Notices

OF THE AMERICAN MATHEMATICAL SOCIETY

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August, 1975
Issue No. 163

Calendar

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the *Notices* was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting *in person* must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

Meeting Number	Date	Place	Deadline for Abstracts* and News Items
727	October 25, 1975	Cambridge, Massachusetts	Sept. 2, 1975
728	November 1, 1975	Chicago, Illinois	Sept. 2, 1975
729	November 7–8, 1975	Blacksburg, Virginia	Sept. 23, 1975
730	November 15, 1975	Los Angeles, California	Sept. 23, 1975
731	January 22–26, 1976 (82nd Annual Meeting)	San Antonio, Texas	Nov. 5, 1975
732	March 4–5, 1976	Tallahassee, Florida	
733	March 15–20, 1976	Urbana, Illinois	
	April 23–24, 1976	Reno, Nevada	
	June 18–19, 1976	Portland, Oregon	
	November 19–20, 1976	Columbia, South Carolina	
	November 26–27, 1976	Albuquerque, New Mexico	
	January 27–31, 1977 (83rd Annual Meeting)	St. Louis, Missouri	

* Deadline for abstracts not presented at a meeting (by title)
October 1975 issue: August 26
November 1975 issue: September 16

OTHER EVENTS

February 1976	Symposium on Some Mathematical Questions in Biology	November 5, 1975

Please affix the peel-off label on these *Notices* to correspondence with the Society concerning fiscal matters, changes of address, promotions, or when placing orders for books and journals.

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Notices
OF THE
AMERICAN MATHEMATICAL SOCIETY

Everett Pitcher and Gordon L. Walker, Editors
Hans Samelson, Associate Editor

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The Seventy-Ninth Summer Meeting
Western Michigan University
Kalamazoo, Michigan
August 18 – 22, 1975

PLEASE NOTE THAT AN IMPORTANT CHANGE HAS BEEN MADE UNDER "TRAVEL AND LOCAL INFORMATION" ON PAGE 221

SHORT COURSE ON APPLIED COMBINATORICS, August 16 and 17

On the recommendation of its Committee on Employment and Educational Policy, the American Mathematical Society will present a one and one-half day Short Course on Applied Combinatorics on Saturday and Sunday, August 16 and 17, in Room 1104 of Rood Hall at Western Michigan University. The course is designed to give substantial introductions to several important areas of application of combinatorics and graph theory. It is intended to present both mathematically challenging aspects and connections with problems encountered in other disciplines, industrial practice, or work of government agencies. This short course, which is open to all who wish to participate, will be similar in format to the short courses on computing and operations research recently given at Society meetings (August 1973, January 1974, January 1975).

The program is under the direction of D. R. Fulkerson, Department of Operations Research and Center for Applied Mathematics, Cornell University. The current members of the AMS Committee on Employment and Educational Policy are Michael Artin, Charles W. Curtis, Wendell H. Fleming, Calvin C. Moore, Martha K. Smith, and Daniel H. Wagner.

The program will consist of six seventy-five minute lectures on the following topics: A survey of algebraic coding theory, I and II, Elwyn R. Berlekamp, Department of Mathematics and Department of Electrical Engineering and Computer Science, University of California, Berkeley; Some problems involving graphs, D. R. Fulkerson; Combinatorial scheduling theory, I and II, Ronald L. Graham, Bell Laboratories, Murray Hill, New Jersey; and Integer programming, Ellis J. Johnson, Mathematical Sciences Department, IBM T. J. Watson Research Center.

SEVENTY-NINTH SUMMER MEETING, August 18 – 22

The seventy-ninth summer meeting of the American Mathematical Society will be held at Western Michigan University, Kalamazoo, Michigan, from Monday, August 18, through Friday, August 22, 1975. All sessions of the meeting will take place on the campus of the university.

Two sets of Colloquium Lectures are scheduled. Ellis R. Kolchin of Columbia University will lecture on "Differential algebraic groups". The other set of lectures will be given by Elias M. Stein of Princeton University; his title is "Singular integrals, old and new." The first lecture in each series will be given in Miller Auditorium on Tuesday afternoon, August 19, with Professor Kolchin speaking at 1:00 p.m. and Professor Stein speaking at 2:15 p.m. The second, third, and fourth lectures in each series will be held at 8:30 a.m. on Wednesday, Thursday, and Friday mornings.

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings there will be seven invited one-hour addresses. The names of the speakers, the titles of their addresses, and the times of presentation are as follows: Roy L. Adler, IBM T. J. Watson Research Center, "Ergodic properties of elementary mappings of the unit interval," 11:00 a.m. Friday; Everett C. Dade, University of Illinois at Urbana-Champaign, "Nearly trivial outer automorphisms of finite groups," 9:45 a.m. Wednesday; Bernard Maskit, State University of New York at Stony Brook, "On the classification of Kleinian groups," 11:00 a.m. Wednesday; David Mumford, Harvard University, "Algebraic cycles on algebraic varieties," 9:45 a.m. Friday; Jack H. Silver, University of California, Berkeley, "The singular cardinals problem," 9:45 a.m. Thursday; James D. Stasheff, Temple University, "The continuous cohomology of groups and classifying spaces," 1:30 p.m. Thursday; Wilhelm F. Stoll, University of Notre Dame," Aspects of value distribution theory in several complex variables," 11:00 a.m. Thursday.

The 1975 Leroy P. Steele Prizes in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein and the Norbert Wiener Prize in Applied Mathematics will be awarded at 3:15 p.m. on Thursday, August 21.

There will be fourteen sessions of selected twenty-minute papers. Janos D. Aczel of the University of Waterloo has organized a special session on Functional Equations, to be held Wednesday morning and all day Thursday and Friday; the speakers will be Janos D. Aczel, John A. Baker, Mary K. Bennett, Francis W. Carroll, Thomas M. K. Davison, George T. Diderrich, Jr., Graeme Fairweather, Maurice J. Frank, Jr., Michael A. Golberg, Konrad John Heuvers, T. D. Howroyd, Ih-Ching Hsu, Ignacy I. Kotlarski, Bohdan Lawruk, J. A. Lester, M. H. McKiernan, C. T. Ng, Thomas A. O'Connor, Frank J. Papp, Ludwig Reich, John V. Ryff, Berthold Schweitzer, Kermit N. Sigmon, Abe Sklar, Donald R. Snow, M. A. Taylor, and David Zupnik. Amassa C. Fauntleroy of the University of Illinois and Andy R. Magid of the University of Oklahoma have organized a special session on Affine Algebraic Groups, to be held all day Thursday and Friday afternoon; the speakers will be Melvin Hochster, James E. Humphreys, T. Kambayashi, Andy R. Magid, Robert A. Morris, Takashi Ono, Brian J. Parshall, Joel L. Roberts, Michael R. Stein, and Ferdinand D. Veldkamp. George Fix of the University of Michigan has organized a special session on Scientific Computing, to be held all day Friday; the speakers will be John Barnes, Alvin Bayliss, Melvyn Ciment, Julio Cesar Diaz, Peter Henrici, Stephen H. Leventhal, and Dianne M. Prost O'Leary. Casper Goffman of Purdue University has organized a special session on Aspects of Real Analysis, to be held all day Thursday; the speakers will be Richard J. Fleissner, Kim E. Michener, Togo Nishiura, Richard J. O'Malley, Daniel Waterman, Clifford E. Weil, and Robert E. Zink. W. Charles Holland of Bowling Green State University has organized a special session on Ordered Groups, to be held on Wednesday and Thursday mornings; the speakers will be Richard N. Ball, W. Russell Belding, Paul F. Conrad, Andrew M. W. Glass, Herbert A. Hollister, Justin T. Lloyd, Jorge Martinez, Stephen H. McCleary, Roberto A. Mena, Joe L. Mott, Norman R. Reilly, A. H. Rhemtulla, Motupalli Satyanarayan, Stuart A. Steinberg, and J. Roger Teller. Robert E. Huff and Peter D. Morris of Pennsylvania State University have organized a special session on Banach Spaces with the Radon-Nikodym Property, to be held all day Thursday and Friday; the speakers will be James B. Collier, William J. Davis, Joseph Diestel, Michael Edelstein, Gerald A. Edgar, Heinrich P. Lotz, Peter D. Morris, Terry J. Morrison, Billy J. Pettis, Robert R. Phelps, Haskell P. Rosenthal, Elias Saab, Francis Sullivan, and J. Jerry Uhl, Jr. Paul C. Kainen of Case Western Reserve University has organized a special session on Topological and Chromatic Graph Theory, to be held Tuesday afternoon, Wednesday morning, and all day Thursday; the speakers will be Michael O. Albertson, Seth R. Alpert, Ruth A. Bari, Frank R. Bernhart, Jonathan L. Gross, Gary M. Haggard, Pavol Hell, Daniel J. Kleitman, Roy B. Levow, Edward A. Nordhaus, Richard D. Ringeisen, Allen J. Schwenk, Saul Stahl, Arthur T. White II, and Herbert S. Wilf. Leroy M. Kelly of Michigan State University has organized a special session on Geometry of Metric Spaces, to be held Tuesday afternoon and Wednesday morning; the speakers will be J. Ralph Alexander, Jr., David C. Kay, Leroy M. Kelly, William A. Kirk, Anthony G. O'Farrell, Clinton M. Petty, Kenneth B. Stolarsky, and Hans S. Witsenhausen. Pierre J. Malraison, Jr. of Carleton College has organized a special session on Categorical Methods in Algebraic Topology, to be held all day Friday; the speakers will be John M. Boardman, Martin Fuchs, Dana May Latch, Pierre J. Malraison, Jr. (speaking on a paper of Jonathan M. Beck), Marvin V. Mielke, and R. Neil Vance. Tilla K. Milnor of Rutgers University has organized a special session on Riemannian Geometry, to be held on Wednesday, Thursday, and Friday mornings; the speakers will be Stephanie B. Alexander, Bang-Yen Chen, Chang Shing Chen, Joseph E. D'Atri, Harold G. Donnelly, Patrick B. Eberlein, Robert B. Gardner, Peter B. Gilkey, Herman R. Gluck, Mikhael Gromov, Ravindra S. Kulkarni, Dominic S.P. Leung, William H. Meeks III, David A. Singer, Ann K. Stehney, James R. Wason, and Edward N. Wilson. David E. Muller of the University of Illinois has organized a special session on Theoretical Computer Science, to be held Thursday afternoon; the speakers will be Ellis D. Cooper, Chung Laung Liu, Franco P. Preparata, and Shmuel Winograd. Peter J. Nyikos of the University of Illinois has organized a special session on General Topology, to be held on Wednesday, Thursday, and Friday mornings and on Friday afternoon; the speakers will be Charles E. Aull, Paul J. Bankston, Raymond F. Dickman, Jr., Gary F. Gruenhage, Charles L. Hagopian, Robert W. Heath, M. Jayachandran, Ronnie Fred Levy, Stanislaw G. Mrowka, Louis F. McAuley, Minakshisundaram Rajagopalan, George M. Reed, Mary Ellen Rudin, James C. Smith, Jr., Franklin D. Tall, Eric K. van Douwen, Jerry E. Vaughan, Henry E. White, Jr., Howard H. Wicke, Scott W. Williams, and Philip L. Zenor. Hans Schneider of the University of Wisconsin has organized a special session on Numerical Ranges for Matrices and Other Operators on Normed Spaces, to be held Tuesday afternoon and Wednesday morning; the speakers will be Joel H. Anderson, Charles S. Ballantine, Earl R. Berkson, Moshe Goldberg, James E. Jamison, Charles R. Johnson, Marvin Marcus, Heydar Radjavi, B. David Saunders, and Joseph G. Stampfli. Peter J. Weinberger of the University of Michigan has organized a special session on Efficient Algorithms for Exact Computation, to be held Friday morning; the speakers will be Dan J. Hoey, Gary L. Miller, and Andrew C. Yao.

There will be sessions for contributed ten-minute papers on Tuesday afternoon, Wednesday morning, and all day Thursday and Friday.

There will be a Poster Session for contributed papers in Operator Theory from 3:30 p.m. to 5:00 p.m. on Tuesday, August 19. Poster Sessions provide an alternative method for presenting papers with which we are experimenting at this meeting. At the session participants will display their papers on easels or bulletin boards, and remain in the room set aside for this purpose to expand on the material and answer questions during the one and one-half hour session. The following persons will present contributed papers by this method: Arthur R. Lubin of Northwestern University and Stephen A. McGrath of the United States Naval Academy. Those attending the meeting are urged to visit the Poster Session and observe this experiment first-hand.

Rooms 1110 and 1111 in Brown Hall have been set aside as informal discussion rooms, and will be open daily from 8:00 a.m. to 6:00 p.m. to small groups desiring a quiet room with blackboard space to discuss mathematics. Room 1110 is available on a first-come, first-served basis; Room 1111 is available for one-hour periods only, and must be reserved in advance. A reservation form will be posted on the door to Room 1111 for individuals to sign up for use of this room. It is requested that discussion groups not be planned to conflict with business meetings or major lectures.

The AMS Committee on Employment and Educational Policy (CEEP) will sponsor a panel discussion on "The role of applications in Ph.D. programs in mathematics" on Thursday evening, August 21, at 8:00 p.m. Members of the panel will include Richard D. Anderson, former chairman of CEEP; Lipman Bers, president of the American Mathematical Society; Henry O. Pollak, president of the Mathematical Association of America; and Wendell H. Fleming, current chairman of CEEP, who will serve as moderator. An open meeting is planned for 4:30 p.m. on Monday, August 18, consisting of a brief report on the state of the job market and an open discussion with comments and suggestions welcomed from the audience. Wendell H. Fleming will moderate. Charles W. Curtis, chairman of the AMS Committee on the Emergency Employment Situation in Mathematics; Richard D. Anderson, chairman of the Data Subcommittee of CEEP; and others will participate in the discussion.

This meeting of the Society will be held in conjunction with the annual meetings of the Mathematical Association of American and Pi Mu Epsilon. The Mathematical Association of America will meet from Monday, August 18, through Wednesday, August 20. The twenty-third series of the Earle Raymond Hedrick Lectures, sponsored by the Association, will be given by Frederick J. Almgren, Jr., Princeton University. The title of his lectures is "Geometric measure theory and the calculus of variations." At the Business Meeting of the Association at 10:00 a.m. on Tuesday, August 19, the Lester R. Ford Awards will be presented.

J. Sutherland Frame, Michigan State University, will address Pi Mu Epsilon on Tuesday, August 19, at 8:00 p.m.; the title of his lecture will be "Matrix functions: a powerful tool."

The Association for Women in Mathematics will hold a panel discussion on "Noether to now—the woman mathematician" on Tuesday, August 19, at 3:30 p.m. The Mathematicians Action Group will hold a discussion on "Unemployment: an exchange of experiences" at 4:30 p.m. on Wednesday, August 20. All mathematicians who have recently experienced unemployment or who anticipate being unemployed in the near future are especially welcome.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet at 5:00 p.m. on Tuesday, August 19, in the Dean's Conference Room (Room 2010) Friedmann Hall (not in the Green Room of Miller Auditorium as previously announced).

The Business Meeting of the Society will be held in Miller Auditorium at 4:00 p.m. on Thursday, August 21. The secretary notes the following resolution of the Council: Each person who attends a Business Meeting of the Society shall be willing and able to identify himself as a member of the Society. In further explanation, it is noted that "each person who is to vote at a meeting is thereby identifying himself as and claiming to be a member of the American Mathematical Society."

In accord with Article X, Section 1, of the bylaws of the Society, the Business Meeting of January 24, 1975, in Washington, D.C. has directed that each of the following motions be placed on the agenda of the Business Meeting of August 21, 1975, in Kalamazoo:

Motion I. In the face of the deepening economic crisis, the rapidly rising unemployment from which mathematicians are not exempt, the ominous nature of massive budget cuts for education and other social services, and the Presidential request for an immediate increase of 300 million for military aid to the Thieu regime augmenting the already huge military budget, it is the sense of this meeting that the federal government must: (1) Fund a national open admissions program at institutions of higher education (2) Fund a massive public works program which will use the skills of the presently and soon-to-be unemployed (including mathematicians) for sorely needed socially useful tasks (3) Transfer massive funds from the military budget to accomplish these aims. We call upon the officers of the Society to work towards effecting the implementation of the above.

Motion II. Resolved that the AMS will not cooperate with UNESCO until such time as the ruling removing Israel from any regional grouping is rescinded.

Motion III. That the officers of the Society be requested to arrange that an early meeting of the Society schedule an open session intended to discuss the possible establishment of an independent ASSOCIATION OF MATHEMATICIANS FOR SOCIAL ACTION.

This announcement constitutes the notice to the full membership required in the bylaws. At the meeting in August, each motion is subject to substantive changes, such as germane amendment or substitution, and is subject to subsidiary resolutions concerning the disposition of the main motion.

Panel discussions have been scheduled on Motions I and II. Motion I will be discussed at 9:30 p.m. on Monday, August 18. Motion II will be discussed at 3:30 p.m. on Tuesday, August 19. Time has been allotted for audience participation in both cases.

AMENDMENT TO THE BYLAWS

The Council at its meeting of April 11, 1975, recommended the following amendment to Article XI, Section 2, of the bylaws:

> The editorial management of the *Notices* shall be in the hands of a committee [consisting of the executive director and the secretary] *chosen in a manner established by the Council.*

Here the words in brackets are to be deleted and the words in italics added. The intent is to make it possible to replace the two existing editors with a larger and more ecumenical committee to consider, in particular, articles and Letters to the Editor. The amendment becomes effective if adopted by a two-thirds vote of the members present at the Business Meeting. This paragraph constitutes the required notice of action.

MEETING PREREGISTRATION AND REGISTRATION

Registration for the short course only will begin on Friday, August 15. Lecture notes and other short course material will be distributed before the first session at the short course registration desk. Those individuals who do not preregister for the short course are strongly urged to register and pick up their material Friday evening in the dormitory, so as not to miss the start of the lecture on Saturday morning. General meeting registration will commence on Sunday, August 17, at 10:30 a.m. Participants who are not attending the short course are advised that no general meeting information (or registration material) will be available prior to the time listed below for the Joint Mathematics Meetings registration. Upon arrival at the Western Michigan University campus, participants should proceed directly to the reception desk, Harvey Garneau Hall, Goldsworth Valley Residence Hall Complex #2, in order to check in to their accommodations and purchase meal tickets, if desired.

Following are the hours that the desks will be open as well as the respective locations:

Applied Combinatorics Short Course

Date and Time	Location
Friday, August 15 4:30 p.m.-7:30 p.m.	Reception Desk Lobby, Harvey Garneau Hall, Goldsworth Valley Residence Hall Complex #2
Saturday, August 16 8:00 a.m.-4:00 p.m.	1104 Rood Hall Lobby
Sunday, August 17 12:00 noon-2:00 p.m.	1104 Rood Hall Lobby

Joint Mathematics Meetings

Date and Time	Location
Sunday, August 17 10:30 a.m.-4:30 p.m.	Miller Auditorium Lobby, 2nd level
Monday, August 18 8:00 a.m.-4:30 p.m.	Miller Auditorium Lobby, 2nd level
Tuesday, August 19 to Thursday, August 21 8:30 a.m.-4:30 p.m.	Miller Auditorium Lobby, 2nd level
Friday, August 22 8:30 a.m.-1:30 p.m.	Miller Auditorium Lobby, 2nd level

Please note the change in hours of operation on Sunday of the Joint Mathematics Meetings registration desk from 2:00 p.m.-8:00 p.m.

Please note that separate registration fees are required for the Short Course and the Joint Meetings. These fees are as follows:

Applied Combinatorics Short Course

	At Meeting
All participants	$15

Joint Mathematics Meetings

	At Meeting
Member	$12
Student or unemployed member	$ 2
Nonmember	$20

There will be no extra charge for members of the families of registered participants except that all professional mathematicians who wish to attend sessions must register independently.

The unemployed status refers to any member currently unemployed and actively seeking employment. It is not intended to include members who have voluntarily resigned or retired from their latest position. Students are considered to be only those currently working toward a degree who do not receive an annual compensation totaling more than $7,000 from employment, fellowships, and scholarships.

A fifty percent refund of the preregistration fee will be made for all cancellations received in Providence prior to August 16. There will be no refunds granted for cancellations received after that date or to persons who do not attend the meetings.

MATHEMATICAL SCIENCES EMPLOYMENT REGISTER

An experimental variant of the open register will be operated on a limited basis during the meeting, providing an opportunity for applicants and employers to arrange interviews at their mutual convenience.

Employment Register headquarters, located in the Green Room of Miller Auditorium, will be open on Tuesday, Wednesday, and Thursday (August 19, 20 and 21) from 8:30 a.m. to 4:30 p.m. The room will be closed from noon to 1:15 p.m.

There will be no interviews scheduled by the staff. Instead, facilities will be provided for applicants and employers to display resumes and listings. Message boxes will be set up for individuals to leave messages for one another requesting interviews. Tables and chairs will be provided in the room for interviews. Employers are encouraged to attend the meetings and participate if possible. Applicants should recognize that the Mathematical Sciences Employment Register cannot guarantee that any employers will in fact attend the meeting or be able to participate. The AMS-MAA-SIAM Committee on Employment Opportunities has requested employers listing in the June and July 1975 issues of Employment Information for Mathematicians to signify in their listing their intention of participating in the open register at the summer meeting.

EXHIBITS

The book and educational media exhibits will be displayed on the second lobby level of Miller Auditorium at the following times: August 18 (Monday), noon to 4:30 p.m.; August 19–20 (Tuesday and Wednesday), 8:30 a.m. to 4:30 p.m.; and August 21 (Thursday), 8:30 a.m. to noon. All participants are encouraged to visit the exhibits sometime during the meeting.

RESIDENCE HALL HOUSING

The deadline for making reservations for dormitory accommodations was August 1, 1975. Those hoping to make late reservations are advised to call or write Mr. William S. Binning, Conference Center, Harrison-Stinson Residence Hall, Western Michigan University, Kalamazoo, Michigan 49008. A reservation form will be found on the last page of the June issue of these *Notices*. No guarantee can be made that late requests will be filled.

The Goldsworth Valley Residence Hall Complex #2 has been set aside for the exclusive use of the Mathematics Meetings participants and for participants of the Applied Combinatorics Short Course. The dormitories are within a ten to fifteen minute walk from the Miller Auditorium and the meeting rooms which will be used during the meetings. Frequent shuttle bus service will be provided between the dormitories and the meeting area. There is a twenty-five cent ($0.25) fare for each one-way trip. The cash fare will be collected by the bus driver; it would be appreciated if exact fares are paid by participants to avoid the necessity of having drivers carry excess money.

Most of the sleeping rooms are in suites of two double rooms which share a bath. Linens, towels, and daily maid service (beds made only) are provided with all rooms. Each room contains the following furniture: two twin beds, a chest of drawers, a lounge chair, a desk, and two study chairs. Keys, curtains, glasses, and soap are also provided. Each dormitory has a fully equipped laundry room with coin-operated washers and dryers. Ironing facilities are also available. A limited number of irons are available at dormitory desks.

Residence hall rooms can be occupied from noon on Friday, August 15, to noon on Sunday, August 17, for participants in the Applied Combinatorics Short Course; and from noon on Sunday, August 17, to noon on Saturday, August 23, for Joint Meeting participants. Clerks will be available on call in each housing unit twenty-four hours a day. Parking for residents will be available free of charge in lots near the dormitories. The daily rate per person is as follows:

Singles	$8.00 per person per day
Doubles	$5.75 per person per day

Children will be housed at the regular rates in rooms adjacent to their parents. Cribs and cots are not available from the university and sleeping bags are not permitted. An infant may occupy the parents' room at no extra charge if the parents supply a crib and bedding. A limited number of cribs are available for rent by writing in advance to United Rent-All, 403 Balch Street, Kalamazoo, Michigan 49003. Pets are not permitted in the dormitories.

HOTELS AND MOTELS

There are a number of hotels and motels in the area which are listed below. All prices are subject to change without notice; a six percent tax should be added to the room rates listed. Participants should make their own reservations. The following codes apply: FP - Free Parking; SP - Swimming Pool; AC - Air Conditioned; TV - Television; CL - Cocktail Lounge; RT - Restaurant.

HOLIDAY INN - CROSSTOWN (616) 349-6711
220 E. Crosstown (8 blocks south of downtown Kalamazoo)
147 rooms

Singles			$14.00
Doubles	1 Bed	2 Persons	18.00
	2 Beds	2 Persons	19.00
Studios		1 Person	15.00
		2 Persons	19.00
Twins	2 Beds	2 Persons	19.00
Extra person in room			4.00

Code: FP-SP-AC-TV-CL-RT
1-1/2 miles from campus

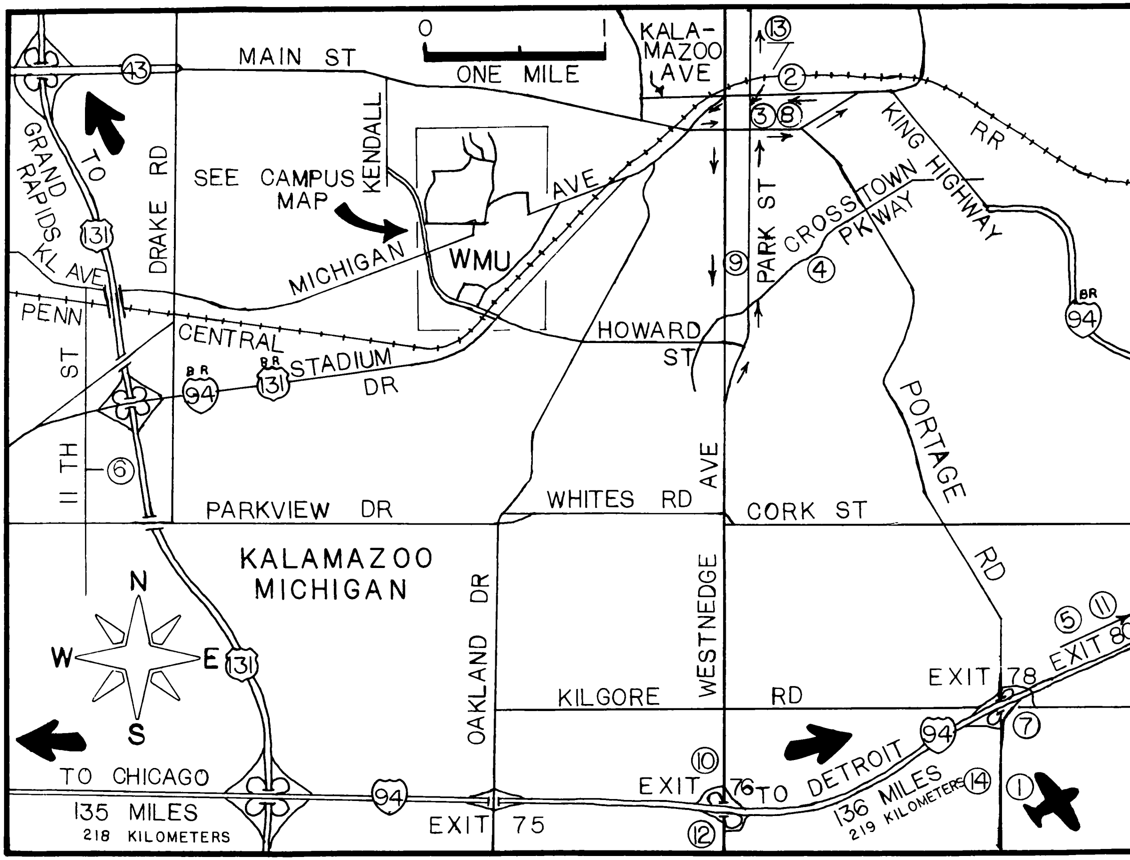
HOLIDAY INN - EXPRESSWAY (616-381-7070)
3522 Sprinkle Road (Use Exit 80 at I-94, turn south).
146 rooms

Singles	1 Bed	1 Person	$16.00
	1 Bed	2 Persons	20.00
Doubles	2 Beds	2 Persons	21.00
	2 Beds	3 Persons	25.00
Twins	2 Beds	4 Persons	29.00
Suites	1 Person		20.00
	(two available only)		
	2 Persons		24.00

Code: FP-SP-AC-TV-CL-RT
7 miles from campus

HOLIDAY INN - WEST (616) 375-6000
2747 - 11th Street (Use Oshtemo Exit at U.S. 131, turn west)
118 rooms

Singles	1 person	$17.00
	2 persons	21.00
Doubles	1 person	17.00
	2 persons	22.00
Twins	Same as a double	



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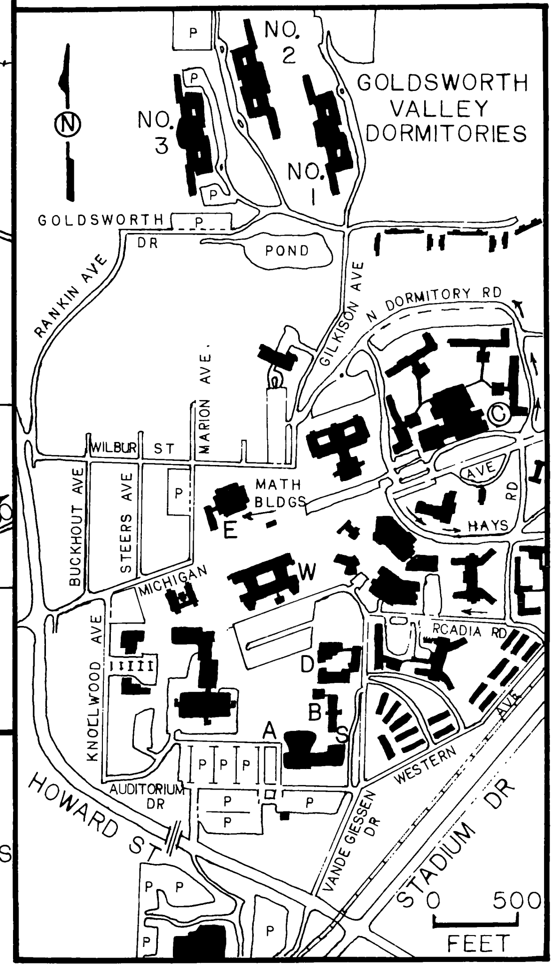

- ① AIRPORT
- ② AMTRAK STATION
- ③ BUS STATION
- ④ HOLIDAY INN-CROSSTOWN
- ⑤ HOLIDAY INN-EXPRESSWAY
- ⑥ HOLIDAY INN-WEST
- ⑦ HOWARD JOHNSON'S
- ⑧ KALAMAZOO CENTER HOTEL
- ⑨ KALAMAZOO TRAVELODGE
- ⑩ RAMADA INN
- ⑪ RED ROOF INN
- ⑫ SOUTHGATE MOTOR INN
- ⑬ VALLEY INN MOTEL
- ⑭ Y-MASTER MOTOR INN
- Ⓐ MILLER AUDITORIUM
- Ⓑ BROWN HALL
- Ⓒ STUDENT CENTER
- Ⓓ DUNBAR-FRIEDMANN-KNAUSS
- Ⓔ EVERETT-ROOD HALLS
- Ⓢ SHAW THEATRE
- Ⓦ WOOD HALL

Suites	1 person	18.00
	2 persons	23.00
Extra person in room		4.00

Code: FP-SP-AC-TV-CL-RT
3 miles from campus

HOWARD JOHNSON'S (616) 382-2303
1912 East Kilgore (Exit 78 off I-94; turn south)
70 rooms

Singles		$15.00
Doubles		19.00
Twins	1 person	16.00
	2 persons	21.50
Extra person in room		4.00

Code: FP-SP-AC-TV-CL-RT
6 miles from campus

KALAMAZOO CENTER HOTEL (616) 381-2130
100 West Michigan (Downtown Kalamazoo)
288 rooms

Singles	$22.00
Doubles	30.00
Suites	30.00- 76.00
Extra person in room (12 years and older)	6.00

Code: FP-SP-AC-TV-CL-RT
2 miles from campus

KALAMAZOO TRAVELODGE (616) 381-5000
(Toll free number 1-800-255-3050)
1211 S. Westnedge (Exit 76-B off I-94; turn north)
57 rooms

Singles	$13.00
Doubles	15.00
Twins	17.00
Extra person in room	2.00
Rollaway bed	3.00

Code: FP-AC-TV
1-1/2 miles from campus

RAMADA INN (616) 382-1000
5300 S. Westnedge (Exit 76 off I-94; turn north)
102 rooms

Singles	$15.50
Doubles	18.50
Twins	20.50
Suites (one available only)	24.50
Extra person in room	3.00

Code: FP-SP-AC-TV-CL-RT
5 miles from campus

RED ROOF INN (616) 382-6350
3701 E. Cork (Exit 80 off I-94, Sprinkle Road)
79 rooms

Singles	$ 9.50
Doubles	12.50
Twins	13.50
Extra person in room	3.00

Code: FP-AC-TV
4-1/2 miles from campus

SOUTHGATE MOTOR INN (616) 343-6143
5630 S. Westnedge (Exit 76 off I-94)
125 rooms

Singles	$14.00
Doubles	16.50
Twins	18.00
Extra person in room	3.00

Code: FP-SP-AC-TV-CL-RT
5 miles from campus

VALLEY INN MOTEL (616) 349-9736
200 N. Park (Downtown Kalamazoo)
107 rooms

Singles	$14.50
Doubles	20.00
Twins	20.00
Suites	25.00 and 30.00
Extra person in room	4.00

Special group rates available
Code: FP-SP-AC-TV-CL-RT
2 miles from campus

Y-MASTER MOTOR INN (616) 345-8603
2333 Helen (1/4 mile south of I-94, off Portage Road opposite airport)
50 rooms

Singles	$10.50
Doubles	14.50
Twins	15.50
Studios	20.00
Suites	25.00
Extra person in room	3.00

Code: FP-SP-AC-TV
10 miles from campus

FOOD SERVICES

The Goldsworth Valley #2 Residence Hall Cafeteria will be open starting with breakfast on Monday, August 18. The cafeteria will continue serving through lunch on Friday, August 22, with the exception of the evening meal on Wednesday, August 20. Hours of service and prices for individual meals are:

		Children	
Hours	Adults	Ages 7-12	Age 6 and Under
Breakfast 7:15 a.m. - 8:30 a.m.	$2.00	$1.50	$1.00
Lunch 11:30 a.m.-12:45 p.m.	2.50	2.00	1.25
Dinner 5:00 p.m. - 6:30 p.m.	4.50	3.25	2.25

A package plan including all meals served from breakfast on Monday, August 18, to lunch on Friday, August 22, (with the exception of the evening meal on Wednesday, August 20) will be available at the dormitory check-in desk at a price of $32 for adults, $24.75 for children from seven to twelve years of age, and $16 for children six years of age and under.

Light snacks and beverages will be available at the snack bar in the University Student Center; the hours of operation will be posted in appropriate areas during the meeting.

For participants in the Short Course, the University Student Center Cafeteria and/or the snack bar will be open for meals. The hours of operation will be posted in the dormitories.

PARKING

Parking permits will be required for parking on all areas of the campus, with the exception of metered lots. Parking will be free of charge in the lots near the dormitories for persons residing there, and in the lots near Miller

Auditorium. Maps showing the location of the various college parking lots will be available at the Local Information Desk along with the permits. There is no charge for the permits.

CAMPING

The following campgrounds are located approximately one-half hour or less from Western Michigan University:

1. Klines Resort, Route 2, Box 257, Three Rivers, 49093
   Telephone: 616-649-2514
   Travel Time: 35 minutes from W. M. U.
   Facilities: Electric hook-ups, bathrooms, lakeside swimming, boat rentals, 40 sites
   Cost: $4.00 per night, $24.00 per week, reservations accepted

2. Oak Shores Resort, Route 3, 28th Street, Vicksburg, 49007
   Telephone: 616-649-1310
   Travel Time: 30–25 minutes from W. M. U.
   Facilities: 80 acre lake, swimming, boating, water and electricity at sites, also sewer hook-up available, club area, 93 sites available
   Cost: Water and electricity $4.00/day, sewer hook-up $4.50/day

3. Schnable Lake Campground, 11th Street, Martin, 49070
   Telephone: 616-672-7524
   Travel Time: 30 minutes from W. M. U.
   Facilities: 45 acre lake, swimming, canoeing, water and electricity are provided, 93 camp sites
   Cost: $4.00 per night, deposits required

4. Shady Bend Park, 15320 Augusta Drive, Augusta, 49012
   Telephone: 616-731-4503
   Travel Time: 25 minutes from W. M. U.
   Facilities: The pavillion houses showers and toilets, spring fed pond, canoeing, 62 sites available
   Cost: $3.50 per night

5. Willow Lake Campground, Box 295, Three Rivers, 49093
   Telephone: 616-279-7920
   Travel Time: 20 minutes from W. M. U.
   Facilities: Tent camping, water, campfires, toilet facilities, private fishing lake on 110 acres, 51 sites available
   Cost: $3.00 per day

BOOKSTORES

The Campus Bookstore is located in the University Student Center. Its hours of operation are from 8:00 a.m. to 5:00 p.m., Monday through Friday. The University Bookstore (private) at 2529 W. Michigan is open from 9:00 a.m. to 5:00 p.m. Monday through Friday, and from 10:00 a.m. to 3:00 p.m. on Saturday. The Book Raft (private) at 2624 W. Michigan is open from 10:00 a.m. to 7:30 p.m. Monday through Thursday, from 10:00 a.m. to 9:30 p.m. on Friday and Saturday, and from 10:00 a.m. to 5:00 p.m. on Sunday. The latter two are also situated near the campus area.

LIBRARIES

The mathematics library, including current mathematical journals and books, is part of the Physical Sciences Library on the third floor of Rood Hall, and is open from 8:00 a.m. to midnight Monday through Thursday, from 8:00 a.m. to 5:00 p.m. on Friday, from 10:00 a.m. to 5:00 p.m. on Saturday, and from 1:00 p.m. to midnight on Sunday. Information can also be obtained in the mathematics library regarding the location of books in other areas. The main collection of other books is in the Waldo Library, which is open from 8:00 a.m. to 11:00 p.m., Monday through Thursday, 8:00 a.m. to 5:00 p.m. on Friday, 10:00 a.m. to 5:00 p.m. on Saturday, and from 1:00 p.m. to 11:00 p.m. on Sunday.

The Kalamazoo Public Library has its main branch located at 315 S. Rose. The hours of operation are from 9:00 a.m. to 9:00 p.m. Monday through Friday, from 9:00 a.m. to 6:00 p.m. on Saturday, and from 2:00 p.m. to 6:00 p.m. on Sunday.

MEDICAL SERVICES

Kalamazoo is served by Borgess Hospital and Bronson Methodist Hospital. The emergency rooms there are staffed around the clock. The Kalamazoo Academy of Medicine can also make referrals Monday through Friday, from 8:00 a.m. to noon, and from 1:00 p.m. to 5:00 p.m. (Telephone: 342-8502). Referrals to dentists can be made by calling 381-0400 during usual office hours. For dental emergency service, the dentist on call may be reached through the Bronson Methodist Hospital. Additional information will be available at the Mathematics Meetings Registration Desk.

ENTERTAINMENT AND RECREATION

Western Michigan University has planned a program of recreation and entertainment for mathematicians and their families. It is hoped that as many people as possible will take advantage of these activities.

A picnic is planned for Wednesday, August 20, at 6:00 p.m. The menu will be barbecued chicken and baked ham; no alcoholic beverages will be served. The cost will be $5 per adult, $4 for children between the ages of seven and twelve, and $2.50 for children six years of age and under.

A beer party has been arranged to follow the picnic from 9:00 p.m. to midnight at the Holiday Inn-West. Soft drinks in cans will also be available for those who prefer them. Potato chips and similar snacks will be served. The cost is $2.50 per person. Bus service from the campus to the Holiday Inn-West and back will be available; the fare will be $0.25 each way.

Tickets to both the picnic and beer party will be sold in advance at the Local Information Desk in the Miller Auditorium Lobby, but can also be purchased at each event.

A tour of the Upjohn pharmaceutical plant, whose home office is in Kalamazoo, is planned for Tuesday morning, August 19. Space on this tour is limited, and it will be necessary to make

(text continued on page 221)

TIMETABLE

(Eastern Daylight Time)

	AMERICAN MATHEMATICAL SOCIETY	
FRIDAY, August 15	SHORT COURSE ON APPLICATIONS OF COMBINATORICS	
4:30 p.m. - 7:30 p.m.	REGISTRATION (Short Course Only) Reception Desk Lobby, Harvey Garneau Hall Goldsworth Valley Residence Hall Complex #2	
SATURDAY, August 16		
8:00 a.m. - 4:00 p.m.	REGISTRATION (Short Course Only) - 1104 Rood Hall Lobby	
	1104 ROOD HALL	
9:00 a.m. - 10:15 a.m.	Some problems involving graphs D. R. Fulkerson	
10:45 a.m. - 12:00 noon	Combinatorial scheduling theory I Ronald L. Graham	
2:00 p.m. - 3:15 p.m.	A survey of algebraic coding theory I Elwyn R. Berlekamp	
3:45 p.m. - 5:00 p.m.	Combinatorial scheduling theory II Ronald L. Graham	
SUNDAY, August 17		
12:00 noon - 2:00 p.m.	REGISTRATION (Short Course Only) - 1104 Rood Hall Lobby	
	1104 ROOD HALL	
2:00 p.m. - 3:15 p.m.	A survey of algebraic coding theory II Elwyn R. Berlekamp	
3:45 p.m. - 5:00 p.m.	Integer programming Ellis J. Johnson	

	AMS — MAA SUMMER MEETINGS	
SUNDAY, August 17	American Mathematical Society	Mathematical Association of America
9:00 a.m. - 4:00 p.m.		MAA - BOARD OF GOVERNORS Dean's Conference Room (2010), Friedmann Hall
10:30 a.m. - 4:30 p.m.	REGISTRATION - 2nd Level, Miller Auditorium Lobby	
7:00 p.m. - 9:30 p.m.		MAA - FILM PROGRAM Shaw Theatre
7:00 p.m. - 7:25 p.m.		Logic II: a BBC Broadcast as part of the Open University Foundation Course in Mathematics (black and white)
		Films of the College Geometry Project
7:30 p.m. - 7:43 p.m.		Dihedral kaleidoscopes
7:45 p.m. - 8:01 p.m.		Curves of constant width
8:05 p.m. - 8:21 p.m.		Projective generation of conics
		Films of the MAA Calculus Film Series
8:25 p.m. - 8:45 p.m.		What is area?
8:50 p.m. - 9:04 p.m.		Area under a curve
9:08 p.m. - 9:30 p.m.		Shapes of the future I: Some unsolved problems in geometry—two dimensions with Victor Klee, a film of the MAA Individual Lectures Film Project
MONDAY, August 18	AMS	MAA
8:00 a.m. - 4:30 p.m.	REGISTRATION - 2nd Level, Miller Auditorium Lobby	
9:00 a.m. - 9:15 a.m.	WELCOME ADDRESS John Bernhard, President Western Michigan University Miller Auditorium	
9:15 a.m. - 10:15 a.m.		THE EARLE RAYMOND HEDRICK LECTURES I: Geometric measure theory and the calculus of variations Frederick J. Almgren, Jr. Miller Auditorium

MONDAY, August 18	American Mathematical Society	Mathematical Association of America
10:30 a.m. - 11:30 a.m.	AMS-MAA PANEL DISCUSSION: The mathematics accreditation question Wade Ellis Daniel T. Finkbeiner II (moderator) Shirley A. Hill Calvin T. Long Miller Auditorium	
11:30 a.m. - 12:00 noon	General discussion by panel and audience	
12:00 noon - 4:30 p.m.	EXHIBITS - 2nd Level, Miller Auditorium Lobby	
1:30 p.m. - 2:30 p.m.		THE EARLE RAYMOND HEDRICK LECTURES II Frederick J. Almgren, Jr. Miller Auditorium
3:00 p.m. - 4:00 p.m.		INVITED ADDRESS Mathematical models in cancer research James R. Thompson, Miller Auditorium
4:30 p.m. - 5:30 p.m.	AMS Committee on Employment and Educational Policy OPEN MEETING: The state of the job market Richard D. Anderson Charles W. Curtis Wendell H. Fleming (moderator) Miller Auditorium	
7:00 p.m. - 9:40 p.m.		MAA - FILM PROGRAM Shaw Theatre
7:00 p.m. - 8:03 p.m.		NIM and other oriented graph games (Andrew Gleason), a film of the MAA Mathematics Today Series (black and white)
8:10 p.m. - 8:35 p.m.		Space filling curves, a film of the Topology Film Project
8:40 p.m. - 8:55 p.m.		The theorem of the mean, a film of the MAA Calculus Film Series
9:00 p.m. - 9:10 p.m.		Linear programming
9:15 p.m. - 9:40 p.m.		Fourier Series—a B. B. C. Broadcast as part of the Open University's course Linear Mathematics
9:30 p.m. - 10:30 p.m.	AMS PANEL DISCUSSION: Motion I Chandler Davis (moderator) Shaw Theatre	

TUESDAY, August 19	AMS	Other Organizations
8:30 a.m. - 4:30 p.m.	REGISTRATION - 2nd Level, Miller Auditorium Lobby	
8:30 a.m. - 4:30 p.m.	EXHIBITS - 2nd Level, Miller Auditorium Lobby	
8:30 a.m. - 4:30 p.m.	EMPLOYMENT REGISTER - Green Room, Miller Auditorium	
9:00 a.m. - 10:00 a.m.		THE EARLE RAYMOND HEDRICK LECTURES III Frederick J. Almgren, Jr. Miller Auditorium
10:15 a.m. - 11:15 a.m.		MAA - BUSINESS MEETING Presentation of Lester R. Ford Awards Miller Auditorium
11:30 a.m. - 12:30 p.m.		MAA - INVITED ADDRESS Some recent applications of graph theory Ronald L. Graham, Miller Auditorium
12:00 noon		Pi Mu Epsilon - COUNCIL LUNCHEON Cafeteria, Goldsworth Valley Residence Hall #2
12:00 noon - 1:00 p.m.		Mathematicians Action Group Steering Committee - OPEN MEETING 3750 Knauss Hall
1:00 p.m. - 2:00 p.m.	COLLOQUIUM LECTURES: Differential algebraic groups, Lecture I Ellis R. Kolchin, Miller Auditorium	
2:15 p.m. - 3:15 p.m.	COLLOQUIUM LECTURES: Singular integrals, old and new, Lecture I Elias M. Stein, Miller Auditorium	
3:00 p.m. - 5:00 p.m.		ΠΜΕ - CONTRIBUTED PAPERS 1118 Rood Hall

TUESDAY, August 19	American Mathematical Society	Other Organizations
3:30 p.m. - 4:30 p.m.	AMS PANEL DISCUSSION: Motion II John W. Milnor (moderator) Miller Auditorium	
	POSTER SESSION	
3:30 p.m. - 5:00 p.m.	Operator Theory 2750 Knauss Hall	
	SPECIAL SESSIONS	
3:30 p.m. - 4:50 p.m.	Topological and Chromatic Graph Theory I 1104 Rood Hall	
3:30 p.m. - 5:20 p.m.	Numerical Ranges for Matrices and Other Operators on Normed Linear Spaces I 3770 Knauss Hall	
3:30 p.m. - 5:20 p.m.	Geometry of Metric Spaces I 3750 Knauss Hall	
	SESSIONS FOR CONTRIBUTED PAPERS	
3:30 p.m. - 4:40 p.m.	Algebra 2480 Dunbar Hall	
3:30 p.m. - 4:40 p.m.	Classical Analysis 3480 Dunbar Hall	
3:30 p.m. - 4:55 p.m.	Differential Geometry and Algebraic Topology 3760 Knauss Hall	
3:30 p.m. - 4:40 p.m.	General Topology 1114 Brown Hall	
3:30 p.m. - 4:25 p.m.	Applications 2520 Dunbar Hall	
3:30 p.m. - 5:00 p.m.		Association for Women in Mathematics PANEL DISCUSSION: Noether to now—the woman mathematician Miller Auditorium Superstar syndrome Lenore Blum Black and female Vivienne Mays The transition between graduate school and a permanent job M. Susan Montgomery Twenty-five years a woman mathematician Mary Ellen Rudin Remarks on some irrelevant experience Jane Cronin Scanlon Alice T. Schafer (moderator)
		AWM - BUSINESS MEETING Miller Auditorium
5:00 p.m.	COUNCIL MEETING Dean's Conference Room (2010), Friedmann Hall	
6:30 p.m.		ΠME BANQUET Room Series #157, University Student Center Cafeteria
7:00 p.m. - 9:15 p.m.		MAA - FILM PROGRAM Shaw Theatre Films produced by C. B. Allendoerfer
7:00 p.m. - 7:25 p.m.		Gauss-Bonnet theorem
7:30 p.m. - 7:52 p.m.		Cycloidal curves or Tales from the Wanklenberg Woods
8:00 p.m.		Allendoerfer Films of the MAA Arithmetic Films
8:00 p.m. - 8:10 p.m.		Area and pi
8:13 p.m. - 8:20 p.m.		Associative property

TUESDAY, August 19	American Mathematical Society	Other Organizations
		MAA - FILM PROGRAM
8:25 p.m. - 8:35 p.m.		Binary operations and commutative property
8:37 p.m. - 8:45 p.m.		Distributive property
8:50 p.m. - 9:00 p.m.		Geometric concepts
9:05 p.m. - 9:15 p.m.		Geometric transformations
8:00 p.m.		ΠΜΕ - INVITED LECTURE Matrix functions: a powerful tool J. Sutherland Frame 1104 Rood Hall

WEDNESDAY, August 20	AMS	Other Organizations
8:00 a.m.		ΠΜΕ DUTCH TREAT BREAKFAST Room Series #157, University Student Center Cafeteria
	SPECIAL SESSION	
8:00 a.m. - 12:00 noon	Ordered Groups I 2480 Dunbar Hall	
8:30 a.m. - 4:30 p.m.	REGISTRATION - 2nd Level, Miller Auditorium Lobby	
8:30 a.m. - 4:30 p.m.	EXHIBITS - 2nd Level, Miller Auditorium Lobby	
8:30 a.m. - 4:30 p.m.	EMPLOYMENT REGISTER - Green Room, Miller Auditorium	
8:30 a.m. - 9:30 a.m.	COLLOQUIUM LECTURES II Ellis R. Kolchin, 3750 Knauss Hall	
8:30 a.m. - 9:30 a.m.	COLLOQUIUM LECTURES II Elias M. Stein, 3770 Knauss Hall	
	SPECIAL SESSIONS	
9:00 a.m. - 12:00 noon	Topological and Chromatic Graph Theory II 1104 Rood Hall	
9:00 a.m. - 11:50 a.m.	Numerical Ranges of Matrices and Other Operators on Normed Linear Spaces II 170 Wood Hall	
9:00 a.m. - 12:00 noon	Riemannian Geometry I 3760 Knauss Hall	
9:00 a.m. - 11:50 a.m.	General Topology I 1114 Brown Hall	
	SESSIONS FOR CONTRIBUTED PAPERS	
9:30 a.m. - 10:40 a.m.	Complex Analysis 2520 Dunbar Hall	
9:45 a.m. - 10:40 a.m.	Probability 3480 Dunbar Hall	
9:45 a.m. - 10:45 a.m.	INVITED ADDRESS: Nearly trivial outer automorphisms of finite groups Everett C. Dade, Miller Auditorium	
10:00 a.m.		MAG - BUSINESS MEETING Shaw Theatre
10:00 a.m. - 12:00 noon		ΠΜΕ - CONTRIBUTED PAPERS 1118 Rood Hall
	SPECIAL SESSIONS	
10:00 a.m. - 11:50 a.m.	Functional Equations I 3770 Knauss Hall	
10:00 a.m. - 11:50 a.m.	Geometry of Metric Spaces II 3750 Knauss Hall	
	SESSION FOR CONTRIBUTED PAPERS	
10:45 a.m. - 11:55 a.m.	Functional Analysis 3480 Dunbar Hall	
11:00 a.m. - 12:00 noon	INVITED ADDRESS: On the classification of Kleinian groups Bernard Maskit, Miller Auditorium	
	SESSION FOR CONTRIBUTED PAPERS	
11:00 a.m. - 11:40 a.m.	Group Theory 2520 Dunbar Hall	
1:30 p.m. - 2:30 p.m.		MAA - INVITED ADDRESS: Error correcting codes: Practical origins and mathematical applications Vera T. Pless, Miller Auditorium

WEDNESDAY, August 20	American Mathematical Society	Other Organizations
2:45 p.m. - 3:45 p.m.		MAA - PANEL DISCUSSION: Training of nondoctoral mathematics students for non-academic employment David C. Bossard Alan Karr Dale W. Lick (moderator) Werner Ulrich Miller Auditorium
3:45 p.m. - 4:15 p.m.		General discussion by panel and audience
4:30 p.m. - 6:00 p.m.		MAG - DISCUSSION: Unemployment: an exchange of experiences Paul Green (moderator) 2480 Dunbar
6:00 p.m.	PICNIC - Goldsworth Valley	
9:00 p.m. - 12:00 midnight	BEER PARTY - Holiday Inn-West	

THURSDAY, August 21	AMS	Other Organizations
	SPECIAL SESSIONS	
8:00 a.m. - 11:50 a.m.	Ordered Groups II 2480 Dunbar Hall	
8:00 a.m. - 11:50 a.m.	Functional Equations II 3760 Knauss Hall	
8:30 a.m. - 9:30 a.m.	COLLOQUIUM LECTURES III Ellis R. Kolchin, 3750 Knauss Hall	
8:30 a.m. - 9:30 a.m.	COLLOQUIUM LECTURES III Elias M. Stein, 3770 Knauss Hall	
8:30 a.m. - 4:30 p.m.	REGISTRATION - 2nd Level, Miller Auditorium Lobby	
8:30 a.m. - 12:00 noon	EXHIBITS - 2nd Level, Miller Auditorium Lobby	
8:30 a.m. - 4:30 p.m.	EMPLOYMENT REGISTER - Green Room, Miller Auditorium	
	SPECIAL SESSIONS	
8:30 a.m. - 12:00 noon	General Topology II 1114 Brown Hall	
9:00 a.m. - 12:00 noon	Topological and Chromatic Graph Theory II 156 Wood Hall	
9:00 a.m. - 11:50 a.m.	Banach Spaces with the Radon-Nikodym Property I 1118 Wood Hall	
9:00 a.m. - 11:50 a.m.	Riemannian Geometry II 170 Wood Hall	
	SESSIONS FOR CONTRIBUTED PAPERS	
9:30 a.m. - 10:10 a.m.	Semigroups 3480 Dunbar Hall	
9:45 a.m. - 10:40 a.m.	Numerical Analysis 2520 Dunbar Hall	
9:45 a.m. - 10:45 a.m.	INVITED ADDRESS: The singular cardinals problem Jack H. Silver, Miller Auditorium	
	SPECIAL SESSIONS	
10:00 a.m. - 12:00 noon	Affine Algebraic Groups I 3750 Knauss Hall	
10:00 a.m. - 11:50 a.m.	Aspects of Real Analysis I 3770 Knauss Hall	
10:00 2.m. - 4:00 p.m.		Conference Board of the Mathematical Sciences COUNCIL MEETING Dean's Conference Room (2010), Friedmann Hall
	SESSIONS FOR CONTRIBUTED PAPERS	
10:15 a.m. - 11:55 a.m.	Matrix Theory 3480 Dunbar Hall	
10:45 a.m. - 11:40 a.m.	Computer Science and Information Theory 2520 Dunbar Hall	
11:00 a.m. - 12:00 noon	INVITED ADDRESS: Aspects of value distribution theory in several complex variables Wilhelm F. Stoll, Miller Auditorium	

THURSDAY, August 21	American Mathematical Society	Other Organizations
	SPECIAL SESSION	
1:00 p.m. - 2:50 p.m.	Theoretical Computer Science 2480 Dunbar Hall	
	SESSION FOR CONTRIBUTED PAPERS	
1:00 p.m. - 2:55 p.m.	Number Theory 2520 Dunbar Hall	
1:30 p.m. - 2:30 p.m.	INVITED ADDRESS: The continuous cohomology of groups and classifying spaces James D. Stasheff, Miller Auditorium	
	SPECIAL SESSIONS	
1:30 p.m. - 3:00 p.m.	Topological and Chromatic Graph Theory III 1104 Rood Hall	
1:30 p.m. - 2:50 p.m.	Affine Algebraic Groups II 3750 Knauss Hall	
1:30 p.m. - 2:50 p.m.	Aspects of Real Analysis II 1114 Brown Hall	
1:30 p.m. - 3:00 p.m.	Functional Equations III 3760 Knauss Hall	
1:30 p.m. - 2:50 p.m.	Banach Spaces with the Radon-Nikodym Property II 3770 Knauss Hall	
	SESSIONS FOR CONTRIBUTED PAPERS	
1:30 p.m. - 2:55 p.m.	Differential Equations 2750 Knauss Hall	
1:30 p.m. - 2:55 p.m.	Geometry 3480 Dunbar Hall	
3:15 p.m. - 4:00 p.m.	STEELE PRIZE SESSION WIENER PRIZE SESSION Miller Auditorium	
4:00 p.m.	BUSINESS MEETING Miller Auditorium	
7:30 p.m. - 10:00 p.m.		CBMS - COUNCIL MEETING Dean's Conference Room (2010), Friedmann Hall
8:00 p.m.	AMS Committee on Employment and Educational Policy PANEL DISCUSSION: The role of applications in Ph.D. programs in mathematics Richard D. Anderson Lipman Bers Wendell H. Fleming (moderator) Henry O. Pollak Shaw Theatre	

FRIDAY, August 22	AMS	
	SPECIAL SESSION	
8:00 a.m. - 11:50 a.m.	Functional Equations IV 3760 Knauss Hall	
8:30 a.m. - 1:30 p.m.	REGISTRATION - 2nd Level, Miller Auditorium Lobby	
8:30 a.m. - 9:30 a.m.	COLLOQUIUM LECTURES IV Ellis R. Kolchin, 3750 Knauss Hall	
8:30 a.m. - 9:30 a.m.	COLLOQUIUM LECTURES IV Elias M. Stein, 3770 Knauss Hall	
	SPECIAL SESSIONS	
8:30 a.m. - 11:50 a.m.	General Topology III 170 Wood Hall	
9:00 a.m. - 11:50 a.m.	Riemannian Geometry III 1114 Brown Hall	
9:00 a.m. - 11:50 a.m.	Categorical Methods in Algebraic Topology I 2480 Dunbar Hall	
9:00 a.m. - 12:00 noon	Scientific Computing I 2520 Dunbar Hall	

FRIDAY, August 22	American Mathematical Society	
	SESSION FOR CONTRIBUTED PAPERS	
9:00 a.m. - 9:40 a.m.	Operator Theory 3480 Dunbar Hall	
9:45 a.m. - 10:45 a.m.	INVITED ADDRESS: Algebraic cycles on algebraic varieties David Mumford, Miller Auditorium	
	SPECIAL SESSIONS	
10:00 a.m. - 11:50 a.m.	Banach Spaces with the Radon-Nikodym Property III 3770 Knauss Hall	
10:00 a.m. - 11:50 a.m.	Efficient Algorithms for Exact Computation 3750 Knauss Hall	
	SESSION FOR CONTRIBUTED PAPERS	
10:00 a.m. - 11:40 a.m.	Combinatorics 3480 Dunbar Hall	
11:00 a.m. - 12:00 noon	INVITED ADDRESS: Ergodic properties of elementary mappings of the unit interval Roy L. Adler, Miller Auditorium	
	SPECIAL SESSION	
1:00 p.m. - 4:00 p.m.	Affine Algebraic Groups III 3750 Knauss Hall	
1:00 p.m. - 1:10 p.m.	GENERAL SESSION 170 Wood Hall	
	SPECIAL SESSIONS	
1:30 p.m. - 4:20 p.m.	Functional Equations V 3760 Knauss Hall	
1:30 p.m. - 4:00 p.m.	Banach Spaces with the Radon-Nikodym Property IV 3770 Knauss Hall	
1:30 p.m. - 4:00 p.m.	General Topology IV 1114 Brown Hall	
1:30 p.m. - 4:00 p.m.	Categorical Methods in Algebraic Topology II 2480 Dunbar Hall	
1:30 p.m. - 4:00 p.m.	Scientific Computing II 2520 Dunbar Hall	
	SESSION FOR CONTRIBUTED PAPERS	
1:30 p.m. - 3:25 p.m.	Real Analysis 3480 Dunbar Hall	

reservations in advance at the Local Information Desk. Bus service to the plant will be available for a nominal fee.

Michigan is the third largest wine producing state (after California and New York). The wineries are centered in an area 15–20 miles west of Kalamazoo, and all welcome visitors. A tour of several of the wineries, with ample tasting privileges, is planned for Tuesday afternoon, August 19. Again, transportation will be available at a moderate cost, and interested parties can obtain further information and sign up for the tour at the Local Information Desk.

Several other local industries, such as the Kellogg cereal company in Battle Creek, offer tours of their facilities. Information about these tours will be available at the Local Information Desk.

Nearby Colon, Michigan, is the "Magic Capital of the World." It is the home of the major sources of professional magicians' supplies. From August 20–23 magicians from all over the continent will be convening in Colon. Magic demonstrations by professionals for professionals, and for the general public, will be held every evening and on Saturday afternoon. Further information about this unique event will be available at the Local Information Desk.

The recreational and athletic facilities of Western Michigan University will be available to all participants. Among these are badminton, basketball, billiards, bowling, handball, paddleball, indoor and outdoor track, softball, swimming (no rental suits available), table tennis, tennis, and volleyball. In addition, the bathing beaches of Lake Michigan are within an hour's drive from the campus. There are several smaller lakes in and around Kalamazoo which have public bathing and picnicking facilities. Interested parties should inquire at the Local Information Desk.

A block of seats has been reserved for the Thursday evening (August 21) performance at the nearby Augusta Barn Summer Stock Theatre. Reservations to attend the performance and other theatre information can be obtained at the Local Information Desk.

A tour of the Kalamazoo Nature Center suitable for children of all ages will be scheduled. Further information about this tour and the facilities at the Nature Center will be available at the Local Information Desk. In addition, there will be information about several other attractions in the area which are of special interest to young people.

There will be daily supervised arts and crafts for elementary age children at the dormitory complex. Information about the class hours and location will be available at the Local Information Desk.

There are also several commerical day care centers in Kalamazoo. Listed below are two which will be in operation during the meetings. They will take children on a short-term basis. Interested parties should write directly for further information and/or registration forms.

1. Child Development Center
   Western Michigan University
   1401 Cherry Street
   Kalamazoo, Michigan 49008
   Telephone: (616) 383-4076
   (Will take children two and one-half years to five years of age
   Hours of operation: 6:30 a.m. to 6:30 p.m.)
2. Michigan Young World
   110 W. Cork Street
   Kalamazoo, Michigan 49001
   Telephone: (616) 349-2445
   (Will take children two and one-half years to eight years of age
   Hours of operation: 6:30 a.m. to 6:30 p.m.)

TRAVEL AND LOCAL INFORMATION

Kalamazoo is situated on two major expressways, I-94 (east-west), and US-131 (north-south), and is approximately halfway between Chicago and Detroit. It is also served by North Central Airlines, Greyhound and Indian Trails Bus Lines, and Amtrak. United Air Lines also serves the area through Grand Rapids, 50 miles north of Kalamazoo. Car rentals from Avis, Hertz, and National are available at the airports, but prior reservations are advisable. The Kalamazoo airport is within the city limits, and regular cab service is available for transportation between the airport and campus. An information desk will be maintained at the Kalamazoo airport at the times of the most frequently used incoming flights to assist arrivals. During the summer, Michigan is on Eastern Daylight Time.

Please note that the Kalamazoo airport may be closed for runway resurfacing during most of the month of August. At this printing, the Local Arrangements Committee is attempting to arrange for a change in the work schedule of the contractors, if at all possible. Otherwise, plans are being made for rerouting of the Kalamazoo flights to a nearby airport where ground transportation to Kalamazoo and the university will be provided.

If you have already made airline reservations for your trip to Kalamazoo, it is suggested that you consult your travel agent or the airline with which you made your reservations regarding this change.

WEATHER

The normal daytime high temperature during this period is 84° F. Normal nighttime low is 60° F. Rainfall in August averages 2.78 inches, with a twenty percent to thirty percent probability of precipitation each day. Humidity normally ranges from a daytime high of eighty-two percent to a nighttime low of fifty-five percent. The record high and low temperatures for August are 101° F and 41° F respectively. Light jackets and sweaters are advised for evening wear.

MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematics Meetings, Western Michigan University, Kalamazoo, Michigan 49003. Mail and telegrams so addressed may be picked up at the Mail and Information Desk located at the registration area in the lobby area of Miller Auditorium.

A message center will be located in the same area to receive incoming calls for registrants during the hours the registration desk is open, cf. the section entitled MEETING PREREGISTRATION AND REGISTRATION, on a previous page. Messages will be taken down, and the name of any member for whom a message has been received will be posted until the message is picked up at the Message Center. The telephone number of the Message Center is (616) 383-1610.

LOCAL ARRANGEMENTS COMMITTEE

Yousef Alavi (chairman), Paul T. Bateman (ex officio), Jean M. Calloway, Gary Chartrand, A. Bruce Clarke, Florence M. Clarke, S. F. Kapoor, Don R. Lick, John W. Petro, James H. Powell, David P. Roselle (ex officio), Gordon L. Walker (ex officio), and Alden H. Wright.

PRESENTORS OF TEN-MINUTE PAPERS

Following each name is the number corresponding to the speaker's position on the program. Speakers at the Short Course on Applied Combinatorics are indicated by (sc)

• Invited one-hour lecturers *Special session speakers

*Aczél, J. #116
Adeniran, T. M. #36
•Adler, R. L. #266
Akemann, C. A. #93
*Albertson, M. O. #187
Alder, H. L. #179
Al-Daffa, A. #177
*Alexander, R. #90
*Alexander, S. #143
Al-Moajil, A. H. #94
*Alpert, S.R #6
*Anderson, J. #60
Andrushkiw, R. I. #150
Arkin, J. #264
Artzy, R. #206
*Aull, C. E. #70
*Baker, J. A. #216
*Ball, R. N. #42
*Ballantine, C. S. #8
*Bankston, P. #120
*Bari, R. A. #53
*Barnes, J. #244
*Bayliss, A. #287
*Beck, J. M. #240
Beem, R. P. #30
*Belding, W. R. #108
Bellenot, S. F. #95
*Bennett, M. K. #117
Berkowitz, H. W. #168
*Berkson, E. #58
Berlekamp, E. R. (sc)
Berndt, B. C. #184
*Bernhart, F. #52
*Boardman, J. M. #238
Bromberg, N. #83
*Buoni, J. J. #249
Burrell, B. A. #35
Buxton, M #183
Cannon, J. W. #29
Cargo, G. T. #79
*Carroll, F. W. #212
Cavior, S. R. #171
Chak, A. M. #24
*Chen, B.-Y. #230
*Chen, C. S. #144
Chen, E. #81
*Ciment, M. #241
*Collier, J. B. #198
*Conrad, P. #105
*Cooper, E. D. #173
•Dade, E. C. #80
Dankel, T., Jr. #38
Dashiell, F. K., Jr. #296
Dastrange, N. #169
*D'Atri, J. E. #232
Davenport, W. H. #19
*Davis, W. J. #134
*Davison, T. M. K. #217
Dey, S. K. #152
*Diaz, J. C. #242
*Dickman, R. F., Jr. #121
*Diderrich, G. T. #219
*Diestel, J. #133
Dlab, V. #167
Dobbins, G. #146
*Donnelly, H. #140
*van Douwen, E. K. #125
*Eberlein, P. #63
*Edelstein, M. #199
*Edgar, G. A. #252
Evans, M. J. #290
*Fairweather, G. #276
Feinberg, R. B. #161
Ferguson, H. #25
Feser, V. G. #211
*Fleissner, R. J. #157
Fora, A. A. #33
Foran, J. #289
Fox, D. W. #203
*Frank, M. J. #111
Fryant, A. #76
*Fuchs, M. #237
Fulkerson, D. R. (sc)
Gandhi, J. M. #178
*Gardner, R. B. #229
*Gilkey, P. B. #142
Gilpin, M. #260
*Glass, A. M. W. #103
*Gluck, H. #65
*Golberg, M. A. #275
Goldberg, M. #207
*Goldberg, M. #61
Graham, R. L. (sc)
Graves, W. H. #295
*Gromov, M. #141
*Gross, J. L. #5
*Gruenhage, G. #282
Hager, W. W. #204
*Haggard, G. #131
*Hagopian, C. L. #122
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*Heath, R. W. #124
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*Henrici, P. #286
*Heuvers, K. J. #114
*Hochster, M. #269
*Hoey, D. #258
Hoffman, W. C. #40
Höft, M. #16
*Hollister, H. A. #43
*Howroyd, T. D. #274
*Hsu, I.-C. #115
Hsü, I.-D. #200
*Humphreys, J. E. #153
Hursch, J. L., Jr. #262
Infantozzi, C. A. #18
*Jamison, J. #9
*Jayachandran, M. #73
*Johnson, C. R. #10
Johnson, E. J. (sc)
Jones, J., Jr. #151
*Kambayashi, T. #190
Karr, A. F. #82
*Kay, D. C. #91
Kazarinoff, N. D. #201
*Kelly, L. M. #92
Kharaghani, H. #147
Kimberling, C. #20
*Kirk, W. A. #12
*Kleitman, D. J. #127
Knoebel, R. A. #170
Koehler, A. B. #17
*Kotlarski, I. I. #273
Krzywoblocki, M. Z. v. #37
*Kulkarni, R. S. #139
*Latch, D. M. #236
*Lawruk, B. #277
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*Leventhal, S. #245
*Levow, R. B. #186
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*Liu, C. L. #174
*Lloyd, J. T. #47
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Longyear, J. Q. #259
*Lotz, H. P. #137
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*McAuley, L. F. #123
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*McCleary, S. H. #41
McGrath, S. A. #4
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Macchia, R. #292
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*Marcus, M. #57
*Martinez, J. #102
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•Maskit, B. #98
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*Meeks, W. H., III #67
*Mena, R. A. #45
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*Michener, K. E. #192
*Mielke, M. V. #235
*Miller, G. L. #256
Morley, T. D. #165
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*Mott, J. L. #106
*Mrowka, S. #74
•Mumford, D. #250
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*Ng, C. T. #112
*Nishiura, T. #193
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*O'Farrell, A. G. #15
*O'Leary, D. P. #243
*O'Malley, R. J. #158
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*Parshall, B. #270
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*Phelps, R. R. #253
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*Radjavi, H. #11
*Rajagopalan, M. #222
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*Roberts, J. L. #268
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*Saunders, B. D. #59
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*Schweizer, B. #110
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*Sigmon, K. #113
•Silver, J. H. #148
Sine, R. C. #34
*Singer, D. #66
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*Sklar, A. #85
*Smith, J. C. #283
*Snow, D. R. #214
Snyder, A. K. #22
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•Stoll, W. #172
*Sullivan, F. #197
*Tall, F. D. #225
*Taylor, M. A. #195
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*Uhl, J. J., Jr. #280
*Vance, R. N. #239
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*Veldkamp, F. D. #155
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Verner, L. #31
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Wagner, D. H. #210
*Wason, J. R. #234
*Waterman, D. #159
*Weil, C. E. #160
*White, A. T. #128
*White, H. E., Jr. #69
*Wicke, H. H. #228
*Wilf, H. S. #51
*Williams, S. #72
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*Wilson, E. N. #231
*Winograd, S. #176
*Witsenhausen, H. S. #13
*Yao, A. C. #255
*Zenor, P. #71
Zimmer, J. A. #263
*Zink, R. E. #194
*Zupnik, D. #86

PROGRAM OF THE SESSIONS

SHORT COURSE ON APPLICATIONS OF COMBINATORICS

All Sessions in Room 1104, Rood Hall, Western Michigan University

SATURDAY, August 16, 9:00 a.m. - 5:00 p.m.

9:00-10:15	Some problems involving graphs, Professor D.R. FULKERSON, Cornell University
10:45-12:00 noon	Combinatorial scheduling theory I, Dr. RONALD L. GRAHAM, Bell Laboratories, Murray Hill, New Jersey
2:00- 3:15	A survey of algebraic coding theory I, Professor ELWYN R. BERLEKAMP, University of California, Berkeley
3:45- 5:00 p.m.	Combinatorial scheduling theory II, Dr. RONALD L. GRAHAM

SUNDAY, August 17, 2:00 p.m. - 5:00 p.m.

2:00- 3:15	A survey of algebraic coding theory II, Professor ELWYN R. BERLEKAMP
3:45- 5:00 p.m.	Integer programming, Dr. ELLIS J. JOHNSON, IBM T. J. Watson Research Center, Yorktown Heights, New York

THE SEVENTY-NINTH SUMMER MEETING

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, <u>the time limits will be strictly enforced.</u>

TUESDAY, 1:00 P.M.

Colloquium Lectures: Lecture I, Miller Auditorium

(1) Differential algebraic groups. Professor ELLIS R. KOLCHIN, Columbia University

TUESDAY, 2:15 P.M.

Colloquium Lectures: Lecture I, Miller Auditorium

(2) Singular integrals, old and new. Professor ELIAS M. STEIN, Princeton University

TUESDAY, 3:30 P.M.

Poster Session on Operator Theory, 2750 Knauss Hall

3:30- 5:00 (3) Isometries induced by composition operators. Professor ARTHUR LUBIN, Northwestern University (726-47-10)

3:30- 5:00 (4) An Abelian ergodic theorem for semigroups in L_p space. Professor STEPHEN A. McGRATH, U.S. Naval Academy (726-47-5)

TUESDAY, 3:30 P.M.

Special Session on Topological and Chromatic Graph Theory I, 1104 Rood Hall

3:30- 3:50 (5) An infinite family of octahedral crossing numbers. Professor JONATHAN L. GROSS, Columbia University (726-05-2)

4:00- 4:20 (6) Current maps and nonorientable graph imbeddings. Professor SETH R. ALPERT, State University of New York, Downstate Medical Center (726-05-3)

4:30- 4:50 (7) New derivations of spectral bounds for the chromatic number. Dr. ALLEN J. SCHWENK, Michigan State University (726-05-13)

TUESDAY, 3:30 P.M.

Special Session on Numerical Ranges for Matrices and Other Operators on Normed Linear Spaces I, 3770 Knauss Hall

3:30- 3:50 (8) Conjunctivity and the numerical range. Professor C. S. BALLANTINE, Oregon State University (726-15-4)

4:00- 4:20 (9) Hermitian operators and groups of isometries on some Banach spaces. Professor EARL BERKSON, University of Illinois, Professor RICHARD FLEMING and Professor JAMES JAMISON*, Memphis State University (726-47-3)

4:30- 4:50 (10) Estimation and computation of the numerical range. Professor CHARLES R. JOHNSON, Institute for Fluid Dynamics and Applied Mathematics, University of Maryland (726-65-2)

5:00- 5:20 (11) On the boundary of numerical ranges. Preliminary report. Dr. M. RADJABALIPOUR and Professor H. RADJAVI*, Dalhousie University (726-47-7)

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*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.

TUESDAY, 3:30 P.M.

Special Session on Geometry of Metric Spaces I, 3750 Knauss Hall

3:30- 3:50 (12) Caristi's fixed point theorem and metric convexity. Preliminary report. Professor W. A. KIRK, University of Iowa (726-52-3)

4:00- 4:20 (13) A support characterization of zonotopes. Preliminary report. Dr. H.S. WITSENHAUSEN, Bell Laboratories, Murray Hill, New Jersey (726-52-2)

4:30- 4:50 (14) Nearly uniform distribution of points on a sphere. Preliminary report. Professor KENNETH B. STOLARSKY, University of Illinois at Urbana (726-52-4)

5:00- 5:20 (15) Sometimes, you've got the wrong metric. Professor ANTHONY G. O'FARRELL, University of California, Los Angeles (726-54-20)

TUESDAY, 3:30 P.M.

Session on Algebra, 2480 Dunbar Hall

3:30- 3:40 (16) The fixed point property in partially ordered sets. Dr. MARGRET HÖFT*, University of Michigan, Dearborn, and Dr. HARTMUT HÖFT, Eastern Michigan University (726-06-14)

3:45- 3:55 (17) Pre-self-injective duo rings. Preliminary report. Dr. ANNE B. KOEHLER, Miami University (726-16-1)

4:00- 4:10 (18) The lattice of the integral-domains, id., of the Cayley-Dickson's integers, CDi. Professor CARLOS A. INFANTOZZI, Universidad le la República, Montevideo, Uruguay (726-17-1)

4:15- 4:25 (19) Malcev ideals in alternative rings. Preliminary report. Dr. W. HAROLD DAVENPORT, University of Petroleum & Minerals, Dhahran, Saudi Arabia (726-17-2)

4:30- 4:40 (20) Semipermutability of Chebyshev polynomials of the second kind. Professor CLARK KIMBERLING, University of Evansville (726-12-1)

TUESDAY, 3:30 P.M.

Session on Classical Analysis, 3480 Dunbar Hall

3:30- 3:40 (21) An integral representation for the product of two Jacobi polynomials. Preliminary report. Professor H. M. SRIVASTAVA*, University of Victoria, and Dr. REKHA PANDA, University of Victoria and Ravenshaw College, Orissa, India (726-33-1)

3:45- 3:55 (22) The bounded consistency and sequential completeness theorems. Professor A.K. SNYDER* and Professor A. WILANSKY, Lehigh University (726-40-1)

4:00- 4:10 (23) Rational approximation. III. Professor D. J. NEWMAN, Yeshiva University, and Professor A. R. REDDY*, Michigan State University (726-41-1)

4:15- 4:25 (24) Some generalizations of Laguerre polynomials. III. Professor A.M. CHAK, West Virginia University (726-44-1)

4:30- 4:40 (25) Reconstruction of plane objects by Farey dissection. Dr. HELAMAN FERGUSON, Brigham Young University (726-44-2)

TUESDAY, 3:30 P.M.

Session on Differential Geometry and Algebraic Topology, 3760 Knauss Hall

3:30- 3:40 (26) Generic G-structures. Preliminary report. LARRY LIPSKIE, University of Illinois at Urbana (726-53-6)

3:45- 3:55 (27) Applications of an equivariant universal coefficient theorem. Dr. STEPHEN J. WILLSON, Iowa State University (726-55-5)

4:00- 4:10 (28) On a structural property of the groups of alternating links. Professor E.J. MAYLAND, Jr.*, York University, and Professor KUNIO MURASUGI, University of Toronto (726-55-8)

4:15- 4:25 (29) Taming codimension-one generalized submanifolds of S^n. Professor J.W. CANNON, University of Wisconsin (726-57-1)

4:30- 4:40 (30) Extensions and reductions of equivariant bordism. Dr. R. PAUL BEEM, Indiana University at South Bend (726-57-3)

4:45- 4:55 (31) A mean value formula for the Spin group. LAWRENCE VERNER, Baruch College, City University of New York (726-57-4)

TUESDAY, 3:30 P.M.

Session on General Topology, 1114 Brown Hall

3:30- 3:40 (32) Closure, interior, and union in finite topological spaces. Professor LOUISE MOSER, California State University, Hayward (726-54-22)

3:45- 3:55 (33) Still on modified Sorgenfrey spaces. Preliminary report. Mr. ALI A. FORA, State University of New York at Buffalo (726-54-24)

4:00- 4:10 (34) Ergodic properties of homeomorphisms. ROBERT C. SINE, University of Rhode Island (726-54-25)

4:15- 4:25 (35) The mountain-climbing problem on the closed 2-cell. Preliminary report. Professor BENJAMIN A. BURRELL, Ohio State University, Marion (726-54-26)

4:30- 4:40 (36) Absolute homology (monotone) union property for X, with coefficient Z. Preliminary report. TINUOYE M.ADENIRAN, College of Science and Technology, Port Harcourt, Nigeria (726-54-27)

TUESDAY, 3:30 P.M.

Session on Applications, 2520 Dunbar Hall

3:30- 3:40 (37) Wave mechanics-macroscopic fluid dynamics approach to turbulence. Professor MARIA Z. v. KRZYWOBLOCKI, Michigan State University (726-81-1)

3:45- 3:55 (38) Derivation of the charge current of the Pauli equation using velocity operators. Preliminary report. Professor THAD DANKEL, Jr., University of North Carolina at Wilmington (726-81-2)

4:00- 4:10 (39) A pricing game with elastic demand. Preliminary report. Professor JOHN R. SORENSON, Valparaiso University (726-90-1)

4:15- 4:25 (40) In defense of the LTG/NP. Professor WILLIAM C. HOFFMAN, Oakland University (726-92-1)

WEDNESDAY, 8:00 A.M.

Special Session on Ordered Groups I, 2480 Dunbar Hall

8:00- 8:20 (41) Which lattice-ordered groups are isomorphic to some A(S)? Professor STEPHEN H. McCLEARY, University of Georgia (726-06-13)

8:30- 8:50 (42) Ideals of the lattice ordered group of order-preserving permutation of the long line. Preliminary report. Dr. RICHARD N. BALL, Boise State University (726-06-11)

9:00- 9:20 (43) Structure of nilpotent ℓ-groups. Preliminary report. Professor HERBERT A. HOLLISTER, Bowling Green State University (726-06-2)

9:30- 9:50 (44) Some sufficient conditions for a group to be orderable. Dr. A. H. RHEMTULLA* and Dr. R. BOTTO MURA, University of Alberta (726-06-9)

10:00-10:20 (45) Retractions and lattice-orderings on groups. Professor ROBERTO A. MENA, University of Wyoming (726-06-6) (Introduced by W. Charles Holland)

10:30-10:50 (46) Some properties of retractable groups. Preliminary report. Professor J. ROGER TELLER, Georgetown University (726-06-5)

11:00-11:20 (47) Extensions of group retractions. Preliminary report. Professor RICHARD D. BYRD and Professor JUSTIN T. LLOYD*, University of Houston (726-06-4)

11:30-12:00 noon (48) Informal problem session

WEDNESDAY, 8:30 A.M.

Colloquium Lectures: Lecture II, 3750 Knauss Hall

(49) Differential algebraic groups. Professor ELLIS R. KOLCHIN, Columbia University

WEDNESDAY, 8:30 A.M.

Colloquium Lectures: Lecture II, 3770 Knauss Hall

(50) Singular integrals, old and new. Professor ELIAS M. STEIN, Princeton University

WEDNESDAY, 9:00 A.M.

Special Session on Topological and Chromatic Graph Theory II, 1104 Rood Hall

9:00- 9:20 (51) An algorithm for the chromatic polynomial. Professor HERBERT S. WILF, University of Pennsylvania (726-05-11)

9:30- 9:50 (52) Flattening equations in the theory of chromatic polynomials. Preliminary report. Dr. FRANK BERNHART* and Dr. W. T. TUTTE, University of Waterloo (726-05-12)

10:00-10:20 (53) Chromatic polynomials and Whitney's broken circuits. Preliminary report. Professor RUTH A. BARI, George Washington University (726-05-4)

10:30-10:50 (54) Some open questions on orientable embeddings. Professor RICHARD D. RINGEISEN, Purdue University (726-05-10)

11:00-11:20 (55) Bounds for graphical parameters. Preliminary report. Professor E. A. NORDHAUS, Michigan State University (726-05-17)

11:30-12:00 noon (56) Informal problem session

WEDNESDAY, 9:00 A.M.

Special Session on Numerical Ranges of Matrices and Other Operators on Normed Linear Spaces II, 170 Wood Hall

9:00- 9:20 (57) Bilinear functionals on the Grassmann manifold. Professor MARVIN MARCUS, University of California, Santa Barbara (726-15-1)

9:30- 9:50 (58) One-parameter groups of isometries on Hardy spaces of the torus. Professor EARL BERKSON* and Professor HORACIO PORTA, University of Illinois (726-47-2)

10:00-10:20 (59) Convexity of the norm-numerical range. Preliminary report. Mr. B. DAVID SAUNDERS, University of Wisconsin (726-15-8)

10:30-10:50 (60) Which sets are numerical ranges? Professor JOEL ANDERSON, California Institute of Technology (726-47-8)

11:00-11:20 (61) On the characterization of spectral matrices. Professor MOSHE GOLDBERG, University of California, Los Angeles (726-15-2)

11:30-11:50 (62) The maximal numerical range. Professor JOSEPH G. STAMPFLI, California Institute of Technology (726-47-4)

WEDNESDAY, 9:00 A.M.

Special Session on Riemannian Geometry I, 3760 Knauss Hall

9:00- 9:20 (63) Surfaces of nonpositive curvature. Professor PATRICK EBERLEIN, University of North Carolina (726-53-14)

9:30- 9:50 (64) Riemannian manifolds with many killing vector fields. Professor ANN K. STEHNEY*, Wellesley College, and Professor RICHARD S. MILLMAN, Southern Illinois University (726-53-10)

10:00-10:20 (65) Deformations of geodesic fields. Preliminary report. Professor HERMAN GLUCK*, University of Pennsylvania, and Professor DAVID SINGER, Cornell University (726-53-13)

10:30-10:50 (66) The existence of nontriangulable cut loci. Preliminary report. Professor DAVID SINGER*, Cornell University, and Professor HERMAN GLUCK, University of Pennsylvania (726-53-16)

11:00-11:20 (67) Periodic minimal surfaces in R^n. Preliminary report. Dr. WILLIAM H. MEEKS, III, University of California, Los Angeles (726-53-9) (Introduced by Professor Tilla K. Milnor)

11:30-12:00 noon (68) Informal discussion; scheduling of extra talks or problem session

WEDNESDAY, 9:00 A.M.

Special Session on General Topology I, 1114 Brown Hall

9:00- 9:20 (69) First countable spaces having special pseudo-bases. Dr. H. E. WHITE, Jr., Columbus, Ohio (726-54-4)

9:30- 9:50 (70) The separation axioms of Van Est and Freudenthal. Professor C. E. AULL, Virginia Polytechnic Institute and State University (726-54-10)

10:00-10:20 (71) Continuous PN-operators. PHILLIP ZENOR, Auburn University (726-54-6)

10:30-10:50 (72) Covering properties in products and yet another cardinal function. Preliminary report. SCOTT WILLIAMS, State University of New York at Buffalo (726-54-12)

11:00-11:20 (73) A common method of attack to some open problems in scattered spaces. Preliminary report. Mr. M. JAYACHANDRAN, Memphis State University (726-54-14)

11:30-11:50 (74) Concerning the covering dimension. Preliminary report. Professor S. MROWKA, State University of New York at Buffalo (726-54-8)

WEDNESDAY, 9:30 A.M.

Session on Complex Analysis, 2520 Dunbar Hall

9:30- 9:40 (75) The Lagrange-Bürmann formula for systems of formal power series. Preliminary report. Mrs. MARIE-LOUISE HENRICI* and Professor PETER HENRICI, Eidgenössische Technische Hochschule, Zürich, Switzerland (726-32-2)

9:45- 9:55 (76) Growth of entire harmonic functions in R^3. Dr. ALLAN FRYANT, University of Wisconsin, Milwaukee (726-30-1)

10:00-10:10 (77) Inequalities for a special class of bounded analytic functions. Professor DOROTHY BROWNE SHAFFER, Fairfield University (726-30-2)

10:15-10:25 (78) Convolutions of holomorphic functions. Preliminary report. Professor HARI SHANKAR*, Ohio University and Mr. RICHARD A. BOGDA, Dupont, Parkersburg, West Virginia (726-30-4)

10:30-10:40 (79) Blaschke products with prescribed boundary values. Professor G.T.CARGO, Syracuse University (726-30-5)

WEDNESDAY, 9:45 A.M.

Invited Address, Miller Auditorium

(80) Nearly trivial outer automorphisms of finite groups. Professor EVERETT C. DADE, University of Illinois (726-20-3)

WEDNESDAY, 9:45 A.M.

Session on Probability, 3480 Dunbar Hall

9:45- 9:55 (81) Existence of interacting Markov processes in quantum lattice systems. Professor ETANG CHEN, University of Rochester (726-60-1)

10:00-10:10 (82) Lévy random measures. Professor ALAN F. KARR, Johns Hopkins University (726-60-3)

10:15-10:25 (83) Boundary layer analysis of equations of Brownian motion in a force field. Professor NEIL BROMBERG, Courant Institute, New York University (726-60-4)

10:30-10:40 (84) Stochastic processes with independent increments taking values in a Hilbert space. JEFFREY L. SPIELMAN, Bowling Green State University (726-60-5)

WEDNESDAY, 10:00 A.M.

Special Session on Functional Equations I, 3770 Knauss Hall

10:00-10:20 (85) Zupnik families and their applications. Professor ABE SKLAR, Illinois Institute of Technology (726-39-19)

10:30-10:50 (86) Cayley sets. Mr. DAVID ZUPNIK, Chicago, Illinois (726-39-17) (Introduced by Professor Abe Sklar)

11:00-11:20 (87) Analytic iteration and roots of biholomorphic and formally biholomorphic mappings. Professor LUDWIG REICH, Universität Graz, Austria (726-39-23)

11:30-11:50 (88) The equation $af(ax) + bf(bx + a) = bf(bx) + af(ax + b)$. Preliminary report. Professor JOHN V.RYFF, University of Connecticut (726-39-4)

WEDNESDAY, 10:00 A.M.

Special Session on Geometry of Metric Spaces II, 3750 Knauss Hall

10:00-10:20 (89) Characterization of geodesic arcs. Professor C.M.PETTY, University of Missouri (726-50-1)

10:30-10:50 (90) Metric averaging in ℓ^2. Preliminary report. Professor RALPH ALEXANDER, University of Illinois at Urbana (726-52-5)

11:00-11:20 (91) Arc curvature in metric spaces. Professor DAVID C.KAY, University of Oklahoma (726-50-7)

11:30-11:50 (92) Order embeddings in E^n. Preliminary report. Professor PAUL ERDÖS, Hungarian Academy of Science, Budapest, Hungary and Professor L.M.KELLY*, Michigan State University (726-50-4)

WEDNESDAY, 10:45 A.M.

Session on Functional Analysis, 3480 Dunbar Hall

10:45-10:55 (93) The spectrum of a derivation of a C*-algebra. Professor CHARLES A.AKEMANN, University of California, Santa Barbara (726-46-1)

11:00-11:10 (94) The commutants of relatively prime powers of elements in operator algebras. Dr. ABDULLAH H.AL-MOAJIL, University of Petroleum & Minerals, Dhahran, Saudi Arabia (726-46-2) (Introduced by Dr. Harold Davenport)

11:15-11:25 (95) Compact operators into Hilbert space. Preliminary report. Professor STEVEN F.BELLENOT, Florida State University (726-46-10)

11:30-11:40 (96) Properties of locally convex spaces generated by classes of bounded sets. Professor DAGMAR HENNEY, George Washington University (726-46-17)

11:45-11:55 (97) Locally convex spaces generated by bounded sets. Professor DAGMAR HENNEY and W. LINCOLN*, George Washington University (726-46-18)

WEDNESDAY, 11:00 A.M.

Invited Address, Miller Auditorium

(98) On the classification of Kleinian groups. Professor BERNARD MASKIT, State University of New York at Stony Brook (726-30-3)

WEDNESDAY, 11:00 A.M.

Session on Group Theory, 2520 Dunbar Hall

11:00-11:10 (99) The isomorphism problem for a class of one-relator groups. STEPHEN MESKIN, University of Connecticut (720-20-7)

11:15-11:25 (100) Small actions of generalized symmetric groups. Dr. DONALD McCARTHY*, St. John's University, Dr. ANDREW WOHLGEMUTH and Dr. GARY HAGGARD, University of Maine, Orono (726-20-8)

11:30-11:40 (101) Algorithms for presenting certain commutator subgroups. Dr. JOSEPH S. VERRET, Tulane University (726-20-9)

THURSDAY, 8:00 A.M.

Special Session on Ordered Groups II, 2480 Dunbar Hall

8:00- 8:20 (102) Pairwise splitting lattice-ordered groups. Preliminary report. Dr. JORGE MARTINEZ, University of Florida (726-06-1)

8:30- 8:50 (103) Problems in lattice-ordered groups. Preliminary report. Professor A.M.W. GLASS*, Bowling Green State University, and Professor STEPHEN H. McCLEARY, University of Georgia (726-06-12)

9:00- 9:20 (104) Ordered groups with compatible tight Riesz orders. Professor NORMAN R. REILLY, Simon Fraser University (726-06-7)

9:30- 9:50 (105) Torsion radicals of lattice-ordered groups. Professor PAUL CONRAD, University of Kansas (726-06-15)

10:00-10:20 (106) The $D + XD_S[X]$ construction. DOUG COSTA, University of Virginia, JOE L. MOTT*, Florida State University, M. ZAFRULLAH, University of Manchester Institute of Science and Technology, United Kingdom (726-13-2)

10:30-10:50 (107) Superunits in lattice-ordered rings. STUART A. STEINBERG, University of Toledo (726-06-8)

11:00-11:20 (108) Bases for the positive cone of a partially ordered module. Preliminary report. Dr. W.R. BELDING, U.S. Naval Academy, Anapolis, Maryland (726-06-3)

11:30-11:50 (109) Some problems in fully ordered semigroups. M. SATYANARAYANA, Bowling Green State University (726-06-10)

THURSDAY, 8:00 A.M.

Special Session on Functional Equations II, 3760 Knauss Hall

8:00- 8:20 (110) The associativity equation on a space of probability distribution functions. Professor B. SCHWEIZER, University of Massachusetts (726-39-8)

8:30- 8:50 (111) On the simultaneous associativity of $C(x,y)$ and $x + y - C(x,y)$. Preliminary report. M.J. FRANK, University of Wisconsin, Milwaukee (726-39-10)

9:00- 9:20 (112) Structured measurement of algorithms. Preliminary report. Professor C.T. NG, University of Waterloo (726-39-13) (Introduced by J.A. Aczél)

9:30- 9:50 (113) Continuously factorable groupoids. Preliminary report. Dr. H.T. HU and Professor KERMIT SIGMON*, University of Florida (726-39-15)

10:00-10:20 (114) A class of associative algebras generated by a unitary space. Preliminary report. Dr. KONRAD JOHN HEUVERS, Michigan Technological University (726-39-12)

10:30-10:50 (115) A fundamental functional equation for vector lattices. Professor IH-CHING HSU, St. Olaf College (726-39-9)

11:00-11:20 (116) On the harmonic product and a resulting functional equation. Professor J. ACZÉL*, University of Waterloo, and Professor W. BENZ, University of Hamburg, Federal Republic of Germany (726-39-20)

11:30-11:50 (117) Axioms for metric affine geometry. Professor MARY KATHRINE BENNETT, University of Massachusetts (726-39-1)

THURSDAY, 8:30 A.M.

Colloquium Lectures: Lecture III, 3750 Knauss Hall

(118) Differential algebraic groups. Professor ELLIS R. KOLCHIN, Columbia University

THURSDAY, 8:30 A.M.

Colloquium Lectures: Lecture III, 3770 Knauss Hall

(119) Singular integrals, old and new. Professor ELIAS M. STEIN, Princeton, University

THURSDAY, 8:30 A.M.

Special Session on General Topology II, 1114 Brown Hall

8:30- 8:50 (120) Ultraproducts in general topology. Preliminary report. Dr. PAUL BANKSTON, McMaster University (726-54-23)

9:00- 9:20 (121) Multicoherent spaces. Preliminary report. Professor R. F. DICKMAN, Jr., Virginia Polytechnic Institute and State University (726-54-2)

9:30- 9:50 (122) Nonseparating plane continua. Professor CHARLES L. HAGOPIAN, California State University, Sacramento (726-54-7)

10:00-10:20 (123) Open mappings and group actions. Professor LOUIS F. McAULEY, State University of New York at Binghamton (726-54-15)

10:30-10:50 (124) ANE's and ANR's in monotonically normal spaces. Preliminary report. Professor R. W. HEATH*, University of Pittsburgh and Professor R. B. SHER, University of North Carolina at Greensboro (726-54-18)

11:00-11:20 (125) Maps onto Baire spaces. ERIC K. van DOUWEN, University of Pittsburgh (726-54-16)

11:30-12:00 noon (126) Informal session

THURSDAY, 9:00 A.M.

Special Session on Topological and Chromatic Graph Theory II, 156 Wood Hall

9:00- 9:20 (127) Another proof of Brooks' theorem. Preliminary report. Professor DANIEL J. KLEITMAN* and Professor CURTIS GREENE, Massachusetts Institute of Technology (726-05-8)

9:30- 9:50 (128) Block designs in graph imbeddings. Preliminary report. Professor ARTHUR T. WHITE, Western Michigan University (726-05-05)

10:00-10:20 (129) Generalized embedding schemes. Preliminary report. Dr. SAUL STAHL, Western Michigan University (726-05-15)

10:30-10:50 (130) On achromatic numbers and graphs with forbidden quotients. Preliminary report. Dr. PAVOL HELL*, Simon Fraser University and Dr. DONALD J. MILLER, University of Victoria (726-05-14)

11:00-11:20 (131) The nonorientable genus of graphs. GARY HAGGARD, University of Maine, Orono (726-05-9) (Introduced by Paul Kainen)

11:30-12:00 noon (132) Informal problem session

THURSDAY, 9:00 A.M.

Special Session on Banach Spaces with the Radon-Nikodým Property I, 1118 Wood Hall

9:00- 9:20 (133) The Radon-Nikodým theorem for Banach space-valued measures. Historical perspective. Mr. J. DIESTEL, Kent State University (726-01-2) (Introduced by T. J. Morrison)

9:30- 9:50 (134) Spaces having and spaces failing the Radon-Nikodým property. WILLIAM J. DAVIS, Ohio State University (726-46-5)

10:00-10:20 (135) Geometric characterizations of the Radon-Nikodým property. Professor PETER MORRIS, Pennsylvania State University (726-46-12)

10:30-10:50 (136) Informal session

11:00-11:20 (137) The Radon-Nikodým property in dual Banach lattices. Preliminary report. Professor HEINRICH P. LOTZ, University of Illinois (726-46-20) (Introduced by Professor J. J. Uhl, Jr.)

11:30-11:50 (138) The Radon-Nikodým property in spaces of operators. Mr. T. J. MORRISON, Kent State University (726-28-2)

THURSDAY, 9:00 A.M.

Special Session on Riemannian Geometry II, 170 Wood Hall

9:00- 9:20 (139) Infinite regular coverings, space forms and Kleinian groups. Preliminary report. Mr. RAVINDRA S. KULKARNI, Columbia University (726-53-4)

9:30- 9:50 (140) Eta invariant of a fibered manifold. Dr. HAROLD DONNELLY, Massachusetts Institute of Technology (726-53-2)

10:00-10:20 (141) Topology of Riemannian manifolds with small curvature and diameter. Preliminary report. MIKHAEL GROMOV, State University of New York at Stony Brook (726-58-1) (Introduced by Professor Tilla K. Milnor)

10:30-10:50 (142) The local invariants of a Riemannian manifold. Mr. PETER B. GILKEY, Princeton University (726-53-3)

11:00-11:20 (143) Hypersurface immersions between hyperbolic spaces. Preliminary report. Dr. S. ALEXANDER* and Dr. E. PORTNOY, University of Illinois at Urbana (726-53-11)

11:30-11:50 (144) On classification of tight surfaces in R^4. Preliminary report. Professor C. S. CHEN* and Professor W. F. POHL, University of Minnesota (726-53-8)

THURSDAY, 9:30 A.M.

Session on Semigroups, 3480 Dunbar Hall

9:30- 9:40 (145) Quotients of uniform semigroups. Dr. DONALD MARXEN, Marquette University (726-22-1)

9:45- 9:55 (146) Simple semigroups in certain locally compact groups. Professor GREG DOBBINS, Wheaton College, Illinois (726-20-6)

10:00-10:10 (147) The support of an invariant mean. Dr. H. KHARAGHANI, Pahlavi University, Shiraz, Iran (726-46-6)

THURSDAY, 9:45 A.M.

Invited Address, Miller Auditorium

(148) The singular cardinals problem. Professor JACK H. SILVER, University of California, Berkeley (726-02-1)

THURSDAY, 9:45 A.M.

Session on Numerical Analysis, 2520 Dunbar Hall

9:45- 9:55 (149) Some extensions of Prony approximation. Dr. HERBERT E. SALZER, Brooklyn, New York (726-65-3)

10:00-10:10 (150) Inclusion theorems for the eigenvalues of a quadratic operator pencil. R. I. ANDRUSHKIW, New Jersey Institute of Technology (726-65-7)

10:15-10:25 (151) Zeros of continuous real-valued functions. II. Preliminary report. Dr. JOHN JONES, Jr., Air Force Institute of Technology (726-65-8)

10:30-10:40 (152) Accelerated iterative scheme for a system of nonlinear equations. Preliminary report. Dr. SUHRIT K. DEY, Eastern Illinois University (725-65-9)

THURSDAY, 10:00 A.M.

Special Session on Affine Algebraic Groups I, 3750 Knauss Hall

10:00-10:20 (153) A family of indecomposable modules. Preliminary report. Professor J. E. HUMPHREYS, University of Massachusetts (726-20-4)

10:30-10:50 (154) Algebraic groups and diophantine equations. TAKASHI ONO, Johns Hopkins University (726-14-2)

11:00-11:20 (155) Regular elements in anisotropic tori. Professor FERDINAND D. VELDKAMP, Ohio State University (726-20-1)

11:30-12:00 noon (156) Informal session

THURSDAY, 10:00 A.M.

Special Session on Aspects of Real Analysis I, 3770 Knauss Hall

10:00-10:20 (157) Products of derivatives. Dr. RICHARD J. FLEISSNER, University of Wisconsin, Milwaukee (726-26-3) (Introduced by Professor Donald W. Solomon)

10:30-10:50 (158) Approximate maxima. Preliminary report. Professor RICHARD J. O'MALLEY, University of Wisconsin, Milwaukee (726-26-5)

11:00-11:20 (159) On generalized bounded variation. Preliminary report. Professor DANIEL WATERMAN, Syracuse University (726-26-1)

11:30-11:50 (160) The oscillatory behavior of certain derivatives. Professor RICHARD J. O'MALLEY, University of Wisconsin, Milwaukee, and Professor CLIFFORD E. WEIL*, Michigan State University (726-26-2)

THURSDAY, 10:15 A.M.

Session on Matrix Theory, 3480 Dunbar Hall

10:15-10:25 (161) Similarity of partitioned matrices. Dr. ROBERT B. FEINBERG, National Bureau of Standards, Washington, D.C. (726-15-3)

10:30-10:40 (162) Lower bounds on the dimension of the spaces of matrices transposed and skew-transposed by a similarity transformation. Professor DONALD W. ROBINSON, Brigham Young University (726-15-5)

10:45-10:55 (163) Orthogonal and unitary circulants. Preliminary report. Professor KENNETH A. BYRD and Dr. THERESA P. VAUGHAN*, University of North Carolina at Greensboro (726-15-6)

11:00-11:10 (164) The Hadamard-Fischer inequalities for a class of matrices defined by eigenvalue monotonicity. Professor GERNOT M. ENGEL, IBM, Owego, New York and Professor HANS SCHNEIDER*, University of Wisconsin (726-15-7)

11:15-11:25 (165) A vectorial generalization of Kirchhoff's laws. Professor R.J. DUFFIN and Mr. T.D. MORLEY*, Carnegie-Mellon University (726-15-9)

11:30-11:40 (166) The relationship of the graph of a singular M-matrix to its Weyr characteristic. DANIEL J. RICHMAN* and HANS SCHNEIDER, University of Wisconsin (726-15-10)

11:45-11:55 (167) Classification of real linear transformations between two complex vector spaces. V. DLAB*, Université de Paris, and C. M. RINGEL, Universität Bonn (726-15-11)

THURSDAY, 10:45 A.M.

Session on Computer Science and Information Theory, 2520 Dunbar Hall

10:45-10:55 (168) The consecutive retrieval property. Professor H. W. BERKOWITZ, State University of New York, College at New Paltz (726-68-5)

11:00-11:10 (169) On the reconstruction of bandlimited signals from sampled values. Preliminary report. Dr. NASSER DASTRANGE, Pahlavi University, Shiraz, Iran (726-94-1) (Introduced by A. Fattahi)

11:15-11:25 (170) Maximal sets of compatible threshold functions. R. ARTHUR KNOEBEL, New Mexico State University (726-94-2)

11:30-11:40 (171) An upper bound associated with errors in gray code. Professor STEPHAN R. CAVIOR, State University of New York at Buffalo (726-94-3)

THURSDAY, 11:00 A.M.

Invited Address, Miller Auditorium

(172) Aspects of value distribution theory in several complex variables. Professor WILHELM STOLL, University of Notre Dame (726-32-1)

THURSDAY, 1:00 P.M.

Special Session on Theoretical Computer Science, 2480 Dunbar Hall

1:00- 1:20 (173) The syntax and semantics of computer science. Preliminary report. Dr. ELLIS D. COOPER, Herbert H. Lehman College, City University of New York (726-68-6) (Introduced by Professor David E. Muller)

1:30- 1:50 (174) Approximation algorithms for discrete optimization problems. Professor C. L. LIU, University of Illinois at Urbana (726-68-3) (Introduced by David E. Muller)

2:00- 2:20 (175) Convex hulls of planar and spatial sets of points. Professor F. P. PREPARATA* and S. J. HONG, University of Illinois at Urbana (726-68-1) (Introduced by David E. Muller)

2:30- 2:50 (176) Some results in arithmetic complexity. Dr. SHMUEL WINOGRAD, IBM Thomas J. Watson Research Center, Yorktown Heights, New York (762-68-7)

THURSDAY, 1:00 P.M.

Session on Number Theory, 2520 Dunbar Hall

1:00- 1:10 (177) Thabit ibn Qurra. Dr. ALI AL-DAFFA, University of Petroleum & Minerals, Dhahran, Saudi Arabia (726-01-1) (Introduced by Dr. Rudolph Festa)

1:15- 1:25 (178) Fermat's last theorem IV: A new circulant condition for the first case for primes of the form $6m + 1$. Professor J. M. GANDHI, Western Illinois University (726-10-1)

1:30- 1:40 (179) Identities relating the number of partitions into an even and odd number of parts. Mr. DEAN R. HICKERSON and Professor HENRY L. ALDER*, University of California, Davis and Professor AMIN A. MUWAFI, American University of Beirut (726-10-2)

1:45- 1:55 (180) On composite n for which $\phi(n) \mid n - 1$, II. Professor CARL POMERANCE, University of Georgia (726-10-3)

2:00- 2:10 (181) On a generalization of a prime generating function. Preliminary report. Professor S. VERMA, University of Nevada, Las Vegas (726-10-4) (Introduced by Mr. L. J. Simonoff)

2:15- 2:25 (182) Polynomial Pell's equations. Professor MELVYN B. NATHANSON, Institute for Advanced Study (726-10-6)

2:30- 2:40 (183) On odd perfect numbers. Preliminary report. Ms. M. BUXTON*, Ball State University, and Professor B. STUBBLEFIELD, NOAA, Boulder, Colorado (726-10-7)

2:45- 2:55 (184) G. H. Hardyand Dedekind sums. BRUCE C. BERNDT, University of Illinois, Urbana (726-10-8)

THURSDAY, 1:30 P.M.

Invited Address, Miller Auditorium

(185) Continuous cohomology of groups and classifying spaces. Professor JAMES D. STASHEFF, Temple University (726-55-7)

THURSDAY, 1:30 P.M.

Special Session on Topological and Chromatic Graph Theory III, 1104 Rood Hall

1:30- 1:50 (186) On chromatic numbers of nearly planar graphs. Preliminary report. Professor ROY B. LEVOW, Florida Atlantic University (726-05-16)

2:00- 2:20 (187) Noncontractible cycles in nonplanar graphs. Preliminary report. Professor MICHAEL O. ALBERTSON*, Smith College and Dr. JOAN P. HUTCHINSON, Dartmouth College (726-05-6)

2:30- 3:00 (188) Informal problem session

THURSDAY, 1:30 P.M.

Special Session on Affine Algebraic Groups II, 3750 Knauss Hall

1:30- 1:50 (189) Stability theorems for algebraic K-functors based on Chevalley groups. Preliminary report. Professor MICHAEL R. STEIN, Northwestern University (726-20-5)

2:00- 2:20 (190) Remarks on the action of G_a on affine schemes (after Miyanishi). Professor T. KAMBAYASHI, Northern Illinois University (726-14-4)

2:30- 2:50 (191) Analytic left algebraic groups. Professor ANDY R. MAGID, University of Oklahoma (726-14-3)

THURSDAY, 1:30 P.M.

Special Session on Aspects of Real Analysis II, 1114 Brown Hall

1:30- 1:50 (192) A weak convergence theorem for nonparametric, area-type functionals. Professor KIM E. MICHENER, Wayne State University (726-28-1)

2:00- 2:20 (193) Interval functions in area theory. Professor TOGO NISHIURA, Wayne State University (726-26-4)

2:30- 2:50 (194) Unconditional bases for certain Banach function spaces. Professor ROBERT E. ZINKI, Purdue University, and Professor JAMES E. SHIREY, Ohio University (726-46-13)

THURSDAY, 1:30 P.M.

Special Session on Functional Equations III, 3760 Knauss Hall

1:30- 1:50 (195) Functional equations on closure conditions. Preliminary report. Dr. M.A. TAYLOR, Acadia University (726-39-7) (Introduced by Dr. J. Aczél)

2:00- 3:00 (196) Informal remarks and open problems

THURSDAY, 1:30 P.M.

Special Session on Banach Spaces with Radon-Nikodým Property II, 3770 Knauss Hall

1:30- 1:50 (197) Some geometrical relatives of the Radon-Nikodým property. Preliminary report. Professor FRANCIS SULLIVAN, Catholic University of America (726-46-3)

2:00- 2:20 (198) The dual of a space with the Radon-Nikodým property. Preliminary report. Dr. JAMES B. COLLIER, University of Southern California, Los Angeles (726-46-7)

2:30- 2:50 (199) Convex hulls of strongly exposed points. Mr. MICHAEL EDELSTEIN, Dalhousie University (726-46-15)

THURSDAY, 1:30 P.M.

Session on Differential Equations, 2750 Knauss Hall

1:30- 1:40 (200) Global existence of periodic solutions of the Glass-Kauffman model. Preliminary report. Mr. IN-DING HSÜ, State University of New York at Buffalo (726-34-1) (Introduced by Dr. N. D. KAZARINOFF)

1:45- 1:55 (201) Existence and stability of periodic solutions of a third order nonlinear autonomous system simulating immune response in animals. Mr. IN-DING HSÜ and Dr. N.D. KAZARINOFF*, State University of New York at Buffalo (726-34-2)

2:00- 2:10 (202) Solution in the large of a certain nth order differential equation. T. K. PUTTASWAMY, Ball State University (726-34-3)

2:15- 2:25 (203) Transient solutions for stratified fluid flow. Dr. DAVID W. FOX, Applied Physics Laboratory, Johns Hopkins University (726-35-1)

2:30- 2:40 (204) Convergence and stability properties of the discrete Riccati operator equation. Professor WILLIAM W. HAGER*, University of South Florida and Dr. LARRY HOROWITZ, Johns Hopkins University (726-49-1)

2:45- 2:55 (205) On the sufficient conditions for control problems with time delay. Dr. M. RAZ-ZAGHI, Pahlavi University, Shiraz, Iran (726-49-2) (Introduced by Dr. M. H. Afghahi)

THURSDAY, 1:30 P.M.

Session on Geometry, 3480 Dunbar Hall

1:30- 1:40 (206) Coordinates for Minkowski planes. Professor R. ARTZY, University of Haifa, Israel (726-50-2)

1:45- 1:55 (207) On the space-filling hexahedra. Mr. MICHAEL GOLDBERG, Washington, D. C. (726-50-3)

2:00- 2:10 (208) Linear isotopies. R. H. BING and MICHAEL STARBIRD*, University of Texas at Austin (726-50-5)

2:15- 2:25 (209) A foundation of hyperbolic geometry. Professor HELEN SKALA, University of Massachusetts, Boston (726-50-6)

2:30- 2:40 (210) Semi-compactness with respect to a Euclidean cone. Preliminary report. Dr. DANIEL H. WAGNER, Daniel H. Wagner Associates, Paoli, Pennsylvania (726-52-1)

2:45- 2:55 (211) Construction of triangular polyhedra. Preliminary report. Dr. VICTOR G. FESER, Mary College (726-52-6)

THURSDAY, 3:15 P.M.

Steele Prize and Wiener Prize Sessions, Miller Auditorium

THURSDAY, 4:00 P.M.

Business Meeting, Miller Auditorium

THURSDAY, 8:00 P.M.

Panel Discussion on the Role of Applications in Ph.D. Programs in Mathematics, Shaw Theatre
Professor WENDELL H. FLEMING, Brown University (Moderator)

FRIDAY, 8:00 A.M.

Special Session on Functional Equations IV, 3760 Knauss Hall

8:00- 8:20 (212) The difference property for Fréchet-space-valued functions. Professor F. W. CARROLL, Ohio State University (726-39-11)

8:30- 8:50 (213) Standard and non-standard solutions of the Cauchy equation. Dr. F. J. PAPP, University of Lethbridge (726-39-5)

9:00- 9:20 (214) Summation of sequences by functional equations. Professor DONALD R. SNOW, Brigham Young University (726-39-3)

9:30- 9:50 (215) The solution of D'Alembert's functional equation on a locally compact Abelian group. Mr. THOMAS A. O'CONNOR, Bowling Green State University (726-39-14)

10:00-10:20 (216) On the functional equation $f(x)g(y) = p(x+y)q(x/y)$. Professor JOHN A. BAKER, University of Waterloo (726-39-22)

10:30-10:50 (217) The Hosszú group of a ring. Dr. T.M.K. DAVISON, McMaster University (726-39-21) (Introduced by J. Aczél)

11:00-11:20 (218) A characterization of certain multiplicative homomorphisms. Ms. J. LESTER, University of Waterloo (726-39-18) (Introduced by Professor J. Aczél)

11:30-11:50 (219) Local boundedness and the Shannon entropy. Preliminary report. Dr. GEORGE T. DIDERRICH, University of Waterloo (726-39-6)

FRIDAY, 8:30 A.M.

Colloquium Lectures: Lecture IV, 3750 Knauss Hall

(220) Differential algebraic groups. Professor ELLIS R. KOLCHIN, Columbia University

FRIDAY, 8:30 A.M.

Colloquium Lectures: Lecture IV, 3770 Knauss Hall

(221) Singular integrals, old and new. Professor ELIAS M. STEIN, Princeton University

FRIDAY, 8:30 A.M.

Special Session on General Topology III, 170 Wood Hall

8:30- 8:50 (222) Scattered spaces. III. Professor M. RAJAGOPALAN, Memphis State University (726-54-5)

9:00- 9:20 (223) The continuum hypothesis. Professor MARY ELLEN RUDIN, University of Wisconsin (726-54-19)

9:30- 9:50 (224) Almost-P-spaces and Lusin's hypothesis. RONNIE FRED LEVY, Goucher College (726-54-1)

10:00-10:20 (225) Some results concerning the hereditarily Lindelöf-hereditarily separable problem. Professor FRANKLIN D. TALL, University of Toronto (726-54-11)

10:30-10:50 (226) A generalization of Tychonoff's product theorem. Professor JERRY E. VAUGHAN, University of North Carolina at Greensboro (726-54-13)

11:00-11:20 (227) Some Moore examples. Professor GEORGE M. REED, Ohio University (726-54-3)

11:30-11:50 (228) Primitive structures and diagonal conditions. Professor HOWARD H. WICKE, Ohio University at Athens (726-54-21)

FRIDAY, 9:00 A.M.

Special Session on Riemannian Geometry III, 1114 Brown Hall

9:00- 9:20 (229) Algebra of the second fundamental form. Professor ROBERT B. GARDNER, University of North Carolina (726-53-7)

9:30- 9:50 (230) Chern classes and Bochner curvature tensor. Professor BANG-YEN CHEN, Michigan State University (726-53-1)

10:00-10:20 (231) Homogeneous manifolds with negative curvature. Professor ROBERT AZENCOTT, Université de Paris, France, and Professor EDWARD N. WILSON*, Washington University (726-57-2)

10:30-10:50 (232) Geodesic spheres and symmetries. Preliminary report. Professor JOSEPH E. D'ATRI, Rutgers University (726-53-15)

11:00-11:20 (233) Riemann curvature tensors and critical values of sectional curvature functions. Professor D.S.P. LEUNG*, Purdue University and Professor T. NAGANO, University of Notre Dame (726-53-5)

11:30-11:50 (234) Metrics of nonnegative curvature on $S^3 \times S^3$. Preliminary report. Dr. JAMES R. WASON, Massachusetts Institute of Technology (726-53-12)

FRIDAY, 9:00 A.M.

Special Session on Categorical Methods in Algebraic Topology I, 2480 Dunbar Hall

9:00- 9:20 (235) Iterative processes. Preliminary report. Professor M. V. MIELKE, University of Miami (726-18-1)

9:30- 9:50 (236) A (weak) homotopy inverse for the functor nerve. Professor DANA MAY LATCH, Lawrence University (726-55-1)

10:00-10:20 (237) Homotopy equivalences in equivariant topology. Professor MARTIN FUCHS, Michigan State University (726-55-3)(Introduced by Professor J. Adney)

10:30-10:50 (238) Localization theory and splittings of MU. Preliminary report. J. M. BOARDMAN, Johns Hopkins University (726-55-4)

11:00-11:20 (239) Homotopy equivariant group actions. Preliminary report. Dr. R. NEIL VANCE, Institute for Advanced Study (726-55-6)

11:30-11:50 (240) Homology of group completions. Preliminary report. Professor JON M. BECK, University of Puerto Rico (726-55-2) (Presented by Professor Pierre J. Malraison, Jr.)

FRIDAY, 9:00 A.M.

Special Session on Scientific Computing I, 2520 Dunbar Hall

9:00- 9:20 (241) Numerical solution of free boundary value problems for parabolic equations. Preliminary report. Dr. MELVYN CIMENT, Naval Surface Weapons Center, White Oak Laboratory, Silver Spring, Maryland (726-65-11)

9:30- 9:50 (242) Hybrid collocation-Galerkin method for Laplaces equation on a rectangle. JULIO CESAR DIAZ, University of Kentucky (726-65-1)

10:00-10:20 (243) Iterative methods for structured systems of equations. Preliminary report. Dr. DIANNE PROST O'LEARY, University of Michigan (726-65-13) (Introduced by Dr. George Fix)

10:30-10:50 (244) Two-dimensional simulation of semiconductor devices using finite element methods. Mr. JOHN BARNES* and Professor R. J. LOMAX, University of Michigan (726-65-10) (Authors introduced by Professor George Fix)

11:00-11:20 (245) On the numerical solution of equations of hyperbolic-elliptic type. Dr. STEPHEN LEVENTHAL*, Naval Surface Weapons Center, White Oak, Silver Spring, Maryland, and Professor A. K. AZIZ, University of Maryland (726-65-12)

11:30-12:00 noon (246) Problem Session

FRIDAY, 9:00 A.M.

Session on Operator Theory, 3480 Dunbar Hall

9:00- 9:10 (247) A note on compact classes of ℓ^2 operators. Preliminary report. Professor RONALD RIETZ, Gustavus Adolphus College (726-47-1)

9:15- 9:25 (248) Existence of large solutions to nonlinear operator equations. Preliminary report. Professor JOHN MACBAIN, Air Force Institute of Technology, Wright-Patterson AFB, Ohio (726-47-6)

9:30- 9:40 (249) A fractional mapping theorem for essential spectra. Dr. JOHN J. BUONI, Youngstown State University (726-47-9)

FRIDAY, 9:45 A.M.

Invited Address, Miller Auditorium

(250) Algebraic cycles on algebraic varieties. Professor DAVID MUMFORD, Harvard University (726-14-6)

FRIDAY, 10:00 A.M.

Special Session on Banach spaces with the Radon-Nikodým Property III, 3770 Knauss Hall

10:00-10:20 (251) On the Radon-Nikodým property in Fréchet spaces and applications. Mr. ELIAS SAAB, Université de Paris, France (726-46-11) (Introduced by R. E. Huff)

10:30-10:50 (252) Noncompact extremal integral representations. Professor G. A. EDGAR, Northwestern University (726-46-4)

11:00-11:20 (253) Geometric characterizations of the RNP. II. Professor ROBERT R. PHELPS, University of Washington (726-46-9)

11:30-11:50 (254) Comments on Radon-Nikodým and allied propositions. Professor B. J. PETTIS, University of North Carolina (726-46-19)

FRIDAY, 10:00 A.M.

Special Session on Efficient Algorithms for Exact Computation, 3750 Knauss Hall

10:00-10:20 (255) Recent developments on minimum spanning trees. Professor ANDREW C. YAO, Massachusetts Institute of Technology (726-68-4)

10:30-10:50 (256) Riemann's hypothesis and tests for primality. GARY L. MILLER, University of California, Berkeley (726-10-5)

11:00-11:20 (257) Informal Session

11:30-11:50 (258) Efficient computations in geometry. DAN HOEY* and MICHAEL IAN SHAMOS, Yale University (726-68-2)

FRIDAY, 10:00 A.M.

Session on Combinatorics, 3480 Dunbar Hall

10:00-10:10 (259) Common transversals for any number of partitioning families. Professor JUDITH Q. LONGYEAR, Wayne State University (726-05-1)

10:15-10:25 (260) Three Stirling number identities from the stabilizing character. Preliminary report. Dr. MICHAEL GILPIN, Michigan Technological University (726-05-7)

10:30-10:40 (261) A generalized measure of dependence and its relation to the Kronecker product of graphs. Preliminary report. Dr. KAREN E. MACKEY*, State University of New York at Binghamton, and Dr. BRUCE H. BARNES, NSF, Washington, D.C. (726-05-18)

10:45-10:55 (262) Edge-3-colorability of certain graphs. II. Dr. J. L. HURSCH, Jr., Boulder, Colorado (726-05-19) (Introduced by Jan Mycielski)

11:00-11:10 (263) Isomorphisms of hypergraphs and graphs. Dr. J. A. ZIMMER, Pahlavi University (726-05-20)

11:15-11:25 (264) A nonregular Latin 3-cube solution of Euler's 6×6 Officers Problem. Professor JOSEPH ARKIN, Spring Valley, New York (726-05-21)

11:30-11:40 (265) The reconstruction conjecture for tournaments is false. Preliminary report. Professor PAUL K. STOCKMEYER, College of William & Mary (726-05-22)

FRIDAY, 11:00 A.M.

Invited Address, Miller Auditorium

(266) Ergodic properties of elementary mappings of the unit interval. Dr. ROY L. ADLER, IBM T. J. Watson Research Center, Yorktown Heights, New York (726-28-3)

FRIDAY, 1:00 P.M.

Special Session on Affine Algebraic Groups III, 3750 Knauss Hall

1:00- 1:20 (267) Embedding finite group schemes in p-divisible groups. ROBERT MORRIS, Institute for Advanced Study (726-14-1)

1:30- 1:50 (268) Rings of invariants and rational singularities. Preliminary report. Professor JOEL L. ROBERTS, University of Minnesota (726-14-5)

2:00- 2:20 (269) Cohen-Macaulay and Gorenstein rings of invariants. Professor MELVIN HOCHSTER, Purdue University (726-13-1)

2:30- 2:50 (270) Simple subgroups of simple algebraic groups. Professor BRIAN PARSHALL, University of Virginia (726-20-2)

3:00- 4:00 (271) Informal Session

FRIDAY, 1:00 P.M.

General Session, 170 Wood Hall

1:00- 1:10 (272) Errors in mathematics. Preliminary report. Mr. PRESTON C. HAMMER, Grand Valley State Colleges (726-00-1)

FRIDAY, 1:30 P.M.

Special Session on Functional Equations V, 3760 Knauss Hall

1:30- 1:50 (273) On characterization of probability distributions by conditional expectations. Dr. IGNACY I. KOTLARSKI, Oklahoma State University (726-60-2)

2:00- 2:20 (274) Some functional equations defined by the infimum. Preliminary report. Professor JOHN E. MAXFIELD, Kansas State University, and Professor T. D. HOWROYD*, University of New Brunswick, Fredericton (726-39-16)

2:30- 2:50 (275) The use of functional equations in numerical analysis. Professor MICHAEL A. GOLBERG, University of Nevada, Las Vegas (726-39-2)

3:00- 3:20 (276) Numerical methods for parabolic partial integro-differential equations. Dr. GRAEME FAIRWEATHER, University of Kentucky (726-65-5)(Introduced by Dr. J. Aczél)

3:30- 3:50 (277) One partial differential equation with transformed argument. Professor BOHDAN LAWRUK, McGill University (726-35-2)

4:00- 4:20 (278) Characterization of null cone preserving maps by functional equations. Professor M. A. McKIERNAN, University of Waterloo (726-39-24) (Introduced by Professor J. Aczél)

FRIDAY, 1:30 P.M.

Special Session on Banach Spaces with the Radon-Nikodým Property IV, 3770 Knauss Hall

1:30- 1:50 (279) Some problems concerning subspaces of L^1, quotients of C[0,1] and the Radon-Nikodým property. Preliminary report. Professor HASKELL P. ROSENTHAL, University of Illinois at Urbana (726-46-14)

2:00- 2:20 (280) Weak compactness in the space of Bochner integrable functions, a sad state of affairs. Professor J. J. UHL, Jr., University of Illinois at Urbana (726-46-8)

2:30- 4:00 (281) Informal discussion of open problems

FRIDAY, 1:30 P. M.

Special Session on General Topology IV, 1114 Brown Hall

1:30- 1:50 (282) Spaces with bases of countable rank. GARY GRUENHAGE*, Auburn University and PETER NYIKOS, University of Illinois at Urbana (726-54-17)

2:00- 2:20 (283) A remark on irreducible spaces. Preliminary report. Professor J. C. SMITH, Virginia Polytechnic Institute and State University (726-54-9)

2:30- 4:00 (284) Informal problem session

FRIDAY, 1:30 P. M.

Special Session on Categorical Methods in Algebraic Topology II, 2480 Dunbar Hall

1:30- 4:00 (285) Informal session dealing with problems, conjectures, etc.

FRIDAY, 1:30 P. M.

Special Session on Scientific Computing II, 2520 Dunbar Hall

1:30- 1:50 (286) Fast algorithms for rational powers of formal power series. Professor PETER HENRICI, Eidgenössische Technische Hochschule, Zürich, Switzerland (726-65-4)

2:00- 2:20 (287) How to make your algorithm conservative. Preliminary report. Dr. ALVIN BAYLISS* and Professor EUGENE ISAACSON, Courant Institute, New York University (726-65-6)

2:30- 4:00 (288) Informal Session

Session on Real Analysis, 3480 Dunbar Hall

1:30- 1:40 (289) On functions whose graph is of linear measure 0 on sets of measure 0. Professor JAMES FORAN, University of Missouri, Kansas City (726-26-6)

1:45- 1:55 (290) Directional cluster sets and essential directional cluster sets of real functions defined in the upper half plane. Preliminary report. Professor MICHAEL J. EVANS* and Professor PAUL D. HUMKE, Western Illinois University (726-26-7)

2:00- 2:10 (291) On functions with summable approximate Peano derivative. Preliminary report. Professor CHENG-MING LEE, University of Wisconsin, Milwaukee (726-26-8)

2:15- 2:25 (292) On roots of differentiable functions. Preliminary report. ROBERTO MACCHIA, Stevens Institute of Technology (726-26-9)

2:30- 2:45 (293) Rohlin theorem in the noninvertible case. STUART P. LLOYD, Bell Laboratories, Murray Hill, New Jersey (726-28-4)

2:45- 2:55 (294) Strong liftings in topological measured spaces. Preliminary report. Dr. RICHARD J. MAHER, Loyola University of Chicago (726-28-5)

3:00- 3:10 (295) Extending vector measures. Preliminary report. Professor WILLIAM H. GRAVES, University of North Carolina (726-28-6)

3:15- 3:25 (296) A theorem on weak compactness of measures with application to the Baire classes. Preliminary report. Professor FREDERICK K. DASHIELL, Jr., California Institute of Technology (726-46-16)

Paul T. Bateman
Associate Secretary

Urbana, Illinois

ORGANIZERS AND TOPICS OF SPECIAL SESSIONS

Abstracts of contributed papers to be considered for possible inclusion in special sessions should be submitted to Providence by the deadlines given below and should be clearly marked "For consideration for special session on (title of special session)." Those papers not selected for special sessions will automatically be considered for regular sessions unless the author gives specific instructions to the contrary.

	Deadline
Chicago, Illinois, November 1975	August 12, 1975
Bruce C. Berndt, Number Theory	
Philip Dwinger, Lattice Theory	
Paul Fong, Finite Groups	
Saunders Mac Lane, Category Theory	
Mark A. Pinsky, Stochastic Analysis	
R. Clark Robinson, Global Analysis	
Paul J. Sally, Jr., Harmonic Analysis on Locally Compact Groups	
Philip D. Wagreich, Algebraic Geometry	
Los Angeles, California, November 1975	September 2, 1975
Theodore W. Gamelin, Function Algebras and Related Areas	
Nathaniel Grossman, Differential Geometry	
Blacksburg, Virginia, November 1976	September 2, 1975
John Burns, Differential Equations and Control Theory	
R. E. Hodel, Point Set Theory	
Eugene M. Norris, Binary Relations	

PRELIMINARY ANNOUNCEMENTS OF MEETINGS

The Seven Hundred Twenty-Seventh Meeting
Massachusetts Institute of Technology
Cambridge, Massachusetts
October 25, 1975

The seven hundred twenty-seventh meeting of the American Mathematical Society will be held at the Massachusetts Institute of Technology, Cambridge, Massachusetts, on Saturday, October 25, 1975.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be two one-hour addresses. The lecturers will be Professor C. Herbert Clemens of Columbia University and Professor Bernard Shiffman of Johns Hopkins University. The titles of their talks will be announced in the October issue of these *Notices*, which will also contain the program of the meeting.

There will be sessions for contributed ten-minute papers both morning and afternoon. Abstracts should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island, 02940, so as to arrive prior to the deadline of September 2, 1975. No provision will be made for late papers.

Walter H. Gottschalk
Associate Secretary

Middletown, Connecticut

The Seven Hundred Twenty-Eighth Meeting
University of Illinois at Chicago Circle
Chicago, Illinois
November 1, 1975

The seven hundred twenty-eighth meeting of the American Mathematical Society will be held at the University of Illinois at Chicago Circle, Chicago, Illinois, on Saturday, November 1, 1975. All sessions of the meeting will be held in the Lecture Center of the university. The university is located approximately one mile west and one-half mile south of the intersection of State and Madison Streets, the origin of coordinates in the Chicago street numbering system.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Professor Jonathan L. Alperin of the University of Chicago will speak at 11:00 a.m. on the topic "Finite groups viewed locally." Professor R.O. Wells, Jr. of Rice University will speak at 1:45 p.m.; his subject will be "Poincaré's equivalence problem for real hypersurfaces in C^n."

By invitation of the same committee there will be eight special sessions of selected twenty-minute papers. Professor Bruce C. Berndt of the University of Illinois at Urbana-Champaign is organizing a special session on Number Theory; the tentative list of speakers includes Michael N. Bleicher, Thomas W. Cusick, Hiroshi Gunji, Marvin I. Knopp, Erik A. Lippa, John L. Selfridge, Kenneth B. Stolarsky, and Peter J. Weinberger. Professor Philip Dwinger of the University of Illinois at Chicago Circle is organizing a special session on Lattice Theory; the tentative list of speakers includes M. E. Adams, Raymond Balbes, Alan Day, George Epstein, George A. Grätzer, and David Kelly. Professor Paul Fong of the University of Illinois at Chicago Circle is organizing a special session on Finite Groups; the tentative list of speakers includes I. Martin Isaacs, Fred Smith, Louis Solomon, and John H. Walter. Professor Saunders Mac Lane of the University of Chicago is organizing a special session on Category Theory; the tentative list of speakers includes Peter Johnstone, F. William Lawvere, John L. MacDonald, Michael Makkai, and Donovan H. Van Osdol. Professor Mark A. Pinsky of Northwestern University is organizing a special session on Stochastic Analysis; the tentative list of speakers includes D. J. Hebert, Jr., Thomas G. Kurtz, George C. Papanicolaou, Mark A. Pinsky, Walter A. Rosenkrantz, Thomas H. Savits, and Daniel W. Stroock. Professor R. Clark Robinson of Northwestern University is organizing a special session on Global Analysis; the tentative list of speakers includes Robert L. Devaney, Brian H. Marcus, Richard P. McGehee, Dennis G. Pixton, and Carl P. Simon. Professor Paul J. Sally, Jr. of the University of Chicago is organizing a special session on Harmonic Analysis on Locally Compact Groups; the tentative list of speakers includes Dan Barbasch, Rebecca A. Herb, Philip C. Kutzko, Mark Novodzorsky, Richard C. Penney, and R. Ranga Rao. Professor Philip D. Wagreich of the University of Illinois at Chicago Circle is organizing a special session on Algebraic Geometry; the tentative list of speakers includes Eric M. Friedlander, Gerald L. Gordon, M. Pavaman Murthy, Peter P. Orlik, and J. Wood.

There will be sessions for contributed ten-minute papers both morning and afternoon. Abstracts should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of September 2, 1975. Those having time preferences for the presentation of their paper should indicate them clearly on their abstracts. There will be a session for late papers if needed, but late papers will not be listed in the printed program of the meeting.

REGISTRATION

The registration desk will be located in the Illinois Room Lobby on the third level of the Chicago Circle Center, the student center of the University of Illinois at Chicago Circle. The Center is located on the west side of Halsted Street, opposite the point at which Polk Street comes to a dead end. The registration desk will be open from 8:00 a.m. to 3:00 p.m. The registration fee will be two dollars.

ACCOMMODATIONS

The hotel headquarters for the meeting will be the Holiday Inn (Kennedy Expressway). Detailed information is given below. The rates quoted are subject to an 8.1 percent tax. Those making reservations should simply specify that they will be attending the meeting of the American Mathematical Society as guests of the University of Illinois at Chicago Circle. There is a free garage.

HOLIDAY INN (Kennedy Expressway)
1 South Halsted Street
Chicago, Illinois 60606
(One-half mile north of the university.)

Singles	$18.00
Doubles	$27.00

FOOD SERVICE

The Pier Room Cafeteria in the Chicago Circle Center will be open for lunch. There are also several Greek restaurants on Halsted Street just north of the campus.

TRAVEL AND LOCAL INFORMATION

The University of Illinois at Chicago Circle is named after, served by, and located at the Chicago Circle cloverleaf formed by the three major expressways into the downtown area of Chicago from the north, south, and west. Those coming from the west on the Dwight D. Eisenhower Expressway should use the Racine Avenue Exit. Those coming from the north on the John F. Kennedy Expressway should turn west on the Eisenhower Expressway and take the Morgan Street Exit. Those coming from the south on the Dan Ryan Expressway should exit at Taylor Street. There will be parking for those attending the meeting in the two parking lots at the corner of Polk and Halsted Streets (across Halsted Street from the Chicago Circle Center).

There is direct limousine service between O'Hare Airport and the Loop area of downtown Chicago. To get from the Loop to the Chicago Circle Campus either (a) take the Dearborn Street subway southbound to the Halsted Street stop, or (b) take the Number 7 bus from its starting point on the west side of State Street between Harrison and Congress Streets, or (c) take a taxi to the intersection of Polk and Halsted Streets.

ENTERTAINMENT

The Lyric Opera of Chicago will perform Mozart's opera The Marriage of Figaro on Friday, October 31, at 8:00 p.m. and Beethoven's opera Fidelio on Saturday, November 1, at 8:00 p.m. The opera house is located at the corner of Madison Street and Wacker Drive. The Chicago Symphony Orchestra will give a program under Rafael Kubelik on Friday, October 31, at 2:00 p.m. and Saturday, November 1, at 8:30 p.m. Orchestra Hall is located on Michigan Avenue between Adams Street and Jackson Boulevard.

The Chicago Circle Campus contains the original site of the Jane Addams' Hull House and Residents' Dining Room. Both have been restored by the University of Illinois and are designated as National Historic Landmarks by the U.S. Department of the Interior. They are open to visitors from 10:00 a.m. to 4:00 p.m. every day but Sunday.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

The Seven Hundred Twenty-Ninth Meeting
Virginia Polytechnic Institute and State University
Blacksburg, Virginia
November 7–8, 1975

The seven hundred twenty-ninth meeting of the American Mathematical Society will be held at the Virginia Polytechnic Institute and State University in Blacksburg, Virginia, from noon Friday, November 7 until noon Saturday, November 8, 1975.

By invitation of the Committee to Select Hour Speakers for the Southeastern Sectional Meetings there will be three one-hour addresses presented. The speakers will be Professor James D. Buckholtz of the University of Kentucky, Professor Robert J. Daverman of the University of Tennessee, and Professor William Jaco of Rice University.

There will be three special sessions at this meeting. Professor John Burns of Virginia Polytechnic Institute and State University is organizing a special session on Differential Equations and Control Theory; Professor R. E. Hodel of Duke University is organizing a special session on Point Set Theory; and Professor Eugene M. Norris of the University of South Carolina is arranging a special session on Binary Relations. Any member of the American Mathematical Society who would like to have his or her paper considered for inclusion in one of the special sessions should have his or her abstract so marked and in Providence at least three weeks before the regular closing date for contributed papers, or by September 2, 1975.

There will also be sessions for contributed papers on Friday and Saturday. Abstracts for contributed papers should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of September 23, 1975.

O. G. Harrold, Jr.
Associate Secretary

Tallahassee, Florida

The Seven Hundred Thirtieth Meeting
University of California
Los Angeles, California
November 15, 1975

The seven hundred thirtieth meeting of the American Mathematical Society will be held at the University of California, Los Angeles, California, on Saturday, November 15, 1975.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited hour addresses. Professor Jerry L. Kazdan, University of California at Berkeley and University of Pennsylvania, will lecture on "Applications of partial differential equations to differential geometry." The other invited address will be given by Professor Robert Osserman, Stanford University; the title of his lecture will appear in the October issue of these *Notices*.

There will be two special sessions of selected twenty-minute papers. Professor Theodore W. Gamelin of the University of California, Los Angeles, is organizing a special session on Function Algebras and Related Areas. Among the speakers will be Alice Chang, Irving L. Glicksberg, Hugo Rossi, and Donald E. Sarason. Professor Nathaniel Grossman of the University of California, Los Angeles, is organizing a special session on Differential Geometry. Among the speakers will be Mark Green and Joel L. Weiner.

As usual, there will be sessions for contributed ten-minute papers. Abstracts for contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of September 23, 1975. Late papers will be accepted for presentation at the meeting, but will not be listed in the printed program of the meeting.

Information about travel and accommodations will appear in the October issue of these *Notices*, and the final program of the meeting will appear in the November *Notices*.

Kenneth A. Ross
Associate Secretary

Eugene, Oregon

INVITED SPEAKERS AT AMS MEETINGS

This section of these *Notices* lists regularly the individuals who have agreed to address the Society at the times and places listed below. For some future meetings, the lists of speakers are incomplete.

Cambridge, Massachusetts, October 1975

C. Herbert Clemens — Bernard Shiffman

Chicago, Illinois, November 1975

Jonathan L. Alperin — R. O. Wells, Jr.

Blacksburg, Virginia, November 1975

James D. Buckholtz — William Jaco
Robert J. Daverman

Los Angeles, California, November 1975

Jerry Kazdan — Robert Osserman

Tallahassee, Florida, March 1976

Joe Ball — Rudolf E. Kalman
Leonard Carlitz

CASE STUDIES
Some Mathematicians with Nonacademic Employment

This is the fourth collection of case studies of mathematicians with nonacademic or nontraditional employment assembled by Professor Martha K. Smith of the University of Texas at Austin on behalf of the Society's Committee on Employment and Educational Policy. The first case studies appeared in the November 1974 issue of the *Notices*, and others were published in the February and June 1975 issues.

GERALD J. DECKER

Since July of this year, I have been employed by INCO, Inc., a moderately sized private consulting firm specializing in systems analysis, research, and development utilizing advanced computer technology for a wide range of customers in government and industry. My position in the company is that of a senior systems analyst. In particular, I provide technical expertise of a mathematical nature and provide general analytical expertise useful in the analysis, design, and development of computer oriented systems.

In the short time I have been with the company, I have had the opportunity to work on only one project. This project, while it does not require any particular mathematical experience, does require the ability to approach a problem logically and the ability to develop creative analysis and solutions. The central problem is the development of a collection of procedures and computer programs that will provide a substantial automation of budget analysis tables used by Congressional appropriations committees in developing budget legislation. Currently, these tables are manually prepared and maintained; however, properly designed application of the considerable power of computer data processing techniques will result in substantial time savings. The challenge is to develop such techniques so that they can be effectively utilized by noncomputer oriented individuals.

I received my Ph.D. in mathematics from the University of Chicago in June, 1974. My dissertation title is, "The Integral Homology of an Eilenberg-Mac Lane Space," so that my area of major interest is algebraic topology. I have not yet used, and do not expect to use, my specific mathematical training while working as a systems analyst. Nonetheless, this training has given me some ability to formulate problems intelligently, search out reasonable approaches for determining solutions, and finally present feasible solutions for potential implementation.

I joined INCO directly after completing my graduate work at Chicago. The company was brought to my attention by a close friend, not a mathematician, who went to work for the firm several years ago and was pleased with the type of work and quality of personnel in the organization. I interviewed with one of the company's directors and explained that I was interested in leaving the academic environment and felt that my training and experience would be a useful asset to their firm. I am currently the only mathematician on the staff and, consequently, have a unique opportunity to provide, in some measure, the fruit of my training in analyzing problems, mathematical or not, that arise in INCO's contractual work. I would say that my primary usefulness to the company is the flexibility, logical approach to problems, and ability to seek new or alternate solutions that I have learned through my formal training.

Thus far, I have found my association with INCO to be interesting and rewarding in that I find that I can actively participate in the design of computer oriented systems to achieve fixed goals under specified constraints. I enjoy working with other people in the environment of free interchange of ideas, something that is certainly necessary to "get along" in the consulting environment.

My advice to others who might be interested in seeking such a job is to show a prospective employer that you are not a "dreamer" but interested in tackling real world problems relevant to the company, using at least your ability to logically analyze a problem, and perhaps some of your formal mathematical training as the occasion arises. It is particularly important to be able and willing to communicate with various people at varying levels of technical experience, and to have training or experience in some complementary area to mathematics (computer science in my case).

DOUGLAS B. PRICE

I have been working for NASA at Langley Research Center since 1965. At the time I was hired, I had a B.S. in mathematics from Millsaps College, Jackson, Mississippi, and was taking graduate work in mathematics at the University of Mississippi. My M.S., at the College of William and Mary, and Ph.D., at North Carolina State University, were completed under NASA's very generous graduate study program which allowed me to receive my salary while going to school and to work on my dissertation on the job. My dissertation research, which involved generalized functions and integral transforms, was not immediately applicable to a NASA project, but was of sufficient general interest to fit within NASA's charter for research.

My work assignment at NASA, which now involves research into control, filtering and estimation theory, especially as these relate to flight control systems, has evolved over the past nine years, as opposed to changing abruptly as I received advanced degrees in mathematics. Still, as far as NASA is concerned, I am not a research mathematician, but a research engineer with a strong background in mathematics. This situation leads to occasional frustrations when engineering questions are not interesting mathematically, and vice-versa. There is a tendency here to relegate theory to appendices in technical reports and concentrate on applications or specific examples, whereas my inclination is to concentrate on theory and put applications and examples in appendices. In spite of this difference in viewpoint, which is certainly not unique to NASA, there is mathematics that can be done here.

NASA sponsors the Institute for Computer Applications in Science and Engineering (ICASE), through which a lot of research in numerical analysis is performed. Other areas of mathematics which are of interest here include solutions of partial differential equations, finite element methods, probability and statistics, stochastic processes, boundary value problems, etc.

As for getting a job here now, the situation does not look good. NASA, and the government in general, is cutting back rather than hiring, and I suspect when hiring does start again, Ph.D. mathematicians will not have very high priority. In order to be considered for employment by NASA, a mathematician will have to convince the employer that he is flexible, and can apply his knowledge of mathematics to a broad range of problems.

ANONYMOUS

I am working in the Numerical Analysis Group of Boeing Computer Services, Inc. My present assignment is to develop different methods (direct and iterative) to solve a large system of linear equations in an engineering research project of the Boeing Commercial Airplane Company.

I wrote my thesis in linear algebra and defended it last November. This is my first full time job. I first started at BCS as a scientific application programmer in July 1973. I heard of the position through a friend who was working at BCS. Because of my educational background and the nature of the project I was working on, I was later transferred to my present group.

Most of my programming knowledge has been acquired through the BCS Training Division and the very experienced programmer I am working with. I never had a course in Numerical Analysis either, but my general mathematical background allows me to pick up the subject without much difficulty. And, very obviously, I do have to use basic matrix theory at my work. Having the continuous opportunity to learn and apply what I have learned does make life interesting. I also appreciate the fact that my client (the engineer who is in charge of the research project) defines his problem and requirements meticulously and is concerned about what I am doing. My job could be difficult otherwise.

My primary responsibility is to meet my client's requirements satisfactorily. Yet publications are encouraged in our group. Popular mathematical journals are routed to members of the group. I was very pleased that I was given a budget to write a paper from my thesis.

Although sometimes I envy my friends who are teaching for their opportunities to work with young people, their flexible schedule, their long vacation, and their much quieter working environment, I think our group, headed by an energetic, understanding, and extremely competent numerical analyst, is ideal for mathematicians who do not have to work on their pet research and who are flexible and like to gain a variety of experiences other than their major field (as long as BCS doesn't make one a supervisor!).

NEWS ITEMS AND ANNOUNCEMENTS

AMS RESEARCH FELLOWSHIP HONORABLE MENTION LIST FOR 1975-1976

The Committee on Postdoctoral Fellowships has previously announced the award of three fellowships for 1975-1976 (these *Notices* 22(1975), 187). The committee now announces that the awards of Honorable Mention have been given to the following fourteen applicants: Marion D. Cohen, Newark College of Engineering; Michael Cwikel, Université de Paris - Sud; Charles T. Fulton, Northern Illinois University; Wayne R. Jones, University of Minnesota; Erwin Lutwak, Columbia University; Larry M. Mannevitz, Yale University; Jorn B. Olsson, Harvard University; Simeon Reich, Tel Aviv University; Ahmad Shafaat, Australian National University; Jack W. Silverstein, Brown University; Jon A. Sjogren, University of California, Berkeley; Carol L. Tretkoff, New York University; William Weiss, University of Toronto; Noriko Yui, University of Copenhagen.

The Council and the Trustees of the Society have voted to continue the Research Fellowship Program on the same terms as at present, except that candidates for an AMS Postdoctoral Fellowship shall be citizens or permanent residents of a country in North America.

The survival of the Research Fellowship Program depends on the contributions the Society receives. It is hoped that every member of the Society will be willing to contribute to the Fund. All members have the opportunity to designate their contribution on the dues billing from the Society. A contribution of at least $3.00 from each employed member would make this program a very successful one. Contributions are, of course, tax deductible. Checks should be made payable to the American Mathematical Society, clearly marked "AMS Research Fellowship Fund" and sent to the American Mathematical Society, P. O. Box 1571, Annex Station, Providence, Rhode Island 02901.

MATHEMATICIANS AWARDED NATO SENIOR FELLOWSHIPS

Five mathematicians are among seventy-two scientists awarded North Atlantic Treaty Organization Senior Fellowships in Science. The fellowships, awarded by the National Science Foundation and the Department of State, carry tenures of one to three months for study in fourteen foreign countries and are designed to strengthen the scientific and research potential of United States institutions.

The following are the five mathematicians, their affilations, and their fellowship institutions abroad: Vipinchandra L. Shah (University of Wisconsin at Milwaukee), the Imperial College of Science and Technology; Martin A. Moskowitz (CUNY Graduate Center), the University of Paris; Donald J. Lewis (University of Michigan at Ann Arbor), the University of Cambridge, England and the University of Pisa, Italy; Leo Katz (Michigan State University at East Lansing), the Israel Institute of Technology, Haifa, Israel and the University of Leeds, England; Jay H. Wolkowisky (University of Colorado at Boulder), the University of Cologne, Germany and Oxford University, England.

MATHEMATICIAN AWARDED NATIONAL MEDAL OF SCIENCE

Kurt Gödel, Professor of Mathematics at the Institute for Advanced Study in Princeton, New Jersey, and a member of the American Mathematical Society, was one of thirteen recipients of the 1974 National Medal of Science awarded by the President of the United States. Established by the Eighty-Sixth Congress in 1959 to acknowledge individuals deserving of recognition for their outstanding contributions in the fields of science, mathematics and engineering, the Medal is the Nation's highest award for scientific and mathematical achievements.

ELEVEN MATHEMATICIANS HONOURED BY ACADEMY

Eleven persons from various fields of mathematics are among the 121 newly elected fellows of the American Academy of Arts and Sciences. They include five members of the American Mathematical Society: George B. Dantzig, Professor of Operations Research and Computer Science at Stanford University; Richard E. Bellman, Professor of Mathematics at the University of Southern California; Phillip A. Griffiths, Professor of Mathematics at Harvard University; Marshall Hall, Jr., Professor of Mathematics at the California Institute of Technology and Edward Nelson, Professor of Mathematics at Princeton University. The remaining six new fellows are Thomas E. Cheatham, Jr., Professor of Computer Science at Harvard University; Julian D. Cole, Professor of Engineering and Applied Science at the University of California (Los Angeles); Erich L. Lehmann, Professor of Statistics at the University of California (Berkeley); Herbert E. Robbins, Professor of Statistics at Stanford University and John G. Thompson, Professor of Mathematics at Churchill College, the University of Cambridge, England.

NSF AWARDS SCIENCE FACULTY FELLOWSHIPS

Among the ninety-three Faculty Fellowships in Science awarded by the National Science Foundation, eight went to the following mathematicians, the first three of whom are members of the Society: Lawrence T. Gurley (Mills College, California), Homer T. Hayslett, Jr. (Colby College, Maine), Edward B. Wright (Linn-Benton Community College, Oregon), Adnan M. Haider (University of Maryland), Virginia S. Muraski (Grand Valley State Colleges, Michigan), Stephen M. Pollock (University of Michigan), Tom K. Ford (Central State University, Oklahoma) and Frederick D. Tabbutt (Evergreen State College, Washington). The awards total nearly $1.5 million and were made to faculty members to help them broaden their perspective in applying science to societal problems.

NSF received 598 applications which were considered on the basis of merit and were evaluated by panelists appointed by the American Council on Education. Applicants were divided into three categories: those with bachelors and masters degrees; those with a doctorate degree; and faculty from two-year or community colleges. Each faculty member has five years or more of full-time, college level teaching experience in science, mathematics, or engineering.

Fellowship stipends are based on salary paid during the preceding year. The tenure period varies from three to nine months. In addition to the stipend, NSF provides the fellowship institution with an allowance to assist the institution in meeting tuition and other costs. Further information and applications may be obtained by writing to the NSF, 1800 G Street, Washington, D. C. 20550.

SALEM PRIZE

The Salem Prize for 1975 was awarded to Dr. William Beckner, of the University of Princeton, for his work on basic inequalities in Fourier analysis. The prize, established in 1968, is given every year to a young mathematician who is judged to have done an outstanding work in the field of interest of Salem, primarily on Fourier series and related topics. The recipient was Dr. Nicholas Varopoulos in 1968, Dr. Richard Hunt in 1969, Dr. Yves Meyer in 1970, Dr. Charles Fefferman in 1971, Dr. Thomas Körner in 1972, Dr. E. M. Nikišin in 1973 and Dr. Hugh Montgomery in 1974. The jury consisted in Professor A. Zygmund, Professor L. Carleson, Professor J.-P. Kahane and Professor Ch. Pisot.

INDIAN INSTITUTE
OF OPERATIONS MANAGEMENT

The International Center for Applied Analysts of the Indian Institute of Operations Management is facing a financial crisis. Collaboration between the Institute and Rice University's Division of Mathematical Sciences can begin only after the necessary funds are raised. Interested individuals and organizations are urged to send contributions to S. Ghoshal, Director, Indian Institute of Operations Management, 11/24 Jheel Road, Newland, Calcutta, India.

NEW AMS COMMITTEES

Committee on Principles and Procedures. Edwin E. Moise has agreed to serve as the chairman for the newly formed Committee on Principles and Procedures. Appointed to the committee are: Chandler Davis, James A. Donaldson, Paul R. Halmos, Robion C. Kirby, and Barbara L. Osofsky.

Committee on Affirmative Action Procedures. An ad hoc Committee on Affirmative Action Procedures has been appointed by President Lipman Bers. Murray Gerstenhaber will serve as chairman and its members are: A. T. Bharucha-Reid, Raoul H. Bott, Mary Elizabeth Hamstrom, Alice T. Schafer and Hans F. Weinberger. This committee has been authorized to prepare recommendations concerning procedures to be used by mathematics departments in implementing affirmative action programs.

Advisory Committee on the Editorial Policy of the *Notices*. An Advisory Committee on the Editorial Policy of the *Notices* has been appointed by President Lipman Bers. Everett Pitcher will serve as chairman and Gordon Walker will be an ex officio member. The remaining members on the committee will be: Ed L. Dubinsky, Joseph B. Keller, Robion C. Kirby, Edward J. McShane, Barbara Osofsky, and Scott Williams.

Joint Projects Committee for Mathematics. The following have been designated to be the Joint Projects Committee for Mathematics: Frederick J. Almgren, Jr., Dorothy L. Bernstein, George F. Carrier, Hirsh G. Cohen, Paul R. Garabedian, Paul R. Halmos, William H. Kruskal, Saunders Mac Lane (chairman), and Jacob T. Schwartz. The appointment was made jointly by Lipman Bers, President of the American Mathematical Society, Henry O. Pollak, President of the Mathematical Association of America, and Herbert B. Keller, President of the Society for Industrial and Applied Mathematics.

PERSONAL ITEMS

PETER J. HILTON of the Battelle Seattle Research Center and Case Institute of Technology has been awarded the Silver Medal for 1974 by the University of Helsinki, Finland.

HOWARD W. LAMBERT of the University of Iowa has been appointed a Latin American Teaching Fellow by Tufts University. He will serve the academic year 1975–1976 at the Universidad de Oriente, Cumaná, Venezuela.

EUGENE H. LEHMAN of the University of Quebec, Trois-Rivieres has received a grant given by the National Research Council of Canada for research on Coding Theory at the Summer Research Institute, University of Calgary.

ELLIOTT H. LIEB has been appointed to a professorship at Princeton University where he is currently teaching as a visiting faculty member while on leave from the Massachusetts Institute of Technology.

MORRIS MARDEN of the University of Wisconsin, Milwaukee, has retired after 45 years on its faculty.

JOHN N. MATHER of Harvard University has been appointed to a professorship at Princeton University.

IVAR STAKGOLD of Northwestern University has been appointed to the chairmanship of the Department of Mathematics at the University of Delaware.

BILL WATSON of the Universidad Simon Bolivar, Sartenejas, Venezuela, has been appointed to an assistant professorship at Case Western Reserve University.

PROMOTION

To Director. Mathematics Division, U.S. Army Research Office: JAGDISH CHANDRA.

To Chairman. Department of Statistics, University of Wisconsin-Madison: JOHN R. VAN RYZIN.

To Professor. University of Nevada, Reno: GERALD W. KIMBLE.

To Associate Professor. Mount Holyoke College: ROBERT J. WEAVER; Pennsylvania State University at McKeesport: D. JAMES SAMUELSON.

DEATHS

Professor BURNS W. BREWER of North Texas State University died on March 24, 1975, at the age of 62. He was a member of the Society for 35 years.

Professor W. E. BYRNE of Rockbridge Baths, Virginia, died on November 4, 1973, at the age of 75. He was a member of the Society for 28 years.

Professor WILLIAM C. CHEWNING, Jr., of the University of South Carolina, Columbia, died on March 23, 1975, at the age of 29. He was a member of the Society for 5 years.

Professor H. A. HEILBRONN of the University of Toronto died on April 28, 1975, at the age of 66. He was a member of the Society for 27 years.

BACKLOG OF MATHEMATICS RESEARCH JOURNALS

Information on the backlog of papers for research journals is published in the February and August issues of these *Notices* with the cooperation of the respective editorial boards. Since all columns in the table are not self-explanatory, we include further details on their meaning.

Column 3. This is an estimate of the number of printed pages which have been accepted but are not necessary to maintain copy editing and printing schedules.

Column 5. The first (Q_1) and third (Q_3) quartiles are presented to give a measure of normal dispersion. They do not include misleading extremes, the result of unusual circumstances arising in part from the refereeing system.

The observations are made from the latest issue of each journal received at the Headquarters Offices before the deadline for the appropriate issue of these *Notices*. Waiting times are measured in months from receipt of manuscript in final form to receipt of final publication at the Headquarters Offices. When a paper is revised, the waiting time between an editor's receipt of the final revision and its publication may be much shorter than is the case otherwise, so these figures are low to that extent.

	1	2	3		4	5		
JOURNAL	Number Issues per Year	Approximate Number Pages per Year	BACKLOG 5/31/75	12/31/74	Estimated Time for Paper Submitted Currently to be Published (In Months)	Observed Waiting Time in Latest Published Issue (In Months) Q_1	Q_2	Q_3
Acta Informatica	4	790-860	0	0	8		*	
American J. of Math.	4	1000	1500	2000	24	28	29	30
Annals of Math.	6	1200	300	600	6	18	19	23
Annals of Probability	6	1200	0	100	14		*	
Annals of Statistics	6	1200	0	200	NR**	12	13	15
Arch. History of Exact Scis.	5	452	0	0	9-10	7	11	13
Arch. Rational Mech. Anal.	12	1170	0	0	10-11	9	12	20
Canad. J. Math.	6	1440	1200	1000	12	12	16	20
Comm. Math. Physics	16	1610	210	96	6-7	6	8	9
Duke Math. J.	4	800	125	170	8	6	8	9
Illinois J. of Math.	4	700	0	0	9-12	12	15	17
Indiana Univ. Math. J.	12	1200	1000	1200	17	12	13	13
Inventiones Math.	12	1152	NR**	NR**	10	5	8	19
J. Amer. Stat. Assoc.	4	1050	201	65	10	10	12	13
J. Assoc. for Comp. Mach.	4	700	0	0	6-9	8	10	13
J. Comp. and Sys. Scis.	6	800	NR**	350	12	19	25	29
J. Diff. Geometry	4	650	900	850	18	16	18	20
J. Math. Physics	12	2300	0	0	6	5	6	10
J. Symbolic Logic	4	900	0	0	10	17	17	18
Linear Algebra and Appl.	9	870	100	200	9	12	15	18
Manuscripta Math.	12	1200	400	98	6-7	3	5	7
Math. Biosciences	10	1900	300	500	9		#	
Math. Systems Theory	4	384	NR**	NR**	6	19	24	28
Math. of Comp.	4	1200	0	0	11	17	20	22
Math. Annalen	15	1440	68	118	9	8	9	13
Math. Zeitschrift	14	1350	0	0	6-7	6	7	8
Memoirs of AMS	6	2000	800	200	9	6	-	11
Michigan Math. J.	4	400	100	120	11	8	10	14
Numerische Math.	9	790-860	0	0	10	8	12	19
Operations Research	6	1200	500	400	12-15	8	9	11
Pacific J. of Math.	12	3600	NR**	NR**	17	13	22	26
Proceedings of AMS	12	3640	1100	1500	12	14	16	16
Quarterly of Appl. Math.	4	500	500	500	12	16	18	19
Semigroup Forum	8	750	0	69	4-6	5	7	8
SIAM J. of Appl. Math.	8	1600	0	0	8-11	13	13	16
SIAM J. on Computing	4	600	0	0	8-10	10	11	11
SIAM J. on Control	6	1250	0	0	9-11	13	16	17
SIAM J. on Math. Anal.	6	1050	0	0	9	10	11	13
SIAM J. on Numer. Anal.	6	1400	0	0	7-9	11	12	17
SIAM Review	4	800	0	0	9	13	16	19
Transactions of AMS	12	5600	400	740	12	15	16	19
Z. Wahrscheinlichkeitstheorie	12	1050	NR**	NR**	7-8	5	7	13

*Not computable for this journal.
**NR means that no response was received to a request for information.
#Date of receipt of manuscript not given in this journal.

LETTERS TO THE EDITOR

Editor, the *Notices*

We would like to criticize the evolving nature of the special sessions at the meetings of the Society. At the recent meeting in Washington it was almost impossible to distinguish between these sessions and the regular sessions. It seems that the original purpose of the special sessions was to supply an opportunity for like-minded mathematicians of all ranks to get together informally to talk mathematics. In contrast to the regular sessions which are highly structured permitting formal presentation of new results with accompanying abstracts, the special sessions should remain as free as possible of administrative and organizational constraints. Possibly a chairman is needed, but otherwise the Society should adopt some rule that no name can be listed and no credits given.

The first such session we attended was at the Oregon meeting in 1969 where various people got together to talk, and listen to others talk, mathematics. Because people wanted to, the sessions continued over three days and were amongst the most successful we have attended. Let's keep them this way and keep them out of the hands of the organizers and in the hands of the people. If people want to put forth incomplete ideas, ask questions, make conjectures and make mistakes, let them, without threat of being published. Those who want to publish can still attend the regular sessions.

J. C. Abbott
R. J. Greechie

EDITORIAL COMMENT: Special sessions are intended to be of several types. In one, the papers are invited by a chairman. In another, the papers accumulate being offered by a speaker and accepted by the chairman or being selected from ten minute contributed papers by the chairman or the Associate Secretary. The invited papers are the more frequent. In any event there is an advance schedule as well as published abstracts.

In addition, there are informal sessions, with no program publicly presented in advance but formed among enthusiasts who wish to get together to talk.

The letter above may be taken as supporting the informal sessions.

Editor, the *Notices*

Once again it is necessary to bring to the attention of the membership the insensitivity of many of its elected and appointed officials in regards to issues that affect AMS members.

That the announcement of a position at the University of Rhodesia appeared in a recent issue of Employment Information for Mathematicians is lamentable that the AMS Council in January tabled a motion to accept no further announcements from this institution is outrageous. It need not be said that many members of the AMS, solely because of their skin color, would not qualify for this position. How long before the editors of our various publications will notice that many of their actions have discriminatory effects on many of our members?

As an observer at the same Council meeting, I was appalled by the vote of the Council which indicated its intent to cancel all reciprocity agreements. This action was an immediate response to permit South Africa to escape individual shame and censure for its racially biased practices.

Such continued insensitivity by the policy makers of the AMS may well force institutional and individual members who value morality to resign, or at the very least, to curtail their use of various publications.

The argument that we must avoid taking political actions is in fact a political action. By taking no action, we maintain, and perpetuate a situation in which many of our members are discriminated against on political, not academic, basis.

M. Solveig Espelie

EDITORIAL COMMENT: The policy for acceptance of announcements of positions in EIM was enunciated by the Council of October 25, 1974 as follows:

> The Council of the AMS adopts the principles that all positions in the mathematical sciences shall insofar as practicable be advertised, and that the standard place for the advertisements to appear is the publication Employment Information for Mathematicians.

The Council of January 22, 1975 agreed in principle with a proposal, made by an individual member to the Committee on Principles of Reciprocity Agreements, that such agreements be cancelled and authorized a committee to study the implications of such a possible action.

Editor, the *Notices*

I am aware of some of the categories (like students, institutional exchange or nominees, etc.) in which persons don't have to pay the regular full dues for the AMS membership. Let me suggest or campaign for one more through the forum of this column. That persons be allowed to remain or to become the members of the AMS while subscribing for the *Notices* only. To a good majority the Bulletin does not serve any purpose whereas the *Notices* means a lot to a lot! This will definitely mean more revenue to the AMS besides the fact that the Society will be more meaningful to a larger section in the professions. Thanks.

S. C. Bhatnagar

Editor, the *Notices*

The well known Russian mathematician Ilya I. Pjatetsky-Shapiro applied for an exit visa from the Soviet Union sometime in the first half of 1974. His application was refused and several appeals were rejected, once on the ground that he possessed information important to the State, another time on the ground that he had had access to classified information. There is no justification for either of these claims, but as is common with applicants for exit visa from the USSR, Professor Pjatetsky-Shapiro was "requested" to give up his position at the Inst. for Applied Mathematics at Moscow University, and he is presently without a job. Since Professor Pjatetsky-Shapiro participated in hunger strikes he has been questioned and accused of "parasitism." The charges have not been pressed so far, but there seems to be a real risk that he will be jailed on this or some other charge. Professor Pjatetsky-Shapiro is cut off from most scientific contacts and it is important that American mathematicians visiting Moscow try to get in touch with him. I urge everyone planning a visit to Moscow to do so, and shall be glad to provide them with Professor Pjatetsky-Shapiro's address.

Harry Kesten

Editor, the *Notices*

The article by Fleming in the April 1975 issue of the *Notices* on the employment situation shows an admirable common sense and realism that has been sadly lacking in most such material printed in the *Notices* since Hard Times arrived four or so years ago. I even more strongly emphasize the importance of the observation that academic mathematicians are not aware that "applied mathematics" in the non-academic sense has been taken over by those trained in electrical engineering, computer science, statistics, etc.

We must recognize that this situation is now irreversible. Too much time was wasted in the 1960's quibbling over definitions of Applied Mathematics and making self-fulfilling prophecies that applied mathematicians were bound to be inferior. Since 1970 neither university administrations nor agencies that support mathematics have shown interest in or understanding of the problems or development of Applied Mathematics as part of the mathematics profession.

As one who has been involved with these problems for a long time, I suggest that thought and discussion could now most usefully be oriented to the following theses:

A) Graduate enrollment in mathematics (pure or applied) must be discouraged and reduced. For example, the physicists did this several years ago, and it now appears to have improved prospects for those entering the profession.

B) Much more attention should be paid to interaction with those in such disciplines as computer science, electrical engineering and physics who have taken over the role of educating the Applied Mathematicians of the future. Perhaps some modest new interdisciplinary programs might be attempted to take up the loss in enrollment strictly within mathematics, although even this I would not recommend unless our so-called Prestigious Universities take the whole problem more seriously than they took the Applied Mathematics problem in the 1960's.

C) At a research level, an attempt could be made to understand that mathematics as an intellectual discipline involves more than blindly piling up Theorems, that motivation and interconnection are more important, and that Applications seem to have historically played an important role in giving meaning and direction to mathematical research. Finally, my personal concern is that a place exist within mathematics for work on the mathematical structure of other disciplines. If someone interested in mathematics can flourish within an electrical engineering department, why cannot someone interested in the mathematics of (say) systems theory, biology, economics or physics be comfortable within mathematics?

Robert Hermann

Editor, the *Notices*

We read with interest the letter by F. G. Asenjo in the February, 1975 *Notices*, and would like to express our agreement with his suggestion that members be allowed to choose between receiving the Bulletin or sections of the Math Reviews. We find that the *Notices* and the Reviews are valuable to us, but the Bulletin is not. Would you please see that our views are passed on to the appropriate Editorial Committee, or let us know where to express our views directly? Also, some discussion in the *Notices* of the considerations that have gone into the rejection of similar proposals in the past would be appreciated.

Thank you.

Douglas D. Smith
Delano P. Wegener
Edwin H. Kaufman, Jr.
Robert A. Chaffer
Richard St. André
Maurice Eggen
Thomas J. Miles
Douglas W. Nance

EDITORIAL COMMENT: Two similar letters have been received. There is an ad hoc committee of the Executive Committee and the Board of Trustees examining the Bulletin and the *Notices*. Comments may be addressed to Professor Hugo Rossi, who is the Chairman of the Committee.

SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, meetings of other associations, and to notify the organizers if conflicts in subject matter, dates, or geographical area become apparent. An announcement will be published in these *Notices* if it contains a call for papers, place, date, subject (when applicable), and speakers; a second announcement will be published only if changes to the original announcement are necessary, or if it appears that additional information should be announced.

In general, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry slightly more detailed information. Information on the pre-preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society. Deadlines for particular issues of the *Notices* are the same as the deadlines for abstracts which appear on the inside front cover of each issue.

August 11-15, 1975
NSF REGIONAL CONFERENCE ON CLASS GROUPS AND PICARD GROUPS OF GROUP RINGS AND ORDERS
Carleton College, Northfield, Minnesota
Program: Irving Reiner (University of Illinois) will deliver a series of ten lectures. There will also be short talks by participants and informal discussions.
Support: Travel and subsistence allowance for 25 participants will be provided by the National Science Foundation.
Information: Steven Galovich, Department of Mathematics, Carleton College, Northfield, Minnesota 55057.

August 18-23, 1975
EVOLVING A MATHEMATICAL ATTITUDE IN THE SECONDARY EDUCATION (14-18 years)
Nyiregyháza, Hungary
Sponsors: Bolyai János Mathematical Society.
Information: János Surányi, Bolyai János Mathematical Society, H-1061 Budapest, Anker köz 1-3, Hungary.

August 25-28, 1975
COMPUTATIONS IN ALGEBRA AND NUMBER THEORY
University of New Brunswick, Fredericton, New Brunswick
Program: Some of the topics which may be included are: use of the computer in algebra, computation of special invariants, integer programming, improving algorithms, new and useful computations in graph theory, useful programming languages for algebraic computations, computations interesting and accessible to high school students. In addition to regular sessions for contributed papers, computing demonstrations and informal seminars, there will be a series of invited lectures.
Guest Speakers: Jack Edmonds (University of Waterloo), Kenneth Iverson (IBM, Philadelphia), Victor Klee (University of Washington), and John McKay (Concordia University).
Sponsors: Atlantic Provinces Inter-University Committee on the Sciences (A.P.I.C.S.), I.B.M. (Canada) Ltd., and the University of New Brunswick.
Call for Papers: Contributed papers (25 minutes) relevant to the conference are invited. Abstracts (not more than one page) and subject titles should be submitted.
Information: The Secretary, Computations Conference, Department of Mathematics, University of New Brunswick, Fredericton, New Brunswick E3B 5A3, Canada.

August 25-28, 1975
SYMPOSIUM ON COMBINATORICS AND PROBABILITY IN PRIMARY SCHOOLS
Warsaw, Poland
Sponsors: ICMI and Polish Ministry of Education.
Information: Z. Semadeni, Institute of Mathematics, Polish Academy of Sciences, ul. Sniadeckich 8, 00-950 Warszawa, Poland.

September 8-13, 1975
SEVENTH IFIP CONFERENCE ON OPTIMIZATION TECHNIQUES
Nice, France
Program: Twenty-one parallel sessions have been scheduled (provisionally) in the areas of medicine and biology, human environments, operational research, optimal design, games, computational techniques, mathematical programming, control deterministic, control stochastic, associated software problems.
Invited Speakers: B. Fraeijs de Veubeke, N. N. Krassovski and G. Marchuk.
Sponsor: IFIP (International Federation for Information Processing), Technical Committee on Optimization (TC 7).
Information: Seventh IFIP Conference, Faculté des Sciences, Parc Valrose, 06034 Nice Cedex, France.

October 3-4, 1975
MINI-CONFERENCE IN NUMERICAL ANALYSIS
State University of New York at Binghamton, Binghamton, New York
Program: Six one-hour lectures (invited) by distinguished researchers, discussion sessions and presentations of short papers in study group sessions.
Principal Speaker: Richard S. Varga (Kent State University).
Sponsor: As of June 6, 1975, there is no support assured except for the invited lecturers. Application is being made for additional funding.
Contributed Papers: Those wishing to submit a paper are invited to do so. Abstracts should be submitted no later than September 1, 1975. The short period of the conference will necessitate acceptance of a limited number of papers. Notification of acceptance will be made by September 10, 1975.
Information: Louis F. McAuley, Chairman, Department of Mathematical Sciences, State University of New York at Binghamton, Binghamton, New York 13901.

October 11-15, 1975
CBMS/NSF REGIONAL CONFERENCE ON THE THEORY OF INFINITE DIMENSIONAL MANIFOLDS AND ITS APPLICATIONS TO TOPOLOGY
Guilford College, Greensboro, North Carolina
Program: Thomas A. Chapman will present ten lectures on infinite dimensional manifolds along with their more important applications, e.g., applications to ANR's, simple homotopy theory, shape theory and hyperspaces. Also, there will be hour talks by the invited lecturers and half-hour talks by invited participants.
Principal Lecturer: Thomas A. Chapman, University of Kentucky.
Invited Lecturers: R.D. Anderson, Marshall Cohen,

Ross Geoghegan, Richard Schori and J. E. West.
Sponsors: Jointly sponsored by University of North Caroline at Greensboro, Duke University, and Guilford College.
Support: (Subject to approval by NSF) There is a limited amount of support for travel and subsistence.
Information: J. R. Boyd, Department of Mathematics, Guilford College, Greensboro, North Carolina 27410.

October 13-17, 1975
CONFERENCE ON BRAUER AND PICARD GROUPS OF RINGS
Northwestern University, Evanston, Illinois
Program: Lectures and informal discussions on Azumaya algebras over commutative rings and recent results on the resulting Brauer groups.
Information: Daniel Zelinsky, Department of Mathematics, Northwestern University, Evanston, Illinois 60201.

December 28-29, 1975
1975–1976 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC
Statler Hilton Hotel, New York, New York
Program: The meeting will be held in conjunction with the annual meeting of the Eastern Division of the American Philosophical Association. The program will include a joint symposium.
Deadline for Abstracts: October 15, 1975.
Information: Paul Benacerraf, Department of Philosophy, Princeton University, Princeton, New Jersey 08540.

January 12-16, 1976
INTERNATIONAL CONFERENCE ON ALGEBRAIC K-THEORY
Northwestern University, Evanston, Illinois
Program: Invited lectures and informal seminars on various topics of current research interest, including L-groups, higher K-groups and connections with algebraic geometry.
Sponsor: Department of Mathematics, Northwestern University.
Participants: Limited support for 10-15 participants, including some Europeans. Those with their own support will be welcomed, up to a total of 40. Those wishing to speak should send a description of their proposed topic.
Information: Michael R. Stein, Department of Mathematics, Northwestern University, Evanston, Illinois 60201.

March 30-April 2, 1976
CONFERENCE ON THE THEORY OF ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS
University of Dundee, Dundee, Scotland
Program: K. P. Hadeler (Tubingen), J. K. Hale (Brown University), J. Serrin (Minnesota) will each deliver two lectures. There will be nine additional invited lecturers. Participants may also contribute lectures.
Information: Organizing Secretaries, Differential Equations 1976 Conference, Department of Mathematics, The University, Dundee, DD1 4HN, Scotland, United Kingdom.

April 20-23, 1976
THIRD EUROPEAN MEETING ON CYBERNETICS AND SYSTEMS RESEARCH
University of Vienna, Vienna, Austria
Program: The following topics and speakers are being prepared: general systems methodology (G. Klir, United States); biocybernetics and theoretical neurobiology (L. Ricciardi, Italy); cybernetics of cognition and learning (G. Pask, United Kingdom); structure and dynamics of socio-economic systems (Chairman to be announced); health care systems (J. Milsum, Canada); cybernetics in organization and management (F. de P. Hanika, United Kingdom); engineering systems methodology (F. Pichler, Austria); system simulation languages (G. Chroust, Austria); computer linguistics (W. Dressler, Austria); and computer performance control and evaluation (N. Rozsenich and L. Heinrich, Austria).
Full Papers: (One paper only from each contributor) should be typed (double-spaced) on A4 size sheets (8 1/2 by 11 with 4 cm margin) on one side of the paper only and sent in triplicate. Typed symbols are to be used; if handwritten ones are unavoidable, specify precise shape and meaning on a separate sheet. Photographs should be suitable for reproduction without further treatment. Diagrams, maps, line drawings etc. should be drawn in Indian or draughtsmen's ink on hard glossy paper. Length of 3000-4000 words is suggested.
Abstracts: Three copies should be sent including the full title of the paper, the author's name and the author's affiliation. It must not exceed one A4 page (250-300 words), and may be single-spaced.
Information: Robert Trappl, Österreichische Studiengesellschaft für Kybernetik, Schottengasse 3, Wien I., Austria.

Spring, 1976
SECOND CANADIAN SYMPOSIUM ON FLUID DYNAMICS
University of British Columbia
Suggestions and Inquiries: B. R. Seymour, Chairman, Organizing Committee, Second Canadian Symposium on Fluid Dynamics, Department of Mathematics, University of British Columbia, Vancouver, British Columbia V6T 1W5.

QUERIES

Edited by Hans Samelson

This column welcomes questions from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published conjectures. When appropriate, replies from readers will be edited into a composite answer and published in a subsequent column. All answers received to questions will ultimately be forwarded to the questioner. The queries themselves, and responses to such queries, should be typewritten if at all possible and sent to Professor Hans Samelson, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

QUERIES

69. Bertram Ross (Mathematics Department, School of Arts and Sciences, University of New Haven, West Haven, Connecticut 06516). 1. The Riemann-Liouville operator of a fractional integration ${}_cD_x^{-\nu} f(x)$ is defined by

$$\frac{1}{\Gamma(\nu)} \int_c^x (x - t)^{\nu - 1} f(t)\, dt, \qquad \mathrm{Re}\ \nu > 0.$$

When $c = 0$ we have Riemann's definition and when $c = -\infty$ we have Liouville's definition. The law of indices is ${}_cD_x^{-u}\,{}_cD_x^{-v} f = {}_cD_x^{-u-v} f$. The operation ${}_aD_x^{-u}\,{}_bD_x^{-v} f$ might be considered as a measure of deviation from the law of indices since the lower terminals of integration are unequal. What are some theorems concerning this operation? What significance can this measure of deviation be?

2. Does anyone know of any application that stems from the physical sciences where the derivative or integration

of complex order is useful as for example $d^{1+i}f(x)/dx$?

3. Does anyone know of a case of the use of the Riemann-Liouville integral that stems from the physical sciences where the lower terminal of integration is neither 0 nor $-\infty$?

70. F. H. Northover (Mathematics Department, Carleton University, Ottawa, Canada). Is there a continuous function which has a fractional derivative but which has no ordinary derivative?

71. Michael A. Goldberg (Department of Mathematics, University of Nevada, Las Vegas, Nevada 89154). If $U(t)$ is the fundamental matrix for the system $x_t = A(t)x$, then it is well known that $(\det U(t))_t = (\operatorname{trace} A(t)) \det U(t)$. Suppose that one looks at the submatrix $V(t)$ of $U(t)$ obtained by crossing out the first k rows and k columns of $U(t)$, $k = 1, \ldots, n-1$. Does anyone know of a reference where a formula can be found for the derivative of the determinant of $V(t)$? In general, one expects this derivative to be obtained in terms of the derivatives of the determinants of other submatrices and I am interested in knowing the coupled differential equations that result from differentiating these.

ACKNOWLEDGEMENTS

The Society acknowledges with gratitude the support rendered by members during the past year. In addition to the contributing members who pay a minimum of $48 per year in dues, mathematicians also contributed to the Mathematical Reviews Fund, the AMS Research Fellowship Fund, the AMS Research Fellowship Fund in Memory of Lillian R. Casey, and made general contributions; some contributors have requested that their names remain anonymous. These extra funds paid by members provide vital support to the work of the Society.

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TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

INTRODUCTION TO SPECTRAL THEORY: SELFADJOINT ORDINARY DIFFERENTIAL OPERATORS by B. M. Levitan and I. S. Sargsjan

Volume 39
526 pages; list price \$48.50; member price \$36.38
ISBN: 0-8218-1589-X
To order, please specify MMONO/39

This monograph is devoted to the spectral theory of the Sturm-Liouville operator $-d^2y/dx^2 + q(x)y$ and to the spectral theory of the Dirac system $dy_2/dx - \{\lambda + p(x)\}y_1 = 0$, $dy_1/dx + \{\lambda + r(x)\}y_2 = 0$. In addition, some results are given for nth order ordinary differential operators. Those parts of this book which concern nth order operators can serve as simply an introduction to this domain, which at the present time has already had time to become very broad.

For the convenience of the reader who is not familar with abstract spectral theory, the authors have inserted a chapter (Chapter 13) in which they discuss this theory, concisely and in the main without proofs, and indicate various connections with the spectral theory of differential operators.

At the request of the authors, V. A. Sadovničiĭ wrote part of Chapter 1, and A. G. Kostjučenko wrote part of Chapters 7 and 12.

This work was translated from the Russian by Amiel Feinstein, who introduced a number of improvements in the original text.

The titles of the chapters follow: I. Expansion in a finite interval; II. Eigenfunction expansions for a Sturm-Liouville operator for the case of an infinite interval; III. Expansion in the singular case for a Dirac system; IV. Investigation of the spectrum; V. Examples; VI. Solutions of the Cauchy problem for the one-dimensional wave equation; VII. Eigenfunction expansion of a Sturm-Liouville operator; VIII. Differentiation of an eigenfunction expansion; IX. Solution of the Cauchy problem for a one-dimensional Dirac system; X. Asymptotic behaviour of the spectral kernel and its derivatives for the case of a Dirac system; XI. Expansion, and differentiation of an expansion, with respect to the eigenfunctions of a Dirac system; XII. Asymptotic behaviour of the number of eigenvalues of a Sturm-Liouville operator; XIII. Elements of the spectral theory of linear operators in Hilbert space, Relation to differential operators; and XIV. Some theorems of analysis. A bibliography of more than one hundred forty listings is also included.

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

ALMOST SURE INVARIANCE PRINCIPLES FOR PARTIAL SUMS OF WEAKLY DEPENDENT RANDOM VARIABLES by Walter Philipp and William Stout

Number 161
140 pages; list price \$5.50; member price \$4.13
ISBN: 0-8218-1861-9
To order, please specify MEMO/161

Let $\{x_n\}$ be a sequence of random variables, centered at expectations with finite $(2 + \delta)$ moments where $\delta > 0$. For $t \geqq 0$ let $S_t = S(t) = \sum_{n \leqq t} x_n$. Assume that $\lim_{n\to\infty} n^{-1} E\, S_n^2 = 1$ (*). The authors establish the following almost sure invariance principle for lacunary trigonometric, several kinds of mixing, Gaussian, functionals of certain Markov sequences, and for what they call retarded asymptotic martingale difference sequences: Without changing its distribution they redefine the process $\{S(t), t \geqq 0\}$ on a new probability space together with standard Brownian motion $\{X(t), t \geqq 0\}$ such that $S(t) - X(t) = 0(t^{\frac{1}{2}-\lambda})$ a.s. where $\lambda > 0$ only depends on the given sequence $\{x_n\}$. This result implies the usual upper and lower class results for partial sums and for maxima of partial sums, the functional versions of the law of the iterated logarithm for partial sums and for maxima of partial sums, and distribution type invariance principles. They do not make any stationarity assumptions. As a matter of fact they also obtain similar results when (*) is not satisfied.

The following chapters are included: 1. Introduction, 2. Description of the method, 3. Lacunary trigonometric series with unweighted summands, 4. Stationary φ-mixing sequences, 5. Gaussian sequences, 6. Lacunary trigonometric series with weights, 7. Functions of strongly mixing random variables, 8. Nonstationary mixing sequences, 9. A refinement of the Shannon-McMillan-Breiman theorem, 10. Markov sequences, 11. Retarded asymptotic martingale difference sequences, and 12. Continuous parameter stochastic processes.

Two appendixes are also included: 1. The Gaal-Koksma strong law of large numbers, and 2. An example.

PROCEEDINGS OF THE STEKLOV INSTITUTE

BOUNDARY VALUE PROBLEMS OF MATHEMATICAL PHYSICS. VIII, edited by O.A. Ladyženskaja

Number 125(1973)
218 pages; list price \$31.30; member price \$23.48
ISBN: 0-8218-3025-2
To order, please specify STEKLO/125

This volume contains thirteen papers on Boundary Value Problems in Mathematical Physics, and is a cover-to-cover translation of the Proceedings of the Steklov Institute of Mathematics, Number 125, for 1973.

In the paper by A.B. Venkov, Expansions in automorphic eigenfunctions of the Laplace-Beltrami operator in classical symmetric spaces of rank one and the Selberg trace formula, spectral characteristics are considered of the Laplace-Beltrami operator on a fundamental region of discrete groups Γ acting on classical symmetric spaces of rank $I: S = SO_0(1,n)/SO(n)$, $SU(1,n)/S(U(1) \times U(n))$, $Sp(1,n)/Sp(1) \times Sp(n)$. The discrete groups satisfy the following conditions: 1) $\Gamma \backslash S$ is noncompact but has finite invariant volume; 2) the nonparabolic elements of Γ are essentially distinct from the parabolic elements. The main result consists in the proof of the eigenfunction expansion theorem. These themes were provided by the famous paper by A. Selberg on the trace formula. Venkov's results permit him to justify this formula completely for the cases of S and Γ he considers, and also to calculate fully all contributions in this formula for several examples. The corresponding arguments are presented in detail in the paper.

In the paper by M.M. Skriganov, On the spectrum of the Schrödinger operator with a rapidly oscillating potential, Schödinger operators $\Delta u + q(x)u$ with rapidly oscillating potentials $q(x)$ unbounded at infinity are studied. For such operators sufficiency criteria are given assuring selfadjointness, semiboundedness, finiteness of the number of negative eigenvalues, and the absence of positive eigenvalues. A theorem on expansion in eigenfunctions of such operators is proved.

Five of the papers are devoted to quasilinear elliptic equations. In the paper by A.V. Ivanov, On the question of an admissible limiting growth of the right side of a quasilinear elliptic equation, from equations of the form

$$(1) \qquad A_{ij}(s,u,u_x)U_{x_ix_j} = B(x,u,u_x)$$

the author selects a certain class (having nonempty intersection with the class of uniformly elliptic equations) for which the Dirichlet problem has a solution for right sides $B(x,u,p)$ increasing as $o(E_1 \ln |p|)$ when $|p| \to \infty$, where $E_1(x,u,p) = A_{ij}(x,u,p)p_ip_j$. In his other paper, On the solvability of the Dirichlet problem for some classes of second order elliptic systems, Ivanov studies elliptic systems of the form

$$A^s_{ij}(x,u^t,u^s_x)u^s_{x_ix_j} = B^s(x,u^t,u^s_x), \quad s = 1,\dots,N$$

and the form

$$A_{ij}(x,u^t,|u_x|)u^s_{x_ix_j} = B^s(x,u^t,u^t_x), \quad s = 1,\dots,N,$$

where $|u_x| = (\sum_{t,\ell}|u^t_{x\ell}|^2)^{1/2}$, under the first boundary-value condition. To solve them, subject to a number of assumptions on the generators of their functions, a priori estimates of $\max_\Omega |u_x|$ are established that, together with known previous results, make it possible to investigate the question of solvability of the Dirichlet problem.

This paper generalizes the corresponding results on systems established by O.A. Ladyženskaja, N.N. Ural'ceva, N.M. Ivočkina and A.P. Oskolkov. In her paper, A priori estimates for solutions of the Dirichlet problem for multidimensional quasilinear equations of elliptic type, N.M. Ivočkina singles out a class of nonuniformly elliptic quasilinear equations in divergence form $da_i/dx_i = a$, for which, satisfying the first boundary-value condition, it is possible to give a priori estimates of $\max_\Omega |u_x|$ and the Hölder norms $|u_x|^{(\alpha)}_{\Omega'}$, $\overline{\Omega}' \supset \Omega$, that do not depend on $\partial a_i/\partial x_k$ for $k \neq i$.

In the paper by I.N. Krol', On the behavior of the solutions of a quasilinear equation near null salient points of the boundary, it is proved that solutions of the Dirichlet problem for a quasilinear elliptic equation of the form

$$(2) \qquad (|u_x|^{p-2}u_{x_i})_{x_i} = 0, \quad p > 1,$$

having finite "energy" norm assume their boundary values at points where there are "null salients outside", with higher than power speed. To prove this a "barrier" function is constructed that is a nonnegative solution of (2) in the spherical cone $K(\ell)$ of angle ℓ, equal to zero on $\partial K(\ell)$ and having finite energy norm near the vertex of $K(\ell)$. This function has the form $u(x) = |x|^\lambda f_\lambda(x|x|^{-1})$, $\lambda = \lambda(\ell)$. It is proved that $\lambda(\ell) = \pm L\ell^{-1} + O(1)$ as $\ell \to 0$, where L is the first zero of the solution of the Cauchy problem for a certain ordinary differential equation.

In his other paper, On solutions of the equation $D_{x_i}(|Du|^{p-2}D_{x_i}u) = 0$ with a singularity at a boundary point, Krol' studies an increasing nonnegative solution of (2), equal to zero on $\partial K(\ell)$ and having the same form

$$u(x) = |x|^\lambda f_\lambda(\arccos(x_n|x|^{-1})).$$

The asymptotic behavior of $\lambda = \lambda(\ell)$ as $\ell \to \pi$ is determined.

Four papers are devoted to the Navier-Stokes equations. V.A. Solonnikov and V.E. Ščadilov, On a boundary value problem for a stationary system of Navier-Stokes equations, study the Stokes linearized steady state problem under boundary conditions of the form

$$vn|_{S_1} = \alpha(s), \quad t - n(tn)|_{S_1} = b(s), \quad v|_{S_2} = a(s)$$

in the domain Ω, where $S_1 \cup S_2 = \partial\Omega$, n is the unit normal vector to $\partial\Omega$, t is the vector with components $t_i = \sum_{k=1}^{3} t_{ik}(s)n_k(s)$, and $t_{ik} = \delta_{ik}\, p + \nu(\partial v_i/\partial x_k + \partial v_k/\partial x_i)$. This problem can be considered as being modeled on the motion of a fluid in the presence of a free boundary S_1. Unique solvability of this problem is established, and the existence of second-order derivatives of v is investigated.

In the paper by V. Ja. Rivkind, The grid method of solving problems of the dynamics of a viscous incompressible fluid, new difference schemes for the Navier-Stokes equations are constructed and their convergence is studied. The explicit scheme turns out to be convergent for t-steps that are larger than in the corresponding papers of Temam and Ladyženskaja (namely for $\Delta t \sim (\Delta x)^2$). This is achieved at the expense of lopping off nonlinear terms, thanks to knowledge of a priori estimate of $\max_{Q_T} |v|$ and $\max_{Q_T} |v_x|$ and certain other modifications of previous schemes.

In his paper, On the asymptotic behavior of solutions of certain systems with a small parameter approximating the systems of Navier-Stokes equations, Oskolkov continues the investigation of systems he has proposed of elliptic and parabolic types passing as $\epsilon > 0$ into the steady state and nonsteady state Navier-Stokes system, respectively. Estimates are established for their solutions, depending on ϵ. In his second paper, On some convergent difference schemes for the Navier-Stokes equations, an explicit difference scheme is constructed on the basis of these approximations, permitting one in the limit to obtain a solution of an initial-boundary value problem for the Navier-Stokes equations.

The paper by N.K. Korenev, On the strong convergence of solutions of difference equations to generalized solutions of second-order linear equations of three classical types, is also devoted to the method of finite differences. For linear second-order equations of three classical types he proves strong convergence in the energy norm of solutions of difference schemes studied earlier by Ladyženskaja to generalized solutions of these equations having finite energy norm.

Finally, in the paper by L. Stupjalis, Boundary value problems for elliptico-hyperbolic equations, unique solvability is established for an initial-boundary value problem involving a linear second-order equation that is elliptic in one part of the domain and hyperbolic in another. This is done under assumptions on the free terms that are broader than in previous work by Stupjalis and Ladyženskaja devoted to the same equation.

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NEWS ITEMS AND ANNOUNCEMENTS

RECENT AMS APPOINTMENTS

Committee to Select Hour Speakers for Summer and Annual Meetings. The Committee to Select Hour Speakers for the Summer and Annual Meetings has recently been expanded to include the following new members: William K. Allard, Everett C. Dade, Richard J. Duffin, Joseph J. Kohn, Haskell P. Rosenthal, and Harold M. Stark. Everett Pitcher will continue temporarily as chairman and Alexandra Ionescu Tulcea and Richard M. Dudley will continue as members through 1975.

Committee on Mathematical Models as Used in Government Policy Decisions. Burton H. Colvin has been appointed to be a member of the Committee on Mathematical Models as Used in Government Policy Decisions. Mark Kac is the chairman and David Gale is the vice chairman for the committee. Its continuing members are: Harold Grad, Jack Kiefer, and Halsey Royden.

Committee on Translations. President Lipman Bers has appointed Kevin M. McCrimmon to serve as the chairman of the Committee on Translations effective July 1975. In addition, Allen Shields will be a new member of the committee; Ivo Babuska will continue to serve in his same capacity.

Associate Editor of the Proceedings. Larry Zalcman has recently been elected to the position of Associate Editor of the Proceedings of the American Mathematical Society for a four-year term beginning July 1, 1975.

Committee on External Membership. As an update to the information given in the April *Notices*, Arthur Mattuck will serve as the chairman of the Committee on External Membership. Tilla Klotz Milnor and Murray Protter are the committee members.

AMS-SIAM Committee on Applied Mathematics. The AMS-SIAM Joint Committee on Applied Mathematics has selected Richard DiPrima as its chairman. Earl A. Coddington, Lester E. Dubbins and J. Barkley Rosser will continue to represent the Society; and Donald S. Cohen and W. Gilbert Strang are SIAM representatives.

ABSTRACTS PRESENTED TO THE SOCIETY

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Invited addresses are indicated by •

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The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the *Notices* but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

*75T-A165 D. Suryanarayana, University of Toledo, Toledo, Ohio 43606.
R. Sita Rama Chandra Rao, Andhra University, Waltair, India
On The True Maximum Order of a Class of Arithmetical Functions

Let $f(n)$ be an arithmetical function which is positive and satisfies the condition that $f(n) = O(n^{\beta})$ for some fixed $\beta > 0$. Define the arithmetical function $F(n)$ by setting $F(1)=1$ and $F(n)=f(a_1)\, f(a_2) \dots f(a_r)$ if $1 < n = p_1^{a_1} p_2^{a_2} \dots p_r^{a_r}$. In this paper we prove the following theorem which gives a useful and easy way of obtaining the "true maximum order" of $F(n)$: Theorem. $\lim_{n \to \infty} \operatorname{Sup} \frac{\log F(n) \log \log n}{\log n} = \operatorname{Sup}_m \frac{\log f(m)}{m}$. Using this theorem, true maximum orders of several divisor functions have been established.
(Received March 24, 1975.)

*75T-A166 D. SURYANARAYANA, The University of Toledo, Toledo, Ohio 43606, Andhra University, Waltair, India. Quasiperfect numbers - II.

A positive integer N is called a Quasiperfect (QP) number, if $\sigma(N) = 2N + 1$, where $\sigma(N)$ is the sum of all the positive divisors of n. It is not known whether there is at least one QP number. However, it is known that a QP number, if it exists, must be the square of an odd integer. Recently, H.L. Abbott, C.E. Aull, Ezra Brown and the author (Acta Arithmetica 22 (1973), 439-447) proved that a QP number must have at least five distinct prime factors and must be $> 10^{20}$. In this paper we obtain bounds for $\prod_{p|N} (1 - \frac{1}{p})$, where the product is taken over all the prime factors of a QP number N.
(Received March 24, 1975.)

75T-A167 MASAO KISHORE, University of Toledo, Toledo, Ohio 43606
On the Equation $k\phi(M) = M-1$

D. H. Lehmer considered the equation (1) $k\phi(M) = M-1$, where k and M are positive integers, M is not a prime, and ϕ is the Euler's totient function [On Euler's totient function. Bull. Am. Math. Soc. 38 (1932), 745-751]. E. Lieuwens proved [Nieuw Archief voor Wiskunde (3), XVIII, 165-169 (1970)] that if M is a solution of (1) then M is the product of at least eleven distinct primes. Using computer, we prove that if M satisfies (1) then M is the product of at least thirteen primes. (Received April 7, 1975.)

*75T-A168 J. M. GANDHI and MARK STUFF, Western Illinois University, Macomb, Illinois 61455. Comment on Certain Results about Fermat's Last Theorem.

Kapferer (Heidel. Akad. Math. Nat. Class Sitzung. (1933) 32-37) allegedly proves that (1) $x^p + y^p + z^p = 0 \Leftrightarrow Z^3 - Y^2 = 3^3\, 2^{2p-2}\, X^{2p}$. Yahya [Complete proof of FLT (privately published by the author, Pakistan Air Force, Kohat, West Pakistan 1958) and Portugalie Math. 32, (1973) 157-170] using Kapfere 's result claims to have proved FLT. We remark that Kapferer equivalent statement for FLT does not appear to be correct even for n=3. For n=3 he proves that (2) $(6AC)^3 = (6C^3+B)^3 + (6C^3-B)^3 \Leftrightarrow (6A)^3 = 2^4\, 3^3\, C^6 + (6B)^2$, but then he has considered a restricted equation and not the general equation (3) $x^3 + y^3 = z^3$. In view of this Yahya's claims are not justified. (Received April 14, 1975.)

75T-A169 RONSON J. WARNE, University of Alabama, Birmingham, Alabama 35294. Generalized orthodox locally inverse bisimple semigroups of type ω.

For definitions, see Abstract 75T-A148, these *Notices* 22(1975), A-450. Let S be a regular semigroup and e, f, and g idempotents of S. If $e \geqq f$ and $e \geqq g$ imply $fg = gf$, S is termed a locally inverse semigroup. Let $(I, \circ)$ be a locally inverse ω-chain of left zero semigroups $(I_n : n \in N$, the nonnegative integers). Let $(J,*)$ be a locally inverse ω-chain of right groups $(J_n : n \in N)$. Suppose $I_n \cap J_n = \{e_n\}$, a single idempotent element. Let H_n denote the maximal subgroup of J_n containing e_n. Let $(n,k) \to \beta_{(n,k)}$ be a homomorphism of C, the bicyclic semigroup, into End$(J,*)$, the semigroup of endomorphisms of $(J,*)$, and let $i \to B_s$ be a homomorphism of $(I,\circ)$ into $P_{(J,*)}$, the semigroup of right translations of $(J,*)$ such that $J_r\beta_{(n,k)} \subseteq J_{n+k-\min(r,n)}$; $g\beta_{(s,s)} = g*e_s$; if $j \in J_n$ and $i \in I_k$, $jB_i \in H_{\max(n,k)}$, and if $j \in H_k$, $jB_i = j$; $jB_{e_s} = j*e_s$. Let (I,J,B,β) denote $I \times J$ under the product: if $i \in I_n$, $j \in J_k$, $u \in I_r$, and $v \in J_s$, $(i,j)(u,v) = (i \circ e_{n+r-\min(k,r)}, jB_u\beta_{(r,s)} * v)$. Theorem. (I,J,B,β) is a generalized orthodox locally inverse bisimple semigroup of type ω, and conversely every such semigroup is isomorphic to some (I,J,B,β). (Received April 14, 1975.)

*75T-A170 J. M. Gandhi, Western Illinois University, Macomb, Illinois 61455 Fermat's Last Theorem III. A New Circulant Condition for the First Case for Primes of the Form 6m-1.

In this paper we prove: Theorem 2. The equation $x^p+y^p+z^p=0$ with $(xyz,p)=1$, where p is an odd prime has no integral solution if $M_{p-1}=(K_2,K_3,\ldots,K_{p-4},0,0,0,K_1) \not\equiv 0 \pmod{p^3}$ where M_{p-1} is a circulant. Here the integers K's are given by $K_1 = K_{p-4} = 1$; $K_2 = K_{p-5} = \frac{p-5}{2}$; $K_3 = K_{p-6} = \frac{(p-4)(p-5)}{6}$ and $K_i - K_{i-3} = \frac{1}{p}\left[\binom{p-1}{i} - \binom{p-1}{i-1} + 2(-1)^{i+1}\right]$ for $3 < i < p-6$. Also $K_i = K_{p-i-3}$. For an illustration taking p = 11 we find that $M_{10} = (3,7,9,7,3,1,0,0,0,1) = -29791 \not\equiv 0 \pmod{11^3}$ and the Fermat's Last Theorem is verified for the prime p = 11 by the new method. (Received May 12, 1975.)

*75T-A171 ALBERT A. MULLIN, 1500 Ronstan Drive, Killeen, TX. 76541 Additive problems with prescribed numbers of summands.

Let k be an integer ≥ 2. Definition 1. Let $R_k(n)$ be the number of representations of the positive integer n as a sum of a k th power number of k th powers of strictly positive integers. Definition 2. Let $R_k^*(n)$ be the number of representations of the positive integer n as a sum of a k th power number of strictly positive integers. Lemma. $R_2^*(n) > R_2(n) > 0$ for every integer $n > 14$. Problem 1. Prove that for each integer $k > 2$, $R_k^*(n) > R_k(n) > 0$ for every sufficiently large positive integer n. Problem 2.

For each $k > 2$ determine the exact order of growth of $R_k(n)$ and $R_k^*(n)$.

(Received April 24, 1975.)

*75T-A172 PAMELA A. FERGUSON, University of Miami, Coral Gables, Florida 33124 A Characterization Theorem for 3' Homogeneous Groups

A finite group is said to be π homogeneous if $\frac{N_G(H)}{C_G(H)}$ is a π-group for every non-identity π-subgroup of G, where π is a set of primes. The purpose of this paper is to prove the following theorem:

Theorem. Let G be a finite 3' homogeneous group. If M is a Sylow 3 subgroup of G, assume $C_G(x) \leq M$ for all $x \in M - 1$, then M is normal in G.

The proof uses some previous results of Zvi Arad and this author. (Received April 28,1975.)

75T-A173 RONALD HIRSHON, Polytechnic Institute of N.Y. Brooklyn, N.Y. A cancellation theorem with applications to nilpotent groups: Preliminary report.

Definition: We say that a group E may be cancelled from a group A in direct products if $A \times E \approx A_1 \times E$ implies $A \approx A_1$.

Theorem 1: If an infinite cyclic group can be cancelled from A, then an arbitrary group which obeys the maximal condition for normal subgroups may be cancelled from A.

Theorem 2: An infinite cyclic group (and hence an arbitrary group which obeys the maximal condition for normal subgroups) may be cancelled from a group A if either (a) A is torsion free nilpotent with cyclic center or (b) A is a torsion free nilpotent group of class 2.

Theorem 3: There exists a torsion free nilpotent group of class 3 such that an infinite cyclic group may not be cancelled from it.

Theorem 4: If $A \times B \approx A_1 \times B_1$ and $B \approx B_1$ and B satisfies the maximal condition for normal subgroups, then the direct product of A and an infinite cyclic group is isomorphic to the direct product of A_1 and an infinite cyclic group.

Some observations are made about the direct decompositions of certain groups A, from which an infinite cyclic group may be cancelled, into indecomposable groups. (Received May 1, 1975.)

*75T-A174 ROBERT LAGRANGE, University of Wyoming, Laramie, WY 82071. The Counting Problem for Superatomic Boolean Algebras.

Theorem. Let m be an uncountable cardinal number. There are 2^m pairwise non-isomorphic superatomic Boolean algebras of cardinality m. (Received May 2, 1975.)
(Author introduced by Joe Martin.)

75T-A175 G. T. DIDERRICH, 2973 N. Cramer Street, Milwaukee, Wisconsin 53211. Some historical remarks on Gallai's extension of van der Waerden's theorem.

Our purpose is to provide some background information on Professor T. Gallai's theorem (cf. our Abstract#75T-A13, these *Notices* 22(1975), A-4). The following is an excerpt from the referee's report. "It was first proved by Gallai (= Grünwald) and later proofs were furnished by Rado, Witt, Hales and Jewett, Graham and Rothschild, and others". See our Errata in these *Notices* 22(1975), A-362, for a source reference. Currently, the best results are due to Graham, Leeb, and Rothschild (Advances in Math. 8 (1972), 417-433) who combine in one theorem: van der Waerden's theorem and extensions i.e., k-parameter sets, the vector space analogue of Ramsey's theorem (Rota's conjecture), and Ramsey's theorem itself. I wish to thank Professor R. Rado and the referee for setting me straight on the credits and to thank Professor Hales, Professor Rothschild, Dr. Graham, and Professor Jewett for helpful correspondences. The following inspired our interest in the problem: (1) M. Gardner, Sci. Amer., Sept. 1973, (2) J. Haley and H. Nelson, J. Recreational Math. 6(1973), 234-236, (3) R. Rado, Amer. Math. Monthly, June-July (1974), 617-620. Lastly, we mention that our proof is a direct generalization using a geometric "free" path approach based on the following idea: a long enough straight line path can be "bent" into the shape of an n-dimensional cube of prescribed edge length. (Received June 10, 1975.)

*75T-A176 DAVID K. HALEY, Universität Mannheim, 68 Mannheim, Fed. Rep. of Germany. On a question of Mycielski.

There is a commutative unital ring R which is equationally compact but not retract of any compact topological ring. R is an extension of the (non-unital) ring constructed by M. L. Kleĭner with the same properties. The core of the construction is W.Taylor's atomic compact graph of infinite chromatic number. (Received May 5, 1975.)

*75T-A177 RABINDRA K. PATNAIK, State University of New York, Stony Brook, N.Y., 11794, Isomorphism classes of knot-like groups.

Let G be a knot-like group; that is $G \simeq P=(x,a_1,\ldots,a_n;\ r_1,\ldots,r_n)$ with commutator quotient group, G/G', infinite cyclic. For a given monic integral polynomial $\phi(x)$ of degree d satisfying $\phi(0)=\pm1$, $\phi(1)=\pm1$, how many isomorphism classes of knot-like groups G are there for which the Alexander polynomial is $\phi(x)$ and G' is the free group F_d of rank d? For the cardinality c of the family of isomorphism classes, lower and upper bounds are determined. I call an automorphism of F_d admissible automorphism if it defines a unique presentation of G. These isomorphism classes are shown to be in one to one correspondence with the conjugacy classes of these automorphisms of F_d. It is conjectured that for one-relator knot-like groups c=1; and this is proved modulo the second commutator subgroup G". A necessary and sufficient condition for G to be a one-relator group, modulo G", is determined. A structure theorem for the commutator subgroup of a one-relator knot-like group is determined. The proofs are cmbinatorial except for certain number theoretic results. (Received April 30, 1975.)

75T-A178 JOHN DAVID, IMECC, UNICAMP, CP 1170, 13100 Campinas, SP, BRAZIL. Inert and Strongly Inert Integral Domains II. Preliminary report.

We answer the following question affirmatively, posed in I, these Notices June 1975: If R is strongly inert in A, is R[X] strongly inert in A[X]? To do this we prove the following theorems: Theorem 1: Let K be a field, $K \subsetneq A$. K is strongly inert in A if and only if for every $0 \neq a$, $b \in A$, $K[a,b] \cap \overline{K[1/a]}_{(1/a)} = K$, where $\overline{S}$ indicates the integral closure of S in K(a,b), when $S \subseteq K(a,b)$. Theorem 2: $R \subsetneq A$ is strongly inert if and only if $K \subsetneq S^{-1}A$ is strongly inert and $A \cap K = R$, where $S = R^*$, and $K = q.f.(R)$. (Received May 6, 1975.)

*75T-A179 JUDITH Q. LONGYEAR, Wayne State University, Detroit, Michigan 48202. Tactical constructions.

This paper treats the construction of a class of combinatorial designs known as tactical configurations of rank 2 and girth 6. Although many constructions are known for such configurations with 0 deficiency (that is, when the smaller vertex set is a subset of the skeleton of the configuration), very few constructions exist for configurations of positive deficiency. In this paper we give several constructions for configurations with positive deficiency, and discuss a few of the many problems remaining to be solved. A list of examples is included. (Received May 2, 1975.)

*75T-A180 STEVEN ROMAN, University of Washington, Seattle, Washington 98195. The maximum number of q-cliques in a graph with no p-clique.

Given n nodes, the complete graph on any p of these nodes will be called a p-clique. Let f(n,p,q) denote the maximum number of q-cliques on n nodes with no p-clique. In 1941, Paul Turán determined f(n,p,2), for all n and p, as well as the unique graph which attains this maximum. In this paper we determine f(n,p,q) for all n, p and q. We also give the graph which attains this maximum, and show that it is unique with respect to this property. (Received May 9, 1975.)

*75T-A181 MARGARET MORTON, The Pennsylvania State University, Pennsylvania 16802, Some Results on Quaternion Polynomial Rings. Preliminary Report.

Let K[x] be the quaternion polynomial ring with coordinates in Q[x]. Then the elements of K[x] have a restricted unique factorization and the two-sided ideals have unique factorization. Let a(x) be in Z[x]. Then the properties of K[x] can be utilized to show that if there exist $a_0(x), a_1(x), a_2(x), a_3(x)$ in Z[x] such that a(x) divides $\sum_{\ell=0}^{3} a_\ell(x)^2$ nontrivially, then there exists c in Z such that $ca(x)$ can be expressed as the sum of four squares in $Z[x]$. (This last result has been proved by Fine and Chowla using other techniques).

Let R be the Hurwitzean quaternion ring composed of elements whose coordinates are either all integers or halves of odd integers. The structure of the two-sided ideals in R is examined. Then these results and the ideal structure of $K[x]$ are used to investigate the structure of the two-sided ideals in $R[x]$. Prime and maximal ideals are classified and an ideal canonical basis is developed. (Received May 12, 1975.) (Author introduced by Robert D. Morton.)

*75T-A182 RICHARD MOLLIN, QUEEN'S UNIVERSITY, KINGSTON, ONTARIO, CANADA K7L 3N6, UNIFORM DISTRIBUTION AND THE SCHUR SUBGROUP.

In this paper we continue the investigation into the group of algebras with uniformly distributed invariants $U(K)$, and its relation to the Schur subgroup undertaken in (Algebras with Uniformly Distributed Invariants", A.M.S. notices (to appear)).

We investigate the index $|U(K)_q:S(K)_q|$ where q is an odd prime and $U(K)_q$ (respectively $S(K)_q$) is the q-primary part of $U(K)$ (respectively $S(K)$).

We obtain that $|U(K)_q:S(K)_q|$ is infinite when $q \| K:Q(\varepsilon_{q^a})|$ where ε_{q^a} is the highest q-power root of unity in K, $a>0$; provided $q^{a+b} \nmid |L:K|$ where L is the smallest root of unity field containing K, and $\varepsilon_{q^{a+b}}$ is the highest q-power root of unity in L.

In the case where $q^{a+b} \| L:K|$. The author's conjecture made in (Algebras with Uniformly Distributed Invariants, Jour. of Alg., (to appear)) is validated.

Moreover $|U(K)_2:S(K)_2|$ is shown to be infinite, and we calculate generators of $U(Q(\varepsilon_{p^a}))_2$ for primes $p \not\equiv 1 (\text{mod } 4)$ explicitly, as an illustration. (Received May 12, 1975.)

75T-A183 J.L. BRENNER, 10 Phillips Rd., Palo Alto, California. 94303 and L. CARLITZ, Duke University, Durham, N.C. 27706. Covering theorems for finite nonabelian simple groups, III. Solutions of the equation $\alpha x^2 + \beta t^2 + \gamma t^{-2} = a$ in a finite field.

The number of solutions of the equation of the title is $N(a) = q-1-\psi(-\alpha\gamma) + \psi(-\alpha)\Sigma_y \psi(\beta y^4 - ay^2 + \gamma)$, where $\psi(\delta)$ denotes the quadratic character of δ. This theorem, and others of the paper, can be used to compute the Burnside coefficients (for multiplication of classes) for the groups $PSL(2,q)$, and hence, in principle, the characters of this group. Almost all classes C in $G = PSL(2,q)$ have the covering property $CC \supset G$. Since character tables will likely never be available for $PSL(n,q)$, the method of this paper may be needed to find a class with the covering property in the latter. (Received May 12, 1975.)

*75T-A184 PETER TANNENBAUM, University of California, Santa Barbara, Santa Barbara, CA. 93106. On the nonexistence of certain finite projective planes.

It has been recently shown (Kantor and Pankin, Arch. Math. 23 (1972), 544-547) that the coordinatizing ternary ring for a finite projective plane of Lenz-Barlotti type I-4 is a neofield having abelian multiplication and inverse property addition (abbreviated AIP neofield). Theorem. There exist no AIP neofields of orders $v \equiv 15$ or $21 \pmod{24}$. Corollary. There exist no finite projective planes of Lenz-Barlotti type I-4 having orders $v \equiv 15$ or 21 (mod 24). (Received May 12, 1975.)

*75T-A185 **WALTER TAYLOR, University of New South Wales, Kensington, N.S.W. 2033, Australia.** **Varieties of topological algebras.**

For Ω a collection of topological algebras, let $H\Omega$ (resp. $Q\Omega$), $P\Omega$, $F\Omega$, $S\Omega$ and $E\Omega$ denote the class of all continuous (resp. open continuous) homomorphic images, products, finite products, subalgebras and 1-1 continuous homomorphic pre-images of members of Ω. A variety is a class Ω with $\Omega = S\Omega = Q\Omega = P\Omega$, and a full variety is such an Ω defined by a set of equations. The class of all Hausdorff topological algebras will be denoted T_2. THEOREM 1 If $QSP\Omega$ has permutable congruences, then $T_2 \cap QSP\Omega \subset SPQSF\Omega$. THEOREM 2 If $HSP\Omega$ has 3-permutable congruences, then $T_2 \cap HSP\Omega \subset EPHSF\Omega$. THEOREM 3 A full variety Ω has regular congruences iff whenever $B \subset A^I \in \Omega$ is a subalgebra with an isolated point, then there exists finite $J \subset I$ such that projection to A^J is 1-1 on B. COROLLARY If Ω is a congruence-3-permutable full variety which is either congruence-regular or not residually small, then Ω has a proper class of subvarieties. EXAMPLE Theorems 1 and 2 fail for lattices and for semigroups. The method of proof of Theorems 1 and 2 is essentially that of Brooks, Morris and Saxon [Proc. Edinburgh Math. Soc. (2) 18 (1973), 191-197] for groups. (Received May 16, 1975.)

*75T-A186 JOSEPH ZAKS, University of Haïfa, Haïfa, Israel. Pairs of Hamiltonian circuits in 5-connected planar graphs.

Settling a problem raised by B. Grünbaum and J. Zaks ("The existence of certain planar graphs", Discrete Math. 10(1974),93-115) and by B. Grünbaum and J. Malkevitch ("Pairs of edge-disjoint Hamiltonian circuits", to appear), we have constructed in few different ways 5-valent 5-connected planar graphs that admit no pairs of edge-disjoint Hamiltonian circuits; our smallest example has 176 vertices.

Let d(G) denote the minimum number of edges shared by every pair of Hamiltonian circuits in G, and let r(k) be the lim $\sup\{d(G)/v(G) \mid G$ is a k-valent k-connected Hamiltonian graph$\}$; define similarily $r^*(k)$ for the planar case; k=4,5. We established the following results: $r(4) \geq 1/16$, $r^*(4) \geq 1/20$, $r(5) \geq 1/76$ and $r^*(5) \geq 1/168$.

In addition, we have constructed 3-connected k-valent non-Hamiltonian planar graphs, for k = 4 and k = 5 . (Received May 19, 1975.)

75T-A187 E. M. PALMER and A. J. SCHWENK, Michigan State University, East Lansing, Michigan 48824. On the enumeration of a class of tree-like graphs constructed from polygons.

In the article [E. Palmer, Variations of the cell growth problem, Graph Theory and Applications, Y. Alavi et al, eds., Springer, 1972, 215-224] a number of questions were raised which included the problem of enumerating the number of labeled graphs in the class $\mathcal{B}$ defined as follows. Cycles of all orders $n \geq 3$ are in $\mathcal{B}$. If the graph G is in $\mathcal{B}$ so is the one obtained by indentifying any line of G with a line of a new cycle of any order. Among our recent results is the determination of the generating function $H_p(y)$ which enumerates these tree-like graphs with the number q of lines as an enumeration parameter:

$$H_p(y) = (p-2)\binom{p}{2} \sum_{q=p}^{2p-3} \binom{p-3}{q-p}^2 \frac{(2p-3-q)!}{q-p+1} q^{q-p-1} y^q .$$

Note that when $q = 2p - 3$, the coefficient of y^q is the number of 2-trees of order p . (Received May 22, 1975.)

*75T-A188 **MICHAEL RICH, Ben Gurion University of the Negev, Beer Sheva, Israel and Temple University, Phila., Pa. 19121** The Prime Radical in Alternative Rings

The characterization by J. Levitzki (Prime ideals and the lower radical, Amer. J. Math. 73

(1951), 25-29) of the prime radical of an associative ring R as the set of strongly nilpotent elements of R is adapted here to apply to the class of non-associative s-rings. A ring R is called an s-ring for a positive number s if A^s is an ideal of R whenever A is an ideal of R. As a consequence of this characterization it is shown that the prime radical is hereditary on the class of alternative rings. It is also shown that if R is a 2 and 3-torsion free alternative ring then $P(R) = P(R^+)$. (Received May 21, 1975.)

*75T-A189 HARLAN STEWART, University of Texas at Austin, Austin, Texas A Divisibility Theorem for Higher Wronskians

The h^{th} consecutive higher Wronskian of $V = \langle f_1,\cdots,f_d \rangle$ where $f_i \in K[x,y]$, f_i homogeneous of degree j, and where K is a field, is $H_h(V) = \det(D^{h+u-1} f_v)$, where D^s is an s^{th} derivative such that $Df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$ and $D(dx) = D(dy) = 0$. For each linear form $\ell = ax+by$, we associate with V a partition P_ℓ in the usual Weierstrass-point manner by picking a basis for V, $\{ g_1,\cdots,g_d \}$, for which $\ell^{a_i} \mid g_i$, $a_1 < a_2 < \cdots < a_d$ and in which the a_i's are maximal. Then we let $P_\ell = \{ p_i = a_i+1-i \}$ and $|P_\ell| = \Sigma p_i$. It follows that $\sum_\ell |P_\ell| = d(j+1-d)$. Denote by $(P_\ell - h)^+$ the partition with parts $\max(0,p_i-h)$.

Theorem 1: Let V and P_ℓ be as above and suppose char $K = 0$. Then $\ell^{|(P-h)^+|} \mid H_h(V)$ and $\ell^{|(P-h)^+|+1} \nmid H_h(V)$.

Theorem 2: The set $\{ H_0(V),\cdots,H_{j+1-d}(V) \}$ determines P_ℓ for each ℓ.

Theorem 3: If char $K = p$ or $f_i \in K[x]$, for all i, then $\ell^{|(P-h)^+|} \mid H_h(V)$.

(Received May 22, 1975.) (Author introduced by A. Iarrobino.)

75T-A190 HEINRICH STRIETZ, Technische Hochschule,FB 4, AG 1, D-61 Darmstadt, Finite partition lattices are 4-generated. Preliminary report.

The partition lattice of a finite set $M = \{1,2,\ldots,n\}$, $n \equiv 1 \pmod 4$, $n \geq 10$, and $10 \leq k \leq n-3$, $k \equiv 2 \pmod 4$, is generated by p_1,p_2,p_3,p_4, with

$p_1 = \{1,2\},\{3,4\},\{5,6\},\ldots,\{k-3,k-1\},\{k-2,k\},\ldots,\{n-2,n\},\{n-1\}$,

$p_2 = \{1,6\},\{2,3\},\{4,5\},\ldots,\{k-2,k-1\},\{k-3,k\},\ldots,\{n-1,n\},\{n-2\}$,

$p_3 = \{1,5\},\{2,4\},\{3,7\},\{6,8\},\ldots,\{k-1,k+1\},\{k,k+2\},\ldots,\{n\}$,

$p_4 = \{1,4\},\{2,7\},\{3,6\},\{5,8\},\ldots,\{k,k+1\},\{k-1,k+2\},\ldots,\{n\}$.

If $N = \{1,2,\ldots,j\}$, $n-3 \leq j \leq n$, then p_1,p_2,p_3,p_4 restricted on N will generate the partition lattice of N. (Received May 22, 1975.) (Author introduced by Rudolf Wille.)

75T-A191 SAMUEL S. WAGSTAFF, JR., University of Illinois, Urbana, Illinois 61801 Fermat's Last Theorem is true for all exponents less than 58150. Preliminary report.

A prime has index i of irregularity if it divides exactly i Bernoulli numbers B_{2k} with $1 < 2k < p - 1$. A congruence of Vandiver has been used to find all irregular primes less than 58150, and it was shown using a theorem of Vandiver that the equation $x^n + y^n = z^n$ has no solution in non-zero integers x, y, z if $n < 58150$. The Iwasawa invariant $\mu_p = 0$ for $p < 58150$. The calculations are still in progress and will continue to at least $p < 100000$. The prime 94693 is the first known prime of index 5 of irregularity (but not necessarily the least such prime). (Received May 27, 1975.)

*75T-A192 AMASSA FAUNTLEROY and ANDY MAGID, Department of Mathematics, University of Oklahoma, Norman, Oklahoma 73069. Quasi-affine Quotients of Unipotent Actions.

Let X be a normal quasi-affine variety over an algebraically closed field and let U be a unipotent algebraic group acting on X such that the stability group in U of every point of X is finite. Then the geometric quotient of X by U exists and is also a quasi-affine variety. (Received May 30, 1975.)

*75T-A193 JOSEPH G. ROSENSTEIN, Rutgers University, New Brunswick, N. J. 08903. On the category of countable subrings of $\Pi_{i\in I}F_i(p^n)$.

The categories of countable subrings of $\Pi_{i\in I}F_i(p^n)$ and $\Pi_{j\in J}F_j(q^m)$ are equivalent whenever the lattices of divisors of m and n are isomorphic (where $F_k(t)$ is the field with t elements and I and J are countable.) This generalizes a theorem of R. W. Stringall. (The categories of p-rings are equivalent, Proc. Amer. Math. Soc., Vol. 29 (1971), pp. 229-235.)
(Received June 2, 1975.)

75T-A194 Mr. Glenn Larson and Dr. Daihachiro Sato, Department of Mathematics, University of Regina, Regina, Saskatchewan, Canada. Simultaneous Equal Product and GCD Properties of Sets of Binomial Coefficients, Preliminary Report (I).

Two sequences of Integers, not necessarily containing the same number of elements, are said to have the *equal product property*, if the products of their elements are equal. They are said to have the *equal GCD property*, if the greatest common divisors of their elements are equal. Two sequences of Integers are said to have the *simultaneous equal product and GCD property*, if they have the "equal product" and the "equal GCD properties" at the same time. We can construct many sequences of binomial (and multinomial) coefficients which have the "simultaneous equal product and GCD properties". One of the interesting pairs of sequences of binomial coefficients, having these properties, is the generalization of the STAR OF DAVID THEOREM Δ_p and ∇_p which are given in the Notices 75T-A82 and 75T-A83. The proof of the product equalities for Δ_p and ∇_p is much easier than that of the "GCD property".
Further generalizations of these properties to other configurations in Pascal's triangle or to the generalizations of multinomial coefficients will be reported later.
Corollary. There exists an arbitrarily large number of sequences, each of which contains an arbitrarily large number of integers, so that all of these sequences have the same product and the same greatest common divisor. (Received June 2, 1975.)

75T-A195 DAIHACHIRO SATO, University of Regina, Saskatchewan, Canada. Simultaneous Product and GCD Equalities of Sets of Binomial Coefficients. Preliminary Report (II).

C.T. Long and V.E. Hoggatt, Jr. have given many configurations which have the equal product property in Figures 8 and 9 of page 77, The Fibonacci Quarterly, Vol. 12, No. 1, 1974. Some of these configurations actually have the simultaneous product and GCD properties as defined in the preliminary report (I). In particular, the configurations II, IV, V, and VI in the list have been shown to have these simultaneous properties. Although it is expected that configuration I in the list also has the GCD property, I have not checked out this test in detail. Configuration III does not possess the GCD property. Theorem: Let A and B be two sequences of binomial coefficients (not necessarily containing the same number of elements) that have the translation invariant equal product property in the sense of Hoggatt, Hansell, Moore and Long. Then there exists a STAR OF DAVID configuration $\Delta_p \cup \nabla_p$ such that all four of the combined sequences $A \cup \Delta_p$, $A \cup \nabla_p$, $B \cup \Delta_p$ and $B \cup \nabla_p$ have the translation invariant simultaneous equal product and GCD properties. There are infinitely many such STAR OF DAVID configurations and we can even prescribe its center relative to A and B. The single star of David $\Delta_p \cup \nabla_p$ may be replaced by a finite set of small stars of David of bounded size (e.g. p=2) which have various centers relative to A and B. Corollary: Let $\{A_i\}$ be any finite set of configurations of binomial coefficients. Then there exist configurations $B_i \supset A_i$, each pair of which has the simultaneous equal product and GCD properties which are translation invariant.
(Received June 2, 1975.)

75T-A196 JAMES E. CARRIG, Rutgers University, New Brunswick, NJ 08903. Global Dimension of Symmetric Algebras. Preliminary Report.

Let D be a Dedekind domain and M a rank one torsion-free D-module. An analysis of $A = S_D(M)$, the symmetric algebra of M, yields the following information:

Theorem 1. Tor-dim $A \leq 2$, and $=1$ iff $M = K$ the quotient field of D;

2. A is coherent;

3. Global dim A = 2.

For higher rank modules coherence is not assured, and only rough estimates of the homological dimensions are found. (Received June 9, 1975.)

*75T-A197 PETER GUMM, University of Manitoba, Winnipeg, Canada. Congruence-equalities and Mal'cev conditions in regular equational classes.

Abstract: Freese and Nation have shown that there is no lattice equation holding in all congruence lattices of semilattices. It follows easily that this result remains true if one replaces the variety of semilattices by any variety defined by a set of regular equations. Wille has introduced the notion of a congruence-equality using the binary term o (relational product) in addition to the binary terms $\vee$ (join) and $\wedge$ (meet). We show that the result of Freese and Nation is also true for a certain class of congruence-equalities in o,$\wedge$, and $\vee$, and on the other hand we provide nontrivial congruence-equalities which do hold in semilattices. They give us examples of congruence-equalities which do not imply any lattice equation.

Two such congruence-equalities are characterized in terms of Mal'cev conditions and it turns out that they are within the class of all regular varieties equivalent to the Mal'cev conditions.

$\exists p\ (p(x,x)= x,\ \ p(x,y)= p(y,x))$

resp. $\exists p\ (p(x,x,x) = x,\ p(x,y,z) = p(z,x,y))$

Finally we characterize the above Mal'cev conditions for arbitrary varieties in terms of fixed points of involutions. (Received June 9, 1975.) (Author introduced by Professor R. W. Quackenbush.)

*75T-A198 ESTHER LEE SANDERS, University of Florida, Gainesville, Florida 32611. A Note on the Number of Maximal Independent Sets in a Tree.

A maximal independent set of a tree T (or forest F) is a set M of vertices of T (F) such that no two elements of M are adjacent in T (F), and M is maximal. Let m_T be the total number of maximal independent sets of T. If T is a tree or forest with n vertices, then $m_T \leq 2^{[\frac{n}{2}]}$ and this bound is best possible; however, if T is a tree with 2k vertices, then $m_T \leq 2^{k-1}+1$ and this bound is best possible. (Received June 9, 1975.)

75T-A199 WITHDRAWN

Analysis

*75T-B153 REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2 and Ravenshaw College, Cuttack 3, Orissa, India. A theorem on bilateral generating functions. Preliminary report.

Recently, H. M. Srivastava and J.-L. Lavoie ["*A certain method of obtaining bilateral generating functions*," Nederl. Akad. Wetensch. Proc. Ser. A 78 = Indag. Math. 37 (1975); see also these *NOTICES* 21 (1974), p. A-491, Abstract 74T-B162] presented a systematic introduction to and several applications of a certain method of obtaining bilinear or bilateral generating relations for a large variety of sequences of special functions. In the present sequel to the Srivastava-Lavoie paper the author gives a multivariable extension of one of their theorems on bilateral generating functions. The paper concludes by indicating a number of interesting special cases and possible applications of the main result obtained here. (Received February 20, 1975.) (Author introduced by Professor H. M. Srivastava.)

75T-B154 R. J. LIBERA and E.J. ZLOTKIEWICZ, University of Delaware, Newark, Delaware 19711. Loewner's differential equation for spirallike functions, Preliminary report.

A function $f(z) = z + a_2z^2 + \ldots$ regular in the open unit disk Δ is spirallike if there is a real number α, $|\alpha| < \pi/2$, such that $\mathrm{Re}\{e^{-i\alpha}zf'(z)/f(z)\} > 0$ for z in Δ. For suitable choices of the parameters the regular univalent and bounded function $\phi(z,M) = M^{-1-i\tan\alpha}z + a_2(M)z^2 + \ldots$ is said to be a member of $\check{S}_q$ if it is a solution of the equation $f(z) = M^{1+i\tan\alpha}f(\phi(z,M))$; properties of subordinate functions guarantee existence of such ϕ.

It is shown that $\phi(z,T)$, $T = M/\cos\alpha$, is in $\check{S}_q$ if and only if it satisfies a Loewner-type differential equation of the form

$$\frac{\partial\phi(z,t)}{\partial t} = -\phi(z,t)p(\phi(z,t)) \text{ , where}$$

$\phi(z,0) = z$ and $p(z) = e^{i\alpha} + e_1 z + \ldots$ is a regular function of positive real part in Δ.

This equation is used to study the behavior of functions in $\check{S}_q$ and their coefficients. Choosing $\alpha = 0$ specializes the equation and the consequent results to the case of quasi-starlike functions. (Received February 24, 1975.)

*75T-B155 MAURICE J. DUPRE', Tulane University, New Orleans, La. 70118.
Classifying C_1^*-bundles, Preliminary report.

If X is a metric space, $A \subset X$ is closed, ξ is a simple C_1^*-bundle over X-A having an upper bound on fibre dimension (fibres may vary with base point), and if β is a homogeneous $M_m(C)$-bundle over a neighborhood of A, then $B(\xi,\beta)$ denotes the set of isomorphism classes of simple C_1^*-bundles over X which are isomorphic to β over A and to ξ over X-A. A homotopy theoretic classification of $B(\xi,\beta)$ is given, the formula being very complicated. As a special case, if X is the suspension of the compact space, Y, if A is the upper cone of X, and if ξ is also homogeneous with fibre $M_n(C)$, $m \leq n$, then $B(\xi,\beta) = [Y, \mathrm{Hom}_1(m,n)]$, where $\mathrm{Hom}_1(m,n)$ denotes the set of unital *-homomorphisms of $M_m(C)$ into $M_n(C)$. This in theory classifies a large class of central CCR-algebras with identity in a fashion previously used by the author for continuous trace C*-algebras via Hilbert bundle theory. (Received April 25, 1975.)

*75T-B156 M.S. HENRY and D. SCHMIDT, Montana State University, Bozeman, Montana 59715,
Continuity Theorems for Product Approximation Operators, Preliminary Report.

Let Tf denote the best uniform approximation to $f\in C[a,b]$ from an n-dimensional Haar subspace Φ of $C[a,b]$. If M is a compact subset of $C[a,b]$ and $M\cap\Phi = \phi$, then (*) there is a constant $\lambda > 0$ such that $\|Tf-Tg\|_{[a,b]} \leq \lambda\|f-g\|_{[a,b]}$ for all $f\in M$ and $g\in C[a,b]$. There are compact subsets of $C[a,b]$ which meet Φ and for which (*) does not hold. Let PF denote the product approximation to $F\in C(D)$, $D=[a,b]\times[c,d]$, with respect to the n-dimensional Haar subspace Φ of $C[a,b]$ and the m-dimensional Haar subspace Ψ of $C[c,d]$. The operator P is continuous at each $F\in C(D)$. If for each $y\in C(D)$, $F_y\notin\Phi$, where $F_y(x)=F(x,y)$, then (**) there is a constant $\lambda>0$ such that $\|PF-PG\|_D \leq \lambda\|F-G\|_D$ for all $G\in C(D)$. There exist $F\in C(D)$ for which $F_y\in\Phi$ for some $y\in[c,d]$ and (**) does not hold. With appropriate normality conditions, analogous results hold for rational and rational product approximation.
(Received April 28, 1975.)

*75T-B157 In-Ding Hsü, State University of New York at Buffalo, Amherst, New York 14226.
Existence and Stability of Periodic Solutions for the Glass-Kauffman Model, Preliminary report.

A Turing-like model (*) $\dot{x}_1 = -ax_1 + b(x_2 - x_1) + c[1 - S(x_3)]$, $\dot{x}_2 = -ax_2 + b(x_1 - x_2)$, $\dot{x}_3 = -dx_3 + e(x_4 - x_3)$, $\dot{x}_4 = -dx_4 + e(x_3 - x_4) + fS(x_2)$, was studied by Glass and Kauffman [J. Theo.Biol. 34 (1972) 219-237]. They showed numerically that (*) has a stable limit cycle. We study two cases: Case 1; a=d and b=e. Case 2; $a=\mu b$ and $c=\rho b$. In Case 1, We prove existence and stability of periodic solutions of (*) for an admissible triple, i.e. a triple (a,c,f) such that $\alpha'(b_c)\neq 0$, where $b_c(a,c,f)$ is a critical value of b and $\alpha(b)$ is the real part of the pair of eigenvalues of (*) (linearized about equilibrium) that become purely imaginary at b_c. We obtain: Theorem. Supose S(x) is the Error (or Hill) function and (a,c,f) is an admissible triple such that $c\neq f$ (or $(n+1)c\neq(n-1)f$) and $\min\{S'(\ell), S'(c/a)\}\cdot\min\{S'(S(\ell)), S'(f/a)\} \geq 60a^2/cf$, where $\ell = c[1 - S(f/a)]/3a$. Then there exist critical values $b_1(a,c,f)$ and $b_2(a,c,f)$ of b with $0<b_1<a<b_2<\infty$ such

that (*) has a bifurcating, periodic solution for each $b \varepsilon (b_1, b_1+\eta)$ and each $b \varepsilon (b_2-\eta, b_2)$ for some $\eta>0$. Moreover, these periodic solutions are asymptotically, orbitally stable with asymptotic phase. In Case 2, we prove: Theorem. For any given parameters (μ,ρ,d,e,f) such that $4(\mu+1)^2(d+e)^2 < \delta\rho ef$, there exists $b_c(\mu,\rho,d,e,f)$ such that (*) has a periodic solution for exactly one of the three cases: either for each $b \varepsilon (b_c, b_c+\eta)$ (stable), or for each $b \varepsilon (b_c-\eta, b_c)$ (unstable) for some $\eta>0$, or for $b=b_c$ (undetermined), where $\delta = S'(x_3^e)S'(x_2^e)$.
(Received April 28, 1975.) (Author introduced by Dr. N. D. Kazarinoff.)

75T-B158 M.L. MOGRA, Department of Mathematics, Indian Institute of Technology, Kanpur-208016, India. On a class of starlike functions in the unit disc. Preliminary report.

Let $S(\alpha)$ denote the class of functions $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ analytic in the unit disc $E(|z| < 1)$ and satisfying $|(zf'(z)/f(z)-1)/(zf'(z)/f(z)+1)| < \alpha$, for some $\alpha(0 < \alpha \leq 1)$ and for all $z \varepsilon E$. For the class $S(\alpha)$, Padmanabhan [J. Indian Math. Soc. 32 (1968), 89-103] has obtained representation formula, distortion theorems and the radius of convexity. In this paper we obtain coefficient estimates and a sufficient condition for a function to be in $S(\alpha)$, the analogue of which has not been obtained by Padmanabhan. Further we extend all the above results to the subclass $S_q(\alpha)$ of $S(\alpha)$ of functions whose power series begins $f(z) = z + a_{q+1} z^{q+1} + a_{q+2} z^{q+2} + \dots$. (Received April 24, 1975.) (Author introduced by Dr. O. P. Kapoor.)

75T-B159 P.K. KAMTHAN AND MANJUL GUPTA, Department of Mathematics, Indian Institute of Technology, Kanpur-208016, India. Shrinking Bases in Locally Convex Spaces.

In this note, we introduce the notion of weakly uniform bases in topological vector space (TVS) χ equipped with the topology $\mathfrak{J}$. We show that the concepts of shrinking bases and weakly uniform bases coincide in locally convex TVS $(\chi,\mathfrak{J})$. The characterizations of weakly uniform bases in a barrelled space are proved as follows : Theorem : A base $\{x_n\}$ in a barrelled space χ is weakly uniform if and only if

$$\limsup_n \{|f(y)| : y \varepsilon \chi_n, y \varepsilon D\} = 0 ,$$

where $\chi_n = \overline{sp}\{x_{n+1}, x_{n+2}, \dots\}$ and D is any bounded set in χ, intersecting χ_n at least for all sufficiently large n. Theorem : A base $\{x_i, f_i\}$ for a barrelled space χ is weakly uniform iff every bounded sequence $\{y_k\}$ in χ, for which $\lim_{k\to\infty} f_i(y_k) = 0$, $i = 1,2,\dots$, converges weakly to zero. We also show that if a base $\{x_n;f_n\}$ is shrinking in a Fréchet space χ, then $\{f_n\}$ is a boundedly complete base for χ^* in the strong topology $\beta(\chi^*,\chi)$. (Received May 2, 1975.) (Authors introduced by Professor J. N. Kapur.)

*75T-B160 J. J. BUONI and J. D. FAIRES, Youngstown State University, SPECTRAL PROPERTIES OF PRODUCTS OF OPERATORS

Let X be a Banach Space. For A and B bounded linear operators and $\lambda \neq 0$ the ascent, descent, nullity and defect of $AB - \lambda I$ is equal to that of $BA - \lambda I$. When A, B, AB and BA are closed linear operators, the ascent, nullity and defect of $AB - \lambda I$ agrees with that of $BA - \lambda I$ provided that $\lambda \neq 0$ is such that for some nonzero $\alpha \varepsilon \rho(AB) \cap \rho(BA)$, α^2/λ is also contained in $\rho(AB) \cap \rho(BA)$. When A and B are bounded or if the above condition holds when A, B, AB and BA are closed then

the non-zero essential spectra of AB and BA coincide for the various definitions of the essential spectrum (see, for instance, Gustafson and Wiedman. "On the Essential Spectrum", J. Math. Anal. Appl. 25(1969)).
(Received April 30, 1975.)

*75T-B161 JON C. HELTON, Dept. of Mathematics, A.S.U.,Tempe,AZ 85281. Two generalizations of the Gronwall inequality by product integration.

Definitions and integrals are of the subdivision-refinement type, and functions are from R to R or R × R to R, where R denotes the set of real numbers. Further, c is a nonnegative constant, h is a function from R to R, each of F and G is a nonnegative function from R × R to R, each of $\int_a^b F$ and $\int_a^b G$ exists and all other stated integrals are assumed to exist. Two integral inequalities are considered:

$$(1)\quad h(t) \le c + \int_a^t h(u)G(u,v) + \int_a^t [\int_a^u h(r)F(r,s)]G(u,v)$$

$$(2)\quad h(t) \le c + \int_a^t h(u)G(u,v) + \int_a^t [\int_a^u h(r)G(r,s)]F(u,v).$$

The following solutions are established for (1) and (2), respectively:

$$(3)\quad h(t) \le c\ \{1 + \int_a^t [{}_a\Pi^u(1 + F + G)]G(u,v)\}$$

$$(4)\quad h(t) \le c\ \{1 + \int_a^t G(u,v)\ [{}_v\Pi^t\ (1 + F + G)]\}.$$ (Received May 6, 1975.)

*75T-B162 A.R. REDDY, Michigan State University, E.Lansing, MI. 48824 Approximation by Rational Functions

THEOREM 1: Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k \ge 0$ $(k \ge 0)$ be an entire function of order $\rho(1 \le \rho < \infty)$ type τ and lower type $\omega(0 < \underline{\omega = \tau} < \infty)$. Then for any polynomials $P_n(x)$ and $Q_n(x)$ of degree at most n, we get for all large n

$$\left\|\frac{1}{f(x)} - \frac{P_n(x)}{Q_n(x)}\right\|_{L_\infty[0,\infty)} \ge \exp\left(\frac{-n\ \pi^2\ 2\sqrt{2}}{\log 2}\ (\tfrac{2}{e})^{\rho}\right).$$

THEOREM 2: Let $f(z) = \sum_{k=0}^{\infty} a_k z^k$, $a_k \ge 0$ $(k \ge 0)$ be an entire function of order ρ $(2 \le \rho < \infty)$ type τ and lower type ω $(0 < \underline{\omega < \tau} < \infty)$. Then for any polynomials $P_n(x)$ and $Q_n(x)$ of degree at most n, we get for all large n,

$$\left\|\frac{1}{f(x)} - \frac{P_n(x)}{Q_n(x)}\right\|_{L_\infty[0,\infty)} \ge \exp\left(\frac{-n\,\pi^2}{\log(\frac{\tau+\omega}{\tau-\omega})}\left[1 + \frac{\tau 2(\tau+\omega)^{\rho+1}}{\omega(\tau-\omega)(2\tau)^{\rho}}\right]\right).$$

We can prove the above theorems for $0 < \rho < 1$ with a less precise constant.
(Received May 8, 1975.)

*75T-B163 MICHAEL A. GOLBERG, University of Nevada, Las Vegas, Nevada 89154. The Equivalence of Fredholm Integral Equations and Cauchy Problems for Differential Equations, Preliminary Report.

Consider the class of integral equations (1) $u(t) = \varphi(t) + \int_a^b K(t,s,u(s))ds$, where $u(t)$, $\varphi(t)$ are n-vectors and $K(t,s,u)$ has the representation $K(t,s,u) = \sum_{i=1}^{\infty} A_i(t)B_i(s,u), a \le s < t$, $K(t,s,u) = \sum_{i=1}^{\infty} C_i(t)\ D_i(s,u)$, $t \le s \le b$. The convergence may be taken in various norms. It is shown that (1) is equivalent to the boundary value problem (2) $\alpha_{i,t}(t) = B_i(t,u(t))$, $\beta_{i,t}(t) = -D_i(t,u(t))$, $\alpha_i(a) = 0$, $\beta_i(b) = 0$, $i = 1,2,\dots,\infty$, where $u(t) = \varphi(t) + \sum_{j=1}^{\infty} A_j(t)\,\alpha_j(t) + \sum_{j=1}^{\infty} C_j(t)\ \beta_j(t)$. The theory of invariant imbedding is then used to convert (2) to an equivalent Cauchy system. By specializing to specific forms of K(t,s,u) many interesting algorithms are developed for the solution of (1). Some of these are shown to be equivalent to previous imbedding algorithms, some are new. The method can be viewed as a generalization of the one introduced recently by Bownds and Wood (*Notices*, American Mathematical Society, #75 T - C-20, Feb. 1975.) for Volterra equations and equals theirs when $c_i(t) = 0$, $i, = 1,2,\ldots, \infty$. (Received May 13, 1975.)

75T-B164 **MARZUQ, H. MAHER, Kuwait University, Kuwait.** **Necessary and sufficient conditions for integrability of certain cosine sums.**

Let $g(x) = \frac{1}{2} \sum_{k=1}^{\infty} b_k + \sum_{k=1}^{\infty} \left(\sum_{j=k}^{\infty} b_j \right) \cos kx$ **and** $h(x)/x = \frac{1}{x} \sum_{k=1}^{\infty} b_k \sin(k+\frac{1}{2})x$.

The purpose of this paper is to prove the following theorem which generalizes results of Rees and Stanojevic [J.Math.Anal. Appl. 43 (1973), 579-586].

Theorem. Let b_n **be a quasi-monotone sequence such that** $\sum_{n=1}^{\infty} b_n/n < \infty$ **and** $\sum_{1}^{\infty} (n+1) [|\Delta b_n| - \Delta b_n] < \infty$. **Then (i)** $g(x)$ **exists for** $x \in (0,\pi]$ **and** $g(x) \in L\,[0,\pi]$ **if and only if** $\sum_{k=1}^{\infty} b_k < \infty$. **(ii)** $h(x)/x$ **exists for** $x \in (0,\pi]$ **and** $h(x)/x \in L\,[0,\pi]$ **if and only if** $\sum_{k=1}^{\infty} b_k < \infty$. (Received May 14, 1975.)

(Author introduced by Professor Syed M. Mazhar.)

*75T-B165 JAMES N. HAGLER, Catholic University of America, Washington, D.C. 20064. Examples of Banach spaces which have James type norms.

By modifying a technique of R. C. James we construct the following: (1) A separable Banach space X with nonseparable dual such that every infinite dimensional subspace of X contains an isomorph of c_0. (This extends the result announced in Abstract 72-46-43, these Notices 22(1975), A-185.) (2) A compact Hausdorff space K which has topological weight c (= the cardinality of the continuum), cardinality 2^c and is sequentially compact; however, the Banach space C(K) contains no subspace isomorphic to $\ell^1(A)$ (where A is uncountable), and for any regular Borel measure μ on K, $L^1(\mu)$ is separable. (Received May 16, 1975.)

*75T-B166 **JOAN PLASTIRAS, Department of mathematics, University of California, Berkeley, California 94720.** **Compact Perturbations of C*-Algebras.**

Let $E = (E_i)_{i \in \mathbb{N}}$ **be a sequence of mutually orthogonal, finite dimensional projections on a Hilbert space H whose sum is the identity. D(E) will denote the commutant of E ; i.e. those operators whose matrix representation with respect to E is block diagonal. D(E) + C(H) will denote the C*-algebra consisting of all operators which can be written as sums of operators in D(E) and C(H). We classify these algebras up to *-isomorphism:**

Theorem: Let A = D(E) + C(H) and B = D(F) + C(H). A is isomorphic to B if and only if there exist finite subsets of the positive integers, R and S, and a bijection $q: \mathbb{N} - R \longrightarrow \mathbb{N} - S$ **such that** $\sum_{i \in R} \dim(E_i) = \sum_{i \in S} \dim(F_i)$ **and** $\dim(E_i) = \dim(F_{q(i)})$ **for all i in** $\mathbb{N} - R$.

(Received May 16, 1975.)

*75T-B167 ELEMER E. ROSINGER, Technion-Israel Institute of Technology, Haifa, Israel, 32000. Nonsymmetric Dirac Distributions in Scattering Theory.

Using a nonsymmetric representation for the Dirac δ distribution, solutions for the equation $u''(x) + k^2u(x) + \alpha(\delta(x))^m u(x) = 0$, with $k, \alpha \in R^1$, $m \in (0,\infty)$, are given. The powers $(\delta(x))^m$ of the Dirac distribution are defined, for m integer, within the associative and commutative algebras with unit element, containing the distributions in $\mathcal{D}'(R^1)$, introduced by the author in earlier papers. (Received May 19, 1975.)
(Author introduced by Professor Francois Treves.)

*75T-B168 MARK A. PINSKY, Northwestern University, Evanston, Illinois 60201
The Navier Stokes Approximation to the Linearized Boltzmann Equation

Let $h^\varepsilon(t,x,\xi)$ be the solution of the linearized Boltzmann equation $h_t + \xi \cdot h_x = \varepsilon^{-1}Qh$, $h(0^+,x,\xi) = f(x,\xi)$; Q is the linearized collision operator corresponding to a spherically symmetric hard potential. Let $N_\varepsilon(t)$ be the solution operator for the linearized Navier-Stokes equations. Let $H_0 = L^2(R^3, e^{-|\xi|^2/2}d\xi)$, $K_0 = \{f \varepsilon H_0 : Qf = 0\}$ $H = L^2(R^6, e^{-|\xi|^2/2}d\xi\, dx)$, $K = \{f \varepsilon H : f(x,\cdot) \varepsilon K_0$ for each $x\}$ Let P be the orthogonal projection onto K.

Theorem 1: If $f \varepsilon K$ is sufficiently smooth, then for each $x \varepsilon R^3$, $P h^\varepsilon(t,x,\cdot) - N_\varepsilon(t) f(x) = o(\varepsilon)$ in the norm of H_0, uniformly for $t_0 \leq t \leq t_1/\varepsilon$ $(t_0 > 0)$

Theorem 2: If $N_\varepsilon'(t)$ is the solution operator of some other second order, symmetric, rotationally invariant system of partial differential equations which posess the above approximation property, then $N_\varepsilon'(t) = N_\varepsilon(t)$. (Received May 23, 1975.)

75T-B169 MANFRED KRACHT, University of Düsseldorf, Düsseldorf, Germany, ERWIN O. KREYSZIG, University of Windsor, Windsor, Ontario, Canada. Construction of equations for polynomial kernels

Polynomial kernels for Bergman operators were introduced by the second author in 1968. Corresponding linear partial differential equations of second order and in two variables include various equations of practical importance, some of them related to the wave equation. It is shown that for a given degree n of the kernel, equations Lw = 0 that admit polynomial kernels can be constructed stepwise by starting from an equation $L^{(n)}w_n = 0$ of a similar form with a vanishing Laplace invariant. Applying a certain differential operator to an auxiliary function formed by the coefficients of $L^{(n)}$, one obtains an explicit method of construction for a polynomial kernel as well as general representations of complex solutions w which are holomorphic in a domain G of C^2. This also yields conditions for the existence of minimal polynomial kernels (kernels of smallest possible degree) as defined by the first author and G. Schroeder in 1973. Solutions obtained by an integral operator with such a kernel can be converted to representations generated by a differential operator, and the latter can be used for developing a function theory of solutions.
(Received May 23, 1975.)

*75T-B170 R. S. DAHIYA, Iowa State University, Ames, Iowa 50010. Two dimensional operational calculus.

The following type of operational relations in two variables for various special functions are developed:

$$(1)\quad (pq)^{5/4-v/2} K_{v-1/2}(2pq) \doteqdot 0 \quad , \quad 0 < xy < 1$$

$$\frac{(xy)^{v-3/2}}{2\sqrt{\pi}\,\Gamma(v)}\ {}_2F_1\left[1, \frac{3-v}{2}; \frac{1}{2}; \frac{1}{xy}\right], \quad 1 < xy < \infty.$$

$$(2)\quad (pq)^{5/4-v/2}\left[H_{1/2-v}(2a\sqrt{pq}) - Y_{1/2-v}(2a\sqrt{pq})\right]$$

$$= \frac{(xy)^{v-1}(a^2+xy)^{-1/2}}{\sqrt{\pi}\,\Gamma(1-v)\,[\Gamma(v)]^2}\ {}_2F_1\left[\tfrac{1}{2}, v-1; v+1; \frac{xy}{a^2+xy}\right], \quad 0 < v < 1.$$

(Received May 23, 1975.)

75T-B171 ROBERTO CIGNOLI, Math. Dept., Univ. of Ill., Chgo, Ill., 60680.
A note on interpolation of martingale spaces. Preliminary report.

J. Peetre (this Notices, 22 (1975), p. 125, problem n)) has posed the following question: On a probability space Ω consider a non-decreasing family $A = (A_n)_{n=\infty}^{\infty}$ σ-algebras of measurable subsets. If $X = (X_n)_{n=\infty}^{\infty}$ is a martingale with respect to A we say that $X \in M_p$ $(0<p<\infty)$ if $\sup_n E(|X_n|^p)^{1/p} < \infty$ and define M_{pq} in a similar fashion. Is it true that $(M_{p_0}, M_{p_1})_{\theta q} \cong M_{pq}$? The aim of this note is to point out that the answer is yes if $1 \leq p_0, p_1 < \infty$ and $1 < q < \infty$. A proof can be obtained by first observing that $M_{pq} \cong L_{pq}(A_\infty)$ for $1 < p, q < \infty$ (where $A_\infty = \vee A_n$) and then applying the well-known results on interpolation of L_{pq} spaces. (Received May 23, 1975.)

*75T-B172 ALEXANDER G.RAMM, Institute of Fine Mechanics and Optics, Leningrad, 197101. On some nonlinear problems.

Consider eq. $Tu = Au + Fu = J$ (1), where: 1) $F: H \to H$ is a monotonic, hemicontinuous, bounded mapping in Hilbert space H, $F(0) = 0$, $\mathrm{Re}(F(u) - F(v), u - v) \geq 0$, $\|F(u) - F(v)\| \leq \mu \|u - v\|$, $0 < \mu < \infty$; A is a closed linear operator, $\mathrm{cl}\{\mathcal{D}(A)\} = H$, $\mathrm{Re}(Tu, u) \geq 0$ $(= 0 \Leftrightarrow u = 0)$; there exists a sequence of linear bounded operators A_n, such that $A_n u \to Au$, $\forall u \in \mathcal{D}(A)$, $A_n^* u \to A^* u$, $\forall u \in \mathcal{D}(A^*)$, $\mathrm{Re}(T_n u, u) \geq \gamma(\|u\|)\|u\|$, $T_n = A_n + F$, $0 \leq \gamma(t)$ as $t \geq 0$, $\gamma(t) \to +\infty$ as $t \to +\infty$;
2) $\mathrm{Re}(Tu - Tv, u - v) \geq \nu_R(\|u - v\|)\|u - v\|$, $\|u\| \leq R$, $\|v\| \leq R$, $\nu_R(t) > 0$ as $t > 0$, $\nu_R(0) = 0$, $\nu_R(t)$ is continuous as $t \geq 0$. Theorem 1. If 1) holds and $J \in H$, then the solution of eq. (1) in H exists and is unique. If in addition, 2) holds then the map T^{-1} is continuous. Theorem 2. If the following conditions hold: 1') $\mathrm{Re}(Au, u) \geq \delta \|u\|^2$, $\delta > 0$, $\forall u \in \mathcal{D}(A)$; $\|F(u)\| \leq \varepsilon |u|_A + C(\varepsilon)$, $\forall \varepsilon > 0$, $\forall u \in \mathcal{D}(A)$; F is monotonic; 2') $\|F(u) - F(v)\| \leq C |u - v|_A$ as $|u|_A \leq \rho$, $|v|_A \leq \rho$, $C = C(\rho)$, $0 < \rho < \infty$, where H is the Hilbert space which is the completion of $D(A)$ relative to the metric generated by the form $\mathrm{Re}(Au, u)$; 3') $E \in H_A$, then the sequence $u_{n+1} = (1 - \alpha)u_n - \alpha KFu_n + \alpha E$, $u_0 \in H_A$, $K = A^{-1}$, $0 < \alpha < 1$, converges in H_A to the unique solution of eq. (1) not slower than the geometry progression with the denominator $0 < q < 1$. The map $(I + KF)^{-1}: H_A \to H_A$ is continuous. Theorems 1, 2 solve the problem of stability in large in some nonlinear oscillation theory problems of practical importance. (Received May 27, 1975.)

75T-B173 DAVID L. ROD, University of Calgary, Calgary, Alberta, Canada, T2N 1N4, GIAMPIERO PECELLI and RICHARD C. CHURCHILL, Hunter College (CUNY), 695 Park Avenue, New York New York, U.S.A., 10021, HYPERBOLIC PERIODIC ORBITS IN HAMILTONIAN SYSTEMS

An extension of the techniques of the authors' paper, Isolated Unstable Periodic Orbits [J. Differential Equations, 17 (1975), 329-348] shows that the unstable periodic orbits constructed in that paper are in fact hyperbolic. In particular, the periodic orbits in the legs of the n-saddle potential $W(x_1,x_2) = \Pi(x_2 - \lambda_i x_1)$, λ_i distinct constants, at energy $h>0$ (with differential equation $\ddot{x} = -W_x$, $x = (x_1,x_2)$) are hyperbolic. Corresponding conclusions hold for the Hénon-Heiles potential $W(x) = (1/2)|x|^2 + \varepsilon((1/3)x_1^3 - x_1x_2^2)$ with parameter $\varepsilon>0$ at energies $h>(54\varepsilon^2)^{-1}$; and also for $W(x) = (1/2)|x|^2 - \varepsilon x_1^2 x_2^2$ with $\varepsilon>0$ at energies $(4\varepsilon)^{-1} < h \leq (9/4\varepsilon)$. These results imply that the pathological structure of bounded orbits in these potentials at the above energy levels is invariant under small perturbations [see R.C. Churchill and D.L. Rod, Pathology in Dynamical Systems I and II, to appear in J. of Differential Equations]. (Received May 19, 1975.)

*75T-B174 LAWRENCE FIALKOW, Western Michigan University, Kalamazoo, Michigan 49001
On limits of unitarily equivalent operators

Let U(H) denote the set of all unitary operators on a separable complex Hilbert space H. If T is in L(H), let U(T) denote the unitary orbit of T in L(H) and let $\pi: U(H) \longrightarrow U(T)$ be defined by $\pi(U) = U^*TU$. If π admits a locally defined norm-continuous cross section $\varphi: B \longrightarrow U(H)$ (where $\pi(\varphi(S)) = S$ for each S in the relatively open subset $B \subset U(T)$ and $\varphi(T) = 1$), then T satisfies property (P): if $(U_n) \subset U(H)$ and $\lim \| U_n^* TU_n - T \| = 0$, then there exists $(W_n) \subset U(H)$ such that $\lim \|W_n - 1\| = 0$ and $W_n^*TW_n = U_n^*TU_n$ for each n. It is proved that if T is normal or isometric, then T satisfies (P) if and only if its spectrum is finite; if the spectrum of T is finite, then there exists a local cross section (φ, B) such that for S in B, $\varphi(S)$ is in the norm-closed algebra generated by S, T, and 1. Each operator on a finite dimensional Hilbert space satisfies (P), and examples are given of non-normal operators on an infinite dimensional Hilbert space that satisfy (P) or have local cross sections. Each operator having an irreducible, non-normal, hyponormal direct summand does not satisfy (P). If T satisfies (P), then the norm closure of U(T) in L(H) is path connected. (Received May 27, 1975.)

75T-B175 B. SINGH, University of Wisconsin, Manitowoc, Wisc. 54220 and R. S.DAHIYA, Iowa State University, Ames, Iowa 50010
On the nonoscillation of Lienard type retarded equations.

We study the Lienard type retarded equation

(1) $(r(t)x'(t))'+p(t)x'(t)+q(t)x(t)+a(t)h(x(g(t))) = f(t)$ for the asymptotic nature of nonoscillatory solutions and prove the following type of results:

Lemma 1. Suppose the following conditions hold: $p(t)$ is bounded and $q(t)-p'(t) \geq 0$, $t \in [A,\infty)$; there exist constants $K_0 > 0$ and $\epsilon > 0$ such that

$$\liminf_{t\to\infty} \int_t^{t+K_0} a(s)\,ds \geq \epsilon; \quad \int^\infty |f(t)|\,dt < \infty, \quad \int^\infty 1/_{r(t)}\,dt = \infty.$$

Let $x(t)$ be a nonoscillatory solution of equation (1). Suppose $\lim_{t\to\infty} x(t) \neq 0$. Then $\lim_{t\to\infty} (r(t)x'(t)+p(t)x(t)) = 0$.

Theorem 1. Let $x(t)$ be a nonoscillatory solution of equation (1). Then $x(t) \to 0$ as $t \to \infty$ under appropriate conditions from Lemma 1.

This theorem generalizes the results of Hammett (Proc. Amer. Math. Soc. 30(1971), 92-96). (Received May 27, 1975.)

75T-B176 **JAMES V. PETERS**, Stevens Institute of Technology, Hoboken, N.J. 07030
Radon Transforms on Sobolev Spaces. Preliminary Report

A Paley-Wiener type theorem is proved for the Radon transform of generalized functions F in the Sobolev space $W^{2,k}$. The space $W^{p,k}$ is defined to be the dual of the class of C^k functions such that the derivatives of $f \in C^k$ up to order k are in L^q where $1/p + 1/q = 1$ [c.f. Yosida, Functional Analysis p. 55]. Using this result necessary and sufficient conditions are given to determine if: 1. $F \in L^2$, 2. F rapidly decreasing, 3. supp f contained in a ball, or disjoint of a ball, of radius r about z_0. The support of F is determined in terms of the Radon transform F and generalized Beta functions.
(Received April 10, 1975.)

*75T-B177 JOSEPH G. STAMPFLI, California Inst. of Tech., Pasadena CA. 91109, and WARREN B. WOGEN, Univ. of N. Carolina, Chapel Hill, N.C. 27514, Reducibility in C^* Algebras.

Let $\mathscr{H}$ be a separable Hilbert space. Denote the reducible operators in $\mathscr{L}(\mathscr{H})$ by $\mathfrak{R}$ and their norm closure by $\bar{\mathfrak{R}}$. A famous question of Halmos asks whether $\mathscr{L}(\mathscr{H}) = \bar{\mathfrak{R}}$. Let $\{P_n\}$ be a sequence of non trivial projections in $\mathscr{L}(\mathscr{H})$. We say that $\{P_n\}$ nearly reduces the set $\mathfrak{S} \subset \mathscr{L}(\mathscr{H})$ if $\|P_nS-SP_n\| \to 0$ for all $S \in \mathfrak{S}$. Theorem 1. The following are equivalent; a) $\mathscr{L}(\mathscr{H}) = \bar{\mathfrak{R}}$, b) $\bar{\mathfrak{R}}$ contains all partial isometries, c) $\bar{\mathfrak{R}}$ contains all nilpotents of order 3, d) $\bar{\mathfrak{R}}$ contains all operators similar to self adjoint operators, e) Every operator in $\mathscr{L}(\mathscr{H})$ is nearly red., f) Every set of three projections is nearly red., g) Every countable subset of $\mathscr{L}(\mathscr{H})$ is nearly red., h)

$A \cdot B \in \overline{\mathfrak{R}}$ for every pair of self adjoint operators A,B. Let $\mathfrak{A}$ denote the Calkin algebra $(\mathcal{L}(\mathcal{H})/\mathcal{K})$ and let $\mathfrak{R}_e$ denote the reducible operators in $\mathfrak{A}$(ie $\pi(T) \in \mathfrak{R}_e$ if $TP-PT \in \mathcal{K}$ for a projection P with $\pi(P)$ non trivial). Theorem 2. The following are equivalent: a) $\mathfrak{A} = \mathfrak{R}_e$, b) $\pi(T) \in \mathfrak{R}_e$ for every partial isometry T, c) $\pi(T) \in \mathfrak{R}_e$ for every T which is nilpotent of order 3. d) $\pi(T) \in \mathfrak{R}_e$ for every T similar to a self adjoint operator. e) Every set of three projections in simultaneously reducible in $\mathfrak{A}$. f) Every finite set of operators in simultaneously reducible in $\mathfrak{A}$. Theorem 3. Let $R, S \in \mathcal{L}(\mathcal{H})$ where R and S have orthogonal ranges, $\|Rf\| \geq r \|f\|$ and $\|Sf\| \geq s \|f\|$ for all $f \in \mathcal{H}$. Let P be a finite dimensional projection in $\mathcal{L}(\mathcal{H})$. Let $\|[R,P]\| = \alpha$ and $\|[S,P]\| = \beta$. If $P \neq 0$; then $\alpha^2 r^{-2} + \beta^2 s^{-2} \geq 1$. (Received May 29, 1975.)

*75T-B178 TSAI-SHENG LIU, The University of Oklahoma, Norman, Oklahoma, 73069.
Integrability of nonoscillatory solutions of a delay differential equation.

Consider the second order delay equation

$$y''(t) + my'(t) + a(t)y(g(t)) = f(t) \tag{1}$$

where $m \geq 0$ is a constant and the functions $a(t)$, $g(t)$ and $f(t)$ are continuous on the real line. Theorem. Assume that (i) $a(t) > 0$, $a'(t) \geq 0$ for all large t and (ii) $g(t) \to \infty$ as $t \to \infty$, $0 < g'(t) \leq 1$ for all large t . If $\int^\infty |f(t)|\,dt < \infty$, then all nonoscillatory solutions of (1) existing on $[t_0, \infty)$ are integrable. This extends a result of Dahiya and Singh [Certain results on nonoscillation and asymptotic nature of delay equations, Hiroshima Math. J., 5(1975), 7-15]. Some examples are given.

(Received June 2, 1975.)

75T-B179 S.M. SHAH, University of Kentucky, Lexington, Kentucky 40506, S.Y. TRIMBLE, University of Missouri, Rolla, Missouri 65401. Univalence of Derivatives of Functions defined by Gap Power Series II

Let ρ_n and $\rho_n(c)$ denote the radius of univalence and the radius of convexity of $f^{(n)}$, respectively. Theorem 1. Let f be defined by $f(z) = \Sigma_0^\infty a_n z^n$ and have radius of convergence R, where $0 < R < \infty$. Suppose that $\lim_{n\to\infty} | a_n/a_{n+1} |$ exists. Then $\limsup_{n\to\infty} n\,\rho_n \leqslant 2\sqrt{3}\,R$ and $\limsup n\,\rho_n(c) \leqslant \sqrt{2}\,R$. Theorem 2. Let f, as defined above, be entire, and suppose that $\limsup_{n\to\infty} | a_{n+1} a_{n-1}/a_n^2 | \leqslant 1$. Let $\left.\begin{matrix}\gamma^*\\ \delta^*\end{matrix}\right\} = \lim_{n\to\infty} \left\{\begin{matrix}\sup\\ \inf\end{matrix}\right\} n \left| \frac{a_n}{a_{n-1}} \right|$. Then $\lim_{n\to\infty} \left\{\begin{matrix}\sup\\ \inf\end{matrix}\right\} \rho_n \leqslant \left\{\begin{matrix}2\sqrt{3}/\delta^*\\ 2\sqrt{3}/\gamma^*\end{matrix}\right.$ and $\lim_{n\to\infty} \left\{\begin{matrix}\sup\\ \inf\end{matrix}\right\} \rho_n(c) \leqslant \left\{\begin{matrix}\sqrt{2}/\delta^*\\ \sqrt{2}/\gamma^*\end{matrix}\right.$. If $\{| a_n/a_{n+1} |\}_1^\infty$ is ultimately a non-decreasing sequence then we can replace δ^* and γ^* by δ and γ, respectively, where $\left.\begin{matrix}\gamma\\ \delta\end{matrix}\right\} = \lim_{r\to\infty} \left\{\begin{matrix}\sup\\ \inf\end{matrix}\right\} \frac{\nu(r)}{r}$. Results announced in a preliminary report [Notices 20 (1973), A–371] have been improved. (For paper I with this title see S.M. Shah and S.Y. Trimble Jour. Land. Math. Soc. (2) 9 (1975), 501–512). (Received June 2, 1975.)

75T-B180 A. H. SIDDIQI and M. MOHAMMADZADEH, Azarabadgan University, Tabriz, Iran.
Approximation by Cesàro and B-means of double Fourier series.

We have extended Theorem 1 [Hasegawa, Kōdai Math. Sem. Rep. 15(1963), 226-238] and Theorem 1 [J. Ahmad, Delta (Waukesha) 1(1968/70), no. 4, 11-16, MR 42 #3487] to Lip j(t, s) class and B-mean respectively. Theorem 1 deals with the approximation of periodic continuous function f(x, y) belonging to Lip j(t, s) class by Cesàro mean of its double Fourier series. Theorem 2 is concerned with the approximation of functions belonging to $\mathrm{Lip}(\alpha, \beta)$ class by $B^{p,q}_{m,n}(x, y) = (m + 1)^{-1}(n + 1)^{-1} \Sigma^{m,n}_{i,j} S_{p+i,\,q+j}$, B-mean of its double Fourier series. (Received June 4, 1975.)

75T-B181 A. H. SIDDIQI, Azarabadgan University, Tabriz, Iran and M.A. SHAHABI, Aligarh Muslim University, Aligarh, India. On quasi normed lattices.

A linear lattice X is called quasi normed lattice with power p $(0 < p \leqq 1)$ if a real valued function v is defined over X with the properties (i) $v(x) \geqq 0$, $v(x) = 0$ if and only if $x = 0$, (ii) $v(\alpha x) = |\alpha|^p v(x)$, α scalar, (iii) $v(x + y) \leqq v(x) + v(y)$, (iv) if $|x| \leqq |y|$ then $v(x) \leqq v(y)$. If X is complete with respect to this norm then it is called (QN) lattice. Several examples of (QN) lattices are given and many properties have been obtained. Typical properties are given below. Theorem 1. Let $\{(X_i, v_i)\}$, $i = 1, 2, \ldots, n$, be a family of quasi normed lattices with power $p_1, p_2, \ldots, p_n$, respectively; then $X = \Pi_{i=1}^n X_i$ is a quasi normed lattice with power $p_1, p_2, \ldots, p_n$ (product of the powers of given quasi normed lattices). If (X_i, v_i) are (QN) lattices, then X is a (QN) lattice and the topology of X is identical with the product topology of spaces X_i. Theorem 2. The conjugate space of a quasi normed lattice is a Dedekind complete (QN) lattice. (Received June 4, 1975.)

*75T-B182 ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620. Oscillation for pertubations containing the solution.

Theorem 1. For the equation (*) $x^{(n)} + H(t,x) = Q(t,x)$, n = even, assume the following: (i) $H : J\times R \to R$, increasing in the second variable, continuous, $uH(t,u) \geq 0$ for every $(t,u) \in J\times R$, and the homogeneous equation has all of its bounded solutions oscillatory; (ii) $Q : J\times R \to R$, continuous and such that $|Q(t,u)| \leq Q_o(t)|u|^r$, $(t,u) \in J\times R$, where $r \geq 1$, and $Q_o : J \to R_+$ is continuous and such that $\int_\alpha^\infty t^{n-1} Q_o(t)\,dt < +\infty$. Then every bounded solution of (*) is oscillatory. Theorem 2. For the equation (*) assume that $H(t,u)$ is as in Th. 1, and that all solutions of the homogeneous equation are oscillatory. Moreover, let $Q : J\times R \to R$ be continuous, increasing in u and $uQ(t,u) \geq 0$ for every $(t,u) \in J\times R$. Let $x(t)$ be a positive solution of (*). Then there exists a constant $M > 0$ and a point $t_o \geq \alpha$ such that $x(t) \leq y(t)$, $t \geq t_o$ where $y(t)$ is any solution of $y^{(n)} = Q(t,y)$ such that $y(t_o) \geq M$, and $y^{(i)}(t_o) = 0$, $i=1,\ldots,n-1$. ($J = [\alpha,\infty)$, $R = (-\infty, \infty)$) (Received June 5, 1975.) (Author introduced by Professor Manoug N. Manougian.)

75T-B183 MATTS ESSÉN, Royal Institute of Technology, S-10044 Stockholm 70, Sweden. A generalization of Beurling's inequality for harmonic measure. Preliminary report.

Let D be an open connected subset of the unit disk Δ and set $\alpha = \partial D \cap \{|z| = 1\}$, $\beta = \partial D \cap \Delta$. Let $\omega(z,\alpha,D)$ be the harmonic measure of α with respect to D. Let $F(r) = \{\theta \in [-\pi,\pi] : \omega(re^{i\theta},\alpha,D) > 0\}$, $a(r) = mE(r)$. Let $u(z) = \omega(z,\alpha,D)$, $z \in D$, $u(z) = 0$, $z \in \Delta\setminus D$ and $A(r) = \sup(2\pi/a(t))$, $r < t < 1$. Theorem. $\{\int_{-\pi}^{\pi} u(re^{i\theta})^2 d\theta\}^{1/2} \leqq 2\sqrt{A(r)}\exp\{-\pi\int_{E\cap(r,1)}(ta(t))^{-1}dt\}$, $0 < r < 1$. If $a(r) = 2\pi$, we obtain Beurling's estimate of harmonic measure. In the proof, results of M. Heins (J. Analyse Math. 7, 1959) and Essén (Springer Lecture Notes 467) are used. There are similar results in higher dimensions. Here, a generalization of A. Baernstein's estimate of harmonic measure due to C. Borell (Springer Lecture Notes 467) replaces the result of M. Heins. (Received June 9, 1975.)

75T-B184 CHARLES A. HAYES, University of California, Davis, California 95616. A necessary and sufficient condition for the derivation of some classes of integrals.

It is known that if a derivation basis $\mathcal{B}$ possesses Vitali-like covering properties, with covering families having arbitrarily small $L^{(p)}(\mu)$-overlap, where $1 \leqq p < +\infty$ and μ is a σ-finite measure in an abstract measure space, then $\mathcal{B}$ derives the μ-integrals of all functions $f \in L^{(q)}(\mu)$, where $p^{-1} + q^{-1} = 1$ if $p > 1$; $q = +\infty$ if $p = 1$. The converse is known for the case $q = +\infty$, $p = 1$. A partial converse is known for $1 < p < +\infty$, if $\mathcal{B}$ is a $[\mathcal{U},\delta]$-basis; specifically, if $\mathcal{B}$ derives the integrals of all functions $f \in L^{(q)}(\mu)$, then $\mathcal{B}$ possesses Vitali-like covering families whose $L^{(p')}(\mu)$-overlap is arbitrarily small, for each p', $1 < p' < p$. This is not a full converse because it does not allow $p' = p$. The present paper closes this gap and simultaneously removes the necessity that $\mathcal{B}$ be a $[\mathcal{U},\delta]$-basis. (Received June 9, 1975.)

*75T-B185 TAI-PING LIU, University of Maryland, College Park, Maryland 20742. Solution in the Large for Nonisentropic Gas Equations, Preliminary report.

Consider gas dynamics equations (*) $u_t + p_x = 0$, $v_t - u_x = 0$, and $E_t + (pu)_x = 0$ where u, p, v, e are the velocity, pressure, specific volume and internal energy, respectively, and $E = e + u^2/2$ is the total energy. These equations form a system of 3×3 conservation laws. We assume that the gas is polytropic, that is, $p = a^2\exp((\gamma-1)S/R)v^{-\gamma}$ where S is the entropy and a, R, γ are positive constants, $1 < \gamma \leq 5/3$. Using Glimm's difference scheme, we are able to prove the following theorem. Theorem: *The Cauchy problem* (*) *with initial data* $(u(x,0),v(x,0),S(x,0)) = (u_0(x),v_0(x),S_0(x))$ *has a weak solution if* $(\gamma-1)\times$ total var.$\{u_0(x),v_0(x),S_0(x) : -\infty < x < \infty\}$ *is sufficiently small and* $0 < \underline{v} < v_0(x)$ for all $-\infty < x < \infty$. *Furthermore, the solution is physical, that is, the entropy encreases across each shock.* (Received June 9, 1975.)

*75T-B186 RICHARD BELLMAN, University of Southern California, Los Angeles, California 90007. A Result on Stability. Preliminary report.

The equation $x' + ax = g(x)$ where $x(0)$, is small. A has some characteristic roots with negative real part, and $g(x)$ is nonlinear in the components of x, was studied by Poincare, Lyapunov, Perron, and others. Many methods are available for its treatment. The same is not true when $g(x)$ contains terms involving higher derivatives. The methods appear to founder on the fact that any method of successive approximations appear to involve more derivatives for the previous approximation. The case where the equation is a second order equation was proposed as a problem in the Bulletin and was treated by Massera. We will give a method which can be used in a general case. We will require the results of Levinson in the general case. Here, we will only present the general second order case where one can use either the associated Riccati equation or the Liouville-W. K. B. method.

We first prepared the equation by differentiating twice. Instead of the equation $u' + u = u''^2$, we have the equation $u' + u = 2uu'' + 2u'^2$. Then, we use the method of successive approximations involving the second derivative, $u_{n+1} + u_{n+1} = 2u_n u_n + 1'' + 2u_n'^2$.

The behavior of the solution of the n-th approximation may be obtained in various ways. For the initial approximation we use the function e^{-t}. (Received June 9, 1975.)

*75T-B187 PAUL ERDŐS, Hungarian Academy, DONALD J.NEWMAN, Yeshiva University and A.R.REDDY, Michigan State University, East Lansing, MI 48824
Rational Approximation (II).

Let $f(x)$ be any continuous function defined on $[0,\infty)$. Denote

$$\lambda_{0,n}(f(x)) = \inf_{P\in\pi_n} \left\| f(x) - \frac{1}{P(x)} \right\|_{L_\infty[0,\infty)},$$

where π_n denotes the class of all algebraic polynomials of degree at most n. Then we have the following:

THEOREM 1:

$$\lim_{n\to\infty} \left(\lambda_{0,n}(xe^{-x})\right)^{1/\log n} = \exp(-2).$$

THEOREM 2:

$$\lim_{n\to\infty} \left(\lambda_{0,n}((1+x)e^{-x})\right)^{1/n^{2/3}} = \exp(-2^{2/3}).$$ (Received June 9,1975.)

75T-B188 P.K. [illegible] and S.K. RAY, Indian Institute of Technology, Kanpur-208016 (India). Some characterizations of shrinking decompositions in locally convex spaces.

This note consists of a few characterizations of shrinking decompositions in a locally convex space (l.c.T.V.S.). We have also defined uniform decomposition in a l.c. TVS which generalizes the one given by McArthur & Retherford [cf., Canad. J. Math. (1965) p. 207-212] for Banach space. We have [illegible] weak-uniform Schauder decompositions and shrinking Schauder decompositions are equivalent. Finally, we have included a result giving the sufficient conditions for a weak*-Schauder decomposition to [illegible]. (Received June 9, 1975.) (Authors introduced by Dr. P. Sharma.)

*75T-C39 DAVID E. MULLER and FRANCO P. PREPARATA, University of Illinois, Urbana, Illinois 61801. Upper-bound to the time for parallel evaluation of arithmetic expressions.

Let E be an arithmetic expression involving n variables, each of which appears just once, and the possible operations of addition, multiplication, and division, requiring times τ_a, τ_m, and τ_d respectively. Then, a constructively achievable upper-bound to the time required for parallel evaluation of E is $(\tau_a + \tau_m)\log n/\log \alpha + \tau_d$, where α is the positive root of the equation $z^2 = z + 1$. R. P. Brent (J.A.C.M., 21, 2, pp. 201-206) obtained the upper-bound $\lceil 4 \log_2(n-1)\rceil$ when $\tau_a = \tau_m = \tau_d = 1$, while the present result improves this to yield $2.88 \log_2 n + 1$. (Received March 31, 1975.) (Authors introduced by Professor James Armstrong.)

*75T-C40 ROBERT G. JEROSLOW, Carnegie-Mellon University, Pittsburgh, Pa. 15213. Subadditive Duality for Integer Programs.

Theorem: The consistent mixed-integer program in equality format

$$\text{(P)}\qquad \begin{aligned} &\inf \sum_{j\in J} \pi_j t_j + \sum_{k\in K} \sigma_k r_k \\ &\text{subject to } \sum_{j\in J} a^j t_j + \sum_{k\in K} d^k r_k = h \\ &\qquad t_j \geq 0 \text{ and integer, } j\in J \\ &\qquad r_k \geq 0,\ k\in K \end{aligned}$$

$(a^j, d^k \in R^m,\ \pi_j, \sigma_k \in R)$ and the subadditive program

$$\text{(D)}\qquad \begin{aligned} &\max F(h) \\ &\text{subject to } F(a^j) \leq \pi_j,\ j\in J;\ \overline{F}(d^k) \leq \sigma_k,\ k\in K, \\ &F \text{ subadditive and } F(0) \leqslant 0, F \text{ defined on the} \\ &\text{semi-group of feasible RHS } h \text{ to (P),} \end{aligned}$$

bear the following primal-dual relations to each other:

1) For any pair of solutions, one to (P) and one to (D), we have

$$\sum_{j\in J} \pi_j t_j + \sum_{k\in K} \sigma_k r_k \geq F(h)$$

2) The optimal values in (P) and (D) are the same. (Received April 7, 1975.)

*75T-C41 ROBERT SPIRA and ANTONIO VILLANUEVA, 4515 Chippewa Dr., Okemos, MI 48864, Ultraprecise Function Evaluation

Methods are given of calculating values of the common functions in n-precision. The square root is accomplished in essentially one division by Newton's method and precision doubling. The exponential function is evaluated as $\exp(x) = \exp(x_1)\exp(x_2)$, where x_2 is very small and x_1 is a large integer times a very small quantity. The function $\log x$ is evaluated by using Newton's method on $\exp(y) - x$ and precision doubling. (Received May 1, 1975.)

*75T-C42 T. ERBER, Illinois Institute of Technology, Chicago, Illinois 60616 Magnetic Bremsstrahlung in the Quantum Regime

When $(E/mc^2)^2 (H/H_{cr}) \gtrsim 1$, significant portions of the magnetic bremsstrahlung spectrum extend above $2mc^2$. Quantum effects then modify the spectral shape and enhance the total energy dissipation. Specifically $I(E, \hbar\omega, H) = \alpha c\lambda_c^{-1} (\hbar\omega \times mc^2 \times H/E^2H_{cr})^{1/3} f(\zeta,\chi)$ where

$$f(\zeta,\chi) \cong -2\mathrm{Ai}'(-\zeta) + (\zeta-\chi)\{\tfrac{1}{3} + \mathrm{Ai}_1(\zeta)\} - \chi\zeta\,\mathrm{Ai}(-\zeta); \quad \chi \lesssim 1/3$$

and

$$\zeta = \left(\frac{\hbar\omega}{ET}\right)^{2/3} \left[\left(\frac{\hbar\omega}{2mc^2}\right)^2 - 1\right], \quad \chi = \frac{1}{3}\left(\frac{\hbar\omega}{2mc^2}\right)^2 \left(\frac{\hbar\omega}{ET}\right)^{2/3}.$$ For notation see Rev. Mod. Phys. 38, 626-529 (1966). (Received April 30, 1975.)

*75T-C43 T. KARUNAKARAN, Deptt. Elec.Engg., Indian Instt. of Technology, New Delhi-110029, India. Functorial Approach to Networks, Preliminary report.

Part 1. By choosing suitable sets R_c, R_o of path pairs of a graph A we obtain the groupoids A_o and A_c (Kirchhoff-models) whose presentations are due to (A,R_c) and (A,R_o) respectively. The 'Kirchhoff-decomposition theorem' proves that the free groupoid $A\pi$ is obtainable as the free-product of A_o and A_c. (Related results: characterisation of cutset as a quotient object and its exactness preperty with circuit subobjects of $A\pi$; Kirchhoff-modeling through adjunctable functors K_o, K_c commuting with free product based network connection; primitive networks clarified by left-right adjunctability of edge functor).

Part 2. The functors $Y,X\colon \underline{\underline{G}}^{op} \times \underline{\underline{G}} \to \underline{K}$ model current and voltage respectively ($\underline{\underline{G}}$ is the category of free groupoids with forest reference, $\underline{K}$ is a general category). Dinatural transformation $z\colon Y \dashrightarrow X$ satisfying commutativity property for specified set of arrows of $\underline{\underline{G}}$ (hence 'selective') models impedance. Classical circuit-cutset formulations captured by making Y(X) preserve (preserve dualy) the Kirchhoff-decomposition and by being dummy in the first(second) argument. (Other results: solutions and consistent nets; prelude to realization theory). (Author introduced by Dr. S.K. BAJPAI). (Received April 30, 1975.)

*75T-C44 JON HELTON AND STEPHEN STUCKWISCH, Dept. of Mathematics, A.S.U., Tempe, AZ 85281. Numerical Approximation of Product Integrals.

A technique for the numerical approximation of matrix-valued Riemann product integrals is developed. For $a \leq x < y \leq b$, $I_m(x,y)$ denotes

$$\int_x^y \int_x^{v_m} \cdots \int_x^{v_2} \Pi_{i=1}^m F(v_i)\, dv_1\, dv_2 \cdots dv_m,$$

and $A_m(x,y)$ denotes an approximation of $I_m(x,y)$ of the form

$$(y-x)^m \sum_{k=1}^N a_k \Pi_{i=1}^m F(x_{ik}),$$

where a_k and y_{ik} are fixed numbers for $i=1,2,\cdots,m$ and $k=1,2,\cdots,N$ and $x_{ik}=x+(y-x)y_{ik}$. The following result is established. If p is a positive integer, F is a function from the real numbers to the set of $w\times w$ matrices with real elements and $F^{(1)}$ exists and is continuous on $[a,b]$, then there exists a bounded interval function H such that, if n, r and s are positive integers, $(b-a)/n = h < 1$, $x_i = a + hi$ for $i=0,1,\cdots,n$ and $0 < r \leq s \leq n$, then

$$\prod_{x_{r-1}}^{x_s} (I + F\,dx) - \Pi_{i=r}^s \left[I + \sum_{j=1}^p I_j\,(x_{i-1},x_i)\right]$$
$$= h^p H(x_{r-1},x_s) + O(h^{p+1}).$$

Further, if $F^{(j)}$ exists and is continuous on $[a,b]$ for $j=1,2,\cdots,p+1$ and A_j is exact for polynomials of degree less than $p+1-j$ for $j=1,2,\cdots,p$ then the preceding result remains valid when A_j is substituted for I_j. (Received May 8, 1975.)

*75T-C45 MICHAEL NEUMANN, Technion - Israel Institute of Technology, Haifa, Israel. Subproper splitting for rectangular matrices.

The splitting $A = M-N$ for a rectangular matrix A is subproper when the ranges $R(A) \subseteq R(M)$ and $R(A^*) \subseteq R(M^*)$. Consider the iteration scheme $x_i = M^\dagger N x_{i-1} + M^\dagger b$. We characterize its convergence to a solution of the linear and consistent system $Ax = b$. When A is real, monotonicity and the concept of subproper regular splitting are used to determine some necessary and some sufficient conditions for the convergence of the scheme to a solution of the system. (Received May 28, 1975.) (Author introduced by Professor Irving J. Katz.)

*75T-C46 EGON BALAS, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. A linear characterization of permutation vectors.

A permutation vector is one that can be obtained by a permutation of the components of $v = (1,\ldots,n)$. Permutation vectors $p \in R^n$ and $n \times n$ permutation matrices P are in a one to one correspondence given by $p = Pv$.

Theorem. $p \in R^n$ is a permutation vector if and only if it is an extreme point of the polytope

$$X = \left\{ x \in R^n \;\middle|\; \begin{array}{ll} \sum_{i \in S} x_i \leq ns - \frac{1}{2}s(s-1) , & \forall\ S \subset N \\ \sum_{i \in N} x_i = \frac{1}{2}n(n+1) & \end{array} \right\}$$

where $N = \{1,\ldots,n\}$ and $s = |S|$. (Received June 2, 1975.)

Geometry

*75T-D6 ALI KYRALA, Arizona State University, Tempe, Arizona, 85281 Stokes Theorem derived from Gauss Theorem

Consider a simply-connected portion S of a curved surface $\overline{R} = \overline{R}(u,v)$ with unit normal vector $\overline{N}(u,v)$ bounded by a closed curve C with a tangent vector $\overline{t}$ unique almost everywhere. Apply the Gauss (divergence)Theorem

$$\int \nabla \cdot \overline{F}\, dV = \oint \overline{n} \cdot \overline{F}\, dS$$

to the region bounded by $S_1 : \overline{R} = \overline{R}(u,v) + \epsilon\overline{N}(u,v)$, $S_2 : \overline{R} = \overline{R}(u,v) - \epsilon\overline{N}(u,v)$ (with ϵ constant for $\overline{R}(u,v)$ interior to S) and by the strip $S_3 : \overline{R} = \overline{R}(u,v) + s\overline{N}(u,v)$ with $|s| \leq 1$ for $\overline{R}(u,v)$ on C.

For this thin curved lamina the above theorem applied to $\overline{F} = \overline{G} \times \overline{n}$ yields

$$2\epsilon \int \nabla \cdot \overline{G} \times \overline{n}\, dS \cong \int_{S_1 \cup S_2} \overline{N} \cdot \overline{G} \times \overline{n}\, dS + 2\epsilon \int_{S_3} \overline{N} \cdot \overline{G} \times n\, ds$$

As $\epsilon \to 0$ the first integral on the right will approach zero since $\overline{N}$ is oppositely directed on S_1 and S_2. Generally

$$\nabla \cdot \overline{G} \times \overline{n} = \overline{n} \cdot \nabla \times \overline{G} - \overline{G} \cdot \nabla \times \overline{n}$$

but if $\overline{n}$ be chosen to be the unit normal to S one has $\nabla \times \overline{n} = 0$ since the equation of S can always be given in a form for which the unit normal is a gradient.

Also the integrand over S_3 as $\epsilon \to 0$ may be replaced by $\overline{G} \cdot \overline{t} = \overline{G} \cdot \overline{n} \times \overline{N} = \overline{N} \cdot \overline{G} \times \overline{n}$ so that

$$\int \overline{n} \cdot \nabla \times \overline{G}\, dS = \oint \overline{G} \cdot \overline{t}\, ds = \oint \overline{G} \cdot \overline{dR}$$

in which the integrations on the right are over C. This proves Stokes Theorem.
(Received May 12, 1975.)

*75T-D7 PAUL EHRLICH, Mathematisches Institut der Universitat, 5300 Bonn, West Germany.
Riemannian metrics without conjugate points.

Let M^n, $n \geqq 3$, be a compact orientable manifold. Given a Riemannian metric g for M, denote by A(g) the total scalar curvature integral. We prove Theorem 1. Suppose g_0 is a Riemannian metric for M with zero total scalar curvature integral. Then given any open C^2 neighborhood U of Riemannian metrics about g_0, there is a metric in U with conjugate points. The proof follows from Theorem 2. Suppose g_0 has zero total scalar curvature integral. Then given any open C^2 neighborhood U of g_0, there exist Riemannian metrics g_1 and g_2 in U with $A(g_1) > 0$ and $A(g_2) < 0$ and a generalized version of a theorem of E. Hopf (Proc. Nat. Acad. Sci. 34(1948), 47-57) which asserts that any metric g for M without conjugate points satisfies $A(g) \leqq 0$. (Received May 28, 1975.)

*75T-D8 GEORGE K. FRANCIS, University of Illinois, Urbana, Illinois 61801
Excellent maps, Preliminary report.

In connection with Haefliger's problem (Ann. Inst. Fourier 10(1960) p. 48) we announce the following results. Let f be a smooth map from a finite collection C of circles to the plane or sphere. N.a.s.c. for f to extent to a proper, excellent map (Whitney, Annals Math. 62(1955) p. 375) with general fold curves C in some surface are expressed in terms of certain bichromatic graphs and associated systems of assemblages (Notices Amer. Math. Soc. 22(1974) Abstract 720-05-12). The well known excellent map from the projective plane to the sphere with fold image the cusped pentagram is unique for that curve, up to topological equivalence, while the astroid (four cusped hypocycloid) has several proper excellent extensions to various orientable and nonorientable surfaces. In the latter case the map is from the punctured projective plane to the plane, due to cusp parity conditions of Thom (Ann. Inst. Fourier 6(1956) p. 43) and Whitney (op.cit. p. 409). We invite communications concerning related problems. (Received June 9, 1975.)

*75T-E46 J.N. CROSSLEY, Monash University, Clayton, 3168, Australia and ANIL NERODE, Cornell University, Ithaca, 48540. Sound functors.

For definitions see the preceding abstract, and Crossley-Nerode: Combinatorial Functors (Springer 1974). For the categories of substructures of a recursively presented universal model $\mathfrak{M}$ with inclusions as the only maps algebraic closure induces a functor which preserves recursive equivalence inducing a map $\underline{V}$ on recursive equivalence types of the appropriate categories. Theorem. The map $\underline{V}$ from R.E.T.s of sets to soundly based types is a bijection. It is also a bijection from Dedekind types to soundly based Dedekind types and $\underline{V}$ also induces a bijection from (partial recursive) combinatorial functors on R.E.T.s of sets to (partial recursive) combinatorial functions on sound types which preserves composition. Thus we reduce the theory of R.E.T.s of soundly based structures to the original Dekker theory of R.E.T.s of sets. (Received March 4, 1975.)

75T-E47 S. SHELAH, Dept. of Math., The Hebrew Univ., Jerusalem, Israel, Eherenfeucht conjecture on rigid models disproved

Theorem: For every Σ_2 sentence Θ in pure second order logic, there is a first order sentence ψ is first order logic such that: ψ has a rigid model of cardinality λ iff $\lambda \models \Theta$ and $\lambda = \sum_{\mu<\lambda} 2^{\mu}$ (Received March 12, 1975.)

*75T-E48 BOHUSLAV BALCAR and PETR ŠTĚPÁNEK, Charles University, Prague, Sokolovská 83, 186 00 Praha 8, Czechoslovakia. Invariant elements and embedding of Boolean algebras, Preliminary report.

By B_{rig}, where B is a (complete) Boolean algebra, denote the (complete) subalgebra of B constituted by all elements left fixed by every automorphism of B. Theorem. Any Boolean algebra B can be completely embedded into a Boolean algebra C such that $B = C_{rig}$. If B satisfies the κ-chain condition for an uncountable cardinal κ, the same holds true for C. Moreover, if B is complete, then $B = C^c_{rig}$, where C^c is the completion of C. (Received April 1, 1975.)

75T-E49 Wolfgang RAUTENBERG, University of California, Berkeley, and Martin ZIEGLER, Technische Universitaet Berlin. Recursive Inseparability in graph theory, Prelim. report.

The theorem below strengthens Trachtenbrodt's theorem on recursive inseparability (= r.i.) as well as results on undecidability of planar graphs recently established by different authors. A graph $G = (A,K)$ is a square net if $A = \{a_{ik} \mid i,k \in n\}$, some $2 \le n < \omega$, the a_{ik}'s pairwise distinct and $K \subseteq K_o := \{\{a_{ik}, a_{i\,k+1}\} \mid {}^{i\in n}_{k\in n-1}\} \cup \{\{a_{ik}, a_{i+1k}\} \mid {}^{i\in n-1}_{k\in n}\}$. G is complete if $K = K_o$. If $C \subseteq A$ then (G,C) is called a 1-colored square net. L denotes the first-order language of a binary predicate and L' the extension obtained from L by adding a unary predicate symbol.

Theorem. (a) The set of L-tautologies is r.i. from the set of sentences refutable in the class of square nets. (b) The set of L'-tautologies is r.i. from the set of sentences refutable in the class of 1-colored complete square nets.

As corollaries we mention: (1) the theory of any class of L-structures containing all square nets (e.g. the class of finite planar graphs) is undecidable; (2) the class of L-tautologies is r. i. from the set of sentences refutable in the class of planar 3-regular graphs; (3) there is a planar, 3-regular, strongly undecidable graph M. One gets the results by coding the action graphs of a universal Turing machine in an appropiate way. (Received April 24, 1975.)

*75T-E50 DR. HEINRICH HERRE, Zentralinstitut Math/Mech Akad der Wissen. der DDR, 108 Berlin, Mohrenstrasse 39, Decidability of the Theory of One Unary Function With the Added Quantifier "There Exist at Least $\aleph_1$ Many"

Let $F(\mathfrak{m})$ be the theory of one unary function with the added quantified "There exist $\mathfrak{m}$ many".

Theorem 1 $F(\aleph_1)$ is decidable.

Theorem 2 a) $F(\aleph_\omega) \subsetneqq F(\aleph_1)$

b) $F(\aleph_1) = F(\mathfrak{m})$ for $\mathfrak{m}$ regular $> \aleph_1$.

Theorem 3 For any sentence φ of $L(\aleph_1)$ which is consistent with $F(\aleph_1)$ and any cardinal $\mathfrak{m} > \aleph_1$ there is a model $\underline{A}$ of $F(\aleph_1)$, card $A = \mathfrak{m}$, s.t. $\underline{A} \models^1 \varphi$.

All theorems hold also for n-separated graphs, i.e. graphs, i.e. graphs in which there is no pair of circles with more than n points in common.

Remark. 2 b) was first proved by S. Vinner. (Received May 9, 1975.) (Author introduced by Professor Kenneth A. Bowen.)

*75T-E51 HUGO VOLGER, Univ.Tübingen,74 Tübingen.Fed.Rep.Germany, The Feferman-Vaught theorem revisited, Preliminary report

The theorem of Feferman and Vaught on generalized products in Fund.Math.47(1959),57-103 can be extended to a certain class of boolean-valued structures, which includes reduced products, limit reduced powers, boolean reduced powers, boolean limit reduced powers and the structures of sections of Comer. Thus the result covers all the known results of the Feferman-Vaught type. A boolean-valued structure $\langle A,B,E,R\rangle$ is called finitely complete if for every pair a_1,a_2 in A and every b in B with $E(a_1,a_1)\geq b$ and $E(a_2,a_2)\geq -b$ there exists a in A with $E(a,a_1)\geq b$ and $E(a,a_2)\geq -b$. $\langle A,B,E,R\rangle$ satisfies the maximum principle if for every formula $\psi(x,a_1,\dots,a_n)$ there exists a in A with $[\exists x\psi(x,a_1,\dots,a_n)]=[\psi(a,a_1,\dots,a_n)]$. The quotient $\langle A,B,E,R\rangle/D$ with respect to a filter D on B is obtained by identifying a_1 and a_2 in A if $E(a_1,a_2)$ is in D.

Now the result can be stated as follows: For every formula ϕ there exists a finite sequence of formulas $\psi_1,\dots,\psi_m$ and a formula Φ of the language of BA's such that for every boolean-valued structure $\langle A,B,E,R\rangle$ which is finitely complete and satisfies the maximum principle and for every filter D on B the following two statements are equivalent:

(i) $\langle A,B,E,R\rangle/D \models \phi(a_1,\dots,a_n)$

(ii) $B/D \models \Phi([\psi_1(a_1,\dots,a_n)]/D,\dots,[\psi_m(a_1,\dots,a_n)]/D)$. (Received May 15, 1975.)

75T-E52 ANDREAS BLASS, University of Michigan, Ann Arbor, Mich. 48104
Two algebraic equivalents of the axiom of choice, Preliminary report.

In Zermelo-Fraenkel set theory without the axiom of choice (and with the axiom of extensionality weakened to allow urelements), the following are equivalent:

(1) All free abelian groups are projective.

(2) All divisible abelian groups are injective.

(3) The axiom of choice.

(Received May 19, 1975.)

75T-E53 RICHARD A. SHORE, Cornell University, Ithaca, NY 14853. Some more minimal pairs of α-r.e. degrees. Preliminary report.

Lerman and Sacks (Ann. Math. Logic $\underline{4}$ (1972) 415-442) showed that there is a minimal pair of α-r.e. degrees unless $\sigma 2p(\alpha) = gc(\alpha) < t\sigma 2p(\alpha) \leq \alpha$. ($\sigma 2p(\alpha)$ is the Σ_2 projectum of α; $gc(\alpha)$ is the greatest cardinal of L_α and $t\sigma 2p(\alpha)$ is the tame Σ_2 projectum of α.) We give another proof which also succeeds if $\sigma 2p(\alpha) = gc(\alpha) < t\sigma 2p(\alpha) = \alpha$ (and so α is Σ_2 admissible). We introduce a priority argument which allows us to use the $\sigma 2p(\alpha)$ to list both the positive and negative requirements to handle the new case. (Received May 29, 1975.)

75T-E54 BRIAN MOORE, 28 Sheraton Drive, Poughkeepsie, New York 12601. Enumeration Reducibility Index Sets

A is a set of integers, $\emptyset_z$ are enumeration operators, $A^* = \{2^z 3^x : x \varepsilon \emptyset_z(A)\}$. A' indicates the jump, $A^{(n)}$ the n^{th} jump and A^c the complement. V is the join operator. $\leq e$, ≤ 1, and $\leq T$ are enumeration, one-one and Turing reducibilities. A is pseudo-enumerable (PE) if $\exists B[A \equiv_e B']$. $A^{[E]} = \{z : \emptyset_z(A) = \emptyset\}$, $A^{[TF]} = \{z : \emptyset_z(A)$ is a unary total function$\}$, and $A^{[PR]} = \{z : \emptyset_z(A)^c \leq_e \emptyset_z(A)\}$. Theorem. $A^{[E]c} \equiv_1 A^*$. Theorem. (1) $A^{[TF]c} \equiv_e A^{*\prime}$, (2) $A^{*\prime} \vee \emptyset^{(3)} \leq_e A^{[TF]} \leq_e A^{*(2)}$, (3) $A \varepsilon PE \rightarrow A^{[TF]c} \equiv_1 A^{*\prime}$ and $A^{[TF]} \equiv_e A^{*(2)}$, (4) A generic $\rightarrow A^{[TF]} \equiv_e A^{*(2)}$ and $A^{[TF]} \equiv_T A^{*\prime}$. Theorem. (1) $A^{*\prime} \vee \emptyset^{(4)} \leq_e A^{[PR]c} \leq_e A^{*(3)}$, (2) $A^{*\prime} \vee \emptyset^{(3)} \leq_e A^{[PR]} \leq_e A^{*(2)}$, (3) $A \varepsilon PE \rightarrow A^{*(2)} \vee \emptyset^{(4)} \leq_e A^{[PR]c} \leq_e A^{*(3)}$ and $A^{[PR]} \equiv_e A^{*(2)}$, (4) A generic $\rightarrow A^{[PR]} \equiv_e A^{*(2)}$ and $A^{[PR]c} \equiv_e A^{*(3)}$ and $A^{[PR]} \equiv_T A^{*(2)}$.
(Received June 3, 1975.) (Author introduced by Kenneth A. Bowen.)

75T-E55 WILLIAMS FORREST, Mathematics Department, Simon Fraser University, Burnaby, B. C. Existentially Closed Structures in Universal Classes

Let Σ be a universal class with the amalgamation property. We assume also that if $A \varepsilon \Sigma$ then $|\{\Gamma : \Gamma$ is an open type over $A\}| \leq |A| + \omega$. Theorem 1 If $A \varepsilon \Sigma$ then there is an existentially closed extension $\hat{A}$ of A such that any embedding $f : A \rightarrow B$ where B is existentially closed in Σ, can be lifted to an embedding $\hat{f} : A \longrightarrow B$. If A is countable then $\hat{A}$ is unique up to isomorphism. Theorem 2 Suppose that $A \varepsilon \Sigma$ is existentially closed. There is a unique existentially closed structure $B \varepsilon \Sigma$ such that if $[\varepsilon \Sigma$ is existentially closed then there is an embedding $f : B \longrightarrow [$ iff A and $[$ have a mutual extension in Σ. Theorem 3 Suppose there is $\lambda \geq \omega_1$ such that for all $A \varepsilon \Sigma$ of power λ if A is algebraically closed then A is existentially closed. Then every uncountable algebraically closed structure in Σ is existentially closed. Theorem 4 Suppose there is $\lambda \geq \omega_1$ such that Σ has exactly one existentially closed structure of power λ. Then Σ has exactly one existentially closed structure in each uncountable power. Theorem 5 Suppose that $A \varepsilon \Sigma$ is existentially closed. Let $\psi(v_0)$ be an open formula over A. If ψ splits into $< \omega$ infinite open formulas over any extension B of A in Σ then ψ is strongly minimal. Theorem 5 is valid for any universal class with AP.
(Received June 9, 1975.)

Statistics and Probability

*75T-F11 R.KANNAN, H.SALEHI, Michigan State University, E.Lansing, Mich. 48824 Random Solutions of Nonlinear Operator Equations of Monotone or Compact Type.

The question of the existence of measurable solutions of random nonlinear integral or differential equations involving random nonlinearities is discussed. When the nonlinearity is of the monotonic type we establish the measurability by an iterative technique. When the nonlinearity is non-monotonic we study the measurability of the solution by rewriting the nonlinear problem into an equivalent system of two operator equations and studying the graph of the coupled system. The methods employed in the latter case can be used to show the measurability of the fixed point set of a stochastically continuous, separable, compact random operator in a separable Banach space, thereby obtaining a random analogue of Schauder fixed point theorem. Several models are provided. (Received May 15, 1975.)

Topology

*75T-G67 V.KANNAN, Madurai University, Madurai-625021, INDIA.
On closed Continuous images of scattered spaces

Answering a question posed by R.Telgarsky (in 'Topics in Topology, Proc.Berlin Topology Conf.) and reposed by M.E.Rudin, we show by construction that a closed continuous image of a scattered space need not be scattered. However, we show that under some other mild additional conditions, it has to be scattered.
(Received February 28, 1975.) (Author introduced by Professor Rajagopalan.)

*75T-G68 R. PAUL BEEM, University of Pennsylvania, Philadelphia, Pa. 19174
Almost free Z_{2^k} actions

Let M_k^* denote the unoriented bordism of Z_{2^k} actions with isotropy subgroups in Z_2. Let $e : M_1^* \longrightarrow M_k^*$ and $p : M_k^* \longrightarrow M_1^*$ be the natural homomorphisms. Let x be the class of the circle with standard free Z_{2^k} action, which acts on M_k^* via a twisted product. Then M_k^* is generated, as an algebra, by $e(M_1^*)$, $xe(M_1^*)$ and a certain ideal of zero divisors. The sequence:

$$M_k^* \xrightarrow{p} M_1^* \xrightarrow{e} M_k^*$$

is exact. (Received April 24, 1975.)

*75T-G69 J. G. HOLLINGSWORTH, University of Georgia, Athens, GA. 30602, and T. B. RUSHING, University of Utah, Salt Lake City, Utah 84112. Embeddings of shape classes of compacta in the trivial range.

This improves work announced by the second author in Abstract 721-G16, these Notices 22(1975) A-345. We say that a compactum X in a manifold M satisfies the small loops condition (SLC) if for any neighborhood U of X, there is a neighborhood V of X in U and an $\varepsilon > 0$ such that each map of S^1 into $V - X$ of diameter less than ε is null homotopic in $U - X$. THEOREM. Let $X, Y \subset R^n$, $n \geqslant 5$, be compacta satisfying SLC whose dimensions are in the trivial range w.r.t. n. Then, $Sh(X) = Sh(Y)$ if and only if $R^n - X \approx R^n - Y$. COROLLARY 1. Let $X, Y \subset R^n$, $n \geqslant 5$, be compact ANR's whose dimensions are in the trivial range w.r.t. n and which satisfy the SLC. Then, X and Y have the same homotopy type if and only if their complements are homeomorphic. COROLLARY 2. Let $X, Y \subset R^n$, $n \geqslant 5$, be homeomorphic compacta whose dimensions are in the trivial range and which satisfy SLC. Then, $R^n - X \approx R^n - Y$. (Received April 24, 1975.)

75T-G70 W. E. L. CLARKE, University of Iowa, Iowa City, Iowa 52242, Homomorphic Images of subgroups of knot groups, Preliminary report.

Theorem Each finitely generated group is the homomorphic image of a normal subgroup, of finite index, of the group of a knot in S^3. Montesinos [Bull, A.M.S. 80(1974), 845-6] showed that each closed orientable 3-manifold M is an irregular 3-sheeted covering of S^3 branched over a knot k. The covering is obtained from a homomorphism w taking $\pi_1(S^3-k)$ onto the symmetric group on 3 elements. By considering the 6-sheeted unbranched covering of S^3-k corresponding to the kernel of w, we obtain a homomorphism from a normal subgroup, of index 6, of $\pi_1(S^3-k)$ onto $\pi_1(M)$. Since Montesinos' 3-manifolds include connected sums of copies of $S^2 x S^1$, we can obtain, for each n, a homomorphism onto the free group of rank n. (Received May 5, 1975.)

75T-G71 MEI-CHIN KU, University of Massachusetts, Amherst, MA 01002
Topological degree of symmetry of a product manifold. Preliminary Report.

The topological degree of symmetry $N_T(M)$ of a compact connected (topological) m-manifold M^m is the maximum of the dimensions of the compact Lie groups which can act effectively on M. Let $\langle n \rangle = n(n+1)/2$ for a non-negative integer n. Theorem. Let M^{m_i}, i = 1,2,...,k, be compact connected m_i-manifolds (k > 2). Suppose $m_{k-1} + m_k \geq 19$, $m_k \geq 6$ and

$$m_i \geq \langle m_{i+1}+\ldots+m_k \rangle - \sum_{j=i+1}^{k} \langle m_j \rangle, \quad i = 1,2,\ldots,k-2.$$

Then $N_T(M^{m_1} \times \ldots M^{m_k}) \leq \sum_{i=1}^{k} \langle m_i \rangle$. This result is obtained as a corollary of a more general theorem which can be proved by induction on k. (Received May 5, 1975.)
(Author introduced by Professor Hsu-Tung Ku.)

*75T-G72 **P.T.DANIEL THANAPALAN, Thiagarajar College of Engineering, Madurai - 625 015 - India.** **On Bitopological (P) - Spaces.**

In this paper we study properties of bitopological (P) - Spaces. A b.t.s. $(X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be bitopological (P) - space iff $\mathcal{T}_1$ and $\mathcal{T}_2$ are P-Spaces. In this paper we prove that a pairwise-Lindelof pairwise-Hausdorff b.t.(P)-space to be pairwise normal. Also it is proved that a locally pairwise-Lindelof pairwise-Hausdorff(P)-Space admits one point pairwise Lindelof extension, also a locally pairwise-Lindelof pairwise-Hausdorff (P)-Space is pairwise completely regular. (Received May 6, 1975.)
(Author introduced by Dr. T. G. Raghavan.)

75T-G73 RICHARD SADY, University of Notre Dame, Notre Dame, Indiana 46556
Involutions on Complex Projective Spaces. Preliminary report

Let Y^N denote a compact manifold of the same homotopy type as CP^N. An easy consequence of the Lefschetz Fixed Point Theorem is: Theorem. If a finite group G acts freely on Y^N, then $G \cong Z_2$ and N = 2n+1. This leaves unsettled the classification of free involutions on Y^N. Using non-simply connected surgery methods, we prove the following (PL and TOP denoting the piecewise linear and topological categories): Theorem A. On CP^{2n+1}, $n \geq 0$, there are exactly 2^n distinct free PL (resp. TOP) involutions. Theorem B. On Y^{2n+1}, $n \geq 0$, there exist 0, 2^{n-1}, or 2^n free PL (resp. TOP) involutions. Theorem C. For any $n \geq 1$, there exists a compact manifold of the same homotopy type as CP^{2n_1} on which no finite group can act freely (PL or TOP). In Theorem B, whether or not there are 2^{n-1} or 2^n free involutions depends on the existence of a homeomorphism: $Y \to Y$ not homotopic to the identity. Proofs of these results and answers to related questions will appear elsewhere.
(Received May 12, 1975.)

*75T-G74 ALLAN CALDER, Birkbeck College, University of London, London WC1E 7HX, England. Cech Cohomology of Stone-Cech Compactifications.

Let $H^q(\ ;G)$ denote the qth Cech cohomology functor based on locally finite open covers and with coefficient group G. THEOREM: *If* (X,A) *is a finite dimensional normal pair of spaces (i.e.* X *is a normal space,* A *is a closed subspace and dim*(X/A) *is finite) and* G *is finitely generated, then the embedding* $\beta : X \to \beta X$ *of* X *into its Stone-Cech compactification induces isomorphisms between* $H^q(\beta X, \beta A; G)$ *and* $H^q(X,A;G)$ *for* $q \neq 1$ *and between* $H^1(\beta X, \beta A; G)/D$ *and* $H^1(X,A;G)$, *where* D *is a divisible subgroup of* $H^1(\beta X, \beta A; G)$. (Received May 12, 1975.)

*75T-G75 TEODOR PRZYMUSIŃSKI, Instytut Matematyczny PAN, 00-950 Warszawa, Poland. Extensions of continuous functions and pseudometrics.

Let $\mathfrak{m}$ be an infinite cardinal number. A subset A of a topological space X is P-embedded / resp. $P^{\mathfrak{m}}$-embedded / iff every continuous pseudometric on A /resp. pseudometric of weight $\leq \mathfrak{m}$ / is continuously extendable onto X / R. Arens and H.L.Shapiro/. Denote by $J/\mathfrak{m}/$ the hedgehog with $\mathfrak{m}$ spikes. The theorems below give affirmative answers to two questions of R.A.Alò and L.I. Sennott /Fund. Math. 76/1972/, 231-243/. THEOREM 1. For a subset A of a completely regular space X the following conditions are equivalent: /i/ A is P-embedded in X, /ii/ $A \times \beta X$ is C^*-embedded in $X \times \beta X$, /iii/ $A \times \beta X$ is C-embedded in $X \times \beta X$. THEOREM 2. For $A \subset X$ and a cardinal $\mathfrak{m}$ the following conditions are equivalent: /i/ A is $P^{\mathfrak{m}}$-embedded in X, /ii/ Every continuous function $f: A \longrightarrow J/\mathfrak{m}/$ is continuously extendable onto X, /iii/ Every continuous function $f: A \longrightarrow M$ into an absolute retract for metric spaces M, which is complete and has weight $\leq \mathfrak{m}$, is continuously extendable onto X, /iv/ $A \times I^{\mathfrak{m}}$ is C^*-embedded in $X \times I^{\mathfrak{m}}$. (Received June 10, 1975.)

*75T-G76 D. K. BURKE, Miami University, Oxford, Ohio, 45056 and R. E. HODEL, Duke University, Durham, N. C. 27706. On the number of compact subsets of a topological space.

Theorems are obtained which give an upper bound on the number of compact subsets of a topological space in terms of other cardinal invariants. For simplicity we state the countable version of the main results. Theorem 1. A separable T_2 space has at most $2^{2^{\aleph_0}}$ compact subsets. Theorem 2. Let X be a topological space. Then X has at most $2^{\aleph_0}$ compact subsets if X satisfies any of the following conditions: (1) X is T_2, hereditarily CCC, and every compact subset is a G_δ ; (2) X is T_2 and hereditarily Lindelöf; (3) X is $\aleph_1$-compact and has a point-countable separating open cover. There is an example of a compact, T_2, first countable, separable space with $2^{2^{\aleph_0}}$ compact subsets. (Received May 8, 1975.)

*75T-G77 LESLIE P. JONES, Rutgers University, The Signature of Symplectic Manifolds, Preliminary Report

Let Ω_*^{Sp} denote the (graded) symplectic cobordism ring, $\tau : \Omega_*^{Sp} \to Z$ the signature homomorphism into the integers and $\sigma_{4n} = \text{Image}(\tau : \Omega_{4n}^{Sp} \to Z)$. We compute the image by proving two divisibility theorems and providing examples of manifolds with minimal signature. Floyd has defined a subalgebra P_* contained in unoriented cobordism and we show the top tangential Stiefel-Whitney number of an element of P_{4k+2} is zero, the signature of any

$[M] \in \Omega^{Sp}_{32k+16}$ is even and the signature of any $[M] \in \Omega^{Sp}_{16k+8}$ is divisible by 4. These results and examples constructed using the symplectic Hattori-Stong map and manifolds given by Ray and Segal show that

$$\sigma_{8k+4} = 16Z, \ \sigma_{16k+8} = 4Z, \ \sigma_{32k+16} = 2Z \ \text{ and } \ \sigma_{32k} = Z.$$

These are the results of a thesis written under Prof. Peter Landweber.
(Received May 22, 1975.) (Author introduced by Professor Peter S. Landweber.)

*75T-G78 ERIC E. ROBINSON, State University of New York, Binghamton, New York 13901.
A Characterization of Certain Branched Coverings as Group Actions.

The question of when light-open mappings are the orbit maps of group actions is in general unsolved. Recently, A. Edmonds (Branched coverings and the geometry of n-circuit, submitted) obtained some partial results with light-open PL maps between compact normal n-circuits using some elaborate techniques. McAuley (A characterization of light-open mappings and the existence of group actions, submitted) also gives an answer in some special cases. We give a partial answer for a large class of light-open mappings which includes Edmonds results as a special case: Theorem: Let $f:X \Rightarrow Y$ be a finite-to-one proper open map where X and Y are locally compact, locally connected, connected, complete metric spaces. Suppose that $f^{-1}(f(B_f))$ does not separate X locally at any point, where B_f is the set of points in X at which f fails to be a local homeomorphism. Then f is the orbit map of a group action if and only if $f|_{X - f^{-1}(f(B_f))}$ is a regular covering.

A corollary of this theorem is that if f is any finite-to-one proper open map between separable connected complete n-manifolds without boundary, then the conclusion of the theorem holds true. (Received May 16, 1975.)

75T-G79 MARK MOSTOW, Harvard University, Cambridge, Massachusetts 02138.
Continuous cohomology of spaces with two topologies, Preliminary report.

We study the concept of continuous cohomology of spaces with two topologies as defined by R. Bott and A. Haefliger ("Some remarks on continuous cohomology", Proc. Int. Conf. on Topology, U. of Tokyo, 1973). We examine other possible definitions of continuous cohomology and present four axioms satisfied by all definitions. The axioms are: 1. Homotopy invariance (in the category of pairs); 2. Taking $\coprod$ to Π 3. $\exists$ a spectral sequence $E_2^{p,q} = H^p_\delta T^q(\mathfrak{U}_*!) \Rightarrow T^{p+q}(X!)$, where X! is a space with two topologies (X' fine and X coarse), T* is a cont. coh. theory, $\mathfrak{U}$ an open cover of X, $\mathfrak{U}_p$ the sum of the (p+1)-fold intersections of $\mathfrak{U}$, and δ the Cech coboundary; 4. Normalization: $T^q(X_D) = 0$, $q>0$, and $C(X)$, $q=0$, where X is paracompact, X_D is X with its given and its discrete topologies, and $C(X) = \{\text{cont. fns.}: X \to \mathbb{R}\}$. We also give smooth and Borel modifications of these axioms and theories. Our main examples of spaces with two topologies are foliated manifolds and (Milnor) classifying spaces $B(G_D)$. THEOREM. All cont. (resp. smooth, Borel) coh. theories satisfying the axioms agree on foliated manifolds. We compute the continuous, smooth and Borel coh. of product foliations, fibrations, foliations of bundles with flat connection, and the Reeb foliation of S^3. We also present a singular foliation on which two smooth theories disagree. THEOREM. (Gen. of Bott-Haefliger). The cont. (resp. Borel) coh. of $B(G_D)$ = the cont. (resp. Borel) coh. of the group G, coeffs. in $\mathbb{R}$.
(Received May 22, 1975.)

*75T-G80 PETER KAHN, Cornell University, Ithaca, New York 14850.
Homotopy-dimension and Localization. Preliminary report.

We define $\dim_h X$, the homotopy-dimension of a connected CW complex X, to be the minimum dimension of all CW complexes homotopy equivalent to X, allowing ∞ as a possible value. Let P be a set of primes, let X be nilpotent, and let X_P be the P-localization of X. Theorem: $\dim_h X_P \leq \dim_h X + e$, where a) e= 1, when $\pi_1 X$ is finite, b) e= 2, when $\pi_1 X$ is finitely-generated, and c) e= 3, when $\pi_1 X$ is countable. The result is straightforward for 1-connected X, but it is non-trivial otherwise because there is no adequate notion of homology decomposition for non-1-connected spaces. In case a), equality is achieved when X is an n-sphere, $n \geq 2$. (Received May 15, 1975.)

*75T-G81 CHARLES D. BASS, Pembroke State University, Pembroke, North Carolina 28732. Piercing connected sets with tame arcs.

A subset X of E^3 is called a pierced set if there exists a 2-sphere S in E^3 containing X such that S can be pierced by a tame arc at each point of X. A set X is said to be universally pierced if each 2-sphere containing X can be pierced by a tame arc at each point of X. In this paper it is shown that a non-degenerate continuum is universally pierced if it is pierced. This result is used to obtain an example of a pierced set which has a degenerate component and is universally pierced.
(Received June 2, 1975.)

*75T-G82 R. GRANT WOODS, University of Manitoba, Winnipeg, Canada
Characterizations of Separable Extremally Disconnected Tychonoff Spaces

Theorem: The following conditions on a separable Tychonoff space X are equivalent:
(1) X is extremally disconnected
(2) X is basically disconnected
(3) X is an F-space
(4) X is an F'-space
(5) X is homeomorphic to a subspace of $\beta\underline{N}$ (the Stone-Čech compactification of the countable discrete space).
(Received June 2, 1975.)

*75T-G83 HOWARD LAMBERT, University of Iowa, Iowa City, Iowa 52242, Longitude surgery on genus 1 knots, Preliminary report.

Let k be a piecewise linear simple closed curve in S^3 and let K be the complement of an open regular neighborhood of k. Let $\ell(K)$ be the resulting closed 3-manifold obtained by identifying the boundary of a solid torus T with the boundary of K in such a way that the meridian of T is identified with the longitude ℓ of K. Let $(S^1 \times S^2)'$ be any 3-manifold which is the connected sum of $S^1 \times S^2$ and a homology 3-sphere. Let S be an orientable spanning surface for k and C the cube with holes obtained by removing an open regular neighborhood of S. Let $(\Pi_1(C))_\omega$ be the intersection of all groups in the lower central series of $\Pi_1(C)$. Theorem 1. If K is of genus 1 and $\ell \notin (\Pi_1(C))_\omega$, then $\ell(K) \neq (S^1 \times S^2)'$. Corollary. If k is a pretzel knot, then $\ell(K) \neq (S^1 \times S^2)'$. (Received June 3, 1975.)

75T-G84 JOSEPH AUTH, College of Our Lady of the Elms, Chicopee, Massachusetts 01013.
Symbolic transformation groups are point transitive. Preliminary report.

Let P be a set with the discrete topology such that $2 \leq$ card $P < \infty$ and let T be a discrete group. Then the symbolic transformation group with symbol set P and phase group T is defined to be the topological transformation group (P^T, T, π) where P^T has the product topology and $\pi(f, t) = f\lambda_t$ where $f \in P^T$ and $\lambda_t : T \to T$ is the left translation $s \to ts$. It is proved that if T is an arbitrary infinite group, then (P^T, T, π) is point-transitive (i.e., there exist $f \in P^T$ such that fT is dense in P^T).
(Received June 6, 1975.)

75T-G85 Diane D. Schmidt, Ithaca College, Ithaca, N.Y. 14850. The Bing Sphere Characterization, and applications of brick partitionings

J.R. Kline conjectured that a non-degenerate Peano space which is separated by every simple closed curve but by no pair of points is a 2-sphere. In 1945, R.H. Bing proved the conjecture, which became known as the Bing Sphere characterization. The present paper is an attempt to organize, clarify, and explucate the difficult original work of Bing, using some modifications of his methods. (Received June 9, 1975.)

The August Meeting in Kalamazoo, Michigan
August 18 – 22, 1975

00 General

*726-00-1 PRESTON C. HAMMER, Grand Valley State Colleges, Allendale, Michigan 49401. Errors in mathematics, Preliminary report.

I have established that a large number of basic errors persist in mathematics and logic. Examples include mistreatment of functions, identities, empty sets, and continuing abuse of the undefinable "random." These have serious educational, scientific and socio-economic implications. The time has come to weed out gross errors to pave the way for a better appreciation of our ignorance and hence for the better development of mathematics. I suggest specific methods to accomplish these ends. (Received June 9, 1975.)

01 History and Biography

*726-01-1 ALI AL-DAFFA', University of Petroleum & Minerals, Dhahran, Saudi Arabia. Thabit ibn Qurra.

Thabit ibn Qurra Abu-Hasan Al-Harrani (836-911 A.D.) of Harran, Mesopotamia, is often regarded as the greatest Arabian geometer. In this paper the author presents and discusses Thabit's solution of cubic equation [of the form $x^3 + a^2b = cx^2$] by the geometric method, as well as his formula [If p,q, and r are prime numbers, and if they are of the form $p = (3.2^n)-1$, $q = (3.2^{n-1})-1$, $r = (9.2^{2n-1})-1$, then p,q, and r are distinct primes and $2^n pq$ and $2^n r$ are a pair of amicable numbers] for finding amicable numbers.
(Received May 6, 1975.) (Author introduced by Dr. Rudolph Festa.)

*726-01-2 J. DIESTEL, Kent State University, Kent, OH 44242. The Radon-Nikodým Theorem for Banach Space-Valued Measures. Historical Perspective.

This talk will concentrate upon problems related to the Radon-Nikodým property which have their origins in the earlier development of vector-valued integration and measure theory. The history of progress (or lack thereof) on these problems will be outlined.
(Received May 8, 1975.) (Author introduced by T. J. Morrison.)

02 Logic and Foundations

726-02-1 JACK H. SILVER, University of California, Berkeley, California 94720. The singular cardinals problem.

• We prove that if $2^{\aleph_\alpha} = \aleph_{\alpha+1}$ for every $\alpha < \omega_1$, then $2^{\aleph_{\omega_1}} = \aleph_{\omega_1+1}$. It had previously been supposed by most set-theorists that the value of $2^{\aleph_{\omega_1}}$ is essentially independent of the values of $2^{\aleph_\alpha}$ for smaller α (by analogy with regular cardinals). More generally: If $\varkappa$ is a singular limit cardinal of uncountable cofinality λ, $\beta < \lambda$, and $\{\mu < \varkappa \mid 2^\mu \leqq \mu^{(\beta)}\}$ is a stationary subset of $\varkappa$, then $2^\varkappa \leqq \varkappa^{(\beta)}$ (where $\mu^{(\beta)}$ is the βth cardinal $> \mu$). A number of extensions of this result have been obtained, notably by Jensen, Galvin, and Hajnal. (Received June 17, 1975.)

726-05-1 JUDITH Q. LONGYEAR, Wayne State University, Detroit, Michigan 48202. Common transversals for any number of partitioning families.

A family of finite sets is a partitioning family if every two sets are disjoint. A transversal of such a family is a set of distinct elements, one in each set of the family. If there are several partitioning families on the same elements, each with the same number of sets, then a set, X, of elements is a common transversal of all the families if X is a transversal of each family.

An obvious graph-theoretical necessary and sufficient condition is noted for a set of t partitioning families to have a common transversal. The condition grows easier to apply as t gets larger, and can be modified to include the case where one of the families is not necessarily a partitioning family.

A uniform cardinality condition is given which is sufficient for t partitioning families to have a common transversal. (Received May 14, 1975.)

*726-05-2 JONATHAN L. GROSS, Columbia University, New York, N.Y. 10027. An infinite family of octahedral crossing numbers.

Suppose that the positive integer p is congruent to 1 modulo 4 and that p is either prime or a power of a prime. Then the crossing number of the p-dimensional octahedral graph in the orientable surface of genus $(p-1)(p-4)/4$ is $(p^2-p)/2$. The key step is the construction of a self-dual imbedding of the complete graph on p vertices such that no face boundary contains a repeated vertex. (Received May 15, 1975.)

*726-05-3 SETH R. ALPERT, SUNY-Downstate Medical Center, Brooklyn, N.Y. 11203. Current Maps and Nonorientable Graph Imbeddings.

The theory of current maps offers a powerful method for generating imbeddings of graphs into surfaces regardless of their orientability. In actual applications, current maps are used in much the same way and with similar convenience as current graphs, a technique developed for use in the orientable case. As an illustration of the usefulness and simplicity of the method, a current map construction is used to compute the nonorientable genus of the 3s-dimensional octahedral graph. (Received May 27,1975)

726-05-4 RUTH A. BARI, George Washington University, Washington, D.C. 20006. Chromatic Polynomials and Whitney's Broken Circuits. Preliminary report.

The theorem of Hassler Whitney, which gives the chromatic polynomial of a graph in terms of spanning subgraphs which contain no broken circuits, is related to other methods of studying chromatic polynomials, including the Tutte polynomial, or dichromate.

Using the principle of inclusion and exclusion, we express the coefficients of a chromatic polynomial as a linear combination of binomial coefficients. (Received June 13, 1975.)

*726-05-5 ARTHUR T. WHITE, Western Michigan University, Kalamazoo, Michigan 49008. Block designs in graph imbeddings. Preliminary report.

The generation of block designs by graph imbeddings was introduced by Heffter in 1897 and studied further by Alpert recently; both authors focused on triangulations of complete graphs. These always give two-fold Steiner triple systems ((v,b,r,k,λ)-BIBD's, with $k = 3$ and $\lambda = 2$) and, in the event that the imbedding has bichromatic dual, two (one-fold) Steiner triple systems ($\lambda = 1$). Triangular imbeddings of strongly regular (not complete) graphs yield partially balanced incomplete block designs on two association classes. Several infinite families of such designs are obtained, using the voltage graph construction of Gross in conjuction with the strong tensor product operation of Garman, Ringeisen, and White to generate the desired imbeddings. In each case the dual is bichromatic, and a PBIBD with ($k = 3$ and) $\lambda_1 = 2$, $\lambda_0 = 0$ splits into two PBIBD's, each having $\lambda_1 = 1$, $\lambda_0 = 0$. Investigations are begun for the cases $k > 3$, for both BIBD's for PBIBD's.
(Received June 2, 1975.)

726-05-6 MICHAEL O. ALBERTSON, Smith College, Northampton, Ma. 01060 and JOAN P. HUTCHINSON, Dartmouth College, Hanover, N.H. 03755. Non-Contractible Cycles in Non-Planar Graphs. Preliminary report.

Suppose G is a triangulation of the torus with V vertices.

Theorem. G contains a non-contractible cycle with no more than $(2V)^{\frac{1}{2}}$ vertices. The theorem and proof are based on ideas presented in Neil Sider's Ph.D. dissertation("Partial Colorings and Limiting Chromatic Numbers", Syracuse Univ. 1971). Denote by L(n) the set of limit points of independence ratios of graphs that embed on S_n, the surface of genus n. In Bull. A.M.S. 81, 1975, p.554, we conjectured that L(n) = L(0) for all n. An immediate consequence of the theorem is that L(1) = L(0). We have a family of triangulations of the torus that show that the upper bound is sharp except for a possible improvement in the constant. (Received June 4, 1975.)

*726-05-7 MICHAEL GILPIN, Michigan Technological University, Houghton, Michigan 49931. Three Stirling number identities from the stabilizing character. Preliminary report.

Let χ be the stabilizing character of the action of the finite group G on the finite set X. Let χ_n denote $[G:1]^{-1}\sum\chi^n(\sigma)$, the sum taken over all σ in G. It is well known that χ_n is the number of orbits of the induced action of G on the Cartesian product $X^{(n)}$. We show if G is at least (n - 1)-fold transitive on X, then χ_n can also be expressed in terms of Stirling numbers of both kinds. Thus some identities between Stirling numbers are developed. (Received May 19, 1975.)

726-05-8 DANIEL J. KLEITMAN and CURTIS GREENE, Massachusetts Institute of Technology, Cambridge, Mass. 02139. Another Proof of Brooks' Theorem. Preliminary Report.

A seventh short proof of Brooks' theorem is presented; it is shown to lead to easy proofs of some extensions. First, a complete characterization of graphs that are not $\leq$ d chromatic when all but one vertex has degree d or less (originally obtained by Dirac); second some results if the induced graph on vertices of degree d+1 or more is d colorable (mainly those described by Berge).

The Dirac result is as follows: let v_0 be the vertex of higher degree. Then the graph must have an induced subgraph that consists of j disjoint K_d's connected together by (j-1) arcs each of which cannot be part of a cycle in $G-v_0$ along with an arc from v_0 to every vertex not on one of these arcs, if it is not d colorable for $d \geq 4$. (Received May 28, 1975.)

726-05-9 GARY HAGGARD, Unversity of Maine at Orono, Orono, Maine 04473. The non-orientable genus of graphs.

The non-orientable genus of a graph G, denoted by $\tilde{\gamma}(G)$, is the least integer n such that G can be embedded on a non-orientable surface of genus n. THEOREM. Let $B_1 = K_7 - 3K_2$ and $B_2 = K_7 - C_4$. Let H be a non-planar subgraph of K_7. Then

$$\tilde{\gamma}(H) = \begin{cases} 1 & \text{if } H \text{ contains neither } B_1 \text{ nor } B_2 \\ 2 & \text{if } H \text{ contains either } B_1 \text{ or } B_2 \text{ but } H \nsupseteq K_7 \\ 3 & \text{if } H = K_7. \end{cases}$$

Various other embedding problems associated with non-orientable surfaces will also be presented. (Received June 5, 1975.) (Author introduced by Paul Kainen.)

726-05-10 Richard D. Ringeisen, Purdue University, Fort Wayne, Indiana, 46805. Some Open Questions on Orientable Embeddings.

The largest and smallest of the genera amongst all orientable surfaces upon which a connected graph G has 2-cell embeddings are called the maximum genus, $\gamma_m(G)$, and the genus, $\gamma(G)$, of G, respectively. We define the embedding range of G, R(G), by $R(G) = \gamma_m(G) - \gamma(G)$. The embedding range for a class of graphs may tend to infinity with the number of vertices. An earlier result by the author may be restated as Theorem: Given positive integers m and n, there exists a graph G with $\gamma(G) = m$ and $R(G) > n$. How small may R(G) be? Given a positive integer k, define $\Gamma(k)$, by, $\Gamma(k) = \min \{R(G) | \gamma(G) = k\}$. A theorem by Nordhaus, Ringeisen, Stewart, and White implies the following result. Theorem: a.) $\Gamma(0) = 0$. b.) For each $k \geq 1$, $\Gamma(k) \geq 1$. Corollary: $\Gamma(1) = 1$. Question: Find an expression for $\Gamma(k)$, $k \geq 2$. Does $\Gamma(2) = 2$? If the maximum genus of a graph G obtains the upper bound for the genera of all 2-cell embeddings of G, $[\frac{1}{2}\beta(G)]$, then G is called upper embeddable. Theorem: The complete graphs, the complete bipartite graphs, and the n cube graphs are upper embeddable. Not all blocks are upper embeddable. Theorem: If the vertices of a cycle are replaced by certain "special" blocks, then the resultant block is non upper embeddable. The "special" category is infinite. Question: Find a characterization for upper embeddable blocks. (Received June 5, 1975.)

726-05-11 HERBERT S. WILF, University of Pennsylvania, Philadelphia, PA 19174. An Algorithm for the Chromatic Polynomial.

An algorithm for finding the chromatic polynomial is described. Basically it is the "delete-and-identify" algorithm, organized as a binary search tree with a stack of graphs. Each graph has a spanning tree and the deleted edge is one not in that tree. Deleted graph is written on the stack. Identification destroys the spanning tree but one can be rapidly reconstructed. Process halts when the graph is a tree, hence coefficients of the Tutte polynomial form are produced directly (proving that these coefficients are nonnegative). A complete description will appear in "Combinatorial Algorithms" by A. Nijenhuis and H. S. Wilf. (Received May 27, 1975.)

726-05-12 FRANK BERNHART* and W.T. TUTTE, University of Waterloo, Ontario. Flattening Equations in the Theory of Chromatic Polynomials, Preliminary Report.

Suppose G is an arbitrary planar graph with outer boundary Q_n (an n-circuit). Let

(Π_i); $i = 1,2,\ldots,f(n)$ be the set of partitions induced by colorings of Q_n, and let $C_i(G,\lambda)$ be the constrained chromial (C-chromial) giving the number of λ-colorings of G associated with Π_i. Let (G_j); $j = 1,2,\ldots,g(n)$ be the set of planar graphs without loops or interior vertices which have Q_n, possibly degenerate, as a boundary, and let $F_j(G,G_j)$ be the free chromial (F-chromial) obtained by joining G and G_j along Q, and counting all λ-colorings.

Each F_j is easily expressed as an integer linear combination of the C_i. In Chapters V, VI of Birkhoff and Lewis' monograph [Chromatic Polynomials, AMS Trans. 60 (1946), 355-451] the problem of expressing the C_i in terms of the F_j is solved for $n = 4$ and $n = 5$ ($n = 6$ was done later by Lewis and Hall). A major difficulty is obtaining certain necessary identities among the C_i. The difficulty is here solved by showing that one special subset of the Π_i, the planar partitions, correspond in a natural way to a basis among the C_i, and to a basis among the F_j. The equations which express non-planar C-chromials in terms of planar C-chromials are new. These identities, called flattening equations, are constructed and proved for arbitrary n.

(Received June 9, 1975.)

726-05-13 ALLEN J. SCHWENK, Michigan State University, East Lansing, Michigan 48824 New derivations of spectral bounds for the chromatic number.

The eigenvalues of a (p,q) graph G are denoted by $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$. The chromatic number $\chi(G)$ has been spectrally bounded above by Wilf and below by Hoffman. We provide new proofs for both bounds to verify that

$$1 - \lambda_1/\lambda_p \leq \chi(G) \leq 1 + \lambda_1 .$$

Estimates for λ_1 may be used in conjunction with these bounds. Wilf found that $\lambda_1 \leq \sqrt{2q - \bar{d}}$ where $\bar{d}$ is the average degree in G. For connected graphs, we improve this to obtain $\lambda_1 \leq \sqrt{2q - p + 1}$. Current research with E. M. Palmer is expected to yield further improvements. (Received June 9, 1975.)

726-05-14 PAVOL HELL, Simon Fraser University, Burnaby, B. C., and DONALD J. MILLER, University of Victoria, Victoria, B. C. On achromatic numbers and graphs with forbidden quotients. Preliminary report.

An irreducible (or point-determining) graph is one in which distinct vertices have distinct neighborhoods. It was proved by the authors (these Notices 22(1975), 75T - A38) that the number of vertices of an irreducible graph of achromatic number k is bounded by a function $v(k)$. Let $Z(X)$ denote the class of all irreducible graphs Y of chromatic number not exceeding the chromatic number of X and such that X is not a quotient (homomorphic image) of Y. The above theorem implies that each $Z(K_n)$ is finite. We describe all graphs X for which $Z(X)$ (respectively a related class $\check{Z}(X)$) is finite. Some estimates of $v(k)$ are discussed. (Received June 9, 1975.)

*726-05-15 SAUL STAHL, Western Michigan University, Kalamazoo, Michigan 49008. Generalized embedding schemes. Preliminary report.

This report is concerned with the theoretical aspects of the combinatorial schemes which describe embeddings of graphs on surfaces which are not necessarily orientable. The theoretical tools used are the classical 2:1 orientable covers of non-orientable surfaces and the voltage (or covering) graph with $\Gamma = Z_2$. The problem of just when two such schemes define the "same" embedding is solved by showing that the embeddings defined by these schemes are in a natural one-to-one correspondence with the orbits of a certain switch group which acts on these schemes. The various character-

izations, obtained by several authors, of schemes which yield orientable embeddings follow as corollaries to this classification theorem. Finally, a close analysis of the effect of "small perturbations" on such schemes is carried out and the following interpolation theorem is derived: If the graph G has non-orientable genus γ, then G has a 2-cell embedding on S_n (the sphere with n cross-caps) if and only if $q-p+1 \geq n \geq \gamma$ (p and q denote, respectively, the number of vertices and edges of G).
(Received June 9, 1975.)

726-05-16 ROY B. LEVOW, Florida Atlantic University, Boca Raton, FL 33432, On Chromatic Numbers of Nearly Planar Graphs--Preliminary Report.

Let G be a planar graph. Let H be a subgraph of G. Let $f: H \to H'$ be a homomorphism with $V(H') \subset V(H)$. Let G' be the graph obtained from G by replacing H by H' (identifying some vertices and adding edges.) For $|V(H)|$ small we obtain sharp bounds on $\chi(G')$ in terms of f and of the subgraph induced by H' together with the vertices adjacent to H'. In particular, if $V(H) = 4$ and $H' = K_4 \setminus e$ then $\chi(G') \leq 5$. Thus up to 5 edges may be added to a planar graph without increasing the chromatic number beyond 5 so long as the lines connect at most 4 points, improving a result of Kainen.

This problem is equivalent to determining the chromatic numbers for spheres with pinch points and/or added lines, or to determining a generalized relative chromatic number for planar graphs. (Received June 13, 1975.)

726-05-17 E.A. NORDHAUS, Michigan State University, East Lansing, Mich. 48824 Bounds for graphical parameters. Preliminary Report.

For a given graph G, parameters useful in studying the quantitative aspects of certain fundamental concepts are defined with the aid of certain subsets of the vertex set V(G) or of the edge set E(G). These subsets have prescribed properties and one usually restricts attention to those which have maximum cardinality or minimum cardinality. This is the case for the parameters associated with covering sets, matching sets, dominating sets, and cut sets. We show that many of the above restrictions are unnecessary and that it is frequently advantageous to consider a wide range of parameter values. Another extension employs certain subsets of the union of V(G) and E(G), leading to such parameters as the total chromatic number, total covering numbers, total matching numbers, and total connectivity numbers. Often exact parameter values can be found for certain special classes of graphs and can be used to establish sharp upper and lower bounds for the corresponding parameters in the class of all graphs. (Received June 13, 1975.)

726-05-18 KAREN E. MACKEY, SUNY-Binghamton, Binghamton, N.Y., 13901 and BRUCE H. BARNES, NSF, Washington, D.C., 20550. A generalized measure of dependence and its relation to the Kronecker product of graphs. Preliminary report.

Let 1,2,...,n denote the vertices of an undirected finite graph G, and let $\{I_j | j=1,\ldots,m\}$ be the set of all maximal independent sets of G. To obtain the generalized measure of dependence $\omega_k(G)$, assign a non-negative weight y_i to each vertex i such that for all j (1) no more than k vertices are weighted non-zero in I_j and (2) $\sum_{i \in I_j} y_i \leq 1$. The k^{th} dependence number $\omega_k(G)$ is then defined to be the maximum $\sum_{i \in G} y_i$ for all weightings $\{y_i | i=1,\ldots,n\}$. Clearly $\omega_1(G)$ is identical to the maximum clique size of G. The largest possible value that this measure can become for a graph, denoted ω_∞, is the complement of the measure defined by M. Rosenfeld in Proc. Amer. Math. Soc., vol. 18, 1967. It is shown that for a graph G, $\omega_\infty(G\times H)$, where $\times$ is the Kronecker product, is equal to $\omega_1(G)\cdot\omega_\infty(H)$ for all graphs H if and only if $\omega_1(G) = \omega_\infty(G)$. (Received June 16, 1975.)

*726-05-19 J.L. HURSCH JR., 2425 University Hts. #4, Boulder, Colorado 80302. Edge - 3 - colorability of certain graphs II.

If G is any graph, a path-factor of G is any subgraph H of G such that every component of H is a path and G - H is a collection of non-adjacent edges. Let G_1 be the graph with vertices v_i, $i = 1,2,\ldots,7$ such that the edges of G_1 are v_1v_5, v_2v_5, v_3v_6, v_4v_6, v_5v_7 and v_6v_7. Let L be the class of all those finite graphs G having a subgraph P which is a path such that, if C is any component of G - P then there exists an isomorphism f of C onto G_1 satisfying $f(C \cap P) = \{v_1,v_2,v_3,v_4\}$. Theorem 1. Every graph in L is edge three colorable. Example. There exists a graph in L with 115 vertices which does not have a path-factor. Theorem 2. Every graph in the class K except G_0 (K and G_0 are defined in A.M.S. Notices 22(1975), A3 - A4) has a path-factor. (Author introduced by Professor Jan Mycielski.) (Received June 16, 1975.)

*726-05-20 J.A. Zimmer, Pahlavi Univ.,Shiraz,Iran. Isomorphisms of Hypergraphs and Graphs.

Isomorphisms of hypergraphs are defined and characterized in two ways. These characterizations, the vertex-edge duality of hypergraphs and three relationships between graphs and hypergraphs are used to provide several new characterizations, in the theory of finite graphs, of isomorphisms and of other mappings which induce isomorphisms. (Received June 16, 1975.)

*726-05-21 JOSEPH ARKIN, 197 Old Nyack Turnpike, Spring Valley, New York 10977. A nonregular Latin 3-cube solution of Euler's 6 x 6 Officers Problem.

In this paper, along with higher dimensional concepts, we consider the systems of triples throughout the nonregular Latin 3-cube of order n $(n > 2)$. We show that it is always possible to place n systems (where each system consists of n^2 three pairwise orthogonal numbers of order n) of three pairwise orthogonal numbers where each 3-digit number in a particular system is on a different file, column and row. The author wishes to thank the great mathematician Professor E. G. Straus of U.C.L.A., whose patient teaching and unstinted advice gave him insight into problems of this nature. (Received June 16, 1975.)

726-05-22 PAUL K. STOCKMEYER, College of William & Mary, Williamsburg, Virginia 23185. The reconstruction conjecture for tournaments is false. Preliminary report.

The reconstruction conjecture for tournaments asserts that except for a few small values of n, every n-point tournament is uniquely determined by its n subtournaments on n - 1 points, each obtained by deleting one point and its incident edges. A brief history of the conjecture was sketched in Abstract 75T-A70, these *Notices* 22(1975), A-305. The discovery of counterexamples among the 9 and 10-point tournaments has led to the construction of n-point counterexamples for all n of the form $2^k + 1$ or $2^k + 2$. (Received June 17, 1975.)

06 Order, Lattices, Ordered Algebraic Structures

*726-06-1 JORGE MARTINEZ, University of Florida, Gainesville, Fl. 32611 Pairwise splitting lattice-ordered groups, preliminary report.

An ℓ-group is pairwise splitting if for each pair $0 < x,y \in G$ there is a decomposition $x = x_1 + x_2$, so that $x_1 \wedge x_2 = 0$, $x_1 \in G(y)$ and $x_2 >> x_2 \wedge y$. All finite valued ℓ-groups and all hyper-archimedean ℓ-groups are pairwise splitting, and each ℓ-group with the property is normal valued. The proof of the latter involves an application of a characterization of normalvaluedness, via permutation groups, due to John Read. The class of all pairwise splitting ℓ-groups is a torsion class. (Received May 9, 1975.)

*726-06-2 HERBERT A. HOLLISTER, Bowling Green State University, Bowling Green, Ohio, 43403. Structure of nilpotent ℓ-groups. Preliminary report.

Theorem 1. Nilpotent ℓ-groups are representable.

Theorem 2. If the center of a nilpotent ℓ-group is totally ordered, then the group is totally ordered. (Received May 19, 1975.)

726-06-3 W. R. Belding, U. S. Naval Academy, Annapolis, MD 21402. Bases for the positive cone of a partially ordered module, Preliminary report.

R is a commutative ring with identity and M an R-module. $(R,\leq)$ is a partially ordered ring and $(M,\leq^1)$ a po group. Call $(M,\leq^1)$ a po $(R,\leq)$-module if $0 < r$ and $0 <^1 m$ implies $0 <^1 rm$. An R^+-basis for M^+ is a subset B of M^+ with (1)(spanning) $0 \leq^1 m$ implies there are r_i in R^+, b_i in B and $m = \Sigma r_i b_i$ and (2) (independence) $0 \leq^1 rb \leq^1 b$ with $b \in B$ implies rb cannot be expressed as an R^+ linear sum of members of $B-\{b\}$.

If B and C are two such bases they have the same cardinality and to within a permutation $c_i = x_i b_i$ for units x_i of R. Other results will be given. (Received May 21, 1975.)

726-06-4 RICHARD D. BYRD and JUSTIN T. LLOYD, University of Houston, Houston, Texas 77004. Extensions of group retractions. Preliminary report.

Let G be a group and F(G) the semigroup of finite complexes of G. Then G is said to be retractable if there is a homomorphism σ from F(G) to G such that $\sigma(\{a\}) = a$ for all elements a of G. A subgroup H of G is a σ-subgroup if $\sigma(A) \in H$ for all $A \in F(H)$. We observe that if G is abelian, then nG is a σ-subgroup of G for each natural number n. Using this result, we prove that if H is a subgroup of a torsion free abelian group G, and if σ is a retraction of H, then σ can be extended to a retraction of G if and only if $nG \cap H$ is a σ-subgroup of H for each natural number n. (Received May 27, 1975.)

726-06-5 J. ROGER TELLER, Georgetown University, Washington, D.C., 20057 Some properties of retractable groups. Preliminary Report

Let G be a group and F(G) the set of all finite non-empty subsets of G. For $A, B \in F(G)$ let $AB = \{ab | a \in A, b \in B\}$. Then F(G) is a subsemigroup. A homomorphism $\sigma: F(G) \to G$ such that $\{g\}\sigma = g$ for all $g \in G$ is a retraction of G. If there is a retraction $\sigma: F(G) \to G$ then G is said to be retractable. It will be shown that the class of retractable groups is a proper subclass of the class of torsion free groups and that not every retractable group can be lattice ordered. Additional properties will be demonstrated. (Received June 2, 1975.)

*726-06-6 ROBERTO A. MENA, University of Wyoming, Laramie, Wyoming 82071. Retractions and lattice-orderings on groups.

Let G be a multiplicatively written group. Let F(G) be the set of finite nonempty subsets of G. Under complex multiplication, F(G) becomes a monoid. Also F(G) is naturally ordered by inclusion, becoming a join-semilattice. A function $\sigma : F(G) \to G$ is called a retraction (of G) if it is multiplicative and $\{g\}\sigma = g$ for all $g \in G$. Let $\langle G,\vee,\wedge\rangle$ be an ℓ-group. If $\sigma : F(G) \to G$ is defined by $A\sigma = \bigvee A$ for all $A \in F(G)$, then σ is a retraction. Such a retraction is called an ℓ-retraction. Theorem. Let σ be a retraction of G.

Then the following are equivalent: i) σ is an ℓ-retraction; ii) $\{A \in F(G) | A\sigma = 1\}$ is a convex subsemilattice of $F(G)$; iii) for all $A, B, C \in F(G)$, if $A\sigma = B\sigma$ then $(A \cup C)\sigma = (B \cup C)\sigma$; iv) for $n \geq 2$, $g_1,\ldots,g_n \in G$, $\{g_1,\ldots,g_n\}\sigma = \{\{g_1,\ldots,g_{n-1}\}\sigma, g_n\}\sigma$. One can also show by considering the retractions of the rationals that the conditions of ii) in the theorem cannot be weakened. (Received June 2, 1975.) (Author introduced by W. Charles Holland.)

*726-06-7 NORMAN R. REILLY, Simon Fraser University, Burnaby 2, B.C., Canada.
Ordered groups with compatible tight Riesz orders.

The order relation on a partially ordered group $(G,\leq)$ is a tight Riesz order if, for any $a,b,c \in G$ such that $a < b, c$, there exists an element $d \in G$ with $a < d < b,c$. Let $\{A_\alpha : \alpha \in A\}$ be a set of totally ordered groups, $\{B_\beta : \beta \in B\}$ be a set of dense totally ordered groups $(B \neq \phi)$ and H be their product, $\preccurlyeq$ the cardinal order on H and $\leq$ the order defined by $f > g$ if and only if $f(\alpha) \geq g(\alpha)$ for all $\alpha \in A$ and $f(\beta) > g(\beta)$, for all $\beta \in B$. Then $\leq$ is a compatible tight Riesz order for $\preccurlyeq$; that is, a tight Riesz order such that $\{h \in H: o < a \Rightarrow o < a + h\} = \{h \in H: o \preccurlyeq h\}$. J.B. Miller (to appear) introduced and employed these hybrid products in the study of compatible tight Riesz orders.

It is shown that any abelian lattice ordered group with a compatible tight Riesz order is a subdirect product of a hybrid product with inherited orders. The result is extended to non-abelian partially and lattice ordered groups using hybrid products of permutation groups of ordered sets. (Received June 2, 1975.)

*726-06-8 STUART A. STEINBERG, University of Toledo, Dept. of Mathematics 2801 W. Bancroft St., Toledo, Ohio 43606.
Superunits in lattice-ordered rings

The positive element e in the lattice-ordered ring (l-ring) R is called a left superunit (after Henriksen and Isbell) if $ex \geq x$ for each positive element $x \in R$. Let S be the convex l-subring of R generated by the left superunit e, and suppose that ${}_SR_S$ is an f-bimodule. Generalizing a result of Birkhoff and Pierce we have the Theorem. R satisfies the identity $x^+x^-=0$ if and only if e is a weak order unit in R. Corollary. A right f-ring with a left identity element satisfies $x^+x^-=0$. There exists such an l-ring which is not an f-ring. Corollary. If R is archimedean, then R is an f-ring if and only if it has squares positive. Theorem. If e is a left identity element and R is semi-perfect with squares positive, then R satisfies the identities $|xy|=|x||y|$ and $x^+x^-=0$. If $x \in R^+\setminus l(R)$ implies $x > l(R)$, then R is an f-ring. $(l(R) = \{ x \in R \mid xR = 0 \}$.) (Received June 9, 1975.)

*726-06-9 A.H. RHEMTULLA and R. BOTTO MURA, University of Alberta, Edmonton, Canada T6G 2G1.
Some Sufficient Conditions for a Group to be Orderable.

Various group theoretical means of deciding whether a group is orderable are discussed. A typical example is the following generalization of results of G. Baumslag and D.M. Smirnov. Theorem: Let F be a free group, A a normal subgroup of F, V a fully invariant subgroup of A and $\underline{V}$ the variety generated by A/V. If F/A has an infrainvariant system with factors that are right orderable $\underline{V}$-groups and A/V is orderable then so is F/V.
(Received June 12, 1975.)

726-06-10 M. SATYANARAYANA, Bowling Green State University, Bowling Green, Ohio 43403.
Some problems in fully ordered semigroups.

Holder and Clifford completely characterized archimedean naturally fully ordered (f.o.) semigroups. Other than the theorem that archimedean naturally f.o. semigroups are o-isomorphic to a subsemigroup of real numbers and a partial result on positively ordered archimedean f.o. semigroups by Hion, very little is known about the structure of archimedean f.o. semigroups and naturally f.o. semigroups. There exist positively ordered archimedean semigroups which are not naturally ordered. It may be observed that every ideal is convex is the necessary and sufficient condition for positively ordered archimedean semigroups to be naturally ordered. Since every f.o. monoid is either positively ordered or contains properly a negatively ordered submonoid, it is natural to expect that some classes of f.o. monoids can be extensions of positively ordered or negatively ordered monoids. In this paper we shall discuss the above problems, in particular, the structures of positively ordered f.o. semigroups and archimedean semigroups. Every commutative cancellative f.o. monoid is contained in a f.o. group and every f.o. group contains a Prufer f.o. monoid. Here we shall investigate how the properties of Prufer f.o. monoid can be transferred to f.o. groups and vice versa. Finally we shall present some new results about the structure of o-simple semigroups. (Received June 16, 1975.)

*726-06-11 RICHARD BALL, BOISE STATE UNIVERSITY, BOISE, IDAHO 83725

Ideals of the Lattice Ordered Group of Order-Preserving Permutation of the Long Line. Preliminary Report.

Let L denote the long line and A(L) the ℓ-group of order-preserving permutations of L. The purpose of this paper is to describe the ideal lattice of A(L). Theorem. Let A be the lattice of filters of the countably complete Boolean algebra P/F, where P is the power set of ω_1 and F is the filter of closed unbounded subsets of ω_1. Let F_L and F_R be the ideals of all permutations with support bounded on the left and right, respectively. For ideals A and B, let (A,B) be the lattice of ideals between A and B.

Conclusions. a. $(1, F_L \cap F_R)$, $(F_R, A(L))$, and $(F_R \cap F_L, F_L)$ are two element lattices,

b. $(F_L, A(L))$ and $(F_R \cap F_L, F_R)$ are lattice isomorphic to A,

c. every ideal of A(L) falls in one of the above five sublattices.

(Received June 16, 1975.)

726-06-12 **A.M.W.GLASS*, Bowling Green State University, Bowling Green, Ohio 43403 and STEPHEN H. McCLEARY**, **Problems in Lattice-ordered groups, Preliminary Report.**

1. Three new ℓ-simple o-primitive lattice-ordered groups are given. 2. A discussion of possible methods for solving the word problem for lattice-ordered groups is given as well as the difficulties that arise. (Received June 16, 1975.)

*726-06-13 STEPHEN H. McCLEARY, University of Georgia, Athens, Georgia 30602.
Which Lattice-Ordered Groups are Isomorphic to some A(S) ?

Every ℓ - group G is isomorphic to an ℓ - subgroup of the ℓ - group A(S) of all order-preserving permutations of some totally ordered set S . Here we find necessary and sufficient conditions that G be isomorphic to the entire ℓ - group A(S) for some S . These conditions are most useful when G is itself an ℓ - group of o - permutations of some totally ordered set T . For example, if (G,T) is depressible, or is transitive and complete in A(T) , then G is isomorphic to some A(S) if and only if G consists of all o - permutations of T which respect the orbits of $(G,\overline{T})$. (Received June 16, 1975.)

726-06-14 MARGRET HÖFT, University of Michigan Dearborn, Dearborn Mich 48128, HARTMUT HÖFT, Eastern Michigan University, Ypsilanti Mich 48197.
The fixed point property in partially ordered sets

A partially ordered set has the fixed point property if every order-preserving map $f : P \longrightarrow P$ has a fixed point, i.e. there is $a \in P$ such that $f(a) = a$. Among others, the following theorem is proved.

Theorem: Suppose P has finitely many minimal elements and satisfies the ascending chain condition as well as the descending chain condition. Assume further that for every non-empty set S of minimal elements in P, the supremum of S exists. Then P has the fixed point property.
(Received June 16, 1975.)

*726-06-15 PAUL CONRAD, University of Kansas, Lawrence Kansas 66045. Torsion Radicals of Lattice-ordered Groups.

A torsion class of ℓ-groups is a class T that is closed with respect to convex ℓ-subgroups, ℓ-homomorphic images, and joins of families of convex ℓ-subgroups in T. For a torsion class T and an ℓ-group G the torsion radical T(G) is the join of all the convex ℓ-subgroups of G that belong to T. We study various torsion radicals (for example, hyperarchimedean, property F, finite valued, DCC on convex ℓ-subgroups) and the relationship between them. Most of the useful torsion radicals are completely determined by the lattice of convex ℓ-subgroups of G. The torsion radicals of a laterally complete ℓ-group G are very useful in determining the structure of G. (Received June 17, 1975.)

10 Number Theory

*726-10-1 J. M. Gandhi, Western Illinois University, Macomb, Illinois 61455 Fermat's Last Theorem IV: A New Circulant Condition for the First Case for primes of the Form 6m+1.

We prove Theorem 3. The equation $x^p+y^p+z^p=0$ with $(xyz,p)=1$, where p is an odd prime of the form 6m+1 has no integral solution if $J_{p-1}=(D_2,D_3\ldots D_{p-6},0,0,0,0,0,D_1)\not\equiv 0 \pmod{p^3}$ where J_{p-1} is a circulant and D_i's are given by $D_1=D_{p-6}=1$; $D_2=D_{p-7}=\frac{p-7}{2}$; $D_3=D_{p-8}=\frac{(p-5)(p-7)}{6}$; and $D_i-D_{i-3}=K_i-K_{i-1}$, $3<i<p-9$ and K's are defined in the last paper. Also $D_i=D_{p-i-5}$. Considering p=13, we have $J_{12}=(3,8,11,8,3,1,0,0,0,0,0,1)$ $=10976,000\not\equiv 0 \pmod{13^3}$ and the first case of FLT is verified for p=13. (Received May 12, 1975.)

726-10-2 DEAN R. HICKERSON and HENRY L. ALDER, University of California, Davis, California 95616, AMIN A. MUWAFI, American University of Beirut. Identities Relating the Number of Partitions into an Even and Odd Number of Parts.

If $i\geq 0$ and $n\geq 1$, let $q_i^e(n)$ be the number of partitions of n into an even number of parts, where each part occurs at most i times. Let $q_i^o(n)$ be the number of partitions of n into an odd number of parts, where each part occurs at most i times. If $i\geq 0$, let $q_i^e(0)=1$ and $q_i^o(0)=0$. If $i\geq 0$ and $n=0$, let $\Delta_i(n)=q_i^e(n)-q_i^o(n)$. If s, t, u are positive integers with s odd and $1\leq s<t$ and n is an integer, let $f_{s,t,u}(n)$ be the number of partitions of n in which each odd part occurs at most once and is $\not\equiv\pm s \pmod{2t}$ and in which each even part is divisible by 2t and occurs $<$ u times. Theorem. If s, t, u are positive integers with s odd and $1\leq s<t$, and n is an integer, then

$$\Delta_{2tu-1}(n)=(-1)^n\sum_j f_{s,t,u}(n-tj^2-(t-s)j).$$

This generalizes the well-known case t = u = 1 proved by Euler, for which the right hand side is $(-1)^j$ if $n=(3j^2\pm j)/2$ for some $j=0,1,2,\cdots$ and 0 otherwise. (Received May 16, 1975.)

*726-10-3 CARL POMERANCE, University of Georgia, Athens, Georgia 30602, On composite n for which $\varphi(n)|n-1$, II.

The problem of whether there exists a composite n for which $\varphi(n)|n-1$ (φ is Euler's function) was first posed by D. H. Lehmer in 1932 and still remains unsolved. In this paper we prove that the number of such n not exceeding x is $O(x^{\frac{1}{2}+\epsilon})$ for every $\epsilon > 0$. The proof is elementary, but fairly intricate and with a combinatorial flavor. The result has an appropriate generalization to integers n for which $\varphi(n)|n-a$, for an arbitrary integer a. The results here better a previous estimate by the author (On composite n for which $\varphi(n)|n-1$, to appear in Acta Arith.) in which the bound $O(x^{(2/3)+\epsilon})$ is obtained. (Received June 9, 1975.)

*726-10-4 VERMA, SADANAND, University of Nevada, Las Vegas, Nevada 89154. On a Generalization of a Prime Generating Function, Preliminary report.

Attempts to invent formulas that produce only prime numbers have been fashionable for quite some time. After Mills produced a striking prime generating function $[\theta^{3^n}]$ in 1947 that takes on prime values only for each positive integral values of n, several other investigators were inspired by the result and came up with several other prime-generating formulas. Mills' generating function does not produce all the primes and much less is known about the real number θ. Later, in 1952, Sierpinski produced a formula that actually represents the n^{th} prime in terms of the integral part function $[x]$ and a quite explicit real number θ. Among other investigators in this area are: Ansari, Bang, Knipers, Niven, Srinivasan, Williams, Wright. The attempt of this presentation is to explore a generalization of a prime generating function. As a result, the following astounding and theoretically most powerful looking theorem precipitates: For each positive integer $k \geq 2$, there is a real number θ_k $(0 < \theta_k < 1)$ such that $f(n,k) = [10^{k^n}\theta_k] - 10^{(k-1)k^{n-1}}[10^{k^{n-1}}\theta_k]$ is the n^{th} prime for every positive integer n. (Received June 11, 1975.) (Author introduced by Mr. L. J. Simonoff.)

726-10-5 GARY MILLER, University of California, Berkeley, California 94720. Riemann's Hypothesis and Tests for Primality.

Two classic computational problems are 1) finding a quick method for deciding if a given integer is prime or composite, 2) finding a quick method for computing the prime factorization of an integer. Towards a solution of the first problem we obtain the following two results:

A) There exists a method for testing primality of an integer n (deciding whether n is prime or composite) that runs in $O(n^{1/7})$ steps.

B) Assuming the Extended Riemann's Hypothesis, there exists a method for testing primality of an integer n that runs in $O(\log^4 n \operatorname{logloglog} n)$ steps.

Using the techniques developed for A) and B) we show that a class of functions are computationally equivalent to the problem of computing the prime factorization of an integer, assuming the Extended Riemann's Hypothesis. In particular, if the Euler phi function can be computed in $O(f(n))$ steps, then we can factor an integer in $O(f(n)+\log^5 n)$ steps on the ERH. (Received June 16, 1975.)

*726-10-6 MELVYN B. NATHANSON, The Institute for Advanced Study, Princeton, New Jersy 08540. Polynomial Pell's equations.

Let d be an integer. The polynomial Pell's equation $P(x)^2 - (x^2 + d)Q(x)^2 = 1$, where

$P(x)$ and $Q(x)$ are polynomials with integer coefficients, has nonconstant solutions only for $d = \pm 1, \pm 2$, and in these four cases all solutions are determined.
(Received June 16, 1975.)

726-10-7 M. BUXTON, Ball State U., Muncie, Indiana 47303 and B. STUBBLEFIELD, NOAA, Boulder, Colorado 80302. On Odd Perfect Numbers. Preliminary report.

Stubblefield proved first that there is no odd perfect number less than 10^{50}, the Notices, 20(1973), A-61. He has extended this result, replacing 10^{50} by 10^{100}. The same techniques are now used to further extend these results, replacing 10^{100} by 10^{150} using the following additional proposition.

PROPOSITION: Suppose

(1) that N is an odd perfect number,

(2) for some prime p and natural number e, $p^e || N$,

(3) $\sigma(p^e)$ has a factor $\underset{\sim}{O}$ that is not a perfect square, and

(4) there is no prime factor q of $\underset{\sim}{O}$ and natural number x, where $x \pmod 4 = 1$, such that $q^x || N$.

Then, for some prime factor r of $\underset{\sim}{O}$ it is true that $r \cdot \underset{\sim}{O}$ divides N. In particular, if in addition $\underset{\sim}{O}$ has no prime factor less than its cube root, then $\underset{\sim}{O}^2$ divides N. (Received June 17, 1975.)

726-10-8 BRUCE C. BERNDT, University of Illinois, Urbana, Illinois 61801. G. H. Hardy and Dedekind sums.

Let h and k be coprime, positive integers, and let $s(h,k)$ denote the ordinary Dedekind sum. It appears to have been forgotten that G. H. Hardy was the first person to give a proof of the reciprocity theorem for Dedekind sums that is independent of the transformation formulae of the Dedekind eta-function. We give a variant of Hardy's proof. A new representative for Dedekind sums is also obtained. If we let $\theta = h/k$, this representation may be extended to define a function $f(\theta)$ for all real θ. It is shown that f is continuous for all irrational θ and discontinuous for all rational θ. (Received June 16, 1975.)

12 Algebraic Number Theory, Field Theory and Polynomials

*726-12-1 CLARK KIMBERLING, University of Evansville, Evansville, Indiana 47702. Semipermutability of Chebyshev polynomials of the second kind.

Let $\{t_n\}_{n=0}$ and $\{u_n\}_{n=0}$ be the sequences of Chebyshev polynomials of the first and second kinds, respectively. Put $\bar{u}_{-1}(x) \equiv 0$ and $\bar{u}_n(x) = u_n(x)\sqrt{1-x^2}$ for $n \geqslant 0$. Then for nonnegative m and n, $t_m(t_n) = t_{mn}$, $\bar{u}_m(t_n) = \bar{u}_{mn+n-1}$,

$$t_m(\bar{u}_n) = \begin{cases} (-1)^{m/2} t_{mn+m}, & \text{even } m \\ (-1)^{(m-1)/2} \bar{u}_{mn+m-1}, & \text{odd } m, \end{cases} \quad \text{and} \quad \bar{u}_m(\bar{u}_n) = \begin{cases} (-1)^{m/2} t_{(m+1)(n+1)}, & \text{even } m \\ (-1)^{(m-1)/2} \bar{u}_{mn+m+n}, & \text{odd } m. \end{cases}$$

The first of these four identities underlies certain well known results on semi-permutable chains of polynomials (e.g., pp. 215-218 of Kuczma, Functional Equations in a Single Variable, Polska Akademia Nauk, Warszawa, 1968) and on polynomials which are permutable with Chebyshev polynomials of the first kind (e.g., pp. 160-164 of Rivlin, The Chebyshev Polynomials, Wiley, 1974.) The present work includes analogous results for the functions $\bar{u}_n$. Also, identities for composites of Lucas and Fibonacci polynomials are found.
(Received June 9, 1975.)

13 Commutative Rings and Algebras

726-13-1 MELVIN HOCHSTER, Purdue University, West Lafayette, Indiana 47907. Cohen-Macaulay and Gorenstein rings of invariants.

Let G be a linearly reductive algebraic group over a field K acting on a regular Noetherian K-algebra S. A number of results have been obtained recently concerning whether the ring of invariants $R = S^G$ is Cohen-Macaulay and, if so, whether R is also Gorenstein. In the case where R is Cohen-Macaulay and not Gorenstein it becomes of interest to ask for a good characterization of the canonical module. The talk will present the current state of knowledge of these questions. (Received May 12, 1975.)

726-13-2 DOUG COSTA, U. of Virginia, Charlotte, Va. 22903, JOE L. MOTT, Florida State Univ. Tallahassee, Fl. 32306, M. ZAFRULLAH, Univ. of Manchester Institute of Sci. and Tech., Manchester, United Kingdom. The $D + XD_S[X]$ Construction.

Suppose that D is a commutative integral domain and that S is a multiplicative system in D. Let $T^1 = D + XD_S[X]$ and $T = D + XK[X]$, where K is the quotient field of D. A domain is a GCD-domain if its group of divisibility is lattice ordered. T^1 is a GCD-domain if and only if D is a GCD-domain and GCD(d,X) exists for each $d \in D$. But T^1 is not Prüfer except when $D_S = K$ and D is Prüfer. An analysis of the Krull dimemsion and the prime filter dimension of T^1 lead to a counterexample to a conjecture of Sheldon [Can J. Math. 26(1974), 98-107]. Many properties (for example, GCD, coherence, elementary division domain, and v-domain and Prüfer v-multiplicative) hold for T if and only if these properties hold for D. (Received June 9, 1975.)

14 Algebraic Geometry

*726-14-1 ROBERT MORRIS, Institute for Advanced Study, Princeton, N. J. 08540
Embedding finite group schemes in p-divisible groups.

The points of order p^n on an abelian variety over a field k of char. p form a p-torsion finite commutative group scheme. If k is perfect,results of Oort and Manin show that this torsion can be arbitrarily unpleasant in the sense that any such group scheme, G, can lie on an abelian variety: Oort showed G lies in a p-divisible group Γ and Manin showed that any such Γ lies on an abelian variety. The former remains true if k is imperfect,but two difficulties arise: The embedding of the "local-local" part of G requires the use of non-perfect Dieudonne Theory,and the existence of non-split extensions of unipotent by multiplicative groups (which cannot occur if k is perfect) requires the study of Ext^2 for p-divisible groups. (Received May 22, 1975.)

*726-14-2 TAKASHI ONO, The Johns Hopkins University, Baltimore, Maryland 21218. Algebraic groups and diophantine equations.

Let (G,X) be a special homogeneous space of Witt type defined over Q in the sense of my paper (A mean value theorem in adele geometry, J. of Math. Soc., Japan, 20(1968), 275-288). We have then the mean value theorem of type:

$$(*) \qquad \tau(G)^{-1} \int_{G_A/G_Q} \sum_{\xi \in X_Q} \phi(\sigma\xi)d(G_A/G_Q) = \rho(G,X) \int_{X_A} \phi(x)dX_A.$$

When G = orthogonal group and X = sphere, (*) is the Siegel's theorem on quadratic forms. In many cases, one derives from (*) explicit formulas which give the number of representations of integers by integral quadratic forms. In this talk, we take other examples of (G,X) including the one related to the Hopf map from sphere to sphere and discuss the similar problems.
(Received May 22, 1975.)

*726-14-3 ANDY R. MAGID, Department of Mathematics, The University of Oklahoma, Norman, Oklahoma 73069. Analytic Left Algebraic Groups.

A left algebraic group is an affine algebraic variety over an algebraically closed field whose set of points is an (abstract) group such that the left multiplication by any element is a morphism of varieties. The Lie algebra of a left algebraic group is the Lie algebra of all right invariant tangent vector fields on the group. A complex analytic left algebraic group is a complex analytic group which carries a structure of algebraic variety so that the group structure is left algebraic. The Lie algebra as left algebraic group of a complex analytic left algebraic group is shown to be the Lie algebra of an algebraic subgroup. This can be regarded as a characterization of affine complex algebraic groups: such a group is a complex analytic left algebraic group whose Lie algebras as analytic and left algebraic groups coincide. Some consequences of this characterization are given. (Received June 2, 1975.)

726-14-4 T. KAMBAYASHI, Northern Illinois University, DeKalb, IL 60115. Remarks on the action of G_a on affine schemes (after Miyanishi).

Let k be the ground field, and suppose given an action of the one-dimensional vector k-group G_a on $X = \mathrm{Spec}_k A$. The situation is interpreted as equivalent to having an infinite iterative higher derivation $\mathcal{D} = (D_0, D_1, \ldots, D_n, \ldots)$ on A such that, for each $a \in A$, $D_i(a) = 0$ for all $i \gg 0$. Using the interpretation, M. Miyanishi was able to describe all G_a-actions on the affine plane (Nagoya Math. J., 41(1971), 97-100). As another application of the same idea, one can prove the equivalence of the following conditions: (a) the action of G_a on X is free and the geometric quotient X/G_a exists; (b) there exists an element $f \in A$ such that $D_1(f) = 1$ and $D_i(f) = 0$ for all $i > 1$. (Cf. M. Miyanishi, J. Math. Kyoto Univ., 11(1971), 399-414; Unipotent Algebraic Groups, Vol. 414 (1974), Springer Lecture Notes in Mathematics.) There are other applications as well, both actual ones and potential ones. Among other things, an important possible application to the Jacobian problem on polynomial endomorphisms of C^2 will be discussed. (Received June 5, 1975.)

726-14-5 JOEL L. ROBERTS, University of Minnesota, Minneapolis, Minnesota 55455. Rings of invariants and rational singularities. Preliminary report.

Let K be a field of characteristic 0, and let G be a reductive linearly algebraic group over K which acts K-rationally on a regular K-algebra R, with ring of invariants R^G. We discuss the problem of whether $X = \mathrm{Spec}\,(R^G)$ has a rational desingularization.

Rationality of the singularities of determinantal loci has applications to the problem of finding explicit minimal resolutions of determinantal ideals. Results of this type will be outlined briefly. (Received June 16, 1975.)

726-14-6 DAVID MUMFORD, Science Center, Harvard University, Cambridge, Massachusetts 02138. Algebraic cycles on algebraic varieties.

• We review the facts about algebraic cycles of codimension one and the counterexamples which show the difficulties in extending these results to higher dimensions. We discuss some recent progress made by Spencer Bloch and suggest some areas in which further progress may be possible. (Received June 17, 1975.)

15 Linear and Multilinear Algebra; Matrix Theory (finite and infinite)

726-15-1 MARCUS, MARVIN, University of California, Santa Barbara, California 93106
Bilinear Functionals on the Grassmann Manifold

Let V be an n-dimensional unitary space with inner product (x,y) and let $A: V \to V$ be an hermitian operator with eigenvalues $\lambda_1 \geq \cdots \geq \lambda_n$. For each m, $1 \leq m \leq n$, let $\overset{m}{\wedge} V$ be the m^{th} exterior space over V and let G_m denote the Grassmann manifold, i.e., all decomposable elements of $\overset{m}{\wedge} V$ of norm 1. The map $C_m(A): \overset{m}{\wedge} V \to \overset{m}{\wedge} V$ is the induced compound of A. This paper is concerned with the structure of the set $\mathcal{P}_m(A)$ consisting of all complex numbers $(C_m(A)x^\wedge, y^\wedge)$ where $x^\wedge, y^\wedge$ vary over all orthonormal pairs in G_m. The first result states that $\mathcal{P}_m(A)$ is a closed disc centered at the origin. If $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of A then the maximum and minimum eigenvalues of $C_m(A)$ are respectively λ_α and λ_β where both $\lambda_\alpha = \lambda_{\alpha_1} \cdots \lambda_{\alpha_m}$ and $\lambda_\beta = \lambda_{\beta_1} \cdots \lambda_{\beta_m}$ are products taken m at a time of the λ_i, $i = 1, \ldots, n$. The radius of $\mathcal{P}_m(A)$ is always at most $\rho = \frac{1}{2}(\lambda_\alpha - \lambda_\beta)$. If $1 \leq m \leq n-2$ then $\mathcal{P}_m(A)$ has radius ρ iff α and β agree in $m-1$ places. For hermitian matrices A this implies that the values of a non-principal subdeterminant of U^*AU, as U runs over unitary matrices fills up a disc whose radius depends on the position of the subdeterminant. (Received April 1, 1975.)

*726-15-2 MOSHE GOLDBERG, University of California, Los Angeles, California 90024
On the characterization of spectral matrices

We give several characterizations of spectral matrices, i.e., matrices whose spectral and numerical radii are equal. Some of these characterizations depend upon the structure of the matrix, and some are critical-power-characterizations. In particular we show that an n-square matrix A is spectral if and only if $r(A^n) = r^n(A)$, where r is the numerical radius. Here n is a critical power in the sense that in general $m = n$ is the least power for which an equality of the form $r(A^m) = r^m(A)$ implies spectrality. In order to demonstrate this assertion we study some techniques for the computation of the numerical radius of positive matrices. (Received April 7, 1975.)

*726-15-3 ROBERT B. FEINBERG, National Bureau of Standards, Washington, D.C. 20234.
Similarity of Partitioned Matrices.

Suppose that A, B, and T are matrices of order r x r, s x s, and r x s respectively over a field F. We prove that $\begin{bmatrix} A & T \\ 0 & B \end{bmatrix}$ is similar to $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ iff AX - XB = T, for some matrix X. We also give some corollaries and a simple generalization. (Received May 22, 1975.)

726-15-4 C. S. BALLANTINE, Oregon State University, Corvallis, Oregon 97331.
Conjunctivity and the numerical range.

In this note are surveyed properties of the classical numerical range $W(S)$ of a square complex matrix S that depend only on the conjunctivity class of S. Generally (and roughly) speaking, one may say that these properties fall under the heading of "local first-order behavior of $W(S)$ at zero". They include such properties as whether or not (1) 0 is an interior point of $W(S)$, (2) 0 is a boundary point of $W(S)$, (3) 0 is an extreme point of $W(S)$, (4) 0 is a bare point of $W(S)$, and (5) 0 is a vertex point of $W(S)$.
(Received May 23, 1975.)

*726-15-5 DONALD W. ROBINSON, Brigham Young University, Provo, UT 84602. Lower bounds on the dimension of the spaces of matrices transposed and skew-transposed by a similarity transformation.

Let B be a real, n-by-n, invertable matrix with transpose B'. Sharp lower bounds for the dimensions of the spaces $\{A : A' = B^{-1} A B\}$ and $\{A : A' = -B^{-1} A B\}$ are determined as functions of n and the ranks of the symmetric $(B + B')/2$ and skew-symmetric $(B - B')/2$ parts of B. In particular, it is shown that these dimensions are at least the greatest integers in $(n + 1)/2$ and $n/2$, respectively. (Received June 2, 1975.)

726-15-6 KENNETH A. BYRD and THERESA P. VAUGHAN, University of North Carolina at Greensboro Greensboro, North Carolina, 27412. Orthogonal and Unitary Circulants, Preliminary report.

The original motivation for this work was the problem of finding the orders of the groups of $n \times n$ orthogonal and unitary circulant matrices over a finite field. We have been able to restate and solve this problem in more general settings.

Here, however, we consider the problem from a matrix-theoretic point of view and work explicitly with finite fields. The problem is reduced to finding solutions of families of bilinear equations; after this is done the procedure is straightforward. The coefficient matrices of these families of equations are themselves of some interest, and we discuss some of their properties.

This method lends itself to generalization in several different directions, and we plan to report further on this at a later date. (Received June 6, 1975.)

*726-15-7 GERNOT M. ENGEL, IBM, Owego, NY 13827 and HANS SCHNEIDER, University of Wisconsin, Madison, WI 53706. The Hadamard-Fischer inequalities for a class of matrices defined by eigenvalue monotonicity.

For $A \in \mathbb{C}^{nn}$ and $\phi \subset \mu \subseteq \langle n\rangle = \{1,\dots,n\}$ let $A[\mu] = (a_{ij})$, $i,j \in \mu$ and $A(\mu) = (a_{ij})$, $i,j \in \langle n\rangle \backslash \mu$. We define a subset $\omega_{\langle n\rangle}$ of $\mathbb{C}^{nn}$ by $A \in \omega_{\langle n\rangle}$ if

(1) $\operatorname{Spec} A[\mu] \cap \mathbb{R} \neq \phi$, for $\phi \subset \mu \subseteq \langle n\rangle$

(2) $\ell(A[\mu]) \subseteq \ell(A[\nu])$, if $\phi \subset \nu \subseteq \mu \subseteq \langle n\rangle$

where $\ell(A[\mu]) = \min(\operatorname{Spec} A[\mu] \cap \mathbb{R})$. For $A, B \in \omega_{\langle n\rangle}$ define $A \leq_\tau B$ by

$\ell(A[\mu]) \subseteq \ell(B[\mu])$, for $\phi \subset \mu \subseteq \langle n\rangle$.

By definition, $A \in \tau_{\langle n\rangle}$ if $A \in \omega_{\langle n\rangle}$ and $0 \leq_\tau A$. For $0 \leq_\tau A \leq_\tau B$ $(A, B \in \omega_{\langle n\rangle})$ it is shown that

(3) $0 \leq \det A \leq \det B - \det(B - \ell(A)I) \leq \det B$.

For $A \in \tau_{\langle n\rangle}$, $A \leq_\tau A[\mu] \oplus A(\mu)$, and hence we obtain the Hadamard-Fischer inequality

(4) $0 \leq \det A \leq \det A[\mu] \det A(\mu)$

for the class $\tau_{\langle n\rangle}$ which includes the positive semi-definite, totally non-negative and M-matrices. Cases of equality in (3) are treated in detail and are related to the cyclic structure of A and B. (Received June 9, 1975.)

*726-15-8 B. DAVID SAUNDERS, University of Wisconsin, Madison, WI 53706. Convexity of the norm-numerical range. Preliminary report.

Let $\mathcal{H}$ denote the norm-Hermitian operators on a finite-dimensional Banach space. The numerical range of an operator A is convex if A is in $\mathcal{J} = \mathcal{H} + i\mathcal{H}$. This is a Generalization of the Hausdorff-Toeplitz theorem.
(Received June 9, 1975.)

726-15-9 R. J. DUFFIN and T. D. MORLEY*, Carnegie-Mellon University, Pittsburgh, Pa. 15213. A Vectorial Generalization of Kirchhoff's Laws

We propose certain analogs of Kirchhoff's laws in which the currents and voltages become vectors, and the "resistors" become certain types of linear operators (usually finite dimensional, but no assumption of invertability is made). This gives rise to matrix operations with interesting properties. For example, if $\Phi(Z_1,\dots,Z_n)$ is the "resistance" obtained by connecting together the Hermitian semi-definite resistors $Z_1,\dots,Z_n$, then $\|\Phi(Z_1,\dots,Z_n)\| \le \Phi(\|Z_1\|,\dots,\|Z_n\|)$. These operations are special cases of the operations studied by Anderson, Duffin, and Trapp (SIAM J. Control 13 (1975), 446-461), but in this special case one can relate the graphical structure of the connection to the properties of the operation. (Received June 9, 1975.)

*726-15-10 DANIEL J. RICHMAN and HANS SCHNEIDER, University of Wisconsin, Madison, WI 53706. The relationship of the graph of a singular M-matrix to its Weyr characteristic.

Let A be an $(n \times n)$ singular M-matrix in standard lower block triangular form with diagonal blocks A_{ii} irreducible. Let Γ be the directed graph given by $i \to j$ if $A_{ij} \neq 0$, $(i \neq j)$, and let S be the subgraph $\{i \in \Gamma : A_{ii}$ is singular$\}$. Let Λ_1 be the set of minimal elements of S, and define the k-th level Λ_k, $k = 1, 2, \dots$ inductively as the set of minimal elements of $S \backslash (\Lambda_1 \cup \dots \cup \Lambda_{k-1})$, let λ_k be the number of elements in Λ_k. The Weyr characteristic of 0 for A is defined to be $\omega(A) = (\omega_1, \omega_2, \dots)$ where $\omega_1 + \dots + \omega_k = \dim \ker A^k$, $k = 1, 2, \dots$. Using a special type of non-negative basis for the generalized eigenspace $E(A)$ of A, we associate a certain rectangular matrix D_k with Λ_k. We show that $\omega(A) = (\lambda_1, \lambda_2, \dots)$ if and only if the matrices D_k, $k = 1, 2, \dots$ have full column rank. In this case there exists a Jordan basis for $E(A)$ of non-negative vectors. The conditions for $\omega(A)$ is satisfied when S is an inverted rooted tree. (Received June 16, 1975.)

*726-15-11 V. DLAB, Université de Paris, and C.M. RINGEL, Universität Bonn. Classification of real linear transformations between two complex vector spaces.

Using some recently developed functorial methods, one can establish the following classification. Theorem. A non-zero indecomposable real linear transformation between two complex vector spaces can be brought, by a suitable choice of bases, to one of the following types:

$$\text{(i)} \begin{bmatrix} E_\infty & & & 0 \\ E_o & E_\infty & & \\ & E_o & \ddots & \\ & & \ddots & E_\infty \\ 0 & & & E_o \end{bmatrix}, \quad \text{(ii)} \begin{bmatrix} E_a & E_\infty & & 0 \\ & E_a & E_\infty & \\ & & \ddots & \ddots \\ & & & E_\infty \\ 0 & & & E_a \end{bmatrix}, \quad \text{(iii)} \begin{bmatrix} F_{bc} & F_\infty & & 0 \\ & F_{bc} & F_\infty & \\ & & \ddots & \ddots \\ & & & F_\infty \\ 0 & & & F_{bc} \end{bmatrix}, \quad \text{and}$$

the transpose of (i), where $|a| \le 1$, and either $c > o$, or $c = o$ and $b < o$. Here, $E_a = \begin{bmatrix} 1 & o \\ o & a \end{bmatrix}$, $E_\infty = \begin{bmatrix} o & o \\ o & 1 \end{bmatrix}$ are real 2x2 matrices and $F_\infty = \begin{bmatrix} 0 & 0 \\ 0 & E \end{bmatrix}$, $F_{bc} = \begin{bmatrix} E_1 & E_{bc} \\ E_{-1} & E_1 \end{bmatrix}$ with $E_{bc} = \begin{bmatrix} b & c \\ c & -b \end{bmatrix}$ are real 4x4 matrices. Every real linear transformation between two complex vector spaces is a product (unique up to the order of factors) of the transformations of these types and a zero transformation. (Received June 6, 1975.)

16 Associative Rings and Algebras

726-16-1 ANNE B. KOEHLER, Miami University, Oxford, Ohio 45056. Pre-self-injective duo rings, Preliminary Report.

A ring R is duo if every one sided ideal is two sided, and it is pre-self-injective if every proper homomorphic image of R is self-injective. A characterization of commutative pre-self-injective rings by Klatt and Levy [Trans. Amer. Math. Soc. 137 (1969), 407-419] is extended to duo rings. One theorem is that when R is a duo domain, R is pre-self-injective iff 1. every proper ideal is contained in only finitely many maximal ideals, and 2. for every maximal ideal M in R and every proper ideal A_M in the localization R_M, R_M/A_M is a maximal rank 0 duo valuation ring. These rings are a special case of the PCQI-rings (proper cyclics are quasi-injective) studied by Jain, Singh, and Symonds, but they did not completely characterize the domain case. (Received June 16, 1975.)

17 Nonassociative Rings and Algebras

726-17-1 CARLOS A. INFANTOZZI, Universidad de la República, Atlántico 1514, Montevideo, Uruguay.
"The lattice of the integral-domains, id., of the Cayley-Dickson's integers, CDi."

If we drop the id. L, similar to the Lipschitz's id. for quaternions, then the lattice of the id. of the CDi., $H \subset D_n \subset I_m$, is analogous to the lattice of Fano plane; here $H=(L,w)$ is the id. generated by L and $w=\frac{1}{2}(1+i+j+k+e+f+g+h)$ (the last letters are the units; $k=ij, f=ie, g=je, h=ke$); $D_n=(L,T_n)$, where T_n $(n=1,2\ldots 7)$ is defined by means of the triads ijk, ief, ihg, jeg, jfh, keh and kgf as follows $T_1=\frac{1}{2}(1+i+j+k)$, ... etc., and $I_n=(L,Z_n)$ where $Z_1=\frac{1}{2}(1+i+e+g)$ and $Z_n=S_n(Z_1)$, being $S_1,\ldots S_7$ the substitutions which change (ijkefgh) to (ijkefgh), (jkieghf), (ihgjkfe), (fiehjgk), (kijehfg), (jfhkige) and (iefhgkj) respectively.//Let $I_m=(L,T_{n'},T_{n''})$ be, where $n' \neq n''$ are values of n corresponding to m as follows: n=1,6,7 if m=1, n=1,2,3 if m=2, n=3,4,7 if m=3, n=2,4,6 if m=4, n=1,4,5 if m=5, n=3,5,6 if m=6 and n=2,5,7 if m=7. Then: $L \subset H \subset D_n \subset I_m$. The I_m are the unique maximal id. of CDi. There are no other id. different from L, H, D_n, I_m. The I_m are isomorphic among themselves. The D_n as well. The automorphisms of an I_m subordinate isomorphisms among its subsets D_n. Only I_1, I_2 and I_5 contain the Hurwitz's integer quaternions.//Putting $M'=w-M$, $Y_1=\frac{1}{2}(1+i+f+h)$ and $Y_n=S_n(Y_1)$, $X_1=\frac{1}{2}(1+j+e+f)$ and $X_n=S_n(X_1)$, $U_1=\frac{1}{2}(1+j+g+h)$ and $U_n=S_n(U_1)$ we have: $D_n=(L,T'_n)$, $I_n=(L,Y_n)=(L,X_n)=(L,U_n)=(L,Z'_m)=(L,Y'_m)=(L,X'_n)=(L,U'_n)$, $I_n=(L,T'_{n'},T_{n''})=(L,T_{n'},T'_{n''})=(L,T'_{n'},T'_{n''})$. (Received January 30, 1975.)

726-17-2 W. HAROLD DAVENPORT, University of Petroleum & Minerals, Dhahran, Saudi Arabia.
Malcev ideals in alternative rings. Preliminary report.

In this article, we define the concept of a Malcev ideal in an alternative ring in a manner analogous to Lie ideals in associate rings. Let Z denote the center of a ring R. For a ring R define $[R, R]$ to be the additive subgroup of R generated by all the elements of the form $[x, y] = xy - yx$ where x and y are elements of R. By using a result of Kleinfield's that: a simple alternative ring which is not a nil ring is either a Cayley-Dickson algebra over its center or associative, we can prove the following: Theorem. Let R be a simple non-associative alternative ring of characteristic not 2 or 3. If R is not a nil ring then $R = Z \oplus [R, R]$ as a ring direct sum of the minimal Malcev ideals Z and $[R, R]$.
(Received April 7, 1975.)

18 Category Theory, Homological Algebra

726-18-1 M. V. MIELKE, University of Miami, Coral Gables, Florida 33124
Iterative Processes. Preliminary report.

An iterative process is a pair of functors $R:S \to C$, $\Phi:S \to S^{\Sigma}$, where S is a subcategory of the functor category C^{Σ}. The process applied to an object D_1 of S defines a sequence $\{RD_n\}$ where $D_n = R^{\Sigma}\Phi(D_{n-1})$, $n \geq 2$. The sequence (process) will not terminate if $D_n \in S$ for each $n \geq 1$. This can be guaranteed by putting conditions on D_1 or by extending Σ, S, R, Φ to $\bar{\Sigma}, \bar{S}, \bar{R}, \bar{\Phi}$ with $\bar{R}^{\bar{\Sigma}}\bar{\Phi}(\bar{S}) \subset \bar{S}$. The first approach is used, for example, when D_1 is the simplicial topological space associated to a topological category D and R is topological realization. The process is endless if D is a compactly generated, abelian, group or group bundle. The sequences $\{RD_n\}$ are spectra for various cohomology theories. This paper is concerned with general conditions under which the extentions in the second approach exist and consequently with the construction of "spectra" for topological categories, group bundles and Γ-spaces which are not necessarily compactly generated or abelian. (Received May 30, 1975.)

20 Group Theory and Generalizations

726-20-1 FERDINAND D. VELDKAMP, The Ohio State University, Columbus, Ohio, 43210. University of Utrecht, The Netherlands. Regular elements in anisotropic tori.

Let G be a quasisimple linear algebraic group, σ an endomorphism with finite G_{σ}, T a σ-invariant maximal torus, σ^* the action induced by σ on the real linear space V spanned by the character group of T. Then $\sigma^{*} = q\tau$ with τ an isometry of V w.r.t. the Killing form (cf. R. Steinberg, Endomorphisms of linear algebraic groups, A.M.S. Memoir 80(1968)). An element of T having T as its centralizer in G is called regular. For large enough q, T_σ contains regular elements (T.A. Springer, Proceedings Budapest, 1971). It is shown that in all cases there exist anisotropic T_σ containing regular elements for all q, e.g., the Coxeter and twisted Coxeter tori with two exceptions $({}^1G_2(2), {}^2A_2(2^2))$. The existence of regular characters, i.e., characters whose stabilizer in the Weyl group is 1, can also be proved in these cases. (Received June 2, 1975.)

*726-20-2 BRIAN PARSHALL, University of Virginia, Charlottesville, Va. 22903
Simple subgroups of simple algebraic groups.

Let G be a simple algebraic group over an algebraically closed field K. We will discuss some recent theorems concerning the conjugacy in G of certain isomorphism classes of simple closed subgroups. The first of these results treats the case when the root systems of the subgroups are isomorphic to one obtained from the fixed-points of a graph isometry of the root system of G; these extend to general characteristic known theorems in the complex case. Next let k be a subfield of K and suppose G is defined over k. The second of our results concerns the conjugacy of maximal k-split simple subgroups. For example, when k is local we discuss the number of G_k-conjugacy classes of the maximal k-split subgroups of Borel and Tits [Publ. Math. I.H.E.S. no. 27 (1965), 55-151]. The approach here is cohomological and it uses earlier results of the author on the centralizer of unipotent elements in G [J. Algebra 35 (1975), to appear]. (Received June 2, 1975.)

726-20-3 EVERETT C. DADE, University of Illinois, Urbana, Illinois 61801. Nearly trivial outer automorphisms of finite groups.

• Outer automorphisms α of a finite group G are rather hard objects to grasp. They only form a factor group Out(G) of the full automorphism group of G. So each individual α is a whole coset of the inner automorphism subgroup. Generally it is difficult to select significant automorphisms inside this coset. For example, it may be impossible to find a trivial automorphism in α (and thus show α to be trivial in Out(G)) starting just from local conditions of the form that certain subgroups or sections of G are each centralized by some element of α. Experience has shown that such local conditions, while not sufficient to guarantee the triviality of α, very often force α to lie in a normal subgroup of Out(G) of a fairly restricted sort (e.g., an abelian normal subgroup or a nilpotent one). We shall discuss several different types of local conditions and the restrictions they are known or conjectured to place on the normal subgroup of Out(G) generated by the α satisfying them. (Received June 2, 1975.)

726-20-4 J.E. HUMPHREYS, University of Massachusetts, Amherst, MA 01002.
A family of indecomposable modules. Preliminary report.

A family of indecomposable modules for a semisimple algebraic group G over an algebraically closed field of characteristic $p > 0$ has been constructed by D.-N. Verma and the author [Bull. Amer. Math. Soc. 79 (1973), 467-468] (proofs to appear in the author's forthcoming paper, Ordinary and modular representations of Chevalley groups). These occur as summands in tensor product modules $M_\mu \otimes St_n$, involving the highest weight, where M_μ is an irreducible G-module of highest weight μ and St_n is the Steinberg module $M_{(p^n-1)\delta}$ (δ = half sum of positive roots). The question arises of how to characterize intrinsically this family of modules by covering or embedding

properties. When $\mu = (p^n-1)\delta$, this might provide an interesting interpretation of W.J. Haboush's proof of the Mumford conjecture.
(Received June 6, 1975.)

726-20-5 MICHAEL R. STEIN, Northwestern University, Evanston, Illinois 60201, Stability theorems for algebraic K-functors based on Chevalley groups, Preliminary report.

Let Φ_n be a root system of type A_n, B_n, C_n or D_n. Analogous to the functors K_1 and K_2 of algebraic K-theory there are functors $K_1(\Phi_n\ ,\ )$ and $K_2(\Phi_n,\ )$ (ordinary K_1 and K_2 correspond to the case $\Phi_n = A_n$; the other cases include the symplectic functors considered by Vaserstein). The stability theorems of Bass, Vaserstein and Dennis may be generalized to these functors; among the relevant techniques are certain basic representations used by Matsumoto in solving the congruence subgroup problem. Partial results concerning such functors based on the exceptional Chevalley groups are also obtained. (Received June 6, 1975.)

*726-20-6 GREG DOBBINS, Wheaton College, Wheaton, Illinois 60187. Simple Semigroups in Certain Locally Compact Groups.

In this paper, G denotes a locally compact connected solvable group and $\mathcal{F}(G)$ is the collection of open simple subsemigroups F such that bd(F) is a subgroup H of G. Theorem 1: If $F \in \mathcal{F}(G)$ with corresponding boundary subgroup H then $G = F \cup H \cup F^{-1}$. Theorem 2: If $F \in \mathcal{F}(G)$ with boundary subgroup H then G/Core(H) is iseomorphic to RP, the noncommutative group on the half-plane. Corollary 3: Under the above hypothesis, we have the double coset decomposition $G = HxH \cup H \cup Hx^{-1}H$ for $x \notin H$. Theorem 4: Let M be an open subsemigroup of G such that bd(M) is a subgroup. Then M is simple and hence $\mathcal{F}(G) \neq \phi$ if and only if L is not a normal subgroup. Theorem 4 is helpful in identifying examples. Moreover, if M is as in the hypothesis then it is in one of two disjoint classes: normal or simple semigroups.

These results extend those announced in Abstract 701-20-16, these Notices 20(1973), A-90. (Received June 9, 1975.)

*726-20-7 STEPHEN MESKIN, University of Connecticut, Storrs, Connecticut 06268. The isomorphism problem for a class of one-relator groups.

It is shown that two groups of the form

$$\langle c_1, c_2, \ldots, c_t;\ c_1^{n_1} c_2^{n_2} \cdots c_t^{n_t} \rangle,\ t \geq 1,\ n_i \geq 2\ (i = 1,\ldots,t)$$

are isomorphic if and only if the n_i's of one group are a permutation of the n_i's of the other group (the restriction that $n_i \geq 2$ is for convenience - the general result follows easily). Use is made of a result of Gerhard Rosenberger on generating systems for such groups. (Received June 12, 1975.)

726-20-8 DONALD McCARTHY, St. John's University, Jamaica, N.Y. 11439 and ANDREW WOHLGEMUTH and GARY HAGGARD, University of Maine, Orono, Maine 04473 Small Actions Of Generalized Symmetric Groups

In the present context, a generalized symmetric group is a complete monomial group $S_k[C_p]$, i.e. the (permutational) wreath product of a cyclic group C_p by the symmetric group S_k, where p is prime and $k > 1$. A small action of a finite group G is a permutation representation of G in which both the kernel K and the degree n are "small"; when $G = S_k[C_p]$, "small" will mean that $K \subseteq D$, the diagonal of G, and $n < d$ where $d = 2kp$ (except when $k=5$, $p=2$ or $k=6$, p odd where we take $d = 2(k-1)p$). Interest in these matters arose from a problem in graph theory, and from that viewpoint the main result is: Given a small action of $G = S_k[C_p]$ on a set, if $k > 4$ there necessarily exists a subset of size kp on which G acts in a fairly natural manner (in that X can be partitioned into mutually disjoint sets $X_1,\ldots,X_k$ such that the top group S_k permutes the X_i by acting on subscripts in the usual way, the base group B leaves each X_i invariant and the stabilizer of X_i in B acts transitively on X_j for $i \neq j$). The proof involves a close examination of the subgroups having small index and core in G, and employs combinatorial as well as group theoretic arguments. (Received June 16, 1975.)

726-20-9 JOSEPH S. VERRET, Tulane University, New Orleans, Louisiana 70118. Algorithms for Presenting Certain Commutator Subgroups, Abstract.

In many cases the commutator subgroup of a finitely generated group with one relator is free. In the case of the fundamental groups of the compact two manifolds, the commutator subgroup is free on a countably infinite number of generators if the two manifold is not the sphere, torus, projective plane or Klein bottle.

We present two algorithms for determining a presentation of the commutator subgroup of a finitely generated group with one relation. These algorithms give presentations which have a countably infinite number of generators and a countably infinite number of relations. In particular, we have that for $n \geq 1$ and $m > 1$, the commutator subgroup of $\langle a,b\colon [a^n,b^m] = 1\rangle$ is the free product of n isomorphic groups each of which is an infinite free product with amalgamation. (Received June 16, 1975.)

22 Topological Groups, Lie Groups

*726-22-1 DONALD MARXEN, Marquette University, Milwaukee, Wisconsin 53233
Quotients of uniform semigroups

Let S be a semigroup and R a congruence on S. It is well known that if S is topological, the induced operation on S/R need not be continuous with respect to the quotient topology. A pair $(S, \mathcal{U})$ is a uniform semigroup if the operation is uniformly continuous with respect to $\mathcal{U}$. Theorem. If $(S, \mathcal{U})$ is a uniform semigroup then S/R is a uniform semigroup with respect to the quotient uniformity. A necessary and sufficient condition is given for the quotient of a uniform semigroup to be Hausdorff. An explicit formulation of the quotient uniformity is used in obtaining these results.
(Received June 16, 1975.)

26 Real Functions

726-26-1 DANIEL WATERMAN, Syracuse University, Syracuse, New York 13210
On generalized bounded variation. Preliminary report.

A survey is presented of recent results on functions of Λ-bounded variation and Φ-bounded variation. The relationship of these classes to the class of regulated functions is discussed. The convergence and summability of Fourier series of functions of Λ-bounded variation is reviewed. Various open questions concerning the behavior of Λ-BV functions themselves and of their Fourier series are discussed. (Received May 5, 1975.)

*726-26-2 RICHARD J. O'MALLEY, University of Wisconsin, Milwaukee, WI 53201 and CLIFFORD E. WEIL, Michigan State University, East Lansing, MI 48824. The oscillatory behavior of certain derivatives.

The derivatives considered are the approximate derivative and the kth Peano derivative. It is known that if one of these derivatives exists on an interval, then 1) it has the Darboux property, 2) it is a function of Baire class one, 3) if it is bounded above or below on a subinterval, then it is an ordinary derivative, and 4) from 2) and 3) there is an open, dense subset of the interval on which it is an ordinary derivative. The main theorems of the paper are the following: Theorem 1. If the approximate derivative, f'_{ap}, of f exists on an interval and if, for $M \geq 0$, f'_{ap} attains both M and $-M$, then there is an open subinterval where $f'_{ap} = f'$ and on which f' attains both M and $-M$. Theorem 2. If the kth Peano derivative, f_k, of f exists on an interval and if, for $M \geq 0$, f_k attains both M and $-M$, then there is an open subinterval where $f_k = f^{(k)}$ and on which $f^{(k)}$ attains both M and $-M$. (Received June 6, 1975.)

*726-26-3 RICHARD J. FLEISSNER, University of Wisconsin, Milwaukee, Wisconsin 53201. Products of derivatives.

Let A denote the class of real valued functions defined on $[0,1]$ whose product with every derivative is a derivative. A function f is said to be of distant bounded variation at a point x if the Lebesgue-Stieltjes integral of $s(t) = t - x$ with respect to the measure induced by the total variation of f exists and satisfies a Lipschitz condition at x. Theorem. A function f belongs to A if and only if it is of distant bounded variation at each point x in $[0,1]$. Theorem. The multiplier class for approximate derivatives of continuous functions consists of the bounded variation functions whose total variation satisfies a Lipschitz condition at each point of $[0,1]$. (Received May 27, 1975.) (Author introduced by Professor Donald W. Solomon.)

726-26-4 TOGO NISHIURA, Wayne State University, Detroit, Michigan 48202. Interval functions in area theory.

In the study of area of continuous mappings, $f: X \to R^m$, $X \subset R^k$, $k \leq m$, set functions involving the topological index play a natural role. When $k = 2$, the sets can be chosen to be topological two cells. (See Cesari, Surface Area, Princeton University Press, 1956.) In studying the equality of area and the integral of "Jacobians," oriented k-cells are the natural choice of sets. Examples are given to show that, for $k > 2$, topological k-cells are not adequate to give a reasonable area theory. These examples are used to show the nonexistence of Fréchet representations which are absolutely continuous and which are differentiably absolutely continuous. (Received May 27, 1975.)

*726-26-5 RICHARD J. O'MALLEY, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201. Approximate maxima, Preliminary report.

It is well-known that if $f: [0, 1] \to R$ is continuous, then f has an absolute maximum. An analogous, not so simple, property is proven for approximately continuous functions. Applications include a new characterization of convex functions. It is further shown that this property does not extend to approximately continuous functions of several variables.
(Received June 5, 1975.)

*726-26-6 JAMES FORAN, University of Missouri-Kansas City, Kansas City Missouri 64110. On Functions Whose Graph is of Linear Measure 0 on Sets of Measure 0.

Continuous real valued functions whose graph is of linear measure 0 on sets of Lebesgue measure 0 are considered. Such functions are shown to satisfy Lusin's condition (N), to have graphs of σ-finite linear measure, and to include the class of Generalized Absolutely Continuous functions (primitives for the Denjoy integral). The sum, product, and composition of a function from this class with a Lipschitz function is again a member of the class. Examples are given to show that this is not true if Lipschitz is replaced by absolutely continuous. Here, use is made of the function Φ defined by Mazurkiewicz (Fund. Math. 16 (1930) p. 348-352) which satisfies Lusin's condition (N) while $\Phi(x) + mx$ does not satisfy condition (N) for every $m \neq 0$.
(Received June 11, 1975.)

726-26-7 MICHAEL J. EVANS and PAUL D. HUMKE, Western Illinois University, Macomb, Illinois 61455. Directional cluster sets and essential directional cluster sets of real functions defined in the upper half plane. Preliminary report.

Let f be a real valued function defined in the open upper half plane. Let x represent a point on the x axis and θ_1, θ_2 be two directions, i.e., angles strictly between 0 and π. The following relationship is established between $C_e(f,x,\theta_1)$, the essential cluster set of f at x along the ray terminating at x in the direction θ_1, and $C(f,x,\theta_2)$, the cluster set of f at x along the ray terminating at x in the direction θ_2. Theorem. If f is measurable, then $C_e(f,x,\theta_1) \subseteq C(f,x,\theta_2)$ for almost every x. If, in addition, f is continuous, then the exceptional set is also of first category. Several examples are presented showing the sharpness of the result, and applications are made to various types of derivates of a real valued function of one real variable. (Received June 13, 1975.)

726-26-8 CHENG-MING LEE, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201. On functions with summable approximate Peano derivative. Preliminary report.

For an integer $n \geq 1$, let the n^{th} approximate Peano derivative of a function F at x, if it exists, be denoted as $F_{(n)}(x)$. For $n = 0$, the existence of $F_{(n)}(x)$ will simply mean that the function $F\ (\equiv F_{(0)})$ is approximately continuous at x. Then the following theorem is proved. THEOREM: Let $n \geq 1$ and suppose that $F_{(n-1)}(x)$ exists for all x in a closed interval I. If $F_{(n)}(x)$ exists for "nearly" every x in I and is summable in I, then $F_{(n-1)}$ is absolutely continuous in I. This includes as a particular case the well-known theorem that a function whose ordinary derivative exists everywhere and is summable is absolutely continuous. A short and transparent proof of this particular case can be found in a recent note by Goffman in AMS Monthly (1971). The proof of the general theorem to be given is short but not as transparent since two recent deep monotonicity theorems are involved, one for the case $n = 1$ and another for $n > 1$. (Received June 17, 1975.)

726-26-9 ROBERTO MACCHIA, Stevens Institute of Technology, Hoboken, New Jersey, 07030. On roots of differentiable functions. Preliminary report.

Theorem: Let $f \in C^{n+rp+v}[0,h)$ $(h>0)$, where $n \geq 0, r>0, p>0$ and $v \geq 0$ are integers such that $0 \leq v < r$, $f^{(j)}(0)=0$ for $0 \leq j \leq rp+v-1$ and $f^{(rp+v)}(0)>0$. Let $Y=\sqrt[r]{f}$, $m=[1-v]^+$; then there exist $\delta>0$ and $K>0$ such that: (i) if $r=2$, then $Y \in C^{nm+p}[0,\delta)$, $Y^{(j)}(0)=0$ for $0 \leq j \leq p-1$, $Y^{(p)}(0)=mK$ and, in general, $Y \notin C^{nm+p+1}[0,\delta)$. (ii) if $r>2, p=1$, then $Y \in C^{nm+1}[0,\delta)$, $Y^{(1)}(0)=mK$ and, in general, $Y \notin C^{nm+2}[0,\delta)$. Corollary: Let $f \in C^{n+rp+v}(-h,h)$, $h>0$; (i) if $r=2$, p is even and $v \geq 0$ is even, then $Y \in C^{nm+p}(-\delta,\delta)$. (ii) if $r>2$ is odd, $p=1$ and $v \geq 0$ is even, then $Y \in C^{nm+1}(-\delta,\delta)$. All the other properties in the conclusion of the above theorem still hold in the respective cases. Remark: This corollary complements earlier results by J.Dieudonné (J.Analyse Math. Jerusalem, XXIII, pp. 85-88) and G.Glaeser (Ann.Inst.Fourier, 13, pp.203-207) on roots of differentiable function under weaker hypotheses. It is also related to our result announced in Abstract 75T-H3, p.A-483, the statement of which has been somewhat improved. (Received June 17, 1975.)

28 Measure and Integration

*726-28-1 KIM E. MICHENER, Wayne State University, Detroit, Michigan 48202. A weak convergence theorem for non-parametric, area-type functionals.

The main purpose of this work is to obtain a generalization of a theorem of L. C. Young (Duke

Math. J. 11, 1944, 43-57) for the area of a non-parametric, continuous surface to the Lebesgue area of non-parametric L_1-surfaces.

Let $z = f(x,y)$ where f is an L_1-function on $Q = (0,1) \times (0,1)$ having finite Lebesgue area. Then f has a distribution derivative which is a finite (Borel) vector measure $\bar{\sigma} = (\sigma_1,\sigma_2)$. If in fact f is A.C. we have that the area is given by: $\iint_Q \left(1 + \left|\frac{d\bar{\sigma}}{dL}(x,y)\right|^2\right)^{\frac{1}{2}} dydx$, where $\frac{d\bar{\sigma}}{dL}$ is the derivative of $\bar{\sigma}$ with respect to Lebesgue measure L. Similar to Young's approach we replace the derivative $\frac{d\bar{\sigma}}{dL}$ with the "difference quotient" $\Delta_{h,k}\bar{\sigma}(x,y) = (h^{-2}\sigma_1([x,x+h] \times [y,y+h]),$ $k^{-2}\sigma_2([x,x+k] \times [y,y+k]))$ and write $A_{h,k}(f) = \iint_Q \left(1 + |\Delta_{h,k}\bar{\sigma}(x,y)|^2\right)^{\frac{1}{2}} dydx$. We then obtain necessary and sufficient conditions for $\lim_{h,k\to 0,0} A_{h,k}(f) = \text{Area}$. If $\sigma_i = \alpha_i + \beta_i$ is the Lebesgue decomposition of σ_i with respect to L with $\alpha_i \ll L$ and $\beta_i \perp L$; then the condition is that $\beta_1 \perp \beta_2$. Weak convergence theorems for a wider class of functionals are also obtained.
(Received May 27, 1975.)

726-28-2 T. J. MORRISON, Kent State Univ., Kent, Ohio. The Radon-Nikodym Property in Spaces of Operators.

Necessary conditions and sufficient conditions for spaces of operators to possess the Radon-Nikodym property in various ideal norms are discussed. (Received June 2, 1975.)

*726-28-3 ROY L. ADLER, IBM T.J. Watson Research Center, Box 218, Yorktown Heights, New York 10598. Ergodic properties of elementary mappings of the unit interval.

• Let f be a function mapping of the unit interval onto itself. We wish to study iterates of f from the point of view of ergodic theory. This usually means proving theorems about frequencies with which sequences of points $f^n(x)$ hit subintervals of the unit interval. We shall consider only simply defined functions, ones that are piecewise smooth but not invertible. Very little is known in general and unsolved problems abound. However there is a class for which an extensive theory can be developed, the most interesting example of which is the function $f(x) = 1/x \pmod 1$. For this function if $f^n(x) \in [1/k+1, 1/k)$ for k a positive integer, then k is the nth digit in the continued fraction expansion of x, $0 < x < 1$. The frequency problem of digits in continued fraction expansions was attacked by Gauss and has attracted many mathematicians since. We will survey some old and new results in this area and discuss the relation between the ergodic behavior of $f^n(x)$ and derivative conditions on f. (Received June 6, 1975.)

*726-28-4 STUART P. LLOYD, Bell Laboratories, Murray Hill, N. J. 07974 Rohlin theorem in the noninvertible case.

If T is an ergodic measure preserving transformation of atomless probability space $(X,\mathcal{B},m)$ then for each $\varepsilon > 0$, $n > 1$ there exists $F \in \mathcal{B}$ such that $F, T^{-1}F, \ldots, T^{-(n-1)}F$ are disjoint and $m(\bigcup_{j=0}^{n-1} T^{-j}F) \geq 1-\varepsilon$. Moreover, if a finite partition $\mathcal{P} \subset \mathcal{B}$ of X is given then there exists N(large) such that F can be chosen to be independent of $T^{-N}\mathcal{P}$. (Received June 11, 1975.)

726-28-5 RICHARD J. MAHER, Loyola University of Chicago, Chicago, Illinois 60626, Strong Liftings in topological measured spaces, Preliminary report

In previous papers(Jour. Math. Anal. Appl., 29(3), 1970, 633-639; and Adv. in Math., 13(1), 1974,55-72) the author has obtained various results on strong liftings in locally compact spaces. We now extend some of these results, together with those of Ann. Inst. Fourier(Grenoble) 21,2(1971), 35-41(coauthored with Prof. C. Ionescu Tulcea) to the case of topological measured spaces. We also obtain new results concerning the strong lifting property for image measures and use these results to extend the disintegration theorem of Bourbaki. (Received June 11, 1975.)

*726-28-6 WILLIAM H. GRAVES, University of North Carolina, Chapel Hill, NC 27514
Extending Vector Measures. Preliminary report.

Let R be a ring of subsets of a set X. Let $M(R)$ be the space of simple functions generated by the characteristic functions χ_A, $A \in R$. A finitely additive set function $\mu : R \to W$, W any locally convex space (ℓcs), is a measure if $\{\mu(A_i)\}$ is Cauchy for every increasing sequence $A_1 \subseteq A_2 \subseteq \ldots$ in R and converges to $\mu(\cup A_i)$ when $\cup A_i \in R$. $\tilde{\mu} : M(R) \to W$ is the associated linear map $\tilde{\mu}(f) = \int f d\mu$. Let $\tau(R)$ be the coarsest locally convex topology on $M(R)$ making all $\tilde{\mu}$ continuous ($\mu : R \to W$ any measure and W any ℓcs). Then the map $\chi : R \to (M(R), \tau(R))$ given by $A \mapsto \chi_A$ is a measure and is a universal measure in the sense that every measure $\mu = \tilde{\mu} \circ \chi$. Theorem: $\chi(R)$ is relatively weak *-compact in $M(R)''$, the bidual of $(M(R),\tau(R))$. From this follows the Theorem: The universal measure $\chi : R \to (\hat{M}(R),\hat{\tau}(R))$ (into the completion of $(M(R),\tau(R))$) extends to a measure on the σ-ring generated by R. This yields the classical extension theorem. Corollary: Every measure $\mu : R \to W$, W any ℓcs, extends to the σ-ring generated by R. (Received June 17, 1975.)

30 Functions of a Complex Variable

726-30-1 ALLAN FRYANT, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201.
Growth of Entire Harmonic Functions in R^3

Let $H(r,\theta,\varphi) = \Sigma_{n=0}^{\infty} r^n \Sigma_{m=0}^{n} (a_{nm}^{(1)} \cos m\varphi + a_{nm}^{(2)} \sin m\varphi) P_n^{(m)}(\cos\theta)$, and $h(u,e^{it}) = \Sigma_{n=0}^{\infty} c_n(e^{it})u^n$ be the Bergman B_3 associate of H. Introduce $g(u) = \Sigma_{n=0}^{\infty} \max_t |c_n(e^{it})|u^n$. If the order ρ_H of H is $<\infty$, we show g is entire and $\rho_H = \rho_g = \overline{\lim_{n\to\infty}} \max_{m,i} \dfrac{n \log n}{.5 \log (n-m)!/(n+m)! - \log|a_{nm}^{(i)}|}$. Further the types satisfy $L \le \tau_H \le \tau_g \le 2^{\rho_H} L$ where $L = \dfrac{1}{e\rho_H} \overline{\lim_{n\to\infty}}\, n\left[\max_{m,i} \sqrt{\dfrac{(n+m)!}{(n-m)!}}\, |a_{nm}^{(i)}|\right]^{\rho_H/n}$ and these bounds are best possible. For entire axisymmetric harmonic functions (A.H.F.'s) we show that a function $\rho(r)$ is a proximate order for H if and only if it is a proximate order for its B_3 associate h, and that the types τ_H, τ_h with respect to $\rho(r)$ are equal. Application is made to generating complete sequences of A.H.F.'s from a single entire A.H.F. (Received April 28, 1975.)

*726-30-2 DOROTHY BROWNE SHAFFER, Fairfield University, Fairfield, Conn. 06430
Inequalities for a Special Class of Bounded Analytic Functions

This paper deals with the class of functions $H_{A,n}$ defined as follows: $h(z) \in H_{A,n}$ if $h(z)$, analytic in the unit disc D, has expansion $h(z) = 1 + c_n z^n + c_{n+1} z^{n+1} + \ldots$, and the image of $h(z)$ is contained in the disc with diameter $\left(\frac{A}{A+1}, \frac{A}{A-1}\right)$, $A \geqslant 1$. For $A = 1$ this reduces for the known class $\operatorname{Re} h(z) \geqslant \frac{1}{2}$. The min $\operatorname{Re}\{zh'(z)/h(z)\}$, $h(z) \in H_{A,n}$ is determined. Typical results are the determination of the order and radius of starlikeness for meromorphic functions in the form $g(z) = A/z - \phi(z)$, $\phi(z)$ majorized by z^n, the radius of convexity for meromorphic functions $F(\zeta) = \zeta + a_0 + \frac{a_m}{\zeta^m} + \ldots$, $|F'(\zeta) - 1| < \frac{1}{A}$ and applications to functions $f(z) \in S$, $g(z) \in S$, $f(z)/g(z) \in H_{A,n}$. (Received June 9, 1975.)

*726-30-3 BERNARD MASKIT, State University of New York, Stony Brook, New York 11794. On the classification of Kleinian groups.

• A Kleinian group is a discrete subgroup of $PSL(2;C)$ which acts discontinuously at some point of the Riemann sphere. We classify those Kleinian groups which, in their action on hyperbolic 3-space have a finite-sided fundamental polyhedron, and which have an invariant component of the set of discontinuity;

these groups include all of the classical uniformizations of compact Riemann surfaces. These groups are classified in the same sense that finitely-generated Fuchsian groups of the first kind are classified; i.e., there are countably many topologically distinct classes of groups, each class can be described by a finite collection of numbers, and the set of groups in each class - when marked by a set of generators - can be parametrized as a complex manifold. (Received June 9, 1975.)

*726-30-4 HARI SHANKAR, Ohio University, Athens, Ohio 45701 and RICHARD A. BOGDA, Dupont, Parkersburg, W. Va. 26101. Convolutions of Holomorphic Functions. Preliminary report.

H(R) denotes the class of all holomorphic functions f, $f(z) = \Sigma_0^\infty a_n z^n$ whose radius of convergence is R, $0 < R < \infty$. Let $f \in H(R)$ and $g \in H(R')$, $0 < R' < \infty$, $g(z) = \Sigma_0^\infty b_n z^n$. Define the convolution $f*g$ of f and g, by $F(z) = (f*g)(z) = \Sigma_0^\infty a_n b_n z^n$. The function F(z) is holomorphic in the disc whose radius is at least equal to RR'. For the definitions of order ρ, lower order λ, of type T and lower type t of an holomorphic function $f \in H(R)$, refer Abstract 711-30-8, these Notices 21 (1974), A-120. The following results are proved. Theorem 1. Let $f*g$ be in $H(R'')$ and of type T and lower type t, then $(f*g)'$ and $f'*g'$ are in $H(R'')$ and are of type T, lower type t, where prime denotes the derivative of the function, $f \in H(R)$, $g \in H(R')$ and $R'' \geq R \cdot R'$. Theorem 2. Let f and g be, as in Theorem 1, and of types, respectively T, T'. Let $f*g$ be in $H(R'')$ such that $R'' = R \cdot R'$, and of type T". If orders of f, g, and $f*g$ are all equal then (a) $T'' \leq T + T'$. (b) If $T = T' = 0$, then $T'' = 0$. (c) If $T'' = \infty$, then $T = \infty$ or $T' = \infty$. With more restrictive conditions on coefficients a_n, b_n, $a_n b_n$, similar results hold for lower types. (Received June 12, 1975.)

726-30-5 G.T. CARGO, Syracuse University, Syracuse, NY 13210. Blaschke products with prescribed boundary values.

Theorem: Let n be a positive integer; let $(z_1, z_2, \ldots, z_n)$ be an n-tuple of distinct complex numbers of modulus one; and let $(w_1, w_2, \ldots, w_n)$ be an n-tuple of (not necessarily distinct) complex numbers of modulus at most one. Then there exists a Blaschke product, B, such that $B(z_k) = w_k$ if $1 \leq k \leq n$. (In this theorem, B may contain a multiplicative constant of modulus one; and, as usual, $B(z_k)$ denotes the radial limit of B at z_k.) Extensions and applications of the theorem are given. Each of the two proofs of the theorem depends upon an earlier (interpolation-theory) version of the theorem due to D. G. Cantor and R. R. Phelps [Proc. A. M. S. 16 (1965), 523-525] (and, more recently, to S. J. Poreda) in which each w_k $(1 \leq k \leq n)$ is required to have modulus one and B is a finite Blaschke product. (Received June 16, 1975.)

32 Several Complex Variables and Analytic Spaces

*726-32-1 WILHELM STOLL, University of Notre Dame, Notre Dame, Indiana 46556. Aspects of value distribution theory in several complex variables.

• At first the classical two main theorems of Nevanlinna in one variable are recalled. Then a history of the status of these theorems in several variables is given. As an example, the theory of Carlson-Griffiths-King is outlined and extended to parabolic spaces. This leads to an intrinsic formulation and to the introduction of the ramification defect, the Ricci defect and the dominator effect into the second main theorem. (Received May 22, 1975.)

726-32-2 MARIE-LOUISE HENRICI, ETH, 8006 Zürich, and PETER HENRICI, ETH, 8006 Zürich. The Lagrange-Bürmann formula for systems of formal power series. Preliminary report.

We consider the integral domain of formal power series $P = \sum_{\underline{k} \geq \underline{0}} a_{\underline{k}} \underline{x}^{\underline{k}}$ in N indeterminates $\underline{x} = (x_1, \ldots, x_N)$, where $\underline{k} = (k_1, \ldots, k_N)$ has integer components, $\underline{x}^{\underline{k}} := x_1^{k_1} \ldots x_N^{k_N}$, inequalities between index vectors are componentwise, and the coefficient field

has characteristic zero. A system of non-units $\underline{P}=(P_1,\dots,P_N)$ is called non-singular if the map defined by the linear terms is invertible. The non-singular systems form a group under composition "o"; denote the inverse of $\underline{P}$ by $\underline{P}^{[-1]}$. The system $\underline{P}$ is called admissible if P_i has the factor x_i. $\underline{P}'$ denotes the formal Jacobian of the system $\underline{P}$. The residue of a formal Laurent series $L=\sum_{\underline{k}\geq \underline{k}_0} a_{\underline{k}}\underline{x}^{\underline{k}}$ is $\mathrm{res}L:=a_{-\underline{e}}$ where $\underline{e}:=(1,\dots,1)$. THEOREM: If the system $\underline{P}$ is admissible and non-singular and if $\underline{F}$ is any system, then $\underline{F}o\underline{P}^{[-1]}=\sum_{\underline{k}\geq 0} \mathrm{res}(\underline{F}\ \underline{P}'\ \underline{P}^{-\underline{e}-\underline{k}})\underline{x}^{\underline{k}}$. The proof requires the LEMMA: If $\underline{L}$ is any system of formal Laurent series, then $\mathrm{res}\underline{L}'=0$.
(Received May 23, 1975.)

33 Special Functions

*726-33-1 H. M. SRIVASTAVA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2 and REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2 and Ravenshaw College, Cuttack-3, Orissa, India. An integral representation for the product of two Jacobi polynomials. Preliminary report.

Recently, H. M. Srivastava and C. M. Joshi ["Integral representation for the product of a class of generalized hypergeometric polynomials", *Acad. Roy. Belg. Bull. Cl. Sci.(5)* 60 (1974), 919-926; see also *Collect. Math.* 24 (1973), 117-121] gave a multiple integral representation for the polynomial product (*) $\Phi_m[(a_p),\lambda;\ (b_q)+1;\ x]\ \Phi_n[(c_p),\mu;\ (d_q)+1;\ y]$, where, for the sake of brevity, (a_p) denotes the sequence of p parameters $a_1,\dots,a_p$, with similar interpretations for (b_q), etc., and (**) $\Phi_m[(a_p),\lambda;\ (b_q);\ z]=\binom{\lambda+n-1}{n}\ {}_{p+2}F_q[-n,\lambda+n,(a_p);\ (b_q);\ z]$, in terms of the generalized hypergeometric functions. The object of the present paper is to first derive an integral representation for the product $P_m^{(\alpha,\beta)}(x)\ P_n^{(\gamma,\delta)}(y)$, which does not seem to be an easy consequence of the general integral representation in the Srivastava-Joshi paper *[op. cit., p. 923, Eq. (1.3)]*. It is then shown how this result can be extended to hold for the product of two generalized hypergeometric polynomials, for instance, of type (**) above, and for polynomials associated with Kampé de Fériet's hypergeometric functions of two variables. (Received June 16, 1975.)

34 Ordinary Differential Equations

*726-34-1 **In-Ding Hsü, State University of New York at Buffalo, Amherst, N. Y. 14226.** **Global Existence of Periodic Solutions of the Glass-Kauffman Model, Preliminary report.**

Recently Hastings and Murray have refined a method of Pliss for proving the existence of "large" periodic solutions of higher-dimensional, nonlinear autonomous systems. This method uses the Brouwer Fixed Point Theorem. Hastings and Murray prove the existence of periodic solutions for the Field-Noyes model. We extend their method to study the 4-dimensional Glass-Kauffman model which is (*) $\dot{x}_1=-ax_1+b(x_2-x_1)+c[1-S(x_3)]$, $\dot{x}_2=-ax_2+b(x_1-x_2)$, $\dot{x}_3=-ax_3+b(x_4-x_3)$, $\dot{x}_4=-ax_4+b(x_3-x_4)+dS(x_2)$, where a, b, c and d are positive parameters and S(x) is the Hill (or Error) function. We obtain the following theorems:

Theorem 1. The Box which is defined by $0\leq x_1,\ x_2\leq c/a$ and $0\leq x_3,\ x_4\leq d/a$ is an invariant set for the system (*). Indeed, any trajectory originating in R^4_+ enters into the interior of this box eventually. Moreover, there are "small" boxes B_1, B_2, B_3 and B_4 such that $B_1\cup B_2\cup B_3\cup B_4$ is an invariant subset and every trajectory originating in each B_i proceeds from box to box in the sequential order: $\to B_1\to B_2\to B_3\to B_4\to B_1\to\cdots$.

Theorem 2. For any given parameters (a, b, c, d) such that the linear part of (*) has two eigenvalues with positive real part, the system (*) has at least one nontrivial periodic solution which lies in an invariant torus in which all trajectories are oscillatory.
(Received April 28, 1975.) (Author introduced by Dr. N. D. Kazarinoff.)

*726-34-2 In-Ding Hsü and N. D. Kazarinoff, State University of New York at Buffalo, Amherst, N. Y. 14226. Existence and Stability of Periodic Solutions of a third order nonlinear Autonomous system simulating Immune Response in Animals.

We study the system (*) $\dot{x}_1 = x_1(\lambda_1 - k\alpha_1 x_2/z)$, $\dot{x}_2 = x_2(-\lambda_2 - k\alpha_2 x_1/z) + k\gamma x_1 x_3/z$, $\dot{x}_3 = x_3[-\lambda_3 + (k\alpha_3 x_1/z)(1 - x_z/\theta) + S$, where $z = 1 + k(x_1 + x_2 + nx_3)$ and α_i, λ_i, k, γ, θ, n and s are positive constants. This was introduced by G. I. Bell and studied by G. H. Pimbley [Arch. Rat. Mech. and Anal. 55(1974), 93-123] as a model for the immune response in an animal to invasion by active, self-replicating antigens. We obtain: Theorem 1. If $\alpha_1 > \lambda_1$, $\theta\lambda_3 > s$, $\alpha_2 = c_1\gamma$, $\lambda_2 = c_2\gamma$ where c_1 and c_2 are positive constants, and λ_1 is sufficiently small, then there exists a one-parameter family of periodic solutions of (*) bifurcating from the steady state x^{1e}. These periodic solutions are asymptotically, orbitally stable with asymptotic phase if $\mu_2\alpha'(\gamma_c) > 0$, but are unstable if $\mu_2\alpha'(\gamma_c) < 0$, where a formula for $\mu_2\alpha'(\gamma_c)$ is explicitly given. Theorem 2. If $S = \lambda_3 = 0$ (i.e. reduced 2×2 system), $\alpha_1 > \lambda_1$, $\alpha_2 = c_1\gamma$, $\lambda_2 = c_2\gamma$, and if λ_1 is sufficiently small, then (*) has a one-parameter family of unstable, periodic solutions bifurcating from the steady state x^{1e}. We also give sufficient conditions for the 3×3 and 2×2 systems each to have two steady states. We give a simple criterion for stability of periodic solutions that bifurcate from each steady state. We study numerical examples for the 2×2 and 3×3 systems. These show stability in the 3×3 case if $(\alpha_1, c_1, \alpha_3, \lambda_1, c_2, \lambda_3, \theta, k, n, s) = (1, 3, 5, 10^{-1}, 10^{-1}, 5/4, 2, 1, 1, 1)$ and $\gamma \to .040188^-$ and instability in the 2×2 case if $(\alpha_1, c_1, \lambda_1, c_2, \theta, k, n) = (10, 10^{-2}, 10^{-2}, 2 \cdot 10^{-5}, 1, 1, 10^2)$ and $\gamma \to .089^-$.
(Received April 28, 1975.)

*726-34-3 T. K. PUTTASWAMY, Ball State University, Muncie, Indiana 47306, Solution in the large of a certain nth order differential equation.

In this paper, the author has solved in the large the differential equation

(1) $\sum_{j=0}^{n} z^j(a_j + b_j z)\frac{d^j y}{dz^j} = 0$. Here, the variable z is regarded complex, as likewise the constants a_j, b_j $(j = 0,1,2,\ldots n)$ with $a_n \neq 0$ and $b_n \neq 0$. If $u = -\frac{a_n}{b_n}$, then the differential equation (1) will have in the language of Fuch's theory three regular singular points at $z = 0$, $z = u$ and $z = \infty$. The indicial equation about $z = 0$ is found to be

(2) $a_0 \sum_{i=0}^{n-1} a_{n-i} \prod_{j=0}^{n-i-1} (h-j) = 0$. It is also assumed that the roots $h_i (i=1,2,\ldots,n)$ of (2) are such that no two of them differ by an integer. (Received June 16, 1975.)

35 Partial Differential Equations

*726-35-1 DAVID W. FOX, The Johns Hopkins University Applied Physics Laboratory, Laurel, Maryland 20810. Transient Solutions for Stratified Fluid Flow.

The system (1) $w^*_{tt} + w^* + \partial\varphi/\partial x_3 = 0$ and $\Delta\varphi + \partial w^*/\partial x_3 = \mu$ arises from a linearization about equilibrium of the equations of motion of an incompressible inviscid stratified fluid in which the buoyancy frequency and the density are treated as constant. φ is a potential for the horizontal velocities, $w^* + \partial\varphi/\partial x_3$ gives the vertical velocity, Δ is the 3-dimensional Laplacian, and $\mu(x,t)$ is a given source distribution. Correct initial conditions for (1) are (2) $w^*(x,0) = f(x)$ and $w^*_t(x,0) = g(x)$. The general initial value problem (1), (2) is solved in $R^3 x(0,T)$ under appropriate conditions on μ, f, and g by use of a fundamental solution, which is explicitly given. Properties of the fundamental solution and of the solution of the initial value problem are obtained. (Received May 16, 1975.)

726-35-2 BOHDAN LAWRUK, McGill University, Montreal, Quebec, Canada. One partial differential equation with transformed argument.

A case of p.d.e. with transformed argument is considered in a class of distributions $D'(R^n)$ and it is shown that the solutions of the form $p(x)e^{ix-\xi}$, where $x \in R^n$, $\xi \in \mathbb{C}^r$ and $p(x)$ is a polynomial, are dense in the set of solutions of the equation. This follows from the fact that the equation is equivalent to a finite number of linear partial differential equations with constant coefficients. (Received June 4, 1975.)

39 Finite Differences and Functional Equations

*726-39-1 PROFESSOR MARY KATHRINE BENNETT, Dept. of Math/Stat, Univ. of MA, Amherst, MA 01002
Axioms for Metric Affine Geometry

The idea of metric affine geometry needs no justification of its importance in classical and modern mathematics. We intend here to give one axiom - which, in the presence of affine space, gives all of the results of metric affine geometry. We shall illustrate the role of the parallelogram law in such geometry, and develop a generalization thereof for general quadrilaterals. We will give the rudimentary notion of orthogonality in our setting and axiomatize the various special metric affine geometries such as Euclidean and Minkowski space.

(Received April 11, 1975.)

*726-39-2 MICHAEL A. GOLBERG, University of Nevada, Las Vegas, Las Vegas, Nevada 89154. The Use of Functional Equations in Numerical Analysis.

In this paper we discuss how certain functional equations satisfied by the solutions of boundary value problems for ordinary differential equations and integral equations are useful in developing numerical algorithms for the solution of such problems. For boundary value problems the relations generalize and unify certain results of Bellman, Kalaba and Scott and give rise to various algorithms commonly called the method of invariant imbedding. For integral equations these relations constitute a novel use of the Hilbert resolvent equation. Recent theoretical results will be discussed concerning the group structure of boundary value problems and some unsolved problems will be presented. (Received April 7, 1975.)

726-39-3 DONALD R. SNOW, Brigham Young University, Provo, Utah 84602.
Summation of Sequences by Functional Equations

A method will be presented for summing sequences by writing a functional equation and appropriate initial condition which describes the sum and then solving it to get the desired formula. Examples of various sequences for which this method is applicable will be shown and the functional equations will be derived. Various methods of solution of the functional equations will be considered including one which the author calls "continuous combinatorics" which consists of extending the discrete variable to a continuous variable. Examples of the resulting formulas will be presented and possible generalizations will be discussed. (Received May 13, 1975.)

*726-39-4 JOHN V. RYFF, University of Connecticut, Storrs, Connecticut 06268.
The equation $af(ax) + bf(bx + a) = bf(bx) + af(ax + b)$. Preliminary report.

This report contains both the genesis and partial solutions to the functional equation

$$\text{(E)} \quad af(ax) + bf(bx + a) = bf(bx) + af(ax + b).$$

Conditions on the function and its domain are optional although the original problem requires that f be bounded in $[0,1]$ and that $a + b = 1$, where a and b are nonnegative.

If f is linear, then f is a solution. Furthermore, if a (hence b) is rational, then a linear function plus a suitable periodic function will solve the equation. But if a is irrational, then the periodic part must disappear and the question remains whether only linear functions satisfy the equation in this circumstance.

In order to apply Fourier analysis it is essential that one be able to extend the function while maintaining (E). This is shown to be possible, but not uniquely. Good growth conditions on the extensions are still lacking (e.g. if the original f is bounded, then an extension exists which is locally in L^p, $0 < p < 1$.

Each side of (E) represents the action of the adjoint of a linear transformation given by composition with certain measure preserving transformations of $[0,1]$.
(Received May 8, 1975.)

*726-39-5 F. J. PAPP; University of Lethbridge; Lethbridge, Alberta T1K 3M4; Canada. Standard and Non-Standard Solutions of the Cauchy Equation.

If f is a real-valued function of a real variable and satisfies Cauchy's functional equation $f(x + y) = f(x) + f(y)$ for all real x and y and if the natural extension, f^*, of f is (a) non-negative for all positive infinitesimals, or (b) non-positive for all positive infinitesimals, or (c) bounded on the monad of some point (the point may but need not be finite), then f is continuous and so of the form $f(x) = cx$ for all real x (f^* is then necessarily of the same form and satisfies the Cauchy equation for all non-standard x). The preceeding generalizes the well-known results of Darboux (1880) and provide the non-standard analysis analogue of his results. The Ostrowski (1929) and Kestelman (1947) condition that f be bounded either above or below and not necessarily both can be similarly extended.

In the case of the finite non-standard real numbers the function $st(x)$, "standard part of x", is a continuous non-standard solution of the Cauchy equation not of the form cx. It is easily shown, however, that the domain of definition cannot be extended to all the non-standard real numbers. The only continuous solutions on all the non-standard reals are of the form cx.
(Received May 21, 1975.)

726-39-6 GEORGE T. DIDERRICH, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1. Local boundedness and the Shannon entropy. Preliminary report.

Theorem. Let f be an information function. If f is bounded on a nonvanishing interval contained in $(0,1)$, then $f \equiv S$ where $S(x)$ is Shannon's measure of entropy on a 2-event space. Corollary. If $|f|$ is measurable on $(0,1)$, then $f \equiv S$. Corollary. If f is bounded a.e. on $[0,1]$, then $f \equiv S$. The theorem answers a question of Aczél and Daróczy ("Measures of information and their characterizations", Academic Press, New York, 1975, Chapter 3), the first corollary implies Lee's theorem, and the second corollary extends the author's previous result ("The role of boundedness in characterizing Shannon entropy", Information and Control (to appear)). (Received May 23, 1975.)

726-39-7 M. A. TAYLOR, Acadia University, Wolfville, Nova Scotia, Canada B0P 1X0. Functional Equations and Closure Conditions. Preliminary Report.

The Thomsen and Reidemeister closure conditions can be used to some effect on certain functional equations. The method is demonstrated on the equation $(xy)\cdot(xz) = zy$, and the generalized bisymmetry equation. The application of the method to equations where the operations are of arity greater than two is discussed, together with some of the limitations of the method.
(Received May 23, 1975.) (Author introduced by Dr. J. Aczel.)

*726-39-8 B. SCHWEIZER, Univ. of Mass., Amherst MA 01002. The Associativity Equation on a Space of Probability Distribution Functions.

Let D be the space of all one-dimensional probability distribution functions concentrated on $[0, \infty]$. The study of the triangle inequality for probabilistic metric spaces leads naturally to the question of finding functions $T : D \times D \to D$ satisfying the conditions: (a) $T(F,\varepsilon_0) = F$, where $\varepsilon_0(x) = 1$ for $x > 0$ and 0 for $x \leqq 0$; (b) $T(F,G) \leqq T(F,H)$, whenever $G \leqq H$; (c) $T(F,G) = T(G,F)$; (d) $T(T(F,G),H) = T(F,T(G,H))$. Generally it is also required that T is jointly continuous with respect to the topology of weak convergence on D, in which case (D,T) is a topological semigroup. In the past several years, a variety of such functions T, i.e. semigroups, have been found and the arithmetic structure of some of them has been studied in detail. This talk is devoted to a survey of some of the major results obtained. (Received May 27, 1975.)

*726-39-9 IH-CHING HSU, Saint Olaf College, Northfield, Minnesota 55057.
A Fundamental Functional Equation for Vector Lattices.

The following functional equation is constructed $u + F[x, \lambda F(y, z)] = F[u + \lambda y, F(u + \lambda z, u + x)]$ so that any (binary-operation) solution F on a vector space V with $F(x, x) = x$ has all the following needed properties to make V a vector lattice: 1) $F(y, z) = F(z, y)$ (commutativity) 2) $F[x, F(y, z)] = F[F(x, y), z]$ (associativity) 3) $\lambda F(y, z) = F(\lambda y, \lambda z)$, λ a positive scalar. (positive homogeneity) and 4) $u + F(y, z) = F(u + y, u + z)$ (additive homogeneity). Conversely, on a given vector lattice, the above functional equation is solvable by F with $F(x, y) = \sup\{x, y\}$. (to appear in Aequationes Mathematicae) (Received May 29, 1975.)

726-39-10 M. J. FRANK, University of Wisconsin, Milwaukee, Wisconsin 53201. On the simultaneous associativity of $C(x,y)$ and $x + y - C(x,y)$. Preliminary report.

Let $\mathcal{C}$ denote the set of monotonic two-place functions $C : [0,1]^2 \to [0,1]$ satisfying $C(x,0) = C(0,x) = 0$ and $C(x,1) = C(1,x) = x$. Let $\mathcal{C}_A = \{ C \in \mathcal{C}: C(x,x) < x$ for every $x \in (0,1) \}$. For $C \in \mathcal{C}$, define the function $C^\wedge : [0,1]^2 \to [0,1]$ by

$$C^\wedge(x,y) = x + y - C(x,y) .$$

We say that $C \in \mathcal{C}$ is bi-associative if both C and $C^\wedge$ are solutions of the associativity equation $F[F(x,y),z] = F[x,F(y,z)]$.

THEOREM 1. The only bi-associative elements of $\mathcal{C}_A$ are $C_1(x,y) = x \cdot y$ and $C_2(x,y) = \max(x+y-1,0)$.

Clearly $C_3(x,y) = \min(x,y)$ is bi-associative. Theorem 1, together with the fact that each associative element of $\mathcal{C}$ is representable as an ordinal sum of C_3 and elements of $\mathcal{C}_A$, then yields:

THEOREM 2. $C \in \mathcal{C}$ is bi-associative iff it is an ordinal sum of C_1 , C_2 , and C_3 .

[This result has some implications in probability, information theory, and topological semigroups.] (Received May 30, 1975.)

*726-39-11 PROFESSOR F. W. CARROLL The Ohio State University, 231 W. 18th Ave., Columbus, OH 43210
The Difference Property for Frechet-Space-Valued Functions

The question, asked by Erodős in 1951, of whether or not measurability has the weak difference property, remains open. Neither is it known whether $L^p[0, 1]$ has the weak difference property in case $0 < p < 1$. The available proofs for the cases $1 \leq p \leq \infty$ depend on local convexity of $L^p[0, 1]$ and the theory of Banach-valued integration. The following theorem is connected with these problems. Let $\mathcal{V}$ be a (not necessarily locally convex) Frechet space, and let the metric ρ for $\mathcal{V}$ satisfy (*) $\rho(1/2u, 1/2v) \leq a\, \rho(u, v)$ for some a , $0 < a < 1$, and all u,v in $\mathcal{V}$. Let F be a function from R , of period 1 , into $\mathcal{V}$, each of whose differences is continuous. Then $F = G + H$, where G is continuous and $H(x + y) = H(x) + H(y)$. (Application: $\mathcal{V} = L^p[0, 1]$. For $0 < p < 1$, the result is new; for $1 \leq p \leq \infty$, the proof is new). The proof depends on two facts: that $(x, y) \to F(x + y) - F(x) - F(y)$ is jointly continuous, and that (*) implies that the functional equation $g(2x) - 2G(x) = F(2x) - 2F(x)$ has a continuous solution G .
(Received May 22, 1975.)

726-39-12 KONRAD JOHN HEUVERS, DEPARTMENT OF MATHEMATICS, MICHIGAN TECHNOLOGICAL UNIV., HOUGHTON, MICHIGAN 49931, A Class of Associative Algebras Generated by a Unitary Space. Preliminary report.

An investigation is made of the associative algebras generated by an n-dimensional unitary space V with the inner product $\langle x|y\rangle$. The algebras are subject to the relation $\Phi_1 x * \Phi_2 y + \Phi_3 y * \Phi_4 x = F(\langle x|\Phi_5 y\rangle)$ where Φ_k: $V \to V$ (k=1,2,3,4,5) are invertible linear or pseudo-linear operators and F: $V \to \mathbb{C}$ is a function. If $\lambda \varepsilon \mathbb{C}$ and $x \varepsilon V$ the condition $\lambda * x = x * \lambda$ in the algebras is replaced by $\lambda * x = x * \phi(\lambda)$, where ϕ: $\mathbb{C} \to \mathbb{C}$ is a function which leaves the reals invariant. This leads to complex algebras when $\phi(\lambda) = \lambda$ and real algebras when $\phi(\lambda) = \bar{\lambda}$ (if V is viewed as a 2n-dimensional vector space over the reals). It is shown that $F(\zeta) = \lambda\zeta + \mu\bar{\zeta}$ (where $\lambda, \mu, \zeta \varepsilon \mathbb{C}$) and that the original relation is equivalent to $x * \Psi_1 y + y * \Psi_2 x = F(\langle x|\Psi_3 y\rangle)$. The pseudo-linear operator Π: $V \to V$, defined by $\langle \Pi x|\Pi y\rangle = \langle y|x\rangle$, is a conjugation on V. If $\langle y|\Psi_3 x\rangle = \langle \Psi_3 y|x\rangle$ and if $\Pi\Psi_3 = \Psi_3\Pi$ then $(x|y) = \langle x|\Psi_3\Pi y\rangle$ is a non-singular symmetric bilinear-metric on V. The relations $x * y + y * x = 2\langle x|\Psi_3\Pi y\rangle$ and $\lambda * x = x * \lambda$ determine the Clifford algebra generated by V with the bilinear-metric $(x|y)$. Similarly, a real algebra is determined by the relations $x * \Pi y + y * \Pi x = 2\langle x|\Psi_3\Pi y\rangle$ and $\lambda * x = x * \bar{\lambda}$. (Received May 19, 1975.)

*726-39-13 C.T. Ng, University of Waterloo, Waterloo, Ontario N2L 3G1. Structured measurement of algorithms, Preliminary report.

Measurement of some parameters of an algorithm are to be considered. Such measurement will be based on the tree that represents the algorithm, and on the branching probability distributions. The consistency of the measurement with the serial joint operation and with the parallel joint operation on trees is of major concern. Representation of such measurement is derived by solving related functional equations. (Received June 6, 1975.)
(Author introduced by J. A. Aczel.)

726-39-14 THOMAS A. O'CONNOR, Bowling Green State University, Bowling Green, Ohio 43403 The Solution of D'Alembert's Functional Equation on a Locally Compact Abelian Group

D' Alembert's functional equation $2f(x)\,f(y) = f(x+y) + f(x-y)$ was solved by Cauchy in 1821, when f is a continuous function on R. This result is extended to the case when f is a continuous, bounded, real valued function defined on a connected locally compact abelian group. The positive definiteness of all such functions is first established. Then, upon application of the general Bochner Theorem and other techniques from Probability Theory, the solution is found to be $f(g) = \mathrm{Re}\,\chi(g)$ where χ is a character of the group.
(Received June 9, 1975.)

726-39-15 H. T. HU and KERMIT SIGMON, University of Florida, Gainesville, Fl. 32611. Continuously factorable groupoids, Preliminary report

A topological groupoid m: $X \times X \to X$ is called continuously factorable if there is such a continuous function μ: $X \to X \times X$ that $m\mu = 1_X$. It is first shown that if A is a closed ideal of a compact connected topological groupoid X admitting a continuous factorization μ such that $\mu[A] \subset A \times X \cup X \times A$, then $\check{H}^*(X) \cong \check{H}^*(A)$. A complete characterization is then given of all continuously factorable semigroups on a closed real interval. (Received June 9, 1975.)

726-39-16 JOHN E. MAXFIELD, Kansas State University, Manhattan, Kansas, KS 66506 and T. D. HOWROYD, University of New Brunswick, Fredericton, N.B., Canada E3B 5A3. Some functional equations defined by the infimum. Preliminary Report.

Functional equations of the form $u(x) = \inf_v f(x,v) + u(v)$ in which u is the unknown function occur in dynamic programming. If there exists a continuous function h(x) such that $h(x) - f(x,v)$ is convex in x, then $h(x) - u(x)$ is convex, so $u(x)$ is continuous, and if $h(x)$ is differentiable except on at most a countable set then $u(x)$ has the same property. The case when $f(x, v)$ is quadratic in x and in v is of interest. A typical example is given by $f(x, v) = (2x - v)^2$; then $h(x) = 4x^2$ will suffice. Furthermore, if S is a closed set and $S = 2S$, then $u(x) = \inf\{3(x-s)^2; s \in S\} + C$ is a solution for arbitrary constant C. The lower envelope of a family of such solutions is also a solution; the family may be chosen in such a way that the solution is specified by its restriction to a compact interval; the restriction may belong to a wide class of functions. (Received June 10, 1975.)

726-39-17 DAVID ZUPNIK, 5437 N. Bernard St., Chicago, Illinois 60625, Cayley Sets

A family F of functions each of which maps a set S into itself is called a Cayley set if there exists a semigroup F on S for which F is the Cayley (i.e., regular) representation. If F has a (one-sided) identity then F is said to admit a (one-sided) identity. The problem of characterizing Cayley sets that admit (one-sided) identities is completely solved. Some partial results are given for the general characterization problem, and the possible connection of this problem with the word problem is discussed. (Received May 27, 1975.)
(Author introduced by Professor Abe Sklar.)

*726-39-18 J. A. LESTER, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1. A characterization of certain multiplicative homomorphisms.

For a field F of characteristic > N, let $(\mathfrak{F},+,\circ)$ be a commutative associative algebra over F, with identity e. Also, let * denote a binary operation with identity on a nonempty set G, and assume all $\alpha \in G$ have inverses. This paper characterizes all multiplicative homomorphisms from G to $\mathfrak{F}$, i.e. all functions $g: G \to \mathfrak{F}$ such that $g(\alpha * \beta) = g(\alpha)\circ g(\beta)$. It is first shown that the general case may be reduced to that where e is the only nonzero idempotent of $\mathfrak{F}$. In this case, then, $g(\alpha) = \pi(\alpha)\exp[a(\alpha)]$ where (i) $\pi: G \to F$ satisfies $\pi(\alpha*\beta) = \pi(\alpha)\pi(\beta)$; (ii) $a: G \to \mathfrak{F}$ satisfies $a(\alpha*\beta) = a(\alpha) + a(\beta)$; (iii) the range of a is the hyperplane of nilpotents of $\mathfrak{F}$ (i.e. $[a(\alpha)]^k = 0$ for some $k \leq N$) and so a is essentially N - 1 additive functions from G to F. (Received June 2, 1975.) (Author introduced by Professor J. Aczél.)

726-39-19 ABE SKLAR, Illinois Institute of Technology, Chicago, Illinois 60616, Zupnik Families and their Applications

An S-function is a function that maps a non-empty set S into itself. A family Z of S-functions is a Zupnik family if: (1) $f \circ g = g \circ f$ for all f,g in Z: (2) there is an element i in S such that, for any element a in S there is a function g_a in Z such that $g_a(i) = a$. Basic Lemma (special case of a lemma of D. Zupnik in Aequat. Math. 6, 1971, 141-148): If an S-function f commutes with every member of a Zupnik family Z on S, then $f = g_{f(i)}$. Theorem: Any Zupnik family on S is the set of inner translations of some commutative monoid on S, and conversely. These results have wide-ranging applications, e.g. to the construction and characterization of flows. (Received May 29, 1975.)

*726-39-20 J. ACZÉL, Dept. of Pure Math., University of Waterloo, Ontario, Canada N2L3G1 and W. BENZ, Mathematisches Institut, Univ. Hamburg, 2 Hamburg, West Germany. On the harmonic product and a resulting functional equation.

Let A be a field, define $a\rho b := (a^{-1}+b^{-1})^{-1}$ on $G := (A\setminus\{0\})\cup\{\infty\}$. Then (G,ρ) is an abelian group related to the harmonic mean, $n/(a_1^{-1}+\ldots+a_n^{-1}) = n(a_1\rho\ldots\rho a_n)$. We are interested in new associative additions α on G such that $a\rho(b\alpha c) = (a\rho b)\alpha(a\rho c)$ (the law of distributivity) holds. This question leads to the functional equation

(*) $$f(x+f(y)) = f(x)+f(x+y-f(y))$$

for all $x,y \in A$, f being a function of A into A. The general solution of (*) can be given by means of semigroups. In the case A = R all continuous and injective solutions f of (*) are given by $f(x) = (1/\gamma)\ln(1+e^{\gamma x})$, $0\neq\gamma\epsilon R$, if $x \mapsto x-f(x)$ is also assumed to be injective. The corresponding operations α are of form $x\alpha y \Rightarrow \gamma/\ln(e^{\gamma/x}+e^{\gamma/y})$. They are characterized as continuous and associative binary operations such that distributivity and the two laws of cancellation hold. (Received May 16, 1975.)

*726-39-21 T.M.K. DAVISON, McMaster University, Hamilton, Ontario, L8S-4K1
The Hossźu group of a ring

Let R be a ring, and G an abelian group (written additively). A function f: $R \to G$ satisfies Hossźu's equation if, for all pairs $x,y\epsilon R$, $f(x+y)+f(xy) = f(x)+f(y)$. We say f is normalised if $f(0) = 0$. The Hossźu group of R, denoted $\mathcal{H}(R)$ is the abelian group with generators $\langle x\rangle$, for $x\epsilon R$ and relations $\langle x+y-xy\rangle+\langle xy\rangle = \langle x\rangle+\langle y\rangle$, and $\langle 0\rangle = 0$. The mapping $x\mapsto\langle x\rangle$ of R of $\mathcal{H}(R)$ is normalised and satisfies Hossźu's equation. Then there is a unique homomorphism. $\bar{f}: \mathcal{H}(R) \to G$ of abelian groups such that $f = \bar{f}\circ\langle\ \rangle$. Thus to solve Hossźu's equation over R it suffices to know $\mathcal{H}(R)$. Known results are: if R is a field with at least 5 elements then $\mathcal{H}(R) \cong R$ (Blanuša, Daróczy, Świątak, Davison). If $R = \mathbb{Z}$, the rational integers then $\mathcal{H}(\mathbb{Z}) \cong \mathbb{Z}^3\oplus \mathbb{Z}/2\mathbb{Z}$ (Davison). $\mathcal{H}(\mathbb{Z}[i]) \cong \mathbb{Z}^3\oplus \mathbb{Z}/2\mathbb{Z}$. (Received June 3, 1975.)
(Author introduced by J. Aczél.)

*726-39-22 JOHN A. BAKER, University of Waterloo, Ontario, Canada. On the Functional Equation $f(x)g(y) = p(x+y)q(x/y)$.

The problem of solving the equation of the title was raised by I. Olkin in connection with a problem in Statistics concerning the characterization of distributions.

The main results are as follows.

Suppose $f,g,p,q:(0,\infty) \to R$ such that the equation of the title holds for all real $x,y > 0$. Suppose also that there exist subsets S and T of $(0,\infty)$ such that $f(x) \neq 0$ for $x \in S$, $g(y) \neq 0$ for $y \in T$ and either

(i) S and T have positive Lebesgue measure or

(ii) S and T are of second category and satisfy the condition of Baire.

Then $f(x)g(x)p(x)q(x) \neq 0$ for all $x > 0$. Moreover there exist constants $A,B,C,D \in R\backslash\{0\}$ and functions $a: R \to R$ and $m_1,m_2:(0,\infty) \to R$ such that $AB = CD$ and

$$a(x+y) = a(x) + a(y), \qquad m_i(xy) = m_i(x) + m_i(y),$$
$$f(x) = A\exp\{a(x) + m_1(x)\}, \qquad g(x) = B\exp\{a(x) + m_2(x)\},$$
$$p(x) = C\exp\{a(x) + m_1(x) + m_2(x)\}, \quad q(x) = D\exp\{m_1(\tfrac{1}{1+x}) + m_2(\tfrac{1}{1+x})\},$$

for all $x,y > 0$, $i = 1,2$. (Received June 17, 1975.)

*726-39-23 LUDWIG REICH, Universität Graz, Graz, Austria. Analytic iteration and roots of biholomorphic and formally biholomorphic mappings.

A) Let $F:x\to Ax+P(x)$, $x={}^t(x_1,\ldots,x_n)$, where det $A\neq0$, $P(x)$ is a power series vector, ord $P\geq2$, be a contraction, i.e.the eigenvalues of A fulfill the condition $0<|\rho_i|<1$. Then, using certain normal forms of F and the theory of "complete linearization" of F, algebraic criteria are developed characterizing these F's which possess an analytic iteration, i.e.an embedding in an analytic 1-parametric group $F_t:x\to A(t)x+P(t,x)$. Connections with the existence of m-th roots of F are discussed.

B) Let now $F:x\to Ax+P(x)$ be a formally biholomorphic map, i.e.an automorphism of the ring of formal power series in x over $\mathbb{C}$. Then again, using the concept of Lie-series, criteria for the existence of analytic iterations of F can be proved. By an alternative method the F's with an analytic iteration are characterized as those ones which can be transformed to a well defined normal form. From this result some important consequences are derived on the set of all F's with analytic iterations, considered as an algebraic set in a certain space. (Received June 6, 1975.)

726-39-24 M. A. McKIERNAN, University of Waterloo, Waterloo, Ontario, Canada. Characterization of null cone preserving maps by functional equations.

Although the analysis carries over to $\mathbb{R}^n$, for simplicity consider $\mathbb{R}^3$, as a real vector space with $\|x\| = x_1^2 + x_2^2 - x_3^2$. By the light cone with vertex at $a = (a_1, a_2, a_3)$ is meant $\mathcal{C}(a) = \{x \mid \|x - a\| = 0\}$. A map $g: \mathbb{R}^3 \to \mathbb{R}^3$ is null cone preserving if $g\{\mathcal{C}(a)\} \subseteq \mathcal{C}\{g(a)\}$. The following generalizes well-known results due to Zeeman and others, by weakening their assumptions. Let L_i and L_i', for $i = 1,2,3,4$, denote two sets of four distinct generators of $\mathcal{C}(0)$, and set $\mathcal{L} = \bigcup_{i=1}^4 L_i$, $\mathcal{L}' = \bigcup_{i=1}^4 L_i'$. Assume that $g: \mathcal{L} \to \mathcal{L}'$ is such that: (H - 1) $g(L_i) \subset L_i'$ for each i; (H - 2) $g(a) = 0$ iff $a - 0$; (H - 3) for each $a \in \mathbb{R}^3$ there exists $b \in \mathbb{R}^3$ $(a \neq 0 \neq b)$ such that $g\{\mathcal{L} \wedge \mathcal{C}(a)\} \subset \mathcal{L}' \wedge \mathcal{C}(b)$. These assumptions lead to the functional equation $A(\alpha + \beta + \gamma) = B(\alpha) + C(\beta) + D(\gamma)$ for four functions $A, B, C, D: \mathbb{R}\setminus\{0\} \to \mathbb{R}\setminus\{0\}$, subject to the nonvanishing of two quadratic forms. This is shown equivalent to Cauchy's equation on $\mathbb{R}$, thereby characterizing all such maps. (Received June 2, 1975.) (Author introduced by Professor J. Aczél.)

40 Sequences, Series, Summability

*726-40-1 A.K. SNYDER and A. WILANSKY, LEHIGH UNIVERSITY, Bethlehem, Pa.18015
The Bounded Consistency and Sequential Completeness Theorems

The Bounded Consistency theorem of Mazur and Orlicz is given an elementary and easy proof. For an arbitrary sequence u we replace a matrix A by $A^* = (a_{nk}u_k)$. Then for $c_B \supset c_A \cap m$ the assertion "$\lim_B u = \lim_A u$" becomes "c_{B^*} is conull if c_{A^*} is". Since the latter is known (and easy) the result follows. The sequential completeness theorem of Bennett and Kalton (Duke Math J. 39(1972), p. 568, Theorem 3) is well known to be an equivalent result. (Received May 19, 1975.)

41 Approximations and Expansions

*726-41-1 D.J. NEWMAN, Yeshiva University, New York and A.R. REDDY, Michigan State University, East Lansing, MI 48824
Rational Approximation (III)

We announce here the following:

THEOREM 1: For every polynomial $P_n(x)$ of degree at most n, and all large n

$$\left\| |x|e^{-|x|} - \frac{1}{P_n(x)} \right\|_{L_\infty(-\infty,\infty)} \geq c_1 \ (\log n)\, n^{-1} .$$

THEOREM 2: There is a rational function $r_n^*(x) = \frac{P_n(x)}{Q_n(x)}$ of degree at most n for which for all large n

$$\left\| |x| e^{-|x|} - r_n^*(x) \right\|_{L_\infty(-\infty,\infty)} \leq e^{-c_2\sqrt{n}} .$$

THEOREM 3: For every polynomial $P_n(x)$ of degree at most n, and all large n

$$\left\| |x|e^{-x^2} - \frac{1}{P_n(x)} \right\|_{L_\infty(-\infty,\infty)} \geq c_3 \ (\log n)^{1/2}\, n^{-1} .$$

THEOREM 4: There is a rational function $r_n^*(x)$ of degree at most n for which for all large n

$$\left\| |x|e^{-x^2} - r_n^*(x) \right\|_{L_\infty(-\infty,\infty)} \leq e^{-c_4\sqrt{n}} .$$

c_1, c_2, c_3, and c_4 are suitable positive constants. (Received June 16, 1975.)

44 Integral Transforms, Operational Calculus

*726-44-1 A. M. CHAK, West Virginia University, Morgantown, West Virginia, 26506
Some Generalizations of Laguerre Polynomials - III.

A. Erdélyi [Akad. Wiss. Wien. Natur. Kl., IIa, 146 (7 und 8) (1937), 431-467]

used the methods of Laplace transforms to study a generalization of Laguerre polynomials to n variables. This is the third paper of a series of papers in which the author [Chak, A.M.; Mat. Vesnik (7), 22 (1970), 7-13 and 14-18] gives another generalization of Laguerre polynomials in one and in two variables. Recurrence relations, differential equation, a generating function and integral representations have been found. It is interesting to find that the generalized Bessel-Maitland function studied by the author [Chak, A.M.; Ann. Soc. Sci. Bruxelles, 68 (1954), 145-156] plays the same role with respect to these polynomials which the Bessel function does in relation to the classical Laguerre polynomials $L_m^{(\alpha)}(x)$.
(Received April 24, 1975.)

*726-44-2 HELAMAN FERGUSON, Brigham Young University, Provo, Utah 84602.
Reconstruction of plane objects by Farey dissection.

Radon's complete double integral solution to the problem of reconstruction of a real valued function with plane domain from its associated line integrals is taken as a starting point. Certain fairly general and reasonable physical assumptions are made and these combined with the Hardy-Ramanujan-Rademacher circle method (Farey dissection) yield an explicit evaluation of the Radon double integral at lattice points as a limit of finite views. These particular finite views are optimal in the sense that at each lattice point every view is represented. That is, all evaluated points are equally weighted with physical observation strips (this is typically not at all the case with current reconstruction techniques). Specific applications are to multiple view parallel beam discrete sensor diagnostic experiments, for example, laser reconstruction of plasma (ionized gas) cross sections. (Received June 2, 1975.)

46 Functional Analysis

*726-46-1 CHARLES A. AKEMANN - Department of Mathematics, University of California, Santa Barbara, CA 93106. Title: "The spectrum of a derivation of a C^*-algebra."

Let δ be a $*$-derivation of a C^*-algebra A. Let Γ be a family of factor representations of A with distinct kernels and faithful direct sum. For each $\pi \in \Gamma$ let a_π be an operator on the Hilbert space associated with π such that $\pi(\delta(b)) = a_\pi \pi(b) - \pi(b) a_\pi$. The main result is that the spectrum of δ (as an operator on A) contains the closure of $\cup \{\lambda - \gamma : \lambda, \gamma \in \text{spectrum } (a_\pi) \text{ and } \pi \in \Gamma\}$ and that the reverse inclusion holds if Γ is finite. Several results on derivations and automorphisms are immediate corollaries of this result. (Received June 10, 1975.)

*726-46-2 DR. ABDULLAH H. AL-MOAJIL, University of Petroleum & Minerals, Dhahran, Saudi Arabia. The Commutants of Relatively Prime Powers of Elements in Operator Algebras.

Let R be a ring and $A(R) = \{x \in R : x \text{ belongs to the second commutant of } \{x^n, x^{n+1}\} \text{ for all integers } n > 1\}$. It is shown that in a prime ring R, $A(R) = R$ if and only if R has no nilpotent elements. The set $A(U)$ is studied for some special $*$-algebras. It is shown that the normal elements of a proper $*$-algebra U belong to $A(U)$. If U is also prime then $A(U) = \{x \in U : x \text{ belongs to the second commutant of } \{x^n, x^{n+1}\} \text{ for some } n > 1\}$. The set $A(B(H))$ is studied, where $B(H)$ is the algebra of bounded operators on a Hilbert space H. Necessary and sufficient conditions for some special types of operators to belong to $A(B(H))$ are obtained. (Received May 5, 1975.) (Author introduced by Dr. Harold Davenport.)

*726-46-3 FRANCIS SULLIVAN, The Catholic University of America, Washington, D.C. 20064. Some geometrical relatives of the Radon-Nikodym property. Preliminary report.

Geometrical conditions on Banach spaces which imply, are

are implied by or are equivalent to the Radon-Nikodym property are discussed. Sample result: Let X be a Banach space and $C \subseteq X$ a closed bounded convex subset. For $f \in X^*$ define $M(f,C) \equiv \sup \{f(c) | c \in C\}$. We say that X^* is <u>malleable</u> iff for each C and $\epsilon > 0$ there is an $f \in X^*$ with $M(f,C) > 0$ such that for some $\delta(\epsilon) > 0$ and all $0 < \lambda \leq \delta$

$$\frac{M(f + \lambda g, C) + M(f - \lambda g, C) - 2M(f,C)}{\lambda} < \epsilon$$

for all $||g|| \leq 1$.

<u>Theorem</u>. X^* is malleable iff X has the Radon-Nikodym property.

(Received May 12, 1975.)

*726-46-4 G. A. EDGAR, Northwestern University, Evanston, Illinois 60201
<u>Noncompact extremal integral representations</u>.

Let X be a closed bounded convex subset of a Banach space E. Define an ordering $\prec$ for tight Borel probability measures on X as follows: $\mu \prec \lambda$ iff there is a dilation T on X such that $T(\mu) = \lambda$. If X has the Radon-Nikodym property, then for every $a \in X$ there is a probability measure μ there is a probability measure μ which is maximal in this ordering such that $\int x \, d\mu(x) = a$. If X is separable, then $\mu(\mathrm{ex}X) = 1$ for every maximal probability measure μ. For nonseparable X, maximal measures need not be on the extreme points in this sense. However, if X is weakly compact, then a maximal measure μ <u>is</u> on ex$X$ in the sense that $\mu(B) = 1$ for every weak Baire set $B \supseteq \mathrm{ex}X$. (Received May 19, 1975.)

726-46-5 WILLIAM J. DAVIS, The Ohio State University, Columbus, Ohio 43210.
<u>Spaces having and spaces failing the Radon-Nikodym property</u>.

This is intended to be a brief catalog of spaces with and spaces without the Radon-Nikodym property. A complete soft description of spaces having the Radon-Nikodym property is still missing. It has been conjectured that X has RNP if and only if its separable subspaces embed into separable conjugate spaces. The complete geometric characterization of spaces having RNP will be left to other speakers. (Received May 27, 1975.)

*726-46-6 H. KHARAGHANI, Pahlavi University, Shiraz, Iran.
<u>The support of an invariant mean.</u>

Let S be a topological semigroup with separately continuous multiplication. It is shown that the support of a left invariant mean on a left translation invariant left introverted closed subalgebra X of $CB(S)$ is a left ideal of S_1, the maximal ideal space of X under Arens multiplication. This generalizes Theorem 4.3 of wilde and witz in ["Invariant mean and the Stone-Cech compactification", Pacific J. of Math. 21 (1967), pp. 577-586].

(Received June 3, 1975.)

726-46-7 JAMES B. COLLIER, University of Southern California, Los Angeles, Ca. 90007
The dual of a space with the Radon-Nikodym Property. Preliminary report.

Let X be a Banach space. It is known that X^* has the Radon-Nikodym Property (RNP) if X is an Asplund space. We show that X has RNP if and only if X^* is a weak*-Asplund space; that is, each weak* lower semi-continuous convex function on X^* is Fréchet differentiable on a dense G_δ subset of its domain of continuity. This characterization is used to prove several other properties of spaces with RNP. (Received June 5, 1975)

726-46-8 J. J. UHL, JR., University of Illinois, Urbana, Illinois 61801. Weak compactness in the space of Bochner integrable functions, a sad state of affairs.

In the late thirties, Dunford and Pettis characterized relatively weakly compact subsets of $L_1(\mu)$ (μ finite) as the bounded uniformly integrable sets. Much later several people noted that their proof also works in the context of $L_1(\mu,X)$ when X is a reflexive B-space. Rather recently some other people reworked the Dunford-Pettis proof in the context of $L_1(\mu,X)$ when X and X^* have the Radon-Nikodym property (RNP). Here we shall look at some known counter-examples and try to isolate the inherent difficulties. The main troubles: when X^* lacks RNP, the dual of $L_1([0,1],X)$ is not $L_\infty([0,1],X^*)$; when X^* has RNP and X lacks RNP, then there is always a weakly Cauchy sequence (hence uniformly integrable, etc.) that converges to a measure without a derivative. (Received June 11, 1975.)

726-46-9 Robert R. Phelps, University of Washington, Seattle, WA 98195. Geometric characterizations of the RNP (Part II).

A number of characterizations of the Radon-Nikodym property have been obtained (by many authors) which have the following general form: The Banach space E has the RNP if and only if every bounded closed convex subset of E is the closed convex hull of its strongly exposed points (extreme points, denting points). This lecture (which will complement an earlier one by P. D. Morris) will be a survey of such results. (Received June 12, 1975.)

726-46-10 STEVEN F. BELLENOT, Florida State University, Tallahassee, Florida 32306. Compact operators into Hilbert space, Preliminary report.

Let X, Y be Banach spaces, let ℓ_2 be Hilbert space and let $\mathcal{B}$ be the class of all infinite dimensional Banach spaces. Let $B(X,Y)$ be all bounded linear operators from X to Y and let $\Pi_1(X,Y)$ be the subset of all absolutely summing operators. Let $\nu(X)$ be the variety (TAMS 172(1972) 207-230) generated by $X \in \mathcal{B}$; and let SH be the variety of Schwartz Hilbertian spaces. Theorem If $T \in B(X,\ell_2)$ is compact, then there exists a subspace $Z \subset Y$ and $U \in B(X,Z)$, $V \in B(Z,\ell_2)$ such that $T = VU$. Corollary 1. $SH = \cap\, \nu(X)(X \in \mathcal{B})$. Corollary 2. If $B(X,Y) = \Pi_1(X,Y)$, then $B(X,\ell_2) = \Pi_1(X,\ell_2)$. (Received June 12, 1975.)

*726-46-11 ELIAS SAAB, Université de Paris, 4 Place Jussieu, 75230 Paris-Cedex 05, France. On the Radon-Nikodym property in Fréchet spaces and applications.

Rieffel introduced the notion of dentability and proved that a Banach space has the Radon-Nikodym property (R.N.P.) whenever it is dentable. From results due to Davis, Phelps, Maynard and Huff, it appears that a Banach space E has the R.N.P. iff E is dentable, iff E is s-dentable, iff every bounded closed convex subset of E is the closed convex hull of its strongly exposed points. In this work we prove that all these results remain true if E is a Fréchet space except the last assertion in which strongly exposed points must be replaced by strongly extreme points. Several applications are given, namely we obtain an extension of two results due to Namioka and Peck by replacing in their theorems extreme points by strongly extreme points. We prove also that the space $\ell^1_I\{G\}$ of absolutely summable family in a Fréchet space G having the R.N.P. has the R.N.P. As consequences we obtain a partially affirmative answer to a question posed by Diestel and we obtain the following theorem: If F is a Banach space and K a compact Hausdorff space then the dual of $C(K,F)$ has the R.N.P. iff K is scattered and the dual of F has the R.N.P. (Received June 12, 1975.) (Author introduced by R.E. Huff.)

*726-46-12 PETER MORRIS, The Pennsylvania State University, University Park, Pennsylvania 16802. Geometric Characterizations of the Radon-Nikodym Property.

We shall be concerned with surveying some characterizations of the Radon-Nikodym Property (RNP) for a Banach space X in terms of geometric properties of

its bounded, closed (convex) subsets. The original theorem of this type was that X has the RNP iff all its closed bounded subsets are dentable in the sense of Rieffel. (The contributors to proving this theorem were Rieffel, Maynard, Davis, Phelps, and Huff.) More recent results have involved the extremal structure of subsets of X and were motivated by the well known question of Diestel whether the Krein-Milman Property implies the RNP. (The converse was established by Lindenstrauss.)
(Received June 13, 1975.)

726-46-13 ROBERT E. ZINK, Purdue University, W. Lafayette, IN 47906 and JAMES E. SHIREY, Ohio University, Athens, Ohio 45701. Unconditional bases for certain Banach function spaces.

According to a theorem of antiquity, the Haar system constitutes an unconditional Schauder basis for each of the spaces $L^p[0,1]$, $p > 1$. The proof, due to Marcinkiewicz, amounts to a demonstration that a certain norm on L^p defined in terms of the Haar functions is equivalent to the ordinary norm. Motivated by this result, Gaposhkin proved that a Schauder basis for L^p, $p > 1$, is unconditional precisely when the norm generated by that basis in the same way that the Marcinkiewicz norm is engendered by the Haar system is equivalent to the customary norm. Subsequently, Gaposhkin and others generalized this theorem to include the reflexive Orlicz spaces as well. In the present note it is shown that the analogue of the Marcinkiewicz-Gaposhkin result continues to hold in a very broad class of function spaces. (Received June 16, 1975.)

726-46-14 HASKELL P. ROSENTHAL, University of Illinois, Urbana, Illinois 61801. Some problems concerning subspaces of L^1, quotients of $C[0,1]$ and the Radon-Nikodym property. Preliminary report.

Some open problems will be discussed which connect the isomorphic structure of Banach subspaces of L^1 and quotients of $C[0,1]$ to the possession of the Radon-Nikodym property. (Received June 16, 1975.)

726-46-15 MICHAEL EDELSTEIN, Dept.of Math., Dalhousie Univ., Halifax, N.S., Canada, B3H 3J5. Convex Hulls of Strongly Exposed Points.

A Banach space X is said to have property (σ) if every closed and bounded convex subset is the closed-convex-hull of its strongly exposed points; a conjugate Banach space $X = E^*$ is said to have property (σ^*) if every weak*-compact convex subset is the weak*-closed-convex-hull of those of its points which are strongly exposed by funtionals form E .

Proposition. A conjugate Banach space has property (σ) whenever it has property (σ^*).

(If E is an SDS then $X = E^*$ has (σ^*) by a result of E. Asplund, Acta Math. 121(1968), 31-49.) (Received June 16, 1975.)

726-46-16 FREDERICK K. DASHIELL, Jr., California Institute of Technology, Pasadena, California 91109. A theorem on weak compactness of measures with application to the Baire classes, Preliminary report.

An extension of the σ-Stonian version of Phillips' lemma is given which produces all the Baire classes of finite or countable order over any topological space as new examples of Grothendieck spaces (which are Banach spaces B having every w* convergent sequence in the dual B^* be weakly convergent). (Received June 17, 1975.)

*726-46-17 DAGMAR HENNEY, George Washington University, Washington, D.C. 20052. Properties of locally convex spaces generated by classes of bounded sets.

One considers a locally convex space X and a collection $U_n \ni 0$ (n-arbitrary natural number) representing a closed absolutely convex neighborhood of X. X is said to be a (DF) space if whenever the intersection $\cap U_n$ $(n = 1,2,\ldots)$ absorbs every bounded set of X then the intersection $\cap U_n$ is a neighborhood of zero as well. It is known that bounded sets of (DF) spaces can be replaced by arbitrary collections of bounded sets and still yield (DF) spaces. Using that approach several new results are obtained. (Received June 2, 1975.)

726-46-18 DAGMAR HENNEY and W. LINCOLN, George Washington University, Washington, D.C. 20052. Locally convex spaces generated by bounded sets.

One considers a locally convex space X and a collection $U_n \ni 0$ (n-natural number) representing a closed absolutely convex neighborhood of X. X is a (DF) space if whenever $\cap U_n$ $(n = 1,2,\ldots)$ absorbs every bounded set of X then $\cap U_n$ yields a zero-neighborhood as well. It is known that bounded sets of (DF) spaces can be replaced by any class of bounded sets and still yield (DF) spaces. This approach yields several new results. (Received June 2, 1975.)

726-46-19 Pettis, B. J., University of North Carolina, Chapel Hill 27514

Comments on Radon-Nikodym and allied propositions

Historical and speculative comments on Radon-Nikodym, Price-Krein-Milman, etc. in various settings. (Received June 17, 1975.)

726-46-20 HEINRICH P. LOTZ, University of Illinois, Urbana, Illinois 61801. The Radon-Nikodym property in dual Banach lattices. Preliminary report.

We characterize Banach lattices whose dual have the Radon-Nikodym property in terms of subspaces. Theorem: For a Banach lattice E the following assertions are equivalent: (a) E' has the Radon-Nikodym property; (b) $L^1([0,1])$ does not embed in E'; (c) ℓ^1 does not embed in E; (d) no closed sublattice of E' is lattice isomorphic to c_0 or $L^1([0,1])$. A similar theorem holds for subspaces of Banach lattices whose order intervals are weakly compact or whose duals have weak order units. (Received June 17, 1975.) (Author introduced by Professor J. J. Uhl, Jr.)

47 Operator Theory

726-47-1 RONALD RIETZ, Gustavus Adolphus College, St. Peter, Minnesota 56082. A note on compact classes of ℓ^2 operators. Preliminary report.

It has been observed that a bounded linear map from ℓ^q to ℓ^p is compact whenever $q > p$ [see for instance, the recent Springer monograph "Classical Banach spaces" by Lindenstrauss and Tzafriri]. We expand upon this result as follows. Definition. Let $T: \ell^2 \to \ell^2$ be compact. (All operators here are bounded and linear.) The $\mathfrak{S}_r$ norm of T is $|T|_r = (\Sigma \lambda_j^r)^{1/r}$ where $\lambda_1 \geq \lambda_2 \geq \ldots \geq 0$ are the eigenvalues, repeated according to multiplicity, of the (positive definite, compact) operator $(T^*T)^{\frac{1}{2}}$. If the sum diverges, then $|T|_r = \infty$. $\mathfrak{S}_r$ is the space of all T for which $|T|_r$ is finite. Some theorems are: A. If $T: \ell^2 \to \ell^2$ has norm 1 as a map from ℓ^2 to ℓ^p for some $1 \leq p < 2$, then T has norm $\leq$ 1 in $\mathfrak{S}_{2p/(2-p)}$. B. If T has norm ≤ 1 in $\mathfrak{S}_{2p/(2-p)}$ for some $1 \leq p < 2$, then there exists a unitary operator $U: \ell^2 \to \ell^2$ such that UT has norm ≤ 1 as a map from ℓ^2 to ℓ^p. C. If $T: \ell^2 \to \ell^2$ has norm ≤ 1 as a map from c_0 to ℓ^p, for some $1 \leq p \leq 2$, then T has norm $\leq 3^{(2-p)/2p}$ in $\mathfrak{S}_p$. (Received June 17, 1975.)

726-47-2 EARL BERKSON, University of Illinois, Urbana, Illinois 61801 and HORACIO PORTA, University of Illinois, Urbana, Illinois 61801. One-parameter groups of isometries on Hardy spaces of the torus.

The strongly continuous one-parameter groups of isometries on H^p of the torus $(1 \leq p < \infty,\ p \neq 2)$, as well as their infinitesimal generators, are classified and concretely described. (Received May 19, 1975.)

*726-47-3 EARL BERKSON, University of Illinois, Urbana, Illinois 61801, RICHARD FLEMING and JAMES JAMISON, Memphis State University, Memphis, Tennessee 38152, Hermitian operators and groups of isometries on some Banach spaces.

Let $L^p(\Omega\text{-}H)$ denote the Banach space of measurable functions F on a finite measure space Ω taking values in a separable Hilbert space H such that $||F(x)||^p$ is integrable. The Hermitian operators are shown to be multiplications by operator valued functions $A(t)$, when $A(t)$ is a self adjoint operator on H for almost all t. The theorem of Cambern [Pac.J.Math. 55(1974),1-18] on isometries of $L^p(\Omega,H)$ can be obtained as a corollary.

Next consider the space X to be one of the Banach spaces $C(H)$(= the sapce of compact operators on H), $C_1(H)$(= the space of trace-class operators with norm $||A||_1 = tr(|A|)$), $B(H)$(= the space of all bounded operators on H). The uniformly continuous as well as the strongly continuous one-parameter groups of isometries of X are described in terms of action on H. Every such strongly continuous group on $B(H)$ has a bounded infinitesimal generator. For $X = C(H)$ or $C_1(H)$, the infinitesimal generator of an arbitrary strongly continuous one-parameter group of *-automorphisms of X (regarded as an algebra) is described in terms of action on H.

The results concerning $L^p(\Omega,H)$ represent work of the latter two authors while the remainder is the result of joint work by all three authors. (Received May 27, 1975.)

*726-47-4 **JOSEPH G. STAMPFLI, California Institute of Technology, Pasadena, California 91109.** **The maximal numerical range.**

Let $\mathcal{H}$ be a finite dimensional Hilbert space. Then $W_0(T)$, the maximal numerical range of T, is defined to be

$$\{(Tx,x) : x \in \mathcal{H}, \|x\| = 1 \text{ and } \|Tx\| = \|T\|\}$$

If $\mathcal{H}$ is infinite dimensional $W_0(T)$ can be defined in an analogous fashion using limits, or by means of states on the C^* algebra $\mathcal{L}(\mathcal{H})$. Properties of $W_0(T)$ are discussed. Applications are given to problems in operator theory and to derivations on operator algebras.
(Received May 27, 1975.)

*726-47-5 STEPHEN A. MCGRATH, US Naval Academy, Annapolis, MD 21402 An Abelian Ergodic Theorem for Semigroups in L_p Space

Let (X,Σ,μ) be a σ-finite measure space and $L_p = L_p(X,\Sigma,\mu)$, $1 < p < \infty$, the usual Banach space. Let $\{T_t : t \geq 0\}$ be a strongly continuous semigroup of positive L_p contractions for some $1 < p < \infty$. Denote $\int_0^\infty e^{-\lambda t}\, T_t f\, dt$ by $R_\lambda f$, $\lambda > 0$, and set $f^* = \sup_{\lambda>0} |\lambda R_\lambda f|$. By suitably modifying a dilation theorem due to M.A. Akcoglu [A Pointwise Ergodic Theorem in L_p Spaces, Canad. J. Math.(to appear)], we obtain the estimate $\| f^*\| \leq (p/p-1)\| f||$, $f \in L_p$. This extends a result of R. Sato [An Abel-Maximal Ergodic Theorem for Semi-Groups, Pacific J. Math. 51 (1974), pp. 543-547]. We use this estimate of f^* to show $\lim_{\lambda\to 0+} \lambda R_\lambda f(x)$ exists and is finite a.e. on X. (Received June 2, 1975.)

*726-47-6 JOHN MACBAIN, Air Force Institute of Technology, Wright-Patterson AFB, Ohio 45433. Existence of Large Solutions to Nonlinear Operator Equations, Preliminary Report.

In a real Banach space B consider the equation $Lu = \lambda u + H(\lambda,u)$ where L is linear (bounded or unbounded), λ is real, and H is continuous and $o(||u||)$ uniformly on bounded λ intervals. Let λ_0 be an isolated normal eigenvalue of L having odd algebraic multiplicity. The author showed [Notices, April 1975] that $(\lambda_0,0)\varepsilon RxB$ is a bifurcation point for our equation possessing a maximal continuous branch C_{λ_0} which obeys an alternative theorem. The following result can now be established. A function $f(\lambda_0,L,H) \to (0,\infty)$ exists such that C_{λ_0} contains a point (λ,u) with $||u|| \geq f(\lambda_0,L,H)$. In particular, if H is sufficiently small or λ_0 is sufficiently far from the rest of the spectrum of L, then large solutions of our equation exist. (Received June 6, 1975.)

*726-47-7 M. RADJABALIPOUR and H. RADJAVI, Dalhousie University, Halifax, N.S. On the Boundary of Numerical Ranges. Preliminary report.

Some results on the geometry of the numerical range $W(T) = \{(Tx,x): ||x|| = 1\}$ of a bounded operator T on a separable Hilbert space are discussed. In particular, the following question raised by Joel Anderson is answered in the affirmative: can a closed half-disc be attained as such a numerical range?
(Received May 30, 1975.)

*726-47-8 JOEL ANDERSON, California Institute of Technology, Pasadena, California 91125 Which sets are numerical ranges?

Let H be a complex Hilbert space of dimension $n \leq \aleph_0$. For a bounded linear operator T acting on H, let $W(T) = \{(Tx,x) : x \in H, \|x\| = 1\}$. Question : If δ is a (necessarily convex and bounded) subset of the complex plane, is there an operator T such that $W(T) = \delta$?

Theorem. If $\dim(H) = n < \aleph_0$, $\delta = \{z : |z| \leq 1\}$, $W(T) \subset \delta$ and $W(T) \cap \partial\delta$ contains at least $n + 1$ distinct points, then $W(T) = \delta$. Similar results hold if $\dim(H) = \aleph_0$ and T is a compact operator. However, an example due to Radjabalipour and Radjavi shows that if T is an arbitrary operator and $\dim(H) = \aleph_0$, the situation is more complicated. (Received June 13,1975.)

*726-47-9 JOHN J. BUONI, Youngstown State University, Youngstown, OH 44555. A Fractional Mapping Theorem for Essential Spectra

Let T be a closed operator on a Hilbert space H, such that $\alpha \in \rho(T)$, the resolvent of T. Set $A = (T - \alpha I)^{-1}$ a bounded operator on H. For $\mu \neq 0$, define λ such that $(\lambda - \alpha)\mu = 1$. A. Taylor, in "Spectral Theory of Closed Distributive Operators" Acta. Math. 84(1950), shows that $\lambda \in \sigma(T)$ iff $\mu \in \sigma(A)$, where $\sigma(S)$ denotes the spectrum of S. The above statement can be generalized to the various definitions of essential spectrum.

Theorem. $\lambda \in$ essential spectrum of T iff $\mu \in$ essential spectrum of A for the various definitions of the essential spectrum.

A number of immediate corollaries can then be derived. (Received June 13,1975.)

*726-47-10 ARTHUR LUBIN, Northwestern University, Evanston, Illinois 60201, Isometries Induced by Composition Operators.

For $c > 0$, $-\infty < t < \infty$, let $\alpha = (t + i(c-1))(t + i(c+1))^{-1}$ and $\phi_{c,t}(z) = (1 - \bar{\alpha})(1 - \alpha)^{-1}(z - \alpha)(1 - \bar{\alpha}z)^{-1}$, and define $V_{c,t}: H^p \to H^p$ by $(V_{c,t}f)(z) = (\phi'(z))^{1/p}f(\phi(z))$. We consider relations between the set of isometries $\{V_{c,t}\}$ and $\{\Delta_r(z)H^p\}$, where $\Delta_r(z) = \exp[-r(1 + z)(1 - z)^{-1}]$.

Theorem 1. $V_{c,t}$ is unitarily equivalent to $T_{c,t}$ on $H^p(\Pi^+)$, where $\Pi^+ = \{w|\text{Re } w > 0\}$ and $(T_{c,t}f)(w) = c^{-1/p}f((w - it)c^{-1})$.

Corollary. $V_{c,t}$ is a bilateral shift of infinite multiplicity on $H^2(D)$.

Theorem 2. Let $\mathscr{A}$ denote the weakly closed algebra generated by $\{V_{1,t}\}_{t\in\mathbb{R}}$ on $H^2(D)$. $\mathscr{A}$ is a reflexive algebra.

In the proof of Theorem 2, we give a spectral resolution of $\mathscr{A}$. (Received June 16, 1975.)

49 Calculus of Variations and Optimal Control

*726-49-1 WILLIAM HAGER, University of South Florida, Tampa, Florida 33620 and LARRY HOROWITZ, John Hopkins University, Silver Spring, Maryland 20901. Convergence And Stability Properties Of The Discrete Riccati Operator Equation

The convergence properties for the solution of the discrete time Riccati matrix equation

are extended to Riccati operator equations such as arise in a gyroscope noise filtering problem. Stabilizability and detectability are shown to be necessary and sufficient conditions for the existence of a positive semi-definite solution to the algebraic Riccati equation which has the following properties: (1) it is the unique positive semi-definite solution to the algebraic Riccati equation, (2) it is converged to geometrically in the operator norm by the solution to the discrete Riccati equation from any positive semi-definite initial condition, and (3) the associated closed loop system converges uniformly geometrically to zero and solves the regulator problem. The stability of the steady state Kalman-Bucy filter is also proved whenever there exists a solution to the algebraic Riccati equation and a detectability condition holds. These stability results are then generalized to time varying problems; also it is shown that even in infinite dimensions, controllability implies stabilizability.
(Received May 19, 1975.)

*726-49-2 M. RAZZAGHI, Pahlavi University, Shiraz, Iran. On the sufficient conditions for control problems with time delay.

In optimal control theory a known necessary condition which together with the maximum principle and Legendre-Clebsch condition constitute sufficient conditions for optimality is the Jacobi or conjugate point condition. This paper is concerned with the necessary and sufficient conditions for the linear quadratic optimization problem with input delay. The proposed method reduces the Jacobi condition to the solution of the matrix differential equations of the type that have been investigated in the field of aircraft flutter analysis and whose solutions can be either obtained analytically or computationally.
(Received June 3, 1975.) (Author introduced by Dr. M.H. Afghahi.)

50 Geometry

*726-50-1 C. M. PETTY, University of Missouri, Columbia, Missouri 65201. Characterization of geodesic arcs.

There exist metric arcs on which the usual metric first curvatures vanish identically and yet no subarc is a metric segment. A basic problem for a given metric first curvature is to find conditions on a metric space M so that a geodesic arc in M is characterized by the vanishing of its first curvature. A number of different constraints on M are given in the literature which solve this problem for the Menger curvature. Here, we show that all such known constraints on M imply the validity locally of a certain generalized ptolemaic inequality and that this inequality alone suffices to solve this problem.
(Received May 9, 1975.)

*726-50-2 R. ARTZY, University of Haifa, Haifa 31999, Israel. Coordinates for Minkowski planes.

In a Minkowski plane M with "right parallelism" and "left parallelism of points, we postulate the transitivity of right-parallelism-preserving translations of points onto left-parallel points. Then M may be coordinatized by means of a quaternary ring (S,Q), where S is a set containing elements 0 and 1, and Q a quaternary operation. Conditions are established for Q to characterize M. For the coordinatization, let X be a distinguished point. Then there is a one-one correspondence between all points not parallel to X and the elements of S^2. Each circle not through X contains all points (x,y) such that y=Q(x,a,b,c) with constant a, b, c in S. Each circle through X contains all points (x,y) with x=Q(t,0,0,1), y=Q(t,0,n,m), which is equivalent to y=T(x,m,n) in a Hall ternary ring (S,T) coordinatizig the affine plane that is the internal structure of M with regard to X. If M is miquelian, y=Q(x,a,b,c) becomes (x-a)(y-b)=c. (Received May 12, 1975.)

726-50-3 MICHAEL GOLDBERG, 5823 Potomac Ave., N.W., Washington, D.C., 20016.

On the space-filling hexahedra.

A space-filling polyhedron is one whose replications can be packed to fill three-space completely. The space-filling tetrahedra and pentahedra have been previously investigated, [J. Combinatorial Theory, Sect. A, 13(1972)437-443; 16(1974)348-354; 17(1974)375-378.] The search is here extended to the convex space-filling hexahedra. These are obtained by the division of known space-fillers into congruent parts, and by the combination of two or more space-fillers.
(Received May 30, 1975.)

726-50-4 PAUL ERDÖS, Hungarian Academy of Sciences, Budapest, Hungary and L.M. KELLY, Michigan State University, East Lansing, Michigan 48824. Order embeddings in E^n. Preliminary report.

If (S,d) is a distance space, C a subset of E^n and T a mapping; $T: S \to C$ such that $d(x,y) \geq d(u,v)$ iff $e(T(x),T(y)) \geq e(T(u),T(v))$ then S is said to be order embedded in E^n by T. We make a number of preliminary observations about the class of distance spaces order embeddable in E^n.

Holman has shown that $(S,d), |S| = n + 1$ is order embeddable in E^n if and only if (S,d) is not ultrametric.

Calling an (S,d) scalene if $d(x,y) = d(u,v)$ implies $\{x,y\} = \{u,v\}$ we prove

1. Any scalene $(S,d), |S| = n + 2$ is order embeddable in E^n.
2. There exists a scalene $(S,d), |S| = n + 3$ not order embeddable in E^n for $n = 1,2,3$.
3. There exists a scalene $(S,d), |S| < \infty$ not order embeddable in E^n.

We hope to show that in some sense most (S,d)'s are not order embeddable in E^n. (Received June 12, 1975.)

*726-50-5 **R.H. BING and MICHAEL STARBIRD, University of Texas at Austin, Austin, Texas 78712. Linear Isotopies.**

Let C be a complex with a triangulation T. A linear isotopy $h_t: C \to E^n$ $(t \in [0,1])$ is a continuous family of simplicial embeddings of C into E^n. It is shown that for every simplicial embedding f of a triangulated disk D into E^2, there is a linear isotopy $h_t: D \to E^2$ $(t \in [0,1])$ such that 1) $h_0 = f$, 2) $h_1(D)$ is a small convex disk contained in $f(D)$, and 3) $h_t(D)$ $(t \in [0,1])$ is monotonically decreasing. A consequence of this theorem is that for any two simplicial embeddings f and g of a triangulated disk D into a connected open subset U of E^2 with the same orientations, there is a linear isotopy $h_t: D \to E^2$ $(t \in [0,1])$ so that $h_0 = f$, $h_1 = g$, and for every t in $[0,1]$ $h_t(D)$ is contained in U.

It is also shown that if C is a complex and f and g are two PL embeddings of C into E^3 so that there is an orientation preserving homeomorphism G of E^3 with $G \circ f = g$, then there is a triangulation of C with respect to which there is a linear isotopy $h_t: C \to E^3$ $(t \in [0,1])$ so that $h_0 = f$ and $h_1 = g$. (Received June 16, 1975.)

726-50-6 HELEN SKALA, University of Massachusetts, Boston, Massachusetts 02125. A foundation of hyperbolic geometry.

Using only the lattice operations of join and meet and postulates concerning them, F. Jenks and H. DeBaggis developed the theory of linear and planar order in the hyperbolic plane. J. Abbott completed a foundation for hyperbolic geometry with two additional postulates, developing the theories of congruency and perpendicularity. One of his assumptions, a restricted form of the fundamental law of projective geometry can be replaced by the assumptions of the laws of Pappus and Desargues(restricted to the case where the lines in question meet). (Received June 17, 1975.)

*726-50-7 DAVID C. KAY, University of Oklahoma, 601 Elm, Room 423, Norman, Oklahoma 73069. Arc curvature in metric spaces.

In the author's paper (*) Arcs of vanishing metric curvature, Journal für die reine und angewandte Mathematik, Vol. 219 (1965), pp 214-220 (with L. M. Kelly) the transverse curvature $\kappa_T(p)$ of an arc A in a metric space having locally unique segments is defined at a point p on A by $\lim \kappa_T(q,r,s) = \kappa_T(p)$, the limit taken with respect to q,r,s on A converging to p such that $qr = rs$, where $\kappa_T(q,r,s) = 8rm_{qs}/qs^2$, with m_{qs} denoting the metric midpoint between q and s and xy denoting the metric. Here, comparisons are made between κ_T and the previously introduced Menger and Haantjes-Finsler curvatures κ_M and κ_M in various metric spaces (see (*)).

THEOREM 1: For arcs in euclidean space the existence of κ_M implies that of κ_T and the functions agree, and the existence of κ_T implies that of κ_H, and the functions agree.

COROLLARY: For arcs in the euclidean plane the curvature functions κ_M, κ_H, and κ_T are equivalent.

THEOREM 2: In a Minkowski plane with strictly convex unit circle U, the existence of $\bar{\kappa}_M$ relative to the base euclidean metric implies that of κ_T and of κ_M. The functions have distinct values at points on a given arc, depending on U.

The sharp distinction between the formulas obtained in Theorem 2 suggests that the curvature functions κ_T and κ_M are independent in abstract metric spaces. (Received June 17, 1975.)

52 Convex Sets and Geometric Inequalities

*726-52-1 DANIEL H. WAGNER, Daniel H. Wagner, Associates, Station Square One, Paoli, Penn. 19301, Semi-Compactness with Respect to a Euclidean Cone, Preliminary Report.

Fix a closed convex cone Γ in R^n. Say $A \subset R^n$ is Γ semi-closed if cl $A \subset A + \Gamma$, Γ semi-bounded if Γ contains the asymptotic cone of cl co A, i.e., $\Gamma \supset \Delta \equiv \{\gamma : \text{cl co } A + \gamma \subset \text{cl co } A\}$, strongly Γ semi-bounded if interior $\Gamma \supset \Delta \setminus \{0\}$, and Γ semi-compact if every covering of A by open sets of the form $R^n \setminus (C - \Gamma)$, for some $C \subset R^n$, has a finite subcovering. Theorem 1. If $A \subset R^n$ is Γ semi-compact, it is Γ semi-closed and Γ semi-bounded. Theorem 2. If $A \subset R^n$ is Γ semi-closed and strongly Γ semi-bounded, then A is Γ semi-compact. Proof of the following lemma needed for Theorem 2 was supplied by V. Klee: If A is strongly Γ semi-bounded and $\gamma \in$ interior Γ, there exists $r \geq 0$ such that $A \subset -r\gamma + \Gamma$. This lemma and Theorem 2 fail if "strongly" is omitted. Let T be a topological space and say $f: T \to R^n$ is Γ semi-continuous if $f^{-1}(C - \Gamma)$ is closed whenever $C - \Gamma$ is closed. Theorem 3. If T is compact and $f: T \to R^n$ is Γ semi-continuous, then f(T) is Γ semi-compact.
(Received January 31, 1975.)

726-52-2 H. S. WITSENHAUSEN, Bell Laboratories, Murray Hill, New Jersey 07974. A support characterization of zonotopes. Preliminary report.

A convex polytope is a zonotope if and only if its support function satisfies Hlawka's inequality. It follows that, for Minkowski spaces with piecewise linear norm, the quadrilateral and hypermetric properties are equivalent.
(Received May 8, 1975.)

726-52-3 W.A. KIRK, University of Iowa, Iowa City, Iowa 52242

Carisit's fixed point theorem and metric convexity. Preliminary report.

Recently J.Caristi proved that if (M,d) is a complete metric space, f an arbitrary mapping of M into M, and ϕ a lower semicontinuous mapping of M into the non-negative real numbers for which $d(x,f(x)) \leq \phi(x) - \phi(f(x))$, $x \in M$, then f has a fixed point in M. In this paper we discuss some of the implications of this theorem. In particular we observe that a critical part of K. Menger's original proof that each two points of a complete and metrically convex metric space are joined by a metric segment of the space is implicit in Caristi's result. Caristi's theorem is characteristic of complete metric spaces in the sense that if M is not complete, then there exists a fixed-point free function f mapping M into M along with a continuous function ϕ mapping M into non-negative real numbers such that for each $x \in M$, $d(x, f(x)) \leq \phi(x) - \phi(f(x))$. (Received May 12, 1975.)

726-52-4 KENNETH B. STOLARSKY, University of Illinois, Urbana, Illinois 61801. Nearly uniform distribution of points on a sphere. Preliminary report.

Problem. Let $F(v_1, \ldots, v_N)$ be a real function of N unit vectors in E^m. When does extremality of F imply that the $\pm v_i$ are nearly uniformly distributed?
Example. If $m = N - 1$ then the sum of the squares of all inner products of the v_i is minimal if and only if the $\pm v_i$ are the vertices of a regular simplex for some choice of signs. (Received May 15, 1975.)

726-52-5 RALPH ALEXANDER, University of Illinois, Urbana, Illinois 61801 Metric averaging in ℓ^2, Preliminary report.

Let $\{\tau_t : t \in T\}$ be a family of homeomorphisms of a topological space X into ℓ^2; furthermore suppose that T is a probability space. Define a metric d on X by $d^2(x,y) = \int |\tau_t(x) - \tau_t(y)|^2 dt$. The new metric space may be isometrically embedded in ℓ^2. Various geometric properties of X as a subset of ℓ^2 are studied. For example, if X consists of $k+1$ points, then the k-volume V of the convex hull of X satisfies $\log V \geq \int \log V_t dt$ where V_t is the k-volume of the convex hull of $\tau_t[X]$. Applications are indicated. (Received June 13, 1975.)

*726-52-6 VICTOR G. FESER, Mary College, Bismarck, N.D. 58501. Construction of Triangular Polyhedra. Preliminary Report.

A method is presented for constructing convex polyhedra with triangular faces: as in all methods already known, the main problem is limiting duplications. A polyhedron is k-valent if its minimal valence is k. For k = 3: faces of a polyhedron (the ancestor) are capped to produce descendants. Two restrictions are observed: i. each 3-valent vertex of the ancestor is destroyed by some cap; ii. two sets of faces equivalent under some symmetry of the ancestor are equivalent for capping. Then we have: every 3-valent polyhedron except the tetrahedron can be produced by capping, without duplication. Further, the symmetry group of the descendant is isomorphic to the invariance group of the set of capped faces. For k = 4 or 5, no analogous method works efficiently. A synthetic method is presented, which begins with maximal vertices: duplications are few, a method exists for identifying them readily, and as a bonus the symmetry group of each polyhedron can be deduced. (Received June 16, 1975.)

53 Differential Geometry

*726-53-1 BANG-YEN CHEN, Michigan State University, East Lansing, Michigan 48824. Chern classes and Bochner curvature tensor.

Let M be a Kaehler manifold and c_k the k-th Chern class of M. Let w be the cohomology class represented by the fundamental 2-form of M. Then we have the following theorems. Theorem 1. Let M be an n-dimensional compact Einstein-Kaehler manifold. Then we have $w^{n-2}c_2 \geq \{n/2(n+1)\}w^{n-2}c_1^2$. The equality holds if and only if M is a complex space form. Theorem 2. Let M be an n-dimensional

$(n > 1)$ compact Kaehler manifold with vanishing Bochner curvature tensor and let $w^{n-2}c_2 = a\, w^{n-2}c_1^2$. If M is not flat and $n/(n+2) \geq a \geq n/2(n+1)$, then either (i) M is a complex space form and $a = n/2(n+1)$, or (ii) $a = n/(n+2)$, n is even and M is locally product space of two $\frac{n}{2}$-dimensional complex space forms of constant holomorphic sectional curvatures $H > 0$ and $-H$. These results will appear in a joint of the author with K. Ogiue entitled "Some characterizations of complex space forms in terms of Chern classes" to appear in Quart, J. Math. (Oxford). (Received May 2, 1975.)

*726-53-2 Harold Donnelly, M.I.T., Cambridge, MA 02139
Eta Invariant of a Fibered Manifold

The multiplicative behavior of the eta invariant of Atiyah-Patodi-Singer in certain types of fiber bundles is investigated. (Received May 7, 1975.)

*726-53-3 PETER B GILKEY, Princeton University, Princeton N.J. 08540
The Local Invariants of a Riemannian Manifold

Let M be a compact Riemannian manifold with metric tensor G. Let P(G) be an invariant polynomial in the derivatives of the metric tensor. If $P(G)(M)=\int_M P(G)\,dvol$ is independent of G for any M of dimension m, then P(G)(M) is a constant multiple of the Euler characteristic. In other words, the only diffeomorphism invariant in the category of Riemannian manifolds which is obtained by integrating a local formula in the derivatives of the metric is the Euler characteristic. There is a corresponding result for the Pontrjagin numbers in the category of oriented Riemannian manifolds. This result was conjectured by I.M.Singer .
(Received May 8, 1975.)

*726-53-4 Ravindra S. Kulkarni, Columbia University, New York, N.Y.10027
Infinite regular coverings, space forms and Kleinian groups. Preliminary report.

Theorem 1 Let M be a noncompact manifold admitting an infinite properly discontinuous group of homeomorphisms. Let $H_c^*(M)$ be the cohomology ring with compact supports with coefficients in a field. Assume that dim $H_c^*(M) < \infty$. Then the multiplication in $H_c^*(M)$ is trivial.

This purely topological theorem has several consequences of interest in the classical problem of space forms and in more or less natural generalizations of the classical theory of Kleinian groups in in higher dimensions. We indicate a typical consequence of theorem 1.

Theorem 2 Let M^n be the interior of a compact manifold with boundary which is homeomorphic to S^{n-1} . If M is a regular covering space of a compact manifold then M is homeomorphic to a disk.

Examples A Mobius band which is a homogeneous space of a Lie group does not have compact "space forms". There exists a linear group acting on the real projective plane $\mathbb{RP}^2$ whose limit set is $\neq \mathbb{RP}^2$ but nevertheless has an interior, in particular has nonzero measure. (Received May 9, 1975.)

*726-53-5 D.S.P. LEUNG, Purdue University W. Lafayette, IN 47907 and T. NAGANO, U. of Notre Dame, South Bend, IN 46556. Riemann Curvature Tensors and Critical Values of Sectional Curvature Functions.

Let C be the space of Riemann curvature tensors on an n-dimensional vector space V with an inner product. For $R \in C$, its sectional curvature function $S(R)$ is a function on $G_{2,n}(V)$, the Grassmannian manifold of 2-dimensional subspaces of V. Theorem 1: There is an open and dense subset W of C such that for $R \in W$, $S(R)$ is a Morse function on $G_{2,n}(V)$. We will also investigate to what extent an element $R \in C$ is determined by the critical points and critical values of $S(R)$, in particular when $R \in W$. (Received May 21, 1975.)

726-53-6 LARRY LIPSKIE, University of Illinois, Urbana, IL 61801.
Generic G-Structures. Preliminary report.

Let G be a closed Lie subgroup of GL(n,R) and M be a smooth manifold which admits G-structures. On the linear space W in which the structure functions of G-structures take their values, a finite G-invariant stratification is constructed, by means of an equivariant version of Whitney's splitting of a real algebraic set into manifolds. The single open stratum consists of the points belonging to G-orbits in W of maximal dimension. This splitting induces a partition of the orbit set W/G.

For a residual set of G-structures, this partition pulls back to a finite stratification on M under the structure function of the G-structure, viewed as a map from M to W/G. The codimensions of the strata in M match the codimensions of the strata in W from which they arise. The non-open strata comprise a subset of M that is singular with respect to a technique which uses the structure function to seek reductions of the G-structure.
(Received June 5, 1975.)

*726-53-7 ROBERT B. GARDNER, University of North Carolina, Chapel Hill, North Carolina 27514, Algebra of the Second Fundamental Form

Basic integer invariants of the vector valued second fundamental form are the rank, the dual dimension and the type number. This report will develop an algebraic setting for the analysis of inequalities between these integer invariants. As applications we give short proofs with some improvements of Allendorfer's theorems on type number and rigidity.
(Received June 9, 1975.)

*726-53-8 C. S. CHEN and W. F. POHL, University of Minnesota, Minneapolis, MN 55455.
On classification of tight surfaces in R^4. Preliminary Report.

Let $f: M^2 \to R^4$ be an immersion of a compact closed differentiable surface into a euclidean space of dimension four. The immersion f is said to be tight if for every hyperplane H of R^4, $f^{-1}(R^4 - H)$ has at most two connected components. The immersion f is substantial in case $f(M)$ is not contained in any hyperplane of R^4. The collection of all hyperplanes closed by some ideal hyperplane is $\tilde{P}^4$. The classification of tight substantial immersions f is done by a certain dual curve C in $\tilde{P}^4$ and depends on whether the true outer part M^{to} is orientable or not. Theorem 1: If M^{to} is orientable, C is the disjoint union of C_1 and C_2. Both C_1 and C_2 consist of finite sets, D_1 and D_2 respectively, joined together by a finite number of convex open plane arcs. For each point a of D_1 or D_2, the number of these open arcs emanating from a is bounded by the first Betti number of the surface. Theorem 2: If M^{to} is not orientable, C is a simple closed curve of class four. (Received June 9, 1975.)

726-53-9 WILLIAM H. MEEKS III, U.C.L.A., Los Angeles, California 90024.
Periodic minimal surfaces in R^n. Preliminary report.

Working primarily within the conformal category, we develop complementary existence and rigidity theories for periodic minimal surfaces in R^n. Applying this theory, we find (1) necessary and sufficient conformal conditions for a compact Riemann surface of genus

g to conformally minimally immerse in a flat 3- or $(2g-1)$-torus

(2) families of distinct isometric minimal surfaces in flat tori

(3) special results on the geometry of minimal surfaces of genus 3, and classical examples of minimal surfaces in flat 3-tori

(4) the determination of the group of self-congruences of a minimal surface in $\mathbb{R}^3$.

(Received June 12, 1975.) (Author introduced by Professor Tilla K. Milnor.)

*726-53-10 ANN K. STEHNEY, Wellesley College, Wellesley MA 02181 and RICHARD S. MILLMAN, Southern Illinois University, Carbondale, IL 62901. Riemannian manifolds with many Killing vector fields.

We consider connected Riemannian manifolds with many Killing vector fields (infinitesimal isometries), specifically those for which the Killing fields span the tangent space at each point (almost-Killing spaces) and those parallelized by Killing fields (Killing spaces). Results include Theorem A: If M is a complete almost-Killing space, then the isometry group $G(M)$ of M is transitive on M (i.e., M is Riemannian homogeneous), Theorem B: If M is a complete Killing space, then the connected subgroup of $G(M)$ whose Lie algebra is generated by the parallelizing Killing fields is transitive on M, Theorem C: If M is parallelized by commuting Killing fields, then M is flat. Examples will be given. (Received June 12, 1975.)

*726-53-11 S. ALEXANDER and E. PORTNOY, University of Illinois,Urbana, Illinois, 61801. Hypersurface immersions between hyperbolic spaces, Preliminary report.

It is well known that every isometric immersion $i: E^n \to E^{n+1}$ between Euclidean spaces takes the form of a Riemannian cylinder built over a plane curve. Our main result is an analogous theorem for isometric immersions $i: H^n \to H^{n+1}$ without umbilic points between hyperbolic spaces. Such an immersion turns out to take the form of a Riemannian cylinder built over a uniquely determined curve, which is generally neither planar nor orthogonal to the generators and which may take values "at infinity". The idea is that a foliation of H^n by hyperbolic (n-1)-planes possesses a uniquely determined parallelizing curve, which coincides with the "striction curve" of the foliation. (A different characterization of isometric immersions $i: H^n \to H^{n+1}$ without umbilics has been given by D. Ferus, Isometric immersions between hyperbolic spaces, Math. Ann. 205 (1973) 193-200.)

(Received June 16, 1975.)

726-53-12 JAMES R. WASON, Massachusetts Institute of Technology, Cambridge, Massachusetts, 02139. Metrics of Non-negative Curvature on $S^3 \times S^3$. Preliminary report.

Let S^3 be the sphere of dimension three. Then $S^3 \times S^3$ may be regarded as the double covering of the frame bundle on S^3. We use this characterization to construct Riemannian metrics on $S^3 \times S^3$ with more widespread positive sectional curvature than the standard product metric. (Received June 16, 1975.)

*726-53-13 HERMAN GLUCK, Univ. of Pennsylvania, Philadelphia, Pa. 19174
DAVID SINGER, Cornell Univ., Ithaca, N.Y. 14850
Deformations of geodesic fields, preliminary report.

We consider the following general situation: two geodesic fields G and G' on a Riemannian manifold N cross a compact hypersurface M transversally. The problem is to decide whether or not it is possible to deform the metric in a neighborhood of M so as to gradually deflect the field G until it coincides

with G'. We answer this by obtaining a "cohomology obstruction" whose vanishing is necessary and sufficient for success. We apply this result to prove: on each smooth manifold of dimension at least 2, there is a Riemannian metric and a point p for which the cut locus C_p from p is nontriangulable. Other applications to Riemannian foliations and geometric optics are also given.
(Received June 16, 1975.)

726-53-14 PATRICK EBERLEIN, University of North Carolina, Chapel Hill, N. C. 27514
Surfaces of nonpositive curvature

Theorem Let M denote a complete (possibly noncompact) two dimensional Riemannian manifold with Gaussian curvature $K \leq 0$ such that every vector in the unit tangent bundle of M is nonwandering relative to the geodesic flow. If M is not the torus or Klein bottle, then the geodesic flow T_t is topologically mixing; that is, for any two open sets U,V in T_1M there exists a number $A > 0$ such that $T_t(U) \cap V$ is nonempty for $|t| \geq A$. In particular T_t has a dense orbit in T_1M. The method is to show that under these conditions the manifold M satisfies a "uniform Visibility" axiom previously considered. The results then follow from previous work. The method is strongly dependent on two dimensions and is different for the compact and noncompact cases. (Received June 9, 1975.)

*726-53-15 JOSEPH E. D'ATRI, Rutgers University, New Brunswick, NJ 08903.
Geodesic spheres and symmetries. Preliminary report.

We examine properties of geodesic spheres and symmetries. For example, in a naturally reductive homogeneous Riemannian space, the geodesic spheres have antipodally symmetric mean and scalar curvatures and (*) the geodesic symmetries are volume-preserving up to sign. As a consequence of a result of U. Simon and joint work of the author and H. K. Nickerson, a homogeneous Riemannian space satisfying (*) and having non-positive sectional curvatures will have covariant constant Ricci tensor and so will be locally the Riemannian product of Einstein spaces. Some open questions and related results will be given. (Received June 16, 1975.)

726-53-16 DAVID SINGER, Cornell Univ., Ithaca, NY 14853, and HERMAN GLUCK, Univ. of Pennsylvania, Philadelphia, Pennsylvania 19104. The Existence of Nontriangulable Cut Loci. Preliminary Report.

In 1936, Myers showed that for a compact analytic surface the cut locus is always a finite graph [Connections Between Differential Geometry and Topology, II. Closed Surfaces, Duke Math. J., 2 (1936), pp.95-102]. That such a theorem is peculiar to analytic manifolds is demonstrated by the following result.

Theorem: There exists a strictly convex closed surface of revolution M in $\mathbb{R}^3$ such that, for a non-empty open set of points p in M, the cut loci $C(p)$ is not a triangulable set.

The technique used to construct M involves deformation of rotationally symmetric Riemannian metrics to achieve preassigned deformation of a field of geodesics. This requires the solution of a certain integral equation by means of the holomorphic Fourier transform. (Received June 17, 1975.)

54 General Topology

726-54-1 RONNIE FRED LEVY, Goucher College, Towson, Maryland 21204
Almost-P-spaces and Lusin's hypothesis.

A completely regular space X is called an almost-P-space if every non-empty zero-set of X has non-empty interior. Lusin's hypothesis (LH) is the statement that $2^{\aleph_1}=2^{\aleph_0}$. Theorem 1. There is a dense-in-itself compact almost-P-space of cardinal $2^{\aleph_c}$ if and only if LH holds. There are almost-P-spaces without P-points; in fact, there is a totally-ordered countably compact almost-P-space without P-points. Theorem 2. Suppose X is a compact almost-P-space, and suppose that the density of C(X) in the uniform norm topology is at most $\aleph_1$. Then X has a dense set of P-points. Among the best known of the infinite compact almost-P-spaces are the spaces βD-D where D is an infinite discrete space. Theorem 3. Suppose α is a cardinal number such that $2^{\alpha}=2^{\aleph_0}$. Then if D is the discrete space of cardinal α and N is the space of natural numbers, $\beta D \subsetneq \beta N-N$. Corollary. LH holds if and only if there is an uncountable discrete subset of βN-N which is C*-embedded.
(Received April 28, 1975.)

*726-54-2 R. F. DICKMAN, JR., Virginia Polytechnic Institute and State University, Blacksburg Virginia 24061. Multicoherent Spaces. Preliminary Report.

Let X denote a locally connected, connected normal space. By a continuum we mean closed and connected subset of X. For $A\subset X$, $b_0(A)$ denotes the number of components of X less one (or ∞ if this number is infinite). The degree of multicoherence, r(X), of X is defined by $r(X) = \sup \{b_0(H\cap K): X = H\cup K$ and H and K are subcontinua of X$\}$. If r(X) = 0, X is said to be unicoherent and we say that X is multicoherent otherwise. If $0<r(X)<\infty$, we say that X is finitely multicoherent and if $0<r(X)\leq\infty$ but $b_0(H\cap K)<\infty$ for any representation $X = H\cup K$, where H and K are continua, we say that X is weakly-finitely multicoherent. Let $n>2$ be an integer and let S(n) denote the following statement: S(n):X is multicoherent iff there exists non-empty continua $A_1,\ldots,A_n$ such that (i) $X = \bigcup_{i=1}^{n} A_i$, (ii) no three of the A_i's have a point in common, and (iii) $A_i\cap A_j \neq \emptyset$ iff $|i(\text{mod } n) - j(\text{mod } n)|\leq 1$. In a private communication A. H. Stone conjectured that S(n) is true for all $n>2$ and he stated that he had established S(n) for all $n>2$ whenever X is finitely multicoherent. In this paper we offer s sequence of conjectures, T(n), $n\geq 2$, characterizing multicoherent spaces. We show that (i) for $n>1$, S(2n) implies T(n), (ii) S(4) is equivalent to T(2), (iii) S(6) is equivalent to T(3), (iv) if X is weakly-finitely multicoherent or compact, then T(n) holds for all $n\geq 2$ and r(X) is obtained and we show that Stone's conjecture follows from ours whenever X is weakly-finitely multicoherent.
(Received May 7, 1975.)

*726-54-3 George M. Reed, Ohio University, Athens, Ohio 45701
Some Moore examples.

In this paper, the author discusses open questions in the literature concerning the existence of counterexamples in the theory of Moore spaces. Three such questions are answered by considering spaces previously constructed for different purposes. Example 1. There exists a base-compact Moore space with a closed subspace that fails to be base-compact. Example 2. There exists a Moore space which admits a strongly complete semi-metric but which is not metacompact. Example 3. There exists a completely regular Moore space with a regular G_δ-diagonal that is not continuously semi-metrizable.

The above examples answer questions raised in [J. M. Aarts and D. J. Lutzer, Diss. Math. 116 (1974)], [G. M. Reed, TOPO'72 Proc. of Sec. Pitt. Int. Top. Conf.], and [H. Cook, Proc. of 1967 Ariz. St. Univ. Top. Conf.], respectively. Example 1 is a slight modification of an example due to F. D. Tall. Example 2 is a space to appear in joint work by S. W. Davis, G. M. Reed, and M. L. Wage. Example 3 is an example due to S. Mrówka. (Received May 1, 1975.)

726-54-4 H. E. WHITE, JR., Columbus, Ohio 43204, First countable spaces having special pseudo-bases.

(1) A first countable Hausdorff space has a σ-disjoint pseudo-base if and only if it has a dense metrizable subspace. (2) If either of the following statements is true, then every subspace of X contains a dense metrizable subspace. (a) X is collectionwise Hausdorff and either X is semi-metrizable or it is perfect and has a base of countable order. (b) X is hereditarily collectionwise Hausdorff and either X is quasi-developable or it has a base of countable order. (3) Suppose X is a Hausdorff space which either is quasi-developable or has a base of countable order. (*) If X has a σ-disjoint pseudo-base, then there is a dense G_δ subset of X which is metrizable. (*) is false for completely regular, Hausdorff, semi-metrizable spaces. (4) Let $\mathfrak{C}$ denote the class of quasi-regular, Hausdorff, quasi-developable (resp. semi-metrizable) spaces. If $X \in \mathfrak{C}$, then X has a Baire space extension which is in $\mathfrak{C}$ if and only if X has a σ-disjoint (resp. σ-discrete) pseudo-base. (Received May 7, 1975.)

726-54-5 M. RAJAGOPALAN, Memphis State University, Memphis, Tennessee 38152. Scattered Spaces III.

A completely regular T_2 space X is called 0-dimensional if open and closed sets of X form a base for the topology of X. Such a space X is called strongly 0-dimensional if βX is 0-dimensional. In a (1974) topology conference, Mary Ellen Rudin asked whether every completely regular, scattered space is 0-dimensional? The same problem was also raised by Z. Semadeni in (1959).

In this paper we construct a scattered, locally compact, T_2, I-countable, separable, sequentially compact, 0-dimensional space X which is not strongly 0-dimensional. By taking the subspace $X \cup \{p\}$ of βX where p belongs to a non-trivial component of βX we get a completely regular, scattered, T_2 space which is not 0-dimensional. We also construct locally compact, T_2, spaces Y_n so that $\operatorname{ind} Y_n = 0$ and $\operatorname{Ind} Y_n = n$ for all integers $n > 0$ and also a completely regular space Y which is 0-dimensional but βY is countably infinite dimensional. (Received May 2, 1975.)

726-54-6 PHILLIP ZENOR, Auburn University, Auburn, Alabama 36830. Continuous PN-Operators

If X is a space, then 2^X will denote the set of closed subsets of X. A function p from $2^X \times X$ into $[0, \infty)$ is a PN-operator if, for each $H \in 2^X$, $\{x \mid p(H, x) = 0\} = H$. Various classes of spaces can be characterized by placing continuity conditions on PN-operators (e.g., the classes of developable spaces, M_1-spaces, M_2-spaces, and M_3-spaces, and metrizable spaces). Recent developments and open questions concerning continuous PN-operators are discussed. (Received May 12, 1975.)

726-54-7 CHARLES L. HAGOPIAN, California State University, Sacramento, California 95819. Nonseparating plane continua.

Several interesting concepts have their origins in problems involving plane continua that do not separate the plane. For example, Professor Borsuk first used retracts to show that locally connected nonseparating plane continua have the fixed point property. Still unsolved is the problem: Does every nonseparating plane continuum have the fixed point property? We shall describe some of the basic properties of these continua. Outstanding problems involving homogeneity, hereditary equivalence, and fixed point properties will be related to indecomposability and chainability. (Received May 12, 1975.)

726-54-8 S. MROWKA, State University of New York at Buffalo, Amherst, New York 14226, Concerning the covering dimension . Preliminary report.

We shall discuss results and open questions related to the following two problems on covering dimension (dim): 1. Is it true that $\dim(X \times X) \leq \dim X + \dim Y$? 2. Does $X = X_1 \cup X_2$, X_1 - closed, X_2 - open, $\dim X_1 = \dim X_2 = 0$, $\operatorname{ind} X = 0$ imply that $\dim X = 0$? (Spaces are assumed to be completely regular; ind stands for the small inductive dimension.) (Received by May 20, 1975.)

*726-54-9 J. C. SMITH, Virginia Polytechnic Institute and State University, Blacksburg, Va. 24061, A Remark on Irreducible Spaces, preliminary report.

A topological space X is called irreducible if every open cover of X has an open refinement which covers X minimally. In this paper we show that weak $\overline{\theta}$-refinable spaces are irreducible. A modification of the proof of this result then yields that $\aleph_1$-compact, weak $\overline{\delta\theta}$-refinable spaces are Lindelöf. It then follows that perfect, $\aleph_1$-compact weak $\delta\theta$-refinable spaces are irreducible. A number of known results follow as corollaries. Several results in this paper were obtained independently by J. R. Boone as announced in the NOTICES of the AMS Jan. 1975, *75T-G1. (Received May 19, 1975.)

726-54-10 C. E. AULL, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. The Separation Axioms of Van Est and Freudenthal.

Van Est and Freudenthal studied separation axioms based on separation by continuous functions. Two disjoint sets A and B are functionally separated if there is a continuous function f to the real line such that $f(A) \cap f(B) = \emptyset$. A space is functionally Hausdorff (functionally regular) (functionally normal) if points (points and closed sets) (closed sets) are functionally separated. A survey of results concerning these axioms is given with particular emphasis on functionally regular spaces. We note that the functionally regular condition is particularly important in regard to the C-embedding of pseudo-compact sets, for instance the following are equivalent for a functionally regular T_1-space $(x,\mathcal{T})$:

(a) Given 2 disjoint zero sets one has the property that every $\mathcal{T}$-cozero cover has a finite subcover. (b) weak topology for $(x,\mathcal{G})$ is almost compact (c) [d] every embedding in a functionally regular space is a (C-embedding) [C*-embedding].

Note: [d] $\rightarrow$ (b) is due to Jack Porter. Stephenson has shown that every functionally Hausdorff space has a functionally Hausdorff closed extension. The question discussed here is "Does every functionally regular space have a functionally regular closed extension?" (Received May 19, 1975.)

*726-54-11 **FRANKLIN D. TALL, University of Toronto, Toronto, Canada M5S 1A1.** **Some results concerning the hereditarily Lindelöf-hereditarily separable problem.**

The hereditarily Lindelöf-hereditarily separable problem is discussed, in particular the following result due jointly to E. K. van Douwen, W. A. R. Weiss, and the author.

Theorem. The continuum hypothesis entails the existence of a non-metrizable hereditarily Lindelöf 0-dimensional space with a point-countable base. (Received May 23, 1975.)

*726-54-12 SCOTT WILLIAMS, SUNY/Buffalo, Amherst, New York 14226. Covering Properties in Products and Yet Another Cardinal Function, Preliminary report.

Let X be a space and m any cardinal. We say that $SLC(X) = m$, iff, m is the supremum of cardinalities of closed subspaces of X whose compact sets have void interior.

This cardinal function arose from investigations of paracompactness and the lindelöf property in products. We survey past results, present new results, and give some examples for the cases of $SLC(X) = 0$, $SLC(X) = \aleph_0$, $SLC(X) = \aleph_1$, and $SLC(X) = 2^{\aleph_0}$ while making an occasional use of various set-theoretic assumptions. A simply stated sample: if X is hereditarily paracompact and $SLC(X) = \aleph_0$, then X^n is paracompact for all finite ordinals m.
(Received May 23, 1975.)

*726-54-13 JERRY E. VAUGHAN, University of North Carolina at Greensboro, Greensboro, North Carolina 27412. A generalization of Tychonoff's product theorem.

For a directed set D, let D' stand for the directed set obtained from D by adding one new element as the smallest (zero) element. The restricted product of a family of directed sets $\{D'_\alpha : \alpha \varepsilon A\}$ is defined to be the subset of the Cartesian product whose members have only a finite number of non-zero coordinates (ordered by the usual product order). Let Ω be a class of directed sets. A family F of subsets of a set X is called an Ω-filter base if there is a $D \varepsilon \Omega$ and a surjection $f:D \to F$ such that $d \leq e$ in D implies $f(d) \supset f(e)$ in F. A space is called (totally) Ω-compact if for each Ω-filter base F there is a finer (Ω-filter base) filter base G which is total (i.e., every filter base finer than G has an adherent point). Theorem. Let Ω be a class of directed sets and m an infinite cardinal. If for every restricted product D of fewer than m members of Ω, there is an $E \varepsilon \Omega$ and an increasing, cofinal map $f:E \to D$, then (A) every product of fewer than m totally Ω-compact spaces is totally Ω-compact, and (B) every product of no more than m totally Ω-compact spaces is Ω-compact. Corollaries: (1) Tychonoff's Theorem, (2) A. H. Stone's Theorem: Every product of $\leq \aleph_1$ sequentially compact spaces is countably compact, (3) N. Noble's Theorem: A countable product of Lindelöf P-spaces is Lindelöf. (Received May 27, 1975.)

726-54-14 M. JAYACHANDRAN, Memphis State University, Memphis, Tennessee 38152. A common method of attack to some open problems in scattered spaces. Preliminary report.

Partial or complete solutions to some outstanding problems in scattered spaces raised by M.E. Rudin, Telgarsky, Semadeni, A.H. Stone, Nyikos, and J. J. Schäffer are presented here. This is an expository talk. They are all solved by taking suitable partitions of βN. The main theorems are:

(i) There exists a locally compact, I countable, countably compact space which is not strongly Zero-dimensional.
(ii) There exists a completely regular scattered space which is not Zero-dimensional. Consequently, there exists a scattered, completely regular space that does not admit a scattered compactification. However,
(iii) The well-known scattered space S_2 (in fact S_n for all natural numbers n) admit scattered T_2 compactifications. Also $N \cup \{p\}$ admits such a compactification for a large class of points $p \varepsilon N^*$.
(iv) There exists a locally compact, I countable, sequentially compact scattered T_2 space X which can be mapped onto [0,1] by a closed continuous map.
(v) There exists a family of locally compact, I countable, separable, sequentially compact, scattered spaces whose product is not countably compact.
(vi) Every topological space is a closed continuous image of a scattered space. (Received May 27, 1975.)

726-54-15 LOUIS F. McAULEY, State University of New York at Binghamton, 13901 Open Mappings and Group Actions.

A brief survey of recent results concerning open mappings which are the orbit maps of some group actions is given (including some work of my students Fintushel and Robinson). Also, a characterization of light open mappings in terms of closed coverings is presented along with an application. Related problems and results are also discussed.
(Received May 27, 1975.)

*726-54-16 ERIC K. van DOUWEN, University of Pittsburgh, Pittsburgh, Pa. 15260 Maps onto Baire spaces.

A space is totally nonmeagre if every closed subspace is a Baire space. Theorem

1 A first countable Hausdorff space is totally nonmeagre if it admits a closed continuous map with totally nonmeagre fibers onto a totally nonmeagre space. We don't know if first countability is essential. The converse of Theorem 1 is false: Example [CH] There exists a first countable locally compact (regular) space that admits a closed continuous map onto the rationals. The following corollary to Theorem 1 partially answers a question of Aarts and Lutzer (Proc. AMS 38 (1973) 198-200). Corollary The product of a totally nonmeagre Hausdorff space and a compact Hausdorff space is totally nonmeagre provided both factors are first countable. Theorem 2 A quasi-regular space is a Baire space if it admits an open continuous map with countably compact fibers onto a Baire space. The method of proof can be used to give a simpler proof of Aarts and Lutzer's theorem that the product of a quasi-regular Baire space and a pseudo-complete space is a Baire space (Pacific J. Math. 48 (1973) 1-10). (Received May 28, 1975.)

*726-54-17 GARY GRUENHAGE, Auburn University, Auburn, Alabama 36830 and PETER NYIKOS, University of Illinois, Urbana, Illinois 61801. Spaces with bases of countable rank.

A collection G of sets has sub-infinite rank (rank n) if whenever $G' \subset G$ with $\bigcap G' \neq \emptyset$, and $|G'| \geq \aleph_0$ ($|G'| \geq n+1$), then there are two members of G' related by set inclusion. G is Noetherian if ascending sequences are finite. A space X has a well-ranked base if it has a base which is the union of countably many Noetherian collections of sub-infinite rank. Theorem. A space which has a base of finite rank is metacompact. Theorem. A compact space, or a regular separable space, is metrizable if it has either a well-ranked base or a base of finite rank. Theorem. A regular CCC space with a well-ranked base has a point-countable base. (Received May 27, 1975.)

726-54-18 R.W. HEATH, University of Pittsburgh, Pittsburgh, Pa. 15260, R.B. SHER, University of North Carolina at Greensboro, N.C. 27412; ANE's and ANR's in monotonically normal spaces, Preliminary report.

A space is an AE (absolute extensor) for spaces in the class C provided that, for every X ε C and every closed subset A of X, every continuous function f: A → Y has a continuous extension uf: X → Y. If every f: A → Y can be extended over a neighborhood of A in X for all such A and X, then Y is an ANE for the class. Every ANE in the class C is clearly an ANR for C. In the class C of metric spaces every ANR is an ANE. Many results concerning AE's and ANE's for metric spaces as well as more general classes of spaces are due to Michael (Pac. J. Math 3(1953) 784-806) and Hanner (Ark. Math 2 (1952), 315-360) - e.g. a metric space that is locally an ANE is an ANE. The authors investigate the possibility of extending certain of these known results for metric spaces to certain classes of generalized metric spaces, such as the monotonically normal spaces. Some of these carry over nicely. For example, for monotonically normal spaces, the ANR's and ANE's coincide. Other results, particularly some of those obtained by Cauty (Bull. Soc. Math. France 102 (1974), 129-149) for stratifiable spaces are shown not to carry over to monotonically normal spaces. (Received May 27, 1975.)

*726-54-19 MARY ELLEN RUDIN, The University of Wisconsin, Madison, Wisconsin 53706. The Continuum Hypothesis.

Kunen has used the Continuum Hypothesis to construct a topology on the line which is not Lindelöf but has most of the other topological properties of the line preserved: especially it is hereditarily separable. Zenor and I have used this idea to construct a separable perfectly normal nonmetrizable manifold. The technique can also be used to construct Dowker spaces. (Received May 30, 1975.)

*726-54-20 ANTHONY G. O'FARRELL, Mathematics Department, UCLA, Los Angeles, CA 90024. Sometimes, you've got the wrong metric.

Some metric spaces admit metrics that are in a certain sense smaller than the original metric. This is related to the existence of certain Lipschitz functions and to connectedness properties of the space. (Received June 4, 1975.)

726-54-21 HOWARD H. WICKE, Ohio University, Athens, Ohio 45701. Primitive structures and diagonal conditions.

This paper concerns primitive topological structures; in particular, spaces having primitive bases and primitively quasi-complete spaces. (Definitions of these spaces may be found in Wicke-Worrell, Primitive structures in general topology, Studies in Topology, Academic Press, 1975, 581-599.) Theorems involving diagonal conditions, especially the condition of having a primitive diagonal, are emphasized and some characterizations of spaces having bases of countable order, of primitive bases, and of metrizable spaces are given. Interactions of conditions defining primitive structure with other conditions arising in the theory of generalized metric spaces such as β-spaces in the sense of Hodel are discussed also.
(Received May 27, 1975.)

*726-54-22 LOUISE MOSER, California State University, Hayward, California 94542. Closure, interior, and union in finite topological spaces.

A variation on a problem of Kuratowski, which appeared recently in the Advanced Problems of the American Mathematical Monthly (vol. 81 (1974), p. 1034, Problem 5996 proposed by Arthur Smith), was to show that at most 13 sets can be constructed from a subset of a topological space by application of the closure, interior, and union operators. In this paper the sets obtainable from a subset A of a topological space X by closure, interior, and union are called associates of A. It is shown that if a finite topological space X has a subset A with 13 distinct associates, then the cardinality of X is greater than or equal to 9. For a topological space X of cardinality less than 9, the maximum number of associates of A is determined. Further, it is shown that if X is a finite T_0-space, then the number of distinct associates of A is less than or equal to 7. Moreover, if A has 7 associates, then the cardinality of X is greater than or equal to 6. For a T_0-space X of cardinality less than 6, the maximum number of associates of A is determined.
(Received June 6, 1975.)

726-54-23 PAUL BANKSTON, McMaster University, Hamilton, Ontario, Ultraproducts in General Topology, Preliminary Report

Let $(X_i)_{i\in I}$ be a family of topological spaces with U an ultrafilter on I. The topological ultraproduct $\Pi_U X_i$ is that quotient of the familar box product formed by identifying I-tuples f,g whenever they agree on a member of U. Let T_0, T_1, T_2, T_3, $T_{3\cdot 5}$, T_4 denote the usual separation axioms (e.g. $T_{3\cdot 5}$ denotes the class of Tichonov spaces). Then T_n (as well as its complement) is closed under ultraproducts for $0\leq n\leq 3$; $T_{3\cdot 5}$ (but not its complement) is closed under ultraproducts; and neither T_4 nor its complement is closed under ultraproducts.

Let κ be an infinite cardinal. If (i) U is κ-regular then $\Pi_U X_i$ is κ^+-open (i.e. its topology is closed under $\leq\kappa$ intersections); and (ii) if U is κ-good then $\Pi_U X_i$ is κ-Baire (i.e. intersections of $<\kappa$ dense open sets are dense).

If U is countably incomplete then the imposition of innocuous properties on the X_i's (e.g. regularity, metrizability) yields various total disconnectedness properties on $\Pi_U X_i$ (e.g. strong zero dimensionality, ultrametrizability, non-Archimedeanness).

Every two perfect regular spaces have homeomorphic ultrapowers. (Received May 28, 1975.)

726-54-24 ALI A. FORA, State University of New York at Buffalo, Amherst, N.Y. 14226 Still on Modified Sorgenfrey Spaces. Preliminary Report

We announced in our previous abstract ["The Dimension Conjecture for products of Modified Sorgenfrey Spaces", these Notices, Abstract number 75T-G65, June 1975, vol. 22] that $S_*^{\aleph_0}$ is strongly zero-dimensional, where S_* is the set $R\times[0,1]$ ordered Lexicographically and equipped with the Sorgenfrey topology.

We now prove that:

1. Any ordered set X with the Sorgenfrey topology is strongly zero-dimensional.
2. If Y is any metrizable strongly zero-dimensional space, then $S_*^{\aleph_0} \times Y$ is also strongly zero-dimensional. (Received June 12, 1975.)

*726-54-25 R. C. SINE, University of Rhode Island, Kingston, R. I. 02881. Ergodic Properties of Homeomorphisms.

Let ϕ be a homeomorphism of a compact Hausdorff space X. Let T be the induced isometry of C(X). We say T (and ϕ) is strongly ergodic if the Cesaro averages of the iterates of T converge in the strong operator topology. Using Lorentz's theory of almost convergent sequences we construct an example of a strongly ergodic homeomorphism ϕ so that ϕ^2 is not strongly ergodic. (Received June 13, 1975.)

726-54-26 BENJAMIN A. BURRELL, Ohio State University, Marion, Ohio 43302. The mountain-climbing problem on the closed 2-cell. Preliminary report.

Let I^n denote the closed unit n-cube and $\mathfrak{F}^n$ the class of all mappings on I^n into itself that are pointwise fixed on the boundary. The mountain-climbing problem considers the question (when $n = 1$): Given $f,g \in \mathfrak{F}^n$, when can $i,j \in \mathfrak{F}^n$ be found such that $fi = gj$? Equivalently, when is the (n-1)-sphere $S^{n-1} = \{(z,z) \mid z \in \text{bdy}\, I^n\}$ null-homotopic in $E = (f \times g)^{-1}(\Delta)$, where Δ is the diagonal in $I^n \times I^n$? For arbitrary n, Sikorski (Fund. Math., Vol. 41 (1954), 345-350) has shown S^{n-1} is homologous to 0 in E. Here the case when $n = 2$ is considered. Examples are given where S^1 is and is not null-homotopic in E. Conditions are given for S^1 to be null-homotopic for monotone and non-alternating maps. If f and g are arbitrary normal (interior maps onto interior and boundary onto boundary) light open maps (or g is light open and a homeomorphism on the boundary) of I^2 onto itself, then the set E is characterized and necessary and sufficient conditions are given for a component of $\{(z_1,z_2) \in \text{bdy}\, I^2 \times \text{bdy}\, I^2 \mid f(z_1) = g(z_2)\}$ to be null-homotopic in E in terms of the local degree of the maps. (Received June 9, 1975.)

726-54-27 TINUOYE M. ADENIRAN, College of Science and Technology, Port Harcourt, Nigeria. Absolute homology (monotone) union property for X, with coefficient Z. Preliminary report.

Definition. Let X be a set and let $\{A_i | A_i \subset A_{i+1}\}$ be a monotone sequence of sets in a topological space C such that $X \underset{=}{\tau} A_i$ and $Y = \bigcup_1^\infty A_i$. If the singular homology group of X equals that of Y for all $j \in Z$, i.e. if $H_j(X) = H_j(Y)\ \forall j$, say that X has the absolute homology union property in C, denoted by AH(X, C). By constructing a monotone increasing union of the rationals in E^1, we prove that the set Q of rationals has the AH(X, C) property, the irrationals does not, and the Cantor set has. (Received June 4, 1975.)

55 Algebraic Topology

*726-55-1 DANA MAY LATCH, Lawrence University, Appleton, Wisconsin 54911. A (weak) homotopy inverse for the functor nerve.

We outline a proof of the fact that the homotopic category of the functor category K of simplicial sets [P. Gabriel and M. Zisman, Calculus of Fractions and Homotopy Theory, Springer Verlag, New York, 1967; IV; 2.3] is equivalent to the "corresponding" homotopic category of Cat, the category of small categories. A functor $\Gamma: K \to$ Cat, which occurs in a construction of G. Segal [G. Segal, "Categories and cohomology theories", Topology 13(1974), 293-312], is shown to be a weak homotopy inverse of the functor nerve N:Cat$\to K$ by defining natural transformations $\eta'': N\Gamma \dot{\to} 1_K$ and $\eta': N\Gamma \dot{\to} 1_{Cat}$ such that the morphism corresponding to each object are equivalences in the respective homotopic categories. To show η'' is a weak homotopy equivalence (whe) a "folkloric" result on pushouts of whe is used and to show η' is a whe we apply Quillen's "Theorem A" [D. Quillen, "Higher algebraic K-theory:I", LNIM, 341, pp 85-147]. (Received May 16, 1975.)

*726-55-2 JON M. BECK, Department of Mathematics, Natural Sciences, University of Puerto Rico, Rio Piedras, Puerto Rico 00918. Homology of group completions. Preliminary report.

Let H_* denote ordinary homology with field coefficients and $^-$ the functor "group completion." Let X be a simplicial (or topological) monoid, and $\overline{X}^L$ the left derived of its group completion. There is a natural map of Hopf algebras $H_*X \to H_*(\overline{X}^L)$. These Hopf algebras can be considered as monoid or group objects in the category of cocommutative coalgebras.

Theorem. Group completion commutes with homology, i.e., the induced map $(H_*X)^- \xrightarrow{\sim} H_*(\overline{X}^L)$ is an isomorphism of Hopf algebras.

Proof. There is a homomorphism and weak equivalence $P \to X$, where P is cofibrant. The theorem is proved first for free monoids, then for P. But $\overline{P} = \overline{X}^L$. It is not assumed that the elements of $\pi_0(X)$ are central in the Pontrjagin ring.

Corollary. The natural inclusion of (weak) homotopy categories

$$\text{Ho (grouplike monoids)} \xleftarrow{\sim} \text{Ho (groups)}$$

is an equivalence. (Received May 28, 1975.)

*726-55-3 MARTIN FUCHS, Michigan State University 48824. Homotopy equivalences in equivariant topology

The paper describes the category $\mathfrak{G}X$ of topological spaces with a topological group (or H-space) action as objects and strongly homotopic equivariant maps as morphisms. Examples are mentioned. The concept of homotopy in this category is discussed and homotopy equivalences are characterized as morphisms in $\mathfrak{G}X$ which come from homotopy equivalences in $\mathfrak{T}$op. (Received June 2, 1975.) (Author introduced by Professor J. Adney.)

*726-55-4 J.M. BOARDMAN, The Johns Hopkins University, Baltimore, Maryland 21218. Localization theory and splittings of MU. Preliminary report.

It has generally escaped notice that MU splits even before localization. We have developed localization techniques that readily construct for each set M of primes a M-local spectrum BP(M) (equivalently, cohomology theory BP(M)*) such that: (i) BP({p}) is the usual Brown-Peterson spectrum; (ii) BP($\emptyset$) = K(Q); (iii) MU = BP(all)$\otimes$E; (iv) $BP(M)_L = BP(L) \otimes H(M-L)$; where H(M) and E are the expected graded polynomial rings and the isomorphisms are best possible multiplicative splittings.

Additively, things are more complicated. If M contains more than one prime, BP(M) splits as a graded sum of copies of a quotient ring. Look for M-local indecomposables having free homology and homotopy modules of finite type, in the simplest case when $M = \{p,q\}$, where $p < q$. Theorem. Either (a) there is only one such, as when p-1 divides q-1 and $q > p^4$, or (b) there are infinitely many, as when p-1 does not divide q-1 or $M = \{2,3\}$. Calculations in degrees up to 19,000,000 suggest that $M = \{2,5\}$ belongs to case (a).
(Received June 9, 1975.)

*726-55-5 STEPHEN J. WILLSON, Dept. of Math. Iowa State Univ. Ames, Ia. 50010
Applications of an equivariant universal coefficient theorem

Let G be a finite group and $\mathcal{H}$ be a collection of subgroups of G. A ring $\mathcal{J}$ is defined, relevant to the study of cellular actions of G on CW complexes such that each isotropy subgroup lies in $\mathcal{H}$. If ${}_Gh_*(X)$ is an equivariant homology theory satisfying that ${}_Gh_i(G/H) = 0$ for $i > 0$ and $H \in \mathcal{H}$, there exists an easily computed left $\mathcal{J}$ module M and a spectral sequence converging to ${}_Gh_*(X)$ with $E^2_{p,q} = \mathrm{Tor}^{\mathcal{J}}_p({}_GH_q(X;\mathcal{J}),M)$. Here ${}_GH_q(X;\mathcal{J}) = \bigoplus_{K \in \mathcal{H}} H_q(X^K)$ is an appropriate right $\mathcal{J}$ module. The spectral sequence is used to compute the homology with Z_{p^r} coefficients of the orbit space of an action of Z_{p^s} on a Z_{p^r} - homology sphere. (Received June 10, 1975.)

726-55-6 R. NEIL VANCE, Institute for Advanced Study, Princeton, NJ 08540.
Homotopy Equivariant Group Actions. Preliminary Report.

Let G be a finite group. A space Y which is homotopy equivalent to a G-space X is a simple example of an algebraic structure up to homotopy, whose general theory is due to Boardman and Vogt. Such a space Y is said to admit a WG action. In contrast to the general situation for algebraic-topological theories, the forgetful functor from G-spaces to WG-spaces admits both a left and a right adjoint. The cotriple (on G-spaces) associated to the left adjoint is $X \longmapsto EG \times X$; the triple associated to the right adjoint is $X \longmapsto \mathrm{Map}(EG,X)$. An analogue of the right adjoint for categories associated a category with G action to a category with coherent G action. Together with G. Segals construction of K-theory spectra, this gives a construction of equivariant algebraic K-theory spectra which is an (equivalent) alternative to the constructions of D. W. Anderson or A. Froelich and C.T.C. Wall. (Received June 16, 1975.)

*726-55-7 JAMES D. STASHEFF, Temple University, Philadelphia, Pennsylvania 19122. Continuous cohomology of groups and classifying spaces.

• The cohomology theory of abstract groups has two major motivations–formal algebraic relations and topology related to fundamental groups. The resulting theory then has two interpretations which give rise to radically different generalizations. The continuous cohomology of a topological group, denoted $H_c(G)$, uses the formal relational approach, subject only to the condition of continuity. The second approach constructs a "classifying space" BG reflecting both the topology and algebra of G and then looks at H(BG). I hope to motivate and introduce both these constructions and then survey their relationship, showing how $H_c(G)$ is an approximation to H(BG). The relationship is particularly well worked out in the case of Lie groups: There is a spectral sequence due to Bott converging from $H_c(G;S(g^*))$ to H(BG) where g^* is the dual to the Lie algebra of G. On the other hand, $H_c(G)$ can be computed using van Est's isomorphism with an appropriate Lie algebra cohomology. Recent developments have justified a generalization to topological groupoids, especially Lie groupoids. The spectral sequence now has a radically different form and Haefliger's generalization of van Est's theorem provides a surprising relation with the Gelfand-Fuks cohomology. Returning closer to the origins of the theory, I intend to complete the survey by bringing out the relation of $H_c(G)$ to topological group extensions. (Received June 16, 1975.)

726-55-8 E. J. MAYLAND, JR., York University, Downsview, Ontario, Canada M3J 1P3, and KUNIO MURASUGI, University of Toronto, Toronto, Ontario, Canada M5S 1A1. On a structural property of the groups of alternating links.

Theorem A. The group of an alternating knot, with prime power leading polynomial coefficient, is residually finite and solvable. Theorem B. The commutator subgroup of the group of an alternating knot, which has leading polynomial coefficient a power of the prime p, is residually a finite q-group for all primes $q \neq p$. Theorem C. The augmentation subgroup of an alternating link is an iterated generalized free product of free groups X_i in which the amalgamated subgroups H_i and K_i are free factors of respective subgroups of finite index equal to (the leading coefficient of the reduced Alexander polynomial). These results are special cases of results for classes of links generalizing alternating links. Theorem C uses Brown and Crowell's (J. Math. Mech. 15 (1966), 1065-1074) description of link group structure and results on the relationship between the link matrix and an alternating link, e.g., K. Murasugi (Osaka J. Math. 12 (1960), 277-303, and Trans. Amer. Math. Soc. 117 (1965), 387-422). The results on residual finiteness use theorems announced in E. J. Mayland, Jr. (Geometric Topology, Springer Lecture Notes #438 (1975), 339-342) which will appear in Canad. J. Math. (Received June 16, 1975.)

57 Manifolds and Cell Complexes

*726-57-1 J. W. CANNON, University of Wisconsin, Madison, Wisconsin 53706. Taming Codimension - One Generalized Submanifolds of S^n.

A generalized n-manifold M is a Euclidean neighborhood retract such that, for each $x \in M$, $H_*(M, M-\{x\}; \mathbb{Z}) \approx H_*(E^n, E^n-\{0\}; \mathbb{Z})$. Conjecture. A space M is a generalized manifold if and only if there is a space N such that M x N is a manifold (or at least, such that M x N is a cell-like image of a manifold). Theorem 1. If M is a compact, codimension-one generalized submanifold of S^n (no restriction on n), then M has a neighborhood in S^{n+1} which has M x E^2 as a cell-like image; if M x E^2 is a manifold, M has an E^2 bundle neighborhood in S^{n+1}. Theorem 2. If M is a compact, codimension-one generalized submanifold of S^n ($n \neq 4$) and S^n - M is 1-ULC, then M has a neighborhood in S^n which has M x E^1 as a cell-like image; if M x E^1 is a manifold, then M is bicollared in S^n. The proofs are like Cernavskii's proof that an (n - 1) -sphere S in S^n ($n>4$) is flat if S^n - S is 1-ULC; complications arise since M has no nice local geometric structure. (Received April 14, 1975.)

*726-57-2 Robert Azencott, Université de Paris 7, Paris (5eme), France and Edward N. Wilson, Washington University, St. Louis, Missouri 63130. Homogeneous Manifolds with Negative Curvature.

Our focus of attention is the structure of the full isometry group I(M) for M a connected, simply connected, homogeneous, Riemannian manifold with non-positive sectional curvature. We prove that M determines in a canonical way a conjugacy class of simply transitive subgroups of I(M) with optimal structural simplicity and show how all simply transitive subgroups may be constructed from a member of the canonical class.

We establish a list of Lie algebra structural conditions which characterizes the class of connected, simply connected Lie groups S for which there exists a manifold of the above type on which S acts simply transitively by isometries. A second list of Lie algebra structural conditions governs the class of connected Lie groups G for which there exists such a manifold M with G isomorphic to the connected component of the identify in I(M). (Received May 23, 1975.)

726-57-3 R. PAUL BEEM, Indiana University at South Bend, South Bend, Indiana 46615. Extensions and reductions of equivariant bordism,

Suppose G is a finite subgroup of S^1, the circle group, with Z_2 a subgroup of G. Consider the following submodules of the unoriented bordism of unrestricted involutions:

A = kernel of extension to G bordism;

B = kernel of extension to S^1 bordism;

C = kernel of (twisted) multiplication by $(S^1,-1)$;

D = image of reduction of S^1 bordism;

E = image of reduction of G bordism.

Theorem. a. B = C = D.

b. If $Z_4 \subset G$, then A = B = C = D = E.

c. If $Z_4 \not\subset G$, then $A = \{0\}$ and E is all of Z_2 bordism.

(Received June 16, 1975.)

726-57-4 LAWRENCE VERNER, Baruch College, City University of New York, New York, New York 10010. A mean value formula for the Spin group.

Let (G, X) be a homogeneous space defined over a number field k. Suppose X is 2-connected as a complex manifold, and G is 1-connected as a complex Lie group. Then for $f \in C_c(X_A)$, $\int_{G_A/G_k} \sum_{x \in X_k} f(gx)\,dg = \int_{X_A} f(x)\,dx$. G = the Spin group, and X = the generalized sphere furnishes an example. (Received April 17, 1975.)

58 Global Analysis, Analysis on Manifolds

*726-58-1 MIKHAEL GROMOV, State University of New York, Stony Brook, New York 11794. Topology of Riemannian manifolds with small curvature and diameter. Preliminary report.

Let n, d and $|K|$ denote dimension, diameter and maximum of absolute values of sectional curvature of a Riemannian manifold V. Theorem 1. For given n there exists such a positive constant A_n that if $d^2|K| \leq A_n$ then V may be covered by a nil-manifold; in particular V cannot be simply connected. Theorem 2. For given n and N there exists such a positive $B_n(N)$ that if $d^2|K| \leq B_n(N)$ then ranks of all homology groups of V (with arbitrary coefficients) are bounded by N. (Received June 13, 1975.) (Author introduced by Professor Tilla K. Milnor.)

60 Probability Theory and Stochastic Processes

*726-60-1 ETANG CHEN, University of Rochester, Rochester, N. Y. 14627
Existence of Interacting Markov Processes in Quantum Lattice Systems

An existence theorem for interacting Markov Processes in classical lattice systems is extended to quantum lattice systems with a class of finite range interactions. This result is then applied to show that (i) the system preserves strong-clustering property of states and (ii) ergodicity of primary states. Finally, the invariance of Gibbs states are also proved. (Received May 23, 1975.)

*726-60-2 DR. IGNACY I. KOTLARSKI, Oklahoma State University, Stillwater, Oklahoma, 74074
On Characterization of Probability Distributions by Conditional Expectations

D.N. Shanbhag [Jour. Am. Stat. Assoc., 65, No. 331, p. 1256-1259, 1970] proved the following characterization theorem for distributions of real random variables: Let X be a positive random variable with a continuous distribution and possessing its expectation. Then X has an exponential distribution with mean m if and only if $\forall\, y > 0$, $E[X|X > y] = y + m$. There were many different efforts to generalize this theorem to more general cases. The most general statement of the problem seems to be the following:

Let $(\Omega, \mathfrak{F})$ be a measurable space, P and P_0 - two probability measures on $(\Omega, \mathfrak{F})$, $\mathfrak{F}_0$ a sub-collection of random events from $\mathfrak{F}$, X a real random variable on $(\Omega, \mathfrak{F})$, such that for each $A \in \mathfrak{F}_0$, $E_P[X|A]$ and $E_{P_0}[X|A]$ do exist. Under what additional conditions the property $\forall\, A \in \mathfrak{F}_0$, $E_P[X|A] = E_{P_0}[X|A]$, implies that $P = P_0$ on $\mathfrak{F}$? There are two theorems in this subject presented as well as some of their applications. The method of functional equations for unknown probability measures is used in the proofs of the theorems. (Received June 13,1975.)

*726-60-3 ALAN F. KARR, Johns Hopkins University, Baltimore, Maryland 21218.
Lévy Random Measures.

A Lévy random measure is distinguished by a conditional independence structure reminiscent of the Markov property: the behaviors of the random measure on the components of certain pairs of sets are conditionally independent given the behavior on the interaction of the two sets. The Lévy property is preserved under a number of transformations; these results generalize known properties of Markov processes and additive random measures. Submeasures are constructed and shown to be Lévy if and only if the defining functional is multiplicative in an appropriate sense; this is analogous to two important results concerning multiplicative functionals of Markov processes. (Received June 16, 1975.)

726-60-4 NEIL BROMBERG, Courant Institute of Mathematical Sciences, New York University, New York, New York 10012. Boundary layer analysis of equations of Brownian motion in a force field.

The asymptotic analysis of equations of Brownian motion as treated by Ilin and Khasminskiĭ is extended to include boundaries. By probabilistic methods the boundary layer is analyzed and the boundary condition for the interior solution is derived. It is proven that the interior solution is a uniform approximation inside a compact subset of the region and for times bounded away from zero and infinity. This result is proven for half plane and convex polygonal regions. (Received June 16, 1975.)

726-60-5 JEFFREY L. SPIELMAN, Bowling Green State University, Bowling Green, Ohio 43403.
Stochastic processes with independent increments taking values in a Hilbert space.

Let $X(t)$, $t \in [0,1]$, be a separable, stochastically continuous, stochastic process with independent increments taking values in a real, separable Hilbert space. Then $X(t)$ can be decomposed into a uniformly convergent sum of independent processes. In this decomposition one of the processes is Gaussian with continuous sample functions, and the remainder of the processes have sample functions whose discontinuities correspond to those of certain real-valued Poisson processes. It is shown that the result is motivated by the structure of infinitely divisible random variables. (Received June 17, 1975.)

65 Numerical Analysis

*726-65-1 JULIO CESAR DIAZ, University of Kentucky, Lexington, Kentucky 40506.
Hybrid Collocation-Galerkin Method for Laplaces Equation on a Rectangle

A procedure that requires fewer quadratures than the Galerkin method using tensor product of piecewise polynomial spaces of degree r is presented for Laplaces equation on Ω the unit square with zero boundary conditions.

The collocation points are based on the roots of the Jacobi polynomial on [0,1] of degree r-1 with respect to the weight function x(1-x). A related variational procedure, based on a semi-discrete inner product defined on Ω using the collocation points, is used to obtain optimal L^2 and H^1_0 rates of convergence. (Received May 2, 1975.)

726-65-2 CHARLES R. JOHNSON, University of Maryland, College Park, Maryland 20742.
Estimation and Computation of the Numerical Range

Let $F(A) \equiv \{x^*Ax : x^*x = 1 , x \in C^n\}$ be the field of values (numerical range) of an $n\times n$ complex matrix A. Thus F is a map from $M_n(C)$ into the convex subsets of the complex plane. First of all, a Gersgorin approach to estimation of the location of F(A) is given. This may be sharpened using diagonal congruences and translations. Secondly,

an approach to explicit numerical computation of F(A) is presented. A method is given for computing a mesh of boundary points which, when combined with the fact that F(A) is convex, allows arbitrarily close approximation of the outline of F(A) with tight bounds on the possible error. Finally, a simple application is made to stability analysis.
(Received April 9, 1975.)

726-65-3 HERBERT E. SALZER, 941 Washington Ave., Brooklyn, New York 11225, **Some Extensions of Prony Approximation.**

Prony's method of approximating a function $f(x)$ by a sum of exponential terms $\sum_{i=1}^{n} A_i\ e^{a_i x}$, where both a_i and A_i are to be determined, is extended to osculatory interpolation of any order, and also to the direct global approximation to the solution of the linear differential equation $\sum_{r=0}^{k} \phi_r(x)\ \ y^{(r)}(x)=f(x)$, where $f(x) \not\equiv 0$.

(Received May 2, 1975.)

*726-65-4 PETER HENRICI, ETH, 8006 Zürich. Fast algorithms for rational powers of formal power series.

Let $P=1+a_1x+\dots$ be a formal power series (fps) with complex coefficients, and let $S=1+b_1x+\dots$ be the unique fps such that $S^2 = P$. Classical algorithms for computing the first n coefficients of S require $O(n^2)$ complex multiplications μ. We present algorithms that compute the first n coefficients of S or of S^{-1} (n is a power of 2) in no more than $24n\log_2 n\ \mu$. The computation of P^r for fixed rational r likewise can be done in $O(n\log_2 n)\ \mu$. (Received May 23, 1975.)

*726-65-5 GRAEME FAIRWEATHER, University of Kentucky , Lexington, Kentucky 40506
Numerical methods for parabolic partial integro-differential equations.

Consider the quasilinear parabolic integro-differential $u_t - \nabla.a(u)\nabla u = \int_0^t f(x,t,s,u(x,s), \nabla u(x,s))ds$. In $\Omega_T \equiv \Omega \times [0,T]$, where Ω is a bounded domain in the plane, a solution u is sought which satisfies appropriate initial and boundary conditions. Such problems arise in many areas, for example, in the theory of viscoelasticity. In this paper existing finite difference procedures for the approximate solution of this problem are reviewed, and several discrete-time Galerkin procedures formulated. For these methods a priori error estimates are derived under reasonable assumptions on the functions a and f. The results of this paper can be extended to the case of n-space variables and to systems of equations. The numerical solution of the "hyperbolic" analogue of this problem is also mentioned.
(Received May 27, 1975.) (Introduced by Dr. J. Aczel.)

726-65-6 ALVIN BAYLISS and EUGENE ISAACSON, Courant Institute, New York University, 251 Mercer Street, New York, New York 10012. How to make your algorithm conservative. Preliminary report.

Given the inital-boundary value problem $u_t = B(u)$ with the vector solution u, construct functionals $g_k\{u\} = 0$, $k = 1,\dots,K$, satisfied by u. Given an algorithm for finding W(n) as an approximation to $u(n\Delta t)$, say (*) $W(n+1) = C\{W(n),\dots, W(n-s)\} \equiv CW(n)$, a modification of (*) produces the approximation U(n) as (**) $U(n+1) = CU(n) + V(n+1)$. Here (i) the truncation errors of (*) and (**) are of the same order, say $(\Delta t)P$; (ii) functionals G_k that approximate g_k to within $(\Delta t)^{p+1}$ are constructed, e.g. by numerical integration; (iii) G_k is linearized about CU(n) as $L_k\{U(n+1)\} \equiv G_k\{CU(n)\} + [\partial G_k/\partial U(n+1)][U(n+1) - CU(n)]$, with the gradient, ξ_k, evaluated at CU(n); (iv) $\|V(n+1)\|$ is minimized subject to the K constraints $L_k = 0$, for some norm $\|\cdot\|$, e.g. ℓ_2 ($L_k = 0$ is a hyperplane with normal vector ξ_k); (v) hence $V(n+1) = \sum\alpha_k \xi_k$ where the α_k are determined by solving K linear equations $L_k\{U(n+1)\} = 0$. This modification method has improved the "stability" properties of some standard difference schemes for the heat equation; it has made possible the long term calculation of "atmospheric" flows with the use of overlapping polar stereographic rectangular meshes. A similar method appears in an unpublished paper of Y.K. Sasaki. (Received June 16, 1975.)

*726-65-7 R. I. Andrushkiw, New Jersey Institute of Technology, Newark, NJ 07102 Inclusion theorems for the eigenvalues of a quadratic operator pencil.

Consider the pencil $L(\lambda) = A_0 - \lambda A_1 - \lambda^2 A_2$ where A_i $(i=0,1,2)$ are linear, in general unbounded and nonhermitian operators, densely defined in a Hilbert space H. Inclusion theorems for the eigenvalues of the nonlinear problem $L(\lambda)u = 0$ are derived in the case when the operators A_i are K-positive and satisfy sufficient conditions to ensure the validity of a generalized Hilbert-Schmidt expansion theorem. The present work extends certain results of the author obtained for K-positive (non-selfadjoint) eigenvalue problems $Tu - \lambda Su = 0$ (J. Math. Anal. Appl. 50 (1975), 511 - 529). Estimates of the eigenvalues of a quadratic pencil with ordinary differential operators are given, and some extensions of the results to polynomial pencils of a higher degree are indicated. (Received June 17, 1975.)

*726-65-8 JOHN JONES JR, Air Force Institute of Technology, Dayton, Ohio 45433 Zeros of continuous real-valued functions II. Preliminary report.

Let $f(x)$ be a solution of $(*)\ \mathcal{L}[f] = [r(x)f'(x)]' + 2q(x)f'(x) + p(x)f(x) = 0$ where $p(x)$, $q(x)$, $r(x) > 0$ are continuous on $\alpha < x < \beta$. Let $u(x,h) \in C'[a,b]$, $[a,b] \subset (\alpha,\beta)$, $u(a,h) = u(b,h) = 0$, h real, and $G(x)$ be a continuous function of x on $\alpha \leq x \leq \beta$ such that $Q(x) = \begin{pmatrix} r(x) & -q(x) \\ -q(x) & G(x) \end{pmatrix}$ is positive semi-definite on $\alpha \leq x \leq \beta$. Let the quadratic functional $J[u]$ be given by

$$J[u] = \int_a^{a+h} \left\{ r(x)\left[u'(x,h)\right]^2 - 2q(x)u(x,h)u'(x,h) + \left[G(x) - p(x)\right]u^2(x,h) \right\} dx.$$

Theorem. If there exists a, $h > 0$ such that $J[u] < 0$ for $u(x,h)$ as above then $f(x)$ has at least one zero on $[a, a+h]$. (Received June 17, 1975.)

*726-65-9 SUHRIT K. DEY, Eastern Illinois University, Charleston, Illinois 61920. Accelerated iterative scheme for a system of nonlinear equations. Preliminary report.

In this work a technique has been developed to solve a set of nonlinear equations with the assumption that they admit a unique solution. The algorithm involves nonlinear Gauss-Seidel iterations and at each iteration the value of the iterate is added to a predetermined acceleration parameter. If $\mathbf{x} = (x_1 x_2 \ldots x_n)^T$ is the unknown vector in a normed vector space $\mathbb{V}$, then the acceleration vector $\mathbf{w} = (w_1 w_2 w_3 \ldots w_n)^T \in \mathbb{V}$. At some $(k+1)$th iteration, the value of $x_i^{(k+1)}$ is obtained with the knowledge of w_i where $w_i = w_i(x_0^{(k+1)}, x_1^{(k+1)}, \ldots, x_{i-1}^{(k+1)}, x_i^{(k)}, \ldots, x_n^{(k)})$. The acceleration vector $\mathbf{w}$ has two properties: (i) it accelerates the rate of convergence of iterations; (ii) it determines the mode of convergence, that means it shows how much more computations are required so that convergence may be achieved. Thus $\mathbf{w}$ gives an estimate of the error in the algorithm. The algorithm is computationally simple and easy to implement. Several nonlinear equations have been studied. The results seem to be very encouraging. (Received June 16, 1975.)

*726-65-10 JOHN BARNES and RON LOMAX, Univ. of Michigan, Ann Arbor, MI 48104. Two-Dimensional Simulation of Semiconductor Devices Using Finite Element Methods

In order to examine in detail the operation of semiconductor devices it is necessary to solve Poisson's equation and the current continuity equation simultaneously in a closed region bounded by, typically, four conducting (Dirichlet) and four insulating (Neumann) segments. Since the operation of a transistor, and in particular a field-effect transistor, involves the flow of particles in a rather complicated two-dimensional manner, this device is being simulated with two-dimensional finite element methods. Although the equations describing this device have much in common with those of fluid flow, there are differences which require careful construction of the approximation in order to assure exact continuity in the formulation. For exact continuity, an element in C' is required, although several aspects of device performance have been studied with linear elements. Specific results for both a linear and an Hermite bicubic element simulation will be discussed and compared with the classical one-dimensional analytical equations of this device first proposed by Shockley. However, the numerical simulation is needed in order to understand the operation under conditions where the Shockley model is invalid, to include the effects of diffusion current and to study the dynamic properties of the transistor. (Received June 17, 1975.) (Authors introduced by Professor George Fix.)

716-65-11 DR. MELVYN CIMENT, Naval Surface Weapons Center, White Oak Laboratory Silver Spring, MD 20910 NUMERICAL SOLUTION OF FREE BOUNDARY VALUE PROBLEMS FOR PARABOLIC EQUATIONS
Preliminary report

Free boundary problems for parabolic equations occur in the mathematical formulation of many diverse problems including, for example; flow through porous media, diffusion of oxygen in tissue, and heat transfer with phase change as in solidification processes. The numerical solution of several such problems will be discussed. Special focus will be placed on numerical methods which succeed in two or more space dimensions. In particular, we will indicate where fixed domain algorithms (no front tracking) have been successful and where current applications are underway. (Received June 17, 1975.)

*726-65-12 STEPHEN LEVENTHAL, Naval Surface Weapons Center, White Oak, Silver Spring, MD20910 and A. K. AZIZ, University of Maryland, College Park, MD 20742
On the Numerical Solution of Equations of Hyperbolic-Elliptic Type

A survey of the numerical methods for the solution of boundary value problems of hyperbolic-elliptic type will be presented. Included in the survey will be finite difference methods which employ one type of differencing in the hyperbolic region and one type of differencing in the elliptic region, finite difference methods which use a first order system form of the equation, and finite element methods which also use a first order system form of the equation. Numerical results from each of these methods will also be included. (Received June 17, 1975.)

726-65-13 DIANNE PROST O'LEARY, University of Michigan, Ann Arbor, Michigan 48104.
Iterative Methods for Structured Systems of Equations. Preliminary report.

This paper discusses the solution of systems of equations for which direct numerical solution is impractical because of the size or sparsity structure. The solution must be obtained through iterative methods, which construct successive approximations to the solution without inverting the full operator. In cases such as discretization of elliptic partial differential equations we may have some information about the problem such as eigenvalues, sparsity of the Jacobian, or the analytical inverse of related operators. This paper presents algorithms which exploit this information to give convergence far more rapid than general iterative techniques. Examples for linear and nonlinear systems will be presented.
(Received June 2, 1975.) (Author introduced by Dr. George Fix.)

68 Computer Science

*726-68-1 F. P. PREPARATA and S. J. HONG, University of Illinois, Urbana, Illinois 61801.
Convex hulls of planar and spatial sets of points.

The convex hull of planar and spatial sets of n points can be determined with $O(n \log_2 n)$ operations. This complexity result was known for planar sets (R. L. Graham, IPL, 1, 132-133, 1972), but it is new for spatial sets. Algorithms are presented, which use the "divide and conquer" technique and recursively apply a merge procedure for two nonintersecting convex hulls. It is also shown that any convex hull algorithm requires at least $O(n \log_2 n)$ operations, so that the time complexity of the proposed algorithms is optimal within a multiplicative constant. (Received May 21, 1975.) (Authors introduced by David E. Muller.)

*726-68-2 DAN HOEY and MICHAEL IAN SHAMOS, Department of Computer Science, Yale University, New Haven, CT 06520. Efficient Computations in Geometry.

For solving problems in combinatorial geometry we seek characterizations that are not only constructive but computationally efficient. We show how to recast into useful algorithmic form such classical theorems as Kirchberger's criterion for two finite point sets to be linearly separable. Metric properties can often be exploited to yield fast solutions to problems that are usually formulated in graph-theoretic or purely combinatorial terms. Upper and lower bounds are given for the number of operations that are required to determine various properties of geometric objects. Given n points in the plane, $O(n \log n)$ operations are necessary and sufficient to find the two closest. A more surprising result is that $O(n \log n)$ operations also suffice to construct a minimum-length spanning tree on n points in the plane, even though the underlying graph has $O(n^2)$ edges. Several problems are presented for which no finite algorithms can exist. (Received June 10, 1975.)

*726-68-3 C. L. LIU, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801. Approximation algorithms for discrete optimization problems.

A topic of significant interest in the study of optimization problems is the design of efficient algorithms which yield a solution that maximizes or minimizes a certain cost function. Unfortunately, there are many optimization problems for which no algorithm for producing a desired result is known. Furthermore, there are also many instances in which although such an algorithm is available, to carry out the algorithmic steps to obtain an optimal result is prohibitively tedious or expensive. Consequently, a reasonable compromise is to search for efficient approximation algorithms that produce good, although not necessarily best possible results. In this paper, we shall attempt to identify some general concepts and techniques related to these algorithms. (Received June 16, 1975.) (Author introduced by David E. Muller.)

*726-68-4 ANDREW C. YAO, Department of Mathematics, Mass. Inst. of Technology, Cambridge, Mass. 02139. Recent Developments on Minimum Spanning Trees.

Recent progress in the development of efficient algorithms finding minimum spanning trees (MST) is surveyed. It will be shown how to construct MST-algorithms that run in time $O(e \log \log v)$. The questions of testing MST and related problems are also examined. (Received June 16, 1975.)

*726-68-5 H.W. BERKOWITZ, SUNY, New Paltz, N.Y. 12561. The consecutive retrieval property.

Let $F = \{x_1, \ldots x_m\}$, and $A_1, \ldots, A_m$ a collection of non-empty subsets of F. Let $S = \{F, A_1, \ldots, A_n\}$. S has the consecutive retrieval property, CRP, if there exists a function f from F into the real numbers such that if $x_i, x_j \in A_k$, and $f(x_p)$ is between $f(x_i)$ and $f(x_j)$, then $x_p \in A_k$. F is partially ordered by: $x_i < x_j$ iff $x_i \in A_k$ implies that $x_j \in A_k$. Eswaran [SIAM J. on Computing 4(1975), 56-68] investigated the CRP by means of Hamiltonian paths in a certain family of graphs. In the present note the CRP is examined by means of the extreme points of the above partial order. It is shown that Lemma 8 part (ii) of Eswaran is not correct. Theorem. If S has the CRP then $m \leq 2n - 1$. (Received June 16, 1975.)

*726-68-6 ELLIS D. COOPER, Herbert H. Lehman College, City University of New York, Bronx, New York 10468. The syntax and semantics of computer science. Preliminary report.

Abstract definitions of hierarchy and modularity are proposed; their "orthogonality" is noted. Software (programs) and hardware (computers) exemplify both concepts. Our ultimate goal is to distill the essence of the software/hardware relationship. So, the graph-theoretic/combinatorial notion of flowchart is shown to be equivalent to a purely algebraic notion: flow formula. Flow formulas participate in various identities, of which some are reminiscent of commutative and distributive laws. Futhermore, there is a compelling analogy between syntax/semantics (mathematical logic) and software/hardware (computer science). Some of our concepts are illustrated by an interpretation of propositional formulas as their own "evaluation procedures". (Received May 23, 1975.) (Author introduced by Professor David E. Muller.)

*726-68-7 SHMUEL WINOGRAD, IBM Thomas J. Watson Research Center, Yorktown Heights, New York, 10598. Some Results in Arithmetic Complexity.

This talk will describe some recent results in arithmetic complexity, concentrating on the effect of the field of scalars on the number of multiplications. (Received June 17, 1975.)

81 Quantum Mechanics

*726-81-1 MARIA Z. v. KRZYWOBLOCKI, Michigan State University, East Lansing, Michigan 48824, Wave Mechanics-Macroscopic Fluid Dynamics Approach to Turbulence.

The macroscopic approach to the mathematical aspects of turbulence (discovered by O. Reynolds, 1883) through the Navier-Stokes System cannot claim to be very successful. The N.S. system, particularly the coefficients of viscosity (first and second) are very crude, they cannot solve delicate problems (transfer, conduction, convection). On the other side, the mathematical approach to the quantum theory by John von Neumann (at 29) is accepted as a very successful approach. The author proposes an approach to macroscopic turbulent fluid system based upon the Feynman diagrams (Nobel Prize) and E. Madelung (1926) "Quantum Theory in Hydrodynamic Form". The first one starts with operations with one-particle concepts and with the aid of correspondence rules determines the structure of any approximation at the molecular level. The second one proposes an association between the electron level (10^{-27}) and macroscopic fluid dynamics. That all allows one to start with the Schroedinger equation (for one electron), to pass to a cluster of molecules and the concept of classical mass in macroscopic fluid, viscosity and all heat transfer phenomena being included (diabatic flow) through "hidden variables" and to end up with the mean value pattern--of a boundary layer.
(Received June 13, 1975.)

726-81-2 THAD DANKEL, JR., University of North Carolina, Wilmington, North Carolina 28401. Derivation of the Charge Current of the Pauli Equation Using Velocity Operators. Preliminary report.

There is a single procedure, using velocity operators, for finding the charge current for both the Schroedinger and Dirac equations. However, this procedure applied directly to the Pauli equation does not yield the charge current of the Pauli theory; this current is usually derived via non-relativistic approximation of the Dirac current. We show how the canonical velocity-operator method does lead to the current of the Pauli theory when applied to a spin model of Bopp and Haag [Z. Naturforschg. 5a, 644-653 (1950)]. (Received June 16, 1975.)

90 Economics, Operations Research, Programming, Games

*726-90-1 JOHN R. SORENSON, Valparaiso University, Valparaiso, Indiana, 46383. A Pricing Game with Elastic Demand. Preliminary Report.

Let N be the set of possible consumers of a good with cost function $C(x)$ for which $C'(x)$ is piecewise continuous. Assume each consumer i has a demand function f_i that is decreasing with a finite number of discontinuities. Denote the composite demand function for the coalition S by f_S and its integral by F_S. The characteristic function v is defined by setting $v(S)$ equal to consumer surplus; that is, the maximum value of $F_S(x) - C(x)$. As in earlier models for inelastic demand, each imputation $(z_1, z_2, \ldots, z_n)$ determines a price $F_i(q_i) - z_i$ for the quantity q_i allocated to consumer i at the general welfare maximum. The main results are:

1. If $C(x)/x$ is non-increasing, then the core is non-empty.

2. If $C'(x)$ is non-increasing, then the game is convex.

(Received June 16, 1975.)

92 Biology and Behavioral Sciences

726-92-1 WILLIAM C. HOFFMAN, Oakland University, Rochester, Michigan 48063
In defense of the LTG/NP.

The Lie transformation group approach to neuropsychology (LTG/NP) has recently been called into question by D. A. Smith (these Notices 22 (1975) A-255) in connection with the author's model for "visual illusions of angle" (S.I.A.M. Review 13 (1971) 169-184). Certain definitional and logical flaws in Smith's analysis are noted and two counter-examples to Smith's "Principle" are given. The matter of canonical variables on the visual manifold and the corresponding local structure are discussed. The empirical scope of the LTG/NP is reviewed: Not only visual illusions of angle, but also the spiral aftereffect, motional aftereffects, orthogonal afterimages, the developmental sequence of infant vision, the phi phenomenon (as an instance of the stroboscopic differential equation), and auditory perception as a relaxation oscillation upon the cochlea, all follow directly and naturally from the well known perceptual constancies (= invariances) and their appropriate Lie subalgebras. It follows that Smith's approach also fails on the basis of Occam's razor. (Received May 14, 1975.)

94 Information and Communication, Circuits, Automata

726-94-1 NASSER DASTRANGE, Pahlavi University, Shiraz, Iran. On the Reconstruction of Bandlimited Signals from Sampled Values, Preliminary report.

The sampling theorem states that any frequency bandlimited singal can be exctly reconstructed from its sampled values.

Although the sampling theorem for sequency bandlimited signals was proved by the aid of Kramer's generalized sampling theorem we will show that this theorem can be proved directly or by the method of Shannon's sampling theorem in Fourier analysis. This is in agreement with the result of Campbell, that for some cases generalized theorem does not enlarge the class of functions to which sampling theorems can be applied. (Received May 29, 1975.)
(Author introduced by A.Fattahi.)

*726-94-2 R. ARTHUR KNOEBEL, New Mexico State University, Las Cruces, New Mexico 88003.
Maximal Sets of Compatible Threshold Functions.

Fix n as a positive integer. A *threshold function* is defined to be any n-ary operation on the set $\{0,1\}$ for which there are a *weight vector* $w = \langle w_1,\ldots,w_n\rangle$ of real numbers and a *threshold* t, also a real number, such that for all vectors $x = \langle x_1,\ldots,x_n\rangle$ with values in $\{0,1\}$, one has

$$f(x) = \begin{cases} 0 & \text{if } w\cdot x < t, \\ 1 & \text{if } w\cdot x \geq t. \end{cases}$$

A set of threshold functions is called *compatible* if there is a common weight vector (but variable threshold) for all functions in the set.

Theorem. Any maximal compatible set of threshold functions has $2^n + 1$ *elements.*

This is proven by showing that the direction of the weight vector of any compatible set may always be shifted slightly so that one obtains a new compatible set of $2^n + 1$ functions containing the old. (Received May 30, 1975.)

726-94-3 STEPHAN R. CAVIOR, Department of Mathematics, State University of New York at Buffalo, Buffalo, New York 14226. An Upper Bound Associated with Errors in Gray Code.

Let j be a binary n-tuple $(j_{n-1}\cdots j_0)_2$. Define the binary Gray codeword $g(j)$ of j by $g(j) = (g^j_{n-1}\cdots g^j_0)_2$, where $g^j_k \equiv j_k + j_{k+1} \pmod 2$ for $k = 0,1,\ldots,n-2$, and $g^j_{n-1} = j_{n-1}$. It has been proved by C. K. Yuen [IEEE Trans. Information Theory IT-20 (1974), 668] that if $0 \leq i,j \leq 2^n - 1$ and i,j are encoded as binary Gray codewords whose Hamming distance is $m \geq 1$, then $|i - j| > 2^m/3$. We prove here that under the same conditions, $|i - j| < 2^n - 2^m/3$. (Received June 16, 1975.)

ERRATA
Volume 22

KENNETH C. LOUDEN, Torsion theories, ring extensions, and group rings. Preliminary report. Abstract 75T-A86, Page A-378.

Line 1, for "β^*: Mod-R → Mod-S" read "β^*: Mod-S → Mod-R".

JOHN M. HOLTE, Exponential limit law for critical general branching processes. Abstract 722-F4, Page A-359.

Line 7, after "E[N(•)]" insert "has finite positive second moment and".

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MATHEMATICS TEACHING AND RESEARCH. Ph.D. Wisconsin, 1974, algebra. Age 28. Four papers accepted, 2 submitted. Research on incidence algebras, matrix theory and applications, graph theory. Background in combinatorial mathematics, logic. Teaching experience, industrial work, including computing, and postdoctoral experience. Available September 1975. Resume and references on request. Robert Feinberg, Applied Math. Div., National Bureau of Standards, Washington, D.C. 20234.

COLLEGE OR JUNIOR COLLEGE TEACHER. 9 years of college teaching, including graduate math courses; FORTRAN programming; quantitative methods for business majors; and remedial math for open-admissions students. Ph.D. Courant Institute; 2 papers in Partial Differential Equations. James M. Newman, 435 West 23rd Street, #14-D, New York, New York 10011.

[redacted]

N. Bogolubov, A. A. Logunov, I. T. Todorov

Authorized translation from the Russian by Stephen A. Fulling and Ludmila G. Popova

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