)	Restrictions of analytic functions. JAMES ROVNYAK*, and MARVIN ROSENBLUM, University of Virginia (720-30-25)
	(471)	Withdrawn

FRIDAY, 8:45 A.M.

Session on Functional Analysis II, Assembly Room, Sheraton-Park (Lobby Level)

8:45- 8:55	(472)	The strict dual of B*-algebras. Preliminary report. Dr. JOHN W. DAVENPORT, Madison College (720-46-4)
9:00- 9:10	(473)	On commutative B*-equivalent algebras. Mr. JOSEF WICHMANN, Texas Christian University (720-46-19)
9:15- 9:25	(474)	The p class in a dual B*-algebra. Preliminary report. Dr. PAK-KEN WONG, Seton Hall University (720-46-20) (Introduced by Professor Charles Franke)
9:30- 9:40	(475)	Representation of C* algebras. Preliminary report. Mr. C. W. BAKER, University of Kentucky (720-46-51)
9:45- 9:55	(476)	Compact groups of automorphisms of von Neumann algebras. Preliminary report. Professor WILLIAM L. GREEN, Georgia Institute of Technology (720-46-28)
10:00-10:10	(477)	Renorming a normed algebra having a semigroup of near isometries. Preliminary report. Professor JOHN A. LINDBERG, Jr., Syracuse University (720-46-29)
10:15-10:25	(478)	On functors from compact pairs to Banach algebras. Professor DONALD HARTIG, Ohio University (720-46-35)
10:30-10:40	(479)	On perturbing bases of complex exponentials in $L^2(-\pi, \pi)$. Professor ROBERT M. YOUNG, Oberlin College (720-46-3)

10:45-10:55 (480) Linear and nonlinear functionals on the spaces L^p, $0<p<1$. Preliminary report. JUDITH PALAGALLO, Colorado State University (720-46-33)

11:00-11:10 (481) On the fine structure of the spectra of certain classes of operators on the ℓ_p spaces. Preliminary report. Professor C.J.A. Halberg, Jr., University of California, Riverside (720-46-47) (Introduced by Professor S.R. Gordon)

(482) Withdrawn

FRIDAY, 8:45 A.M.

Session on Groups, Senate Room, Sheraton-Park (Lobby Level)

8:45- 8:55 (483) The automorphism group of a p-group of maximal class with an abelian maximal subgroup. Dr. ALPHONSE H. BAARTMANS, Southern Illinois University, and Dr. JAMES J. WOEPPEL*, Indiana University Southeast (720-20-17)

9:00- 9:10 (484) On a paper of Richman and Walker. Ms. ROBIN KUEBLER* and Professor J.D. REID, Wesleyan University (720-20-28)

9:15- 9:25 (485) Fully invariant subgroups of primary abelian groups. Professor RONALD C. LINTON, University of South Alabama (720-20-23) (Introduced by Professor Richard Vinson)

9:30- 9:40 (486) The conjugacy problem for free products of sixth groups with cyclic amalgamation. Preliminary report. Dr. LEO COMERFORD*, Michigan State University and Dr. BERNARD TRUFFAULT, Université de Nantes, France (720-20-29)

9:45- 9:55 (487) A characterization of the maximal monoids and maximal groups in β_X. Professor D.J. HARTFIEL and Professor C.J. MAXSON*, Texas A&M University (720-20-7)

10:00-10:10 (488) On permutations of groups. Preliminary report. Dr. ALFRED P. MANEKI, Department of Defense, Fort George G. Meade, Maryland (720-05-41)

FRIDAY, 8:45 A.M.

Session on Manifolds and Global Analysis, Madison Room, Sheraton-Park (First Floor)

8:45- 8:55 (489) Torsion in the bordism of oriented involutions. Dr. RUSSELL J. ROWLETT, University of Tennessee (720-57-1)

9:00- 9:10 (490) PL actions and equivariant general position. Preliminary report. Dr. ROBERT SCHWARTZ, Burlington County College (720-57-9)

9:15- 9:25 (491) Close piecewise linear codimension one embeddings. Preliminary report. Dr. MARVIN ISRAEL, Goucher College (720-57-10)

9:30- 9:40 (492) Embedding contractible 2-complexes in E^4. Mr. BENJAMIN M. FREED, Kent State University (720-57-14)

9:45- 9:55 (493) Brick partitionings and 2-cells. Preliminary report. Dr. RICHARD SLOCUM, University of Tennessee at Martin (720-57-5)

10:00-10:10 (494) A theory of higher order derivatives for H-differentiability on locally convex spaces. Dr. F.R. MILLER*, and Dr. W.D. CURTIS, Kansas State University (720-58-1) (Introduced by Professor R.B. Burckel)

10:15-10:25 (495) Truncations of vector field flows. Preliminary report. Dr. WILLIAM PERRIZO, North Dakota State University (720-58-2)

10:30-10:40 (496) Hamiltonian systems. Professor SHELDON E. NEWHOUSE, University of North Carolina (720-58-3)

10:45-10:55 (497) An irreversible analogue of the Denjoy theorem. Preliminary report. ZBIGNIEW NITECKI, Tufts University (720-58-4)

11:00-11:10 (498) Unique ergodicity of some flows related to Axiom A diffeomorphisms. Mr. BRIAN H. MARCUS, University of California, Berkeley (720-58-6)

11:15-11:25 (499) The $(H^1, 1)$ rectifiable subsets of a homogeneous space without rotation. Mr. SAMIR KAR, Indiana University (720-58-5)

FRIDAY, 8:45 A.M.

Session on Probability II, Alexandria Room, Sheraton-Park (Ground Level)

8:45- 8:55 (500) On the square variation of martingales. Professor ITREL MONROE, University of Arkansas (720-60-21)

9:00- 9:10 (501) A smoothing index and equiconvergence of martingales. Professor LOUIS H. BLAKE, Worcester Polytechnic Institute (720-60-22)

9:15- 9:25 (502) On investigation of sequential testing procedures for comparison of stochastic branching processes. Preliminary report. Mr. T. SMITH*, and Professor C.P. TSOKOS, University of South Florida (720-60-32)

9:30- 9:40 (503) A generalization of a theorem of Cramér. Professor G.D. ALLEN, Texas A&M University (720-60-33)

9:45- 9:55 (504) The Petersburg paradox. Professor HELEN SKALA, University of Massachusetts, Boston (720-60-17)

10:00-10:10 (505) The determinant of a random matrix. F. ALBERTO GRUNBAUM, University of California, Berkeley (720-60-25) (Introduced by Professor Jacob Feldman)

10:15-10:25 (506) On the convolution iterates of a probability measure. ARUNAVA MUKHERJEA, and JAMES GARD*, University of South Florida (720-60-27)

10:30-10:40 (507) Zero-one laws in finite W*-algebras (noncommutative probability). Professor E. C. GOOTMAN, and Professor D. KANNAN*, University of Georgia (720-60-29)

10:45-10:55 (508) On the Erdös-Feller criterion. Professor D.G. KOSTKA, Texas A&M University (720-60-34)

FRIDAY, 9:00 A.M.

Informal Special Session on Color Theory, Arlington Room, Sheraton-Park (Ground Level)

9:00-12:00 (509) Informal. Conducted by Professor RUTH BARI, George Washington University

FRIDAY, 9:00 A.M.

Invited Address, Regency Ballroom, Shoreham

(510) General relativity and cosmology. Professor RAINER K. SACHS, University of California, Berkeley (720-83-1)

FRIDAY, 9:00 A.M.

Special Session on Fourier Integral Operators, Continental Room, Sheraton-Park (Lobby Level)

9:00- 9:30 (511) On Kostant quantization and partial differential equations. Professor VICTOR W. GUILLEMIN, Massachusetts Institute of Technology

9:45-10:15 (512) Fourier integral operators with complex phase. Professor FRANCOIS TREVES, Rutgers University (720-47-22)

10:30-11:00 (513) Periodic Fourier integral operators. Professor ALAN WEINSTEIN, University of California, Berkeley

11:15-11:45 (514) Withdrawn

FRIDAY, 9:00 A.M.

Special Session on Interpolation of Operators and Applications II, Palladian Room, Shoreham (Lower Lobby Level)

9:00- 9:20 (515) Weighted norm inequalities. Professor BENJAMIN MUCKENHOUPT, Rutgers University (720-46-24)

9:25- 9:45 (516) Extensions of the Hausdorff-Young theorem. Professor COLIN BENNETT, California Institute of Technology (720-42-7)

9:50-10:10 (517) Convolution operators of weak type. JOHN J. F. FOURNIER, University of British Columbia (720-43-1)

10:15-10:35 (518) Tensor products, interpolation theory and convolution operators. JOHN E. GILBERT, University of Texas (720-43-13)

10:40-11:00 (519) Interpolation of operators and a.e. convergence of Fourier series. Preliminary report. Professor RICHARD A. HUNT, Purdue University (720-42-11)

11:05-11:25 (520) The theory of ultraspherical multipliers and the idea of local stability. WILLIAM CONNETT*, and ALAN SCHWARTZ, University of Missouri, St. Louis (720-42-4)

11:30-11:50 (521) Some applications of interpolation theory. Professor YOREM SAGHER, University of Minnesota, and Weizmann Institute of Science, Israel

FRIDAY, 9:00 A.M.

Special Session on Interrelations Between Computation and Number Theory, Richmond Room, Sheraton-Park (Ground Level)

9:00- 9:25 (522) Improved bounds for the Chebyshev functions with applications. Professor LOWELL SCHOENFELD, State University of New York at Buffalo (720-10-41)

30- 9:45	(523)	Zeros of Fekete polynomials. Preliminary report. Professor PAUL T. BATEMAN, University of Illinois (720-10-42)
:50-10:15	(524)	The distribution of pseudo-random numbers generated by the linear congruential method. Dr. HARALD NIEDERREITER, Institute for Advanced Study (720-10-4)
):20-10:35	(525)	On the number of normal number fields. P.J. WEINBERGER, University of Michigan (720-12-18)
0:40-11:05	(526)	Factorization and prime testing. Professor J.L. SELFRIDGE and Professor M.C. WUNDERLICH*, Northern Illinois University (720-10-45)
1:10-11:20	(527)	On the permanent of Schur's matrix. R.L. GRAHAM*, Bell Laboratories, Murray Hill, New Jersey, and D.H. LEHMER, University of California, Berkeley (720-10-44)
11:25-11:45	(528)	Analysis and improvement of the continued fraction method of factorization. Dr. DANIEL SHANKS, Naval Ship Research and Development Center, Bethesda, Maryland (720-10-43)

FRIDAY, 9:00 A.M.

Special Session on Probabilistic Analysis III, Heritage Room, Shoreham (Lower Lobby Level)

9:00- 9:20	(529)	An infinite dimensional Laplacian. Preliminary report. M. ANN PIECH, State University of New York at Buffalo (720-60-7)
9:30- 9:50	(530)	Potential theory associated with Uhlenbeck-Ornstein process. Professor HUI-HSIUNG KUO, University of Virginia (720-31-1)
10:00-10:20	(531)	Measurability of generalized inverses of random linear operators in Banach spaces and random generalized Green's functions. Preliminary report. Professor M. ZUHAIR NASHED, Georgia Institute of Technology (720-60-13)
10:30-11:00	(532)	The law of the iterated logarithm for Banach space valued random variables. Professor JAMES KUELBS, University of Wisconsin, Madison (720-60-3)
11:10-11:40	(533)	Informal Discussion

FRIDAY, 9:00 A.M.

Special Session on Singular Cauchy Problems, Wilmington Room, Sheraton-Park (Ground Level)

9:00- 9:10	(534)	Opening remarks by Professor BERNARD FUSARO, Salisbury State College
9:10- 9:30	(535)	The Riemann-Liouville integral and parameter shifting in a class of linear abstract Cauchy problems. Preliminary report. Professor LOUIS R. BRAGG, Oakland University (720-35-2)
9:35- 9:55	(536)	Analogous function theories in the theory of heat conduction and singular Cauchy problems. Preliminary report. Professor LOUIS R. BRAGG and Professor JOHN W. DETTMAN*, Oakland University (720-35-1)
10:00-10:20	(537)	Growth properties of solutions to a nonlinear Euler-Poisson-Darboux equation. Professor HOWARD A. LEVINE, University of Rhode Island (720-35-19)
10:25-10:45	(538)	Nonlinear degenerate evolution equations and partial differential equations of mixed type. R.E. SHOWALTER, University of Texas (720-35-8)
10:50-11:10	(539)	Comparison and oscillation theorems for singular hyperbolic equations. Professor EUTIQUIO C. YOUNG, Florida State University, Tallahassee (720-35-18)
11:15-12:00	(540)	Eisenstein integrals and singular Cauchy problems. Professor ROBERT CARROLL, University of Illinois (720-35-16)

FRIDAY, 10:00 A.M.

Special Session on Set-Theoretic and Combinatorial Methods in Topology I, The Forum, Shoreham (Upper Lobby Level)

10:00-10:20	(541)	A perfectly normal, nonmetrizable manifold. Professor MARY ELLEN RUDIN, University of Wisconsin (720-54-8)
10:30-10:50	(542)	A solution to the Blumberg problem. Preliminary report. Mr. W.A.R. WEISS, University of Toronto (720-54-42) (Introduced by Professor Franklin D. Tall)
11:00-11:20	(543)	On nonmetrizable spaces in which every subspace of small cardinality is metrizable. Preliminary report. Professor ISTVÁN JUHÁSZ, University of Wisconsin and Mathematical Institute, Budapest, Hungary (720-54-25) (Introduced by M.E. Rudin)

11:30-11:50 (544) Continuous pseudometrics. Professor STELIOS NEGREPONTIS, Athens University and McGill University

FRIDAY 10:15 A.M.

Invited Address, Regency Ballroom, Shoreham

(545) A representation theoretic proof of a formula of Max Noether. Professor NOLAN R. WALLACH, Rutgers University (720-22-14)

FRIDAY, 1:00 P.M.

Colloquium Lectures: Lecture II, Regency Ballroom, Shoreham

(546) New directions in model theory. Professor H. JEROME KEISLER, University of Wisconsin-Madison

FRIDAY, 2:15 P.M.

Retiring Presidential Address, Regency Ballroom, Shoreham

(547) Topology and logic as sources of algebraic ideas. Professor SAUNDERS MAC LANE, University of Chicago (720-99-6)

FRIDAY, 3:15 P.M.

Cole Prize and Steele Prize Sessions, Regency Ballroom, Shoreham

FRIDAY, 4:30 P.M.

Business Meeting, Regency Ballroom, Shoreham

FRIDAY, 8:30 P.M.

Panel Discussion on Seeking Employment Outside Academia: Views from some who have recently succeeded.

Professor MARTHA K. SMITH, University of Texas, Austin (Moderator)

SATURDAY, 12:00 NOON

Special Session on Hyperbolic Conservation Laws I, Palladian Room, Shoreham (Lower Lobby Level)

12:00-12:30 (548) The generic theory of regularity for a single convex conservation law. Professor DAVID G. SCHAEFFER, Massachusetts Institute of Technology (720-35-42)

12:35- 1:05 (549) Regularity for a single conservation law. JOHN GUCKENHEIMER, University of California, Santa Cruz (720-35-4)

1:10- 1:40 (550) The semigroup approach to a single conservation law. MICHAEL G. CRANDALL, University of Wisconsin-Madison (720-35-31)

SATURDAY, 12:00 NOON

Special Session on Some Directions of Current Progress in Commutative Algebra II, Arlington/Alexandria Rooms, Sheraton-Park (Ground Level)

12:00-12:45 (551) Ideals of low codimension. Professor ARTHUR OGUS, Princeton University

1:00- 1:45 (552) The method of Tschirnhausen-Newton-Puiseux. Expository report. T. T. MOH, University of Minnesota (720-13-12)

SATURDAY, 1:00 P.M.

Colloquium Lectures: Lecture III, Regency Ballroom, Shoreham

(553) New directions in model theory. Professor H. JEROME KEISLER, University of Wisconsin-Madison

SATURDAY, 1:00 P.M.

Special Session on Advances in Graph Theory I, The Forum, Shoreham (Upper Lobby Level)

1:00- 1:20 (554) On the expected packing and covering numbers of a tree. Preliminary report. Professor J. W. MOON, University of Alberta (720-05-36)

1:30- 1:50 (555) The isomorphism disease. Preliminary report. Professor R. C. READ*, University of Waterloo, and Professor D. G. CORNEIL, University of Toronto (720-05-6)

2:00- 2:20 (556) The maximum size of an independent set in a graph. Dr. MICHAEL O. ALBERTSON, Smith College, and Dr. JOAN P. HUTCHINSON*, Dartmouth College (720-05-8)

2:30- 2:50 (557) Map color theorem. Professor GERHARD RINGEL, University of California, Santa Cruz (720-05-4)

3:00- 3:30 (558) Informal Discussion

3:30- 3:50	(559)	On moments and coefficients in spectral graph theory. Professor ALLEN J. SCHWENK, Michigan State University (720-05-37)
4:00- 4:20	(560)	A theorem about planar graphs in that there are two adjacent vertices such that the sum of their degrees is at least 13 more or less. Professor ANTON KOTZIG, University of Montreal
4:30- 5:00	(561)	Chairman's remarks. Professor W. T. TUTTE, University of Waterloo
5:00- 6:00	(562)	Unsolved problems and recent results

SATURDAY, 1:00 P.M.

Special Session on Stucture and Representation of Lie Algebras over General Fields II, Executive Room, Shoreham

1:00- 2:00	(563)	Irreducible representations of Lie algebras. Professor RICHARD E. BLOCK, University of California, Riverside (720-17-9)
2:10- 2:30	(564)	On the analogues of Verma modules in characteristic p. Preliminary report. Professor J. E. HUMPHREYS, University of Massachusetts (720-17-12)
2:35- 2:55	(565)	Existence of conical vectors in induced modules. Professor JAMES LEPOWSKY, Yale University (720-17-16)
3:00- 3:20	(566)	Representation theory for GL_2 over local rings. Professor P. C. KUTZKO, University of Iowa (720-22-10)
3:30- 3:50	(567)	Algebras of type E_7 over number fields. Dr. JOSEPH C. FERRAR, Ohio State University (720-17-3)
3:55- 4:15	(568)	Lie algebras of type BC_1. Professor BRUCE N. ALLISON, University of Alberta (720-17-19)
4:20- 4:40	(569)	Coordinatization of some quadrilateral geometries. Preliminary report. Professor JOHN R. FAULKNER, University of Virginia (720-17-1)
4:45- 5:05	(570)	On inner ideals and ad-nilpotent elements of Lie algebras. DR. GEORGIA BENKART, University of Wisconsin (720-17-17)
5:10- 5:30	(571)	Some results and open questions concerning Lie algebras defined by Cartan matrices. STEPHEN BERMAN, University of Saskatchewan-Saskatoon (720-17-5)
5:35- 5:55	(572)	Representations of infinite dimensional Lie algebras and formulas of Weyl-Macdonald type. R. V. MOODY, University of Saskatchewan (Introduced by Professor George B. Seligman) (720-17-18)

SATURDAY, 2:00 P.M.

Session on Analysis IV, Madison Room, Sheraton-Park (First Floor)

2:00- 2:10	(573)	Convergence of generalized power series. Preliminary report. Dr. BILL ANDERSON*, East Texas State University and Mr. KENNY ZUBER, New Mexico State University (720-40-2)
2:15- 2:25	(574)	An extension of Parseval's equation. Professor CHULL PARK, Miami University (720-42-1)
2:30- 2:40	(575)	On the summability of Fourier series of functions of Λ-bounded variation. Professor DANIEL WATERMAN, Syracuse University (720-42-3)
2:45- 2:55	(576)	On the degree of approximation by partial sums of Fourier series in certain Orlicz space norms. Preliminary report. Dr. ELAINE COHEN, University of Utah (720-42-5)
3:00- 3:10	(577)	Existence, uniqueness and an algorithmic solution for a dual cosine series. Preliminary report. Mr. TIMOTHY SIMPSON*, and Professor ROBERT B. KELMAN, Colorado State University (720-42-6)
3:15- 3:25	(578)	On integrability and L^1 convergence of certain cosine sums. Dr. J. W. GARRETT*, and Professor C. V. STANOJEVIC, University of Missouri-Rolla (720-42-8)
3:30- 3:40	(579)	On p-weights for even functions. Professor LAWRENCE GLUCK, DePaul University (720-42-12)
3:45- 3:55	(580)	On the existence of a random solution of a nonlinear perturbed random integral equation. Dr. SUSAN MILTON*, Radford College, and Dr. CHRIS TSOKOS, University of South Florida (720-45-1)

4:00- 4:10 (581) On existence criteria and approximation procedures for integral equations of the first kind. Dr. CHARLES GROETSCH, University of Rhode Island (720-45-2)

4:15- 4:25 (582) Resolvents of Volterra integral equations with differentiable kernels. Preliminary report. Professor STANLEY I. GROSSMAN, University of Montana (720-45-3)

4:30- 4:40 (583) Asymptotic behavior of unbounded solutions of linear Volterra integral and integro-differential equations. Professor G.S. JORDAN*, University of Tennessee, and Professor ROBERT L. WHEELER, Iowa State University (720-45-4)

4:45- 4:55 (584) Local stability properties for a nonlinear Volterra integral equation. Professor TERRY L. HERDMAN, Virginia Polytechnic Institute and State University (720-45-5)

5:00- 5:10 (585) Invariant imbedding applied to a class of Fredholm integral equations. Preliminary report. Dr. JOSEPH F. McGRATH, Dikewood Industries, Albuquerque, New Mexico (720-45-6)

SATURDAY, 2:00 P.M.

Session on Complex Analysis III, Senate Room, Sheraton-Park (Lobby Level)

2:00- 2:10 (586) Zeros of strongly annular functions. Professor K.F. BARTH*, Syracuse University, Professor D.D. BONAR, Denison University, and Professor F.W. CARROLL, Ohio State University (720-30-17)

2:15- 2:25 (587) Strongly annular functions with small Taylor coefficients. Preliminary report. Dr. D.D. BONAR*, Denison University, Dr. F.W. CARROLL, Ohio State University, and Dr. GEORGE PIRANIAN, University of Michigan (720-30-19)

2:30- 2:40 (588) Zeros of an annular function. Preliminary report. Dr. I-LOK CHANG, American University (720-30-31)

2:45- 2:55 (589) Level sets of polynomials in n real variables. Professor MORRIS MARDEN, University of Wisconsin, and Professor PETER A. McCOY*, United States Naval Academy (720-30-9)

3:00- 3:10 (590) The minimum modulus of polynomials with restricted coefficients. Professor F.W. CARROLL, Professor DAN EUSTICE*, and Professor T. FIGIEL, Ohio State University (720-30-12)

3:15- 3:25 (591) A characterization of the complete polar of a polynomial. Professor JOHN R. QUINE, Jr., Florida State University (720-30-30)

3:30- 3:40 (592) An extremal entire function. S. CHAKRAVARTY, University of Kentucky (720-30-23)

3:45- 3:55 (593) Components of characteristic functions of Gaussian and Poisson distributions and means. Professor S.M. SHAH, University of Kentucky (720-30-26)

4:00- 4:10 (594) Holomorphic functions on cyclic group algebras. Professor HERBERT H. SNYDER, Southern Illinois University (720-30-18)

SATURDAY, 2:00 P.M.

Session on Differential Equations, Continental Room, Sheraton-Park (Lobby Level)

2:00- 2:10 (595) The topology of the Stokes phenomenon. Dr. BRIAN KELLY, Pennsylvania State University, Worthington Scranton Campus (720-34-5)

2:15- 2:25 (596) A geometric oscillatory theory for the equation $L_n y=q$. Dr. J. MICHAEL DOLAN, Western Carolina University and Dr. GENE A. KLASSEN, University of Tennessee (720-34-34)

2:30- 2:40 (597) Asymptotically autonomous multi-valued differential equations. Professor JAMES P. FOTI, U.S. Naval Academy (720-34-20) (Introduced by Professor Peter McCoy)

2:45- 2:55 (598) Continuity of the limit set maps in semidynamical systems. Dr. STEPHEN H. SAPERSTONE, George Mason University, and Mr. MASAHIRO NISHIHAMA*, University of Maryland (720-34-22)

3:00- 3:10 (599) Separatrices in solitude. RICHARD H. ELDERKIN, Pomona College (720-34-25)

3:15- 3:25 (600) Comparison and uniqueness results for a class of quasilinear elliptic partial differential equations. Preliminary report. Dr. LOUIS B. BUSHARD, Aerospace Research Laboratories, Wright-Patterson AFB, Ohio (720-35-5) (Introduced by Dr. Dennis W. Quinn)

3:30- 3:40 (601) Local solutions for quasi-linear semielliptic differential equations. Preliminary report. Dr. R.E. WHITE, North Carolina State University (720-35-29)

3:45- 3:55 (602) An existence theorem for first-order elliptic operators in R^n. Preliminary report. Professor HOMER F. WALKER, University of Houston (720-35-9)

4:00- 4:10 (603) A note on a theorem by Landesman and Lazer. Preliminary report. Mr. HOWARD C. SHAW, University of Michigan (720-35-24)

4:15- 4:25 (604) Existence of fundamental solutions for degenerate or singular, second order, linear, elliptic partial differential equations, Dr. WILLIAM F. MOSS, Georgia Institute of Technology (720-35-39)

SATURDAY, 2:00 P.M.

Session on Functional Analysis III, Adams/Hamilton Room, Sheraton-Park (First Floor)

2:00- 2:10 (605) Order arguments on the dimension of vector lattices. Dr. JOHN ANNULIS, University of Arkansas at Monticello (720-46-9)

2:15- 2:25 (606) A common fixed point theorem of the alternative type. Preliminary report. Dr. MARIJEAN SEELBACH, State University of New York, College at Brockport (720-46-13)

2:30- 2:40 (607) Sequential convergence in locally convex spaces. Professor TERRENCE S. McDERMOTT, Loyola Marymount University (720-46-52)

2:45- 2:55 (608) Probabilistic foundations of quantum theories and Rubin-Stone spaces. Mr. S.J. GUCCIONE* and Professor C.V. STANOJEVIC, University of Missouri-Rolla (720-46-14)

3:00- 3:10 (609) Packing spheres in Orlicz spaces. Professor CHARLES E. CLEAVER, Georgia Institute of Technology and Kent State University (720-46-16)

3:15- 3:25 (610) LMC and LUC semigroups of operators. Preliminary report. Dr. J.F. BERGLUND, Virginia Commonwealth University (720-46-18)

3:30- 3:40 (611) Extreme Lipschitz functions. JERRY A. JOHNSON, Oklahoma State University (720-46-21)

3:45- 3:55 (612) Images of linear pseudocomplete spaces. AARON R. TODD, City University of New York, Brooklyn College (720-46-49)

4:00- 4:10 (613) A special class of function metrics. Preliminary report. Dr. KLAUS BICHTELER, University of Texas at Austin (720-46-15)

4:15- 4:25 (614) Unique topologies in duality. Preliminary report. Professor A.C. COCHRAN*, University of Arkansas, and Professor T.K. MUKHERJEE, Jadavpur University, India (720-46-30)

SATURDAY, 2:00 P.M.

Session on Manifolds, Wardman Room, Sheraton-Park, (Lobby Level)

2:00- 2:10 (615) Diffeomorphisms of 3-manifolds which are homotopy equivalent to S^1. Professor LAWRENCE S. HUSCH, University of Tennessee (720-57-2)

2:15- 2:25 (616) Surfaces in noncompact 3-manifolds. Preliminary report. Professor THOMAS W. TUCKER, Colgate University (720-57-6)

2:30- 2:40 (617) A 3-manifold with no PL involutions. Professor JEFFREY L. TOLLEFSON, University of Connecticut (720-57-13)

2:45- 2:55 (618) Local combinatorial pontrjagin classes. Professor P.L. KING, University of North Carolina at Chapel Hill (720-57-4)

3:00- 3:10 (619) Involutions of 2-connected 6-manifolds. Dr. IAN HAMBLETON, University of Chicago (720-57-12)

3:15- 3:25 (620) Building subbundles. Professor JEROME DANCIS, University of Maryland, College Park (720-57-3)

3:30- 3:40 (621) Homology of finite cyclic coverings. Preliminary report. Dr. JOHN M. WOODS, Oklahoma Baptist University (720-57-8)

3:45- 3:55 (622) Indices of framefields with finite singularities. Professor ULRICH KOSCHORKE, City University of New York, Queens College (720-57-11)

4:00- 4:10 (623) Bounded weak-* manifolds. Dr. RICHARD E. HEISEY, Vanderbilt University (720-57-7)

SATURDAY, 2:00 P.M.

Session on Number Theory II, Richmond Room, Sheraton-Park (Ground Level)

2:00- 2:10 (624) Every odd perfect number has at least eight prime factors. Preliminary report. Professor PETER HAGIS, Jr., Temple University (720-10-14)

2:15- 2:25 (625) On left-handed, right-handed and two-sided primes. Professor JOHN E. MAXFIELD, Kansas State University and University of New Brunswick (720-10-19)

2:30- 2:40 (626) Interprimed vectors in two dimensions. Professor ALAN H. STEIN, University of Connecticut (720-10-26)

2:45- 2:55 (627) Bounds on positive solutions of linear Diophantine equations. Dr. I. BOROSH* and Dr. L.B. TREYBIG, Texas A&M University (720-10-18)

3:00- 3:10 (628) On a conjecture of S. Chowla. Dr. D. SURYANARAYANA, University of Toledo (720-10-20) (Introduced by Professor J. Chidambaraswamy)

3:15- 3:25 (629) A generalization of a cyclotomic polynomial. Preliminary report. Dr. K. NAGESWARA RAO, North Dakota State University (720-10-21)

3:30- 3:40 (630) Unitary harmonic numbers. Dr. PETER HAGIS, Jr. and Dr. GRAHAM LORD*, Temple University (720-10-22)

3:45- 3:55 (631) Some remarks concerning quasi-perfect numbers. Dr. NEVILLE ROBBINS, San Francisco, California (720-10-23)

4:00- 4:10 (632) Trinomials with Galois group contained in A_n. Preliminary report. PHYLLIS LEFTON, Columbia University (720-10-24)

SATURDAY, 2:00 P.M.

Session on Operator Theory I, Franklin Room, Sheraton-Park (First Floor)

2:00- 2:10 (633) Equivalence for groups and semigroups of operators. ALAN LAMBERT, University of North Carolina at Charlotte (720-47-3)

2:15- 2:25 (634) Canonical models for contraction operators with unitary part. Preliminary report. Dr. JOSEPH A. BALL, Virginia Polytechnic Institute and State University (720-47-4)

2:30- 2:40 (635) Subnormal operators quasisimilar to an isometry. Preliminary report. Mr. WILLIAM HASTINGS, University of Michigan (720-47-7)

2:45- 2:55 (636) An iterative process for nonlinear monotonic nonexpansive operators in Hilbert space. Dr. W.G. DOTSON, Jr., North Carolina State University (720-47-10)

3:00- 3:10 (637) Reductive operators with a spectral separation property. Professor FRANK GILFEATHER, University of Nebraska (720-47-13)

3:15- 3:25 (638) Numerical ranges of powers of operators. Mr. ELIAS S. W. SHIU, California Institute of Technology (720-47-14)

3:30- 3:40 (639) Indecomposable Hilbert-Schmidt operators. GARY WEISS, University of Michigan (720-47-17) (Introduced by Professor Allen L. Shields)

3:45- 3:55 (640) The absolute continuity of phase operators. JOANNE DOMBROWSKI* and GERD H. FRICKE, Wright State University (720-47-19)

4:00- 4:10 (641) Closures of unitary equivalence classes. Preliminary report. Mr. DONALD W. HADWIN, Indiana University (720-47-20)

4:15- 4:25 (642) Products of hermitian operators. Preliminary report. Dr. L.J. GRAY, Union Carbide Corporation, Nuclear Division, Oak Ridge, Tennessee (720-47-21)

SATURDAY 2:00 P.M.

Session on Operator Theory II, Assembly Room, Sheraton-Park (Lobby Level)

2:00- 2:15 (643) Product integration in reflexive Banach spaces. Preliminary report. Dr. W.E. FITZGIBBON, University of Houston (720-47-6)

2:15- 2:25 (644) Strongly decomposable unbounded operators. Professor IVAN ERDELYI, Temple University (720-47-15)

2:30- 2:40 (645) An abstract averaging theorem. Professor THOMAS G. KURTZ, University of Wisconsin (720-47-23)

2:45- 2:55 (646) Positive eigenvalues for nonlinear operators. Dr. P.M. FITZPATRICK*, University of Chicago, and Dr. W.V. PETRYSHYN, Rutgers University (720-47-27)

3:00- 3:10 (647) On a conjecture of Istratescu. Professor R.J. FLEMING, and Professor J.E. JAMISON*, Memphis State University (720-47-31)

3:15- 3:25 (648) Weakly closed abelian algebras which are not selfadjoint need not be singly generated. Preliminary report. Dr. JOHN MICHAEL McVOY, Penn State University, Ogontz Campus (720-47-11)

3:30- 3:40 (649) A limit theorem for functional Sturm-Liouville problems. Professor WILLIAM R. DERRICK, University of Montana (720-47-18)

3:45- 3:55 (650) Commutativity properties of continuous operators on the space of entire functions. Dr. DAVID P. MATHER, American University (720-47-25)

4:00- 4:10 (651) Tensor products of p-nuclear, quasi p-nuclear, and p-integral maps. Dr. RONALD C. ROSIER, Georgetown University (720-47-26)

4:15- 4:25 (652) Operators on modulared spaces. Professor R.J. FLEMING*, and Professor J. E. JAMISON, Memphis State University (720-47-29)

4:30- 4:40 (653) A note on Frechet's definition of polynomial operators. Professor D.H. HYERS, University of Southern California (720-47-30)

SATURDAY, 2:00 P.M.

Session on Ordinary Differential Equations III, Dover Room, Sheraton-Park (Ground Level)

2:00- 2:10 (654) Applications of the Drazin inverse to linear systems of differential equations. Dr. STEPHEN L. CAMPBELL*, Dr. C.D. MEYER, Jr., and Dr. N.J. ROSE, North Carolina State University (720-34-3)

2:15- 2:25 (655) An algebraic approach to homogeneous matrix differential equations. Professor ROBERT E. BECK*, Villanova University, and Professor BERNARD KOLMAN, Drexel University (720-34-10)

2:30- 2:40 (656) Integration of ordinary linear differential equations by Laplace-Stieltjes transforms. Dr. JAMES D'ARCHANGELO*, U.S. Naval Academy, and Dr. PHILIP HARTMAN, Johns Hopkins University (720-34-11)

2:45- 2:55 (657) On operational methods via matrix operators for a system of ordinary differential equations. Preliminary report. Professor SADANAND VERMA, University of Nevada (720-34-33) (Introduced by L.J. Simonoff)

3:00- 3:10 (658) Block diagonal dominance and reducibility for linear differential systems. Professor DENNIS D. BERKEY, Boston University (720-34-28)

3:15- 3:25 (659) Maximal and minimal solutions and comparison results for differential equations in abstract cones. Preliminary report. Professor V. LAKSHMIKANTHAM, Professor A. RICHARD MITCHELL, and Professor ROGER W. MITCHELL*, University of Texas at Arlington (720-34-12)

3:30- 3:40 (660) Differential equations on closed subsets of a Banach space. Preliminary report. Professor V. LAKSHMIKANTHAM, Professor A. RICHARD MITCHELL*, and Professor ROGER W. MITCHELL, University of Texas at Arlington (720-34-13)

3:45- 3:55 (661) On the zeros of nonlinear operators on closed subsets of a Banach space. Preliminary report. Professor V. LAKSHMIKANTHAM, University of Texas at Arlington, and Professor S. LEELA*, State University of New York, College at Geneseo (720-34-14)

4:00- 4:10 (662) Oscillation of second order differential systems in Banach spaces. Professor GARRET J. ETGEN, University of Houston (720-34-23)

4:15- 4:25 (663) Discrete spectra criteria for singular differential operators with middle terms. Dr. DON B. HINTON, University of Tennessee, and Dr. ROGER T. LEWIS*, Slippery Rock State College (720-34-17)

SATURDAY, 2:00 P.M.

Session on Probability and Statistics, Wilmington Room, Sheraton-Park (Ground Level)

2:00- 2:10 (664) On the limiting behavior of infinitely many queues in tandem with identical service time distributions. HAROLD D. TAYLOR, Indiana University (720-60-20)

2:15- 2:25 (665) SLLNs and CLTs for infinite particle systems. Professor S.C. PORT, and Professor C.J. STONE, University of California, Los Angeles, and Professor N.A. WEISS*, Arizona State University (720-60-26)

2:30- 2:40 (666) An epidemic branching process. Preliminary report. Dr. DAVID B. MILLER, Virginia Commonwealth University (720-60-30) (Introduced by Dr. James Deveney)

2:45- 2:55 (667) An error estimate for the diffusion approximation in population genetics. Preliminary report. Mr. STEWART N. ETHIER, University of Wisconsin (720-60-31)

3:00- 3:10 (668) Nonstationary stochastic gold-mining. Professor GAINEFORD J. HALL, Jr., University of Texas (720-62-1)

3:15- 3:25 (669) Bayesian estimation of life parameters and reliability function in the Weibull distribution. Preliminary report. Professor A.N.V. RAO*, and Professor C.P. TSOKOS, University of South Florida (720-62-2)

3:30- 3:40 (670) Scoring multiple-answer multiple-choice test items: Bayes and minimax strategies. Professor G.T. DUNCAN, Carnegie-Mellon University, and Professor E.O. MILTON*, University of California, Davis (720-62-3)

3:45- 3:55 (671) The multivariate moments of weight 6 of moment-statistics for samples drawn from a finite population. Dr. D.S. TRACY*, and Dr. N.N. MIKHAIL, University of Windsor, and Mrs. C.E. COX DWYER, Simon Fraser University (720-62-4) (Introduced by Dr. F.W. Lemire)

4:00- 4:10 (672) On a classification model. Professor ARJUN K. GUPTA, University of Michigan (720-62-5) (Introduced by Professor J. Sethuraman)

4:15- 4:25 (673) The sampling moments of the generalized variance for finite populations. Preliminary report. Dr. N.N. MIKHAIL*, and Dr. D.S. TRACY, University of Windsor (720-62-6) (Introduced by Dr. F.W. Lemire)

SATURDAY, 2:15 P.M.

Invited Address, Regency Ballroom, Shoreham

(674) Fibrations and geometric realizations. Professor DONALD W. ANDERSON, University of California, San Diego (720-55-12)

SATURDAY, 2:30 P.M.

Special Session on Set-Theoretic and Combinatorial Methods in Topology II, Tudor Room, Shoreham (Upper Lobby Level)

2:30- 2:50 (675) On the cardinalities of compact spaces. Preliminary report. Professor KENNETH KUNEN, University of Wisconsin (720-54-21)

3:00- 3:20 (676) Nonextendable families of continuous functions, Preliminary report. KAREL PRIKRY, Institute for Advanced Study, and University of Minnesota (720-54-14)

3:30- 3:50 (677) On the structure of open subsets of β N-N. Preliminary report. Professor STEPHEN H. HECHLER, City University of New York, Queens College (720-54-54)

4:00- 4:20 (678) The density topology, Preliminary report. Professor FRANKLIN D. TALL, University of Toronto (720-54-43)

4:30- 4:50 (679) Topological properties of Specker types. Preliminary report. Professor JAMES E. BAUMGARTNER, Dartmouth College (720-54-46)

SATURDAY, 3:15 P.M.

Special Session on Some Directions of Current Progress in Commutative Algebra III, Arlington/Alexandria Rooms, Sheraton-Park (Ground Level)

3:15- 4:00 (680) Projective modules over affine algebras. M.P. MURTHY, University of Chicago (720-13-19) (Introduced by Professor David Eisenbud)

4:15- 5:00 (681) K-theory and cycles. Professor STEPHEN GERSTEN, University of Illinois

SATURDAY, 8:30 P.M.

Panel Discussion on Public Science Policy, Regency Ballroom, Shoreham
Professor R.H. BING, University of Texas at Austin (Moderator)

SUNDAY, 1:00 P.M.

Colloquium Lectures: Lecture IV, Regency Ballroom, Shoreham
(682) New directions in model theory. Professor H. JEROME KEISLER, University of Wisconsin-Madison

SUNDAY, 1:00 P.M.

Special Session on Advances in Graph Theory II, The Forum, Shoreham (Upper Lobby Level)

1:00- 1:20 (683) Faithful trivalent Cayley diagrams. Preliminary report. Professor H.S.M. COXETER, University of Toronto (720-05-7)

1:30- 1:50 (684) Extremal problems in graph theory. Preliminary report. Dr. BÉLA BOLLOBAS, University of Cambridge, England (720-05-48) (Introduced by Professor F. Harary)

2:00- 2:20 (685) Some recent results in hamiltonian graphs. Dr. LINDA LESNIAK, Louisiana State University (720-05-35)

2:30- 2:50 (686) Enumeration of self-dual configurations. E.M. PALMER*, Michigan State University, and R.W. ROBINSON, University of Newcastle, New South Wales, Australia (720-05-18)

3:00- 3:30 (687) Informal Discussion

3:30- 3:50 (688) How not to prove the four color conjecture. Dr. FRANK R. BERNHART, University of Waterloo (720-05-5)

4:00- 4:20 (689) The enumeration of Boolean graphs. Preliminary report. Professor PAUL K. STOCKMEYER, College of William and Mary (720-05-46)

4:30- 5:00 (690) Chairman's remarks. Professor FRANK HARARY, University of Michigan

5:00- 6:00 (691) Unsolved problems and recent results

SUNDAY, 1:00 P.M.

Informal Special Session on Finite H-Spaces, Executive Room, Shoreham

1:00- 6:00 (692) Informal Session. Conducted by Professor JAMES D. STASHEFF, Temple University

SUNDAY, 1:00 P.M.

Special Session on Hyperbolic Conservation Laws II, Palladian Room, Shoreham (Lower Lobby Level)

1:00- 1:30 (693) Survey of problems in the theory of shock waves. Professor PETER D. LAX, New York University (720-35-40)

1:35- 2:05 (694) Hyperbolic conservation laws and the theory of functions of bounded variations. RONALD J. DiPERNA, University of Michigan (720-35-33)

2:10- 2:40 (695) Mixed problems for a class of nonlinear conservation laws. TAKAAKI NISHIDA, Kyoto University, Japan, and JOEL SMOLLER*, University of Michigan (720-35-37)

2:45- 3:15 (696) Decay theorems for the quasilinear wave equation. Professor JAMES M. GREENBERG, State University of New York at Buffalo (720-35-35)

3:20- 3:50 (697) Layering methods for parabolic systems. Preliminary report. Professor AVRON DOUGLIS, University of Maryland (720-35-34)

3:55- 4:25 (698) Numerical methods for conservation laws. Dr. BURTON WENDROFF, Los Alamos Scientific Laboratory (720-35-38)

4:30- 5:00 (699) Formation and decay of shock waves in several dimensions. Professor E.D. CONWAY, Tulane University (720-35-46)

5:05- 5:35 (700) Entropy condition for general systems of hyperbolic conservation laws. Professor TAI-PING LIU, University of Maryland (720-35-36)

5:40- 6:00 (701) Admissibility criteria for solutions of conservation laws. CONSTANTINE M. DAFERMOS, Brown University (720-35-32)

SUNDAY, 1:00 P.M.

Special Session on Operator Theory, Tudor Room, Shoreham (Upper Lobby Level)

1:00- 1:20	(702)	On the structure and characterization of piecewise continuous functions. Professor SUN-YUNG CHANG, State University of New York at Buffalo (720-46-39)
1:30- 1:50	(703)	A trace theory for integral operators. Professor WILLIAM HELTON, University of California, San Diego
2:00- 2:20	(704)	Algebras of one-dimensional singular integral operators. Professor IZRAIL GOHBERG, Tel-Aviv University, Israel
2:30- 2:50	(705)	Norms of compact perturbations of operators. Professor CATHERINE OLSEN, State University of New York at Buffalo
3:00- 3:20	(706)	The reducing essential matricial spectra of an operator. Professor CARL PEARCY, University of Michigan, and Professor NORBERTO SALINAS*, University of Kansas (720-47-8)
3:30- 3:50	(707)	Interpolation problems in nest algebras. Professor WILLIAM ARVESON, University of California, Berkeley

SUNDAY, 2:00 P.M.

General Session III, Madison Room, Sheraton-Park (First Floor)

2:00- 2:10	(708)	On countably extending ring topologies. Professor JOHN O. KILTINEN, Northern Michigan University (720-22-1)
2:15- 2:25	(709)	On the centralizer conjecture for monoids on manifolds. PAUL S. MOSTERT, University of Kansas (720-22-3)
2:30- 2:40	(710)	The deficiency index of a third order operator. Professor RICHARD C. GILBERT, California State University, Fullerton (720-34-8)
2:45- 2:55	(711)	On the mean ergodic theorem of Sine. Dr. STUART P. LLOYD, Bell Laboratories, Murray Hill, New Jersey (720-46-7)
3:00- 3:10	(712)	On strong triples for closed graph theorem. Preliminary report. Professor P.P. NARAYANASWAMI, Memorial University of Newfoundland (720-46-36)
3:15- 3:25	(713)	Openness for homomorphisms via somewhere denseness. Professor B.J. PETTIS, University of North Carolina (720-46-26) (Introduced by Professor R. L. Davis)
3:30- 3:40	(714)	Starshaped sets and the Hausdorff metric. Professor GERALD BEER, California State University, Los Angeles (720-52-1)
3:45- 3:55	(715)	Tiling convex sets by translates. Preliminary report. Dr. G. T. SALLEE, University of California, Davis (720-52-2)
4:00- 4:10	(716)	Construction of disjoint arcs through finite sets within given disks. Professor JOHN C. OXTOBY, Bryn Mawr College (720-52-3)
4:15- 4:25	(717)	Characterization of spheres among compact 3-bodies. II. Preliminary report. Dr. HANS-HEINRICH HERDA, Boston State College (720-52-4)
4:30- 4:40	(718)	General decomposition theorems for m-convex sets in the plane. Professor DAVID C. KAY*, and Professor MARILYN BREEN, University of Oklahoma (720-52-5)

SUNDAY, 2:00 P.M.

Session on Algebraic Topology, Wardman Room, Sheraton-Park (Lobby Level)

2:00- 2:10	(719)	Groups and manifolds characterizing links. Professor WILBUR WHITTEN, University of Southwestern Louisiana (720-55-1)
2:15- 2:25	(720)	On homotopy inverse limits and the vanishing of $\lim^S$ for stable pro-groups. Professor D.A. EDWARDS*, Professor R. GEOGHEGAN, and Professor H.M. HASTINGS, State University of New York at Binghamton (720-55-2)
2:30- 2:40	(721)	Epimorphism plus monomorphism implies equivalence in the homotopy category. Professor DAVID HANDEL, Wayne State University (720-55-3)
2:45- 2:55	(722)	A uniqueness theorem for homology in $\mathscr{C}$at, the category of small categories. Professor DANA MAY LATCH, Lawrence University (720-55-4)

3:00- 3:10 (723) On the fixed point indices and Nielsen numbers. Preliminary report. Professor S.K. KIM*, University of Connecticut, Professor D. McGAVRAN, University of Connecticut, Waterbury, and Professor J. PAK, Wayne State University (720-55-5)

3:15- 3:25 (724) On the problem of Hilbert cube factors. Preliminary report. Professor RAYMOND Y. T. WONG, University of California, Santa Barbara (720-55-6)

3:30- 3:40 (725) The wall obstruction in shape and prohomotopy, with applications. Professor DAVID A. EDWARDS, and Professor ROSS GEOGHEGAN*, State University of New York at Binghamton (720-55-7)

3:45- 3:55 (726) Relations among characteristic classes. Preliminary report. Professor STAVROS G. PAPASTAVRIDIS, Brandeis University (720-55-8)

4:00- 4:10 (727) Localizations and $G_n(X)$. Professor GEORGE E. LANG, Jr., Fairfield University (720-55-9)

4:15- 4:25 (728) Knots invariant under switching an essential crossing. Professor R.M. GILLETTE, Montana State University, and Professor J.M. VAN BUSKIRK*, University of Oregon. (720-55-10)

4:30- 4:40 (729) Presentations for proper fundamental groups. Preliminary report. Dr. J.C. CHIPMAN, Oakland University (720-55-11)

SUNDAY, 2:00 P.M.

Session on Analysis V, Franklin Room, Sheraton-Park (First Floor)

2:00- 2:10 (730) Square-integrability wrt nonnegative hermitian measures and spectral integrals of operator-valued functions. Preliminary report. Dr. MILTON ROSENBERG, Rockaway Beach, New York (720-47-1)

2:15- 2:25 (731) Unitary parts of Hankel contractions. Dr. JEFFREY R. BUTZ, Johns Hopkins University (720-47-5)

2:30- 2:40 (732) Commutants of analytic Toeplitz operators. Preliminary report. Professor JAMES E. THOMSON, Virginia Polytechnic Institute and State University (720-47-9)

2:45- 2:55 (733) Approximating the shift with Toeplitz operators. Professor RICHARD BOULDIN, University of Georgia (720-47-24)

3:00- 3:10 (734) Polyhomogeneous maps. Professor DAVID F. FINDLEY, University of Cincinnati (720-47-12)

3:15- 3:25 (735) Stable and nonstable elastica equilibria. Professor S.D. FISHER* and Professor J.W. Jerome, Northwestern University (720-49-1)

3:30- 3:40 (736) A new proof of Rayleigh's principle for eigenvalue approximations. Preliminary report. Dr. DONALD R. SNOW, Brigham Young University (720-49-2)

3:45- 3:55 (737) Perturbation analysis of a class of time optimal control problems. MARVIN I. FREEDMAN, and JAMES L. KAPLAN*, Boston University (720-49-3)

4:00- 4:10 (738) Boundary arcs for integral equations. Professor V.L. BAKKE, University of Arkansas (720-49-4) (Introduced by Professor James E. Scroggs)

SUNDAY, 2:00 P.M.

Session on Applied Mathematics III, Wilmington Room, Sheraton-Park (Ground Level)

2:00- 2:15 (739) On the foundations of integer linear programming. I. Professor JACK E. GRAVER, Syracuse University (720-90-1)

2:15- 2:25 (740) Covering and matching problems as integer linear programs. Mr. CHARLES J. LESKA, Syracuse University (720-90-2)

2:30- 2:40 (741) Stable assignments. Preliminary report. Dr. ISSIE RABINOVITCH, University of Waterloo (720-90-3) (Introduced by Professor H. Shank)

2:45- 2:55 (742) A functional differential equation approach to solving infinite games. Professor ROBERT G. UNDERWOOD, University of South Carolina (720-90-5) (Introduced by Professor William C. Chewning)

3:00- 3:10 (743) Reducing the number of constraints in covering problems. Preliminary report. URI N. PELED, University of Waterloo (720-90-8) (Introduced by Dr. P.L. Hammer)

3:15- 3:25 (744) The role of master polytopes in the unit cube. Professor P. L. HAMMER*, Professor E. L. JOHNSON, and Mr. U. N. PELED, University of Waterloo (720-90-9)

3:30- 3:40 (745) A model for perception of optical illusions. Preliminary report. Professor DAVID A. SMITH, Duke University (720-92-1)

3:45- 3:55 (746) Mathematical model of a cochlea (inner ear) with variable geometry. The low frequency threshold. Dr. ALFRED INSELBERG, IBM Scientific Center, Los Angeles (720-92-2)

4:00- 4:10 (747) Optimal sustainable yield of a renewable resource divided into classes. Professor CHRIS RORRES*, and Professor WYMAN FAIR, Drexel University (720-92-3)

4:15- 4:25 (748) Minimal recognition of biodevelopmental languages. Preliminary report. Professor FRED SPRINGSTEEL*, University of Montana, and PAUL BENNETT, Holyoke College (720-92-4) (Introduced by Stanley Grossman)

4:30- 4:40 (749) Enzymatic neurons. Dr. ROBERT SCHWABAUER, Virginia Commonwealth University (720-92-5)

SUNDAY, 2:00 P.M.

Session on Functional Analysis IV, Alexandria Room, Sheraton-Park (Ground Level)

2:00- 2:10 (750) On Nakano's theorem and the completeness of locally solid Riesz spaces. Professor C. D. ALIPRANTIS, Occidental College and STD Research Corporation, Arcadia, California (720-46-2)

2:15- 2:25 (751) Concerning σ-homomorphisms of Riesz spaces. Professor CHARLES TUCKER, University of Houston (720-46-12)

2:30- 2:40 (752) Hyperarchimedean Riesz spaces. Professor L. C. MOORE, Jr., Duke University (720-46-34)

2:45- 2:55 (753) Interpolating bases in $C[0,1]$ are not Besselian. RONN CARPENTER, University of Houston (720-46-1)

3:00- 3:10 (754) Sequential convergence in $C(X)$. Mr. GARY D. RICHARDSON, East Carolina University (720-46-17)

3:15- 3:25 (755) Extreme operators in the unit ball of $L(C(X), C(Y))$. Dr. ALAN M. GENDLER, California State University, Long Beach (720-46-46)

3:30- 3:40 (756) Extreme linear operators on H_∞. Professor JOHN N. McDONALD, University of Arizona (720-46-10)

3:45- 3:55 (757) Representation of harmonic functions. Preliminary report. Professor W. A. FELDMAN*, and Professor J. F. PORTER, University of Arkansas (720-46-41)

4:00- 4:10 (758) Order and distributions. Preliminary report. Professor J. F. PORTER*, and Professor W. A. FELDMAN, University of Arkansas (720-46-42)

4:15- 4:25 (759) A new algebra of distributions on R^n. Dr. HARRIS S. SHULTZ, California State University, Fullerton (720-46-6)

4:30- 4:40 (760) Jordan-Hahn decomposition of signed weights on finite orthogonality spaces. Professor G. T. RUTTIMANN, University of Massachusetts (720-46-37) (Introduced by Professor T. A. Cook)

4:45- 4:55 (761) The Hahn-Jordan decomposition theorem in infinite quantum logics. Professor THURLOW COOK, University of Massachusetts (720-46-38)

SUNDAY, 2:00 P.M.

Session on General Topology IV, Arlington Room, Sheraton-Park (Ground Level)

2:00- 2:10 (762) Cardinal functions on compact dispersed spaces. Professor PAUL R. MEYER, City University of New York, Lehman College (720-54-6)

2:15- 2:25 (763) Scattered compactifications. Mr. M. JAYACHANDRAN*, Madurai College, India and Professor M. RAJAGOPALAN, Memphis State University (720-54-13)

2:30- 2:40 (764) Some properties related to [a,b]-compactness. II. Professor JERRY E. VAUGHAN, University of North Carolina at Greensboro (720-54-24)

2:45- 2:55 (765) The lattice structure of the set of $(E, \beta E)$-compactifications. Dr. GARY M. BRADY, North Carolina State University (720-54-40)

3:00- 3:10 (766) β-like compactifications and 0-1 measures. Preliminary report. Dr. MICHAEL D'AMBROSA, Seton Hall University (720-54-51)

3:15- 3:25 (767) On the existence of compact metric subspaces. DANIEL J. RANDTKE, University of Georgia (720-54-41)

3:30- 3:40 (768) Point-countability and compactness. Professor H. H. WICKE*, and Professor J. M. WORRELL, Jr., Ohio University (720-54-49)

3:45- 3:55 (769) The character of ω_1 in first countable spaces. WILLIAM G. FLEISSNER, McGill University (720-54-20) (Introduced by M. Makkai)

4:00- 4:10 (770) Some results about spread. Preliminary report. Professor JUDITH ROITMAN, Wellesley College (720-54-29)

4:15- 4:25 (771) Products with Lindelöf spaces. Preliminary report. Professor SCOTT WILLIAMS*, and Ms. MARLENE GEWAND, State University of New York at Buffalo (720-54-59) (Introduced by Professor Lee Mohler)

4:30- 4:40 (472) Ultrafilter invariants in topological spaces. Professor VICTOR SAKS, University of Costa Rica, San Jose, Costa Rica (720-54-44) (Introduced by Professor W. W. Comfort)

SUNDAY, 2:00 P.M.

Session on General Topology V, Richmond Room, Sheraton-Park (Ground Level)

2:00- 2:10 (773) Relative collaring. Professor LEONARD R. RUBIN, University of Oklahoma (720-54-15)

2:15- 2:25 (774) Some applications of Landweber-Novikov operations. DAVID M. SEGAL, City University of New York, City College (720-54-62)

2:30- 2:40 (775) The height of the lattice of finite topologies. Professor ROLAND E. LARSON, Pennsylvania State University, Behrend College (720-54-37)

2:45- 2:55 (776) An embedding theorem for seminearness spaces. Preliminary report. MARYAM SHAYEGAN HASTINGS, University of Toledo (720-54-47) (Introduced by H. L. Bentley)

3:00- 3:10 (777) Boundary of a fuzzy set in a fuzzy topological space. Dr. RICHARD H. WARREN, Aerospace Research Laboratories, Wright-Patterson AFB, Ohio (720-54-48)

3:15- 3:25 (778) Closure space structures for probabilistic metric spaces. Mr. ROBERT M. TARDIFF, University of Massachusetts (720-54-50) (Introduced by B. Schweizer)

3:30- 3:40 (779) On the absolute Baire property. Preliminary report. Professor JOHN C. MORGAN II, Syracuse University (720-54-53) (Introduced by Professor John C. Oxtoby)

3:45- 3:55 (780) Extensions, retracts, and absolute neighborhood retracts in proper shape theory. Professor R. B. SHER, University of North Carolina at Greensboro (720-54-58)

4:00- 4:10 (781) Dimension in metric spaces. Preliminary report. Professor STANISLAW MROWKA, State University of New York at Buffalo (720-54-61)

SUNDAY, 2:00 P.M.

Session on Logic, Dover Room, Sheraton-Park (Ground Level)

2:00- 2:10 (782) Nonextendible bases and maximal vector spaces. Preliminary report. Professor GEORGE METAKIDES*, University of Rochester, and Professor ANIL NERODE, Cornell University (720-02-1)

2:15- 2:25 (783) Stability, $\aleph_0$-categoricity and finiteness conditions in rings. Dr. JOHN BALDWIN, University of Illinois at Chicago Circle and Mr. BRUCE ROSE*, University of Chicago (720-02-3)

2:30- 2:40 (784) Some second-order transfer theorems. Preliminary report. FRED HALPERN, Rider College (720-02-12)

2:45- 2:55 (785) Complete and model-complete theories of monadic algebras. Professor STEPHEN D. COMER, Clemson University (720-02-4)

3:00- 3:10 (786) The weak topology on logical calculi. Professor A. R. VOBACH, University of Houston (720-02-9)

3:15- 3:25	(787)	A new compactness theorem for infinitary logic. Dr. WILLIAM RICHARD STARK, University of Texas (720-02-15)
3:30- 3:40	(788)	Simultaneous embeddings into special subclasses of Λ_{ZR}. Professor T. G. McLAUGHLIN, Texas Tech University (720-02-2)
3:45- 3:55	(789)	Definable real numbers, Preliminary report. LEO HARRINGTON, and DAVID GUASPARI*, State University of New York at Buffalo (720-02-5)
4:00- 4:10	(790)	Beth's theorem in Δ-logics. Dr. J.A. MAKOWSKY, Simon Fraser University (720-02-6)
4:15- 4:25	(791)	A complete representation theorem for varieties categorical in power. Mr. STEVEN GIVANT, University of California, Berkeley (720-02-7)
4:30- 4:40	(792)	Types realized in all models of power $\aleph_2$. Dr. JULIA F. KNIGHT, State College, Pennsylvania (720-02-11)

SUNDAY, 2:00 P.M.

Session on Number Theory III, Continental Room, Sheraton-Park (Lobby Level)

2:00- 2:10	(793)	On the Scholz-Brauer problem in addition chains. Professor A.A. GIOIA, Western Michigan University and Professor M.V. SUBBARAO*, University of Alberta (720-10-27)
2:15- 2:25	(794)	On Fibonacci numbers of the form x^2+1. RAPHAEL P. FINKELSTEIN, Bowling Green State University (720-10-28)
2:30- 2:40	(795)	A problem involving simultaneous binary compositions, an improvement in error term. Professor NINA SPEARS, University of Nebraska (720-10-29) (Introduced by Professor Walter E. Mientka)
2:45- 2:55	(796)	Elimination sequences. Preliminary report. Professor CHARLES R. WALL, University of South Carolina (720-10-30)
3:00- 3:10	(797)	Toward a Kronecker limit formula for the Siegel modular group of dimension two. Preliminary report. JUDITH S. SUNLEY, American University (720-10-31)
3:15- 3:25	(798)	The asymptotic behavior of sums of multiplicative functions. Preliminary report. Dr. SAMUEL D. LAWN, Penn Yan, New York (720-10-32)
3:30- 3:40	(799)	On detecting a periodic event by means of period observations. I. Dr. STEFAN A. BURR, Bell Laboratories, Madison, New Jersey (720-10-33)
3:45- 3:55	(800)	A composition theory for decomposable forms. Preliminary report. Dr. CARTER WAID, University of Delaware (720-10-35)
4:00- 4:10	(801)	On the distribution of quadratic residues. Professor R. WELLS JOHNSON, Bowdoin College (720-10-36)
4:15- 4:25	(802)	Generalized Dedekind's ψ-functions with respect to a polynomial. Professor J. CHIDAMBARASWAMY, University of Toledo (720-10-37)
4:30- 4:40	(803)	On the Littlewood conjecture. Professor GERALD A. BOTTORFF, Pennsylvania State University, Mont Alto Campus (720-10-38)

SUNDAY, 2:00 P.M.

Session on Numerical Analysis, Senate Room, Sheraton-Park (Lobby Level)

2:00- 2:10	(804)	A method of continuation for finding Brouwer fixed points. Dr. R. BRUCE KELLOGG and Dr. JAMES YORKE, University of Maryland and Dr. TIEN-YIEN LI*, University of Utah (720-65-1)
2:15- 2:25	(805)	A direct numerical procedure for the Cauchy problem for the heat equation. Preliminary report. Professor JOHN R. CANNON, University of Texas at Austin and Professor RICHARD E. EWING*, Oakland University (720-65-2)
2:30- 2:40	(806)	Computer symbolic integration of nonlinear differential equations: Progress report. Ms. PATRICIA S. WILLIS, American University and Computer Sciences Corporation, Virginia (720-65-3)
2:45- 2:55	(807)	Zeros of continuous real-valued functions. Preliminary report. Dr. JOHN JONES, Jr., Air Force Institute of Technology (720-65-5)
3:00- 3:10	(808)	Numerical solutions of differential equations. Preliminary report. Professor JOHN GREGORY, Southern Illinois University (720-65-6) (Introduced by Dr. George Parker)

3:15- 3:25 (809) Convergence rates for conjugate direction methods. Preliminary report. Professor JUNIOR STEIN, University of Toledo (720-65-4)

3:30- 3:40 (810) An example concerning the Ritz method. Preliminary report. Professor J. L. LARDY*, Syracuse University, and Professor J. E. OSBORN, University of Maryland (720-65-7)

3:45- 3:55 (811) A quadratically convergent iterative procedure for solutions of equations of the second kind. Preliminary report. Professor GEORGE F. VOTRUBA*, and Professor ROBERT P. BANAUGH, University of Montana (720-65-8)

4:00- 4:10 (812) Approximate integration with built-in error bounds. Dr. DIRAN SARAFYAN*, and Dr. CURTIS OUTLAW, University of New Orleans (720-65-9)

4:15- 4:25 (813) Constrained optimization of nonlinear functions without derivatives. Preliminary report. Dr. JAMES V. BLOWERS, U.S. Air Force, Eglin AFB, Florida (720-65-10)

SUNDAY, 2:00 P.M.

Session on Topological Groups, Adams/Hamilton Room, Sheraton-Park (First Floor)

2:00- 2:10 (814) (CA) topological groups. Dr. DAVID ZERLING, Philadelphia College of Textiles and Science (720-22-4)

2:15- 2:25 (815) Irreducibility of induced representations. Preliminary report. Professor LAWRENCE CORWIN, Yale University (720-22-5)

2:30- 2:40 (816) A note on unitary duals of C(X,G). Preliminary report. Professor L. H. CRABTREE, The Citadel and Professor J.S. YANG*, University of South Carolina (720-22-6)

2:45- 2:55 (817) Neighborhoods of the identity of the free abelian topological group. Dr. DONALD MARXEN, Marquette University (720-22-7)

3:00- 3:10 (818) On equicontinuous transformation group. II. Professor TER-JENQ HUANG, State University of New York, College at Cortland (720-22-8)

3:15- 3:25 (819) Spherical functions on compact wreath products. Preliminary report. Professor JOHN R. DURBIN, University of Texas (720-22-9)

3:30- 3:40 (820) Density characterization of locally compact groups. Preliminary report. Professor GERALD L. ITZKOWITZ, City University of New York, Queens College (720-22-13)

SUNDAY, 2:15 P.M.

Invited Address, Regency Ballroom, Shoreham

(821) Martingale methods in analysis. Professor D. L. BURKHOLDER, University of Illinois at Urbana-Champaign (720-46-53)

SUNDAY, 8:00 P.M.

AMS-MAA Panel Discission on Schemes for training Graduate Students to Teach: Four Reports
Professor James H. Wells, University of Kentucky (Moderator)

S - Shoreham
SP - Sheraton-Park

TIMETABLE
(Eastern Standard Time)

AMERICAN MATHEMATICAL SOCIETY

TUESDAY, January 21	SHORT COURSE ON MATHEMATICS IN OPERATIONS RESEARCH
11:00 a.m. - 3:00 p.m.	REGISTRATION - Main Lobby (S)
	PALLADIAN ROOM (S)
1:45 p.m.	Introductory remarks Alan J. Goldman
2:00 p.m. - 3:15 p.m.	Mathematicians in the practice of operations research Gordon Raisbeck
3:45 p.m. - 5:00 p.m.	Linear programming and flows in networks Christoph Witzgall
WEDNESDAY, January 22	
8:30 a.m. - 1:00 p.m.	REGISTRATION - Main Lobby (S)
	PALLADIAN ROOM (S)
9:00 a.m. - 10:15 a.m.	Stock cutting and its ramifications: Mathematical operations research in industry Ralph E. Gomory
10:45 a.m. - 12:00 noon	Mathematical theory of reliability, with applications Frank Proschan
2:00 p.m. - 3:15 p.m.	Lattice programming and inventory theory Arthur F. Veinott, Jr.
3:45 p.m. - 5:00 p.m.	Queueing theory and applications: Some mathematical frontiers Carl M. Harris

AMS - MAA ANNUAL MEETINGS

WEDNESDAY, January 22	American Mathematical Society	Other Organizations
12:00 noon - 2:00 p.m.		Mathematicians Action Group STEERING COMMITTEE - Open meeting Colonial Room (S)
2:00 p.m.	COUNCIL MEETING Diplomat Room (S)	
2:00 p.m. - 8:00 p.m.	REGISTRATION - Main Lobby (S)	
THURSDAY, January 23	**AMS**	**Other Organizations**
8:00 a.m. - 5:00 p.m.	REGISTRATION - Main Lobby (S)	
	SPECIAL SESSION	
8:30 a.m. - 12:00 noon	Mathematics and Games I Empire Room (S)	
	SESSIONS FOR CONTRIBUTED PAPERS	
8:45 a.m. - 11:30 a.m.	Combinatorics Dover Room (SP)	
8:45 a.m. - 12:00 noon	Commutative Rings Wilmington Room (SP)	
8:45 a.m. - 11:30 a.m.	Complex Analysis I Alexandria Room (SP)	
8:45 a.m. - 11:15 a.m.	Fields Franklin Room (SP)	
8:45 a.m. - 11:45 a.m.	Functional Analysis I Richmond Room (SP)	
8:45 a.m. - 11:45 a.m.	Graphs I Continental Room (SP)	
8:45 a.m. - 11:30 a.m.	General Topology I Arlington Room (SP)	
8:45 a.m. - 11:30 a.m.	Geometry Wardman Room (SP)	
8:45 a.m. - 11:30 a.m.	Harmonic Analysis Assembly Room (SP)	
8:45 a.m. - 11:30 a.m.	Lattices Adams/Hamilton Room (SP)	
8:45 a.m. - 11:30 a.m.	Real Analysis Senate Room (SP)	
8:45 a.m. - 11:30 a.m.	Semigroups Madison Room (SP)	

THURSDAY, January 23	American Mathematical Society	Other Organizations
9:00 a.m. - 10:00 a.m.	INVITED ADDRESS: Invariant differential equations on homogeneous manifolds Sigurdur Helgason, Regency Ballroom (S)	
9:00 a.m. - 10:15 a.m.		Association for Symbolic Logic CONTRIBUTED PAPERS I Tudor Room (S)
	SPECIAL SESSIONS	
9:00 a.m. - 11:30 a.m.	Analytic Number Theory I Diplomat Room (S)	
9:00 a.m. - 12:00 noon	Expository Session on Some Directions of Current Progress in Commutative Algebra Palladian Room (S)	
9:00 a.m. - 12:00 noon	Probabilistic Analysis I Heritage Room (S)	
10:00 a.m. - 12:00 noon	Topological Dynamics I Executive Room (S)	
10:15 a.m. - 11:15 a.m.	INVITED ADDRESS: Representations of semisimple Lie groups (a nontechnical survey) Wilfried Schmid, Regency Ballroom (S)	
10:30 a.m. - 11:30 a.m.		ASL - INVITED ADDRESS: α-Recursion theory Richard Shore, Tudor Room (S)
	SESSIONS FOR CONTRIBUTED PAPERS	
12:00 noon - 2:30 p.m.	General Session I Madison Room (SP)	
12:00 noon - 2:30 p.m.	Algebra I Wardman Room (SP)	
12:00 noon - 2:15 p.m.	Analysis I Senate Room (SP)	
12:00 noon - 2:45 p.m.	Applied Mathematics I Richmond Room (SP)	
12:00 noon - 2:30 p.m.	Associative Rings I Franklin Room (SP)	
12:00 noon - 2:30 p.m.	Finite Groups I Assembly Room (SP)	
12:00 noon - 2:30 p.m.	General Topology II Alexandria Room (SP)	
12:00 noon - 2:15 p.m.	Linear Algebra Adams/Hamilton Room (SP)	
12:00 noon - 2:15 p.m.	Number Theory I Dover Room (SP)	
12:00 noon - 2:45 p.m.	Ordinary Differential Equations I Wilmington Room (SP)	
12:00 noon - 2:30 p.m.	Partial Differential Equations I Arlington Room (SP)	
12:00 noon - 2:30 p.m.	Probability I Continental Room (SP)	
12:00 noon - 5:00 p.m.	EXHIBITS - Ambassador Room (S)	
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURES: New directions in model theory, Lecture I H. Jerome Keisler, Regency Ballroom (S)	
	SPECIAL SESSIONS	
2:00 p.m. - 6:00 p.m.	Interpolation of Operators and Applications I Palladian Room (S)	
2:00 p.m. - 5:45 p.m.	Mathematics and Games II Empire Room (S)	
2:00 p.m. - 4:30 p.m.	Probabilistic Analysis II Heritage Room (S)	
2:00 p.m. - 5:30 p.m.	Topological Dynamics II Executive Room (S)	
2:15 p.m. - 3:15 p.m.	INVITED ADDRESS: On real Teichmuller spaces and their modular groups Linda Keen, Regency Ballroom (S)	
2:15 p.m. - 3:30 p.m.		ASL - CONTRIBUTED PAPERS II Tudor Room (S)
2:15 p.m. - 4:15 p.m.		MAG - BUSINESS MEETING The Forum Room (S)

THURSDAY, January 23	American Mathematical Society	Other Organizations
	SESSIONS FOR CONTRIBUTED PAPERS	
2:30 p.m. - 5:15 p.m.	General Session II Madison Room (SP)	
2:30 p.m. - 5:00 p.m.	Algebra II Wardman Room (SP)	
2:30 p.m. - 5:00 p.m.	Analysis II Adams/Hamilton Room (SP)	
2:30 p.m. - 5:15 p.m.	Associative Rings II Franklin Room (SP)	
2:30 p.m. - 4:45 p.m.	Finite Groups II Assembly Room (SP)	
2:30 p.m. - 4:15 p.m.	Categories Arlington Room (SP)	
2:30 p.m. - 4:45 p.m.	General Topology III Alexandria Room (SP)	
2:30 p.m. - 5:00 p.m.	Graphs II Continental Room (SP)	
2:30 p.m. - 5:15 p.m.	Ordinary Differential Equations II Dover Room (SP)	
2:30 p.m. - 5:15 p.m.	Partial Differential Equations II Senate Room (SP)	
2:45 p.m. - 5:15 p.m.	Applied Mathematics II Richmond Room (SP)	
2:45 p.m. - 5:00 p.m.	Differential Geometry Wilmington Room (SP)	
3:30 p.m. - 4:30 p.m.	INVITED ADDRESS: Some recent discoveries in the isomorphic theory of Banach spaces Haskell P. Rosenthal, Regency Ballroom (S)	
	SPECIAL SESSION	
3:30 p.m. - 6:30 p.m.	Analytic Number Theory II Diplomat Room (S)	
3:45 p.m. - 4:45 p.m.		ASL - INVITED ADDRESS: Descriptive set theory Alexander Kechris, Tudor Room (S)
4:30 p.m.	OPEN REGISTER ORIENTATION SESSION The Forum (S)	
5:00 p.m. - 6:30 p.m.		ASL - SOCIAL HOUR Club Room, Sections A and B (S)
8:00 p.m.		ASL - COUNCIL MEETING Colonial Room (S)
8:30 p.m.	GIBBS LECTURE A priori estimates, geometric effects, and asymptotic behavior Fritz John, Regency Ballroom (S)	

FRIDAY, January 24	AMS	Other Organizations
8:00 a.m. - 4:00 p.m.	REGISTRATION - Main Lobby (S)	
	SPECIAL SESSION	
8:30 a.m. - 12:40 p.m.	Structure and Representations of Lie Algebras over General Fields I Dover Room (SP)	
	SESSIONS FOR CONTRIBUTED PAPERS	
8:45 a.m. - 12:00 noon	Analysis III Wardman Room (SP)	
8:45 a.m. - 11:00 a.m.	Approximation Adams/Hamilton Room (SP)	
8:45 a.m. - 11:30 a.m.	Complex Analysis II Franklin Room (SP)	
8:45 a.m. - 11:30 a.m.	Functional Analysis II Assembly Room (SP)	
8:45 a.m. - 10:30 a.m.	Groups Senate Room (SP)	
8:45 a.m. - 11:30 a.m.	Manifolds and Global Analysis Madison Room (SP)	
8:45 a.m. - 11:00 a.m.	Probability II Alexandria Room (SP)	
9:00 a.m. - 10:00 a.m.	INVITED ADDRESS: General relativity and cosmology Rainer K. Sachs, Regency Ballroom (S)	

FRIDAY, January 24	American Mathematical Society	Other Organizations
9:00 a.m. - 10:15 a.m.		ASL - CONTRIBUTED PAPERS III Tudor Room (S)
	SPECIAL SESSIONS	
9:00 a.m. - 12:00 noon	Fourier Integral Operators Continental Room (SP)	
9:00 a.m. - 12:00 noon	Interpolation of Operators and Applications II Palladian Room (S)	
9:00 a.m. - 11:50 a.m.	Interrelations Between Computation and Number Theory Richmond Room (SP)	
9:00 a.m. - 11:40 a.m.	Probabilistic Analysis III Heritage Room (S)	
9:00 a.m. - 12:00 noon	Singular Cauchy Problems Wilmington Room (SP)	
	INFORMAL SPECIAL SESSION	
9:00 a.m. - 12:00 noon	Color theory Ruth Bari, Arlington Room (SP)	
9:00 a.m. - 4:00 p.m.	EMPLOYMENT REGISTER - Empire Room (S)	
9:00 a.m. - 4:00 p.m.		Mathematical Association of America BOARD OF GOVERNORS MEETING Diplomat Room (S)
9:00 a.m. - 5:00 p.m.	EXHIBITS - Ambassador Room (S)	
	SPECIAL SESSION	
10:00 a.m. - 12:00 noon	Set-theoretic and Combinatorial Methods in Topology I The Forum Room (S)	
10:00 a.m. - 12:00 noon		Association for Women in Mathematics PANEL DISCUSSION: Action programs Executive Room (S) An action program in progress Lenore Blum Reasons qualified women avoid mathematics: The role of the educator in developing mathematical confidence and interest Maita Levine Why are there not more women mathematicians? Edith H. Luchins Metamorphosis of women in mathematics Carolyn T. MacDonald
10:15 a.m. - 11:15 a.m.	INVITED ADDRESS: A representation theoretic proof of a formula of Max Noether Nolan R. Wallach, Regency Ballroom (S)	
10:30 a.m. - 11:30 a.m.		ASL - INVITED ADDRESS: Model theoretic algebra Gregory Cherlin, Tudor Room (S)
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURES II H. Jerome Keisler, Regency Ballroom (S)	
2:15 p.m. - 3:15 p.m.	RETIRING PRESIDENTIAL ADDRESS Topology and logic as sources of algebraic ideas Saunders Mac Lane, Regency Ballroom (S)	
3:15 p.m. - 4:30 p.m.	COLE PRIZE SESSION STEELE PRIZE SESSION Regency Ballroom (S)	
4:30 p.m.	BUSINESS MEETING Regency Ballroom (S)	
4:30 p.m. - 6:10 p.m.		ASL - CONTRIBUTED PAPERS IV Tudor Room (S)
7:00 p.m. - 10:20 p.m.		MAA - FILM PROGRAM Empire Room (S) Allendoerfer Films (in color)
7:00 p.m. - 7:25 p.m.		Gauss-Bonnet theorem

FRIDAY, January 24	American Mathematical Society	Other Organizations
		MAA - FILM PROGRAM (continued) Empire Room (S)
7:26 p.m. - 7:48 p.m.		Cycloidal curves or tales from the Wanklenburg Woods
8:00 p.m. - 8:22 p.m.		A film of the Topology Films Project (in color): How to turn a sphere inside out
8:25 p.m. - 8:50 p.m.		Topology - a B.B.C. Broadcast as part of The Open University Foundation Course in Mathematics (in black and white)
8:55 p.m. - 9:10 p.m.		Mathematics of the honeycomb
9:15 p.m. - 9:45 p.m.		Numerical analysis - Excerpts from a video-taped course produced by Ben Noble at Oberlin College (in black and white)
9:55 p.m. - 10:20 p.m.		Statistics at a glance (in color)
7:30 p.m. - 9:00 p.m.		PANEL DISCUSSION: Computer oriented supplements to the undergraduate mathematics curriculum (Report of an N.S.F. workshop group) Richard Alo, Lyle Mauland Noal Harbertson, Joseph Mayne Carl Leinbach, Harold Weinstock J. C. Mathews Heritage Room (S)
8:30 p.m.	AMS Committee on Employment and Educational Policy PANEL DISCUSSION: Seeking employment outside academia: Views from some who have recently succeeded Steve Bravy John Matherne Martha K. Smith (moderator) Michael Weiss Regency Ballroom (S)	

SATURDAY, January 25	AMS	Other Organizations
8:00 a.m. - 4:00 p.m.	REGISTRATION - Main Lobby (S)	
9:00 a.m. - 5:00 p.m.	EXHIBITS - Ambassador Room (S)	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER - Empire Room (S)	
9:00 a.m.		MAA JOINT SESSIONS with National Council of Teachers of Mathematics Regency Ballroom (S)
9:00 a.m. - 9:50 a.m.		A personalized system of instruction B. A. Green, Jr.
10:15 a.m. - 10:45 a.m.		The first U.S. participation in the International Mathematical Olympiad S. L. Greitzer
11:00 a.m. - 11:50 a.m.		Ordering of fields and prime ideals of Witt rings Alex Rosenberg
12:00 noon	LUNCHEON - Honoring Henry L. Alder - Diplomat Room (S)	
	SPECIAL SESSIONS	
12:00 noon - 2:00 p.m.	Expository Session on Some Directions of Current Progress in Commutative Algebra Arlington/Alexandria Rooms (SP)	
12:00 noon - 6:00 p.m.	Hyperbolic Conservation Laws I Palladian Room (S)	
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURES III H. Jerome Keisler, Regency Ballroom (S)	
	SPECIAL SESSIONS	
1:00 p.m. - 6:00 p.m.	Advances in Graph Theory I The Forum Room (S)	
1:00 p.m. - 6:00 p.m.	Structure and Representations of Lie Algebras over General Fields II Executive Room (S)	
	SESSIONS FOR CONTRIBUTED PAPERS	
2:00 p.m. - 5:15 p.m.	Analysis IV Madison Room (SP)	

SATURDAY, January 25	American Mathematical Society	Other Organizations
	SESSIONS FOR CONTRIBUTED PAPERS	
2:00 p.m. - 4:15 p.m.	Complex Analysis III Senate Room (SP)	
2:00 p.m. - 4:30 p.m.	Differential Equations Continental Room (SP)	
2:00 p.m. - 4:30 p.m.	Functional Analysis III Adams/Hamilton Room (SP)	
2:00 p.m. - 4:15 p.m.	Manifolds Wardman Room (SP)	
2:00 p.m. - 4:30 p.m.	Number Theory II Richmond Room (SP)	
2:00 p.m. - 4:30 p.m.	Operator Theory I Franklin Room (SP)	
2:00 p.m. - 4:45 p.m.	Operator Theory II Assembly Room (SP)	
2:00 p.m. - 4:30 p.m.	Ordinary Differential Equations III Dover Room (SP)	
2:00 p.m. - 4:30 p.m.	Probability and Statistics Wilmington Room (SP)	
2:00 p.m. - 4:00 p.m.		Conference Board of the Mathematical Sciences PANEL DISCUSSION: Wide-ranging applications of statistics Joan R. Rosenblatt (moderator) Palladian Room (S) You have to really like applications to do them well! Joseph B. Kruskal Multi-dimensional economic time series Mervin E. Muller Statistical science and biomedical applications Marvin Zelen
2:15 p.m. - 3:15 p.m.	INVITED ADDRESS: Fibrations and geometric realizations Donald W. Anderson, Regency Ballroom (S)	
2:15 p.m. - 4:45 p.m.		Rocky Mountain Mathematics Consortium BOARD OF DIRECTORS MEETING Club Room, Section A (S)
	SPECIAL SESSIONS	
2:30 p.m. - 5:00 p.m.	Set-theoretic and Combinatorial Methods in Topology II Tudor Room (S)	
3:15 p.m. - 5:15 p.m.	Expository Session on Some Directions of Current Progress in Commutative Algebra Arlington/Alexandria Rooms (SP)	
3:30 p.m. - 4:30 p.m.		National Association of Mathematicians BUSINESS MEETING Heritage Room (S)
5:00 p.m. - 6:30 p.m.	NO-HOST COCKTAIL PARTY - Palladian Room (S)	
7:00 p.m.		MAA - FILM PROGRAM Empire Room (S)
7:00 p.m. - 8:15 p.m.		An application of computer graphics to teaching mathematics: Exhibition of computer-animated super-8 movies and slide sequences on calculus, statistics, pure and applied analysis, by Professors R. B. Kirchner and R. W. Nau of Carleton College
8:30 p.m.	PANEL DISCUSSION: Public Science Policy R. H. Bing (moderator) Regency Ballroom (S) The role of Office of Technology Assessment in shaping public science policy Emilio Q. Daddario Basic research in a problem plagued world H. Guyford Stever	

SUNDAY, January 26	AMS	Other Organizations
8:00 a.m. - 4:00 p.m.	REGISTRATION - Main Lobby (S)	
9:00 a.m. - 5:00 p.m.	EXHIBITS - Ambassador Room (S)	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER - Empire Room (S)	

SUNDAY, January 26	American Mathematical Society	Other Organizations
9:00 a.m.		MAA-NCTM - JOINT SESSION Regency Ballroom (S)
9:00 a.m. - 9:50 a.m.		PANEL DISCUSSION: Teaching mathematics to the beginning undergraduate C. A. Lathan M. H. Protter Andrew Sterrett, Jr. Benjamin Volker A. B. Willcox (moderator)
10:00 a.m. - 11:00 a.m.		MAA - BUSINESS MEETING Fourteenth Award for Distinguished Service to Mathematics; Presentation of the Lester R. Ford Awards; Award of the Chauvenet Prize; Presentation of a Resolution of Appreciation to a Member of the MAA Regency Ballroom (S)
11:10 a.m.		MAA-NCTM - JOINT SESSION Regency Ballroom (S)
11:10 a.m. - 12:00 noon		The Open University in Great Britain—a progress report R. J. Wilson
	SPECIAL SESSIONS	
1:00 p.m. - 6:00 p.m.	Advances in Graph Theory II The Forum Room (S)	
1:00 p.m. - 6:00 p.m.	Hyperbolic Conservation Laws II Palladian Room (S)	
1:00 p.m. - 3:50 p.m.	Operator Theory Tudor Room (S)	
	INFORMAL SPECIAL SESSION	
1:00 p.m. - 6:00 p.m.	Finite H-spaces James D. Stasheff, Executive Room (S)	
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURES IV H. Jerome Keisler, Regency Ballroom (S)	
1:30 p.m. - 2:45 p.m.		NAM - PANEL DISCUSSION: Today's student and the mathematics curriculum Willie Black Vivienne Mayes Theodore Sykes (moderator) Irvin Vance Charles Williams Eddie Williams Diplomat Room (S)
	SESSIONS FOR CONTRIBUTED PAPERS	
2:00 p.m. - 4:45 p.m.	General Session III Madison Room (SP)	
2:00 p.m. - 4:45 p.m.	Algebraic Topology Wardman Room (SP)	
2:00 p.m. - 4:15 p.m.	Analysis V Franklin Room (SP)	
2:00 p.m. - 4:45 p.m.	Applied Mathematics III Wilmington Room (SP)	
2:00 p.m. - 5:00 p.m.	Functional Analysis IV Alexandria Room (SP)	
2:00 p.m. - 4:45 p.m.	General Topology IV Arlington Room (SP)	
2:00 p.m. - 4:15 p.m.	General Topology V Richmond Room (SP)	
2:00 p.m. - 4:45 p.m.	Logic Dover Room (SP)	
2:00 p.m. - 4:45 p.m.	Number Theory III Continental Room (SP)	
2:00 p.m. - 4:30 p.m.	Numerical Analysis Senate Room (SP)	
2:00 p.m. - 3:45 p.m.	Topological Groups Adams/Hamilton Room (SP)	
2:00 p.m. - 6:00 p.m.		CBMS - COUNCIL MEETING Heritage Room (S)
2:15 p.m. - 3:15 p.m.	INVITED ADDRESS: Martingale methods in analysis Donald L. Burkholder, Regency Ballroom (S)	

SUNDAY, January 26	American Mathematical Society	Other Organizations
8:00 p.m. - 10:30 p.m.		CBMS - COUNCIL MEETING Heritage Room (S)
8:00 p.m. - 10:00 p.m.	AMS-MAA PANEL DISCUSSION: Schemes for training graduate students to teach: Four reports Anthony L. Peressini G. Thomas Sallee David I. Schneider Philip Treisman James H. Wells (moderator) Regency Ballroom (S)	
MONDAY, January 27		**Mathematical Association of America**
8:30 a.m. - 2:30 p.m.	REGISTRATION - Main Lobby (S)	
9:00 a.m. - 5:40 p.m.	EMPLOYMENT REGISTER - Empire Room (S)	
9:00 a.m.		SESSIONS OF THE MAA Regency Ballroom (S)
9:00 a.m. - 9:50 a.m.		The challenges to a discipline in an era of interdisciplinary emphasis L. J. Paige
10:00 a.m. - 10:50 a.m.		The National Institute of Education J. M. Mays
11:00 a.m. - 11:50 a.m.		What is an automorphic form? L. J. Goldstein
1:30 p.m.		SESSIONS OF THE MAA Regency Ballroom (S)
1:30 p.m. - 2:20 p.m.		On measuring things, Eudoxus revisited A. M. Gleason
2:30 p.m. - 3:20 p.m.		The early days of probability theory Mark Kac
3:30 p.m. - 4:20 p.m.		Intuitive geometry is alive and well Branko Grünbaum

PRESENTORS OF TEN-MINUTE PAPERS

Following each name is the number corresponding to the speaker's position on the program

Speakers at the Short Course on Mathematics in Operations Research are indicated by (sc)

● Invited one-hour lecturers *Special session speakers

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Springsteel, F. #748
Stahl, S. #374
Starbird, M. #94
*Stark, H.M. #153
Stark, W.R. #787
Stecher, M.J. #262
Stein, A.H. #626
*Stein, E.M. #278
Stein, J. #809
Stephenson, R.M., Jr. #219
*Stockmeyer, P.K. #689
Stoll, M. #38
Stone, H.E. #318
*Strade, H. #432
Strauss, M.J. #393
Strommer, T. #89
Subbarao, M.V. #793
Suffridge, T.J. #462
Sullivan, F. #62
Sunley, J.S. #797
Suryanarayana, D. #628
Szeto, G. #337
Taft, E.J. #319
*Tall, F.D. #678
Tardiff, R.M. #778
Taylor, H.D. #664

*Teply, M. L. #339
*Terras, A. #427
*Terras, R. #429
Thompson, L.D. #314
Thompson, R.C. #231
Thomson, J.E. #732
*Thorp, E. #291
Todd, A.R. #612
Tollefson, J.L. #617
Tong, M. #121
Toralballa, L.V. #90
*Torchinsky, A. #281
Torrance, E. #237
Tortorella, M. #406
Tracy, D.S. #671
Trench, W.F. #385
*Treves, F. #512
Trotter, W.T., Jr. #307
*Tsokos, C.P. #145
Tucker, C. #751
Tucker, T.W. #616
*Tutte, W.T. #561
*Ulam, S. #1
Underhill, W.V. #333
Underwood, R.G. #742
Van Buskirk, J.M. #728
Vanderjagt, D.W. #71
Van Osdol, D.H. #358
Vaughan, J.E. #764
Vaughan, N.H. #29
Veinott, A.F., Jr. (sc)
Verma, S. #657
Vobach, A.R. #786
Votruba, G.F. #811
Waid, C. #800
Walker, H.F. #602
Walker, P.W. #250
Wall, C.R. #796
•Wallach, N.R. #545
Ward, J.D. #447
Ward, L.E., Jr. #80
Warner, H.D. #48
Warren, J. #36
Warren, R.H. #777
Waterman, D. #575
Watkins, M.E. #72
Weaver, J.R. #351
*Weinberger, P.J. #525
*Weinstein, A. #513
Weiss, Gary #639
*Weiss, Guido #280
Weiss, N.A. #665
*Weiss, W.A.R. #542
Wen, S.-L. #190
*Wendroff, B. #698
White, R.E. #601
Whitesides, S. #87
Whitley, W.T. #363
Whitten, W. #719
Wichmann, J. #473
Wicke, H.H. #768
Wiegand, R. #23
Wiegand, S. #21
Wiener, J.C. #468
Wiggs, J.F. #390
Wigner, D. #83
Williams, D.L. #467
Williams, L.K. #244
Williams, S. #771
Williamson, J.E. #378
Willis, P.A. #413
Willis, P.S. #806
Wilson, R.L., Jr. #138
*Wilson, R. Lee #431
Wilson, R.R. #107
Witzgall, C. (sc)
Woeppel, J.J. #483
Wolfe, J. #56
Wolfe, S.J. #270
Wolff, H. #360
Wong, P.-K. #474
Wong, R.Y.T. #724
Woods, J.M. #621
Wright, D. #22
Wulczyn, G. #235
*Wunderlich, M.C. #526
Yang, J.S. #816
Yasuhara, M. #158
Yellen, J. #207
*Young, E.C. #539
Young, R.M. #479
*Zafran, M. #282
Zame, W.R. #444
Zenor, P. #367
Zerling, D. #814
Zitarelli, D.E. #131

NEWS ITEMS AND ANNOUNCEMENTS

ELECTION OF 1974

The Tellers of the election of 1974, Richard E. Sours and Bing Kuen Wong, have reported that the following persons were elected to contested offices:

Vice President	John W. Milnor
Member-at-Large of the Council	David Gale Phillip A. Griffiths Judy Green J. Ernest Wilkins, Jr. Karl K. Norton

On January 1, 1975, Lipman Bers assumed the post of president.

The complete results of the election will appear in the report of the meeting of the Council of January 1975.

Everett Pitcher
Secretary

Symposium on Some Mathematical Questions in Biology
New York, New York
January 29 – 30, 1975

The ninth annual symposium on Some Mathematical Questions in Biology will be held for one and one-half days on January 29-30, 1975, in the LaLoire Room 2 and 3 of the Americana Hotel in New York City. The symposium is being held in conjunction with the annual meeting of the American Association for the Advancement of Science. It will be cosponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics. The support of the National Science Foundation is anticipated. Registration and local arrangements were announced in the September 27, 1974, issue of Science.

The program has been arranged by the AMS-SIAM Committee on Mathematics in the Life Sciences, whose members are Hans J. Bremermann, Jack D. Cowan, Murray Gerstenhaber, Alston S. Householder, Simon Levin (chairman), and Richard C. Lewontin.

PROGRAM

January 29, 9:00 a.m.

Structure in Ecological and Evolutionary Models

Chairman: Simon A. Levin, Cornell University

9:00 a.m. Spatial heterogeneity in ecological systems. SIMON A. LEVIN

10:00 a.m. Dynamics of age-structured populations. GEORGE F. OSTER, University of California, Berkeley

11:00 a.m. Natural selection in age-structured populations. BRIAN CHARLESWORTH, University of Sussex, England

January 29, 2:00 p.m.

Chairman: George F. Oster, University of California, Berkeley

Session for Short Papers

2:00 - 2:15 p.m. Nonlinear waves in reacting systems. H. G. OTHMER, Rutgers University (Bio 75-1) (Introduced by K. Wolfson)

2:20 - 2:35 p.m. The phase problem in crystallography. F. ALBERTO GRUNBAUM, University of California, Berkeley (Bio 75-2) (Introduced by Jacob Feldman)

2:40 - 2:55 p.m. Predator/prey dynamics with age distributions. Preliminary report. D. J. SIMANAITIS* and WILLIAM P. MacLEAN, College of the Virgin Islands, St. Thomas (Bio 75-3)

3:00 - 3:15 p.m. Some mathematical aspects of the ecological principle of competitive exclusion. Preliminary report. ROBERT ARMSTRONG*, RICHARD McGEHEE, and JOHN ZICARELLI, University of Minnesota (Bio 75-4)

3:20 - 3:35 p.m. Stability of model ecosystems with time-delay. Preliminary report. GANGARAM S. LADDE, State University of New York, College at Potsdam (Bio 75-5)

3:40 - 3:55 p.m. Oscillations in stable ecosystems. P. T. SAUNDERS* and M. J. BAZIN, Queen Elizabeth College, London, England (Bio 75-6) (Introduced by D. A. Brannan)

4:00 - 4:15 p.m. Random selection forces in infinite diploid populations. BENNY LEVIKSON, Purdue University (Bio 75-7)

4:20 - 4:35 p.m. A mathematical model of immunoglobulin evolution. J. MYRON HOOD*, Occidental College, and LEROY HOOD, California Institute of Technology (Bio 75-8)

4:40 - 4:55 p.m. Age structured models in population biology. LLOYD DEMETRIUS, Rutgers University (Introduced by Simon A. Levin)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

January 30, 9:00 a.m.

Mathematical Theory of Nerve-Signal Propagation and Other Topics in the Mathematics of the Nervous System

Chairman: Hirsh G. Cohen, T. J. Watson Research Center, IBM

9:00 a.m. Mathematical developments in Hodgkin-Huxley theory and its approximation. HIRSH G. COHEN

10:00 a.m. Simple model equations for active nerve conduction and passive neuronal integration. JOHN RINZEL, Division of Computer Research and Technology, National Institute of Health

11:00 a.m. Is neurobiology totally unmathematical? HORACE B. BARLOW, Trinity College, Cambridge, England

This symposium is the ninth in a series intended to acquaint mathematicians with current biological problems possessing substantial mathematical content, and to encourage research partnerships between mathematicians and biological scientists.

Simon A. Levin, Chairman
Organizing Committee
Ninth Annual AMS-SIAM
Symposium on Some Mathematical Questions in Biology

Ithaca, New York

Symposium on Theory vs. Practice in the Finite Element Method New York, New York January 31, 1975

A symposium on Theory vs. Practice in the Finite Element Method will be held on the morning of January 31, 1975, at the Americana Hotel in New York City, in conjunction with the annual meeting of the American Association for the Advancement of Science. The symposium will be sponsored by the American Mathematical Society and will be supported by the Air Force Office of Scientific Research. Registration and local arrangements were announced in the September 27, 1974, issue of SCIENCE.

The program, which will be held in the La Loire Rooms 4 and 5 at the Americana Hotel, is being arranged by Professors Ridgway Scott and Gilbert Strang (co-chairmen). The finite element method has been applied to such diverse problems as the study of earthquakes, calculation of the modes of vibration of the human skull, design of nuclear reactors and airplanes, and guiding secondary recovery techniques in oil wells. First developed by structural engineers, the finite element method is now widely studied mathematically by numerical analysts. The goal of this symposium is to introduce its audience to the finite element method from both the engineering and mathematical points of view, and to stimulate new areas of application.

PROGRAM

January 31, 9:00 a.m.

Chairman: Gilbert Strang, Massachusetts Institute of Technology

9:10 a.m. Can mathematics be useful? Professor GILBERT STRANG

9:45 a.m. Modeling wave propagation with finite element methods. Professor TODD DUPONT, Department of Mathematics, University of Chicago

10:20 a.m. Computer program construction and maintenance—the future of centralized finite element activity. Dr. ROBERT E. NICKELL, Department of Applied Mechanics, Sandia Laboratories, Albuquerque, New Mexico

10:55 a.m. The interplay between the mathematical and engineering approaches. Professor RIDGWAY SCOTT, Department of Mathematics, University of Chicago

11:30 a.m. The impact of the finite element method on structural engineering. Professor ROBERT L. TAYLOR, Department of Civil Engineering, University of California, Berkeley

Ridgway Scott
Gilbert Strang
Co-chairmen

INVITED SPEAKERS AT AMS MEETINGS

This section of these *Notices* lists regularly the individuals who have agreed to address the Society at the times and places listed below. For some future meetings, the lists of speakers are incomplete.

Mobile, Alabama, March 1975

Richard F. Arenstorf
R. C. Lacher
Charles R. Hobby

St. Louis, Missouri, April 1975

A. O. L. Atkin
K. T. Chen
Kenneth Kunen
Guido L. Weiss

Monterey, California, April 1975

Isaac Namioka
Kennan T. Smith

Pullman, Washington, June 1975

David W. Barnette
Theodore E. Harris

Chicago, Illinois, November 1975

Jonathan L. Alperin
R. O. Wells, Jr.

ORGANIZERS AND TOPICS OF SPECIAL SESSIONS

Abstracts of contributed papers to be considered for possible inclusion in special sessions should be submitted to Providence by the deadlines given below and should be clearly marked "For consideration for special session on (title of special session)." Those papers not selected for special sessions will automatically be considered for regular sessions unless the author gives specific instructions to the contrary.

Mobile, Alabama, March 1975 — January 7, 1975

B. J. Ball, Shape Theory and Related Topics
William H. Ruckle, Functional Analysis

St. Louis, Missouri, April 1975 — January 28, 1975

David Drasin, Classical Function Theory
David L. Elliott, Differential Geometric Problems in Control Theory
Franklin Haimo, Applications of Ring Theory to Groups
Richard P. Jerrard, Geometric Topology
Rangachary Kannan, Nonlinear Functional Analysis
Walter Leighton, Ordinary Differential Equations: Oscillation Theory, Boundary Value Problems
Marian Boykan Pour-El, Recursion Theory
Grant V. Welland, Classical Harmonic Analysis
David J. Winter, Finite Dimensional Field Extensions

Chicago, Illinois, November 1975

Saunders Mac Lane, Category Theory

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Seven Hundred Twenty-First Meeting
University of South Alabama
Mobile, Alabama
March 20 – 21, 1975

The seven hundred twenty-first meeting of the American Mathematical Society will be held at the University of South Alabama in Mobile, Alabama, on Thursday and Friday March 20–21, 1975. The meeting of the American Mathematical Society will be followed by a meeting of the Southeastern Section of the Mathematical Association of America.

By invitation of the Committee to Select Hour Speakers for Southeastern Sectional Meetings, there will be three one-hour addresses. Professor Richard F. Arenstorf of Vanderbilt University will give an address entitled "Some recent developments in celestial mechanics." An address entitled "Finite p-groups" will be given by Professor Charles R. Hobby of the University of Alabama, and Professor R. C. Lacher of Florida State University will present an address entitled "Cell-like mappings and their generalizations."

There will be two special sessions in addition to the regular sessions. Professor B. J. Ball of the University of Georgia is arranging a session on Shape Theory and Related Topics. The speakers will include David Edwards, Ross Geoghegan, James E. Keesling, George Kozlowski, Christopher Lacher, Daniel R. McMillan, T. Benny Rushing, Jack Segal, Richard B. Sher, and James E. West. Professor William H. Ruckle of Clemson University is organizing a session on Functional Analysis; further information will be announced in the February *Notices*. Any member of the AMS who would like to have his or her paper considered for inclusion in one of the special sessions should have his or her abstract so marked and in Providence at least two weeks before the regular closing date for contributed papers (by January 14, 1975).

There will also be sessions for contributed papers Thursday afternoon and Friday morning. The closing date for abstracts for contributed papers is January 28, 1975.

ACCOMMODATIONS

RAMADA INN
600 Beltline Highway

Singles	$16.00
Doubles	$22.00
Extra Person	$ 4.00

RODEWAY INN
1724 Michigan Avenue

Singles	$11.00
Doubles	$15.00
Extra Person	$ 3.00

HOWARD JOHNSON'S
3132 Government Boulevard

Singles	$15.00
Doubles	$17.00
Extra Person	$ 3.00

QUALITY INN
3650 Airport Boulevard

Singles-downstairs	$17.12
Singles-upstairs	$16.05
Doubles-downstairs	$20.00
Doubles-upstairs	$19.33

HOLIDAY INN
Airport Boulevard at I-65

Singles	$14.00
Doubles	$18.00
Extra Person	$ 2.00

ERDC
Brookley

2 Bedroom Suite	$20.00
1 Bedroom Suite	$15.00

(Include sitting room and kitchenette)

RAMADA INN
1705 Dauphin Island Parkway

Singles	$13.00
Doubles	$19.00
Extra Person	$ 2.00

RODEWAY INN
1500 Government

Singles	$12.00
Doubles	$18.00
Studio couch and 1 Double bed	$17.00
Extra Person	$ 4.00

DAYS INN
3675 Government Boulevard

Singles	$ 9.88
Doubles	$12.88
Extra Person	$ 3.00

HOLIDAY INN–West
3939 Highway 90 West

Singles	$13.00
Doubles	$18.00
Twins	$15.00
Extra Person	$ 2.00

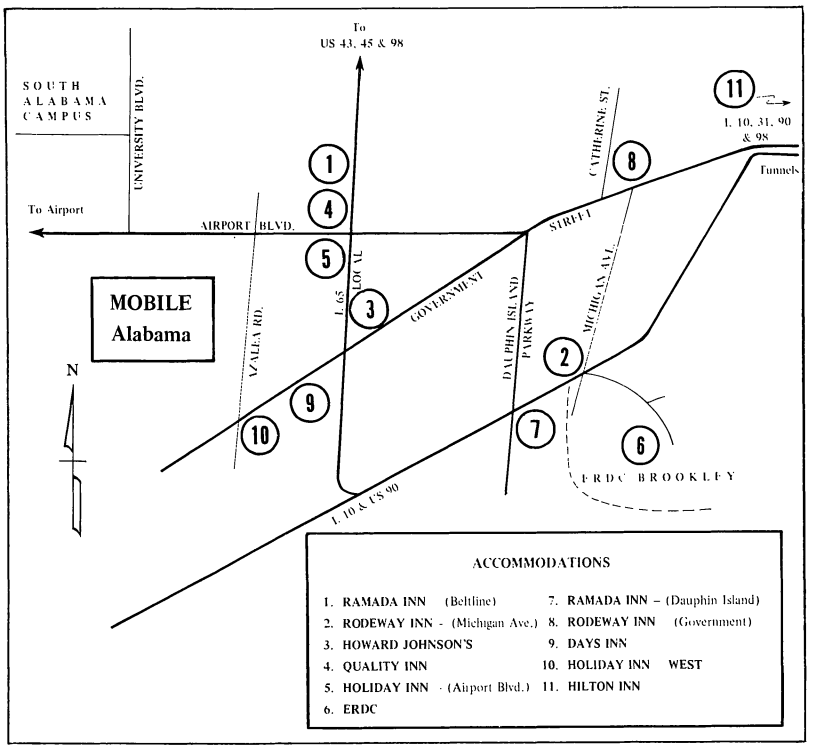
HILTON INN
5050 Battleship Parkway

Singles	$18.00
Doubles	$23.00
Extra Person	$ 5.00

Mobile is served by National, Eastern, and Southern Airlines, Greyhound and Trailways Bus Lines. It is accessible by highway on US 90, 45, 43, 31, and Interstate 10 and 65.

O. G. Harrold
Associate Secretary

Tallahassee, Florida



The seven hundred twenty-second meeting of the American Mathematical Society will be held at the Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York, from Sunday, March 23, through Wednesday, March 26, 1975.

On Tuesday, March 25, there will be a program entitled "A day of differential equations" which will be concerned with the qualitative properties of both ordinary differential equations and partial differential equations, exploring the similarity of their methods and results. This program is being organized by Professor Jack K. Hale of Brown University and will consist of several half-hour addresses in the morning and afternoon. By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be two one-hour addresses on Wednesday, March 26.

Sessions for contributed ten-minute papers will be scheduled in the morning and afternoon on Tuesday and Wednesday, with papers deemed relevant to the "Day" program appearing in regular sessions for contributed ten-minute papers on the same day. No provision will be made for late papers. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of January 28, 1975. The final program of the meeting will appear in the February issue of these *Notices*.

SYMPOSIUM ON NONLINEAR PROGRAMMING

With the expected support of the National Science Foundation and the Atomic Energy Commission, a symposium on Nonlinear Programming is scheduled to be held on Sunday and Monday, March 23 and 24. This topic was selected by the AMS-SIAM Committee on Applied Mathematics whose members are Earl A. Coddington, Richard C. DiPrima (chairman), Lester E. Dubins, Harold Grad, J. Barkley Rosser, and W. Gilbert Strang.

The purpose of the symposium is to help bring this vigorous field of activity to the attention of a larger mathematical audience. The organizing committee, comprised of Richard W. Cottle, Stanford University (chairman), Carlton E. Lemke, Rensselaer Polytechnic Institute, Stephen M. Robinson, University of Wisconsin, and J. Ben Rosen, University of Minnesota, has organized the symposium into four sessions, each with two lecturers. The subject will be treated from the perspectives of theory, computation, and applications. To the greatest extent possible, the lectures will be pedagogically oriented and will identify the principal lines of current research. It is hoped that the lectures will help the audience to identify and appreciate some areas where they can use their own background

and the information they acquire at the symposium to begin research in nonlinear programming. Lecturers and the titles of their talks are: John E. Dennis, Jr. (Cornell University) who will speak on "Some methods for unconstrained minimization of nonlinear functionals"; Burchet C. Eaves (Stanford University) "Path generation in piecewise-linear structures for solving equations"; Harold W. Kuhn (Princeton University) "Nonlinear programming: A historical view"; Olvi L. Mangasarian (University of Wisconsin) "Unconstrained methods in nonlinear programming"; G. P. McCormick (George Washington University) "Optimality criteria in nonlinear programming"; M. J. D. Powell (Atomic Energy Research Establishment, Harwell, England) "Global convergence properties of a variable metric minimization algorithm without line searches when the objective function is convex"; R. Tyrrell Rockafellar (University of Washington) "Lagrange multipliers in nonlinear programming"; and Philip Wolfe (IBM T. J. Watson Research Center) "Difficult problems in nonlinear programming." Ample time will be provided for discussion and questions after each lecture.

REGISTRATION

The registration desk will be located in the Key Room of the Biltmore Hotel on the nineteenth floor adjacent to the Grand Ballroom. The desk will be open from 8:30 a.m. to 4:30 p.m. on Sunday, March
and from 8:30 a
March 26.

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as follows:

Member
Student an
Nonmember 5

ACCOMMODATIONS

Persons intending to stay at the Biltmore Hotel should make their own reservations with the hotel. A reservation form and a listing of room rates will be found on the last page of these *Notices*. The deadline for receipt of reservations is March 5, 1975.

TRAVEL

The Biltmore Hotel is located on Madison Avenue at 43rd Street on the east side of New York City. Walkways to Grand Central Station are located under the hotel and signs are posted directing persons to the lobby of the hotel.

Those arriving by bus may take the Independent Subway System from the Port Authority Bus Terminal. There is shuttle bus service from LaGuardia and Kennedy Airports directly to Grand Central Station. Starters can direct participants to the correct bus.

Air passengers arriving at Newark Airport can take a shuttle bus to the East Side Terminal and take either a subway, taxi, or bus to the hotel.

Those arriving by car will find many parking facilities in the neighborhood in addition to those at the hotel. Parking service can be arranged through the hotel doorman at a cost of $9 for the 24-hour period. There will be an additional charge for extra pickup and delivery service if it is required. The parking fee is subject to New York City taxes.

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otel, Madison
, New York

Walter H. Gottschalk
Associate Secretary

Middletown, Connecticut

The Seven Hundred Twenty-Third Meeting University of Missouri St. Louis, Missouri April 11 – 12, 1975

The seven hundred twenty-third meeting of the American Mathematical Society will be held at the University of Missouri, St. Louis, Missouri, on Friday and Saturday, April 11 and 12, 1975. The University of Missouri at St. Louis is located about ten miles northwest of downtown St. Louis and about four miles east of St. Louis Lambert Airport. All sessions will be held in the J.C. Penney Building of the university.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be four one-hour addresses. Professor A.O.L. Atkin of the University of Illinois at Chicago Circle will speak on Friday, April 11, at 11:00 a.m.; the title of his talk will be announced later. Professor Kuo-Tsai Chen of the University of Illinois at Urbana–Champaign will address the Society on Friday, April 11, at 1:45 p.m.; his subject will be "Iterated path integrals." Professor Kenneth Kunen of the University of Wisconsin at Madison will speak on Saturday, April 12, at 11:00 a.m.; the title of his talk will be announced later. Professor Guido L. Weiss of Washington University at St. Louis will address the Society on Saturday, April 12, at 1:45 p.m.; his topic will be "The use of Hardy spaces and their generalizations in harmonic analysis."

There will be sessions for contributed ten-minute papers on both Friday and Saturday. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of February 18, 1975. Those having time preferences for the presentation of their papers should indicate them clearly on their ab-

stracts. There will be a session for late papers if one is needed, but late papers will not be listed in the printed program of the meeting.

There will be nine special sessions of selected twenty-minute papers. The subjects of these special sessions and the names of the mathematicians arranging them are as follows: Classical Function Theory, Professor David Drasin of Purdue University; Differential Geometric Problems in Control Theory, Professor David L. Elliott of Washington University; Applications of Ring Theory to Groups, Professor Franklin Haimo of Washington University; Geometric Topology, Professor Richard P. Jerrard of the University of Illinois at Urbana–Champaign; Nonlinear Functional Analysis, Professor Rangachary Kannan of the University of Missouri at St. Louis and Michigan State University; Ordinary Differential Equations: Oscillation Theory, Boundary Value Problems, Professor Walter Leighton of the University of Missouri at Columbia; Recursion Theory, Professor Marian Boykan Pour-El of the University of Minnesota; Classical Harmonic Analysis, Professor Grant V. Welland of the University of Missouri at St. Louis; Finite Dimensional Field Extensions, Professor David J. Winter of the University of Michigan. Most of the papers to be presented at these sessions will be by invitation. However, anyone contributing an abstract for the meeting who feels that his paper would be particularly appropriate for one of these special sessions should indicate this conspicuously on his abstract and submit it three weeks earlier than the above deadline, namely by January 28, 1975, in order to allow time for the additional handling necessary.

The Council will probably meet at 5:00 p.m. on Friday, April 11; further details will be given in the February issue of these *Notices*.

ACCOMMODATIONS

The following four inns are located between St. Louis Lambert Airport and the University of Missouri at St. Louis. They are listed in the order of increasing distance from the university.

ROYAL INN*
9600 Natural Bridge Road 63134
(314) 428-9732

Single	$17.00
Double	$20.00

RAMADA INN
9636 Natural Bridge Road 63134
(314) 426-4700

Single	$17.00
Double	$21.00
Twin	$23.00
Extra person	$ 4.00

EIGHT DAYS INN*
4545 Woodson Road 63131
(314) 423-6770

Single	$ 9.88
Double	$12.88
Extra person	$ 3.00

(Double rooms have two beds.)

STAN MUSIAL & BIGGIE'S
ST. LOUIS HILTON INN
10330 Natural Bridge Road 63134
(314) 426-5500

Single	$16.00
Twin/Double	$22.00
Triple	$28.00

Requests for room reservations at the Eight Days Inn should be made directly with the Inn using the room reservation form which will be found on the last page of these *Notices*. Reservations at other motels should be made directly, and mention should be made of this meeting in order to obtain the quoted rates.

Further information will appear in the February issue of the *Notices*; the final program of the meeting will appear in the April *Notices*.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

*N. B. The Eight Days Inn and the Royal Inn will provide free courtesy-van shuttle service between their respective motels and the campus on April 11 and 12.

The Seven Hundred Twenty-Fourth Meeting
Naval Postgraduate School
Monterey, California
April 19, 1975

The seven hundred twenty-fourth meeting of the American Mathematical Society will be held at the Naval Postgraduate School, Monterey, California, on Saturday, April 19, 1975. Invited hour addresses will be given by Professor Isaac Namioka, University of Washington, and by Professor Kennan T. Smith, Oregon State University. The titles of their lectures will be given in the February *Notices*.

There will be sessions for contributed papers. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of February 18, 1975. Late papers will be accepted for presentation at the meeting, but will not appear in the printed program of the meeting.

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

1975 Summer Seminar in Applied Mathematics

The ninth AMS Summer Seminar in Applied Mathematics will be held on the campus of Rensselaer Polytechnic Institute, Troy, New York, from July 7 through July 18, 1975, and will be sponsored jointly by the American Mathematical Society and the Society for Industrial and Applied Mathematics. The topic for the seminar will be Modern Modeling of Continuum Phenomena. This was proposed by the AMS-SIAM committee on Applied Mathematics which at the time consisted of Earl A. Coddington, Hirsh G. Cohen (chairman), Lester E. Dubins, Harold Grad, J. Barkley Rosser and Richard S. Varga. The organizing committee will consist of George F. Carrier, Harvard University; Hirsh G. Cohen, IBM T.J. Watson Research Center; Stephen H. Davis, Johns Hopkins University; Richard C. DiPrima, Rensselaer Polytechnic Institute (chairman); Joseph B. Keller, Courant Institute of Mathematical Sciences of New York University; and Lee A. Segel, Weizmann Institute of Science and Rensselaer Polytechnic Institute. Support is expected from the National Science Foundation and Office of Naval Research.

Continuum problems in the physical sciences have always provided a rich area of study for the applied mathematician. More recently continuum models have been developed for important problems in the biological sciences and ecology.

The primary purposes of the seminar are (i) to introduce the participants to selected mathematical research areas of high current interest and relevance, (ii) to present the underlying fundamental laws of continuum model building, and (iii) to present selected mathematical topics particularly useful in solving modern mathematical problems of continuum phenomena. This will be accomplished through core series of lectures in continuum model building and mathematical methods, and four in-depth case studies concerned with a problem of current interest in fields such as ocean circulation, population dynamics, cell motions, and seismology. A partial list of principal lecturers includes Lee Segel, Don Cohen, George Carrier, George Oster, Gary Odell and George Papanicolau. In addition there will be an opportunity for informal seminars in which all who attend may directly participate.

Allowance will be made for faculty members whose principal background is in pure mathematics but who wish to enlarge their understanding of applications and to find opportunities for research on applied problems. Thus, there will be some introductory material but taking account of the fact that the audience will have a high level of general mathematical sophistication. However, a research level will be maintained in almost all parts of the seminar.

Dormitory accommocations, and food service will be available on the campus. A complete brochure of information will be sent to the participants early in the spring.

Individuals may apply for admission to the seminar. Application blanks for admission and financial assistance can be obtained from the Meeting Arrangements Department, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940. The application deadline is March 17, 1975. An applicant will be asked to indicate his scientific background and interest; he should have completed at least one year of graduate school. A graduate student's application must be accompanied by a letter from his faculty advisor concerning his ability and promise. Those who wish to apply for a grant-in-aid should so indicate. However, funds available to the Seminar are limited and so individuals who can obtain support from other sources should do so.

1975 Summer Institute on Several Complex Variables July 28 – August 15, 1975

The twenty-second Summer Research Institute of the American Mathematical Society will be devoted to the topic "Functions of Several Complex Variables". The site of this conference has not been selected yet, but will most likely be somewhere in the northeastern United States. The Organizing Committee consists of Professors H. Grauert, R. C. Gunning (co-chairman), D. Lieberman, J. Morrow, R. Narasimhan, H. Rossi (co-chairman), Y. T. Siu, and R. O. Wells. It is expected that the institute will be supported by a grant from the National Science Foundation.

The program will consist of three types of seminars: a series of expository talks on recent developments in special areas; a running seminar featuring half-hour talks; a collection of special one hour talks on topics of special significance. Tentatively, the topics will be "Deformation of complex spaces", "Partial differential equations in complex analysis", "Boundary values of holomorphic functions", "Analysis on noncompact Kahler manifolds", "Compact complex manifold", "Complex integral geometry", "Kernel functions and integral formulae", "Direct image theorems", and "Approximation theorems".

Information on location, travel, and accommodations will be available in subsequent announcements.

Funds for participant support will be limited; it is anticipated that participants will have to find their own sources of travel support. The institute is open to all mathematicians specializing in complex analysis in several variables and related topics, and to advanced graduate students in this field. Those wishing to participate should write to Dr. Gordon L. Walker, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940. Recent Ph.D.'s and advanced graduate students who wish to be considered for support should write before March 1, 1975.

NEWS ITEMS AND ANNOUNCEMENTS

COMMENTS ON AN INVITATION TO V. I. ARNOL'D

Invitations to present expository addresses at the International Congress of Mathematicians included one to Professor V. I. Arnol'd of Moscow University. When no reply was received from Professor Arnol'd, Dr. Philip Handler, President of the National Academy of Sciences, wrote a letter on July 17, 1974 to Academician Yu. A. Ovchinnikov, Vice President of the Academy of Sciences of the USSR. A portion of the text of the letter was as follows:

> A number of Soviet mathematicians have been invited as speakers at this Congress...; of the proposed 17 major invited expository hour-long addresses, two Soviet mathematicians have been named: V. I. Arnold and A. G. Vitushkin. The difficulty arises because the Organizing Committee has not yet had acceptances from a number of the invited speakers, one of whom yet to respond is V. I. Arnold.
>
> Professor Arnold enjoys an outstanding reputation among the mathematicians in the U.S. and in the world mathematical community as well. His presence at the Vancouver meeting and the presentation of his talk would contribute much to the eventual success of the Congress.

The reply, dated August 9, 1974 came from Academician V. A. Kotelnikov, Vice President of the Academy of Sciences of the USSR and is reproduced in translation in entirety.

> We acknowledge receipt of your letter of July 17 of this year.
>
> We should like to inform you that in the Soviet Union there is a National Committee of Soviet Mathematicians which decides questions concerning participation of Soviet mathematicians in international colloboration.
>
> This Committee did not find it possible to recommend Professor V. I. Arnold as one of the Soviet participants in the International Congress of Mathematicians to be held in Vancouver in August of this year.
>
> At the same time we should report that Doctor of Physico-Mathematical Sciences V. I. Arnold is a professor at Moscow University and does not work in the system of the AS USSR.

An address by Professor Arnol'd was read at the Congress by Professor David Gale.

ALLAHABAD MATHEMATICAL SOCIETY

Individuals interested in joining the Allahabad Mathematical Society should direct their inquiries to the Society's new address: Dr. S.R. Sinha, Secretary; Allahabad Mathematical Society, 'Parijata,' 5, C.Y. Chintamani Rd., Allahabad - 211002 - India.

Dues and privileges remain the same.

AMS RESEARCH FELLOWSHIP FUND
Request for Contributions

The AMS Research Fellowship Fund is intended to support research fellows for one year with awards being made on the basis of mathematical promise. Serving on the Committee appointed by President Mac Lane to administer the Fund are C. B. Bell, Walter Feit, Leonard Gillman, Peter J. Hilton, Mark Kac, and Alice T. Schafer, Chairman. The Committee hopes to award for 1975–1976 several partially tax-exempt fellowships of approximately $10,000 each, a sum equivalent to the salary which a research person with a recent Ph.D. in mathematics might expect. Two such awards were made in August 1974. Although the number of fellowships awarded may be small, the existence of the Research Fellowship Fund demonstrates the importance the Society and its members attach to research. The Committee is happy to report that many individuals have already contributed to the Fund and that among the recent contributions was an anonymous one for $6,000.

The survival of the Research Fellowship program depends on the contributions the Society receives. It is hoped that every tenured member of the Society will be willing to contribute at least $100 to the Fund, but any contribution is welcome. The Society itself has pledged to contribute a minimum of $9,000 and will match one-half the funds in excess of $18,000 raised from other sources. However, the Society can contribute no more than $20,000. Contributions are, of course, tax deductible. Checks should be made payable to the American Mathematical Society, clearly marked "AMS Research Fellowship Fund" and sent to the American Mathematical Society, Post Office Box 1571 Annex Station, Providence, Rhode Island 02901.

AMS RESEARCH FELLOWSHIPS

The American Mathematical Society invites applications for the AMS Research Fellowships. These are postdoctoral fellowships to be awarded for research in mathematics during the year 1975–1976, and are open to individuals who have recently received the Ph.D. The stipend will be in the range of $9,000–$10,000, depending on the amount in the Fund. Of the award, $3,600 plus $500 for travel expenses will be tax deductible.

Completed applications must be received by March 15, 1975. A small number of fellowships will be awarded, the number depending on the amount contributed to the AMS Research Fellowship Fund. Notification of awards will be made by April 15, 1975.

For further information and application forms write to Dr. Gordon L. Walker, Executive Director, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

The American Mathematical Society
REPORT OF THE PRESIDENT FOR 1974

This is the year when our Providence office moved into its fine, new and somewhat austere building in Providence, and also the year when our Society has started to move in on a number of difficult professional and scientific problems.

1. The Joint Projects Committee. Problems of teaching and understanding mathematics also concern many of our closely related Societies and so require joint committees and joint efforts. Initially, steps to oil these joints started with the establishment of an overall Committee representing the Association, SIAM, and the Society (see the Report of the President for 1973, these *Notices*, January, 1974). The major recommendation of this Committee called for the establishment of a Joint Projects Committee with explicit instructions and authority to tackle matters of joint concern. This Joint Projects Committee has been approved by the respective Councils for SIAM and for the Society, and is in process of consideration by the Association. Moreover, the Conference Board of the Mathematical Sciences has agreed to provide administrative supervision for the Joint Projects Committee.

The first major joint project is to be a Mini-Study: A survey of the development and use of selected mathematical ideas in a volume of 10 or 12 appropriate essays. These essays, to be prepared under the oversight of a Planning Committee, can be viewed as a sequel to the earlier COSRIMS Study (= Committee on the Support of Research in the Mathematical Sciences). They are intended to clarify for other scientists the meaning and use of mathematics by carrying out an examination of the development of quantitative and qualitative concepts; their implications, applications, and meaning for other sciences, and for the world at large. It is hoped that such a study will illuminate the many-sided relationships between core mathematics and its applications.

2. The Proceedings of our Society is a meticulously edited and wide ranging journal, publishing short articles on practically every aspect of mathematics. It clearly serves a vital purpose in communicating new ideas from active mathematicians, but there are times and occasions when criticisms of the Proceedings arise. For example, when I looked carefully at a couple of recent issues, I was struck by the facts that the good papers greatly outnumber the excellent papers, and that a large number of papers seemed to be piecemeal contributions: Papers proving just a single result which might have been better set in a larger context and a fuller paper. After some discussion of these points the Editors of the Proceedings have agreed to add to their regular editorial tasks that of looking again at the general purposes and objectives of the journal. We hope that the Proceedings can thereby contribute more to the effective presentation of research.

3. Mathematical Reviews continues to be our premier publication; it is even more useful now that the Eight-Year Index, 1965–1972 is available. Other new features include a subject-matter index every 6 months and a reconstitution of the publication, "Contents of Contemporary Mathematical Journals" (CCMJ), now renamed "Current Mathematical Publications" and consisting of a listing, classified by subject, of articles in the full range of journals covered by Mathematical Reviews. A Joint Committee with SIAM is making progress on the related and perennial problem of reviews in applied fields—perhaps initially by way of a publication similar to CCMJ. This joint consideration has been sparked by the Conference Board of the Mathematical Sciences. Through all this, the main work of Mathematical Reviews goes on and continues to depend on the expert services of reviewers all over the world. I would like to take this occasion to thank them for their effective contributions.

4. Expository Journal. The Committee to consider the establishment of a research expository journal has reported with enthusiasm, suggesting that the basic coverage of such a journal should extend over all fields of pure and applied mathematics to provide expository articles in the strict sense, and also critical surveys of the literature in major fields of research, biographies of distinguished mathematicians, and groups of related papers of a research expository character. The Trustees are still in the process of considering how to provide the support for such a journal.

5. Financial Problems. As with many organizations, the Society has had difficulties in getting its income to balance its expenses. Both the staff of the Providence office and the Board of Trustees have worked extensively on these matters, especially for the 1974 budget. After valiant efforts, that budget was finally put into satisfactory shape. For the future, progress has been made both with new controls and new information: The new controls stemming from the analysis on the AMS computer of financial operations of the Society and the new information consisting of five-year displays of past and future budgets. With these means the Trustees hope to have better planning so that our membership dues and subscription fees can be used to provide maximum service for the members of the Society.

6. Employment. The uncertain economic situation continues to make grave employment difficulties for many of our members. The Society has a variety of Committees working on aspects of this problem. The long-established Committee on Employment and Educational Policy (CEEP) analyzes these policies and collects statistics, especially careful statistics each summer for the number of new Ph.D.'s placed (see "Doctorates and Jobs, 1974 Report" by R. D. Anderson, these *Notices* 21,

November 1974, pp. 335–340). The Joint AMS-MAA-SIAM Committee on Employment Opportunities manages the Employment Register at each January meeting and presses for more complete listings of positions available in the publication, "Employment Information for Mathematicians." The Joint Committee on Training of Graduate Students to Teach is concerned with getting more effective training in graduate schools for prospective teachers. The Joint Committee on the Employment of Mathematicians in Two-Year Colleges is examining all the aspects of this part of the employment situation, as is indicated in the preliminary report in these *Notices*, November 1974, p. 349.

At the San Francisco business meeting of the Society in January, Resolution C called on the Society to emphasize the importance for good education of keeping teaching loads and class size down. In May a Committee was appointed to examine ways and means for doing this. The means are obviously not easy because of the economic forces involved. In another resolution (Bulletin AMS, 80 (1974), p. 656) the San Francisco meeting declared its categorical opposition to discrimination against mathematicians...on the basis of race, sex, politics, religion, ethnic origin, age or other non-professional characteristics. In response to this resolution, a new Society Committee on Legal Aid has been set up and has decided upon a mechanism for providing legal aid in cases of mathematicians bringing suit because of cases of discrimination.

More recently, the May meeting of the Council of the Society called for creation of a new Committee on the Emergency Employment Situation. This Committee is already hard at work. Three of its recommendations were adopted at the October meeting of the Council. To wit:

(1) The development of a series of summer institutes to provide continuing education and familiarity with research problems in such fields as statistics, computing, and operations research for people holding Ph.D. degrees in other fields.

(2) The principle that, in so far as practical, all positions in the mathematical sciences be advertised and that the standard place for the ads to appear is the publication, "Employment Information for Mathematicians."

(3) A feasibility study looking to the publication of a list of non-academic employers of mathematicians.

We hope to enlist the help of MAA and SIAM in further work in this complex of problems. They concern economic issues of demand and supply for mathematicians and therefore would involve ways of increasing the demand, ways of controlling the supply, and ways of improving communications in the market for mathematicians. The bigger questions of the whole state of the U.S. economy are involved here, but are largely beyond our control.

7. Science Policy. In the past, mathematicians have had relatively little influence on the various determinations of national science policy. Today our first steps in these directions are still incomplete. The Society has recommended to appropriate quarters the granting of more predoctoral NSF fellowships and the restoration of the postdoctoral fellowships, but so far these recommendations have not been adopted. Among other developments, our Committee on Relations with Government has endorsed the idea that the National Science Foundation might establish several research institutes in mathematics which would provide effective ways for young mathematicians to come into contact with the core of the subject and its applications. At the San Francisco meeting one resolution called for "the massive transfer of funds from the federal military budget to the support of education, including higher education." This resolution was tabled. In my judgment, such global attacks on everything at once are likely to be so global as to miss the connectivity and the local maxima, and so to fail to persuade anyone, while dividing our membership along lines which are hardly mathematical. On the larger political scene, some recent American experience may have highlighted actions that are raucously aggressive, rather than specifically directed. Even when these actions seem to succeed, they may be counterproductive, at least for mathematics, in that they offer the wrong models for mathematical activity and that they constitute a distraction from the main objective of pressing forward aggressively in the always difficult and exciting task of discovering new mathematical facts and theorems.

This task does continue, as the following incident may indicate. In San Francisco, just after the close of the business meeting, complete with parliamentarian, resolutions, chairs, and a table, a young mathematician of my acquaintance buttonholed me in the hall of the Hilton, to complain about a refusal to arrange a meeting for his/her seminar that day at 4:00 p.m. I tried to explain the refusal: 4:00 p.m. was exactly the time of the business meeting, which was not a routine one, since it had been instructed a year ago to deal with three important, well advertised policy resolutions. He/she wasn't convinced, hadn't heard of the resolutions, didn't care about them but evidently did very much care about that seminar. Mathematical research matters.

8. State of Mathematics. As this incident indicates, our meetings and publications continue to show that mathematical research really advances. The long run health of our science will surely depend both on direct conceptual advances and on effective relations to the applications. Of late I have been troubled to read many extreme formulations of this issue. On the one hand, some appear to say that only applicable mathematics matters, while on the other hand, there are firm suggestions that pure mathematics could continue to be healthy for a couple of hundred years without having any contact whatever with its applications. Both statements are vigorous and extreme but neither extreme seems sensible. My positive observation is rather to the effect that mathematics is concerned with understanding the deep structure of the universe as may be revealed by straight abstract throught, by building on previous notions, and by careful examination of examples and applications. When mathematics is truly directed at this deep structure, then it fits. This is the ultimate reason why mathematics does work, and why all aspects of the endeavor are important. Today mathematical methods and

mathematical models are indeed used in very many connections, especially in very many policy studies, and of late I have noted too many occasions when some of these uses were careless, superficial, or incomplete. My fundamental hope is that the high conceptual standards and real understanding, traditionally available in the mathematical community, can be brought effectively to bear on these methods. A more thorough use of mathematics and its deeper techniques on policy questions should be of vital assistance to society—and at the same time, should provide some real and substantive correction for the present difficulties in the employment of mathematicians.

9. Mathematical Events. This year there have been two notable meetings. One was the International Congress of Mathematicians in Vancouver; notable for the award of two Fields Medals (to Bombieri and Mumford) and for many effective lectures. The second meeting was the AMS meeting in DeKalb in May with expert lectures devoted to summarizing the state of each of the Hilbert problems. Because of its breadth and focus, this meeting provided a most effective presentation of a large view of mathematics and so was a special stimulus to those fortunate enough to attend. We need more such meetings.

I remind you that the American Mathematical Society will again provide its own postdoctoral fellowships, the number depending on the generosity of your contributions, which can be sent to the Providence office or to Professor Alice Schafer, Chairman of the Committee.

10. A Program. As the end of my term approaches, I took the occasion to reread my earlier article, "A Program for the American Mathematical Society", these *Notices*, May 1973. Most of the proposals there made still seem approriate. Some have been realized and others are in process, though some of the suggestions about strengthening and varying our style of meeting still require study. I hope these and other improvements will come about in the future, and that the meetings and our publications can continue to convey the difficulty and excitement of mathematical research and discovery. In any event, I would like to take this occasion to say that I deeply appreciate the opportunity I have had to serve the Society as its President.

Saunders Mac Lane

NEWS ITEMS AND ANNOUNCEMENTS

FULBRIGHT-HAYS AWARDS FOR SENIOR AMERICAN AND FOREIGN SCHOLARS

Specialists in mathematics who are U.S. citizens and have a doctorate or college teaching experience are invited to apply for lecturing and advanced research awards under the senior Fulbright-Hays program. A simple registration form may be obtained and returned to the Senior Fulbright-Hays Program, 2101 Constitution Avenue, N.W., Washington, D.C. 20418. Registrants will be sent a detailed announcement of the 1976–1977 program in April. July 1, 1975 is the deadline for applying for research awards and it is also the suggested date for filing for lectureships.

Each year Fulbright-Hays agencies abroad forward to the Council application of senior foreign scholars who are interested in remunerative appointments for lecturing and postdoctoral research at American colleges or universities for temporary periods. The scholars are eligible for Fulbright-Hays travel grants if arrangements are confirmed for remunerative appointments.

The Council welcomes information regarding appointments available at American educational institutions for foreign scholars for temporary periods of three months to one year. Approximately 20 foreign mathematicians are presently in the United States under the sponsorship of the Fulbright-Hays program. They are listed in the 1974–1975 Directory of Visiting Lecturers and Research Scholars; many are available for lectures or attendance at special conferences. Information about the scholars and copies of the Directory which lists a total of 500 scholars in several different disciplines is available from the Council for International Exchange of Scholars, 2101 Constitution Avenue, N.W., Washington, D.C. 20418. Many of the mathematicians have been listed in the October and November *Notices* under Visiting Mathematicians; this issue of the *Notices* includes those not previously listed.

ZENTRALBLATT FÜR MATHEMATIK

In cooperation with Springer-Verlag, the American Mathematical Society offers Zentralblatt für Mathematik, Volume 62, Parts I/II, a cumulative author index for Volumes 1–25. This is an index of all the reviews contained in these volumes with full title and bibliography. Thus a comprehensive bibliography for 1931–1940 is available for the first time. (Mathematical Reviews began publication in 1940 and contains only the subsequent period.)

When ordered from Springer the price is DM 136 plus shipping. The index may be ordered from the AMS for $60 list, this is the December 11, 1974 currency equivalent plus a shipping charge, or $45 member's price. Present subscribers of Zentralblatt will receive the index and will be billed by Springer.

Orders must be accompanied by payment and sent to the American Mathematical Society, P.O. Box 1571, Annex Station, Providence, Rhode Island 02901.

CHANGE IN SUBSCRIPTION PRICE

Mathematica Scandinavica has increased subscription price because of the increased cost of production to 75 Danish crowns per volume. The new price applies to Volumes 36 and 37 which are to be published in 1975. Subscription price for members in 1975 is 150 Danish crowns. Orders should be sent directly to Mathematica Scandinavica, NY Munkegade, 8000 Aarhus C, Universitetsparken, Denmark.

DOCTORATES CONFERRED IN 1973-1974
Supplementary List

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during the interval July 1973—June 1974. This is a supplement to the list printed in the October 1974 issue of these *Notices*. The numbers appearing in parentheses after each university indicate the following: the first number is the total number of degrees listed for that institution; the next seven numbers are the number of degrees in the categories of 1. Pure Mathematics, 2. Statistics, 3. Operations Research, 4. Computer Science, 5. Applied Mathematics, 6. Mathematics Education, 7. Other. Each entry contains the dissertation title. 9 universities are listed with a total of 53 individual names. This total, combined with the other list, includes doctorates from 154 universities with a total of 1,201 individual names; 241 departments granting doctorates.

CALIFORNIA

STANFORD UNIVERSITY (16;0,0,0,16,0,0,0)

Department of Computer Science

Erman, Lee Daniel
An environment and system for machine understanding of connected speech

Gabow, Harold Neil
Implementations of algorithms for maximum matching on nonbipartite graphs

Gips, James Elliot
Shape grammars and their uses

Goldman, Neil Murray
Computer generation of natural language from a deep conceptual base

Hannah, Marsha Jo
Computer matching of areas in stereo images

Meyers, William James
Linear representation of tree structure: a mathematical theory of parenthesis- free notations

Nelson, Edward Clark
Resource allocation for free running and resource limited program graphs

Palmer, John Franklin II
Conjugate direction methods and parallel computing

Rieger, Charles Joseph III
Conceptual memory: a theory and computer program for processing the meaning content of natural language utterances

Riesbeck, Christopher Kevin
Computational understanding: analysis of sentences and context

Rivest, Ronald Linn
Analysis of associative retrieval algorithms

Shapiro, Gerald Norris
A functional approach to structured combinational-logic design

Sites, Richard Lee
Proving that computer programs terminate cleanly

Swinehart, Daniel Carl
Copilot: a multiple process approach to interactive programming systems

Vuillemin, Jean Etienne
Proof techniques for recursive programs

Yakimovsky, Yoram
Scene analysis using a semantic base for region growing

ILLINOIS

NORTHWESTERN UNIVERSITY (3;0,0,0,0,3,0,0)

Committee on Applied Mathematics

Ben-Tal, Aharon
Contributions to geometric programming and generalized convexity

Patarra, Cyro
Discretization methods for neutral ordinary differential equations

Schmeichel, Edward
The cycle structure and planarity of graphs under degree constraints

MICHIGAN

UNIVERSITY OF MICHIGAN (2;0,0,0,2,0,0,0)

Department of Computer and Communication Sciences

Martin, Nancy
Convergence properties of class of probabilistic adaptive schemes called sequential reproductive plans

Springer, Allen Lewis
A macro-oriented programming language extension facility

NEW MEXICO

NEW MEXICO STATE UNIVERSITY (2;2,0,0,0,0,0,0)

Department of Mathematical Sciences

Daly, James Edward
Algebras of certain singular integrals over local fields

Downey, Charles Patrick
Classification of singular integrals over a local field

NEW YORK

COLUMBIA UNIVERSITY (1;0,0,0,0,0,1,0)

Department of Mathematical Education

Kalish, Aida
Training in mathematics appropriate for the field of operations research

CORNELL UNIVERSITY (1;1,0,0,0,0,0,0)

O'Malley, Thomas J.
The geometry of S-subgroups of the hyperbolic groups

RENSSELAER POLYTECHNIC INSTITUTE (5;0,4,1,0,0,0,0)

Department of Operations Research and Statistics

Monaco, Salvatore John
A Bayesian approach to the design and analysis of experiments for regression models

Nicklin, Edmund Harold
A constrained regression model for rational warrant pricing

Utter, Merlin Lester
Robustness of experimental designs to various optimality criteria

Vescovacci, Miguel Antonio
Modal estimation in pert networks

Wiebking, Rolf
Deterministic and stochastic geometric programming models for optimal engineering design problems in electric power generation and computer solutions

STATE UNIVERSITY OF NEW YORK AT STONY BROOK (14;7,0,0,0,7,0,0)

Adler, Allan Russell
Complex conjugations of Kuga's varieties
Ehrlich, Paul Ewing
Metric deformations of Ricci and sectional curvature on compact riemannian manifolds
Heisler, David
On the cohomology of modules over the Klein-4 group
Kiefe, Catarina Isabel
On the rationality of the zeta-function of a set definable over a finite field
Poor, Walter Andrew
Some result on non-negatively curved manifolds
Roy, Ranjan
Deformations of Fuchsian groups
Topping, Ira Miles
Free generators and the free differential calculus

Department of Applied Mathematics and Statistics
Bose, Ramendra K.
Realizability theory on groups
Easton, Malcolm
Spanning vectors of a subspace of R^n
Goldstein, Jacobo D.
Absolute continuity of measures of Hilbert-space-valued additive processes
Ichikawa, Ikira
Differential games in abstract space
Liu, Jason
Manpower scheduling in transportation systems and multiple set covering problems
Narain, Prem
Solutions of linear equations and generalized inverses
Surmont, Johan
Approximate solutions of non-linear problems with inadequate data

OKLAHOMA

OKLAHOMA STATE UNIVERSITY (1;0,1,0,0,0,0,0)

Department of Statistics
Chen, Jin-Tze
Sequential and non-sequential confidence interval of constant width for a simple linear regression mean

TEXAS

UNIVERSITY OF HOUSTON (8;7,0,0,0,1,0,0)

Epps, Billy B.
A classification of continua and confluent transformations
Gibson, William
Stieltjes and Stieltjes-Volterra integral equations
Giese, Robert
Selected topics in addition chains
Giese, Sharon Mae
Measures and integration in Riesz spaces
Houston, Raymond
Cancellative semigroups on manifolds
Oberhoff, Kenneth
Semilattice structures on certain non-metric continua
Quirein, John A.
Measurable transformations and related variational techniques in pattern classification
Whitley, William
Infinite dimensional microbundles

QUERIES

Edited by Wendell H. Fleming

The QUERIES column is published in each issue of these *Notices*. This column welcomes questions from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published conjectures. When appropriate, replies from readers will be edited into a definitive composite answer and published in a subsequent column. All answers received to QUERIES will ultimately be forwarded to the questioner. Consequently, all items submitted for consideration for possible publication in this column should include the name and complete mailing address of the person who is to receive the replies. The queries themselves, and responses to such queries, should be typewritten if at all possible and sent to Professor Wendell H. Fleming, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

QUERIES

52. Claude W. Anderson (Department of Mathematics, University of California, Berkeley, California 94720). I would appreciate receiving any and all references concerning multiply perfect numbers that have been discovered in the twentieth century. There is no complete list that is up to date, and I am currently compiling one.

53. Gerald A. Edgar (Department of Mathematics Northwestern University, College of Arts and Sciences, Evanston, Illinois 60201). I have been shown an ingenious proof of the Theorem: Let G be a group, and let d be a complete metric on G with respect to which group multiplication is separately continuous. Then G is a topological group. This theorem was stated by Banach, but his proof was correct only in the separable case. Many similar results have been proved since that time (see p. 889 of Reviews of Papers in Algebraic and Differential Topology, Topological Groups, and Homological Algebra, ed. by N.E. Steenrod). None of these results seems to imply the theorem stated. I have been told that the ingenious proof was buried in a paper published many years ago. Can anyone provide the reference?

54. Kwangil Koh (Department of Mathematics, North Carolina State University, Raleigh, North Carolina 27607). Does any one have an example of two non-isomorphic fields, say K_1 and K_2 such that K_1 is isomorphic to a subfield of K_2 and K_2 is isomorphic to a subfield of K_1?

55. David Shelupsky (Department of Physics, The City College of The City University of New York, New York, New York 10031). According to a theorem of Frobenius if G is a finite group of order n and d is a positive integer dividing n then the number of solutions in G of the equation $x^d = 1$ (1 the group identity) is a multiple of d. Can anyone tell me if the following additional statement is a conjecture or a theorem or false, and if a theorem where a proof can be found: if the number of solutions is equal to d then these form a subgroup of G?

56. Daniel Zwillinger (5700 Arlington Avenue, Bronx, New York 10471). The largest prime number known now is $2^{19937} - 1$ (Guinness Book of Records, 1974). What is the largest prime number known not of the form $2^k - 1$ or $2^k + 1$?

57. S. Zaidman (Department of Mathematics, Université de Montréal, Montréal (P.Q.) Canada). I would appreciate any information concerning validity or falsity of the following statement: " $u \equiv 0$ is the only solution with zero boundary values of the equation $\Delta u + A(x,y)\, u = 0$ in a simply connected domain $\mathcal{D}$, provided that $(\text{Area } \mathcal{D}) \cdot \iint_{\mathcal{D}} A^2(x,y)\, dx\, dy \leqq 1$ ".

58. S. Zaidman (Department of Mathematics, Université de Montréal, Montréal (P.Q.) Canada). I would like to ask for a proof (or references) connected with the following inequality (which I found in "An algebra of pseudo-differential operators" by Kohn and Nirenberg, Comm. Pure Appl. Math., 18, No 1/2(1965), 269-305). (Inequalities of this kind appear twice in the paper, essentially in estimate (5.9), §5 and in §9 near the end.) "Let $f(x_1 \ldots x_n)$ be a real-valued function, defined on $R^n - \{0\}$, of class C^p, homogeneous of order zero; then, if $x, x+h \in R^n - \{0\}$, we have

$$\left| f(x+H) - f(x) - \left(\sum_{i=1}^{n} h_i \frac{\partial}{\partial x_i}\right) f(x) - \frac{1}{2!}\left(\sum_{i=1}^{n} h_i \frac{\partial}{\partial x_i}\right)^2 f(x) \ldots - \frac{1}{p!}\left(\sum_{i=1}^{n} h_i \frac{\partial}{\partial x_i}\right)^p f(x) \right| \leqq C \frac{|h|^{p+1}}{(|x+h|+|x|)^{p+1}}$$

Also: what about the best possible constant C?

59. V. Gopala Rao (Department of Mathematics, Lehigh University, Bethlehem, Pennsylvania 18015). I am interested in obtaining a copy of the lectures and articles presented at the 1974 Summer Institute (proceedings of which will not be published) on inverse problems. Would the people who have participated in the institute please respond?

RESPONSES TO QUERIES

Replies have been received to queries published in recent issues of these *Notices*, as follows:

26. (Small, Oct. 1973) The query sought a simple algebraic proof of the decomposition $H = BB^*$ in the notation below with a, b, c, d integers, a, b positive, $\det(A) = 1$, B with entries in $\mathbb{Z}[i]$. Previous responses appear in the April, June, and August 1974 *Notices*. V.C. Nanda communicated the following solution: "The following argument which I believe must be in the literature somewhere, does not use the theory of quadratic forms over $\mathbb{Z}$. Since

$$H = \begin{pmatrix} a & c+id \\ c-id & b \end{pmatrix}$$

is the matrix of a positive definite hermitian form which represents integers in $\mathbb{Z}$, it represents the smallest number, min H. Moreover the column X such that $X^* HX = \min H$ must be primitive and can therefore be completed to a unimodular matrix U. The query is therefore equivalent to: Can U^*HU be written as B^*B?

We may thus assume without loss of generality that $a = \min H$. Now

$$H = \begin{pmatrix} 1 & 0 \\ a^{-1}(c-id) & 1 \end{pmatrix} \begin{pmatrix} a & 0 \\ 0 & e \end{pmatrix} \begin{pmatrix} 1 & a^{-1}(c+id) \\ 0 & 1 \end{pmatrix} .$$

Thus it suffices to prove that $a = 1$ (since $ae = |H| = 1$). Consider

$$(\bar{x}\ 1)\, H \begin{pmatrix} x \\ 1 \end{pmatrix} = a\,|x + a^{-1}(c + id)|^2 + e.$$

Now choose a Gaussian integer $x = x_1 + ix_2$ in such a way that both $x_1 + a^{-1}c$ and $x_2 + a^{-1}d$ lie between $-1/2$ and $+1/2$. Thus since a is the minimum (and $e = a^{-1}$), $a \leqq a(1/4 + 1/4) + a^{-1} \Rightarrow a^2 \leqq 2 \Rightarrow a = 1$."

36. (Cunningham, April 1974) J. Lambek provided the following references:
(1) B. Brainerd, Introduction to the mathematics of language study, Elsevier, New York, 1971.
(2) M. Gross, Mathematical models in linguistics, Prentice-Hall, Englewood Cliffs, 1972.
(3) R. Jacobson (editor), Structure of language and its mathematical aspects, Proc. Symposia in Applied Math., Vol. XII, Amer. Math. Soc., Providence, 1961.
(4) S. Marcus, Algebraic linguistics; analytical models, Academic Press, New York, 1967.

43. (Shelupsky, August 1974) D. Mauldin responded as follows:
" In addition to the responses given in the October 1974 *Notices*, I would like to add my own paper, A representation theorem for the second dual of $C[0,1]$, Studia. Math 46(1973), 198-200.
" In this paper matters are considered as follows: let us consider an element of the bidual of $C(S)$ to be a linear functional T on $ca(S,\Sigma)$, where $ca(S,\Sigma)$ is the space of all regular Borel measures defined on the σ-algebra Σ of all Borel subsets of S.
"Then, assuming the continuum hypothesis and $|ca(S,\Sigma)| \leqq 2^{\aleph_0}$, the functional T may be represented as: $T(\psi) = \int_S \psi\, d\mu$, where ψ is a function from Σ into R.
"I would also like to point out that generalization of this result will appear in a paper of mine entitled "The continuum hypothesis, integration and duals of measure spaces" in Illinois J. of Math.
"Finally, similar representations can be obtained using Martin's Axiom or the assumption that $2^{\aleph_0}$ is a real-valued measurable cardinal."

47. (Hayes, October 1974) R. Cowen responded as follows. "Both results (1) 'The product of finite topological spaces is compact' and (2) 'A power of a finite space is compact' quoted by Hayes are equivalent to the prime ideal theorem, P.I., which is known to be weaker than the axiom of choice. E.S. Wolk, "On theorems of Tychonoff, Alexander and R. Rado", Proc. AMS 18(1967), 113-115, has shown that in the presence of ACF, the axiom of choice for families of finite sets, (2) is equivalent to a combinatorial theorem of R. Rado. R. Cowen, "Some combinatorial theorems equivalent to the prime ideal theorem", Proc. AMS 1(1973), 268-273, has shown that P.I. implies Rado's theorem and it is well known that P.I. implies ACF(e.g. see Los and Nardzewski, "Effectiveness of the representation theory for Boolean algebras," Fund. Math. XLI, 49-56, where this is stated although not proved). Therefore P.I. implies (1) implies (2).
"On the other hand, J. Mycielski, "Two remarks on Tychonoff's product theorem", Bull. Acad. Polon. Sci. 12(1964), 439-441, proves that P.I. is equivalent to the special case of (2) where the finite space equals $\{0,1\}$."

51. (Heller and Beyer, October 1974) The query was as follows: Suppose $\sum_{n=0}^{\infty} a_n z^n$ has radius of convergence R, and $r_N(z) \equiv |\sum_{n=N}^{\infty} a_n z^n|$. With $|z_1| < |z_2| < R$, is it possible that $r_N(z_1) > r_N(z)$ for all z in some neighborhood of z_2, (a) for infinitely many values of N? (b) for all N except a finite number? G. H. Fricke responded as follows: (a) is possible for any radius of convergence including infinity and (b) is impossible for any radius of convergence including infinity even for a single point $z = z_2$. Fricke also supplied proofs of these statements.

Problems in Commutative Harmonic Analysis No. 16 (K.A. Ross, August 1973) Ross has supplied the following information: "The answer is 'yes,' and is an immediate consequence of a theorem of Bonami. She has not published the theorem, as far as I know, but it is referred to by Déchamps-Gondim in Remarque 6.2., Ensembles de Sidon topologiques, Ann. Inst. Fourier (Grenoble) 22, fasc. 3(1972), 51-79. The result will be amply treated in the forthcoming Sidon Sets, by López and Ross."

PERSONAL ITEMS

RICHARD J. BAGBY of New Mexico State University will be on sabbatical leave during the year 1974–1975 at Washington University, St. Louis.

EMMANUEL N. BARRON of Northwestern University has been appointed to an assistant professorship at Georgia Institute of Technology.

EDWIN F. BECKENBACH of the University of California, Los Angeles, has retired with the title of Professor Emeritus.

DOROTHY L. BERNSTEIN on sabbatical leave from Goucher College will spend the winter and spring quarters at the University of Tennessee.

Z. SAMUEL BERNSTEIN of Northwestern University has been appointed to an assistant professorship at Georgia Institute of Technology.

ELDON C. BOES of New Mexico State University will be on sabbatical leave during the year 1974–1975 at the Sandia Laboratories in Albuquerque.

ARMAND BOREL of the Institute for Advanced Study has been appointed to a visiting professorship at the University of California, Berkeley.

ITSHAK BOROSH has been appointed to an assistant professorship at Texas A & M University.

DAVID T. BROWN of the University of Pittsburgh has been appointed to an assistant professorship at Bethany College.

WILLIAM J. BROWNING of Purdue University has accepted a position as associate with Daniel H. Wagner, Associates.

ELOISE H. CARLTON of the University of Colorado has been appointed to a lecturership at the University of California, Berkeley.

MOU-HSIUNG CHANG of the University of Rhode Island has been appointed to an assistant professorship at the University of Alabama in Huntsville.

CHARLES E. CLEAVER of Kent State University has been appointed to a visiting associate professorship at Georgia Institute of Technology.

PAUL E. COHEN of the University of Tennessee and the Institute for Advanced Study has been appointed to an assistant professorship at Lehigh University.

DONALD M. DAVIS of Northwestern University has been appointed to an assistant professorship at Lehigh University.

JOHN DE PREE of New Mexico State University will be on sabbatical leave during the year 1974–1975 at the University of British Columbia.

EDWARD EFFROS of the University of Pennsylvania has been appointed to a visiting professorship at the University of California, Berkeley.

EDWARD F. EISENBERG of Cornell University has been appointed to a visiting assistant professorship at Vassar College.

RICHARD ELDERKIN of Brown University has been appointed to an assistant professorship at Pomona College.

JOHN A. FEROE of the University of California, San Diego, has been appointed to an assistant professorship at Vassar College.

ROBERT J. FRANK of Philadelphia, Pennsylvania has been named a lecturer in Lehigh University's department of mathematics.

EVARIST GINE-MASDEU of the Universidad Metropolitana, Caracas, has been appointed to a lecturership at the University of California, Berkeley.

SANDY GRABINER of the Claremont Graduate School has been appointed to an associate professorship at Pomona College.

WILLIAM L. GREEN of the University of Oslo has been appointed to an assistant professorship at Georgia Institute of Technology.

DONALD LEE GREENWELL of Emory University has been appointed to an assistant professorship at Iowa State University of Science and Technology.

LEO HARRINGTON of SUNY at Buffalo has been appointed to a lecturership at the University of California, Berkeley.

C. WARD HENSON of Duke University has been appointed to an associate professorship at New Mexico State University.

JOHN HOLTE of the University of Wisconsin has been appointed to a visiting assistant professorship at Rensselaer Polytechnic Institute.

JOHN T. HURT has been named Professor Emeritus at Texas A & M University.

WILLIAM F. KEIGHER of the University of Illinois has been appointed to an assistant professorship at the University of Tennessee.

MICHAEL KLASS of the University of California, Los Angeles, has been appointed to a lecturership at the University of California, Berkeley.

MORRIS KLINE of Courant Institute of Mathematical Sciences, New York University, has been appointed visiting distinguished professor at Brooklyn College (CUNY).

ANDRZEJ S. LELEK of the University of Houston has been appointed to a professorship at Wayne State University.

GUNAR LIEPINS of Dartmouth College has been appointed to a lecturership at Texas Tech University.

RAPHAEL LOEWY of the California Institute of Technology has been appointed to an assistant professorship at the University of Tennessee.

JUDITH Q. LONGYEAR of Dartmouth College has been appointed to an assistant professorship at Wayne State University.

WILHELMUS A. J. LUXEMBURG of the California Institute of Technology has been elected a corresponding member of the Royal Academy of Sciences of Amsterdam.

THOMAS MCLAUGHLIN of the University of Illinois has been appointed to an associate professorship at Texas Tech University.

MICHAEL MILLER of the University of California, Los Angeles, has been appointed to a lecturership at the University of California, Berkeley.

WILLIAM F. MOSS of the University of Delaware has been appointed to an assistant professorship at Georgia Institute of Technology.

ARTHUR OGUS of Princeton University has been appointed to an assistant professorship at the University of California, Berkeley.

GEORGE B. PURDY of the University of Illinois has been appointed to an assistant professorship at Texas A & M University.

RONALD S. RIVLIN of Lehigh University has been elected secretary of the Committee of Scientific Society Presidents.

JUDITH ROITMAN of the University of California, Berkeley has been appointed to an assistant professorship at Wellesley College.

RUSSELL J. ROWLETT of Princeton University has been appointed to an assistant professorship at the University of Tennessee.

ARTHUR SAGLE of the University of Minnesota has been appointed to a professorship at the University of Hawaii at Hilo.

BRUCE E. SCRANTON of Purdue University has accepted a position as associate with Daniel H. Wagner Associates.

LEE SEGEL of Rensselaer Polytechnic Institute has been appointed to a professorship and to the chairmanship of the Department of Applied Mathematics at the Weizmann Institute of Science. He will also continue as a research professor at Rensselaer Polytechnic Institute.

ALAN SHUCHAT of Mount Holyoke College has been appointed to an assistant professorship at Wellesley College.

NANCY K. STANTON of the Massachusetts Institute of Technology has been appointed to a lecturership at the University of California, Berkeley.

MICHAEL B. TAMBURRO of the University of California, Los Angeles, has been appointed to an assistant professorship at Georgia Institute of Technology.

WESLEY E. TERRY of the University of Saskatchewan has been appointed to an assistant professorship at Georgia Institute of Technology.

IRVIN VANCE of New Mexico State University is currently on leave at the Educational Development Corporation in Newton, Massachusetts.

JAMES WHITE of the University of California, Los Angeles, has been appointed to a visiting associate professorship at the University of California, Berkeley.

JEFFREY C. WIENER of Emory University has been appointed to an assistant professorship at Georgia Institute of Technology.

PROMOTION

To Provost. Purdue University: FELIX HAAS.

To Vice President for University Planning. St. Mary's University, San Antonio, Texas: JAMES F. GRAY, S. M.

To Assistant Dean in the College of Letters and Science. University of California, Berkeley: H. BLAINE LAWSON.

To Assistant to the President for University Affairs. University of Alabama in Huntsville: F. LEE COOK.

To Coordinator, Department of Applied Science, Red Deer College: VED P. MADAN.

To Vice Chairman for Personnel, Department of Mathematics, University of California, Berkeley: DAVID GALE.

To Chairman. Department of Mathematics, University of Alabama in Huntsville: PETER G. CASAZZA.

To Professor. California State University, Hayward: RANJIT S. SABHARWAL; Pomona College: PAUL B. YALE; Texas A & M University: CHARLES K. CHUI, CARLTON J. MAXSON; University of Alabama in Huntsville: PETER M. GIBSON; University of California, Berkeley: ROBERT C. HARTSHORNE; University of Saskatchewan: BIKKAR S. LALLI; University of Tennessee: DON B. HINTON; Wellesley College: TORSTEN NORVIG.

To Associate Professor. Florida International University: WILLIAM T. KRAYNEK, ANTHONY C. SHERSHIN; New Mexico State University: DAVID M. ARNOLD, RICHARD J. BAGBY; Texas A & M University: JAMES R. BOONE, LAWRENCE F. GUSEMAN, Jr., DARALD J. HARTFIEL; University of California, Berkeley: PAUL R. CHERNOFF; University of Tennessee: GENE A. KLAASEN, CARL G. WAGNER; Wayne State University: DAVID W. JONAH, JINGYAL PAK.

To Assistant Professor. Georgia Institute of Technology: CATHERINE C. AUST, ALAN D. SLOAN.

DEATHS

Dr. CARL B. ALLENDOERFER of the University of Washington died on September 29, 1974, at the age of 63. He was a member of the Society for 39 years.

Professor Emeritus I. ALBERT BARNETT of the University of Cincinnati died on September 27, 1974, at the age of 79. He was a member of the Society for 55 years.

Professor Emeritus RAYMOND W. BRINK of the University of Minnesota died on December 27, 1973, at the age of 83. He was a member of the Society for 55 years.

Professor Emeritus CECILIA K. DUNAJ of the University of Toronto died on August 4, 1974. She was a member of the Society for 40 years.

Professor Emeritus EDWARD S. HAMMOND of Bowdoin College died on March 21, 1972, at the age of 78. He was a member of the Society for 23 years.

Professor Emeritus LOUIS MELVILLE MILNE-THOMPSON died on August 21, 1974 at the age of 83. He was a member of the Society for 17 years.

Vice-President Emeritus WEBSTER G. SIMON of Case Western Reserve University died on August 17, 1974, at the age of 81. He was a member of the Society for 56 years.

Professor Emeritus RALPH S. UNDERWOOD of Texas Tech University died on May 18, 1974, at the age of 82. He was a member of the Society for 42 years.

LETTERS TO THE EDITOR

Editor, the *Notices*

Since the American Mathematical Society plays a major role in the Mathematical Sciences Employment Register, the Society should encourage the use of the Employment Register at the Annual Meetings. The need for this action was apparent at the San Francisco Meeting. Many people attended the meeting in order to find jobs, but there were only about 50 employers who used the Register.

The officers of the Society should encourage their own departments to use the Register at the Annual meetings whenever their departments have a job opening.

Harriet M. Lord

Editor, the *Notices*

The current eloquent letter from Professor Lord about the Employment Register ends in a statement urging certain actions by officers of the Society. This seems to me to be based on a misapprehension. A mathematician, when elected to any Society office, is expected to act in the interests of the Society, but is not expected to press, especially upon his or her own institution, policies which are not addressed to all institutions.

The Employment Register held at each January meeting, provides an organized forum for interviews for positions. Interviews of this character are useful for many positions and wholly inappropriate for other positions in mathematics. For this and other reasons, there is at present no Society policy urging that all available positions be represented by interviews at this Register. Mathematicians searching for positions might bear this in mind. At the same time, the Society Committees and staff responsible for the Register encourage voluntary moves of potential employers to participate in the Register.

It is now (Council meeting, October 25, 1974) a matter of Society policy that all available positions in the mathematical sciences be advertised, insofar as practicable, and that the standard place for the advertisements to appear is the publication, "Employment Information for Mathematicians." This publication (appearing six times each academic year with a weekly summer supplement) can become even more helpful if it is even more complete. I hope that all potential employers will list positions there.

Saunders Mac Lane

With the approbation of the Council of October 25, 1974, President Lipman Bers has presented the following open letter for publication as a Letter to the Editor.

E. P.

We appeal to the world public opinion concerning the threat against the well known Moscow mathematician and philosopher Yuri Gastiev. Gastiev is a "candidate" of philosophical sciences, the author of several dozen papers, a member of several foreign scientific societies. His work is well known in the U.S.S.R. and abroad.

During the past three months Yuri Gastiev was subject to three house searches and he was called in many times for interrogations. He is in danger of being arrested. It is amazing that the formal reason for this harassment is the so-called "File 38" of the KGB in Vladimir about the Russian nationalist journal Veche affair, which, in itself, is groundlessly overblown.

Yuri Gastiev, as a young man, spent five years in a Stalin labor camp after World War II. His father, Alexei Gastiev, a well known worker-revolutionary, scholar and poet, a director of the Central Labor Institute, was shot by the NKVD in 1939. His mother and his brother spent many years in camps and prisons. Another brother was killed at the front fighting the Germans.

Yuri Gastiev has been persecuted since 1968 when he defended the mathematician A. Yessenin-Volpin. Since then he has had to change jobs several times.

We call upon the scientific world public opinion to defend Yuri Gastiev. We appeal in the first place to mathematicians and philosophers.

I. Shafarevich
V. Turchin
M. Ogurski
A. Tverdokhlebov
Y. Orlov
A. Levitin-Krasnov
V. Albrecht
A. Lavut
G. Rosenstein

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates, or geographical area become apparent. An announcement will be published in these *Notices* if it contains a call for papers, place, date, subject (when applicable), and speakers; a second announcement will be published only if changes to the original announcement are necessary, or if it appears that additional information should be announced.

In general, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry slightly more detailed information. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society. Deadlines for particular issues of the *Notices* are the same as the deadlines for abstracts which appear on the inside front cover of each issue.

February 17–20, 1975
SIXTH SOUTHEASTERN CONFERENCE ON COMBINATORICS, GRAPH THEORY AND COMPUTING
Florida Atlantic University, Boca Raton, Florida
Program: Instructional lecture series will be given by Paul Erdös, Hungarian National Academy; Richard Karp, University of California, Berkeley, Ernest E. Shult, Kansas State University, and Richard M. Wilson, Ohio State University.
Contributed papers: There will be fifteen minute sessions for contributed papers throughout the conference. Title and abstract of papers must be submitted by January 21, 1975.
Proceedings: The proceedings of the conference will be refereed, and complete texts must be submitted by May 1, 1975.
Information: Frederick Hoffman or Roy B. Levow, Department of Mathematics, Florida Atlantic University, Boca Raton, Florida 33432

March 3–14, 1975
ADVANCED COURSE ON COMPILER CONSTRUCTION
Technical University of Munich, Munich, Germany
Topics: Compiler Construction, LL(1) and LR Grammars and Analysers, Transformational Grammars, Two-Level Grammars, Semantic Analysis and Synthesis, Run-Time Organization, Symbol Table Access, Code Generation, Compiler-Compilers, Portability and Adaptability, Structuring Compiler Development, Programming Language Design, Optimization, Economics
Deadline for applications: January 17, 1975. Participants will be limited to 80. All applications will receive notice by February 4, 1975.
Information: Mathematisches Institut der Technischen Universität München, Advanced Course, Congress Office-D-8000 München 2, Arcisstrasse 21, Germany

March 10–14, 1975
CONFERENCE ON GEOMETRY
University of Haifa, Israel
Program: Invited addresses and contributed papers dealing with convexity and foundations of geometry. The foundations portion of the conference will be led by Walter Benz of the University of Hamburg.
Information: Department of Mathematics, University of Haifa, Haifa 31999, Israel

March 13–15, 1975
ANNUAL SPRING TOPOLOGY CONFERENCE
Memphis State University, Memphis, Tennessee
Deadline for abstracts: February 14, 1975
Information: M. Rajagopalan, Chairman of the Committee on Topology Conference, Memphis State University, Memphis, Tennessee 38152

March 31–April 2, 1975
ADVANCED SEMINAR ON SPECIAL FUNCTIONS
Mathematics Research Center, University of Wisconsin, Madison, Wisconsin
Program: 13 invited lectures dealing with various properties of special functions and applications to scientific fields. A detailed program of the seminar and information on registration and accommodations will be available about February 1, 1975.
Information: R. Askey, Mathematics Research Center, University of Wisconsin-Madison, 610 Walnut Street, Madison, Wisconsin 53706

April 3–5, 1975
ELEVENTH SYMPOSIUM ON BIOMATHEMATICS AND COMPUTER SCIENCE IN THE LIFE SCIENCES
Marriott Motor Hotel, Houston, Texas
Program: The program will emphasize mathematical, statistical bioengineering and computing applications in biology and medicine.
Abstracts: Abstracts must be submitted before January 1, 1975. Papers of an interdisciplinary nature and appropriate papers on education and communication will be welcomed. Submit abstracts to the program chairman, Birger Jansson, The National Large Bowel Cancer Project, The University of Texas Cancer System at Houston, M.D. Anderson Hospital and Tumor Institute, Prudential Building, Room 1801, Houston, Texas 77025.
Information: Stuart O. Zimmerman, c/o Office of the Director, The University of Texas Health Science Center at Houston, Division of Continuing Education, P.O. Box 20367, Houston, Texas 77025

April 8–12, 1975
TWENTY-SEVENTH BRITISH MATHEMATICAL COLLOQUIUM
University of Newcastle upon Tyne, England
Program: The morning program will be divided into three pairs of concurrent lectures. In the afternoon there will be meetings of splinter groups on various branches of mathematics; members are invited to contribute short papers and there will be opportunity for discussion.
Membership fee: £2. This will be doubled for applications received after February 17, 1975. In addition, all residential members are required to pay a nonreturnable deposit of £3 toward the cost of accommodation.
Information: R.M. White, School of Mathematics, The University, Newcastle upon Tyne, England NE1 7RU

April 18–19, 1975
SECOND ANNUAL MATHEMATICS AND STATISTICS CONFERENCE
Miami University, Oxford, Ohio
Program: The conference will emphasize teaching and applications of statistics. Topics of interest to professional statisticians and college teachers (Friday) and high school teachers (Saturday).
Principal skeaker: R.V. Hogg, University of Iowa
Deadline for abstracts: February 15, 1975.
Information: Vasant B. Waikar, Department of Mathematics and Statistics, Miami University, Oxford, Ohio 45056

April 24–25, 1975
1975 SPRING MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC
Palmer House, Chicago, Illinois
Program: This meeting will be held in conjunction with the annual meeting of the Western Division of the American Philosophical Association, and the program will include a joint symposium on Church's Thesis. The program also will include invited lectures by Anil Nerode and A. H. Lachlan
Speakers: Harvey Friedman, Hilary Putnam, and Joseph Shoenfield.
Deadline for abstracts: February 1, 1975
Information: Robert I. Soare, Department of Mathematics, University of Illinois at Chicago Circle, Chicago, Illinois 60680

June 10–14, 1975
CONFERENCE ON STOCHASTIC SYSTEMS: ANALYSIS, OPTIMIZATION, ESTIMATION, COMPUTATIONS AND APPLICATIONS
University of Kentucky, Lexington, Kentucky
Program: Sessions on stochastic processes, stochastic integration, stochastic optimization modeling, estimation and filtering, identification computations, control of queus, applications to economics, engineering, and population genetics
Sponsor: University of Kentucky Mathematics Department
Information: Raymond Rishel, Department of Mathematics, University of Kentucky, Lexington, Kentucky 40506

July 7–18, 1975
SEMINAR ON MODERN MODELING OF CONTINUUM PHENOMENA
Rensselaer Polytechnic Institute, Troy, New York
Application deadline: March 17, 1975
Information: Gordon L. Walker, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940

July 15–26, 1975
SÉMINAIRE INTERNATIONAL D'ÉTÉ ET COLLOQUE INTERNATIONAL DE LOGIQUE (SÉMINAIRE July 15–26; COLLOQUE July 18–25)
Université de Clermont, Clermont-Ferrand, France
Program: Mathematical Logic
Sponsors: Association for Symbolic Logic, Centre National pour la Recherche Scientifique (Colloque); Université de Clermont (Séminaire)
Contributed papers: Members of the ASL and others sponsored by members may send abstracts of no more than 300 words to Professor Guillaume (see below) by May 1, 1975.
Information: M. Guillaume, Mathématiques Pures, Faculté des Sciences, F63037 Clermont-Ferrand Cédex, France

July 21–August 1, 1975
DURHAM SYMPOSIUM ON CONVEXITY
Program: It is intended that the symposium should study the relationships between various finite dimensional problems and the geometry of Banach spaces.
Support: The Royal Society and the Science Research Council
Information: Dr. D. A. Brannan, Secretary, London Mathematical Society, Queen Elizabeth College, London W8 7AH, England

July 28–August 15, 1975
DISCRETE GROUPS AND AUTOMORPHIC FUNCTIONS
University of Cambridge, England
Speakers: A.F. Beardon, L. Bers, J. Birman, C. Earle, L. Greenberg, J. Lehner, O. Lehto, A.M. Macbeath, A. Marden, B. Maskit, R.A. Rankin, H.P.F. Swinnerton-Dyer
Sponsor: London Mathematical Society
Support: London Mathematical Society, NATO Advanced Study Institute
Information: A.F. Beardon, Department of Pure Mathematics and Mathematical Statistics, 16 Mill Lane, Cambridge CB2 1SB, England

Early August 1975
INTERNATIONAL SYMPOSIUM ON INFINITE DIMENSIONAL HOLOMORPHY
Universidade Estadual de Campinas, Brasil
Information: Mario C. Matos, Instituto de Matematica, Universidade Estadual de Campinas, Caixa Postal 1170, 13100 Campinas, S.P., Brasil

August 27–29, 1975
FOURTH AUSTRALIAN CONFERENCE ON COMBINATORIAL MATHEMATICS
University of Adelaide, Adelaide, South Australia
Information: L.R.A. Casse, Department of Pure Mathematics, University of Adelaide, Adelaide, 5001, South Australia

VISITING MATHEMATICIANS
Supplementary List

American and Canadian Mathematicians Visiting Abroad

Name and Home Country	Host Institution	Field of Special Interest	Period of Visit
Gasper, George (U.S.A.)	Technische Hochschule Aachen, Fed. Rep. of Germany	Analysis and Special Functions	10/74 - 6/75
Goldstein, Jerome A. (U.S.A.)	Universidade de Brasilia Universidade Federal do Rio de Janeiro	Differential Equations	1/75 - 6/75 7/75 - 8/75
McAllister, Gregory T. (U.S.A.)	University of Florence	Partial Differential Equations	1/75 - 8/75

Foreign Mathematicians Visiting in the United States

Aarts, Johannes (Netherlands)	Carnegie-Mellon University	Topology	1/75
Behzad, Mehdi (Iran)	Michigan State University	Graph Theory, Mathematics Teaching Methodology	9/74 - 6/75
Bravo, Raul (Chile)	University of California, Berkeley	Algebra	9/74 - 6/75
Ciobanu, Gheorghe (Romania)	Johns Hopkins University	Mechanics of Continua, Theory of Viscoelasticity	9/74 - 6/75
Dembinski, Piotr (Poland)	University of California, Los Angeles	Mathematical Foundations of Computer Science	9/74 - 6/75
Diederich, Klas (Germany)	Princeton University	Complex Analysis	8/74 - 7/75
Gladysz, Stanislaw (Poland)	Texas Tech University	Probability, Ergodic Theory	12/74 - 2/75
Glod, Florica (Romania)	Stanford University	Modern Methods of Teaching Mathematics and Fluid Mechanics	9/74 - 6/75
Gori, Francesco (Italy)	University of California, Berkeley	Statistical Methodology	11/74 - 5/75
Grassman, Johan (Holland)	Rensselaer Polytechnic Institute	Applied Mathematics, Singular Pertubation Theory	8/74 - 8/75
Kuku, A.O. (Nigeria)	University of Chicago	Algebraic K-theory	10/74 - 10/75
Magidor, Menachem (Israel)	University of California, Berkeley	Mathematical Analysis, Algebra	7/73 - 6/75
Manselli, Paolo (Italy)	University of California, Berkeley	Applied Mathematics	9/74 - 7/75
Marcus, Morel M. (Israel)	Carnegie-Mellon University	Functional Analysis, Variational Problems	9/74 - 6/75
Nield, Donald Arthur (New Zealand)	University of Minnesota	Mathematics Applied to Fluid Mechanics	1/75 - 6/75
O'Brian, Nigel Robert (United Kingdom)	Institute for Advanced Study	Complex Analysis, Algebraic Geometry	9/74 - 6/75
Orman, Gabriel (Romania)	University of Pennsylvania	Formal Languages, Automata Theory	9/74 - 6/75
Smyrl, Joseph Logan (United Kingdom)	University of Maryland	Undergraduate Mathematics, Teaching Methodology	8/74 - 6/75
Syrjanen, Markku (Finland)	Stanford University	Documentation of Information Systems	9/74 - 6/75
Uziemblo, Bogdan (Poland)	Carnegie-Mellon University	Mechanics-Thermodynamics	10/74 - 6/75
Vinner, Shlomo (Israel)	University of California, Berkeley	Logic	9/74 - 8/75

NEW AMS PUBLICATIONS

SIAM-AMS PROCEEDINGS

COMPLEXITY OF COMPUTATION, edited by Richard M. Karp

Volume VII
180 pages; list price $14.20; member price $10.65
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This volume contains the proceedings of a symposium in applied mathematics on Complexity of Computation held in New York City on April 18–19, 1973. The symposium was cosponsored by the American Mathematical Society and the Society for Industrial and Applied Mathematics.

During the last decade computational complexity has become one of the most active research areas within the mathematical theory of computation. Workers in computational complexity seek to derive efficient algorithms for computational problems of practical interest, to prove the optimality of particular algorithms relative to well-defined measures of computational efficiency, and to derive general lower bounds on the time or space intrinsically necessary for the performance of computational tasks. The specific problems considered are drawn from diverse areas, including numerical computation, symbolic algebraic computation, combinatorics, computational logic and the manipulation of data structures. The mathematical tools called upon range from algebraic geometry to computability theory.

Two of the papers in the present volume concern the relation between deterministic and nondeterministic computing devices. J. Hartmanis and H.B. Hunt III in "The LBA problem and its importance in the theory of computing" discuss the so-called LBA problem—whether nondeterministic Turing machines operating within linear space have more power as recognizers than deterministic linear-space Turing machines. While the problem remains unsolved, this paper reduces it to determining whether certain specific decision problems can be solved deterministically in linear space; one such decision problem is the equivalence of regular expressions. The paper by Ronald Fagin, "Generalized first-order spectra and polynomial-time recognizable sets", defines generalized spectra and shows that they are essentially coextensive with languages recognized by nondeterministic Turing machines operating in polynomial time. Using a mixture of techniques from logic and automata theory, he exhibits specific "complete" generalized spectra. These results are motivated in part by the open $\mathcal{P}$ vs. $N\mathcal{P}$ question: whether nondeterministic Turing machines operating in polynomial time can recognize languages not recognizable by deterministic polynomial-time Turing machines.

Michael J. Fischer and Michael O. Rabin in their paper, "Super-exponential complexity of Presburger arithmetic," derive lower bounds on the computational complexity of the decision problem and on the inherent length of proofs for two classical decidable theories of logic: the first-order theory of the real numbers under addition, and Presburger arithmetic. Their results may be construed as implying that no decision procedure for either of these theories can be guaranteed to operate within a practical time bound, and no proof format can guarantee proofs of convenient length.

The papers by S.O. Aanderaa, "On k-tape versus (k - 1)-tape real time computation," and by Michael S. Paterson, Fischer and Albert R. Meyer, "An improved overlap argument for on-line multiplication," employ an "overlap" technique for deriving lower bounds on computing time. Aanderaa settles a long-standing conjecture by showing that, for real-time computation, k-tape Turing machines are more powerful than (k - 1)-tape Turing machines, for all $k \geqq 2$. Paterson, Fischer and Meyer derive new lower bounds on the time required for the multiplication of numbers by on-line multi-tape Turing machines.

The paper by Fischer and Paterson, "String-matching and other products," exploits a formal similarity between pattern matching in strings and integer multiplication to derive a new, asymptotically efficient algorithm for a version of the former problem.

The last three papers in the volume study the computational complexity of fundamental processes in numerical computation. W.M. Gentleman and S.C. Johnson show that no method of expanding a determinant by minors requires fewer multiplications than expanding by column minors in their paper "The evaluation of determinants by expansion by minors and the general problem of substitution." Martin H. Schultz in his paper on "The complexity on linear approximation algorithms" studies the worst-case error bounds that can be achieved by certain linear approximations to continuous functions. H.T. Kung and J.F. Traub in "Computational complexity of one-point and multi-point iterations" derive lower and upper bounds on the efficiency of iterative algorithms for approximating real numbers. Author and subject indexes are provided at the end of the volume.

MATHEMATICAL ASPECTS OF CHEMICAL AND BIOCHEMICAL PROBLEMS AND QUANTUM CHEMISTRY, edited by Donald S. Cohen

Volume VIII
156 pages; list price $13.80; member price $10.35

ISBN 0-8218-1328-5
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This volume contains lectures presented at the symposium on Mathematical Aspects of Various Problems in Chemical and Biochemical Reactions, Chemical Reactor Theory, and Quantum Chemistry, held in New York City, April 10-11, 1974.

The aim of the symposium was to identify some currently active problems arising within the general fields of chemical reactor theory, chemical reaction theory, quantum chemistry and biochemical problems bringing mathematical tools to bear for the synthesis and solution of these practical problems and, of equal importance, generating new mathematical ideas and points of view which arise from the physical problems.

The titles of the papers appearing in this volume and the names of their authors follow: "Nonlinear problems in chemical reactor theory" by Neal R. Amundson, "The dynamics of open reaction systems" by F. Horn, "Wave trains, shock fronts, and transition layers in reaction-diffusion equations" by L. N. Howard and N. Kopell, "Semi classical theory of atomic and molecular excitation and dissociation" by James C. Keck, "Tubular chemical reactors with recycle" by Herbert B. Keller, "Patterns of spatio-temporal organization in chemical and biochemical kinetics" by G. Nicolis, "An analysis of the counter-current moving bed reactor" by Subbaraman Viswanathan and Rutherford Aris, and "Rotating solutions to reaction/diffusion equations in simply-connected media" by Arthur T. Winfree.

The volume is both expository and of current research interest. The reader should know some advanced differential equations (including asymptotic methods) and have some background in deriving the equations of motion for some aspect of continuum mechanics.

LECTURES ON MATHEMATICS IN THE LIFE SCIENCES

SOME MATHEMATICAL QUESTIONS IN BIOLOGY. VI, edited by Simon A. Levin

Volume 7
232 pages; list price $20.00; member price $15.00

ISBN 0-8218-1157-6
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This volume contains lectures given at the eighth symposium on Some Mathematical Questions in Biology, held in San Francisco on February 25-26, 1974, in conjunction with the annual meeting of the American Association for the Advancement of Science. The symposium was co-sponsored by the American Mathematical Society and by the Society for Industrial and Applied Mathematics under the auspices of Section A, Mathematics, of the AAAS.

The first two papers in this volume, "Some models of allelic mutation in molecular population genetics" by Motoo Kimura and "Mathematical and statistical problems arising in the non-Darwinian theory" by Warren Ewens, argue two sides of the continuing debate over the relative importance of selection and chance in evolution. Random processes, first stressed by Sewall Wright, are regarded as predominant by those who view evolution at the molecular level, and form the basis for the neutral gene theory as developed largely by Kimura. On the other hand, those (such as ecological geneticists) who prefer to deal at the level of the whole organism are consistently impressed by the adaptedness of organisms to specific environmental conditions, and tend to regard as unimportant most point mutations. For them, selection is the essential force in evolution.

The third paper, "Competition, selection and evolution in chemical networks" by John J. Tyson, is an extension of evolutionary reasoning to biological macromolecules, and represents the work of Manfred Eigen, Tyson, and other co-workers. These ideas are in a very early stage of development and, as Tyson states, "the theory would benefit by closer contact with theoretical ecology and population genetics." The paper interfaces not only the preceding two papers, but also the fourth paper, an elegant treatise by E. C. Zeeman, "Primary and secondary waves in developmental biology," which is an excellent exposition of some applications of the catastrophe theory of René Thom.

The question of the development of pattern, one of the fundamental questions in theoretical biology, has received attention in previous volumes in this series. It is the focus not only of Zeeman's paper, but also of three other contributions: "Biological pattern formation involving lateral inhibition" by Alfred Gierer and Hans Meinhardt, "A diffusion model of pattern formation in the insect cuticle" by Stephen Blomfield, and "Pattern formation in the Belousov reaction" by Nancy Kopell and L. N. Howard. Gierer and Meinhardt, emphasizing the approach of Turing to pattern formation, concentrate on the problem of regeneration in Hydra. Patterns arise, in the theory, through the interplay of short-range activation and long-range inhibition. In the paper of Stephen Blomfield, which relates to joint work with Peter Lawrence, the patterns of interest are in the cuticle of the insect Rhodnius, and a simple gradient model is employed. The emphasis in both papers on diffusing morphogens embedded in reaction schemes sets the stage for the paper of Kopell and Howard, which examines in detail a somewhat different aspect of diffusion-reaction systems, the existence of oscillating solutions.

In the final paper in the volume, "On the geometry of color perception," H. L. Resnikoff presents some aspects of the mathematical theory of color perception. The techniques are from differential geometry; the basic axioms a "distillation of results" of Maxwell, Grassmann,

Helmholtz, and others.

This volume is expository in nature. The background of the reader should include some knowledge of ordinary and partial differential equations.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

PIECEWISE LINEAR CONCORDANCES AND ISOTOPIES by Kenneth C. Millet

Number 153
74 pages; list price $3.10; member price $2.33
ISBN 0-8218-1853-8
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In this Memoir several geometrical techniques which have had substantial application in piecewise linear topology are extended to the situation where one has a projection which must be preserved. These include a new refinement of the general position property, engulfing and two sunny collapsing procedures. They are then employed to study the relative homotopy groups of the simplicial spaces of concordances and isotopies and, consequently, give a proof of a parameterized version of Hudson's "concordance implies isotopy" theorem.

PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS

PROCEEDINGS OF THE TARSKI SYMPOSIUM, edited by Leon Henkin with John Addison, C.C. Chang, William Craig, Dana Scott, and Robert Vaught.

Volume XXV
498 pages; list price $40.00; members of AMS and ASL $30.00
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This volume contains the proceedings of the Tarski Symposium held at the University of California, Berkeley, June 23-30, 1971. The Symposium was co-sponsored by the University of California, Berkeley; the Association of Symbolic Logic; and the International Union for History and Philosophy of Science–Division of Logic, Methodology and Philosophy of Science. The volume is intended for mathematicians and philosophers with some background of understanding in one or more areas of foundations of mathematics or philosophy of science.

Volume 25 is the first volume in a subseries of PROCEEDINGS OF SYMPOSIA IN PURE MATHEMATICS. The subseries is published for the Association of Symbolic Logic by the American Mathematical Society. Standing orders are being accepted for the subseries alone as well as for the whole series.

The volume is dedicated to Alfred Tarski, known to four generations of logicians and students as a scholar of extraordinary breadth and depth. His influence on the development of foundational studies in logic, mathematics, and the philosophy of science is due not only to his own investigations and numerous writings, but also to his influence as a teacher and a source of energy and organization in the international scientific community during the past half century. The Symposium papers deal with the development and current state of research in the various areas in which Tarski has worked.

The arrangement of papers within this volume may be described roughly as an ordering which moves from algebraic to set-theoretical aspects of the foundations of mathematics, then proceeds to philosophical questions, and finally arrives at deductive aspects of empirical sciences. A more detailed description now follows.

The volume begins with Jónsson's survey of recent work in the general theory of algebraic structures. Gaifman next deals metamathematically with general operations on algebraic structures. The Birkhoff/Lipson paper deals with structures having several domains of elements, there follow papers by McKenzie/Shelah, Hanf, Craig, and Henkin/Monk, dealing with special classes of algebraic structures, and then come papers by Szmielew and Schwabhäuser/Szczerba on geometric structures. A re-examination of the decision problem for elementary algebra and geometry, by A. Robinson, then follows.

Several papers on model theory are introduced by an historical survey, split at the year 1945 between the papers of Vaught and Chang. Shelahs's paper deals with a problem of first-order model theory, while Feferman's is concerned with a variety of languages and those of Karp, Morley, and Fraïssé are involved in part or in whole with models of infinitary languages. Eršov deals with models of equational logic, and Ehrenfeucht deals with highly non-classical logics arising from the ultra-intuitionistic studies of Essenin-Volpin.

The first of a series of papers on set theory is a survey of problems and results by Erdös/Hajnal. Then comes axiomatic studies by Bernays, Church, and Levy, followed by examinations of models of set theories by Cohen, Keisler, Mostowski, Silver, and Solovay.

Philosophical papers by Quine, Mates, and Popper are focused on Tarski's theory of truth, and then Adams, Jan Tarski, and Suppes deal with the employment of the methodology of deductive sciences in empirical theories. The last paper is a personal tribute to Tarski by Woodger, to whom all readers of this volume are grateful for his translations of Tarski's early papers as well as for his own foundational research.

The volume closes with a list of Tarski's doctoral students, and a bibliography of his published work to date, more than one hundred fifty references.

The titles of the papers in this volume and their authors follow: "Some recent trends in general algebra" by B. Jónsson, "Operations on relational structures, functors and classes. I" by H. Gaifman, "Universal algebra and automata" by G. Birkhoff and J.D. Lipson, "The cardinals of simple models for universal theories" by R. McKenzie and S. Shelah, "Primitive Boolean algebras" by W. Hanf, "Diagonal relations" by W. Craig, "Cylindric algebras and related

structures" by L. Henkin and J.D. Monk, "The role of the Pasch axiom in the foundations of Euclidean geometry" by W. Szmielew, "An affine space as union of spaces of higher dimension" by W. Schwabhäuser and Ł. Szczerba, "A decision method for elementary algebra and geometry—revisited" by A. Robinson, "Model theory before 1945" by R.L. Vaught, "Model theory 1945-1971" by C.C. Chang, "Categoricity of uncountable theories" by S. Shelah, "Applications of many-sorted interpolation theorems" by S. Feferman, "Infinite-quantifier languages and ω-chains of models" by C. Karp, "Applications of topology to $L_{\omega,\omega}$" by M. Morley, "Isomorphisme locale et équivalence associés à un ordinal; utilité en calcul des formules infinies à quanteurs finis" by R. Fraïssé, "Theories of nonabelian varieties of groups" by Ju. L. Eršov, "Logic without iterations" by A. Ehrenfeucht, "Unsolved and solved problems in set theory" by P. Erdös and A. Hajnal, "Zu den Maximalprinzipien der Mengenlehre" by P. Bernays, "Set theory with a universal set" by A. Church, "Parameters in comprehension axiom schemas of set theory" by A. Levy, "Automorphisms of set theory" by P. Cohen, "Models with tree structures" by H.J. Keisler, "Observations concerning elementary extensions of ω-models" by A. Mostowski, "Indecomposable ultrafilters and $0^{\#}$" by J. Silver, "Strongly compact cardinals and the GCH" by R. Solovay, "Truth and disquotation" by W.V. Quine, "Austin, Strawson, Tarski, and truth" by B. Mates, "Some philosophical comments on Tarski's theory of truth" by K. Popper, "Completeness and axiomatizability in many-valued logic" by D. Scott, "Model-theoretic aspects of fundamental measurement theory" by E. Adams, "Quantum field theory: an unusual discipline" by J. Tarski, "The axiomatic method in the empirical sciences" by P. Suppes, and "Thank you, Alfred" by J.H. Woodger.

TRANSACTIONS OF THE MOSCOW MATHEMATICAL SOCIETY

Volume 24 (1971)
247 pages; list price $35.40; member price $26.55
ISBN 0-8218-1624-1
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This volume contains the transactions of the Moscow Mathematical Society for the year 1971. The translation was prepared jointly by the American Mathematical Society and the London Mathematical Society.

The authors and the titles of their papers follow: "Canonical transformations and pseudodifferential operators," by Ju. V. Egorov; "On necessary conditions for solvability of pseudodifferential equations of principal type," by Ju. V. Egorov; "Pseudodifferential operators with holomorphic symbols and Gevrey classes," by L.R. Volevič; "Spaces defined by means of local approximations," by Ju. A. Brudnyĭ; "Generalized functions and differential equations in linear spaces. I. Differentiable measures," by V.I. Averbuh, O.G. Smoljanov, and S.V. Fomin; "The matrix method for continuous systems in classical statistical mechanics," by Ju. M. Suhov; and "o-metrizable spaces," by S. I. Nedev.

NEWS ITEMS AND ANNOUNCEMENTS

COMPUTER SCIENCE EMPLOYMENT REGISTER

The Third Annual Computer Science Employment Register will be conducted at the Washington, D.C. Computer Science Conference, February 18–20, 1975. This unique Register aids in matching computer scientists and data processing specialists with potential employers. The Conference is sponsored by the Association for Computing Machinery, the Computer Science Departments of many universities, and a number of industrial organizations. Previous Registers have attracted thousands of applicants and hundreds of employers from all parts of the United States as well as a number of foreign participants. In the 1973 Register the Eastern States accounted for 40% of the listings, Central States 41%, Western States 15% and foreign 4%.

Application deadline is February 3, 1975. Both applicants and employers must file their registration on official forms which may be obtained from and returned to Orrin E. Taulbee, Computer Science Employment Register, Department of Computer Science, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. Employers should request one form for each type of position available (only one form is needed in the case of several identical positions). Employers may use this opportunity to list summer or temporary positions. Forms must be typewritten since they will be reproduced exactly as submitted. Photocopies will not be accepted.

NOMINATING COMMITTEE FOR 1975

The Chairman of the Nominating Committee for 1975 is Professor Frederick W. Gehring, whose address is Department of Mathematics, University of Michigan, Ann Arbor, Michigan 48104. The other members of the committee are J. J. Kohn, Kevin M. McCrimmon, Mary E. Rudin, and John Wehausen.

The Nominating Committee will be glad to consider suggestions from the membership. Among the information already available to them is a tabulation of the write-in votes in the 1974 election and a tabulation of the suggestions returned on the "pink sheet" with the 1974 ballot. Such lists usually contain on the order of one thousand names.

ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred.
Invited addresses are indicated by •

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the *Notices* but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

75T-A1 JOYCE LONGMAN, Villanova University, Villanova, Pennsylvania 19085
Scalar Dependent Algebras in the Alternative Sense. Preliminary report.

An algebra A over a field F of characteristic $\neq 2,3$, will be called scalar dependent in the alternative sense if there exists a map $g: A\times A\times A \to F$ such that for every x,y,z in A (1) $(xy)z + (yx)z = g(x,y,z)\ [x(yz) + y(xz)]$ and (2) $x(yz) + z(yx) = g(x,y,z)\ [(xy)z + (zy)x]$.

Theorem 1. Every scalar dependent algebra in the alternative sense with identity element is alternative.

Theorem 2. If A is scalar dependent in the alternative sense and contains a nonzero idempotent e then A is alternative.

These results generalize those of Coughlin, Kleinfeld, and Rich (Proceedings) AMS, V39, No 1, June 1973, pp 69-72. (Received September 23, 1974.)

75T-A2 E.M. WRIGHT, University of Aberdeen, Aberdeen, U.K.
The proportion of unlabelled graphs which are Hamiltonian.

An (n,q) graph has n nodes and q edges, no slings and no multiple edges. We write $N = \frac{1}{2}n(n-1)$. Let $\rho = \rho(n,q)$ (resp. $\sigma = \sigma(n,q)$) be the proportion of labelled (resp. unlabelled) (n,q) graphs which are Hamiltonian. It is obvious that, for fixed n, the proportion ρ increases (in the non-strict sense) as q increases. On the other hand, σ can decrease as q increases; in particular, for any fixed s, we have $\sigma(n,q-1) > \sigma(n,q)$ when $N-n-s \leqslant q \leqslant N-n+2$ and $n > n_0(s)$, but $\sigma = 1$, if $q \geqslant N-n+3$. However, if q is any function of n, we have $\rho - \sigma \to 0$ as $n \to \infty$. (Received September 23, 1974.)

75T-A3 JUTTA STRECKER, 56 Wuppertal, Hofkamp 79, W-Germany. Cancellation for free products of bounded distributive lattices, Preliminary report.

Suppose L, L_1, L_2, L_3 are bounded distributive lattices and L_1, L_2, L_3 are sublattices of L. $L = L_1 * L_2$ if $L_1 \cup L_2$ generates L and L_1 and L_2 are independent. Theorem I. Each $x \in L_1 * L_2$ can be written in the form $x = \sum_{i=1}^{n} (a_i \cdot b_i)$ where $a_i \in L_1 - \{o\}$, $b_i \in L_2$ and for any $\{i,k\}$, $N \subseteq \{1,\ldots,n\}$ the following properties hold: (1) $a_i \leq a_k$ iff $b_k \leq b_i$, (2) $a_k \cdot b_k \leq \sum_{i \in N} (a_i \cdot b_i)$ implies $N = \emptyset$ or $k \in N$, and (3) b_k is minimal in $\{b_i \mid i \in N\}$ implies $a_k \nleq \sum_{i \in N}^{i \neq k} (a_i)$. The number n and the coefficients b_i are uniquely determined by (1)-(3). Uniqueness of the coefficients a_i is accomplished if the b_i's satisfy certain conditions. Theorem II. $L_1 * L_2 = L_1 * L_3$ implies that any $x \in L_2$ can be expressed as $x = \sum_{i=1}^{n} (a_i \cdot f_i(b))$ where $\{a_i \mid i = 1, \ldots, n\}$ is a partition of the unit in L_1, $b \in L_3$ and f_i, $i=1,\ldots,n$, are automorphisms of L_3. Theorem II and a result of Comer (AMS Notices, Abstract 711-06-10, 21(1974), A-44) show that the following two conditions are equivalent: (A) L_1 has no non-trivial complemented element and (B) $L_1 * L_2 = L_1 * L_3$ implies $L_2 = L_3$ for all L_2 and L_3. Theorem II also yields algebraic proofs for (C) $L_2 \simeq L_3$ whenever $L_1 * L_2 = L_1 * L_3$ and (D) $L_2 = L_3$ if $L_1 * L_2 = L_1 * L_3$ and $|\mathrm{Aut} L_3| = 1$.
(Received September 23, 1974.) (Author introduced by Professor S.D.Comer.)

*75T-A4 TREVOR EVANS, Emory University, Atlanta, Georgia 30322. The Construction of Orthogonal k-Skeins and Latin k-Cubes.

A k-skein is the k-dimensional analogue of a quasigroup and in the finite case (order n), its operation table consists of a k-dimensional latin cube based on $\{1, 2, \ldots, n\}$. A set of k latin k-cubes of order n is orthogonal, if when superimposed, each of the n^k different k-tuples from $\{1, 2, \ldots, n\}$ occurs exactly once in the resulting array. A number of procedures are given for constructing k-sets of orthogonal k-skeins (or latin k-cubes) one of which leads to the construction, for every $k > 2$, of a k-set of orthogonal latin k-cubes of every order $n \neq 2, 6$. This last result has also been obtained by J. Arkin and E. G. Straus (Latin k-cubes, Fibonacci Quarterly, to appear). (Received September 24, 1974.)

*75T-A5 Michael Rich, Temple University, Phila., Pa. 19121 and Ben Gurion University of the Negev, Beer Sheva, Israel. The Levitzki radical in Jordan and associative algebras. Preliminary report.

Let A be an associative algebra over a field F of characteristic $\neq 2$. If A is the attached jordan algebra and $L(A)$, $L(A^+)$ the Levitzki radical of A and A^+, respectively, we have: Theorem 1: $L(A) = L(A^+)$.

If $*$ is an involution on A and S is the jordan algebra of $*$-symmetric elements then: Lemma 1: If S is nilpotent of index n then A is nil of bounded index $\leq 2n$. Lemma 2: If S is nil of index n and F contains at least n elements then A is nil of bounded index $\leq 2n$. Finally, we have: Theorem 2: If A satisfies a polynomial identity then $L(S) = S \cap L(A)$. (Received September 26, 1974.)

75T-A6 EMANUEL VEGH, Naval Research Laboratory, Washington, D. C. 20375. Addition chains. Preliminary report.

Let $1 = a_0 < a_1 \leq \cdots < a_r = n$ be a sequence of integers such that every member can be written as a sum of two preceeding members. Such a sequence is called an addition chain for n. Let $\ell(n)$ denote the number of terms in a shortest addition chain for n. Sholz

conjectured that $\ell(2^m-1) \leq (m-1) + \ell(m)$. It is shown here that every value of m for which the conjecture is now known to hold, may be obtained by using the special chains of A. Brauer [Bull. Am. Math. Soc. 45(1939), 736-737]. In the case $m = 2^{\alpha_1} + 2^{\alpha_2} + 2^{\alpha_3} + 2^{\alpha_4}$, $\alpha_1 > \alpha_2 > \alpha_3 > \alpha_4 \geq 0$, the Sholz conjecture is shown to hold in at least one case not considered by Gioia, Subbarao and Sugunamma [Duke Math. J. 29 (1962), 481-487].
(Received September 27, 1974.) (Author introduced by Dr. C. Osgood.)

*75T-A7 ALBERT A. MULLIN, 9213 Kristin Lane, Fairfax, Virginia 22030.
On geometric methods in number theory. Preliminary report.

Let B(n, r) be an origin-centered n-dimensional ball of radius r in real Euclidean space E^n, $n \geq 2$. Lemma 1. For each $n \geq 2$, $B(n, n^2)$ contains a lattice point with distinct prime-number coordinates. Definition. For each $n \geq 2$, let r(n) be the least r such that B(n, r) contains a point with distinct prime coordinates; e.g., $r(7) = (666)^{\frac{1}{2}}$. Lemma 2. For each $n \geq 3$, $n < r(n) < n^2/\log\log n$. Problem. Determine the exact asymptotic order of growth of r(n). Finally, lattice-point modifications of basic theorems of combinatorial geometry are studied (e.g., theorems of Helly, Carathéodory, and Grünbaum) as well as their application to problems in the theory of diophantine approximations. E.g., Corollary. In E^n, let F be a family of rectangular parallelopipeds having edges parallel to the coordinate axes and such that the intersection of each pair in the family contains a lattice point. Then there is a lattice point common to all members of F. Scholium. The corollary for the case n = 1 entails Helly's theorem for n = 1.
(Received October 2, 1974.)

*75T-A8 DAVID K. HALEY, Universität Mannheim, 68 Mannheim A,5, Fed. Rep. of Germany.
Equationally compact rings with A.C.C.

Let R be an associative ring satisfying the ascending chain condition on left ideals. Theorem 1. The following are equivalent: (i) R is equationally compact. (ii) R is a compact topological ring. (iii) R is an ideal of a compact topological unital noetherian ring. (iv) R is a subdirect product of a family of finite subdirectly irreducible rings, closed with respect to the product of the discrete topologies on the factors. Theorem 2. R possesses at most one compact topology compatible with its structure, namely, its (Jacobson) radical topology. (Received September 25, 1975.)

75T-A9 RAYMOND G. SEPETA, University of Notre Dame, Notre Dame, Indiana 46556. A characterization of the adjoint Chevalley group of type E_7 over fields of odd characteristic. Preliminary report.

Let G* denote the adjoint Chevalley group of type E_7 over a field of q elements with $q \equiv 1 \bmod 4$ and let G be an arbitrary finite group with involution t satisfying $G \neq O(G)C_G(t)$. If $C_G(t)$ has the structure of the centralizer of a central involution in G* then G is isomorphic to G*.
(Received October 9, 1974.)

75-A10 J.L. HURSCH, JR., 2995 Eagle Way, Boulder, Colorado 80302
Edge - 3 - colorability of certain graphs.

Let K be the class of all connected graphs G having the following properties:

(1) G has no loops or double edges. (2) Every vertex in G is of degree two or three.

(3) Every edge in G is incident to a vertex of degree three. (4) If H is obtained from G by removing all vertices of degree two and their incident edges, then every vertex in H is of degree two. Let G_0 be the graph obtained from Peterson's graph by removing one vertex v, and the three edges incident to v (all such graphs are isomorphic).

Theorem. Every graph in K except G_0 may be edge - 3 - colored. (Received October 21, 1974.)

(Introduced by Professor Jan Mycielski.)

*75T-A11 ARTHUR SAGLE, University of Hawaii at Hilo, Hilo, Hawaii 96720 and J. SCHUMI, St. Paul Companies, St. Paul, Minn. Anti-commutative algebras and homogeneous spaces with multiplications.

By generalizing results on Lie groups it is shown that an n-dimensional analytic H-space (M, μ) with identity e has, in general, a coordinate system at e so that the multiplication μ induces a bilinear anti-commutative multiplication α on R^n. Thus an algebra (R^n, α) analogous to the Lie algebra of a Lie group is obtained and all such algebras are isomorphic. If M = G/H is a reductive homogeneous space these results generalize the Lie group - Lie algebra correspondence and the algebra (R^n, α) induces a G-invariant connection on G/H. Relative to this connection it is shown that an automorphism of $(G/H, \mu)$ is an affine map and induces an algebra automorphism of (R^n, α) however, the converse is false. Also the connection is irreducible if $(G/H, \mu)$ has no proper invariant subsystems (the analog of normal subgroups). If G/H has a Riemannian structure and the coordinate functions are to be local isometries, then there need not exist this type of coordinate system which gives an anti-commutative algebra. (Received October 10, 1974.)

75T-A12 CLAUDE W. ANDERSON, Department of Mathematics, University of California, Berkeley, California 94720. A New $\mathcal{G}(n)$ in the Waring Problem. Preliminary report.

We define $\mathcal{G}(n) = \min\left\{k \mid d\left(\sum_{i=1}^{k} P_n\right) = 1\right\}$, where $P_n = \left\{m^n \mid m=0,1,2,3,\ldots\right\}$ are the nth powers and d is the natural density function. g(n) reflects the situation for all of N, while G(n) reflects the situation for all but a finite subset of N. $\mathcal{G}(n)$ reflects the situation for almost all of N in which peculiarities of zero density subsets are suppressed. $d(P_2+P_2+P_2) = 5/6$, so $\mathcal{G}(2) = 4 = G(2) = g(2)$. $d(P_3+P_3+P_3) \leq (\Gamma(4/3))^3/6 < 1/7$, where $\Gamma(x)$ is the gamma function, and $d(P_3+P_3+P_3+P_3) = 1$. Thus, $\mathcal{G}(3) = 4$. $G(3) \in \{4,5,6,7\}$. $d\left(\sum_{i=1}^{14} P_4\right) = 7/8$, and $d\left(\sum_{i=1}^{15} P_4\right) = 1$. Hence, $\mathcal{G}(4) = 15 < 16 = G(4)$. While the situation for G(n) is difficult for n = 3 and $n \geq 5$, the determination of $\mathcal{G}(n)$ for $n \geq 5$ may be more accessible to suitable analysis. The determination of $\mathcal{G}(n)$ for n = 3 and 4 follows from H. Davenport's work on the Waring Problem: (1) Acta Math., v. 71, 123-143 (1939), and (2) Annals of Math., v. 40, 731-747 (1939). (Received October 4, 1974.)

75T-A13 G. T. DIDERRICH, 2973 N. Cramer Street, Milwaukee, Wisconsin 53211. Van der Waerden's theorem extended to N dimensions. Preliminary report.

Let L be a n-dimensional lattice whose lattice points are colored with a finite number of colors. We show that L contains arbitrarily large, in number of lattice points on an edge, monochromatic cubes in any suitably large cube of L. The one dimensional case is equivalent to van der Waerden's famous theorem. Thus, finite monochromatic "anything", up to similarity, occurs in L. (Received October 15, 1974.)

*75T-A14 JOE V. PETTY, Texas Instruments, Inc., MS 907, Dallas, Texas 75222. Some Closure Properties of Series Determined Classes of Groups.

Definitions not given here can be found in Robinson["Finiteness Conditions and Generalized Soluble Groups",Part 1,Springer-Verlag,Berlin,1972]. Given a variety of groups V, let $\chi(V)$ be the subgroup theoretical property V-marginal. Define an operator $\hat{P}_v$ on the class of all varieties of groups by $G \in \hat{P}_v V$ if G has a $\chi(V)$-series. If V is a variety of groups, define an operator $Q_{\chi(V)}$ on the class of all group theoretic classes by $G \in Q_{\chi(V)}X$ if there exist $K \in X$ and a homomorphism σ of K such that $G = (K)\sigma$, and the kernel of σ is contained in the $\chi(V)$-hypercenter of K. THEOREM. If V is a variety of groups, then each of the classes $\hat{P}_v V$, $\hat{P}_n V$, $\hat{P}V$ is $Q_{\chi(V)}$-closed. This generalizes a result of the author[Abstract 74T-A229,these Notices 21(1974),A-531]. Define an operator Q_R by $G \in Q_R X$ if G is isomorphic to a reduced direct product of X-groups. THEOREM. If V is a variety of groups, then each of the classes $\hat{P}_v V$, $\hat{P}_n V$, $\hat{P}V$ is Q_R-closed. EXAMPLE. For each natural number k, let D_k be a dihedral group of degree 2^k. If F is any free filter on the natural numbers, then $\pi D_k/F$ is neither a $\bar{Z}$-group nor a ZD-group. (Received October 21, 1974.)

*75T-A15 KEVIN E. OSONDU and DOV TAMARI, State University of New York at Buffalo, Buffalo, New York 14226. Embedding a semigroup in a group.

Every ordered couple of elements of a multiplicative system uniquely determines a right [left] "ray" and a corresponding "homogeneous quotient" as a union of connected rays. This construction is a natural generalisation of that of fractions in the multiplicative semigroup of the natural numbers. Homogeneous quotients of semigroups S embeddable in a group are necessarily one-to-one partial transformations on S . This natural, necessary, but in general not sufficient condition, called the "generalised (right [left]) quotient condition", has numerous connections with conditions in the earlier literature (Malcev 1939, Lambek and Tamari 1951, Clifford-Preston 1961-67): it turns out to be equivalent to the "complete cancellativity" of Bouleau (1973).

If a semigroup S satisfies the generalised (right [left]) quotient condition, one can construct a "group of right [left] homogeneous quotients" which together with an appropriate mapping forms a universal group on S . A semigroup is embeddable in a group if and only if it is embeddable in its group of right [left] homogeneous quotients. This can be used to obtain explicit, necessary, and sufficient embedding conditions. (Received October 21, 1974.)

*75T-A16 C. B. GARCIA, University of Chicago, Chicago, Illinois 60637. Computing Ramsey Numbers.

Isbell (J. Comb. Theory, 6 (1969), 210) showed that the Ramsey number $N(4,4;3) \geq 13$. Using the method of implicit search (Balas, Opns. Res., 13 (1965), 517-546), we show $N(5,5;2) \geq 42$ and $N(5,4;3) \geq 23$. To show the former: let $\{i,j\}$ be a 2-subset of the set of integers between 1 and 41. Color $\{i,j\}$ if and only if $\min\{|j-i|, 41 - |j-i|\} = 1,2,3,5,7,10,13,15,16$, or 17. To show the latter: let $\{i,j,k\}$ be a 3-subset of the set of integers between 1 and 22. Then, color $\{i,j,k\}$ if and only if $(\min\{|j-i|, 22 - |j-i|\}, \min\{|k-j|, 22 - |k-j|\}, \min\{|k-i|, 22 - |k-i|\}) = (1,1,2), (1,2,3), (1,3,4), (1,4,3), (1,4,5), (1,5,6), (1,6,5), (1,6,7), (1,7,8), (1,8,7), (1,8,9), (1,9,10), (1,10,9), (2,6,4), (2,8,6), (2,9,11), (2,10,10), (3,7,4), (3,7,10), (3,8,11), (3,9,10), (3,10,7), (4,5,9), (4,9,5), (4,11,7), (5,8,9), (5,10,7)$, or $(6,8,8)$. This last result may be utilized for substantially improving Table 1 of (Abbott and Williams, J. Comb. Theory, A16 (1974), 12-17) using Kalbfleisch's theorem (Theorem 1 of the aforementioned paper). (Received October 29, 1974.)

75T-A17 CATHERINE STAHL, Lake Erie College, Painesville, Ohio, 44077. Systems of additive equations over algebraic number fields, Preliminary Report.

By an extension of the Hardy-Littlewood circle method, the following theorem is established. Theorem. Let k be an odd prime and let K be an algebraic number field in which k is not wildly ramified. The equations

$$F_1(\underline{x}) = a_{11}x_1^k + a_{12}x_2^k + \dots\dots\dots\dots\dots + a_{1m}x_m^k = 0$$
$$\vdots$$
$$F_R(\underline{x}) = a_{R1}x_1^k + a_{R2}x_2^k + \dots\dots\dots\dots\dots + a_{Rm}x_m^k = 0$$

whose coefficients lie in K, have a non-trivial solution $x_1 \dots x_m$ with x_j in K, if

$$m > \frac{R^2(R+1)2^{k-1}k}{k-2} .$$

Of interest is the fact that the necessary number of variables does not depend on the degree of K over the rationals. (Received October 21, 1974.) (Author introduced by Stanley Stahl.)

*75T-A18 EDWARD G. THURBER, Biola College, La Mirada, California 90639. On addition chains concerning the equality $l(2n) = l(n)$.

Let $l(n)$ denote the minimal length for an addition chain for a positive integer n, and let $h(x)$ denote the number of integers n less than or equal to x for which $l(2n) = l(n)$. There exist infinitely many infinite classes of integers for which $l(2n) = l(n)$. In powers of two these integers are expressed as $n = 2^{2m+k+7} + 2^{2m+k+5} + 2^{m+k+4} + 2^{m+k+3} + 2^{m+2} + 2^{m+1} + 1$ where $m \geqq 1$ and $k \geqq 3$. Also, $\log x = o(h(x))$. (Received October 15, 1974.)

75T-A19 G.E. HARDY, Department of Mathematics, University of Alberta, Edmonton, Alberta Canada T6G 2G1. Schnirelmann density for k-free integers for $7 \leq k \leq 12$.

An integer n is called k-free if it is not divisible by p^k for any prime p . We define d(k) , the Schnirelmann density of the k-free integers, by: $d(k) = \inf_{1 \leq n < \infty} (Q_k(n)/n)$, where $Q_k(n)$ is the number of k-free integers less than or equal to n . Rogers (Proc. Am. Math. Soc., 15 (1964), 515-516) found d(2) while Orr (J. London Math. Soc., 44 (1969), 313-319) found d(3) through d(6). For $k \geq 4$, Orr showed, $d(k) = Q_k(n_o)/n_o$ for some n_o satisfying $5^k \leq n_o < 6^k$; and for no n_o outside this interval. Using this result and several new (unpublished) theoretical results, the following unique values of n_o for which $d(k) = Q_k(n_o)/n_o$ are found: d = 7: $n_o = 236288$, $Q_7(n_o) = 234331$; d = 8: $n_o = 1174528$, $Q_8(n_o) = 1169758$; d = 9: $n_o = 7814151$, $Q_9(n_o) = 7798488$; d = 10: $n_o = 48833536$, $Q_{10}(n_o) = 48785015$; d = 11; $n_o = 293001216$, $Q_{11}(n_o) = 292856489$ and d = 12: $n_o = 1709645824$, $Q_{12}(n_o) = 1709225206$. P.H. Diamanda recently obtained n_o for d = 7 independently, but it has not been published. (Received October 25, 1974.)

*75T-A20 BRUNO J. MÜLLER, McMaster University, Hamilton, Ontario. Localization in non-commutative rings.

Attach to any semiprime ideal S the finite set of prime ideals minimal over S . Call such S , as well as the attached set of prime ideals, classical if $\mathcal{C}(S) = \{c \in R : \bar{c} \text{ is regular in } R/S\}$ is left- and right-Ore, and if the Jacobson radical of the quotient ring R_S has the AR-property. Let a cycle be a minimal classical set. (1) The Jacobson radical is the only classical semiprime ideal, for an indecomposable artinian ring. (2) Every classical set of prime ideals is uniquely the union of cycles, for any noetherian

ring. (3) For an HNP ring, these cycles coincide with the cycles in the sense of D.Eisenbud and J.C.Robson[J.Algebra 16(1970),86-104]. (4) If a noetherian ring is finitely generated as module over its center, then the cycles are just the sets of prime ideals lying over the various primes of the center. (5) For the group ring AG of a finite group G over a commutative noetherian ring A, the cycles correspond to the blocks of KG, where K varies over the quotient fields of A/Q for the primes Q of A. All prime ideals of AG are classical, iff G is p-nilpotent for all p not invertible in A. (Received October 25, 1974.)

*75T-A21 L. M. CHAWLA and ELLEN TORRANCE, Kansas State University, Manhattan, Ks. 66502
Three Restricted Product-sum Partition Functions.

Preliminary Report.

In On the Classification and Evaluation of Some Partition Functions and Their Tables, Conference W. Mich. Univ. 1971, Lecture Notes in Mathematics 251, Springer-Verlag, 103-112, Chawla and Maxfield studied the product-sum partition function $p(n,m)$, defined as the number of otherwise unrestricted partitions of n such that the product of the summands is m. In the present paper we study three restricted product-sum partition functions $P_S(n,m)$, $P_R(n,m)$ and $P_Q(n,m)$, namely, the number of partitions of n such that the product of the summands is m and further for $P_S(n,m)$ the summands come from the set S of non-negative integer powers of primes; for $P_R(n,m)$ the summands are pairwise relatively prime; and for $P_Q(n,m)$ at least one of the summands divides all the others. Let $f(x)$ be the sum of the prime factors of x counting multiplicities, and $h(x)$ the sum of the terms p^a in the prime factorization of x. Let $m = q_1^{b_1} \cdot \ldots \cdot q_s^{b_s}$, and $p(b)$ be the number of unrestricted partitions of b. We show that $P_S(n,m) \geq 1$ if and only if $f(m) \leq n$; and $P_S(n,m) \leq p(b_1) \cdot \ldots \cdot p(b_s)$, with equality holding if and only if $h(m) \leq n$. We also show that $P_R(n,m) \geq 1$ if and only if $h(m) \leq n$; and $P_R(n,m) \leq A(s)$, the number of partitions of a set with s distinguishable elements; equality holds if and only if $m \leq n$. We obtain a similar condition for $P_Q(n,m)$ to be ≥ 1, and find an explicit expression for $P_Q(n,m)$. (Received October 25, 1974.)

75T-A22 LOUIS HALLE ROWEN, University of Chicago, Chicago, Illinois 60637. Monomial conditions on rings.

Let R be a ring; let R' be R with 1 adjoined formally. $\mathbb{Z}\{X;t\}$ is the free ring (without 1) generated by noncommuting indeterminates $X_1, \ldots, X_t$. Let $\pi(t)$ {monic monomials $h \in \mathbb{Z}\{X;t\} | h \neq X_1 \ldots X_t$ and $\deg h \geq t$}, $\pi(t,m) = \{h \in \pi(t) | \deg h \leq m\}$. Call y R-regular if $yr \neq 0$, $\forall\, r \neq 0$ in R; y is strongly R-regular if ($\forall\, r \neq 0$ in R) ($yry \neq 0$ and ($\exists\, a_1, a_2 \neq 0$ in R) ($a_2 y = a_1 r$)). Modifying a notion of Drazin, say $X_1 \ldots X_t$ is R-pivotal (resp. almost R-pivotal) if ($\forall$ hom $\varphi: \mathbb{Z}\{X;t\} \to R$)($\exists$ strongly R-regular (resp. R-regular) $y \in R'$) ($y\varphi(X_1 \ldots X_t) \in R'\varphi(\pi(t))$). R is (t,n)-elementary if ($\forall\, \varphi: \mathbb{Z}\{X;t\} \to R$) ($\exists$ strongly R-regular $y \in R'$)($y\varphi(X_1 \ldots X_t) \in R'\varphi(\pi(t,n))$). Theorems. TFAE for R prime: (1) R is a left order in a simple artinian ring of index $< t$; (2) R is (t,n)-elementary for some n; (3) $X_1 \ldots X_t$ is R-pivotal. If R is semiprime with almost pivotal monomial, then all nil subrings are nilpotent and the singular ideals are 0. Let $R\{X;t\}$ = free product of R and $\mathbb{Z}\{X;t\}$. A monomial $r_1 X_{\mu 1} \ldots X_{\mu m} r_{m+1}$ has fingerprint $X_{\mu 1} \ldots X_{\mu m}$ and coefficients $r_1, \ldots, r_{m+1}$; a generalized monomial (gen. mon.) is a sum of monomials with the same fingerprint. Given finite $W \subset R$, let $\pi_1(t;W)$ = {gen. mon. with fingerprints in $\pi(t,t)$ and coefficients in W}. A gen. mon. h with fingerprint $X_1 \ldots X_t$ is R-pivotal if, for some W, ($\forall\, \varphi: R\{X;t\} \to R$) ($\varphi(h) \in R'\varphi(\pi_1(t;W))$). Theorem. The socle of a primitive ring is the set of evaluations of its pivotal gen. mons. (Received October 15, 1974.)

*75T-A23 L. N. CHILDS, State University of New York, Albany, New York 12222, G. S. GARFINKEL, Oakland University, Rochester, Michigan 48063 and M. ORZECH, Queen's University, Kingston, Ontario, K7L 3N6 Canada. Factoriality of normal noetherian domains and the Brauer group.

Let R be a normal noetherian local domain with maximal ideal m, quotient field K. Brauer group $B(R)$, divisor class group $Cl(R)$ and henselization R^h. We analyze the relationship between the kernel $B(K/R)$ of the natural map $B(R) \to B(K)$ and factoriality of R vis-a-vis separable extensions of R. If every finite Galois extension of R is factorial then $B(K/R) = 0$. If $B(R/m) = 0$ then there is a monomorphism from $B(K/R)$ to $Cl(R^h)/Cl(R)$. We construct a family of examples R, each a local ring of a normal singularity of a complex surface, such that factoriality of R is equivalent to nontriviality of $B(K/R)$. One of these examples is in fact factorial, with $Cl(R^h) = \mathbb{Z} = B(K/R)$. (Received October 17, 1974.)

*75T-A24 KIM KI-HANG BUTLER, Alabama State University, Montgomery, Alabama 36101. New regular subsemigroup of semigroup of binary relations, Preliminary report.

A binary relation is called an *acyclic binary relation* if it does not contains a permutation relation of order two as a subrelation. Let A_n denote the set of all such relations defined on a set containing n elements. Then (i) A_n is a regular semigroup. Note: A_n is not an inverse semigroup.

(ii) $|A_n| = \sum_{r=1}^{n} (r!S(n+1, r+1))^2$, where $S(m, k)$ is the Stirling number of the second kind. (Received October 29, 1974.)

*75T-A25 ESMOND E. DeVUN, Wichita State University, Wichita, Kansas 67208. U-product. Preliminary report.

A U-semigroup is a semigroup which is iseomorphic to $[0,1]$ with the usual multiplication. A semigroup S is said to be the unique product of U-semigroups $S_1, S_2, \ldots, S_n$ if each S_i is a U-semigroup and for every $\sigma \in G_n$ (G_n is the symmetric group on n elements, $S = S_{\sigma(1)}S_{\sigma(2)}\cdots S_{\sigma(n)}$ and for every nonzero element s of S with $s = x_{\sigma(1)}x_{\sigma(2)}\cdots x_{\sigma(n)} = y_{\sigma(1)}y_{\sigma(2)}\cdots y_{\sigma(n)}$ and $x_{\sigma(i)}, y_{\sigma(i)} \in S_{\sigma(i)}$ we have $x_{\sigma(i)} = y_{\sigma(i)}$. In this note we will assume S is the unique product of U-semigroups, S has no zero divisors, and $E(S) = H = \{0,1\}$. Theorem. S is uniquely divisible. A semigroup T is said to be left (right) reversible if every pair of right (left) ideals have nonempty intersection. Now let C be the convex hull of the $(n \times n)$ matricies $\{X_0, X_1, \ldots, X_k, \ldots, X_{n-1}\}$ where X_0 is the identity matrix and X_k is the matrix (x_{ij}) where $x_{nj} \in (0,1]$, $x_{nn} = 1$ and $x_{ij} = 0$ otherwise. Theorem. If each subsemigroup $S_iS_j - \{0\}$ of S, $i \neq j$, is left reversible and not right reversible, then S is iseomorphic to the semigroup formed by the one point compactification of the semigroup C with matrix multiplication. (Received October 24, 1974.)

75T-A26 ALBRECHT WOLF, Technische Hochschule, Darmstadt, Fb. 4, AG 1, D-61 Darmstadt, Germany. Sheaves of rings and arithmetical algebras. Preliminary report.

Theorem. Let A be a universal algebra. Let L be a zero distributive lattice of mutually permuting congruences of A, containing ω and ι. Suppose that $x_1 \vee \ldots \vee x_n = \iota$ implies $(y_1 \vee z_1) \wedge \ldots \wedge (y_n \vee z_n) \doteq (x_1 \wedge y_1) \vee \ldots \vee (x_n \wedge y_n) \vee (x_1 \wedge z_1) \vee \ldots \vee (x_n \wedge z_n)$ for all $x_i, y_i, z_i \in L$. Then there is a sheaf F over the space of all prime ideals of L such that A is canonically isomorphic to the algebra ΓF of all

sections of F. The above hypotheses are satisfied in the lattice of all ideals of a semiprime ring with 1 as well as in the lattice of all congruences of an arithmetical algebra. Thus, the theorem unifies known representation theorems for these two cases; see K.H. Hofmann, Bull. Amer. Math. Soc. 78(1972), 291-373, Mem. Amer. Math. Soc. No. 148(1974), 87-93. (Received October 30, 1974.)

*75T-A27 CARL BUMILLER, Yale University, New Haven, Ct, 06520
Partial Geometries and Rank Three Groups.

Let Ω be an (r,s,t) Partial Geometry [Bose, Pacific J. of Math 13, (1963), pp 389-419] which has a rank 3 permutation group and such that r is prime. Theorem Assume the dual of Ω satisfies Pasch's axiom: any two transversals of a pair of intersecting lines intersect. (1) If t odd < 30, then r and s are bounded by a function of t; (2) if t = 3 or 7, then r = t; (3) if t = 2 and r < 200, then r = 2 or 3; (4) if t = 2 and r > 200, then s = 2 + r(r-1)(r-5)/6. The proof uses a thorough description of $\Delta(0)$ for 0 in Ω, analysis of possible subgeometries derived from certain subgroups, elementary restrictions of the parameters, and various well-known theorems on permutation groups. The partial geometries with r = t or t+1 whose dual satisfies Pasch's axiom have been previously determined [Higman, Atti del Convegno di Geometria Combinatoria (Perugia) 1971, pp263-293]. A characterization of certain rank 3 groups follows easily. Theorem Suppose G is a rank 3 permutation group with subdegrees k = r(s-1) and ℓ = (r-1)(s-1)(s-t)/t such that r is prime and t is sufficiently large. Then (1) through (4) hold. (Received October 30, 1974.)

Analysis

*75T-B1 REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2 and Ravenshaw College, Cuttack-3, Orissa, India. Some multiple series transformations.

Recently, H. M. Srivastava [On the reducibility of Appell's function F_4, *Canad. Math. Bull.* 16 (1973), 295-298] gave a number of cases of reducibility of certain double series with arbitrary terms. Of concern in the present paper is one of his results which leads to a multiple series transformation. A simple proof by induction is given of this multiple series transformation and it is shown how this transformation may be applied to derive various reduction formulas for the generalized Lauricella function of several complex variables, which was introduced in an earlier paper by H. M. Srivastava and M. C. Daoust [Certain generalized Neumann expansions associated with the Kampé de Fériet function, *Nederl. Akad. Wetensch. Proc. Ser. A* 72 = *Indag. Math.* 31 (1969), 449-457; see also *Math. Nachr.* 53 (1972), 151-159]. This paper is scheduled to appear in *Jñānabha Sect. A* 4 (1974).

(Received September 19, 1974.)

75T-B2 S.ZAIDMAN, Université de Montréal: Linear operators on Sobolef spaces.

We consider a family of bounded complex-valued functions("symbols"),a(t),defined on an abstract set $\mathcal{T}$ and satisfying several postulates.To any such symbol a we associate a couple of linear operators $\{A_a, \mathcal{A}_a\}$ mapping $H^{-\infty} = \bigcup_{s\in R^1} H^s$ into itself,where $H^s(R^n)$ is the well-known Sobolef space.A second list of postulates concerns these ope-

rators. Then, various results, including several inequalities, are proved.

The paper is an abstract version of some results in the well-known Kohn-Nirenberg paper on pseudo-differential operators. (Received September 23, 1974.)

75T-B3 S. CHAKRAVARTY, University of Kentucky, Lexington, Kentucky 40506

The set where a transcendental entire function is large

Let $f(z)$ be a transcendental entire function of finite order, λ (say). Define $E(r) \equiv \{z: |z| = r, |f(z)| > |, r > 0\}$, $M(r, f) = \max_{|z|=r} |f(z)|$ and, let $|E(r)|$ denote the angula measure of $E(r)$. When $|E(r)|$ is small, we have

Theorem (1): $\lim_{r \to \infty} \{\log M(rR, f)|E(r)|^2\} = +\infty$, $(\varepsilon > 0)$ and $R \geq 1$ but fixed.

Corollary 1 : $\lim_{r \to \infty} \{r^{\lambda/2 + \varepsilon} |E(r)|\} = +\infty$; $\liminf_{r \to \infty} \{r^{\lambda/2 - \varepsilon} |E(r)|\} = 0$.

Theorem (1) is best possible. In fact, theorem (1) is complementary to Theorem of Arima [Journ. Math. Soc. Japan, 1952]. Its close analogue for meromorphic functions is the "spread - relation" of A. Edrei. (author introduced by Professor S.M. Shah) (Received September 23, 1974.)

75T-B4 A. R. REDDY, Michigan State University, East Lansing, MI 48824

A note on rational approximation on $[0, \infty)$

THEOREM: Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$ $(> 0$ on $[0, \infty))$ be an entire function of order $\rho (0 < \rho < \infty)$, type τ and lower type ω $(0 < \omega \leq \tau < \infty)$ satisfying the further assumption that $f(z)$ grows on $[0, \infty)$ as fast as anywhere else in the complex plane. Then for any polynomial $P_n(x)$ of degree at most n, we have

$$\liminf_{n \to \infty} \left\| \frac{1}{f(x)} - \frac{1}{P_n(x)} \right\|_{L_\infty[0,\infty)}^{1/n} \geq \left(2^{2 + \frac{1}{\rho}} \tau^{\frac{1}{\rho}} \omega^{-\frac{1}{\rho}} - \frac{7}{4} \right)^{-1}.$$

(Received September 23, 1974.)

*75T-B5 C.M. JOSHI, University of Jodhpur and M.L. PRAJAPAT, Defence Laboratory, Jodhpur, India. On some properties of generalized Hermite polynomials. Preliminary report.

In pointing out that Maya Lahiri's generalization of Hermite polynomials [Proc. Amer. Math. Soc. 27(1971), 117-121] is a particular case of the more general representation of Hermite polynomials studied by Gould and Hopper [Duke Math. J. 29(1962), 51-69] and in fact connected with it by relation $H_{n,m,\nu}(x) = g_n^m(\nu x, -i)$, we have obtained new generating relations and finite summation theorems concerning the latter. In the sequel, it is also pointed out that relations (20), (21) of "Higher transcendental functions", Vol. II, 1953, p. 119 are incorrect. (Received September 23, 1974.)

75T-B6 IN-DING HSÜ, State University of New York at Buffalo, Amherst, New York 14226. An Applicable Hopf Bifurcation Formula. Preliminary report.

The original direction of bifurcation formulas of Hopf [Ber. Verh. Sachs. Akad. Wiss. Leipsig. Math.-Nat. 94 (1942), 3-22] and Friedrichs [N. Y. Gordon and Breach, 1965] are difficult to use even in the case of two dimensions, see, for example [Poore, Arch. Rat. Mech. Anal., 52 (1973) 358-388]. We derive a new, readily applicable form of Hopf's formula for the direction of bifurcation for systems of the form $\dot{X} = A(\mu)X + F(X,\mu)$ where $X = (x_1,\ldots,x_n)$, F is a real analytic function on a domain $G \times (-c,c)$ in R^{n+1}, $c > 0$, A is a real $n \times n$ analytic matrix with exactly two purely imaginary exponents $\alpha(\mu)$, $\bar{\alpha}(\mu)$ at $\mu = 0$ such that $\mathrm{Re}(\alpha'(0)) \neq 0$. The assumption of analyticity can be relaxed to C^N for suitable N. Our result reduces to that of Bruŝlinskaya [Dokl. Akad. Nauk SSSR, 139 (1961), 9-12] in the special case she considers. Using this formula we are able to determine the direction of bifurcating periodic solutions of some $n \times n$ systems $(n > 2)$ of that arise in chemistry and biochemistry. (Received October 1, 1974.) (Author introduced by Professor N. D. Kazarinoff.)

75T-B7 IN-DING HSÜ and NICHOLAS D. KAZARINOFF, State University of New York at Buffalo, Amherst, New York 14226. Instability of Small Periodic Solutions of the Field-Noyes Model for the Belousov-Zaikin-Zhabotinskiĭ reaction. Preliminary report.

Using an applicable Hopf bifurcation formula of Hsü, we prove the following theorem for the system (*) $\dot{x}_1 = s(x_1 + x_2 - x_1x_2 - qx_1^2)$, $\dot{x}_2 = s^{-1}(fx_3 - x_2 - x_1x_2)$, $\dot{x}_3 = w(x_1 - x_3)$. Theorem: Corresponding to any given $s \geq 34$, $q \in [0,(30s^2)^{-1}]$ and $f \in (2-r,2+r)$, for some $r > 0$, which satisfy the condition $2q(x_1^0)^2 + x_1^0(q-1) + f < 0$, where (x_1^0,x_2^0,x_3^0) is the corresponding critical point, of (*) in the first orthant, there exists a $w_0 = w_0(f,s,q) > 0$ such that the system (*) has a periodic solution for each $w \in (w_0, w_0 + \epsilon_0)$ for some $\epsilon_0 > 0$, where $w_0 = \frac{1}{2\alpha}\{-[\alpha^2 + f(1-x_1^0)] + \{\alpha^2 + f(1-x_1^0)]^2 - 4\alpha^2[2q(x_1^0)^2 + x_1^0(q-1) + f]\}^{\frac{1}{2}}\}$. Moreover these small bifurcating periodic solutions are unstable. The system (*) is due to Field and Noyes [J. Chem. Phys. 60 (1974) 1877-1884]. It models oscillations in the concentrations of Br^- and $HBrO_2$ and Ce(IV) in the Belousov-Zaikin Zhabotinskiĭ reaction. Hastings and Murray [SIAM J. Appl. Math., to appear] proved that a (probably large) periodic solution exists for each w on $(0,w_0)$ but they did not study the stability of these solutions. Hsü [J. of Differential Equations, to appear] showed existence of bifurcating periodic solutions for w near w_0 but he neither determined the direction of bifurcation nor determined their stability. (Received October 1, 1974.)

*75T-B8 CHARLES DIMINNIE and ALBERT WHITE, St. Bonaventure University, St. Bonaventure, N.Y., 14778. 2-inner product spaces. Preliminary report.

Let $(L,(\cdot,\cdot|\cdot))$ be a 2-inner product space. (''2-Inner Product Spaces'', Diminnie, Gähler, White, Demonstrato Mathematica, to appear) Theorem: For $a,b,c \in L$ and $f(x,y) = \frac{1}{2}\|x,y\|^2$, $[a,b|c]$ is a 2-inner product and $[a,b|c] = (a,b|c)$ where $[a,b|c] = \lim_{t \to 0} \frac{1}{t}[f(a+tc,b) - f(a,b)]$.

(Received October 2, 1974.)

*75T-B9 H. M. SRIVASTAVA and REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2. Some analytic or asymptotic confluent expansions for functions of several variables. Preliminary report.

In recent years several writers have contributed to the theory of analytic or asymptotic confluent expansions for functions of one and two variables. A reasonably detailed account

of this theory is contained in the works of Jerry L. Fields [Math. Comp. 21 (1967), 189-197], Yudell L. Luke ["The special functions and their approximations", Vol. I, Academic Press, New York and London, 1969, especially Chapter III], and V. L. Deshpande [Math. Comp. 28(1974), 605-611]. The object of the present paper is to discuss extensions of these results to hold for a fairly large variety of functions of several variables. In an attempt to illustrate the usefulness of these extensions, an asymptotic confluent expansion is obtained for a certain class of the generalized Lauricella functions which were introduced and studied earlier by H. M. Srivastava and M. C. Daoust [Nederl. Akad. Wetensch. Proc. Ser. A 72 = Indag. Math. 31 (1969), 449-457; see also Math. Nachr. 53 (1972), 151-157, especially Section 5]. It is also demonstrated how this last expansion can be further specialized to yield the corresponding asymptotic confluent expansions for the ordinary Lauricella functions $F_A^{(r)}$, $F_B^{(r)}$ and $F_D^{(r)}$ of r variables [G. Lauricella, Rend. Circ. Mat. Palermo 7 (1893), 111-158]. (Received October 3, 1974.)

*75T-B10 Siamak Khalili, IBM, Dept 523, Bldg 032, 740 New Circle Road, Lexington, KY. 40507. Independently Scattered Measures.

Let $(\Omega, \mathcal{B}, P)$ be a probability space and $L_2 = L_2(\Omega, \mathcal{B}, P; \mathcal{F})$, where $\mathcal{F} = \mathbb{R}$ or $\mathbb{C}$. DEF. Let $\mathcal{P}$ be a pre-ring over a set Λ. We say that ξ is a L_2-valued countably additive independently scattered(c.a.i.s) measure on $\mathcal{P}$ iff (i) ξ is a function on $\mathcal{P}$ to L_2 (ii) $\forall\, k \geq 1$, $A_k \in \mathcal{P}$, A_k pairwise disjoint, $\bigcup_1^\infty A_k \in \mathcal{P}$ $\Rightarrow \sum_k \xi(A_k)$ converges unconditionally in L_2 to $\xi(\bigcup_1^\infty A_k)$, (iii) $n \geq 1, A_1, A_2, \ldots, A_n \in \mathcal{P}$, A_i pairwise disjoint $\Rightarrow \{\xi(A_i) : i = 1, 2, \ldots, n\}$ is a collection of independent random variables on $(\Omega, \mathcal{B}, P)$. THM. Let (i) ξ be a L_2-valued c.a.i.s measure on a pre-ring $\mathcal{P}$ over a set Λ such that $\forall\, A \in \mathcal{P}$, $E[\xi(A)] = 0$ (ii) $\forall\, A \in \mathcal{P}$, $m(A) =_d \|\xi(A)\|^2$. Then (a) m is a non-negative countably additive measure on $\mathcal{P}$, (b) ξ can be uniquely extended to a c.a.i.s measure on the δ-ring $\mathcal{A}$ of sets of finite $\tilde{m}$ measure, where $\tilde{m}$ is the Hahn extension of m to the σ-ring generated by $\mathcal{P}$. Moreover $\forall\, A \in \mathcal{A}$, $\|\xi(A)\|^2 = \tilde{m}(A)$. (Received October 4, 1974.)

*75T-B11 RICHARD R. GOLDBERG, University of Iowa, Iowa City, Iowa 52242. Multipliers from L^1 to a Segal algebra, Preliminary report.

Let S be a Segal algebra on the compact abelian group G. Denote by $\hat{S}$ the set of all measures $\mu \in M(G)$ for which there exists a sequence $\{f_n\}$ in S such that $\sup_n \|f_n\|_S < \infty$ and such that $f_n \to \mu$ in the weak* topology for M(G). THEOREM. The set (L^1, S) of multipliers from L^1 to S is identical with $\hat{S}$. The proof of the theorem involves the following LEMMA. Let G be a compact abelian group. If $\mu_n \to \mu$ in the weak* topology, and $h \in L^1$, then $\|\mu_n * h - \mu * h\|_1 \to 0$. (The lemma does not hold if G is not compact.) (Received October 9, 1974.)

75T-B12 ERWIN O. KREYSZIG, University of Windsor, Windsor, Ontario, Canada.

Chebychev polynomials as Bergman generators.

$Lu = u_{zz^*} + au_z + bu_{z^*} + cu = 0$ (a, b, c complex analytic, $c \neq 0$) is said to be of class Π_0, written $L \varepsilon \Pi_0$, if there is a Bergman operator for $Lu = 0$ with a generator $g(z, z^*, t) = \Sigma_{\mu=0}^{\infty} k_\mu h(z, z^*, t)^{\mu+\sigma}$ (σ complex). Theorem 1. (1) $L = \partial^2/\partial z \partial z^* + c(z, z^*) \varepsilon \Pi_0$ iff there is a $\psi(z^*)$ such that $c = -\nu(\nu + 1)\psi'/(z + \psi)^2$. Then corresponding generators are $g_1 = (1 - h)^{-1/2} \times \cos H$, $g_2 = (1 - h)^{-1/2} \sin H$ ($\nu \neq -1/2$), $g_3 = (1 - h)^{-1/2} \arcsin \sqrt{h}$ ($\nu = -1/2$), where $h = zt^2/(z + \psi)$, $H = (2\nu + 1) \arcsin \sqrt{h}$. Corollary. If 2ν in Theorem 1 is integral, generators for (1) are $g_1 = (1 - h)^{-1/2} T_m(\sqrt{h})$, $g_2 = (1 - h)^{-1/2} \times U_m(\sqrt{h})$, where T_m, U_m, $m = 2\nu + 1$, are Chebychev polynomials.- This and a result of M. Kracht and the Author (Manuscripta Math. 1, 1969, 369-376) imply that the differential operator used by K. W. Bauer (J. Reine Angew. Math. 221, 1966, 48-84) is a Bergman operator with a Chebychev polynomial as generator.

(Received October 10, 1974.)

75T-B13 STEFAN BERGMAN, Stanford University, Stanford, California 9[illegible].

Bounds for distortion in pseudo-conformal mapping. Preliminary report.

The invariant $J = J_{\mathfrak{R}}(z)$ has been introduced in Survey [illegible] of the Amer. Math. Soc., p. 18[illegible]. Let $z_k = f_k(\zeta_1, \zeta_2)$, $k = 1, 2$, be a pseudo-conformal transformation by a pair of analytic functions of two complex variables, with a nonvanishing Jacobian. Suppose that r is the maximum distance of the point $z = (z_1, z_2)$, $z \in \mathfrak{R}$, to the boundary $\partial\mathfrak{R}$, and [illegible] is the corresponding minimum distance. Then $H([illegible], r) \leq J_{\mathfrak{R}}(z) \leq H(r, [illegible])$ where $H([illegible], r) = (\frac{[illegible]}{9})\rho^{-[illegible]}$ [illegible] and $H(r, [illegible])$ is an analogous expression which one obtains by replacing [illegible] by r and r by [illegible]. Let $\mathfrak{R}$ be a Reinhardt circular domain, and let $K_{\mathfrak{R}}(\zeta, \bar{\zeta}) = B_{00} + B_{10}\zeta_1\bar{\zeta}_1 + B_{01}\zeta_2\bar{\zeta}_2 + \cdots$ be its development at the center, which is assumed to be the origin [illegible] of the coordinate system. $B_{mp}^{-1} = \int_{\mathfrak{R}} |\zeta_1|^{2m} |\zeta_2|^{2p} d\omega$ ($d\omega$ volume element) are the moments of $\mathfrak{R}$. Then $H([illegible], r) \leq B_{00}^{-1} B_{10}^{-1} B_{01}^{-1} \leq H(r, [illegible])$. Here z is the image of the center [illegible] and r and ρ are the maximum, respectively minimum, distances of z from $\partial\mathfrak{R}$.

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75T-B1[illegible] M. J. FRANK, University of Wisconsin, Milwaukee, Wisconsin 53201.

On associative solutions of a distributivity equation. Preliminary report.

A copula is a two-place function $C : [0,1]^2 \to [0,1]$ satisfying

$$C(x,0) = C(0,x) = 0, \quad C(x,1) = C(1,x) = x, \qquad \text{for every } x \text{ in } [0,1],$$
$$C(x_1,y_1) - C(x_2,y_1) - C(x_1,y_2) + C(x_2,y_2) \geq 0 \qquad \text{whenever } x_1 \leq x_2 \text{ and } y_1 \leq y_2.$$

If C is strictly increasing in each variable over $(0,1]^2$, it is said to be strict.

Consider the system of functional equations

$$C(C(x,y),z) = C(x,C(y,z)),$$
$$C(x+y-C(x,y),z) = C(x,z) + C(y,z) - C(C(x,y),z).$$

THEOREM A. $C_1(x,y) = x \cdot y$ is the only strict copula that is a solution of (*).

The proof rests on the well-known representation for solutions of the associativity equation and involves reduction to differential and functional equations in one variable. Theorem A and some previous results of the author can then be used to prove the general

THEOREM B. A copula C is a solution of the system (*) if and only if it is representable as an ordinal sum of C_1 and the copula $C_2(x,y) = \text{minimum}\ (x,y)$.

This result has some applications to semigroups of probability distribution functions.
(Received October 4, 1974.

*75T-B15 Herbert Halpern, University of Cincinnati, Cincinnati, Ohio 45221.
Essential central range of a von Neumann algebra.

Let A be a von Neumann algebra with center Z, let I be a central ideal of A, and let x be an element of A. (For terminology refer to H. Halpern, Pacific J. Math. 43(1972), 349-380.) Then an element z in Z is in the essential central range of x modulo I if and only if, given $\varepsilon > 0$, there is a projection e in A with dim e > dim I such that $||e(z - x)e|| < \varepsilon$. For properly infinite A, one obtains the following corollaries. (i) The intersection of Z with the weak closure of the convex hull of $\{uxu^* | u \text{ unitary in } A\}$ equals the intersection of Z with the weak closure of $\{uxu^* | u \text{ unitary in } A\}$. (ii) An element x of A is a self-adjoint commutator in A, i.e., there are $y = y^*$ and z in A with $x = yz - zy$, if and only if the essential central range of x modulo the strong radical of A contains 0. (Received October 15, 1974.)

75T-B16 C.J. Mozzochi, Box 1315, Hartford, Connecticut, 06101
Another approach to Some Recursion Theorems of Landau.

In this note a new proof is provided for Theorem 4, Theorem 28, Theorem 275 and Theorem 276 in Landau's Grundlagen Der Analysis. Landau's original proofs of these theorems although very elementary are nevertheless rather subtle. My proofs are equally elementary, but I have found that by combining portions of Landau's original proofs with a very straightforward technique of elementary set theory all subtleties can be eliminated. (Received October 16, 1974.)

75T-B17 S.K. SINGH and G.P. BARKER, University of Missouri-Kansas City, Kansas City, Missouri 64110. Slowly Changing Functions and Their Applications, Preliminary report.

A function of $L(r)$ positive and differentiable for $r > 0$ is called a slowly changing function if $\frac{rL'(r)}{L(r)} \to 0$ as $r \to \infty$. Utilizing the properties of slowly changing functions and by taking $L(r)r^{\rho}$ as the comparison function, the growth of an entire function $f(z)$, its derivative $f'(z)$ and the distribution of a-points have been studied. A typical result is the following. If $f(z)$ is an entire function of non-integral order ρ, then

$$\sigma = \limsup_{r\to\infty} \frac{\log M(r,f)}{r^{\rho}L(r)} \text{ is } 0, \infty \text{ or } 0 < \sigma < \infty \quad \text{if and only if}$$

$$\gamma = \limsup_{r\to\infty} \frac{n(r,a)}{r^{\rho}L(r)} \text{ is } 0, \infty \text{ or } 0 < \gamma < \infty \quad \text{respectively.}$$

(Received October 18, 1974.)

*75T-B18 ROBERT E. HUFF, and PETER D. MORRIS, The Pennsylvania State University, University Park, Pennsylvania 16802. Geometric Characterizations of the Radon-Nikodym Property.

Let X be a Banach space. Theorem 1. X fails to have the Radon-Nikodym Property (RNP) if and only if there is an equivalent norm on X such that the open unit ball contains a closed set whose closed convex hull is the closed unit ball.

Define a point x in a (not necessarily convex) set A to be an extreme point of A if x is not in the convex hull of $A\setminus\{x\}$. Theorem 2. The following are equivalent. (a) X has the RNP; (b) Every closed bounded subset of X has an extreme point; (c) Every closed bounded subset contains an extreme point of its closed convex hull. (For definitions and further information on geometric criteria for the RNP, see R.R. Phelps, "Dentability and Extreme Points in Banach Spaces", J. Fcnl. Anal. 16(1974), 78-90). (Received October 18, 1974.)

*75T-B19 R. S. DAHIYA, Iowa State University, Ames, Iowa 50010. Nonoscillation generating delay terms in even order differential equation.

The purpose of this paper is to consider a general equation

$$y^{(2n)}(t) - \sum_{i=1}^{n} p_i(t)y(g_i(t)) = f(t), \quad (n \geq 1 \text{ an integer})$$

to find conditions which will ensure that bounded non-oscillatory solutions approaches zero and to find conditions such that nonoscillatory solutions do not approach zero. (Received October 21, 1974.)

75T-B20 R. S. DAHIYA, Iowa State University, Ames, IA 50010 and BHAGAT SINGH, University of Wisconsin, Manitowoc, WI 54220. Certain results on nonoscillation and asymptotic nature of delay equations.

In the first part of this paper the authors find conditions for the nonoscillatory solutions of the delay equation

$$y''(t) + a(t)y_{\tau}(t) = f(t) \quad , \quad y_{\tau}(t) = y(t-\tau(t))$$

to be asymptotic to a linear function of t. This result is further generalized. The second part of the paper is devoted to proving a sort of Liapunov inequality for the above equation.

In the third part conditions have been found to ensure that non-oscillatory solutions of this equation are integrable. (Received October 21, 1974.)

75T-B21 M. D. GUAY, University of Maine at Portland-Gorham, Portland, Maine, 04103 and S. A. NAIMPALLY, Lakehead University, Thunder Bay, Ontario, P7B 5E1, Canada
Hahn-Banach theorem implies linear topological space.

Let X be a set having both a topology and a convexity. We give necessary and sufficient conditions that X is isomorphic to a convex subspace of a LTS or isomorphic to a LTS. Essentially these conditions are those which are necessary and sufficient for the linearization of a convexity space (obtained recently by P. Mah, S. A. Naimpally and J. H. M. Whitfield) together with the Hahn-Banach theorem. (Received October 21, 1974.)

*75T-B22 GERALD ANDERSON, Massachusetts Institute of Technology, Cambridge, Massachusetts. Klein Manifolds.

A Klein manifold of (complex) dimension n is a manifold defined locally by coordinate transformations $\mathbb{C}^n \to \mathbb{C}^n$ which are either holomorphic or anti-holomorphic in each variable. The idea of a Klein surface (n=1) originated with Klein, and has been studied by Schiffler and Spencer in their book "Functionals of Finite Riemann Surfaces", Princeton (so called non-orientable Riemann surfaces) and by Alling and Greenleaf ("Foundations of the Theory of Klein Surfaces", Springer) and applied to real function fields.
Theorem. If M is a Klein n-manifold, then M has a 2^n-fold covering $\tilde{M} \to M$, where $\tilde{M}$ is a complex manifold. This reduces the study of Klein manifolds to that of complex manifolds. (Received October 22, 1974.)

*75T-B23 ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620.
Equations possessing at most one positive solution. Preliminary report.

Theorem. Assume that the equation (*) $x^{(n)} + H(t,x) = 0$, n even, has no eventually positive solutions. Moreover, assume that (i) P is defined, positive and n times continuously differentiable on $[0,\infty)$ with $P^{(n)}(t) \equiv Q(t)$ and $\liminf_{t\to\infty} P(t) = 0$; (ii) $H(t,u)$ is defined and continuous on $[0,\infty) \times R^n$, and is continuously differentiable there w.r.t. u, and $H_1(t,u) \equiv (\partial/\partial u)H(t,u)$ is nonnegative and increasing w.r.t. u. Moreover, $uH(t,u) > 0$ for $u \neq 0$ and $\int^\infty t^{n-1} H_1(t,P(t))dt < \infty$. Then the equation $x^{(n)} + H(t,x) = Q(t)$ can have at most one positive solution. The second order version of this result has been proved by Atkinson in a paper to be published. Atkinson made use of Sturm's comparison theorem which does not hold for nth order equations. Here use is made of certain recent results of the author [Proc. Amer. Math. Soc., 33 (1972), 377-383]. (Received October 24, 1974.)

*75T-B24 JANG-MEI GLORIA WU, University of Illinois, Urbana, Illinois 61801,
Convexity of Harmonic Functions.

Let G be an n-ply connected domain on which the Dirichlet problem is solvable; m be a harmonic measure on G, continuous on $\overline{G}$; u be a harmonic function on G. We use ℓ_α to denote the level set of m on which m takes the value α, S to denote the set $\{m(z): \frac{\partial}{\partial x} m(z) = \frac{\partial}{\partial y} m(z) = 0\}$ and $V(\alpha)$ to denote the total variation of u on ℓ_α. Conclusion: $V(\alpha)$ is

convex on $(0,1)\backslash S$. The convexity can not be extended across S as we see in the example $m(z) = \log |z^2-1|$, $u(z) = \operatorname{Re} z$. (Received October 24, 1974.)

*75T-B25 MARK J. CHRISTENSEN, Wayne State University, Detroit, Michigan 48202. Kolmogorov extension theorems and algebraic models for operator-valued measures.

The results of Benioff (J. Mathematical Phys. 13 (1972), 231-242) on Kolmogorov extension theorems for normalized operator-valued measures on the directed family of spaces (R_n, B_n), $n = 1,2,\ldots$, are generalized to include all Polish spaces. A theorem of Bochner-type for operator-valued measures is obtained. The notion of an algebraic model for a measure space (Dinculeanu and Foiaş, Illinois Math. J. 12 (1968), 340-351) is generalized to the operator-valued case; and the results of Schreiber, et al. (Trans. Amer. Math. Soc. 158 (1971), 93-105) are extended to the case of operator-valued probability measures.

(Received October 24, 1974.)

*75T-B26 R. P. SINGAL, Guru Nanak College, Ferozepore Cant, 152001, India. Some transformations of hypergeometric series.

Starting from a known result ("Higher transcendental functions", Vol. I, p. 112(18)) and using Euler's first integral, a transformation formula connecting ${}_3F_2$ with ${}_4F_3$ without any conditions on the parameters is obtained. From this relation a sum for ${}_4F_3$ and a Saalchutzian sum for nonterminating series are deduced. Another relation between ${}_3F_2(\frac{1}{2})$ and ${}_3F_2(1)$ is also obtained. A corresponding transformation for double hypergeometric series connecting $F^{2,2}_{2,1}$ with $F^{1,2}_{1,1}$ is given. A sum of $F^{2,2}_{2,1}$ is then obtained. (Received October 24, 1974.) (Author introduced by Dr. Brij M. Nayyar.)

*75T-B27 URI FIXMAN and G.K.R. RAO, Queen's U., Kingston, Canada, K7L 3N6. The numerical range of compact operators in L_p-spaces. Preliminary report.

Let X be a complex normed linear space with a unique duality map J. Suppose that $A \in B(X)$, with numerical range $W(A)$, satisfies (i) $\Lambda(A) = \sup \operatorname{Re} W(A)$ is attained as a maximum. A variational equation for $\Lambda(A)$ is derived under mild regularity assumptions on J. The latter hold for $X = L_p(\Omega,\Sigma,\mu)$, $\mu \geq 0$, $2 < p < \infty$. For $x \in L_p$ put $K(x) = |x|^{p-2}\bar{x} \in L_q$, $q = p/(p-1)$. Then, provided (i) holds, $\Lambda(A)$ is the largest λ for which the equations (1) $A^*(K(x))(w) = 0$ a.e. in $\{w \in \Omega : x(w) = 0\}$; (2) $\operatorname{Re}(q^{-1}A(x)K(x) + p^{-1}xA^*(K(x))) = \lambda|x|^p$; (3) $\operatorname{Im}(A(x)K(x) - xA^*(K(x))) = 0$ have a solution $0 \neq x \in L_p$. If $1 < p < 2$ $(p=2)$, (1) is replaced by a dual (self dual) equation. A sufficient condition for (i) is that A be compact with $\Lambda(A) \neq 0$ and (ii) K is weakly sequentially continuous. For $p \neq 2$, (ii) holds iff every measurable $E \subset \Omega$ with $0 < \mu(E) < \infty$ contains an atom. $\Lambda(A)$, and in some cases $\overline{\operatorname{conv}}\, W(A)$, is computed for certain operators in ℓ_p^n and ℓ_p. (Received October 25, 1974.)

*75T-B28 LES A. KARLOVITZ, University of Maryland, College Park, Maryland 20742. On nonexpansive mappings.

A generalized Hilbert space property is used to analyze nonexpansive mappings in certain situations. In particular it is shown that in ℓ_1 and in an interesting new space defined recently by R. C. James, a nonexpansive self-mapping of a bounded weak* closed convex subset has a fixed point. (Received October 29, 1974.)

75T-B29 JERRY LESTER, Western Illinois University, Macomb, Illinois 61455. Well-capped structure of $L_1^+(S,u)$.

Let S be a σ-finite measure space with positive measure u. It is shown that $L_1^+(S,u)$ is well capped in the weak-topology if and only if u is purely atomic. In particular, if u equals the counting measure and $S = \{1,2,3,\ldots,n,\ldots\}$, we have that ℓ_1^+ is well capped in the norm-topology. This result also implies that every norm-closed, bounded, convex subset of $L_1^+(S,u)$ is the closed convex-hull of its extreme points. This completes the analysis of the $L_p^+(S,u)$ cones for $1 < p \leqq \infty$, begun by L. Asimow in 1968. By similar methods, it can be shown that every norm-closed, bounded, convex subset of c_0^+ or ℓ_∞^+ has an extreme point if it contains a point of ℓ_1^+. (Received September 30, 1974.)

75T-B30 MOSTAFA A. ABDELKADER, 25 Sh. Champollion, Alexandria, Egypt. Circular and hyperbolic functions satisfying $f(x) = f(p - x)$.

The functional equation $g(x) = g(p + x)$, where p is real, characterizes real periodic functions, such as combinations of the circular functions. The companion functional equation (E) $f(x) = f(p - x)$ is satisfied by $\sin x$ (with $p = \pi$), and by $\cos x$ (with $p = 2\pi$). It is shown that certain combinations of the hyperbolic functions (or the exponential function) also satisfy (E). Two examples are: (1) $f(x) = a\sinh x + b\cosh x$, with $p = \log((b - a)/(b + a))$, $b^2 > a^2$, and (2) $f(x) = a\sinh^2 x + b\sinh x\cosh x + c\cosh^2 x$, with $2p = \log((a - b + c)/(a + b + c))$, $(a + c)^2 > b^2$. Moreover, the generalized Euler expression: $a\cos x + ib \cdot \sin x$ where $a^2 > b^2$, is shown to satisfy the functional equation $f(x) = f(iq - x)$, where $q = \log((a + b)/(a - b))$. (Received October 7, 1974.)

*75T-B31 H. BANILOWER, Baruch College of CUNY, New York, N.Y. 10010. Isomorphisms and simultaneous extensions in C(S), Preliminary report.

For notation see Proc. Amer. Math. Soc. 36 (1972), 451-455. Suppose h maps a subspace A continuously into the completely regular space S so that A and h(A) are completely separated in S. Let Q be the quotient space of S gotten by identifying a with h(a) for all a in A. Consider: (1) there exists a simultaneous extension from C(A) into C(S); (2) h has a continuous extension mapping a clopen subset $C \supset A$ into $S - C$; there exists an isomorphism of C(S) onto itself, taking C(Q) onto $C(S \| A)$, which is the identity on $C(S \| h(A))$. Then (1) implies (3) (whence C(Q) is complemented in C(S)) and (2) implies (3). (3) implies (1) providing A and h(A) are normally embedded in S and h is a homeomorphism. (Received October 30, 1974.)(Author introduced by Tsu-Chih Wu)

75T-B32 MOHAMED ABDEL-HAMEED, University of North Carolina, Charlotte, 28223 and F. PROSCHAN, Florida State University, Tallahassee, 32306, Total Positivity Properties of Generating Functions

In this paper, we strengthen results obtained in Keilson (1972), *Annals of Mathematical statistics* 43, 1702-1708. Our main result is: Let $P_0(z) = \sum_{i=0}^{\infty} p_i z^i$ be the generating function of the sequence $\{p_i\}_{i=0}$, with p_i real for $i = 0, 1, ---, N-1$, $p_N > 0$, and $p_i = 0$ for $i = N+1, N+2, \ldots$. Let $p_i(t)$ be defined by $P_0(z+t) = \sum_{i=0}^{\infty} p_i(t) z^i$. Then (a) there exists a smallest nonnegative value t_r^* such that $p_{i+j}(t_r^*)$ has the sign reverse rule property of order r (RR_r) in $i,j = 0,1,2,\ldots$ (see Karlin, 1968, *Total Positivity*, Vol. 1, p. 12, Stanford University Press) for $r = 1,2,\ldots$; (b) $p_{i+j}(t)$ is RR_r in $i,j = 0,1,2,\ldots$ for each fixed $t \geq t_r^*$, $r = 1,2,\ldots$; and (c) $t_1^* \leq t_2^* \ldots$.

(Received October 30, 1974.)

Applied Mathematics

*75T-C1 LOKENATH DEBNATH, East Carolina University, Greenville, North Carolina 27834. Asymptotic Analysis of the Unsteady Hydromagnetic Boundary Layer Flow. Preliminary Report.

This paper presents an asymptotic analysis of the unsteady hydromagnetic boundary layer flow generated in a semi-infinite expanse of an incompressible electrically conducting viscous rotating fluid bounded by a non-conducting infinite rigid disk in the presence of an external magnetic field. The effect of the pressure gradient is included in this paper and the solution is calculated asymptotically for small and large times. It is shown that the initial motion for small times consists of the two distinct boundary layers which remain unaffected by the external magnetic field and rotation. It is also shown that the ultimate steady-state boundary layer flow is established through the propagation of diffused hydromagnetic waves and the inertial oscillations which decay eventually within the ultimate steady-state boundary layers. The effects of the external magnetic field and rotation on the unsteady flow are also examined. (Received September 23, 1974.)

*75T-C2 GAIL A. CARPENTER, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 Traveling Wave Solutions of Nerve Impulse Equations

Isolating block techniques give sufficient conditions for the existence of homoclinic, heteroclinic, and periodic solutions of first order autonomous systems. These results prove the existence of homoclinic (single-impulse wave train), heteroclinic, and periodic traveling wave solutions of the Nagumo equation. In addition, they prove the existence of homoclinic solutions of the Hodgkin-Huxley equations which are wave trains of any length ≥ 1; and of periodic solutions when $\tau_n \gg \tau_h$ or $\tau_h \gg \tau_n$.

(Received September 27, 1974.)

*75T-C3 H.M. CHAWLA, Indian Institute of Technology, Hauz Khas, New Delhi-29, India and T.R. RAMAKRISHNAN, Indian Institute of Technology, Hauz Khas, New Delhi-29, India. Numerical evaluation of Cauchy principal value integrals.

A systematic development of quadrature formulas for Cauchy principal value integrals is given using the method of subtracting out the singularity; a known quadrature formula (for the proper integral) is modified for the evaluation of the Cauchy principal value integral. Several particular modified quadrature formulas are discussed for Cauchy principal value integrals with finite and infinite ranges of integration, and those for periodic functions. The results of various previous papers, derived through special techniques, come out as particular cases of our modified formulas.

(Received October 2, 1974.) (Authors introduced by Dr. S.K. Bajpai)

*75T-C4 J. M. Bownds, University of Arizona, Tucson, Arizona 85721 and J. M. Cushing, University of Arizona, Tucson, Arizona 85721. On the behavior of solutions of predator-prey equations with hereditary terms. Preliminary report.

Some global properties of solutions of the classical integrodifferential systems introduced by Volterra in his study of two species predator-prey populations are studied. It is shown for large logistic loads that the predator goes to extinction and the prey tends to its carrying capacity. By use of a nonlinear approximation it is shown that for smaller logistic loads a "critical point" is asymptotically stable while for sufficiently small logistic loads this point is unstable. These cases are demonstrated numerically for the original integrodifferential system using parameters which were computed on the basis of experimental data of S. Utida for Bean Weevil vs. Braconid Wasp interactions. Moreover, numerical solutions suggest further varied behavior of solutions of this system.
(Received October 7, 1974.)

75T-C5 R. S. FALK, Rutgers Univ.,New Brunswick, N. J. 05903; J. T. KING, Univ. of Cincinnati, Cincinnati, Ohio 45221. An Analysis of the Penalty Method and Extrapolation for the Stationary Stokes Equations.

A major difficulty in the finite element method for approximating the solution of stationary Stokes equations is the treatment of the incompressibility condition div v = 0. In this paper we use a penalty method approach to eliminate this difficulty. In addition we show how extrapolation can be used to compute higher order accurate solutions using matrices of lower condition number than arise in the simple penalty method. This approach is motivated by the work of King (Abstract 74T-C12, these Notices 21(1974)) and relies on a regularity result of Temam (Lecture note #9, University of Maryland (1973)). (Received October 21, 1974.)

75T-C6 J WOLFOWITZ, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS 61801 Signaling over a Gaussian channel with feedback and autoregressive noise II

The author proves the conjecture announced in his previous abstract with the same title namely, the unique optimal linear signaling scheme is shown to be optimal among all signaling schemes whether linear or not Thus C of the previous abstract is the capacity of the channel.
(Received October 21, 1974.)

*75T-C7 H. K. VERMA, Punjab Agricultural University, Indhiana, India. Deep-bed grain drying—a mathematical model.

Here we consider coupled differential equations given by Crank ("The mathematics of diffusion", Oxford Univ. Press, New York) characterising radial moisture and heat diffusion which arise during drying spherical porous bodies in deep-bed grain drying systems. Using Laplace transforms, solutions of the system have been obtained in a closed form which is valid for both long and short times. The problem concerns food-grain storage which is of national and international importance. (Received October 7, 1974.)

75T-C8 DAVID S. LAWRENCE, Courant Institute, New York University, New York, New York 10012. Continuous dilemma games. Preliminary report.

Continuous versions of Prisoner's Dilemma can be defined as modified pursuit games between $[0,1]$-valued real functions. Here we can only indicate a generous response function p_1 equal to $\sup_{0 \leq T < t}(\int_T^t \varphi dt/(t-T)) = \sup_T \hat{\varphi}(t;T)$, which picks up the history of any partner-strategy φ, secured by damping opponent's advantage in terms of $p_2 = \inf_T \hat{\varphi}$, which solves some versions by playing $p = p_2 + e^{-kt}(p_1 - p_2)$, for example. As readers will also see, opponent has nothing better to aim for than cooperation in the specialized case where play is governed by choice of initial condition and a team of other constant parameters defining p' up to the constraint $p \in [0,1]$, linearly in terms of $\pm(p-\varphi)$, or p if $p = \varphi$. We have assumed that noncooperative payoff for p is of the order of $\$_1 = \pm\{\lim T^{-1} \int_0^T (p-\varphi)^2\}^{\frac{1}{2}}$, for $S = \{p \lesseqgtr \varphi\}$, or $\{\lim T^{-1} \int_0^T p^2\}^{\frac{1}{2}} = \$_2$ cooperative payoff when $\$_1$ vanishes, for both sides; and ignored time lag. (Received November 5, 1974.)

*75T-C9 ALI KYRALA, Arizona State University, Tempe, Arizona 85281
Product of general Lorentz transformations

The relativistic Thomas precession is based upon a limit of successive Lorentz transformations. The product of two general Lorentz transformations is useful in this and other problems of relativity. This transformation product may be written dyadically.

$$\bar{L}\, L = \begin{bmatrix} \underline{a} & -a\,\underline{\bar{\mu}} \\ -\underline{a}\,\underline{\bar{\mu}} & I + (\underline{a} - 1)\,\underline{\bar{\nu}}\,\underline{\bar{\nu}} \end{bmatrix} \begin{bmatrix} a & -a\,\bar{\mu} \\ -a\,\bar{\mu} & I + (a-1)\,\bar{\nu}\,\bar{\nu} \end{bmatrix} = \begin{bmatrix} f & \bar{F} \\ \underline{\bar{F}} & \Phi \end{bmatrix}$$

with a, $\bar{\mu}$, $\bar{\nu}$ defined by $\bar{\mu}\, c = \bar{v} \neq \bar{v}$, $\mu = |\bar{\mu}|$, $\mu\, \bar{v} = \bar{\mu}$, $a^2(1 - \bar{\mu} \cdot \bar{\mu}) = 1$ mutatis mutandis for $\underline{a}$, $\underline{\bar{\mu}}$, $\underline{\bar{\nu}}$ and

$$f = \underline{a}\, a\, (1 + \underline{\bar{\mu}} \cdot \bar{\mu})$$

$$\bar{F} = -\underline{a}\, (a\, \bar{\mu} + \underline{\bar{\mu}}) + (a - 1)\, (\underline{\bar{\mu}} \cdot \bar{\nu})\, \bar{\nu}$$

$$\underline{\bar{F}} = -a\, (\underline{a}\, \underline{\bar{\mu}} + \bar{\mu}) + (\underline{a} - 1)\, (\bar{\mu} \cdot \bar{\nu})\, \underline{\bar{\nu}}$$

$$\Phi = I + \underline{a}\, a\, \underline{\bar{\mu}}\, \bar{\mu} + (\underline{a} - 1)\, \underline{\bar{\nu}}\, \underline{\bar{\nu}} + (a - 1)\, \bar{\nu}\, \bar{\nu} + (\underline{a} - 1)(a - 1)(\underline{\bar{\nu}} \cdot \bar{\nu})\, \underline{\bar{\nu}}\, \bar{\nu}$$

and naturally contains no restriction on the directions or magnitudes of $\bar{v}$ and $\underline{\bar{v}}$.
(Received October 30, 1974.)

Geometry

*75T-D1 KINETSU ABE, University of Connecticut, Storrs, Connecticut 06268 and JOSEPH ERBACHER, University of Connecticut, Storrs, and U. of So. Calif., L. A.
Some examples of non-regular contact structures on Brieskorn manifolds

In a previous paper, we showed that all Brieskorn manifolds admit non-regular normal almost contact structures. See Notices 74T-D19. We announce here that a three dimensional Brieskorn manifold admits a non-regular contact structure if its fundamental group is of finite order but not cyclic. The proof consists of the following two steps. First, we show that every Brieskorn manifold admits a contact structure. This structure is the same as the one constructed by S. Sasaki and C-J Hsu. See "On a property of Brieskorn manifold", preprint. Next, we show that such a Brieskorn manifold cannot admit a Boothby-Wang fibration (Ann. of Math. 68 (1958), pp. 721-734) if its fundamental group satisfies the above conditions. A typical example is the Brieskorn manifold B defined by the polynomial $f(z_0, z_1, z_2) = z_0^2 + z_1^3 + z_2^5$. In this case, the fundamental group is isomorphic to $SL(2, Z_5)$.

(Received September 18, 1974.)

*75T-D2 VASANT A. UBHAYA, Box 1045, Washington University, St. Louis, Missouri, 63130
Duality in approximation and conjugate cones in normed linear spaces.

Let $(X, ||\cdot||)$ be a real or complex normed linear space and L be a nonempty subset of its continuous dual X* such that $L \neq \{0^*\}$. Let $K=\{f \varepsilon X: \ Re(x^*(f)) \leq 0$ for all $x^* \varepsilon L\}$. Let cc(L) denote the smallest convex cone containing L, co(L), the convex hull of L, $S^*=\{x^* \varepsilon X^*: ||x^*|| \leq 1\}$ and $S_1^*=\{x^* \varepsilon X^*: \ ||x^*||=1\}$. Let Cl and Cl' denote respectively the weak* and strong closure in X*. Theorem 1: For all spaces X, the following conditions (a), (b), (c) are equivalent and (d) implies (a). If X is a Hilbert space then all four are equivalent.

(a) $\inf_{f \varepsilon K} ||g-f|| = \sup_{x^* \varepsilon co(L), x^* \neq 0^*} Re(x^*(g)/||x^*||)$ for all $g \varepsilon X-K$. (Note that $K \neq X$)

(b) $S^* \cap Cl(cc(L)) = Cl(S^* \cap cc(L))$ (c) $S_1^* \cap Cl(cc(L)) \subset Cl(S_1^* \cap cc(L))$ (d) $Cl(cc(L)) = Cl'(cc(L))$.

Theorem 2: If X is a Hilbert space and L is orthonormal in X* then (a), (b), (c), (d) of Theorem 1 hold. Theorem 3: Let X be a normed linear space. Suppose that the equivalent conditions (a), (b), (c) hold and, given every $x^* \varepsilon co(L)$ there exist x_i^*, $i=1,2,\ldots,n$ in L and $\mu_i \geq 0$ such that $x^* = \Sigma_{i=1}^n \mu_i x_i^*$ and $||x^*|| = \Sigma_{i=1}^n \mu_i ||x_i^*||$, then $\inf_{f \varepsilon K} ||g-f|| = \sup_{x^* \varepsilon L, x^* \neq 0^*} Re(x^*(g)/||x^*||)$ for all $g \varepsilon X-K$. (Examples illustrate results).

(Note: $\{x^* \varepsilon X^*: \ Re(x^*(f)) \leq 0$ for all $f \varepsilon K\} = Cl(cc(L))$ is called the conjugate cone of K.)

(Received October 24, 1974.)

Logic and Foundations

*75T-E1 HARVEY FRIEDMAN, State University of New York at Buffalo, Amherst, New York 14226.
The Complexity of Explicit Definitions.

The Beth definability theorem cannot be refined to provide bounds on the complexity of an explicit definition effectively from the given implicit definition, even if the implicit definitions are assumed to be universal. There is no bound on the complexity of an explicit definition even if the implicit definition is assumed to consist of four universalized equations. (Received August 5, 1974.) (Author introduced by Mr. Dallas Webster.)

75T-E2 JOHN R. COWLES, The Pennsylvania State University, University Park, Pa. 16802
Real closed fields, Archimedean fields, and logics extending first order logic, Preliminary report.

(1) L(Q), (2) L(R), (3) L(W) are the logics obtained from first order logic with equality by adding respectively (1) the quantifier "There exist infinitely many...", (2) the Ramsey quantifier (ie. $Qxy\varphi$ means "There exist an infinite set, X, such that for distinct $x,y \in X$, $\varphi(x,y)$ holds."), (3) second order variables and quantifiers for finite sets. Theorem. The theory of real closed fields in the logic L(Q) is complete. The proof is by elimination of quantifiers. Corollary. Archimedean fields do not form an elementary class in L(Q). Theorem. Archimedean fields do form a basic elementary class in the logic L(R) [L(W)]. Corollary. The theory of real closed fields is not complete in the logic L(R) [L(W)]. It is well known that L(R) and L(W) are extensions of L(Q). The above results show that they are both proper extensions of L(Q). (Received September 18, 1974.)

75T-E3 WILLIAMS FORREST, Mathematics Department, Simon Fraser University, Burnaby, B. C. V5A 1S6. A Definability Result for Strongly Minimal Sets

Let T be a first order theory and $\mathfrak{A}$ a model of T. Suppose that $\psi(v_0) \in L(\mathfrak{A})$ is strongly minimal. If $\mathfrak{A}_1 < \mathfrak{A}$ then there is $\psi_1(v_0) \in L(\mathfrak{A}_1)$ such that $\psi(\mathfrak{A}) \cap \mathfrak{A}_1 = \psi_1(\mathfrak{A}_1)$. (Received September 23, 1974.)

75T-E4 SAHARON SHELAH, The Hebrew University, Jerusalem, Israel. Various results in mathematical logic

Th. 1: If λ is singular, D a λ-good filter then D is $\lambda^{\pm}$ good.
Th. 2: In Keisler's ordering of theories, the theory of any infinite ordering is maximal (see Keisler, J. Symb. Logic 32 (1967) 23-46).
Th. 3: Let L be a finite language, with only one one-place function and any predicates. Then we can effectively decide whether a sentence of the form $(\exists \bar{z})(\forall x)\exists \bar{y}\varphi$ (φ quantifier free) has an infinite model [a finite model].
Th. 4: For a countable model M the following are equivalent (A) The number of (M,P) ($P \subseteq |M|$) up to isomorphism is $< 2^{\aleph_0}$ (B) The number of (M, P) ($P \subseteq |M|$) up to isomorphism is $\aleph_0$ (C) M is a reduct of a definable expansion of $N_0 + \sum_n N$, where N_0, N are finite.
Th. 5: There is an ordered set I, $|I| = \aleph_1$, so that I^2 is the union of $\aleph_0$ chains (in the natural partial order)
Th. 6: For singular λ, any λ - free algebra is λ^+ - free, and any λ^+ - free algebra is $L_{\infty,\lambda}$ - equivalent to a free one (Received October 4, 1974.)

75T-E5 STEVEN GARAVAGLIA, Yale University, New Haven, Connecticut 06520. Two results in nonstandard analysis. Preliminary report.

(1) Let R be the ring of integers, and let *R be an ultrapower of R with respect to an ω-incomplete ultrafilter on an infinite index set. Then there is a compact Hausdorff space X such that the Čech homology group $\check{H}_0(X, {}^*R)$ is not isomorphic to the McCord homology group $\bar{H}_0(X, {}^*R)$. (2) Let p(z) be an internal polynomial of finite rank s in some enlargement *C of the complex plane. Let g(z) be an internal univalent conformal mapping such that p(g(z)) is defined and finite for all finite z in *C. Then there are standard complex numbers $a_1, \ldots, a_s$ and a standard polynomial $f(z_1, \ldots, z_{s+1})$ such that ${}^o(p(g(z))) = f(e^{a_1 z}, \ldots, e^{a_s z}, z)$ for all standard complex numbers z. (Received October 29, 1974.)

75T-E6 WILLIAM J. LENIHAN, University of Calgary, Calgary, Alberta, Canada. The ordered reachability problem is unsolvable. Preliminary report.

Let I, N be the set of integers, nonnegative integers respectively. Definition. (1) $W = \{\vec{w}_i | 1 \leq i \leq m \ \& \ \vec{w}_i \in I^n \ \& \ m \in \omega \ \& \ i \in \omega\}$ is an ordered n-vector addition system. (2) If $\vec{a} = \langle a_1, \ldots, a_n \rangle$ we say that $\vec{a} \geq 0$ if and only if $a_i \geq 0$ for all $i, 1 \leq i \leq n$. (3) Given $\vec{a}_0 \in N^n$ let $\vec{a}_{k+1} = \vec{a}_k + \vec{w}^*$ where $\vec{w}^*$ is the first $\vec{w}_i \in W$ such that $\vec{a}_k + \vec{w}_i \geq 0$. (4) The reachability set for the ordered vector addition system, W, starting at $\vec{a}$, is $R_W^{\vec{a}} = \{\vec{a}_k | k \in \omega\}$. (5) The ordered reachability problem is the question of deciding for a given $(\vec{a}, W)$ and $\vec{x} \in N^n$, whether or not $\vec{x} \in R_W^{\vec{a}}$. Theorem. The ordered reachability problem is recursively unsolvable. That is, there exist systems $(\vec{a}, W)$ such that there is no algorithm to decide whether, given $\vec{x} \in N^n$, $\vec{x}$ is in $R_W^{\vec{a}}$. (Received October 23, 1974.)

75T-E7 BARRY E. JACOBS, Courant Institute, New York University, New York, New York 10012. α-complexity classes.

For α Σ_1 admissible, Φ^α is an α-complexity measure and C_t is the α-complexity class bounded by t. A set of functions $\{f_\epsilon\}$ which can be α-recursively enumerated and for which we can α-recursively decide for all ϵ, β and γ whether $f_\epsilon(\beta) = \gamma$ is called an α-measured set. Theorem 1 (α-Union Theorem). Let $\{f_\epsilon | \epsilon < \alpha\}$ be an α-recursively enumerable set of α-recursive functions such that for each ϵ and β, $f_\epsilon(\beta) > f_\tau(\beta)$ for $\tau < \epsilon$. Then there exists an α-recursive function $t(\beta)$ such that $C_t = \bigcup C_{f_\epsilon}$. Theorem 2 ($\alpha$-Naming Theorem). For each α-measure Φ^α there exists an α-measured set naming every α-complexity class. These results are proved using methods like those of Sacks and Simpson for the α-Post problem (Ann. Math. Logic 4, 343-367). (Received October 30, 1974.)

*75T-E8 Robert W. Button, Carnegie-Mellon University, Pittsburgh, Pa., 15213 $\mathfrak{U}$-Filters and $\mathfrak{J}$-Filters.

Let $\mathfrak{U} \subseteq 2^X$ be non-empty and closed under finite intersections. A $\mathfrak{U}$-filter $\mathfrak{F}$ on the space X is a non-empty subset of $\mathfrak{U}$ such that: (i) if $G, H \in \mathfrak{F}$, then $G \cap H \in \mathfrak{F}$, and (ii) if $G \in \mathfrak{F}$ and $G \subseteq H$ for some $H \in \mathfrak{U}$, then $H \in \mathfrak{F}$. A $\mathfrak{J}$-filter is defined in the same way except that $\mathfrak{J}$ is required to be closed under finite unions as well as intersections.

In this paper we extend the non-standard theory of $\mathfrak{U}$-filters and $\mathfrak{J}$-filters, characterizing $\mathfrak{U}$-ultrafilters, prime $\mathfrak{J}$-filters, examining nuclei of $\mathfrak{U}$-filters, etc. (Received October 30, 1974.)

75T-E9 ROBERT I. SOARE, University of Illinois, Chicago, Illinois 60680. Incomplete recursively enumerable sets.

One of the fundamental open questions of recursive function theory is to determine the relationship between the structure of an r.e. set and its degree. (See our "Automorphisms of the lattice of recursively enumerable sets," Bull. Amer. Math. Soc. 80(1974), 53-58.) The following result constitutes a further step in this general program. Theorem. For any r.e. set A and nonrecursive r.e. set C there is an automorphism Φ of the lattice of r.e. sets $\mathcal{E}$ such that $C \not\equiv_T \Phi(A)$. Corollary 1 (Sacks). There exists an incomplete maximal set. Corollary 2. No property of r.e. sets which guarantees completeness (such as creativeness or effective simplicity) is invariant under automorphisms of $\mathcal{E}$. The proof combines our

automorphism machinery with a variation of the Sacks "infinite-injury" priority method, where we must use a series of finite-injury priority arguments within the main infinite-injury argument. This insures that for each $e \in \omega$ the set I_e of "injuries" to the negative requirement $N_e : \{e\}^{\Phi(A)} \neq C$ is large enough to accommodate the opposing positive requirements, but small enough to satisfy N_e. (Received October 30, 1974.)

*75T-E10 A. H. BRADY, University of Nevada, System Computing Center, Reno, Nevada 89507. The solution to Rado's "Busy Beaver Game" is now decided for $k = 4$. Preliminary report.

The well-defined but noncomputable functions $\Sigma(k)$ and $S(k)$ defined by T. Rado (Bell System Tech. J., May 1962) as the "score" and "shift number" for the "Busy Beaver Game" have heretofore been known only for $k \leqq 3$ (Lin and Rado, JACM 12, April 1965). The largest known lower bounds yielding the relations $\Sigma(4) \geqq 13$ and $S(4) \geqq 106$ were discovered by this author (these *Notices* 12, June 1965, p.476), and it was conjectured on the basis of significant progress toward a solution for $k = 4$ that $\Sigma(4) = 13$ and $S(4) = 106$ (Brady, IEEE Trans. EC-15, October 1966). The blank tape halting problems for the remaining subset consisting of 5,820 4-state Turing machines were submitted for decision to a set of computer programs which searched for observed general patterns of behavior to which explicitly mechanized inductive proofs could be applied. A final remaining 218 cases not decided by the computer were decided by direct examination. The key to the computerized technique used was the treatment of each machine and its tape as a representation of a growing chain of self-replicating cellular automata. It is now proved that $\Sigma(4) = 13$ and $S(4) = 106$ (or 107 counting the "halt"). (Received October 21, 1974.)

Statistics and Probability

*75T-F1 TEPPER GILL and A.T. BHARUCHA-REID, Wayne State University, Detroit, Michigan 48202. Tensor products of contraction semigroups associated with some Markov processes.

In this paper results on the tensor products of contraction semigroups on Hilbert spaces (Gill and Bharucha-Reid, Notices Amer. Math. Soc. 20 (1973), A-581) are applied in the operator theory of Markov processes. In particular, we study the tensor products of contraction semigroups on ℓ_2 associated with Markov processes with denumerable state spaces, and on H_2 associated with certain branching processes. (Received October 22, 1974.)

75T-F2 THEODORE E. HARRIS, University of Southern California, Los Angeles, California, 90007. Reciprocal processes II. (Countable case). Preliminary Report.

Notation as in I (these NOTICES August, 1974, A-502) except now Z is the d-dimensional integer lattice. Let $\mathcal{A}_0$ be the generator of an interaction process moving only on a finite subset V of Z and affected only by the condition of V. Let $\mathcal{A}_x$ be $\mathcal{A}_0$ translated to x, $\mathcal{A} = \sum_{x \in Z} \mathcal{A}_x$. Suppose $\mathcal{A}$ has a reciprocal. Then $\mathcal{A}$ determines a process ξ_t having a reciprocal ξ_t^*, where

$$P_\xi\{\xi_t \cap \eta = \phi\} = P_\eta^*\{\xi_t^* \cap \xi = \phi\}$$

if $|\xi|$ or $|\eta| < \infty$.

Assume : (a) $\sup_{|\xi| = n} P_Z\{\xi_t \cap \xi \neq \phi\} < 1$, $t > 0$, $n = 1, 2, \ldots$, and (b) $P_\xi\{\xi_t \cap x \neq \phi\} > 0$, $t > 0$, $x \in Z$, $\xi \neq \phi$. Then every invariant probability measure for $\{\xi_t\}$ has the form $\mu = (1-c)\,\delta_\phi + c\,\nu$ for some $c \in [0,1]$, where $\nu = \text{weak} \lim_{t \to \infty} P(t, Z, \cdot)$.

(Received October 29, 1974.)

Topology

*75T-G1 JAMES R. BOONE, Texas A&M University, College Station, Texas 77843.
On the Cardinality Relationships Between Discrete Collections and Open Covers

This paper unifies and extends various theorems which deal with the relationship between the cardinality of discrete collections and the cardinality of open coverings. For this purpose, the class of spaces which are irreducible of order α is defined. This class includes the $\delta\theta$-refinable and the $[\alpha,\infty)$-refinable spaces. Some examples of applications of this class are: a space is $[\alpha,\infty)$-compact if and only if it is irreducible of order α and has the α-BW property and if X is irreducible of order $\Delta(X)$, then $\Delta(X) \leq L(X) \leq \Delta(X)^+$. The open ordinal space $[0,\Omega)$ serves as a model to generate examples of spaces which are irreducible of order α but not irreducible of order β, if $\beta > \alpha$.

(Received September 3, 1974.)

*75T-G2 ROBERT CONNELLY, CORNELL UNIVERSITY, ITHACA, NEW YORK, 14850, AN ATTACK ON RIGIDITY, II. PRELIMINARY REPORT.

This is a continuation of Part I where we have set up the notation and defined the fundamental equations for our attack on rigidity. We show the following: Theorem: If a polyhedral suspension is embedded in R^3, then it is rigid. This is the first non-trivial example, that we know of, where all embeddings of a surface are shown to be rigid. Previously the surfaces were assumed, usually, to be convex.

First we define the notion of generalized volume, which assigns to any orientable polyhedral surface mapped into R^3 a number which is the volume enclosed by the surface when it is embedded. The idea then is to show that if the polyhedral suspension flexes, then it is not an embedding near the suspension points for a trivial kind of flexing, or it has a generalized volume of zero for a non-trivial kind of flexing, and thus it is not embedded in either case. It turns out that the generalized volume is easily defined in terms of variables set up in Part I.

We then give a fairly detailed description of the structure of a flexible suspension. This involves the group operation defined on a non-singular cubic from classical algebraic geometry, and a flow graph where the flow involves these group elements.

(Received September 17, 1974.)

*75T-G3 **RONNIE FRED LEVY, Goucher College, Towson, Maryland 21204.** **A countable space with no point of first countability, Preliminary report.**

A topological space which is not the union of complementary dense subsets is called non-resolvable. Theorem. There is a countable, non-resolvable, extremally disconnected Hausdorff space with no point of first countability. (Received September 20, 1974.)

*75T-G4 DOUGLAS E. MILLER, U.C.B., Berkeley, California 94720. The Invariant $\underset{\sim}{\Pi}^0_\alpha$ Separation Principle in Topology and Logic.

Let X be a Polish space, E an equivalence relation on X which is induced by the continuous action on X of a non-meager topological group. For any collection Γ of subsets of X we write "invΓ" to denote the collection of E-invariant members of Γ. THEOREM 1: For each countable $\alpha > 1$, the collection inv$\underset{\sim}{\Pi}^0_\alpha$ has the first separation property, i.e., disjoint invariant $\underset{\sim}{\Pi}^0_\alpha$ sets can be separated by a set belonging to inv$\underset{\sim}{\Gamma}^0_\alpha \cap$ inv$\underset{\sim}{\Sigma}^0_\alpha$. Theorem 1 is a consequence of the following sharper result; here "$\mathcal{D}_\gamma(\Gamma)$" denotes the γth level of the difference hierarchy based on Γ, "$\underset{\sim}{\Pi}^0_{(\alpha)}$" denotes $\cup\{\underset{\sim}{\Pi}^0_\beta : \beta<\alpha\}$, and "$D^+$" denotes $\{x : (\exists y)(yEx \ \& \ y \in D)$. Theorem 2: For each $\alpha > 1$, each γ, and each $D \in \mathcal{D}_\gamma(\underset{\sim}{\Pi}^0_{(\alpha)})$ there is a set $D^\# \in \mathcal{D}_\gamma(\text{inv}\underset{\sim}{\Pi}^0_{(\alpha)})$ such that $\sim(\sim D)^+ \subset D^\# \subset D^+$; hence inv$\mathcal{D}_\gamma(\underset{\sim}{\Pi}^0_{(\alpha)}) = \mathcal{D}_\gamma(\text{inv}\underset{\sim}{\Pi}^0_{(\alpha)})$ and inv$\underset{\sim}{\Delta}^0_\alpha = \cup\{\mathcal{D}_\gamma(\text{inv}\underset{\sim}{\Pi}^0_{(\alpha)}) : \gamma < \omega_1\}$. For $\alpha = 2$, theorems 1 and 2 hold under the weaker hypothesis that E is any equivalence such that the canonical map $X \to X/E$ is an open or closed map. Our proof uses techniques introduced by R. Vaught in "A Borel Invariantization", AMS Bulletin 79(1973)p.1292. Applying his 3.2 we obtain results in model theory (We have learned that 3(a) is an unpublished result of G. Reyes.): THEOREM 3: For each $\alpha > 1$, (a) The collection of $\underset{\sim}{\Pi}'^0_\alpha$ classes of models (cf. Vaught) has the first separation property and $\underset{\sim}{\Delta}'^0_\alpha = \cup\{\mathcal{D}_\gamma(\underset{\sim}{\Pi}'^0_{(\alpha)}) : \gamma < \omega_1\}$; (b) If X is a logic space then for each γ,m,n, inv$\mathcal{D}_\gamma(\underset{\sim}{\Pi}^0_{(\alpha)}) = \mathcal{D}_\gamma(\underset{\sim}{\Pi}'^0_{(\alpha)})$ and $\mathcal{D}_m(\underset{\sim}{\Pi}^0_n) \cap L_{\omega\omega} = \mathcal{D}_m(\Psi^0_n)$. (Received October 29, 1974.)

*75T-G5 O.T.ALAS, C.A.E.C.E., Calle Belgrano 2211, Buenos Aires, Argentine. Three theorems on sigma-m-expandable spaces. Preliminary report.

Let E be a Hausdorff space and m be an infinite cardinal number. Definition. E is compact-m-expandable (sigma-m-expandable) if foe every locally finite family of compact (respectively, closed sigma compact) subsets of E, $(F_i)_{i\in I}$, with $|I| \leqslant m$, there is a locally finite family of open subsets of E, $(U_i)_{i\in I}$ such that $F_i \subset U_i$, for every $i \in I$. Theorem. If E is countably paracompact or sigma-m-normal, and compact-m-expandable, then it is sigma-m-expandable. Problem: Are sigma-m-normality (respectively, sigma-m-expandability) and compact-m-normality (respectively, compact-m-expandability) equivalent notions?
(Received October 30, 1974.)

*75T-G6 IRA N. ROSENHOLTZ, University of Wyoming, Laramie, Wyoming 82071. *More* evidence of a conspiracy among fixed point theorems.

When we last left John and Martha, they had just learned of a conspiracy between contractions and local contractions: namely, that on compact, connected metric spaces local contractions are actually contractions after a change of metric. Apparently, the cancer is more widespread than had been suspected. It now seems that similar collusion exists between contractive and locally contractive maps. Furthermore, the techniques of the proof have uncovered a new fact in the case: that open locally expansive maps defined on compact, connected metric spaces have fixed points. Details must be withheld for now, for reasons covered by the "national security" blanket.
(Received September 30, 1974.)

*75T-G7 R.F. DICKMAN, JR., Virginia Polytechnic Institute, Blacksburg, Virginia 24061 and J.R. PORTER, University of Kansas, Lawrence, Kansas 66045.
Θ-closed Subsets of Hausdorff Spaces.

A subset is Θ-closed in a space if each point in the complement of the subset has a closed neighborhood disjoint from the subset. Let X be a Hausdorff space, $A \subseteq X$, σX the Fomin H-closed extension of X, and EX the absolute of X with the associated irreducible, Θ-continuous, perfect surjection $\pi: EX \to X$. Theorem. The following are equivalent: (1) $\pi^{-1}(A)$ is compact, (2) A is Θ-closed in some H-closed extension of X, and (3) A is Θ-closed in every H-closed extension of X. Theorem. $E(\sigma X) = \sigma(EX)$ if and only if the subset of non-isolated points of X is Θ-closed in σX. Theorem. An H-closed space is not the countable union of Θ-closed nowhere dense subsets. Theorem. Martin's Axiom is equivalent to every H-closed space with the countable chain condition is not the union of less than c (= cardinality of real numbers) Θ-closednowhere dense subsets. Theorem. X has the property that every continuous function from X to a Hausdorff space is closed if and only if X has the property that every Θ-continuous function from X to a Hausdorff space is closed. (Received October 7, 1974.)

*75T-G8 CLARK, GOTTLIEB, LEACH, & ROSENHOLTZ, Univ. of Wyoming, Laramie, 82071.
An unholey version of the irrationals.

The "Sorgenfrey line" or "E^1-bad" or the "half-open interval space" is the topological space having the real line as underlying set and having the collection of all half-open intervals, $\{[a,b) \mid a,b \text{ reals}\}$, as a basis for the topology. A natural modification of this space is gotten by again taking the real line for underlying set but this time letting the collection of all half-open intervals having rational endpoints, $\{[a,b) \mid a,b \text{ rational}\}$, be the basis for the topology. Perhaps this space should be called "E^1-not-so-bad". It turns out that "E^1-not-so-bad" is homeomorphic to the irrationals (with the usual topology), and so we have a nice picture of the irrationals having no "holes". Furthermore, there is a metric for "E^1-not-so-bad" so that the metric balls are all half-open intervals with rational endpoints. (Received October 17, 1974.)

*75T-G9 PHILIP BACON, University of Florida, Gainesville, Florida 32611.
Continuous functors.

The concepts of continuity and Čech continuity for functors on the homotopy category of topological spaces are so defined as to be equivalent, thereby extending a theorem already known for the homotopy category of compact Hausdorff spaces. Every paracompact Hausdorff space is an inverse limit of polyhedra. (Received October 21, 1974.)

*75T-G10 ROBERT C. HOOPER, University of Nevada, Reno 89507
Locally compact subgroups of topological groups

Let G be the quotient group of the Banach space c_0 modulo the subgroup of all integer sequences. If H is a non-trivial, closed subgroup of G, then H has a non-trivial subgroup which is either torsion or is infinite cyclic and discrete.

Let $\{u_n\}_{n=1}^{\infty}$ be the standard basis for the Banach space ℓ_1 and let $v_n = \frac{1}{n} u_n$. Define A to be the closed subgroup of ℓ_1 generated by the v_n. Let D be the subgroup of the

quotient group ℓ_1/A consisting of those elements which are images under the quotient map of elements of the form $\sum_{n=1}^{\infty} \frac{m_n}{2^n} v_n$ with the m_n integers and $m_{n+1} \equiv m_n (\mathrm{mod}\ 2^n)$. It is proved that D is closed and the only locally compact subgroup of D is the identity subgroup.

It is still not known if E is a separable Banach space, K a discrete subgroup of E, and H a non-trivial, closed subgroup of E/K, whether H must have a non-trivial locally compact subgroup. (Received October 21, 1974.)

74T-G11 ZVONKO T. CERIN, Louisiana State University, Baton Rouge, Louisiana 70803. Cellular approximations in the topology of manifolds.

If X, Y are compact metric spaces and $\epsilon > 0$, let CE(X, Y) [$CE_\epsilon(X, Y)$] denote the set of all maps f from X onto Y such that the preimage under f of each point in Y has trivial shape [and diameter less then ϵ]. X is CE-Y-like if for every $\epsilon > 0$, $CE_\epsilon(X, Y)$ is nonempty. Theorem 1. Let Y be a compact n-manifold ($n \neq 4$) or a compact Q-manifold (i.e., locally homeomorphic to an open subset of the Hilbert cube Q). Then every CE-Y-like compactum X such that $CE(Y, X) \neq \emptyset$ is homeomorphic to Y. A number of applications especially in connection with the problem of identifying Hilbert cube factors and Hilbert cube manifold factors include Theorem 2. Let $X \times Y$ be a compact Q-manifold with Y contractible. Then $X \times Q$ is homeomorphic to $X \times Y$. (Received October 23, 1974.)

75T-G12 KENNETH A. PERKO, JR., One Chase Manhattan Plaza, New York, New York 10005. Remark on 2-bridged knots.

The dihedral linking numbers of a knot with two bridges [Knotentheorie, III, §15] can be easily reckoned from its normal form (α, β) as follows: for $\beta > 0$, each $v_{i0}^{\alpha} = 2(-1)^j$ where $j\alpha < i\beta < (j+1)\alpha$. Cf. these Notices 20 (1973), A-598, and 21 (1974), A-327. (Received October 23, 1974.)

*75T-G13 S. Broverman, Dept. of Mathematics University of Manitoba, Winnipeg, Man., "The structure of continuous {0,1}-valued functions on a products space" Preliminary Report.

All spaces are assumed to be completely regular and Hausdorff. Let {0,1} denote the two point discrete space. A continuous {0,1} - valued function on a product space $X \times Y$ is called finitely decomposable if there exist finite decompositions $\{A_i\}_{i=1}^{n}$ and $\{B_j\}_{j=1}^{m}$ of X and Y respectively into clopen sets such that f is constant on each set of the form $A_i \times B_j$.

Theorem 1. A continuous {0,1} - valued function on $X \times Y$ admits a continuous extension to $\beta X \times \beta Y$ iff f is finitely decomposable. A space X is 0 - dimensional if it has a base of clopen sets. In the sense of Mrowka's E- compactness (Acta Mathematica 120(1968, 161-185), let $\beta_o X$ denote the {0,1} - compactification of a 0-dimensional space.

Theorem 2. A continuous {0,1} - valued function or a 0-dimensional space $X \times Y$ admits a continuous extension to $\beta_o X \times \beta_o Y$ iff f is finitely decomposable.

Theorem 3. Every continuous {0,1} - valued function on a 0-dimensional space $X \times Y$ is finitely decomposable iff $X \times Y$ is pseudocompact. (Received October 24, 1974.)

*75T-G14 WILLIAM G FLEISSNER, McGill University, Montreal, Quebec, Canada.

A Collectionwise Hausdorff, not Collectionwise Normal Moore Space.

Under an assumption that is a consequence of V = L, and consistent with MA + not CH, there is a collectionwise Hausdorff, not collectionwise normal Moore space; Assuming MA + not CH, this space is normal, assuming V = L, it is not.

(Received October 25, 1974.) (Author introduced by M. Makkai.)

75T-G15 Donald M. Davis, Lehigh University, Bethlehem, Pa. 18015 and R. James Milgram, Stanford University, Stanford, Calif. 94305

Correction to "Symmetries and Operations in Homotopy Theory"

If h_i^2 represents an infinite cycle in the Adams spectral sequence, let θ_i denote a corresponding element in the stable homotopy of spheres. In [R. J. Milgram, Proc. Symp. Pure Math., 22, 1971, 203-210] it was announced that if θ_i exists, $2\theta_i = 0$, and $\theta_i^2 = 0$, then θ_{i+2} exists. This would imply the existence of θ_6 and hence of a framed 126-manifold of Kervaire invariant 1.

Davis discovered a difficulty in the proof of this theorem, so that we know of no proof of the existence of θ_6. $H_*(R\Gamma_4(X_n); \mathbb{Z}_2)$ has more classes than indicated in the proof of Theorem 6.2, and there is no element $\alpha \in \pi_*(Y)$ having non-trivial Hurewicz image for which $\tau_*(\alpha)$ could possibly be θ_{i+2}. (Received October 30, 1974.)

The November Meeting in Los Angeles, California November 23, 1974

718-A2 WALTER DEUBER, University of California, Los Angeles, California 90024

Partition theorems for Abelian groups

Let A be a finite matrix with integral entries and G be an Abelian group. Define A to be partition regular in G if for every partition of $G\setminus\{0\}$ into finitely many classes there exist elements $x_1,\ldots,x_m$ contained in one class such that $A(x_1,\ldots,x_m)^T = 0$. Theorem: A is partition regular in G iff at least one of the following statements holds. (i) There is $x \in G\setminus\{0\}$ such that $A(x,\ldots,x)^T = 0$. (ii) A is partition regular in $\mathbb{Z}_p^{\aleph_0}$ (p prime) and $\mathbb{Z}_p^{\aleph_0} \subset G$. (iii) A is partition regular in $\mathbb{Z}$ and the set of orders of elements in G is unbounded.

(Received November 29, 1974.)

The January Meeting in Washington, D. C.
January 21 – 26, 1975

00 General

*720-00-1 PRESTON C. HAMMER, Grand Valley State Colleges, Allendale, Michigan 49401. Identities. Preliminary report.

There can be no use of language without use of identifications. There has been global abuse of identities in mathematics. This abuse seems to derive from the confusion between objects and verbs in sentences revealed in the use of "transformation" rather than "transformer" as a synonym for function. In this paper there is presented a case study of this particular form of aberration and its possible effects. This is part of an extensive study of language usage in mathematics. (Received October 21, 1974.)

02 Logic and Foundations

*720-02-1 GEORGE METAKIDES, University of Rochester, Rochester, New York 14627 and ANIL NERODE, Cornell University, Ithaca, New York 14850. Nonextendible bases and maximal vector spaces. Preliminary report.

We investigate the effective content of constructions from linear algebra as part of an investigation of recursively enumerable substructures of recursively presented models. Let U be a recursively presented ω-dimensional vector space over a recursive field, let $I_0, I_1, \ldots$ be an effective list of all r.e. independent subsets of U, and let V_n be the subspace that I_n spans. An independent set I is nonextendible if for all n, $I_n \supseteq I$ implies $I_n - I$ is finite.

THEOREM: There exists an n such that I_n is infinite, $U \bmod (V_n)$ is infinite dimensional, and I_n is nonextendible.

DEFINITION: A subspace V of U is maximal if V is r.e. and infinite dimensional, $U \bmod (V)$ has infinite dimension and $V_n \supseteq V \Rightarrow ((\dim V_n \bmod (V) < \infty) \vee (\dim U \bmod (V_n) < \infty))$.

THEOREM: Maximal subspaces exist.

The proofs use priority arguments which mix well with algebraic constructions. (Received July 1, 1974.)

*720-02-2 T.G. MCLAUGHLIN, Texas Tech University, Lubbock, Texas, 79409 Simultaneous embeddings into special subclasses of Λ_{ZR}.

E. Ellentuck has defined an infinite retraceable set α to be meager in case $(\forall e)[\alpha \subseteq \mathrm{dom}(\sigma_e) \Rightarrow (\exists m)(\forall n > m)[p_\alpha(n) > q_e(p_\alpha(n-1))]]$; here p_α is the order-of-magnitude enumeration function for α, and q_e is the e-th partial recursive function. He showed (Pacific J. Math., 42 (1972), 629-638, MR47, #3157) that arbitrary countable posets can be embedded into special classes of regressive isols ordered by $\leq$ (isolic

predecessor), by $\leq_\Lambda$, or by $<_\Lambda$. We obtain, by procedures not unrelated to his, the following result on "simultaneous embedding." Theorem. Let $\underset{\sim}{a}$ be an r.e. degree such that $\underset{\sim}{a}' = \underset{\sim}{0}''$, and let $\Lambda^{\underset{\sim}{a}}_{ZRM}$ denote the class of co-simple isols I such that $(\forall\beta \in I)[\beta$ is a meager retraceable set of degree $\underset{\sim}{a}]$. Then there is a subclass $\mathcal{A}$ of $\Lambda^{\underset{\sim}{a}}_{ZRM}$ such that: if Q is an arbitrarily given countable poset then there exists an embedding of Q into $\mathcal{A}$, simultaneously with respect to the three orderings $\leq$, $\leq_\Lambda$, $<_\Lambda$. Moreover, it can be required that every finite sum of elements of $\mathcal{A}$ be a regressive isol. (Received September 30, 1974.)

*720-02-3 John Baldwin, Illinois at Chicago, 60680, Bruce Rose, U. of Chicago, 60637, Stability, $\aleph_0$-Categoricity and Finiteness Conditions in Rings

Let R be an associative ring (possibly without unit and non-commutative). For ring theoretic usages, see Herstein's: Noncommutative rings; for logic, see Shelah, Stability, the f.c.p. etc. Annals of Mth. Logic.(1971). Theorem 1. a) A semiprime (i.e. no nilpotent ideals) stable ring is Artinian and thus a finite direct sum of complete matrix rings over a division ring. b) If in addition, R is commutative then R is a finite direct sum of fields,(b) improves Reineke, These Notices Apr. (1973) pg. A340.) Corollary. If R is an infinite semiprime $\aleph_0$-categorical ring then Th(R) has 2^λ models in every uncountable cardinal λ. Theorem 2. An $\aleph_0$-categorical ring which satisfies the a.c.c. (or d.c.c) is finite. The class $\mathcal{S}$ of $\aleph_0$-categorical rings with unit and no nilpotent elements has been completely described by Macintyre and Rosenstein (unpublished). Theorem 3. If R is $\aleph_0$-categorical commutative ring with unit then J(R) (the Jacobson radical of R) is nilpotent and $R/J(R) \in \mathcal{S}$. Theorem 4. (Baldwin and Berman). There is an $\aleph_0$-categorical, commutative, Jacobson semisimple ring which does not have an identity. Theorem 5. If R is stable then J(R) is nilpotent. More precise results are possible if R is w-stable. (Received October 8, 1974.)

*720-02-4 STEPHEN D. COMER, Clemson University, Clemson, South Carolina 29631. Complete and model-complete theories of monadic algebras.

Let C consist of all sectional structures $\Gamma(X,S)$ where S is a sheaf of models of T and X is a Boolean space with no isolated points. A. Macintyre [Fund. Math 81(1973), 73-89] gave a sufficient condition for Th(C) to be model-complete where T is a theory of rings. Macintyre's result is extended and the generalization is applied to theories of monadic algebras. It is shown that the class of nontrivial members of each variety of monadic algebras has a model companion. These theories are axiomatizable, ω-complete, and decidable. Examples are also produced to show there are 2^ω elementary types of monadic algebras. (Received October 21, 1974.)

720-02-5 LEO HARRINGTON and DAVID GUASPARI, State University of New York at Buffalo, Amherst, New York 14226. Definable real numbers, Preliminary report.

A real is a subset of ω. We consider the question: which reals are definable in models of (fragments of) set theory? If $\mathfrak{M}$ is an ω-structure for set theory, write "$x \in \mathfrak{M}$" for "x occurs in $\mathfrak{M}$".

Define, for any sentence φ of the language of set thoery: $\mathrm{Const}(\varphi) = \{x \mid \forall \mathfrak{M}(\mathfrak{M} \models \varphi \ \& \ x \in \mathfrak{M} \Rightarrow \mathfrak{M} \models \underline{x} \in L)\}$; $\mathrm{Def}(\varphi) = \{x \mid \forall \mathfrak{M}(\mathfrak{M} \models \varphi \ \& \ x \in \mathfrak{M} \Rightarrow \exists \psi(\mathfrak{M} \models \underline{x} = \iota v \psi))\}$; $u - \mathrm{Def}(\varphi) = \{x \mid \exists \psi \ \forall \mathfrak{M}(\mathfrak{M} \models \varphi \ \& \ x \in \mathfrak{M} \Rightarrow \mathfrak{M} \models \underline{x} = \iota v \psi)\}$. C_1, the maximum thin π^1_1 set, is defined in [1], [2].

Call φ adequate if: i) $\varphi \vdash$ admissible set theory and ii) φ is true.

Thm. 1. φ adequate & provable $\Rightarrow$ $(\mathrm{Const}(\varphi) = C_1)$; φ adequate $\Rightarrow$ $(u - \mathrm{Def}(\varphi) = \{x \mid \{x\} \in \pi^1_1\})$.

Unlike Const and u - Def, Def is not so stable. It depends on φ (since, e.g., possibly $\varphi \vdash (\mathrm{Def}(\psi)$ is countable)). Regain invariance by putting $\mathrm{DEF} = \bigcup\{\mathrm{Def}(\varphi) \mid \varphi$ is adequate$\}$.

Thm. 2. $\mathrm{DEF} = C_1 \Leftrightarrow$ there exists a definable enumeration of the constructible reals; $\mathrm{DEF} = C_1 \cap (\Delta^1_3)_L \Leftrightarrow$ otherwise.

Some of these are probably known. One was mentioned in [2].

[1] Kechris, A. S., "Theory of Countable Analytical Sets" to appear in TAMS.

[2] Guaspari, D., "The largest constructible π^1_1 set", Notices, Aug., 1973, 73T-E83.

(Received November 1, 1974.)

*720-02-6 DR. J.A. MAKOWSKY, Department of Mathematics, Simon Fraser University, Burnaby 2, B.C. Canada. Beth's Theorem in Δ-logics.

We define interpolation $C(L_1,L_2)$, Δ-interpolation $\Delta(L_1,L_2)$, Beth, $B(L_1,L_2)$ and Weak Beth, $WB(L_1,L_2)$ for logics L_1 and L_2 analogously to [Gregory, Beth definability in infinitary languages, JSL 39 No. 1, 1974]. For Δ-logics, see [K.J. Barwise, Axioms for abstract model theory, Annals of Math. Logic, 1975]. Theorem 1: not $WB(L_{\omega_1}, L_{\infty\infty})$. Theorem 2: not $B(LQ_\alpha, \Delta(L_{\infty\omega}))$ for $\alpha \geq 1$ and $\aleph_\alpha$ regular. Corollary 3: Neither $\Delta(LQ_1)$ nor $\Delta(L_{\infty\omega})$ satisfy Beth. This solves problems 6 and 7 in [H. Friedman, 94 problems in math. logic, to appear]. Corollary 4: $\Delta(L_1,L_2) \neq B(L_1,L_2)$ and $WB(L_1,L_2) \neq B(L_1,L_2)$. As S. Shelah observed, $B(L_1,L_2) \neq C(L_1,L_2)$, but the same argument yields $B(L_1,L_2) \neq \Delta(L_1,L_2)$. This settles all the possible implications between the definability properties defined above. Theorem 5: $L_{\infty\omega}$ does not satisfy Beth even for finite similarity type and no constants. This improves [Gregory, l.c.]. (Received October 29, 1974.)

720-02-7 STEVEN GIVANT, University of California, Berkeley, California 94720. A complete representation theorem for varieties categorical in power.

Let n-ary d^A_n and unary $p^A_{n,i}$, $1 \leq i \leq n$, be operations defined on ${}^nA$ by: $d^A_n(\langle x_{1,1},\ldots,x_{1,n}\rangle,\ldots,\langle x_{n,1},\ldots,x_{n,n}\rangle) = \langle x_{1,1},x_{2,2},\ldots,x_{n,n}\rangle$; $p^A_{n,i}(\langle x_1,\ldots,x_n\rangle) = \langle x_i,x_i,\ldots,x_i\rangle$. For a class K of algebras put $K_n = \{\mathfrak{B} : \exists_{\mathfrak{A}}(\mathfrak{A} \in K \wedge \mathfrak{B} \cong \langle {}^n\mathfrak{A}, d^A_n, p^A_{n,i}\rangle_{1 \leq i \leq n})\}$. If $\sigma(v_1)$ is a term of the language of K_n let $T(\sigma)$ be the set of terms $\gamma(v_1,\ldots,v_m)$ such that $K_n \models \gamma(\sigma,\ldots,\sigma) = \sigma$, and set $K_n(\sigma) = \{\langle B, \gamma^{(\mathfrak{B})}\rangle_{\gamma \in T(\sigma)} : \mathfrak{B} \in K_n\}$. Thus the $\gamma^{(\mathfrak{B})}$ (the realization of γ into $\mathfrak{B}$) are the polynomial oper-

ations of $\mathfrak{P}$ which are idempotent on the range of $\sigma^{(\mathfrak{P})}$. Theorem. K is a variety categorical in power iff K is polynomially equivalent to one of the following: (i) $V_n(\sigma)$ where V is a variety of vector spaces over some skew field, $n \geqq 1$, and σ is a term of the language of V_n; (ii) S_n where S is the variety of sets and $n \geqq 1$; (iii) C_n where C is the variety of "pointed sets" $\langle A, a\rangle$, $a \in A$, and $n \geqq 1$. F. Lawvere observed that when V is the variety of vector spaces over $\mathfrak{J}$, then V_n is polynomially equivalent to the class of modules over the ring of n by n matrices with coefficients in $\mathfrak{J}$. Thus each of the examples in (i) has a module interpretation. The theorem completes the partial representation announced by the author [Abstract 705-E1, these *Notices* 20(1973), A-461. (Received October 29, 1974.)

720-02-8 MITSURU YASUHARA, University of Kentucky, Lexington, Kentucky 40506
The axiom of choice in Church's type theory, Preliminary report.

Let $\mathfrak{T}$ be Church's theory of types with the axiom of description, as in [Andrews, J.S.L., vol. 37 (1972), 385-394]. Call a type propositional if it is composed of o only. Let the relation '$\geq$' among the types be the transitive closure of $(\alpha\beta) \geq \alpha$, $(\alpha\beta) \geq \beta$, and $\alpha \geq \alpha$; when $\alpha \geq \beta$, call α higher than β. Finally, let $(AC)_\alpha$ be the axiom of choice formulated for the domain of type α. Theorem 1. If $\alpha \geq \beta$, then $\mathfrak{T} \vdash (AC)_\alpha \rightarrow (AC)_\beta$. Conversely, Theorem 2. Given a set T of types, none of which is propositional, there is a model of $\mathfrak{T}$ in which $(AC)_\alpha$ fails if and only if α is higher than a member of T. [This extends Theorem 4 in the above paper of Andrews.] Also, some conjectures of Büchi in J.S.L., vol. 18 (1953), 125-135, are verified. For instance, Zorn's lemma formulated at the type α does not imply $(AC)_\alpha$. (Received November 4, 1974.)

*720-02-9 A. R. VOBACH, University of Houston, Houston, Texas 77004.
The weak topology on logical calculi.

Let $\mathfrak{L}$ be the Sierpinski space with $\{0\}$ open. Let P be an infinite set of propositional variables, Prop (P) the propositional calculus on P and Hom(Prop (P), $\mathfrak{L}$) its set of realizations in $\mathfrak{L}$. Theorem. The weak topology, $\mathfrak{W}$ on Prop (P), induced by its realizations in $\mathfrak{L}$, is the smallest topology for which consequence closed sets are closed. Given a first order predicate language L, with F_L and At_L its sets of formulas and atomic formulas, respectively, F_L can be given a weak topology, determined by the canonical realizations of L, such that (Prop (At_L), $\mathfrak{W}'$) embeds in F_L. (Received November 4, 1974.)

720-02-10 Robert W. Button, Carnegie-Mellon University, Pittsburgh, Pa. 15213
$\mathfrak{U}$-Filters and $\mathfrak{J}$-Filters.

Let $\mathfrak{U} \subseteq 2^X$ be non-empty and closed under finite intersections. A $\mathfrak{U}$-filter $\mathfrak{F}$ on the set X is a non-empty subset of $\mathfrak{U}$ such that: (i) if $G, H \in \mathfrak{F}$, then $G \cap H \in \mathfrak{F}$, and (ii) if $G \in \mathfrak{F}$ and $G \subseteq H$ for some $H \in \mathfrak{U}$, then $H \in \mathfrak{F}$. A $\mathfrak{J}$-filter is

defined in the same way except that $\mathfrak{J} \subseteq 2^X$ is required to be closed under finite unions as well as intersections.

$\mathfrak{V}$-filters and $\mathfrak{J}$-filters play an important role in non-standard analysis and topology. We extend here the non-standard theory of $\mathfrak{V}$-filters and $\mathfrak{J}$-filters with some very recent results, characterizing $\mathfrak{V}$-ultrafilters and prime $\mathfrak{J}$-filters, examining nuclei of $\mathfrak{V}$-filters, etc. (Received November 4, 1974.)

*720-02-11 JULIA F. KNIGHT, 1000 West Beaver, State College, Pennsylvania 16801. Types realized in all models of power $\aleph_2$.

Let T_0 be either first order Peano arithmetic or the set theory $ZF + V = L$. There is a type Σ, in the language of T_0, such that Σ is realized in all models of T_0 of power $\aleph_2$, but for any consistent extension T of T_0 (in the same language), Σ is omitted in a model of T of power $\aleph_1$. The type Σ says of the variables v_1, v_2 that neither is definable from the other. In the course of the proof, forcing is used to show that if $\mathfrak{A}$ is a countable model of T which omits Σ, then $\mathfrak{A}$ has a proper elementary extension which omits Σ. The author wonders whether for $n \geqq 3$, there is a type which is realized in all models of T of power $\aleph_n$ but is omitted in a model of power $\aleph_{n-1}$. In particular, if Σ_n is the type which says that none of the variables $v_1, \ldots, v_n$ is definable from the others, is Σ_n omitted in a model of T of power $\aleph_{n-1}$? (Received November 6, 1974.)

720-02-12 FRED HALPERN, Rider College, Trenton, New Jersey 08602. Some second-order transfer theorems.

Let L be a language for group theory with the additional predicate $\in$ denoting membership of an individual in a (normal) subgroup. L has variables for individuals, subgroups, and normal subgroups and permits quantification ($\forall$ or $\exists$) of each of these variables. $\sigma \in L$ is preserved under substructures (homomorphic images) if $G \vdash \sigma$ implies $G' \vdash \sigma$ where G' is a substructure (homomorphic image) of G. We provide a class of sentences preserved under substructure (homomorphic image). The preserved sentences are described in terms of a complicated relation between their prefix and matrix (which we hope to simplify). It is somewhat surprising that (normal) subgroup variables may be existentially quantified in sentences preserved under subgroup and that $x \notin H$ may appear in sentences preserved under homomorphic image. The main tool of this investigation is the notion that $\sigma(v, w)$ is preserved under the pair of relations (R, S) which is defined as: $G \vdash \sigma(a, b)$, aRa', and bSb' imply $G' \vdash \sigma(a', b')$. This notion allows treatment of image and inverse image simultaneously. (Received November 6, 1974.)

*720-02-13 GEORGE McNULTY, Dartmouth College, Hanover, New Hampshire 03755. Undecidable properties of finite sets of equations, II.

A sentence is an equation if it is the universal closure of $\phi \approx \Psi$ for some terms ϕ and Ψ. A collection G of finite sets of equations is closed under equivalence if $\Delta \varepsilon G$ whenever Δ is a finite set of equations logically equivalent with some member of G. A set Σ of equations is a definitional reduct of a set Δ of equations just in case every infinite model of Σ can be expanded to a model of Δ in which all the fundamental operations are polynomials in the language of Δ. Theorem 1. If G is a collection of finite sets of equations such that G is non-empty, closed under equivalence, and for every $\Gamma \varepsilon G$ there is a term τ in which

both the variables x and y occur with $\Gamma \vdash \tau \approx x$, then G is not recursive. Theorem 2. If G is a collection of finite sets of equations in a language with at least two unary operation symbols or some operation symbol of rank at least two such that $\{\forall xy[x \approx y]\} \in G$, G is closed under equivalence, and there is a finite set Σ of equations so that $\Delta \notin G$ whenever Σ is a definitional reduct of Δ, then G is not recursive.
(Received November 6, 1974.)

720-02-14 STANLEY E. HAYES, State University of New York, College at New Paltz, New Paltz, New York 12561. Natural deduction and lattice models of some weak propositional logics.

Some weak systems of natural deduction for propositional logic are examined. The weakest of them generalizes both intuitionist logic and Lukasiewicz logic. Axiomatic formulations of these logics are given and their corresponding lattice models are studied. (Received November 5, 1974.)

720-02-15 W. RICHARD STARK, The University of Texas, Austin, Texas 78712. A New Compactness Theorem for Infinitary Logic.

A new kind of compactness theorem for $L_A = A \cap L_{\infty\omega}$ (A is a not-necessarily-countable admissible set) is described which: 1 is a generalization of Barwise's compactness theorem for countable admissible sets, 2 for many uncountable A (such as the constructable admissible sets) is consistent with ZFC. The theories to which it applies are not all of the $\Sigma_1^0(A)$-theories, but a non-absolute subset of these theories. The non-absoluteness of the definition allows the structure of the universe to be taken into consideration. It is this sensitivity to the structure of the universe which allows this compactness theorem to be natural in the uncountable case.
(Received November 6, 1974.)

720-02-16 WITHDRAWN

*720-02-17 DONALD A. MARTIN, The Rockefeller University, New York, N.Y. 10021. Borel determinacy.

For each set X let X^ω be the collection of all infinite sequences of elements of X. Give X the discrete topology and, thinking of X^ω as a product, give X^ω the product topology. For each $A \subseteq X^\omega$ define a two person game of perfect information $G(A)$ as follows. I plays $x_0 \in X$, II plays $x_1 \in X$, etc. I wins if $\langle x_i : i \in \omega \rangle \in A$. Theorem. If A is Borel, either I or II has a winning strategy for $G(A)$. (Received November 27, 1974.)
(Author introduced by Professor William Mitchell.)

04 Set Theory

720-04-1 STEVEN GLAZER, University of Kansas, Lawrence, Kansas 66044. The indiscrete ultrafilters problem. Preliminary report.

A collection of k distinct k-complete nonprincipal ultrafilters $\{U_a : a \in k\}$ over measurable cardinal k is called discrete if there exist $A_a \in U_a$ for all $a \in k$ such that $A_a \cap A_b = \emptyset$ for all $a, b \in k$ with $a \neq b$. It is called strongly indiscrete if it is not the union of fewer than k discrete collections. We have Theorem 1. There exists a strongly indiscrete collection of Rudin-Frohlik minimal untrafilters over ω. Theorem 2. If $V = L(D)$ then ω is the only cardinal with this property. Theorem 3. If there exists a maximal k-independent set of subsets of k, then there is a strongly indiscrete collection of k-complete ultrafilters over k. Theorem 1 has already been shown by Kunen assuming CH. Our construction is different and does not assume anything more than ZFC. (Received November 1, 1974.)

05 Combinatorics

*720-05-1 GARY CHARTRAND, Western Michigan University, Kalamazoo, Michigan 49001, and ALBERT D. POLIMENI, SUNY, College at Fredonia, Fredonia, New York 14063. Ramsey theory and chromatic numbers

Let $\chi(G)$ denote the chromatic number of a graph G. For positive integers $n_1, n_2, \cdots, n_k$ $(k \geq 1)$ the chromatic Ramsey number $\chi(n_1, n_2, \cdots, n_k)$ is defined as the least positive integer p such that for any factorization $K_p = \bigcup_{i=1}^{k} G_i$, $\chi(G_i) \geq n_i$ for at least one i, $1 \leq i \leq k$. It is shown that $\chi(n_1, n_2, \cdots, n_k) = 1 + \prod_{i=1}^{k} (n_i - 1)$. Let a(G) denote the vertex-arboricity of a graph G. For positive integers $n_1, n_2, \cdots, n_k$ $(k \geq 1)$ the vertex-arboricity Ramsey number $a(n_1, n_2, \cdots, n_k)$ is defined as the least positive integer p such that for any factorization $K_p = \bigcup_{i=1}^{k} G_i$, $a(G_i) \geq n_i$ for at least one i, $1 \leq i \leq k$. It is shown that $a(n_1, n_2, \cdots, n_k) = 1 + 2k \cdot \prod_{i=1}^{k} (n_i - 1)$. (Received October 23, 1974.)

720-05-2 AVIEZRI S. FRAENKEL and YAACOV YESHA, The Weizmann Institute of Science, Rehovot, Israel. Theory of annihilation games. Preliminary Report.

Two players play with m stones placed on distinct vertices $u^m = (u_1, \ldots, u_m)$ of a finite digraph. Each player at his turn moves one stone from a vertex u to a neighboring vertex v along a directed edge. If there was already a stone at v, both stones get annihilated. The player making the last move wins. If there is no last move, the game is a tie. Let G denote the generalized Sprague-Grundy function, $\oplus, \Sigma'$ Nim-sum. Theorem: $G(u^m) < \infty \Rightarrow G(u^m \cup v^n - u^m \cap v^n) = G(u^m) \oplus G(v^n)$. Corollary: Suppose u^m is of type k, i.e., $G(u_i) = \infty$ $(1 \leq i \leq k)$, $G(u_i) < \infty$ $(k < i \leq m)$. Then $G(u^m) = G(u^k) \oplus \Sigma'_{i>k} G(u_i)$. (Combinatorial games with an annihilation rule, Proc. Symp. Appl. Math. 20, AMS, 1974.) A graph is simple if $\text{type}(u^m) > 0 \Rightarrow G(u^m) = \infty$. A characterization of simple digraphs is given, yielding a polynomial algorithm for testing simplicity. The theory was applied to the annihilation

game on the game-graph of the "Generalized Battle of Numbers" (Constructions in Combinatorial games with cycles, Proc. Intern. Colloq. Infinite & Finite Sets, Keszthely, 1973), which is a game with cycles depending on three parameters which control the tie-positions, resulting in a complete strategy for the annihilation game. (Received October 25, 1974.)

*720-05-3 Joseph Arkin, 197 Old Nyack Turnpike, Spring Valley, New York 10977
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Ernst G. Straus, University of California, Los Angeles 90024
TITLE. Systems of magic Latin k-cubes.

We use a method which starts from two given orthogonal Latin squares of order n to construct 4 Latin cubes of order n which are orthogonal by threes, that is the superposition of any three cubes yields a cube in which all n^3 ordered triples occur. Starting with these Latin cubes we can construct examples of magic 4-digit Latin cubes of certain orders n, that is a superposition of 4 Latin cubes so that the sums of the 4-digit numbers in every row, column, file and major diagonal are the same. As an example the construction of such a cube of order 10 is given. There also is a partially magic 4-cube of order 10 where the sums in each of the 4 directions and along 6 of the 8 major diagonals are the same. (Received October 23, 1974.)

720-05-4 GERHARD RINGEL, University of California Santa Cruz,
Santa Cruz, California 95064 Map Color Theorem

P.J. Heawood published a formula for the chromatic number of the orientable surfaces of genus $\neq 0$ about 85 years ago. However, he neglected to prove it. The proof was completed much later by Youngs and the author using combinatorial methods. Recently these methods were improved in certain respects. At present the proof of the Map Color Theorem consists of a 2-dimensional part (surfaces, maps, embedding theorems) and a 1-dimensional one (graph theory and combinatorics). The latter has proved the more interesting part. For the purposes of this lecture we therefore concentrate on the combinatorial methods such as rotations of graphs and current graphs. Moreover the 2-dim part of the problem will be reformulated in graph theoretical terms. The Map Color Theorem will be developed in a combinatorial way, circumventing the unwieldy embedding theory. (Received October 29, 1974.)

*720-05-5 FRANK R. BERNHART, University of Waterloo, Waterloo, Ontario, N2L 3G1
How Not to Prove the Four Color Conjecture

What makes the four color problem hard? Part of this question can best be answered by the analysis of invalid proofs. To demonstrate that this is not a waste of time, we consider several recent and a few older attempts to prove the four color conjecture, including proofs by Kempe, Tait, Thomas, Trent, Shimamoto which are available as published literature, and a few other 'nameless' contenders.

In each case the lesson to be learned has more than a "warning" value, as we hope to show, hence each of the authors makes a positive contribution to the theory. In some cases, we can make several suggestions for further research.

A rough and ready classification of false proofs is as follows: false inference from a) Semantics b) Neighborhoods c) Kempe chain structure d) Coloring Algorithms e) Reduction operations f) Algebra. (Received October 29, 1974.)

720-05-6 R.C. READ, University of Waterloo, Ontario, and D.G. CORNEIL University of Toronto, Ontario, Canada. The Isomorphism Disease. Preliminary Report.

The graph isomorphism problem -- to devise a good algorithm for determining if two graphs are isomorphic -- is of considerable practical importance. It is also of great theoretical interest due to its relationship to the concept of NP-completeness. No efficient (i.e. polynomial-bound) algorithm for graph isomorphism is known, and it has been conjectured that no such algorithm can exist. Recently many papers on the subject have appeared, but progress has been slight. In fact, the intractable nature of the problem and the way that many graph theorists have been led to devote much time to it, recall those aspects of the four-colour conjecture which prompted Harary to rechristen it the "four-colour disease".

This paper surveys the present state of the art of isomorphism testing, discusses the relationship to NP-completeness, and indicates some of the difficulties inherent in this particularly elusive and challenging problem. (Received October 29, 1974.)

*720-05-7 H.S.M. COXETER, University of Toronto, Toronto, Ontario M5S 1A1. Faithful Trivalent Cayley diagrams, Preliminary Report.

A group H of order g with k given generators has a Cayley diagram consisting of a directed graph with g vertices. The edges have k colors, one for each generator, coming and going at each vertex. For each generator of period 2, the digon formed by 2 oppositely directed edges is conveniently collapsed to form a single undirected edge. Let Γ denote the group of the ordinary graph derived by disregarding the colors and arrows. H is, of course, a subgroup of Γ, usually a proper subgroup. (For instance, when $k=1$ and $g>2$, H is cyclic while Γ is dihedral.) M.E. Watkins and Roberto Frucht have found several instances in which H, having three involutory generators, is the whole of Γ. It now appears that there are some interesting families of two-generator groups whose Cayley diagrams are likewise faithful. For instance, the group

$$S^4 = T^2 = (ST)^4 = (STS)^b\ (S^2T)^c = 1\ ,$$

of order $4(b^2 + c^2)$, has a faithful Cayley diagram whenever $b>c>0$. (Received October 29, 1974.)

*720-05-8 MICHAEL O. ALBERTSON, Smith College, Northampton, MA., 01060 and JOAN P. HUTCHINSON, Dartmouth College, Hanover, NH 03755. The maximum size of an independent set in a graph.

Let $\alpha(G)$ be the maximum number of vertices in any independent set of G, a simple graph with V vertices, and let $\mu(G) = \alpha(G)/V$, the independence ratio. Let $U(n) = \{\mu(G)\colon G$ embeds on $S_n\}$ and $L(n) = \{$limit points of $U(n)\}$. It is known that $U(0) \subset (2/9,1]$ and that $U(n) \subset [1/\chi,1]$ where χ is the chromatic number of $S_n (n\geq 1)$; Erdős has conjectured that $U(0) \subset [1/4,1]$.

Suppose G embeds on $S_n (n \geq 1)$ and χ is the chromatic number of S_n. Theorem 1. If $G \neq K_\chi$, then $\mu(G) \geq \frac{1}{\chi - 1}$. Theorem 2. If $G \neq K_\chi, K_{\chi-1}$ and $n \geq 4$, then $\mu(G) > \frac{1}{\chi - 1}$. More precise results for the torus have been obtained. Theorem 3. $L(n) \subset [1/5, 1]$ for all n. Conjecture. $L(n) = L(0)$ for all n. (Received October 29, 1974.)

*720-05-9 MARTIN CHARLES GOLUMBIC, Dept. of Mathematics, Columbia Univ., New York, N.Y. Comparability Graphs and a new Matroid

An undirected graph (V,E) is transitively orientable if each edge can be assigned an orientation so that the resulting oriented graph has the property that whenever $\vec{ab}$, $\vec{bc}$ are oriented edges then $\vec{ac}$ is also. Such an orientation is a partial ordering of the vertices V whose comparability relation is precisely E.

While extending the results of past researchers, we discover a new matroid structure associated with an undirected graph, (not just TRO graphs.) We characterize this matroid and give a formula for the number of transitive orientations of an undirected graph. (Received October 29, 1974.)

720-05-10 E. MAHMOODIAN, U. of P., Philadelphia, Pa. 19174, On a method of attacking some combinatorial problems, Preliminary report.

A method employed by P. R. Halmos and H. E. Vaughn to prove P. Hall's theorem on "representatives of subsets", in Amer. Jour. Math., 72 (1950), pp. 214-215 has been found very useful in proving some combinatorial theorems involving certain inequalities as necc. and sufficient conditions.

The application of this method has simplified the proofs of four standard theorems of graph theory:

1) 1-factor theorem of Tutte (1947)
2) The necc. and suff. condition for the existence of a (0,1)-matrix, with given row and column sum (Ryser, Gale 1957)
3) The necc. and suff. theorem for a graph to have a matching of deficiency d (a theorem of Berge 1970)
4) Erdős-Galli theorem (1960, The existence of a graph with given degree sequence). (Received October 29, 1974.)

*720-05-11 VERA PLESS, Massachusetts Institute of Technology, Cambridge, Massachusetts, 02139. Mathematical Foundations of Interconnected J-K Flip-Flops.

Shift registers composed of simple flip-flops have had many practical applications and an extensive mathematical basis. We introduce here the study of n-counters which consist of more complicated J-K flip-flops whose interconnections are quite general. Although used practically for many years no mathematical theory has been developed. This paper sets the foundations for such a theory which also appears to be interesting mathematically and potentially useful.

It is shown that the outputs of an n-counter are determined by its inputs and an easily computed matrix M. Also associated with an n-counter is a transformation N which determines each output from the preceeding one. The specific form that N can take is determined. N is easily computed from M and vice-versa. The graph of an n-counter is defined. An n-counter is shown to be cyclic if there is an i so that $N^i = I$. The group of an n-counter is defined. These notions are used to classify all 2-counters and linear 3-counters of a special type. (Received October 30, 1974.)

*720-05-12 GEORGE K. FRANCIS, University of Illinois, Urbana, Illinois 61801. The combinatorial topology of bordered Riemann surfaces.

By a bordered Riemann surface F we mean a properly interior (= topologically holomorphic) map from a compact, connected, oriented surface with regular boundary to the Gaussian sphere whose restriction ∂F to the boundary maps to a finite collection of regular closed oriented curves f lying in general position in the complement of the critical values (= branchpoints) of F. An assemblage for F is, roughly speaking, a cell decomposition of the surface on whose open edges (=1-cells) and sheets (=2-cells) F is univalent, hence the critical points are among the vertices. Given the curves f and a finite number of points a_j in their complement we construct assemblages for all bordered Riemann surfaces F with $\partial F = f$ and branched over the a_j. The assemblages, which are represented in terms of certain permutation groups, classify the bordered Riemann surfaces up to sense preserving topological equivalence. In particular, we obtain simple necessary and sufficient conditions for a plane immersion of the boundary of a compact oriented surface to extend to a plane immersion of the entire surface. (Received October 29, 1974.)

720-05-13 MARIANNE LEPP GARDNER, Carleton College, Northfield, Minnesota 55057. Brooks' Theorem is True for Hypergraphs. Preliminary report.

Define a hypergraph $H = (X,\mathcal{E})$ to be a finite set of nodes X and a nonempty family $\mathcal{E} = (E_1,\ldots,En)$ of subsets of X (called edges). For $x\varepsilon X$, define deg x to be the number of edges containing the node x. Two edges E_i and E_j are quasi-disjoint if $E_i \cap E_j = \{x\}$. Define $\phi(x)$ to be the maximum number of edges containing x which are quasi-disjoint in pairs. Let $\max_{x\varepsilon X} \deg x = n$ and $\max_{x\varepsilon X} \phi(x) = d$. Define the chromatic number of H, $\gamma(H)$, to be the least number of colors needed to color the nodes of H so that no edge has all its nodes colored one color. It is well-known that $\gamma(H) \leq d + 1$. The following two generalizations of Brooks' theorem will be presented. Theorem 1. Let $H = (X,\mathcal{E})$ be a hypergraph. Then $\gamma(H) \leq n + 1$ and equality holds if and only if H is a graph and H is K_{n+1} (the complete graph on n+1 nodes) or an odd cycle. This result contains Brooks' theorem. Theorem 2. Let $H = (X,\mathcal{E})$ be a hypergraph with $\gamma(H) = d + 1$. Suppose every node of H with $\phi(x) = d$ satisfies the property that if $E_1,\ldots,E_d$ are pairwise quasi-disjoint edges containing x, then $\bigcup_{i=1}^{d} E_i = \bigcup_{x\varepsilon E} E$. Then (1) $\phi(x) = d$ for all $x\varepsilon X$. (2) H contains K_{d+1} $(d>2)$ or an odd cycle with edges of cardnality two (d = 2) as a subhypergraph. (Received October 31, 1974.)

*720-05-14 R.D. RINGEISEN, Purdue Univ., Ft. Wayne, Ind. 46805; A.T. WHITE, W. Mich. Univ., Kalamazoo, Mich. 49001. On Pseudosurface Imbeddings of Graphs.

Let S be a compact orientable 2-manifold. Let $A(i,j)$, $i = 1,\ldots,t$, $j = 1,\ldots,n(i)$ be a collection of $\Sigma n(i)$, i = 1 to t, mutually disjoint sets of points of S, with $|A(i,j)| = m(i), i=1,\ldots,t$, and $1<m(1)<m(2)<\ldots<m(t)$. Identify the points of $A(i,j)$, for each i and j. The resulting topological space is defined to be a pseudosurface, S'. Let $g = \Sigma n(i)(m(i)-1)$, i = 1 to t. Petrolje("Imbedding Graphs in Pseudosurfaces", Thesis, W. Mich. Univ.) shows that, if G is a graph imbedded on a pseudosurface, S', then $F-E+V=2-2\gamma-g$, where F,E, and V are the number of faces, edges, and vertices of G, and γ is the genus of the surface S from which S' was created. The psuedocharacteristic, $\chi'(S')$, of a pseudosurface S' is defined by $\chi'(S')=2-2\gamma-g$. The pseu-

docharacteristic, $\chi'(G)$, of a graph G is the largest integer $\chi'(S')$ among all pseudosurfaces S' in which G has an imbedding. Theorem 1: $\chi'(K(m,n)) = 2-\{\frac{1}{2}(m-2)(n-2)\}$. Theorem 2: $\chi'(Q_n+\overline{K}_n) = n-(n-2)2^{n-1}$. The imbeddings given for K(m,n) are, in one-half the cases, more efficient (in terms of maximizing characteristic) than those possible in the genus situation. (Received October 25, 1974.)

*720-05-15 LINDA LESNIAK, Louisiana State University, Baton Rouge, Louisiana 70803, ALBERT D. POLIMENI, SUNY, College at Fredonia, Fredonia, New York 14063 and DONALD W. VANDERJAGT, Grand Valley State Colleges, Allendale, Michigan 49401. Degree sets and traversability.

For a graph G, the degree set is the set of degrees of the vertices of G. For a finite, nonempty set S of positive integers, if the minimum element of S is at least two, there is a hamiltonian graph whose degree set is S; if S contains at most two odd integers, there is a traversable graph (a connected graph in which there exists an open trail containing all its edges) whose degree set is S. The minimum order of many such graphs is determined. (Received October 31, 1974.)

720-05-16 FRANK ALLAIRE, University of Manitoba, Winnipeg, Manitoba R3T 2N2. Reducible Configurations for the Four Colour Conjecture, Preliminary Report.

The classical approach to the 4 CC is through the determination of reducible configurations, i.e. configurations that cannot occur in a 5-chromatic graph with a minimum number N of vertices. Using these restrictions, a lower bound for N can be determined. The current bound is $N \geq 52$.

The Heesch algorithm with several enhancements has been implemented on a computer. A complete analysis of configurations bounded by a 10-circuit or an 11-circuit has shown almost 200 new configurations to be reducible. An improvement on the lower bound for N is in progress. (Received October 24, 1974.) (Author introduced by Professor R.C.Mullin.)

*720-05-17 R. J. FAUDREE and R. H. SCHELP, Memphis State University, Memphis, Tenn., 38152. A Class of Panconnected Graphs.

A graph G which is already embedded in a plane is called a plane graph. For G a plane graph V(G), E(G), and F(G) denote the sets of its vertices, edges, and faces, respectively. Two distinct vertices (edges, faces) of G are adjacent if they share a common edge (vertex, edge). A vertex and an edge, a vertex and a face, or an edge and a face, are adjacent if they are incident (in the obvious sense). The entire graph of G, denoted e(G), is the graph with vertex set $V(G) \cup E(G) \cup F(G)$, with two vertices of e(G) adjacent if and only if they are adjacent in G. A graph is panconnected if each pair of vertices is connected by paths of all lengths greater than or equal to the distance between the vertices. Theorem. The entire graph e(G) of a bridgeless connected plane graph G is panconnected.
(Received October 31, 1974.)

*720-05-18 E.M. Palmer, Mich. State Univ.; R.W. Robinson, U of Newcastle, New South Wales, Australia. Enumeration of self-dual configerations.

A variety of combinatorial structures are self-dual in the sense that opposite elements have opposite properties. We provide a general enumeration theorem for these which has a number of interesting applications including the enumeration of self-dual boolean functions and 2-colorings of the vertices of polyhedra in which opposite vertices have different colors. Our method involves a modification of Pólya's enumeration theorem. (Received October 31, 1974.)

*720-05-19 MARK PANKIN, Marshall University, Huntington, West Virginia 25701, Row Admissible Latin Squares.

Definition. Two latin squares $A = (a_{ik})$ and $B = (b_{ik})$ based on the same set of n elements are called row admissible if and only if for each i,j ($1 \leq i, j \leq n$), there exists a unique k such that $a_{ik} = b_{jk}$. We show that row admissibility is equivalent to orthogonality in the following sense: Theorem. There exist two row admissible latin squares of order n if and only if there exist two orthogonal latin squares of order n.

The relationship of row admissible squares to finite projective planes is also discussed. (Received November 1, 1974.)

*720-05-20 CHARLES C. LINDNER, Auburn University, Auburn, Alabama 36830. A partial Steiner triple system of order n can be embedded in a Steiner triple system of order $6n + 3$.

A (partial) Steiner triple system (or more simply a triple system) is a pair (S, t) where S is a finite set and t is a collection of three element subsets of S (called triples) such that every pair of distinct elements of S belong to (at most) exactly one triple of t. The number $|S|$ is called the order of the (partial) triple system. The partial triple system (P, p) is said to be embedded in the triple system (S, t) provided that $P \subseteq S$ and $p \subseteq t$. A well-known theorem of Treash ["The completion of finite incomplete Steiner triple systems with applications to loop theory", J. Combinatorial Theory 10(1971), 259-265] states that a partial triple system of order n can be embedded in a triple system of order less than 2^{2n}. Not only does Treash's theorem give a very large containing triple system, but the proof is quite complicated (using induction). This paper very much improves Treash's theorem by proving that a partial triple system of order n can be embedded in a triple system of order $6n + 3$. An added benefit of the construction used is that it is an astonishingly simple direct construction. (Received November 1, 1974.)

*720-05-21 MURRAY HOCHBERG, Brooklyn College, City University of New York, Brooklyn, New York 11210. Doubly incomparable s-systems.

A map f from the first r natural numbers to the power set of a set Ω of cardinality n is called an "r-decomposition" if $f(i) \cap f(j) = \emptyset$, $i \neq j$, and $\bigcup f(i) = \Omega$. A set σ of r-decompositions is an "s-system of order r" if, for each index i, no two sets in the set $\{f(i) : f \in \sigma\}$ satisfy an inclusion relation. Meshalkin proved that the cardinality of σ cannot exceed the maximum multinomial coefficient. This generalizes an earlier result of Sperner, who had studied the case $r = 2$. We call the set $\{f(i) : f \in \sigma\}$ the

"ith column" of the s-system. The s-system is said to be "doubly incomparable" if no two elements in the ith column have the same cardinality, $i = 1, 2, \ldots, r$. We prove Theorem. In a doubly incomparable s-system with $r \geqq 3$, there are at most $n - k$ r-decompositions, where k is the smallest positive integer such that $1 + [n(r-2)]/r \leqq k$. Many of the lemmas used in proving the above result are also of independent interest. (Received November 1, 1974.)

*720-05-22 Y. EDMUND LIEN, University of Kansas, Lawrence, Kansas 66045. A theorem on strongly connected digraphs. Preliminary report.

An assignment to a directed graph $\langle N,A\rangle$, where N is the set of nodes and A is the set of arcs, is a function δ from $N \cup A$ to positive integers such that for each node n, we have

$$\sum_{\langle n_i,n\rangle \text{ in } A} \delta(\langle n_i,n\rangle)\cdot\delta(n_i) = \sum_{\langle n,n_j\rangle \text{ in } A} \delta(\langle n,n_j\rangle)\cdot\delta(n_j).$$

The main result in this note is the following: A connected directed graph is strongly connected if and only if there exists an assignment to the graph.

(Received November 1, 1974.) (Author introduced by Professor Richard Hetherington)

720-05-23 L.W. BEINEKE, Purdue University, Fort Wayne, Indiana, 46805 and S. FIORINI, The Open University, Milton Keynes, Bucks, England. On The Chromatic Index of Small Graphs.

Let $\chi'(G)$ denote the chromatic index (edge chromatic number) of graph G, and let $\rho(G)$ denote the maximum vertex degree. A theorem of Vizing states that for a simple graph G, $\chi'(G)$ equals $\rho(G)$ or $\rho(G) + 1$. A graph G is called critical if $\chi'(G) = \rho(G) + 1$ and $\chi'(H) < \chi'(G)$ for all proper subgraphs H. It has been conjectured that all critical graphs have odd order. The truth of this conjecture is shown for all graphs with up to ten vertices. Some structural properties on degrees and 1-factors in critical graphs are derived, and all critical graphs with fewer than nine vertices are constructed. (Received November 4, 1974.)

*720-05-24 WILLIAM T. TROTTER, JR. and JOHN I. MOORE, JR., University of South Carolina, Columbia, S. C. 29208, The Dimension of Planar Posets, Preliminary Report.

A partially ordered set (poset) is said to be planar if it has a Hasse diagram in which edges do not cross. It is known that a planar poset is a lattice iff it has both a greatest element and a least element. The dimension of a poset X is the smallest positive integer n for which X is isomorphic to a subposet of Euclidean n-space equipped with the product ordering. It is also known that the dimension of a planar lattice is at most two. In this paper we develop a fundamental theorem on extensions of partial orders. We obtain as corollaries several well known theorems in the dimension theory of posets. We then use this theorem to prove that the dimension of a planar poset which has a greatest element (or least element) is at most three. We also produce an infinite family of four dimensional planar posets. However, we have not been able to construct planar posets with dimension five or greater nor have we been able to bound the dimension of planar posets.

(Received November 4, 1974.)

*720-05-25 WILLIAM T. TROTTER, JR. and JOHN I. MOORE, JR., University of South Carolina, Columbia, S. C. 29208, The Dimension of A Tree is at Most Three, Preliminary Report.

If x and y are distinct points in a finite partially ordered set (poset) X, y is said to cover x when $x < y$ in X but there is no point $z \in X$ for which $x < z < y$ in X. The Hasse graph G(X) of X is the ordinary graph whose vertex set is X and whose edge set contains all two element sets of the form $\{x,y\}$ where y covers x. In this paper, we call a finite poset X a tree if G(X) is a tree in the graph theoretic sense. We then prove that the poset obtained from a tree by attaching a least element is planar and conclude that the dimension of a tree is at most three. We construct four trees J_1, J_2, $\hat{J}_1$, and $\hat{J}_2$ each of which has dimension three and then prove that a tree has dimension at most two unless it contains one or more of these three dimensional trees as subposets. (Received November 4, 1974.)

720-05-26 PAUL C. KAINEN, Case Western Reserve University, Cleveland, Ohio 44106 Coloring maps via branched coverings. Preliminary report.

Alpert, Gross, and Tucker have applied the technique of branched covering spaces to embeddings of complete graphs and, hence, to the Heawood map-coloring problem (see, e.g., Gross and Alpert, The topological theory of current graphs, J. Comb. Th., Ser. B., to appear). The following theorems indicate a direct connection between branched coverings and coloring. Theorem A. If M is a map on the sphere, then the 2-colorings of M are in 1-1 correspondence with the branched coverings of M(2), the map determined by a single vertex and a single loop. Theorem B. If M is a cubic bridgeless map on the sphere, then the 3-colorings of M are in 1-1 correspondence with the branched coverings of M(3), the map determined by two points and three parallel edges. These theorems depend on the appropriate local characterizations of 2-colorable maps and 3-colorable cubic maps, and they also use an interesting formulation of colorability in terms of digraphs due to Gallai. (Received November 4, 1974.)

*720-05-27 Michael O. Albertson, Smith College, Northampton, Mass. 01060 and Joan P. Hutchinson, Dartmouth College, Hanover, N. H. 03755. The Independence Ratio of a Toroidal Graph.

Suppose G is a graph with V vertices that embeds on the torus. Let $\alpha(G)$ be the maximum number of vertices in any independent set and $\mu(G) = \alpha(G)/V$. Since any toroidal graph can be seven colored $\mu(G) \geq 1/7$. Theorem. If $G \neq K_7$ and $V \not\equiv 1 \pmod 5$ then $\mu(G) \geq 1/5$. Theorem. If $V = 5K + 1$ then $\mu(G) \geq K/(5K + 1)$. There exists a graph J on 11 vertices with the properties that J is regular of degree 6, J does not contain K_5, yet $\mu(J) = 2/11$. (Received November 4, 1974.)

*720-05-28 RALPH FAUDREE and RICHARD SCHELP, Memphis State University, Memphis Tennessee 38152. Ramsey Numbers for Linear Forests.

An (N,j)-linear forest L is the disjoint union of nontrivial paths, j of which have an odd number of vertices, and such that the union has N vertices. If L_i is an (N_i,j_i)-linear forest $(i = 1,2)$, then we show that the Ramsey number of L_1 and L_2 is

$$r(L_1,L_2) = \max\{N_1 + (N_2 - j_2)/2 - 1,\ N_2 + (N_1 - j_1)/2 - 1\}.$$

This establishes the conjecture of S. Burr and J. Roberts given in the article "On Ramsey Numbers for Linear Forests" which is in Vol. 8 of Discrete Mathematics. (Received November 4, 1974.)

720-05-29 Gene A. Berg, Colorado State University, Fort Collins, Colorado 80523
An enumeration problem in finite geometries. Preliminary report.

Let X_i be the point of PG(N,S) for which $x_i = 1$ and for which the rest of the coordinates are zero ($i = 0,1,2,\ldots,N$). The simplex formed by $X_0, X_1, \ldots, X_n$ may be called the fundamental simplex. The points X_i may be called its vertices or 0-cells, and the (k-1)-flat formed by any k of the n + 1 vertices may be called a (k-1) cell of the simplex. A (k-1)-flat in PG(N,S) is said to have minimum weight $\geq$ m if every point has at least m nonzero coordinates. A k-flat has minimum weight $\geq$ m if and only if it intersects no d-cell of the fundamental simplex for any $d \leq m-1$. Using this fact together with lattice theory we count the number of k-flats of minimum weight $\geq$ m for m = 2 and 3, and in the case m = 4 for small values of N. This count has applications in coding theory and in the packing problem. For example, if $n_m(r,s)$ equals the maximum number of points in PG(r-1,s) such that no m are dependent, then $n_m(r,s) = n$ if and only if k = n-r is the largest integer for which there exists a (k-1)-flat in PG(n-1,s) of minimum weight $\geq$ m + 1. (Received November 4, 1974.)

720-05-30 JAMES E. WILLIAMSON, Dept. of Math., Southern Illinois University, Carbondale, Ill., 62901, Maximally Nonhamiltonian-connected Graphs. Preliminary Report.

If G is a graph which is not hamiltonian-connected and has the property that G + uv is hamiltonian-connected for each pair u,v of nonadjacent vertices of G, then G is called a maximally nonhamiltonian-connected graph. If G is a graph of order $p \geq 4$ whose vertex connectivity $\kappa(G)$ satisfies $2 \leq \kappa(G) \leq p/2$ and if c(G) denotes the number of components of the graph $G \backslash S$ where S is a minimal cut set of G, then all graphs G of order $p \geq 4$ which are maximally nonhamiltonian-connected and satisfy $2 \leq \kappa(G) \leq c(G) \leq p/2$ are known. In addition, examples of maximally nonhamiltonian-connected graphs G of order $p \geq 4$ satisfying $2 \leq c(G) < \kappa(G) \leq p/2$ are presented. (Received November 4, 1974.)

*720-05-31 WILLIAM B. POUCHER, Auburn University, Auburn, Alabama 36830. Finite embedding theorems for partial pairwise balanced designs. Preliminary report.

The pair (P, B) is a (partial) (n, b)-PBD if (P, B) is a (partial) pairwise balanced design with the property that $|P| = n$ and each block in B has exactly b elements. The following theorems are proved.
Theorem. If (P, B) is an (n, b)-PBD and $n > b \geq 4$, then (N, B) has an isomorphic disjoint mate.
Theorem. Suppose k and b are positive integers and $b > 4$. There is a constant C(k, b) such that if (P, B) is an (n, b)-PBD and $n > C(k,b)$, then there exist k mutually disjoint isomorphic mates of (P, B).
Theorem. Suppose k and b are positive integers, $k \geq 2$ and $b > 4$. If $(P,p_1), (P,p_2), \ldots, (P,p_k)$ is a collection of partial $(|P|, b)$-PBD's, there exist k (n, b)-PBD's, $(X,x_1), (X,x_2), \ldots, (X,x_k)$ such that (P,p_i) is embedded in (X,x_i) and for $i \neq j$, $p_i \cap p_j = x_i \cap x_j$. Additionally the existence of certain collections valuable in embedding is explored. (Received November 4, 1974.)

720-05-32 WILFRIED IMRICH, Montanistische Hochschule Leoben, 8700 Leoben, Austria and MARK E. WATKINS, Syracuse University, Syracuse, New York 13210, On automorphism groups of Cayley graphs, preliminary report.

If G is a group and H is a generating subset of G, let $X_{G,H}$ denote the Cayley graph of G w.r.t. H. Let $A(X_{G,H})$ denote the automorphism group of $X_{G,H}$, and define $c(G) = \min\{|A(X_{G,H})|/|G| : H \text{ generates } G\}$. Lemma: If $|G_1| > |G_2|$ and G_1 is not one of five "forbidden" groups, then $c(G_1 \times G_2) \leq c(G_1)c(G_2)$. Theorem 1: If G is a finite abelian group but not an elementary abelian 2-group, then $c(G) = 2$ with at most seven exceptions. Theorem 2: $c(G)$ is determined for all finite generalized dicyclic groups G; it is usually 2, occasionally 4, but never more than 16. (Received November 4, 1974.)

720-05-33 DONALD J. McCARTHY, St. John's University, Jamaica, New York 11439. Unicyclic graphs with cyclic automorphism group. Preliminary report.

A graph whose automorphism group is isomorphic to the group Γ is termed a Γ-graph. Throughout, Γ_k denotes a cyclic group of order k, and U is a unicyclic graph on p vertices. If m is a natural number, $U(m)$ denotes the m-fold replication of U; this is a unicyclic graph on mp vertices obtained from U in a natural way and having U as a factor graph. Theorem. If U is asymmetric then $U(m)$ is a Γ_m-graph. Moreover, if $m \neq 2$ every Γ_m-graph is of the form $U(m)$ where U is asymmetric. Corollary. If $A(X)$ is the generating function for asymmetric unicyclic graphs, the generating function for unicyclic Γ_m-graphs is $A(X^m)$ when $m \neq 2$. The Theorem is also applied to the following minimum-edge problem. Given a group Γ, it is known (c.f. these *Notices* 19(1972), A-620) that for n sufficiently large, the minimum number of edges for a Γ-graph on n vertices is attained by adding standard asymmetric forests to a fixed Γ-graph M. A description of M can be given whenever Γ is commutative. Direct factors of order 2 are easily dealt with; if Γ is commutative and has no direct factors of order 2, then M is a minimum-vertex Γ-graph whose components are mutually nonisomorphic and each of which is unicyclic with cyclic automorphism group. (Received November 4, 1974.)

720-05-34 DENNIS P. GEOFFROY, University of South Carolina, Columbia, South Carolina 29208. Nuclei of a point determining graph. Preliminary report.

The nucleus of a point determining graph G is the set $G^0 = \{v \in V(G) | G - v \text{ is point determining}\}$ and similarly the edge nucleus is the set $E^0(G) = \{e \in E(G) | G - e \text{ is point determining}\}$. (See D. P. Sumner, Discrete Math. 5(1973), 179-187 and Abstract 74T-A193, these *Notices* 21(1974), A-519.) In this paper the investigation of $E^0(G)$ (begun in the abstract above) is continued with particular emphasis on the relation of $E^0(G)$ to G^0. For G a connected, noncomplete, point determining graph we have the following. Theorem 1. $G^0 \cap V(E^0(G)) \neq \emptyset$. Theorem 2. Every vertex of G is adjacent to a vertex in $V(E^0(G))$. Theorem 3. For $v \in G^0 - V(E^0(G))$, $\delta(v) \leq |V(E^0(G))|$. Theorem 4. There exists only a finite number of graphs with a given edge nucleus. Theorem 5. If G is totally point determining then every odd cycle contains an edge in $E^0(G)$. (Received November 4, 1974.)

720-05-35 LINDA LESNIAK, Louisiana State University, Baton Rouge, Louisiana, 70803. Some recent results in hamiltonian graphs.

A variety of recent developments in hamiltonian theory are reviewed. In particular, several sufficient conditions for a graph to be hamiltonian, certain hamiltonian properties of line graphs, and various hamiltonian properties of powers of graphs are discussed. Furthermore, the concept of an n-distant hamiltonian graph is introduced and several theorems involving this special class of hamiltonian graphs are presented. (Received November 4, 1974.)

720-05-36 J.W. MOON, University of Alberta, Edmonton, Alberta
On the expected packing and covering numbers of a tree. Preliminary report.

The packing number P of a tree T is the size of any largest subset of nodes of T no two of which are joined by an edge; the covering number C is the size of any smallest subset of nodes of T such that each remaining node of T is joined to at least one node in the subset. We discuss a method for determining the expected values of P and C for various types of trees. In particular, it follows that the expected value of $P + C$ over the n^{n-2} trees T with n labelled nodes is approximately $.927\ n$, when n is large.

(Received November 4, 1974.)

*720-05-37 ALLEN J. SCHWENK, Michigan State University, East Lansing, Michigan 48824. On moments and coefficients in spectral graph theory.

The coefficients and monents of the characteristic polynomial of a p-point graph are obviously related by means of the intermediate step of calculating the eigenvalues. However, this often tedious intermediate step can be avoided by various possible approaches. In fact, it is even possible to determine the first $n - p$ coefficients from the first n moments (or vice-versa), whereas the routine method must fail because the eigenvalues are not even determined. The oldest formula that accomplishes this is a recurrence known as "Newton's Relation". The solution of this relation for the n th coefficient is best expressed by means of cycle indices as presented in Read [Canad. J. Math. 20 (1968) 808-841]. We present the corresponding solution for the n th moment. To express this formula, we introduce the concept of circular partitions of the integer n. (Received November 4, 1974.)

*720-05-38 WILLIAM SPEARS and BARBARA JEFFCOTT, Kansas State University, Manhattan, Kansas 66506. A construction of combinatorial geometries.

We give a method for constructing rank $k + 1$ combinatorial geometries from rank k combinatorial geometries. Specifically, given a geometry $G(B)$, a set V disjoint from B, and a descriptor Q (a descriptor is a function from $L(B)$ to $P(P(V))$), we construct a geometry $G[Q]$ on $B \cup V$ which, as a lattice, contains B as a copoint and contains $L(B)$ as the closed interval from 0 to B. Conversely, given a combinatorial geometry $G(S)$, we may find a copoint $B \in L(S)$ and a descriptor Q such that $G(S) = G[Q]$. Given $G(B)$, V and a descriptor Q for $G(B)$ and V, Q determines a geometry $G(V)$. We characterize all those descriptors where the rank of $G(V)$ is 1 or 2. (Received November 4, 1974.)

*720-05-39 Frank Harary, University of Michigan, Ann Arbor, Michigan 48104 and CARSTEN THOMASSEN, Aarhus University, Aarhus, Denmark, Anticritical graphs

The term "critical graph" has been used most frequently to mean a graph such that the removal of any line decreases the chromatic number. This can be generalized to define a graph G to be "critical" (μ)" for an arbitrary invariant μ, if for each line $e, \mu(G - e) \neq \mu(G)$. Analogously we may consider any line e which joins two points not already adjacent in G, and define G as "anticritical (μ)", if $\mu\ (G = e) \neq \mu(G)$. Anticritical graphs are investiga-

ed here with respect to the following nine invariants: chromatic number, line-chromatic umber, point-connectivity, line- connectivity, point-independence number, line- chromatic umber, diameter, radius, and cyclability number. (Received November 4, 1974.)

*720-05-40 JEAN DUNBAR and RENU LASKAR, Clemson University, Clemson, S. C. 29631. Finite nets of dimension d. Preliminary report.

This paper introduces the concept of a finite net of dimension d. A d-class association scheme is defined on this system which is then shown to form a partially balanced incomplete block design. This generalizes results of Bruck [Finite Nets I. Numerical Invariants, Can. J. Math., (1951), 96-107] and Laskar [Finite Nets of Dimension Three I, to appear].

(Received November 4, 1974.)

720-05-41 ALFRED P. MANEKI, Dept. of Defense, Fort George G. Meade, MD 20755, On permutations of groups, Preliminary report.

Let G be a group of order n and let π be a permutation of G. Let Y be the n-multiset of G given by $Y = \{g^{-1}\pi(g):g\varepsilon G\}$. It is easy to see that Y must have the property

(*) $y_1y_2 \ldots y_n = 1$ for some ordering of the elements of Y.

Marshall Hall, Jr. (Proc. A.M.S. 3 (1952)) has shown that property (*) characterizes such multisets Y in the case that G is abelian.

We investigate this characterization problem without the abelian assumption on G. In particular, we show that Hall's theorem fails for nonabelian groups.

(Received November 4, 1974.)

720-05-42 JOHN F. DILLON, Dept. of Defense, Fort George G. Meade; MD 20755. Difference sets in 2-groups. Preliminary report.

Let H denote the class of groups which contain a Hadamard difference set (i.e. v=4n). We generalize a construction of R. L. McFarland to obtain the

THEOREM. *H contains any group of order 2^{2m} whose center contains Z_2^m.*

COROLLARY. *H contains any group of order $2^{2\pi}$ which is the direct product of m nontrivial subgroups.*

We thus obtain difference sets in the groups $Z_2^{m-1} \times Z_{2^{m+1}}$ which demonstrate the sharpness of the exponent bound established by R. J. Turyn. That the condition of the theorem is not necessary is shown by the

REMARK. *Of the fourteen groups of order 16 only the cyclic and dihedral groups fail to belong to H.* (Received November 4, 1974.)

*720-05-43 TERRENCE J. BROWN, State University of New York, College at Oswego, Oswego, New York 13126. A rank equality for matroids.

Let J and K be matroids on the same finite set. The union $J \cup K$ is the matroid whose indepenc sets are unions of J and K independent sets. The meet $J \cap K$ is the matroid whose spanning sets ar intersections of J and K spanning sets. Then the rank functions satisfy: $r(J \cup K) + r(J \cap K) = r(j) + r(K)$. Moreover, the maximal cardinality of an independent set shared by J and K is $r(J) - r(J \cap K^*)$ (Received November 5, 1974.)

720-05-44 LADNOR GEISSINGER, University of North Carolina, Chapel Hill, N.C. 27514 Combinatorics and an Algebra of Divided Powers, Preliminary Report

In his notes λ-Rings and the Representation Theory of the Symmetric Group, Lecture Notes in Math 308, Springer 1973, D. Knutson systematically developes the correspondence between characters of the finite symmetric groups and symmetric functions of an infinite number of variables, and does it in the context of the free λ-ring on one generator. This allows for a nice description of much of the classical combinatorial theory associated with symmetric groups and functions. By considering the λ-ring as a Hopf algebra we are led to new interpretations of some symmetric function identitites and to the identification of the primitive and group-like elements and sequences of divided powers. We relate these to properties of characters of the symmetric groups. A natural inner product and duality are shown to correspond to P. Hall's inner product on symmetric functions. There are further connections with the representation rings of the hyperoctahedral groups. (Received November 6, 1974.)

*720-05-45 A.E. HURD, University of Victoria, Canada. Nonstandard Methods in Combinatorics and Graph Theory, Preliminary report.

The methods of nonstandard analysis are employed to extend known results in finite combinatorics and graph theory to the infinite case. As examples we consider the Rado Selection Lemma and generalizations, factorization of graphs, and results of the Folkman-Fulkerson type on flows in infinite graphs. (Received November 6, 1974.)

720-05-46 PAUL K. STOCKMEYER, College of William and Mary, Williamsburg, Virginia 23185. The enumeration of Boolean graphs. Preliminary report.

A Boolean graph on $2n$ vertices is a graph whose automorphism group is the permutation group $S_2 + S_2 + \ldots + S_2$ of order 2^n. A one-to-one correspondence is established between Boolean graphs on $2n$ vertices and pseudo-graphs on n vertices with no nontrivial atuomorphisms. Appropriate modifications of the author's results for counting graphs with only the identity automorphism lead to an enumeration of Boolean graphs by number of vertices and edges. (Received November 6, 1974.)

720-05-47 F. D. PARKER, St. Lawrence University, Canton, New York 13617. Cycles of length four in chromatic graphs.

Given a set of n points, we join each pair of points by a colored line, using any one of k colors. How large can n be if the complete graph is to contain no cycle of length four? Using the adjacency matrix and its characteristic values, it is proved that $n \leq k^2 + k + 1$. (Received November 6, 1974.)

*720-05-48 BÉLA BOLLOBÁS, University of Cambridge, England. Extremal problems in graph theory. Preliminary report.

Denote by G^n a graph with n vertices and by $G(n,e)$ a graph with n vertices and e edges. The complete graph with p vertices is denoted by K_p. The author investigates the following functions. The minimal number of K_p's a $G(n,e)$ must contain. The minimal number of edge disjoint K_p's and edges sufficient to cover every G^n. The minimum of e ensuring that every $G(n,e)$ contains a given graph H. The minimal number of edges a G^n must have if the addition of every edge creates a new K_p. The minimum of e such that every $G(n,e)$ contains a topological copy of a given graph. The minimum of δ such that every G^n (if n is sufficiently large) with minimal degree δ contains d vertex disjoint edges, cycles or K_p's. The minimal number of pairs of vertices one has to check in order to decide whether or not a graph with a given vertex set has a given property. The author also investigates some corresponding problems about r-graphs and states a number of unsolved problems. (Received November 21, 1974.) (Author introduced by Professor F. Harary.)

06 Order, Lattices, Ordered Algebraic Structures

*720-06-1 ROBERT R. WILSON, California State University, Long Beach, California 90840 Lattice Orderings on the Real Field.

Since every total order is a lattice order, the real field with the usual total order is a lattice-ordered field. In 1956 Birkhoff and Pierce raised the question of whether the real field can be made into a lattice-ordered field in any other way. The answer is affirmative: Theorem: Every subfield L of the real field $\underset{\sim}{R}$, except the rational field, admits a non-total lattice order. Moreover, $\underset{\sim}{R}$ admits 2^c such orders, where c is the cardinality of the continuum. (Received August 2, 1974.)

720-06-2 TAEN-YU DAI, York College, City University of New York, Jamaica, New York 11432. RALPH E. DeMARR, University of New Mexico, Albuquerque, New Mexico 87131. Isotone functions and diagonal projection operators.

Let A be a Dedekind σ-complete partially ordered (real, associative) linear algebra (dsc-pola) which contains $1 \geq 0$ as a multiplicative identity. Let $I = \{y \in A : y \geq 1 \text{ and } y^{-1} \geq 0\}$ and $A_1 = I - I$. It is known that A_1 is a commutative algebra and is called the diagonal or functional part of A. (If $A =$ the algebra of all real $n \times n$ matrices with componentwise ordering, then $A_1 =$ all the diagonal matrices.) Let $H = \{x \in A : x \geq 1\}$. Definition. An isotone function $f : H \to A$ is a function with logarithm property if: (1) $f(xy) = f(x) + f(y)$; (2) $f(x) \leq x - 1$; (3) there exists some $a \in H$ such that $f(a) \neq 1$. An analogous definition for a function with root property can also be similarly defined on H. Theorem. If a dsc-pola A which is also a lattice has a function with logarithm or root property, then there exists an isotone linear diagonal projection map $\Delta : A \to A_1$ such that: (i) $\Delta(1) = 1$; (ii) $\Delta \leq \Delta(x) \leq x$ for $x \geq 0$; (iii) $\Delta(z) = z$ for $z \in A_1$; (iv) Δ is multiplicative, i.e., $\Delta(xy) = \Delta(x)\Delta(y)$ for all x, y. (Received October 2, 1974.)

720-06-3 MICHAEL KEISLER, Randolph-Macon College, Ashland, Virginia 23005. A characterization of a type of nearest point set in complete ℓ-groups.

Let G be a complete ℓ-group. $S \subset G$ will be said to be an ℓ-nearest point set iff $x \in G$ implies

there is an $\bar{x} \in S$ such that $|x - \bar{x}| < |x - y|$, for every $y \in S$ such that $y \neq \bar{x}$. Lemma. If G is a UMB-lattice, then every ℓ-nearest point set is a nearest point set with respect to the norm on G, but not the converse. Let P be the positive cone of G, $\mathcal{L} = \{ \alpha \mid \alpha : P \to P$ such that α is isotone, $\alpha(x) \leq x$, for $x \in P$, and $\alpha(x + y) \leq \alpha(\alpha(x) + \alpha(y))$, for $x,y \in P\}$, $\alpha + \beta = \{ (x, \alpha(x) + \beta(x - \alpha(x))) \mid x \in P\}$, for $\alpha,\beta \in \mathcal{L}$, and $P^* = \{ \alpha \in \mathcal{L} \mid \alpha + \beta = 1$ implies $\beta = 1\}$. P^* is the positive cone of a lateral completion of G, denoted G^*. For $\alpha \in \mathcal{L}$, let $\bar{\alpha}(x) = \alpha(x^+) - \alpha(x^-)$, for $x \in G$. Theorem. $\bar{\alpha}(G)$ is an ℓ-nearest point set, for $\alpha \in \mathcal{L}$, and there is an ℓ-nearest point set S such that $S = -S$ and $S \neq \bar{\alpha}(G)$, for $\alpha \in \mathcal{L}$. Theorem. If G is a complete vector lattice, then S is an ℓ-nearest point set iff there is $\lambda \in G^*$ and $\alpha \in \mathcal{L}$ such that $T(S) = (\lambda + [-\alpha,\alpha])\circ\Gamma(G)$, where Γ is the canonical map from G to G^*, and $[-\alpha,\alpha] = \{ \beta \in G^* \mid |\beta| \leq \alpha\}$. Corollary. S is an ℓ-nearest point set such that $S = -S$ iff $S = \bar{\alpha}(G)$, for some $\alpha \in \mathcal{L}$.
(Received October 21, 1974.)

720-06-4 EUGENE M. NORRIS, DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE, UNIVERSITY OF SOUTH CAROLINA, COLUMBIA 29208. Galois Connection Algorithms. Preliminary Report

Let R be a relation from a set X to a set Y. A maximal rectangle in R is such a set $A\times B \subseteq R$ that if $A\times B \subseteq C\times D \subseteq R$ then $A = C$ and $B = D$. When $Y = X$ we say a set C is an R-chain if $C\times C\ \Delta_X \subseteq R\cup R^{-1}$. As in Bednarek and Taulbee's paper [Rev. Roum. Math. Pures App XI (1966), 23-25] we say an R-chain is maximal if it is contained in no other R-chain. It is known that R and its reflexive symmetric extension $R^* = R\cup R^{-1}\cup\Delta_X$ have the same (maximal) chains. One verifies that if C is a maximal R-chain then $C\times C$ is a maximal rectangle in R^*. In the general case ($X\neq Y$) R generates a Galois connection (GC) between the "closed" subsets of X and Y; see Birkhoff's lattice theory [AMS Colloquium Publication XXV Ch. 4 Sections 5 and 6 (1948)] for details. The GC operates to produce the maximal rectangles in R. In An Algorithm for Finite Galois Connections [Technical Report of KGM ISZSZI, Budapest, Hungary (1973)] G. Fay produces an algorithm for finding the GC when X and Y are finite, which is quite cumbersome computationally. We show how to use the algorithm of Bednarek and Taulbee for finding maximal R-chains to find GC's. The link is this: Proposition. $A\times B$ is a maximal rectangle in R iff (1) A is a maximal $R\cdot R^{-1}$ chain (2) B is a maximal $R^{-1}\cdot R$ chain and (3) $A \not\times B$. (Received October 25, 1974.)

720-06-5 SIEMION FAJTLOWICZ, University of Houston, Houston, Texas. Means in lattices and semilattices.

If L is a complete lattice, $\mathcal{F}$ an ultrafilter of subsets of a set X and $f \in L^X$ we put $M^{\mathcal{F}}(f) = \bigcap_{F\in\mathcal{F}} \bigcup_{t\in F} f(t)$ and $M_{\mathcal{F}}(f) = \bigcup_{F\in\mathcal{F}} \bigcap_{t\in F} f(t)$. Theorem 1. A complete distributive lattice is bi-Brouerian iff the algebra $(L; \vee, (a\wedge x)_{a\in L})$ is equationally compact. Corollary (Baezer). A complete distributive lattice is bi-Brouerian iff every system of polynomial equations in one variable is solvable in L whenever it is finitely solvable in L. Theorem 2. If $M^{\mathcal{F}} = M_{\mathcal{F}}$ then L is equationally compact iff both $(L;\vee)$ and $(L;\wedge)$ are equationally compact. Corollary. If L is a complete completely distributive lattice then L is equationally compact. The proofs of Theorems 1 and 2 are based on the following Lemma. If L is a join complete semilattice in which every down-directed set has a meet then L is equationally compact iff for every ultrafilter $\mathcal{F}$, $M^{\mathcal{F}} : L^X \to L$ is a homomorphism. (Received October 30, 1974.)

*720-06-6 RICHARD GREECHIE, Kansas State University, Manhattan, Kansas 66506
Some Results from the Combinatorial Approach to Quantum Logic

A cubic structure space is a pair $(X,\mathcal{E})$ where X is a non-empty set and $\mathcal{E}$ is the set of all maximal cliques for a graph on X such that each element of $\mathcal{E}$ is a three-element subset of X. A cubic structure space is wide (tight) in case, for all $x,y \in X$ with $x \neq y$, x and y have at most (exactly) one common neighbor in the graph determined by $(X,\mathcal{E})$. Each wide cubic structure space $(X,\mathcal{E})$ can be embedded in a tight cubic structure space $\pi(X,\mathcal{E})$. Each tight cubic structure space $\pi(X,\mathcal{E})$ determines an orthocomplemented projective plane $\mathcal{L}(\pi(X,\mathcal{E}))$ called the free orthocomplemented projective plane over $(X,\mathcal{E})$. Most of the $\mathcal{L}(\pi)$'s constructed are non-desarguesian projective planes. Thus there exist non-desarguesian orthocomplemented projective planes. These correspond to 3-dimensional Proposition Systems chez Jauch and Piron. By properly selecting $(X,\mathcal{E})$ we produce proposition systems which are not quantum logics, or which are and which admit dispersion free states. It follows that the quantum logical axiom "$\sigma(a) = \sigma(b) = 1$ implies $\sigma(a \wedge b) = 1$" is not deducible from the other axioms in a general proposition system (unlike the special cases of a Boolean σ-algebra or a Hilbert space). We also construct a π_0 such that $\mathrm{Aut}_\perp(\mathcal{L}(\pi_0))$ contains a copy of the general quantum mechanical group. (Received November 1, 1974.)

*720-06-7 STEPHEN H. McCLEARY, University of Georgia, Athens, Georgia 30602.
Groups of homeomorphisms of the real line with manageable automorphism groups.

Let R be the real numbers, M the group of monotone permutations of R, A the increasing permutations, and B the increasing permutations of bounded support. Let $D = \{g \in M \mid g' \text{ exists and is never } 0\}$ and $PL = \{g \in M \mid g \text{ is piecewise linear}\}$. Let K be a doubly transitive subgroup of M containing a positive (in the pointwise order) element of bounded support, and let $K^* = K \cap A$. Thm. Every group automorphism of K or K^* is conjugation by an element of M, i.e., $\mathrm{Aut}(K) \leq M(R)$. Thus every automorphism either preserves or reverses order, and this is the key to the proof. Cor. Every automorphism of M is inner (Fine and Schweigert), and $\mathrm{Aut}(A) = \mathrm{Aut}(B) = M$; and analogously for D and PL. B is an infinite counterexample to Schreier's conjecture that every simple group has a solvable outer automorphism group. (Received November 4, 1974.)

*720-06-8 JORGE MARTINEZ, University of Florida, Gainesville, Fla. 32611.
Doubling chains, singular elements and hyper-χ ℓ-groups.

In an ℓ-group G a (descending) doubling chain is a sequence $a_1 > a_2 > \dots > a_n > \dots$ of positive elements of G such that $a_n \geq 2a_{n+1}$. An element $0 < s \in G$ is singular if $0 \leq g \leq s$ implies that $g \wedge (s - g) = 0$. The main theorems are as follows: 1. The following two statements are equivalent; a) every doubling chain in G is finite; b) $G = \bigcup_{\tau<\alpha} G^\tau$ (τ ranging over all ordinals less than α), where each G^τ is an ℓ-ideal of G, $\sigma < \tau$ implies that

$G^{\sigma} \subseteq G^{\tau}$, and $G^{\tau+1}/G^{\tau}$ is generated by its singular elements, (or is a Specker group, à la Conrad). 2. If G is hyper-archimedean as well then either of the above conditions is equivalent to c): G is hyper-$\mathbb{Z}$, ie. every totally ordered ℓ-homomorphic image of G is cyclic. (Received November 4, 1974.)

*720-06-9 ERIC G. R. GERELLE, Kansas State University, Manhattan, Kansas 66506.
Representation of Finite Orthomodular Posets, Preliminary report.

Let P be a finite orthomodular partially ordered set and S the convex set of states of P. If S is full on P then there is a faithful embedding $\hat{P}$ of P into the unit interval $[0,1_S]$ of the order unit space of affine functionals on S. Let X denote the atoms of P.

Theorem: If $A \subseteq X$ and $s|_A \neq 0$ for every s in S imply that A contains a maximal orthogonal subset of X, then $\hat{P}$ coincides with the extreme points of $[0,1_S]$. (Received November 4, 1974.) (Author introduced by Dr. R. J. Greechie.)

*720-06-10 C. HERRMANN, Vanderbilt University, Nashville, Tennessee 37235 and R. WILLE, Technische Hochschule, Darmstadt, W. Germany. On modular lattices with four generators II. Preliminary report.

For definitions see A. Day et al., Algebra Universalis 2 (1972), 317-323.

Theorem. The subdirectly irreducible modular lattices which are generated by elements e_1,e_2,e_3,e_4 subject to the relations $e_1 \cdot e_2 = e_3 \cdot e_4 = 0$ and $e_1 + e_2 = e_3 + e_4 = 1$ are exactly the lattices M_4, $S(k,4)$ $(k < \infty)$, $FM(J_1^4)$ and its dual. This solves a problem stated by G. Birkhoff. The proof is based on the Lemma. If M is a subdirectly irreducible modular lattice with generators e_1,e_2,e_3,e_4 and none of the above then either (a) $e_i \cdot e_j = 0$ for all $i < j$ or (b) $e_i + e_j = 1$ for all $i < j$. Moreover, if (a) holds then there is a $k < \infty$ such that $m'_k = m'_{k+1}$ where the m'_k are defined recursively by: $m'_0 = 1$, $m'_{k+1} = e_1 m'_k + e_2 m'_k$ if k is even and $m'_{k+1} = e_3 m'_k + e_4 m'_k$ if k is odd. The key to the proof of the Lemma is a recent result of R. Freese: any subdirectly irreducible modular lattice can be embedded in an upper continuous subdirectly irreducible modular lattice.
(Received November 4, 1974.)

*720-06-11 ARMANDO R. GINGRAS, IBM Corporation, Boulder, Colorado 80302.
Compact convergence lattices. Preliminary report.

A *convergence lattice* is a complete lattice in which order convergence coincides with topological convergence w.r.t. the order topology Θ (Abstract 74T-A118, these *Notices* 21(1974), A433). *Theorems*. 1. A convergence lattice is Θ-compact iff the interval topology is Hausdorff. 2. A completely-distributive complete lattice is a compact convergence lattice. 3. The order and interval topologies agree on a complete Boolean algebra iff it is a convergence lattice.
(Received November 4, 1974.)

*720-06-12 BARBARA JEFFCOTT and WILLIAM SPEARS, Kansas State University, Manhattan, Kansas 66506. Semimodularity in the completion of a poset.

Theorem. Let L be a complete lattice and let P be a subset of L containing 0 and 1. Provide P with the induced ordering. Let $\overline{P}$ denote the MacNeille completion by cuts of P. For $X \subseteq L$ define $X^u = \{p \in P : p \geq x \forall x \in X$, define X^l dually, and define $X^{ul} = (X^u)^l$. Define $\sigma : \overline{P} \to L$ by $\sigma(X^{ul}) = \bigvee X^{ul}$. Then σ is an isomorphism iff $\forall a \in L$, $a = \bigwedge a^u = \bigvee a^l$. M.D. MacLaren proved that if L is a semimodular, atomic, orthocomplemented lattice, then $\overline{L}$ is semimodular. We use the above theorem to show that this result does not generalize to posets. Theorem. Let P be an atomic, orthomodular, orthocomplete poset with the covering condition. Then the following conditions are equivalent: (1) $\overline{P}$ satisfies the covering condition; (2) P is a lattice; (3) P is a complete lattice; (4) P is orthoisomorphic to $\overline{P}$; (5) $\overline{P}$ is orthomodular. Furthermore, if P has no infinite chains, then the above conditions and the following conditions are all equivalent: (6) $\overline{P}$ is modular; (7) $\overline{P}$ is M-symmetric. (Received November 4, 1974.)

720-06-13 MICHAEL E. DETLEFSEN, Slippery Rock State College, Slippery Rock, PA 16057 Products of symmetric elements in the Noether lattice, $RL(A_i)$, Preliminary report.

If $\mathfrak{M} = (k_1, \ldots, k_m)$ is an m-tuple of non-negative integers, a matrix (a_{ij}) of non-negative integers with m rows is an $\mathfrak{M}$-matrix iff $0 \leq a_{ij} \leq k_i$ for $i = 1, \ldots, m$. An mXn $\mathfrak{M}$-matrix is maximal iff each row is maximum with respect to lexicographic order on n-tuples. Using a modification of [Gale, Thm. p1080, Pacific J. Math 7(1957) 1073-1082] we have Theorem. Let B be the maximal mXn $\mathfrak{M}$-matrix with row sum vector (a_i) and column sum vector (s_j). For a monotone decreasing n-tuple (b_j) with $\sum b_j = \sum a_i$, there exists an $\mathfrak{M}$-matrix with column sum vector (b_j) iff (b_j) is majorized by (s_j); i.e., for $i = 1, \ldots, n$, $\sum_1^t b_j \leq \sum_1^t s_j$. Now, if $RL(A_i)$ is the distributive regular local Noether lattice with regular parameters $A_1, \ldots, A_n$ let a_i be the join of all principal elements whose exponents sum to i and are bound by 0 and 1 $(i = 1, \ldots, n)$. Theorem. $a_1^{t_1} \cdots a_n^{t_n}$ is the join of all compositions of $\sum it_i$ whose exponent n-tuples are majorized by (t_i^*) where $t_i^* = t_i + \ldots + t_n$. (Received November 6, 1974.)

*720-06-14 RONALD P. MORASH, University of Michigan, Dearborn, Michigan 48128. The hyperoctant property in orthomodular AC-lattices.

The complete atomic orthomodular lattice L is said to have the hyperoctant property iff for every orthogonal family of atoms $\{a_\alpha\}$ in L with cardinality ≥ 2, there exists an atom q s.t. $q \leq \bigvee_\alpha a_\alpha$ and $q \not\subset a_\alpha$ for each α. The projection lattice of any separable Hilbert space has the hyperoctant property and any atomic orthomodular lattice possessing the hyperoctant property is irreducible, but it is not known whether every complete, atomic, irreducible, orthomodular lattice (with no orthogonal families of atoms of cardinality $> \aleph_0$) has the hyperoctant property. In this paper we show that under the additional assumptions on L of M-symmetry and the angle bisection property the hyperoctant property does obtain. (Received November 6, 1974.)

08 General Mathematical Systems

720-08-1 AWAD A. ISKANDER, University of Southwestern Louisiana, Lafayette, Louisiana, 70501. Locally finite ring variety.

A variety is called locally finite if every finitely generated member is finite. If U and V are ring varieties, then $U \cdot V$ is the variety of all rings possessing an ideal belonging to U whose quotient belongs to V. Theorem 1: A ring variety is locally finite iff its free member of rank 1 is finite. Theorem 2: A ring variety contains only finitely many subvarieties iff it is generated by a finite ring. Theorem 3: Let U and V be ring varieties. Then $U \cdot V$ is generated by a finite ring (locally finite) iff each of U and V is generated by a finite ring (locally finite). (Received October 7, 1974.)

10 Number Theory

*720-10-1 DONALD HAZLEWOOD, Southwest Texas State University, San Marcos, Texas 78666. On ideals having only small prime factors. Preliminary report.

Let K be an arbitrary algebraic number field of degree n. Let $\mathcal{O}$ denote the ring of integers in K, a an integral ideal in $\mathcal{O}$, p a prime ideal in $\mathcal{O}$, and Na the norm of a. For real numbers, $x \geqq 1$, $y \geqq 1$, and an arbitrary integral k, $k \neq (0)$, we define the function $\Psi_k(x,y)$ to be the number of ideals a with $Na \leqq x$, $(a,k) = (1)$, and if $p|a$, then $Np \leqq y$. After the manner of B.V. Levin and A.S. Fainleib [Russ. Math. Surveys. 22: 3(1967), 119-204], we prove a general theorem giving a functional equation satisfied by sums over the ideals a with $Na \leqq x$ for certain classes of completely multiplicative functions. For the special sum $\Psi_k(x,y)$, using the functional equation, we derive an asymptotic estimate with arbitrarily many leading terms and a bonafide error term that is uniform with respect to x, y, and k within a restricted range. In the special case that $k = (1)$, our asymptotic estimate improves previous estimates for x and y within the restricted range. (Received September 19, 1974.)

*720-10-2 CARL POMERANCE, University of Georgia, Athens, Georgia 30602. On multiply perfect numbers and Mersenne primes.

Say $n(1) < n(2) < \ldots$ is an infinite sequence of integers, each with precisely k distinct prime factors, and $\sigma(n(t))/n(t) = \alpha$ for all t, where α is a fixed rational. Theorem (Artuhov, Acta Arith. 23 (1973), 249-255): For all but finitely many t, n(t) is even. Theorem (Kanold, Math. Ann. 132 (1956), 246-255): For all but finitely many t, $n(t) = e(t)m(t)$ where $(e(t),m(t))=1$ and e(t) is an even perfect number. In this paper we present a much quicker proof of Kanold's theorem and also prove some new corollaries. Say that a set S of integers has property D if for each k there are only finitely many members of S with precisely k distinct prime factors. Corollary 1. The set of all n for which $\sigma(n)/n$ is an odd integer has property D. Corollary 2. The set of all non-perfect multiply perfect numbers fails to have property D if and only if (1) there are infinitely many Mersenne primes and (2) there exists an odd multiply perfect number $m \neq 1$. Important in the proof of the theorem is a well-known result of Gelfond concerning rational approximations. Note that Corollary 2 ties together two famous unsolved problems.
(Received November 6, 1974.)

*720-10-3 Craig M. Cordes, Louisiana State University, B.R., La., Quadratic forms over non-formally real fields.

Kaplansky defined the radical, R, of a field, K, as $R = \{a \in \dot{K} \mid [a,b] = 1 \text{ for all } b \in \dot{K}\}$ where $[a,b]$ is the Hilbert symbol. R is a subgroup of $\dot{K}$ containing $\dot{K}^2$. Non-formally real fields with exactly two quaternion algebras have u-invarient equal to four and thus have level s equal to 1, 2, or 4. These notions are used to obtain a characterization of the Witt ring (and hence of the quadratic form structure) over such fields. Theorem. Let K be a non-formally real field with exactly two quaternion algebras. Suppose $|\dot{K}/\dot{K}^2| = 2^r < \infty$ and $|R/\dot{K}^2| = 2^k$. Then $r = k \pmod 2$ if $s = 1,2$ and $r \not\equiv k \pmod 2$ if $s = 4$. Moreover, k and the Witt group structure determine the Witt ring. (Received October 9, 1974.)

*720-10-4 HARALD NIEDERREITER, The Institute for Advanced Study, Princeton, New Jersey 08540. The distribution of pseudo-random numbers generated by the linear congruential method.

For $m \geq 2$, let $y_0, y_1, \ldots$ be a sequence in the least residue system mod m generated by $y_{n+1} \equiv \lambda y_n + r \pmod m$, $n = 0,1,\ldots$ where $(\lambda, m) = 1$ and $\lambda \not\equiv 1 \pmod m$, and let τ be its period. Results concerning the equidistribution test for the associated sequence $x_0 = y_0/m$, $x_1 = y_1/m, \ldots$ of pseudo-random numbers are established. If m is prime, then for $1 \leq N \leq \tau$ the discrepancy D_N of $x_0, x_1, \ldots, x_{N-1}$ satisfies $D_N < X \log(1 + 4X^{-1}) + X$, where $X = (4m^{1/2}/\tau N)(\frac{2}{\tau} \log \tau + \frac{2}{5} + \frac{N}{\tau})$, and a similar result holds for prime powers m. Analogous estimates are shown for arbitrary linear congruential generators mod primes, the results being particularly interesting for pseudo-random numbers generated by maximal period sequences mod primes. The proofs depend heavily on bounds for character sums involving linear recurring sequences established by the author ["Some new exponential sums with applications to pseudo-random numbers." Colloquium on Number Theory (Debrecen, 1974), North-Holland, to appear]. It is also shown that the discrepancy estimates are best possible apart from logarithmic factors. (Received October 24, 1974.)

*720-10-5 L. M. CHAWLA and ELLEN TORRANCE, Kansas State Univ., Manhattan, Ks. 66502 More Additive Arithmetic Functions Inversely Associated with Partition Functions, Preliminary Report.

Three classes of additive arithmetic functions $\{F(x)\}$, $\{G(x)\}$, $\{H(x)\}$, in each of which each member is inversely associated with a partition function $\overline{F}(n)$, $\overline{G}(n)$, $\overline{H}(n)$ in the sense that $\overline{F}(n)$ is the number of integers x such that $F(x) = n$, etc., were studied by Chawla, L. M., in On Additive Arithmetic Functions Inversely Associated with Partition Functions, Jour. Natur. Sci. and Math. 12 (1972) 413-420. Here we sharpen its Theorems A and C concerning respectively $\{F(x)\}$ and $\{H(x)\}$ and widen the class $\{G(x)\}$, and then sharpen Theorem B on the new class $\{G(x)\}$. We next establish three classes of additive arithmetic functions of two or more variables which are inversely associated with partition functions. These arise as the sum of two or more additive arithmetic functions belonging to

the same class $\{F(x)\}$, $\{G(x)\}$, or $\{H(x)\}$, and in each case the associated partition function picks up at least one summand from each of a given number of disjoint subsets of positive integers. Finally we set up three classes $\{(F,A)\}$, $\{(G,B)\}$, $\{(H,C)\}$ of couples in which F, G, and H are additive and A, B and C are multiplicative arithmetic functions, and prove that these arithmetic functions are inversely associated with certain product-sum partition functions which now arise as the number of integers x which simultaneously satisfy $F(x) = n$, $A(x) = m$; and similarly for the other two couples. (Received October 24, 1974.)

*720-10-6 RIHO TERRAS, 399 Stratford Ct., Del Mar, CA. 92014.
A stopping time problem on the positive integers.

Define a function T on the positive integers by $Tn = (3n + 1)/2$ if n is odd and by $Tn = n/2$ if n is even. If there exists a positive integer k such that $T^k n < n$ then choose the smallest such k and set $X(n) = k$, otherwise set $X(n) = \infty$. Let μ denote the counting function. Theorem. $F(k) = \lim_{m\to\infty} (1/m)\mu\{n \leq m \mid X(n) \geq k\}$ exists and $\lim_{m\to\infty} F(k) = 0$. Let $p(0,1) = 1/2$ and let $p(a,1)$ if a is an integer different from 0. Let k be a positive integer and let $c(a,k) = 1/2$ if a is an integer satisfying $k \leq (k - a) > k(\ln 2/\ln 3)$ and let $c(a,k) = 0$ otherwise. Compute $p(a,k)$ from the recursion $c(a-1,k)p(a-1,k) + c(a,k)p(a,k) = p(a,k+1)$. Theorem. $F(k) = \sum_{a=0}^{k} p(a,k)$. The determination of the values of X is discussed. The distribution function $F(k)$ is tabulated up to $k = 1000$. (Received October 25, 1974.)

*720-10-7 A. TERRAS, University of California, San Diego, Department of Mathematics, La Jolla, California 92037. A Formula Relating $\zeta_K(s)$ and $\zeta_K(2-s)$ for K totally real with class number one.

If K is a totally real algebraic number field with class number one, we obtain a formula relating the Dedekind zeta function $\zeta_K(s)$ with $\zeta_K(2 - s)$ (or $\zeta_K(s - 1)$), via a series of Bessel functions $K_\nu(z)$. Here s is any complex number (with suitable interpretation at poles). For integer s, the result is similar to results of Grosswald [Proc. Symp. in Pure Math, 24, A.M.S.] which generalize formulas of Ramanujan for K = the rational numbers. For example, $\zeta(3)$ and $\zeta(4)$ are related in this fashion. The proof is via the Fourier expansion of the Epstein zeta function for the field K [T. Tamagawa, Proc. Internatl. Symp. on Alg. No. Theory, Tokyo-Nikko (1955); or T. Asai, Nagoya Math J. 40 (1970)]. If K = the rationals, the Fourier expansion is often called the Selberg-Chowla formula [Crelle, 227 (1967), 86-110]. It is analogous to the Fourier expansion of modular forms. (Received October 18, 1974.)

*720-10-8 LARRY J. GOLDSTEIN and PILOE DE LA TORRE, University of Maryland, College Park, MD. 20742.
Dedekind Sums for a Totally Real Algebraic Number Field, Preliminary Report.

A definition of a Dedekind sum for a totally real algebraic number field K is proposed. The connection between these generalized Dedekind sums and the law of transformation of certain generalized ζ-functions is described. These Dedekind sums can be used to state class number formulas for a certain family of algebraic number fields of complex-multiplication type. (Received October 25, 1974.)

*720-10-9 GREGORY WULCZYN, Bucknell University, Lewisburg, Pennsylvania 17837. Isomorphic Pellian quadratic forms and their associated continued fraction expansions. Preliminary report.

From the primitive solution $x = x_0, y_0 = y$ of $x^2 - Dy^2 = \pm 1$, D a nonsquare integer, are built up $\sqrt{D_1} = \langle(\frac{1}{2})yt + a_1, a_2, \ldots, a_{n-1}, 2(\frac{1}{2})yt + 2a_1\rangle$, $\sqrt{D_2} = \langle(\frac{1}{2})y(t-1) + r - 1, 1, a_2 - 1, a_3, \ldots, 1, 2(\frac{1}{2})y(t-1) + 2r - 2\rangle$, and the associated quadratic forms $D_1 = (\frac{1}{4})y^2t^2 + (1)2x_0t + D$, $D_2 = (\frac{1}{4})y^2t^2 - (1)2x_0t + D$. Two parameter quadratic forms and c.f. expansions are found for periods 3–5 and 4–6. (Received October 29, 1974.)

*720-10-10 J. M. GANDHI, Western Illinois University, Macomb, Illinois 61455
On sums analogous to Dedekind's sums

Let h,k be integers, (h,k)=1, $k \geq 1$ and set $s'(h,k) = \sum_{\mu=1}^{k} (-1)^{\mu} ((\frac{h\mu}{k}))((\frac{\mu}{k}))$ where the symbol ((x)) is defined by

$$((x)) = \begin{cases} x-[x]-\frac{1}{2} & \text{if } x \text{ is not an integer} \\ 0 & \text{if } x \text{ is an integer} \end{cases}$$

and [x] the greatest integer not exceeding x. In this paper we prove

Theorem 1.

$s'(h,k) = 0$ if k is odd and $s'(h,k) = -\frac{1}{4k} \sum_{m=1}^{k-1} \tan\frac{\pi m}{k} \cot\frac{\pi hm}{k}$ when k is even.

Theorem 2.

The denominator of s'(h,k) is a divisor of 2k. (Received October 30, 1974.)

*720-10-11 GORDON L. SIPP, California State University, Los Angeles, 90032.
The Spinor Genus of Quaternion Orders

Let D be a global domain whose quotient field F does not have characteristic 2, let A be a quaternion algebra over F, and let O be an order on A over D. A right O-module M which is simultaneously a lattice on A over D is said to be right O-generic if there exists $a \in A$, $N(a) \neq 0$, such that $a^{-1}M \in \text{gen } O$. Our main result is that every right O-generic module is cyclic if and only if every class in the spinor genus of O represents a unit in D. One consequence is that O is in a spinor genus of one class if and only if O-generic modules are cyclic and O represents every unit represented by its spinor genus. In addition, it is shown that a necessary and sufficient condition that an integral ternary lattice L be in a spinor genus of one class is that every right O_L-generic pair be equivalent to a two-sided O_L-generic pair, where O_L is the quaternion order associated with L. (Received October 29, 1974.)

720-10-12 LAWRENCE KUIPERS, Southern Illinois University, Carbondale, IL. 62901.
Asymptotic distribution mod m and the notion of independence.

Let $m \geq 2$ be a fixed modulus. Let (a_n), $n = 1,2,\ldots$, be a given sequence of integers. For integers $N \geq 1$ and j, let $A(N;j,a_n)$ be the number of n, $1 \leq n \leq N$, with $a_n \equiv j \pmod m$. If $\alpha(j) = \lim_{N\to\infty} A(N;j,a_n)/N$ exists for each j, then

(a_n) is said to have α as its asymptotic distribution function mod m (abbreviated a.d.f. mod m). We denote $\alpha(j)$ also by $\|A(a_n \equiv j)\|$.

If (b_n) is another sequence of integers, then for $N \geq 1$ and $j,k \in Z$ we define $A(N;j,a_n;k,b_n)$ as the number of n, $1 \leq n \leq N$, such that simultaneously $a_n \equiv j \pmod m$ and $b_n \equiv k \pmod m$. We write $\|A(a_n \equiv j,\ b_n \equiv k)\| = \lim_{N\to\infty} A(N;j,a_n;k,b_n)/N$ in case the limit exists.

Definition. The sequences (a_n) and (b_n) are called independent mod m if for all $j,k = 0,1,\ldots,m-1$ the limits $\|A(a_n \equiv j,\ b_n \equiv k)\|$ exist and we have

$$\|A(a_n \equiv j,\ b_n \equiv k)\| = \|A(a_n \equiv j)\| \cdot \|A(b_n \equiv k)\|.$$

A criterion for independence mod m in terms of exponential sums has already been established. We prove a criterion of a different type.

(Received October 30, 1974.)

720-10-13 CHARLES F. OSGOOD, Mathematics Research Center, Naval Research Laboratory, Washington, D.C. 20375. Finding good rational approximations to power series.

Let K be any field of characteristic zero, y any formal power series in z^{-1} which is not the expansion of a rational function but which is a solution of an algebraic differential equation with coefficients in $K[z]$, and $p(z)$ and $q(z)$ any two elements of $K[z]$ with $q(z) \neq 0$. Kolchin [Proc. Amer. Math. Soc. 19(1959), 238-244] showed how one could obtain an upper bound of the form $d(\deg q(z)) + c$ on the order of vanishing at $z = \infty$ of any difference $y - p(z)(q(z))^{-1}$ where d and c are independent of both $p(z)$ and $q(z)$. Kolchin's value of d is given explicitly in terms of the differential equation satisfied by y. It will be shown, in several cases, how to give a smaller value of d than that given by Kolchin. (Received October 30, 1974.)

720-10-14 PETER HAGIS, JR., Temple University, Philadelphia, Pa. 19122. Every Odd Perfect Number has at Least Eight Prime Factors, Preliminary report.

Since the time of Sylvester there has been interest in the problem of determining a lower bound for the number of prime divisors of an odd perfect number. The most recent work in this area is due to Pomerance [Acta Arithmetica, XXV (1974)] and Robbins [doctoral dissertation, Polytechnic Institute of Brooklyn (1972)] who independently and simultaneously showed that every odd perfect number is divisible by at least seven distinct primes. Using an extensive computer based case study the present author has shown that every odd perfect number has at least eight prime factors. (Received October 31, 1974.)

*720-10-15 EZRA BROWN, Virginia Polytechnic Institute & State University, Blacksburg, VA 24061. Diophantine Equations of the Form $x^2 + D = y^p$. II

The author makes a study of diophantine equations of the form $x^2 + dA^2 = y^p$, where d and A are integers and p is a prime. In particular, he studies the cases $d = 2$, $A = 2,3,4,5$. Sample results are as follows: Theorem. The diophantine equation $x^2 + 8 = y^p$ has only the following solutions for $p \not\equiv 7 \pmod 8$: $(p,x,y) = (2,\pm 1,\pm 3)$ and $(3,0,2)$; the diophantine equation $x^2 + 32 = y^p$ has only the following solutions for $p \not\equiv 7 \pmod 8$: $(p,x,y) = (2,\pm 2,\pm 6)$, $(2,\pm 7,\pm 9)$, $(5,0,2)$ and $(5,\pm 88,6)$.
(Received October 31, 1974.)

720-10-16 BURTON W. JONES, University of Colorado, Boulder, Colorado, 80302

Quasi-genera of quadratic forms.

Let f be a quadratic form in n variables with integer coefficients and determinant d not zero. A prime p is called exceptional if every automorph of f whose matrix has rational elements and denominator prime to 2d, has a denominator which is a quadratic residue of p. Every exceptional prime is a factor of 2d. For $n > 2$, and in most cases for $n = 2$, each exceptional prime induces a splitting of the genus of f into two quasi-genera. Here it is proved that there are exceptional primes only if $n \leq 4$ and for $n = 4$ only if $p^k \| d$ implies that d/p^k is a square. It is also shown, with some exceptions possibly for $p = 2$, that if p is symmetry-exceptional, it is exceptional for all products of two symmetries. (Received October 30, 1974.)

*720-10-17 ELMER K. HAYASHI, Wake Forest University, Winston-Salem, North Carolina 27109.

Factoring integers whose digits are all ones in base b.

Let b be a positive integer greater than one. For each positive integer k, define $I_k = b^{k-1} + b^{k-2} + \cdots + b + 1$, the integer whose base b representation consists of k ones. If n and b are relatively prime, we define c(n) to be the smallest positive integer k such that n divides I_k. It is not hard to show that c(n) exists and furthermore that $c(n) \leq n$. We prove that $c(n) = n$ if and only if $n = p_1^{a_1} \cdots p_r^{a_r}$ where p_i is a prime divisor of b-1 and $a_i > 1$ only if $p_i \| I_{c(p_i)}$ for $i = 1, \cdots, r$ ($p \| m$ means p divides m but p^2 does not divide m). In particular, if b = 10. the theorem asserts that $c(n) = n$ if and only if n is a power of three. We also show that $c(n) = n-1$ if and only if n is a prime which does not divide b-1 and b is a primitive root of n. (Received November 1, 1974.)

*720-10-18 I. Borosh and L. B. Treybig, Texas A&M University, College Station, TX. 77843

Bounds on positive solutions of linear Diophantine Equations

Assuming that a system of linear equations with integer coefficients has a nontrivial solution in non negative integers, bounds B depending only on the system coefficients are found, such that the existence of such a solution bounded by B is guaranteed. For a single equation, B is the maximum of the coefficients and it is clear that this bound is sharp. (Received November 4, 1974.)

*720-10-19 JOHN E. MAXFIELD, Kansas State University and University of New Brunswick, Fredericton, N.B., CN. On left-handed, right-handed and two-sided primes.

A positive integer $N = a_n \cdots a_2 a_1$ written to base r is a right- (left-) handed prime to base r iff

(1) $a_1, a_2a_1, a_3a_2a_1, \ldots, N$ ((1)' $a_n, a_na_{n-1}, a_na_{n-1}a_{n-2}, \ldots, a_1$)

are each primes. A positive integer is a two-sided prime iff it is both a left- and a right-handed prime. A two-sided prime N is a uniformly two-sided prime to base r iff each of the primes in (1) and (1)' is a two-sided prime.

A table of all left-handed and two-sided primes to the bases 2,3,...,11 is given, and the following theorems are proved.

(A) There are no two-sided primes to an odd base having more than 3 digits.

(B) If the base r is even and p is the smallest prime such that $(p,r) = 1$, then the largest uniformly two-sided prime to base r has fewer than 2p digits.

(Received November 4, 1974.)

*720-10-20 DR. D. SURYANARAYANA, University of Toledo, Toledo, Ohio 43606.
On a Conjecture of S. Chowla.

Let $\psi(x) = x - [x] - \frac{1}{2}$. It has been conjectured by S. Chowla that $\sum_{n \le \sqrt{x}} \{\psi^2(\frac{x}{n}) - \frac{1}{12}\} = O(x^{\frac{1}{4}+\varepsilon})$, for every $\varepsilon > 0$. In this paper we show that this conjecture is equivalent to $\sum_{n \le \sqrt{x}} n^2\{\psi^2(\frac{x}{n}) - \frac{1}{12}\} = O(x^{\frac{5}{4}+\varepsilon})$ by proving that

$$\sum_{n \le \sqrt{x}} \{\psi^2(\frac{x}{n}) - \frac{1}{12}\} + \frac{1}{x} \sum_{n \le \sqrt{x}} n^2\{\psi^2(\frac{x}{n}) - \frac{1}{12}\} = O(x^{\frac{1}{4}}).$$

(Received November 4, 1974.)

(Author introduced by Professor J.Chidambaraswamy.)

720-10-21 K. NAGESWARA RAO, North Dakota State University, Fargo, ND 58102
A generalization of a cyclotomic polynomial, Preliminary report.

A(n) is a set of positive divisors of an integer $n \ge 1$ satisfying the conditions of Narkiewiez [Math. Colloq. 10 (1963), 81-94]. If m is any integer, then the A-greatest common divisor (A-g.c.d.) of m and n is defined as the largest member of A(n) that divides m and is denoted by $(m,n)_A = 1$. Let ρ be any primitive n^{th} root of unity and $Q_A^{(n)}(x) = \pi(x - \rho^m)$, where the product runs over all integers m such that $0 < m \le n$ and $(m,n)_A = 1$. The above polynomial generalizes the cyclotomic polynomial. In this report, various properties of this polynomial are obtained together with its relation to a polynomial whose roots are the products of the roots of two or more generalized cyclotomic polynomials. Finite Fourier series and character sums play an important role in this study. (Received November 4, 1974.)

*720-10-22 PETER HAGIS, JR. and GRAHAM LORD, Temple University, Philadelphia, Pa. 19122.
Unitary Harmonic Numbers.

If $d^*(n)$ and $\sigma^*(n)$ denote the number and sum, respectively, of the unitary divisors of the natural number n then the harmonic mean of the unitary divisors of n is given by $H^*(n) = nd^*(n)/\sigma^*(n)$. Here the properties of $H^*(n)$ are investigated with particular attention being paid to the set, UH, of those natural numbers for which $H^*(n)$ is an integer.

Among the results obtained are: (1) there exist at most a finite number of elements in UH with a specified number of distinct prime factors; (2) UH has density zero.
(Received November 4, 1974.)

*720-10-23 NEVILLE ROBBINS, 1665 Grant Avenue, San Francisco, California 94133. Some remarks concerning quasi-perfect numbers.

A natural number n is called quasi-perfect (or almost perfect) if $\sigma(n) = 2n + 1$, where $\sigma(n)$ denotes the sum of the divisors of n. P. Cattaneo showed that if n is quasi-perfect, then n is not divisible by 2 or 3. He also obtained a lower bound for n. Here we show that if $n = \Pi_{i=1}^{k} q_i^{e_i}$ is quasi-perfect, where the q_i are distinct primes, then (1) each $q_i \equiv 1 \pmod{16}$; (2) each $e_i \equiv 0$ or $2 \pmod 8$; (3) at least one $e_i \equiv 2 \pmod 8$. From (1), one can obtain a much-improved lower bound for n. (Received November 4, 1974.)

720-10-24 PHYLLIS LEFTON, Department of Mathematics, Columbia University, New York, N.Y. Trinomials with Galois Group Contained in A_n. Preliminary report.

Let $J_k(N)$ be the number of trinomials $f(x) = x^n + ax^k + b$ with integer coefficients bounded in absolute value by N whose Galois group, considered as a permutation group on the roots of $f(x)$, is a subgroup of A_n.

We derive, with certain conditions on n and k, a non-trivial upper bound for $J_k(N)$. We also give formulas for the discriminant of the general trinomial $f(x) = x^n + ax^k + b$.

Our estimate for $J_k(N)$ relates to a conjecture of van der Waerden on the number of polynomials with integer coefficients bounded in absolute value by N whose Galois group is a proper subgroup of S_n. (Received November 4, 1974.)

720-10-25 WITHDRAWN

*720-10-26 ALAN H. STEIN, University of Connecticut, Waterbury, Connecticut 06710. Interprimed vectors in two dimensions.

A vector (m,n) with positive integer components is called interprimed iff m and n have the same set of distinct prime factors. Let $N(x)$ be the number of interprimed vectors with both components less than or equal to x. Motzkin and Erdos conjectured that $N(x) \sim \alpha x$, which was proven by H. N. Shapiro. Here it is shown that $N(x) = \alpha x + \beta x^{2/3} + O(x^{3/5} \log\log x)$. The proof involves Dirichlet series in two complex variables. (Received November 4, 1974.)

720-10-27 A.A. GIOIA, Western Michigan University, Kalamazoo, Michigan 49001 and
M.V. SUBBARAO, University of Alberta, Edmonton, Alberta, Canada T6G 2G1
On the Scholz-Brauer problem in addition chains.

The Scholz conjecture on addition chains is $\ell(2^n-1) \leq n - 1 + \ell(n)$. For definitions see Brauer (Bull. AMS, 45 (1939), 736-739) and Hansen (J. Reine Angew. Math., 202 (1959), 129-136). Write each term as $a_i = b_i + c_i$, $b_i \geq c_i$ $(i = 1,\ldots,r)$. If $b_i = a_{i-1}$ $\forall i$, define $m = 0$ and T_o to be the multiset $\{c_1,\ldots,c_r\}$. For each (if any) $j \ni b_j \neq a_{j-1}$ define

$R_j = \{c_k : b_k = b_j\}$. Since $i \neq j \Rightarrow R_i = R_j$ or $R_i \cap R_j = \emptyset$, let m be the number of distinct R_j , and relabel so $T_1,\ldots,T_m$ are the distinct sets; let T_o be the complement of $T_1 \cup \ldots \cup T_m$ relative to $\{c_1,\ldots,c_r\}$. Define $\theta = \Sigma_1 c_i + \Sigma_2 d_k$, where Σ_1 denotes the sum over $c_i \in T_o$, Σ_2 is summed over k, $1 \leq k \leq m$, and $d_k = \max\{c \in T_k\}$. Principal results: $\ell(2^n-1) \leq \theta + r$; and, $\theta \geq n - 1$, $= n - 1$ iff $b_i = a_{i-1}$ or b_{i-1} $(i = 1,\ldots,r)$. As special cases these results include Brauer's, and most of Hansen's results on the Scholz conjecture. An extension of Hansen's work is obtained by proving that for n which do not satisfy above necessary and sufficient conditions, a refinement of the construction yields a shorter chain, which in some cases has length $n - 1 + \ell(n)$. (Received November 4, 1974.)

720-10-28 RAPHAEL P. FINKELSTEIN, Bowling Green State University, Bowling Green, Ohio 43403.
On Fibonacci numbers of the form $x^2 + 1$.

Finkelstein ["On Fibonacci numbers which are one more than a square", J. Reine Angew. Math. 262/263(1973), 171-178] showed that the only solutions of (1) $F_n = x^2 + 1$, where F_n is the nth Fibonacci number and n is positive, are $F_1 = 1$, $F_2 = 1$, $F_3 = 2$ and $F_4 = 5$. However, the proof given there was quite deep and depended upon properties of units in quartic fields. Recently, Williams ["On Fibonacci numbers of the form $k^2 + 1$", to appear] has found a simpler solution of (1) which employs some very pretty identities of Lucas. I present a completely elementary solution of (1) which involves neither algebraic number theory nor the identities used by Williams. This is accomplished by showing that (1) leads to an equation of the form $F_m = x^2$ if m is odd and $F_m = 3x^2$ if n is even and then solving these equations. (Received November 5, 1974.)

*720-10-29 NINA SPEARS, University of Nebraska, Lincoln, Nebraska, 68508, A problem involving simultaneous binary compositions, an improvement in error term.

Let $m \leq n$ be positive integral variables, and let $2 \leq h, k \leq \ell$ be fixed integral constants. An asymptotic formula is obtained for $S(m,n)$, the number of positive integral solutions to the simultaneous equations

$$m = x_1 + y_1,\ n = x_2 + y_2,$$

subject to the conditions

$$(x_1,x_2) \in Q_\ell,\ y_1 \in Q_k,\ y_2 \in Q_h,$$

where Q_k is the set of k-free integers. The work is elementary in nature and yields a significant improvement in error term over previous results. (Received November 5, 1974.) (Author introduced by Professor Walter E.Mientka.)

720-10-30 CHARLES R. WALL, University of South Carolina, Columbia, South Carolina 29208.
Elimination sequences, Preliminary report.

An *e-elimination sequence* $\{x_n\}$ of positive integers starts with two arbitrary integers $x_1<x_2$; an integer larger than x_2 is a term of the sequence if it cannot be expressed e or more ways as a sum of two distinct sequence elements ($e\geq 1$). *Conjecture*: Any elimination sequence has positive density. *Theorem*: Any double elimination ($e=2$) sequence whose first term is larger than 1 has positive density. (Received November 5, 1974.)

720-10-31 JUDITH S. SUNLEY, American University, Washington, D. C., 20016. Toward a Kronecker Limit Formula for the Siegel Modular Group of Dimension Two. Preliminary Report.

The nonanalytic Eisenstein series for the Siegel modular group of dimension two is considered in an effort to derive a formula comparable to the Kronecker first limit formula. The series $E(Z,s) = |Y|^s \sum_{\{C,D\}} ||CZ+D||^{-2s}$ has been shown to converge for $\mathrm{Re}(s) > 3/2$. It has a pole of order one at $s = 3/2$ and an analytic continuation to the rest of the complex plane. The function has a Fourier expansion and the coefficients of this expansion have been investigated for $s = 3/2$. Evaluating these coefficients (for the pieces which are entire) means evaluating a certain integral over a complicated matrix-space domain. This can now be done in terms of products of the gamma function and functions related to the gamma function. (Received November 5, 1974.)

720-10-32 SAMUEL D. LAWN, 408 Court Street, Penn Yan, New York 14527. The asymptotic behavior of sums of multiplicative functions. Preliminary report.

Some results of Wirsing [Das asymptotische Verhalten von Summen über multiplicative Functionen II, Acta Mathematica Academiae Scientiarum Hungaricae 18(1967), 411-467] are specialized and sharpened. A typical result is the following. Theorem. Let f be a non-negative, real-valued, multiplicative function bounded by Λ on the set of primes and satisfying $\sum_{p\leq x} f(p)\frac{\log p}{p} = \tau \log x + O((\log x)^{1-\delta})$ as $x\to\infty$, where τ and δ are positive constants. Suppose that $a<1/2$, $A<1$ and c are absolute constants such that for every integer $k\geq 2$, $f(p^k)\leq cp^{ak}$ for all but a finite number of primes p, and $f(p^k)\leq cp^{Ak}$ for all remaining primes. Then there exists a positive constant B such that $\sum_{n\leq x} f(n) = Bx(\log x)^{\tau-1} + O(x(\log x)^{\tau-1-\rho})$ for all $\rho<\alpha$, where α is a suitable positive absolute constant dependent upon τ, δ and Λ. A similar result specializes and sharpens a portion of Satz 1.2 of the above paper of Wirsing, from which a form of the prime-number theorem with error term is obtained. Other applications are made to estimate the summatory functions of various classical number-theoretic functions. All proofs are elementary. (Received November 5, 1974.)

*720-10-33 STEFAN A. BURR, Bell Laboratories, Madison, N. J. 07940. On Detecting a Periodic Event by Means of Period Observations. I.

The following situation is considered. A certain event is observable only intermittently according to some period. An observer attempts to detect an occurrence of the event by making observations according to some other period. We wish to determine the probability P(s) that the event is first detected by the sth observation. The period of the event may be known or unknown. This paper discusses the number-theoretic calculations necessary to solve the problem; computational aspects will be discussed in a future paper. (Received November 6, 1974.)

720-10-34 PAUL ERDÖS, Hungarian National Academy, and IAN RICHARDS, Universit of Minnesota, Minneapolis, MN 55455. Density functions for prime an relatively prime numbers.

Let $r^*(x)$ denote the maximum number of relatively prime integers which can exist in an interval $(y,y+x]$ of length x, and let $\rho^*(x)$ denote the maximum number of prime integers in any interval $(y,y+x]$ where $y \geq x$. (Our definition of $\rho^*(x)$ assumes the "prime k-tuples conjecture". There are definitions of $\rho^*(x)$ which avoid this conjecture, but they are more technical.) We investigate the difference $r^*(x) - \rho^*(x)$: i.e. we ask, how many more relatively prime integers can exist on an interval of length x than the maximum possible number of prime integers ? As a lower bound we obtain $r^*(x) - \rho^*(x) > x^c$ for some $c > 0$ (when $x \to \infty$). This improves the previous lower bound of $\log x$. As an upper bound we get $r^*(x) - \rho^*(x) = o[x/(\log x)^2]$. It is known that $\rho^*(x) - \pi(x) > \text{const}[x/(\log x)^2]$; thus the difference between $r^*(x)$ and $\rho^*(x)$ is negligible compared to $\rho^*(x) - \pi(x)$. The function $r^*(x)$ was studied by Erdös and Selfridge (who used a different notation; cf. Proc. Manitoba Conference, Univ. of Manitoba, 1971). To mention briefly some further results: The functions $r^*(x)$ and $\rho^*(x)$ are related to the "maximizing" or "saving" sieve. Similar comparisons are made between two variants of the "minimizing" or "killing" sieve. (Received November 6, 1974.)

720-10-35 CARTER WAID, University of Delaware, Newark, Delaware 19711. A Composition Theory for Decomposable Forms. Preliminary Report.

A theory of composition of the classes of primitive decomposable forms in a given rational class is developed which resolves the orientation problem inherent in Dedekind's theory. The product is multivalued in general but does satisfy the associative law in a certain sense. (Received November 6, 1974.)

720-10-36 R. WELLS JOHNSON, Bowdoin College, Brunswick, Maine 04011. On the distribution of quadratic residues.

For a prime p and any subinterval I of $(0,p)$, let $R(I)$ and $NR(I)$ denote the number of quadratic residues and nonresidues which occur in I. If $p \equiv 3 \pmod 4$, it is well known that $R(I) > NR(I)$ for $I = (0,p/2)$, the difference being a positive multiple of the class number h of the imaginary quadratic field $\mathbb{Q}(\sqrt{-p})$. By relating h to the Bernoulli number $B_{(p+1)/2}$ and using Voronoi's congruences for the Bernoulli numbers, the differences $R(I) - NR(I)$ are evaluated on the intervals $I = (0,p/6)$, $(p/6,p/4)$, $(p/4,p/3)$, and $(p/3,p/2)$. The results are in terms of h and depend upon the residue of $p \pmod{24}$. (Received November 6, 1974.)

720-10-37 J. CHIDAMBARASWAMY, University of Toledo, Toledo, Ohio, 43606, Generalized Dedekind's ψ-functions with Respect to a Polynomial

For any polynomial $f = f(x)$ of positive degree with integer coefficients and positive integers t and k, let

$$\Psi_{f,t}^{(k)}(n) = \sum_{d^k|n} \frac{n^t}{d^{kt}} \frac{\Phi_{f,t}^{(k)}(d^k, n/d^k)\, \nu_{f,t}^{(k)}(d^k)}{(d^k, n/d^k)^t};$$

here $\Phi_{f,t}^{(k)}(n)$ is the totient function defined and studied by the author (Notices: Jan. 1972, p. A53 where $\varphi_f^{(k)}(t;n)$ is written for $\Phi_{f,t}^{(k)}(n)$) and

$\nu_{f,t}(n) = \prod_{p|n} (N_f(p^k)^{\alpha t}$ if n is a kth power and $p^{\alpha k}$ is the highest power of p dividing

n and 0 otherwise and let $\psi_{f,t}^{(k)}(n) = \Psi_{f,t}^{(k)}(n^k)$. It turns out $\Psi_{x,1}^{(1)}(n) = \psi_{x,1}^{(1)}(n)$ is the Dedekind's ψ-function and $\Psi_{x,1}^{(k)}(n) = \Psi_k(n)$ and $\psi_{x,1}^{(k)}(n) = \psi_k(n)$, the functions $\Psi_k(n)$ and $\psi_k(n)$ being the extensions of Dedekind's ψ-function studied recently by D. Suryanarayana (Math. Scand. 26(1970), 107-118. Among other things all the results concerning $\Psi_k(n)$ and $\psi_k(n)$ in the paper mentioned above are extended to $\Psi_{f,t}^{(k)}(n)$ and $\psi_{f,t}^{(k)}(n)$.

(Received November 6, 1974.)

720-10-38 GERALD A. BOTTORFF, Pennsylvania State University, Mont Alto, Pennsylvania 17237. On the Littlewood conjecture.

The "Littlewood conjecture" is as follows: Given arbitrary algebraically independent real numbers α,β; then for arbitrary $\epsilon > 0$ there exists p,q,r so that $q|q\alpha - p||q\beta - r| < \epsilon$. The author considers the function $f(n)$ defined as follows: $f(n) = \min(q)$ so that for every pair α,β (alg. indep.) $q|q\alpha - p||q\beta - r| < 1/n$. If L.C. is true then for all n, $f(n)$ is finite, but if L.C. is false then some N exists so that $f(N) = \infty$. The author first tried to calculate $f(1), f(2), f(4), f(8), f(16), \ldots$; but $f(16)$ must be so large (> 2000) as to be entirely out of hand computation range—there appears to be no easy way to program a computer for these calculations. If the L.C. is true, it is certainly interesting why $f(n)$ is increasing at so great a rate. $f(1) = f(2) = f(3) = f(4) = 1$, $f(5) = f(6) = f(7) = 2$, $f(8) = 3$, $f(9) = 7$, $f(10) = 15$, $f(11) = 49$, etc.

(Received November 6, 1974.)

720-10-39 H. M. STARK, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Analytic estimates of discriminants.

Let D be the absolute value of the discriminant of an algebraic number field of degree $n > 1$. An old result of Minkowski is that there is a constant $c_n > 1$ such that $D^{1/n} > c_n$. Recently an analytic method was developed which gives, at least for large n, larger values of c_n than Minkowski. This method has been refined by Odlyzko to the point that, for large n, c_n is larger than the best previously obtained results in the geometry of numbers. The method has interesting applications to some problems in number theory which will be discussed. (Received November 6, 1974.)

720-10-40 HUGH L. MONTGOMERY, University of Michigan, Ann Arbor, Michigan 48104. Assorted research problems in analytic number theory. Preliminary report.

A selection of problems of current interest in multiplicative number theory will be mentioned, and possible approaches to these problems will be discussed. The problems will be primarily concerned with Dirichlet series, L-functions, character sums, lattice points, and the distribution of prime numbers.

(Received November 6, 1974.)

720-10-41 LOWELL SCHOENFELD, SUNY at Buffalo, Amherst, N.Y. 14226. Improved bounds for the Chebyshev functions with applications.

In a forthcoming paper by Rosser and Schoenfeld, certain bounds are given for $\theta(x) - x$ where $\theta(x) = \Sigma_{p \leq x} \log p$. These results are now applied to get better bounds for various other functions of primes such as $\pi(x)$. In addition, the following better bound for $\theta(x)-x$ is given:

$|\theta(x) - x| < U(x) \{8X/(17\pi)\}^{\frac{1}{2}} e^{-X}$ provided $\log x \geq 1711$ and $X = \{(\log x)/9.645908801\}^{\frac{1}{2}}$. Here $U(x)$ is an explicitly given function

which tends to 1 as $x \to \infty$. Further, under the assumption of the Riemann hypothesis, $|\theta(x) - x| < (8\pi)^{-1}\sqrt{x}\log^2 x$ for all $x \geq 599$.
(Received November 6, 1974.)

720-10-42 PAUL T. BATEMAN, University of Illinois, Urbana, Illinois 61801. Zeros of Fekete polynomials. Preliminary report.

If χ is a real residue character modulo k, the corresponding Fekete polynomial is defined as $f(u,\chi) = \sum_{n=1}^{k} \chi(n)u^n$. The behavior of $f(u,\chi)$ for u in (0,1) determines the behavior of the correspondi L-function. Here we examine the effects of replacing χ by the product of χ with a suitable principal character. It is easy to prove that for any given χ there is a character X which is the product of χ by a suitable principal character and is such that $f(u,X)$ has n changes of sign in (0,1). Here we give some numerical evidence for the dual conjecture that for any given χ there is a character X which is the product of χ by a suitable principal character and is such that $f(u,X)$ has no changes of sign in (0,1).
(Received November 6, 1974.)

*720-10-43 DANIEL SHANKS, Naval Ship Research & Development Center, Bethesda, MD 20084. Analysis and Improvement of the Continued Fraction Method of Factorization.

The continued fraction method of factoring N, by Morrison and Brillhart, Math. Comp., v. 29, 1975, is based upon $N = q_0 + \frac{1}{q_1 +} \frac{1}{q_2 +} \cdots + \frac{1}{q_{n-1} +} \frac{1}{(\sqrt{N} + P_n)/Q_n}$. One has $(-1)^n Q_n = A_n^2 - B_n^2 N$ for each n. A subset of the $(-1)^n Q_n$ having a square product Q^2 leads to $N|A^2-Q^2$. If (*) $N \nmid A-Q$ and $N \nmid A+Q$, the GCD(A-Q,N) is a proper factor of N. In this paper, we analyze the conditions that lead to a failure of (*). Secondly, while $P_n, Q_n = O(\sqrt{N})$, $A_n \pmod N = O(N)$. But we show that the A_n are not needed at all: the P_n, Q_n generate the period of reduced forms; that suffices to determine the ambiguous forms and thereby the factors of N. (Received November 6, 1974.)

*720-10-44 R. L. GRAHAM, Bell Laboratories, Murray Hill, New Jersey 07974 and D. H. LEHMER, University of California, Berkeley, California 94720. On the permanent of Schur's matrix

Let M_n denote the n by n matrix (m_{jk}) where $m_{jk} = \varepsilon^{jk}$ and $\varepsilon = \exp(2\pi i/n)$. We denote the permanent of M_n by P_n. The quantity P_n occurs on a variety of contexts in number theory and combinatorics, e.g., as the coefficient $A(1,1,\ldots,1)$ in the expansion of the determinant of the circulant matrix C given by $\det(C) = \sum A_{i_0 \ldots i_{n-1}} X_0^{i_0} \ldots X_{n-1}^{i_{n-1}}$ where the $(i,j)^{th}$ entry of C is x_{i-j} with the subscript reduced modulo n.

It is known that P_n is a rational integer, $P_{2n} = 0$ and $P_n \equiv 0 \pmod n$. In this note we give several new results for P_n. In particular, we show:

(i) $P_p \equiv p! \pmod{p^3}$ for p prime;

(ii) $P_n \equiv n \pmod 2$;

(iii) $N \equiv 0 \pmod{p^\alpha}$ implies $P_n \equiv 0 \pmod{p^{(p^\alpha-1)n/p^\alpha(p-1)}}$ for p prime.

We also compute P_n for $n \leq 21$. (Received November 22, 1974.)

*720-10-45 J. L. SELFRIDGE, Northern Illinois University, DeKalb, Illinois 60115 and M. C. WUNDERLICH, Northern Illinois University, DeKalb, Illinois 60115. Factorization and Prime Testing

Several algorithms have been recently implemented on the IBM 360/65 at Northern Illinois University for factoring and prime testing large numbers. Among these are: a) Pollard's method which searches for prime factors p of N for which $p - 1$ has only small prime factors, b) an improved automatic version of the Morrison-Brillhart method of factorization which uses the continued fraction expansion of $\sqrt{N}$, and c) an algorithm which attempts to prove the primality of N by finding and using factors of $N - 1$ and $N + 1$. (Received November 20, 1974.)

12 Algebraic Number Theory, Field Theory and Polynomials

*720-12-1 CLARK KIMBERLING, University of Evansville, Box 329, Evansville, Indiana 47702. Generalized cyclotomic polynomials and the irreducible factors of Fibonacci polynomials.

Let $L_n(x, z)$ be the nth generalized Lucas polynomial, which is determined recursively by $L_0(x, z) = 2$, $L_1(x, z) = x$, and $L_n(x, z) = xL_{n-1}(x, z) + zL_{n-2}(x, z)$ for $n \geqq 2$. Then the polynomials $\ell_n(x, y, z) = [L_n(x, z) - L_n(y, z)]/(x - y)$ for $n \geqq 0$ include the Fibonacci polynomials and modified Lucas polynomials as special cases. From the $\ell_n(x, y, z)$ polynomials $C_n(x, y, z)$ are obtained having the following properties: (1) C_n is irreducible; (2) ℓ_n equals the product of C_d's for which $d|n$; (3) the polynomial $C_n(x, 1, 0)$ is the nth cyclotomic polynomial. Special choices for x, y, z in $C_n(x, y, z)$ provide the irreducible factors of the modified and unmodified Lucas polynomials, and also, in answer to a question raised by Hoggatt and Long (Fibonacci Quart. 12(1974), 113-120), the irreducible factors of the Fibonacci polynomials. (Received September 3, 1974.)

720-12-2 FRANK GERTH III, The University of Texas, Austin, Texas 78712. 3-class groups of cubic fields.

One of the classical results of algebraic number theory is the specification of the ranks of the 2-class groups of quadratic fields. This paper describes algorithms for computing the ranks of the 3-class groups of cubic fields. These algorithms use the number of ramified primes, the ranks of the 3-class groups of certain quadratic fields, and norm-residue symbol computations. Certain examples of cubic fields provide interesting analogs to the classical results on quadratic fields. (Received September 23, 1974.)

*720-12-3 VICTOR ALBIS-GONZALES, University of Cincinnati, Cincinnati, Ohio 45221. Maximal abelian extensions viewed as Kummer extensions.

Let L be a local field of characteristic 0, containing the q-th roots of 1, q a power of a prime p; if L_a is the maximal abelian extension of L, then it may be considered as a cyclic (kummerian) extension, of degree q, of the fixed field of a cyclic subgroup, with q elements, of Gal (L_a/L). We wish to determine which elements $\beta \in L^x = L - \{0\}$ satisfy the con-

dition $L_q(\sqrt[q]{\beta}) = L_a$. Our main results are contained in the following theorems: Theorem 1. Let ζ_q be a primitive root of 1. If $L(\sqrt[q]{\zeta_q})/L$ is not totally ramified, then $L_q(\sqrt[q]{\beta}) = L_a$ if and only if $L(\sqrt[q]{\beta}) = L(\sqrt[q]{\pi})$ where π is a uniformizing parameter of L. Moreover, there are exactly q^{n+1} such extensions. Theorem 2. If $L(\sqrt[q]{\zeta_q})/L$ is totally ramified then there are $\phi(q)q^{n+1}$ distinct extensions $L(\sqrt[q]{\beta})/L$ such that $L_q(\sqrt[q]{\beta}) = L_a$ and they are completely determined modulo a specific basis of $L^x \bmod L^{xq}$. (Received October 18, 1974.)

*720-12-4 Thomas Callahan, University of Toronto, Toronto, Canada.
Morris Newman, National Bureau of Standards, Washington, D.C. 20234.
Mark Sheingorn, Baruch College - CUNY, New York, New York 10010
Fields with Maximal Kronecker Constant

In 1857 Kronecker showed that if an algebraic integer Θ and its conjugates Θ_i lie within the unit disc then Θ is a root of unity (r.o.u.). Many authors have improved this theorem, most recently Blanksby and Montgomery who showed in 1971 that if $|\Theta_i| \leq 1 + 0(1/30n^2\log 6n)$ then Θ is an r.o.u. The best possible constant is log2/n. This paper: (i) analyzes a class of "trivial" examples that have the best possible constant (ii) produces a class of non-trivial examples having that constant and (iii) proves a related embedding theorem. (Received October 30, 1974.)

720-12-5 R. MINES and F. RICHMAN, New Mexico State University, Las Cruces, New Mexico 88003.
Effective procedures in algebraic number theory.

There are finite procedures for obtaining a basis for the group of units and the elements of the finite class group of an algebraic number field. (Received October 30, 1974.)

720-12-6 RONALD P. INFANTE, Seton Hall University, South Orange, New Jersey 07079.
The Structure of Strongly Normal Difference Field Extensions. Preliminary Report

An inversive difference field M is called a strongly normal extension of F if (i) $C_M = C_F$ is algebraically closed, (ii) F is relatively closed in M, (iii) for every isomorphism σ of M over F, $M < \sigma M > = M < C_\sigma > = \sigma M < C_\sigma >$ where C_σ is the field of constants of $M < \sigma M >$. Theorem 1: If M is strongly normal over F and the Galois group of M over F is linear, then $F \subset E \subset M$, where E is a Picard-Vesiot extension of F and M is a finite algebraic, monadic extension of E. A similar result holds if the Galois group is an abelian variety. (Received November 1, 1974.)

720-12-7 HOYT D. WARNER, Vanderbilt University, Nashville, Tennessee 37235. Group, Round, and Pfister forms over a linearly compact field. Preliminary Report.

These results are applications of the structure theorems for quadratic forms over a field K with char(K) $\neq 2$, linearly compact at a valuation v, reported in Abstract 711-12-5 of these Notices. For an anisotropic quadratic form f over K let $G_f = \{a \in K^\times | af \cong f\}$,

$D_f = \{a \in K^\times | a = f(x_1, \cdots, x_n), x_i \in K\}$, $\Lambda_f = \{v(a) + 2\Gamma_v | a \in D_f\} \subseteq \Gamma_v/2\Gamma_v$ (Γ_v = value group of v); call f a group form (resp. round form) if D_f is a subgroup of $K^\times$ (resp. $G_f = D_f$) --Thus round $\Rightarrow$ group. Assume further that the residue field k_v of v is perfect if $\mathrm{char}(k_v) = 2$, that k_v has a unique quadratic extension k, and that $N_{k \to k_v}(k) = k_v$. If f is anisotropic then (1) f a group form $\Rightarrow \dim f = 2^k$, $k \geq 0$; indeed either $\dim f = \mathrm{card}(\Lambda_f)$ or $f \cong \langle 1, a_1 \rangle \otimes \cdots \otimes \langle 1, a_k \rangle$, a Pfister form. (a) f a round form $\Rightarrow \dim f = 2^k$, $k \geq 0$, and $\mathrm{discr}(f) = 1$ if $k > 1$; $k = 2 \Rightarrow f$ is Pfister. (3) if $(\Gamma_v : 2\Gamma_v) = 4$ then equivalent conditions for f with $\dim f = 4$ are : Pfister, round, group, $1 \in D_f$ and $\mathrm{discr}(f) = 1$. Question: Does group, or even round, always imply Pfister? (Received November 4, 1974.)

*720-12-8 JACOB T. B. BEARD, JR., University of Texas at Arlington, Arlington, TX 76019 and ROBERT M. McCONNEL, University of Tennessee, Knoxville, TN 37916. Rings containing division rings or fields.

Let R be a ring with identity and let $(R)_n$ denote the complete matrix ring over R. $A \in (R)_n$ is called diagonable over R if A is similar over R to a diagonal matrix in $(R)_n$. The study of matrix fields of $(R)_n$, that is, subrings of $(R)_n$ which are fields, led to a number of general results concerning rings containing division rings or fields. In particular, if each idempotent in $(R)_n$ is diagonable over R, then R contains a division ring (field) if and only if $(R)_n$ contains a division ring (field). (Received November 4, 1974.)

*720-12-9 JACOB T. B. BEARD, JR., University of Texas at Arlington, Arlington, Texas 76019, and ROBERT M. McCONNEL, University of Tennessee, Knoxville, Tennessee 37916. Matrix fields over the integers modulo m.

Let Zm denote the ring of integers modulo m, with $(Zm)_n$ the complete matrix ring of all $n \times n$ matrices over Zm under normal matrix addition and multiplication. All subfields of the ring $(Zm)_n$ are enumerated and constructively characterized, with related results given on field extensions in $(Zm)_n$ of subfields of $(Zm)_n$ and a rational canonical form over Zm for subfields of $(Zm)_n$. We also obtain partial results on subfields of $(R)_n$ when R is an arbitrary finite commutative ring with identity. The techniques used are similar to those used earlier by the first author [Duke Math. J. 39(1972), 313-322, 475-484], [Acta Arith. 25 (1974), 315-329, 331-335] and rely on results of R. W. Davis [Duke Math. J. 35(1968), 49-59] and A. Steger [Pacific J. Math. 19(1966), 535-542]. (Received November 4, 1974.)

*720-12-10 R.RUDMAN, Wayne State University, Detroit, Michigan 48202 and R.FINKELSTEIN, Bowling Green State University, Bowling Green, Ohio 43403. A generalization of Berwick's unit algorithm.

Let K be an algebraic number field, O an order of K. An algorithm is given which determines a maximal set of independent units of O. When K is real quadratic or complex cubic, the single unit obtained is fundamental. (Received November 4, 1974.)

*720-12-11 MURRAY A. MARSHALL, University of Saskatchewan, Saskatoon, S7N 0W0 Canada. Local-global principles for formally real fields. Preliminary report.

Let F be a formally real field, W its Witt ring, M the augmentation ideal of W, and T the

torsion ideal of W. Let S be the set of sums of squares of F, X the set of orderings of F, and F_δ the real closure of F at $\delta \in X$. Pfister showed that $W \to \Pi_{\delta \in X} W_\delta$ has kernel T. Conjecture. The kernel of $M^k/M^{k+1} \to \Pi_{\delta \in X} M^k_\delta / M^{k+1}_\delta$ is $[M^k \cap T + M^{k+1}]/M^{k+1}$. Artin showed that if $x \in F^*$ is totally positive, then $x \in S^*$. Since $M/M^2 = F^*/F^{*2}$, this affirms the conjecture in the case $k = 1$. The conjecture is true for $k = 2$ implies: (1) The Clifford map $M^2/M^3 \to \mathrm{Quat}(F)$ is injective if the generalized u-invariant is ≤ 12. (2) The map $\mathrm{Quat}(F) \to \Pi_{\delta \in X} \mathrm{Quat}(F_\delta)$ has kernel $\{\Pi_i (a_1, s_i) | a_i \in F^*, s_i \in S^*\}$. The main result is Theorem. The conjecture is true for all $k \geq 2$ if X if finite. (Received November 4, 1974.) (Author introduced by Dr. Stephen Berman.)

720-12-12 GARY L. MULLEN, The Pennsylvania State University, University Park, Pa. 16802
Local Permutation Polynomials over a Finite Field, Preliminary report.

A polynomial $f(x_1,\ldots,x_r)$ with coefficients in $GF(q)$ is a local permutation polynomial (LPP) in $r \geq 1$ variables over $GF(q)$ if for each $i = 1,\ldots,r$, f is a permutation in x_i when the remaining $r-1$ variables are replaced by arbitrary elements of $GF(q)$. Various general properties of LPP's are discussed. The number of LPP's in 2 variables over $GF(q)$ equals the number of Latin squares of order q. If q is an odd prime, a set of necessary and sufficient conditions is obtained for the coefficients of a polynomial $f(x_1,x_2)$ in order that it be an LPP. Thus the number of Latin squares of order q an odd prime equals the number of sets of coefficients satisfying the given set of conditions. The conditions however are too complicated to state here. (Received November 4, 1974.)

720-12-13 LARRY J. GERSTEIN, University of California, Santa Barbara, California 93106.
Indecomposable hermitian forms over quadratic number fields. Preliminary report.

It is well known that there exist indecomposable, positive definite, even, unimodular quadratic forms over the rational integers in each dimension $n \equiv 0 \pmod 8$. A result of this sort is now given for hermitian forms. More precisely: Let $E = Q(\sqrt{\lambda})$, where $\lambda < 0$ is a square-free rational integer such that $\lambda \not\equiv 1 \pmod 4$, and let R be the ring of algebraic integers in E. Then there exists a definite, even, unimodular, indecomposable hermitian R-lattice for each dimension $n \equiv 0 \pmod 4$. Moreover, an explicit construction is given for an infinite family of these lattices. (Received November 4, 1974.)

720-12-14 THOMAS RIGO, Indiana University Purdue University at Indianapolis, Indianapolis, Indiana, 46205. On Pseudo Valuations, Preliminary report.

Let F be a division ring and G a partially ordered directed homogeneous group. A mapping σ from F onto $G \cup \{0\}$ is a pseudo-valuation if $\sigma(0)=0$, $\sigma(1)=1$, $\sigma(ab)=\sigma(a)\sigma(b)$ and $\sigma(a+b) \leq g^{-1}h$ for all $a,b \in F$ for some fixed $g \leq 1$ and all h such that $\sigma(a) \leq h$ and $\sigma(b) \leq h$. If σ satisfies only $\sigma(ab) \leq \sigma(a)\sigma(b)$ then σ is a weak pseudo-valuation. If σ_1 and σ_2 are pseudo-valuations with $\sigma_1(F)=G_1 \cup \{0\}$ and $\sigma_2(F)=G_2 \cup \{0\}$ then $\sigma_1 \ll \sigma_2$ if there is a bounded subgroup $N \subseteq G_1$ and a homomorphism $\gamma: G_2 \cup \{0\} \to (G_1/N) \cup \{0\}$ such that $\gamma\sigma_2(x) = j\sigma_1(x)$ for all $x \in F$ where j is the canonical mapping $j: G_1 \to G_1/N$ and $h \in G_2$ with $h \leq 1$ implies that $\gamma(h) \leq g$ for some fixed $g \in G_1$. σ_1 is equivalent to σ_2 ($\sigma_1 \approx \sigma_2$) if $\sigma_1 \ll \sigma_2$ and $\sigma_2 \ll \sigma_1$. The notion of equivalency in terms of the image groups has not previously been available.

Theorem 1: If F is a field, $\sigma_1 \approx \sigma_2$ iff σ_1 and σ_2 give rise to the same topology (locally bounded ring topology) on F. Theorem 2: If F is a field, any weak pseudo-valuation is equivalent to a pseudo-valuation. Theorem 3: The topology on a field given by a pseudo-valuation is a field topology iff the topology induced on G makes G a topological group.

(Received November 5, 1974.)

*720-12-15 DAVID A. GAY, New College, Florida, and University of Arizona, Tucson, Arizona 85721. A new proof in the Galois theory of function fields.

Let K be a field and G a finite group. Then G can be realized over K means that there exists a Galois extension of K with Galois group G. Douady (C.R. Acad. Sc. Paris, vol. 258, 5305-5308 (1964)) proved a theorem one of whose immediate consequences is: If K is an algebraically closed field of characteristic zero and x is an indeterminant over K, then any finite group can be realized over $K(x)$. This theorem follows from (1) Any finite group can be realized over $\mathbb{C}(x)$ (L. Greenberg, BAMS, vol. 69, 569-573 (1963); M. Tretkoff, Comm. Pur. & App. Math., vol. 24, 491-497 (1971)) and (2) If K and L are algebraically closed fields of characteristic zero, then, if G can be realized over $K(x)$, G can also be realized over $L(x)$.

A proof of (2) will be offered using standard Galois Theory and basic commutative algebra.

(Received November 6, 1974.)

720-12-16 ROBERT A. MORRIS, Dept. of Defense, Fort George G. Meade, MD, 20755.
An Enumeration of the Orders in Cubic Number Fields, Preliminary Report.

Let O be an order whose index in R, the ring of integers of a cubic field, is a power of some rational prime p, and let F be its conductor, namely, the largest ideal of R contained in O. Dedekind gave a characterization of the ideals which can serve as the conductor of some order. We give the following alternative characterization: F is either of the form p^r, or there exists a prime ideal $\mathfrak{p}$ of first degree over p such that F is of the form $p^{r+s}\mathfrak{p}^{-s}$, for non-negative integers r and s, with the restriction $s = 1$ if $\mathfrak{p}$ is ramified. We can also show that O/F is generated as an abelian group by 1 and at most one other element of O, and has type (p^{r+s}, p^t), where $0 \leq 2t \leq r$. Moreover, for F of the given form, there is a unique order having F as conductor and $t = 0$ (namely $Z + F$), and precisely $W_F p^{t-1}$ distinct orders having F as conductor with $t > 0$, where W_F is $\{p+1$ - number of 1st degree ideals over p, $p-1$, $p\}$ according as $s = 0$, $s \geq 1$ and $\mathfrak{p}$ is unramified, $s = 1$ and $\mathfrak{p}$ is ramified$\}$. These results together with some primary decomposition theory allow us to enumerate the orders of general composite index in R. (Received November 6, 1974.)

*720-12-17 D.Brizolis, University of Southern California, University Park, California 90007. A local version of the nullstellensatz in rings of integer-valued polynomials.

A Local Version of the Nullstellensatz in
Rings of Integer-Valued Polynomials

Let K be any algebraic number field with ring of integers D and let $D^{(n)} = D \times \cdots \times D$.

$R_D^{(n)} = \{f \in K[X_1, \ldots, X_n] \mid f(D^{(n)}) \subseteq D\}$, the ring of integer-valued

polynomials over K. The following is true:

Theorem Let P be any prime ideal of D and let $\bar{D}_P$ be the completion of D with respect to the P-adic valuation on D. For any ideal J of $R_D^{(n)}$ containing P let $V_{J,P} = \{\alpha \in \bar{D}_P \mid |f(\alpha)|_P < 1$ for all $f \in J\}$. If $g \in R_D^{(n)}$ and $|g(\alpha)|_P < 1$ for all $\alpha \in V_{J,P}$ then $g \in \sqrt{J}$.

(Received November 6, 1974.)

720-12-18 P.J. WEINBERGER, University of Michigan, Ann Arbor, Michigan 48104
On the number of normal number fields.

Let G be a finite group. Let $N(T,G)$ be the number of number fields K with $\mathrm{Gal}(K) = G$ and $|\mathrm{disc}(K)| \leq T$. If G is Abelian then $N(T,G)$ is asymptotically of the form $cT^a(\log T)^b$ for certain constants c,a,b too complicated to give here. There are similar results for some small solvable groups.

(Received November 20, 1974.)

13 Commutative Rings and Algebras

720-13-1 MELVIN HOCHSTER, Purdue University, West Lafayette, Indiana 47907.
Homological questions in commutative rings and Cohen-Macaulay modules.

The talk will survey the status of several open questions in the homological theory of modules over commutative Noetherian rings. There will be particular emphasis on problems and results connected with proving the existence of maximal Cohen-Macaulay modules and on questions which can be settled, wholly or partially, using their existence. (Received October 25, 1974.)

720-13-2 HENRY J. SCHULTZ, University of Michigan-Dearborn, Dearborn, Michigan 48128.
Higher Derivations and Automorphisms of $C[[X]]$. Preliminary report.

Let A be the algebra of all formal power series with zero constant term over the complex numbers. If $(D_i)_{i\geq 0}$ is a higher derivation of A (i.e., the D_i are linear with $D_0 = I$ and $D_k(ab) = \sum_{i=0}^{k} D_i(a)D_{k-i}(b)$ for all $k \geq 1$ and all a,b belonging to A), we define $e^{(D_i)} = \sum_{i\geq 0} D_i/i!$. For a large class K of higher derivations (D_i) on A (which include all those of the form (D^i) where D is a derivation), $e^{(D_i)}$ is a well defined automorphism of A. Theorem. Every automorphism of A is of the form $e^{(D_i)}$ for some (D_i) belonging to the class K. This extends a result of Scheinberg (J. Math. Anal. Appl., 31 (1970), p. 330) where the (non-empty) class of automorphisms of A which are not of the form $e^{(D^i)}$ for any derivation D of A are characterized. (Received October 29, 1974.)

720-13-3 JOHN FUELBERTH, Univ. of Northern Colorado, Greeley, CO 80631, JAMES KUZMANOVICH, Wake Forest Univ., Winston-Salem, NC 27109, and THOMAS SHORES, Univ. of Nebraska, Lincoln, NB 68508. Splitting torsion theories over commutative rings.

Let R be a commutative ring with identity and mod-R the category of unitary R-modules. If $M \in$ mod-R and I is an ideal of R, let $Soc(M,I) = \{x \in M | xI=0\}$. Theorem. Let e be an idempotent of R and Y a finite set of maximal ideals of R containing e such that for each $P \in Y$ the localization R_P of R at P is a field. Define a functor τ on mod-R by the formula (1) $\tau(M) = Me + (\Sigma\{Soc(M,P) | P \in Y\})$, for all $M \in$ mod-R. Then (i) the sums in (1) are direct, (ii) formula (1) defines a splitting hereditary torsion functor on mod-R, and (iii) every splitting hereditary torsion functor on mod-R is so obtained. Corollary. Every splitting hereditary torsion theory on mod-R is TTF; moreover if R is Noetherian then such theories are centrally splitting. The known theorems about the splitting of the Goldie and simple torsion theories (for commutative rings) can be derived from our Theorem.

(Received October 29, 1974.)

*720-13-4 IRA J. PAPICK, Rutgers University, New Brunswick, New Jersey 08903 Topologically defined classes of going-down domains

Let R be an integral domain. Our purpose is to study GD (going-down) domains [Abstract 74T-A151, these Notices 21(1974), A-473] which arise topologically; i.e., we investigate how certain going-down assumptions on R and its overrings relate to the topological space Spec(R). Many classes of GD domains are introduced topologically, and a systematic study of their behavior under homomorphic images, localization and globalization, integral change of rings, and the "D+M construction" is undertaken. Before stating two typical theorems, we make the following definitions. R is called an open (respectively, LH-) domain if for each overring T of R, $Spec(T) \longrightarrow Spec(R)$ is open (respectively, a local homeomorphism). R is said to have finite fibers if for each overring T of R, $Spec(T) \longrightarrow Spec(R)$ has finite fibers. If M is a maximal ideal of R, the set $\{P \in Spec(R) : P \subset M\}$ is called a branch of R. Theorem R is open $\Longleftrightarrow$ R is GD, R is semilocal, and each branch of R is well-ordered under inclusion. Theorem R is an LH-domain $\Longleftrightarrow$ every overring of R is open $\Longleftrightarrow$ R is open, R has finite fibers, and every overring of R is treed. (Received October 30, 1974.)

720-13-5 JOSEPH E. CICERO, Clayton J.C., Morrow, Georgia 30260. Prime Ideals and Pseudo Boolean Valuations.

Let R be a commutative ring with identity. A pseudo Boolean valuation on R is a mapping v from R into a Boolean algebra B which satisfies for all x and y in R: V1) $v(0) = 0$, V2) $v(xy) = v(x) \cap v(y)$, and V3) $v(x + y) \leq v(x) \cup v(y)$. THEOREM 1: For every commutative ring R with identity, there is a Boolean algebra B and a pseudo valuation v from R into B. THEOREM 2; Every set of prime ideals of R induces a pseudo valuation on R. For each b in B the set $I_b = \{x \text{ in } R : v(x) \leq b\}$ is an ideal in R. The ideal I_b is the intersection of all the prime ideals which contain it. Two pseudo valuations v and v' from

R into B are said to be equivalent if the sets of all I_b for v and v' are equal. THEOREM 3: Every pseudo valuation on a commutative ring with identity is equivalent to a pseudo valuation on R induced by some set of prime ideals of R. (Received October 31, 1974.)

720-13-6 PAUL A. FROESCHL III, Saint Mary's College, Winona, Minnesota 55987. Chained Rings. Preliminary report.

A chained ring is a ring in which the set of ideals are linearly ordered under inclusion. A chained ring with Noetherian total quotient ring is the homomorphic image of a valuation domain. Moreover, a valuation ring with chained total quotient ring is a chained ring if and only if the zero divisors of the total quotient ring are contained in the ring. An example is given of a valuation ring with chained total quotient ring that is not a chained ring. Every valuation overring of a ring R is a chained ring if and only if the ideal of zero divisors of T(R) is the conductor of $\overline{R}$, the integral closure of R, in T(R). (Received October 31, 1974.)

720-13-7 E. GRAHAM EVANS, JR., University of Illinois, Urbana, Illinois 61801 Invariant Theory, Macaulay Rings, and Gorenstein Rings, Preliminary Report.

If k is a field, $R=k[X_1,\ldots,X_n]$, and G is a finite subgroup of GL(n,k), then we can define a G action on R by extending the linear action on the X_i. Using theorems of Hochster and Serre, Watanabe has given necessary and sufficient conditions for R^G to be Gorenstein if the order of G is prime to the characteristic of k. The above theorems will be discussed and a sketch of the proof of Watanabe's Theorem will be given. (Received October 31, 1974.)

720-13-8 MICHAEL SCHLOSSER, University of Kentucky, Lexington, Kentucky 40506
Entropy of Ring Endomorphisms, Preliminary Report

Rings are commutative with identity. Definition: A cover of a ring R is a finite collection $\alpha = \{a_1, a_2, \ldots, a_k\} \subset R$ such that $\sum_{i=1}^{k} a_i R = R$. Definition: For covers α_i of R, $\bigvee_{i=1}^{n} \alpha_i = \{\prod_{i=1}^{n} a_i \mid a_i \in \alpha_i\}$. Definition: For α a cover of R, $N(\alpha)$ is the number of elements in a subcover of α of least cardinality. Definition: For α a cover of R, T: R → R a ring endomorphism (unital), $h(T,\alpha) = \lim_{n\to\infty} \frac{1}{n} \log N(\bigvee_{i=0}^{n-1} T^i_\alpha)$. The ring entropy of T is $\sup_\alpha h(T,\alpha) = h_{ring}(T)$. Propositions analogous to those in Adler, Konheim, McAndrew, Topological Entropy, Trans. Amer. Math. Soc. 114(1965), 309–319, are obtained. Theorem: If X is compact Hausdorff, T: X → X is continuous and onto, $\hat{T}: C_R(X) \to C_R(X)$ via $\hat{T}(f) = f \circ T$, then $h_{topology}(T) = h_{ring}(\hat{T})$. Theorem: If R is a ring, spec R its prime ideal spectrum, T: R → R an endomorphism, then $h_{ring}(T) = h_{topology}(\text{spec } T)$. Theorem: If R is a commutative ring with the property that any nonzero ideal is contained in at most finitely many maximal ideals and T is an automorphism of R, then h(T) = 0. Corollary: Automorphisms of PID's, Dedekind Domains, and dim 1 Noetherian Domains will have zero entropy. (Received November 1, 1974.)

720-13-9 ROGER WIEGAND and SYLVIA WIEGAND, University of Nebraska, Lincoln, Nebraska 68508. The maximal spectrum of a Noetherian ring. Preliminary report.

We describe a general method of constructing a Noetherian ring with a given maximal ideal space. (Received November 1, 1974.)

*720-13-10 ROGER WIEGAND, University of Nebraska, Lincoln, Nebraska 68508. Dimension functions on the prime spectrum.

We define an invariant $\Gamma(R) \in \{0,1,2,\ldots,\infty\}$ associated to each commutative ring R. We have $\Gamma(R) \leq \dim(R)$ for rings with Noetherian spectrum, and examples exist with $\dim(R) - \Gamma(R)$ arbitrarily large. This invariant allows one to prove sharpened versions of Serre's theorem on free summands in large projectives, Bass' stable range theorem, and several related theorems. We prove that $\Gamma(R[X]) = \Gamma(R) + 1$ for every ring R. (Received November 1, 1974.)

720-13-11 DAVID WRIGHT, Columbia University, New York, N.Y. 10027. Algebras Which Resemble Symmetric Algebras, preliminary report.

Let k be a commutative noetherian ring, and A a commutative k-algebra. A is said to be locally polynomial if $A \otimes_k k_g$ is a polynomial algebra for all $g \in \operatorname{spec}(k)$. A is said to be invertible if there exists a k-algebra B such that $A \otimes_k B$ is k-isomorphic to a polynomial k-algebra. (All k-algebras here are assumed to be finitely generated over k.) Symmetric algebras of finitely generated projective modules are both locally polynomial and invertible.

Assume one of the following two conditions holds for A: 1) A is invertible. 2) A is locally polynomial and augmentable (i.e. there exists a k-algebra homomorphism $A \to k$). If I is the kernel of an augmentation, I/I^2 is a finitely generated projective k-module. Denote by S the symmetric algebra of I/I^2. Then $\hat{A} \cong \hat{S}$, where $\hat{A}$ denotes the completion of A with respect to the I-adic topology, and $\hat{S}$ denotes the completion of S with respect to its canonical augmentation ideal. In particular if I/I^2 is free of rank n, then A is isomorphic to the ring of formal power series in n variables over k. Furthermore if spec (k) is connected, we have $\dim A = \dim k + \operatorname{rank}_k(I/I^2)$. If we view k as an A-module, we have either $\operatorname{hd}_A(k) = \operatorname{rank}_k(I/I^2)$ or $\operatorname{hd}_A(k) = \infty$.
(Received November 1, 1974.)

*720-13-12 T. T. MOH, University of Minnesota, Minneapolis, Minnesota 55455. The Method of Tschirnhausen-Newton-Puiseux. Expository report.

Let k be a field of characteristic zero, k[x] a polynomial ring of one variable over k. Let $f(x) \in k[x]$ be with $\deg f(x) = mn$. Then there is an unique monic polynomial $g(x)$ of degree m such that $\deg(f(x) - g(x)^n) < (n-1)m$. The polynomial $g(x)$ is called the approximate n-th root of $f(x)$ or a generalized Tschirnhausen transformation of $f(x)$. In case $k \subset K((T))$ field of meromorphic functions of one variable, the equation $f(x) = 0$ can be solved in $K((T^{1/S}))$ by Newton-Puiseux expansions as in complex variable. Important properties of Newton-Puiseux expansions can be deduced from the approximate n-th roots of a polynomial. One advantage is to have all information provided by Newton-Puiseux expansion while staying in k[x]. Several applications will be discussed, open problems will be mentioned. This talk is based on the author's works in collaboration with S. S. Abhyankar (c.f. Crélle vol. 260, 261 and two preprints). (Received November 1, 1974.)

*720-13-13 JIMMY T. ARNOLD AND PHILIP B. SHELDON, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061. Integral Domains which satisfy Gauss's Lemma.

Let D be a commutative integral domain with unity. For a polynomial f(X) in D[X], let A_f denote the ideal of D generated by the coefficients of f. The polynomial f(X) is said to be primitive in case A_f is contained in no proper principal ideal of D, and superprimitive in case $A_f^{-1} = D$. In this paper two classes of domains are studied, the GL-domains, those over which the product of two primitive polynomials is primitive (i.e. those in which Gauss's Lemma holds), and the PSP-domains, those over which every primitive polynomial is superprimitive. Some ideal-theoretic characterizations of these classes of domains are obtained, along with the relations between them and some related classes of domains. It follows from results of Tang ["Gauss's Lemma", Proc. Amer. Math. Soc. 35 (1972), 372-376] that every GCD-domain is a PSP-domain, and every PSP-domain is a GL-domain. It is shown that the GL-domains are contained in the class of all domains in which irreducible elements are prime, and it is shown by examples that all of these containments are proper. Finally a characterization is obtained for all those domains D for which D[X] is a GL-domain. (Received November 4, 1974.)

720-13-14 JOONG HO KIM, East Carolina University, Greenville, North Carolina 27834. Completion of local ring which is not noetherian. Preliminary report.

Let A be a semilocal ring with $\bigcap_{n=1}^{\infty} J(A)^n = (0)$ where J(A) is the Jacobson radical of A. Let d be the metric on A derived from the J(A)-adic topology in the obvious way, and let $(\hat{A}, d')$ be the completion of a metric space (A, d). Then $\hat{A}$ forms a semilocal ring with $\bigcap_{n=1}^{\infty} J(\hat{A})^n = (0)$. Let $\hat{d}$ be the metric derived from the $J(\hat{A})$-adic topology. Then if J(A) is finitely generated, d' and $\hat{d}$ are equivalent; therefore, $\hat{A}$ is a complete semilocal ring. If J(A) is not finitely generated, d' and $\hat{d}$ are not necessarily equivalent. Proposition. Let B be a semilocal ring with $J(B)^n = (0)$. Let $B[[X_1, \ldots, X_n]]$ be a formal power series ring in n indeterminates over B, and let $A = \bigcup_{n=1}^{\infty} B[[X_1, \ldots, X_n]]$. Then A is a semilocal ring with $\bigcap_{n=1}^{\infty} J(A)^n = (0)$, but a semilocal ring $\hat{A}$ is not complete ($(\hat{A}, \hat{d})$ is not complete). (Received November 4, 1974.)

*720-13-15 JOHN W. PETRO, Western Michigan University, Kalamazoo 49001. Concerning filtrations on commutative rings.

Let $f = \{\mathfrak{a}_n\}_{n=0}^{\infty}$ be a (multiplicative) filtration of ideals on a commutative ring R with identity. (For example, $f_{\mathfrak{a}} = \{\mathfrak{a}^n\}$ for an ideal $\mathfrak{a}$ of R.) The power type of f is $\tau(f) = \inf\{k/m \mid \mathfrak{a}_{kn} \subseteq \mathfrak{a}_m{}^n \ \forall n\}$, provided such pairs k, m exist, and $\tau(f) = \infty$, otherwise. (Thus, $\tau(f_{\mathfrak{a}}) \leq 1$.) General properties of $\tau(f)$ are discussed. For instance, what real numbers can $\tau(f)$ assume? Lemma. $\tau(f) \leq 1$ implies that $\tau(f) = 0$ or 1. Example 1: A noetherian ring R such that for each $r \geq 1$ there exists a filtration f on R with $\tau(f) = r$. Theorem 1. If R is a Dedekind domain and $\tau(f) < \infty$, then $\tau(f) = 0$ or 1. Example 2. A noetherian ring R with ideals $\mathfrak{a}$ and $\mathfrak{b}$ such that $\tau(f_{\mathfrak{a}} \cap f_{\mathfrak{b}}) = 2$, where $f_{\mathfrak{a}} \cap f_{\mathfrak{b}} = \{\mathfrak{a}^n \cap \mathfrak{b}^n\}$, Theorem 2. If R is a Prüfer domain, $f = \{\mathfrak{a}_n\}$, $g = \{\mathfrak{b}_n\}$ are filtrations on R with $\tau(f) \leq 1$, $\tau(g) \leq 1$, then $\tau(f \cap g) \leq 1$, where $f \cap g = \{\mathfrak{a}_n \cap \mathfrak{b}_n\}$. (Received November 4, 1974.)

720-13-16 JOHN W. PETRO and M. EDWARD PETTIT, JR , Western Michigan University, Kalamazoo, Michigan 49001. Filtrations with finite power type on Prüfer domains, Preliminary report.

Let $f = \{\mathfrak{a}_n\}_{n=0}^{\infty}$ be a (multiplicative) filtration of ideals on a commutative ring R. Let rad f denote the common radical of $\mathfrak{a}_n$ for $n \geq 1$. The power type of f is $\tau(f) = \inf\{k/m \mid \mathfrak{a}_{kn} \subseteq \mathfrak{a}_m^{\,n}$ for all $n\}$ provided such pairs k, m exist, and $\tau(f) = \infty$, otherwise. Theorem 1. If R is a valuation ring and $\tau(f) < \infty$, then $\tau(f) = 0$ or 1. Theorem 2. If R is a Prüfer domain, $\tau(f) < \infty$ and rad $f = p_1 \cap \cdots \cap p_t$, for $p_i \in \operatorname{Spec} R$, then $\tau(f) = 0$ or 1. Remark. There exist Prüfer domains with filtrations f such that $1 \leq \tau(f) < \infty$, For example, the ring of entire functions. (Received November 4, 1974.)

*720-13-17 ANNE GRAMS, Univ. of Tennessee at Nashville, Nashville, Tn. 37203 and HOYT WARNER, Vanderbilt Univ., Nashville, Tn. 37235, Irreducible divisors in domains of finite character, Preliminary report.

A domain D is called an (idf)-domain if each nonzero element of D has only a finite number of nonassociate irreducible divisors. Theorem. A domain $D = \bigcap_\alpha V_\alpha$ of finite character is an (idf)-domain if one of the following holds: (1) each V_α, except possibly one of them, is rank one discrete, (2) D is a GCD-domain, or (3) for some Bezout domain D_0 with quotient field K, $D_0 \subseteq D \subseteq K[\{x_\lambda\}]$. Several examples of non-(idf)-domains of finite character are constructed, each an overring of a polynomial ring over Q or Z and each of the form $V \cap W \cap (\bigcap_{i \geq 1} V_i)$, where V and W are rank one and each V_i is rank one discrete. Included is an example of a one-dimensional Prüfer domain of finite character that is not an (idf)-domain. (Received November 5, 1974.)

720-13-18 N. H. VAUGHAN, North Texas State University, Denton, Texas and R. W. YEAGY, Stephen F. Austin State University, Nacogdoches, Texas. Factorization of ideals into semiprime ideals in an almost Dedekind domain.

Let D be a domain with $1 \neq 0$ and quotient field K. An ideal A of D is said to be semiprime if $A = \sqrt{A}$. We prove Theorem 1. If every ideal A of D is a finite product of semiprime ideals then an ideal A of D is a prime power iff A is primary. Theorem 2. If every ideal of D is a finite product of semiprime ideals and the A.C.C. holds for prime ideals of D, then D is almost Dedekind. Theorem 3. Let $R_1 \subseteq R_2 \subseteq R_3 \subseteq \ldots \subseteq R$ be a Dedekind tower-construction. Suppose all the residue fields of R_1 are perfect and that R is an almost Dedekind domain. Then every proper ideal of R is a product of semiprime ideals iff R has no critical maximal ideals. Theorem 4. There exists an almost Dedekind domain D (which is not Dedekind) such that every ideal of D is a finite product of semiprime ideals. (Received November 6, 1974.)

720-13-19 M. P. MURTHY, University of Chicago, Chicago, Illinois 60637. Projective modules over affine algebras.

A survey of results known and problems, with reference to Serre's problem, complete intersections, and structure of projective modules over two dimensional affine algebras. (Received November 6, 1974.) (Author introduced by Professor David Eisenbud.)

*720-14-1 GUSTAVE A. EFROYMSON, University of New Mexico, Albuquerque, New Mexico 87131. Substitution in Nash functions. Preliminary report.

Let D be a domain in R^n defined by a finite number of polynomial inequalities: $p_i(x) > 0$. Let $A_D = \{f : D \to D | f$ is analytic and there exists $p_f(z,x)$ in $R[z,x]$ with $p_f(f(x),x) = 0\ \forall x$ in $D\}$. Then there exists a polynomial relation $B(z,x)$ so that $f(a) = b$ iff $B(b,a)$. Let L be a real closed field containing R and define $D_L = \{a \in L^n | p_i(a) > 0$ all $i\}$. Then one can define $f_L : D_L \to L$ by $f_L(a) = b$ if $a \in D_L$ and $B(b,a)$. Let $\varphi : A_D \to L$ be a homomorphism. Since $R[x_1, \ldots, x_n] \subset A_D$, one can consider $\varphi x = (\varphi x_1, \ldots, \varphi x_n) \in L^n$. Moreover $p_i(\varphi x) > 0$, all i, so $\varphi x \in D_L$. Theorem. $f_L(\varphi x) = \varphi f$ for all f in A_D. This is proved using T. Mostowski's separation theorem, which he used to prove the nullstellensatz for A_D. The nullstellensatz also follows from the above theorem. (Received October 31, 1974.)

720-14-2 BASIL GORDON, University of California, Los Angeles, California 90024 and LORNE HOUTEN, Roswell Park Memorial Institute, Buffalo, New York 14203. On cubic curves over finite fields. Preliminary report.

The authors will discuss the following theorem and related results. Theorem. Let F be the finite field $GF(q)$ where q is a prime power. Let $N(*)$ be the number of solutions of $*$ over F. Then if v is any nonsquare of F we have $N(Y^2 = X^3 + AX + B) + N(Y^2 = X^3 + v^2AX + v^3B) = 2q$. (Received November 5, 1974.)

*720-14-3 FRANK SERVEDIO, Dalhousie University, Halifax, Nova Scotia, Canada. Singular orbits for linear algebraic groups. Preliminary report.

Let P be a form of degree r in $k[X_1, \ldots, X_n] = \Gamma(\mathbb{A}_k^n)$ where k is algebraically closed of characteristic 0. Let $GL(n,k)$ act naturally on $\mathbb{A}_k^n$ and on $\Gamma(\mathbb{A}_k^n)$ and take $S_P \subset GL(n,k)$ to be the full subgroup fixing P and S_P^0 to be the identity component of S_P. S_P^0 is a linear algebraic group defined over K. Finally, for each $j = 1, 2, \ldots, r-1$ define $(\partial^{(j)}P)$ to be the ideal in $\Gamma(\mathbb{A}_k^n)$ generated by the set of all jth partial derivatives of P with $(\partial^{(0)}P) = (P)$, and let $Z(P^{(j)})$ be the Zariski closed algebraic subset of zeros of $(\partial^{(j)}P)$ in $\mathbb{A}_k^n$ for $j = 0, 1, \ldots, r-1$. Theorem. If S_P^0 is reductive, then the orbits of S_P^0 in $Z(P)$ are the open subsets of nonsingular points of the irreducible components of $Z(P^{(j)})$ as j varies over $0, 1, \ldots, r-1$. The techniques involve characterizing $L(S_P)$, the Lie algebra of S_P, by a split exact sequence and use of the Lemma. If a connected G acts on Z in the category of affine algebraic spaces over k with X a closed point in Z, then the G orbit of X is open in Z iff $L(G)X = T_X(Z)$, the tangent to Z at X. (Received November 6, 1974.)

720-15-1 MARVIN MARCUS, University of California, Santa Barbara, California 93106
Adjoints and the numerical range.

All operators are on a finite dimensional unitary space. W. A. Beck and C. R. Putnam (J. London Math. Soc. 31 (1956), 213-216) proved that if $AN = N^*A$ for normal N, A is non-singular, $A = PU$, $P > 0$, U unitary, and $0 \notin W(U)$, the numerical range of U, then $N = N^*$. Let z be any complex number and define the range multiplicity, $\nu_z(A)$, to be the largest integer k for which there exist k o.n. vectors $x_1,\ldots,x_k$ such that $(Ax_j,x_j) = z$, $j = 1,\ldots,k$. The first result develops a normal form under unitary similarity for arbitrary A satisfying $AN = N^*A$, of the type $A = B \dot{+} C$, where C has only 0's in the main diagonal. It is then proved that $\nu_0(U) \geq \text{rank } A - \text{rank } B$. Using this inequality the Beck-Putnam result is generalized as follows. If $0 \notin W(U)$ then there exists an orthogonal decomposition of the space, $V = W \dot{+} W^\perp$, such that W and $W^\perp$ are invariant subspaces of both A and N, $A|W^\perp = 0$, $N|W$ is hermitian, and $\dim W \geq \text{rank } A$. (With A singular, U is an arbitrary but fixed unitary factor in the polar decomposition.) Various connections with and generalizations of subsequent work of S. K. Berberian, P. A. Fillmore, C. A. McCarthy, C. R. MacCluer, I. H. Sheth, O. Taussky, R. C. Thompson, and J. P. Williams are discussed. (Received September 23, 1974.)

720-15-2 JAMES W. BURGMEIER, University of Vermont, Burlington, Vt. 05401
RONALD E. PRATHER, University of Denver, Denver, Colo. 80210
Polynomial Calculus with D-like operators.

The expansion capabilities of rather general "derivative-like" linear operators are investigated. The class of all such operators is characterized. Imposing familiar differential properties leads to those possessing "Taylor-like" expansions and ultimately the derivative is characterized among all such operators. (Received September 27, 1974.)

720-15-3 CARL D. MEYER, JR., North Carolina State University, Raleigh, North Carolina 27607 The role of the group generalized inverse in the theory of finite Markov chains.

For an m-state homogeneous Markov chain whose one-step transition matrix is T, the group inverse, $A^\#$, of the matrix $A = I - T$ is shown to play a central role. For an ergodic chain, it is demonstrated that virtually everything that one would want to know about the chain can be determined by computing $A^\#$. Furthermore, it is shown that the introduction of $A^\#$ into the theory of ergodic chains provides not only a theoretical advantage, but it also provides a definite computational advantage that is not realized in the traditional framework of the theory. (Received October 15, 1974.)

*720-15-4 W.C. Pye, University of Southern Mississippi, Hattiesburg, Mississippi 39401 and Melvyn W. Jeter, University of Southern Mississippi, Hattiesburg, Mississippi A Minimal Permanent-Like Func.

This paper contains a study of the smallest n-homogeneous, superadditive, lower semicontinuous function $T(S) = \sup\{\sum_{i=1}^{n!}\lambda_i^n : S \leq \sum_{i=1}^{n!}\lambda_i A_i\}$ from

the n-th order, nonnegative real matrices into the nonnegative reals, which agrees with the permanent function on each n-th order permutation matrix A_i. Several results for this function analogous to those for the permanent function are derived, and a particular effort is made to determine when T is convex. For example, if S is a doubly stochastic matrix and $\alpha \varepsilon [1-1/(n-1)!,1]$, then $T(\alpha A_i+(1-\alpha)S) \leq \alpha+(1-\alpha)T(S)$.
(Received October 29, 1974.)

720-15-5 R. C. THOMPSON, Department of Mathematics, University of California, Santa Barbara. Singular values and diagonal elements.

Necessary and sufficient conditions will be given for the existence of a matrix with prescribed singular values and prescribed elements in its main diagonal.
(Received November 1, 1974.)

*720-15-6 B.E.CAIN, Iowa State University, Ames Iowa 50010, and B.D.SAUNDERS and H.SCHNEIDER, University of Wisconsin, Madison, Wisconsin 53706, On the geometry of dual pairs.

If ν is a norm equivalent to the Hilbert norm on a Hilbert space H with inner product $\langle\cdot,\cdot\rangle$, then the numerical range with respect to ν of an operator A is the image under the continuous mapping $(x,y) \to \langle Ax,y\rangle$ of the set of dual pairs, $\Pi(\nu) = \{(x,y) \in H \times H : (x,y) = \nu(x)\nu^*(y) = 1\}$. Here ν^* is the norm dual to ν. The same numerical range results when $\Pi(\nu)$ is replaced by its subset, $\Pi_0(\nu) = \{(x,y) \in H \times H : (x,y) = \nu(x) = \nu^*(y) = 1\}$. We develop natural homeomorphisms between $\Pi_0(\nu)$ and the unit sphere S of H, and between $\Pi(\nu)$ and the cylinder $S + \mathbb{R}$. These sets are arcwise connected when $\dim_{\mathbb{R}} H > 1$, consequently, the numerical range is arcwise connected. In particular, this holds for all norms on $\mathbb{C}^n$. We use a correspondence between positive homogeneous, convex functionals on H and cones in $H \oplus \mathbb{R}$. Our techniques are related to those used in the theory of monotone operators. (Received November 4, 1974.)

*720-15-7 EDWIN K. GORA and JAMES J. TATTERSALL, Departments of Physics and Mathematics, Providence College, Providence, Rhode Island 02918. On the reduction of functions of matrices to matrix polynomials.

A previously reported method [Gora, "On the use of Lucas polynomials in the theory of matrix functions", Abstract 703-A26, these *Notices* 20(1973), A-354] permitting the reduction of functions of matrices to matrix polynomials without explicit knowledge of the characteristic roots is shown to be equivalent to the familiar Lagrange-Sylvester method. Procedures utilizing a combination of the two methods are now proposed, and first applied to circulant matrices. Adaptation of the results thus obtained to matrices in general leads to substantial simplifications in the expansions representing the coefficients appearing in the matrix polynomials. These simplifications make it possible to reduce such expansions to expressions in closed form for any matrix of rank $N \leq 4$, and for many matrices of higher rank. The proposed procedures are applicable to a wide range of problems, and have proved particularly useful in the treatment of linear canonical transformations in quantum mechanics. (Received November 4, 1974.)

*720-15-8 JIN BAI KIM, West Virginia University, Morgantown, West Virginia 26505. Induced transformations on exterior product spaces.

Let E be a vector space of dimension $n(\geq 2)$ over a field. Let L(E) denote the set of all linear transformations of E. Let $\wedge E = \wedge^0 E \oplus \wedge^1 E \oplus \wedge^2 E \oplus \wedge \ldots \wedge \oplus \wedge^n E$ be the exterior algebra generated by E. h in L(E) induces $h^{(q)}: \wedge^q E \to \wedge^q E$ and $h^\wedge$. ($h^\wedge$ is called the induced transformation of h.) r(h) denotes the rank of h. N(h) and R(h) denote respectively the null space and the range space of h. Theorem 1. Let $h, g \in L(E)$. Then $r((gh)^\wedge) = 2^{r(h)-\dim(R(h) \cap N(g))}$. In Theorem 2, we generalize a theorem by A. P. Stone ["Induced transformations on exterior product spaces", Tensor 23(1972), 147-150]. (Received November 6, 1974.)

720-15-9 STEPHEN BANCROFT, Gordon College, Wenham, Mass. 01914
A Note on the Jordan Canonical Form

Let $\sigma: V^n \to V^n$ be a linear transformation of an n-dimensional space over the complex field C. The classical matrix representation theorem for such a transformation is known as "The Jordan Canonical Form". In this paper an elegant geometric approach is taken which motivates the construction of this form in a truly "natural" way. The proof is especially interesting because it makes no use of the Hamilton-Cayley theorem, minimal polynomials, or any "premature" triangular matrix forms. The proof displays, through an "unforgettable" geometric discussion, natural interpretations of the number of blocks and block dimensions appearing in the final form. The Hamilton-Cayley theorem and construction of the minimal polynomial for the transformation follow as immediate consequences of the Jordan Form, rather than as integral steps in the achievement of that form. (Received November 6, 1974.) (Author introduced by Professor Thomas Goulding.)

16 Associative Rings and Algebras

*720-16-1 CARTER LYONS, Madison College, Harrisonburg, Virginia 22801, Finite groups with semisimple endomorphism rings.

If the near ring E(G) generated by the endomorphisms of a finite group G is a semisimple ring, or more generally, has square-free characteristic then it is shown that G is abekian, a direct sum of elementary p-groups. (Received October 4, 1974.)

*720-16-2 TOM CHEATHAM, Samford University, Birmingham, Alabama 35209
Regular Modules

A left R-module is called regular (see Fieldhouse [Math. Ann. 184(1969), 1-18])if each of its submodules is (Cohn) pure. Theorem 1. The class of regular modules is closed under taking products if and only if R/J is a von Neumann regular ring. (J denotes the Jacobson radical of R.) Theorem 2. Every regular (left) R-module is semisimple (i.e. a sum of simple modules) if either of the following conditions is satisfied: (i) all maximal left ideals of R are finitely generated, (ii) R is a commutative ring with a noetherian maximal spectrum. An example is given to show that neither condition is necessary. (Received October 11, 1974.) (Author introduced by Dr. W. D. Peeples.)

*720-16-3 JOE W. FISHER, University of Texas, Austin, Texas, 78712. Rings generated by their units. Preliminary report.

Let R be an associative ring with unity. Theorem. If R is π-regular of bounded index n in which 2 is a unit, then R is generated by its units. In fact, each element of R is a sum of at most 2n units. This theorem extends Theorem 17 of Henriksen [J. Algebra 31 (1974), 182-193]. R. L. Snider and the author have shown that if R is von Neumann regular of bounded index, then R is unit regular and hence, if 2 is a unit in R, then each element of R is a sum of at most 2 units. (Received October 4, 1974.)

*720-16-4 ZOLTAN PAPP, George Mason University, Fairfax, Virginia 22030 On Stable Noetherian rings.

P. Gabriel calls a localizing subcategory stable if it is closed under taking injective envelopes and proves that a commutative Noetherian ring R is stable, that is, every localizing subcategory of ${}_RM$ is stable. (Bull. Soc. Math. France 90 (1962) 323-448, MR 38 #1144.) The following theorem is a generalization of this result. Theorem: A Noetherian ring R is stable if and only if for every pair of indecomposable injective R-modules E and F, $\mathrm{Hom}_R(F, E) \neq 0$ implies that $\tau_E \leq \tau_F$. τ_S is the associated kernel functor to the R-module S as defined by O. Goldman. (J. of Algebra 13 (1969) 10-47, MR 39 #6914.) It can be shown that the stability of Noetherian rings is Morita invariant property. A number of applications of these results are also given in the paper. (Received October 15, 1974.)

*720-16-5 K. R. GOODEARL, University of Utah, Salt Lake City, Utah 84112. Completions of simple regular rings.

This paper is concerned with the structure of the completion of a simple, von Neumann regular ring R with respect to the metric induced by a rank function. The set $\mathbb{P}(R)$ of all pseudo-rank functions on R is given a geometric structure, and it is proved that the completion of R with respect to a rank function N is a direct product of k (or ∞) simple rings if and only if N is a positive convex combination of k (or ∞) vertices of $\mathbb{P}(R)$. Examples are constructed to show that the completion of a simple regular ring with respect to a rank function may be any of these possibilities. (Received October 17, 1974.)

720-16-6 GEORGE SZETO, Bradley University, Peoria, Illinois 61606. On rings with no non-trivial annihilators. Preliminary report.

Let R be a ring. We call an element r in R a proper left zero divisor if there exist non-zero elements r', r" in R such that $rr' = 0$ and $rr'' \neq 0$. Clearly, if R does not contain proper left zero divisors, then $R = A \cup S$ where

$A = \{r \,/\, rR = 0\}$ and $S = \{r \,/\, rt = 0$ implies $t = 0$ for t in $R\}$. Theorem 1. By keeping the above notations, if R does not contain proper left zero divisors such that every element in S has a right identity, then (1) either $R^2 = 0$ or $R \cong Re \oplus A$ as left R-modules, where e is a right identity of an element in S, (2) Re is an integral domain (called an integral component of R) and A is a torsion free left Re-module. Theorem 2. Let C' be the category of all rings satisfying the hypothesis of Theorem 1 and with a same integral component D. Let C" be the category of all torsion free left D-modules. Then $F: R \to A$ is a bijective and full functor from C' to C". (Received October 21, 1974.)

*720-16-7 ROBERT GORDON, Temple University, Phila., Pa. 19122 and EDWARD L. GREEN, University of Pennsylvania, Phila., Pa. 19104. A Representation Theory for Noetherian Rings.

The representation theory of noetherian rings developed in this paper resembles the representation theory of artinian rings from which it is derived. It involves a notion of strongly indecomposable modules; and noetherian modules are decomposed in terms of them. Certain envelopes of the strongly indecomposable modules which decompose the factors of the module's submodule sequence are determined up to order and isomorphism by the module. Moreover, if the ring is finite over a commutative noetherian ring then the same strongly indecomposable modules decompose the module itself, and are determined by it up to subisomorphism. For such rings a notion of finite representation type is introduced. Various characterizations are obtained by exploiting well known theorems of M. Auslander and A. Roiter concerning the representation type of Artin algebras. (Received October 23, 1974.)

*720-16-8 JITENDRA N. MANOCHA, Kent State University, East Liverpool, OH 43920
σ-noetherian rings and σ-artinian rings. II.

We use the notation of Abstract 74T-A69, these Notices 21 (1974), A-297. Theorem 1. Let R be noetherian relative to E(R)-torsion theory. (We shall call R a (right) Lambek-noetherian ring.) Then (i) R is finite dimensional. Thus condition (2): R has the a.c.c. on right E(R)-annihilators implies condition (1): R is finite dimensional in Jans [J. Algebra, 7 (1967), 35-43]. (ii) Finitely generated flat modules are projective. (iii) A projective submodule of a finitely generated free module is finitely generated. Theorem 2. (i) R is a (right) Lambek-noetherian ring iff each ring between R and the (right) maximal ring of quotients $Q_m(R)$ is a (right) Lambek-noetherian ring. (ii) If R is σ-noetherian then $R/\sigma(R)$ and $Q(R)$ are (right) Lambek-noetherian rings. Define M to be σ-artinian if M has the d.c.c. on σ-closed submodules. Then Theorem 3. A necessary condition for $Q(R)$ to be a (right) noetherian (artinian) ring is that R is σ-noetherian (σ-artinian). If σ has property (T), the above condition is sufficient. Theorem 4. $Q(R)$ is a quasi-Frobenius ring iff $Q(R)$ is right R-injective and R is σ-noetherian or σ-artinian. Further, $Q(R)$ is a semi-simple ring with minimum conditions iff the $\bar{R}$-singular ideal of $\bar{R}$, $Z_{\bar{R}}(\bar{R}) = 0$ where $\bar{R} = R|\sigma(R)$. Applications to classical ring of quotients are discussed, improving a result of Jans; and Mewborn and Winton. (Received October 24, 1974.)

720-16-9 PROFS. D. E. RADFORD, Lawrence University, Appleton, Wi. 54911, E. J. TAFT AND R.L. WILSON, Rutgers University, New Brunswick, N.J. 08903. Some Forms of Certain Hopf Algebras.

One of the authors has given a construction of a finite-dimensional Hopf algebra H over a field F whose antipode S has order $2n$, where F contains a primitive n-th root ω of unity. We show here that the restriction that ω be in F can be dropped, so that F is required only to be of characteristic zero or of prime characteristic not dividing n. One forms the field extension $K = F(\omega)$ with Galois group G. Over K, one constructs a Hopf algebra L by tensoring certain Hopf algebras whose antipodes have order $2n$. G acts on L by semi-linear Hopf automorphisms, and the space of G-fixed points of L is a Hopf algebra over F whose antipode has order $2n$. H is an F-form of L in the obvious Hopf algebra sense.

(Received October 30, 1974.)

*720-16-10 JAY SHAPIRO, Rutgers University, New Brunswick, New Jersey 08903
A non-commutative analog to prime ideals

We work with a non-commutative analog of prime ideals in a commutative ring. Call $I_R \subseteq R$ subcritical if R/I is monoform. Theorem: For any T-torsion theory σ, $\exists$ a bijection: subcritical ideals of $R_\sigma \longleftrightarrow \sigma$ closed subcritical ideals of R. Theorem: Let $R \longrightarrow S$ be a left flat, epi ring extension, let $\{M_i\}$ be the maximal right ideals of S, then $R \cap M_i$ are subcritical ideals of R and S is ring $\simeq$ to R localized at $\Pi E(R/M_i \cap R)$. We give a candidate for a topological Spec R, which reduces to the Zariski topology in a commutative ring. We define a presheaf of rings on Spec R, whose sheafification agrees with the structure sheaf if R is commutative. In certain instances (e.g. prime goldie rings) this presheaf is a sheaf, with the stalk at a point $E \in$ Spec R being R_E. If A = center R, we get an induced map Spec $R \longrightarrow$ Spec A which is always continuous, and onto if R is integral over A. Theorem: Let R be a ring which is either f.g. over A or has right krull dimension and is integral over A, then $\exists$ a family F of semiprime ideals $\ni$, if $N \in F$, the localization σ at $E(R/N)$ is T, $N_\sigma = J(R_\sigma)$, R_σ/N_σ is artinian. Moreover, M_P is flat $\Longleftrightarrow$ M_σ is flat over R_σ $\forall N \in F$, and w. dim R = sup w. dim R_σ. (Received October 30, 1974.)

720-16-11 SUSAN C. GELLER, Cornell University, Ithaca, New York 14853. Presentations of $GE(n,R)$ and computation of $K_2(n,R)$. Preliminary report.

Recall that $GE(n,R)$ is the subgroup of $GL(n,R)$ generated by elementary and diagonal matrices. Let $GEU(n,R)$ be the group with generators $X_{ij}(r)$, $r \in R$, $1 \leq i \neq j \leq n$, and $[u_1,\ldots,u_n]$, $u_i \in R^*$ (units of R), with relations the Steinberg relations, $[u_1,\ldots,u_n][v_1,\ldots,v_n] = [u_1v_1,\ldots,u_nv_n]$, $X_{ij}(r)[u_1,\ldots,u_n] = [u_1,\ldots,u_n]X_{ij}(u_i^{-1}ru_j)$, and $X_{ij}(u-1)X_{ji}(1) = [u]_{ij}X_{ji}(u)X_{ij}(1-u^{-1})$, where $[u]_{ij} = [v_1,\ldots,v_n]$ where $v_i = u$, $v_j = u^{-1}$, $v_k = 1$ for $k \neq i,j$. Let $KG(n,R)$ be the kernel of the natural projection $GEU(n,R) \to GE(n,R)$. Then for all n there is an exact sequence $1 \to H(n,R) \cap K_2(n,R) \to K_2(n,R) \to KG(n,R) \to R^* \cap E(n,R)/[R^*,R^*] \to 1$. Note that, if R is commutative, this sequence is $1 \to$ [symbols] $\to K_2(n,R) \to KG(n,R)$

$\to 1$. It can be shown that $KG(n,R)$ is central in $GEU(n+1,R)$, and hence $KG(R) = \lim KG(n,R)$ is abelian. If R satisfies $SR_m(R)$, then $KG(n,R) \to KG(n+1,R)$ is surjective for $n > m$. By adding more relations to $GEU(n,R)$, one can get corresponding exact sequences. Despite its similarity to $K_2(R)$, however, $KG(R)$ is not Morita invariant. (Received October 30, 1974.)

720-16-12 GARY F. BIRKENMEIER, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53211. A Decomposition of Rings. Preliminary Report.

Let R be an associative ring with unity. A proper ideal B in R is semicompletely prime if for $r \in R$ s.t. $r^n \in B$, for some integer n, then $r \in B$. Let N denote the set of nilpotents of R and J is the Jacobson radical of R. A right ideal X is densely nil, DN, if $X = 0$ or if $X \neq 0$ then every non-zero right ideal of R that is contained in X has non-zero intersection with N. Theorems: 1. If $N \neq 0$ then N is contained in tR where $t^2 = t \neq 1$ iff tR contains every element of nilpotent index 2 iff there exists an $e \neq 0$ s.t. $e^2 = e \neq 1$ and $N \cap eR = 0$. 2. If $N \neq 0$ and N is contained in tR where $t^2 = t$ then tR is a two sided ideal which is semicompletely prime and the right singular ideal of R is contained in tR. 3. If R is semiperfect with J nil or if R is right self-injective then $R = eR \oplus (1-e)R$ where $e^2 = e$, $N \subseteq (1-e)R$ and $(1-e)R$ is DN, and if $e \neq 0$ then, for R semiperfect with J nil, eR will be a finite direct sum of division rings, and for R right self-injective then eR will be a right self-injective strongly regular ring with unity. Furthermore if R has another decomposition, $R = A \oplus B$, where $N \subseteq B$ and B is DN then $B = (1-e)R$ and A is R-isomorphic and ring isomorphic to eR. (Received October 31, 1974.) (Author introduced by Professor Edmund Feller.)

*720-16-13 MARJORY J. JOHNSON, University of South Carolina, Columbia, South Carolina 29208. Chain conditions on regular near-rings.

Let R be a regular near-ring with a two-sided zero. A subgroup H of $(R,+)$ is an R-subgroup of R if $HR \subset H$. A principal R-subgroup of R is an R-subgroup of R of the form xR, where $x \in R$. Theorem 1. If R satisfies d.c.c. on R-subgroups of R, then R satisfies a.c.c. on R-subgroups of R. Theorem 2. If R satisfies a.c.c. on right ideals of R, then R satisfies d.c.c. on principal R-subgroups of R. (Received October 31, 1974.)

720-16-14 J. J. MALONE, Worcester Polytechnic Institute, Worcester, Mass. 01609. Endomorphism near rings that are rings. Preliminary report.

R. J. Faudree [Proc. Amer. Math. Soc. 27(1971), 236-240] has given a class of p-groups such that the distributively generated near rings generated by the endomorphisms of the groups are rings. These groups are of exponent p^2 and have the property that $\Phi = G' = Z$. In this note the necessity of the condition $\Phi = G' = Z$ is explored and it is shown that at least the condition $\Phi = G' \subseteq Z$ is necessary. It is also shown that, for $p = 2$, the group in Faudree's class has its endomorphism near ring fail to be a ring. (Received November 1, 1974.)

*720-16-15 JOHN LAWRENCE, Carleton University, Ottawa, Canada. Semiperfect Group Rings, Preliminary report.

Let $C(p)$ denote the class of (not necessarily commutative) fields of characteristic p. Theorem Suppose G is a group and p a prime. Then the group ring $F[G]$ is semiperfect for all $F \in C(p)$ if and only if G is a finite extension of a locally finite p-group. Examples of nonlocal group rings $F[G]$, where $F \in C(p)$ is a commutative field and G is a finitely generated residually finite p-group, are constructed. These answer, in the negative, a conjecture due to Goursaud. P-adic valuations are used to give properties that G must possess if $Q[G]$ is semiperfect (Q = the rationals).

(Received November 1, 1974.)

720-16-16 S. C. Goel and S. K. Jain, Ohio University, Athens, Ohio 45701. Rings with a class of modules projectives or quasi-injectives. Preliminary report.

It was shown that for a ring R every cyclic right R-module is injective or projective if and only if $R = S \oplus T$ where S is semi-simple artinian and T is a simple right semi-hereditary right Öre-domain whose every proper cyclic right module is injective. (Goel - Jain - Singh; Rings whose cyclic modules are injective or projective, to appear Proc. A.M.S.) We show here that the above conditions are also equivalent to: every cyclic J-semi-simple module ($J(M) = 0$) is injective or projective. Other equivalences in terms of a class of modules over a ring R being injective or projective which imply R is semi-simple artinian or right V-ring are also established. The condition that each singular right R-module is quasi-injective is studied and results analogous to SI-rings investigated by Goodearl (Mem. A.M.S. #124, 1972) are obtained. For example, under this condition for any ideal A which is essential as a right ideal, R/A is semi-simple artinian ring. (Received November 4, 1974.)

*720-16-17 MARK L. TEPLY, University of Florida, Gainesville, Florida 32611. Semi-prime and prime splitting rings. Preliminary report.

A ring R is called a splitting ring if the singular submodule $Z(M)$ is a direct summand of each right R-module M. A semiprime splitting ring with zero right socle contains no infinite direct sum of two-sided ideals. This result is used as a tool to reduce the study of splitting rings to the case where R is a prime ring. The center of a prime splitting ring is a field. Prime splitting rings with finiteness conditions on left and/or right ideals are examined. (Received November 4, 1974.)

*720-16-18 DON E. EDMONDSON, The University of Texas, Austin, Texas 78712. Reductivity in Rings.

Some properties of a ring are studied which follow from a lattice theoretic property in the lattice of ideals. A ring R is reductive if and only if for each pair A, B of ideals there exist ideals M,N such that $A \subseteq M$, $B \subseteq N$, $M+N = R$ and $A \cap B = M \cap N$. It is proved that for a reductive ring the lattice of ideals is distributive and that the same result holds

for certain weak forms of reductivity. It is also noted that a domain is principally reductive if and only if it has a certain factorization property and its finitely generated ideals are principal. (Received November 4, 1974.)

*720-16-19 JOHN McCONNELL, University of California, Berkeley, California 94704.
On simple associative algebras arising from solvable Lie algebras.

The simple algebras which arise from a completely solvable Lie algebra of characteristic zero may each be viewed as an algebra $\mathcal{A}(V,\delta,G)$ constructed from a finite dimensional vector space V, an alternating bilinear form δ on V and a finitely generated subgroup of the dual space V^*. In "Representations of solvable Lie algebras, II," (to appear in Ann.Sci. Ecol. Norm. Sup.) we conjectured that two such algebras $\mathcal{A}(V,\delta,G)$ and $\mathcal{A}(V',\delta',G')$ are isomorphic if and only if there exists a linear isomorphism from V to V' compatible both with δ and δ' and with G and G' and we showed that this was true if $\delta|V^G = 0$, where $V^G = \cap \operatorname{Ker} \lambda, (\lambda \in G)$. If $n = \operatorname{rank}(\delta|V^G)$ then we show that this conjecture is equivalent to a conjecture on the automorphism group of the Weyl algebra A_n and that both conjectures are true when $n = 1$. (Received November 4, 1974.) (Author introduced by Professor G.Seligman.)

720-16-20 SNIDER, ROBERT L. Mathematics Department, V.P.I. & S.U., Blacksburg, Va. 24061
Rings Whose Ideals Have Projective Covers

Let R be a commutative ring with no nontrivial idempotents. Theorem: All ideals of R have projective covers if and only if R is one of the following three types (1) perfect, (2) Local Noetherian, (3) Dedekind. (Received November 4, 1974.)

*720-16-21 JOHN FUELBERTH, University of Northern Colorado, Greeley, Colorado 80639 and
J. KUZMANOVICH, Wake Forest University, Winston-Salem, North Carolina 27109.
Primary Direct Sum Decomposition.

If P is a prime ideal in a Noetherian ring, then a module N is called P-primary if every element of N is annihilated by some power of P. It is well known that the Dedekind domains are precisely the integral domains for which the P-primary submodule is a direct summand for each finitely generated module and prime ideal P. In this paper we determine the (noncommutative) Noetherian rings for which this property holds. Roughly speaking, they are the Artinian primary rings and the Asano prime rings with the restricted minimum condition. (Received November 4, 1974.)

720-16-22 H. E. STONE, University of Pittsburgh, Pittsburgh, Pennsylvania 15260. Matrix representation of simple halfrings.

The right and left simple halfrings described in Abstract 672-275, these *Notices* 17(1970), 161 are used to obtain a matrix representation result. A right unital pair (u, v) of a halfring H is primitive if $v - u$ is the only nonzero idempotent of $(v - u)\overline{H}(v - u)$; for present purposes a parafield is a right and left simple halfring having a primitive unital pair. Two halfrings are called isodiffric if they have isomorphic rings of differences. Theorem. Let H be a simple halfring which is the strong direct sum of minimal subtractive right ideals having primitive right unital pairs. Then H is isodiffric to the full halfring of matrices over a parafield. (Received November 4, 1974.)

*720-16-23 KLAUS E. ELDRIDGE and SAMSON HSIA, Ohio University, Athens, Ohio 45701. Hereditary and semihereditary group rings. Preliminary report.

A ring with unity is said to be left (semi-) hereditary if all of its (finitely generated) left ideals are projective. Let G be a group of order n and let R be a ring with unity and characteristic $m > 1$. Among other results, we establish: (a) The group ring $R[G]$ is left semihereditary iff R is left semihereditary and n is a unit in R. (b) The group ring $R[G]$ is left hereditary iff R is left hereditary and n is a unit in R. The proofs depend on the fact that left $R[G]$-modules which are projective as left R-modules must be projective as R-modules, and the fact that if n is not a unit in R, then $R[G]$ contains a central nilpotent element. (Received November 5, 1974.)

*720-16-24 DAVID E. RADFORD, Lawrence University, Appleton, Wisconsin 54911. The antipode of a Hopf algebra. Preliminary report.

Let A be a finite dimensional Hopf algebra over a field k with antipode s. For a nonzero left integral x in A let $\alpha \in G(A^*)$ satisfy $xh = \alpha(h)x$, all $h \in A$, and let $\alpha \in G(A)$ be the corresponding element for A^*. Then $s^4(h) = a^{-1}(\alpha \rightharpoonup h \leftharpoonup \alpha^{-1})a$. Theorem. The order of the antipode of a finite dimensional Hopf algebra is finite. If x is any left integral of A then $s(x) = \alpha \rightharpoonup x$. The scalar $\alpha(a)$ plays a significant role in the structure of s^2. For any integral (left or right) x of A one can prove $s^2(x) = \alpha(a)x$. For $0 \neq \lambda \in k$ the eigenspaces of s^2 belonging to λ and $\lambda^{-1}\alpha(a)$ have the same dimension. Thus the eigenvalues $\lambda_1, \ldots, \lambda_r$ of s^2 and also $\lambda_1^{-1}\alpha(a), \ldots, \lambda_r^{-1}\alpha(a)$. In general there is no connection between existence of roots of unity in the ground field on the order of s. One can show that there is a close connection between the invariant factors of s^2 and the coradical under fairly general conditions. For any Hopf algebra A the injectivity (bijectivity) of the antipode s is determined on the coradical. The antipode is always "locally bijective" in the following sense: if D is a finite-dimensional subcoalgebra invariant under s, then $s: D \to D$ is bijective. Various conditions can be given to insure that s is bijective; for example the coradical is finite-dimensional, or a Hopf subalgebra. (Received November 5, 1974.)

*720-16-25 KLAUS E. ELDRIDGE and SAMSON HSIA, Ohio University, Athens, Ohio 45701. Group rings which are domains. Preliminary report.

A ring with unity is a domain if it has no nontrival zero divisors, and a group is torsion free if no element different from the identity has finite order. Among other results, we establish: (a) If R is a domain and G is a finite direct sum of infinite cyclic groups, the group ring $R[G]$ is a domain: (b) Let G be an FC-group. The group ring $R[G]$ is a domain iff R is a domain and G is torsion free. In this case, $R[G]$ has only trivial units: (c) If G is a torsion free FC-group and if R is a domain, the cross product ring of R with G is a domain. The proofs depend on the fact that an infinite cyclic group, hence a finite direct sum of such, is an ordered group and the fact that a finitely generated, torsion free FC-group is a finite direct sum of infinite cyclic groups. These results provide a partial answer to Problem 28 of Passman ["Infinite group rings," Dekker, New York, 1971]. For a definition of ordered and FC-groups consult Passman's book; for a definition of cross product rings see A. A. Bovdi [Soviet Math. Dokl. 2(1961), 438-440]. (Received November 5, 1974.)

720-16-26 RICHARD K. MOLNAR, Oakland University, Rochester, Michigan 48063. On the coradical of a Hopf algebra. Preliminary report.

Let H be a Hopf algebra over a field k and let H_o be the coradical of H. We wish to determine when H_o is a sub Hopf algebra of H; and if so, whether there is a Hopf algebra retract for the canonical inclusion $H_o \to H$. Such a retract induces a semi-direct product decomposition of H as a Hopf algebra under certain conditions (Abstract 711-16-27, these Notices 21(1974), A-72).

Theorem 1. Let H be a Hopf algebra with cocommutative coradical. Then H_o is a sub Hopf algebra if and only if H_o is separable as a coalgebra.

Theorem 2. Let H be a cocommutative Hopf algebra over k. If H_o is a sub Hopf algebra (e.g. when k is perfect) then there is a Hopf algebra retract $H \to H_o$ and $H \cong H^1 \# H_o$ as Hopf algebras.

Theorem 3. Let H be a finite dimensional commutative Hopf algebra over $k = \bar{k}$. If H_o is a sub Hopf algebra then there is a Hopf algebra retract $H \to H_o$.

(Received November 6, 1974.)

720-16-27 ROBERT W. MILLER, College of William and Mary, Williamsburg, Virginia 23185.

Finitely generated projective modules. Preliminary report.

Let A be an associative ring with unit. Let P_A be a finitely generated projective right A-module with A-endomorphism ring B, and let T be the trace ideal associated with P_A. Let ${}_AQ = \mathrm{Hom}_B({}_BP, {}_BW)$ where ${}_BW$ is an injective cogenerator. Let $\mathcal{J} = \{{}_AX \mid P \otimes X = 0\}$, a hereditary torsion class. Characterizations of the conditions that (1) ${}_BP$ is projective, (2) ${}_AT$ is projective, (3) ${}_BP$ is flat, and (4) ${}_AT$ is flat are obtained via the P_A-dominant codimension and ${}_AQ$-dominant dimension of certain classes of A-modules, and the $\mathcal{J}$-global dimension of the ring A. (Received November 6, 1974.)

720-16-28 JOHN H. COZZENS, Rider College, Lawrenceville, NJ and Kent State University, Kent, OH 44242; FRANCIS L. SANDOMIERSKI, Kent State University, Kent, OH 44242

Maximal Orders, Preliminary report.

If Q is a simple Artin-ring and R a subring of Q, then R is an order in Q, if Q is the classical (two-sided) quotient ring of R. Orders R, S in Q are equivalent, $R \sim S$, if there are units α, β, α', $\beta' \in Q$ such that $\alpha R \beta \subseteq S$, $\alpha' S \beta' \subseteq R$. R is a maximal order in Q if whenever $R \subseteq S$, $R \sim S$, then $R = S$. Throughout R denotes a prime left and right Noetherian ring which is a maximal order in Q. An ideal I of R is invertible if $II^{-1} = R = I^{-1}I$ where $I^{-1} = \{q \in Q / qI \subseteq R\}$. If P is a prime invertible ideal of R, then R_P, the Goldie local envelope of R exists and $R_P = \{rc^{-1} / r \in R, c \in C(P)\} = \{c^{-1}r / r \in R, c \in C(P)\}$, where $C(P) = \{c \in R / cx \in P \Rightarrow x \in P\}$ is a set of regular elements of R with left and right Ore conditions as shown byHajarnavis-Lenagan, Localication in Asano Orders, J. of Alg., 21(1972), 441-449. Theorem: If $\mathrm{glb}(R) \leq 2$, then $R = (\bigcap_{\mathcal{P}} R_P) \cap S$, where $\mathcal{P}$ consists of all prime maximal invertible ideals of R. $(\bigcap_{\mathcal{P}} R_P) = T$ and S are R-flat ring epimorphisms, T is bounded and S has no non-trivial invertible ideals. Other observations about these rings will be made. (Received November 6, 1974.)

720-17-1 JOHN R. FAULKNER, University of Virginia, Charlottesville, Va. 22903
Coordinatization of some quadrilateral geometries
Preliminary report.

An elation in a quadrilateral geometry (generalized quadrangle) is a collineation fixing all objects incident to a,b, or c where a,b,c is an incident chain. The group generated by all elations of a quadrilateral geometry which has all possible elations has Steinberg relations of type B_2 or BC_2; i.e., the group is generated by subgroups X_α, $\alpha\varepsilon\Sigma$, where Σ is a root system of type B_2 or BC_2, with $(X_\alpha, X_\beta)\subseteq \Pi X_{i\alpha+j\beta}$, for $1\neq x\varepsilon X_\alpha$, there is $w\varepsilon xX_{-\alpha}X_\alpha$ with $X_\beta{}^w = X_{\beta w_\alpha}$, where w_α is the reflection in the Weyl group determined by α, and $\langle X_\alpha | \alpha>0\rangle \cap \langle X_\alpha | \alpha<0\rangle = 1$. Groups with Steinberg relations of type B_2 may be parameterized by a pair (A,H) where either A is an associative division algebra with involution and H is the Jordan algebra of symmetric elements or H is a field and A is a Jordan division algebra of a quadratic form. This yields a corresponding coordinatization of certain quadrilateral geometries, paralleling the coordinatization of projective planes which admit all elations by alternative division algebras. The BC_2 case is incomplete.
(Received October 17, 1974.)

*720-17-2 MICHAEL A. GAUGER, University of Massachusetts, Amherst, Massachusetts 01002
Borel and Borel-like solvable Lie algebras.

Observing that the Borel subalgebras of semi-simple Lie algebras are rather special among all solvable algebras, we seek a characterization. A generator-relation presentation for Borel and parabolic subalgebras is obtained, and the derivations of these subalgebras are shown to be inner. Investigating solvable Lie algebras S whose derivations are inner (a D-algebra), one finds they have much of the structure associated to a Borel algebra through a Cartan decomposition: S = H + N where H is a maximal abelian subalgebra of semi-simple elements and N is the maximal nilpotent ideal. Results on H-roots in N are insufficient to make then the positive part of a "root system" (Bourbaki). Thus many non-Borel sovable D-algebras are created for which the H-roots satisfy: twice a root can be a root, root spaces can be more than 1-dimensional, and root strings can have length equal to the length of N. These results suggest axioms R_1 - H-root spaces are 1-dimensional, and R_2 - the H-roots $\alpha_1,\ldots,\alpha_g$ occuring in N/N^2 are a basis of H* and $\{(k_1,\ldots,k_g) | \Sigma k_i\alpha_i$ is a root$\}$ is the set of coefficient sequences of the positive part of a "root system". D, R_1 and R_2 characterize the Borels as they allow construction of generators and relations obtained for Borels at the start.
(Received October 22, 1974.)

*720-17-3 JOSEPH FERRAR, Ohio State University, Columbus, Ohio 43210. Algebras of type E_7 over number fields.

J. Tits has shown that with suitably defined product the vector space $\mathfrak{L}(\mathfrak{A}, \mathfrak{J}) = \mathrm{Der}\,\mathfrak{A} + \mathfrak{A}_o \otimes \mathfrak{J}_o + \mathrm{Der}\,\mathfrak{J}$, where $\mathfrak{A}$ is a quaternion algebra over a field k, $\mathfrak{J}$ an exceptional central simple Jordan algebra over k, Der X the derivation algebra of X, and X_o the kernel of the generic trace map, is a simple Lie algebra of type E_7 over k. We assume here that k is an algebraic number field, S the set of primes of k, and denote by k_p the completion of k at

$p \in S$. As a consequence of Harder's Hasse principle for simply connected algebraic groups of type E_7 we obtain Lemma: If $\mathfrak{L}$ and $\mathfrak{L}'$ are algebras of type E_7 over k, then $\mathfrak{L} \cong \mathfrak{L}'$ if and only if $\mathfrak{L}_{k_p} \cong \mathfrak{L}'_{k_p}$ for all $p \in S$. Combined with the local classification results of Kneser and Tits, this yields Theorem: If $\mathfrak{L}$ is of type E_7 over k, there exist algebras $\mathfrak{A}$, $\mathfrak{J}$ as above such that $\mathfrak{L} \cong \mathfrak{L}(\mathfrak{A},\mathfrak{J})$. Applied to an investigation of the forms of the 56-dimensional irreducible module for the split E_7 this yields Corollary: Every exceptional Freudenthal triple system over k is reduced. (Received October 25, 1974.)

*720-17-4 JOHN SCHUE, Macalester College, St. Paul, Minnesota 55105
Representations of Solvable p-algebras

Let L be a solvable p-algebra over an algebraically closed field F of characteristic $p > 2$ and suppose ρ is a non-trivial irreducible p-representation of L on the vector space V. A p-subalgebra K of L will be called reducing for L on V if $\rho(K)$ has a unique minimal subspace U(K) and $\dim V = p^{[L:K]} \dim U(K)$ where $[L:K] = \dim L/K$. We prove the existence of a reducing subalgebra K such that U(K) is one-dimensional and show that K and (U(K) determine ρ up to equivalence. Further results are obtained relating to invariant subspaces and composition series for p-ideals of L. (For example, if J is a p-ideal then in any composition series for $\rho(J)$ all composition factors are J-isomorphic.) A consequence is that if $\rho(L)$ is nilpotent then for any maximal ideal p-ideal J containing the kernel of ρ, J is a reducing subalgebra. (Received October 25, 1974.)

720-17-5 STEPHEN BERMAN, University of Saskatchewan, Saskatoon, Sask., Canada
Some results and open questions concerning Lie Algebras defined by Cartan matrices.

Some known results concerning Lie Algebras defined by Cartan matrices will be presented, (for definitions see S. Berman, "On the construction of simple Lie Algebras," J. Algebra, Vol. 27, No. 1, 1973; or R. Moody, "A new class of Lie Algebras," J. Algebra, Vol. 10, No. 2, 1968), as well as some new results about these algebras. Also, associated automorphism groups, the possibility of a generalized MacDonald formula, some relations with algebras of Cartan type, and other problems, will be discussed.
(Received October 29, 1974.)

*720-17-6 DENIS R. FLOYD, IRVIN ROY HENTZEL, Iowa State Univ., Ames, IA 50010.
An Application of Group Rings to Associator Dependent Algebras.

A nonassociative algebra A over a field F is called an associator dependent algebra if, for some fixed scalars $\alpha,\beta,\Gamma,\delta,\epsilon,\eta$ in F, the identity (1) $\alpha(a,b,c) + \beta(c,a,b) + \Gamma(b,c,a) + \delta(b,a,c) + \epsilon(a,c,b) + \eta(c,b,a) = 0$ holds for all choices of a,b,c in A, where the associator (a,b,c) is defined by $(a,b,c) = (ab)c - a(bc)$.

Identities of the form (1) correspond to left ideals of the group ring of S_3. We compute the basic types of associator identities; all others consist of assuming one or more of the basic ones. We classify various

identities found in the literature. Our group ring approach not only tells which identities are equivalent but presents a method of deducing one identity from another. (Received October 29, 1974.)

720-17-7 MARGUERITE FRANK, 115 Broadmead, Princeton, New Jersey 08540. On certain classes of Jacobian generated algebras. Preliminary report.

Several classes of simple graded modular Lie algebras of known dimensions are obtained by constructing the Cartan prolongation of L_{-1} by L_0 for restricted or nonrestricted representations $\Gamma : L_0 \to \operatorname{End} L_{-1}$. In most cases L_0 is chosen as a Witt-Jacobson or generalized Witt algebra. In the particular case $p = 3$, the solvable algebra L_0 which previously led to a new simple restricted 18-dimensional algebra (Proc. Amer. Math. Soc. 38(1973)) is now used to define a class of simple $2 \cdot 3^n$-dimensional algebras, nonrestricted for $n > 2$. (Received November 4, 1974.) (Author introduced by Professor George Seligman.)

720-17-8 HELMUT STRADE, Universität Hamburg, Hamburg, West Germany. Simple modular Lie algebras and Lie algebras over valuation rings.

Let k be a field of characteristic $p > 0$. Then there exists a field K of characteristic 0 with valuation ring R, such that $k \cong R/\mathfrak{M}$ ($\mathfrak{M}$ the maximal ideal in R). Let L be a simple finite dimensional Lie algebra over k. By representing L as a matrix algebra in $sl(k, n)$ we construct a Lie algebra M over R. Theorem. (1) M has a maximal ideal G. (2) $M/G \cong L$. (3) $M \otimes_R K$ is a simple Lie algebra over the field K of characteristic 0. (4) $M \otimes_R k$ has a maximal ideal which is nilpotent. If L is classical but not of type A_{np-1} one can regard this construction as a reversal of the Chevalley construction. There are some applications especially for graded algebras. (Received November 4, 1974.)

720-17-9 RICHARD E. BLOCK, University of California, Riverside, California 92502. Irreducible representations of Lie algebras.

The author surveys some recent results on irreducible representations of finite dimensional Lie algebras, especially concerning infinite dimensional representations at characteristic 0 (mainly over $\mathbb{C}$) and (finite dimensional) representations (not necessarily restricted) of Lie p-algebras. Emphasis is given to results and methods which apply in both the characteristic 0 and characteristic p cases. A principal method involves the inducing and coinducing constructions. Included in the results are embedding theorems for irreducible representations of nilpotent Lie algebras and of semisimple Lie algebras (over $\mathbb{C}$), and generalizations of theorems of Blattner, Dixmier, Lemire, Veisfeiler-Kats, Wallach and others. (Received November 4, 1974.)

*720-17-10 ROBERT LEE WILSON, Rutgers University, New Brunswick, New Jersey 08903. Recent advances in classification of simple Lie algebras.

We will discuss the classification of finite dimensional simple Lie algebras over algebraically closed fields of characteristic ≥ 5. Recent progress on this problem has resulted largely from the study of the Lie algebras of Cartan type defined by Kostrikin and Šafarevič [Izv. Akad. Nauk SSSR Ser. Mat. 33(1969), 251-322; English transl., Math. USSR-Izv. 3(1969), 237-304] (and subsequent generalizations). We will also discuss the classification of finite dimensional simple Lie algebras over arbitrary fields of characteristic zero. (Received November 4, 1974.)

720-17-11 HASAN A. ÇELIK, California State Polytechnic University, Pomona 91768, Semi-prime Antiflexible Rings, preliminary report.

Let R be an antiflexible ring whose commutators are in the middle nucleus M(R). The author (J. Algebra 21 (1972) 428-440) proved structure theorems if R is prime. Theorem 1. Let R be a semi-prime ring. Then one of the following holds: (1) R is associative, (2) N(R) = Z(R), (3) R is a subdirect sum of a semi-prime associative ring R_1 and a semi-prime antiflexible ring R_2 with $N(R_2) = Z(R_2)$. Theorem 2. Let R be a semi-prime ring with $[R,R] \subset N(R)$. Then one of the following holds: (1) R is associative, (2) N(R) = Z(R) and R^+ is commutative and associative, (3) R is a subdirect sum of a semi-prime associative ring R_1 and a semi-prime antiflexible ring R_2 whose plus ring R_2^+ is commutative and associative. Corollary. If R is prime with $[R,R] \subset N(R)$ then, either R is associative or R^+ is commutative and associative.
(Received November 4, 1974.)

720-17-12 J.E. Humphreys, University of Massachusetts, Amherst, MA 01002 On the analogues of Verma modules in characteristic p. Preliminary report.

Let $\underline{g}$ be the Lie algebra of a simply connected Chevalley group over a field of characteristic p>0, $\underline{u}$ its restricted universal enveloping algebra. For each restricted weight λ, Z_λ denotes the universal $\underline{u}$-module of highest weight λ, with composition factors M_μ occurring d_μ times. Besides the properties of these modules studied earlier [J. Algebra 19 (1971), 51-79], it can be shown that $d_\mu=1$ when μ is maximal in its "linkage" class (subject to a further condition). More generally, it can be conjectured that d_μ is essentially independent of p, depending just on the position of μ in its linkage class. This is supported by recent work of S.G. Hulsurkar, D.N. Verma, J.C. Jantzen, which allows the numbers d_μ to be computed for types A_1, A_2, A_3, B_2. (Received November 4, 1974.)

720-17-13 JOHN B. JACOBS, University of Oregon, Eugene, Oregon 97403. Triple forms and related Lie algebras. Preliminary Report.

Let V be a finite-dimensional vector space over F admitting a non-degenerate triple form, ψ, either skew or symmetric. Let $q(V,\psi) = \{T \in \mathrm{Hom}_F(V,V) \mid \psi(Ta,b,c) + \psi(a,Tb,c) + \psi(a,b,Tc) = 0 \quad \forall a,b,c \in V\}$. Then $q(V,\psi)$ is a Lie algebra, and, if (U,θ) is vector space and triple form with the above properties, $q((V,\psi) + (U,\theta)) = q(V,\psi) + q(U,\theta)$.

THEOREM: Let $\mathcal{L}$ be classical Lie algebra over F with Killing form B. If $\theta(a,b,c) = B([a,b],c)$, then $q(\mathcal{L},\theta) \cong \mathcal{L}$.

COROLLARY: Let $(\mathcal{L},\theta)$ be as above. Then dim $\mathcal{L}$ and θ uniquely determine $\mathcal{L}$.

THEOREM: Let $\mathfrak{A} = \mathrm{Aut}\ (\mathcal{L},\theta) = \{T \in \mathrm{Hom}_F(\mathcal{L},\mathcal{L}) \mid \theta(Ta,Tb,Tc) = \theta(a,b,c)\ \forall a,b,c \in \mathcal{L}\}$ and $\mathcal{J} = \{T \in \mathfrak{A} \mid Ta = \rho a$ where $\rho^3 = 1\}$. Then $\mathfrak{A}/\mathcal{J} \cong \mathrm{Aut}\ (\mathcal{L})$.

(Received November 4, 1974.)

720-17-14 ELENA M. MEDINA and BYOUNG-SONG CHWE, University of Alabama, University, Alabama 35486. A note on idealizers of terms of the ideal closure series of subalgebras of Lie algebras. Preliminary report.

If H is a subalgebra of a Lie algebra L of finite or infinite dimension over a fixed, but arbitrary, field K, the ideal closure series of H in $L, \ldots H_i \lhd H_{i-1} \lhd \ldots \lhd H_0 = L$, is defined inductively by $H_0 = L$ and $H_{i+1} = \langle H^{H_i} \rangle$. By $\langle H^{H_i} \rangle$ is meant the smallest subalgebra containing H and invariant unde Lie multiplication by the elements of H_i. We consider $I(H_i)$, the idealizer of H_i in L, for each i, and show that if H is the largest subalgebra which decides $\{H_i\}$, then $H = \bigcap H_i$ and $\bigcap I(H_i) = I(\bigcap H_i)$. It is then possible to show that $\ldots I(H_i) \subsetneq I(H_{i-1}) \subseteq \ldots \subseteq I(H_j) = L$. (Received November 5, 1974.)

720-17-15 DAVID KOPCSO, Babson College, Babson Park, Massachusetts 02157. Forms of Certain Generalized Witt Algebras. Preliminary report.

Let $\emptyset$ be a field of characteristic $p > o$. Following Wilson [Trans. Amer. Math. Soc. 153 (1971), 191-210] we consider the generalized Witt algebras over $\emptyset$ as certain Lie algebras of derivations denoted $W(m:n_1,\ldots,n_m)$ where $m,n_1,\ldots,n_m$ are integers ≥ 1. Let Γ be an extension field of $\emptyset$ and let $p > 2$. The $\emptyset$-forms of the Jacobson-Witt algebras $W(m:1,1,\ldots,1)_\Gamma$ are known [see Allen and Sweedler, J. Alg. 12 (1969), 242-249]. Let $n=n_1$. Here we determine the $\emptyset$-forms of $W(1:n)_\Gamma$ for $\Gamma/\emptyset$ Galois or simple purely inseparable exponent one. For $n > 1$, $W(1:n)$ is non-restricted. In particular, if $\Gamma/\emptyset$ is Galois then any $\emptyset$-form of $W(1:n)_\Gamma$ is $\emptyset$-isomorphic to $W(1:n)_\emptyset$. If $\Gamma/\emptyset$ is simple purely inseparable exponent one, let $A = \emptyset\,[x_1,\ldots,x_n]/(x_1^p - \gamma_1^p,\ldots,x_n^p - \gamma_n^p)$ and $F = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} x_1^{p-1} + \ldots + \frac{\partial}{\partial x_n} x_1^{p-1} \ldots x_{n-1}^{p-1}$. For E a derivation of A, let $L(A;E)$ denote the subalgebra $\{Ef \mid f \in A\}$ of Der A. Then, for suitably defined parameterizing sets X and Y, we have any $\emptyset$-form of $W(1:n)_\Gamma$ is $\emptyset$-isomorphic to one and only one algebra $L(A;E)$ where either $E=F$ with $(\gamma_1,\ldots,\gamma_n) \in X$ or $E = F + (-1)^n \left(\sum_{i=1}^{n-1} F^{p^i} (\gamma_{n-i})^{p^i} \right) \cdot x_1^{p-1} \ldots x_n^{p-1}$ with $\gamma_1 = \ldots = \gamma_n = 0$ and $(\gamma_1, \ldots, \gamma_{n-1}) \in Y$. (Received November 5, 1974.)

*720-17-16 JAMES LEPOWSKY, Yale University, New Haven, Connecticut 06520. Existence of conical vectors in induced modules.

Let $\mathfrak{g} = \mathfrak{k} + \mathfrak{a} + \mathfrak{n}$ be an Iwasawa decomposition of a semisimple symmetric Lie algebra with splitting Cartan subspaces $\mathfrak{a}$ over a field k of characteristic zero, and let $\mathfrak{m}$ be the centralizer of $\mathfrak{a}$ in $\mathfrak{k}$. Given a linear functional ν on $\mathfrak{a}$, extend ν to a character of $\mathfrak{m} + \mathfrak{a} + \mathfrak{n}$ which is trivial on $\mathfrak{m} + \mathfrak{n}$, and let X^ν be the $\mathfrak{g}$-module (algebraically) induced from the resulting one-dimensional $\mathfrak{m} + \mathfrak{a} + \mathfrak{n}$-module. Certain conical vectors (nonzero $\mathfrak{m} + \mathfrak{n}$-invariant vectors) are shown to exist in the $\mathfrak{g}$-modules X^ν, and it is conjectured that with a certain natural restriction, these are all the conical vectors in the X^ν. The result generalizes the main theorem in D.-N. Verma's thesis [Yale University, 1966], which essentially deals with the case $\mathfrak{m} = 0$, in the present notation. The rank 1 case ($\dim \mathfrak{a} = 1$) has been studied in the author's "Conical vectors in induced modules" [Trans. Amer. Math. Soc., to appear]. (Received November 5, 1974.)

*720-17-17 GEORGIA BENKART, University of Wisconsin, Madison, Wisconsin 53706. On inner ideals and ad-nilpotent elements of Lie algebras.

An inner ideal of a Lie algebra L over a field F is an F-subspace B of L such that $[B[B\,L]] \subseteq B$. This paper investigates properties of inner ideals and obtains results relating ad-nilpotent elements and inner ideals. For example, let L be a simple Lie algebra in which $\mathrm{ad}_y^2 = 0$ implies $y = 0$. If L satisfies the descending chain condition on inner ideals and has proper inner ideals, then L contains a

subalgebra $S = \langle e, f, h \rangle$, isomorphic to the split 3-dimensional simple Lie algebra, such that $ad_e^3 = ad_f^3 = 0$. Lie algebras having such 3-dimensional subalgebras decompose into the direct sum of two copies of a Jordan algebra, two copies of a Jordan bimodule, and a Lie subalgebra of transformations of the Jordan algebra and bimodule. In the special case when L is a finite dimensional simple Lie algebra over an algebraically closed field of characteristic $p > 5$ this decomposition yields Theorem. L is classical iff there is an $x \neq 0$ in L such that $ad_x^{p-1} = 0$ and if $ad_y^2 = 0$ implies $y = 0$. The proof involves actually constructing a Cartan subalgebra which has 1-dimensional root spaces for nonzero roots and then using the Block axioms (Trans. Amer. Math. Soc. 121(1966), 378-392). (Received November 6, 1974.)

720-17-18 R. V. MOODY, University of Saskatchewan, Saskatoon, Saskatchewan, Canada. Representations of infinite dimensional Lie algebras and formulas of Weyl-Macdonald type.

Using representations of Lie algebras based on abstract root systems with invariant form, V.G. Kato has proved and generalized Macdonald's identities (Invent. Math. 15(1972), 91-143). We will discuss this work and its implications. (Received November 6, 1974.) (Author introduced by Professor George B. Seligman.)

720-17-19 BRUCE N. ALLISON, University of Alberta, Edmonton, Alberta, Canada. Lie algebras of type BC_1.

Let L be a central simple Lie algebra of type BC_1 over a field k of characteristic zero. Suppose the highest root space of L has dimension greater than 1. Theorem. L is isomorphic to one of the following: (a) The derived algebra of the skew transformations of a skew hermitian form of index 1. (b) The Lie algebra constructed from an algebra with involution (A,K), where $(A \otimes_k F, K)$ is isomorphic to the tensor product of two composition division algebras for some extension F/k of degree 1 or 2. The construction in (b) is based on a construction developed in the author's "A construction of Lie algebras from J-ternary algebras". Amer. J. Math. (to appear). (Received November 5, 1974.)

18 Category Theory, Homological Algebra

720-18-1 JEAN MARIE McDILL, California Polytechnic State University, San Luis Obispo, California 93407. Epireflective subcategories in concrete categories. Preliminary report.

A definition will be presented for $\mathcal{A}$-regular and $\mathcal{A}$-compact objects in concrete categories. In consequence, a characterization theorem for epireflective subcategories, which has been proved previously only for certain categories of topological spaces can be extended to a variety of concrete categories. Theorem. Let $(\mathcal{C}, \mathcal{U})$ be a concrete category, which is complete, well-powered and co-(well-powered) and for which $\mathcal{U}$ preserves monomorphisms, and let $\mathcal{A}$ be a full replete subcategory of $\mathcal{C}$. Then $\mathcal{A}$ is epireflective in $\mathcal{C}$ if and only if each $\mathcal{A}$-regular object has a concrete-embedding extension (w, W) in $\mathcal{A}$ for which w is $\mathcal{A}$-extendible. (Received October 8, 1974.)

720-18-2 CHARLES CHING-AN CHENG, Rutgers University, New Brunswick, N.J. 08903. Finite Partially Ordered Sets of Cohomological Dimension One. Preliminary Report.

Let $\mathbb{C}$ be a small category and R a ring with identity. Define the R-cohomological dimension of $\mathbb{C}$ by $cd_R\mathbb{C}=\sup\{k|\lim^k\neq 0\}$, where $\lim^k$ is the k-th right derived functor of the limit functor $\lim:(\text{mod } R)^{\mathbb{C}}\longrightarrow \text{mod } R$. Let $\mathbb{C}$ be a finite poset from now on. If $p,q\in\mathbb{C}$ such that $p<q$ and there is no $k\in\mathbb{C}$ with $p<k<q$, then call p a cocover for q and q a cover for p. An element of $\mathbb{C}$ is superfluous if it is minimal with only one cover and if it is of height one with only one cocover. By the process of eliminating superfluous elements, we obtain a finite poset $E(\mathbb{C})$ which has no superfluous elements. It can be shown that $E(\mathbb{C})$ is independent of the order in which superfluous elements are removed. Theorem 1. Let $\mathbb{C}$ be a finite poset with terminal element and $R\neq 0$. Then $cd_R\mathbb{C}\leq 1$ iff $E(\mathbb{C})$ is a singleton. A poset $\mathbb{C}$ is initially finite if $\mathbb{C}_q=\{p\in\mathbb{C}|p<q\}$ is finite for each $q\in\mathbb{C}$. Corollary 2. Let $\mathbb{C}$ be an initially finite poset and $R\neq 0$. Then $cd_R\mathbb{C}\leq 1$ iff $E(\mathbb{C}_q)$ is a singleton for each $q\in\mathbb{C}$. Using this, one can prove Mitchell's theorem which characterizes all finite posets $\mathbb{C}$ such that $gl\ dim\ A^{\mathbb{C}}\geq 3+gl\ dim\ A$, where A is any abelian category with finite global dimension. ("On the dimension of objects and categories II", J. Algebra 9 (1968), 341-368).

(Received October 15, 1974.)

720-18-3 DONOVAN VAN OSDOL, University of New Hampshire, Durham, New Hampshire, 03824. Principal Homogeneous Objects as Representable Functors. Preliminary report.

Let $\underline{A}$ be a cocomplete category, $G: \underline{A}\to\underline{A}$ a functor, and ε a natural transformation from G to the identity functor on $\underline{A}$ such that ε is the coequalizer of εG and $G\varepsilon$. Let X be an object of $\underline{A}$ and Π an abelian group in $\underline{A}$, and assume that $\underline{A}$ has finite products. A Π-principal homogeneous object over X relative to G is a morphism $Y\xrightarrow{p}X$ in $\underline{A}$, an action $\theta: Y\times\Pi\to Y$, and a morphism $s: GX\to Y$ such that $pos=\varepsilon X$ and $Y\times\Pi \underset{\theta}{\overset{p\pi_1}{\rightrightarrows}} Y\xrightarrow{p} X$ is a coequalizer kernel-pair diagram in $\underline{A}$. Such an object gives rise to a morphism $f: G^2X\to\Pi$ such that $fo\varepsilon G^2X - foG\varepsilon GX + foG^2\varepsilon X = 0$, ie to a one-cocycle. Given a one-cocycle $f: G^2X\to\Pi$ let $D(-,f): \underline{A}^{op}\to\underline{Sets}$ be the functor defined by $D(Z,f)=\{(g,h)|g: Z\to X,\ h: GZ\to\Pi,\ \text{and } foG^2g = hoG\varepsilon X - ho\varepsilon GX\}$. If this functor is representable, then the representing object is actually part of a Π-principal homogeneous object over X. In particular if $\underline{A}$ is tripleable over a base category and G is the generated cotriple then the elements of $H^1_G(X,\Pi)$ are equivalence classes of certain representable functors $\underline{A}^{op}\to\underline{Sets}$. Such relationships form the basis for this talk.

(Received November 4, 1974.)

720-18-4 HARRY GONSHOR, Dept. of Math., Rutgers University, New Brunswick, N. J. 08903. An application of nonstandard analysis to category theory.

We prove Manes' theorem that the category of compact Hansdorff spaces is tripleable over sets by nonstandard analysis. This method permits the use of second enlargements rather than ultrafilters over the set of ultrafilters. The corresponding left adjoint and ingrediants of the corresponding triple translate very succinctly into nonstandard analysis.

(Received October 31, 1974.)

*720-18-5 HARVEY WOLFF, The University of Texas, Austin, Texas 78712, Free triples and the orthogonal subcategory problem.

We are concerned with the relationship between free triples and the orthogonal subcategory problem (i.e., when the full subcategory of objects orthogonal to a given class of morphisms is reflective). Our main result deals with the following: suppose $F \dashv U: \mathcal{A} \to \mathcal{B}$ and let $\underline{T} = (T,\eta,\mu)$ be any idempotent triple on $\mathcal{A}$. Let $\mathcal{C}$ be the full subcategory of $\mathcal{B}$ whose objects are those B such that $\eta UB: UB \to TUB$ is an isomorphism. We ask when is $\mathcal{C}$ a reflective subcategory of $\mathcal{B}$. If $\mathcal{B}$ has suitable completness properties, we characterize when $\mathcal{C}$ is reflective in terms of the existence of a certain free triple. Immediate consequences are the orthogonal subcategory theorem of Gabriel and Ulmer (Lect. Notes in Math No. 221) and a slight generalization of the Freyd-Kelly orthogonal subcategory theorem (J. Pure and Applied Algebra 2(1972), 169-191). In addition, we generalize a result of Gabriel and Ulmer (op. cit.) by showing the following: If $\mathcal{A}$ is a small category, Σ is a set of morphism in $[\mathcal{A}^o,\text{Sets}]$ and $\mathcal{D}$ is a category in which certain limits and colimits commute with each other, then the full subcategory of $L \in [\mathcal{A}^o,\mathcal{D}]$ such that $\mathcal{D}(D,L)$ is orthogonal to Σ for all D in $\mathcal{D}$ is reflective. (Received October 31, 1974.)

*720-18-6 WILLIAM F. KEIGHER, University of Tennessee, Knoxville, Tennessee 37916. Adjunctions and comonads in differential algebra.

Let $< S,T,\sigma,\tau > : A \to A'$ be an adjunction, let G and G' be comonads on A and A' respectively and let $U : A_G \to A$ and $U' : A'_{G'} \to A'$ denote the forgetful functors. If T commutes with G' and G, there is an adjunction $< \overline{S},\overline{T},\overline{\sigma},\overline{\tau} > : A_G \to A'_{G'}$ such that $U'\overline{S} = SU$, $U\overline{T} = TU'$, $U\overline{\sigma} = \sigma U$ and $U'\overline{\tau} = \tau U'$. Hence if one takes as the base adjunction, for example, the ring of fractions adjunction, or the inverse image - direct image of sheaves of rings adjunction, or the stalk adjunction, or the affine scheme - global sections adjunction, and as comonads ones whose coalgebras are the differential analogues of the base categories, then the above result gives extensions of these ring-theoretic constructions to their differential analogues, i.e., differential rings of fractions, etc. (Received November 4, 1974.)

720-18-7 V. S. KRISHNAN, Temple Univ., Philadelphia, Pa. 19122; Structured Categories and Projection Functors, Preliminary Report

The forgetful functor from a topological or uniform algebra to the base algebra was generalized to a form of "base functor" in a paper by the author in 1969; in 1971 O. Wyler defined a top-category C^S over a category C by means of a contravariant functor $s:C \to$ Cord, associating to each object of C a complete lattice of superposed structures. This paper combines the two forms of approach to present a type of "projective functor" from a "structured category over C^{o} to $C^{\text{*}}$. The two notions are interrelated closely. This includes as examples not only semiuniform, topological or preordered algebras over the algebras, but also semiuniform algebras over topological ones, or preordered ones, and topological spaces over convergence spaces. The projection functor also raises bicategory structures. (Received November 6, 1974.)

*720-20-1 JOHN R. DURBIN, K. BOLLING FARMER, University of Texas at Austin, Austin, Texas 78712. Projectively Monomial Groups.

Transitive monomial projective representations of finite groups are shown to be the same as those induced from linear characters of subgroups. It is proved that all projective representations of a finite group G are monomial if and only if G has a representation group G^* all of whose linear representations are monomial. The class of all such groups G is then shown to lie strictly between the classes of finite supersolvable and finite monomial groups. It is also shown that Dade's proof [Huppert, Endliche Gruppen I, V, 18.11] that every solvable group can be embedded in a monomial group cannot be extended to projectively monomial groups. (Received August 12, 1974.)

*720-20-2 JANET E. AULT, Madison College, Harrisonburg, Virginia 22801. Simple inverse semigroups whose $\mathcal{D}$-classes are ω-semigroups.

An ω-semigroup is a semigroup whose idempotents form an ω-chain $e_0 > e_1 > \dots$. Such semigroups occur naturally within regular semigroups, for if a $\mathcal{D}$-class of a regular semigroup is not a Rees matrix semigroup, then it contains a subsemigroup which is an ω-semigroup. Considered here are simple inverse semigroups whose $\mathcal{D}$-classes are in fact ω-semigroups. A construction is given of all such semigroups which are also fundamental, and the main result provides a method of locating all simple inverse monoids whose $\mathcal{D}$-classes are ω-semigroups, a description given explicitly in terms of a semilattice, a semilattice of groups V and a partial homomorphism of V. The multiplication in the construction is essentially the same as that of the Bruck-Reilly semigroups, with the only complication arising from the semilattice multiplication. An isomorphism theorem demonstrates that the construction is a very natural one. (Received September 16, 1974.)

*720-20-3 JAMES C. BEIDLEMAN, Department of Mathematics, University of Kentucky, Lexington, Ky. On Fitting Classes

All groups considered are finite and soluble. Let $\mathfrak{X}$ and $\mathfrak{Y}$ be Fitting classes of finite soluble groups. Then $\mathfrak{X}$ is said to be **strictly normal** in $\mathfrak{Y}$, denoted by $\mathfrak{X} \lhd < \mathfrak{Y}$, provided that for each finite soluble group G an $\mathfrak{X}$-injector of G is contained as a normal subgroup of some $\mathfrak{Y}$-injector of G. **Theorem 1.** Let $\mathfrak{X}_1, \mathfrak{X}_2$ and $\mathfrak{Y}$ be Fitting classes such that $\mathfrak{X}_i \lhd < \mathfrak{Y}$, i = 1,2. Then

(a) $\mathfrak{X}_1 \cap \mathfrak{X}_2 \lhd < \mathfrak{Y}$.

(b) If X is an $\mathfrak{X}_1 \cap \mathfrak{X}_2$ - injector of the finite soluble group G, then there exists an $\mathfrak{X}_1$ - injector X_1 of G and an $\mathfrak{X}_2$ - injector X_2 of G such that $X_1 \cap X_2 = X$.

(c) $\mathfrak{X}_1 \cap \mathfrak{X}_2 \lhd < \mathfrak{X}_1$.

Theorem 2. Let $\mathfrak{X}_1, \mathfrak{X}_2$ and $\mathfrak{Y}$ be Fitting classes such that $\mathfrak{Y} \lhd < \mathfrak{X}_i$, i = 1,2. Then $\mathfrak{Y} \lhd < \mathfrak{X}_1 \cap \mathfrak{X}_2$.

Let $\mathfrak{X}$ and $\mathfrak{Y}$ be Fitting classes. Let $\mathfrak{X} \cdot \mathfrak{Y}$ denote the class $\{G \mid G/G_{\mathfrak{X}} \in \mathfrak{Y}\}$. Then $\mathfrak{X} \cdot \mathfrak{Y}$ is a Fitting class. Let $\mathcal{N}$ denote the class of finite nilpotent groups. **Theorem 3.** Let $\mathfrak{X} \neq \{1\}$ be a Fitting class. Then $\mathfrak{X}$ is a normal Fitting class if and only if $\mathfrak{X} \lhd < \mathfrak{X} \cdot \mathcal{N}$. (Received October 4, 1974.)

*720-20-4 JOSEPH BUCKLEY, Western Michigan University, Kalamazoo, Michigan 49001. Automorphism groups of isoclinic p-groups.

This paper is related to the conjecture that $|G|$ divides $|\mathrm{Aut}\, G|$ for all finite, non-cyclic p-groups G of order greater than p^2. This has been proved for p-groups of class 2 by Faudree (Proc. Amer. Math. Soc., 19 (1968), 1379-1382) and for p-abelian p-groups by Davitt (Illinois J. Math. 16 (1972), 76-85). Recently, Hummel (Proc. Amer. Math. Soc. - to appear) has shown that if the finite p-group G is a central product of non-trivial subgroups N and A where A is abelian and if $|N|$ divides $|\mathrm{Aut}\, N|$, then $|G|$ divides $|\mathrm{Aut}\, G|$.

Theorem. If the finite p-group G is a central product of non-trivial subgroups N and S where $N < G$, $S \leq G$ and $N \cap [S,S] = 1$, then $|\mathrm{Aut}\, G|_p \geq p|\mathrm{Aut}\, N|_p|\mathrm{Aut}\, G/N|_p$. (Here $|X|_p$ denotes the p-share of $|X|$ the order of X.) As corollaries we obtain Hummel's result as well as a dual result. The concept of isoclinism, first introduced by P. Hall, enters into these corollaries. Hummel's hypothesis on G is equivalent to the statement that G is isoclinic with N. The hypothesis of our theorem is satisfied if G is isoclinic with either N or G/N. (Received October 10, 1974.)

*720-20-5 JAMES A. ANDERSON, Northern Arizona University, Flagstaff, 86001 Tensor Product of Commutative Semigroups.

The purpose of this paper is to describe the tensor product of two commutative semigroups in terms of their archimedean components.

Given commutative semigroups U and V with maximal idempotent images A and B resp., let $\phi(A,B)$ be the set of finitely generated bifilters of A x B. For $\Delta \in \phi(A,B)$, designate M_Δ to be the union of all finite minimal generating sets of Δ. Given $m \in M$, let $\Lambda^{\Delta}_{m,\gamma} = \bigcup_{\alpha\times\beta \in c_{m\gamma}} U_\alpha \otimes V_\beta$ where $\{c_{m,\gamma}\}_{\gamma\in\Gamma}$ is the set of all chains in A x B with least element m, which are maximal with respect to this property; and U_α and V_β are maximal archimedean components of U and V resp. for each α,β. Define W_Δ to be the subsemigroup of $\bigoplus_{m,\gamma}\Lambda^{\Delta}_{m,\gamma}$ such that if $(\sum_{i,\gamma}\lambda_{i\gamma}) \in W_\Delta$ and $M = \{(\alpha,\beta) \mid \lambda_{i\gamma} \in U_\alpha \otimes V_\beta$ for some $i,\gamma\}$ then $[M] = \Delta$.

Denote the ρ, the finest congruence on W_Δ such that $(\sum_{m,\gamma}\lambda_{m\gamma})\rho(\sum_{m,\gamma}\lambda'_{m\gamma})$ iff $\lambda_{pq} = \lambda'_{pq} + U_\alpha \otimes V_\beta, \lambda'_{rs} = \lambda_{rs} + U_\alpha \otimes V_\beta$ for some p,q,r,s,α,β and $\lambda_{m\gamma} = \lambda'_{m\gamma}$ otherwise. Let $T = \bigcup_\Delta W_{\Delta/\rho}$. Define addition on T as follows: For $(\sum_{m,\gamma}\lambda_{m\gamma})\rho$, $(\sum_{n,\delta}\lambda_{n\delta})\rho \in T$ and $\Delta(M,N) = [M \cup N]$, $(\sum_{m,\gamma}\lambda_{m\gamma})\rho + (\sum_{n,\delta}\lambda_{n\delta})\rho = (\sum_{m,\gamma}\lambda_{m\gamma} \oplus \sum_{n,\delta}\lambda_{n\delta})\rho$ where $\oplus$ denotes addition in $W_{\Delta(M,N)}$. T is the tensor product of U and V. (Received October 10, 1974.)

*720-20-6 JAMES R. WEAVER, The University of West Florida, Pensacola, Florida 32504. The Existence of a Normal Sylow p-subgroup or a Normal p-complement.

If H is a subgroup of a finite group G then $A(G;H) = \{\alpha \in A(G) \mid g^{-1}(g)\alpha \in H$ for each g in G$\}$ and $\Gamma(G;H) = \{\alpha \in A(G) \mid (h)\alpha \in H$ for each h in H$\}$ where A(G) represents the automorphism group of G. A(G;H) is a normal subgroup of $\Gamma(G;H)$. It is shown that if $\Gamma(G;H)/A(G;H)$ is a p-group for each p-subgroup H of G for p a prime dividing the order of G then G has a normal Sylow p-subgroup P or a normal p-complement. (Received October 15, 1974.)

*720-20-7 D. J. HARTFIEL, Texas A&M University and C. J. MAXSON, Texas A&M University, College Station, Tx 77801, A characterization of the maximal monoids and maximal groups in β_X.

In this paper we give a unified characterization of the maximal monoids and maximal groups in β_X, the semigroup of binary relations on a set X. Our characterization is presented for the isomorphic semigroup B_X of X×X Boolean matrices. Let I be an idempotent in B_X, $M(I) = \{A \in B_X | AI = IA = A\}$, G(I), the group of units of M(I), C(I), the cone generated by the columns of I, and End C(I) the semigroup of cone endomorphisms. Theorem: M(I) is isomorphic to End C(I). This characterization is then refined, and applied to obtain the Clifford, Plemmons, Schein result. Corollary: If I is a partial order on X, G(I) is isomorphic to the group of order automorphisms of (X,I).
(Received October 21, 1974.)

*720-20-8 KEVIN E. OSONDU, State University of New York at Buffalo, Amherst, New York 14226. Homogeneous quotients of an inverse semigroup.

Let S be an inverse semigroup. On $S \times S$, define the relation θ_R by the rule $(a,b)\ \theta_R\ (c,d)$ if there exists $x,y \in S$ such that $ax = cy$ and $bx = dy$. Theorem. θ_R is a left congruence on $S \times S$. The equivalence class of (a,b) is called the right homogeneous quotient containing (a,b). Theorem. The right [left] homogeneous quotients of an inverse semigroup S, together with a suitably defined binary operation, form a group which contains a subgroup isomorphic to the maximal group homomorphic image of S. This is the analogue for inverse semigroups of the theorem of Ore that a left [right] reversible, cancellative semigroup is embeddable in a group. (Received October 21, 1974.)

720-20-9 K. D. MAGILL, JR. and S. SUBBIAH, SUNY at Buffalo, Amherst, New York 14226. Regularity and Green's relations for sandwich semigroups, Preliminary report.

Let X and Y be nonempty sets, let α be a function with domain contained in Y and range contained in X and let $T(X,Y,\alpha)$ be any collection of functions with domains contained in X and ranges contained in Y subject to the condition that $f \circ \alpha \circ g$ belongs to $T(X,Y,\alpha)$ whenever both f and g belong to $T(X,Y,\alpha)$. Then $T(X,Y,\alpha)$ is a semigroup with the product fg of two functions f and g defined by $fg = f \circ \alpha \circ g$. This is what is meant by a sandwich semigroup with sandwich function α. The regular elements of sandwich semigroups are characterized and it is determined precisely when two such elements are $\mathcal{L},\mathcal{R},\mathcal{H},\mathcal{D}$ and $\mathcal{J}$-equivalent where the latter are Green's relations. The regular $\mathcal{D}$-classes are characterized and the maximal subgroups are determined. These results are then applied to such semigroups as sandwich semigroups of continuous functions from one topological space into another and sandwich semigroups of Boolean ring homomorphisms. (Received October 23, 1974.)

*720-20-10 DAVID E. ZITARELLI, Temple University, Philadelphia, Pennsylvania 19122. Compatible extensions of semigroups.

An ideal extension V of a semigroup S is called compatible if for every congruence σ on S, $a \sigma b$ implies $va \sigma vb$ and $av \sigma bv$ for all $v \in V$. Equivalent conditions for a congruence σ to be extended to V are given in terms of the identity extension of σ and the translational hull of S. It is shown that every extension of an inverse semi-

group is compatible. This notion is used to describe the least congruence on a subdirectly irreducible semigroup V, where V is a compatible extension of its kernel.

(Received October 29, 1974.)

*720-20-11 GEORGE AKST and THOMAS P. DENCE, New Mexico State University, Las Cruces, New Mexico 88003. Linear projection operators on standard threads.

All standard threads are constructed, rather basically, from three particular semigroups on $[0,1]$. Each is a topological semigroup under the ordinary topology. The space of all linear projections on each semigroup is characterized, and, using the structure theorem for standard threads, the general linear projection operator on a standard thread is exhibited. (Received October 30, 1974.)

720-20-12 J. C. Higgins, Brigham Young University, Provo, Utah 84602. Semilattices of Topological Semigroups. Preliminary report.

Let S be a commutative semigroup and let $S = \{S_\alpha\}$ be the archimedean components of the greatest semilattice decomposition of S. Further, let each S_α be a topological semigroup where each topology is obtained from a convergence class C_α relative to the same directed set D. We then obtain a proposition:

Theorem. The class $\cup C_\alpha$ is a convergence class for S. S endowed with this topology is a topological semigroup if and only if $\lim \mu \lim \nu = \lim \mu\nu$ for any $\mu, \nu \in \cup C_\alpha$.

If the S_α are N- semigroups (Cancellative and idempotent free) a standard topology based on the Tamura representation is given such that if multiplication in S is constant on the structure groups of the S_α then S is a topological semigroup.

(Received October 29, 1974.)

*720-20-13 LARRY E. KNOP, The University of Texas, Austin, Texas 78712. Aut(G) of a Semidirect Product of Cyclic p-Groups, Preliminary report.

Let p be an odd prime and let $G = \langle a\rangle\langle b\rangle$ where $\langle b\rangle$ G, $\langle a\rangle \cap \langle b\rangle = 1$, $|p| = p^s$, and $|a| = p^r$. Let p^u be the order of a as an automorphism of $\langle b\rangle$. Characterizing the automorphisms of G in terms of the generators a and b, it can be shown that the order of $\mathrm{Aut}(G)$ is $[p^{r-u}][\min(p^s,p^r)][\min(p^{s-u},p^r)][(p-1)p^{s-1}]$. Hence $|G| \mid |\mathrm{Aut}(G)|$, and $|G|$ equals the p-order of $\mathrm{Aut}(G)$ if and only if $r = u = s-1$ or $r = u = 1$.

(Received October 31, 1974.)

720-20-14 JOHN R. RASMUSSEN, Bowdoin College, Brunswick, Maine 04011. The generalized Artin exponent of a finite group. Preliminary report.

Let $P(G)$ and $Q(G)$ denote, respectively, the ring of all integer combinations of permutation

characters and rationally valued characters of a finite group G. We define the generalized Artin expon n(G) as $n(G) = |Q(G) : P(G)|$. Theorem. For every natural number n, there exists a finite group G su that $n(G) = n$. The theorem is proved by constructing a metacyclic group G depending on n, and then calculating n(G) by investigating the irreducible characters of G. (Received October 31, 1974.)

720-20-15 GARY L. PETERSON, Michigan State Univ., East Lansing, MI 48823. On the Automorphism Group of an Integral Group Ring. Preliminary report.

Let G be a finite group and let Z(G) denote the integral group ring of G. A ring automorphism of Z(G) is called a normalized automorphism if f(g) has augmentation 1 for all $g \in G$. The group of normalized automorphisms is denoted by NA(G). $f \in NA(G)$ is said to have an elementary representation if $f(x) = u\sigma(x)u^{-1}$ for all $x \in Z(G)$ where u is a unit in the group ring of G over the rationals and $\sigma \in Aut(G)$, Aut(G) the automorphism group of G. G is called an E.R. group if every $f \in NA(G)$ has an elementary representation. Several E.R. groups are obtained, some of which include: (1) groups G of the form $G = AB$ where A is a cyclic normal subgroup of G and B is an abelian subgroup of G. (2) groups in which $|G'| = p$. (3) groups G where $G/\mathcal{Z}(G)$ is metacyclic, $\mathcal{Z}(G)$ the center of G, (4) p-groups of the form $G = AB$ where A is an abelian normal subgroup with $A \supseteq \mathcal{Z}(G)$ and $A/\mathcal{Z}(G)$ elementary abelian of order p^2, and (5) all the symmetric groups. Previously known results on when Aut(G) equals NA(G) and when Aut(G) has a normal complement in NA(G) are extended. Finally, normalized automorphisms of direct products are studied. One of the results obtained is that if $G = G_1 \times \ldots \times G_n$ where $(|G_i|, |G_j|) = 1$ for $i \neq j$ and each G_i is an E.R. group, then G is an E.R. group. (Received November 1, 1974.)

*720-20-16 Jay Yellen, Colorado State University, Fort Collins, Colorado 80523
On Solvability of Certain Groups of Central Type, Preliminary report.

Let G be a finite group with center Z having an irreducible character χ such that $\chi(1)^2 = [G:Z]$, then G is called a group of central type. K. Iwahori and S. Matsumoto [J. Fac. Sci. Univ. Tokyo, 10 (1964)] noted that G is a group of central type if and only if there is a central simple projective group algebra associated with G. They conjectured that groups of central type are solvable.

Suppose G is a counterexample to the conjecture. We show that G has a homomorphic image of the form $H \wr T$, where T is solvable and H is an automorphism group of a simple group. If $32 \nmid |G/Z|$ then $|T| \leq 2$ and G is solvable. This is done by using the well-known classifications of some finite simple groups. This strengthens a result of F. R. DeMeyer and G. J. Janusz [Math. Zeit., 108 (1969), Theorem 7] and also supplements the following unpublished theorem of S. Gagola: Let G be a group of type f.r. Assume that G has an abelian p-Sylow subgroup for every prime p satisfying $p^6 / |G|$. Then G is solvable.

(Received November 1, 1974.)

*720-20-17 ALPHONSE H. BAARTMANS, Southern Illinois University, Carbondale, Illinois 62901 and JAMES J. WOEPPEL, Indiana University Southeast, New Albany, Indiana 47150. The automorphism group of a p-group of maximal class with an abelian maximal subgroup.

We give a detailed description of the automorphism group of a p-group of maximal class with exponent p having a maximal subgroup which is abelian. We first prove Theorem 2.1. If G is a p-group of maximal class with exponent p having a maximal subgroup (a subgroup of index p) which is abelian, then the p'-complement H of the normal Sylow p-subgroup P of the group $\mathcal{A}$ of automorphisms of G is isomorphic to $C_{(p-1)} \oplus C_{(p-1)}$. Using the methods of Blackburn ("On a special class of p-groups," Acta Math. 100(1958), 45-92), we study the normal Sylow p-subgroup P of $\mathcal{A}$, obtaining Theorem 3.1.

G is a p-group of maximal class with exponent p having a maximal subgroup which is abelian, then a normal Sylow p-subgroup P is metabelian of class $n-2$ of order $p^{(2n-3)}$. The commutator subgroup P' of P is the subgroup of inner automorphisms $\mathcal{J}_1$ induced on G by the maximal subgroup of G which is abelian. (Received October 30, 1974.)

0-20-18 WITHDRAWN

720-20-19 THOMAS KRAFCIK, Miami University, Oxford, Ohio 45056. The subgroup lattice for groups of order p^2q and p^3.

In a paper by Holmes (Sem. Mat. Univ. Padova 45(1971), 71-80), the idea of the projective representation for a lattice of subgroups was used. Here, we have extended this notion to groups of order p^2q and p^3 in the process of classifying groups of such orders. Niegel (J. Reine Angew. Math. 258 (1973), 1-22) classifies these same groups, but does so by using the standard Hasse diagram to graphically represent the lattice of subgroups. The projective representation gives an alternative visual representation of the lattice of subgroups which in some cases can lead to a much clearer understanding of the relationships among the subgroups. (Received November 4, 1974.) (Author introduced by Professor Charles Holmes.)

720-20-20 JOHN KARLOF, University of Nebraska at Omaha, Omaha, Nebraska 68101
The Subclass Algebra Associated with a Finite Group and Subgroup II

Let G be a finite group and let H be a subgroup of G. If $g \varepsilon G$, the subclass of G containing g is the set $E_g = \{hgh^{-1} \mid h \varepsilon H\}$ and the subclass sum containing g is $B_g = \sum_{x \varepsilon E_g} x$. The algebra over the complex numbers, K, generated by these subclass sums is called the subclass algebra (denoted by S) associated with G and H. Form the group $\bar{G} = H \times G$, subgroup $\bar{H} = \{(h,h) \mid h \varepsilon H\}$, and double cosets $\bar{H}(h,g)\bar{H}$ for $h \varepsilon H$, $g \varepsilon G$. Let $\overline{(h,g)} = \sum_{x \varepsilon \bar{H}(h,g)\bar{H}} x$ and let D be the subalgebra of $K(H \times G)$ generated by the sums $\{\overline{(h,g)}\}$. We show that S is isomorphic to D as K-algebras. Also, the irreducible modules and characters of S and D are demonstrated. (Received November 4, 1974.)

*720-20-21 ROBERT L. WILSON, JR., University of Wisconsin, Madison, Wisconsin 53706
Quasidirect Products of Quasigroups

A sub-quasigroup is normal if it is the preimage of a single element for some homomorphism. This specializes to the usual definitions of normality in loops and groups. A construction is given, analogous to the semidirect product for groups, which produces all quasigroups with a specified normal sub-quasigroup and quotient. (Received November 4, 1974.)

*720-20-22 MARSHALL SAADE, University of Georgia, Athens, Georgia 30602.
A note on some varieties of point algebras.

For definitions see these Notices 16 (1969), Abstract # 663-27. Theorem. Let n be an integer > 1 and (*) a generating operation. Then the class V (n,*) of all point algebras of the form $(S^n,*)$ is a variety. (Received November 4, 1974.)

*720-20-23 RONALD C. LINTON, University of South Alabama, Mobile, Alabama 36688. Fully invariant subgroups of primary abelian groups.

F is fully invariant in G if $\phi(F) \subseteq F$ for all $\phi \in \mathrm{End}(G)$. It is known that if F is fully invariant in the totally projective G, then F and G/F are also totally projective (see Abstract 709-A16, these *Notices* 20(1973), A-650). If λ is a limit ordinal cofinal with ω, then the reduced p-primary group G is a C_λ-group if $G/p^\alpha G$ is totally projective for all $\alpha < \lambda$. Theorem: Let F be fully invariant in the C_λ-group G of length λ. Then (1) G/F is also a C_λ-group if F is bounded, and (2) if F is unbounded, then F is a C_μ-group where μ denotes the length of F, and G/F is totally projective of length not exceeding the length of G. (Received November 4, 1974.) (Introduced by Professor Richard Vinson.)

720-20-24 DEAN C. MORROW. State University of New York, Oneonta, New York 13820. A characterization of SL(3, 8). Preliminary report.

Let G be a finite group. A maximal element of the set of normalizers of 2-subgroups of G is called a maximal 2-local subgroup of G. The special linear group $SL(3,8)$ has a maximal 2-local subgroup M consisting of matrices of the form

$$\begin{pmatrix} a & b & c \\ d & e & f \\ 0 & 0 & g \end{pmatrix}$$

with determinant 1. M is a split extension of an elementary abelian group of order 64 by $GL(2,8)$. Theorem. If G is a finite simple group with a maximal 2-local subgroup isomorphic to M, then G is isomorphic to $SL(3,8)$. Actually, this theorem is a corollary of a more general theorem classifying finite groups which have a maximal 2-local subgroup involving $SL(2,8)$ in a specified way. (Received November 4, 1974.)

720-20-25 DENNIS. F. KINCH, University of North Carolina, Chapel Hill, N.C. 27514, Partitions and Representations of the Hyperoctahedral Groups, Preliminary Report

To each partition π of an integer n we can associate an irreducible representation $\{\pi\}$ and a transitive permutation representation $\Delta(\pi)$ of the symmetric group $S(n)$. If we linearly order the partitions, then the transition matrix for these representations is lower triangular with 1's on the diagonal.

We call $(\pi; \lambda)$ a double partition of n if π is a partition of $p \le n$ (denoted $\pi \vdash p$) and $\lambda \vdash n - p$. Each irreducible representation $\{\pi; \lambda\}$ of the Hyperoctahedral group $B(n)$ corresponds uniquely to a double partition $(\pi; \lambda)$. Let $\Delta(\pi; \lambda)$ be the transitive permutation representation with stabiliser $S(\pi) \times B(\lambda) = S(\pi(1)) \times \ldots \times B(\lambda(1) \times \ldots$, then we prove the analogous result for the transition matrix. Further the inner product $\langle \Delta(\alpha; \beta), \{\pi; \lambda\}\rangle$ is a sum of $\langle \Delta(\rho), \{\pi\}\rangle \langle \Delta(\tau), \{\lambda\}\rangle$ for appropriate $\rho \vdash p$, $\tau \vdash n - p$. Using these results we then show that the representation ring of the Weyl group $D(n)$ has as an integral basis the permutation representations with stabiliser $[S(\pi) \times B(\lambda)] \cap D(n)$ for $(\pi; \lambda)$ a double partition of n with $\pi \vdash p \le n/2$, along with (if $n = 2m$) all $S(\rho) \times S(\rho)$ for $\rho \vdash m$.

Some of the $B(n)$ results were obtained by S. Mayer by different methods. (Received November 4, 1974.)

720-20-26 D. R. LATORRE, Clemson University, Clemson, S. C. 29631, A Construction of the idempotent-separating congruences on a bisimple orthodox semigroup.

In this article we show how to construct all idempotent-separating congruences on a bisimple orthodox semigroup, by making use of the construction theorem for all such semigroups due to Clifford [Semigroup Forum 5(1972), 127-136]. The process yields an immediate one-one correspondence between these congruences and certain anti-isomorphic pairs (V,V') of normal subgroups of some of the components in Clifford's construction. (Received November 4, 1974.)

*720-20-27 FRANCIS E. MASAT, Glassboro State College, Glassboro, New Jersey 08028. A generalization of right simple semigroups. Preliminary report.

An element s in a semigroup S will be called a right simple element if $sS = S$. Note that a semigroup is right simple (RS) iff each of its elements is a right simple element. Non-RS semigroups with right simple elements are denoted by RSE; the subset of S of right simple elements is denoted by R, and the nonright simple elements by N. If R is a RS subsemigroup (subgroup, right group) of S, then S is called a partial RS semigroup (partial group, partial right group) and denoted by PRS (PG, PRG). A PRS semigroup is by definition an RSE semigroup; the converse is shown to be false. The structure of RSE semigroups is determined, and a decomposition is developed for R, with the results extending those of Abstract 713-A1, these *Notices* 21(1973), A-386. The structure theorems are then applied to RSE semigroups where other conditions, such as the DCC on right ideals of N, finiteness, or left (right) cancellativity, are also present. Theorem. If S is a RSE semigroup and left simple (left cancellative), then S is a PG (PRG). For right cancellativity, the development follows that of the Baer-Levi theory. (Received November 4, 1974.)

*720-20-28 ROBIN KUEBLER and J. D. REID, Wesleyan University, Middletown, Connecticut 06457. On a paper of Richman and Walker.

In [Math. Z. 89 (1965), 77-81] F. Richman and E. A. Walker give a construction of a p-group F from its endomorphism ring E, as E-module, in case G is bounded or has unbounded basic subgroup. A different determination of the E-module G, for nonreduced G (thus covering the remaining case) is contained in Theorem. If $0 \neq D$ is the maximal divisible subgroup of G then the E-modules D and R in $0 \to D \to G \to R \to 0$ are determined by E. The group $\mathrm{Ext}_E^1(R, D)$ is cyclic p-adic module-free of rank 1 if R is unbounded, of order p^k if R has exponent k—with generator determined by the above sequence. Two E-exact sequences $0 \to D \to X_i \to R \to 0$ have isomorphic modules X_i iff the corresponding elements of $\mathrm{Ext}_E^1(R, D)$ generate the same p-adic submodules. (Received November 4, 1974.)

*720-20-29 LEO COMERFORD, Michigan State University, East Lansing, MI 48824 and BERNARD TRUFFAULT, Université de Nantes, Nantes, France. The Conjugacy Problem for Free Products of Sixth-Groups with Cyclic Amalgamation. Preliminary Report.

We call a group G a sixth-group if G has a finite presentation $\langle X; R \rangle$ where R is symmetrized, that is, each element of R is cyclically reduced and R is closed under the taking of inverses and cyclic permutations, and R has the property that if r_1 and r_2 are distinct elements of R with $r_1 = bc_1$ and $r_2 = bc_2$, then $|b| < \frac{1}{6}|r_1|$ and $|b| < \frac{1}{6}|r_2|$. In this paper the

authors show that the conjugacy problem for the free product of a collection o: sixth-groups with cyclic subgroups of the same order amalgamated is solvable. This generalizes a result of S. Lipschutz who showed (Proc. Amer. Math. Soc. 17 (1966), 759-762) that the free product of a collection of free groups with infinite cyclic subgroups amalgamated has solvable conjugacy problem.
(Received November 5, 1974.)

720-20-30 Malcolm Ottaway, SUNY Binghamton, Binghamton, New York 13901. Maximal subgroups which determine the properties of a finite group. Preliminary Report.

Let E be a class of finite groups. $M(E)$ is the class of those finite groups G such that if the prime p divides g, the order of G, then G has a subgroup H in the class E and the index of H in G is p.

Theorem. $E = M(E)$ if and only if E is the class of supersolvable groups.
(Received November 6, 1974.) (Introduced by Ben Brewster.)

720-20-31 MICHAEL A. KLEMBARA, University of Cincinnati, Cincinnati, Ohio45221 On the canonical sets of subgroups induced by a Fitting class F. Preliminary report.

An arbitrary Fitting class F, produces the following three canonical sets of subgroups in any given finite solvable group G : $I_F(G)$ the set of F-injectors of G; $II_F(G)$ the set of Fischer F-subgroups; and $III_F(G)$ the set of F-maximal subgroups of G which contain G_F the F-radical of G. It is clear from the definitions that $I_F(G) \subseteq II_F(G) \subseteq III_F(G)$. Given a Fitting class F, F. P. Lockett [see Math. Z. 137, (1974), 131-136] defined a new Fitting class $F^* = \{G \mid (G \times G)_F$ is subdirect in $G \times G\}$. The purpose of this note is to examine various conditions on F and F^* which given $I_F(G) = II_F(G) = III_F(G)$ implies $I_{F^*}(G) = II_{F^*}(G) = III_{F^*}(G)$ and conversely. (Received November 6, 1974.)

*720-20-32 DAVID M. ROCKE, Governors State University, Park Forest South, Illinois 60466. p-Groups in which centralizers of non-central elements have the same order: Part I. Preliminary report.

Let G be a finite group in which $[G:C(x)] = n$ if x is not in the center of G. We say then that G is of conjugacy type (n). In such a case, Ito has proved that $n = p^a$ and that G is the direct product of a p-group and an abelian group (Nagoya Math J., Vol. 6, 1953). Thus, consideration may be restricted to p-groups of type (p^a). In this paper, we derive certain facts about the structure of p-groups of types (p) and (p^2). Let G_i be the i'th term of the lower central series for G with $G_2 = G'$. Let Z_i be the i'th term of the upper central series for G with $Z_1 = Z(G)$. THEOREM G is of type (p) if and only if $|G_2| = p$. THEOREM Let G be of type (p^2) with class $G > 2$. Then: a) class $G = 3$, b) $G^p \leq Z_1$ and $G_2^p = 1$, c) $|G_3| = p^2$, d) $[G:Z_2] = p^2$, e) $[G_2:Z_1 \cap G_2] = p$ and f) Z_2 is abelian or Z_2 is of type (p^2) and class 2 with $|Z_2'| = p^2$. (Received November 6, 1974.)

*720-20-33 CHARLES FORD Washington University, St. Louis, Missouri 63130. Connection between the Schur Index of a representation and the structure of the group.

Theorem A Let G be a finite group with an irreducible complex character χ whose Schur index over an algebraic number field is m. Let p^n be the p-part of m for an odd prime divisor p of m. Then one of the following alternatives holds: (1) p^{2n} divides the exponent of G (2) p^n divides the exponent of G', the commutator subgroup of G.

This generalizes a recent result by B. Fein and T. Yamada. We have proved a general result which extends the Brauer-Witt theorem. It determines for each odd prime p a section of G whose structure explicitly determines p^n. Any faithful irreducible character of this section has Schur index p^n. One such character is related to χ. Theorem A is a consequence of this result. We state a special case of the general result.

Theorem With the hypotheses of Theorem A suppose G has Abelian p-Sylow subgroups. Then there is a prime q where p^n divides q-1 and a section M of G which is a semi-direct product M = Q P where Q has order q and is normalized by P a cyclic p-group. For some integer c and any generator x of P x^{q^c-1} centralizes Q and has order p^n. M has a faithful irreducible character with Schur index p^n which, when regarded as a character on a subgroup of G, induces a character of G which contains χ. (Received November 6, 1974.)

*720-20-34 Ben Brewster, State University of New York at Binghamton, Binghamton, N.Y. 13901 The formation generated by a Φ-free group.

An attempt to explicitly characterize the formation generated by a finite group can be reasonably successful if restrictions on Frattini chief factors are included. For instance, let G be a finite solvable group. Then 1.) If there are no Frattini chief factors below F(G), then $QR_0(G)$ can be explicitly described in terms of formations of smaller Fitting length, and 2.) If G has no Frattini chief factors, $QR_0(G)$ is completely determined by primitive epimorphic images of G. (Received November 6, 1974.)

*720-20-35 CHARLES S. HOLMES, Miami University, Oxford, Ohio 45056. Groups of order p^3q with identical subgroup structures.

It is shown that there are two nonisomorphic groups G and H both of order p^3q with identical subgroup structures; i.e., there is a lattice isomorphism from the lattice of subgroups of G onto the lattice of subgroups of H which preserves conjugacy and indices, and maps proper subgroups of G to isomorphic subgroups of H. Here p and q are odd prime numbers with p dividing q - 1. The smallest order of G and H in these examples is 189. The smallest order of such G and H in previously known examples is 260. In addition, for the groups G and H of order p^3q, various direct products $G \times K$ and $H \times K$ are shown to have isomorphic subgroup lattices. This is true if K is a group which is p,q-nilpotent in the sense that if $h \in K$ and the order of h is infinite or divisible by p and $k \in K$ and the order of k is infinite or divisible by q, then the subgroup generated by h and k is nilpotent. In particular K can be locally nilpotent, infinite dihedral, or a torsion group in which every element has order relatively prime to p and q. In this latter case $G \times K$ and $H \times K$ have the same situation of subgroups, i.e., $G \times K$ and $H \times K$ have isomorphic subgroup lattices in which the lattice correspondence preserves indices and conjugacy. (Received November 6, 1974.)

720-20-36 RAY SHEPHERD, Palm Beach Atlantic College, W. Palm Beach, Florida 33401. Construct of p-groups of maximal class with group order $p^n \leqq p^{2k+6}$.

Let P be a p-group of maximal class with lower central series $P, P_2, P_3, \ldots, P_n = 1$, where $|P| = p^n$, $n \geqq 4$. Set $P_1 = C(P_2/P_4)$, choose $s \notin P_1$, $s_1 \in P_1 - P_2$ and write $s_i = [s_{i-1}, s]$ for $i \geqq 2$. For P_1' abelian, consider P_1' as a $\mathbb{Z}[D]$-module where $Dx = [x, s]$ and $(\sum_i a_i D^i)\, x = \Pi_i [x^{a_i}, is]$. Theorem. Let $p \geqq 5$ be a prime and $k \geqq 1, \alpha, \beta$, and γ be given integers. For $a = (p-1)/2$ and $1 \leqq i \leqq a - 1$, let V_i be any word in the symbols s_j for $j \leqq k + 2i + 1$. Then for any $n \leqq 2k + 6$ there is a unique p-group of maximal class $\langle s, s_1 \rangle$ of order p^n and degree of commutativity at least k such that (1) $s^p = s_{n-1}^{\alpha}$, (2) $(ss_1)^p = s_{n-1}^{\beta}$, (3) $[s_i, s_{i+1}] = V_i$ for $1 \leqq i \leqq a - 1$ and (4) $[s_a, s_{a+1}] = \sum_{i=1}^{a-1} (-1)^{a+i+1} q_i(D) V_i + \gamma s_{n-1}$, where

$$q_i(D) = \sum_{r=0}^{a-i} \binom{r+i}{i} \binom{a+i+1}{r+2i+1} D^r + \sum_{r=0}^{a-i-1} \binom{a-r-1}{i} \binom{r+i}{r} (D+1)^{p-i-r-1}.$$

(Received November 7, 1974.)

22 Topological Groups, Lie Groups

*720-22-1 JOHN O. KILTINEN, Northern Michigan University, Marquette, Michigan 49855 On countably extending ring topologies.

Sufficient conditions are obtained for putting a natural ring topology on a union of a countable chain of topological rings, each of whose topologies extends those lower in the chain, in such a way that the topology on the union also extends each one in the chain. Three applications of this result are then given. First, it is shown to yield a new proof of a result of L. A. Hinrichs that locally bounded field topologies can always be extended to countably dimensioned algebraic overfields. Second, together with a recent theorem of K.-P. Podewski, it leads to a proof that a field topology can always be extended from a countable field to any purely transcendental extension field of countable transcendence degree. Third, it implies that a countable union of locally compact rings can always be ring-topologized, although not necessarily with a locally compact topology. Combining the first and second results, one can get the following THEOREM: A first countable, locally bounded field topology on a countable field can always be extended to any countable overfield.
(Received October 7, 1974.)

720-22-2 MOHSEN PAZIRANDEH, The University of Texas, Austin, Texas 78712, Radial component of invariant differential operators at Principal nilpotent points.

Let X_0 be a principal nilpotent element of a complex semi-simple Lie algebra $\mathfrak{g}$, $\mathfrak{L}$ a TDS containing X_0 with a basis $\{X_0, Y_0, H_0\}$ and U the centralizer of Y_0 in $\mathfrak{g}$ For any invariant differential operator D on $\mathfrak{g}$ with polynomial coefficients, the existence of radial components $\delta(D)$ and $\Delta(D)$ on a Cartan subslgebra $\mathfrak{h}$ and the hyperplane $X_0 + U$, respectively, has been shown by Harish-Chandra. $\delta(D)$ has been calculated explicitly when $D = \partial(p)$, where p is an invariant element of the symmetric

algebra of $\mathfrak{g}$. We will use this result and a result of Kostant to calculate $\Delta(\partial(p))$ on X_0+U. (Received October 10, 1974.) (Author introduced by Professor Roger Bleier.)

*720-22-3 PAUL S. MOSTERT, University of Kansas; Lawrence, Kansas 66045, On the Centralizer Conjecture for Monoids on Manifolds.

Let M be a topological monoid whose underlying space is a compact connected manifold with boundary, and let G be a connected closed subgroup of the group of units. Let Ψ denote the action of G on M under inner automorphisms, $\Omega'(\Psi)$ denote the system of non-zero weights of the action and $\Delta'(G)$ the system of non-zero roots of G. Suppose that for each idempotent e in the centralizer of G, Ψ is equivalent to a linear action at e.
Theorem: $\Delta'(G) \subseteq \Omega'(\Psi)$. Moreover, the centralizer $Z_M(G) = \{x \in M: gx = xg,\ g \in G\}$ is connected (modulo the minimal ideal) iff there is a connected submanifold T invariant under G which contains G, meet the minimal ideal, and for which $\Omega'(\Psi|T) = \Delta'(G)$.
As a corollary, it follows, for example, that if G is simple and $\dim M - \dim G < 2r$, $r =$ rank G, then the centralizer $Z_M(G)$ is connected (modulo) the minimal ideal of M). As another corollary, if G is simple and $Z_G(x) = \{g \in G: gx = xg\}$ has positive dimension, then $Z_M(G)$ is connected (modulo the minimal ideal). Proofs are based on the techniques of W.-C. and W.-Y. Hsiang. (Received October 25, 1974.)

*720-22-4 DAVID ZERLING, Philadelphia College of Textiles and Science, Philadelphia, Pennsylvania 19144 (CA) Topological Groups.

A locally compact topological group G is called (CA) if the group of inner automorphisms of G is closed in the group of all bicontinuous automorphisms of G. The author's previous results [Abstract 711-22-20, these Notices 21(1974) A-111] are generalized by showing that each non-(CA) locally compact connected group G can be written as a semi-direct product of a (CA) locally compact connected group by a vector group. This decomposition yeilds a natural dense imbedding of G into a (CA) locally compact connected group P, such that each bicontinuous automorphism of G can be extended to a bicontinuous automorphism of P. This imbedding and extension property enables us to derive a sufficient condition for the normal part of a semi-direct product decomposition of a (CA) locally compact connected group to be (CA). (Received October 25, 1974.)

720-22-5 LAWRENCE CORWIN, Yale University, New Haven, Connecticut 06520. Irreducibility of Induced Representations. Preliminary report.

Let G be a locally compact group, H a closed subgroup, π an irreducible unitary representation of H, and σ the induced representation on G. Mackey (Am. J. Math. 73 (1951) gave necessary and sufficient conditions for the irreducibility of G in the case where G is finite. We shall discuss various extensions of this result to infinite groups. The basic tool (for necessity) is a method of constructing intertwining operators of σ with itself. (Received October 29, 1974.)

720-22-6 L. H. CRABTREE, the Citadel, Charleston, S. C., 29409, and J.S. YANG, University of South Carolina, Columbia, S. C. 29208. A note on unitary duals of C(X,G). Preliminary report.

For a topological group H, let H' and H'' denote the first and second unitary dual spaces of H, respectively. If X is a topological space and G a topological group, let Γ be the topological group of all continuous functions from X into G endowed with the compact-open topology and the pointwise multiplication. It is shown that Γ can be continuously imbedded into Γ'' if X is hemicompact and G is maximally almost periodic metrizable group, and that Γ can be (topologically) imbedded into Γ'' as a retract subgroup if either (1) X is a first countable hemicompact space, and G is a direct product of a vector group and a compact metrizable group admitting faithful representation, or (2) X is a hemicompact k-space, and G is a compactly generated Lie group such that G/Z is compact. (Received October 31, 1974.)

*720-22-7 DONALD MARXEN, Marquette University, Milwaukee, Wisconsin 53233. Neighborhoods of the identity of the free abelian topological group.

The free abelian uniform group $(AG(X), \mathcal{U}_G)$ on a uniform space $[X, \mathcal{U}]$ is constructed as a quotient of the direct product of the free uniform semigroup (on $[X, \mathcal{U}]$) with itself. If X is a completely regular space and $\mathcal{U}$ is its largest admissible uniformity, the uniform topology $T(\mathcal{U}_G)$ is precisely the topology of the free abelian topological group. A base for the neighborhood systems relative to $T(\mathcal{U}_G)$ is described in terms of the gage of $\mathcal{U}$. It is shown that no sequence of words in $(AG(X), \mathcal{U}_G)$, whose lengths increase without bound, can have a limit. Corollaries of this result include some familiar theorems regarding the countability axioms and local compactness on free (abelian) topological groups. (Received November 1, 1974.)

*720-22-8 TER-JENQ HUANG, State University of New York, College at Cortland, Cortland, New York 13045. On Equicontinuous Transformation Group II.

In this note, we intend to study more applications of the main theorem (On Equicontinuous Transformation Group, Notices of AMS, vol. 19 (1972) p. A-102 (abstract)) and we prove the following theorem. Theorem. Let T be a topological group and S be a closed syndetic subgroup of T. Then T is maximally almost periodic if and only if S is maximally almost periodic. In the proof, we apply a recent result of W. H. Gottschalk (Some General Dynamical Notions, Recent Advances in Topological Dynamics, Lecture Notes in Mathematics, Springer-Verlag 1973, p. 102 - 125) and the main theorem cited. As consequences, we have Corollary 1. (Murakami) Let G be a topological group and let N be a normal subgroup of G with finite index. Then G is maximally almost periodic if (and only if) N is maximally almost periodic. Corollary 2. (Murakami, Kuranishi) Let G be a locally compact group and let G^o be the connected component of the identity of G such that G/G^o is compact. Then G is maximally almost periodic if and only if G^o is maximally almost periodic. (Received November 5, 1974.)

720-22-9 JOHN R. DURBIN, The University of Texas, Austin, Texas 78712. Spherical functions on compact wreath products. Preliminary Report.

Explicit formulas are derived, in terms of the characters of A, for the zonal spherical functions and associated spherical functions on a wreath product (monomial group) A wr S_n,

where A is any compact Abelian group and S_n is the symmetric group of degree n. This extends results on the zonal spherical functions on Z_k wr S_n (Z_k the cyclic group of order k), which are known to be the generalized Krawtchouk polynomials orthogonal on a symmetric multinomial distribution (D. Vere-Jones, Quart. J. Math. Oxford (2) 22 (1971) 247-270).
(Received November 4, 1974.)

*720-22-10 P. C. KUTZKO, University of Iowa, Iowa City, Iowa 52242. Representation theory for GL_2 over local rings.

A complete set of complex continuous irreducible representations of the two dimensional general linear group over the ring of integers in a p-field is constructed using finite group-theoretical techniques, notably those of Clifford and Mackey. The restrictions to SL_2 of these representations are then decomposed thus yielding a complete set of irreducible representations for this latter group. Since the methods used are independent of the residual characteristic, complete results are obtained, in particular, for residual characteristic 2 which, even for SL_2, have apparently not been previously obtained.
(Received November 4, 1974.)

720-22-11 SIGURDUR HELGASON, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Invariant differential equations on homogeneous manifolds.

• Motivated in part by the historical contact between Lie groups and differential equations, a number of general problems for group-invariant differential operators will be raised. Solvability questions for such operators will be discussed in some detail. (Received November 4, 1974.)

720-22-12 LEONARD SHAPIRO, University of Minnesota, Minneapolis, Minnesota 55455. Generalized Bohr Compactifications, Preliminary report.

In this joint work with Professor Shmuel Glasner, we extend the results of Ellis and Keynes [Israel J. of Math, 12 (1972), pp. 314-330] to apply to arbitrary topological groups T. In this way we obtain information about a compact group which generalizes the concept of a Bohr compactification of T. (In case T is nilpotent this group reduces to the ordinary Bohr compactification, but for noncompact semisimple Lie groups the group is nontrivial). For example, we prove that if T/Γ is compact, then for any neighborhood U of the identity in T there is a (generalized) Bohr neighborhood W of e such that $W \subseteq \Gamma U \Gamma V \Gamma$ for all neighborhoods V of e in T. These results promise to be of use in deciding for certain groups T whether the Furstenberg-Satake boundary is the same as the universal minimal proximal flow for T, and whether certain solvable Lie groups can admit nontrivial proximal minimal flows.(Received November 4, 1974.)

*720-22-13 GERALD L. ITZKOWITZ, Queens College, Flushing, New York 11367. Density character of locally compact groups. Preliminary report.

Let $d(X)$ denote the least cardinal of a dense subset of the topological space X. Theorem 1. Let G be a locally compact topological group and let H be a closed subgroup; then $d(H) \leq d(G)$. Theorem 2.

Let G be a σ-compact locally compact topological group and let H be a compact normal subgroup such that G/H is a separable metric group. Then d(G) = d(H). This theorem shows that the density character of all locally compact topological groups may be described in terms of the density character of an open σ-compact subgroup. This answers a question of S. Kakutani ["On cardinal numbers related with a compact abelian group", Proc. Japan Acad. 19(1943), 366-372]. (Received November 4, 1974.)

*720-22-14 NOLAN R. WALLACH. Rutgers University, New Brunswick, New Jersey 08903. A representation theoretic proof of a formula of Max Noether.

Let X be a symmetric space of noncompact type and let G be the identity component of the isometry group of X. Let $\Gamma \subset G$ be a discrete group acting freely on X. We will study techniques for computing the multiplicities of certain unitary representations of G in $L^2(\Gamma\backslash G)$ and relate these multiplicities to certain topological and analytic invariants of $\Gamma\backslash X$ using formulas of Matsushima and Matsushima and Murakami. As an application we prove the Max Noether formula for $\Gamma\backslash B^2$ where B^2 is the unit ball in C^2. (Received November 6, 1974.)

26 Real Functions

720-26-1 GOODMAN, A.W., UNIVERSITY OF SOUTH FLORIDA, DEPARTMENT OF MATHEMATICS, TAMPA, FLORIDA 33620.
Remarks on the Gauss-Lucas Theorem in Higher Dimensional Space

A recent paper by J.B. Diaz and Dorothy B. Shaffer extends the Gauss-Lucas Theorem (on the location of zeros of the derivative of a polynomial) to n-dimensional Euclidean space. The authors leave open certain natural questions concerning the existence of "zeros of the derivative". This paper answers three such questions and suggests several other questions for further investigation. (Received October 29, 1974.)

720-26-2 STEVE AGRONSKY, University of California, Santa Barbara. Some analogues of the Lusin-Menchoff Theorem and applications, Preliminary report.

If A and B are sets of real numbers, we let $A \mathrel{\underline{d}} B$ denote "every point of A is a point of Lebesque density of B." The Lusin-Menchoff Theorem states: If A is perfect, $A \subset E \mathrel{\underline{d}} E$, then there exists a perfect set A' such that $A \mathrel{\underline{d}} A' \subset E$. Analogues of this theorem are obtained by replacing "d" above by other suitable properties. Maximoff (Bull. Amer. Math. Soc. 1959 pg. 264-267) has used the Lusin-Menchoff Theorem to give a rather complicated characterization of the class of approximately continuous functions. The question considered is whether other classes of functions have similar characterizations. A general theorem is given which states (roughly) that a class of functions has a Maximoff-type characterization if certain rather weak conditions are met, and most importantly, if the appropriate analogue of the Lusin-Menchoff Theorem is valid. Several analogues of the Lusin Theorem are proven, each having as a corollary that a certain class of functions has a Maximoff-type characterization. Among the classes having such characterizations are Zahorski's classes $\mathfrak{M}_i$, $i = 0,1,\ldots,5$ (Sur la première derivée, Trans. Amer. Math. Soc. 69 (1950), pg. 1-54.) (Received November 6, 1974.) (Author introduced by Professor A. M.Bruckner.)

*720-26-3. L. V. TORALBALLA and LOUIS I. ALPERT, Bronx Community College, Bronx, New York 10453. On the existence of the differential of a function of several real variables.

The concept of deviation as introduced by Toralballa in his work on surface area has been suitably modified here to define $D(f:x_0)$, the deviation of f at x_0 where f is a general real valued function defined on a connected open subset of the real line. $D(f:x_0)$ is, in a sense, a measure of the nondifferentiability of f at x_0. We further define $\overline{f'}(x_0)$, the derivative mean of f at x_0, a generalization of $f'(x_0)$. Differentiability is related to the definitions cited above as follows: (1) If f is differentiable at x_0 then $f'(x_0) = \overline{f'}(x_0)$. (2) f is differentiable at x_0 iff $D(f:x_0) = 0$. Consider now functions $f(x,y)$ of two real variables x and y. We define the partial deviation functions $D_x(f)$ and $D_y(f)$ and the partial derivative means $\overline{f}_x$ and $\overline{f}_y$ as functions of x and y in the natural manner. We state a set of conditions for f to have a differential at (x_0,y_0) which we believe is milder than those currently in the literature. Principal theorem. Let $f(x,y)$ be defined on $N((x_0,y_0),\delta)$. If, as functions of x and y (1) f is continuous at (x_0,y_0), (2) f_x and f_y exist at (x_0,y_0), (3) $D_x(f)$ is continuous at (x_0,y_0), (4) $\overline{f}_x$ is continuous at (x_0,y_0), then f has a differential at (x_0,y_0). The above concepts and theorem may be extended to functions of more than two real variables. (Received November 4, 1974.)

28 Measure and Integration

720-28-1 MONBILL TONG, California State University, Sacramento, California 95819. Trjitzinsky Theorem in the product space of two abstract measure spaces.

The late professor Trjitzinsky generalized a version of the density theorem in sense of Denjoy in a general abstract measure space as follows: Definition. Given the sets $\Delta(F)$ of the completely regular family F, and the set X in $\Delta(F)$; $\Delta(F)$ and X are positive measurable-Q. If $0 < \alpha < 1$, and $0 < \delta < 1$, $\mathcal{T}_{\alpha,\delta}(X) = \sum E$, such that $E \in F$, $\frac{Q(XE)}{Q(E)} > \alpha$, $Q(E) < \delta$. Let $S(F)$ be the union of all sets in F. Theorem. The density theorem, relative to a completely regular family $F = \{s\}$ with $S(F)$ of finite measure-Q, takes place if and only if for all $0 < \alpha < 1$, for every sequence $0 < \delta_m < 1$, $\delta_m \to 0$ as $m \to \infty$, and for every sequence of sets X_n, which is in $\Delta(F)$ and is measurable-Q, such that $X_1 \supset X_2 \supset X_3 \supset \cdots$, $Q(\prod_n (X_n)) = 0$, we have $\lim_{m \to \infty} Q(\mathcal{T}_{\alpha,\delta_m}(X_m)) = 0$. (W.J. Trjitzinsky, Théorie métrique dans les Espaces ou il y a une mesure, Gauthier-Villars, Paris, 1960) We extend the various forms of the theorem to the product space of two abstract measure spaces. An example to illustrate the concepts is also given. (Received October 21, 1974.) (Author introduced by Professor Hugo Sun.)

720-28-2 PARFENY SAWOROTNOW, Catholic University, Washington, D.C. 20064. Riesz theorem for H*-algebra valued positive linear mappings. Preliminary report.

Let A be a proper H*-algebra and $\tau(A)$ its trace-class [Proc. Amer. Math. Soc. 26(1970), 95-100]; let L be a family of real-valued functions defined on a space S. Assume that L is closed under lattice and vector space operations and satisfies Stone's condition: if $f \in L$ then $f \cap 1 \in L$. Let I be a positive linear mapping of L into $\tau(A)$ such that $f_n \downarrow 0$ implies $(I(f_n)a,a) \downarrow 0$ for all $a \in A$ (f_n is a sequence of members of L). Then there exists a $\tau(A)$-valued measure μ on S, defined on a ring of subsets of S, such that $I(f) = \int f d\mu$ for all $f \in L$. (Received October 30, 1974.)

*720-28-3 G. Y. H. CHI, University of Florida, Gainesville, Florida 32611 and University of Pittsbur Pittsburgh, Pennsylvania 15260. The Radon-Nikodym theorem in locally convex spaces. Preliminary report.

Let (Ω,Σ,μ) be a complete probability space, where Σ is a σ-algebra. Let $L_0^1(\Omega,\Sigma,\mu;F)$ be the locally convex space of all strongly integrable Borel measurable functions with the Egoroff property. Let F be a quasi-complete locally convex space with the following property: for every bounded subset $B \subset \ell_N^1\{F\}$, the space of absolutely summable sequences, there exists an increasing sequence of absolutely convex compact metrizable sets, $M_n \subset F$, such that $M = \bigcup_{n=1}^{\infty} M_n$ is bounded and $\sum_{i=1}^{\infty} p_M(x_i) \leq 1$, $\forall (x_i)_{i=1}^{\infty} \in B$. Let $m:\Sigma \to F$ be a vector measure. Theorem. There exists an $f \in L_0^1(\Omega,\Sigma,\mu;F)$ such that $m = \mu_f$ iff (i) $m \ll \mu$, (ii) m has bounded variation, and (iii) m has locally relatively compact average range. This theorem extends the results presented in Abstract 712-B23, these *Notices* 21(1974), A-342. However, the result here covers in addition the precompact duals of separable β^*-barreled l.c.s.' with property (B) of Pietsch, and hence, in particular, the precompact duals of separable metrizable spaces. (Received November 6, 1974.)

*720-28-4 ROY A. JOHNSON, Washington State University, Pullman, Washington 99163. A decomposition of measures. Preliminary report.

Suppose μ and ν are measures on the same sigma-ring. We say that μ is quasi-dominant over ν if given measurable $E, \exists F$ s.t. $\mu(E) = \mu(E \cap F)$ and s.t. $\nu(E \cap F \cap G) = 0$ whenever $\mu(E \cap F \cap G) = 0$. For example, any measure is quasi-dominant over a sigma-finite measure. We say that μ is strongly dominated by ν if the only measure less than or equal to μ and quasi-dominant over ν is the zero measure. Theorem. If μ and ν are measures on the same sigma-ring, then $\exists$ measures π and ρ s.t. (1) μ is the sum of π and ρ, (2) π is strongly dominated by ν, and (3) ρ is quasi-dominant over ν. An application is given to decompositions of product measures. (Received November 4, 1974.)

*720-28-5 WILLIAM GRAVES, University of North Carolina, Chapel Hill, North Carolina 27514 An Example of a Universal Measure. Preliminary Report.

Let R be a ring of subsets of a set X, $M(R)$ the corresponding space of real-valued simple functions on R, and $\chi : R \to M(R)$ the natural map which associates to each $A \in R$ its characteristic function χ_A. Let LCS be the category of locally convex Hausdorff topological vector spaces. For any $W \in$ LCS, a finitely additive function $\mu : R \to W$ is a measure if $\{\mu(A_i)\}$ is Cauchy for every sequence $A_1 \subseteq A_2 \subseteq \ldots$ from R and converges to $\mu(A)$ when, in addition, $A = \cup A_i \in R$. Let τ be the coarsest locally convex topology on $M(R)$ such that $f \mapsto \int f d\mu$ is continuous for every $W \in$ LCS and every measure $\mu : R \to W$.

Theorem. $(M(R),\tau) \in$ LCS, and $\chi : R \to (M(R),\tau)$ is a universal measure (i.e. for any measure $\mu : R \to W$, $\mu = \tilde{\mu} \circ \chi$ where $\tilde{\mu}(f) = \int f d\mu$ is linear continuous).

Example. Let $R = P(\mathbb{N})$ = the power set of the positive integers. Functions in $M(P(\mathbb{N}))$ may be identified with sequences (α_i) which assume only finitely many values. For any positive sequence (η_i) such that $\eta_i \to \infty$, let $U(\eta_i) = \{(\alpha_i) \in M(P(\mathbb{N})) : |\alpha_i| < \eta_i \ \forall i\}$. The $U(\eta_i)$'s are a fundamental system of neighborhoods of 0 for the topology τ on $M(P(\mathbb{N}))$. That τ is a Mackey topology is equivalent to the Orlicz-Pettis Theorem on subseries convergence.

Many other aspects of the theory of vector-valued measures have interesting formulations in terms of $(M(R),\tau)$ and the universal measure χ. (Received November 6, 1974.)

*720-28-6 W.M. Bogdanowicz & J.P. McCloskey, Catholic University of America, Washington, D.C., 20064. A relation of measurability with respect to a sigma-ring V to measurability with respect to the smallest sigma-algebra extending V.

Let V be a sigma-ring of subsets of a space X such that $X \notin V$. Let W be the smallest sigma-algebra extending V. Denote by M(V,R) the space of all V-measurable functions $f: X \to R$, where $R=(-\infty,\infty)$. Let B denote the collection of all sets being complements of sets in V. Notice that B is filter base.

Theorem. A function $f \in M(W,R)$ belongs to M(V,R) if and only if the limit of f with respect to the filter base is zero. (Received November 6, 1974.)

*720-28-7 JON C. HELTON, University of Arizona, Tempe, Arizona 85281. Mutual existence of product integrals in normed rings.

Let R be the reals and N a normed ring. If G is a function from $R \times R$ to N, then $G \in OM^*$ on [a,b] only if (1) ${}_x\Pi^y(1+G)$ exists for $a \leqq x < y \leqq b$ and (2) if $\epsilon > 0$, then $\exists$ a subdivision D of [a,b] $\ni$, if $\{x_i\}_{i=0}^n$ is a refinement of D and $0 \leqq p < q \leqq n$, then $|{}_{x_p}\Pi^{x_q}(1+G) - \Pi_{i=p+1}^q(1+G_i)| < \epsilon$; and $G \in OM^o$ on [a,b] only if (i) ${}_x\Pi^y(1+G)$ exists for $a \leqq x < y \leqq b$ and (ii) $\int_a^b |1+G-\Pi(1+G)|$ exists and is zero. Further, $G \in OP^o$ on [a,b] only if $\exists$ a subdivision D of [a,b] and a number B $\ni$, if $\{x_i\}_{i=0}^n$ is a refinement of D and $0 < p \leqq q \leqq n$, then $|\Pi_{i=p}^q(1+G_i)| < B$. If F and G are functions from $R \times R$ to N, $F \in OP^o$ on [a,b], each of $\lim_{x,y\to p^+} F(x,y)$ and $\lim_{x,y\to p^-} F(x,y)$ exists and is zero for $p \in [a,b]$, each of $\lim_{x\to p^+} F(p,x)$, $\lim_{x\to p^-} F(x,p)$, $\lim_{x\to p^+} G(p,x)$ and $\lim_{x\to p^-} G(x,p)$ exists for $p \in [a,b]$, and G has bounded variation on [a,b], then any two of the following statements imply the other (on [a,b]): (1) $F + G \in OM^*$, (2) $F \in OM^*$, and (3) $G \in OM^*$. In addition, with the same restrictions on F and G, any two of the following statements imply the other (on [a,b]): (1) $F + G \in OM^o$, (2) $F \in OM^o$, and (3) $G \in OM^o$. (Received November 6, 1974.)

*720-28-8 PAUL LEWIS, North Texas State University, Denton, Texas 76203. Absolutely summing and mapping properties of representing measures.

Let H be a compact Hausdorff space, let each of E and F be a Banach space, and let C(H,E) be the space of all continuous E-valued functions on H re sup norm. Proposition 1. (a) If (x_i) is weakly p-summable in E and (y_i) in F_1 (= closed unit ball of F), then $\sum x_i \otimes y_i$ is weakly p-summable in $E \check{\otimes} F$. (b) If $L: C(H,E) \to F$ is absolutely p-summing (unconditionally convergent), then $L_1: C(H) \to B(E,F)$ is absolutely p-summing (unconditionally convergent), where L_1 is the naturally induced operator. Proposition 2. Suppose that $L: C(H,E) \to F$ is an operator with representing measure $m: \Sigma \to B(E,F)$. If L is absolutely p-summing (unconditionally converging) (completely continuous) (nuclear) (quasinuclear) (strictly singular), then m(A) is absolutely p-summing (unconditionally converging) (completely continuous) (nuclear) (quasinuclear) (strictly singular) for each $A \in \Sigma$. Proposition 3. If $m \leftrightarrow L: C(H,E) \to F$ is absolutely p-summing, then $\pi_p^p(m) < \infty$. Conversely, if $\pi_p(m) < \infty$, then L is absolutely p-summing. Here $\pi_p(m)$ denotes the p-summing total variation of m. (Received November 4, 1974.)

*720-30-1 F.R. KEOGH, University of Kentucky, Lexington, Kentucky 40506
A characterisation of convex domains in the plane

Let $f(z) = \sum_0^\infty a_n z^n$ be analytic in the open unit disc Δ, and let $s_n^f(z) = \sum_0^n a_k z^k$, $\sigma_n^f(z) = \sum_0^n s_k(z)/(n+1)$. T. Basgöze, J.L. Frank and F.R. Keogh [Can. J. Math., Vol. XXII, No. 1(1970), 123-127] showed that if $f(\Delta) \subset D$, where D is a convex domain, then $\sigma_n^f(\Delta) \subset D$ for all n and the values assumed by $s_n^f(z)$ for $|z| < \frac{1}{2}$ lie in D for all n. Theorem. If D is a nonempty non-convex domain, then there exists a function f analytic in Δ such that $f(\Delta) \subset D$ and $\sigma_1^f(\Delta) - D \neq \phi$ (or, equivalently, $s_1^f(\frac{1}{2}z)$ assumes values outside D). (Received October 1, 1974.)

*720-30-2 HERB W. Silverman, University of Delaware, Newark Delaware 19711 and EVELYN M. SILVIA, University of California, Davis, California 95616. On Linear Combinations of Convex Functions of Order β. Preliminary report.

Given $f(z) = z + \sum_2^\infty a_n z^n$ analytic and univalent in V ($|z| < 1$), α ($0 \leq \alpha < 1$), $f(z)$ is said to be starlike of order α denoted $f \in S(\alpha)$ if $\mathrm{Re}\,\frac{zf'}{f} > \alpha$. We denote $f \in K(\alpha)$ if $\mathrm{Re}\,\{1 + \frac{zf''}{f'}\} > \alpha$ and we say that $f(z)$ is convex of order α. Let $f,g \in K(\alpha)$ and $F = \lambda f + (1 - \lambda)g$ where $z \in V$ and $0 \leq \arg \frac{\lambda}{1-\lambda} < \pi$. The authors obtain bounds for the radii of starlikeness and convexity of F. This generalizes the work of R. K. Stump [Can. J. Math. (23) 4(1971), 712-717]. (Received September 27, 1974.)

*720-30-3 DALE H. MUGLER, Syracuse University, Syracuse, New York 13210. Completely Convex and Positive Harmonic Functions.

A completely convex function is a positive real-valued function on a real interval whose even derivatives alternate in sign. In this report, I show that every completely convex function is the restriction to the real line of a positive harmonic function in a vertical strip which is completely convex in x for each y. I present an integral representation for those extensions whose first order partial derivatives with respect to y vanish for y = 0, and discuss its relation to the integral representation of a completely convex function found by R. P. Boas, Jr. (Amer. J. Math., 81(1959),pp. 709-714). Finally, conformal transformations are used to derive analogous differentiability conditions providing similar extensions for functions defined on other curves in the plane. (Received October 17, 1974.)

*720-30-4 TED J. SUFFRIDGE, Department of Mathematics, University of Ky., Lexington, Kentucky 40506 Starlike functions as limits of polynomials

Let $C_k(\alpha)$, $k = 1,2,\cdots$, be defined by the equation $z/(1-z)^{2(1-\alpha)} = \sum_{k=1}^\infty C_k(\alpha) z^k$ where $\alpha \leq 1$. Assume $f(z) = \sum_{k=1}^\infty C_k(\alpha) A_k z^k$ and $g(z) = \sum_{k=1}^\infty C_k(\alpha) B_k z^k$ are starlike of order α for some fixed $\alpha \leq 1$. Then $f*g(z) = \sum_{k=1}^\infty C_k(\alpha) A_k B_k z^k$ is starlike of order α. Further, if f and g are analytic and satisfy $\mathrm{Re}(zf'(z)/f(z)) = \alpha$ and $\mathrm{Re}(zg'(z)/g(z)) = \alpha$ on open arcs of length γ_f and γ_g respectively on the unit circle $|z| = 1$ and $\gamma_f + \gamma_g > 2\pi$ then

$\mathrm{Re}(zf*g'(z)/(f*g(z))) = \alpha$ on an open arc of length $\gamma_f+\gamma_g - 2\pi$ on $|z| = 1$. Also, if $1 \geq \beta > \alpha$ then $\Sigma_{k=1}^{\infty} C_k(\beta)A_k z^k$ is starlike of order β.

The proof consists of characterizing the class of function which are starlike of order $\alpha \leq 1$ as being limits of polynomials of a certain type. Then theorems which can be proved about the classes of polynomials translate into theorems about functions starlike of order α. (Received October 21, 1974.)

*720-30-5 Herb Silverman, University of Delaware, Newark, Delaware 19711. Univalent Functions with Negative Coefficients.

Let $T^*(\alpha)$ and $C(\alpha)$ denote, respectively, the subclasses of univalent functions that are starlike of order α and convex of order α for which the coefficients are all negative. We obtain coefficient bounds, distortion, and covering theorems for these classes. We also find the order of starlikeness of $C(\alpha)$ and the radius of convexity of $T^*(\alpha)$. Finally, we determine the closed convex hulls and the set of extreme points for these classes. All results are sharp. (Received October 21, 1974.)

720-30-6 JEFFREY C. WIENER, Georgia Institute of Technology, Atlanta, Georgia 30332. Quadratic Differentials, Preliminary Report.

If R $\hat{R}$ are Riemann surfaces, we will say that z_0 is an isolated point boundary component of R if there exists a neighborhood U of z_0 in $\hat{R}$ with $U - \{z_0\}$ R. We shall prove that the quadratic differential $Q(z)\,d\,z^2$ obtained from solving the module problem $p(a_1,a_2,\cdots,a_k)$ for a free family of k homotopy classes on R (see On Analytic Self-Mappings of Riemann Surfaces II by J. A. Jenkins and N. Suita) can be extended to $z_0 \in \hat{R}$ so that either $Q(z)$ is regular at z_0 or $Q(z)$ has a simple pole at z_0. (Received October 21, 1974.)

*720-30-7 MAXWELL O. READE, University of Michigan, Ann Arbor, Michigan 48104 and THOMAS ARMSTRONG, Princeton, New Jersey. On functions of bounded boundary rotation. Preliminary report.

Let $P(M)$ denote the set of functions $f(z) = z + \cdots$ that are analytic in the unit disc, with $f'(z) \neq 0$ and boundary rotation at most $M\pi$ there. The authors find the (sharp) radius $R(M,N)$ such that each $f \in P(M)$ has boundary rotation at most $N\pi$ in the disc $|z| < R(M,N)$, where $2 \leq N \leq M$. The authors' proof depends upon solving a suitable extremal problem in a suitable Banach space. (Received October 24, 1974.)

720-30-8 ROGER W. BARNARD, Texas Tech University, Lubbock, Texas 79409. On the Radius of Starlikeness of $(zf)'$ for f Univalent. Preliminary report.

Let S be the standard class of normalized univalent functions. For a given function f, $f(z) = z + a_2z^2+\ldots$, regular for $|z| < 1$, let $r(f)$ be the radius of starlikeness of f. In 1947 R. M. Robinson considered the combination

$g_f(z) = (zf)'/2$ for $f \in S$. He found a lower bound of .38 for $r(g_f)$ for all $f \in S$. He noted that the standard Koebe function k, $k(z) = z(1-z)^{-2}$ has its $r(g_k)$ equal to 1/2. A consequence of A. Livingston's result in 1966 is that minimum $r(g_f)$ for all close to convex functions is 1/2. A question that has been asked since Robinson's paper is whether 1/2 is the minimum $r(g_f)$ for all f in S. It is shown that this is not the case by giving examples of functions f whose $r(g_f)$ is less than 1/2. (Received October 25, 1974.)

720-30-9 MORRIS MARDEN, University of Wisconsin - Milwaukee, Milwaukee, Wisc., 53201 and PETER A. MCCOY, U.S. Naval Academy, Annapolis, Md., 21403
Level Sets of Polynomials in n Real Variables

The methods used in studying the zeros of a polynomial in a single complex variable are here adapted to investigating the level surfaces of a real polynomial in E^n, with respect to their intersection and finite or asymptotic tangency with certain cones. Special attention is given to the equipotential surfaces generated by an axisymmetric harmonic polynomial in E^3. For example, if (x,ρ,ϕ) are the cylindrical coordinates in E^3, then an equipotential surface generated from a real harmonic polynomial of degree m is unbounded if and only if it is asymptotic to a cone $\theta = \theta_j$ where $P_m(\cos\theta_j) = 0$, $1 \le j \le m$. The $P_m(u)$ is the Legendre polynomial of degree m and $\cos\theta = x/\sqrt{x^2 + \rho^2}$. (Received October 30, 1974.)

*720-30-10 JOSEPH CIMA, University of North Carolina, Chapel Hill, North Carolina 27514
On the log of a schlicht function. Preliminary Report.

We give an elementary proof of the fact that if f is a normalized schlicht function in the unit disk then $\log\left(\frac{f(z)}{z}\right)$ is in $\bigcap_{p>1} H^p$. Further, it is shown that for each such f the function $|\log|f(e^{it})||$ has its averages over intervals of length h bounded by a constant multiple of $h|\log h|$. (Received October 30, 1974.)

720-30-11 DOUGLAS M. CAMPBELL, Brigham Young University, Provo, Utah 84602
The limiting behavior of $zf''(z)/f'(z)$ and two conjectures on univalent functions. Preliminary Report.

There is a univalent analytic function in $|z| < 1$ for which $\limsup (1 - |z|^2)\,|zf''(z)/f'(z)| = 0$ but such that 1) f(z) cannot be quasiconformally extended to the entire complex plane and 2) $f(e^{-t}z) + (e^t - e^{-t})\,zf'(e^{-t}z)$ is not univalent for small t. This settles in the negative two questions of J. Becker appearing in Ch. Pommerenke's Problems in Complex Function Theory, Bull. London Math Soc. 1972, 354-364. (Received October 30, 1974.)

*720-30-12 F. W. CARROLL, DAN EUSTICE AND T. FIGIEL, The Ohio State University, Columbus, Ohio 43210. The Minimum Modulus of Polynomials with Restricted Coefficients.

For $n = 1,2,\ldots$, let $\mathfrak{F}_n$ denote the class of polynomials with ± 1 as coefficients. For each P in $\mathfrak{F}_n$, let $m(n,P)$ denote its minimum modulus on the circle $T = \{|z|=1\}$, and let $m(n)$ be the largest of the $m(n,P)$, as P ranges over $\mathfrak{F}_n$. Let $L = \sup\{\log m(n)/\log(n+1): n = 1,2,\ldots\}$. J. E. Littlewood [JLMS 16(1966), 367-376], has found the extremal polynomials for $n \leq 20$, and has shown in particular that $\log m(12)/\log 13 = .431$. Hence, we know that $L \geq .431$, while elementary considerations show that $L \leq .5$. Our most striking result is:

Theorem 2. The sequence $\{\log m(n)/\log(n+1)\}$ converges to L.

To prove this result we first show that $m(n) \to \infty$ with n. Specifically, we prove that $m(n) \geq C(n+1)^{.431}$ for a positive constant C by using the polynomials found by Littlewood to generate polynomials in $\mathfrak{F}_n$ for $n = 13^k - 1$. These polynomials have this order of growth. An interpolation technique enables us to complete the argument. At a crucial step in the proof we need that $m(n) \geq 2$ for $13 \leq n \leq 168$. For certain values of n in this range we find it necessary to use a high speed computer to verify that polynomials in $\mathfrak{F}_n$ exist with minimum modulus greater than 2 on T. (Received October 31, 1974.)

720-30-13 J.W. NOONAN, Holy Cross College, Worcester, Massachusetts 01610. Powers of p-valent Functions, Preliminary report.

For $p > 0$, denote by $S(p)$ the class of functions regular and areally mean p-valent in the unit disc γ, and denote by $C(p)$ the class of functions regular and circumferentially areally mean p-valent in γ. It is known that if $f \in S(p)$ and $0 < \lambda \leq 1$, then $f^\lambda \in S(p\lambda)$, and that if $f \in C(p)$, then $f^\lambda \in C(p\lambda)$ for all $\lambda > 0$. Theorem 1. If $f \in S(p)$ and $\lambda > 1$, then $f^\lambda \in S(p\lambda^2)$. The valence $p\lambda^2$ is best possible. Theorem 2. Let $p > 0$. Then $f^\lambda \in S(p\lambda)$ for all $\lambda > 0$ if and only if $f \in C(p)$.

(Received October 31 1974.)

720-30-14 JOSEPH WARREN. Fordham University, New York, New York 10023. Rate of growth of spiral functions.

Let D be the unit disk and $S = \{r(\theta)\exp(i\theta) \mid r(\theta)$ positive, continuous and monotonically increasing to 1 as θ tends to $\infty\}$ a spiral in D. Let $w = f(z)$ be holomorphic in D with asymptotic value zero on S. Define $K(\theta) = \operatorname{Sup}|f(r\exp(i\theta))|$, $r(\theta - 2\pi) \leq r \leq r(\theta)$, and $S(\theta) = \operatorname{Sup}|f(r(t)\exp(it))|$, $t \geq \theta - 2\pi$. It is known that $K(\theta)$ has limit infinity as θ tends to infinity. Theorem. Given $\epsilon > 0$ and $0 < \delta < 1$, there is a θ_0 such that for $\theta > \theta_0$, $S(\theta - \epsilon) = K(\theta + \epsilon)^{-\delta}$. The spiral S may be replaced by a general spiral in D and the δ may be replaced by a function of θ which tends to zero at a rate which depends on the shape of this spiral. Moreover, in the stated theorem, $K(\theta)$ may be replaced by the maximum modulus $M(r(\theta),f)$. The proof uses Ahlfor's distortion theorem and principles of harmonic measure. (Received November 1, 1974.)

*720-30-15 STEVEN JACOBSON, Yale University, New Haven, Connecticut 06520. Analytic capacity of open sets.

It has been conjectured that the set function, analytic capacity, is subadditive for disjoint compact sets in the complex plane. Using results of A. Davie, this can be shown to be equivalent to the following question. Given a bounded open set V and a compact set E of zero analytic capacity, lying in the boundary of V, is it always true that V and $E \cup V$ have the same analytic capacity? One can show that this is so in many cases, and, in fact, that all of the boundary can be added on in these cases without changing the value of the analytic capacity. Another class of open sets is described next; the sets of this class are seen to be the crucial ones for comparing the capacities of V and $E \cup V$; and it is shown that for members of this class a certain portion of the boundary can always be added on. The technique of proof is A.G. Vitushkin's approximation procedure, as modified by T. Gamelin and J. Garnett. (Received October 30, 1974.)

720-30-16 SANFORD S. MILLER, State University of New York, Brockport, New York 14420. A Class of Differential Inequalities Implying Boundedness. Preliminary report.

Let $w(z)$ be regular in the unit disc with $w(0)=0$, and let $h(u,v)$ be a continuous function defined in a domain of $\mathbb{C} \times \mathbb{C}$. With some simple restrictions on $h(u,v)$ the author proves that $|h(w(z),zw'(z))| < 1$ implies $|w(z)| < 1$. Generalizations of this result for other domains are considered. (Received November 1, 1974.)

720-30-17 K.F. BARTH, Syracuse Univ., Syracuse, NY 13210, D.D. BONAR, Denison Univ., Granville, Ohio 43023, and F.W. CARROLL, Ohio State Univ., Columbus, Ohio 43210, Zeros of Strongly Annular Functions.

In his monograph, On Annular Functions (VEB Deut. Verlag der Wissen., Berlin 1971), the second author raised the question: "Can the set of limit points of the zeros of a strongly annular function consist of exactly one point?" This was shown to be the case by Barth and Schneider (J. reine angew. Math. 234 (1969), 179-183). Using an extension of the techniques in the above paper, the authors answer the question affirmatively. (Received November 1, 1974.)

*720-30-18 HERBERT H. SNYDER, Southern Illinois University, Carbondale, IL 62901. Holomorphic functions on cyclic group algebras.

Let $C_n = \{1, j, j^2, \ldots, j^{n-1}\}$ $(j^n = 1 = j^0)$ be the cyclic group of order n, and let $\mathfrak{A} = \{Z: Z = x_1 + jx_2 + \ldots + j^{n-1}x_n,\ x_i \in \mathfrak{R}\}$ the group algebra of C_n over the real field $\mathfrak{R}$. We study functions $f: \mathfrak{A} \to \mathfrak{A}$, i.e., $f(Z) = \Sigma\, j^{k-1} f_k(x_1, \ldots, x_n)$ where the (real-valued) component functions $f_k \in C^{(1)}(\mathfrak{D})$ ($\mathfrak{D}$ = a domain of $\mathfrak{A}$ in the topology of the Euclidean norm on $\mathfrak{A}$). We call $f(Z)$ S-regular on $\mathfrak{D}$ if it is differentiable on $\mathfrak{D}$ in the classical sense, and F-regular on $\mathfrak{D}$ if $\int_S f(Z)d\sigma = 0$ for every smooth closed (n-1)-dimensional hypersurface in $\mathfrak{D}$. We obtain such theorems as (e.g.): (i) $f(Z)$ is S-regular on $\mathfrak{D}$ iff the line integral $\int_C f(Z)dZ = 0$ for every smooth closed path $C \subset \mathfrak{D}$; (ii) both F- and S-regularity are characterized by sets of (generalized) Cauchy-Riemann equations satisfied on $\mathfrak{D}$ by the f_k; (iii) let f be S-regular; then there exist infinitely many linear transformations carrying f to a function g which is F-regular on $\mathfrak{D}$. The matrices of these transformations are given explicitly. (Received November 1, 1974.)

720-30-19 D.D. BONAR, Denison Univ., Granville, OH 43023, F.W. CARROLL, Ohio State Univ., Columbus, OH 43210 and GEORGE PIRANIAN, Univ. of Michigan, Ann Arbor, MI 48104. Strongly annular functions with small Taylor coefficients, Preliminary report.

A function f analytic on the unit disk is strongly annular if there exists a sequence $\{r_m\}$ such that $r_m \nearrow 1$ and the minimum modulus of f on the circle $|z| = r_m$ tends to ∞ as $m \to \infty$. Various examples of strongly annular functions have been constructed by means of gap series with unbounded coefficients. We use the function $b(z) = (1/2 - z)/(1 - z/2)$ as a building block and establish the following result. For appropriate sequences $\{c_m\}$ and $\{k_m\}$, the function $f(z) = \sum c_m [b(z)]^{k_m} = \sum a_n z^n$ is strongly annular, satisfies the condition $a_n \to 0$ as $n \to \infty$, and is nonlacunary in a strict sense: if $f = g + h$, where $g(z) = \sum b_\nu z^{\lambda_\nu}$ and $\lambda_{\nu+1} - \lambda_\nu \geq 2$ for all ν, then h does not belong to the Hardy class H^2. The proof depends on careful estimates of the Taylor coefficients of the function $[b(z)]^k$.

(Received November 4, 1974.)

*720-30-20 A. L. SHIELDS, University of Michigan, Ann Arbor, Michigan 48104 and D. L. WILLIAMS Syracuse University, Syracuse, New York 13210. Duality and Multipliers in spaces of harmonic functions.

Let ϕ be a positive continuous function on $[0,1)$ with $\phi(r) \leq 1$, $\phi(0) = 1$, and $\phi(r) \to 0$ as $r \to 1^-$. We study the Banach spaces $h_\infty(\phi)$ of functions u harmonic for $|z| < 1$ with $\|u\| = \sup\{|u(z)|\phi(|z|) : |z| < 1\}$. L. A. Rubel and A. L. Shields have shown that $h_\infty(\phi)$ is the second dual of its subspace $h_0(\phi) = \{u \in h_\infty(\phi) : |u(z)|\phi(|z|) \to 0, |z| \to 1^-\}$. For weight functions ϕ such that $1/\phi(r)$ grows more slowly than some power of $1/(1-r)$, we show there exists a finite positive Borel measure μ on $[0,1)$ such that $h^1(\mu) = \{v \in L^1(d\mu(r)d\theta) : v$ harmonic for $|z| < 1\}$ is isomorphic to the dual of $h_0(\phi)$. We also characterize the algebras of coefficient multipliers of these spaces of harmonic functions, and we generalize some results of Hardy and Littlewood on fractional integration. (Received November 6, 1974.)

*720-30-21 JAMES W. ROBERTS and MANFRED STOLL, University of South Carolina, Columbia, South Carolina 29208. Maximal ideals in the algebra N^+.

Let N denote the Nevanlinna class of functions of bounded characteristic in the unit disc U. A function $f \in N$ is in N^+ if $\lim_{r\to 1} \int_0^{2\pi} \log^+|f(re^{it})|dt = \int_0^{2\pi} \log^+|f(e^{it})|dt$. The space N^+ with the metric ρ given by $\rho(f,g) = \lim_{r\to 1} \frac{1}{2\pi} \int_0^{2\pi} \log(1 + |f(re^{it}) - g(re^{it})|dt$ is an F-algebra. For each $\lambda \in U$, $\gamma(f) = f(\lambda)$ defines a continuous multiplicative linear functional on N^+. Conversely, if γ is a nontrivial multiplicative linear functional on N^+, then γ is continuous and there exists $\lambda \in U$ such that $\gamma(f) = f(\lambda)$ for all $f \in N^+$. Consequently, for each $\lambda \in U$, $M_\lambda = \{f \in N^+ : f(\lambda) = 0\}$ is a closed maximal ideal in N^+. The following converse is proved.

Theorem. If I is a nonzero prime ideal in N such that I is not dense in N^+, then $I = M_\lambda$ for some $\lambda \in U$.

Corollary. Every closed maximal ideal in N^+ is the kernel of a multiplicative linear functional on N^+. We also show by example that there exist maximal ideals in N^+ which are not the kernel of a multiplicative linear functional. (Received November 4, 1974.)

*720-30-22 DOROTHY BROWNE SHAFFER, Fairfield University, Fairfield, Connecticut. Analytic functions in class $H_{\frac{1}{2},n}$ and their applications.

Let $H_{\alpha,n}$ denote the class of functions $h(z)$ with expansion $h(z) = 1 + c_n z^n + \ldots, n \geqq 1$, in $|z| < 1$ satisfying the condition $\mathrm{Re}\{h(z)\} > \alpha$, $0 \leqq \alpha < 1$. Lemma 1. Let $H(z) \in H_{\frac{1}{2},n}$, then $R(zh'(z)/h(z)) \geqq$

$-nr^n/(1+r^2)$, for $r = |z| \leq \gamma$, $\mathrm{Re}(zh'(z)/h(z)) \geq \{2[1 - r^{2n} + n(1 - r^2)r^{n-1}]^{\frac{1}{2}} - 2 - n(1 - r^2)r^{n-1}\}/(1 - r^2)r^{n-1}$ for $|z| > \gamma$; γ solution of $2\gamma(1 + \gamma^n) = n(1 - \gamma^2)$. Typical applications of this lemma are the following results: (1) the determination of the radius of convexity of the meromorphic function $F(\zeta) = \zeta + \alpha_0 + \alpha_1/\zeta^{n-1} + \alpha_2/\zeta^n t \ldots$, $n \geq 2$, anal. $1 < |\zeta| < \infty$ satisfying the condition $|F'(\zeta) - 1| < 1$. (2) Let $g(z)$ be convex, $f(z)/g(z) \in H_{\frac{1}{2},1}$ then the radius of starlikeness r_s of $f(z)$ is given by the solution of $r^3 - 3r^2 + 8r - 4 = 0$. (Received November 4, 1974.)

720-30-23 S. CHAKRAVARTY, Dept. of Mathematics, University of Kentucky, Lexington, Kentucky 40506 An extremal entire function

Description: Let $f(z)$ be a transcendental entire function of order, $\lambda(<+\infty)$. Define, $M(r) = \max_{|z|=r} |f(z)|$, $E(r) \equiv \{z: |z| = r, |f(z)| > 1\}$ and $|E(r)|$ denotes angular measure of the set $E(r)$. Earlier, I had announced a theorem about $|E(r)|$. Now we show that the theorem is best possible through,

Theorem (2): Given λ, $(0<\lambda<+\infty)$, there exists an entire function $f(z)$ of order λ, having all its zeros placed on a very rapidly expanding sequence of circles $\{|z| = R_K\}$ so that,

(1) $|E(R_K)| \to 0$, $(K \to \infty)$.

(2) $\lim_{K\to\infty} \{R_K^{\lambda/2+\varepsilon}|E(R_K)|\} = +\infty$.

(3) $\lim_{K\to\infty} \{R_K^{\lambda/2-\varepsilon}|E(R_K)|\} = 0$ $(\varepsilon>0)$.

Proofs will appear shortly. (Received November 4, 1974.)

*720-30-24 JAMES W. ROBERTS, University of South Carolina, Columbia, South Carolina 29208
The component of the origin in the Nevanlinna class.

The Nevanlinna class N is the class of analytic functions on the disc U whose characteristic $T(r,f) = \frac{1}{2\pi} \int_0^{2\pi} \log^+ |f(re^{it})|dt$ is bounded. The metric d defined by $d(f, g) = \lim_{r\to 1} \frac{1}{2\pi} \int_0^{2\pi} \log(1 + |f(re^{it}) - g(re^{it})|dt$ is complete and translation invariant on N. J.H. Shapiro and A. L. Shields have shown that N is not connected and they have posed the question of characterizing the component of the origin in N. The component of the origin is described as follows; For any $f \in N$, $\lim_{r\to 1} f(re^{it}) = f(e^{it})$ for almost every t in the unit circle T. A function f is said to be in N^+ if $\lim_{r\to 1} \int \log^+ |f(re^{it})|dt = \int \log^+ |f(e^{it})|dt$. If μ is a nonnegative singular Borel measure on T, then the function $S_\mu(z) = \int \frac{e^{it}+z}{e^{it}-z} d\mu(t)$ is called a singular inner function. The measure μ is said to be continuous if $\mu(\{w\}) = o$ for every $w \in T$. We let K denote all functions of the form, f/S_μ where $f \in N^+$ and μ is continuous. Theorem: K is the component of the origin and K is locally connected. (Received November 4, 1974.)

*720-30-25 MARVIN ROSENBLUM and JAMES ROVNYAK, University of Virginia, Charlottesville, Virginia 22903. Restrictions of analytic functions.

Let P denote the class of functions $w(z)$ analytic and having nonnegative imaginary part in the upper half-plane $y > 0$. Let Δ be a Borel subset of the real line. For any $f \in L^2(\Delta)$, let $\tilde{f}$ denote the Hilbert transform of f. By a Cayley inner function we mean a function $\xi(z)$ in P whose boundary

function $\xi(x) = \xi(x + i0)$ is real a.e. New results are obtained for the class P which solve, for example, the following two problems. Problem 1 (generalized Loewner problem). Characterize all real valued functions $u(x)$ on Δ such that $\pi^{-1}\int\int'_{\Delta\times\Delta}[(u(x) - u(t))/(x - t)]f(t)f(x)^*dt\,dx \geq 0$ for all functions f such that both f and uf belong to $L^2(\Delta)$ (the integral is taken as $\langle(uf)^\sim, f\rangle - \langle\tilde{f}, uf\rangle$). Problem 2. Characterize all nonnegative functions $v(x)$ on Δ such that $\int_\Delta |\tilde{f}|^2 v\,dt \leq \int_\Delta |f|^2 v\,dt$ for all functions f such that both f and vf belong to $L^2(\Delta)$. A real valued function u on Δ is a solution to Problem 1 iff there exists $w \in P$ such that $u(x) = w(x + i0)$ a.e. on Δ. A nonnegative function v on Δ is a solution to Problem 2 iff there exists $w \in P$ such that $iv(x) = w(x + i0)$ a.e. on Δ. A theory of Cayley inner functions is developed with an application to rational approximation on a finite union of real intervals. (Received November 4, 1974.)

720-30-26 S.M. SHAH, University of Kentucky, Lexington, Kentucky 40506

Components of Characteristic functions of Gaussian and Poisson distributions and means

Let f be entire and denote by $M_0(r, f)$, $M_p(r, f)$ $(0 < p < \infty)$ the means of f and by $M(r, f)$ the maximum modulus. Theorem 1. Let $f(z)$ be the characteristic function (c.f.) of Gaussian distribution, f_1 any component of f, c_0, c_1 constants and $|c_0| + |c_1| \neq 0$. If $F = c_0 f + c_1 f_1$ then (i) $\lim_{r\to\infty} M_0(r, F)/M_p(r, F) = 0$; (ii) $\lim_{r\to\infty} M_p(r, F)/M(r, F) = 0$. Theorem 2. Let $f_i(z) = z^{k_i}\exp(Q_i(z))P_i(z)$ $(i = 1, 2)$ be entire functions of integer order ρ, $(1 \leq \rho < \infty)$. Let $Q_i(z) = a_i z^\rho + b_i z^{\rho-1} + \ldots$, $\log M(r, P_i) = o(r^\rho)$ $(i = 1, 2)$, and $F = c_1 f_1 + c_2 f_2$ $(c_1 \neq 0)$. Assume that $|a_2| < |a_1|$. Then (*) $\lim_{r\to\infty} M_0(r, F)/M_p(r, F) = 0$. If f_2 be any entire function such that $\limsup_{r\to\infty} \log M(r, f_2)/r^\rho = |a_2|$ and if $\pi|a_2| < (\pi-1)|a_1|$ then also (*) holds. Analogous results have been proved for c.f. of Poisson distributions and for functions of infinite order. (Received November 4, 1974.)

*720-30-27 JOHN PFALTZGRAFF, University of North Carolina, Chapel Hill, North Carolina 27514.

Univalence of the integral of $f'(z)^c$.

For $f(z) = z + \ldots$ analytic and locally univalent in the open unit disk E, let $f(z;c) = \int^z f'(w)^c dw$, c complex, and let ord(f) denote the order of the linear invariant family generated by f. Theorem. If $\text{ord}(f) = a < \infty$ then $f(z;c)$ is univalent in E for all c in $|c| \leq 1/(2a)$. Corollary 1. If $f(z)$ is univalent in E and if $|c| \leq \frac{1}{4}$ then $f(z;c)$ is univalent in E. Corollary 2. If $\text{Re}\{e^{ib}(1 + zf''(z)/f'(z))\} > 0$ in E and if $0 < \cos b \leq \frac{1}{2}$ then $f(z)$ is univalent in E. Proof of the theorem is elementary and depends on a recent result of Ahlfors. Corollary 1 extends the known information on a problem of Duren, Shapiro and Shields (Duke Math. J. 33(1966), 247-254). Corollary 2 completes the answer to a question raised by M.S. Robertson (Michigan Math. J. 16(1969), 97-101). (Received November 5, 1974.)

720-30-28 David Horowitz, University of South Florida, Tampa, Florida 33620

Coefficient Estimates for Univalent Polynomials

Let $\mathcal{S} = \{f(z) = z + a_2z^2 + a_3z^3 + \cdots : f$ is analytic and univalent on the open unit disc$\}$, and define the subclasses $\mathcal{T} = \{f(z) \in \mathcal{S} : f$ is a polynomial$\}$ and $\mathcal{T}_m = \{f(z) \in \mathcal{S} :$ f is a polynomial of degree $\leq m\}$ for $m = 1,2,3,\ldots$. The Bieberbach conjecture asserts that

if $f(z) = z + a_2z^2 + a_3z^3 + \cdots$ is in $\mathscr{S}$, then $|a_n| \leq n$ for $n = 2,3,4,\ldots$. Moreover it is well-known that it suffices to prove this conjecture for the class $\mathscr{T}$, i.e. if there is a counterexample to the Bieberbach conjecture, then there is a polynomial counterexample. Using a lemma of Dieudonné (Ann. École Norm. (3) 48(1931), 247-358), an inequality of FitzGerald (Arch. Rat. Mech. Anal. 46(1972), 356-368), and the fact that it is known that $|a_n| \leq n$ for $n \leq 6$, the author has shown that the Bieberbach conjecture is true for the subclass $\mathscr{T}_{27}$. (Received November 6, 1974.)

720-30-29 WITHDRAWN

720-30-30 JOHN R. QUINE, JR., Florida State University, Tallahassee, Florida, 32304. A characterization of the complete polar of a polynomial.

Let $p(x)$ be a polynomial of degree n and $i:\mathbb{C}\to\mathbb{C}^n$ be defined by $i(x) = (x, \ldots, x)$. Then there exists a unique polynomial $\tilde{p}(z_1, \ldots, z_n)$ such that (i) $\tilde{p}$ is symmetric (ii) $\tilde{p}\circ i = p$ (iii) $\tilde{p}$ is linear as a function of the elementary symmetric functions of $z_1, \ldots, z_n$. $\tilde{p}$ is in fact the complete polar of p and properties (i), (ii), and (iii) characterize $\tilde{p}$. (Received November 6, 1974.)

720-30-31 I-LOK CHANG, American University, Washington, D.C. 20016. Zeros of an annular function. Preliminary report.

A function f, analytic in the unit disc D, is called annular if there exists a sequence $\{J_n\}$ of closed Jordan curves about $z = 0$, converging outward to the unit circle K, such that the minimum modulus of f on J_n tends to infinity. Let $Z'(f)$ be the subset of K consisting of all the limit points of the zeros of f in D. Let $m(r)$ denote the minimum modulus of f on $|z| = r$, and let $T(r)$ be the Nevanlinna characteristic function of f. We show Theorem. Let f be an annular function in D. Let $\{R_k\}$ be a positive sequence increasing to 1. Suppose there exist positive constants A, α, and c, where $0 \leq \alpha < 1$, $0 < c < 1$, such that $\log(m(R_k)) \leq A(R_k - R_{k-1})^{\alpha} \int_c^{R_k} T(r)(R_k - r)\,dr$, for $k = 1,2,\ldots$, then $Z'(f) = K$. (Received November 6, 1974.)

31 Potential Theory

*720-31-1 HUI-HSIUNG KUO, University of Virginia, Charlottesville, Va. 22903 Potential theory associated with Uhlenbeck-Ornstein process.

Parallel results of Gross' paper (Potential theory on Hilbert space, J. Func. Anal. 1 (1967), 123-181) are obtained for Uhlenbeck-Ornstein process $U(t)$ in an abstract Wiener space (H,B). Generalized number operator N is defined by $Nf(x)=\lim_{r\downarrow 0} \{E[f(U(\tau_x^{(r)}))]-f(x)\}/E[\tau_x^{(r)}]$, where $\tau_x^{(r)}$ is the first exit time of $U(t)$ starting at x from the ball centered at x of radius r. It is shown that $Nf(x)=\text{trace } D^2f(x)-(Df(x),x)$ for a large class of functions f. Let $q_t(x,\cdot)$ be the transition probabilities of $U(t)$. The λ-potential $G_\lambda f(\lambda > 0)$ and normalized potential Rf of f are defined by $G_\lambda f(x)= \int_0^\infty e^{-\lambda t} q_t f(x)dt$ and $Rf(x)= \int_0^\infty [q_t f(x)-q_t f(0)]dt$. It is shown that if f is a Lip-1 function then trace

$D^2G_\lambda f(x)-(DG_\lambda f(x),x)=-f(x)+\lambda G_\lambda f(x)$ and trace $D^2Rf(x)-(DRf(x),x)=\int_B f(y)p_1(dy)-f(x)$ Where p_1 is the Wiener measure in B with parameter 1. (Received October 7, 1974.)

*720-31-2 MYRON GOLDSTEIN, Arizona State University, Tempe, Arizona 85281
Subharmonic Functions Possessing Local Harmonic Majorants.

Let $\overline{R}$ be a compact bordered Riemann surface and R the interior of $\overline{R}$. Let V be a non-negative subharmonic function on R having the property that each boundary point is the center of a half disk in which V possesses a harmonic majorant. The V possesses a globally defined harmonic majorant. This extends a recent result of Gauthier and Hengartner. (Received October 23, 1974.)

*720-31-3 N.V. RAO, University of Toledo, Toledo, Ohio 43606 and
DAN F. SHEA, University of Wisconsin, Madison, Wisconsin 53706
Growth problems for subharmonic functions of finite order in space.

Let u be a subharmonic function in R^m; $T(r,u) = \sigma_m^{-1}\int u^+(r\omega)d\omega$ the characteristic function and its order $\overline{\lim_{r\to\infty}} \frac{\log T(r,u)}{\log r} = \lambda$ where $d\omega$ denotes the (m-1) dimensional surface area on the unit sphere and σ_m its total area. The growth of T(r,u) with that of $N(r,u) = \sigma_m^{-1} \quad u(r\omega)d\omega$.

Let $k(u) = \overline{\lim_{r\to\infty}} \frac{N(r,u)}{T(r,u)}$. We prove that

$$\pi\lambda k(u) \geq |\sin \pi\lambda|(\lambda+1)^{-\frac{m}{2}+1} \qquad (0 \leq \lambda < \infty\ ;\ m = 2, 3, 4)$$

and

$$k(u) \geq A_m\ |\sin \pi\lambda|(\lambda+1)^{-\frac{m+1}{2}} \qquad (0 < \lambda < \infty\ ;\ m \geq 5)$$

where A_m depends only on m.

We further show that there exist subharmonic functions $u_{\lambda,m}$ of order λ in R^m with

$$\pi\lambda k(u_{\lambda,m}) \leq C_m\ |\sin \pi\lambda|(\lambda+1)^{-\frac{m}{2}+1}$$ where C_m depends only on m. (Received October 23, 1974.)

*720-31-4 NORMAN MIRSKY, Texas Tech University, Lubbock, Texas 79409. Polyharmonic Green's functions and null classes, Preliminary report.

On a Riemannian manifold R, let γ_n be the Green's function for the polyharmonic equation $\Delta^n u = 0$, and $u = \Delta u = \ldots = \Delta^{n-1}u = 0$ on the ideal boundary. For ω the harmonic measure of ∂R_0, $\overline{R}_0$ contained in a regular subregion of R, we have: γ_n exists if and only if $\omega \in L^n(R_0)$. This generalizes Sario's result (A criterion for the existence of biharmonic Green's functions (To appear)). The above condition is used to find relations between the existence of γ_n and polyharmonic null classes. (Received October 30, 1974.)

*720-31-5 J.R. DIEDERICH, University of California, Davis, California 95616
Natural limits for harmonic and superharmonic functions. Preliminary report.

Let D be an open, connected set in R^n. Let $\tilde{B}(y,r) = \{x : |x-y| < r \cap D\}$: $|E|$, the measure of E: $\fint_E g(x)\,dx = |E|^{-1}\int_E g(x)\,dx$. Set $u_f(y,r) = \fint_{\tilde{B}(y,r)} |u(x)-f(y)|\,dx$ for $0 < r$. Theorem 1. If D is a Lipschitz domain and u(x) is harmonic and bounded in D, then there is a function f(y) on ∂D such that $u_f(y,r) = o(1)$ for a.e. y on ∂D. Theorem 2. If u(x) is an L_1 Green potential in $|x| < 1$, then $u_o(y,r) = o(1)$ for a.e. $|y| = 1$. Theorem 3. As in Theorem 1, if u(x) is bounded from below in almost every cone with vertex on ∂D, then there is an f(y) on ∂D such that $u_f(y,r) = o(1)$ for a.e. y, where the integral is over A(y,r) which contains a cone with vertex at y and $\lim_{r\to 0} |A(y,r)|\,|B(y,r)|^{-1} = 1$. Theorem 3 extends the main results of Hunt and Wheeden, Trans. Amer. Math. Soc., 133(1968), 307-322. Theorems 1 and 2 show that mean continuous limits, which are stronger than nontangential limits for harmonic functions in Lipschitz domains, are natural for harmonic and superharmonic functions vis-a-vis nontangential limits for superharmonic functions which do not exist in general, see Widman, Ark. Mat., 6(1966), 485-533.

(Received November 1, 1974.)

*720-31-6 EDWIN BRAITHWAITE, University of Illinois, Urbana, Illinois 61801. A converse to Fatou's general theorem. Preliminary report.

Let n be an integer with $n \geq 2$; let m denote the (n - 1)-dimensional Lebesgue measure on the $x_n = 0$ plane in the real Euclidean space R^n, and let A be a Borel subset of $x_n = 0$ which, in the relative topology for $x_n = 0$, has nonempty interior and Lebesgue-measure-zero boundary (that is, $m(\partial A) = 0$). Theorem. If u is a positive harmonic function on the half-space $x_n > 0$ in R^n, if μ is the Borel measure in the Poisson representation for u, and if $u(\lambda x) \to L$ as $\lambda \to 0+$ for each x in $x_n > 0$, where L is a constant, then to each $\epsilon > 0$ there corresponds a number $r > 0$ such that $|\mu(B)/m(B) - L| < \epsilon$ whenever B is a subset of $x_n = 0$ which satisfies the following conditions: (1) B is geometrically similar to A; (2) B contains the origin; and (3) B is a subset of the ball of radius r with center at the origin. Observe that this theorem generalizes the converse to Fatou's general theorem in a half-space as given by Loomis in two dimensions (Trans. Amer. Math. Soc. 53(1943), 239-250) and as given by Gehring in three dimensions (Proc. London Math. Soc. (3) 8(1958), 149-160). Generalizations and related theorems are also treated. (Received November 4, 1974.)

*720-31-7 PETER A. FOWLER, California State University, Hayward California 94542
Infimum and domination principles in vector lattices

Let L be a vector lattice, L' a subspace of the algebraic dual of L and $L'^+ = \{f \in L' | \langle x, f\rangle \geq 0$ for all $x \in L$ with $x \geq 0\}$. DEF. A map $T: L \to L'$ is coercive if for all $w \in L$, $\langle v-w, Tv\rangle / \|v\| \to \infty$ as $\|v\| \to \infty$. DEF. The triple (L,T,L') satisfies the infimum principle if for all $x, y \in L$ and $z' \in L'$, $Tx - z' \in L'^+$ and $Ty - z' \in L'^+$ imply $T(x \wedge y) - z' \in L'^+$; it satisfies the domination principle if $Tx - z' \in L'^+$ and $Ty - z' \in L'^+$ and $((x-y)^+, Tx - z') = 0$ imply $x \leq y$. DEF. If L is a Banach space and L' the continuous dual space, a map $T: L \to L'$ is hemi-continuous if it is continuous from each line segment of L to the weak* topology of L'.
THEOREM 1. Let $T: L \to L'$ be strictly monotone. Then the infimum principle implies the domination principle. THEOREM 2. Let L be a reflexive Banach space with closed positive cone, L' the continuous dual space, $T: L \to L'$ a monotone, hemi-continuous and coercive map. Then the domination principle implies the infimum principle. (Received November 4, 1974.)
(Author introduced by Professor George B. Pedrick.)

720-32-1 L.R. HUNT, Texas Tech University, Lubbock, Texas 79409. Uniqueness of Analytic Continuation on A Real Hypersurface in $\mathbb{C}^2$, Preliminary report.

Let M be a $\mathcal{C}^\infty$ real hypersurface in $\mathbb{C}^2$. If the Levi form on M either vanishes identically on M or does not vanish anywhere on M, then necessary and sufficient conditions for M to have uniqueness of analytic continuation for its C-R functions are known. We wish to find necessary and sufficient conditions for M to have uniqueness of analytic continuation for these functions if the Levi form on M is allowed to vanish only on a nowhere dense subset S of M. If S contains a complex submanifold of M which disconnects M, then uniqueness of analytic continuation does not exist (at least locally). If S does not contain such a manifold, then M does have the desired property of uniqueness of analytic continuation. (Received October 15, 1974.)

*720-32-2 JOHN PESEK, JR., University of Michigan, Ann Arbor, Michigan 48104, Runge domains and lacunary power series in several variables. Preliminary Report.

Theorem: Let $f(z) = \Sigma a_\alpha z^\alpha$ define a function holomorphic in the unit polydisc U^n. Let $(m_k, m_k')_{k=1}^\infty$ be a sequence of pairs of integers such that $m_k'/m_k \to \infty$ and $m_{k+1} \geq m_k'$. Assume that $a_\alpha = 0$ if $m_k \leq |\alpha| \leq m_k'$ for some k. Let $p:\Omega \to C^n$ be the (possibly many-sheeted) domain of existence of the function $f(z)$. Then the sequence $s_{m_k}(z) = \sum_{|\alpha| \leq m_k} a_\alpha z^\alpha$ converges uniformly on compact sets to $f(z)$ in Ω. Corollary 1: Ω is single-sheeted. Corollary 2: Ω is a Runge domain. This result is a generalization of a classical theorem of Ostrowski for a single variable (J. London Math. Soc. 1 (1926) 251-263). Our proof is based on an estimate of the remainders of the Taylor series outside the domain of convergence. The estimate uses the Bergman-Weil integral formula and some algebraic lemmas. (Received October 31, 1974.)

*720-32-3 JOHN MURRAY, Texas Tech University, Lubbock, Texas 79409. A Second Main Theorem on Stein Manifolds with Pseudoconvex Exhaustion, Preliminary report.

A second main theorem, and, with restrictions on the map under consideration, a defect relation are derived for a meromorphic map from a Stein manifold to a complex projective space; this is done without the assumption that the deficit term which appears in the first main theorem vanishes. The proof is a modification of the Ahlfors-Weyl argument for meromorphic curves; it utilizes Stoll's deficit estimate, and also his generalization to higher dimensional manifolds of the idea of associated map. (Received November 1, 1974.) (Author introduced by Professor L. R. Hunt.)

720-32-4 WITHDRAWN

720-32-5 WILLIAM R. ZAME, State University of New York at Buffalo, Amherst, New York 14226. Some analytic uniform algebras. Preliminary report.

Let Ω be an open subset of the complex plane, let $\mathcal{O}(\Omega)$ be the algebra of all functions analytic on Ω and let A be a closed subalgebra of $\mathcal{O}(\Omega)$ which contains the polynomials. It is shown that, if Ω has only a finite number of connected components, then $\mathrm{Sp}\,A$ (the spectrum of A) admits the structure of a one-dimensional analytic space and that A is represented as the algebra of all analytic functions on $\mathrm{Sp}\,A$. Analysis of the singularities of $\mathrm{Sp}\,A$ leads to classification theorems for subalgebras of $\mathcal{O}(\Omega)$. If Ω has infinitely many connected components, analogous results are obtained by restricting the algebra A to open and closed subsets of Ω which have only finitely many connected components, and passing to a limit. (Received November 4, 1974.)

*720-32-6 ANDREW G. MARKOE, Department of Mathematics, University of Washington, Seattle, Washington, 98195. Invariance of Holomorphic Convexity Under Proper Maps.

A counter-example is presented disproving the conjecture that the proper holomorpic image of a holomorphically convex analytic space is again holomorphically convex. This is quite surprising since it is known that the conjecture is true under the additional hypothesis of holomorphic point separation.

In a positive direction, it is shown that holomorphic convexity does persist under a proper map provided that the quotient of the image by the partition of maximal connected compact subvarieties is an analytic space. Also holomorphic convexity persists under a proper map whenever the image is normal and the fibers are connected.
(Received November 6, 1974.)

*720-32-7 LINDA KEEN, City University of New York, Lehman College and Graduate School & University Center, New York, New York 10036. On real Teichmuller spaces and their modular groups.

• Let S be a compact Riemann surface of genus g from which n points and m closed disks have been removed. Characterizing the space of conformal structures such a surface admits is the classical problem of moduli. In this paper we approach this problem by first looking at a covering space of this moduli space, the Teichmuller space. We describe the Teichmuller space real analytically in terms of lengths of closed geodesics on the surface S. The group of conformal homeomorphisms of S acts on the Teichmuller space and is called the Teichmuller modular group. We study the action of this group on the parameters of the Teichmuller space and indicate how to represent the factor space which is the moduli space. The constructions of the Teichmuller space v involve a study of Fuchsian groups and rest upon the theory of quasiconformal mappings as developed by Ahlfors and Bers. The construction of the fundamental domain for the modular group depends upon the classical work of Fricke and Dehn as well as some recent results of Bers. (Received November 6, 1974.)

*720-33-1 JOAQUIN BUSTOZ, University of Cincinnati, Cincinnati, Ohio 45221
A sum of Jacobi polynomials.

Let $P_k^{(\alpha,\beta)}(x)$ denote the Jacobi polynomial of degree k with parameters α, β, and set $R_k^{(\beta)}(x) = \dfrac{P_k^{(\beta,\beta)}(x)}{P_k^{(\beta,\beta)}(1)}$. R. Askey conjectured that $\sum_{k=0}^{n} R_k^{(\beta)}(x)z^k \neq 0$ if $|z| < 1$, $-1 \leq x \leq 1$, $\beta \geq 0$. We conjecture that if $\alpha \geq 0$, $\beta \geq 0$, $-1 \leq x \leq 1$, $|z| < 1$ then $\sum_{k=0}^{n} \binom{n-k+\alpha}{n-k} R_k^{(\beta)}(x)z^k \neq 0$. We prove that this conjecture is true for $\beta = 1/2$ and $\alpha \geq 0$, and is also true for $\beta \geq 1/2$, $\alpha \geq 1$.

(Received October 23, 1974.)

*720-33-2 MOURAD E.H. ISMAIL, Mathematics Research Center, 610 Walnut Street, Madison, Wisconsin 53706 and DOUGLAS KELKER, University of Alberta, Edmonton, Alberta, Canada.
The Bessell polynomials and the student t-distribution.

Trlifaj (Apl. Mat. 19(1974), 1-5) studied the asymptotic expansion of $K_\nu(x)/xK_{\nu+1}(x)$. This quotient of Bessel functions appeared in connection with the Schrodinger equation of a rectangular potential well. The infinite divisibility of the student t-distribution for odd degrees of freedom is equivalent to the complete monotonicity of $K_{n-\frac{1}{2}}(\sqrt{x})/\sqrt{x}K_{n+\frac{1}{2}}(\sqrt{x}) = y_{n-1}(1/\sqrt{x})/\sqrt{x}y_n(1/\sqrt{x})$, $n = 1,2,\ldots$, $y_n(x)$ being the nth Bessel polynomial. Let $\alpha_{n,j}$, $1 \leq j \leq n$, be the zeros of $y_n(x)$. We show that $\sum_{j=1}^{n}\alpha_{n,j}^k$ vanishes for $k = 3,5,\ldots,2n-1$ and is $(-4)^n(n!)^2\{(2n)!\}^{-2}$ and $(2n-1)^{-1}(-4)^n(n!)^2\{(2n)!\}^{-2}$ for $k = 2n+1$ and $2n+3$ respectively. This is then used to show that $K_\nu(\sqrt{x})/\sqrt{x}K_\nu(\sqrt{x})$ is infinitely divisible for $\nu = 3\frac{1}{2}, 4\frac{1}{2}, 5\frac{1}{2}$, and, in general, there exists $\theta_n > 0$ such that the inverse Laplace transform of $K_\nu(\sqrt{x})/\sqrt{x}K_{n+\frac{1}{2}}(\sqrt{x})$ is positive on $[\theta_n, \infty]$. We conjecture that $K_\nu(\sqrt{x})/\sqrt{x}K_{\nu+1}(\sqrt{x})$ is completely monotonic for $\nu > 0$.

(Received October 30, 1974.)

*720-33-3 B. C. CARLSON, Ames Laboratory-USAEC, Iowa State University, Ames, Iowa 50010.
Quadratic transformations of Appell functions.

In the accepted classification of hypergeometric power series in two variables, Appell's functions F_1, F_2, F_3, F_4 are series of order two. Doubts raised by Erdélyi in 1948 about the significance of this classification are revived by a quadratic transformation of F_4 with equal denominator parameters into a series of order three. From a different point of view (that of double integral averages instead of power series) this transformation resembles five known cases in which F_4 is reduced to something simpler by a single restriction on the parameters. Comparison of the six reduction formulas yields eight quadratic transformations with two free parameters, all but one of them involving F_1 or F_2. They include a generalization of Landen's transformation of incomplete elliptic integrals and a transformation which contains Gegenbauer's formula for the product of two ultraspherical polynomials. (Received November 1, 1974.)

*720-34-1 KURT KREITH, University of California, Davis, California 95616.
Nonselfadjoint Fourth Order Differential Equations with Conjugate Points.

Let $\eta(\alpha)$ be the smallest $\beta > \alpha$ such that the real nonselfadjoint fourth order differential equation $\ell[y] \equiv (p_2(t)y'' - q_2(t)y')'' - (p_1(t)y' - q_1(t)y)' + p_0(t)y = 0$ has a nontrivial solution satisfying $y(\alpha) = y'(\alpha) = 0 = y(\beta) = y'(\beta)$. It is relatively easy to establish criteria for the non-existence of the conjugate point $\eta(\alpha)$, but the problem of showing the existence of conjugate points is substantially more difficult in the nonselfadjoint case. Generalizing upon a technique due to W. M. Whyburn (Amer. J. Math. 52(1930), 171-196) one can represent $\ell[y] = 0$ in the form of a second order system $y'' = a(t)y + b(t)x$; $x'' = c(t)y + d(t)x$ and establish the following criteria for the existence of $\eta(\alpha)$: (i) $c(t) \geq a(t) > 0$ and $b(t) \geq d(t) > 0$; (ii) $u'' + \min\{b(t)-d(t), c(t)-a(t)\}u = 0$ is oscillatory at $t = \infty$; and (iii) $\int^{\infty} tb(t)\,dt = \int^{\infty} tc(t)\,dt = \infty$. These criteria can be translated into criteria involving the original coefficients. While these results establish criteria for the existence of conjugate points in the nonselfadjoint case, they fail to provide specific upper bounds for $\eta(\alpha)$. (Received August 15, 1974.)

*720-34-2 JOYCE R. MCLAUGHLIN, Rensselaer Polytechnic Institute, Troy, N.Y.,
An inverse eigenvalue problem of order four.

Suppose that 2n positive real numbers $\lambda_1<\lambda_2<\cdots<\lambda_n$ and $\rho_1,\rho_2,\ldots,\rho_n$ are given. A method is determined for finding coefficients $A(s)$, $B(s) \in C^{\infty}[0,1]$ such that $\lambda_1<\lambda_2<\cdots<\lambda_n$ are the first n eigenvalues for $y^{(4)}+(Ay^{(1)})^{(1)}+By-\lambda y = 0$, $y(0) = y^{(1)}(0) = y(1) = y^{(1)}(1) = 0$; and if y_{λ_i} is the eigenfunction corresponding to λ_i, then $\rho_i = \int_0^1 [y_{\lambda_i}(s)]^2 ds$. The proof is constructive. The functions $A(s)$ and $B(s)$ are determined to be $A(s) = -4\frac{d}{ds}K(s,s)$ and $B(s) = -AK_s|_{s=t} + (K_{ss}-K_{tt})_t|_{t=s} - 2\frac{d^3}{ds^3}K(s,s)$, where $K(s,t) = \sum_{i=1}^{n} [F_i Z_{\lambda_i} - G_i Z_{\lambda_i^*}]$. The functions Z_{λ_i} and $Z_{\lambda_i^*}$, $i=1,2,\ldots,n$, are known solutions to a related eigenvalue problem, and F_i and G_i, $i=1,\ldots,n$ are solutions of a set of 2n nonhomogeneous, linear equations. (Received October 4, 1974.)

*720-34-3 S. L. CAMPBELL, C. D. MEYER, JR., AND N. J. ROSE, North Carolina State University, Raleigh, N. C. 27607. Applications of the Drazin inverse to linear systems of differential equations.

Let A,B be $n \times n$ matrices, f a vector valued function. A and B may both be singular. The differential equation $Ax' + Bx = f$ is studied utilizing the theory of the Drazin inverse. A closed form for all solutions of the differential equation is given when the equation has unique solutions for consistent initial conditions. (Received October 15, 1974.)

*720-34-4 LLOYD K. WILLIAMS, Atlanta University, Atlanta, Georgia 30314. $Y' = G(X,Y)$, GEC'.

Much previous work, from the time of Newton, has centred on the situation where G is a polynomial in Y of degree at most three (3).

Those of degree one (1) are first order linear equations. By a well known result, those of degree two (2) are equivalent to second order linear equations. Collectively called Abel equations, those of degree three (3) include Van der Pol, Emden, Thomas-Fermi and Langmuir equations.

One is ordinarily restricted to particular solutions if the degree exceeds one (1) and uses series, numerical methods or successive approximations. By the procedure given here we can find the general solution whenever G is continuous together with its first partial derivatives. It is not necessary that G be a polynomial in Y.
(Received September 23, 1974.)

*720-34-5 BRIAN KELLY, The Pennsylvania State University, Worthington Scranton Campus, Dunmore, Pennsylvania 18512. The Topology of the Stokes Phenomenon.

Topological methods of function theory are particularly effective in determining domains in which an asymptotic solution to a differential equation is valid and across whose boundaries the solution may undergo an abrupt change in asymptotic character. This change is known as the Stokes Phenomenon. The main analysis first gives a necessary and sufficient condition for identifying those domains, called admissible, which arise naturally in the asymptotic theory; and then indicates how this condition leads to a geometric interpretation of the Stokes Phenomenon. (Received October 9, 1974.)

*720-34-6 MARK FEY
The Pennsylvania State University, The Capitol Campus, Middletown, Pa. 17057
"On the equation $4x^{n+2}y'' + y^{2n+1} = 0$."

It will be shown that the given equation has both oscillatory and nonoscillatory solutions. (Received October 17, 1974.) (Author introduced by Dr. G. Diantonio.)

*720-34-7 GANGARAM S. LADDE, State University of New York, College at Potsdam Potsdam, New York 13676, Time-Delay Interconnected Systems and Extensions of Lyapunov's Direct Method. Preliminary Report

Consider a time-delay interconnected system described by the system of functional differential equations

(1) $x' = f(t,x,x_t)$, $x_{to} = \phi_o$, where $x, f \varepsilon R^n$, $\phi_o \varepsilon C^n[-\tau,o]$.

System (1) is composed of s subsystems described by the systems of differential equations

(2) $x_i' = g_i(t,x_i) + h_i(t,x) + k_i(t,x_t)$, where $i=1,2,\ldots s$, $x_i \varepsilon R^{n_i}$ and represents the i-th component of the state vector

$x = [x_1^T, x_2^T, \ldots, x_i^T, \ldots, x_s^T]^T$, where T denotes the transpose of a vector.

When the interaction $h_i(t,x) \equiv o \equiv k_i(t,x_t)$, the unforced subsystems described by

(3) $x_i' = h_i(t,x_i)$.

By employing Lyapunov-like functions and the theory of functional differential inequalities, sufficient conditions are derived for connective stability of the time-delay interconnected system (1) under structural perturbations, on the basis of stability of each unforced system (3).
(Received October 18, 1974.)

*720-34-8 RICHARD C. GILBERT, California State University, Fullerton, California 92634
The deficiency index of a third order operator

Consider the formally selfadjoint operator $Ly = iy^{(3)} + (a_1y')' + ib_1y' + (2^{-1}ib_1' + a_o)y$, where $b_1 = ax^{\alpha}$, $a_1 = bx^{\gamma/3}$, $a_o = cx^{\gamma}$, $c \neq 0$, $1 \leq x < \infty$. Let $d = b/c^{1/3}$.

Theorem. If $d \neq 3/2^{2/3}$, $d \neq 3/2^{1/3}$, $\gamma > 0$, $\alpha < 2\gamma/3$, $\sigma \neq 0$, $\sigma \neq a\alpha/2$, then $Ly = i\sigma y$ has three linearly independent solutions $y_k = [1 + o(1)]\, x^{-\gamma/3} \exp \int_{x_o}^{x} \lambda_k(t)\, dt$, where the $\lambda_k(t)$ are solutions of $i\lambda^3 + a_1\lambda^2 + ib_1\lambda + (2^{-1}ib_1' + a_o - 2\sigma) = 0$. The proof depends on asymptotic formulas for the $\lambda_k(t)$ and a method of Fedorjuk ("Asymptotic methods in the theory of one-dimensional singular differential operators", Trans. Moscow Math. Soc. 15 (1966), 333-386). Using this theorem and previous work of other authors the deficiency index of L is determined, for example, for all values of α, γ except $\alpha = 2\gamma/3$ when $\gamma \geq 0$, provided $d \neq 3/2^{2/3}$, $d \neq 3/2^{1/3}$. (Received October 29, 1974.)

*720-34-9 VADIM KOMKOV, Texas Tech University, Lubbock, Texas 79409. A generalization of the Sturm-Piccone comparison theorem.

The behavior of solutions of (1) $(a(t)x')' + c(t)x = 0$ is compared with solutions of (2) $(A(t)u') + C(t)u = g(t)$. Suppose a solution of (1) has zeros at $t = \alpha$ and $t = \beta > \alpha$, and $\exists$ a function $\varphi(t) \in C^1[\alpha,\beta]$ s.t. $a(t)\varphi(t) > 0$, $(a\varphi')' \leq 0$, $g(t)\varphi(t) \leq 0$ on $[\alpha,\beta]$ and either $g(t)\varphi(t)$ or $(a\varphi')'$ does not vanish identically on $[\alpha,\beta]$. If $A \leq a\varphi$ and $C \leq c\varphi$ $\forall$ $t \in [\alpha,\beta]$ then every solution of (2) will have at least one zero on $[\alpha,\beta]$. (It was not asserted that either $A \leq a$, or that $C \leq c$). (Received October 29, 1974.)

720-34-10 ROBERT E. BECK, Villanova University, Villanova, PA 19085 and BERNARD KOLMAN, Drexel University, Philadelphia, PA 19104. An Algebraic Approach to Homogeneous Matrix Differential Equations.

One type of system of polynomial differential equations arises from the matrix differential equation $\dot{X} = X(\Gamma X)^{m-1}$, $m \geq 2$, where X is a $p \times q$ matrix and Γ is a $q \times p$ matrix. Levin and Shatz [Duke Math. J. 30(1963), 579-594] have studied the case where m=2 from an algebraic point of view. This paper treats the case where $m \geq 3$. Following Markus' construction, one can associate to each such matrix differential equation, a pq dimensional m-algebra A (ie. a pq dimensional real vector space with an m-ary multilinear operation). For certain principal indempotents of A, a Pierce-like decomposition of A is obtained, and properties of this decomposition are investigated. (Received October 30, 1974.)

720-34-11 JAMES D'ARCHANGELO, U.S. Naval Academy, Annapolis, Md. 21402 and PHILIP HARTMAN The Johns Hopkins University, Baltimore, Md. 21218, Integration of ordinary linear differential equations by Laplace-Stieltjes transforms.

Let R be a constant N x N matrix and g(t) an N x N matrix of functions representable as absolutely convergent Laplace - Stieltjes transforms for $t > 0$. Sufficient conditions are given for the existence of certain solutions of the linear, first order system $y' = (R + g(t))y$ which are representable as Laplace - Stieltjes transforms or linear combinations of such transforms with coefficients of the form $e^{\lambda t}t^{\nu}$. The results are applied to the Bessel equation to obtain for the Hankel functions $H_{\mu}^{(1)}$, $H_{\mu}^{(2)}$, new integral representations involving Legendre functions. (Received October 30, 1974.)

*720-34-12 V. LAKSHMIKANTHAM, A. RICHARD MITCHELL, and ROGER W. MITCHELL, The University of Texas at Arlington, Arlington, TX 76019. Maximal and Minimal Solutions and Comparison Results for Differential Equations in Abstract Cones. Preliminary Report.

In this paper properties of abstract cones, the extended definition of quasimonotone function, and the Kuratowski measure of non-compactness of a set were used to prove existence of extremal solutions in Banach spaces. From the existence of extremal solutions comparison, theorems are obtained and as an application of the comparison technique a general uniqueness theorem was considered. All the results in the paper have as special cases results for finite systems of differential equations. (Received October 31, 1974.)

*720-34-13 V. LAKSHMIKANTHAM, A. RICHARD MITCHELL, and ROGER W. MITCHELL, University of Texas, Arlington, TX 76019. Differential Equations on Closed Subsets of a Banach Space. Preliminary report.

The problem of existence of solutions to the initial value problem $x' = f(t,x)$, $x(t_0) = x_0 \in F$, where $f \in C[[t_0,t_0 + a] \times F, E]$, F is a locally closed subset of a Banach space E. Nonlinear comparison functions and dissipative type conditions in terms of Lyapunov-like functions are employed. A new comparison theorem is established which helps in surmounting the difficulties that arise in this general setting. (Received October 31, 1974.)

*720-34-14 V. LAKSHMIKANTHAM, University of Texas, Arlington, TX 76019 and S. LEELA, State University College, Geneseo, N.Y. 14454. On the Zeros of Nonlinear Operators on Closed Subsets of a Banach Space. Preliminary report.

One of the techniques of proving the existence of zeros of nonlinear operators in a Banach Space is intimately connected with proving the existence of solutions of the corresponding differential equation and considering the solution operators. In this paper, Lyapunov-like functions and the theory of differential inequalities are employed to obtain some general results in this direction. (Received October 31, 1974.)

*720-34-15 T.K. PUTTASWAMY, Ball State University, Muncie, Indiana 47306, Two Point Connection Problem for A Certain Differential Equation with An Irregular Singular Point of Rank Two

The purpose of this paper is to solve in the large the linear homogeneous differential equation (1) $a_0z^3 \frac{d^3y}{dz^3} + z^2(b_0 + b_1z + b_2z^2) \frac{d^2y}{dz^2} + z(c_0 + c_1z + c_2z^2)\frac{dy}{dz} + (d_0+d_1z+d_2z^2)y = 0.$

The variable z is complex, the coefficients a_0, b_i, c_i, d_i (i = 0, 1, 2) are real or complex with $a_0 \neq 0$ and $b_2 \neq 0$.

Then (1) will have a regular singular point at z = 0 and an irregular singular point at $z = \infty$. The rank of $z = \infty$ is two. The indicial equation about z = 0 is found to be

$$(2) \quad a_0 h(h-1)(h-2) + b_0 h(h-1) + c_0 h + d_0 = 0.$$

It is also assumed that no two roots of (2) differ by an integer.

(Received November 1, 1974.)

720-34-16 BHAGAT SINGH, University of Wisconsin Center, Manitowoc WI, Asymptotic Nature of Nonoscillatory Solutions of nth Order Retarded Equations

Theorem 1: For the delay equation $r(t)y^{(n)}(t) + r'(t)y^{(n-1)}(t) + a(t)y_\tau(t) = f(t)$, let (i) $r(t) > 0$, $\int^\infty 1/r(t)dt < \infty$; (ii) for sequences $\{a_n\}$, $\{b_n\}$, $b_n > a_n$ for all n, and $b_n - a_n \to \infty$ as $n \to \infty$ let $\int_{a_n}^{b_n} a(t)dt = \infty$, $a(t) > 0$; (iii) $\int^\infty |f(t)|dt < \infty$ then $y^{(n-2)}(t) \to$ a finite limit as $t \to \infty$; where y(t) is a nonoscillatory solution of the above equation. Application of this result is shown for second and third order equations of the above type which constitute Theorems 2 and 3. Theorem 4: Suppose n = 2 and conditions (ii) and (iii) of theorem (1) hold. Further suppose that r(t) is positive bounded function that satisfies the following:

$$\text{If} \quad \lim_{K\to\infty} \int_{a_K}^{b_K} r(t)dt = \infty$$

then $(b_K - a_K) \to \infty$ as $K \to \infty$. Suppose $0 < \tau'(t) < L < 1$. Let y(t) be a nonoscillatory solution of the above equation, then $y(t) \to 0$ as $t \to \infty$.

Some known results of this type are generalized. (Received November 4, 1974.)

*720-34-17 JON B. HINTON, University of Tennessee, Knoxville, Tennessee 37916 and ROGER T. LEWIS, Slippery Rock State College, Slippery Rock, Pennsylvania 16057. Discrete spectra criteria for singular differential operators with middle terms.

Let ℓ be the differential operator of order 2n defined by $\ell(y) = \sum_{i=0}^{n}(-1)^i(p_i y^{(i)})^{(i)}$ $(0 \leq x < \infty)$, where the coefficients are real continuous functions and $p_n > 0$. The formally self-adjoint operator ℓ determines a minimal closed symmetric linear operator L_0 in the Hilbert space $L_2(0,\infty)$ with domain dense in $L_2(0,\infty)$. The operator L_0 has a self-adjoint extension L which is not unique, but all such L have the same continuous spectrum. We are concerned here with conditions on the p_i which will imply that the spectrum of such an L is bounded below and discrete. A. M. Molchanov gave a necessary and sufficient criterion for boundedness below and discreteness of the spectrum of self-adjoint operators generated by $(-1)^n y^{(2n)} + q(x)y$ on $[0,\infty)$. We make an extension of the sufficiency criterion of Molchanov to ℓ. A necessary criterion for ℓ is also given. (Received November 4, 1974.)

*720-34-18 R. W. MITCHELL and M. E. MOORE, University of Texas, Arlington, TX 76019. Vector Lyapunov Functions and Perturbation Theory. Preliminary report.

A new comparison result is given that connects the solutions of perturbed and unperturbed differential systems in terms of solutions of comparison system. Stability and asymptotic behavior of solutions is considered. The study involves vector Lyapunov functions and the theory of vectorial differential inequalities. (Received November 4, 1974.)

720-34-19 Ronald Grimmer, Southern Illinois University, Carbondale, Illinois 62901. Nonlinear Boundary Value Problems.

The boundary value problem (B V P) $x'' + f(t,x) = 0$, $x(0) = x(1) = 0$ is studied. Assume that the solutions of the initial value problem are unique, $f(t,x)/x \to 0$ as $x \to 0$ uniformly in t , $0 \le t \le 1$, and assume that the solutions of $u'(t) = (t - a)f(t,u)$, $0 \le a < 1$, have uniform escape time. Theorem. For each n , $n = 0,1,\ldots$ there is a solution to (B V P) that has n zeros in $(o,1)$. (Received November 4, 1974.)

*720-34-20 JAMES P. FOTI, U. S. Naval Academy, Annapolis, Maryland 21402. Asymptotically Autonomous Multi-valued Differential Equations.

For multi-valued functions F and G whose values are non-empty subsets of d-dimensional Euclidean space, R^d , the generalized differential equation $x' \in F(x) + G(t,x)$ is said to be asymptotically autonomous if $G(t,x)$ becomes small in some sense as $t \to \infty$. The main result of this investigation establishes the relationship of the asymptotic behavior of solutions of the above equation to that of solutions of the autonomous equation $x' \in F(x)$.

Main result: Let F be a positive-homogeneous upper semi-continuous mapping from R^d to the non-empty, compact, convex subsets of R^d such that all solutions of $x' \in F(x)$ approach zero as $t \to \infty$. Let G be a mapping from R^{1+d} to the non-empty subsets of R^d such that $G(t,\cdot) \to 0$ as $t \to \infty$ uniformly on compact subsets of R^d . If ϕ is a bounded solution of the equation $x' \in F(x) + G(t,x)$ on $[0,\infty)$ then $\phi(t)$ approaches zero as $t \to \infty$. (Received November 4, 1974.) (Author introduced by Professor Peter McCoy.)

*720-34-21 M. E. LORD and A. RICHARD MITCHELL, University of Texas, Arlington, TX 76019. A New Approach to the Method of Nonlinear Variation of Parameters. Preliminary report.

Parallel to the classical variation of constants formula for linear systems, a technique for nonlinear equations is presented which offers a variety of variation of constants formula. These results include various existing results, including Alekseev formula. As applications of the methods, asymptotic equilibrium and asymptotic behavior of solutions are considered. (Received November 4, 1974.)

720-34-22 Stephen H. Saperstone, George Mason University, Virginia 22030 and Masahiro Nishihama, University of Maryland, Maryland 21228. Continuity of the Limit-set maps in Semi-Dynamical Systems.

Let a semi-flow π be given on a metric space (X, ρ). We investigate both the upper and lower semi-continuity of the set valued maps $x \to K(x)$, $x \to L(x)$, $x \to J(x)$ and $x \to D(x)$, where $K(x)$ is the positive orbit closure, $L(x)$ is the positive limit set, $J(x)$ is the positive prolongational limit set and $D(x)$ is the positive Prolongational set of x.

The question of contuity of the map L defined on probability measures arising from diffusion processes was prompted by A. Boyarsky, J. Diff. Equations 14 (1973), 559-567. We establish that for the map K, the following are equivalent. (i) Each K(x) is positively stable, (ii) Each L(x) is positively stable, (iii) The map K is continuous on X. We further show that the map L is continuous on X if and only if each L(x) is eventually stable. The notion of eventual stability is weaker than the one of stability. We prove several results for prolongations. Some results are closely related to the ones given by N.P. Bhatia, Symposia Mathematics vol. VI (pp. 151-166), Academic Press, New York-London 1971. and Ordinary Differential Equations 1971 NRL-MRC Conference, edited by L. Weiss (pp. 365-370), Academic, New York-London 1972. (Received November 4, 1974.)

*720-34-23 GARRET J. ETGEN, University of Houston, Houston, Texas 77004. Oscillation of second order differential systems in Banach spaces.

Let $\mathcal{K}$ be a real Hilbert space and denote by $\mathcal{A} = \mathcal{A}(\mathcal{K},\mathcal{K})$ the Banach algebra of bounded linear operators from $\mathcal{K}$ to $\mathcal{K}$ with the standard operator norm. Let S be the subspace of $\mathcal{A}$ consisting of the self-adjoint operators and consider the second order differential equation (1) $Y'' + P(t)Y = 0$ on $R^+ = [0,\infty)$, where $P: R^+ \to S$ is continuous. A solution $Y = Y(t)$ of (1) is nonsingular at $t = a$ if $Y(a)$ has a bounded inverse and the range of $Y(a)$ is $\mathcal{K}$; otherwise Y is singular at $t = a$. A solution Y is conjoined if $Y^*Y' \equiv (Y^*)'Y$ (* denotes adjoint) on R^+. A solution Y is oscillatory if $Y(a)$ is nonsingular for at least one $a \in R^+$ and for each $b \in R^+$ there exists $c \geq b$ such that $Y(c)$ is singular. The equation (1) is oscillatory if it has at least one conjoined oscillatory solution; otherwise (1) is nonoscillatory. Let $\mathcal{G}$ be the set of bounded linear functionals on S such that to each $g \in \mathcal{G}$ there corresponds a positive number ρ_g such that $g(A^2) \geq \rho_g[g(A)]^2$ for all $A \in S$. The set $\mathcal{G}$ is non-empty. THEOREM: If there exists $g \in \mathcal{G}$ such that $\lim_{t\to\infty} g[\int_0^t P(s)ds] = \infty$, then (1) is oscillatory. This result includes a number of well-known oscillation criteria for (1) as special cases. (Received November 4, 1974.)

720-34-24 J. F. WIGGS, North Carolina State University, Raleigh, N. C. 27607. Boundedness of solutions to ordinary differential equations using Liapunov functions. Preliminary report.

Consider the system $x' = f(t,x)$ where $f : [0,\infty) \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous. By using two Liapunov-type functions U and V, with U being a Liapunov-type function on a set $E \subset \mathbb{R}^n$ and V being a Liapunov-type function on $\mathbb{R}^n$ and there being assumed a relationship between their Dini derivatives on E, we obtain results guaranteeing that no noncontinuable solution to the initial value problem $x' = f(t,x)$ with $x(t_o) = x_o$ has finite escape time and that all solutions to the initial value problem are bounded in the future (provided V is sufficiently well behaved with respect to (t_o,x_o) outside of E when $|x|$ is large). These results extend results due to J. Haddock [Applicable Analysis, 1972, Vol. 1, pp. 321-330]. (Received November 4, 1974.)

*720-34-25 RICHARD H. ELDERKIN, Pomona College, Claremont, California 91711. Separatrices in solitude.

A compact invariant set I of a flow φ is solitary if it has a compact neighborhood U such that any half-trajectory contained in U has its limit set contained in I. A maximum trajectory segment σ in U - I is a primary separatrix generated by I if for some $x \in \sigma$ there are sequences $x_n \to x$, $t_n \to \infty$ such that $\varphi(x_n,[0,t_n]) \subset U$ and $\varphi(x_n,t_n)$ converges to a point in U - I, or the analogous statement holds

for $t_n \to -\infty$. The separatrix set $\mathscr{S}$ is the closure of the union of all such segments. When one tries to find $\mathscr{S}$ using numerical methods in a computer, the propagation by φ of external tangencies to some subneighborhood $V, I \subset V \subset U$, appears to approximate $\mathscr{S}$. This view is justified to the following extent: For such V, let $e(V)$ be the union of all trajectory segments in $U - V$ which contain an external tangency to V. If $\nu = \{V_n\}$ is a fundamental sequence of neighborhoods, let $e(\nu) = \limsup e(V_n)$ taken as $n \to \infty$. Theorem. $\mathscr{S}$ is the intersection $\bigcap e(\nu)$ taken over all fundamental systems ν. (Received November 4, 1974.)

*720-34-26 ALLAN PETERSON, University of Nebraska, Lincoln, Nebraska 68508
On the sign of Green's functions.

We are concerned with the sign of Green's functions for n-th order linear differential equations. The result is well known for disconjugate differential equations. Here we do not assume that we have an interval of disconjugacy. A corollary of the main result is that if our equation is $(k,n-k)$-disconjugate for $k = p-1,\ldots,n-1$, then the Green's function for the (pq)-BVP is of constant sign. (Received November 4, 1974.)

*720 34-27 PHILIP W. WALKER, University of Houston, Houston, Texas 77004. Certain second order boundary value problems.

The second order boundary value problem, $-y'' = \lambda y$, $\alpha_1 y(0) + \alpha_2 y'(0) + \alpha_3 y(1) + \alpha_4 y'(1) = 0$, $\beta_1 y(0) + \beta_2 y'(0) + \beta_3 y(1) + \beta_4 y'(1) = 0$ is considered in the case where the Green's functions has double poles or perturbed double poles of large modules. Asymptotic formulae are given for the eigenvalues. The nature of the related expansion in eigenfunctions is studied. (Received November 4, 1974.)

*720-34-28 DENNIS D. BERKEY, Boston University, Boston, Massachusetts 02215
Block Diagonal Dominance and Reducibility for Linear Differential Systems

Let the nxn matrix A have block form $A = \begin{bmatrix} B & C \\ D & F \end{bmatrix}$ where B is kxk. Let $m = \max\left\{\|C\|, \|D\|\right\}$. We say A is block diagonally dominant if for some $\delta > 0$ $\operatorname{Re} b_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{k} |b_{ij}| + m \leq -\delta < 0$ for all $i=1,\ldots,k$ and $\operatorname{Re} f_{ii} - \sum_{\substack{j=1 \\ j\neq i}}^{n-k} |f_{ij}| - m \geq \delta > 0$ for all $i=1,\ldots,n-k$. Theorem. If $A(t)$ is block diagonally dominant and $m(t)$ is bounded then the differential system $x' = A(t)x$ (a) admits an exponential dichotomy and (b) is kinematically similar to an equation of the form $y' = \begin{bmatrix} G(t) & 0 \\ 0 & H(t) \end{bmatrix} y$. If $A(t)$ is almost periodic the similarity is via an almost periodic change of variables. This theorem is used to give conditions under which the a.p. system $x' = A(t)x$ is kinematically similar to the a.p. system $y' = D(t)y$ with D diagonal a.p. (Received November 5, 1974.)

720-34-29 FREDERICK SCOTT, Comcon, Inc. Cinnaminson, New Jersey 08077 On a Partial Asymptotic Stability Theorem of Willett and Wong

A theorem of Willett and Wong (p.p. 15, Journal of Mathematical Analysis and Applications, Vol. 23, No. 1, July 1968) which gives sufficient con-

ditions for all solutions of the differential equation $(p(t)x')' + q(t)f(x) = 0$ to tend to zero as t approaches infinity is generalized. The generalization illustrates a procedure which the author has used in the development of new results on partial asymptotic stability. In addition, an analysis of Willett and Wong's results is presented which includes a counterexample to three "corollaries" of the theorem given by them. (Received November 5, 1974.)

720-34-30 HERMAN GOLLWITZER, Drexel University, Philadelphia, Pa. 19104.

Stability properties of a differential equation

Some recent results concerning the asymptotic stability properties of the ordinary differential equation $(py')' + q\ f(y) = e$ will be presented. Special reference will be made to some recent and unpublished work of Dr. Frederick Scott. (Received November 5, 1974.)

*720-34-31 WILLIAM F. TRENCH, Drexel University, Philadelphia, Pa. 19056.

Linear Differential Equations Subject to Integral Smallness Conditions Involving Ordinary Integrability.

Theorem. If $p_1, \ldots, p_n$ are continuous on $[0,\infty)$, $\int^{\infty} |p_1(t)|\ t^q\ dt < \infty$, and the integrals $\int^{\infty} p_k(t)\ t^{q+k-1}\ dt$ $(2 \leq k \leq n)$ converge - perhaps conditionally - for some $q > 0$, then the scalar equation $x^{(n)} + p_1(t)x^{(n-1)} + \cdots + p_n(t)x = 0$ $(t \quad 0,\ n = 2)$ has solutions $x_0, \ldots, x_n$ such that $x_j^{(r)}(t) = \frac{t^{j-r}}{(r-j)!}(1 + o(t^{-q}))$ $(0 \leq j \leq r)$ and $x_j^{(r)}(t) = o(t^{r-j-q})$ $(r + 1 \leq j \leq n - 1)$. (P. Hartman has obtained the same conclusion by assuming that $\int^{\infty} |p_k(t)|\ t^{q+k-1}\ dt < \infty$ $(1 \leq k \leq n)$ for some $q \geq 0$.) The special case applicable to $y'' + p(t)y = 0$, together with a suitable transformation, yields results on asymptotic relationships between solutions of $(r(t)x')' + g(t)x = 0$ (assumed nonoscillatory on $[0,\infty)$) and $(r(t)y')' + f(t)y = 0$, under integral smallness conditions on f - g which involve ordinary - rather than absolute integrability at ∞. (Received November 6, 1974.)

720-34-32 Kent Foster, Southern Illinois University, Carbondale, Ill. 62901

Oscillations of a Forced Even Order Differential Equation.

Consider the equation $x^{(n)} + f(t,x) = Q(t)$, n = even, where f is nondecreasing in x, $xf(t,x) > 0$ for $x \neq 0$, and $Q(t) = r^{(n)}(t)$, $t \geq 0$. If $x(t)$ is a positive (negative) solution, then $y(t) = x(t) - r(t)$ is a positive (negative) solution of $y^{(n)} + f(t,y + r(t)) = 0$, for sufficiently large t, and satisfies $y^{(n-1)}(t) \leq (\geq)\ v(t,T,y^{(n-1)}(T))$, $t \geq T$ where $v(t,a,v_0)$ is the maximal (minimal) solution of $v' = -f(t,dt^{n-1}v + r(t))$ when $v_0 > 0$ $(v_0 < 0)$ and $d^{-1} = (n-1)!2^{n(n-1)}$.

Theorem. Assume that for each $v_0 > 0$ ($v_0 < 0$), and $a > 0$, the solution $v(t,a,v_0)$ has a zero at some $T > a$. Then each solution of $x^{(n)} + f(t,x) = Q(t)$ oscillates on its maximum interval of existence. (Received November 6, 1974.)

*720-34-33 SADANAND VERMA, University of Nevada, Las Vegas, Nevada 89154. On operational methods via matrix operators for a system of ordinary differential equations. Preliminary report.

While solving ordinary differential equations with constant coefficients, one introduces the operator D, the nth order linear operator with constant coefficients $L \equiv a_0 D^n + \ldots + a_n$, and their inverses in some sense to obtain particular solutions in a convenient fashion. All books on the subject illustrate through their exercises that such operational methods fail in many cases. The intent of this paper is to devise generalized operators and their inverses in order to show that the operational method never fails, and then to extend the operational methods for solving systems of ordinary differential equations with constant coefficients $M(D)X(t) = F(t)$ via the matrix operator $M(D)$ and its inverse $M(D)^{-1}$. The generalized operator $F(D)$ and its inverse $F(D)^{-1}$, associated with an arbitrary function $F(x)$ representable by its Maclaurin's series, are considered. (Received November 6, 1974.) (Author introduced by L. J. Simonoff.)

*720-34-34 J. MICHAEL DOLAN, Western Carolina University, Cullowhee, North Carolina 28723 and GENE A. KLASSEN, University of Tennessee, Knoxville, Tennessee 37916. A geometric oscillatory theory for the equation $L_n y = q$.

By identifying the solution set of $L_n y = q$ with Euclidean n-space, the geometric properties of the set of oscillatory and nonoscillatory solutions of $L_n y = q$ are examined when $L_n y = 0$ is nonoscillatory. It is shown that there are 2 hyperplanes which decompose n-space into three disjoint regions with the set of oscillatory solutions "sandwiched" between the positive and negative solution sets. Simplification of these results is obtained if $L_n y = 0$ is disconjugate. (Received November 6, 1974.)

720-34-35 JOHN V. BAXLEY, Wake Forest University, Winston-Salem, North Carolina 27109. On Singularly Perturbed Initial Value Problems. Preliminary Report.

In his investigation of two-parameter singularly perturbed initial value problems, O'Malley [J. Math. Mech. 16 (1967), 1143-1164] showed that the initial value problem $\varepsilon R'' + \mu a(x)R' + b(x)R = O(\varepsilon^{1/2})$, $R(0) = 0$, $R'(0) = O(\varepsilon^{-1/2})$, where $a(x)$ and $b(x)$ are positive and ε and μ are inter-related small positive parameters satisfying $\varepsilon/\mu \to 0$, $\mu^2/\varepsilon \to 0$ as $\varepsilon \to 0$, has a unique global solution $R(x,\varepsilon)$ on $[0,1]$ which is uniformly bounded as $\varepsilon \to 0$. We obtain a non-linear version of this result. Conditions are formulated which guarantee the global existence on $[a,b]$ of a unique solution $y(x,\varepsilon)$ of the problem $\varepsilon y'' + f(x,y,y',\varepsilon) = 0$, $y(a) = \alpha(\varepsilon)$, $y'(a) = \beta(\varepsilon)$ for $\varepsilon > 0$ sufficiently small. Explicit bounds are obtained on $y(x,\varepsilon)$ and $y'(x,\varepsilon)$ which imply that $y(x,\varepsilon)$ is uniformly bounded on $[a,b]$ as $\varepsilon \to 0$. The linear result mentioned above then appears as a corollary. (Received November 6, 1974.)

720-34-36 J. S. BRADLEY, University of Tennessee, Knoxville, TN 37916 and W. N. EVERITT, The University, Dundee DD1 4HN, Scotland. Singular Integro-Differential Inequalities. Preliminary report.

An inequality of the form $\int_a^b [p|f'|^2 + q|f|^2] \geq \mu_0 \int_a^b f^2$, $(f \in D)$ is established, and the cases of equality are determined, where p, q are real-valued coefficients, with

$p(x) > 0$, μ_0 is a real number, and f is a complex-valued function in a set D so chosen that both sides of the inequality are finite. The interval of integration is of the form $-\infty < a < b < \infty$. The problem is singular in that q is not integrable on (a,b) .
(Received November 6, 1974.)

*720-34-37 DAVID LOWELL LOVELADY, Department of Mathematics; Florida State University Tallahassee, Florida 32306. The Structure of Oscillatory Solutions of Third Order Linear Differential Equations

Let q be a continuous function from $[0,\infty)$ to $(0,\infty)$. THEOREM 1: If u is an oscillatory solution of $u''' - qu = 0$ and $\{f,g\} \subseteq \{u,u',u''\}$ then the zeros of f and g separate each other. If $a<b$, $u(a) = u(b) = 0$, $u(t) \neq 0$ if $a<t<b$, c and d are in $(a,b,)$ and $u'(c) = u''(d) = 0$, then $c<d$. THEOREM 2: If u is an oscillatory solution of $u''' + qu = 0$, and α is the second zero of u, and $\{f,g\} \subseteq \{u,u',u''\}$, then the zeros of f and g on $[\alpha,\infty)$ separate each other. If $\alpha \leq a < b$, $u(a) = u(b) = 0$, $u(t) \neq 0$ if $a<t<b$, c and d are in (a,b), and $u'(c) = u''(d) = 0$, then $d<c$. (Received November 6, 1974.)

720-34-38 ABOLGHASSEM GHAFFARI, Arya-Mehr University of Technology, Teheran, Iran and NASA. On a nonlinear differential equation occurring in astrodynamics.

The integration of (1) $y'' = (1 + y'^2)f(x,y,y')$, which is encountered in astrodynamics (A. Ghaffari, Celestial Mech. 4(1971), 49-53), can be carried out either by quadrature or reduced to a first-order differential equation provided the function $f(x,y,y')$ is linear in y' and an exact total differential. Physical interpretations of (1) are described. (Received November 5, 1974.)

35 Partial Differential Equations

720-35-1 Louis R. Bragg and John W. Dettman, Oakland University, Rochester, Michigan 48063. Analogous Function Theories in the Theory of Heat Conduction and Singular Cauchy Problems. Preliminary Report.

In 1966, Widder pointed out various analogies between the theory of heat conduction and classical analysis. These connections can be made more precise by considering various integral transformations which carry solutions of the heat equation into solutions of Laplace's equation. The basic building blocks in the case of the heat equation are the heat polynomials of Widder and Rosenbloom and their corresponding Appell transforms. The expansions of solutions in terms of these functions hold respectively in time strips and half-planes. The corresponding expansions in the case of Laplace's equation hold respectively inside and outside of a circle in the complex plane. A similar analysis can be carried out for the radial versions of these equations and for the GASPT problem. In these cases, the basic solutions involve Jacobi polynomials, and in the case of GASPT certain transforms of the associated heat functions produce pseudo analytic functions which can be used to solve Dirichlet and Newmann problems for the disk. There are corresponding analogies between the heat equation and the E-P-D equation. (Received October 9, 1974.)

*720-35-2 Louis R. Bragg, Oakland University, Rochester, Michigan 48063. The Riemann-Liouville Integral and Parameter Shifting in a Class of Linear Abstract Cauchy Problems. Preliminary report.

Numerous abstract Cauchy problems, both singular and non-singular, can be transformed into abstract hypergeometric problems involving one or more parameters. The structure of a solution of such a problem corresponding to one set of values of these parameters can differ substantially from a solution corresponding to a different set of values for these parameters. In this paper, the author uses the properties of the Riemann-Liouville integral to develop an extensive class of parameter shifting formulas from a pair of basic integrals connecting solutions of these hypergeometric problems. The results are applied to solutions of a variety of singular Cauchy problems and to some special functions. (Received October 11, 1974.)

720-35-3 L.R. HUNT and M.J. STRAUSS, Texas Tech University, Lubbock, TX 79409. On Holmgren's Theorem. Preliminary Report.

Let M be a real hypersurface in $\mathbb{C}^n$, $n \geq 2$, and assume that the Levi form on M vanishes at most on a nowhere dense subset of M. Let N be a real (2n-2)-dimensional C^∞ submanifold of M. Sufficient conditions are given for Cauchy-Riemann functions on M that vanish on N to vanish on an open subset of M containing a given point of N. (Received October 15, 1974.)

*720-35-4 JOHN GUCKENHEIMER, UNIVERSITY OF CALIFORNIA, SANTA CRUZ, Regularity for A Single Conservation Law

Existence and uniqueness theorems for conservation laws generally rely upon techniques which allow one to say little about the geometric structure of the discontinuity set of a solution. This paper deals with this problem. A regularity theorem for "generic" solutions of a single "generic" conservation law in one space dimension is stated. Examples are given and problems stated concerning the structure of the discontinuity set for a single conservation law in two space dimensions. (Received October 15, 1974.)

*720-35-5 LOUIS B. BUSHARD, Aerospace Research Laboratories, Wright-Patterson AFB, Ohio 45433. Comparison and uniqueness results for a class of quasilinear elliptic partial differential equations, Preliminary report.

A comparison theorem and a uniqueness corollary for positive solutions to the equation $\sum_{i=1}^{n} \left(p_i(x,u)u_{x_i}\right)_{x_i} + q(x,u)u = 0$ on the closure of a bounded open set are found. The important hypotheses on the nonlinear coefficients are that each p_i is positive and monotone increasing in u and q is monotone decreasing in u. (Received October 17, 1974.)
(Author introduced by Dr. Dennis W. Quinn.)

*720-35-6 RENE P. SPERB, Dept. of Math., University of Tennessee, Knoxville, Tenn. 37916
Extension of two Theorems of Payne to some Nonlinear Dirichlet Problems.

It is shown that under the appropriate convexity and symmetry assumption on a domain $D \subset R^2$ the solution u of $\Delta u + \lambda f(u) = 0$ in D, $u = 0$ and D has only one interior critical point. This extends results of L. E. Payne. (Received October 21, 1974.) (Author introduced by Professor P. W. Schaefer.)

*720-35-7 PATRICK M. BROWN, Department of Mathematics, Gonzaga University, Spokane, Washington 99202; ROBERT P. GILBERT, Department of Mathematics, Indiana University, Bloomington, Indiana 47405; and GEORGE C. HSIAO, Department of Mathematics, University of Delaware, Newark, Delaware 19711. Constructive function theoretic methods for fourth order pseudoparabolic and meta-parabolic equations in two space variables.

Two problems are discussed: an initial-boundary value problem for a fourth order pseudoparabolic equation, and a non-characteristic Cauchy problem for a fourth order meta-parabolic equation in two space variables with analytic, time independent coefficients. Integral representations for the solutions are obtained in terms of fundamental solutions. In both problems, a fundamental solution for the corresponding equation is constructed by recursively solving an infinite family of complex Goursat problems. (Received October 23, 1974.)

*720-35-8 R. E. SHOWALTER, The University of Texas at Austin, Austin, Texas 78712.
Nonlinear Degenerate Evolution Equations and Partial Differential Equations of Mixed Type.

The Cauchy problem for the evolution equation $Mu'(t) + N(t,u(t)) = 0$ is studied, where M and $N(t,\cdot)$ are, respectively, possibly degenerate and nonlinear monotone operators from a vector space to its dual. Sufficient conditions for existence and for uniqueness of solutions are obtained by reducing the problem to an equivalent one in which M is the identity but each $N(t,\cdot)$ is multivalued and accretive in a Hilbert space. Applications include weak global solutions of boundary value problems with quasilinear partial differential equations of mixed Sobolev-parabolic-elliptic type, boundary conditions with mixed space-time derivatives, and those of the fourth or fifth type. Similar existence and uniqueness results are given for the semilinear and degenerate wave equation $Bu''(t) + F(t,u'(t)) + Au(t) = 0$, where each non-linear $F(t,\cdot)$ is monotone and the non-negative B and positive A are self-adjoint operators from a reflexive Banach space to its dual. (Received October 24, 1974.)

720-35-9 HOMER F. WALKER, University of Houston, Houston, Texas 77004. An existence theorem for first-order elliptic operators in R^n. Preliminary report.

Suppose $A = A_\infty + \sum_{i=1}^{n} b_i(x) \frac{\partial}{\partial x} + b_0(x)$ is an elliptic operator in R^n which is the sum of a constant-coefficient elliptic operator A_∞ and a perturbation term satisfying the decay conditions $\lim_{|x|\to\infty} |b_i(x)| = 0$, $i = 1,\ldots,n$, and $\lim_{|x|\to\infty} |x||b_0(x)| = 0$. Then the

following Fred holm-type existence theorem holds: a function $f \in L_2$ is in the range of A if and only if f is in the range of A_∞ and orthogonal to the (finite-dimensional) null-space of A^* in L_2. This is an extension of earlier work of the author and is based on results obtained jointly with L. Nirenberg. (Received October 25, 1974.)

*720-35-10 C. V. PAO, North Carolina State University, Raleigh, North Carolina 27607.
On A Uniform Parabolic Equation With Mixed Boundary Condition

Let Ω be a bounded domain and let $\Gamma = \Gamma_1 \cap \Gamma_2$ be the boundary surface of Ω. Consider the parabolic equation $Lu \equiv b(t,x)u_t - \sum_{i,j=1}^{n} (a_{ij}(t,x)u_{x_j})_{x_i} + c(t,x) = f(t,x)$ subjecting the initial condition $u(0,x) = u_0(x)$ $(x \in \Omega)$ and the boundary condition $\partial u/\partial\nu + \beta(t,x)u = 0$ $(x \in \Gamma_1)$, $u(t,x) = 0$ $(x \in \Gamma_2)$, where $\beta \geq 0$ and $\partial/\partial\nu$ denotes the conormal derivative on Γ_1. The operator L is uniformly parabolic if the matrix $A \equiv (a_{ij})$ is positive definite and the function b is positive on $[0,T] \times \bar{\Omega}$. In this paper we study a nonuniformly parabolic equation in the sense that A is only positive semi-definite and b is not necessarily positive. It is shown that if $b(0,x) \geq 0$ and for some $\delta > 0$, $c - 1/2b_t \geq \delta$ then the system has a unique weak solution in some appropriate sense. When $b(t,x) \equiv 0$ the above system is reduced to a degenerate elliptic system. The existence of a unique weak solution for the elliptic problem is then deduced from the corresponding parabolic problem. (Received October 29, 1974.)

720-35-11 HAROLD D. MEYER, Texas Tech University, Lubbock, Texas 79409. A representation for distributional solutions of parabolic problems. Preliminary Report.

A boundary-integral representation is derived for certain distributional solutions of the parabolic problem

$$\sum_{|p|,|q|\leq m} (-1)^{|p|} D_x^p(a_{pq}(x,t)D_x^q u) + \frac{\partial u}{\partial t} = 0$$

in a finite cylinder, where the base of the cylinder is an analytic manifold. The method involves use of a duality principle at the boundary and extends results in [H. D. Meyer, A representation for a distributional solution of the heat equation, to appear in SIAM J. Math. Anal.]. A corollary provides a version of the result suitable for practical applications. (Received October 29, 1974.)

720-35-12 LEONARD J. LIPKIN, University of North Florida, Jacksonville, Florida 32216
On a singular parabolic operator. Preliminary report.

We study the operator $L[u] = \frac{\partial u}{\partial t} - \Delta r^2 u$, $r^2 = \sum x_i^2$, which is parabolic except at an isolated interior point of the x-space. In the class of functions $u(t,x)$ which are analytic in (t,x) for t in some interval in $\mathbb{R}^1$ and $x \in B_\rho$ (the ball of radius $\rho > 0$ with center at the origin in $\mathbb{R}^n$), we exhibit all solutions of $L[u]=0$ using the series $u(t,x) = \sum_{k=0}^{\infty} r^{2k} e^{2(k+1)(2k+n)t} h_k(e^{4(k+1)t}x)$, where each h_k is harmonic and suitable conditions are required for convergence. A representation is also obtained for solutions of $L[u]=f(t,x)$, $f(t,x)$ analytic. As a consequence we find contrasts to the "regular" parabolic case. Among them are: (1) a solution of $L[u]=0$ analytic in x at $t=t_0$ is analytic for all $t<t_0$, and examples are given to show that there may be no forward analyticity; (2) for prescribed analytic initial values at $t=t_0$ and no prescribed boundary con-

ditions there exists a unique local analytic solution of $L[u]=0$ valid for $t<t_0$, but the "sections" $u(t_1,x)$, $t_1<t_0$, must be in a special subclass of analytic functions; (3) for given initial values $u(t_0,x)$, $x \in B_\rho$, the domain of analyticity in x-space of $u(t_1,x)$ may vary in t_1; (4) there exist solutions which possess certain "interior" maxima. The same methods may be employed in the study of the singular hyperbolic operator $L[u]=\frac{\partial^2 u}{\partial t^2}-\Delta_r^2 u$.
(Received October 30, 1974.)

720-35-13 BARBARA KEYFITZ: Columbia University, New York, N.Y. 10027. Formation of a Shock from a compression wave. Preliminary Report.

For a system of two conservation laws $u_t + f(u,v)_x = 0$, $v_t + g(u,v)_x = 0$, such that Riemann invariants are defined, consider Cauchy data such that one Riemann invariant is initially constant. Under certain restrictions on the equations and on the data it is shown that a unique, smooth shock forms and at a later time, a rarefaction wave of the opposite family, enclosing between them a region of smooth, fully-developed flow. This makes rigorous some of the results of K. O. Friedrichs [Comm. Pure Appl. Math 2 (1949)] on approximate shock paths and profiles arising from a compression wave, and extends, in this special case, the local results of S. R. Goldner on nonlinear hyperbolic equations with a discontinuity along a curve Thesis, N. Y. U., 1949] to all t. The method of proof employs the hodograph transformation.
(Received October 30, 1974.)

720-35-14 EDWIN T. HOEFER, Rosary Hill College, Buffalo, New York 14226. Harmonic Faber polynomials. Preliminary report.

Solutions of the three dimensional Laplace equation may be obtained by the method of Bergman integral operators. Let $\bar{b}$ be a compact, connected set in E^3 which is related by the method of the B_3-integral operator to the set $K \subset C$. Suppose that K has a set of Faber polynomials, $\{p_k(u)\}$, $k = 1,2,3,\ldots$ Apply the B_3-integral operator to the set $\{p_k(u)t^m\}$, $m = 0, \pm 1, \pm 2, \ldots, \pm k$, to obtain $H_{k,m}(X)$ $B_3(p_k(u)t^m, X, L)$. The set of all such $H_{k,m}(X)$ is called the set of harmonic Faber polynomials belonging to $\bar{b}$. Theorem. Let m be fixed. If $\{a_k\}$ is an infinite sequence of complex numbers such that $\overline{\lim}|a_k|^{1/k}$ $1/R$, then the series $\sum_{k=1}^{\infty} a_k H_{k,m}(X)$ converges absolutely and uniformly on $b_R \supset \bar{b}$ and represents a harmonic function there. Theorem. The harmonic Faber polynomials $\{H_{k,m}(X)\}$ belonging to $\bar{b}$ satisfy a recursion formula analogous to the formula for the recursion of the Faber polynomials belonging to K. (Received October 30, 1974.) (Author introduced by G. Georgantas.)

*720-35-15 GIDEON PEYSER, New Jersey Institute of Technology, Newark, N.J. 07102
On the corner problem for symmetric positive systems.

Consider the symmetric positive system $A\frac{\partial u}{\partial x}+B\frac{\partial u}{\partial y}+\sum C_i\frac{\partial u}{\partial z_i}+Du = f$ $(i=1,\ldots,m)$, in the corner domain: $x>0$, $y>0$, $-\infty<z_i<\infty$, with homogeneous data on $x=0$ and $y=0$. The $n\times n$ matrices A,B,C_i are symmetric and D is sufficiently positive. The matrix elements and their first order derivatives are continuous and uniformly bounded. On the boundary surfaces the matrix coefficients A,B,C_i satisfy certain positivity conditions, referred to as "torsion" conditions. For f with square integrable first order derivatives, the strong

solution with strong first order derivatives is derived. The principal tools are spaces with norms of negative order, adapted to include boundary values. From this the strong solution is obtained for the case that f is only square integrable. Also, subject to a possibly stronger positivity condition on D, the identity of the weak and strong solutions is established. For partially differentiable f, that is, possessing derivatives with respect to only some of the variables, the partially differentiable strong solution is derived, provided more severe torsion conditions are satisfied on the boundaries. Further, the partially differentiable strong solution is obtained for the case that the torsion conditions are satisfied only on one side of the corner.

(Received October 30, 1974.)

*720-35-16 Professor Robert Carroll, Math. Dept., University of Illinois
Eisenstein integrals and singular Cauchy problems

For previous results see R. Carroll, Notices, AMS, Vol 20, No. 6, Oct. 1973, p. A-615 and R. Carroll and H. Silver, Jour. Applicable Analysis, 3 (1973), 247-266, where references to earlier work are cited. The idea is to characterize, solve, and obtain properties of solutions of singular Cauchy problems, generalizing the classical EPD equations of Weinstein, on homogeneous spaces G/K in terms of Lie theoretic Fourier machinery in the sense of Harish-Chandra and Helgason. For G a real connected noncompact semisimple Lie group and K a maximal compact subgroup with G/K of split rank 1 the problem is completely solved and the objects of interest turn out to be certain Eisenstein integrals. Explicit formulas, recursion relations, etc., are obtained in terms of hypergeometric functions, using recent results of Helgason. The Euclidean case can also be handled, and some higher rank cases.

(Received October 30, 1974.)

*720-35-17 JEFFERY COOPER, University of Maryland, College Park, Maryland, 20742, and
WALTER A. STRASS, Brown University, Providence, R.I. 02906
Scattering of Waves Reflected off a Moving Obstacle.

We consider finite energy solutions of the wave equation $\Box u = 0$ in the exterior of a moving obstacle $O(t)$ with the boundary condition $u = 0$ on $\partial O(t)$ for all t. Let $\nu = (\nu_x, \nu_t)$ be the interior unit normal to the space time boundary $\Sigma = \cup\, \partial O(t) \times \{t\}$. Assume that the obstacle remains in a bounded set and that there exists α, $0 \leq \alpha < 1$ such that

$$\nu_t + \alpha \nu_x \cdot \frac{x}{r} \leq 0 \quad \text{on } \Sigma.$$

When the number n of space dimensions is odd and $n \geq 3$, we prove that the total energy of solutuons is bounded, and that for solutions with initial data of compact support, that the local energy decays exponentially. It is then possible to show the existence of a family of scattering operators $S(t)$, depending on t. The condition on Σ is interpreted as an illumination condition, and this condition (modified slightly) is shown to be invariant under Lorentz transformations which preserve the positive t orientation. As an example, our results apply to the case of a star shaped, pulsating body centered at the origin, assuming only that the body stays within a bounded set, and that Σ is strictly time-like.

(Received October 31, 1974.)

*720-35-18 EUTIQUIO C. YOUNG, Florida State University, Tallahassee, Florida 32306. Comparison and oscillation theorems for singular hyperbolic equations.

Let $Lu \equiv u_{tt} + k(u_t/t) - (a_{ij}u_{x_i})_{x_j} + pu = 0$ and $Mv \equiv v_{tt} + k(v_t/t) - (b_{ij}v_{x_i})_{x_j} + qv = 0$, where k is a real parameter. Comparison theorems are obtained for solutions of $Lu = 0$, $Mv = 0$ subject to certain boundary conditions for various ranges of k in the cylindrical domain $R_\tau = \{(x,t)|x \in D, \tau \leq t \leq T\}$, where D is a bounded domain in E^n. When all the coefficients of the differential equations are independent of t, it is shown that under certain conditions on a solution of $Lu = 0$ every solution of $Mv = 0$, $\partial v/\partial n + sv = 0$ has a zero in the interior of R_0. Oscillation theorems are also obtained for solutions of $Mv = 0$. When $0 \leq p \leq q$ and $0 < r \leq s$, it is shown that every solution of $Mv = 0$, $\partial v/\partial n + sv = 0$ has a zero in $\{(x,t)|x \in D, \tau \leq t \leq \infty\}$ for every τ. More generally, every solution of $Mv = 0$, $v = 0$ on $\partial D \times [0,\infty)$ is oscillatory if the equation $Z'' + k(Z'/t) + CZ = 0$ is oscillatory, where C is some continuous function. (Received October 25, 1974.)

720-35-19 HOWARD A. LEVINE, University of Rhode Island, Kingston, Rhode Island, 02881. Growth properties of solutions to a nonlinear Euler-Poisson-Darboux Equation.

We study the initial and initial boundary value problems for a nonlinear EPD equation of the form $u_{tt}+kt^{-1}u_t-\Delta_n u=f(u)$ where Δ_n is the n dimensional Laplacian and u is a function of n+1 variables. Here f is a convex point function satisfying

$$\liminf_{s\to+\infty} s^{-p-1}f(s)>0$$

for some $p>0$. We show that weak solutions cannot exist for all times for some choices of initial datum $u(x,0)$. The result holds for all $k>-1$ and any space dimension. This improves the result of J. Keller for $k>1$, $n=2$ and classical (twice continuously differentiable) solutions. (Received November 6, 1974.)

720-35-20 J.B. RAUCH and B.A. TAYLOR, University of Michigan, Ann Arbor, Michigan, 48104. The Dirichlet Problem for the Equation of Zero Gauss Curvature.

If $D \subset \mathbb{R}^2$ is an open set then the smooth surface $z = u(x,y)$ has zero Gauss curvature iff $u_{xy}^2 - u_{xx}u_{yy} = 0$. This nonlinear partial differential equation is not elliptic.

Theorem. If D is bounded and strictly convex then for any continuous function $f:\partial D \to \mathbb{R}$ there is a unique convex function u such that $u = f$ on ∂D and $u_{xy}^2 - u_{xx}u_{yy} = 0$ in a generalized sense. In general u will not be smooth and the generalized sense of zero curvature is that the set of unit vectors which are normals to support planes of the convex surface $z = u(x,y)$ over D is a set of measure zero in S^2. (Received November 1, 1974.)

720-35-21 DENNIS W. QUINN, Aerospace Research Laboratories, Wright-Patterson AFB, Ohio 45433. Exterior Dirichlet and Neumann Problems in Generalized Bi-Axially Symmetric Potential Theory, Preliminary report.

Existence and uniqueness theorems for boundary value problems are obtained for the equation of generalized bi-axially symmetric potential theory (GBASPT): $\sum_{i=1}^{n} U_{x_i x_i} + (p/x_{n-1})U_{x_{n-1}} + (q/x_n)U_{x_n} = 0$. For $p = 0$, this equation reduces to Weinstein's generalized axially symmetric potential theory. The problems considered differ from previous results in that exterior Dirichlet, exterior Neumann and interior Neumann boundary value problems are solved. For the exterior Dirichlet problem, the boundary data are allowed to become unbounded near the singular hyperplanes $x_{n-1} = 0$, $x_n = 0$. The existence proofs are constructive in that potential theoretic methods are employed. (Received November 4, 1974.)

720-35-22 NATHANIEL CHAFEE, Georgia Institute of Technology, Atlanta, Georgia, 30332. Saddle Point Behavior for a Nonlinear One-Dimensional Parabolic Equation, Preliminary report.

We shall consider an initial-boundary value problem governed by a nonlinear parabolic partial differential equation in one space variable. This space variable x is allowed to vary on the interval $[0,\pi]$. The boundary conditions are of homogeneous Neumann type. We shall introduce an appropriate phase space for this problem and we shall investigate the phase portrait near an equilibrium solution. Under appropriate hypotheses we shall show that this phase portrait has a saddle point structure. We shall also point out some related aspects of the phase portrait. (Received November 1, 1974.)

*720-35-23 NARENDRA L. MARIA, California State College, Stanislaus, Turlock, California 95380. Elementary Solutions of P(D).

Let $\alpha = (\alpha_0,\alpha_1,\ldots,\alpha_n)$, α_j non-negative integers, $|\alpha| = \sum_{j\le n} \alpha_j$, $D_j = \frac{\partial}{\partial x_j}$, $D^\alpha = D_0^{\alpha_0} D_1^{\alpha_1} \ldots D_n^{\alpha_n}$, $\check{x} = (x_1,x_2,\ldots,x_n)$, $x = (x_0,x_1,\ldots,x_n)$, $P(D) = \sum_{|\alpha|\le m} a_\alpha D^\alpha$, a_α constants, with coefficient of D_0^m unity. Let $E(x_0,\check{x})$ be a distribution in $\check{x}$ which depends on x_0 as a parameter, be a solution of the equation $P(D)E = 0$ for $x_0 > 0$, and be such that $\lim_{x_0\to 0} E(x_0,\check{x})$, $\lim_{x_0\to 0} \frac{\partial E}{\partial x_0}(x_0,\check{x}) = 0,\ldots,$ $\lim_{x_0\to 0} \frac{\partial^{m-2} E}{\partial x_0^{m-2}}(x_0,\check{x}) = 0$, $\lim_{x_0\to 0} \frac{\partial^{m-1} E}{\partial x_0^{m-1}}(x_0,\check{x}) = \delta(\check{x})$; then an elementary solution of P(D) with support in the half-space $x_0 \ge 0$ is given by $e(x) = 0$, for $x_0 < 0$, $= E(x_0,\check{x})$, for $x_0 \ge 0$. The result is used in writing down the elementary solutions in a number of specific cases of P(D) with support in a half-space.
(Received November 1, 1974.)

720-35-24 HOWARD C. SHAW, University of Michigan, Ann Arbor, Michigan 48104. A note on a theorem by Landesman and Lazer. Preliminary report.

Consider the boundary value problem $Lu = Nu$, where L is a second order, selfadjoint, uniformly elliptic operator, with homogeneous Dirichlet boundary conditions and nontrivial null space. Let N be a nonlinear operator of the form $Nu = h(x) - g(u)$, with g real-valued and continuous, h in $L^2(G)$, and G a bounded domain in R^n. Then under sutiable conditions on g, including that $g(\infty)$ and $g(-\infty)$ exist and that (*) $g(-\infty) < g(t) < g(\infty)$, a solution is guaranteed to the nonlinear boundary value problem. (See Landesman and Lazer, J. Math. Mech. 19(1970), 609-623.) We give a much shorter proof of the sufficiency of these conditions that also permits generalization of the operators L and N; in particular, the condition (*) is dropped and N may involve derivatives of u. As in the paper of Landesman and Lazer, use is made of the alternative schemes of Cesari and Hale. (Received November 4, 1974.)

*720-35-25 NEIL EKLUND, Centre College of Kentucky, Danville, Kentucky 40422. Parabolic Green functions in open sets.

The potential theoretic definition of a Green's function is generalized to parabolic operators L with discontinuous coefficients in divergence form. Green's functions are shown to exist for bounded domains D in E^{n+1} and, in the case where $D = \Omega \times (0, T)$ for some bounded domain $\Omega \subset E^n$, coincide with that obtained by D.G. Aronson [Ann. Scuola Norm. Sup. Pisa 22(1968), 607-694]. (Received November 4, 1974.)

*720-35-26 CLIFFORD O. BLOOM, SUNY at Buffalo, Amherst, N. Y. 14226. An Algebraic Rate of Decay of Local Energy of Solutions of Hyperbolic Systems on Exterior Regions, Preliminary Report.

We establish an algebraic rate of decay for the local energy of solutions of initial-boundary value problems for general systems of 2nd order hyperbolic equations with variable coefficients. The solutions we study are defined on exterior regions with star-shaped boundaries. The solutions are required to vanish on the boundary, and have compact initial data with continuous first derivatives. Our decay estimate is derived assuming a piecewise smooth boundary, and that the coefficients in the differential equations have continuous first and second derivatives. We also require hypotheses on the rate of approach of the coefficients to those of a system of uncoupled wave equations as either $r \to \infty$ or $t \to \infty$, and on the magnitude of the first derivatives of the coefficients. The total energy of the solutions may increase algebraically with time. Our results apply to solutions of a general class of symmetric first order hyperbolic systems. The argument we use to obtain energy decay is modeled after that used by Bloom and Kazarinoff to obtain decay estimates for the local energy of solutions of single 2nd order hyperbolic equation with variable coefficients (Cf. Bull. Amer. Math. Soc. 79 (1973), 969-972.). (Received November 4, 1974.)

720-35-27 DANIEL J. DEIGNAN, University of Kentucky, Lexington, Kentucky 40506 Boundary Regularity of Weak Solutions to a Quasilinear Parabolic Equation

Second order nonlinear parabolic equations of the form $u_t = \operatorname{div} A(x, t, u, u_x) + B(x, t, u, u_x)$ are studied in cylinders in $n + 1$ space. I investigate the boundary behavior of weak, or distributional solutions to the first boundary value problem

for this equation. The main result is a Wiener-like condition, involving capacity, which implies continuity of the weak solution at a point on the lateral boundary of the cylinder. (Received November 4, 1974.)

*720-35-28 LETITIA SEESE, University of New Mexico, Albuquerque, New Mexico 87106. Uniqueness in the Cauchy problem for a doubly characteristic operator.

Let P be a second order differential operator doubly characteristic at the origin and strictly hyperbolic on the remaining points of the initial surface t=0. A simple example is $(x^2+y^2)\,\partial_t^2 - \partial_x^2 - \partial_y^2 - c\,\partial_t$. A uniqueness theorem is obtained subject to a certain factorization of the principal symbol if $c \neq -n$, $n \in Z^+$. Counterexamples are given when this condition fails. (Received November 4, 1974.)

720-35-29 R. E. WHITE, North Carolina State University, Raleigh, North Carolina 27607 Local Solutions for Quasi-Linear Semi-Elliptic Differential Equations. Preliminary.

Let $P(x,p_j(D),D)u = \sum_{0<|\alpha:m|\leq 1} a_\alpha(x,p_j(D)u)D^\alpha u$ where $p_j(D)$ $j = 1,\ldots,\ell$ are differential operators with complex constant coefficients, $|\alpha:m| = \sum_{i=1}^{n} \frac{\alpha_i}{m_i}$, $\alpha = (\alpha_1,\ldots,\alpha_n)$, $m = (m_1,\ldots,m_n)$ with α_i non-negative integers and m_i positive integers, and $a_\alpha(x,p) \in C(\Omega \times \mathbb{C}^\ell)$ with $\Omega \subset \mathbb{R}^n$ open and bounded. $P(x,p_j(D),D)$ is defined to be semi-elliptic on Ω if and only if for all $x \in \Omega$ and for all $p \in \mathbb{C}^\ell$ $P_0(x,p,\xi) = \sum_{|\alpha:m|=1} a_\alpha(x,p)\xi^\alpha = 0$ implies $\xi = 0$ where $\xi \in \mathbb{R}^n$. Let $\tilde{p}_j(\xi)^2 = \sum_{|\alpha|\geq 0} i^{|\alpha|}|D^\alpha p_j(\xi)|^2 \leq C(1 + \sum_{i=1}^{n} |\xi_i|^{m_i})^2/(1 + |\xi|^2)$ for all $\xi \in \mathbb{R}^n$ where C is a constant depending only on j. Let $a_\alpha(x,p) \in C^{M+s+2+[n/2]}(\Omega \times \mathbb{C}^\ell)$ and $f(x,q) \in C^{s+1+]n/2]}(\Omega\times\mathbb{C})$ where $M = \max_{1\leq i\leq n} m_i$ and s is a positive integer. Using the Schauder fixed point theorem we prove for all $x_0 \in \Omega$ there exist a compact neighborhood $K \subset \Omega$ of x_0 such that $P(x,p_j(D),D)u = f(x,u)$ has a local solution in $H^c_{s+m+[n/2]}(K)$. (Received November 4, 1974.)

*720-35-30 WILL Y. LEE, Rutgers University, Camden, New Jersey 08102. On a correctness class of the Bessell type differential operator S_μ. Preliminary report.

A correctness class of the Cauchy problem of the Bessell type differential operator S_μ was found among smooth functions with compact supports. It is left as an open problem to find a correctness class among nonsmooth functions with certain boundary conditions. (Received November 4, 1974.)

720-35-31 MICHAEL G. CRANDALL, University of Wisconsin - Madison, Madison, Wisconsin 53706. The Semigroup Approach to a Single Conservation Law.

S. N. Kruzkov (Math. U.S.S.R. - Sb. 10(1970), 217-243) defined a notion of the solution of a single conservation law and proved existence and uniqueness of solutions of the associated Cauchy problem. The flow so

obtained in fact agrees with the evolution governed by an m-accretive operator in $L^1(R^N)$. This relationship, developed in the work of the author, Ph. Benilan and others, is described. (Received November 5, 1974.)

720-35-32 CONSTANTINE M. DAFERMOS, Brown University, Providence, R.I. 02912. Admissibility criteria for solutions of conservation laws.

We survey various admissibility criteria for weak solutions of hyperbolic conservation laws and we discuss the interrelationship, established or conjectured, of these criteria. (Received November 5, 1974.)

720-35-33 RONALD J. DiPERNA, University of Michigan, Ann Arbor, Michigan 48104
Hyperbolic conservation laws and the theory of functions of bounded variations.

One of the natural function spaces in the theory of conservation laws is the space BV of functions which have bounded variation in the sense of Tonelli-Cesari. The purpose of this talk is to give a short introduction to BV theory from the point of view of conservation laws and to discuss certain open problems in the regularity of solutions which are naturally posed in the framework of BV theory. (Received November 5, 1974.)

720-35-34 AVRON DOUGLIS, University of Maryland, College Park, Md., 20742. Layering methods for parabolic systems, Preliminary report.

Methods of layering, originally designed to obtain weak solutions of scalar, quasilinear, first order conservation laws [Kuznetsov, Math. Zametki 2 (1967), 401-410; Douglis, Ann. Inst. Fourier 22 (1972), 141-227] are here extended to parabolic problems of certain kinds. The principal results so far refer to a system of equations of the form

$$(1)_\mu \quad \frac{\partial u_i}{\partial t} + \frac{\partial f_i(x,t,u(x,t))}{\partial x} + g_i(x,t,u) = \mu_i \frac{\partial^2 u_i}{\partial x^2}, \quad i = 1,\cdots,n, \quad \text{where } \mu_i > 0,$$

and the matrix $(\partial f_i/\partial u_j)$ has real, distinct eigenvalues. In initial-value problems for $(1)_\mu$ in the upper-half xt-plane, layering permits the existence, smoothness, and other properties of solutions to be established by working with smooth solutions of the hyperbolic system

$$(1)_o \quad \frac{\partial u_i}{\partial t} + \frac{\partial f_i(x,t,u(x,t))}{\partial x} + g_i(x,t,u) = 0, \quad i = 1,\cdots,n,$$

only. The layering method also leads to proofs of convergence of certain finite difference schemes with discontinuous intitial data. These results of course carry over to boundary-value problems that are reducible to pure initial-value problems, say by reflection. Generalizations to multi-dimensional x are made in the case $n = 1$. (Received November 5, 1974.)

*720-35-35 JAMES M. GREENBERG, State University of New York at Buffalo, Buffalo, New York 14226. Decay theorems for the quasilinear wave equation.

We shall confine our attention to piston type problems for the quasilinear wave equation (WE) $\frac{1}{c(q)}\frac{\partial q}{\partial t} - \frac{\partial v}{\partial x} = 0$ and $\frac{\partial v}{\partial t} - c(q)\frac{\partial q}{\partial x} = 0$, where $c(q) > 0$ and $dc(q)/dq < 0$ for $-\infty < q < \infty$; that is to solutions of (WE) in $\{x > 0 \text{ and } t > 0\}$ which satisfy a homogeneous initial condition and one of the following boundary conditions: $(BC)_1$ $q(0,t) = q_o(t)$ where $q_o(t) < 0$, $0 < t < T_1$ and $q_o(t) \equiv 0$, $t > T_1$ or $(BC)_2$ $v(0,t) = v_o(t)$ where $v_o(t) > 0$, $0 < t < T_1$ and $v_o(t) \equiv 0$, $t > T_1$. We shall demonstrate how the vanishing of the forms $U(q)dx + vd\tau$ and $vdx + \Sigma(q)d\tau$ with $U(q) = \int_o^q ds/c(s)$ and $\Sigma(q) = \int_o^q c(s)\,ds$ around

certain judiciously chosen contours in $\{x > 0$ and $t > 0\}$ together with an integrated form of the "entropy" inequality may be exploited to produce the estimate that solutions to the above problems decay as $1/t^{\frac{1}{2}}$ as $t \to +\infty$. (Received November 5, 1974.)

*720-35-36 TAI-PING LIU, University of Maryland, College Park, Maryland, 20742. Entropy Condition for General Systems of Hyperbolic Conservation Laws.

We consider general systems of conservation laws

$$(1) \qquad u_t + f(u)_x = 0, \qquad u = (u_1, \cdots, u_n), \quad t \geq 0, \quad -\infty < x < \infty.$$

Assume that (1) is strictly hyperbolic and that along any shock curve, the shock speed is unequal to the characteristic speed of different family. An extended condition (E) is proposed for (1). Condition (E) extends Oleinik's condition (E) to systems and reduces to Lax's shock inequalities (L) when (1) is genuinely nonlinear. For 2×2 general conservation laws and 3×3 gas dynamics equations, we prove that there exists a unique solution to the Riemann problem in the class of centered simple waves which satisfy condition (E). We also justify condition E for 2×2 conservation laws by viscosity method. For gas dynamics equations, it is shown that the thermodynamics entropy increases across any shock satisfying condition (E).

(Received November 5, 1974.)

*720-35-37 TAKAAKI NISHIDA, Kyoto University, Kyoto Japan and JOEL SMOLLER, University of Michigan, Ann Arbor, Michigan 48104. Mixed problems for a class of nonlinear conservation laws.

We consider mixed problems for hyperbolic systems of conservation laws of the form $v_t - u_x = 0$, $u_t + p(v)_x = 0$, where $p(v) = v^{-(1+\varepsilon)}$, $0 \leq \varepsilon < 1$, in regions a) $t \geq 0$, $x \geq x_1 = 0$, and in regions b) $t \geq 0$, $x_1 \leq x \leq x_2$. On the vertical boundaries we prescribe the velocity $u = u_i(t)$, $i = 1,2$, and on the remaining boundaries we prescribe the Cauchy data $u_o(x)$, $v_o(x)$. We assume (1) $0 < A \leq v_o(x) \leq B < +\infty$, $|u_o(x)| \leq C < +\infty$ and $u_1(t)$, $u_2(t)$, $v_o(x)$, $u_o(x)$ have finite total variation. For problems in regions a), global solutions exist if $\varepsilon \cdot TV\{u_1, v_o, u_o\}$ is sufficiently small. In regions b), we set $Q(t) = \int_{x_1}^{x_2} v_o(x)\,dx + \int_0^t (u_2(s) - u_1(s))\,ds$, and we assume (2) $0 < \delta \leq Q(t) \leq V < +\infty$ for all $t \geq 0$, where δ and V are constants. Then global solutions exist if $\varepsilon \cdot TV\{u_1, u_2, v_o, u_o\}$ is sufficiently small. Assumption (2) has a natural physical interpretation. We construct an example violating (2) for which global solutions fail to exist.

(Received November 5, 1974.)

720-35-38 BURTON WENDROFF, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87544. Numerical Methods for Conservation Laws.

A short survey of computational methods is presented, with particular emphasis on the equations of compressible gas dynamics. Unresolved questions of convergence are discussed.

(Received November 5, 1974.)

*720-35-39 WILLIAM F. MOSS, Georgia Institute of Technology, Atlanta, Georgia 30332. Existence of Fundamental Solutions for Degenerate or Singular, Second Order, Linear, Elliptic Partial Differential Equations.

Let G denote an open, connect set in R^n and L the partial differential operator. Under suitable hypotheses on the coefficients of L, there exists a fundamental solution for L in G. If certain "barrier" functions exist at each point of $\dot{G}$, the boundary of G, then this fundamental solution is continuous up to $\dot{G}$. The ellipticity of L may degenerate on $\dot{G}$, or the coefficients of L may be discontinuous on $\dot{G}$. This class of equations includes, for example, Tricomi's equation in the upper half-plane, the equation of Weinstein's generalized axially symmetric potential theory, and Schrödinger's equation with a singular potential. (Received November 5, 1974.)

720-35-40 PETER D. LAX, New York University, New York, N.Y. 10012 Survey of Problems in the Theory of Shock Waves

Existence, uniqueness, decay and numerical computation of solutions of hyperbolic systems of conservation laws. Steady and unsteady flows with shocks in two space dimensions. (Received November 6, 1974.)

*720-35-41 J.B.DIAZ, Rensselaer Polytechnic Institute, Troy, N.Y. 12180 and R.B.Ram, State University College, Oneonta, N.Y. 13820. Reflection principles for the iterated Helmholtz equation.

The well known reflection principle for harmonic functions, due to Schwarz, has been extended in many directions. Diaz & Ludford (Reflection principles for linear elliptic 2nd order p.d.es. with constant coefficients, Annali di Matematica pura ed applicata, Serie IV, XXXIX 1955, pp.87-95) have proved the reflection principles for Helmholtz equation with 3 kinds of b.cs.: (b_1) of Dirichlet type, (b_2) involving a linear combination of the normal derivative and the function, (b_3) involving a linear combination of a directional derivative and the function. The reflection principle for the biharmonic equation with Dirichlet b.cs. is given in the work of Poritsky (Application of analytic functions to two dimensional biharmonic analysis, Trans. of the AMS, vol 59 (1946) pp.248-279), Duffin (Continuation of biharmonic functions by reflection, Duke Math J. 22 (1955) pp.313-324), Huber (The reflection principle for polyharmonic functions, Pacific J. of Math, vol 5, No.3 (1955) pp.433-439, On the reflection principle for polyharmonic functions, Comm. on Pure and Appl. Math, vol IX (1956) pp.471-78). Their results were used by Diaz & Ram (A reflection principle for a 4th order p.d.e. which is related to the biharmonic eqn.- to appear) to obtain a reflection principle for iterated Helmholtz eqn with Dirichlet b.cs. This result of Diaz & Ram has been used in this paper to obtain reflection principles for iterated Helmholtz eqn. with b.cs. of the type: (b_4) the simultaneous vanishing, on the boundary, of a linear comb. of the function and its normal derivative, and, also, of the normal derivative of the same lin. comb., (b_5) the simultaneous vanishing, on the boundary, of a linear comb. of the function and its directional derivative in a fixed direction, and, also, of the directional derivative, in the same fixed direction, of the same linear combination. (Received November 6, 1974.)

*720-35-42 DAVID G. SCHAEFFER, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. The generic theory of regularity for a single convex conservation law.

The author applies the theory of singularities of differentiable mappings to prove that generically the solution of a conservation law is piecewise smooth. Only a single convex conservation law is considered, and the method makes essential use of the Hopf-Lax formula for the solution in this case. (Received November 6, 1974.)

*720-35-43 FRANK J. MASSEY, III, University of Kentucky, Lexington, Kentucky 40506
Analyticity of solutions of nonlinear evolution equations

We establish analyticity in t of solutions $u(t)$ of nonlinear evolution equations of the form $du/dt + A(u)u = f(u)$, $t \geqslant 0$, $u(0) = u_0$. The unknown $u(t)$ is a function of t with values in a complex Banach space X, while $A(u)$ is a linear operator in X and $f(u) \in X$. **Theorem.** Assume the following. The resolvent of $A_0 \equiv A(u_0)$ contains the left half plane and there exists C such that $\|(\lambda - A_0)^{-1}\| \leqslant C(1 + |\lambda|)^{-1}$, $\operatorname{Re}\lambda \leqslant 0$. The domain, D, of A_0 is dense in X. There exists $0 \leqslant \alpha < \beta \leqslant 1$, $R > 0$ such that $u_0 \in D(A_0^{\beta})$ and for $\| w - A_0^{\alpha} u_0 \| < R$, one has $f(A_0^{-\alpha} w) \in X$ and $A(A_0^{-\alpha} w)$ is a linear map from D into X. The maps $w \mapsto A(A_0^{-\alpha} w)A_0^{-1}$ and $w \mapsto f(A_0^{-\alpha} w)$ are analytic with values in $B(X)$ and X respectively. Under these assumptions there exists $T > 0$, $\varphi > 0$ and a unique function mapping $\Sigma = \{t: |\arg t| < \varphi, 0 < |t| < T\}$ analytically into X such that $u(t) \in D$, $\|u(t) - A_0^{\alpha} u_0\| < R$, $u'(t) + A(u(t)) = f(u(t))$, $t \in \Sigma$ and $u(t) \to u_0$ as $t \to 0$. This theorem can be generalized to the case where A and f depend on t. Applications are given to parabolic mixed problems of the form $\partial u/\partial t + \Sigma a_{ij}(t,x,u,\nabla u)\, \partial^2 u/\partial x_i \partial x_j = f(t,x,u,\nabla u)$, $t \geqslant 0$, $x \in \Omega \subset \mathbf{R}^n$, $u(x,0) = u_0(x)$, $x \in \Omega$, $u(x,t) = 0$, $x \in \partial\Omega$, $t \geqslant 0$. (Received November 6, 1974.)

*720-35-44 W. RUNDELL, Department of Mathematics, Texas A&M University, College Station, Texas, 77843, The Solution Of Initial-Boundary Value Problems For Equations Of Sobolev Type.

Using an analogue of Hadamard's method a fundamental solution for linear, third order equations of Sololev type is constructed. This fundamental solution is then used to solve initial-boundary value problems for this class of equations. (Received November 6, 1974.) (Author introduced by Dr. Michael Stecher)

720-35-45 M. J. STECHER, Department of Mathematics, Texas A&M University, College Station, Texas, 77843, Construction Of Solutions Of Parabolic Partial Differential Equations.

A constructive approach to solving initial-boundary value problems for certain types of linear parabolic partial differential equations in three space variables is developed. The approach is based upon constructing Bergman type integral operators and then reducing the original problem to solving an integral equation. (Received November 6, 1974.)

720-35-46 E. D. CONWAY, Tulane University, New Orleans, Louisiana 70118, Formation and decay of shock waves in several dimensions.

We consider the initial value problem for a single conservation law in several dimensions:

$$u_t + \nabla \cdot F(u) = 0, \quad u(\circ,x) = \phi(x).$$

Here $F: I \subset \mathbb{R} \to \mathbb{R}^n$, $\phi(x) \in I$ for all x in $\mathbb{R}^n$. If ϕ is C^1 then a shock forms iff $F''(\phi) \cdot \nabla\phi$ is negative somewhere. The time at which a shock first appears is τ_0, $\tau_0^{-1} = \sup\{-F''(\phi) \cdot \nabla\phi\}$. Let u be the unique generalized solution satisfying Kruzkor's entropy condition. Theorem if $F''(s) \cdot \hat{e} > 0$ for all $s \in I$ where $\hat{e}$ is some unit vector then $\sup\{|u(t,x)| : x \in \mathbb{R}^n\} = O(t^{-\frac{1}{2}})$ provided that the support of ϕ is bounded in the direction $\hat{e}$. If ϕ has compact support then the support of $u(t,\cdot)$ is asymptotically translated along the vector $F'(0)$. Its dimension in the direction of $F''(0)$ is $O(t^{\frac{1}{2}})$ while its dimension in directions normal to $F''(0)$ is bounded. (Received November 6, 1974.)

720-39-1 C.S. DURIS, Drexel University, Philadelphia, PA 19104. Interpolating with Discrete Natural Polynomial Spline Functions.

Discrete natural polynomial spline functions of degree 2n-1 with nodes $\Lambda_k = \{m_0, m_1, \ldots m_k\}$, $m_0 = 0$, $m_k = N$, are functions $S(x)$ defined for integers $x \in [-n+1:N+n-1]$ and satisfying $\nabla^n \Delta^n S(x) = 0$ for $x \in [0:N] - \Lambda_k$ and $\Delta^n S(x) = 0$ for $x \leq -1$ and $x \geq N-n+1$. In a recent paper, Astor and Duris [to appear in Numer. Math (1974)] have shown that an analog of the minimum norm property for continuous splines is also satisfied by these discrete natural splines. This minimum norm property causes the discrete natural splines to be desirable for interpolation. In this paper some further results are given concerning discrete B-splines developed by Schumaker [In Approximation Theory, G.G. Lorentz, ed. New York. Academic Press 1973]. These results are then used to develop a procedure for carrying our discrete natural spline interpolation, which is analogous to a procedure for continuous splines presented by Anselone and Laurent [Numer. Math. 12, 66-82 (1968)]. The method is based on the use of discrete B-splines to form a basis for the class of n-th differences of discrete natural splines of degree 2n-1. The actual computer algorithm for this interpolation procedure is reasonable in terms of computational stability and efficiency. (Received November 5, 1974.)

*720-39-2 VLADIMIR DROBOT, University of Santa Clara, Santa Clara, Cal.,95050 On the speed of convergence of iterations of a function.

Let $f(x)$ be a continuous function such that $0 < f(x) < x$, $f(0) = 0$. Put $f_1(x) = f(x)$, $f_{n+1}(x) = f(f_n(x))$. It was shown by A.Ostrowski that if $f(x) = x - Ax^p + o(x^p)$, $p > 1$, then (*) $\lim_n n^b f_n(x) = B$, where $b(p-1) = 1$ and $B^{p-1}(A(p-1)) = 1$. If $f(x)$ is concave, we prove the following converse: If (*) holds then $x - x^{p-\epsilon} < f(x) < x - x^{p+\epsilon}$ for arbitrary $\epsilon > 0$ and x sufficiently small. (Received November 6, 1974.)

720-39-3 H. T. BANKS, Brown University, Providence, Rholde Island 02912, and J. A. BURNS, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. A note on linear hereditary systems in L^p-spaces. Preliminary report.

Working in the space of continuous functions, Hale and Shimanov developed a method of finite-dimensional projections for linear retarded functional differential equations. By an appropriate choice of state space, the equation may be treated as an equivalent abstract equation in a Banach space. In this note we exploit this equivalence and indicate how functional analytic techniques may be applied to develop a theory which parallels that developed by Hale and Shimanov. (Received November 6, 1974.)

720-39-4 BERNARD A. ASNER, JR., University of Dallas, Irving, Texas 75060. Representation of the solution of a delay-differential equation. Preliminary report.

It is known that the solutions to the linear vector delay-differential equation $\dot{x}(t) = Ax(t) + Bx(t-h)$ may be expressed by using the fundamental matrix $X(t)$. In situations where the equation is pointwise degenerate (perhaps strongly), a Z matrix is known to exist and satisfy the matrix equations $B = AZ - ZA$, $ZB = 0$. In this case, we prove the following representation for the fundamental matrix

$$X(t) = e^{At} + \sum_{j=1}^{\infty} (-Z)^{j-1} (e^{A(t-jh)} Z - Ze^{A(t-jh)}) u(t-jh).$$

where $u(t-jh)$ is the unit step function. (Received November 6, 1974.)

720-40-1 DAVID F. DAWSON, North Texas State University, Denton, Texas 76203. A Tauberian theorem for stretchings.

I. J. Maddox [Bull. London Math. Soc. 2(1970), 63-65] proved the following Tauberian theorem for subsequences: If x is a sequence such that some non-Schur matrix sums every subsequence of x, then x is convergent, and the set of non-Schur matrices is maximal for this statement. Let D denote the set of all complex matrices $A = (a_{pq})$ such that for some N, $\lim_p a_{pq} = c_q$ for $q > N$, $\sum_{q=N+1}^{\infty} c_q$ converges, and $\lim_p \sum_{q=N+1}^{\infty}(a_{pq} - c_q) = 0$. In the present paper the following Tauberian theorem for stretchings is proved: If x is a complex sequence such that some non-D matrix sums every stretching of x, then x is convergent, and the set of non-D matrices is maximal for this statement. (Received August 19, 1974.)

720-40-2 BILL ANDERSON, East Texas State University, Commerce, Texas 75428 and KENNY ZUBER, New Mexico State University, Las Cruces, N. M. 88001. Convergence of generalized power series. Preliminary report.

A generalized power series is a series of the form $\Sigma a_n x^{\phi(n)}$ where $\phi(n)$ is a function satisfying the conditions: 1) $0 < \phi(1) < \phi(2) < \ldots < \phi(n) < \ldots$ 2) $\lim \phi(n) = \infty$. V. Murgescu and A. Climescu (Gaz. Mat. Ser. A 12(65) (1960), 517-522) studied the radius of convergence of a generalized power series and gave the formula $R = \lim \left|\frac{a_n}{a_{n+1}}\right|^{1/[\phi(n+1)-\phi(n)]}$ for the radius of convergence. B. Anderson and R. Ward (Gaz. Mat. Ser. A 11(67) (1972), 406-408) gave a counterexample for this formula in case $\lim [\phi(n+1) - \phi(n)] = 0$. In this paper sufficient conditions are given which imply the validity of this formula for the radius of convergence. (Received September 30, 1974.)

720-40-3 J.K. SHAW, Virginia Polytechnic Institute and State University, Blacksburg, Va. 24061. Series Expansions and Linear Differential Operators. Preliminary report.

Let L be the nth order linear differential operator given by $Ly = a_0 y^{(n)} + a_1 y^{(n-1)} + \ldots + a_n y$, where $a_j \in C^{n-j}[a,b]$ and $a_0(t) \neq 0$, $a \leq t \leq b$. Let $B_1, B_2, \ldots, B_n$ be linearly independent boundary forms and assume that the eigenvalue problem $Ly = \lambda y$, $By = 0$, is self-adjoint. With each such problem there is associated a series expansion whose coefficients are boundary values. If 0 is not an eigenvalue, the expansion takes the form $f(x) = \Sigma_{k=0}^{\infty}\Sigma_{j=1}^{n}(B_j L^k f)p_{nk+j}(x)$, $a \leq x \leq b$, where the functions $\{p_k\}$ are defined in terms of a fundamental system of solutions of the homogeneous equation $Ly = 0$ and the Green's function for the problem. If 0 is a simple eigenvalue, with normalized eigenfunction y_0, the expansion is $f = (f_1 y_0)y_0 + \Sigma_{k=0}^{\infty}\Sigma_{j=1}^{n}(U_j L^k f)\, q_{nk+j}$, where each U_j is a linear combination of $B_1, \ldots B_n$, and the functions $\{q_k\}$ are developed by means of generalized Green's functions. In each instance, necessary and sufficient conditions for absolute convergence are given in terms of the non-zero eigenvalue nearest the origin. For the case $Ly = -y''$, $B_1 y = y(0)$, $B_2 y = y(1)$, the first expansion is the Lidstone series. If $Ly = iy'$, $B_1 y = y(1) - y(0)$, the latter series reduces to the well-known expansion in terms of Bernoulli polynomials. (Received October 24, 1974.) (Author introduced by George W. Crofts.)

*720-40-4 J.A. FRIDY, Kent State University, Kent, OH 44242
Summability of Rearrangements of Sequences

The sequence y is called a rearrangement of the sequence x provided that there is a one-to-one function π from the positive integers onto themselves such that for each k, $y_k = x_{\pi(k)}$. In 1943, R. C. Buck gave a characterization of convergent sequences by proving that if x is a nonconvergent sequence, then no regular summability matrix can sum every subsequence of x [Bul.A.M.S. 49 (1943), 898-899]. The basic result of the present paper is an analogue of Buck's theorem in which "subsequence" is replaced by "rearrangement." This result is then extended to show that for any x and any regular matrix A, there exists a rearrangement y of x such that every limit point of x is a limit point of the transform Ay. Other variations show that one can weaken the assumption that A be regular. Similar results are proved for absolute summability: e.g., the null sequence x is in ℓ^1 if and only if there exists a sum-preserving ℓ-ℓ matrix that maps every rearrangement of x into ℓ^1. (Received October 18, 1974.)

*720-40-5 HELAMAN FERGUSON, Brigham Young University, Provo, Utah 84602.
Generalization of Fibonacci Numbers useful in Dynamic Memory Allocation.

A binary splitting of memory blocks into a pair of blocks gives block sizes corresponding to certain generalized Fibonacci numbers. In general a splitting sequence of block sizes $\{B_n: n \geq 1\}$ has the property that $B_n = B_r + B_m$ where $1 \leq m \leq r < n$. We consider analytically the case of $r = n - 1$, $m = n - k$, $k \geq 1$: generalized Fibonacci numbers with structure polynomial $z^k - z^{k-1} - 1$. Location of the roots gives complete information about limits of quotients for each k and limits of those limits as $k \to \infty$. For example we have the Two Annuli Theorem: All of the complex roots of the above structure polynomial lie in the intersection of two annuli containing the unit circle.
(Received October 31, 1974.)

*720-40-6 W. VANCE UNDERHILL, East Texas State University, Commerce, Texas 75428.
Matrix Summability Theorems Involving a Certain Class of Sequences.

By the limiting set of a sequence is meant the set of all limits of subsequences of the sequence. A closing sequence $x = \{x_n\}$ is a bounded, complex sequence such that $\lim|x_{n+1}-x_n|=0$. It is known that every closing sequence has a connected limiting set. The following partial converse is proved: if x is any bounded sequence having a connected limiting set, then x is a rearrangement of some closing sequence. Let l_∞, c, c_0, and CL denote the Banach Space of all bounded, convergent, null, and closing sequences, respectively. If an infinite, complex matrix A is regarded as a sequence-to-sequence transformation, necessary and sufficient conditions on A are found in order that: (1) A: $c_0 \to$ CL, (2) A: c $\to$ CL, (3) A: $l_\infty \to$ CL.
(Received November 4, 1974.)

*720-40-7 KUSUM SONI, University of Tennessee, Knoxville, Tennessee 37916. A Tauberian Theorem for the Generalized Logarithmic Method of Summation.

Let $L(t)$ be a slowly varying function in the sense of Karamata and let $L^*(t) = \int_0^t (u+1)^{-1} L(u)\, du \to \infty$ as $t \to \infty$. We define the generalized logarithmic method of summation (A, p_n) as follows. If $p_n = L(n)$ and $(\sum_{n=0}^{\infty} p_n s_n x^n)/(\sum_{n=0}^{\infty} p_n x^n) \to s$ as $x \to 1-$, then $s_n \to s\ (A, p_n)$. Theorem. If $\liminf (s_n - s_m) \geq 0$ when $n > m$, $m \to \infty$ and $\log L^*(n) - \log L^*(m) \to 0$, then $s_n \to s\ (A, p_n)$ implies $s_n \to s$. This includes similar results given recently by Kwee [J. Math. Anal. Appl. (1971)] and Phillips [Canad. J. Math. (1973)]. (Received November 6, 1974.)

41 Approximations and Expansions

*720-41-1 SHELDON M. EISENBERG, University of Hartford, West Hartford, Connecticut 06117 and BRUCE WOOD, University of Arizona, Tucson, Arizona 85721. Error estimates in approximation by extended Hermite-Fejer operators.

The authors present an analogue to an estimate of Bojanic ("a note on the precision of interpolation by Hermite-Fejer polynomials", Proceedings of the Conference on the Constructive Theory of Functions, Budapest, 1969, pp. 69-76) for the approximation of unbounded functions on the real line by extended Hermite-Fejer operators. (Received October 15, 1974.)

*720-41-2 MICHAEL BRANNIGAN, Iowa State University, Ames, Iowa 50010. H-sets, an approach to linear Tchebycheff approximation.

We show that H-sets can be defined in a simple manner. Using this definition of H-sets, in terms of singular matrices, many of the theorems of linear Tchebycheff approximation, where the Haar condition is supposed, can be extended to the non-Haar case. Thus a general framework for linear Tchebycheff approximation can be constructed. The maximal linear functional of the dual problem can be written in terms of H-sets and with a strong H-set uniqueness follows.

We also consider a geometric definition for H-sets as the non-trivial intersection of balanced and convex hulls of certain sets. Developing this idea we show that this intersection is a line when we have minimal H-sets and for basic minimal H-sets the intersection of simplices in a single point interior to each simplex. (Received October 24, 1974.) (Author introduced by Richard K. Miller.)

*720-41-3 MICHAEL P. CARROLL, Virginia Polytechnic Institute and State University, Math. Dept., Blacksburg, Virginia 24061, An Algorithm for L_1 Approximation Using Near Best Approximates. Preliminary Report.

Let I be a nondegenerate real compact interval, $[a,b]$. Let f be a continuous real valued function on I and H a Haar system consisting of n functions, $p_1, p_2, \ldots, p_n$. Let $A, (a_1, a_2, \ldots, a_n)$,

be a point in E^n and $L(A,x) = \sum_{i=1}^{n} a_i p_i(x)$. An algorithm is given to construct a sequence $\{A_n\}$ such that $\int_I |f-L(A_n,x)|dx \le \inf_{A \in E^n} \int |f - L(A,x)|dx + \frac{1}{n}$. Further it is shown that $\{A_n\} \to \bar{A}$, where $\int_I |f-L(\bar{A},x)|dx = \inf_{A \in E^n} \int_I |f-L(A,x)|dx$. (Received October 31, 1974.)

*720-41-4 ALBERT EDREI, Syracuse University, Syracuse, New York 13210, The complete Padé tables of certain series of simple fractions II

Let the meromorphic function $A(z) = \kappa + \sum_{j=1}^{\infty} \frac{r_j}{\beta_j - z} = \sum_{k=0}^{\infty} a_k z^k$ be such that (i) all the β's are real, no β is zero and $\beta_j \to \infty$ as $j \to \infty$; (ii) $\kappa \geqq 0$, $r_j/\beta_j > 0$ for all j; (iii) $\Sigma(r_j/\beta_j) < +\infty$; (iv) for some integer $s \geqq 0$ $\Sigma\, 1/\beta_j|^{s+1} < +\infty$. Let $\{m(\lambda)\}_\lambda$, $\{n(\lambda)\}_\lambda$ $(\lambda = 1,2,3,\ldots)$, be two sequences of positive integers such that $m(\lambda) + n(\lambda) =$ odd integer and $m(\lambda) \to \infty$, $n(\lambda) \to \infty$, as $\lambda \to \infty$. Consider the Pade approximant P_λ/Q_λ of the entry $(m(\lambda), n(\lambda))$ of the table of $\sum_{k=0}^{\infty} a_k z^k$ [degree $(P_\lambda) \leqq m(\lambda)$, degree $(Q_\lambda) \leqq n(\lambda)$, $Q_\lambda(0) = 1$]. Then, with each λ, it is possible to associate polynomials $V_\lambda(z)$, $W_\lambda(z)$, both of degree $\leqq s$ and such that, as $\lambda \to \infty$, $P_\lambda(z) \exp(V_\lambda(z)) \to G(z)$, $Q_\lambda(z) \exp(W_\lambda(z)) \to H(z)$, $\{W_\lambda(z)-V_\lambda(z)\} \to \Omega(z)$, uniformly on every compact set.

The entire functions G and H have no common zeros and $A(z) = \exp(\Omega(z))G(z)/H(z)$. The polynomial $\Omega(z)$ is of degree $\leqq s$; the functions G and H are both of order $\leqq s+1$.

(Received October 31, 1974.)

*720-41-5 P. ERDÖS, Stanford University, Stanford, Cal. 94305 and A.R. REDDY, Michigan State University, East Lansing, MI. 48824, Rational approximation on certain unbounded domains of the complex plane.

Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_0 > 0$, $a_k \geq 0$ $(k \geq 1)$ be an entire function. Define $\psi_{0,n} = \psi_{0,n}(1/f) = \inf_{P \in \pi_n} \left\|\frac{1}{f(z)} - \frac{1}{P(z)}\right\|_{L_\infty(G)}$, where π_n denote the class of all complex polynomials in the variable z having degree at most n, G denotes a (suitably defined) certain unbounded domain in the complex plane. We obtained in this note upper and lower bounds for $\psi_{0,n}$, for some class of entire functions of order less than 2. (Received November 1, 1974.)

*720-41-6 T. J. RIVLIN, T. J. Watson Research Center, IBM Corp., Box 218 Yorktown Heights, New York, NY 10598 and E. W. CHENEY, University of Texas at Austin, Austin, Texas 78712. On Some Polynomial Approximation Operators.

A linear projection operator introduced by de La Vallée Poussin produces the polynomial of degree n-1 which best approximates a given function on a discrete set of n+1 points ("nodes"). If the nodes are taken to be the extrema of the nth Tchebycheff polynomial, then the operator norms grow like $(2/\pi)\log n$, and it is conjectured that this is the minimal order of growth for such operators. The paper also discusses the norm of this operator for other choices of nodes, and establishes a theorem to the effect that the projection constant c_n of the nth-order polynomial subspace in $C[-1,1]$ is strictly less than the projection constant ρ_n of the nth order trigonometric polynomials in $C_{2\pi}$. The best current estimate of c_n is $\frac{1}{2}(\rho_n - 1) \leq c_n < \rho_n$.

(Received November 1, 1974.)

720-41-7 W. H. LING, Union College, Schenectady, N. Y. 12308, H. W. MCLAUGHLIN and M. L. SMITH, Rensselaer Polytechnic Institute, Troy, N. Y. 12181, Approximation of Random Functions, Preliminary Report.

Let $f_1(x)$ and $f_2(x)$ be real valued functions defined on $[0,1]$ and occur with probability w_1 and w_2 respectively, where $w_1 + w_2 = 1$. We seek an approximating polynomial p with degree at most n which will minimize the expected value of the error, that is, will minimize $||w_1(f_1-p)||+||w_2(f_2-p)||$, where the norm is the supremum norm. Such a polynomial will be called a best approximation. We show that every best approximation has an alternating property under certain smoothness conditions. Characterization of the set of best approximations and an algorithm for computing a best approximation are presented. Generalizations for more than two functions and for other approximating families are studied. (Received November 4, 1974.)

*720-41-8 C.K. CHUI, E.R. ROZEMA, P.W. SMITH and J. D. WARD, Dept. of Mathematics, Texas A&M University, College Station, Texas, Simultaneous Spline Approximation and Interpolation Preserving Norm.

Let $\mathscr{S}_k$, $k \geq 2$, be the space of all splines of order k and continuity class $C^{k-2}[a,b]$ with a finite number of knots in $[a,b]$. Given any function f in $C[a,b]$, along with interpolation points $\{t_i\}$, $i = 1,\cdots,n$, and $\varepsilon > 0$. We prove that there exists a spline $s \in \mathscr{S}_k$ such that $s(t_i) = f(t_i)$, $i = 1,\cdots,n$, $||f-s|| < \varepsilon$, and $||s|| = ||f||$. (Received November 4, 1974.)

720-41-9 C. K. CHUI, P. W. SMITH, and J. D. WARD, Department of Mathematics, Texas A&M University, College Station, Texas, Limits of $H^{k,p}$-splines as $p \to 1$.

The weak-star cluster points of $H^{k,p}$-splines as $p \to 1$ are partially characterized. In particular, they are solutions of a natural minimization problem in the space of functions whose k^{th} derivatives are regular Borel measures. Furthermore, if s is such a cluster point then $s^{(k)} = \mu + \nu$ where μ is purely atomic and ν is piecewise an exponential of a polynomial spline. (Received November 4, 1974.)

*720-41-10 C. K. CHUI, P. W. SMITH, & J. D. WARD, Department of Mathematics, Texas A&M University, College Station, Texas, Favards Solution is the Limit of $H^{k,p}$ Splines.

Let $s_p \in H^{k,p}$ satisfy $||s_p^{(k)}||_p \leq ||f^{(k)}||_p$ for all $f \in H^{k,p}(I)$ such that $f(t_i) = s_p(t_i) = \gamma_i$ for $i = 1,\ldots,n$ and $t_i \in I$. Then $s_p \to s$ in $H^{k,1}$ as $p \to \infty$, where s is the Favard solution satisfying $||s^{(k)}||_\infty \leq ||f^{(k)}||_\infty$ for all $f \in H^{k,\infty}$ such that $f(t_i) = s(t_i) = \gamma_i$. (Received November 4, 1974.)

*720-41-11 DAVID W. KAMMLER, Southern Illinois University, Carbondale, Illinois 62901. Chebychev approximation of completely monotonic functions by sums of exponentials.

Let $V_n = \{Y \in C^n[0,\infty): [(D + \lambda_1)\ldots(D + \lambda_n)]Y(t) \equiv 0$ for some choice of $\lambda_1,\ldots,\lambda_n \in \mathbb{R}\}$, $n=1,2,\ldots$, (where $D=d/dt$ is the differential operator) and let

$b \in (0,\infty]$. A function $F \in C[0,b]$ is completely monotonic on $[0,\infty]$ (i.e., $(-D)^r F(t) \geq 0$ for $t > 0$, $r = 0,1,\ldots$) if and only if for each $n=1,2,\ldots$ there exists a best uniform approximation, Y_n, to f on $[0,b]$ from V_n having the form $Y_n(t) = a_{n1}\exp(-\lambda_{n1}t)+\ldots+ a_{nn}\exp(-\lambda_{nn}t)$ with $0 \leq \lambda_{n1} < \ldots < \lambda_{nn}$ and $a_{ni} \geq 0$ for $i=1,\ldots, n$. If F is completely monotonic on $[0,\infty]$ with $F(\infty) = 0$ if $b = \infty$, and if $F \notin V_n$, then the best approximation Y_n is unique, the error $F-Y_n$ alternates exactly $2n$ times on $[0,b]$ with $F(0) - Y_n(0) = \|F - Y_n\|$, and the exponents λ_{ni} and partial sums $A_{ni} = a_{n1}+\ldots+ a_{ni}$ for Y_n strictly interlace those for Y_{n+1}. (Received November 5, 1974.)

720-41-12 R. D. LARSEN and W. R. MADYCH, Texas A&M University, College Station, Texas, 77843, Walsh-Like Expansions and Hadamard Matrices. Preliminary report.

We develop general methods for generating complete orthonormal systems of Walsh-like functions from certain permuted Kronecker products of Hadamard matrices.

(Received November 6, 1974.)

720-41-13 URI FIXMAN, Queens University, Kingston, Ontario, Canada and LEE A. RUBEL, University of Illinois, Urbana, Illinois 61801, Uniform approximation by rational functions and their derivatives. Preliminary report.

A subset E of the complex plane is called a D-set if for every sequence $\{f_n\}_{n=1}^{\infty}$ of rational functions holomorphic on E which converges uniformly to 0 on E, such that the derivatives $\{f'_n\}$ converge uniformly on E to a finite function g, one must have $g = 0$ identically on E. It is easy to see that a totally disconnected set is not a D-set. It is proved here that a rectifiable curve is a D-set. An example of E. Bishop provides a continuous (but nowhere rectifiable) arc that is not a D-set. It is asked whether the closed D-sets are just the closures of unions of rectifiable arcs. A pseudometric Δ is defined, by means of ϵ-chains, on any closed set $E \subseteq \mathbb{C}$, such that x is joined to y by a rectifiable arc $J \subseteq E$ iff $\Delta(x,y) < \infty$. In particular, two points lying on a rectifiable curve lie on a rectifiable arc contained in that curve. (Received November 6, 1974.)

*720-41-14 B. GOPINATH and R. P. KURSHAN, Bell Laboratories, Murray Hill, New Jersey 07974. Extremal polynomials in bounded T-systems.

For the classical T-system $u_i(t) = t^i$, $i = 0,1,\ldots,n$, on $[a,b]$ a theorem of Karlin and Shapley [Mem. Amer. Math. Soc. No. 12(1953)] states that every nonnegative polynomial admits a unique representation as a sum of two nonnegative polynomials having n zeroes counting multiplicities. In Karlin and Studden ["Tchebycheff systems; with applications in analysis and statistics", Pure and Appl. Math., vol. 15, Interscience, New York, 1966] an analog of the result for continuous T-systems on $[a,b]$ is proved. We extend the relevant results, in particular Theorem 10.2, Chapter II of the second reference to bounded T-systems on bounded subsets of the real line. (Received November 6, 1974.)

720-41-15 STEPHEN RODI, Mary Baldwin College, Staunton, Virginia 24401. Stone-Weierstrass Properties in Some Commutative Banach Algebras. Prelimnary Report.

The paper establishes sufficient conditions (namely, X and Y both totally disconnected) which assure that every point-separating, self-adjoint, non-vanishing subalgebra of $V^a(X,Y)$

$= C_o(X) \otimes_a C_o(Y)$ is dense whenever $a = a_{p,q}$, the Lapresté norm. For $4/3 < p \leq 2$, a counter-example is given to show that not every closed subalgebra of $V^a(Z, Z)$, $a = a_{p,p}$, is determined by its constancy sets. The same is true for the closed subalgebras of the companion spaces $\mathcal{V}^a(Z)$. These latter examples answer in part a question posed by Kahane regarding the closed subalgebras of the intermediate algebras in a chain of algebras whose smallest member is $A(Z)$ and whose largest member is $C_o(Z)$.

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720-41-16 FRITZ JOHN, Courant Institute of Mathematical Sciences, New York University, New York, New York 10012. A priori estimates, geometric effects, and asymptotic behavior.

• Physical processes are often described by solutions $u(x) = u(x_1, \ldots, x_n)$ of a system (S) of partial differential equations. Ordinarily, the solutions are well behaved and do not display any striking qualitative features. Such features usually only appear in extreme situations, brought about by some degeneracy in the differential equations or in the domain of the solutions. Certain terms in (S) then become negligible, while the remaining ones, sometimes combined with boundary conditions, lead to a new asymptotic system (S') of differential equations. For a rigorous derivation of (S'), one has to decide on the orders of magnitude of the various derivaties of u occuring in (S). Here, it is useful to introduce the notion of wave length L of a function $u(x)$ with domain R as a measure for the distance required for u to change by an appreciable percentage. We define the wave length at a point x of R as the largest L such that (1) $|D_k u(x)| \leq (1/L) \sup_R |u|$ for $k = 1, \ldots, n$; $(D_k = \partial/\partial x_k)$. The larger L, the smoother the function u. Fundamental for many asymptotic results is the fact that for solutions u of an elliptic system there exist predictable lower bounds for the wave lengths of solutions u of the form (2) $L \geq \lambda d(x)/(\mu + d(x))$; here λ and μ are certain characteristic lengths associated with system (S) (found by dimensional analysis in the simplest cases), while $d(x)$ denotes the distance of the point x from the boundary B of R. For significant applications, a further observation is needed. If on a portion B' of B homogeneous boundary conditions (of a type implying suitable coerciveness properties) are satisfied, we can largely ignore the location of x relative to B' and replace $d(x)$ in (3) by the distance of x from the remaining portion $B'' = B - B'$ of the boundary. As a typical example, such estimates for L can be applied to the equations of equilibrium for a thin elastic plate of thickness h, loaded only at the edge. There they lead to "interior" equations, valid asymptotically at points whose distance from the edge is large compared to h. No boundary conditions at the edge need to specified, as long as we are given an upper bound for $\sup_R |u|$. While boundary layers forming near the edge might possibly depend on the precise nature of the edge conditions, such boundary effects are largely filtered out in the interior. The effects of constraint that have the form of a priori inequalities are of importance in other contexts. Their use is widespread in controlling the size of errors in numerical solutions of problems, especially of ill-posed problems. Proceeding in a different direction, we observe that for a fixed solution u of an elliptic system (S) in a fixed domain R, the estimate (2) implies that (3) $|D_k u(x)| \leq M/d(x)$, $\quad k = 1, \ldots, n; \quad x \in R$, with a fixed M. Generally, the converse does not hold, that is, the u satisfying (3) need not be bounded. However, (3) always implies that u is of bounded mean oscillation, i.e., there exist an N and, for each cube γ in R, a constant c_γ such that $|u - c_\gamma| < N$ in at least 2/3 (by measure) of the points of γ. In some cases this yields a connection between solutions of partial differential equations with certain a priori bounds and functions of bounded mean oscillations. Other instances of such a connection occur in geometry. (Received November 11, 1974.)

*720-42-1 CHULL PARK, MIAMI UNIVERSITY, OXFORD, OHIO 45056
An Extension of Parseval's Equation

Let $\{\alpha_k(x)\}_1^\alpha$ be any real complete orthonormal system on [a,b]. Then Parseval's equation asserts that $\Sigma_{k=1}^\infty \int_a^b \int_a^b f(x)g(y)\alpha_k(x)\alpha_k(y)dxdy = \int_a^b f(x)g(x)dx$ holds for any square integrable functions f and g on [a,b]. This paper gives conditions on k(x,y) under which $\Sigma_{k=1}^\infty \int_a^b \int_a^b k(x,y)\alpha_k(x)\alpha_k(y)dxdy = \int_a^b k(x,x)dx$ holds. Also shown is that for a large class of complete orthonormal systems $\{v_k(x)\}_1^\infty$ on [a,b] the identity $\Sigma_{k=1}^\infty \int_a^b \int_a^b k(x,y)v_k(x)\int_a^x v_k(u)dudxdy = (1/2)\int_a^b \int_a^b k(x,y)dxdy$ holds. The result and others are used to introduce a certain type of stochastic integral as an application.

(Received October 15, 1974.)

*720-42-2 HAROLD E. BENZINGER, Department of Mathematics, University of Illinois, Urbana, IL 61801. Biorthogonal Systems Equivalent To Fourier Series

Using the interpolation theorems of Marcinkiewicz and Riesz-Thorin, we prove that the system $\{U_k\}$, $k = 0, \pm 1, \ldots$ of root functions of an arbitrary order Birkhoff regular boundary value problem on the interval [0,1], is a basis for $L^p(0,1)$, $1 < p < \infty$, which is equivalent to the Fourier system $\varphi_k(x) = \exp(2k\pi ix)$, that is, there exists a bounded invertible linear operator $A : L^p(0,1) \longrightarrow L^p(0,1)$ such that $A\varphi_k = U_k$.

(Received October 21, 1974.)

*720-42-3 DANIEL WATERMAN, Syracuse University, Syracuse, New York 13210. On the summability of Fourier series of functions of Λ-bounded variation.

For f in ΛBV with $\Lambda = \{n^{\beta+1}\}$, it is shown that the Fourier series of f, S[f], is everywhere (C,β) bounded, $-1 \leq \beta < 0$, and everywhere (C,α) summable for $\alpha > \beta$. If f is continuous in $\{n^{\beta+1}\}$-variation, $-1 < \beta < 0$, S[f] is everywhere (C,β) summable. These results hold uniformly on each closed interval of continuity of f. If $\Lambda BV \supset \{n^{\beta+1}\}BV$ properly, there is a continuous function f in ΛBV such that S[f] is not (C,β) bounded at some point. A lemma on the continuity of the Λ-variation and a summability test similar to the Lebesgue convergence test are the principal tools. A new proof is indicated for the fact that functions of harmonic bounded variation satisfy the Lebesgue test. (Received October 31, 1974.)

*720-42-4 WILLIAM CONNETT AND ALAN SCHWARTZ, University of Missouri-St. Louis, St. Louis, Missouri 63121. The Theory of Ultraspherical Multipliers and The Idea of Local Stability.

The complex method of interpolation of Calderón is adapted so that it can be used for non-reflexive Banach spaces. In particular, it is shown that various families of multipliers for the ultraspherical polynomials can be interpolated, and although the scale of spaces is not stable, it is locally stable, and that is sufficient to prove that every space in the scale of spaces is a space of multipliers. This method has applications to other orthogonal families. For example if $\mathcal{L}^{q}_{\alpha}$ are the Bessel potential spaces on R and $\varphi_k(x) = \varphi(x-k)$ is a cut off function, then define $B^i = \{f \mid \sup_k \|\varphi_k f\|_{\mathcal{L}^{q_i}_{\alpha_i}} < C < \infty\}$ for $i = 0,1$. Then $[B^0, B^1]_s \neq B_s$, but B_s is a space of multipliers if B^0, B^1 are. (Received October 31, 1974.)

720-42-5 ELAINE COHEN, University of Utah, Salt Lake City, Utah 84112. On the degree of approximation by partial sums of Fourier series in certain Orlicz space norms, Preliminary report.

Let Φ,Ψ be N-functions, V_Φ the set of functions of Φ-bounded variation and for $r\geq 0$, $f^{(r)}$ the Weyl fractional derivative of order r. Suppose $f^{(r)} \in V_\Phi$, we prove a generalization of a result of Salem [Essais sur les series trignometriques, Actualite, Sci. Ind., #862] and B.I. Golubov[Math. USSR Izv. 2(1968),pp 803-5]. Theorem: If $\Sigma\Phi^*(1/k)<\infty$ where Φ^* is the Young's complement to Φ, $\|f-S_n(f)\|_C = o(n^{-r})$ if $f^{(r)}$ is continuous and $\|f-S_n(f)\|_{L^\infty} = o(n^{-r}) + O(n^{-r}\|f^{(r)}\|^*_{\Phi v})$, if $f^{(r)}$ is not continuous. We also generalize another result of Golubov [Ibid., p. 802]. Under the assumption that Ψ is Δ', and other restrictions on Ψ, Theorem: If $f^{(r)}$ is continuous, $\|f-S_n(f)\|_\Psi = o(\frac{1}{n^{r}\Psi^{-1}(n)})$. Otherwise, $\|f-S_n(f)\|_\Psi$

$$= o\left(\frac{1}{n^{r}\Psi^{-1}(n)}\right) + \frac{1}{n^{r}\Psi^{-1}(n)\Psi^{-1}(1/(k\sum^{\infty}\Psi(\sigma_j)))}$$

, where $f^{(r)}$ is discontinuous as x_i and $\sigma_i = f^{(r)}(x_i+) - f^{(r)}(x_i-)$. (Received November 1, 1974.)

*720-42-6 TIMOTHY SIMPSON and ROBERT B. KELMAN, Colorado State University, Ft. Collins 80523 Existence, Uniqueness and an Algorithmic Solution for a Dual Cosine Series. Preliminary report.

An algorithm (roughly speaking, a precisely-defined finite set of rules involving only a finite number of elementary arithmetic operations) is presented for the solution of the dual cosine series

$$\tfrac{1}{2} a_0 \cos\tfrac{1}{2}x + \textstyle\sum_{n=1}^{\infty} a_n \cos(n+\tfrac{1}{2})x = f(x) \quad (0<x<c)$$

$$\tfrac{1}{2}\alpha a_0 \cos\tfrac{1}{2}x + \textstyle\sum_{n=1}^{\infty} a_n\, n(n+1)/(2n+1) \cos(n+\tfrac{1}{2})x = g(x) \quad (c<x<\pi)$$

In support of the algorithm, computational results are presented, and an existence proof is described. Approaches to the problem of uniqueness are discussed. (Received November 4, 1974.)

*720-42-7 COLIN BENNETT, California Institute of Technology, Pasadena, California 91125. Extensions of the Hausdorff-Young theorem.

Variants of Peetre's J- and K-interpolation methods are used to characterize the rearrangement-invariant (r.i.) spaces that are weak-interpolation spaces between L^p and L^q $(1 \leq p < q \leq \infty)$. Application to the Fourier transform leads to a generalized Hausdorff-Young theorem for r.i. spaces. This result contains as special cases previous extensions of the Hausdorff-Young theorem due to Hardy-Littlewood, Zygmund, Calderón and the author (Pacific J. Math. 47(1973), 311-328). (Received November 4, 1974.)

*720-42-8 J.W. GARRETT and C.V. STANOJEVIC, University of Missouri-Rolla, Rolla, Missouri 65401. On Integrability and L^1 Convergence of Certain Cosine Sums.

The cosine sums in [C.S. REES and C.V. STANOJEVIC, Necessary and Sufficient Conditions for Integrability of Certain Cosine Sums, J. Math. Anal. Appl. 43 (1973), 579-586] motivated the following. Theorem 1. Let $g_n(x) = \frac{1}{2}\phi_0^n + \sum_{k=1}^{n}\phi_k^n \cos kx$ where $\{\phi_k^n\}$, $0 \leq k \leq n$, is a double sequence of real numbers satisfying the following properties: (1) $\lim_{n\to\infty} \phi_n^n = 0$; (2) $\lim_{n\to\infty} \phi_{n-1}^n = 0$; (3) $\lim_{n\to\infty} (\phi_0^n - \phi_1^n) < \infty$; (4) for $0 \leq k \leq n-2$, $\Delta_k^2 \phi_k^{n+1} \geq \Delta_k^2 \phi_k^n$; and (5) for $n \geq 2$, $\Delta_k^2 \phi_k^n]_{k=n-2} \geq 0$. Then (i) $g(x) = \lim_{n\to\infty} g_n(x)$ exists for $x \in (0,\pi]$; and (ii) $g \in L^1[0,\pi]$ if and only if $\lim_{n\to\infty} [\phi_0^n - n(\phi_{n-1}^n - \phi_n^n)] < \infty$. Theorem 2. Let $g(x) = \frac{1}{2}a_0 + \sum_{k=1}^{\infty} a_k \cos kx$ where $\lim_{n\to\infty} a_n = 0$ and $\sum_{k=1}^{\infty} (k+1)|\Delta^2 a_k| < \infty$. Let $g_n(x) = \frac{1}{2}\sum_{k=0}^{n} \Delta a_k + \sum_{k=1}^{n} \sum_{j=k}^{n} \Delta a_j \cos kx$. Then g_n converges to g in L^1-norm. (Received November 5, 1974.)

720-42-9 A.P. CALDERON, Massachusetts Institute of Technology, Cambridge, Mass. 02139 and A. TORCHINSKY, Cornell University, Ithaca, N.Y. 14850. Interpolation between HP spaces, the complex method. Preliminary Report.

Let $F(x,t) = (f * \varphi_t)(x)$, where f is a tempered distribution in R^n and φ_t is an approximate identity of the form $\varphi_t(x) = (\det A_t)^{-1}\varphi(A_t^{-1}x)$, φ in , $\int \varphi(x)dx \neq 0$. We say that $f \in H^p$ if $\sup_{t>0}|F(x,t)| \in L^p$, $0 < p \leq \infty$, and $f \in \tilde{H}^p$ if $\iint |F(y,t)| t^{\gamma(1/p^{-1})} dy \frac{dt}{t} < \infty$, where γ = trace infinitesimal generator of A_t and $0 < p < 1$. We discuss the interpolation properties, via the complex method, of the spaces H^p and $\tilde{H}^p$ and give applications to the problems of fractional integration and differentiation. Fefferman, Riviere and Sagher have obtained results in this direction for the spaces H^p using the real method of Lions-Petree and approximate identies $\varphi_t(x) = t^{-n}\varphi(x/t)$, $\gamma = n$. (Received November 5, 1974.)

*720-42-10 MISHA ZAFRAN, Stanford University, Stanford, Ca. 94305. Measures as convolution operators on H^1 and Lip α.

If μ is a finite regular Borel measure on the real line, let T_μ denote the operator defined as convolution by μ. We study the spectral theory of the operators T_μ on the spaces H^1 and Lip α, $0 < \alpha < 1$. The analogue of the classical Wiener-Pitt theorem for the algebra of measures is obtained in this context; specifically, we show that there exists a measure $\mu \in M_0(R)$ so that $sp(T_\mu, H^1) \neq \hat{\mu}(R) \cup \{0\}$ and $sp(T_\mu, \text{Lip } \alpha) \neq \hat{\mu}(R) \cup \{0\}$. Here

$sp(T,X)$ denotes the spectrum of the operator T on the Banach space X. The object of our study is an infinite Bernoulli convolution of the form $\mathop{*}_{k=1}^{\infty} (\frac{1}{2}\delta_0 + \frac{1}{2}\delta_{t_k})$, where the sequence $\{t_k\}$ is subject to certain arithmetic constraints. We prove our results by carefully analyzing the j-fold sum of the Cantor set $\{\sum_{k=1}^{\infty}\varepsilon_k t_k \mid \varepsilon_k = 0 \text{ or } 1\}$, $j = 1,2,\ldots$. Some applications of this theory are also given. (Received November 5, 1974.)

720-42-11 RICHARD A. HUNT, Purdue University, West Lafayette, Indiana 47907. Interpolation of operators and a.e. convergence of Fourier series, Preliminary report.

Interpolation of restricted weak type inequalities for the maximal function $Mf = \sup|S_n f|$ can be used to obtain a.e. convergence of Fourier series. Endpoint results are obtained by extrapolation of the L^p estimates. Estimates of $\tilde{f}$, the conjugate function of f, play an important role in the basic results. If one could show that $||f||_\infty \leq 1$ implies $m\{|\tilde{f}| > 1\} \leq C \int \exp\{-C/|f(x)|\}dx$, a duality argument could be used to prove that lacunary subsequences of the partial sum of the Fourier series of f converge a.e. for $f \in L \log\log L$. This would be a best possible result. Some partial results are known. (Received November 6, 1974.)

*720-42-12 LAWRENCE GLUCK, DePaul University, Chicago, Illinois 60624. On p-weights for even functions.

Necessary and sufficient conditions on a weight w are given for the Hilbert transform restricted to even functions to be continuous on the weighted ℓ^p space, $\hat{\ell}^p((0,\infty);w)$. (Received November 6, 1974.)

*720-42-13 Charles Fefferman, Nestor M. Rivière and Y. Sagher. University of Chicago, University of Minnesota and Weizmann Institute of Sciences. Interpolation between H^p spaces, $0 < p < \infty$.

Using the characterization of H^p spaces due to Fefferman and Stein and a decomposition of Calderon-Zygmund type we show that $(H^{p_0,q_0}, H^{p_1,q_1})_{\theta,q} = H^{p,q}$ where $\frac{1}{p} = \frac{1-\theta}{p_0} + \frac{\theta}{p_1}$, $0 < \theta < 1$, $0 < q \leq \infty$. (Received November 6, 1974.)

43 Abstract Harmonic Analysis

*720-43-1 JOHN J.F. FOURNIER, University of British Columbia, Vancouver, Canada V6T 1W5
Convolution operators of weak type

Let G be an infinite, locally compact, group, and let $1 < p < 2$. It is shown that there exists a convolution operator on G that is of weak type (p,p) but is not of strong type (p,p). This generalizes previous results of the author (these Notices 21(1974), 74T-B115), and M. Zafran (these Notices 20(1973), 73T-B261). The counterexample is the sum of a series of Rudin-Shapiro measures. The key tool in proving that the operator is of weak type (p,p) is Zafran's multilinear interpolation theorem for the real method.
(Received October 21, 1974.)

*720-43-2 FREDERICK W. KEENE, University of Miami, Coral Gables, Florida 33124. A Plancherel theorem for AN.

Let $G = KAN$ be the Iwasawa decomposition of a semisimple Lie group. The nilpotent subgroup N has L_2-representations iff the reduced restricted root system is of type A_1 or A_2. By combining the Mackey machinery for induced representations and the Kirillov theory, we prove Theorem. Let $G = KAN$ so that G/K has symmetric space rank 1. Let Z be the center of N so that $Z = \mathbb{R}^k$ is a manifold. Then, for $\gamma \in C_c^\infty(NA)$, $\gamma(1_{NA}) = \int_{S^{k-1}} \text{trace } \pi_\lambda(D\gamma)\, d\sigma(\lambda)$ where S^{k-1} is the unit sphere in Z. $d\sigma(\lambda)$ is Lebesgue measure on S^{k-1}. $\pi_\lambda \in (NA)^\wedge$ parametrized by λ and D is an operator on $C_c^\infty(\mathbb{R}^k)$ given by $D = i(\Delta)^{\frac{1}{2}}/2\pi$ if $Z = N$, and $D = (i/2\pi)^q(\Delta)^{\frac{1}{2}q}$ if $Z \neq N$, where $q = \frac{1}{2}(n + 2k)$, where $n = \dim(N/Z)$ and Δ is the Laplace operator on $C_c^\infty(\mathbb{R}^k)$. (Received October 29, 1974.)

*720-43-3 HARVEY A. SMITH, Oakland University, Rochester, Michigan, 48063. Characteristic Principal Bundles.

Characteristic principal bundles are the duals of commutative twisted group algebras. A principal bundle with locally compact second countable (Abelian) group and base space is characteristic iff it supports a continuous eigenfunction for almost every character measurably in the characters, also iff it is the quotient by Z of a principal E-bundle for every E in $\mathrm{Fxt}(G, Z)$ and a measurability condition holds. Although characteristic G-bundles need not be locally trivial, the class of characteristic G-bundles is a homotopy invariant of the base space. The isomorphism classes of commutative twisted group algebras over G with values in a given commutative C^*-algebra A are classified by the extensions of G by the integer first Cech cohomology group of the maximal ideal space of A. (Received October 29, 1974.)

*720-43-4 ROGER A. HORN, The Johns Hopkins University, Baltimore, Md. 21218. Quadratic Forms in Harmonic Analysis and the Bochner-Eberlein Theorem.

Let φ and f be complex valued functions on a locally compact Abelian group G, and suppose that φ and f are continuous at the identity element of G. Consider the inequality
(*) $\Sigma\, \varphi(x_i - x_j)\, c_i\bar{c}_j \geq \left| \Sigma\, f(x_i + x_j)\, c_i c_j \right|$ where the sum is over all finite choices of group elements x_i and complex numbers c_i. Then: (1) (*) is satisfied if and only if φ is a positive definite function (i.e., $\varphi \in P(\hat{G})$) and f is a Fourier transform of a bounded regular complex measure on the dual group $\hat{G}$ (i.e., $f \in B(\hat{G})$); (2) if $\varphi \in P(G)$ then (*) is satisfied with $f = \varphi$; (3) if (*) is satisfied then the measure of which f is the Fourier transform has certain properties of the positive measure of which φ is the Fourier transform, e.g., having compact support or being an L_p function; (4) the relation (*) is closed under convex combinations and products; (5) φ and f are bounded and continuous. These results are alternatives to well-known characterizations of Bochner, Eberlein, Weil, and Ryan.
(Received October 31, 1974.)

*720-43-5 CHARLES P. DOWNEY, University of Nebraska at Omaha, Omaha, Nebraska 68101. Reisz Transforms for Vector Spaces over a Local Field.

Reisz transforms R_j, $1 \leq j \leq d$, are defined on $L_p(K^d)$, $1 < p < \infty$, as singular ntegral operators with kernals w_j, $w_j(x) = w(x_j)$, $x_j \neq 0$, and $w_j(x) = 0$ f $x_j = 0$, where $x = (x_1, \ldots, x_d)$ and w is a homogenous degree zero multiplicative character of K^*. These transforms are shown to satisfy a classification theorem analogous to that satisfied by the Hilbert transform over the local field K. (Received November 1, 1974.)

720-43-6 GUNAR E. LIEPINS, Texas Tech University, Lubbock, Texas 79409
A Paley-Wiener Theorem, Preliminary report.

A generalized Paley-Wiener theorem is proved in the setting of locally compact abelian groups. Let G be a l.c.a. group, Γ its character group, Γ' the space of images of the one-parameter subgroups of Γ, and $U \subset \Gamma'$ an open set. A function F is analytic on $U \times \Gamma$ if

i) $$F_{(x_1,y_1)}(x,y) = \lim_{h \to 0} \frac{F(x+hx_1, y+hy_1) - F(x,y)}{h}$$

is continuous for all $(x_1,y_1) \in \Gamma' \times \Gamma'$ and $(x,y) \in U \times \Gamma$, and

ii) $$(u+iv)F_{(x_1,y_1)}(x,y) = F_{(ux_1-vy_1,\, uy_1+vx_1)}(x,y)$$

The notion of analyticity is further generalized for L^2 functions and an L^2 Paley-Wiener theorem is proved. This proves affirmatively a conjecture of Mackey ["The Laplace Transform for Locally Compact Abelian Groups", Proc. Nat. Acad. Sci. U.S.A., 34(1948), 156-162]. (Received November 1, 1974.)

*720-43-7 J. Edward Kerlin, University of Kentucky, Lexington, Kentucky 40506
Generalized multipliers of spaces on locally groups II, Preliminary report.

If G, H and K are locally compact groups, $\theta : K \to G$ and $\psi : K \to H$ are continuous homomorphisms, and X(G) and Y(H) are left translation invariant normed linear spaces of functions or measures on G and H, respectively, then a (θ, ψ) – *multiplier* $T : X(G) \to Y(H)$ is bounded linear map such that $L_{\psi(z)} \circ T = T \circ L_{\theta(z)}$ for all $z \in K$. The basic properties and results of (θ, ψ)–multipliers are introduced in an earlier paper by this author [Bull. A.M.S. **79** (1973), 1223–27]. By an extension of the results of Bachelis and Gilbert on compact multipliers, we have **Theorem 1.** *If* G *and* H *are compact and if* X(G) *and* Y(H) *are left homogeneous Banach spaces such that either both are reflexive or one is reflexive and the other contains the continuous functions and has its dual identifiable with a subspace of bounded measures, then the dual of the compact* (θ, ψ)*– multipliers of* X(G) *into* Y(H) *is* $X(G) \otimes_{L^1(K)} Y(H)$ *and the double dual is all the* (θ, ψ)*– multipliers.* **Theorem 2.** $X(G) \otimes_K Y(H)$ *and* $X(G) \otimes_{L^1(K)} Y(H)^*$ *are identifiable concretely with function spaces on* $G \otimes_K H$. The next theorem is the strongest result known in the event G and H are possibly non-compact. **Theorem 3.** *If* $\{(\theta(z), \psi(z)) \mid z \in K\}$ *is precompact in* $G \times H$ (e.g., if K is compact), *then* $L^p(G) \otimes_K L^q(H)$ *is identifiable concretely for* $1 \leq p, q < \infty$ *with the function space* $\mathcal{A}_p^q(G \otimes_K H)$. **Corollary.** $\mathrm{Hom}_K(L^p(G), L^{q'}(H)) = \mathcal{A}_p^q(G \otimes_K H)^*$. **Corollary.** *The space of bounded linear maps of* $L^p(G)$ *into* $L^q(G)$ *commuting with left translation by elements of a compact subgroup* K *is identifiable concretely as the dual of a function space on* $(G \times G)/\mathrm{diag}\, K \times K$. (Received November 4, 1974.)
(Author introduced by Professor A. Lambert.)

720-43-8 ALAN ARMSTRONG, Kansas State University, Manhattan, Kansas 66506. Multipliers or compact groups with unconditionally converging Fourier series. Preliminary repor

Let G be a compact group with dual object Γ. Let $M_p^q(\Gamma)$ denote the multipliers of type (p,q) and $m_p^q(\Gamma)$ the compact multipliers of type (p,q). A set $E \subset \Gamma$ is said to be local Λ_p if there exists an $M < \infty$ such that $||Tr(AU_\alpha(x))||_p \leq M||Tr(AU_\alpha(x))||_r$ for all $\alpha \in E$ and some $r<p$. It is shown that for $1 \leq p \leq 2$ and $1 \leq q \leq 2$ then an element $\phi \in m_p^q(\Gamma)$ and having an unconditionally converging Fourier series is equivalent to $\phi \in m_p^2(\Gamma)$ if the support of ϕ is local Λ_2. If also $p \leq q$, then the derived algebra of $m_p^q(\Gamma)$ is $M_p^2(\Gamma) \cap m_p^q(\Gamma)$ if Γ is local Λ_2. It follows that if $1 \leq p \leq q \leq 2$ then the derived algebra of $M_p^q(\Gamma)$ is the double dual of (p,q) multipliers with unconditionally converging Fourier series if Γ is local Λ_2. Similar results are obtained when $1 \leq p < \infty$ and $q > 2$. This generalizes results of Bachelis and Pigno(Can. J. Math. 24(1972), 477-484.) (Received November 4, 1974.)

*720-43-9 WILLIAM H. BARKER, Dartmouth College, Hanover, New Hampshire 03755. The Spherical Bochner Theorem on Semi-Simple Lie Groups.

Let G be a connected semi-simple Lie group with finite center and K a maximal compact subgroup. Denote (i) Harish-Chandra's Schwartz spaces by $C^p(G)$ $(0<p\leq 2)$, (ii) the K-biinvariant elements in $C^p(G)$ by $I^p(G)$, (iii) the positive definite (zonal) spherical functions by P, and (iv) the spherical transform on $C^p(G)$ by $\phi \to \hat{\phi}$. For T a positive definite distribution on G it is established that (i) T extends uniquely onto $C^1(G)$, (ii) there exists a unique measure μ of polynomial growth on P such that $T[\phi] = \int_P \hat{\phi} d\mu$ for all $\phi \in I^1(G)$, (iii) all measures μ of polynomial growth on P are obtained this way, and (iv) T may be extended to a particular $I^p(G)$ space $(1\leq p\leq 2)$ if and only if the support of μ lies in a certain easily defined subset of P. These results generalize a well known theorem of Godement, and the proofs rely heavily on the recent harmonic analysis results of Trombi and Varadarajan. (Received November 4, 1974.)

720-43-10 JIA-ARNG CHAO, University of Texas, Austin, Texas 78712. A characterization of H^p-spaces on a local field, Preliminary report.

Let K be a local field. For any positive p, $H^p(K)$ is the space of all distributions on K whose maximal functions are in $L^p(K)$. It is shown that for $0 < p < \infty$, a distribution is in H^p if and only if its Lusin area function is in L^p. An immediate consequence is that nice singular integral transforms preserve H^p for $0 < p < \infty$. (Received November 4, 1974.)

720-43-11 DANIEL M. OBERLIN, Florida State University, Tallahassee, Florida 32306. Multipliers of closed ideals of $L^p(D^\infty)$. Preliminary report.

Let G be a compact abelian group with character group X. For a subset E of X, let M_E^p be the set of complex-valued functions on E which multiply $\widehat{L_E^p(G)}$ into itself, and let $M^p = M_X^p$. Let G be the Cantor group $D^\infty = Z(2)^N$. Theorem 1. There exists $E \subseteq X$ such that for $1 \leq p \leq 2$, there is a multiplier of $\widehat{L_E^p}$ into $\widehat{L_E^2}$ which is not in $M^p|_E$. Theorem 2. For $p = 4, 6, 8, \ldots$, $M^p|_E$ is a proper

subset of M_E^p. Now let $G = Z(n)^N$, for some $n = 3,4,\ldots$. Theorem 3. With G and n as above, there exist $E \subseteq X$ and $m \in M_E^{2(n-1)}$ such that $m \in M_E^p$ for only a finite number of $p \in (2, 2(n-1))$. Thus m is in no space $M^p|_E$ for $p \neq 2$. (Received November 6, 1974.)

*720-43-12 P. R. MASANI, University of Pittsburgh, Dept. of Math., Pittsburgh, Pa. 15260
Generalizations of P. Levy's inversion formula.

Let Γ, $\hat{\Gamma}$ be dual l.c.a. groups with Haar measures m, $\hat{m}$ on the σ-algebras $\mathcal{B}$, $\hat{\mathcal{B}}$ generated by their topologies. Let μ be a bounded complex-valued c.a. measure on $\mathcal{B}$ and ϕ be its Fourier-Stieltjes (FS) transform on $\hat{\Gamma}$. Write $\eta_B(\alpha) = \int_B \overline{\alpha(t)}\, m(dt)$, $\forall \alpha \in \hat{\Gamma}$ & $\forall B \in \mathcal{B} \ni m(B) < \infty$.
Thm. 1. Let Γ be first countable and $(V_k)_1^\infty$ be a base of pre-compact open symmetric sets $V_{k+1} \subseteq V_k$ & $\bigcap_1^\infty V_k = \{0\}$. Then $\forall$ pre-compact $D \in \mathcal{B} \ni |\mu|\,(\bar{D} \setminus D) = 0$,

$$\mu(D) = \lim_{n\to\infty} \frac{1}{m(V_n)} \int_{\hat{\Gamma}} \eta_{D+V_n}(\alpha)\, \eta_{V_n}(\alpha)\, \phi(\alpha)\, \hat{m}(d\alpha).$$

Thm. 2. Let R be a non-degenerate closed bounded interval in $\mathbb{R}^q$, and ∂R, $\partial^2 R.,,,.$ $\partial^q R$ be the union of its $q-1$, $q-2,\ldots,$ 0 dimensional closed faces. Then

$$\mu(R) - \sum_1^q \mu(\partial^k R) = \lim_{A\to\infty} \int_{[-A,A]^q} \eta_R(\alpha)\, \phi(\alpha)\, m(d\alpha), \qquad \wedge\ 2^{-k}$$

where m is $(1/\sqrt{2\pi})^q$ times q-dimensional Lebesgue measure. These and other similar theorems extend to Hilbert space-valued orthogonally scattered measures and for spectral (projection-valued) measures and their FS transforms. (Received November 6, 1974.)

720-43-13 JOHN E. GILBERT, University of Texas, Austin, Texas 78712. Tensor products, interpolation theory and convolution operators.

Using tensor product techniques involving factorization of linear operators through L^p-spaces we introduce pointwise Banach algebras $W^{pq}(X,Y)$ of functions arising as bilinear forms on $L^1(X,\mu) \times L^1(Y,\nu)$. Connections with group representation theory and with both strong type (p,p) and weak type (p,p) convolution operators are established when G acts as a transformation group on X and Y (cf. Bull. Amer. Math. Soc. Sept. 1974). Together with interpolation space theory these techniques solve a number of problems in convolution operator theory and suggest new approaches for solving others. (Received November 6, 1974.)

45 Integral Equations

*720-45-1 SUSAN MILTON, Radford College, Radford, Virginia 24141 and CHRIS TSOKOS, University of South Florida, Tampa, Florida 33620. On the Existence of a Random Solution of a Nonlinear Perturbed Random Integral Equation

In the past few years much attention has been focused on developing the theory and investigating the applications of those random integral equations which can be considered as probabilistic analogs of classical deterministic integral equations of the Hammerstein,

Volterra, and Fredholm type. The systematic study of the theoretical aspects of these equations is a relatively new endeavor which has developed rapidly in the last ten years using as a basis the techniques of probabilistic functional analysis. The object of this study is to develop conditions under which there exists a unique random solution of a nonlinear perturbed random integral equation of the form

$$x(t;\omega) = h(t,x(t;\omega)) + \int_0^\infty k_0(t,\tau;\omega)e(\tau,x(\tau;\omega))d\tau, \quad t \geq 0.$$

(Received October 2, 1974.)

*720-45-2 CHARLES GROETSCH, Univ. of Rhode Island, Kingston, R. I. 02881. On Existence Criteria and Approximation Procedures for Integral Equations of the First Kind.

These results are motivated by a paper of Diaz and Metcalf [Math. Comp. 24 (1970)]. Suppose that A is a compact, self adjoint, positive semidefinite operator on a real Hilbert space. Let $\{\phi_t\}$ be a net of continuous functions on $[0,a]$ $\sigma(A)$ such that $\{\lambda\phi_t(\lambda)\}$ is uniformly bounded and $\phi_t(\lambda)\to\lambda^{-1}$ for each non-zero eigenvalue λ. If $Ax=y$ has a solution x then $y \perp N(A)$ and $\phi_t(A)y\to x-x_N$. A formally weaker converse holds; if $y \perp N(A)$ and $\{\phi_t(A)y\}$ has a weak limit point x then $Ax=y$ and $\phi_t(A)y\to x$. The results are easily extended to the case in which A is only compact. Special choices for $\{\phi_t\}$ yield results for Fridman's iteration, Faddeev's iteration, a Newton-type iteration and Tihonov's regularization. (Received November 4, 1974.)

720-45-3 STANLEY I. GROSSMAN, University of Montana, Missoula, Montana 59801. Resolvents of Volterra Integral Equations with Differentiable Kernels. Preliminary Report.

We consider the Volterra Integral Equation

$$x(t) = f(t) + \int_0^t a(t-s)x(s)ds$$

and define its resolvent by

$$r(t) = -a(t) + \int_0^t a(t-s)r(s)ds.$$

In this paper we give conditions which guarantee that the resolvent $r(t)$ will be in $L^1[0, \infty)$ when some derivative of the kernel $a(t)$ is integrable. (Received November 4, 1974.)

*720-45-4 G. S. JORDAN, University of Tennessee, Knoxville, TN 37916 and ROBERT L. WHEELER, Iowa State University, Ames, IA 50010. Asymptotic behavior of unbounded solutions of linear Volterra integral and integrodifferential equations.

The asymptotic behavior as $t \to \infty$ of solutions of the linear Volterra integral equation $\int_0^t u(t-s)dA(s) = f(t)$ $(0 \leq t < \infty)$ is described when the Laplace-Stieltjes transform $A^\sim(z) \equiv \int_0^\infty e^{-zt}dA(t)$ has finitely many zeros in its closed half plane of convergence. The results extend a theorem of Pitt [Proc. London Math. Soc. (2) 47 (1942), 248-267] who considers the case of finitely many zeros in the open half plane of convergence. In addition,

some of the results of Miller [J. Differential Equations 15 (1974), 129-157] concerning the integrodifferential equation $x'(t) = Ax(t) + \int_0^t x(t-s)B(s)ds + f(t)$, $x(0) = x_0$, are somewhat sharpened. (Received November 4, 1974.)

*720-45-5 TERRY L. HERDMAN, Mathematics Department, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Local Stability Properties for a Non-Linear Volterra Integral Equation.

We consider an n-dimensional non-linear Volterra integral equation (*) $x(t) = f(t) + \int_0^t g(t,s,x(s))ds$, $t \in [0,\infty)$. It is shown that whenever (*) possesses a unique solution, a certain type of local stability holds for the set of solutions of a corresponding equation involving "nearby data". (Received November 6, 1974.)

720-45-6 JOSEPH F. McGRATH, Dikewood Industries, Albuquerque, 87106. Invariant Imbedding Applied to a Class of Fredholm Integral Equations. Preliminary Report.

An existence and uniqueness result is obtained for the integral equation $\phi(z) = g(z) + \gamma \int_0^\infty K(|z-z'|)\, \phi(z')\, dz'$ for $z \geq 0$. The solution is shown to be approximated almost uniformly on the finite interval $y \leq z \leq x$. For appropriate functions K, the techniques of invariant imbedding are employed to write the solution of the finite interval problem in the form $\phi(z,x,y) = \int_0^\infty R(x-z,\ z-y,\ s)\ f(s)ds$ for any g of the form $g(z,x,y) = \int_0^\infty f(s)\ e^{-a(s)(z-y)} ds$ where R is the solution of a set of four initial value imbedding equations. (Received November 6, 1974.)

46 Functional Analysis

*720-46-1 RONN CARPENTER, University of Houston, Houston, Texas Interplating bases in C[0,1] are not Besselian

Many of the Schauder bases which have been constructed for C[0,1] consist of sequences (perhaps doubly subscripted) of piecewise linear functions satisfying the following condition. Corresponding to the basis $[f_{i,j}]$ is a sequence of points $[x_{i,j}]$ such that $f_{i,j}(x_{k,l}) = 0$ for $k < i$ and the matrix whose i,j^{th} entry is $f_{k,j}(x_{k,i})$ is non-singular for all k. It is shown that a bounded basis of this type cannot be p-Besselian. (Received October 3, 1974.)

*720-46-2 C. D. ALIPRANTIS, Occidental College, Los Angeles, CA 90041 and STD Research Corporation, Arcadia, CA 91006. On Nakano's Theorem and the completeness of locally solid Riesz spaces.

A locally solid Riesz space (L, τ) is said to have the Fatou property if τ has a basis for zero consisting of solid and order closed sets. If (L, τ) has the Fatou property and L is Dedekind complete then (L, τ) is called locally order complete. Nakano's theorem states that: If (L, τ) is locally order complete then every order interval of L is τ-complete. We show that Nakano's theorem is equivalent to Fremlin's theorem: If (L, τ) is a Hausdorff locally solid Riesz space with the Fatou property then L is order dense in its topological completion $\hat{L}$. Using the above equivalence a new proof of the following result (due to D. H. Fremlin) is obtained: If (L, τ) is Hausdorff with the Fatou property then its topological completion $(\hat{L}, \hat{\tau})$ has also the Fatou property.

Finally, some characterizations of the topological completeness of locally solid Riesz spaces will be presented and a conjecture will be stated which if true will provide a nev proof of the Nakano theorem. (Received October 15, 1974.)

*720-46-3 ROBERT M. YOUNG, Oberlin College, Oberlin, Ohio, 44074.
On perturbing bases of complex exponentials in $L^2(-\pi,\pi)$.

A sequence of complex exponentials $\{e^{i\lambda_n t}\}$ is said to be a Riesz basis for $L^2(-\pi,\pi)$ if each function in the space has a unique representation $f = \sum c_n e^{i\lambda_n t}$, with $A\sum |c_n|^2 \leqslant \|f\|^2 \leqslant B\sum |c_n|^2$. It is known that if each λ_n is real and $|\lambda_n - n| \leqslant L < 1/4$ $(-\infty < n < \infty)$, then $\{e^{i\lambda_n t}\}$ is a Riesz basis. The purpose of this note is to show that not only the orthonormal basis $\{e^{int}\}$ but any Riesz basis can be suitably perturbed. (Received September 20, 1974.)

720-46-4 JOHN DAVENPORT, Madison College, Harrisonburg, Virginia 22801. The strict dual of B -algebras, Preliminary report.

Let X be a B* -algebra and $A \subset X$ be a closed two-sided ideal. Denote by β_A the strict topology on X induced by the semi-norms $(\lambda_a)_{a\varepsilon A}$ and $(\rho_a)_{a\varepsilon A}$. Define the strong topology on $(X,\beta_A)^*$ to be the topology of uniform convergence on the β_A-bounded subsets of (X,β_A). Using the representation $X^* = \{x\cdot f : x\varepsilon X, f\varepsilon X^*\}$ for the dual space of X and the existance of a bounded approximate identity on X, we have the following result. Theorem. If A is a closed two-sided ideal in the B* -algebra X, then $(X,\beta_A)^*$ with the strong topology is isomorphic to A*. (Received October 18, 1974.)

*720-46-5 G. A. EDGAR, Northwestern University, Evanston, Illinois 60201
A NONCOMPACT CHOQUET THEOREM

The proof of the following representation theorem, which is analogous to Choquet's theorem, uses vector-valued martingales. Theorem: Let E be a Banach space with the Radon-Nikodym property, and let C be a closed, bounded, separable, convex subset of E. Then for every point $x \varepsilon C$, there is a probability measure μ on the sigma-algebra of universally measurable subsets of C such that the barycenter of μ (as a Bochner integral) is the point x and μ is concentrated on the set of extreme points of C.
(Received October 21, 1974.)

*720-46-6 HARRIS S. SHULTZ, California State University, Fullerton, California 92634.
A New Algebra of Distributions on R^n.

An algebra of distributions on R^n is introduced; it contains all the distributions having support in the positive cone $\{(t_1,\cdots,t_n) : t_i \geqq 0, i=1,\cdots,n\}$ and all the functions which are locally integrable on R^n. For $n = 2$, the algebra consists of those distributions which are regular in the sets $\{(x,y) : \eta < x < 0\}$ and $\{(x,y) : \eta < y < 0\}$ for some

$\eta < 0$. The multiplicative operation is convolution, defined so that there are no growth restrictions nor are there any restrictions on the supports. For those distributions having support in the positive cone, the given definition of convolution is consistent with the standard one. (Received October 22, 1974.)

*720-46-7 STUART P. LLOYD, Bell Laboratories, Murray Hill, New Jersey 07974.
On the mean ergodic theorem of Sine.

Robert Sine has shown that the ergodic averages $(I+T+\ldots+T^n)/(n+1)$ converge in the strong operator topology iff $\{x \in \mathfrak{X} : Tx = x\}$ separates $\{\xi \in \mathfrak{X}^* : T^*\xi = \xi\}$, $T : \mathfrak{X} \to \mathfrak{X}$ being a Banach space contraction with adjoint $T^* : \mathfrak{X}^* \to \mathfrak{X}^*$ [Proc. Amer. Math. Soc. 24, 438-439.] We prove a generalization in which $r(T) \leq 1$ (spectral radius) replaces $\|T\| \leq 1$, and any bounded averaging sequence converging uniformly to invariance replaces the ergodic averages. Conditions for the existence of such sequences are discussed.
(Received October 24, 1974.)

720-46-8 W. L. BYNUM, College of William and Mary, Williamsburg, VA 23185.
Weak parallelogram laws for Banach spaces. Preliminary report.

Theorem 1. If $2 \leq p$, if $p - 1 \leq b$, and if x and y are in ℓ_p, then

$$\|x + y\|^2 + b\,\|x - y\|^2 \geq 2\,\|x\|^2 + 2\,\|y\|^2 ;$$

moreover, $p - 1$ is the smallest possible value of b.

Bynum and Drew showed in Amer. Math. Monthly 79(1972), 1012-1015, that for $1 < p \leq 2$, the inequality in Theorem 1 is reversed. These two results motivate the following definitions: A Banach space V is an upper weak parallelogram space (or an upper w.p. space) with constant b if b is a number such that the inequality of Theorem 1 holds for each x and y in V. A lower w.p. space with constant b is a space V for which there is a number b such that the inequality of Theorem 1 is reversed for each x and y in V.

Theorem 2. If V is a lower w.p. space with constant b, then its dual V* is an upper w.p. space with constant $(16/b) + 1$. (Received October 29, 1974.)

*720-46-9 JOHN ANNULIS, University of Arkansas at Monticello, Monticello, Arkansas 71655.
Order Arguments on the Dimension of Vector Lattices.

Theorem: If V is an infinite dimensional Dedekind complete space with unit, then there exists an infinite set of positive disjoint components. Corollary. Let V be a Dedekind sigma-complete space with unit then dim (V) $\geq$ c. The Corollary is false if we do not assume that V has a unit. The Corollary may be established by Banach space arguments, but here we only use order arguments. (Received October 29, 1974.)

*720-46-10 JOHN N. MCDONALD, University of Arizona, Tempe, AZ 85281
Extreme Linear Operators on H_∞.

Let H_∞ denote the algebra of bounded analytic functions on the unit disk D. Let B denote the unit ball of H_∞. Let $P = \{T : T:H_\infty \to H_\infty,\ T \text{ linear},\ T(B) \subseteq B, \text{ and } T1 = 1\}$. Theorem. Suppose $F, G \in B$ and G is an extreme point of B. If there is a $T \in P$ such that $TF = G$, then there exists an extreme point S of P such that $SF = G$. Theorem. If F is inner and G is an extreme point of B with $G(D) \subseteq F(D)$, then there is an extreme point S of P such that $SF = G$. (Received October 30, 1974.)

*720-46-11 S.J. BERNAU AND H. ELTON LACEY. The University of Texas, Austin, Texas 78712,
Complex Banach lattices with order continuous norm.

Let X be a complex Banach space. In an earlier announcement (these Notices 74T-B186) the authors claimed a necessary and sufficient condition for X to admit the structure of of a complex Banach lattice with order continuous norm. In this talk we will prove the necessity part of our condition and show that in the case of L_p spaces $(1 \leq p < \infty)$ our condition reduces to X is an $\mathcal{L}_{p,\lambda}$ space for every $\lambda > 1$. (Received October 30, 1974.)

*720-46-12 CHARLES TUCKER, University of Houston, Houston, Texas, 77004. Concerning
σ-homomorphisms of Riesz spaces.

If L is a Riesz space (lattice ordered vector space), a Riesz homomorphism φ of L is an order preserving linear map which preserves the finite operations "$\vee$" and "$\wedge$". Theorem Suppose L is a σ-complete Riesz space with the property that every Riesz homomorphism of L onto an Archimedean Riesz space preserves the order limits of sequences. Then every Riesz homomorphism of L into an Archimedean Riesz space preserves the order limits of sequences. Theorem Suppose L is the αth Baire class of functions on the interval $[0,1]$, where $\alpha > 0$. Then every Riesz homomorphism of L into an Archimedean Riesz space preserves the order limits of sequences (Received October 30, 1974.)

*720-46-13 MARIJEAN SEELBACH, State University College at Brockport, Brockport, New York 14420. A common fixed point theorem of the alternative type. Preliminary report.

Let (E,T) denote a locally convex Hausdorff topological vector space whose topology is generated by a family P of continuous seminorms. Theorem: Let S be a compact subset of E. Let $f,g : S \to S$ be functions (not necessarily continuous) which satisfy

(1) $p[f(x) - g(y)] \leq \phi_1[p(x - f(x))] + \phi_2[p(y - g(y))] + \phi_3[p(x - y)]$, and

(2) $p[g(y) - f(x)] \leq \phi_1[p(y - g(y))] + \phi_2[p(x - f(x))] + \phi_3[p(x - y)]$, for every $x,y \in S$, $p \in P$, where ϕ_1, ϕ_2, and ϕ_3 are real valued functions which are continuous at 0 and either ϕ_1 or ϕ_2 satisfies the condition $\phi_i(r) < r$ if $r > 0$. Then either

(i) $\inf_{x \in S} \{M_p(x)\} \neq 0$ for some $p \in P$, where $M_p(x) = \max\{p(x - f(x)), p(x - g(x))\}$ or

(ii) f and g have a common fixed point in S. (Received October 22, 1974.)

720-46-14 S.J. GUCCIONE and C.V. STANOJEVIC, University of Missouri-Rolla, Rolla, Missouri 65401. Probabilistic Foundations of Quantum Theories and Rubin-Stone Spaces

Let R and N denote generalized inner product spaces in the sense of Rubin-Stone (PAMS, vol.4,1953) and Nagumo (Jap.J.Math.,vol.12,1936) respectively. Let (S,p) denote a probability space in the sense of Mielnik (Comm.Math.Phys.,vol.9,1968) and (S,p*) a probability space as modified by Stanojevic (TAMS,vol.183,1973). The following results are established: Theorem 1: Let R be a topological linear space with a Rubin-Stone functional q. Then (R,p) is a probability space of dimension 2, where $p(x,y) = q(x+y)/2[q(x)+q(y)], x,y \neq \theta$, $p(\theta,\theta) = 1$. Theorem 2: Let R be a real T_1-topological linear space. Let U be a bounded, absolutely convex neighborhood of "0" and q(x) its Minkowski functional. Define $p: U \times U \to [0,1]$ by $p(x,y) = q^2(x+y)/2[q^2(x)+q^2(y)]$, $x,y \neq \theta$, $p(\theta,\theta) = 1$. If (U,p) is a probability space of dimension 2, then R is a generalized inner product space in the sense of Rubin-Stone. Theorem 3: Let N be a generalized inner product space in the sense of Nagumo. Let $F^* = \{f \mid f \in C_R[0,\infty),\ f \geq 0,\ f(0) = 0,\ f(1) = 1,\ f(x) = 0 \iff x = 0\}$. If $p^*(x,y) = f(||x+y||)/2[f(||x||) + f(||y||)]$, $x,y \neq \theta$, $p^*(\theta,\theta) = 1$, where $f \in F^*$, then (N,p^*) is a probability space of dimension 2. The notion of a probability space arises in attempts to represent quantum states in a more general setting than Hilbert space. (Received October 30, 1974.)

*720-46-15 KLAUS BICHTELER, The University of Texas at Austin, Austin, Texas 78712 A special class of function metrics. Preliminary report.

Let X be a set, $\mathcal{R}$ a vector lattice of functions closed under the operation $f \to f \wedge 1$, and $G : [0,\infty]^X \to [0,\infty]$ an increasing countably subadditive function with $G(rf) \to 0$ as $r \downarrow 0$ $(f \in \mathcal{R})$. The G-mean closure $L^1(\mathcal{R},G)$ is complete, and satisfies the monotone and dominated convergence theorems provided (*) $G(f_n) \to 0$ for disjoint order bounded sequences of $\mathcal{R}$. There is a notion of G-measurability satisfying all the usual properties, which enables one to identify the dual, multipliers etc. of L^1 in terms of measurable functions.

Applications: The topological vector lattices in which disjoint order bounded sequences converge have order continuous topology and are exactly the spaces $L^1(\mathcal{R},G)$, which can be regarded as 'known'. Gauges G with (*) replace control measures for non-locally convex valued vector measures. They unify the theories of Lorentz spaces, Orlicz spaces, Koethe sequence spaces, and stochastic integrals. They appear naturally in weak type inequalities, martingale theory, and various interpolation schemes. (Received October 30, 1974.)

*720-46-16 CHARLES E. CLEAVER, Georgia Institute of Technology, Atlanta, Georgia 30332 and Kent State University, Kent, Ohio 44242, Packing Spheres in Orlicz Spaces.

Given an Orlicz space $L_M(X,a,\mu)$, let Λ_M be the largest number such that for $r \leq \Lambda_M$, an infinite number of non-overlapping balls of radius r can be packed in the unit ball of L_M. Using an interpolation theorem and examples, upper and lower bounds are found for Λ_M. These results generalize known results for L_p spaces. (Received October 30, 1974.)

*720-46-17 GARY D. RICHARDSON, Department of Mathematics, East Carolina University, Greenville, N. C. 27834, Sequential Convergence in C(X)

Let X be a sequential convergence space as defined by Frechet. Let C(X) denote the set of all continuous real-valued functions on X, equipped with following structure: $f_n \to f$

in $C(X)$ iff $f_n(x_n) \to f(x)$ in R whenever $x_n \to x$ in X, which is known as continuous convergence in $C(X)$. The pretopological modification (neighborhood space) of X is denoted by πX.

Proposition 1. $C(X)$ is complete (in the sense of Dudley).

Proposition 2. X is embedded in $C^2(X)$ iff X is sequentially regular (in the sense of Novak).

Proposition 3. $\pi C(X)$ is a Banach space whenever X is sequentially compact.

(Received October 30, 1974.)

720-46-18 JOHN BERGLUND, Department of Mathematical Sciences, Virginia Commonwealth University, Richmond, Virginia 23284. LMC and LUC Semigroups of Operators, Preliminary Report.

Let T be an equicontinuous semigroup of operators on a Mackey space E. If the topological dual E' of E contains an element λ_0 such that the weak* closed convex circled hull $\overline{\text{cco}}(T'\lambda_0)$ is absorbing, then one may define analogues to the Banach subspaces $LMC(S)$ and $LUC(S)$ of the space $C(S)$ of bounded continuous complex valued functions on a semitopological semigroup S. If right translation operators on $C(S)$ are considered, definitions of $LMC(S)$ and $LUC(S)$ may be given demonstrating that they are generalized almost periodic functions. Using a suitable anti-homorphic image $\hat{T}$ of T, one may do the same in the abstract setting. The groundwork is then laid to work with LMC- and LUC-semigroups of operators in a way reminiscent of deLoeew and Glicksberg's work with weakly amost periodic semigroups of operators. (Received October 31, 1974.)

*720-46-19 JOSEF WICHMANN, Texas Christian University, Fort Worth, Texas 76129. On commutative B^*-equivalent algebras.

A complex Banach $*$-algebra is said to be B^*-equivalent if it possesses a B^*-norm equivalent to the given norm. It is well known that a Banach $*$-algebra A is B^*-equivalent iff there exists a constant M such that $||\exp(ih)|| \leq M$ for all Hermitian elements h of A (Glickfeld's characterization). In this note we show that a commutative Banach $*$-algebra A is B^*-equivalent iff for every Hermitian element h in A there exists a constant M, depending on h, such that $||\exp(ith)|| \leq M$ for all real t. As an immediate consequence we obtain a new characterization of locally B^*-equivalent algebras (B.A. Barnes, Locally B^*-equivalent algebras, Trans. Amer. Math. Soc. 167(1972), 435-442, MR 45 # 5763). (Received November 1, 1974.)

720-46-20 PAK-KEN WONG, Seton Hall University, South Orange, New Jersey 07079. The p class in a dual B*-algebra. Preliminary Report.

Let A be an arbitrary dual B*-algebra. Suppose a is a nonzero element in A. Let $a^*a = \sum_n r_n e_n$ be a spectral representation of a^*a and $k_n = \sqrt{r_n}(>0)$. Define $|a|_p = (\sum_n k_n^p)^{1/p}$ for $(0 < p < \infty)$ and $|a|_\infty = \max\{k_n : n = 1, 2 \ldots\}$. For $a = 0$, we define $|a|_p = 0$ $(0 < p \leq \infty)$. Let $A_p = \{a \in A : |a|_p < \infty\}$. Theorem 1. For $1 \leq p \leq \infty$, A_p is a dual A*-algebra which is a dense two-sided ideal of A. Theorem 2. For $1 < p < \infty$, A_p

is uniformly convex and hence reflexive. Theorem 3. For $1 \leq p < \infty$, and $1/p + 1/q = 1$, the conjugate space of A_q can be identified with A_p. This is a generalization of the class C_p of compact operators on a Hilbert space. (Received November 1, 1974.) (Author introduced by Professor Charles Franke.)

*720-46-21 JERRY A. JOHNSON, Department of Mathematics, Oklahoma State University, Stillwater, OK Extreme Lipschitz Functions

If (S,d) is a metric space, let $E(S)$ denote the set of extreme points of $\{f: S \to R \mid |f(s)| \leq 1$ and $|f(s) - f(t)| \leq d(s,t)$ $\forall s, t \in S\}$. A. K. Roy characterized $E([0,1])$. The problem of characterizing $E(S)$ in general is discussed and partial results presented. (Received November 1, 1974.)

720-46-22 WITHDRAWN

*720-46-23 STEVEN F. BELLENOT, Florida State University, Tallahassee, Florida, 32306 On nonstandard hulls of convex spaces

Let E be a locally convex TVS with dual E'. The nonstandard hull $\hat{E}$ of E is a (standard) TVS which contains E as a subspace (see Trans. AMS, 172(1972),405-435.) Let $\perp_{E'}$ be the subspace of $\hat{E}$ which is mapped to zero by elements of E'. THEOREM. E is inductive semi-reflexive (Sovt. Math. Dok., 9 (1968), 1080 - 1082) iff $\hat{E} \oplus \perp_{E'}$. THEOREM. A F or DF space has invariant nonstandard hulls iff its strong dual has invariant nonstandard hulls iff bounded sets are precompact.

Proposition. A DF space is a Schwartz space iff bounded sets are precompact. The proposition is proved standardly. (Received November 4, 1974.)

720-46-24 BENJAMIN MUCKENHOUPT, Rutgers University, New Brunswick, NJ 08903 Weighted norm inequalities

The general weighted norm inequality problem to be discussed is the following. Given an operator T and p satisfying $1 \leq p < \infty$, determine all pairs of non-negative functions $U(x)$,

$V(x)$ such that $\int_{-\infty}^{\infty} |Tf(x)|^p U(x)dx \leq C\int_{-\infty}^{\infty} |f(x)|^p V(x)dx$, where C is a constant independent of f. For some operators such as the Hilbert transform or the Hardy-Littlewood maximal function it is not hard to obtain a necessary condition for such an inequality. In these cases when $U(x)$ is assumed to equal $V(x)$ the necessary condition for a given value p_0 of p can be shown to imply weak type inequalities for values of p both less than and greater than p_0 so that a standard interpolation argument shows that the strong type norm inequality is true. If $U(x)$ is not assumed equal to $V(x)$, the conjectured condition for a given p does not imply weak type for a smaller p. It seems that a new type of interpolation theorem with change of measure is needed. The suggested generalization of the Marcinkiewicz theorem would assume that for $p_0 \leq p \leq p_1$ that T maps $L^p(V)$ into weak $L^p(U)$ for any pair U, V satisfying a condition, C_p, depending on p, and conclude that for $p_0 < p < p_1$ T maps $L^p(V)$ into $L^p(U)$ for any pair U, V satisfying C_p. (Received November 4, 1974.)

720-46-25 JOHN WOLFE, Oklahoma State University, Stillwater, OK 74074, Equivalent strictly convex increasing norms on $C(S)$ spaces, Preliminary report

A norm on a Banach space is strictly convex if the surface of the unit ball contains no line segments. A norm $\|\cdot\|$ on the Banach space $C(S)$ of continuous functions as a compact Hausdorff space S is increasing if $0 \leq f \leq g \Rightarrow \|f\| \leq \|g\|$. The point $s \in S$ has the neighborhood countable intersection property if the family of neighborhoods of s is closed under countable intersections. An adaption of an argument of Day [Trans. Amer. Math. Soc. 78 (1955), 516-528] gives: Theorem: If S has a point with the neighborhood countable intersection property, then $C(S)$ has no equivalent strictly convex increasing norm. (Received November 6, 1974.)

*720-46-26 B. J. PETTIS, University of N.C., Chapel Hill, N.C., Openness For Homomorphisms Via Somewhere Denseness

A homomorphism T on a subsemigroup S' of a topological group G to a Hausdorff group H is "somewhere dense" if $T(N \cap S')$ is somewhere dense for every neighborhood N of the origin. Two theorems, elaborations of known results and essentially consequences of Weston's bijection open mapping theorem and Kelley's uniform space lemma, are given; these assert if T is somewhere dense then under mild restrictions on G and T the latter is open on S' to H. These imply certain open mapping results of Katznelson, Badé and Curtis, Kaufman, Pták, Bernard, Ng, and Glicksberg, and indicate the possible importance of characterizations of somewhere denseness for sets and maps. (Received November 4, 1974.) (Author introduced by Professor R.L. Davis.)

*720-46-27 CLIFFORD KOTTMAN, Oregon State University, Corvallis, Or. 97330 and BOR-LUH LIN, University of Iowa, Iowa City, Iowa 52240. On the other side of nearest points.

Let A be a closed convex bounded subset of the Banach space X and for each x in X, let $P_A(x)$ denote the (possibly empty) set of points of A nearest to x. For an element y of $P_A(x)$, consider the ray $R(x,y) = \{tx + (1-t)y: t \leq 0\}$

on the "other side" of the nearest point. Our question concerns whether the union $OS(A) = \cup\{R(x,y): y \in P_A(x) \text{ and } x \in X\}$ is all of X. Conjecture: $OS(A) = X$ for all closed convex bounded subsets A of X only if X is finite dimensional. The converse of the conjecture is easy to establish. Theorem: For each p with $1 \leq p \leq \infty$, l_p contains a closed convex bounded set A such that $OS(A) \neq l_p$. (Received November 4, 1974.)

720-46-28 WILLIAM L. GREEN, Georgia Institute of Technology, Atlanta, Georgia 30332. Compact groups of automorphisms of von Neumann algebras, Preliminary report.

Let A be a von Neumann algebra, let B be its unit ball, and let Aut(A) be the group of all automorphisms of A. We consider Aut(A) in any of several natural topologies obtained by viewing it as a set of maps of B into B. (These topologies have recently been considered by Araki, Connes, Haagerup, and others.) We then derive necessary and sufficient conditions that a subgroup of Aut(A) have compact closure in Aut(A). A special case of these results concerns the von Neumann algebra L generated by the left regular representation of a locally compact unimodular group $\tilde{G}$. If G is the group of all inner automorphisms of $\tilde{G}$, then G may be identified with a subgroup of Aut(L), and G has compact closure in Aut(L) iff $\tilde{G}$ is an $[FIA]^-$ - group. (Received November 4, 1974.)

720-46-29 JOHN A. LINDBERG, JR., Syracuse University, Syracuse, New York, 13210. Renorming a normed algebra having a semi-group of near isometries. Preliminary Report.

Let $(A, ||\cdot||)$ be a normed commutative algebra with identity e and let $\varepsilon \in (0,1]$. A multiplicative semi-group S of non-zero elements with $e \in S$ is called a semi-group of ε-near isometries if $||sa|| \geq \varepsilon||s||\,||a||$ holds for all $(s,a) \in S\times A$, and a semi-group of isometries if the preceeding inequality holds with $\varepsilon = 1$. Our main result states that if S is a semi-group of ε-near isometries in A, then there is an equivalent norm on A with respect to which S is a semi-group of isometries. The equivalent norm constructed is shown to be the largest such norm less than the given norm. In the second part of the paper, our results are concerned with norms $||\cdot||_0$ on [A,S], the usual algebra of quotients, that extend $||\cdot||$. One of our results states that if S is a semi-group of ε-near isometries and a set of generators for A, then the $||\cdot||_0$-completion of [A,S] is a regular Banach algebra whenever $||\cdot||_0$ satisfies $||s^{-1}||_0 \leq \beta||s||^{-1}$ for all $s \in S$ and some positive β. As a corollary we have that, under the above hypotheses, every non-topological divisor of zero in A has an inverse in the $||\cdot||_0$-completion of [A,S]. (Received November 4, 1974.)

*720-46-30 A. C. COCHRAN, University of Arkansas, Fayetteville, Arkansas 72701 and T. K. MUKHERJEE, Jadavpur University, Calcutta, India. Unique topologies in duality. Preliminary report.

The main results of this paper give conditions which insure the existence of a unique topology in dual pairing. Any subspace of the arbitrary product of complex fields which contains the corresponding direct sum has this property. In addition, related questions for topological algebras are considered. In particular, the direct sum of scalar fields (R or C) is shown to have a unique locally m-convex topology in duality. It is also shown that A-convex topologies may have locally m-convex topologies in the same dual pairing. (Received November 4, 1974.)

720-46-31 ROBERT A. FONTENOT and IRWIN SCHOCHETMAN, Oakland University, Rochester, Michigan 48063. Compact operators and induced representations. Preliminary report.

Let G be a locally compact group and π a representation in a Banach space E. Under certain fairly general conditions, we define a representatio of G induced from π. Besides obtaining basic results on induced representations, we obtain Theorem: Let the quotient G/H be compact. If π is CCR (Hilbert-Schmidt), so is the induced representation of G. Representations of type C_p for $p \geq 1$ are defined and studied; trace-class and Hilbert-Schmidt representations are special cases. (Received November 4, 1974.)

720-46-32 JAMES R. HOLUB, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. Strict convexity and orthogonality in Banach spaces. Preliminary report.

Several characterizations of strictly convex Banach spaces suggested by various notions of orthogonality are given. Extensions of these results to characterizations of inner product spaces are also obtained. (Received November 4, 1974.)

720-46-33 JUDITH PALAGALLO, Colorado State University, Fort Collins, Colorado 80523 Linear and Nonlinear Functionals on the Spaces L^p, $0 < p < 1$. Preliminary report

Let (X,β,μ) be a measure space where μ is a finite (σ-finite) nonnegative measure on the members of β. The effect of the presence of atoms in X on the existence of linear and nonlinear functionals on $L^p(X)$, $0 < p < 1$, is studied. Then, under various continuity requirements on a nonlinear additive functional F, we obtain integral representations for F of the form

$$F(x) = \int_X (f \circ x)\, d\mu ,$$

for all x in $L^p(X)$, $0 < p < 1$, where f is a real-valued function on R with properties corresponding to those imposed on F. (Received November 4, 1974.)

*720-46-34 L. C. MOORE, JR., Duke University, Durham, North Carolina 27706. Hyperarchimedean Riesz spaces.

A Riesz space L is called hyperarchimedean if for every order ideal A in L the quotient Riesz space L/A is archimedean. Theorem. An archimedean Riesz space L is hyperarchimedean if and only if every finite dimensional subspace of L is contained in a finite dimensional Riesz subspace of L. (Received November 4, 1974.)

*720-46-35 DONALD HARTIG, Ohio University, Athens, Ohio 45701. On functors from compact pairs to Banach algebras.

Certain natural properties of the functor taking a compact pair (X, A) to the Banach algebra $C(X, A)$ of continuous complex-valued functions on X vanishing on A are shown to characterize it. (Received November 4, 1974.)

720-46-36 P. P. NARAYANASWAMI, Memorial University, St. John's, New Foundland, Canada A1C 5S7. On strong triples for closed graph theory. Preliminary report.

Let $\mathcal{K}$ denote a class of linear maps between locally convex spaces, CG the class of linear maps with a closed graph, and $\mathfrak{B}$ the category of locally convex spaces and continuous linear maps. For $\mathcal{A},\mathcal{B} \subseteq |\mathfrak{B}|$ define $\langle \mathcal{A},\mathcal{B},\mathcal{K} \rangle$ to be a strong triple for CGT iff (i) $E \in \mathcal{A}$, $F \in \mathcal{B}$, $f: E \to F$, $f \in \mathcal{K} \cap CG \Rightarrow f \in$ Morph $\mathfrak{L}$; (ii) $E \in \mathcal{A}$ iff for each F in $\mathcal{B}$, $f: E \to F$, $f \in \mathcal{K} \cap CG \Rightarrow f \in$ Morph $\mathfrak{B}$; and (iii) $F \in \mathcal{B}$ iff for each E in $\mathcal{A}$, $f: E \to F$, $f \in \mathcal{K} \cap CG \Rightarrow f \in \mathfrak{B}$. If we relax either (ii) or (iii), we obtain a weak (domain) or a weak (range) triple for CGT. For a large collegtion $\mathcal{A}$ of spaces, we provide a method to construct strong triples $\langle \mathcal{A},\mathcal{B},\mathcal{K} \rangle$, and choosing subclasses $\mathcal{B}'$ of $\mathcal{B}$, we obtain several generalizations of the class $\mathcal{A}$. Specifically choosing the range as the class of all separable Banach spaces, or separable Infra-Pták spaces, several generalizations of countably barrelled and countably infrabarrelled spaces are obtained. The case when $\mathcal{B}$ reduces to a singleton is interesting. We study two such instances, where F is either c_0 or $C[0,1]$, and describe the corresponding domain classes. In the analogous situation for the open mapping theorem, we are able to obtain only weak triples. (Received November 4, 1974.)

*720-46-37 G. T. RÜTTIMANN, Univ. of Mass. Math/Stat Dept., Amherst, Mass. 01002 Jordan-Hahn Decomposition of Signed Weights on Finite Orthogonality Spaces.

Let $(X,\#)$ be a finite orthogonality space (X a non empty set; $\#$ a non-reflexive, symmetric relation) with at least one weight, and Δ a non-empty, convex, closed (in the topology of pointwise convergence) subset of weights in the signed weight space of $(X,\#)$. Then $(\mathrm{lin}\,\Delta,\Delta)$ is a finite dimensional base normed space. $\{\mathfrak{L}(X,\#),\underline{c}\}$ denotes the logic, and $((\mathrm{lin}\,\Delta)^*,\leq,e)$ the order unit normed space associated with $(\mathrm{lin}\,\Delta,\Delta)$. There is an order monomorphism f from $\{\mathfrak{L}(X,\#),\underline{c}\}$ into the order interval $\{[0,e],\leq\}$ as follows: $f_M(\nu) = 0$ if $M = \emptyset$, $f_M(\nu) = \Sigma_{x\in D}\nu(x)$ if $M \neq \emptyset$ $(M\varepsilon\mathfrak{L}(X,\#))$ where D is an orthogonal set such that $D^{\#\#} = M$. Main Theorem: f is an orderisomorphism from $\{\mathfrak{L}(X,\#),\underline{c}\}$ onto $\{\mathrm{ext}[0,e];\leq\}$ if and only if Δ has the Jordan-Hahn decomposition property (in the sense of N. Zierler) and is ultrafull. If it is so then (i) $(X,\#)$ is a Dacey space, (ii) Δ is unital and has finitely many pure weights, (iii), for any $M\varepsilon\mathfrak{L}(X,\#)$ $\Delta \subseteq \mathrm{aff}\,\{\omega\varepsilon\ \mathrm{ext}\Delta \mid \omega(N) = 0 \text{ or } = 1\}$ holds. This result is of interest for the operational approach (Foulis-Randall) to the Foundations of Empirical Sciences. (Received September 26, 1974.) (Author introduced by Professor T. A. Cook.)

720-46-38 PROFESSOR THURLOW COOK - UMass, Math/Stat Dept., Amherst, MA 01002 The Hahn-Jordan decomposition theorem in infinite quantum logics

Based on the theory of manuals of operations as developed in the works on Empirical Logics by C. Randall and D. Foulis, we give a necessary and sufficient condition that each signed state on such a logic can be written canonically as the difference of two positive states. The result is then applied to the traditional model of quantum logic, namely, the duality of projections and self-adjoint trace class operators on a separable Hilbert space and is also applied to countably additive measures on a compact Hausdorff space. The latter case gives the connection with the classical Hahn-Jordan decomposition for measures. (Received September 26, 1974.)

*720-46-39 SUN-YUNG CHANG, SUNY at Buffalo, Amherst, New York 14226. On the Structure and Characterization of Piecewise Continuous Functions.

Let $C_{\{\infty\}}$ be the subalgebra of L^∞ of the real line $\mathbb{R}$ consisting of functions f continuous on R, such that $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ both exist but are not necessarily equal, and let

$B_{\{\infty\}}$ be the closed subalgebra of L^∞ generated by H^∞ and $C_{\{\infty\}}$. The following results are proved: (i) Let BULC (for bounded uniformly logarithmically continuous) be the set of all bounded uniformly continuous functions f on $\mathbb{R}$ satisfying the condition: given $\epsilon > 0$, there exists a $\delta > 0$ such that $|f(hx) - f(x)| < \epsilon$ for all $x \in R$ whenever $|h - 1| < \delta$. Then BULC is the closed algebra generated by quotients of Blaschke products invertible in $B_{\{\infty\}}$. (ii) $B_{\{\infty\}} = H^\infty$ + BULC the linear hull of H^∞ and BULC. (iii) If B is any subalgebra of L^∞ containing H^∞ which has the same maximal ideal space as $B_{\{\infty\}}$, then $B = B_{\{\infty\}}$. For, E an arbitrary subset of $\mathbb{R} \cup \{\infty\}$, let C_E denote the space of functions in L^∞ that are continuous at each point of $\mathbb{R} \cup \{\infty\} - E$ and have one-sided limits at each point of E. Let B_E be the closed subalgebra of L^∞ generated by C_E and H^∞. All above results could be generalized to corresponding statements in B_E. (Received November 5, 1974.)

*720-46-40 CHARLES W. NEVILLE, Central Connecticut State College, New Britain, Connecticut 06050. Conditionally injective Banach spaces.

Let $\mathcal{C}$ be a subcategory of the category $\mathcal{B}$ of Banach spaces and linear maps of norm ≤ 1. We call a Banach space E (not necessarily an object of $\mathcal{C}$) $\mathcal{C}$ injective if, given objects A and B in $\mathcal{C}$, an into isometry $i \in \mathrm{Hom}_{\mathcal{C}}(A, B)$, and a map $f \in \mathrm{Hom}_{\mathcal{B}}(A, E)$, there exists $g \in \mathrm{Hom}_{\mathcal{B}}(B, E)$ such that $g \circ i = f$. If $\mathcal{C}$ is the full subcategory of all Banach spaces with a dense set of cardinality $\leq \aleph_0$, then each $\mathcal{C}$ injective is the direct limit of its subspaces in $\mathcal{C}$ of type $C(X)$. Further, $C(X)$ is $\mathcal{C}$ injective iff it satisfies a certain conditional completeness property as a vector lattice. These two results generalize the Goodner-Nachbin-Kelley characterization of injectives in $\mathcal{B}$. Finally, we give an example of a $\mathcal{C}$ injective E which is far from being a space of type $C(X)$. (Received November 5, 1974.)

*720-46-41 W.A. FELDMAN and J.F. PORTER, University of Arkansas, Fayetteville, Arkansas 72701. Representation of harmonic functions, Preliminary report.

The space E of real functions harmonic on the complex plane in the topology of compact convergence is realized as a space S of continuous functions on the unit circle with a locally convex topology stronger than the sup norm topology. The dual S' is an extension of the measures on the circle analogous to the Schwartz distributions. The space E' of functions harmonic on some neighborhood of zero then has representation via the members of S'. This includes the Herglotz representation of harmonic functions on the open unit disc which are pointwise positive. (Received November 5,1974.)

*720-46-42 J.F. PORTER and W.A. FELDMAN, University of Arkansas, Fayetteville, Arkansas 72701. Order and distributions, Preliminary report.

The Schwartz distributions on the unit circle with their usual topology are obtained as a quotient space of a generalized base norm space V'. Here V' is the dual of a generalized order unit space which is an extension of the infinitely differentiable functions on the circle with their usual topology.
(Received November 5, 1974.)

720-46-43 JAMES N. HAGLER, Catholic University of America, Washington, D.C. 20064. A cousin of a space of R.C. James. Preliminary report.

By modifying an example of James and using techniques of Lindenstrauss and Stegall, we construct a separable Banach space X with basis (e_n) satisfying: 1. If F is the closed span of the functionals

X^* biorthogonal to the e_n's, then X^*/F is isometric to $c_0(A)$ (A uncountable). 2. If Y is a ace of X with Y^* nonseparable, then Y contains a subspace isomorphic to c_0. (Received nber 6, 1974.)

-46-44 FRANCIS SULLIVAN, Catholic University of America, Washington, D.C. 20064. Upstream geometry. Preliminary report.

Let X be a Banach space and X, X', X'', X''' and $X^{(4)}$ be the successive dual spaces. A sequence of a well-known theorem of Dixmier is that if X is not reflexive then $X^{(4)}$ is not rotund. act, Dixmier's theorem shows that if X is not reflexive then $X^{(4)}$ fails to have a property which is ictly weaker than rotund involving only the behavior of X' in X'''. This observation motivates the finition of Banach spaces which are very smooth (VS) or very rotund (VR). The local reflexivity inciple is used to obtain connections between VS, VR and other geometric conditions for reflexivity. ne results are "upstream" in the sense that properties of X imply conditions in X''. Local reflexivity also applied to Dixmier's condition to obtain a new proof of a criterion of R.C. James for irreflexivity. Received November 6, 1974.)

*720-46-45 ROBERT SHARPLEY, III. Oakland University, Rochester, Michigan 48063. Counterexamples in interpolation theory.

Various indices have been considered for rearrangement invariant function spaces in order to obtain interpolation theorems. Shimogaki ["A note on norms of compression operators on function spaces," Proc. Japan Acad. 46 (1970), 239-242.] has given an example of a space X where two types of indices (Boyd and fundamental) are unequal. Using this space, one can show that several theorems appearing in the literature are incorrect. For example, the fundamental indices of X $(\underline{\gamma}_X, \overline{\gamma}_X)$ are both equal to 1/2 but X is not an interpolation space for L^p and L^q when $1<p<2<q<\infty$. X also provides a counterexample to a conjecture of the author. X satisfies

$$L^{2,1} \hookrightarrow X \hookrightarrow L^{2,\infty}$$

but X is not an interpolation space for $\Lambda(X) = L^{2,1}$ and $M(X) = L^{2,\infty}$.
(Received November 6, 1974.)

*720-46-46 ALAN M. GENDLER, California State U., Long Beach, CA 90840. Extreme operators in the unit ball of $L(C(X),C(Y))$.

X, Y are compact Hausdorff spaces. $C(X)$, $C(Y)$ are the Banach spaces of continuous complex-valued functions on X and Y, respectively. $L(C(X),C(Y))$ is the space of bounded linear operators from $C(X)$ to $C(Y)$. If E is a Banach space, then $S(E)$ is the closed unit ball in E. An operator T which is extreme in $S(L(C(X),C(Y)))$ is nice if $T^*(\text{ext } S(C(Y)^*)) \subset \text{ext } S(C(X)^*)$. For each $y \in Y$, ϵ_y denotes point mass at y. Theorem 1. If T is extreme in $S(L(C(X),C(Y)))$ and $\|T^*(\epsilon_y)\| = 1$ for each $y \in Y$, then T is nice. Corollary. If T is extreme in $S(L(C(X),C(Y)))$ and T is compact, then T is nice. Theorem 2. If X is metric, Y is basically disconnected, and T is extreme in $S(L(C(X),C(Y)))$, then T is nice. Now assume that the functions in $C(X)$ and $C(Y)$ are real-valued. The next theorem is a special case of a theorem proved by Blumenthal, Lindenstrauss, and Phelps (Pacific J. Math., 15(1965), 747-756). However, the proof does not use any selection theorem. Theorem 3. If X is the Cantor set and T is extreme in $S(L(C(X),C(Y)))$, then T is nice. (Received November 6, 1974.)

720-46-47 C.J.A. HALBERG, JR., University of California, Riverside, California 92502. On structure of the spectra of certain classes of operators on the ℓ_p spaces. Prelimin report.

We introduce some elementary techniques for the determination of the fine structure of the sp of certain classes of bounded linear operators on the sequence spaces ℓ_p, including some of those studied by Stafney (Pacific J. Math. 42(1972), 515-525). In particular we compute the fine structur the spectra of the operators defined on the ℓ_p spaces, $1 \leqq p \leqq \infty$, by the lower triangular matrix A (a_{ij}), where $a_{ij} = a_i$, $i \geqq j > 0$, $\sum_i |a_i| < \infty$. This result, together with other lemmas, is used to determine the fine structure of the spectra of operators defined by more general classes of infinite matrices. (Received November 6, 1974.) (Author introduced by Professor S.R. Gordon.)

720-46-48 WITHDRAWN

720-46-49 AARON R. TODD, Brooklyn College, Brooklyn, New York 11210. Images of linear pseudocomplete spaces.

Definitions. A topological space has a continuous metric iff there is a metric which induces on X a topology weaker then the topology of X. A function f from X to a topological space Y is almost open iff, for each U open in X, fU is contained in $\mathrm{int}_{fX}((fX)^- \cap fX)$. For pseudocompleteness of X, s Oxtoby [Fund. Math. 49(1961), 157-166]. The following extends the theorem in Abstract 711-46-55, these *Notices* 21(1974), A-189: The image of a linear pseudocomplete space under a linear continuous almost open map is complete if it has a continuous metric. The corollaries follow immediately: 1. The linear continuous image of a convex pseudocomplete space is complete if it is barrelled and has a continuous metric. 2. The linear continuous image of a linear pseudocomplete space is complete if it is Baire and has a continuous metric. As each uncountable product of nontrivial (linear) pseudocomplete spaces contains proper dense (linear) pseudocomplete subspaces, a linear pseudocomplete space need not be complete, and a convex pseudocomplete space need not be a Pták space. (Received November 6, 1974.)

720-46-50 HASKELL P. ROSENTHAL, University of Illinois, Urbana, Illinois 61801. Some recent discoveries in the isomorphic theory of Banach spaces.

• We shall discuss recent progress made by several researchers concerning both infinite and finite dimensional Banach spaces. In particular, we shall consider two fundamental problems. The first concerns infinite dimensional subspace structure, and is as follows: Does every infinite dimensional Banach space contain a subspace which is either isomorphic to c_0, isomorphic to ℓ^1, or reflexive and infinite dimensional? (Two Banach spaces are said to be isomorphic if there exists a bicontinuous linear bijection between them; c_0 denotes the space of all sequences of scalars vanishing at infinity; ℓ^1 the space of all sequences of scalars with absolutely convergent sum.) The recent work on this problem involves combinatorial and real-variable techniques, in particular a basic method and theorem of Baire on functions of the first Baire class. This work reduces the problem to the case of spaces not containing ℓ^1. For example, it is proved that if a separable Banach space does not contain a subspace isomorphic to ℓ^1, then it is "sequentially dense" in its double dual, and also every bounded sequence in its double dual has a weak* convergent subsequence. The second problem concerns "local" or finite dimensional subspace structure, and is as follows: Does every infinite dimensional Banach space contain uniformly complemented ℓ_n^p's for some $p \in \{1,2,\infty\}$? Precisely, given such a space B, is there a $\lambda < \infty$ and a $p \in \{1,2,\infty\}$ so that for all n there exist linear operators $u_n : \ell_n^p \to B$ and $v_n : B \to \ell_n^p$ with $v_n u_n = I$ and $\|v_n\| \, \|u_n\| \leqq \lambda$? The progress on this problem uses some rather powerful finite dimensional inequalities concerning subspaces of L^1, and more generally, operators from finite dimensional spaces into L^1. (Received November 6, 1974.)

)-46-51 CRUMP BAKER, Dept. of Mathematics, University of Kentucky, Lexington Kentucky 40506 Representation of C*algebras, Preliminary Report

Let A be a C*algebra. It is known from the Dauns-Hofmann representation theory that there exist a uniform field of C*algebras such that A is isometric and isomorphic to the ring of global sections which vanish at infinity. Under the assumption that the set $\{za \mid z$ is in the center of A, $a \in A\}$ is dense in A, it is shown that K, the Pedersen ideal of A, is isometric and isomorphic to the ring of sections with compact support, that the algebra of multiplications on A (double centralizer algebra), M(A), is isomorphic to the ring of all bounded sections, and finally that the algebra of multiplications on K, M(K), is isomorphic to the ring of all (continuous) sections. (Received November 6, 1974.)

720-46-52 TERRENCE S. McDERMOTT, Loyola Marymount University, Los Angeles, California 90045. Sequential convergence in locally convex spaces.

Let (E,T) be a Hausdorff, locally convex, topological linear space, T_1 a locally convex topology for E satisfying $\sigma(E,E') \leqq T_1 \leqq T$, and T_0 the topology for E' of uniform convergence on all T_1-null sequences of E. Theorem. Every T_1-null sequence is T-null iff every T-equicontinuous subset of E' is T_0-precompact. Corollary 1. Every T_1-null sequence is T-null iff every T_1-null sequence is T-precompact. Corollary 2. If (E,T) is metrizable, every T_1-null sequence is T-null iff T_0 coincides with $\lambda(E',E)$, the topology of uniform convergence on T-precompact subsets of E. (Received November 6, 1974.)

720-46-53 D. L. BURKHOLDER, University of Illinois, Urbana, Illinois 61801. Martingale methods in analysis.

• Martingale theory, long a standard tool in probability, is now being used with increasing frequency and effectiveness in such fields as differential equations, the structure theory of Banach spaces, and harmonic analysis. Some of the recent developments in both the theory and its applications will be sketched. New work to be described includes information on the problem of the existence of an unconditional basis for H^1. (Received November 6, 1974.)

47 Operator Theory

720-47-1 MILTON ROSENBERG, 8400 Shore Front Pkwy, Rockaway Beach, New York 11693. Square-integrability wrt nonnegative hermitian measures and spectral integrals of operator-valued functions. Preliminary report.

We extend the results of Rosenberg [J. Multivariate Anal. 2(1974), 166-209] by making use of facts "On operator ranges" [Fillmore and Williams, Advances in Math. 7(1971), 254-281]. Let M, N be $q \times q$ nonnegative hermitian measures of finite trace on $\mathcal{B}$, a σ-algebra over Ω. Let Φ be a $p \times q$ linear operator (not necessarily densely-defined or closed) -valued function on Ω. Let A be a $p \times q$ operator. Let $1 \leqq p,q,r \leqq \infty$. For $\Phi \in L_{2,M}$, let $\int \Phi\, dM\, \Phi^* = \int_\Omega (\Phi\sqrt{M'})(\Phi\sqrt{M'})^*\, d\nu$ (Bochner) where $M \ll \nu$. Theorem. (a) $\Phi \in L_{2,M}$ and $\Phi \in L_{2,N} \Rightarrow \Phi \in L_{2,M+N}$ and $\int \Phi\, d(M+N)\, \Phi^* = \int \Phi\, dM\, \Phi^* + \int \Phi\, dN\, \Phi^*$; (b) $M(B) \leqq N(B)\ \forall\, B \in \mathcal{B}$ and $\Phi \in L_{2,N} \Rightarrow \Phi \in L_{2,M}$ and $\int \Phi\, dM\, \Phi^* \leqq \int \Phi\, dN\, \Phi^*$; (c) [$\mathcal{R}$= range]

$\mathcal{R}\sqrt{M'(\omega)} \subseteq \mathcal{R}\sqrt{M(\Omega)}$ a.e. (ν) and $A\sqrt{M(\Omega)}$ is Hilbert-Schmidt $\Rightarrow \Phi(\omega) \equiv A \in L_{2,M}$ and $\int A \cdot dM \cdot A^{\prime}$ $A\sqrt{M(\Omega)}(A\sqrt{M(\Omega)})^*$; (d) $\mathcal{R}\sqrt{M(\Omega)} = \{y \mid y = \int (\sqrt{M'(\omega)})x(\omega)\, d\nu,\ x \in L_{2,\nu}(\ell^2_q)\}$; (e) E, a spectral measure for a Hilbert space $\mathcal{H} \Rightarrow$ (i) $\int \Phi\, dE$ is a linear operator from $\mathcal{H}^q$ to $\mathcal{H}^p$; (ii) $M_z(B) \leqq M_y(B)\ \forall\, B \in \mathcal{B}$ $z = L^q_x y$ where $x \in \mathcal{H}^r$; (iii) $\forall x \in \mathcal{H}^r$, $L^p_x \int \Phi\, dE \subseteq \int \Phi\, dE\, L^q_x$. Etc. (Received October 30, 1974.)

*720-47-2 JOSEPH BALL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and ARTHUR LUBIN, Northwestern University, Evanston, Illinois 60201. Contractive perturbations of restricted shifts.

We study a class of contractive perturbations of restricted shifts on infinite dimensional Sz.-Nagy-Foiaş spaces. In special cases, we determine the spectra and characteristic functions of the perturbed operators and relate this to the original operators; when the perturbation is unitary, we give a unitary equivalence to multiplication by $e^{i\theta}$ in $L^2(\mu)$ for an operator-valued measure μ. (Received October 21, 1974.)

*720-47-3 ALAN LAMBERT, University of North Carolina, Charlotte, North Carolina 28223. Equivalence for groups and semi-groups of operators.

Let H be a complex Hilbert space. Theorem 1. If $\{S_g\}$ and $\{T_g\}$ are strongly continuous unitary group representations of the σ-compact, locally compact abelian group G and $\sup_{g \in G} \|S_g - T_g\| < 1$ then $\{S_g\}$ and $\{T_g\}$ are unitarily equivalent. Theorem 2. If $\{S_t\}$ and $\{T_t\}$ are strongly continuous semi-group representations of $\mathbb{R}_+$ and $\sup_{t \in \mathbb{R}_+} \|S_t - T_t\| < 1$ then $\{S_t\}$ and $\{T_t\}$ are unitarily equivalent. The proof of Theorem 2 uses the following lemma: If U and V are isometries on H and for some integer $n \geq 1$ $\|U^n - V^n\| < 2$ the dim Ker U^* = dim Ker V^*. This generalizes the fact that every unitary operator is the distance 2 from every non-unitary isometry (P. R. Halmos, A. Hilbert space problem book, Van Nostrand, Princeton, 1967, problem 119.). (Received October 23, 1974.)

*720-47-4 JOSEPH A. BALL, Virginia Polytechnic Institute and State University, Dept. of Mathematics, Blacksburg, Virginia 24061. Canonical models for contraction operators with unitary part. Preliminary Report.

Let T be a contraction operator acting on a complex separable Hilbert space H. We give a method of recapting the spectral invariants of T_u, the unitary part of T, from the behavior of T on its invariant subspaces. The methods are based on the canonical models (especially overlapping spaces) of de Branges-Rovnyak and the characteristic functions of Brodskii and Svarcman. A representation of T_u is obtained in terms of characteristic functions of restrictions of T, from which the spectral invariants for T_u may be read off.

Sample Theorem: Suppose $\mathcal{L}$ is a collection of invariant subspaces of T such that H is spanned by the vectors $P_M\,(I-wT^*)^{-1}(I-T^*T)x$ and $P_N(I-wT)^{-1}(I-TT^*)x$ for $x \in H$, $|w| < 1$, $M \in \mathcal{L}$, $N^{\perp} \in \mathcal{L}$, P_M (P_N) = projection onto M(N). Then T_u is absolutely continuous. If θ and θ_M are respective characteristic functions of T and $T|M$, the spectrum of T_u is $\Gamma \backslash J$ (Γ the unit circle) where J is the largest open subset of Γ such that the induced factorization $\theta(t) = \psi(t)\ \theta_M(t)$ is regular (in the sense of Sz.-Nagy-Foias) a.e. on J.

spectral multiplicity function is sometimes easy to interpret. In case $T = U(I+K)$ is a unitary multiplication operator and K is a Volterra type integral operator on , the spectral invariants of T_u are described in terms of U and K. These results (but methods) parallel results of T. L. Kriete (Notices 74T-B189 Vol.21, no.6) for the self part of a dissipative operator. (Received October 24, 1974.)

7-5 JEFFREY R. BUTZ, The Johns Hopkins University, Baltimore, Md. 21218.
Unitary Parts of Hankel Contractions.

a Hankel matrix $H = (c_{j+k})$ which is a contraction, necessary and sufficient conditions obtained for the existence of a non-trivial unitary part, and an explicit description this unitary part is given. In particular, a Hankel contraction H defined by $e^{i\theta}) = P_+f(e^{i\theta})x(e^{-i\theta})$, where $x \in H^2$, $f \in L^\infty$ with $\|f\|_\infty = \|H\|$, and $P_+ : L^2 \to H^2$ is orthogonal ojection, has a non-trivial unitary subspace M if and only if $|f(e^{i\theta})| = 1$ and $(e^{i\theta})f(e^{-i\theta}) = k^2$ a.e. for some constant k^2, $k = 1$. In such a case, the subspace M s given by $M = \{x \in H^2 : \bar{k}Hx = x\} \oplus \{x \in H^2 : \bar{k}Hx = -x\}$. Analogous results are obtained for abstract Toeplitz and Hankel contractions defined on any Hilbert space X having a unilateral shift. (Received October 15, 1974.)

*720-47-6 W.E. FITZGIBBON, University of Houston, Houston, Texas 77004
Product Integration in Reflexive Banach Spaces, Preliminary report.

X denotes a reflexive Banach space and $\{A(t) \mid t \in [0,T]\}$ a time dependent family of accretive operators defined in X. Def. $\{A(t) \mid t \in [0,T]\}$ is said to be product stable provided that: $D(A(t))$ is independent of t; $R(I + \lambda A(t)) \supseteq D(A(0))$ for $\lambda > 0$; there exists a subset N of $[0,T]$ having Lebesque measure zero so that $A(\cdot):[0,T] \sim N \times X \to X$ is weakly closed; and if $x \in D(A(0))$ there exists an $M(x) > 0$ so that for any portition $\{t_i\}$ $[0,T]$ we have $||A(u)\Pi_{i=1}^k(I + (t_i-t_{i-1})A(t_{i-1}))^{-1} x|| \le M(x)$. We have the following theorems: Theorem 1. Let $\{A(t) \mid t \in [0,T]\}$ be a product stable family in X, then for each $x \in D(A(0))$ there exists a unique strong solution to the Canchy initial value problem; $u'(t,x) + A(t)u(t,x) = 0$, $u(o,x) = x$ $t \in [0,T]$. Theorem 2. Let $\{A(t) \mid t \in [0,T\}$ satisfy the above conditions and $\varepsilon_n \downarrow 0$, then the solution to Canchy problem has representation $u(t, x) = \omega\text{-lim } \Pi^{[t/\varepsilon_n]}(I + \varepsilon_n A(i\varepsilon_n))^{-1} x$ where ω-lim denotes the weak limit. (Received October 30, 1974.)

720-47-7 WILLIAM HASTINGS, University of Michigan, Ann Arbor, Michigan, 48104. Subnormal operators quasisimilar to an isometry, Preliminary report.

Let $V = V_0 \oplus V_1$ be a Hilbert space isometry, where V_0 is unitary and V_1 is a unilateral shift of finite multiplicity. Theorem. Let S be a subnormal contraction, and let $S = S_0 \oplus S_1$ be its decomposition into a unitary operator and a completely nonunitary (c.n.u.) contraction. Then S is quasisimilar to V if and only if S_0 is unitarily equivalent to V_0 and S_1 is quasisimilar to V_1. We also give a complete description of when a c.n.u. subnormal contraction S_1 is quasisimilar to V_1. This generalizes a result of S. Clary [Abstract 684-B38, these Notices 18 (1971), 522]. (Received October 30, 1974.)

*720-47-8 Carl Pearcy, University of Michigan, Ann Arbor, Michigan 48104 and Norberto Salinas, University of Kansas, Lawrence, Kansas 66044
The reducing essential matricial spectra of an operator.

If T is an operator on a separable infinite dimensional Hilbert space $\mathscr{H}$ and n is a i integer, then the reducing essential $n \times n$ spectrum $R_e^n(T)$ of T is by definition the of all $n \times n$ complex matrices L such that there exists an identity preserving $*$-hom morphism φ from the C^*-algebra generated by T and $1_{\mathscr{H}}$ into the algebra of all $n \times$ complex matrices satisfying $\varphi(T) = L$ and such that φ can be factored through the Calkin algebra. The set $R_e^1(T)$ coincides with the reducing essential spectrum of the operator T, c.f. "Reducing essential eigenvalues" by Norberto Salinas, Duke Mathemat Journal, Vol. 40, No. 3, 1973. The following generalization of the Weyl-Von Neumann-Berg theorem is obtained. If T and S are n-normal operators on $\mathscr{H}$ (i.e. if each of them is unitarily equivalent to an $n \times n$ matrix of commuting normal operators on $\mathscr{H}$), t T is unitarily equivalent to a compact perturbation of S if and only if $R_e^n(T) = R_e^n(S)$. It is also shown that if T is an operator on $\mathscr{H}$ such that $R_e^n(T) \neq \emptyset$ and S is an n-normal operator on $\mathscr{H}$ such that $R_e^n(S) \subset R_e^n(T)$, then $T \oplus S$ is unitarily equivalent to a compact perturbation of T. (Received October 30, 1974.)

*720-47-9 JAMES E. THOMSON, Virginia Polytechnic Institute and State University, Blacksbu Virginia, 24061. Commutants of analytic Toeplitz operators. Preliminary repor

For φ in H^∞, let T_φ denote the analytic Toeplitz operator on H^2 defined by $T_\varphi f = \varphi f$. Le $\{T_\varphi\}'$ denote the commutant of T_φ. We characterize the commutants of certain analytic Toeplit operators in terms of the well-known commutants of unilateral shifts. Theorem. If the inner part of $\varphi - \varphi(\lambda)$ is a finite Blaschke product for uncountably many λ in the open unit disk, then $\{T_\varphi\}' = \{T_\beta\}'$ where β is a finite Blaschke product of which φ is a function. Furthermore, if $\varphi = \chi F$ is the inner-outer factorization of φ, then $\{T_\varphi\}' = \{T_\chi\}' \cap \{T_F\}'$. In particular, if φ is univalent and nonvanishing, then $\{T_{\varphi^k}\}' = \{T_z\}'$ for every positive integer k. (Received October 29, 1974.)

*720-47-10 W. G. DOTSON, JR., North Carolina State University, Raleigh, N. C. 27609
An iterative process for nonlinear monotonic nonexpansive operators in Hilbert space.

The following theorem is proved: Suppose H is a complex Hilbert space, and $T : H \to H$ is a monotonic, nonexpansive operator on H, and $f \in H$. Define $S : H \to H$ by $Su = -Tu + f$ for all $u \in H$. Suppose $0 \leq t_n \leq 1$ for all $n = 1, 2, 3, \ldots$, and $\sum_{n=1}^{\infty} t_n(1 - t_n)$ diverges. Then the iterative process $v_{n+1} = (1 - t_n)v_n + t_n S v_n$ converges to the unique solution $u = p$ of the equation $u + Tu = f$.
(Received October 31, 1974.)

720-47-11 JOHN MICHAEL MCVOY, Ogontz Campus of Penn State U., Abington, PA 19001. Weakly closed abelian algebras which are not self-adjoint need not be singly generated. Preliminary report.

The role of the adjoint operation in the study of von Neumann algebras is critical; its use underlies much of this theory. A result of von Neumann showed that any self-adjoint weakly closed abelian algebra is singly generated. However, if the weakly closed abelian algebra is not self-adjoint, the algebra need not be singly generated. Let A denote the algebra of

lytic Toeplitz operators. Let H be the direct sum of countably many copies of H^2 and M be family of all analytic Toeplitz matrices over A which are in B(H). Theorem: The family M an antisymmetric maximal abelian algebra of operators on H. Although M is naturally gener-ed by two isometries, it is not singly generated. Let K be the Hilbert space which is the rect sum of two copies of H^2 and N the algebra of 2 by 2 matrices over A with $A_{11} = A_{22}$ and $_2 = 0$. Theorem: The family N is an antisymmetric maximal abelian algebra over K which is ot singly generated. Nevertheless, N is naturally generated by an isometry and a nilpotent perator. (Received November 1, 1974.)

*720-47-12 DAVID F. FINDLEY, University of Cincinnati, Cincinnati, Ohio 45221
Polyhomogeneous Maps.

E and F are topological vector spaces; $0 \leq \alpha_0 < \alpha_1 < \cdots \alpha_n$ are real numbers. A map $P : E \to F$ is polyhomogeneous of degrees $(\alpha_0, \cdots, \alpha_n)$ in $x-x_0$ if it has the form $P(x) = \sum_{i=0}^{n} A_{\alpha_i}(x - x_0)$, where the maps $A_{\alpha_i} : E \to F$ satisfy $A_{\alpha_i}(tx) = t^{\alpha_i} A_{\alpha_i}(x)$ for all $t > 0$ and all $x \in E$. We have obtained theorems about such maps which generalize most of the results of Bochnak and Siciak (Polynomials and multilinear mappings between topological vector spaces, Studia Math 39(1971) Fasc.1, 59-76). Also, suppose $\sum$ is a cover of E by bounded sets, P is sequentially continuous, and $\alpha \geq \alpha_n$ is given. If $f : E \to F$ is such that $\lim_{t \downarrow 0}(f(x_0+th)-f(x_0)-P_\alpha(x_0+th))/t^\alpha = 0$ uniformly in h on each $S \in \sum$, then P is said to be the best local approximation to f of degree α at x_0. The results of Moore and Nashed (Local and asymptotic approximations of nonlinear operators by $(k_1, \ldots, k_N)$-homogeneous operators, TAMS 178(1963), 293-305) can be obtained for $\sum$-precompact maps when F is locally convex. Finally, the familiar "permanence" properties regarding products, quotients, composites and integrals of Taylor polynomials can be established for these approximations.
(Received November 1, 1974.)

720-47-13 FRANK GILFEATHER, University of Nebraska, Lincoln, Nebraska 68508
Reductive operators with a spectral separation property.

An operator on a Hilbert space H is called reductive if it is reduced by its invariant subspaces. Under a spectral separation hypothesis (given below) the structure of a reductive operator is determined as the direct sum of spectral operators with normal scalar parts. This extends similar results for quasi-decomposable or decomposable reductive operators. The spectrum of an operator T is said to be separated if whenever $z_1 \neq z_2$ belongs to $\sigma(T)$, then there exists hyperinvariant subspaces $\mathcal{M}_{in}$, $i = 1,2$, $n = 1,2,\ldots$, for T so that their closed span is H and disjoint neighborhoods N_i of z_i, $i = 1,2$, so that $N_i \cap \sigma(T|\mathcal{M}_{in}) = \phi$ while $N_i \cap \sigma(T|\mathcal{M}_{jn}) \neq \phi$, $j \neq i$, $i = 1,2$, $n = 1,2,\ldots$. (Received November 4, 1974.)

720-47-14 ELIAS S.W. SHIU, California Institute of Technology, Pasadena, California 91125. Numerical Ranges of Powers of Operators.

Let T be a bounded operator on a Hilbert Space. Let $W(T)$, $\sigma(T)$ denote the numerical range spectrum of T, respectively. Using von Neumann's theory of spectral sets, we prove

Theorem. Let $\sigma(T) \subseteq [m,\infty)$, $m > 0$ and let D be an operator commuting with T, $\sigma(D) \cap (-\infty,0] = \emptyset$. Then either $T = T^*$ or for each $\epsilon \in (0,m)$ there is an integer $N(\epsilon)$ such that $\{z \in \mathbb{C} \mid |z| \leq (m - \epsilon)^n\} \subseteq W(T^n D)$ $\forall n \geq N(\epsilon)$. The theorem readily implies the following results in C. R. DePrima & B. K. Richard, A Characterization of the Positive cone of $B(\mathcal{H})$, Indiana Univ. Math. J. 23(1973), 163-172. Corollary 1. If T^n is accretive, $n = 1,2,3\ldots$, then $T = T^*$. Corollary 2. If Re $\overline{W(D)} > 0$, $TD = DT$, $T^n D$ accretive, $n = 1,2,3\ldots$, then $T = T^*$. (Received November 4, 1974.)

*720-47-15 IVAN ERDELYI, Temple University, Philadelphia, Pennsylvania 19122. Strongly decomposable unbounded operators.

A strong version of spectral capacity (sc) in a Banach space X is applied to investigate strong decomposability in the unbounded case. A strong spectral capacity ssc in X is an application $E : \underline{F} \to \underline{S}($ from the collection $\underline{F}$ of closed subsets of the complex plane π into the set $\underline{S}(X)$ of subspaces of X which satisfies (i) $E(\emptyset) = \{0\}$, $E(\pi) = X$; (ii) $\bigcap_{n=1}^{\infty} E(F_n) = E(\bigcap_{n=1}^{\infty} F_n)$ for every sequence $\{F_n\} \subset \underline{F}$; (iii) for every $F \in \underline{F}$ and every finite open cover $\{G_i\}_1^n$ of F, $E(F) = \sum_{n=1}^{\infty} E(F \cap \overline{G}_n)$; (iv) for every $F \in \underline{F}$ the linear manifold $E_0(F) = \{x \in E(K) : K$ is compact and $K \subseteq F\}$ is dense in $E(F)$. A linear operator $T : D_T (\subseteq X) \to X$ has ssc E if T: is closed, has a nonvoid resolvent set, and satisfies: (v) $E(K) \subseteq D_T$ for every compact $K \subset \pi$; (vi) $T(E(F) \cap D_T) \subseteq E(F)$ $\forall F \in \underline{F}$; (vii) the restriction $T_F = T|(E(F) \cap D_T)$ has the spectrum $sp(T_F) \subseteq F$ for each $F \in \underline{F}$. (i), (iv) and (v) imply T is densely defined on X. The following properties of T with ssc E are obtained: (I) for every compact $K \subset \pi$, $T_K = T|E(K)$ is a strongly decomposable bounded operator on $E(K)$ with sc E_K defined by $E_K(F) = E(K \cap F)$ $\forall F \in \underline{F}$. (II) All operators with ssc satisfy the single valued extension property. (III) $E(K)$ is a spectral maximal space of T for every compact $K \subset \pi$, and $E(F)$ is a weak spectral manifold of T for each $F \in \underline{F}$. (IV) Every T under consideration has a unique ssc. (Received November 4, 1974.)

*720-47-16 GUIDO WEISS, Washington University, St. Louis, Missouri 63130. Interpolation on H^p spaces and some of their generalizations. Preliminary report.

On spaces of homogeneous type (Coifman-Weiss, Lecture Notes in Math., no. 242, Springer) one can define H^p spaces in terms of atoms (Coifman, "A real variable characterization of H^p," Studia Math., to appear); several interpolations of operator techniques can be extended to obtain new results concerning operators acting on these H^p and corresponding L^p spaces. One obtains Marcinkiewicz type theorems as well as extensions of interpolation of analytic families of operators. These results are applied to study Fourier multipliers since Fourier transforms of these H^p spaces are easily characterized. (Received November 6, 1974.)

*720-47-17 GARY WEISS, University of Michigan, Ann Arbor, Michigan 48104. Indecomposable Hilbert-Schmidt Operators.

In 1973, L. G. Brown, R. G. Douglas, and P. A. Fillmore characterized the set of all operators of the form $N + K$ where N is a normal operator and K is a compact operator

they asked whether or not every Hilbert-Schmidt operator is the sum of a normal operator i a trace class operator. They later asked if, for every Hilbert-Schmidt operator A, ere exists a normal operator N for which $A \oplus N$ is the sum of a normal operator and a :ace class operator. We produce a large class of Hilbert-Schmidt operators A none of which s the sum of a normal operator and a trace class operator, and furthermore, for each .rbitrary operator Q, $A \oplus Q$ is not the sum of a normal operator and a trace class operator. Ve then use this to show that their characterization of the operators $N + K$ does not hold true if we replace the class of compact operators by the trace class or by any ideal I for which $I \neq I^{1/2}$. (Received November 4, 1974.) (Author introduced by Professor Allen L. Shields.)

*720-47-18 WILLIAM R. DERRICK, University of Montana, Montana 59801.
A limit theorem for Functional Sturm-Liouville Problems.

Let $C^n=\{C^n[0,1],\|\cdot\|_n\}$, C_0^n the subspace of functions vanishing at 0, and $D_0^0=\{C_0^0[-1,1],\|\cdot\|\}$. Let $\varphi(\geq 0)\varepsilon D_0^0$ be positive on $(-\delta,0)$ and vanish on $[0,1]$. Define $T_r:C_0^1\to D_0^0$ by $T_r u=\varphi(-r)$ on $[-1,-r]$, $T_r u=\varphi$ on $[-r,0]$, and $T_r u=u$ on $[0,1]$. Consider the functional Sturm-Liouville eigenvalue problem $(*)(-u''=)Lu=\lambda F(\lambda,T_r u),u(0)=u'(1)=0$, where $F:R^1 x D_0^0\to C^0$ is a bounded continuous operator positive with respect to the cones of nonnegative functions. If F is superlinear there exists a continuum Γ_r of nontrivial positive solutions (λ,u) of $(*)$ joining $(0,0)$ to ∞ in C^1, for all $r>0$. This is not the case when $r=0$. Theorem. The continuum $\Gamma_r(0<r<<1)$ of nontrivial positive solutions (λ,u) of $(*)$ satisfies (1). $\|u\|_0\leq(\alpha/2)\|T_r 0\|^b\lambda$, for $\|u\|_0\leq\|T_r 0\|<\varepsilon$, and (2). $(2/\alpha)\leq\lambda\|u\|_0^{b-1}$, if $\|T_r 0\|\leq\|u\|_0$, $\alpha>0$, $b>1$. Thus two eigenfunctions correspond to each $\lambda<2/(\alpha\|T_r 0\|^{b-1})$, and the eigenfunction satisfying (1) tends to 0 as $r\to 0$. (Received November 4, 1974.)

*720-47-19 JOANNE DOMBROWSKI and GERD H. FRICKE, Wright State University, Dayton, Ohio 45431. The absolute continuity of phase operators.

Let H be a separable Hilbert space with orthonormal basis $\{\varphi_n\}$, V the unilateral shift operator on H so that $V\varphi_n = \varphi_{n+1}$, and A be defined by $A\varphi_n = a_{n-1}\varphi_n$ with $a_0 = 0$. Consider the operator $C = 1/2(V^*A + AV)$. If the sequence $\{a_n\}$ converges monotonically to 1 and is so chosen that the spectrum of C is exactly the interval $[-1,1]$, the operator C is called a phase operator. It was previously known that certain phase operators were absolutely continuous and that all phase operators had an absolutely continuous part. The present work completes the discussion by showing that all phase operators are absolutely continuous. (Received November 5, 1974.)

720-47-20 DONALD W. HADWIN, Indiana University, Bloomington, Indiana 47401. Closures of unitary equivalence classes. Preliminary report.

The author has determined (Abstract 74T-B55, these *Notices* 21(1974), A-306) the closure of the unitary equivalence class $U(T)$ for each operator T in a certain large class of operators on a separable infinite-dimensional Hilbert space. This class of operators has been enlarged to include the following: (1) the class of operators which generate a CCR (liminal) C^*-algebra (this class includes all n-normal operators), (2) the class of essentially n-normal operators, (3) the class of operators T for which T^*T

- 1 is compact, (4) the class of irreducible operators whose image in the Calkin algebra generates a C*-algebra. It is also shown that if T generates a CCR (postliminal) C*-algebra, then U(T) is cl iff T is unitarily equivalent to $A \oplus B \oplus B \oplus B \oplus \ldots$, where A, B act on finite-dimensional Hilbert s The techniques involved make use of the direct integral decompositions of representations of C*-alge (Received November 5, 1974.)

*720-47-21 L.J. GRAY, Union Carbide Corporation, Nuclear Division, P. O. Box Y, Oak Ridg Tenn. 37830. Products of Hermitean Operators. Preliminary report.

Let A and B be hermitean operators on a Hilbert space, $T = AB$. Williams and Radjavi (Michigan Math J. 16 (1969) 177) have shown that, contrary to the finite dimensional case, T is not necessarily similar to T^*. They have asked whether $\dim N(T) = \dim N(T^*)$ ($\dim N(X)$ is the dimension of the null space of X) is a sufficient condition to guarantee similarity; by means of a counter example it is shown to be insufficient. The situation when A and B are Fredholm operators is investigated, and it is shown that T and T^* are similar under various additional assumptions. (Received November 5, 1974.)

720-47-22 FRANCOIS TREVES, Rutgers University, New Brunswick, New Jersey 08903
Fourier integral operators with complex phase.

Summaries of the works of Melin and Sjostrand. Possible extensions and problems. Some applications. (Received November 6, 1974.)

*720-47-23 THOMAS G. KURTZ, University of Wisconsin, Madison, WI 53706
An Abstract Averaging Theorem

For each $t \geq 0$, let $A(t)$ generate a contraction semigroup on a Banach space L. Suppose the solution $u_t = \varepsilon A(t)u$ is given by an evolution operator $V_\varepsilon(t, s)$. Conditions are given under which $V_\varepsilon((t + s)/\varepsilon, s/\varepsilon)$ converges strongly as $\varepsilon \to 0$ to a semigroup $T(t)$ generated by the closure of $\overline{A}f \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T A(t)f\,dt$.

This result is applied to the following situation: Let A generate a contraction group $S(t)$ and the closure of $A + \varepsilon B$ generate a contraction semigroup $S_\varepsilon(t)$. Conditions are given under which $S(-t/\varepsilon)S_\varepsilon(t/\varepsilon)$ converges strongly to a semigroup generated by the closure of $\overline{B}f \equiv \lim_{T\to\infty} \frac{1}{T} \int_0^T S(-t)BS(t)f\,dt$. This work was motivated by and generalizes a result of Pinsky and Ellis for the linearized Boltzmann Equation. (Received November 6, 1974.)

*720-47-24 Professor RICHARD BOULDIN, Univ. of Georgia, Athens, Georgia 30602
Approximating the Shift with Toeplitz Operators

The distance from a (bounded linear) operator T to the set of non-negative operators is called the modulus of positivity, denoted $\delta(T)$. If U_n denotes the n dimensional shift then we prove that $\{\delta(U_n)\}$ is an increasing sequence which converges to 1; this verifies a conjecture of P. R. Halmos. For the unilateral shift, as an element of the Toeplitz operators, we exhibit all positive approximants (best approximating non-

negative operators) and all extreme points of the set of positive approximants. We extend those results to any nonnormal subnormal operator for which the zero operator is a positive approximant. (Received November 6, 1974.)

*720-47-25 DAVID P. MATHER, The American University, Washington, D.C. 20016, Commutativity Properties of Continuous Operators on the Space of Entire Functions.

In 1904, J. Schur published a paper asserting that if two finite order differential operators A and B commute with a third operator C, then AB = BA. His proof was not completely clear. We are concerned with a general examination of the algebra $\mathcal{L}(\mathcal{E})$ of continuous linear operators on the space $\mathcal{E}$ of entire functions; by a representation theorem of Buck, each of these can be represented by an infinite order differential operator with entire coefficients. The study concentrates on $\mathcal{F}$ the subalgebra of finite order operators, and on the nature of the centralizer $\mathcal{C}(A)$ in $\mathcal{L}(\mathcal{E})$ of a single operator $A \in \mathcal{F}$. Using a theorem of Delsarte and Lions, this is reduced to the study of $\mathcal{C}(D^N)$ and the complete structure of the latter can be described in terms of certain specific nilpotent operators. The basic techniques use the entire functions in λ and z associated with an operator T by $\hat{T}(z,\lambda) = e^{-\lambda z}Te^{\lambda z}$; analysis of the nature of these entire functions of λ yield decompositions of T. These imply $\mathcal{C}(D^N)$ is itself not commutative, but $\mathcal{C}(D^N) \cap \mathcal{F}$ is, in confirmation of Schur's assertion. A further application of the Delsarte and Lions theorem gives that in general $\mathcal{C}(A)$ is not commutative. (Received November 6, 1974.)

*720-47-26 RONALD C. ROSIER, Georgetown University, Washington, DC 20057. Tensor products of p-nuclear, quasi p-nuclear, and p-integral maps.

This paper extends the results of two earlier papers of Holub [Tensor product mappings, Math. Ann. 188 (1970), 1-12. MR 44 # 2052. Tensor product mappings II, Proc. Amer. Math. Soc. 42 (1974), 437-441]. The main results are:

(1) Let $1 \leq p < \infty$. If S and T are both p-nuclear, or quasi p-nuclear, or p-integral, then $S \otimes_\varepsilon T$ is of the same type and the appropriate norm of $S \otimes_\varepsilon T$ is bounded by the product of the corresponding norms of S and T.

(2) If α is in c_0 but not in ℓ^p for any $p < \infty$, then the diagonal map $D_\alpha : \ell^1 \to \ell^2$ is p-nuclear for every p, $1 < p < \infty$ (so also quasi p-nuclear, p-integral, and absolutely p-summing), but $D_\alpha \otimes_\pi D_\alpha$ is not absolutely p-summing for any p, $1 \leq p < \infty$ (hence not p-nuclear, quasi p-nuclear, or p-integral).

Combined with the results of Holub, this shows that p-nuclear, quasi p-nuclear, p-integral, and absolutely p-summing maps are preserved by the ε tensor product for all p, $1 \leq p < \infty$, but that only 1-nuclear and 1-integral maps are preserved by the π tensor product.
(Received November 6, 1974.)

720-47-27 P. M. FITZPATRICK, University of Chicago, Chicago, Ill. 60637 and W. V. PETRYSHYN, Rutgers University, New Brunswick, N. J. 08903. Positive eigenvalues for nonlinear operators.

Let X be a Banach space. If $K \subset X$, K is called a wedge if it is closed and if $\lambda x + \beta y \in K$ when $x, y \in K$ and $\lambda, \beta \in R$ with $\lambda, \beta \geq 0$; a

wedge K with the property that $K \cap \{-K\} = \{0\}$ is called a cone. Theorem: Let $K \subset X$ be a wedge, with $U \subset X$ a bounded neighborhood of 0. Suppose $C: \overline{K \cap U} \to K$ is compact and $\alpha > 0$ is such that $\|C(x)\| \geq \alpha$ if $x \in \partial_K(U \cap K)$. Let $T: \overline{U \cap K} \to K$ be condensing, with $i_K(T, U \cap K) \neq 0$. Then, if either K is a cone or $K \cap \bar{B}(0,1)$ is noncompact, there exists $\lambda \in R$ with $\lambda > 0$ and $x \in \partial_K(U \cap K)$ such that $x - T(x) = \lambda C(x)$. (See R. D. Nussbaum [Ann. Mat. Pura Appl. 89 (1971)]) for the definition of the fixed point index, $i_K(T, U \cap K)$.) A multivalued version of the above result is also proven, and applications are given to differential equations.

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*720-47-28 RALPH GELLAR, North Carolina State University, Raleigh, North Carolina.
Circularly symmetric normal and subnormal operators.

A bounded operator T in Hilbert space will be called rotationally invariant if and only if T is unitarily equivalent to $e^{i\theta}T$ for all real θ. T is circularly symmetric (c. s.) if and only if $U_\theta T U_\theta^{-1} M = e^{i\theta}T$ for U_θ a 2π-periodic unitary group. (A c. s. operator is an operator-weighted shift). Are these concepts equivalent? Yes for finite dimensional operators and normal operators on separable spaces. No on non-separable spaces. ν is a c. s. measure on the complex plane if and only if $\nu(e^{i\theta}E) = \nu(E)$ for all real θ, all measurable E. A subnormal weighted shift is multiplication by z, M_z, on $H^2(d\nu_1)$, a cyclic c. s. normal operator is M_z on $L^2(d\nu_2)$ for c. s. ν_1 and ν_2. Main Theorem: A cyclic subnormal c. s. operator is $M_z \oplus M_z$ on $H^2(d\nu_1) \oplus L^2(d\nu_2)$. The converse question "which operators of this form are cyclic?" is answered except in those cases where ν_1 and ν_2 are "kissing" and ν_2 is "outside" ν_1, for instance when ν_1 is Lebesgue area measure on the unit disc and ν_2 is Lebesgue linear measure on the unit circle.

(Received November 6, 1974.)

720-47-29 R. J. FLEMING AND J. E. JAMISON, Memphis State University, Memphis, Tennessee 38152, Operators on Modulared Spaces.

Let (Ω, Σ_1, μ) be a measure space, let $\varphi(\xi, t)$ be a real valued nondecreasing function of $\xi > 0$ for each $t \in \Omega$ and a measurable function of t for each $\xi > 0$, and let $\Phi(\xi, t) = \int_0^\xi \varphi(\eta, t)d\eta$. Let L_Φ denote the space of functions determined by the modular $M(f) = \int_\Omega \Phi(|f(t)|, t)d\mu$. The class of modulared spaces L_Φ includes the Orlicz spaces and the $L^{p(t)}$ spaces. [Nakano, Topology and Linear Topological Spaces, Tokyo 1951]. With suitable (though fairly general) restrictions on φ, it is possible to identify the Hermitian operators and therefore the onto isometries on L_Φ. In general, the spaces contain a Hilbert subspace (possibly trivial) as a direct summand and the operators of interest necessarily are direct sums of operators on the summands. In particular, Hermitian operators are essentially

multiplications and adjoint abelian operators are either multiples of reflections or simply a projection onto the Hilbert subspace followed by a self adjoint operator.

(Received November 6, 1974.)

*720-47-30 D. H. HYERS, University of Southern California, Los Angeles, California 90007. A note on Frechet's definition of polynomial operators.

In 1929, M. Frechet defined "polynomes abstraits" as those operators P from a real vector space X to another, Y (where X and Y were "espaces algebrophiles") which have the following properties. (1) The operator P is continuous. (2) $\Delta^m_{h_1 \ldots h_m} P(x) \equiv 0$, for all x and h_j in X $(j = 1,2,\ldots,m)$. Here the symbol in the left member of (2) represents the mth forward difference of P with the increments $h_1, h_2, \ldots, h_m$ (J. Math. Pures Appl. (9) 8(1929), 71-92). We modify Frechet's definition as follows. The spaces X and Y are taken to be topological vector spaces with real or complex scalars. Condition (1) of continuity is replaced by the weaker condition that P be Gateaux differentiable everywhere. The resulting definition turns out to be equivalent, in the present context, to either of the two standard definitions of polynomial operators (Prenter, Nonlinear Functional Anal. and Appl. (Proc. Advanced Sem., Math. Res. Center, Univ. of Wisconsin, Madison, Wisconsin, 1970), Academic Press, New York, 1971, pp. 362-363 and Hille, "Functional analysis and semigroups," Amer. Math. Soc. Colloq. Publ., vol. 31, Amer. Math. Soc. Providence, R.I., 1948, p. 66) (Received November 6, 1974.)

720-47-31 R. J. FLEMING and J. E. JAMISON, Memphis State University, Memphis Tennessee 38152, On a Conjecture of Istratescu.

In this note we give an example of an adjoint abelian operator A on a uniformly convex Banach space X with the property that $A^2 + \lambda I$ fails to be adjoint abelian for every nonzero scalar λ. This answers a question of V. I. Istratescu in the negative. We also show that the conjecture of Istratescu is valid in many of the classical Banach spaces.

(Received November 6, 1974.)

49 Calculus of Variations and Optimal Control

*720-49-1 S. D. FISHER AND J. W. JEROME, Department of Mathematics, Northwestern University, Evanston, Illinois 60201. Stable and Non-stable elastica equilibria.

Let points $p_1, \ldots, p_m$ be given in the plane and let L be a fixed positive number. Let C consist of all curves $(x(t), y(t))$, $0 \le t \le 1$, which pass through the points $\{p_i\}_1^m$; which have length no more than L and for which $\ddot{x}, \ddot{y} \in L^2(0,1)$. Assume C is non-empty. Then it is known that C contains at least one curve $(x_0(t), y_0(t))$ for which $\int_0^1 k^2(t)dt$ is minimal over the class C where k is the curvature. Let $\{t_i\}_1^m$ be points in $[0,1]$ for which $(x_0(t_i), y_0(t_i)) = p_i$, $i = 1,\ldots,m$. Then each of x_0, y_0 is C^∞ on (t_i, t_{i+1}) for $i = 1,\ldots,m-1$

and C^2 on [0,1]. Further, if k is the curvature of (x_0, y_0) then k satisfies the differential equation

$$\frac{d^2k}{ds^2} + \frac{k^3}{2} - \lambda k = C_i$$

on (t_i, t_{i+1}) where s is arc-length, $\lambda \geq 0$, and C_i is a constant. Other results are also given. (Received October 15, 1974.)

*720-49-2 DONALD R. SNOW, Brigham Young University, Provo, Utah 84602. A New Proof of Rayleigh's Principle for Eigenvalue Approximations. Preliminary report.

Rayleigh's Principle gives a characterization of the eigenvalues and eigenfunctions of a differential equation as the minimum value and minimizing function, respectively, of a related variational problem. This gives information on the eigenvalues and a simple means of obtaining upper bounds for them. This paper presents a new proof of this principle in the case of Sturm-Liouville differential equations. The method shows both the necessity and sufficiency of the condition by using a modification of Caratheodory's method of equivalent problems in the Calculus of Variations. The method is simple both conceptually and computationally and does not use the classical variational techniques. It shows immediately that any function satisfying the boundary conditions, when substituted into the Rayleigh ratio, yields an upper bound to the eigenvalue, and the eigenfunction used therein yields exactly the eigenvalue. It also shows the positivity properties of the eigenvalues under appropriate assumptions on the coefficients of the differential equation. It appears that the proof can be generalized to handle other kinds of problems, such as eigenvalue problems for partial differential equations, as well as the one independent variable case presented here. (Received October 7, 1974.)

*720-49-3 MARVIN I. FREEDMAN and JAMES L. KAPLAN, Boston University, Boston, Massachusetts 02215. Perturbation analysis of a class of time optimal control problems.

The authors phrase a regular perturbation problem for a time optimal control problem in which the optimal controller is bang-bang. Formal procedures for computing the asymptotic series expansions of the variables and then establishing the uniform validity of these asymptotic series are given. The technique is based upon the idea of introducing a nonlinear change of variables which freezes the switch times and the terminal time of the perturbed problem, and allows for an asymptotic analysis in the new variables. This is necessitated by the fact that a perturbation analysis in the original variables does not, in general, exist because of the discontinuous nature of the control. (Received November 5, 1974.)

*720-49-4 V. L. BAKKE, University of Arkansas, Fayetteville, Arkansas 72701. Boundary arcs for integral equations.

Behavior of boundary arcs for control systems is investigated when the systems are governed by integral equations of the Volterra type. The main result, in the form of a maximum principle, is used to obtain necessary conditions for a minimum control problem. (Received November 6, 1974.) (Author introduced by Professor James E. Scroggs.)

50-1 LYNN E. GARNER, Brigham Young University, Provo, Utah 84602
Proper lineations on real spaces are collineations, Preliminary Report.

Jsing the corresponding place to characterize a proper lineation (see L. Garner, ıeations between projective spaces," Abstract 711-50-3, these Notices 21(1974), 02), a proper lineation from a real projective space into itself is shown to be a jective collineation. Applications to collinearity-preserving functions between affine nes are discussed. (Received October 25, 1974.)

0-50-2 THOMAS STROMMER, University of Washington, Seattle, Washington 98195.

Arrangements Generated by Polygons, Preliminary report.

one defines a polygon P to be a set of points $x_1, \ldots, x_n$ in the plane together with the ɜgments $x_i x_{i+1}$ and $x_n x_1$ such that no three consecutive x_i are collinear, one may consider the et of lines A determined by these segments. A forms a special type of what is known as a ;uclidean arrangement of lines. We show that if P is generated by n points, then A may contain as few as $\sqrt{1+2n}$ distinct lines. Further, A may have any number of vertices (places where two or more lines meet) between n and (1/2)n(n-1) inclusive whenever $n \neq 6$. (When n=6, the only exception is that there is no 6-gon which generates an arrangement with only 6 vertices.) (Received October 30, 1974.)

*720-50-3 LEOPOLDO V. TORALBALLA, Bronx Community College, Bronx, New York 10453. A topological measure-theoretic approach to dimension theory.

Our work on surface area has led to a general definition of p-dimensional measure in q-dimensional Euclidean space, $p \leqq q$. With this definition, dimension is always an integer and is topological. Let $P \in E \subset S$, a Euclidean space. Let n be any positive integer. If there exists a ball $B(P, \epsilon)$ and an open set $A \subset E \cap B(P, \epsilon)$, $P \in A$, such that $\mu_n(A) = 0$, where $\mu_n(A)$ is our n-dimensional outer measure of A, we say that the dimension of E at P is less then n. If m is the least positive integer such that the dimension of E at P is less than m, then we say that the dimension of E at P is m - 1. If for all $P \in E$, the dimension of E at P is d, then we say that the dimension of E is d. Theorem 1. Let the dimension of E at P be $n > 0$. Then there exists a ball $B(P, \epsilon)$, and an open set $A \subset E \cap B(P, \epsilon)$, $P \in A$, such that the dimension of Bdy A is $\leqq n - 1$. Theorem 2. Let dimension of E at P be m. Let A be an open set in $E \cap B(P, \epsilon)$ such that $\mu_m(A) > 0$. Then there exists $Q \in$ Bdy A such that dim(Bdy A) at Q = m - 1. Theorem 3. Dimension, as we defined it, is topological and agrees with the Menger-Urysohn definition. Through imbedding, our definition is extended to separable metric spaces in general. (Received November 1, 1974.)

*720-50-4 S. E. PAYNE, Miami University, Oxford, Ohio 45056 and J.A. THAS, University of Ghent, Ghent, Belgium. Generalized quadrangles with symmetry.

Let S be a generalized quadrangle with order (s, s). A symmetry θ about a line L is a collineation of S fixing each line meeting L. Then L is said to be a center of symmetry if the group of symmetries about L is of order s (the maximum possible). Theorem. If three lines that are centers of symmetry meet at a point x_∞, then S is a translation generalized quadrangle. So s is a prime

power with x_∞ regular when s is even and antiregular when s is odd. Theorem. A generali quadrangle S of order s with an antiregular point is equivalent to a finite Laguerre plane of odd s. (Received November 1, 1974.)

*720-50-5 Branko Grünbaum, University of Washington GN-50, Seattle, Washington 98195. Arrangements of colored lines.

An arrangement of n lines in the real projective plane is k-colored if ea line is assigned one of the "colors" 1,2,...,k. We denote by n_j the number of lines having color j, by s_j the number of vertices all lines through which have color j, and we put $d = n_2 - n_1$. It is well known tha $\Sigma_j s_j > 0$ (T. S. Motzkin, Notices A.M.S. 14(1967), 837; G. D. Chakerian, Ame Math. Monthly 77(1970), 164-167.). A 2-colored arrangement is biased if $s_2 = 0$. Theorem 1. There exist (two) infinite families of 2-colored biased arrangements with $d = 4$. Conjecture 1. If the lines of color 2 of a biase 2-colored arrangement do not form a pencil then $d \le 4$; moreover, if $s_1 = 1$ then $d \le 1$. A k-coloring of an arrangement is nice if no vertex is incident with lines of precisely two different colors. Theorem 2. For arbitrarily large k there exist (nontrivial) arrangements that have nice k-colorings. Conjecture 2. In each nice k-coloring $n_j \le 4$ for some j.
(Received November 4, 1974.)

720-50-6 J. W. FREEMAN, Department of Mathematical Sciences, Virginia Commonwealth University, Richmond, Virginia 23284. Relations Between Two Baer Subplanes of $PG(2,q^2)$, Preliminary Report.

Using the representation of $PG(2,q^2) = \pi(q^2)$ in $PG(5,q) = \Sigma(5)$ and 4-dimensional sections of $\Sigma(5)$, the possible relations between two distinct Baer subplanes of $\pi(q^2)$ are examined. The number of subplanes in a particular intersection relation with a given subplane is found in each of 6 possible relations. (Received November 5, 1974.)

720-50-7 Gary L. Ebert, University of Wisconsin, Madison, Wisconsin 53706. Disjoint circles in finite miquelian inversive planes. Preliminary report.

Let $C_1,---,C_n$ be any n mutually disjoint circles in a finite miquelian inversive plane IP(q), where q is an odd prime power and $n \ge 4$. Assume that there does not exist a circle D orthogonal to the given n circles, and that we do not have one circle of our set orthogonal to the other n-1 circles. Also assume that all subtriples are non-linear. Let G be the group of collineations of IP(q) generated by the inversions and the collineations induced by the projective linear group of the line $PG(1,q^2)$. Let $H = \{\theta \in G / \theta$ permutes the C_i's$\}$ and $K = \{\theta \in G / \theta$ fixes the C_i's$\}$. Theorem: $K = 1$ and H is isomorphic to a subgroup of S_n. A classification of the possibilities for H is given when $n = 4$ and $n = 5$. The possible orders for H are discussed when $n = 6$, 7, or 8. Theorem: H cannot be non-abelian simple for $4 \le n \le 8$. The exceptional case when $n = 5$ and one of the circles is orthogonal to the other 4 circles is also considered. It should be noted that the classification of sets of n disjoint circles of IP(q) where $0 \le n \le (q-1)/2$ is equivalent to the classification of subregular spreads of index n, and hence each equivalence class of disjoint circles corresponds to an equivalence class of translation planes.
(Received November 5, 1974.)

720-50-8 SUE WHITESIDES, University of Wisconsin, Madison, Wisconsin 53706.
Collineations of order 10 planes. Preliminary report.

The existence of a projective plane P of order 10 has been investigated by studying its collineations. An unpublished result of E. T. Parker, that a collineation of P of prime order must be non-central and of order 3 or 5 (see p. 177 of Dembowski), has been verified; a somewhat stronger result stated without proof by D. R. Hughes (On homomorphisms of projective planes. Proc. Symp. Appl. Math. 10, 45 - 52.) has not been verified. Results: 1)The full collineation group of P has order 3^e or $3^e \cdot 5$, where e may be 0; 2)A collineation of P of order 5 must have a unique fixed point and a unique fixed line. Such a collineation determines much of the structure of P; 3)Any collineation group of P which maps a set of 12 points, no 3 collinear, upon itself must have order 1 or 3. (Received November 5, 1974.) (Author introduced by Professor R. H. Bruck.)

720-50-9 T. F. BANCHOFF, Mathematics Department, Brown University
C. M. STRAUSS, Applied Mathematics, Brown University
"Cusps of Simplicial Mappings to the Plane (With Computer Graphics Analysis)"

For an "excellent" smooth mapping of a closed surface M^2 to the plane (such that the only singularities are folds and a finite number N of cusps), Thom has shown that $N \equiv \chi(M^2) \pmod 2$. A more general theorem for simplicial mappings involving multiple cusps and ramification indices is proved which specializes to the above result if the singularities correspond to those of a smooth mapping, and this result is extended to simplicial mappings of an n-manifold to n-space. Computer graphics illustrations will include projections of the Veronese surface and unfoldings of simplicial approximations of the complex singularities of $z \to z^n$. (Received November 6, 1974.)

*720-50-10 R.H. BING and MICHAEL STARBIRD, University of Texas at Austin, Austin, Texas 78712. Linear Isotopies.

Let P be a polygon simplicially embedded in E^n. An isotopy $h_t: E^n \to E^n$ ($t \in [0,1]$) is a linear isotopy taking P to $h_1(P)$ if h_0 is the identity map and for each t in [0,1], h_t is linear on each simplex of P.

It is proved that if P is a finite graph simplicially embedded in E^2 and $h: E^2 \to E^2$ is an orientation preserving homeomorphism such that h(P) is also a simplicial embedding, then there is a linear isotopy of E^2 taking P to h(P). An example is presented to show that the analogous statement for a finite graph embedded in E^3 is false. (Received November 6, 1974.)

720-50-11 RODNEY T. HOOD, Franklin College, Franklin, Indiana 46131.
Rest points, tangent circles, and the rational number line.

We consider two externally tangent circles lying on the same side of a common tangent line. Let S be the set of all circles that can be generated from these, such that each generated circle is tangent to the given line and to two touching circles of the family. If the radii of the two original circles are each of the form kr^2, where k

is a non-square integer and r is rational, then every circle in S has a radius of the same form, and the coordinates of points of tangency of the members of S with the given line may be taken to be rational. We show that the set of these rest point coordinates is precisely the set of rationals on a given closed interval. We further show that every rational number is the coordinate of the rest point of a circle of an enlarged family S^*, obtained by an easy extension of the definition of S.
(Received November 6, 1974.)

52 Convex Sets and Geometric Inequalities

*720-52-1 GERALD BEER, California State University, Los Angeles, California 90032.
Starshaped sets and the Hausdorff metric.

Let C be a compact set in R^n. The r-parallel body of C, $B_r(C)$, is the union of the family of closed r-balls whose centers lie in C. If C is starshaped with respect to the origin, then the gauge of $B_r(C)$ is a Lipschitz function; this observation in conjunction with the Arzela-Ascoli theorem yields a Blaschke selection theorem for starshaped sets. In addition, each parallel body is a union of a finite collection of parallel bodies of starshaped sets. From this decomposition, we show that Lebesgue measure is continuous on the metric space of parallel bodies of a fixed radius in R^n relative to the Hausdorff metric. (Received October 24, 1974.)

*720-52-2 G. T. SALLEE, University of California, Davis, California 95616,
Tiling Convex Sets by Translates. Preliminary report.

Let K be a compact, convex subset of E^d such that K can be tiled by a finite number of translates of some compact set Y; that is, $K = X + Y$ where X is finite and the $\{X_i + Y\}$ are disjoint on interiors. The possible structures of K, X and Y are completely determined under these conditions. This result extends earlier ones of H. Groemer [Arch. d. Math. 19(1968), 445-448], and S. K. Stein ["Factors of some direct products", Duke Math. J., to appear]. (Received October 25, 1974.)

*720-52-3 JOHN C. OXTOBY, Bryn Mawr College, Bryn Mawr, Pennsylvania 19010. Construction of disjoint arcs through finite sets within given disks.

Let U be an open subset of the plane, $F_1, \ldots, F_m$ disjoint finite sets, and $D_1, \ldots, D_m$ open disks. $\exists$ disjoint arcs $A_1, \ldots, A_m \ni F_j \subset A_j \subset D_j \cap U$ iff for each j, F_j is contained in a single component of $D_j \cap U$. If U contains a convex neighborhood of each F_j, if $\operatorname{diam} F_j > 0$, and if $c > 2/\sqrt{3}$, then $\exists$ disjoint arcs $\ni F_j \subset A_j \subset U$ and $\operatorname{diam} A_j \leqq c \operatorname{diam} F_j$, but not necessarily if $c \leqq 2/\sqrt{3}$. Among other applications, this result completes the proof, in the case $n = 2$, of a lemma of C. Goffman [Acta Math. 89(1953), 261-278, Lemma 4] as modified by H. E. White, Jr. [Proc. Amer. Math. Soc. 44(1974), 391-394]. (Received November 4, 1974.)

*720-52-4 HANS-HEINRICH HERDA, Boston State College, Boston, Massachusetts 02115. Characterization of spheres among compact 3-bodies. II. Preliminary report.

Let K be a compact subset of E^3 having finite surface area s. Let q be the infimal surface-area-bisecting cross-section of K. If K is convex, then $s \geqq 4q$, and $s = 4q$ implies that K is a sphe [H. Herda, "A characterization of spheres among compact 3-bodies", Colloq. Math. 33 (1) in press]. It is shown here that the result still holds if K is only assumed to be the closure of a bounded open set B such that ∂B is rectifiable. (Received November 4, 1974.)

*720-52-5 DAVID C. KAY and MARILYN BREEN, University of Oklahoma, Norman, Oklahoma 73069. General Decomposition Theorems for m-Convex Sets in the Plane.

A set S is m-convex if for each m points $x_1,\ldots,x_m$ in S at least one pair x_i,x_j exists such that the segment joining x_i and x_j lies in S. Let C(S) = the smallest integer m such that S is the union of convex sets $C_1,\ldots,C_m$. Obviously, if C(S) = m - 1 then S is m-convex, $m \geq 2$. The main result of this paper settles in the affirmative the conjecture that $C(S) < \infty$ for any closed, planar, m-convex set S. A bound obtained for such a set is $C(S) < m^m$.

In particular it can be shown: If a line of support for S exists at a point $p \in \ker S$ then $C(S) = m - 1$; if $\mathrm{bd}\, S \cap \ker S \neq \emptyset$, then $C(S) \leq 2m - 3$; if S is 4-convex then $C(S) \leq 6$ (best possible bound here is $C(S) = 5$); and if $\mathrm{conv}\, Q \subset S$, where Q = l.n.c. points of S, then $C(S) \leq 3m - 2$. In general, if S is closed, planar, m-convex then $C(S) \leq 2(m-1)^m - 2(m-1)^2$. (Received November 6, 1974.)

53 Differential Geometry

*720-53-1 GEORGE PARKER, Southern Illinois University, Carbondale, Illinois 62901. On Results of Bochner and Nomizu.

Nomizu ("Holonomy, Ricci tensor and Killing vector fields," Proc. AMS 12 (1961), 594-597) gave an invariant proof of Bochner's Theorem relating Killing fields and the sign of the Ricci tensor, basing his proof on the Lemma: If X is a Killing field, then $\mathrm{div}\,(A_X X) = -\mathrm{Ric}(X,X) - \mathrm{trace}\,(A_X^2)$. We generalize this lemma and use the generalization to give an invariant proof of Bochner's Theorem relating harmonic fields and the sign of the Ricci tensor. (Received October 24, 1974.)

*720-53-2 THOMAS E. CECIL, Vassar College, Poughkeepsie, N. Y. 12601, Taut Immersions of Non-compact Surfaces into R^3.

Let f be a C^∞ immersion of a smooth surface M into R^3. For $p \in R^3$, $x \in M$, the function $L_p(x) = d(f(x),p)^2$, where d is the Euclidean distance in R^3. The immersion f is called taut if f is proper and if every Morse function L_p has the minimum number of critical points on M required by the Morse inequalities.

The cyclides of Dupin are precisely those surfaces M in R^3 for which both sheets of the focal surface of M are degenerate. A compact cyclide is either a Euclidean sphere, a standard torus of revolution or a surface obtained by inverting a torus of revolution in a sphere whose center is not on the torus.

A non-compact complete cyclide is either a plane, a circular cylinder, or a surface obtained by inverting a compact toroidal cyclide in a sphere whose center is on the compact cyclide. We prove the following:

Theorem: let $f: M \to R^3$ be a taut immersion of a connected, non-compact surface. Then f(M) is a complete cyclide of Dupin. (Received October 29, 1974.)

*720-53-3 ROBERT S. CAHN, University of Miami, Coral Gables, Florida 33124 and JOSEPH A. WOLF, University of California, Berkeley, California 94720. The asymptotic expansion of the zeta-function of compact symmetric spaces of rank one.

Let $M = G/K$ be a compact symmetric space with Lie algebra $\mathfrak{G} = \mathfrak{K} + \mathfrak{A} + \mathfrak{N}$ with $\mathfrak{A}$ of dimension one. Then by classification M is $\mathbb{R}P^n$, $\mathbb{C}P^n$, $\mathbb{H}P^n$ or $F_4/\mathrm{Spin}(9)$. Using the results of Cartan, $L^2(M) = \bigoplus \pi$ where π is a representation of G with a K-fixed vector. If M is made a Riemannian manifold with G invariant metric at the coset eK given by the projection of $-\Omega$, with Ω the Casimir element of $\mathfrak{G}$, then every representation space is an eigenspace of Δ and the zeta-function of M is $\zeta(t) = \sum_\pi \mathrm{degree}(\pi) \exp(-t\pi(\Omega))$. Using the dominant weight theory of class one representations $\zeta(t)$ can be analyzed using a summation formula of Mulholland (Proc. Cambridge Philos. Soc. 24(1928), 280-289) and the coefficients in the asymptotic expansion of $\zeta(t)$ can be computed. The formulas involve the convolution of $\exp(|\delta|^2 t)$ with a Laurent series. (Received October 30, 1974.)

720-53-4 RICHARD H. ESCOBALES, JR., Canisius College, Buffalo, New York 14208. Submersions from complex projective space.

Let f be a Riemannian submersion from complex projective r space, CP(r), onto another Riemannian manifold B. Assume f has connected, complex, totally geodesic fibers with $2 \leq$ dimension of fiber $\leq 2r - 2$. The following result then obtains, if the fibers are complete. Theorem. Any submersion $f: CP(r) \to B$ satisfying the above conditions must fall into one of the following classes: (1) $f: CP(2n+1) \to QP(n)$ or (2) $f: CP(7) \to S^8(\frac{1}{2})$ with $1 \leq K^* \leq 4$, where K^* denotes the sectional curvature of quaternionic n space QP(n) and where $S^8(\frac{1}{2})$ denotes the eight sphere of radius $\frac{1}{2}$. In fact, class one is not empty and if $n \geq 2$ then any two submersions from a fixed CP(2n + 1) onto QP(n) are equivalent. (Received October 30, 1974.)

*720-53-5 GARY JENSEN, Mathematics Department, Washington University, St. Louis, MO 63130 Imbeddings of Stiefel manifolds into Grassmannians

There are natural imbeddings of the Stiefel manifold $S_{n,p}$ of unitary p-frames in $\mathbb{F}^n$ (where $F = \mathbb{R}$ or $\mathbb{H}$) and $0 < p < n$) into the Grassmannian $G_{n+p,p}$ of (oriented) p-planes in $\mathbb{F}^{n+p}$. $S_{n,p}$ is imbedded as an orbit of the isotropy of the homogeneous space $G_{n+p,p}$ at a specified origin. Thus $S_{n,p}$ is contained in a distance sphere of $G_{n+p,p}$ centered at the origin. There is a choice of radius of this distance sphere such that the metric on $S_{n,p}$ induced from the canonical metric on $G_{n+p,p}$ is Einstein. For example, when $\mathbb{F} = \mathbb{H}$, $p = 1$, $S_{n,1}$ is the unit $4n - 1$ sphere in $\mathbb{H}^n$

and $G_{n+1,1}$ is the quaternionic projective space $\mathbb{H}P^n$. If ds^2 is the metric on $\mathbb{H}P^n$ of sectional curvature K, $\frac{1}{4} \leq K \leq 1$, then for any r, $0 < r < \pi$, $S_{n,1}$ can be imbedded as the distance sphere of radius r centered at the origin of $\mathbb{H}P^n$. When r satisfies $\cos^2 \frac{r}{2} = \frac{1}{1+2n}$, then the induced metric on $S_{n,1}$ is an Einstein metric of positive sectional curvature with pinching $\delta = \left(\frac{1}{1+2n}\right)^2$. (Received November 4, 1974.)

*720-53-6 CALABI, Eugenio, Math. Dept., University of Pennsylvania, Philadelphia, Pa. 19174, On Manifolds with nonnegative Ricci curvature II.

Let M be an n-dimensional Riemannian manifold with nonnegative Ricci curvature. Then the exponential mapping $\exp_p$ for any $p \in M$, restricted to the domain bounded by the cut locus, is everywhere volume decreasing. From this fact one deduces the following THEOREM. Let M be a Riemannian, n-dimensional, complete manifold with nonnegative Ricci curvature. Then, if r denotes the injectivity radius and D the geodesic diameter of M, the volume V of M satisfies $V \geq c_n r^{n-1} D$, where c_n is a positive constant depending only on n. In particular, if M is not compact (i. e. if $D = \infty$), the volume of M, under the same assumptions, is infinite. (Received November 6, 1974.)

720-53-7 GEORGE DEBNEY, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. On weakly asymptotically simple space-times. Preliminary Report

Consider a Lorentz 4-manifold (M,g) which is time-orientable, possesses no closed timelike trajectories, and the metric g satisfies the Einstein equations without cosmological constant. An asymptotically simple (a.s.) space-time is, roughly, any such manifold which allows "compactification" by extension to a conformally-related manifold with boundary $(\hat{M},\Omega^2 g)$ using an appropriate smooth conformal factor Ω^2 to "bring in" the infinities of null geodesics. The conformal infinity so produced is $\partial\hat{M}$ and $\hat{M} = M \cup \partial\hat{M}$. In that this is quite a great global restriction, one is usually concerned with weakly asymptotically simple (w.a.s.) space-times, which might possess large open regions with the a.s. property but might also have other regions where that property does not hold. It is well-known that under certain rather general conditions (e.g., the vacuum Einstein equations) $\partial\hat{M}$ is a union of null hypersurfaces for a w.a.s. space-time. We prove in this instance that the null generator of $\partial\hat{M}$ is both a geodesic and a conformal Killing vector field on $\partial\hat{M}$. In general this vector field on the boundary is not divergence-free; otherwise $\partial\hat{M}$ would be a so-called Killing horizon. Some of this approach is discussed in the wider context of "asymptotic symmetries" in general and "asymptotically stationary" in particular. (Received November 6, 1974.)

*720-53-8 JOHN K. BEEM, University of Missouri, Columbia, Missouri 65201. Pseudo-Riemannian manifolds with totally geodesic bisectors.

Let M be a pseudo-Riemannian manifold. Locally a distance function may be defined. The bisector of two points is the set of points equidistant from these two points. The space M is shown to have constant curvature if and only if whenever two points are not zero distance apart the bisector of the points is a totally geodesic submanifold of M. (Received November 6, 1974.)

720-53-9 HOWARD JACOBOWITZ, Rice University, Houston, Texas 77001. Kaehler Gauss-Codazzi Equations, Preliminary report.

The Gauss-Codazzi equations for a Kaehler manifold can be given as a Hirsh-Smale type statement.

Theorem. A simply connected Kaehler manifold M can be isometrically embedded in $\mathbb{C}^N$ by means of a holomorphic map if and only if there is a holomorphic bundle map $TM \to M \times \mathbb{C}^N$ which is isometric on each fibre. The results of Calabi [Ann. Math 58(1953), 1-23] follow simply from this formulation of the isometric embedding problem. (Received November 6, 1974.)

54 General Topology

*720-54-1 PETER F. MAH and S. A. NAIMPALLY, Lakehead University, Thunder Bay, Ontario, Open and Uniformly open mapping theorems

The following results were obtained in a search for a common refinement of theorems of J.L. Kelley (Theorem 6.36 of *General Topology*) and J.D. Weston (Theorem 8 of J. London Math. Soc. 32 (1957), 342-354) as proposed by B.J. Pettis at the Charlotte Topology Conference, March 1974. If (X,δ) is an Efremovič proximity space and Y a topological space, then a relation $R \subset X \times Y$ is called *weakly open* iff $y \in R[A]^-$ implies $R^{-1}[y]\delta A$.

(1) If R is weakly open and nearly open, then R is open.

(2) If X is metric, Y a Morita uniform space, and $R \subset X \times Y$ is weakly open and uniformly nearly open, then R is uniformly open. Generalization of Weston's result to an injective relation and Kelley's result to Morita uniform space follow. These in turn give two results on relations which are refinements of two pairs of results of Pettis. We finally discuss applications to topological groups and topological vector spaces. (Received May 8, 1974.)

720-54-2 BEVERLY L. BRECHNER, University of Florida, Gainesville, Florida 32611. Almost periodic homeomorphisms of E^2 are periodic.

In this paper it is shown that every almost periodic (a.p.) homeomorphism of E^2 onto itself is periodic. This improves similar results for the disk by Foland [A Characterization of Almost Periodic Homeomorphisms on the Closed 2-cell, PAMS, 16 (1965) pp. 1031-1034] and Hemmingsen [Plane Continua Admitting non-periodic autohomeomorphisms with equicontinuous iterates, Math. Scand. 2 (1954) 119-141]. Foland proves the result for the disk directly, while it follows from Hemmingsen's work when one uses the fact that a.p. homeomorphisms on a compact metric space have equicontinuous families of iterates. The proof of the main theorem in the present paper uses Bing's ε-growth technique to obtain invariant disks and thus re-does a portion of Foland's and Hemmingsen's papers in a particularly nice way. (Received August 19, 1974.)

720-54-3 CHARLES E. AULL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, 24061. Initial topologies, k-spaces and accessibility spaces

Every T_2 continuous image of a T_4 space X is a k or k' space iff the space is compact. Every T_3 continuous image of a T_4 space X is an accessibility space iff X is a compact Fréchet space. Every T_4 continuous image of a T_4 space X is a sequential (Fréchet) space. A space that is accessible by countably compact sets such that every sequentially compact subset is closed is a Fréchet space.

(Received August 19, 1974.)

*720-54-4 W. T. WHITLEY, Marshall University, Huntington, West Virginia, 25701. Another characterization of semiprime ideals in C(X).

Text. Let C(X) denote the ring of continuous real-valued functions on a topological space X. Definition. An ideal I in a commutative ring R is an FP-ideal if for each x in I, there exist y, z in I such that x = yz. Theorem. An ideal I in C(X) is an FP-ideal if and only if I is semiprime. Neither implication in the theorem is true for arbitrary commutative rings.

(Received October 7, 1974.)

*720-54-5 LUDVIK JANOS, University of Montana, Missoula, Montana 59801. On the Edelstein contractive mapping theorem.

A mapping f from a metric space (X, d) into itself is said to be contractive if $x \neq y$ implies $d(f(x), f(y)) < d(x,y)$. It was proved by M. Edelstein that if f is contractive and for some $x \in X$ the sequence of iterates $\{f(x)^n\}$ has a convergent subsequence, then f has a unique fixed point. This theorem proved relevant to many situations concerning differential equations where the Banach fixed point principle is not applicable as is seen from the following Theorem 1. Let C denote the space of all continuous functions on the real line endowed with the topology of uniform convergence on compacta and let $T: C \to C$ be the operator of integration, i.e., $Tg(x) = \int_0^x g(t)\,dt$ for $g \in C$. Then there is a metric on C, compatible with the above topology, relative to which T is contractive, but in contrast to this there is not such a metric relative to which T is a contraction in the Banach sense. Recently J. Bryant and L. F. Guseman [Canad. Math. Bull. 16(1973), 185-192] investigated the case when the hypotheses in the Edelstein theorem imply that for every $y \in X$ the sequence $\{f(y)^n\}$ converges. We have the following topological characterization of this case. Theorem 2. Let X be a metrizable topological space, $f: X \to X$ continuous and such that for every $x \in X$ the sequence $\{f(x)^n\}$ converges. Then the following two statements are equivalent: 1. There is a metrization of X relative to which f is contractive. 2. For every nonempty compact f-invariant subset Y of X the intersection of all iterates $f^n[Y]$ is a one-point set. (Received September 23, 1974.)

*720-54-6 PAUL R. MEYER. Lehman College C.U.N.Y., Bronx, N.Y. 10468. Cardinal functions on compact dispersed spaces.

I. Juhász (Cardinal functions in topology, Math. Centre Tracts #34, Amsterdam, 1971) gave a partial ordering for the cardinal functions on a compact space X (all spaces are T_2). It is shown here that, under the further hypothesis that X is dispersed, this ordering reduces to:

i) $k \leq c = d = \pi \leq \psi = \chi = s = h = z = w = |X| \leq \inf\{d^{\tau}, \exp d\}$, and ii) $k \leq \sigma = \tau = \rho \leq |X|$. Examples show that no further reductions are possible. Notation follows Juhász, with the following exceptions: τ = tightness; σ = net character, as defined by the author (Indag. Math. 34 (1972) 210-211); and ρ = dispersal character. Portions of the proof are established under less restrictive hypotheses, including: If X is a chain net space, then $\sigma = \tau \leq \psi$ and $|X| \leq d^{\tau}$. If X is dispersed, then $c = d = \pi \leq s = z$ and $|X| \leq w$. If X is regular and dispersed, then $|X| \leq \exp d$. (Received September 27, 1974.)

*720-54-7 W.F. PFEFFER, University of California, Davis, California 95616.
On some subspaces of Helly space, Preliminary report.

Helly space H is defined in Kelley's General Topology, chapter V, problem M. We show that every separable metrizable space can be imbeded into H and we characterize all separable metrizable subspaces of H. We also characterize compact G_δ subspaces of H and by means of this characterization we prove that every compact metrizable subspace of H is G_δ. This result is used to exhibit a certain class of compact first countable Hausdorff spaces which are not imbedable into H.
(Received October 18, 1974.)

*720-54-8 MARY ELLEN RUDIN, University of Wisconsin, Madison, Wisconsin 53706
A perfectly normal, nonmetrizable manifold

$\diamondsuit$, a combinatorial statement which holds in Gödels constructible universe, implies the existence of a perfectly normal, nonmetrizable manifold. The example given is hereditarily separable and 2-dimensional. (Received October 18, 1974.)

*720-54-9 H.B. KEYNES, University of Minnesota, Mpls, MN 55455 and D. NEWTON, University of Sussex, England. Real Prime Flows.

We continue the study of prime flows initiated by Furstenberg, Keynes, and Shapiro in two directions. First, we enlarge the class of examples of prime (actually POD) flows by constructing real-time prime flows. These examples are obtained by suspending under a 2-valued function which is constant over 2 "intervals", and imposing mild assumptions on the heights of the function. Next, we examine general properties of prime transformation groups by studying various proximality conditions and properties of automorphisms. We show that there are classes of prime flows (the purely prime flows) which are distinct from POD flows, and that such flows arise quite naturally. Purely prime flows are studied in detail, and results such as the "unique-factorization" of minimal n-fold products, and pairwise disjointness yielding minimality of the n-fold product are proved. Finally, automorphisms of prime flows are shown to be weakly mixing and regular-prime (no regular factors).
(Received October 21, 1974.)

*720-54-10 Ethan M. Coven, Wesleyan University, Middletown, Ct. 06457 and Michael E. Paul, University of Maryland Baltimore County, Baltimore, Maryland 21228. Sofic Systems.

A symbolic flow is called a sofic system if it is a homomorphic image (factor) of a subshift of finite type. We show that every sofic system can be realized as a finite-to-one factor of a subshift of finite type with the same entropy. From this

it follows that sofic systems share many properties with subshifts of finite type. We concentrate especially on the properties of TPPD (transitive with periodic points dense) sofic systems. (Received October 21, 1974.)

*720-54-11 J.C. SMITH, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. θ-Characterizations of Generalized Paracompact Spaces, Preliminary report.

The notion of θ-refinability, introduced by Worrell and Wicke in 1969, has played an important role in the study of a number of classes of topological spaces. In 1970 D. Burke provided a θ-type characterization for the class of subparacompact spaces. In this paper it is shown that certain generalizations of paracompactness such as metacompactness, mesocompactness, sequential mesocompactness and screenability have θ-type characterizations. In particular, we have the following:

Definition: A space X is said to have property M provided every open cover $\mathcal{G}$ of X has a refinement $\bigcup_{i=1}^{\infty} \mathcal{G}_i$ satisfying (i) each $\mathcal{G}_i$ is an open cover of X, and

(ii) for each $x \in X$, there exists a neighborhood $U(x)$ of x and an integer $n(x)$ such that $\mathrm{ord}(y, \mathcal{G}_{n(x)}) < \infty$ for $y \in U(x)$.

Theorem: A space X is metacompact iff X satisfies property M. (Received October 21, 1974.)

*720-54-12 Dr. David Wigner, University of Michigan at Ann Arbor. Inverse limits and the completeness of quotient groups.

It is shown that the quotient of a complete abelian topological group by a closed metrizable subgroup is complete. It is further shown that the incompleteness of an arbitrary quotient is related to the derived functors of the inverse limit. (Received October 21, 1974.) (Author introduced by Peter Weinberger.)

*720-54-13 M. JAYACHANDRAN, Madurai College, Madurai, India, and M. RAJAGOPALAN, Memphis State Univ., Memphis, Tn. 38152. Scattered compactifications.

N is the discrete space of integers. A P-point of order 1 in $\beta N-N$ is an ordinary P-point. Let a P-point of order α have been defined for an ordinal α. Let $x \in \beta N-N$. x is called a P-point of order $\alpha + 1$ if there is a sequence $x_1, x_2, \ldots x_n, \ldots$ such that each x_n is a P-point of order α and x is a P-point of the subspace $\overline{\{x_1, x_2, \ldots x_n, \ldots\}} - \{x_1, x_2, \ldots x_n, \ldots\}$. Let α be an ordinal and let a P-point of order γ have been defined for all $\gamma < \alpha$. Let $x \in \beta N-N$. x is called a P-point or order α if there is a sequence $x_1, x_2, \ldots x_n, \ldots$ and an increasing sequence $\gamma_1 < \gamma_2 < \cdots < \gamma_n <$ of ordinals such that $\gamma_n \to \alpha$ and x is a P-point of $\overline{\{x_1, x_2, \ldots x_n \ldots\}} - \{x_1, x_2, \ldots x_n \ldots\}$ and x_n is a P-point of order γ_n.

In this paper it is shown that if p is a P-point of order α for some ordinal α in $\beta N-N$ then $N \cup \{p\}$ has a scattered compactification. We also show that the space S_α of Arkangelskii and Franklin have scattered compactifications for each ordinal α. This is done by introducing the idea of the $N \cup \{p\}$ sum of a sequence of known topological spaces and showing that this process preserves scattered compactifications. This answers a problem of Semadeni partially. (Received October 21, 1974.)

720-54-14 KAREL PRIKRY, Institute for Advanced Study, Princeton, NJ 08540. Non-extendable families of continuous functions, Preliminary report.

Let Q denote the space of rationals. Mrówka Acta Math. 120 (1968), 161-185, defined $\mathrm{def}_N Q$. Let λ be the least cardinal such that there is a sequence U_ξ $(\xi \in \lambda)$ of open sets of reals with $Q = \bigcap\{U_\xi : \xi \in \lambda\}$.

Theorem. (a) $\mathrm{def}_N Q = \lambda$.

(b) If Con ZF, then $\mathrm{Con}(\mathrm{ZFC} + 2^{\aleph_0} = \aleph_2 + \lambda = \aleph_1)$ and $\mathrm{Con}(\mathrm{ZFC} + 2^{\aleph_0} = \aleph_2 = \lambda)$.

(c) If $2^{\aleph_0}$ is real-valued measurable, then $\lambda < 2^{\aleph_0}$. (Received October 21, 1974.)

These results answer a question of Mrówka. (Received October 21, 1974.)

*720-54-15 LEONARD R. RUBIN, Department of Mathematics, The University of Oklahoma, 601 Elm Ave., Rm. 423, Norman, Oklahoma 73069. Relative collaring.

Robert Connelly published an alternate proof of Morton Brown's collaring theorem, and T. Benny Rushing in his book, Topological Embeddings, used Connelly's method to prove a relative collaring theorem for admissible manifold pairs (N,M). However, Rushing's proof does not work in case $\partial M \cap \partial N$ is not closed in ∂N. Under a revised definition of relative local collaring, we prove in general that relative local collaring implies relative collaring in paracompact spaces. This theory embraces the previous set of results as corollaries, while correcting the difficulty encountered in Rushing's setting. (Received October 21, 1974.)

720-54-16 BRUCE LOATMAN, Georgetown University, Washington, D.C. 20007. Functions continuous in some compact metric topology. Preliminary report.

Let (E,T) be an uncountable compact metric space and $F:(E,T) \to R^n$ $(n \leq \infty)$ be a B-measurable mapping with compact image. Lorch ("Continuity and Baire functions", Amer. Math. Monthly. 78 (1971), 748-762) has asked whether there always exists another compact metric topology T' such that $F:(E,T') \to R^n$ is continuous. The answer is no, even for functions of Baire class 1. More generally, if $F:(E,T) \to R^n$ is continuous (T compact metric) then for each finite k, the set X_k of all points of R^n whose preimage under F has at least k points, must be an F_σ. Also (Kuratowski, Topology, vol. 1, Academic Press, New York, 1966, p.496) the set A_F of points of uncountable order is analytic. The converse has not been resolved. However, if F has compact image, each X_k is an F_σ, points of uncountable order are of order c, and in addition, the set A_F is an F_σ then there exists a compact metric topology T on E such that F is T-continuous and also T has the same dimension as the image of F. Kuratowski (ibid. p.497) has remarked that A_F need not be borelian. (Received October 21, 1974.)

*720-54-17 DOUGLAS MCMAHON, Dept. of Mathematics, University of Oregon, Eugene, OR 97403 and T. S. WU, Dept. of Mathematics, Case WRU, Cleveland, OH 44106. On the Connectedness of Homomorphisms in Topological Dynamics. Preliminary Report.

Let (X,T) be a minimal transformation group with compact Hausdorff phase space. In his paper on distal transformation groups, Ellis proved that a distal minimal set (X,T) is equicontinuous if X is totally disconnected and T is generative, i.e., T is an Abelian group generated by some compact neighborhood of the identity. We have generalized this result by requiring that T be the direct product of a compact group with a compactly generated separable group rather than that T be generative. More generally, we have shown that if $\phi : (X,T) \to (Y,T)$ is a distal homomorphism and has a structure similar to the structure Furstenberg derived for distal minimal sets, then for T belonging to a class of topological groups $\mathcal{J}$, the homomorphism $X \to X/S(\phi)$ has connected fibers, where $S(\phi)$ is the relativized equicontinuous structure relation. The class $\mathcal{J}$ is defined by Sacker and Sell in their paper "finite extensions of minimal transformation groups" as consisting of all groups T with the property that there is a compact set $K \subseteq T$ such that T is generated by each open neighborhood of K. They show that for such T, a distal minimal set which is a finite-to-one extension of an almost periodic minimal set is itself an almost periodic minimal set. We have provided an example that shows that the restriction on T cannot be dropped.
(Received October 24, 1974.)

*720-54-18 ALLAN JAWORSKI, Department of Mathematics, University of Texas, Austin, Texas 78712. The discrete Bebutov dynamical system.

Suppose (X,G) is a transformation group, where X is a locally compact separable metric space and G is a finitely generated discrete group. Consider the shift dynamical system $\Sigma(G)$, the transformation group consisting of the continuous real-valued functions on G equipped with the topology of pointwise convergence, where G is acting by left translation. We give necessary and sufficient conditions for certain (X,G) to admit equivariant embeddings in $(\Sigma(G),G)$ in terms of torsion in the action of G. (Received October 24, 1974.)

*720-54-19 FRANK SIWIEC, John Jay College of CUNY, New York, NY 10019. Characterizations of some countable, T_1-, k-spaces having exactly one non isolated point. Preliminary report.

There are essentially only seven distinct examples of topological spaces which satisfy the conditions of the title and which additionally are quotient images of complete separable metric spaces. Quotient images of separable metric spaces are also considered, but with an incomplete analysis.
(Received October 25, 1974.)

*720-54-20 WILLIAM G FLEISSNER, McGill University, Montreal, Quebec Canada.
The Character of ω_1 in First Countable Spaces.

The character of Y in X is the minimal cardinality of a neighborhood basis for Y as a subset of X. We seek the least upper bound, β, for this when $Y \cong \omega_1$ and X is first countable. We show that it is consistent that $\beta > \omega_1$, and it is consistent that $\beta = \omega_1$, using techniques developed in considering Kurepa's Hypothesis.

Further, it is consistent that in a first countable space, discrete collections of sets homeomorphic to ω_1 can be separated whenever the space is normal. (Received October 25, 1974.) (Author introduced by M. Makkai)

720-54-21 KENNETH KUNEN, University of Wisconsin, Madison, WI 53706
On the cardinalities of compact spaces, Preliminary report.

Let P_1 be the statement: Whenever X is compact Hausdorff of weight $\leq \omega_1$, if $|X| > \omega_1$ then $|X| = 2^{\omega_1}$.

Let P_2 be the statement: Whenever X is compact Hausdorff of character $\leq \omega_1$, if $|X| > \omega_1$ then $|X| = 2^{\omega_1}$.

Thus, $P_2 \rightarrow P_1$ trivially. However each of $\neg P_1$, $P_1 \wedge \neg P_2$, and P_2 is consistent with $CH \wedge 2^{\omega_1} > \omega_2$.

Note that if in P_1 and P_2, ω_1 is replaced everywhere by ω_0, then both statements are always true. (Received October 30, 1974.)

720-54-22 JOHN C. MARTIN, North Dakota State University, Fargo, North Dakota 58102
Generalized Morse Flows on n Symbols

The definition of a generalized Morse sequence given by Keane in Z. Wahrscheinlichkeitstheorie und verw. Geb. 10 (1968), pp. 335-353, is extended to include sequences on n symbols, which are almost periodic points in the shift transformation group on n symbols, and the topological structure of the resulting symbolic minimal flows is investigated. The maximal equicontinuous factor of such a flow is computed, and the symbolic flow is shown to be an AI extension of its maximal equicontinuous factor. Finally, it is shown that every endomorphism of such a flow is simply the composition of the shift transformation with a certain permutation of the symbols.

(Received October 29, 1974.)

*720-54-23 GARY GRUENHAGE, Auburn University, Auburn, Alabama 36830. Compact Basically Screenable Spaces are Metrizable, Preliminary report.

A collection Γ of subsets of a space X is rank 1 if whenever $g, g' \in \Gamma$ and $g \cap g' \neq \emptyset$, then $g \subset g'$ or $g' \subset g$. A collection T of open subsets of X is a tree of open sets if whenever $t \in T$, the set $\{t' \in T \mid t' \supset t\}$ is well-ordered by reverse inclusion. X is said to be basically screenable if X has a basis B which is the union of countably many rank 1 trees of open sets. Theorem. A compact basically screenable space is metrizable. This answers a question of Peter Nyikos. We say a cover $\mathcal{U}$ of X by elements of B is B-minimal if it is a minimal cover in the usual sense, and no element of $\mathcal{U}$ can be replaced by one of its predecessors in a tree without destroying the minimality of the cover. The major part of the proof is showing that number of B-minimal covers is countable. (Received October 29, 1974.)

*720-54-24 JERRY E. VAUGHAN, University of North Carolina at Greensboro, Greensboro, North Carolina 27412. Some properties related to [a,b]-compactness,II.

In these NOTICES 21 (1974) A-621, we defined for infinite cardinals $a \leq b$ the properties N[a,b], S[a,b], and G[a,b], and announced that N[a,b] $\rightarrow$ [a,b]-compact $\rightarrow$ S[a,b] $\rightarrow$ G[a,b] $\rightarrow$ [a,b]-compact^r, and that none of these implications can be reversed for every value of a and b. Here we are concerned with conditions on the cardinals a and b under which some reverse implications hold. Theorem: (1). (Alexandroff & Urysohn, and Mrówka) If $a = \aleph_0$, then [a,b]-compact^r $\rightarrow$ N[a,b]. (2). If a is regular and $b^{\underline{a}} = b$, then G[a,b] $\rightarrow$ N[a,b]. (3). If b is regular or if $cf(b) \geq a$, then S[a,b] $\rightarrow$ [a,b]-compact. (4). [GCH] If a is regular then G[a,b] $\rightarrow$ S[a,b]. In particular, if [GCH] holds and both a and b are regular, then G[a,b], S[a,b], [a,b]-compact, and N[a,b] are equivalent regardless of the number of singular cardinals between a and b. (Received October 30, 1974.)

720-54-25 ISTVAN JUHASZ, University of Wisconsin, Madison, WI 53706 On non-metrizable spaces in which every subspace of small cardinality is metrizable, Prelim. report.

Let $M(\kappa)$ be the statement: Whenever X is a first countable normal space of cardinality κ such that every subspace $Y \subset X$ with $|Y| < \kappa$ is metrizable, then so is X. P. Hamburger raised the question whether $M(\kappa)$ was true for $\kappa \geq \omega_2$. The following two results concerning this question were obtained by A. Hajnal and the present author: a) $M(\kappa)$ is true if κ is a weakly compact cardinal; b) if V = L then $M(\kappa)$ is false for every non-weakly compact regular κ. (Received October 30, 1974.) (Author introduced by Professor M. E. Rudin.)

*720-54-26 Roman Frič, University of Transport Engineering, Žilina, Czechoslovakia, On E-sequentially regular spaces

Using the theory of sequential envelopes for convergence spaces in the sense of Y. Novak we describe the categories of E-sequentially regular and E-sequentially complete sequential spaces. Definition. A sequential space X is said to be E-sequentially regular if it is homeomorphic with the sequential modification sY (in the sense of S. P. Franklin) of a subspace Y of a power E^m of E. If Y is closed in sE^m, then we say that X is E-sequentially complete. Theorem. The categories of [o,1] - sequentially complete and R-sequentially complete spaces, resp. {0,1} - sequentially complete and Q-sequentially complete spaces coincide. We also indicate further connections between the categories of convergence spaces and sequentially spaces. (Received October 30, 1974.) (Author introduced by Professor Darrell C. Kent.)

720-54-27 CARLOS A. INFANTOZZI, Universidad de la República, Atlántico 1514, Montevideo, Uruguay. "A note on the Foundations of SEQUENTIAL TOPOLOGY".

Notations: X' derived set; $\{x_n\}$ sequence ($n \in N$, integers ≥ 0); (x_n) range of $f(n) = x_n$.
Axioms for (L)-spaces: L_1), L_2), L_3), Kuratowski; L_0).-(uniqueness of the limit); L_0').-(uniqueness of the limit only for "eventually constant" sequences); L_0'').-(distinctness of points).- If $p \in X'$, then $\exists \{x_n\}$ such that: $\{x_n\} \rightarrow p$, $x_n \neq p$, $\overline{(x_n)} \subseteq X$ and all the sequences $\{y_n\} \not\rightarrow p$ if $y_n \in (x_n)$; L_4).- If $\{x_{mn}\} \rightarrow x_m$ and $\{x_m\} \rightarrow x$, then $\exists$ a sequence $\{x_{m_i n_i}\} \rightarrow x$, with $x_{m_i n_i} \in (x_{m_n})$.
Axioms for the spaces: (1) (L^*)-Kuratowski: L_1, L_2, L_3; (2) (L)pre-T_1: L_0', L_1, L_2, L_3; (3) (L)pre-$T_{1\frac{1}{2}}$ ($\overline{(L_t)}$-Urysohn): L_0'', L_1, L_2, L_3; (4) (L)pre-T_2: D, L_1, L_2, L_3 (D of Hausdorff).

Adding $\underline{L}_4$ to spaces (1),(2),(3),(4) we characterize, respectively, the spaces ($\underline{1}'$) (S^*)-spaces; ($\underline{2}'$) (L)-T_1 sp.; ($\underline{3}'$) (L)-$T_{1\frac{1}{2}}$ sp.((S$_t$)-Urysohn sp.); ($\underline{4}'$) (L)-T_2 sp.
With similar axioms $\Lambda_1, \Lambda_2, \Lambda_3, \Lambda_c, \Lambda_c', \Lambda_c'', \Lambda_4$ for Moore-Smith convergence, we characterize, respectively, the spaces: ($\underline{1}''$) "closure-sp." (Cech); ($\underline{2}''$) pre-T_1 sp. ("gestufte Räume"-Hausdorff); ($\underline{3}''$) pre-$T_{1\frac{1}{2}}$ ("Classes (R)"-Fréchet); ($\underline{4}''$) pre-T_2, and adding the axiom Λ_4, the spaces (1"), (2"), (3"), (4") become, respectively: ($\underline{1}'''$) topological-sp.; ($\underline{2}'''$) T_1-sp.; ($\underline{3}'''$) $T_{1\frac{1}{2}}$-sp.; ($\underline{4}'''$) T_2-sp.
If the spaces (1"), (2"), (3"), (4"), (1'''), (2'''), (3'''), (4''')fulfill the condition $\underline{L}$).- $p \in X' \rightleftarrows \exists \{x_n\} \longrightarrow p,\ x_n \neq p,\ (x_n) \subseteq X$, then they are, respectively, the spaces (1), (2), (3), (4), (1'), (2'), (3'), (4').
(Remark: $D \rightarrow L_c \rightleftarrows L_c''$, but $D \rightleftarrows \Lambda_c \rightarrow \Lambda_c''$). (Received October 30, 1974.)

*720-54-28 LEWIS LUM, Salem College, Winston-Salem, N. C. 27108
Weakly Smooth Continua.

We define and investigate a class of continua called weakly smooth. Smooth dendroids, weakly smooth dendroids, generalized trees, and smooth continua are all examples of weakly smooth continua. We generalize characterizations of the above mentioned examples to weakly smooth continua. In particular, we characterize them as compact Hausdorff spaces which admit a quasi order satisfying certain properties. (Received October 31, 1974.)
(Author introduced by G. R. Gordh, Jr.)

720-54-29 JUDITH ROITMAN, Wellesley College, Wellesley, Mass. 02181. Some results about spread. Preliminary report.

All spaces are Hausdorff. The spread of a space is the supremum of the cardinalities of its discrete subspaces. 1. If $cf(\kappa)=\xi>\omega$ and for some $\beta<\xi$, $2^\beta \geq \xi$, κ not a successor, then $\exists X$ whose spread is κ such that X has no discrete subspace of cardinality κ. 2. For any κ there is a space whose spread is κ which is not the union of a hereditarily κ-separable and a hereditarily κ-Lindelof space. 3. Assume κ-CH. Then there is a space of cardinality κ^+ which is hereditarily κ-separable and hereditarily κ-Lindelof such that, if Y is a subspace of cardinality κ^+, then any basis for Y has cardinality κ^+. 4. Define a downward space as the disjoint set union $\sum_{i<\omega} X_i$ of discrete subspaces X_i such that each $\bigcup_{i<n} X_i$ is open. Then there is a model of set theory in which some space has spread of cofinality ω and no discrete subspaces of the cardinality of the spread iff in this model there is a downward space with the same spread and no discrete subspaces of the cardinality of the spread.
(Received October 31, 1974.)

*720-54-30 ROBERT A. McCOY, Virginia Polytechnic Institute & State University, Blacksburg, VA 24061. First Category function spaces under the topology of pointwise convergence.

The space $C_\pi(X,Y)$ of continuous functions from X into Y under the topology of pointwise convergence turns out to be of first category for most non-pathological spaces X and Y. Space X will be called completely Hausdorff with respect to Y if for every finite set $\{x_1,\ldots,x_n\}$ of distinct points of X and every finite set $\{V_1,\ldots,V_n\}$ of nonempty open subsets of Y, there exists a continuous function $f: X \to Y$ such that $f(x_i) \in V_i$ for every $i=1,\ldots,n$. Let X contain a convergent sequence which is infinite as a subset of X, and let X be completely Hausdorff with respect to Y. If Y satisfies any of the following three conditions, then $C_\pi(X,Y)$ is of first category: (i) Y is of first category, (ii) Y contains two nonempty open subsets with disjoint closures, or (iii) Y contains a sequence $\{W_i\}$ of nonempty open subsets such that for each sequence $\{y_i\}$ in Y with $y_i \in W_i$ for every i, no subsequence of $\{y_i\}$ converges. As a corollary to (ii), if X is a non-descrete T_1-space which is completely Hausdorff with respect to Hausdorff space Y, then $C_\pi(X,Y)$ is of first category. For example, $C_\pi(I,I)$ is of first category, where I is the closed unit interval.
(Received October 31, 1974.)

*720-54-31 **L. E. Ward, Jr., University of Oregon, Eugene, Oregon 97403.**
The Hahn-Mazurkiewicz theorem for rim-finite continua.

A continuum is a compact connected Hausdorff space. An arc is a continuum with exactly two non-cutpoints. A space is rim-finite if its topology admits a base whose members have finite boundaries.

Theorem. If X is a rim-finite continuum, then there exists an arc L and a continuous mapping $f(L) = X$. (Received October 31, 1974.)

720-54-32 MANUEL P. BERRI and CARROLL F. BLAKEMORE, University of New Orleans, New Orleans, Louisiana 70122. The regular continuous image of a minimal regular space is not necessarily minimal regular.

Herrlich has shown that the regular continuous image of a regular-closed space is regular-closed. An example is given to show that Herrlich's result cannot be extended to a corresponding result for minimal regular spaces. A modification of this example shows that a continuous function from a minimal regular space onto a regular space is not necessarily a closed function. (Received November 1, 1974.)

*720-54-33 W. HOLSZTYNSKI, Southern Illinois University, Carbondale, Illinois 62901, Extending cross-sections onto countable unions.

Let $g : A \to E$ be a cross-section of $p : E \to X = A \cup X_1 \cup X_2 \cup \ldots$ such that g and any extension of g onto $A \cup X_1 \cup \ldots \cup X_n$ can be extended to a cross-section $g_{n+1} : A \cup X_1 \cup \ldots \cup X_{n+1} \to Y$ $(n = 0,1,\ldots)$.

The main theorem of this paper asserts that if X is compact and p satisfies some conditions then g can be extended to a global cross--section $G : X \to E$ of p. It has several applications to universal functions, dimension theory, compactifications and bijective continuous functions. The main theorem specialized to an extension theorem for maps holds even for countably compact spaces.

On the other hand, the class of the "fiberations" $p : E \to X$ considered in the paper is interesting from the geometrical and algebraical points of view. (Received October 30, 1974.)

*720-54-34 PHILLIP ZENOR, Auburn University, Auburn, Alabama 36830. Countable paracompactness of F_σ-sets.

For each cardinal number m, there is an m-paracompact space X which contains an F_σ-set F such that F is not countably paracompact. This follows from Theorem. Each F_σ-subset of $X \times [0,1]$ is countably paracompact iff X is normal and countably paracompact. (Received November 1, 1974.)

*720-54-35 SAM B. NADLER, JR., University of Georgia, Athens, Georgia. Arcwise accessibility in hyperspaces.

Let X be a metric continuum, 2^X the space of all nonempty compact subsets of X with the Hausdorff metric, and C(X) the space of nonempty subcontinua of X canonically embedded in 2^X. Arcwise accessibility of points of C(X) from $2^X - C(X)$ is investigated. It is shown that every nondegenerate member of C(X) is arcwise accessible from $2^X - C(X)$ beginning with a two-point set. The rest of the paper will focus on arcwise accessibility of singletons. Specific examples are included and unsolved problems are stated. A thorough study is made of the theory of points of 2^X which arcwise disconnect 2^X. (Received November 4, 1974.)

720-54-36 PETER S. SHOENFELD, National Bureau of Standards, Washington, D.C. 20234. Generalized almost finite homomorphisms of minimal sets. Preliminary Report.

Suppose $\pi: X \to Y$ is a homomorphism of minimal sets (R.Ellis, Lectures on Topological Dynamics, Benjamin, New York, 1969). Define the flow $2^\pi = \{A \subset X \mid A$ closed and $A \subset \pi^{-1}(y)$ for some $y \in Y\}$, with the Hausdorff topology. We say that π is generalized almost finite (G.A.F.) if some, or equivalently every, almost periodic element of 2^π is finite. We say that π is highly proximal if these elements are singletons. Theorem. π is regular (these NOTICES 20 (1973), A-378, Abstract 703-G1) and G.A.F. iff $\pi = \gamma\psi$ where γ is highly proximal and ψ is a finite group extension. This and related results extend to products of G.A.F.'s and to homomorphisms which satisfy (1) $\pi^{-1}(y)u = \pi^{-1}(y) \circ u$ for all $y \in Y$ and $u \in J$ with $yu = y$; (2) $(Au) \circ u = \overline{Au}$ whenever $A \in 2^\pi$, $A \subset \pi^{-1}(y)$ for some $y \in Y$, and $u \in J$ with $yu = y$. Theorem. π satisfies (1) and (2) iff there exist minimal set homomorphisms δ, γ, and ϕ such that ϕ is almost periodic, γ is highly proximal, and $\pi\delta = \gamma\phi$. We may take δ to be highly proximal as well. (Received November 4, 1974.)

*720-54-37 ROLAND E. LARSON, Pennsylvania State University, Behrend College, Erie, Pennsylvania 16510. The height of the lattice of finite topologies.

The height of a finite lattice is the maximum length of a chain in the lattice. In this paper we show that the height of the lattice of topologies on a set of finite cardinality n is $(n^2 + n)/2$. (Received November 4, 1974.)

*720-54-38 George M. Reed, Ohio University, Athens, Ohio 45701
On normality and countable paracompactness

It was established by C. H. Dowker in 1951 that in spaces with closed sets G_δ-sets, normality implies countable paracompactness. However, the validity of the converse remains an open question. In [Bull. Pol. Acad. Sci. 8 (1970), 179-181], C. W. Proctor defined a space to be pseudo-normal provided each two mutually exclusive closed sets one of which is countable can be separated by open sets, and he noted that countably paracompact T_3-spaces are pseudo-normal. Also, Proctor gave an example of a separable, pseudo-normal Moore space which is not metrizable. Another such example is now known due to F. Tall. However, it is not known if either of these spaces is countably paracompact (or even normal without additional set theory). Theorem 1. There exists a separable, pseudo-normal Moore space that is not countably paracompact. Theorem 2. There exists a collectionwise Hausdorff, pseudo-normal, first countable T_3-space that is neither countably paracompact nor normal.

In addition, new concepts are defined and other examples are given which relate to the above question. (Received November 4, 1974.)

720-54-39 WILLIAM REDDY, Wesleyan University, Middletown, Connecticut 06457. Expansive properties of maps. Preliminary report.

Positively expansive maps and expanding endomorphisms of manifolds are quite similar while Anosov diffeomorphisms and expansive homeomorphisms are not. If the restriction to manifolds is lifted, the dissimilarity increases. Work of M. Eisenberg, E. Hemmingsen, T. O'Brien, W. Reddy, R.K. Williams, M. Shub and others will be cited in illustrating this theme. Expansive homeomorphisms arising as inverse limits of positively expansive maps of 3-dimensional nilmanifolds will be classified and general theorems suggested by this family of examples will be stated. (Received November 4, 1974.

720-54-40 GARY BRADY North Carolina State University, Raleigh, North Carolina 27607. The lattice structure of the set of $(E,\beta E)$-compactifications.

A brief discussion of $(E,\beta E)$-compactness as defined by H. Gonshor [Fun. Math. 1972] is followed by the development of a "closure" type operator designed to operate in Tychonoff spaces and replace the closure operator in defining an $(E,\beta E)$-compactification of a Tychonoff space X. In order to guarantee an $(E,\beta E)$-compact superspace of a space X we simply restrict our consideration to spaces which are Tychonoff. Also, mappings will be continuous functions. This operator has most of the useful properties of the usual closure operator. Next, an equivalence relation "$\approx$" and a partial order "$\leq$" are defined in the usual way on the class of all $(E,\beta E)$-compactifications of X where the equivalence relation generated by "$\leq$" is equivalent to "$\approx$". This enables us to consider the lattice structure induced by "$\leq$" on the set $K_E(X)$, of all non-equivalent $(E,\beta E)$-compactifications of X. The main results are (1) that $K_E(X)$ is a complete upper semi-lattice with a largest element which can be exhibited, (2) that $K_X(X) = K(X)$ the set of all non-equivalent compactifications of X and (3) that K(X) is a lattice iff $K_E(X)$ is a lattice for all Tychonoff E.
(Received November 4, 1974.)

*720-54-41 DANIEL J. RANDTKE, University of Georgia, Athens, Georgia 30602 On the existence of compact metric subspaces.

A topological space X has property σ-CM if for every countable family F of continuous functions on X there is a compact metrizable subspace M of X such that $f(X) = f(M)$ for every f in F. Clearly every space having property σ-CM is pseudocompact. Every compact metric space, every closed ordinal space and every weakly compact subset of a Banach space has property σ-CM. Our main result implies that every Hausdorff continuous image of an arbitrary product of spaces having property σ-CM also has property σ-CM. The one point compactification of a locally compact metric space always has property σ-CM; the Stone-Čech compactification of a non-compact locally compact metric space never has property σ-CM. If X is a completely regular space having property σ-CM, then X is a Baire space and every closed G_δ subset of X has property σ-CM. Every normal space having property σ-CM is countably compact. For compact spaces property σ-CM is not comparable with sequential compactness. (Received November 4, 1974.)

*720-54-42 W.A.R. WEISS, University of Toronto, Toronto, Canada M5S 1A1. A solution to the Blumberg Problem, Preliminary report.

A topological space X is called Blumberg if every real-valued function on X is continuous

on a dense subspace of X. The Blumberg problem is "Is every compact Hausdorff space Blumberg?" An internal characterization of the Blumberg property for linearly ordered spaces is given, as well as the following Theorem. If $2^{\aleph_0} \geq \aleph_2$, then there exists a compact linearly ordered space X which is not Blumberg. Let Y be the Stone space of the Boolean algebra of Lebesgue measurable sets modulo sets of measure zero. Theorem. If $2^{\aleph_0} = \aleph_1$, then Y is not Blumberg. This leads to Corollary. $X \times Y$ is a compact Hausdorff space which is not Blumberg. (Received November 4, 1974.) (Author introduced by Professor Franklin D.Tall.)

720-54-43 FRANKLIN D. TALL, University of Toronto, Toronto, Canada M5S 1A1. The density topology, Preliminary report.

The density topology on the real line consists of those measurable sets S with metric density 1 at each point of S. It is known to be completely regular, but not normal, first countable, separable, or Lindelöf. It follows from known results that it is perfect, that it satisfies the countable chain condition, and that its projective cover is the Stone space of the reduced measure algebra. The following new results are established concerning this topology. It is not metacompact. It is hereditarily subparacompact. It is not of point-countable type. Any collectionwise normal or metacompact subspace is the union of a Lindelöf subspace and a closed discrete subspace, and (hence) is hereditarily paracompact. Assuming $2^{\aleph_0} < 2^{\aleph_1}$, the same holds true for normal subspaces. Assuming Martin's Axiom, the density topology is neither Blumberg nor metalindelöf, but has a subspace of power $2^{\aleph_0}$ such that any subset of it of power $< 2^{\aleph_0}$ is closed, while any open cover of any subset of it has a subcover of power $< 2^{\aleph_0}$. The usual axioms of set theory do not decide the density character of the space, nor whether it has caliber $\aleph_1$. (Received November 4, 1974.)

*720-54-44 VICTOR SAKS, Dept. of Math., University of Costa Rica, San José, Costa Rica

Ultrafilter invariants in topological spaces

Theorem. Let $m \geq \aleph_0$ and $X = \prod_{i \in I} X_i$. Then X is $[\aleph_0, m]$-compact if and only if $\prod_{i \in J} X_i$ is $[\aleph_0, m]$-compact for all $J \subset I$ with $|J| \leq 2^{2^m}$. Definitions. Let $m \geq \aleph_0$, $\{x_\xi : \xi < m\}$ a net in X, $p \in X$, and $\mathcal{D} \in \beta m$. Then $p = \mathcal{D}\text{-}\lim_{\xi < m} x_\xi$ if $\{\xi < m : x_\xi \in U\} \in \mathcal{D}$ for every neighborhood U of p; a subset A of X is $\mathcal{D}$-closed if $p = \mathcal{D}\text{-}\lim_{\xi < m} x_\xi$ and $x_\xi \in A$ implies $p \in A$; finally, a function $f : X \to Y$ is $\mathcal{D}$-closed if $f[A]$ is $\mathcal{D}$-closed in Y for all $\mathcal{D}$-closed $A \subset X$. Theorem. Every topological space is characterized by its $\mathcal{D}$-limits. Theorem. A function $f : X \to Y$ is perfect if and only if f is $\mathcal{D}$-closed for every $\mathcal{D}$. Theorem. If X is a Hausdorff space and D is a discrete space equipotent with a dense subset of X, then X is a continuous perfect image of a subspace of βD if and only if X is regular. (Received November 4,1974.) (Author introduced by Professor W.W.Comfort.)

*720-54-45 S.D. Shore, University of New Hampshire, Durham, N.H. 03824 and S.A. Kenton, Eastern Connecticut State College, Willimantic, Conn. 06226
Completeness in semimetric spaces. Preliminary report

If d is an admissible semimetric for a topological space (X,T) (i.e., $cl_T A = \{x \mid \inf \{d(x,a) \mid a \in A = 0\}$ for each $A \subseteq X$), then several completeness concepts are possible: every d-cauchy sequence converges; every d-cauchy filterbase of closed sets converges; strongly complete and weakly complete (in the sense of McAuley, A relation between perfect separability, completeness, and normality, Pacific J. Math. 6(1956), 315-326); complete development (in the sense of R.L. Moore). Relationships among these are considered in the cases that d is a semimetric, a developable semimetric (i.e., a semimetric that is continuous on the diagonal, $\{(x,x) \mid x \in X\}$) and a continuous semimetric. THEOREM. For any admissible semimetric d for (X,T), (X,T) is d-weakly complete iff every d-cauchy filterbase of closed sets converges. THEOREM. If d is an admissible developable semimetric for (X,T) and (X,T) is d-weakly complete, then (X,T) admits a complete development. (Received November 4, 1974.)

720-54-46 JAMES E. BAUMGARTNER, Dartmouth College, Hanover, New Hampshire 03755.
Topological properties of Specker types, Preliminary report.

A Specker type is an uncountable linear ordering with the property that no uncountable subset is well-ordered, conversely well-ordered, or embeddable in the reals. A Specker type is small if it is the countable union of discrete subspaces (relative to the order topology); otherwise it is large. Theorem 1. There are 2^{ω_1} Specker types none of which is homeomorphic to a subspace of any other. Theorem 2. There are metrizable and nonmetrizable large Specker types. Theorem 3. There are metrizable small Specker types. Theorem 4. $\diamondsuit$ implies that there are nonmetrizable small Specker types. Theorem 5. Martin's Axiom implies that every small Specker type is metrizable. Theorem 6. There are metrizable and nonmetrizable Specker types with no non-trivial homeomorphisms. (Received November 4, 1974.)

720-54-47 MARYAM SHAYEGAN HASTINGS, University of Toledo, Toledo, Ohio 43606
An embedding theorem for seminearness spaces. Preliminary report.

The concept of nearness spaces was introduced by Horst Herrlich (Gen. Topol. Appl. 4(1974)). Let S-Near denote the category of seminearness spaces. Let $(X,\xi) \in$ S-Near, then (X,ξ) is topological iff $\mathcal{A} \in \xi \Rightarrow Cl_\xi \mathcal{A} \neq \phi$.

Theorem. The epireflective hull of topological S-Near spaces in S-Near is S-Near. (Received November 4, 1974.) (Author introduced by H. L. Bentley.)

*720-54-48 RICHARD H. WARREN, Aerospace Research Laboratories, ARL/LB, Wright-Patterson AFB, Ohio 45433. Boundary of a fuzzy set in a fuzzy topological space.

G. J. Nazaroff (J. Math. Anal. Appl. 41 (1973), 478-485) has shown that the boundary of a fuzzy set in a fuzzy topological space is useful in the study of optimality in fuzzy systems. Definition 1. Let X be a set. A fuzzy set in X is a function from X into [0,1]. A fuzzy topology T on X is a family of fuzzy sets in X such that $\mu_\emptyset$, $\mu_X \in T$; if $g_i \in T$, then $\bigvee g_i \in T$; and if g, h $\in T$, then $g \wedge h \in T$. Definition 2. If a is a fuzzy set in (X,T), then $\bar{a} = \bigwedge\{f : a \leq f \text{ and } 1 - f \in T\}$. Definition 3. If a is a fuzzy set in (X,T), then the boundary of a, denoted by a^b, is defined as follows. If $\bar{a} \wedge \overline{1 - a} = \mu_\emptyset$, then $a^b = \mu_\emptyset$. Other-

wise, $a^b = \bigwedge\{\bar{d} : \bar{d}(x) = \bar{a}(x) \text{ if } (\bar{a} \wedge \overline{1 - a})(x) > 0\}$. Theorem. If a° denotes the interior of the fuzzy set a, then $\bar{a} = a^\circ \vee b^b$ and $a^b \geq \bar{a} \wedge \overline{1 - a} \geq \bar{a} - a^\circ$. Other properties of the fuzzy boundary are proved. A fuzzy boundary operator is introduced and it is shown that spaces so defined are identical to fuzzy topological spaces. (Received November 4, 1974.)

*720-54-49 H. H. WICKE and J. M. WORRELL, JR., Ohio University, Athens, Ohio 45701.
Point-countability and compactness.

A proof is given, partly in response to certain requests, of the following theorem, stated for the T_1 case as Theorem (iv) of Worrell-Wicke, Can. J. Math. 17 (1965) 820-830. Theorem 1. Suppose X is a countably compact space and $\mathcal{U}$ is an open covering of X which is the union of a countable collection $\{\mathcal{V}_n : n \in N\}$ of collections of open sets such that each $x \in X$ is in at least one but not in more than countably many elements of some $\mathcal{V}_n$. Then some finite subcollection of $\mathcal{U}$ covers X. Call a space X weakly $\delta\theta$-refinable if and only if every open covering of X has a refinement $\mathcal{U}$ with the property of the hypothesis of Theorem 1. A collection $\{\mathcal{V}_n : n \in N\}$ as in Theorem 1 (whether or not it covers X) is called σ-distributively point-countable. Theorem 2. A Hausdorff M-space is a paracompact p-space if and only if it is weakly $\delta\theta$-refinable. Theorem 3. Every weakly $\delta\theta$-refinable regular T_0 wΔ-space satisfies Burke's p-space criterion. Theorem 4. Suppose X is a countably compact Hausdorff space. Then the following are equivalent: (a) X is compact metric. (b) X is quasi-developable. (c) X is weakly $\delta\theta$-refinable and has a G_δ-diagonal. (d) X is weakly $\delta\theta$-refinable and its diagonal is locally a primitive set of interior condensation. (e) X can be covered by a σ-distributively point-countable collection of open sets each of which is weakly $\delta\theta$-refinable and has a primitive base. (Received November 5, 1974.)

*720-54-50 ROBERT M. TARDIFF, University of Massachusetts, Amherst, Massachusetts 01002, Closure space structures for probabilistic metric spaces.

Let $(S,\mathcal{F},\tau)$ be a probabilistic metric space, and let $\phi:R\to[0,1]$ be a profile function, so that for any $x>0$, $\phi(x)$ is the maximum probability with which one can make statements about distances less than x. For any $\delta>0$, the ϕ-δ neighborhood of p is the set $N_p(\phi,\delta) = \{q\varepsilon S \mid F_{pq}(x+\delta)+\delta\geq\phi(x)$, for $x\varepsilon[0,\frac{1}{\delta})\}$, where F_{pq} is the distance distribution function of p and q. (If $F_{pq}(x)\geq\phi(x)$ for all x, then p and q are indistinguishable relative to ϕ.) For any any $A\subseteq S$ let $C(A)= \{p\varepsilon S \mid N_p(\phi,\delta)\cap A\neq\square$ for all $\delta>0\}$. Then (S,G) is a closure space in the sense of Čech. If, furthermore, C is idempotent (which is the case when τ is continuous and $\tau(\phi,\phi)=\phi$), then C is a Kuratowski closure operator. These results improve earlier results due to E. Thorp and R. Fritsche where, in general, $C(A\cup B)\neq C(A)\cup C(B)$. Products and quotients of these closure space structures for PM spaces are also discussed.
(Received November 5, 1974.) (Author introduced by Professor B. Schweizer.)

*720-54-51 MICHAEL D'AMBROSA, Seton Hall University, South Orange, New Jersey 07079. β-like Compactifications and 0-1 Measures. Preliminary report.

We begin with a set X and a certain algebra of functions, Σ, on X. The weak topology generated by Σ is put on X. A Hausdorff compactification $\beta(\Sigma,X)$ is

derived and we show $\beta(\Sigma,X)$ is equivalent to the Wallman compactification derived from the normal base $\Lambda = Z(\Sigma)$. $f \in \Sigma$ iff f is bounded and Λ-continuous iff f is Λ-uniformly continuous iff f can be extended to $\beta(\Sigma,X)$. $\beta(\Sigma,X)$ is called a β-like compactification (as in Mrowka) because of its similarity to βX. We show $\beta(\Sigma,X)$ is equivalent to the compactification generated by all Λ-regular 0-1 measures defined on the algebra generated by Λ, thus getting an integral representation for the extension of f. Finally we show a connection with inverse-closed subalgebras of $C(X)$. (Received November 5, 1974.)

720-54-52 H. L. SHAPIRO, Northern Illinois University, DeKalb, IL, and F. A. SMITH, Kent State University, Kent, OH 44242 On Extending Locally Finite Collections

We prove the following theorems: THEOREM: If X is a normal expandable space then for every closed subset S of X, every locally finite co-zero-set cover of S can be extended to a locally finite co-zero-set cover of X. We also show the following THEOREM: If X is a countable union of closed paracompant P-embedded subsets, then X is paracompact.

(Received November 5, 1974.)

720-54-53 JOHN C. MORGAN II, Syracuse University, Syracuse, New York 13210, On the absolute Baire property. Preliminary report.

A unification of the concepts of absolute measurability and the Baire property in the restricted sense is given, within the author's abstract theory of Baire category (see Abstract 72T-B95, these Notices 19(1972), p.A-436), in the case of complete separable metric spaces. The relationship between this unification and that given earlier for the real line (see Abstract 711-04-2, these Notices 21(1974), p.A-29) is clarified by the existence of analogies between order isomorphic mappings of sets and homeomorphic mappings of sets. (Author introduced by Professor John C. Oxtoby.) (Received November 5, 1974.)

*720-54-54 STEPHEN H. HECHLER, Queens College of the City University of New York, Flushing, New York 11367. On the structure of open subsets of $\beta N - N$. Preliminary report.

We consider the structure of open subsets of $\beta N - N$ in terms of the clopen subsets used to construct them. Thus, following Negrepontis, we define the *type* of an open set U (abbr. $tp(U)$) to be the cardinality of the smallest family of clopen subsets of U which covers U. We also define an open set to be *monic* iff it can be expressed as an increasing union of clopen sets and to be *disjointed* iff it can be expressed as a disjoint union of clopen sets. Theorem. a) An open set is both monic and disjointed iff it is clopen or of type $\aleph_0$. b) An open set U is monic iff it contains the closure of each open subset $V \subset U$ of type strictly less than $tp(U)$. c) (J. Isbell) An open set is disjointed iff it is paracompact. d) Martin's Axiom implies that if $tp(U) < c$, then U is regular. Next, define the *inclosure* of an open set U (abbr. $inc(U)$) to be the interior of its closure. Since the inclosure operator is in a sense a "smoothing" operator, it is reasonable to expect that while it might lower the type of an open set, it would not raise it. This is in fact true given Martin's Axiom as can be seen from d

above, but we also prove: Theorem. It is consistent with the negation of the Continuum Hypothesis that there exist an open set U such that $tp(U) = \aleph_1$, but $tp(inc(U)) = c$. We conclude by considering pairs of disjoint open sets with intersecting closures, and we note that many of our results hold for arbitrary compact zero-dimensional spaces. (Received November 5, 1974.)

720-54-55 EUGENE S. BALL, The American University in Cairo, Cairo, Egypt. α-weak normality and related properties. Preliminary report.

S is α-weakly normal if for any well-ordered monotone decreasing family $\{H_a | a \in A\}$, cardinality of $A \leq \alpha$, with no common part and a closed set H not intersecting H_a, $a \in A$, then for some $b \in A$ there is a domain D containing H_b and the $cl(D)$ does not intersect H. This property is considered in its relationship to regularity, normality and paracompactness. (Received November 6, 1974.)

*720-54-56 PETER W. HARLEY III and R.M. STEPHENSON, JR., University of South Carolina, Columbia, South Carolina 29208. Products of symmetrizable spaces.

A point p of a space X is called a regular G_δ if there is a sequence $\{V_n\}$ of neighborhoods of p such that $\{p\} = \bigcap\{Cl\ V_n\}$. Theorem. Let X be a symmetrizable space in which each point is a regular G_δ. The following are equivalent. (a) For every symmetrizable Hausdorff space Y, $X \times Y$ is symmetrizable. (b) X is locally compact. (Received November 6, 1974.)

720-54-57 JOSEPH W. GOLDSTON, N. C. Central Univ., Durham, N.C., 27707. Generalized sequential spaces, Preliminary report.

Let Ω be a class of directed sets. J. E. Vaughan ["Convergence, closed projections, and compactness," to appear Proc. Amer. Math. Soc.] defines a topological space, X, to be an Ω-net space if $U \subset X$ is open if and only if every net, with domain in Ω, which converges to a point in U is eventually in U. This extends the concept of sequential space ($\Omega = \{\omega_0\}$) due to S. P. Franklin [Fund. Math. 57(1965), 107] and m-sequential space (Ω = the class of all directed sets of cardinality $\leq m$) due to P. R. Meyer [Coll. Math. 23(1971), 223]. Analogously, Ω-neighborhood and Ω-Fréchet spaces are defined. We study the properties of these spaces. Theorem. The category of Ω-net spaces and continuous functions is coreflective in TOP. Theorem. Every Ω-net (Ω-Fréchet) space is the quotient (pseudo-open) image of an Ω-neighborhood space. These results unify and extend those of Franklin, Meyer, and others. Also, for a broad class of Ω's, a necessary and sufficient condition for a regular Ω-net space to have Ω-net product with any Ω-net space is given. This extends and improves the sequential case due to E. Michael [Ann. Instit. Fourier Gren. 18 2(1968), 281]. (Received November 6, 1974.)

*720-54-58 R. B. SHER, University of North Carolina at Greensboro, Greensboro, North Carolina 27412. Extensions, Retracts, and Absolute Neighborhood Retracts in Proper Shape Theory.

The notion of an extension of a proper fundamental net is defined and studied. Various results concerning this notion are obtained; these include a homotopy extension theorem and results relating the idea of extension to the concept of proper fundamental retraction. We also define absolute neighborhood proper shape retract (ANPSR), and show that the property of being an ANPSR is a hereditary proper shape invariant. (Received November 6, 1974.)

*720-54-59 S. WILLIAMS and M. GEWAND, SUNY at Buffalo, Amherst, New York 14226. Products with Lindelöf spaces. Preliminary report.

The authors give some new, but simple sufficient conditions for the product of (1) two Lindelöf spaces to be Lindelöf, (2) two Lindelöf spaces to be c-Lindelöf, (3) a paracompact and a Lindelöf space to be paracompact, and (4) two paracompact spaces to be paracompact. Two techniques are used: (a) investigating a special closed subspace determined by an open cover, (b) investigating a special quotient space determined by an open cover. Technique (a) is used in (1), (3), and (4). Technique (b) is used in (1), (2), and (3). (Received November 6, 1974.) (Authors introduced by Professor Lee Mohler.)

*720-54-60 Peter W. Harley,III, University of South Carolina, Columbia, South Carolina 29208. G_δ points in symmetrizable spaces

A symmetric d for a space X is said to satisfy (*) provided that for every point x in X and sequence $\{s_n\}$ in X, if $s_n \to x$, then $d(s_n,x) \to 0$. Here it is shown that a set of cardinality $\aleph_1$ with the cofinite topology is symmetrizable. This provides a simple example of a symmetrizable space in which no point is a G_δ and for which there exists no symmetric satisfying (*). (Received November 6, 1974.)

720-54-61 STANISLAW MROWKA, State University of New York at Buffalo, Buffalo, N.Y. 14226 Dimension in metric spaces, Preliminary Report.

We offer some simplications and extensions of P. Roy's construction of metric spaces X with $\operatorname{ind} X < \dim X$. (Received November 6, 1974.)

*720-54-62 DAVID M. SEGAL, City College, New York, New York 10031. Some applications of Landweber-Novikov operations.

Previous results on the characteristic numbers of Sp-manifolds are extended in three different ways. I. It is shown that the primitive symplectic Pontrjagin class evaluated on a $4(2^j - 1)$ dimensional Sp-manifold always gives a number divisible by 8. This forms an analogue to a well-known result of Milnor concerning U-manifolds. II. It is shown that some of the results of Floyd as well as an analogue of the previous result can be obtained for 'pseudo-symplectic' manifolds. III. Results are generalised to (Sp,fr) manifolds. (Received November 6, 1974.)

*720-54-63 G. R. GORDH, JR., Guilford College, Greensboro, North Carolina 27410 and SIBE MARDEŠIĆ, University of Zagreb, Zagreb, Yugoslavia. Characterizing local connectedness in inverse limits.

Let X denote the limit of an inverse system $\underline{X}$ of locally connected Hausdorff continua. The main purpose of this paper is to define a notion of local connectedness for inverse systems, and to prove that if $\underline{X}$ is locally connected, then so is the limit X . If the bonding maps of the system $\underline{X}$ are surjective, then X is locally connected if and only if $\underline{X}$ is. A number of corollaries are also obtained. For example, if $\underline{X}$ is a well-ordered inverse system of arcs, then X is metrizable or X is a generalized arc. (Received November 6, 1974.)

720-54-64 EDWARD BECKENSTEIN, St. John's University, Staten Island, New York, LAWRENCE NARICI, St. John's University, Jamaica, New York, CHARLES SUFFEL, Stevens Institute of Technology, Hoboken, New Jersey and SETH WARNER, Duke University, Durham, North Carolina. Maximal ideals in algebras of continuous functions.

Let T be a topological space, F a topological field, C(T,F) the F-algebra of all continuous F-

valued functions on T. Conditions are given on T and F insuring that every subspace of codimension 1 in C(T,F) containing only singular elements is a maximal ideal. This result is shown to be true for all completely regular spaces T when F is the real numbers, and all ultraregular spaces T when F is a complete rank one nontrivially nonarchimedean valued field.

55 Algebraic Topology

*720-55-1 WILBUR WHITTEN, University of Southwestern Louisiana, Lafayette, La. 70501. Groups and manifolds characterizing links

Let L denote a tame link $K_1 \cup \ldots \cup K_\mu$ in S^3, and let ρ and η be fixed integers; ρ, arbitrary; $\eta = \pm 2$. For each of $i = 1,\ldots,\mu$, let V_i be a tubular neighborhood of K_i; assume that $V_i \cap V_j = \emptyset$ when $i \neq j$. If $D(K_i;\rho,\eta)$ is a (ρ,η)-double of K_i lying in $\mathrm{Int}(V_i)$ in the obvious "nice" way, we call $\cup D(K_i)$ the (ρ,η)-double of L and denote it by $D(L;\rho,\eta)$.

Let W_i be a tubular neighborhood of $D(K_i;\rho,\eta)$ in $\mathrm{Int}(V_i)$, set $C^3(L;\rho,\eta) = S^3 - \mathrm{Int}(W_1 \cup \ldots \cup W_\mu)$, and let $\{L\}$ denote the ambient isotopy type of L in S^3.

Theorem. *If L_1 and L_2 are tame links in S^3, then $\{L_1\} = \{L_2\}$ iff $\pi_1(C^3(L_1)) \approx \pi_1(C^3(L_2))$.*

Corollary 1. *We have $\{L_1\} = \{L_2\}$ iff $C^3(L_1)$ is homeomorphic to $C^3(L_2)$.*

Corollary 2. *If K is a knot in S^3 and if K^* is K's mirror image, then K is amphicheiral iff $\pi_1(C^3(K)) \approx \pi_1(C^3(K^*))$.*

An outline of the proofs is given in [W. Whitten, Bull. Amer. Math. Soc. 80 (1974)]. In case L is a knot, complete proofs are in [W. Whitten, Invent. Math. 26 (1974), 259-270]. (Received October 29, 1974.)

*720-55-2 D. A. EDWARDS, R. GEOGHEGAN and H. M. HASTINGS, SUNY - Binghamton, N. Y. 13901
On Homotopy Inverse Limits and the Vanishing of $\lim^s$ for stable Pro-Groups.

Let SS be the category of simplicial sets and Pro-SS be the category whose objects $\{X_\alpha\}$ are inverse systems in SS and whose set of morphisms is given by $\text{Pro-SS}(\{X_\alpha\},\{Y_\beta\}) = \lim \operatorname{colim} SS(X_\alpha,Y_\beta)$. A strong homotopy category Ho(Pro-SS) is defined by inverting level weak equivalences.

Theorem 1: The natural inclusion $\mathrm{Ho}(SS) \longrightarrow \mathrm{Ho}(\text{Pro-SS})$ has an adjoint

Holim: $\mathrm{Ho}(\text{Pro-SS}) \longrightarrow \mathrm{Ho}(SS)$.

Let I be a directed set and $\lim^s$ the s-th derived functor of the inverse limit functor

lim: $(\text{Abelian Groups})^I \longrightarrow (\text{Abelian Groups})$.

Theorem 2: $\lim^s$ factors through Pro-(Abelian Groups).

Corollary: If $\{A_\alpha\}$ is stable (i.e., isomorphic in Pro-(Abelian Groups) to a group), then $\lim^s\{A_\alpha\} = 0$ for $s \neq 0$. (Received November 1, 1974.)

*720-55-3 DAVID HANDEL, Department of Mathematics, Wayne State University, Detroit, Michigan 48202. Epimorphism plus monomorphism implies equivalence in the homotopy category.

A continuous map $f: X \to Y$ is epic (resp. monic) if for any two maps $g,h: Y \to Z$ (resp. $g,h: Z \to X$), $gf \simeq hf$ (resp. $fg \simeq fh$) implies $g \simeq h$, where $\simeq$ denotes free homotopy.

Theorem. If X and Y are path-connected and $f: X \to Y$ is both epic and monic, then f is a weak homotopy equivalence. (Received November 1, 1974.)

*720-55-4 DANA MAY LATCH, Lawrence University, Appleton, Wisconsin 54911. A uniqueness theorem for homology in $\mathcal{C}$at, the category of small categories.

Laudal [Ann. Sci. Ecole. Norm. Sup. (3)82 (1965), 241-296] has shown that derived functors of colimit (resp. limit) define a relative homology (cohomology) theory for pairs (X, X') of posets, where X' is initial in X. The author has shown that these derived functors define relative homology (and cohomology) theories in the sense of Eilenberg-Steenrod-Milnor (infinite additivity) for a category $\mathcal{A}$d of admissible pairs $(\mathbb{C}, \mathbb{C}')$ in $\mathcal{C}$at. $(\mathbb{C}, \mathbb{C}') \in |\mathcal{A}\mathrm{d}|$ IFF $\mathbb{C}'$ is an initial subcategory of $\mathbb{C}$, i.e., $p \in \mathbb{C}'$ implies $\{\alpha : q \to p\} \subseteq \mathbb{C}'$. This theory has absolute homology $(L_* \operatorname{colim})\Delta A = H_*(N(\mathbb{C}); A)$, where $N(\mathbb{C})$ is the nerve of $\mathbb{C}$ in simplicial sets, and ΔA in a AB4 category $\mathcal{A}$. The author will outline a proof of the uniqueness of these theories with respect to the Eilenberg-Steenrod-Milnor axioms. (Received November 4, 1974.)

720-55-5 S. K. KIM, University of Connecticut, Storrs, Connecticut 06268, D. McGAVRAN, University of Connecticut, Waterbury, Connecticut 06710 and J. PAK, Wayne State University, Detroit, Michigan 48202. On the fixed point indices and Nielsen numbers. Preliminary report.

Let $\mathfrak{F} = (E, p, B)$ be a locally trivial orientable fibering in the category of compact connected ANR spaces. A fiber map $f: E \to E$ induces maps $f': B \to B$ and $f_b: p^{-1}(b) \to p^{-1}(b)$ for each $b \in B$. If the spaces involved satisfy the Jiang condition then it is known that the Nielsen numbers satisfy the relation $N(f) \cdot P(f) = N(f') \cdot N(f_b)$ and the fixed point indices satisfy the relation $i(f) = i(f') \cdot i(f_b) \cdot P(f)$, where $P(f)$ is defined algebraically. We show that the number $P(f)$ is a fiber homotopy invariant and if $\mathfrak{F}$ is a principal circle bundle over n-dimensional complex projective space then $P(f)$ depends only on the classification of the bundle and the degree of f. We also develop some algorithms for the computation of $P(f)$ and show that the formula $N(f) = N(f') \cdot N(f_b)$ may not hold in some cases. (Received November 4, 1974.)

720-55-6 RAYMOND Y. T. WONG, University of California, Santa Barbara, California 93106. On the problem of Hilbert cube factors. Preliminary Report.

Suppose $X \subset Y \subset Z$ are metric spaces, a homotopy $\lambda_t : Y \to X$ is a straight deformation retraction if $\lambda_0 = \mathrm{id}$, λ_1 is a retraction of Y onto X and for $t < 1$, λ_t is an imbedding of Y into Z such that $\lambda_t|_X = \mathrm{id}$. We show that if $\lambda : Y \times [0,1] \to Z \times [0,1]$ is defined by $\lambda(y,t) = (\lambda_t(y), t)$, then $\lambda(Y \times \{1\})$ is a Z-set in M. Theorem. The product of any compact metric absolute retract with the Hilbert cube is homeomorphic to the Hilbert cube. Theorem. The product of any locally compact metric absolute neighborhood retract with the Hilbert cube is a Hilbert cube manifold. (Received November 4, 1974.)

*720-55-7 David A. Edwards and Ross Geoghegan, State University of New York at Binghamton, Binghamton, New York 13901, The Wall obstruction in shape and pro-homotopy, with applications.

A series of papers is described, in which the interplay between shape, pro-homotopy and geometric topology is used. Among the results are: (in homotopy theory) a splitting theorem for homotopy idempotents; (in differential topology) a new method of constructing open manifolds of dimension ≥ 5 with strange ends in the sense of Siebenmann; (in pro-homotopy and shape) algebraic

criteria for a pro-complex [resp. compactum] to be homotopy [resp. shape] equivalent to a complex; (in shape theory) two-dimensional compacta shape dominated by finite complexes but not shape equivalent to finite complexes. The principal tools are the Bousfield-Kan homotopy limit and the Wall obstruction to finiteness. (Received November 4, 1974.)

720-55-8 STAVROS G. PAPASTAVRIDIS, Brandeis University, Waltham, Massachusetts 02154.
Relations among characteristic classes. Preliminary report.

Let B be BO or BSO or BU, M an n-dimensional closed, compact, connected C^∞ manifold, which is orientable in the theory B and $t_M : M \to B$ the map which classifies the stable tangent bundle of M. Let q,k be nonnegative integers s.t. $q + k \leq n$, and all cohomology groups have coefficients Z_p where p is a prime number. Let q,k,n,p be fixed throughout. Let $S \subseteq H^*(B)$. Let $I_n^q(B,S,k,p)$ be the set of all elements $x \in H^q(B)$ s.t. $t_m^*(x) \cdot H^k(M) = \{0\}$ for all manifolds which can be oriented in B and have $t_M^*(S) = \{0\}$. The problem is to compute $I_n^q(B,S,k,p)$. We have the following two results. Theorem A. If all elements of S have degree $> n/2$ then $I_n^q(B,S,k,p) = I_n^q(B,\emptyset,k,p) + \{S\}^q$ where $\{S\}$ is the smallest ideal of $H^*(B)$ which contains S and which is a module over A_p (the mod-p Steenrod algebra). Theorem B. Except for the case $B = BSO$, $p = 2$ for all the other cases $I_n^q(B,\emptyset,k,p) = I_n^q(B,\emptyset,n-q,p)$. There is an algebraic analogue for $I_n^q(B,S,k,p)$ (see Brown and Peterson, Topology 3(1964), 39-52); it is a little bit more complicated to describe $I_n^q(BSO,\emptyset,k,2)$, nevertheless it is true that $I_n^q(BSO,\emptyset,k,2) = \operatorname{alg} I_n^q(BSO,\emptyset,k,2)$ although usually $I_n^q(BSO,\emptyset,k,2) = I_n^q(BSO,\emptyset,n-q,2)$. The proofs follow the ideas of Brown and Peterson. (Received November 6, 1974.)

720-55-9 GEORGE E. LANG, JR., Fairfield University, Fairfield, CT. 06430
Localizations and $G_n(X)$.

Let X^X be the identity component of the space of functions from a finite, simple, connected CW complex X to itself, $\omega : X^X \longrightarrow X$ the evaluation map, and $G_n(X) = \omega_*(\pi_n(X^X))$. If $G_n(X)_p$ is the localization of the group $G_n(X)$ at the prime p and X_p the homotopy localization of X then $G_n(X_p) \cong G_n(X)_p$. Corollaries on cross sectioning fibrations with fiber localized at p and conditions for X to be a G-space in terms of $G_n(X_p)$ follow from this result. (Received November 6, 1974.)

720-55-10 R. M. GILLETTE, Montana State University, Boseman, MT 59715 and J. M. VAN BUSKIRK, University of Oregon, Eugene, OR 97403. Knots Invariant under Switching an Essential Crossing.

A knot crossing is essential if it can be encircled by an unknotted simple closed curve which is homologically, but not homotopically, trivial in the complement of the knot, and is ambiguous if, further, it can be switched (from an over-crossing to an under-crossing) without changing knot types. Examples are given of: a knot having an ambiguous crossing among the minimal number of crossings for its type; a knot having an ambiguous crossing and the Alexander polynomial of any given amphicheiral knot; a knot having an ambiguous crossing and unit Alexander polynomial; a trivial knot which retains its unit Alexander polynomial on switching an essential crossing. A purely algebraic version is given of J. M. Martin's question of the existence of a trivial knot with ambiguous crossing, and it is noted than an affirmative answer to this question would imply that each knot type contains a knot having an ambiguous crossing. (Received November 6, 1974.)

720-55-11 J. C. CHIPMAN, Oakland University, Rochester, Michigan 48063. Presentations for Proper Fundamental Groups. Preliminary report.

This paper is concerned with devising a framework for the development of computational means of determining the proper fundamental group, $\underline{\pi}_1(K,\underline{a})$, of the end $[a]$ of K based at $\underline{a}$. (The introduction of proper homotopy groups was made by Brown in Topology Conference, Lecture Notes in Math. 375, Springer (1974).) Now $\underline{F} = \underline{\pi}_1(\underline{S}^1, \underline{*})$ forms a natural set of operators on $\underline{\pi}_1(K,\underline{a})$, and it is necessary to know these actions as well as the group $\underline{\pi}_1(K,\underline{a})$ for many applications. These requirements can be satisfied by exploiting the fact that for any $\underline{\pi}_1(K,\underline{a})$ there exists an epimorphism $\eta: \underline{F} \to \underline{\pi}_1(K,\underline{a})$. We define a presentation of $\underline{\pi}_1(K,\underline{a})$ to be a set of generators for Ker η. Then we have that: i) The action of $\underline{F}$ on $\underline{\pi}_1(K,\underline{a})$ is reduced to the action of $\underline{F}$ on $\underline{F}$ modulo the Ker η, ii) Any homomorphism $\theta: \underline{\pi}_1(K,\underline{a}) \to \underline{\pi}_1(L,\underline{b})$ commuting with the action of $\underline{F}$ lifts to a homomorphism $\bar{\theta} : \underline{F} \to \underline{F}$ commuting with the action of $\underline{F}$, and iii) If $K = K_1 \cup K_2$ where $K_o = K_1 \cap K_2$ and $[a]$ is an end of K_o then a presentation for $\underline{\pi}_1(K,\underline{a})$ can be given in terms of presentations for the $\underline{\pi}_1(K_i,\underline{a})$ and the appropriate inclusion maps between them. For i) and ii) above the problems reduced to $\underline{F}$ are solved explicitly.
(Received November 6, 1974.)

*720-55-12 DONALD W. ANDERSON, University of California, San Diego, La Jolla, California 92037. Fibrations and geometric realizations.

• There is a folk theorem in homotopy theory which has been particularly useful in the study of loop spaces, classifying spaces, etc. This theorem states that if $E \to B$ is a map of simplicial spaces, which is a degreewise fibration, then $R(E) \to R(B)$ is a fibration where R is geometric realization. However, along with this folk theorem comes some folk wisdom—extra hypotheses are needed and the terms need suitable definition or anyone will be able to produce counterexamples. The first version of this theorem to be proved and published is due to Peter May, who requires among other things that all B_n be connected, and who substitutes "quasi-fibration" for "fibration". His proof is difficult and his hypotheses are not of a homotopy invariant nature, but his version of the theorem has proved to be very useful. However, it is desirable to have other versions of so useful a theorem, as well as a sufficient understanding of the geometric realization functor to deduce homotopy invariant versions of this theorem. Chris Reedy showed that certain of the mysterious technical assumptions which naturally appear in any attempt to prove the folk theorem are in fact very reasonable conditions from the point of view of Quillen's axiomatic homotopy theory. Also, Reedy developed sufficient tools within axiomatic homotopy theory to understand the homotopy theoretic nature of the geometric realization functor. Reedy's work will be explained, and a new version of the folk theorem will be proved which does not require that the B_n be connected. While this is not exactly a generalization of May's version, it can be used to extend some of May's applications of the folk theorem. (Received November 6, 1974.)

57 Manifolds and Cell Complexes

720-57-1 RUSSELL J. ROWLETT, Department of Mathematics, University of Tennessee, Knoxville, TN 37916. Torsion in the bordism of oriented involutions.

We give a new geometric proof of an old result: a class in the oriented bordism ring Ω_* has order 2 iff it is represented by a manifold that admits an orientation-reversing involution. The method generalizes, so that one obtains results on the bordism $\Omega_*(Z_2^k)$ of smooth, oriented Z_2^k-actions. Theorem: All torsion in $\Omega_*(Z_2^k)$ is of order 2. If P is

the family of proper subgroups, all torsion in $\Omega_*(Z_2^k,P)$ maps to 0 in $\Omega_*(Z_2^k)$. A class in $\Omega_*(Z_2^k)$ is of order 2 iff it is represented by an action that commutes with some orientation-reversing involution. In particular the Witt invariant of Conner and Raymond vanishes on the 2-torsion of $\Omega_*(Z_2)$. (Received October 21, 1974.)

*720-57-2 LAWRENCE S. HUSCH, University of Tennessee, Knoxville, Tennessee 37916. Diffeomorphisms of 3-manifolds which are homotopy equivalent to S^1.

Let h be a diffeomorphism of an open 3-manifold M onto itself and let $\Gamma(h)$ be the closed semigroup generated by h in the space of smooth mappings of M into itself with the C^1-topology. If $\Gamma(h)$ is compact, then $\Gamma(h)$ contains a unique idempotent π; let $I(h)$ be the image of π. THEOREM. If M is homotopy equivalent to the 1-sphere and if g and h are diffeomorphisms of M onto itself such that $\Gamma(g)$ and $\Gamma(h)$ are compact and such that $I(h) \neq M \neq I(g)$, then h and g are topologically equivalent if and only if (i) there exists a homeomorphism k of M onto itself such that $k(I(h)) = I(g)$; (ii) $h|I(h)$ is topologically equivalent to $k^{-1}gk|I(h)$ and (iii) if $(h|M - I(h))_*$ is the homomorphism of $H_*(M - I(h))$ induced by h, then $(h|M - I(h))_* = (k^{-1}gk|M - I(h))_*$. (Received October 21, 1974.)

*720-57-3 JEROME DANCIS, University of Maryland, College Park, Maryland, 20742. Building Subbundles.

Theorem 1. Let M and Q be topological m- and q-manifolds, resp. $m \leq (2/3)q - 2$, (M compact and $\partial Q = \emptyset$). Let $M \hookrightarrow Q$ be a locally flat embedding. There is a locally flat embedding $h: M \times [-1,1] \hookrightarrow Q$, with $h \mid M \times 0$ the inclusion map, if and only if there is a homotopy

$$\{f_t: M \to Q, \quad t \in [0,1]\}, \qquad f_t(M) \cap M = \emptyset, \qquad t \in (0,1]$$

and f_0 is the inclusion map.

Theorem 2. Let $\xi^r \mid M$ and $\eta^q \mid M$ be PL r- and q-block bundles over a compact PL m-manifold M, $\partial M = \emptyset$, with associated projection maps $\rho_1: \xi^r \mid M \twoheadrightarrow M$ and $\rho_2: \eta^q \mid M \twoheadrightarrow M$. Let $3r + m \leq 2q - 5$. Suppose $h: \dot{\xi}^r \mid M \to \eta^q \mid M$ is an embedding of the sphere block bundle $\dot{\xi}^r \mid M$ associated with $\xi \mid M$ such that $\rho_2 n$ is homotopic to $\rho_1 \mid \dot{\xi}^r \mid M$. Then there is a PL (q-r)-block bundle $\theta^{q-r} \mid M$ such that $\eta^q \mid M = \xi^r \oplus \theta^{q-r} \mid M$ (mod subdivision).
(Received October 25, 1974.)

*720-57-4 P.L.KING, University of North Carolina, Chapel Hill, N.C. 27514. Local combinatorial pontrjagin classes.

Theorem. A 4-manifold M with polyhedral structure can be given a riemannian structure in which the pontrjagin data vanish outside arbitrarily small neighborhoods of the vertices, and near a vertex depend only on the polyhedral structure at the vertex.

The idea of the proof is that if a sufficiently natural metric is constructed, the pontrjagin form will vanish nearly everywhere. Here "sufficiently natural" means "flat in enough directions." Roughly, the metric is built thus. A flat metric is canonically constructed on $M - Sk^2M$. Near $Sk^2M - Sk^1M$ a natural metric arises, because the link of a 2-face of M is a triangulated circle, and there is a natural flat metric on the disc which is the

cone on a triangulated circle. A metric is determined near the edges and vertices of M by making (non-constructive) choices related to combinatorial structure. The local metrics are then patched with a suitable partition of unity, and the pontrjagin form computed explicitly.

(Received October 30, 1974.)

720-57-5 RICHARD SLOCUM, University of Tennessee at Martin, Martin, Tennessee 38238. Brick partitionings and 2-cells. Preliminary report.

(Zippen) Theorem. Let M be a Peano continuum which has a 1-sphere J such that a) there is an arc which spans J b) every arc which spans J separates M and c) no closed proper subset of an arc spanning J separates M . Then M is a closed 2-cell with boundary J . A new proof is given which uses brick partitionings. The properties of the brick partitionings are similar to properties of brick partitions used to prove characterizations of 2-manifolds. (Received November 4, 1974.)

720-57-6 THOMAS W. TUCKER, Colgate University, Hamilton, N. Y. 13346. Surfaces in noncompact 3-manifolds, Preliminary report.

A 3-manifold M is a missing-boundary manifold (abbreviated mbm) if it is homeomorphic to N-K where N is a compact manifold and K is a closed nonempty subset of ∂N. A surface F in M is trivial if F is in ∂M and $\pi_1(F) \to \pi_1(M)$ is trivial or M-F has two components and at least one component C has $\pi_1(C) \to \pi_1(M)$ trivial. Removing the wild point from the Fox-Artin sphere (Ann. Math 49 (1948), 979-990), one obtains a trivial plane F in R^3 such that the closure, C, of one component of $R^3 - F$ is not a mbm while the interior of C is. In contrast, we prove: Theorem 1 Let F be a nontrivial separating surface in the mbm M where $\pi_1(F)$ is finitely generated. Then the closures of the components of M-F are mbm's. Theorem 2 Let M be a 3-manifold such that the interior of M is a mbm and every component of ∂M is nontrivial in M. Then M is a mbm. The first result implies that any plane in R^3 invariant under a standard Z-action splits R^3 into components whose closures are half-space, and corrects an error in A. Marden's recent paper on kleinian groups (Ann. of Math 99 (1974), 383-462). (Received November 4, 1974.)

720-57-7 RICHARD E. HEISEY, Vanderbilt University, Nashville, Tennessee 37235. Bounded weak- manifolds.

Let F be of (1) a separable, reflexive, infinite-dimensional Banach space with its bounded weak topology, (2) the conjugate of a separable, infinite-dimensional Banach space with its bounded weak-* topology, or (3) $Q^\infty = \text{dir lim } Q^n$, where Q is the Hilbert cube. Let M and N denote paracompact connected F-manifolds. Theorem 1. $M \times F$ is homeomorphic to M. Combined with previous work of the author (see Abstract 699-G30, these Notices 19 (1972), A-811) this theorem yields the following as corollaries. Theorem 2. M embedds as an open subset of F. Theorem 3. If M and N have the same homotopy type, then they are homeomorphic. (Received November 4, 1974.)

*720-57-8 JOHN M. WOODS, Oklahoma Baptist University, Shawnee, Oklahoma 74801. Homology of finite cyclic coverings. Preliminary report.

This paper gives several new results concerning the rational homology for finite cyclic coverings of smooth codimension 2 submanifolds of S^{n+2}, $n \geq 1$. Particular emphasis is given to periodicity phenomena of the homology. The resulting theory is a broad generalization of part of C. Gordon's paper (Trans. Amer. Math. Soc. 168(1972), 357-370), which deals only with classical knots and their branched

cyclic covers. The author's results include a complete description of the rational homology of finite branched and/or unbranched cyclic coverings of knots and links in all dimensions. Some of the results also apply to more general submanifolds. The principal method used is that of Wang sequences. (Received November 4, 1974.)

*720-57-9 ROBERT SCHWARTZ, Burlington County College, Pemberton, New Jersey 08068. P.L. Actions and Equivariant General Position. Preliminary Report.

If G is a closed subgroup of SO(n) and K is a simplicial complex, a P.L. action is defined to be a topological transformation group $G \times K \longrightarrow K$ such that (i) for each $x \in K$, $G(x)$ is a P.L. subspace of K (ii) for each $g \in G$, $g: K \longrightarrow K$ is a P.L. map (iii) at each $x \in K$, there is a slice which is a P.L. subspace of K.

Let K be a simplicial complex and M be a P.L. manifold, each with a P.L. action under G. If $f: K \longrightarrow M$ is an equivariant P.L. map, f is defined to be in equivariant general position if f is non-degenerate and $\dim(f(K-K^H) \cap M) \leq \dim(K-K^H) + \dim M^H - \dim M$ for each isotropy subgroup H of G. Theorem: Let $f: K \longrightarrow M$ be an equivariant P.L. map and suppose that for each isotropy subgroup H of G, $\dim K^H \leq \dim M^H$, and M^H is a P.L. submanifold of M whenever $K^H \neq \phi$. Then f is equivariantly homotopic to a map in equivariant general position. This theorem is subsequently used to show that certain bounds on the dimension of the double point set can be achieved through equivariant homotopy. Hence, under certain conditions, $f: K \longrightarrow M$ is equivariantly homotopic to an embedding. (Received November 4, 1974.)

*720-57-10 MARVIN ISRAEL, Goucher College, Towson, Maryland 21204. Close piecewise linear codimension one embeddings. Preliminary report.

Let M, N be PL manifolds with M a closed codimension 1 TOP submanifold of N. Let $n = \dim(N)$. Theorem. Suppose $n < 4$ or $n > 4$ and $H^2(M;Z_2) = 0$ and $\epsilon > 0$. $\exists$ a $\delta > 0 \ni$ if $g_i : M \to N$, $i = 0,1$, are PL locally flat embeddings within δ of the inclusion, then there is an ϵ PL isotopy $G: N \times I \to N \times I$, $G_0 = \mathrm{id}$ and $G_1 g_0 = g_1$. Counterexamples. If $n > 4$, $H^2(M;Z_2) \neq 0$, $M = M \times [0] \subset M \times R = N$, then the theorem is false. (Received November 5, 1974.)

*720-57-11 ULRICH KOSCHORKE, Queens College, Flushing, N.Y. 11367. Indices of framefields with finite singularities.

Assume $n>2q$. Let $\pi(n,q)$, resp. $\pi(n,q)^{or}$, denote the set of elements in $\pi_{n-1}(V_{n,q})$ which occur as indices of q-framefields with finite singularities on arbitrary, resp. oriented, closed smooth (connected) n-manifolds. Let $\Theta: \pi_{n-1}(V_{n,q}) \longrightarrow \Omega_{q-1}(P^{q-1} \times BO(n); \phi)$ be the homomorphism defined in the author's papers "Framefields and non-degenerate singularities", to appear in Bull. AMS. Theorem 1. $\pi(n,q)$ is a subgroup of $\pi_{n-1}(V_{n,q})$ and contains the kernel of Θ. — Next define condition $C(n,q)$ to mean that an n-manifold M is bordant to a manifold with (stable) span $\geq q$ whenever all Stiefel-Whitney numbers divisible by $w_j(M)$, $n-q < j$, vanish. This holds e.g. for $q \leq 4$ and arbitrary n. Theorem 2. Assume $C(n,q)$ holds. If n is odd, then $\pi(n,q) = \mathrm{Ker}\ \Theta$. If n is even, then $\pi(n,q) = \mathrm{Ker}\ \Theta \oplus \mathbb{Z} \cdot i$, where i is the index of a q-field on S^n (if $q > \mathrm{span}\ (S^{n-1})+1$), resp. on P^n (if $q \leq \mathrm{span}\ (S^{n-1})$). — Similar results hold for $\pi(n,q)^{or}$. Theorem 3. Let $s \equiv -n \not\equiv o(4)$, and let θ^s be the homomorphism of Atiyah and Dupont (Acta Math. 128 (1972), p. 3). If $C(n,q)$ holds for oriented manifolds and bordisms, then $\theta^s(\pi(n,q)^{or})$ is the largest homomorphic image of $\pi(n,q)^{or}$ in which the index always becomes independent of the q-field (and dependent only on the underlying oriented manifold). (Received November 6, 1974.)

*720-57-12 IAN HAMBLETON, University of Chicago, IL 60637
Involutions on 2-Connected 6-Manifolds

We classify smooth, closed, orientable 6-manifolds M with $\pi_1 M = Z_2$ and $\pi_2 M = 0$. Consider $H = H_3\widetilde{M}$ as a module over $Z[Z_2]$ and $\lambda: H \times H \to Z$, the intersection pairing, as a Z_2-form. Let $\Psi: H \longrightarrow Z_2$ be the Browder-Livesay quadratic map. The form $\theta = (\lambda, \Psi, H)$ is restricted if $\theta \simeq \theta_0 \oplus \theta_1$, where θ_0 is the form on $S^3 \times RP^3$ and θ_1 is a non-singular form on a free $Z[Z_2]$-module.
Theorem: Diffeomorphism classes of closed orientable 6-manifolds M with $\pi_1 M = Z_2$ and $\pi_2 M = 0$ correspond bijectively with isomorphism classes of invariants: (1) a free abelian group H of rank $\equiv 2(4)$, (2) a restricted form (λ, Ψ, H), (3) a class $w_2 \in Z_2$, and (4) a splitting invariant $s_2 \in Z_2$.
(Received November 6, 1974.)

*720-57-13 JEFFREY L. TOLLEFSON, University of Connecticut, Storrs, Connecticut 06268.
A 3-manifold with no PL involutions.

We present an example of a closed, orientable 3-manifold M (fibered over S^1) with the property that 1_M is the only PL homeomorphism h of M for which $h^2 = 1_M$. We conjecture that M admits no non-trivial, periodic, PL maps. So far, we have only been able to show the nonexistence of free periodic maps if the period is greater than two.
(Received November 6, 1974.)

*720-57-14 FREED, BENJAMIN M., Kent State University, Kent, OH 44242
Embedding Contractible 2-Complexes in E^4

Let L be either a figure eight complex with sewing words $\alpha^r\beta^s\ \alpha^m\beta^n$, where $rn - sm = \pm 1$ or a (1,1,1) complex with sewing word $\alpha^{\varepsilon(1)}\alpha^{\varepsilon(2)} \cdots \alpha^{\varepsilon(n)}$ where $\varepsilon(i) = \pm 1$ and $\sum_{i=1}^{n} \varepsilon(i) = \pm 1$. Then there are infinitely many different embeddings of L in E^4. We do this by showing the embeddings have distinct non-simply connected complements.
(Received November 6, 1974.)

58 Global Analysis, Analysis on Manifolds

720-58-1 F. R. MILLER and W. D. CURTIS, Kansas State University, Manhattan, Kansas 66506.
A Theory of Higher Order Derivatives for H-differentiability on Locally Convex Spaces

A theory of calculus for locally convex spaces is given which uses the strong(H) differentiability but gives higher order chain rules. This is accomplished by requiring that the derivative of $f:E \to F$ take values in the space $L(E,F)_\sigma$ where σ is the mapping from the continuous seminorms of F into the continuous seminorms of E with respect to which f is differentiable. It is shown that this is the good theory to use for the infinite dimensional varieties of mappings and its nice properties are developed.
(Received October 23, 1974.) (Authors introduced by Professor R. B. Burckel.)

720-58-2 WILLIAM PERRIZO, North Dakota State University, Fargo, North Dakota 58102
Truncations of Vector Field Flows, Preliminary Report

If a vector field flow admits an isometric tangent lift which commutes with its derivative lift, the derivative displays a linearity with respect to length. Dynamical properties of the original flow can be studied using this linearity. If a vector field flow admits a tangent lift, which commutes with its derivative and is "nearly isometric," the behavior of the derivative flow with respect to length is sufficiently linear to allow a similar analysis. Various definitions of "nearly linear" can be used. The method is roughly similar to truncation of the nonlinear part in ordinary differential equations.

(Received October 29, 1974.)

720-58-3 SHELDON E. NEWHOUSE, University of North Carolina, Chapel Hill, North Carolina 27514, Hamiltonian Systems

We extend results of Pugh, Robinson, and Takens on the existence of special motions in Hamiltonian systems. Let M be a (not necessarily compact) symplectic manifold and X_H be a Hamiltonian vector field with Hamiltonian function H. If x is a non-wandering point for X_H whose orbit is bounded, then H may be C^2 perturbed to H_1 so that X_{H_1} has homoclinic oribts near x. If X_H has a second independent integral, then H_1 may be found so that X_{H_1} has infinitely many quasi-periodic and homoclinic motions as well as periodic motions near x. (Received November 4, 1974.)

*720-58-4 ZBIGNIEW NITECKI, Tufts University, Medford, Massachusetts 02155. An irreversible analogue of the Denjoy theorem. Preliminary report.

The classical dynamical theory of homeomorphisms of the circle, initiated by Poincare, can be symmarized in three statements. There is an invariant, τ, of (oriented) topological conjugacy, called the rotation number. It is rational iff the system has periodic points. If τ is irrational, there is a perfect, closed invariant subset with dense orbit, which is either the whole circle (in which case the system is conjugate to rotation by τ) or is a nowhere-dense set which can be mapped 2-to-1 onto the rotation. Denjoy showed that a sufficiently smooth (continuous derivative of bounded variation) homeomorphism with irrational τ must be a rotation, but that C^1 systems of the second type exist. It was shown ("Factorization of nonsingular circle endomorphisms", Dynamical Systems, Academic Press, 1973, 367-374) that, in general, a map f of the circle to itself of degree $n > 1$ which is locally a homeomorphism possesses a bi-invariant, perfect, closed set K_f which is either the whole circle (in which case f is conjugate to the map $\exp i\theta \to \exp ni\theta$) or a cantor set. Here, an argument of A.J. Schwartz (Amer. J. Math. 85(1963), 453-458) is adapted to show that the map on K_f, for $f \in C^2$, is an inverse limit of one-sided shifts of finite type, but examples show that this fails in the C^1 case.

(Received November 5, 1974.)

*720-58-5 SAMIR KAR, Indiana University, Bloomington, Indiana 47401. The $(H^1,1)$ rectifiable subsets of a homogeneous space without rotation.

On his paper on the (φ,k) rectifiable subsets of a homogeneous space [Acta Math, Vol. 122, 1969, 197-229] John Brothers gave an interesting characteriza-

tion of purely (H^k,k) unrectifiable subsets of Euclidean space $\mathbb{R}^n$ under the assumption that the full group of isometries was acting on the space. Our investigation was directed towards relaxing the condition on the group of isometries and as a significant step in this direction we present the following theorem:

Let B be a proper (n-1) dimensional submanifold of class ∞ of $\mathbb{R}^n$ with non-zero Gaussian curvature at every point. If $E \subset \mathbb{R}^n$ with $H^1(E)<\infty$ and if $g \in \mathbb{R}^n$, then there exists a countably 1-rectifiable Borel subset R of $\mathbb{R}^n$ such that $(E\sim R)$ is purely $(H^1,1)$ unrectifiable and $(E\sim R)\cap(g+B) = \emptyset$ for H^n almost all $g\in\mathbb{R}^n$. Conversely, if $E\subset\mathbb{R}^n$ such that for H^n almost all $g\in\mathbb{R}^n$ $E\cap(g+B) = \emptyset$, then E is purely $(H^1,1)$ unrectifiable. For definitions and notations see Federer [Geometric Measure Theory]. (Received November 5, 1974.)

*720-58-6 BRIAN H. MARCUS, University of California, Berkeley, California 94720. Unique ergodicity of some flows related to Axiom A diffeomorphisms.

A continuous flow on a compact metric space is called uniquely ergodic if it has a unique invariant Borel probability measure. Now it is possible to define a continuous flow whose orbits are the unstable manifolds of a connected attractor of an Axiom A diffeomorphism, provided that the unstable bundle is one-dimensional and orientable. Such a flow is called a W^u flow. Theorem. W^u flows are uniquely ergodic. This is related to, but does not include, H. Furstenberg's result (Recent Advances in Topological Dynamics, Lecture Notes in Math., no. 318, Springer-Verlag, pp. 95-114) that the horocycle flow for a two dimensional compact riemannian manifold of constant negative curvature is uniquely ergodic (the horocycles are the unstable manifolds for the geodesic flow). We also observe that if two continuous flows have the same orbits and no fixed points and one is uniquely ergodic then so is the other. Finally we give an example to show that the no fixed point assumption above is essential. (Received November 6, 1974.)

60 Probability Theory and Stochastic Processes

*720-60-1 William N. Hudson, University of Utah, SAlt Lake City, Utah 84112 and Howard G. Tucker, University of California at Irvine, Irvine California 92664
On Admissible Translates of Infinitely Divisible Distributions.

The density of an absolutely continuous infinitely divisible distribution function F is positive a.e. (Lebesgue measure) over the support of F which is necessarily an unbounded interval. (Received September 9, 1974.)

*720-60-2 W. N. HUDSON, J. D. MASON, University of Utah, Salt Lake City, Utah 84112.
More on equivalence of infinitely divisible distributions.

Any infinitely divisible distribution on R^n, $n \geq 1$, with infinite absolutely continuous Lévy spectral measure and no Gaussian component has a density which is positive a.e. over its support. This extends the result for $n = 1$ of W. N. Hudson and H. G. Tucker [Abstract 711-60-3, these *Notices* 21(1974), A-234]. (Received September 30, 1974.)

*720-60-3 JAMES KUELBS, University of Wisconsin, Madison, Wisconsin 53706, The law of the iterated logarithm for Banach space valued random variables.

Let B denote a real separable Banach space and assume $X_1, X_2, \ldots$ are i.i.d. B-valued random variables such that $E(X_k) = 0$ and $E(\|X_k\|^2) < \infty$. We provide necessary and sufficient condition for the sequence $\{X_k : k \geq 1\}$ to satisfy the law of the iterated logarithm on B. As an application of this result the law of the iterated logarithm is established for $C[0, 1]$ valued random variables under conditions related to those used to prove the central limit theorem in this setting. (Received October 7, 1974.)

*720-60-4 AVNER FRIEDMAN, Northwestern University, Evanston, Illinois 60201 Existence of densities for degenerate diffusion processes.

A system of n stochastic differential equations $d\xi = b(\xi)dt + \sigma(\xi)dw(t)$ determines a diffusion Markov process provided, say, $b(x)$ and $\sigma(x)$ are uniformly Lipschitz continuous. If the matrix $\sigma(x)$ is non-degenerate, then the transition density function exists. In this work the existence of the transition density function is established in some cases where $\sigma(x)$ is degenerate. (Received October 7, 1974.)

*720-60-5 WALTER ROSENKRANTZ, University of Massachusetts, Amherst, MA 01002 Limit Theorems for solutions to Stochastic differential equations.

Let $x(t)$ be the solution to the stochastic differential equation

(1) $dx(t) = dw(t) + b(x(t))dt$, where $w(t)$ is the Wiener process and $\int_{-\infty}^{\infty} b(x)dx = \alpha$. Set $x_n(t) = x(n^2 t)/n$.

Theorem: As $n \to \infty$ the family of processes $x_n(t)$ converge weakly to a Markov process $Y_\alpha(t)$. If $\alpha = 0$ then $Y_0(t) = w(t)$ but in all other cases $Y_\alpha(t)$ is a Markov process whose infinitesimal generator is a non-classical generalized second order differential operator of the Feller type $D_m D_p^+$. The implications of these results for the general theory of Markov processes will be discussed. These results will appear in the Dec. 1974 issue of the Indiana University Journal of Mathematics. (Received October 7, 1974.)

*720-60-6 MARK A. PINSKY, Department of Mathematics, Northwestern University, Evanston, Illinois 60201. Asymptotic analysis of the linearized Boltzmann equation

For the linearized Boltzmann equation with finite cross section, the solution is represented as an integral over the paths of a Markov jump process. The integral is only shown to converge conditionally, where the limiting process is defined by an increasing sequence of stopping times. The integrand is a local martingale, in the sense of K. Itô and S. Watanabe. Using purely ana-

lytic methods, we outline the proofs of the following limit theorems (see these Notices, Abstract 74T-B144, 21(1974), A-487); $\lim_{\epsilon \to 0} E(-t/\epsilon)T_\epsilon(t/\epsilon)f = \overline{N}(t)f$ where $E(t)$ denotes the semigroup corresponding to the linear inviscid Euler equations and $T_\epsilon(t)$ denotes the semigroup corresponding to the linearized Boltzmann (or Navier-Stokes) equations. Finally we discuss the accuracy of the Navier-Stokes approximation in relation to the approximation obtained through the limit theorems. (Received October 10, 1974.)

*720-60-7 M. Ann Piech, SUNY at Buffalo, Amherst, N. Y. 14226.
An infinite dimensional Laplacian, Preliminary report.

The transition measures associated with the Brownian motion process provide probability measures which are particularly well suited to the study of differential operators over infinite dimensional spaces. We discuss the properties of a promising candidate L for the role of "Laplacian" on a real separable Hilbert space. L is a stochastically defined infinite-dimensional second order differential operator, whose closure coincides with the infinitesimal generator of the Ornstein - Uhlenbeck velocity process for Brownian motion - i.e. with the number of particles operator of quantum field theory. (Received October 15, 1974.)

720-60-8 MURRAY ROSENBLATT, University of California, San Diego, La Jolla, California 92037.
Random solutions of Burgers equation. Preliminary report.

A class of stationary random solutions of the Burgers equation $u_t + u\,u_x = \mu\,u_{xx}$, $\mu > 0$, are considered. The asymptotic distribution of these solutions is determined up to first and second order as $t \to \infty$. Information about the spectra of the asymptotic approximations is also obtained. (Received October 9, 1974.)

720-60-9 PAO-LIU CHOW, Wayne State University, Detroit, Michigan 48202.
Integration of Stochastic Partial Differential Equations.

A class of linear partial differential equations with random coefficients is considered. In general, the coefficients involved are multi-parameter stochastic processes (or random fields). To determine the solution measures, it is expedient to establish the connection between the stochastic P.D.E.s and the corresponding differential equations in function space governing certain expectation-functionals for solution processes. Integration of stochastic P.D.E.s is thereby reduced to a problem of integration in function space. Concrete examples are provided for the purpose of illustration. (Received October 10, 1974.)

*720-60-10 GEORGE C. PAPANICOLAOU, Courant Institute, New York University, N. Y. 10012, Fluctuations in the Navier-Stokes equations.

We consider viscous incompressible motion of a fluid in a two-dimensional bounded region in the presence of random body forces. The statistical analysis of the velocity process is formulated as a limit theorem for a nonlinear abstract stochastic equation. We show in particular that the fluctuation of the velocity converges weakly, when properly scaled, as a process in the space of continuous functions with values in a Sobolev space with negative index, to a generalized Ornstein-Uhlenbeck process (joint work with CIPRIAN FOIAS).

(Received October 17, 1974.)

*720-60-11 CHRIS P. TSOKOS and A.N. RAO, University of South Florida, Tampa, Florida 33620. Existence and Stability Behavior of a Class of Stochastic Functional Integral Equations. Preliminary report.

The purpose of the study is to investigate the existence and stability properties of solutions of a random, Hilbert space valued integral equation of the form

$$x(t;\omega) = h(t;\omega) + \int_0^t k_1(t,\tau;\omega)f_1(\tau,x(\tau;\omega))d\tau + \int_0^t k_2(t,\tau;\omega)f_2(\tau,x(\tau;\omega))d\beta(\tau).$$

Using functional analytic techniques and admissibility theory, sufficient conditions are obtained to assure the existence of a random solution to the above equation. Also, frequency criteria are developed for the *almost sure* and *mean-square absolute* sample path stabilities, when the kernel is of the convolution type. (Received October 17, 1974.)

720-60-12 HARRY KESTEN, Cornell University, Ithaca, N.Y. 14850. Recurrence of d-dimensional random walks and the exterior Dirichlet problem. Preliminary report

e_i denotes the d-vector with components δ_{ij}, $1 \leq j \leq d$. The random walks under consideration are Markov chains $\{S_n\}_{n \geq 0}$ on $\mathbb{Z}^d$ with (*) $P\{S_{n+1} = x + e_i | S_n = x\} = P\{S_{n+1} = x - e_i | S_n = x\} = p_i(x) > \delta > 0$, $2\sum_1^d p_i(x) \equiv 1$ (so that $|S_{n+1} - S_n| = 1$ w.p.1.). It is well known that if there exists a function f, $f(x) \geq 0$ $f(x) \to \infty$ (respectively $\to 0$) as $|x| \to \infty$ and such that $f(S_n)$ is a supermartingale as long as S_n is outside a finite set, then S_n is recurrent (respectively transient). Meyers and Serrin, J. Math and Mech. 9 (1960) 513-528 in their study of the exterior Dirichlet problem considered the existence of such functions f in the analogous situation on $\mathbb{R}^d$. Their method yields examples of transient 2-dimensional random walk satisfying (*) and even $p_i(x) \to \frac{1}{4}$ ($|x| \to \infty$). It also seems that even under (*) there exist recurrent random walks in any dimension provided $\delta = \delta(d)$ is sufficiently small. (Received October 17, 1974.)

720-60-13 M. ZUHAIR NASHED, Georgia Institute of Technology, Atlanta, Georgia 30332. Measurability of generalized inverses of random linear operators in Banach spaces and random generalized Green's functions. Preliminary report.

Let $(\Omega, \mathcal{B}, \mu)$ be a complete probability space. Let X, Y be Banach spaces. Let T be an almost surely (a.s.) bounded or densely defined closed linear random operator on $\Omega \times X$ into Y with range $\mathcal{R}_{T(\omega)}$ and nullspace $\mathcal{N}_{T(\omega)}$. Suppose that a.s. the closed subspaces $\mathcal{R}^{-}_{T(\omega)}$ and $\mathcal{N}_{T(\omega)}$ have topological complements, say $\mathcal{S}(\omega)$ and $\mathcal{M}(\omega)$, respectively. Several results on the measurability of the generalized inverse of T, relative to $\mathcal{S}(\omega)$ and $\mathcal{M}(\omega)$, are established under mild technical conditions. In particular if X and Y are separable Hilbert spaces and T is an a.s. closed densely defined linear separable random operator from $\Omega \times X$ into Y such that $\bigcap_{\omega} \mathcal{D}_{T(\omega)}$ is dense in X, $\bigcap_{\omega} \mathcal{D}_{T^*(\omega)}$ is dense in Y, and $T(\omega)T^*(\omega)x$ is separable on $\Omega \times X_0$, where $X_0 = \{x : x \in X \text{ and } x \in \bigcap_{\omega} \mathcal{D}_{T(\omega)T^*(\omega)}\}$, then the (Moore-Penrose) generalized inverse of T is a random operator. Our setting also provides extensions and refinements of results of Nashed and Salehi [SIAM J. Appl. Math. 25(1973), 681-692]. We give applications to generalized random Green's functions and matrices from the viewpoint of generalized inverses.
(Received October 21, 1974.)

*720-60-14 T. F. LIN, Louisiana State University, Baton Rouge, La. 70803 Existence of Markov processes associated with noncontraction semigroups.

Let B be the ordinary Banach space of all bounded, measurable, C-valued functions on C. If the semigroup S_t, $0 \leq t \leq T$, of linear operators on B satisfies conditions (i) $S_t 1 = 1$, (ii) w-lim $f_n = f$ implies w-lim $S_t f_n = S_t f$, (iii) $\| S_t \| \leq \exp(bt)$ for some $b \geq 0$, then there exists a (generalized) Markov process $x(t)$, $0 \leq t \leq T$, such that $E\{f[x(t+s)] \mid \sigma[x(u), 0 \leq u \leq t]\} = S_s f[x(t)]$ for each $f \in B$ and $0 \leq t \leq t+s \leq T$. (Received October 21, 1974.)

*720-60-15 PAUL C. SHIELDS, Department of Mathematics, The University of Toledo, Toledo, Ohio 43606 How well can one fit two processes together

We use the terminology of Furstenberg's disjointness paper (Math. Systems Theory 1(1967 pp. 1-49). Call U a pairing of S and T if S and T are factors of U. We show that if S and T are ergodic measure preserving transformations then there is a pairing U such that $h(U) = \max\{h(S), h(T)\}$. The proof makes use of the Sinai-Ornstein Theorem (see Shields, The theory of Bernoulli shifts) and the theory of skew products. An analogous topological result seems unlikely for no topological analogue of the Sinai-Ornstein theorem is known.
(Received October 23, 1974.)

*720-60-16 MARK J. CHRISTENSEN and A.T. BHARUCHA-REID, Wayne State University, Detroit, Michigan 48202. An algebraic model for Wiener measure.

In this paper we construct an algebraic model (in the sense of Dinculeanu and Foias, Illinois J. Math. 12 (1968), 340-351) for Wiener measure. Let D be any dense subset of [0, 1]. Then, for every $x \in D$, n an integer, and k a positive integer, we define a mapping from C[0, 1] to the unit circle as follows: $f \to \exp\{2\pi i(nf(x))/k\}$, which we denote by $F(x,n,k,\cdot)$. The collection of all such F generates a group G if we consider multiplication to be

$F(x,n,k,\cdot)F(y,m,l,\cdot)(f) = \exp\{2\pi i[nf(x)/k + mf(y)/l]\} = F(x,n,k,f)F(y,m,l,f)$. G, with integration on C defining a positive-definite function, is an algebraic model for Wiener measure on $C[0, 1]$. (Received October 24, 1974.)

*720-60-17 HELEN SKALA, University of Massachusetts, Boston, Massachusetts. The Petersburg paradox.

The Petersburg Game is a two-person game in which one person A flips a coin until a head turns up. If this happens on the nth throw (i.e., the first $n - 1$ throws turn up tails and the nth throw heads), A receives 2^{n-1} dollars from the other person B. What is the maximum amount A should pay B to play this game? Theorem. If A has wealth w, he should pay no more than the amount s where $2^{2s-3} + s/2 = w$. If A pays more than the amount s, his chance of going broke, never having attained more than w, is greater than $\frac{1}{2}$. (Received September 19, 1974.)

*720-60-18 S.P. LLOYD, Bell Laboratories, Murray Hill, N.J., 07974, and R.C. SINE, University of Rhode Island, Kingston, R.I., 02881. Lorentz manifolds and retarded Markov operators. Preliminary report.

A single stable Markov operator on C(X) is considered. If T has reasonable asymptotic behavior then the convergence can be improved upon by slowing the process down (averaging with the identity operator so that T is replaced by $T_\varepsilon = \varepsilon I + (1-\varepsilon)T$ for $0<\varepsilon\leq 1$). If T is norm ergodic then T_ε has norm convergent iterates (with the aid of Karlin's ergodic theorem). If T is strongly ergodic then T_ε has iterates convergent in the strong operator topology (with the aid of Lorentz's theory of almost convergence). Conditions are given on convex power series $\Sigma a_n T^n$ for similar results to hold. (Received November 5, 1974.)

*720-60-19 DAVID H. NASH, Transportation and Urban Analysis, General Motors Research Laboratories, General Motors Technical Center, Warren, Michigan 48090 and MURRAY S. KLAMKIN, University of Waterloo, Waterloo, Ontario, N2L 3G1 Canada. A spherical characterization of the normal distribution.

An elementary functional equation proof of the following classic theorem is given: Theorem. Assume (1) $f: R^1 \to [0,\infty)$ is Lebesgue (μ) measurable, (2) $\int_{R^1} f\,d\mu = 1$, (3) $n > 1$ and (4) $\Pi f(x_i)$ is constant on each $n - 1$ sphere $S(r) \equiv \{(x_1,\ldots,x_i,\ldots,x_n) | \Sigma x_i^2 = r^2\}$, $r > 0$. Then f is a normal density with zero mean. The theorem remains true for $n > 2$ if (4) is replaced by the condition that there is a sequence $\{r_j\}$ of positive reals such that $r_j \to +\infty$ and $\Pi f(x_i)$ is nonzero and constant on each $S(r_j)$. (Received November 1, 1974.)

720-60-20 HAROLD D. TAYLOR, Indiana University, Bloomington, Indiana 47401. On the limiting behavior of infinitely many queues in tandem with identical service time distributions.

Masterson and Sherman [Ann. Math. Statist. 34(1963), 300-307] have shown that, in the case of infinitely many queues in tandem with identical service time distribution, the probability that the interarrival time between the first and second customers is less than any fixed t tends to zero as time approaches infinity. They also raise the question of whether the same behavior is exhibited by the interarrival time of customers n and $n + 1$. We answer this question in the affirmative. An analogous result is that if the input distribution is the same as the service time distribution, the probability that the length of the nth queue is less than k tends to zero as time approaches infinity. (Received November 4, 1974.)

*720-60-21 ITREL MONROE, University of Arkansas, Fayetteville, Arkansas 72701. On the square variation of martingales.

Let (X_t, G_t), $t \leq 1$, be a uniformly integrable martingale with continuous paths. Let P_n be an increasing sequence of partitions of the interval $[0,1]$. Letting $P_n = \{t_0 < t < \ldots < t_m\}$, define $S = \sum_{i=1}^{m} (X_{t_i} - X_{t_{i-1}})^2$. It is well known that as $n \to \infty$, S_n converges in measure. An example is given to show that S_n need not converge almost surely. (Received November 4, 1974.)

*720-60-22 LOUIS H. BLAKE, Worcester Polytechnic Institute, Worcester, Mass. 01609. A smoothing index and equiconvergence of martingales.

Let $(\Omega, \mathfrak{A}, P)$ be a probability space with $\mathcal{L}$ and $\mathcal{C}$ sub σ-fields of $\mathfrak{A}$. A smoothing index ρ is given by $\rho(\mathcal{C}, \mathcal{L}) \equiv \inf_{\varepsilon > 0} \{\sup_{C \in \mathcal{C}} P[|E(I_C|\mathcal{L}) - I_C| > \varepsilon| \leq \varepsilon\}$ which indicates the amount of smoothing done on $\mathcal{C}$ by $\mathcal{L}$. ρ is used to reprove Boylan's equiconvergence theorem for martingales (Ann. Math. Statist., 42(1971), 552-559). It is also proved that the usual metric on sub σ-fields of $\mathfrak{A}$, the "Neveu conditions" (as set forth in "A note on the tightness of the metric on the set of complete sub σ-algebras of a probability space", Ann. Math. Statist., 43, 1369-1371), equiconvergence of martingales, and the smoothing index ρ are all equivalent concepts. (Received November 4, 1974.)

*720-60-23 STEPHEN J. WOLFE, University of Delaware, Newark, Delaware 19711. On the unimodality of spherically symmetric stable distribution functions.

Several theorems are obtained concerning the unimodality of spherically symmetric distribution functions. These theorems are used to show that a class of spherically symmetric infinitely divisible distribution functions that contains the class of spherically symmetric stable distribution functions is unimodal. (Received November 4, 1974.)

*720-60-24 ROBERT P. KERTZ, Georgia Institute of Technology, Atlanta, Georgia 30332. Random Evolutions and Limit Theorems for Initial Value Problems.

We give a new representation in terms of random evolutions for the solution $(u_i(t))$ $i = 1,\cdot\cdot,N$ of the Cauchy problem

$$\frac{\partial u_i}{\partial t} = A_i u_i + \sum_{j=1}^{N} a_{ij} u_j$$

$$u_i(0) = f_i \qquad i = 1,\cdot\cdot,N$$

where for each $i = 1,\cdot\cdot,N$, A_i is the infinitesimal generator of a strongly continuous semi-group $T_i(t)$; $t \geq 0$ on some Banach space B and (a_{ij}) $i,j = 1,\cdot\cdot,N$ are real numbers (or operators). We construct this representation through techniques of renewal theory and semi-Markov processes. We then use this representation together with extensions of the Hersh-Papanicolaou renewal-type convergence arguments to prove singular perturbation-type theorems. A special case of interest was mentioned by M. Miyamoto [Proc. Japan Academy, 42 (1966), 70-74]. (Received November 4, 1974.)

720-60-25 F.Alberto Grunbaum, University of California,Berkeley,California 94720. The determinant of a random matrix

Let M(x) be a random matrix with independent Gaussian entries M_{ij}.The means $EM_{ij}=m_{ij}$ are unknown and the variances are all equal to one.Then the distribution of the determinant of M suffices to determine the singular values of E M=m plus the value of det m.
(Received November 4, 1974.) (Author introduced by Professor Jacob Feldman.)

720-60-26 S.C.PORT,C.J.STONE,U.C.L.A.,Los Angeles,CA & N.A.WEISS,Arizona State U.,Tempe,AZ 85281. SLLNs and CLTs for Infinite Particle Systems

Suppose that particles are distributed in Z^2 according to a point process A_0 where the r.v.'s $A_0(x)$, $x\in Z^2$ are i.i.d. with finite fourth moments. The particles initially present are translated independently via two-dimensional recurrent random walks all with the same transition function. For a finite nonempty subset B let $S_n(B)$ denote the total occupation time of B by time n and $L_n(B)$ the number of distinct particles in B by time n. Let g_n equal the expected number of visits to 0 by time n of a particle starting at 0, q_n the probability that such a particle does not hit 0 by time n, and $\lambda = EA_0(x)$. Then $ES_n(B) = \lambda|B|n$, $\mathrm{Var}\, S_n(B)\sim 2\lambda|B|^2 ng_n$, $EL_n(B)\sim\lambda nq_n$ and $\mathrm{Var}\, L_n(B)\sim\lambda nq_n$. Moreover, both $S_n(B)$ and $L_n(B)$ satisfy the strong law of large numbers and central limit theorem. (Received November 4, 1974.)

*720-60-27 ARUNAVA MUKHERJEA and JAMES GARD, University of South Florida, Tampa, Florida 33620. On the Convolution Iterates of a Probability Measure.

In this paper we study the limit behavior of convolution iterates of a probability measure on certain groups and semigroups. Let S be a locally compact non-compact group that is either (i) nilpotent, or (ii) non-unimodular, or (iii) connected maximally almost periodic. Suppose S is generated by the support of a probability measure μ. Then $\mu^n\to 0$ vaguely ($\int f d\mu^n\to 0$ for every f with compact support).

We also studied properties of the limit of μ^n where μ is a probability measure with two point support on a certain compact semigroup. (Received November 4, 1974.)

*720-60-28 Richard A. Moynihan, Univ. of Mass., Amherst, MA 01002. Conjugate transforms for τ_T semigroups of probability distribution functions.

Let T: $[0,1]\times[0,1] \to [0,1]$ be continuous, non-decreasing, associative with T(a,1)=a (so that T is a t-norm or I-semigroup). Let Δ^+ be the set of p.d.f.'s F such that F(0)=0. For any F,G in Δ^+ and any T the mapping τ_T defined by $\tau_T(F,G)(x)=\sup_{u+v=x}T(F(u),G(v))$ is a semigroup on Δ^+ (such semigroups arise in connection with the triangle inequality in probabilistic metric spaces). Using the conjugate function concept of convex analysis, one can define a conjugate transform C_T on Δ^+ for any suitable T. Such transforms and their inverses yield a theory of "characteristic functions" for τ_T semi-

groups. Thus they can conveniently be used to develop a structure theory for τ_T semigroups. With their aid we have studied algebraic questions such as the cancellation law, roots and powers (i.e., "infinite divisibility"), solvability of linear equations as well as various convergence questions and limit theorems. (Received November 5, 1974.) (Author introduced by Professor Berthold Schweizer.)

720-60-29 E. C. GOOTMAN and D. KANNAN, University of Georgia, Athens, Georgia 30602. Zero-One Laws in Finite W-Algebras (noncommutative probability).

Let A be a W*-algebra, τ a normal, finite, normalized, faithful trace on A, and $\{A_i, i \geq 1\}$ a sequence of W*-subalgebras of A. Definition: (a) $\{A_i\}$ is called π-independent (resp. $\wedge$-independent) iff for every family $\{P_i;\ P_i \in A_i\}$ of projections $\tau(\pi_{j=1}^n P_{ij}) = \pi_{j=1}^n \tau(P_{ij})$ (resp. $\tau(\wedge_{j=1}^n P_{ij}) = \pi_{j=1}^n \tau(P_{ij})$). We consider only the $\wedge$-independent A_i. Then, (1) A_i are mutually commuting. In what follows A is generated by $\wedge$-independent A_i. (2) $A = \otimes A_i$. (1) and (2) open up several structure problems of A. (3) If $A_F = W^*[A_i, i \notin F]$, F a finite subset of indices, and $B = \cap_F A_F$, then $B = \{\alpha I\}$. Definitions: (b) A_i's are identically distributed iff for each pair (i,j), $i \neq j$, there is a trace preserving isomorphism $\varphi_{ij} : A_i \to A_j$. (c) $T \in A$ is a symmetry operator if $\sigma(T) = T$ for all finite permutations σ of integers. (4) If A_i's are also identically distributed, the only symmetry operations in A are scalar multiples of I. (Received November 5, 1974.)

720-60-30 DAVID B. Miller, Department of Mathematical Sciences, Virginia Commonwealth University, Richmond, Virginia 23284. An Epidemic Branching Process, Preliminary Report.

A critical Bellman-Harris branching process with generating function $h(s)$ and lifetime distribution function $G_\ell(t)$ is amended by permitting cells to experience a sterile existence after reproduction according to a different lifetime distribution function $G(t)$. In an epidemic context, this is the isolated stage following detection of the disease and separation from the population. Asymptotic moments and limit laws for the number of isolated cells alive at time t, the total born by time t, and other quantities of interest are found via integral equations. Immigration of fertile cells and correlation of offspring lifetimes are also introduced into the model, both separately and jointly. Asymptotic moments and limit laws for functions similar to the above are then derived in each of these three models. (Received November 6, 1974.) (Author introduced by Dr. James Deveney.)

720-60-31 STEWART N. ETHIER, University of Wisconsin, Madison, WI 53706 An error estimate for the diffusion approximation in population genetics. Prelim report.

Let $\{Y_n(k) : k = 0,1,\ldots\}$ be the homogeneous Markov chain in $Q_n = \{(i,j) :$ i,j integers ≥ 0, $i + j \leq 2n\}$ whose transition function starting from $(i,j) \in Q_n$ is trinomially distributed with parameters $2n$, $i/2n$, $j/2n$, and define $X_n(t) = Y_n([2nt])/2n$ for $t \geq 0$. Let $\{X(t) : t \geq 0\}$ be the diffusion process on $Q = \{(x,y) : x \geq 0,\ y \geq 0,\ x + y \leq 1\}$ associated with the closure (in $C(Q)$) of the degenerate elliptic operator A on $C^2(Q)$ given by $Af(x,y) = \frac{1}{2}x(1-x)f_{xx}(x,y) - xyf_{xy}(x,y) + \frac{1}{2}y(1-y)f_{yy}(x,y)$. Then for $f \in C^4(Q)$,

$E[f(X_n(t))|X_n(0) = (x_n, y_n)] - E[f(X(t))|X(0) = (x,y)] = O(\|f\|_{C^4(Q)}/n)$ as $n \to \infty$ uniformly for $(x,y) \in Q$ and $t \geq 0$, where $x_n = [2nx]/2n$, $y_n = [2ny]/2n$, and $\|f\|_{C^m(Q)} = \Sigma_{\substack{i+j \leq m \\ i,j \geq 0}} \sup_{(x,y)\in Q} |D_x^i D_y^j f(x,y)|$. Weak convergence follows from this. Remark: $Y_n(\cdot)$ is Wright's model for random genetic drift at a tri-allelic locus in a monoecious diploid population of size n. Analogous results hold for one or two multi-allelic loci with the effects of mutation, migration, selection, and recombination taken into account. (Received November 6, 1974.)

*720-60-32 T. SMITH and C.P. TSOKOS, University of South Florida, Tampa, Florida 33620. On Investigation of Sequential Testing Procedures for Comparison of Stochastic Branching Processes. Preliminary Report.

It has been previously suggested the implementation of an inexact form of Wald's sequential probability test (Girshick's procedure) for comparing two branching stochastic processes characterized by a probabilistic law of an exponential class and whose trajectory values, $X_i(t)$, $i = 1, 2$, are sufficient statistics. When $X(t)$ is Markovian, stationary or ergodic or if the transition probabilities are stationary, then the exact test is developed and compared with the inexact test. A modified form of the test is given for use when $X(t)$ is not a sufficient statistic. (Received November 6, 1974.)

720-60-33 G. D. ALLEN, Texas A&M University, College Station, Texas 77843. A Generalization of a Theorem of Cramér.

Consider the stochastic process defined by the spectral representation $x(t) = \int_0^t g_1(t,u)d\, b_1(u) + \int_0^t g_2(t,u)d\, b_2(u)$, where $b_i(u)$, $i = 1,2$, are independent Weiner processes and $0 \leq t \leq 1$. If, for $i = 1,2$, the functions $g_i(t,u)$ satisfy the regularity conditions: (1) $g_i(t,u)$ is continuous for $0 \leq u \leq t \leq 1$; (2) $|g_i(t,u) - g_i(s,u)| < M\,|t-s|$ for $u \leq \min(s,t)$; and (3) $g(t,t) \neq 0$, it is shown that the spectral representation cannot be canonical in the sense of the Hida-Cramér theory. This result is a generalization of a theorem of Cramér. In his result (2) is replaced by (2)' $g(t,u) \in C_1$ for each u and $dg(t,u)/dt$ is $L_2(0 \leq u \leq t \leq 1)$. The techniques employed here involve establishing the existence of a linear operator T satisfying $x_1(t) = Tx_2(t)$. (Received November 6, 1974.)

*720-60-34 D. G. KOSTKA, Department of Mathematics, Texas A&M University, College Station, Texas, 77843, On The Erdös-Feller Criterion

Let $S_n = X_1 + \cdots X_n$ where $\{X_k\}_{k \geq 1}$ is a sequence of independent, identically distributed random variables with mean zero and variance one. Suppose $E(|X_k|^2(\lg^+\lg^+|X_k|^3) < \infty$ and for $\phi(n)$ positive and non-decreasing $\sum_{n=1}^{\infty} \frac{\phi(n)}{n} e^{-\phi^2(n)/2} < \infty$. Then we show that $\sum_{n=1}^{\infty} \frac{\phi^2(n)}{n} P(S_n \geq \sqrt{n}\,\phi(n)) < \infty$.

The proof relies on the Skorohod representation of S_n and a judicious use of Chebyshev's inequality. As with the Erdös-Feller criterion this result is used to find a sequence $\psi(n)$ such that $P(S_n > \psi(n)$ infinitely often$) = 0$. (Received November 6, 1974.)

62 Statistics

*720-62-1 GAINEFORD J. HALL, JR., The University of Texas, Austin, Texas 78712, Non-Stationary Stochastic Gold-Mining.

This paper treats an extension of gold-mining problems formulated by R. Bellman and J. Kadane. There are N gold mines, each mine i having an initial amount of gold $z_i \geq 0$. There is a gold-mining machine which can be used in only one mine per day. We assume that the fraction of gold mined on the jth excavation of mine i is a random variable, α_{ij}, and that $\{\alpha_{ij}\}_{j\geq 1}$ are independent processes for $1 \leq i \leq N$. The probability that the machine will break down and be irreparably damaged during the jth excavation of mine i is given by $1-\beta_{ij}(\alpha_{i1},\ldots,\alpha_{i,j-1})$ where β_{ij} are known functions. Kadane treated the case where $\{\alpha_{ij}\}_{j\geq 1}$ and $\{\beta_{ij}\}_{j\geq 1}$ are constant, $1 \leq i \leq N$. The problem is to find an optimal gold-mining policy to maximize the expected amount of gold excavated before breakdown of the machine. Conditions are given to guarantee the existence of an optimal policy which is the appropriate analog of the Bellman-Kadane rules proven to be optimal in the case where the r.v.'s are constant. (Received November 4, 1974.)

720-62-2 A.N.V. RAO and C.P. TSOKOS, University of South Florida, Tampa, Florida 33620. Bayesian Estimation of Life Parameters and Reliability Function in the Weibull Distribution, Preliminary Report

This paper develops a Bayesian analysis of the scale and shape parameters in the Weibull probability distribution and the corresponding reliability function with respect to the usual life-testing procedures. In the present analysis we consider both the scale and shape parameters to behave as random variables. The joint probability density function which characterizes their simultaneous behavior is being approximated by using generalized Laguerre polynomials. Previous works on this type of problem were based on independent characterization of the two parameters by assuming specific classical probability distributions for each. Bayesian estimates are also given through a conjunction of a Monte Carlo simulation and numerical-integration techniques. (Received November 6, 1974.)

*720-62-3 G.T. DUNCAN, Carnegie-Mellon University, Pittsburgh, PA 15213, and E.O. MILTON, University of California, Davis, CA 95616. Scoring Multiple-Answer Multiple-Choice Test Items: Bayes and Minimax Strategies,

A multiple-answer multiple-choice test item has a certain number of alternatives, any number of which might be keyed. The examinee is also allowed to mark any number of alternatives. This increased flexibility over the one keyed alternative case is useful in practice

but raises questions about appropriate scoring rules. In this article a certain class of scoring rules is considered and the concepts of proper scoring rules and equivalence among these scoring rules are introduced. The examinee's strategy with respect to a scoring rule is examined and the critical role of a quantity called the scoring ratio is emphasized. In the case of examinee uncertainty about the number of correct alternatives on the item, a Bayes and a minimax strategy for the examinee are developed. Also an appropriate response for the examiner to the minimax strategy is outlined. (Received November 6, 1974.)

720-62-4 D. S. TRACY and N. N. MIKHAIL, University of Windsor, Windsor, Ontario and C. E. COX DWYER, Simon Fraser University, Burnaby, B.C. The multivariate moments of weight 6 of moment-statistics for samples drawn from a finite population.

The work of Sukhatme (1943) for moments and product moments of moment-statistics in univariate finite sampling was extended by Mikhail (1970) for bivariate case and Tracy (1974) for multivariate case through weight 5. In this paper, we use Dwyer's (1972) and Skellam's (1949) techniques, using partitions, to derive all moments of weight 6 in the bivariate case. (Received November 6, 1974.) (Author introduced by Dr. F.W. Lemire.)

*720-62-5 ARJUN K. GUPTA, Dept. of Statistics, The University of Michigan, Ann Arbor, Michigan, 48104, On a Classification Model

A model for classification into one of two populations has been studied when multiple observations are available on the same variable for each individual. The distribution of the classification statistic, and exact and approximate probabilities of misclassification are investigated. (Received November 6, 1974.) (Author introduced by Professor J. Sethuraman.)

720-62-6 N.N. MIKHAIL and D.S. TRACY, University of Windsor, Windsor, Ontario, N9B 3P4. The sampling moments of the generalized variance for finite populations. Preliminary report.

For p-variate X distributed with mean vector μ and variance-covariance matrix Σ, we obtain the moments of the generalized variance for samples of size n, drawn from a finite population of size N, in terms of population generalized variance $|\Sigma|$. The first and second moments for p=2, and the first moment for p=3, have been derived using a partitional approach and a theorem on products of power sums (Tracy and Dwyer, 1973). Carver functions in $e_i^{-1} = N^{(i)}/n^{(i)}$ are made use of. The case for general p is under consideration. The moments will be useful for testing multivariate normality. (Received November 6, 1974.) (Author introduced by Dr. F.W. Lemire.)

65 Numerical Analysis

720-65-1 R. BRUCE KELLOGG, JAMES YORKE, Institute of Fluid Dynamics and Applied Mathematics, University of Maryland, College Park, Maryland 20783 and TIEN-YIEN LI, University of Utah, Salt Lake City, Utah 84112. A method of continuation for finding Brouwer fixed points.

Let D be a bounded convex domain in R^n, and let $F : \overline{D} \to \overline{D}$ be continuous. The Brouwer fixed theorem guarantees the existence of a fixed point, a point $x^0 \in \overline{D}$ such that $F(x^0) = x^0$. The broad

applicability of this theorem makes the associated numerical problem important, namely "give a general applicable algorithm for finding the fixed point." The first constructive method for finding such fixed points is due to H. Scarf in 1967. The method has been substantively improved over the recent years. In this paper we introduce a different method for computing a fixed point of a C^1 function F. Our algorithm is based on a different proof of the Brouwer fixed point theorem given by M. Hirsch and J.W. Milnor. In this proof one reduces the existence of a fixed point to the nonexistence of a retraction H of D onto ∂D. The retraction is shown not to exist by assuming that it does and studying the curves $H^{-1}(x^0)$, $x^0 \in \partial D$. To formulate our algorithm, we construct a retraction $H : D \backslash C \to \partial D$, where C is the set of fixed points. We then show that the curves $H^{-1}(x^0)$ lead from a point $x^0 \in \partial D$ to the set C. The algorithm involves following these curves. The numerical results are also given. (Received September 30, 1974.)

720-65-2 JOHN R. CANNON, University of Texas at Austin, Austin, Texas 78712 and RICHARD E. EWING, Oakland University, Rochester, Michigan 48063. A direct numerical procedure for the Cauchy problem for the heat equation. Preliminary report.

For the Cauchy problem, $u_t = u_{xx}$, $0 < x < 1$, $0 < t \leq T$, $u(0,t) = f(t)$, $0 < t \leq T$, $u_x(0,t) = g(t)$, $0 < t \leq T$, a direct numerical procedure involving the elementary solution of $v_t = v_{xx}$, $0 < x$, $0 < t \leq T$, $v_x(0,t) = g(t)$, $0 < t \leq T$, $v(x,0) = 0$, $0 < x$, and a Taylor's series computed from $f(t) - v(0,t)$ is studied. Continuous dependence better than any power of logarithmic is obtained. Some numerical results are presented.

(Received October 29, 1974.)

720-65-3 PATRICIA S. WILLIS, American University and Computer Sciences Corporation, 2824 W. George Mason Road, Falls Church, Virginia 22042 Computer Symbolic Integration of Non-Linear Differential Equations : Progress Report

$A_n = D^n + a_1 D^{n-1} + \ldots + a_{n-1} D + a_n I$ for $n = 1,2,3,\ldots$, is a differential operator with entire function coefficients which operates on the space of entire functions; B_m is defined similarly. The relation $A_n B_m - B_m A_n = 0$ generates a coupled system of differential equations which may be solved for the coefficients of B_m in terms of the coefficients of A_n using integration and substitution. A computer program has been developed for doing symbolic algebraic manipulation on the system of equations. Results obtained by using the program on the cases $n = 2$, $m = 1,2,\ldots,7$, were presented at the International Congress of Mathematicians, Vancouver, B.C., 1974. Results of using the procedure on higher order differential operators will be presented. (Received November 6, 1974.)

720-65-4 JUNIOR STEIN, The University of Toledo, Toledo, Ohio 43606. Convergence rates for conjugate direction methods, Preliminary report.

Gram-Schmidt conjugate direction methods are used to minimize a real valued function of n real variables. By restarting every n steps, convergence is quadratic under appropriate conditions. This is verified computationally. The main idea is that one Newton step is roughly equivalent to n iterations using conjugate directions. (Received November 6, 1974.)

*720-65-5 JOHN JONES, JR., Air Force Institute of Technology, Dayton, Ohio 45431. Zeros of continuous real-valued functions. Preliminary report.

Let $f(x)$ be a solution of the selfadjoint d.e. $\mathcal{L}(f) = [r(x)f'(x)]' + p(x)f(x) = 0$ where $r(x) > 0$, $p(x)$ are continuous on $\alpha < x < \beta$. Let $v(x,h)$, $[r(x)v'(x,h)] \in C'[a,b]$, $[a,b] \subset (\alpha,\beta)$, $v(a,h) = v(b,h) = 0$. Theorem 1. If $\exists\ h > 0 \ni \int_a^{a+h} v(x,h)\mathcal{L}[v(x,h)]\,dx > 0$, then $f(x)$ has a zero on $(a, a+h)$. Let the trial function $v(x_1,x_2;h,k)$ be a solution of the elliptic equation $\mathcal{L}[v] = \sum_{i=1}^2 \sum_{j=1}^2 \frac{\partial}{\partial x_i}(a_{ij}\frac{\partial v}{\partial x_j}) + c(x_1,x_2)v(x_1,x_2;h,k) = 0\ \forall\ (x_1,x_2) \in R = \{(x_1,x_2): x_{10} < x_1 < x_{10} + h, x_{20} < x_2 < x_{20} + k\}$ and vanish on the boundary ∂R of R. Let $f(x_1,x_2)$ be a solution of the elliptic equation $\mathcal{M}[f] = \sum_{i=1}^2 \sum_{j=1}^2 \frac{\partial}{\partial x_i}(A_{ij}\frac{\partial f}{\partial x_j}) + C(x_1,x_2)f(x_1,x_2) = 0$ for $(x_1,x_2) \in \overline{R}$. Theorem 2. If $\exists\ h,k > 0 \ni$

$$\int_{x_{10}}^{x_{10}+h}\int_{x_{20}}^{x_{20}+k}\left\{\sum_{i=1}^{2}\sum_{j=1}^{2}(a_{ij} - A_{ij})\frac{\partial v}{\partial x_i}\frac{\partial v}{\partial x_j} + \left[C(x_1,x_2) - c(x_1,x_2)\right] v^2(x_1,x_2;h,k)\right\} dx_1\,dx_2 \geqq 0$$

then $f(x_1,x_2)$ has at least one zero in $\overline{R}$. (Received November 4, 1974.)

*720-65-6 John Gregory, Southern Illinois University, Carbondale, Illinois 62901. Numerical Solutions of Differential Equations. Preliminary Report.

Numerical algorithms are given for problems involving second order differential equations which are the Euler-Lagrange equations for a quadratic form. The quadratic form is approximated by finite dimensional quadratic forms which "preserve the negative and zero dimensions" of the original quadratic form. Focal points and Euler-Lagrange equations are derived for the finite dimensional quadratic forms; the resulting equations converge to the given differential equation in the space W_2^1. The results are the "final" step in an abstract theory of quadratic forms given by the author. Thus, they can be generalized to include the numerical solutions of $2n^{th}$ order, selfadjoint differential systems (including partial and integral-differential equations of Fredholm type) which occur in eigenvalue problems, oscillation problems, and other extremal problems. (Received November 4, 1974.)(Author introduced by Dr. George Parker.)

720-65-7 L.J. LARDY, Syracuse University, Syracuse, N.Y., 13210 and J.E. OSBORN, University of Maryland, College Park, MD., 20742, An example concerning the Ritz method. Preliminary Report.

Let H be a separable Hilbert space and let A be an operator on H which is densely defined, self-adjoint, and positive bounded below. If the Ritz method is used to obtain approximations x_n to the exact solution x^* of the equation $Ax = y$, then x_n converges to x^* in the energy space H_A provided the sequence of linearly independent coordinate functions belongs to and is complete in H_A. Lucka in [Ukrainian Math. J. 20(1968), 713-717] asserts that if the linearly independent coordinate functions are taken from the domain of A and are complete in H, then x_n converges to x^*. We show that this assertion is not valid. (Received November 4, 1974.)

720-65-8 GEORGE F. VOTRUBA and ROBERT P. BANAUGH, University of Montana, Missoula, MT 59801. A quadratically convergent iterative procedure for solutions of equations of the second kind. Preliminary report.

We consider the equation $(I - \lambda K)x = y$ in a Banach space, where K is bounded and linear, and we search for solutions in the form $x = (I + \lambda R)y$. A well known iterative scheme for matrix inversion suggests the use of the iteration $R_{n+1} = R_n + (I + \lambda R_n)C_n$ where $C_n = K - (I - \lambda K)R_n$. The convergence of the method depends upon the choice of the initial approximation R_0, and the method may converge even if $I - \lambda K$ is not invertible. Theorem 1. If the spectral radius of C_0 is $< 1/|\lambda|$, then $\{R_n\}$ converges to some R_* where $I - \lambda R_*$ is a right inverse for $I - \lambda K$. Theorem 2. If, for the initial choice R_0, $\{R_n\}$ converges, say to R_*, and if $\{C_n(I - \lambda K)\}$ converges to 0, then $I + \lambda R_*$ is a generalized inverse of $I - \lambda K$. Methods for the selection of R_0 are discussed and error estimates are obtained. The results are applied to the case where K is compact and to Fredholm integral equations.
(Received November 5, 1974.)

*720-65-9 DIRAN SARAFYAN and CURTIS OUTLAW, University of New Orleans, Lakefront, New Orleans, Louisiana 70122. Approximate integration with built-in error bounds.

A three-point formula for numerical integration is presented which can be combined with Simpson's rule in one way to produce a highly accurate approximate integral, and in another way to yield a running error estimate, usually with upper and lower bounds for the error. The error estimate and bounds are obtained from simple combinations of the available ordinates; no evaluations of or bounds for derivatives are required. (Received November 1, 1974.)

720-65-10 JAMES V. BLOWERS, U.S. Air Force, AFATL (DLYA), Eglin AFB, Florida 32542. Constrained optimization of nonlinear functions without derivatives. Preliminary report.

In recent years several methods for converting constrained nonlinear optimization problems into unconstrained problems have been published. Some do not require evaluation of derivatives, e.g., Powell ("A method for nonlinear constraints in minimization problems," Optimization (Sympos., Univ. Keele, Keele, 1968), Academic Press, London, 1969, 283-298), Bandler and Charalambous (J. Optimization Theory Appl. 13(1974), 607-619), and Kowalik, Osborne, and Ryan (Operations Res. 17(1969), 973-983). Any of these methods will be completely derivativeless if combined with a nonderivative unconstrained optimization method such as that of Powell (Comput. J. 7(1964), 155-162). In this presentation the three constrained methods mentioned above combined with Powell's 1964 method are compared for a selection of test problems. (Received November 1, 1974.)

68 Computer Science

*720-68-1 KENNETH O. LELAND, Navy Personnel Research and Development Center, San Diego, California 92152. An interactive program for several algorithms for winding numbers.

Several definitions of winding numbers for closed polygonal paths in the plane are given as well as several algorithms for their computation. One of these has been discussed by this author in Abstract 73T-B177 in these NOTICES 20 (1973), A-434. An on-line interactive computer program has been written which allows one to use all of

the algorithms and to test their efficiency. Applications of winding numbers include complex function theory, the location of points in polygonal domains and locating minima of functions. (Received November 4, 1974.)

70 Mechanics of Particles and Systems

720-70-1 GERALD ROSKES, Queens College, Flushing, N.Y. 11367. On the Propagation of Energy in Multiphase Wavetrains. Preliminary report.

The motion of a broad class of slowly varying nonlinear dispersive waves can be shown to satisfy the equation $a_t = i\gamma a_{xx} + i\beta a^2 a^*$. In the unstable case $(\beta\gamma > 0)$, solitary envelope solutions can be found which show that energy can be transmitted over long distances in a concentrated packet of waves. For unstable multiphase wavetrains with overlapping group velocity projections, we derive conditions under which solitary envelope solutions can be found. The results indicate that multiphase waves consisting of non-energy trapping modes can be used to concentrate energy. A physical explanation is given.

*720-70-2 RAY F. SNIPES, Bowling Green State University, Bowling Green, Ohio 43403. The Rotational Gradient Operator. Preliminary report.

The total torque, about the center of mass of a rigid body, acting on that rigid body in a conservative force field is usually written in terms of cross-products of vectors. In this note, we introduce a rotational gradient operator associated with the Eulerian angles specifying the orientation of a rigid body. The total torque acting on the rigid body can then be expressed as the negative of the rotational gradient of the potential energy of the rigid body. We also introduce a rotational Laplacian operator and briefly indicate the properties of these operators as they "operate" on the (spherical) surface harmonics. Finally, a simple example is given showing an application to electrostatics. (Received November 1, 1974.)

*720-70-3 PETER J. MELVIN, LAS College, University of Illinois, Urbana, Illinois, 61801. On a New Analytical Method of Solution of Nonlinear Differential Equations: I. The Nonlinear Oscillator Equation. Preliminary report.

The equation of motion in a potential energy well $Y''=F(Y)$ is formally integrated by a combined power and Fourier series. A new notation is used to reduce the integration problem to the algebraic problem of the solution of two recursion relations in a finite form. Two general algorithms are obtained from the recursion relations for motions in asymmetrical and symmetrical, parabolic wells. A third algorithm is presented for the periodic solutions of a form of the Emden-Fowler equation. The computer versions of the algorithms for parabolic wells are checked against independent analytical solutions of the equation for time dependent radial motion in the Newtonian two-body problem and the equation of Blasius. The limitations of the

solutions to small to moderate amplitudes are found by the analytical-computer solution of the radial part of the orbital and scattering motions in a Lennard-Jones six-twelve potential. (Received November 4, 1974.) (Author introduced by Professor Paul T. Bateman.)

720-70-4 DONALD G. SAARI, Northwestern University, Evanston, Illinois 60201 On the asymptotic nature of the n-body problem.

Bounds on the growth properties of some random type behavior in the n-body problem are found. They permit a sharper picture of general evoling n-body systems. (Received November 4, 1974.)

76 Fluid Mechanics

*720-76-1 SHIH-LIANG WEN, Ohio University, Athens, Ohio 45701 and L.K. CHI, Sperry Univac Division, Sperry Rand, Cinnaminson, N.J. 08077. Higher order corrections to the boundary layer equations.

The boundary layer equations are usually derived from the Navier-Stokes equations. They can also be regarded as the zeroth order asymptotic solutions to the Bhatnagar-Gross-Krook model of the Boltzmann equation [Chi, Phys. Fluids, 8, 5, 991 (1965)] when the Chapman-Enskog development is coupled with the Prandtl boundary layer analysis. We obtain the first and second order asymptotic solutions which are higher order correction terms to the usual boundary layer equations. (Received October 3, 1974.)

*720-76-2 LOKENATH DEBNATH, University of Maryland, College Park, Maryland, and East Carolina University, Greenville, North Carolina 27834. On a Microcontinuum Model of Pulsatile Blood Flow.

Based on a microcontinuum model, an initial value investigation is made of the pulsatile blood flow through a rigid circular tube with entrance effects due to the impulsive action of an arbitrary as well as particular pressure gradient. The solutions for the unsteady velocity field as well as the microrotation velocity of the blood red cells are explicitly determined. The asymptotic behavior of the solutions is studied with physical significance. Special attention is given to the investigation of the entrance effects on the structure of the blood flow. It is shown that the entrance effects are reflected in the solutions of the problem through the terms containing the exponential factor $\exp(-\frac{zb\delta}{V})$, and the solutions are significantly different from those of the fully developed flow. It is also predicted that the present theory gives rise to a boundary layer phenomenon not present in the previous works in the microcirculation of blood. This new boundary layer is introduced by the influence of the entrance effects. This analysis reveals the generation and propagation of waves in the blood vessels traveling downstream with velocity $\frac{V}{b}$. These waves are primarily caused by the entrance effects and eventually decay within the boundary layer distance of the order $\frac{V}{b\delta}$. An exact as well as an approximate equation for the oscillatory inlet length is derived. (Received October 2, 1974.)

720-76-3 M. BALARAM, Grambling State University, Grambling, Louisiana 71245. Heat generated by MHD Couette flow with porous walls.

The simultaneous effects of mass injection and magnetic field on the heat generation due to viscous

dissipation is investigated analytically. The present findings are in general agreement with the temperature measurements of Chang-Yi Wang [J. Lubrication Technology, Trans. ASME 95(1973), 539-541]. Numerical results indicate larger reductions in maximum temperature and a significant increase in the heat take off by porous boundaries, if magnetic field is present. The most striking feature of this investigation is the marked improvement in the temperature reduction with the application of the electromagnetic force. (Received October 29, 1974.)

*720-76-4 DAVID H. HOITSMA, Wesleyan University, Middletown, Connecticut 06457. Existence and uniqueness of axisymmetric free boundary flows.

This paper gives a proof for the existence and uniqueness of certain steady axisymmetric flows with free boundaries. We consider the flow of an ideal fluid about an infinite cylinder of radius r $(r \geqq 0)$ when a small, nonconstant pressure disturbance acts on the free surface. The existence of a solution to the problem is established by using a general perturbation theorem on a nonlinear elliptic equation obtained by a change of variables. Uniqueness can be shown under more general conditions, and proofs are given when either a large pressure disturbance or an obstacle is present. (Received November 6, 1974.) (Author introduced by Professor J.D. Reid.)

78 Optics, Electromagnetic Theory

*720-78-1 JOHN I. BOBBITT, Purdue University, Hammond, Indiana 46323. The Asymptotic Expansion Near a Focus For Almost Plane Waves. Preliminary Report.

A technique is developed for obtaining an approximate differential equation valid near a focus or a caustic. The asymptotic expansion is based on a parameter, ε, measuring the thinness of the ray system. Higher order terms of both the phase and the amplitude are considered in order to verify the consistency of the technique. The solution to the equation is obtained and compared to the known solution.

The technique presented systematizes methods used by other authors to obtain analogous equations. The importance of the phase variable, in particular, is stressed. The technique as presented is easily generalized to nonlinear problems. (Received October 18, 1974.)

720-78-2 RICHARD C. MORGAN, St. John's University, Jamaica, New York 11439 and SAMUEL N. KARP, New York University, Courant Institute of Mathematical Sciences, New York, New York 10002. Multi-mode surface wave phenomena.

We show, by means of an explicitly solvable problem, use of a slightly singular solution to demonstrate certain aspects of surface wave propagation. The solution is required to be bounded. However, one of the first derivatives is allowed to be singular at a junction of the structure. The surface wave guidance action of the structure is modelled by a two-mode phenomenological boundary condition (multiple impedance condition). For simplicity of illustration a plane structure that is terminated by a perfectly conducting plane vertical to it is selected. The solution to the resulting mathematically formulated two-dimensional quarter plane problem is obtained exactly in an elementary fashion. Thus, it is possible to simply observe certain properties of surface wave propagation. The main result is that the "subsurface" power flow at the junction depends directly on the amplitude of the slightly singular solution, leading us to conjecture that such fields represent leakage or excitation through the junction. Additionally, we show that control over the form of the excited outward travelling surface waves may be exercised by inclusion of this term or by a slot imbedded in the vertical termination, i.e., mode conversion is possible. Finally, by simultaneously adjusting the termination and the slot, the structure can be made to be only cylindrically radiating. (Received November 5, 1974.)

81 Quantum Mechanics

:0-81-1 MICHAEL TORTORELLA and JAMES A. LEISE, U. Wisconsin - Milwaukee 53211
The Unitarity Equations for Matrices and the Problem of Determining the Scattering Amplitude from the Differential Cross-Section.

n this article we discuss the problem of finding a unitary matrix with pre-.cribed diagonal and each of whose elements has prescribed modulus. We show ıow this problem arises from the discretization of a nonlinear integral equation (the "generalized optical theorem", Newton, J Math Phys 9, 2050) arising in the inverse scattering problem for elastic scattering in quantum mechanics. Sufficient conditions for the solution of this problem for a symmetric circulant unitary matrix are given. (Such matrices arise, for example, in spherically symmetric scattering in two dimensions). We give several examples of nonuniqueness and a decomposition theorem for circulant matrices. This theorem is used to deduce necessary and sufficient conditions for solution of the problem for an 8×8 symmetric circulant unitary matrix, and necessary conditions for solution for $2^k \times 2^k$ such matrices. As an illustration, we give an example in which the prescribed modulus of each element is $n^{-1/2}$ and a connection of this example with the Gauss sums $G_n = \sum_{k=0}^{n-1} \exp(2\pi i k^2/n)$. Finally, two other equivalent formulations of the problem are presented.
(Received October 23, 1974.)

82 Statistical Physics, Structure of Matter

*720-82-1 WILLIAM GREENBERG, Virginia Polytechnic Institute & State University, Blacksburg, Virginia 24060. Spectral Analysis of the Multigroup Transport Operator.

The self-adjoint kernel of the multigroup linear transport equation is represented as a multiplication operator on L^2 by decomposing the Hilbert space into non-orthogonal cyclic subspaces. Then Hille-Yosida semigroup theory provides a solution of the inhomogeneous transport equation. An explicit representation of the isomorphism generated by the functional calculus is derived. The method generalizes the approach of R. Hangelbroek.
(Received November 4, 1974.)

83 Relativity

*720-83-1 RAINER K. SACHS, University of California, Berkeley, California 94720. General relativity and cosmology.

• A broad survey, for mathematicians, of current nonquantum general relativity and cosmology, with a few examples discussed in detail. Standard definition of a spacetime with remarks on: the unification of space, time and gravity; the multiple roles a Lorentzian metric plays in physics; time-orientability; paracompactness; etc. Basic cosmology: observed time scales; the microwave radiation; the hot Einstein deSitter model; matter now and in the early universe. Astrophysical versus mathematical cosmology. Lumps in the universe and the need for general theorems. A theorem of S. W. Hawking on geodesic incompleteness; its application to the early universe. The theorem is representative of various recent results on: the big bang; gravitational collapse; the evolution of black holes; and the structure of stationary black holes. Its assumptions will therefore be put into context, mathematically and physically, as follows.

1. Chronology theory is the study of $\{\exp P_q | q \in M\}$, where M is a spacetime, $\exp_q$ is the expon map and P_q is the set of past-timelike vectors in the domain of $\exp_q$. One gets two main global structures: the transitive chronology relation $\ll$ (a subset of $M \times M$; "$q \ll r$ iff q can signal to a speed less than the speed of light"); and the proper time function $t: \ll \to (0, \infty]$ (somewhat simila the distance function of Riemannian geometry). Technical restrictions on spacetimes are conditions how well $\ll$, t, and the spacetime topology T cooperate. In particular, M is defined as strongly causal iff the interval topology for $\ll$ is T ("even near-ancestorcide is impossible"). More space of current interest are strongly causal and Hawking's result assumes strong causality. 2. It also as restrictions, suggested by current observations, on the Ricci tensor of spacetime. Such conditions a related to: the local measurement of gravity by measuring relative accelerations ("tidal forces"), modelled by the Jacobi equation of geometry; the Einstein field equation; and the properties of stress-energy. Roughly: positive energy and mass $\to$ Ricci$(Y, Y) > 0$ $\forall$ timelike vector $Y \to$ gravity tends to p things together. If time permits, Hawking's theorem will be proved in detail and some similar theore will be stated (an expanded version of the talk is being prepared for the Bulletin). Concluding remarks mathematics versus physics. (Received October 18, 1974.)

*720-83-2 M. HUMI, J. A. LEBRITTON, Worcester Polytechnic Institute, Worcester Mass. 01609. Interior solutions to plane symmetric Einstein-Maxwell equations.

For zero pressure we obtain an interior solution to plane symmetric Einstein-Maxwell equations. For the general case we assume a functional relationship between the coefficients of the metric and show that the solution can be written in an integral form. (Received November 5, 1974.)

90 Economics, Operations Research, Programming, Games

720-90-1 JACK E. GRAVER, Syracuse University, Syracuse, New York 13210. On the Foundations of Integer Linear Programming I.

Every integer linear programming problem can be put in the following form: maximize $g \cdot x$ subject to $b \cdot x = \beta(b)$ for all $b \in R$, and to $b \cdot x \geq \beta(b)$ for all $b \in S$, where $x \in Z^n$ (the set of all n-dimensional integral vectors), where R and S are disjoint finite subsets of Z^n, and where $\beta: R \cup S \to Z$. The set D of vectors satisfying the above constraints is called the domain of the problem; the vector g, considered as a function from D into the reals by $x \to g \cdot x$, is called the gain function of the problem.

Let $V = \{x: b \cdot x = 0 \text{ for all } b \in R, x \in Z^n\}$. For $v, u \in V$, we say $v \geq u$ if $(b \cdot v)(b \cdot u) \geq 0$ and $|b \cdot v| \geq |b \cdot u|$ for all $b \in S$. Under the assumption that $R \cup S$ spans Z, which is not a very restrictive assumption, the relation $\geq$ is a partial order with 0 as unique minimum element. The minimal non-zero elements in this partial order are called indecomposable vectors of this problem. It is proved that the set of all indecomposable vectors is finite.

The main result is that the algorithm described below converges to a an optimal solution in a finite number of steps whenever g is bounded from above on D. Starting with a vector $x \in D$, search through I for a v such that $x + v \in D$ and $g \cdot v > 0$. If such a v is found, replace x by $x + v$, otherwise stop - x is optional. (Received October 30, 1974.)

*720-90-2 CHARLES J. LESKA, Syracuse University, Syracuse, New York 13210. Covering and Matching Problems as Integer Linear Programs.

Let $G = (V,E)$ be a connected graph. Let $F \subseteq E$ and $p \in V$, then $\rho_F(p)$ is the number of edges of F incident with p. Let b be a function, $b: V \to Z^+ \cup \{0\}$. A b-covering of G is a subset $C \subseteq E$ which satisfies $\rho_C(p) \geq b(p)$ for all $p \in V$. Let w be a function, $w: E \to Q^+ \cup \{0\}$. The weight of a b-covering $C \subseteq E$ is given by $\sum_{e \in C} w(e)$. A b-covering C^* is a minimal weighted b-covering if $w(C^*) = \min \{w(C): C \text{ is a b-covering of } G\}$. A subset $M \subseteq E$ is a b-matching if $\rho_M(p) \leq b(p)$ for all $p \in V$.

In this paper we are interested in finding:

1. a maximal weighted b-matching of G, and
2. a minimal weighted b-covering of G.

Considering these problems as integer linear programs, we use the theory presented by J. E. Graver in "The Foundations of Linear and Integer Programming I" to produce natural algorithms for these integer linear programs and for the special cases: $w(e) = 1$ for all $e \in E$ or $b(p) = 1$ for all $p \in V$. The algorithms for these matching problems are compared to known graph theoretic algorithms. (Received October 30, 1974.)

720-90-3 Issie Rabinovitch, University of Waterloo, Waterloo, Ontario N2L 3G1. Stable Assignments, Preliminary report.

Given a collection of n men and n women. Each individual ranks those of the opposite sex, with no ties allowed. A marriage assignment is just a bijective function from the set of men to the set of women. An assignment is unstable if there is a man and a woman who are not assigned to each other but prefer each other to their assigned partners. Gale and Shapley showed that there always exists at least one stable assignment.

In this paper the following questions are treated. Necessary and sufficient conditions for the existence of a unique stable assignment, bounds on the number of distinct stable assignments, the structure of the set of stable assignments relative to certain natural partial orders and binary relations, and convergence to stable assignments from arbitrary assignments under various criteria for partner swapping. The roommate problem is similar in structure to the above marriage problem and yet, as Gale and Shapley showed, does not in general have a stable solution. Sufficient conditions for the roommate problem to have stable assignments are given and many of the above questions are posed for the roommate problem and generalizations of it. (Received October 30, 1974.) (Author introduced by Professor H. Shank.)

720-90-4 JAN MYCIELSKI, Mathematics Department, University of Colorado 80302. Discrete theories of pursuit and evasion

This talk will be about (1) approximations of some differential games of pursuit and evasion by positional discrete games with perfect information, (2) discrete analogs of some theorems of differential game theory, (3) some new existential theorems on optimal strategies, and (4) open problems about pursuit and evasion in the circle. (Received November 4, 1974.)

720-90-5 Robert G. Underwood University of South Carolina Columbia, S. C.
A Functional Differential Equation Approach to Solving Infinite Games

This paper develops a theoretical foundation for the numerical solution of two classes of infinite zero sum games, namely continuous and L-infinity games. Our approach is to intro-

duce a dynamical model (a functional differential equation) for non-dynamic L-infinity games and then to show that approximate solutions to a symmetric L-infinity game can be obtained by examining the limiting behavior of the game's dynamical model. By viewing a continuous game as an L-infinity game, it is shown that exact solutions to a symmetric continuous game can be found by examining the limiting behavior of the corresponding L-infinity game dynamical model. Finally a symmetrization is described for continuous games, and thus, the theory provides a general method for solving games of this class. (Received November 4, 1974.) (Author introduced by Professor William C. Chewning.)

720-90-6 RICHARD K. GUY, The University of Calgary, Alberta, T2N 1N4.
Combinatorial game theory.

What is a game? Nim-like games and Sprague-Grundy theory for normal play in impartial games. Octal games. How to play several games at once in a dozen different ways; disjunctive, conjunctive and selective compounds. How to lose when you must; the misere game. Conway's Sylver Coinage. Numerous open questions will be mentioned. This to complement a talk of E.R. Berlekamp on unimpartial games and other aspects of combinatorial game theory. (Received November 4, 1974.)

720-90-7 EDWARD THORP, University of California, Irvine, California 92664
Recent results and open questions for some particular games, Preliminary report.

Blackjack: A simple formula for approximately evaluating an arbitrary point count system. Allows rapid estimates, relative merits of various count systems without lengthy calculations. Leads to discovery of new more nearly optimal systems. Also prove that a specified simulation procedure will determine approximate maximum theoretical advantage using all cards seen.

Backgammon: Approximate win probabilities for endgame from computer simulation. Recursive procedure for exact solution, endgame.

Baccarat and Chemin de Fer: Proof that for none of the many variants is there a card counting procedure which yields a significant advantage. Negates many recent papers. Proof uses Fund. Thm. Card Counting [Intl. J. Game Theory, 1973, pp. 109-119].

Also discuss Go, chess, bridge, Trente et Quarante, connections with economics (theories of risk, utility), computer applications. (Received November 4, 1974.)

720-90-8 URI N. PELED, Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Ontario N2L 3G1. Reducing the Number of Constraints in Covering Problems, Preliminary Report.

Given a covering problem in 0-1 variables, it is desired to represent the feasible set as the solution set of a small number of linear inequalities. An algorithm based on a variant of the dual simplex algorithm is presented. In particular, if the feasible set is the solution set of a single linear inequality, the algorithm always finds one; thus it solves the synthesis problem of threshold logic. (Received November 6, 1974.)

*720-90-9 P.L. HAMMER, E.L. JOHNSON and U.N. PELED, Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Ontario N2L 3G1. The Role of Master Polytopes in the Unit Cube.

Many 0-1 programming problems can be reduced to monotone problems in the same variables, i.e. ones whose set of feasible solutions is monotone. A class of special monotone sets, called master sets is introduced. To each monotone set S one can associate a family F of master sets S*, so that all the facets of (the convex hull of) S can be obtained (by means of a straight forward transformation) from those of any S*εF. (Received November 6, 1974.)

720-90-10 EMMETT B. KEELER, Rand Corporation, Santa Monica, California 90406 and JOEL H. SPENCER, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Backgammon doubling strategy.

Using a theoretic result on Markov processes the optimal doubling strategy in backgammon is determined. The technique is applied to a variety of other doubling rules. (Received November 27, 1974.)

92 Biology and Behavioral Sciences

720-92-1 DAVID A. SMITH, Duke University, Durham, North Carolina 27706. A model for perception of optical illusions. Preliminary report.

The Lie-theoretic model of Hoffman (SIAM Rev. 13 (1971), 169-184) is reformulated in terms of elementary differential equations and is shown to be based on the well-established empirical observation that the eye tends to see acute angles as slightly larger than they really are. Examples are given which show that Hoffman's model fails to explain adequately some illusions of angle. A modification is proposed which corrects these difficulties and resolves some ambiguities in the cited paper. The adequacy of the modified model is demonstrated by computer plots of its predicted apparent curves without the distortion patterns that give rise to the illusions. (Received October 15, 1974.)

*720-92-2 ALFRED INSELBERG, IBM Scientific Center, Los Angeles, California 90067. Mathematical model of a cochlea (inner ear) with variable geometry. The low frequency threshold.

A mathematical model of the cochlea is studied at low input frequencies. The cochlear shell is represented by a conical surface of revolution. Internally it is partitioned into two chambers (scalae) by a plane consisting of a wedge shaped basilar membrane—modelled as a plate—and its rigid supports. An opening in this plane—the helicotrema—at the narrow end of the cochlea (where the basilar membrane is the widest) allows the viscous and imcompressible fluid filling the two chambers to communicate. At the other end (where the cochlea is wide and the basilar membrane is narrow) the system is driven by periodic forcing. The equations of motion of the system consist of three linear partial differential equations with variable coefficients. Previous models of the cochlea utilizing uniform geometry showed good response at high frequencies but poor low frequency response. A previous conjecture that the opposing tapers of the cochlea and basilar membrane as well as the boundary conditions at the helicotrema are needed to provide good low frequency response is proved. Preliminary results obtained on a 3-chambered cochlear model (that is with the scalae tympani vestibuli and the cochlear duct) will also be presented. (Received November 1, 1974.)

*720-92-3 CHRIS RORRES, Drexel University, Philadelphia, PA and WYMAN FAIR, Drexel Univ. Optimal Sustainable Yield of a Renewable Resource Divided into Classes.

A renewable resource is divided into classes (animals according to age or stage, trees according to height, fish according to location, etc.) and the effect of harvesting different portions of different classes is investigated. The yield of the harvest is said to be sustainable if the distribution of the resource after each harvest remains unchanged. If a discrete linear deterministic model governs the growth of the resource between harvests, linear programming techniques will determine the optimal sustainable yield. The optimal harvesting policy is shown to be one in which each class is either completely harvested or not harvested at all, with possibly one exceptional class for which some fraction is harvested. (Received November 5, 1974.)

720-92-4 FRED SPRINGSTEEL, University of Montana, Missoula, Montana 59801 and PAUL BENNETT, Holyoke College, Holyoke, Massachusetts. Minimal recognition of biodevelopmental languages. Preliminary report.

With the definitions of biological (cellular) developmental languages of A. Lindenmayer (e.g., J. Theoretical Biology 30(1971), 455-484), one can compare the usual hierarchy of context-sensitive (CS), etc., languages and deduce recognition space bounds, possibly deterministic ones, for the various systems. Roughly, an i - L system ($i = 0,1,2$) is a linear array of cells where the next state of each cell is influenced by i of its neighbors. We obtain: Theorem 1. If M is a propagating 1 - L system, $L(M)$ is left CS. Theorem 2. A language L is CS iff $L = L(M)$ for some propagating 2 - L system M. Corollary. Minimal recognition space, on offline Turing machines, for 2 - L languages is a linear function of input length. Theorem 3. A deterministic, propagating 2 - L system requires linear storage only for the deterministic recognition of its generated language. Thus, linear bounded automata recognize such languages deterministically. (Received November 6,1974.) Author introduced by Stanley Grossman.)

*720-92-5 ROBERT SCHWABAUER, Virginia Commonwealth University, Richmond, Virginia 23284 Enzymatic neurons

By enzymatic neuron we mean the model neuron introduced by Michael Conrad (J. Theor. Biol. 46(1974) 167-188; see also feature article by Conrad in SIAM News, August 1974). Let p be a propositional function. Then p can be performed by an enzymatic neuron with k enzymes, where k is the number of conjunctions in the disjunctive normal form of p. The weighting coefficents can be taken as 1 or -1 and the activation parameter of each enzyme can be the number of non-negated variables in its corresponding conjunction.

The concepts of simplicity (Of the 2-ary functions, only "iff" and "exclusive or" require enzymes.) and evolvability (This leads to new problems in logic.) are discussed with examples of Conrad circuits, i.e. nets of McCulloch-Pitts and enzymatic neurons. In some instances, these concepts suggest the value hence biological possibility of dual (or conjunctive) enzymatic neurons. (Received October 30, 1974.)

93 Systems, Control

*720-93-1 N.U. AHMED, University of Ottawa, Ottawa, Canada, On the existence of Optimal control for systems governed by Ito stochastic differential and functional equations.

In this paper a critical and up to date review of the existence of optimal controls for systems governed by stochastic differential and functional differential equations of Ito type will be presented. Existence theorems for optimal controls with or without state (information) feedback will be discussed.

A new result on the existence theorem for optimal control of partially observed diffusion, with control constraint set a variable compact convex subset of a metric space, will be presented. The proof of the theorem, based on Martingale theory and some recent selection theorems in metric spaces, will be briefly outlined.

A number of unresolved problems will be discussed.

(Received October 9, 1974.) (Author introduced by Professor A.T. Bharucha-Reid.)

*720-93-2 ROBERT M. GOOR, University of Kentucky, Lexington, Kentucky 40506
Existence of an Optimal Control for Systems with Jump Markov Disturbances, Preliminary Report

Consider the optimal control of stochastic systems in the form $x = f(t, r(t), x, u)$, where $r(t)$ is a countable state Markov process and the control u is to be chosen from a suitable class $\mathfrak{B}$ of uniformly bounded, non-anticipative functions taking values in a fixed set U. The performance criterion to be minimized is $J(u) = E\{\tau \mid x(t_0) = x_0,\ r(t_0) = i_0\}$, where τ is the smaller of $\bar{\tau}$, the first time $x(t)$ reaches a compact target set M, and T, a fixed terminal time. If the function f is uniformly bounded and $f(t, r, x, U)$ is closed and convex for each (t, r, x); if suitable closure properties hold for the space constraint set, then there exists an optimal control u in $\mathfrak{B}$ for $J(u)$. (Received November 4, 1974.)

*720-93-3 Professor William C. Chewning, University of South Carolina, Columbia, S.C.29208
Null Controllability For Parabolic Non-Linear Equations

Consider the parabolic boundary control problem $(0 \le x \le 1)$

(1) $u_t(x,t) = u_{xx}(x,t) + f(u);\ u(x,0) \in L^2[0,1];\ u(0,t) = 0;\ u(1,t) = c(t),\ 0 \le t \le T.$

We say that (1) is locally approximately null controllable if for each positive integer N, there is a constant $K(N) > 0$ such that if $||u(x,0)|| \le K(N)$, then there is a smooth control $c(t)$ for which (1) has a solution $u^c(x,t)$ with $< u^c(x,T), \sin n\pi x > = 0$ for $n = 1,2,\ldots,N$.

The proof that (1) is locally approximately null controllable if f satisfies mild assumptions will be presented. Some numerical examples and the extension of (1) to more general equations and more space variables also will be discussed.

(Received November 4, 1974.)

*720-94-1 HENRY G. GORDON, Allentown College of St. Francis DeSales, Center Valley, Pa. 18034. Complete Degrees of Finite-State Transformability

Let (Q,Σ,δ,ω) be a finite state automaton with input-output alphabet Σ. Let Σ^+ be the free semigroup generated by Σ, and let Σ^N be the set of infinite sequences of elements of Σ. G. Rayna, in "Degrees of Finite-State Transformability", Information and Control (1974), calls a sequence $x \in \Sigma^N$ complete if and only if every $A \in \Sigma^+$ is contained in x. Rayna also defines a partial ordering on the class of degrees of finite-state transformability. In this paper we present several results concerning this partial ordering as it pertains to the complete degrees of finite-state transformability, i.e., degrees generated by a complete sequence.

(Received November 1, 1974.)

720-94-2 PAUL A. WILLIS, Teledyne Geotech Laboratories, P.O.Box 334, Alexandria, Virginia 22313 Distortion Mappings III : Progress Report

An algorithm for recovering an original discrete lattice spectrum from its N-aliased sampled version was presented at the International Congress of Mathematicians, Vancouver, B.C., 1974. (A spectrum of width W cycles-per-second is N-aliased when the corresponding time series is sampled at a rate of R samples-per-second, where 2W/(N+1) .LT. R .LE. 2W/N.) This algorithm is applied to detect and estimate the component frequency locations of a narrow-bandwidth discrete lattice signal spectrum imbeded in a wide-bandwidth signal space. (Received November 6, 1974.)

98 Mathematical Education, Collegiate

*720-98-1 JAMES D. FABREY, University of North Carolina, Chapel Hill, N.C. 27514. Integration of Polynomials without Evaluation of Antiderivatives.

Various such methods have been proposed (for example, Abstract 74T-B14, these Notices 21 (1974), A-9). We present yet an easier proof which avoids mathematical induction and requires only the first two terms of the binomial expansion. The problem easily reduces to the calculation of $I = \int_0^1 x^r dx$. If we set $J = \int_0^1 x^{1/r} dx$, then $I + J = 1$ (the area of the unit square). If we select partition points of the form i/n for I and $(i/n)^r$ for J, then, without evaluating either integral directly, we readily obtain a second equation $J = rI$. Solving the two equations for I and J, we obtain $I = 1/(r + 1)$, the desired result.

(Received October 21, 1974.)

*720-98-2 WILLIAM E. HARTNETT, State University College, Plattsburgh, New York 12901. The Mathematization of Analysis.

There is at present and will be for the future a strong need for economical ways to learn to do Mathematics - economical, that is, for the learner. One possible aid to economical learning would be (in lecturing and writing) to "make things which are the same look the same". In particular, this would mean that functions, wherever and whenever they

appear, should be recognizable as such. Analysis has traditionally done much to obscure what is really happening in Analysis; most of what is done in elementary Analysis involving functions doesn't even resemble what is done in other parts of Mathematics. Functions which do appear don't look like functions, say, that the learner encounters in Algebra or Geometry and the most interesting functions are never even identified as functions - indeed, they are carefully disguised. Examples are given of the kind of thing which might improve the situation and hence lead to economical learning. One might hope that the ubiquitous "new Ph.D.'s" might help cure our long-standing functional schizophrenia. (Received November 4, 1974.)

720-98-3 THERON ROCKHILL, State University of New York, College at Brockport, Brockport, New York 14420. Media assisted self study lessons in calculus.

We discuss the development and evaluation of eleven media assisted self study lessons in beginning calculus. Each lesson is designed to replace one fifty minute calculus lecture and consists of a super-8 film cassette synchronized with an audio cassette. The delivery system enables the student to stop a lesson at any time to review segments of the lesson or to work the exercises in the accompanying lesson guide. Upon completion of each lesson the student provides a brief evaluation of the lesson and the delivery system. The paper summarizes these evaluations and the overall student evaluation of media assisted self study lessons in calculus. (Received November 4, 1974.) (Author introduced by Dr. E.O. Stephany.)

720-98-4 LEE D. THOMPSON, Loyola University, Chicago, Illinois 60626. Method for teaching introduction to computer science. Preliminary report.

An introductory course in computer science has been taught using, in part, a technique which might be called PROGRAMMING: A DISCOVERY APPROACH. Students are given a series of source language program listings to keypunch and submit to the computer. Output from each program includes a list of questions produced by a call to a stored subroutine. Example: From the output X = .5; J = 0; K = 0; L = 1 produced by instructions X = 1./2.; J = 1/2; K = 1/2*2; L = 2*1/2 the student is led by the questions to conclude that there are different types of numbers from the computer's view and integer arithmetic is not commutative. Curiosity about results produces class discussions leading to language independent issues concerning the computer and its operation. Advantages and disadvantages of the method will be discussed. (Received November 4, 1974.)

*720-98-5 SISTER RITA EHRMANN, Villanova University, Villanova, Pennsylvania 19085. Minimal surfaces rediscovered.

The present"paper" is a ten minute 16mm. mathematical movie at the collegiate level. The subject is minimal surfaces and minimal linkages -- the solution of Plateau's and Steiner's problems. The soap films are captured rather clearly on black-and-white film, and the experiments are accompanied by expository narration. The paper indicated as available on request is of the nature of a teacher's manual for the movie. This manual elaborates some of the mathematical aspects of the film and suggests (and solves) several classroom problems. The latter include a compass and straight edge construction of a "Steiner point", computation of the minimal spanning surface of a tetrahedral framework, and a calculus minimization of the linkage for the vertices of a square. (Received November 5, 1974.)

99 None of the Above

720-99-1 SAUL STAHL, Western Michigan University, Kalamazoo, Michigan 49001. Self-dual embeddings of graphs. Preliminary report.

All embeddings discussed here are 2-cell embeddings on surfaces (2-manifolds). An embedding of a graph G is said to be self-dual if G is isomorphic to its dual on that surface. By an embedding of a group on a surface is meant an embedding of some Cayley graph of that group on that surface. Finally, the regular complete m-partite graph of order mn is denoted by $K_{m(n)}$. Theorem 1: $K_{m(n)}$ has a self-dual embedding on an orientable surface provided that both m and n-1 are divisible by 4. Theorem 2: $K_{m(n)}$ has a self-dual embedding on a non-orientable surface if any of the following four conditions holds, a) $n\equiv 0 \pmod 4$ and $m>2$; b) $n\equiv 2 \pmod 4$ and $m\not\equiv 2 \pmod 4$; c) n is odd, $m\not\equiv 2 \pmod 4$, m is not a power of 2, and $K_{m(n)}$ is not K_3, K_5, K_{12}, or $K_{3(3)}$; d) $n=1$, and $m\equiv 0 \pmod 4$. Theorem 3: Every finitely generated abelian group, except Z_2 and Z_3, has a self-dual embedding on an orientable surface. Theorem 4: Every finitely generated abelian group of order at least 6 has a self-dual embedding on a non-orientable surface. (Received October 17, 1974.)

720-99-2 R. HECHT-NIELSEN and A. P. WANG, Arizona State University, Tempe, AZ 85281 Transport Operators

Let $D=\{(x,y)\varepsilon R^2 | x<y\}$ and B be a Banach Space. A Transportor is a continuous map $S:D\to L(B)$ that satisfys $S(x,x)=I, S(x,y)*S(y,z)=S(x,z)$. Given a transportor S define $\tau=S(D)\cup\{[S(x,y)]^{*-1} | (x,y)\varepsilon D\}$. Then τ is a weakly associative local topological group. Also, S satisfys a Riccati operator differential equation. A formula for solving this equation is developed by means of a $*$-integral. These results connect the local physical properties of a system with the bulk response of the system. The formulation makes the results pertinent to Radiative Transfer, Neutron Transport, and Wave Propagation.

(Received October 21, 1974.)

*720-99-3 **JOHN ROBERTS, Western Michigan University, Kalamazoo, Michigan, 49001. Vertex Cyclic Graphs.**

A graph G having only cyclic blocks is vertex cyclic if it has at least one vertex which belongs to every cycle of G. This generalizes the notion of a graph being randomly eulerian from a vertex. In addition to showing that many properties of randomly eulerian graphs hold in this more general setting, it is also shown that the center of a vertex cyclic graph induces a connected graph.

(Received October 21, 1974.)

*720-99-4 T. J. LANOUE and V. L. BOAZ, Westinghouse Electric Corp., MIR M. ALI, Ball State University, Muncie, Indiana 47305. A Mathematical Method For Approximating the Potential and Gradient At Any Point Between An Insulated Cable to Plane.

An approximate mathematical model is developed for the determination of the potential and gradient at any point between an insulated cylinder to plane. The solution is based on an approximated boundary system of the exact system so that uniform field boundary conditions can be used after an analytic transformation of the modified boundaries. This method assumes that the interface of the two insulating mediums lie on an equipotential contour. With this assumption, only one analytic transformation is necessary for obtaining the solution to this problem. The accuracy of this method is checked by comparing the results to an electrostatic analog field plot, since an exact solution to the problem is not available. The comparison shows that the derived mathematical model can be used as a reliable means for determining minimum spacing between pairs of high voltage leads and high voltage leads to grounded planes in large high voltage transformers. (Received October 30, 1974.) (Authors introduced by Dr. T.K. Puttaswamy.)

*720-99-5 H. W. Gould, West Virginia University, Morgantown, W. Va. 26506. Limiting and asymptotic behavior of the Bondurant-Hoskold formulas for valuation of coal mine parcels for equitable tax assessment. Preliminary Report.

Donald M. Bondurant (unpublished engineering of mines thesis, W. Va. Univ.) has defined the factors $H(i_p,i_s,n) = 1/(i_p + i_s/((1 + i_s)^n - 1))$ (Hoskold factor), $A(i_p,i_s,k,n) = (1 + i_p)^{-k}H(i_p,i_s,n-k)$, $B(i_p,i_s,c_t,k,n) = c_t(1 + c_t)^k A(i_p,i_s,k,n)$, n = number of years, $A_n = \sum_{k=0}^{n-1} A(i_p,i_s,k,n)$, $B_n = \sum_{k=0}^{n-1} B(i_p,i_s,c_t,k,n)$, $C_n = \sum_{k=0}^{n-1}(k+1)B(i_p,i_s,c_t,k,n)$, $D_n = \sum_{m=1}^{n-1} \sum_{k=1}^{m} c_t(1+c_t)^{k-1}A(i_p,i_s,m,n)$, $E_n = C_n + D_n$, where i_p, i_s, c_t are certain investment, sinking fund, and production interest rates relative to valuation of coal mine parcels. The Hoskold factor H was introduced in 1877 by Henry D. Hoskold and has been a standard method of valuation ever since. In this paper we make an analysis of the behavior of A_n, B_n, C_n, D_n, E_n, for general values of the parameters involved and obtain limiting and asymptotic values as $n \to \infty$. For example, it is proved that: $A_\infty = (1 + i_p)/(i_p)^2$; $B_\infty = c_t(1 + i_p)/i_p(i_p - c_t)$ if $0 < c_t < i_p$; $B_n \sim n$ as $n \to \infty$ if $c_t = i_p$; $B_n = O(z^{-n})$ as $n \to \infty$ if $c_t > i_p > 0$ and where $z = (1 + i_p)/(1 + c_t)$ so that $0 < z < 1$; $C_\infty = c_t(1 + i_p)^2/i_p(i_p - c_t)^2$ if $0 < c_t < i_p$; etc. The results are useful and the series are a kind of generalized Lambert series requiring great care to estimate for practical applications.
(Received November 1, 1974.)

720-99-6 SAUNDERS MAC LANE, University of Chicago, Chicago, Illinois 60637. Topology and logic as sources of algebraic ideas.

• Research interests in algebra which depend upon suggestions and problems in geometry, topology and mathematical logic are summarized. Such dependence emphasizes the sense in which mathematics is a web of ideas with many surprising connections between fields. Several specific examples will be discussed. The first, dealing with independence relations, arose from geometric ideas of linear dependence in a vector space, particularly from work of Hassler Whitney, which greatly assisted the study of arithmetic questions concerning fields: The so-called inseparable extensions of prime characteristic p used a geometrically inspired idea of p-dependence which led to the notion of a separable extension of fields which was later

applied to algebraic geometry. The second example concerns cohomology of groups, arising from arithmetic work on fields with Schilling and topological work with Eilenberg, which has applications in topology and number theory. Closely related is the geometric idea of splitting a topological space into component pieces each with one homotopy group (Eilenberg-Mac Lane spaces). These pieces demanded algebraic analysis, however, which led to many new algebraic constructions, e.g., bar construction and divided powers introduced by Cartan. Finally the rise of the concepts of category theory from geometric questions is described — specifically why certain formulas for homology groups in topology involve isomorphisms which are "natural". Specific recent theorems about coherence in categories, which assert that all diagrams of certain types commute, are also useful in the study of infinite loop spaces in topology. (Received November 21, 1974.)

Symposium on Some Mathematical Questions in Biology
New York, New York, January 29 – 30, 1975

Bio 75-1 H. G. OTHMER, Rutgers University, New Brunswick, New Jersey 08903
Nonlinear Waves in Reacting Systems.

It has frequently been suggested that propagating chemical waves can serve as a means of communication and control in developing biological systems. We consider systems described by the scalar equation $u_t + f(u)u_x = Du_{xx} + g(u)$ where u is chemical concentration, $f(u)u_x$ describes active or convective transport, Du_{xx} diffusive transport and $g(u)$ reaction and exchange. When $f(u) = 0$ it is known that transition (change of state) waves exist for appropriate $g(u)$. We show that if $f(u) \neq 0$ both periodic and solitary waves also exist for a large class of $f(u)$. The asymptotic stability (or instability) of these waves and their significance in the Cauchy problem is as yet undetermined. (Received November 1, 1974.)
(Author introduced by Professor K. Wolfson.)

*Bio 75-2 F. ALBERTO GRUNBAUM, University of California, Berkeley, California 94720
The phase problem in crystallography

A finite number of unknown masses m_k are located at the unknown positions r_k in R^3. The problem is that of determining the masses and the positions from the modulus of the Fourier transform of this measure in R^3. As such the problem has a nonunique solution. We show that if the r_k are taken as independent Gaussian vectors with unknown means $\langle r_k \rangle$, $\sigma_k^2 \equiv 1$, then both the m_k's and $\langle r_k \rangle$'s can be determined in an essentially unique fashion. This is a very simple model of the effect of random thermal motion of the atoms on the diffraction pattern they produce. (Received October 31, 1974.)(Author introduced by Professor Jacob Feldman.)

*Bio 75-3 D.J. SIMANAITIS and WILLIAM P. MACLEAN, College of the Virgin Islands, St. Thomas, USVI 00801. Predator/prey dynamics with age distributions. Preliminary Report.

Two-species predator/prey dynamics are studied in a model which reflects varying age distributions of each species. The mechanism makes use of Leslie population matrices superimposed on interactive landscapes of the sort considered by Rosenzweig and MacArthur. Simulations yield equilibria, extinctions, and the limit cycle behavior suggested by May. The approach appears amenable to modeling interactions of more than two species without affecting accessibility. (Received October 23, 1974.)

Bio 75-4 ROBERT ARMSTRONG, RICHARD McGEHEE, and JOHN ZICARELLI, University of Minnesota, Minneapolis 55455. Some mathematical aspects of the ecological principle of competitive exclusion. Preliminary report.

A model proposed by S. Levin of n species competing for p limiting factors is studied. Certain linearity assumptions allow one to conclude that exclusion must occur if $p < n$. However, it can be shown that exclusion need not occur if the linearity assumptions are relaxed. This limiting factors model is related to certain predator-prey models. It is shown that n predators can coexist on fewer than n prey. (Received October 30, 1974.)

*Bio 75-5 GANGARAM S. LADDE, State University of New York, College at Potsdam Potsdam, New York 13676, Stability of Model Ecosystems with Time-Delay, Preliminary Report.

In this work, we attempt to formalize and partially resolve the "time-delay vs. stability as well as "complexity vs. stability"problems in model ecosystems, in the framework of Lyapunov's stability theory. Sufficient conditions are given for stability of models with time-lag under structural perturbations caused by nonlinear interactions among species in the community. As a byproduct of this analysis, we will show important structural properties of the density-dependent models with time-delay, and establish tolerance of community stability to a broad class of nonlinear perturbations and to a class of time-delays. (Received October 18, 1974.)

Bio 75-6 P.T. SAUNDERS and M.J.BAZIN, Queen Elizabeth College, London W8 7AH, England. Oscillations in stable ecosystems.

It is commonly held that the observed absence of large-scale population oscillations in tropical ecosystems is due to their complexity. A simple analytical argument indicates that the degree of complexity may have relatively little effect on the rate at which the oscillations of a stable system damp, and this is borne out by the results of computer simulations. A possible alternative explanation is suggested by the analysis of a general n-species food chain: it is shown that under conditions of great enrichment such a system will, providing that it remains stable, pass to an overdamped phase in which any oscillations rapidly die out. (Received November 4, 1974.)
(Authors introduced by D. A. Brannan.)

*Bio 75-7 BENNY LEVIKSON, Purdue University, West Lafayette, IN 47907 Random selection forces in infinite diploid populations.

We study the effects of varying selection intensities on the evolutionary process of gene frequencies in infinite diploid populations. Thus we re-introduce a discrete time model having random selection forces, obtain its limiting diffusion and analyze its properties. Conditions for fixation, quasi-fixation, convergence to a polymorphic point or to a stationary

polymorphic distribution are given in terms of the means and variances of the selection coefficients. Comparing our results (for the diploid case) with those of the haploid case shows some important differences. For example while quasi-fixation in the haploid case implies recurrency of the gene frequency process this is not so in the diploid case. (Received October 10, 1974.)

Bio 75-8 J. MYRON HOOD, Occidental College, Los Angeles, Ca. 90041 and LEROY HOOD, Caltech Pasadena, Ca.91109. A Mathematical Model of Immunoglobulin Evolution.

One important area in the study of immunology centers around determining the pattern of evolution of immunoglobulin polypeptides. Such studies are greatly enhanced by the availability of immunoglobulins from animals representing many stages of the evolutionary spectrum. Our study compares the immunoglobulin polypeptides from two inbred strains of mice (NZB and BALB/c) whose evolutionary paths separated only a few hundred years ago, as well as those of man and mouse which have been separated for at least 75 million years. The study of immunoglobulin evolution of these species is based on an analysis of the nature and extent of diversity in the amino acid sequences of their polypeptide chains. In order to determine how sets of polypeptides vary, we have constructed a mathematical model that assigns to each chain, a matrix based on which of 20 amino acids are present in each of the 23 N-terminal residues. Next we define a diversity measure on sets of these matrices. By calculating the index of diversity of different sets, we can analyze the degrees of relatedness of either different populations of polypeptides occurring within a single species or populations from different species. Preliminary evidence from our model suggests that immunoglobulin evolution proceeds very rapidly.
(Received November 4, 1974.)

Symposium on Theory vs. Practice in the Finite Element Method New York, New York, January 31, 1975

(1) GILBERT STRANG, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Can mathematics be useful?

The finite element method has been a tremendous success in engineering calculations, opening up the possibility of solving more complicated problems with much greater accuracy and speed then ever before. But, from the viewpoint of contributions by mathematicians, the story is in part one of missed opportunities. The basic idea was originally presented by a mathematician, Richard Courant, in 1941—and, as far as one can see, it was immediately forgotten. It was only after a group of structural engineers had reinvented and developed the method that its coincidence with Courant's idea was recognized. The question now is: What contributions can mathematicians make to the extension of the idea to new classes of problems—the safety of nuclear engineering designs, and a whole range of other applications? I plan to discuss this question with the other panelists and with the audience. The finite element method has provided an unusual opportunity to numerical analysts, and I hope we will be able to accept it.

(2) ROBERT L. TAYLOR, Department of Civil Engineering, University of California, Berkeley, California 94720. The impact of the finite element method on structural engineering.

Although the origins of the finite element method can be traced to Courant (1943), in Structural

Engineering it was independently developed and published by Turner et al (1956) and Argyris (1955). This early work was published under the titles of discrete element and matrix displacement methods; the earliest mention of the term "Finite Element" can be found in a paper by Clough (1960). During the 1960's the method expanded rapidly in the field of structural engineering and many computer programs based upon the method were made available to practicing structural engineers. Analyses on many structural systems ranging from gravity dams to solid propellant rocket systems were performed necessitating further developments for transient heat conduction, flow in porous media, incompressible elasticity, etc., including both transient and nonlinear effects. As the finite element method nears its twentieth anniversary in structural engineering, its use in analysis has become routine. Its impact has been profound and much of the required analysis for certification of structures safe against earthquakes, blast, wind, as well as other common dead and live loads can be traced to the growth of the finite element method.

(3) TODD DUPONT, University of Chicago, Chicago, Illinois 60637. Modeling wave propagation with finite element methods.

Finite element methods, developed by structural engineers in response to the necessity of being able to treat problems with complex geometry, are now being intensively studied by numerical analysts. One of the effects of the interest of mathematicians in these methods has been the acceleration of their successful application in many fields other than structural mechanics. These applications, in turn, stimulate questions that would not be asked if one merely considered modeling structures. I shall discuss some of the uses of finite element methods to problems that can be loosely grouped under the heading of wave propagation. In particular, I shall try to compare several finite element schemes for use on first order hyperbolic equations and diffusion-convection equations. For some of these problems, one factor in the choice of a particular method is that increasing the freedom in the trial space (without making it nonconforming) can actually increase the error; this is something that one would not expect in the context of structural engineering.

(4) RIDGWAY SCOTT, University of Chicago, Chicago, Illinois 60637. The interplay between the mathematical and engineering approaches.

Mathematical questions about the finite element method began to be studied in earnest in the late 1960's. By then, there was already a considerable body of successful techniques (developed by structural engineers) that had no theoretical basis. The study of these techniques by numerical analysts has yielded new mathematical insights in addition to making the engineering ideas mathematically precise. The mathematical theory now encompasses isoparametric elements, nonconforming elements, the use of numerical integration, mixed methods, hybrid methods, the facet approximation of a shell, and more. Rather than try to describe all of these ideas, we will illustrate how mathematics and engineering have interacted in these problems by describing in detail the evolution of the displacement model of the bending problem for a simply supported plate with curved boundary. This is a typical example of how engineering ideas led to significant mathematics that we hope, in turn, will have a productive influence on engineering thinking.

(5) ROBERT E. NICKELL, Applied Mechanics Division, Sandia Laboratories, Albuquerque, New Mexico. Computer program construction and maintenance—the future of centralized finite element activity.

Finite element software that was developed ten years ago was often ancillary to other concerns in physical and mathematical research. Little effort was ever given to writing programs that could be used

easily by others; instead, the method of operation was geared toward the training of finite element specialists—each with at least some ability to generate functional programs. Later in the decade, entrepreneurial forces began to presage a new format for program development, generally built around a temporary and often accidental collection of gifted programmer-analysts. The fundamental demarcation of these later groupings from earlier workers, however, was the ultimate aim of producing software for analysts to use who were not specialists in finite element methodology. The initial investment in the computational tools and adjunct equipment can be a deterrent to the development or acquisition of finite element software by companies and organizations with relatively small analysis staffs. However, the vast market potential represented by such firms collectively has not gone unnoticed and a number of hardware suppliers and software suppliers have formed loose alliances to serve these customers. While the development cost of a general purpose program may be about $500,000 and be prohibitively expensive to the small user, a surcharge of 15-20 percent on computer time for the same program used in a service bureau environment may only cost the small user a few hundred dollars per month, well within economic reason. Since the majority of the general purpose programs available today (and the special purpose programs as well) were developed, at least initially, at government (taxpayer) expense—through the subsidy of contract research for programmer support—there is a growing movement that seeks to establish a nationally-based software activity. In this presentation I will try to indicate the various steps that are taken during the inception and development of a general purpose finite element program, with some examples of portability constraints and modular construction; then, the marketing principles that currently apply will be examined in order to test the climate for establishment of a centralized software activity of the type described above.

ERRATA
Volume 21

JEFFREY B. REMMEL, Co-r.e. cohesive vector spaces, Abstract 74T-E90, Page A-595.

Line 6, for "$cl(W \cup S) \subseteq V$" read "$cl(W \cup S) \supseteq V$".

Line 7, for "generates a finite dimensional subspace" read "contains no infinite dimensional subspace".

FREDRIC T. HOWARD, Factors and roots of the van der Pol polynomials, Abstract 717-A34, Page A-608

Line 4, for "234(1969), 45-64" read "260(1973), 35-46".

Line 7, after "number," insert " $p > 3m$,".

WILLIAMS K. FORREST, A property of uncountable free algebras. Preliminary report, Abstract 74T-A-107, Page A-431.

Line 3, for "$|A|$" read "$|A|'$".

J. M. PLOTKIN, ZF and Boolean algebras. Preliminary report, Abstract 74T-E69, Page A-551.

Line 2, for "homogeneous" read "weakly homogeneous".

DAVID ZEITLIN, Identities for integer sequences involving the greatest integer function. VI, Abstract 74T-A203, Page A-523.

Line 4, for "$C(i,j)/T^2$" read "$C(i,j)/T$ ".

FRED GALVIN, Bounds for power sets of singular cardinals. Preliminary report, Abstract 74T-E80, Page A-554.

This result was obtained independently by Andras Hajnal and the author, and it will be published jointly.

KENT R. FULLER, On rings of finite module type, Abstract 74T-A236, Page A-588.

The "Theorem" stated has not been proved.

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MATHEMATICS TEACHING-UNDERGRADUATE, SERVICE AREAS. Ph.D. 1967, Illinois, Algebra. M.S. in Operations Research expected 1975. Male, age 34, 7 years experience. 6 papers. Prefer west, midwest. Available Fall 1975. J. Stephen Montague, 514 Rockingham Drive, Knoxville, Tennessee 37919.

APPLIED MATHEMATICIAN. Ph.D. 1967. Four years nuclear power engineering; eight years teaching graduate/undergraduates, emphasis on numerical analysis, differential equations; several publications; computer related course development. Desires position at a technical university; or industrial position developing/applying numerical methods (finite element, splines, etc.), differential equations, control theory. Robert B. Grafton, 16 Fairfield Rd., West Hartford, Connecticut 06117.

PROFESSORSHIP-VISITING OR PERMANENT. Currently active interests, in which direction (of research, seminars, graduate courses) could be furnished include universal algebra, ordered structures, and model theory. Work well with students and colleagues; poorly with administrators or in administration. Isidore Fleischer, Centre de Recherches Mathematiques, Universite de Montreal Case Postale 6128, Montreal H3C 3J7, Quebec, Canada.

RESEARCH AND/OR TEACHING MATHEMATICIAN. Ph.D. 1974. Age 25. Speciality: Elasticity and Numerical Analysis. Four published articles. Five years teaching and research experience. References and resume upon request. Available immediately. Dr. Raj Rani Bhargava, Indian Institute of Technology, Bombay 400076, India.

TEACHING AND/OR RESEARCH MATHEMATICS EDUCATION. Ph.D. Cornell University 1973. Age 29. 4 years experience teaching math and supervising student teachers at Cornell. 2 years experience in teacher preparation at Haile Sellassie I University, Ethiopia. Resume and references upon request. Rita Cantor, P.O. Box 5329, Addis Ababa, Ethiopia.

MATHEMATICS TEACHING AND RESEARCH. Ph.D. 1971, Princeton University, Hacettepe University, Ankara. 4 papers on algebra and geometric topology. Available now. Dr. Mehmet Emin Bozhuyuk, Visiting Fellow, Department of Mathematics, Dartmouth College, Hanover, New Hampshire 03755.

ANONYMOUS

APPLIED MATHEMATICIAN. 25 years experience since Ph.D. from top institution. Equally at home in industrial and academic environments. Effective at interacting with members of other disciplines. Have taught at all levels; supervised numerous Ph.D. students. Publication record: over 50 papers in open literature, 3 books. Currently self-employed. Seek stimulating position in industry, college, or university. Salary considerations secondary. SW39

ADMINISTRATIVE POSITION. Ph.D. in Analysis - 1967, twelve published papers, one forthcoming graduate level book, excellent teacher, held several administrative assignments. SW40.

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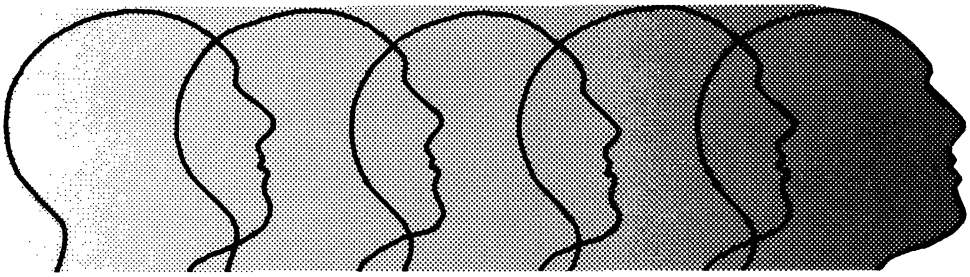

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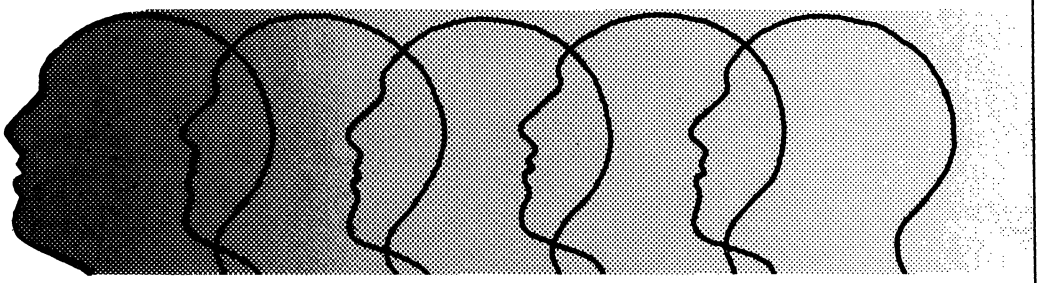

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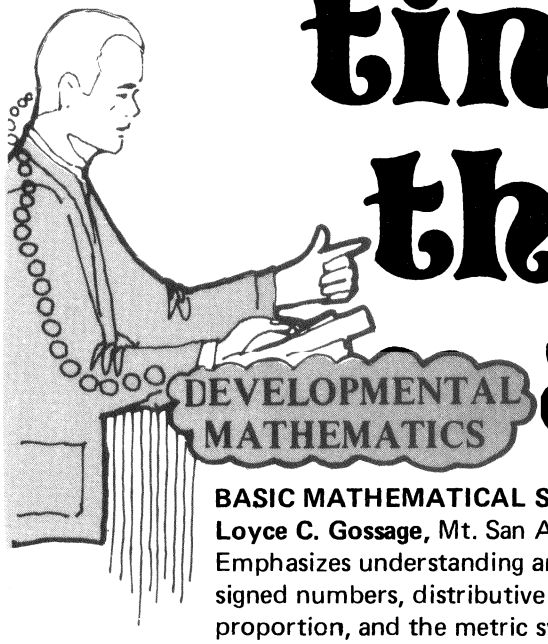

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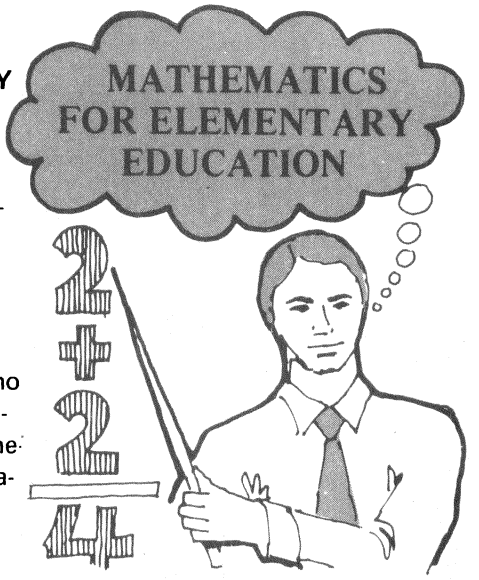

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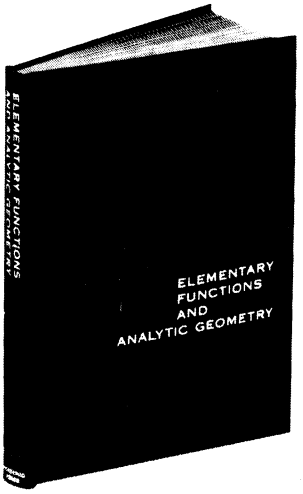

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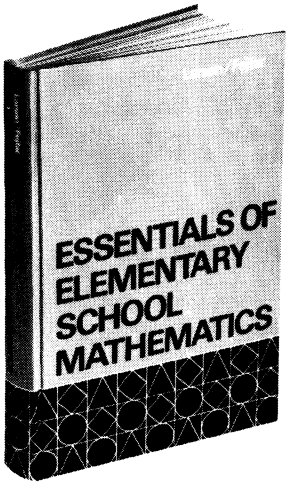

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