# Calendar

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting *in person* must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

<table>
<thead>
<tr>
<th>Meeting Number</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts* and News Items</th>
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<tbody>
<tr>
<td>723</td>
<td>April 11–12, 1975</td>
<td>St. Louis, Missouri</td>
<td>Feb. 18, 1975</td>
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<tr>
<td>724</td>
<td>April 19, 1975</td>
<td>Monterey, California</td>
<td>Feb. 18, 1975</td>
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<td>725</td>
<td>June 20–21, 1975</td>
<td>Pullman, Washington</td>
<td>Apr. 29, 1975</td>
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<td>726</td>
<td>Aug. 18–22, 1975</td>
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<td>June 17, 1975</td>
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<td>727</td>
<td>October 25, 1975</td>
<td>Cambridge, Massachusetts</td>
<td>Sept. 2, 1975</td>
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<td>728</td>
<td>November 1, 1975</td>
<td>Chicago, Illinois</td>
<td>Sept. 2, 1975</td>
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<td>729</td>
<td>November 7–8, 1975</td>
<td>Blacksburg, Virginia</td>
<td>Sept. 23, 1975</td>
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<tr>
<td>730</td>
<td>November 15, 1975</td>
<td>Los Angeles, California</td>
<td>Sept. 23, 1975</td>
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<tr>
<td>731</td>
<td>January 22–26, 1976</td>
<td>San Antonio, Texas</td>
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<td></td>
<td>(82nd Annual Meeting)</td>
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<tr>
<td>March 15–20, 1976</td>
<td>Urbana, Illinois</td>
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<tr>
<td>April 23–24, 1976</td>
<td>Reno, Nevada</td>
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<td>June 18–19, 1976</td>
<td>Portland, Oregon</td>
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<tr>
<td>November 26–27, 1976</td>
<td>Albuquerque, New Mexico</td>
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*Deadline for abstracts not presented at a meeting (by title).*  
June 1975 issue: April 22  
August 1975 issue: June 10

Please affix the peel-off label on these *Notices* to correspondence with the Society concerning fiscal matters, changes of address, promotions, or when placing orders for books and journals.

The *Notices* of the American Mathematical Society is published by the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, in January, February, April, June, August, October, November, and December. Subscription per annual volume is $10. Member subscription of $5 is included in annual dues. Price per copy $3. Special price for copies sold at registration desks of meetings of the Society, $1 per copy. Orders for subscriptions or back numbers (back issues of the last two years only are available) should be sent to the Society at P. O. Box 1571, Annex Station, Providence, Rhode Island 02901. All orders must be accompanied by payment. Other correspondence should be addressed to P. O. Box 6248, Providence, Rhode Island 02940. Second class postage paid at Providence, Rhode Island, and additional mailing offices.

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NOTICES
OF THE
AMERICAN MATHEMATICAL SOCIETY

Everett Pitcher and Gordon L. Walker, Editors
Hans Samelson, Associate Editor

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The Seven Hundred Twenty-First Meeting  
University of South Alabama  
Mobile, Alabama  
March 20 – 21, 1975

The seven hundred twenty-first meeting of the American Mathematical Society will be held at the University of South Alabama in Mobile, Alabama, on Thursday and Friday, March 20–21, 1975. The meeting of the American Mathematical Society will be followed by a meeting of the Southeastern Section of the Mathematical Association of America.

By invitation of the Committee to Select Hour Speakers for the Southeastern Sectional Meetings, there will be three one-hour addresses in Room 170, Auditorium of the Humanities Building. Professor Richard F. Arenstorf of Vanderbilt University will give an address entitled "Some recent developments in celestial mechanics." An address entitled "Finite p-groups" will be given by Professor Charles R. Hobby of the University of Alabama, and Professor R. C. Lacher of Florida State University will present an address entitled "Cell-like mappings and their generalizations."

There will be two special sessions in addition to the regular sessions. Professor B. J. Ball of the University of Georgia is arranging a session on Shape Theory and Related Topics. The speakers will include David Edwards, Ross Goghegan, James E. Keesling, George Kozlowski, W. Kuperburg, Christopher Lacher, Daniel R. McMillan, T. Benny Rushing, Jack Segal, Richard B. Sher, and James E. West. Professor William H. Ruckle of Clemson University is organizing a session on Functional Analysis. Participants in this session include M. Altman, P. G. Casazza, T. A. Cook, Mary R. Embry, R. Knowles, Charles W. McArthur, D. J. Randtke, Stephen A. Saxon, and S. Simons.

There will also be sessions for contributed papers Thursday afternoon and Friday morning.

The registration desk will be located in the Humanities Building. Registration hours will be from noon to 5:00 p.m. on Thursday, March 20, and all day Friday, March 21.

The registration fee for the Society meeting will be $1 (noon Thursday, March 20, through noon Friday, March 21). For the regular annual meeting of the Association (noon Friday, March 21, through Saturday, March 22) the registration fee will be an additional $2. It is hoped that registrants will sign up for both meetings.

ACCOMMODATIONS

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84
HILTON INN
5050 Battleship Parkway
Singles $18.00
Doubles $23.00
Extra Person $5.00

Mobile is served by National, Eastern, and Southern Airlines, Greyhound and Trailways Bus Lines. It is accessible by highway on US 90, 45, 43, 31, and Interstate 10 and 65.

MOBILE Alabama

ACCOMMODATIONS
1. RAMADA INN - (Belling)
2. ROYAL INN - (Michigan Ave.)
3. HOWARD JOHNSON'S
4. QUALITY INN
5. HOLIDAY INN - (Airport Blvd.)
6. ERDC
7. RAMADA INN - (Dauphin Island)
8. ROYAL INN - (Government)
9. DAYS INN
10. HOLIDAY INN - WEST
11. HILTON INN

PROGRAM OF THE SESSIONS

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, the time limits will be strictly enforced.

THURSDAY, 1:00 P.M.

Invited Address, Room 170, Auditorium, Humanities Building
(1) Finite p-groups. Professor C. HOBBY, University of Alabama (721-A7)

THURSDAY, 2:15 P.M.

Special Session on Shape Theory I, Room 150, Humanities Building
(2) Compact ANR's have finite homotopy type. Preliminary report. JAMES E. WEST, Cornell University (721-G5) (Introduced by Professor Joe Ball)
(3) Movability in three-manifolds. Professor DANIEL R. McMILLAN, Jr., University of Wisconsin (721-G3)
(4) A cellularity criterion based on codimension. Professor R. C. LACHER, Florida State University (721-G11)
(5) Embeddings of shape classes of compacta. Preliminary report. Professor T. BENNY RUSHING, University of Utah (721-G16)
(6) Docility at infinity and characterizations of APR's. Professor R. B. SHER, University of North Carolina at Greensboro (721-G6)
(7) Shapes of finite dimensional continua have shape irreducible representatives. Dr. HOWARD COOK, Dr. GARY FEUERBACHER, University of Houston, and Dr. WLODZIMIERZ KUPERBERG*, Auburn University (721-G9)
(8) WITHDRAWN

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
### Special Session on Functional Analysis I, Room 160, Humanities Building

#### Thursday, 2:15 P.M.

**2:15-2:35 (9)**  
Subnormal semigroups of operators. Professor MARY R. EMBRY* and Professor ALAN LAMBERT, University of North Carolina at Charlotte (721-B2)

**2:40-3:00 (10)**  
Contractor directions and directional contractions for solving equations. Professor M. ALTMAN, Louisiana State University (721-B1)

**3:05-3:25 (11)**  
The order topology of an ℱ-lattice. Preliminary report. Professor CHARLES W. McARTHUR, Florida State University (721-B3)

**3:30-3:50 (12)**  
The geometry of infinite quantum logics. Preliminary report. Professor THURLOW A. COOK, University of Massachusetts (721-B5)

**3:55-4:15 (13)**  
On the embedding of Schwartz spaces into product spaces. Preliminary report. DANIEL J. RANDTKE, University of Georgia (721-B4)

### Session on Algebra and Number Theory, Room 142, Humanities Building

#### Thursday, 2:15 P.M.

**2:15-2:25 (14)**  
Structure properties of cones of positive operators. Preliminary report. Mr. MICHAEL W. POOLE, Auburn University (721-A1)

**2:30-2:40 (15)**  
More intersection preserving embedding theorems for collections of partial PBD's. Preliminary report. WILLIAM B. POUCHER, Auburn University (721-A2)

**2:45-2:55 (16)**  
A maximal rectangle algorithm. Preliminary report. Professor EUGENE M. NORRIS, University of South Carolina (721-A3)

**3:00-3:10 (17)**  
Enumeration of regular topologies. Professor KIM KI-HANG BUTLER and Professor WALLACE MARYLAND*, Alabama State University (721-A4)

**3:15-3:25 (18)**  
Locally nilpotent injectors and Plotkin groups. JOHNNY L. HOUSTON, Savannah State College (721-A5)

**3:30-3:40 (19)**  
Monic free k-ideals in polynomial semiring. Preliminary report. Professor LOUIS DALE, University of Alabama in Birmingham (721-A6)

**3:45-3:55 (20)**  
Hypergraph coloring. Preliminary report. Mr. WILLIAM FRYE, Clemson University (721-A8)

**4:00-4:10 (21)**  
Fully invariant subgroups of primary abelian groups. Professor RONALD C. LINTON, University of South Alabama (721-A9) (Introduced by Professor Richard Vinson)

### General Session, Room 144, Humanities Building

#### Thursday, 2:15 P.M.

**2:15-2:25 (22)**  
Church's thesis and the foundations of relativistic quantum mechanics. M.C. GOOD­ALL, University of Alabama in Birmingham (721-C1)

**2:30-2:40 (23)**  
A numerical technique for obtaining approximations to the solution of certain functional equations arising in the theory of epidemics. Preliminary report. Dr. W. ROBERT BOLAND and Mr. MORRIS W. POWERS*, Clemson University (721-C2)

**2:45-2:55 (24)**  
On the intersection of regressive sets. Dr. JOSEPH BARBACK, State University of New York, College at Buffalo (721-E1) (Introduced by Kyung Barback)

**3:00-3:10 (25)**  
Boundary value problems for second order nonhomogeneous differential systems. Professor S. C. TEFTELLER, University of Alabama (721-B7)

**3:15-3:25 (26)**  
Universal operators and invariant subspaces. Preliminary report. Mr. M. L. HOWARD, University of Houston (721-B10)

**3:30-3:40 (27)**  
Boundness of value regions and convergence of continued fractions. Professor F. A. ROACH, University of Houston (721-B11)

**3:45-3:55 (28)**  
On the regularity of the image of an ordinary point of a surface under a differentiable mapping. Professor M. O. GONZALEZ, University of Alabama-Tuscaloosa (721-D1)

**4:00-4:10 (29)**  
Connectivity (B) functions, Darboux (B) functions and functions of Baire class 1. Professor KENNETH R. KELLUM*, Miles College and Professor RICHARD G. GIBSON, Alabama State University (721-G21)
THURSDAY, 4:30 P.M.

Invited Address, Room 170, Auditorium, Humanities Building
(30) Some recent developments in celestial mechanics. Professor R. F. ARENSTORF, Vanderbilt University (721-C3)

FRIDAY, 9:00 A.M.

Invited Address, Room 170, Auditorium, Humanities Building
(31) Cell-like mappings and their generalizations. Professor R. C. LACHER, Florida State University (721-G22)

FRIDAY, 10:15 A.M.

Special Session on Shape Theory II, Room 150, Humanities Building

10:40-11:00 (33) Local behavior and the Vietoris and Whitehead theorems in shape theory. Professor JACK SEGAL* and Professor GEORGE KOZLOWSKI, University of Washington (721-G2)

11:05-11:25 (34) Algebraic invariants in shape theory. Professor JAMES KEESLING, University of Florida (721-G1)


11:55-12:15 (36) When has a space the shape of a CW complex? Professor DAVID A. EDWARDS, State University of New York at Binghamton and Professor ROSS GEOGHEGAN*, University of Georgia (721-G13)

FRIDAY, 10:15 A.M.

Special Session on Functional Analysis II, Room 160, Humanities Building
10:15-10:35 (37) On James' space. Preliminary report. Professor PETER G. CASAZZA*, University of Alabama in Huntsville, Professor B. L. LIN, University of Iowa, and Professor B. LOHMAN, Kent State University (721-B8)

10:40-11:00 (38) Some aspects of bornology. Preliminary report. Professor ROBERT KNOWLES, University of Connecticut, Waterbury (721-B6)

11:05-11:25 (39) Dense, nonbarrelled subspaces of Banach spaces. Professor STEPHEN A. SAXON, University of Florida (721-B12)

11:30-11:50 (40) Convergence theorems and minimax theorems. Professor STEPHEN SIMONS, University of California, Santa Barbara (721-B9)

FRIDAY, 10:15 A.M.

Session on Topology, Room 160, Humanities Building
10:15-10:25 (41) Characterizations of finitely Suslinian continua. Mr. J. GRISPOLAKIS*, Professor A. LELEK, and Professor E. D. TYMCHATYN, Wayne State University (721-G7)

10:30-10:40 (42) Uniform free topological groups. Professor ERIC C. NUMMELA, University of Florida (721-G10)

10:45-10:55 (43) On the Baire order of concentrated spaces and $L_1$ spaces. Professor JACK B. BROWN, Auburn University (721-G14)

11:00-11:10 (44) Epics in the category of $T_2$ k-groups need not have dense range. W. F. LaMARTIN, University of New Orleans (721-G15)


11:45-11:55 (47) Embedding 1-dimensional continua in the product of an arc and a simple triod. Preliminary report. Mr. CARL H. STUCKE, Emory University (721-G19) (Introduced by J. W. Rogers, Jr.)

12:00-12:10 (48) A note on weakly uniform bases. Preliminary report. Professor GEORGE M. REED and Mr. SHELDON W. DAVES*, Ohio University (721-G20)

O. G. Harrold, Jr.
Associate Secretary

Tallahassee, Florida
The Seven Hundred Twenty-Second Meeting
Biltmore Hotel
New York, New York
March 23–26, 1975

The seven hundred twenty-second meeting of the American Mathematical Society will be held at the Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York, from Sunday, March 23, through Wednesday, March 26, 1975.

On Tuesday, March 25, there will be a program entitled "A day of differential equations" which will be concerned with the qualitative properties of both ordinary differential equations and partial differential equations, exploring the similarity of their methods and results. This program was organized by Professor Jack K. Hale of Brown University and will include half-hour lectures by Professors Donald Aronson, Nathaniel Chafee, Shui-Nee Chow, Daniel Henry, and Jerrold Marsden.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be two one-hour addresses on Wednesday, March 26. Professor Herman Chernoff of the Massachusetts Institute of Technology will speak on "Identifying a member of a large population using noisy data" at 11:00 a.m. Professor Peter J. Freyd of the University of Pennsylvania will speak on "Topoi: Applications to and from logic" at 2:00 p.m.

Sessions for contributed ten-minute papers will be scheduled in the morning and afternoon on Tuesday and Wednesday, with papers deemed relevant to the "Day" program appearing in regular sessions for contributed ten-minute papers on the same day. No provision will be made for late papers. Each meeting room will be equipped with an overhead projector.

SYMPOSIUM ON NONLINEAR PROGRAMMING

With the expected support of the National Science Foundation and the Energy Research and Development Administration (formerly Atomic Energy Commission), a symposium on Nonlinear Programming is scheduled to be held on Sunday and Monday, March 23 and 24. This topic was selected by the AMS-SIAM Committee on Applied Mathematics whose members are Earl A. Coddington, Richard C. DiPrima (chairman), Lester E. Dubins, Harold Grad, J. Barkley Rosser, and W. Gilbert Strang.

The purpose of the symposium is to help bring this vigorous field of activity to the attention of a larger mathematical audience. The organizing committee, comprised of Richard W. Cottle, Stanford University (chairman), Carlson E. Lemke, Rensselaer Polytechnic Institute, Stephen M. Robinson, University of Wisconsin, and J. Ben Rosen, University of Minnesota, has organized the symposium into four sessions, with a total of nine lecturers. The subject will be treated from the perspectives of theory, computation, and applications. To the greatest extent possible, the lectures will be pedagogically oriented and will identify the principal lines of current research. It is hoped that the lectures will help the audience to identify and appreciate some areas where they can use their own background and the information they acquire at the symposium to begin research in nonlinear programming.

The list of speakers includes John E. Dennis, Jr. (Cornell University), B. Curtis Eaves (Stanford University), Harold W. Kuhn (Princeton University), D. G. Luenberger (Stanford University), Olvi L. Mangasarian (University of Wisconsin), G. P. McCormick (George Washington University), M. J. D. Powell (Atomic Energy Research Establishment, Harwell, England), R. Tyrrell Rockafellar (University of Washington), and Philip Wolfe (IBM T. J. Watson Research Center). Ample time will be provided for discussion and questions. The symposium will be held in the Grand Ballroom of the Biltmore Hotel both days.

ASSOCIATION FOR WOMEN IN MATHEMATICS

There will be a concurrent meeting of the Association for Women in Mathematics organized by Professor Linda Keen.

REGISTRATION

The registration desk will be located in the Key Room of the Biltmore Hotel on the nineteenth floor adjacent to the Grand Ballroom. The desk will be open from 8:30 a.m. to 4:30 p.m. on Sunday, March 23, through Tuesday, March 25; and from 8:30 a.m. to 3:30 p.m. on Wednesday, March 26.

The registration fees for the meeting are as follows:

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ACCOMMODATIONS

Persons intending to stay at the Biltmore Hotel should make their own reservations with the hotel. A reservation form and a listing of room rates will be found on the last page of the January Notices. The deadline for receipt of reservations is March 5, 1975.

TRAVEL

The Biltmore Hotel is located on Madison Avenue at 43rd Street on the east side of New York City. Walkways to Grand Central Station are located under the hotel and signs are posted directing persons to the lobby of the hotel.

Those arriving by bus may take the
Independent Subway System from the Port Authority Bus Terminal. There is shuttle bus service from LaGuardia and Kennedy Airports directly to Grand Central Station. Starters can direct participants to the correct bus.

Air passengers arriving at Newark Airport can take a shuttle bus to the East Side Terminal and take either a subway, taxi, or bus to the hotel.

Those arriving by car will find many parking facilities in the neighborhood in addition to those at the hotel. Parking service can be arranged through the hotel doorman at a cost of $9 for the 24-hour period. There will be an additional charge for extra pickup and delivery service if it is required. The parking fee is subject to New York City taxes.

MAIL ADDRESS
Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, The Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York 10017.

PROGRAM FOR THE SYMPOSIUM
SUNDAY, MARCH 23

First Session, Grand Ballroom, Nineteenth Floor
Chairman: Richard W. Cottle, Stanford University
10:30 a.m. Nonlinear programming: A historical view. HAROLD W. KUHN, Princeton University
11:30 a.m. Lagrange multipliers in nonlinear programming. R. TYRRELL ROCKAFELLAR, University of Washington

Second Session, Grand Ballroom, Nineteenth Floor
Chairman: Carlton E. Lemke, Rensselaer Polytechnic Institute
2:30 p.m. Optimality criteria in nonlinear programming. G. P. MCCORMICK, George Washington University
3:30 p.m. Coffee
4:00 p.m. Path generation in piecewise-linear structures for solving equations. B. CURTE EAVES, Stanford University

MONDAY, MARCH 24

Third Session, Grand Ballroom, Nineteenth Floor
Chairman: J. Ben Rosen, University of Minnesota
9:00 a.m. Some methods for unconstrained minimization of nonlinear functionals. JOHN E. DENNIS, Jr., Cornell University
10:00 a.m. Coffee
10:30 a.m. Methods for optimization under constraint. DAVID G. LUENBERGER, Stanford University
11:30 a.m. Global convergence properties of a variable metric minimization algorithm without line searches when the objective function is convex. M. J. D. POWELL, Atomic Energy Research Establishment, Harwell, England

Fourth Session, Grand Ballroom, Nineteenth Floor
Chairman: Stephen M. Robinson, University of Wisconsin
2:30 p.m. Unconstrained methods in nonlinear programming. OLVI L. MANGASARIAN, University of Wisconsin
3:30 p.m. Coffee
4:00 p.m. Difficult problems in nonlinear programming. PHILIP WOLFE, IBM T. J. Watson Research Center

THE SEVEN HUNDRED TWENTY-SECOND MEETING
PROGRAM OF SESSIONS
The time limit for each contributed paper in the general sessions is ten minutes. Each paper in "A day of differential equations" is twenty-five minutes. To maintain this schedule, the time limits will be strictly enforced.

TUESDAY, 9:00 A.M.

Session on Analysis I, French Suite, First Floor
9:00-9:10 (1) Invariance of holomorphic convexity under proper maps. Dr. ANDREW G. MARKOE, University of Washington (720-32-6)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
9:15–9:25 (2) Some sets obeying harmonic synthesis. Professor YITZHAK KATZNELSON, The Hebrew University and O. CARRUTH McGHEE*, Louisiana State University (722-B10)

9:30–9:40 (3) On complete semicardinal quadrature formulae. Professor SHERWOOD D. SILLIMAN, Cleveland State University (722-B14)

9:45–9:55 (4) A theorem on Cesaro summability for fractional orders of integrals. Mr. SANTIRANJAN MUKHOTI, University of Pennsylvania (722-B20)

10:00–10:10 (5) Barnes type integrals via fractional differentiation. Preliminary report. Professor THOMAS J. OSLER, Glassboro State College (722-B21)

(6) Withdrawn

TUESDAY, 9:00 A.M.

Session on Applied Mathematics I, Vanderbilt Suite, First Floor

9:00–9:10 (7) Schemes for fast matrix multiplication. Preliminary report. Professor JOHN de PILLIS, University of California, Riverside (722-C1)


9:30–9:40 (9) Linear programming via a nondifferentiable penalty function. Dr. A. R. CONN, University of Waterloo (722-C6) (Introduced by Professor R. C. Read)

9:45–9:55 (10) On the convergence of a class of derivative-free minimization algorithms. Dr. ANN-SHENG CHIEN TING, Virginia Polytechnic Institute and State University (722-C7) (Introduced by Professor John E. Dennis, Jr.)

TUESDAY, 9:00 A.M.

Session on Ordinary Differential Equations, Music Room, First Floor

9:00–9:10 (11) Bifurcations in symmetric systems. Dr. J. HENRARD, Facultes Universitaires de Namur, Belgium, and Dr. K. R. MEYER*, University of Cincinnati (722-B3)

9:15–9:25 (12) An integral representation for the solution Wkm of Whittaker's differential equation. Dr. JAMES D'ARCHANGEOLO, U. S. Naval Academy (722-B6)


9:45–9:55 (14) Two point connection problem for a certain differential equation with an irregular singular point of arbitrary rank. T. K. PUTTASWAMY, Ball State University (722-B15)

10:00–10:10 (15) Two-term linear differential operators. Preliminary report. Professor GEORGE JOHNSON, University of South Carolina (722-B17)


TUESDAY, 10:30 A.M.

A Day of Differential Equations I, Grand Ballroom, Nineteenth Floor

10:30–10:55 (17) Invariant manifolds for parabolic equations. DANIEL HENRY, Northwestern University (722-B16) (Introduced by Professor N. Chafee)

11:00–11:25 (18) Hopf bifurcation and results of Ruelle-Takens. Mr. JERROLD E. MARSDEN* and Ms. M. M. McCracken, University of California, Berkeley (722-B7)

11:30–11:55 (19) Averaging and bifurcation from equilibrium or periodic solutions. Professor SHUI-NEE CHOW*, Michigan State University and Brown University and Professor JOHN MALLET-PARET, Brown University (722-B6)

TUESDAY, 2:00 P.M.

A Day of Differential Equations II, Grand Ballroom, Nineteenth Floor

2:00–2:25 (20) Stability for nonlinear parabolic equations via the maximum principle. Professor D. G. ARONSON, University of Minnesota and Rice University (722-B23)

2:30–2:55 (21) Liapunov methods and parabolic equations. Professor NATHANIEL CHAFEE, Georgia Institute of Technology (722-B4)
### Session on Analysis II, French Suite, First Floor

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<tr>
<th>Time</th>
<th>Session</th>
<th>Speaker/Institution</th>
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<tbody>
<tr>
<td>3:15-3:25</td>
<td>A factorization theorem for compact operators. Professor GREGORY BACHELIS</td>
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<td>(22) Wayne State University (722-B22)</td>
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<tr>
<td>3:30-3:40</td>
<td>Ergodic theory and Grothendieck spaces. Professor ROBERT E. ATALLA, Ohio</td>
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<td>University (722-B24)</td>
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<td>3:45-3:55</td>
<td>A class of regular matrices. Preliminary report. Professor GODFREY L. ISAACS</td>
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<td>(City University of New York, (724-T-B23)</td>
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<td>4:00-4:10</td>
<td>Asymptotic distribution of lattice points in hyperbolic space. Preliminary report.</td>
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<td>Mr. WILLIAM WOLFE, City University of New York, Graduate Center (722-D1)</td>
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<td>Professor KENNETH B. HANNSGEN, Virginia Polytechnic Institute and State University (722-B5)</td>
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### Session on Applied Mathematics II, Vanderbilt Suite, First Floor

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<th>Speaker/Institution</th>
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<tbody>
<tr>
<td>3:15-3:25</td>
<td>A limit form of the $E_p$-transformation. JOE B. THRASH, University of Southern Mississippi (722-C3)</td>
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<tr>
<td>3:30-3:40</td>
<td>Structure of symmetric shock reflection with an isothermal energy release hypothesis. Dr. S. MANICKAM, Western Carolina University (722-C4)</td>
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<td>3:45-3:55</td>
<td>On the geometry of steady circulation preserving geodesic flow of a gas. Dr. E. R. SURYANARAYAN, University of Rhode Island (722-C8)</td>
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<tr>
<td>4:00-4:10</td>
<td>An asymptotic formula for the twisted product of distributions. Preliminary report. Professor KUANG CHI LIU, State University of New York, College at Oswego (722-C5)</td>
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### Session on Partial Differential Equations, Music Room, First Floor

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<th>Speaker/Institution</th>
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<tbody>
<tr>
<td>3:15-3:25</td>
<td>Bessel functionals and solutions of hyperbolic differential equations in Hilbert space. Dr. NAzar ABDelaZiz, Virginia Polytechnic Institute and State University (722-B1)</td>
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<tr>
<td>3:30-3:40</td>
<td>Lie theory of the heat equation. Mr. STEVEN I. ROSECRANS, Tulane University (722-B9)</td>
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<td>3:45-3:55</td>
<td>Uniqueness of Dirichlet, Neumann, and mixed boundary value problems for Poisson's equation $u_{xx} + u_{yy} = f$ for a rectangle. Professor J. B. DIAZ, Polytechnic Institute of Technology and Professor R. B. RAM*, State University of New York at Oneonta (722-B11)</td>
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<td>4:00-4:10</td>
<td>On the zeros of solutions of elliptic partial differential equations $\nabla^2 \psi + A(r^2)x \cdot \nabla \psi + C(r^2)\psi = 0$ in three real variables. Preliminary report. Professor PETER A. McCoy, U. S. Naval Academy (722-B19)</td>
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### Session on Flows, Music Room, First Floor

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<tr>
<th>Time</th>
<th>Session</th>
<th>Speaker/Institution</th>
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<tr>
<td>4:30-4:40</td>
<td>On the existence of limit cycles following peeling. Dr. OKAN GUREL, IBM</td>
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<td>Corporation, White Plains, New York (722-G9)</td>
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<td>4:45-4:55</td>
<td>Generalized Hartman's theorem for degenerate rest points of a flow. Preliminary report. JOHN T. MONTGOMERY, University of Rhode Island (722-G10)</td>
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<tr>
<td>5:00-5:10</td>
<td>Stepanoff flows on compact manifolds. MICHAEL LIN, Ohio State University and ROBERT SINE*, University of Rhode Island (722-B13)</td>
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<tr>
<td>5:15-5:25</td>
<td>Stepanoff-like flows on compact orientable surfaces of genus $h \geq 2$. Mr. DENNIS A. MARCHETTO, Virginia Commonwealth University (722-G7) (Introduced by Professor William L. Reddy)</td>
<td></td>
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<tr>
<td>5:30-5:40</td>
<td>On the index sum of a vector field. Preliminary report. Professor DENIS BLACKMORE, New Jersey Institute of Technology (722-G4)</td>
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</table>
WEDNESDAY, 9:00 A.M.

Session on Algebra I, Vanderbilt Suite, First Floor
9:00– 9:10 (41) Trebly-magic systems in a Latin 3-cube of order eight. Professor JOSEPH ARKIN*, Spring Valley, New York, and Professor PAUL SMITH, University of Victoria (722-A1)

9:15– 9:25 (42) The convex polyhedron of doubly stochastic matrices. I. Applications of the permanent function. Professor RICHARD A. BRUALDI, University of Wisconsin, and Professor PETER M. GIBSON*, University of Alabama in Huntsville and University of Wisconsin (722-A3)

9:30– 9:40 (43) Matroids of prime power index and associated RBBD's. Preliminary report. Mr. B. KESTENBAND* and Professor H. P. YOUNG, City University of New York, Graduate Center (722-A1)

9:45– 9:55 (44) Automorphism groups of the \((q+1, (q+1)/2)\) extended QR codes for \(q\) and \((q+1)/2\) prime, \(5 \leq q \leq 4079\). Preliminary report. Professor EDWARD P. SHAUGHNESSY, Lafayette College (722-A5)

10:00–10:10 (45) Nilgroups of finite Abelian groups, Preliminary report. ROBERT MARTIN, Columbia University and City University of New York, Hunter College (722-A9)


WEDNESDAY, 9:00 A.M.

Session on Probability and Statistics, Music Room, First Floor
9:00– 9:10 (47) On the absorption probabilities and absorption times of finite homogeneous birth-death processes and its application to a genetic model of Moran. Dr. W. Y. TAN, Washington State University (722-F1) (Introduced by Professor C. T. Long)


9:30– 9:40 (49) Exponential limit law for critical general branching processes. Dr. JOHN M. HOLTE, Rensselaer Polytechnic Institute (722-F4)

9:45– 9:55 (50) Asymptotic analysis of deterministic and stochastic equations with rapidly varying components. Professor G. C. PAPANICOLAOU, Courant Institute, New York University and Professor W. KOHLER*, Virginia Polytechnic Institute and State University (722-F5)

WEDNESDAY, 11:00 A.M.

Invited Address, Grand Ballroom, Nineteenth Floor
(51) Identifying a member of a large population using noisy data. Professor HERMAN CHERNOFF, Massachusetts Institute of Technology (722-F2)

WEDNESDAY, 2:00 P.M.

Invited Address, Grand Ballroom, Nineteenth Floor
(52) Topoi: Applications to and from logic. Professor PETER J. FREYD, University of Pennsylvania

WEDNESDAY, 3:15 P.M.

Session on Algebra II, Vanderbilt Suite, First Floor
3:15– 3:25 (53) On ultrafilters of distributive lattices. Preliminary report. Dr. SIMEON FAJTLOWICZ, University of Houston (722-A4)


3:45– 3:55 (55) Skew symmetric biderivations of a two-generator purely inseparable field. Professor FRANCIS P. CALLAHAN, Pennsylvania State University, King-of-Prussia Campus (722-A6)

4:00– 4:10 (56) Strictly local solutions of Diophantine equations. Professor M. J. GREENBERG, University of California, Santa Cruz (722-A8)


4:30– 4:40 (58) Set productive and arithmetically set productive sets. Preliminary report. Mr. BRUCE M. HOROWITZ, City University of New York, Queens College (722-E1)
WEDNESDAY, 3:15 P.M.

Session on Topology, Music Room, First Floor


3:30 - 3:40 (60) Convergence of sequence of iterates. Preliminary report. Professor S. P. SINGH, Memorial University of Newfoundland (722-G3)


4:00 - 4:10 (62) On topological methods in homological algebra. Professor H. M. HASTINGS* and Professor D. A. EDWARDS, State University of New York at Binghamton (722-G2)

4:15 - 4:25 (63) On the topological structure of algebraic surface in CP^3. Dr. R. MANDELBAUM*, Weizmann Institute of Science, Rehovot, Israel, and Dr. B. MOISHEZON, Tel Aviv University, Ramat Aviv, Israel (722-G5)

4:30 - 4:40 (64) Infinite mapping cylinders are Hilbert cube factors. Professor LEON STAGG NEWMAN, Jr., City University of New York, Baruch College (722-G8)

Middletown, Connecticut

PRESENTORS OF PAPERS

Walter H. Gottschalk
Associate Secretary

Following each name is the number corresponding to the speaker's position on the program

*Day of Differential Equations

Abdelaziz, Nazar #31
Arkin, Joseph #41
*Aronson, D. G. #20
Atalla, Robert E. #23
Bachelis, Gregory #22
Blackmore, Denis #40
Callahan, Francis P. #55
*Chafee, Nathaniel #21
Chernoff, Herman #51
Chow, Shui-Nee #19
Conn, A. R. #9
D'Archangelo, James #12
Ellis, Richard S. #48
Fajtlowicz, Simeon #53
*Freyd, Peter J. #52
Gibson, Peter M. #42
Greenberg, M. J. #56
Gurel, Okan #36
Haanen, Kenneth B. #26
Hastings, H. M. #62
*Henry, Daniel #17
Holte, John M. #49
Horowitz, Bruce M. #58
Hsu, Pao-Sheng #54
Isaacs, Godfrey L. #24
Johnson, George #15
Johnson, R. Warren #57
Kestenband, B. #43
Kohler, W. #50
Kulkarni, Ramesh M. #16
Liu, Kuang Chi #30
McCoy, Peter A. #34
McGehee, O. Carruth #2
Magill, Jr., K. D. #46
Man德尔baum, R. #63
Manickam, S. #28
Marchetto, Dennis A. #39
Markoe, Andrew G. #1
*Marsden, Jerrold E. #18
Martin, Robert #45
Meyer, K. R. #11
Montgomery, John T. #37

● Invited one-hour lectures

Mrowka, S. #61
Mukhoj, Santiranjan #4
Newman, Jr., Leon Stagg #64
Obi, Wilson C. #13
Osler, Thomas J. #5
de Pillis, John #7
Puttaswamy, T. K. #14
Rajagopalan, M. #59
Ram, R. B. #33
Rosencrans, Steven I. #32
Salzer, Herbert E. #8
Shaughnessy, Edward P. #44
Silliman, Sherwood D. #3
Sine, Robert #38
Singh, S. P. #60
Slemrod, Marshall #35
Suryanarayan, E. R. #29
Tan, W. Y. #47
Thrish, Joe B. #27
Ting, Ann-Sheng Chien #10
Wolfe, William #25

ERRATA

Walter H. Gottschalk of Wesleyan University was the Associate Secretary in charge of the program for the Eighty-First Annual Meeting in Washington, D.C. His name should have appeared at the end of the program, page 45, of these Notices, January 1975.

In the December 1974 (Notices), p. 423, the number of Ph.D.'s granted (1972–1974 incl.) should read: A&NT 1, G&T 1, A&FA 1, CS 1. Total: 4

The last sentence of "Comment on an Invitation to V. I. Arnol'd", Notices, January 1975, p. 65, should read "An address by Professor Arnol'd was read at the Congress by E. Brieskorn." David Gale presented a paper by E. B. Dynkin.
PRELIMINARY ANNOUNCEMENTS OF MEETINGS
The Seven Hundred Twenty-Third Meeting
University of Missouri
St. Louis, Missouri
April 11 – 12, 1975

The seven hundred twenty-third meeting of the American Mathematical Society will be held at the University of Missouri, St. Louis, Missouri, on Friday and Saturday, April 11 and 12, 1975. The University of Missouri at St. Louis is located about ten miles northwest of downtown St. Louis and about four miles east of St. Louis Lambert Airport. All sessions will be held in the J. C. Penney Building of the university.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be four one-hour addresses. Professor A. O. L. Atkin of the University of Illinois at Chicago Circle will speak on Friday, April 11, at 11:00 a.m.; the title of his talk "Supersingular games". Professor Kuo-Teai Chen of the University of Illinois at Urbana-Champaign will address the Society on Friday, April 11, at 1:45 p.m.; his subject will be "Iterated path integrals". Professor Kenneth Kunen of the University of Wisconsin at Madison will speak on Saturday, April 12, at 11:00 a.m.; the title of his talk will be "What good are ultrafilters"? Professor Guido L. Weiss of Washington University at St. Louis will address the Society on Saturday, April 12, at 1:45 p.m.; his topic will be "The use of Hardy spaces and their generalizations in harmonic analysis".

By invitation of the same committee there will be nine sessions of selected twenty-minute papers. Professor David Drasin of Purdue University is arranging a special session on Classical Function Theory to be held Friday afternoon and Saturday morning; the tentative list of speakers includes Albert Baernstein II, Burgess J. Davis, Frederick W. Gehring, Dennis A. Hejhal, Simon Hellerstein, James A. Jenkins, John L. Lewis, Richard H. Rochberg, Glen E. Schober, Ted J. Suffridge, and Allen W. Weiss. Professor David L. Elliott of Washington University is arranging a special session on Differential Geometric Problems in Control Theory to be held all day Friday and Saturday morning; the tentative list of speakers includes John B. Bell, William M. Boothby, Roger W. Brockett, Jan M. Gronska, Robert Hermann, Henry G. Hermes, Ronald M. Hirschorn, Velimir Jurdjevic, Arthur J. Krener, Jan Kucera, Deborah Rebhuhn, Jackson L. Sedwick, Jr., M. B. Suryanarayana, and Hector J. Sussman. Professor Franklin Haimo of Washington University is arranging a special session on Applications of Ring Theory to Groups, to be held both Friday and Saturday; the tentative list of speakers includes Paul F. Conrad, Vance Faber, Barton I. Fein, Charles E. Ford, Brian Hartley, Jutta Hausen, Israel N. Herstein, Arun V. Jategaonkar, Everett L. Lady, Charles P. Lanski, Donald S. Passman, Gary L. Peterson, Richard E. Phillips, Derek J. S. Robinson, Martha Kathleen Smith, Julian S. Williams, and Cleon R. Yohe. Professor Richard P. Jerrard of the University of Illinois at Urbana-Champaign is arranging a special session on Geometric Topology, to be held Friday morning; the tentative list of speakers includes John E. Connett, Robert J. Daverman, Mary-Elizabeth Hamstrom, and Jan Jawrowski. Professor Rangachary Kannan of the University of Missouri at St. Louis and Michigan State University is arranging a special session on Non-Linear Functional Analysis, to be held all day Saturday; the tentative list of speakers includes Lambert Cesari, Michael G. Crandall, Jack K. Hale, George J. Minty, Roger D. Nussbaum, Paul H. Rabinowitz, Duane P. Sather, Luc Tartar, and Hans F. Weinberger. Professor Walter Leighton of the University of Missouri at Columbia is arranging a special session on Ordinary Differential Equations; Oscillation Theory, Boundary Value Problems, to be held all day Friday and Saturday morning; the tentative list of speakers includes William J. Coles, Arlington M. Fink, Louis J. Grimm, Heinrich W. Guggenheimer, Don B. Hinton, Lloyd K. Jackson, Marvin S. Keener, Kurt Kreith, Alan C. Lazer, William T. Reid, Jerry R. Ridenhour, Curtis C. Travis, and W. Roy Utz, Jr. Professor Marian Boykan Pour-El of the University of Minnesota is arranging a special session on Recursion Theory, to be held Saturday; the tentative list of speakers includes Harvey Friedman, Carl G. Jockusch, Jr., Manuel Lerman, Thomas G. McLaughlin, Amil Nerode, Hilary Putnam, Wayne H. Richter, Gerald E. Sacks, and Robert I. Soare. Professor Grant V. Wendland of the University of Missouri at St. Louis is arranging a special session on Harmonic Analysis and Related Topics, to be held Friday; the tentative list of speakers includes David R. Adams, Richard J. Bagby, Richard A. Hunt, Alexander J. Nagel, Victor L. Shapiro, Elias M. Stein, Mitchell H. Taibleson, Alberto Torchinsky, S. Vagi, Woe-Sang Young, and William P. Ziemer. Professor David J. Winter of the University of Michigan is arranging a special session on Finite Dimensional Field Extensions, to be held Friday; the tentative list of speakers includes Stephen U. Chase, Lindsay N. Childs, Raymond T. Hoobler, Herbert F. Kreiner, Jr., Andy R. Magid, Moss E. Sweedler, and David J. Winter.

There will be sessions for contributed ten-minute papers on both Friday and Saturday. Abstracts should have been sent to the American
Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of February 18, 1975. Those having time preferences for the presentation of their papers should indicate them clearly on their abstracts. There will be a session for late papers if one is needed, but late papers will not be listed in the printed program of the meeting.

On Thursday, April 10, the day before the meeting itself, the University of Missouri at St. Louis and Washington University will sponsor a brief symposium on Harmonic Analysis and Related Topics, which will supplement Professor Welland's special session. The speakers at this symposium will be Ronald R. Coifman, Charles L. Fefferman, Nestor M. Riviere, and Stephen Wainger.

COUNCIL MEETING

The Council will meet at 5:00 p.m. on Friday, April 11, 1975, in the Champagne Room of Stan Musial & Biggie's St. Louis Hilton Inn, 10330 Natural Bridge Road, St. Louis 63134. The Champagne Room is located on the first level of the Inn.

REGISTRATION

The registration desk will be located in the lobby of the J. C. Penney Building. The desk will be open from 9:00 a.m. to 5:00 p.m. on Friday, April 11 and from 8:00 a.m. to 4:00 p.m. on Saturday, April 12. The registration fee for the meeting will be $2.00.

ACCOMMODATIONS

The following four inns are located between St. Louis Lambert Airport and the University of Missouri at St. Louis. They are listed in the order of increasing distance from the university. The first two are essentially at Natural Bridge Road and Brown Road; the last two are essentially at Natural Bridge Road and Woodson Road. All four are within a mile of the airport.

ROYAL INN*
9600 Natural Bridge Road 63134
(314) 428-9732

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RAMADA INN
9636 Natural Bridge Road 63134
(314) 426-4700

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EIGHT DAYS INN*
4545 Woodson Road 63131 (just south of 10232 Natural Bridge Road)
(314) 423-6770

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* N. B. The Eight Days Inn and the Royal Inn will provide free courtesy-van shuttle service between their respective motels and the campus on April 11 and 12.

STAN MUSIAL AND BIGGIE'S
ST. LOUIS HILTON INN
10330 Natural Bridge Road 63134
(314) 426-5500

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Requests for room reservations at the Eight Days Inn should be made directly with the Inn using the room reservation form which appeared on the last page of the January Notices. Reservations at other motels should be made directly, and mention should be made of this meeting in order to obtain the quoted rates.

FOOD SERVICE

Food service will be available in the cafeteria of the J. C. Penney Building. A list of nearby restaurants will also be available.

TRAVEL AND LOCAL INFORMATION

The University of Missouri at St. Louis is located on Natural Bridge Road approximately four miles east of St. Louis Lambert Airport and about three miles east of the inns listed above. Natural Bridge Road is just south of and roughly parallel to Interstate Route 70, the university being in the vicinity of mileage marker 240 on the Interstate 70. Automobile drivers should leave Interstate 70 as follows:

1. Those coming from the west who wish to go directly to the University should use Exit 238 (Natural Bridge Road and Brown Road) and then go three miles east on Natural Bridge Road to the University.

2. Those coming from the west who wish to go to one of the inns listed should use Exit 237 southbound (Lambert Airport) and then turn east on Natural Bridge Road, which is the connection to the Interstate at that point.

3. Those coming from the east should use Exit 241 southbound (Lucas and Hunt Road), go one mile south on Lucas and Hunt Road (to the first traffic light), and then go 1 1/2 miles west on Natural Bridge Road to reach the University. The inns listed are about three miles farther west on Natural Bridge Road.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

95
The Seven Hundred Twenty-Fourth Meeting  
Naval Postgraduate School  
Monterey, California  
April 19, 1975

The seven hundred twenty-fourth meeting of the American Mathematical Society will be held at the Naval Postgraduate School, Monterey, California, on Saturday, April 19, 1975. All functions will be held in Ingersoll Hall. Registration will begin at 8:30 a.m. in the lobby.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two one-hour addresses. Professor Kenman T. Smith of Oregon State University will lecture at 11:00 a.m. on "Practical and mathematical aspects of the problem of reconstructing objects from radiographs." Professor Isaac Namioka of the University of Washington will lecture at 3:30 p.m. He will speak on "Some topological questions concerning Banach spaces." Both hour addresses will be in Room I-122 of Ingersoll Hall.

There will be sessions for contributed papers. Abstracts of papers to appear in the program should have been submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of February 18, 1975. Late papers will be accepted for presentation at the meeting, but will not appear in the printed program of the meeting.

The Visitors and Convention Bureau of Monterey has blocked off rooms in the following motels at the specified rates (rates in parentheses are for Saturday night). All motels except the last are within a 10-minute walk of the meeting.

| DEL MONTE HYATT HOUSE  |  | STAGE COACH LODGE  |
|------------------------|  |--------------------|
| Single                 | $24 | Single             | $15 (20) |
| Twin                   | $34 | Double/Twin        | $18 (22) |

| ROYAL INN  |  | CALIFORNIAN  |
|------------|  |-------------|
| Single     | $20 | Single      | $12 (19) |
| Double     | $26 | Double/Twin | $14 (22) |
| Twin       | $30 |             |

| MONTEREY MOTOR LODGE  |  |
|-----------------------|  |
| Single                | $15 |
| Twin                  | $22 |

Requests for reservations should be addressed to Visitors and Convention Bureau, P.O. Box 1770, Monterey, California 93940; phone (408) 375-2252. Requests should indicate type of accommodation, arrival and departure time, and order of preference of motels. They should refer to the American Mathematical Society meeting, must be accompanied by one night's deposit, and it is suggested that you ask for a confirmation. Deadline for requests is March 24, 1975.

Less expensive accommodations are available at the Motel 6, 2124 Fremont Avenue (old Highway 1), Monterey, California 93940 (phone [408] 373-3500). Reservations at the Motel 6 must be made directly with the motel and should be made well in advance of the meeting. Their rates range from $6.95 to $12.95.

Noon meals will not be available on campus, but several restaurants are within a 10-minute walk of the meeting. There are over fifty restaurants in the Monterey area. More detailed information will be available at the meeting.

The Monterey Airport is served by United and Hughes-AirWest. Greyhound buses have frequent service to Monterey, and Amtrak serves nearby Salinas daily. By car, one should first get on Highway 1. As you approach downtown, look for signs reading "Naval Postgraduate School." Follow signs to the school, thence to Ingersoll Hall. Parking will be available in three lots adjacent to Ingersoll Hall.

Kenneth A. Ross  
Associate Secretary  
Eugene, Oregon
The Seven Hundred Twenty-Fifth Meeting
Washington State University
Pullman, Washington
June 21, 1975

The seven hundred twenty-fifth meeting of the American Mathematical Society will be held at Washington State University in Pullman, Washington, on Saturday, June 21, 1975. The Mathematical Association of America and the Society for Industrial and Applied Mathematics will hold Northwest Sectional Meetings in conjunction with this meeting of the Society. Some of their sessions will be held on Friday, June 20.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, invited hour addresses will be given by Professor David W. Barnette of the University of California, Davis, and by Professor Theodore E. Harris of the University of Southern California. The titles of their addresses will be given in the April issue of these Notices.

There will be sessions for contributed papers. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of April 29, 1975. Late papers will be accepted for presentation at the meeting, but will not appear in the printed program of the meeting.

Further information will appear in the April issue of the Notices, and the final program will appear in the June Notices.

Kenneth A. Ross
Eugene, Oregon
Associate Secretary

TO MATHEMATICIANS IN FAR WESTERN STATES

The scheduling and format of Regional Society Meetings in the Far West have been the same for over twenty years. The attendance and interest in recent meetings, especially in California, has dropped off; it seems appropriate to re-examine these meetings. I would appreciate suggestions on possible changes in the organization or content of the Far West Regional Meetings, which might make them more useful to members. Please send them to me at the Department of Mathematics, University of Oregon, Eugene, Oregon 97403.

Kenneth A. Ross
Associate Secretary
1975 Summer Seminar in Applied Mathematics

The ninth AMS Summer Seminar in Applied Mathematics will be held on the campus of Rensselaer Polytechnic Institute, Troy, New York, from July 7 through July 18, 1975, and will be sponsored jointly by the American Mathematical Society and the Society for Industrial and Applied Mathematics. The topic for the seminar will be Modern Modeling of Continuum Phenomena. This was proposed by the AMS-SIAM committee on Applied Mathematics which at the time consisted of Earl A. Coddington, Hirsh G. Cohen (chairman), Lester E. Dubins, Harold Grad, J. Barkley Rosser and Richard S. Varga. The organizing committee will consist of George F. Carrier, Harvard University; Hirsh G. Cohen, IBM T.J. Watson Research Center; Stephen H. Davis, Johns Hopkins University; Richard C. DiPrima, Rensselaer Polytechnic Institute (chairman); Joseph B. Keller, Courant Institute of Mathematical Sciences of New York University; and Lee A. Segel, Weizmann Institute of Science and Rensselaer Polytechnic Institute. Support is expected from the National Science Foundation and Office of Naval Research.

Continuum problems in the physical sciences have always provided a rich area of study for the applied mathematician. More recently continuum models have been developed for important problems in the biological sciences and ecology.

The primary purposes of the seminar are (i) to introduce the participants to selected mathematical research areas of high current interest and relevance, (ii) to present the underlying fundamental laws of continuum modeling, and (iii) to present selected mathematical topics particularly useful in solving modern mathematical problems of continuum phenomena. This will be accomplished through core series of lectures in continuum model building and mathematical methods, and four in-depth case studies concerned with a problem of current interest in fields such as ocean circulation, population dynamics, cell motions, and seismology. A partial list of principal lecturers includes Lee Segel, Don Cohen, George Carrier, George Oster, Gary Odell and George Papanicolaou. In addition there will be an opportunity for informal seminars in which all who attend may directly participate.

Allowance will be made for faculty members whose principal background is in pure mathematics but who wish to enlarge their understanding of applications and to find opportunities for research on applied problems. Thus, there will be some introductory material but taking account of the fact that the audience will have a high level of general mathematical sophistication. However, a research level will be maintained in almost all parts of the seminar.

Dormitory accommodations, and food service will be available on the campus. A complete brochure of information will be sent to the participants early in the spring.

Individuals may apply for admission to the seminar. Application blanks for admission and financial assistance can be obtained from the Meeting Arrangements Department, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940. The application deadline is March 17, 1975. An applicant will be asked to indicate his scientific background and interests; he should have completed at least one year of graduate school. A graduate student's application must be accompanied by a letter from his faculty advisor concerning his ability and promise. Those who wish to apply for a grant-in-aid should so indicate. However, funds available to the Seminar are limited and so individuals who can obtain support from other sources should do so.

1975 Summer Institute on Several Complex Variables
July 28 – August 15, 1975

The twenty-second Summer Research Institute of the American Mathematical Society will be devoted to the topic "Functions of Several Complex Variables". The Institute will be held on the campus of Williams College, Williamstown, Massachusetts. The Organizing Committee consists of Professors H. Grauert, R. C. Gunning (co-chairman), D. Lieberman, J. Morrow, R. Narasimhan, H. Rossi (co-chairman), Y. T. Siu, and R. O. Wells. It is expected that the Institute will be supported by a grant from the National Science Foundation.

The program will consist of three types of seminars: a series of expository talks on recent developments in special areas; a running seminar featuring half-hour talks; a collection of special one hour talks on topics of special significance. Tentatively, the topics will be "Deformation of complex spaces", "Partial differential equations in complex analysis", "Boundary values of holomorphic functions", "Analysis on noncompact Kahler manifolds", "Compact complex manifold", "Complex integral geometry", "Kernel functions and integral formulae", "Direct image theorems", and "Approximation theorems".

Information on travel, accommodations, and a complete brochure of information will be sent to participants in late spring.

Funds for participant support will be limited; it is anticipated that participants will have to find their own sources of travel support. The Institute is open to all mathematicians specializing in complex analysis in several variables and related topics, and to advanced graduate students in this field. Those wishing to participate should write to Dr. Gordon L. Walker, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940. Recent Ph.D.'s and advanced graduate students who wish to be considered for support should write before March 15, 1975.
INVITED SPEAKERS AT AMS MEETINGS

This section of these *Notices* lists regularly the individuals who have agreed to address the Society at the times and places listed below. For some future meetings, the lists of speakers are incomplete.

**Mobile, Alabama, March 1975**
- Richard F. Arenstorf
- R. C. Lacher
- Charles R. Hobby

**Monterey, California, April 1975**
- Isaac Namioka
- Kennan T. Smith

**New York, New York, March 1975**
- Herman Chernoff
- Peter J. Freyd

**St. Louis, Missouri, April 1975**
- A. O. L. Atkin
- Kenneth Kunen
- K. T. Chen
- Guido L. Weiss

**Pullman, Washington, June 1975**
- David W. Barnette
- Theodore E. Harris

**Chicago, Illinois, November 1975**
- Jonathan L. Alperin
- R. O. Wells, Jr.

ORGANIZERS AND TOPICS OF SPECIAL SESSIONS

Abstracts of contributed papers to be considered for possible inclusion in special sessions should be submitted to Providence by the deadlines given below and should be clearly marked "For consideration for special session on (title of special session)." Those papers not selected for special sessions will automatically be considered for regular sessions unless the author gives specific instructions to the contrary.

**Deadlines**

**Mobile, Alabama, March 1975**
- B. J. Ball, Shape Theory and Related Topics
- William H. Ruckle, Functional Analysis

**St. Louis, Missouri, April 1975**
- David Drasin, Classical Function Theory
- David L. Elliott, Differential Geometric Problems in Control Theory
- Franklin Haimo, Applications of Ring Theory to Groups
- Richard P. Jerrard, Geometric Topology
- Rangachary Kannan, Nonlinear Functional Analysis
- Walter Leighton, Ordinary Differential Equations: Oscillation Theory, Boundary Value Problems
- Marian Boykan Pour-El, Recursion Theory
- Grant V. Welland, Harmonic Analysis and Related Topics
- David J. Winter, Finite Dimensional Field Extensions

**Chicago, Illinois, November 1975**
- Saunders Mac Lane, Category Theory
CASE STUDIES
Some Mathematicians with Nonacademic Employment

Following are three more in the series of case studies of mathematicians with nonacademic or
nontraditional employment (see these Notices, November 1974). Case studies have been selected and
edited to minimize duplication (for example, almost all case studies emphasize communication skills),
and thus the series is best considered as a unit.

The Committee on Employment and Educational Policy seeks more case studies, particularly those
involving less traditional areas of employment for mathematicians. Members who wish to submit case
studies for consideration, or who have suggestions to make, are encouraged to communicate with
Professor Martha K. Smith, Department of Mathematics, University of Texas, Austin, Texas 78712.

ANONYMOUS

I have been employed for the past year and
a half as a state Educational Coordinator of Com­
proller's Data Processing. My primary respon­
sibility has been to develop and implement a data
processing training and education curriculum for
state data processing personnel. This has includ­
ed the development and presentation of courses
ranging from programmer and operator through
advanced systems analysis and management
training. Part of my duties are now managerial
since I have a staff of one full time assistant and
two half-time assistants. In addition to the above
teaching and curriculum development duties, I
act as a consultant to the director of data proces­
sing on special topics ranging from personnel re­
cruitment to short and long term planning for our
installation. I am also asked to help produce cur­
riculum recommendations in data processing for
the state educational institutions.

I completed my doctoral work in November,
1972, at a state university. My dissertation was
in module theory. Fortunately, my university re­
quires all graduate students to pursue a strong
minor outside of mathematics, which in my case
was computer science. During my last two years
of graduate school I did additional study in com­
puter science while completing my dissertation.
At the time I left graduate school, my course­
work background in computer science was nearly
equal that of doctoral students in that area.

I learned of my present position quite hap­
hazardly through my wife (a DP systems analyst
with a Master's degree in computer science). The
academic job market at the time I was seeking
employment (primarily through letters in re­
sponse to advertisements) was particularly dis­
appointing. The State was looking for someone
special, but they were not quite sure of the ne­
necessary requirements. After the initial interview
I had little difficulty becoming the top candidate.
Needless to say, the computer science back­
ground was the prime factor.

The two qualities most valuable in my work
are communication skills and the ability to work
with a wide variety of people from highly varied
educational backgrounds.

My work is highly challenging in terms of
organizing and developing massive amounts of in­
formation into material that is accessible and
meaningful to practicing DP professionals. The
work is rewarding in that I have been able to
bring increased performance to our installation
and I have been able to bring greater personal
satisfaction to many of our staff. I am also very
fortunate in that my mathematical training (cate­
gory theory and universal algebra) is now be­
coming useful in the formalization of theoretical
computer science. Unfortunately, my work al­
 lows almost no time for what might be called
'pure' research.

My main value to my employer is my ability to
attack problems logically, to organize material and
to communicate the results of my work. The work for
the doctorate and writing of my dissertation helped
to develop these skills, but it was the computer sci­
ence that gave me the leverage to enter the field.

My main advice to those seeking nonaca­
demic mathematics-related positions concerns
their orientation and outlook. Be fully prepared
to sell yourself by being prepared to tell a pros­
pective employer what you can do for him. Keep
your background reasonably broad and remember
the social and communicative skills. Be prepared
to go from the relative freedom of the academic
environment in planning and using your time to
the somewhat restrictive 40 to 50 hour work
week. You may also find it necessary to make a
financial sacrifice (relatively speaking) until
you can show your abilities and usefulness to
your employer.

GERALD A. MARIN
Center for Naval Analyses

The Center for Naval Analyses is an af­
fliliate of the University of Rochester that pro­
vides the Navy, Marine Corps and other spon­
sors with scientific support in the form of opera­
tions research and systems analysis. CNA con­
ducts an extensive study program in its home of­
cices at Arlington, Virginia and provides analyti­
cal services at operational and test commands
in the United States and overseas. Currently, I
am one of two company representatives on the
staff of the Commander Patrol Wings Pacific at
Moffett Field, California. Initially, most of my
work at CNA involved the assessment of the
relative strengths of forces on a theater level,
I led the second phase of a study of Total Allied
Forces in the Mediterranean. The study con­
siders the ability of U.S. Naval and Air Forces
to meet potential threats in the Mediterranean
region and measures the contribution of our al­
lies to U.S. capability in these potential con-
filets. More recently, I have been analyzing search problems in the antisubmarine warfare area.

I received the Master's and Ph.D. degrees from North Carolina State University at Raleigh, North Carolina. My major interest was probability theory and my thesis dealt with branching processes. I have not had occasion to do further work in my thesis area but the groundwork laid by the mathematics, probability, and statistics courses I took has been invaluable.

The following are examples of projects I have been able to undertake at CNA and during previous Oil Company experience:

- The development of a computer program to simulate competitive bidding for oil leases.
- The development of a straightforward algorithm to generate correlated n-dimensional normal noise on seismic traces (for signal detection research).
- The development of simple, computer-assisted Markov techniques to allow the analysis of the engagement of naval forces when traditional Markov techniques were unusable because more than a million "states" were possible.
- The development of simple geometrical techniques that can be used to approximate the shape of a target probability area after time lapse since contact has made an initial two-dimensional normal distribution inaccurate.

None of these projects can be said to have pushed back the frontiers of mathematics or statistics, but they are examples of how a mathematician/statistician in industry can contribute by analyzing and "making things work."

I was helped in my job seeking efforts (in late 1970) by a variety of summer jobs that I was fortunate enough to find. When I finished graduate school, my work experience included teaching calculus, teaching a computer lab, working as a telephone reporter for the Winston-Salem Journal, working as a summer actuarial trainee with Pilot Life Insurance Company, and doing analysis/programming for IBM for four consecutive summers. The summers that I spent away from school gave me an exposure to the industrial environment that was helpful when the time came to select courses and pick a major.

It is impossible to write a case history like this without stressing the importance of communication skills. Having now had a chance to review the work of other analysts, I find that the ability to write for and speak to people who know very little about a given problem (or solution techniques) is of paramount importance. Most graduate work stresses the production of elegant, concise technical articles that are written for one's mathematical peers. However, the people who will use most industrial analysis want to know what difference it will make to them and are content to have the mathematical developments relegated to appendices. It is not unusual for the industrial analyst to feel frustrated at doing work that his academic training has led him to feel is mundane. It is also not unusual for the academician to have difficulty in writing for a different kind of audience than his training prepared him for. A student can improve his writing skills by such basic things as working equations into complete, punctuated sentences, and professors can help by insisting on these basics.

My advice to student mathematicians who may be interested in industry is to obtain the broadest possible course background. If possible, pick a major and a thesis topic that you can discuss with nonmathematicians (who comprise the largest part of the personnel interviewers).

Teaching and academic research provide an opportunity for self-satisfaction that is quite different from that realized in industry. The academic mathematician struggles at the frontiers of his own intellect and knowledge to achieve the best result of which he is capable. The satisfaction for the industrial mathematician comes from making things work and from bringing life to mathematical techniques by applying them. I believe that most mathematics departments expose good students to the challenges of the academic researcher and teacher but that few departments are able to guide students toward nonacademic development. The interested student should thus independently look for mathematical problems in places other than the exercises at the end of a chapter. He should seek professors that will stress the importance of tackling "real" problems and of writing them up in a manner that assumes little.

EDWARD T. ORDMAN
Office of Business Development and Government Services
University of Kentucky

I received a Ph.D. from Princeton University in 1969. My thesis concerned subgroups of amalgamated free products of groups. I accepted an appointment as Assistant Professor of Mathematics at the University of Kentucky; during the following five years I wrote about a dozen papers (mainly about infinite groups and topologies on infinite groups) plus some shorter notes, book reviews, and so on. In 1973 I held a research grant (Fulbright) to the University of New South Wales, Sydney, Australia. During this time graduate enrollment in the Mathematics Department at Kentucky declined sharply, and I was one of a number of Assistant Professors who did not get tenure.

As a student I had worked for several summers as a computer programmer; while on the mathematics faculty at Kentucky I did some computer programming, e.g., writing economics-oriented "games" to play at a computer terminal, games designed principally to generate problems which students could then attempt to solve using (for instance) linear programming.

While I was job-hunting, the chairman of Kentucky's Computer Science Department told me that he had been approached by the Office of Business Development and Government Services, an extension (e.g., business consulting) and research (primarily contract research for government agencies) unit of the College of Business and Economics. I spoke to a representative of this office and found that their problems were in fact sufficiently complex and interesting to
require more than the computer programmer they
had been looking for. Accordingly, I was hired in
July 1974 with the title "Research Associate and
Computer Programmer".

The office has several quite large projects:
econometric modelling, income estimates, an
unusually elaborate developing cost-of-living
study. While at present none appear to me to be
near current areas of active mathematical re­
search, they are certainly close enough to the
fringes of econometric research and have definite
mathematical possibilities not previously ex­
plored here.

While I spend a large amount of time actu­
ally discussing mathematical questions with
economists, both in the Office of Business De­
velopment and Government Services and the De­
partment of Economics, it will be obvious to the
reader that few of these relate to my previous
published mathematical research. Part of my
value to the Office lies in the fact that I spent
five years in the Mathematics Department here
and am able to interact as necessary with this
university's applied mathematicians, computer
scientists, and statisticians. Part of the attrac­
tiveness of the position is that I remain on the
same campus, interacting with many of the same
people (both in connection with my new duties,
and in continuing on my own my former line of
research).

Being "the" mathematician in an organiza­
tion with a very large range of activities leads
to quite a few demands on my time, but there is
adequate opportunity to pick and choose which
aspects I want to get deeply involved in or pursue
as research opportunities. Four months on the
job, of course, is too short a time to make per­
manent judgements: I do not yet know whether I
will find this a satisfactory situation to carry on
mathematical work over the long term, or wheth­
er I will eventually return to a more traditional
position with the added experience of having held
an essentially nonacademic position that was,
however, located on campus. My teaching ex­
perience, incidentally, has in my view been es­
sential to this position; a great deal of my time
is spent explaining a large variety of mathemati­
cal phenomena to nonmathematicians.
The Council of the AMS instructed the secretary to initiate the following sequence of correspondence and has requested that it be published.

June 3, 1974

His Excellency Walter Heitmann, Amb. E. and P.
The Embassy of Chile
2305 Massachusetts Avenue, N. W.
Washington, D. C. 20008

Your Excellency:

Members of the Council of this Society have heard distressing news of the situation of Professor Galo Gomez Oyarzun, formerly Vice-rector of the University of Concepcion, whose work is well known to some of them.

The Council has therefore instructed me to request that you tell me where he now is and what is the state of his health. I shall then report on these matters to the Council.

Yours respectfully,

Everett Pitcher

M. Everett Pitcher
Secretary
American Mathematical Society
Lehigh University
Bethlehem, Pennsylvania 18015

Dear M. Pitcher:

I acknowledge receipt of your letter dated June 3 by which you inquire about the status of Professor Galo Gomez Oyarzun, formerly Vice-rector of the University of Concepcion.

As the information you received undoubtedly belongs to the organized campaign to mislead the opinion of the american citizens and create adverse feelings and impressions of the latest social and political events that have taken place in Chile, I would like to know the source of the "distressing news of the situation of Professor Galo Gomez Oyarzun".

I would recommend you to propose to the Council to write directly to the Director of the University of Concepcion, as I feel he will be able to give you complete and detailed information.

Sincerely,

Walter Heitmann
Ambassador

July 12, 1974

The Director
The University of Concepcion
Concepcion, CHILE

Dear Sir:

At the suggestion of His Excellency Walter Heitmann, I am writing for the Council of the American Mathematical Society to inquire about the location and the state of health of Professor Galo Gomez Oyarzun. The ambassador tells me that you will be able to give us complete and detailed information.

Yours sincerely,

Everett Pitcher

Concepcion, 29 de Julio de 1974

Señor

Everett Pitcher
Secretario de la Lehigh University
Bethlehem, Pennsylvania 18015

Señor Secretario:

Se ha recibido en esta Universidad su comunicación de fecha 12 de Julio del año en curso, por la que solicita antecedentes sobre el profesor Galo Gomez Oyarzun, a sugerencias del señor Embajador don Walter Heitmann.

Al respecto, puedo informarle que el profesor Gomez presentó su renuncia indeclinable al cargo que servía en la Universidad con fecha 30 de Diciembre de 1973, la que le fue aceptada por la autoridad universitaria con fecha 9 de Enero de 1974, y a partir del día 15 del mismo mes y año.

Por informaciones periodísticas y de amigos del profesor Gomez se supo que fue detenido por orden de la autoridad pública, que se encuentra relegado en el pueblo de Chacabuco, provincia de Antofagasta, y en buenas condiciones de salud.

Es todo cuanto puedo informarle, por no tener la Universidad antecedentes oficiales en su documentación sobre el particular.

(English Translation)

We have received at this university your letter of July 12, 1974 in which, at the suggestion of Ambassador Walter Heitmann, you request information concerning Professor Galo Gomez Oyarzun.

With regard to this matter, we can inform you that Professor Gomez resigned irrevocably from his position at this university on December 30, 1973. His resignation was accepted by the university authorities on January 9, 1974, to take effect January 15.

Via the newspapers and from friends of Professor Gomez, it has been learned that he was arrested by the public authorities, exiled to the town of Chacabuco in the province of Antofagasta, and is in good health.

This is all we can communicate, since the university does not have any official information in its file on this matter.

Saluda atentamente a Ud.

Gustavo Villagrán Cabrera
SECRETARIO GENERAL
Persons in whose information I have complete trust have assured me that this town is in fact an abandoned mining camp which was re-inhabited by the current authorities in Chile for use as a prison camp. At present many political (and perhaps other) prisoners are held there, and the "town" is used for no other purpose.

Sincerely yours,

C. Herbert Clemens
NATIONAL SCIENCE FOUNDATION BUDGET

On February 3, 1975, the National Science Foundation released its annual budget report. Accounts of previous such reports, with observations on the trend of NSF support for mathematics, have appeared in these Notes in February 1972 (pages 126–127), April 1973 (pages 135–137), and April 1974 (pages 152–153).

Last year the NSF budget request for the fiscal year 1975 (July 1, 1974, through June 31, 1975) was increased at the last minute from $672.1 million to $788.2 million by addition of a so-called 'energy amendment' amounting to $116.1 million. Fiscal 1975 funding was subsequently reduced to $768.8 million, and on January 1, 1975, $51.7 million of this amount was transferred to the newly created Energy Research and Development Administration. Therefore, the estimated total for fiscal 1975 is $697.1 million, only 8% above the actual expenditures for fiscal 1974.

The accompanying tables give comparative figures for fiscal years 1971 through 1976. Actual figures are available for 1971 through 1974, current estimates are given for fiscal 1975 (now half over) and budget requests are given for fiscal 1976.

Table I presents data showing the portion of NSF funds going for support of mathematics. Table II compares support for the Mathematical Sciences Section with the other sections comprising the Division of Mathematical and Physical Sciences, and Table III compares the fraction of the total of SRPS funds going to the Mathematical Sciences Section and to the Division of Mathematical and Physical Sciences.

Line (6) of Table I shows that support for mathematics research, as a fraction of the total research support, will be lower in fiscal 1976 than it has been for the past five years. Tables II and III show that on a relative basis support for the Mathematical Sciences Section is steadily decreasing, both as a fraction of the Division budget and as a fraction of total SRPS funds.

### TABLE I: NATIONAL SCIENCE FOUNDATION

(Millions of Dollars)

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<tbody>
<tr>
<td>(1) Mathematics Research Support</td>
<td>$ 12.9</td>
<td>6.2%</td>
<td>$ 13.7</td>
<td>2.9%</td>
<td>$ 14.1</td>
<td>2.8%</td>
<td>$ 14.5</td>
<td>14.5%</td>
<td>$ 16.7</td>
<td>8.4%</td>
<td>$ 18.0</td>
</tr>
<tr>
<td>(2) Other Research Support (Note A)</td>
<td>315.5</td>
<td>37.9%</td>
<td>435.0</td>
<td>11.1%</td>
<td>483.5</td>
<td>1.7%</td>
<td>491.5</td>
<td>12.2%</td>
<td>551.5</td>
<td>15.0%</td>
<td>634.4</td>
</tr>
<tr>
<td>(3) Education, Information, Foreign Currency Program (Note B)</td>
<td>145.9</td>
<td>-12.7%</td>
<td>127.4</td>
<td>-40.0%</td>
<td>84.1</td>
<td>24.1%</td>
<td>104.4</td>
<td>-19.3%</td>
<td>84.3</td>
<td>0.0%</td>
<td>84.3</td>
</tr>
<tr>
<td>(4) Program Development and Management (&quot;Overhead&quot;) (Note C)</td>
<td>21.8</td>
<td>12.8%</td>
<td>24.6</td>
<td>16.3%</td>
<td>23.6</td>
<td>23.1%</td>
<td>35.2</td>
<td>27.0%</td>
<td>44.7</td>
<td>2.2%</td>
<td>45.7</td>
</tr>
<tr>
<td>(5) Totals</td>
<td>$496.1</td>
<td>21.1%</td>
<td>$600.7</td>
<td>1.6%</td>
<td>$610.3</td>
<td>5.8%</td>
<td>$645.6</td>
<td>8.0%</td>
<td>$697.1</td>
<td>11.2%</td>
<td>$775.4</td>
</tr>
<tr>
<td>(6) (1) as % of (1) &amp; (2)</td>
<td>3.93%</td>
<td>3.05%</td>
<td>2.78%</td>
<td>2.28%</td>
<td>2.87%</td>
<td>2.39%</td>
<td>2.39%</td>
<td>2.39%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7) (1) as % of (5)</td>
<td>2.60%</td>
<td>2.25%</td>
<td>2.27%</td>
<td>2.25%</td>
<td>2.36%</td>
<td>2.32%</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**NOTE A:** Scientific research and facilities (excluding mathematics), national and special research programs excluding science information activities, national research centers, and research applied to national needs. Support for mathematics has been excluded, cf. Items (1) and (3).

**NOTE B:** The programs in this group are ones in which there is support for projects in every field, including mathematics. The foreign currency program involves both cooperative scientific research and the dissemination and translation of foreign scientific publications. Foreign currencies in excess of the normal requirements of the U.S. are used.

**NOTE C:** This heading covers the administrative expenses of operating the Foundation; the funds involved are not considered to constitute direct support for individual projects. The 1974, 1975 and 1976 figures include science advisory activities ($3.5, $6.0, $4.0 million, respectively).

### TABLE II: MATHEMATICAL AND PHYSICAL SCIENCES DIVISION

(Millions of Dollars)

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Sciences</td>
<td>$ 12.5 (19.6%)</td>
<td>$ 15.7 (17.2%)</td>
<td>$ 14.1 (17.0%)</td>
<td>$ 14.5 (16.6%)</td>
<td>$ 16.6 (16.0%)</td>
<td>$ 18.0 (15.2%)</td>
<td></td>
</tr>
<tr>
<td>Physics</td>
<td>26.5 (40.3%)</td>
<td>35.3 (41.1%)</td>
<td>34.9 (42.1%)</td>
<td>36.7 (43.2%)</td>
<td>42.6 (41.2%)</td>
<td>49.2 (45.1%)</td>
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</tr>
<tr>
<td>Chemistry</td>
<td>19.6 (28.8%)</td>
<td>24.5 (30.8%)</td>
<td>25.1 (30.5%)</td>
<td>26.6 (30.5%)</td>
<td>35.5 (32.4%)</td>
<td>39.5 (33.3%)</td>
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<tr>
<td>Astronomy</td>
<td>6.7 (10.2%)</td>
<td>8.0 (10.1%)</td>
<td>8.5 (10.6%)</td>
<td>9.3 (10.7%)</td>
<td>10.8 (10.4%)</td>
<td>11.8 (10.0%)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$ 65.7</td>
<td>$ 79.5</td>
<td>$ 82.9</td>
<td>$ 87.1</td>
<td>$ 103.5</td>
<td>$ 115.5</td>
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</tr>
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</table>

### TABLE III: SCIENTIFIC RESEARCH PROJECT SUPPORT

(Millions of Dollars)

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics Sciences</td>
<td>$ 12.9 (7.2%)</td>
<td>$ 15.7 (5.5%)</td>
<td>$ 14.1 (6.1%)</td>
<td>$ 14.5 (5.0%)</td>
<td>$ 16.6 (4.9%)</td>
<td>$ 18.0 (4.7%)</td>
<td></td>
</tr>
<tr>
<td>Physics</td>
<td>65.7 (36.4%)</td>
<td>79.5 (32.0%)</td>
<td>89.9 (29.9%)</td>
<td>87.1 (30.1%)</td>
<td>103.5 (30.4%)</td>
<td>118.5 (31.2%)</td>
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</tr>
<tr>
<td>Total</td>
<td>$180.4</td>
<td>$248.6</td>
<td>$277.3</td>
<td>$289.8</td>
<td>$340.6</td>
<td>$380.0</td>
<td></td>
</tr>
</tbody>
</table>
LETTERS TO THE EDITOR

Editor, the Notices

Last year I wrote to AMS officials suggesting that the Bulletin, the Proceedings, and the Transactions be merged into a single monthly journal with each issue devoted exclusively to one specific area of mathematics—logic, algebra, topology, etc. AMS members would then have the option of choosing two issues of the journal free of charge, paying extra for any additional issues they might wish to order. This arrangement would reduce publication costs and at the same time provide more rewarding service to members. It is a delusion to think that we can keep abreast of developments in all mathematical areas by publishing comprehensive mathematical journals. The era of such an approach is over forever. Let us face it, the amount of material that an individual reader finds interesting or at least readable, let alone usable, in each comprehensive journal is minimal.

As an alternative proposal last year, just in case the AMS administration were to find it impossible to abandon a format that has been operative for so long, I also suggested that members might at least be allowed to choose between receiving a subscription to the Bulletin or receiving instead two sections of the Mathematical Reviews that are now offered separately at extra cost. Personally, I would rather have reviews of papers in my own and related fields than most of the articles in the Bulletin.

My suggestions drew courteous responses from AMS officers in which I was told that in the past proposals such as mine had been considered and rejected, that the matter had been raised at the 1974 Council meeting and no action taken here to write individually or collectively to society officials requesting a poll of the members. It is thus a very valuable service which the journal "Linear and Multilinear Algebra" has performed by publishing an article entitled Six Letters from Alexander C. Aitken [Vol. 2, pp. 1-12] arranged by Professor Henryk Minc. These letters, written by the late Professor Aitken to his colleague Professor I. M. H. Etherington, deal with a problem suggested by Professor Marvin Marcus and give a "blow-by-blow" account of Professor Aitken's attempts at a solution, culminating in final success. As such, they provide a rare and stimulating glimpse of the workings of a first-rate mathematical mind.

The mathematical community should be thankful to Professor Minc for preserving these letters and arranging for their publication. It ought likewise to encourage others to do the same. Perhaps the American Mathematical Society could take the initiative of establishing a depository for similar examples of mathematics in action and arranging for their eventual publication in some suitable framework.

Jonathan S. Golan

Editor, the Notices

As we all know, the process of doing mathematics involves, in the end, concealment of one's tracks. Published mathematical papers rarely hint at the method by which the results were obtained or at the process of thought behind them. It is thus a very valuable service which the journal "Linear and Multilinear Algebra" has performed by publishing an article entitled Six Letters from Alexander C. Aitken [Vol. 2, pp. 1-12] arranged by Professor Henryk Minc. These letters, written by the late Professor Aitken to his colleague Professor I. M. H. Etherington, deal with a problem suggested by Professor Marvin Marcus and give a "blow-by-blow" account of Professor Aitken's attempts at a solution, culminating in final success. As such, they provide a rare and stimulating glimpse of the workings of a first-rate mathematical mind.

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Jonathan S. Golan

Editor, the Notices

I am writing as a mathematician and as a member of the Committee of Concerned Scientists. The Committee, a New York City based organization (505 Park Avenue), is concerned with the (lack of) rights of certain groups of Soviet scientists, including the right to emigrate. A disturbing pattern has emerged over the past few years. A large number of scientists, mostly Jewish, have lost their jobs and have been ostracized from the scientific community after they had applied for exit visas. In order to alleviate their professional isolation, several of these scientists in Moscow are attempting to organize a weekly seminar in their homes and thereby establish urgently needed contacts with visiting scientists. The organizers of the seminar (who include Drs. M. Zabel and A. Voronel) have asked that the following announcement be made.

THE MOSCOW SEMINAR ON COLLECTIVE PHENOMENA cordially invites the participation of visitors interested in a broad range of topics including physics, mathematics, chemistry, economics, cybernetics, linguistics and biophysics. The seminars are held every Sunday at noon, in the apartment of I. and V. Brailovsky, pr. Vernadskogo 99, korp (building) No. 1, kv (apt.) 123. The address may be reached by taxi or by 10 minutes walk from Metro Station Yugo-zapadnaya. Visitors are invited to present papers, and

F. G. Asenjo

These suggestions and other proposals for restyling the journals of the Society, particularly the Proceedings and the Transactions, continue to be studied by the respective Editorial Committees and other interested groups.

E. P.
are asked to visit the above address the day before the seminar to communicate title and abstract.

Richard S. Ellis

Editor, the Notices

I strongly object to having different fees for people on different salaries. This is not an income tax!

Nathan J. Divinsky

Editor, the Notices

In November 1974, the United Nations Educational, Scientific, and Cultural Organization decided to exclude Israel from any of its regional groupings, as well as depriving Israel of some UNESCO funds. These actions violate the stated purpose of UNESCO, which is "to promote collaboration among nations through education, science and culture in order to further justice, rule of law, and human rights and freedoms without distinction of race, sex, language, or religion." Many people actively concerned with the purposes stated above have strongly protested the UNESCO action and refused to work with UNESCO until it rescinds its actions against Israel. I urge members of the mathematical community to join in this protest and withhold their cooperation from UNESCO until it again treats the nation of Israel the same as all other nations.

Barbara L. Osofsky

EDITORIAL COMMENT: The letter above is published by request of the Council. The Secretary notes that a motion protesting the action of UNESCO was introduced, with the required advance notice, into the meeting of the Council of 22 January 1975. Article IV, Section 8, of the bylaws provides that the vote of the Council shall be taken by mail not earlier than one month after the debate and requires a two thirds vote for passage. The vote on the procedural question of instructing the Secretary to conduct the mail ballot failed, with 13 11/12 votes in favor and 14 1/12 votes opposed. The fractions arise in the voting mechanism described in Article IV, Section 4.

A procedural resolution before the Council to put a similar motion of protest before the Business Meeting of 24 January for final action at that meeting, in the manner described in the exception in Article X, Section 1, of the bylaws, was tabled. (On the other hand, a procedural resolution at the Business Meeting of 24 January to put a motion of protest on the agenda of the Business Meeting of August 1975 in Kalamazoo was passed.)

An account of the dispute is published in Science, 186, 1100. The reader is cautioned that there is not agreement among opposing partisans that the account correctly represents the situation.

The Secretary has been requested to seek information about the dispute from the Director General of UNESCO and from the Chairman of the Israel Mathematical Union. When and if both are received, consideration is to be given to publishing them in these Notices.

Editor, the Notices

The economic "problem in a new era of slow (or no) growth", as phrased in Recommendation 6 of the report of the National Board on Graduate Education (these Notices 21 (1974), 221), suggests to me an opportunity which might be responsive not only to that problem, but also to an undesirable situation of long standing:

The fresh Ph.D., eager to exploit the ideas which culminated in his dissertation, is often forced into an unattractive choice as he begins his full-time teaching career—either he does his undergraduate teaching in an off-hand way, or (more likely, I think) he puts his research "on the back burner" for a couple of years while he wrestles conscientiously with the difficult task of developing into an effective teacher.

A program of two- or three-year "postdoctoral teaching fellowships" or "research instructorships", during which the instructors would teach approximately half-time loads while devoting the remainder of their energies to vigorous prosecution of their research, might help develop some outstanding scholar-teachers. Indeed, at a number of universities there are named instructorships which provide essentially this opportunity. If the AMS Research Fellowship Fund were to be used to provide the "research half" of the fellows' stipends, leaving the colleges and universities to provide the "teaching half", the Fund could be used to provide leverage for the productive employment of more able young mathematicians than the institutions can hire without such financial help.

R. A. Rosenbaum

Editor, the Notices

If a part of the AMS Research Fellowship Fund, or other sources of support, were available for the purpose described above, would your institution in principle be interested in participating? If so, the AMS Committee on Postdoctoral Fellowships would be very happy to hear from you and to receive any suggestions you may have. It is understood, of course, that any changes which the Committee might recommend be made in the use of the Research Fellowship Fund would not affect the use of the Fund for 1975-1976.

Alice T. Schafer, Chairman
AMS Committee on Postdoctoral Fellowships

Editor, the Notices

In view of the fact that many mathematicians feel a moral obligation to contribute to the development of mathematics in developing countries and that there are indications of improving U.S.-Cuban relations it may be interesting for members of the AMS to get some information on mathematics in Cuba. The following information was obtained during a 7-week stay in La Habana in 1973, when I was there to teach a course on qualitative theory of dynamical systems. The visit was partially financed by the International Mathematical Union and organized by a committee for scientific exchange with Cuba.
The general situation of mathematics in Cuba is characterized by the following facts. In 1959, the year of the revolution, there was practically no mathematics in Cuba. Since then mathematics departments have been built from scratch in all three universities. They train mathematicians for the tertiary educational system and for employment in industry, administration, computation centers, etc. The department which functions best is the one in La Habana. About 150 students have graduated from there since 1966. A curriculum has been set up and young lecturers have been trained who are able to give the students a solid and moderately modern basic mathematical training. Research is just beginning. The important role of scientific research for the cultural and economic development of the country is increasingly being recognized. This holds also for basic research, which must be combined with applied research and must be oriented in accordance with the strategy of development of the country. Priorities for the research in mathematics are being chosen in agreement with these conditions.

Since the department of mathematics of the University of La Habana (Escuela de Matematica de la Universidad de La Habana) is the one which functions most efficiently it is also the one for which foreign aid will be most efficient. Therefore I would like to mention some details on this department. At this time there are about 300 students of mathematics. The teaching staff has about 40 members. The head of the department at this time is Francisco Guerra. The department is divided into 4 groups: (i) Pure mathematics, (ii) numerical analysis, (iii) statistics, (iv) operations research. There exists a library which has a very small but reasonably good selection of textbooks—most of them American and donated by visiting mathematicians. There is a good selection of journals, but for most of them there are big gaps or only the latest volumes exist.

The department teaches a 5-year course, and the students who finish it are "licenciados". Most members of the staff are holding this degree. The emphasis in the course is on analysis and algebra—geometry is only represented by analytic geometry, topology and a course on manifolds. The teaching staff has about 40 members. The head of the department at this time is Francisco Guerra. The department is divided into 4 groups: (i) Pure mathematics, (ii) numerical analysis, (iii) statistics, (iv) operations research. There exists a library which has a very small but reasonably good selection of textbooks—most of them American and donated by visiting mathematicians. There is a good selection of journals, but for most of them there are big gaps or only the latest volumes exist.

The progress made so far in the development of mathematics in Cuba would have been impossible without the help of foreign mathematicians, mostly from socialist countries and Western Europe, especially France and West Germany. For the future development, Cuba will have to rely mostly on help from socialist countries. But mathematicians from other countries can also make a contribution. One thing they can do is to send books and journals, and if somebody wants to do this I shall be glad to get him lists of material needed in Cuba. The other possibility for a mathematician would be to go to Cuba in order to teach students and/or faculty or to do research on one of the projects of the department. However, although this kind of aid is particularly valuable, Cuba has only a very limited capacity for receiving such aid. Moreover, scientific exchange with the U.S. is extremely difficult under the present circumstances. Finally, such a visit will only be useful if a number of conditions is satisfied: The visiting mathematician should be a specialist in one of the fields mentioned above. He must have a very good general knowledge of his field, plenty of research experience and a good general mathematical culture. He should be willing to communicate, to give guidance to those who make their first steps in research, and to continue the work begun in this department. Too frequent reorientations by foreign mathematicians will be counterproductive. He should be willing to work hard under conditions he is not used to, and he should have sympathy for the revolutionary effort of the Cuban people. It would be good if he speaks Spanish, but English might do in the department of mathematics. If a mathematician with these qualifications is willing to go to Cuba, I shall be glad to try to establish contacts with the Cuban mathematicians.

E. Brieskorn
Mathematisches Institut
der Universität Bonn

Editor, the Notices

In case people really want to publish mathematics more cheaply, I suggest a simple method of economizing: stop trying to impose artificial and pedantic standards on authors. Nobody except some eagle-eyed precisians in the editorial office will ever notice whether the author says "two-dimensional" or "two-dimensional" or "2-dimensional", or whether the bibliography has the date before or after the page numbers, or whether cited authors' initials come before or after their names. One journal insists on numbering theorems, lemmas, definitions, and formulas in separate series, whereas I prefer to number everything consecutively in one series (which makes it easier to locate things); its copy editor spent much (expensive, I hope) time changing my typescript to suit the journal's style, introducing numerous errors in the process. My suggestion is just that journals restrict copy editing to the necessary minimum of identifying symbols and simplifying typography, but otherwise leave the authors' work strictly alone. The difference will not be noticed by the readers, and the savings will be appreciable.

R. P. Boas
PERSONAL ITEMS

JAMES D. HALPERN of the University of Toledo has been appointed to a professorship at the University of Alabama, Birmingham.

B. FRANK JONES of Rice University has been named Noah Harding Professor of Mathematics. He has three times won the George R. Brown Award for Superior Teaching.

SAMUEL KOTZ of Temple University has been elected a member of the International Statistical Institute.

KENNETH O. LELAND of the Air Force Aerospace Research Laboratories, Dayton, Ohio, has been appointed a mathematician at the Navy Personnel Research and Development Center, San Diego.

MORDECHAI LEWIN of the Israel Institute of Technology has been named the first recipient of the Mahler Prize for research in Pure Mathematics.

M. HUGH MILLER, JR., of the University of Alabama, Tuscaloosa, has been appointed to a visiting assistant professorship at the University of Mississippi.

VIJAY K. ROHATGI of Bowling Green State University has been elected to an ordinary membership in the International Statistical Institute.

ALEXIA SONTAG of the University of Minnesota, Morris, has been appointed to an assistant professorship at Mount Holyoke College.

ELIAS M. STEIN of Princeton University has been named to hold the Albert Baldwin Dod Professorship of Mathematics.

VENKATA R.R. UPPULURI of Union Carbide, Oak Ridge, Tennessee, was appointed to a visiting professorship at the Istituto di Matematica, Università di Genova, Italy, for the month of November, 1974.

W. ROY WESSEL of Florida State University has been appointed a visiting scientist at the National Center for Atmospheric Research, Boulder, Colorado.

PROMOTION

To Chairman, Department of Mathematics and Associate Professor, Mount Holyoke College: HARRIET S. POLLATSEK.

To Professor, Mount Holyoke College: LESTER J. SENECHAL; Stevens Institute of Technology: STEPHEN L. BLOOM.

To Associate Professor, Stevens Institute of Technology: ROBERT H. GILMAN, CHARLES L. SUFFEL, MARVIN D. TRETKOFF; University of Bridgeport: HERBERT KOLLER.

DEATHS

Dean Emeritus EDWIN B. ALLEN of Rensselaer Polytechnic Institute, Graduate School, died in October, 1974, at the age of 76. He was a member of the Society for 45 years.

Professor ZIPORA ALTERMAN of the Tel-Aviv University died in April, 1974, at the age of 48. She was a member of the Society for 16 years.

Professor Emeritus MYRTIE COLLIER of the Immaculate Heart College died on June 25, 1974, at the age of 96. She was a member of the Society for 58 years.

Professor MARY J. COX of Chevy Chase, Maryland, died on October 9, 1974. She was a member of the Society for 41 years.

Professor HAROLD T. DAVIS of Northwestern University died on November 14, 1974, at the age of 82.

Mr. RALPH KEFFER of West Hartford, Connecticut, died on August 23, 1974, at the age of 83. He was a member of the Society for 57 years.

Professor K. V. RAJESWARA RAO of Purdue University died on July 29, 1974, at the age of 37. He was a member of the Society for 11 years.

Professor JOHN H. RAYMOND of Saint Martin's College died on December 7, 1974, at the age of 64. He was a member of the Society for 31 years.

Professor ROGER F. SCOTT of West Lafayette, Indiana, died on October 4, 1974, at the age of 69. He was a member of the Society for 12 years.

Dr. LAWRENCE SHULMAN of New York City, formerly of Georgetown University, died on January 6, 1975, at the age of 93. He was a member of the Society for 7 years.

Professor Emeritus HAROLD SIMPSON of Bedford College, London, England, died on April 4, 1974, at the age of 97. He was a member of the Society for 50 years.

ERRATUM

In the November 1974 issue of these Notices it was announced that GARY E. STEVENS had been appointed to an instructorship at Hartwick College. The item should have read that Professor Stevens had been appointed to an assistantship professorship at Hartwick College.
NEWS ITEMS AND ANNOUNCEMENTS

CALL FOR TOPIC SUGGESTIONS AND EDITORS
FOR COLLECTIONS OF REVIEWS

The AMS Committee to Monitor Problems in Communication is interested in receiving topic suggestions and names of editors and compilers for collections of reviews to be considered for publication by the Society. To date, review collections covering the fields of algebraic topology, finite groups, infinite groups, and number theory have appeared; and a collection on differential geometry, to be edited by Michael Spivak, has been approved by the Council of the AMS. Any well defined field that would be valuable to research, and especially to applied mathematicians, will be considered. The field may be much more narrowly defined than those collections so far published. Suggestions of topics and of people capable of editing and compiling such collections should be sent to Leonard Gillman, Chairman of the Committee to Monitor Problems in Communication, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

Listed below is price information regarding collections that have already appeared. Orders may be sent to the Order Department, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

<table>
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<tr>
<th>Reviews of Papers in Number Theory edited by William J. LeVeque</th>
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<td>50</td>
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<td>Volume 6</td>
<td>40</td>
<td>28</td>
<td>16</td>
<td>8</td>
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<tr>
<td>Complete set Volumes 1-6</td>
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<td>133</td>
<td>76</td>
<td>38</td>
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</table>

| Reviews of Papers in Finite Groups edited by Daniel Gorenstein | 50 | 35 | 20 | 10 |

| Reviews of Papers in Infinite Groups edited by Gilbert Baumslag Parts I and 2 | 70 | 49 | 28 | 14 |

| Combined set of three volumes (Infinite Groups 1 and 2 and Finite Groups) | 100 | 30 | 40 | 20 |

| Reviews of Papers in Algebraic and Differential Topology, Topological Groups, and Homological Algebra edited by Norman E. Steenrod | 27.10 | 20.33 | 20.33 | 10.50 |

NOTICE CONCERNING THE PROCEEDINGS

In action of January 22, 1975, the Council of the Society directed the Proceedings Editorial Committee to upgrade the content of the journal. Effective immediately, Proceedings referees and editors will be asked to act in accord with the following statement of policy.

To be published in the Proceedings, a paper must be correct, new, nontrivial and significant. Further, it must be well written and of interest to a substantial number of mathematicians. Piecemeal results, such as an inconclusive step toward an unproved major theorem or a minor variation on a known result, are in general not acceptable for publication. Proceedings Editors shall solicit, and encourage publication of, worthy papers of length not exceeding twelve typed pages.

In further action, the Council selected the Proceedings for a two-year experiment with "blind refereeing". Effective with the publication of these Notices, authors submitting a paper to the Proceedings shall send two manuscripts: one containing all pertinent personal data and the other not containing the author's name or institutional affiliation. This second copy will be refereed, and the Editor shall not reveal the name or institution of the author to the referee. It shall not be the responsibility of the Editor to suppress evidence internal to the paper from which the referee might guess the author's identity.

ELBERT F. COX SCHOLARSHIP FUND

The Department of Mathematics of Howard University announces with pride the establishment of the Elbert F. Cox Scholarship Fund for undergraduate mathematics majors. Dr. Elbert F. Cox (1895-1969) was the first Black to receive a Doctor of Philosophy degree in mathematics. He received the Ph.D. from Cornell University in 1925. Dr. Cox was a member of the Howard mathematics department from 1929 through 1966 and was chairman of the department from 1947-1961. Dr. Cox was a member of the Society for 42 years, 1925-1967.
NEW AMS COMMITTEES

Committee on Editorial Policy. Edwin Moise has accepted the appointment of Chairman of the Ad Hoc Committee on Editorial Policy. Members of this newly formed committee are: John W. Green, Paul R. Halmos, Irwin Kra, and Everett Pitcher (ex officio).

Committee on Science Policy. President Saunders Mac Lane, as authorized by the Council, has appointed R. H. Bing, Edwin E. Moise, and George D. Mostow (chairman) as the Ad Hoc Committee on Science Policy. However, reconstitution of the Committee is planned.

Committee on Committees. An Ad Hoc Committee on Committees has been formed by President Lipman Bers to examine the selection, activity and function of Society committees. Its members, Jonathan L. Alperin, Phyllis J. Cassidy, Chandler Davis, F. Reese Harvey, Nathan Jacobson, Everett Pitcher (ex officio), P. Emery Thomas (chairman), Karen Uhlenbeck, and J. Ernest Wilkins, Jr., have been charged to report their findings to the Council in August, 1975.

Committee on Budgeting Procedure and Cost Effectiveness. In response to a recommendation by the Committee on Employment and Educational Policy, President Lipman Bers charged a Committee on Budgeting Procedure and Cost Effectiveness. Appointed to this committee are: Felix Browder, Calvin C. Moore (chairman), Everett Pitcher, Halsey L. Royden, and Kenneth G. Wolfson.

AMS-SIAM Committee to Select the Winner of the Wiener Prize for 1975. President C. C. Lin of the Society for Industrial and Applied Mathematics and President Saunders Mac Lane have jointly appointed Gustave A. Hedlund, Jurgen K. Moser (chairman), and Robert D. Richtmyer to the AMS-SIAM Committee to Select the Winner of the Wiener Prize for 1975.

RECENT AMS APPOINTMENTS

Trustees Committee on Russian Translations. The President Saunders Mac Lane, in 1974, appointed Alex Rosenberg as a member of the Trustee Ad Hoc Committee on Russian Translations. Abraham Taub will serve as chairman and Hyman Bass as Vice Chairman. The other members are Henry A. Antosiewicz, Ronald Douglas, and Garth Warner.

Committee on The Emergency Employment Situation in Mathematics. John R. Isbell has been appointed by the President of the Society to the Ad Hoc Committee on the Emergency Employment Situation in Mathematics. The Chairman is Charles W. Curtis and the other members of the Committee are: Michael Artin, David Gale, William J. Gordon, Judy Green, Judah Rosenblatt, and William Singer.

AMS-IMS Committee on Translations from Russian. Felix R. Albrecht and Izaak Wajszup have been appointed to the AMS-IMS Joint Committee on Translations from Russian for a term extending through June 30, 1976. Continuing members are Edwin Hewitt, Chairman (1975), Ivo Babuška (1975), Kevin M. McCrimmon (1976), Donald L. Burkholder (indefinite), and Judah I. Rosenthal (indefinite).

Committee on Steele Prizes. Joseph Shoenfield and Frank Spitzer have been appointed to the Committee on Steele Prizes. Hans F. Weinberger is serving as chairman with Edward B. Curtis, Harley Flanders, Paul R. Halmos, H. A. Helmborn and Edwin Hewitt as continuing members.

Committee to Select Hour Speakers for Western Sectional Meetings. The President of the Society has appointed Richard G. Swan to the Committee to Select Hour Speakers for Western Sectional Meetings. The chairman of the Committee is Paul Bateman and the continuing member is Richard A. Askey.

COMMITTEE ON COMMITTEES

President L. Bers has appointed a Committee on Committees, whose charge is to consider: (1) the activity and function of Society committees; (2) the way the Society nominates, selects, and appoints its officers and the members of its committees. Members of the Society are invited to send in written comments concerning the above topics. Such comments may be sent (preferably by April 1, 1975) either to the following address or to any one of the committee members (listed below): Committee on Committees, American Mathematical Society, P.O. Box 6245, Providence, Rhode Island 02940.

The committee will present its recommendations to the Council of the Society. Members of the committee are: Johnathan L. Alperin (Department of Mathematics, University of Chicago); Chandler Davis (Department of Mathematics, University of Toronto); Phyllis J. Cassidy (Department of Mathematics, Smith College); F. Reese Harvey (Department of Mathematics, Rice University); Nathan Jacobson (Department of Mathematics, Yale University); Everett Pitcher (Department of Mathematics, Lehigh University); P. Emery Thomas, Chairman (Department of Mathematics, University of California, Berkeley); Karen Uhlenbeck (Department of Mathematics, University of Illinois, Urbana); J. Ernest Wilkins, Jr. (Department of Physics, Howard University).

ONR RESEARCH GRANTS

Unfortunately the Office of Naval Research was not included in the Grants and Stipends section of the December Notices. The Office of Naval Research does support research over a wide range of areas including mathematics, operations research, statistics and probability, information systems, fluid dynamics and structural mechanics. Proposals for research contracts and requests for information on ONR Programs should be addressed to: The Director, Mathematical and Information Sciences Division, Office of Naval Research, Arlington, Virginia 22217.
AMS RESEARCH FELLOWSHIPS

The American Mathematical Society invites applications for the AMS Research Fellowships. These are postdoctoral fellowships to be awarded for research in mathematics during the year 1975–1976, and are open to individuals who have recently received the Ph.D. The stipend will be in the range of $9,000–$10,000, depending on the amount in the Fund. Of the award, $3,600 plus $500 for travel expenses will be tax deductible.

Completed applications must be received by March 15, 1975. A small number of fellowships will be awarded, the number depending on the amount contributed to the AMS Research Fellowship Fund. Notification of awards will be made by April 15, 1975.

For further information and application forms write to Dr. Gordon L. Walker, Executive Director, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

AMS RESEARCH FELLOWSHIP FUND
Request for Contributions

The AMS Research Fellowship Fund is intended to support research fellows for one year. Awards are made to individuals who have recently received the Ph.D. degree and who show unusual promise in mathematical research. Serving on the Committee appointed by President Mac Lane to administer the Fund are C. B. Bell, Walter Feit, Leonard Gillman, Peter J. Hilton, Mark Kac, and Alice T. Schafer, Chairman. The Committee hopes to award for 1975–1976 several partially tax-exempt fellowships of approximately $10,000 each, a sum equivalent to the salary which a research person with a recent Ph.D. in mathematics might expect. Two such fellowships were awarded for 1974–75. The Committee hopes that this number can be increased for 1975–1976. Although the number of fellowships awarded may, of necessity, be small, the existence of the Research Fellowship Fund demonstrates the importance the Society and its members attach to research.

The survival of the Research Fellowship program depends on the contributions the Society receives. It is hoped that every tenured member of the Society will be willing to contribute at least $100 to the Fund, but any contribution is welcome. The Society itself has pledged to contribute a minimum of $9,000 and will match one-half the funds in excess of $18,000 raised from other sources. However, the Society can contribute no more than $20,000. Contributions are, of course, tax deductible. Checks should be made payable to the American Mathematical Society, clearly marked "AMS Research Fellowship Fund" and sent to the American Mathematical Society, Post Office Box 1571 Annex Station, Providence, Rhode Island 02940.

NBS–NSF SOFTWARE ENGINEERING HANDBOOK

The National Bureau of Standards and the National Science Foundation are jointly attempting to develop a Software Engineering Handbook which would be produced with wide participation of the professional computing community. The Handbook, which will have the format of a desk reference, will provide guidance for programmers and systems managers by gathering widely scattered reference material and presenting it in a concise and factual manner. The NBS Institute for Computer Sciences and Technology is organizing the editorial staff to complete pilot technical material for the Handbook. Potential contributors can obtain the Prospectus by writing to: Software Engineering Handbook, National Bureau of Standards, Institute for Computer Science Section (TECH A265), Washington, D.C. 20234.

STEVER ANNOUNCES FCST PAGE CHARGE POLICY

Dr. H. Guyford Stever, Science Adviser and Chairman of the Federal Council for Science and Technology (FCST), has announced a revised FCST policy on page charges for publication of research results stemming from Federally funded research projects.

The new policy statement says: "Scientific policy representatives of Federal agencies that constitute the Federal Council for Science and Technology have established the following criteria for honoring page charge bills submitted by journal publishers: (1) The research papers report work supported by the Government. (2) Mandatory or voluntary page charge policies are acceptable, provided that the page charge policy for the publication is administered impartially for Government and non-Government sponsored research reports. (3) The journals involved are not operated for profit."

This policy represents a change from the original 1961 policy in the following areas: (1) the original criterion number 3 has been deleted (this criterion stated "payment of such charges is in no sense a condition for acceptance of manuscripts by the journals.") (2) Original criterion number 2 has been modified as stated above to reflect the acceptability of mandatory page charge policies. (3) The original criterion number 4 has been renumbered to become criterion (3) in the revised statement.

HOUSTON JOURNAL OF MATHEMATICS

The Houston Journal of Mathematics is a new international journal that will appear quarterly. The scope of the journal is intended to be broad, covering all areas of mathematics. The journal is being supported in part by the American Mathematical Society.

The members of the Editorial Board are R. H. Bing, David G. Bourgin, Jutta Hausen (Associate Managing Editor), Gordon G. Johnson (Managing Editor), Johnny A. Johnson (Associate Managing Editor), Bjarni Jonsson, Andrzej Lesk, John S. Mac Nerney, J. W. Neuberger, B. H. Neumann and Jurgen Schmidt.

The subscription price is $35 per volume ($40 if sent outside the United States and Canada). Subscriptions received prior to September 1975 will receive all issues published in 1975 and 1976 for $35.

For information write to Gordon G. Johnson, Houston Journal of Mathematics, University of Houston, Houston, Texas 77004.
Information on the backlog of papers for research journals is published in the February and August issues of these journals with the cooperation of the respective editorial boards. Since all columns in the table are not self-explanatory, we include further details on their meaning.

Column 3. This is an estimate of the number of printed pages which have been accepted but are not necessary to maintain copy editing and printing schedules.

Column 5. The first (Q1) and third (Q3) quartiles are presented to give a measure of normal dispersion. They do not include misleading extremes, the result of the case otherwise, so these figures are low to that extent.

<table>
<thead>
<tr>
<th>JOURNAL</th>
<th>Number Issues per Year</th>
<th>Approximate Number Pages per Year</th>
<th>BACKLOG 12/31/74</th>
<th>Estimated Time for Paper Submitted Currently to be Published (in Months)</th>
<th>Observed Waiting Time in Latest Published Issue (in Months)</th>
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<tr>
<td><strong>Acta Informatica</strong></td>
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<td><strong>Annals of Statistics</strong></td>
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<td>2064</td>
<td>118</td>
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<td><strong>Math. Zeitschrift</strong></td>
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<td>2114</td>
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<td>900</td>
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<td>NR*</td>
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*NR means that no response was received to a request for information.
**The latest issue of this journal consisted of just one article.
***Only six issues to be published in 1976.
#Date of receipt of manuscript not indicated in this journal.
ASSISTANTSHIPS AND FELLOWSHIPS IN MATHEMATICS IN 1975 – 1976

Supplementary List
FOR GRADUATE STUDY AT UNIVERSITIES

<table>
<thead>
<tr>
<th>TYPE</th>
<th>STIPEND</th>
<th>TUITION</th>
<th>SERVICE REQUIRED</th>
<th>DEGREES AWARDED</th>
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<td>of financial assistance (with number anticipated 1975-1976)</td>
<td>amount in dollars</td>
<td>9 or 12 months</td>
<td>if not included in stipend (dollars)</td>
<td>hours per week</td>
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**ALABAMA**

Alabama A & M University, Normal 35762

DEPARTMENT OF MATHEMATICS

Teaching Assistantship (1) 2300 9 30/cr.hr. 25 Teaching, tutoring

University of Alabama in Birmingham 35294

DEPARTMENT OF COMPUTER AND INFORMATION SCIENCES

Fellowship (2) 3000 12 Varies

**CALIFORNIA**

University of California, Los Angeles 90024

DEPARTMENT OF BIOSTATISTICS

Applications due: 3/15/75

Fellowship (1) 1200 12

Teaching Assistantship (2) 4401-4896 9 Yes 20 Calculator Lab

University of California, Santa Barbara 93106

DEPARTMENT OF MATHEMATICS

Applications due: 1/15/75

Fellowship (4) 2200-2500 9 1500* 6 Teaching

Private Teaching Fellowship (4) 4401 9 6 Teaching

*For all out-of-state students.

**COLORADO**

University of Denver, Denver 80210

DEPARTMENT OF MATHEMATICS

Applications due: 4/30/75

Fellowship (1) 3000 9

Teaching Assistantship (12) 2500-2900 9 10

**MASSACHUSETTS**

Boston College, Chestnut Hill 02167

DEPARTMENT OF MATHEMATICS

Applications due: 3/15/75

G. G. Bilodeau, Chairman

Teaching Assistantship (9) 2400-2600 9 6 Teaching

Scholarship (2) Remitted 3

Southeastern Massachusetts University, North Dartmouth 02747

DEPARTMENT OF MATHEMATICS

Applications due: 3/15/75

Anthony J. John, Chairman

Teaching Assistantship (2) 3000 9 Remitted 9–12 Teaching

University of Nevada, Reno 89507

DEPARTMENT OF MATHEMATICS

Applications due: 3/1/75

E. M. Beesley, Chairman

Teaching Assistantship (2) 2800-3000 9 20 Teaching, preparing

NEVADA

Applications due: 4/15/75

Bachelor's by inst. 350

Bachelor's by dept. 17

Master's by dept. 6

Bachelor's by inst. 668

Master's by dept. 2

Bachelor's by inst. 2302

Bachelor's by dept. 52

Master's by dept. 13

Ph.D. (1972-1974 incl.) 10. Total: 10

Ph.D. (1972-1974 incl.) 16


Total: 16

Applications due: 3/15/75

Bachelor's by inst. 1226

Bachelor's by dept. 20

Master's by dept. 7

Ph.D. (1972-1974 incl.) 7

A&NT 1, A&FA 3, AM 2.

Total: 6

Bachelor's by inst. 744

Bachelor's by dept. 23

Master's by dept. 2
NEW YORK

New York University, New York 10003

DEPARTMENT OF COMPUTER SCIENCE
Jacob T. Schwartz, Chairman
Fellowship (10) 4710 9
Research Assistantship (3) 4710 9

State University College at Plattsburgh 12901

DEPARTMENT OF MATHEMATICS
Harold J. Perkins, Chairman
Teaching Assistantship (1) 2500 9 *

* Tuition waivers usually available.

NORTH CAROLINA

Duke University, Durham 27706

DEPARTMENT OF COMPUTER SCIENCE
Donald W. Loveland, Chairman
Fellowship (3) 5000-5700 9 2880
Teaching Assistantship (3) 4700 9 2300 10 Teaching
Research Assistantship (4) 4700 9 1728-2300 10-15 Research

OKLAHOMA

University of Tulsa, Tulsa 74104

DEPARTMENT OF MATHEMATICAL SCIENCES
Thomas W. Cairns, Chairman
Teaching Assistantship (6) 2500 9 20 Teaching
Research Assistantship (3) 2500 9 20 Research

TEXAS

University of Texas at Austin 78712

DEPARTMENT OF COMPUTER SCIENCE
J. C. Browne, Chairman
Fellowship (3) 6000-7476 9 150 20 Teaching
Research Assistantship (10) 2961-3528 12 20

West Texas State University, Canyon 79105

DEPARTMENT OF MATHEMATICS
H. L. Cook, Head
Teaching Assistantship (6) 2700 9 * 6 Teaching

* In-state (GAS): $130/semester or $260/year; out-of-state students (GAS): no tuition.

CANADA

Université de Montréal, Montréal, Québec

DEPARTMENT OF COMPUTER SCIENCES
Paul Bradley, Chairman
Teaching Assistantship (20) 2500-3500 9 550 9 Tutoring, teaching, correcting programming, research
Research Assistantship (8-10) 2500-3500 9 550 9
Governmental Agencies (15) 3500-5000 12 550

* Application to graduate studies for teaching assistantship, for National Research Council awards: 12/1/74; Quebec Ministry of Education awards: 2/1/75 at the latest.

University of British Columbia, Vancouver, British Columbia

DEPARTMENT OF MATHEMATICS
Donald Baras, Head
Teaching Assistantship (25) 1650-1750 9 510 6 Teaching, grading
Research Assistantship (25) 3300-3500 9 510 12 Teaching, grading

Applications due: 1/1/75
Bachelor's by inst. 1202
Bachelor's by dept. 14
Master's by dept. 10
Ph.D. (1972-1974 incl.) CS 8. Total: 8

Applications due: 2/1/75
Master's by dept. 1

Applications due: 2/15/75
Bachelor's by inst. 1300
Bachelor's by dept. 6
Master's by dept. 5

Applications due: 3/1/75
Bachelor's by inst. 5325
Bachelor's by dept. 10
Master's by dept. 17
Ph.D. (1972-1974 incl.) CS 17. Total: 17

Applications due: 3/1/75
Bachelor's by inst. 1300
Bachelor's by dept. 26
Master's by dept. 3

Applications due: *
Bachelor's by dept. 4
Master's by dept. 17
Ph.D. (1972-1974 incl.) CS 2, Other 10, Total: 12

Applications due: 3/1/75
Bachelor's by inst. 2943
Bachelor's by dept. 51
Master's by dept. 12
Ph.D. (1972-1974 incl.) A&NT 7, G&T 4, L 1, A&FA 4, P&S 1, CS 2, AM 1. Total: 20
SUMMER GRADUATE COURSES

The following is a list of graduate courses being offered in the mathematical sciences during the summer of 1975. Another list will appear in the April issue of these Notices.

ALABAMA
SAMFORD UNIVERSITY
Birmingham, Alabama 35209
Application deadline: Open
Information: W. D. Peeples, Jr., Chairman, Department of Mathematics
June 2 – July 9
Theory of Numbers
July 10 – August 5
Advanced Calculus
Functions of Several Variables

ARIZONA
NORTHERN ARIZONA UNIVERSITY
Flagstaff, Arizona 86001
Application deadline: May 1
Information: Box 5717, Department of Mathematics
June 9 – July 12
Elements of Algebraic Systems
Introduction to Higher Algebra
Advanced Calculus I
Seminar in Mathematics Education
Theory of Numbers
July 14 – August 15
Elements of Analysis
Finite Dimensional Vector Spaces
Numerical Analysis
Point Set Topology

CONNECTICUT
FAIRFIELD UNIVERSITY
Fairfield, Connecticut 06430
Application deadline: June 30
Information: R. Bolger, Director, MAT-M Program
July 1 – August 1
Set Theory
Development of Mathematical Thought
Algebraic Topology

GEORGIA
ARMSTRONG STATE COLLEGE
Savannah, Georgia 31406
Application deadline: May 1
Information: R. M. Summerville, Mathematics Coordinator
June 13 – August 15
Foundations of Mathematics
Algebra I
Modern Mathematics for Elementary Teachers

GEORGIA STATE UNIVERSITY
University Plaza, Atlanta, Georgia 30303
Application deadline: May 1
Information: Jan List Boal, Chairman, Department of Mathematics
June 13 – August 21
College Geometry
Complex Analysis
Linear Algebra
Theory of Numbers
Foundations of Mathematics I*
The Real Number System
Computer Oriented Mathematics
Mathematical Models
*For Elementary School Teachers.

ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA–CHAMPAIGN
Urbana, Illinois 61801
Information: J. W. Armstrong, Department of Mathematics, 259 Altgeld Hall
June 9 – August 1
Undergraduate and Graduate Students
History of Calculus
Combinatorial Mathematics
Abstract Algebra
Linear Algebra
Applied Modern Algebra
Advanced Calculus
Differential Equations
Complex Variables
Real Analysis
Transformational Geometry
Theory of Numbers
Probability Theory
Statistics
Linear Programming
Mathematical Methods in Engineering
Switching Theory
Graduates Only
Partial Differential Equations
Mathematical Methods of Physics

INDIANA
INDIANA UNIVERSITY
Bloomington, Indiana 47401
Application deadline: January 1
Information: Graduate Secretary, Department of Mathematics
June 20 – August 15
Introduction to Modern Algebra I
Mathematical Models and Applications II*
Introduction to Probability Theory I
General Topology
Seminar in the Teaching of College Mathematics I–II
*This course does not ordinarily carry credit toward the M.A. or Ph.D. degrees in mathematics. They may, however, be taken by M.A.T. candidates and graduate students in other departments for graduate credit.

LOUISIANA
TULANE UNIVERSITY
New Orleans, Louisiana 70118
Application deadline: April 15
Information: F. B. Mullin, Department of Mathematics
June 9 – August 1
Probability and Statistics
Analysis

MASSACHUSETTS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge, Massachusetts 02139
Information: Director of Summer Session, Room E19–356
June 16 – June 27
Design Analysis of Scientific Experiments
MICHIGAN
EASTERN MICHIGAN UNIVERSITY
Ypsilanti, Michigan 48197
Application deadline: May 30
Information: Graduate School
June 30 – August 8
Modern Mathematics Content, K-6
New Topics in Modern Mathematics, K-8
Foundations of Mathematics
Theory of Integers
Elements of Set Theory
Modern Mathematics Content, Junior High School
Analytic Projective Geometry
Graph Theory
Modern Mathematics Methods, Junior High School

NORTHERN MICHIGAN UNIVERSITY
Marquette, Michigan 49855
Application deadline: May 1
Information: C. B. Stortz, Chairman, Department of Mathematics
June 16 – August 11
Linear Algebra
Mathematical Logic
Computers for Teachers
Seminar in Mathematics

MISSISSIPPI
UNIVERSITY OF SOUTHERN MISSISSIPPI
SS Box 45, Hattiesburg, Mississippi 39401
Information: James Caveny, Chairman, Department of Mathematics
June 2 – August 13
Differential Equations
Number Theory
Complex Variables
Geometry
Topics in Analysis
Seminar

MISSOURI
SAINT LOUIS UNIVERSITY
221 North Grand Boulevard, St. Louis, Missouri 63103
Application deadline: May 1
Information: Graduate School
June 18 – July 25
Analytic Geometry and Calculus for High School Teachers
Introduction to Complex Variables
Fortran Programming
Combinatorial Topology
Computer Science

UNIVERSITY OF MISSOURI
Columbia, Missouri 65201
Application deadline: May 1
Information: J. L. Zemmer, Department of Mathematics
June 9 – July 3
Theory of Equations
June 9 – August 1
Advanced Calculus with Applications
Differential Equations
Matrix Theory
Foundations of Geometry
Topics in Analysis (Approximation Theory)

NEBRASKA
KEARNEY STATE COLLEGE
Kearney, Nebraska 68847
Application deadline: June 16
Information: L. M. Larsen, Department of Mathematics, Statistics and Computer Science
June 9 – August 4
Higher Algebra
Linear Algebra
Theory of Fields
Numerical Analysis
Regression Analysis (Statistics)
History of Mathematics
Elementary Mathematics Workshop
Secondary Mathematics Workshop

UNIVERSITY OF NEBRASKA, LINCOLN
Lincoln, Nebraska 68508
Application deadline: May 15
Information: E. Halfar, Acting Chairman, Department of Mathematics and Statistics
June 9 – July 11
Number Theory
Probability
Topics in Statistics
Topics in Applied Mathematics
July 14 – August 15
Topics in Algebra

NEW JERSEY
MONMOUTH COLLEGE
West Long Branch, New Jersey 07764
Information: Department of Mathematics
June 2 – July 11
Number Theory

RUTGERS–THE STATE UNIVERSITY OF NEW JERSEY
Hill Center–Busch Campus, New Brunswick, New Jersey 08903
Application deadline: June 15
Information: Mrs. Mary Anne Jablonski, Department of Mathematics
June 30 – August 20
Elementary Set Theory and Topology
Functional Analysis

SETON HALL UNIVERSITY
South Orange, New Jersey 07079
Application deadline: June 13
Information: Michael J. D’Ambrosia, Department of Mathematics
June 30 – July 24
Mathematical Methods of Operations Research

NEW YORK
C. W. POST COLLEGE
Greenvale, New York 11548
Application deadline: First Session: June 23; Second Session: July 28
Information: Wendell L. Jones, Department of Mathematics
June 23 – July 25
Mathematics for Elementary School Teachers
Set Theory
Introduction to Abstract Algebra
Probability
July 28 – August 29
Mathematics for Elementary School Teachers
Mathematical Logic
Euclidean Geometry
Digital Computer Programming for Teachers

NEW YORK UNIVERSITY
Courant Institute of Mathematical Sciences
251 Mercer Street, New York, New York 10012
Information: Ruth Shor, Department of Mathematics and Computer Science
June 9 – July 18
Computer Science: Fundamental Algorithms I
Mathematics: Elementary Numerical Methods
Linear Algebra I
Program for Teachers: Groups and Graphs
July 21 – August 29
Mathematics: Linear Algebra II
NORTH CAROLINA

EAST CAROLINA UNIVERSITY
Greenville, North Carolina 27834
Application deadline: May 1
Information: Graduate School
June 16 - July 10
Theory of Numbers I
Historical Development of Mathematics
Advanced Calculus I
Matrix Algebra
Modern Algebra I

July 14 - August 8
Theory of Numbers II
Advanced Calculus II
Matrix Algebra
Modern Algebra II

OHIO

JOHN CARROLL UNIVERSITY
University Heights, Ohio 44118
Application deadline: June 1
Information: J. Moreno, Department of Mathematics
June 16 - July 18
Limits and Series *
Mathematical Logic
July 21 - August 21
Probability and Statistics *
Topics from Real Analysis
*These courses carry undergraduate or graduate credit.

MIAMI UNIVERSITY
Oxford, Ohio 45056
Information: S. E. Bohn, Chairman, Department of Mathematics
June 16 - July 18
Probability and Statistics I
Advanced Euclidean Geometry
Abstract Algebra I
Modern Algebra for the Social Scientist
Real Analysis I
Mathematical Models and Applications
Computing for Secondary School Teachers
Geometry for Secondary School Teachers
Functions of a Real Variable I
July 21 - August 22
Probability and Statistics II
Abstract Algebra II
Real Analysis II
Computing for Elementary School Teachers
Functions of a Real Variable II

PENNSYLVANIA

BUCKNELL UNIVERSITY
Lewisburg, Pennsylvania 17837
Application deadline: March 1
Information: Coordinator of Graduate Studies
June 23 - August 1
Topics in Algebra
Topics in Analysis
Topics in Applied Mathematics
Seminar

EDINBORO STATE COLLEGE
Edinboro, Pennsylvania 16444
Information: Ronald G. Butler, Chairman, Department of Mathematics
June 2 - June 20*
Advanced Topics in Field Work
Topics in Mathematics for Elementary and Middle School Teachers
June 23 - August 1**
History of Mathematics
Intermediate Analysis I
Linear Algebra
Mathematical Statistics I
Computer Education

SHIPPENSBURG STATE COLLEGE
Shippensburg, Pennsylvania 17257
Application deadline: Open
Information: Dean of Graduate Studies
June 9 - June 27
Graph Theory
June 30 - August 8
Advanced Calculus I
Geometry II
Modern Algebra II
Logic for Mathematics Teachers
August 11 - August 29
Mathematics Discovery

UNIVERSITY OF PITTSBURGH
Pittsburgh, Pennsylvania 15260
Application deadline: April 15
Information: Orrin E. Taulbee, Department of Computer Science
April 28 - June 18
Social Implications of the Computer
Artificial Intelligence: Learning and Adaptive Systems
June 24 - August 13
Economics of Computing
April 28 - August 13
Automatic Text Processing

SOUTH CAROLINA

CLEMSON UNIVERSITY
Clemson, South Carolina 29631
Information: Department of Mathematical Sciences
May 19 - June 26
Introduction to Stochastic Processes
Linear Algebra
Advanced Calculus I
Operational Mathematics
Special Topics in Numerical Processes
Peripherals and File Design
June 30 - August 7
Statistical Inference
Partial Differential Equations
Advanced Calculus II
Advanced Linear Algebra
Special Topics in Analysis
Special Topics in Operations Research
Introduction to Assembler Language

TENNESSEE

VANDERBILT UNIVERSITY
Nashville, Tennessee 37232
Information: Charles F. Federspiel, Summer Session of Statistics, Division of Biostatistics, Department of Preventive Medicine
June 22 - August 1
Tentative course offerings include elementary and intermediate biostatistics, actuarial statistics, demography, sampling methods, research design, design of experiments, Bayesian inference, categorical data, and health facility statistics.

TEXAS

UNIVERSITY OF HOUSTON
Houston, Texas 77004
Application deadline: May 15
Information: Professor Benner, Department of Mathematics
June 2 - July 8
Boolean Algebra
Differential Geometry
Topics in Topology
Point Sets and Transformations

July 10 - August 16
Functional Completion

UNIVERSITY OF TEXAS AT ARLINGTON
Arlington, Texas 76019
Application deadline: May 16
Information: Graduate Advisor

June 2 - August 15
Topology
Engineering Mathematics
Experimental Designs

June 2 - July 10
Applied Complex Variables
Topics in Differential Equations

July 14 - August 20
Numerical Analysis
Functional Analysis
Topics in Analysis and Applications

UNIVERSITY OF TEXAS AT ARLINGTON
Arlington, Texas 76019
Application deadline: May 15
Information: Graduate Advisor, R. W. Walker, Department of Computer Science

June 2 - August 10
Advanced Information Structures
Operating Systems
Information Storage and Retrieval

UNIVERSITY OF TEXAS AT DALLAS
Box 688, Richardson, Texas 75080
Application deadline: June 1
Information: John W. Van Ness, Head, Graduate Program in Mathematical Sciences

June 2 - July 11
Computer Science I
Computer Science II
Higher Geometry
Modern Algebra I
Computer-assisted Instruction
Statistical Inference I

July 14 - August 22
Computer Science III
Non-Euclidean Geometry
Modern Algebra II
Systems Programming
Statistical Inference II

WISCONSIN

MARQUETTE UNIVERSITY
Milwaukee, Wisconsin 53233
Application deadline: May 12
Information: W. E. Lawrence, Chairman, Department of Mathematics and Statistics

June 23 - August 2
History of Mathematical Ideas*
Linear Algebra and Matrix Theory*
Modern Computer Languages*
Complex Variables*
Advanced Geometry II
Seminar in Mathematics Curriculum Development and Material I

*Undergraduate courses carrying graduate credit.

VISITING MATHEMATICIANS
Supplementary List
American and Canadian Mathematicians Visiting Abroad

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<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Special Interest</th>
<th>Period of Visit</th>
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<tr>
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<td>University of Bucharest, Romania</td>
<td>Vector Measures</td>
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<tr>
<td>Friedberg, Stephen H. (U.S.A.)</td>
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<td>Harmonic Analysis</td>
<td>8/74 - 8/75</td>
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<td>Arya-Mehr University of Technology, Iran</td>
<td>Celestial Mechanics</td>
<td>1/75 - 6/75</td>
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<td>Giambalvo, Vincent (U.S.A.)</td>
<td>Chinese University of Hong Kong</td>
<td>Algebraic Topology</td>
<td>2/75 - 4/75</td>
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SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates, or geographical area become apparent. An announcement will be published in these Notices if it contains a call for papers, place, date, subject (when applicable), and speakers; a second announcement will be published only if changes to the original announcement are necessary, or if it appears that additional information should be announced.

In general, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry slightly more detailed information. Information on the pre-processed planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society. Deadlines for particular issues of the Notices are the same as the deadlines for abstracts which appear on the inside front cover of each issue.

March 31–April 1, 1975
SYMPOSIM IN HONOR OF R. H. CAMERON
University of Minnesota, Minneapolis, Minnesota
Program: After three decades of distinguished service to the University of Minnesota, Robert H. Cameron retired on June 15, 1974. Professor Cameron’s fundamental contributions to several branches of mathematics, and especially his papers on Wiener and Feynman integrals, have established him as a mathematician of world renown. Among his students are many who have become established mathematicians.
Participants: In addition to many of Professor Cameron’s former students, participants will include S. Bochner, M. Kac, W. T. Martin, and I. E. Segal.
Information: Johannes C. C. Nitsche, Head, School of Mathematics, University of Minnesota, 127 Vincent Hall, Minneapolis, Minnesota 55455

April 4–5, 1975
KENT STATE NON-COMMUTATIVE RING THEORY CONFERENCE
Kent State University, Kent, Ohio
Program: Five principal lectures by mathematicians in varied areas of non-commutative ring theory. There also will be informal problem sessions including brief expository talks following each lecture.
Speakers: George Bergman, University of California, Berkeley; Kenneth Goodearl, University of Utah; Robert Gordon, Temple University; Arun Jategaonkar, Fordham University; J. C. Robson, University of Leeds
Information: John Cozam and Frank Sandomierski, Kent State University, Kent, Ohio 44242

April 5, 1975
ALGEBRA DAY
Carleton University, Ottawa, Canada
Speakers: Hyman Bass, K. W. Gruenberg, Donald Passman
Information: Luis Ribe, Department of Mathematics, Carleton University, Ottawa, Canada K1S 5B6

April 6, 1975
ILLINOIS NUMBER THEORY CONFERENCE
Illinois State University, Normal, Illinois
Deadline for abstracts: March 20, 1975. 15- to 20-minute talks, 1-3 sentence abstracts.
Information: E. F. Ecklund, Jr., Mathematics Department, Illinois State University, Normal, Illinois 61761

April 7–8, 1975
ANALYTIC COMPUTATIONAL COMPLEXITY SYMPOSIUM
Computer Science Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania
Program: Invited lectures will be presented by R. Brent, Australian National University; B. Kacwicz, University of Warsaw; H. T. Kung, Carnegie-Mellon University; J. R. Rice, Purdue University; M. H. Schultz, Yale University; J. F. Traub, Carnegie-Mellon University; S. Winograd, IBM Research; and H. Wozniakowski, University of Warsaw.
Contributed papers: Contributed papers are welcome. Write for additional information.
Proceedings: Conference proceedings will be available.
Information: J. F. Traub, Computer Science Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213

April 11–12, 1975
EIGHTH GEORGE HUDSON SYMPOSIUM ON SYSTEMS; APPROACHES, THEORIES, APPLICATIONS
State University College at Plattsburgh, New York
Invited Speakers: Michael Arbib, University of Massachusetts (Computer Science); Walter Buckley, University of New Hampshire (Sociology); Mario Bunge, McGill University (Foundations of Science); George Kliir, SUNY at Binghamton (Computer Science); Ernest Manes, University of Massachusetts (Mathematics); Arthur Milgram, University of Pennsylvania (Systems Engineering); Peter Miler, McGill University (Psychology); Robert Rosen, SUNY at Buffalo (Mathematical Biology)
Information: William E. Hartnett, Department of Mathematics, State University College, Plattsburgh, New York 12901

April 24–25, 1975
THE JOHN H. BARRETT MEMORIAL LECTURES
University of Tennessee, Knoxville, Tennessee
Principal speaker: Jack K. Hale, Brown University
Contributed papers: A session of 30 minute contributed papers on ordinary differential equations will be held on April 25.
Abstracts and information: Gene A. Klassen, Department of Mathematics, University of Tennessee, Knoxville, Tennessee 37916

April 30–May 2, 1975
MATHEMATICAL OPTIMIZATION AND ENGINEERING DESIGN SYMPOSIUM
Palmer House Hotel, Chicago, Illinois
Sponsors: Operations Research Society of America and Office of Naval Research.
Deadline for abstracts: March 1, 1975
Program: This symposium will be held in conjunction with the 47th National Meeting of the Operations Research Society of America and the Institute of Management Sciences. Its objectives will be to stimulate search for engineering design principles derived from mathematical optimization, and to selectively examine usage of mathematical optimization in various design contexts.
Information: Neal Glassman, Symposium Co-Chairman, Office of Naval Research, Operations Research Program (Code 404), 800 North Quincy Street, Arlington, Virginia 22217
April 12–14, 1975
THIRD SYMPOSIUM ON THE NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS
The University of Maryland, College Park, Maryland
Support: National Science Foundation
Information: David Chittenden, Conferences & Institutes Division, University of Maryland University College, University Boulevard at Adelphi Road, College Park, Maryland 20742

May 26–29, 1975
ABRAHAM ROBINSON MEMORIAL CONFERENCE
Yale University, New Haven, Connecticut
Program: Approximately 16 invited addresses on model theory and nonstandard analysis. The program is structured to give a comprehensive survey of topics pioneered by Robinson.
Information: Angus Macintyre, Department of Mathematics, Yale University, New Haven, Connecticut 06520

June 9, 1975
JOINT SYMPOSIUM ON EXCITEMENT IN MATHEMATICS
Churchill College, Cambridge, England
Program: The symposium will be held in celebration of the 90th birthday of J. E. Littlewood, F.R.S., and is a tribute to the contributions to mathematics which gave Professor Littlewood a worldwide reputation. Professor Littlewood hopes to be present for at least part of the meeting.
Organizers: The Institute of Mathematics and its Applications and the London Mathematical Society
Information: Secretary and Registrar, The Institute of Mathematics and its Applications, Mainland House, Warrior Square, Southend-on-Sea, Essex SS1 2YF, England

June 9–13, 1975
FIFTH CONFERENCE ON STOCHASTIC PROCESSES AND THEIR APPLICATIONS
University of Maryland, College Park, Maryland
Sponsors: Committee for Conferences on Stochastic Processes and the University of Maryland Department of Mathematics
Program: Invited papers on current research on various aspects of stochastic processes and their applications, and review papers in these areas. There will be daily sessions for contributed papers.
Information: R. Syski, Department of Mathematics, University of Maryland, College Park, Maryland 20742

June 10–11, 1975
CONFERENCE ON CURRICULUM ALTERNATIVES IN THE MATHEMATICAL SCIENCES
University of Regina, Saskatchewan
Program: Curriculum levels from high school to doctoral, and teaching methods (particularly via applications), will be studied. The program will include invited talks as well as panel discussions.
Information: P. Binding, Mathematics Department, The University of Calgary, Calgary T2N 1N4, Alberta, Canada

June 17–July 18, 1975
SEMINAR ON TOPICS IN TOPOLOGY
University of Toledo, Toledo, Ohio
Information: American Mathematical Society, Editorial Department, P.O. Box 6248, Providence, Rhode Island 02940

June 30–July 4, 1975
RESEARCH APPLICATIONS CONFERENCE ON ECOLOGY AND STATISTICS
Alta Lodge, Alta, Utah
Sponsors: SIAM Institute for Mathematics and Society and the International Statistical Ecology Program
Program: 18–20 invited participants will make presentations on ecological problem areas in which statistics is one of the principal disciplines involved.
Information: SIMS, 93 South 17th Street, Philadelphia, Pennsylvania 19103

June 30–July 21, 1975 (approx.)
C.M.C. SUMMER RESEARCH INSTITUTE
University of Calgary, Alberta, Canada
Program: The Institute's Applied Mathematics Workshop will have singular perturbations and control as its theme. It will consist of invited series of talks together with informal seminars contributed by participants.
Invited speakers: P. Kokotovic, R. O'Malley, and D. Willett.
Information: P. Binding, Mathematics Department, University of Calgary, Alberta, Canada

July 7–11, 1975
RESEARCH APPLICATION CONFERENCE ON ENERGY
Alta Lodge, Alta, Utah
Program: 18–20 invited participants will make presentations on various energy topics such as supply and resources, demand and consumption, the environment, and general policy, regulation and decision-making.
Information: Fred S. Roberts, Chairman of the Conference on Energy, Rutgers University, New Brunswick, New Jersey 08903

July 7–18, 1975
SEMINAR ON MODERN MODELLING OF CONTINUUM PHENOMENA
Rensselaer Polytechnic Institute, Troy, New York
Application deadline: March 17, 1975
Information: American Mathematical Society, Editorial Department, P.O. Box 6248, Providence, Rhode Island 02940

July 28–August 1, 1975
FOURTH ANNUAL SOUTHERN CALIFORNIA SYMPOSIUM IN ALGEBRAIC AND GEOMETRIC TOPOLOGY
California State University, Long Beach, California
Program: Contributed lectures and papers in algebraic and/or geometric topology
Information: Robert E. Mosher, Department of Mathematics, California State University, Long Beach, California 90840

August 3–9, 1975
INTERNATIONAL SYMPOSIUM ON INFINITE DIMENSIONAL HOLOMORPHY
Universidade Estadual de Campinas, Brazil
Program: A series of one-hour lectures and sessions for short communications.
Information: Mario C. Matos, Instituto de Matemática, Universidade Estadual de Campinas, Caixa Postal 1170, 13100 Campinas, S.P., Brazil

August 10–23, 1975
CANADIAN MATHEMATICAL CONGRESS’ FIFTEENTH BIENNIAL SEMINAR ON MATHEMATICS AND THE LIFE SCIENCES
The University of Sherbrooke, Sherbrooke, Québec, Canada
Application deadline: April 1, 1975
Information: C. McQueen, Program Committee Chairman, Department of Mathematics, Laval University, Québec, G1K 7P4, Canada

September 8–14, 1975
SEVENTH IFIP CONFERENCE ON OPTIMIZATION TECHNIQUES (Modelling and optimization in the service of man)
Nice, France
Program: The aim of the conference is to present recent advances in theory and practical applications in the broad area of system modelling and optimization.
Deadline for abstracts: April 1, 1975
Information: Jean Caz, Parc Valrose, 06034 Nice Cedex-France

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September 8–19, 1975  
NATO ADVANCED STUDY INSTITUTE ON NONLINEAR OPERATORS AND THE CALCULUS OF VARIATIONS  
Université Libre de Bruxelles  
Lecturers: H. Amann (Bochum), H. Brezis (Paris), U. Mosco (Rome), R. T. Rockafellar (Seattle), R. Temam (Orsay).  
Deadline for applications: April 15, 1975. The application should mention clearly whether a grant is necessary and whether the grant should cover all or part of the travel or living expenses.  
Information: Enrique Lami Dozo, Département de Mathématique de l’U. L. B., Chaussée de la Hulpe, 166, B-1170 Bruxelles, Belgium

September 9–11, 1975  
SYMPOSIUM ON SPARSE MATRIX COMPUTATIONS  
Argonne National Laboratory, Argonne, Illinois  
Support: (anticipated) Office of Naval Research and U.S. Atomic Energy Commission  
Program: The symposium will be conducted in cooperation with the Society for Industrial and Applied Mathematics. Topics include theoretical aspects of sparse elimination, implementation of sparse matrix codes, fast direct methods, iterative methods for linear equations, sparse eigenvalue problems, sparse least squares, sparse linear programming, sparse non-linear equations, and applications to electrical network analysis, ocean models, and reservoir simulation.  
Contributed papers: James R. Bunch, Department of Mathematics, University of California, San Diego, LaJolla, California 92037, before May 30, 1975.  
Information: James C. T. Pool, Applied Mathematics Division, Argonne National Laboratory, Argonne, Illinois 60439

October 13–15, 1975  
SIXTEENTH ANNUAL SYMPOSIUM ON FOUNDATIONS OF COMPUTER SCIENCE (formerly called the Annual Symposium on Switching and Automata Theory)  
Berkeley, California  
Sponsors: Mathematical Foundations of Computing Committee of the IEEE Computer Society and the Department of Electrical Engineering and Computer Sciences of the University of California, Berkeley, in cooperation with the ACM Special Interest Group on Automata and Computability Theory.  
Program: Papers describing original research in the theoretical aspects of computer science are being sought. Typical, but not exclusive, topics of interest include analysis of algorithms, computational complexity, formal languages, mathematical theory of computation, switching and automata theory, and theory of programming, compiling, and formal semantics.  
Abstracts: Authors are requested to send eight copies of an extended abstract (not a complete paper) by May 12, 1975, to Daniel J. Rosenkrantz, General Electric Company, Research and Development Center, Schenectady, New York 12345.  

October 29–November 1, 1975  
SECOND NATIONAL CONVENTION OF TWO-YEAR COLLEGE MATHEMATICS EDUCATORS  
Sheraton-Chicago, Chicago, Illinois  
Information: William L. Drezdzon, Convention Committee Chairman, Oakton Community College, 7900 North Nagle, Morton Grove, Illinois 60053
QUERIES

Edited by Hans Samelson

The QUERIES column is published in each issue of these Notices. This column welcomes questions from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of expositions of folk theorems, or the state of current knowledge concerning published conjectures. When appropriate, replies from readers will be edited into a definitive composite answer and published in a subsequent column. All answers received to QUERIES will ultimately be forwarded to the questioner. Consequently, all items submitted for consideration for possible publication in this column should include the name and complete mailing address of the person who is to receive the replies. The queries themselves, and responses to such queries, should be typewritten if at all possible and sent to Professor Hans Samelson, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

60. Paul M. Cere (15-43 Bell Boulevard, Bayside, New York 11360). I am compiling a list of textbooks and pamphlets (in English) entirely devoted to problems (with solutions) in Probability and Statistics (such as I. Toda­hunter, A History of Mathematical Theory of Prob­ability, Chelsea, New York, or W.A. Whitworth, DCC Exercises in Choice and Chance, Hafer, Riverside, New Jersey), and would like to get references.

61. Josef Weichmann (Department of Mathematics, Texas Christian University, Fort Worth, Texas 76129). A B*-norm on a *-algebra A is an algebra norm on A such that \(||x* \cdot x|| = ||x||^2\) for all elements x in A. Question: Does anyone have an example of a *-algebra with two different (necessarily incomplete) B*-norms?

62. A. Wilansky (Department of Mathematics, Lehigh University, Bethlehem, Pennsylvania 18015). Give an example of a Banach space \(E\) and a bounded operator \(T\) from \(E\) to \(E\) with any one of the following three properties (1, 2 are equivalent):

1. \(T\) is onto but has no right inverse,
2. \(T\) is onto and the null-space of \(T\) is not comple­mented,
3. \(T\) is range closed but \(T^2\) is not.

RESPONSES TO QUERIES

Replies have been received to queries published in recent issues of these Notices. The editor wishes to thank those who have replied.

54. (vol. 22, p. 71, Jan., 1975, K. Koh): Example of two nonisomorphic fields, each isomorphic to a subfield of the other. Many people sent in the following answer and related ones: \(K_1 = \mathbb{C}, K_2 = \mathbb{C}(X)\) (\(X\) transcendental), \(K_2\) embeds in \(K_1\), since the algebraic closure of \(K_2\) is isomorphic to \(\mathbb{C}\) (it has the same transcendence degree over \(Q\) as \(\mathbb{C}\)). W.C. Waterhouse points out a deeper example (negative solution of the "Luroth problem"):\n\[\mathbb{C}(X_1, X_2, X_3)\] has a subfield that has transcendence degree 3 but is not purely transcendental over \(\mathbb{C}\).

55. (vol. 22, p. 71, Jan., 1975, Z. Zwillinger): Largest primes not of the form \(2^k + 1\). J.L. Hlavca supplies \(5, 2591, 2^{346} + 1\) (with 206 digits), from an unpublished paper by J. Brillhart-D. H. Lehmer-J. L. Selfridge.

58. (vol. 22, p. 71, Jan., 1975, S. Zaldman): L. Nirenberg pointed out that the formula in the query can't hold for large \(h\) and was quoted incorrectly; the right side should read \(c \cdot ||P^{p+1} \cdot (1/x) + h||^2 + 1/||x||^{p+1}\). On p. 285, line 9 of the cited paper by J. J. Kohn and L. Nirenberg the wrong formula appears, by mistake; the correct form should be substituted and estimated by (3.6) instead of (3.10). W.B. Houston, Jr., showed that the (corrected) formula holds for \(f\) in \(C_{p+1}\) and with \(||h|| < ||x||\), with the constant \(c = C_{p+1} K/(p+1)!\), where \(K\) is the maximum of \(||P_h^{p+1}\) \(f(x)||\) for all \(x\) and all unit vectors \(h\) (here \(D_h\) means \(\sum h_i x_i\). In fact the right side can be taken as \(K ||P_h^{p+1}/(p+1)!||^{p+1}\) where \(d\) (assumed > 0) is the distance from 0 to the segment from \(h\) to \(x + h\).

PROBLEM LISTS

PROBLEMS IN HARMONIC ANALYSIS

The following were presented at the Special Session on Harmonic Analysis, at the meeting of the Society at Houston, Texas, November 23, 1974.

1. A.K. Varma (Department of Mathematics, University of Florida, Gainesville, Florida 32611) Do there exist, for each \(f\) in \(C[-1, 1]\), algebraic polynomials \(p_n\) (of degree \(n\)) such that \(|f(x) - p_n(x)| \leq \omega_n(n^{-1/2} - x^n)\)? Here \(\omega_n(\delta)\) is the 2nd modulus of continuity defined as the supremum of \(|f(x + h) - 2f(x) + f(x - h)|\) as \(-1 \leq x \leq 1, ||h|| < \delta\).

2. B.D. Anderson (Mathematics Department, East Texas State University, Commerce, Texas 75428) Let \(S\) be a closed subset of a normal space \(T\), and let \(C(S)\) and \(C(T)\) denote the spaces of bounded continuous functions with sup norm. If closed subspaces \(G \subset C(S)\) and \(H \subset C(T)\) have jointly the bounded extension property \((E.A. Michael and A. Pełczyński, A linear extension theorem, Illinois J. Math 11(1967), 563-579]\), what are necessary and sufficient conditions for the equivalence of \((1) G\) is an E-space in \(C(S)\); (2) \(H\) is an E-space in \(C(T)\) ? A subspace is an "E-space" if each point outside has a nearest point inside. See also [B.D. Anderson, Projections and extension maps in \(C(T)\), Illinois J. Math 17(1973), 513-517].

3. A.K. Varma (Department of Mathematics, University of Florida, Gainesville, Florida 32611) Recently I proved (in the Proceedings of the Conference on Approximation Theory, Poznan, Poland, 1972) the following result. If \(\delta_{kn} = 2k^n/n\) and \(G(\delta) = 2n^{-1} \sum_{j=0}^{n-1} (n-j)^m (n-j)^m + jm^{-1}\cos(j\delta)\), then for every \(f \in C_{2m}\) and for every \(\theta\),
Here \( m \) is any even integer and \( \omega_m \) is the generalized modulus of continuity of \( f \). What are the best values for the constants \( \omega_m \)?

4. B.D. Anderson (Mathematics Department, East Texas State University, Commerce, Texas 75428) Let \( X \) denote a topological measure space, i.e., a \( T_0 \)-space with measure structure such that nonvoid open sets are measurable with positive measure, and finite sets have finite measure. Let \( C_1(X) \) denote the collection of continuous real functions \( f \) on \( X \) for which \( \| f \|_\infty < \infty \). Norm \( C_1(X) \) by \( \| f \| = \| f \|_1 \). What precisely are the closed subsets \( Y \) of \( X \) such that a bounded linear multiplicative extension map exists from \( C_1(Y) \) to \( C_1(X) \)?

5. C.K. Chui (Mathematics Department, Texas A & M University, College Station, Texas 77843) A formal power series \( f(z) = \sum_{k=0}^{\infty} a_k z^k \) is called a St"ettelsje Series if \( a_k = 0^\infty k^d du \) for some positive measure \( \mu \) whose support consists of infinitely many points. If \( a_n = O((2n) R^{2n}) \) with \( R > 1 \) and if \( f \) is unique, then the \([n + 1]n \) Pad"e approximants of \( f \) converge uniformly on compact subsets of \( C([0, \infty) \) to \( f \). What can be said about the convergence or divergence of subsequences of \([n + 1]n \) if \( f \) is unique?

6. C.K. Chui (Mathematics Department, Texas A & M University, College Station, Texas 77843) For a function \( f \) in the disc algebra \( A \), let \( a_0(f) = \sum_{k=0}^{\infty} a_k z^k \) and let \( a_n(f) = f(0)/a! \). It is known that \( f \) is uniquely determined by the sequence \( a_n(f) \) if \( f \) is "smooth" (for example, if \( f(0) \) is of bounded variation on \( [\pi, \pi] \)), or if \( a_n(f) = O(n^p) \) for some \( p > 1 \). On the other hand, there are nontrivial \( f \) in \( A \) for which \( a_n(f) \) are all zero. Let \( \epsilon_n \to 0 \) for all \( \epsilon_n > 0 \). Does there exist a nontrivial \( f \) in \( A \) such that \( a_n = O(a^n\epsilon_n) \) and \( a_n(f) = 0 \)?

7. P.W. Smith (Mathematics Department, Texas A & M University, College Station, Texas 77843) If \( M \) is a linear subspace of \( E = L^1 \cap L^\infty \) and if \( f \) is an element of \( E \), denote by \( f_L \) the element of \( M \) closest to \( f \) in \( L^\infty \)-norm. What conditions (on \( M \) or \( f \)) guarantee the convergence of \( f_L \)? Reference: [7].

8. D.E. Wulbert (Mathematics Department, University of California at San Diego, La Jolla, California 92037) Let \( \mathcal{L} \) be a set of bounded linear operators on a Banach space \( E \), and let \( K \subseteq E \). The \( \mathcal{L} \)-shadow of \( K \) is the set of all \( f \) in \( E \) with the property that the conditions \( L_k \in \mathcal{L} \) and \( \lim \| f - L_k f \| = 0 \) for all \( k \in K \) imply that \( \lim \| f - L f \| = 0 \). If \( \mathcal{L} \)-shadow of \( K \) is all of \( E \), then \( K \) is termed a "Korovkin" set for \( \mathcal{L} \). Problem: What are the Korovkin sets for the family of positive linear operators on \( L^1[0,1] \)?

9. E.W. Cheney (Mathematics Department, University of Texas, Austin, Texas 78712) For a subspace \( Y \) in a Banach space \( X \), let \( p(Y,X) \) be the projection constant, defined as \( \inf \| P \| \), where \( P \) ranges over all the bounded linear projections of \( X \) onto \( Y \). What is the projection constant of \( L^2 \) (the quadratic polynomials) in \( C([-1,1]) \)? Is it true for each \( n \) that \( p(L^1_n, C([-1,1])) = \lim p(L^1_n, C([-1,1])) \)? Is it true that the "next-to-interpolation" operators \( A_n \) of Sharma and Motzkin [5], Approximation Theory 5(1972) 176-198] satisfy \( p(L^1_n, C([-1,1])) > 2\pi \log n - C \) for some constant \( C \)?
f. Determine all functions $h$ such that the \textquote{\textit{Slobodeckij space}} $W^p_0(b)$, defined by the condition
\[
\left( \int \frac{f(x) - f(y)}{|x - y|^p} \, dx \, dy \right)^{1/p} < \infty
\]
is an (exact) interpolation space between $L^p_p$ and $L^p_p$.\hfill (Such functions could also be termed interpolation functions but in a different sense (Voncèll). The case $p = 2$ can in view of Parseval’s relation be reduced to the case above (see e). The limiting case $p = \infty$ has also been treated (cf. [19]).)

g. Determine the best constant in the \textit{\textit{lemma}}, the latter says for any $a$ in the closure of $A_0 \cap A_1$, $A_0 + A_1$ and any $r > 0$ we can find a function $u = u(t) \in L^p_p(0 < t < \infty)$ such that $a = \int_0^r u(t) \, dt/t$ and $J(t,u(t)) \leq \gamma + e K(t,a)$, where $\gamma$ is a universal constant.\hfill (This is needed in the proof of the \textit{\textit{equality theorem}} (cf. [1]). We ask this for the exact value of $\gamma$. Other related problems are stated in my talk at the Budapest meeting (in 1969) on \textit{Constructive Function Theory}.)

h. Interpolation between $L^1_1$ and $W^m_1$. In particular is $W^1_0$ an interpolation space between $L^1_1$ and $W^2_2$? Our guess is \textit{\textit{no}}.\hfill i. The dual of $H^\infty$. The work of Fefferman-Rivièr-Sagher has indicated the interest of the \textit{\textit{Hardy-Lorentz-Sobolev}} spaces of $H^\infty$.\hfill It is easy to see that the dual of $H^\infty$ if $0 < p < 1$ or $1 < p < \infty$ is \textit{\textit{a Lorentz-Besov space}}, \hfill (Other properties of the \textit{\textit{Hardy-Lorentz-Sobolev}} spaces. But the situation is at the outset purely real so \textit{\textit{real variable}} approach. (The problem is meaningful also for fixed $p$, the spaces $L^p_p$ have the same inclusion properties, and the same (complex) interpolation properties as the $L^p_p$ spaces. If $G$ is a noncompact, locally compact group and $\rho < \infty$, then there are no positive multipliers from $L^p_p(G)$ to $L^p_p(G)$ (A. P. Bloisinski, \textit{\textit{Convolution of $L_p$, q}} functions, Proc. Amer. Math. Soc. 32 (1972), 237-)

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8. __________, Ark. Mat. 10 (1972), 235-249.
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2. W. CONNELL (University of Missouri at St. Louis, St. Louis, Missouri 63121)

a. (A small question.) Construct a counterexample to show that the complex method is not stable for the space of (nonreflexive) spaces $L^p_p$. It is true that $\lim_{n \to \infty} [f,_{n}(L^p_p)] \subseteq \lim_{n \to \infty} [f,_{n}(L^q_q)]$, but the inverse inclusion is probably false.

b. (A hard and strange question.) There is obvious topological content to the complex method. In one way it is similar to homotopy, in another way to the theory of fibrations. Since P. Grisvard’s thesis (Commutativité de deux foncteurs d’interpolation et applications, J. Math. Pures Appl. (9) 45 (1966), 143-206), no one has attempted to place the properties of interpolation in a larger topological or algebraic context. Someone with this knowledge should try to classify the scales of spaces which are stable under a method of interpolation in terms of other algebraic or topological properties.

c. (A problem of Calderón.) There are many methods of interpolation. Some methods have richer families of intermediate spaces than others. For example, the complex method gives only the $L^p_p$ spaces, the K method gives all the Lorentz spaces, Can all of the spaces that are intermediate between two spaces by \textit{\textit{any}} method of interpolation be determined?

3. J. FOURNIER (University of British Columbia, Vancouver, B.C., Canada)

a. For fixed $p$, the spaces $L^p_q$ have the same inclusion properties, and the same (complex) interpolation properties as the $L^p_q$ spaces. If $G$ is a noncompact, locally compact group and $\rho < \infty$, then there are no positive multipliers from $L^p_p(G)$ to $L^p_p(G)$ (A. P. Bloisinski, \textit{\textit{Convolution of $L_p$, q}} functions, Proc. Amer. Math. Soc. 32 (1972), 237-250.)
Let \( q' \) denote the conjugate index to \( q \). If \( G \) is Abelian, must the space of multipliers from \( L_{p_1}^q(G) \) to \( L_{p_2}^r(G) \) coincide with the space of multipliers from \( L_{p'}^{q'}(G) \) to \( L_{p''}^{r'}(G) \)?

c. Let \( L_p^G \) denote the space of multipliers from \( L^p \) to \( L^p(G) \). If \( p/(p-1) + q/1 \), then \( \|L_p^G\| \subseteq L_p^G \). Is this inclusion strict?

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4. J. E. Gilbert (University of Texas at Austin, Austin 78712).

a. Peetre \( p \)-spaces, Herz \( \mathcal{P}_p \)-spaces. Peetre defines a Banach space \( Z \) to be a \( p \)-SPACE \( 1 \leq p \leq 2 \) if the Fourier transform \( \mathcal{F}: L^p(\mathbb{R}) \to L^p(\mathbb{R}) \) extends to a bounded mapping \( \mathcal{F} \otimes I: L^p(Z) \to L^p(Z) \). Herz defines a \( \mathcal{P}_p \)-space as one such that for any \( T: L^p(X;G) \to L^p(Y;\mathcal{G}) \), the extension \( T \otimes I: L^p(Z;G) \to L^p(Z;\mathcal{G}) \) is bounded. The connection of \( p \)-spaces with interpolation theory arises because if \( Z_0 \) is a \( p_0 \)-space and \( Z_1 \) is a \( p_1 \)-space, then \( \mathcal{P}_p \)-spaces are important in convolution operator theory, and \( L^p \)-factorizable linear transformations. What are the connections between these two classes?

b. Characterization of weak type \((p,p)\) convolution operators. For \( G \) compact a linear operator \( T: L^p(G) \to L^{p_0}(G) \), \( 1 < p < 2 \), is a weak type \((p,p)\) convolution operator if and only if the restriction \( T: L^p(G) \to L^p(M(G)) \) is \( p \)-summing. Is there an analogous characterization for \( G \) noncompact (remember \( C_0(G) \neq L^p(G) \)).

c. Interpolation between Banach spaces. If \( (\mathcal{A}_0, \mathcal{A}_1) \) is a compatible couple of Banach algebras such that the multiplication operation coincides on \( \mathcal{A}_0 \cap \mathcal{A}_1 \), then \( (\mathcal{A}_0, \mathcal{A}_1, \mathcal{B}_0, \mathcal{B}_1) \) is a Banach algebra. The real bilinear interpolation theorem ensures that \( (\mathcal{A}_0, \mathcal{A}_1, \mathcal{B}_0, \mathcal{B}_1) \) is a Banach algebra and \( (\mathcal{A}_0, \mathcal{A}_1, \mathcal{B}_0, \mathcal{B}_1) \) is a module over \( (\mathcal{A}_0, \mathcal{A}_1, \mathcal{B}_0, \mathcal{B}_1) \). Is it true that \( (\mathcal{A}_0, \mathcal{A}_1, \mathcal{B}_0, \mathcal{B}_1) \) may fail to be a Banach algebra, \( 1 < q \leq \infty \), \( 0 < \theta < 1 \)?


I. Linear Theory

a. Let \( A_0, A_1 \) be an "interpolation couple\(^1\) of Banach spaces. What are the conditions on \( A_0, A_1 \) such that there exists an interpolation space \"between\" \( A_0 \) and \( A_1 \) which is a Hilbert space (more precisely, whose interpolation norm is equivalent to a Hilbert norm)?

b. Let \( C^M(T) \) be the space of real valued, \( m \) times continuously differentiable functions on the circle; \( C^1(T) \) an interpolation space between \( C^0(T) \) and \( C^2(T) \)? (The answer is probably "no").

c. Let \( (\mathcal{A}_0, \mathcal{A}_1, \mathcal{B}_0, \mathcal{B}_1) \) be two interpolation couples. We suppose that \( \mathcal{A}_1 \) is a closed subspace of \( \mathcal{B}_1 \) and \( \mathcal{A}_0, \mathcal{A}_1 \) are not necessarily closed in the corresponding spaces between \( \mathcal{B}_0 \) and \( \mathcal{B}_1 \). But in all examples which are known, the spaces of interpolation are indeed closed for all extremal values of the parameters. Is it possible to obtain a precise result along these lines, saying that this is "in general" the case?

II. Nonlinear Theory

d. Let \( A_0, A_1, B_0, B_1 \) be Banach spaces and \( A_0 \subset A_1, B_0 \subset B_1 \). Let \( \pi \) be a nonlinear operator from \( A_1 \) to \( B_1 \), which maps \( A_0 \) into \( B_0 \) and has the properties

\[
\| \pi a_0 - \pi a_1 \|_{B_0} \leq C \| a_0 - a_1 \|_{A_0}, \quad a_0, a_1 \in A_0,
\]

\[
\| \pi a_1 \|_{B_1} \leq C \| a_1 \|_{A_1}, \quad a_1 \in A_1,
\]

for every \( \theta \). Remark. Such a construction would prove that there exist situations where the trace spaces and the complex interpolation spaces cannot be made identical for suitable parameters. This is known to be true (an example of Kahane and Stein).

c. In the notations of problem d, let \( \pi \) satisfy (2) and

\[
\| \pi a_0 - \pi a_1 \|_{B_0} \leq C \| a_0 \|_{A_0} \| a_1 \|_{A_1}, \quad a_0, a_1 \in A_0.
\]

What can be said about the image by \( \pi \) of the trace spaces between \( A_0 \) and \( A_1 \)?

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6. Benjamin Muckenhoupt (Rutgers University, New Brunswick, New Jersey 08903).

a. We will say that a pair of nonnegative functions \( U(x), V(x) \) on \([0,1] \) satisfies the condition \( A_p^* \) with constant \( K \) if for every subinterval \( I \) of \([0,1] \)

\[
\left( \int_0^1 \left[ \int_{x-I}^{x+I} |U(y) V(y)| \right]^{p-1} \frac{dy}{I} \right)^{1/(p-1)} \leq K,
\]

where \( x \) denotes the center of \( I \) and \( I \) is the length of \( I \). Assume that \( C \) is a fixed constant and \( T \) is a sublinear operator such that for two values of \( q \) in \((1,\infty)\) and \( a > 0 \)

\[
\int_{|\eta|<\theta} \left| \int_{|\xi|<\theta} |\hat{u}(\eta,\xi)\xi|^p d\xi \right| d\eta \leq C K \eta^a \theta^p |\hat{u}(\eta,\xi)|^p d\eta \theta^p d\xi
\]

for each pair \( U(x), V(x) \) satisfying \( A_p^* \) with constant \( K \). Does this imply the existence of a number \( C_p \) such that

\[
\left( \int_0^1 \int_{|x-\theta|^p}^{x+\theta} |U(y) V(y)| \right)^{p-1} \frac{dy}{I} \leq C_p \theta^p K \theta^p |\hat{u}(\eta,\xi)|^p d\eta \theta^p d\xi
\]

for all pairs \( U(x), V(x) \) satisfying \( A_p^* \) with constant \( J \) for \( p \) between the two values of \( q \), where \( C_p \) is independent of \( f \) but can depend on \( C, p, J \) and the two values of \( q \)?

b. If the answer to problem a is no, will (2) be true for all pairs \( U(x), V(x) \) satisfying \( A_p^* \) for \( p \) in the interval \((b,d)\) if (1) is assumed for every pair \( U(x), V(x) \) satisfying \( A_p^* \) with constant \( K \) for every \( q \) in \([b,d]\) or \([b,d]\) if \( C_p \) is independent of \( f \) but can depend on \( C, p, J \) and the two values of \( q \)?

c. Determine conditions other than \( A_p^* \) for which theorems like those in a and b hold and do this over other spaces.
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Algebra & Theory of Numbers

*75T-A28 GEORGE F. XEROUDAKES, 2 Korae St., Iraklion Crete, Greece. On Fermat's Last Theorem.

**Theorem** -- For every natural number \( n \equiv 1 \pmod{3} \) the equation: \( x^n + y^n + z^n = 0 \)
is satisfied in integers \( x, y, z \) of the field \( K(\sqrt[3]{3}) \) by:
\[
x = (2a - b) + c\theta^{\sqrt[3]{3}}, \quad y = (a + b) + c(a - b), \quad z = (2b - a) - c\theta^{\sqrt[3]{3}}
\]
where \( a, b \) are rational integers and \( c = \pm 1 \).

Consequently we conclude that Fermat's last theorem is not true in associated integers of the field \( K(\sqrt[3]{3}) \) for \( n \equiv 1 \pmod{3} \). (Received October 31, 1974.) (Introduced by Professor N. C. Petridis.)

75T-A29 RICHARD M. GRASSL, University of New Mexico, Albuquerque, New Mexico, 87131
Levi Structures For Polynomial Ideals

Let \( R \) be the ring of polynomials in a denumerable set of independent indeterminates over a field \( F \) and let \( I \) be an ideal \( (x_0, x_1, x_2, \ldots) \) in \( R \). A Levi structure for \( I \) provides bases for \( I \) and \( R \) as vector spaces over \( F \) and an associated algorithm for determining whether an element of \( R \) is in \( I \).

Such structures have been developed previously for certain principal differential ideals [i.e., ideals \( (x_0, x_1, \ldots) \) with \( x_j \) the \( j \)-th derivative of \( x_0 \)] in which the generator is homogeneous and isobaric and for a family of related ordinary ideals. The present paper extends this to cases in which the generator need not be homogeneous and need not be isobaric and also presents Levi structures for ideals generated by subsets of the \( x_j \). The family of ordinary ideals is enlarged similarly.

These generalizations are obtained with the aid of a new ordering of power products in the indeterminates. In particular, a Levi structure is given for the differential ideal \( (z g) \) generated by the \( g \)-th derivative of the product \( z = y_1 y_2 \cdots y_n \) of \( n \) independent differential indeterminates. (Received November 1, 1974.) (Introduced by Dr. Abraham P. Hillman.)

75T-A30 ROBERT GILMER, Department of Mathematics, Florida State University, Tallahassee, Florida 32306. The group of units of a commutative semigroup ring.

Let \( R \) be a commutative ring with identity and let \( S \) be a torsion-free, cancellative abelian semigroup with zero. An element \( f = a_1 S^1 + a_2 S^2 + \ldots + a_n S^n \) of the semigroup ring \( R[S] \) is a unit if and only if there is a positive integer \( k \) such that
\[
R = (a_1^k) \oplus (a_2^k) \oplus \ldots \oplus (a_n^k), \quad s_i \text{ is invertible in } S \text{ for each } i \text{ such that } a_i \text{ is not nilpotent.}
\]
(Received November 1, 1974.)

75T-A31 E. E. DEVUN, Wichita State University, Wichita, Kansas 67208, U-Products Preliminary Report.

A U-semigroup is a semigroup which is isomorphic to the unit interval.
[0, 1] with the usual multiplication. A semigroup \( S \) is said to be the unique product of U-semigroups \( S_1, S_2, \ldots, S_n \) if each \( S_i \) is a U-semigroup and for every \( \sigma \in G_n \) ( \( G_n \) is the symmetric group on \( n \) elements) \( S = \sigma_0(S_0(1)) \sigma_2(S_0(2)) \ldots \sigma_n(S_0(n)) \) and for every non-zero element \( s \) of \( S \) with \( s = x_\sigma(1) x_\sigma(2) \ldots x_\sigma(n) = y_\sigma(1) y_\sigma(2) \ldots y_\sigma(n) \) and \( x_\sigma(i), y_\sigma(i) \in S_\sigma(i) \) we have \( x_\sigma(i) = y_\sigma(i) \). In this note we will assume \( S \) is the unique product of U-semigroups, \( S \) has no zero divisors, and \( \mathbb{F}(S) = \mathbb{F} = \{0, 1\} \). Theorem \( S \) is uniquely divisible. A semigroup \( T \) is said to be left (right) reversible if every pair of right (left) ideals have non-empty intersection. Now let \( C \) be the convex hull of the \( (n \times n) \) matrices \( \{X_0, X_1, \ldots, X_k, \ldots, X_{n-1}\} \) where \( X_0 \) is the identity matrix and \( X_k \) is the matrix \( (x_{ij}) \) where \( x_{nk} \in \{0, 1\}, \ x_{nn} = 1 \) and \( x_{ij} = 0 \) otherwise. Theorem If each subsemigroup \( S_i \) of \( S \) is left reversible and not right reversible, then \( S \) is isomorphic to semigroup formed by the one point compactification of the semigroup \( C \) with matrix multiplication. (Received November 4, 1974.) (Author introduced by William M. Perel)

A ring \( R \) containing more than one element is called a generalized quasi-Jacobson ring (gJ ring) if, for every nonzero element \( x \in R \), there exist integers \( n > 1, a, b \) such that \( ax^n = bx \neq 0 \) and \( (a, b) = 1 \). Particularly, if such \( a \) can be chosen to be 1 for every \( x \in R \) then \( R \) is called a quasi-Jacobson ring (qJ ring). Let \( R \) be a gJ ring and \( R^{+} \) be the additive group of \( R \). Theorem 1. \( R^{+} \) is either torsion or torsion free. Theorem 2. If \( R^{+} \) is torsion then \( R \) is a Jacobson ring (i.e. for each \( x \in R \), there exists an integer \( n > 1 \) such that \( x^n = x \)). Theorem 3. If \( R^{+} \) is torsion free then \( R \) is isomorphic to a subring of the rational field. Corollary. If \( R \) is a qJ ring with \( R^{+} \) torsion free then \( R \) is isomorphic to a subring of the ring of integers. (Received November 11, 1974.)

Let \( s = \sigma + it, \ 0 < a \leq 1 \). The author shows that, except for a small bounded region near the origin, all complex zeros of \( \zeta(s,a) = \sum_{n=0}^{\infty}(n+a)^{-s} \) lie in the vertical strip \(-1 \leq \sigma \leq 1 + a\), and that all the real zeros, except for this bounded region, lie on the negative real axis one in each interval \(-2n - 4a + 1, n \geq 1 - 2a\). Zeros are calculated for \( a = \frac{1}{3}, \frac{2}{3}, \ |t| < 100, \ \sigma \geq -5 \). (Received November 11, 1974.)

An Algorithm to expand the Star of David Theorem (GCD properties of binomial coefficients) Preliminary report. An Algorithm, named pennant closure process has been developed in order to test several sets of binomial or multinomial coefficients to have the same greatest common divisor.

Applying this algorithm to the special geometric patterns of binomial or multinomial coefficients such as triangles, squares, hexagons, etc., many generalizations of the Star of David Theorem, which was conjectured by H.W. Gould in 1972 [Notices AMS 72T-A248, A-685 (1972)] can be obtained.

This algorithm is based on an identity of S. Hitotumatu and easily carried out on a paper with a pencil. Also this process can be programmed for the computers or manually practiced on a chinese checker board with abacus like operations.

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The machine production of many combinatorial identities involving binomial, or multinomial coefficients, which leads to the new greatest common divisor property is possible. (Received November 11, 1974.)

Let \( \binom{n}{k} \) be binomial coefficients. The greatest common divisor property i.e.; \( \text{GCD}(n-1\binom{k}{k-1}, n+1\binom{k+1}{k}) = \text{GCD}(n+1\binom{k+1}{k+1}, n-1\binom{k-1}{k}) \), was conjectured and named as the Star of David Property by H.W. Gould in 1972 [Notices AMS 72T-A248, (19) (1972) A-689, Fibonacci Q.J. 10 (1972), 579-584, 628]. So far, three solutions appeared, but all three proofs were based on the determination of the exponents of primes in various binomial coefficients [Fibonacci Q.J. 10 (1972), 565-568, 598, 11 (1973), 25-26, 11 (1973), 282-284, Journal für die Reine und Angewandte Mathematik, 262-263 (1973), 375-380]. Let \( X = [n-1\binom{k-1}{k-1}, n+1\binom{k+1}{k}, n-1\binom{k+1}{k}] \), \( Y = [n+1\binom{k+1}{k+1}, n-1\binom{k-1}{k}] \), \( A_1 = [k+1,-k,k+1] \), \( A_2 = [k-n-1,n-k+1,k-n] \), \( A_3 = [-n-1,n,-n] \), \( B_1 = [-n,n,-n] \), \( B_2 = [-k,k+1,-k-1] \), \( B_3 = [n-k+1,k-n,n-k+1] \), where \( T \) indicates the transposition of each integer row vectors to column vectors. Now define 3x3 integer matrices as \( A = [A_1, A_2, A_3] \) and \( B = [B_1, B_2, B_3] \), then it is easy to verify that \( X = AY, Y = BX, B = A^{-1} \). This shows that a common factor of numbers that appear in one of the vectors \( X \) and \( Y \) also divides each number of the other vector. This proves the Star of David Property in its original form. It is to be noted that the same techniques can be applied to the expansion of the Star of David Property of larger sizes, or to the analogous property in multinomial coefficients. These results will be reported separately. (Received November 11, 1974.)

Theorem. There exists infinitely many primes \( p \) such that for every non-negative integer \( n \), the equation \( \phi(n) = k \) has no solution. (Received November 11, 1974.)

Further characterizations and properties of exactly covering congruences.

If every nonnegative integer occurs in exactly one of the \( m \) integer sequences \( a_i n + b_i \) \( (n \geq 0, 1 \leq i \leq m) \), then the system \( a_i n + b_i \) is called an exactly covering congruence (ECC). Three characterizations of ECC's in terms of exponential functions, Bernoulli polynomials and Euler polynomials are given from which several properties of ECC's are deduced, including a method of obtaining from an ECC with \( m \) moduli \( a_i \) several ECC's with \( < m \) moduli.

An irreducible (or point-determining) graph is one in which distinct vertices have distinct neighborhoods. The achromatic number of a graph \( X \) is the largest integer \( k \) such that \( X \) admits a complete \( k \)-colouring (if such an integer exists). Let \( X^* \) be obtained from \( X \) by identifying all vertices of \( X \) that have the same neighborhood; if \( X^* \) admits a complete \( k \)-colouring, then so does \( X \). Hence irreducible graphs are instrumental for the study of the achromatic number. We prove that there are only finitely many irreducible graphs with a given achromatic number. This permits us to describe all graphs with achromatic number less than four. In the proof we use a lemma yielding the existence of a system of distinct representatives for the edges of a special hypergraph, and a calculation determining the achromatic number of all paths and cycles. Possible generalizations of the main theorem to homomorphisms other than colourings are discussed, and an application to the "almost-colourings" of G. Sabidussi is mentioned.

(Received November 18, 1974.)
Let $R$ be a ring of characteristic $\neq 2$ which satisfies (1) $(x,y,x) = 0$ and (2) $(w,x^2,z) = x \cdot (w,x,z) + (x,x,[w,z])$ where $x \cdot y = xy + yx$, $[x,y] = xy - yx$ and the associator $(x,y,z) = (xy) z - x (yz)$.

**Theorem.** If $R$ is a simple ring satisfying (1) and (2) with idempotent $e \neq 1$ such that $(e,e,R) = 0$ then $R$ is alternative.

In the case of power associative algebra $A$ over an algebraically closed field of characteristic $\neq 2,3$, we can show the following: **Theorem.** If $A$ is simple with idempotent $e \neq 1$ such that (1) and (2) hold in $A$, then $A$ is a noncommutative Jordan algebra.

Also, as a partial converse to the result that any noncommutative Jordan ring of characteristic $\neq 2$ satisfies $(w,x^2,z) = x \cdot (w,x,z)$ (Journal of Algebra, V23, No 3, Dec., 1972, p.503), we can establish the following: **Theorem.** If $R$ is a flexible, power associative ring of characteristic $\neq 2$ which satisfies $(w,x^2,z) = x \cdot (w,x,z)$ and $(x,x,[w,x]) = 0$ for all $x,w,z$ in $R$ then $R$ is a noncommutative Jordan ring. (Received November 21, 1974.)

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A finite quasigroup of order greater than one is plain if it has no nontrivial proper subquasigroups and no nontrivial congruence relations. Every variety that contains a nontrivial finite quasigroup contains a plain quasigroup, and every equationally complete variety that contains a nontrivial finite quasigroup is generated by a plain quasigroup. Nonisomorphic plain quasigroups generate distinct varieties. Hence, equationally complete varieties which contain nontrivial finite quasigroups can be characterized in terms of equationally complete plain quasigroups. Let $Q$ be a plain quasigroup. If $Q$ has an idempotent element, then $Q$ is equationally complete. If $Q$ does not have an idempotent element, then $Q$ is equationally complete if and only if the diagonal is not a congruence class for some congruence relation $\Theta$ on $Q \times Q$. Finally, if $Q$ is not equationally complete, then $(Q \times Q)/\Theta$ is plain and generates the only nontrivial proper subvariety of $V(Q)$. (Received November 26, 1974.) (Author introduced by Professor Trevor Evans.)

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Let $p$ be a prime and assume that $G$ is a finite simple group with a Sylow $p$-subgroup $P$ of order $p^2$. Assume also that $|G| < p^3$. Then the following are proved: **Theorem 1.** The Sylow $p$-subgroups of $G$ form a trivial intersection set. **Theorem 2.** Either $p = 3$ and $G$ is isomorphic to the group $SL_2(8)$, or else $|N_G(P):C_G(P)| > p$. **Corollary.** If $p$ is a prime and $G$ is a finite simple group whose order is divisible by $p^2$ then the order of $G$ is greater than $p^5$. These results partially generalize a theorem of Brauer and Reynolds (Ann. Math. 68 (1958), 713-720) who assume that $P$ has order $p$, and a theorem of Herzog (J. Algebra 15 (1970), 408-416) who assumes that $P$ is cyclic. (Received December 2, 1974.)

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products, homomorphic images, and group extensions; and proved there is a one-to-one correspondence between idempotent ideals and TTF classes. We discuss TTF classes which are closed under injective envelopes--stable TTF classes. Theorem 1. For a ring $R$, there is a one-to-one correspondence between stable TTF classes and idempotent ideals $I$ such that $R/I$ is flat as a (left) $R$-module. We classify stable TTF classes over certain types of rings: e.g. Theorem 2. For a commutative perfect ring, each hereditary torsion class is a stable TTF class. Theorem 3. Suppose the (right) socle is essential. Then the class of torsion modules with respect to Goldie torsion theory is a stable TTF class with $(\text{socle})^2$ as the minimal ideal in the corresponding filter, and $(\text{socle})^2$ is projective and non-singular. Among other applications we improve some results of Gordon and Storrer. (Received December 2, 1974.)

**75T-A43** CHONG-KEANG LIM, Department of Mathematics, University of Malaya, Kuala Lumpur, Malaysia. On graphical regular representations of direct product of groups.

A graph $X$ is called a graphical regular representation (GRR) of the given group $G$ if the automorphism group $\text{A}(X)$ of $X$ is regular, as a permutation group, and isomorphic to G. M.E. Watkins [J. Combinatorial Theory Ser. B 11 (1971), 95-104] has shown that the direct product of two finite groups $G_1$ and $G_2$ has a GRR if both $G_1$ and $G_2$ have GRR and are different from the cyclic of order two. Later, W. Imrich [Israel J. Math. 11 (1972), 258-264] generalises this result to infinite groups. A new proof of this result for finite and infinite groups is given. The new proof rests heavily on the following result. Theorem. If $X$ is a graphical regular representation of the group $G$ (which may be finite or infinite) then $X$ is not self-complementary. (Received December 3, 1974.)

**75T-A44** MENDELSOHN ERIC, University of Toronto, Toronto, Ont. M5S 1A1 Almost perfect one-factorizations of $K_{2n}$ ($2n + 1$ prime)

It is shown that there exists a family of one factorizations of $K_{2n}$ so that the factors fall into two classes $H, H'$ where $|H| = 2n-3$, $|H'|=2$ such that if two factors are in the same class their union is a Hamiltonian circuit, however all but 6 of the pairs where one factor is taken from each class have a 4-cycle in their union. (Received December 4, 1974.)

**75T-A45** CURTIS GREENE, M.I.T., Cambridge, Mass. 02139 and GEORGE MARKOWSKY, IBM T.J. Watson Res. Center, P.O. Box 218, Yorktown Heights, N.Y. 10598. A Combinatorial Test for Local Distributivity

The second author proved (Proc. Univ. of Houston Lattice Theory Conf., Houston Lattice Theory Conf., Houston 1973, p. 39), that if $L$ is a finite lattice in which all maximal chains have the same length, $L$ is distributive if and only if $h(L) = |J(L)| = |M(L)|$, where $h(L)$ is the length of $L$, $J(L)$ the set of join-irreducibles in $L$, and $M(L)$ the set of meet-irreducibles in $L$. In this note, the authors show that if $L$ is a finite lattice in which all maximal chains have the same length, $L$ is locally distributive if and only if $h(L) = |J(L)|$ (and dually). In the course of proving this result they show that if $f: L_1 \rightarrow L_2$ is an injective meet-preserving map, $|J(L_1)| \leq |J(L_2)|$.

(Received December 4, 1974.)

**75T-A46** ROLF JELTSCH, University of California, Los Angeles, CA 90024. On universally uniform bounds for the matrix exponential. Preliminary report.

A constant $K > 0$ is said to be a universally uniform bound for the matrix exponentials A-299.
on $\Omega$ if $\|\exp(-tA)\|$ is bounded by $K$ for all $t \geq 0$ and all $A \in \Omega$. $\Omega$ is a set of complex $n \times n$ matrices. For four sets it is shown that such a bound exists. For the set $
abla^+ = \{ A | \lambda_1 \text{ eigenvalues of } A, \text{Re} \lambda_1 \geq 0 \text{ and if } \text{Re} \lambda_1 = 0 \text{ then the corresponding elementary divisor is linear} \}$ such a constant $K$ does not exist. However, two semi universal results are proved. (i) For each $A \in \nabla^+$ exists a constant $K_A$, dependent on $A$, such that $\|\exp(-tA)\| < K_A$ for all $t \geq 0$. (ii) There exists a $K$, independent of $A$, and for each $A$ a norm $\| \|_A$, dependent on $A$, such that $\|\exp(-tA)\|_A < K$ for all $t \geq 0$.

(Received December 9, 1974.)

75T-A47 MARVIN TRETKEFF, Stevens Institute of Technology, Hoboken, New Jersey, 07030, Covering Spaces and HNN Extensions, Preliminary Report.

The present work is devoted to a topological proof of a few basic results about HNN extensions. Namely, we establish (i) the injectivity of the "obvious" map of the base into an HNN extension, (ii) Britton's Lemma, and (iii) Collins' Lemma. Our proofs employ the standard theory of covering spaces and, for expository purposes only, a tree associated with special covering complexes. The latter simplifies the description of our key construction. Our methods can also be applied to amalgamated free products. (Received December 11, 1974.)

*75T-A48 E. RAPAPORT STRASSER, SUNY at Stony Brook, and W.H. Levinson, Rutgers, New Brunswick, NJ. PLANARITY OF CAYLEY DIAGRAMS: PLANAR PRESENTATIONS

Let $G$ be a Cayley diagram in the plane, $X = (x_1, x_2, \ldots, x_k)$ its edge labels, and $R = (r_1, \ldots, r_m)$ words in the $x$-symbols spelled out by the boundaries of the faces. Then $(x; R)$ is a presentation whose graph is $G$. It is special planar if $G$ has no loops, no accumulation points of vertices, and if the succession of edges in a given direction is the same at each vertex. Let $P = (x_1, \ldots, x_k, r_1^{d_1}, \ldots, r_m^{d_m}) = (x; r^d)$ denote a presentation of a group for which the product $r_1 \cdots r_m$ is a permutation of $X$. A certain other permutation, $S_r$, of the set $X$ is uniquely assigned to $r$. It is shown that the conditions (1) $r$ is of minimal $x$-length under automorphisms of the free group $F_t$ of rank $t$, (2) $S_r$ is a single cycle of length $2t$, and (3) $P$ is special planar, are equivalent. Let $P^* = (a_1, b_1, \ldots, b_n, c_1^{d_1}, \ldots, c_q^{d_q}, (c_1^{d_1}, c_2^{d_2}, \ldots, c_q^{d_q})^{d+1})$. It is shown that $S_r$ consists of $h+1$ disjoint cycles if and only if $P$ is automorphic image in $F_t$ of a free product of the form $P^* \ast F_n$. The values of $q$, $n$, and $h$ are read off the set of words $r_1, \ldots, r_m$. For given $t$ and (compatible) $n$ all special planar presentations and their isomorphism classes are found. The methods are combinatorial. (Received December 16, 1974.)


A norm $\sigma$ on $C^n$ and a direct-sum decomposition $\sigma$ of $C^n$ induce a vectorial norm of order $k$: $p(\sigma) = (\sigma(E_1), \ldots, \sigma(E_k)) \in C^k$, where the $E_i$'s are the projections associated with $\sigma$. We write $p = (\sigma, \pi)$ and we call $p$ equilibrated if $\|E_j\| = 1$ for all $j$. Given an equilibrated vectorial norm $p = (\sigma, \pi)$ of order $k$, one defines the mapping $\beta_p: C^{mn} \rightarrow R^{nk}$ by $\beta_p(\lambda) = \lim M(\lambda, I_n + hA) - I_k)/h$ as $h \rightarrow 0$ ($A \in C^{mn}$), where $I_q$ is the identity matrix of order $q$ and $M_p(\lambda) = (\|E_jE_k\|)^{\pi}$. This generalizes the concept of a logarithmic norm. We denote by $\alpha(X)$ the maximal real part of the eigenvalues of the matrix $X$. Theorem 1. Let $A \in C^{mn}$ and let $k \in [1, \ldots, n]$. Then $\alpha(A) = \inf \alpha(\beta_p(\lambda))$, the
Theorem 2. Let \( A \in \mathbb{C}^{n \times n} \), \( \lambda_1, \ldots, \lambda_s \) the eigenvalues of \( A \) with maximal real part, \( k_j \) the multiplicity of \( \lambda_j \) in the minimal polynomial of \( A \) and \( k = \max k_j \). Then, there exists an equilibrated vectorial norm \( p = (\sigma, \pi) \) of order \( k \) such that \( \alpha \| \! \! p \|_p^2(A) = \alpha(A) \). (Received December 12, 1974.)

A homeomorphically irreducible graph is defined as a graph with no loops or multiple edges such that no vertex has valency two. The exponential counting series

\[
H(x, y) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{1}{i!} \frac{1}{j!} \frac{1}{(1+xy)^{i+j}} \cdot \exp \left( -\frac{x^2}{4} + \frac{xy}{2} - \frac{y^2}{4} \right) \cdot \sum_{k=0}^{\infty} \frac{1}{k!} \left( (1+x) \exp \left( -\frac{x^2}{4(1+xy)} \right) \right)^k \cdot \left( y \exp \left( \frac{x^2}{2(1+xy)} \right) \right)^k \cdot \left( (1+xy)^{i+j} \right).
\]

The exponential counting series for connected graphs of this type is the logarithm of this expression. Similar results can be found for such graphs with loops and multiple edges. (Received December 16, 1974.) (Authors introduced by Professor Leroy J. Dickey.)

A commutator-power presentation has been obtained (with the help of a computer) for the free group \( B(4,4) \) of rank four of the variety of groups of exponent four. The group \( B(4,4) \) has order \( 2^{422} \) and nilpotency class precisely ten. The lower central factors of \( B(4,4) \) have orders \( 2 \exp 8, 6, 20, 55, 99, 84, 80, 40, 20, 10 \) respectively. The presentation has the form

\[
\langle a_1, \ldots, a_{422} : a_1^2 = a_5, \ldots, a_{422}^2 = 1, [a_2, a_1] = a_9, \ldots, [a_{422}, a_{421}] = 1 \rangle.
\]

It is about one hundred and eighty pages long. (Received January 2, 1975.)

Let \( V \) be a variety (equational class) of algebras all of whose congruence lattices are modular. Then there is an identity \( \varepsilon \) such that the congruence lattices of the members of \( V \) satisfy \( \varepsilon \), but \( \varepsilon \) fails in all non-desargian projective planes. (Received December 12, 1974.)*

A pair \( (a, b) \) of elements of \( R \) will be called a semi-regular pair iff \( \exists \alpha \in R \) such that \( \alpha a' = a \) and \( \text{ann}(a) \subseteq \text{ann}(a') \). A pair \( (a, b) \) of elements of \( R \) will be called a semi-regular pair iff \( \exists \alpha \) a 2x2 matrix

\[
\begin{pmatrix}
\alpha & \gamma \\
\beta & \delta
\end{pmatrix}
\]

with elements in \( R \) such that

\[
(a, b)(\begin{pmatrix}
\alpha & \gamma \\
\beta & \delta
\end{pmatrix}) = (a, b)
\]

and

\[
\left\{ \left( \begin{pmatrix}
\gamma \\
\beta
\end{pmatrix} \right) \mid ax + by = 0 \right\} \subseteq \left\{ \left( \begin{pmatrix}
\gamma \\
\beta
\end{pmatrix} \right) \mid \left( \begin{pmatrix}
\alpha & \gamma \\
\beta & \delta
\end{pmatrix} \right) = (0) \right\}
\]

The following theorems are then proved:
Theorem 1. A principal ideal \( aR \) is projective as \( R \)-module iff \( a \) is semi-regular.

Theorem 2. An ideal \( aR + bR \) is projective iff the pair \( (a,b) \) is semi-regular. (Received November 15, 1974.)

*75T-A54 E. G. WHITEHEAD, Jr., University of Pittsburgh, Pittsburgh, Pennsylvania 15260.

Difference sets and sum-free sets in groups of order 16.

Transversals for sum-free sets in the nine nonabelian order 16 groups are given. It is shown that exactly eight of the order 16 groups have difference sets \( D \) with \( \lambda = 2 \) and \( D = -D \). It is proven that the only nontrivial \( (v,k,\lambda) \)-difference sets with \( v = 2^7 \) and \( \lambda = 2 \), have parameters \((16,6,2)\). It is shown that exactly four of the order 16 groups can be partitioned into three sum-free sets. (Received December 20, 1974.)

75T-A55 S.F. WILLIS, Ridgetown, Ontario, Canada. Dualities between Categories of Locally Compact Modules. Preliminary report.

A duality is a contravariant category equivalence between the category of all locally compact Hausdorff right modules over a discrete ring \( R \) and left modules over a discrete ring \( S \). Such a duality is naturally equivalent to the composition of the Pontryagin duality and an extension of a Morita Equivalence. (Received December 2, 1974.)

*75T-A56 JIMMY ARNOLD, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and ROBERT GILMER, Florida State University, Tallahassee, Florida 32306. Krull dimension of commutative semigroup rings. Preliminary report.

Let \( R \) be a commutative ring with identity and let \( S \) be a cancellative additive abelian semigroup with zero and with quotient group \( G \). The (Krull) dimension of the semigroup ring \( R[S] \) of \( S \) over \( R \) is the same as the dimension of \( R[G] \); moreover, it is known that \( R[G] \) has the same dimension as the ring \( R[X, X^{-1}] \), where \( |A| \) is the torsion-free rank of \( G \). (Received December 23, 1974.)

*75T-A57 M. BHASKARAN, 9-18, Butterick Place, Girrawheen, W. Australia 6064.

On the genus field of a Galois extension.

The theory of genus group and genus field for an arbitrary number field was developed by Fröhlich. He defined the genus field \( \mathbb{K} \) of a finite algebraic number field \( K \) as the maximal unramified extension of \( K \) of form \( K \mathcal{O} \), where \( \mathcal{O} \) is an abelian extension of \( \mathbb{Q} \). Let \( \mathbb{K} \) be a Galois extension of \( \mathbb{Q} \) and \( \overline{\mathbb{K}} \) its genus field over \( \mathbb{Q} \). For a rational prime \( p \), let \( e_p \) denote the ramification index of any \( K \)-prime divisor of \( p \) and \( \zeta_p \) denote a primitive \( p^m \) root of unity. Then \( \overline{\mathbb{Q}} \) contains the subfield of \( \mathbb{Q} \) ( \( \zeta_p \) ) of degree \( (e_p, \mathcal{O}_p, \mathcal{O}_\mathbb{Q}) \). Using this, it is possible to describe the genus field of a tamely or wildly ramified Galois extension at least when the degree of the extension is odd. Proof of the theorem depends on the author's previous result (to appear, these Notices, Aug. 1974), some Ideas of Parry in his application of Cebotarev density theorem (Acta Arith. 19 (1971), Th. 1) and class field theory. (Received December 26, 1974.)

75T-A58 C. J. MOZZOCHI, Box 1315, Hartford, Connecticut 06101. A remark on Goldbach's conjecture.

The notation here is the same as that in "Modern prime number theory" by T. Estermann. It is well known that \( \int_{\mathbb{P}_1} x^2 \mathbb{Z}(x, n) e(-nx) dx = O(n \log \log n) \) and that if \( (b, q) = 1, \left| y \right| \leq x_q \) and \( q \leq \log_{15} n \), then \( f(h/q + y, n) - \mu(q) \varphi^{-1}(q)(g(y, n) = O(n \log^{-69} n) \). In this paper I first show that there exists an \( M \) and a sequence \( e(n) \) such that for each \( n, M > e(n) > 0 \) and \( (b/q + y, n) - \mu(q) \varphi^{-1}(1+e(n)) \varphi(g(y, n) = O(n \log^{-69} n). \)

I then show that if (A): the estimate on the above integral could be improved to \( O(n \log^{-A} n) \), for some
$\Delta > 2$; and if (B); my result could be improved (and in a sense weakened) to: there exists $M, \alpha$ and a sequence $e(n)$ such that for each sufficiently large (even) $n$ we have $M > e(n) > \alpha > \Delta/30$ and $f(h/q + y, n) - \mu(q)^{-1}e(n)(g(y, n) = O(\log^{-45\gamma} n),$ then it would follow that every sufficiently large even integer can be expressed as the sum of two primes. (Received December 27, 1974.)

G. J. RIEGER, Technical University, D-3 Hannover. On a theorem of Deshouillers concerning sums of fractional powers.

Denote by $[a]$ the integer part of the real number $a$. Using estimates of van der Corput, Deshouillers (Acta Arithmetica 25 (1974), 393-403) proved an interesting result: For real $c$ with $1 < c < 4/3$, every large natural number $n$ can be written as $n = [x^c] + [y^c]$ with natural numbers $x, y$. By a similar method we prove: For real numbers $c > 1, d > 1, t > 2$ with $c \neq d, c^{-1} + d^{-1} > 16/9$ the number $A(t; c, d)$ of natural numbers $x, y$ with $[x^c] = [y^d] < t$ satisfies $A(t; c, d) = (1 - (c-1)(d-1))^{-1} t^{1+c^{-1}d^{-1}} (1 + o_1((\log t)^{-1})).$ The range of $c, d$ can easily be extended. The method is also applicable to more general equations of the type $[f(x)] + [g(y)] = n.$ (Received December 30, 1974.)

EDWARD SPENCE, University of Glasgow, Glasgow G12 8QW, Scotland. Skew-Hadamard matrices of order $2(q+1)$.

Using a cyclic relative difference set with parameters $(q+1, 4, q, 1, q(q-1)/8)$ it is shown that if $q \equiv 5 \pmod{8}$ is a prime power then there exists a skew-Hadamard matrix of the Goethals-Seidel type of order $2(q+1).$ (Received January 7, 1975.)

DAVID E. DOBBS and IRA J. PAPICK, Rutgers University, New Brunswick, New Jersey 08903. When is $D + M$ coherent?

Let $V$ be a valuation ring of the form $K + M$, where $K$ is a field and $M(\neq 0)$ is the maximal ideal of $V$. Let $D$ be a proper subring of $K$; let $k$ be the quotient field of $D$; let $R$ denote $D + M$. Recall (cf. Appendix 2 of R. Gilmer, Multiplicative ideal theory, Queen's University, 1968) that $R$ is Noetherian iff $D = k, [K:k] < \infty$ and $V$ is Noetherian. 

Theorem. $R$ is coherent iff one of the following two conditions holds: (1) $k = K$ and $D$ is coherent; (2) $M$ is a finitely generated ideal of $R$. Moreover, in case (2), $D = k$ and $[K:k] < \infty$. Corollary. $R$ is integrally closed and coherent iff $k = K$ and $D$ is integrally closed and coherent. Greenberg and Vasconcelos have recently studied coherence of pullbacks under riding assumptions which, when specialized to our context, amount to $k = K$ and $M$ flat over $R$. Theorem. A nonzero ideal $I$ of $V$ is $R$-flat iff at least one of the following two conditions holds: (1) $k = K$ and $I/MI$ is $D$-flat; (2) $I = MI$. Corollary. $R$ is coherent and $M$ is $R$-flat iff $k = K$, $D$ is coherent and $M/N^2$ is $D$-flat. The proofs of both theorems use a result of Ferrand, C. R. Acad. Sci., Paris 269 (1969), A946-A949, on the descent of flatness. Related examples are also given. (Received December 30, 1974.)

IRWIN S. PRESSMAN, Dept. of Mathematics, Carleton Univ, Ottawa K1S 5B6 Canada. The bivariant long exact sequence for the Ext Functor.

Given two short exact sequences $\xi$: $0 \to A \to B \to C \to 0$ and $\Omega$: $0 \to X \to Y \to Z \to 0$ in some abelian category, then there is a functorial construction of a long exact sequence

$$0 \to \text{Hom}(Z, A) \to \text{Hom}(Y, B) \to \text{Hom}(Y, E) \to \text{Ext}^1(Z, A) \to \text{Ext}^1(Y, B) \to \text{Ext}^1(Y, E) \to \text{Ext}^2(Z, A) \to \cdots.$$ 

This generalizes the usual 2 long exact sequences in that for proper choices of $\xi$ or $\Omega$ one obtains respectively the contravariant or covariant Ext-sequences together with commutative diagrams to these from the new bivariant sequence. The groups $\text{Ext}(Y, E)$ are computed in the category of morphisms. The connecting homomorphisms can be explicitly described and give interesting applications to the computation of obstructions to factoring certain diagrams. A generalization of the usual Mayer-Vietoris sequence is also obtained. (Received January 8, 1975.)
The late Professor I.A. Barnett posed the following problem: Are there infinitely many primitive Pythagorean triples \((x,y,z)\) for which \(x+y\) and \(x-y\) are simultaneously prime. The purpose of this note is to report that if \(m\) and \(n\) denote the relatively prime integers of opposite parity so that \(x = 2mn, y = m^2 - n^2,\) and \(z = m^2 + n^2,\) then for each positive integer \(m \leq 46000 --\) with the exception of \(m = 1, 2, 5,\) and \(30 --\) there is an integer \(n\) which generate a primitive Pythagorean triple with \(x+y\) and \(x-y\) prime. The tests have been conducted on the IBM System 360/44 and 370/145 at Ohio University using the multiprecision package MPARITH developed by D.H. Lehmer and C. Weinberger and a multi-method prime test whose principal ingredient is based upon a converse to Fermat's Theorem (Bull. AMS, 33(1927), 327-340). An assembler version of the multi-precision package is currently being tested to improve the speed of the primality checks. (Received January 10, 1975.)

A commutative ring \(R\) is said to be locally noetherian if for each prime ideal \(p\) of \(R, R_p\) is a noetherian ring. \(R\) is said to be a weak inertial coefficient ring if for each \(R\)-algebra \((\text{The sum need not be direct.})\) E.C. Ingraham has proved in (On the existence and conjugacy of inertial subalgebras, J. Algebra 31(1974), 547-556) that every noetherian ring is a weak inertial coefficient ring. Using Theorem 1 of the same paper and the topological properties of Spec\((R),\) the following theorem is proved: Theorem: A locally noetherian ring \(R\) is a weak inertial coefficient ring for which the uniqueness statement holds. (Received January 10, 1975.)

It was shown recently by the author that \(-\text{det}(AB - BA)\) is a norm in the quadratic field generated by the characteristic roots of \(A,\) assumed irrational, where \(A\) and \(B\) are \(2 \times 2\) rational matrices. Previously, it was shown that in the relation \(S^{-1}AS = A'\) (transpose), \(S\) also a rational matrix, \(\text{det} S\) is a negative norm in the same field. Both facts lead to the representation of zero by an integral ternary quadratic form, a problem studied by Gauss. Such a representation of zero also occurs in connection with ideal classes of order \(4\) in a quadratic field (theorems of Rédei and Rédei and Reichardt). Links between the above facts are studied leading to connections between \(q(\sqrt{m})\) and \(q(\sqrt{-m}).\) It is further shown that the numbers whose norms appear as \(-\text{det}(AB - BA)\) can be chosen to form an ideal linked to the square of the ideal class connected with \(A.\) (Received November 29, 1974.)

Theorem 5 for both cases: Let \(x = F_n/2 + SQ\) \(2k+1,\) \(y = \text{det} L(14)^{1/2}((/2)Q_{4k}^3)\) \(3k+1,\) \(z = u_{2k} + p\). Then, for \(n \geq 1\),

\[
\left[ F_{2k} x_n^2 + 4z^2 (SQ) (R_2 x_n^2 - (SQ) x_{2k} x_{2k+1}) + 2Tz^2 x_{2k+1} \right] = \begin{cases} y_{2k+1}^2 + 4z_{2k+1} & \text{if } n = 1 \\
F_{2k+1} x_{2k+3}^2 - 4z_{2k+3}^2 - 3z_{2k+3} + 3z + 5.875 & \text{if } n \geq 1.
\end{cases}
\]
1.075 = \frac{1}{2n+1} - \frac{1}{2n+2}, \quad W_{n+2} = 4W_{n+1} - W_n, \quad W_0 = 1, W_1 = 3, \quad p > 1 \text{ is a root of } x^2 - 4x + 1 = 0. \quad \text{Let } B = \sqrt{3}.

For n \geq 1, \quad \left[ 4W_n^3 + (2W_{n+1} - (1122-30)W_n^2)^3 + 3B(28/9) + (1234/195) \right] = W_{n+1}^3. \quad \text{Remark: Theorem 2 (for cubes) was given in V, Abstract 74T-A121, these Notices, 21(1974), A-433.} \quad \text{(Received January 9, 1975.)}

75T-A67 SEUNG AHN PARK, University of Illinois, Urbana, Illinois 61801. A characterization of the unitary groups \( U_4(q), q = 2^n. \)

The four-dimensional projective unimodular unitary group \( U_4(q) \) and the four-dimensional projective unimodular linear group \( L_4(q) \), where \( q = 2^n > 2 \), can be characterized by the structure of the centralizer of a noncentral involution. \textbf{Theorem.} Let \( H_q \) be the centralizer in \( U_4(q) \) of a noncentral involution of \( U_4(q) \), where \( q = 2^n > 2 \). Let \( G \) be a finite simple group which contains a subgroup \( H \) such that \( H = H_q \) and \( H = C_G(z) \) for any involution \( z \) in the center of \( H \). Then \( G \) is isomorphic to either \( U_4(q) \) or \( L_4(q) \). Both cases occur because of the \textbf{Proposition}. The group \( H_q \) is isomorphic to the centralizer in \( L_4(q) \) of a noncentral involution of \( L_4(q) \). As usual we can generalize the theorem by removing the assumption that the group is simple. \textbf{(Received January 13, 1975.)}

*75T-A68 RICHARD C. O'BRIEN, Queen's University, Kingston, Ontario, Canada K7L 3N6, An Upper Bound on the Path Number of a Digraph.

\textbf{Theorem.} If \( G \) is a digraph of order \( n \geq 4 \), then the path number of \( G \) is at most \( \left\lfloor \frac{n^2}{4} \right\rfloor \). This result was first conjectured by Alspach and Pullman [Bull. Austral. Math. Soc. 10(1974), 422]. \textbf{(Received January 13, 1975.)}

*75T-A69 JOE V. PETTY, Texas Instruments, Inc., MS 907, Dallas, Texas 75222. A Generalization of the Local Property for Groups.

Let \( G \) be a group and suppose \( P = \{ H_a, K_a : a \in A \} \) is a set of pairs of subgroups of \( G \) with \( K_a H_a \) and for each finite subset \( F \) of \( G \setminus \{ 1 \} \), there exists \( a \in A \) such that \( F \subseteq H_a \). Then \( P \) is called a \textbf{local factor system} of \( G \). For each \( a, H_a/K_a \) is called a \textbf{factor} of \( P \). If \( K_a = \{ 1 \} \), for each \( a \in A \), then \( \{ H_a : a \in A \} \) is a local system of \( G \) in the usual sense. Define operators \( L_F, L \) by \( G \subseteq L_F X \) if \( G \) has a local factor system with \( X \)-factors; \( G \subseteq L X \) if \( G \) has a local system of \( X \)-subgroups. Clearly \( L \leq L_F \). Moreover, \( L \) is nilpotent, where \( N \) is the class of all nilpotent groups, because any nonabelian free group is in \( L_N \), but not in \( L \). Define an operator \( H_u \) by \( G \subseteq H_u X \) if \( G \) is isomorphic to an ultraproduct of a set of \( X \)-groups. \textbf{THEOREM.} If \( X \) is a class of groups which is subgroup closed and \( U \)-closed, then \( L \subseteq L_F \). As a consequence of a result of the author [Abstract 75T-A14, these Notices, 22(1975), A-5], each of the classes \( \hat{P}_u, \hat{P}_y, \hat{P}_u, \hat{P}_y \), where \( V \) is a variety of groups, is \( U \)-closed. These classes are subgroup closed, so we have that each is \( L \)-closed, hence \( L \)-closed. Thus, the local theorems of Mal'cev for the \( SN \)-groups, \( SI \)-groups, \( Z \)-groups can be obtained as corollaries to the above theorem. \textbf{(Received January 13, 1975.)}

75T-A70 PAUL K. STOCKMEYER, College of William and Mary, Williamsburg, Va., 23185. The reconstruction conjecture for tournaments. Preliminary report.

The famous graph reconstruction conjecture asserts that except for small trivial cases, every \( n \)-point graph is uniquely determined by its \( n \) subgraphs on \( n-1 \) points, each obtained by deleting one point and its incident edges. The corresponding conjecture for tournaments was first considered by Harary and Palmer [Monatsh. Math. 71 (1967) 14-23], who verified the conjecture for non-strong tournaments with more that 4 points. The truth of the conjecture for strong tournaments was cast into serious doubt by Beineke and Parker [J. Combinatorial Theory 9 (1970) 324-326], who displayed a pair of 5-point tournaments with the same collection of subtournaments, and three pairs of 6-point counterexamples. (Counterexample pairs on 3 and 4 points were already known.)

An exhaustive computer search has now determined that

1) the conjecture is true for 7-point tournaments, and

2) there are exactly two distinct pairs of counterexamples among the 6,880 tournaments on 8 points. In both cases, each tournament is the converse of the other, a property noticed by Beineke and Parker of all counterexamples on 4 and 6 points. \textbf{(Received January 13, 1975.)}
Let $n_i$, $1 \leq i \leq 4$, be positive integers such that $(n_1, n_2, n_3, n_4) = 1$. Let $S = \langle n_1, n_2, n_3, n_4 \rangle$ be the semigroup of all elements $z = \sum_{i=1}^{4} z_i n_i$, $z_i$ nonnegative integers, and assume no proper subset of $(n_1, n_2, n_3, n_4)$ generates $S$. If $c \notin S$, $c$ maximal, and $\forall z \in S$, $c - z \in S$, then $S$ is said to be symmetric. Let $P(n_1, n_2, n_3, n_4)$ be the prime ideal of all polynomials $f(x_1, x_2, x_3, x_4)$ such that $f(t_1, t_2, t_3, t_4) = 0$, where $t$ is transcendental over the arbitrary field $K$. It is shown:

1. $S$ symmetric $\Rightarrow$ $P(n_1, n_2, n_3, n_4)$ is generated by 3 or 5 elements.
2. Constructing the generators explicitly gives two types of prime ideals with 3 and one type with 5 generators.
3. Some arithmetical conditions on $n_i$, $1 \leq i \leq 4$, are given for $S$ to be symmetric. (Received January 13, 1975.)

**A Fixed Point Theorem for Finite Partially Ordered Sets.**

An order-preserving map $f$ of a partially ordered set $P$ to itself has a fixed point if there is an element $a$ in $P$ such that $f(a) = a$. $P$ is said to have the fixed point property if every order-preserving map of $P$ to itself has a fixed point; otherwise, $P$ is fixed point free. $P$ is dismantlable by irreducibles if there is a chain $P = P_0 \supset P_1 \supset \ldots \supset P_n = \emptyset$ such that $P_{i-1} \setminus P_i = I(P_{i-1})$ for each $i = 1, 2, \ldots, n$, where $I(P)$ denotes the set of all elements in $P$ which have precisely one upper cover or precisely one lower cover in $P$. Theorem. Let $P$ be a finite, connected, partially ordered set of length one. Then $P$ has the fixed point property if and only if $P$ is dismantlable by irreducibles. In addition, we prove a decomposition theorem about finite, fixed point free, partially ordered sets in which the basic ingredients are connected, fixed point free subsets of length one. Finally, for a finite, connected, partially ordered set $P$ and an order-preserving map $f$ of $P$ to itself we show that either there is a $c \in P$ such that $f(a) = a$ or there exists a connected subset $C$ of $P$, of length one, with $I(C) = \emptyset$, such that $f(C) = C$. (Received January 14, 1975.)

**Proper splittings of rectangular matrices.**

Proper splittings $A^N = N_1$ of a rectangular matrix $A$ of rank $r$ are characterized in terms of splittings $A_1 = N_1$ where $A_1$ is a nonsingular submatrix of $A$ of order $r$. It is shown that $\rho(N^N) = \rho(N_1^N)$. Conditions for $M^N$ to converge are given, including monotonicity type conditions under regularity assumptions. (Received December 6, 1975.)

**Goldie centralizers of separable subalgebras.** Preliminary report.

The relationships between the structure of an algebra $R$ over a field $F$ and the centralizer of an appropriate subset $A$ of $R$, denoted by $C_R(A)$, is studied in some recent papers (to appear).

Susan Montgomery has proven that when $A$ is a finite dimensional separable subalgebra of $R$ and $R$ is semiprime then $C_R(A)$ is semisimple Artinian if and only if $R$ is semisimple Artinian.

We have shown that for $R$ and $A$ as above $R$ is a Goldie ring if and only if $C_R(A)$ is a Goldie ring. (Received January 17, 1975.) (Author introduced by I. N. Herstein.)

**A theorem and a consistency theorem on bqo's.**

Let $\eta_{w+1}$ be the tree of height $w+1$ consisting of the binary tree of height $w$.
with an additional \( N_0 \) nodes densely situated on level \( w \). If \( T_1, T_2 \) are trees (all trees here taken to be well-founded), let \( T_1 \preceq T_2 \) mean that \( T_1 \) is homeomorphically embeddable in \( T_2 \) (as in Nash-Williams, On well-quasi-ordering infinite trees, Proc. Camb. Phil. Soc. 61 (1965), 697-720, where it is shown that the class of trees of height \( < w \) is better-quasi-ordered under \( \preceq \)). Let \( \mathcal{T} \) be the class of trees \( T \) such that \( T \) can be written \( \bigcup_{n<w} T_n \), each \( T_n \) closed downwards in \( T \), \( \mathcal{T}_{<w} \) \( \not\preceq T_n \). Theorem \( \mathcal{T} \) is better-quasi-ordered under \( \preceq \).

Let \( \mathcal{M}^{+} \) be the (order types of) the smallest class of linear orderings which contains the singletons, is closed under well ordered and converse well ordered sums, contains \( (L, \leq_L) \) whenever \( L = \bigcup_{n<w} L_n \) and it contains each \( (L_n, \leq_{L_n}) \), and is closed under sums indexed by a set of reals of power \( < 2^{N_0} \). Let \( \preceq \) be the embeddability relation between order types. Theorem It is relatively consistent that \( 2^{N_0} = N_2 \) and \( \mathcal{M}^{+} \) is better-quasi-ordered under \( \preceq \). (Received January 17, 1975.)

### *75T-A76* AVELZRI S. FRAENKEL and UZI TASSA, The Weizmann Institute of Science, Rehovot, Israel. STRATEGY FOR A CLASS OF GAMES WITH DYNAMIC TIES.

Let \( 0 < 2^b < a < a \) be integers. Two players play alternately with a pile of stones. Each player at his turn selects one move from the following two: (i) Remove \( k \) stones from the pile subject to \( 1 < k < a \) or \( a + k < a < a + e \). (ii) If the number \( m \) of stones in the pile satisfies \( m = 2^b \) (mod \( 2^a \)), add \( a \) stones to the pile. The player making the last move wins. If there is no last move, the game is a (dynamic) tie. The Generalized Sprague-Grundy function \( G \) is determined thus giving the strategy of play for the game and its disjunctive compound. An algorithm requiring \( O(a^2) \) steps for computing \( G \) is given. It turns out that \( G = G(a, b, c) \) is of a rather complicated form. The main interest of the paper is in presenting a complete strategy for a class of games with dynamic ties. [To appear in Computers and Mathematics.] (Received January 17, 1975.)

### *75T-A77* DAVID E. DOBBS, Rutgers University, New Brunswick, N.J. 08903. On the Global Dimensions of \( D + M \).

A family of quasilocal noncoherent going-down domains of global dimension 3 is constructed, thus answering affirmatively a question raised by the author, Comm. in Algebra 1 (1974), 439-458. Let \( V \) be a valuation ring of the form \( K + M \), where \( K \) is a field and \( M (\neq 0) \) is the maximal ideal of \( V \). Let \( D \) be a proper subring of \( K \); let \( K \) be the quotient field of \( D \); let \( R = D + M \). Call \( R \) a solution in case \( R \) (but not \( D \)) is an integrally closed quasilocal nonvaluation domain of global dimension 3, each of whose overrings is a going-down ring. Proposition 1. If \( R \) is a solution, then \( k \not\in \mathcal{K}, M = M^2 \) and \( k + M \) is a solution. Proposition 2. If \( D = K (\not\in \mathcal{K}) \) and \( M = M^2 \), then \( w, \text{gl. dim} (R) = 2 \) and \( \text{gl. dim} (R) \geq 3 \). Theorem. Solutions exist. In addition to the paper cited above, the proofs of the propositions depend upon Theorem 4.3 of B. Greenberg, J. Algebra 32 (1974), 31-43; Theorem A of Appendix 2 of R. Gilmer, Multiplicative ideal theory, Queen's University, 1968; and some recent work of I. J. Papick and the author on flatness. After some cardinal arithmetic, the theorem follows from Proposition 2 and a bound of B. Osofsky. (Received January 20, 1975.)

### *75T-A78* Brian A. DAVEY & Bill SANDS, Univ. of Manitoba, Winnipeg, Canada. An Application of Whitman's Condition (W) to Lattices with no Infinite Chains. Preliminary Report.

Let \( \mathcal{FC} \) denote the class of all lattices in which all chains are finite. We investigate lattices in \( \mathcal{FC} \) which are subject to the condition \( \mathcal{FC} \). Whenever there exists \( M \in \mathcal{FC} \)
and a homomorphism \( \varphi \) of \( M \) onto \( L \), there exists an embedding \( \bar{\varphi} \) of \( L \) into \( M \) such that \( \varphi \circ \bar{\varphi} = 1_L \).

**Theorem:** Let \( L \in \mathcal{F}_C \). Then \( L \) satisfies Whitman's condition \((W)\) if and only if \( Q_{\mathcal{F}_C} (L) \) holds.

**Corollary 1:** Let \( L \in \mathcal{F}_C \). If \( Q_{\mathcal{F}_C} (L) \) holds, then \( Q_{\mathcal{H}_S} (K) \) holds for every \( K \in \text{Sub}(L) \).

**Corollary 2:** Let \( L \) be a finite lattice and let \( K \) be a finite subdirectly irreducible lattice satisfying \((W)\). Then \( K \in \mathcal{H}_S \mathcal{P}(L) \) if and only if \( K \in \text{Sub}(L) \).

**Corollary 3:** Let \( L \) be a finite lattice satisfying \((W)\). Then \( L \) is projective in \( \mathcal{F}_C \).

We give an example of a finite lattice \( L \) such that \( Q_{\mathcal{F}_C} (L) \) fails although \( L \) is embeddable into every lattice in \( \mathcal{F}_C \) of which it is a homomorphic image. (Received January 30, 1975.)

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All solutions to the equations:

\[
(2N+1)^K = 2^M + 1 \quad (N,K,M \text{ integers } \geq 0)
\]

are identified.

In the case \((-1)\) solutions exist only when \( N = 2^{M-1} - 1 \) and \( K = 1 \). In the case \((+1)\) solutions are \( N = 1, K = 2, M = 3 \); and \( N = 2^{M-1}, K = 1 \). **Remark.** These results are obtained through elementary methods. **Scholium.** These exponential Diophantine equations played a motivating role in characterizing \( \log_2 N \) as an additive arithmetic function, c.f. "The role of boundedness in characterizing Shannon entropy", G. T. Diderrich, to appear. (Received January 20, 1975.)

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**Analysis**

This is the first of a series of papers in which the author proposes to study a new generalization of several known polynomial systems belonging to (or providing extensions of)

JANG-MEI GLORIA WU, University of Illinois, Urbana, Illinois 61801, An Integral Problem For Positive Harmonic Functions Preliminary report.

Let \( u \) be a positive harmonic function on the unit disk and \( u(0) = 1 \). We are mainly concerned with finding the limit inferior of the integral of \( u \), with respect to arc length, along closed rectifiable curves tending to the unit circle. The conclusion is strongly dependent on the boundary measure \( u \) in the Poisson-Stieltjes sense. By a conformal mapping to a half plane, we see that the limit inferior is at least 4. The limit inferior is exactly 4 for purely discontinuous measures and a special kind of singular continuous measure; we prove this by estimations of the Poisson kernel and close examination of specially constructed curves. On the other hand, when \( u \) is an absolutely continuous measure, the limit inferior is 2; the main tool is Fatou's theorem. Extension is possible in two directions. The theorems remain true if we replace the disk by an annulus; this can be proved by a close study of the Poisson formula for an annulus. Also the result is true for a certain type of subharmonic function by a theorem of Littlewood. (Received October 24, 1974.)

A. R. BLASS, University of Michigan, Ann Arbor, Michigan 48104 and C. V. STANOJEVIC, University of Missouri, Rolla, Missouri. Partial Mielnik spaces and uniformly convex spaces.

**Theorem.** A normed linear space \( E \) is uniformly convex (Clarkson, Trans. Amer. Math. Soc. 40(1936), 396) iff there is a continuous strictly increasing \( f:[0,2] \rightarrow [0,1] \), with \( f(0) = 0 \) and \( f(2) = 1 \), such that for all unit vectors \( a, b \in E \), \( f(||a + b||) + f(||a - b||) \leq 1 \). This functional inequality specializes to Clarkson's inequality (take \( f(x) = (x/2)^p \)) for \( L_p \) spaces \( (p \geq 2) \). It can be interpreted as saying that the function \( f(||a + b||) \) makes the unit sphere of \( E \) a partial Mielnik space (defined as in Mielnik, Comm. Math. Phys. 9(1968), 55, except that \( \sim \) is changed to \( \leq \) in Axiom C). Thus, the theorem is analogous to the characterization of inner product spaces by Stanojevic (Trans. Amer. Math. Soc. 183(1973), 441). We also have some results on embeddability of partial Mielnik spaces into (total) Mielnik spaces. (Received November 4, 1974.)

S. ZAIDMAN, Math. Dept., Univ. of Montreal, Canada; Convexity result for weak differential inequalities.

Let \( H \) be a Hilbert space and \( B \) be a self-adjoint unbounded operator in \( H \); then \( K_B [a,b] \) is a natural class of test-functions associated to \( B \) and to a given interval \([a,b]\). Consider a couple of \( H \)-valueg functions \( u(t), f(t) \) continuous on \([a,b]\) such that

\[
\int_a^b \langle u(t), \Psi'(t) \rangle_H dt = \int_a^b \langle u(t), B \Psi(t) \rangle_H dt + \int_a^b \langle f(t), \Psi(t) \rangle_H dt
\]

for any test-function. Assume furthermore that

\[
\| f(t) \|_H \leq \Phi(t) \| u(t) \|_H \]

for any \( t \in [a,b] \) where \( \Phi(t) \) is a everywhere defined on \([a,b]\) non-negative integrable function such that

\[
\int_a^b \phi(s) ds \leq \frac{1}{2||u||_H^2}
\]

Then the estimate

\[
\| u(t) \| \leq 2 \sqrt{2} \| u(a) \|_{K_B} \| u(b) \|_{K_B}, \quad a \leq t \leq b
\]

is verified.
The result is slightly more general than one by Agmon-Nirenberg (Comm. Pure Appl. Math., May 1963, p. 139) where \( u(t) \) is differentiable and belongs to the domain of \( B \) for any \( t \) in the given interval \([a,b]\) (Received November 6, 1974.)

*75T-B37 JAN MYCIELSKI, Mathematics Department, University of Colorado 80302. Invariant measures in metric spaces.

Theorem 1. For every compact metric space there exists a Borel probability measure which is equal on isometric open sets. Theorem 2. For every metric space which has compact perfect subsets there exists a Borel measure vanishing on points, positive and finite on some compact sets and equal on isometric Borel sets. Theorem 3. If \( M \) is a metric space, \( M_0 \) is a Borel set in \( M \) and \( \mu_0 \) is a Borel measure over \( M_0 \) which is equal on isometric Borel sets then there exists an extension \( \mu \) of \( \mu_0 \) which is a Borel measure over \( M \) and is equal on isometric Borel sets. The proof of Theorem 1 is similar to the construction of Haar's measure. Theorem 2 is based on Theorem 3 and a construction of J. C. Oxtoby, Trans. Amer. Math. Soc. 60 (1946), p. 220 ff. (Received November 8, 1974.)

*75T-B38 J. DAVID LOGAN and JOHN S. BLAKESLEE, Kansas State University, Manhattan, Kansas 66506. An Invariance Theory for Second-Order Variational Problems, Preliminary Report.

The authors investigate the invariance properties of a second-order variational problem \( J = \int [L(t,x,x',x'')]dt \) under an \( r \)-parameter local Lie group of transformations of the form \( \bar{t} = \Phi(t,x), \bar{x} = \Psi(t,x) \). A set of fundamental invariance identities are obtained which extend the work of H. Rund for first-order problems. These invariance identities lead to a simple proof of the Noether theorem for second-order problems as well as provide a method for determining invariance groups and therefore conservation theorems. Interpreted differently, the identities, which involve the Lagrangian \( L \) and the generators of the infinitesimal group, may be used to characterize Lagrangians which possess given invariance properties under a given group. Both the single and multiple integral cases are discussed and the Zermelo conditions for each are shown to follow as a direct consequence of the identities. (Received November 8, 1974.)

*75T-B39 ELEMER R. ROSINGER, Technion-Israel Institute of Technology, Haifa, Israel 3200 A Distribution Multiplication Theory.

Associative, commutative algebras \( A_{p,\lambda}, p=0,1,\ldots,m, \lambda \in \Lambda \), with unit element, containing \( D^p(\mathbb{R}^n) \) are constructed. The multiplication in \( A_{p,\lambda} \) induces on \( C^\infty(\mathbb{R}^n) \) the usual multiplication and \( f(x) = 1, x \in \mathbb{R}^n \), is the unit element in \( A_{p,\lambda} \). There exist linear mappings \( D_{p,\lambda} : A_{p,\lambda} \rightarrow A_{p-1,\lambda} \), satisfying the 'rule of product derivative'. For the distributions in \( C^\infty(\mathbb{R}^n) \) or with finite support, \( D_{p,\lambda} \) is the distribution derivative. For each \( t \in \mathbb{R}^n \), not a solution of any equation \( a_0 T + \cdots + a_n D^n T = 0, a_0, \ldots, a_n \in \mathbb{R} \), \( D_{\infty}(\mathbb{R}^n) \), supp \( S \) finite, there exists \( \lambda \in \Lambda \) that \( D_{p,\lambda} \) is the distribution derivative on \( \{D^q T \mid q=0,1,\ldots \} \). The following relations hold in the algebras:\( \Psi(x-x_0) e^{\delta x} \delta x_0 = 0 \) for \( \lambda, \mu \), \( x_0, x_1, x_2, \ldots \)

An inversion formula for \( S_2 \) transform given by D.V. Widder and R.P. Boas is extended to a

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class of generalized functions. Define \( S_2 \) transform of a member \( f \) of a certain space of
generalized functions as
\[ F(x) = (f(t), K(t, x)) \]
where
\[
K(t, x) = \begin{cases} 
\log x/t, & x \neq t \\
1/x, & x = t.
\end{cases}
\]
\((0 < t < \infty, 0 < x < \infty)\).
It is shown that
\[
\lim_{n \to \infty} H_{n,x}[F(x)] = f(x)
\]
in the weak distributional sense. Here \( H_{n,x} \) is a certain linear generalized differential
operator. Some related results are also established. (Received November 11, 1974.)

**75T-B41**
S.J. POREDA, Clark University, Worcester, MA 01610
Interpolation by Rational Functions With Restricted Zeros.
Preliminary report.
Several theorems dealing with the existence of interpolating
functions with restricted zeros are given. For example, if \( (e^{i\theta_k})_k \) are
n distinct points on the unit circle and \( \{\psi_k\}_k \) are prescribed arguments
then it is shown that there exists a finite Blaschke product \( B(z) \) all of
whose zeros lie in the open unit disc and such that \( B(e^{i\theta_k}) = e^{i\psi_k} \) for
\( k = 1, 2, \ldots, n \). (Received November 15, 1974.)

**75T-B42**
SHIMSHON ZIMERING, Ohio State University, Columbus, Ohio 43210. A generalization of
Mercer's theorem for regularly varying sequence.

**Theorem 1.** Let \( R(r) \) be a regularly varying sequence in the sense of Karamata of order \( r \) \((r > -1)\),
I.e., \( R(t)/t^r \to -1 \) \((n \to \infty)\) for every \( t > 0 \), and let \( R(0) > -1 \). Then, \( S_n = R_n \) \((n \to \infty)\) such that
\[
S_n = (1 + r)n^{-1}R_n \to -1 \quad \text{as} \quad n \to \infty.
\]
We have shown that Theorem 1 may fail to hold for slowly oscillating sequences in
the sense of Schmidt by proving Theorem 2. Let \( S_n \) be slowly oscillating in the sense of Schmidt, i.e.,
\( n \to 0 \) when \( m + \alpha \) and \( m/n \to 1 \). Then \( S_n \) is not necessarily a slowly varying sequence
(I.e., a regularly varying sequence of order \( 0 \)) in the sense of Karamata, and conversely. (Received
November 18, 1974.)

**75T-B43**
WALEED A. AL-SALAM, University of Alberta, Edmonton, Alberta, Canada
and M. E. H. ISMAIL, University of Wisconsin, Math. Dept., Madison, Wis. 53706
Polynomials Orthogonal With Respect to Discrete Convolution
With every formal power series \( \phi(x) \) we associate the transform
\[
L[f; \phi; x] = \frac{1}{n!} (-x)^n \phi^{(n)}(x)f(n)/n!,
\]
defined on sequences \( f(0), f(1), \ldots \). This transform maps a
polynomial of the discrete variable \( n \) to a polynomial of the same degree in the continuous
variable \( x \). Pre-images, under the above transform, of orthogonal polynomials are called
orthogonal with respect to a discrete convolution. We study some properties of such poly­
nomials. As applications of our study, we derive connection and bilinear relations for discrete
orthogonal polynomials that are also convolution orthogonal. This leads to new connection rela­tions and bilinear generating functions for the Charlier polynomials. We also prove that if
for \( x \in (0, \infty) \), \( \phi(x) \) is the Laplace transform of a non-negative function, then the pre-images,
under our transform, of polynomials orthogonal on a subset of \([0, \infty)\), will have real and simple
zeros. In particular, this shows that the zeros of the Rice polynomials \( H_n(\xi, P, v) \), as
polynomials in \( \xi \), are real and simple for \( p > 0 \) and \( v < 0 \). (Received November 22, 1974.)

**75T-B44**
ROBIN HART, University College of Cork, Cork, Ireland.
Singularity, exponentials and spectral permanence.

If \( A \) and \( B \) are complex Banach algebras with identity, if \( T : A \to B \)
is a homomorphism which preserves identity, and if \( a \in A \) has spectrum \( \sigma_A(a) \), then obviously
Conversely we show for hounded \( T \) that \( \bigcap_{T' \in \mathcal{A}} \sigma_{T'}(a+a') \subseteq \eta_{\mathcal{A}}(Ta) \), where for compact \( K \subseteq \mathbb{C}, \eta_K \) is the union of \( K \) and the bounded components of \( \mathbb{C} \setminus K \). If \( T \) is one-to-one the argument is that \( \tau_{\mathcal{A}}(a) \subseteq \eta_{\mathcal{A}}(Ta) \), where \( \tau_{\mathcal{A}}(a) \) is the set of \( s \in \mathbb{C} \) for which \( a - s \) is not a topological zero-divisor. If \( T \) is onto then we show \( \bigcup_{T' \in \mathcal{A}} \sigma_{T'}(a+a') = \epsilon_{\mathcal{A}}(Ta) \), where \( \epsilon_{\mathcal{A}}(a) \) is the set of \( s \in \mathbb{C} \) for which \( a - s \) is not in the connected component of \( 1 \) in \( \mathbb{A}^{-1} \).

(Received December 2, 1974.)


A set of sufficient conditions is established for a strong minimum in the following canonical problem in optimal control.

Let \( C \) be the class of arcs \( \alpha: x(t), u(t) b \) \( \in \mathbb{R}^n \) (with \( x, u, b \) vectors) whose elements \( (t,x(t),u(t),b) \) lie in a region \( R \) in \( txub \)-space, have \( u(t) \) piecewise continuous and satisfy: i) the differential equation constraints \( x'(t) = f(t,x(t),u(t),b) \)

\[
1 \leq i \leq N
\]

ii) the state constraints \( \psi_i(t,x,b) \leq 0 \), \( 1 \leq i \leq m \), \( \psi_i(t,x,b) = 0 \) \( m' < a < m \); iii) the isoperimetric constraints \( I_{\gamma}(\mathcal{A}) \leq 0 \), \( 1 \leq \gamma \leq p' \), \( I_{\gamma}(\mathcal{A}) = 0 \) \( p' < \gamma \leq p \) (where \( I_{\gamma}(\mathcal{A}) = g_{\gamma}(b) + \int_0^1 f_{\gamma}(t,x(t),u(t),b)dt \)) and iv) the end point constraints \( x^B(t) = x^B(b) \) \( s = 0 \).

It is desired to minimize the functional \( I_0(\mathcal{A}) = g_0(b) + \int_0^1 f_0(t,x(t),u(t),b)dt \) on the class \( C \). (Received December 2, 1974.)

75T-B46 ALDO J. LAZAR, Tel-Aviv University, Tel-Aviv, Israel. Extreme points of convex bodies in \( R_2 \).

Any uncountable separable complete metric space is homeomorphic to the set of extreme points (in the weak topology) of a bounded closed convex body in \( R_2 \). This substantiates a conjecture of J. Lindenstrauss and R.R. Phelps (Israel J. Math., 6(1968), 39-48). (Received December 5, 1974.)

75T-B47 L. M. PERKO, Northern Arizona University, Flagstaff, Arizona 86001. Second Species Periodic Solutions of the Restricted Three-Body Problem

A solution \( x(t,\mu) \) of the restricted three-body problem with mass ratio \( \mu \), which is periodic in rotating coordinates with period \( T \), and which for \( 0 < t < T/2 \) approaches \( n \) arcs of intersecting Keplerian ellipses as \( \mu > 0 \) is called a periodic solution of the second species of Poincare of type \( n \). Theorem: For each positive integer \( n \), there is a \( \mu_1 > 0 \) such that for \( 0 < \mu < \mu_1 \) there exist one-parameter families of periodic solutions of the second species of Poincare of type \( n \). The existence proof and 1st order asymptotic approximation for type 1 families is given in the S.I.A.M. J. on Appl. Math., 27 (1974), 200-237. As was conjectured by Poincare in his Methodes Nouvelles, p. 364, for small \( \mu > 0 \) a second species solution is near one of the Keplerian ellipses until it approaches a point where two Keplerian ellipses intersect (an "angular point"); these points of intersection occur at the position of the perturbing body for \( \mu = 0 \), and for \( \mu > 0 \) the second species solution in an \( O(\mu^{1/2}) \)-neighborhood of one of these points is near a hyperbolic arc which "approaches an angular point" as \( \mu \to 0 \); after each such passage near the perturbing body, the second species solution is near another one of the Keplerian ellipses. (Received December 13, 1974.) (Author introduced by Dr. Richard Meyer.)


In this note we consider the space \( X \) of all entire Dirichlet functions \( f \), of finite order less than or equal to \( p \), \( f(z) \) denotes the Frechet topology on \( X \) generated by the family \( \{||f_n||_p: n \geq 0\} \) of semi-norms on \( X \), where for each \( \eta > 0 \)

\[
||f + \eta \delta|| = \sum_{n=1}^{\infty} a_n \lambda^{-n/(\eta+\delta)}, f = \sum_{n=1}^{\infty} a_n \lambda^{-n}, s = \eta + \delta \text{ it.}
\]

Our main results are as follows:

Theorem 1: Let \( A \subseteq X \). Then \( A \) is \( G \)-bounded if and only if for each \( \eta > 0 \), there

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exists a constant C such that
\[ |\varphi_n(t) - \varphi_{n+1}(t)| \leq C|\lambda_n - \lambda_{n+1}| \]
uniformly in \( t \in A \) and for all \( n \geq 1 \).

Theorem 2: X equipped with \( G \) is an infrabarrelled space in which every \( G \)-bounded set is relatively compact, that is, \( X \) is a Fréchet space.

(Received November 4, 1974.) (Authors introduced by Dr. S.K. Bajpai.)

William D. L. Applking, North Texas State University, Denton, Texas 76203, An Integrability and Measurability Decomposition Theorem

U, F, P, \( p \), \( p^* \) and the notions of set function integrability, summability and measurability are as in previous abstracts of the author. For each \( v \in p^* \), \( I(v) \), \( M(v) \), \( W(v) \) and \( s(v) \) denote, respectively, the set of all elements of \( p \) that are \( v \)-integrable, the set of all elements of \( p \) that are \( v \)-measurable, \( v \)-summable, and the \( v \)-summability operator. If each of \( A \) and \( B \) is a subset of \( p \), then "\( A \subset B \)" means that if \( r \) is in \( A \), then there is \( t \) in \( B \) such that \( r(V) = t(V) \). \( \Box \)

Theorem. If each of \( w \) and \( x \) is in \( p^* \), then \( I(\min\{w, x\}) \) \( \in \) \( I(w) \cup \{0\} \cup I(x) \), and \( M(\min\{w, x\}) \) \( \in \) \( M(w) \cup \{0\} \cup M(x) \).

(Received November 6, 1974.)

Edward Beckenstein, St. John's University, Staten Island, New York, Lawrence Narici, St. John's University, Jamaica, New York, Charles Suffel, Stevens Institute of Technology, Hoboken, New Jersey and Seth Warner, Duke University, Durham, North Carolina. Subspaces of codimension 1 in rings of continuous functions. Preliminary report.

Gleason has shown that if \( X \) is a commutative Banach algebra with identity, a subspace of codimension 1 is a maximal ideal if and only if it contains no invertible elements. This is shown to be true for the ring \( C(T, R) \) of continuous functions on the completely regular space \( T \) taking values in the real numbers \( R \). It is also shown to be true for the ring \( C(T, F) \) where \( T \) is any ultraregular space and \( F \) any complete rank one nontrivially nonarchimedean valued field. When \( T \) is compact, the above rings are Banach algebras (with supremum norm). Elementary proofs (not using complex variables) are presented in these two cases using measure theoretic methods. An elementary proof of Gleason's Banach algebra theorem is not yet known. (Received November 11, 1974.)

Mostafa A. Abdelkader, 25 Sh., Champollion, Alexandria, Egypt. Circular and hyperbolic functions satisfying \( f(x) = f(p - x) \).

The functional equation \( g(x) = g(p + x) \), where \( p \) is real, characterizes real periodic functions, such as combinations of the circular functions. The companion functional equation \( f(x) = f(p - x) \) is satisfied by \( \sin x \) (with \( p = \pi \)), and by \( \cos x \) (with \( p = 2\pi \)). It is shown that certain combinations of the hyperbolic functions (or the real exponential function) also satisfy \( \varphi(x) = \varphi(p - x) \). Two examples are: (1) \( f(x) = \sinh x + b \cosh x \), with \( p = \cosh((b-a)/(b+a)) \), \( b^2 > a^2 \), and (2) \( f(x) = \sinh^2 x + b \sinh x \cosh x + c \cos^2 x \), with \( 2p = \cosh((a-b)/(a+b)) \), \( (ab)^2 > b^2 \). Moreover, the generalized Euler expression : \( a \cos x + ib \sin x \), where \( a^2 > b^2 \), is shown to satisfy the functional equation \( f(x) = f(iq - x) \), where \( q = \log((ab)/(a-b)) \). (Received December 4, 1974.)

Mourad El-Houssieny Ismail, Mathematics Research Center, 610 Walnut Street, Madison, Wisconsin 53706. Preliminary report. Connection Relations and Bilinear Formulas for the Classical Orthogonal Polynomials

A linear operator that maps at least one set of orthogonal polynomials to a set of orthogonal polynomials is called, by Al-Salam and Verma, "Orthogonality Preserving Operator". Ob-
vious examples are \((T_a f)(x) = f(x+a)\) and \((S_\lambda f)(x) = f(\lambda x)\). Less trivial examples are the fractional derivatives and integrals, Bateman’s transform
\[
(T_{\lambda, \mu} f)(x) = \frac{x^{\lambda-\mu}}{\beta(\lambda+1, \mu)} \int_0^x t^\lambda (x-t)^{\mu-1} f(t) dt, \quad \lambda > -1, \quad \mu > 0,
\]
and the \((L, \phi)\) transforms of Al-Salam and Ismail (see their earlier announcement entitled “Polynomials orthogonal with respect to a discrete convolution” in Notices. The present paper shows how the orthogonality preserving operators can be used to derive connection relations and bilinear formulas for certain orthogonal polynomials. The method is carried out in details for the Jacobi, Laguerre and Hahn polynomials. In the treatment of Hahn polynomials, the \((L, \phi)\) transforms were generalized. The Meixner and Charlier polynomials have been treated in the above mentioned work of Al-Salam and Ismail. (Received December 12, 1974.)

*75T-B53 MAX MLYNARSKI, Kingsborough Community College, Oriental Blvd., Brooklyn, N. Y. 11235. The Zeros of Polynomials Preliminary report.

Let \(f(z)\) be a polynomial with complex coefficients. Let \(\alpha(f) = \text{max}\{\text{Re}(xe^{-i\theta}): f(x) = 0, 0 \leq \theta \leq 2\pi\}.\) Investigating upper bounds for \(\alpha(f)\) we obtained Theorem 1. Let \(f(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_0\) be a polynomial with complex coefficients. Then
\[
i) \alpha(f) \leq \sqrt{\left[1 - \text{Re}(e^{-i\theta} a_{n-1})\right] + \left[1 + \text{Re}(e^{-i\theta} a_{n-1})\right] + 4M},
\]
\[
ii) \alpha(f) \leq \text{max}\{1 + M, 1 - \text{Re}(e^{-i\theta} a_{n-1})\},
\]
\[
iii) \alpha(f) \leq 2M - \text{Re}(e^{-i\theta} a_{n-1})\}
\]
where \(M = \text{max}\{|a_0|, \ldots, |a_{n-2}|\}.\) (Received December 16, 1974.) (Author introduced by Professor Richard Staum.)

*75T-B54 DONALD SWANTON, Roosevelt University, Chicago, Ill., 60605, Compact composition operators on \(B(D)\).

Let \(D\) be domain in the complex plane, \(\phi: D \to D\) be analytic, and \(B(D)\) be the uniform algebra of bounded analytic functions on \(D\) with maximal ideal space \(M\). The composition operator \(C_{\phi}(f) = f \circ \phi\) is compact if and only if the weak* and norm closures of \(\phi(D)\) coincide if and only if whenever the Euclidean closure of \(\phi(D)\) contains a point \(\lambda\) of the boundary of \(D\) then each \(f \in B(D)\) extends continuously from \(\phi(D)\) to \(\lambda\). If \(C_{\phi}\) is compact, then either \(\phi\) fixes a point of \(D\) or else the adjoint of \(C_{\phi}\) fixes a point of \(M\). (Received December 16, 1974.)

75T-B55 V. KARUNAKARAN, University of Madras, Madras-5, India. A subclass of functions of positive real part. Preliminary report.

Let \(p(z)\) be regular in \(E = \{z: |z| < 1\}\). \(p(z) \in P(A,B)\) if and only if \(p(z) = (1 + Aw(z))/(1 + Bw(z))\) where \(w(z)\) is regular in \(E\), \(w(0) = 0\), and \(-1 < A < B < 1\). Sharp lower bounds are obtained for \(\text{Re}(Cp(z) + A/p(z))\), \(p(z) \in P(A,B)\) and \(C \geq B\), and sharp upper bounds are obtained for \(\text{Re}(zw'(z))/(1 + Aw(z))(1 + Bw(z))\) where \(w(z)\) is regular in \(E\) and \(w(0) = 0\). Both results are obtained without appeal to variational techniques and can be used to solve a variety of extremal problems over \(P(A,B)\) and related classes of functions. (Received December 16, 1974.) (Author introduced by M. R. Ziegler)

75T-B56 ALLEN M. RUSSELL, University of Melbourne, Parkville 3052, Victoria, Australia. Stieltjes Type Integrals. Preliminary Report.

We will be concerned with \(k\)-th order Riemann-Stieltjes integrals of the form
\[
\int_a^b f(x) \frac{d\xi(x)}{x^{k-1}}\]
when \(k > 2\). The integral is linear in both \(f\) and \(g\), but additivity over adjoining intervals requires restrictions upon \(f\) and \(g\). The \(k\)-th order integral exists when \(f\) is con-
tinuous and $g$ is of bounded $k$-th variation - see Russell (Proc.London Math.Soc.(3), 26(1973), 547-563). The existence of a modified integral is established under the less restrictive condition of $f$ being quasi-continuous. Other related integrals are also discussed, one of which exhibits properties of Dirac's delta function. Integration by parts results are also obtained. (Received December 16, 1974.) (Author introduced by Professor T. Price.)

75T-B57 F. JAVIER THAYER, Tulane University, New Orleans, Louisiana 70118; Complete reducibility of representations, Preliminary report.

Call a representation $\phi$ of a C*-algebra $A$ completely reducible iff there is an increasing sequence of finite dimensional projections $\{p_i : i \geq 0\}$ which converges weakly to $1$ and such that $\pi(A) \subseteq \pi(\{p_i : i \geq 0\})$, where $\pi$ is the Calkin map $B(H) \to A(H)$. Let $A$ be a separable C. C. R. algebra; $\tilde{\Lambda}_c$ is the set of $\lambda \in \Lambda$ such that $\dim \lambda = \omega$. Proposition. If $A$ is separable, C. C. R. and $\tilde{\Lambda}_c$ is a countable union of closed Hausdorff sets on which the canonical bundle of $A$ is locally trivial, then every representation of $A$ is completely reducible. Corollary. If $A$ is separable, C. C. R. and $\tilde{\Lambda}_c$ is a countable union of closed Hausdorff sets of finite dimension, every representation of $A$ is completely reducible. (Received December 20, 1974.) (Author introduced by Professor Jerome A. Goldstein.)

75T-B58 BELL, WAYNE CHARLES, North Texas State University, Denton, Texas 76203, Some Normability Conditions for $H_{\mu\omega}$.

Setting and notions are as in previous abstracts of W.D.L. Appling. If $\mu \in F^+_A$ and $p > 1$ denote $\{\xi \in \Phi_{\mu} | \sum_D |\xi|^{1-p}_p D < \{u\} \}$ is bounded with $\| \xi \|_p = \sup_{D < \{u\}} \sum_D |\xi|^{1-p}_p$ by $H_{\mu p}$. Theorem 1. If $\lambda \in F^+_A$ then $\lambda \in \text{Lip}(\mu)$ iff $H_{\lambda} \subseteq H_{\mu}(= H_{\mu\mu})$. Theorem 2. If $\mu \in F^+_A$ then these are equivalent: 1) $H_{\mu\mu} (= \bigcap_{p=2}^\infty H_{\mu\mu})$ admits a norm stronger than each $p$ norm, 2) $H_{\mu\mu} = H_{\mu\mu\mu}$ for some $p > 1$, 3) $\Phi_{\mu} = H_{\mu\mu}$, 4) Lip$(\mu) = \Phi_{\mu}$, 5) $\Phi_{\mu}$ is finite dimensional, 6) Lip$(\mu) = H_{\mu\mu\mu}$. (Received December 23, 1974.)

75T-B59 LOUISE RAPHAEL, Clark College Atlanta, Georgia, 30314. Summable integrals, Preliminary report.

A. N. Tihonov (Dokl. Akad. Nauk SSSR 151 (1963), 501-504) while working on ill posed problems, defined a method of regularization which is a generalized summability procedure for divergent series. The summability methods of Tihonov and Poisson-Abel have been extended to divergent integrals. It has been proved (following Hudak (Dokl. Akad. Nauk SSSR (1972) 304-308) under which conditions one method implies the other. It is assumed that $z(t)$ is continuous on $[0, \infty)$. Definition 1: The integral $\int z(t)dt$ is summable by the method $T(t)$ to the sum $Z$ if the integral $\int z(t)(1+bt)^{-1}dt$ converges for $b > 0$ as $b \to 0$. Definition 2: The integral $\int z(t)dt$ is summable by the method $PA(t)$ to the sum $Z$, if the Laplace transform of $z(t)$, $\int [z(t)] \exp(-ts)dt$ converges for $s > 0$ and $\lim_{s \to 0} [z(t)\exp(-ts)]dt = Z$ as $s \to 0$. Theorem 1: If $\int z(t)dt$ is summable by the $PA(t)$ method and $\int z(t)(1+bt)^{-1}dt$ converges for $b > 0$, then the $z(t)dt$ is summable by the $T(t)$ method. Theorem 2: If $\int z(t)dt$ is summable by the $PA(t)$ method and $\int z(t)(1+bt)^{-1}dt$ converges for $b > 0$, then $\int z(t)dt$ is summable by the $PA(t)$ method to the same sum. (Received December 23, 1974.)

*75T-B60 GRAHAME BENNETT, Indiana University, Bloomington, Indiana 47401. Freshman products of infinite matrices.

For $1 \leq p, q \leq \infty$, the matrix transformations from $t^p$ to $t^q$, endowed with the usual operator norm, form a commutative Banach algebra under coordinatewise multiplication. (Received December 27, 1974.)
In a classical paper S. Thcklind (Nova Acta Soc. Sci. Upsalienes 10(1936),1-57) closed the uniqueness question for the Cauchy problem for the heat equation with a general growth hypothesis which was both necessary and sufficient. Thcklind's proof of the sufficiency involved an ingenious bootstrapping comparison technique employing the maximum principle and a comparison function constructed from the Green's function for a half cylinder. G. N. Zolotarev (Izvestia Vys. Uch. Zared. Mat. 2(1958), 118-135) has extended this result using essentially the same technique to show that Thcklind's uniqueness condition remains sufficient for a general second order parabolic equation provided the coefficients are regular enough to permit the existence and estimation of a Green's function. We have now shown, using a new approach which replaces the construction based upon a Green's function by an appropriate comparison solution of the maximizing equation (C. Pucci, Ann. Mat. Pura Appl. 72(1966), 141-170), that Thcklind's condition is sufficient without any regularity conditions on the coefficients. (Received January 6, 1975.)

An entire function $F(z)$ is said to be prime iff every factorization $F(z) = f(g(z))$ implies that one of the entire functions $f(z)$ or $g(z)$ is linear. In this paper, we prove that every entire function of the form $az^m + p(e^{iz}, e^{-iz})$ is prime, where $a$ is a nonzero constant, $m$ is an integer, and $p(u, v)$ is a polynomial in $u$ and $v$. We also obtain all the possible forms for entire factors $f(z)$ and $g(z)$ in the factorization $z + e^{H(z)} = f(g(z))$, where $H(z)$ is a periodic entire function. (Received January 8, 1975.) (Introduced by Dr. Charles Osgood.)

The Field-Noyes model of the Belousov-Zhabotinski reaction is of considerable interest as it is apparently the only such model that is derived directly from detailed knowledge of a real oscillating chemical reaction. The values of most of the parameters in the model are directly assignable by analogy to the chemistry occurring in the real system. However, two parameters are only qualitatively related to this chemistry and the behavior of the model as the values of these parameters are varied is of interest. Earlier numerical and qualitative work has shown that the model exhibits limit cycle oscillations for certain values of these parameters. The present qualitative work shows that for other values limit cycles do not appear as the physically significant steady state is globally asymptotically stable. However, this steady state may be excitable in that a finite perturbation is greatly amplified before the system returns to rest. The size of the excursion depends upon the value of a parameter related to an autocatalytic portion of the model. The relationship between this excitability and phenomena actually observed in the reaction is discussed and an analogy is drawn between the behavior of this model and the Hodgkin-Huxley model for nerve impulse transmission. (Received December 27, 1974.)

Let $R(a, \beta)$ denote the class of functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ analytic in the unit disc $E(\{z\} < 1)$ and satisfying $\left| \frac{f'(z) - 1}{f'(z) + (1-2a)} \right| < \beta$, for some $a, \beta(0 < a < 1, 0 < \beta < 1)$ and for all $z \in E$. In this paper we obtain sharp coefficient estimates, distortion theorems and the radius of convexity for functions in $R(a, \beta)$. A
sufficient condition for a function to belong to $P(n,B)$ has also been obtained. The results extend the corresponding work of McGregor (Trans. Amer. Math. Soc. 104 (1962), 532-537), Padmanabhan (Ann. Polon. Math. 23 (1970/71), 73-81) and Caplinger and Causey (Proc. Amer. Math. Soc. 39 (1973), 357 - 361). (Received January 13, 1975) (Authors introduced by Professor J. N. Kapur.)

GUTTALU R. VISWANATH, Howard University, Washington, D. C. 20001, ON CONVEX UNIVALENT FUNCTIONS, (Preliminary Report.)

Let $B$ be the class of functions $f$ analytic and univalent in the unit disc $D: |z| < 1$ and normalized by the conditions $f(0) = 0, f'(0) = 1$. A function $f \in B$ is said to be convex univalent in $D$, if $f(D)$ is a convex set. Let $S$ be this subclass of $B$. We study a subclass $K(m,M)$ of functions $f \in S$ for which

$$|z| + |zf ''(z)f'(z)| < M, |z| < 1,$$

where $(m,M) \ni (m,M): 1 > m > 1/2, m \geq M > m - 1$.

Theorem 1: Let $f(z) \in S$. A necessary and sufficient condition that $f(z) \in K(m,M)$ is that $f(z)$ has the representation

$$f(z) = \int_0^z \exp \left\{ \int_0^t I(n) \text{d}n \right\} \text{d}t,$$

where $I(z) = (M^2 + (m-1)^2 P(z)) |M + (m-1)zP(z)|$. $P(z)$ is analytic and $|P(z)| \leq 1$ in $|z| < 1$.

Theorem 2: If $f(z) \in K(m,M)$, then $a_n \leq (M^2 + (m-1)^2) |m-n-1|$ for $n \geq 2$.

We have derived certain inequalities involving the derivatives of $f(z) \in K(m,M)$.

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Let $E(\tau)$ be a locally convex Riesz space with dual $E'$. A subset $A$ of $E$ will be called solid if it has the property: $x \in A, |y| \leq |x|$ implies $y \in A$. A directed ($\subset$) set $A$ will be denoted by $A \uparrow$. Two elements $x, y$ in $E$ will be called disjoint if $\inf (|x|, |y|) = 0$. The main result is a characterization, in terms of the order structure, of the weak compact subset of $E$ which are convex and solid. The following is proven.

Theorem. Let $E(\tau)$ be a locally convex Riesz space. For a convex solid set $B$ in $E$ the following are equivalent.

1. $B$ is weakly compact.
2. Each sequence $\{x_n\} \subset B$ of mutually disjoint elements converge weakly to $0$; if $A \uparrow B$ then $x = \text{sup} A$ exist, $x$ is an element of $B$, and the filter of sections $\mathcal{F}(A)$ converges to $x$ in $\tau$. (Received December 30, 1974.)

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Extreme points of the shell of a linear relation. Preliminary report.


For each $u = (\xi, h) \in s(A)$, where $\xi$ is a complex number, $h$ a real, let $Y_u = \{(y, x) \in A : 2 \langle y, x \rangle = \xi \langle |x|^2 + |y|^2 \rangle \} - \{h(|x|^2 + |y|^2)\}$. Suppose that $u$ is a boundary point of $s(A)$. Then $u$ is an extreme point of $s(A)$ if and only if $Y_u$ is a linear subspace of $H$. The assertion is an analogy of Theorem 1 (i) obtained by M. Embry in Pacific J. Math. 32 (1970) 647-660. (Received December 30, 1974.)

LOKENATH DEBNATH and JOHN G. THOMAS, JR., East Carolina University, Greenville, North Carolina 27834. On Finite Laplace Transformation with Applications.

A theory of the finite Laplace transformation $\mathcal{L}(s, T)$ of $f(t)$ defined by the integral

$$\mathcal{L}(s, T) = \int_0^T e^{-st} f(t) \text{d}t$$

is presented with applications. Several operational properties including the finite Laplace transform of derivatives of functions, periodic functions and of some integrals are proved.

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The shifting properties and the Tauberian theorems for the finite Laplace transform are discussed. A theorem concerning the inverse transformation is proved. Some simple initial and boundary value problems are solved to demonstrate applications of the finite Laplace transform. Two general theorems concerning the solutions of the initial and final value problems are established. The mean, standard deviation, variance, and higher order moments of a random variable are expressed in terms of the derivatives of the finite Laplace transform of the probability density function. A short table of the finite Laplace transform of some particular functions of interest is included. (Received January 17, 1975.)

75T-B69 WALTER A. STRAUSS, Brown University, Providence, Rhode Island 02912. Energy of waves reflecting off a moving body.

Let $Q$ be an unbounded domain in 4-dimensional space-time with $C^\infty$ timelike boundary $\Sigma$. Let $u(x) = u(x_0, \mathbf{x})$ be a $C^\infty$ solution of the wave equation $\Box u = \Delta u - u_{x_0}x_0 = 0$ in $Q$ which has compact support in $\mathbf{x}$ for each $x_0$ and vanishes on $\Sigma$. Let $(v, v) = -v_0^2 + \mathbf{v}^2$. Theorem. The total energy of $u$ is bounded for all time provided there exists a $C^\infty$ vector field $\mathbf{f}(x)$ in $\overline{Q}$ such that (a) $\overline{Q}$ is invariant under $\mathbb{R}$; (b) $(x - y, x - y)$ does not increase along trajectories of $\mathbf{f}$; (c) $\mathbf{f}$ is strictly timelike; (d) $\Box u_{x_0} = \mathbf{f}_0(x_0)$ is bounded for each $x_0$; (e) $\mathbf{f}_1(x_0 + \eta(r))$ for $r = |x| \geq R$ for some function $\eta$. Also the local energy of $u$ decays exponentially if (b) is slightly strengthened. (Received January 20, 1975.)

#75T-B70 JAMES C.S. WONG, University of Calgary, Calgary, Alberta. Generalised Functions on Locally Compact Semigroups and Invariant Means. Preliminary report.

Let $S$ be a locally compact semigroup with jointly continuous multiplication and $M(S)$ its measure algebra with convolutions as multiplication. We show that the conjugate $M(S)*$ is isometrically order isomorphic to the space $GL(S)$ of generalised functions on $S$ introduced by Sreider [The structures of maximal ideals in rings of measures with convolution (Russian), Mat. Sbornik (N.S.), 27 (69) (1950) 297-318. AMS Transl. (First Series) 81 (1953) 365-391]. Moreover, convolutions in $M(S)*$ and $GL(S)$ by measures in $M(S)$ can be defined so that the isomorphism commutes with convolutions. $GL(S)$ can also be made into a Banach algebra and is isomorphic (under the same isomorphism above) to the second conjugate Banach algebra $M(S)**$ of the Banach algebra $C_b(S)$, the continuous functions on $S$ which vanish at infinity (pointwise product, supremum norm). Many applications to invariant means are obtained and this practically opens a new chapter in abstract harmonic analysis. (Received January 20, 1975.)

#75T-B71 LANE YODER, University of Hawaii, Honolulu, Hawaii 96822. Functions of different strong and weak $\phi$-variation.

This paper shows by example how different the strong $\phi$-variation can be from the weak $\phi$-variation. Let $\phi$ be a convex function on $[0, \infty)$ with $\phi(0) = 0$. A continuous function $f$ on $[a, b]$ is of bounded strong $\phi$-variation if $\sup \Sigma \phi(|f(x) - f(x_{i-1})|) < \infty$, for the partitions of $[a, b]$. Since $\inf \Sigma \phi(|f(x) - f(x_{i-1})|) = 0$ if $\lim_{x \to 0} x^{1/\phi}(x) = 0$, the weak $\phi$-variation is defined as $\inf \Sigma \phi(|f(x) - f(x_{i-1})|)$, where $f(x; c, d)$ is the oscillation of $f$ on $[c, d]$. Of special interest is the case $\phi(x) = x^p$, $p \geq 1$, in terms of which strong and weak variation dimensions are defined. They are denoted by $\dim_S(f)$ and $\dim_W(f)$, respectively. By a lemma of Goffman and Loughlin, the Hausdorff dimension $d$ of the graph of $f$ provides a lower bound for $\dim_W(f)$: $1/(2 - d) \leq \dim_W(f)$. A Lipschitz condition of order $\alpha$ provides an upper bound for $\dim_S(f)$: $\dim_S(f) \leq 1/\alpha$. Besicovitch and Ursell showed that $1 < d < 2 - \alpha$ and gave examples to show that $d$ can take any value in this interval. It turns out that these examples provide the extreme cases for variation dimensions; i.e., $\dim_W(f) = 1/(2 - d)$ and $\dim_S(f) = 1/\alpha$. (Received January 20, 1975.)
Let $E$ and $F$ be two non-zero Banach spaces. We equip $C(E,F)$. 2) $u \in A$ for every compact $A$ is called a pseudo-algebra of defined by the family of semi-norms finite-dimensional range. 3) There exists a non-zero continuous bilinear map $p: F \times C_0(E) \to \mathbb{R}$ belonging to $A$.

**Theorem** Let $X$ be a Banach space. The following sentences are equivalent: 1) $A$ is a pseudo-algebra of $C(E,F)$, contained in $C_0(E,F)$, satisfying the Nachbin's conditions (*) and such that $g \in A$ for

The proof of Theorem 1 in [T. C. Lim, A fixed point theorem for multi-valued nonexpansive mappings in a uniformly convex Banach space, Bull. Amer. Math. Soc. 80 (1974), 1123-1126] can be simplified by using the following set theoretic result: Let $(x_n)$ be a sequence in a set $X$ and $r$ a real-valued function corresponding to $u$ and $\delta$ is $L^1$ uniformly caratheodory. This is somewhat the analog of the representation valid for Hammerstein Operators (See Mizel Can. J. Math. 22(1970) 449-471) (Received January 20, 1975.)

**Theorem** Let $X$ be a Banach space and let $\gamma$ be a finitely additive set function with finite variation and values in $X$. Assume $\gamma$ is absolutely continuous with respect to $\mu$. Then there is a sequence of simple functions $f_n$ such that $f_n \gamma$ converges to $\gamma$ in the variation norm. A sufficient condition is obtained for $\gamma$ to be the limit in the variation norm of set functions of the form $f_n \mu$, this condition involves the property that $\mu_n$ are dominated in some sense where $\mu(A) = \sum_n \mu_n(A)x_n$. (Received January 20, 1975.)
each $g \in A$ and for each continuous linear map $u : E \to E$ with finite-dimensional range, then $A$ is dense in $C^\infty(E,F)$.

(*) L. Nachbin, Comptes Rendus, 228 (1949), 1549. (Received January 21, 1975.)

(Author introduced by Professor Alfonso Casal.)

Applied Mathematics


The constructive half of Krohn-Rhodes decomposition theorem uses irreducible semi-automata that are complete. This assumption often requires the basic building semiautomata to be more complex than necessary or desirable. Considering the length of the input strings we obtain the relationship of a semiautomation to its completion for group-machines. It is shown that a cyclic-machine with a single input is strictly weaker than its completion. However, for a cyclic-machine with two distinct inputs, the completion can be generated using series-parallel connections. For a set $\Gamma$ of cyclic-machines, just one machine with two distinct inputs is sufficient to generate the complete machines for all members of $\Gamma$ in the series-parallel closure of $\Gamma$. Using the above results a stronger constructive result is obtained for semiautomata whose semigroups are solvable groups. (Received November 8, 1974.)

(Authors introduced by Professor Robert K. Wright.)

75T-C11 CHARLES BLAIR, Carnegie-Mellon University, Pittsburgh, Pa. 15213. Solutions to mixed-integer problems

We consider solutions to

(*) $\sum_{i} t_i q_i + \sum_{i} r_i p_i = v \quad t_i, r_i \geq 0 \quad t_i \text{ integer}$

where $v, q_i, p_i \in \mathbb{Q}$.

Fix $q_i, p_i$. Let $C = \{ v \mid (*) \text{ has a solution} \}$

Theorem. There are $K_1, K_2$ such that if $v \in C$ with solution $(t_i, r_i)$ and $v + w \in C$ then $(*)$ has a solution $(t'_i, r'_i)$ with $v = v + w$ such that

$$\sum_{i} |t_i - t'_i| + \sum_{i} |r_i - r'_i| \leq K_1 + K_2 |w|.$$  If $n = 0$ or $m = 0$ we may take $K_1 = 0$.

Corollary. Let $F(v)$ be the function defined by

$$\inf\left( \sum_{i} t_i c_i + \sum_{i} r_i d_i \right) \text{ subject to } (*) \text{. Then } F \text{ is subadditive on } C,$n and $F$ can be extended to a subadditive function defined on all of $\mathbb{R}^k$.

(Received November 18, 1974.)

*75T-C12 Joe B. Thrash, University of Southern Mississippi, Hattiesburg, MS 39401. On improving the convergence of improper integrals.

A nonlinear transformation, called the A-transformation, which improves the rate of convergence of some improper integrals will be introduced.

The definition of the A-transformation was motivated by a simplification of the generalized G-transformation as given in H.L. Gray and T.A. Atchison, The generalized G-transformation, Math. of Comp., No. 103 (1968), 595-606.

Theorems are established which give conditions under which the A-transformation will either converge to the value of the improper integral, converge more rapidly than the truncated improper integrals, or converge uniformly better than the truncated improper integral. (Received November 21, 1974.)
Call $F$ **conical** if $F(v+w) \leq F(v) + F(w)$ and $F(0,v) = \lambda F(v)$ for all $v, w$ and $\lambda \geq 0$. Consider a propositional logic with propositions $A, B, \ldots$, such that every atomic proposition is of the form $F(t) \geq f$, with $F$ conical. In what follows, we always assume that $t = (t_1, \ldots, t_n) \geq 0$.

Assign to each proposition $A$ a co-proposition $F(A,t,-)$.

**Theorem 1.**

The same conclusion holds if $A$ is true, so are all cuts of $CT(A)$.

**Theorem 2.**

If $A = B \cup D$, $F(A,t) = F(B,t) + F(D,t)$, $F(A,t) = \max (F(B,t), F(D,t))$ and $f(B) = \min (f(B), f(D))$.

**Theorem 3.**

If $A$ is true, so are all cuts of $CT(A)$.

**Theorem 4.**

If $A = B \cup D$, $F(A,t) = \max (F(B,t), F(D,t))$ and $f(B) = \min (f(B), f(D))$.

We give a procedure for strengthening cutting planes obtained from subadditive functions.

In particular, this can be used to strengthen cutting planes from disjunctive constraints.

Let $y \in \mathbb{R}^q$ satisfy (1) $y \in T$ for some arbitrary $T \subseteq \mathbb{R}^d$; and (2) $y = a - \sum_{j} a_j t_j$, $t_j \geq 0$, $j \in J$. Let $J_2 = J - J_1$. An inequality implied by (1) and (2) will be termed a valid cut. It has been established earlier that, if $F$ is a subadditive function on $\mathbb{R}^d$ (i.e., such that $F(u + v) \leq F(u) + F(v)$, $\forall u, v \in \mathbb{R}^d$), then $\sum_{j \in J_1} F(a_j t_j) + \sum_{j \in J_2} F(-a_j t_j) \geq f_0$ is a valid cut for all $f_0$ such that (3) $\inf_{y \in T} F(y-a) \leq f_0$.

**Theorem.**

If $M \subseteq \mathbb{R}^q$ is a monoid (i.e., $0 \in M$ and $u, v \in M \Rightarrow u + v \in M$), such that (3) holds with $T$ replaced by $T + M$, then $\sum_{j \in J_1} F(a_j t_j) + \sum_{j \in J_2} F(-a_j t_j) \geq f_0$ is a valid cut.

**Corollary.**

If $T = \{y \in \mathbb{R}^q \mid y \geq 0; \sum_{j} y_j \geq 1\}$ and $M = \{m \in \mathbb{R}^d \mid \sum_{j} m_j \geq 0, m_i \text{ integer}, i \in \mathbb{Q}\}$, where $Q = \{1, \ldots, q\}$, then $\sum_{j \in J_1} a_j t_j \geq 1$ is a valid cut for $\sum_{j \in J_1} \inf_{y \in T} F(y-a_j t_j) = \inf_{e \in T} (1-a_j t_j)$.

In every voting situation, the voters consider a set of, say $m$, alternatives, labeled $l, \ldots, m$. For each such situation, there is an $m \times m$ "election matrix" (e.m.) $E \Rightarrow e_{ij}$ which is the fraction of voters who prefer the $i$th to the $j$th alternative. Let $E^k$ be the e.m. derived from $E$ by deleting row $r$ and column $k$. A binary social decision function (s.d.f.) is a rule for assigning to each $E^k$ a "winning" subset of $F(\partial E)$. Let $e_{ij} = e_{jik}$ be derived from $e_{i,j,m-1}$ by relabeling as $j+1$ all $j \in k$ and then inserting $k$ in position $i$. If $F$ is "contractible" if $f(E) = \sup_{e \in T} \inf_{e \in E} F(e_{i,j,m})$, whenever the $k$th row of $E$ has all entries of $\frac{1}{2}$. If $F$ is "continuous" if, whenever $\{E_{n} \Rightarrow E \text{ and } \inf_{e \in T} F(e) \}$ $\forall n$, then $\inf_{e \in T} F(E)$. Let $A_n$ be the e.m. that results when all voters order the alternatives according to $\pi$. The Kemeny s.d.f. (see Daedalus, 88 (1959), 577-591) is defined by: $F(E) = \sum_{A_n \Rightarrow E, \pi \in \pi}$, the anti-Kemeny s.d.f., $F^\pi$, reverses the permutations chosen by $F$. Theorem: The only 2 binary, continuous, symmetric, consistent (see Young, Abstract 74T-C10 of these Notices), and contractible s.d.f.s that are nontrivial are $F$ and $F^\pi$. (Received January 6, 1975)
\[ \frac{n}{i=1} c_i y_i, \quad \dot{y}_i = a_i y_i (x - \beta_i), \quad i = 1, \ldots, n, \] where \( a, b, c_i, a_i, \beta_i \) are positive constants. The system is a model for \( n \) predators competing for 1 prey. The behaviour as \( t \to \infty \) is studied, for all solutions satisfying initial conditions \( x(0) > 0, y_i(0) > 0, i = 1, \ldots, n \). Essentially, the sizes of the parameters \( \beta_i \)'s, relative to \( a/b \), determine the limiting situation. (Received January 10, 1975)

In this paper we give easily computable best upper and lower bounds on the coefficients of facets of the knapsack polytope associated with minimal covers. For some coefficients the upper bounds are equal to the lower bounds; for the others the two bounds differ by 1. We give a necessary and sufficient condition for all lower bounds to be equal to the corresponding upper bounds, i.e., for the facet associated with the given minimal cover to be unique. Also, we define a partial order on the set of minimal covers and show that all facets associated with minimal covers can be obtained from weak covers; but that each facet obtainable from several ordered minimal covers is easiest to compute from the strongest one.

Further, we characterize the class of all facets associated with minimal covers, and show that the facets obtainable by Padberg's sequential lifting procedure are precisely those members of the class which have integer coefficients for a certain given right-hand side. We then give a procedure for generating the other (simultaneously) lifted facets, which have fractional coefficients for the same right-hand side. (Received January 13, 1975.)

Under one dimensional particle and rocket frame motions, it is shown that special relativity has an arithmetic basis in that symmetry, conservation of linear momentum, conservation of energy, the Einstein energy equation, and the direct relationship between the displacement vector and the momentum-energy vector can all be deduced using only arithmetic formulas for basic physical quantities. The resulting model is not only completely compatible with modern computer technology, but it yields dynamical equations which can be solved directly by such computers. Velocity and acceleration are defined by forward difference quotients. The dynamical equation is \( F_k = c^2 m a_k / ((c^2 - v_k^2)(c^2 - v_{k+1}^2)) \dot{v}_k \), which is invariant in form under the classical Lorentz transformation. Conservation of momentum is assumed and is shown to imply conservation of energy. Energy is defined by \( E = mc^2 \), from which rest energy is implied. Mass varies in the classical way with particle velocity. (Received January 6, 1975.)

A computer simulation of two stochastic automata with variable structure interacting in feedback loops is developed to model the game "Prisoner's Dilemma" (Prisoner's Dilemma, Rapoport and Chammah, U. of Mich. Press, 1965). The states of each automaton are the decisions of each player whether to cooperate, C, or defect, D; the input is the payoff from the preceding play which reflects the two independent decisions. A modification of a linear reinforcement algorithm (Fu, K.S.; IEEE Trans. on Automatic Control, April 1970, 210-221) is used to vary the transition probabilities after each play. Comparisons are made with experimental data (Rapoport and Chammah, page 99). (Received January 10, 1975.)

A method is described whereby certain nonlinear Volterra integral equations may be nu-
merically solved by computing the solution to a system of ordinary differential equations. In the case the kernel is not finitely decomposable, certain two dimensional approximating techniques are employed. In such a case, there is a trade-off between computational effort and accuracy. Several explicit error estimates are given and numerical examples illustrate the applicability of the method and its comparison with other literature. (Received January 8, 1975.)


Stability of a density stratified layer of a Boussinesq fluid confined between two parallel planes heated from either side is discussed. The basic density stratification of the fluid, due to mixing of some chemical of negligible diffusivity, depends on the vertical coordinate(z), is assumed to be of the form \( \rho_M = \rho_0 (1 + \delta z) \), \( \delta \) being a real number of the order of \( \alpha \beta \) where \( \alpha \) is the coefficient of volume expansion of the fluid, and \( \beta \) is the uniform temperature gradient which is maintained. For the inviscid case, if either \( \delta \) is positive or \( \beta \) is negative the system is found to be unstable. For the viscous case it is shown that the principle of exchange of stabilities is not valid and neutral modes can not exist if \( \delta \) is positive. The characteristic value problem is solved for the cases of two non-deformable free boundaries, and two rigid boundaries when \( \delta \) and \( \beta \) both are negative. (Received January 16, 1975)

(Received January 16, 1975) (Authors introduced by Dr. B. A. Fusaro.)


A system of two stieltjes integrodifferential equations serving as a model of two interacting species is studied. The model is essentially that originally introduced by Volterra in his well known work on population dynamics except that Stieltjes integrals are used in order to include in the analysis the classical differential models and models with multiple time lags as well as Volterra's model with continuously distributed hereditary effects. Logistic terms are also allowed. Using a parameter which measures the magnitude of the inherent, unrestrained birth rates of each species the existence of periodic solutions is studied in a general setting by means of bifurcation theory. Necessary conditions and sufficient conditions for the existence of periodic solutions which bifurcate from equilibrium are given in terms of the eigenvalues of a certain \( 4 \times 4 \) matrix. Many examples are worked out. Of interest among these is the competing species model where examples show that bifurcation does occur when time lags or continuous hereditary effects are present (but not when they are absent) leading to the conclusion that the Law of Competitive Exclusion is invalid for birth rates larger than some critical value when hereditary effects are taken into consideration. Also included are examples of predator-prey interactions where the results for nonhereditary problems are shown to be consistent with Volterra's periodicity theorem concerning the classical Volterra-Lotka model and where it is shown that hereditary or lag effects tend to destroy the existence of bifurcating branches of periodic solutions which are present in the nonhereditary model. (Received January 17, 1975.)

ROBERT SCHWABAUER, Virginia Commonwealth University, Richmond, Virginia 23284, Enzymatic neurons II

The enzymatic gap (E gap) between Conrad circuits is defined. A sequence of Conrad circuits that makes successively finer distinctions is given. The sequence is such that the E gap between consecutive elements is 1.

The concept of evolvability leads to new problems in logic (Enzymatic neurons, AMS NOTICES, Vol. 22 (1975), page A-256), and also in algebra. The evolvability of a circuit for the group (ring) \( \mathbb{Z}_2 \) into a circuit for the
Boolean semilattice (lattice) on \( n \) generators is discussed. Distributive lattices are given by suitable input combinations. (Received January 20, 1975.)

**Geometry**


In the previous abstract [A-499, 1974], the author announced that all the generalized Brieskorn manifolds admit non-regular almost contact structures whose associated foliations have closed curves as their leaves. In particular, all the exotic spheres bounding parallelizable manifolds belong to this category. In this abstract the author announces that a slight modification of the above structures gives rise to normal almost contact structures, i.e., their torsion tensors vanish. Furthermore, he has shown that the almost complex structures induced naturally on products of two generalized Brieskorn manifolds from these almost contact structures have vanishing torsion tensors. Thus, by the well known theorem of Newlander and Nirenberg, these almost complex structures are actually complex structures. These complex structures, in a way, generalize those of Hopf and Calabi-Eckmann, and they seem to be different from those of Brieskorn and Van de Ven. (Received January 3, 1975.)

**Logic and Foundations**

STEVEN GARAVAGLIA, Yale University, New Haven, Connecticut, 06520. *Ultrapowers and Čech homology, Preliminary Report.*

Let \( R \) be a commutative ring with unit, and let \( R^* \) be an ultrapower of \( R \) with respect to any \( \omega \)-incomplete ultrafilter on an infinite index set. Then (1.) the Čech homology theory with coefficients \( R^* \) satisfies the exactness axiom on the category of compact pairs; (2.) if \( (A,B) \) is a pair of compact metric spaces, then the homology groups \( H_n(A,B; R^*) \) defined by McCord in *Fundamenta Mathematicae*, 1972, page 21, are isomorphic to the Čech homology groups \( H_n(A,B; R) \) for all \( n \geq 0 \). (Received October 25, 1974.)


Let \( L \) be a universal class with the amalgamation property and suppose \( L(\Sigma) \) contains no function symbols. Let \( \Sigma(\lambda) \) be the set of isomorphism types of structures in \( L \) of power \( \lambda \). If there is \( \lambda \geq \omega \) such that \( |\Sigma(\lambda)| < \omega \) then there is \( k < \omega \) such that for all \( \lambda \geq k \), \( |\Sigma(\lambda)| = |\Sigma(\lambda)| \). (Received November 1, 1974.)

WALTER BAUR, Yale University, New Haven, Connecticut 06520. *Undecidability of the theory of abelian groups with a subgroup, Preliminary report.*

Let \( L = \{0, +, P\} \) be the first order language of abelian groups with a unary predicate \( P \) and let \( p \) be an arbitrary prime. Let \( T \) be the \( L \)-theory of the class of structures \( <A,B>\) where \( A \) is an abelian group, \( p^A = 0 \), and \( B \) is an arbitrary subgroup of \( A \). Theorem: \( T \) is undecidable. Corollary 1: There are finite commutative rings \( R \) such that the theory of \( R \)-modules is undecidable. Corollary 2: The theory of abelian groups with a predicate selecting a subgroup is undecidable. (Received January 17, 1975.)

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A functional $F(f)$ is said to be totally computable if there exists a Turing machine that computes the values of $F$ from the values of $f$ wherever $F(f)$ is defined and the value "undefined" where $F(f)$ is not defined. Theorem: The range of a totally computable functional is always finite. If we think of the process of making inductive inferences as representable by the process of evaluating a functional, then this theorem suggests that the role of the totally computable functional as a model for this process is limited or, intuitively, that computers can't do induction very well. (Received November 4, 1974.)

Let $m$ be any regular uncountable cardinal. 1. Theorem: If $0^\#$ does not exist, then every uniform ultrafilter over $m$ is $(n,m)$-regular for some $n$ less than $m$. In particular, every uniform ultrafilter over the first uncountable cardinal is regular. 2. Theorem: Suppose that $m$ is weakly inaccessible and $D$ a uniform ultrafilter over $m$ which is not $(n,m)$-regular for any infinite cardinal $n$ less than $m$. Then there is a weakly normal ultrafilter over $m$ below $D$ in the Rudin-Keisler order. (Received November 4, 1974.)

A well-known Steinitz theorem says that any 1-1 map $f: A \to B$ can be extended (in many ways) to a field monomorphism $F(f): \bar{A} \to \bar{B}$, where $\bar{A}, \bar{B}$ are algebraically closed fields with transcendence bases $A, B$. For fixed characteristic $m$, to what extent may this $F$ be functorial? We show by Artin-Schreier theory that from the category of sets with 1-1 maps no such functor $F$ exists (nor for full subcategories with a two-element set if $m > 0$ or with a three-element set if $m = 0$). For trivial reasons such a functor exists from the category of sets with only inclusions; by an Ehrenfeucht-Mostowski argument there is such a functor from the category of order-monomorphisms between linearly ordered sets.

The argument shows that the canonical Skolemization of the ($\omega$-stable) theory of algebraically closed fields of characteristic $m$ has no complete extension which is stable.

These questions (except the last) arise naturally in the study of combinatorial functors. (A. Nerode partly supported by Australian A.R.G. C. grant B73/15116.) (Received November 8, 1974.)

In propositional modal logic, a formula is said to be of modal degree one if no occurrence of $\Box$ or $\Diamond$ is within the scope of another. If $L$ is a propositional modal logic, let $L^*$ be the set of $L$-theorems of modal degree one. Theorem: If $T \subseteq L \subseteq S_5$, then $T^* = L^* = S_5^*$. Corollary: $T, B, S_4$ and $S_5$ agree on modal degree one theorems. The method of proof is manipulation of Kripke models. (Received November 21, 1974.)
if there is an \((A,X) \in \Gamma\) with \(B \subseteq A\). We assume i) \(\Gamma\) is isomorphism closed ii) if \((A,X) \in \Gamma\) and \(Y \subseteq X\) then \((<Y>,Y) \in \Gamma\) and iii) if \(A\) is \(\tau\)-algebra, \(A = \langle X, X, |X| = \kappa\), and for each \(Y \in P_X(X)\) \((<Y>,Y) \in \Gamma\) then \((A,X) \in \Gamma\). Theorem D a \(\tau\)-algebra \(|D| = \kappa\). If every subalgebra \(S \in P_A(D)\) is \(\Gamma\)-embeddable then \(D\) is \(\Gamma\)-embeddable. A class of \(\tau\)-algebras \(\Sigma\) is weak \(\kappa\)-local if whenever \(|A| = \kappa\) and for each subalgebra \(S \in P_A(A)\) \(S \in \Sigma\) then \(A \in \Sigma\). \(J\) is a universal class of groups. Some consequences of the theorem are: \(\Sigma_0\) is class of all free (abelian) groups, \(\Sigma_1\) is class of all subdirect products of \(\tau\)-groups, and \(\Sigma_2\) is class of all subgroups of free products of \(\tau\)-groups are all weak \(\kappa\)-local. This result for \(\Sigma_0\) is due to Mekler and Gregory. (Received November 29, 1974.)

**Theorem.** The set of prime numbers is identical with the set of positive values of the polynomial

\[
(k+2) \cdot \left[1 - \left(\varepsilon w + h + j - q\right)^2 + \left(\varepsilon w^2 + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^4 + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^6 + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^8 + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^{10} + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^{12} + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^{14} + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^{16} + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^{18} + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^{20} + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^{22} + h + j \cdot k + 2\right)^2 + \left(\varepsilon w^{24} + h + j \cdot k + 2\right)^2\right] \cdot \left(\varepsilon w^2 + h + j \cdot k + 2\right)^2.
\]

Here it is understood that the 26 variables \(a, b, c, \ldots, z\) run through the nonnegative integers. The number of variables can be reduced to 12. (Received December 2, 1974.)

**THEOREM.** If \(B = \langle A, R_1, R_2, \ldots \rangle\) is a 1-embeddable complete Boolean algebra with \(D = \langle A, R_1, R_2, \ldots \rangle\) as its set of free generators, then \(\Sigma\) is weak \(\kappa\)-local.\(\Sigma_0\) is class of all free (abelian) groups, \(\Sigma_1\) is class of all subdirect products of \(\tau\)-groups, and \(\Sigma_2\) is class of all subgroups of free products of \(\tau\)-groups are all weak \(\kappa\)-local. This result for \(\Sigma_0\) is due to Mekler and Gregory. (Received November 29, 1974.)

**Theorem.** Any subset of an algebraically closed field of characteristic \(0\), which is definable in \(L(R)\) by a formula with one free variable is either finite or co-finite. The proof is by elimination of quantifiers which also shows that the theory of algebraically closed fields of fixed characteristic in the logic \(L(W)\) is complete. It is, no doubt, well known: Theorem. In the logic \(L(W)\) there are subsets of algebraically closed fields of characteristic \(0\) which are definable by a formula with one
algebras $A, B$ whose free squares $A^*A, B^*B$ are isomorphic. The common refinement property for free products of Boolean algebras is also shown to fail. (Received January 9, 1975.)

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equivalence of countable and uncountable orderings. Preliminary report.

Let $< a$ be a linear ordering of a set $A$. $(A, <)$ is scattered if it does not contain a copy of the rational numbers, otherwise nonscattered. M. Makkai has shown that if $(A, <)$ is scattered and countable, then it is not $L_{\omega_1\omega}$ equivalent to any uncountable structure. We provide an extension of Makkai's theorem which characterizes those countable orderings which are $L_{\omega_1\omega}$ equivalent to uncountable structures.

Let $\mathcal{U}$ be of the form $(A, <, \pi_i)_{i \in I}$, where $A$ and $I$ are countable, each $\pi_i$ is a unary predicate, and $< \subseteq A$ linearly orders $A$.

Let $a$ be an element of $A$, then the orbit of $a$ is the set of elements $b$ of $A$ such that there is an automorphism of $\mathcal{U}$ carrying $a$ to $b$. Each orbit of $\mathcal{U}$ is a subordering of $\mathcal{U}$.

**Theorem.** $\mathcal{U}$ is $L$ equivalent to an uncountable structure if and only if it has a nonscattered orbit. (Received January 9, 1975.)


**Theorem 1.** Assuming the consistency of Peano arithmetic, there exist two polynomial terms $P(x_1, \ldots, x_n)$ and $Q(x_1, \ldots, x_n)$ in the first-order-language of Peano arithmetic such that the existential sentence $\exists x_1 \ldots \exists x_n (P(x_1, \ldots, x_n) = Q(x_1, \ldots, x_n))$ is undecidable from the axioms of Peano arithmetic.

**Theorem 2.** In every consistent recursive extension of Peano arithmetic there exists an existential sentence in the language of Peano arithmetic such that the sentence is undecidable from the axioms of the extended arithmetic, i.e. Peano arithmetic is essentially-existentially-incomplete.

**Corollary 1.** Every subtheory of Peano arithmetic, with the same language as Peano arithmetic, is essentially-existentially-incomplete.

Note 1. Undecidable existential sentences are false in the standard model of Peano arithmetic; and thus are true in some non-standard models.

(Received January 17, 1975.)

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Joins and Complements Below High Degrees.

In "Degrees Joining to $0'$" (to appear) R. W. Robinson showed that given $0 < a < 0'$ there is a $b < 0'$ such that $a \lor b = 0'$. Here $\lor$ is the join operation in the upper semilattice of degrees of unsolvability; $\land$ is the meet operation. He showed that if also $a'' = 0'$ then there is a $b < 0'$ such that $a \lor b = 0'$ and $a \land b = 0$. That is, $a$ has a complement in the degrees $\leq 0'$ (i.e. $D(\leq 0')$). Subsequently in Minimal Degrees of Unsolvability and the Full Approximation Construction (Memoirs of the A.M.S., to appear) we showed that given $0 < a$ r.e. there is a minimal degree $m < 0'$ such that $m \lor a = 0'$, thus exhibiting a complement for $a$ in $D(\leq 0')$. We have now shown that if $h$ is a high degree (i.e. $h \leq 0'$ and $h' = 0'$) then these results hold in $D(\leq h)$. (Received January 20, 1975.)


$A[8]^*$ denotes the bounded Boolean power of $A$ by $8$, and $A[8]$ the Boolean power (when it is defined). **Theorem 1.** $A_0 \leq A_1$ and $B_0 \leq B_1$ implies $A_0[B_0]^* \leq A_1[B_1]^*$ and, if $B_0$ is complete, $A_0[B_0] \leq A_1[B_0]$. **Theorem 2.** Free products in a variety generated by a primal algebra preserve elementary equivalence and substructure. **Theorem 3.** Suppose $A$ is finite. If $B$ is complete then $A[8]$ is equationaly compact. If $B$ is $\kappa$-saturated, $\kappa = \omega$, then $A[8]$ is $\kappa$-saturated. **Theorem 4.** The theory of countable m-rings with quantification over ideals is decidable, for each $m \geq 2$. (Received January 20, 1975.)

free individual variable but are neither finite nor co-finite (e.g. the set of integers). The
above results show that the logic \( L(R) \) is not an extension of the logic \( L(W) \).
(Received December 2, 1974.)

**75T-E22** JUDITH ROITMAN, Wellesley College, Wellesley, Mass. 02181. Almost disjoint
strong refinements. Preliminary report.

If \( V \) is a refinement of \( U \), we say that \( V \) is a strong refinement iff there is a one-one
function \( f: U \to V \) such that \( f(u) \leq u \), for every \( u \in U \). Let \( E \) be the statement: every maximal
almost disjoint family on \( \omega \) has cardinality \( 2^{\omega} \). 1. Assume \( E \). Then every ultrafilter on
\( \omega \) has a strong refinement into almost disjoint sets. 2. Assume \( E \) and \( 2^\kappa = 2^{\omega} \). Then the
uncountable subsets of \( \kappa \) have a strong refinement into countable almost disjoint sets.
(Received December 12, 1974.)

**75T-E23** ALLAN K. SWETT, McGill University, Montreal, Quebec, Canada H3C 3G1
Topological Spaces Elementarily Equivalent to the Line. Preliminary report.

In their manuscript "First Order Topology", Jockusch, Rubel, and Takeuti identify with each
\( T_1 \) topological space \( X \) the model-theoretic structure \( X^* = \langle LX, \subseteq \rangle \) where \( LX \) is the lattice of
closed subsets of \( X \). Considering the finitary first order theory of such structures \( X^* \), they inquire:
for what \( T_1 \) topological spaces \( X \) is \( X^* \) elementarily equivalent (i.e.) to \( R^* \) (or to
\( I^* \), where \( R \) denotes the real line and \( I \) the compact unit interval)? We prove for any \( T_1 \)
topological space \( X \): Theorem 1 If \( X^* \) is e.e. to \( R^* \) then \( X \) is first countable, \( \sigma \)-compact,
and its topology is induced by a dense, complete linear order without endpoints. Theorem 2
If \( X^* \) is e.e. to \( R^* \) then \( X \) has precisely continuum-many points. Theorem 3 If \( X \) is locally
separable and \( X^* \) is e.e. to \( R^* \) then \( X \) is homeomorphic to \( R \). Theorem 4 If \( X^* \) is e.e. to \( (R^\alpha)^* \)
for some \( \alpha > 1 \) then \( X \) is path connected. Theorem 5 If \( X^* \) is e.e. to \( R^* \) and \( (X^3)^* \) is e.e. to
\( (R^\alpha)^* \) for some \( \alpha > 1 \) then \( X \) is homeomorphic to \( R \).

However Conjecture If \( S \) is a Souslin line then \( X^* \) is elementarily equivalent to \( R^* \).
In fact, we suspect that the converse to Theorem 1 is true.

These results carry over in complete analogy for the compact interval \( I \) rather than the
line \( R \). (Received December 19, 1974.)

**75T-E24** RICHARD A. SHORE, Cornell University, Ithaca, NY 14853. Automorphism
bases for the recursively enumerable sets, Preliminary report.

Call a class \( \mathcal{C} \) of recursively enumerable sets an automorphism basis for
\( \mathcal{E}^* \) (the lattice of r.e. sets modulo finite sets) if the action of any
automorphism \( \varphi \) of \( \mathcal{E}^* \) is completely determined by its action on \( \mathcal{C} \) (or
equivalently, if \( \varphi \) is the identity on \( \mathcal{C} \) then it is the identity on \( \mathcal{E}^* \)).

Theorem: The classes of recursive sets, maximal sets and sets of any
fixed r.e. degree are all automorphism bases for \( \mathcal{E}^* \). (Received December 23, 1974.)

**75T-E25** PHILIP OLIN, York University, Downsview, Ontario, Canada M3J1P3. Countably
saturated free products of Boolean algebras. Preliminary report.

Using Tarski's elimination-of-quantifiers result for Boolean algebras, an algebraic
characterization of the countably saturated Boolean algebras is given and used, together with
previous work (see Notices A.M.S. 21(1974), A-554), to show that if \( A \) and \( B \) are countably
saturated Boolean algebras then so is the free product \( A*B \). As a corollary of this, a question
of P.R. Halmos (Lectures on Boolean algebras, Van Nostrand, Princeton, 1963, page 125) is
answered by getting non-isomorphic (in fact, non-elementarily equivalent) denumerable Boolean
A, B and C are sets of integers, \( \emptyset_z \) are enumeration operators, \( A^* = \{ 2^z x : x \in \emptyset_z(A) \} \). \( A' \) is the jump of A and \( A^0 \) its complement. \( A(n) \) denotes iterations of the jump and \( \vee \) is the join operator. \( \ll_e \) and \( \ll_T \) are enumeration and Turing reducibilities. A is pseudo-recursive (PR) if \( A^* \ll_e A \). A is pseudo-enumerable (PE) if \( \exists B[A \ll_e B] \). Theorem 1. \( A \in \text{PR} \iff A' \ll_e A' \) \( \ll_{\text{PE}} \).

Statistics and Probability

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Regularity properties of second order processes.

Let \( \{X(t) : t \in T\} \) be a mean zero, continuous covariance, second order process on some probability space, where \( T \) is a separable metric space, with the following property: There exists a continuous, nonnegative, nonincreasing function \( \psi \) defined on \([0, +\infty)\) such that \( \lim_{u \to +\infty} \psi(u) = 0 \) and for \( \lambda > 0 \), \( s, t \in T \), \( F(\left| X(s) - X(t) \right| > \lambda) \leq \psi(\lambda/\tau(s,t)) \);

where \( \tau \) denotes the increments variance of the process. We show that many interesting processes satisfy this property and find conditions in terms of \( \psi \) and \( \tau \) in order to establish the existence of a version of \( X \) with continuous sample paths. One such condition when \( T = [0, 1] \) and there exists a nondecreasing function \( \varphi \) which majorizes \( \tau \) is \( \int_0^\infty \varphi \left( 1 - \varepsilon(u) \right) du < +\infty \) for some \( 0 < \varepsilon < 1 \). This condition in the Gaussian case is equivalent to a well known condition of Fernique. Furthermore, certain of these conditions imply the almost sure uniform convergence of orthogonal expansions for \( X \) and also central limit theorems for \( \mathbb{C}(T) \)-valued random variables. (Received November 18, 1974.)

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Some recent results in age-dependent processes, Preliminary report.

Let \( X(\omega) \) be a classical Bellman-Harris age-dependent process with lifetime distribution \( G \) and generating function \( \pi(s) = \sum_{k=0}^{\infty} G_k s^k \). We assume \( G \) is non-lattice and \( G(0^+) = 0 \). We consider the supercritical case \( m = \pi'(1) > 1 \) and assume that \( \pi' \) is a continuous, nonnegative, nonincreasing function. Then it is known that the nonnegative martingale \( W_t = e^{-\lambda t} \psi(X_t) + W \) a.s. with \( E_0(W) = 1 \). Theorem 1. If \( Z(x,t) \) is the number of particles alive at time \( t \) and of age \( \leq x \), then \( Z(x,t)/e^{\lambda t} + A(x)W \) in probability, where \( A(x) \) is the limiting age distribution. Theorem 2. If in addition, \( a(x) = \inf_{y < x} \psi(y) > 0 \), we have convergence in \( L^1 \). Theorem 3. \( Z(x,t)/Z(t) + A(x) \) in probability and \( L^1 \) off \( \{ W = 0 \} \). Analogous results are valid for the more general age-dependent process considered in Savits, AMS Notices 21 (1974), abstract 74T-F6. (Received November 18, 1974.)
We present a self-contained proof of the Griffiths-Hurst-Sherman and related inequalities for spin-1/2 Ising ferromagnets with pair interactions. Fixing \( N \geq 1 \), we consider configurations \( \sigma = (\sigma_1, \ldots, \sigma_N) \), where each \( \sigma_i \) may take on the values \( \pm 1 \). The partition function \( Z(h) \) of such a system is defined by the formula:

\[
Z(h) = \sum_{\sigma} \exp(-\beta H(\sigma)),
\]

where \( \beta \) is a positive constant, \( H(\sigma) = -2^{-1} \sum_{1 \neq i,j} \sigma_i \sigma_j \rho_{ij} + \sum_i h \sigma_i \) with \( \rho_{ij} \geq 0, h \geq 0 \), and the sum is taken over all configurations. The Griffiths-Hurst-Sherman inequality states that \( 3 \ln Z(h) / \partial h \) is non-negative. The proof is obtained by representing \( 3 \ln Z(h) / \partial h \) as an expectation over Gaussian random variables. The inequality has the interpretation that the average magnetization per site in an Ising system is a convex function of \( h \), a parameter which denotes the external magnetic field. (Received December 23, 1974.)

The power spectrum of a mean zero complex Gaussian random signal process \( \{y(t;\theta)\} \) is assumed to be known except for the parameter vector \( \theta \). Using pulse-pair samples of \( y(t;\theta) + v(t) \) where \( v(t) \) is discrete white noise, a closed form solution has been obtained for the error probabilities of the likelihood ratio test that distinguishes between the hypotheses that the unknown power spectrum parameters are \( \theta_0 \) rather than \( \theta_1 \). A complete exact solution that optimally distinguishes between stochastic signals, and is independent of the observed sample signal power as well as the signal power, has also been determined. The errors are expressed in terms of the incomplete Beta function ratio. The results are valid even when the power spectra (under the hypotheses) are nonseparable. (Received January 9, 1975.)

A subgroup \( H \) of a topological group \( G \) is closed if it is non-meager in itself and either Borel in \( G \) or almost open (Kelley's General Topology, p. 211) in \( \overline{H} \); the proposition fails if "in \( \overline{H} \)" is replaced by "in \( G \)." The proof is a brief exercise in the use of Banach's subgroup theorem (Kelley, p. 211). Hence among Hausdorff groups the Cech complete (and therefore the metrically complete) are absolutely closed and so must coincide with their bilateral completions. A first countable group is metrically complete iff it is bilaterally complete. (Received November 4, 1974.)

Let \((X, U, V)\) be a bitopological space. We say the space is \( d \)-compact if \( X \) contains a \( U \) compact, \( V \) dense set, and a \( V \) compact, \( U \) dense set, and if \((X, U) \) and \((X, V) \) are \( T_1 \) spaces. If we list properties desirable with any definition of compactness of \((X, U, V)\), we would include the following: a) compactness should be
productive; b) the space may be compact and pairwise Hausdorff without $U$ coinciding with $V$; c) compactness should not force $(X, U)$ and $(X, V)$ to be compact; and d) compactness should imply all real-valued, $U$ upper semi-continuous, $V$ lower semi-continuous functions on $X$ are bounded. Other definitions of compactness have enjoyed no more than two of the above properties. In fact, it is unknown whether a definition is possible that enjoys a) and b). Note that b) implies c). The notion of d-compactness has all of these except b). There are reasons to believe that b) may, in fact, be too strong, and a weakened form of pairwise Hausdorff should be substituted in b). If this is done, d-compactness has all four of the desirable properties. Examples are given to indicate the connections between this definition and the others. (Received November 4, 1974.)


Let $M^n$ be a closed PL $n$-manifold and $Z^{n-1}$ a compact PL $(n-1)$-manifold, $n \leq 4$. A PL open $f: M^n \to Z^{n-1}$ whose inverses are circles or points is the orbit map of a local $S^1$-action on $M^n$. Examples are constructed of $S^1$-actions on $M^n$'s with PL orbit maps such that any such action with a manifold as orbit space is equivalent to one of the constructions. (Received November 6, 1974.) (Author introduced by Louis F. McAuley.)

*75T-G19 T. THRISSURKRISHNAN, Mar Athanasius College, Kothamangalam, 686666, Kerala, India. A net characterization of Tychonoff's $k$-compactness

A net $\mathfrak{N}$ in a Tychonoff space $X$ is said to be a $k$-net if given any zero-set $E$, the net $\mathfrak{N}$ is either eventually in $E$ or eventually in the complement of $E$. Also $\mathfrak{N}$ is said to be $k$-directed for an infinite cardinal $k$, if the directed set $\mathfrak{N}$ on which $\mathfrak{N}$ is defined, is $k$-directed, i.e., if given any $\{\mathfrak{N}_i\}_{i \in J} \subseteq \mathfrak{N}$ such that $|J| < k$, there exists an element $d \in D$ such that $d \geq d_i$ for every $i$ in $J$. Theorem 1 If $(X, f)$ is $k$-directed for an infinite cardinal $k$ (cf. Math. Scand. 22(1968), 79-83.), if and only if every $k$-directed $\mathfrak{N}$-net in $X$ is convergent. (Received December 31, 1974.)

75T-G20 LUDVIK JANOS, University of Montana, Missoula, Montana. 59801
Compactifications of self-bijections.

By a self-bijection (s.b.) we understand a pair $(X, f)$ consisting of a set $X$ and a bijection $f:X \to X$. Given a s.b. $(X, f)$ a question arises whether there is a nice nontrivial topology on $X$, having some prescribed properties, and rendering $f$ a homeomorphism. Assuming that card $(X) \leq c$ (cardinality of continuum) we say that $(X, f)$ is of compact type iff there is a compact metric topology on $X$ relative to which $f$ is a homeomorphism. It has been proved by H. de Vries (compactification of a set which is mapped onto itself, Bull. Acad. Polonaise des Sci. III, 5(1957) 943-945) that if card $(X) = c$ then the statement: "$(X, f)$ is of compact type for every $f$" is equivalent to the continuum hypothesis. In contrast to the case of cardinality $c$ the case of countably infinite cardinality is more complicated. Let $A$ denote the class of all $(X, f)$ with $X$ countably infinite and let $C \subseteq A$ be the subclass of those s.b. which are of compact type. Theorem 1 If $(X, f) \in A$ then either $(X, f) \in C$ or $(X, f)$ can be equivariantly embedded in some $(Y, g) \in C$ such that card $(Y-X) = 1$. Theorem 2 There is a $(X, f) \in A$ with $f$ pointwise periodic, such that $(X, f)$ is not in $C$. (Received November 15, 1974.)

*75T-G21 PAUL BANKSTON, Dept. of Mathematics, McMaster University, Hamilton

Clopen Sets in Hyperspaces, Preliminary Report.

Let $X$ be a space and let $H(X)$ denote its hyperspace (= All nonempty closed subsets of $X$ topologized via the Vietoris topology). Then $X$ is Boolean (= totally disconnected compact
Hausdorff) iff \( H(X) \) is Boolean; and if \( B \) denotes the characteristic Algebra of clopen sets in \( X \) then the corresponding Algebra for \( H(X) \) is the free Algebra generated by \( B \) modulo the ideal which "remembers" the upper semilattice structure of \( B \). (Received November 15, 1974.)

*75T-G22 S. BROVERMAN, Dept. of Mathematics, University of Manitoba, Winnipeg, Manitoba, The topological extension of a product, Preliminary report.

Let \( B(X) \) denote the \( E \)-compactification of an \( E \)-completely regular space in the sense of Mrowka (Acta Mathematica 120(1968), 161-185). Let \( E = \{0,1\} = 2 \) the two point discrete space. The \( E \)-completely regular spaces are the 0-dimensional spaces, i.e. those spaces with a base of clopen sets. Theorem. Let \( \{X_\alpha\} \in A \) be a family of 0-dimensional spaces. Then \( \bigwedge_{\alpha} X_\alpha = H_0 X \iff \bigwedge_x X_\alpha \) is pseudocompact. A property \( P \) of topological spaces which is closed hereditary, productive, and enjoyed by all compact, \( T_\alpha \) spaces is called an extension property. Let \( P(X) \) denote the maxima; \( P \)-extension of a \( T_\alpha \), space on \( X \). Theorem . \( P(X \times Y) = P(X) \times P(Y) \iff X \times Y \) is \( C \)-embedded in \( PX \times PY \). Theorem. Let \( P \) be contained in pseudocompactness. Then \( P(X \times Y) = PX \times PY \iff \bigwedge_x X_\alpha \) is pseudocompact. An example is given of a \( P \) contained in pseudocompactness and an infinite pseudocompact product \( \bigwedge_x X_\alpha \), such that \( P(\bigwedge_x X_\alpha) \neq \bigwedge_x P(X_\alpha) \). (Received November 20, 1974.)


Let \( cd(X,G) \) be the cohomology dimension over the group \( G \). A Peano semilattice is a compact connected metric locally connected space which supports a continuous associative operation satisfying \( xy = yx \) and \( x^2 = x \). Theorem: If \( X \) is a Peano semilattice and \( cd(X,G) = n \), then \( X \) contains a topological \( n \)-cell. This answers problem 39 in Brown and Stralka's "Problems on compact semilattices," Semigroup Forum 6, No. 3 (1973), 265-270. (Received November 21, 1974.)

75T-G24 HAROLD R. BENNETT and HAROLD W. MARTIN, Texas Tech University, Lubbock, Texas, Metrizability of \( M \)-spaces, II, Preliminary Report

A collection \( \mathcal{A} \) is said to be \( \text{pair-finite} \) (\( \text{pair-countable} \)) provided that whenever \( x \) and \( y \) are distinct points, at most finitely (countably) many members of \( \mathcal{A} \) contain both the point \( x \) and the point \( y \).

Proposition 1. Let \( \mathcal{B} \) be a pseudo-base for a separable space \( X \). Then, \( \mathcal{A} \) is countable if and only if \( \mathcal{B} \) is pair-countable.

Proposition 2. Let \( X \) be a first countable space and \( \mathcal{A} \) be a pair-countable pseudo-base for \( X \). If \( x \in X \) such that \( \{x\} \) is not an open set, then \( \mathcal{B} \) is point-countable at \( x \).

Proposition 3. Every first countable, countably compact Hausdorff space with a pair-countable pseudo-base is metrizable.

Proposition 4. A regular \( M \)-space is metrizable if and only if it has a pair-finite pseudo-base. (Received November 29, 1974.)

75T-G25 RONNIE FRED LEVY, Goucher College, Towson, Maryland 21204. Path connectedness properties of remainders of compactifications. Preliminary report.

I denotes the set of non-negative real numbers. \( BX \) denotes the Stone-Cech compactification of \( X \). If \( T \) is a totally-ordered compact connected space, a space is \( T \)-path-connected if whenever \( p \) and \( q \) are elements of \( X \), there is a continuous function from \( T \) to \( X \) which maps the endpoints of \( T \) to \( p \) and \( q \). Theorem 1. If \( T \) is any totally ordered compact connected space, \( BI-I \) fails to be \( T \)-path-connected. Theorem 2. If \( X \) is compact and contains a dense, separable, path-connected subset, then \( X \) is a continuous image of \( BI-I \). Theorem 3. If \( BX-X \) is path-connected, \( X \) is pseudocompact. (Received January 8, 1975.)

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GLORIA J. TASHJIAN, Wesleyan University, Middletown, Connecticut 06457. Cozero and Baire maps on products of uniform spaces.

The main result is Theorem. A uniform Baire function from a product of uniform spaces into a metric space depends on countably many coordinates. The proof uses a set-theoretic result: Let $X$ be a cartesian product and $\mathcal{U}$ denote the collection of subsets of $X$ which depend on countably many coordinates. Lemma. If $\mathcal{U}$ is a family of subsets of $X$ such that $\bigcup \mathcal{U} \in \Sigma$ for each $\mathcal{U}' \subset \mathcal{U}$, then $\mathcal{U}$ depends on countably many coordinates. Corollaries describe the cozero and Baire-fine coreflections of a product of uniform spaces in terms of its countable subproducts, and it follows in particular that metric-fine coreflections of products of metric spaces and measurable coreflections of products of complete metric spaces are proximally fine. (The various classes of uniform spaces mentioned above are discussed in recent papers of A. W. Hager, Z. Frolik, and M.D. Rice.) (Received December 9, 1974.)

PHILIP BACON, University of Florida, Gainesville, Florida 32611. Finite dimensional shape theory.

Maps $f, g: X \to Y$ are $n$-homotopic if, for every map $m$ of a polyhedron $P$ of dimension $\leq n$ into $X$, $fm \approx gm$. In the same way that the homotopy theory of polyhedra extends (by the construction of Čech or Vietoris) to shape theory, the $n$-homotopy theory of polyhedra extends to $n$-dimensional shape theory. Since the $n$-homotopy theory of polyhedra is essentially the same as the homotopy theory of CW complexes homotopically trivial in all dimensions $> n$, the $n$-dimensional shape category can be constructed by the method of Holsztynski. (Received January 9, 1975.)

MARK D. MEYERSON, Department of Mathematics, Stanford University, Stanford, California 94305. Projections of Cantor Sets, Simple Closed Curves, and Spheres in $E^3$.

The main theorem in this paper is that in $E^3$ there is a tame 2-sphere with a Cantor subset such that all projections of the two sets are the same. There are several interesting related results. One corollary is that every tame knot class contains a knot which cannot be linked by a straight line. Compare this to the well-known result that every knot can be linked by a polygonal loop. Another implication solves a problem posed to the author by R. H. Bing, namely there does exist a simple closed curve in $E^3$ with Cantor subset such that all projections of the two sets are the same. The chief previous result in the area of this paper is due to K. Borsuk who showed that there is an arc whose projection in every plane has interior points. (Received January 6, 1975.)

M.RAJAGOPALAN, Memphis State Univ., Memphis, TN. 38152 & R. Grant WOODS, Univ. of Manitoba, Winnipeg, Canada R3T 2N2 PRODUCTS OF SEQUENTIALLY COMPACT SPACES. Preliminary Report.

Assuming the continuum hypothesis, the authors give an example of a family of $\exp(\exp \aleph_0)$ locally compact sequentially compact Hausdorff spaces whose product is not countably compact.

This answers a question raised by C. T. Scarborough and A. H. Stone in [Trans. Amer. Math. Soc. 124 (1966), 131 - 147]. (Received January 14, 1975.)


In this paper we show how model theory can be used to construct example spaces in topology. A $Q$-topological space in an enlargement $*\mathbb{N}$ is a pair $(X, J)$, where $X$ is an internal set and $J \subseteq 2^X$ is an internal family of sets closed under internal unions and $*\text{finite}$ intersections and which contains $\emptyset$ and $X$. The $Q$-topology on $X$ is the topology on $X$ for which $J$ is a base and is denoted by $Q$. This extends Abraham Robinson’s definition to non-standard *topological spaces. These are the major two results:

If $(X, J)$ is a $Q$-Urysohn space in an enlargement $*\mathbb{N}$, then $(X, J)$ is a totally separated space. Moreover, if $(X, J)$ is totally separated, then $(X, J)$ must be *completely Hausdorff.
If \((X, \mathcal{J})\) is a *completely regular space in an enlargement *\(\mathbb{M}\), then \((X, \mathcal{J})\) is a zero dimensional space. Moreover, if \((X, \mathcal{J})\) is zero dimensional, then \((X, \mathcal{J})\) must be *regular.

\((X, \mathcal{J})\) is a *discrete space iff \((X, \mathcal{J})\) is discrete, and if \((X, \mathcal{J})\) is extremally disconnected, then \((X, \mathcal{J})\) is *extremely disconnected.

Thus if \((X, \mathcal{J})\) is a connected \(T_3\)-space but not \(T_3 1/2\) (e.g. the Smirnov topology on \(\mathbb{R}\), \((X, \mathcal{J})\) is an example of a totally separated space which is neither zero dimensional nor extremally disconnected. If \(\mathcal{J}\) is the Euclidean topology on \(\mathbb{R}\), then \((\mathbb{R}, \mathcal{J})\) is a zero dimensional space which is not extremally disconnected, etc.

We also deal with some of the net convergence properties of these spaces. (Received January 15, 1975.)

75T-G31 FRANKLIN D. TALL, University of Toronto, Toronto, Canada M5S 1A1. Hereditarily separable spaces and a theorem of Weiss, Preliminary report.

W.A.R. Weiss has proven that, assuming Martin's Axiom plus the negation of the continuum hypothesis, countably compact perfectly normal spaces are compact. Several corollaries follow, e.g. Theorem. Assume Martin's Axiom plus the negation of the continuum hypothesis. If \(X\) is \(\mathbb{M}\) but not paracompact p—in particular if \(X\) is countably compact but not compact—then \(X^\mathcal{J}\) is not hereditarily normal. \(X\) is metrizable if and only if \(X\) is a perfectly normal \(\mathcal{M}\)-space with either a \(G_\delta\)-diagonal or a point-countable separating open cover. An analysis of Weiss' proof, combined with techniques of K. Kunen and the author, leads to Theorem. The existence of a hereditarily separable, normal, non-Lindelof space does not imply the existence of a countably compact, non-compact, perfectly normal, hereditarily separable space. In a different direction, the following improvement of results of M. E. Rudin has been obtained by elementary means. Theorem. If there is a hereditarily separable, regular, non-Lindelof space, there is a hereditarily separable, regular, non-completely regular space. (Received January 15, 1975.)

75T-G32 W. A. R. WEISS, University of Toronto, Toronto, Canada M5S 1A1. Countably compact, perfectly normal spaces may or may not be compact, Preliminary report.

Assuming \(V = L\), Ostaciewski has constructed a countably compact, perfectly normal, non-compact space. Theorem. Assume Martin's Axiom plus the negation of the continuum hypothesis. Then every countably compact, completely regular space in which closed sets are \(G_\delta\)'s is compact. (Received January 15, 1975.) (Author introduced by Professor Franklin D. Tall.)

75T-G33 ROBERT D. EDWARDS, University of California, Los Angeles 90024. The double suspension of a certain homology 3-sphere is \(S^5\).

Let \(H^3\) denote the non simply-connected Mazur homology 3-sphere, gotten as the boundary of a regular neighborhood of Mazur's noncellular embedding of the duncehat into \(\mathbb{h}\) (see [B. Mazur, Ann. Math. 73 (1961)] or [E. C. Zeeman, Topology 2 (1964)])]. In this paper it is proved that the twofold (double) suspension \(2^vH^3\) of \(H^3\) is topologically homeomorphic to the 5-sphere \(S^5\). This answers, for a specific nontrivial homology 3-sphere, one of the two remaining open questions of Milnor from 1963 [Ann. Math. 81 (1965), p. 779]. It establishes the existence of manifold triangulations which are not combinatorial, that is, are not locally piecewise linearly homeomorphic to euclidean spaces. Also, it gives an example of an embedding of a circle (the suspension circle) into \(S^5\), such that the image has a mapping cylinder neighborhood, but is not locally flat. (Received January 20, 1975.)

75T-G34 Dennis K. Burke, Miami University, Oxford, Ohio 45056. On point-countable separating open covers in Moore spaces. Preliminary report.

An example is constructed of a locally compact Moore space that does not have a point-countable, point-separating open cover. (Received January 20, 1975.)

75T-G35 WITHDRAWN

Miscellaneous Fields


We say a digraph \(D\) (having neither loops nor multiple edges) of order \(p\) is of Ore-type \((k)\) if \(d(u) + d(v) \geq p + k\) whenever \(u\) and \(v\) are distinct vertices for which \(u\) does not dominate \(v\). A digraph \(D\) is hamiltonian-connected if for every pair of vertices \(u\) and \(v\), \(D\) has a hamiltonian
u;v path. Also, D is strongly hamiltonian if it has arcs and each one lies on a hamiltonian cycle. The digraph D is k path-hamiltonian (resp., k path-traceable) if every path of length at most k can be extended to a hamiltonian cycle (resp., hamiltonian path). **THEOREM:** If D is nontrivial and of Ore-type (k), k > 0, then D is k path-hamiltonian. **COROLLARY:** If D is of Ore-type (k), k > 1, then D is (k + 1) path-traceable. **COROLLARY** [Woodall]: If D is nontrivial and of Ore-type (0), then D is hamiltonian. **COROLLARY:** If D is of Ore-type (1) and nontrivial, then D is both strongly hamiltonian and hamiltonian-connected.

A graph G (having neither loops nor multiple edges) of order p is of Ore-type (k) if \( \deg u + \deg v \geq p + k \) for every pair of distinct, non-adjacent vertices u and v of G. **COROLLARY** [Kapoor and Theckedath]: If G is of Ore-type (k), k > 1, then G is (k + 1) path-traceable. **COROLLARY** [Ore]: If G is of order at least 3 and of Ore-type (0), then G is hamiltonian. **COROLLARY** [Ore]: If G is of Ore-type (1), then G is hamiltonian-connected. **COROLLARY** [Kronk]: If G is of order at least 3 and of Ore-type (k), k > 0, then G is k path-hamiltonian. (Received November 11, 1974.)

**The March Meeting in Mobile, Alabama**

**March 20 – 21, 1975**

**Algebra & Theory of Numbers**


Let \( K_1 \) and \( K_2 \) be polyhedral cones in \( \mathbb{R}^n \) and \( \mathbb{R}^m \) respectively, with dual cones \( K_1^* \) and \( K_2^* \). Let \( \mathcal{T}(K_1, K_2) \) denote the set of \( m \times n \) real matrices that map \( K_1 \) into \( K_2 \). It was shown by Schneider and Vidyasagar (SIAM J. Numer. Anal. (7) 1970, 508-519) that \( \mathcal{T}(K_1, K_2) \) is a polyhedral cone in the vector space of real \( m \times n \) matrices. The following statements are shown to be equivalent. (1) \( K_1 \) or \( K_2 \) is simplicial. (2) \( \mathcal{T}(K_1, K_2) \) is generated by rank one matrices. (3) \( \mathcal{T}(K_1^*, K_2^*) \) is generated by rank one matrices. (4) \( (\mathcal{T}(K_1, K_2))^* = \mathcal{T}(K_1^*, K_2^*) \).

Suppose \( K_1 = K_2 = K \) is a full, pointed, polyhedral cone and A is a nonsingular positive operator on K. A necessary and sufficient condition for \( A^{-1} \) to belong to \( \mathcal{T}(K) \) is that A map extremal elements into extremal elements. In this case A is similar to a diagonal matrix. (Received December 16, 1974.)


The ordered pair \((P, \alpha)\) is a (partial) \((n, B)\)-PBD provided that \((P, \alpha)\) is a (partial) pairwise balanced design, \(|P| = n\), and if \( b \) is in \( \alpha \) then \( |b| \leq B \) is in B. The following result generalizes a result of C. C. Lindner and Alex Rosa for partial Steiner triple systems and my earlier generalization of their result for partial \((n, B)\)-PBD's where \(|B| = 1\) and \( \mathcal{P} \neq \mathcal{Q} \).

**Theorem.** If \( B \) is a set of positive integers \( \geq 2, |B| \geq 2, \) and \((P, p_1), (P, p_2), \ldots, (P, p_k)\) are k partial \((|P|, B)\)-PBD's then there exist \( k \) \((|P|, B)\)-PBD's, \((X, x_1), (X, x_2), \ldots, (X, x_k)\), with the property that for each \( i, (P, p_i) \) is embedded in \((X, x_i)\) and for \( i \neq j, p_i \cap p_j = x_i \cap x_j \).

A simple construction for pairs of isomorphic PBD's valuable in making such embeddings is also given. (Received January 20, 1975.)


We present a simple recursive algorithm and its supporting theorems for computing the maximal rectangles \( A \times B \) contained a given finite relation R. (Received January 24, 1975.)
The purpose of this paper is to extend the concept of injectors and Plotkin groups.

We shall discuss a variety of results about finite p-groups. For n greater than or equal to three, let T(n) be the hypergraph of all three-element subsets (or lines) of an n-element set. Define R(n) to be the maximum number of colors with which the lines of T(n) can be colored so that, for each vertex v and for each color r, v is adjacent to a line colored r. The following theorem is proven and used to find R(n). Theorem. Every T(R) is one-factorizable. (Received January 24, 1975.)
Ronald C. Linton, University of South Alabama, Mobile, Alabama 36688. Fully invariant subgroups of primary abelian groups.

F is fully invariant in G if φ(F) = F for all φ ∈ End(G). It is known that if F is fully invariant in the totally projective G, then F and G/F are also totally projective (see Abstract 709-A16, these Notices 20(1973), A-650). If λ is a limit ordinal cofinal with ω, then the reduced κ-primary group G is a Cλ-group if G/κκ'G is totally projective for all κ < λ. Theorem. Let F be fully invariant in the Cλ-group G of length λ. Then (1) G/F is also a Cλ-group if F is bounded, and (2) if F is unbounded, then F is a Cλ-group where μ denotes the length of F, and G/F is totally projective of length not exceeding the length of G. (Received November 4, 1974.) (Author introduced by Professor Richard Vinson.)

Analysis

Analysis

M. Altman, Louisiana State University, Baton Rouge, La. 70803. Contractor directions and directional contractions for solving equations.

Definition. Let X be an abstract set and P: X → Y a mapping of X into a Banach space Y and x ∈ X. Then Γx(P) ⊆ Y is a set of contractor directions for P at x if there is a positive q < 1 such that for each y ∈ Γx(P) there exists a positive number ℓ = ℓ(x,y) ≤ 1 and an element x ∈ X yielding ∥Px - ℓy∥ ≤ q ∥y∥. This concept is closely related to the notion of directional contractors (Studia Math., 46(1973), 101-110) and generalizes a concept of asymptotic directions by F.E. Browder(*), Bull. Amer. Math. Soc. 76(1970), 993-998. Sufficient conditions are presented for a nonlinear operator to be a mapping onto a Banach space. New proofs eliminate a geometric theorem by Browder(*) and a theorem by P.P. Zabreiko and M.A. Krasnoselskii, Funktion. Analiz i Ego Prilozhen., vol. 5, No. 3(1971), 42-44. A mapping F: X → X (Banach space) is a directional contraction if there is a positive q < 1 such that for each x, y ∈ X there is a 0 < ℓ(x,y) ≤ 1 yielding ∥F(x+ℓy) -Fx∥ ≤ qℓ∥y∥. If F is a directional contraction and P is closed, then P is onto, where PX = X - FX. (Received December 9, 1974.)

Mary R. Embry, Alan Lambert, University of North Carolina, Charlotte, N. C. 28223. Subnormal semi-groups of operators.

Let {At} be a strongly continuous semi-group representation of R+ on a Hilbert space X.

Theorem 1. The following are equivalent: a) each At is subnormal, b) there exists a Hilbert space Y containing X and a strongly continuous semi-group (Nt) of normal operators such that Nt|X = At and ∥Nt∥ = ∥At∥, c) \[ \int_0^a \int_0^a \langle A_t x_s, A_t x_r \rangle \, dr \, ds \geq 0 \] for every interval [0,a] and every continuous mapping x_t of [0,a] into X, d) \[ \int_0^a \int_0^a \langle A_{t+r} x_s, A_{t+r} x_r \rangle \, dr \, ds \geq 0 \] for every interval [0,a] and every continuous x_t of [0,a] into X. Theorem 2. Let {Nt} be the minimal normal extension of a strongly continuous subnormal semi-group (At). Let N and A be the infinitesimal generators of (Nt) and (At) respectively. Then N is the minimal normal extension of A, \( \sigma(N) \subseteq \sigma(A) \), and \( \sigma(N_t) \subseteq \sigma(A_t) \) for each t ≥ 0. (Received December 20, 1974.)

Charles W. McArthur, Florida State University, Tallahassee, Florida 32306. The order topology of an F*-lattice. Preliminary report.

In the following, F-lattice is used as defined in Yosida's "Functional analysis", New York, 1965, and order topology as in Luxemburg and Zaanen's "Riesz spaces". I, Amsterdam-London, 1971 and F*-lattice is an F-lattice in which a sequence (x_n) converges to x in the order topology if (x_n) star converges to x. The following are shown, Theorem 1. A necessary and sufficient condition for the

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topology of an \(F^*-\)lattice to be equivalent to its order topology is that the condition \(u_n \downarrow 0\) (i.e. \((u_n)\) is a monotone decreasing sequence whose infimum is \(0\)) implies \(\|u_n\| \to 0\) holds. Theorem 2. If \(X\) is an \(F^*-\)lattice whose order intervals are compact then \(u_n \downarrow 0\) implies \(\|u_n\| \to 0\) so its topology is equivalent to the order topology. It is a corollary of Theorem 1 that for all \(p > 0\) the pseudo norm topology and the order topology coincide for the spaces \(L^p[0,1]\) and \(L^p\) with their natural orders. This seems only to have been known for \(p \geq 1\). (Received January 6, 1975.)


A Banach space \(E\) contains \(\ell^\infty_n\) uniformly if for every \(\varepsilon > 0\) and every positive integer \(n\) there is an \(n\)-dimensional subspace \(H\) of \(E\) such that

\[
d(\ell^\infty_n, H) < 1 + \varepsilon, \quad \text{where} \quad d(\ell^\infty_n, H)\]

denotes the Banach-Mazur distance between \(\ell^\infty_n\) and \(H\) and where \(\ell^\infty_n\) denotes the vector space of all \(n\)-tuples \(\lambda = (\lambda_1, \ldots, \lambda_n)\) of scalars equipped with the norm \(\|\lambda\| = \sup |\lambda_k|\). For a Banach space \(E\) the following are equivalent: (a) \(E\) contains \(\ell^\infty_n\) uniformly.

(b) Every Schwartz space is embeddable into a suitable power \(E^1\) of \(E\).

(c) \(E\) equipped with the topology of uniform convergence on null sequences in the norm dual \(E'\) of \(E\) is a universal Schwartz space. This result can be used to answer certain questions concerning the embedding of Schwartz spaces into product spaces that were raised in [Mich. Math. J. 20 (1973), 39-44] and [Proc. Amer. Math. Soc. 37 (1973), 185-188]. (Received January 10, 1975.)

**721-B5** THURLOW A. COOK, University of Massachusetts, Amherst, MA 01002. The Geometry of Infinite Quantum Logics, Preliminary report.

This paper completes the work reported in these Notices [Vol. 22, no. 1, p. A-183, 1975]. Specifically, we prove that the propositions of a quantumlogic \(\pi\) with a strong set of states \(\Lambda\) are extreme points of the positive order unit interval in the order unit Banach space \(S^*\) dual to the base-normed state space \(S\), provided \(S\) has a special property called the generalized Jordan decomposition property. All the definitions of these concepts will be provided and will be related to the traditional Hilbert space formalism of quantum mechanics. (Received January 13, 1975.)


Suppose that \(E\) is a real (infinite dimensional) vector space. A family of subsets of \(E\) is called a bornology for \(E\) if it is an hereditary covering of \(E\) which contains the vector sum of any two members, the scalar multiples of any member, and the circled hull of the union of any finite subfamily. The theory of these structures has been studied most recently by H. Hogbe-Nlend and V. Moscatelli. We shall indicate the nature of their basic results and report on the direction of our own investigations. A connection will be made with our earlier work on finite dimensional weakly bounded sets [Abstract 717-B11, these Notices 21(1974), A-612]. (Received January 13, 1975.)


This paper is a study of second order nonhomogeneous differential systems involving a parameter with boundary conditions specified at two points. By means of a polar coordinate transformation for this system, the existence of eigenvalues is established. The results of this study extend earlier results of the author and those of Max Mason, in that self-adjointness of the problem is not necessary to insure a solution. (Received January 22, 1975.)
We will discuss some recent results on James' Space done individually and collectively by P. Casazza, B. L. Lin, and B. Lohman. The authors have undertaken a careful study of the structure of this interesting and important example from Banach space theory. Many results have already been obtained. For example, it has been shown that if \( \{e_n\} \) is the unit vector basis of \( J \) and if \( J^{nd} \) denotes the closed linear span of \( \{e_1, e_2, \ldots, e_n\} \), then every constant coefficient block basic sequence of \( J \) spans a complemented subspace of \( J \) which is isomorphic to either \( J, J_1, 2, \infty \) for some sequence of natural numbers \( \{n_i\} \). It follows that \( J \) contains a continuum of mutually non-equivalent unconditional basic sequences each of which spans a complemented subspace of \( J \). Many other results will be discussed.

(Received January 23, 1975.)

We first discuss a convergence theorem for certain sequences of bounded functions on an abstract set. This theorem contains Lebesgue's bounded convergence theorem for continuous functions on a pseudocompact space and Rainwater's theorem on weak convergence in a normed linear space as well as other more technical results. Secondly we discuss some minimax theorems and their converses and the connection between these results and R. C. James' sup criterion for a set in a locally convex space to be weakly compact. (Received January 24, 1975.

An operator \( S \) in the Hilbert space \( K \) is \( 0 \)-universal if and only if \( 0 \) is a set of operators in a Hilbert space \( H \) and, for each member \( A \) of \( 0 \), there is a linear homeomorphism \( \sigma \) from \( H \) into \( K \) such that \( \sigma A \sigma^{-1} = S \sigma (H) \). Suppose \( H \) is a Hilbert space with norm \( \| \cdot \| \), let \( K = \{ \{ x_p \}_{p=1}^{\infty} : \sum_{p=1}^{\infty} \| x_p \| \leq \infty \} \) (with the usual inner product), and let \( c \) be a real number with \( 0 < c < 1 \). If \( c = 0 \), let \( O_c \) consist of all operators in \( H \) of operator norm less than \( 1 \), and, if \( 0 < c \), let \( O_c \) consist of those invertible members \( A \) of \( O_o \) such that \( A^{-1} \) has operator norm less than \( c^{-1} \). Define \( S_c \) in \( K \) by letting \( [S_c x_p]_p = \sum_{p=1}^{\infty} x_p \) or \( x_p \) when \( p \geq 0 \) or \( p < 0 \) respectively. Then \( S_c \) is \( 0 \)-universal, and \( S_c^{-1} \) is \( 0 \)-universal if \( 0 < c \) and \( O^{-1}_c = \{ A^{-1} : A \) belongs to \( O_c \} \). This generalizes a result of G.C. Rota [Comm. Pure Appl. Math., Vol. 33, pp. 469-472].

(Received January 27, 1975.)

Continued fractions of two types are considered: (A) \( a_1 + a_2/1 + \ldots \) and (B) \( 1/b_1 + 1/b_2 + \ldots \). If \( E \) is a set of complex numbers, the symbol \( V_A(E) \), [the symbol \( V_B(E) \) denotes the set of all values of finite continued fractions of type (A) [type (B)] with each \( a_p \) [each \( b_p \)] in \( E \). Theorem. Suppose that either (i) \( E \) is a set of complex numbers such that \( -1 \) is not a limit point of the set of all possible sums \( x + y \) with \( x \) and \( y \) in \( V_A(E) \), [all possible products \( xy \) with \( x \) and \( y \) in \( V_B(E) \) or else (ii) \( E \) is a set of real numbers. Then \( E \) is a convergence region for continued fractions of type (A) [type (B)] if and only if \( V_A(E) \), [\( V_B(E) \) is bounded. Applications, improvements, and extensions of these results are discussed. (Received January 27, 1975.)
Bourbaki and Grothendieck gave curiously similar examples of a special class of Banach spaces containing dense, barrelled, non-Baire subspaces: in normed spaces, Baire $\not\subset$ barrelled. A very simple argument (recent article in Math. Ann., also joint paper with Prof. Wilansky, to appear) shows that if $E$ is any (infinite-dimensional) Banach space, a dense, barrelled, non-Baire subspace is spanned by the set $A$ of all $x$ in $E$ such that $f_i(x)$ is a rational scalar for all $i$, where $(x_i,f_i)$ is any biorthogonal sequence in $E$.

The joint effort with Prof. Wilansky shows that $E$ has an (infinite-dimensional) separable quotient (an unconfirmed conjecture of long standing) if and only if $E$ has a dense, non-barrelled subspace. To produce a dense, non-Baire subspace was so easy (sigh)!! (Received January 28, 1975.)

Applied Mathematics

721-C1 M. C. GOODALL, University of Alabama, Birmingham, Alabama 35294. Church's thesis and the foundations of relativistic quantum mechanics.

Quantum theory contains a logical flaw preventing discussion of existence, rather than interaction, of matter (interaction involves contradiction however, e.g. Haag's theorem). Central issue here is the Born statistical postulate, acceptable neither in the objective (Copenhagen) interpretation (because of the above) nor in the subjective (hidden variable) form, since this implies counterfactual observables. To evade this dilemma a physical version of Church's thesis, undecidable -- unobservable, is proposed, implying the existence of an (undecidable) object language $\Omega$ over which current theory operates as a metalanguage $M$. The 2-step nilpotent $H(Z)$, Heisenberg-Weyl group restricted to $Z$, is a candidate for $\Omega$ in the nonrelativistic case with $M \sim Sp(6, Z)$, since it is known from work on Hilbert's 10th problem that $\text{Th}(H, Z)$ is undecidable, therefore finite words in $H(Z)$ (histories in the sense of Feynman) generated by an universal algorithm are r.e. of degree $0'$. The Born postulate is then a consequence of summing over unobservable elements of this set. However, only in the relativistic case, dominated by the Study-Cartan principle of triality, can essentially new consequences be derived from this representation. With $M \sim F_4$, $\Omega = H^*(\hat{K})$, a 3-step nilpotent extension of $H(K)$, where $\hat{K}$ is an infinite abelian extension of $K$. By Golod-Shafarevich's theorem on class field towers there is uniquely a smallest $K(D^3)$ permitting such extension; $\hat{K}$ then, rather than $H$, is the field of definition of observed physical quantities over which observables appear as recursively enumerable sets. (Received January 24, 1975.)


An efficient numerical technique for approximating the solution to a system of nonlinear integral equations arising in the epidemic theory described by Hoppensteadt and Waltman [Math. Biosci. 12 (1971), 133-145] is developed. The algorithm is based upon the product-type quadrature formulas of Boland and Duris [BIT 11(1971), 139-158]. It is shown that for a finite interval $[0,a]$ and for $\epsilon > 0$ there exists a partition $P$ of $[0,a]$, with norm less than $\epsilon$, and corresponding polygons (i.e. polygons with nodes the points of $P$) which satisfy the system of equations at the points of $P$. The sequence of polygons generated by letting $\epsilon$ tend to zero is shown to converge to the solution of the system of equations. Implementation of this method on a digital computer is discussed and several examples given. (Received January 27, 1975.)


- Considering the astronomical $N$-body problem, and in particular the three-body problem and its specializations, a survey will be given of important results achieved during the last decade or so. These
are results on periodic and almost periodic motions, stability, asymptotic behavior near collisions or for large time, regularization and perturbation methods, etc. Basic ideas in this regard and advances in mathematical techniques will be discussed and some open problems pointed out. (Received January 27, 1975.)

**Geometry**

721-D1 M. O. GONZALEZ, University of Alabama, University, Alabama 35486

On the regularity of the image of an ordinary point of a surface under a differentiable mapping.

Let $S$ be a surface in $E^3$ defined by $\mathbf{x} = \mathbf{x}(u,v) = x^i(u,v)e_i$, where the summation convention is used, $(u,v) \in D =$ parametric domain in $E^2$, $\{e_1, e_2, e_3\}$ is the standard orthonormal basis in $E^3$, and the $x^i(u,v)$, $i = 1, 2, 3$, are differentiable in $D$. Let $\mathbf{F} = (f^1, f^2, f^3)$ be a vector function defined in some region $\Omega$ containing $S$ with differentiable components $f^i$ at each point of $S$, and let $\mathbf{F}(S) = S': \mathbf{y} = \mathbf{F}(\mathbf{x}) = y^i(u,v)e_i$. With the notations $J = |\partial f^i/\partial x^j|$ for the Jacobian of $\mathbf{F}$, and $\nabla f^i$ for the gradient vectors of the components of $\mathbf{F}$, the following proposition holds. **Theorem.** Suppose that $\mathbf{n} = \mathbf{n}_u \times \mathbf{n}_v \neq \mathbf{0}$ at $p \in S$, and let $\mathbf{n}' = \mathbf{n}_u \times \mathbf{n}_v$ at $p' = F(p)$. Then (1) $J \neq 0$ implies $\mathbf{n}' \neq \mathbf{0}$, (2) $J = 0$, rank$\left(3f^i/3x^j\right) = 2$, and $\mathbf{n} \neq \alpha_1 \nabla f^i$ (for some scalars $\alpha_i$) imply $\mathbf{n}' \neq \mathbf{0}$ also, (3) $J = 0$, rank$\left(3f^i/3x^j\right) = 2$, and $\mathbf{n} = \alpha_1 \nabla f^i$ imply $\mathbf{n}' = \mathbf{0}$, and (4) $J = 0$ and rank$\left(3f^i/3x^j\right) < 2$ imply $\mathbf{n}' = \mathbf{0}$. Thus, $p'$ is an ordinary point if either $J \neq 0$ or $J = 0$ with rank = 2 and $\mathbf{n}$ does not lie in the plane of the gradient vectors $\nabla f^i$. (Received January 9, 1975.)

**Logic and Foundations**

*721-E1 JOSEPH BARBACK, SUNY College at Buffalo, Buffalo, New York 14222.

On the intersection of regressive sets.

The following result was proved in the paper: J.C.E.

Dekker, Infinite Series of Isols, AMS - Proceedings of Symposia in Pure Mathematics, volume 5 (1962). **Theorem.** (Dekker) Let $A$ and $B$ be regressive sets that are recursively equivalent. Then their intersection $A \cap B$ will be either a finite set or an infinite regressive set. In this announcement we would like to give one generalization of this theorem. **Theorem.** Let $a$ and $b$ be regressive isols and let $A \in a$ and $B \in b$. If $a + b$ is also a regressive isol then $A \cap B$ will be either a finite set or an infinite regressive set. (Received January 27, 1975.) (Author introduced by Kyung Barback.)

**Topology**

*721-G1 JAMES KEESSLING, University of Florida, Gainesville, Florida 32611.

Algebraic Invariants in Shape Theory.

Shape theory could properly be called Čech homotopy theory. In homotopy theory there are algebraic invariants: homology, cohomology, and the homotopy groups. In shape theory the analogous invariants are the Čech homology and Čech cohomology groups and in place of the homotopy groups one uses the shape groups. The Čech homology and shape groups are particularly useful for movable compacta. More recently it has been found that the homology pro-groups and homotopy pro-groups are useful for nonmovable spaces. In this report the more important theorems and examples in this area of shape theory are stated and discussed. The author also presents his recent work on the structure of the Čech cohomology of movable and n-movable spaces. 

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THEOREM: Let X be a movable space, then there is a subgroup $E \subset H^n(X)$ such that $E$ is the union of all the algebraically compact subgroups of $H^n(X)$. $H^n(X)/E$ is an $S^1$-free abelian group. The structure of the Čech cohomology of an $LC^n$ paracompactum has also been determined. (Received December 20, 1974.)

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Local Behavior and the Vietoris and Whitehead Theorems in Shape Theory

Theorem: Any $LC^n$ paracompactum X of dim $n$ is shape dominated by a polyhedron of dim $n$. (Such a space X need not be an ANR.) Movability is generalized to arbitrary topological spaces as an extension of uniform movability and is strictly stronger than movability for ANR-systems. Theorem: Every $LC^{n-1}$ paracompactum of dim $n$ is movable. The strength of this definition is illustrated by the theorem: If $(X,x)$ is a movable pointed continuum with trivial shape groups then $(X,x)$ has trivial shape. Theorem: For $LC^n$ paracompacta the shape groups and the homotopy groups are naturally isomorphic. As an application of this we have the theorem: For metric spaces a proper map $f : X \to Y$ such that $f^{-1}(y)$ is approximately $k$-connected $(0 < k < n+1)$ for all $y \in Y$ induces for each $x \in X$ an isomorphism of the $n$th shape group of $(X,x)$ with that of $(Y,f(x))$. Finally we construct a movable continuum $X*$ (based on the Kahn space $X$, one of its ANR-sequences, and the map recently described by J.L. Taylor of $X$ onto the Hilbert Cube) which can be mapped onto a movable continuum of different shape by a Vietoris (or CE) map. (Received December 23, 1974.)

DANIEL R. MCMILLAN, JR., University of Wisconsin, Madison, Wisconsin 53706. Movability in $\mathbb{R}$-manifolds.

K. Borsuk has introduced and studied the useful concept of movability (a shape property) for compacta. Examples of movable compacta include compact ANR's and compacta embeddable in 2-manifolds. The author will discuss some questions and results related to the general problem of detecting movable continua in Euclidean 3-space, $E^3$. In particular, when is a locally connected continuum in $E^3$ movable? Borsuk has given an example of a locally connected, non-movable continuum in $E^3$. His example separates $E^3$ into two components. He has also raised the question that is answered by Theorem. There exists a locally connected, non-movable continuum $X$ in $E^3$ such that $E^3 - X$ is connected. (Received January 2, 1975.)


For convenience $X'$, $X$, and $Y$ in the statements here are assumed to be metric compacta. Analogous statements hold for $Z$-sets in Hilbert cube manifolds. $Q$ is the Hilbert cube $[-1,1]^\infty$ and $s = (-1,1)$. A map $f : X' \to X$ is an hereditary shape equivalence (h.s.e.), if for every closed subset $A$ of $X$ the map $f \mid f^{-1}A : f^{-1}A + A$ is a shape equivalence. Recall that a near-homeomorphism is a uniform limit of homeomorphisms. Theorem 1. Let $X'$ and $X$ be subsets of $s$. A map $f : X' \to X$ is an h.s.e. if and only if there is a near-homeomorphism $g : Q + Q$ such that (1) $g$ maps $Q \setminus X'$ homeomorphically onto $Q \setminus X$ and (2) $g(u) = f(u)$ for all $u \in X'$.

Corollary. If $f : X \to Y$ is an arbitrary map between subsets of $s$, then there is a near-homeomorphism $g : Q + Q$ such that $g(x) = f(x)$ for all $x \in X$. Theorem 2. A map $f : X' \to X$ such that $f^{-1}(x)$ has trivial shape for all $x \in X$ is an h.s.e. proved any one of the following conditions is satisfied: (1) $X'$ and $X$ are ANR's, (2) $X$ is countable-dimensional, (3) the set of all points $x$ of $X$ such that $f^{-1}(x)$ is non-degenerate is contained in a subset (of $X$) which has strong transfinite dimension. These theorems transform to statements for upper semi-continuous decompositions of $Q$. (Received January 6, 1975.) (Author introduced by Professor Jack Segal.)

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Compact ANR's have finite homotopy type, Preliminary Report.

Every locally compact ANR (for metric spaces), $A$, admits a cell-like mapping from a Hilbert cube manifold. A corollary is that if $A$ is compact, then it is homotopy-equivalent to a compact polyhedron.

(Received January 7, 1975.) (Author introduced by Professor Joe Ball.)

Various conditions of contractibility and extensibility at $\infty$ for locally compact metric spaces are studied. These are shown to be equivalent if the space under consideration is an absolute neighborhood retract (ANR) and an ANR satisfying them is called docile at $\infty$.

Docility at $\infty$ is a hereditary proper homotopy invariant. The ANR $X$ is docile at $\infty$ if and only if $FX$ (the Freudenthal compactification of $X$) is an ANR and $FX-X$ is unstable in $FX$; the inclusion of $X$ into $FX$ is a homotopy equivalence. These facts are applied to obtain results concerning the absolute proper retracts (these NOTICES 21(1974), A-212, Abstract 711-54-13). (Received January 13, 1975.)

We say that a topological space is $\sigma$-connected provided it is connected and cannot be decomposed into countably many mutually separated non-empty subsets. A (metric) continuum $X$ is called finitely Suslinian provided every infinite sequence of mutually disjoint subcontinua of $X$ is a null-sequence. Theorem 1: A hereditarily locally connected continuum $X$ is finitely Suslinian if and only if every connected subset of $X$ is $\sigma$-connected.

Theorem 2: A continuum $X$ is finitely Suslinian if and only if every connected $F_\sigma$-subset of $X$ is arcwise connected. Some other characterizations of finitely Suslinian continua are also discussed. (Received January 13, 1975.)

Shapes of finite dimensional continua have shape irreducible representatives.

A compactum $X$ is shape irreducible if no proper subcompactum of $X$ has the same shape as $X$. The following question of K. Borsuk is considered: Do all shapes of continua have shape irreducible representatives? Theorem. If $X$ is a continuum of finite shape dimension, $Sd X = n < \infty$, then there is a continuum $Y$ with $Sh Y = Sh X$ and such that every proper closed subset of $Y$ is of shape dimension less than $n$. This provides a partial solution of Borsuk's problem. In the infinite dimensional case, the problem remains open. (Received January 13, 1975.)

Uniform free topological groups.

Let $U$ denote the category of pointed uniform spaces and base-point preserving uniformly continuous...
functions, and let \( \mathcal{G} \) denote the category of topological groups and continuous group homomorphisms. By assigning each topological group its two-sided uniformity and taking its identity element as base-point, \( \mathcal{G} \) can be considered to be a subcategory of \( \mathcal{U} \). The forgetful functor from \( \mathcal{G} \) to \( \mathcal{U} \) has a coadjoint called the uniform free topological group (u-free group) functor. The Samuel compactification of a uniform space plays a central role in the investigation of the theory of u-free groups, which differs from the theory of free topological groups in several interesting ways. For example, the u-free group \( F\mu X \) over a discrete uniform space \( \mu \) need not be discrete; and if \( \alpha: \nu Y \to \mu X \) is a uniform embedding, then the induced morphism \( \alpha: F\nu Y \to F\mu X \) is a (uniform) isomorphism onto \( \alpha(F\nu Y) \).

(Received January 17, 1975.)


Consider a manifold \( M^n \), where \( n \) is a positive integer or \( n = \infty \), and a compact set \( X \) in the interior of \( M^n \). When \( n = \infty \), D. W. Henderson has shown the equivalence of the following three statements: (a) \( M^n/X \) is homeomorphic to \( M^n \); (b) \( M^n/X \) is a manifold; (c) \( X \) has the shape of a point. The implications (a) \( \Rightarrow \) (b) and (a) \( \Rightarrow \) (c) hold in the finite-dimensional case as well, but the others fail in general. The failure of (c) \( \Rightarrow \) (a) or (b) is due to the existence of wild embeddings. The failure of (b) \( \Rightarrow \) (a) is easier to see: Let \( M^n = S^k \times S^{n-k} \) and \( X = S^k \cup S^{n-k} \); plainly \( M^n/X \sim S^n \neq M^n \).

Following the philosophy of Chapman and Geoghegan and Summerhill, among others, we examine the implication (b) \( \Rightarrow \) (a) with regard to the dimension of \( X \). Note that \( 2 \dim X \geq n \) in the above example. We show that (b) \( \Rightarrow \) (a) assuming \( 2 \dim X < n \neq 4 \). An essential step in the proof is to show that the usual cellularity criterion applies assuming that \( X \) is acyclic and that \( \dim X + 2 \leq n \neq 4 \). (Received January 17, 1975.)

*721-G12 DAVID A. EDWARDS, State University of New York, Binghamton, N.Y., 13901. On Homotopy Theories of Pro-Spaces: Strong versus Weak.

A strong homotopy theory of Pro-Spaces \( \text{Ho}(\text{Pro-SS}) \) is described and compared with the weak Artin-Mazur theory of \( \text{Ho} \)(SS). Applications to shape theory are given. (Received January 20, 1975.)

*721-G13 DAVID A. EDWARDS, State University of New York, Binghamton, N.Y., 13901, and ROSS GEOGHEGAN, University of Georgia, Athens, GA. 30602. When has a space the shape of a CW complex?

Shape is in the sense of Mardesic. If "CW complex" is replaced by ANR, and "space" is replaced by "metric space" the theorems hold in Fox's theory.

Theorem 1 (pointed or unpointed): A space \( Z \) is shape equivalent to a CW complex iff \( Z \) is shape dominated by a CW complex; a compact space is shape equivalent to a CW complex iff \( Z \) is shape dominated by a finite complex. Theorem 2: A "finite-dimensional" pointed connected space \( Z \) is pointed shape equivalent to a CW complex iff each pro-group \( \pi_\ast(Z) \) is dominated in the category pro-Groups by a group. Corollary 3: If a compact connected space \( Z \) is shape equivalent to a CW complex and \( \pi_1(Z,z) \) is its first (inverse limit) shape group, then there is a Wall obstruction in \( \mathcal{K}^0(\pi_1(Z,z)) \) which vanishes iff \( Z \) is shape equivalent to a finite complex; similarly in the pointed case. (Received January 20, 1975.)

*721-014 JACK B. BROWN, Auburn University, Auburn, Alabama 36830. On the Baire order of concentrated spaces and \( L_1 \) spaces.

"K40" refers to §40 of Kuratowski's Topology, where certain "singular" metric space properties are considered. It is shown in K40 that (1) \( \sigma \to \lambda \) always 1st category \( \Rightarrow \) totally imperfect and (2) \( \nu \to \text{concentrated} \Rightarrow \text{C} \to \text{C} \Rightarrow \text{totally imperfect} \). An \( L_1 \) space (Fund. Math. 84 (1974) pp. A-344
(35-45) is one which can be decomposed into countably many \( \mathfrak{v} \) subspaces, and this property fits into (2) as follows: \( \mathfrak{v} \to L_1 \to \) concentrated. \( \sigma \) spaces have "Baire order" 0 or 1, but Mauldin has shown (Notices AMS 21 \#3 p. A-376) that CH implies the existence of a \( \mathfrak{y} \) subspace of the reals with Baire order \( \omega_1 \). \( \mathfrak{v} \) spaces are known to have Baire order 0, 1, or 2 (see VII Th. 2 of K40), and \( L_1 \) spaces also have Baire order 0, 1, or 2 because of the following. Theorem 1: Every set which has the Baire property in the restricted sense in an \( L_1 \) space \( X \) is a \( G_{60} \) in \( X \). On the other hand, the following is true. Theorem 2: CH implies the existence of a subspace \( X \) of the reals such that \( X \) is concentrated about the rationals but in which there exists an \( F_{60} \) set which is not a \( G_{60} \) set. (Received January 20, 1975.)

W. F. LAMARTIN, University of New Orleans, New Orleans, La. 70122. Epics in the category of \( T_2 \) k-groups need not have dense range.

In a category \( C \) an epic is a morphism \( f : X \rightarrow Y \) such that for each pair of morphisms \( g, h : Y \rightarrow Z \) in \( C \), whenever \( gf = hf \), then \( g = h \). In most of the familiar categories arising from topological algebra, epics are precisely those morphisms with dense range. This, however, is not the case for either \( T_2 \) k-groups or \( T_2 \) abelian k-groups. A counterexample demonstrating this is provided as follows. An epic \( X \rightarrow Y \) in the category of functionally \( T_2 \) k-spaces which has non-dense range is given; for the free k-group \( FX \), it is then shown that \( FX \rightarrow FY \) is an epic in \( T_2 \) k-groups with non-dense range. This method does not carry over to \( T_2 \) topological groups, where, to the best of our knowledge, epics are still unidentified. (Received January 20, 1975.)

T. BENNY RUSHING, University of Utah, Salt Lake City, Utah 84112. Embeddings of Shape Classes of Compacta. Preliminary report.

The talk will begin with a brief survey of various results in the area of embeddings of shape classes and will conclude with an outline of the proof of the trivial range theorem stated below. Let \( X \subseteq S^n \) be a compactum whose dimension \( k \) is in the trivial range \( w.r.t. \) \( n \), i.e., \( 2k + 2 \leq n \). Then, \( X \) is homotopically stable in \( S^n \) if given a neighborhood \( U \) of \( X \), there is a neighborhood \( V \subseteq U \) of \( X \) such that \( \pi_i(V, V - X) = 0 \) for \( 0 \leq i \leq \left[ \frac{n}{2} \right] \). (It follows from a result of Bryant [Proc. Amer. Math. Soc., 23(1969), 46-51] that if \( S^n - X \) is \( 1\)-ULC, then \( X \) is homotopically stable. Clearly any cellular \( X \subseteq S^n \) is also homotopically stable). Theorem. Let \( X, Y \subseteq S^n, n \geq 5 \), be homotopically stable compacta whose dimensions are in the trivial range \( w.r.t. \) \( n \). Then, \( Sh(X) = Sh(Y) \) if and only if \( S^n - X \approx S^n - Y \). Corollary. (Geoghegan and Summerhill) Let \( X, Y \subseteq S^n, n \geq 5 \), be compacta whose dimensions are in the trivial range \( w.r.t. \) \( n \) such that \( S^n - X \) and \( S^n - Y \) are 1-ULC. Then, \( Sh(X) = Sh(Y) \) if and only if \( S^n - X \approx S^n - Y \). (Received January 23, 1975.)

S. A. STRICKLEN JR, Southern Technical Institute, Marietta, GA 30060, Closed Mappings Of Nowhere Locally Compact Metric Spaces

We present a method of construction which allows us to construct many examples of nowhere first countable spaces which are images of metric spaces under closed mappings, and which allows us to prove (with a certain natural restriction on the mapping) that there is a closed mapping from a metric space \( M \) onto a space with a dense nowhere first countable subspace if and only if \( M \) is nowhere locally compact. (Received January 24, 1975.)
George M. Reed, Ohio University, Athens, Ohio 45701, On subspaces of separable first countable $T_2$-spaces

In this paper, the author considers embedding questions related to the work of J. W. Ott [Proc. Auburn Univ. Top. Conf. (1969)], J. W. Green [Gen. Top. 4 (1974), 297-313], and G. M. Reed [Gen. Top. 4 (1974), 255-267]. The following main results are established. (1) (M.A.) Each $T_2$-space of card. $\leq \aleph_1$ can be embedded in a separable $T_2$-space. (2) Each connected Souslin space can be embedded in a compact, separable first countable $T_2$-space. (3) (CH) Each Souslin space can be embedded in a compact, separable first countable $T_2$-space. (4) There exists a CCC, nonseparable Moore space which can be embedded in a separable Moore space. (5) Each normal Moore space of card. $\leq \aleph_1$ can be embedded in a separable developable $T_2$-space. (6) Each locally compact, first countable $T_2$-space of weight $\leq \aleph_1$ can be embedded in a locally compact, separable Moore space. (7) Each locally compact Moore space of card. $\aleph_1$ can be embedded in a locally compact, pseudo-compact Moore space. (9) (CH) Each locally compact Moore space of card. $\leq \aleph_1$ can be embedded in a locally compact, pseudo-compact (hence, separable and Moore-closed) Moore space. Results (2), (3), and (6) are obtained with techniques used by S. P. Franklin and M. Rajagopalan [Trans. Amer. Math. Soc. 155 (1971), 305-314]. (Received January 27, 1975.)

CARL H. STUCKE, Emory University, Atlanta, Georgia 30322. Embedding 1-dimensional Continua in the Product of an Arc and a Simple Triod, Preliminary report.

R. H. Bing [Embedding Circle-Like Continua in the Plane, Canad. J. Math. 14 (1962), 113-128] has shown that each circle-like continuum can be embedded in the cartesian product of a triod and an arc. A continuum $T$ is said to be a simple triod iff it is the sum of three arcs $O_X$, $O_Y$ and $O_Z$ such that no two of them have any point in common except $O$. Theorem. If $M$ is a 1-dimensional continuum such that the set of all points of $M$ at which $M$ fails to be locally planar has dimension less than 1, then $M$ can be embedded in the product of an arc and a simple triod. (Received January 28, 1975.) (Author introduced by J. W. Rogers, Jr.)

A note on weakly uniform bases, Preliminary report.

A base $B$ for a space $X$ is said to be uniform (resp., weakly uniform) if, for each $x \in X$ and each infinite subcollection $H$ of $B$, each member of which contains $x$, $H$ is a local base for $x$ (resp., $\bigcap H = \{x\}$). R. W. Heath has shown in [Canad. J. Math. 16 (1964), 763-770] that a $T_3$-space has a uniform base if and only if it is a metacompact Moore space. In [These Notices 21 (1974), A620-A621], R. W. Heath and W. F. Lindgren have introduced the study of weakly uniform bases. They raised the question as to whether each first countable $T_3$-space with a weakly uniform base has a point-countable base. With respect to this question, they presented a certain class $C$ of Moore spaces of card. = $\aleph_1$ with weakly uniform bases, and they asked if there was a non-metrizable member of $C$. In this paper, the authors show that the class $C$ does contain non-metrizable spaces. However, the following result is also established. Theorem. Each first countable $T_3$-space with a weakly uniform base and with no more than $\aleph_1$ isolated points has a point-countable base. (Received January 28, 1975.)

Suppose $X$ is a Euclidian space and $B$ is a base of connected open sets for $X$. We say that $f: X \to Y$ is a Darboux (B) function (resp., connectivity (B) function) if $f(cl(U))$ (resp. $f(cl(U)) = \{(x, f(x)): x \in cl(U)\}$) is connected for each $U \in B$. We show that, under suitable restrictions on $B$, a Baire class 1 function $f: X \to Y$ is connectivity (B) if and only if $f$ is Darboux (B). Several examples are given to show that our restrictions on $B$ cannot be dropped. (Received January 23, 1975.)
*721-G22 R. C. LÄCHER, Florida State University, Tallahassee, Florida 32306. 
Cell-like mappings and their generalizations.

- If P is a property of spaces (or subspaces), let the phrase "P map" mean a mapping each point-inverse of which has property P. Thus cell-like maps are those whose point-inverses are cell-like spaces (as subspaces of the domain). A space is cell-like if it is homeomorphic to a cellular subset of some manifold. This definition was given in 1968, at which time I began to study proper, cell-like maps between euclidean neighborhood retracts (ENR's). At the time, I pointed out that such maps form a category which includes proper, surjective, contractible maps between polyhedra and proper, cellular maps from a manifold to an ENR. S. Smale had previously studied contractible maps between ANR's, proving a Vietoris-like theorem; M. Cohen had shown that PL contractible maps between finite polyhedra are simple homotopy equivalences; and of course proper, cellular maps on manifolds had been studied extensively (often under the label "point-like decompositions"). This unification seemed worthwhile at the time because of the equivalence of cell-like (for proper, surjective maps between ENR's) and a condition studied by D. Sullivan in connection with the Hauptvermutung: the restriction to the inverse of any open set is a proper homotopy equivalence. Using Sullivan's work it followed that a cell-like map between PL manifolds is often homotopic to a PL homeomorphism. In 1971, L. C. Siebenmann identified the red herring nature of PL in the above sentence: The set of cell-like maps \( M \rightarrow N \) (where M and N are closed n-manifolds, \( n \neq 4 \)) is precisely the closure of the set of homeomorphisms \( M \rightarrow N \) in the space of maps \( M \rightarrow N \). (The case \( n = 3 \) requires also that M contains no fake cubes and was done earlier by S. Armentrout. The case \( n = 2 \) was done by R.L. Moore in 1925.)

We will conclude with a discussion of the infinite-dimensional versions of these topics, including J. West's solution of the Borsuk conjecture and J. Taylor's example of a non-ANR which is the cell-like image of the Hilbert cube. (Received February 3, 1975.)

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Algebra & Theory of Numbers

*722-A1 JOSEPH ARKIN, 197 Old Nyack Turnpike, Spring Valley, New York 10977 and PAUL SMITH, University of Victoria, British Columbia, Canada.
Treble-magic systems in a Latin 3-cube of order eight.

In this paper we superimpose three Latin cubes of order 8 to form an orthogonal Latin 3-cube. That is, three Latin cubes of order 8 are orthogonal if, when superimposed, each ordered triple will occur. In addition to orthogonality we construct into the cube certain treble-magic systems. In this paper we define a treble-magic system as sets of integers that have the same sum of kth powers for \( k = 1, 2 \) and 3.
(Received January 16, 1975.)

Schützenberger groups of \( \mathbb{N} \)-classes containing finite-to-one functions.
Preliminary report.

A topological space is reversible if each continuous bijection of the space is a homeomorphism and it is irretractable if it is not homeomorphic to any proper subspace which is a retract of the space. Reversible irretractable spaces include all Euclidean \( \mathbb{N} \)-spaces.

THEOREM: Let \( X \) be any reversible irretractable space and let \( S(X) \) denote the semigroup, under composition, of all continuous selfmaps of \( X \). Let \( f \) be any finite-to-one function in \( S(X) \) which maps \( X \) onto \( X \) but does not map any proper subspace onto \( X \) which is the range of an element in \( S(X) \). Then the Schützenberger group of the \( \mathbb{N} \)-class of \( S(X) \) contain
Some geometrical properties of the set $\Omega_n$ of $n \times n$ non-negative doubly stochastic matrices are investigated. If $\mathfrak{F}$ is a face of $\Omega_n$, then $\mathfrak{F}$ corresponds to an $n \times n$ $(0,1)$-matrix $A$ where per $A$ is the number of vertices of $\mathfrak{F}$. If $A$ is fully indecomposable, then $\dim \mathfrak{F}$ equals $\sigma(A) - 2n + 1$ where $\sigma(A)$ is the number of 1's in $A$. The only 2-dimensional faces of $\Omega_n$ are triangles and rectangles. For $n \geq 6$, $\Omega_n$ has 4 types of 3-dimensional faces. The facets of the faces of $\Omega_n$ are characterized. Faces of $\Omega_n$ which are simplices are determined. If $\mathfrak{F}$ is a face of $\Omega_n$, which is 2-neighborly but not a simplex, then $\mathfrak{F}$ has dimension 4 and 6 vertices. All $k$-dimensional faces with $k + 2$ vertices are determined. The maximum number of vertices of a $k$-dimensional face is $2^k$. All $k$-dimensional faces with at least $2^{k-1} + 1$ vertices are determined. (Received January 22, 1975.)


Theorem 1. Let $L$ be a complete distributive lattice such that for every $x > 0$ there are $x_1$ and $x_2$ such that $x_1 > 0$, $x_2 > 0$ and $x_1 \wedge x_2 = 0$. Then there is an ultrafilter $\mathcal{F}$ of subsets of $L$ such that for every $F \in \mathcal{F}$ we have $\bigvee F = 1$ and $\bigwedge F = 0$. Corollary 2. Let $B$ be a complete Boolean algebra such that every ultrafilter of subsets of $B$ order converges. Then $B$ is isomorphic to a Boolean algebra of all subsets of a set. Corollary 3 (I. Kaplansky, D Strauss) Every compact Hausdorff Boolean algebra is isomorphic to a $2^X$ with product topology. Corollary 4. Let $(X, J)$ be a Hausdorff topological space without isolated points. Then there is an ultrafilter $\mathcal{F}$ of subsets of $J$ such that for every $F \in \mathcal{F}$ we have $\bigvee F = X$ and $\bigwedge F = \emptyset$. (Received January 22, 1975.)

*722-A5* EDWARD P. SHAUGHNESSY, Lafayette College, Easton, Pennsylvania 18042, Automorphism groups of the $(k + 1, (k + 1)/2)$ extended QR codes for $k$ and $(k - 1)/2$ prime, $3 < k < 4079$, Preliminary report.

The automorphism group $\mathcal{A}$ of the $(k + 1, (k + 1)/2)$ extended quadratic residue code over GF($q$), where $k$ and $(k - 1)/2$ are prime and $5 \leq k \leq 4079$, is $\text{PSL}_2(k)$ except in the following four cases: $k = 5$, $q = 4$ and $\mathcal{A} = \text{Alt}(6)$; $k = 7$, $q = 2$ and $\mathcal{A} = \text{Alt}(3)$; $k = 11$, $q = 3$ and $\mathcal{A} = \text{M}_{12}$; $k = 23$, $q = 2$ and $\mathcal{A} = \text{M}_{24}$. (Received January 24, 1975.)

722-A6 FRANCIS P. CALLAHAN, Pennsylvania State University, King-of-Prussia, Pennsylvania 19406, Skew symmetric biderivations of a two-generator purely inseparable field.

Notation. Let $x$ and $y$ generate purely inseparable field $F$ (having groundfield $k$; char $k = p$), and let $f = \sum_{i,j} a_{ij} x^i y^j (0 \leq i,j \leq p - 1)$ and $\partial f / \partial x$. Define general skew symmetric biderivations from $F \times F$ to $F$, $\sigma$, by $\phi(u,v) = (x u_1 y - x u_2 y)/f$, $\phi_x$ is the partial derivative of $u$ with respect to $x$, etc.

Theorem. If $p - 1, p - 1 > 0$, then a separable element can always be adjoined to $k$ so that the enlarged $F$ now has a pair $u, v$ for which $\phi(u,v) = 1$. Application will be made to results in F. Callahan, these Notice 18 (1971), 511. (Received January 27, 1975.)

*722-A7* PAO-SHENG, HSU, Department of Mathematical Sciences, Virginia Commonwealth University, Richmond, Virginia 23284, An Application of Compactifications: Some Theorems on Maximal Ideals.

$C^r(S, A)$ denotes all continuous functions from a topological space $(S, T)$ to a topological division ring $A$ with relatively compact ranges, and $C^b(S, A)$ all continuous bounded functions on $S$. 

A-348
The division ring $A$ is hausdorff, zero-dimensional, and $S$ is $A$-completely regular ($\mathcal{T}$ is the weak hausdorff topology associated with $C^r(S,A)$). If a Stone-Cech type compactification $\beta_k S$ of $S$ are viewed as completions of certain uniform structures $U$ and $U^*$ on $S$ respectively, then $U = U^*$ in this case and they generate $\mathcal{T}$. Hence $\beta_k S = \beta_k S$.

These compactifications provide a unifying framework for proving some theorems on maximal ideals in $C^r(S,A)$ and $C^b(S,A)$. Theorem (Staum): Every maximal ideal in $C^b(S,k)$ ($k$ is a non-archimedean rank-one valued field) is of the form $M_s = \{ f : f(s) = 0 \}$ for some $s \in S$ if and only if $S$ is mildly compact. Stone's Theorem holds for $C^b(S,k)$ (the only quotient field in $C^b(S,k)$ is $k$) if $k$ is locally compact or a non-archimedean rank-one valued field) is of the form $M_s = \{ f : f(s) = 0 \}$ for some $s \in S$ if and only if $S$ is mildly countably compact. Theorem (Correl and Henriksen): If $A$ is hausdorff zero-dimensional division ring and $S$ is $A$-completely regular, then Stone's Theorem holds for $C^r(S,A)$. [This generalizes the classical result in which $A$ is hausdorff, non-discrete, locally compact, $S$ is $A$-completely regular, and the space is $C^b(S,A)$]. (Received January 27, 1975.)

722-A8 M. J. GREENBERG, University of California, Santa Cruz, California 95064. Strictly local solutions of Diophantine equations.

For any system $f$ of Diophantine equations, there exist positive integers $C(f)$, $D(f)$ with the following properties: For any nonnegative integer $n$, for any prime $p$, if $v$ is the $p$-adic valuation, $p$-adic integral solution $y$ to the system $f$ such that $v(x - y) =$ and if a vector $x$ of integers satisfies the inequality $v(f(x)) > C(f)n + v(D(f))$ then there is an algebraic group ring. We investigate the comportment of the groups $NK_1(R)$. (Received January 24, 1975.)


Let $\pi$ be a finite Abelian group, $R_n$ the $n$th cyclotomic extension of the integers and $R_n \pi$ the group ring. We investigate the comportment of the groups $NK_1(R_n \pi)$. Here $NK_1(R) = \text{Ker}(K_1(R)[t]) \rightarrow K_1(R)$ by augmentation, and $K_1$ is the functor introduced by Bass. It is well known for regular rings $R$ that $NK_1(R) = 0$. Theorem I. If $[R]$ is squarefree and $N, [R] = 1$ or $[R] = 2$ and $N, [R] = 2$, then $NK_1(R \pi) = 0$. This theorem provides interesting examples of nonregular rings $R$ with $NK_1(R) = 0$. Theorem II. Let $\pi$ be cyclic of prime power order $p$ and assume $m \equiv 3$ or $m \equiv 2$ and $p$ is odd, then $NK_1(2\pi$) is infinite $p$-torsion. (Received January 27, 1975.)


Let $\#_1$ denote the Lie algebra of traceless endomorphisms of a vector space $V$ of dimension $n$. We investigate the comportment of the groups $NK_1(R_n \#_1)$. Here $NK_1(R) = \text{Ker}(K_1(R)[t]) \rightarrow K_1(R)$ by augmentation, and $K_1$ is the functor introduced by Bass. It is well known for regular rings $R$ that $NK_1(R) = 0$. Theorem I. If $[R]$ is squarefree and $N, [R] = 1$ or $[R] = 2$ and $N, [R] = 2$, then $NK_1(R \#_1) = 0$. This theorem provides interesting examples of nonregular rings $R$ with $NK_1(R) = 0$. Theorem II. Let $\#_1$ be cyclic of prime power order $p$ and assume $m \equiv 3$ or $m \equiv 2$ and $p$ is odd, then $NK_1(2\pi \#_1)$ is infinite $p$-torsion. (Received January 27, 1975.)


A Matroid Design on a finite set $E$ is a matroid all of whose hyperplanes have the same cardinality $k$. The cardinality of the cocircuits, $I = \# - k$, is called the index of the matroid design. The prime factorization of $I$ is the key to analyzing the structure of matroid designs. Matroid designs of prime power index include the known projective geometries, but are more general.

Every rank 3 matroid design with prime power index is essentially either:

(i) $a (1 + q + q^2, 1 + q, 1) - \text{BIBD}$, or

(ii) $a (1 + q + q^{2p+1}, 1 + q, 1) - \text{BIBD}$, or

(iii) a group divisible design with $\lambda = 1$, consisting of $1 + q^{2p+1}$ groups, each of size $p^k (p^2 - 1 = q^k)$ and having $k = 1 + q$. $p$ is a prime and $q$ is a prime power.

The first two classes of rank 3 matroid designs always exist, they being special.
cases of the larger family of \(1 + q^n_1 + q^n_2 + \ldots + q^n_r, 1 + q, 1\) - BIBD's, where \(r \geq 1\), \(n_i \equiv i \mod 2\) and \(n_i < n_{i+1}\) for all \(i\); the latter BIBD's exist for any prime power \(q\).

The existence of type (iii) matroid designs is an unsolved problem.

One can say that rank 3 matroid designs of prime power index are "almost" perfect matroid designs (matroids in which for any \(h\), all flats of rank \(h\) have the same cardinality).

Similar results hold for ranks higher than 3. (Received January 28, 1975.)

**Analysis**


Let \(\mathcal{H}\) be a complex Hilbert space, \(\mathcal{L}(\mathcal{H})\) be the Banach space of bounded operators of \(\mathcal{H}\); let \(A, B, C \in \mathcal{L}(\mathcal{H})\) and \(X(t,s); F(t,s; X(t,s))\) be functions from \(\mathbb{R}^2\) to \(\mathcal{H}\). In this paper we discuss the existence and uniqueness of bounded (strongly) continuous solutions to the equation (*)

\[
\frac{d^2 X}{dt^2} + A \frac{dX}{dt} + B X(t,s) + C X(t,s) = F(t,s; X(t,s)).
\]

The classical method of successive approximations is used here; the application of this method to the present problem depends on obtaining upper bounds on the norms of the semi-groups generated by \(A\) and \(B\) (this is known when \(A, B\) satisfy appropriate conditions), as well as on the norm of a Bessel functional \(J_\nu(tE)\) from the reals to \(\mathcal{H}\), which we define as \(J_\nu(tE) = \sum_{r=0}^{\infty} (-1)^r (1/2\pi)^{2r} r!^2 (r!)^2\).

**Theorem** If \(E\) is self-adjoint, then \(\| J_\nu(tE) \| \leq 1\) for all real \(t\). The existence and uniqueness of solutions of equation (*) is proved when \(A, B, C, F(\ldots; X(\ldots))\) satisfy:

- \(AB = BA\);
- the spectra of \(A, B\) and \(C - AB\) are all contained in the right half-plane; \(C - AB\) is self-adjoint and \(F(\ldots; X(\ldots))\) satisfies a Lipschitz condition. (Received December 12, 1974.)


Let \(D\) be the closed unit disk in the complex plane, \(C(D) = \) all continuous complex valued functions on \(D\). For \(f\) in \(C(D)\), denote by \(P_f\) the uniform closure on \(D\) of polynomials in \(z\) and \(f\). **Theorem.** If \(g\) is a quasiconformal mapping of a neighborhood of \(D\), then \(P_g = C(D)\). This result is related to results which the author announced recently (see Abstract 74T-B235, these Notices 21 (1974), A-593). A feature of this theorem is that \(f\) is not required to be differentiable inside \(D\). (Received December 13, 1974.) (Author introduced by Professor Joseph A. Sullivan.)

*722-B3* J. HENRARD, Facultes Universitaires de Namur, Belgium and K. R. MEYER, University of Cincinnati, Cincinnati, 45221. Bifurcations in symmetric systems.

The generic bifurcation of periodic orbits of systems of equations of the form \(\dot{x} = f(t,x,\lambda)\) where \(f(t + \tau, -x, \lambda) = -f(t, x, \lambda)\) are considered. (\(\lambda\) is a parameter). It is shown that such a system is equivalent to a \(\tau\)-periodic system. A computer program which performs in literal form the method of averaging to a high order is used to discuss several non trivial examples. (Received January 10, 1975.)

*722-B4* NATHANIEL CHAFEE, Georgia Institute of Technology, Atlanta, Georgia 30332, Liapunov Methods and Parabolic Equations.

In recent years several mathematicians have used Liapunov methods to study the asymptotic behavior of solutions for parabolic differential
equations. This lecture is a report on their work. The problems we will discuss involve quasilinear one-dimensional parabolic equations of the form \( u_t = u_{xx} + f(x,u) \). We will discuss these problems from a dynamical systems point of view. We will indicate the use of the Hale-LaSalle invariance principle and we will exhibit an interesting Liapunov functional. Also, we will describe some of the results obtained by these methods. (Received January 13, 1975.)

722-B5
KENNETH B. HANNSGEN, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. The resolvent kernel of an integrodifferential equation in Hilbert space. Preliminary report.

Let \( y(t,x,f) \) denote the solution of \( y'(t) + \int_0^t a(t-s) L y(s) \, ds = f(t), \ y(0) = x \), \( t \geq 0 \), where \( L \) is a self-adjoint densely defined operator on a Hilbert space \( H \), with \( L \geq \mu > 0 \). Let \( U(t)x = y(t,x,0) \). **THEOREM:** If \( a(t) \) is continuous \( (t \geq 0) \) and completely monotonic \( (t > 0) \), then \( U(t) \) is a bounded operator on \( H \) with \( \|U(t)\| \leq 2 \), \( \|U(t)\| \to 0 (t \to \infty) \), and \( \int_0^\infty \|U(t)\| \, dt < \infty \).

This result is useful when the representation \( y(t,x,f) = U(t)x + \int_0^t U(t-s)f(s) \, ds \) holds. The proof starts with the inequality \( \|U(t)\| \leq \sup_{\lambda \geq \mu} |u(t,\lambda)| \), where \( u_t(t,\lambda) + \lambda \int_0^t a(t-s) u(s,\lambda) \, ds = 0, \ u(0,\lambda) = 1 \). (Received January 20, 1975.)

722-B6
Shui-Nee Chow, Michigan State Univ. and Brown Univ., John Mallet-Paret, Brown Univ., Averaging and Bifurcation from Equilibrium or Periodic Solutions

The method of integral averaging is used to obtain Hopf bifurcation theorems for ordinary differential equations and more general systems such as functional differential equations and certain partial differential equations. Computable criteria are given to determine the qualitative nature of the bifurcations. The method also extends to the situation of a periodic orbit bifurcating into an invariant torus. Examples are given to illustrate these theorems. (Received January 21, 1975.)

722-B7
JERROLD E. MARSDEN and MARJORIE M. MCCracken, University of California 94720 Hopf Bifurcation and Results of Ruelle - Takens.

Bifurcation to periodic orbits and invariant tori are well established for ordinary differential equations through the work of Hopf, Ruelle and Takens. We give conditions on evolutionary partial differential equations under which the same qualitative conclusions can be drawn. This involves certain smoothness results on the time \( t \) maps of the evolution operator (due to J. R. Dorroh and J.E. Marsden) which enable one to prove invariant manifold theorems as in the case of ordinary differential equations and hence to deduce the bifurcation theorems. Explicit formulae for testing stability of the periodic orbit are given. Reference: "The Hopf Bifurcation and its Applications" by J. Marsden and M. McCracken, Applied Math. Sciences, Springer (submitted). (Received January 22, 1975.)

722-B8
JAMES D'ARCHANGELI, U.S. Naval Academy, Annapolis, Maryland 21402. An integral representation for the solution \( W_{km} \) of Whittaker's differential equation.

Recent results of Philip Hartman concerning the existence of solutions of ordinary differential equations with Laplace - Stieltjes transforms for coefficients are applied to Whittaker's equation, \( W' + \left\{ -1/4 + k/t - (m^2 - 1/4)/t^2 \right\} W = 0 \). As a result, new integral representations involving in certain cases the Legendre functions are obtained for the solutions \( W_{km} \) to Whittaker's equation and their products. (Received January 20, 1975.)

722-B9
STEVEN I. ROSECRANS, TULANE UNIVERSITY, NEW ORLEANS, LA. 70118 LIE THEORY OF THE HEAT EQUATION

We explain a new derivation of the group of the heat equation (and other diffusion equations), one which is more direct than the classical method.
Instead of treating \( u(\text{solution}), \ t(\text{time}), \) and \( \mathbf{x} (\text{space}) \) on the same footing, as classically, we consider multiplier representations of groups on \( (t, \mathbf{x}). \) From this point of view some new results emerge. Consider for example, the heat equation in one space dimension. We obtain simple closed-form formulas for the semigroup \( \exp(tM) \) in terms of \( \exp(t\frac{\partial^2}{\partial x^2}) \), where \( M = \frac{\partial^2}{\partial x^2} + (ax+b)\frac{\partial}{\partial x} + (cx^2+dx+e). \) Further, similarity solutions of the heat equation can be derived simply when this viewpoint is taken.

(Received January 20, 1975.)

Let \( PM \) be the space of distributions \( S \) on the circle group \( T \) such that \( \mathbb{S} \in L^\infty \). For a closed set \( E \subset T \), let \( M(E) \) be the set of bounded Borel measures whose supports are contained in \( E \). When \( X \) is a linear submanifold of \( L^\infty \), let \( X_1 \) denote the set of all weak* limits of sequences in \( X \). Define \( X_s \) for every ordinal number \( s \) by the inductive rule, \( X_s = \{ u_{<s}X_{<t} \} \). The first countable ordinal number \( s \) such that \( X_s \) is the weak* closure of \( X \) is called the order of \( X \). Theorem: For every countable ordinal number \( s \), there is a subset \( E \) of the circle group such that \( E \) obeys harmonic synthesis and the order of \( M(E) \) is \( s \). (Received January 20, 1975.)

We consider sixteen boundary value problems for a plane rectangle and prove this uniqueness theorem which is stronger than those generally known: THEOREM: Let \( D \) be the interior of a finite plane rectangle and suppose that the real valued, twice continuously differentiable function \( u \) is defined on \( D \), and satisfies Laplace's equation \( u_{xx}+u_{yy}=0 \) throughout \( D \). Suppose that \( u \) is bounded in absolute value on \( D \). Let \( R \) denote the rectangular boundary of \( D \) (\( R \) is then the union of four open straight line intervals, call them \( I_1,I_2,I_3,I_4 \), plus four vertices). Suppose, further, that \( u \) satisfies, on each open interval \( I_i \), a boundary condition of the following form: either the \( \lim_{(x,y) \in I_i} (u_{xx},u_{yy}) = 0 \), whenever \( (x,y) \in I_i \) tends to a point of an open interval \( I_i \), or else the \( \lim_{(x,y) \in R} \frac{\partial u}{\partial n} (x,y) = 0 \), whenever \( (x,y) \in R \) tends to a point of an open interval \( I_i \), where \( 3/\partial \) denotes the directional derivative in the direction normal to the open interval \( I_i \). Then, unless, for every open interval \( I_i \), the boundary condition is \( \lim_{(x,y) \in R} \frac{\partial u}{\partial n} (x,y) = 0 \), it can be concluded that \( u \) must be identically zero on \( D \), while, in this excluded case, it can only be concluded that \( u \) must have a constant value throughout \( D \). The uniqueness proof uses the geometry of the domain considered, whose boundary is piece-wise rectilinear. The Schwarz reflection principle, the Riemann isolated singularity theorem for harmonic functions, and the Liouville theorem for harmonic functions are the main tools used in the proof. This theorem readily implies corresponding uniqueness theorems for Poisson's equation \( u_{xx}+u_{yy}=F(x,y) \) with b.c.s. of the form \( u=f \) and \( (u_{xx}+u_{yy})=g \).

(Received January 22, 1975.)

By developing the theory of systems of functional differential inequalities we establish the existence of maximal and minimal solutions for systems of functional differential equations. As a byproduct of this we obtain a very general comparison theorem which will play an important role in the qualitative study of functional differential systems. (Received January 22, 1973)

Let \( A \) be a \( C^0 \) vector field on a \( C^1 \) compact manifold \( X \). Assume that \( A \) induces a flow. Let \( m \geq 0 \) have isolated zeros disjoint from the critical points of \( A \).
Then there are conditions (essentially Osgood's criterion) involving the non-integrability of $1/m$ on orbits which imply that $mA$ induces a flow. Further conditions are given in terms of the non-integrability of $1/m$ with respect to certain ergodic measures for the strong ergodicity of $A$ flow to imply the strong ergodicity of the $mA$ flow. The flow is strongly ergodic if the integral
\[
\left( \frac{1}{T} \right) \int_{0}^{T} f \circ \phi_{t} \, dt
\]
has a uniform limit as $T \to \infty$ for all continuous real valued functions $f$ on $X$. (Received January 23, 1975.)

SHERWOOD D. SILLMAN, The Cleveland State University, Cleveland, Ohio 44115.

On Complete Semicardinal Quadrature Formulae.

We consider so-called complete semicardinal quadrature formulae (q.f.), as opposed to Euler-Maclaurin or natural semicardinal q.f. By requiring our q.f. to be exact for a particular sequence of B-splines, we are led by a generating function approach to a characterization of these q.f. and are able to determine their coefficients very accurately. (Received January 23, 1975.)

T.K. PUPPASAMY, Ball State University, Muncie, IN 47306. Two Point Connection Problem For a Certain Differential Equation With An Irregular Singular Point of Arbitrary Rank.

In this paper, the author has solved in the large the linear homogeneous ordinary differential equation of third order.

\[
(z^{3} \frac{d^{3}y}{dz^{3}} + z^{2} \left[ K_{1} + K_{2} z^{m} \right] \frac{dy}{dz} + z \left[ K_{3} + K_{4} z^{m} \right] \frac{d^{2}y}{dz^{2}} + \left[ K_{5} + K_{6} z^{m} \right] y = 0,
\]

with $z$-complex the constants $K_{i}$ ($i=1,2,3,4,5,6$) real or complex and $m$ a positive integer. Then (1) will have a regular singular point at $z = 0$ and an irregular singular point at $z = \infty$. In general, the rank of $z = \infty$ will be $m$. The indicial equation about $z = 0$ is found to be
\[
h(h-1)(h-2) + K_{1} h(h-1) + K_{2} h + K_{3} = 0.
\]
It is also assumed that the roots $h_{i}$, $i=1,2,3$ of (2) are such that the difference of no two of them is congruent to zero modulo $m$. (Received January 24, 1975.)

DANIEL HENRY, Northwestern University, Evanston, Illinois 60201, Invariant manifolds for parabolic equations

In analogy with ODE theory, define local stable and unstable manifolds near an equilibrium point (or time-periodic solution) of a nonlinear parabolic equation. The unstable manifold $U_{loc}$ is finite dimensional and extends to an invariant manifold, using a backward uniqueness theorem. The local stable manifold $S_{loc}$ is infinite dimensional and its global extension seems less well-behaved, but any manifold contained in this stable "manifold" has codimension at least as large as codim $S_{loc}$: here we use backward uniqueness for the adjoint equation. Thus we show, for example, that an unstable equilibrium point of the Navier-Stokes equation has region of attraction nowhere dense. Also, for many gradient systems we prove the maximal bounded invariant set is finite dimensional. (Received January 27, 1975.)

(George Johnson)
L is of Type II if no nontrivial solution has a 1 - 3 or 3-1 distribution of zeros on \([0, \infty)\). It is well known that if \(L y = D y + p(x)y\), where \(D\) is disconjugate on \([0, \infty)\) and \(p(x) \neq 0\), then \(L\) is Type I or Type II when \(p(x) > 0\) or \(p(x) < 0\) respectively. We show that the converse is also true; that is if \(L\) is either Type I or Type II, then \(L\) may be put in the form \(L y = D y + p(x)y\) with \(D\) disconjugate on \([0, \infty)\) and \(p(x) \neq 0\). (Received January 27, 1975.)


We deal with the equation \((1 + t) y'' + 2 t y' + y = 0\) and show how a set of nonlinear time scales may be derived from it. Using these time scales multitime expansions of the solution of the equation satisfying the initial conditions \(y(0) = 0, y'(0) = 1\) are found. Proof of the uniform asymptotic convergence of the expansions is given. The technique used is an extension of the method used by Reiss (SIAM Rev. 13(1971), 189-196), and has been successfully applied to a broad class of second order linear differentials with slowly varying coefficients. (Received January 27, 1975.)

722-B19 Peter A. McCoy, U. S. Naval Academy, Annapolis, Maryland 21402. On the zeros of solutions of elliptic partial differential equations \(\phi^2 + A(x) \phi^2 + \phi = 0\) in three real variables. Preliminary report.

Under certain regularity conditions, S. Bergman studied series developments

\[
\psi(x) = \sum_{n=0}^{\infty} \frac{B_n}{n!} (\cos x e^{imx})
\]

via the operators \(\psi = D(H), H = B_2(f)\). Within a sphere of radius \(r_0\) center at the origin, \(r_0\) being the smallest positive zero of a polynomial associated with bounds on \(A\) and \(C\), the zeros of \(\psi\) are characterized by sequences of (and) positive (semi-) definite matrices \(M_n\) related to the subsequences \((B_n^{(mu)})\). If \(A = C = 0 (D(H) = B_2(f))\), NASC on \(M_0\) imply that \(\psi\) is a finite linear combination of Newtonian potentials of positive mass distributed on the unit sphere. In axisymmetric problems, special attention is given to global relations between the zeros of \(\psi\) and the \(M_n\) where certain conditions imply \(B_0^{(mu)} = \int_1 \exp(-ivt) du(t), I = (-\pi, \pi)\) for unique \(\mu\).

In higher dimensions, equations with radially dependent coefficients can be considered using R. P. Gilbert’s operators. (Received January 27, 1975.)

*722-B20* SANTIRANJAN MUKHOTI, Graduate School of Arts and Sciences, University of Pennsylvania, Philadelphia, Pennsylvania 19174. A theorem on Cesàro summability for fractional orders of integrals.

We consider the Cesàro summability for fractional orders of integrals \(\int_0^{\infty} \phi(t) dA(t)\) and establish Theorem 1 (The case \(k\) fractional). Suppose that \(\phi(t)\) is positive, \(\phi^{[k]}(t)\) is absolutely continuous for \(t \geq 1\); (i) \(\Phi_k(x) = o(x^k)\) as \(x \to \infty\), where \(\Phi_k(x) = (C(k))^{-1} \int_1^x (x - u)^{k-1} \phi(u)du\); (ii) \(\int_1^x \phi(t) (t - w)^k \phi(u) du = O(x^k)\). Then necessary and sufficient conditions for \(I \int_0^1 \phi(t) dA(t)\) to be summable \((C,k)\) to \(B\) are that \((II) - \int_0^1 \phi(t) dA(t)\) should be summable \((C,k)\) to \(B\), and (III) \(A(x) \phi(x) = o(1)\) as \(x \to \infty\). (Received December 11, 1974.)


The central item in this paper is the formula

\[
\Gamma(a) \Gamma^{(p)}(z) \Gamma(z + p) = \Gamma(\omega - a) \Gamma(- \omega) \Gamma^{(p)}(z) D^{a-\omega} u(z) D^{\omega} v(z) \omega, \tag{1}
\]

where the symbol \(D^{\beta} f(z)\) denotes the fractional derivative defined by \(D^{\beta} f(z) = \Gamma(- a)^{-1} \int_z^\infty (t - x)^{a-1} f(x) dx\). By selecting specific functions for \(u(z)\) and \(v(z)\) Barnes type integrals are found. For example, when we set \(u(z) = z^p\) and \(v(z) = z^q\) we get

\[
\Gamma(a) \Gamma(a - p - q) \Gamma(a - p - q) \Gamma(a - p - q) = (2\pi i)^{-1} \int_0^{\infty} (\omega - A) \Gamma(\omega - q) \Gamma(\omega) \Gamma(a - p - q) \omega, \tag{2}
\]

which is known as Barnes’ first lemma [L. J. Slater, "Generalized hypergeometric functions", Cambridge Univ. Press, 1966, pp. 109-111]. Other examples are considered. (Received January 28, 1975.)
**GRIDORY BACHELIS, Wayne State University, Detroit, Michigan 48202. A factorization theorem for compact operators.**

If $X$ and $Y$ are Banach spaces, let $K(X,Y)$ denote the compact operators from $X$ to $Y$ and $F(X,Y)$ the uniform closure of the bounded finite rank operators from $X$ to $Y$. Theorem 1.

Let $Y$ be a Banach space with the bounded approximation property and $E$ a closed subspace of $Y$. Then for any Banach space $X$ and $T \in K(X,E)$ there exists a closed subspace $Z$ of $Y$ such that $T = UZ$. As a consequence of Theorem 1, we have:

**Theorem 2.** Let $Y$ be a Banach space with the bounded approximation property, and let $E$ be a closed subspace of $Y$ which fails the approximation property. Then there exists a closed subspace $Z$ of $Y$ such that $F(Z,E) \neq K(Z,E)$. In particular, if in addition $Y$ is isomorphic to $Y \oplus Y$, then there exists a closed subspace $S$ of $Y$ such that $F(S,S) \neq K(S,S)$. The above theorems generalize results obtained by Freda Alexander when $Y = L^p, 1 \leq p < \infty$.

(Received January 28, 1975.)

**D. G. ARONSON, University of Minnesota, Minneapolis, MN 55455 and Rice University, Houston, TX 77001. Stability for Nonlinear Parabolic Equations via the Maximum Principle.**

The maximum principle and its variants provide one of the classical methods for the study of the behavior of solutions of elliptic and parabolic differential equations. The ideas which underlie the method are embarrassingly simple and its utility in any given problem depends upon the ability to construct suitable comparison functions. Such constructions cannot be codified but must be tailored to the particular problem.

We illustrate these observations by a detailed study of the behavior for large $t$ of solutions of the so-called Fisher equation

$$\frac{2u}{t} = \frac{\partial u}{\partial x^2} + f(u). \quad (*)$$

Using comparison functions constructed from the steady state solutions of $(*)$, we examine the stability of equilibria and the occurrence of threshold phenomena. (Received January 28, 1975.)

**ROBERT E. ATALLA, Ohio University, Athens, Ohio 45701. Ergodic theory and Grothendieck spaces.**

A contraction $T$ on a Banach space $B$ is strongly ergodic if the averages $(1/n)(T + \ldots + T^n)$ converge in the strong operator topology. The following is a consequence of Sine's criterion [Proc. Amer. Math. Soc. 24(1970), 438-439]. **Theorem 1.** If $B$ is a G-space, then $T$ is strongly ergodic iff the norm-closure of $(I - T^*)(B^*)$ is equal to its weak-* closure. Examples show that this can fail for non-G-spaces. **Definition.** A sequence $\{f_n\} \subset B^*$ is said to have disjoint supports if for any scalars $\{t_1, \ldots, t_n\}$,

$$\sum_{i=1}^n |t_i| f_i \rightarrow 0,$$

and $\inf \|f_i\| > 0$. **Theorem 2.** If $T$ is a contraction on G-space $B$, and for some $f \in B^*, \{T^n f\}$ have disjoint supports, then $T$ is not strongly ergodic. This follows from the Lemma. If $B$ is a G-space and $\{f_n\} \subset B^*$ have disjoint supports, then there exists $x \in B$ such that the sequence $\{f_n(x)\}$ is not Cesaro summable. From this we derive the mean divergence theorem for $C(X)$, where $X$ is a compact F-space, due to W. Rudin [Duke Math. J. 25(1958), 197-204] and J. Gait and S. Koo [Math. Systems Theory 6(1972), 23-25]. (Received January 28, 1975.)

**Applied Mathematics**

**JOHN DE PILLIS, University of California, Riverside 92502. Schemes for fast matrix multiplication, Preliminary report.**

We say we have an $(m,n,p)$ set-up whenever we consider multiplication of $m \times n$ matrix $A$ with $n \times p$ matrix $B$, whose (scalar) entries are elements of a (not

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necessarily commutative) ring of characteristic \(\neq 2\). The number of (scalar) products sufficient to produce all \(mp\) entries of the matrix \(AB\), by means of a scheme \(P\), say, is denoted \(P(m,n,p)\). For example, the classical definition or scheme \(Cl\), of matrix multiplication produces \(AB\) with at most \(Cl(m,n,p) = mnp\) products. We develop a theory which recasts the scheme \(Str\) of V. Strassen [Numer. Math. 13(1969), 354-356] where \(Str(2,2,2) = 7\), and the more general scheme \(H-K\) of J. Hopcroft and L. Kerr [SIAM J. Appl. Math. 20(1971), 30-36], where \(H-K(m,2,p) = [(3mp + \max(m,p))/2]\); in particular, \(H-K(6,2,6) = 57\). We further present schemes \(Q_1, Q_2\) where \(Q_1(6,2,6) = 56\), and \(Q_2(72,72,72) = 56^3\). (Received December 30, 1974.)

In the numerical calculation of \(f(t)\), the inverse Laplace transform of \(F(p)\), where \(f(t) = (1/2\pi i) \int_{c-i\infty}^{c+i\infty} e^{pt}F(p)dp\), sufficient accuracy is usually obtainable when \(pF(p)\), \(s > 0\), is replaced by an interpolating polynomial in \(1/p\). From the values of \(F(p)\) with \(F'(p)\), or with \(F''(p)\), for \(p\) at points equally spaced on the real axis, an osculatory or hypersculatory interpolation polynomial for \(pF(p)\), namely \(L_{2n-1}(x)\) or \(L_{3n-1}(x)\), where \(x = 1/p\), is obtained in barycentric form. Then \(f(t)\) is calculated by a Gaussian-type quadrature formula employing complex values of \(L_{2n-1}\) or \(L_{3n-1}\), instead of \(pF(p)\) which may be unknown or more difficult to compute. For calculating \(L_{2n-1}\) and \(L_{3n-1}\), auxiliary coefficients, suitable for economical storage in the program, are given exactly for \(n = 2(1)11\) and \(n = 2(1)7\), furnishing up to 21st and 20th degree accuracy respectively. (Received December 31, 1974.)

If in the \(E_p\)-transform \(R(t,k,p) \rightarrow R(k,p)\) as \(t \rightarrow +\infty\), then a limit form of the \(E_p\)-transformation, denoted by \(H_p\), is defined by

\[
H_p[F,t,k] = \left[\frac{F(t)}{1-R(k,p)}\right] \left[\frac{R(k,p)}{1-R(k,p)}\right] \text{ provided } R(k,p) \neq 1.
\]

Here \(F(t) = \int_a^t f(x)dx\), \(R(t,k,p) = \frac{f(t)}{1-p}g_p(t)^{1-1/p}\), and \(g_p(t) = \frac{(t^1-p+k)^{1-p}}{t^1-p}\)

for \(0 < p < 1\). Conditions on \(R(k,p)\) are given which establish when the \(H_p\)-transformation will converge more rapidly than \(F(t)\) as \(t \rightarrow +\infty\), or converge uniformly better than \(F(t)\). (Received January 17, 1975.)

It is shown that in an imploding shock wave problem, the hypothesis of an isothermal energy release in the reflection region allows solutions that remain uniformly bounded in that region including the center of symmetry, which is a singularity of the governing partial differential equations. The hypothesis enables one to uncouple the continuity and the momentum equations from the other equations in an approximate theory. (Received January 24, 1975.)

The expansion theorem of the twisted product associated with the Weyl form of CCR for \(n\) degrees of freedom (Abstract 706-81-1, these \(\mathcal{C} N\) vol. 20(1973), A-543) is generalized to involve tempered distributions. (Received January 24, 1975.)
A numerically stable form of an algorithm that is closely related to the work of Gill and Murray [1] and Conn [2] is presented.

Among other reasons, the penalty function approach has never been available for linear programming in a viable sense because of the inherent non-linearities introduced. The non-differentiable penalty function is unique in that it is a piecewise linear function and hence maintains a computational efficiency comparable with, and in general, better than, the standard form.

The method admits non-simplex steps and this feature enables it to be readily generalised to quadratic programming.


ANN-SHENG CHIEN TING - VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY

On the convergence of a class of derivative-free minimization algorithms

A convergence analysis is presented for a general class of derivative-free algorithms for minimizing a function \( f(x) \) whose analytic form of the gradient and the Hessian is impractical to obtain. The class of algorithms accepts finite difference approximation to the gradient with step-sizes chosen according to the following rule: if \( x_1, x_2 \) are two successive iterate points and \( h, \tilde{h} \) are the corresponding step-sizes, then the following two conditions are required:

1. \( \frac{1}{2} \| h \|_2 \leq \min \left( \frac{1}{2} \| h \|_2, \frac{1}{2} \| \tilde{h} \|_2 \right) \)
2. \( \frac{1}{2} \| h \|_2 \leq c_2 \| x_2 - x_1 \|_2 \)

The algorithms also maintain an approximation to the second derivative matrix and require the change in \( x \) made by each iteration is subject to a bound that is also revised automatically. The convergence theorems have the features that the starting point \( x_1 \) needs not be close to the true solution and \( f(x) \) needs not be convex. Furthermore, despite of the fact that the second derivative approximation may not converge to the true Hessian at the solution, the rate of convergence is still \( Q \)-superlinear. The theory is also shown to be applicable to a modification of Powell's dog-leg algorithm. (Received January 27, 1975.)

(Author introduced by Professor John E. Dennis, Jr.)


A steady circulation preserving isentropic gas flow is shown to be a geodesic flow. A counter example is provided to show that a steady homentropic flow is not a geodesic flow, even though a steady homentropic flow is a circulation preserving flow. A necessary and sufficient condition is derived for a circulation preserving flow to be a geodesic flow. The geometric properties of steady, circulation preserving geodesic flows for which the flow lines form geodesics on Lamb surfaces (surfaces containing the flow lines and the vortex lines) are studied. (Received January 28, 1975.)


The relationship between complete controllability and stabilizability for distributed boundary control systems will be discussed. Applications are given to hyperbolic partial differential equations. (Received January 28, 1975.)

(Author introduced by Professor J. K. Hale.)
Geometry

WILLIAM WOLFE, Graduate Center, City University of New York, New York, New York 10036. Asymptotic distribution of lattice points in hyperbolic space. Preliminary report.

Suppose \( x \) and \( y \) are two points in the upper half-plane \( \mathbb{H}^+ \), and suppose \( \Gamma \) is a discontinuous group of conformal automorphisms of \( \mathbb{H} \) having compact fundamental domain \( \mathcal{F} \). Denote by \( N_{\mathcal{F}}(x,y) \) the number of points of the form \( \gamma y \ (\gamma \in \Gamma) \) in the closed disc of hyperbolic radius \( T \) centered about \( x \), and set \( R_{\mathcal{F}}(x,y) = N_{\mathcal{F}}(x,y) - V(T)/A \), where \( V(T) \) is the hyperbolic area of the disc, and \( A \) is the hyperbolic area of \( \mathcal{F} \). The asymptotic behavior of the quantity \( \int_{\mathbb{H}^+} (R_{\mathcal{F}}(x,y))^2 \, dx \, dy \) is estimated in terms of small eigenvalues of the Laplacian on functions automorphic under \( \Gamma \). (Received January 27, 1975.) (Author introduced by Professor Burton Randol.)

Logic and Foundations


The concept of an arithmetically productive set was introduced to the Association for Symbolic Logic at the January, 1975 meeting. (See my forthcoming abstract in the Journal of Symbolic Logic.) Let \( \omega^1, \omega^2, \ldots \) be a standard enumeration of the \( \Sigma^0_n \) sets. Let \( J(x,y) \) be a recursive pairing function. Enumerate the arithmetical sets \( \alpha^1, \alpha^2, \ldots \) be \( \omega^n = J(n, \alpha^n) \). Call a set \( A \) a set productive (arithmetically set productive) iff there exists a partial recursive function \( f(x) \) such that \( \omega^m \subseteq A \ (\alpha^m \subseteq A) \Rightarrow f \neq \emptyset \). Theorem. \( A \) is \( \Sigma^0_n \)-set productive iff \( A \) is \( \Sigma^0_n \)-productive. Theorem. \( A \) is arithmetically set productive iff \( A \) is arithmetically semiproductive. Theorem. \( A \) is arithmetically productive implies \( A \) is arithmetically set productive, but not conversely. (Received January 28, 1975.)

Statistics and Probability


Consider a finite homogeneous birth-death process \( X_t, t \in [0,\infty) \), state space \( S = \{0,1,2,\ldots,M\} \), birth rate \( b_j \) and death rate \( d_j \), \( j = 0,1,2,\ldots,M \), \( d_0 = b_M = 0 \). We assume that \( b_0 = d_M = 0 \) and \( b_j > 0 \), \( d_j > 0 \), \( j = 1,2,\ldots,M-1 \) so that \( 0 \) and \( M \) are absorbing states. In this paper we derive some formulas for the ultimate absorption probabilities, the first time absorption probabilities, the mean absorption times and the variances of the first absorption times of the state \( k \), \( 1 \leq k \leq M-1 \). The results are then applied to a genetic model of Moran (1958) in evolution theory. The problems studied in this paper do not seem to have been considered before for birth-death processes in the literatures. (Received November 22, 1974.) (Author introduced by Professor C. T. Long.)

Herman Chernoff, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Identifying a member of a large population using noisy data. Preliminary report.

The FBI has a large collection of fingerprints on file. When a new set arrives, the file must be checked to see if it contains another set for the same individual. What characteristics of the fingerprints should be recorded to facilitate the necessary comparisons and how does one measure closeness of two sets being compared? Although Galton concerned himself with fingerprint identification, classical statistical theory, dealing mainly with estimation and hypothesis testing, does not confront this problem directly. Here one is concerned with choosing one among many possible hypotheses and the problem seems to lie between one of hypothesis testing and estimation. A reformulation reduces a major aspect of this problem to that of testing a simple hypothesis against a simple alternative. Large deviation results become relevant and provide measures of the relevant information in specific attributes or measurements associated with fingerprints. This formulation has applications to the identification of chemical compounds through mass spectrograph, classification of species, character recognition information retrieval, cluster analysis, etc. (Received January 15, 1975.)

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Consider real-valued random variables $X_1, \ldots, X_N$ with joint distribution $d\mu$ on $\mathbb{R}^N$ defined by $d\mu = Z^{-1} \exp\left(-\frac{1}{4} \sum_{i \neq j} J_{ij} X_i X_j - \frac{1}{2} \sum_i h_i X_i \right) \int d\nu_1(x_1) \cdots d\nu_N(x_N)$, where $Z$ is a normalization constant, $J_{ij} \geq 0$, $h_i \geq 0$, $\rho_i$ even exponentially bounded measures. Consider $m_k = E[X_k^2]$, $k = 1, \ldots, N$, which is a function of the $h_i$ and the $J_{ij}$.

**Theorem.**

a) If the $\rho_i$ are as above, then $m_k \geq 0$, $\delta m_k / \delta h_i \geq 0$, $\delta m_k / \delta J_{ij} \geq 0$. b) If the $\rho_i$ are as above and if certain matrix moments of the $\rho_i$ are negative, then $\delta^2 m_k / \delta h_i \delta h_j < 0$.

Part a) is known, but our proof is new as is the condition in part b). Proofs of these and other inequalities will be discussed as well as applications to statistical mechanics.

(Received January 21, 1975.)

*722-F5


Some of the results obtained are used to study the initial value problem:

$$\frac{d}{dt} X = \frac{1}{\epsilon} \left( A^{(1)} w^{-1}(A^{(2)}) \right) X + \frac{1}{\epsilon} F(X,t,w) + G(X,t,w), \quad X(0) = x,$$

where $A^{(1)}$ is an $n \times n$ skew symmetric matrix, $A^{(2)}$ is an $m \times m$ positive definite matrix while $X$, $F$ and $G$ are random vector functions in $\mathbb{R}^{n+m}$. The asymptotic behavior as $\epsilon \downarrow 0$, $0 \leq t \leq T$ is found to consist of an initial layer and a diffusion process in $\mathbb{R}^n$. For the specific case of one undamped oscillator stochastically coupled to $m$ damped oscillators, mean square stability information is obtained.

(Received January 23, 1975.)

**Topology**

M. JAYACHANDRAN, Madurai College, Madurai, India and M. RAJAGOPALAN, Memphis State University, Memphis, Tennessee 38152. On two problems of M. E. Rudin and J. J. Schaffer in scattered spaces, Preliminary report.

In a topology conference held in Wyoming in 1974, M. E. Rudin has raised the following problem:

"Can a scattered, Tychonoff space be mapped by a closed continuous map onto $[0,1]$?"

J. J. Schaffer has announced the following problem as Problem No. 45 in the Problem book of the Department of Mathematics of Carnegie-Mellon University:

"Can a scattered, countably compact, Tychonoff space be mapped by a continuous function onto $[0,1]$?"

(Received January 24, 1975.)
Both the above problems are answered in affirmative by constructing suitable scattered, locally compact, seq. compact, I countable, Tychonoff spaces $X$ and $Y$. The methods employed in constructing these spaces can also be used to solve an outstanding problem of C. Scarborough and A. H. Stone on products of sequentially compact spaces raised in Transactions of the A.M.S. (1966). (Received January 13, 1975.)

On topological methods in homological algebra.

A series of papers is described, in which the interplay between the topological and algebraic properties of the category $\text{Pro-SSAG}$ of inverse systems of simplicial abelian groups is developed.

Theorem 1. There are natural closed model structures on $\text{Pro-SS}$ ($\text{SS} = \text{simplicial sets}$) and $\text{Pro-SSAG}$. Hence the homotopy categories $\text{Ho}(\text{Pro-SS})$ and $\text{Ho}(\text{Pro-SSAG})$ exist.

Theorem 2. A flasque inverse system of abelian groups is a fibrant object of $\text{Pro-SSAG}$.

Theorem 3. $\text{Ext}_{\text{Pro-SSAG}}^{s}(\{G_i\}, \{H_j\}) = \text{Ho}(\text{Pro-SSAG})(\{G_i\}, \{\overline{H}_j\})$, where $\overline{W}$ is the simplicial classifying space.

Theorem 4. $\lim^8 (H_j) = \text{Ext}^8(\mathbb{Z}, \{H_j\})$, for $\{H_j\}$ in $\text{Pro-AG}$.

Theorem 5. $\lim^8$ vanishes on fibrant (i.e., flasque) inverse systems of abelian groups for $s > 0$.

Theorem 6. $\lim^1$ does not vanish on all inverse systems of surjections, hence not on all Mittag-Leffler systems. (Received January 16, 1975.)

S. P. SINGH, Memorial University, St. John's, Nfld., Canada, Convergence of sequence of iterates, Preliminary report.

Let $C$ be a closed, bounded, convex subset of a strictly convex Banach space $X$. Let $T : C \to C$ be a densifying mapping satisfying

$$||Tx - Ty|| \leq \alpha ||x - y|| + \beta(||x - Tx|| + ||y - Ty||) + \gamma(||x - Ty|| + ||y - Tx||)$$

with $\alpha$, $\beta$ and $\gamma$ positive and $\alpha + 2\beta + 2\gamma < 1$.

Then for $x_0 \in C$, the sequence of iterates $\{T^n x_0\}$ converges to a fixed point of $T$, where $T^\lambda : C \to C$ is a mapping defined by $T^\lambda = \lambda T + (1 - \lambda)I$, $0 < \lambda < 1$.

The known results due to Diaz and Metcalf, and Petryshyn follow as corollaries. (Received January 17, 1975.)

DENIS BLACKMORE, New Jersey Institute of Technology, New Jersey, 07102. On the index sum of a vector field, Preliminary report.

Let $M$ be a smooth $n$-manifold with boundary $\partial M$, and let $X$ be a $C^1$ vector field on $M$. We first prove some theorems about the index sum of a vector field having no zeros on $\partial M$; primary among them is the following: Let $X$ and $Y$ be vector fields on $M$ having isolated zeros none of which lies on $\partial M$. If $X$ and $Y$ are never opposite on $\partial M$, their index sums are equal. The theorems are then applied to prove a generalization of some Poincaré-Hopf type index formulas due to Morse [Amer. J. Math. 51 (1929), 165-178] and Pugh [Topology 7 (1968), 217-226]. We also obtain some analogous index formulae for the case when $X$ has isolated zeros on $\partial M$ (Received January 22, 1975.)

Dr. R. Mandelbaum, Weizmann Institute of Science, Rehovot, Israel

On the Topological Structure of Algebraic Surface in $\mathbb{C}P^3$.

The following theorem is proven: Theorem: Let $V_n$ be a non-singular algebraic surface of degree $n$ in $\mathbb{C}P^3$. Let $\kappa_n = \frac{n}{3} (n^2 - 6n + 11)$, $\lambda_n = \frac{n}{3} (2n^2 - 4n + 3)$, $P$ = The complex projective plane with its usual orientation, $Q$ = The complex projective plane with orientation opposite to the usual, $\# -$ The connected sum operation on manifolds. Then $V_n \# P$ is diffeomorphic to $\kappa_n P \# \kappa_n Q$. Corollary: Suppose $V$ is a non-singular $K3$ surface. Then $V \# P$ is diffeomorphic to $4P \# 19Q$.

(Received January 23, 1975.)
We examine the covering dimension of several "classical" product spaces.

(Received January 24, 1975.)

DENNIS A. MARCHETTO, Department of Mathematical Sciences, Virginia Commonwealth University, Richmond, Virginia 23284. Stepanoff-Like Flows on Compact Orientable Surfaces of Genus \( h \geq 2 \).

By adding more fixed points to the Stepanoff flows of J. C. Oxtoby (Proc. Amer. Math. Soc. 4 (1953), 982-987), we obtain flows \((R^2/Z^2, \psi_t)\) that are essential. The flows lift via the pseudo-covering map \( \psi \) defined by \( \psi(z,x) = (z^n,x) \) of \( R^2/Z^2 \) by \( M_h \) (a sphere with \( h \geq 2 \) handles attached) to flows on \( M_h \) belonging to the class of Stepanoff-Like flows. Definition: Let \( \varphi: M_h \to M_h \) be defined by \( \varphi(x,y) = (\exp(\theta),x) \) where \( \theta = (x_1,\ldots,x_n) \). A flow \((M_h, \psi_t)\) \((h \geq 2)\) is Stepanoff-Like if (i) \((0,0,\pm 1) \in M_h\) are fixed points, (ii) \( \varphi \circ \psi = \psi \circ \varphi \), and (iii) \((M_h, \psi_t)\) projects \( \psi \) to \((R^2/Z^2, \psi_t)\).

All topological and measure-theoretic properties of \((R^2/Z^2, \psi_t)\) essentially lift to \((M_h, \psi_t)\) such that each point is fixed or transitive and number of normalized ergodic measures admitted is finite. However, \((M_h, \psi_t)\) may admit either 1 or \( h \) ergodic measures \( \mu \) for which the set of fixed points is \( \mu \)-null while \((R^2/Z^2, \psi_t)\) may admit exactly one. To show this, we extend Kryloff-Bogoliouboff theory to \( R^2/Z^2 \) action on \( M_h \).

(Received January 27, 1975.)

(Received January 24, 1975.)

LEON STAGG NEWMAN, Jr., City University of New York, Baruch College, New York, New York 10010. Infinite mapping cylinders are Hilbert cube factors.

For each \( i \), let \( X_i \) be a compact Hilbert cube factor manifold and \( f_i: X_i \to X_{i+1} \). Let \( Y \cup \infty \) denote the one-point compactification of the space \( Y \) by the point \( \infty \). Define the infinite mapping cylinder of the \( F \)'s to be \( M(f) = \bigcup_{i=1}^{\infty} X_i \times [i-1,i) / \sim \) where \( (x,0) \sim (f(x),1) \), \( x \in X_i \). Theorem 1. \( M(f) \) is a topological space homeomorphic to \( Q \). Application. For \( K \) any locally finite CW-complex, \( \exists \) an embedding \( e: Q \to Q, s.t. Q - e(Q) \) has the homotopy type of \( Q \). Example. Let \( S_i \) be a copy of the two sphere for each \( i \) and \( g_i: S_i \to S_{i+1} \) be a degree two map. By Theorem 1, \((M(g_i)), \cup \infty, \times Q)\), but in contrast to the finite dimensional case, it can be shown that \( \infty \times Q \) does not have regular open neighborhoods even though it is an embedded submanifold of trivial shape.

(Received January 27, 1975.)

OKAN GUREL, IBM Corporation, 1133 Westchester Avenue, White Plains, New York 10604.

On the existence of limit cycles following peeling.

Determining the existence of periodic solutions, in particular limit cycles, for a given system, \((X_m, f)\) has long been the subject of investigations. It is clear that peeling (Gurel, Collective Phenomena 2(1974)) may or may not yield periodic solutions unless certain conditions are satisfied. A theorem is stated which can be argued as an extension of the global stability theorem stated in the reference above: For the appearance of at least one periodic solution with the stability property (SP) same as \( SP \) of the singular solution \( s_i \) of a system \((X_m, f)\) prior to peeling, it is necessary that all (or some of) characteristic values of the characteristic equation at \( s_i \) alter in sign (peeling (partial peeling)), and sufficient that the number of singular solutions remains unaltered, or the global stability property of \( S[X_m] - s_i \) remains unaltered. The theorem also points out that the parameters, \( m \), are the sum of two sets, \( p^C \oplus p^I \), corresponding to parameters introducing complete and incomplete topological transformations.


(Received January 27, 1975.)


A flow \( f \) in a neighborhood of a degenerate rest point is examined and found to be topologically equivalent to the product of the restrictions of \( f \) to the invariant manifolds.
\( x^+, \, M^-, \, \) and \( M^0 \) where \( x^+ \) is the unstable manifold of the rest point, \( M^- \) the stable manifold, and \( M^0 \) is any center manifold of the rest point. This result generalizes theorems of Hartman and Bliss, and is applied to the structure problem for weak shock waves of higher order theories of gas dynamics. (Received January 24, 1975.)

**ERRATA**

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Line 4 should read "... and \( \rho \) is the corresponding minimum...".

Line 5 should read "... Then \( H(\rho, r) \equiv J_\rho(z) \equiv H(r, \rho) \) where \( H(\rho, r) = \frac{2}{\rho} \rho^{-6} (r^2 - z_1 \bar{z}_1 - z_2 \bar{z}_2)^{-9} r^6 (\rho^2 - z_1 \bar{z}_1 - z_2 \bar{z}_2)^9 \)".

Line 6 should read "... by replacing \( \rho \) by \( r \) and \( r \) by \( \rho \)".

Line 7 should read "... let \( K_\rho(z, \bar{z}) = B_{10} \bar{z}_1 + B_{01} z_1 \bar{z}_2 + ... \)".

Line 9 should read "\( B_{\text{mp}}^{-1} = \int_{\mathcal{R}} \ldots H(\rho, r) \equiv B_{00}^3 B_{10}^{-1} B_{01}^{-1} \)".


The right side of the inequality for \( |B(x) - x| \) should be multiplied by \( x \).


Line 2, for "Non-RS semigroups" read "Semigroups".

Line 11, for "PG" read "group".


The abstract should read: A method of ascent is developed for parabolic partial differential equations.


Line 9, read "of \( x \), then \( x \) is convergent."
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SPECTRAL THEORY SYMPOSIUM
The Royal Irish Academy have published the Proceedings of a Spectral Theory Symposium held in Trinity College, Dublin, in March 1974. The volume, Proceedings of the Royal Irish Academy Volume 74. Section A, 18-36, consists of 19 papers on Spectral Theory and the authors include Douglas, Arveson, Berkson, Rosenthal, Sz-Nagy, Schaefer and Shields. Copies may be purchased from:
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