Notices
OF THE
AMERICAN
MATHEMATICAL
SOCIETY

CODEN: AMNOAN
Volume 23, Number 3
Pages 133–182, A-349–A418

April 1976
Issue 169
Calendar

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the Notices was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting in person must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

<table>
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<th>Meeting Number</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts*</th>
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<tr>
<td>736</td>
<td>June 18, 1976</td>
<td>Portland, Oregon</td>
<td>April 27, 1976</td>
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<td>737</td>
<td>August 24-28, 1976 (80th Summer Meeting)</td>
<td>Toronto, Canada</td>
<td>June 15, 1976</td>
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<td>November 6, 1976</td>
<td>Ann Arbor, Michigan</td>
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<td>November 19-20, 1976</td>
<td>Columbia, South Carolina</td>
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<td>November 19-20, 1976</td>
<td>Albuquerque, New Mexico</td>
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<td>January 27-31, 1977 (83rd Annual Meeting)</td>
<td>St. Louis, Missouri</td>
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<td>March 31-April 1, 1977</td>
<td>Huntsville, Alabama</td>
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<td>April 22-23, 1977</td>
<td>Hayward, California</td>
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<td>August 14-18, 1977 (81st Summer Meeting)</td>
<td>Seattle, Washington</td>
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<td>November 11-12, 1977</td>
<td>Memphis, Tennessee</td>
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<td>January 17-22, 1978 (84th Annual Meeting)</td>
<td>Atlanta, Georgia</td>
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<td></td>
<td>January 11-15, 1979 (85th Annual Meeting)</td>
<td>Milwaukee, Wisconsin</td>
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<td>January 8-12, 1981 (87th Annual Meeting)</td>
<td>San Francisco, California</td>
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*Deadline for abstracts not presented at a meeting (by title)

June 1976 issue: April 20
August 1976 issue: June 8

OTHER EVENTS

April 11-12, 1976  Symposium on Asymptotic Methods and Singular Perturbations, New York, New York
April 12, 1976  Short Course on Introduction to Computer Science for Mathematicians, New York, New York
August 22-23, 1976  Short Course on Mathematical Economics, Toronto, Canada
OF THE
AMERICAN MATHEMATICAL SOCIETY

Everett Pitcher and Gordon L. Walker, Editors
Hans Samelson, Associate Editor

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April, 1976
The seven hundred thirty-fourth meeting of the American Mathematical Society will be held at the Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York, from Sunday April 11, through Wednesday, April 14, 1976.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be three one-hour addresses. John S. Lew of the IBM T. J. Watson Research Center will speak on "Some applications of structural concepts in asymptotic analysis" at 11:00 a.m. on Tuesday, April 13. W. Wistart Comfort of Wesleyan University will speak on "Ultrafilters: some old and some new results" at 2:00 p.m. on Tuesday, April 13. Robert MacPherson of Brown University will speak on "Characteristic classes for singular spaces" at 11:00 a.m. on Wednesday, April 14.

JOHN F. KENNISON of Clark University has organized a special session on Sheaves and other topics to be held Tuesday morning and afternoon; the speakers will be Norman Auspitz, J. F. Kennison, Christiane Rousseau, and Lawrence Stout. G. S. LADDE of SUNY College at Potsdam has organized a special session on Applicable differential equations to be held Tuesday morning and afternoon; the speakers will be S. R. Bernfeld, Thomas G. Hallam, Stuart Hastings, James L. Kaplan, William Kilmer, G. S. Ladde, S. Lee-la, and Robert Rosen. EDGAR R. LORCH of Columbia University has organized a special session on Functional analysis and allied topics to be held Tuesday morning and afternoon; the speakers will be Lewis A. Coburn, R. G. Douglas, Edward G. Effros, James Glimm, Roger Howe, Shizuo Kakutani, Peter D. Lax, and Bertram Yood.

There will be sessions for contributed ten-minute papers in the morning and afternoon on Tuesday and Wednesday. No provision will be made for late papers.

Each meeting room will be equipped with an overhead projector.

The Council of the Society will meet on Sunday, April 11, at 5:00 p.m. in Suite L & M of the Biltmore Hotel.

SYMPOSIUM ON ASYMPTOTIC METHODS AND SINGULAR PERTURBATIONS

With the support of the Energy Research and Development Administration and the National Science Foundation, a symposium on Asymptotic Methods and Singular Perturbations is scheduled to be held on Sunday and Monday, April 11-12, 1976. This topic was selected by the AMS-SIAM Committee on Applied Mathematics, whose members are Earl A. Coddington, Donald S. Cohen, Richard C. DiPrima, Lester E. Dubins, J. Barkley Rosser, Stephen Smale, and W. Gilbert Strang.

The purpose of the symposium is to help bring this active and productive field of applied analysis to the attention of a larger mathematical audience, and emphasis will be put on recent advances in analytical techniques and problems emerging from particular physical situations featuring singular perturbation phenomena. The organizing committee, comprised of Donald S. Cohen, California Institute of Technology; Joseph B. Keller, New York University; Courant Institute of Mathematical Sciences; Robert E. O'Malley, Jr., University of Arizona (chairman); and M. D. Van Dyke, Stanford University, has organized the symposium into four sessions. To the greatest extent possible the lectures will identify the principal lines of current research. It is hoped that the lectures will enable the audience to appreciate some areas where they could use such techniques in their own research. The broad spectrum of applications to be discussed should make the symposium valuable to both specialists and the general mathematical audience.

The list of speakers includes Paul C. Fife (University of Arizona), Frank C. Hoppensteadt (New York University, Courant Institute of Mathematical Sciences), David R. Kassoy (University of Colorado, Boulder), Petar Kokotovic (University of Illinois, Urbana-Champaign), H. O. Kreiss (University of Uppsala, Sweden, and New York University, Courant Institute of Mathematical Sciences), Donald Ludwig (University of British Columbia), Frank W. J. Olver (University of Maryland, College Park), Mark A. Pinsky (Northwestern University), Keith Stewartson (University of California, London), and Wolfgang R. Wasow (University of Wisconsin, Madison). A novel feature of the symposium will be a session on open problems. The speakers in this session will include Ellis Cumberbatch (Purdue University), Marvin L. Friedman (Boston University), G. W. Hedstrom (University of California, Lawrence Livermore Laboratory), David J. Wollkind (Washington State University), and Chen H. Wu (University of Illinois at Chicago Circle). The symposium will be held in the Windsor Room of the Biltmore Hotel both days.

SHORT COURSE ON INTRODUCTION TO COMPUTER SCIENCE FOR MATHEMATICIANS

The American Mathematical Society will present a one-day Short Course on Introduction to Computer Science for Mathematicians on Monday, April 12, 1976 in the Grand Ballroom of the Biltmore Hotel. The course is designed to give
stantial introductions to three important topics in computer science. It is intended primarily for mathematicians trained in other areas who wish to get a concentrated introduction to this field. However, the course is open to all who wish to participate, provided payment of the registration fee.

The program is under the direction of Samuel Winograd of the IBM T. J. Watson Research Center.

There will be three lecturers, each of whom will give two fifty-minute talks. Robert L. Constable (Department of Computer Science, Cornell University) will speak on "Proving the correctness of programs"; Nicholas Pippenger (IBM T. J. Watson Research Center) will speak on "The complexity of algebraic and order-theoretic computations"; and Michael Rabin (Hebrew University and Massachusetts Institute of Technology) will speak on "Exponential explosions in computations."

Summaries of these talks and accompanying reading lists appeared on pages A-333 through A-335 of the February 1976 issues of these Notices.

ASSOCIATION FOR WOMEN IN MATHEMATICS

There will be a concurrent meeting of the Association for Women in Mathematics, organized by Lenore Blum.

ACCOMMODATIONS

Persons intending to stay at the Biltmore Hotel should make their own reservations with the hotel. A reservation form and a listing of room rates can be found on the last page of the February Notices. The deadline for receipt of reservations is March 28, 1976.

REGISTRATION

The registration desk will be located in the Key Room of the Biltmore Hotel on the nineteenth floor adjacent to the Grand Ballroom. The desk will be open from 8:30 a.m. to 4:30 p.m. on Sunday, April 11, through Tuesday, April 13; and from 8:30 a.m. to 3:30 p.m. on Wednesday, April 14.

The registration fees for the meeting are as follows:

- Member $3
- Student and unemployed 1
- Nonmember 5
- Short Course on Computer Science 12

TRAVEL

The Biltmore Hotel is located on Madison Avenue at 43rd Street on the east side of New York City. Walkways to Grand Central Station are located under the hotel and signs are posted directing persons to the lobby of the hotel.

Those arriving by bus may take the independent Subway System from the Port Authority Bus Terminal. There is shuttle bus service from LaGuardia and Kennedy Airports directly to Grand Central Station. Starters can direct participants to the correct bus.

Air passengers arriving at Newark Airport can take a shuttle bus to the Port Authority Bus Terminal and take a subway, taxi, or bus to the hotel.

Those arriving by car will find many parking facilities in the neighborhood, in addition to those at the hotel. Parking service can be arranged through the hotel doorman at a cost of $9 for the 24-hour period. There will be an additional charge for extra pickup and delivery service if it is required. The parking fee is subject to New York City taxes.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed in care of the American Mathematical Society, The Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York 11017.

PROGRAM FOR THE SYMPOSIUM ON ASYMPTOTIC METHODS AND SINGULAR PERTURBATIONS

All sessions will be held in the Windsor Room, 18th Floor

SUNDAY, APRIL 11

First Session - Chairman: Joseph B. Keller, Courant Institute of Mathematical Sciences

10:00 a.m. Adiabatic invariants. WOLFGANG R. WASOW, University of Wisconsin, Madison

11:15 a.m. On numerical methods and singular perturbations. HEINZ-Otto KREISS, University of Uppsala, Sweden, and Courant Institute of Mathematical Sciences

Second Session - Chairman: Donald S. Cohen, California Institute of Technology

1:30 p.m. Uniform asymptotic expansions and singular perturbations. FRANK W. J. OLVER, University of Maryland, College Park and The National Bureau of Standards

2:30 p.m. Extremely rapid transient phenomena in combustion: ignition and explosion. DAVID R. KASSOY, University of Colorado, Boulder

3:45 p.m. Multi-time stability analysis of a problem from population biology. FRANK C. HOPPENSTEADT, Courant Institute of Mathematical Sciences

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
MONDAY, APRIL 12

Third Session - Chairman: Milton D. Van Dyke, Stanford University

9:00 a.m. How singular perturbation ideas apply to control and how control concepts could be useful in singular perturbations. PETAR KOKOTOVIĆ, University of Illinois, Urbana-Champaign

10:15 a.m. Some asymptotic problems in fluid mechanics. SUSAN N. BROWN and KEITH STEWARTSON*, University College, London

11:15 a.m. Asymptotic analysis of the Linearized Boltzmann equation. MARK A. PINSKY, Northwestern University

Session on Open Problems - Chairman: Robert E. O'Malley, Jr., University of Arizona

1:00 p.m. On the possibility of carrying out a perturbation analysis for control problems involving switch points that coalesce. MARVIN I. FREEDMAN, Boston University

1:20 p.m. Singular perturbation and certain open problems in fracture mechanics. CHIEN H. WU, University of Illinois at Chicago Circle

1:40 p.m. The use of singular perturbation techniques as a tool for modeling ecosystems. DAVID J. WOLLKIND* and JESSE A. LOGAN, Washington State University

2:00 p.m. Nonlinear theory of a caustic. ELLIS CUMBERBATCH, Purdue University

2:20 p.m. Uniform asymptotic expansions for numerical solutions of hyperbolic equations. G. W. HEDSTROM* and R. CHIN, University of California, Lawrence Livermore Laboratory

MONDAY, APRIL 12

Fourth Session - Chairman: Martin B. Kruskal, Princeton University

3:00 p.m. Singular perturbation and wave-front techniques applied to reaction-diffusion problems. PAUL C. FIFE, University of Arizona

4:00 p.m. The validity of the diffusion approximation to discrete stochastic processes. DONALD LUDWIG, University of British Columbia

SHORT COURSE ON INTRODUCTION TO COMPUTER SCIENCE FOR MATHEMATICIANS

All Sessions will be held in the Grand Ballroom, 19th Floor

MONDAY, APRIL 12

9:00-9:50 a.m. Complexity of algebraic and other theoretic computations I. NICHOLAS PIPPENGER, IBM T. J. Watson Research Center

10:00-10:50 a.m. Exponential explosions in computation I. MICHAEL O. RABIN, Hebrew University and Massachusetts Institute of Technology

Break

11:10-noon Proving the correctness of programs I. ROBERT L. CONSTABLE, Cornell University

2:00-2:50 p.m. Complexity of algebraic and other theoretic computations II. NICHOLAS PIPPENGER

3:00-3:50 p.m. Exponential explosions in computation II. MICHAEL O. RABIN

Break

4:10-5:00 p.m. Proving the correctness of programs II. ROBERT L. CONSTABLE

PROGRAM OF THE SESSIONS

THE SEVEN HUNDRED THIRTY-FOURTH MEETING

The time limit for each contributed paper in the general sessions is ten minutes and in the special sessions is twenty minutes. To maintain this schedule, the time limits will be strictly enforced.

TUESDAY, 8:30 A.M.

Special Session on Applicable Differential Equations I, Music Room, First Floor

8:30-9:50 (1) Pattern formation, coding, and self-organization in neural networks. STEPHEN GROSSBERG, Boston University (734-C7)

9:00-9:20 (2) Structural sensitivity of grazing formulations in nutrient controlled plankton models. Professor THOMAS G. HALLAM, Florida State University (734-C3)
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<tr>
<th>Time</th>
<th>Session</th>
<th>Speaker(s)</th>
<th>Institution(s)</th>
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<tr>
<td>9:30-9:50</td>
<td>(3) Mathematical problems arising from the</td>
<td>Professor J. EISENFELD</td>
<td>University of Texas at Arlington (734-C1)</td>
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<td>theory of synovial joint disease (arthritis)</td>
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<td>10:00-10:20</td>
<td>(4) Competitive exclusion and nonequilibrium</td>
<td>JAMES L. KAPLAN*, LEWIS A. COBURN*, and ARNOLD LEBOW</td>
<td>University of Texas at Arlington, University of</td>
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<td>coexistence.</td>
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<td>Maryland</td>
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<td>10:30-10:50</td>
<td>(5) Analysis of competitive processes in</td>
<td>V. LAKSHMIKANTHAM, S. LEELA*</td>
<td>College at Geneseo (734-B10)</td>
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<td>abstract cones. Preliminary report.</td>
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<td>TUESDAY, 8:45 A.M.</td>
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<td>8:45-9:05</td>
<td>Special Session on Functional Analysis and</td>
<td>CHARLES A. BERGER, LEWIS A. COBURN*, and ARNOLD LEBOW</td>
<td>Yeshiva University</td>
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<td>Allied Topics I, Vanderbilt Suite, First</td>
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<td>University</td>
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<td>9:45-10:05</td>
<td>(7) Representation and index theory for C*-</td>
<td>Dr. EDWARD G. EFFROS*, Dr. MAN-DUEN CHOI, University of</td>
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<td>algebras generated by commuting isometries.</td>
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<td>10:15-10:35</td>
<td>(8) Operator theory and complex geometry.</td>
<td>Professor M. J. COWEN and Professor R. G. DOUGLAS*</td>
<td>State University</td>
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<td></td>
<td>Preliminary report.</td>
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<td>at Stony Brook</td>
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<td>TUESDAY, 9:00 A.M.</td>
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<td>9:00-9:20</td>
<td>Special Session on Sheaves and Other Topoi</td>
<td>JOHN KENNISON</td>
<td>Clark University</td>
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<td>I, French Suite, First Floor</td>
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<td>9:30-9:50</td>
<td>(10) A topological spectrum of a commutative</td>
<td>LAWRENCE NEFF, University of</td>
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<td>topological ring.</td>
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<td>Pennsylvania</td>
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<td>10:00-10:20</td>
<td>(12) Topos theory and complex analysis.</td>
<td>CHRISTIANE ROUSSEAU, Universite de Montreal</td>
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<td>Preliminary report.</td>
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<td>TUESDAY, 9:00 A.M.</td>
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<td>9:00-9:10</td>
<td>General Session I, Suite G, First Floor</td>
<td>FRANK HARARY*, University of Michigan, Robert W. ROBINSON, University of</td>
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<td>antioxidant</td>
<td>Newcastle, Australia</td>
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<td>9:15-9:25</td>
<td>(14) Reversed digit orthogonality.</td>
<td>JOSEPH ARKIN*, Spring Valley, New York, Professor PAUL SMITH, University</td>
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<td>of Victoria</td>
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<td>10:00-10:10</td>
<td>(17) A characterization of the complement</td>
<td>BRUCE M. HOROWITZ, Adelphi University and State University of New York,</td>
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<td>of a creative set.</td>
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<td>Maritime College</td>
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<td>10:15-10:25</td>
<td>(18) On Tarski's paradox and the</td>
<td>RIMAM LIPSCHUTZ-YEVICK, Rutgers University, New Brunswick</td>
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<td>complementarity between sequential (aural,</td>
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<td>descriptive) and holistic (optical,</td>
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<td>holographic) recognition (denotation).</td>
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<td>TUESDAY, 9:00 A.M.</td>
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<td>9:00-9:10</td>
<td>Session on Algebra, Suite I, First Floor</td>
<td>HANS SCHNEIDER, University of Wisconsin</td>
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<td>9:15-9:25</td>
<td>(19) Necessity and sufficient conditions</td>
<td>EDWIN K. GORA* and Professor JAMES J. TATTER-SALL, Providence College</td>
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<td>for the existence of a common zero of several polynomials in several variables.</td>
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<td>9:45-9:55</td>
<td>(22) Semiprime rings with finite length</td>
<td>S. K. JAIN*, Ohio University, and Professor SURJEET SINGH, Guru Nanak</td>
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<td>w.r.t. an idempotent kernel functor.</td>
<td>Dev University, Amritsar, India</td>
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<td>10:00-10:10</td>
<td>(23) Algorithmic entropy of sets.</td>
<td>V. K. GOEL* and Professor S. K. JAIN, Ohio University, and Professor</td>
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<td>SURJEET SINGH, Guru Nanak Dev University, Amritsar, India</td>
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10:15-10:25 (24) Symmetric functions on direct powers. Preliminary report. Professor JOHN R. DURBIN, University of Texas at Austin (734-A8)

TUESDAY, 11:00 A. M.

Invited Address, Grand Ballroom, Nineteenth Floor
(25) Some applications of structural concepts in asymptotic analysis. Dr. JOHN S. LEW, IBM T. J. Watson Research Center, Yorktown Heights, New York (734-B26)

TUESDAY, 2:00 P. M.

Invited Address, Grand Ballroom, Nineteenth Floor
(26) Ultrafilters: some old and some new results. Professor W. Wistar Comfort, Wesleyan University (734-G5)

TUESDAY, 3:15 P. M.

Special Session on Functional Analysis and Allied Topics II, Vanderbilt Suite, First Floor
3:15- 3:35 (27) On a connection between nilpotent groups, singularities, and oscillatory integrals. Mr. ROGER HOWE, Yale University (734-H1)

3:45- 4:05 (28) Quantum theory and probability theory. Professor JAMES GLIMM, Rockefeller University (734-F1)

4:15- 4:35 (29) Scattering theory for automorphic functions, Professors PETER D. LAX* and RALPH S. PHILLIPS, New York University (734-B1)

4:45- 5:05 (30) Classification of ergodic transformations. Professor SHIZUO KAKUTANI, Yale University (734-B33)

TUESDAY, 3:15 P. M.

Session on Analysis I, Suite G, First Floor
3:15- 3:25 (31) Infinitesimal generators for general semigroups on a Banach space. Preliminary report. Dr. YEN TZU FU, Indiana State University (734-B11)

3:30- 3:40 (32) Multipliers and the Nevanlinna–Pick theorem, Professor A. K. SNYDER, Lehigh University (734-B20)

3:45- 3:55 (33) Bounds for solutions to nonlinear wave equations in Hilbert space with applications to nonlinear elastodynamics. Dr. FREDERICK BLOOM, University of South Carolina (734-B2) (Introduced by Professor M. Slemrod)

4:00- 4:10 (34) Time (space)-invariant subordinate (multivariate-) linear operators are spectral integrals. Preliminary report. Dr. MILTON ROSENBERG, Rockaway Beach, New York (734-B9)

4:15- 4:25 (35) Direct summands of systems of continuous linear transformations. Professor URI FIXMAN, Queen's University, and Professor FRANK ZORZITTO*, University of Waterloo (734-B3)

4:30- 4:40 (36) Spherical functions on compact groups. Preliminary report. Ms. MARGOT SMALL, Columbia University (734-A7)

4:45- 4:55 (37) Inner–outer factorizations of functions whose Fourier series vanish off a semigroup. Dr. HOWARD L. PENN, United States Naval Academy (734-B16)

5:00- 5:10 (38) Fourier transforms and their Lipschitz classes, Preliminary report, G. SAMPSON* and H. TUY, State University of New York at Buffalo (734-B4)

TUESDAY, 3:15 P. M.

Session on Topology, Suite I, First Floor
3:15- 3:25 (39) Sequentially complete mappings. Professor HOWARD H. WICKE, Ohio University (734-G3)


3:45- 3:55 (41) Adams type spectral sequence and stable homotopy modules II. Preliminary report. Professor T. Y. LIN, University of South Carolina (734-G7)

4:00- 4:10 (42) Homeotopy groups of 2-dimensional manifolds with one boundary component. Preliminary report. Dr. DAVID J. SPROWS, Villanova University (734-G11)

4:15- 4:25 (43) Poincaré's bifurcation analysis. Dr. OKAN GUREL, IBM Corporation, White Plains, New York (734-G2)

4:30- 4:40 (44) The composition closing lemma. Preliminary report. Professor DENIS BLACKMORE, New Jersey Institute of Technology (734-G4)

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TUESDAY, 3:30 P. M.

Special Session on Sheaves and other Topoi II, French Suite, First Floor
3:30- 3:50 (45) Q-sheaves and Q-fields of Banach spaces. Mr. NORMAN E. AUSPITZ, McMaster University (734-A14) (Introduced by Mr. John F. Kemmison)

4:00- 5:00 (46) Informal Discussion

TUESDAY, 3:45 P. M.

Special Session on Applicable Differential Equations II, Bowman Room, Lobby
3:45- 4:05 (47) Dynamical realizability. Professor ROBERT ROSEN, Dalhousie University (734-C8) (Introduced by Professor G. S. Ladde)

4:15- 4:35 (48) Some analytic observations on the pressure field in a gas-lubricated slider bearing. Preliminary report. Dr. JAGDISH CHANDRA, United States Army Research Office, Research Triangle Park, North Carolina, and Professor PAUL WILLIAM DAVIS*, Worcester Polytechnic Institute (734-B17)

4:45- 5:05 (49) Periodic solutions of some higher order systems from biology and chemistry. Preliminary report. STUART HASTINGS, State University of New York at Buffalo (734-B12)

5:15- 5:35 (50) The equations of neutral type for a two-body problem of electrodynamics. Dr. R. D. DRIVER, University of Rhode Island (734-B24)

5:45- 6:05 (51) A basic qualitative analysis on cancer chemotherapy. Professor KUANG-HO CHEN, University of New Orleans (734-B23)

WEDNESDAY, 8:30 A. M.

Special Session on Applicable Differential Equations III, Music Room, First Floor
8:30- 8:50 (52) Renewable resource depletion models for grazing populations. Preliminary report. Professor WILLIAM KILMER, University of Massachusetts, Amherst (734-C6) (Introduced by Dr. Michael Arbib)

9:00- 9:20 (53) Characterization of length distributions of macromolecules from electro-optical decay. Preliminary report. Dr. S. R. BERNFELD*, University of Texas at Arlington, and Drs. M. M. JUDY, R. M. DOWBEN, and G. A. CAMPBELL, University of Texas Health Science Center, Dallas (734-C10)

9:30- 9:50 (54) Error catastrophe in and the evolution of the protein synthesizing machinery. Preliminary report. NARENDRA S. GOEL* and SIRAUL ISLAM, State University of New York at Binghamton (734-C11) (Introduced by Professor G. S. Ladde)

10:00-10:20 (55) Competitive processes III: stability of \( n \)-type stochastic systems. Professor G. S. LADDE, State University of New York, College at Potsdam (734-B19)

10:30-10:45 (56) Informal Discussion

WEDNESDAY, 9:00 A. M.

General Session II, French Suite, First Floor
9:00- 9:10 (57) On simplical volumes. Preliminary report. Professor FRANCINE ABELES, Kean College of New Jersey (734-D2)

9:15- 9:25 (58) Morley polygons. Professor FRANCIS P. CALLAHAN, Pennsylvania State Graduate Center, King of Prussia (727-D2)


9:45- 9:55 (60) Integrability conditions of a structure of a differentiable manifold. Professor JIN BAI KIM, West Virginia University (734-D1)

10:00-10:10 (61) Some generalizations of a theorem of Schl"{u}milch. Professor CARLOS A INFANTOZ- ZI, Universidad de la Rep"{u}blica, Montevideo, Uruguay (734-B21)

WEDNESDAY, 9:00 A. M.

Session on Analysis II, Vanderbilt Suite, First Floor
9:00- 9:10 (62) The integral in a Boolean algebra: Some applications. Professor WILLIAM J. AMADIO, Rider College (734-B6)

9:15- 9:25 (63) Integrals of a function with respect to a function pair and the mean Stieltjes \( \sigma \)-integral. Professor R. A. STOKES, University of Mississippi, and Professor D. B. PRIEST* and Mr. JERRY LEWIS, Harding College (734-B13)

9:30- 9:40 (64) Holomorphic functionals on open Riemann surfaces. Professor PAUL M. GAUTHIER, Université de Montréal, and Professor LEE A. RUBEL*, University of Illinois at Urbana-Champaign (734-B8)
9:45- 9:55 (65) Maximum term of a power series and its derivatives. Professor HARI SHANKAR, Ohio University (734-B18)
10:00-10:10 (66) Nonanalytic automorphic forms and Dirichlet series. Professor V. V. RAO, University of Regina (734-B32)
10:15-10:25 (67) Increasing unions of Stein spaces. Professor ANDREW MARKOE, University of Connecticut (734-B27)
10:30-10:40 (68) A spectral theory for partial isometries. Preliminary report. Professor IVAN ERDELYI, Temple University (734-B22)

WEDNESDAY, 11:00 A.M.

Invited Address, Bowman Room, Lobby
(69) Characteristic classes for singular spaces. Professor ROBERT MacPHERSON, Brown University (734-G8)

WEDNESDAY, 1:30 P. M.

Session on Applied Mathematics, French Suite, First Floor
1:30- 1:40 (70) Convergence of weak amarts: a probabilistic characterization of reflexivity. Professor ANTOINE BRUNEL, Université P. et M. Curie, Paris, France, and Professor LOUIS SUCHESTON*, Ohio State University (734-F3)
1:45- 1:55 (71) On a limit distribution theorem of linear order statistics. Professor BELA GYIRES, L. Kossuth University, Debrecen, Hungary (734-F2) (Introduced by Professor J. Galambos)
2:00- 2:10 (72) A variable coefficient extension of a formula for series conversion. Dr. HERBERT E. SALZER, Brooklyn, New York (734-C4)
2:15– 2:25 (73) A remark on the generalized G-transform. Dr. JOE B. THRASH, University of Southern Mississippi (734-C5)
2:30– 2:40 (74) A generalized complex potential in fluid dynamics. Professor MARIO O. GONZALEZ, University of Alabama (734-C9)
2:45– 2:55 (75) An approach to semantic innovation. Preliminary report, Mr. DAVID LAWRENCE, Courant Institute of Mathematical Sciences, New York University (734-C2)
3:00– 3:10 (76) A boundary-value problem with a turning point. GILBERT N. LEWIS, University of Wisconsin, Milwaukee (734-C12) (Introduced by Professor Robert E. O'Malley, Jr.)

WEDNESDAY, 1:30 P. M.

Session on Differential Equations, Vanderbilt Suite, First Floor
1:30– 1:40 (77) A nonself-adjoint, fourth order, inverse eigenvalue problem. Dr. JOYCE R. McLAUGHLIN, Rensselaer Polytechnic Institute (734-B15)
1:45– 1:55 (78) Two point connection problem for a certain ordinary differential equation. Dr. T. K. PUTTASWAMY, Ball State University (734-B28)
2:00– 2:10 (79) The shape of stable invariant sets of continuous dynamical systems. Professor HAROLD M. HASTINGS, Hofstra University (734-B29)
2:15– 2:25 (80) Reflection principles for Laplace's equation when boundary data involves derivatives of higher order. Professor J. B. DIAZ, Rensselaer Polytechnic Institute, Professor R. B. RAM*, State University of New York, College at Oneonta, and Professor G. R. VERMA, University of Rhode Island (734-B7)
2:30– 2:40 (81) Groups of parabolic equations. Preliminary report, Professor STEVE I. ROSENCRANS, Tulane University (734-B14)

Middletown, Connecticut

Walter H. Gottschalk
Associate Secretary
PRESENTORS OF PAPERS

Following each name is the number corresponding to the speaker's position on the program

o Invited one-hour lecturers

Abeles, F. #57
Amadio, W. J. #62
Arkin, J. #14
*Ausmitz, N. E. #45
*Bernfield, S. R. #53
Blackmore, D. #44
Bloom, F. #33
Callahan, F. P. #58
Chaitin, G. J. #16
*Chen, K.-H. #51
*Coberly, L. A. #26
*Davis, P. W. #48
*Douglas, R. G. #9
*Driver, R. D. #50
Durbin, J. R. #24
*Effros, E. G. #8
*Eisenfeld, J. #3
Erdelyi, I. #68
Fu, Y. T. #31
*Glimm, J. #28
*Goel, N. S. #54
*Goel, V. K. #23
*Gonzalez, M. O. #74
*Gora, E. K. #20
*Grossberg, S. #1

Gurel, O. #43
Gyires, B. #71
*Hallam, T. G. #2
Harary, F. #13
Hastings, H. M. #79
*Hastings, S. #49
Horowitz, B. M. #17
*Howe, R. #27
Infantinozzi, C. A. #61
Jain, S. K. #22
*Kakutani, S. #30
*Kaplan, J. L. #4
*Kennison, J. #10
*Kim, J. B. #60
*Ladde, G. S. #55
Lawrence, D. #75
*Lax, P. D. #29
*Leela, S. #5
*Lew, J. S. #25
Lewis, G. N. #76
Lin, T. Y. #41
Lipschutz-Yevick, M. #18
McLaughlin, J. R. #77
*MacPherson, R. #69
Markoe, A. #67

Meyers, P. R. #15
Olliker, V. #59
Penn, H. L. #37
Priest, D. B. #63
Puttaswamy, T. K. #78
Ram, R. B. #80
Rao, V. V. #66
*Rosen, R. #47
Rosenberg, M. #34
Rosencrans, S. I. #81
*Rousseau, C. #1
Rubel, L. A. #64
Salzer, H. E. #72
Sampson, G. #38
Schneider, H. #19
Shankar, H. #85
Small, M. #86
Snyder, A. K. #32
Sprows, D. J. #42
*Stout, L. N. #11
Sucheston, L. #70
Thrax, J. B. #73
Tsai, J. H. #40
Wicke, H. H. #39
*Yood, B. #7
Zorzitto, F. #35

*Special session speakers

INVITED SPEAKERS AT AMS MEETINGS

This section of these Notices lists regularly the individuals who have agreed to address the Society at the times and places listed below. For some future meetings, the lists of speakers are incomplete.

Portland, Oregon, June 1976
Thomas M. Liggett
R. James Milgram

Toronto, Canada, August 1976
Michael Aschbacher
Jurgen K. Moser (Colloquium Lecturer)
Edward Nelson
Marian B. Pour-El

ORGANIZERS AND TOPICS OF SPECIAL SESSIONS

Abstracts of contributed papers to be considered for possible inclusion in special sessions should be submitted to the Providence office by the deadlines given below and should be clearly marked "For consideration for special session on (title of special session)." Those papers not selected for special sessions will automatically be considered for regular sessions unless the author gives specific instructions to the contrary.

Deadline
May 25, 1976

Toronto, Canada, August 1976

James McCool, Combinatorial group theory
Z. H. Nitecki, Differentiable dynamical systems (tentative)
Anatol Rapoport, Mathematical psychology
Ray C. Shiflett, Doubly stochastic measures and their operators
Joseph R. Shoenfield, Recursively enumerable sets and degrees (tentative)
W. T. Tutte, Chromatic polynomials and related topics
C. T. Yang, Transformation groups
The Seven Hundred Thirty-Fifth Meeting
University of Nevada
Reno, Nevada
April 23 – 24, 1976

The seven hundred thirty-fifth meeting of the American Mathematical Society will be held at the University of Nevada in Reno, Nevada, on Friday and Saturday, April 23 and 24, 1976. The Association for Symbolic Logic will hold a meeting in conjunction with this meeting of the Society. The Association for Symbolic Logic has scheduled two invited one-hour talks. Gabriel Sabbagh of the University of Paris VII and the University of California, Berkeley, will lecture on "First order properties of linear groups" at 2:00 p.m. on Saturday, April 24. Yiannis N. Moschovakis of the University of California, Los Angeles, will give a survey lecture at 9:50 a.m. on Saturday entitled "Inductive definability." The Association has also scheduled four invited one-half hour talks on Saturday as follows: Michael Beeson of Stanford University will lecture at 11:00 a.m. on "The principle of local continuity." John S. Schlipf of the California Institute of Technology will lecture at 11:40 a.m. on "Ordinal spectra of first order theories," Andreas R. Blass of the University of Michigan will lecture at 3:10 p.m. on "Ultra product proofs of ultrafilter theorems," and Richard J. Laver of the University of Colorado will lecture at 3:50 p.m. on "Strong saturation properties of ideals." There will also be sessions of contributed twenty-minute papers. All Association sessions will be held in Room 2 of the Lecture Building.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be one invited hour address. Robert T. Powers of the University of Pennsylvania and the University of California, Berkeley, will lecture on Saturday at 11:00 a.m. in Room 101 of the Scroggham Engineering and Mines Building. The title of his lecture will be "KMS states of UHF algebras and their application to quantum statistical mechanics."

There will be four special sessions. BRUCE E. BLACKADAR of the University of Nevada, Reno, is organizing a special session on C*-Algebras and related topics. Participants will include Bruce A. Barnes, Bruce E. Blackadar, Lawrence G. Brown, Man-Deon Choi, John A. Ernest, Jacob Feldman, Ramesh Gangolli, Philip P. Green III, Peter Florin Hahn, J. William Helton, Calvin C. Moore, Marc A. Rieffel, Jonathan Rosenberg, and William M. Scruggs. PETER A. GRIFFIN of California State University, Sacramento, is organizing a special session of thirty-minute talks on Mathematics of gambling. The speakers will be Thomas M. Cover, William H. Cutler, Thomas S. Ferguson, and Peter A. Griffin. DARRELL C. KENT of Washington State University is organizing a special session on Convergence spaces. Among the speakers will be Allan C. Cochran, William Alan Feldman, Roman Frič, Ray J. Gazik, James M. Irwin, Darrell C. Kent, James F. Porter, Gary D. Richardson, M. Schroder, and Edwin F. Wagner. The proceedings of these sessions on Convergence Spaces will be published by the University of Nevada. RALPH N. McKENZIE of the University of California, Berkeley, is organizing a special session on Varieties of algebras. The speakers will be Kirby A. Baker, John T. Baldwin, George M. Bergman, Alan Day, George A. Grätzer, Bjarni Jónsson, Peter H. Krauss, George McNulty, Walter David Neumann, Philip Olin, A. Regev, Walter F. Taylor, and Paul M. Welchel.

There will be sessions for contributed ten-minute papers on Saturday. Late papers will be accepted for presentation at the meeting, but will not be listed in the printed program of the meeting.

The registration area will be on the first floor of the Scroggham Engineering and Mines Building. Registration on Friday will be from 1:00 p.m. until 3:00 p.m. and on Saturday from 8:30 a.m. until noon and from 1:00 p.m. until 2:30 p.m. The registration area for the Association for Symbolic Logic will be on the first floor of the Physics Building.

There are many hotels and motels not far from the university campus. The following is a selection of a few convenient ones. The rates are projected for spring 1976 and are subject to change. They do not include the six percent motel tax. In some instances special rates may be allowed for participants in university conventions.

MOTEL CAPRI (702) 323-8398
895 North Virginia Street (89501)
Single $18 up
Double 18 up

COED LODGE (702) 329-2742
800 North Virginia Street (89501)
Single $14 up
Double 18 up

FLYING J MOTEL (702) 329-3464
1651 North Virginia Street (89503)
Single $10 up
Double 12 up

ELDORADO HOTEL (702) 786-5700
345 North Virginia Street (89501)
Single $25
Double 32

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GOLDEN WEST MOTOR LODGE (702) 329-2192
530 North Virginia Street (89501)
Single $20 up
Double 22 up

HEART O'TOWN MOTEL (702) 322-4066
520 North Virginia Street (89501)
Single $14 up
Double 16 up

JACKPOT MOTEL (702) 329-2591
730 North Virginia Street (89501)
Single $16
Double 18

SHOWBOAT INN (702) 786-4032
660 North Virginia Street (89501)
Single $12
Double 14

SUNDOWNER HOTEL (800) 648-5490 (toll free)
450 North Arlington (89501)
Single $20
Double 24

TINY'S MOTEL (702) 329-9248
850 North Virginia Street (89501)
Single $18
Double 20

UPTOWN MOTEL (702) 323-8906
570 North Virginia Street (89501)
Single $18
Double 18

The following motel is one-and-one-half miles from both downtown and the airport.

MOTEL 6 (702) 825-8401
1901 South Virginia Street (89502)
Single $8.95
Double 10.95 up

The only eating facility on campus which will be open on Saturday is the Jot Travis Student Union cafeteria. A list of restaurants off and near campus will be available at the registration desk.

The southwest corner of the campus is at Ninth Street and North Virginia Street, one block north of the Interstate 80 east-west freeway. Persons driving from the west should take the Sierra Street-Virginia Street exit (Exit 13), and from the east, the Virginia Street-Business exit (Exit 13). Persons driving from either north or south on Highway 395 should travel on Virginia Street which is Business Highway 395. Parking will be available in the campus lot accessible from about 1600 North Virginia Street.

Reno International Airport is approximately three miles from the campus. Taxi fare to the campus and motel area is about $3.

PROGRAM OF THE SESSIONS
THE SEVEN HUNDRED THIRTY-FIFTH MEETING

All talks are scheduled in the Scrugham Engineering and Mines Building

The time limit for each contributed paper in the general sessions is ten minutes. In the special sessions the time varies from session to session and within sessions. To maintain the schedule, the time limits will be strictly enforced.

FRIDAY, 2:00 P. M.

Special Session on C*-Algebras and Related Topics I, Room 326
2:00–2:20 (1) Square integrable primary representations. Professor CALVIN C. MOORE, University of California, Berkeley (735–B13)

2:30–2:50 (2) Extensions of C*-algebras. Preliminary report. Professor LAWRENCE G. BROWN, Purdue University and University of California, Berkeley (735–B12)

3:00–3:20 (3) C*-algebras of transformation groups with smooth orbit spaces, and applications to the computation of some group C*-algebras. Mr. PHILIP GREEN and Mr. JONATHAN ROSENBERG*, University of California, Berkeley (735–B14)

3:30–3:50 (4) Morita equivalence of C*-algebras. Mr. PHILIP GREEN, University of California, Berkeley (735–B20)

4:00–4:20 (5) Operator algebras and the propagation of singularities in solutions to a differential equation. J. WILLIAM HELTON, University of California, San Diego (735–B11)

4:30–4:40 (6) The thin operators relative to an ideal in a von Neumann algebra. Professor BRUCE A. BARNES, University of Oregon (735–B1)

4:50–5:10 (7) Irreducible unbounded representations of *-algebras. Preliminary report. Mr. WILLIAM M. SCRUGGS, University of Denver (735–B3)

FRIDAY, 2:00 P. M.

Special Session on Convergence Spaces I, Room 234
2:00–2:20 (8) Regular convergence spaces. Preliminary report. Professor D. C. KENT*, Washington State University, and Mr. G. D. RICHARDSON, East Carolina University (735–G9)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
2:30- 2:50  (9) Singly generated convergence spaces. Preliminary report. Professor ALLAN C. COCHRAN, University of Arkansas (735-G7)

3:00- 3:20  (10) A Marinescu structure for vector lattices. Professor W. A. FELDMAN* and Professor J. F. PORTER, University of Arkansas (735-G5)

3:30- 3:50  (11) An inductive limit of order partition spaces. Professor J. F. PORTER* and Professor W. A. FELDMAN, University of Arkansas (735-G6)

4:00- 4:20  (12) Sequential Cauchy spaces. Dr. JAMES M. IRWIN, El Dorado Hills, California (735-G4) (Introduced by Professor Darrell C. Kent)

(13) and (14) cancelled

FRIDAY, 2:00 P. M.

Special Session on Varieties of Algebras I, Room 351

2:00- 2:30  (15) Congruence-valued logic in varieties of algebras. Preliminary report. Professor KIRBY A. BAKER, University of California, Los Angeles (735-A11)

2:40- 3:10  (16) Polynomial and rational identities in associative algebras, Professor GEORGE M. BERGMAN, University of California, Berkeley (735-A16)

3:20- 3:50  (17) Some results in topological algebra. WALTER TAYLOR, University of Colorado, Boulder (735-A2)

4:00- 4:20  (18) Discussion Period

4:20- 4:50  (19) Para-primal algebras. PETER H. KRAUSS, State University of New York, College at New Paltz

5:00- 5:30  (20) The T-ideal generated by the standard identity $s_3[x_1,x_2,x_3]$. AMITAI REGEV, University of Chicago (735-A17)

SATURDAY, 8:30 A. M.

Special Session on Convergence Spaces II, Room 234

8:30- 8:50  (21) CO-embedded spaces. Preliminary report. Mr. G. D. RICHARDSON, East Carolina University (735-G10)

9:00- 9:20  (22) Minimal uniform convergence space. Professor R. J. GAZIK*, Arkansas State University, and Professor DARRELL C. KENT, Washington State University (735-G8)

9:30- 9:50  (23) Convergence structures for Mikusiński operators. Preliminary report. Professor EDWIN F. WAGNER, University of Nevada, Reno (735-B24) (Introduced by Professor Darrell C. Kent)

10:00-10:20  (24) On continuous characters of Borel sets. Professor ROMAN FRIČ, Transport College, Žilina, Czechoslovakia (735-G2) (Introduced by Professor Darrell C. Kent)

10:30-10:50  (25) Marinescu structures and c-spaces. Dr. M. SCHRODER, University of Waikato, Hamilton, New Zealand (735-G1) (Introduced by Professor Darrell C. Kent)

SATURDAY, 8:30 A. M.

Special Session on Varieties of Algebras II, Room 351

8:30- 9:00  (26) The lattice of equational classes of weakly associative lattices. Professor GEORGE A. GRÄTZER, University of Manitoba (735-A13)

9:10- 9:30  (27) Elementary properties of free products. Preliminary report. Professor PHILIP OLIN, York University (735-E1)

9:40-10:00  (28) Structural diversity in the lattice of equational theories. III. GEORGE McNULTY, University of South Carolina (735-E2)

10:10-10:30  (29) Regular p-groups and their varieties. Preliminary report. Professor PAUL M. WEICHSSEL, University of Illinois (735-A5)

10:35-10:55  (30) WALTER DAVID NEUMANN, University of Maryland

SATURDAY, 9:00 A. M.

Session on Analysis, Room 349

9:00- 9:10  (31) On improving the rate of convergence of power series solutions of differential equations. Preliminary report. Dr. EUGENE M. FRIEDMAN, Los Angeles, California (735-B22)
9:15-9:25 (32) On a calculus for nuclear kernels defined on a rigged Hilbert space. Preliminary report. Professor JOHN B. BUTLER, Portland State University (735-B16)

9:30-9:40 (33) Dual orthogonal series with applications in cryptology. Professor ROBERT P. FEINERMAN, Lehman College, and Professor ROBERT B. KELMAN*, Colorado State University (735-B21)

9:45-9:55 (34) Generators for some subalgebras of $t^1$. Preliminary report. Mr. RAYMOND C. ROAN, University of Michigan (735-B9)

10:00-10:10 (35) Tensor product representing measures. Professor J. K. BROOKS, University of Florida, and Professor P. W. LEWIS*, North Texas State University (735-B10)


10:30-10:40 (37) An implicit function theorem in Banach spaces. IAIN RAEBURN, University of Utah (735-B5)

10:45-10:55 (38) A generalization of Oka’s hypersurface normality criterion. Professor ANDREW MARKOE, University of Connecticut (735-B18)

SATURDAY, 9:30 A. M.

Special Session on Mathematics of Gambling I, Room 346

9:30-10:00 (39) Hedging in favorable games. T. FERGUSON, University of California, Los Angeles (735-C3)

10:10-10:40 (40) Use of bivariate normal approximations to evaluate card-counting systems in blackjack. Professor PETER A. GRIFFIN, California State University, Sacramento (735-F2) (Introduced by Professor Fred Krakowski)

SATURDAY, 11:00 A. M.

Invited Address, Room 101

(41) KMS states of UHF algebras and their application to quantum statistical mechanics. Professor ROBERT T. POWERS, University of California, Berkeley and University of Pennsylvania (735-C2)

SATURDAY, 2:00 P. M.

Session on Algebra, Room 349

2:00-2:10 (42) Equational theory of algebras with a majority polynomial. R. PADMANABHAN, University of Manitoba (735-A14)

2:15-2:25 (43) Cyclic identities and associativity. V. R. CHANDRAN* and R. PADMANABHAN, University of Manitoba (735-A15)

2:30-2:40 (44) On the associativity of products of ring varieties. Preliminary report. Professor AWAD A. ISKANDER, University of Southwestern Louisiana (735-A8)

2:45-2:55 (45) Axioms for quaternions. Preliminary report. Mr. RALPH HOWARD and Professor D. H. POTTS*, California State University, Northridge (735-A10)

3:00-3:10 (46) A duality for Boolean spaces. Preliminary report. Dr. SIEMION FAJTLOWICZ, University of Houston (735-A7)

3:15-3:25 (47) Some relations between classes of ideals in a domain. Professor NICK H. VAUGHAN, North Texas State University (735-A9)

3:30-3:40 (48) Inequalities in the tensor product of semilattices. Professor GRANT A. FRASER, University of Santa Clara (735-A4)

3:45-3:55 (49) The range multiplicity of an Hermitian matrix. Professor M. MARCUS, University of California, Santa Barbara, and Professor M. ISHAQ*, Laval University (735-A12)

4:00-4:10 (50) Operators on regular maps. Preliminary report. Mr. STEPHEN E. WILSON, University of Washington (735-A3)

SATURDAY, 2:00 P. M.

Special Session on C*-Algebras and Related Topics II, Room 326

2:00-2:20 (51) Charting the operator terrain. Professor JOHN ERNEST, University of California, Santa Barbara (735-B19)

2:30-2:50 (52) The extensions of nuclear C*-algebras. Dr. MAN-DUEN CHOI, University of California, Berkeley (735-B2)

3:00-3:20 (53) Tensor products of C*-algebras. Professor BRUCE E. BLACKADAR, University of Nevada, Reno (735-B7)
3:30- 3:50 (54) Tomita-Takesaki theory via bounded operators. Professor MARC A. RIEFFEL*, University of California, Berkeley, and Dr. ALFONS VAN DAELE, University of Leuven, Belgium (735-B4)

4:00- 4:20 (55) The regular representations of measure groupoids. Dr. PETER F. HAHN, University of California, Berkeley (735-B15)

4:30- 4:50 (56) Symmetry of group algebras and a Tauberian theorem. RAMESH GANGOLLI, University of Washington (735-B23)

5:00- 5:20 (57) Measure groupoids coming from group actions. Professor J. FELDMAN*, Dr. P. HAHN, and Professor C. C. MOORE, University of California, Berkeley (735-B17)

SATURDAY, 2:00 P. M.

Special Session on Mathematics of Gambling II, Room 346
2:00- 2:30 (58) Optimal gambling systems. Professor THOMAS M. COVER, Stanford University (735-F4)

2:40- 3:10 (59) Selected topics in the mathematics of poker. Dr. WILLIAM H. CUTLER, Louisiana State University (735-C1)

SATURDAY, 2:00 P. M.

Special Session on Convergence Spaces III, Room 234
(60) Informal Session (details will be available at the meeting)

SATURDAY, 3:10 P. M.

Special Session on Varieties of Algebras III, Room 351
3:10- 3:40 (61) The variety covering the variety of all modular lattices. Professor BJARNI JONSSON, Vanderbilt University (735-A6)


4:30- 5:00 (63) A model theoretic approach to Malcev conditions. Dr. JOHN T. BALDWIN* and Dr. JOEL Berman, University of Illinois at Chicago Circle (735-A1)

SATURDAY, 3:30 P. M.

Session on Probability, Room 346
3:30- 3:40 (64) Central limit theorems for D[0,1]-valued random variables. Preliminary report. Dr. MARJORIE G. HAHN, University of California, Berkeley (735-F3)

3:45- 3:55 (65) A version of the Erdős-Rényi law of large numbers for triangular arrays. Professor STEPHEN A. BOOK, California State College, Dominguez Hills (735-F1)

Eugene, Oregon

Kenneth A. Ross
Associate Secretary

PRESENTERS OF PAPERS
Following each name is the number corresponding to the speaker’s position on the program
• Invited one-hour lectures

*Baker, K. A. #15
*Baldwin, J. T. #63
*Barnes, B. A. #6
*Bergman, G. M. #16
*Blackadar, B. E. #53
*Book, S. A. #65
*Brown, L. G. #2
*Butler, J. B. #32
*Chandran, V. R. #43
*Choi, M.-D. #52
*Cochran, A. C. #9
*Cover, T. M. #58
*Cutler, W. H. #59
*Day, R. A. #62
*Ernest, J. #51
*Fajtlowicz, S. #46
*Feldman, J. #57
*Feldman, W. A. #10
*Ferguson, T. #39
*Fraser, G. A. #48
*Frič, R. #24

Friedman, E. M. #31
*Gangolli, R. #56
*Gazik, R. J. #22
*Gilliam, D. #36
*Grätzer, G. A. #26
*Green, P. #4
*Griffin, P. A. #40
Hahn, M. G. #64
*Hahn, P. F. #55
*Helton, J. W. #5
*Irwin, J. M. #12
*Ishaq, M. #49
*Iskander, A. A. #44
*Jönsson, B. #61
*Kelman, R. B. #33
*Kent, D. C. #8
*Krauss, P. H. #19
*Lewis, P. W. #35
*McNulty, G. #28
Markoe, A. #38

*Moore, C. C. #1
*Neumann, W. D. #30
*Olin, P. #27
*Padmanabhan, R. #42
*Porter, J. F. #11
Potts, D. H. #45
*Powers, R. T. #41
Raeburn, I. #37
*Regev, A. #20
*Richardson, G. D. #21
*Rieffel, M. A. #54
*Roan, R. C. #34
*Rosenberg, J. #3
*Schröder, M. #25
*Scruggs, W. M. #7
*Taylor, W. #17
*Vaughan, N. H. #47
*Wagner, E. F. #23
*Weichsel, P. M. #29
Wilson, S. E. #50

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The seven hundred thirty-sixth meeting of the American Mathematical Society will be held at Portland, Oregon, on Friday, June 18, 1976. The Mathematical Association of America and the Society for Industrial and Applied Mathematics will hold Northwest Sectional Meetings in conjunction with this meeting of the Society. All of their sessions will be held on Saturday, June 19. In addition, Pi Mu Epsilon will sponsor a session of fifteen-minute papers on Friday afternoon. Abstracts of papers to be presented at the Pi Mu Epsilon session should be sent to John R. Reay, Western Washington State College, Bellingham, Washington 98225, so as to arrive before May 20.

The Mathematical Association of America will sponsor two or three hour talks. Richard M. Koch of the University of Oregon will lecture at 9:00 a.m. on Saturday, June 19. The title of his lecture is "Invariant functions on matrices."

The Featured SIAM Lecturer will be Philip M. Anselone of Oregon State University. His fifty-minute address is entitled "Nonlinear operator approximation theory and applications to integral equations," and will be given at 10:00 a.m. on Saturday, June 19. SIAM will also sponsor contributed talks until 3:30 p.m. on Saturday and will schedule a no-host lunch at 12:30 p.m.

By invitation of the Committee to Select Hour Speakers for Far Western Section Meetings, there will be two invited addresses. R. James Milgram of Stanford University will lecture at 11:00 a.m. on Friday on "Generalized cohomology theories and their applications." Thomas M. Liggett of the University of California, Los Angeles, will lecture at 2:00 p.m. on Friday. The title of his talk is "The stochastic evolution of infinite systems of interacting particles." Both hour addresses will be given in Room 71 of Cremer Hall.

There will be sessions for contributed papers scheduled Friday afternoon. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of April 27, 1976. Late papers will be accepted for presentation at the meeting, but will not appear in the printed program of the meeting.

The registration desk will be located in Room 334 of Neuberger Hall, and will be open during the following periods: 10:00 a.m. to noon and 1:00 p.m. to 4:00 p.m. on Friday; 8:30 a.m. to noon on Saturday. The registration fee will be $2.

There are numerous hotels and motels in Portland. Reservations should be made directly with the hotel or motel; early reservations are advised. The following are located near the Portland State campus:

<table>
<thead>
<tr>
<th>Hotel Name</th>
<th>Address</th>
<th>Rates $</th>
</tr>
</thead>
<tbody>
<tr>
<td>JAMAICA MOTOR HOTEL</td>
<td>415 S. W. Montgomery (97201)</td>
<td>Single: $12.50 up, Double: $13.50 up, Twin: $14.50 up</td>
</tr>
<tr>
<td>RAMADA INN-PORTLAND CENTER</td>
<td>(503) 221-0450</td>
<td>Single: $19.50 up, Double: $24.50 up</td>
</tr>
<tr>
<td>SHERATON MOTOR INN</td>
<td>Lloyd Center (97232)</td>
<td>Single: $17.00-$23.00, Double: $23.00-$29.00</td>
</tr>
</tbody>
</table>

Meals can be obtained at a variety of establishments near the campus. More information will be available in the registration area. The campus food service will not be in operation during the meetings.

Portland International Airport is served by several major airlines. Both limousine service and taxis are available between the airport and downtown. Neuberger Hall is located on Broadway thirteen blocks south of Morrison Street. Greyhound and Trailways Bus Lines serve Portland.

Persons driving to the meetings may park free in the parking structure across the street from Neuberger Hall. The freeway system in Portland can be very frustrating for people unfamiliar with it. Here are instructions on how to get to the parking structure mentioned above.

From the south on Interstate 5: At milepost 299 get in the right lane. Take the exit with the sign "City Center-Beaverton." On the ramp look for the sign "Portland State 2nd Right." This ramp joins highway 26. You must move over two
lanes to the right. Skip the 4th Avenue exit and take the 6th Avenue exit. The entrance to the parking structure is two-and-one-half blocks north, on the left side of the street. From the north on Interstate 5: Stay on the freeway all the way to the Marquam Bridge and keep in the center lane. The left lane disappears as you approach the Marquam Bridge. The two lanes on the ramp change to four lanes on the bridge. Take the second lane from the left, marked on the signs above by highway 26. Skip the 4th Avenue exit and turn right on the 6th Avenue exit. The entrance to the parking structure is two-and-one-half blocks north, on the left side of the street. From the east on Interstate 80N: After you pass the 21st Avenue viaduct, get in the left lane. (The left lane is restricted up to that point.) Stay in this lane, marked by "Salem-Beaverton" overhead signs. This will join Interstate 5. Get in the middle lane. The left lane disappears as you approach the Marquam Bridge. The two lanes on the ramp change to four lanes on the bridge. Take the second lane from the left, marked on the signs above by highway 26. Skip the 4th Avenue exit and turn right on the 6th Avenue exit. The entrance to the parking structure is two-and-one-half blocks north on the left side of the street.

The final program of the meeting will appear in the June issue of these Notices.

Kenneth A. Ross
Associate Secretary
Eugene, Oregon

The Eightieth Summer Meeting
University of Toronto
Toronto, Canada
August 24 – 27, 1976

The eightieth summer meeting of the American Mathematical Society will be held at the University of Toronto, Toronto, Ontario, Canada from Tuesday, August 24, through Friday, August 27, 1976. All sessions of the meeting will take place on the campus of the University.

A set of Colloquium Lectures, consisting of four one-hour talks, will be presented by Jurgen K. Moser of the Courant Institute of Mathematical Sciences, New York University. The first lecture in the series will be given on Tuesday morning, August 24. The second, third, and fourth lectures will be on Wednesday morning, Thursday afternoon, and Friday afternoon.

By invitation of the Society's Program Committee, there will be six invited one-hour addresses. The names of the speakers, titles of their addresses, and times of presentation will appear in a later issue of these Notices.

The following special sessions are being organized: JAMES McCOOL, University of Toronto, Combinatorial group theory; Z. H. NITECKI, Tufts University, Differentiable dynamical systems (tentative); ANATOL RAPOPORT, University of Toronto, Mathematical psychology; RAY SHIFLETT, California State University, Fullerton, Doubly stochastic measures and their operators; JOSEPH R. SHOENFIELD, Duke University, Recursively enumerable sets and degrees (tentative); W. T. TUTTE, University of Waterloo, Chromatic polynomials and related topics; and C. T. YANG, University of Pennsylvania, Transformation groups.

There will be sessions for contributed ten-minute papers on Tuesday afternoon, Wednesday morning, early Wednesday afternoon, Thursday afternoon, and Friday afternoon. Abstracts of contributed papers should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940; the deadline for receipt of abstracts is June 15, 1976. No provision will be made for late papers.

If a sufficient number of requests is received, a poster session will be organized. This represents an alternative method for presenting papers. At the poster session, individuals display their papers on an easel and remain in the room set aside for this purpose to expand on the material and answer questions during the session. Individuals who wish to have their papers considered for a poster session should so indicate on their abstracts, clearly in large block letters, and submit the abstract to the Society by June 1, 1976.

Rooms 2050 and 2054 in Sidney Smith Hall (A on map on page 152) have been set aside as informal discussion rooms, and will be open daily from 8:00 a.m. to 10:00 p.m. to small groups desiring a quiet room with blackboard space to discuss mathematics. Room 2050 is available on a first-come, first-served basis; room 2054 is available for one-hour periods only, and must be reserved in advance. A reservation form will be posted on the door to room 2054 for individuals to sign up for use of this room. It is requested that discussion groups not be planned to conflict with business meetings or major lectures.

The AMS Committee on Employment and Educational Policy (CEEP) will sponsor an open meeting on the job market on Tuesday, August 24, at 8:30 p.m. The meeting will consist of a brief report on the state of the job market, followed by an open discussion with comments and suggestions welcomed from the audience.

This meeting of the Society will be held in conjunction with the annual summer meeting of the Mathematical Association of America (August 26–28), and Pi Mu Epsilon. Participants should note that the Society and Association are meeting in order opposite from the usual schedule for a summer meeting. The twenty-fourth series of Earle Raymond Hedrick Lectures, sponsored by the Association, will be given by Martin D. Davis, of the
The American Mathematical Society will present a one-and-one-half day short course on Mathematical Economics on Sunday and Monday, August 22 and 23, in Room 2135 of Sidney Smith Hall on the campus of the University of Toronto. The course will present an introduction to several problems (existence and computation of equilibria, structure and dependence on parameters of the set of equilibria, game models and the theory of the core, dynamic economics) illustrating the application to economics of various fields of mathematics (in particular, algebraic topology, measure theory, and differential topology). It is intended to present both mathematically challenging aspects and their applications in current economic theory.

The program is under the direction of Gerard Debreu, Departments of Economics and Mathematics, University of California, Berkeley, in cooperation with Hugo Sonnenschein, Department of Economics, Northwestern University, as codirectors. This short course was recommended by the Society's Committee on Employment and Educational Policy (CEEP), whose members are David Blackwell, Charles W. Curtis, Wendell H. Fleming (chairman), Martha K. Smith, and Daniel H. Wagner.

The program will consist of six seventy-five minute lectures, as follows: Hugo Sonnenschein will speak on "Price formation: The theory of general economic equilibrium"; Werner Hildenbrand, Department of Economics, University of Bonn, will speak on "Measure spaces of economic agents"; David Gale, Departments of Economics and Mathematics, University of California, Berkeley, and Center for Advanced Study in the Behavioral Sciences, Stanford, California, will speak on "The role of prices and interest rates in dynamic economics"; Andreu Mas-Colell, Departments of Mathematics and Economics, University of California, Berkeley, will speak on "The theory of economic equilibrium from the differentiable point of view"; Robert J. Aumann, Departments of Economics, Hebrew University and Stanford University, will speak on "Some game models in economics"; and Stephen Smale, Department of Mathematics, University of California, Berkeley, will speak on "Computation and existence of Walras equilibria."

Summaries of these talks and accompanying reading lists appear on pages A-405 through A-407 of these Notices.

This short course is open to all who wish to participate upon payment of the registration fee. This fee has been reduced for students and unemployed individuals, with a modest increase for other registrants. Please refer to the section entitled MEETING REGISTRATION AND PREREGISTRATION for details.

<table>
<thead>
<tr>
<th>Date and Time</th>
<th>Location</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Saturday, August 21</td>
<td>4:30 p.m. -7:30 p.m.</td>
<td>Hallway outside of Room</td>
</tr>
<tr>
<td>Sunday, August 22</td>
<td>8:00 a.m. -5:00 p.m.</td>
<td>2135, Sidney Smith Hall</td>
</tr>
<tr>
<td>Monday, August 23</td>
<td>8:00 a.m. -noon</td>
<td>(Saturday through</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Monday)</td>
</tr>
<tr>
<td>Monday, August 23</td>
<td>2:00 p.m. -8:00 p.m.</td>
<td>Entrance Lobby, Sidney</td>
</tr>
<tr>
<td>Tuesday, August 24</td>
<td>8:00 a.m. -5:00 p.m.</td>
<td>Smith Hall (Monday</td>
</tr>
<tr>
<td>Wednesday, August 25</td>
<td>8:30 a.m. -4:30 p.m.</td>
<td>through Saturday)</td>
</tr>
<tr>
<td>Saturday, August 28</td>
<td>8:30 a.m. -1:30 p.m.</td>
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</tbody>
</table>
Participants who wish to preregister for the meetings should complete the Meeting Preregistration Form on the last page of these Notices. Those who preregister will pay lower registration fees than those who register at the meeting, as indicated in the schedule below. Preregistrants will be able to pick up their badges and programs when they arrive at the meeting after 2:00 p.m. on Monday, August 23, at the Joint Mathematics Meetings registration desk. Complete instructions on procedures for making hotel or dormitory reservations are given in the sections entitled RESIDENCE HALL HOUSING and HOTELS.

Meeting registration and preregistration fees partially cover expenses of holding the meetings. The preregistration fee does not represent an advance deposit for lodgings.

Please note that separate registration fees are required for the short course and for the Joint Meetings. These fees are as follows:

<table>
<thead>
<tr>
<th>Mathematical Economics Short Course</th>
<th>Preregistration</th>
<th>At Meeting</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(by mail prior to 8/6/76)</td>
<td></td>
</tr>
<tr>
<td>Student or unemployed</td>
<td>$3</td>
<td>$5</td>
</tr>
<tr>
<td>All other participants</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>One day fee for second day</td>
<td>-</td>
<td>10</td>
</tr>
</tbody>
</table>

Joint Mathematics Meetings

<table>
<thead>
<tr>
<th></th>
<th>Member</th>
<th>Student or unemployed</th>
<th>Nonmember</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$12</td>
<td>1</td>
<td>20</td>
</tr>
</tbody>
</table>

There will be no extra charge for members of the families of registered participants except that all professional mathematicians who wish to attend sessions must register independently.

The unemployed status refers to any participants currently unemployed and actively seeking employment. It is not intended to include participants who have voluntarily resigned or retired from their latest position. Students are considered to be only those currently working toward a degree who do not receive an annual compensation totaling more than $7,000 from employment, fellowships, and scholarships.

Checks for the preregistration fee(s) should be mailed to arrive in Providence not later than August 6, 1976. Participants should make their own reservations directly with hotels in the area (cf. section titled HOTELS). It is essential, however, to complete the Meeting Preregistration Form on the last page of these Notices to take advantage of the lower preregistration fee(s) and to obtain dormitory accommodations.

A fifty percent refund of preregistration fees will be made for all cancellations received in Providence prior to August 20. There will be no refunds granted for cancellations received after that date or to persons who do not attend the meetings.

**MATHEMATICAL SCIENCES EMPLOYMENT REGISTER**

At last summer's meeting at Western Michigan University, an experimental variant of the Employment Register was operated (successfully) on a limited basis. No interviews were scheduled by the staff. Instead facilities were provided for applicants and employers to display resumes and job listings. Message boxes were set up for individuals to leave messages for one another requesting interviews. Tables and chairs were provided in the room for interviews.

It is planned to repeat this form of the Employment Register at the University of Toronto. Employers are encouraged to attend the meetings and participate, if possible. Applicants should recognize that the MSER cannot guarantee that any employers will, in fact, attend the meeting or be able to participate in the Employment Register. The AMS-MAA-SIAM Committee on Employment Opportunities will, however, request employers listing in the July and August 1976 issues of Employment Information for Mathematicians to signify in their listing their intention to participate in the Employment Register at the summer meeting.

**EXHIBITS**

The book and educational media exhibits will be displayed in Room 2096 of Sidney Smith Hall at the following times: August 24 (Tuesday), 1:00 p.m. to 5:00 p.m.; August 25 and 26 (Wednesday and Thursday), 8:30 a.m. to 4:30 p.m.; August 27 (Friday), 8:30 a.m. to noon. All participants are encouraged to visit the exhibits sometime during the meeting.

**RESIDENCE HALL HOUSING**

Several residence hall facilities have been set aside for the use of participants in the Joint Mathematics Meetings and the Mathematical Economics Short Course. According to regulations set by the university's housing office, these facilities are divided into sections for male participants only, female participants only, couples, and families accompanied by children under 10 years of age or over, and families with any children under 10 years of age. In order for your dormitory assignments to be made correctly, you must be explicit when completing the preregistration form on the last page of these Notices.

The rates quoted below are in Canadian funds, and are subject to a seven percent Provincial Sales Tax. Also, these rates are those in effect as of the press deadline for this announcement, and it is anticipated that they will be increased slightly within the next month or so. It is hoped the new rates will be available for announcement in the June issue of these Notices.

**Student, Unemployed, or Children Age 10 or Over**

<table>
<thead>
<tr>
<th></th>
<th>Single or Double</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5/night per person*</td>
<td>$8/night per person*</td>
</tr>
<tr>
<td>$7/night per person**</td>
<td>$9/night per person**</td>
</tr>
</tbody>
</table>

*with air conditioning
**without air conditioning

Please note that payment in full for your dormitory accommodations must be made at the time of check-in in Canadian funds. Participants coming to the meeting from countries other than Canada are advised to exchange their currency for Canadian money either before they leave for the
meeting or at the airport immediately upon arrival in Toronto, since clerks in the dormitories will not be authorized to accept U.S. funds in payment, nor to allow occupancy of the room for even the first night until a participant can have his or her money exchanged. There are several banks nearby which will exchange U.S. for Canadian funds. The Canadian Imperial Bank of Commerce on the southwest side of Bloor and St. George Streets is open from 10:00 a.m. to 4:30 p.m. Monday through Thursday, and from 10:00 a.m. to 6:00 p.m. on Friday. There is also a branch of the Bank of Montreal on the northwest corner of Bloor and St. George. No banks are open on Saturday or Sunday.

Participants accompanied by children over 10 years of age should be aware of the fact that the children must occupy separate although adjacent rooms from the parents (i.e, parents in a double and one child over 10 in a single; or single parent and one child in a double; or two children in a double with parents in a double, etc.), and that the parents will be charged the appropriate rate (depending on whether students, unemployed, or ordinary participants), while the children will be charged the student/unemployed rate. Room occupancy is limited to the number of regular beds in the standard rooms; that is one in a single and two in a double.

Participants accompanied by children under 10 years of age should be aware of the fact that cribs without sides are available free of charge. For small infants, however, cribs with sides will have to be rented by the parent(s). Information on availability of these cribs will be published in the June issue of these Notice. Firm room rates for families with children under 10 are not yet available, but will be similar to the rates quoted above. Participants with small children will be requested to sign a waiver on property damage, and are advised to bring plastic sheets.

Clerks will man the check-in desks in the dormitories from 8:00 a.m. until midnight only. Participants arriving after midnight will not be able to occupy their rooms until 8:00 a.m. the next morning. Residence hall rooms may be occupied from 8:00 a.m. on Saturday, August 21, until 10:00 a.m. on Saturday, August 28. Under no circumstances will participants be allowed to occupy rooms past 10:00 a.m. on Saturday, since the university staff must begin shortly thereafter to prepare these rooms for students arriving the next week. Facilities for checking baggage will be available at the Joint Meetings Registration Desk.

Pay telephones are located on each floor. Each dormitory has a fully equipped laundry room with coin-operated washers and dryers. Ironing facilities are also available. There are no private baths; generally speaking, there are two large bathroom facilities on each floor. Toilet paper and soap will be provided. Light kitchen facilities are available in some dormitories. Beds will be made daily, Monday through Friday; however, only one set of sheets and towels will be furnished for the meeting period. Pets are not allowed in the residence halls.

To be assured of a room, guests should register in advance. Please use the Residence Hall Reservation form provided on the last page of these Notice. Residence hall reservation requests will be acknowledged by the Mathematics Housing Bureau. Do not include payment for dormitory accommodations with your preregistration form, since this will only cause a delay in the processing of your preregistration and housing request.

HOTELS

Blocks of rooms have been set aside for use by participants at the Park Plaza Hotel, the Hotel Plaza II, and the Chelsea Inn. Participants should make their own reservations with these hotels directly, and should identify themselves as participants in either the Mathematical Economics Short Course or the Joint Mathematics Meetings. Also listed below are several other hotels in the area of the campus. All prices are subject to change without notice and a seven percent Provincial Sales Tax. The extra person charge is for a roll-away cot. The age limit for children under which there is no charge, provided an extra cot is not required, is shown in parentheses. The following codes apply: FP - Free Parking; SP - Swimming Pool; AC - Air Conditioned; TV - Television; CL - Cocktail Lounge; RT - Restaurant. The numbers at the end of the first line correspond to the numbers on the maps on pages 152 and 153. Hotels numbered 3, 6, 7, 10, and 11 are within a 10-15 minute walk of the campus. Hotels numbered 4, 5, 8, 9, 12, 13, and 14 are within a 20-30 minute walk, or short subway ride.

HOLIDAY INN (Downtown) - 3
89 Chestnut Street
Single $33.50 Double $40.00 Twin $43.00
Extra person (12 years) $6.00
Code: RT CL AC TV SP FP
Telephone: 416-367-0707

KING EDWARD SHERATON - 4
37 King Street East
Single $25.00 Double $28.00 Twin $30.00
Extra person (17 years) $6.00
Code: RT CL AC TV FP
Telephone: 416-368-7474

ROYAL YORK HOTEL - 5
Front and York Streets at Union Station
Single $32.00 Double $41.00
Extra person (14 years) $7.00
Code: RT CL AC TV
Parking $3.50 (24 hours)
Telephone: 416-368-2511

WINDSOR ARMS HOTEL - 6
22 St. Thomas Street at Bay and Bloor
Single $24.00 Twin $30.00
Extra person (12 years) $6.00
Code: RT CL AC TV FP
Telephone: 416-921-5141

Y, M, C.A. (men only) - 7
40 College Street at Yonge
Single $10.00 Double $6.36 per person per day.
These prices include 7% sales tax.

FOUR SEASONS MOTOR HOTEL - 8
415 Jarvis Street at Carlton
Single $29.00 Double $37.00
Extra person (14 years) $6.00
Code: RT CL AC TV SP FP
Telephone: 416-924-6631
LORD SIMCOE HOTEL - 9
150 King Street West at University
Single $19.00 Double $25.00
Extra person (12 years) $6.00
Code: RT CL AC TV SP FP (overnight)
Telephone: 416-362-1848

WESTBURY HOTEL - 10
475 Yonge Street at College
Single $29.50 Double $36.50
Extra person (14 years) $7.00
Code: RT CL AC TV Parking $3.50 (24 hours)
Telephone: 416-924-0611

SUTTON PLACE HOTEL - 11
Bay at Wellesley Street
Single $32.50 Double $40.50
Extra person (12 years) $8.00
Code: RT CL AC TV SP Parking $3.00 (24 hours)
Telephone: 416-924-9221

PARK PLAZA HOTEL - 12
Bloor at Avenue Road
Single $35.00 Double $43.00
Extra person (14 years) $8.00
Code: RT CL AC TV Parking $3.00 (24 hours)
Telephone: 416-924-5471

HOTEL PLAZA II - 13
Bloor at Yonge Street
Single $37.00 Double $44.00
Extra person (10 years) $5.00
Code: RT CL AC TV Parking $3.75 (24 hours)
Telephone: 416-961-8000

CHELSEA INN - 14
Gerrard at Bay Street
Single $22.00 Double $27.00
Extra person (13 years) $5.00
One Bedroom Suite with Kitchen:
$47/day based on 2 adults and 2 children
$54/day based on 4 adults
Code: RT CL AC TV Parking $1.75 (24 hours)
Telephone: 416-595-1975

The following hotels are in the suburbs, and are within an hour's drive of the campus:

CANADIANA MOTOR HOTEL - 20
Kennedy Road and Highway 401
Single $23.00 Double $31.00
Extra person (12 years) $7.00
Code: RT CL AC TV SP FP
Telephone: 416-291-1171

HOLIDAY INN WEST - 21
Highway 427 at Burnhamthorpe
Single $27.50 Double $32.50 Twin $37.00
Extra person (12 years) $4.00
Code: RT CL AC TV SP FP
Telephone: 416-621-2121

HOLIDAY INN EAST - 22
Highway 401 at Warden Avenue
Single $28.50 Double $33.50 Twin $38.00
Extra person (12 years) $4.00
Code: RT CL AC TV SP FP
Telephone: 416-293-8170
FOOD SERVICES

Two cafeterias will be in operation on campus during the meetings. New College cafeteria (B on the map on page 152) will be open for breakfast from 7:00 a.m. to 8:30 a.m., lunch from 11:30 a.m. to 1:30 p.m., and dinner from 5:30 p.m. to 7:30 p.m. throughout the meetings, beginning on Sunday, August 22, through Friday, August 27. New College cafeteria will be open for breakfast and lunch only on Saturday, August 28. The cafeteria in the Medical Sciences Building (C on the map) will be open from 7:30 a.m. to 4:00 p.m. for breakfast and lunch, Monday through Friday. The average costs of meals in both these facilities are $1.70 breakfast; $2.60 lunch; $3.45 dinner. Individual meal tickets or daily tickets ($7.75) will be on sale at the Joint Meetings Registration Desk. Participants may also pay for meals on the spot; however, a slight saving may be realized if tickets are purchased in advance.

A snack bar, selling soup, sandwiches, beverages, light desserts, etc., will be operated in Room 5025 of Sidney Smith Hall, and will be open from 8:30 a.m. to 3:30 p.m., Monday through Friday. In addition, there are a number of catering trucks which park outside Sidney Smith Hall; picnic tables are provided.

It is also anticipated that a continental breakfast will be offered for a nominal charge in the residence hall assigned to families with children under 10 years of age. More details on this will be published in the June issue of the University of Toronto Newsletter. Grille.

CAMPING

There are no suitable camping sites located near the University of Toronto. Those persons wishing to camp should contact their local KOA office for the current issue of "Handbook and Directory for Campers."

BOOKSTORES

There are three bookstores located on campus. The University of Toronto Bookroom, located on King's College Circle (D on the map) and the University of Toronto Textbook store, located on Huron Street (E on the map), are both open from 8:45 a.m. to 4:30 p.m., Monday through Friday. The Student Christian Movement Bookstore, located on the edge of campus at the southwest corner of Bloor and St. George, is open 9:00 a.m. to 6:00 p.m., Monday through Friday, and 10:00 a.m. to 6:00 p.m., on Saturday.

LIBRARIES

The Mathematics Department Library, located on the second floor of Sidney Smith Hall, will be open from 9:00 a.m. to 9:00 p.m. for the duration of the meetings. Information concerning books located in other libraries is available from the Department Library. The main collection of books is in the Science and Medicine Library, located on King's College Circle (F on the map). The Robarts Library (G on the map) houses the Humanities Collection. Summer hours for the university libraries have not yet been fixed. The Metro Toronto Central Public Library is located at the southwest corner of St. George and College Streets, and has summer hours of 9:00 a.m. to 8:00 p.m., Monday through Friday, and 9:00 a.m. to 5:00 p.m., Saturday.

MEDICAL SERVICES

The University Health Service (H on the map) is open from 9:00 a.m. to 4:30 p.m. daily for medical attention. Emergencies occurring during the evening or weekends can be handled at the Emergency Department of any of the local hospitals: Toronto General Hospital, College at University; Women's College Hospital, 76 Grenville Street (College at Bay); The Hospital for Sick Children, 555 University Avenue. In addition, the Academy of Medicine can advise of local doctors who are on emergency call. Their telephone number is 922-1134. Dental service can be arranged through the University Health Centre.

DAY CARE CENTERS

Information is still being developed on day care centers. As of this moment, the Campus Cooperative will accept up to fourteen children. The cost here is $11 per day, plus two hours time donated by a parent. Interested parties should write direct to Ms. Marilyn Wilcoxen, Campus Cooperative, 12 Sussex Avenue, Toronto, Canada and include a deposit equal to one day's fee. Margaret Fletcher Daycare Centre will accept up to fifteen children, but the daily cost and terms are not yet fixed. More information will appear in the June issue of the Newsletter. Participants interested in utilizing day care facilities are asked to check the appropriate box on the preregistration form.

ENTERTAINMENT

The University of Toronto is planning entertainment for mathematicians and their families during the meetings. At 8:00 p.m. on Wednesday, August 25, there will be an evening beer party at the university at a nominal charge to those attending. Tickets to this event will be sold in advance at the Joint Meetings Registration Desk. During the week of the meeting, there will be many enter-
entertainment events in Toronto and vicinity. A number of theatres will be giving regular performances at this time. At Niagara-on-the-Lake there is a Shaw festival, and in Stratford, a Shakespeare festival. Both of these places are relatively close to Toronto and return transportation for any evening is easy to arrange. In addition to theatrical events, Toronto has a wide variety of musical activities in the summer. The Canadian National Exhibition will be in progress during the week of the meeting, and it is easily accessible from the university by public transportation. Tours can be arranged during the daytime to local places of interest such as the Ontario Science Centre, the McMichael Collection, and the large, new Metropolitan Zoo. Participants interested in these events should check with the Local Information section of the Joint Meetings Registration Desk.

TRAVEL AND LOCAL INFORMATION

Toronto is served by Air Canada, Allegheny, American, C. P. Air, Eastern, Great Lakes, Nordair, North Central, Quebecair, Transair, and United airlines to Toronto International Airport. There are several ways of getting from the airport to campus: (1) There is bus service available to the Islington subway station. From there, subway and surface transportation is available into the campus area. (2) The airport bus goes to the Royal York Hotel and the Sutton Place Hotel. The cost is about $2.50. (3) Cab service from the airport costs up to $10 per cab. (4) There is also a limousine service which costs up to $12 per air-conditioned luxury car.

Rail service to Toronto is by Canadian National and Canadian Pacific Railways, with good connections from Detroit, Buffalo, and Montreal. Limited access highways (#401, #427, and Queen Elizabeth Way) connect Toronto with Detroit, Buffalo, Kingston, or Montreal.

Entering Canada is usually no problem for American citizens, and involves nothing more than answering questions about where they were born, where they are going, and how long they will stay. To be assured of entry, however, it is advised that participants bring with them some proof of citizenship, such as a voter’s, baptismal, or birth certificate. Permanent U.S. residents who are not citizens are required to bring their alien registration receipt card (U.S. form 1-151). Entry requirements vary for people coming to Canada from countries other than the United States. As a general rule, the visitor should have a valid national passport.

PARKING

Parking throughout the campus is extremely limited as the campus was not designed for motor traffic. Parking stickers for pay lots will be on sale at the Joint Meetings Registration Desk; the price is not yet known. There are also several areas where on-street parking is free from 9:00 a.m. to 4:00 p.m. Maps indicating these parking areas will also be available at the Registration Desk. Participants, however, are urged to drive as little as possible between dormitories and the meeting area.

WEATHER

The normal daytime high temperature during this period is 79°F. Normal night-time low is 61°F. Rainfall in August averages 2.65 inches, with a 30 percent probability of precipitation each day. Humidity ranges from a daytime high of 79 percent to a night-time low of 61 percent. The record high and low temperatures for August are 102°F and 39°F, respectively. Light sweaters and jackets are advised for evening wear. Temperatures in Canada are now given in the Celsius scale, so the preceding temperatures would read: normal high 26°C; normal low 16°C; record high 39°C; record low 4°C.

MAIL AND TELEPHONE MESSAGES

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematics Meetings, Department of Mathematics, University of Toronto, Toronto, Ontario, Canada M5S 1A1. Mail and telegrams so addressed may be picked up at the Joint Meetings Registration Desk located in the entrance lobby of Sidney Smith Hall.

A telephone message center will be located in the same area to receive incoming calls for registrants during the hours the desk is open. The name of any participant for whom a message has been received will be posted until the message is picked up at the Message Center. The telephone number of the Center will be listed in a later issue of these Notices.

LOCAL ARRANGEMENTS COMMITTEE

SUMMARY OF ACTIVITIES

The purpose of this summary is to provide assistance to registrants in the selection of arrival and departure dates. The program, as outlined below, is based on information available at press time.

<table>
<thead>
<tr>
<th>SATURDAY, August 21</th>
<th>AMERICAN MATHEMATICAL SOCIETY</th>
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<tbody>
<tr>
<td>4:30 p.m. - 7:30 p.m.</td>
<td>SHORT COURSE ON MATHEMATICAL ECONOMICS</td>
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<tr>
<td>4:30 p.m. - 7:30 p.m.</td>
<td>REGISTRATION (Short Course Only)</td>
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<tr>
<th>SUNDAY, August 22</th>
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<tbody>
<tr>
<td>8:00 a.m. - 5:00 p.m.</td>
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<tr>
<td>9:00 a.m. - 10:15 a.m.</td>
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<tr>
<td>10:45 a.m. - noon</td>
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<tr>
<td>2:00 p.m. - 3:15 p.m.</td>
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<td>3:45 p.m. - 5:00 p.m.</td>
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<tr>
<th>MONDAY, August 23</th>
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<tbody>
<tr>
<td>8:00 a.m. - noon</td>
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<tr>
<td>9:00 a.m. - 10:15 a.m.</td>
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<tr>
<td>10:45 a.m. - noon</td>
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<thead>
<tr>
<th>MONDAY, AUGUST 23</th>
<th>AMS - MAA SUMMER MEETINGS</th>
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<tbody>
<tr>
<td>2:00 p.m. - 8:00 p.m.</td>
<td>American Mathematical Society</td>
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<tr>
<td>2:00 p.m.</td>
<td>Council Meeting</td>
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<tr>
<td>2:00 p.m.</td>
<td>REGISTRATION</td>
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<tr>
<th>TUESDAY, August 24</th>
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<tbody>
<tr>
<td>8:00 a.m. - 5:00 p.m.</td>
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<tr>
<td>10:30 a.m. - 11:30 a.m.</td>
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<tr>
<td>noon - 6:00 p.m.</td>
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<tr>
<td>1:00 p.m. - 5:00 p.m.</td>
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<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
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<tr>
<td>2:45 p.m. - 3:45 p.m.</td>
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<tr>
<td>4:00 p.m. - 5:00 p.m.</td>
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<tr>
<td>5:15 p.m. - 6:15 p.m.</td>
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<tr>
<td>7:00 p.m. - 8:00 p.m.</td>
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<tr>
<th>WEDNESDAY, August 25</th>
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<tbody>
<tr>
<td>8:00 a.m. - noon</td>
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<tr>
<td>8:00 a.m. - 4:30 p.m.</td>
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<td>8:30 a.m. - 4:30 p.m.</td>
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<td>8:30 a.m. - 4:30 p.m.</td>
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<td>8:30 a.m. - 4:30 p.m.</td>
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# SUMMARY OF ACTIVITIES

<table>
<thead>
<tr>
<th>WEDNESDAY, August 25</th>
<th>American Mathematical Society</th>
<th>Other Organizations</th>
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<tbody>
<tr>
<td>9:00 a.m. - 4:00 p.m.</td>
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<tr>
<td>10:15 a.m. - 11:15 a.m.</td>
<td>COLLOQUIUM LECTURE II</td>
<td>Jurgen K. Moser</td>
</tr>
<tr>
<td>11:30 a.m. - 12:30 p.m.</td>
<td></td>
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<tr>
<td>noon - 2:45 p.m.</td>
<td>Sessions for Contributed Papers</td>
<td>Special Sessions</td>
</tr>
<tr>
<td>4:00 p.m.</td>
<td></td>
<td>Business Meeting</td>
</tr>
<tr>
<td>5:00 p.m. - 6:30 p.m.</td>
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<tr>
<td>8:00 p.m.</td>
<td>BEER PARTY</td>
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<thead>
<tr>
<th>THURSDAY, August 26</th>
<th>AMS</th>
<th>Mathematical Association of America</th>
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<tbody>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>REGISTRATION</td>
<td></td>
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<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>EXHIBITS</td>
<td></td>
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<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>EMPLOYMENT REGISTER</td>
<td>Hedrick Lecture I Martin D. Davis</td>
</tr>
<tr>
<td>noon - 6:00 p.m.</td>
<td>Sessions for Contributed Papers</td>
<td>Special Sessions</td>
</tr>
<tr>
<td>2:45 p.m. - 3:45 p.m.</td>
<td>COLLOQUIUM LECTURE III</td>
<td>Jurgen K. Moser</td>
</tr>
<tr>
<td>4:00 p.m. - 5:00 p.m.</td>
<td>INVITED ADDRESS</td>
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<tr>
<td>5:15 p.m. - 6:15 p.m.</td>
<td>INVITED ADDRESS</td>
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<tr>
<td>evening</td>
<td>Film Program</td>
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<tr>
<th>FRIDAY, August 27</th>
<th>AMS</th>
<th>Mathematical Association of America</th>
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<tbody>
<tr>
<td>8:30 a.m. - noon</td>
<td>EXHIBITS</td>
<td></td>
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<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>REGISTRATION</td>
<td></td>
</tr>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>EMPLOYMENT REGISTER</td>
<td>Hedrick Lecture II Martin D. Davis</td>
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<tr>
<td>9:00 a.m. - 10:00 a.m.</td>
<td>Business Meeting</td>
<td></td>
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<tr>
<td>10:00 a.m. - 11:00 a.m.</td>
<td>Retiring Presidential Address</td>
<td>Henry O. Pollak</td>
</tr>
<tr>
<td>11:00 a.m. - noon</td>
<td>Sessions for Contributed Papers</td>
<td>Special Sessions</td>
</tr>
<tr>
<td>2:45 p.m. - 3:45 p.m.</td>
<td>COLLOQUIUM LECTURE IV</td>
<td>Jurgen K. Moser</td>
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<tr>
<td>evening</td>
<td>Film Program</td>
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<tr>
<th>SATURDAY, August 28</th>
<th>REGISTRATION</th>
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<tbody>
<tr>
<td>8:30 a.m. - 1:30 p.m.</td>
<td>Hedrick Lecture III</td>
<td>Martin D. Davis</td>
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<tr>
<td>9:00 a.m. - 10:00 a.m.</td>
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Middletown, Connecticut

Walter H. Gottschalk
Associate Secretary
THE ROLE OF APPLICATIONS IN PH.D. PROGRAMS IN MATHEMATICS

On Thursday evening, August 21, 1975, the AMS Committee on Employment and Educational Policy sponsored a panel discussion on "The Role of Applications in Ph.D. Programs in Mathematics" at the summer meeting in Kalamazoo, Michigan. The members of the panel were: Lipman Bers, Davies Professor at Columbia University and President of the AMS; Henry O. Pollak, Director of The Mathematics and Statistics Research Center at Bell Laboratories, Murray Hill, New Jersey, and President of the MAA; and Richard D. Anderson, Boyd Professor at Louisiana State University and a past chairman of the Committee on Employment and Educational Policy. The moderator was Wendell H. Fleming, Brown University.

The texts of the panel members' talks follow.

LIPMAN BER

There is one overwhelming reason why I should not participate in this panel: lack of competence. There are several other reasons why I should, and I let those reasons override the reason why I should not. (As I look at the audience and see so many distinguished applied mathematicians who know much more about the subject than I do, I feel that I have made a mistake; but it's too late now.)

The first reason is that I personally profited immensely from my relatively brief experience in applied mathematics. During World War II, many of us worked in applied mathematics as part of the war effort. I tried to do something about compressible flow; in fact, I worked on subsonic flow which was not a fashionable chapter of this fashionable topic. What I did with the problem I worked on is unimportant, but my thinking concerning this problem formed the basis of my future mathematical career. The research which I later did in "pure mathematics" can be traced back to my work involving gas dynamics, though in gas dynamics I was by no means successful. So I owe a debt of personal gratitude to applied mathematics.

The second reason why I think I should participate in this panel is that, being the oldest member, I represent the past and will bring in an old-fashioned conservative point of view, which you might want to hear out of curiosity, if not out of sympathy.

The third reason is related to my experience as chairman of COSRIMS.* The COSRIMS report is sometimes accused of having enticed mathematicians to rethink graduate education, this is good. But everybody is thinking along these lines mainly because we face an economic crisis in the profession. If the poor employment situation forces us to rethink graduate education, this is good. But in designing a graduate program, we should think primarily of long range effects. We are educating people not for a short-term commitment, but of mathematicians did not turn out to be right, and I think that the COSRIMS experience should teach us to be very careful in making predictions. The number of positions for mathematicians and the amount of money available for support of mathematical endeavor depend, in my opinion, largely on social processes over which we have no control, and which we do not know how to predict. (When I was offered the chairmanship of COSRIMS I at first turned it down, saying that while the Vietnam war was going on I did not want to give any advice to the government. I was told that the report would not appear before 1967 and by that time the war would certainly be over. Actually, the report appeared in 1968.) Since our prediction about the shortage of mathematicians turned out to be incorrect, I am afraid that the other parts of the report did not receive the attention they perhaps deserved. I reread the report before coming here, and I think that our description of the structure of mathematical sciences was basically correct; so indeed was our statement that we live in a time of mathematization of culture. Our whole society, our whole civilization is being mathematized. Mathematics penetrates everywhere; mathematical literacy is an important social need.

The COSRIMS report also contains a plea for a less specialized, less narrow, training in mathematics. Maybe this point was not emphasized as strongly as it should have been. Now everybody is thinking along these lines mainly because we face an economic crisis in the profession. If the poor employment situation forces us to rethink graduate education, this is good. But in designing a graduate program, we should think primarily of long range effects. We are educating people not for a short-term commitment, but of mathematicians did not turn out to be right, and I think that the COSRIMS experience should teach us to be very careful in making predictions. The number of positions for mathematicians and the amount of money available for support of mathematical endeavor depend, in my opinion, largely on social processes over which we have no control, and which we do not know how to predict. (When I was offered the chairmanship of COSRIMS I at first turned it down, saying that while the Vietnam war was going on I did not want to give any advice to the government. I was told that the report would not appear before 1967 and by that time the war would certainly be over. Actually, the report appeared in 1968.) Since our prediction about the shortage of mathematicians turned out to be incorrect, I am afraid that the other parts of the report did not receive the attention they perhaps deserved. I reread the report before coming here, and I think that our description of the structure of mathematical sciences was basically correct; so indeed was our statement that we live in a time of mathematization of culture. Our whole society, our whole civilization is being mathematized. Mathematics penetrates everywhere; mathematical literacy is an important social need.

*The Committee on Support of Research in the Mathematical Sciences (COSRIMS) was appointed by the Division of Mathematical Sciences of the National Research Council at the instigation of the Committee on Science and Public Policy of the National Academy of Sciences. Its task was to assess the status and projected future needs, especially fiscal needs, of the mathematical sciences. The results of their study were published as a three-volume series:

hopefully, for a life-long professional involvement.

We talk now about the necessity for young mathematicians to broaden their interests, to take some courses in applied areas of mathematics, to learn some physics, or some economics, or some biology. It is worthwhile to note that such expectations were taken for granted many years ago. In 1935 or 1936 when the Nazis ruled Germany and one had to do something about the many German-Jewish mathematicians who lost their jobs, a plan developed in Prague, and I think in Switzerland (where there were German-speaking mathematicians in safe positions), to form industrial consulting firms, staffed by these dismissed mathematicians. I heard about the plan from my late teacher, Charles Loewner, who was involved in its preparation. (Needless to say, the plan did not materialize.) The initiators were worried about the economic crisis in Europe at that time, and they thought they might not be able to make contacts with industry. But it never occurred to them to ask whether Professor X or Professor Y who lost his chair of geometry or function theory, could act, if necessary, as an applied mathematician. It was taken for granted, that an educated mathematician could do anything that was required, and in particular that he knew basic physics.

I come now to the fourth reason for accepting the invitation to speak here. I was thinking about a plan, called the Six Year Plan, involving use of the present difficulties in the job situation to reform graduate education. I circulated this plan only privately, among a few friends. Briefly speaking, I wanted to make the Ph.D. harder to get, and to make holders of this degree better qualified; and I wanted to say that the optimal time for getting the Ph.D. should be six years and not four years. I now see that this plan is unrealistic: Degrees, like money, are always devalued. If we want to raise standards we cannot upgrade an existing degree, we must put a super degree on top of it. My plan for the super degree, for the upgraded Ph.D., had several parts. One of them was to require from every Ph.D. candidate, regardless of his or her specialty, a certain precisely described amount of general mathematical knowledge. Some people consider such a requirement to be impossible, since mathematics develops too fast and is already too specialized. I think that while certain foundational parts of mathematics do change very rapidly, there still exists a common ground of examples, devices, tricks, and concepts which constitutes, so to speak, our common intellectual heritage. (If you attended the excellent colloquium lectures by Eli Stein, you must have noticed that, in order to understand this report on his latest work, you did not have to know every last thing about Heisenberg groups and homogeneous groups, etc. But you did have to know, quite thoroughly, some simple examples: What it means to solve the equation \( \Delta u = 0 \) in the plane and what properties the solution has, and also what it means to solve the equation \( \partial t/\partial x + i\partial t/\partial y = \phi \). At one time I thought of designing a course "What every gentleman or lady should know about ordinary and partial differential equations" which would consist not of general theorems but of certain typical examples.)

The other part of the plan was to demand a minor. (This probably shows my provincialism: I remember my own days as a student in Europe— the term graduate student didn't exist—when it was required to choose a minor and eighty or ninety percent of the mathematics students chose physics. The result was that, as a student, I took as many physics courses as mathematics courses, while I did almost all of my independent reading in mathematics.) The minor could be physics or some parts of engineering, biology, economics, or computer science. I don't mean one or two courses, but systematic study which does not necessarily culminate in a research thesis. For somebody getting a Ph.D. in applied mathematics, or in computer science or in mathematical statistics, I would accept topology, or analysis as a minor.

The third requirement would be at least a year of supervised teaching. I think this would be useful, not only for future Ph.D.'s who will gain employment in colleges and universities, but also for those who will work in industry. Because, I am told—I am not speaking from my own experience—that one responsibility of the industrial mathematician is to talk to people who do not necessarily have the same mathematical background as he or she has. This, after all, is also what the eighty percent of our students who enter college teaching are supposed to do. I understand some of them are not necessarily completely expert in this.

I do not think that we can foresee what qualifications will provide jobs five years from now, or ten years from now. But we can be sure that a broad scientific culture and a broad mathematical culture—and I don't think that one is possible without the other—will result in a flexibility which will enable people to do whatever will be in their interest and in society's interest.

I was told by the organizer of this panel that I should talk about classical applied mathematics, physical mathematics. I maintain that those applications of mathematics which have proved spectacularly successful, which shaped mathematics, which shaped our world as it is today, should be part of mathematical education. They should be part of the mathematical curriculum regardless of the job situation. I have in mind primarily classical Newtonian mechanics which developed together with calculus. On the graduate level, and certainly on the Ph.D. level, the same is true of the applications of differential geometry to relativity, and of Hilbert space theory to quantum mechanics. At one point I wanted COSRIMS to affirm that knowledge of Newtonian mechanics should be expected from every mathematician, COSRIMS had nine members—two voted for this proposal, all others against it. I still make this motion here today. I know that there are, for instance, important applications of combinatorics to computer science where what you deal with has nothing to do with Newtonian mechanics. I still say that the man or woman who does not know how Newton derived universal gravitation from Kepler's laws is not mathematically educated. I also think that even people who plan to work entirely in so-called new applications should always have before their eyes, at least in order to evaluate their own
efforts realistically, this unbelievable success story which has not yet been repeated in any other discipline, the application of mathematics to physics.

HENRY O. POLLAK

Let me start out by giving a very brief picture of what I mean by applications of mathematics. Let us take a region and label it mathematics. Within it there is a certain subset which is classical applied mathematics: mathematical physics, those parts of mathematics that are traditionally recognized as applied. There is another subset of mathematics which I will call applicable mathematics, that is mathematics for which people know significant practical applications. When you include it on the picture, applicable mathematics has a fair amount of overlap with classical applied mathematics, although it by no means includes all of it, and includes a lot of other mathematics as well. Both of these subsets are used as definitions of applied mathematics by people, which is part of the reason that we have so many arguments.

Let us now draw another region, disjoint from mathematics, which we shall call the rest of the world. We can now state a third definition of applied mathematics which people use—namely that you start out with a problem in the rest of the world, you make your way over to mathematics and make a mathematical model of the problem, and you do some mathematics. With this mathematics you develop some understanding, and then you go back over again into the rest of the world. Applied mathematics in this sense is not just a subset of mathematics, it also includes two mappings, one from and one to the rest of the world. If you think of some of the "applied mathematics" courses that are typically taught, this is the definition they try to use, although they often do a very poor job of explaining the two mappings. I am afraid that in too many cases, the professor just mouths a few words from another discipline, and then says "consider the following partial differential equation", without explaining anything about how you got from the real world to the equation, what you kept and what you threw away, and so forth, or what the results mean physically. This ignores the mapping from and to the real world, and is not what I had in mind.

The fourth definition, and the one that I am going to use for the rest of this talk, is what I see actually happening when people at Bell Laboratories and in many other industries actually practice mathematics. They start out with a situation in the real world, map it into a mathematical model, do some mathematics, come back to the real world—and find out that it is all nonsense! It doesn't make sense in terms of the original physical, or economic, or engineering, or whatever situation, from which they started. So they try again, they go around the loop between the real world and the mathematics many, many times, until a satisfactory model is reached.

To analyze this process in more detail, what are the "real world" problems like? They may, for example, be well-formulated problems in some other field, but not well-formulated mathematically as yet. In this case, the applied mathematician's job will be to try to find a mathematical formulation. For example, someone may come in and ask what purely resistive circuits can be made by printed circuit techniques and, after you answer that, what R-C circuits, and so on. Or else: Which parts of this composition were probably written by Mozart, and which were probably written by one of his graduate students? Or again: How do you best lay out a running track? In these cases, we start with quite a specific question in some other field, but nobody knows ahead of time exactly how to make a mathematical model: This is part of the mathematician's job. There is a second variety of real world situations, in which, rather than a specific question in some other field, all you have to begin with is someone's vague uneasiness. Someone comes in, sits down, plunks his feet on your desk and essentially says "Help". You then have a very interesting job ahead of you of trying to determine exactly what it is that is bothering him—before you can ever hope to invent a mathematical model. Typical example: "I don't understand FM. Help." It is true that radio, TV, and satellite communication all work by frequency modulation, but there are lots of aspects of FM that we don't fully understand. When people come in with a question like that, the first n hours are spent trying to understand more precisely what's bothering them. Only then can you go on to think about modelling.

There is yet another way in which a piece
of work in applied mathematics in industry can start, and that there are simply questions in the air. In industry, the mathematician lives in an environment of hundreds of people doing interesting things. Sometimes no one comes in either with a specific question, or with some uneasiness; you yourself realize that if you could only understand so and so, everyone might be a lot better off. Much interesting applied mathematics begins in this way.

In this spectrum of activities we have called applied mathematics, what branches of mathematics are involved? You can get a first approximate description in terms of some of the mathematical activities in Bell Laboratories. Certainly classical applied mathematics, linear and non-linear systems problems, stochastic processes, combinatorial and discrete mathematics, data analysis and statistical problems intermingled with numerical analysis, computer programming and computer science research generally, operations research, mathematical economics are all important to the communications industry. Other industries use still other mathematical areas.

As an interesting sidelight on the mathematics and the computer science research areas at Bell Laboratories, here is a count on the backgrounds of the seventy-four doctorate holders in these two organizations. (The separate operations research organization is not included in this count.) Among these seventy-four Ph.D.'s there are twenty-eight doctorates in mathematics, fifteen in one or another form of engineering, eight in economics or business administration, eight in physics or chemistry, ten in statistics, three in computer science, and one each in philosophy and psychology. They are all doing clearly recognizable, interesting research in the applied mathematical sciences. Furthermore, you would have a difficult time looking at the work many of them do now and making an easy association between that and the field in which they got their degree.

If this is what applied mathematicians do, what would I recommend with regard to graduate education? What aspects seem right, and what aspects would it be advantageous to work toward changing? One thing that Professor Bers touched on and I completely agree with: Make sure that the student has the fundamentals of all the fields of mathematics. You just never know what is going to come up. In particular, this applies also to the undergraduate level: There are half a dozen fundamental areas in the mathematical sciences that all mathematics majors should have. In present curricula, in one form or another, a large portion of these: Mathematics majors will always have beginning analysis and linear algebra. You cannot be sure that they have had any probability, any computer science, any statistics, or any experience in modelling, or (one more that shouldn't be forgotten) that they have had any experience in geometry. All of these areas come up frequently and fundamentally in an applied setting.

Actually, a compelling argument can be made that all mathematics majors ought to have the basics of all these fields, no matter what their destinations are—and that they have to be made up in graduate school if they are missing from the undergraduate program. I won't go into that point further since it is not the main topic of this discussion.

Next, we must make sure that courses involve questions of both mathematical understanding and technique. Don't communicate the feeling that technique is unimportant. Going back to my own education for some examples, the linear algebra that I learned contained fine theory, but too little technique. My partial differential equations course had much too much technique without understanding, and was therefore rather poor preparation for a mathematician. If you want to see what can be done in mixing technique and theory, see Professor Henrici's new book Applied and Computational Complex Analysis.

Professor Bers mentioned another item that I also feel very strongly about; teaching experience, well supervised, is terribly important. What he said is absolutely correct: The primary difference between university and industry is not the difference between teaching and not teaching. What corresponds, in industry, to teaching, is the tremendous amount of time spent helping other people—with this difference, that most of the time these "other people" really want to learn, which is not necessarily true of all undergraduates. The real differences between mathematics in industry and in academia are in the motivation of many of the problems you work on, and in the sequence of theorems—the theories—that you try to develop.

Let me further urge another aspect of graduate education: Try, as much as possible, to maintain a broad interest. I observe that there is little difficulty in persuading the brightest undergraduate to participate in the Putnam competition. The Putnam exam cuts broadly across undergraduate mathematics, and yet everybody enjoys it. Look at the same undergraduate four years later, and all too often there is just one little branch of mathematics which is interesting, and everything else is bunk. It is true that the process of getting a research degree requires you to become an approximate world expert in some particular topic. But that does not imply that you therefore have to believe that all other topics are worthless and uninteresting. I am afraid that sometimes this attitude is even adopted in imitation of more senior mathematicians. Particularly for applications it is essential to keep up a broad interest in the mathematical sciences.

Let me describe the kind of person who I find is happiest doing mathematical work in industry. It is a person who does, indeed, have a major research field, but who also can't resist a good problem, even if it isn't in the same field. If something interesting comes along you try it because it is an exciting challenge; you come back to the major field in which you work frequently, but you also do a lot of other things.

We have talked about fundamentals of all mathematical fields, about courses that emphasize both understanding and technique, and about maintaining a broad interest. Another useful component of mathematics education is some model building experience. I myself don't care very much in what applied field it happens. The basic point is to look into some other area, get a situation that isn't understood, and experience what is involved when you go through the iterative
model building process. Certainly the fundamentals of physical science are very good things to have and provide a good possible field of applications, but physics is only one of many. At the moment, I get the feeling that among all the areas of application, it is the people on the borderline between mathematics and economics who seem to have the most fun. I have tried to recall some of the research papers in mathematical economics that have recently crossed my desk. Among them, there were papers trying to make mathematical models of the following topics: franchising, criminality, altruism, sizes of juries, zoning, and utility regulation. These were written by people with a lot of background in both mathematics and economics, and they seemed to be having one heck of a good time. So whatever the field, please get some experience in this model building process. Quite independently of any question of the job market, I think that it is part of being a mathematician to know something about how the subject is applied. You simply don't have a complete education for becoming a professional mathematician if you don't have some feeling about how the subject interacts with the rest of the world.

One more suggestion about graduate work. In applications, you frequently find yourself doing joint work, either with people in other fields, or together with other mathematicians often with backgrounds different from your own. Typically, mathematicians in industry write a great many joint papers. However, it is difficult to get some graduate schools in mathematics to tolerate joint work. For example, I remember being told that R. L. Moore forbade students from ever discussing mathematics with each other! He wanted to develop each person's individual creativity, and that is very important. But it is a mistake to give students a value system which deprecates joint work, because cooperation is going to be terribly important in the applied field. Thus you should try not only to tolerate joint work, but even to encourage it, both with other mathematicians and particularly with people in other fields. For purposes of working in industry you give students the wrong instinct if you make it quite clear that working by yourself is better than working together with somebody else.

In this line, there is one possible suggestion for education, but unfortunately it is expensive. People have thought a great deal about the best way of teaching courses where very large numbers of students are involved, such as calculus, statistics, linear algebra, and probability. One pattern which is used in many places is large lecture sections plus small recitations. As an additional feature of this system, you could have recitation sections run jointly by a graduate student in mathematics and a graduate student in some other field. You might expect undergraduates to go to two of these weekly, one in mathematics and biology and one in mathematics and electrical engineering, or mathematics and physics, and so on. Thus the student who is taking calculus, or statistics, or probability, or linear algebra, with the hope of applying it to some other field, can actually get some motivation and encouragement to be in applications as well as mathematics, in the recitation sections. Such sections would, incidentally, also be good for the graduate students involved in the teaching, because here graduate students in mathematics and in other disciplines would talk to each other, and see what each other's problems are.

Let me close by mentioning one other obvious question, but one on which I am really terribly uncertain, and that is the question of requiring a major amount of experience in another field. It is very clear to me that it is terribly desirable. But when I look at the background of people who are most successful in industrial applied mathematics, I get no simple answer. There are people whose background is pretty pure in mathematics, but who love to talk to other people and can't turn a good problem down, and who in the course of a few years find themselves to be extremely successful in doing interesting applied mathematics. On the other hand, we have had the experience of people with equal backgrounds in, say, mathematics and electrical engineering, but really not well enough trained in either one in order to become successful researchers. There seems to be no easy answer to that one.

RICHARD D. ANDERSON

The question of doctoral programs in mathematics ought to be viewed in terms of the historical role of such programs in the United States and the almost inevitable evolving newer role. In particular, the doctoral programs in this century have been training researchers; in the process they have trained other people, but basically they have been training people in and for research. The programs have had two destinations for graduates in mind: the primary one, higher education, involving teaching and/or research in academic institutions; the secondary one, work in industry, business, or government, using mathematical knowledge and ability in various areas of applications. Unlike liberal arts undergraduate training, doctoral training has been professionally oriented. The faculty and the students have viewed it as directed toward professional work and involvement.

In the recent past, and still at the present time, about 80% of employed doctorate level mathematicians are in academic life and about 20% in nonacademic positions. It is almost inevitable that, for the class of the younger doctoral–level mathematicians now being produced, those numbers are likely almost to reverse over the next ten or fifteen years. That is, we are very likely to have a large degree of our new doctoral mathematicians, including some of our ablest young mathematicians, ending up spending most of their productive lives working outside the academic domain. The demographic facts of life are just overwhelming in this direction. Furthermore, most of those outside academia will also be working outside the traditional domain of research and development in industry, business and government. That is, the actual research and development activities to which doctoral–level mathematicians might, or would naturally, go are very definitely limited. If there is to be employment for mathematicians over the next generation outside of academia, it has to be in nontraditional ways; in ways for which we have not had much experience identifying the requisite training.
Now, one possible answer is that we should simply cut down the number of doctoral students who are being trained, to bring this in balance with anticipated traditional employment demand; then we would have a state of equilibrium. But, I believe, as Professor Bers has mentioned, that there are societal forces that affect us that are far stronger than the temporary forces which face us with respect to immediate employment. Among these forces are the increasing complexity of society, the need and patterns of more education for more people. We are already at the level where over 50% of our young people start to college (we are certainly not going to get to a situation where 100% of them go to college). There has to be a kind of stretch-out in some people's education, as a social phenomenon. That fact almost ensures that, in the long term if not in the short, doctoral-level training is going to be on the rise rather than on the decline—in spite of the current adverse employment prospects. That being the case, we must somehow come to grips with accommodating our graduate training generally, and our doctoral-level training specifically, to the ultimate destinations of most of our students. In my own view, the heavy emphasis on research as the primary or sole aspect of doctoral-level training is going to have to give way. We will be training a good many doctoral-level students essentially as practitioners in much the same sense that most medical doctors are trained as practitioners, while relatively few medical doctors are trained as research people. It is true that we will not be able to hang out shingles and get people to come to us for the same kind of activity as M.D.'s; the individual practitioner will be working in a company or group of some sort, rather than simply as an individual mathematician selling his wares to other individuals in society.

A practitioner's degree does not necessarily mean something other than a Ph.D.; it may well be called a Ph.D., or it may well ultimately be called something else. But, in any event, doctoral-level training is very likely (and, inevitably, in the long-range future) going to involve a different emphasis from that which we have known in the past. One of our tasks in the mathematical community is to find devices to accommodate to this changing direction of our students with the resultant changing direction of graduate education. We must find an accommodation which preserves the virtues of the healthy research emphasis that now exists. Indeed, our research program over the past fifteen or twenty years has been fantastically successful in terms of good theorems being proved and in terms of a large number of people being active, energetic mathematicians. We must act to maintain or improve it.

What are the details of the kind of training at the undergraduate and graduate level which will give us the opportunity to train people both ways? At the undergraduate level, it probably will result in a somewhat more rigid program than we have. If a person is going to be a mathematician, he is going to have to have calculus; he is certainly going to have to have some algebra, indeed, a good bit of linear algebra and some abstract algebra; he is going to have to have some advanced calculus or baby real variables, but he is also going to have to have probability and statistics. He is going to have to have a good bit of computing, perhaps not in the undergraduate mathematics program, but somewhere. And, he is certainly going to have to have one or more courses in modelling, or in something which is akin to modelling in applying mathematics to other areas. And, realistically, if we are to have that much content in an undergraduate program, there is not much room left for very many other courses. We are going to have to accept the fact that our beginning graduate students are going to come with less explicitly directed training toward research mathematics, but with broader undergraduate training. This will, in fact, produce a lesser amount of research maturity than we have been expecting, but it also will produce a greater amount of breadth in the broader sense of mathematics.

At the first-year graduate level, commonly, but not universally, there are three kinds of core courses: a course in analysis, a course in algebra, and a course in topology in some form. I think it is almost inevitable that we are going to have to give up at least one of those. My own tentative suggestion as to what we should be providing at the beginning graduate level is an offering of four or five courses from which students will choose three, with advice. And, among courses that might well be offered in addition to the three standard courses, are courses in combinatorics, or probability, or computational theory, or in modelling. Hopefully, by the end of the second year, the student should have picked up second-year courses in some of these disciplines, as well as a further broadening in the sense of other first-year courses taught from a somewhat more sophisticated viewpoint. The student would have the equivalent of three or four year courses in core mathematics and three or two year courses in modern applicable or applied mathematics.

In any event, a student who would go through a program like that conceived here would, at the end of his second year, be sizably more employable than many of our present Master's-level students. Furthermore, he would have a background in the mathematical sciences which would offer him more options in the long run.

We should not be prescriptive and say that every student should have just such a program; we ought to operate with advice and encouragement of students, not with arbitrary regulations. I would be opposed to introducing minors into programs, because minors may well simply become obstacles to be overcome, instead of opportunities to learn. The program ought to be organized in such a way that students will obviously want to be exposed to other disciplines as well as to applications of mathematics, rather than requiring it to be so. Furthermore, there may be occasional individuals who should go the pure route itself; I would have no objection to a few individuals doing so. There is a big distinction between the training of an individual and the training of the group. The group of mathematicians must be much more broadly trained; the individual himself should seek to be more broadly trained, but the breadth of training will inevitably reduce the depth of his training in a particular specialty.
What does that mean with respect to post-Master's-level doctoral programs? Assuming we continue to have a four or five year post-Baccalaureate type program for the doctorate, it means the student has sizably less time to specialize, he will probably be at least one year back in mathematical maturity toward specific research than our present students are. I do not believe it is feasible, or right, to extend the length of time he must spend in graduate school. Indeed, to train a person for a long time in graduate school, and then to have very limited opportunities for him to practice his profession, seems most unrealistic. Therefore, it would appear to me almost inevitable that we are going to be facing a situation in which, for those who choose the research line, the doctoral degree is going to represent a lesser research type contribution than our present program customarily involves.

I am not sure in my own mind what a practitioner's degree is going to be. It probably involves further breadth beyond the Master's level and some dissertation, but probably a less deep dissertation than we are now getting. What one would hope to do would be to get a sort of core training that would offer the student the options at this stage to become either a pure mathematician or an applied one. And, at the same time, provide most, or almost all, of the pure mathematicians with a background that gives them alternative opportunities if academic employment is not available to them. I disagree with Lipman Bers that it will be possible to identify, or even that there is, a sort of core at the graduate level that all mathematicians should have. Mathematics is getting too broad for that. It seems to me we can identify a number of topics within mathematics from which an individual should be choosing several and getting reasonable depth in these, and that his training should include a dispersion of these topics across various sub-disciplines. But, I think it is essentially impossible to identify a rigid core that must be taken by almost anybody who should be called a mathematician.

As discussed earlier by Henry Pollak, I think another of the sort of practical aspects of training for nonacademic life is going to have to be the infusion of group or cooperative activity, somewhat contrary to the traditions of current graduate training. I do not know exactly how this can or should be designed, but it is something which, in the long run, we in the mathematics community will have to face. Furthermore, the nature of exposition that we expect students to produce is going to have to be different. Some engineers are more employable than some mathematicians because they have learned to write reports. Mathematicians, generally, are not forced to do expository writing. In a dissertation, a student does do some exposition, but the expository writing is frequently secondary to the research mathematics itself. I think, in the nature of graduate training, we should be emphasizing more expository-type writing, and perhaps correspondingly less oral or verbal exposition. If you are in the academic community, then your basic contact with other people, at least students, is through verbal exposition. But, in much of the business or professional world, there is a kind of descriptive written exposition that is needed and required. There are also, of course, important components of verbal exposition in such areas.

If, as appears almost inevitable, we are going to give a degree with less depth of research training, then we have to provide some other mechanism for producing the kind of research vitality and depth of training for those who are going to be, in fact, active researchers. A super degree is one possibility. Another possibility is to get into a pattern, a much broader pattern than we now have, of active postdoctoral training. We must select from among those who get degrees, those who are to be researchers, and give them specific postdoctoral training as part of the overall training program. This implies somewhat more supervision than now exists in much mathematical postdoctoral training. Who would fund this, under what conditions; I do not know. But, it seems to me that expanded post-doctoral training would be both necessary and realistic in an attempt to maintain and expand the research vitality of mathematics.

It is important that we should, in fact redefine mathematics to mean the mathematical sciences. Some fifteen or twenty years ago there were important reports about mathematics that never mentioned computing. Somehow, that was a mistake. What we should be doing is thinking of mathematics in the broad sense and not the narrow sense. It is in the broad sense that mathematics is going to be used in the future, and that mathematics of the future will be identified, if we have this broadened definition, then I think mathematics, including pure mathematics, has a very real and positive future. In fact, pure mathematics will benefit from the extension of the applications of mathematics to more fields, because ideas will ultimately come in from outside. If we restrict ourselves to thinking of mathematics in a narrow sense, as core mathematics, then I think we are going to be in much more trouble with respect to justification of our field as well as the intellectual vitality of it.

I wish to close my comments in a spirit of genuine optimism. The next generation offers real prospects for significant and useful changes; there is a lot of hope in the future. For the time being, we must learn to accommodate to very different employment patterns, very different uses of mathematics. But, when you put talented and well-trained individuals out in society in jobs for which mathematicians have not traditionally been used, you are providing people with an opportunity to use their minds and their training in nontraditional ways and thus to extend the domains in which mathematics is used by society. I think the next generation is going to see a much broader conception of how mathematics can be applied and this, in turn, will generate the further mathematical ideas which will make mathematics even more effective in the generation beyond that.
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GRADUATE SCHOOLS OF ORIGIN OF FEMALE PH.D.'s
By I. N. Herstein

While this is not a report of the AMS-MAA Committee on Women, the suggestion to write this note was made in January 1975 at a Washington meeting of that committee (of which the author is a member).

In order for the reader to be able to assess the accuracy of the statistics presented here, let me describe how the statistics were compiled. What I attempted to do initially was to break down the statistics on Ph.D.'s in mathematics published by the AMS to see from where, and in what areas of mathematics, the female Ph.D.'s originated in the period July 1, 1961 through June 30, 1974.

I went through these lists of the AMS and picked out what seemed to be the female names; for these names, from the titles of the dissertations, I tried to assign the Ph.D. to a particular area of mathematics.

This turned out to be much more difficult than I would have anticipated. To begin with, schools sometimes report their Ph.D.'s giving only the surnames and initials of the recipients. Here, clearly, there was no earthly way of guessing the sex of the new Ph.D. Even when the given names were furnished it was sometimes difficult, even impossible, to make a reasonable guess. There are many names which could be either male or female; there are many foreign names for which I could not make a decent guess. Although I had the help of Chinese and Japanese graduate students with the Chinese and Japanese names, they were often stumped, for the names often became meaningless to them in the transliteration to English. There were similar difficulties with Indian and Korean names.

In attempting to judge the field of a thesis by its title, naturally enough many uncertainties cropped up. I assigned the theses to six large groupings: algebra, analysis, statistics, geometry, computer science, and education.

Under the heading of "algebra" I put all theses on the usual algebraic subjects; in addition, I included there logic, combinatorics and number theory (except if the title sounded like analytic number theory, which I put under "analysis"). Into "analysis" I put the usual analytic subjects as well as most of applied mathematics, especially that part that referred in any way to differential equations. Into "geometry" I put all topology (of any sort: general, algebraic or differential), algebraic geometry, differential geometry, finite geometries, and the few theses on Euclidian or non-Euclidian geometry. Under the heading of "statistics" went, naturally enough, all the theses reported as statistics; in addition I put there some doctorates obtained in mathematics departments whose titles suggested a topic in statistics, a variety of theses in biostatistics, public health and allied topics. Into "computer science" went the theses reported as computer science; in addition, I included those on operations research, management science, and mathematical economics. Into "education" went both the theses clearly on education and those on the history of mathematics.

After I compiled these preliminary statistics, the AMS-MAA Committee on Women sent the information I had, for each department, to that department for its corrections. When these departments answered with corrections, I, of course, used this new information for the final data. If a department did not answer, I assumed that my data, as far as they were concerned, were correct. Since many departments other than the mathematics departments themselves report Ph.D.'s to the AMS (for instance, statistics, computer science, operations research, educational, biometrics, applied mathematics, etc.) this entailed a great deal of work for the committee, and especially for its head, Professor Alice Schafer. I should like to express my gratitude to her and the other members for their help.

Finally, one might ask: what purpose can the information gathered serve? Clearly, it may be of interest in its own right. It could serve as a guide to prospective female graduate students trying to decide on an appropriate graduate school for them. This was the rationale for the break-down of the Ph.D.'s by fields. Finally, it could give a given department an idea of how well it is doing with its female graduate students in comparison to its peers.

There is no doubt that the data, with all their uncertainties and unreliabilities, do show definite patterns. I will make no analysis of such patterns, leaving it to the readers to form their own impressions of what has been taking place.
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**Leading Producers of Female Ph.D.'s**

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**Leading Producers, by Percentage of Female Ph.D.’s (five or more Women Ph.D.’s)**

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**SUMMER GRADUATE COURSES. Supplementary List**

The following graduate courses are being offered in the mathematical sciences during the summer of 1976. These are in addition to the courses listed in the February issue of these *Clavis*.

**MASSACHUSETTS**

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**
Cambridge, Massachusetts 02139

Information: Director, Summer Session, Rm E19-356

- June 14–June 25
- Design and Analysis of Scientific Experiments

**OREGON**

**OREGON COLLEGE OF EDUCATION**
Monmouth, Oregon 97861

Application deadline: June 21

- Information: Summer Session 1976
- June 22–August 13
- Foundations of Mathematics
- Modern Geometry
- Analysis
- Modern Algebra
- Special Methods in Mathematics
- Diagnosis and Correction in Mathematics
- Computer Literacy
NEWS ITEMS AND ANNOUNCEMENTS

NSF SEeks NOMINATIONS
FOR ALAN T. WATERMAN AWARD

Applications are currently being sought for the first National Science Foundation Alan T. Waterman Award. Established in 1975, the award marks the twenty-fifth anniversary of the National Science Foundation and is named in honor of its first Director.

The award will be given annually to an outstanding scientist. In addition to a medal, the recipient will receive up to $50,000 per year for three years for scientific research or advanced study at the institution of his or her choice.

Applications must include a description of the nominee's predoctoral and doctoral work and accomplishments since obtaining an advanced degree, a summary of the nominee's career and potential for accomplishment, three references attesting to the candidate's work to date, and any material which may be helpful to the nominating committee. All materials must be submitted to the Award Committee, National Science Foundation, Washington, D.C. 20550, no later than March 19, 1976. The first award will be made on May 20, 1976.

GEORGE S. RINEHART-TELLURIDE CHAIR

A chair is being established at Deep Springs College in honor of George S. Rinehart, late mathematician of Cornell University, who was killed in an automobile accident in November 1972. Rinehart was an alumnus of both Deep Springs College and Cornell University. His friends and the Telluride Association, of which he was a member, have pledged more than two-thirds of the required amount, but approximately $55,000 is still required to complete the chair. All contributions will be matched. Donations are tax deductible under both Federal and State income tax laws.

Contributors should make their checks payable to the Trustees of Deep Springs College and send them in care of Miss Rachael Schrock, Deep Springs College, Deep Springs, California via Dyer, Nevada 89010.

AAUP WRITERS AWARD

Entries are presently being accepted for the Seventh Annual Higher Education Writers Award, sponsored by the American Association of University Professors (AAUP). The purpose of the awards program, established in 1969, is to recognize outstanding interpretive reporting of issues in higher education through newspapers, magazines, radio, television and films.

Work submitted for consideration should have appeared between March 1, 1975 and March 1, 1976, and must be accompanied by a brief biographical statement. Announcement of the award will be made at the AAUP Annual Meeting in Santa Barbara, California, June 25–26, 1976. Entries should be sent to the Office of Information, American Association of University Professors, Suite 500, One Dupont Circle, Washington, D.C. 20036. The deadline is April 1, 1976.

FIFTEEN PERCENT CREDIT FOR LIFE INSURANCE PLAN

Those members who are insured under the AMS-MAA-SIAM Life Insurance Plan will receive a credit equal to fifteen percent of the semianual contribution due April 1, 1976 provided they were insured on September 30, 1975.

The credit is based on favorable experience during the previous policy year. While it is hoped that credits of this type will continue to be available in future years, they cannot be promised or guaranteed, nor will they be paid in cash.

All inquiries regarding the Life Insurance Plan or any other plan in the Group Insurance Program should be made to the SAM Insurance Administrator, 1707 L Street, N.W., Washington, D.C. 20036. Telephone: (202) 296-3000.

NEW AMS COMMITTEES

Committee on Page Charges. President Lipman Bers has appointed Robert M. Baer, Leonard Gillman (chairman), George B. Seligman, and Allen L. Shields to the new ad hoc Committee on Page Charges.

RECENT AMS APPOINTMENTS

Committee on Employment and Educational Policy. David Blackwell has been appointed by the President of the Society to the Committee on Employment and Educational Policy. The chairman is Wendell Fleming; continuing members are Charles W. Curtis, Martha K. Smith, and Daniel H. Wagner.

Committee to Select Hour Speakers for Eastern Sectional Meetings. President Lipman Bers has appointed Joan S. Birman to the Committee to Select Hour Speakers for Eastern Sectional Meetings. The chairman of the committee is Walter Gottschalk and the continuing member is George B. Seligman.

Committee on Postdoctoral Fellowships. Daniel Gorenstein and William P. Thurston have been appointed by the President of the Society to the Committee on Postdoctoral Fellowships. Continuing members are Leonard Gillman, Peter Hilton, Mark Kac and Alice Schafer (chairman).

Committee on Committees. President Lipman Bers has appointed a standing Committee on Committees consisting of Richard D. Anderson, R. H. Bing, Phyllis Cassidy, W. W. Comfort, Chandler Davis, and J. Ernest Wilkins, Jr., Everett Pitcher, Secretary, is a member ex-officio. This committee replaces an ad hoc committee of the same name.

President Lipman Bers has named Herbert B. Keller (chairman), Max M. Schiffer, and
TRAVEL GRANTS AWARDED FOR THIRD INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION

The National Research Council announces that travel grants will be awarded to approximately thirty qualified individuals in mathematical education for attendance at the Third International Congress on Mathematical Education to be held August 16-21, 1976, in Karlsruhe, Germany.

Awards will be given to individuals who have made significant contributions to the field of mathematical education. Applicants will also be expected to present plans for disseminating and interpreting the results of the Congress to their colleagues.

Applications may be obtained from the Math Office, National Research Council, Washington, D. C., 20418 and must be received in that office on or before April 30, 1976.

NOTE: Gail S. Young is incorrectly listed in the 1976 Administrative Directory as a member of the Ad Hoc Committee on Academic Freedom, Tenure, and Employment Security. Members of the committee are Murray Gerstenhaber and Paul J. Sally, Jr.

LETTERS TO THE EDITOR

Editor, the 

Some thoughts on blind refereeing: (1) Of course, we are all convinced that our own papers are excellent, so if they are rejected somebody must be biased. But let's not be paranoid about it. (2) Most referees, as far as my experience goes, tend to be soft-hearted toward beginners and much tougher with established mathematicians. (3) It makes little difference whether the name is on the manuscript or not, since there are now so many mathematicians that nobody can know most of them. (4) The editor can have a lot of influence by his/her choice of referee and the tone of her/his letter of transmittal to the referee; hence papers ought to be submitted anonymously. (5) To prevent biased reviews we ideally ought to publish anonymously. (If we could only live anonymously...)

R. P. Boas

Editor, the 

The content of the first two letters in the October, 1975, 

suggests to me the need for additional instruction in the History of Mathematics to include possibly the cyclic economic situation in mathematics during recent times; say, e.g., from 1930 to 1941 and from 1957 to 1965. Special ad hoc lectures at the AMS Meetings are not needed (unemployed can't afford to go), but rather more historical lectures at and around the Schools possibly supplemented by AMS lectures on the road at the request of Schools.

To place things in perspective, I vaguely recall that Professors N. Wiener and E. L. Post enjoyed heavy teaching loads at various times in their careers. I'm sure many other heroes and non-heroes did likewise when such work was even available to them; other examples of heavy teaching loads might be worth recording for future mathematicians. I do not intend to argue that excessive workloads are "good" for one's character, although they may be for some. Rather, they can, in some cases, be "bad" for the community in the long run, since they may produce some people so adept at overcoming adversity and obstacles that succeeding people, not exposed to such adversities, cannot easily acquire leadership positions from them; i.e., the mathematical baton is not passed and the system fails. It is my hypothesis that the academic leadership problems closely associated with the classical unemployment situations of N. H. Abel, E. Galois, and C.S. Peirce were due, in no small part, to an entrenched and inflexible academic leadership mistrained on excessive adversity.

On the other hand, unless I'm mistaken, adversity provides some people the opportunity to excel rather than to give up.

Albert A. Mullin
NATIONAL SCIENCE FOUNDATION BUDGET

On January 21, 1976, the National Science Foundation described its budget request for fiscal 1977 (July 1, 1976–June 30, 1977) in a press release. Based on previous such press releases, with observations on the trend of NSF support for mathematics, have appeared in these (Note 1) in February 1972 (pages 126–127), April 1973 (pages 135–137), April 1974 (pages 152–153), and February 1975 (page 106).

The accompanying tables give comparative figures for fiscal years 1973 through 1977. Actual figures are available for 1973 through 1975, current estimates (this year the customary estimates are consistently referred to as "plans") are given for fiscal 1976 (now half over) and budget requests are given for fiscal 1977.

Table I presents data showing the portion of total NSF funds going for support of mathematics. Table II compares support for the Mathematical Sciences Section with the other sections comprising the recently organized Division of Mathematical and Physical Sciences and Engineering.

In the summary report published in the February 1975 issue of these (Note 1), figures for science advisory activities were included with overhead, line (4) of Table I. This year it was necessary to include this item in line (2) because of a change in the way the NSF presents its figures. This item is now grouped together with others under the heading "Science Assessment, Policy, and Planning" (including Science Resource Studies, Policy Research and Analysis, and Planning and Evaluation). The following quotation is taken from the paragraph in the press release which describes this group of activities: "NSF Planning and Evaluation supports research and analysis designed to strengthen the Foundation's ability to plan and improve its program and management activities. Although there is a $0.3 million reduction in the Evaluation section, the overall agency effort in this area will increase because evaluation studies will be supported by funds from the programs being evaluated in FY 1977."

Line (6) of Table I shows that support requested for research in the mathematical sciences, as a fraction of the total research support, is higher for fiscal 1977 than what was actually spent or planned in any of the previous four years.

Table II shows that on a relative basis support for the Mathematical Sciences Section continues to decrease monotonically as a fraction of the Division budget.

### TABLE I: NATIONAL SCIENCE FOUNDATION
(Millions of Dollars)

<table>
<thead>
<tr>
<th>Year</th>
<th>Actual</th>
<th>Change</th>
<th>Year</th>
<th>Actual</th>
<th>Change</th>
<th>Year</th>
<th>Change</th>
<th>Year</th>
<th>Change</th>
<th>Year</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1973</td>
<td>$14.1</td>
<td>2.8%</td>
<td>1974</td>
<td>$14.5</td>
<td>13.1%</td>
<td>1975</td>
<td>$16.4</td>
<td>6.7%</td>
<td>1976</td>
<td>$17.5</td>
<td>19.4%</td>
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<tr>
<td>(1)</td>
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<td>(2)</td>
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<td>(3)</td>
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<td>(4)</td>
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<td></td>
</tr>
<tr>
<td>(1) Mathematical Sciences Research Support</td>
<td>$14.1</td>
<td>2.8%</td>
<td>$14.5</td>
<td>13.1%</td>
<td>$16.4</td>
<td>6.7%</td>
<td>$17.5</td>
<td>19.4%</td>
<td>$20.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) Other Research Support (Note A)</td>
<td>483.5</td>
<td>2.17%</td>
<td>494.0</td>
<td>11.7%</td>
<td>551.9</td>
<td>8.0%</td>
<td>595.4</td>
<td>12.6%</td>
<td>670.6</td>
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</tr>
<tr>
<td>(3) Education, Information, Foreign Currency Program (Note B)</td>
<td>84.1</td>
<td>24.1%</td>
<td>104.4</td>
<td>-20.5%</td>
<td>83.0</td>
<td>-8.3%</td>
<td>76.1</td>
<td>1.2%</td>
<td>77.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(4) Program Development and Management (&quot;Overhead&quot;) (Note C)</td>
<td>28.6</td>
<td>10.8%</td>
<td>31.7</td>
<td>19.6%</td>
<td>37.9</td>
<td>12.4%</td>
<td>42.6</td>
<td>2.1%</td>
<td>43.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totals</td>
<td>$610.3</td>
<td>5.5%</td>
<td>$645.6</td>
<td>7.4%</td>
<td>$693.2</td>
<td>5.5%</td>
<td>$731.6</td>
<td>11.0%</td>
<td>$812.0</td>
<td></td>
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</tr>
<tr>
<td>(6) (1) as % of (1) &amp; (2)</td>
<td>2.5%</td>
<td></td>
<td>2.56%</td>
<td></td>
<td>2.59%</td>
<td></td>
<td>2.56%</td>
<td></td>
<td>2.02%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7) (1) as % of (6)</td>
<td>2.31%</td>
<td></td>
<td>2.25%</td>
<td></td>
<td>2.37%</td>
<td></td>
<td>2.39%</td>
<td></td>
<td>2.57%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

NOTE A: Scientific research and facilities (excluding mathematics), national and special research programs (excluding science information activities), national research centers, and research applied to national needs. Support for mathematics has been excluded, cf. items (1) and (3). The 1974, 1975, 1976 and 1977 figures include science advisory activities.

NOTE B: The programs in this group are ones in which there is some support for projects in every field, including mathematics. The foreign currency program involves both cooperative scientific research and the dissemination and translation of foreign scientific publications. Foreign currencies in excess of the normal requirements of the U.S. are used.

NOTE C: This heading covers the administrative expenses of operating the Foundation; the funds involved are not considered to constitute direct support for individual projects.

### TABLE II: MATHEMATICAL AND PHYSICAL SCIENCES AND ENGINEERING
(Millions of Dollars)

<table>
<thead>
<tr>
<th></th>
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<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>Mathematical Sciences</td>
<td>$14.1 (9.7%)</td>
<td>$14.5 (9.5%)</td>
<td>$16.4 (9.0%)</td>
<td>$17.5 (9.0%)</td>
<td>$20.9 (8.9%)</td>
</tr>
<tr>
<td>Computer Research</td>
<td>$9.9 (6.5%)</td>
<td>$9.8 (6.4%)</td>
<td>$11.8 (6.5%)</td>
<td>$12.6 (6.51%)</td>
<td>$15.8 (6.77%)</td>
</tr>
<tr>
<td>Physics</td>
<td>$34.9 (24.0%)</td>
<td>$36.7 (24.2%)</td>
<td>$42.2 (23.3%)</td>
<td>$44.9 (23.1%)</td>
<td>$55.5 (23.79%)</td>
</tr>
<tr>
<td>Chemistry</td>
<td>$28.1 (17.30%)</td>
<td>$26.8 (17.5%)</td>
<td>$32.7 (18.0%)</td>
<td>$35.5 (18.50%)</td>
<td>$42.3 (18.13%)</td>
</tr>
<tr>
<td>Engineering</td>
<td>$26.1 (17.9%)</td>
<td>$28.1 (18.5%)</td>
<td>$34.3 (18.9%)</td>
<td>$36.3 (18.77%)</td>
<td>$44.7 (19.16%)</td>
</tr>
<tr>
<td>Materials Science</td>
<td>$35.0 (24.12%)</td>
<td>$35.6 (23.52%)</td>
<td>$43.5 (24.0%)</td>
<td>$46.8 (24.20%)</td>
<td>$54.1 (23.19%)</td>
</tr>
<tr>
<td>Total</td>
<td>$145.1</td>
<td>$151.3</td>
<td>$180.9</td>
<td>$195.4</td>
<td>$233.3</td>
</tr>
</tbody>
</table>

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NEW AMS PUBLICATIONS

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

SUR LES SECTIONS ANALYTIQUES DE LA COURBE UNIVERSELLE DE TEICHMULLER by John Hamal Hubbard

Number 166
137 pages; list price $7.60; member price $5.70
Publication date: February 29, 1976
To order, please specify MEMO/166

In this Memoir the following theorem is proved:

Let \( p : \Xi \to \mathbb{C} \) be the universal curve with \( n \) punctures over Teichmüller space. If \( g > 2 \), or if \( g = 2 \) and \( n > 0 \), \( p \) admits no analytic sections. If \( g = 2 \) and \( n = 0 \), \( p \) has exactly six analytic sections, given by the Weierstrass points.


Many mathematicians have contributed to the theory of Teichmüller spaces, from many points of view; in order to state their results in a coherent language, the author has rewritten the theory from scratch, from the point of view of analytic geometry.

The author also proves a few results of independent interest; there is a proof of Grauert's direct image theorem and of the representability of Isom in the smooth case, using \( \delta \) techniques; there is a (slightly) new description of the Finsler Teichmüller metric; some details have been filled in of the proof of Teichmüller's theorem omitted in the paper of L. Bers ("Quasiconformal mappings and Teichmüller's theorem," pp. 89-119 in Analytic functions, R. Nevanlinna et al., Princeton University Press, Princeton, N.J., 1960); and the construction of Teichmüller spaces with punctures is carried out.

SINGULAR PERTURBATIONS AND DIFFERENTIAL INEQUALITIES by Frederick A. Howes

Number 168
75 pages; list price $7.20; member price $5.40
ISBN 0-8218-1868-6
Publication date: April 30, 1976
To order, please specify MEMO/168

In this Memoir differential inequalities of Nagumo type are used to study the existence and the asymptotic behavior of solutions of the singularly perturbed boundary value problem

\[ \frac{d^2y}{dt^2} = f(t, y, y', \epsilon), \quad 0 < t < 1, \]

\[ a_1 y(0, \epsilon) + a_2 y'(0, \epsilon), \quad b_1 y(1, \epsilon) + b_2 y'(1, \epsilon) \]

prescribed, for small, positive values of the perturbation parameter \( \epsilon \). The principal asymptotic phenomenon investigated is the familiar boundary layer phenomenon which arises from the loss of a boundary condition in formally setting \( \epsilon \) equal to zero in the original problem. A stable solution of the reduced \( \epsilon = 0 \) differential equation which satisfies one of the original boundary conditions is used to construct upper and lower solutions of the full \( \epsilon > 0 \) problem. These bounding functions contain explicit boundary layer terms which reveal the exact nature of the nonuniform behavior of the solution as a function of \( t \) and \( \epsilon \). Other approaches to the study of these and related phenomena are also discussed. Most notably, results of Coddington and Levinson, Erdélyi, Willett, O'Malley, Chang and Dorr, Parter and Shampine are compared with those obtained in the present treatment.

CBMS REGIONAL CONFERENCE SERIES IN MATHEMATICS

RELATION MODULES OF FINITE GROUPS by Karl W. Gruenberg

Number 25
82 pages; list price $8.40; member price $6.30
ISBN 0-8218-1675-6
Publication date: April 30, 1976
To order, please specify CBMS/25

This book reproduces a course of ten lectures given by the author, Karl W. Gruenberg, at an NSF Regional Conference at the University of Wisconsin-Parkside in July 1974. The aim of the lectures was twofold: to show group theorists the presentation theory of finite groups can nowadays be successfully approached with the help of integral representation theory,
and to persuade ring theorists that here was an area of group theory well suited to applications of integral representation theory. The course was constructed so that only a modicum of either group theory or module theory would be presupposed of the audience. He has also drawn on lectures that he gave at the Australian Summer Research Institute held at the University of Sydney in 1971 and at the Australian National University at Canberra in 1974. The author hopes that the present account will be of use to someone wishing to learn the subject.

In Lecture 1 the author describes the group-theoretic setting from which our subject arises. Lecture 2 contains a complete discussion of relation modules over a field. The results here (but not the proofs) go back to a paper of Gaschütz in 1954 ("Uber modulare Darstellungen endlicher Gruppen, die von freien Gruppen induziert werden," Math. Z. 60(1954), 274-286. MR 16, p. 446). Lecture 3 collects together elementary material for ease of reference later. Lecture 4 contains a proof of Swan’s structure theorem for projective modules. This is given completely modulo only the nonsingularity of the Cartan matrix. In Lecture 5 the study of relation modules begins by discussing projective summands and introducing the notion of relation cores and the presentation rank. The latter is studied further in Lecture 6. In Lecture 7 he discusses the question of the number of the abelianised relations: a basic result of Swan is proved and applied to these problems. In the next two lectures he explains the decomposition properties of relation cores. The last lecture places relation cores into the broader context of group theory and connects his results with general facts about extension theory.

TRANSLATIONS OF MATHEMATICAL MONOGRAPHS

MATRIX GROUPS by D. A. Suprunenko

Volume 45
252 pages; list price $31, 20; member price $23, 40
ISBN 0-8218-1595-4
Publication date: April 30, 1976
To order, please specify MMONO/45

This volume is a translation from the Russian of D. A. Suprunenko’s book which was published in the Soviet Union in 1972. The book gives an account of the classical results on the structure of normal subgroups of the general linear group over a division ring, of Burnside’s and Schur’s theorems on periodic linear groups, and of the theorem on the normal structure of $SL(n, Z)$ for $n > 2$. The theory of solvable, nilpotent, and locally nilpotent linear groups is also discussed.

The chapter headings are as follows:
"Elements of the theory of permutation groups."
"The general linear group."
"The normal structure of the groups $GL(\Delta)$ and $GL(n, Z)$, $n > 2."
"Reducibility and imprimitivity."
"Solvable matrix groups."
"Periodic linear groups."
"Nilpotent and locally nilpotent matrix groups."

The book also contains a subject index and one hundred thirty-three references.

TRANSACTIONS OF THE MOSCOW MATHEMATICAL SOCIETY

Volume 29 (1973)
290 pages; list price $36, 80; member price $27, 60
Publication date: April 30, 1976
To order, please specify MOSCOW/29

This volume is a cover-to-cover translation of Volume 29 of Transactions of the Moscow Mathematical Society for the year 1973. The volume is dedicated to the famous Russian algebraist Aleksandr Gennadievich Kuroš. It was translated from the Russian by K.A. Hirsch.

The following articles are included:
"In memoriam: Aleksandr Gennadiievich Kuroš,”
"Torsions and Kuroš chains in algebras,” by V.A. Andrunakievich and Ju. M. Rjabuhin,
"Chain varieties of linear algebras,” by V.A. Artamonov,
"On free decompositions of algebras with a binary operation and arbitrary operations,” by T.M. Baranović,
"On representations of a group of type $D_∞$ over a field of characteristic 2,” by S.D. Berman and N.I. Višnjakova,
"Subalgebras of free products of linear $Ω$-algebras,” by M.S. Burgin,
"On dense extensions,” by L.M. Gluskin,
"Nilpotency of ideals in alternative rings with minimal condition,” by K.Z. Ževlakov,
"On wreath products of abelian groups,” by L.A. Kalužnin and V.I. Šuščanskii,
"Isomorphisms of special direct sums,” by A.H. Livšic and M.S. Calenko,
"Remarks on stable representations of nilpotent groups,” by B.I. Plotkin,
"On the lattice determination of wreath products,” by L.E. Sadovskil and M.N. Aršinov,
"Almost direct products,” by L.A. Skornjakov,
"Minimal irreducible soluble linear groups of prime degree,” by D.A. Suprunenko,
"Semigroups with an isotope Idealizer function,” by L.N. Ševrin and A.S. Prosvirnov,
"Wreath products of Lie algebras and their application in the theory of groups,” by A.L. Šmel’kin,

and "Bivarieties and Galois connections in a functor category,” by E.G. Šul’geifer.

Volume 30 (1974)
260 pages; list price $28, 40; member price $21, 30
ISBN 0-8218-1630-6
Publication date: May 30, 1976
To order, please specify MOSCOW/30

This is a cover-to-cover translation of Volume 30 of Transactions of the Moscow Mathematical Society for the year 1974. This translation was prepared jointly by the London Mathematical Society and the AMS.

The following articles are included:
"A theorem on the connectedness of a subgroup of a simply connected Lie group commuting with any of its automorphisms,” by P.K. Raševiski,
"Continuous images of complete spaces,” by M.M. Čoban,
"On the solutions of the Cauchy problem for Laplace’s equation

SIAM-AMS PROCEEDINGS

NONLINEAR PROGRAMMING, edited by Richard W. Cottle and Carlton E. Lemke

Volume IX
200 pages; list price $15.60; member price $11.70
ISBN 0-8218-1329-3
Publication date: May 30, 1976
To order, please specify SIAMS/9

These Proceedings are based on lectures delivered at the symposium on Nonlinear Programming held March 23 and 24, 1975, as part of the American Mathematical Society's annual New York meeting. This event was the ninth in a series of Symposia in Applied Mathematics jointly sponsored by the Society for Industrial and Applied Mathematics and the American Mathematical Society with financial support from the Energy Research and Development Agency (formerly the Atomic Energy Commission) and the National Science Foundation.

The organizing committee for the symposium consisted of R.W. Cottle (chairman), C.E. Lemke, S.M. Robinson, and J.B. Rosen. The committee's intent was to help bring to the attention of a larger mathematical audience some of the history, theory, applications and vigorous research activity of the Nonlinear Programming field. The editors feel that the results included in these Proceedings can be recommended as well to the worker in the field as to the interested initiate.

Titles of the lectures included in these Proceedings are "Nonlinear programming: A historical view" by Harold W. Kuhn, "Optimality criteria in nonlinear programming" by Garth P. McCormick, "Algorithmic analysis in constrained optimization" by David G. Luenberger, "Some global convergence properties of a variable metric algorithm for minimization without exact line searches" by M.J.D. Powell, "A short course in solving equations with PL homotopies" by B. Curtis Eaves, "Lagrange multipliers in optimization" by R. Tyrrell Rockafellar, "Unconstrained methods in nonlinear programming" by O.L. Mangasarian, and "A brief survey of convergence results for quasi-Newton methods" by J.E. Dennis, Jr.

PERSONAL ITEMS

HEINZ BAUER of the University of Erlangen-Ntirnberg has been elected to regular membership in the Bavarian Academy of Sciences in Munich. He has also been elected to President of the German Mathematical Society for the year 1976.

WILFRED H. CROCKCROFT of the University of Hull has been appointed Vice-Chancellor at the New University of Ulster, Coleraine, Northern Ireland.

SAMUEL M. RANKIN III of the Florida Institute of Technology has been appointed to an assistant professorship at Murray State University.

NEVILLE ROBBINS has been appointed a visiting lecturer at Beloit College.

GIAN-CARLO ROTA has been elected a fellow of the Academia Argentina de Ciencias Exactas, Fisicas y Naturales. He will be the corresponding member for Cambridge, Massachusetts.

DEATHS

Dr. AUGUSTUS H. FOX of Silver Spring, Maryland, died on June 15, 1975, at the age of 72. He was a member of the Society for 42 years.

Professor MORRIS HALPERIN of Brooklyn, New York, died in January, 1973, at the age of 64. He was a member of the Society for 39 years.

Professor R. L. JEFFERY of Acadia University died on December 12, 1975, at the age of 85. He was a member of the Society for 51 years.

Professor SAMUEL D. LAWN of Penn Yan, New York, died on December 3, 1975, at the age of 83. He was a member of the Society for 7 years.

Dr. HARVEY A. SIMMONS of Mission, Texas, died on November 27, 1975, at the age of 81. He was a member of the Society for 48 years.
SPECIAL MEETINGS INFORMATION CENTER

The purpose of this center is to maintain a file on prospective symposia, colloquia, institutes, seminars, special years, and meetings of other associations, and to alert the organizers if conflicts in subject matter, dates, or geographical area become apparent. An announcement will be published in these Notices if it contains a call for papers, place, date, subject (when applicable), and speakers; a second full announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in each issue until it has been held and a reference will be given in parentheses to the volume and page of the issue in which the complete information appeared.

In general, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry slightly more detailed information. Information on pre-published meetings will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society. Deadlines for particular issues of the Notices are the same as the deadlines for abstracts which appear on the inside front cover of each issue.

1976 Euromech Kolloquium. Fifteen colloquia in Europe on together mathematicians, physicists and engineers to discuss turbulence theory. A series of expository lectures will be given. All interested persons are invited to attend. Except for the speakers, no support money is available. Information: Michael G. Berg. Sponsor: Duke University, Durham, North Carolina
Information: Michael C. Reed, Department of Mathematics, Duke University, Durham, North Carolina 27706.
26-May 1. REGIONAL SYMPOSIUM ON MATHEMATICS, Universiti Malaya, Kuala Lumpur, Malaysia. Organizer: Department of Mathematics, University of Malaya and Southeast Asian Mathematical Society in cooperation with the Malaysian Mathematical Society. Program: The theme of the symposium is "Recent trends in mathematics and its applications to other disciplines". There will be about six invited lectures of one hour each. It is also hoped that there will be a public forum on "The role of mathematics education in developing countries". The remainder of the program will consist of short talks of fifteen minutes each offered by participants. Information: Regional Symposium on Mathematics, Department of Mathematics, University of Malaya, Kuala Lumpur, Malaysia.
MAY
3-5. Eighth Annual ACM Symposium on Theory of Computing, Pennsylvania (22, p. 297)
3-5. Advanced Seminar on Classification and Clustering, Wisconsin (23, p. 94)
3-7. Conference on Optimal Control Theory of Systems Governed by Partial Differential Equations, Maryland (23, p. 121)
6-7. Optimization Days 1976, Canada (23, p. 84)
10-14. 47 ANZAAS Congress, Section 8 (Mathematical Sciences), Tasmania (23, p. 84)
11-15. International Graph Theory Conference, Michigan (22, p. 368)
17-21. ALGORITHMIC ASPECTS OF COMBINATORICS, Qualicum Beach, British Columbia. Speakers: Tentatively scheduled: C. Berge, V. Chvatal, D. Cornick, P. Erdos, R. Graham, P. Hamburg, A. Hoffman, D. S. Johnson, E. Johnson, R. Karp, V. Klee, G. Lawler, D. Matula, R. Read, R. Stanton, R. Tarjan, and R. Wilt. There will be no contributed talks, but poster sessions will be arranged. Sponsors: Partially supported by Simon Fraser University, University of Victoria and University of British Columbia. Information: Pavol Hell or Brian Alsip, Department of Mathematics, Simon Fraser University, Burnaby, B.C., Canada V5A 1S6.
19-22. Complex Analysis, Kentucky (23, p. 122)
20-22. SYMPOSIUM ON WAVE PROPAGATION AND SPECIAL FUNCTIONS, Oregon State University, Corvallis, Oregon. Speakers: C. H. Wilcox, University of Utah; P. Henrici, Zurich; P. Werner, Stuttgart; G. Barr, Sandia Corporation, and two others to be selected. Information: P. M. Anselone, Mathematics Department, Oregon State University, Corvallis, Oregon 97331.
21-24. ELEVENTH NEW ZEALAND MATHEMATICS COLLOQUIUM, Massey University, Palmerston North, New Zealand. Program: There will be an opening session on Friday evening followed by a social get-together. There will be sessions of contributed and invited papers on Saturday and Monday. It is intended that Sunday morning will be kept free and Sunday afternoon devoted mainly to discussion sessions. The colloquium will be held in conjunction with the Annual Meeting of the New Zealand Mathematics Society. Contributed Papers: Papers in any area of mathematics and its applications may be submitted for inclusion in the program, and suggestions for special interest sessions and invited speakers will be welcomed. It is also intended that sessions will be organized on various aspects of mathematics education (in schools, technical institutes, teachers' colleges and universities) including curricula, teaching approaches and career opportunities. Information: The Colloquium Secretary, Department of Mathematics, Massey University, Palmerston North, New Zealand.
Applications, Israel (23, p. 84)

Objectives: The mathematical approach to philosophy (or exact philosophy as it is termed) goes back to Russell, in recent years there has been growing concern for and work in exact philosophy, particularly as it influences the foundations and philosophy of science. These conversations provide interested scholars in a variety of disciplines with the opportunity to live, in close intellectual contact with one of the prominent, active workers in exact philosophy. The worker is Mario Bunge, Head of the Foundations and Philosophy of Science Unit, McGill University, Montreal, Quebec, Canada.

Program: M. Bunge will give two lectures on each of three days. Each will be followed by discussion. Informal sessions may also be held.

Sponsor: State University of New York.

Information: William E. Hartnett, Department of Mathematics, State University College, Plattsburgh, New York, 12901.

31-June 4. REGIONAL CONFERENCE ON TRANSFERENCE METHODS IN HARMONIC ANALYSIS, University of Nebraska, Lincoln, Nebraska

Lecturer: Guido Weiss (Washington University).

Support: NSF and CBMS.

Information: Frank G. Hicken or Gerald Johnson, Department of Mathematics and Statistics, University of Nebraska, Lincoln, Nebraska 68588.

31-June 5. APPLICATION OF TOPOLOGICAL METHODS IN ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS, University of Colorado, Boulder, Colorado

Principal Lecturer: Charles Conley.

Information: NSF Regional Research Conference sponsored by CBMS.

Information: Jerrold Bershow, Department of Mathematics, University of Colorado, Boulder, Colorado 80309.

31-June 11. SECOND ADVANCED COURSE ON FOUNDATIONS OF COMPUTER SCIENCE, University of Amsterdam, The Netherlands

General Information: This course is organized in cooperation with the European Communities, sponsored by the European Association for Theoretical Computer Science and financed through the Netherlands Organization for the Advancement of Pure Research.

Lectures: E. L. Lawler, University of California, Berkeley; T van Leeuwen, State University of New York, Buffalo; Z. Manna, Stanford University; R. Milner, University of Edinburgh; A. Salomaa, University of Turku; and W. J. Storer, University of California, San Diego/Mathematical Centre.

Application: Previous training and experience in the programming of computers are prerequisite as well as a good background in mathematics. Preference will be given to participants having some experience in teaching computer science.


JUNE

7-8. SUNY Conference on Complex Analysis, New York (23, p. 122)


7-11. Thirteenth Yugoslav Congress on Rational Mechanics, Yugoslavia (22, p. 197)

7-11. Sixth Conference on Stochastic Processes and their Applications, Israel (23, p. 84)

7-July 2. FIFTEENTH SESSION OF THE SEMINARE DE MATHEMATIQUES SUPERIEURES, Université de Montréal, Montréal, Québec, Canada

Program: Group theoretical methods in physics.

Sponsors: National Research Council of Canada, the Department of Education of the Government of Quebec and the Université de Montréal.


Information: Aubert Daiglemont, Department of Mathematics, Université de Montréal, C.P. 6128, Montréal, Québec, Canada H3C 3J7.

9-11. Mathematical Language and Mathematical Thought, Luxembourg (22, p. 122)

14-16. Seventh Annual Conference on Computers in the Undergraduate Curriculum, New York (23, p. 84)


14-18. SYMPOSIUM ON APPLICATIONS OF STATISTICS, Wright State University, Dayton, Ohio (23, p. 84)

Sponsors: The Air Force Flight Dynamics Laboratory.

Speakers: H. Akaline (On entropy maximization principle), S. T. Arafat (Stochastic stability of structures), A. V. Balakrishnan (Stochastic problems in control and optimization), J. S. Bendat (Statistical problems in vibrations), R. D. Bock (Psychometrics), R. C. Bose (topic to be announced), G. E. P. Box (Some applications of time series analysis), J. Chipman (Statistical problems arising in the theory of aggregation), A. Cohen, R. Canadesian, J. R. Kettering, and J. M. Landwehr (Methodological and theoretical questions arising from some applications of clustering), R. Elashoff (Two stage Bemmoulli screening designs with applications to carcinogenesis bioassay program), K. S. Fu (Some applications of stochastic language), H. L. Harter (Statistical problems in the size effect on material strength), L. Kanal (Pattern recognition), N. Keyfitz (Population problems), P. R. Krishnaiah (topic to be announced), N. Lanberg, A. J. Quinzi, and F. Proschan (Transformations yielding reliability models based on independent random variables), P. A. W. Lewis (Statistical analysis of transaction processing in a data-base system), F. M. Lord (Reliability of mental tests), V. K. Murthy (The effect of air pollution on mortality in Los Angeles), J. Neyman (Weather modification problems), C. R. Rao and C. P. Paul (Weighted discrete distributions), F. J. Rohlf (Taxonomy), S. Saunders (Reliability of structures), L. N. Shimi (topic to be announced), M. Sobel (Predicting effect of future nuclear power plants on ocean life), G. Wahlster (Smoothing, F. J. Rohlf (Stochastic problems in water resources), M. Zelen (Statistical problems in cancer research), and A. Zellner (Econometric models). The list of persons who have already accepted the invitations to preside over the sessions includes K. S. Fu, H. Hughes, G. S. Maddala, J. N. Srivastava, C. P. Tsokos, V. M. R. Tummalia, and J. R. Van Ryzin.

Attendance: Open to anyone interested. Persons interested in presenting contributed papers should submit abstracts not exceeding 200 words to the address below.

Information: P. R. Krishnaiah, AFFDL/FYS Building 125, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio 45433.

14-18. SYMPOSIUM ON THE USE OF OPTIMIZATION IN STATISTICS, Maryland (23, p. 122)

14-18. BIOLOGICAL AND BIOMATHEMATICS, Milton School, Tilson, New Hampshire

Speakers: Irving Adler, Peter Bryant, James Crow, Ralph Erickson, Judah Folkman, Sidney Fox, Joseph Frankel, Leon Glass, Steven Gould, Harvey Greenspan, Daniel Hartline, Lorne Houton, Steven Hubbell, Simon Levin, Frederick Melis, Ronald Smykow, Kenneth Trabert, Stanislaw Ulam, Michael Wade, and Hugh Wilson.

Information: Alexander M. Cruckshank, Director, Pastore Chemical Laboratory, University of Rhode Island, Kingston, Rhode Island 02881.

14-July 9, Conference/Summer school on Applications of Differential and Algebraic Geometry to Systems Theory, California (23, p. 123)

16-20. IFAC Symposium on Large Scale Systems, Theory and Applications, Italy (22, p. 161)

18-26. CENTRO INTERNAZIONALE MATEMATICO ES­ TIVO 1976. FIRST SESSION. STATISTICAL MECHANICS, Bressanone, Italy

Sponsors: Ministero della Pubblica Istruzione and Consiglio Nazionale della Ricerca del Ministero della Pubblica Istruzione.

Lecturers: P. Cartier (I.H. E.S. Bures-sur-Yvette), O. E. Lanford (University of California, Berkeley), and E. H. Lieb (Princeton University).
Information: A. Moro, C.I.M.E. Secretary, Istituto Matematico "U. Dini", Viale Morgagni 67/A, 50134 Firenze, Italy.

24–July 2. CENTRO INTERNAZIONALE MATEMATICO ESTIVO 1976, SECOND SESSION. HYPERBOLICITY, Cortona, Tuscany, Italy

Sponsors: Ministero della Pubblica Istruzione and Consiglio Nazionale della Ricerca of Italy, Scientific Affairs Division of N.A.T.O., Laboratory L. Garding (University of Lund), J. Chazarain and A. Piron (University of Nice), T. Kato (University of California, Berkeley), and K. W. Morton (University of Reading).

Information: Antonio Moro, Secretary C.I.M.E., Istituto Matematico "U. Dini", Viale Morgagni 67/A, 50134 Firenze, Italy.

28–July 2. REGIONAL CONFERENCE ON NONLINEAR DIFFUSION, University of Houston, Houston, Texas

Program: Donald G. Aronson (University of Minnesota) will deliver a series of ten lectures on the theory and applications of nonlinear diffusion equations. There will also be informal discussions and sessions for a limited number of contributed papers.

Support: (Anticipated) National Science Foundation; travel and subsistence allowances for twenty-five invited participants.

Deadline: Inquiry should be received by May 10, 1976. Every effort will be made to notify selected participants by June 1, 1976.

Information: W. E. Fitzgibbon or Homer F. Walker, Department of Mathematics, University of Houston, Houston, Texas 77004.

28–July 2. COLING 76: International Conference on Computational Linguistics, Canada (22, p. 101)

28–July 3. Fifth Hungarian Colloquium on Combinatorics, Hungary (22, p. 297)

29–July 1. Third Symposium on Machine Processing of Remotely Sensed Data, Indiana (23, p. 84)

JULY


5-15. SUMMER SCHOOL ON NONLINEAR FUNCTIONAL DIFFERENTIAL AND VOLterra INTEGRAL EQUATIONS, Université de Louvain, Institut de Mathématique Pure et Appliquée, Louvain-la-Neuve, Belgium

Lecturers: C. Corduneanu (Iasi), J. K. Hale (Providence), and R. K. Miller (Ames).

Program: A series of lectures will be delivered and a similar program will also be organized. It is hoped that financial assistance will be given to those who could not obtain support elsewhere.

Information: Jean Mawhin, Université de Louvain, Institut Mathématique, Chemin du Cyclotron, 2, B-1348 Louvain-la-Neuve, Belgium.

11-17. Third Latin American Symposium on Mathematical Logic, Brazil (23, p. 123)

11-31. COURSE ON BANACH ALGEBRAS AND OPERATOR THEORY, University of Newcastle upon Tyne, England

Program: This meeting is intended for postgraduate students. It will provide short lecture courses on the following topics, at a level suitable for students who have completed one year of postgraduate study: Theory of a single Hilbert space operator (15 lectures), several complex variables and commutative Banach algebra theory (12 lectures), \( C^* \)-algebras (8 lectures), von Neumann algebras (8 lectures), and group representations (8 lectures).

Lecturers: G. R. Allan (Leeds), P. R. Halmos (Indiana), E. C. Lance (Manchester), G. W. Mackey (Harvard), J. R. Ringrose (Newcastle).

Support: Science Research Council.

Application and Information: R. M. White, School of Mathematics, The University, Newcastle upon Tyne, NE1 7RU England.


18-31. 1976 European Summer Meeting of the ASL, England (23, p. 123)

19-23. Nonlinear Systems and Applications to Life Sciences, Texas (23, p. 84)

19-30. NATO Advanced Study Institute on Computer-Based Science Instruction, Belgium (23, p. 85)

25-31. Conference on Harmonic Analysis of Functions, Measures and Convolution Operators on Groups, Poland (25, p. 85)

26–August 20. MAA Workshop on Modules in Applied Mathematics, New York (22, p. 297)

AUGUST


11-13. Second Cape Town Symposium on Categorical Topology, Republic of South Africa (22, p. 368)

16-20. Conference on Numerical Analysis, Ireland (23, p. 85)

16-21. THIRD INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION (ICME), Karlsruhe, Federal Republic of Germany (22, p. 137)

Sponsors: So far, the following invited speakers have accepted an invitation to present a main paper: G. Th. Gilbaud (Ecole Pratique des Hautes Etudes, Paris, France), P. J. Hilton (University of Washington, Seattle), A. Kirsch (Gesamthochschule Kassel, Germany), and Sir J. Lighthill (University of Cambridge, Great Britain).

Information: E. F. an Huef, Secretary, Third International Congress on Mathematical Education, Hertzstr. 16, D 75 Karlsruhe, Federal Republic of Germany.

17-20. THIRD AUSTRALIAN STATISTICAL CONFERENCE, Memmies College, La Trobe University, Melbourne, Australia

Program: The conference is being organized by the Victorian Branch of the Australian Statistical Society and will be held in conjunction with the annual meeting of the CSIRO Division of Mathematics and Statistics. Sessions of the CSIRO Division of Mathematics and Statistics meeting will be held on August 16-17 and all participants in the Statistical Conference are invited to attend.


Information: The Organizing Committee, Third Australian Statistical Conference, C. P. Brockwell, Department of Statistics, La Trobe University, Bundoora, Victoria 3083, Australia.


24-27. Fifth Australian Conference on Combinatorial Mathematics, Australia (22, p. 369)

26–September 1. SECOND LOS ALAMOS WORKSHOP ON MATHEMATICS IN THE NATURAL SCIENCES, Los Alamos Scientific Laboratory, Los Alamos, New Mexico

Program: Invited lectures and lecture series giving non-specialized exposition of some areas in mathematics at a level accessible to the average scientist.


Information: N. Metropolis, 22-7, Mail Stop 233, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545.

27–September 4. CENTRO INTERNAZIONALE MATEMATICO ESTIVO 1976, THIRD SESSION. DIFFERENTIAL TOPOLOGY, Varenna, Italy

Sponsors: Ministero della Pubblica Istruzione and Consiglio Nazionale della Ricerca of Italy.

Lecturers: A. Haefliger (University of Geneva), J. N. Mather (Princeton), and W. P. Thurston (Princeton).
Information: Antonio Moro, Secretary C.I.M.E., Istituto Matematico "U. Dini", Viale Morgagni 67/A, 50134 Firenze, Italy.


SEPTEMBER

1-10. ADVANCED STUDY INSTITUTE ON COMBINATORICS, Freie Universität Berlin, Germany
Program: Five sections on counting theory, combinatorial order theory and set theory, combinatorial geometries, designs, groups and coding theory. There will be seminars and a few sessions on contributed papers.
Participation: Participation is limited to about 100 persons. Some grants (partially covering accommodation expenses) are available. Please submit application for a grant (including current position, fields of interest, list of papers) and for enrollment as early as possible but no later than April 30, 1976.
Information and Application: M. Aigner, Kombinatorik, II. Math. Institut, Freie Universität Berlin, Klugh-Luise-Str. 24-26, D-1000 Berlin 33, Germany.
5-10. Conference on Finite Groups and Geometries, Belgium (23, p. 123)
6-11. European Meeting of Statisticians, France (23, p. 85)
6-11. Eighth International Congress on Cybernetics, Belgium (22, p. 288; 23, p. 124)
6-17. Eighth International Congress on Cybernetics, Belgium (22, p. 288; 23, p. 124)
6-17. NATO Advanced Study Institute on Boundary Value Problems for Evolution Partial Differential Equations, Belgium (22, p. 369)
20-24. The Second Compstat Symposium on Computational Statistics, Germany (22, p. 298)
24-25. FOURTH ANNUAL MATHEMATICS AND STATISTICS CONFERENCE, Miami University, Oxford, Ohio
Program: Recreational Mathematics. This conference will explore some of the many facets of recreational mathematics including its use as a motivating tool and its relationship with significant problems. Emphasis will also be placed on helping college and secondary teachers use recreational mathematics in their classrooms. There will be sessions for contributed papers and abstracts should be sent to Stanley Payne, Department of Mathematics and Statistics, Miami University, Oxford, Ohio 45056. The deadline for abstracts is August 1, 1976.
Information: Donald O. Koehler, Department of Mathematics and Statistics, Miami University, Oxford, Ohio 45056.
25. PI MU EPSILON STUDENT CONFERENCE, Miami University, Oxford, Ohio
Program: The Ohio Delta Chapter at Miami University is sponsoring a student conference in conjunction with the conference on recreational mathematics. Papers are invited from any students, undergraduate or graduate. Papers are not limited to recreational mathematics and may be on any topic related to mathematics and statistics. Abstracts and Information: Please send a brief abstract of your paper by August 1, 1976 to Milton D. Cox, Department of Mathematics and Statistics, Miami University, Oxford, Ohio 45056.

OCTOBER

20-22. ACM 1976 Annual Conference, Texas (23, p. 85)
25-27. SEVENTEENTH ANNUAL IEEE SYMPOSIUM ON FOUNDATIONS OF COMPUTER SCIENCE, Houston, Texas
Sponsor: IEEE Computer Society Technical Committee on Mathematical Foundations of Computing, in cooperation with the ACM Special Interest Group on Automata and Computability Theory, and with Rice University and the University of Houston.
Call for Papers: Papers describing original research in the theoretical aspects of computer science are being sought. Topics of interest include analysis of algorithms, computational complexity, formal languages and semantics, mathematical theory of computation, switching and automata theory, theoretical studies of computer systems, and theory of programming and compiling.
Abstracts and Information: Authors are requested to send seven copies of a detailed abstract (not a complete paper) by May 17, 1976 to the Program Chairman at the address below. The abstract must provide sufficient detail to allow the program committee to apply uniform criteria for evaluation, and should include appropriate references and comparisons with extant work. It is strongly suggested that abstracts be limited to between five and ten typewritten pages, as each member of the program committee reads each abstract. Authors will be notified of acceptance or rejection by June 25, 1976. For inclusion in the Conference Record, a copy of each accepted paper, typed on special forms, will be due at the address below by August 16, 1976.
Information: Michael J. Fischer, Department of Computer Science, FR-36, University of Washington, Seattle, Washington 98195.

* * * 1977 * * *

APRIL

18-23. UPPSALA 1977 INTERNATIONAL CONFERENCE ON DIFFERENTIAL EQUATIONS, Uppsala University, Uppsala, Sweden
Program: There will be ten plenary speakers giving lectures on subjects related to the deficiency index problem. Other mathematicians will be invited to give talks on connected subjects, such as general operator theory and general differential problems. Conference proceedings will be published.
Support: Uppsala University, the International Mathematical Union and the Government of Sweden.
Speakers: Those who have already indicated that they are likely to participate include W. N. Everitt, J. K. Hale, R. Conti, R. Nevanlinna, B. M. Levitan, L. S. Pontrjagin, B. Szokefalvi-Nagy, S. G. Krein, E. A. Coddington, and W. A. Coppel.
Information: Gunnar Berg, Secretary, 1977 Conference in Mathematics, University of Uppsala, Box 256, 751 05 Uppsala, Sweden.

AUGUST

7-13. Eighth International Conference on General Relativity and Gravitation, Canada (23, p. 85)
16-27. International Conference on Combinatorial Theory, Australia (23, p. 85)
QUERIES
Edited by Hans Samelson

This column welcomes questions from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published conjectures. When appropriate, replies from readers will be edited into a composite answer and published in a subsequent column. All answers received to questions will ultimately be forwarded to the questioner. The queries themselves, and responses to such queries, should be typewritten if at all possible and sent to Professor Hans Samelson, American Mathematical Society, Post Office Box 6248, Providence, Rhode Island 02940.

QUERIES

91. Robert S. Wolf (Department of Mathematics, California Polytechnic State University, San Luis Obispo, California 93407), a. I would like to know of any results, or any mention, at all in the literature, of the σ-algebra of sets which are “Borel modulo a σ set smaller than the continuum” (i.e., sets (B - A₁) U A₂, where B is Borel and A₁ < 2^N), in a topological space.

b. For (X, d) a metric space, let X* be the set of compact subsets of X, and d* the derived Hausdorff metric on X*:

\[ d^*(A, B) = \text{SUP}\{d(a, B) : a \in A \cup (d(b, A) : b \in B)\}. \]

Each Y \subset X gives us a Y* \subset X*. It is known that if X = [0,1] and Y = X \cap Q, then Y* is not even analytic in X*.

My questions involve the case where (X, d) is the usual Cantor set. In that case, does Y* Borel imply that Y* is Borel (or even analytic)? Or at least that Y* \cap \{Perfect, nowhere dense subsets of X\} is analytic? More weakly still, can it be shown that this latter set does not have cardinality \(2^\aleph_0\)?

92. James Murdock (Department of Mathematics, City College (CUNY), New York, New York 10031), Do there exist accurate numerical tables, published or unpublished, for the period of the limit cycle of Van der Pol's equation \( x + k(x^2 - 1)x + x = 0 \) for numerous values of k, especially k that are neither large nor small and for which asymptotic formulas are invalid?

93. S. Caron (Poste Responde, Bureau 91, 13 Rue Cujas, Paris, 92, France). Does there exist a shorter proof of Sinaï's theorem on Boltzmann's ergodic hypothesis or something like that?

RESPONSES TO QUERIES

Replies have been received to queries published in recent issues of this column, as follows: The editor would like to thank all who have replied.

58. (vol. 22, p. 71, Jan., 1975, and p. 123, Feb., 1975, Zaidman, Hitoshi Kumanogo (Department of Mathematics, Osaka University, Toyonaka, Osaka, 560, Japan) writes the following:

We have a sharper and more general result than W. B. Houston's. In our result we can replace the term \(|h|^{p-1}(1/|x| + h)^{p-1} + 1/|x|^{p-1}\) in Nirenberg's comments by \(|h|/|x|^{p-1}\) for any \(x, x + h \in \mathbb{R}^n - \{0\}\), and the result is obtained by applying the formula (on p. 82 and 83) of my paper (Comm. Pure Appl. Math., 22(1969), 73-129).

1. Main result. “Let \(f(x)\) be defined in \(\mathbb{R}^n - \{0\}\), homogeneous of order zero, and of class \(C^p\). Set

\[ F_p(x, h) = f(x + h) - \sum_{j=1}^{p-1} \frac{1}{j!} \int_0^1 \int_0^1 \cdots \int_0^1 f^{(j)}(x) \]

Then, we have

\[ (*) \quad F_p(x, h) = \frac{(2p+1)!}{p!} \int |x|^{p-1} \quad \text{for} \quad x, x + h \in \mathbb{R}^n - \{0\}, \]

where \(K_p = \text{Max}_{0 \leq j \leq p} |A_j B_j| \) and \(A_j, B_j\) are defined by

\[ A_j = \text{Max}_{|x|=1} \int |f(x, h)| \quad B_j = \text{Max}_{|x|=1} \int |f(x, h)| \]

Furthermore, let \(f(x)\) be defined in \(\mathbb{R}^n - \{0\}\), homogeneous of order \(m(=1,2,\ldots)\), and of class \(C^{p+m}\). Then, we have

\[ (*) \quad F_p(x, h) = \frac{(2p+1)!}{(p+m)!} \int \frac{|x|^{p-m}}{|h|^{m}} \quad \text{for} \quad x, x + h \in \mathbb{R}^n - \{0\}, \]

where \(K_{p+m} = \text{Max}_{0 \leq j \leq p+m} |A_j B_{j+m}| \).

The proof is a bit too long to be published here. Write to the AMS Queries editor for a copy of the proof.

80. (vol. 23, p. 80, Jan., 1976, Raphael), Concerning the relation \(ab = a^b\), R. Raphael writes in correction of his query that the semigroup should have no nontrivial nilpotents. F. R. McMorris writes the following:

I have characterized (not necessarily commutative) semigroups such that the relation \(a \leq b \iff a^2 = ab\) is a partial order. These are precisely left separative semigroups. This result along with some others is contained in a paper under preparation. The relation is separative \(a^2 = ab = b^2\) implies \(a = b\). Hence for an example, take any commutative semigroup with zero having a non-zero nilpotent element.

Melvin Henriksen notes that an article by A. Abian, "Order in a special class of rings and a structure theorem," Proc. Amer. Math. Soc. 52(1975), 45-49, gives sufficient conditions on a ring in order that the given relation be a partial order.
ABSTRACTS PRESENTED TO THE SOCIETY

Preprints are available from the author in cases where the abstract number is starred.
Invited addresses are indicated by •

Abstracts for papers presented at
734 meeting in New York, April 11–14, 1976 A-370
735 meeting in Reno, April 23–24, 1976 A-389
Short Course in Toronto, August 22–23, 1976 A-405
733 meeting in Urbana, March 15–20, 1976 A-408

The papers printed below were accepted by the American Mathematical Society for presentation by title. The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form. The miscellaneous group includes all abstracts for which the authors did not indicate a category.

An individual may present only one abstract by title in any one issue of the Notices but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for an issue.

Algebra & Theory of Numbers

76T-A74 Mary J. Cowles, The Pennsylvania State University, University Park, Pennsylvania 16802.
A case when the class number of an imaginary quadratic field is divisible by 3. Preliminary report.

Let \( d = -(c^2 - 4q^3) \) be negative, odd, square free, and not equal to -1 or -3; with \( q \) an odd prime \( (q, d) = 1 \), then if \( 4q \neq s^2 - dt^2 \)
for any integers \( s \) and \( t \), the class number of \( \mathbb{Q}(\sqrt{d}) \) is divisible by 3. (Received December 12, 1975.)


Let \( L = \{0, +, -, \cdot, 1, \cdot, 1\} \) be the language of lattice-ordered groups together with a new constant-symbol 1 and a unary function-symbol \( \cdot \). Let \( \Sigma \) be the class of all \( L \)-structures \( M \) such that:
(i) \( M \) is a lattice-ordered abelian group. (ii) \( 1 \) is a weak unit in \( M \). (iii) The interval \( [0, 1] \) forms a Stone algebra under the operations \( \cdot, u, \cdot \). (iv) For all \( a \in M, a^* = ((a \cdot 0) + 1)^* \).

Let \( T = \text{Th}(\Sigma) \) and let \( T' = T \cup \{ \forall x \exists y(y = x^*: n < \omega) \cup \{ \forall x(x^* > 0 \rightarrow \exists y(0 < y^* < x^*)) \} \cup \{ \forall x \forall y \exists z \exists z'(x^* > 0 \rightarrow z > 0 \land z^* > 0 \land x > z + z' \land z^* y^* \land z'^* y^* \land z'^* y^* > 0) \} \).

THEOREM (i) \( T' \) is the model-completion of \( T \) and \( T' \) is complete.
(ii) Every model \( M \) of \( T \) has a unique prime model extension to a model \( M' \) of \( T' \). \( M' \) is a minimal extension of \( M \) if and only if \( \text{Im}(\text{im}(M)) \) is atomless. (Received November 28, 1975.)

76T-A76 GONG-HWAI CHEN, William Paterson College, Wayne, New Jersey 07470. On products of continuous self-maps on topological spaces. II.

This continues Abstract 76T-A50, these Notices 23(1976), A-269. Theorem 4. All nonsurjective periodic self-maps on \( E \), all real numbers, are idempotent generable. Theorem 5. A nondemptotent self-map \( f \) on the interval \([a, b]\) is idempotent generable if and only if it has a proper right unit self-map and \( R(f) \) is a closed interval.

Theorem 6. \( f \) is a nonidentity self-map on either \([a, b] \) or \([a, b] \) implies that: (i) \( f \) is idempotent generable if \( f \) is not surjective and \( f \) is constant on a neighborhood of either \( a \) or \( b \). (ii) \( f \) is not idempotent generable if either \( \lim_{x \to a^+} f(x) \neq \lim_{x \to b^-} f(x) \) for all \( a < x < b \) or \( \lim_{x \to a^-} f(x) \neq \lim_{x \to b^+} f(y) \) for all \( a < x < b \). Theorem 7. Let \( X \) be an RE-space, \( f \) a continuous self-map on \( X \), and \( R(f) \) a proper retract of \( X \) such that \( f: R(f) \to R(f) \) is a homeomorphism. Then \( f \) is a product of three idempotent self-maps if \( Z(f) \) contains a retract subset which is homeomorphic to \( R(f) \). Theorem 8. Suppose \( X \) is an RE-space such that the complement of each retract \( A \) of \( X \) contains a retract subset of \( X \) which is homeomorphic to \( A \). Then each nonsurjective self-map is idempotent generable if it is in a subgroup of the semigroup of all continuous self-maps on \( X \). (Received December 15, 1975.) (Author introduced by Professor C.K. Pfau.)
We have shown that a pair \( A, B \) of \( n \times n \) complex matrices are simultaneously similar to upper triangular matrices if any one of the following conditions is satisfied: (1) \( A \) has distinct eigenvalues and \( p(AB - BAJ)^2 \) is nilpotent for all polynomials \( p \).

(2) the pair \( A, B \) has property \( L \) and either (a) \( \text{rank } A = 1 \) or (b) \( n < 6 \) and \( A^2 = B^3 = 0 \). (3) \( n = 3 \) and \( (AB - BA)^2 = 0 \). The following result is related to (3): Let \( A, B \) be \( 3 \times 3 \) matrices which are not simultaneously triangularizable. Then \( A, B \) have a common left eigenvector and a common right eigenvector if and only if \( (AB - BA)^2 \) commutes with \( A \) and \( B \). (Received December 15, 1975.)

\[ \text{Theorem on the diophantine equation } f(x, y) = 0 \]

In this abstract, we extend A. Baker's theorem on explicit bounds for the solutions of the diophantine equation \( f(x, y) = 0 \) to those cases where the splitting field, \( K \), of \( f(x, y) \) over \( \mathbb{Q}(y) \) contains subfields generated by elliptic and hyper-elliptic equations. THOMAS LAFFEY, Queen's College Dublin, Ireland. Simultaneous triangularization of complex matrices. Preliminary report.

Let \( L(x) \) denote the number of square-full integers \( \leq x \). By a square-full integer, we mean a positive integer \( n \) such that \( p^2 \mid n \) for every prime \( p \). It is well-known that \( L(x) = \frac{x}{k(3/2)} \frac{\zeta(3)}{\zeta(2)} x^{1/2} + \frac{x}{k(2/3)} \frac{\zeta(3)}{\zeta(2)} x^{1/3} \), where \( \zeta(s) \) is the Riemann Zeta function. Let \( \Delta(x) \) denote the error function in the asymptotic formula for \( L(x) \). On the assumption of the Riemann Hypothesis (R.H.), it has been shown that \( \Delta(x) = O(x^{(13/81)+\epsilon}) \) for every \( \epsilon > 0 \). In this paper, we prove the following results on the assumption of R.H. (1) \( \frac{1}{X} \int_{1}^{X} \Delta(t) dt = O(x^{1/12}) \) (2) \( \int_{1}^{X} \frac{\Delta(t)}{t} \log t^{1-1} \left( \frac{x}{t} \right) = o(x^{1/12}) \) for any integer \( \nu \geq 1 \). (3) \( \int_{1}^{X} \Delta(t)^p dt = O(x^{12p/12\epsilon}) \) for any \( p \) such that \( 1 \leq p < 12/7 \).

On the basis of (1), (2) and (3) above, we conjecture that \( \Delta(x) = O(x^{(1/12)+\epsilon}) \) under the assumption of R.H. (Received January 12, 1976.)

Let \( R \subseteq X \times X \) be a reflexive, locally finite binary relation, \( k \) a field, \( C(X) \) the \( k \) vector space with basis \( R \). Define on \( C(X) \) a (not necessarily coassociative) comultiplication by \( \Delta_{\lambda}(x, y) = \sum_{z \in X} \lambda(z) \cdot \delta_{X,Y}(x, z) \otimes (z, y) \), where the sum is taken over all \( z \) with \( (x, z) \) and \( (z, y) \) in \( R \) and \( \lambda^*(z) \) is arbitrarily selected in \( k^* \). \( (C(X), \Delta) \) is the \( \lambda \)-weighted incidence coalgebra \( C_{\lambda}(X) \). In the event \( R \) is a partial ordering and \( \lambda^*(z) = 1 \), this is the usual incidence coalgebra \( C(X) \).
Theorem: Every cofinite ideal in $C_\lambda(X)^*$ is closed in the finite topology if and only if the group-like coalgebra on $X$, $kX$, is coreflexive. Corollary 1: An incidence coalgebra of a partially ordered set is coreflexive if and only if its coradical is coreflexive. Corollary 2: If $\varphi: C_\lambda(X)^* \rightarrow C_\lambda(X')^*$ is an algebra morphism, and if $kX$ is coreflexive, then $\varphi = \varphi^*$ for some coalgebra morphism $\psi: C_\lambda(X') \rightarrow C_\lambda(X)$. If $C_\lambda(X)$ is coassociative (equivalent to a weak form of transitivity of $R$ and "cohomological triviality" of $\lambda$) one can recover the set $X$ and relation $R$ from $C_\lambda(X)$ via the coradical filtration. Theorem: If $C(X)$ is coassociative, then $C(X) \cong C(X')$ if and only if $X$ is isomorphic to $X'$ as ordered sets. (Received January 12, 1976.)


If $L$ is a lattice, $J(L)$ denotes the lattice of ideals of $L$. Let $L$ and $M$ be lattices and let $\varphi: M \rightarrow L$ be a homomorphism of $M$ onto $L$. Then there is a natural homomorphism $\hat{\varphi}$ of $J(M)$ onto $J(L)$ defined by $\hat{\varphi} = \{x\varphi | x \in I\}$. Theorem: Let $L$ and $M$ be lattices with $L$ satisfying Whitman's condition (W), and let $\varphi: M \rightarrow L$ be a lower bounded epimorphism. Then there is a homomorphism $\hat{\varphi}: J(L) \rightarrow J(M)$ such that $\hat{\varphi} \hat{\varphi} = id_{J(L)}$. As a corollary we get the following result of R. Freese: If the lattice $L$ has no infinite chains then $L$ satisfies (W) if and only if $L$ is a retract of the lattice of ideals of the lattice of dual ideals of a free lattice. (Received January 15, 1976.)

76T-A82 VERNER E. HOGGATT, Jr., San Jose State University, San Jose, California 95192. A variation on a problem of B. W. Jones. Preliminary report.

Following B. W. Jones in Abstract 731-10-8, these Notices (23(1976), A-47, call a set of integers a p-set if the product of any two increased by 1 is a square. The set 1, 3, 8, 120 is such a set. Notice that 1, 3, and 8 are $F_2$, $F_4$, and $F_6$, where $F_n$ is the nth Fibonacci Number. A generalization of the above set is the p-set $F_{2n}$, $F_{2n+2}$, $F_{2n+4}$, $x$ where $x = 4(F_{2n+1}, F_{2n+2}, F_{2n+3})$. For $n=1$, $x = 120$ is known to be unique. For $n=2$, $x = 2080$ is the only solution < 100,000. For $n=3$, $x = 37,128$ is also the only solution < 100,000.

Conjecture. For each $n$, $x$ is unique. (Received January 12, 1976.)

*76T-A83 Withdrawn.


Let $S$ be a regular semigroup and let $T$ denote the union of the maximal subgroups of $S$. Assume $T$ is a semigroup (i.e. $T$ is a semilattice of completely simple semigroups ($T_y$: $y \in Y$)). If, furthermore, for each $y \in Y$, there exists $e_y \in T_y$ such that $e_y e_y = e_y$, for all $y, z \in Y$, we term $S$ a normal regular semigroup. Let $t = \{(a, b) \in S \times S \mid aa', bb' \in T_y$ and $a'a, b'b \in T_z$ for some $y, z \in Y$ and some inverses $a', b'$ of $a$ and $b$, respectively}. We show $t$ is a congruence on $S$, $S/t$ is an inverse semigroup with semilattice $Y$, each subgroup of $S/\tau$ is trivial, and ker $t = T$. Using this congruence, we give structure theorems for the following classes of normal regular semigroups: (1) regular, (2) locally inverse, (3) $\Lambda$ congruence on $T$, (4) orthodox, (5) $\Lambda$-unipotent, (6) split extensions of $T$ by $S/\tau$, (7) regular bisimple of type $\omega$. The various building blocks are $S/\tau$, upper partial chains and semilattices $Y$ of right groups, semilattices $Y$ of left zero semigroups, and normal $\Lambda$-unipotent semigroups with trivial subgroups (just for (3)). The structure theorems are given by explicit multiplications of ordered "triples" or "pairs". Schreier-type conditions are used. (Received January 26, 1976.)

76T-A85 MICHAEL SINGER, The Ohio State University, Columbus, Ohio 43210. Augmentation terminals of finite abelian groups, II. Preliminary report.

Conclusion of the work described in Notices, January 1976, *73l-13-7. We determine the structure of the augmentation terminal for all finite abelian groups, using developments of our earlier techniques. We give closed formulae for the structure, as well as a natural description by
generators and relations. The crux of the argument is an induction step for which two proofs are given: one direct, the other based on Stallings' identification of the augmentation factors with the terms of a certain spectral sequence. (Received January 19, 1976.)


A. Rosenfield (J. Math. Anal. Appl. 35(1971), 512-517) introduced the concept of fuzzy subgroups on a given group. As a sort of generalisation, the authors introduce in this paper, a more general concept of fuzzy group and other algebraic structures on any given set X by considering the class of all fuzzy sets on X over a Boolean Algebra. (Received January 29, 1976.) (Authors introduced by Dr. M. Sitaramayya.)

Kenneth McDowell, Wilfrid Laurier University, Waterloo, Ontario, Canada.


For a commutative ring R and non-zero module M, \( \text{Pc Spec } R \) is called a strong Krull prime of M (denote \( \text{Pc}(M) \)) if, for any finitely generated ideal \( I \subseteq P \), there exists \( 0 \neq m \in M \) with \( I \subseteq \text{ann}(m) \subseteq P \). \( \text{sk}(M) \) shares many of the properties possessed by other sets of associated primes and in a sense generalizes the set of weak Bourbaki primes, \( \text{Ass}_{f}(M) \), which it contains. In the coherent situation where these primes originated the concept of faithful flatness often arises. e.g. Theorem If \( M \neq 0 \) is a finitely presented module over a coherent ring \( R \) and \( \text{Pc Spec } R \), then \( \text{Pc}(M) \) if and only if there exists a (faithfully) flat module \( F \) with \( \text{Pc Ass}_{f}(M \oplus F) \). Theorem Suppose \( A \rightarrow R \) is a faithfully flat ring extension, \( R \) is coherent, and \( M \neq 0 \) is a finitely presented \( A \)-module. Then \( 0_{*}[\text{sk}(M \oplus R)] = \text{sk}_{A}(M) \) where \( 0_{*} : \text{Spec } R \rightarrow \text{Spec } A \) represents the contraction map. Of special interest in the coherent situation are those finitely presented modules whose maximal primes are strong Krull primes. Many of the results in the Noetherian theory of depth and \( R \)-sequences remain valid when all finitely presented modules have this property. (Received January 19, 1976.)

Kenneth McDowell, Wilfrid Laurier University, Waterloo, Ontario, Canada.


For a commutative ring \( R \) and non-zero module \( M \), \( \text{Pc Spec } R \) is called a strong Krull prime of \( M \) if, for any finitely generated ideal \( I \subseteq P \), there exists \( 0 \neq m \in M \) with \( I \subseteq \text{ann}(m) \subseteq P \). \( \text{sk}(M) \) shares many of the properties possessed by other sets of associated primes and in a sense generalizes the set of weak Bourbaki primes, \( \text{Ass}_{f}(M) \), which it contains. In the coherent situation where these primes originated the concept of faithful flatness often arises. e.g. Theorem If \( M \neq 0 \) is a finitely presented module over a coherent ring \( R \) and \( \text{Pc Spec } R \), then \( \text{Pc}(M) \) if and only if there exists a (faithfully) flat module \( F \) with \( \text{Pc Ass}_{f}(M \oplus F) \). Theorem Suppose \( A \rightarrow R \) is a faithfully flat ring extension, \( R \) is coherent, and \( M \neq 0 \) is a finitely presented \( A \)-module. Then \( 0_{*}[\text{sk}(M \oplus R)] = \text{sk}_{A}(M) \) where \( 0_{*} : \text{Spec } R \rightarrow \text{Spec } A \) represents the contraction map. Of special interest in the coherent situation are those finitely presented modules whose maximal primes are strong Krull primes. Many of the results in the Noetherian theory of depth and \( R \)-sequences remain valid when all finitely presented modules have this property. (Received January 19, 1976.)

Steven Roman, University of Washington, Seattle, Washington 98195

A Problem on Multi-colorings of Graphs.

Let \( K_{m} \) be the complete graph on \( m \) vertices, and let \( c_1, \ldots, c_n \) be \( n \) distinct labels or colors. For each \( i=1, \ldots, n \) color some complete subgraph of \( K_{m} \) with color \( c_i \). It is permissible to color an edge of \( K_{m} \) with more than one color. If there is no cycle of length \( r \) in \( K_{m} \) with the property that we can choose a different color, from the coloring, for each edge of the cycle, we call the coloring \( r \)-admissible. We denote by \( E(m,n,r) \) the maximum, taken over all \( r \)-admissible colorings, of the number of incidences of edges of \( K_{m} \) and colors. In this paper we will show that
\[
2\left(\frac{m-1}{2}\right) + n - 1 \leq E(m,n,3) \leq 2\left(\frac{m}{2}\right) + n; \\
3\left(\frac{m-2}{2}\right) + 3n - 6 \leq E(m,n,4) \leq 3\left(\frac{m}{2}\right) + 3n; \\
4\left(\frac{m-3}{2}\right) + 6n - 18 \leq E(m,n,5) \leq 4\left(\frac{m}{2}\right) + 6n.
\]
We also discuss some related problems. (Received January 20, 1976.)

Peter Hagis, Jr., Temple University, Philadelphia, Pennsylvania 19122 and Graham Lord, Naval University, Quebec, Canada. Quasi-aliquot sequences.

If \( J(N) = \sigma(N) - N = 1 \), a quasi-aliquot sequence with leader \( n_0 \) is a sequence of integers \( n_0, n_1, n_2, \ldots \) such that \( n_{i+1} = J(n_i) \). If \( J(m) = n \) and \( J(n) = m \), \( m \) and \( n \) are said to be quasi-amicable numbers. All quasi-aliquot sequences with leader \( < 10^5 \) have been generated and their behavior is discussed in this paper. A search was made for quasi-amicable numbers in the range \([1, 10^6]\). Eighteen pairs were found and are listed here. (Received January 23, 1976.)

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The following two results are proved: (i) if \( q \equiv 1 \pmod{4} \) is a prime power, then there exists a Hadamard matrix of Williamson type of order \( 2q^r(q+1) \) for all \( r \geq 0 \), (ii) if \( q \equiv 3 \pmod{4} \) is a prime power, then there exists a Hadamard matrix of order \( q^r(q+1) \) for all \( r \geq 0 \). (Received January 26, 1976.)

SHEILA M. WOODS, University of Wisconsin, Madison, Wisconsin 53706.

A ring \( R \) with \( 1 \) is semi-local if \( R/JR \) is artinian, where \( JR \) denotes the Jacobson radical. Theorem:

Let \( K \) be a field of characteristic zero and let \( G \) be a group. If the group ring \( KG \) is semi-local, then \( G \) is finite. It follows easily that if \( A \) is a ring such that \( A/JA \) has characteristic zero, then \( AG \) is semi-local if and only if \( A \) is semi-local and \( G \) is finite. (Received January 26, 1976.)

Man-Duen Choi and Tsiu-Yuen Lam, University of California, Berkeley, CA 94720.


In 1888, Hilbert proved that there exist positive semi-definite (psd) homogeneous polynomials that are not sums of squares of polynomials. However, Hilbert's idea does not throw any light on explicit construction. The first example in print is a sextic form given by T. S. Motzkin (1967), and the other known examples are due to R. M. Robinson (1969). In a forthcoming work, a systematic investigation on extremal psd homogeneous polynomials is pursued. In particular, it is easily shown that the quartic form, \( x^2 y^2 + y^2 z^2 + z^2 x^2 + w^4 - 4xyzw \), and the sextic form, \( x^4 y^2 + y^4 z^2 + z^4 x^2 - 3x^2 y^2 z^2 \), are (extremal) psd but not sums of squares of polynomials. Apparently, these two forms are the simplest examples for Hilbert's result. (Received January 26, 1976.)

DAVID ZEITLIN, 1650 Vincent Ave., North Minneapolis, MN., 55411. Parametric solutions for two equal sums of 17 biquadrates.

Let \( W_{k+2} = W_{k+1} + P W_k \), \( k=0,1, \ldots \), where \( P, W_0, \) and \( W_1 \) are integers. Then, for \( k=0,1, \ldots \), we have

\[
(*) \quad (28W_{k+5})^4 +(14P^2 W_{k+3})^4 +(49^2 W_{k+3})^4 +(28^2 W_{k+2})^4 + (42^2 W_{k+2})^4 + (14P^2 W_{k+2})^4 + (21P^2 W_{k+2})^4 + (56P^2 W_{k+2})^4 + (12P^2 W_{k+2})^4 + (20P^2 W_{k+2})^4 + (32P^2 W_{k+2})^4 + (28P^2 W_{k+2})^4 + (12P^2 W_{k+2})^4 + (20P^2 W_{k+2})^4 + (28P^2 W_{k+2})^4 + (32P^2 W_{k+2})^4 = (28W_{k+4})^4 + (12W_{k+4})^4 + (20W_{k+4})^4 + (28W_{k+4})^4 + (32W_{k+4})^4 + (12W_{k+4})^4 + (20W_{k+4})^4 + (28W_{k+4})^4 + (32W_{k+4})^4.
\]

Remark. For \( P = 1 \), \( W_k = H_k \), a general Fibonacci seq., and \((*)\) gives \((1) \quad (2H_{k+5})^4 + H_{k+4}^4 + (2H_{k+3})^4 + (4H_{k+2})^4 + (3H_{k+1})^4 = (3H_{k+4})^4 + (4H_{k+3})^4 + (2H_{k+2})^4 + H_{k+1}^4 + (2H_{k+1})^4 \). For \( P = 2, 3, \) and \( 4 \), \((*)\) is reducible to two equal sums of 9, 14, and 13 biquadrates, respectively. If \( W_{k+2} = P^2 W_{k+1} + W_k \), then \( W_k \) gives again a parametric solution for two equal sums of 17 biquadrates (to be announced). Parametric solutions for two equal sums of 7, 8, 10 biquadrates has been obtained (to be announced) for varied seqs. (Received January 26, 1976.)

John Mitchem, San Jose State University, San Jose, California 95192.

Counterexamples to a conjecture by R. H. Fox.

Let \( F_n \) denote the set of all planar graphs of order \( 12 + n \) which have 12 vertices of degree 5 and \( n \) vertices of degree 6. Theorem. For a non-negative integer \( n \), \( F_n \)
The following results are proved: THEOREM A. Let $G$ be a doubly transitive permutation group of degree $p^2 + 1$, $p$ a prime. Then either $G$ is doubly primitive or $p = 2$ and $G$ is a Frobenius group of order 20. THEOREM B. Let $G$ be a doubly transitive permutation group of degree $p^2 + 1$, $p$ a prime. Assume that $G$ contains two Sylow $p$-subgroups with nontrivial intersection. Then either a) $G$ is triply transitive, or b) the stabilizer of a point is a rank 3 permutation group of degree $p^2$ with subdegrees $1, 2(p-1), (p-1)^2$. (Rank 3 permutation groups with degree and subdegrees as in b) are discussed in Higman, Arch. Der. Math. 21, 1970. They are subgroups of $S_p \, S_2$. ) (Received February 2, 1976.)

Ralph McKenzie, University of California, Berkeley, California 94720. On minimal, locally finite varieties with permuting congruences.

R. Quackenbush has proved that finite algebra has small fine spectrum if and only if it is simple, has only trivial subalgebras, and belongs to a congruence permutable variety. Clark and Krauss have characterized this same class of algebras from another direction, and call them plain para-primal algebras. A para-primal is quasi-primal if it generates a congruence distributive variety. We have proved that every plain para-primal algebra is either quasi-primal or is isomorphic to an affine algebra over a prime field. Corollaries: (1) Every minimal, locally finite, variety with permuting congruences is defined by a finite set of equations. (2) Every simple finite algebra in a variety with permuting congruences is either functionally complete or affine. (Received February 2, 1976.)

Frank W. Owens, Ball State University, Muncie, Indiana 47306. Complete d-nary trees are graceful. Preliminary report.

Theorem. All complete d-nary trees are graceful. A complete d-nary tree always has a graceful numbering with the largest (smallest) number at the apex of the tree. Corollary. If $n = (d^k-1)/(d-1)$, where $d > 1$ and $k > 1$, or if $n > d = 1$, then $K_{2n-1}$ has a cyclical decomposition into $2n-1$ copies of the complete d-nary tree having $n$ vertices. The theorem answers the questions on complete binary trees posed by I. Cahit (Amer. Math. Monthly 83 (1976), 35-37). (Received February 2, 1976.)


John Shuck (Ph.D. Dissertation, Northeastern 1969) and more recently G.I. Gusev (Math. Notes 14 (1973) 817-822) have considered a conjecture of Borevich and Shafarevich (Number Theory, Academic Press 1966 p. 47). For a polynomial $h(X)$ in $s$ variables with integral coefficients let $c_k$ be the number of solutions of $h(X) \equiv 0 (\mod p^k)$ for $k$ a positive integer. Borevich and Shafarevich conjecture that the Poincaré series $\phi(t) = 1 + \sum_{k=1}^{\infty} c_k t^k$ is a rational function of $t$. Shuck and Gusev have affirmed the conjecture for polynomials in one variable, forms in two variables and certain other cases. A
method based on an idea of David Hayes using a p-adic change of variable theorem (Ph.D. Dissertation, U. Mass. 1974) settles a somewhat generalized form of the conjecture affirmatively for a wider class of polynomials including those above, diagonal polynomials and any others built up from them by sums and products with separate variables i.e. the conjecture is true for any $h(X) = \sum_{i=1}^{s} a_i x_i^{d_i}$ for integral $a_i, d_i$ and $d_i > 0$; if the conjecture is true for $h_1$ and $h_2$, it is true for $h_1(X)h_2(Y)$ and $h_1(h_2(Y))$. (Received February 2, 1976.)

As far as is known, every element of any finite nonabelian simple group $G$ is a commutator $xyx^{-1}y^{-1}$ ($x, y \in G$). For the finite alternating groups, a stronger theorem is: some conjugacy class $C$ in $A_n$ satisfies $C \supset A_n$ (Bertram, J. Comb. Theory 1972; Xu, Sci. Sinica 1965). Theorem 1. If $C$ has period 3, then $C \not\supset A_{4m+2}$. Theorem 2. $\exists C \subset A_n$ with $C \supset A_n$, and $\lim_n |C_n^2|/|A_n|^{1+\delta} = 0$ for every $\delta > 0$. This is the best possible result. $(C_{4m} \supset 3^{m+1}m, C_{4m+1} \supset 3^{m+1}m1, C_{4m+2} \supset 3^{m+2}m2, C_{4m+3} \supset 3^{m+1}m3)$. Theorem 3. The square of the class $C^2$ covers $A_{2r}$ if $r > 3$. Theorem 4. $C \supset A_{4p}$ where $C$ has type $4^k$ (in $S_{4k}$). Theorem 5. Let $T$ be any type in $A_n$. Then for all $k > k_0 = f_k(k)$ ($\geq f_1(T)$), the square of the class of type $T \otimes 2^k$ in $A_{n+2k}$ does not cover $A_{n+2k \setminus 1}$. Theorem 6. If $C \supset 3^3$, then $C \not\supset A_{3^k}$. Conjecture: Almost all classes $C$ in $S_n$ satisfy $C \supset A_n$. (Received February 2, 1976.)

Let $k$ be a field and $n$ be a positive integer prime to the characteristic of $k$. We construct a field $K$ containing $k$ and a cyclic division algebra $D$ of index $n$ over its center $K$. We prove that if for each prime $p$ dividing $n, k$ does not contain a primitive $p$-th root of unity, then every subfield of $D$ which is Galois over $K$ is cyclic. Using this construction we show that if $n$ is divisible by the square of a prime $p$ different from the characteristic of $k$ and $k$ does not contain a primitive $p$-th root of unity, then $k(x)$, the generic division ring over $k$ of index $n$ as defined by Amitsur, is not a crossed product. Our proof does not depend on the previously known case $k$ a global field. (Received February 4, 1976.)

The pragmatic geometry of numbers is a constructive geometry whose proofs rely exclusively on the use of straight lines (linear segments) in much the sense that J. Steiner reduces constructive Euclidean geometry to the use of only the "ruler" granted a single fixed circle with known center. This approach is consistent with, e.g., the ideas used in the definitions of convexity and stellarity. Lemma 1 (n=2). Let $K$ be an origin-symmetrical convex set in $E^2$ containing a triangle $T$ of area $A(T) > 2$. Then $K$ contains at least one pair of visible lattice points. Note 1. There is no direct need to determine the Lebesgue measure of $K$ itself. Note 2. More generally, in $E^n$, the constant for the enclosed-simplex volume is $2^n/(\binom{n}{k!})$. Lemma 2. Let $A_1, A_2, \ldots, A_n$ be real homogeneous linear forms in the real variables $x_1, x_2, \ldots, x_n$ with determinant $D \neq 0$. Then there exist integers $x_1, x_2, \ldots, x_n$ whose mosaics have no prime in common such that $|A_i| \leq |D|^{1/n}$ for every integer $i \leq n$. Lemma 3. Let $S$ be any stellar body in $E^n$ containing the Origin. Then $S$ contains a lattice point distinct from the Origin iff $S$ contains a visible lattice point. (Received February 4, 1976.)

Recently, several new categories defined by filter axioms which arise from general
topology have been shown to be cartesian closed and either "topological" in the sense of Herrlich (Math. Colloq. Univ. Cape Town 4(1974), 1-16) or "initially structured" in the sense of Nel (to appear Canad. J. Math.). Both Herrlich's theory and Nel's more general theory yield interesting and efficient treatments of cartesian closedness. In this paper, the connection between the notions of union and coproduct is investigated in initially structured categories. It is shown that intersection and, in the case of cartesian closedness, product distribute across the union of embedding subobjects. This gives rise to some nice categorical relation theoretic characterizations. The calculus of such relations becomes completely analogous to that in sets. (Received February 5, 1976.)

*76T-A103
Hilbert Levitz, Florida State University, Tallahassee, Florida 32306. Ordinal bounds for well ordered extensions of the coordinatewise partial ordering II.

Let \( A \) be a well ordered set with order type \( \alpha \). The coordinatewise partial ordering of \( A^n \) is defined by \( (x_1, x_2, \ldots, x_n) \leq (y_1, y_2, \ldots, y_n) \) if and only if \( x_i \leq y_i \) for all \( 1 \leq i \leq n \). Our result is that for any subset of \( A^n \), any extension of this ordering to a well ordering has order type which does not exceed \( \alpha^n \); product here being understood as the Hessenberg "natural product" of ordinals. (Received February 5, 1976.)

*76T-A104
H. TURNER LACKEY, University of New Mexico, Albuquerque, N. M. \( 97131 \). Values of Circulants with Integer Entries.

Let \( C_n(x_1, x_2, \ldots, x_n) \) be the determinant of the circulant matrix \( (a_{ij}) \) in which \( a_{ij} \equiv x_i \pmod{n} \). Let \( \mathcal{V}_n \) be the set of values of \( C_n \) on the domain of all ordered \( n \)-tuples \( (x_1, x_2, \ldots, x_n) \) with integer entries \( x_i \). It is proved that for primes \( p \), \( \mathcal{V}_p \) consists of the integers \( m \) with either \( p \mid m \) or \( p^2 \mid m \). For odd primes \( p \), \( \mathcal{V}_p \) consists of the integers \( m \) satisfying \( p \mid m \) or \( p^2 \mid m \) and also satisfying \( 2 \nmid m \) or \( 4 \mid m \), i.e., \( \mathcal{V}_p = \mathcal{V}_2 \cap \mathcal{V}_4 \). (Received February 6, 1976.) (Author introduced by Abraham P. Hillman.)

*76T-A105
BRUCE BERNDT, University of Illinois, Urbana. Illinois 61801 and RON EVANS, University of California, San Diego, California, 92093, Gauss and Jacobi Sums. Preliminary report.

Let \( k \in \mathbb{N} \) be \( > 1 \). For any prime \( p = 1 \pmod{k} \) and Dirichlet character \( \chi \pmod{p} \) of order \( k \), consider the Jacobi sum \( K(X) = \sum_{n=0}^{p-1} \frac{\chi(1-n)}{x_1^{n+1}} \) (where \( \left( \frac{n}{p} \right) \) is the Legendre symbol) and the Gauss sum \( G_k = \sum_{n=0}^{p-1} e^{2\pi i n^k/p} \). In the case \( k = 12 \), it is shown that \( K(X) = a + bi \) where \( p = a^2 + b^2 \), and \( a = \left( \frac{2}{p} \right) \pmod{4} \) if \( 3 \not| a \), \( a = \left( \frac{2}{p} \right) \pmod{4} \) if \( 3 \mid a \). Also, \( G_{k=12} \) is evaluated in terms of \( G_3 \) and \( a \). Jacobi and/or Gauss sums are also evaluated in cases \( k = 6, 8, 20, \) and \( 24 \).

The formulae for \( G_6 \), \( G_8 \), and \( G_{12} \) are applied to determine power residue difference sets. The formulae for Jacobi sums are applied to evaluate Jacobsthal-Whiteman and Brewer character sums. Our proofs are elementary except in the cases \( k = 20 \) and \( k = 24 \), where we make use of unique factorization in certain cyclotomic fields (as do Leonard and Williams [Rocky Mountain J. 5 (1975), 301-308] in studying octic Jacobi sums). (Received February 6, 1976.)

*76T-A106
IRA J. PAPICK, Adelphi University, Garden City, N. Y., 11530. A remark on coherent overrings.

In ["Overrings of commutative rings I, Noetherian overrings," Trans. Amer. Math. Soc., 104(1962), 52-61.], as a corollary of a more general
theorem, Davis showed that if each overring of a (commutative integral) domain R is Noetherian, then the Krull dimension of R is at most 1. (This corollary is the converse of a version of the Krull-Akizuki Theorem) It is our purpose to prove the following:

**Theorem:** If R is a Noetherian domain and each overring of R is coherent, then the Krull dimension of R is at most 1. We also indicate some related questions and examples. (Received February 9, 1976.)


Emma Lehmer (On the quartic character of quadratic units, J. reine angew. Math., 268/269 (1974), 294-301) has made a number of conjectures concerning the quartic character of certain fundamental units $\varepsilon_m$ of real quadratic fields $\mathbb{Q}(\sqrt{m})$. In this direction the authors have proved the following results: (1) Let $p \equiv 1 \pmod{56}$ be prime, so that there are integers $x, y$ with $p = x^2 + 112y^2$. Then $\left(\frac{\varepsilon_m}{p}\right)_4 = (-1)^y$. (2) Let $q = 11, 19, 43, 67, 163$. Let $p \equiv 1 \pmod{8q}$ be prime, so that either $p = x^2 + 16qy^2$ or $4p = x^2 + qy^2$ for integers $x, y$. If $p = x^2 + 16qy^2$, then $\left(\frac{\varepsilon_m}{p}\right)_4 = (-1)^y$. If $4p = x^2 + qy^2$, $x \equiv 1 \pmod{4}$, Then $\left(\frac{\varepsilon_m}{p}\right)_4 = (-1)^{(x-1)/4 + m}$ where $p = 8qm + 1$. (Received January 20, 1976.)

Dwight Duffus and Ivan Rival, University of Calgary, Calgary, Canada. Path length in the covering graph of a lattice.

Let $L$ be a lattice. The covering graph $C(L)$ of $L$ is the graph whose vertices are the elements of $L$ and whose edges are those pairs $(a, b)$, $a, b \in L$, satisfying $a$ covers $b$ or $b$ covers $a$. We apply certain elementary properties concerning path length in $C(L)$ to the study of the possible orientations of $C(L)$ as the diagram of a graded lattice. Theorem. Let $L$ and $L'$ be graded lattices with graph isomorphic covering graphs. Every element of $L$ is the join of atoms and the meet of coatoms if and only if every element of $L'$ is the join of atoms and the meet of coatoms. Moreover, there are sublattices $A$ and $B$ of $L$ such that $L = A \times B$ and $L' \cong A^d \times B^d$, where $A^d$ denotes the dual of $A$. For example, every geometric lattice is graded; moreover, every element of a geometric lattice is the join of atoms and the meet of coatoms. Occasionally, important properties of $L$ can be derived from an analysis of the subgraphs of $C(L)$. Theorem. Let $L$ and $L'$ be graded lattices with graph isomorphic covering graphs. If $L$ is semimodular and dismantlable then $L'$ is dismantlable. (Received February 9, 1976.)

D.P. CHOUDHURY and K. TEWARI, Department of Mathematics, Indian Institute of Technology, Kanpur-208016, India. On generators, cogenerators, faithful and cofaithful in $\mathbb{M}$. Preliminary report.

Anderson and Fuller defined a $*$-faithful module as one which generates a cogenerator (dualizing the property that a module is faithful if and only if it cogenerates a generator) and asserted that a $*$-faithful module is faithful. In this note it has been proved that any faithful module generates a cogenerator and hence the two notions coincide. Also it has been observed that every cogenerator of $R$ is cofaithful if and only if $R$ is finitely cogenerated but every cofaithful is a cogenerator if and only if $R$ itself is a cogenerator, which is also a necessary and sufficient condition for every generator to be a cogenerator. The converse of this last condition is a strictly stronger property in the sense that every cogenerator is a generator if and only if $R$ is an injective cogenerator (hence also finitely cogenerated). But it has also a characterizing property dual to its converse, viz, the minimal cogenerator $C_0 = \oplus E(V_1)$ is a generator where $V_1$ ranges over
all nonisomorphic simple modules and \( E(V_i) \) denotes the injective envelope of \( V_i \).

(Received February 9, 1976.) (Author introduced by Dr. O. P. Kapoor.)

Free lattices in some locally finite varieties.

We have developed a method of determining some large free lattices within reasonable amounts of computer time. Let \( F_{n_5}(n) \) (resp. \( F_{n_k}(n) \)) denote the free lattice on \( n \) generators in the variety generated by the non-modular pentagon (resp. by the \( k \)-element lattice of length 2). It is known that \( | F_{n_5}(3)| = 99 \) and that \( | F_{n_k}(3)| = 28 \) for \( k \geq 5 \). We show that \( | F_{n_5}(4)| = 19982 \), \( | F_{n_k}(4)| = 56694 \) and that \( | F_{n_k}(4)| = 540792672 \). These results are obtained by considering the canonical mapping onto the free distributive lattice on \( 4 \) generators and determining the cardinalities of the 166 congruence classes.


Professor R. Padmanabhan has given an interesting axiomatic characterization of semi-lattices. (Reference. Canad. Math. Bull. 9(1966), pp. 357-358). In this paper we generalize his theorem for \( C-n \) semi-groups, where \( n \) is a positive integer greater than or equal to 2 and give an axiomatic characterization of \( C-n \) semi-groups. Definition. A semi-group \((S,*)\) is a \( C-n \) semi-group if \( S \) is commutative and \( x*x*x*...n \) times = \( x \). Note that \( C-2 \) semi-group is a semi-lattice and vice versa. We prove the following theorem. Theorem. A set \( S \) with a binary operation \( * \) defined in it is a \( C-n \) semi-group if and only if the following axioms hold:

1. \( S \) is power associative w.r.t. elements of \( S \) i.e. \( x*(x*x) = (x*x)*x \).
2. \( x*x*x*...n \) times = \( x \) \((n \geq 2)\).
3. \( (x+y)*z = (y+z)*x \).

(Received January 8, 1976.) (Author introduced by Professor G. Grätzer.)

Analysis

Almost automorphic solutions of some abstract evolution equations.

In this paper we discuss vector-valued almost-automorphic functions (in the sense of Bochner) and give several results concerning almost-automorphic solutions of abstract evolution equations as in a recent paper by M. Zaki (Annali di Matem., 1974). (Received November 24, 1975.)

Consider the equation: \( x^{(n)} + H(t,x) = Q(t) \), where \( n \) is an even integer, \( H \) is continuous on \([T, \infty)\times(-\infty, \infty)\), increasing in its second variable and such that \( uH(t,u) > 0 \) for every \((t,u)\) with \( u \neq 0 \). Moreover, \( Q \) is continuous on \([T, \infty)\). Here \( T \) is a nonnegative number. Theorem. Let the equation \( x^{(n)} + mH(t,x) = 0 \) be oscillatory for every \( m > 0 \). Let the inequality \( x^{(n)} + H(t,x) \leq Q(t) \) have an eventually positive solution \( x_1(t) \), and the inequality \( x^{(n)} + H(t,x) \geq Q(t) \) have an eventually negative solution \( x_2(t) \) such that

\[
\lim_{t \to \infty} x_1(t) = \lim_{t \to \infty} x_2(t) = 0.
\]

Then (\( * \)) is oscillatory if the same is true for the homogeneous equation. A similar theorem holds for the bounded solutions of (\( * \)). The existence of the function \( x_1(t) \) \((x_2(t))\) as above ensures the nonexistence of negative (positive) solutions of (\( * \)). Several other results are given concerning the behaviour of nonoscillatory solutions of (\( * \)) under certain conditions on certain intermediate antiderivatives \( P(t) \) \((P^{(j)}(t) = Q(t), 0 < j < n)\) of the function \( Q(t) \). These results suggest further oscillation criteria. (Received December 11, 1975.) (Author introduced by Professor M. N. Manougian.)
We have proved the following theorem: A necessary and a sufficient condition for a function $B$ to conserve the asymptotic equality, i.e. to have the property that $g(x) \equiv f(x)$ (i.e. $g(x)/f(x) \to 1$, $x \to \infty$) implies $B(g(x)) \subseteq B(f(x))$ when $x \to \infty$ for every couples of functions $f$ and $g$ such that $f(x) \to \infty$ when $x \to \infty$, is that $s(x) = \ln B(x)$ will be a slowly oscillating function in the sense of Schmidt, i.e. $s(y) - s(x) + o(1)$ for every $y > x$ such that $y \notin x (x = \infty)$. Denoting now by $S$ the set of the functions which conserve the asymptotic equality, and by $R$ the set of regularly varying functions in the sense of Karamata, i.e. $r \in R$ iff $\lim_{x \to \infty} (r(x)/r(x))$ exists for every $x > 0$.

we have proved that $R$ is a proper subset of $S$. (Received January 8, 1976.)

**Recent results on special classes of p-valent functions.**

Let $V_k^p(x)$ ($k \geq 2$, $|\lambda| < \pi/2$, $p \geq 1$) denote the class of functions $f$ analytic in $\{|z| < 1\}$ having (p-1) critical points there and satisfying

$$\lim_{r \to 1^-} \int_0^{2\pi} |f(r^{1/p}(\lambda))| d\lambda = k \pi p \cos \lambda.$$

From $V_k^p(x)$, we can obtain many interesting known subclasses including the class of functions of bounded boundary rotation and the class of p-valent functions $f(z)$ for which $z f'(z)$ is $\lambda$-spiral-like. In the present paper, the results obtained for $f \in V_k^p(x)$ include a domain of values for $1 + z f'(z) f(z)$, a distortion theorem for $Re f^{(p)}(z)/f(z)$, and the Hardy classes to which $f'$ and $f$ belong. (Received January 12, 1976.)

**The set of totally ambiguous points of planar functions.** Preliminary report.

Let $f$ be a function from the plane to the complex sphere. Denote the range of $f$ by $R(f)$. A point $f$ is a point of ambiguity of $f$ with $\Theta \in [0, \pi]$ as direction of ambiguity if there are two linear arcs, $\Lambda_1$, and $\Lambda_2$, at $f$ with direction $\Theta$ and $-\Theta$, resp., such that $Cl(\Lambda_1, S) \cap Cl(\Lambda_2, S)$ is empty. A linear arc at $S$ consists of a line segment terminating at $S$ and $Cl(\Lambda_1, S)$ is the cluster set of $f(z)$ at $S$ along the arc $\Lambda_2$. Let $T(f)$ be the set of points of the plane that have every direction as a direction of ambiguity of $f$. Let $fA$ if every point of the plane is an ambiguous point of $f$ with some direction $\Theta$ as direction of ambiguity. Theorem. If $fA$ such that $|T(f)| = 2^{1/2}$. Corollary 1. If $fA$ such that $|T(f)| = 2^{1/2}$ and $|R(f)| = \infty$. Corollary 2. If $|T(f)| = 2^{1/3}$, then $fA$ such that $|T(f)| = 2^{1/2}$ and $|R(f)| = 2$. (Received January 12, 1976.)
A study of a spectral synthesis theory for Banach modules over certain commutative Banach algebras is considered. The spectrum of an element (belonging to the module) is defined as the hull of its annihilator ideal. Fundamental properties of spectra are used to develop a spectral synthesis theory in the spirit of the spectral synthesis theory of bounded functions. Sets of synthesis for Banach modules are defined and related spectral problems are formulated, notably (1) a spectral synthesis problem, (2) spectral analysis problem, and (3) closure problems. The impact of the particular topology imposed on the module is apparent in a duality condition introduced: bi-annihilation invariance. This notion is analogous to Kaplansky's dual ring concept. The study includes extension of the notions of "local membership" and "Wiener-Ditkin condition" to a module context. The investigation culminates in a Banach module formulation of a "Wiener-Ditkin-Shilov Theorem" and applications to almost periodicity. In particular, Loomis' characterization of almost periodic elements and a theorem of Beurling regarding convolution equations is obtained for a wide class of Banach modules. (Received January 16, 1976.)
For a bounded linear operator $T$ on a complex Hilbert space let $\{T\}$, $\{T\}^\circ$ and $\text{Alg } T$ denote the commutant, the double commutant and the weakly closed algebra generated by $T$ and $1$, respectively. Assume that $T$ is a completely non-unitary contraction with a scalar-valued characteristic function $\Phi(e^{it})$. In this note we show that the condition that $|\Phi(e^{it})|=1$ on a set of positive Lebesque measure implies that $\text{Alg } T = \{T\}'$. Moreover, if $\Phi(e^{it})$ is assumed to be outer, then these two conditions are equivalent.

We also show that in the latter case $T$ has the same lattice of hyperinvariant subspaces as that of the bilateral shift on $L^2(E)$, where $E = \{ e^{it}, |\Phi(e^{it})| < 1 \}$. Our main result generalizes the well-known fact, due to D. Sarason, that the compression of the shift satisfies $\text{Alg } T = \{T\}'$. (Received January 26, 1976.)

Let $G$ be a non-discrete LCA group, and let $B$ be a Segal algebra on $G$. We prove two results: (1) If $B$ has weak factorization, then $B \subseteq \bigcup_{\alpha \in \mathbb{R}} \mathbb{L}^p(G)$; and (2) if $B$ is a character Segal algebra and if $B$ has weak factorization, then $B = \mathbb{L}^1(G)$. These results had previously been proved under the additional hypothesis that $G$ is compact. (Received January 29, 1976.)

Our main purpose in this paper is to prove characterisations of strict domination of sequences in locally convex spaces, for instance we have Theorem:

Let $\{x_n\}$ and $\{y_n\}$ be sequences respectively in two l.c. TVS's $X$ and $Y$ such that $Y$ is complete and sp $\{x_n\}$ is bornological. Then $\{x_n\}$ strictly dominates $\{y_n\}$ iff to each $g \in \{y_n\}^*$ there exists a unique $f \in \{x_n\}^*$ such that $f(x_n) = g(y_n)$, $n \geq 1$. After proving that each non-zero sequence in a complete l.c. TVS is equivalent to a Schauder base in a certain sequence space, we derive a few results concerning the Schauder character of topological bases in l.c. TVS's. (Received January 29, 1976.) (Authors introduced by Professor J. N. Kapur.)

A mapping $T:K \to X$, $K$ a convex subset of a 2-normed space $(X, \| \cdot \|, \| \cdot \|)$, is non-expansive if $\|T(x)-T(y)\| + \|z\| \leq \|x-y\| + \|z\|$ for all $x, y \in K$ and all $z \in X$. A set $L = \{x_1 + \alpha x_2 : x_1, x_2 \in X, \alpha \in \mathbb{R} \}$ is called a line. Theorem. Let $K$ be a convex set which contains at least two elements and is not a subset of a line. Then, $T$ is non-expansive if and only if there is a $\alpha \in \mathbb{R}$ and there is a point $z_0 \in X$ such that $\|z_0\| = 1$ and $T(x) = \alpha x + z_0$, for every $x \in K$. If $K \subseteq L$ with $x_1 \in K$ and $L = \{ \alpha : x_1 + \alpha x_2 \in K \}$, then $T$ is non-expansive if and only if there is a function $g: A \to \mathbb{R}$ such that for every $\alpha, \gamma \in A$, $\|g(\alpha) - g(\gamma)\| \leq \|x_1 - x_2\|$, $g(0) = 0$, and $T(x_1 + \alpha x_2) = g(\alpha)x_2 + T(x_1)$. See Diminnie, Gähler, White (Math Nachr. 59(1974)p319-324) and Iséki (Math. Sem. Notes, Kobe University V3 no.1 (1975)) for definitions. (Received February 2, 1976.)

The first six chapters of this book deal with the theoretical background of multiple hypergeometric functions and the final two chapters give a number of examples of their occurrence in the field of statistics and in a variety
of areas of theoretical physics. This is the first book in English to devote more than a few sections to the topic in question, and is an attempt to collect and unify a large number of results scattered throughout the literature together with results of the author's own researches. The chapter headings are: 1. Introduction. Single and double hypergeometric functions. 2. The Lauricella functions. 3. Other hypergeometric functions of several variables. 4. Multiple hypergeometric transformations. 5. Systems of partial differential equations. 6. Generating relations and recurrence relations. Analytical continuation. 7. Statistical applications. 8. Examples of other applications. In addition to the fruitful nature of multiple hypergeometric theory, this work is motivated by the fact that the advent of big computers has presented the opportunity actually to use these complicated functions in a numerical environment. To appear 1976. Publisher: Ellis Horwood Ltd., Chichester, Sussex, United Kingdom. (Received February 2, 1976.)

GIANCARLO MAUCERI, University of Genoa, Genoa, Italy. Square integrable representations and the Fourier algebra of a locally compact unimodular group.

Let \( G \) be a locally compact unimodular group and denote by \( A(G) \), \( B(G) \), \( VN(G) \) the Fourier algebra the Fourier Stieljes algebra and the von Neumann algebra of \( G \) respectively. Let \( A_d(G) \) be the closed subspace of \( A(G) \) generated by the coordinates of the square integrable irreducible representations of \( G \). Theorem 1. \( A_d(G) \) is contained in \( L^2(G) \) if and only if the formal degrees of the square integrable representations of \( G \) are bounded away from zero. Theorem 2. Let \( G \) be a group such that \( A_d(G)=A(G) \), then \( G \) has no infinite discrete subgroups and for every \( x \in G \) there is a compact open subgroup of \( G \) containing \( x \). Theorem 3. If \( G \) is a locally compact group such that its dual space is discrete, then \( G \) is compact. The latter result was proved by L. Bagget assuming \( G \) separable and by G.A. Elliott assuming that \( G \) is type I. The following example is due to Fell: let be \( G \) the semidirect product \( K \otimes \mathbb{N} \), of the additive group \( \mathbb{N} \) of the \( p \)-adic numbers, and the multiplicative group \( K \) of the \( p \)-adic numbers of valuation one, acting on \( \mathbb{N} \) by multiplication, then \( A_d(G)=A(G) \). For this group we have. Theorem 4. \( B(G)=A(G) \Leftrightarrow AP(G) \), where \( AP(G) \) is the Banach involution algebra of almost periodic functions on \( G \). In particular \( u \in B(G) \) vanishes at infinity if and only if \( u \in A(G) \). Theorem 5. The natural Dirichlet kernel is a sumability kernel for \( L^p(G) \), \( 1 < p < \infty \) but not for \( L^1(G) \). (Received February 3, 1976.) (Author introduced by A. Figà-Talamanca.)

Ira W. Herbst, Princeton University, Princeton, New Jersey 08540 and Alan D. Sloan, Georgia Institute of Technology, Atlanta, Georgia 30339. An Approximation Theorem for Singular Perturbations.

The unperturbed operators \( K \) on \( L^2(\mathbb{R}^n) \), are of the form \( F^{-1} \), where \( F \) is the Fourier transform and \( M \) is the selfadjoint operator given a multiplication by a non-negative, continuous function \( k \) such that \( \exp(-tk) \) is integrable and is also positive-definite. The perturbations \( Z \), are the form sum, \( V+\cdot \cdot \cdot \cdot \cdot \), of a non-negative function \( V \), which has dense form domain intersection with the form domain of \( K \) and of a Hermitian form \( W \), which agrees with a distribution and which is a small form perturbation of \( K \). Examples of \( K \)'s and \( Z \)'s are given. Theorem - Let \( H \) be the form sum of \( K \) and \( Z(n) \) converges strongly to those of \( H \). As an immediate corollary we show that \( \exp(-tH) \) is positivity preserving. We further show that \( \exp(-tH) \) is an integral operator with a non-negative kernel which is locally \( p \)-integrable for certain \( p \). Regularity properties of eigenfunctions are investigated. For example, it is shown that all square integrable eigenfunctions of \( H \) are \( p \) integrable for certain \( p \). For certain \( K \)'s, including those representing the relativistic and non-relativistic energy of a free particle, \( \exp(-t(K+(Z/\gamma))) \) is shown to be a Carleman operator and functions \( w \) are found so that the generalized eigenfunctions are square integrable with respect to \( w \cdot dx \) where \( \gamma \) denotes Lebesgue measure. In particular, if \( K \) is minus Laplacian \( w \) may be \( w(x)= \left(1+ \|x\|^2 \right)^{-1} \) for \( \|x\|>N \). (Received February 6, 1976.)

FRANK N. HUGGINS, University of Texas at Arlington, Arlington, TX 76019. A property of functions having bounded slope variation.

If \( m \) is an increasing function on \( [a,b] \), an \( m \)-polygonal function on \( [a,b] \) is a generalization of a polygonal function obtained by replacing straight line segments with arcs of the curve \( y = m(x) \). Theorem. If the function \( f \) has bounded slope variation with respect to \( m \) over \( [a,b] \), there exists an in-

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finite sequence \( \{a_n\} \) of m-polygonal functions on \([a,b]\) which converges to \(f\) on \([a,b]\) (if \(f\) is continuous on \([a,b]\), the convergence is uniform.) and such that as \(n \to \infty\), the infinite sequences \( \{v^b_a \frac{(d a_n)}{d m}\} \), \( \{v^b_a (a_n)\} \), \( \{v^b a \frac{(d a_n)}{d m}\} \) and \( \int_a^b \frac{(d f)}{d m} \frac{(d a_n)}{d m}\) converge respectively to \(v^b_a \frac{(d f)}{d m}\), \(v^b_a (f)\), \( \int_a^b \frac{(d f)}{d m}\) and \( \frac{(d f)}{d m}\). (Here, for example, \(v^b_a (\frac{(d f)}{d m})\) is the slope variation of \(f\) with respect to \(m\) over \([a,b]\), \(v^b_a (t)\) is the total variation of \(f\) on \([a,b]\), \(\int_a^b \frac{(d f)}{d m}\) is the mean Stieltjes integral.)

(Received February 6, 1976.)

*76T-B75* WILLIAM D. L. APPLING, North Texas State University, Denton, Texas, 76203. *A Nonintegrable Bochner-Radon-Nikodym Theorem for the Lipschitz Case.*

Setting and notions are as in previous abstracts of the author. Suppose \(A\) is in \(p_B\), \(m\) is in \(p^*_1\), \(D\) is a subdivision of \(U\), and for each \(V\) in \(D\), \(a(V)\) is in \(A(V)\). Very elementary considerations involving the Hellinger integral and Schwarz's Inequality imply the following: Theorem. If \((W,Q)\) is \((\max,L)\) or \((\min,G)\) and \(h = \int Q(Am)\), then \(\sum D l h(V) - \int W h(I) a(V) m(I) \leq \int h(U) - \sum D Q(Am) h(I)\) \(+ \{\int h(U) (\frac{1}{m(I)} - \sum D Q(Am))\}^{1/2} (m(U))^{1/2} \). Corollary. If, for each \(V\) in \(D\), \(E(V)\) is a subdivision of \(V\) and for each \(I\) in \(E(V)\), \(a(I)\) is in \(A(I)\) and \(f = \int L(Am)\) and \(g = \int G(Am)\), then \(\sum D \sum E(V) a(V) - a(I) m(I) \leq 2\{\sum D L(Am) - G(Am)\} - \{g(U) - f(U)\} + \{\int f(I) \frac{1}{m(I)} - \sum D E(V) \frac{1}{m(V)}\}^{1/2} + \{\int G(E(V)) \frac{1}{m(V)} - \sum D G(V) \frac{1}{m(V)}\}^{1/2}\). (Received February 9, 1976.)

76T-B76 A. P. BLOZINSKI, Ball State University, Muncie, Indiana 47306. *The modulus of continuity of a function and \(L(p, q)\) spaces.* Preliminary report.

The function \(f(x)\) is Lebesgue measurable, complex valued and defined on \((-\infty, \infty)\). \(f^*(t)\) is the non-increasing rearrangement of \(f(x)\) on \((0, \infty)\) and \(f^{**}(t) = (1/t) \int_0^t f^*(u) \, du\). Define: \(\Delta f(x,t) = [f(x + t) - f(x)]\). The \(L^p\)-modulus of continuity of \(f(x)\) is \(\omega_p f(t) = \sup_0 < \chi \Phi_f \sup_{0 < \chi < t} \int_0^t \Delta f(x,t) \, dx\). Put: \(F(t) = \sup_0 \int_0^t \frac{f^*(t)}{t^p} \, du\). Then \(F(t) \leq t^{-1/p} \omega_p f(t)\). We have as our primary Theorem. For \(t > 0\) and \(\lambda > t\), \(f^{**}(t) \leq 2(F(t) + \frac{1}{\lambda} F(\lambda) ds/s) + (1/\lambda) \int_0^t f^*(u) \, du\). Consequences. The expression on the right in the theorem is a function norm which is equivalent to the function norm \(f^{**}(t)\) with respect to the \(L(p, q)\)-spaces of Lorentz. The Theorem has applications to the modulus of continuity of a function, Lipschitz spaces and the Lorentz \(L(p, q)\)-spaces. (Received February 10, 1976.) (Author introduced by Professor Frank W. Owens.)

76T-B77 YAU-CHUEN WONG, The Chinese University of Hong Kong, Hong Kong. *On a theorem of Grothendieck.*

Let \((E, \Phi)\) be a locally convex space with the topological dual \(E'\). Denote by \(\beta^*(E, E')\) (resp. \(\beta(E, E')\)) the topology on \(E\) of uniform convergence on \(\beta(E, E')\)-bounded (resp. \(\sigma(E, E')\)-bounded).
Theorem: If $\{(x,y)|Ax + By \geq d, x \geq 0, y \geq 0\}$ is bounded and non-empty, then any valid cutting-plane for the complementarity constraints $Ax + By \geq d, x \geq 0, y \geq 0$, is obtained by starting from the linear defining inequalities $Ax + By \geq d, x \geq 0, y \geq 0$, and applying, finitely often, the following two rules (the second for $j = 1, \ldots, n$): (i) Take non-negative combinations of given inequalities, and possibly weaken the right-hand-side; (ii) Having already obtained two inequalities $\alpha_1 x_1 + \ldots + \alpha_n x_n + \beta_1 y_1 + \ldots + \beta_n y_n \geq \alpha_0$ and $\alpha_1' x_1 + \ldots + \alpha_n' x_n + \beta_1' y_1 + \ldots + \beta_n' y_n \geq \alpha_0'$, deduce $\alpha_1 x_1 + \ldots + \alpha_n x_n + \beta_1 y_1 + \ldots + \beta_n y_n \geq \alpha_0$. Conversely, any inequality thus obtained is valid for the complementarity constraints. (The proof of this result is constructive, and it supplies a finitely-convergent cutting-plane algorithm for this generalization of the linear complementarity problem. (Received January 13, 1976.)

Applied Mathematics

Goal programming is designed to optimally attain conflicting goals for the case in which some goals have higher priority than others. We develop an iterative approach for solving general goal programming problems. This approach is not only applicable to linear systems (which have received considerable attention) but also to nonlinear and integer goal programming problems. A dual for linear problems is developed and methods of sensitivity analysis with respect to changes in goals and constraints are considered. It is also shown that the iterative approach makes possible new modeling techniques in goal programming, such as least squares attainability of goals and priority dependent constraints. (Received January 13, 1976.)

*76T-C23 Jerald F. Dauer and Robert J. Krueger, University of Nebraska, Lincoln, Nebraska 68588. An Iterative Approach to Goal Programming.


It is known from experiments that the wake behind a circular cylinder in the flow of a viscous incompressible fluid, becomes unstable after a critical Reynolds number. A numerical study of this phenomenon has been made at Reynolds numbers 100 and 200, under natural boundary conditions by using a numerical method in which a very fine finite difference mesh is constructed in the neighbourhood of the cylinder and a coarse one for away from it. The numerical results have been presented through a series of the figures showing stream lines and equivorticity lines at different times. The figures are also drawn to exhibit the variations of the flow characteristics like the pressure distribution, the vorticity distribution, the frictional drag, the pressure drag and the total drag coefficients on the surface of the cylinder. A horizontal S-shaped pattern of the streamlines is seen in the wake with the start of the process of the vortex shedding. The computed Strouhal number is found to be in good agreement with the empirical results. (Received January 21, 1976.)

*76T-C25 PADAM C. JAIN and B.S. GOEL, Indian Institute of Technology, Powai, Bombay-40007, INDIA. Shedding of Vortices Behind a Circular Cylinder

A nonlinear mathematical model for the propagation of tides in interlacing channels is presented. The problem is solved with the help of a high speed digital computer using the
explicit finite difference method with leap-frog operator. A grid scheme is developed to simulate the propagation of tides in the confluences of the channels. It is shown that the new grid scheme can incorporate any number of junctions of a single river as well as the junction of any number of tidal rivers. The model is studied both for the proving stage as well as for application to the interaction between the incoming tide from the downstream end and abnormal freshet discharges from the upward ends of the different tributaries. It is shown that the computational results are in good agreement with the data observed in the model. (Received February 9, 1976.)


By an unified simple treatment theorems for deciding whether a code is uniquely decipherable, decipherable with a finite delay or synchronizable are provided. (Received February 10, 1976.)

Geometry

DAVID FOLEY, Math. Dept., Illinois Institute of Technology, Chicago, IL 60616

Projective Planes and Superassociative Algebras.

Let \( \pi \) be a projective plane, \((0,1,U,V)\) a quadrangle, \( j = 01 \) and \( \omega = (01)(UV) \). For any points \( P \) and \( Q \) on \( j \), not both \( \omega \), write \( P^*Q = (PV)(QU) \). For any point \( A \) on \( j \) the projective substitution of \((B,0)\) into \( A \) is defined to be the point \( A(B,0) \) into which \( A \) is mapped by the projectivity 
\[
  j + CU \rightarrow B\stackrel{C}{\rightarrow} j \quad \text{with centers} \ V, \quad 0\stackrel{C}{\rightarrow} D \quad \text{and} \quad j + BU \rightarrow 0V \quad \text{with centers} \ V, \quad 1\stackrel{C}{\rightarrow} D \quad \text{and} \quad U \text{if} \ C \neq \omega, \quad \text{and} \quad j + BU \rightarrow 0V \quad \text{with centers} \ V, \quad 1\stackrel{C}{\rightarrow} D \quad \text{and} \quad U \text{if} \ C = \omega.
\]

The superassociative law reads:
\[
  A(B,C,D)(E,F,G) = A(B(E,F,G), C(E,F,G), D(E,F,G))
\]

Seall [J. Geometry 4(1974), 11-33] has characterized Pappus, Desargues and U,V-Desargues planes in terms of the superassociative law. Theorem 1. \( \pi \) is a U,V-Pappus plane [Burn, Math. Z. 105(1968), 351-364] iff \( A(0,1,0)(0,F,1) = A(0,C(F,1), 0) \) for all \( A,C,F \) on \( j \). Theorem 2. \( \pi \) is a little Desargues plane iff
\[
  1) \quad A(B,C,F)(E,F,A)(B,C,F) = A(B(E,F,A), C(E,F,A), D(E,F,A)),
  
  2) \quad A(0,1,0)(0,F,1) = A(0,F,0) \quad \text{and}
  
  3) \quad A(0,1,0)(0,F,1) = A(0,0,1) \quad \text{for all} \ A,B,C,D,E,F \ \text{on} \ j.
\]

(Received January 30, 1976.) (Author introduced by Robert Seall.)

Logic and Foundations


Def. 1. \( F \) denotes free lattice logic, modeled by \( H \), matrix algebra (NOTICES, 22(1975), A647), with juncture (NOTICES), 23(1976), A-8); \( H_1 \), universal upper, lower bounds; \( G \), sublogic of \( F \), restricted such that precedent in juncture is never \( L; \ (F) \), statement logic isomorph of \( F(G) \).

Th. 1. \( G \) represents minimal statement logic; the image in \( T \) of \( G \)-juncture is strict implication, and minimally satisfying. Def. 2. \( F(M) \)-juncture (juncture): \( e \) true (false) in all models; \( F(M) \)-contingent, iff true (false) in some model, its juncture (juncture) \( S \)-necessary (impossible), if true (false) in some model, and, in some \( S \)-sublogic, its juncture (juncture) is relatively \( S \)-necessary (impossible). Def. 3. \( F(M) \)-Modos: An element of \( F(M) \) is \( F(M) \)-necessary (impossible) if it maximizes (minimizes) \( F(M) \), \( F(M) \)-contingent, if maximizing (minimizing) some sublattice (subalgebra), and, in \( F(M) \), its juncture (juncture) is \( F(M) \)-necessary (impossible), \( F(M) \)-possible, if maximizing (minimizing) some sublattice (subalgebra), and, in some sublattice (subalgebra), its juncture (juncture) is relatively \( F(M) \)-necessary (impossible), properly, improperly, according to subsystem. Th. 2. The \( F(M) \)-modes are isomorphs of the \( G \)-modes. Deontic logic (modes: obligatory, forbidden, meritorious, permitted) can be interpreted in modal logic. Th. 3. Modal and deontic logic find representation in \( F(M) \).

(Received January 7, 1976.)
Invariant sentences were defined in these $\mathcal{N}$ote$\mathcal{C}$, 22(1975), A-473. Theorem 1. If $R$ is any fixed ring, then two topological $R$-modules satisfy the same invariant sentences if and only if they satisfy the same invariant sentences with only one alternation of individual quantifiers. Theorem 2. If $n \in \omega$ or $n = \omega$, let $T_n$ be the invariant theory of the class of real topological vector spaces of dimension $n$ whose topologies are induced by norms. Then $T_n$ is complete and recursively axiomatizable for each $n \in \omega \cup \{\omega\}$. Theorem 3. Let $\mathcal{C}$ be a distinguished individual constant symbol. Then there is a recursive set $M(\mathcal{C})$ of invariant sentences such that two topological structures satisfy the same sentences in $M(\mathcal{C})$ if and only if they have ultrapowers which are locally isomorphic at $\mathcal{C}$.

(Received January 29, 1976.)


Bounded arithmetic predicates and the bounded arithmetic hierarchy.

The class of bounded arithmetic predicates (BA) is the smallest class containing the polynomial predicates and closed under quantification bounded by a simple variable $(\exists w) \leq y R(x,y,w)$ or $(\forall w) \leq y R(x,y,w)$). The bounded arithmetic predicates are a small subset of the recursively enumerable, but they include most of the standard examples from recursive function theory, and form a basis for the r.e. sets. BA is closed under Boolean operations, and quantification bounded by a polynomial, but it is not closed under quantification bounded by $x^y$. In analogy with Kleene's arithmetic hierarchy, there is a bounded arithmetic hierarchy of predicate classes within BA, based on the number of alternations of bounded quantifiers. The closure properties of these classes are also studied. Although the existence of a strict hierarchy is not shown, necessary and sufficient conditions for the hierarchy to be strict are established. (Received January 28, 1976.)

Iraj Kalantari and Allen T. Retzlaff, Cornell University, Ithaca, New York 14853.

Maximal vector spaces and automorphisms.

We reveal more contrast between $\mathcal{A}$ and $\mathcal{L}(V_\omega)$. Let $V_\omega$ be a recursively presented infinite dimensional vector space over a recursive field $F$ with a dependence algorithm. Let $\mathcal{L}(V_\omega)$ be the lattice of r.e. subspaces of $V_\omega$. (See these $\mathcal{N}$ote$\mathcal{C}$, April 1975, Abstract 723-E6.) For $A \in \mathcal{L}(V_\omega)$ let $\mathcal{H}(A) = \{X \in \mathcal{L}(V_\omega) | A \subseteq X \}$.

Theorem 1. If $A \in \mathcal{L}(V_\omega)$ and $A$ has an r.e. basis extendable to an r.e. basis of $V_\omega$, then $\mathcal{H}(A)$ is complemented. Corollary 1. There exists a maximal space $M$ such that $\mathcal{L}(M)$ is complemented. This follows since in G. Metakides-A. Nerode, R.E. Vector Spaces (to appear) a maximal space with an r.e. extendable basis was constructed, and furthermore, in the case that $M$ is maximal, $\mathcal{L}(M) = \mathcal{H}(M)$. Definition. Given $A \in \mathcal{L}(V_\omega)$, we say $\mathcal{L}(A)$ is k-uncomplemented if (i) $[V \text{ is r.e.}, \wedge A \subseteq V \wedge \text{ dimension of } V/A > k] = V$ is not complemented in $\mathcal{L}(A)$, (ii) $k$ is smallest such number. Theorem 2. For any $k > 0$, $k \in \omega$, there exists a maximal space $M_k$ such that $\mathcal{L}(M_k)$ is k-uncomplemented.

Corollary 2. There are $\aleph_0$ distinct maximal spaces, no pair of which is exchangeable by an automorphism of $\mathcal{L}(V_\omega)$. This result accentuates the contrast between $\mathcal{A}$ and $\mathcal{L}(V_\omega)$ as R.I. Soare has shown that any pair of maximal sets can be exchanged by an automorphism of $\mathcal{A}$. (Received January 30, 1976.)

Suppose that $\Sigma$ is a class of $L$-structures and $\Sigma$ satisfies the following five algebraic conditions: (A) $\Sigma$ is closed under isomorphism (B) $\Sigma$ is closed under substructure (C) $\Sigma$ is closed under ultraproducts (D) if $A \in \Sigma$ has power $\leq n$ then $A$ has $\leq \aleph_n$ finitely generated extensions in $\Sigma$ (E) $\Sigma$ has the amalgamation property. Then there is a structure theory defined on the class of open formulas over structures $A$ in $\Sigma$ which corresponds via the following dictionary to the structure theory of basic
algebraic geometry:  
(A) Rank ↔ Dimension  
(B) Open formula ↔ Affine Variety  
(C) Degree ↔ Number of Irreducible Components of the Same Dimension over the  
Algebraic Closure  
(D) Open Type ↔ Prime Ideal in the Polynomial Ring  
(E) Independent Point ↔ Generic Point  
(F) Independent Subsets ↔ Free Subsets of Generic Points.  

Received February 2, 1976.)


A model-theoretic framework for the study of empirical theories is described. Although the models considered are models of two or more formal languages \( K_i \), they are characterized independently of any particular one of them. This is accomplished by taking the models to be models of set theory and by interpreting the constants, relations, functions, and sorts as fixed elements of the set-theoretic universe. The intention is to think of the languages \( K_i \) as formal scientific languages. The language of set theory, \( \epsilon \), is added so that statements in any \( K_i \) as well as extra-linguistic observations (sentences in the \( \epsilon \)-diagram of the models) can be expressed in some neutral formal language. Formal analogues of the theses of Kuhn and Whorf are derived as well as further model-theoretic applications. (Received February 4, 1976.) (Author introduced by Professor Louis Raymon.)

*76T-227  MATATAHU RUBIN The Hebrew Univ., Jerusalem, Ben Gurion Univ., Beer Sheva. 
SAHARON SHELAH The Hebrew Univ., Jerusalem, Israel. Downward Skolem Lowenheim theorems for automorphism groups of Boolean algebras.

For a model \( M \) let \( \text{Aut}(M) \) be the group of automorphisms of \( M \), and \( M^* = (M, \text{Aut}(M)) \) (with the relation \( \{<h,a,b> : f \in \text{Aut}(M), f(a) = b, \text{and the relations of } M.\} \) (we can replace below automorphisms by complete endomorphisms.) Theorem 1: (CH) Let \( B \) be an infinite Boolean algebra. Then there are \( 2^{N_1} \) non-isomorphic Boolean algebras \( B_1, B_2 \) such that \( B_1, B_2 \) are elementarily equivalent. Theorem 2: (CH) Let \( B \) be an atomic Boolean algebra which is of the second category (as a family of subsets of its set of atoms) \( \psi \in \text{L}_{\text{omega}1}\text{omega}1 \). Theorem 3: (CH) Let \( B \) be a free Boolean algebra of cardinality \( N_1 \). Then there are \( 2^{N_1} \) non-isomorphic \( B_1, B_2 \) of cardinality \( N_1 \) satisfy \( \psi \). The theorems are complementary to results of M. Rubin announced in these Notices Vol. 21, 5, 76T-E64 and Vol. 22, 6, 76T-E63. We in fact prove that portions of set theory have non-standard models with standard concepts. Sufficient conditions on theories to have models with "inner" automorphisms and transfer theorems to other cardinals will appear subsequently. (Received February 9, 1976.)

*76T-228  MIRIAM LIPSCHUTZ-YEVICK, Rutgers The State University, University College, New Brunswick, New Jersey 08903. On the complementarity between sequential and holographic identification (denotation).

Given a white-on-black pattern on a finite transparency, i.e. an "object" \( A(x, y) = 1 \), \((x, y) \in S = 0 \) elsewhere; \( |x|, |y| < M \) and a collection of such objects among which one wishes to recognize \( A \). Recognition can proceed sequentially or holographically; the first method matches the description within a fixed mesh, of the sequential (binary) representation of the object, the second projects the description over this mesh and a complex object is identified by its hologram up to parameters \( \epsilon, \eta, \delta \) by the correlation spot yielded by holographic filtering. This is of relevance to the discussion of Quine, Ontological Relativity, Columbia University Press 1969, p. 55. (Received February 4, 1976.)

A-367
Statistics and Probability

JIN Y. CHANG, Case Western Reserve University, Cleveland, Ohio 44106. Theory of dams with inputs forming a sequence of dependent random variables.

This dissertation considers the dams of infinite capacity in which the inputs form a Pólya interchangable random sequence and a stationary (moving average) sequence and the demand is continuous at a constant rate. The main object is to determine the distribution of the dam content. In the case where the inputs form a Pólya sequence, explicit solutions are found for the distribution of the dam content for any constant release exceeding the mean input. An extension of Takacs' (1975) results for a semi-Markov input to a moving average input is given by using the methods of matrix factorization. This approach provides a unification of the solutions for different patterns of inputs. Its difficulty lies in factorizing the resultant matrix into two, each of which must satisfy the properties of having inverse, regularity, boundedness and continuity in a suitable domain. Toward this end, we provide two algorithms for the factorization of matrices confronted in the solution of discrete dam problems with demonstrations. (Received January 9, 1976.) (Author introduced by Professor Lajos F. Takács.)

A Probability and Game Theoretic Example of a Set of Second Category

Theorem. The set of all real numbers of the unit interval determined by all sequences of zeros and ones for which there is a subsequence of the sequence of "averages" whose limit is 1/2 is a set of second category. (Received February 5, 1976.)

Topology

J.M. van Wouwe, Free University, Amsterdam, The Netherlands. Finite-to-one, open preimages of $\sigma$-spaces.

Theorem 1: If the finite-to-one, open preimage of a regular $\sigma$-space is a Hausdorff-space, it is a regular $\sigma$-space. In order to prove this theorem, we first show:

Theorem 2: If the finite-to-one, open preimage of a semi-stratifiable space is a Hausdorff-space, it is a semi-stratifiable space. (A somewhat stronger form of the latter theorem has been announced by J.M. Atkins [Notices 20 (1973) A.533]: in this theorem it suffices to assume the preimage to be $T_1$-space. (Received November 11, 1975.)

One point near-compactifications.

All spaces are Hausdorff and for $(X,T)$ we let $T_s$ be its semiregularization.

Lemma 1. $(X,T)$ is locally nearly-compact iff $(X,T_s)$ is locally compact. Hence, locally nearly-compact $\rightarrow$ completely regular $\rightarrow$ completely Hausdorff, almost-regular $\rightarrow$ Urysohn. Thm. 1. Let $(Y,\tau)$ be a one point near-compactification of $(X,T)$. Then $(X,T)$ is locally near-compact, $X \in \tau$, $Y - X \notin \tau$ iff $(X,T)$ is nearly-compact. Thm. 2. Let $(X,T)$ be locally nearly-compact, but not nearly-compact. Then there exist one point near-compactifications $(Y,\tau)$, $(Y^*,\tau^*)$ of $X$ such that (i) $(Y,\tau)$ is a proj. min. in the class of all extensions of $X$ in which $X$ is open, (ii) if $(Z,\sigma)$ is a one point near-compactification of $X$, then there exists an almost-continuous bijection $f:Y^* \rightarrow Z$ such that $f|X = \text{identity on } X$. Moreover, if $Z - X$ is semiregular, then $f$ is continuous on $Y^*$ and strongly $\theta$-continuous on $Z - X$. Thm. 3. A space is locally nearly-compact iff it is locally H-closed and almost-regular. Thm. 4. If $(X,T)$ is locally $H$-closed, not $H$-closed and $(X^*,T^*)$ is a one point $H$-closed extension of $X$ which is a proj. max. in its class, then $(X^*,T^*)$ is isomorphic to $(Y^*,\tau^*)$ iff $X$ is almost-regular and $X^* - X$ is a regular point in $(X^*,T^*)$. (Received December 8, 1975.)

The following questions were raised in 1969 by J. W. Ott and later repeated by B. Fitzpatrick and G. M. Reed (all spaces are regular): 1. Can each Moore space of cardinality ≤ \(2^{<\aleph_0}\) be embedded into a separable Moore space? 2. Can each first countable space of cardinality ≤ \(2^{\aleph_0}\) be embedded into a separable first countable space? The following example, which is consistent with ZFC axioms for set theory, answers both questions in the negative. Example. A (submetrizable) Moore space of cardinality continuum, which cannot be embedded into any separable first countable space.

**Theorem.** Every regularly submetrizable Moore (resp. first countable) space can be embedded into a separable Moore (resp. first countable) space. (Received December 24, 1975.)


The paper assigns to an arbitrary topological space \((X,\mathcal{T})\) an extension topological space \((X^*,\mathcal{T}^*)\), in the sense of Abraham Robinson, which is constructed by the use of ultrafilters and sequences. Then the paper investigates the interdependency between \((X,\mathcal{T})\) and \((X^*,\mathcal{T}^*)\) by using the topological properties first and second countability, Hausdorffness, separability, compactness, and connectedness. For example, the following is proved: Theorem. The topological space \((X,\mathcal{T})\) is connected iff the topological space \((X^*,\mathcal{T}^*)\) is connected. (Received January 12, 1976.) (Author introduced by Brad E. Clark.)


Overton has shown that if \((X,A)\) is a movable pair of compacta then the \(\check{C}\)ech homology sequence for the pair is exact. We use the fundamental groups defined for shape theory by Borsuk and prove the following theorem:

**Theorem:** Let \((X,A,x_0)\) be a movable pointed pair of compacta. Then there is an exact sequence

\[
\cdots \to \pi_{n+1}(X,A,x_0) \to \pi_n(A,x_0) \to \pi_n(X,x_0) \to \pi_n(X,A,x_0) \to \cdots \to \pi_1(X,x_0)
\]

where \(\pi_n(X,x_0)\) and \(\pi_n(A,x_0)\) denote the \(n\)th (shape) fundamental groups of Borsuk, and \(\pi_n(X,A,x_0)\) is the corresponding \(n\)th (shape) relative fundamental group. (Received January 12, 1976.)

SATYA DEO, Allahabad University, Allahabad, 211002, India. The cohomological dimension of an \(n\)-manifold is \(n+1\).

For a topological space \(X\), the largest integer \(n\) (or \(\infty\)) for which there exists a sheaf \(\mathcal{O}\) and a family \(\varphi\) of supports on \(X\) such that the Grothendieck cohomology group \(H^n(\mathcal{O}(X,\mathcal{O})) = 0\), is called the cohomological dimension of \(X\). Here we announce the following Theorem. The cohomological dimension of a topological \(n\)-manifold is \(n+1\). This generalizes a previous result of the author and proves his conjecture also (Proc. Amer. Math. Soc. 52(1975), 445-447). (Received January 5, 1976.)

THOMAS M. PHILLIPS, Auburn University, Auburn, Alabama 36830. Some observations on semicompletable Moore spaces.

In his doctoral dissertation at Auburn University, J. W. Ott, assuming the continuum hypothesis, proved the existence of a separable non-semicompletable Moore space. Later G. M. Reed [Proc. A.M.S. 36 (1972), 591-596] constructed a rather complicated example without the use of the continuum hypothesis. The purpose of this paper is to observe that R. W. Heath's example of a non-quasi-metrizable Moore space \(X\) [Notices A.M.S. 19 (1972), A-338] is a very simple example of this phenomenon. (As is true of the other two examples, \(X\) is not a subspace of any complete Aronszajn space.) However \(X\) is a dense subspace of a Moore space having the Baire property but \(X\) does not have a development which satisfies Reed's Axiom C* [TOP0'72, Springer-Verlag (1974), 368-384] at a dense subspace. Thus the converse of Reed's Theorem 11 [ibid.] is not true. In fact, an example is given here of a Moore space...
space which itself has the Baire property but has no development satisfying Axiom C* at a dense subspace. In a different direction, it is observed that Whipple's property [Pac. J. Math. 18 (1966), 191-199] characterizing completable Moore spaces has a natural generalization which characterizes semicompletable Moore spaces. (Received January 29, 1976.)

Using ♦ we show how one may modify a construction of A. Ostaszewski to obtain a family of perfectly normal, sequentially compact spaces whose product is not countably compact. This may be compared with the result of W.A.R. Weiss: Assume Martin's Axiom and the negation of the continuum hypothesis. Every sequentially compact, perfectly normal space is compact (so every product of such spaces is compact). By using only CH instead of ♦, the same proofs yield a family of sequentially compact spaces whose product is not countably compact. The first example of this phenomenon was announced by M. Rajagopalan and R. Grant Woods in these NOTICES 22, No. 2 (1975) A-333. (Received February 4, 1976.)


Let $d, \chi$, and $c$ denote the density, character, and cellularity of a topological space. **Theorem.** If $X$ is regular, then $d(X) \leq \chi(X)c(X)$. (Received February 9, 1976.)

**Miscellaneous Fields**

76T-H3 Gerald J. Porter, University of Pennsylvania, Philadelphia, PA 19174. Computing and Undergraduate Mathematics at the University of Pennsylvania. A survey of the interactions between computing and the undergraduate mathematics curriculum at Penn. Among the topics discussed are computer sections of freshman calculus, the major program in computational mathematics, a course on simulation methods, a course on combinatorial algorithms, the use of APL, and the use of computer generated graphics for use in the classroom. Examples of the graphics are given. (Received February 2, 1976.)

**The April Meeting in New York, New York**

**April 11 – 14, 1976**

**Algebra & Theory of Numbers**


To obtain the cyclic decomposition of modules $M$ over a ring $R$, Faith in his paper, 'When are proper cyclics injective?' Pacific J. Math. (1973), 97-112, proposed to study the class of rings with the condition:

(f): Every proper cyclic right $R$-module $R/A$ is injective modulo its annihilator ideals.

It is shown that these rings are either right Ore-domains or semi-perfect, and in the latter case, they coincide with the semi-perfect rings each of whose cyclic module $C \cong_R R$ is quasi-injective (called right PCQI-rings). (Received November 13, 1975.)

734-A2 Frank Harary, University of Michigan, Ann Arbor, Michigan 48109 and Robert W. Robinson, University of Newcastle, Australia 2308. The number of achiral trees. A nontrivial plane tree is achiral if its mirror image is the same plane tree. We enumerate achiral plane trees which are (a) planted, (b) rooted, (c) unrooted and find surprisingly that the number
of such n-point trees is the same for (a) and (c), while the answer for (b) is $2^{n-2}$.

A tree is called partially achiral if some plane embedding is achiral, and it is (totally) achiral if every plane embedding is achiral. We count both partially achiral trees and achiral trees.

(Received January 22, 1976.)

734-AS

Hans Schneider, University of Wisconsin, Madison, Wisconsin 53706. The irreducibility of matrices and determinants.

THEOREM: Let D be a commutative integral domain and let A be an n x n matrix with elements in D. Let $X = \text{diag}(x_1, \ldots, x_n)$, where $x_1, \ldots, x_n$ are distinct indeterminates. Then $\det(A + X)$ (or $\text{per}(A + X)$) is irreducible as a polynomial over D if and only if A is an irreducible matrix.

This result contains a classical result of Frobenius, in which it is assumed that all non-zero elements of the matrix A are distinct indeterminates. (Received February 9, 1976.)

734-A4

V. K. Goel, S. K. Jain, Ohio University, Athens, Ohio 45701 and Surjeet Singh, Guru Nanak Dev University, Amritsar, India. Semiprime rings with finite length w.r.t. an idempotent kernel functor.

Theorem. Let R be a semiprime ring and $\sigma$ be an idempotent kernel functor on Mod-R. Then the following are equivalent:

(i) R has finite $\sigma$-length
(ii) $\sigma(R) = (0)$, R is a right Goldie ring, and the classical quotient ring Q coincides with the ring of quotients $Q_{\sigma}(R)$.

In each case $\sigma = Z$ where Z is the idempotent kernel functor corresponding to the Goldie torsion theory. (Received February 12, 1976.)

734-A5

JOSEPH ARKIN, 197 Old Nyack Turnpike, Spring Valley, New York 10977

PAUL SMITH, University of Victoria, Victoria, British Columbia, Canada

Reversed Digit Orthogonality

We have found a new way of constructing $4n+2(n=2,3,4,\ldots)$ pairwise orthogonal squares by reversing each pair of digits on each row (for example on a certain row in a 10 x 10 square you will find the two digit number 73 as well as 37). In combination with the reversing process we have systematically constructed the square so that the numbers on each row are interrelated (mod 4n+2) (for example on a certain row in a 10 x 10 square you will find the numbers 73, 37, 28 and 82). It is also observed that the pairwise orthogonal numbers in this new construction always yield a common transversal.

We conclude the paper by extending the concept into a k dimensional "Arkin-Straus Latin k-cube" where each three dimensional cube contains a bisecting plane with a common transversal.

Throughout the paper we illustrate our purpose by constructing squares and cubes of order 10. (Received February 12, 1976.)

734-A6

EDWIN K. GORA and JAMES J. TATTERSALL, Providence College, Providence, Rhode Island 02918. An alternative to the use of the inverse of the Vandermonde matrix for the reduction of matrix functions.

A number of authors [Vidyasagar, "A novel method of evaluating $e^{At}$ in closed form," IEEE Trans. Automat. Contr. (Corresp.) AC-15 (1970), 600-601, et al.] have proposed procedures utilizing the inverse of a generalized Vandermonde matrix to reduce functions $f(A)$ of a square matrix A. A method based on the use of generalized Lucas polynomials has been developed independently for the same purpose [Gora and Tattersall, "On the reduction of matrix functions," Proc. IEEE (Lett.) 63 (1975), 1357-1358]. It is now shown that the two methods are related and complementary. One has thus a choice but the Lucas polynomial

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A continuous function \( f : G \to \mathbb{C} \) is an \( H \)-class function if \( f(\alpha \alpha^{-1}) = f(\alpha) \), \( \alpha \in G, \alpha \in H \). The space \( \mathcal{D} \) of continuous \( H \)-class functions is an algebra under convolution. Let \( \mathcal{W} \) be the center of \( \mathcal{D} \).

**THEOREM 1.** Each \( h \in \mathcal{W} \) is uniformly approximated by finite linear combinations of the spherical functions.

**THEOREM 2.** Let \( F = \frac{\phi}{\chi} \psi(1) \psi(1) \). Then \( F \) satisfies

\[
(1) \quad F(\sigma)F(\tau) = \int_{G \cap H} F(\sigma \rho \tau^{-1} \rho^{-1}) d\rho d\sigma.
\]

Conversely, if a nonzero continuous \( F \) satisfies the functional equation (1), it is of the above form for some \( \chi, \psi \), with \( \frac{\phi}{\chi}(1) \neq 0 \). (Received February 16, 1976.)
The problem solved is that of selecting $n$ subsets of the unit interval, each of measure $\alpha$ so as to minimize the maximum of the measures of the $p$-fold intersections. This is achieved by minimizing the sum of the measures of these $p$-fold intersections. (Received February 17, 1976.)

General methods are used to prove: Theorem 1: A lattice-ordered ring $R$ is isomorphic to the ring of all sections of a sheaf of totally ordered rings iff $R$ is an $f$-ring. Theorem 2: $R$ is isomorphic to the ring of all sections of a sheaf of ordered integral domains iff $R$ is (i) An $f$-ring (ii) Commutative and unitary (iii) Nilpotent free and (iv) Whenever $a^2b = b^2$ in $R$ then there is an $r$ in $R$ with $rb = ab$ and $r^2 = b$. Sheaves of integral domains and representations in other topoi are also considered. (Received February 17, 1976.)

Given a commutative topological ring $(R,T)$ we wish to construct a sheaf $\tilde{R}$ of local rings which is a local topological ring in an appropriate topos with the property that $\tilde{R}$ is isomorphic to $R$ not only as a ring but also as a topological space.

This is done by taking the whole process of localization into the topos and constructing the topology on $\tilde{R}$ as a quotient topology internally. (Received February 17, 1976.)

Let $\text{Ban}_1$ denote the category of Banach Spaces and linear maps bounded by 1.

Given a topological space, one can define a full reflective subcategory of $\text{Ban}_1$ presheaves over the space which are separable and $Q$-patching. We call this the category of $Q$-sheaves over the topological space. This category is naturally equivalent to the category of $Q$-fields or Banach Bundles over the same topological space. (Received February 17, 1976.) (Author introduced by Mr. John F. Kennison)

**Analysis**

**734-BL** PETER D. LAX, New York University, New York, N.Y. and RALPH S. PHILLIPS, Stanford University, Stanford, Calif. Scattering Theory for Automorphic Functions.

Description of recent work on the spectral theory of the Laplace operator over the fundamental domain of the Poincaré plane modulo discrete sub-group of $\text{SL}(2,\mathbb{R})$. In particular the authors have an operator theoretic proof of the meromorphic continuation of the Eisenstein series. Details will appear in a monograph in the Annals of Math Studies, Princeton. (Received January 9, 1976.)

**734-B20** Frederick Bloom, University of South Carolina, Columbia, S. C. 29208. Bounds for Solutions to Nonlinear Wave Equations in Hilbert Space with Applications to Nonlinear Elastodynamics.

For the nonlinear wave equation in Hilbert space, $u_{tt} - Nu + G(t,u,u_x) = f$, with associated homogeneous initial data, we show how an a priori bound of the form

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A C*-system (V,W) is determined by a representation T of C^N as a vector space of continuous linear transformations from V to W; the latter being separated locally convex vector spaces. Let (X,Y) have the representation S. A homomorphism (ψ,η):(X,Y)→(V,W) satisfies Tψ = S, e ∈ C^N. The space V⊗X ⊗_W Y is given the inductive topology. Let R be its subspace spanned by \{(ψ⊗_e ω', -T ω) : ω ∈ V, ω' ∈ Y, e ∈ C^N}\). Theorem. Let X, Y be finite-dimensional with bases (x_i), (y_j) and dual bases (x_i^*), (y_j^*). Then the above (ψ,η) is left-invertible iff \(\forall i, j : β_{k,l} ∈ ℂ, \sum_{i,j} (ψx_i ⊗ x_j^* + \sum_{k,l} ω_k e_{k,l} ω_l^* e_{l,j})e_{i,j} = 0\). This trace condition is used to generalize the "chain" conditions of Studia Math. 30, 273-335, Thm. 6.6 to "broken chain" conditions characterizing finite-dimensional topological direct summands in C*-systems. An example involving a C^3-system is also treated. En route, adjoint internal hom and ⊗ functors for C*-systems are defined. (Received January 19, 1976.)


For each \(α > 0\), \(A_α\) is the set of all locally integrable functions \(k\) for which \(\sup \bigg| \int_1^{1/2h} k(t)e^{itx}dt \bigg| \). Theorems 2 and 3. If \(k ∈ A_α\), (i) \(\int_1^{2/h} e^{itx}dt = 0\) for some \(0 < t < 1\), (ii) \(\sum_{i,j} (ψx_i ⊗ x_j^* + \sum_{k,l} ω_k e_{k,l} ω_l^* e_{l,j})e_{i,j} = 0\). Application: Theorem 4. If for some \(α, β > 0\), (i) \(k ∈ A_β\), (ii) \(\text{ess sup} \bigg| k(t) \bigg| \). (iii) \(\text{ess sup} \bigg| \int_1^{2/h} k(t)e^{itx}dt \bigg| \). Theorem. If for each \(1 < p < \infty\), \(\sup \bigg| \int_1^{2/h} k(t)e^{itx}dt \bigg| \). (iv) \(\text{ess sup} \bigg| \int_1^{2/h} k(t)e^{itx}dt \bigg| \). The above \(k\) is bounded function with compact support. (Received January 21, 1976.)


A C*-algebra \(A\) is said to be nuclear if it has only one C*-algebraic tensor product with an arbitrary C*-algebra \(B\). A map \(Φ : A → B\) is said to be nuclear if it is a point-norm limit of compositions of completely positive contractions \(A → M_∞ → B\), with \(M_∞\) the n × n matrices (n varying). Theorems. The following are equivalent: A is nuclear, the identity map \(A → A\) is nuclear, \(A^{**}\) is injective (in the completely positive sense). Corollary (Tomiyama's conjecture). If \(J\) is a closed two-sided ideal in \(A\), then \(A\) is nuclear if and only if \(J\) and \(A/J\) are nuclear. Remark. If \(A\) is separable, Connes' work shows that \(A\) is nuclear if and only if its factor representations are hyperfinite. Theorem. If \(B\) is separable and \(Φ : B → A\) is nuclear, then \(Φ\) has a nuclear lifting. The latter result may be used for noncommutative generalization of the Brown-Douglas-Fillmore Theorem that Ext X is a group. (Received January 22, 1976.)


We begin with a generalization and extension of a result of Sikorski concerning the inducing

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of homomorphisms by point mappings. The integral of these generalized homomorphisms is defined, and this new construction is used to develop the theory of weak convergence for measures defined on a Boolean algebra. Among the results proven are several necessary and sufficient conditions for weak convergence and a Prohorov type theorem. In addition, existing treatments of the expectation of a random variable defined on a Boolean algebra are unified using the integration theory developed above.

The initial construction of this paper also has applications to non-Boolean measure theory. For example, the existence of a spectral measure defined on all Borel sets which is equal to the resolution of the identity associated with a self-adjoint operator on the half rays is obtained as an easy consequence of our construction. (Received January 23, 1976.)


734-B8  Paul M. Gauthier, Université de Montréal, Montréal 101, Canada and Lee A. Rubel, University of Illinois at Urbana-Champaign, Urbana, IL 61801. Holomorphic functionals on open Riemann surfaces.

Let \( \mathcal{O}(R) \) denote the space of functions holomorphic on an open Riemann surface \( R \), where \( \mathcal{O}(R) \) has the topology of uniform convergence on compact sets. A new characterization is given of the dual space \( \mathcal{O}(R)^* \). Let \( D(R) \) denote the space of holomorphic differentials on \( R \) and let \( D(\infty) \) denote the space of germs of holomorphic differentials "at \( \infty \). It is proved that \( \mathcal{O}(R)^* \cong D(\infty)/D(R) \). This representation has the advantage over the classical representation \( \mathcal{O}(R)^* \cong \mathcal{O}(\infty)/\mathcal{O}(R) \) of being both concrete and canonical. (Received January 30, 1976.)


We discuss here representation and Fredholm theory for C*-algebras generated by commuting isometries. More particularly, for \( n \) commuting isometries \( \{V_j; 1 \leq j \leq n\} \) on a separable Hilbert space we give a representation resembling the well-known representation for a single isometry. Our representation permits an analysis of the C*-algebra \( \mathcal{A} = \mathcal{A}(V_j; 1 \leq j \leq n) \) generated by the \( \{V_j\} \). The commutator ideal in \( \mathcal{A} \) is identified precisely and, under certain additional hypotheses, the Fredholm operators in \( \mathcal{A} \) are also precisely determined. Finally, we obtain formulas in terms of topological data for the index of Fredholm operators in some interesting algebras of the type \( \mathcal{A}(V_j; 1 \leq j \leq n) \).

(Received February 9, 1976.)


Differential equations in a Banach Space which can be employed as comparison equations through
cones, represent abstract models for competitive processes that exist in several areas. The most striking property of such equations is that the solutions must belong to a positive cone. The discussion of stability properties of such equations is important since the only solutions that can predict the actual behavior of the physical system are asymptotically stable solutions. If the equation displays a periodic behavior, the question of its stability is useful and is of interest. Some of these questions are discussed in an abstract setting.

(Received February 11, 1976.)

734-B21 Yen Tzu Fu, Indiana State University, Evansville, Indiana 47712. Infinitesimal generators for general semigroups on a Banach space. Preliminary report.

Most perturbation theorems for strongly continuous semigroups (linear or nonlinear) deal with contraction semigroups of continuous transformations on a Banach space. There are cases such that the sum of two infinitesimal generators for strongly continuous nonlinear, noncontraction semigroups of transformations with discontinuities on a Banach space is the infinitesimal generator for the semigroup obtained by applying the Trotter's product formula to the original semigroups. Some examples on $L_p$ are given. (Received February 12, 1976.)


Autonomous ordinary differential equations of order three or more arise in a variety of settings where periodic oscillations are expected. In this context the Brouwer fixed point theorem has been used by a number of authors to prove existence of a periodic solution, often by showing that a cross section of a solid torus is mapped continuously into itself by the flow. Recently a number of examples have been found where the torus can be constructed by dividing the relevant region in $n$-dimensional phase space into $2^n$ rectangular "boxes", all intersecting at an equilibrium point, and showing that a particular set of $2^n$ of these boxes forms an invariant subset of "oscillating" solutions. Examples include the three-dimensional Field-Noyes model for the Zhabotinsky reaction (joint with J. Murray), a class of $n$th order systems describing negative feedback cellular control systems (joint with J. Tyson and D. Webster), a three dimensional system modelling a nuclear reactor (studied by W. Troy), and the four dimensional system suggested by Glass and Kauffman to describe oscillatory cellular dynamics (work of I. D. Hsu). (Received February 12, 1976.)

734-B13 R.A. Stokes, University of Mississippi, University, Mississippi 38677, and D.B. Priest and J.W. Lewis, Harding College, Searcy, Arkansas 72143. Integrals of a function with respect to a function pair and the mean Stieltjes $c$-integral.

In Mean Stieltjes Type Integral (Pacific J. Math. Vol. 44, No. 1, 1973, pp. 291-297) we presented $A, L_D(g, h), N_D(g, h), m\int_A f \, dg \, dh$ and partition $D$ of $A$. Let $m(c)\int_A f \, du$ be as in Abstract 664-59, these Notices 16 (1969), 517. Defining $U_D(g, h)$ as an analogue to $L_D(g, h)$ and $N_D(g, h)$ and using techniques similar to those used for $m\int_A f \, dg \, dh$, we improve the existence theorem for the integral $\int_A f \, dg \, dh$ presented in Abstract 677-566 these Notices 14 (1967), 825 and show: Proposition 1 If $f$ is quasicontinuous on $A$, $g$ and $h$ are of bounded variation in Hardy's sense, then $\int_A f \, dg \, dh$ exists whenever each horizontal contour map of $g$ or $h$ and each vertical contour map of $g$ or $h$ is continuous on $A$. Proposition 2 There does not exist a general integration-by-parts formula for $m(c)\int_A f \, du$ as there does with the ordinary mean Stieltjes integral. Applications for $m\int_A f \, dg \, dh$ are also given. (Received February 12, 1976.)
In the Lie-Ovsjannikov transformation theory of partial differential equations $F(x,u,Du,D^2u,...) = 0$ (or systems of such) one considers (local) one-parameter groups of $(x,u)$-space which slightly deform every (local) solution manifold $u = u(x)$ (in the space) into another solution manifold $\tilde{u} = \tilde{u}(x)$. For many linear equations these turn out to be of the form $u + v(x,g)u(g(x))$ where $g$ is an element of a transformation group acting on $x$-space alone, and $v$ is some multiplier depending on $g$. In effect the transformations are in these cases multiplier representations of transformation groups acting on $x$-space. (The trivial transformations $u + u$ (another solution have been eliminated by taking quotients against them.) We give a few simple examples to show that there are linear equations $F = 0$ for which not every transformation is a multiplier representation. They can even be nonlinear. We prove however, that for linear second-order parabolic equations $F = 0$, every transformation is necessarily of multiplier type. (Received February 13, 1976.)

Given $n$ distinct complex numbers $\lambda_1, \lambda_2, \ldots, \lambda_n$ with $|\lambda_1| \leq |\lambda_2| \leq \ldots \leq |\lambda_n|$ and $n$ non-zero numbers $\rho_1, \rho_2, \ldots, \rho_n$, a constructive procedure is determined for finding complex-valued functions $a(s) \in C(2)[0,1]$, $b(s) \in C(1)[0,1]$, and $c(s) \in C[0,1]$, so that $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the first $n$ eigenvalues and $\rho_1, \rho_2, \ldots, \rho_n$ are the corresponding normalization constants (weight numbers) for the non-self-adjoint eigenvalue problem $y^{(4)} + ay^{(2)} + by = 0$, $y(0) = y(1) = y(2) = y(3) = 0$. The functions $a(s), b(s)$ and $c(s)$ are determined as finite sums of functions which in turn are solutions of a set of $2n$ linear, non-homogeneous, equations. (Received February 13, 1976.)

Let $G$ be a compact, connected, Abelian group. Its dual group, $\Gamma$, is discrete and can be ordered by a positive semigroup, $\Gamma_+$. Let $\Gamma_1$ be a semigroup which is strictly smaller than $\Gamma_+$. Define $H^p(\Gamma_1)$ to be the subset of $L^p$ consisting of those functions whose Fourier coefficients vanish off $\Gamma_1$. A function $f \in H^p(\Gamma_1)$ is said to be outer if $\int_\Gamma \log|f(x)|dx = \log|\int_\Gamma f(x)dx| > -\infty$. The function is said to be inner if $|f| = 1$ a.e. $dx$. Since $\Gamma_1$ is strictly smaller than a half plane we cannot assume every function has an inner-outer factorization. If $\Gamma_1$ is the intersection of half planes than a function, $f$, has an inner-outer factorization if and only if the Fourier coefficients for $\log|f|$ vanish off $\Gamma_1 \cup \Gamma_1$. (Received February 16, 1976.)

The pressure field $p(x)$ in an infinitely long, gas-lubricated slider bearing is governed by Reynold's equation, $(h^3 pp')' - \Lambda (hp')' = 0$, in which $h(x)$ denotes the separation between the bearing surfaces and $\Lambda$ is a parameter proportional to the relative speed of the bearing surfaces. Qualitative features of the pressure field, such as bounds and stability, are obtained by applying monotone iteration techniques and nonlinear maximum principles. These same tools also permit comparison of the true pressure with asymptotic estimates for large $\Lambda$ obtained by R. C. DiPrima. The net result is a reasonably complete characterization of the pressure field in certain types of bearings. (Received February 16, 1976.)
Let $\mathcal{T}_0$ be defined by $f(z) = \sum_{n=0}^{\infty} a_n z^n$, and have radius of convergence $R$, $0 < R < \infty$, and suppose that $a_0^{-1} |a_n| R^n = \infty$, and let $f$ be of order $\rho$ and of lower order $\lambda$, (see Abstract 711-30-B, these Notices 21(1974), A-120.). For any $r$, $0 < r < R$, denote maximum term of the power series as $u(r,f) = \max |a_n| R^n = u(r,f)$, and the rank of this term as $\nu(r,f) = \max \{ n : |a_n| R^n = u(r,f) \}$. As usual $\|f(z)\| = \max |f(z)|$. Theorem 1. For $f$ as above, and if $\liminf_{r \to R} \frac{\log u(r,f)}{\log (R/(R-r))} = 1 + \lambda \ (*)$, then for all $r$, $0 < r < R$, $u(r,f) < \ldots < r^k u(r,f(k)) < \ldots$. Theorem 2. For $f$ as above $\limsup_{r \to R} \frac{\log u(r,f)}{\log (R/(R-r))} \leq (1 + \lambda) \leq (1 + \rho) = \liminf_{r \to R} \frac{\log u(r,f)}{\log (R/(R-r))}$. Equality holds for $\liminf$ if $\ (*)$ holds, Theorem 3. For $f$ as above, and for $r$ such that $u(r,f) > 1$, $M(r,f) < u(r,f) \left( \log M(r,f) + 1 \right) \frac{2t}{t - r}$ where $r < t < R$. (Received February 16, 1976.)

734-B39 G.S. LADDE, State University of New York at Potsdam, New York 13676

COMPETITIVE PROCESSES III: STABILITY OF İTO TYPE STOCHASTIC SYSTEMS

In this work, by employing the concept of vector Lyapunov functions and the theory of differential inequalities, the stability analysis of competitive systems by using Ito type stochastic differential equations is initiated in a systematic and unified way. Furthermore, an attempt has been made to formulate and partially resolve the "deterministic vs. stochastic", and the "complexity vs. stability" problems in stochastic competitive processes. Finally, the usefulness of the stability analysis of the competitive systems has been demonstrated by exhibiting several well-known examples in competitive processes in biological, medical, physical, and social sciences in a coherent way. (Received February 16, 1976.)


The Nevanlinna-Pick theorem is examined in the context of norms of induced operators on quotients of Banach spaces and of restrictions of multiplication operators on function spaces. The main result characterizes those Banach spaces of functions for which a Nevanlinna-Pick theorem is valid. Specifically, let $E$ be a Banach space of functions on a set $S$. Assume further that $E$ is a dual space with weak-star continuous point evaluations. For any subset $W$ of $S$ let $E|W$ be the space of restrictions to $W$ as a quotient of $E$. A Nevanlinna-Pick theorem is obtained if and only if each multiplier $u$ of $E|W$ has an extension to a multiplier of $E$ with norm arbitrarily close to that of $u$. It is then shown that multipliers on the Hardy spaces $H^p$ have the required extension property. The principle tool in the latter demonstration is an identification of the dual of $H^p|W$ for subsets $W$ of the unit disk. (Received February 16, 1976.)

734-B21 CARLOS A. INFANTOZZI, Universidad de la República, Atlántico 1514, Montevideo, Uruguay. Some generalizations of a theorem of Schlömilch.

Let $f(x)$ be a function such that: (a) $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$; (b) $f(x) \equiv 1 + x$; (c) $x < 1 \Rightarrow f(x) \equiv 1/(1 - x)$. If $\sum u_m$ is a series of positive terms, $L_{m,n} = L_{m-1,n} + L_{n-1,n} + L_{n,n} = n$ (L natural logarithm) and we put $S_n = L_{0,n} R - L_{0,n} u_m$, $S_n = L_{m,n} (S_{m-1,n} + 1)$ when $m \equiv 1/(k-1)$ is the inverse function of $f(x)$, then it results:

$$S_m = \sum_{i=1}^{m} \sigma_i \geq 1$$

in $C$ and

$\equiv 1$ in $D$ (notations of K. Knopp "Theorie und Anwendung der Unendlichen Reihen").

The same test can be written thus:

$\sum u_m / u_0 = S_{m,n} / u_m \sigma_i \geq 1$ in $C$ and

$\equiv 1$ in $D$, where $P_m(n) = L_{0,n}, L_{1,n}, \ldots, L_{m,n}, P_0(n) = L_{0,n}, \sum_{i=0}^{m} 1 / P_i(n), \sum_{i=0}^{m} 1 / P_i(n) = 0$. If $m = 0$ and $f(x) = e^x$, then one obtains the Schlömilch's test. With the same hypotheses one can prove the following test:

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$L_m^2 \ldots L_2^1 n \{ n^{-1}(u_n/u_{n+1}) - 1 \} \ldots - 1 \} \geq \alpha > 1$ in $C$ and $\leq 1$ in $D$, or briefly $P_m(u)[r^{-1}(u_n/u_{n+1}) - \Sigma] \geq \alpha > 1$ in $C$ and $\leq 1$ in $D$. If the function $f(x)$ has the property $f(1/x) = -f(x)$, then this second test coincides with the former. (Received February 16, 1976.)

734-B22 IVAN ERDELYI, Temple University, Philadelphia, Pa. 19122  

A spectral theory is developed for partial isometries on a Hilbert space which admit a duality theory of type 3 in the sense of E. Bishop [Pacific J. Math. 9 (1959), 379-397]. The spectrum of such an operator coincides with the approximate point spectrum, having an element of absolute value 1. The first order rate of growth condition, the single-valued extension property and some duality properties of the operators under investigation produce a spectral decomposition. In the special case the spectrum lies on a Jordan curve and it contains the origin, the partial isometry has a resolution of the identity. (Received February 16, 1976.)

734-B23 KUANG-HO CHEN, University of New Orleans, Louisiana 70122. A basic qualitative analysis on cancer chemotherapy.

This paper presents an analysis of the three models which Chuang obtained by modifying a tumor model first proposed by Skipper and Zubrod. For the three cancer chemotherapeutic model systems formulated on the basic Von Foerster's population balance equation and the compartment equation, the well-defined expected cell population of each phase in proliferating state, of resting state, and of nondividing state in each tumor mass is provided through the existence of the uniquely defined related population density functions which are continuous in both the age and time variables for certain class of the transfer coefficients for cycle phases in the proliferating state, of the initial age distributions, and of the loss functions for each phase before and after drug treatment. The initial-boundary value problem for population density functions is reduced through the standard device for a hyperbolic equation into the related integral equations for the birth rate functions. The contraction mapping theorem is used to prove the existence of the unique solution in a suitable space. (Received February 9, 1976.)

734-B24 R. D. DRIVER, University of Rhode Island, Kingston, RI 02881.  
The equations of neutral type for a two-body problem of electrodynamics.

For the delay differential equation of neutral type (1) $x'(t) = x'(t - x(t))$ ($t \geq 0$), it is unclear how to describe a well-posed initial-data problem. However, for the equation (2) $x''(t) = x''(t - x(t))$ ($t \geq 0$), a well-posed problem can be defined using initial data $x(t) = \phi(t)$ for $t \leq 0$, where $\phi$ has an absolutely continuous first derivative, $\phi(0) > 0$, and $\phi'(0) < 1$.

The classical equations of motion for two charged particles (without radiation reaction) can be written as a system of delay differential equations of neutral type with state-dependent delays. The system is complicated, but fortunately it belongs to a class represented by (2) rather than by (1). For these equations of the two-body problem of electrodynamics, one then obtains reasonable theorems on existence and uniqueness of solutions and continuous dependence on the initial data. (Received February 16, 1976.)

734-B25 Bertram Yood, Pennsylvania State University, University Park, Pennsylvania 16802.  
Decomposition of Hilbert Algebras.

For a Hilbert algebra $A$ let $P(A)$ denote its partially ordered set of projections (self-
adjoint idempotents). Every full Hilbert algebra \( A \) shown to be the direct sum of two full Hilbert algebras \( A_1 \) and \( A_2 \) where \( P(A_1) \) has no atoms and each \( p \) in \( P(A_2) \), \( p \neq 0 \) dominates an atom in \( P(A_2) \). Call a \(^\ast\)-ideal in a full Hilbert algebra almost full. If \( A \) is almost full it has the decomposition \( A_1 \oplus A_2 \) just described where \( A_1, A_2 \) are almost full. Hilbert algebras of the form \( A_2 \) are analyzed. (Received February 16, 1976.)

*734-B26 JOHN S. LEW, Mathematical Sciences Department, IBM T.J. Watson Research Center, Yorktown Heights, New York 10598. Some applications of structural concepts in asymptotic analysis.

Powerful techniques for relatively restricted problems have yielded many important advances in asymptotic analysis; however we review some alternative developments which are also rewarded by specific results, but which are partially motivated by more abstract questions: the existence of smooth functions with given expansions, the generalization of standard definitions for asymptotic series, the closure properties of function systems with certain expansions, the relation of integral transforms to many series. In particular, we discuss the use of Mellin transform methods, and indicate some of their various applications. (Received February 17, 1976.)

*734-B27 ANDREW MARKOE, University of Connecticut, Storrs, Connecticut, 06268. Increasing \( \{X_n\} \) of Stein Spaces.

Many partial results are known about the question of whether \( \bigcup X_n \) is Stein when each \( X_n \) is a Stein open subset of \( X_{n+1} \). For example, if each \( X_n \subset X_{n+1} \) is a Runge pair or if the union is open in a Riemann domain the answer is affirmative. Recently J. Fornaess has produced an increasing union of Stein manifolds which is not Stein. Therefore the entire story about the above question appears to be contained in the following results.

Definition A countable family of analytic spaces \( \cdots \subset X_n \subset X_{n+1} \subset \cdots \) is said to be a Runge family if for every compact \( K \subset \bigcup X_n \), \( \exists j = j(K) \) such that \( K \subset X_j \) and \( \Theta (\bigcup X_n) \) uniformly approximates \( \Theta (X_j) \) on \( K \).

Theorem If \( \cdots \subset X_n \subset X_{n+1} \subset \cdots \) is a countable increasing family of Stein spaces then the following are logically equivalent: \( \bigcup X_n \) is Stein \( \iff \cdots \subset X_n \subset \cdots \) is a Runge family \( \iff \Theta (\bigcup X_n) \) is smooth on \( K \).

(Received February 17, 1976.)

*734-B28 T.K. PUTTASWAMI, Ball State University, Muncie, Indiana 47306. Two Point Connection Problem for a certain ordinary differential equation

This paper is devoted to the solution in the large of the differential equation.

\[
\sum_{i=0}^{3} a_i x^{i} \frac{d^i y}{dx^i} = 0.
\]

(1)

Here, \( m \) is a positive integer, the variable \( x \) is regarded as complex and the constants \( a_i, b_i \) \( (i = 0, 1, 2, 3) \) are real or complex with \( a_0 \neq 0, b_0 \neq 0 \). If \( u_i (i = 1, 2, \ldots, m) \) are the roots of \( a_0 + b_0 x^m = 0 \), then (1) will have regular singular points at \( x = 0, x = u_i (i = 1, 2, \ldots, m) \) and \( y = \infty \). The indicial equation about \( x = 0 \) is found to be

\[
\sum_{i=0}^{3} a_i x^{i} (h-1) (h-2) = \sum_{i=0}^{3} a_i x^{i} + a_{i+1} x^{i-1} + a_{i+2} x^{i-2} + a_{i+3} x^{i-3} = 0.
\]

(2)

It is also assumed that the roots \( h_i (i = 1, 2, 3) \) of (2) are such that no two of them by an integer. (Received February 17, 1976.)

*734-B29 HAROLD M. HASTINGS, Hofstra University, Hempstead, New York, 11550. The shape of stable invariant sets of continuous dynamical systems.

We obtain an analogue of the Poincaré-Bendixson theorem in Euclidean \( n \)-space with the use of shape theory. (Received February 17, 1976.)
We extend results of Masani and Rosenberg (where \( p = q = 1 \)) [J. Funct. Anal., to appear] and Rosenberg [Abstract 731-47-23, these Notices 23(1976), A-167]. Let \( E \) be a spectral measure on a \( \sigma \)-algebra \( S \) over a set \( \Omega \) for a Hilbert space \( \mathcal{H} \). Let \( T \) be a closed densely-defined operator from \( \mathcal{H}^q \) to \( \mathcal{H}^p \) \((1 \leq p, q \leq \infty)\). For \( x = (x_i)_{i=1}^{s} \in \mathcal{H}^s \), denote \( \mathcal{L}_x^f = \text{[subspace of } \mathcal{H} \text{ spanned by } E(B)x_i, B \in S, i = 1, \ldots, s\). \( L_x^f \) is projection on \( \mathcal{H}^f \) with range \( \mathcal{H}^f \). Theorem I. Let \( E \) be of countable multiplicity for \( \mathcal{H} \). Then: (A) When \( p \) or \( q \) is \( < \infty \), the following 3 conditions are equivalent: (1) \( (i) \) \( Tx \in \mathcal{H}_X \text{ for } x \in \mathcal{H}_S \) and \( (ii) \) \( E^T(B)T \subseteq TE^S(B) \forall B \in S \). (2) \( L_p^T \subseteq L_q^T \forall X \mathcal{H} \subseteq \mathcal{H} \text{ (r fixed, } 1 \leq r \leq \infty \text{).} \) \text{(3)} \( T = \int \Phi dE \) where \( \Phi \) is a \( p \times q \) function. (B) When \( p = q = \infty \), \((2) \circ (3) \circ (1)\), but \((1) \circ (3) \). Let \( U_t = \int \omega \mathbb{E}(d\omega) \) \((t \in S)\) be a weakly continuous group of unitary operators, where \( S \) is an i.c.a. group and \( \Omega \) is its dual group, cf., Rosenberg [Z. Wahr. verw. Geb. 12(1969), 333-343]. For \( x \in \mathcal{H}^s \), denote \( \mathcal{M}_x = \text{[subspace of } \mathcal{H} \text{ spanned by } U_t x_i, t \in S, i = 1, \ldots, s\}. \text{Lemma. (A)} \mathcal{M}_x = \mathcal{N}_x. \text{(B)} U^{(p)}_t \subseteq U^{(q)}_t \forall t \in S \subseteq E^S(B) \subseteq TE^S(B) \forall B \in S. \text{Thm. I} \circ \text{Theorem I condition (1) may now be replaced by: (1'1) \( Tx \in \mathcal{H}_X \text{ for } x \in \mathcal{H}_S \) (T is } U_t \text{-subordinate) and (ii) } U^{(p)}_t \subseteq U^{(q)}_t \forall t \in S. \text{(T is } U_t \text{-invariant).} \) (Received February 13, 1976.)

*734-B31 James L. Kaplan and James A. Yorke, Boston University, Boston, Massachusetts 02215. Competitive exclusion and nonequilibrium coexistence.

The widely accepted competitive exclusion principle states that no stable equilibrium (either point or periodic) is possible in ecosystems in which some \( n \) species are limited by less than \( n \) resources, but which nevertheless has an asymptotically stable periodic solution. Thus the usual statement of the competitive exclusion principle is not robust. An alternate version of this principle is formulated whose conclusions are considerably weaker than those of previous authors but which does apply to a broad spectrum of mathematical models. In particular, we have exhibited a system in which \( n \) species are limited by less than \( n \) resources, but which nevertheless has an asymptotically stable periodic solution. Thus the usual statement of the competitive exclusion principle is not robust.

An alternate version of this principle is formulated whose conclusions are considerably weaker than those of previous authors but which does apply to a broad spectrum of mathematical models. Specifically, we show that in an ecological community in which \( n \) species are limited by less than \( n \) resources, there is probability zero that the system tends to equilibrium. (Received February 11, 1976.)

734-B32 V.V. RAO, University of Regina, Regina, Saskatchewan, Canada S4S 0A2. Nonanalytic automorphic forms and Dirichlet series.

In this paper we consider a class of nonanalytic automorphic forms of the type considered by C. L. Siegel [Math. Zeit. 44(1938), pp. 398-426] and consider Dirichlet series arising from the Fourier expansion of the nonanalytic automorphic form, study their analytic behaviour and obtain a functional equation. The results represent generalizations of the results of Siegel for the zeta functions of indefinite quadratic forms with rational coefficients. (Received February 17, 1976.)

734-B33 Shizuo Kakutani, Yale University, New Haven, Connecticut 06520 Classification of Ergodic Transformations.

Let \((X, \mathcal{B}, m)\) be a Lebesgue measure space with \( m(X) = 1 \), and let \( \mathcal{G}(X) = \mathcal{G}(X, \mathcal{B}, m) \) be the group of all \( \mathcal{B} \)-measurability preserving, \( m \)-non-singular, one-to-one transformations \( T \) of \( X \) onto itself. Various notions of equivalence relations between elements of \( \mathcal{G}(X) \) are introduced, and elements of \( \mathcal{G}(X) \) are classified according to these equivalence relations. Most of the arguments are concerned with the case when \( T \) is ergodic. The notions of "induced transformations" and "full group" are also discussed. (Received February 17, 1976.)


Techniques from complex geometry are used to study the class \( \mathfrak{a}_n(\mathfrak{A}) \) of operators on a separable Hilbert space \( \mathfrak{A} \) for \( \mathfrak{A} \) a bounded domain in \( \mathfrak{A} \), where \( T \in \mathfrak{a}_n(\mathfrak{A}) \) if

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1) \( \Omega \subset \sigma(T) \), 2) \((T-w)\mathbb{M} = \mathbb{M} \ \forall \ w \in \Omega \), 3) \( \forall \ ker(T-w) = \mathbb{M} \), and 4) \( n=\dim ker(T-w) \). A hermitian holomorphic bundle \( E_T \) is defined on \( \Omega \) by the map \( w \mapsto ker(T-w) \) which determines the bundle up to unitary equivalence. Three sample results are:

**Theorem 1:** \( S, T \in \mathcal{B}_1(\Omega) \) are unitary equivalent iff \( S|ker(S-w)^2 \) is unitarily equivalent to \( T|ker(T-w)^2 \) for \( w \in \Omega \).

**Theorem 2:** The curvature of \( E_T \) is strictly negative and uniformly approaches \( -\infty \) when \( \Omega \) is smooth and \( T \) is rationally bounded.

**Theorem 3:** There is a monomorphism from the commutant of \( T \) into the algebra of bundle maps on \( E_T \). (Received February 17, 1976.)

**Applied Mathematics**

*734-C1* J. EISENFELD, Department of Mathematics, University of Texas at Arlington, Arlington, TX 76019. Mathematical problems arising from the theory of synovial joint disease (arthritis).

A synovial joint may be modeled as the interaction between synovial fluid, cartilage, and bone. This gives rise to a boundary value problem for a system of PDE's (or an ODE in Banach space). Each configuration (lying, walking, etc.) and each experimental situation requires special consideration. Thus a multitude of interesting problems abound concerning existence, stability, and duration of solutions. Several such problems (some unsolved) will be presented. (Received February 9, 1976.)


We extrapolate from the idea of Hamburger and Wexler (A mathematical theory of learning transformational grammar, J. Math. Psych, 12(1975), pp. 137-177), a framework for empirical and theoretical semantics. Namely we equip the parties \( T \& R \) of a channel with memories \( M_j \ (j = T,R) \) subdivided into linguistic and empirical parts \( M_j^L + M_j^E = M \), and further into parts called idiolectal, dialectal, and residual: \( M_j^p(T,R), \ p = L,D,U \), resp., and \( M_k^p(T), \ (k = L,E) \). We call principal means of innovation "congruences", "instructive analogies", and "adaptations" (wherein instructive analogy is modified to suit disparate features of a situation compared with past similar innovative events). Infinitary sememic alphabets (countable or continuous) form the basis for the study. Subject only to limitations, if any, of motivation, each such has the advantage of capacity to support the discovery of infinitely many theses for any given length bound \( B \). (Received December 8, 1975.)

*734-C3* THOMAS G. HALLAM, Florida State University, Tallahassee, Florida 32306. Structural Sensitivity of Grazing Formulations In Nutrient Controlled Plankton Models

Stability and persistence properties of three nonspatial plankton models, each differentiated by its herbivore grazing term, are analytically compared. The dynamical persistence function in the ecomodel is shown to operate uniformly even though stability configuration characteristics of the model may be topologically distinct. The persistence threshold for each model indicates that total nutrient is a fundamental biological control. In the parameter space, all of the models are structurally unstable; however, an important bifurcation mechanism associated with this instability governs persistence. While, topologically, ecomodel transfiguration through parameter modulation is noncontinuous, the biological populations evolve in a continuous or a lower semicontinuous manner. A basic conclusion of the paper is that fundamental problems for marine ecomodels remain unresolved since each of the models is a structurally unstable system for a fixed dynamically persistent ecology. (Received January 29, 1976.)


A previously given five-term recurrence formula for converting a series of polynomials \( \sum_{m=0}^{n} a_m q_m(x) \) into \( \sum_{m=0}^{n} A_m Q_m(x) \), when \( q_m(x) \) and \( Q_m(x) \) satisfy three-term linear recurrence formulas, is extended to the more general case where, instead of the constant coefficients \( a_m \), there are now polynomial co-
coefficients $a_m(x)$ of any degree $d_m \geq 0$. An application is made to the conversion of a product of series. (Received January 30, 1976.)

In this paper a results of H. L. Gray and T.A. Atchison given in A Note on the G-transformation, J. Res. Nat. Bur. Standards, 72B (1968), pp. 29-31, is extended to the generalized G-transform which was presented in their paper The Generalized G-transform, Math. Comp., Vol. 22 (1968), pp. 595-606. These transformations are used to accelerate the convergence of some improper integrals of the form $\int_a^b f(x)dx$. (Received February 9, 1976.)

Populations of grazing animals frequently go unstable or exhibit unexplained oscillations following depletions of the renewable resources supporting them. We have developed population dynamic equations that include an explicit resource depletionary term and appropriate time lags, and have modelled the above phenomena with our equations. The basic equation is

$$(I) \frac{dN(t)}{dt} = rN(t)\left[1 - \frac{N(t)/K(t)}{1 - D(t)}\right],$$

for population $N$ and intrinsic rate of increase $r$. $K$ is a parameter, and $D$ is given by the convolution integral

$$(II) H(t) = \int_{t-T}^{t} F[N(r)/K(r)] \frac{1}{1 - D(r)} P(t-r) \, dt,$$

with appropriate rate-of-depletion function $F$ and depletion-persistence function $P$. Equations (I) and (II) can yield $N$ instabilities sufficient to nearly extinguish $N$. A second definition of $D$ is obtained from

$$(III) \frac{dD(t)}{dt} = aN(t)K(t) - \int_{t-T}^{t} \frac{N(r)/K(r)}{1 - D(r)} g(t-r) \, dr,$$

where $g(x)$ approximates an impulse at $x = (3/4)T$, and $T$ is the period of large antiphasic $N$ and $D$ oscillations. Study of the above equations should contribute to a better ecological understanding of wildlife population data for grazing fauna, and the design of better harvesting and feeding, and insect control practices in wildlife sanctuaries and food farms. (Received February 10, 1976.) (Author introduced by Dr. Michael Arbib.)

Neural networks can be derived from first principles that express environmental constraints to which the networks adapt. Some networks can process patterns in the presence of noise and saturation. Their properties include adaptation, or normalization, of total activity, contrast enhancement, short term memory, stable and unstable limit cycles, global existence of asymptotic steady states, pattern transformations akin to visual illusions, etc. Other networks are capable of learning arbitrary space-time patterns in essentially arbitrary anatomicities and given arbitrary data preprocessing. Such networks can adapt to arbitrary evolutionary specializations. By joining short term memory and long term memory mechanisms, a network unit is found that is capable of reclassifying, or recoding, arbitrary sets of spatial patterns into convex regions through experience. This universal recoding, or adaptive pattern classification, property is relevant to data on the development of visual cortex in infants and the self-organization of codes in adults. (Received February 11, 1976.)

Many important biological systems, such as the central nervous system and the cellular genetic control networks, admit alternate discrete and continuous descriptions. The former fall under the context of automata theory; the latter are systems of ordinary and partial differential equations. For many reasons, it is important to have some kind of "correspondence principle" for relating these alternate descriptions of the same system. A general notion of dynamical realizability is introduced for this purpose, applicable to any abstract mapping process $f : A \rightarrow B$. In particular, we can consider the dynamical realization of the mappings which define a finite automaton, thereby replacing a discrete description by a much more powerful continuous description. Applications
of this concept to biological modelling problems, to discrimination and control systems, and to purely mathematical questions, will be discussed.
(Received February 11, 1976.) (Author introduced by Professor G. S. Ladde.)

734-C9 MARIO O. GONZALEZ, University of Alabama, University, AL 35486. A generalized complex potential in Fluid Dynamics.

For a stationary plane-parallel flow of an ideal fluid which is solenoidal but rotational in some domain D, it is shown that there exist a nonanalytic potential function \( F(z) = U + iW \) with differentiable components \( U \) and \( V \), having the property \( F'(z) = W \), the notation \( F'(z) \) meaning the complex directional derivative of \( F \) in the direction specified by \( \theta \). THEOREM. Suppose that the velocity field is given by \( \mathbf{W} = U + iV \), where \( U \) and \( V \) have continuous partial derivatives in \( D \), and that \( \text{div} \mathbf{W} = 0 \) while \( \mathbf{W} \not= 0 \) in \( D \). Then the function \( F(z) = \int_{z_0}^{z} \frac{\mathbf{W}(\zeta)}{i} d\zeta \), where \( z_0 \) is a fixed point of \( D \), \( z \) a variable point in \( D \), and \( C \) any regular arc in \( D \) containing \( z_0 \) and \( z \), is such that \( F'(z) = W \), the directional derivative at each point being taken in the direction of the arc. Furthermore, if we write \( \mathbf{F} = U + iV \), then \( D_0 U = w_t \) and \( D_0 V = w_v \), the symbol \( D_0 \) denoting the real directional derivative in the direction of the arc, and \( w_t, w_v \) the tangential and normal components of the flow with respect to \( C \). (Received February 16, 1976.)

*734-C10 S. R. BERNEFIELD, University of Texas at Arlington, Arlington, TX 76019, M. M. JUDY, R. M. DOWDEN, and G. A. CAMPBELL, University of Texas Health Science Center, Dallas Texas 75235, Characterization of length distributions of macromolecules from electro-optical decay. Preliminary report.

Electro-optical decay measurements, \( \Delta n(t) \), depend on the length of macromolecules in solution; and, thus, contain information about the molecular length distribution. Values of the time integral and derivative of \( \Delta n(t) \) at \( t = 0 \) depend explicitly on the parameters, \( s, \beta, \) and \( \alpha \) of the length distribution, \( n(l) = k^\beta \exp[-(l-\alpha)/\beta] \), where \( k \) is the polymer length and \( \alpha \) is the minimal polymer length. Using a nonlinear fit, we obtain the values of the physical parameters \( s \) and \( \beta \) and use these values to obtain important information about the mechanism of polymerization of biological macromolecules, such as the fibrous contractile proteins of heart muscle. (Received February 16, 1976.)


A theory of propagation of errors in the system of enzymes translating genetic information into proteins is presented. The formulation, which is non-statistical, makes it possible to determine the quantities of individual components of a system at any time. These quantities satisfy linear and non-linear differential equations depending upon the model. These equations are explicitly solved and the results are biologically interpreted in terms of error catastrophe and the evolution of the protein synthesizing machinery. (Received February 16, 1976.) (Authors introduced by Professor G. S. Ladde.)

734-C12 GILBERT N. LEWIS, University of Wisconsin, Milwaukee, Wisconsin 53201. A boundary-value problem with a turning point.

Consider the boundary-value problem:

\[ \epsilon^2 y'' + (p(x) + \epsilon^2 f(x, \epsilon))y' + g(x, \epsilon) y = 0, \quad y(a, \epsilon) = A, \quad y(b, \epsilon) = B, \]

where \( p(x) < 0 \), and \( p, f, \) and \( g \) are analytic. If \( -g(0, \epsilon)/p(0) \not= c + O(\epsilon^2) \), where \( c \in N = \{0, 1, 2, \ldots\} \), then so-called resonance does not occur, and \( y = o(\epsilon^n) \) on \( (a, b) \), for any \( n \in N \), with expected boundary layer behavior at the endpoints. If \( -g(0, \epsilon)/p(0) = c + O(\epsilon^2) \), \( c \in N \), then further transformations (a procedure is developed) may still expose nonresonance. These transformations are performed sequentially on the original differential equations, and, as a result, a higher order nonresonance criterion is readily obtained. This is done without having to rely on expanding the solution in powers of \( \epsilon \) and solving at each step for related functions to find higher order criterion for resonance or nonresonance (see O'Malley, SIAM J. Math. Anal., 1970, and Ackerberg and O'Malley, Stud. in Appl. Math., 49(1970), and their discussion of \( \sigma(\epsilon) \) in relation to their
Uniform Reduction Theorem), and without having to rely on matching at each step (see Cook and Eckhaus, Stud. in Appl. Math., 52(1973)). In the case of nonresonance, an existence proof is given, which is a modification of a method used by Fraenkel, Proc. Camb. Phil. Soc., 65(1969), for a related problem. (Received February 17, 1976.) (Author introduced by Professor Robert E. O'Malley, Jr.)

**Geometry**

*734-D1*  
Jin Bai Kim, West Virginia University, Morgantown, W. Va. 26505

Integrability conditions of a structure of a differentiable manifold.

Let q be an odd positive integer. Let $\Omega^{m}_{20}$ be a differentiable manifold of class $C^{r+1}$. If there exists a mixed tensor $f_{i}^{1}h_{i}^{1}$ of class $C^{r}$ which satisfies

$$f_{i}^{1}f_{i}^{2}...f_{i}^{h} = -\epsilon_{i}^{h},$$

then we say that the structure $f_{i}^{h}$ gives an extended almost complex structures of degree $q$ to the space $\Omega^{h}$ and we call $\Omega^{h}$ an extended almost complex space.

We now consider the integrability of this structure $f_{i}^{h}$.

(See: Jin B. Kim, Notes on f-manifolds, Tensor, N. S. 29(1975), 299-302.) (Received January 26, 1976.)

*734-D2*  
Francine Abeles, Kean College of New Jersey, Union, New Jersey 07083


The absolute volume ratio of a geometric n-simplex $\sigma_{n}$ in $\Omega^{n} \Omega$, the first barycentric subdivision of the complex $C_{1} (V_{n})$, to the geometric n-simplex $V_{n}$ is the inverse of the number of n-simplices in $\Omega^{n} \Omega$.

We extend the notion of $\Omega^{n} \Omega$ to the first centroidal subdivision of a complex by considering general centers of mass of the subsimplices. The ratio $|\sigma_{n}|/|V_{n}|$ can then be expressed in terms of products of masses located at the vertices. (Received February 12, 1976.)

*734-D3*  

Let $S'$ and $S''$ be twice differentiable strictly convex surfaces in Euclidean space $E^{n}$ satisfying the following conditions: a) they have common spherical image $\omega$; b) their products of principal radii of curvature have equal values at the points with the same normal; c) their support functions coincide on the boundary of the spherical image. It is known that when $\omega$ contains a hemisphere or is contained in a hemisphere, the surfaces coincide. However the following theorem is true: there exists a region on a unit hypersphere in $E^{n}$ such that two strictly convex surfaces ($C^{2}$ satisfying the conditions a), b), and c) differ on a non-trivial transformation. (Received February 16, 1976.) (Author introduced by Professor Ivan Erdelyi.)

**Logic and Foundations**

*734-EX*  
G. J. CHAITIN, IBM Research Division, P. O. Box 218, Yorke N. Heights, NY 10598.

Algorithmic entropy of sets.

In a previous paper a theory of program size formally identical to information theory was developed. The entropy of an individual finite object was defined to be the size in bits of the smallest program for calculating it. It was shown that this is $-\log_{2} p$ of the probability that the object is obtained by means of a program whose successive bits are chosen by flipping an unbiased coin. Here a theory of the entropy of recursively enumerable sets of objects is proposed which includes the previous theory as the special case of sets having a single element. The primary concept in the generalized theory is the probability
that a computing machine enumerates a given set when its program is manufactured by coin flipping. The entropy of a set is defined to be $-\log_2$ of this probability. (Received January 19, 1976.)

*MIRIAM LIPSCHUTZ–YEVICK, University College, Rutgers The State University, New Brunswick, New Jersey 08803. On Tarski’s paradox and the complementarity between sequential (aural, descriptive) and holistic (optical, holographic) recognition. (Denotation.)

We refer to the discussion p. 158, Tarski: “Logics, Semantics, Metamathematics”, Oxford 1958. The symbol ‘c’ denotes ‘the sentence printed on this page line 5 from the top’ which we observe to be ‘c is not a true sentence.’ Hence the symbol ‘c’ denotes the visual pattern corresponding to this sentence (note, for instance, that c is not identical with the same sentence written in characters so small as to become invisible), whereas the quotation name “c is not a true sentence” denotes the sequential, verbal content of the sentence ‘c is not a true sentence’, i.e. that which is asserted. The truth of the sentence as a pattern must be established by holographically recognizing whether or not the pattern is contained in the set of patterns representing true sentences. However the only pattern which can be recognized unequivocally by holographic recognition is that which is totally random. (For this pattern the only sequential description possible is the one identical with the visual pattern). Similar considerations apply to self-referential systems in that the two names entering into the paradox must be interpreted in the two complementary modes (languages) of perception. For instance in the relation $\varphi N*\varphi N*$ of Smullyan, J. Symb. Logic 22(1957), is true in $S_p$, $\varphi N*\varphi N^*$ is false in $P_p$, the expression $\varphi N*\varphi N^*$ on the left expresses the sequential meaning of the sentence, whereas the expression on the right (the expression named by $N*\varphi N^*$) represents the pattern in its entirety, ("the sense determined by its form", Tarski p. 166; "scratches on paper", Kleene p. 70). See Lipschutz-Yevick, Holographic or Fourier Logic, Pattern Recognition, Dec. 1975. (Received February 5, 1976.)


A well-known result of Myhill states that a set is productive iff it is completely productive. It is also known that every productive set is productive via a recursive permutation. However, as we have shown at the December, 1975 meeting of the Association for Symbolic Logic, a set is completely productive via an onto recursive function iff its complement is creative. We now strengthen the "if" part of this result to obtain the following: Theorem. If a set is creative then its complement is completely productive via a recursive permutation.

The notion of set-productivity was explored at the March, 1975 meeting of the AMS. (See Notices, vol. 22, pp. A-358, A-490, 1975.) A corresponding notion of complete set-productivity may be defined. We have the following results: Theorem. If A is set-productive then A is set-productive via a recursive permutation. However: Theorem. No set may be completely set-productive via an onto function. But we do have the following: Theorem. If A is completely set-productive, then A is completely set-productive via a |) recursive function. Furthermore, this may be done via a function which assigns different recursively enumerable sets to different natural numbers. (Received February 16, 1976.)

Statistics and Probability

James Glimm, The Rockefeller University, New York City, NY 10021. Quantum Theory and Probability theory.

The analytic continuation of the expression $e^t$ from real to imaginary times transforms quantum mechanics into probability theory. At imaginary times, the dynamics is a solution of the heat equation, represented by a path space integral. By this Feynmann-Kac construction, Euclidean quantum fields in d-dimensional space time define measures on the path space $\mathcal{D}(\mathbb{R}^d)$. Questions concerning existence, uniqueness, nonuniqueness, energy spectrum, particle structure, bound states and critical point can be answered in favorable cases. (Received December 17, 1975.)

BELA GYIRES, L. Kossuth University, Debrecen, Hungary. On a limit distribution theorem of linear order statistics.

In a previous paper (which appeared in Publ. Math. Debrecen, Vol. 22), the author investigated linear rank statistics for the case of dependent sample elements. In the present paper we give a necessary and sufficient condition for a so-called "symmetric linear order statistic" to have an asymptotic normal distribution. The proof is based on a previous result of the present author on the asymptotic behaviour of the characteristic function of the
above mentioned statistics. As a special case, among others, one gets back a well known limit theorem on the Wilcoxon statistic. (Received February 16, 1976.) (Author introduced by Professor J. Galambos.)

**734-F3**  
ANTOINE BRUNEL, Université P. et M. Curie, 4, pl. Jussieu, 75230 Paris and LOUIS SUCHESTON, Ohio State University, Columbus, Ohio 43210. Convergence of weak amarts: a probabilistic characterization of reflexivity.

Let $E$ be a Banach space, $(X_n)$ a sequence of $E$-valued random variables adapted to a sequence of increasing $\mathcal{F}_n$'s. Let $T$ be the collection of bounded stopping times of the $(X_n)$ is said to be of class $(B)$ if $\sup_T \|X_n\| < \infty$. $(X_n)$ is called a weak amart if there exists an element $z$ of $E$ such that for each $f$ in the dual of $E$, $\lim_T f(X_n) = f(z)$.

**Theorem 1.** A Banach space $E$ is reflexive if and only if every weak amart of class $(B)$ converges weakly almost surely to a random variable.

**Theorem 2.** This is the descending amart analogue of Theorem 1.


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**Topology**

**734-G1**  
DAVID J. SPROWS, Villanova University, Villanova, PA 19085. Homeotopy groups of 2-dimensional manifolds with one boundary component. Preliminary report.

Let $X$ be a compact connected 2-dimensional manifold with one boundary component. Let $Y$ denote the closed manifold obtained by sewing a disc to the boundary of $X$. By considering the isotopy classes of those homeomorphisms of $Y$ which keep a given point $p$ in $Y$ fixed, it is shown that if $X$ is not a Moebius band or disc, then the homeotopy group of $X$ is isomorphic to the group of automorphisms of $\pi_1(Y,p)$. (Received February 12, 1976.)

**734-G2**  
OKAN GUREL, IBM Corporation, 1133 Westchester Avenue, White Plains, New York 10604. Poincaré's bifurcation analysis.

H. Poincaré defined bifurcations (Acta Math. 7(1885)256-380)) such that, 1. Hessian must vanish, 2. Stability change must take place, 3. The number of equilibrium forms must alter. Poincaré's definition as applied to the analysis of differential equations can be used in a. Determining bifurcation parameters, b. Detecting bifurcations of a singular point to equilibrium forms, i.e., multiple singular points or periodic solutions, c. Analyzing a system qualitatively.

A complete study of van der Pol equation shows that vanishing of the parameter does not correspond to a bifurcation, because neither 1 nor 3 are satisfied. The bifurcation study of the system of E. N. Lorenz (J. Atmos. Science 20(1963)130-141) shows that i. There is only one bifurcation parameter out of three, ii. The two types of bifurcations, the first leading to multiple solutions, the second to a limiting surface, exist. Stability changes are limited, and can be classified completely following (O. Gurcl, Math. System Theory 7(1973)154-163). The types of equilibrium forms are classified according to (O. Gurel, Collective Phenomena 2(1975)89-97). (Received February 16, 1976.)

**734-G3**  
Howard H. Wicke, Ohio University, Athens, Ohio 45701. Sequentially complete mappings.

Let $X$ be a space. A mapping $f : X \to Y$ is called sequentially complete iff there exists a sequence $\langle B_n : n \in \mathbb{N} \rangle$ of bases for $X$ such that, for all $y \in Y$, if $B_n \subseteq B_{n+1} \subseteq B_n$, and $f^{-1}(y) \cap B_n \neq \emptyset$ for all $n \in \mathbb{N}$, then $f^{-1}(y) \cap \bigcap_n (B_n : n \in \mathbb{N}) \neq \emptyset$.

**Theorem 1.** The following kinds of maps are sequentially complete: (a) countably compact, (b) maps with $T_1$ range and countably Čech complete domain, (c) uniformly monotonically complete [Wicke-Worrell, Duke Math. J. 34(1972), p. 257] with $T_1$ range, (d) uniformly $\lambda$-complete [Wicke,

**Theorem 2.** Open continuous sequentially complete maps with regular domains preserve the following properties: primitive base, primitive quasi-completeness, base of countable order, monotone p-space (= $\beta_b$-space), monotone countable p-space (= $\beta_c$-space), primitive diagonal, diagonal a set of interior condensation, set of interior condensation, primitive set of interior condensation.

(Received February 16, 1976.)

#734-G4

DENIS BLACKMORE, New Jersey Institute of Technology, New Jersey 07102

The composition closing lemma, Preliminary report.

Let $M$ be a closed, smooth, finite-dimensional, riemannian manifold, and let $Diff(M)$ be the space of diffeomorphisms of $M$ carrying the uniform $C^1$ topology.

In giving sufficient conditions for structural stability [Notices Amer. Math. Soc. 22 (1975), 728-G5], and more recently for $C^*$-stability, we prove the following result which we call the "composition closing lemma".

**Theorem** - Given a tangent vector $u$, there exist arbitrarily small neighborhoods $U$ and $V$ of $g$ in $Diff(M)$ such that if $g_1, \ldots, g_n, \in U$, there exists an $f \in V$ satisfying

$$Df^n(u) = D(g_1 \ldots g_n)(u).$$

We show how, among other things, this leads to a shorter and more intuitive proof of the closing lemma than that given by Pugh [Amer. J. Math. 89 (1967), 956-1021]. (Received February 16, 1976.)

734-G5

W. W. COMFORT, Wesleyan University, Middletown, Connecticut 06457. Ultrafilters: some old and some new results.

- The "old" results will be selected from among those thought by the speaker to be the most important, or interesting, or elegant, or useful, or readable, in his book with S. Negrepontis "The Theory of Ultrafilters" [Springer-Verlag, 1974]. Samples may include: (a) determination of some natural cardinal invariants (density character, Souslin number, etc.) associated with certain spaces of ultrafilters; (b) the fact that for $\alpha \geq \omega$ there is $T \subset U(\alpha)$ with $|T| = 2^\omega$ such that if $S \subset T$ and $p \in S$ there is $A_p \in p$ so that $|A_p \cap A_{p'}| < \alpha$ whenever $p, p' \in S$ and $p \neq p'$, then $|S| \leq \omega_1$ (c) Kunen's theorem that the space $U(\alpha)$ of uniform ultrafilters on $\alpha$ has a subset of cardinality (at least) $2^\alpha$ of elements which are pairwise incomparable in the Rudin-Keisler pre-order; (d) directedness properties of some sets of ultrafilters. The "new" results to be discussed appeared, for the most part, after the above-mentioned book went to press. Examples are: (a) Louveau's characterization of (homeomorphs of) the closed subspaces, and Woods' characterization of the $C^*$-embedded subspaces, of $B(\omega)\setminus \omega$ (b) related results of Glazer; (c) the statement (see(c) above) that no infinite, compact $F$-space is homogeneous, (d) applications of Bernstein's $D$-compact concept ($D \in B(\omega)\setminus \omega$) by Ginsburg-Saks, and Saks, to compactness-like properties of products; (e) Hindman's theorem that if $\omega = \bigcup_{i \leq n} A_i$ with $n < \omega$ then there are $i < n$ and infinite $B \subset \omega$ such that every $\sum_{k=1}^m b_k$ is an element of $A_i$ (with $1 \leq m < \omega$, $b_k$ distinct elements of $B$). (Received February 16, 1976.)

734-G6


For definitions of $E$-compact spaces, $E$-non-extendable classes of spaces and $E$-defects of a space $X$ ($def_EX$), see S. Mrowka ["Further Results on $E$-compact Spaces I", Acta Math. 120(1968) 161-185]. We deal with several theorems on preservation of $E$-compactness and their corresponding rules concerning the $E$-defect. For instance, corresponding to the theorem "If $X$ contains a compact subspace $X_0$ such that every closed subspace of $X$ disjoint from $X_0$ is $E$-compact, then $X$ is $E$-compact" we have the rule "$def_EX \leq \Sigma (\text{card} \ FC(A) \cdot def_EA : A \in D(X_0))$ where $D(X_0)$ denotes the class of all closed subsets of $X$ which are disjoint from $X_0$ and $FC(A)$ denotes the class of closed subsets of $X$ which are $E$-functionally contained in $X_0$. (Received February 16, 1976.)
Let $\pi_*$ be the "2-primary component" of the stable homotopy ring of spheres, that is, to be precise, the ring localized at the maximal ideal generated by 2 and positive degree elements. Then, there is an Adams type spectral sequence

$$E_2^{*,*} = \text{Ext}_{\pi_*}^{*,*}(\mathbb{Z}_2, \mathbb{Z}_2) \Longrightarrow \mathcal{A} = \text{Steenrod algebra mod 2}.$$ 

In this paper, we give a detailed study of this spectral sequence and obtain some results on the Kervaire invariant problem, which are analogous to Barratt-Mahowald's theorems. (See Mahowald, M., Some remarks of the Kervaire invariant problem from the homotopy point of view, Proc. Sympos. Pure Math., vol. 22, Amer. Math. Soc., Providence, R. I., 1971) (Received February 17, 1976.)

Robert MacPherson, Brown University, Providence, Rhode Island 02912. Characteristic classes for singular spaces.

There is a general program to extend the techniques and results of differential topology to singular spaces. One aspect of this program which has been successful recently is finding singular analogues of characteristic classes. There exist singular versions of Whitney classes, Chern classes, Todd classes, and $Z$ classes. These give extension to singular spaces of the Riemann-Roch theorem and the Hirzebruch signature theorem. Although the various classes are defined for different categories of spaces and by completely different methods, they follow strikingly similar patterns of formal properties. (Received February 17, 1976.)

Miscellaneous Fields

Roger Howe, Yale University, New Haven, Connecticut 06520, On a connection between nilpotent groups, singularities, and oscillatory integrals.

A construction is given of certain nilpotent groups which are related to degenerate singularities in the same way the Heisenberg group is related to Morse singularities. These groups have square integrable representations. Their harmonic analysis is related to the study of oscillatory integrals. (Received February 11, 1976.)

The April Meeting in Reno, Nevada April 23 - 24, 1976

Algebra & Theory of Numbers


We first give a model theoretic version of Taylor's theorem characterizing classes of varieties defined by Malcev conditions (Alg. Univ. 3(1973) pp. 351-397). In analogy to $EC_\delta$, $EC_\sigma$ etc. we define $MC$ (strong Malcev conditions), $MC_\sigma$ (roughly, weak Malcev conditions), $MC_\delta$, $MC_\sigma\delta$.

Theorem 1. There is an $MC_\sigma\delta$ class of varieties which is not $MC_\delta$. (This answers questions of Neumann and Taylor.) We also give two additional examples answering a question (raised and answered by Taylor). Theorem 2. There are $2^{2^6}$ distinct $MC_\sigma$ classes. (Received January 19, 1976.)
**735-A2** Walter Taylor, Mathematics Department, University of Colorado, Boulder, Colorado 80302. Some results in topological algebra.

This talk will survey recent results in topological algebra, especially some from the author's abstracts 735-A158 and 735-A227. The emphasis is on varieties of topological algebras, for which we have an analog of Birkhoff's theorem. (Received February 5, 1976.)


The ideas of Petrie paths and "holes" introduced by Coxeter may be used to define operators D, P, and Hj over the class of regular maps in the following way: If M is a regular map, D(M) is the familiar dual of M, P(M) is the map whose faces are the Petrie paths of M, and Hj(M) is the map whose faces are the j-th order holes of M. The product DPD=PDP acting on M yields a map whose faces are the faces of M, but oppositely matched. We describe in detail the nature of each operation, and the use of the operations in the construction and taxonomy of regular maps. (Received February 12, 1976.)

**735-A4** GRANT A. FRAZER, University of Santa Clara, Santa Clara, CA 95053. Inequalities in the tensor product of semilattices.

In the tensor product A ⊗ B of semilattices A and B we consider inequalities of the form a ⊗ b ≤ Σ(a_i ⊗ b_i). For n = 1,2, we characterize such inequalities in terms of relations that hold among a, a_i in A and b, b_i in B. For n ≥ 3 we show that such a characterization is not possible. We have the following result for arbitrary n. If x is an element of a semilattice, we denote by (x) the principal ideal determined by x. If p is a lattice polynomial, we denote by p* the polynomial obtained by interchanging the lattice operations in p. Theorem. a ⊗ b ≤ Σ(a_i ⊗ b_i) if and only if there is an n-ary lattice polynomial p such that a ∈ p((a_1),..., (a_n)) and b ∈ p*((b_1),..., (b_n)). (Received February 13, 1976.)


The concept of regularity plays a central role in the theory of finite p-groups. A finite p-group G is called regular if for each pair a,b ∈ G, (ab)p = aPbPcP with c contained in the commutator subgroup of <a,b>. If p ≥ 5, then the property of regularity cannot be characterized by laws. In particular, the variety generated by all regular p-groups for a fixed p ≥ 5 is the variety of all groups. The proof of this fact requires that infinite cartesian products be employed in an essential way. Since some irregular p-groups arise from the direct product of finitely many regular p-groups we have the natural question: Do all irregular p-groups arise in this way?

Among the techniques used in recent years to shed some light on regular p-groups and the varieties they generate are strengthenings of the definition and consideration of irregular p-groups which are "minimal" in some sense. In this paper we will discuss recent results of J. R. J. Groves, A. Mann and the author which bear on this and related questions. (Received February 13, 1976.)

**735-A6** Bjarni Jónsson, Vanderbilt University, Nashville, Tennessee 37235. The variety covering the variety of all modular lattices.

The variety in question, call it M⁺, is known to be finitely based (B. Jónsson, Advances in Mathematics 14 (1976), 454-468). We obtain a set of identities in six variables or less that characterize...
Thus a lattice \( L \) belongs to \( M^+ \) iff every sublattice of \( L \) generated by six elements or less belongs to \( M^+ \). It is not known whether six can be replaced by a smaller number (five or perhaps four). (Received February 13, 1976.)

*735-A7 SIRMION FAJTLOWICZ, University of Houston, Houston, Texas 77004  A duality for Boolean spaces  Preliminary Report

For a Boolean space \( X \) let \( X^* \) denote the lattice of all clopen partitions of \( X \). If \( g: X \to Y \) is a continuous map of Boolean spaces, then \( g \) induces (by taking inverse images) the meet-homomorphism \( g^* \) of \( Y^* \) into \( X^* \). Moreover, \( \text{Im}^* \) is a filter in \( X^* \). A lattice \( L \) is said to be locally complete if, for every \( x \in L \), the principal filter \( (x) \) is isomorphic to a lattice of all partitions of a finite set. Theorem: Let \( f:L_1 \to L_2 \) be a meet-homomorphism of locally complete lattices such that \( \text{Im} f \) is a filter. Then there are Boolean spaces \( X_i, i = 1,2 \) and a continuous map \( g:X_2 \to X_1 \) such that \( L_1 \cong X_1 \) and \( g^* = f \). (Received February 13, 1976.)


The set of all subvarieties of the variety of associative rings (not necessarily with unity) is a groupoid \( G \) under the product: \( A \cdot B \) is the class of all rings that posses an ideal from \( A \) whose quotient belongs to \( B \). Let \( C \) be the variety of all commutative rings. Theorem 1. \( (C \cdot C) \cdot C \) properly contains \( C \cdot (C \cdot C) \), i.e., \( G \) is not power associative. Theorem 2. \( P \in G \) is idempotent if, and only if \( P \cdot A = P \lor A \) for all \( A \in G \). Theorem 3. If \( P \in G \) is idempotent, then \( (P \cdot A) \cdot B = P \cdot (A \cdot B) \) and \( A \cdot (P \cdot B) = (A \cdot P) \cdot B \) for all \( A,B \in G \). (Received February 11, 1976.)

*735-A9 Professor Nick H. Vaughan, North Texas State University, Denton, Texas, 76203. Some relations between classes of ideals in a domain.

Let \( D \) be an integral domain with \( 1 \neq 0 \) and quotient field \( K \). The set of primary ideals of \( D \) will be denoted by \( \mathcal{P} \) and the set of semi-primary ideals (i.e., ideals with prime radical) will be denoted by \( \mathcal{I} \). If \( \pi \) is a general ring property then an ideal \( A \) of \( D \) is called a \( \pi \)-ideal provided there exists a \( \pi \)-domain \( J \) such that \( D \subset J \subset K \) and \( A = AJ \). The classes of Dedekind, Krull, and Principal ideal domain ideals are denoted by \( \mathcal{D}, \mathcal{K}, \text{ and } \mathcal{P} \text{DD} \) respectively. Theorem 1. If prime ideals are almost Dedekind ideals, then \( \mathcal{D} \subset \mathcal{I} \) iff primes are chained.

Theorem 2. \( D \) is an almost Dedekind domain iff \( \mathcal{D} \subset \mathcal{P} \text{DD} \) and proper prime ideals of \( D \) are maximal. Theorem 3. \( \mathcal{I} \subset \mathcal{D} \) in the domain \( D \). (Received February 16, 1976.)

*735-A10 Ralph Howard and D. H. Potts, California State University, Northridge, California 91324. Axioms for quaternions. Preliminary report.

Let \( D \) be a division ring in which each element has a square root and for which there is an anti-automorphism \( * \) such that \( x** = x \) and \( yxx = xx \) for each \( x \) and \( y \). Let \( F \) be the set of elements of \( D \) which are invariant under \( * \). Then (i) \( F \) is an ordered field; (ii) if \( F \) is completely ordered then \( D \) is the field of complex numbers if \( D \) is commutative and is the quaternions otherwise. (Received February 16, 1976.)


Let \( A \) be an arbitrary algebra and let \( C(A) \) denote the congruence lattice of \( A \). Regard \( 0 \in C(A) \) as truth. For \( a,b \in A \), regard the principal congruence relation \( \Theta(a,b) \) as the truth value of the formula \( x = y \) when \( x \) is evaluated at \( a \) and \( y \) at \( b \). Truth values for arbitrary atomic formulas are similarly defined. From this starting point, a truth value in the
lattice \( C(A) \) can be assigned to every negation-free sentence \( S \) in the first-order language of \( A \), by a standard recursion on complexity of formulas. If this truth value is 0, let \( A \) be called a "congruence model" of \( S \). A typical theorem: If \( V \) is a congruence-distributive variety and \( S \) is a positive universal sentence, then the congruence models of \( S \) in \( V \) form a subvariety of \( V \), in fact, the one generated by the ordinary models of \( S \) in \( V \).

(Received February 17, 1976.)

735-A12 M. MARCUS, University of California, Santa Barbara, CA 93106 and M. ISHAQ, Laval University, Quebec, G1K 7M4, Canada. **The Range Multiplicity of an Hermitian Matrix.**

Let \( A \) be a linear operator on a finite dimensional unitary space. The range multiplicity, \( v_z(A) \), of an arbitrary complex number \( z \), associated with \( A \), is defined to be the largest integer \( k \) for which there exist \( k \) ortho-normal vectors \( x_1, \ldots, x_k \) such that \( (Ax_j, x_j) = z \), \( j=1, \ldots, k \) (M. Marcus: Adjoints and the Numerical Range, Linear and Multilinear Algebra, 3 (1975), 81-89). The purpose of the present paper is to compare \( v_0(A) \) and \( v_0(U) \) where \( A = PU \) is a polar factorization of \( A \).

**Theorem:** (a) If \( A \) is arbitrary, then \( v_0(A) \geq 0 \) implies \( v_0(U) = 0 \). (b) If \( A \) is hermitian and non-singular and \( \text{In}(A) = [p,q,0] \), then \( v_0(U) = 2 \min [p,q] \) and \( v_0(U) \leq 2 v_0(A) \). (c) If \( A \) is hermitian and \( \text{In}(A) = [p,q,n] \), then there exists \( U \), a unitary polar factor of \( A \) such that \( v_0(U) = 2 \min [p,q] + n \) if \( \max [p,q] > \frac{n}{2} \), and \( v_0(U) = 2 \frac{n}{2} \) if \( \max [p,q] < \frac{n}{2} \). We conjecture that in fact the inequality \( v_0(U) \geq 2 v_0(A) \) holds for arbitrary \( A \), but the proof, even for \( A \) normal, seems to pose substantial difficulties. (Received February 17, 1976.)

735-A13 George A. Grätzer, University of Manitoba, Winnipeg, Manitoba R3T 2N2. **The lattice of equational classes of weakly associative lattices.**

A review of recent results by E. Fried, Vera T. So's, R.W. Quackenbush, the author, and others dealing with covers of \( \mathcal{P} \), distributive identities, and so on. (Received February 17, 1976.)

*735-A14 R. PADMANABHAN, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2 **Equational Theory of Algebras with a Majority Polynomial.**

R.N. McKenzie (Math. Scand. 27 (1970) 24-38) has proved that a subvariety of the variety of all lattices is one-based iff it is improper or trivial. Call an identity of the form \( f(x_1, \ldots, x_n) = x \) an absorption identity. Both these varieties are definable by absorption identities and lattices admit a majority polynomial i.e. a ternary polynomial \( p(x,y,z) \) such that \( p(x,x,z) = p(x,y,x) = p(y,x,x) = x \). In this note we give a very simple proof of the result (mentioned in the above reference without proof) that any finitely based variety of algebras which admit a majority polynomial and definable by absorption identities is one-based. For the case of lattices, this yields a 'short' identity with only seven variables (Cf. Problem 17 of G. Grätzer, Lattice Theory, San Fransisco (1971)). (Received February 17, 1976.)

*735-A15 V. R. CHANDRAN AND R. PADMANABHAN, University of Manitoba, Winnipeg, Manitoba **Cyclic Identities and Associativity.**

It is well-known that in presence of commutativity and associativity the binary word \( x_1 x_2 \ldots x_n \) is equal to \( x_1 x_2 \ldots x_{n+1} \) for any permutation \( \sigma \) of \((1,2, \ldots, n)\). In this note we prove that the cyclic identity \( x_1 x_2 \ldots x_{n+1} = x_2 x_3 \ldots x_n x_{n+1} x_1 \) (where 'abc' stands for '(ab)c') implies \( x^n y^n = (yx)^n \), \( n \geq 2 \). Under the further assumption that \( x^n = x \) we get both commutativity and associativity. (Received February 17, 1976.)
George M. Bergman, University of California, Berkeley, CA 94720. Polynomial and rational identities in associative algebras.

(a) The theory of rings with polynomial identity is dominated by a certain small family of subvarieties of the variety of all associative rings. I shall state for universal algebraists the basic results characterizing these, and discuss some related points. (Summary available, 2 3 pp.) (b) I shall then discuss rational identities in division algebras. (Because ( )-1 is a partial operation we are no longer quite within the theory of varieties of algebras.) Amitsur has shown (J. Alg. 3(1966)304-359) that the rational identities of a division algebra D are determined by its polynomial identities (except conceivably when D has finite center and no polynomial identities). Yet the inclusions among the "varieties" of division algebras do not correspond to the inclusions among the corresponding varieties of associative algebras! (Preprints available: 23 and 45 pp.) (Received February 17, 1976.)

Amitai Regev, University of Chicago, Chicago, Illinois 60637. The T-ideal generated by the standard identity \( s_3[x_1, x_2, x_3] \).

Let \( F \) be a field, \( \text{Char } F = 0 \), and let \( F\langle x \rangle \) be the free (non-commutative) associative algebra on infinitely many generators \( \{x\} \). Let \( Q \subseteq F\langle x \rangle \) be a T-ideal, \( V_n = F[S_n] \) the group algebra of the symmetric group \( S_n \), identify \( eS_n \) with \( x_{e_1} \cdots x_{e_n} \) and let \( V_n \cap Q = Q_n \). Then \( Q_n \) is a left-ideal in \( V_n' \). \( V_n = Q_n \oplus J_n \) for some left-ideal \( J_n \) which determines a unique character \( \chi_n = \chi(I_n) \) of \( S_n \). We call \( \{x_n\} \) the co-character sequence of \( Q \) and \( \{c_n\} \), \( c_n = \dim J_n \), the co-dimensions of \( Q \). In the present paper we study the sequence \( \{x_n\} \) for \( Q = T_0(s_3[x_1, x_2, x_3]) \), where \( s_3[x] = \sum_{\sigma \in S_3} (-1)^\sigma \sigma \). By applying the theory of Young diagrams, of co-dimensions and Capelli identities, we prove Theorem: For \( n \geq 3 \), \( x_n = [n] + 2(n-1, 1) + \sigma[n-2, 2], \sigma = 0 \text{ or } 1, c_n = 2n-1 \) or \( c_n = \frac{n(n+1)}{2} - 1 \). Here \( [\lambda] \) is the character determined by \( \lambda \in \text{Par}(n) \). (Received February 17, 1976.)


For a variety of algebras, \( K \), define its congruence variety: \( \text{Con}(K) = \text{HSP}(\text{Con}(A) : A \in K) \).

The problems that arise in the relationships between varieties and their congruence varieties are of the following nature. Problem 1 (Mal'cev Condition characterization) For which lattice identities \( \delta \) can \( \text{Con}(K) \vdash \delta \) be characterized? Problem 2: If \( \delta_1 \equiv \delta_2 \) is defined by \( (\forall K)(\text{Con}(K) \vdash \delta_1 \iff \text{Con}(K) \vdash \delta_2) \), can one describe the equivalence classes of \( \equiv \)? Problem 3: What lattice varieties \( V \) are congruence varieties (i.e. \( \mathfrak{A}(K)(V = \text{Con}(K)) \)? The status of each of these problems will be given and possible methods for solutions discussed. (Received February 17, 1976.) (Author introduced by Professor Ralph N. McKenzie.)

Analysis

Bruce A. Barnes, Department of Mathematics, University of Oregon, Eugene, Oregon 97403. The thin operators relative to an ideal in a von Neumann algebra.

Let \( \mathcal{A} \) be a von Neumann algebra, let \( \mathcal{Z} \) be the center of \( \mathcal{A} \), and let \( \mathcal{R} \) be a closed ideal of \( \mathcal{A} \) with the property that if \( T \in \mathcal{A} \) and \( T\mathcal{R} = \{0\} \), then \( T = 0 \). The set of thin operators of \( \mathcal{A} \) relative to \( \mathcal{R} \), denoted by \( \mathcal{Z}_\mathcal{R} \), is the set of operators of the form \( Z + K \) where \( Z \in \mathcal{Z} \) and \( K \in \mathcal{R} \). Let \( \mathcal{R}_\mathcal{R} \) be the directed set of all projections in \( \mathcal{R} \).

We define a function \( \tau \) on \( \mathcal{A} \) which is a nonspatial form of the \( \tau \) function of Brown and Pearcy. Then we prove
THEOREM. For $T \in \mathcal{H}$, $\eta(T) = \limsup_{P \in \mathcal{P}} \|TP - PT\| = \text{dist}(T, J)$.


735-B2 Man-Duen Choi, University of California, Berkeley, CA 94720. The extensions of nuclear $C^*$-algebras.

This is a joint work with Edward Effros.

A nuclear $C^*$-algebra has an intrinsic characterization: the identity map can be approximated in the point-norm topology by finite-rank completely positive linear maps. We note that all type I $C^*$-algebras, AF $C^*$-algebras are nuclear.

Main Theorem: Let $A$, $B$ be $C^*$-algebras, $J$ be an ideal of $B$, and $\eta: B \to B/J$ be the quotient map. If $A$, or $B$, or $B/J$ is nuclear, then each completely positive linear map $\varphi: A \to B/J$ can be lifted to a completely positive linear map $\psi: A \to B$ such that $\eta \circ \psi = \varphi$.

A special case of the above theorem, in addition to a recent result of Voiculescu, shows that $\text{Ext}(A)$ is a group whenever $A$ is nuclear. (Received January 22, 1976.)


Three notions are considered for closed unbounded representations other than $\{0\}$ and $D(\pi)$ that are closed in the induced topology. (1) The representation $\pi$ has no reducing subspaces other than $\{0\}$ and $D(\pi)$ is closed in the induced topology. (2) Every vector $f \in D(\pi)$ is strongly cyclic, i.e., $\{\pi(\alpha)f\}$ is dense in $D(\pi)$ in the induced topology. (3) The weak commutant $\pi(\mathcal{M})' = \{A \in \mathcal{C}\}$. Powers (Commun. math. Phys. 21 (1971), 85-124) demonstrated the existence of a self-adjoint representation for which (3) was true but (2) was not true. Thus, the equivalence of these three notions for bounded representations of $C^*$-algebras does not carry over to the unbounded case. We have shown that for self-adjoint unbounded representations these notions are in the logical relation (1) $\iff$ (2) $\implies$ (3). Sufficient conditions for other logical implications between these three notions are given. (Received January 28, 1976.)

735-B4 Marc A. Rieffel, University of California, Berkeley, Cal. 94720 and Alfons Van Daele, University of Leuven, Leuven, Belgium. Tomita-Takesaki theory via bounded operators.

Let $\mathcal{M}$ be a von Neumann algebra on a Hilbert space $H$, with cyclic and separating vector $\omega$, and commutant $\mathcal{M}'$. Let $\mathcal{M}_H$ denote the collection of self-adjoint operators in $\mathcal{M}$. Let $K$ denote the closure of $\mathcal{M}_H \omega$, so $K$ is a real subspace of $H$. Equip $H$ with the real part of its inner product, and let $P$ and $Q$ be the orthogonal projections on $K$ and $iK$ with respect to this real inner-product. Let $JT$ be the polar decomposition of $P - Q$. It will be indicated how $J$ is the involution on $H$ from Tomita-Takesaki theory with the property that $JM = \mathcal{M}'$, and how the other main results of Tomita-Takesaki theory can be obtained in a similar way, using only bounded operators. (Received January 28, 1976.)
We prove the following theorem:

**THEOREM:** Suppose $X$, $Y$ and $Z$ are complex Banach spaces, $U$ and $V$ are open sets in $X$ and $Y$ respectively, and $x \in U$, $y \in V$. Suppose $f : U \to V$ and $K : V \to Z$ are holomorphic maps with $f(x) = y$, $K \circ f$ constant and range $f'(x) = \ker K'(y) \neq \{0\}$. Let $D$ be a domain in $\mathbb{C}^n$, $z \in D$ and $g : D \to Y$ be a holomorphic map with $g(z) = y$ and $K \circ g$ constant. Then there is an open neighborhood $W$ of $z$ and a holomorphic map $h : W \to X$ such that $h(z) = x$ and $g \mid W = f \circ h$.

We use this result to prove an Oka principle for sections of a class of holomorphic fibre bundles on Stein manifolds whose fibres are orbits of actions of a Banach Lie group on a Banach space.

(Received February 2, 1976.)
sequences of functions, under certain conditions, is also relation-continuous. As a basic tool in
the proof of main theorem is used the generalized Diagonal theorem (ibid.). As special cases of
this general theorem are obtained some theorems in the measure theory on the special lattices and
also the Nikodym type theorem for the countable additive set functions. In the theory of distribu-
tions the general theorem reduces to the known theorem of L. Schwartz, that the space of all distri-
butions is sequential closed. (Received February 10, 1976.) (Author introduced by Professor Darrell
Kent.)

735-B9 Raymond C. Roan, University of Michigan, Ann Arbor, Michigan 48109. Generators for some
subalgebras of \( L^1 \). Preliminary report.

An element \( f \) in a topological algebra \( A \) is said to generate \( A \) if \( \{ P(f) | P \text{ is a polynomial} \} \) is dense
in \( A \). We discuss the generators for the spaces \( D(p,a) = \{ f(z) = \sum a_n z^n | \sum (n+1)a_n P_c = 0 \} \). \( D(p,a) \) is
a Banach algebra if and only if either \( p = 1 \) and \( a > 0 \) or \( p > 1 \) and \( a = p-1 \). We obtain a convenient
characterization for the generators of these spaces by introducing the two Banach algebras \( H^w(p,\beta) = \{ h | h \in D(p,\beta) \text{ for all } f \in D(p,\beta) \} \) with the operator norm and \( A(p,\beta) = \text{the closure of the polynomials in } H^w(p,\beta) \). We show that \( f \) generates \( D(p,a) \) if and only if \( f \) generates \( A(p,a-p) \) and \( f' \) is cyclic in \( D(p,a-p) \), i.e. \( f'A(p,a-p) \) is dense in \( D(p,a-p) \). We use this result to characterize the generators of
many of the algebras \( D(p,a) \). For each \( a \), the space \( D(1,a) \) is a conjugate Banach space. Consequently, one can consider the problem of describing those functions \( f \) for which \( \{ P(f) | P \text{ is a polynomial} \} \) is dense in \( D(1,a) \) in the weak-* topology. One surprising result is that each weak-* generator of
\( L^1 = D(1,0) \) must be univalent on the closed unit disc. Our techniques also give us a description of
the generators of the algebras \( A^R_n = \{ f | f(m) \in A \} \), where \( A \) is the disc algebra, and \( S^P = \{ f | f \in H^w, P > 1 \} \). (Received February 11, 1976.)

735-B10 Professor J. K. Brooks, University of Florida, Gainesville, Florida and Professor P. W.
Lewis, North Texas State University, Denton, Texas. Tensor product representing measures.

Let \( E, X, Y \) be Banach spaces, let \( H \) be a compact \( T_2 \) space, let \( C(H) \) be the Banach space of con-
tinuous real valued functions on \( H \), and let \( E \) be the Borel \( \sigma \)-algebra of \( H \). We shall be concerned
with representing measures of the form \( m \otimes T : E \to B(\mathcal{E},X \otimes Y) \), where \( m : L : C(H) \to X \) is a strongly bounded
operator and \( T : E \to Y \) is an operator. These vector measures naturally occur as representing measures
of operators \( L \otimes T : C(H,E) \to X \otimes Y \), and they may be used to provide short proofs of the fact that
\( L \otimes T \) is absolutely summing (weakly compact) if each of \( L \) and \( T \) is absolutely summing (weakly compact).
Additionally, induced measures of the form \( \nu_f(A) = \int_A f \otimes T, f \in L^1(m) \), are studied. In particular,
if \( T \) is an isomorphism (isometry), then the mapping \( f \mapsto \nu_f \) is an isomorphism (isometry) of \( L^1(m) \).
Further, results on weak compactness of the operators represented by the measures \( \nu_f \) are obtained.
(Received February 13, 1976.)

*735-B11 J. William Helton, University of California, San Diego, La Jolla, California 92093.
Operator algebras and the propagation of singularities in solutions to a differential
equation.

One of the most basic theorems in modern partial differential equations describes the behavior of
singularities of solutions to a partial differential equation. The theorem is not difficult to prove
using standard techniques involving pseudodifferential operators, but it is fundamental. The order
0 pseudodifferential operators are an operator algebra which commutes modulo the compact operators.
The talk describes how a reasonable version of the singularities theorem holds in this abstract set-
If $C$ is a separable $C^*$-algebra, then $\text{Ext}(C)$ is defined as the set of (slightly modified) unitary equivalence classes of *-monomorphisms from $C$ to $A = \mathcal{L}(H)/K(H)$. Here $\mathcal{L}(H)$ is the algebra of bounded operators on the separable infinite-dimensional complex Hilbert space $H$ and $K(H)$ is the ideal of compact operators. $\text{Ext}(C)$ can also be defined by means of extensions of the form $0 \to K(H) \to E \to C \to 0$. $\text{Ext}(C)$ is a semigroup. In the special case $C = C(X)$, $X$ compact metric, R. G. Douglas, P. A. Fillmore, and the author showed that $\text{Ext}(C)$ is a group which has various nice properties, such as homotopy invariance. As a result, $\text{Ext}(C(X))$ can be characterized as $K$-homology. For general (non-commutative) C. D. Voiculescu has shown that $\text{Ext}(C)$ has an identity, M. D. Choi and E. G. Effros have shown that inverses exist if $C$ is nuclear, and the other nice properties are known for large classes of $C^*$-algebras. The talk will describe the properties that one hopes are valid and the extent to which they are presently known to the author. (Received February 13, 1976.)

If $G$ is a locally compact separable group, a primary representation $\pi$ is said to be square integrable if it is quasi contained in the regular representation. We shall define the formal degree of such a $\pi$ and establish orthogonality relations for its matrix coefficients, generalizing known results for irreducible $\pi$. If $\lambda$ is a homomorphism of $G$ to $R^+$, and $\varphi$ a normal semi-finite weight on the von Neumann algebra generated by $\pi$, we say that $\varphi$ is semi-invariant of degree $\lambda$ if $\varphi(\pi(g) \pi(g)^{-1}) = \lambda(g) \varphi(\pi^d)$, $g \in G$. We show that two such $\varphi$ are always proportional, and that $\pi$ is square integrable iff there is a $\varphi$ for $\lambda = \Delta$, the modular function of $G$, so that at least one matrix coefficient $\pi_n(g) = \varphi(\pi(g)^{-1}a)$ is a square integrable function on $G$ (left Haar measure). We show that $\varphi$ may be normalized so that orthogonality relations $\langle \pi_n(g) \pi_m(h) \rangle = \delta_{n,m} \varphi(\pi^d(a))$ hold for inner products of the matrix coefficients of $\pi$. We say that $\varphi$ is the formal degree of $\pi$, and if $\pi$ is irreducible, this reduces to the usual formal degree and orthogonality relations. (Received February 16, 1976.)

Let $(G, X)$ be a locally compact transformation group, $C^*(G, X)$ its associated $C^*$-algebra as defined by E. G. Effros and F. Hahn (Mem. Amer. Math. Soc., no. 75, 1967). When $G$ and $X$ are second countable and $G$ acts freely, $C^*(G, X)$ is a continuous trace algebra if and only if for all compact sets $K$ in $X$, \{ $x \in G : xK \not= \emptyset$ \} is relatively compact in $G$; when this condition holds, $C^*(G, X)$ is defined by a continuous field of Hilbert spaces (and so defines the trivial element of $H^3(X/G, \mathbb{Z})$). $C^*(G, X)$ can also be characterized using results of L. G. Brown, R. G. Douglas, and P. A. Fillmore (Lecture Notes in Math., vol. 395, 1973) in several cases when there is a finite set $Y$ of $G$-fixed points and $X \sim Y$ is a trivial $G$-principal fiber bundle over a compact space. The results are applied to the computation of the group $C^*$-algebras of "ax + b" groups over locally compact fields and of a number of solvable Lie groups. There are 1-parameter families (even containing both unimodular and non-unimodular groups) of non-isomorphic Lie groups with isomorphic group $C^*$-algebras, and only finitely many $C^*$-algebras arise from three-dimensional simply connected groups. (Received February 16, 1976.)

Techniques are developed to study the regular representation and $\alpha$-regular representations of measure groupoids. Convolution, involution, a modular Hilbert algebra, and local and global versions of the regular representation are defined. The associated von Neumann algebras, which are uniquely determined by the groupoid and the cocycle $\alpha$, provide a generalization of the group-measure space construction. When the groupoid is principal and ergodic, these algebras are factors. Necessary and sufficient conditions for the $\alpha$-regular representations of a principal ergodic groupoid to be of type

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I, II, or III are given, as well as a description of the flow of weights; these are independent of \( \sigma \).

To treat non-ergodic groupoids, an ergodic decomposition theorem is provided. (Received February 16, 1976.)

735-B16

JOHN B. BUTLER, Portland State University, Portland, Oregon 97207. On a calculus for nuclear kernels defined on a rigged Hilbert space. Preliminary report.

Let \( T \) be a selfadjoint operator on a rigged Hilbert space \( \Phi = H \subset \Phi' \) with left inverse \( T^{-1} \) and dmn \( T \supset \Phi \). There is a space \( H' \subset H = H' \subset \Phi' \), such that the spectral integral \( \int \psi(s) \otimes \psi(t) \, d\mu(t) \) associated with \( T \) converges on \( H' \otimes H' \). Assume \( \mu(1) \) to be absolutely continuous, \( \mu(0) = 1 \). Given a kernel \( P \in H' \otimes H' \) choose \( \psi_1(s)(t) = \psi_1(s) \otimes \psi(t) \), \( \psi_2(s) \otimes \psi_2(t) \). Let kernels \( \delta, \eta \in H' \otimes H' \) be defined by \( \delta(u)(v) = (v, u), \eta(u)(v) = (v, T^{-1}u) \). Given \( (s, t) \in \Phi \) and define product kernels \( \delta P, \eta P \) by \( \int \psi(s) \otimes \psi(t) \, d\mu(t) \), \( \int 1-J \psi(s) \otimes \psi(t) \, d\mu(t) \).

Let kernels \( \delta, \eta \in H' \otimes H' \) be defined by \( \delta(u)(v) = (v, u), \eta(u)(v) = (v, T^{-1}u) \). Given \( (s, t) \in \Phi \) and define product kernels \( \delta P, \eta P \) by \( \int \psi(s) \otimes \psi(t) \, d\mu(t) \), \( \int 1-J \psi(s) \otimes \psi(t) \, d\mu(t) \).

If \( \nu_{jk} = (T^j \otimes T^k)P \in H' \otimes H', j, k = 0, 1 \), \( \nu_{jk}(s, t) \) are locally Lipschitz in \( s, t \) and \( \int \nu_{jk}(s-1, t+1) \, d\mu(t) \) is square integrable with respect to \( d\mu(s) \). Then (a) \( (T^j \otimes T^k)(\delta P) = (\delta P)^j \), \( (\eta P)^j = \lim_{\epsilon_0} (\eta P)^j \), \( (T^j \otimes T^k)(\eta P) = (\eta P)^j \).

Then in the various generalizations of the Murray-von Neumann group-measure space factors, no examples arise from continuous groups which did not already come from countable groups. For such groupoids, questions concerning equivalence of various notions of similarity (and several natural new notions) can be answered affirmatively. Hyperfiniteness of measure groupoids is studied, and all the natural notions are shown equivalent. (Received February 16, 1976.)

735-B17

J. FELDMAN, P. HAHN, C.C. MOORE, University of California, Berkeley

Measure Groupoids coming from group actions.

Let the separable l.c. group \( G \) act on the analytic Borel space \( (S, \mathcal{G}) \) with quasi-invariant measure \( \mu \). The set \( \mathcal{G} = \{(s, sx) = s \in S, x \in G\} \) is an analytic equivalence relation; there is a measure \( \nu \) on \( \mathcal{G} \), the image of \( \mu \times Haar \) under the map \((s, x) \mapsto (s, sx)) : S \times G \to S \times S \), which makes \((\mathcal{G}, \nu)\) a "principal measure groupoid" in Mackey's sense. Theorem: \( \exists \) a Borel set \( E \subset S \) such that \( \{sx : s \in E, x \in G\} \) is connul and \( E \) intersects each equivalence class in a countable set; this generalizes Forrest's theorem on free ergodic actions. Corollary: Every ergodic such \((\mathcal{G}, \nu)\) is isomorphic to a product of a transitive one and one which comes from a countable group action.

Corollary: In the various generalizations of the Murray-von Neumann group-measure space factors, no examples arise from continuous groups which did not already come from countable groups. For such groupoids, questions concerning equivalence of various notions of similarity (and several natural new notions) can be answered affirmatively. Hyperfiniteness of measure groupoids is studied, and all the natural notions are shown equivalent. (Received February 16, 1976.)

735-B18


In 1951 K. Oka proved that a hypersurface has a normal point at \( x \) provided that the singular set has codimension at least two at \( x \). S. Abhyankar and W. Thimm later showed that one could replace "hypersurface" by "complete intersection". The author presents the following generalization which gives Oka's result for perfect varieties as a corollary.

Theorem A reduced analytic space is normal if \( \dim \text{Sing}(X) \leq \text{codim}(X) - 2 \).

An application of this result gives an easy example of a non-perfect normal variety. (Received February 17, 1976.)

735-B19

John Ernest, University of California, Santa Barbara, 93106. Charting the Operator Terrain.

The purpose of this talk is to suggest a cartographic procedure for bringing some organizational sense to the prodigious task of exploring and describing the vast and varied terrain of bounded operators on a separable Hilbert space. Our cartographic system is essentially an adaptation to single Hilbert space. Our cartographic system is essentially an adaptation to single Hilbert space. Our cartographic system is essentially an adaptation to single Hilbert space. Our cartographic system is essentially an adaptation to single Hilbert space.
tor version is cast into a mold which exhibits it as a natural generalization of the spectral theorem and the spectral multiplicity theory for normal operators. To each bounded operator we associate a certain set of (equivalence classes of) "factor operators," which is called the quasi-spectrum of the operator. An operator is then determined, up to unitary equivalence, by a triplet consisting of its quasi-spectrum, a finite measure class on that quasi-spectrum, and a "spectral multiplicity function" defined on the measure classes absolutely continuous with respect to the first mentioned measure class.

The spectral theorem, and the spectral multiplicity theory for normal operators are (very) special cases of this general theory. We discuss the relationship of the quasi-spectrum of an operator to other recent definitions of operator valued spectra, specifically the reducing matricial spectrum of Pearcy and Salinas and the reducing operator spectrum of Donald Hadwin. (Received February 17, 1976.)

**735-B20** Philip Green, University of California, Berkeley, California 94720. Morita Equivalence of C*-Algebras.

The following results concern N. Rieffel's concept (Advances in Math, 13 (1974), 176-257) of (strong) Morita equivalence of C*-algebras: Morita equivalence classes of approximately finite dimensional C*-algebras are in 1-1 correspondence with isomorphism classes of certain ordered Abelian groups (the "Elliott groups" of the algebras). For any locally compact Hausdorff space X the Morita equivalence classes (over C_0(X)) of continuous trace C*-algebras with spectrum homeomorphic to X form a "generalized Brauer group" under the tensor product operation. If (G, μ) and (H, ρ) are two locally compact transformation groups such that the actions of G and H commute and are free, and such that the subsets {x ∈ G (resp. H) | gK ∩ X ≠ ∅} of G (resp. H) are relatively compact for each compact subset K of X, then the transformation group C*-algebras C*(G, μ/H) and C*(H, ρ/G) are Morita equivalent. (Received February 17, 1976.)

**735-B21** ROBERT P. FEINERMAN, Lehman College, Bronx, NY 10648, and ROBERT B. KELMAN, Colorado State University, Ft. Collins, CO 80523, Dual orthogonal series with applications in cryptology.

A dual orthogonal series problem in which the ratios of the modifiers oscillate between finite, nonnegative limits is considered in an abstract Hilbert space. Existence and uniqueness of the solution are established using arguments based on properties of appropriately constructed linear functionals. Dual orthogonal series with oscillatory modifiers occur in problems of communication theory and an example will be presented showing the use of these series to encode messages for cryptographic purposes. (Received February 17, 1976.)


Let y(x) ≠ 0, y'(n) ≠ f(x) (D) be an nth order linear differential equation with constant coefficients A_j where A_0 ≠ 0. Let the forcing function f(x) be analytic at x = 0 and let the roots r_1, r_2, ..., r_n of the characteristic polynomial be distinct. If y = \sum_{n=0}^{∞} a_n x^n is a particular solution of (D) then conditions are given which allow the application of Runge's transformation of series to the solution y. The particular solution can then be written as y = y_h + \sum_{n=0}^{∞} \alpha_n x^n, where y_h is a solution to the corresponding homogeneous differential equation and \sum_{n=0}^{∞} \alpha_n x^n converges more rapidly than \sum_{n=0}^{∞} a_n x^n. The method requires that the Laplace transform of the forcing function f(x), be analytic at infinity and that the reciprocal of at least one of the roots r_j be within the disk of convergence of the power series expansion of the Laplace transform of f about the point at infinity. (Received February 17, 1976.)

**735-B23** Ramesh Gangolli, University of Washington, Seattle, Washington 98195. Symmetry of group algebras and a Tauberian theorem.

Let G be a locally compact motion group. i.e., G is the semi-direct product of a closed abelian
normal subgroup $A$ and a compact subgroup $K$, acting on $A$ by inner automorphisms. **Theorem 1:** The convolution algebra $L_1(G)$ is symmetric. In other words, for each $f \in L_1(G)$, $f^* \ast f$ has nonnegative spectrum.

As a corollary, one gets the following generalization of Wiener's Tauberian theorem: **Theorem 2:** Every proper closed two-sided ideal in $L_1(G)$ is annihilated by a unitary representation of $G$, lifted to $L_1(G)$ in the usual way.

The proof involves techniques of the theory of spherical functions on $G$, in the manner of Godement, together with a comprehensive theorem of Fell concerning the uniformly bounded Banach representations of $G$. (Received February 17, 1976.)

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**Applied Mathematics**

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**Part I.** The following common end-game situation is completely analyzed: There are two players, $A$ and $B$, left. $A$ has a possible "lock" (his concealed cards could make his hand unbeatable). $B$ assesses that $A$ has the "lock" with probability $p$. If $A$ does not hold the "lock", then $B$ has the best hand (his hand, for all practical purposes, is exposed). It is $A$'s bet, there are $P$ chips in the pot, the betting limit is $L$, and each player has $C$ chips left. Part II. "Betterness" for completed poker hands is well defined, but what about hands in the course of a game? Several possible definitions are given, one of which is intransitive. Several 3-person hands are analyzed, including one which is very paradoxical. (Received February 12, 1976.)

**Part II.** For the description of quantum lattice systems the appropriate mathematical structure seems to be uniformly hyperfinite (UHF) algebras. These algebras, first introduced by Glimm, are generated by an increasing sequence of full hermitian $(n \times n)$-matrix algebras. Systems at thermal equilibrium are described by states satisfying a condition known as the KMS condition. Recently Sakai presented a fairly general and abstract sufficient condition for the uniqueness of a KMS state of a UHF algebra. Then Araki considerably strengthened Sakai's result using the notion of relative entropy which has been recently studied by Araki. These results generalize and make precise the general "folk theorem" (which had been proved by Ruelle and others in special cases) that one dimensional systems do not have phase transitions. (Received February 12, 1976.)

**Part III.** A Bayesian approach to investment involves knowledge of the distribution of future market events (payoffs) and of future world events (extraneous dependent variables). Under certain general conditions on these distributions, the simple one-stage look-ahead rule is optimal for certain well-studied utility functions in finite horizon problems. Generalizing results of J. L. Kelly, Jr. and R. E. Murphy, Jr., one can obtain explicit optimal betting systems for games
in which one and only one outcome wins. The value of hedging is still obvious, but seen to be a
function of the parameters of the general model. (Received February 17, 1976.)

Logic and Foundations

735-EL PHILIP OLIN, York University, Downsview, Ontario, Canada M3J 1P3.
Elementary properties of free products, Preliminary report.
A simple absorption property, exemplified by $A \ast B \trianglelefteq A$ where $A$ is an infinite free power
of $B$, is used to obtain results of rather opposite kinds.

Let $V$ be the variety of all groups of exponent 3 and let $F_\alpha$ denote the $V$-free group on $\alpha$
generators. (So $F_1 = \mathbb{Z}_3$.) A group $G$ is constructed with $G \in V$, $G \triangleright F_\omega$, and $G \ast F_1 \not\triangleright G$. Since
$F_\omega \ast F_1 \not\triangleright F_\omega$, we have $F_1 \ast F_\omega \not\triangleright F_1 \ast G$. Hence the $V$-free product operation does not preserve $\leq$.

A question of G. Grätzer is answered by constructing a distributive lattice $A$ ( of
cardinality $\aleph_\omega$ ) such that for every distributive lattice $B$, $A \ast B \triangleright A$. Hence $(VB_1)(VB_2)$
$\not\triangleright [B_1,B_2 = A \ast B_1 \ast A \ast B_2]$. This latter property of $A$ is a property of finite distributive
lattices but one which seems to happen "infrequently" for infinite distributive lattices.
The proof uses the downward LST theorem for the infinitary logic $L_{\omega,\omega}$ as well as a
generalization of the method of games. (Received January 19, 1976.)

735-B2 George McNulty, University of South Carolina, Columbia, South Carolina 29208.
Structural Diversity in the Lattice of Equational Theories, III.
$\Pi_\kappa$ is the lattice of all equivalence relations on the cardinal $\kappa$. Fix a similarity type of algebras
and let $T$ be an equational theory of this type. A set $\Phi$ of terms satisfies the $T$-term condition
provided $\phi = \psi = \theta$ whenever $\phi, \psi, \theta, T\phi \theta$, and some substitution instance of $\psi$ is a subterm of $\theta$.
Theorem 1. If $T$ is an equational theory, $\Phi$ satisfies the $T$-term condition, and $|\Phi| = \kappa$, then $\Pi_\kappa$ is
isomorphic to an interval in the lattice of equational theories extending $T$. Theorem 2. For each
$k \in \omega$ there is a finite commutative semigroup $S$ such that $\Pi_k$ is isomorphic to an interval in the lattice
of equational theories extending the equational theory of $S$. A similarity type is strong if it induces
an operation symbol of rank at least two. Theorem 3. Let $T$ be an equational theory in a strong
similarity type. If $T$ is not true in every algebra and $k \in \omega$, then $\Pi_k$ is isomorphic to an interval in the lattice
of subtheories of $T$. Example. $\Pi_\omega$ is not isomorphic to any interval in the lattice of subtheories of the equational theory of commutative semigroups. (Received January 23, 1976.)

Statistics and Probability

*735-FL STEPHEN A. BOCK, California State College, Dominguez Hills, Ca. 90747.
A Version of the Erdös-Rényi Law of Large Numbers for Triangular Arrays.

$\{X_{nk} : 1 \leq k \leq N, 1 \leq N < \infty\}$ is a triangular array of row-wise independent random
variables having $E(X_{nk}) = 0, \Sigma k=1^N \text{Var}(X_{nk}) = \Sigma k=1^N \sigma_{nk}^2 = 1$, and possessing moment-generating
functions. Under some uniformity conditions on the variances and the m.g.f.'s and a
relationship between the rows which holds almost surely, the almost sure behavior as
$N \rightarrow \infty$ of

$$
\sum_{k=n+1}^{n+K} X_{nk}
$$

is studied, where $K$ depends on $N$. Here $\psi(x)$ is a function such that, as $x \rightarrow \infty$, $x^{-1} \psi(x) \rightarrow 0$ and $(\log x)^{-1} \psi(x) \rightarrow \infty$. It is shown that, if $K$ is of the order of
$\psi^{-1}(2 \lambda^{-2} \log N)$, then $\sum_{k=n+1}^{n+K} X_{nk}$ is a.s. close to $\lambda$ for large $N$. The theorem
generalizes a 1975 result (Proc. Amer. Math. Soc. 48, 438-446) for i.i.d. random variables.
(Received December 15, 1975.)

Peter A. Griffin, CSU, Sacramento, Sacramento, California 95819. Use of bivariate normal approximations to evaluate card-counting systems in blackjack.

Correlation is concerned with the prediction of one variable from knowledge of another. The card-counter in a game of blackjack wishes to assess favorability for a particular action using his knowledge of a parameter which reflects, usually imperfectly, the composition of the unplayed subset of cards he is confronting. The Gauss-Markov theorem allows an intuitive approximation of multcard interactions by linear effects. A limit theorem and the results of extensive simulations justify the treatment of the distribution of favorability as approximately normal. The efficiency of a card-counting system depends on its ordinary correlation coefficient with the linear effects. Application of the theory permits rapid evaluation of any system without lengthy simulations. The necessity of increasing the number of parameters to improve efficiency and the technique for so doing are understood in this context. (Received January 26, 1976.) (Author introduced by Professor Fred Krakowski.)

MARJORIE G. HAHN, University of California, Berkeley, California 94720. Central limit theorems for [0,1]-valued random variables. Preliminary report.

Let \([0,1]\) be the space of real-valued functions on the unit interval which are right continuous with left limits at each point. Endow \([0,1]\) with the usual Skorohod topology. Let \(\{X_n, n \geq 1\}\) be a sequence of independent identically distributed \([0,1]\)-valued random variables with mean 0 and \(EX^2(t) < \infty\) for all \(t \in [0,1]\). \(X_1\) is said to satisfy the central limit theorem (CLT) in \([0,1]\) if the measures induced by \((X_1 + \ldots + X_n)/n^{1/2}\) converge weakly to a Gaussian measure on \([0,1]\). Sufficient conditions are found for \(X_1\) to satisfy the CLT. As a consequence, all stochastically continuous independent increment processes and all finite state stochastically continuous Markov chains with sample paths in \([0,1]\) satisfy the CLT. (Received February 16, 1976.)

Thomas M. Cover, Stanford University, Stanford, California 94305. Optimal Gambling Systems.

An investigation is made of sequential gambling systems which perform universally well against all stochastic sequences. It can be shown for a fair casino that a random variable capital \(S\) can be achieved by a sequential gambling scheme with initial capital \(S_0 = 1\) if and only if \(S \geq 0\), a.e., and \(E S \leq 1\). Thus all reasonable distributions are possible. The game theoretic question of how to win more money than one's opponent in a gambling casino, given the same initial capital, is solved for the optimal distribution. These results will be applied to the sequential weather prediction game. It is shown that the proportional gambling system is essentially optimal in the sense that the predictor knowing the true statistics will win more, with probability greater than or equal to one half, than his opponent. Finally, the notion of universal algorithmic complexity due to Kolmogorov, Chaitin, and Schnorr will be shown to yield a universal gambling system which causes one's capital to tend to infinity with probability one against any computable ergodic process, where the rate of growth is \(1 - H\), and \(H\) is the entropy of the process. (Received February 17, 1976.)

Topology

M. Schroeder, University of Waikato, Hamilton, New Zealand. Marinescu structures and \(c\)-spaces.

For any \(T_3^{\omega}\) topological space \(X\), the set \(C_X\) of all its continuous real-valued functions was made into a Marinescu space \(C^1_X\) by E. Binz, W. Feldman and others. A more geometrical internal description of \(C^1_X\) is given here, with the advantage of applying to any space \(X\) at all. With one exception, theorems concerning \(C^1_X\) need little alteration in the general case; the exception raises interesting questions about compactness. In addition, one construction allows
two rather different internal characterizations of c-spaces to be reconciled. (Received February 5, 1976.) (Author introduced by Professor Darrell Kent.)

735-G2 Roman Frič, Transport College, Žilina, Czechoslovakia

On Continuous Characters of Borel Sets

We study sequentially continuous complex-valued functions \( \alpha \) on rings of Borel sets of the real line such that \( \alpha(A+B) = \alpha(A) \cdot \alpha(B) \), where \( A + B = (A - B) \cup (B - A) \) is the symmetric difference of \( A \) and \( B \). We determine all such functions. The character group and the second character group are defined and a duality theorem of the Pontryagin - van Kampen type for the Borel field generated by semiclosed intervals is proved. We also show that if \( \mathcal{A} \) is the field of all Borel sets of order \( \omega \), \( 1 \leq \omega \leq \omega_1 \), then the cardinal number of \( \mathcal{A} \) and the cardinal number of the second dual group of \( \mathcal{A} \) are different. (Received February 11, 1976.) (Author introduced by Professor Darrell C. Kent.)

735-03 Petr Kratochvíl, Math. Institute of the Czechoslovak Academy of Science, 115 67 Prague, Czechoslovakia, Multisequences and Measure Theory. Preliminary report.

Let \( N \) denote the discrete space of all natural numbers; let \( \Sigma = N^N \) be the topological power of \( N \) to the abstract set \( N \). Let \( L \) be a set. A mapping \( \theta \) is said to be a multisequence in \( L \), if \( \theta \) is a continuous mapping of \( \Sigma \) into \( L \) provided with the discrete topology. Thus, multisequences are a generalization of double sequences to higher multiplicities. Convergence of multisequences, associated filter of sections, the notion of submultisequence, and other concepts are developed.

The general theory is applied to measure theory. The applications are based on the following statement. Let \( x \) be an element of a \( \sigma \)-algebra generated by a set algebra \( A \). Then there is a multisequence in \( A \) converging to \( x \). (Received February 16, 1976.) (Author introduced by Professor Darrell C. Kent.)

735-G4 James M. Irwin, 952 King Henry Way, El Dorado Hills, California 95630

Sequential Cauchy Spaces.

A sequential Cauchy space \( (X, L) \) is a set \( X \) along with a set \( L \) of sequences on \( X \) subject to certain axioms. A sequential Cauchy structure induces a sequential convergence structure \( (L\text{-space in the sense of Frechet}) \), and is induced in a natural way by a UL structure in the sense of A. Goetz. A completion theory with universal property is developed for a class of sequential Cauchy spaces. This gives rise to a completion theory for an associated class of UL spaces. (Received February 16, 1976.) (Author introduced by Professor Darrell C. Kent.)


Given a vector lattice \( V \), a family of extended real-valued functions is defined on \( V \) lattice-theoretically. Those functions whose restrictions to a principal order ideal \( \langle x \rangle \) are finite generate a locally convex topology on \( \langle x \rangle \). This system results in a Marinescu structure for \( V \) which is interpreted for the vector lattice \( C(X) \). Among the consequences is a lattice-theoretic description of the local uniform convergence structure when \( X \) is Lindelöf.

(Received February 17, 1976.)
Let $C_c(X)$ denote the space of continuous real-valued functions on $X$ with the topology of compact convergence. Let $B$ denote the collection of all solid sublattices containing 1 in $C(X)$ which are complete metrizable order partition spaces. (See Abstract 731-46-63, these Notices 23(1976), for a definition of an order partition space.) A relationship is obtained between the convergence space inductive limit of $B$ and two known Marinescu structures on $C(X)$. This result is used to establish sufficient conditions for the locally convex inductive limit of $B$ to coincide with $C_c(X)$. (Received February 17, 1976.)

This study is a continuation of an earlier paper (to appear in Bull. Calcutta Math Soc) by the same author. A singly generated convergence structure is the convergence structure associated with a uniform convergence structure generated by a single symmetric filter on the product (coarser than the diagonal) and all its compositions. Various sufficient conditions are given which insure that a given convergence space be singly generated. Connections with other properties (e.g. pseudo-convergence, solid, K-regular) are given and applications to compactification and completion are made. A typical result characterizes singly generated spaces by means of an inductive limit of spaces with a "regularity" type condition. Two open questions from the earlier paper are resolved. (Received February 17, 1976.)

There is a natural one-to-one correspondence between minimal Hausdorff uniform convergence structures and minimal Hausdorff convergence structures. The following statements about a Hausdorff uniform convergence space $(X, I)$ are equivalent: (1) $(X, I)$ is minimal uniformly regular and totally bounded; (2) $I$ is the coarsest member of its convergence class, and the convergence structure associated with $I$ is a semi-minimal regular Hausdorff topology. (Received February 17, 1976.)

Let $X$ be a Hausdorff locally convex topological vector space, and let $L_X$ denote the set of all continuous linear mappings from $X$ into the reals. The space $L^0_0X$ denotes $L_X$ endowed with the compact-open topology. A study is made of the spaces, $X$, which have the property that the natural mapping from $X$ into $L^0_0(L^0_0X)$ is an embedding. (Received February 17, 1976.)
An economy is composed of households and firms. For any vector of commodity prices, firms maximize profit, and this determines their demand for inputs and their supply of outputs. Households hold resources, including potential labor service; in addition they own shares in the firms. For any vector of commodity prices, the set of choices available to each household is defined, and so both the demand for consumer goods and the supply of labor services by each household depend on prices. An equilibrium price vector for an economy is a price vector which equates the supply of each commodity (resource, capital good, or consumer good) with the demand for that commodity. If there are \( n \) commodities, then there are \( n \) prices and \( n \) demand and supply functions. Thus, the problem of finding an equilibrium price vector is reduced to solving a system of \( n \) equations in \( n \) unknowns. A major concern of mathematical economics during the past twenty-five years has been to provide conditions under which there exists a solution to the above problem. Extensions of the Brouwer Fixed Point Theorem have played a central role in the analysis. We will review some leading results concerning the existence of general economic equilibrium and suggest areas for further research.

References

G. Debreu (1959), *Theory of value: An axiomatic analysis of economic equilibrium*, Cowles Foundation for Research in Economics at Yale University, Monograph 17, Wiley, New York; Chapman & Hall, London, MR 22 #1447. (This is a classic monograph on the theory of general economic equilibrium.)

K. J. Arrow and F. H. Hahn (1971), *General competitive analysis*, Holden-Day, San Francisco, California. (This is a standard text and reference work on the subject.)

E. Malinvaud (1972), *Lectures on microeconomic theory*, North-Holland, Amsterdam. (Theories of general equilibrium have at their foundation theories of household and firm behavior. Microeconomics is the study of household and firm behavior and this is a standard text on that subject.)


A very selected reading list is:


to the population, or invested, thus producing more of itself. This latter operation is described as follows: if \( x \) units are invested as input in period \( t \) then \( y = f(x) \) units emerge as output in period \( t + 1 \). Here \( f \) is called the production function of period \( t \). We suppose at time \( t = 0 \) we are given \( y_0 \) units of goods. Definition. A program (of consumption and investment) is a sequence \((c_t, x_t, y_t)\) such that (1) \( c_t + x_t = y_t \), (2) \( y_{t+1} = f(x_t) \).

Note that from (1) and (2) the program is completely determined by the consumption sequence \((c_t)\). Definition. The program \((c_t)\) is called efficient if for any program \((c_t')\) (3) if \( c_t \geq c_t' \), then \( c_t = c_t' \). (In words, any other program which supplies more consumption in some period must supply less in some other.)

We will be concerned with determining when a given program is efficient. The key notion is the following: Definition. The programs \((c_t', x_t', y_t')\) will be called competitive if there exists a nonnegative sequence \((p_t')\) called prices such that (4) \( p_{t+1}' f(x) - p_t x \) is maximized at \( x_t' \). The condition means that given prices \( p_t \) and \( p_{t+1}' \) the input to production \( x_t \) is chosen so as to maximize profits. The ratio \( \phi_t = p_t / p_{t+1}' \) is called the interest factor. A typical elementary result is the following: Theorem 1. The program \((c_t)\) is efficient if it is competitive and (5) \( \lim_{t \to \infty} p_t x_t = 0 \). Under suitable conditions on \( f \), notably concavity or "diminishing returns" one can show that conversely any efficient program is competitive. A program is called a steady state if \((c_t', x_t', y_t') = (c_0, x_0, y_0)\) for all \( t \). Theorem 2. The steady state \((c_0, x_0, y_0)\) is efficient if and only if it is competitive with constant interest factor \( \phi \) at most equal to 1. One of our main results is to generalize the above result to more realistic technologies with many goods and many industries. For such cases in the steady state one can again define an interest factor which turns out to be a generalized eigenvalue whose existence is proved in a theorem which is a broad generalization of the Perron-Frobenius theory of nonnegative matrices. The fundamental theorem of the subject states roughly that a steady state is efficient if and only if the associated interest factor \( \phi \) is at least as large as its growth factor \( \gamma_t \). (We include among steady states those in which all quantities grow at the same rate \( \gamma_t \).)

Returning to the nonsteady state theory condition (5) of Theorem 1 is in general not necessary. However, for a wide class of functions \( f \) it has been shown that the following much weaker condition is both necessary and efficient: (6) \( \lim_{t \to \infty} 1/p_t x_t = \gamma \). Generalization of this result to the multi-good, multi-industry case will be described. (Received February 23, 1976.)

Reading List


SC 76-15 ANDREU MAS-COLELL, Department of Mathematics and Economics, University of California, Berkeley, California, 94720. The theory of economic equilibrium from the differentiable point of view.

The lecture will review the uses of differential methods in the theory of resource allocation and price determination. The use of calculus has a long tradition in economic theory (see P. A. Samuelson, Foundations of economic analysis, Harvard University Press, Cambridge, Mass., 1947, MR 10, 555), but in the 1950's and 1960's under the combined influence of the theoretical and practical breakthroughs provided by linear programming, game theory, and the inter-industry Leontief input-output analysis, differential theory was relegated in favor of mathematical techniques grounded on convexity. In the last six or seven years, differential theory has made a steady comeback (of course, it never quite disappeared) as an analytical tool; the availability of the modern techniques of differential topology have been instrumental. The pioneering contributions were by G. Debreu and S. Smale, and the influence of J. W. Milnor's Topology from the differentiable viewpoint (the University Press of Virginia, Charlottesville, Virginia, 1965, MR 37 #2239) was nonnegligible. In the lecture, we will emphasize the new developments, and focus attention on the problems whose successful treatment has required a differentiable viewpoint and has prompted the current flowering of calculus methods in economic theory; this includes among others the problem of determinateness of equilibria which gives rise to Debreu's theory of regular economies, the problem of computation of equilibria which has stimulated Smale's generalized Newton method, the problem of establishing sharp correspon-
dences between economic (competitive equilibria) and game-theoretic concepts (core, value), etc. A selection of topics will unavoidably have to be made; fortunately some of them will be covered from different perspectives in other lectures.

References

For the mathematical background, a book like the following will be quite adequate:


For the economic applications, a good introduction is:


SC 76-16 ROBERT J. AUMANN, Department of Economics, Hebrew University, Jerusalem, Israel and Department of Economics, Stanford University, Stanford, California 94305. Some game models in economics.

Game theory is the analysis of situations involving several people, whose interests are nonidentical but not necessarily diametrically opposed; i.e., of situations involving a mixture of competitive and cooperative elements. These situations are called games, and the people involved are called players. Almost any economic problem, ranging from a bargaining session between two players to a whole economy with millions of players, may be viewed as a game. Other examples come from political life (congress, the UN, an election). One seeks general principles that apply to all such situations; one then goes back and sees what such principles might imply in any given case. In several instances well-known dicta in particular disciplines such as economics have turned out to be special cases of more general game-theoretic principles. In other instances the more generally valid principles have provided fresh insights into the particular applications. Thus game theory serves as a unifying force in the social sciences. In this talk we will introduce three basic game-theoretic notions—the Core, the Shapley Value, and the von Neumann-Morgenstern Solution. These provide three different but complementary approaches to game problems—a highly competitive approach, an approach based on the notion of compromise, and an approach which gives us some insight as to how society might structure itself in a stable fashion. Having defined these concepts, we will apply them to some economic problems, particularly those involving large numbers of players. (Received February 24, 1976.)

Reading List


SC 76-17 STEPHEN SMALE, University of California, Berkeley, California 94720. Computation and existence of Walras equilibria.

Two closely related and central aspects of general equilibrium theory of mathematical economics are treated. One is the existence of equilibria. The definitive exposition of the existence theorems as done traditionally is the book by Debreu [1]. The second subject is the computation of economic equilibria. This has been developed more recently and is the subject of the book by Scarf [2] (in collaboration with T. Hansen). Although in these books differentiability is de-emphasized or eliminated, our account will be based on the calculus. For further background reading, one can see the calculus oriented articles in the Journal of Mathematical Economics. (Received February 23, 1976.)

Reading List

The March Meeting in Urbana, Illinois
March 15 – 20, 1976

733-A30 Carla C. Neaderhouser and George B. Purdy, Texas A&M University, College Station, Texas 77843. Threshold functions for Random Hypergraphs. Preliminary report.

Erdős and Rényi proved the following theorem: The threshold function for the property that a random graph should contain at least one subgraph isomorphic to a connected balanced graph with k vertices and 2k edges is $n^{2-k/t}$. The authors extend this result to random hypergraphs. (Received January 12, 1976.)

733-B35 Vadim Komkov, Texas Tech University, Lubbock, Texas 79409. Accuracy and Variational principles for dissipative elastic systems.

A specific problem of accuracy, of elastic system (beams, plates, axially symmetric shells) arises in machine tool operations, and in weapon systems. The corresponding fifth order system is closely related to the p.d.e. systems of Sobolev type. Assuming the simplest possible form of hysteresis loop, i.e. Voigt type internal dissipation, corresponding equations of motion of statically indeterminate system are treated simultaneously with the adjoint system to derive optimality criteria for maximum accuracy. Variational principle is established and an algorithm developed for identification of near-optimal designs. (Received January 22, 1976.)

733-D1 George B. Purdy, Texas A & M University, College Station, Texas, 77843. Remarks on a conjecture of T. Bang. Preliminary report.

Let K be a convex body in $E_n$ covered by n slabs $s_1, \ldots, s_n$. (A slab is a region bounded by two parallel hyperplanes). Let $W_i$ be the width of $s_i$ divided by the width of K in the direction normal to $s_i$—the so called "relative width" of $s_i$. Various conjectures of T. Bang, Ohmann and Davenport would be proved if it were known that $\sum W_i \geq 1$. We are able to prove this in the special case $-K = K$ and $-s_i = s_i$. (Received January 15, 1976.)

ERRATA

Volume 23

Donald F. Young, Linear hereditary equations in Banach space, Abstract 731-45-8, Page A-144.

Line 12, in place of "$\alpha(t, \tau)x(t)$" read "$\alpha(t, \tau)x(\tau)$".
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Unemployed mathematicians, or those under notice of involuntary unemployment, are allowed two free advertisements during the calendar year; retired mathematicians, one advertisement. The service is not available to professionals in other disciplines, nor to graduate students seeking their first postdoctoral positions; however, veterans recently released from service will qualify. Applicants must provide (1) name of institution where last employed; (2) date of termination of service; (3) highest degree; (4) field. Applications from nonmembers must carry the signature of a member. Free advertisements may not exceed fifty words (not more than six 65-space lines), including address of advertiser; excess words are charged at the rate of $0.15 per word (minimum charge $1). Anonymous listings are carried for an additional fee of $5; correspondence for such applicants will be forwarded to them. Employed members of the Society may advertise at the rate of $0.15 per word; nonmembers, currently employed, will be charged $0.50 per word (minimum charge $15). Deadline for receipt of advertisements is the same as that for abstracts; date appears on the inside front cover of each issue of the Notices. Application forms may be obtained from, and all correspondence should be directed to, the Editorial Department, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940.

Correspondence to applicants listed anonymously should be directed to the Editorial Department; the code which is printed at the end of the listing should appear on an inside envelope in order that correspondence can be forwarded.

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MATHEMATICIAN. Ph. D. 1970. Age 30. Specialty: Optimization and Dynamical Systems. Eleven articles published, three accepted, two being reviewed. Five years experience in teaching and research, Russell D. Rupp, R.D. #1, Blessing Road, Slingerlands, New York 12159

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This textbook is an introduction to the theory of finite spaces, which leads the reader by way of natural analogies from traditional methods to an up-to-date treatment of finite geometries.

Limit theorems of probability theory
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1975 420 pages US $37.95/Dfl. 95.00
This volume is the proceedings of the colloquium on Limit Theorems in Probability and Statistics held in Keszthely, Hungary in 1974. It contains 24 detailed papers on several topics in this field viz., the invariance principle; the asymptotic expansions of the probabilities of trigonometric and other series; limit theorems on algebraic structures; convergence of iterative procedures; limit theorems of point processes and queuing systems. In addition to new results, several papers give a comprehensive review of the present state of the particular subject.

Nonarchimedean fields and asymptotic expansions
by A. H. LIGHTSTONE and A. ROBINSON.
1975 204 pages US $23.95/Dfl. 60.00
For many years mathematicians have been aware of a close connection between nonarchimedean systems and the orders of infinity and of smallness, associated with the asymptotic behaviour of a function. In examining the whole background of this relationship from the viewpoint of nonstandard analysis, the authors demonstrate that infinitessimals and infinitely large numbers form a natural background to asymptotics.

Comparison theorems in Riemannian geometry
by J. CHEEGER and D. G. EBIN.
1975 viii+172 pages US $19.95/Dfl. 50.00
The central theme of this book is the interaction between the curvature of a complete Riemannian manifold and its topology and geometry. The greater part of the material presented here has not appeared before in book form and a number of the proofs are considerably simpler than what have previously appeared in the literature. As an introduction to Riemannian geometry, the book is ideal for a 1-2 term course in topics at a graduate level.

Markov chains
by D. REVUZ.
1975 346 pages US $33.95/Dfl.85.00
The first part of this book is an expository text on the foundations of Markov chains with general state space. Some of the topics of this theory, such as pointwise ergodic theory, transient and recurrent random walks, chains recurrent in the Harris sense and some aspects of transient and recurrent potential theory are developed in the second part.

Volterra Stieljes-integral equations
Functional Analytic Methods; Linear Constraints
by CHAIM SAMUEL HÖNIG.
1975 x+158 pages US $11.25/Dfl. 28.00
This book represents the edited version of a course given by the author in 1974 at the Institute de Matematica e Estatistica, at the University of São Paulo. The work deals with the results obtained in a study of linear Volterra Stieljes-integral equations with linear constraints. As particular instances of such constraints the author has presented the following conditions: initial, boundary, periodicity, di-continuity, multiple point, integral, interface, and conditions at infinite points etc.

Hewitt-Nachbin spaces
by MAURICE D. WEIR.
1975 278 pages US $15.95/Dfl.40.00
The author provides a unified approach to much of the theory of Hewitt-Nachbin complete (i.e. real-compact) spaces from a topological point of view. Beginning with the Embedding Lemma and E-compact spaces, the text systematically develops the Z-filter approach to Hewitt-Nachbin spaces, brings together the known characterizations and properties of these spaces, and the relationships to other important classes of spaces and continuous mappings. An extensive bibliography of research papers directly related to these spaces is also included.

Abstract analytic number theory
by J. KNOPFMACHER.
1975 330 pages US $29.95/Dfl. 75.00
Based on very recent research results many of which can be attributed to the author, this innovative study applies classical analytic number theory to a wide variety of mathematical subjects that are not usually treated in an arithmetical way. Abstract axiomatic methods are used to unify the treatment of certain mathematical phenomena but not for the sake of generalization alone. The monograph includes as special cases. various basic theorems of ordinary analytic number theory, which are in fact, by-products of the axiomatic development and a selection of unsolved questions.

Localization of nilpotent groups and spaces
by P. HILTON, G. MISLIN and J. ROITBERG.
1975 x+156 pages US $11.25/Dfl. 28.00
The present monograph develops the theory of localization of nilpotent groups and nilpotent spaces, using basic group theory and algebraic topology. The first chapters of the monograph are devoted to setting up and obtaining the basic results on the localization functor in the two categories in question. A final chapter discusses various applications of the theory, particularly to H-spaces and non-cancellation phenomena.

Infinite and finite sets
by Paul Erdős on his 60th birthday
edited by A. Hajnal, R. Rado and Vera T. Sós.
1975 1560 pages (in 3 volumes)
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Approximately ninety papers presented at a colloquium in honour of Paul Erdős are contained in these three volumes. They cover aspects of infinite set theory (combinatorial, large cardinals, saturated ideals, admissible sets, etc.), finite combinatorics such as graph and hypergraph theory (factorization, coloring, automorphisms, critical graphs, planar graphs, etc.), arithmetic and other combinatorial topics. About 60 unresolved problems are also included.

The number of numbers
translated by S. IANAGA.
1975 535 pages US $51.95/Dfl.130.00
This volume gives a complete and self-contained exposition of class field theory. It adopts Chevalley's formulation of the Main Theorems of this theory, for the proof of which the cohomology theory of groups is used. A historical development and a future perspective of the theory is expanded in the Appendix.
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Mathematics Meetings Housing Bureau
P. O. Box 6887
Providence, Rhode Island 02940

Please check type of accommodations required. Do NOT include payment for dormitory accommodations, since this will only cause a delay in the processing of your preregistration and housing request.

(A)  TYPE OF RESIDENCE HALL (check one):

- Male only [  ]
- Female only [  ]
- Family (children 10 and over) [  ]
- Family (children under 10) [  ]

TYPE OF ROOM(S) REQUIRED:

<table>
<thead>
<tr>
<th>Number</th>
<th>Type of Room</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>______</td>
<td>Single with air conditioning (Employed)</td>
<td>$10/night</td>
</tr>
<tr>
<td>______</td>
<td>Single without a/c (Employed)</td>
<td>9/night</td>
</tr>
<tr>
<td>______</td>
<td>Single with a/c (Student/Unemployed, Child 10 or over)</td>
<td>8/night</td>
</tr>
<tr>
<td>______</td>
<td>Single without a/c (Student/Unemployed, Child 10 or over)</td>
<td>7/night</td>
</tr>
<tr>
<td>______</td>
<td>Double with a/c (Same rate for all)</td>
<td>8/night per person</td>
</tr>
<tr>
<td>______</td>
<td>Double without a/c (Same rate for all)</td>
<td>7/night per person</td>
</tr>
</tbody>
</table>

All of the above rates are quoted per person per night in Canadian dollars and do not include seven percent Provincial Sales Tax.

I will share a double room with ____________________________ (name)

Participants planning to share a room should provide the name of the person with whom they plan to share. Each person, however, should complete a separate preregistration form. Please remember that occupancy is limited to the number of beds in standard rooms; however, any combination of parents or children is allowed.

(B)  PARTICIPANTS ACCOMPANIED BY CHILDREN UNDER TEN YEARS OF AGE (Rates not yet available)

- Single room/crib without sides [  ]
- Double room/crib without sides [  ]

If additional single or double rooms are required for children 10 years of age or over, please check the appropriate space in Section (A).

Please check here if you plan to utilize Day Care facilities [  ]

(C)  NAME

(Please print) last first middle ________________________________

ADDRESS

(For confirmation) number and street city state zip code ________________________________

I (We) will arrive (date) (time) Via (plane, bus, auto, train) ______

I (We) will depart (date) (time) ________________________________

PLEASE COMPLETE REVERSE SIDE FOR PREREEGISTRATION, REQUIRED FOR OBTAINING RESIDENCE HALL RESERVATIONS
PREREGISTRATION AND RESERVATION FORM

University of Toronto
Toronto, Canada

Short Course on Mathematical Economics
August 22-23, 1976

Joint Mathematics Meetings
August 24-28, 1976

MUST BE RECEIVED NO LATER THAN AUGUST 6, 1976
Please complete these forms and return with your payment to
Mathematics Meetings Housing Bureau
P. O. Box 6887
Providence, Rhode Island 02940

HOUSING BUREAU SERVICES

Persons desiring to make reservations for dormitory accommodations through the Housing Bureau are required to
preregister for the meeting.

Dormitory reservations will not require a deposit in advance. Full payment for rooms at the dormitories must be
made at check-in time in cash or by travelers check in Canadian funds only. DO NOT SEND PAYMENT FOR YOUR
LODGING TO THE HOUSING BUREAU. Requests for residence hall housing will be acknowledged.

PREREGISTRATION ONLY

Those participants who prefer to PREREGISTER ONLY and do not wish to obtain accommodations through the Housing
Bureau should complete ONLY the preregistration section below. Please note that separate registration fees are re­
quired for the Short Course and for the Joint Mathematics Meetings.

REGISTRATION FEES

<table>
<thead>
<tr>
<th></th>
<th>Preregistration (by mail prior to 8/6)</th>
<th>At Meeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>JOINT MATHEMATICS MEETINGS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Member</td>
<td>$12</td>
<td>$15</td>
</tr>
<tr>
<td>* Student or unemployed</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Nonmember</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>MATHEMATICAL ECONOMICS</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SHORT COURSE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student or unemployed</td>
<td>$ 3</td>
<td>$ 5</td>
</tr>
<tr>
<td>All other participants</td>
<td>18</td>
<td>20</td>
</tr>
<tr>
<td>One day fee for second day</td>
<td>--</td>
<td>10</td>
</tr>
</tbody>
</table>

* For definitions of student and unemployed, see section on Meeting Preregistration and Registration.

Make checks payable to AMERICAN MATHEMATICAL SOCIETY

MEETING PREREGISTRATION FORM

MEETING(S) (Please check): Joint Mathematics Meetings [ ] Mathematical Economics Short Course [ ]

NAME (Please print) last first middle

ADDRESS number and street city state zip code

Employing institution or unemployed [ ]

Member of:

AMS [ ] MAA [ ] Pi Mu Epsilon [ ] Nonmember [ ] AMOUNT ENCLOSED $_________ (check or money order)

I am a student at

Name of spouse Accompanying children (name, age, and sex)

(list only if accompanying to meeting)

PLEASE COMPLETE RESIDENCE HALL RESERVATION REQUEST FORM ON REVERSE SIDE
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