CALENDAR OF MEETINGS

This Calendar lists all of the meetings which have been approved by the Council up to the date this issue of the Notices was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

Abstracts should be submitted on special forms which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting in person must be received at the headquarters of the Society in Providence, Rhode Island, on or before the deadline for the meeting.

<table>
<thead>
<tr>
<th>Meeting Number</th>
<th>Date</th>
<th>Place</th>
<th>Deadline for Abstracts* and News Items</th>
</tr>
</thead>
<tbody>
<tr>
<td>739</td>
<td>November 6, 1976</td>
<td>Ann Arbor, Michigan</td>
<td>September 7, 1976</td>
</tr>
<tr>
<td>740</td>
<td>November 19–20, 1976</td>
<td>Columbia, South Carolina</td>
<td>September 25, 1976</td>
</tr>
<tr>
<td>741</td>
<td>November 19–20, 1976</td>
<td>Albuquerque, New Mexico</td>
<td>September 25, 1976</td>
</tr>
<tr>
<td>742</td>
<td>January 27–31, 1977</td>
<td>St. Louis, Missouri</td>
<td>November 3, 1976</td>
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<tr>
<td></td>
<td>(83rd Annual Meeting)</td>
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<tr>
<td></td>
<td>March 31–April 1, 1977</td>
<td>Huntsville, Alabama</td>
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<td></td>
<td>April 15–16, 1977</td>
<td>Evanston, Illinois</td>
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<td></td>
<td>April 22–23, 1977</td>
<td>Hayward, California</td>
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<td></td>
<td>August 14–18, 1977</td>
<td>Seattle, Washington</td>
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<tr>
<td></td>
<td>(81st Summer Meeting)</td>
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<td></td>
<td>November 11–12, 1977</td>
<td>Memphis, Tennessee</td>
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<td></td>
<td>January 18–22, 1978</td>
<td>Atlanta, Georgia</td>
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<td></td>
<td>(84th Annual Meeting)</td>
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<td></td>
<td>March 18–23, 1978</td>
<td>Columbus, Ohio</td>
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<td></td>
<td>January 11–15, 1979</td>
<td>Milwaukee, Wisconsin</td>
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<td></td>
<td>(85th Annual Meeting)</td>
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<td></td>
<td>January 8–12, 1981</td>
<td>San Francisco, California</td>
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<td></td>
<td>(87th Annual Meeting)</td>
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</tr>
</tbody>
</table>

*Deadline for abstracts not presented at a meeting (by title)

October 1976 issue: August 31
November 1976 issue: September 21
January 1977 issue: October 27

OTHER EVENTS

January 25–26, 1977 Short Course on Statistics, St. Louis, Missouri
February 1977 Symposium on Some Mathematical Questions in Biology, Denver, Colorado
OF THE AMERICAN MATHEMATICAL SOCIETY

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ASSOCIATE EDITOR
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The eightieth summer meeting of the American Mathematical Society will be held at the University of Toronto, Toronto, Ontario, Canada, from Tuesday, August 24, through Friday, August 27, 1976. All sessions of the meeting will take place on the campus of the university.

A set of Colloquium Lectures, consisting of four one-hour talks, will be presented by Jürgen K. Moser of the Courant Institute of Mathematical Sciences, New York University, in Convocation Hall. The title of the series is "Recent progress in dynamical systems." The first lecture in the series entitled "A survey," will be given at 9:00 a.m. on Tuesday, August 24. The second lecture entitled "Periodic orbits," will be given at 9:00 a.m. on Wednesday. The third lecture entitled "Integrable Hamiltonian systems," and the fourth entitled "Unstable phenomena and the n-body problem," will be given at 1:30 p.m. on Thursday and Friday. In accordance with a recent Council decision, no other sessions are planned to conflict with the Colloquium Lectures.

On Tuesday, August 24, J. Frank Adams of the University of Cambridge will present the AMS Bicentennial Lecture at 11:00 a.m. in Convocation Hall. The title of his lecture is "Maps between classifying spaces." (Please refer to news item on page 265 of these Notices).

By invitation of the Society's Program Committee, there will be six invited one-hour addresses. All hour talks will be given in Convocation Hall. The names of the speakers, the titles of their addresses, and the times of presentation are as follows: Eugenio Calabi, University of Pennsylvania, "Nearly flat triangulations of Riemannian manifolds," 1:30 p.m. Tuesday; Per Enflo, University of Stockholm, "On the invariant subspace problem for Banach spaces," 2:45 p.m. Tuesday; Lawrence A. Shepp, Bell Telephone Laboratories, "Optimal reconstruction of a function from its projections," 4:00 p.m. Tuesday; Michael Aschbacher, California Institute of Technology, "Determining the finite simple groups," 2:45 p.m. Thursday; Edward Nelson, Princeton University, "Principles and practice of nonstandard analysis," 4:00 p.m. Thursday; and Marian Boykan Pour-El, University of Minnesota, Minneapolis, "Computability revisited—an approach via the continuum," 5:15 p.m. Thursday.

There will be sessions for contributed ten-minute papers on Tuesday afternoon, Wednesday morning, early Wednesday afternoon, Thursday afternoon, and Friday afternoon. No provision will be made for late papers. Overhead projectors and screens will be provided, and each room has ample chalkboard space.

The AMS Committee on Employment and Educational Policy (CEEP) will sponsor an open meeting on the job market on Tuesday, August 24, at 8:30 p.m. in Convocation Hall. The meeting will consist of a brief report on the state of the job market, followed by an open discussion with comments and suggestions welcomed from the audience. Speakers will include Charles W. Curtis, Wendell H. Fleming (moderator), John W. Jewett, and John A. Nothel. Professor Jewett will report on the current survey of the mathematical profession by the Conference Board of the Mathematical Sciences.

This meeting of the Society will be held in conjunction with the annual summer meetings of the Mathematical Association of America (August 26–28), and Pi Mu Epsilon. Participants should note that the Society and Association are meeting in order opposite from the usual schedule for a summer meeting. The twenty-fourth series of Earle Raymond Hedrick Lectures, sponsored by the Association, will be given by Martin D. Davis of the Courant Institute of Mathematical Sciences, New York University. The title of his lecture series is "Some mathematical applications of logic." At the Business Meeting of the Association at 10:00 a.m. on Friday, August 27, the Lester R. Ford Awards will be presented, there will be a tribute to a distinguished Association member, and a gift to the Association will be announced.

There will be a dinner at 6:30 p.m. on Friday, August 27, for those who have been members of the Association for thirty years or more. The dinner will be followed by a short program with Carroll V. Newsom as toastmaster, and H. L. Alder, Burton W. Jones, Henry O. Pollak, and G. Bailey Price as speakers. The dinner has been planned as a pleasant occasion which recalls the past services of the Association's senior members and their spouses, informs them about current activities and future plans, and will provide an opportunity for renewing friendships. Individuals who have been MAA members for thirty years or more and would like to attend this dinner will find tickets on sale at the Joint Meetings registration desk until 5:00 p.m. on Thursday. Tickets for the dinner are $8.50 each, and spouses are invited. The ticket price includes sales tax and service charge.

The J. Sutherland Frame Lecture will be delivered to Pi Mu Epsilon by H. S. M. Coxeter on Wednesday, August 25, at 7:00 p.m. Professor Coxeter will speak on "The Pappus configuration and its groups."

The Association for Women in Mathematics will hold a panel discussion on the "History of women in mathematics" on Thursday, August 26, at noon. Lenore Blum will serve as moderator. Speakers will include Lida K. Barrett, Mary W. Gray, Linda Keen, Emiliana Noether, and Martha K. Smith. The panel discussion will be immediately followed by AWM's Business Meeting. There
Short Course on Mathematical Economics  
August 22–23, 1976

The American Mathematical Society will present a one-and-one-half day short course on Mathematical Economics on Sunday and Monday, August 22 and 23, in Room 2135 of Sidney Smith Hall on the campus of the University of Toronto. The course will present an introduction to several problems (existence and computation of equilibria, structure and dependence on parameters of the set of equilibria, game models and the theory of the core, dynamic economics) illustrating the application to economics of various fields of mathematics (in particular, algebraic topology, measure theory, and differential topology). It is intended to present both mathematically challenging aspects and their applications in current economic theory.

The program is under the direction of Gerard Debreu, Departments of Economics and Mathematics, University of California, Berkeley, in cooperation with Hugo F. Sonnenschein, Department of Economics, Princeton University, as codirectors. This short course was recommended by the Society's Committee on Employment and Educational Policy (CEEP), whose members are David Blackwell, Charles W. Curtis, Wendell H. Fleming (chairman), Martha K. Smith, and Daniel H. Wagner.

The program will consist of six seventy-five minute lectures, as follows: Hugo F. Sonnenschein will speak on "Price formation: The theory of general economic equilibrium"; Werner Hildenbrand, Department of Economics, University of Bonn, will speak on "Measure spaces of economic agents"; David Gale, Departments of Economics and Mathematics, University of California, Berkeley, and Center for Advanced Study in the Behavioral Sciences, Stanford, California, will speak on "The role of prices and interest rates in dynamic economics"; Andreu Mas-Colell, Departments of Mathematics and Economics, University of California, Berkeley, will speak on "The theory of economic equilibrium from the differentiable point of view"; Robert J. Aumann, Departments of Economics, Hebrew University and Stanford University, will speak on "Some game models in economics"; and Stephen Smale, Department of Mathematics, University of California, Berkeley, will speak on "Computation and existence of Walras equilibria."

Summaries of these talks and accompanying reading lists appeared on pages A-405 through A-407 of the April issue of these (Notices). This short course is open to all who wish to participate upon payment of the registration fee. This fee has been reduced for students and unemployed individuals, with a modest increase for other registrants. Please refer to the section entitled MEETING REGISTRATION for details.

will be an open meeting of the AWM Executive Committee at 5:30 p.m. on Wednesday, August 25.
The Council of the Conference Board of the Mathematical Sciences will meet on Friday, August 27, at 2:30 p.m.
The Mathematicians Action Group will hold its Business Meeting on Tuesday, August 24, at 8:00 p.m., and a session on "Other mathematical models in economics" at 8:00 p.m. on Thursday, August 26. Speakers will include Francis R. Buianouckas and Graciela Chichilnisky.

Rooms 2050 and 2054 in Sidney Smith Hall (Aon the map on page 231) have been set aside as informal discussion rooms, and will be open daily from 8:00 a.m. to 10:00 p.m. for small groups desiring a quiet room with blackboard space to discuss mathematics. Room 2050 is available on a first-come, first-served basis; room 2054 is available for one-hour periods only, and must be reserved in advance. A reservation form will be posted on the door to room 2054 for individuals to sign up for use of this room. It is requested that discussion groups not be planned which conflict with the business meetings or major lectures.

SPECIAL SESSIONS

University has organized a special session on Recursively enumerable sets and degrees. The speakers will be Keh-Hsun Chen, Robert A. Di Paola, Sy D. Friedman, Manuel Lerman, G. E. Sacks, Richard A. Shore, Stephen G. Simpson, and Robert Soare, W. T. TUTTTE of the University of Waterloo has organized a special session on Chromatic polynomials and related topics. The speakers will be Ruth A. Bari, Gerald Berman, H. S. M. Coxeter, A. Czerniakiewicz, Stéphane Foulques, Wolfgang Haken, Dick Wick Hall, Paul C. Kainen, Bruce Richmond, and Michael R. Rolle.

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet at 2:00 p.m. on Monday, August 23, in the Council Chamber in the Galbraith Building (on the map on page 231). The Business Meeting of the Society will be held in Convocation Hall (M on the map) at 4:00 p.m. on Wednesday, August 25. The secretary notes the following resolution of the Council: Each person who attends a Business Meeting of the Society shall be willing and able to identify himself as a member of the Society. In further explanation, it is noted that "each person who is to vote at a meeting is thereby identifying himself as and claiming to be a member of the American Mathematical Society."

The Council recommends to the Business Meeting changes in the bylaws establishing a category of "foreign member." Certain individuals, the class to be defined precisely by the Council, may elect to be foreign members, with all privileges accorded to ordinary members except the right to vote. The dues, as set by the Council with the approval of the Trustees, would not exceed two-thirds of the dues of an ordinary member.

The Council recommends to the Business Meeting that Article IV, Section 8, of the bylaws be deleted.

These two paragraphs constitute the "notice of proposed action and of its general nature" as required by Article XIII of the bylaws.

MEETING REGISTRATION

Registration for the short course only will begin on Saturday, August 21. Lecture notes and other short course material will be distributed before the first session at the short course registration desk. Those individuals who did not preregister for the short course are strongly urged to register and pick up their material on Saturday evening so as not to miss the start of the lecture on Sunday morning. General meeting registration will commence on Monday, August 23, at 2:00 p.m. Participants who are not attending the short course are advised that no general meeting information or registration material will be available prior to the time listed below for the Joint Mathematics Meetings registration. Upon arrival at the University of Toronto campus, participants should proceed directly to the dormitory to which they have been assigned in order to check into their accommodations before registering for the meetings.

The hours the registration desks will be open are as follows:

<table>
<thead>
<tr>
<th>Mathematical Economics Short Course Registration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Date and Time</td>
</tr>
<tr>
<td>Saturday, August 21</td>
</tr>
<tr>
<td>Sunday, August 22</td>
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<tr>
<td>Monday, August 23</td>
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</tbody>
</table>

Joint Mathematics Meetings Registration

<table>
<thead>
<tr>
<th>Date and Time</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday, August 23</td>
<td>Entrance lobby, Sidney Smith Hall (Monday through Saturday)</td>
</tr>
<tr>
<td>Tuesday, August 24</td>
<td>8:00 a.m.-5:00 p.m.</td>
</tr>
<tr>
<td>Wednesday, August 25 to Friday, August 27</td>
<td>8:30 a.m.-4:30 p.m.</td>
</tr>
<tr>
<td>Saturday, August 28</td>
<td>8:30 a.m.-1:30 p.m.</td>
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</tbody>
</table>

Meeting registration and preregistration fees partially cover expenses of holding the meetings. The preregistration fee does not represent an advance deposit for lodgings.

Please note that separate registration fees are required for the short course and for the Joint Meetings. These fees are as follows:

<table>
<thead>
<tr>
<th>Mathematical Economics Short Course</th>
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<tbody>
<tr>
<td>At Meeting</td>
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<tr>
<td>Members/Nonmembers</td>
</tr>
<tr>
<td>Student or unemployed</td>
</tr>
<tr>
<td>One day fee for second day</td>
</tr>
</tbody>
</table>

Joint Mathematics Meetings

| Member | $15 |
| Student or unemployed | 2 |
| Nonmember | 24 |

There will be no extra charge for members of the families of registered participants except that all professional mathematicians who wish to attend sessions must register independently.

The unemployed status refers to any participant who is not currently employed and actively seeking employment. It is not intended to include participants who have voluntarily resigned or retired from their latest position. Students are considered to be only those currently working toward a degree who do not receive an annual compensation totaling more than $7,000 from employment, fellowships, and scholarships.

A fifty percent refund of preregistration fees will be made for all cancellations received in Providence prior to August 20. There will be no refunds granted for cancellations received after that date or to persons who do not attend the meetings.

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At last summer's meeting at Western Michigan University, an experimental variant of the Employment Register was operated (successfully) on a limited basis. No interviews were scheduled by the staff. Instead facilities were provided for applicants and employers to display resumes and job listings. Message boxes were set up for individuals to leave messages for one another requesting interviews. Tables and chairs were provided in the room for interviews.

It is planned to repeat this form of the Employment Register at the University of Toronto. Employers are encouraged to attend the meetings and participate, if possible. Applicants should recognize that the MSER cannot guarantee that any employers will, in fact, attend the meeting or be able to participate in the Employment Register. The AMS-MAA-SIAM Committee on Employment Opportunities has, however, requested employers listing in the July and August 1976 issues of Employment Information for Mathematicians to signify in their listing their intention to participate in the Employment Register at the summer meeting.

EXHIBITS

The book and educational media exhibits will be displayed in Room 2096 of Sidney Smith Hall at the following times: August 24 (Tuesday), 1:00 p.m. to 5:00 p.m.; August 25 and 26 (Wednesday and Thursday), 8:30 a.m. to 4:30 p.m.; August 27 (Friday), 8:30 a.m. to noon. All participants are encouraged to visit the exhibits sometime during the meeting.

RESIDENCE HALL HOUSING

Several residence hall facilities have been set aside for the use of participants in the Joint Mathematics Meetings and the Mathematical Economics Short Course. According to regulations set by the university's housing office, these facilities are divided into sections for male participants only, female participants only, couples, families accompanied by children all of whom are ten years of age or over, and families with any children under ten years of age.

The rates quoted below are in Canadian funds, and are subject to a seven percent Provincial Sales Tax. These are now firm rates.

Participants accompanied by children, all of whom are ten years of age or over will be assigned rooms in Wetmore Hall in New College, and Whitney Hall in University College (B and J respectively) on the map on page 231. New College is air-conditioned; University College is not. Only one person may occupy a single room, and only two people may occupy a double room. Sleeping bags, cots, and cribs are not allowed. Families with more than two members will be assigned to adjacent rooms. Children will be charged the Student/Unemployed rate. Single rooms for male participants are in Wetmore Hall, New College; Sir Daniel Wilson Hall, and Whitney Hall, both in University College; and in Devonshire House (K on the map). New College is air-conditioned; University College and Devonshire House are not. Participants requesting air-conditioned rooms in this category should be aware of the fact that the number of these rooms available is extremely limited. Air-conditioned rooms in this category will be assigned on a first-come, first-served basis, and your confirmation will tell you whether you have been successful in obtaining an air-conditioned room. Double rooms for male participants are in Wetmore Hall, New College; and Devonshire House, New College is air-conditioned; Devonshire House is not. Single and double rooms for female participants are in Wilson Hall, New College, and are air-conditioned. Double rooms for couples are in Wetmore Hall, New College; and Whitney Hall, University College. New College is air-conditioned; University College is not. Rates for all of the facilities mentioned above are:

Students, Unemployed, and Children Age Ten or Over

Single or Double
$8/night per person*
$6/night per person**

All Other Participants not Accompanied by Any Children under Ten Years of Age

Single
$11/night* $9/night per person*
$10/night** $8/night per person**

with air-conditioning
**without air-conditioning

Participants accompanied by any children under ten years of age will be assigned rooms in Victoria College (K on the map). These rooms are not air-conditioned. Each room contains two beds. Children ten years of age or over must occupy a bed and will be charged the appropriate rate (single or double). For children between the infant stage and nine years of age, cots without sides are available at $5 per cot per night. Children in this age bracket may also occupy a bed, but will be charged the same rate as an adult. For small infants, cribs with sides can be rented for a flat charge of $12, regardless of the number of nights required. Participants wishing to rent cribs with sides should write to Thomas H. Callahan, Department of Mathematics, University of Toronto, Toronto, Ontario, Canada M5S 1A1, giving specific details as to the number of cribs required, arrival and departure dates. Please mark the outside of the envelope "CRIBS." No deposit is required. At most, one cot or crib will be permitted in a room (maximum room occupancy is three persons). Participants with small children will be requested to sign a waiver on property damage, and are advised to bring plastic sheets. Light kitchen facilities are available in the basement at Victoria College so that families accompanied by small children may prepare light meals, warm bottles, etc. Participants should bring their own cooking utensils. Rates at Victoria College are as follows:

<table>
<thead>
<tr>
<th>Rate</th>
<th>Victoria College</th>
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</thead>
<tbody>
<tr>
<td>Cot</td>
<td>$5/night</td>
</tr>
<tr>
<td>Crib</td>
<td>$12/flat</td>
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<tr>
<td>Single</td>
<td>$10/night*</td>
</tr>
<tr>
<td>Double</td>
<td>$8/night per*</td>
</tr>
</tbody>
</table>

*All participants (regardless of whether student or unemployed) including children ten years of age or over
Please note that payment in full for your dormitory accommodations must be made at the time of check-in in Canadian funds. Participants coming to the meeting from countries other than Canada are advised to exchange their currency for Canadian money either before they leave for the meeting or at the airport immediately upon arrival in Toronto, since clerks in the dormitories will not be authorized to accept U.S. funds in payment, nor to allow occupancy of the room for even the first night until a participant can have his or her money exchanged. There are several banks nearby which will exchange U.S. for Canadian funds. The Canadian Imperial Bank of Commerce on the southwest side of Bloor and St. George Streets is open from 10:00 a.m. to 4:30 p.m. Monday through Thursday, and from 10:00 a.m. to 6:00 p.m. on Friday. There is also a branch of the Bank of Montreal on the northwest corner of Bloor and St. George. No banks are open on Saturday or Sunday.

Clerks will be at the check-in desks in the dormitories from 8:00 a.m. until midnight only. Participants arriving after midnight will not be able to occupy their rooms until 8:00 a.m. the next morning. Residence hall rooms may be occupied from 8:00 a.m. on Saturday, August 21, until 10:00 a.m. on Saturday, August 28. Under no circumstances will participants be allowed to occupy rooms past 10:00 a.m. on Saturday, since the university staff must begin at that time to prepare these rooms for students arriving the next week. Facilities for checking baggage will be available at the Joint Meetings Registration Desk.

Pay telephones are located on each floor. Each dormitory has a fully equipped laundry room with coin-operated washers and dryers. Ironing facilities are also available. There are no private baths; generally speaking, there are two large bathroom facilities on each floor. Toilet paper and soap will be provided. Light kitchen facilities are available in some dormitories. Beds in all dormitories except Victoria College will be made daily, Monday through Friday; however, only one set of sheets and towels will be furnished for the meeting period. Pets are not allowed in the residence halls.

To be assured of a room, participants should have registered in advance. The deadline for requests for dormitory accommodations was August 6. Participants who arrive in Toronto without having previously reserved a room in the residence halls are advised that porters in the halls cannot make room assignments. The university housing office has agreed to supply the Joint Mathematics Meetings registration desk with a list of vacant rooms each morning; assignments to these rooms, however, can be made only during the hours the registration desk is open (see page 228 for hours of operation). Participants without previously reserved rooms are advised to arrange to arrive in Toronto during the period the desk is in operation in order to secure a room assignment.

HOTELS

Blocks of rooms have been set aside for use by participants at the Park Plaza Hotel, the Hotel Plaza II, and the Chelsea Inn. Participants should make their own reservations with these hotels directly, and should identify themselves as partici-
The following hotels are in the suburbs, and are within an hour's drive of the campus:

**WESTBURY HOTEL** - 10
475 Yonge Street at College
Single $29.50 Double $36.50
Extra person (14 years) $7.00
Code: RT CL AC TV Parking $3.50 (24 hours)
Telephone: 416-924-0611

**SUTTON PLACE HOTEL** - 11
Bay at Wellesley Street
Single $32.50 Double $40.50
Extra person (12 years) $8.00
Code: RT CL AC TV SP Parking $3.00 (24 hours)
Telephone: 416-924-9221

**PARK PLAZA HOTEL** - 12
Bloor at Avenue Road
Single $35.00 Double $43.00
Extra person (14 years) $8.00
Code: RT CL AC TV Parking $3.00 (24 hours)
Telephone: 416-924-5471

**HOTEL PLAZA II** - 13
Bloor at Yonge Street
Single $37.00 Double $44.00
Extra person (10 years) $5.00
Code: RT CL AC TV Parking $3.75 (24 hours)
Telephone: 416-961-8000

**CHELSEA INN** - 14
Gerrard at Bay Street
Single $22.00 Double $27.00
Extra person (13 years) $5.00
One Bedroom Suite with Kitchen:
$47/day based on 2 adults and 2 children
$54/day based on 4 adults
Code: RT CL AC TV Parking $1.75 (24 hours)
Telephone: 416-595-1975

**CANADIANA MOTOR HOTEL** - 20
Kennedy Road and Highway 401
Single $23.00 Double $31.00
Extra person (12 years) $7.00
Code: RT CL AC TV SP FP
Telephone: 416-291-1171

**HOLIDAY INN WEST** - 21
Highway 427 at Balmahon Road
Single $27.50 Double $32.50 Twin $37.00
Extra person (12 years) $4.00
Code: RT CL AC TV SP FP
Telephone: 416-621-2121

**HOLIDAY INN EAST** - 22
Highway 401 at Warden Avenue
Single $28.50 Double $33.50 Twin $38.00
Extra person (12 years) $4.00
Code: RT CL AC TV SP FP
Telephone: 416-293-8170

**SKYLINE HOTEL** - 23
655 Dixon Road (near International Airport)
Single $28.00 Double $34.00
Extra person (12 years) $6.00
Code: RT CL AC TV SP FP
Telephone: 416-244-1711

**THE CONSTELLATION HOTEL** - 24
900 Dixon Road (near International Airport)
Single $32.00 Double $40.00
Extra person (16 years) $6.00
Code: RT CL AC TV SP FP
Telephone: 416-677-1500
FOOD SERVICES

Two cafeterias will be in operation on campus during the meetings. Wilson cafeteria in New College ([F] on the map on page 231) will be open for breakfast from 7:00 a.m. to 8:30 a.m., lunch from 11:30 a.m. to 1:30 p.m., and dinner from 5:30 p.m. to 7:30 p.m. throughout the meetings, beginning on Sunday, August 22, through Friday, August 27. New College cafeteria will be open for breakfast and lunch only on Saturday, August 28. The cafeteria in the Medical Sciences Building ([G] on the map) will be open from 7:30 a.m. to 4:00 p.m. for breakfast and lunch, Monday through Friday. The average costs of meals in both these facilities are $1.70 breakfast; $2.60 lunch; $3.45 dinner. Individual meal tickets or daily tickets ($7.75) will be on sale at the Joint Meetings Registration Desk. Participants may also pay for meals on the spot; however, a slight saving may be realized if tickets are purchased in advance.

A snack bar, selling soup, sandwiches, beverages, light desserts, etc., will be operated in Room 5025 of Sidney Smith Hall, and will be open from 8:30 a.m. to 3:30 p.m. Monday through Friday. In addition, there are a number of catering trucks which park outside Sidney Smith Hall; picnic tables are provided.

CAMPING

There are no suitable camping sites located near the University of Toronto. Those persons wishing to camp should contact their local KOA office for the current issue of “Handbook and Directory for Campers.”

BOOKSTORES

There are three bookstores located on campus. The University of Toronto Bookroom, located on King’s College Circle ([H] on the map) and the University of Toronto Textbook Store, located on Huron Street ([I] on the map), are both open from 8:45 a.m. to 4:30 p.m., Monday through Friday. The Student Christian Movement Bookstore, located on the edge of campus at the southwest corner of Bloor and St. George, is open 9:00 a.m. to 6:00 p.m. Monday through Friday, and 10:00 a.m. to 6:00 p.m. on Saturday.

LIBRARIES

The Mathematics Department Library, located on the second floor of Sidney Smith Hall, will be open from 9:00 a.m. to 9:00 p.m. for the duration of the meetings. Information concerning books located in other libraries is available from the Department Library. The main collection of books is in the Science and Medicine Library, located on King’s College Circle ([F] on the map). The Robarts Library ([G] on the map) houses the Humanities Collection. Summer hours for university libraries are 8:00 a.m. to 11:00 p.m., Monday through Friday, and 8:00 a.m. to 6:00 p.m. on Saturday. The Metro Toronto Central Public Library is located at the southwest corner of St. George and College Streets, and has summer hours of 9:00 a.m. to 8:00 p.m., Monday through Friday, and 9:00 a.m. to 5:00 p.m. Saturday.

MEDICAL SERVICES

The University Health Service ([O] on the map) is open from 9:00 a.m. to 4:30 p.m. daily for medical attention. Emergencies occurring during the evening or weekends can be handled at the Emergency Department of any of the local hospitals: Toronto General Hospital, College at University; Women's College Hospital, 76 Grenville Street (College at Bay); The Hospital for Sick Children, 555 University Avenue. In addition, the Academy of Medicine can advise of local doctors who are on emergency call. Their telephone number is 922-1184. Dental service can be arranged through the University Health Centre.

DAY CARE CENTRES

The Campus Cooperative will accept up to fourteen children. The cost there is $11 per day, plus two hours time donated by a parent. Interested parties should write directly to Ms. Marilyn Wilcoxen, Campus Cooperative, 12 Sussex Avenue, Toronto, Canada, giving the dates they will utilize the Centre, and include a deposit equal to one day’s fee. Margaret Fletcher Daycare Centre will accept up to fifteen children. The cost there is $11 per day, but no donated time is required of parents. Participants should write directly to Mrs. N. Lupton, Margaret Fletcher Daycare Centre, University of Toronto, to make reservations, again giving dates. An $11 deposit is required.

N. B.: Day care is not available on weekends.

ENTERTAINMENT

The University of Toronto is planning entertainment for mathematicians and their families during the meetings. At 8:00 p.m. on Wednesday, August 25, there will be an evening beer party in Wetmore Hall Cafeteria, New College. Tickets to this event will be sold in advance at the Joint Meetings Registration Desk. The price per ticket is $5. Light sandwiches and snacks will be served. During the week of the meeting, there will be many entertainment events in Toronto and vicinity. A number of theatres will be giving regular performances at this time. At Niagara-on-the-Lake
there is a Shaw festival, and in Stratford, a Shakespeare festival. Both of these places are relatively close to Toronto and return transportation for any evening is easy to arrange. In addition to theatrical events, Toronto has a wide variety of musical activities in the summer. The Canadian National Exhibition will be in progress during the week of the meetings, and it is easily accessible from the university by public transport. Tours can be arranged during the daytime to local places of interest such as the Ontario Science Centre, the McMichael Collection, and the large, new Metropolitan Zoo. Participants interested in these events should check with the Local Information section of the Joint Meetings Registration Desk.

TRAVEL AND LOCAL INFORMATION

Toronto is served by Air Canada, Allegheny, American, C. P. Air, Eastern, Great Lakes, Nordair, North Central, Quebecair, Transair, and United airlines to Toronto International Airport. There are several ways of getting from the airport to campus: (1) There is bus service available to the Islington subway station. From there, subway and surface transportation is available into the campus area. (2) The airport bus goes to the Royal York Hotel and the Sutton Place Hotel. The cost is about $2.50. (3) Cab service from the airport costs up to $10 per cab. (4) There is also a limousine service which costs up to $12 per air-conditioned luxury car.

Rail service to Toronto is by Canadian National and Canadian Pacific Railways, with good connections from Detroit, Buffalo, and Montreal. Limited access highways (#401, #427, and Queen Elizabeth Way) connect Toronto with Detroit, Buffalo, Kingston, or Montreal.

The Toronto Transit Commission, which operates the subway, buses, and streetcars in Toronto, requires passengers to pay fares in either exact change ($0. 50), or by prepurchased token ($0. 40). Tokens can be purchased at any subway station.

Entering Canada is usually no problem for American citizens, and involves nothing more than answering questions about where they were born, where they are going, and how long they will stay. To be assured of entry, however, it is advised that participants bring with them some proof of citizenship, such as a voter's, baptismal, or birth certificate. Permanent U. S. residents who are not citizens are required to bring their alien registration receipt card (U. S. form I-151). Entry requirements vary for people coming to Canada from countries other than the United States. As a general rule, the visitor should have a valid national passport. Passengers arriving in Toronto by air should be aware that delays of up to one hour may be encountered in being processed through customs at the airport.

PARKING

Parking throughout the campus is extremely limited as the campus was not designed for motor traffic. Parking stickers for pay lots will be on sale at the Joint Meetings Registration Desk for $1.50 per day. There are also several areas where on-street parking is free from 9:00 a.m. to 4:00 p.m. Maps indicating these parking areas will also be available at the Registration Desk. Participants, however, are urged to drive as little as possible between dormitories and the meeting area.

WEATHER

The normal daytime high temperature during this period is 79°F. Normal night-time low is 61°F. Rainfall in August averages 2. 65 inches, with a 30 percent probability of precipitation each day. Humidity ranges from a daytime high of 67 percent to a night-time low of 55 percent. The record high and low temperatures for August are 102°F and 39°F, respectively. Light sweaters and jackets are advised for evening wear. Temperatures in Canada are now given in the Celsius scale, so the preceding temperatures would read: normal high 26°C; normal low 16°C; record high 39°C; record low 4°C.

MAIL AND TELEPHONE MESSAGES

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematics Meetings, Department of Mathematics, University of Toronto, Toronto, Ontario, Canada M5S 1A1. Mail and telegrams so addressed may be picked up at the Joint Meetings Registration Desk located in the entrance lobby of the Royal York Hotel.

A telephone message center will be located in the same area to receive incoming calls for registrants during the hours the desk is open, cf. the section entitled MEETING REGISTRATION, on a previous page. Messages will be written down, and the name of any participant for whom a message has been received will be posted until the message is picked up at the message center. The telephone number of the center is (416) 978-4856.

LOCAL ARRANGEMENTS COMMITTEE


234
## TIMETABLE
(Eastern Daylight Time)

### AMERICAN MATHEMATICAL SOCIETY

#### SATURDAY, August 21

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>4:30 p.m. - 7:30 p.m.</td>
<td><strong>SHORT COURSE ON MATHEMATICAL ECONOMICS</strong></td>
</tr>
<tr>
<td>2135 SIDNEY SMITH HALL</td>
<td>REGISTRATION (Short Course Only)</td>
</tr>
</tbody>
</table>

#### SUNDAY, August 22

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m. - 5:00 p.m.</td>
<td>Price formation: The theory of general economic equilibrium</td>
</tr>
<tr>
<td>9:00 a.m. - 10:15 a.m.</td>
<td>Hugo F. Sonnenschein</td>
</tr>
<tr>
<td>10:45 a.m. - noon</td>
<td>Measure spaces of economic agents</td>
</tr>
<tr>
<td>2:00 p.m. - 3:15 p.m.</td>
<td>The role of prices and interest rates in dynamic economics</td>
</tr>
<tr>
<td>3:45 p.m. - 5:00 p.m.</td>
<td>The theory of economic equilibrium from the differentiable point of view</td>
</tr>
<tr>
<td>2135 SIDNEY SMITH HALL</td>
<td>REGISTRATION (Short Course Only)</td>
</tr>
</tbody>
</table>

#### MONDAY, August 23

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m. - noon</td>
<td>Some game models in economics</td>
</tr>
<tr>
<td>9:00 a.m. - 10:15 a.m.</td>
<td>Robert J. Aumann</td>
</tr>
<tr>
<td>10:45 a.m. - noon</td>
<td>Computation and existence of Walras equilibria</td>
</tr>
<tr>
<td>2135 SIDNEY SMITH HALL</td>
<td>REGISTRATION (Short Course Only)</td>
</tr>
</tbody>
</table>

#### AMS - MAA SUMMER MEETINGS

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>2:00 p.m. - 8:00 p.m.</td>
<td>REGISTRATION - Entrance Lobby, Sidney Smith Hall</td>
</tr>
<tr>
<td>2:00 p.m.</td>
<td>COUNCIL MEETING Council Chamber, Galbraith Building</td>
</tr>
</tbody>
</table>

#### TUESDAY, August 24

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m. - 5:00 p.m.</td>
<td>REGISTRATION - Entrance Lobby, Sidney Smith Hall</td>
</tr>
<tr>
<td>9:00 a.m. - 10:00 a.m.</td>
<td>Recent progress in dynamical systems:</td>
</tr>
<tr>
<td>11:00 a.m. - noon</td>
<td>BICENTENNIAL LECTURE: Maps between classifying spaces</td>
</tr>
<tr>
<td>noon - 2:20 p.m.</td>
<td>Chromatic polynomials and related topics I</td>
</tr>
<tr>
<td>1:00 p.m. - 3:15 p.m.</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS</td>
</tr>
<tr>
<td>1:00 p.m. - 3:00 p.m.</td>
<td>Algebra I - Room #1069 Sidney Smith Hall</td>
</tr>
<tr>
<td>1:00 p.m. - 3:30 p.m.</td>
<td>Applied Mathematics I - Room #1071 Sidney Smith Hall</td>
</tr>
<tr>
<td>1:00 p.m. - 5:00 p.m.</td>
<td>General Topology - Room #1083 Sidney Smith Hall</td>
</tr>
<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>INVITED ADDRESS: Nearly flat triangulations of Riemannian manifolds</td>
</tr>
<tr>
<td>2:45 p.m. - 3:45 p.m.</td>
<td>INVITED ADDRESS: On the invariant subspace problem for Banach spaces</td>
</tr>
<tr>
<td>1:00 p.m. - 5:00 p.m.</td>
<td>EXHIBITS - Room #2096 Sidney Smith Hall</td>
</tr>
<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>INVI... - Convocation Hall</td>
</tr>
<tr>
<td>2:45 p.m. - 3:45 p.m.</td>
<td>INVI... - Convocation Hall</td>
</tr>
<tr>
<td>Time</td>
<td>Event</td>
</tr>
<tr>
<td>---------------</td>
<td>----------------------------------------------------------------------</td>
</tr>
<tr>
<td>3:00 p.m.</td>
<td>SPECIAL SESSIONS</td>
</tr>
<tr>
<td>3:00 p.m. - 6:00 p.m.</td>
<td>Combinatorial identities I</td>
</tr>
<tr>
<td>3:00 p.m. - 5:15 p.m.</td>
<td>Applied mathematics II</td>
</tr>
<tr>
<td>3:15 p.m. - 5:45 p.m.</td>
<td>Algebra II</td>
</tr>
<tr>
<td>3:15 p.m. - 5:00 p.m.</td>
<td>Manifolds</td>
</tr>
<tr>
<td>4:00 p.m. - 5:00 p.m.</td>
<td>INVITED ADDRESS: Optimal reconstruction of a function from its projections</td>
</tr>
<tr>
<td>7:00 p.m. - 8:00 p.m.</td>
<td>Mathematics Action Group Business Meeting</td>
</tr>
<tr>
<td>8:30 p.m. - 10:00 p.m.</td>
<td>Committee on Employment and Educational Policy</td>
</tr>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>REGISTRATION - Entrance Lobby, Sidney Smith Hall</td>
</tr>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>EXHIBITS - Room #2096 Sidney Smith Hall</td>
</tr>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>EMPLOYMENT REGISTER - Room #2042 Sidney Smith Hall</td>
</tr>
<tr>
<td>9:00 a.m. - 4:00 p.m.</td>
<td>COLLOQUIUM LECTURES: Recent progress in dynamical systems</td>
</tr>
<tr>
<td>9:00 a.m. - 10:00 a.m.</td>
<td>Lecture II, Periodic orbits</td>
</tr>
<tr>
<td>10:00 a.m. - 1:00 p.m.</td>
<td>Chromatic polynomials and related topics II</td>
</tr>
<tr>
<td>10:15 a.m. - noon</td>
<td>General session</td>
</tr>
<tr>
<td>10:15 a.m. - 12:45 p.m.</td>
<td>Analysis I</td>
</tr>
<tr>
<td>10:15 a.m. - 11:30 a.m.</td>
<td>Foundations</td>
</tr>
<tr>
<td>10:15 a.m. - noon</td>
<td>Number theory</td>
</tr>
<tr>
<td>10:15 a.m. - 11:30 a.m.</td>
<td>Probability</td>
</tr>
<tr>
<td>10:30 a.m. - 12:30 p.m.</td>
<td>Differentiable dynamical systems I</td>
</tr>
<tr>
<td>noon - 1:00 p.m.</td>
<td>SPECIAL SESSIONS</td>
</tr>
<tr>
<td>12:30 p.m. - 3:00 p.m.</td>
<td>Category theory I</td>
</tr>
<tr>
<td>1:30 p.m. - 4:00 p.m.</td>
<td>Differentiable dynamical systems II</td>
</tr>
<tr>
<td>1:30 p.m. - 2:45 p.m.</td>
<td>Geometry</td>
</tr>
</tbody>
</table>
## Wednesday, August 25

<table>
<thead>
<tr>
<th>Time</th>
<th>Sessions</th>
<th>Location</th>
<th>Other Organizations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:30 p.m. - 3:15 p.m.</td>
<td>Analysis II</td>
<td>Room #1071 Sidney Smith Hall</td>
<td></td>
</tr>
<tr>
<td>1:30 p.m. - 3:45 p.m.</td>
<td>Functional analysis</td>
<td>Room #1069 Sidney Smith Hall</td>
<td></td>
</tr>
<tr>
<td>1:45 p.m. - 3:45 p.m.</td>
<td>Combinatorial identities II</td>
<td>Room #159 Lash Miller Hall</td>
<td></td>
</tr>
<tr>
<td>2:00 p.m. - 4:00 p.m.</td>
<td></td>
<td></td>
<td>IME: Contributed Paper Sessions Room #1072 Sidney Smith Hall</td>
</tr>
<tr>
<td>4:00 p.m.</td>
<td>BUSINESS MEETING</td>
<td>Convocation Hall</td>
<td>Association for Women in Mathematics EXECUTIVE COMMITTEE - Open Meeting Room #2129 Sidney Smith Hall</td>
</tr>
<tr>
<td>5:30 p.m. - 6:30 p.m.</td>
<td></td>
<td></td>
<td>IME - Informal Get-Together Dinner Wilson Hall Cafeteria, New College</td>
</tr>
<tr>
<td>7:00 p.m. - 8:00 p.m.</td>
<td></td>
<td></td>
<td>IME - J. SUTHERLAND FRAME LECTURE The Pappus configuration and its groups H. S. M. Coxeter, #2135 Sidney Smith Hall</td>
</tr>
<tr>
<td>7:00 p.m. - 8:00 p.m.</td>
<td></td>
<td></td>
<td>MAA - Committee on Two-Year Colleges INFORMAL MEETING: Mathematical education at the two-year college level Room #1069 Sidney Smith Hall</td>
</tr>
<tr>
<td>8:00 p.m.</td>
<td>BEER PARTY</td>
<td>Wetmore Hall Cafeteria, New College</td>
<td></td>
</tr>
</tbody>
</table>

## Thursday, August 26

<table>
<thead>
<tr>
<th>Time</th>
<th>Sessions</th>
<th>Location</th>
<th>Other Organizations</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m. - 9:00 a.m.</td>
<td></td>
<td>Wilson Hall Cafeteria, New College</td>
<td></td>
</tr>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>REGISTRATION</td>
<td>Entrance Lobby, Sidney Smith Hall</td>
<td></td>
</tr>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>EXHIBITS</td>
<td>Room #2096 Sidney Smith Hall</td>
<td></td>
</tr>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td>EMPLOYMENT REGISTER</td>
<td>Room #2042 Sidney Smith Hall</td>
<td></td>
</tr>
<tr>
<td>9:00 a.m. - 9:10 a.m.</td>
<td></td>
<td></td>
<td>WELCOME ADDRESS</td>
</tr>
<tr>
<td>9:10 a.m. - 10:00 a.m.</td>
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<td></td>
<td>MAA - THE EARLE RAYMOND HEDRICK LECTURES: Some mathematical applications of logic: Lecture I, Unsolvable problems Martin D. Davis</td>
</tr>
<tr>
<td>10:10 a.m. - 11:00 a.m.</td>
<td></td>
<td></td>
<td>MAA - INVITED ADDRESS: Stamp out math boredom N. G. Gunderson</td>
</tr>
<tr>
<td>11:10 a.m. - noon</td>
<td></td>
<td></td>
<td>MAA - INVITED ADDRESS: Geometrical optics and the singing of whales Cathleen S. Morawetz</td>
</tr>
<tr>
<td>noon - 1:00 p.m.</td>
<td>Combinatorics I</td>
<td>Room #1069 Sidney Smith Hall</td>
<td></td>
</tr>
<tr>
<td>noon - 1:15 p.m.</td>
<td></td>
<td></td>
<td>AWM - PANEL DISCUSSION: History of women in mathematics, Lenore Blum (moderator) Room #2135 Sidney Smith Hall</td>
</tr>
<tr>
<td></td>
<td>An overview</td>
<td></td>
<td>An overview</td>
</tr>
<tr>
<td></td>
<td>Lida K. Barrett</td>
<td></td>
<td>Sophie Germain</td>
</tr>
<tr>
<td></td>
<td>Mary W. Gray</td>
<td></td>
<td>Sonya Kovalevski</td>
</tr>
<tr>
<td></td>
<td>Linda Keen</td>
<td></td>
<td>Emmy Noether—Twentieth century mathematician and woman</td>
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<td></td>
<td></td>
<td></td>
<td>Emiliana Noether</td>
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<td></td>
<td></td>
<td></td>
<td>Emmy Noether—Her work and influence</td>
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<td></td>
<td></td>
<td>Martha K. Smith</td>
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<td></td>
<td></td>
<td></td>
<td>AWM - BUSINESS MEETING</td>
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<td></td>
<td></td>
<td></td>
<td>Room #2135 Sidney Smith Hall</td>
</tr>
</tbody>
</table>
## TIMETABLE

### THURSDAY, August 26

<table>
<thead>
<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Other Organizations</th>
</tr>
</thead>
</table>
| 1:30 p.m. - 2:30 p.m. | **COLLOQUIUM LECTURES:**  
Recent progress in dynamical systems:  
Lecture III, Integrable Hamiltonian systems  
Jürgen K. Moser, Convocation Hall  
**SPECIAL SESSIONS**  
2:30 p.m. - 5:30 p.m. | Combinatorial identities III  
Room #159 Lash Miller Hall  
2:30 p.m. - 6:00 p.m. | Doubly stochastic measures and their operators I  
Room #2118 Sidney Smith Hall  
2:30 p.m. - 6:00 p.m. | Knots and 3-manifolds I  
Room #2117 Sidney Smith Hall  
2:30 p.m. - 5:30 p.m. | **SME Contributed Paper Sessions**  
Room #1072 Sidney Smith Hall  
2:45 p.m. - 3:45 p.m. | **INVITED ADDRESS:**  
Determining the finite simple groups  
Michael Aschbacher, Convocation Hall  
**SESSIONS FOR CONTRIBUTED PAPERS**  
2:45 p.m. - 4:30 p.m. | Analysis III  
Room #1071 Sidney Smith Hall  
2:45 p.m. - 4:30 p.m. | Differential equations  
Room #1069 Sidney Smith Hall  
**SPECIAL SESSIONS**  
3:00 p.m. - 6:00 p.m. | Category theory II  
Room #162 Lash Miller Hall  
3:00 p.m. - 5:25 p.m. | Mathematical psychology I  
Room #2102 Sidney Smith Hall  
Convocation Hall  
4:00 p.m. - 5:00 p.m. | **INVITED ADDRESS:**  
Principles and practice of nonstandard analysis  
Edward Nelson  
5:15 p.m. - 6:15 p.m. | **INVITED ADDRESS:**  
Computability revisited—an approach via the continuum  
Marian Boykan Pour-El  
7:00 p.m. - 9:48 p.m. | **MAA FILM PROGRAM**  
Faculty of Education Auditorium  
All films are from the MAA Mathematics Today Series; unless noted otherwise, all films are in color  
7:00 p.m. - 8:01 p.m. | Let us teach guessing  
8:05 p.m. - 9:00 p.m. | Challenge in the classroom  
9:05 p.m. - 9:48 p.m. | Götingen and New York  
7:00 p.m. - 10:00 p.m. | **MAA SECTION OFFICERS**  
Council Chamber, Galbraith Building  
**MAG SESSION:** Other mathematical models in economics  
Francis R. Bafouzoukas  
Graciela Chichilnisky  
Room #2135 Sidney Smith Hall  
8:00 p.m. - 10:00 p.m.  
|  
### FRIDAY, August 27

<table>
<thead>
<tr>
<th>Time</th>
<th>AMS</th>
<th>Other Organizations</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:30 a.m. - noon</td>
<td><strong>EXHIBITS</strong> - Room #2096 Sidney Smith Hall</td>
<td></td>
</tr>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td><strong>REGISTRATION</strong> - Entrance Lobby, Sidney Smith Hall</td>
<td></td>
</tr>
<tr>
<td>8:30 a.m. - 4:30 p.m.</td>
<td><strong>EMPLOYMENT REGISTER</strong> - Room #2042 Sidney Smith Hall</td>
<td></td>
</tr>
</tbody>
</table>
| 9:00 a.m. - 9:50 a.m. | **MAA THE EARL RAYMOND HEDRICK LECTURES**  
Some mathematical applications of logic: Lecture II, Diophantine sets  
Martin D. Davis |

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<table>
<thead>
<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Other Organizations</th>
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<tr>
<td>10:00 a.m. - 10:50 a.m.</td>
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<td>Convocation Hall</td>
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<tr>
<td>11:00 a.m. - 11:50 a.m.</td>
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<td>MAA - BUSINESS MEETING</td>
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<td>Presentation of Lester R. Ford Awards and Tribute to a distinguished Association member</td>
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<tr>
<td>noon - 1:00 p.m.</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS</td>
<td>MAA - RETIRING PRESIDENTIAL ADDRESS</td>
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<tr>
<td></td>
<td>Combinatorics II</td>
<td>Convergence, divergence, and the computer</td>
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<td>Room #1069 Sidney Smith Hall</td>
<td>Ralph P. Boas</td>
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<tr>
<td>12:10 p.m. - 1:10 p.m.</td>
<td>SPECIAL SESSIONS</td>
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<tr>
<td>1:30 p.m. - 2:30 p.m.</td>
<td>COLOQUIUM LECTURES:</td>
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<td></td>
<td>Recursive enumerable sets and degrees I</td>
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<td>Room #2135 Sidney Smith Hall</td>
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<tr>
<td>2:30 p.m. - 5:20 p.m.</td>
<td>SPECIAL SESSIONS</td>
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<tr>
<td>2:30 p.m. - 6:00 p.m.</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS</td>
<td>Conference Board of the Mathematical Sciences</td>
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<tr>
<td>2:30 p.m. - 6:00 p.m.</td>
<td>Complex Analysis</td>
<td>COUNCIL MEETING</td>
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<tr>
<td>6:30 p.m. - 9:00 p.m.</td>
<td>SPECIAL SESSIONS</td>
<td>Council Chamber, Galbraith Building</td>
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<tr>
<td>7:00 p.m. - 10:03 p.m.</td>
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<tr>
<td>7:00 p.m. - 7:48 p.m.</td>
<td>MAA - Banquet for 30 year members</td>
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<tr>
<td>7:55 p.m. - 8:55 p.m.</td>
<td>Wetmore Hall Cafeteria, New College</td>
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<tr>
<td>9:00 p.m. - 10:03 p.m.</td>
<td>MAA - FILM PROGRAM</td>
<td>All films are from the MAA Mathematics Today Series; unless noted otherwise, all films are in color</td>
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<tr>
<td>8:00 p.m. - 10:30 p.m.</td>
<td>Faculty of Education Auditorium</td>
<td>Pits, peaks, and passes</td>
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<tr>
<td>SATURDAY, August 28</td>
<td>8:30 a.m. - 1:30 p.m.</td>
<td>Fixed points</td>
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<td>REGISTRATION - Entrance Lobby, Sidney Smith Hall</td>
<td>John Von Neumann, A documentary (b &amp; w)</td>
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<td>CBMS - COUNCIL MEETING</td>
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<td>Council Chamber, Galbraith Building</td>
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<td>Convocation Hall</td>
<td>THE EARLE RAYMOND HEDRICK LECTURES:</td>
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<tr>
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<td>Some mathematical applications of logic:</td>
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<td>Lecture III, Nonstandard analysis</td>
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<td>Martin D. Davis</td>
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<td>INVITED ADDRESS</td>
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<tr>
<td></td>
<td></td>
<td>Mathematics of genetics and evolution</td>
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<td>Samuel Karlin</td>
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<th>Time</th>
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| 11:00 a.m. - 11:50 a.m. | INVITED ADDRESS  
Geometrical models  
H. S. M. Coxeter |
| 1:30 p.m. - 2:20 p.m. | INVITED ADDRESS  
The role of mathematics in the design of an  
x-ray Tomographic Human Body Scanner  
Lawrence A. Shepp |
| 2:30 p.m. - 3:20 p.m. | INVITED ADDRESS  
J. C. Fields, the Fields Medal, and mathematical research  
Henry S. Tropp |
| 3:30 p.m. - 4:20 p.m. | INVITED ADDRESS  
Modern algebraic approach to combinatorial designs  
N. S. Mendelsohn |
PROGRAM OF THE SESSIONS

SHORT COURSE ON MATHEMATICAL ECONOMICS

All Sessions in Room 2135 Sidney Smith Hall, University of Toronto

SUNDAY, August 22, 9:00 a.m.-5:00 p.m.

9:00 a.m.-10:15 a.m. Price formation: The theory of general economic equilibrium. HUGO F. SONNENSCHEIN, Princeton University

10:45 a.m.-noon Measure spaces of economic agents. WERNER HILDENBRAND, University of Bonn

2:00 p.m.- 3:15 p.m. The role of prices and interest rates in dynamic economics. DAVID GALE, University of California, Berkeley, and Center for Advanced Study in the Behavioral Sciences, Stanford

3:45 p.m.- 5:00 p.m. The theory of economic equilibrium from the differentiable point of view. ANDREU MAS-COLELL, University of California, Berkeley

MONDAY, August 23, 9:00 a.m.-noon

9:00 a.m.-10:15 a.m. Some game models in economics. ROBERT J. AUMANN, Hebrew University and Stanford University

10:45 a.m.-noon Computation and existence of Walras equilibria. STEPHEN SMALE, University of California, Berkeley

THE EIGHTIETH SUMMER MEETING

The time limit for each contributed paper in the general sessions is ten minutes. In the special sessions the time varies from session to session and within sessions. To maintain the schedule, the time limits will be strictly enforced.

TUESDAY, 9:00 A. M.

Colloquium Lectures: Lecture I, Convocation Hall

(1) Recent progress in dynamical systems: A survey. Professor JÜRGEN K. MOSER, Courant Institute, New York University

Bicentennial Lecture, Convocation Hall

(2) Maps between classifying spaces. Professor J. FRANK ADAMS, University of Cambridge, England (737-55-3)

TUESDAY, 12:00 Noon

Special Session on Chromatic Polynomials and Related Topics I, 159 Lash Miller Hall

12:00-12:20 (3) Chromatic equivalence and constrained polynomials. Preliminary report. Professor RUTH A. BARI, George Washington University (737-05-19)

12:30-12:50 (4) The dichromatic and orientations of a graph. Professor GERALD BERMAN, University of Waterloo (737-05-12)

1:00- 1:20 (5) The asymptotic behaviour of certain chromatic sums. Preliminary report. BRUCE RICHMOND, University of Waterloo (737-05-7)

1:30- 1:50 (6) The regularity of the graphs \(\{6,3\}\)\(_b,c\). Professor H. S. M. COXETER, University of Toronto

2:00- 2:20 (7) Informal Session

TUESDAY, 1:00 P. M.

Session on Algebra I, 1069 Sidney Smith Hall

1:00- 1:10 (8) Counting permutation polynomials in \(n\) indeterminates over \(GF(q)\). Preliminary report. Dr. R. G. Van Meter, State University of New York, College at Oneonta (737-12-2)

1:15- 1:25 (9) Geometric characterizations of lattice orders on fields. Preliminary report. Dr. ROBERT ROSS WILSON, California State University, Long Beach (737-12-3)

1:30- 1:40 (10) Normed fields and topological fields which are not copies of \(C\). GERHARD F. KOHLMAYR, Mathmodel Consulting Bureau, Glastonbury, Connecticut (737-12-4)

1:45- 1:55 (11) Finitely generated projective ideals in commutative rings. Dr. ADIL G. NAOUM, University of Baghdad, Iraq (737-13-1)

2:00- 2:10 (12) Sweedler's two-cocycles and a theorem of Rosenberg and Zelinsky. Preliminary report. Mr. DAVE RIFFELMACHER, Cornell University (737-16-1)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
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<th>Time</th>
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<tr>
<td>2:15-2:25</td>
<td>Rationally complete ring extensions. Preliminary report. Mr. JOHN LAWRENCE, University of Chicago (737-16-2)</td>
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<td>2:30-2:40</td>
<td>S-ideals of a matrix ring. Preliminary report. Dr. H. ZAND, Free University of Iran, Tehran (737-16-3) (Introduced by Mr. M. Razzaghi)</td>
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<tr>
<td>2:45-2:55</td>
<td>Certain generalizations of Boolean rings. Professor HAL G. MOORE*, Brigham Young University, and Professor ADIL YAQUB, University of California, Santa Barbara (737-16-4)</td>
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<td>3:00-3:10</td>
<td>Stem kernels. Preliminary report. Professor BRUCE CONRAD, Temple University (737-18-11)</td>
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**Tuesday, 1:00 P.M.**

**Session on Applied Mathematics I, 1071 Sidney Smith Hall**

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<th>Time</th>
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<tbody>
<tr>
<td>1:00-1:10</td>
<td>Periodic solutions of finite difference approximations. Professor MARTIN BRAUN* and Professor JOSEPH HERSHENOV, City University of New York, Queens College (737-65-1)</td>
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<tr>
<td>1:15-1:25</td>
<td>A new form of trigonometric orthogonality and Gaussian-type quadrature. Dr. HERBERT E. SALZER, Brooklyn, New York (737-65-2)</td>
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<tr>
<td>1:30-1:40</td>
<td>An $O(h^4)$ finite difference analogue for the numerical solution of some differential equations occurring in plate deflection theory. Dr. RIAZ A. USMANI, University of Manitoba (737-65-3) (Introduced by Professor F. M. Arscott)</td>
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<td>2:00-2:10</td>
<td>Analytical vs. implicit numerical solutions of certain time-dependent fluid flow problems. Dr. S. K. DEY, Eastern Illinois University (737-76-1)</td>
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<td>2:15-2:25</td>
<td>A family of free streamline problems with no external forces. VURYL J. KLASSEN, California State University, Fullerton (737-76-2)</td>
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<td>2:30-2:40</td>
<td>A maximum principle for compressible flows on manifolds. Professor ROBERT J. SIBNER, City University of New York, Brooklyn College (737-76-3)</td>
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<td>2:45-2:55</td>
<td>A model of thermal response to radiation. Preliminary report. DAVID K. COHOON, School of Aerospace Medicine, San Antonio, Texas (737-80-1)</td>
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**Session on General Topology, 1083 Sidney Smith Hall**

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<th>Time</th>
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<tbody>
<tr>
<td>1:00-1:10</td>
<td>Hereditarily indecomposable tree-like continua. Professor W. T. INGRAM, University of Houston (737-54-2)</td>
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<td>1:15-1:25</td>
<td>On $\sigma$-connected spaces. Mr. ELSAYED A. ABO-ZEID, University of Saskatchewan (737-54-1)</td>
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<td>1:30-1:40</td>
<td>A note on preparacompactness. Preliminary report. Professor J. C. SMITH, Virginia Polytechnic Institute and State University (737-54-3)</td>
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<td>1:45-1:55</td>
<td>Some hyperspace uniformities. Preliminary report. Dr. ROBERT A. HOVIS, Ohio Northern University (737-54-4)</td>
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<td>2:00-2:10</td>
<td>Stratifiable $k\R$-space which is not $k$-space. Professor CARLOS R. BORGES, University of California, Davis (737-54-5)</td>
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<td>2:15-2:25</td>
<td>A nonstandard characterization of perfect mappings. Dr. ROBERT WARREN BUTTON, Southern Illinois University (737-54-6)</td>
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<td>2:30-2:40</td>
<td>Notes on $Z$-embedding. Preliminary report. C. E. AULL, Virginia Polytechnic Institute and State University (737-54-7)</td>
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<td>3:00-3:10</td>
<td>A mapping theorem basic to open mapping theory. Professor HOWARD H. WICKE, Ohio University (737-54-9)</td>
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<td>3:15-3:25</td>
<td>Weak forms of compactness and B-completeness in topological groups. Dr. D. L. GRANT, College of Cape Breton (737-22-1)</td>
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**Tuesday, 1:30 P.M.**

**Invited Address, Convocation Hall**

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<tr>
<td>3:35-3:45</td>
<td>Nearly flat triangulations of Riemannian manifolds. Professor EUGENIO CALABI, University of Pennsylvania (737-53-4)</td>
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TUESDAY, 2:45 P. M.

Invited Address, Convocation Hall

(36) On the invariant subspace problem for Banach spaces. PER ENFLO, University of Stockholm, Sweden

TUESDAY, 3:00 P. M.

Special Session on Combinatorial Identities I, 159 Lash Miller Hall

3:00–3:20 (37) An overview of combinatorial identities. Professor H. W. GOULD, West Virginia University (737–05–22)

3:30–3:50 (38) Some combinatorial formulas extending theorems of Steiner and Roberts on arrangements in Ed. Preliminary report. Professors G. L. ALEXANDERSON*, University of Santa Clara, and John E. WETZEL, University of Illinois at Urbana–Champaign (737–50–3)

4:00–4:20 (39) Quadratic transformations of hypergeometric series. Professor RICHARD ASKEY, University of Wisconsin, and Professor MOURAD ISMAIL*, McMaster University (737–33–3)

4:30–4:50 (40) On a unimodal sequence of binomial coefficients. II. Preliminary report. Professor STEPHEN M. TANNY*, University of Toronto, and Dr. MICHAEL ZUKER, National Research Council, Ottawa, Canada (737–05–13)

5:00–5:20 (41) Sums associated with containment relations among intervals. Professor CLARK KIMBERLING, University of Evansville (737–05–16)

5:30–5:50 (42) Units in algebraic number fields and combinatorial identities. Professor LEON BERNSTEIN, Illinois Institute of Technology (737–05–6)

TUESDAY, 3:00 P. M.

Session on Applied Mathematics II, 1071 Sidney Smith Hall

3:00–3:10 (43) On the approximation of Wiener–Feynman integrals. Preliminary report. Dr. LEONARD J. GRAY*, Union Carbide Corporation, Oak Ridge, Tennessee, and Dr. THEODORE KAPLAN, Oak Ridge National Laboratory, Tennessee (737–81–1)


3:45–3:55 (46) Some examples of incomplete space-times. Professor JOHN K. BEEM, University of Missouri, Columbia (737–83–1)

4:00–4:10 (47) Mean curvature of gravitational fields. Professor KISHORE B. MARATHE, Brooklyn College (737–83–2)


4:30–4:40 (49) The exponential smoothing model, theory and application. Preliminary report. Mr. I. STEPHAN SIMS, Memphis State University (737–90–1) (Introduced by Ralph J. Faudree)


5:00–5:10 (51) The existence of odd group codes for the Gaussian channel. Preliminary report. Professors J. K. KARLOF* and CHARLES P. DOWNEY, University of Nebraska at Omaha (737–94–1)

TUESDAY, 3:15 P. M.

Session on Algebra II, 1069 Sidney Smith Hall

3:15–3:25 (52) Purity and copurity in systems of linear transformations. Professor FRANK ZORZITTO, University of Waterloo (737–15–1) (Introduced by U. Fixman)


3:45–3:55 (54) On the class of M-matricies with a given singular graph. Preliminary report. Dr. DANIEL J. RICHMAN, University of Wisconsin (737–15–4)

4:15- 4:25 (56) Some counterexamples for infinite dimensional Lie algebras. Professor GARY E. STEVENS, Hartwick College (737-17-1)

4:30- 4:40 (57) Riemann surfaces over regular maps. Preliminary report. Mr. STEPHEN E. WILSON, University of Washington (737-20-3)

4:45- 4:55 (58) The commutator calculus applied to nilpotent products of cyclic groups. Professor A. M. GAGLIONE, City University of New York, City College (737-20-2)

5:00- 5:10 (59) Locally free groups and direct limits of CW-complexes. Preliminary report. Professor BENJAMIN FINE* and Professor GEORGE E. LANG, Jr., Fairfield University (737-20-4)

5:15- 5:25 (60) Some asymptotic formulas in group theory. M. RAM MURTY* and V. KUMAR MURTY, Carleton University (737-20-6)

5:30- 5:40 (61) Some counterexamples for infinite dimensional Lie algebras. Professor GARY E. STEVENS, Hartwick College (737-17-1)

TUESDAY, 3:15 P. M.

Session on Manifolds, 1085 Sidney Smith Hall

3:15- 3:25 (62) Lifts and prolongations of connections. Preliminary report. Dr. JURAJ VIRSIK, Monash University, Clayton, Australia and University of California, Berkeley (737-53-2)

3:30- 3:40 (63) On the infinitesimal action of a Lie group on vector bundles. Preliminary report. Dr. YVETTE KOSMANN-SCHWARZBACH, Université de Lille I, France, and Brooklyn, New York (737-53-3)

3:45- 3:55 (64) Relations among characteristic classes. II. Preliminary report. Professor STAVROS G. PAPASTAVRIDIS, University of Athens, Greece (737-55-4)

4:00- 4:10 (65) Relations between the Alexander polynomial and summit power of a closed braid. Preliminary report. Dr. MARK E. KIDWELL, Yale University (737-55-9)

4:15- 4:25 (66) T^n-actions on simply connected (n + 2)-manifolds. Preliminary report. Professor DENNIS McGAVRAN, University of Connecticut, Waterbury (737-57-1)

4:30- 4:40 (67) On certain problems connected with the homeomorphisms which satisfy the Poincaré recurrence theorem. Professor CHUNG-WU HO, Southern Illinois University (737-58-1)

4:45- 4:55 (68) The homology of literary schema. Professor VINCENT O. McBRIEN, College of the Holy Cross (737-55-1)

TUESDAY, 4:00 P. M.

Invited Address, Convocation Hall

(69) Optimal reconstruction of a function from its projections. Dr. L. A. SHEPP, Bell Laboratories, Murray Hill, New Jersey (737-26-1)

TUESDAY, 8:30 P. M.

Open Meeting: The state of the job market, Convocation Hall

Professor WENDELL H. FLEMING, Brown University (Moderator)

WEDNESDAY, 9:00 A. M.

Colloquium Lectures: Lecture II, Convocation Hall

(70) Recent progress in dynamical systems: Periodic orbits. Professor JÜRGEN K. MOSER, Courant Institute, New York University

WEDNESDAY, 10:00 A. M.

Special Session on Chromatic Polynomials and Related Topics II, 159 Lash Miller Hall

10:00-10:20 (71) Why is the four-color problem difficult? Preliminary report. Professor WOLFGANG HAKEN, University of Illinois (737-05-24)

10:30-10:50 (72) Constrained chromials. Preliminary report. Professor DICK WICK HALL, State University of New York at Binghamton (737-05-8)

11:00-11:20 (73) Higher Euler invariants. Preliminary report. Professor PAUL C. KAINEN* and Mr. MARK THIEL, Case Western Reserve University (737-05-30)

11:30-11:50 (74) On the chromatic polynomial and the rotor effect. Mr. STÉPHANE PÖLDES, Department of Combinatorics and Optimization, University of Waterloo (737-05-32) (Introduced by Professor W. T. Tutte)

12:00-12:20 (75) Counting monochromatic paths and stars. Professor A. CZERNIAKIEWICZ, City University of New York, Queens College (737-05-3)

12:30-12:50 (76) General solutions for free chromatic polynomials on the plane. Mr. MICHAEL R. ROLLE, Department of Combinatorics and Optimization, University of Waterloo (737-05-31) (Introduced by W. T. Tutte)
General Session, 1072 Sidney Smith Hall

10:15-10:25  (77) A graph-theoretical generalization of a Cantor theorem. SIEMION FAJTLOWICZ, University of Houston (737-04-1)

10:30-10:40  (78) On a problem of Peter Fenton and the distance set of the Cantor set. Dr. BENJAMIN LEPSON, U. S. Naval Research Laboratory, Washington, D.C. (737-04-2)

10:45-10:55  (79) Planar sublattices of a free lattice. Dr. IVAN RIVAL and Mr. BILL SANDS*, University of Calgary (737-06-1)

11:00-11:10  (80) Inductive Boolean algebras and special prime ideals. Professor ALEXANDER ABIAN, Iowa State University (737-06-2)

11:15-11:25  (81) Representation theorems for varieties generated by single precomplete algebras. R. ARTHUR KNOEBEL, New Mexico State University (737-08-1)

11:30-11:40  (82) Categories for the beginning mathematician. JOSEPH C. BODENRADER and WILLIAM E. HARTNETT*, State University of New York, College at Plattsburgh (737-98-1)


Session on Analysis I, 1071 Sidney Smith Hall

10:15-10:25  (84) Uniqueness in the theory of variational inequalities. Professor GILBERT STRANG, Massachusetts Institute of Technology (737-47-4)

10:30-10:40  (85) Transformation of differentiable functions. Dr. RICHARD FLEISSNER, University of Wisconsin, Milwaukee, and Professor JAMES FORAN*, University of Missouri, Kansas City (737-26-2)

10:45-10:55  (86) Scalar Radon-Nikodym derivative for an H*-algebra valued measure. Professor PARFENY P. SAWOROTNOW, Catholic University of America (737-28-1)

11:00-11:10  (87) Extension of a continuous strongly additive group-valued set function. GEOFFREY FOX and PEDRO MORALES*, Université de Montréal (737-28-4)


11:30-11:40  (89) On unconditional section boundedness in sequence spaces. JOHN J. SEMBER, Simon Fraser University (737-40-1)

11:45-11:55  (90) Algorithms for rational approximations for a confluent hypergeometric function. II. Professor YUDELL L. LUKE, University of Missouri, Kansas City (737-41-1)

12:00-12:10  (91) On the existence of a class of best nonlinear approximations in Hilbert spaces. Preliminary report. Dr. NIRA RICHTER-DYN, Tel Aviv University, Israel (737-41-3) (Introduced by Ghandour Edmond)

12:15-12:25  (92) On a family of approximation operators. Dr. MOURAD E. H. ISMAIL, McMaster University, and Dr. C. PING MAY*, University of Toronto (737-41-4)

12:30-12:40  (93) On the degree of approximation by monotone polynomials. Professor VASANT A. UBHAYA, Case Western Reserve University (737-41-5)

Session on Foundations, 1085 Sidney Smith Hall

10:15-10:25  (94) Foundations of recursively presented models. Preliminary report. Dr. TERRENCE S. MILLAR, Cornell University (737-02-1) (Introduced by Anil Nerode)

10:30-10:40  (95) Models without indiscernibles. Professor FRED G. ABRAMSON*, University of Wisconsin, Milwaukee, and Professor LEO A. HARRINGTON, University of California, Berkeley (737-02-5)

10:45-10:55  (96) Subspaces of V_4. Preliminary report. Mr. ALLEN T. RETZLAFF, Cornell University (737-02-2) (Introduced by Professor Anil Nerode)

11:00-11:10  (97) Automorphisms of a lattice of recursively enumerable subspaces. Preliminary report, Dr. IRAJ KALANTARI, Cornell University (737-02-3) (Introduced by Professor Anil Nerode)

11:15-11:25  (98) The Gödel sentence and the sequential and holistic modes of recognition (denotation). Professor MIRIAM LIPSCHUTZ-YEVICK, Rutgers University (737-02-6)

Session on Number Theory, 1083 Sidney Smith Hall

10:15-10:25  (99) Proof of certain identities conjectured by Ramanujan. Preliminary report. Mr. DAVID M. BRESSOUD, Temple University (737-10-1)

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10:30-10:40 (100) Waring's problem mod n. Professor CHARLES SMALL, Queens University (737–10–2)

10:45-10:55 (101) The asymptotic behaviour of zeta functions associated to positive definite forms with integer coefficients. Preliminary report. Professor CHUNGMIN AN, Seton Hall University, and Professor ALAN H. STEIN*, University of Connecticut, Waterbury (737–10–3)

11:00-11:10 (102) Nonstandard derivations of Hardy–Littlewood like formulas. Preliminary report. Professor SHUICHI TAKAHASHI, University of Montreal (737–10–4)

11:15-11:25 (103) Special pythagorean triples. Preliminary report. Professor GERALD E. BERGUM* and Miss MARY ANN DAHLQUIST, South Dakota State University (737–10–5)

11:30-11:40 (104) New observations on "Fermat's last theorem". Professor JOSEPH ARKIN, Spring Valley, New York (737–10–6)

11:45-11:55 (105) Coefficients of the cyclotomic polynomial $F_{3qr}(x)$. Preliminary report. Professor MARION BEITER, Rosary Hill College (737–12–1)

WEDNESDAY, 10:15 A.M.

Session on Probability, 1069 Sidney Smith Hall

10:15-10:25 (106) A discrete queueing problem arising in packet switching. Preliminary report. Dr. B. GOPINATH and Dr. JOHN A. MORRISON*, Bell Laboratories, Murray Hill, New Jersey (737–60–1)

10:30-10:40 (107) A discrete queueing problem arising in packet switching. Preliminary report. Dr. B. GOPINATH and Dr. JOHN A. MORRISON*, Bell Laboratories, Murray Hill, New Jersey (737–60–1)

10:45-10:55 (108) Methods of proof for Bonferroni inequalities. Professor JANOS GALAMBOS, Temple University (737–60–3)

11:00-11:10 (109) Unique ergodicity for random maps. ROBERT SINE, University of Rhode Island (737–60–4)

11:15-11:25 (110) Weak convergence of conditioned random walks. Preliminary report. Mr. ROBERT M. ANDERSON, Yale University (737–60–10)

WEDNESDAY, 10:30 A.M.

Special Session on Differentiable Dynamical Systems I, 2118 Sidney Smith Hall

10:30-10:50 (111) Double collisions for non-Newtonian potentials. Preliminary report. Dr. RICHARD McGEHEE, University of Minnesota (737–70–1)

11:00-11:20 (112) Transversal homoclinic orbits in an integrable system. Professor ROBERT L. DEVANEY, Northwestern University (737–58–3)

11:30-11:50 (113) On the structure and stability of local Pareto optima with constraints. Professor YIEH-HEI WAN, State University of New York at Buffalo (737–58–6)

12:00-12:30 (114) Informal Discussion

WEDNESDAY, 12:30 P.M.

Special Session on Category Theory I, 162 Lash Miller Hall

12:30-12:50 (115) Pulling paths, and canonical sheaves of paths. Professor JOHN R. ISBELLS, State University of New York at Buffalo (737–18–4)

1:00-1:20 (116) Closed categories of variable quantities over a base topos. Preliminary report. Professor F. WILLIAM LAWVERE, State University of New York at Albany (737–18–8)

1:30-1:50 (117) Indicial methods for relative categories. Preliminary report. RICHARD J. WOOD, Dalhousie University (737–18–1)

2:00-3:00 (118) Discussion Period

WEDNESDAY, 1:30 P.M.

Special Session on Differentiable Dynamical Systems II, 2118 Sidney Smith Hall

1:30–1:50 (119) When are chain recurrent points on cycles? Preliminary report. Professor JOHN E. FRANKE* and JAMES F. SELGRADE, North Carolina State University (737–58–5)


2:30–2:50 (121) Fuller’s index and global Hopf bifurcation. Professor SHUI-NEE CHOW, Michigan State University, and Professor JOHN MALLET-PARET*, Brown University (737–58–8)
3:00–3:20 (122) Topological entropy at an $\Omega$-explosion. Preliminary report. Dr. LOUIS BLOCK, University of Florida (737-58-2)

3:30–3:50 (123) Constructing flows in $n$ dimensions with prescribed invariant sets. Preliminary report. Dr. RONALD SVERDOLOVE, Stanford University (737-58-7)

WEDNESDAY, 1:30 P. M.

Session on Analysis II, 1071 Sidney Smith Hall

1:30–1:40 (124) Multiplier criteria of Marcinkiewicz type for Jacobi expansions. Professor GEORGE GASPER*, Northwestern University, and Professor WALTER TREVELS, Technische Hochschule, Darmstadt, West Germany (737-42-1)

1:45–1:55 (125) A strengthened form of a theorem of Zygmund on integrability of a function. Preliminary report. Dr. RAFA N. SIDDQI, Université de Moncton (737-42-2)

2:00–2:10 (126) On the convergence of the Fourier integral of a unimodal distribution. Preliminary report. Professor CONSTANTINE GEORGAKIS, DePaul University (737-42-3)

2:15–2:25 (127) Dual orthogonal series: An abstract approach II. Professor ROBERT FEINERMAN*, City University of New York, Lehman College, and Professor ROBERT KELMAN, Colorado State University (737-42-4)

2:30–2:40 (128) Invariant functionals. Preliminary report. Dr. WATSON L. CHIN, Southwestern Union College (737-43-1)

2:45–2:55 (129) Weakly almost periodic measures on locally compact semigroups. Preliminary report. Dr. H. KHARGHANI, Pahlavi University, Shiraz, Iran (737-43-2)

3:00–3:10 (130) On topologies of topological groups. Preliminary report. Professor TER-JENQ HUANG, State University of New York, College at Cortland (737-43-3)

WEDNESDAY, 1:30 P. M.

Session on Functional Analysis, 1069 Sidney Smith Hall

1:30–1:40 (131) $L^q + (l^q \div C) \div \{\text{Blaschke products}\}$. SHELDON AXLER, Massachusetts Institute of Technology (737-46-9)

1:45–1:55 (132) Controllability, stabilization, and mean ergodic theorems. Preliminary report. R. E. O'BRIEN, Goddard Space Flight Center, Greenbelt, Maryland (737-46-1)

2:00–2:10 (133) On uniqueness of weak solutions of abstract differential inequality. Dr. M. A. MALIK, Concordia University (737-46-2)


2:30–2:40 (135) Closure of integrals of set-valued functions. Dr. CHARLES BYRNE, Catholic University of America (737-46-6)

2:45–2:55 (136) Vector measures and scalar operators in locally convex spaces. Professor ALAN SHUCHAT, Wellesley College (737-46-7)

3:00–3:10 (137) Weak convergence of operators. Preliminary report. Professor J. K. BROOKS*, University of Florida, and Professor P. W. LEWIS, North Texas State University (737-46-8)


3:30–3:40 (139) Function space completions. Professor JOHN W. BRACE, University of Maryland, and Professor JOEL D. THOMSON*, Ithaca College (737-46-4)

WEDNESDAY, 1:30 P. M.

Session on Geometry, 1085 Sidney Smith Hall

1:30–1:40 (140) On the space-filling heptahedra. Mr. MICHAEL GOLDBERG, Washington, D.C. (737–50–1)

1:45–1:55 (141) The ninety-one types of isogonal tilings of the plane. Professor BRANKO GRÜN­BAUM*, University of Washington, and Professor G. C. SHEPHARD, University of British Columbia (737–50–2)

2:00–2:10 (142) Conditions for summability for series of convex sets in the space $C(Y)$ of all closed convex and bounded sets of a reflective Banach space. Dr. DAGMAR R. HENNEY, George Washington University (737–52–1)

2:15–2:25 (143) Radon type theorems without independence conditions. Professor JOHN R. REAY, Western Washington State College (737–52–2)

2:30–2:40 (144) On totally real flat surfaces. Professor BANG-YEN CHEN, Michigan State University, and Professor CHORNG-SHI HOUH*, Wayne State University (737–53–1)
Special Session on Combinatorial Identities II, 159 Lash Miller Hall

1:45–2:05 (145) Combinatorial problems arising in the health sciences. Professor ROGER C. GRIMSON, School of Public Health, University of North Carolina, Chapel Hill (737-05-26)

2:15–2:35 (146) Stirling number identities from chromatic polynomials. Professor E. G. WHITEHEAD, Jr., University of Pittsburgh (737-05-5)

2:45–3:05 (147) Combinatorial identities generalized via the fractional calculus. Professor THOMAS J. OSLER, Glassboro State College (737-05-15)

3:15–3:35 (148) The multiple umbral notation as a manipulative aid. ANDREW P. GUINAND, Trent University (737-05-9)

WEDNESDAY, 4:00 P. M.

Business Meeting, Convocation Hall

THURSDAY, 12:00 noon

Session on Combinatorics I, 1069 Sidney Smith Hall

12:00–12:10 (149) Order 24 Hadamard matrices of characteristic at least 2. Professor JUDITH Q. LONGYEAR, Wayne State University (737-05-1)


12:30–12:40 (151) A scheduling problem for tournaments with a constraint on locations. PAUL SMITH, University of Victoria and University of Montana (737-05-10) (Introduced by Dr. H. E. Reinhardt)

12:45–12:55 (152) A connectedness game and the c-complexity of certain graphs. GOPAL DANARAJ, Cleveland State University, and VICTOR KLEE*, University of Washington (737-05-18)

THURSDAY, 1:30 P. M.

Colloquium Lectures: Lecture III, Convocation Hall

(153) Recent progress in dynamical systems: Integrable Hamiltonian systems. Professor JÜRGEN K. MOSER, Courant Institute, New York University

THURSDAY, 2:30 P. M.

Special Session on Combinatorial Identities III, 159 Lash Miller Hall

2:30–2:50 (154) Numbers generated by the reciprocal of a series. Preliminary report. Professor FREDRIC T. HOWARD, Wake Forest University (737-05-4)

3:00–3:20 (155) A combinatorial sum. Professor M. L. GLASSER, University of Waterloo (737-33-4) (Introduced by M. S. Klamkin)

3:30–3:50 (156) Expansions and convolution formulas related to MacMahon's master theorem. Professor I. CARLITZ, Duke University (737-41-2)


5:00–5:20 (159) Informal Problem Session

THURSDAY, 5:00 P. M.

Special Session on Doubly Stochastic Measures and Their Operators I, 2118 Sidney Smith Hall

2:30–3:15 (160) Developments in the theory of doubly stochastic measures and transformations. Professor JOHN V. RYFF, University of Connecticut, Storrs (737-47-3)


4:00–4:20 (162) Bistochastic operators. Professor M. M. RAO, University of California, Riverside (737-46-5)

4:30–5:00 (163) Discussion Period

5:00–5:20 (164) A covariant structure for doubly stochastic measures. Preliminary report. Dr. PAUL N. DE LAND* and Dr. RAY C. SHIPLETT, California State University, Fullerton (737-60-6)

5:30–5:50 (165) N-dimensional distributions with given k-dimensional marginals. Preliminary report. J. H. B. KEMP ERMAN, University of Rochester (737-60-7)
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<th>Session</th>
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<td>2:30- 2:50</td>
<td>Inductive arguments on rational (two-bridge) knots. Professor E. J. Mayland, Jr., York University (737-57-2)</td>
<td>2117 Sidney Smith Hall</td>
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<td>3:00- 3:20</td>
<td>Prime factors of knot manifolds. Professor Wilbur Whitten, University of Southwestern Louisiana (737-55-6)</td>
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<td>3:30- 3:50</td>
<td>Fibered knots in homotopy 3-spheres. Jonathan Simon, University of Iowa (737-55-5)</td>
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<td>4:00- 4:20</td>
<td>Surgery and the calculus of links. Dale Rolfsen, University of British Columbia (737-57-7)</td>
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<td>5:00- 5:20</td>
<td>I-equivalent graphs in $S^3$. Professor Daniel R. McMillan, Jr., University of Wisconsin (737-57-6)</td>
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<td>5:30- 6:00</td>
<td>Discussion Period</td>
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<td>2:45</td>
<td>Determining the finite simple groups. Professor Michael Aschbacher, California Institute of Technology (737-20-1)</td>
<td>2117 Sidney Smith Hall</td>
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<td>2:45- 2:55</td>
<td>Monotonicity and convexity properties of zeros of Bessel functions. Professor J. T. Lewis, Dublin Institute for Advanced Study, Ireland, and Professor M. E. Muldoon*, York University (737-33-5)</td>
<td>1069 Sidney Smith Hall</td>
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<tr>
<td>3:00- 3:10</td>
<td>Operational calculus for functions of two variables. Professor Harris S. Shultz, California State University, Fullerton (737-44-1)</td>
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<td>3:15- 3:25</td>
<td>The Kantorovich-Lebedev transformation of distributions. Preliminary report, Professor R. S. Pathak* and Professor J. N. Pandey, Carleton University (737-44-2)</td>
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<tr>
<td>3:45- 3:55</td>
<td>Some fixed point theorems for multivalued mappings. Preliminary report, Professor V. M. Sehgal, University of Wyoming, and Professor S. P. Singh*, Memorial University of Newfoundland (737-47-1)</td>
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<td>4:00- 4:10</td>
<td>A remark on inward mappings. Dr. Simeon Reich, University of Chicago (737-47-2)</td>
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<td>4:15- 4:25</td>
<td>The Toeplitz-Hausdorff theorem and ellipticity conditions. Alan McIntosh, Macquarie University, Australia (737-47-6)</td>
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<td>2:45- 2:55</td>
<td>Conjugate points of vector-matrix differential equations. Professor Roger T. Lewis, University of Alabama in Birmingham (737-34-1)</td>
<td>1069 Sidney Smith Hall</td>
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<td>3:00- 3:10</td>
<td>A comparison theorem for second order differential systems in Banach spaces. Preliminary report. Professor Garret J. Etgen, University of Houston (737-34-2)</td>
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<tr>
<td>3:30- 3:40</td>
<td>Partial differential equations, Orlicz spaces, and measure functions. Professor Victor L. Shapiro, University of California, Riverside (737-35-2)</td>
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<tr>
<td>3:45- 3:55</td>
<td>$H^p$ space theory of linear elliptic system in the disc. Professor Chung-Ling Yu, Bengazi University, Libya (737-35-3) (Introduced by Chiu Yeung Chan)</td>
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<td>4:00- 4:10</td>
<td>An interior mixed boundary value problem for the two dimensional Helmholtz equation. Preliminary report. Dr. Dennis W. Quinn, Flight Dynamics Laboratory, Wright-Patterson AFB, Ohio (737-35-4)</td>
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<tr>
<td>4:15- 4:25</td>
<td>Growth and complete sequences of generalized axisymmetric potentials. Professor Allan J. Fryant, U. S. Naval Academy, Annapolis, Maryland (737-35-5)</td>
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THURSDAY, 3:00 P. M.

Special Session on Category Theory II, 162 Lash Miller Hall
3:00–3:20 (188) Distributive laws and the lifting of triples. Dr. HARVEY WOLFF, University of Toledo (737-18-3)
4:30–4:50 (191) Internally complete 2-categories. Preliminary report. Professor ROSS STREET, Macquarie University, North Ryde, Australia (737-18-7)
5:00–6:00 (192) Discussion Period

THURSDAY, 3:00 P. M.

Special Session on Mathematical Psychology I, 2102 Sidney Smith Hall
3:00–3:20 (193) A model of cue interaction in retrieval from memory. Preliminary report. Dr. JOHN OGILVIE*, Dr. ENDEL TULVING, and Ms. SHARON PASKOWITZ, University of Toronto (737-92-2) (Introduced by Professor Anatol Rapoport)
3:30–4:10 (194) The INDSCAL model and method for three-way multidimensional scaling. Dr. J. DOUGLAS CARROLL* and Dr. MYRON WISH, Bell Laboratories, Murray Hill, New Jersey (737-92-3) (Introduced by Professor Anatol Rapoport)
4:25–4:45 (195) Going three-dimensional: matrices and three-way arrays. Dr. JOSEPH B. KRUSKAL, Bell Laboratories, Murray Hill, New Jersey (737-15-3)
4:55–5:15 (196) An answer for Socrates. Preliminary report. Professor PRESTON HAMMER, Grand Valley State College (737-00-1)

THURSDAY, 4:00 P. M.

Invited Address, Convocation Hall
(197) Principles and practice of nonstandard analysis. EDWARD NELSON, Princeton University (737-02-15)

THURSDAY, 5:15 P. M.

Invited Address, Convocation Hall
(198) Computability revisited—an approach via the continuum. Professor MARIAN BOYKAN POUR-EL, University of Minnesota (737-02-12)

FRIDAY, 12:00 noon

Session on Combinatorics II, 1069 Sidney Smith Hall
12:00–12:10 (199) An existence theory for group divisible designs. Dr. K. CHANG*, Professors D. K. RAY-CHAUDHURI, and R. M. WILSON, Ohio State University (737-05-2)
12:15–12:25 (200) On the homeomorphic embedding of K_n and K_{n,m} in the t-cube. JEHUDA HART-MAN, University of California, Los Angeles (737-05-17)
12:30–12:40 (201) Growth number and colorability of graphs II. Preliminary report. Dr. A. EHRENFEUCIT, Dr. J. L. HURSCH, Jr.*, and C. MORGENSTERN, University of Colorado (737-05-29)

FRIDAY, 12:10 P. M.

Special Session on Recursively Enumerable Sets and Degrees I, 2135 Sidney Smith Hall
12:10–12:30 (203) The operator gap theorem in α-recursion theory. Professor ROBERT A. DI PAOLA, City University of New York, Queens College (737-02-5)
12:40–1:00 (204) β-recursion. Preliminary report. G. E. SACKS, Harvard University (737-02-10)

FRIDAY, 1:30 P. M.

Colloquium Lectures: Lecture IV, Convocation Hall
(205) Recent progress in dynamical systems: Unstable phenomena and the n-body problem. Professor JÜRGEN K. MOSER, Courant Institute, New York University

FRIDAY, 2:30 P. M.

Special Session on Combinatorial Identities IV, 159 Lash Miller Hall
2:30–2:50 (206) Parameter liberation in binomial and factorial identities. Preliminary report. Professor MICHAEL P. DRAZIN, Purdue University (737-05-20)
3:00–3:20 (207) Survey of q-orthogonal polynomials. Preliminary report. Professor GEORGE E. ANDREWS, Pennsylvania State University, and Professor RICHARD ASKEY*, University of Wisconsin (737–39–2)

3:30–3:50 (208) Combinatorial identities and applications. Preliminary report. Professor S. G. MOHANTY, McMaster University (737–05–14)

4:00–4:20 (209) An identity involving Stirling's number of the second kind. Preliminary report. Professor KI HANG KIM, Alabama State University (737–05–28)

4:30–5:20 (210) Informal session and any late papers

FRIDAY, 2:30 P.M.

Special Session on Knots and 3-Manifolds II, 2117 Sidney Smith Hall

2:30–2:50 (211) The conjugacy problem for 3-manifolds. BENNY EVANS, Oklahoma State University (737–57–9)

2:30–2:50 (212) HNN groups, residual finiteness, and 3-manifolds. Preliminary report. Professor JOHN HEMPEL, Rice University (737–57–5)

2:30–2:50 (213) Centralizers, roots and relations in 3-manifold groups. W. JACO and P. B. SHALEN*, Rice University (737–57–8)

3:00–3:20 (214) Alexander polynomials of amphicheiral knots. Professor JAMES M. VAN BUSKIRK, University of Oregon (737–55–7)

3:30–3:50 (215) A new invariant of link concordance. Professor DEBORAH L. GOLDSMITH, University of Michigan (737–57–4)

5:00–5:20 (216) Covering linkage. Mr. RICARD I. HARTLEY* and Professor KUNIO MURASUGI, University of Toronto (737–55–2)

FRIDAY, 2:30 P.M.

Special Session on Doubly Stochastic Measures and Their Operators II, 2118 Sidney Smith Hall

2:30–2:50 (218) A brief history of Birkhoff’s problem. JAMES R. BROWN, Oregon State University (737–47–5)

3:00–3:20 (219) Doubly stochastic measures with prescribed supports. Professor T. L. SEETHOFF*, Northern Michigan University, and Professor R. C. SHIFLETT, California State University, Fullerton (737–60–5)


4:00–4:20 (221) Doubly stochastic operators in axiomatic potential theory. Professor MYRON GOLDSTEIN, Arizona State University (737–31–1)

4:30–4:50 (222) Discussion Period

5:00–5:20 (223) Prime ergodic transformations. Preliminary report. Professor KENNETH R. BERG, University of Maryland (737–28–3)

5:30–5:50 (224) Some remarks about doubly stochastic operators. Professor W. A. J. LUXEMBURG, California Institute of Technology (737–28–2)

6:00–6:20 Informal Session

FRIDAY, 2:45 P.M.

Session on Complex Analysis, 1069 Sidney Smith Hall


3:00–3:10 (226) Classes of monogenic functions and approximation by exponential polynomials on a rectifiable arc of C. Preliminary report. Professor P. MALLIAVIN, Université Paris VI, France, and Professor JAMIL A. SIDDIQI*, Université Laval (737–30–2)


3:30–3:40 (228) Linear homeomorphisms of some classical families of univalent functions. Dr. FREDERICK W. HARTMANN, Villanova University (733–30–4)


4:00–4:10 (230) Classifying germs of irreducible analytic plane curves. Preliminary report. Dr. LEE RUDOLPH, Brown University (737–32–1)
4:15– 4:25 (231) New generating functions involving several complex variables. Professor H. M. SRIVASTAVA*, University of Glasgow, United Kingdom and University of Victoria, and, Dr. REKHA PANDA, University of Victoria and Revenshaw College, India (737-33-1)

FRIDAY, 3:00 P. M.

Special Session on Recursively Enumerable Sets and Degrees II, 2135 Sidney Smith Hall

3:00– 3:20 (232) Recursive well-founded orderings. Preliminary report. Mr. KEH-HSUN CHEN, Duke University (737-02-14)

3:30– 3:50 (233) d-Simple r.e. sets. Professor MANUEL LERMAN, University of Connecticut, Storrs, and Professor ROBERT SOARE*, University of Chicago (737-02-11)

4:00– 4:20 (234) Determining automorphisms of \( \mathcal{P} \). Preliminary report. Professor RICHARD A. SHORE, Cornell University (737-02-9)

4:30– 4:50 (235) Forcing in \( \beta \)-recursion theory. Preliminary report. Dr. SY D. FRIEDMAN, Massachusetts Institute of Technology (737-02-13)

5:00– 5:20 (236) On the elementary theory of some lattices of \( \alpha \)-r.e. sets. Professor MANUEL LERMAN, University of Connecticut, Storrs (737-02-7)

5:30– 5:50 (237) A hierarchy of formulas in degree theory. STEPHEN G. SIMPSON, Pennsylvania State University (737-02-4)

FRIDAY, 3:00 P. M.

Special Session on Category Theory III, 162 Lash Miller Hall

3:00– 3:20 (238) Some results on the existence of 2-monads. Preliminary report. Mr. ROBERT BLACKWELL, University of New South Wales, Kensington, Australia (737-18-9) (Introduced by Professor J. W. Gray)


5:00– 6:00 (242) Discussion Period

FRIDAY, 3:00 P. M.

Special Session on Mathematical Psychology II, 2102 Sidney Smith Hall

3:00– 3:20 (243) A model for learning global properties. Preliminary report. Professor MANFRED KOCHEN* and Mr. PAUL EITNER, University of Michigan (737-92-4)

3:30– 3:50 (244) Identifiability of theories with sequences of nonobservable underlying states. IRWIN D. NAHINSKY, Department of Psychology, University of Louisville (737-60-9) (Introduced by Professor A. Rapoport)

4:00– 4:20 (245) Applications of categorical algebra to the theory of measurement. Dr. PAUL H. PALMQUIST*, Irvine, California, and Dr. JOHN PAUL BOYD, University of California, Irvine (737-18-12)

4:30– 4:50 (246) Asymptotic distribution theory in paired-comparison experiments. Professor P. V. RAO, University of Florida (737-60-8)

5:00– 5:20 (247) A shift of focus from predictive to structural models. Professor ANATOL RAPOPORT, University of Toronto (737-92-1)

Middletown, Connecticut

Walter H. Gottschalk
Associate Secretary
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• Invited one-hour lectures

• Special session speakers

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Abo-Zeid, E. A. #26
Abramsom, F. G. #95
*Alexander, G. L. #38
Andersen, R. M. #110
Arklin, Joseph #104
*Aschbacher, M. #173
*Andrews, G. E. #207
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Axler, S. #191
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*Bari, R. A. #3
Beek, J. M. #46
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*Singh, S. P. #178
*Small, C. H. #100
*Smith, J. C. #27
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PRELIMINARY ANNOUNCEMENTS OF MEETINGS

738TH MEETING

University of Connecticut
Storrs, Connecticut
October 30, 1976

The seven hundred thirty-eighth meeting of the American Mathematical Society will be held at the University of Connecticut, Storrs, Connecticut, on Saturday, October 30, 1976.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be two invited addresses which will be given in the Physics Lecture Hall (PB36), located on the ground floor of the building connected to the Mathematical Sciences Building. Full details will be included in the October issue of these Notices. Two special sessions are being organized.

JEROME H. NEUWIRTH of the University of Connecticut is organizing a special session on Ergodic theory, and EUGENE SPIEGEL of the University of Connecticut is organizing a special session on Group rings.

The Mathematical Sciences Building is to the right in a large science complex of three buildings located on Hilda Drive. It faces Jorgenson Auditorium, and may be entered through several locations. The ground floor houses the Computer Center, and the next floor (first) contains the Mathematics Department offices, the lobby where the registration desk will be located, and a lounge. The registration desk will be open from 8:30 a.m. to noon, and from 1:30 p.m. to 3:30 p.m.

There will be sessions for the presentation of contributed ten-minute papers, both morning and afternoon. These sessions will be held on the second floor of the Mathematical Sciences Building. Overhead projectors and screens will be provided and each room has ample chalkboard space. Abstracts should be submitted to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, in order to arrive prior to the deadline of September 7, 1976.

ACCOMMODATIONS

A limited number of dormitory-type single rooms may be available on the campus at a cost of $6 per night. Inquiries regarding these accommodations, as well as all other matters, should be sent to: John V. Ryff, Head, Department of Mathematics U-9, University of Connecticut, Storrs, Connecticut 06268.

The University of Connecticut is located in the northeastern sector of Connecticut, about 25 miles east of Hartford. Public transportation to the university is severely limited. Those using public transportation are advised to stay over in Hartford Friday night, or arrive there before 8:00 a.m. on Saturday in order to take a university bus which will leave the main entrance of the Hotel Sonesta (5 Constitution Plaza) at 8:00 a.m. on Saturday to transport people to the campus. The bus will leave the Mathematical Sciences Building at 5:00 p.m. for the return trip to the Hotel Sonesta. There will be no charge for the bus transportation.

Reservations should be made directly at the following motels and hotels:

ASHFORD MOTEL (203) 684–2221
Route I–86, Exit 194, Ashford, Connecticut
(10 miles from campus)
Single $14.50 Triple $20.50
Double 17.50 Quadruple 22.50
Twin 19.00

CONNECTICUT MOTOR LODGE (203) 643–1555
Route I–86, Exit 94, Manchester, Connecticut
(15 miles from campus)
Single $13.00 Twin $19.00
Double 16.00

ESSEX MOTOR INN (203) 646–2300
100 East Center Street, Manchester, Connecticut
(15 miles from campus)
Single $15–$20 Triple $25.00
Double 23.00 Quadruple 27.00

HARTFORD HILTON (203) 278–1880
Ford and Pearl Streets, Hartford, Connecticut
(25 miles from campus—5 blocks from Sonesta) (Has faculty rate; request when making reservation. Some faculty identification is expected.)
Single $16.00 Twin $25.00
Double 25.00

HOWARD JOHNSON'S (203) 875–0781
Route I–86, Exit 96, Vernon, Connecticut
(15 miles from campus)
Single $16–$18 Triple $21–$24

RAMADA INN (203) 528–9703
East River Drive, Route I–84, Exit 3, East Hartford, Connecticut
(20 miles from campus)
Single $21.00 Double $27.00

SHERATON NORWICH MOTOR INN (203) 889–5201
Connecticut Turnpike, Exit 80, Norwich, Connecticut
(25 miles from campus)
Single $21.00 Double $32.00
(Mention AMS meeting when making reservations.)

WILLIMANTIC MOTOR INN (203) 423–8451
Route 195, Mansfield, Connecticut
(7 miles from campus)
Single $15.00 Twin $18.00

HOTEL SONESTA (203) 278–2000
5 Constitution Plaza, Hartford, Connecticut
(25 miles from campus)
(Has faculty rate; request when making reservation. Some faculty identification is expected.)
Single $21.75 Double $29.50
FOOD SERVICE

Refreshments will be available in the lounge during the meeting. In addition, a lunch will be served in the Faculty-Alumni Center at a cost of $1.50 per person, payable at the registration desk. An additional $1.50 will be contributed by the University of Connecticut Research Foundation toward the cost of each meal. The University Commons, located in the Student Union, will serve cafeteria-style meals. This facility will be open from 7:30 a.m. to 6:30 p.m.

TRAVEL AND LOCAL INFORMATION

The university is reached from Hartford by taking Exit 99 off Interstate 86 and following Route 195 south to the campus. From Boston, take Exit 101 off I-86 and follow Route 32 south, turning left at the intersection with Route 195. Those traveling along the Connecticut Turnpike to Norwich should take Route 32 north to Willimantic at Exit 81, and then follow Route 195 to the campus.

Hartford may be reached by Greyhound or Trailways bus, and by the Penn Central Railroad from New Haven on the south, or from Springfield on the north. Most major airlines serve Bradley International Airport, which is midway between Hartford and Springfield, Massachusetts. Limousine service is available to downtown Hartford at a cost of $2.35. There is no limousine service to the University of Connecticut.

The University Cooperative Bookstore is located below the University Commons. The Mathematics Department office will be open during the meeting to receive messages; the telephone number is (203) 486-3923.

Weather during this time of the year can range from a pleasantly warm Indian Summer to cool mid-fall, and from very dry to exceedingly wet.

PARKING

The intersection of Route 195 and North Eagleville Road is easily identified by the Congregational Church with the large white steeple. Persons driving to the meeting should park in Lot #9 located on the campus at the southwest corner of North Eagleville Road and Hillside Drive. There is no parking fee.

Walter H. Gottschalk
Associate Secretary
Middletown, Connecticut

739TH MEETING

University of Michigan
Ann Arbor, Michigan
November 6, 1976

The seven hundred thirty-ninth meeting of the American Mathematical Society will be held at the University of Michigan, Ann Arbor, Michigan, on Saturday, November 6, 1976. All sessions of the meeting will be held in the Auditorium Unit of Angell Hall. Angell Hall is a large building on the east side of State Street in the Central Campus area; the Auditorium Unit is on the ground floor on the far side from State Street.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be two one-hour addresses. Philippe M. Tondeur of the University of Illinois at Urbana-Champaign will address the Society at 11:00 a.m.; his subject is "G-foliations and their characteristic classes." M. Penumam Murthy of the University of Chicago will speak at 1:45 p.m. on the topic, "Serre's problem and complete intersections."

By invitation of the same committee there will be three special sessions of selected twenty-minute papers. HERBERT J. ALEXANDER of the University of Illinois at Chicago Circle is organizing a special session on Some complex variables; the tentative list of participants includes Eric D. Bedford, Frank Forelli, John Erik Fornaess, Walter Rudin, and B. A. Taylor. NOEL J. HICKS of the University of Michigan is organizing a special session on Differential geometry and global analysis; the tentative list of participants includes Harold G. Donnelly, Arthur J. Ledger, and Anthony J. Tromba. JOSEPH LIPMAN of Purdue University is organizing a special session on Stratification of algebraic and analytic varieties; the tentative list of participants includes James N. Damon, Robert M. Hardt, Helsuke Hironaka, Kenneth R. Mount, Augusto Nobile, and Joel Roberts. Most of the papers to be presented at these sessions will be by invitation; however, anyone contributing an abstract for the meeting who feels that his or her paper would be particularly appropriate for one of these special sessions should indicate this clearly on the abstract and submit it by August 24, two weeks before the normal deadline for contributed papers.

There will be sessions for contributed ten-minute papers both morning and afternoon. Abstracts should be submitted to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the abstract deadline of September 7, 1976.

REGISTRATION

The registration desk will be located in the lobby connecting Mason Hall, Haven Hall, and the Auditorium Unit of Angell Hall. This lobby faces the main University library and is known locally as "The Fishbowl". The desk will be open from 8:00 a.m. to 3:00 p.m.

ACCOMMODATIONS

The following two hotels are within walking distance of Angell Hall. Each one has about 200 guest rooms and has set aside 50 of these for persons attending the meeting. It is important to mention the AMS meeting when making reservations.

CAMPUS INN (313) 769-2200
615 E. Huron Street (at State Street), Ann Arbor, Michigan 48108
Four blocks north of Angell Hall
Single rooms $20 Double rooms $26
The seven-hundred fortieth meeting of the American Mathematical Society will be held at the University of South Carolina in Columbia, South Carolina, from noon Friday, November 19, until noon Saturday, November 20, 1976.

By invitation of the Committee to Select Hour Speakers for the Southeastern Sectional Meetings, there will be three one-hour addresses. The speakers will be Frank T. Birtel of Tulane University, Thomas A. Chapman of the University of Kentucky, and Thomas G. Hallam of Florida State University. All of the invited addresses will be presented in the Physical Sciences Building.

There will be five special sessions at this meeting. DOUGLAS N. CLARK of the University of Georgia is organizing a special session on Complex variables and operator theory; speakers will include Arthur Lubin, Thomas L. Kriete, Patrick R. Ahern, Joseph A. Ball, and Donald Sarason. LOUIS F. McAULEY of the State University of New York at Binghamton is organizing a special session on Monotone mappings and open mappings; speakers will include David C. Wilson, John J. Walsh, William E. Haver, Ronald Finthshel, Thomas Chapman, and Steven C. Ferry. CARL D. MEYER, Jr., of North Carolina State University is organizing a special session on Generalized inverses and operator theory; the speakers will include William N. Anderson, Jr., Richard H. Bouldin, Michael P. Drazin, Richard J. Duffin, I. N. Erdélyi, Charles W. Groetsch, Emilie V. Haynsworth, David C. Lay, Thomas L. Markham, and Carl D. Meyer, Jr. WILLIAM T. TROTTER, Jr., of the University of South Carolina is organizing a special session on Graph theory and combinatorics; the speakers will include Richard A. Duke, Marianne L. Gardner, Donald L. Greenwell, Ralph Faudree, Linda Lesniak, Brooks Reid, David P. Roselle, Richard Schelp, Paul Stockmeyer, and Neil White. EUTIQUIO C. YOUNG of Florida State University is organizing a special session on Modeling in biological systems; the speakers will include Steven F. Blumsack, John W. Heidel, and Chris P. Tsokos. Any member of the American Mathematical Society who would like to have his or her paper considered for inclusion in one of these special sessions should have the abstract so marked and in Providence at least three weeks before the regular closing date for contributed papers, which is September 7, 1976.

There will also be sessions for contributed ten-minute papers both Friday and Saturday. Abstracts should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline for contributed papers of September 28, 1976.

Tallahassee, Florida
O. G. Harrold, Jr.
Associate Secretary
The seven hundred and forty-first meeting of the American Mathematical Society will be held at the University of New Mexico, Albuquerque, New Mexico, on Friday and Saturday, November 19 and 20, 1976.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited hour addresses. Gary M. Seitz of the University of Oregon will be one of the invited speakers. The name of the other invited speaker and the titles of the hour addresses will be listed in the October issue of these Notices.

There will be three special sessions of selected twenty-minute papers. DAVID FOX of the Applied Physics Laboratory at Johns Hopkins University is organizing a special session on The estimation of eigenvalues. PRAMAD PATHAK of the University of New Mexico is organizing a special session on Probability and statistics. A special session on Partial differential equations is being organized by STANLY LEE STEINBERG of the University of New Mexico. The deadline for abstracts of papers to be considered for inclusion in one of these special sessions is September 7, 1976.

Sessions for contributed ten-minute papers will be scheduled on Saturday unless there are specific requests to the contrary. Abstracts for contributed papers should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive prior to the deadline of September 28, 1976. Late papers will be accepted for presentation at the meeting, but will not be listed in the printed program of the meeting.

Information about travel and accommodations will appear in the October issue of these Notices and the final program of the meeting will appear in the November Notices.

Kenneth A. Ross
Eugene, Oregon
Associate Secretary
INVITED SPEAKERS AT AMS MEETINGS

This section of these Notices lists regularly the individuals who have agreed to address the Society at the times and places listed below. For some future meetings, the lists of speakers are incomplete.

Ann Arbor, Michigan, November 1976
M. Pavaman Murthy
Philippe M. Tondeur

Columbia, South Carolina, November 1976
Frank T. Birtel
Thomas G. Hallan
Thomas A. Chapman

Albuquerque, New Mexico, November 1976
Gary M. Seitz

ORGANIZERS AND TOPICS OF SPECIAL SESSIONS

Abstracts of contributed papers to be considered for possible inclusion in special sessions should be submitted to the Providence office by the deadlines given below. The latest abstract form has a section for indicating special sessions. Lacking this, be sure your abstract form is clearly marked "For consideration for special session (title of special session)." Those papers not selected for special sessions will automatically be considered for regular sessions unless the author gives specific instructions to the contrary.

Storrs, Connecticut, October 1976
Jerome H. Neuwirth, Ergodic theory
Eugene Spiegel, Group rings

Ann Arbor, Michigan, November 1976
Herbert J. Alexander, Several complex variables
Noel J. Hicks, Differential geometry and global analysis
Joseph Lipman, Stratification of algebraic and analytic varieties

Columbia, South Carolina, November 1976
Douglas N. Clark, Complex variables and operator theory
Louis F. McAuley, Monotone mappings and open mappings
Carl D. Meyer, Jr., Generalized inverses and operator theory
William T. Trotter, Jr., Graph theory and combinatorics
Eutiquio C. Young, Modeling in biological systems

Albuquerque, New Mexico, November 1976
David Fox, The estimation of eigenvalues
Pramad Pathak, Probability and statistics
Stanly Lee Steinberg, Partial differential equations

St. Louis, Missouri, January 1977
Kenneth P. Bogart, Coding theory
William C. Connett and Alan L. Schwartz, Multipliers for series and transforms and their applications
Charles R. Deeter, Discrete analytic function theory
Aviezri S. Fraenkel, Theory of combinatorial games
Michael J. Kallaher, Finite geometries
M. Zuhair Nashed, Functional analysis methods in numerical analysis
Billy E. Rhoades, Summability and related topics
Robert C. Sine, L1 contraction and Doeblin theory of Markov operators
Karen K. Uhlenbeck, Global analysis
John L. Van Iwaarden, Using the computer in teaching undergraduate mathematics courses

N.B. Because of a projected shortage of meeting room space at the St. Louis meeting, it may not be possible to schedule any further special sessions beyond those listed above. For the same reason, those contributing abstracts for the meeting who feel that their papers are particularly appropriate for one of these sessions should submit their abstract as soon as possible.
AMS TO SERVE AS INSTITUTIONAL AFFILIATE
FOR RESEARCH GRANTS

On April 11, 1976, the Council of the American Mathematical Society, upon a recommendation of the Committee on Graduate Education, approved a resolution calling for the Society to serve as institutional affiliate for those members of the Society in need of such affiliation who want to apply for research grants. Pertinent excerpts from the minutes of this meeting appear below.

Members wishing to apply for grants under the Society's auspices should first obtain a copy of the funding agency's brochure of instructions on preparation of the necessary proposal. The originator of the proposal is responsible for preparing the main text of the proposal in a form ready for reproduction and to provide a draft of the budget and cover. This material should be submitted to the Secretary of the Society, Everett Pitcher, Lehigh University, Christmas-Saucon Hall #14, Bethlehem, Pennsylvania 18015, for review and transmittal to the Providence Headquarters for completion of the cover, budget, and signature pages.

A sheet of instructions on the preparation of copy for proposals in a form suitable for reproduction may be obtained from the Providence Division of Research Grants Office, Bethesda, Maryland 20014 (Information and Instructions for Application for Research Grant, Form NIH 398) and (Application for Public Health Service Grant or Award).

Air Force Office of Scientific Research
Building 410, Bolling AFB
Washington, D.C. 20330
(U.S. Air Force Grants for Basic Research)

National Institutes of Health
Office of Grants Inquiries
Division of Research Grants
Westwood Building, Room 448
Bethesda, Maryland 20205
(Application and Instructions for Application for Research Grant, Form NIH 398) and (Application for Public Health Service Grant or Award)

Army Research Office
P. O. Box 12211
Research Triangle Park, North Carolina 27709
(Basic Research Grants and Contracts)

Office of Naval Research
800 North Quincy Street
Arlington, Virginia 22217
(Contract Research and Technology Program, ONR-1)

Energy Research and Development Administration
Division of Physical Research
Washington, D.C. 20545
(Guide for the Submission of Research Proposals from Educational Institutions)

EXCERPTS FROM THE MINUTES OF THE COUNCIL
April 11, 1976

The Committee on Graduate Education presented a report, concerned with a resolution of the Council of April 11, 1975, that the Society "serve as the institutional affiliate for mathematicians in need of such affiliation who want to apply for research grants." The Committee made a recommendation in two parts, consisting of the last two paragraphs of the report, as follows:

The Committee recommends that the AMS headquarters prepare a detailed brochure which describes the steps an individual should take so that his proposal will be processed by the Society. This service should be open to all AMS members. In addition, there should be a detailed announcement, perhaps in the Notices, which describes the limitations the Society will impose on its own activity. For example, the Society will not referee proposals for their scientific merit (this, of course, is done by the NSF), the Society will disburse funds only at the request of the Principal Investigator, the Society will not supply technical typing or other such services under any circumstances, etc.

The Committee further proposes that this initial action be limited to a three-year period at the end of which time a review committee appointed by the President of the AMS will recommend to the Council whether or not the sponsorship of grants by the Society should be continued.

The President noted that the Society planned to charge overhead, although more modest than that charged by universities, so that the proposal should not cost the Society money. Moreover, it was intended that the Society would provide less service than is frequently provided by a University, limiting itself to disbursing funds on the order of the chief investigator, including possibly payment for supporting services. It was understood that the Secretary would screen proposals and sign those offered as an indication that the proposer is a professional mathematician, but not as a judgment of the scientific merit of the proposal.

The Council approved the recommendations of the Committee with the understanding that the issue be brought back to the Council for review in three years, possibly sooner.
EXTRACTS FROM THE BYLAWS OF THE SOCIETY

ARTICLE I
Officers

SECTION 1. There shall be a president, a president-elect (during the even-numbered years only), an ex-president (during the odd-numbered years only), three vice-presidents, a secretary, four associate secretaries, a treasurer, and an associate treasurer.

ARTICLE II
Board of Trustees

SECTION 1. There shall be a Board of Trustees consisting of eight trustees, five trustees elected by the Society in accordance with Article VII, together with the president, the treasurer, and the associate treasurer of the Society ex officio. The Board of Trustees shall designate its own presiding officer and secretary.

SECTION 2. The function of the Board of Trustees shall be to receive and administer the funds of the Society, to have full legal control of its investments and properties, to make contracts, and, in general, to conduct all business affairs of the Society.

ARTICLE III
Publications and Communications Committees

SECTION 1. There shall be eight publications committees, which shall be the eight editorial committees specified in Section 2 of this Article.

SECTION 2. There shall be eight editorial committees as follows: committees for the Bulletin, for the Proceedings, for the Colloquium Publications, for Mathematical Surveys, for Mathematical Reviews; a joint committee for the Transactions and the Memoirs; a committee consisting of the representatives of the Society on the Board of Editors of the American Journal of Mathematics; and a committee for Mathematics of Computation.

SECTION 3. There shall be a communications committee called the Committee to Monitor Problems in Communication.

SECTION 4. The size of each publications committee and communications committee shall be determined by the Council.

*Article V deals with the Executive Committee and is irrelevant to the election. For complete text of bylaws see the Bulletin, Vol. 81, No. 6, November 1975, pp. 1137–1143.

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COUNCIL

OFFICERS (Members of the Council, ex officio)

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<tr>
<th>Position</th>
<th>Name</th>
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<tr>
<td>President</td>
<td>Lipman Bers</td>
<td>1976</td>
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<td>President-elect</td>
<td>R. H. Bing</td>
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MEMBERS-AT-LARGE

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<td>Barry Simon</td>
<td>1978</td>
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PUBLICATION AND COMMUNICATIONS COMMITTEES

AMERICAN JOURNAL OF MATHEMATICS, Representatives of the Society on the Board of Editors

<table>
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<td>Hyman Bass</td>
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BULLETIN Editorial Committee

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<td>Olga Taussky</td>
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COLLOQUIUM Editorial Committee

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<td>Alberto P. Calderón</td>
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<td>Samuel Eilenberg</td>
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<td>S. S. Chern</td>
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MATHEMATICAL REVIEWS Editorial Committee

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<td>D. J. Lewis</td>
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<td>Jacob T. Schwartz</td>
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MATHEMATICAL SURVEYS Editorial Committee

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<td>Robert G. Bartle</td>
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<td>Maxwell A. Rosenlicht</td>
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<td>C. W. Brown, Jr.</td>
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MATHEMATICS OF COMPUTATION Editorial Committee

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PROCEEDINGS Editorial Committee

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<td>Chandler Davis</td>
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TRANSACTIONS and MEMOIRS Editorial Committee

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<td>Alexandra Bellow</td>
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Committee to Monitor Problems in Communication*

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BOARD OF TRUSTEES

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<td>Lipman Bers (ex officio)</td>
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<td>Cathleen S. Morawetz</td>
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NOTE: All terms expire on December 31 of the year listed.

*Only the chairman of this committee is a member of the Council.
RESEARCH PROPOSALS SUBMITTED
TO THE MATHEMATICAL SCIENCES SECTION,
NATIONAL SCIENCE FOUNDATION

by William H. Pell

The Mathematical Sciences Section of the National Science Foundation again announces that in order to insure full consideration, proposals requesting support beginning prior to September 1, 1977, should be in the hands of the cognizant program director at least six months prior to the desired starting date of such support, but not later than November 1, 1976, and earlier if possible.

The Section also wishes to encourage greater attention to the NSF brochure "Grants for Scientific Research" (NSF 73-12) in the writing of proposals. Recent proposals have frequently been deficient in one or more of the following items necessary in the properly written proposal:

1. A full description of all other current research support or pending applications for such for all proposed investigators; in case there is no other support and no other application is pending or contemplated, the proposal must contain such a statement explicitly (for example: None of the listed investigators has any other research support and no other application is pending or contemplated);

2. In requests for renewed support, estimates both of total expenditures and commitments under the existing award to the date at which new funding is desired and of the projected residual balance, if any (see NSF 73-12, pp. 16-17);

3. An abstract of the proposed research (in the entire proposal), about one-half page in length, written in the third person, with a minimal number of symbols not on standard typewriters, and suitable for inclusion in the Science Information Exchange and for transmittal to several Foundation offices for information;

4. Justification for any but the most usual items of support; in particular, this should be done in requests for partial support of sabbatical leaves;

5. Curricula vitae of the proposed investigators, including for each a list of publications relevant to the proposed research;

6. A bibliography of other pertinent publications (one's results are rarely based entirely on one's own work); and

7. It would be helpful if each proposal listed the telephone numbers of the mathematics department and the principal investigator(s).

NEWS ITEMS AND ANNOUNCEMENTS

NEW AMS COMMITTEE

A joint Committee on Arrangements for the Eighty-third Annual Meeting of the Society has been appointed by President Lipman Bers. The members are Paul T. Bateman (ex officio), Carl M. Bruns, William C. Connett, Ronald C. Freiwald, James B. Riles, David P. Roselle (ex officio), Jerrold N. Siegel (chairman), Nadine L. Verderber, and Gordon L. Walker (ex officio). The meeting will be held at St. Louis on January 27-31, 1977.

NSF PROJECT GRANTS ENCOURAGE PARTICIPATION OF WOMEN IN SCIENCE

In an attempt to tap the underutilized scientific resource which women represent, the National Science Foundation (NSF) has made grants totalling $946,171 to support two types of projects, twenty-two Science Career Workshops and eleven Science Career Facilitation Projects. The grants were made under the NSF's Women in Science Program whose objective is to develop and test methods to attract and retain women in scientific careers.

In the Science Career Workshop projects, two will be in the field of mathematics. Each grantee institution for mathematics will select participants in the one- or two-day workshop sessions from undergraduate women from its own student body as well as from those at institutions within a 100-mile radius. The grantee institution, project director, and academic level for the mathematics workshops are as follows: Lenore Blum (AMS member), Department of Mathematics and Computer Science, Mills College, Oakland, California 94613, freshman-sophomore; Etta Falconer (AMS member), Department of Mathematics, Spelman College, Atlanta, Georgia 30314, junior-senior.

Two of the Science Career Facilitation Project (SCFP) awards will be in the field of mathematics and computer science. The SCFP grants are aimed at women who received bachelor's or master's degrees in science between two and fifteen years ago and who are not presently employed in the fields for which they were trained and would like to update their education for graduate training or employment. Project directors and grantee institutions for the SCFP awards in mathematics are: N. B. Dale, Department of Computer Sciences, University of Texas-Austin, Austin, Texas 78712 and Calvin T. Long (AMS member), Department of Pure and Applied Mathematics, Washington State University, Pullman, Washington 99163.

Information about individual projects should be requested from the grantee institutions and not from the National Science Foundation.
E. D. BERGMANN MEMORIAL RESEARCH GRANTS

The United States-Israel Binational Science Foundation has established two special grants in honor of the memory of the late E. D. Bergmann. The grants will be awarded annually to one American and one Israeli scientist, for research to be conducted in Israel. Support will be for a two-year period, and for 1977 will be about 120,000 Israeli pounds (about $15,350). Applicants for the grants must have completed their doctoral degrees within the past five years. Their research project may be in any of the following areas: Agriculture, Natural Sciences, Health Sciences, Science Services, or such technologies as Energy, Arid Zone, or Environmental Research. Application forms and guidelines to applicants are available from the National Science Foundation, Division of International Programs (United States-Israel Binational Science Foundation), Washington, D.C. 20550, or from the United States-Israel Binational Science Foundation, P. O. Box 7677, Jerusalem. Applications should be submitted by November 1 each year; awards will be made April 1 of the following year.

IEEE- USSR COOPERATION AGREEMENT

An agreement of cooperation in the field of information theory has been signed by the Working Group on Information Theory of the USSR Academy of Science and the Information Theory Group of the Institute of Electrical and Electronics Engineers (IEEE). The agreement aims at developing these cooperative activities: scientist exchange for joint work; meetings of working groups of scientists from both groups to conduct joint research and for scientific interchange; joint publications; and facilitation of exchange of literature and research results.

The first joint activity was a scientific workshop help in Moscow in December 1975, in which fifteen scientists from the IEEE and over thirty from the USSR participated. A joint Proceedings of the papers presented will be published.

Their next cooperative venture will be a scientist exchange program of twelve man-months during the academic year 1976-1977, followed by a workshop to take place in the United States in 1977.

Anyone interested in obtaining more information about the IEEE-USSR cooperative program on information theory should contact J. K. Wolf, Department of Electrical and Computer Engineering, University of Massachusetts, Amherst, Massachusetts 01002.

CHARLES L. FEFERMAN RECEIVES WATERMAN AWARD

The National Science Foundation (NSF) has selected Charles L. Fefferman, 27, of Princeton University as the first recipient of the Alan T. Waterman Award. The award, named in honor of the first Director of the NSF, carries with it a medal and a grant of $50,000 a year for three years for research or advanced study at an institution of the recipient's choice. Dr. Fefferman was selected from among 232 nominees on the basis of his research in Fourier analysis, partial differential equations and several complex variables, which has brought fresh insight and renewed vigor to the classical areas of mathematics and contributed significantly to the advancement of modern mathematical analysis.

FIVE HONORED BY ACADEMY

Five members of the American Mathematical Society are among the 122 newly elected fellows of the American Academy of Arts and Sciences. They are: Herbert B. Keller, Professor of Applied Mathematics, California Institute of Technology; Ellis R. Kolchin, Professor of Mathematics, Columbia University; Lucien M. Le Cam, Professor of Statistics, University of California at Berkeley; Jerzy Neyman, Professor of Statistics, University of California at Berkeley; and Gabor Szegö, Professor of Mathematics, Stanford University.

NSF GRANTS AWARDS FOR SCIENCE TEACHING PROJECTS

Among the sixty-six innovative undergraduate science teaching projects for which the National Science Foundation has granted awards are twelve directed by mathematicians, the first seven of whom are members of the Society, as follows: David L. Outcalt (University of California at Santa Barbara), Robert H. Szczerba (Yale University, Connecticut), Leland D. Graher (Central University, Iowa), Ralph N. Johanson (Bentley College, Massachusetts), Robert A. Melter (Long Island University-Southampton College, New York), James A. Murtha (Marietta College, Ohio), Jeane L. Agnew (Oklahoma State University), W. Wesley Peterson (University of Hawaii), Ronald James Bieniek (University of Illinois), Lawrence Verbit (SUNY at Binghamton, New York), William L. Fellinger (University of Vermont), and G. Lansing Blackshaw (West Virginia University).

These faculty-oriented projects are supported under the NSF's Restructuring the Undergraduate Learning Environment (RULE) program with two principal objectives: (1) enhancing the teaching vitality of science faculty, and (2) insuring the rapid dispersal of important new scientific and educational concepts throughout the nation's colleges. These awards, while totaling approximately $900,000, are relatively modest in size, ranging between $2,500 and $25,000, and generally support an individual or a small group of science faculty members, sometimes with student assistance, for intensive work during a specific time period, such as a summer term.

COUNCIL ELECTS REPRESENTATIVE

Louis N. Howard of the Massachusetts Institute of Technology has been elected by the Council to be the representative of the Society on the U. S. National Committee for Theoretical and Applied Mechanics. He begins his four-year term on November 1, 1976, succeeding Joseph B. Keller.

SALEM PRIZE

The Salem Prize for 1976 was awarded to Dr. Michael Robert Herman of the Centre de Mathematiques de l'Ecole Polytechnique for his
work on the rotation numbers of transformations of a circle (Arnold's conjecture). The prize, established in 1968, is given every year to a young mathematician who is judged to have done an outstanding work in the field of interest of Salem, primarily on Fourier series and related topics.

The recipient was Dr. Nicholas Varopoulos in 1968, Dr. Richard Hunt in 1969, Dr. Yves Meyer in 1970, Dr. Charles Fefferman in 1971, Dr. Thomas Körner in 1972, Dr. E. M. Nikis in 1973, Dr. Hugh Montgomery in 1974, and Dr. William Beckner in 1975.

The jury consisted of Professor A. Zygmund, Professor L. Carleson, Professor J. –P. Kahane, and Professor Ch. Pisot.

SCIENCE FACULTY FELLOWSHIPS AWARDED BY NSF

Nine mathematicians are among the seventy-nine persons who have been awarded Faculty Fellowships in Science from the National Science Foundation. The mathematicians, the first six of whom are members of the Society, are: J. Myron Hood (Occidental College, California), Stanley R. Deans (University of South Florida, St. Petersburg), Malcolm D. Tobey (Southwest Minnesota State College), Barry N. Stein (Bronx Community College, CUNY), Edward J. Wegman (University of North Carolina, Chapel Hill), Venice A. Searle (Del Mar College, Texas), Jerry M. Payne (Brunswick Junior College, Georgia), Claire T. Machlin (American University, Washington, D.C.), and Ruth A. Loyd (Central Oklahoma State University).

The awards total almost $1.25 million and are made to faculty members to help them improve their competence in relating the applications of science to problems in society.

NSF received 538 applications which were reviewed and evaluated on a merit basis by panels appointed by the American Council on Education, with final selection being made by the NSF. Applicants were divided into three categories: faculty at four-year colleges and universities who hold bachelor's and master's degrees, holders of doctoral degrees, and faculty at two-year or community colleges. Each eligible faculty member had five years or more full-time, college-level teaching experience in science, mathematics, or engineering.

Each applicant presented a plan of study or research related to a societal need. Selections were made in the following fields: 30 in the physical sciences, mathematics, and engineering; 25 in the life sciences; and 24 in interdisciplinary fields, including the social sciences.

Fellowship stipends cover a period of from three to nine months and are based on salaries paid during the preceding year.

UNITED STATES–INDIA EXCHANGE OF SCIENTISTS

Senior mathematicians of the United States wishing to make short visits to India to work with Indian colleagues on projects of mutual interest may receive travel support under the United States–India Exchange of Scientists Program. American participants in the exchange program are selected by the National Science Foundation with the concurrence of the Council of Scientific and Industrial Research in New Delhi.

The program is based on, but not limited to, visits for periods from two weeks to a few months. NSF pays for the travel of American participants between the United States and India. Travel and subsistence expenses within India are borne by CSIR and the and the Indian host institutions.

For further information and application forms write to: U. S.–India Exchange of Scientists, Division of International Programs, National Science Foundation, Washington, D. C. 20550.

AMS RESEARCH FELLOWSHIP FUND

Request for Contributions

The AMS Research Fellowship Fund was established three years ago because of the scarcity of funds for postdoctoral fellowships. From this Fund AMS Research Fellowships are awarded annually to individuals who have received the Ph. D. degree, who show unusual promise in mathematical research, and who are citizens or permanent residents of a country in North America. Each fellowship for 1977–1978 will carry a partially tax-exempt stipend of approximately $10,500.

For 1976–1977 two Research Fellowships were awarded with nine people receiving Honorable Mention. (See the announcement in the June 1976 Notices.) The Society hopes that the number of fellowships to be awarded for 1977–1978 can be increased. This number, of course, depends on the contributions the Society receives. The Society itself contributes a minimum of $9,000 to the Fund each year, matching one-half of the funds in excess of $18,000 raised from other sources, up to a total contribution by the Society of $20,000. It is hoped that every member of the Society will contribute to the Fund.

Contributions to the AMS Research Fellowship Fund are tax deductible. Checks should be made payable to the American Mathematical Society, clearly marked "AMS Research Fellowship Fund", and sent to the American Mathematical Society, P. O. Box 1571, Annex Station, Providence, Rhode Island 02901.

MATHEMATICIANS RECEIVE NATIONAL MEDAL OF SCIENCE

Two mathematicians, Shing Shen Chern of the University of California, Berkeley and George B. Dantzig of Stanford University, both members of the American Mathematical Society, are among the fifteen recipients of the National Medal of Science for 1975. The medal is the nation's highest award for outstanding achievement in science and engineering. Announcement of the awards was made by President Gerald Ford.

SEED PROPOSALS DUE BY DECEMBER 15

Proposals for the Scientists and Engineers in Economic Development (SEED) program for 1977 may be submitted until December 15, 1976. Under the SEED program, scientists and engineers from U. S. colleges and universities teach and conduct research in developing countries in Africa, Asia, and Latin America.

The program, funded by the Agency for International Development and administered by the National Science Foundation, provides two
types of awards: Research/Teaching Grants that enable recipients to spend five months to a year at an academic institution in a developing country and International Travel Grants for short-term visits.

For additional information contact: Division of International Programs, National Science Foundation, Washington, D.C. 20550.

INDIAN INSTITUTE INVITES VISITORS

The Indian Institute of Operations Management, a non-profit academic organization founded in 1973, invites mathematicians who will be visiting India to spend some time at the Institute for lectures and seminars.

Established with both American and Indian professors as patrons, the Institute concentrates its activities on study and research in different areas of management sciences. It has started conducting membership examinations (of M.B.A. standard) and admits qualified people as members.

For further information write: Dr. Sudhan-shu K. Ghoshal, Director, c/o Udoy Kendra, Indian Chamber of Commerce Building, 4, India Exchange Place, Calcutta-1, or 11/24, Jheel Road, P. O. Jadavpur University, Calcutta-32.

BICENTENNIAL EXCHANGE OF
OF LMS–AMS SPEAKERS

J. Frank Adams of the University of Cambridge, England, has been invited to present the Bicentennial Lecture at the eighthieth summer meeting of the American Mathematical Society at 11:00 a.m. on Tuesday, August 24, at Convocation Hall at the University of Toronto, Toronto, Canada. The title of his talk is “Maps between classifying spaces.”

The London Mathematical Society and the American Mathematical Society agreed to exchange lecturers in recognition of the bicentennial of the United States.

S. S. Chern of the University of California at Berkeley has been invited to be the speaker from the AMS and will visit England in October. His lecture to the London Mathematical Society is scheduled for Friday, October 15.

SIXTEEN MATHEMATICIANS RECEIVE
SLOAN FELLOWSHIPS

Sloan Fellowships for Basic Research totaling $1,559,000 have been awarded for 1976–1977 to ninety-one scientists including sixteen mathematicians. They are: Leo A. Harrington and Arthur E. Ogas (both University of California, Berkeley), Richard Elman (University of California, Los Angeles), James P. Lin (University of California, San Diego), Martin E. Walter (University of Colorado), Linda P. Rothschild (Columbia University), Daniel Rubink (Cornell University), Henry B. Laufer (State University of New York, Stony Brook), Jean E. Taylor (Rutgers University), James Lepowsky (Yale University), John D. Fay (Bowdoin College), Mark L. Green (University of California, Los Angeles), Micheal E. Taylor (University of Michigan), Jurg M. Fröhlich and Allen E. Hatcher (both Princeton University), and Thomas C. Spencer (Rockefeller University). The first ten are members of the American Mathematical Society.

The scientists, whose average age is 30, were selected from among hundreds of nominees on the basis of their potential to make creative contributions to scientific knowledge in the early stages of their careers.

Established in 1955 by the Alfred P. Sloan Foundation, the fellowships run for two years in varying amounts averaging about $8,500 a year. Candidates for fellowships are nominated by senior scientists familiar with their capacities and the nominations are reviewed by a committee of distinguished senior scientists. No formal research proposal is required and the Fellow is free to shift the direction of his research at any time.

SUMMER INSTITUTE OF 1978

Suggestions of topics for a Summer Institute for 1978 are being received by the Committee on Summer Institutes until December 1, 1976. A Summer Institute is intended to be devoted to the state of the art in an active field of research. The topics tend to be in pure mathematics, since there is other provision for applied mathematics. Recent topics are:

Algebraic Geometry, 1974
Several Complex Variables, 1975
Algebraic and Geometric Topology, 1976
Automorphic Forms, Representations, and L-functions, 1977

Suggestions may be sent to any member of the Committee which includes: Daniel Gorenstein, Richard K. Lashof (chairman), Mary E. Rudin, Harold M. Stark, and Elias M. Stein.

RECENT AMS APPOINTMENTS

Richard C. DiPrima has been appointed by President Lipman Bers to the Committee to Select Hour Speakers for Eastern Sectional Meetings. Continuing members are Joan S. Birman, Walter H. Gottschalk (chairman), and George B. Seligman.

The Council has recently elected Robion C. Kirby, Arthur P. Mattuck, and George Piranian to the Editorial Board of the Notices. Their terms will be for two years, beginning January 1, 1977. Continuing members of the Committee are Ed Dubinsky, Barbara L. Osofsky, Everett Pitcher (chairman), Gordon L. Walker (ex officio), and Scott Warner Williams.

President Lipman Bers has appointed Stuart Kaufman to the Joint Committee on Mathematics in the Life Sciences. Continuing members of the Committee are Hans J. Bremerman, Jack D. Cowan, Murray Gerstenhaber, Simon A. Levin (chairman), Robert M. May, George F. Oster, and Sol I. Rubinow.

Two new members of the Committee on Postdoctoral Fellowships have been appointed by President Lipman Bers. They are Myles Tierney and Karen Uhlenbeck, whose three-year terms will begin January 1, 1977. Continuing members of the committee are Leonard Gillman, Daniel Gorenstein (chairman), Mark Kac, and William Thurston.
NSF UNDERGRADUATE RESEARCH PROPOSALS
DUE SEPTEMBER 10

The deadline for 1977 proposals under the Undergraduate Research Participation (URP) program sponsored by the National Science Foundation is September 10, 1976. These programs, held for many years, attempt to involve undergraduates showing unusual promise, usually following their junior year, in a research project.

Competition for awards is between all science and engineering departments, and requests for funding far exceed resources allocated to URP. For 1976 the NSF funded eight proposals for undergraduate research programs in mathematics.

Several years ago NSF required that proposals be related to the energy crisis, a move that sharply curtailed applications in all fields and which particularly limited the number of applications coming from departments of mathematics.

Recent trends have been to interpret these guidelines more broadly, and representatives of the eight mathematics departments receiving grants in 1976 agreed at the national meeting of directors that a letter should be sent to appropriate journals encouraging colleagues to consider applying for URP funds.

The Project Directors agreed to provide descriptions of their programs to interested colleagues. The directors are: Russell Merris, California State University, Hayward, California; Richard Elderkin, Pomona College, Claremont, California; Edwin Stueben, Illinois Institute of Technology, Chicago, Illinois; Carl E. Langenhop, Southern Illinois University, Carbondale, Illinois; George Springer, Indiana University, Bloomington, Indiana; Melvin Maron, University of Louisville, Louisville, Kentucky; Wayne Roberts, Macalester College, Saint Paul, Minnesota; and Robert Norman, Dartmouth College, Hanover, New Hampshire.

REGIONAL RESEARCH CONFERENCE PROPOSALS SOUGHT BY NSF

The National Science Foundation (NSF) is seeking proposals from prospective host institutions in the U.S. for five-day regional conferences, each to feature ten lectures by a distinguished guest lecturer on a subject of current research interest in the mathematical sciences. An applying institution should have at least a minimal research competence in the area of its proposal. The conferences are to be held during the summer of 1977 or during the succeeding fall or winter. The lectures normally appear in one of two monograph series, one of them published by the American Mathematical Society and the other by the Society for Industrial and Applied Mathematics.

Proposals by prospective host institutions should be sent directly to the Mathematical Sciences Section (Attention Dr. William H. Pell), National Science Foundation, 1800 G Street, N.W., Washington, D.C. 20550 by November 15, 1976. Inquiries on details of proposals for the regional conferences should be addressed to the Conference Board of the Mathematical Sciences, 832 Joseph Henry Building, 2100 Pennsylvania Avenue, N.W., Washington, D.C. 20037.

The objective of the project is to stimulate and broaden mathematical research activity, particularly in regions of the country where such activity needs further development. The organization of the conferences, evaluation of proposals, and arrangements for publication of expository papers based on the guest speakers’ lectures are to be carried out by the Conference Board of the Mathematical Sciences under contract with the Foundation.

TWO MATHEMATICIANS AWARDED NATO POSTDOCTORAL FELLOWSHIPS

Among the recipients of the forty-seven North Atlantic Treaty Organization (NATO) Postdoctoral Fellowships in Science are two mathematicians, both members of the American Mathematical Society. They are Steven R. Alpern who will attend the London School of Economics, England, and Donald E. Marshall who will attend the Swedish Academy of Science, Djursholm, Sweden. Both were affiliated with UCLA.

This fellowship program was initiated by NATO in 1959 to advance science and technology and promote closer collaboration among NATO nations or countries that cooperate with NATO. Each country administers the program for its own nationals. At the request of the Department of State NSF administers this NATO-funded program for U.S. citizens.

NATO Fellows will receive a stipend of $10,800 for twelve months or $8,100 for nine months. In addition, dependency allowances for round-trip travel will be provided.

SIX MATHEMATICIANS HONORED BY ACADEMY

Six persons from various fields of mathematics are among the 122 newly elected members of the American Academy of Arts and Sciences. They include five members of the American Mathematical Society: Herbert B. Keller, professor of applied mathematics at the California Institute of Technology; Ellis R. Kolchin, professor of mathematics at Columbia University; Lucien M. Le Cam and Jerzy Neyman, professors of statistics at the University of California at Berkeley; and Gábor Szegő, professor of mathematics at Stanford University. Also elected to the Academy was Frederick P. Brooks, professor of computer science at the University of North Carolina.

EXECUTIVE EDITOR OF MATH REVIEWS

The Editorial Committee of the Mathematical Reviews announces that Robert G. Bartle has accepted an appointment as Executive Editor of Mathematical Reviews, effective July 1. He replaces Jacob BurIak who is assuming the position of Senior Editor.
NEW AMS PUBLICATIONS

MEMOIRS OF THE AMERICAN MATHEMATICAL SOCIETY

CHARTING THE OPERATOR TERRAIN by John Ernest

Number 171
210 pages; list price $8.80; member price $6.60
ISBN 0-8218-1871-6; LC 76-3583
Publication date: August 31, 1976
To order, please specify MEMO/171

This Memoir reformulates and adapts much of the algebraic theory of von Neumann algebras and C*-algebras to the spatial context of single operators. For example, the single operator version of von Neumann's classification and reduc­tion theory takes a classical form (involving a spectral multiplicity function defined for arbitrary operators) which exhibits it as a natural generalization of the spectral theorem and the spectral multiplicity theory for normal operators. The Memoir describes how this theoretical framework can bring some organizational sense to the prodigious task of exploring and charting the vast work can bring some organizational sense to the prodigious task of exploring and charting the vast.

NON-COMMUTATIVE SPECTRAL THEORY FOR AFFINE FUNCTION SPACES ON CONVEX SETS by Erik M. Alfsen and Frederic W. Shultz

Number 172
120 pages; list price $7.60; member price $5.70
ISBN 0-8218-1872-4; LC 76-18309
Publication date: August 31, 1976
To order, please specify MEMO/172

Geometric notions related to self-adjoint projections and one-sided ideals in operator algebras are developed in the context of affine function spaces on convex sets. In terms of these concepts a "spectral axiom" is stated. From this axiom one can prove the spectral theorem: that elements of the affine function space admit a unique spectral decomposition. This in turn yields a satisfactory functional calculus, which is unique under a natural minimality requirement (that it be "extreme point preserving"). Via this functional calculus one can define a product
\[ a \circ b = 1/2((a + b)^2 - a^2 - b^2), \]
and it is shown that if this product is bilinear then the affine function space becomes a normed Jordan algebra of the type called a JB-algebra by Alfsen, Shultz and Störmer.

Examples to which this spectral theory applies include the spaces of bounded affine functions on Choquet simplices, and on "round" compact convex sets, such as the unit ball of L^P(1 < p < \infty). Another example is the space of self-adjoint elements of a von Neumann algebra, considered as the space of all bounded affine functions on its normal state space. Using the enveloping von Neumann algebra one can pass to the case of a C*-algebra considered as the space of all continuous affine functions on its state space. In this context the new concepts reduce to the usual ones, and the general spectral theory is used to obtain generalizations and new (more "geometric") proofs of various results in operator theory (e.g. that the normalized traces on a C*-algebra with identity form a \( w^* \)-compact simplex). As a final application it is shown that the spectral theory of JB-algebras can also be subsumed under the general theory of the present paper.

INDECOMPOSABLE REPRESENTATIONS OF GRAPHS AND ALGEBRAS by Vlastimil Dlab and Claus Michael Ringel

Number 173
57 pages; list price $6.40; member price $4.80
ISBN 0-8218-1873-2; LC 76-18784
Publication date: August 31, 1976
To order, please specify MEMO/173

I. N. Bernstein, I. M. Gelfand, and V. A. Ponomarev have recently shown that the bijection, first observed by P. Gabriel, between the indecomposable representations of graphs ("quivers") with a positive definite quadratic form and the positive roots of this form can be proved directly. Appropriate functors produce all indecomposable representations from the simple ones in the same way as the canonical generators of the Weyl group produce all positive roots from the simple ones.

In this Memoir the method is extended in two directions. In order to deal with all Dynkin diagrams rather than with those having single edges only, the authors consider valued graphs ("species"). In addition, they consider valued graphs with positive semi-definite quadratic form, i.e. extended Dynkin diagrams.

The main result of this Memoir describes all indecomposable representations up to the homogeneous ones, of a valued graph with positive semi-definite quadratic form. These indecomposable representations are of two types: those of discrete dimension type, and those of continuous dimension type. The indecomposable representations of discrete dimension type are determined by their dimension vectors: these are precisely the positive roots of the corresponding quadratic form. The continuous dimension vectors are the positive integral vectors in the radical space of the quadratic form and are thus the positive multiples of a fixed dimension vector. The full subcategory of all images of maps between direct sums of indecomposable representations of continuous dimension type is an abelian exact subcategory, which is called the category of all regular subcategories. It is the product of two categories \( U \) and \( H \), where \( H \) is the largest direct factor containing only representations of continuous dimension type. The representations in \( H \) are called homogeneous and their behaviour depends very strongly on the particular modulation of the valued graph. The study of the category \( H \) can be reduced to the study of the homogeneous representations of a simpler valued
The indecomposable representations which are non-regular can be described in the following way: there are two endofunctors $C^+$ and $C^-$ on the category of all representations, called the Coxeter functors, such that the list of all representations of the form $C^{-1}P$ and $C^{-1}Q$, where $P$ is an indecomposable projective representation and $Q$ is an indecomposable injective representation, is a complete list of all non-regular indecomposable representations. Also there is a numerical invariant, called the defect, which measures the behaviour of the indecomposable representations, and depends only on the dimension type. The defect of a representation is negative, zero, or positive, if and only if it is of the form $C^{-1}P$, regular, or of the form $C^{-1}Q$, respectively.

The paper concludes with tables of all valued graphs with positive semi-definite quadratic form. The tables provide, in condensed form, most of the information which is available about the representation theory of these valued graphs.

**PROCEEDINGS OF THE STEKLOV INSTITUTE**

**PROBLEMS IN THE CONSTRUCTIVE TREND IN MATHEMATICS, VI** edited by V. P. Orevkov and N. A. Sanin

Number 129 (1973)
272 pages; list price $55.60; member price $41.70
ISBN 0-8218-3029-5; LC 75-11951
Publication date: July 31, 1976
To order, please specify STEKLO/129

This volume is the sixth collection of papers on constructive mathematics. The collection consists of papers on the theory of complexity of algorithms, constructive mathematical analysis and constructive mathematical logic. The papers were presented at a seminar on constructive mathematics at the Leningrad Branch of the Steklov Mathematical Institute of the Academy of Sciences of the USSR and at a scientific seminar of the Mathematics-Mechanics Faculty of Leningrad University.

The titles of the papers follow: "Constructive versions of the laws of large numbers" by N. K. Kosovskii, "On the complexity of expansion of algebraic irrationalities in continued fractions" by V. P. Orevkov, "Recognizing a symmetry predicate by multithread Turing machines with input" by A. O. Slisenko, and "On a hierarchy of methods of interpreting propositions in constructive mathematics" by N. A. Sanin.

Part V was published as Number 113 of the Steklov book series and Part IV as Number 93 (MR 49 #4761 and MR 41 #1539).

**TRANSLATIONS OF MATHEMATICAL MONOGRAPHS**

**PROJECTIVE–ITERATIVE METHODS FOR SOLUTION OF OPERATOR EQUATIONS** by N. S. Kurbel

Volume 46
196 pages; list price $24.50; member price $18.60
ISBN: 0-8218-1596-2; LC 76-17114
at the International Congress of Mathematicians in Paris in 1900. The Organizing Committee's basic objective was to obtain as broad a representation of significant mathematical research as possible within the general constraint of relevance to the Hilbert problems. The Committee consisted of P. R. Bateman (secretary), F. E. Browder (chairman), R. C. Buck, D. Lewis, and D. Zelinsky.

The volume contains the proceedings of that symposium and includes papers corresponding to all the invited addresses with one exception. It contains as well the address of Professor G. Stampacchia that could not be delivered at the symposium because of health problems. The volume includes photographs of the speakers (by the courtesy of Paul Halmos), and a translation of the text of the Hilbert Problems as published in the Bulletin of the American Mathematical Society of 1903. The papers are published in the order of the problems to which they are filiated, and not in the alphabetical order of their authors.

An additional unusual feature of the volume is the article entitled "Problems of present day mathematics" which appears immediately after the text of Hilbert's article. The development of this material was initiated by Jean Dieudonné through correspondence with a number of mathematicians throughout the world. The resulting problems, as well as others obtained by the editor, appear in the form in which they were suggested.

The papers follow: R. A. Aleksandrjan, Ju. M. Bere­

lev on the occasion of his sixtieth birthday. It was translated from the Russian by V. P. Besov, V. P. Il'in, and A. G. Kostyuchenko, "Some problem with unknown boundaries"; P. P. Bel­

inskii, "Stability in the Liouville theorem on spatial quasiconformal mappings"; Stefan Berg­


nov, "Huygens' principle"; V. M. Kravtsov, "Hydrodynamic models of explosion in the ground"; B. A. Lugovcov, "Motion of a turbulent vortex ring that carries a passive admixture"; G. I. Marchuk, "On the formation of a thermocline in the ocean"; D. E. Men'sov, "The limits of indeterminacy in measure of the T-means of subseries of a trigonometric series"; R. Nevan­


PARTIAL DIFFERENTIAL EQUATIONS (Proceedings of a Symposium Dedicated to Academician S. L. Sobolev)

Series 2, Volume 105

346 pages; list price $34.40; member price $25.80 ISBN: 0-8218-3054-6; LC 76-8428
Publication date: May 31, 1976
To order, please specify TRANS2/105

This volume contains the papers from a symposium dedicated to Academician S. L. Sobolev on the occasion of his sixtieth birthday. It was translated from the Russian by S. Smith.

The names of the authors and titles of their papers follow: R. A. Aleksandryan, Ju, M. Bere­

zanskii, V. A. Il’in and A. G. Kostyuchenko, "Some problem in spectral theory for partial differ­

ential equations"; V. M. Babič, "Application of the methods of the mathematical theory of dif­

LETTERS TO THE EDITOR

LETTERS INVITON ON BLIND REFEREEING

In January, 1975, the Council of the American Mathematical Society selected the Proceedings for a two-year experiment with blind refereeing and since then all articles sent to referees by the editors of the Proceedings have excluded the author's name and institution. This policy will be reviewed by the Council in January, 1977.

Readers of the Notices are invited to submit letters to the editor expressing their thoughts on the subject of blind refereeing. All letters received in Providence before September 16, 1976, will be considered by the Editorial Board of the Notices for publication in the November, 1976, issue prior to the review by the Council.

Previous letters to the editor from AMS members concerning blind refereeing have appeared in issues of the Notices as follows: October, 1975, p. 292, from Victor L. Klee, Jr., with editorial comment, p. 293; January, 1976, p. 77; from Ed Dubinsky and Chandler Davis; and April, 1976, p. 173, from R. P. Boas.

Editor, the Notices

So much has already been written about the Employment and Applied Mathematics situation that I must apologize for trying once more to focus attention on the main issue—the need for a fundamental psychological reorientation of a substantial part of the mathematical community away from its preoccupation with the internal structure of mathematics and towards mathematical problems that arise in science and technology. I now believe that this cannot be carried out within the present structure of Academia. If it does not occur, all the ghastly predictions that are now being made for unemployment and miserable career prospects in the 1980's will come to pass.

As I understand the situation, the only conceivable way that this can change, given the demographic data which affect college enrollment, is that substantial numbers of mathematicians will be employed in industry and government. A visitor from outer space would wonder why this does not happen, given the overwhelming importance of mathematical thought in today's world, but the facts seem to be that there are about as many professionally trained mathematicians being hired for work outside academia as there are philosophers working in police departments. Why this is so is not completely clear to me (IBM doesn't hire people trained in business administration to design their computers), but obviously, the disdain with which the mandarins of the pure mathematics world have treated Applications plays a major role.

After much thought about these matters (which goes back to my employment in industry fifteen years ago—a very interesting and stimulating experience, in fact), I believe that the only way things might change is that small groups break off from our Titanic in order to work on the mathematical problems of the outside world on a part-time and consulting basis. One feature of the American economy is its decentralized and spasmodic nature. This is often harmful to theoretical science, which needs steady and long-term support, but the Universities and the tenure system are available as a counter-weight. Thus, I could envisage a group of mathematicians with common interests banding together, developing expertise in some applied areas, and finding groups within industry or government which need specific work done.

For example, I have been involved for almost twenty years in trying to adapt the concepts of geometry to various areas of science and technology. People trained in these areas could be extremely useful in many applied problems, but this need has had absolutely no effect on the research or teaching interests of the rest of today's geometers who are within the mainstream of academia. I now see such a group as the only conceivable way that the progress that has been made in geometry (algebraic, differential, topological,...) might be made available in a significant way to the applied world.

The AMS and SIAM could obviously play a key role in leading such a development. I suggest that they move forward from the surveys, symposia, etc., of the last years and consider some such activities.

Robert Hermann
Editor, the Notices

Prospective authors of Research Announcements in the Bulletin, and members of the society in general, may be interested in the following letter. We are sending it to all those members of the Council who are empowered to communicate Research Announcements.

"As you are undoubtedly aware, there has been a great deal of criticism of both the quality and the quantity of Research Announcements. This letter is an appeal to all communicators to help meet the criticisms, in the hope that it will
not be necessary to abolish Research Announcements altogether.

"The editors of the Bulletin ask that from now on you communicate a Research Announcement only if you are sure that the result is not just new, correct, and interesting, but that it is a result of importance that mathematicians have been seeking for some time, and whose communication to the mathematical community is so urgent that it cannot await the normal time of publication.

"Only rarely can you expect to receive a manuscript that meets these standards. It is more likely that you may hear of a result of this kind in your field, in which case you are encouraged to solicit it as a Research Announcement."

"We are not decreasing the quotas of communicators, but we do hope that the above criterion will be so faithfully applied that most quotas will be far from being filled. In order to make it easier for you to enforce the new standard, we are publishing it in the Notice."

Bulletin Editorial Committee
P. R. Halmos (Chairman)
Olga Taussky Todd
Hans Weinberger

PERSONAL ITEMS

STEWART S. CAIRNS of the University of Illinois has been granted a Senior U.S. Scientist Award by the Alexander von Humboldt Foundation of West Germany. The award will support six months of research at the University of Ulm, beginning October 1, 1976.

JOSEPH M. GANI of CSIRO, Canberra, Australia, was elected to a fellowship in the Australian Academy of Science on May 6, 1976.

RICHARD R. GOLDBERG of the University of Iowa has been appointed to the chairmanship of the Department of Mathematics and to a professorship at Vanderbilt University.

LEO A. GOODMAN of the University of Chicago has been elected a member of the American Philosophical Society.

WILLIAM J. GORDON of the General Motors Research Laboratories has been appointed Liaison Mathematician in the London branch of the Office of Naval Research.

JERRY P. KING of Lehigh University has been awarded the Donald B. and Dorothy L. Stabler Foundation Award.

ROBERT L. KRUSE of Emory University has been appointed chairman of the Department of Mathematics at Saint Mary's University, Halifax, Nova Scotia, Canada.

O. TIMOTHY O'MEARA of the University of Notre Dame has been appointed to the Howard J. Kenna Chair in Mathematics at that university.

HARI M. SRIVASTAVA of the Universities of Glasgow and Victoria has been elected a fellow of the Institute of Mathematics and its Applications, United Kingdom.

DAYA-NAND VERMA of the Tata Institute of Fundamental Research, Bombay, was appointed to head of the mathematics department and to a professorship at the North-Eastern Hill University, Shillong, India in September 1975. He was granted leave to take up the appointment.

PROMOTIONS

To Professor. Carnegie-Mellon University: ROBERT G. JEROSLOW; Colorado State University: H. HOWARD FRISINGER, RICHARD P. OSBORNE, and JAMES W. THOMAS; University of Calgary: BADRI N. SAHNEY.

To Associate Professor. Colorado State University: NICHOLAS K. KRIER and ROBERT A. LIEBLER; Hardin-Simmons University: EDWIN J. HEWETT; Texas Christian University: DAVID F. ADDIS.

DEATHS

Professor JOHN Q. ADAMS III of Oregon State University died on May 6, 1976, at the age of 26. He was a member of the Society for 6 years.

Professor JOHN P. MURRAY of Fairfield University died on January 7, 1976, at the age of 64. He was a member of the Society for 28 years.

Professor IVAN S. SOKOLNIKOFF of UCLA died on April 16, 1976, at the age of 75. He was a member of the Society for 48 years.

Professor MINORU URABE of Kyushu University died on September 4, 1975, at the age of 63. He was a member of the Society for 15 years.

Professor JAN G. VANDER CORPUT of the Netherlands, died on September 13, 1975, at the age of 85. He was a member of the Society for 28 years.
"Very important mathematics
is being spoken here."
— Paul R. Halmos

MATHEMATICAL DEVELOPMENTS
ARISING FROM
HILBERT PROBLEMS
EDITED BY
FELIX E. BROWDER

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SPECIAL MEETINGS INFORMATION CENTER

THIS CENTER maintains a file of prospective symposia, colloquia, institutes, seminars, special years, and meetings of other associations, helping the organizers become aware of possible conflicts in subject matter, dates, or geographical area.

AN ANNOUNCEMENT will be published in these Communications if it contains a call for papers, place, date, subject (when applicable), and speakers; a second full announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in each issue until it has been held and a reference will be given in parentheses to the volume and page of the issue in which the complete information appeared.

IN GENERAL, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, time and place of preconference registration (if required), time and place of registration for the meeting, abstracts (or sometimes full papers) to be sent to the organizers and page of the issue in which the complete information appeared.

DEADLINES are the same as the deadlines for abstracts. They appear on the inside front cover of each issue.


1976-1977. Arne Beurling Year, Institut Mittag-Leffler, Sweden (23, p. 82)


Fall-Spring, 1976-1977. NSF CHAUTAUQUA-TYPE SHORT COURSES FOR COLLEGE TEACHERS, Field Centers in Eastern, Central, and Western Circuits. Program: Each class meets twice in two-day sessions, once in the late fall and again in the early spring for the courses which are conducted by the American Association for the Advancement of Science. Teachers: Eastern circuit, Modeling and simulation, William I. Davidson and John Uhran, Notre Dame University; Mathematical modeling and voting, William Lucas, Cornell University; Modeling, mathematics, and computers in biology, Benjamin Jayne and R. Rajagopal, Duke University; Central circuit, Mathematical modeling in the biological sciences, H. T. Banks, Brown University; Microcomputers applied to science education, Robert Tinker, Springfield Technical Community College, Springfield, Massachusetts.

Support: National Science Foundation.

Information: American Association for the Advancement of Science, Chautauqua-Type Short Courses, Box J, 1776 Massachusetts Avenue, N.W., Washington, D.C. 20036.

AUGUST

16-20. CONFERENCE ON NUMERICAL ANALYSIS, Trinity College, Dublin, Ireland (23, p. 85)

Sponsor: The National Committee for Mathematics of the Royal Irish Academy.

Speakers: P. Arminjon (Montréal), G. Capriz (Pisa), P. G. Clariet (Paris), G. Dahlquist (Stockholm), J. Descouls (Lausanne), J. Douglas (Chicago), H. Fujiita (Tokyo), J. W. J. Hoeken (Sibshur-on-Thames), H. B. Keller (Pasadena), N. N. Kuznetsov (Moscow), J. L. Lions (Paris), D. Loewenthal (Tel-Aviv), G. I. Marchuk (Novosibirsk), W. L. Miranker (Yorktown Heights), N. Rasif (Beirut), N. A. Bitsche (Freiburg), R. E. O'Malley, Jr. (Tucson), E. L. Ortiz (London), M. H. Osborne (Cambridge), V. Pereyra (Caracas), P. Robinolnote (Rehovot), P. Roza (Budapest), M. N. Spiller (Leiden), J. Stoer (Würzburg), F. G. Toheremilsson (Moscow), R. Téan (Orsay), S. Treitel (Tusla), A. Wakhutin (Warsaw), and M. Yamaguti (Kyoto).

Information: J. Miller, Conference Director, School of Mathematics, Trinity College, Dublin 2, Ireland.

26-September 1. Second Los Alamos Workshop on Mathematics in the Natural Sciences, New Mexico (23, p. 180)


SEPTEMBER

1-10. Advanced Study Institute on Combinatorics, Germany (23, p. 181)

5-10. Conference on Finite Groups and Geometries, Belgium (23, p. 123)


Program: Major review lectures on the behavior of solitons.

Speakers: M. F. Atiyah, Oxford University; R. K. Bulloch, University of Manchester Institute of Science and Technology, Manchester, England; P. D. Lax, Courant Institute, New York; and A. C. Newell, Clarkson College, New York.

Information: Miss C. Richards, The Institute for Mathematics and its Applications, University of Illinois at Urbana-Champaign, Illinois.

6-11. European Meeting of Statisticians, France (23, p. 85)

6-11. Eighth International Congress on Cybernetics, Belgium (22, p. 296; 23, p. 124)

6-17. NATO Advanced Study Institute on Boundary Value Problems for Evolution Partial Differential Equations, Belgium (22, p. 369)

7-9. SEVENTH ANNUAL SERIES OF ARTHUR B. COBLE MEMORIAL LECTURES, Department of Mathematics, University of Illinois at Urbana-Champaign, Illinois.

Program: One talk each day by Andrew M. Gleason of Harvard University on the general topic "Coding Theory and Combinatorial Accidents".

Support: University of Illinois Foundation through a fund established by the late Professor Coble's family.

Information: Department of Mathematics, University of Illinois at Urbana-Champaign, Illinois.

14-16. CONFERENCE ON RECENT THEORETICAL DEVELOPMENTS IN CONTROL, University of Leicester, Leicester, England.

Sponsor: Institute of Mathematics and its Applications.

Program: H. H. Rosenbrock (University of Manchester Institute of Science and Technology) will lecture on "Algebraic Theory of Linear Systems". Contributed papers will also be presented on such topics as algebraic systems, optimal control, identification, stochastic control, filtering theory, stability of nonlinear systems, and control of systems governed by partial differential equations.

Information: The Secretary and Registrar, Institute of Mathematics and its Applications, Maitland House, Warrior Square, Southend on Sea, Essex, SS1 2JY, England.


22-December 10. APPLICATIONS OF ANALYSIS TO MECHANICS, International Center for Theoretical Physics, Trieste, Italy.

Sponsor: United Nations Development Program.
Program: The course for mathematicians, physicists and engineers will include lectures by an international faculty. It is open to research workers from all countries that are members of the United Nations, IAEA or UNESCO.

Information: The Deputy Director, International Center for Theoretical Physics, P. O. Box 586, I-34100 Trieste, Italy.


25. Pi Mu Epsilon Student Conference, Ohio (23, p. 181)

26-October 1. Conference on Finite Geometries and Groups, Belgium (23, p. 221)


Program: The purpose of the course is to show that the lack of basic mathematical concepts and skills is a general one and to focus attention on short and long term attempts being made to come to grips with some of the consequent problems. Speakers will include J. Crank, W. D. Furneaux, and Mrs. Ruth Rees (all of Brunel University) and Sir Hermann Bondi (Ministry of Defence) as well as some employers at craft and technician levels. There will also be a workshop on how students solve mathematical problems and the learning and teaching of mathematics.

Information: The Secretary and Registrar, Institute of Mathematics and its Applications and the Brunel University Education Liaison Center.

Program: Twelve invited addresses dealing with the applications of bifurcation theory to a diverse range of fields.

Information: P. H. Rabinowitz, Mathematics Research Center, University of Wisconsin–Madison, 610 Walnut Street, Madison, Wisconsin 53706.

OCTOBER


8–9. MIDWEST CONFERENCE ON DIFFERENTIAL AND INTEGRAL EQUATIONS, Southern Illinois University, Carbondale, Illinois.

Program: Approximately fifteen invited papers will be given. In addition, a limited number of contributed papers will be accepted.

Contributed Papers: Abstracts should be received by September 1, 1976.

Information: T. Burton or R. Grimmer, Department of Mathematics, Southern Illinois University, Carbondale, Illinois 62901.

16. ILLINOIS NUMBER THEORY CONFERENCE, Illinois State University, Normal, Illinois.

Contributed papers: Fifteen to twenty-minute papers are desired. Send a one- or two-sentence abstract to the address below by October 4, 1976.

Information: Charles Vandlen Eyden, Department of Mathematics, Illinois State University, Normal, Illinois 61761.

18–20. SIAM 1976 FALL MEETING, Georgia Institute of Technology, Atlanta, Georgia (23, p. 223)

Program: Four symposia will have invited speakers as follows: special symposium honoring the 30th anniversary of the founding of the Office of Naval Research; J. Keller, New York University-Courant Institute; B. Parlett, University of California, Berkeley, and R. Tarjan, Stanford University; symposium on numerical solution of initial value problems for partial differential equations; E. Adams Technische Hochschule, Karlsruhe, Germany; H. Kreiss, University of Upsala, Sweden; J. Oden, University of Texas, Austin, and M. Climent, Naval Ordnance Laboratory, Washington; operator equations, focusing on stochastic equations and two-sided bounded for deterministic problems; W. Boyce, Rensselaer Polytechnic Institute; T. Sei, University of New York, Buffalo; R. Moore, University of Wisconsin, Madison; H. Weinberger, University of Minnesota, and a special lecture by N. Ibragimov, University of Novosibirsk, Soviet Union; mathematics and environmental health; H. W. Hethcote, University of Iowa, and H. A. Tsuchiya, Jr., National Institute of Environmental Health Sciences. A special feature will be a panel discussion on computing practices in industry. Participants will be computer management oriented scientists from industry and government.

Sponsor: Symposia partially sponsored by the Office of Naval Research.

Information: H. B. Hair, SIAM Headquarters, 33 South 17th Street, Philadelphia, Pennsylvania 19103.

20–22. ACM 1976 Annual Conference, Texas (23, p. 85)


25–27. Second ERDA Statistical Symposium, Tennessee (23, p. 222)


Program: Twelve invited addresses dealing with the applications of bifurcation theory to a diverse range of fields.

Information: P. H. Rabinowitz, Mathematics Research Center, University of Wisconsin–Madison, 610 Walnut Street, Madison, Wisconsin 53706.

DECEMBER

15-17. CONFERENCE ON RATIONAL APPROXIMATION WITH EMPHASIS ON APPLICATIONS OF PADÉ APPROXIMANTS, University of South Florida, Tampa, Florida.

Program: The conference will include approximately twenty American and Canadian invited participants as well as J. S. R. Chisholm (England), G. Freud (Hungary and USA), P. Henrici (Switzerland), G. Meinardus (Germany), H. Van Rossum (Netherlands), and Hans Wallin (Sweden). Sessions for contributed papers will be included. Proceedings of the conference will be published.


Information: E. B. Saff, Department of Mathematics, University of South Florida, Tampa, Florida 33620 or R. S. Varga, Department of Mathematics, Kent State University, Kent, Ohio 44242.


27-January 1, 1977. Advanced Mathematical Techniques in Physical Sciences, India (23, p. 222)

*** 1977 ***

January 2–December 17, 1977. MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH (Mathematics Research Institute of Oberwolfach)

Information: Attendance at the sessions is by invitation only. Those wishing to attend should write directly to the chairman of individual sessions requesting an invitation.

January

2–5. Arbeitsgemeinschaft

Chairman: H. Schmalfuss, Tübingen


Chairman: K. Pottthoff, Kiel; A. Prestel, Konstanz

16–22. Kontinuumsmechanik

Chairman: W. Mainz, Karlsruhe; H. Lippmann, München

23–29. Problemgeschichte der Mathematik

Chairman: I. Schneider, München; C. J. Scriba, Hamburg

30–February 5. Mathematische Wirtschaftstheorie

Chairmen: H. Föllmer, Bonn; W. Hildenbrand, Bonn; D. Sondermann, Hamburg

February

6–12. Nichtkommutative Zahlentheorie und ganzzahligere Darstellungen endlicher Gruppen

Chairman: K. W. Roggenkamp, Stuttgart

13–19. Funktionentheorie

Chairmen: J. Gaier, Giessen; K. Strebel, Zürich; H. Wittich, Karlsruhe

20–26. Mathematische Methoden in der Medizin

Chairman: H. lmmich, Heidelberg; S. Schach, Dortmund

27–March 5. Partielle Differentialgleichungen

Chairmen: E. Heinz, Göttingen; G. Hellwig, Aachen
March
6-12. Diophantische Approximationen
Chairman: Th. Schneider, Freiburg
Chairmen: E. Kunz, Regensburg; H.-J. Nastold, Münster; L. Saper, Paris
20-26. Mathematische Stochastik
Chairman: D. Bierlein, Regensburg
27-April 2. Gewöhnliche Differentialgleichungen
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3-9. Arbeitsgemeinschaft Geyer-Harder
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12-16. Distributionen
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24-30. Mathematische Logik
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29-June 4. Gruppen und Geometrien
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12-18. Darstellungstheorie endlich dimensionaler Algebren
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19-25. Riesz Spaces and Ordered Bounded Operators
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26-July 2. Representations of semi-simple Lie groups
Chairmen: W. Casselman, Vancouver; W. Schmid, New York

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Chairman: J. Nitsche, Freiburg

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27-December 3. Didaktik der Stochastik
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December
4-10. Methoden und Verfahren der mathematischen Physik
Chairmen: B. Brosowski, Frankfurt; E. Martensen, Karlsruhe
11-17. Praktische Behandlung von Differentialgleichungen in Anwendungen
Chairmen: R. Ansorge, Hamburg; W. Tünnig, Darmstadt

January
4-7. SECOND CARIBBEAN CONFERENCE ON COMBINATORICS AND COMPUTING, University of the West Indies, Cave Hill, Barbados, West Indies (23, p. 222)
Sponsor: Department of Mathematics of the University of the West Indies
Program: invited lectures and a series of twenty-minute contributed papers each day.
Lecturers: R. C. Read, University of Waterloo, Canada; R. Guy, University of Calgary, Canada; R. F. Churchhouse, University of Cardiff, United Kingdom; P. Hanra, University of Michigan.
Abstracts: Title and abstract due before August 31, 1976, at the address below.
Information: Charles C. Cadogan, Department of Mathematics, University of the West Indies, P. O. Box 94, Bridgetown, Barbados, West Indies.
10-14. SYMPOSIUM ON ALGEBRAIC GEOMETRY, Kyoto University, Kyoto, Japan.
17-19. FOURTH ACM SIGACT-SIGPLAN SYMPOSIUM ON PRINCIPLES OF PROGRAMMING LANGUAGES, Los Angeles, California
Call for Papers: Papers on significant developments in the principles of programming languages and theoretical studies with application to programming languages are solicited. Summaries must be submitted by August 16, 1976, to: Dr. Ravi Sethi, Bell Laboratories, Murray Hill, New Jersey 07974.
Information: Gerald J. Popek, Department of Computer Science, University of California, Los Angeles, California 90024.
27-28. THE 1976-1977 ANNUAL MEETING OF THE ASSOCIATION FOR SYMBOLIC LOGIC, Chase-Park Plaza Hotel, St. Louis, Missouri (in conjunction with the annual meeting of the American Mathematical Society).
Information: Kenneth Kunen, University of Wisconsin, Madison, Wisconsin 53706.
FEBRUARY
6-9. 1977 AUSTRALIAN APPLIED MATHEMATICS CONFERENCE, Terrigal, New South Wales, Australia. Sponsor: Division of Applied Mathematics, Australian Mathematical Society. Program: Papers to be presented will be on research work on applied or applicable mathematics and on the teaching of applied mathematics. Overseas visitors will be welcomed although financial support will not be available. Information: N. G. Barton, School of Mathematics, University of New South Wales, P. O. Box 1, Kensington, New South Wales 2033, Australia.

APRIL

JUNE
3-4. Gauss Bicentennial Symposium, Canada (23, p. 222)

JULY
FIRST WORLD CONFERENCE ON MATHEMATICS AT THE SERVICE OF MAN, Barcelona, Spain. Program: General sessions on the application of mathematical methods to the acquisition of new knowledge in many fields and short informal presentations of ongoing work. Information: Dr. David Cardus, Baylor College of Medicine, P. O. Box 20095, Texas Medical Center, Houston, Texas 77025 or Dr. Nadal Batlle, Secretario General de la Conferencia MASH, Universidad Politecnica de Barcelona, Hectorado, Barcelona, Spain.

AUGUST
1-6. INTERNATIONAL SYMPOSIUM ON APPROXIMATION THEORY, Campinas, State of Sao Paulo, Brazil. Information: J. B. Prolla, Instituto de Matematica, Universidad Estadual de Campinas, Caixa Postal 1170, Campinas, Sao Paulo, Brazil.

7-13. Eighth International Conference on General Relativity and Gravitation, Canada (23, p. 85)

16-27. International Conference on Combinatorial Theory, Australia (23, p. 85)
QUERIES
Edited by Hans Samelson

QUESTIONS WELCOMED from AMS members regarding mathematical matters such as details of, or references to, vaguely remembered theorems, sources of exposition of folk theorems, or the state of current knowledge concerning published conjectures.

REPLIES from readers will be edited, when appropriate, into a composite answer and published in a subsequent column. All answers received will ultimately be forwarded to the questioner.

QUERIES AND RESPONSES should be typewritten if at all possible and sent to Professor Hans Samelson, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

○ QUERIES

101. Guy Châtillon (Département de mathématiques, Université du Québec, Trois-Rivières G9A 5H7, Canada). Has anyone done any work on the multi-(or at least bi-) variate probability gamma function with dependence? For example,

\[ g_i(x_i; \alpha_i) = x_i^{\alpha_i}/\Gamma(\alpha_i), \quad x_i > 0, \quad \alpha_i > -1; \quad i = 1, \ldots, n, \]

= 0 elsewhere.

A bivariate gamma function would then be

\[ f(x, y; \alpha, \beta) = (x^\alpha y^\beta / \Gamma(\alpha) \Gamma(\beta)) \cdot f(x, y), \]

\[ x, y > 0, \quad \alpha, \beta > -1, \]

where

\[ g(x; \alpha) = (x^{\alpha}/\Gamma(\alpha)) \cdot \int_0^\infty x^{\alpha} f(x, y) e^{-y} \, dy = x^{\alpha}/\Gamma(\alpha) \]

is the marginal density function of \( x \), and similarly

\[ h(y; \beta) = (\beta y^{\beta}/\Gamma(\beta)) \cdot \int_0^\infty y^{\beta} f(x, y) e^{-x} \, dx = \beta y^{\beta}/\Gamma(\beta) \]

is the marginal density function of \( y \). Also

\[ (x^\alpha y^\beta f(x, y) \cdot (x^\alpha y^\beta f(x, y)) f(x, y) \]

can be considered as the conditional densities of \( y \) given \( x \) and of \( x \) given \( y \), respectively. This has arisen in my research on sample theory—please write to me.

102. Warren Shepherd (Department of Mathematics, Oklahoma Baptist University, Shawnee, Oklahoma 74801). I am looking for English translations of Kurt Gödel's famous theorems, and the theorem of Poincaré referred to on page 173, column 3 of Scientific American. I do not want articles on the theorems; I want the theorems themselves.

103. A. G. Ramm (prosp. Engelsa 63, korp. 2, kv. 178, 194017 Leningrad K-17, USSR). 1. Let \( A = A^* \) be a linear compact operator in the Hilbert space, \( B \) be an orthoprojector, \( F(\lambda) \) be a continuous function \( -\infty < \lambda < \infty \). Is it true that

\[ \lambda_n(\text{P}(A)P) \rightarrow \lambda_n(F(PAP)) \quad \text{as} \quad n \rightarrow \infty. \]

Here \( \lambda_n(B) \) is the \( n \)th eigenvalue of a selfadjoint operator \( B \).

2. Is it possible to approximate with prescribed accuracy in an appropriate metric an arbitrary positive (nonnegative) definite kernel \( A(x, y) \) (i.e. \( A(f, f) \geq 0, \forall f \in H \)) by a kernel \( R(x, y) \in \mathbb{R} \), where the class \( R \) is defined in the paper (A. G. Ramm, Differential'nye Uravnenija 9 (1973), 931–941, 978 (MR 49 #5749); these Notices 22 (1975), p. A-708; Abstract #75T-8239).

104. Robin Harte (Department of Mathematics, University College, Cork, Ireland). 1. Does there exist a bounded linear operator between Banach spaces \( T : E \rightarrow F \) which is "almost open" (closure \( \{Tx : |x| \leq 1 \} \supset \{y \in F : |y| \leq k \} \) for some \( k > 0 \) but not onto?

2. Does there exist a bounded linear operator \( T : E \rightarrow E \) on a Banach space for which \( \|T^n\| \neq \|T\|^n \) but, for some \( n \), \( \|T^n\| = \|T\|^n \)?

○ RESPONSES

The replies below have been received to queries published in recent issues of these Notices. The editor would like to thank all who have replied.

94. (vol. 23, p. 223, June 1976, Miller). References on elementary antidifferentiation appear in the following:


6. Daniel Richardson, J. Symbolic Logic 33 (1968), 514–520. MR 39 #1330. (From the point of view of logic.)

(Contributed by Leroy F. Meyers, Edward M. Reingold, and Lawrence Washington).

100. (vol. 23, p. 223, June 1976, Leonard). Discussions of the Banach-Tarski paradox appear in the following:


ACKNOWLEDGEMENTS

The Society acknowledges with gratitude the support rendered by members during the past year, in addition to the contributing members who pay a minimum of $48 per year in dues, mathematicians also contributed to the Mathematical Reviews Fund, the AMS Research Fellowship Fund, and made general contributions; some contributors have requested that their names remain anonymous. These extra funds paid by members provide vital support to the work of the Society.

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* NR means that no response was received to a request for information.
** Only two articles in latest issue.
*** Date of receipt of manuscript not indicated for this journal.
ABSTRACTS

The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form and are based on the AMS (MOS) Subject Classification Scheme (1970). Abstracts for which the author did not indicate a category are listed under miscellaneous.

* Indicates that preprints are available from author. • Indicates invited addresses.

Abstracts for papers presented at
736 meeting in Portland, June 18, 1976
737 meeting in Toronto, August 24–28, 1976

Appear on page A-502

Abstracts Presented to the Society

The abstracts printed in this section were accepted by the American Mathematical Society for written presentation. An individual may present only one abstract by title in any one issue of the Notices, but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for the same issue.

Algebra and Theory of Numbers (05, 06, 08, 10, 12–18, 20)

*76T-A159 J. M. Gandhi, Department of Mathematics, Western Illinois University, Macomb, IL 61455

The Genocchi numbers \( G_n \) are defined by \( 2x/(e^{x+1}) = \sum_{n=0}^{\infty} G_n \frac{x^n}{n!} \) and can be calculated by the symbolic formula: \((G+1)^r + G = 1\), with \( G_1 = +1 \). It is known that \( G_{2n+1} = 0 \) and that \( G_{2n} \) is an odd integer. Also \( G_{2n} = -2(2^{2n-1})B_{2n} \), \( B_{2n} \) being Bernoulli numbers. We prove Theorem 7. If Fermat's equation has an integral solution for the first case, then \( G_{p-1} \equiv 0 \pmod{p} \). In fact we prove that the condition \( G_{p-1} \equiv 0 \pmod{p} \) is equivalent to the condition \( 2^{p-1} \equiv 1 \pmod{p^2} \). Thus there are only two primes 1093, 3511 below \( 3 \times 10^9 \) which satisfy the congruence (1). A proof of the above theorem, independent of Wieferich's condition, will be interesting. From Krasner's Theorem [Comptes Rendus, 199(1934), 296] we prove Corollary 1. If Fermat's equation has an integral solution for the first case, then all \( \lceil \frac{3}{\log p} \rceil \) consecutive Genocchi numbers of even index \( G_{p-3}, G_{p-5}, \ldots, G_{p-2i-1} \) \( (1 \leq i \leq \lceil \frac{3}{\log p} \rceil) \) must be divisible by \( p \). The work is supported by the James Vaughn Jr. Foundation. (Received February 26, 1976.)

*76T-A160 Dr. Brian J. Day, Department of Pure Mathematics, University of Sydney, Sydney, N.S.W. 2006 Australia. Density presentations of categories.

The concept of a presentation of a dense (= adequate) functor is formulated (similarly to that described in the author's article "On closed categories of functors II", Springer Lecture Notes, Vol. 420). A list of examples is given and the relationship between the concept of a density presentation and that of a monad with rank is discussed. All categorical algebra employed is relative to a fixed small-complete and small-cocomplete symmetric monoidal closed "ground" category \( y \). (Received April 5, 1976.)

76T-A161 M.E. ADAMS and J. SICHLER, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2. Endomorphism monoids of bounded lattices.

Every monoid is isomorphic to the \([0,1]\)-endomorphism monoid of some bounded lattice \( L \) (G. Grätzer and J. Sichler, Pac. J. Math 35(1970), 639-647). In that construction \( L \) always contains an infinite chain. Let \( \text{End}_{[0,1]}(L) \) denote the \([0,1]\)-endomorphism monoid of the bounded lattice \( L \), and let \( E_n = [M \mid M \subseteq \text{End}_{[0,1]}(L) \) for some \( L \) of finite height less than \( n \). Theorem: There exists a monoid \( M \) such that if \( M \subseteq N \subseteq \text{End}_{[0,1]}(L) \) then \( L \) has an infinite chain. Theorem: \( E_n \supseteq E_{n+1} \) and there exists a finite monoid \( M \) such that \( M \subseteq E_n + 1 \subseteq E_n \). Theorem: If \( M \) is a finite monoid, then there exists a finite lattice \( L \) such that \( M \subseteq \text{End}_{[0,1]}(L) \). (Received April 5, 1976.)

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where $I_{k,j}$ is a principal prime ideal in a partitioned matrix $(A_{ij})$, $i, j = 1, \ldots, k$. Let 

$$f(x, y) = x^n + \sum_{j=1}^{n_k} a_{j} x^{n-j} = 0 \quad (1)$$

where $a_j \in \mathbb{Z}[y]$, $f(x, y)$ is irreducible over $\mathbb{Q}(y)$ and $K$ is defined as above.

Then the following statements hold:

(a) Let $p(y)$, $q(y)$ belong to $\mathbb{Z}[y]$ where both polynomials are of at least degree 3, $p(y)$ is separable, and $q(y)$ has no irreducible factor of multiplicity greater than 2. Then there are at most finitely many equations of form

$$w^2 = p(y) \quad (2)$$

such that a solution of $(2)$ or $(3)$ generates a subfield of $K$.

(b) We can obtain all solutions of form $(2)$ or (3) defined as in (a) in a finite number of steps.

(c) If $K$ contains even one subfield generated by an equation of form (2) or (3) (defined as in (a)), then we can obtain all solutions in integers $(x_0, y_0)$ of (1) in a finite number of steps. (Received April 6, 1976.)


For $\beta \in \mathbb{R} \setminus \mathbb{Q}$, denote by $\overline{\beta}$ the nearest integer of $\beta$ and let $S(\beta) = |\overline{\beta} - \beta|$. Denote by $\nu$ the Lebesgue measure on $\mathbb{R}$. For $0 \leq \nu < 1$, let $H_n(\alpha) = \{ \beta \in \mathbb{R} \setminus \mathbb{Q} : |\beta - \alpha| < 1/2 \wedge 0 < \nu(\mathbb{R} \setminus \mathbb{Q}) \leq \alpha < 1/2 \}$ and let $H(\alpha) = \{ \beta \in \mathbb{R} \setminus \mathbb{Q} : |\beta - \alpha| < 1/2 \wedge 0 < \nu(\mathbb{R} \setminus \mathbb{Q}) \leq \alpha < 1 \}$.

It is easy to verify that for $0 \leq \alpha \leq 1$, $n \geq 0$ we have $H_{n+1}(\alpha) = \sum_{k=1}^{n} \left( H_n(1/k) - H_n(1/(k+\alpha)) \right) + H_n(1-1/k) - H_n(1-1/2)$. Let $G = (1 + \sqrt{5})/2$ and let $R_n(\alpha) = H_n(\alpha) - (\log G)^{-1} \int_{1}^{\infty} (G + t)^{-1} dt \ (0 \leq \alpha \leq 1, n \geq 0)$. Theorem. There exist real numbers $C > 0, D > 1$ such that for $0 \leq \alpha \leq 1, n \geq 0$ we have $|R_n(\alpha)| < C D^{-n}$. This is a continuation of earlier work (see Abstract 76T-A53, these Notices 23(1976), A-270). $S$ is the shift operator for continued fractions by nearest integers. (Received April 19, 1976.)

Raymond Balbes, University of Missouri, St. Louis, Missouri, 63121. On representable partially ordered sets. Preliminary report.

A poset is called representable provided that it is isomorphic with the poset of prime ideals (together with $\phi$ and $\Gamma$) of some distributive lattice $L$. Theorem. The representable posets are those and only those of the form

$$P = 2^I \sim \bigcup_{k \in \mathbb{K}} \left( \bigcup_{i=1}^{k} I_{k,i} \right) \bigcap \left( \bigcup_{j=1}^{m_k} \mathbb{F}_{k,j} \right)$$

where $I_{k,i}$ is a principal prime ideal in $2^I$ and $\mathbb{F}_{k,i}$ is a principal prime filter in $2^I$ for each $k \in \mathbb{K}, 1 \leq i \leq n_k$ and $1 \leq j \leq m_k$. (Received April 22, 1976.)

KI HANG KIM, Box 69, Alabama State University, Montgomery, Alabama 36101. Primitive Nonnegative Matrices

We give a criterion for nonnegative matrix to be primitive.

A nonnegative matrix $A$ will be called an RC-matrix if every row and every column of $A$ contains at least one nonzero entry. An $n$-square RC-matrix will be called an $K$-matrix if (i) there exists a permutation matrix $P$ of order $n$ such that $PAP^T$ is equal to a partitioned matrix $(A_{ij})$, $i, j = 1, \ldots, k$; (ii) there exists a permutation matrix $Q = (q_{ij})$ of order $m$ ($m \geq 2$) which is not equal to an identity matrix such that

$$q_{ij} = \begin{cases} 1 & \text{if } A_{ij} \text{ is an RC-matrix} \\ 0 & \text{if } A_{ij} \text{ is a zero matrix} \end{cases}$$

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THEOREM. An n-square RC-matrix is primitive if and only if A is neither decomposable
nor K-matrix. (Received April 23, 1976.)

*76T-A166 Maureen T. Bleazby, La Trobe University, Bundoora, Victoria, Australia, 3083. Automorphisms of skew polynomial rings. Preliminary report.

Let R be any ring with identity and \( a \) any automorphism of R. Then the map \( x \rightarrow r_0 + r_1x + \ldots + r_nx^n \) induces an automorphism of the skew polynomial ring \( R[x,a] \) which fixes each element of R if and only if

(i) \( r^a_i = r_i \) for all \( r \in R \) and all \( 0 \leq i \leq n \);

(ii) \( r \) is a unit in \( R \); and

(iii) \( r^a_i\) is nilpotent for all \( 1 \leq i \leq n \).

Also these three conditions hold precisely when the above map induces a surjective homomorphism of \( R[x,a] \) which fixes each element of R. (Received April 27, 1976.) (Author introduced by Dr. K. R. Pearson.)

*76T-A167 BONNIE PAGE DANNER, Virginia Commonwealth University, Richmond, Virginia, 23284. Maximal separable subfields. Preliminary report.

Let \( L/K \) be a finitely generated separable field extension of characteristic \( p \neq 0 \). Let \( M \) be an intermediate field such that \( L/M \) is inseparable. We characterize maximal subfields \( S \) of \( M/K \) with respect to the property that \( L/S \) is separable as follows:

Theorem 1. Let \( L/S \) be separable. \( S \) is a maximal subfield for \( L/M \) if and only if \( S(L^P) \supseteq M \) and \( S \) is algebraically closed in \( M \).

Theorem 2. If \( L/S \) is separable, \( S \) is a maximal subfield for \( L/M \) if and only if the inseparability of \( L/M \) is equal to the transcendence degree of \( M/S \).

Theorem 3. There is a unique maximal \( S \) for \( L/M \) if and only if \( S/K \) is algebraic for some \( S \). (Received April 27, 1976.)

76T-A168 RONSON J. WARNE, University of Alabama, Birmingham, Alabama 35294. The congruence "t". Preliminary report.

Let \( S \) be a regular semigroup in which the union \( T \) of the maximal subgroups is a subsemigroup. Hence, \( T \) is a semilattice \( Y \) of completely simple semigroups \( (T_y : y \in Y) \). Let \( t = \{(a,b) \in S \times S : aa', bb' \in T_y \text{, for some } y, z \in Y \text{, and some inverses } a', b' \text{ of } a \text{ and } b \text{ respectively}\} \). We show \( t \) is a congruence relation on \( S, S/t \) is an inverse semigroup with semilattice \( Y \), each subgroup of \( S/t \) is a single element, and \( \ker t = T \). Hence (R.J. Warne, "Normal regular semigroups", I, Notices Amer. Math. Soc, 23(1976), A-351, 76T-A84) will be valid even if we omit the condition "If, furthermore, for each \( y \in Y \), there exists \( e_y \in T_y \) such that \( e_yz = ez = e_zy \) for all \( y, z \in Y \). We have previously shown \( t \) is a congruence in special cases — see, for example, R.J. Warne, "Generalized \( \omega - Z \)-unipotent bisimple semigroups," Pacific J. Math, 51(1974), 631-648. (Received April 30, 1976.)

*76T-A169 B. N. DATTA, DECC-UNICAMP, CAMPINAS-BRASIL. APPLICATION OF HANKEL MATRIX TO THE ROOT-LOCATION PROBLEM

Let \( f(x) = x^n - a_0x^{n-1} - a_2x - a_1 \) be a polynomial of degree \( n \) with real coefficients. Then it is well-known [2] F.R. Gantmacher, The Theory of Matrices, Vol. 2 (1959), pp. 202 that the rank and signature of the associated Hankel matrix of Newton sums specify respectively the number of distinct zeros and the number of distinct real zeros of \( f(x) \). In this paper, it is shown how this Hankel matrix \( H \) can also be employed to obtain information on the location of zeros of \( f(x) \) in a given half plane. As a particular case of
this result, it is shown how \( H \) provides a criterion for testing aperiodicity of \( f(x) \) (a polynomial \( f(x) \) is called aperiodic if all its zeros are distinct and negative real).

(Received May 3, 1976.)

*76T-A170  CARL H. FITZGERALD, University of California at San Diego, La Jolla, California 92039 and ROGER A. HORN, The Johns Hopkins University, Baltimore, Maryland 21218. On fractional Hadamard powers of positive definite matrices.

Let \( A = (a_{ij}) \) be a real symmetric \( n \times n \) positive definite matrix with nonnegative entries. We show that \( A^{(a)} = (a_{ij}^a) \) is positive definite for all real \( a \geq n - 2 \). Moreover, the lower bound is sharp. We give related results for pairs of quadratic forms and discuss partial generalizations to the case in which \( A \) is a complex Hermitian matrix. (Received May 7, 1976.)

*76T-A171  Héctor A. Merklen, Instituto de Matemática, Universidade de São Paulo, São Paulo, Brasil. Automorphisms of nilpotent groups of class two.

Let \( G \) be a finitely generated, nilpotent group of class 2 with commutator group \( G' \). A subgroup \( \text{Aut}(G/G') \) of \( \text{Aut}(G/G') \) is defined and \( \text{Hom}(G/G', G') \) is made into a group by point-wise addition of functions. \textbf{Theorem 1.} There is an exact sequence:

\[
0 \rightarrow \text{Hom}(G/G', G') \rightarrow \text{Aut}(G) \rightarrow \text{Aut}(G/G') \rightarrow 0,
\]

where \( \text{Hom}(G/G', G') \) denotes passage from \( G \) to \( G/G' \) and for each \( t \) in \( \text{Hom}(G/G', G') \), \( t(x) = x + t(x) \).

Let \( F \) be the fixed subgroup of \( G \) under \( \text{Aut}(G) \); \( A \), the set of those \( f \) in \( \text{Aut}(G) \) which reduce to the identity on \( G/F' \); \( F' \), the fixed subgroup of \( G \) under \( A \). \textbf{Theorem 2.} If \( \text{Aut}(G) \) is commutative, \( G' \subseteq F' \subseteq G \), and for each \( f \) in \( \text{Aut}(G) \) there is a \( t \) in \( \text{Hom}(G/F', F') \) such that \( f(x) = x + t(x) \), for all \( x \) in \( G \) (where \( x \) is the class of \( x \) modulo \( F' \)).

An example is given of a non-commutative \( G \) of order 1024 such that \( \text{Aut}(G) \) is commutative. (Received May 10, 1976.)


The groups which are locally finite or locally solvable and satisfy the condition of the title are completely classified: those which are not Černikov groups are just the groups \( G \) for which (a) the set \( R \) of right Engel elements is a locally nilpotent normal subgroup and (b) the factor group \( G/R \) is finite and has all non-cyclic subgroups subnormal. Such groups \( G \) are in fact locally nilpotent by finite cyclic and have no non-serial subgroups which are not locally nilpotent. Strengthening the chain condition in various ways, one obtains a number of known results, for instance (a) the Šunkov-Kegel-Wehrfritz theorem that all locally finite groups satisfying the minimal condition are Černikov groups, and (b) a structure theorem of Černikov for certain groups satisfying the minimal condition on non-Abelian subgroups (in Dokl. Akad. Nauk 154(6), 759-760 (1964)). (Received May 10, 1976.)

*76T-A173  James A. Carlson, University of Utah, Salt Lake City, Utah 84112. A result on mixed Hodge Structures. Preliminary report.

To a mixed Hodge structure are associated the Hodge structures on the graded pieces of the weight filtration. To reconstruct the former from the latter, one needs the extension datum. The extension datum is defined, and the group of such is shown to be a real torus. Applications to degenerations of varieties and their periods are given: Let \( N = \log (\text{monodromy matrix}) \). When \( N^2 = 0 \), the group of extensions is shown to be a complex torus. A boundary component, different from Cattani's, is defined. It is a holomorphic fiber space over a product of period domains with fiber a complex torus. For curve degenerations the fiber is a product of Jacobians; for quartic surfaces, it is the seventeen-fold product of the double curve. In general, when \( N^2 = 0 \), rank \( N \) is minimal, this boundary component is maximal. In particular, its dimension in the quartic surface case is eighteen. (Received May 10, 1976)

*76T-A174  Gunter Bruns, McMaster University, Hamilton, Ontario L8S 4K1. Finitely generated orthomodular lattices with at most three blocks.

\textbf{Theorem 1.} Every orthomodular lattice with at most two blocks (maximal Boolean subalgebras) which is generated by an \( n \)-element set has at most \( 2^n \left( 2^{2n-1} + 2 \right) \) elements and this bound is best possible.

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Theorem 2. Every finitely generated orthomodular lattice with at most three blocks is finite.

(Received May 12, 1976.) (Author introduced by B. Banaschewski.)

Dr. J.L. HURSCH, JR., 2895 Glenwood Drive, Boulder, Colorado 80302, Growth number and colorability of graphs, Preliminary report.

Consider any connected three valent graph $G$ which can be constructed ("grown") as follows: Start with 2 distinct vertices $a, b$, and one edge $ab$. Then at each step either (1) identify two vertices of degree one to get a vertex $x$ of degree two and then add a vertex $y$ and an edge $xy$; or (2), given some vertex $x$ of degree one, add two distinct vertices $y$ and $z$ and edges $xy$ and $xz$. Finally, given edges $tu$ and $vw$ such that $u$ and $w$ are the only vertices of degree one in the whole graph, we can remove $u, w, tu$ and $vw$ and add $tv$. Let the "growth number of $G"$ be the minimum over all possible ways of growing $G$ of the maximum over all steps of the number of vertices of degree one. If $G$ is planar define the planar growth number $(pl(G))$ of $G$ similarly by allowing only those constructions where all vertices of degree one are in the same component of the plane at each step. Theorem: (Proof by computer) If $G$ is planar bridgeless and $pl(G) \leq 7$ then $G$ is edge 3-colorable. Example: Tutte's graph $(T)$ is such that $pl(T) \leq 7$. Remark: there exist arbitrarily large graphs $G$, such that $pl(G) \leq 7$.

(Author presented by Jan Mycielski.) (Received May 14, 1976.)

V.R. Chandran and Harry Lakser, both of The University of Manitoba, Winnipeg, Manitoba, Canada, R3T 2N2. Distributive Lattices Whose Prime Ideals Are Principal.

A well known theorem of I.S. Cohen states that if $R$ is a commutative ring with 1, and every prime ideal in $R$ is principal, then every ideal in $R$ is principal. In this note, the analogue of this theorem is proved for distributive lattices with 1. (Received May 17, 1976.)

Authors introduced by R. Padmanabhan.


For notations see (1) Jönsson-Tarski, Amer. J. Math., vol. 73(1951), pp. 891-939, vol. 74(1952), pp. 127-162 and (2) Monk, Math. Nachr., vol. 46(1970), pp. 47-55. For any RA (i.e. relation algebra) $O$, let $FnO$ be the set of elements $x$ of $O$ that are functional (i.e. $x^2 = 1'$). By Zorn's lemma we obtain Lemma: If a complete RA $B$ satisfies condition (C); for every $x \neq 0$ in $B$ there is a $y \in FnB$ with $0 < y < x$, then it also satisfies (C'). For every $x$ in $B$ there is a $y \in FnB$ with $y < x$ and $y1 = x1$. Theorem 1: Every RA $O$ satisfying (C) is representable (i.e. isomorphic to a proper relation algebra). Proof: Let $\mathcal{B}$ be the completion of $O$ (see (2)), and $\mathcal{L}$ be the perfect extension of $\mathcal{B}$ (see (1)). Clearly $\mathcal{B}$ satisfies (C) and hence also (C'). Let $U$ be the set of all atoms $u$ of $\mathcal{L}$ such that $u < v$ for some $v \in Fn\mathcal{L}$, and let $F_x = \{u, v : u, v \in U$ and $v < u x\}$ for every $x$ in $\mathcal{L}$. Using (C') it is shown that $F$ maps $\mathcal{B}$ onto, and hence $O$ into, a proper RA. An easy consequence of theorem 1 is Theorem 2: $O$ is a representable RA if $O$ is a subalgebra of an RA satisfying (C). Theorems 4.29 and 4.32 of (1) are special cases of theorem 1. Related results also hold for cylindric algebras. (Received May 20, 1976.)

ROGER MADDUX, University of California, Berkeley, California 94720

Another sufficient condition for the representability of relation algebras

For references and notation see preceding abstract. Theorem 1: If $O$ is a relation algebra in which $\sum \{p, q \in FnO : p = 1\}$, then $O$ is representable. Aside from some elementary results in the calculus of relation algebras, the proof of theorem 1 only requires the fact that the class of representable relation algebras is a variety. Corollary 1: (Maddux-Tarski) If $O$ is a relation algebra in which $\sum FnO = 1$, then $O$ is representable. Corollary 1 was first proved by methods completely different from those of theorem 1 (see preceding abstract). Corollary 2: (Tarski, J. Symb. Logic, v. 18 (1953), pp. 188-189) If $O$ is a relation algebra with two elements $p, q \in FnO$ such that $\exists q = 1,$
then $\mathcal{O}$ is representable. The original proof made essential use of some metamathematical results, and can be found in Tarski, A formalization of set theory without variables (to appear). Theorem 1 can also be used to prove the following analogous result for cylindric algebras (for notations see Henkin-Monk-Tarski, Cylindric Algebras). Theorem 2: Suppose $n = 3$, $\mathcal{O} \in \mathcal{C}_n'$, and $L = \Sigma \{ o_0(x, x', y') : x, y' \in A \}$. Then $\mathcal{O}$ is representable if $n$ is infinite, and $\mathcal{P}_n^{-2}\mathcal{O}$ is representable if $n$ is finite.

(Received May 20, 1976.)

76T-A179 Christiane Rousseau, Université de Montréal, C.P. 6128, Montréal 101, Canada. Continuity, holomorphy and logic of topoi.

There exists non boolean topos without axiom of choice, namely the topos $\text{Sh}(X)$ where $X$ is a locally compact space where the following theorem is true.

Theorem: $f : U \to \mathbb{C}$, where $U$ is an open set in $\mathbb{C}$, is continuous iff uniformly continuous on every closed sphere in $U$, $f$ is differentiable iff uniformly differentiable on every closed sphere in $U$.

Also the existence of the integral interprets as follows:

Integral: Let $U$ be an open set in $\mathbb{C}$ and $f : U \to \mathbb{C}$ holomorphic. Let $\gamma$ be a piecewise differentiable path in $\Pi_n(U)$ depending holomorphically upon the parameters $(t_1, \ldots, t_{n-1}) \in \Pi_n(U)$. Then

$$\int_{\gamma(t_1, \ldots, t_{n-1})} f(z_1, t_1, \ldots, t_{n-1}) \, dz$$

depends holomorphically upon $t_1, \ldots, t_{n-1}$.

(Received May 21, 1976.)

76T-A180 SAAD MOHAMED, Kuwait University, Kuwait and THANAA BOUHY, Women's University College, Ain Shams University, Cairo, Egypt. Continuous modules.

Following Utumi, we call a module $M$ continuous if it satisfies the following:

(i) every submodule of $M$ is essential in some direct summand of $M$, and (ii) if a submodule $A$ of $M$ is isomorphic to some direct summand of $M$, then $A$ is a direct summand of $M$. The following results are proved: (1) Every quasi-injective module is continuous. (2) A module $M$ is quasi-injective if and only if $M \times M$ is continuous. (3) A ring $R$ is semisimple artinian if and only if every finitely generated $R$-module is continuous. (4) Let $M$ be a continuous module, $H$ its ring of endomorphisms and $J$ the Jacobson radical of the ring $H$, then $H/J$ is a continuous regular ring.

(Received May 24, 1976.) (Authors introduced by Professor S. K. Jain.)

76T-A181 Ralph Freese, University of Hawaii, Honolulu, HI 96822. The class of Arguesian lattices is not a congruence variety. Preliminary report.

Let $L$ be the Hall-Dilworth lattice obtained by gluing together projective planes of different characteristics, and let $V$ be a variety of algebras all of whose congruence lattices are modular. Then $L$ is not in the variety of lattices generated by the congruence lattices of $V$.

By a result of B. Jónsson, $L$ is Arguesian. (Received May 26, 1976.)


If $a$ is an element and $U$ is a finite set of a lattice $L$, then $U$ is called a cover of $a$ if $V \cup U \geq a$. $U$ is a non-trivial cover if there is no $a \in U$ with $a \geq U$. $V < U$ holds if for all $V \in V$ there is a $u \in U$ with $V \leq u$. $D_0(L)$ is the set of join - prime elements of $L$, $D_{k+1}(L)$ is the set of those elements $a \in L$ such that if $U$ is a non-trivial cover of $a$ then there exists a cover $V \subseteq D_k(L)$ of $a$ with $V < U$. $\mathcal{W}$ is the Whitman condition free of the generators. Onto maps are used in the definition of projective lattices.

THEOREM. A lattice $L$ is projective if and only if each of the following and their duals holds

1. $U \cup D_k(L) = L$
2. $(W)$
3. for each $a \in L$, there is a finite set $\mathcal{S}(a)$ of covers of $a$ such that if $U$ is a cover of $a$, then there is a $V \in \mathcal{S}(a)$ with $V < U$

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4. there is a homomorphism \( f: FL(X) \) onto \( L \) and an order-preserving map \( g: L \rightarrow FL(X) \) such that \( f(g(a)) = a \) for all \( a \in L \).

**COROLLARY.** Countable projective lattices are characterized by 1, 2, 3, above, and their duals.

(Received May 26, 1976.)

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**The thickness of the complete graph.**

The thickness of a graph \( G \) is defined as the minimum number of planar graphs whose union is \( G \). The main theorem of this paper states that the thickness of the complete graph \( K_p \) is equal to the greatest integer less than or equal to \((p+7)/6\) for all positive integer values of \( p \) except when \( p \) is equal to nine or ten. In these two cases, the thickness of the complete graph is three. Constructions are given for a decomposition of any complete graph into a minimum number of planar subgraphs. This paper extends the work of L. W. Beineke and F. Harary to cover the case when \( p \) is of the form \( 6n+4 \). (Received June 1, 1976.)

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**On a norm theorem, Preliminary report.**

Some time ago, Garbanati (Bull. Amer. Math. Soc., May 1975) announced that for an abelian number field \( k \), Hasse norm theorem holds if and only if the narrow genus number and narrow central class number of \( k \) are the same. Here this is improved as:

For any normal number field \( k \), norm theorem holds if and only if the genus number and central class number of \( k \) are the same.

Defining central class field of an arbitrary number field in an appropriate manner, it is worth investigating whether the restriction of normality on \( k \) in (*) could be deleted. In this connection, Furuta’s isomorphism theorem for the relative Galois group of the genus field of \( k \) over \( k \) could be extended to the case when \( k \) is non-normal by using Ex. 8, Cassels and Fröhlich, Alg. Number Theory, p. 364. Possibly Masuda’s isomorphism theorem for the relative Galois group of central class field of \( K \) over \( k \) also could be extended to the case when \( k \) is non-normal.

At present this is a conjecture supported to some extent by an earlier result of the author (Abstract 76T-A58, these Notices, Feb. 1976). If this conjecture is proved, then the normality condition on \( k \) could be deleted from (*). From (*), it follows that a normal number field with a finite class field tower (or central class field tower) has an unramified normal extension for which the norm theorem holds. (An earlier announcement of the author, Abstract 75T-A246, these Notices, Oct. 1975, should be corrected in this manner). Now one may ask the following question: Let \( k \) be a number field for which norm theorem does not hold (with some additional restrictions if necessary). Is it true that \( k \) has no unramified normal extension for which norm theorem holds? (Received May 28, 1976.)

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**On Some m-Fold Summations.**

A number of results on functionals of \( \max(i_1, i_2, \ldots, i_m) \), \( \min(i_1, i_2, \ldots, i_m) \), etc., are...
generalized and lead to previously known results as special cases in a simpler fashion. Some of these results are as follows: If $I$ denotes the set of values $(i_1, i_2, \ldots, i_m)$, then

$$\int F(\max I) = \sum_{I_0}^n \left( (r+1)^m - r^m \right) F(r), \quad \int F(\min I) = \sum_{I_0}^n \left( (r+1)^m - r^m \right) F(n-r),$$

with $J(m,k) = \sum \left( \min I \right)^k = J(k,m)$ (in all the undesignated sums, the $m$ summation indices $i_k$ run from 0 to $n$).

In the next undesignated sum, the indices $i_k$ run from 0 to $n_k$, $k = 1, 2, \ldots, m$.

$$\int F(\min I) = \sum_{I_0}^n F(r) \left( \frac{m}{n} + \frac{m}{n} \right) \left( n - r \right) F(n-r),$$

with $J(m,k) = \sum \left( \min I \right)^k$ (more complicated). (Received June 1, 1976.)

*76T-A187 Yehiel Lehrer-Illamed, Soreq Nuclear Research Centre, Yavne, Israel. On exceptional identities for skew symmetric matrices.

Let $M_n$ be the algebra of $n \times n$ matrices over a field of characteristic zero. Let $K^-$ and $K^+$ be the subspaces of $M_n$ of skew symmetric matrices and symmetric matrices, respectively. Let $s_2(x_1, \ldots, x_k)$ be the standard polynomial of degree $k$; $s_2(x_1, x_2, x_3) = \left[ x_1, x_2 \right] = x_1 x_2 - x_2 x_1$. If $n$ is even $s_2(x_1, \ldots, x_{2n})$ is an identity for $K^-$; this theorem of Kostant was shown to be valid also for odd $n$ by J. Hutchinson, P. Owens and L. Rowen (see Jacobson's lecture in the Proceedings of the Oklahoma Ring Theory Conference, 1973). We have obtained the following result:

**Theorem.**

1. $\left[ s_3(x_1, x_2, x_3), x_4 \right]$ is an identity for $K^-$.
2. $s_3(x_1, x_2, x_3)$ is a central polynomial for $K^+$.

**Remarks:**

a) The identity (1) is an exceptional identity for $K^-$; i.e., it is not the special case, $n=3$, of the above mentioned identity for $K^-$.

b) For $k>1$ $s_{2k+1}(x_1, \ldots, x_{2k+1})$ is a central polynomial only for a subspace of $K_{2k+1}$.

c) The polynomial (1) vanishes for all specializations of $x_1, x_2, x_3, x_4$ in $K^-$ and $x_4$ in $K^+$.

d) $s_3(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = \left[ s_3(x_1, x_2, x_3), [s_2(x_4, x_5), [s_2(x_6, x_7), s_2(x_1, x_2)]] \right]$ is a central polynomial for $K^+$. (Received June 2, 1976.)

*76T-A188 Charles M. Grinstead, University of California, Los Angeles, California, 90024. On Coverings and Packings of the Complete Graph with Cycles. Preliminary report.

The maximum number of pairwise edge-disjoint simple 6-cycles in the complete graph $K_n$ and the minimum number of simple 6-cycles whose union is $K_n$ are determined. The analogous numbers when 6-cycles are replaced by 8-cycles are also determined. For example, the maximum number of pairwise edge-disjoint 6-cycles in $K_n$ is

$$\left\lfloor \frac{n}{6} \left( \frac{n-1}{2} \right) \right\rfloor$$

if $n \not\equiv 5 \pmod{6}$, and

$$\left\lfloor \frac{n}{6} \left( \frac{n-1}{2} \right) \right\rfloor - 1,$$

if $n \equiv 5 \pmod{6}$.

(Received June 4, 1976.) (Author introduced by B. Rothchild).

*76T-A189 Edgar O. Goodaire, Forschungsinstitut für Mathematik, ETH, CH-8006 Zürich, Switzerland (on leave from Memorial University of Newfoundland, St. John's, Canada until July 15, 1976). A Classification of Jordan Bimodules by Weights.

The presence of a Cartan subalgebra in a Jordan algebra allows the classical definition of weighted Lie module to be made also for Jordan bimodules. In this paper the set of weights on the irreducible bimodules of a finite dimensional simple reduced Jordan algebra over a field of characteristic different from 2 are determined. In general (and in total contrast to the situation with Lie algebras) two non-isomorphic irreducible Jordan bimodules can have precisely the same set of weights; however, for those algebras for which this phenomenon does not occur, irreducible modules can be distinguished up to isomorphism by the maximum weight relative to a certain ordering of weights. (Received June 4, 1976.)


Let $K_n$ be the operator on $l^2(\mathbb{Z}^2 \times \{1, 2\})$ defined by $K_n(m,a)(n,b) = R_s(m,a)(n,b) \phi_s(n) g(n)$, where $g(x) = \chi_A \cdot |x|^{-2}$ is compact and where $R_s(m,a)(n,b) = \int_{|k|< \pi/25} \int_{|k|< \pi/25} e^{\mathbf{k} \cdot (m-n)} \delta_{2}(M + 1\gamma/5 \sin(k_n) \theta) \frac{d^2k}{d^2k}$. Then $\text{det}(I+K_n)$ is a
multinomial in the $\phi_0(n)$ all of whose coefficients are positive.

Consequently if $\phi_0(n)$ is the lattice boson field of Guerra, Rosen and Simon, if $\phi_0(n)$ is the free expectation and if
\[
\det_{\text{ren}}(I + K) = \det(I + K) \exp \left\{ -\text{tr} K - \frac{1}{2} \text{tr} K^2 - \frac{1}{2} C_5 \phi_0(n) \phi_0^2(n) \right\}
\]
with $C_5 > 0$, then
\[
\phi_0(n_1) \ldots \phi_0(n_r) \det_{\text{ren}}(I + K) > 0.
\]
(Received June 3, 1976.)

76T-A191 DAVID ZEITLIN, 1650 Vincent Ave., North, Minneapolis, MN., 55411. Parametric solutions for two equal sums of 14 biquadrates. I.

Let $W_{k+2} = P^2 W_k + B W_k$, $k = 0, 1, \ldots$, where $P, B, W_0$, and $W_1$ are integers. Then, for $k = 0, 1, \ldots$, we have
\[
\begin{align*}
(7P W_k + 3) &+ (7B W_k + 3) + (5P W_k + 3) + (5B W_k + 3) + (7P^2 W_k + 2) + (7P B W_k + 2) + (3P W_k + 2) + (3B W_k + 2) + (7P W_k + 2) + (7B W_k + 2) = \\
&= (7P^2 W_k + 4) + (3P B W_k + 4) + (5P^2 W_k + 4) + (5B^2 W_k + 4) + (7P B W_k + 4) + (7B^2 W_k + 4)
\end{align*}
\]
Remark. If $W_{k+2} = P^2 W_{k+1} + B W_{k+1}$, then $W_k$ satisfies two equal sums of 14 biquadrates (to be announced).

(*) is a generalization and reduction of two previously announced results; see these NOTICES 23(1976), A-353, and A-419++. (Received June 4, 1976.)

76T-A192 JOHN CHUCHEL, Montana State University, Bozeman, Montana 59715 and NORMAN EGGERT, Montana State University, Bozeman, Montana 59715. The complete quotient ring of Images of Semilocal Prüfer Domains. Preliminary report.

It is well known that the complete quotient ring of a Noetherian ring coincides with its classical quotient ring. But in general, the structure of the complete quotient ring of a given ring is largely unknown. The authors investigate the structure of the complete quotient ring of certain Prüfer rings. The complete quotient ring of a semilocal Prüfer domain is characterized in terms of complete quotient rings of local rings and a completion of a topological ring. If the kernel of the homomorphism has an irredundant primary decomposition, the elements of the complete quotient ring are characterized. (Received June 7, 1976.)

76T-A193 JOE W. FISHER and JAMES OSTERBURG, University of Cincinnati, Cincinnati, Ohio, 45221, Semiprime ideals in rings with finite group actions.

Let $R$ be a ring, $G$ a finite group of automorphisms acting on $R$ and $R^G = \{ r \in R : r^g = r \text{ for each } g \in G \}$. If $R$ is semiprime with no additive $|G|$-torsion, then $R$ is left Goldie if and only if $R^g$ is left Goldie. By coupling this with an examination of the prime ideal structures of $R^G$ and $R$, we prove that if $|G|$ is invertible in $R$ and if $R^G$ is left Noetherian, then $R$ satisfies the ascending chain condition on semiprime ideals, every semiprime factor ring of $R$ is left Goldie, nil subrings of $R$ are nilpotent, there are only finitely many primes minimal over any ideal of $R$, and every ideal of $R$ contains a product of prime ideals. In examining the prime ideal structures of $R^G$ and $R$, we also study lying over, going up, going down, and incomparability. We show that if each prime ideal of $R$ is maximal, then each prime ideal of $R^G$ is maximal. We prove that if $R$ is semiprime and $R^G$ is a finite direct sum of simple rings, then $R$ is a finite direct sum of simple rings. As an application of this we obtain that if each prime ideal of $R^G$ is maximal and if each semiprime ideal of $R^G$ is a finite intersection of prime ideals, then each prime ideal of $R$ is maximal. (Received June 7, 1976.)
Suppose $G$ is an abelian $p$-group and $B$ is a basic subgroup (s.g.) of $G$. A s.g. $K$ of $G$ will be called $B$-big if $K$ is projection-invariant (P.I.) in $G$ and $K + B = G$. If $K$ is $B$-big for every basic s.g. $B$ of $G$, then $K$ will be called big in $G$. Clearly every large s.g. is big. \textbf{THM. (1)} If $B$ is a basic s.g. of $G$, then the following are equivalent (T.A.E.): (a) $G^1$ is $B$-big; (b) $G = B \oplus G^1$; (c) $B$ is a direct summand of $G$. \textbf{COR. (1)} If $D$ is the maximal divisible s.g. of $G$, then T.A.E.: (a) $G^1$ is big in $G$; (b) $G = B \oplus D$ for every basic s.g. $B$ of $G$; (c) $G = H \oplus D$, where $H$ is bounded; (d) $G$ has finite length. \textbf{THM. (2)}: If $G = B \oplus D$ where $B$ is bounded and $D$ is divisible, then T.A.E.: (a) $K$ is big in $G$; (b) $K$ is P.I. in $G$ and $K \cong D$; (c) $K = P \oplus D$, where $P$ is P.I. in $B$. (Received June 7, 1976.)

For notations see two works of R. McKenzie: (I) Thesis, Univ. of Colorado, Boulder, 1966; (II) Michigan Math. J. 17(1970). By $B, G, P, L, S, C$ we respectively denote the classes of representable, group-representable, permutational, integral, and simple relation algebras (RA's); let $C$ be the class of RA's representable only on infinite domains. For any class $K$ of RA's containing a finite algebra, let $\mu K$ be the smallest number of atoms in any finite algebra in $K$. We consider here all classes $K$ of RA's that are unions of intersections of classes among $B, G, P, L, S, C$ and their complements. Given any such class we may attempt to decide whether $K$ contains a finite algebra and if so, to determine $\mu K$. E.g., if $K$ is $S\cap R$ or $I\cap R$, then $\mu K = 4$; see (I) or (II). If $K$ is $G \cap C$, then $\mu K = 3$. If $K$ is $P \cap G \cap C$, then $4 \leq \mu K \leq 7$, but $\mu K$ is not yet known; cf. (II). Finally, for $K = I \cap R \cap P$ it is not even known whether $K \neq \emptyset$. It turns out that the value of $\mu K$ completely determines the set of cardinalities of finite algebras in $K$. In fact, we have Theorem. Let $K$ be any class of RA's that can be formed as a Boolean combination of the classes $B, G, P, L, S, C$. If $\mu K = m$, then $K$ contains algebras of cardinality $2^m$ for any $n$ with $m \leq n < \omega$ (and clearly no other finite algebras). For the case $K = S\cap R$ the theorem solves a problem of McKenzie in (I), p. 124; we do in fact give integral nonrepresentable one-generated RA's with $n$ atoms, any $4 \leq n < \omega$, and an equation in one variable which fails in all these algebras but holds in $R$. Independent of the theorem, another problem suggested by (I), p. 39, is solved: all finite RA's with 4 atoms are determined. (Received June 8, 1976.) (Author introduced by Alfred Tarski.)

\textbf{76T-A195} ULP WOSTNER, University of California, Berkeley, California 94720. \textbf{Finite relation algebras.}

Two varieties $\mathcal{E}_1$ and $\mathcal{E}_2$ are said to be $n$-weakly independent if whenever $p$ and $q$ are $n$-ary polynomials with $p = q$ in $\mathcal{E}_1 \wedge \mathcal{E}_2$, then there exists a polynomial $r$ such that $p = r$ in $\mathcal{E}_1$ and $q = r$ in $\mathcal{E}_2$. If $\mathcal{E}_1$ and $\mathcal{E}_2$ are $n$-weakly independent for all $n$, then they are called weakly independent (see A. L. Foster and A. F. Pixley, Math. Z. 125 (1972), p. 272). Theorem: Let $\mathcal{E}_1$ and $\mathcal{E}_2$ be varieties of lattices having a finite number of subdirectly irreducible members, each having at most $n - 1$ nonzero join-irreducible elements. If $\mathcal{E}_1$ and $\mathcal{E}_2$ are $n$-weakly independent, then they are weakly independent. Theorem: If $\mathcal{E}$ is any variety of lattices, then the varieties $\mathcal{E}_{\mathbb{N}}$ and $\mathcal{E}$ are weakly independent. Corollary: Let $\mathcal{Y} = \mathcal{E}_{\mathbb{N}} \vee M_5$. Then $|\mathcal{F}(\mathcal{Y})| = 379$ and $|\mathcal{F}(\mathcal{Y}(4))| = 824,356,943,788$. There are varieties of lattices which are not weakly independent, such as $M_5$ and $Q_2$ (where $Q_2$ is defined in R. N. McKenzie, Trans. Amer. Math. Soc. \textbf{174} (1972), 1-, 43). (Received June 8, 1976.)
**CORRECTED ABSTRACTS**

*76T-B92* N.K. GOVIL and V.K. JAIN, Indian Institute of Technology, Hauz Khas, New Delhi-110029. **On Enevirström-Kakeya theorem II.**

Govil and Rahman [Tohoku Math. J., 20(1968), 126-136] have obtained a generalisation of the Enevirström-Kakeya Theorem for polynomials with complex coefficients. As an improvement of this result we prove: Let \( p(z) = \sum_{k=0}^{n} a_k z^k \neq 0 \) be a polynomial of degree \( n \) with complex coefficients such that (i) \( |\arg a_k - \beta| \leq \pi/3 \), \( k = 0, 1, \ldots, n \) for some real numbers \( \alpha \) and \( \beta \), and (ii) \( |a_n| \geq |a_{n-1}| \geq \cdots \geq |a_0| \); then \( p(z) \) has all its zeros in \( R_3 = \{ z : |z| \leq R_3 \} \) where \( R_3 = (2M^2_2 - 1) R_2 \) and \( M_2 = |a_0| R_2 - |a_0|/(|a_0| + \cos \alpha + \sin \alpha) \). \( M_1 = |a_1| R_1 = \cos \alpha + \sin \alpha \) and \( B = 1 - \cos \alpha \). (Received March 9, 1976.) (Authors introduced by Dr. S.K. Bajpai.)

*76T-B108* R. VENKATARAMAN, Madura College, Madurai 625011 and A.M.S. RAMASWAMY, A.V.C. College, Mayuram 609305, India. **Fuzzy measure.** Preliminary report.

Let \( F \) be the class of all fuzzy sets on a given set \( X \) over the set \( J \) of all rational numbers in \( I \), the unit interval. We first define the fuzzy \( x \)-outer measure on a set \( A \) in \( F \) with respect to an interval, \( 0, 1, \ldots, R^2 \), where \( R^2 = \{ (x, y) : x, y \in R \} \). Then by introducing a partial order relation in \( 2^J \) we define the fuzzy set outer measure on \( A \) with respect to \( 2^J = \{ 0, 1, \ldots, R \} \) where \( R = \{ (x, y) : x, y \in R \} \). We then provide a partial order relation in \( 2^J \) and prove: If \( f(B) \leq 2^{n/2}(n/2 + 1) \) then \( F \) contains a common fixed lattice point \( p \neq 0 \). **Note:** If \( \theta \in F \), then \( F \) contains a lattice point \( q \neq \theta \) provided \( \mu(F) \geq 2^n \) and \( F \) is centrally symmetric about \( \theta \). **Lemma 2.** Let \( S \) be a bounded compact set in \( R^n \) and \( \mu \) be a non-expansive mapping of \( S \) into \( S \). If the convex body \( F \) of all common fixed-points over \( C \) is disjoint with \( \theta \) and contains a ball \( B \) with Lebesgue measure \( \mu(B) \geq 2^{n/2}(n/2 + 1) \) then \( F \) contains a common fixed lattice point \( t \neq \theta \). (Received April 5, 1976.) (Author introduced by Dr. T.V. Lakshminarasimhan.)

*Analysis (26, 28, 30–35, 39–47, 49)*

*76T-B120* Albert A. Mullin, 6840 Todd Street, Patton Park, Ft. Hood, TX 76544. **Applications of functional analysis to number theory.**

This note is a synthesis of several results by Minkowski and Blichfeldt on the existence of lattice points in convex bodies and by A. A. Markov, S. Kakutani, and E. Browder on the existence of common fixed-points for commuting families of continuous mappings. **Lemma 1.** Let \( S \) be a compact convex subset of an \( n \)-dimensional linear topological space with origin \( \theta \). Let \( C \) be a non-empty commuting family of affine continuous mappings of \( S \) into \( S \). If the convex body \( F \) of all compatible fixed-points over \( C \) is disjoint with \( \theta \) and contains a ball \( B \) with Lebesgue measure \( \mu(B) \geq 2^{n/2}(n/2 + 1) \) then \( F \) contains a common fixed lattice point \( p \neq 0 \). **Note:** If \( \theta \in F \), then \( F \) contains a lattice point \( q \neq \theta \) provided \( \mu(F) \geq 2^n \) and \( F \) is centrally symmetric about \( \theta \). **Lemma 2.** Let \( S \) be a bounded closed subset of \( R^n \). Let \( C \) be a non-empty commuting family of non-expansive mappings of \( S \) into \( S \). If the convex body \( F \) of all common fixed-points over \( C \) is disjoint with \( \theta \) and contains a ball \( B \) with \( \mu(B) \geq 2^{n/2}(n/2 + 1) \) then \( F \) contains a common fixed lattice point \( t \neq \theta \). (Received May 11, 1976.)

*76T-B121* N.K. GOVIL and V.K. JAIN, Indian Institute of Technology, Hauz Khas, New Delhi-110029. **Indian Institute of Technology, Kharagpur, India. On entire functions of exponential type.**

The following problem was proposed by Prof. R.R. Boas, Jr.: If \( f(z) \) is an entire function of exponential type \( \tau \) such that \( |f(x)| \leq 1 \) for real \( x \), \( h_\tau(x) = 0 \) and \( f(x+iy) \neq 0 \) for \( y > x \), what are the bounds for \( |f'(x)| \) and \( |f(x+iy)| \)? The case \( k < 0 \) was considered by Govil and Rahman [Trans. Amer. Math. Soc. 137(1969) 501-511]. Here we consider the case when \( k \geq 0 \) and prove: If \( f(z) \) is an entire function of exponential type \( \tau \) such that \( h_\tau(x) = 0 \), \( |f(x)| \leq 1 \) for all real \( x \) and \( f(x+iy) \neq 0 \) for \( y > x \), \( k \geq 0 \), then we have, \( |f'(x)| \leq \frac{1}{2} \frac{1}{e^{kx}} \) and \( |f(x+iy)| \leq \frac{1}{2} \frac{e^{kx}}{e^{k|y-x|}} \) for \( -\infty < x < \infty \). (Authors introduced by Dr. J.L. Bajpai.) (Received March 10, 1976.)
S. Zaidman (Neres Notices 22, (1975), p.71) put forward the question: consider the problem $(1) \Delta U + (2) U = 0$ in $B \subset \mathbb{C}^2$. Is it true that $U \equiv 0$ provided that $\partial \overline{\partial} U + \partial \overline{\partial} \chi = 0$ on the boundary of the domain $\Omega$? Proof: if $\partial \overline{\partial} U = 0$ on $\partial \Omega$, then $U \equiv 0$. Hence we obtain the assertion of the theorem. If $\partial \overline{\partial} U \neq 0$ on $\partial \Omega$, then $U \equiv 0$ is possible to prove that $\Lambda = \inf J$. It is clear that $U \equiv 0$ provided that $(3) \partial \overline{\partial} U \neq 0$ on $\partial \Omega$. If $\partial \overline{\partial} U = 0$ on $\partial \Omega$, then it is possible to conclude that $\Lambda \leq \frac{s}{2}$. Hence we obtain the assertion of the theorem. To estimate $\Lambda$ when $\partial \overline{\partial} U = 0$, one starts from the Euler eq. $-\Delta U + \Lambda U = 0$, $\partial \overline{\partial} U = 0$, then obtains $\Lambda = \Delta U$, $\partial \overline{\partial} U = 0$, being the Green function. Further $\partial \overline{\partial} U = 0$, $\partial \overline{\partial} U = 0$, and $\partial \overline{\partial} U = 0$. Using this estimate, one obtains the inequality: $\inf \{2\}$ with the new solutions provided that $U(0,0) = 0$, $U(0,0) = 0$, $U(0,0) = 0$, $U(0,0) = 0$, $U(0,0) = 0$, $U(0,0) = 0$, $U(0,0) = 0$, $U(0,0) = 0$.

(Rceived April 12, 1976.)

A. T. Dash, University of Guelph, Guelph, Ontario, Canada, N1G 2W1. The products of Fredholm operators of indices zero.

It is well known that if $A_1, \ldots, A_n$ are commuting operators on a Banach space such that their product $A = A_1A_2 \cdots A_n$ is Fredholm, then each $A_j (1 \leq j \leq n)$ is Fredholm. The analogous question for Fredholm operators of indices zero is discussed. More precisely, it is shown that if $A_1, \ldots, A_n$ are commuting operators on a Banach space and if their product $A = A_1A_2 \cdots A_n$ is Fredholm, then all $A_j$ are Fredholm of index zero. (Received April 22, 1976.)

P. H. Brownell, Univ. of Washington, Seattle, Wash. 98195. Spherical Harmonic Integrals.

We generalize Stein & Weiss ("Introduction to Fourier Analysis on Euclidean Space", 1971, pp. 159, 3.10) invariance of spherical harmonics under Fourier transformation to show that, subject to $|f| \leq (1+|t^2-y^2|)^{-1}$ for any spherical harmonic $Q_k$ of degree $k$, \[ \int_0^\infty \int_{\mathbb{R}^n} f(x) \overline{Q_k(x)} \, dx \, dy < \infty \] over all $f \in L^1(\mathbb{R}^n)$, where $\overline{Q_k(x)}$ is the complex conjugate of $Q_k(x)$.

We estimate the number $N$ of zeros, in an interval of length $T$, of functions $f(t) = c(t)^x$, where $A$ is an $n$-square and $c, b$ are $n$-vectors (i.e., of linear combinations, with polynomial coefficients, of $e^{x \sin \omega (t-t_0)}$). The estimate is $N \leq x + (2k-1)T + O(1)$; here $x$ is the number of real, and $2k$ of the complex, eigenvalues of $A$; $\omega = \max \lambda$. (with $\omega > 0$ arbitrary if all $\lambda$ are real); and $[x]!$ denotes the smallest integer $x \geq x$.

(Rceived April 21, 1976.)

U. Jajic, Case Western Reserve University, Cleveland, Ohio 44106. Zeros of Exp-Trig Polynomials, Preliminary report.

We generalize Stein & Weiss ("Introduction to Fourier Analysis on Euclidean Space", 1971, pp. 159, 3.10) invariance of spherical harmonics under Fourier transformation to show that, subject to $|f| \leq (1+|t^2-y^2|)^{-1}$ for any spherical harmonic $Q_k$ of degree $k$, \[ \int_0^\infty \int_{\mathbb{R}^n} f(x) \overline{Q_k(x)} \, dx \, dy < \infty \] over all $f \in L^1(\mathbb{R}^n)$, where $\overline{Q_k(x)}$ is the complex conjugate of $Q_k(x)$.

We estimate the number $N$ of zeros, in an interval of length $T$, of functions $f(t) = c(t)^x$, where $A$ is an $n$-square and $c, b$ are $n$-vectors (i.e., of linear combinations, with polynomial coefficients, of $e^{x \sin \omega (t-t_0)}$). The estimate is $N \leq x + (2k-1)T + O(1)$; here $x$ is the number of real, and $2k$ of the complex, eigenvalues of $A$; $\omega = \max \lambda$. (with $\omega > 0$ arbitrary if all $\lambda$ are real); and $[x]!$ denotes the smallest integer $x \geq x$.

(Rceived April 21, 1976.)

A. T. Dash, University of Guelph, Guelph, Ontario, Canada, N1G 2W1. The products of Fredholm operators of indices zero.

It is well known that if $A_1, \ldots, A_n$ are commuting operators on a Banach space such that their product $A = A_1A_2 \cdots A_n$ is Fredholm, then each $A_j (1 \leq j \leq n)$ is Fredholm. The analogous question for Fredholm operators of indices zero is discussed. More precisely, it is shown that if $A_1, \ldots, A_n$ are commuting operators on a Banach space and if their product $A = A_1A_2 \cdots A_n$ is Fredholm, then all $A_j$ are Fredholm of index zero. (Received April 22, 1976.)

We estimate the number $N$ of zeros, in an interval of length $T$, of functions $f(t) = c(t)^x$, where $A$ is an $n$-square and $c, b$ are $n$-vectors (i.e., of linear combinations, with polynomial coefficients, of $e^{x \sin \omega (t-t_0)}$). The estimate is $N \leq x + (2k-1)T + O(1)$; here $x$ is the number of real, and $2k$ of the complex, eigenvalues of $A$; $\omega = \max \lambda$. (with $\omega > 0$ arbitrary if all $\lambda$ are real); and $[x]!$ denotes the smallest integer $x \geq x$.

(Rceived April 21, 1976.)
**WITHDRAWN**

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**76T-B127**


It is fairly easy to observe that one of the Gould-Hopper generalizations [H.W. Gould and A.T. Hopper, Duke Math. J. 29 (1962), 51-63, especially p.58, Equation (6.2)] of the classical Hermite polynomials $H_n(x)$, viz. (1) $\psi_n^{(m)}(x,\lambda) = \sum_{k=0}^{n}[n/k](n/k/l)^k x^{n-k}$, is contained in the more general Brafman polynomials defined earlier [F. Brafman, Canad. J. Math. 9 (1957), 180-187, especially see p.186]. At least two further special cases of the Gould-Hopper polynomials (1) have since appeared in the literature. One of these special cases happens to have formed the subject-matters of a series of papers by M. Lahiri [cf., e.g., Proc. Amer. Math. Soc. 27 (1971), 117-121, especially p.118, Equation (3.2)], who considered the polynomials (2) $H_{n,m,v}(x) = v^{n,m}(x,-1) = v^{n,m}(x,-1)$. The other special case of (1) was considered by L.R. Bragg [Boll. Un. Mat. Ital. (4) 1 (1968), 347-355], who indeed discussed an initial-value problem leading to the polynomials (3) $p_n^P(x) = p_n^P(px,-1), n = 0,1,2,\ldots$. In the present paper several generating-function relations involving the polynomials $p_n^P(x)$, and their natural generalization $v_n^P(x)$, are discussed. A hitherto seemingly unnoticed fact on the equivalence of certain known generating functions is also pointed out. (Received May 4, 1976.)

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**76T-B128**

Myron Goldstein, Arizona State University, Tempe, AZ 85281 and Wellington H. Ow, Michigan State University, E. Lansing, MI 48824. Uniform Approximation of Harmonic Functions on Closed Sets by Global Harmonic Functions.

Let $C$ be the complex plane and $E$ the closure (relative to $C$) of an arbitrary open subset of $C$. Denote by $H(E)$ the set of all harmonic functions on $E$ and by $H_C(E)$ the subset of $H(E)$ consisting of those functions which can be uniformly approximated on $E$ by harmonic functions on $C$, i.e., if $u \in H_C(E)$ then for each $\varepsilon > 0$ there exists a harmonic function $v$ on $C$ such that $|u(p) - v(p)| < \varepsilon$ for all $p \in E$.

Denote by $S^2$ the Riemann sphere with chordal metric $\rho(p,q)$ and $\{F_i\}^{\infty}_{i=1}$ the set of unbounded components of $C - E$.

**Theorem.** $H(E) = H_C(E)$ if and only if i) $C - E$ has no relatively compact components; ii) for every $\varepsilon > 0$ there exists a $\delta > 0$ such that if $p \in F_i$ and $\rho(p,\infty) < \delta$ then there is an open path $\gamma \subset F_i$ joining $p$ to $\infty$ (with limit point $\infty$) such that $\gamma \subset \partial C$ where $\partial C = \{p \in S^2 \mid \rho(p,\infty) < \varepsilon\}$. (Received April 26, 1976.)

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**76T-B129**


Let $m$ denote product Lebesgue measure on $I^n = \prod_{i=1}^{n} [0,1]$.

**Theorem:** If $f : I^n \rightarrow I^n$ is 1-1, onto and both $f^{-1}(E)$ and $f(E)$ are Lebesgue measurable whenever $E$ is, then for any positive number $\varepsilon$, there is a homeomorphism $g$ of $I^n$ onto itself such that $m(\{x : f(x)\neq g(x) \text{ or } f^{-1}(x)\neq g^{-1}(x)\})$ is less than $\varepsilon$.

Goffman proved the analogous result for $I^n$. The proof is an adaptation, to the Hilbert cube, of Goffman's argument (see Acta Math. 89) for the n-dimensional cube. We use $Z$-sets instead of the sectionally

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zero dimensional closed sets employed by Goffman, and then apply R. D.

Anderson's extension theorem that homeomorphisms between $Z$-sets can be extended to homeomorphisms of $I^n$. (Received May 7, 1976.)

Mark A. Smith, Lake Forest College, Lake Forest, Illinois 60045. Rotundity and Smoothness in Conjugate Spaces.

The existence of a non-reflexive Banach space with rotund third conjugate space is established. Stronger notions of rotundity and smoothness in conjugate spaces are investigated. In particular, it is shown that a Banach space is reflexive whenever its conjugate space is weakly uniformly rotund. (Received May 10, 1976.)

ATHANASSIOS G. KARTSATOS, University of South Florida, Tampa, Fla. 33620. Perturbations of m-accretive operators and quasi-linear evolution equations.

Let $X$ be a complex Banach space with uniformly convex dual. Let $D$ be a closed subset of $X$, $T$ a positive number, and consider the Cauchy problem: (I) $x' + A(t,x)x = 0$, $x(0) = x_0$. Theorem 1. Let (I) satisfy the following: i) $A : (t,u,v) \mapsto A(t,u)v$ maps $[0,T) \times X \times X$ into $X$ and $x_0 \in D$; ii) for every $(t,u) \in [0,T) \times X$, $A$ is m-accretive in $v$; iii) $\|A(t,u_1)v - A(s,u_2)v\| \leq r(\|u_1\|, \|u_2\|, \|v\|)(|t-s| + \|A(s,u_2)v\| + \|u_1 - u_2\|)$ for every $t, s \in [0,T)$, $u_1, u_2, v \in D$. Here $r : R^3 \mapsto R_+ = [0, +\infty)$ is increasing in all three variables. Then there exists $T_1 < T$ such that (I) has a unique strong solution $x(t)$, $t \in [0,T_1]$. Theorem 2. Let $A$ with $A(t,u,v) = A(u)v$ satisfy the assumptions of Theorem 1. Let $B : u \mapsto A(u)u$ be demiclosed and accretive. Then $B$ is m-accretive. A theorem is also given extending a well known admissibility result of Massera and Schaffer to unbounded operators. (Received May 12, 1976.) (Author introduced by Professor M. N. Manougian.)

H.M. SRIVASTAVA, University of Glasgow, Glasgow G12 8QW, U.K. and University of Victoria, Victoria, British Columbia, Canada V8W 2Y2, and REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2 and Ravenshaw College, Cuttack-3, India. Some expansion theorems and generating relations for the H function of several complex variables. II. Preliminary report.

In the present sequel to a recent paper [H.M. Srivastava and R. Panda, Comment. Math. Univ. St. Paul. 24 (1975), fasc.2, 119-137] the authors continue their study of expansion theory and generating-function relations associated with a multivariable extension of the familiar H-function. (The multiple H-function was defined in an earlier paper by these authors [J. Reine Angew. Math. 283/284 (1976), 265-274; see also Abstract 74T-B13, these NOTICES 21 (1974), p.A-9] as a natural further generalization of the generalized Lauricella hypergeometric function of several complex variables; a study of its expansion theory was indeed initiated in another paper by the present authors, which is scheduled to appear in the Journal für die reine und angewandte Mathematik.) It may be remarked, among other things, that the main results in two recent papers by B.L. Sharma and R.F.A. Abiodun [Proc. Amer. Math. Soc. 46 (1974), 69-72; ibid. 53 (1975), 379-384], involving the G-function of two variables, are contained in a very special case of one of the generating relations given already in Part I. (In fact, the aforementioned Sharma-Abiodun results are essentially equivalent.) (Received May 14, 1976.)


Let $J$ denote $R^+ = [0,\infty)$ or $R = (-\infty,\infty)$. Let $C_p(J)$ denote the smallest constant for which the classical Hardy-Landau-Littlewood inequality $\|f'\|_p \leq C_p(J) \|f''\|_p \|f\|_p$ holds for all $f \in L^p(J)$ with second derivative $f'' \in L^p(J)$. We have obtained the following upper bounds on $C_p(J)$ which are improvements of the previous best results known to the author.

(I) $C_p(R^+) \leq 2(5/4)^{2/p} - 1$ for $1 < p < 2$;

(II) $C_p(R) \leq 2^{2/p} - 1$ for $1 < p < 2$;

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Consider the system of differential equation (1) \( x'' + A(t)x = 0 \), where \( A(t) \) is an \( n \times n \) continuous matrix. The author, jointly with A. C. Lazer, announced four theorems in the Bulletin of the AMS (vol. 82, No. 2 (1976)). Proofs of these theorems will appear in the SIAM J. of Math. Analysis. In these theorems, which are in the general area of Sturmian Theory of (1), we required \( A(t) \) to be symmetric. In a subsequent paper (to appear by the author and A. C. Lazer) it was shown, as a consequence of some other results, that Theorem 1 of our Bulletin announcement is true without the requirement that \( A(t) \) be symmetric. The author has now shown that all four theorems hold without the assumption that \( A(t) \) be symmetric.

Consider the delay integral equations:

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\begin{align*}
\tilde{x}_1(t) &= \int_{t-\tau_1}^{t-\tau_2} a(s)x_2(s)(1-x_1(s))ds, \\
\tilde{x}_2(t) &= \int_{t-\tau_1}^{t-\tau_2} a(s)x_2(s)(1-x_1(s))ds 
\end{align*}
\]

which can be regarded as the equations governing the S+I+R epidemic model proposed by Hoppensteadt and Waltman (Math. Biosciences 12 (1971), 133-145) where \( \rho = \text{constant, } \rho_0 = 0 \) \( \left( \tau(t)=\tau_1 \right) \), \( S_0 = 1 \), the initial data \( I_0, I_1, I_2 \) are suppressed, and the populations have been scaled so \( S+E+I+R = 1 \). The \( \tau_1, \tau_2, \tau_3 \) represent respectively the fixed time spent in the classes E, I, R and it is assumed \( a(t) \) is \( \omega \)-periodic. It can then be shown that non-zero (positive) \( \omega \)-periodic solutions bifurcate from the trivial solution \( (x_1 = 0, x_2 = 0) \) at a critical value of the parameter \( \tau_2 \). Solutions decay exponentially (S+1), the periodic solutions appearing only if \( \tau_2 > \tau_2^* \). The analysis can be extended to any number of classes.

Let \( p_n > 0 \) \((n=1,2,\ldots)\), \( P_n = p_0 + p_1 + \ldots + p_n \to \infty \) \((n \to \infty)\), and for the series
\[ \sigma_n, \text{ with } a_n = a_0 + a_1 + \ldots + a_n \text{ write: } \sigma_n = \frac{n}{v=0} p_v s_v/p_v; t_n = \frac{n}{v=1} p_v-1 a_v/p_n; \]

\[ p(x) = \frac{n}{v=0} p_v x^n, \quad p_v(x) = \frac{n}{v=0} p_v s_v x^n \quad (0 \leq x < 1) \]

\[ M(w) = \frac{n}{v=1} \frac{p_v}{p_v-1} \left( \frac{e^{v/w}}{\frac{v}{u} \left( 2u-v \right)} p_u \right) \left( \frac{e^{v/w}}{u=v_0(w-v+1)} p_u \right) \]

K is a strictly positive constant. \( \sigma_n \) is summable \( |J_n| \), if \( p_n(x)/p(x) \in BV(c, 1) \), \( 0 < c < 1 \), and it is summable \( \left| \sum p_n \right| \in BV \). The following Tauberian Theorems have already been proved: Theorem 1. If \( \{t_n\} \in BV \) and \( p_n \) is such that (i) \( p_n/p_n-1 < K \) \( n=1,2,\ldots \), (ii) \( M(w) > K \), for \( w \geq 1 \), then \( |J_n| \subseteq |C, 0| \). Theorem 2. If \( t_n \) and \( p_n \) are the same as in Theorem 1, then \( |J_n| \subseteq |C, 0| \). Theorem 3. If \( \{a_n \} \in BV \) and \( p_n \) is such as in Theorem 1, then \( |J_n| \subseteq |C, 0| \). This theorem (Theorem 3) contains, when \( p_n = 1 \), for all \( n \geq 0 \), a result of J.W. HYSLOP (J. London Math. Soc. 12 (1937), 176-180 Theorem 3). (Received May 26, 1976.) (Authors introduced by Professor Surjeet Singh.)

76T-B138 DAVID F. DAWSON, North Texas State University, Denton, Texas 76201. Mapping properties of matrix transformations.

Let \( DV_k = \{ x \in m : \sum |\Delta^k x_p | < \infty \} \), \( BV_k = \{ x \in m : \sum (p^k-1)|\Delta^k x_p | < \infty \} \), \( D = \{ DV_n : n = 0, 1, 2, \ldots \} \), and \( B = \{ BV_n : n = 1, 2, 3, \ldots \} \). In this paper necessary and/or sufficient conditions are given for a complex matrix \( A \) to map \( u \) into \( v \), where \( u \in DUB \) and \( v \in DUB \{ c \} \), extending earlier results of the author in which necessary and sufficient conditions were given for \( A \) to map \( u \) into \( v \), where \( u \in B \) and \( v \in B \{ c \} \). (Received May 27, 1976.)

76T-B139 P. RANGARAJA, The Ramanujan Institute, University of Madras, Madras-600 005(India), The Steinhaus theorem for Toeplitz matrices in non-archimedian fields.

\( \mathbb{K} \) denotes a complete, non-trivially valued non-archimedian field. The main object of this paper is to characterize Schur matrices (viz. matrices which sum bounded sequences) with entries from \( \mathbb{K} \). The classical Steinhaus theorem that a matrix cannot be both regular (viz. a limit preserving matrix) and a Schur matrix is then deduced in this case too. This deduction corrects a wrong statement of N. Varadacharyulu (Publ. Math. Debrecen 21 (1974), 171-177) that the Steinhaus theorem is not generally true in the non-archimedian case. The treatment in the non-archimedian case is neither an adaptation of that of the classical case nor can be adapted to the classical case. (Received May 28, 1976.) (Author introduced by Dr. T.V. Lakshminarayanan.)

76T-B140 ROBERT C. JAMES, Claremont Graduate School, Claremont, California 91711. Banach Spaces Quasi-Reflexive of Order One.

It is known that the \( l_2^2 \) product of \( J \) and \( J^* \) is quasi-reflexive of order two and isometric to its first dual, if \( J \) is isometric to \( J^* \). It is shown that there is a Banach space which is quasi-reflexive of order one and isomorphic to its first dual. It has a basis with several properties similar to properties of the bases for \( J \) and \( J^* \). (Received June 1, 1976.)

76T-B141 HANS W. MEYER, University of Cologne, 5000 Cologne 41, Western Germany. Monotone operators with strongly continuous inverse. Preliminary report.

Let \( F \) be a maximal monotone potential operator with linear and dense domain in a real separable Hilbert space \( H \).

**Theorem:** If \( F = B + T \), where \( B \) is a positive self-adjoint operator with compact inverse and \( T \) monotone, then \( F^{-1} \) is an everywhere defined strongly continuous and strictly positive potential operator in \( H \). If \( T \) is furthermore odd, then Ljusternik-Schnirelmann theory can be applied to \( F^{-1} \), leading directly to the existence of eigenfunctions of \( F \). (Received June 1, 1976.) (Author introduced by Professor N. W. Bazley.)
Consider the DDE \( y'(t) + p(t)y(t-T) = 0 \) where \( t \) is a positive constant and the function \( p(t) \) is \( > 0 \) and continuous for \( t > T_0 \). We proved that (2) \( \lim_{t \to +\infty} \int_{t-T}^t p(s)ds > \frac{1}{e} \) is a sufficient condition for all solutions of Eq. (1) to oscillate. In the special case where \( p(t) \) is a constant \( p \) the condition (2) becomes (3) \( pt \equiv 1 \) and is a necessary and sufficient condition for all solutions of the DDE \( y'(t) + py(t-T) = 0 \) to oscillate. The condition (2) is sharp and cannot be improved. Extensions of the above result to more general DDE's have been obtained. (Received June 3, 1976.)

Let \( f \) be a real valued function defined on \([0,1]\), with \( |f| \leq M \). Except in Theorem 3, all functions below are bounded in absolute value by \( M \). A positive parameter \( \gamma \) is used in the definition.

Definition. The lower integral \( \int_0^1 f \, dx = \lim_{\gamma \to 0^+} \sup \{ \int_0^1 f(s) \, ds \mid s \leq f \text{ except possibly on a set of outer measure } \gamma \} \). The upper integral is defined analogously. It is shown that \( \int_0^1 f \, dx \leq \int_0^1 |f| \, dx \). Then \( f \) is defined as R-integrable if the upper and lower integrals coincide. Their common value is denoted by \( \int_0^1 f \, dx \).

Theorem 1. \( f \) is R-integrable if and only if for each \( \varepsilon > 0 \), and each \( \gamma > 0 \) there are step functions \( s \leq f \leq S \) such that \( s \leq f \leq S \) except possibly on a set of outer measure \( \gamma \) and \( \int_0^1 (S - s) \, dx < \varepsilon \). Theorem 2. (Bounded convergence) Let \( f_n \) be a sequence of R-integrable functions. If for each \( \gamma > 0 \), there is a set \( T \) of outer measure \( \gamma \), and \( f_n \) converges uniformly to \( f \) on \([0,1] - T \), then \( f \) is R-integrable, and \( \int_0^1 f_n \, dx \to \int_0^1 f \, dx \). Theorem 3. (Dominated convergence) The statement is classical, but modified as in Theorem 2. The proof is immediate from Theorem 2.

Theorem 4. \( f \) is R-integrable if and only if \( f \) is Lebesgue integrable; and the integrals are equal. (Received June 1, 1976.)

Let \( G \) be a compact group and let \( B \) be one of the Banach algebras \( L^p(G) \), \( 1 \leq p < \infty \), or \( C(G) \), where convolution is multiplication. Let \( A \) be a closed (Jacobson) semi-simple subalgebra of \( B \). An equivalence relation is defined on the minimal idempotents in the commutant of \( A \) in \( B \). Two such idempotents are equivalent if, roughly speaking, they induce equivalent representations of \( A \). A Rudin class is an equivalence class with respect to this equivalence relation. It is proved that, if \( R \) is a Rudin class for which \( A \in R \) is not nilpotent for some (and hence all) \( q \in \mathbb{R} \), then there is a minimal central idempotent of \( A \) which is the sum of any (necessarily finite) maximal orthogonal subset of \( R \). This generalizes a theorem of Walter Rudin for compact abelian groups. (Fourier Analysis on Groups, Interscience, New York, 1962, page 232). It is shown which sums of minimal idempotents in \( B \) give minimal idempotents of \( A \). Conditions on \( A \) are considered which imply (1) \( A \) is the closed span of its idempotents (the span of the idempotents of \( A \) is the same as the trigonometric polynomials in \( A \)) or (2) \( A \) has no Rudin class \( R \) such that \( A \in R \) is nilpotent, but \( A \notin (0) \), for \( q \notin \mathbb{R} \). Condition (1) implies condition (2). If the degrees of the irreducible representations of \( G \) are unbounded, an example is given of a closed commutative semi-simple subalgebra of \( L^2(G) \) which fails (1) and (2). (Received June 7, 1976.)

Existence of fixed points of nonexpansive mappings in a space without normal structure.

Let \( C \) be a weakly compact convex subset of a Banach space \( X \). Let \( T : C \to C \) be non-expansive, i.e., \( \|Tx-Ty\| \leq \|x-y\| \) for all \( x, y \in C \). An important open question is whether \( T \) has a fixed point in \( C \). If \( X \) is reflexive and uniformly convex or, more generally, if \( X \) is reflexive and has normal structure then the answer is affirmative. (Browder, Göhde, Kirk.) In spite of vigorous efforts, the analysis has been confined to this class of spaces. We show that the answer is affirmative for a classical reflexive space, due to R. C. James, which does not have normal structure. This space is defined by renorming \( l_2^1 \) according to: \( \|x\| = \max \{\|x\|_2, \|x\|_2^{1/2}\} \) where \( \|\cdot\|_2 \) denotes the \( l_2 \) norm and \( \|\cdot\|_2 \) the \( l_2^1 \) norm. The methods employed may also be of more general interest. (Received June 7, 1976.)
A nonnegative, infinitely differentiable function \( \phi \) defined on the real line is called a Friedrichs mollifier function if it has support in \([0,1]\) and \( \int_0^1 \phi(t) \, dt = 1 \). Let \( \mathcal{M} \) denote the class of all the mollifier functions. We consider the following problem: Determine \( \Lambda_k = \inf_{\phi \in \mathcal{M}} \int_0^1 |\phi^{(k)}(t)| \, dt \), where \( \phi^{(k)} \) denotes the kth derivative of \( \phi \). This problem has applications to monotone polynomial approximation problem as shown by this author in "Moduli of Monotonicity with Applications to Monotone Polynomial Approximation," SIAM J. Math. Anal. 7 (1976), 117-130. The problem is reducible to three equivalent optimization problems—a nonlinear programming problem, a problem on the functions of bounded variation and an approximation problem. The principal result of this article shows that \( \Lambda_k = k!2^{2k-1} \), for all \( k = 1, 2, \ldots \). The numerical values of the optimal solutions of the three problems are obtained as a function of \( k \). (Received June 7, 1976.)

Theorem Let \( \mu \) be a probability measure on the closed unit disk \( D = \{ z : |z| \leq 1 \} \) with the property that \( \int_D z^n \, d\mu(z) = 0 \), for all positive integers \( n \). Then the restriction of \( \mu \) to the boundary of \( D \) is absolutely continuous and the restriction of \( \mu \) to the interior of \( D \) is a Carleson measure.

In view of Bram's Theorem, we get the following result.

Corollary If \( T \) is subnormal, \( ||\mu|| = 1 \), and \( T \) has a cyclic vector \( e \) such that \( T^*e = 0 \), then there is a probability measure \( \mu \) on \( D \) which satisfies the conclusion of the theorem such that \( T \) is unitarily equivalent to the operator \( F(z) = zF(z) \) acting on \( H^2(\mu) \), the closure of the polynomials in \( L^2(\mu) \).

An analogous result holds if any nonzero vector, not necessarily \( e \), is in the kernel of \( T^* \). The proof of the Theorem uses the method of "sweeping out a measure" to the boundary.

(Received June 8, 1976.)

Applied Mathematics

(65, 68, 70, 73, 76, 78, 80-83, 85, 86, 90, 92-94)

Nitrogen Transformations in Soil During Leaching - A Theoretical solution.

Three transport simultaneous & coupled partial Differential equations describing the movement and simultaneous oxidation involving sequential reaction by microbial and/or chemical means:

\[
\text{Urea} \quad (c_1) \rightarrow \text{NH}_4^+ (c_2) \rightarrow \text{NO}_3^- (c_3) \rightarrow N_2
\]

(or even \( \text{NH}_4^+ \rightarrow \text{NO}_2^- \rightarrow \text{NO}_3^- \rightarrow N_2 \), if desired) under practical boundary and initial conditions have been solved using Laplace Transform Theory. The transport of Nitrogenous materials in soils containing living micro-organisms responsible for the transformation of these materials plays an important role in both crop production and the quality of surface and ground-water supplies. The work is based on C.Misra, D.R.Nielson, and J.W.Biggar's paper Soil Sci. Soc. Amer. Proc; Vol. 38, 1974, pp. 289-293. (Received March 2, 1976.)

Higher Order Finite Difference Schemes for two Dimensional Heat Equation.

By taking the heat equation in two space variables and using a cubic spline technique,
general implicit finite difference schemes of LOD, ADI and SOD types have been
derived and examined for their stability. For particular values of the parameter,
these schemes turn out to be of higher order and are found to be unconditionally
stable. The formulae due to Fairweather and Mitchell, Peceman and Rachford, Hubbard
and a general explicit scheme are obtained as special cases of the general implicit
schemes. Relative merits of these formulae are discussed and illustrated by using
numerical test examples. (Received May 3, 1976.)

R.A. Mollin & J. McKay, Concordia University, Computer Science Dept., 1455 de Maisonneuve
West, Montreal, P.Q., Canada. Exact Computation of the Rational Canonical Form.

Remarks on the use of a computer for exact methods in linear algebra are followed by a mathematical
description of the rational canonical form of a matrix. A worked example and a complete
APL program (written in easy to follow APL) are presented.

We give an algorithm for computing the Frobenius-Perron normal form of a matrix (also known as
the rational canonical form). In particular we answer the question: Given $X_1$, $X_2$ square matrices
over the integers, does there exist a matrix $A$ such that $A^{-1}X_1A=X_2$? We ask the question for the
cases (i) in which $A$, $A^{-1}$ are matrices over the rationals and (ii) over the integers. The answer
is found by (as usual in this type of problem) converting both $X_1$ and $X_2$ to the aforementioned
canonical form. The given algorithm also converts a general square polynomial matrix to Smith
normal form. (to appear: SIGSAM Bulletin) (Received May 6, 1976.)

E.A. Galperin, NP Research, P.O.Box 24, Station 'CDN', Montreal, Quebec,
Canada. Innovations approach to nonlinear asymptotical observation .
Preliminary report .

Given a system $dx/dt=f(x, u(x,t), t), t>0$, $x \in \mathbb{R}^n$, $x(0)$ unknown, with the output
$y(t)=h(x, u(x,t), t), y \in \mathbb{R}^m$, $m \leq n$, measured on its trajectories, assume that by means of a
non-asymptotic technique (see Engineering Cybernetics, No.1, 1972, pp.165-172) there
is found an estimate $z(0)$ of the unknown $x(0)$, such that $\|z(0)-x(0)\| < \epsilon^0$.

Theorem. If such a function $v(s,t)$ can be chosen that the unperturbed motion $w=0$
of the system
$dw/dt=A(t)w+p(w,t)-v[H(t)w+q(w,t), t]$, $t\geq 0$ (1)
is asymptotically stable, then the model
$dz/dt=f(z, u(z,t), t) + v[H(t)w+q(w,t), t]$, $t\geq 0$, $z(0)$ found (2)
delivers a nonlinear asymptotic observer with the property: $\|z(t)-x(t)\| < \delta$, $z(t) \rightarrow x(t)$
as $t \rightarrow \infty$. Corollary. The model (2) with $v=0$ (a duplicate model) can serve as an obse­
rver if and only if the unperturbed motion of the original system is uniformly asymptotically
stable. Here $\delta, \epsilon^0$ represent the accuracy of observation, $w(t)=x-z$, and
terms in (1) are perturbations of $f(\cdot)$ and $h(\cdot)$ around the unperturbed motion $w=0$ .
(Received May 6, 1976.) (Author introduced by Professor L. Cesari.)

Peter Goorjian, Computational Fluid Dynamics Branch, NASA-Ames,
Moffett Field, Calif. 94035. The Euler Poisson Darboux Equation in
General Relativity.
The Euler Poisson Darboux equation is
$v_{tt} + v_{xx} + k \frac{k}{t} v_t = 0$, where $k$ is a real
parameter. Of particular interest in relativity are the Kasner spacetimes
ds^2 = -dt^2 + t^{2p_1} dx^2 + t^{2p_2} dy^2 + t^{2p_3} dz^2$, where $p_1 + p_2 + p_3 = 1$.
They are prototypes for the mixmaster, i.e. Khalatnikov-Lifshitz generic space­
times, which have a time singularity, i.e. a big bang. Maxwell's equations with
no sources in a curved spacetime are $F_{\mu\nu}; \mu = 0$, where the Faraday tensor
is given in terms of the vector potential $A_\mu$ by $F_{\mu\nu} = A_\nu;_\mu - A_\mu;_\nu$.
Theorem: For electromagnetic plane wave perturbations in a Kasner cosmology,
Maxwell’s equations reduce to the Euler Poisson Darboux equation for the one component of the vector potential $A_\mu$ which is not identically zero.

(Received May 12, 1976.)


Def. 1. Logine L is automaton, each of whose logans (logical organs), at time $t-1$, formulates response $R$ from matrix $M$ over lattice logic $B$ ("Matrices for free lattice logics", NOTICES, 22(1975), A647). A typological- or t-logine has usual logans (disj., etc.); ordinal- or o-logine also has saturable adjunction organ (analogue: powers of factors), for degrees of $R$ intensity, all logans over $o$-matrices. For $b$ atomic response basons, $C(R) = n/2^b = \langle n_1/2^b, n_2/2^b, \ldots \rangle$ is cyberility measure of $R$ (scalar or vector, according to matrix) iff $n_i (n_j)$ measures ones in $M(R)$ (in $i$th column of $M(R)$). Th. 1. (Gen. Boolean Alg.) Each $C(R)$ can be written as disjunction of conjuncted polynomials of adjunctioned powers of atomic terms. Cor. Each $o$-logine can be constructed from associated organ generators. Th. 2. If $U, N, T$ denote, resp., universal, null, transitory responses, $C(U) = 1$, $C(N) = 0$, $0 < C(T) < 1$. Def. 2. $W(i,j)$ is (scalar) wiener function iff $W(i,j) = (\eta_i \Rightarrow \eta_j) \Rightarrow (\eta_i \Rightarrow \eta_j)$, for goal-hypotheses $G_i$, goal-predictors $P_i$. Lemma. $W(i,j) = \eta_j \Rightarrow \eta_i$. Th. 3. $C(W(i,j)) = 1 - \frac{1}{2^j} + \frac{1}{2^j}$, $i,j \geq 1$. Th. 4. $C(W(i+1,j)) \leq C(W(i,j))$, $i,j \geq 1$; $C(W(i+k,j+1)) \leq C(W(i,j))$, $i,j,k \geq 1$. Def. 3. For $C(W(i,j))$, measure of some goal-hypothesis-prediction program (GHP) of logine $L_0$, let $C(W(i,j+1))$ be resulting measure if one more prediction were derived and confirmed. Let $d$ be numerator of $C(W(i,j+1)) - C(W(i,j))$. Then $C(L_0; GHP; W)$ is prediction potential iff $C(L_0; GHP; W) = d + 1$, and $I(L_0; GHP; W)$ is predictive information iff $I(L_0; GHP; W) = \log_2 (L_0; GHP; W)$. Th. 5. $C(L_0; GHP; W) = 2^i$; $I(L_0; GHP; W) = i$. (Received May 13, 1976.)

76T-C48 G. T. DIDERRICH, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1. _Continued fractions_ and the fundamental equation of information. Preliminary report.

It was proved (Abstract 726–39–6, these Notices 22(1975), A-561) (to appear in Information and Control) that an information function $f$ bounded on a subinterval of the unit interval must coincide with Shannon’s measure of entropy on a two event space. This answered some questions in Aczél and Daróczy, "On Measures of Information and Their Characterizations", Academic Press, New York, 1975. The argument provided there was by reduction to Z. Daróczy’s “small for small probabilities” theorem and it involved number theoretic results of either P. Erdős or I. Kátai or N. I. Feldman (A. Baker). Here we give another proof along the lines indicated in Remark 3 of Information and Control 29(1975), 149–161. It turns out that an $n$-th rank interval $[a_1 \ldots a_n]$ from the metrical theory of continued fractions allows one to track the flight of the sequences $x_k$ for $a_1 + \ldots a_n = 1$ time units. Thus, by the author’s iteration argument, $f$ becomes approximately locally Lipschitz a.e., whence $f$ must be measurable, thereby reducing the problem to Lee’s (or Tverberg’s) theorem. (Received May 26, 1976.)

76T-C49 WITHDRAWN


An unsteady analysis is made of the hydromagnetic convective flows in an electrically conducting rotating viscous fluid bounded by an infinite vertical non-porous or porous plate due to the transient...
heating and motion of the plate in the presence of a uniform magnetic field normal to the plate.

Initially, the fluid and the plate are in a state of rigid body rotation with uniform angular velocity \( \Omega \) about the z-axis normal to the plate; and are at the same temperature. A uniform magnetic field \( B_0 \) normal to the plate is applied everywhere in the fluid system. From some instant of time, the plate and its temperature start to oscillate with constant frequency. The temperature and the hydromagnetic convective flow field are determined for both non-porous and porous plate configurations.

It is shown that the ultimate flow field and the associate boundary layers are significantly modified by the thermal effects, magnetic field, rotation and suction or blowing. (Received June 7, 1976.)

**Geometry (50, 52, 53)**

HEIKO HARBORTH, Technische Universität Braunschweig, D 3300 Braunschweig, West Germany. Nonperiodic Tesselations of the Plane.

Consider polygons which tile the plane strict (any common point of the boundaries of two tiles is either a vertex of both or of neither), but not periodically. R.M. Robinson (Inventiones Math. 12 (1971), 177 - 209) found six tiles with this property. R. Penrose (Talk of R. K. Guy, Oberwolfach, 16.7.1975) found two tiles yielding only nonperiodic tesselations. However, not all sides of equal length are allowed to be juxtaposed. - We describe infinitely many pairs of equilateral polygons which tesselate only monperiodically without any further restrictions. (Received April 20, 1976.)


Let \( M^{n+1} \) be a smooth compact Riemannian manifold of constant sectional curvature \( c \), canonical volume form \( \nu \) and total volume \( \int_M \nu = v \). Suppose \( M \) has a \( C^2 \) codimension-one foliation \( \mathcal{F} \). Then each leaf has an induced Riemannian metric and a scalar Gaussian curvature function \( K \). (In the case \( n \) odd, \( K \) is well defined only up to a global choice of sign. See M. Spivak, A Comprehensive Introduction to Differential Geometry, volume IV, p. 104.)

**Theorem.** Define \( \overline{K} = \frac{1}{vM} K \nu \). Then \( \overline{K} \) is independent of \( \mathcal{F} \) in the cases

(a) \( n \) is even, or
(b) \( n \) is odd and \( c = 0 \) (in this case \( \overline{K} = 0 \)). Hence in these cases \( \overline{K} \) is purely a function of \( n \) and \( c \). (Received April 23, 1976.) (Author introduced by W. G. Dwyer.)

Larry Graves, Brown University, Providence, Rhode Island 02912. Null Curves and Flat Lorentz Surfaces in \( \mathbb{R}^3 \). Preliminary report.

A null curve \( \mathbb{R}^3 \) is a curve \( x(s) \) whose tangent vectors are non-zero and have zero length with respect to the metric \( \langle x, x \rangle = -x_0^2 + x_1^2 + x_2^2 \) on \( \mathbb{R}^3 \). A (null) frame for \( x(s) \) is an ordered triple \( \langle A(s), B(s), C(s) \rangle \) such that \( \frac{dx}{ds} \) is a positive scalar multiple of \( A(s) \) and \( \langle A, A \rangle = \langle B, B \rangle = \langle C, C \rangle = 0 \), \( \langle A, B \rangle = -1 \), and \( \langle C, C \rangle = 1 \). A null curve together with a frame is a framed null curve. Framed null curves and transformations of frames are studied, and canonical frames are derived. If \( x(s) \) is a null-curve with such \( \langle A(s), B(s), C(s) \rangle \), then \( f(u,v) = x(v) + uB(v) \) parametrizes a Lorentz surface in \( \mathbb{R}^3 \), called the \( B \)-scroll. The class of canonically framed null curves for which the associated \( B \)-scrolls are flat (i.e., globally isometric to \( \mathbb{R}^3 \) ) is characterized. These \( B \)-scroll immersions \( \mathbb{R}^2 + \mathbb{R}^3 \) are isometric immersions whose relative nullity spaces are light lines, and which are not cylinders over a plane curve in the sense analogous to the Hartman-Wirenberg theorem.
Indeed, I can show that all isometric immersions into isometric immersions $\mathbb{L}^2 \times \mathbb{L}^3$ whose relative nullity spaces are light lines are B-scroll immersions.

(Received May 6, 1976.)

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With the terminology of Gluck, (Springer Lecture Notes #438, Geometric Topology, pp. 225-39) and Laman (J. Engineering Math 4 (1970), 331-340) the classical result of Cauchy states: "a strictly convex polyhedron with rigid faces is rigid." A convex polyhedron is one in which all edges and faces lie in the boundary of the convex hull, and a planar vertex is one for which all the edges at that vertex are coplanar. By an extension of Alexandrov's proof, and other work on plane rigidity we have: Theorem 1:

A convex polyhedron with infinitesimally rigid faces and no planar vertices is infinitesimally rigid.

Theorem 2: If in any convex polyhedron we replace each face by a framework which is infinitesimally rigid within its plane, and the new structure has no planar vertices, then it is infinitesimally rigid. These results have applications in architecture for shell constructions in which the skeleton of a strictly convex polyhedron is erected with faces rigidified by tense cables or other plane frame works as well as prefab constructions with hinged panels. (Received June 7, 1976.)

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**Logic and Foundations (02, 04)**

**76T-D18** Walter J. Whiteley, Champlain Regional College, St. Lambert, Quebec, Canada. Rigid Polyhedra with plane-rigid faces.

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We solve here problems of Keisler; e.g. problem 17 in Chang and Keisler book.

**Th. 1**: For cardinals $\lambda, \mu > \aleph_0$, for some $\aleph_1 < \omega$ $\mu = \Pi \aleph_1 / D$ iff $\mu = \aleph_0 \cdot \aleph_1 \leq 2^{\lambda}$.

**Th. 2**: Over each $\lambda > \aleph_0$ there is a regular non-good ultrafilter $D$ over $\lambda$ such that $\aleph_0 < \mu \leq \aleph_1 / D$ $\mu = 2^{\lambda}$. 

**Th. 3**: There are non-minimal nor-maximal countable theories in Keisler order.

**Th. 4**: If $\aleph_k = \aleph_k^{\aleph_0} \leq 2^{\lambda}(\lambda < \omega)^*$ then for some regular ultrafilter $D$ over $\lambda$, $\{ \Pi \aleph_1 / D : \aleph_1 < \omega \} = \omega \cup \{ \aleph_k : \lambda < \omega \}$. Let $\lambda = \mu(< \omega)$ be the minimal cardinality of a subset of $\{ \nu \in \aleph_0 / D : \aleph_0 < \omega \}$ for $\nu < \omega$. 

**Th. 5**: If $\aleph_k \leq \lambda$,

$\aleph_k \leq \aleph_k \leq 2^{\lambda}$ are regular $(\lambda < \omega)$ then for some regular ultrafilter $D$ over $\lambda$,

$f(\aleph_k, D) = \aleph_k$. 

**Th. 6**: For regular $\mu \leq \lambda$, there is a $\mu$-good, regular ultrafilter over $\lambda$ which is not $\mu^+$ good. We combine 4, 5, 6 and generalize to infinitely many $\aleph_k$.

**Th. 7**: Over $\lambda = 2^{\aleph_0}$ **unbounded from below.** (Received April 16, 1976.)

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**76T-D43** SAHARON SHELAH, Hebrew University of Jerusalem, Jerusalem, Israel. Ultraproducts of finite cardinalities and Keisler order.

We solve here problems of Keisler; e.g. problem 17 in Chang and Keisler book.

**Th. 1**: For cardinals $\lambda, \mu > \aleph_0$, for some $\aleph_1 < \omega$ $\mu = \Pi \aleph_1 / D$ iff $\mu = \aleph_0 \cdot \aleph_1 \leq 2^{\lambda}$.

**Th. 2**: Over each $\lambda > \aleph_0$ there is a regular non-good ultrafilter $D$ over $\lambda$ such that $\aleph_0 < \mu \leq \aleph_1 / D$ $\mu = 2^{\lambda}$. 

**Th. 3**: There are non-minimal nor-maximal countable theories in Keisler order.

**Th. 4**: If $\aleph_k = \aleph_k^{\aleph_0} \leq 2^{\lambda}(\lambda < \omega)^*$ then for some regular ultrafilter $D$ over $\lambda$, $\{ \Pi \aleph_1 / D : \aleph_1 < \omega \} = \omega \cup \{ \aleph_k : \lambda < \omega \}$. Let $\lambda = \mu(< \omega)$ be the minimal cardinality of a subset of $\{ \nu \in \aleph_0 / D : \aleph_0 < \omega \}$ for $\nu < \omega$. 

**Th. 5**: If $\aleph_k \leq \lambda$,

$\aleph_k \leq \aleph_k \leq 2^{\lambda}$ are regular $(\lambda < \omega)$ then for some regular ultrafilter $D$ over $\lambda$,

$f(\aleph_k, D) = \aleph_k$. 

**Th. 6**: For regular $\mu \leq \lambda$, there is a $\mu$-good, regular ultrafilter over $\lambda$ which is not $\mu^+$ good. We combine 4, 5, 6 and generalize to infinitely many $\aleph_k$.

**Th. 7**: Over $\lambda = 2^{\aleph_0}$ **unbounded from below.** (Received April 16, 1976.)

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**76T-D44** PAUL BANKSTON, McMaster University, Math. Dept., Hamilton, Ontario Canada. On the Number of Boolean Algebras.

For any pair $\kappa, \lambda$ of cardinals let $B(\kappa, \lambda)$ be the number of isomorphically distinct atomless Boolean algebras of cardinality $\kappa$ and stability number $\lambda$. We prove the Theorem (GCH):

$$B(\kappa, \lambda) = \begin{cases} 
\kappa^+ & \text{if } \kappa > \omega, \lambda \in \{ \kappa, \kappa^+ \} \\
1 & \text{if } \kappa = \omega, \lambda = \omega_1 \\
0 & \text{otherwise}
\end{cases}$$

The method of proof is entirely topological and answers dually the question of how many homeomorphism types of discontinua (= perfect Boolean spaces) there are of a given cardinality and weight. (Received April 19, 1976.)
There is a uniform and explicit definition of any Boolean ring $B$ in its linear group $\text{GL}(B)$. Let $U$ be a recursively presented vector space over a recursive field with a dependence algorithm. Morphisms of $B$. (Received May 17, 1976.) (Author introduced by Professor G. Sabbagh.)

Produced via a priority argument in R.E. *Holographic logic*, equivalent (resp. isomorphic) only if $B$ and $B'$ are two Boolean rings their linear groups $\text{GL}(B)$ and $\text{GL}(B')$ are "almost" induced by automorphisms of a $\text{GL}(B)$. (Received April 23, 1976.)

**76T-E46**

AMI LITMAN and SAHARON SHELAH, The Hebrew University, Jerusalem, Israel. Independence of the Gap One Conjecture From G.C.H. Preliminary report:

Theorem: $ZFC + \text{G.C.H.} + (N_1, N_o) \not\subset (N_{o+1}, N_o)$ is consistent if $ZFC + \text{there is a supercompact cardinal is consistent. (Remark: Jensen proved from } ZFC + V = L \text{ that } (N_1, N_o) \to (N_{o+1}, N_o) \text{ for every } \alpha.)$ (Received April 28, 1976.)

**76T-E47**

MIRIAM LIPSCHUTZ-YEVICK, Rutgers University, New Brunswick, New Jersey. Holographic recognition that **results from * by substitution of $\bar{m}$ for the free variable $x_1$.**

Referring to Mendelsohn, "Introduction to Mathematical Logic", Van Nostrand, p. 143 and Lipschutz-Yevick, "Holographic logic", Pattern Recognition 7(1975), p. 211: consider the Godel patterns of the formal expressions $B_1(x_1, x_2)$ and $B_1(m, x_2)$. Let $l_1$ be the length of the first pattern and let $l_1$ be its length up to the occurrence of $x_1$ (a in the notation of the above). Filtering the $G$ pattern of $B_1(m, x_2)$ in the hologram of $B_1(x_1, x_2)$ will produce a correlation spot of intensity $l_1$ at $(0, l_1)$ and a correlation spot of intensity $l - l_1 - 1$ at $(0, l_1 + m + 1)$. This indicates that $0^{m+1}$ (m primes) has been substituted for $x_1$ (since the only permissible substitution for variables is of numerals). (Received May 19, 1976.)

**76T-E48**

Chantal Berline, Université de Paris 7, 2 place Jussieu, 75005 Paris, France. Etude du groupe linéaire d'ordre 2 d'un anneau de Boole.

There is a uniform and explicit definition of any Boolean ring $B$ in its linear group $\text{GL}_2(B)$. Hence, if $B$ and $B'$ are two Boolean rings their linear groups $\text{GL}_2(B)$ and $\text{GL}_2(B')$ are elementary equivalent (resp. isomorphic) only if $B$ and $B'$ are. We give some other model-theoretic consequences.

With a new definition of $\text{GL}_2(B)$ we show that the class $\text{GL}_2(B) / B$ is a Boolean ring is an elementary class and that the automorphisms of a $\text{GL}_2(B)$ are "almost" induced by automorphisms of $B$. (Received May 17, 1976.) (Author introduced by Professor G. Sabbagh.)

**76T-E49**

Jeffrey B. Remmel, University of California, San Diego, Department of Mathematics, C-012 La Jolla, California 92039. Degrees of R.E. Subspaces with Nonextendible Bases.

Let $U$ be a recursively presented vector space over a recursive field with a dependence algorithm. (See Notices, April 1975, Abstract 723-E6.) A r.e. independent set $B$ is nonextendible if there is no r.e. independent set $I \supset B$ such that $I - B$ is infinite. G. Metakides and A. Nerode produced via a priority argument in R.E. Vector Spaces (to appear) a r.e. vector space $V \subseteq U$ such that $U \mod V$ is infinite dimensional and every r.e. basis of $V$ is nonextendible. We modify their construction with a permitting argument to prove the following. Theorem: Let $\delta$ be any non-zero r.e. degree. There is a r.e. subspace $V \subseteq U$ of degree $\delta$ such that $U \mod V$ is infinite dimensional and every r.e. basis of $V$ is nonextendible. Since every r.e. subspace $V \subseteq U$ has a r.e. basis $B$ such that $B \equiv V$, and Metakides and Nerode have produced recursive r.e. independent sets which are nonextendible it follows that Corollary: For every r.e. degree $\delta$, there is a r.e. independent set $B$ of degree $\delta$ which is nonextendible. (Received May 27, 1976.)

**76T-E50**


In this paper, we present a set of axioms for an extension ORD* of the
Theory ORD (See Abstract 72T-E15, these Notices 19(1972), A-331). The following results are proven about ORD*:

Theorem 1. Every standard model of ORD* is of the form $A = \langle A, E_A \rangle$, where $A = \aleph_0$ or $A \in \aleph_0$ and where $\aleph_0$ is the class of all ordinals.

Theorem 2. Let $\mathcal{A} = \langle A, R \rangle$ be any well-founded extensional model of ORD*. If $A$ is a set or if $\text{Seg}(A) = \{ b \in A : \langle b, a \rangle \in R \}$ is a set for every $a \in A$, then there exists a standard model $\mathcal{A}^* = \langle A^*, E_{A^*} \rangle$ of ORD* isomorphic to $\mathcal{A}$.

Corollary: Any two extensional well-founded models of ORD* of the same cardinality are isomorphic.

Theorem 3. ORD* is a conservative extension of ORD. (Received June 1, 1976.)


Theorem: If $\mathcal{A} = (A, \prec)$ is an $\mathcal{K}_0$-categorical partially ordered set of width 2, then $\text{Th}(\mathcal{A})$ is finitely axiomatizable. (Received June 3, 1976.)

76T-E52 Friedrich Hebeisen, Mathematisches Institut, Helbelstr. 40, D78 Freiburg, West Germany. Decidability of the "almost all" theory of WT-degrees. Preliminary report.

WT-degrees were introduced as a generalization of degrees of unsolvability to partial functions by L.P. Sasso (Journ. of Symbolic Logic, 40, pp 130-140). Stillwell showed the decidability of the "almost all" theory of degrees of unsolvability with ' (jump), $\cup$ (join), and $\cap$ (meet) (Journ. of Symbolic Logic, 37, pp 501-506).

We modified Stillwell's proof and showed that the "almost all" theory of WT-degrees with $\cup$ and $\cap$ (but without jump) is also decidable. Our proof does not work, if the theory is extended by the jump for WT-degrees or if the theory is the analogous theory for T-degrees or e-degrees (also considered by Sasso).

(Received May 21, 1976.) (Author introduced by Walter Felscher.)


Simulating 2-register machines by tag systems we can answer a question left open by Aanderaa and Belsnes (JSL 1971, pp 229-239). Theorem: To every r.e. set $a$, $a \not\in \mathcal{N}$, there exists a tag system $T$ with deletion number 4 such that the word problem and the halting problem of $T$ are many-one equivalent to $a$. By slight modifications of the proof this result may be extended to each given deletion number which is even and greater than 2.

(Received June 3, 1976.) (Author introduced by Dr. Egon Böger.)


A model $\mathfrak{G}$ is minimal (strictly minimal) if $3 < \mathfrak{G}$ implies $\mathfrak{G} \not\cong \mathfrak{G}$ ($\mathfrak{G} = \mathfrak{G}$). Let $G$ denote a countable torsion free abelian group. For each prime number $p$, let $T_f(p, G)$ be the dimension of $G/pG$ as a vector space over $\mathbb{F}_p$. Theorem 1: The theory $\text{Th}(G)$ has an elementary prime model iff for almost all primes $T_f(p, G) = 0$ and all non-zero $T_f(p, G)$ are equal. Theorem 2: $\text{Th}(G)$ has a strictly minimal model iff the $T_f(p, G)$'s are bounded by some natural number. Theorem 3: There is a torsion free abelian group $3$ such that $\text{Th}(G)$ has exactly one non-prime minimal model. Theorem 4: $G \equiv \mathbb{Z}$ is minimal iff $G$ is strictly minimal. Theorem 4 answers a problem posed by Baldwin, Blass, Glass and Kueker. (Received June 7, 1976.) (Author introduced by Professor O. H. Kegel.)
We give two examples. $T_0$ has nine models and a nonprincipal 1-type which contains infinitely many two types. $T_1$ has four models and an inessential extension $T_2$ having infinitely many models. (Received June 8, 1976.) (Author introduced by A. H. Lachlan.)


Let $\Phi$ be any computational complexity measure and let $A_\Phi = \{ \exists m \in \mathbb{N} : \exists j, |j| = m \text{ and } \phi(j) > m \}$ where $|j|$ denotes the length of $j$. Any infinite subset of $A_\Phi$ will be called a busy beaver set. The following are very easy to verify. $A_\Phi$ is strongly dense simple, effectively speedable, autoreducible. $A_\Phi$ is not finitely strongly hypersimple. $A_\Phi$ is uniformly introreducible. Every r.e. superset of $A_\Phi$ is autoreducible and is not finitely strongly hypersimple. Every co-r.e. subset of $A_\Phi$ is uniformly introreducible. There exists a busy beaver (which can be simply described) set $B_\Phi$ such that $B_\Phi$ is retraceable and $\overline{B_\Phi}$ is strongly uniformly hypersimple. From these properties result many other interesting properties of the sets $A_\Phi$ and $B_\Phi$ all of which are easy to verify directly. (Received June 7, 1976.)

Statistics and Probability (60, 62)

James W. Carter, 853 Carillo, San Gabriel, California, 91776. An Axiom of Information/Entropy. If $H$ is a continuous function on $(\mathbb{R}^+)^N$ into $\mathbb{R}$ which satisfies the following two conditions: 1) $H(p_1, p_2, \ldots, p_n) = \sum H(1, l, \ldots, l, p_i, l, \ldots, l, l)$ where $H(l, \ldots, l, l) = H(l, \ldots, l, l)$. 2) $H(pq) = (\sum q_i) H(p, q) + (\sum p_i) H(q, p)$, where $P = (p_1 \ldots, p_q, \ldots, p_m, \ldots, p_n, \ldots, p_m)$ and $nm = N$, then $H$ is an information function. Axiom: Information function s exist. Theorem: $H(p_1, \ldots, p_n) = \sum p_i \log_b p_i$, where $b = 2$, if $H(a) = 1$. (Received May 24, 1976.)


Let $F$ be the class of one dimensional distribution functions corresponding to the absolutely continuous probability laws. Let $X_1, \ldots, X_n$ be i.i.d. stochastically independent random variables each having the same distribution function $F$ in $F$, and let $E(y) = \bigcup_{r=1}^{N} [x_{rs}] - \bigcap_{r=1}^{N} [x_{rs}]$. The probability of the symmetric difference $E(y)$ defines a distance $D_n(y)$ given by $1 - (1 - F(y))^n - (F(y))^n$. Let $F_D_n(Y) = F_n(Y)$. It is shown that (i) $H_n(\delta)$, which is distribution-free with respect to $F$, can be constructed for each $n (n \geq 2)$, and (ii) $D_n(Y) - (1 - 2^{-n})$ converges to zero in probability. Result (i) is part of statistical decision structures relating the sample space of $X$ to that of $Y$, and (ii) implies the existence of the smallest sample size $n$ needed to ensure with a fixed probability $\gamma_0$ that the chance of the event $E(Y)$ occurring is at least $\delta_0$. (Received June 7, 1976.)

Topology (22, 55, 57, 58)


Professor G. Darbo introduced the concept of weighted maps and defined a singular homology theory on the category of Hausdorff spaces and weighted maps. The
The author defined simplicial homology theory on the category of simplicial complexes and weighted maps and proved its uniqueness on it in two recent papers.

In this paper, the author proves a simplicial approximation theorem in the category of triangulable spaces and weighted maps. (Received March 15, 1976.)

A topological space $X$ is supercompact provided there is a sub basis $S$ for the open subsets of $X$ so that any cover of $X$ by elements of $S$ has a two element subcover. This notion was originally due to de Groot, who conjectured that all compact metric spaces are supercompact. O'Connor claimed to have proven this conjecture, but his proof was only valid in the case when the space was perfect (every point is a limit point). This was an important and difficult special case. However, his work was apparently abandoned, and a new proof of the conjecture was supplied by Strok and Szymanski in a paper that is difficult reading. The purpose of our paper is to fill in the gap in O'Connor's work. We prove that if perfect compact metric spaces are supercompact, then so are all compact metric spaces. (Received April 26, 1976.)

By a "generalized $n$-manifold" we mean a compact euclidean neighborhood retract $X$ that satisfies the "homology $n$-manifold" condition, i.e., $H_i(X, X - x; Z) = H_i(R^n, R^n - 0; Z)$ for all $i \in Z$ and $x \in X$. The term "approximate fibration" was introduced recently by Coram and Duvall and is being studied by several authors. (See Notices AMS 23(Feb.1976),Abs.732-G5,p.A-308) R. Goad is studying a similar concept (ibid.,Abs.732-B2,p.A-307). There is an exact sequence of homotopy groups associated with an approximate fibration like the one for a fibration (except that shape groups of the fiber are used in place of ordinary homotopy groups). Theorem. If $X$ is a generalized $n$-manifold then, for sufficiently large $k$, there exist a closed orientable topological manifold $W^{n+k}$ and an approximate fibration of $W^{n+k}$ onto $X$ with fiber having the shape of $S^k$. (Conversely, the condition that $X$ be a homology manifold is necessary for the existence of the blow up.) The proof uses recent results of Miller, West, Edwards and Siebenmann together with a homotopy-theoretic analysis of the retraction of a mapping cylinder neighborhood onto a generalized manifold core. (Received April 27, 1976.)

The obstruction to mapping smooth to continuous cohomology of spaces with two topologies.

In a recent paper, the author gave an example of a singular foliation on $\mathbb{R}^2$ for which it is impossible to map the deRham cohomology $T_{DR}$ to the continuous singular cohomology $T_C$ (in the sense of Bott and Haefliger's continuous cohomology of spaces with two topologies) compatibly with evaluation of cohomology classes on homology classes. In this paper the obstruction to mapping $T_{DR}$ to $T_C$ is pinpointed by defining the whole family of cohomology theories $T_{kmn}$ which mediate between the two. It is shown that the obstruction vanishes on non-singularly foliated manifolds. These cohomology theories are extended to Haefliger's classifying space $(B^q \to B^q)$, with its germ and jet topologies, by using the machinery of differentiable spaces, as developed by J. W. Smith and K. T. Chen. The author proposes that certain of the $T_{kmn}$ be used instead of $T_C$ to study Bott and Haefliger's conjecture that the continuous cohomology of $(B^q \to B^q)$ equals the relative Gel'fand-Fuks cohomology $H^*(\mathcal{A}_q, \mathcal{O}_q)$. (Received May 6, 1976.)

M-functions were introduced independently by G. Darbo and R. Jerrard as a generaliza-
tion of continuous functions between topological spaces. They are weighted, finitely-valued functions with a property corresponding to that of usual continuity.

M-functions have many of the important properties of continuous functions; in earlier papers an m-homotopy theory and an m-homology theory have been defined. M-homology theory is isomorphic to simplicial homology (on compact polyhedra). However m-functions induce more homomorphisms between homology groups than do continuous functions. The major result of this paper is that m-homotopy theory is the usual homology theory for polyhedra. Hence each element of a homology group can be represented by an m-function. (Received May 7, 1976.)


The box topology on a Cartesian product of topological spaces is the one whose open sets are exactly the sets open in each co-ordinate. For M a model of set theory, let $\mathcal{B}$ be the algebra adding $\lambda^\omega$ many Cohen reals. Then in $\mathcal{B}$, $\Box(\omega_1(\omega_1+1))$ is paracompact if each $\alpha_1$ is countable, hence by a theorem of Kunen $\Box^\omega(\omega_1+1)$ is paracompact in $\mathcal{B}$. Since if $\kappa > \omega_2$ in $\mathcal{B}$ there are no $\lambda$-scales, the conjecture of Williams that $\Box^\omega(\omega_1+1)$ is paracompact $\Rightarrow \exists \lambda$ is shown to be false. (Received May 11, 1976.)


Let K be the 2-complex formed by sewing two 2-cells to the wedge of two circles a and b by the words $a^0b^0$ and $a^1b^1$ respectively. Then $K \times I$ collapses. The techniques appear to extend to the more general $a^n b^m$, $a^n b^m$. In fact, there is some feeling that they may handle the arbitrary case $a^n b^m$, $a^n b^m$. (Received May 10, 1976.)


Suppose L is a finite CW complex, $G = \pi_1 L$, and $0 \neq \sigma_0 \in \text{Wh}(G)$. Let $A(\sigma_0, G)$ be the assertion: "If $H = [G*F(x_1, \ldots, x_n)]/(r_1 = \ldots = r_n = 1)$ where 1) $r_i = n_{i,k}^{-1} x_i^{-1}$, $2) (g_{ik} \in G, n_{ik} \in \mathbb{Z}, x_{ik} \in \{x_1, \ldots, x_n\})$ and 2) the matrix $A = (\rho_{ij})$ with $\rho_{ij} = \pm n_{i,j}^{-1}$ is an invertible matrix with Whitehead torsion $\tau(A) = \tau_0$ then $H$ is a proper split extension of $G$." THEOREM 1: $A(\sigma_0, G)$ is true $\iff \exists$ no homotopically trivial pair $(K, L)$ such that $\dim(K-L) = 2$ and $\tau(K, L) = \tau_0$. Motivated by Theorem 1, O.S.Rothaus has shown (Bull.A.M.S.82 (1976), 281-283) that $A(\sigma_0, G)$ is sometimes true. These results imply: THEOREM 2: $\exists$ group $G$ and an element $\tau_0 \in \text{Wh}(G)$ $\exists$ no h-cobordism $(W, M, M')$ with $\tau(W, M) = \tau_0$ can be presented without handles of index $\geq 3$. THEOREM 3: For every $n \geq 3$ $\exists$ compact contractible polyhedron $X^n \ni X^n \times I$ is not PL collapsible. (Received May 17, 1976.)

76T-G84 Teodor C. Przymusiński, Institute of Mathematics of the Polish Academy of Sciences, Warsaw. Products of paracompact spaces.

The following genuine examples are constructed.

**Example 1.** A (separable and first countable) paracompact space X such that $X^2$ is (collectionwise) normal but not paracompact.

**Example 2.** A (separable and first countable) paracompact space X and a separable metric space M such that $X \times M$ is not subparacompact. (Received May 21, 1976.) (Author introduced by Professor David Lutzer.)

76T-G85 Rastislav Telgársky, Institute of Mathematics, Wrocław Technical University, Wrocław, Poland. Scattered spaces which do not admit scattered compactifications.

We say that X is **extremally disconnected at a point** x if, given disjoint open sets U and V, we have $x \notin U \cap V$. 

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Theorem. Assume that there exists a dense in itself subset $E$ of $\mathbb{R}^n$ such that $E$ is extremally disconnected at a point of $E \cap X$. Then $X$ has no scattered compactification.

Corollary. If $E$ is a dense in itself, extremally disconnected subset of $\mathbb{N} \setminus \mathbb{N}$ and $p \in E$, then $\mathbb{N} \cup \{p\}$ has no scattered compactification. (Received May 24, 1976.) (Author introduced by Andrew Lelek.)

D.L. EVERETT, Utah State University, Logan, Utah, 84322

Embedding Theorems for Decomposition Spaces

Suppose that $G$ is a cell-like upper semicontinuous decomposition of $\mathbb{R}^n$. For $n \geq 3$ many examples are known for which $\mathbb{R}^n/G$ is non-Euclidean; however, it is conjectured that for each such $G$, $\mathbb{R}^n/G \times \mathbb{E}^n = \mathbb{R}^{n+1}$.

A necessary condition for $\mathbb{R}^n/G \times \mathbb{E}^n$ to be Euclidean is that $\mathbb{R}^n/G$ embed in $\mathbb{R}^{n+1}$ with 1-LC complement. Armentrout (Fund. Math. 61 (1967), pp. 1-21) obtains this condition if $G$ is 0-dimensional and elements of $G$ are cellular. We strengthen his result by showing that $\mathbb{R}^n/G$ embeds in $\mathbb{R}^{n+1}$ with 1-LC complement if $G$ is either 0-dimensional or closed 1-dimensional (the closure of the nondegenerate elements of $G$ in $\mathbb{R}^n/G$ is 1-dimensional); we do not require that elements of $G$ be cellular.

Also we show that if $G$ is a finite-dimensional closed cell-like upper semicontinuous decomposition of $\mathbb{R}^n$, the question of whether $\mathbb{R}^n \times \mathbb{E}^n$ is homeomorphic with $\mathbb{R}^{n+1}$ is equivalent in high dimensions ($n+1 \geq 5$) to a finite sequence of taming problems. (Received May 24, 1976.)

(Author introduced by J. W. Cannon.)

Joachim Grispolakis, Wayne State University, Detroit, Michigan 48202. Two theorems on open mappings. Preliminary report.

We say that a topological space $Y$ is hereditarily locally connected (h.l.c.) provided every connected subspace of $Y$ is locally connected. Theorem 1: If $f$ is an open, perfect and 0-dimensional mapping of a regular space onto an h.l.c. space $Y$, $K \subseteq Y$ is a connected set, and $Q \subseteq f^{-1}(K)$ is a quasi-component of order $\alpha$ of $f^{-1}(K)$ (where $\alpha$ is any ordinal number), then $f(Q) = K$ and the mapping $f|Q$ is open. Theorem 2: If $f$ is an open mapping of a continuum onto an h.l.c. continuum $Y$, $K \subseteq Y$ is a connected set, and $C$ is a component of $f^{-1}(K)$, then $f(C) = K$. This theorem generalizes an earlier result (see B.B. Epps, Jr., these Notices 19 (1972), A-807; see also A. Lelek and E.D. Tymchatyn, Canad. J. Math. 27 (1975), p.1344). (Received June 1, 1976.)


Let $X$ be compact $T^2$. If $m \in M^+ = M(X)^+$, $S = S(m)$ is the support of $m$. A closed set $F$ is called a $P_1$-set if it satisfies the following equivalent conditions. Theorem. Suppose the closure of a cozero set in $X$ is always a $P_1$-set. Then every support set in $X$ is extremally disconnected. Hence (as in Seever's paper) $C(X)$ is a Grothendieck space. Problem: find $P_1$-sets which are not $P$-sets - e.g., in $\mathbb{N} \setminus \mathbb{N}$. (Received May 27, 1976.)

Richard Jerrard and Mark Meyerson, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801. Homotopy with M-Functions.

In this paper we investigate some of the homotopy properties of certain multiple-
valued functions called $m$-functions. Previous papers by G. Darbo and the first
author (independently) have shown that ordinary singular homology groups for compact
polyhedra are $m$-homotopy type invariants, and that this is a stronger invariance than
homotopy type. In this paper we define $m$-homotopy groups (actually $R$-modules) and
prove those essential properties one would expect for a homotopy theory. A subsequent
paper by the second author uses this groundwork to show that $m$-homotopy theory is a
homology theory. (Received May 7, 1976.)

*76T-G90  MICHAEL J. CELLINI, Ladycliff College, Highland Falls, New York 10928. Unifying ad hoc
equivariant cohomology theories. Preliminary report.

This paper is an announcement of a few of the results discussed by the author in the area of equivariant
category of $G$-pairs (simplicial) over a fixed simplicial $G$-set $x$ with coefficients in a stack $A$ of abelian groups
over $x$. The construction is made via derived functors of the global section functor $\Gamma_G(-,A) \approx \lim(A)$. Soren
Illman's equivariant singular cohomology theory is a special example of this theory. This theory satisfies the
Eilenberg-Steenrod-Milnor axioms (it is unique in this sense) and is representable by a generalized Eilenberg-
Mac Lane complex (equivariant version). The author's construction of the geometrical realization of a $G$-set
yields Walker's equivariant cellular cohomology theory. (Received May 13, 1976.)

*76T-991  HAROLD BELL, Department of Mathematics, University of Cincinnati, Cincinnati, OH
Two fixed point theorems for non-separating plane continua. 45221

Let $f$ be a map from a non-separating plane continuum $N$ into itself. Let $g$ be an extension
of $f$ to a continuous function defined on a neighborhood $U$ of $N$ such that $g(N - U) \subseteq R^2 - U$.
Theorem 1: If $f$ is 1-1 then $f$ has a fixed point.
Theorem 2: There is a $x$ in $N$ such that $f(f(x)) = x$. (Received May 26, 1976.)

*76T-992  HEIKI J. K. JUNNILA, Lounskatu 12C22, Helsinki 17, Finland.
Every $N_3$ space is an $N_2$ space.

Lemma. Let $X$ be an orthocompact semi-stratifiable space and let $U$ be a reflexive relation
on $X$ such that for each $x \in X$, $U(x) \in \eta_x$. Then there is an interior-preserving open cover $C$ of
$X$ such that for each $x \in X$ there correspond points $y$ and $z$ of $X$ such that $\bigcap C_x \subseteq U(y)$ and
$\{x, y\} \subseteq U(z) \cap U^{-1}(z)$. The following theorems are immediate consequences of the lemma.
Theorem 1. If $X$ is an orthocompact semi-stratifiable space, then the fine quasi-uniformity of
$X$ has a transitive base. Theorem 2. Every $N_3$ space is an $N_2$ space. (Received June 7, 1976.)

(Author introduced by W. F. Lindgren.)

*76T-993  Ulrich Koschorke and Brian Sanderson, Universities of Bonn, Germany, and of Warwick, Coventry, England.
Selfintersections and higher Hopf invariants.

I. James, P. May and others have constructed combinatorial models for iterated
loop spaces $\Omega^n S^m X$, $0 \leq m \leq n$. If $X$ is the Thom complex of a bundle $\xi$, we
can interpret these models in a natural way as "Thom spaces for immersions." Hence bordism of smooth embeddings into $M \times R^m$ ($M$ a manifold), equipped
with a description of the normal bundle as a pullback of $\xi \otimes R^m$, turns out
to be isomorphic to the corresponding bordism of embeddings which project
to immersions into $M$. An analysis of transverse $k$-tuple points of the
resulting immersions leads to bordism invariants which translate into homotopy
operations $\bigodot K$, $k = 1, 2, \ldots$. As special cases we deduce the generalized
and higher Hopf invariants of James, the Hopf ladder of Boardman-Steer as
well as the (un)stable cohomotopy operations of G. Segal. (Received June 8, 1976.)
Several important fluid dynamic problems require the solution of non-linear wave equations. The finite element method (F.E.M.) has been very satisfactory for linear elliptic problems. In trying to use the F.E.M. on non-linear hyperbolic problems there are a variety of ways to handle the non-linear terms. We present here an analysis of the pros and cons of the different possibilities. (Received May 24, 1976.)


If \( f(z) = b(z + a_z^2 + a_3z^3 + \cdots) \), \( 0 < b \leq 1 \), is any analytic and univalent mapping of the unit disc into itself, then by the methods of the calculus of variations the author has shown [Arch. Rational Mech. Anal. 44 (1971), 93-120]

\[
(*) \quad \text{Re}\{a_3^2 - 4a_2^2\} \leq \begin{cases} 
1 - b^2 - 4a^2 \log b, & 0 \leq \sigma \leq b, \\
1 + b^2 - 4a^2 \log \sigma + 6a^2 - 8\sigma b, & b \leq \sigma \leq 1.
\end{cases}
\]

In the limiting case \( b \to 0 \), the inequality reduces to one of Jenkins for the class of (non-bounded) univalent functions \( z + a_z^2 + a_3^3 + \cdots \) on the unit disc. G. B. Leeman, Jr. [Proc. Amer. Math. Soc. 54 (1976), 114-116] recently gave a quite direct new proof of the Jenkins' inequality employing the Loewner theory. We extend his method to give a new proof of \((*)\). (Received May 27, 1976.)


Socrates is said to have rejected mathematics as a means to the truth because he could not justify the statement "\( 1 + 1 = 2 \)," interpreting the process of addition as occurring during a time interval. The same paradox persists today in misinterpretations of equivalence, equality, and identity not only in mathematics but in all forms of communication. The foundations of mathematics and logic are psycho-linguistic and when the psychological nature of mathematical activity is considered, this paradox is partially resolved. The duality principle, observed in other contexts, I state here as follows:

A claim that two or more objects are equal is significant only if there is an interpretation in which the objects are not treated as equal.
A claim that two or more objects are not related can be significant only if there is a context in which the objects are related.

This principle I apply to mathematics education and language and I show that it is psychology which provides insights rather than logic.

(Received April 30, 1976.)

02 ▲ Logic and Foundations


A countable model \( \mathcal{M} \) of a theory \( T \) is recursively presentable (rp) in \( \mathbb{C} \subseteq \omega \) if its satisfaction predicate is Turing computable from \( C \). Theorem. There is a complete decidable theory (CDT) such that all types are recursive (rec.), the prime model is rp and the saturated model is not. Theorem. If a CDT has a prime model (only rec. types), then the prime model is rp in \( \mathbb{O} \). Theorem. If a CDT has a countable saturated model \( \mathcal{M} \) is rp in a hyp set. Definition. Let \( B \) be a set of indices (of rec. types) of a CDT. \( B \) has the effective amalgamation property (eap) if (i) \( f: B \times B \rightarrow B \), (ii) \( \Gamma_f(\bar{c}) \cup \Gamma_f(\bar{s}) \subseteq \Gamma_f(\bar{r}, \bar{s}) \) for \( r, s \in B \), (iii) if \( S_n(T) \) is the space of \( n \)-types over \( T \), then \( \{f(\bar{c}, \bar{x}_1, \ldots, \bar{x}_n) \mid \bar{c} \in \omega \cap S_n(T \cup \Gamma_{\bar{r}}(\bar{c})) \) for all \( r \in B, n \in \omega \), where \( \bar{c} = (c_1, \ldots, c_m) \) are constants not in \( T \). Theorem. Let \( \mathcal{M} \) be a homogeneous model of a CDT realizing only rec. types of \( T \). Then (i) \( \mathcal{M} \) is rp iff \( \mathcal{M} \) has a \( \Sigma^0_2 \) set of types with eap. (ii) If \( \{\Gamma_{\text{rec}}(\mathcal{M}) \text{ realizes } \Gamma \} \) and \( \{\Gamma_{\text{rec}}(\mathcal{M} \text{ omits } \Gamma \} \) are \( \Sigma^0_2 \), then \( \mathcal{M} \) is rp. Theorem. There is a CDT with exactly two rp models. (Received November 12, 1975.) (Author introduced by Anil Nerode.)

737-02-2 • Allen T. Retzlaff, Cornell University, Ithaca, New York 14853. Subspaces of \( \mathcal{H} \). Preliminary report.

We follow Metakides-Nerode (R. e. vector spaces, Ann. Math. Logic. (to appear)) and let \( \mathcal{X} = \mathcal{X}(V_\infty) \) be the lattice of all r.e. (recursively enumerable) subspaces of an infinite dimensional, fully effective vector space \( V_\infty \) over a recursive field \( F \). A \( A \in \mathcal{X} \) is called a recursive subspace (or decidable subspace) if \( A \) is complemented in \( \mathcal{X} \). Theorem. The recursive subspaces of \( V_\infty \) do not form a sublattice of \( \mathcal{X} \), in fact, recursive subspaces \( A, B \) exist with \( A \oplus B \) a creative subspace of \( V_\infty \). Theorem. Any infinite dimensional re. non-recursive subspace of \( V_\infty \) is a direct sum of two infinite dimensional re. non-recursive subspaces of \( V_\infty \). Theorem. There exists an re. subspace of \( V_\infty \) with two re. bases of incomparable Turing degree.

The proof of the second theorem requires an analogue of Friedberg's splitting argument for sets, but does not reduce to the latter. (Received March 1, 1976.) (Author introduced by Anil Nerode.)


We begin investigation of automorphisms of \( \mathcal{L}(V_\infty) \) the lattice of r.e. subspaces of an infinite dimensional fully effective vector space over a recursive field. (See Metakides-Nerode, R.E.Vector spaces, Ann. Math. Logic, to appear.) Theorem. Every automorphism of \( \mathcal{L}(V_\infty) \) is induced by a semi-linear transformation of \( V_\infty \).

Let \( \mathcal{L}^*(V_\infty) \) be the quotient of \( \mathcal{L}(V_\infty) \) mod finite dimensional spaces.

Theorem. Every automorphism of \( \mathcal{L}^*(V_\infty) \) is induced by a semi-linear transformation of \( V_\infty \).

Theorem. There are \( \aleph_0 \) automorphisms of \( \mathcal{L}(V_\infty) \).

These theorems are analogues for r.e.vector spaces of theorems for r.e. sets due to Kent and Soare. For the last theorem a Martin-Soare type priority argument is adapted to \( \mathcal{L}(V_\infty) \), using a new notion of "e-state" different from that in Metakides-Nerode. (Received March 9, 1976.) (Author introduced by Anil Nerode.)
We shall discuss some topics related to our recently circulated paper, "First-order theory of the degrees of recursive unsolvability." In that paper it is proved that the first-order theory of the semilattice of degrees is recursively isomorphic to second-order arithmetic. Copies of the paper are available. (Received March 26, 1976.)

Robert A. Di Paola, Queens College, CUNY, Flushing, New York 11367. The Operator Gap Theorem in $\alpha$-recursion theory.

Let $\alpha$ be any admissible ordinal and $\phi^\alpha$ an $\alpha$-computational complexity measure. Define the $\alpha$-computational complexity class $C^\alpha = (\phi^\alpha)|_\alpha$ as $\alpha$-recursive and $\phi^\alpha(\beta) < f(\beta)$ for all but an $\alpha$-finite set of $\beta$. In his recent doctoral dissertation ("$\alpha$-Computational Complexity," New York University, v + 157 pp., 1975), B. Jacobs has asked whether for admissible $\alpha$, $\alpha$-computational complexity classes can always be extended by total $\alpha$-effective operators $F$; that is, do we have $C^\alpha \subseteq C^\alpha_F$ for all $\alpha$-recursive $F$, or is there an $\alpha$-operator gap?

Theorem (\alpha-Operator Gap Theorem): for all $\phi^\alpha$ and all total $\alpha$-effective operators $F$ there are arbitrarily large increasing $\alpha$-recursive functions $b$ such that for all $\epsilon$ if $b(\beta) < \phi^\alpha(\beta) \leq F(b;\beta)$ for $\beta$ without bound, then $F(b;\gamma) < \phi^\alpha(\gamma)$ for $\gamma$ without bound. Thus, there is no $\alpha$-recursive $\psi^\alpha_C$ in $C^\alpha - C^\alpha_F$. For $\alpha = \omega$, the operator gap theorem was first given by R. L. Constable (J. Association of Computing Machinery, vol. 19, no. 1 (1972), pp. 175-183).

However, Constable's construction is deficient in several respects, and his subsequent argument is to that extent vitiated. Our proof is a lift, with some of the usual complications one encounters for admissible $\alpha > \omega_1$, of a suitably rectified version of Constable's proof, and of course gives a valid proof for $\alpha = \omega_1$ as well. (Received May 3, 1976.)

Miriam Lipschutz-Yevick, Rutgers University, New Brunswick, New Jersey 08903. The Godel sentence and the sequential and holistic modes of recognition (denotation).

Recent advances in optical recognition methods allow some previously inaccessible logical distinctions to be made. We claim that the "human computer" is both a serial computer (Turing machine) and an instantaneous holistic pattern recognizer. Both of these capabilities (modes) are necessary for the construction and interpretation of a formal language or abstract system of representation. These insights applied to the Tarski sentence suggested that empirical considerations indicate that "c" denotes the sentence "c is not a true sentence" as an optical pattern, whereas the expression in quotes represents that which is asserted. Similarly for the Godel sentence.

Following the argument (p. 143) in Mendelsohn: Let ** be provable. We then recognize (observe) that the pattern $m$ has been substituted for $x_1$ in * (this recognition can be achieved holographically). Hence $W_1(m,x_2)$ as defined in the first two lines of 17(a), p. 141, holds for some $x_2$ and for the ostensive numeral $m$ (numeral as a pattern, holistic expression). On the other hand in the formula $\exists_1(m,k)$ which numeralwise expresses $W_1(m,k)$, $m$ is the numeral with $g$ number Num(m). Num(m) is defined recursively from Num(0) = $\omega_1^g$, the g number of $\beta$ as an expression. Thus $m$ is here a "sequential numeral" or the expression resulting from the concatenation of individual symbols as expressions. (Received May 19, 1976.)

Manuel Lerman, University of Connecticut, Storrs, Ct. 06268. On the elementary theory of some lattices of $\alpha$-r.e. sets.

Let $\alpha$ be an admissible ordinal, let $e(\alpha)$ denote the lattice of $\alpha$-r.e. sets, and let $e^*(\alpha)$ denote the lattice of $\alpha$-r.e. sets modulo $\alpha$-finite sets. A.H. Lachlan [Duke Math. J. 35(1968), 123-146] showed for a relatively strong language $I$ suitable for lattice theory, that the $\forall\alpha$ theory of $e^*(\alpha)$ is decidable. These results are extended: Theorem 1: If $\alpha^* = \alpha$ and $t02p(\alpha) = w$, then the $\forall\alpha$ theory of $e^*(\alpha)$ is the same as that of $e^*(\omega)$. Theorem 2: If $\alpha$ is a successor cardinal of $L$, then the $\forall\alpha$ theory of $e^*(\alpha)$ is decidable. Theorem 2 is proved for a wider class of admissible ordinals, roughly those for which r-maximal and hyperhypersimple sets are known not to exist. Recent progress has been made with R.I. Soare which we hope will lead to extending Lachlan's result by adjoining to the language $I$ a predicate which picks out maximal sets, and showing that the $\forall\alpha$ theory of $e^*(\omega)$ in this language is decidable. (Received May 21, 1976.)


For $T$ any completion of Peano Arithmetic and for $n$ any positive integer, there is a
model of $T$ of size $\subseteq_n$ with no $n + 1$-size set of indiscernibles. Hence the Hanf number for omitting types over $T$, $H(T)$, is at least $\subseteq_n$. (Now, using an upper bound previously obtained by Julia Knight, $H(\text{true arithmetic})$ is exactly $\subseteq_n$.) If $T \neq \text{true arithmetic}$, then $H(T) = \subseteq_{n+1}$. If $\delta \rightarrow^*(\omega)^{\subseteq_n}$, then any completion of Peano Arithmetic has a model of size $\delta$ with no infinite set of indiscernibles. There are similar results for theories strongly resembling Peano Arithmetic, e.g., $ZF + V = L$.

(Received May 28, 1976.)


We show that every automorphism of $\mathcal{E}^*$ (the lattice of r.e. sets modulo finite sets) is uniquely determined by its action on any non-trivial class of r.e. sets. To be precise if $\varphi_1$ and $\varphi_2$ are automorphisms of $\mathcal{E}^*$ agreeing on even one recursive equivalence class (other than that of $\emptyset$ or $\mathbb{N}$) then $\varphi_1 = \varphi_2$. We also consider the related question of the extendability of automorphisms of sublattices to ones of $\mathcal{E}^*$. For example there is an automorphism of the lattice generated by the maximal sets which does not extend to one of $\mathcal{E}^*$. We also give an elementary proof of the same result for the recursive sets. (A result originally gotten jointly with R. Soare by applying his "extension lemma". See these Notices 21 (1974) A524-525.) (Received June 3, 1976.)

*737-02-10 G.E. SACKS, 1 Oxford St., Cambridge, Mass. 02138. Recursive Functions, Preliminary Report

Recent work by S. Friedman suggests that combinatory principles weaker than admissibility suffice to solve Post's problem. (Received June 3, 1976.)


Post's program which has predominated for 30 years has been to classify the isomorphism type of an r.e. set $A$ by its lattice of r.e. supersets $\mathcal{L}(A)$, and by r.e. arrays of r.e. sets. More recent results on automorphisms of the lattice $\mathcal{E}$ have shown that this is inadequate for a complete classification of the isomorphism type of $A$. For example, if $A$ and $B$ are low and coinfinite then $\mathcal{L}(A) \cong \mathcal{L}(B) \cong \mathcal{E}$ even though $A$ may be simple and $B$ nonsimple. This suggests classifying r.e. sets which are simple with respect to r.e. arrays of differences of r.e. sets (d.r.e. sets). A coinfinite r.e. set $A$ is d-simple if $(\forall Y)(\exists U)[U \subseteq A & Y - A \subseteq V] \Rightarrow (W - Y) \cap A = \emptyset$, i.e., $A$ intersects each infinite member of the r.e. array of d.r.e. sets $(W - U; W \in \mathcal{E})$. Every $h$-simple set is d-simple and every d-simple set is simple. There are low sets $A$ and $B$ such that $A$ is d-simple, $B$ is simple but not d-simple, and $\mathcal{L}^*(A) \cong \mathcal{L}^*(B)$ by lowness. We have explored the degrees and structure of d-simple sets and have used d-simplicity as a new invariant for classifying isomorphism types of r.e. sets. For example, we conjecture that if $A, B$ are d-simple and $\mathcal{L}(A) \cong \mathcal{L}(B)$ then $<\mathcal{E}, A> \cong <\mathcal{E}, B>$. We have obtained this result for $\forall \exists$ equivalence in place of $\cong$. (Received June 7, 1976.)

737-02-12 MARIAN BOYKAN POUR-EL, University of Minnesota, Minneapolis, Minnesota 55455. Computability revisited — an approach via the continuum.

- We compare computability as defined traditionally using discrete atomic steps (via Turing machines, Herbrand-Gödel equations, etc.) with an approach via the continuum motivated by analog computers. More precisely, we trace the development of the concept, "recursive function of a real variable", obtained from traditional formulations by recursive analysis. We then consider a mathematical definition of "analog generable function" which covers functions generated by existing analog computers — including the original analog computers invented by Lord Kelvin and Vannevar Bush. These are the two concepts which are compared and contrasted. The emphasis
is on open problems concerned with (1) computability theory itself, (2) the possible impact of this approach in formulating a foundation for mathematics in which the continuum is not a derived concept. (Received June 7, 1976.)

737-02-13 SY D. Friedman, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Forcing in \( \varepsilon \)-Recursion Theory. Preliminary report.

Let \( \beta \) be a limit ordinal, \( \beta = \omega \cdot \gamma \). Then \( \beta \)-r.e. = \( \Sigma_1 \) over \( J_\gamma \), \( \beta \)-rec. = \( \Delta_1 \) over \( J_\gamma \). \( \beta \)-finite = member of \( J_\gamma \), \( \beta^* \) and \( \beta \)-cardinals are defined just as \( \alpha^* \) and \( \alpha \)-cardinals for \( \alpha \) admissible, except replace \( L_\alpha \) by \( J_\gamma \). In an earlier abstract we announced:

Thm. \( \beta^* \) a successor \( \beta \)-cardinal \( \implies \exists \beta \)-r.e. \( A \subseteq B, A \uparrow \nsubseteq B \uparrow \Delta_1 \). 

We develop a notion of (iterated) forcing over \( J_\gamma \) to show:

\[ \beta^* \text{ a successor } \beta \text{-card., } \beta^* = \text{greatest } \beta \text{-cardinal} \implies \exists \beta \text{-r.e. } A \subseteq B, B \uparrow \Delta_1 \subseteq J_\gamma [B] \text{, } \varepsilon \text{,} \]

(Received June 9, 1976.)


Let \( \alpha \) be a recursive ordinal, \( k \) a positive integer. Define \( F_{\alpha,k} = \{ x \in \mathcal{P}(\mathbb{N}^k) | x \leq \alpha \} \). \( \mathcal{P}(\mathbb{N}^k) \) is the set of all \( k \)-tuples of natural numbers.

We present a new approach to Abraham Robinson's nonstandard analysis which makes the subject easily accessible to the working mathematician. This approach is based on a theory which we call Internal Set Theory (IST). IST incorporates all of conventional set theory without any change of terminology. In addition to the usual elementhood relation \( \in \), there is a new undefined predicate \( \text{standard} \), and there are three new axiom schemes: the principle of idealization (I), the principle of standardization (F), and the transfer principle (T). This paper is devoted to showing what these principles are and how they may be used in mathematical reasoning. (Received June 21, 1976.)

04 ▶ Set Theory

737-04-1 Siemion Fajtlowicz, University of Houston, Houston, Texas 77004. A graph-theoretical generalization of a Cantor theorem.

By a graph we shall mean here a pair \((X, R)\) where \( X \) is a set and \( R \) a binary relation on \( X \). If \( P \) and \( Q \) are graphs then \( H(P, Q) \) denotes the graph of all homomorphisms from \( P \) into \( Q \) considered as a subgraph of \( Q^P \), i.e. with the relation defined coordinate-wise.

Theorem. If \( Q \) is a reflexive graph with at least two elements, \( P \) an arbitrary graph and \( P_0 \) a subgraph of \( P \) then no antihomomorphism from \( P_0 \) into \( H(P, Q) \) is onto. In the case when \( P \) is a partially ordered set and \( Q = \{0, 1\} \), then the theorem was obtained earlier by Dilworth and Gleason. Both theorems clearly generalize the Cantor cardinality Theorem. (Received June 11, 1976.)
The following problem of potential theory was proposed by Peter Fenton: Does there exist a set of circles, each contained in the unit disk, of Lebesgue area measure zero whose centers cover the unit disk? The author observes that the one-dimensional analog of this problem, the existence of a set of intervals whose endpoints form a set of Lebesgue linear measure zero and whose midpoints cover the unit interval, follows easily from the known result that the distance set of the Cantor middle third set is the unit interval. This latter result was first proved in 1917 by H. Steinhaus by a simple geometric method. A somewhat complicated algebraic proof, in terms of the ternary expansions of the points of the Cantor set, was given in 1940 by J. F. Randolph. This made possible the consideration of the powers of the sets of representations of the points of the unit interval as sums or differences of points of the Cantor set, which was partially solved by N. C. Bose Majumder in 1959. (See Boas, Math. Rev. 24, rev. no. A 2538, for a summary of these and related results.) The author has found a very simple algebraic proof of the above result of Steinhaus, which makes possible a complete and explicit solution of the problem of the powers of the above sets of representations. These results can be extended to certain other Cantor-like sets which will be considered in detail in a forthcoming paper by Charles F. Osgood and the author.

(Received June 15, 1976.)

05 Combinatorics


A Hadamard matrix H of order 4t has associated with it a (4t-1, 2t-1, t-1)-design D in which every pair of blocks intersects in t-1 treatments. It may happen that a triple of blocks intersects in t-1 treatments. The characteristic of D is the number of (t-1)-tuples of treatments which occur as the intersection of three blocks. The characteristic of H is derived from that of D.

All of the non-isomorphic Hadamard matrices of order 24 and characteristic at least 2 are determined by considering both the associated (23, 11, 5)-designs and several (24, 9, 8, 3, 2)-designs which arise from such H. (Received March 22, 1976.)


Let \( n, m \) and \( \lambda \) be given positive integers and \( X \) a given set of positive integers. An \((n, m, K, \lambda)\)-group divisible design is a triple \((X, J, \mathcal{A})\) where (1) \( X \) is a set of \( nm \) elements, called points, (2) \( J \) is a class of \( m \)-subsets of \( X \) which partition \( X \), called groups, (3) \( \mathcal{A} \) is a family of \( m \)-subsets of \( X \) whose cardinalities are in \( K \), called blocks, (4) no block meets a group in more than one point, (5) each pair \((x, y)\) of elements of \( X \), not contained in a group, is contained in exactly \( \lambda \) blocks. For any subset \( K \) of positive integers, define \( \alpha(K) = \gcd \{k(k-1) \mid k \in K\} \) and \( \beta(K) = \gcd \{k(k-1)(k+1)/6 \mid k \in K\} \). The necessary conditions for the existence of an \((n, m, K, \lambda)\)-group divisible design are (a) \( \lambda m(n-1) \equiv 0 \pmod{\alpha(K)} \) and (b) \( \lambda m^2(n-1) \equiv 0 \pmod{\beta(K)} \). It is shown here that if \( \lambda, m \) and \( K \) are given, then there is a constant \( C = C(m, K, \lambda) \) such that an \((n, m, K, \lambda)\)-group divisible design exists for any positive integer \( n > C \), satisfying the necessary conditions (a) and (b) above.

The proof of this theorem requires the construction of a class of group divisible designs using finite fields and pairwise balanced designs. (Received March 31, 1976.)

737-05-3 A. Czerniakiewicz, Queens College of CUNY, Flushing, N.Y. 11367. Counting monochromatic paths and stars.

Given a 2-coloration (of the edges) of the complete graph \( K_n \), we say that the coloration satisfies Property A if the number of lines in each color is as close to equality as possible. The 2-coloration satisfies Property B if the degrees of the points in each of the 2 colors is as close to equality as possible. Theorem 1: The minimum number of monochromatic stars \( K_{1,m} \) in a
2-coloration of \( K_n \) \((n \geq 5)\) occurs in just those 2-colorations of \( K_n \) satisfying property B. **Theorem 2:** The minimum number of monochromatic paths of length 3 in a 2-coloration of \( K_n \) \((n \geq 5)\) occurs in just those 2-colorations of \( K_n \) satisfying property A when \( n = 5 \) and both A and B when \( n \geq 6 \).

(Received May 20, 1976.)

737-05-6 **BERNSTEIN, Illinois Institute of Technology, Chicago, Illinois 60616.** **Units in algebraic number fields and combinatorial identities.**

The author has invented a new technique to establish infinitely many new combinatorial identities of a highly complicated structure. He has used his method in previous papers and will illustrate it by a new example in this paper. The technique proceeds as follows: Let \( w \) be a real root of an irreducible polynomial over \( Q \); \( P(x) = x^n + b_{n-1}x^{n-1} + \ldots + b_1x + b_0 \in \mathbb{Z}, i = 1, \ldots, n \). Let \( e = f_1 + f_2w + \ldots + f_nw^{n-1} \) be a unit in \( Q(w) \), \( f_i = f_i(k_1, \ldots, k_n) \). e must, of course, be explicitly stated. Let \( e^m = g_1 + g_2w + \ldots + g_nw^{n-1}, m \) a natural number, \( g_i = g_i(k_1, \ldots, k_n); e^{-m} = h_1 + h_2w + \ldots + h_nw^{n-1}, h_i = h_i(k_1, \ldots, k_n). \) Both \( g_i \) and \( h_i \) (i = 1, \ldots, n) are found by Euler's generating functions; they are polynomials in \( k_1, \ldots, k_n \) with combinatorial coefficients. From 1 = \( e^m \cdot e^{-m} \) the combinatorial identities are established by comparison of coefficients and involve determinants of order \( n \) with entries of powers of \( k_1 \) with combinatorial coefficients. The coefficients of these powers can also be sums of combinatorial structures. In his example the author starts with the polynomial \( P(x) = x^n - d^nx - d^n, d \) a natural number. (Received May 19, 1976.)

737-05-7 **Bruce Richmond, University of Waterloo, Waterloo, Ontario N2L 3G1.** **The Asymptotic Behaviour of Certain Chromatic Sums.** Preliminary report.

The asymptotic behaviour for large \( n \) of the sum of the chromatic polynomials \( P(T, \lambda) \) over all rooted triangulations \( T \) with \( 2n \) faces is calculated for the cases \( \lambda = \tau^2 \) and \( \tau^{-2} \), where \( \tau \) is the golden ratio. Some conjectures concerning the maximum and minimum values of \( P(T, \lambda) \) when \( \lambda \) is a Beraha number will be stated. (Received June 1, 1976.)
A discussion of methods of attack on problems concerning constrained chromatic
for the f, 7, and n circuit. (Received June 4, 1976.)

Andrew P. Guinand, Trent University, Peterborough, Ontario, Canada.

The multiple umbral notation as a manipulative aid.

The umbral (or Blissard) calculus, in which powers of certain symbols
(or umbrae) are interpreted as terms of sequences, can be regarded as a
notation subject to rules of interpretation and of manipulation. With
these rules it can be used to simplify proofs of various identities
involving Bernoulli and Euler numbers, and other related sequences.
(Received June 7, 1976.)

Paul Smith, University of Victoria, Victoria, B. C., University of Montana, Missoula, MT

A scheduling problem for tournaments with a constraint on locations.

Let \( v = kn \) where \( k \geq 2 \) and \( n \) is a positive integer. Let \( n \leq r \leq (v - 1)/(k - 1) \). Given \( v \) players
and \( r \) locations it is required to construct a schedule subject to the following constraints:

1. On each round a block of \( k \) players is sent to each of \( n \) locations. (2) No two players ever
appear together in more than one block. (3) Each player appears exactly once at each of the \( r \)
locations. For \( k = 2 \) and \( r = n - 1 \) this is the Room square problem. For \( n = r \) it is the problem of
constructing a complete set of mutually orthogonal Latin squares. We present two special solutions
for \( n < r = [(v - 1)/(k - 1)] \) and \( k > 2 \). (Received June 7, 1976.) (Author introduced by Dr. H. E.
Reinhardt.)

J. S. Frame, Michigan State University, East Lansing, Michigan 48824. Power
sums of roots of a trinomial equation.

For integral \( n \neq 0 \), the power sum \( S_n = \sum_{k=1}^{u} z_k^n \) of the \( u \) zeros of the real trinomial
\( az^u - bz^v - c \), with \( abc \neq 0 \) and relatively prime integral exponents \( u > v > 0 \), is ex­
pressible by the finite sum \( \sum_{i,j} [u(i) - v(j)]a^{-i}b^{-j}c^{-j} \), summed for integers \( i,j \)
such that \( ui - vj = n \) and \( j \geq 0 \). Choosing \( a = b = c \), the products \( q_u,v(n) = \Pi_k(1-z_k^n) \)
are integer valued polynomials in the \( S_m \) for \( m = \pm 1, \pm 2, \ldots \) which yield for various
\( u, v \) a rational integral factorization of the determinant \( D_n \) of the \( n \times n \) binomial
circulant matrix \( M_n \) with \((i,j)\)-entry \( \binom{n}{i-j} \). (Received June 7, 1976.)

Gerald Berman, University of Waterloo, Waterloo, Ontario. The dichromate and
orientations of a graph.

Internal and external activities are defined for any orientation of a graph \( G \) relative to a
fixed labeling of its edges. It is shown that the number of such orientations of \( G \) having internal
and external activities \( r, s \) is \( 2^{r+s}x_{rs} \) where \( x_{rs} \) is the coefficient of \( x^r y^s \) in
the dichromate of \( G \). It follows that the number of orientations of \( G \) in which the resulting digraph
\( D \) is acyclic is given by \( P(G, -1) \), where \( P(G, x) \) is the chromatic polynomial associated with \( G \).
In case \( G \) is planar the number of orientations of \( G \) in which \( D \) is strongly connected is equal
to \( P(G', -1) \), where \( G' \) is the planar dual of \( G \). (Received June 8, 1976.)

STEPHEN M. TANNY, University of Toronto, Toronto, Canada, M5S 1A1 and MICHAEL ZUKER,
National Research Council, Ottawa, Canada: On a Unimodal Sequence of Binomial
Coefficients II, Preliminary report.

A variety of properties are proved for the binomial coefficients \( \binom{n-kr}{r} \), where \( k \) is an
arbitrary fixed positive integer. In particular it is shown that for fixed $k$, these coefficients are strongly logarithmically concave, with at most a double maximum. Let $r_{n,k}$ be the least value at which this maximum occurs, and set $r_{n,k} = \delta_{n,k} n$. We show that $\delta_{n,k}$ converges as $n \to \infty$ (for fixed $k$) to the smallest positive root $\delta_k$ of the polynomial $p_k(\lambda) = [1 - (k+1)\lambda]^{k+1} - [1 - k\lambda]^k$ of degree $k + 1$. Further results, including an asymptotic formula for $\delta_k$ and inequalities satisfied by the sequence $\{r_{n,k}\}$, are also developed. (Received June 9, 1976.)


Some combinatorial identities arising from counting lattice paths are generalized. Their applications through path consideration are varied and wellknown. Another set of identities are generated from the enumeration of certain sets of trees. Though the paper basically contains a survey of earlier identities, it also includes some new results. (Received June 11, 1976.) (Author introduced by Professor H. W. Gould.)


In the familiar elementary calculus we encounter "derivatives of order $n", \ D^n f(x) = d^n f(x)/dx^n, \text{ where } n \text{ is a whole number. In the so called fractional calculus, } D^\alpha f(x) \text{ is defined in such a way that } \alpha \text{ can be any ( rational, irrational or complex ) number. In the fractional calculus } [\text{SIAM Review, } 16(1976), \text{ pp. 240-268}], \text{ familiar formulas such as the Taylor’s series, the chain rule and the Leibniz rule are generalized to include derivatives of arbitrary order. Using these generalized formulas, we can obtain extensions of known combinatorial identities. When these methods are applicable, a known identity of the form } \sum \alpha = \sum_{n=0}^\alpha C(n) \text{ will generalize to } L = \sum_{n=0}^\alpha C(n) a, \text{ where } 0 < a \leq 1 \text{ and } \gamma \text{ is arbitrary. (Received June 11, 1976.)}

CLARK KIMBERLING, University of Evansville, Evansville, Indiana 47702. Sums associated with containment relations among intervals.

Let $C(m,a,n)$ be the number of half-open intervals $[p/q, (p+1)/q]$ containing the closed interval $[m/n, (m+a)/n]$, where $n,m$, and $a$ are positive integers. Then $C(m,a,n) = \sum_{p=0}^K \left\{[p+1]/n + (m+a) - \lfloor pn/m \rfloor \right\}$, where $K = \lfloor m/a \rfloor$. For fixed prime $n$ and $a = 1$, $C(m,a,n)$ is invariant for $m = 1,2,\ldots,n-a$, but considerably more complicated for $a > 1$. Maximal containment chains of intervals $[p/q, (p+1)/q]$ are identified, and these provide further expressions for $C(m,a,n)$. The intervals $[p/q, (p+1)/q]$ are assigned various weights which, through the lattice point method of Voronoi, provide identities for the total weight of intervals $[p/q, (p+1)/q]$ containing a prescribed set of intervals $[m/n, (m+a)/n]$. (Received June 11, 1976.)

JEHUDA HARTMAN, University of California, Los Angeles, California 90024. On the homeomorphic embedding of $K_n$ and $K_{n,m}$ in the $L$-cube.

A graph $G'$ is said to be homeomorphically embeddable in $G$ if there exists a homeomorph of $G'$ which is isomorphic to a subgraph of $G$. In this paper we obtain homeomorphic embeddings of $K_n$ and $K_{n,m}$ in $Q^d$, having minimal number of edges, and prove their uniqueness up to isomorphism. These results are applied to achieve bounds on the coarseness of the $n$-cube. (Received June 10, 1976.)

GOPAL DANARAJ, Cleveland State University, Cleveland, Ohio 44115 and VICTOR KLEE, University of Washington, Seattle, Washington 98195. A connectedness game and the c-complexity of certain graphs.

When $Z$ is a finite family of finite sets such that $\bigcup Z \in Z$, there is an associated game $D(Z)$ that a certain player can always win by making enough tests, where a test is a special sort of move in the game. The complexity of $Z$ is defined as the minimum number of tests that suffices to win the game. As a specialization of this notion,
there is associated with each connected graph $G = (V, E)$ a game $C(G)$ that involves detecting, in a dynamic setting, the connectedness of a subgraph of $G$. The number of tests required to win $C(G)$ is called the $c$-complexity of $G$. Both the $c$-complexity and a closely related computational notion are shown to be $O(|V|)$ when $G$ belongs to a class of graphs that includes all paths and circuits, and this is used in the design of a linear-time shelling algorithm. The paper will appear in SIAM J. Appl. Math. (Received June 11, 1976.)


Chromatic equivalence and constrained polynomials. Preliminary report

Let $G$ be a planar graph that contains an $n$-cycle $C_n$. Then $G$ is the union of two subgraphs, the interior $G_{in}$ of $C_n$, and the exterior $G_{ex}$ of $C_n$, whose intersection is $C_n$. G. D. Birkhoff and D. C. Lewis showed that the chromatic polynomial $P(G, \lambda)$ could be expressed in terms of certain constrained polynomials, that give the number of ways in which $G_{in}$ and $G_{ex}$ can be colored in $\lambda$ colors when $C_n$ is assigned a specific color pattern. This suggests that one can construct a graph chromatically equivalent to $G$ by replacing, say, $G_{in}$ by a graph $H_{in}$ that is chromatically equivalent to $G_{in}$. We examine conditions under which this method yields chromatically equivalent graphs, and give examples of such constructions. (Received June 14, 1976.)

737-05-20 Michael P. Drazin, Purdue University, West Lafayette, Indiana 47907.

Parameter liberation in binomial and factorial identities. Preliminary report.

Given a pair of sums of (say) products of binomial coefficients, if it is desired to determine whether these are "essentially equivalent" or whether one is a special case of the other, in view of the peculiarities of binomial notation, such questions can usually be decided only after first translating the binomial coefficients into factorial (or $\Gamma$-function) form. In particular, the most general binomial sum of a given type cannot be recognized as such (and indeed a viable concept of "type" cannot even be defined) without departing from binomial notation. These familiar ideas are illustrated anew by considering the sum

$$\sum_{k=a}^{c} (-1)^{k} \binom{k}{a} \binom{a+b}{b} \binom{a+b+1}{c-k}$$

which is evaluated for arbitrary complex $\beta$ and arbitrary integers $a$, $b$, $c$ with $c \geq a$ (apparently only the cases where $\beta = 0$ or $a = 0$ having been known previously). (Received June 14, 1976.)

737-05-21 WITHDRAWN

#737-05-22 H. W. Gould, West Virginia University, Morgantown, W. Va. 26506.

An overview of combinatorial identities.

The term "combinatorial identity" may be applied in general to any mathematical identity that involves enumerative functions. This includes the usual (finite or infinite) series of factorials or binomial coefficients. But it also includes such things as Euler's formula $V + F = E + 2$, power series and Dirichlet series of certain kinds, number-theoretic relations, sums and products of natural numbers of all kinds, etc. Here we discuss the following topics: Canonical forms (binomial versus
factorial notation); metatheory; current approaches using computers; the role of special functions (hypergeometric, etc.); applications; unsolved problems; directions of current research; methods of proof (combinatorial, algebraic, function-theoretic, computer). (Received June 14, 1976.)


Theorem (Proof by computer). If G is planar, bridgeless, and \( p^e(G) = 8 \) then G is edge 3-colorable. Reference: Growth Number and Colorability of Graphs, preliminary report by J.L. Hursch Jr. to appear these Notices, August 1976. (Received June 14, 1976.)


Probabilistic considerations show that every planar map can be expected to contain a reducible configuration of ring size not exceeding 14, but that some maps can be expected to exist which do not contain any reducible configuration of ring size smaller than 13. This means that a proof of the Four-Color-Conjecture with the reduction method (i.e., a repair of Kempe's 1879 argument) can be expected to be possible, but that there is a (considerably higher) lower bound for the combinatorial complexity of such a proof, and also an upper bound for the complexity of a "reasonably short" proof. If one assumes that there is no other way of proving the Four-color-Conjecture than the reduction method then this explains the difficulty of the Four-Color Problem. In this context it is remarkable that E.P. Moore in 1963 constructed maps which apparently do not contain any reducible configuration of ring size smaller than 12 (private communication). (Received June 14, 1976.)

737-05-25 Donald McCarthy, St. John's University, Jamaica, N.Y. 11439. Minimal edge problems for graphs with generalized symmetric automorphism group. Preliminary report.

A graph on \( n \) vertices with automorphism group isomorphic to the abstract group G is termed a \( (G,n) \)-graph. Consider the following problems: given a finite group G, for each positive integer \( n \) determine \( e(G,n) \) and \( e_c(G,n) \); these denote the minimum number of edges among all (resp. all connected) \( (G,n) \)-graphs. Solutions exist when G is symmetric, dihedral or hyperoctahedral (for references, see Discrete Math 14(1976)139-156). Extensive partial results have also been obtained for certain generalized symmetric groups. These are the complete monomial groups \( S_k \langle F_p \rangle \), i.e. the (permutational) wreath products of the cyclic group \( C_p \) by the symmetric group \( S_k \) where \( p \) is an odd prime and \( k \geq 1 \). In particular, when \( G = S_k \langle F_p \rangle \) we have \( e(G,n) = e(S_1,n-6kp) \) for \( n \geq 6kp + 6 \). Also, \( e_c(G,n) \leq nkp-2 \) whenever \( n > 4kp \), with equality holding in many cases; when \( k=1 \), for example, equality holds except when \( n = mp \geq 6p \), and in these latter cases \( e_c(G,n) = n \). (Received June 14, 1976.)


Many problems that arise in health studies may be naturally expressed in terms of counting and discrete probability problems; this is illustrated in the presentation. The search for solutions and simplifications gives rise to combinatorial identities. Some unsolved problems are introduced. (Received June 15, 1976.)


Let \( f \) denote the asymptotic series \( \sum_{n=0}^{\infty} (-1)^n n! x^n \). Different ways of computing power series coefficients of \( f^p \), \( p \) integral, give rise to a multiple sum generalization of a well-known series identity (H.W. Gould, "Combinatorial Identities", p. 21, line 2,25) which corresponds to \( p = 2 \). The case \( p = 3 \) was discovered by means of recurrence relations obtained from a linear differential equation for \( f^3 \). Generalization to arbitrary integral \( p \) is facilitated by the following Lemma. Define inductively the function sequences \( F_p(y) \) and \( G_p(y) \). \( F_1 = G_1 = (1 + y)^{-1} \). For \( p \geq 2 \), \( F_p(y) = \int_0^y f^{p-1}(z)(1 + y - z)^{-1} dz \) and \( G_p(y) = \int_0^y y + F_p(y)^{-1} f(p-1)(y)^{-1} \). Then \( F_p(y) = G_p(y) \).

Theorem. Let \( f = \sum_{n=0}^{\infty} (-1)^n (n + p - 1)! q_p, n^n, q_{1,n}, 1 \). Then for \( p \geq 2 \), \( q_{p,n} = \sum_{j=0}^{p-1} \sum_{j=0}^{p-1} q_{p-1,j} = 1 \), and alternatively, \( q_{p,n} = (n + p - 1)^{-1} \sum_{j=0}^{p-1} j^{p-1} \). (Received June 15, 1976.)
In this paper we shall discuss various applications of the following
identity: \[ r! \sum_{s \geq r} (-1)^{r-s} \binom{n}{s} n^s S(s, r). \] (Received June 15, 1976).

For any tree \( T \), let \( T(n) \) be the \( n \)-th level of \( T \). Call a set of nodes \( S \subseteq T \) \((h,k)-dense\) if there is a node \( x \) of level \( h \) such that every node of level \( h+k \) which succeeds \( x \) in turn precedes some node in \( S \). Consider a sequence of trees \( T_i, 1 \leq i \leq k \). For any \( n \), a set \( M \subseteq \prod_{1 \leq k \leq k} T_i(n) \) is called an \((h,k)-matrix\) if there are \((h,k)-dense\) \( M_i \subseteq T_i(n) \) such that \( M = \prod_{1 \leq k \leq k} M_i \). We prove the following theorem as a consequence of the partition theorem of Halpern and Lauchli (Trans. Amer. Math. Soc. 124 (1966), 360-367). Theorem. Let \( T_i, 1 \leq i \leq k \), be perfect trees. If \( \bigcup_{n \in \omega} \prod_{1 \leq k \leq k} T_i(n) = \bigcup_{j < p} P_j \), then there exists \( j \) and \( h \) such that for all \( k \), \( P_j \) includes an \((h,k)-matrix\). Results previously obtained by Laver, Pincus, and Milliken using a method of word transformations are obtainable from the theorem of Halpern and Lauchli via our theorem without the use of such a method.

(Received June 15, 1976.)

F. Walder (On pairs of nonintersecting faces of cell complexes, Proc. Amer. Math. Soc., vol. 51 no. 2 (1975), 438-440) has shown that if the underlying space \( |K| \) of \( K \) is a manifold then \( \chi(2)(K) = \chi^2(K) - \chi(K) \). We generalize his result to arbitrary \( n \) when \( |K| \) is a 2-manifold - that is \( \chi(n)(K) = \chi^n(K) - \chi(K) \). Applications to graph-embedding problems are given. (Received June 15, 1976.)

#737-05-33 Michael R. Rolle, Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Ontario, Canada. General solutions for free chromatic polynomials on the plane.

This paper ties in with recent work of D.W. Hall (Trans. AMS 1948 and a paper presented in Kalamazoo, 1975), ideas conceived in the Birkhoff-Lewis paper on chromatic polynomials, and further mentioned in a paper of F. Bergehart at Kalamazoo 1975. There are some interesting applications, according to Biggs and Kasteleyn, of these results to theoretical physics, relating to energy levels in plane crystals.

The present paper is concerned with planar partitions, which can be associated with rooted planar trees. The matrix \( A \), for a particular \( n \), is constructed as follows: For any two partitions \( P \) and \( Q \), the element \( a_{P,Q} \) is defined as the monomial \( x_k \) where \( k \) is the number of parts of the join of the partitions. The second and third partitions above, give the join \((135,26,4)\), the first and third give \((13,2,46,5)\), and the first and second give \((1,246,3,5)\). We prove the following theorem as a consequence of the partition theorem of Halpern and Lauchli (Trans. Amer. Math. Soc. 124 (1966), 360-367). Theorem. Let \( T_i, 1 \leq i \leq k \), be perfect trees. If \( \bigcup_{n \in \omega} \prod_{1 \leq k \leq k} T_i(n) = \bigcup_{j < p} P_j \), then there exists \( j \) and \( h \) such that for all \( k \), \( P_j \) includes an \((h,k)-matrix\). Results previously obtained by Laver, Pincus, and Milliken using a method of word transformations are obtainable from the theorem of Halpern and Lauchli via our theorem without the use of such a method.

(Received June 15, 1976.)

#737-05-32 Stéphane Foldes, Department of Combinatorics and Optimization, University of Waterloo, Waterloo, Ontario, Canada. On the chromatic polynomial and the rotor effect.

The rotor effect is a transformation of a graph \( G \) in which a certain subgraph \( H \) having a rotational symmetry is replaced by its mirror image. Generally the new graph is not isomorphic with \( G \). Unlike the tree number, the chromatic polynomial is generally not invariant in the rotor effect.

The problem of the invariance of the highest coefficients is examined in detail. (Received June 15, 1976.) (Author introduced by W. T. Tutte.)

A-513
Planar sublattices of a free lattice.

A lattice \( L \) is semidistributive if, for every \( a, b, c \in L \), \( ab = ac \) implies \( ab = ac \) and \( ab = ab \). Furthermore, \( L \) satisfies Whitman's condition \((W)\) if, for every \( a, b, d \in L \), \( a \leq b \) implies \( a \leq a \) or \( a \leq a \) or \( a \leq b \). Theorem. A finite planar lattice is embeddable in a free lattice if and only if it is semidistributive and satisfies \((W)\). In addition, we construct a list \( \mathcal{L} \) of finite lattices subject to the following condition.

A finite lattice is a planar sublattice of a free lattice if and only if it contains no sublattice isomorphic to a lattice in \( \mathcal{L} \). Moreover, \( \mathcal{L} \) is the minimum such list. (Received June 7, 1976.)

Inductive Boolean Algebras and Special Prime Ideals.

Let \((A, +, \cdot, \leq)\) be a Boolean Algebra with unit \(1\) and zero \(0\). Let \(k\) be a cardinal number. Definition. \(A\) is called \(k\)-inductive if and only if every well ordered (w.r.t. \(\leq\)) subset \(W\) of nonunit elements of \(A\) with \(W < k\) has a nonunit upper bound. Theorem. Let \(p\) and \(q\) be elements of a \(k\)-inductive Boolean algebra \(A\) such that \((l+p)q \neq 0\), and let \((S_i)_{i \in k}\) and \((H_i)_{i \in k}\) be families of subsets \(S_i\) and \(H_i\) of \(A\) such that \(S_i = \text{lub} S_i\) and \(H_i = \text{glb} H_i\) for every \(i \in k\). Then \(A\) has a prime ideal \(P\) such that \(p \in P\) and if \(S_i \subseteq P\) then \(S_i \subseteq P\) for every \(i \in k\); moreover, \(q \in (A-P)\) and if \(H_i \subseteq (A-P)\) then \(H_i \subseteq (A-P)\) for every \(i \in k\).

(The term \(k\)-inductive is used in contradistinction to \(k\)-inductive since the latter usually refers to a partially ordered set in which every well ordered subset of cardinality less than \(k\) has a least upper bound). This paper extends the results of Abstract *76T-El9, these Notices, 23(1976),A-289. (Received June 10, 1976.)

Representation theorems for varieties generated by single precomplete algebras.

Ivo Rosenberg (MR 45, #1732) determined all precomplete (finitary) algebras with finite carrier, and these algebras split up naturally into six classes. Any algebra \(A\) generates a variety \(V(A)\), the smallest equational class containing \(A\). By a theorem of G. Birkhoff, each algebra of any variety \(V\) is a subdirect product of the set \(Sl(V)\) of subdirectly irreducible algebras of \(V\). For all algebras \(A\) in five of Rosenberg's classes we find \(Sl(V(A))\); each such set has at most three elements. The one class of precomplete algebras \(A\) for which we could not find all the \(Sl(V(A))\) is made up of algebras all of whose operations are preserved by a partial order with largest and smallest elements. (Received June 14, 1976.)

Proof of certain identities conjectured by Ramanujan. Preliminary report.

E.J. Birch (Math. Proc. Cambridge Philos. Soc., 28 (1975), 73-79) published 40 identities conjectured by Ramanujan involving the functions \(G(x) = \prod_{n=1}^{\infty} \frac{1}{(1-x^{5n})} \) and \(H(x) = \prod_{n=1}^{\infty} \frac{1}{(1-x^{5n})} \). L.J. Rogers (Proc. London Math. Soc., 12 (1921), 387-397) and G.N. Watson (J. Indian Math. Soc., 20, A-514)
(1933), 57-69) had proven 16 of these identities. By combining and extending the methods of Rogers and Watson, we additionally prove the identities which Birch has numbered: 12, 13, 14, 17, 19, 20, 21, 22, 25, 26, 27, 28, 32, 38, and a corrected version of 39. We also formalize Rogers' approach and demonstrate how it can be used to generate and prove similar identities. (Received May 24, 1976.)

*737-10-2 CHARLES SMALL, Queens University, Kingston, Ontario, Canada. Waring's problem mod \( n \).

This paper has been presented by title in these Notices 22(1975), A-615, Abstract 75T-A214; the present version includes some new results. The goal is to compute \( g(k, n) \), the smallest \( r \) such that everything is a sum of \( r \) \( k \)th powers mod \( n \); we get complete results for \( k = 2 \) and 3, and partial results for larger \( k \). The heart of the problem is the computation of \( g(k, p) \) where \( p \) is prime, \( k \mid p - 1 \), \( 3k + 1 \leq p \leq (k - 1)^2 \); here we can prove only \( 2 \leq g(k, p) \leq \lceil k/2 \rceil + 1 \). (Received June 7, 1976.)

*737-10-3 CHUNGMING AN, Seton Hall University, So. Orange, N.J. 07079 and ALAN H. STEIN, University of Connecticut, Waterbury, Ct. 06701. The asymptotic behaviour of zeta functions associated to positive definite forms with integer coefficients. Preliminary report.

Let \( F(x) \) be a positive definite form with \( n \) variables \( x=(x_1,\ldots,x_n) \) and let \( a_k \) be the number of integral solutions to \( F(x)=k \). The analytical property of the zeta function \( \zeta(F,s)=\sum \frac{1}{y^s} \), with sum over all non-zero integral vectors, has been established by C. An (Mich. Math. J., vol. 21(1974), pp. 45-68). In this paper we shall consider the following two results. 1) The crude estimate \( \left| \zeta(F,\sigma+it) \right| = O(t^{-\frac{n}{2d}}) \), for \( n-1 \leq \sigma \leq \frac{n+1}{d} \).

2) The asymptotic formula \( \sum_{k \leq y} a_k = (\lambda d/n)y^{n/d} + O(y^{n+\varepsilon}), \) where \( \lambda = \text{Res} \left( \zeta(F,s) \right) \). (Received June 9, 1976.)


Instead of the integral, \( \int_0^1 \sum_{j=0}^{n-1} e(2\pi inj/d \mu) \) one can use the non-standard sum, \( \sum_{j=0}^{n-1} e(2\pi n j \mu) \). To obtain the following formula: \( A(n,k,H) = \lim \sum_{j=0}^{n-1} e(kH) \) where \( A(n,k,H) \) is the number of positive integral solutions of \( x_1^k + x_2^k + \ldots + x_m^k = H \) and \( H \) is given by \( \left( \sum_{x \in I} e(kx/H) \right)^m e(-H^m/n) \). Let us remark that \( A(n,k,H) \) is the number of solutions modulo \( n \) of \( x_1^k + x_2^k + \ldots + x_m^k \equiv H \pmod{n} \) is given by \( \left( \sum_{x \in I} e(kx/H) \right)^m e(-H^m/n) \). (Received June 10, 1976.)

*737-10-5 GERALD E. BERGUM and MARY ANN DAHLQUIST, South Dakota State University, Brookings, South Dakota 57006. Special pythagorean triples. Preliminary report.

**Theorem 1.** If \( n = 2(2^m + 1) \), then there exist positive integers \( A \) and \( B \) relatively prime, \( A \) even, and such that \( A^2 + B^2 \) and \( A^2 + (B - na)^2 \) are both perfect squares. **Theorem 2.** If \( n = 2^m + 3, m \geq 4, \) there do not exist positive integers \( A \) and \( B \), relatively prime, \( A \) even, and such that \( A^2 + B^2 \) and \( A^2 + (B - na)^2 \) are both perfect squares. For \( n = 8 \) it is shown that the \( A \) and \( B \) of Theorem 1 do exist and that the least value of \( A \) is 2,996,760. (Received June 14, 1976.)

*737-10-6 Joseph Arkin, 197 Old Nyack Turnpike, Spring Valley, New York 10977. New observations on "Fermat's last Theorem".

In about 1637 Pierre de Fermat wrote what has become known as "Fermat's last Theorem", namely that there do not exist integers \( x, y, \) and \( z \), none of which are zero, which satisfy \( x^n + y^n = z^n \) for \( n > 2 \). It is not difficult to see, that to prove the impossibility of (1), we
need only consider when \( n > 2 \) is an odd prime. Now, if in (1), \( x, y \) and \( z \) are prime to each other and to \( n \) (where \( n \geq 3 \) is an odd prime), this condition is referred to as case I; if \( x, y \) and \( z \) are prime to each other and one of them is divisible by \( n \), the condition is referred to as case II.

In case II very little has been discovered concerning (1); however, in case I everything indicates that (1) is impossible.

In this paper I have made some new observations concerning the difficult case II. With a new simple approach this author has found several \( n \) (where \( n \geq 3 \) is an odd prime) that can only belong to case II.

The author wishes to thank Professors R. Pollack (NYU) and D.J. Newman (Yeshiva Univ.) for their generous information over the telephone on some of what has and has not been found on "Fermat's Last Theorem". (Received June 15, 1976.)

12 \boldsymbol{\text{Algebraic Number Theory, Field Theory and Polynomials}}

*737-12-1 MARION BEITER, Rosary Hill College, Buffalo, New York 14226. Coefficients of the cyclotomic polynomial \( F_{3qr}(x) \). Preliminary report.

Let \( r \) and \( q \) be odd primes such that \( 3 < q < r \). Let \( r = (6q \pm 1)/h \), \( 0 < h \leq (q - 1)/2 \). In this paper we prove that no coefficient in the cyclotomic polynomial of index \( 3qr \) is greater than one in absolute value when one of the following hold: (a) \( h = 1, k \equiv 0 \) (mod 3), (b) \( h > 1, k \equiv 0 \) and \( h + q \equiv 0 \) (mod 3), (c) \( h > 1, h \equiv 0 \) and \( k + r \equiv 0 \) (mod 3). (Received June 9, 1976.)

737-12-2 E. G. Van Meter, State University College, Oneonta, New York 13820. Counting permutation polynomials in \( n \) indeterminates over \( GF(q) \). Preliminary report.

A polynomial \( f \) in \( n \) indeterminates over \( GF(q) \) is called a \( (q,n) \)-permutation polynomial if the number of solutions in \( GF(q)^n \) of the equation \( f(x_1, \ldots, x_n) = a \) is independent of \( a \), which requires that it be \( q^{n-1} \) for all \( a \in GF(q) \). This definition yields the familiar notion of a permutation polynomial over \( GF(q) \) if \( n = 1 \). In this paper, the number of \( (q,n) \)-permutation polynomials is determined. (Received June 14, 1976.)

737-12-3 ROBERT ROSS WILSON, California State University, Long Beach, California 90840. Geometric Characterizations of Lattice Orders on Fields. Preliminary Report.

Suppose that \( K \) is any proper subfield of the real field \( \mathbb{R} \), a real algebraic over \( K \) with minimal polynomial \( f \) and that \( f \) factors over \( \mathbb{R} \) into \( r \) linear factors and \( s \) quadratic factors. By viewing the real algebra \( \mathbb{A} = \mathbb{R}^r \oplus \mathbb{C}^s \) in terms of idempotents so that \( \mathbb{E} = \mathbb{E}_1 \oplus \mathbb{E}_2 \oplus \mathbb{E}_3 \) and \( \mathbb{K}(a) \) as a subfield of \( \mathbb{A} \), we may characterize the positive cone \( P \) of any lattice order on \( \mathbb{K}(a) \) by: Theorem. The crosssection of \( P \) by the \( a_1 = 1 \) hyperplane must be a simplex inside \([-1,1]^r \times [0,1]^s \), where \( \mathbb{D} \) is the unit disc. Furthermore, the product of any two simplex corners must lie in the simplex. This readily generalizes to any formally real field different from \( \mathbb{R} \). (Received June 14, 1976.)

737-12-4 GERHARD F. KOHLMAYR, Mathmodel Consulting Bureau, Glastonbury, Connecticut 06033. Normed fields and topological fields which are not copies of \( \mathbb{C} \).

Let \( \mathbb{C} \) be the complex number field. By a copy of \( \mathbb{C} \) is meant any field which is ring-isomorphic and homeomorphic to \( \mathbb{C} \). Theorem I. Let \( n \) and \( m \) be positive integers. For every \( n \) there is a normed complex field \( \mathbb{X}_n \) which is not a copy of \( \mathbb{C} \). Further, when \( n \neq m \), \( \mathbb{X}_n \) is not a copy of \( \mathbb{X}_m \). Theorem I provides a countable number of counterexamples to Theorem I by S. Mazur (C.R. Acad. Sci. Paris 207(1938), 1027). A quasi-normed topological field is a quasi-normed linear space which is also a Hausdorff topological field whose topology is metric and determined by a quasi-norm. Theorem II. For every \( n \) there is a quasi-normed complex topological field \( \mathbb{X}_n \) which is not a copy of \( \mathbb{C} \) and such that \( n \neq m \) implies \( \mathbb{X}_n \neq \mathbb{X}_m \). A Frechet topological field is a complete quasi-normed topological field. Theorem III. For every \( n \) there is a separable complex Frechet.
topological field $X_n$ which is not a copy of $C$ and such that $n \neq m$ implies $X_n \neq X_m$. Theorem III provides a countable number of counterexamples to Theorem 3 by R. Arens (Bull. Amer. Math. Soc. 52(1947), 627).

(Received June 14, 1976.)

13 ▶ Commutative Rings and Algebras

*737-13-1 Dr. A.G. NAOUM, Math. Dept., College of Science, University of Baghdad, Baghdad, Iraq. Finently generated projective ideals in commutative rings.

Let $R$ be a commutative ring with 1. An $n$-tuple $x = (x_1, x_2, \ldots, x_n)$ of elements of $R$ will be called semi-regular iff $\exists$ and $n \times n$ matrix $A = (a_{ij})$ with elements in $R$ such that $x A = x$ and

$$\{y = (y_1, y_2, \ldots, y_n) \mid y^T x = 0 \subseteq \{y \mid A y^T = 0^T, 0 = (0, 0, \ldots, 0)\}$$

where multiplication is the usual multiplication of matrices.

Theorem. An ideal generated by $x_1, x_2, \ldots, x_n$ in $R$ is projective iff the $n$-tuple $x = (x_1, x_2, \ldots, x_n)$ is semi-regular. (Received February 24, 1976.)

15 ▶ Linear and Multilinear Algebra; Matrix Theory (finite and infinite)


Consider a system of $N$ linear transformations $A_1, \ldots, A_N : V \to W$, where $V$ and $W$ are complex vector spaces. Denote it for short by $(V,W)$. A pair of subspaces $X \subseteq V, Y \subseteq W$ such that $\sum_{j=1}^{N} A_j X \subseteq Y$ determines a subsystem $(X,Y)$ and a quotient system $(V/X, W/Y)$ (with the induced transformations). The system $(X,Y)$ is of finite codimension in $(V,W)$ iff $V/X$ and $W/Y$ are finite-dimensional. It is a direct summand of $(V,W)$ in case there exist supplementary subspaces $P$ of $X$ in $V$ and $Q$ of $Y$ in $W$ such that $(P,Q)$ is a subsystem. Theorem. If $(X,Y)$ is of finite codimension in $(V,W)$ and for every subsystem $(U,Z)$ of finite codimension in $(X,Y)$, $(X/U,Y/Z)$ is a direct summand of $(V/U,W/Z)$, then $(X,Y)$ is a direct summand of $(V,W)$. Consequently a subsystem is pure iff it is copure.

The proof uses a dual theorem (Aronszajn and Fixman, Algebraic spectral problems, Studia Math. 30(1968), 273-338, Theorem 5.5, in case $N = 2$) and systems of $N$ continuous linear transformations between topological vector spaces. (Received February 4, 1976.) (Author introduced by U. Fixman.)


Capital letters denote $n$ by $n$ matrices with elements belonging to the field $\mathbb{C}$ of complex numbers. Let $f_{\alpha}(\lambda)$ denote a polynomial of degree $s$ with coefficients belonging to $\mathbb{C}$ and defined on the spectrum of $\bar{R}$ where $\bar{f}_{\alpha}(\bar{R})$ are given by

$$\bar{R} = \begin{pmatrix} -A & 0 & -D \\ 0 & I & 0 \\ -C & 0 & A \end{pmatrix}, \quad \bar{f}_{\alpha}(\bar{R}) = \begin{pmatrix} U & \bar{G} & M \\ P & Q & R \\ V & S & N \end{pmatrix}. \quad T = \text{transpose}, \quad D^T = D$$

and $U$ or $M$ and $\bar{G}$ are nonsingular. Theorem If $X$ is a solution of $(X,0,I)f_{\alpha}(\bar{R}) = (0,0,0)$ then $X$ is also a solution of the Riccati Matrix equation (*) $A^T X + XA + C - XDZ = 0$ and (**)$A^T X = -I$. If $X = X^T$ then such a solution is unique. (Received June 8, 1976.)


The long development of matrix theory, which deals primarily with linear mappings and bilinear functions, has never been matched in depth by a similar development...
for three-way arrays. Today that is beginning to change. One source of new problems are the INDSCAL and CANDECOMP models, which pose questions about trilinear functions. Another source is arithmetic complexity theory, which asks questions like "what is the minimum number of multiplications needed to multiply two nxn matrices?" Both of these applications involve the representation of a three-way array as a sum of triads. The minimum number required is the rank of the array, a concept which significantly generalizes the rank of a matrix. Finding the rank of a given array is a challenging task. Two $4 \times 4 \times 4$ arrays of particular significance are known to have rank 7 and 8.

Several interesting theorems have already been proved. The prettiest one generalizes a classical lower bound on matrix rank due to Frobenius (1911). Another gives sufficient conditions for a representation (of an array by triads) to be essentially unique. While several results will be precisely stated, no proofs will be presented, to avoid undue complex detail. An effort will also be made to identify some interesting questions, both pure and applied, which are unsolved but probably within reach. (Received June 14, 1976.)


The association of a finite partially ordered set $S$ (singular graph) with a singular M-matrix $A$ is discussed in Abstract 726-15-10, Notices, August, 1975. Also defined there are $\alpha(A)$, the Weyr characteristic of $A$ (for 0), and $\Lambda_p(A)$ the $p$-th level of $S$ (read "maximal" in place of "minimal"). $\lambda^p$, the cardinality of $\Lambda^p$, is called the $p$-th level number of $S$.

For $\alpha \in \Lambda_p(A), p \geq 2$, let $\Delta(\alpha) = \{ \beta \in S; \beta > \alpha \} \cap \Lambda_{p-1}$. $S$ is said to have SDR if for all $p \geq 2$ and for all subsets $\Lambda \subseteq \Lambda^p(A)$, $\alpha \in \Lambda$, have a system of distinct representatives.

Denote by $\#(S)$ the class of all M-matrices with singular graph $S$.

**Theorem:** For non-empty $S$ with level numbers $(\lambda^1, \ldots, \lambda^h)$, the following are equivalent:

a) there exists $A \in \#(S)$ such that $\alpha(A) = (\lambda^1, \lambda^2, \ldots, \lambda^h)$;

b) $S$ has SDR;

c) $\lambda^1 \geq \lambda^2 \geq \ldots \geq \lambda^h$ and there exists a chain decomposition $C_1, C_2, \ldots, C_k$ of $S$ such that $(C_1, \ldots, C_k)$ and $(\lambda^1, \ldots, \lambda^h)$ are conjugate partitions, where $c_i$ is the cardinality of $C_i$.

(Received June 14, 1976.)


Previous investigative work by the author resulted in the application of the Eigenvalue concept to a physical problem involving a set of linear homogeneous equations. A resulting paper was titled "Study of Weapon Worth Concepts for Determining the Value of Diverse Weapon Systems in Combined Arms Battles," (preliminary report). It was found that the solution obtained by the use of the Eigenvalue concept gave unexpected results which ran contrary to intuitive requirements of the particular physical problem under investigation.

The paper cited above has been offered for presentation to the forthcoming meeting of American Mathematical Society in Toronto, Aug 76. As a result of the unsatisfactory results obtained from the application of the Eigenvalue concept to the specific problem, the author looked for and found a model that did not depend on Eigenvalues. The model is new insofar as the author could determine. It is based upon the concept of adding a constant $b_i A$ to each linear homogeneous equation, where $b_i$ is the number of weapons of type $i$ and $A$ is the value per ith weapon if the weapon did not kill or was not killed. It is shown that the solution for the ratio of the value per weapon for one weapon to another weapon is independent of the value of $A$. As a result of this concept, it has also become possible to avoid dependence on the Eigenvalue method of solution that has been so prominent and subject to criticism in the past.

The new model of the weapon value problem leads to much broader questions of the relationship between the singular solution of a nonhomogeneous linear system and the Eigenvalue solution of a corresponding homogeneous linear system of equations. (Received June 14, 1976.)

(Author introduced by Dr. Robert M. Thrall.)

**16 Associative Rings and Algebras**

737-16-1 Dave Riffel, Cornell University, Ithaca, New York 14853. **Swedler's two-cocycles and a theorem of Rosenberg and Zelinsky.** Preliminary report.

Let $k$ be a commutative ring and $C$ be a k-algebra with nilpotent ideal $I$. $\overline{A}^2(C)$ denotes the set of equivalence classes of Swedler's C-two-cocycles. Equivalence means cohomologous via a vertible element (see Sweedler, Ill. J. of Math., 15(1971), 302-323 for definitions). **Theorem 1.** If $C/I$ is a projective k-module, the map $\overline{A}^2(C) \rightarrow \overline{A}^2(C/I)$ induced by the natural projection $C \rightarrow C/I$ is bijective. **Corollary.** (cf Prop. 3.3, A-518
Rosenberg and Zelinsky, Osaka Math. J., 14(1962), 219-240) If $C$ is commutative and $C/I$ is $k$-projective, the natural map $H^2(C) \rightarrow H^2(C/I)$ on Amitsur cohomology is an isomorphism.

**Theorem 2.** Let $k$ be a perfect field and $\dim_k(C/J(C))<\infty$ with $J(C)$ the Jacobson radical of $C$. Suppose $C/J(C)$ is commutative and $\{x \in C \mid \sum a_i x b_i = 0\}$ is an ideal of square zero for some separability idempotent $\sum a_i \otimes b_i$ of $C/J(C)$. (Received April 22, 1976.)


Let $T$ be a ring extension of $R$, where $T$ is generated over $R$ by normalizing elements and $T$ is a free $R$-module. We investigate conditions under which $T$ may be rationally complete. Particular attention is paid to twisted Laurent polynomial rings and group rings as well as tensor products of algebras. Theorem 1. A countable semiprime right rationally complete ring is left nonsingular. As a consequence we have Theorem 2. If $G$ is a countable group and the group algebra $FG$ is semiprime and rationally complete, then $G$ is finite. Some partial reductions to the semiprime case are given. Theorem 3. A twisted Laurent polynomial ring over the rational closure of a countable commutative ring with dense singular ideal is not rationally complete. There exists a rationally complete twisted Laurent polynomial ring over a commutative ring with dense singular ideal. (Received May 18, 1976.)


Let $R$ be a ring and $S$ a radical property in the sense of Amitsure. The relation $S(R_n) = S(R)_n$, where $R_n$ is the ring of $n$ by $n$ matrices, is known to be true for many radical properties, e.g. Jacobson, Brown-McCoy, etc. The purpose of this talk is prove some results about the $S$-radical ideals of $R_n$ and the computation of $S(R_n)$ using the techniques developed by Amitsure and Sands. (Received June 1, 1976.) (Author introduced by Mr. M Razzaghi.)

*737-16-4* Hal G. Moore, Brigham Young University, Provo, UT 84602 and Adil Yaqub, University of California, Santa Barbara, CA 93106. Certain generalizations of Boolean rings.

Let $R$ be a ring of prime characteristic $p$ and with Jacobson radical $J \neq R$. Suppose further that for every $x$ in $R$, $x$ is nilpotent or $x^{p^n} = x$ for some positive integer $n = n(x)$. Then $R/J$ is a subdirect sum of fields, $R/J$ is commutative, and the commutator ideal of $R$ is nil. If, in addition, the ground ring $R$ has a unity — but not in general otherwise — then $R$ itself is necessarily commutative. Similar results are obtained when $R$ contains a nonzero central idempotent or when $R$ contains a nonzero idempotent and the idempotent elements of $R$ commute with each other. The proofs use the structure theory. (Received June 7, 1976.)

17 ▶ Nonassociative Rings and Algebras

*737-17-1* GARY E. STEVENS, Hartwick College, Oneonta, New York 13820. Some counterexamples for infinite dimensional Lie algebras.

Two theorems of J. S. Wilson (Math. Z. 114(1970), 19-21), useful in the study of chain conditions on normal subgroups of infinite groups, state that the maximal and minimal conditions on normal subgroups are inherited by subgroups of finite index. The analogous statements for Lie algebras are not valid, and in this paper we construct counterexamples to illustrate this. (Received June 14, 1976.)

18 ▶ Category Theory, Homological Algebra


For $V$ a monoidal category, $\text{SET}^{\text{op}}$-categories are discussed from the point of view of categories.
equipped with $V$-indexed families of morphisms using techniques similar to those of Paré and Schumacher. (See these NOTICES, Nov. 1975, Abstract 728-A43.) Various simplifications to the usual theory of $V$-categories are given. SET-$V^O$-categories are identified as category objects of "type: $N + N \leftarrow KV \rightarrow N \rightarrow 0$ in a suitable category with finite limits. $V$-indexed categories are defined as category objects of arbitrary "type", $\hat{N}$, where $\hat{N}$ is a co-category object in the category of finitely presented monoids. Comm objects of $V$-indexed functors are strong enough to discuss $V$-discrete $V$-(co)fibrations. $V$-limits in the sense of Kelly and Borceux can then be studied in the same way as ordinary limits. Further generalizations and applications are discussed. (Received May 21, 1976.)


Let $V$ be a closed category with equalizers and let $G$ be a closed comonad on $V$ satisfying a number of additional conditions. Then the category $V_G$ of $G$-coalgebras in $V$ is closed, the adjoint functors $V_G:V_G \rightarrow V$ and $H_G:V \rightarrow V_G$ are closed and the adjunction transformations $\varepsilon_G:U_G \rightarrow V$ and $\eta_G:V \rightarrow H_G$ are closed. Similarly, if $V$ is symmetric monoidal closed with equalizers and $G$ is symmetric monoidal closed with additional conditions, then $V_G$ is symmetric monoidal closed, $H_G$ and $U_G$ are symmetric closed and $\varepsilon_G$ and $\eta_G$ are closed. For example, if one takes $V = \text{Ab} = [abelian groups]$ with a suitable $G$, one obtains that $V_G = [abelian groups]$ with an endomorphism $G$ is symmetric monoidal closed. Similarly, if one takes $V = \text{R-Mod}$ for $R$ the underlying ring of a differential ring with a suitable $G$, one obtains $V_G = [differential R-modules]$ is symmetric monoidal closed. (Received May 24, 1976.)

737-18-3 Harvey Wolff, University of Toledo, Toledo, Ohio 43606. Distributive laws and the Lifting of Triples.

In Beck's paper on distributive laws (Lect. Notes. in Math. 80, 119-140) he raises the question of the "possible extension of the distributive law formalism to non-tripleable situations." We answer this question by giving a condition, which is automatic in the tripleable case, for lifting a triple $\mathcal{T}$ on $A$ to a triple $\mathcal{L}$ on $B$ where there is an adjoint pair $F:U:A \rightarrow B$ and a distributive law $\lambda:UFT \rightarrow TUF$. While $\lambda$ does not determine a unique lifting in general, the lifting $\mathcal{L}$ of $\mathcal{T}$ does satisfy the property that if $S$ is any lifting of $\mathcal{T}$ which determines the given distributive law $\lambda$ then there is a unique triple map $\Theta: \mathcal{L} \rightarrow S$ such that $U(\Theta)$ is an equivalence. Furthermore, we show that the algebras for $\mathcal{L}$ are in fact a pullback of the algebras for the lifted monad $\mathcal{T}$ on $A^UF$ (the algebras of the triple generated by $F:U$) over the canonical comparison functor $K: B \rightarrow A^UF$. (Received May 24, 1976.)


The sheaves for the canonical topology on the monoid $M$ of continuous selfmaps of a 1-cell are described. Such a sheaf $X$ is an $M$-set with two more types of partial operations. An element $\pi$ of $X$ is a sort of "path"; by applying suitable maps in $M$ one can obtain the first half of $\pi$, the points of $\pi$, etc. Two paths $\pi_1, \pi_2$ in $X$ can be amalgamated if a non-degenerate end segment of $\pi_1$ coincides with a beginning segment of $\pi_2$, but not if only endpoints agree. In the latter case, however, one has splice $\pi_1 \circ \pi_0 \circ \pi_2$ where $\pi_0$ is a constant path, giving a definite well-behaved operation on homotopy classes. Finally for certain $\alpha \in M$, the covering maps, any $\pi$ satisfying all equations $\pi \beta = \pi \gamma$ satisfied by $\alpha$ can be lifted to $\pi$, where $\pi \alpha = \pi$. They are generated by amalgamation from those $\alpha_0$ such that the pullback of $\alpha_0$ along any $\gamma \in M$ contains a path projecting onto the domain of $\gamma$, which are those maps mapping some subinterval $J$ of the cell $I$ lightly onto $I$ so that each interior point $p$ of $J$ has a neighborhood composed of two intervals on each of which $\alpha_0(x) - \alpha_0(p)$ does not change sign. (Received May 24, 1976.)


We pursue the study of the duality of functors (in the sense of Fuks, Mityagin, and Svarc) in the
category of Banach spaces as developed in the recent joint paper of the author with C. Herz (see Abstract # 716-B1, Notices, October 1974). Following the discussion in the above mentioned work of the functor $\text{INT}(A,\cdot)$ (integral operators) as the dual functor of $A\cdot\cdot$, where $A\cdot\cdot$ is the closure in $\text{HOM}(A',X)$ of $A\cdot X$, we examine further properties of $\text{INT}(A,\cdot)$, including its relation to $A\cdot\cdot'$. The latter question is related to the Radon-Nikodym property, equivalent formulations of which are given in the context of dual functors. Weakly compact and compact operators are also discussed in this setting. (Received May 25, 1976.)


Let $A$ be a reflective subcategory of a varietal category, $B$ the category of (a) uniform spaces, (b) topological spaces. An object $I$ "sitting in both categories" establishes a pair of adjoint functors $U: A^{\text{OP}} \rightarrow B$, $F: B \rightarrow A^{\text{OP}}$, and therefore a duality between the full subcategories $\text{Fix } U$ and $\text{Fix } F$. Of interest is the situation when $\text{Fix } U$ is the limit closure of $I$ in $A$ and $\text{Fix } F$ is the limit closure of $I$ in $B$. In the uniform case this certainly happens when $I$ (as a uniform space) is compact, separated, injective in its limit closure and every uniformly continuous mapping $I^n + I$, for finite $n$, can be uniformly approximated arbitrarily closely by operations. When $I$ is discrete, this theorem reduces to a result by Hu. In the topological case a somewhat more complicated set of sufficient conditions has been obtained. The Gelfand and Stone dualities are special cases of both situations. (Received May 27, 1976.)

737-18-7 Ross Street, Macquarie University, North Ryde, Australia 2113. Internally Complete 2-categories. Preliminary report.

It is reasonable to call a category $E$ internally complete when it is finitely complete and, for each object $X$ of $E$, the category $E_X$ of objects over $X$ is cartesian closed. For example, if $E$ is a topos it is well-known that this is the case. Alas, however, the category $\text{Cat}(E)$ of category objects in $E$ need not be internally complete if $E$ is. $\text{Cat}(E)$ is more than a category, it is a 2-category. This suggests that an appropriate notion of internal completeness is required for a 2-category $K$. We define a 2-category $K$ to be internally complete when it is finitely complete and possesses certain internal versions of the indexed limits of Street [Limits indexed by category-valued 2-functors, J. Pure and Applied Algebra 10 (1976)]. One property of such a $K$ is that the 2-category of split 0-fibrations over any object is cartesian closed. (Received May 28, 1976.)


After reviewing the bivariate description from the author's Perugia Notes (December 1972) of those fibrations $Y$ over a base topos $B$ which are $B$-valued and $B$-bicocomplete, the further structure making such a fibration $U$ into an $B$-bicocomplete closed category is explored and compared with the expectation that such pairs $(B,U)$ constitute a useful generalization of "ringed spaces". (Received June 1, 1976.)


A polyad $X$ on a 2-category $A$ is a triple $X = (X, X_1, X_2)$ where $X: M \rightarrow [A,A]$ is a strict monoidal 2-functor and where $X_1$ and $X_2$ are subcategories of the small monoidal 2-category $M$.

For any polyad $X$ on $A$ we define the 2-category $X\text{-Alg}_*$ of algebras for $X$ together with a forgetful 2-functor $V: X\text{-Alg}_* \rightarrow A$.

Theorem If $A$ is a complete and cocomplete 2-category then $X\text{-Alg}_*$ is the 2-category of algebras for a ranked 2-monad on $A$ whenever the 2-functor $X$ factors through $[A,A]_*$ the monoidal 2-category of ranked endo-2-functors $A$.  

As an application of this theorem we can deduce that the category of algebras for a ranked endofunctor of $A$ is the category of algebras for a ranked monad (that is, the free monad exists and has rank); that the lax-algebras for a ranked 2-monad are the algebras for a ranked 2-monad; and that the category of monoids in the monoidal category $A$ is monadic provided $\otimes: A \times A \rightarrow A$ has a rank in each variable. (Received June 1, 1976.) (Introduced by Professor J. W. Gray.)

737-18-10 E. G. WAGNER, J. B. WRIGHT, J. W. THATCHER, Mathematical Sciences Department, IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598.

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Algebraic theories (Lawvere) are equationally described: categories $T$ with objects $\{0,1,\ldots\}$ equipped with tupling operations and injections satisfying certain identities. An ordered theory is an algebraic theory in which each homset is a strict poset and composition and tupling are monotonic. An ordered theory is $\omega$-continuous if each homset is $\omega$-complete and composition is continuous. A rational theory is an ordered theory with less than $\omega$-completeness, but with enough to solve fixed-point equations. Free rational and $\omega$-continuous theories are described using partial trees and illustrated with computer science applications. (Received June 7, 1976.)


Preliminary report.

Let $G$ be a group and $R \rightarrow F \rightarrow G$ be a free presentation, with $R$ not cyclic. An exact sequence (*) $H: K \rightarrow F \rightarrow G$ in which $1: H \rightarrow \text{center}(K)$ and $2: F \rightarrow G$ acts as a group of automorphisms on $K$, with $u(k) \cdot k' = kk'k^{-1}$, gives $K$ the structure of an abstract kernel over $G$. We shall also assume $3: \text{the action of } F \text{ on } H \text{ given by restriction is trivial. Definitions. (a) Let } \Gamma_{K} \subseteq K \text{ be the subgroup generated by elements of the form } (f \cdot k)k^{-1}, f \in F, k \in K. \text{ (b) } (*) \text{ is a stem kernel if } H \subseteq \Gamma_{K}. \text{ Theorem 1. Let } k \in H^{1}(G;H) \text{ be the MacLane obstruction class for the kernel } (*). \text{ Then if } k' \in \text{Hom}(H^{1}(G;H), H) \text{ is the corresponding element (by universal coefficients), } (*) \text{ is a stem kernel if } k' \text{ is epi. Theorem 2. Suppose } H_{1}(G) = 0, i = 1,2. \text{ Then there exists a unique maximal stem kernel } H_{i}(G) \rightarrow K \rightarrow F \rightarrow G: \text{ this kernel maps uniquely up to homotopy into any other kernel of the form } (*). \text{ (Received June 7, 1976.)}

*737-18-12 PAUL H. PALMQUIST, 454 C Sandsburg, Irvine, California 92717 and JOHN PAUL BOYD, University of California, Irvine, California 92717. Applications of categorical algebra to the theory of the constructions, representations and uniqueness theorems in measurement theory are universal in the sense of categorical algebra. This approach unifies and simplifies previous results as well as suggesting some interesting generalizations. Specific examples discussed will include regular scales, scale types, ordinal utility functions, extensive measurement, and conjoint measurement. (Received June 15, 1976.)

20 Group Theory and Generalizations

737-20-1 MICHAEL ASCHBACHER, California Institute of Technology, Pasadena, California 91125. Determining the finite simple groups.

It appears possible that the ongoing program to determine the finite simple groups may be completed during the next five or ten years. This talk will supply an outline of that program, and is intended for the nonspecialist. (Received March 8, 1976.)


Let $G^{n} = G/G^{n+1}$ where $G$ is a free product of a finite number of cyclic groups, and $G^{n+1}$ is the $(n+1)$ subgroup of the lower central series of $G$. Recently the author established an algorithm for completely determining every $G^{n}$ ("On Nilpotent Products of Cyclic Groups" - Reexamined by the Commutator Calculus, with H.V. Waldinger, Canadian Journal of Mathematics Vol. 27 (1975) p.1185-1210) This algorithm is called the "Representation Algorithm," and it is based on known methods of the commutator calculus. The "Representation Algorithm" represents elements of $G^{n}$ by unique words in the basic commutators. This investigation deals with many consequences of the application of the "Representation Algorithm" to the groups $G^{n}$. (Received May 10, 1976.)
Coxeter and Moser gave an example of a regular map which could be considered as a two-sheeted Riemann surface over the octahedron with branch points at the vertices. Both Sherk and Garbe constructed families of regular maps by covering in a similar way maps on the sphere and on the torus. We provide an algorithm for determining all regular coverings of any regular map with any number of sheets, provided only that the projection is one-to-one on the vertices. Further, given defining relations for the base map, the algorithm provides a set of defining relations for the symmetry group of the covering map. (Received May 24, 1976.)


The problem of when locally free and almost free groups are actually free was studied by Takahasi and by Higman. Here a related topological question was examined; when does \( \lim X_a \) a directed system of wedge of circles. First Theorem 1. Let \( \{X_a, f_a\} \) be a "cellularly directed system" of CW-complexes (similar to a cellular equivalence relation). If \( \lim X_a \) is Hausdorff, then \( \lim X_a \) is a CW-complex and \( \Pi_a(\lim X_a) = \lim \Pi_a(X_a) \). These are related to results of Lundell and Weingram. Let \( \{W_a, f_a\} \) be a directed system of wedges of circles based at \( x_0 \). Then Theorem 2. Let \( V = \bigcup_{a \beta} f^{-1}_a (x_0) \); then \( V \cap S_{a}{a} \) is a circle in wedge \( W_{a} \) is finite for each pair \( (a, a) \) and \( f_{a {a}} \) is locally one-to-one, then \( \lim W_{a} \) is a wedge of circles and \( \Pi_a(\lim W_a) = \lim \Pi_a(W_a) \). From this we get Theorem 3. Let \( \{G_n, f_{nm}\} \) be a countable directed system of free groups, if \( f_{nm} : G_n \to G_m \) is an embedding onto a free factor for each pair \( (n, m) \), then \( \lim G_n \) is free. Corollary. Let \( G \) be a group. If \( G \) is the union of countably many free subgroups with each inclusion an embedding onto a free factor, then \( G \) is free. The theorem and the corollary recover results of Takahasi and Higman and also lead to further topological conditions. (Received June 8, 1976.)

G. THOMAS, University of Western Ontario, London, Canada. \( \eta \)-Simple reflective semigroups. In this paper we give a characterization of \( \eta \)-simple reflective semigroups. First, we describe some properties of an arbitrary \( \eta \)-simple reflective semigroup. We then use these properties to obtain the characterization. (Received June 15, 1976.)

M. RAM MURTY and V. KUMAR MURTY, Carleton University, Ottawa, Canada. Some asymptotic formulas in group theory. Let \( G(n) \) be the number of nonisomorphic groups of order \( n \). For each \( k \), let \( F_k(x) \) be the number of \( n \leq x \) such that \( G(n) = k \). Erdős has shown that \( F_1(x) = (1 + o(1))xe^{-Y}/\log \log \log x \), where \( Y \) is Euler's constant. We show \( F_2(x) = (1 + o(1))xe^{-Y}/\log \log \log x \), and we obtain some results for \( F_k \) in general. (Received June 15, 1976.)

22 ▲ Topological Groups, Lie Groups

DOUGLASS L. GRANT, College of Cape Breton, Box 760, Sydney, Nova Scotia, Canada. Weak forms of compactness and \( B \)-completeness in topological groups.

For definitions, see T. Husain, Introduction to Topological Groups (Saunders, 1966). Let \( C_1 \) and \( HPK \) be the categories of first countable and hereditarily paracompact groups, respectively. Theorem. Every locally pseudocompact group is a \( B(HPK) \) and hence a \( B(C_1) \) group. Examples are produced to show that (1) local pseudocompactness does not imply either pseudocompactness or local countable compactness, and (2) that a countably compact \( B(A) \) group need not be compact. The first example consists of the product of a pseudocompact, non-countably compact group with a locally compact, non-compact one. The second consists of the set of elements of an uncountable product of finite, simple, non-Abelian, discrete groups which differ from the identity at at most countably many entries. (Received June 15, 1976.)
26 ▶ Real Functions

*737-26-1 L. A. SHEPP, Bell Laboratories, Murray Hill, New Jersey 07974. Optimal reconstruction of a function from its projections.

I will discuss the work of B. F. Logan and myself on a partially discretized, rigorous formulation of the Radon transform which admits an explicit inversion formula (Duke Math J. 42(1975)). The partially discretized Radon transform of a function \( f \) in \( L^2 \) and supported on the unit circle \( C \) is the set of projections or line integrals of \( f \) along any of \( n \) prescribed directions. These do not determine \( f \) for any \( n \) but there is a unique function with the given projections and smallest \( L_2 \) norm which is given explicitly. A theorem of Logan gives a bound on the fraction of the energy at low spatial frequencies of an invisible function, one that projects to zero in each of the \( n \) directions. A function with most of its energy at spatial frequencies less than \( n \) requires \( n \) "views" or projections to essentially determine it. I will also discuss various related problems in this pure mathematical field which was recently strongly reactivated by the emergence of computerized tomography, an engineering realization due to Godfrey Hounsfieid of the inverse Radon transform. The technique is now routinely used to obtain cross-sectional images of the body using x-ray pencil beams to measure the projections. The engineering or applied mathematical aspects will be discussed in our MAA talk at the same meeting. We will also discuss the nonrigorous but very useful convolution algorithms due to V. Lakshminarayan and others which are in wide current use in inversions of the Radon transform based on the Fourier inversion formula in the so-called fan-beam geometry. (Received April 16, 1976.)

737-26-2 Richard Fleissner, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201 and James Foran, University of Missouri-Kansas City, Kansas City, Missouri 64110. Transformation of Differentiable Functions.

Let \( T \) be the class of all real-valued functions of the form \( h \circ g \) where \( h \) is a homeomorphism and \( g \) is a differentiable function. Then \( T \subseteq C \), where \( C \) is the class of all continuous real-valued functions. Conditions are given for a function to belong to \( T \). (Received June 9, 1976.)

28 ▶ Measure and Integration


Let \( A \) be a proper \( H^* \)-algebra, let \( \tau_A = \{ xy | x, y \in A \} \) be its trace-class [Proc. AMS 26 (1970) 95-100] and let \( \mu \) be a \( \tau_A \)-valued measure defined on some \( \sigma \)-algebra \( \Sigma \) of subsets of a set \( S \). Theorem. A \( \tau_A \)-valued absolutely continuous measure \( \nu \) defined on \( \Sigma \) is of the form \( \nu_E = \int_E f d\mu \) for some real \( \Sigma \)-measurable function \( f \) if and only if for each \( \Delta \in \Sigma \) the (variable) measure \( \nu_{\Delta} : E \rightarrow \nu_E + r \nu(\partial \Delta) \) has a Hahn decomposition for each real number \( r \). The Hahn decomposition of the space \( S \) is defined as a pair \( P, N \) of disjoint measurable sets such that \( S = P \cup N \) and \( \nu_{\Delta} \) is positive (negative) for each measurable subset of \( P, N \). (Received June 7, 1976.)

737-28-2 W. A. J. LUXEMBURG, California Institute of Technology, Pasadena, Calif. 91125 Some remarks about doubly stochastic operators.

Let \( (X, \lambda, \mu) \) and \( (X', \Lambda', \mu') \) be two probability spaces. Following J. Ryff we shall call a linear transformation from \( L'(\mu) \) into \( L'(' \mu') \) doubly stochastic whenever \( Tf = f \) for all \( f \in L'(\mu) \). Some characterizations of doubly stochastic operators will be presented. In particular, the class of \( T \) satisfying \( Tf = f \) for all \( f \in L' \) will be examined. Also generalizations to normed spaces of measurable functions will be discussed. (Received June 10, 1976.)
Let \( T \) be an invertible measure preserving transformation on the probability space \((X,F,m)\). Let \( P \) be a doubly stochastic operator on \( L_1 \), i.e., \( P1 = 1 \), \( \int Pf \, dm = \int f \, dm \). Suppose also \( P(f \circ T) = (Pf) \circ T \). We study the implications of approximability assumptions on \( T \) with regard to the set \( P \) of all such \( P \). In particular, a certain type of cyclic approximability implies that the invariant subspace of such a \( P \) consists only of the constants, unless \( P \) is the identity. This is closely related to some work of Ornstein, and also of Chacon. (Received June 15, 1976.)

30 ▶ Functions of a Complex Variable

Let \( \mathcal{L} \) be a lattice of subsets of a set \( T \) with \( \varphi \in \mathcal{L} \) and \( G \) a sequentially complete Hausdorff commutative topological group. A set function \( \lambda: \mathcal{L} \to G \) is (i) strongly additive if \( \lambda(\varphi) = 0 \) and \( \lambda(E \cap F) = \lambda(E) + \lambda(F) - (E \cup F) \) for all \( E, F \in \mathcal{L} \); (ii) monotonely convergent if, for every monotone sequence \( (E_n) \) in \( \mathcal{L} \), \( (\lambda(E_n)) \) converges; (iii) continuous if, whenever \( (E_n), (F_n) \) are, respectively, decreasing, increasing sequences in \( \mathcal{L} \) s.t. \( \lim_n E_n \subseteq \lim_n F_n \), we have \( \lambda(E_n) - \lambda(F_n) \to 0 \). Let \( \mathcal{Y} \) denote the hereditary ring consisting of all \( E \in \exp(T) \) s.t. \( E \) is contained in at least one set of \( \mathcal{L} \). If \( \lambda \) is strongly additive and monotonely convergent, the extensions of \( \lambda: \exp(T) \to G \), \( \lambda^*: \mathcal{Y} \to G \) are well defined by the formulas \( \lambda_n(E) = \lim_{E \subseteq E n} \lambda(E_n) \), \( \lambda_n^*(E) = \lim_{E \subseteq E n} \lambda(E_n) \), respectively. If, further, \( \lambda \) is continuous, the restrictions \( \lambda_{\sigma} = \lambda_\sigma|_{\mathcal{L}_{\sigma}} \) and \( \lambda_{\delta} = \lambda_\delta|_{\mathcal{L}_{\delta}} \) have the same properties. **Theorem.** A strongly additive continuous set function \( \lambda: \mathcal{L} \to G \) extends to a strongly additive continuous set function \( \bar{\lambda} \) on the \((\sigma, \delta)\)-lattice \( \mathcal{L} \) generated by \( \mathcal{L} \) iff \( \lambda \) is monotonely convergent. In this case, \( \lambda_\sigma \), \( \lambda_\delta \) coincide on \( \mathcal{L} \) and \( \bar{\lambda} = (\lambda_\delta)^{-1}(\lambda_\sigma) \) is the unique strongly additive continuous set function on \( \mathcal{L} \) extending \( \lambda \). This generalizes the extension theorem of Sion (Proc. London Math. Soc. 19(1969), 89-106) as improved by Drewnowski (Bull. Acad. Pol. Sci. 20(1972), 439-445). (Received June 14, 1976.)

30 ▶ Functions of a Complex Variable

**30-1** Sidney L. Hantler, IBM T. J. Watson Research Center, Yorktown Heights, NY 10598.

**Polynomial approximation in certain weighted Hilbert spaces of entire functions.**

Estimates for reproducing kernels are used to prove polynomial approximation results in weighted Hilbert spaces of entire functions. The idea is to show that the polynomials are dense in those spaces whose weight functions are nearly a function of \( |z| \) alone. A consequence of Theorem 4 is that if \( \phi \) is plurisubharmonic and \( |\phi - |z|^2| \leq c(1+|z|)^2 \),

where \( c \) is a constant, then the polynomials are dense in the Hilbert space of entire functions \( f \) with \( \int |f|^2 \exp(-\phi) \, d\lambda < \infty \) (\( d\lambda \) is the Lebesgue measure on \( \mathbb{C}^n \)). Applications to a problem of D. J. Newman and R. S. Shapiro and extensions of our results are also discussed. (Received April 1, 1976.)

**30-2** P. Malliavin, Université Paris VI, Paris, France and Jamil A. Siddiqi, Université Laval, Québec, Canada. **Classes of monogenic functions and approximation by exponential polynomials on a rectifiable.** Preliminary report.

Let \( \gamma \) be a rectifiable in \( \mathbb{C} \). A function \( f \in \mathcal{G}(\gamma) \) is called monogenic, in symbols \( f \in \mathcal{G}(\gamma) \), if there exists a \( g \in \mathcal{G}(\gamma) \) such that \( \lim_{z' \to z, z' \in \gamma} (z'-z)^{-1}(f(z')-f(z)) = g(z) = \frac{df}{dz} (z) \). \( \mathcal{G}(\gamma) \) is then defined recursively. The class \( \mathcal{G}(M_0, \gamma) = \{ f \in \mathcal{G}(\gamma) : |df| \leq A \} \), \( k = k(f) \), where \( \{ M_n \} \) is such that \( \lim M_n(n!)^{-1} = 0 \), is called a monogenic class. \( \mathcal{G}(\gamma) \)

It is shown that if \( \{ \mathcal{G}(\gamma) : \lambda \in \mathcal{A} \} \) is complete in \( \mathcal{G}(\gamma) \) then a certain monogenic class associated with \( \lambda \) and \( \gamma \) is trivial. Conversely, if \( \gamma \) is Lipschitz and satisfies certain geometric condi-
tion then the triviality of a similar monogenic class implies that \( e^{12} \) is complete in \( G(\gamma) \).

These results generalize the Müntz theorem for analytic arcs. (Received May 3, 1976.)

*737-30-3  Dorothy B. Shaffer, Fairfield University, Fairfield, Ct. 06430. Extremal problems for Generalized Equipotential Surfaces.

The results in this paper derive from generalizations to surfaces in \( \mathbb{R}^n \) of properties of lemniscates and level curves of rational functions in the plane. The generalized equipotential surfaces are defined as follows: Let \( x_1 \ldots x_p \) be points in \( \mathbb{R}^n \), \( p \geq 2 \), let \( f_j(x) \in \mathbb{C}, \ r > 0 \), \( f_j'(r) < 0 \) for \( j=1 \ldots p \). The functions considered are \( T(x) = \sum_j f_j(r_j) \) and \( U(x) = \sum_j f_j(r_j) - \sum_{j=p+1}^q f_j(r_j) \), \( r_j \) the Euclidean distance and their level surfaces \( V \) and \( U \). The following theorem will be established:

Let \( N \) denote the normal to \( V \), \( H \) is the convex hull of \( x_1 \ldots x_p \) and \( K \) the convex hull of \( Y_1 \ldots Y_q \). Let \( b \) and \( B \) denote the shortest distances along \( N \) from a point \( X \) to \( b \) and \( B \). Assume \( 0 < f_j'(r) < (\alpha+1)-f_j'(r) \), all \( j=1 \ldots q, \alpha > 0 \).

Let \( G \) be any curve on \( U \), \( N \) principal normal then \( R \) the radius of curvature of \( C \) at \( X \) is numerically greater than \( \frac{1}{\alpha} > \min (b, B) \). (Received May 28, 1976.)

*737-30-4  Frederick W. Hartmann, Villanova University, Villanova, PA 19085. Linear homeomorphisms of some classical families of univalent functions.

Brickman, et al. "Convex hulls of some classical families of univalent functions", Trans. Amer. Math. Soc., 156 (1971) p.91-107] characterized the extreme points of the closed convex hull of some classical families of univalent functions analytic on the unit disk, e.g. the convex, \( K \), and starlike, \( St \), sets. These characterizations are used to determine an explicit representation for the class of linear homeomorphisms of the extreme points of the closed convex hulls of \( K \) and \( St \) and thus of the convex hulls themselves. With the aid of this representation it is shown that the only linear homeomorphisms of \( K \) and \( St \) are rotations, i.e. convolution with \( \exp(in \theta) : n = 0,1, \ldots \). In the way of a positive result: if \( P \) is the convex set of analytic functions with positive real part and \( f(0) = 1 \) and \( L \) is a linear homeomorphism of \( P \), then \( L(St) \subset St \), but \( L(K) \not\subset K \). (Received June 1, 1976.)


The notion of a hypernormal meromorphic function defined on the unit disc was introduced by H. M. Anderson and L. A. Rubel [Notices 23, no.2 (1976), 733-B17]. The present authors observe that such a function must be holomorphic. (Received June 15, 1976.)

31 ▶ Potential Theory


Let \( X \) be a harmonic space satisfying axioms I, II, IV in the Springer lecture notes of Bauer, Brelot's convergence axiom, and the axiom that \( I \) be harmonic. Let \( A \) denote the closure of the complement of an outer regular compact set in \( X \) and \( D \) a regular region containing \( X-A \). Let \( \delta \) denote the boundary of \( D \) and \( \alpha \) the boundary of \( A \). Let \( H(A) \) denote the set of functions continuous on \( A \) and harmonic in the interior of \( A \). A normal operator is a mapping \( L \) from the continuous functions on \( A \) into \( H(A) \) which is linear and satisfies 1) \( L(f)|_A = f \), 2) \( L(1) = 1 \), 3) \( L(f) \geq 0 \) if \( f \geq 0 \), and 4) \( \int_L(f)dv \) is independent of \( \delta \) where \( \nu_{T_L} \) is the measure introduced by Rodin and Sario on page 303 of their book on principal functions.

Let \( \Gamma \) denote the Wiener harmonic boundary of \( X \). We can then regard a normal operator as a linear operator \( L \) from \( C(\alpha) \) into \( C(\Gamma) \) satisfying the conditions (a) \( L(1) = 1 \), (b) \( L(f) \geq 0 \) if \( f \geq 0 \) and (c) \( \int_{\Gamma} L(f)dv = \int_{\alpha} fdv \) where \( \nu \) is a measure induced by \( \nu_{T_L} \) in a natural way. Thus we see that normal operators are doubly stochastic operators. We then proceed to study the extreme points of the set of normal operators. (Received June 10, 1976.)
Several Complex Variables and Analytic Spaces

737-32-1 Lee Rudolph, Department of Mathematics, Box 1917, Brown University, Providence, Rhode Island. Classifying germs of irreducible analytic plane curves. Preliminary report.

The local ring at (0,0) of the germ of an irreducible analytic plane curve is isomorphic to a closed subalgebra R of \( \mathcal{O} \) with two generators. The set of germs of irreducible analytic plane curves is therefore in 1:1 correspondence with the orbits of the group of automorphisms of \( \mathcal{O} \) acting on the set of all such subalgebras R. Let \( \mathbb{M}_R \) be the set of all integers that are orders of series in R; \( \mathbb{M}_R \) is a sub-semigroup-with-0 of \( \mathbb{N} \), briefly, a monoid, and clearly is invariant under \( Aut_{\mathcal{O}} \).

Wolfhardt (Am. J. Math., 68) showed that to any monoid \( M \) there is a (perhaps empty) smooth, finite-dimensional complex space \( X(M) \) whose points are all the local rings R of plane curves with \( \mathbb{M}_R = M \); the action of \( Aut_{\mathcal{O}} \) on \( X(M) \) factors through an algebraic group action by a solvable Lie group G. Wolfhardt found all monoids for which G is transitive on \( X(M) \): one infinite family, two sporadic monoids. I have completely classified "stable" cases, those in which there is an open G-orbit in \( X(M) \): two more infinite classes, two more sporadic examples. In fact, in each stable case there turn out to be only finitely many orbits. Also, no irreducible plane curve-germ with 3 or more Puiseux pairs is stable. (Received June 10, 1976.)

33 ▶ Special Functions

*737-33-1 H.M. SRIVASTAVA, University of Glasgow, Glasgow G12 8QW, U.K. and University of Victoria, Victoria, British Columbia, Canada V8W 2Y2, and REKHA PANDA, University of Victoria, Victoria, British Columbia, Canada V8W 2Y2 and Revenshaw College, Cuttack-3, India. New generating functions involving several complex variables.

In the present paper, which essentially contains two preliminary reports 74T-B237 and 75T-B112 [these NOTICES 21 (1974), p.A-593; ibid. 22 (1975), pp.A-458 to A-459], a number of new classes of generating functions are given for certain sequences of functions of several complex variables. These generating relations involve either Taylor or Laurent series and provide extensions of several known results given recently by H.M. Srivastava and R.G. Buschman [Trans. Amer. Math. Soc. 205 (1975), 360-370], and D. Zeitlin [Scripta Math. 29 (1973), 43-48]. It is also shown how one of the main results can be suitably applied to yield the corresponding generating functions for the generalized Lauricella functions. (Received April 19, 1976.)

737-33-2 GEORGE E. ANDREWS, Pennsylvania State University, University Park, Pa. 16801 and RICHARD ASKEY, University of Wisconsin, Madison, Wis. 53706. Survey of q-orthogonal polynomials. Preliminary report.

In 1846, E. Heine introduced basic hypergeometric functions (also known as q-series or Eulerian series). In 1948 W. Hahn showed that within this class of functions there are orthogonal polynomials which are extensions of the classical polynomials. We use the q-Jacobi polynomials to give a derivation and slight extension of Watson's transformation between a balanced \( \phi_3^\theta \) and a very well poised \( \phi_7^\theta \), the formula which lies behind most of the proofs of the Rogers-Ramanujan identities. Another class of these polynomials has a generating function which gives Schur's mod 6 partition identity. We have a few more results, just enough to make it certain that a very rich theory exists. Some of these will be mentioned. (Received May 24, 1976.)

*737-33-3 RICHARD ASKEY, University of Wisconsin, Madison, Wisconsin 53706 and MOURAD ISMAIL, McMaster University, Hamilton, Ontario, Canada. Quadratic transformations of hypergeometric series

Many combinatorial sums are best considered as special cases of hypergeometric series. See Andrews, SIAM Review, 16, 1974, 441-484. Important special cases of the classical hypergeometric function are those functions which have quadratic transformations. These transformations were discovered by Gauss and Kummer. Goursat gave a list which is supposed to be complete. Actually his list is not quite complete and it is not very well ordered. An ordering
will be given and some of the extensions found in the last one hundred years will be mentioned.

A result of J. Brenner, \[ \sum_{k=0}^{n} \binom{2n}{2k} \binom{2n-2k}{n-k} \left( \frac{x^2 + y^2}{(xy)^2} \right)^k = \sum_{k=0}^{2n} \binom{2n}{k} \left( \frac{x^2}{y^2} \right)^k (xy)^{2n-k} \]

is easily seen to fit into this class of transformations, and it was given by Kummer. Brenner's sum can be generalized to \[ \sum_{k=0}^{2n} \binom{2n}{2k} \binom{2n-2k}{n-k}^{-1} \left( \frac{x^2 + y^2}{(xy)^2} \right)^k (xy)^{2n-2k} \]

which is also a result from the last century.

(Received May 24, 1976.)

*737-33-4 M. L. GLASSER, University of Waterloo, Waterloo, Ontario N2L 3G1

A Combinatorial Sum.

The summation \[ \sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{a^k}{k!} \frac{b^n}{n!} \frac{2n}{n} \]

Which has been evaluated recently by Novoseller and the author in terms of elliptic integrals will be discussed. Some related material having to do with elliptic functions and Watson sums will also be dealt with. (Received June 1, 1976.) (Author introduced by M. S. Klamkin).

*737-33-5 J.T. Lewis, Dublin Institute for Advanced Studies, Dublin 4, Ireland and M.E. Muldoon, York University, Downsview, Ontario M3J 1P3, Canada. 

Monotonicity and convexity properties of zeros of Bessel functions.

It is shown that \( j_{n,k} \) decreases as \( v \) increases, \( 0 < v < \infty \) and that \( j_{v,k} \) and \( \frac{dj_{v,k}}{dv} \) increase with \( v \) for sufficiently large \( v \), where \( j_{v,k} \) is the \( k \)th positive zero of the Bessel function \( J_v(x) \). In particular, \( j_{v,k} \) and \( \frac{dj_{v,k}}{dv} \) increase for \( 3 \leq v < \infty \). This can be used to prove that \( j_{v+1,k} - j_{v+1,k+1} > 0 \), \( n = 0,1,2,\ldots \) a result which was conjectured on physical grounds, and which arose from a study of a rotating ideal Bose gas. Some related results are proved for zeros of \( J_v(x) \), of cross-product Bessel functions and of modified Bessel functions of purely imaginary order. (Received June 3, 1976.)

34 ▶ Ordinary Differential Equations

*737-34-1 Roger T. Lewis, University of Alabama in Birmingham, Birmingham, Alabama 35294. 

Conjugate Points of Vector-Matrix Differential Equations.

The system of equations \[ \sum_{k=0}^{n} (-1)^{n-k} (P_k(x)y^{n-k})(x) = 0 \quad (0 < x < \infty) \]

is considered where the coefficients are real, continuous, symmetric matrices, \( y \) is a vector, and \( P_k(x) \) is positive definite. It is shown that the well-known quadratic functional criterion for existence of conjugate points for this system can be further utilized to extend results of the associated scalar equation to the vector-matrix case, and in some cases the scalar results are also improved. The existence and nonexistence criteria for conjugate points of this system are stated in terms of integral conditions on the eigenvalues or norms of the coefficient matrices. (Received June 14, 1976.)

*737-34-2 GARRET J. ETGEN, University of Houston, Houston, Texas 77004. 


Let \( K \) be a real Hilbert space, let \( \mathcal{Q} = \mathcal{Q}(K,K) \) be the Banach algebra of bounded linear operators from \( K \) to \( K \) with the standard operator norm, and let \( S \) be the set of self-adjoint elements of \( \mathcal{Q} \). We consider second order differential systems of the form

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(1) \( [P(x)Y']' + Q(x)Y = 0 \) on \( \mathbb{R}^+ = [0, \infty) \), where \( P, Q: \mathbb{R}^+ \to \mathbb{R} \) are continuous and \( P(x) \) is positive definite for all \( x \in \mathbb{R}^+ \). A solution \( Y = Y(x) \) of (1) is nonsingular at \( x = a \) if \( Y(a) \) has a bounded inverse and the range of \( Y(a) \) is \( \mathbb{K} \); otherwise \( Y \) is singular at \( x = a \). A solution \( Y \) is conjoined if \( Y^*(PY') = (PY')^*Y \) (\( * \) denotes adjoint) on \( \mathbb{R}^+ \). Let \( \mathcal{A} = \{ g: \mathbb{R}^+ \to \mathbb{R} \mid g(A^*A) \geq 0 \} \) for all \( A \in \mathcal{A} \). Let (2) \( [F(x)Y']' + G(x)Y = 0 \) be a system of the same type as (1). Suppose \( g \in \mathcal{A} \) and \( V \) is a non-identically zero solution of (2) satisfying (i) \( g[V* (Q-G)V] \geq 0 \) on \( [a,b] \); (ii) \( g[V^* (F-P)V] \geq 0 \) on \( [a,b] \); (iii) \( g[V^* (a)V(a)] = g[V^* (b)V(b)] = 0 \); (iv) for any point \( c \in [a,b] \), if \( g[V^* (c)V(c)] = 0 \), then \( g[V^* (c)P(c)V(c)] > 0 \). If \( Y \) is a conjoined solution of (1), then \( Y(x) \) is singular at some point \( x \in [a,b] \).

This theorem extends a result of K. Kreith [Proc. Amer. Math. Soc., 26 (1970), 270-272], and it can be used to develop criteria for the oscillation of (1) which include a number of well-known oscillation criteria as special cases. (Received June 15, 1976.)

35 ▶ Partial Differential Equations


This paper proves that several initial-boundary value problems for a wide class of nonlinear reaction-diffusion equations have solutions \( c_i(x,t) \), \( 1 \leq i \leq N \) (with \( c_i(x,t) \) representing the concentration of the \( i \)-th species at position \( x \) in a set \( \mathbb{R}^n \) at time \( t \geq 0 \)), which exist for all \( t \geq 0 \) and are unique, smooth, non-negative, and strictly positive for \( t > 0 \). The Volterra-Lotka predator-prey model with diffusion (to which the results above are proved to apply) is then studied in more detail. It is proved that any bounded solution of this model loses its spatial dependence and behaves like a periodic function of time alone as \( t \to \infty \). It is proved that if the spatial dimension is one or if the diffusion coefficients of the two species are equal, then all solutions are bounded. (Received April 23, 1976.)


\( h \) is called a measure function if \( h \) is a continuous nondecreasing function on \( [0, \infty) \) with \( h(0) = 0 \) and \( h(t) > 0 \) for \( t > 0 \). The following result is established in Euclidean \( n \)-space, \( n \geq 2 \): Let \( A \) and \( A^- \) be complementary \( N \)-functions and set \( h(t) = t^n A^-(t-m) \) where \( 1 \leq m \leq n-1 \). Suppose that (i) \( h \) is a measure function; (ii) \( Z \) is a relatively closed subset of the bounded open set \( \mathbb{R}^n \); (iii) \( Z = \bigcup_{i=1}^{\infty} Z_i \) where for each \( i \), \( Z_i \) is closed set with \( h(Z_i) < \infty \); (iv) \( P(x,D) = \sum_{|\beta| \leq \infty} b_{\infty}^\beta(x)D^\beta \) is a linear partial differential equation of order \( m \). Then \( Z \) is a removable set for \( P(x,D) \) with respect to \( E_{A}(\cdot) \). In particular, this gives a new result for the class of functions \( u \in L^p(\mathbb{R}^n) \log(1+|u|)dx < \infty \). Also, \( A(t) = t^p \) gives the previous well-known results established in this area. \( \Lambda_h \) is the Hausdorff measure generated by \( h \), and \( c_{\beta}^\beta \) is assumed to be in \( C^m(\Omega) \). (Received June 4, 1976.)

*737-35-3* CHUNG-LING YU, Engineering Faculty, Benghazi University, Benghazi, Libya. \( H^p \) space theory of linear elliptic system in the disc.

Let \( A(z) \), \( B(z) \) be two \( n \times n \) bounded measurable matrix functions. A solution \( W(z) = (W_1, \ldots, W_n) \) of the elliptic system (E) \( W_{\bar{z}} = A(z)W + B(z)W \) in a unit disc \( \{ z \} \leq 1 \) is said to be of class \( H^p \) \( 0 < p \leq \infty \) if \( \| W \| < \infty \), where \( \| W \| = \sup_{0 < \rho < 1} \sum_{j=1}^{n} |W_j(e^{\rho t})| dt \) for \( 0 < p < \infty \), and \( \| W \|_{\infty} = \sup_{|z| < 1} \sum_{j=1}^{n} |W_j(z)| \). The following areas have been investigated: (a) \( H^p \) as a Banach space for \( 1 \leq p \leq \infty \). (b) Boundary behaviour of the \( H^p \) solutions of (E). The generalizations of Fatou’s lemma have been established. (c) The unique continuation and analytic continuation theorems of \( H^p \) solutions of (E). (Received May 27, 1976.) (Author introduced by Chiu Yeung Chan.)
An interior boundary value problem for the Helmholtz equation $U_{xx} + U_{yy} + k^2 U = 0$ with mixed boundary conditions containing complex-valued coefficients depending on $k$ is considered. A potential theoretic approach is adopted, resulting in a Fredholm integral equation of the first kind on part of the boundary and of the second kind elsewhere. Existence and uniqueness theorems for the integral equation and hence existence for the original boundary value problem are shown to depend on the uniqueness of solutions of that boundary value problem. The problem arises in the investigation of sound propagation in ducts. (Received June 14, 1976.)

Gilbert's $A_\mu$ integral operator is used to study the growth of entire generalized axisymmetric potentials i.e., solutions of the equation $\psi^2 u_{\alpha\beta}^2 + \psi^2 u_{\gamma\gamma}^2 + (2\psi u)_{\alpha\gamma} = 0$, $\psi > 0$, which have no finite singularities. Solutions of regular and perfectly regular growth are characterized in terms of their coefficients in an ultraspHERical harmonic expansion. The concept of proximate order is introduced as in function theory, and it is shown that an entire GASP and its $A_\mu$ associate have the same type with respect to a proximate order. A method for generating sequences of GASP's complete in a disk of given radius from single entire GASP's is obtained. The following result is typical.

**Theorem:** Let $u(x,y)$ be an entire GASP where the coefficients of $u(x,0) = \sum_{n=0}^{\infty} a_n x^n$ are all non-zero. If $u(x,y)$ has type at most $\tau$ with respect to the proximate order $\rho(t) + \sigma$, and $(\lambda_n, n = 1,2,\ldots)$ is any sequence of distinct reals, then the sequence of entire GASP's $u_n(x,y) = u(x,\lambda_n y)$ is complete in the space of GASP's on the disk centered at $(0,0)$ of radius $R$, where $R^2 = (ep\tau)^{-1} \lim_{n \to \infty} |\lambda_n|^{\rho(\lambda_n)}$.

(Received June 15, 1976.)

**Finite Differences and Functional Equations**

Existence of solutions with interior zeros of nonlinear Sturm-Liouville problems. Let $L$ be the Sturm-Liouville operator with homogeneous boundary conditions, and consider the equation (*) $Lu = f(x, u)$, where $f$ is a nonlinear continuous function satisfying (i) $tf(x, t) \geq 0$, and (ii) $|f(x, u(x))| \leq h(|u(x)|)$ whenever $0 < |u(x)| \leq A$. Suppose the function $h(t)$ is nonnegative and continuous for $t \geq 0$, $h(t)$ is strictly increasing and $t/h(t)$ tends to $0(< \infty)$ as $t$ approaches zero. Then (*) has two solutions having exactly $n$ interior zeros for every integer $n = 0, 1, 2, \ldots$. The proof is by induction: first a positive solution of (*) is found. From (i) it follows that we also have a negative solution. A solution with $n$ interior zeros is obtained by splicing a positive solution of $Lu = f(x, u)$ on $[0, T]$ (with the right boundary condition being $u(T) = 0$) to a solution of $Lu = f(x, u)$ on $[T, 1]$ having $n - 1$ internal zeros. The splicing is accomplished by making use of Miranda's fixed point theorem. (Received May 21, 1976.)

**Sequences, Series, Summability**

Let $E$ be a locally convex topological sequence space for which the coordinate linear functionals are continuous (a K-space), and suppose that $E$ contains all finite sequences. We say that a sequence $x$ (not necessarily in $E$) has unconditionally bounded sections (UBS) in $E$ in case $H(x) = \{ \sum_{k=0}^{\infty} x_k : x_k \in E_k \}$ is a bounded set in $E$, and has unconditional F infinite (UF) in $E$ in case $H(x)$ is a Cauchy net in $E$ relative to the weak topology $\sigma(E,E')$.
THEOREM 1. Let $E$ be a $K$-space, $x$ any sequence. The following are equivalent:

(i) $x$ has UAB in $E$

(ii) $x$ has UFAK in $E$

(iii) $\sum |x_k|^2 f(x_k) < \infty \forall f \in E'$

THEOREM 2. Let $E$ be an FK-space. Every sequence in $E$ has UAB if and only if $E \simeq E'$.

THEOREM 3. Let $E$ be an FK-space with (ordinary) section boundedness. Every sequence in $E$ has UAB if and only if $E^\alpha = E^\beta = E'$. (Received April 27, 1976.)

41 \hspace{1cm} Approximations and Expansions

*737-41-1 YUDELL L. LUKE, University of Missouri, Kansas City, Mo., 64110. ALGORITHMS FOR RATIONAL APPROXIMATIONS FOR A CONFLUENT HYPERGEOMETRIC FUNCTION II

This is a sequel to a previous paper where rational approximations for the confluent hypergeometric function $z^aU(a;c;z)$ were treated. Here we take up rational approximations for $\genfrac{[}{]}{0pt}{}{a}{c}(a;c;-z)$. The confluent functions are very important in the applications since they include as special cases the incomplete gamma function (special cases of which are exponential, sine and cosine integrals, Fresnal integrals and the error function), Bessel functions, parabolic cylinder functions and Coulomb wave functions. The subject of rational approximations for a wide class of functions including those named above were examined in some detail in my volumes on the special functions. In the special case where $a$ is unity, the confluent function becomes an incomplete gamma function. In this event, complete a priori error analyses for the main diagonal Padé approximations and much more were presented. For general parameters, the rational approximations treated were of the Padé class. It was shown that the rational approximations converge, but a complete a priori analysis was not available. One of the purposes of this report is to correct this deficiency. Further, FORTRAN programs are provided to evaluate the Padé and non-Padé rational approximations by using the appropriate recursion formulas to generate the numerator and denominator polynomials as a number, and to also evaluate the coefficients which define these polynomials. The programming of the routines was done for use by the IBM 370/168 operating under OS/VS Release 1.7 on the FORTRAN IV H-Extended Compiler, Release 2.1. All computer programs are written for quadruple precision and real arithmetic. By making a few simple changes, one can have double or single precision. Further, it is easy to get complex arithmetic along with any of the precisions noted above. (Received May 20, 1976.)


The writer (SIAM Journal on Applied Mathematics 26(1974), 431-436) has applied MacMahon's Master Theorem to prove certain multiple expansions. In the present paper a number of related results are obtained. In particular explicit formulas are obtained for the inverse of the system of equations $u_i = x_i \exp \left(-\sum_{j=1}^{n} a_{ij} x_j\right)$, $i = 1, 2, \ldots, n$ and also of $u_i = x_i \prod_{j=1}^{n} (1+x_j)^{-a_{ij}}$, $i = 1, 2, \ldots, n$. The convolution formulas obtained in the paper are multi-dimensional generalizations of well-known one-dimensional convolution formulas. (Received May 21, 1976.)

*737-41-3 NIRA RICHTER-DYN, Tel-Aviv University, Tel-Aviv, Israel. On the existence of a class of best nonlinear approximations in Hilbert Spaces. Preliminary Report.

A proof is given of the existence of best nonlinear approximations $E^\alpha K(x,x_1)$ to functions of the form $f(x) = \int_{a}^{b} K(x,n) d\sigma(n)$, in a Hilbert space with a reproducing kernel $K(x,y)$ which is extended totally positive on a real interval $I = [a,b]$.

The proof relates the best approximations in the norm of the Hilbert space with best one-sided approximations in the $L_1$ norm $\|f\|_1 = \int_{a}^{b} |f(n)| d\sigma(n)$. The proof is significantly simpler than the proofs given by S. Karlin ("Studies in Spline
functions and approximation theory" Academic Press (1976), p. 19-66) and by R.B. Barrar & el. (Numer. Math. 23 (1974), 105-117), and is not limited to specific families of Hilbert spaces. (Received June 7, 1976.) (Introduced by Chandour Edmond.)

737-41-4

Mourad E.H. Ismail, McMaster University, Hamilton, Ontario, and C. Ping May, University of Toronto, Toronto, Ontario. On a family of approximation operators.

We show that an exponential operator $S_A(f,t) = \int_0^\infty W(A,t,u) f(u) \, du$, $\lambda > 0$, is uniquely determined by the differential equation $\frac{dW}{dA} = \frac{1}{p(t)} W$, and the normalization $\int_0^\infty W(A,t,u) \, du = 1$. The kernel $W(A,t,u)$ may be a generalized function. The domain of $t$ is any component of $\{t: p(t) > 0\}$ and, $p(t)$ is assumed to be an entire function and is real for $t$ real. These exponential operators are approximation operators, $A(t) = A(t-A)$, and include, for instance the Bernstein polynomials, $p(t) = t(1-t)$ and the Szasz operators, $p(t) = t$. We characterize the operators corresponding to quadratic polynomials $p(t)$, which turn out to be all known except when $p(t) = 1 + e^{\lambda u}$, where $\lambda > 0$.

The kernel $W(2A,t,u) = 2^{-1} \lambda (1+t^2)^{-1/2} (\pi^{-1})^{-1} |\gamma(\lambda + i \mu)|^2 \exp(2u \arctan t)$.

As further examples we construct two new operators with cubic $p(t)$.

Finally direct and inverse theorems for exponential operators, with arbitrary $p(t)$, are proved. (Received June 7, 1976.)

737-41-5

Vasant A. Ubhaya, Case Western Reserve University, Cleveland, Ohio 44106. On the Degree of Approximation by Monotone Polynomials.

Let $I = [a,b]$ be a real interval of length $b-a$. Let $C$ be the space of continuous functions on $I$ and $P_n$, the set of nondecreasing (on $I$) polynomials of degree at most $n$. Let $E_n(f) = \inf \{p_{\alpha} \in P_n : p_{\alpha} \leq f\}$, where $\|p\|$ is the uniform norm and $f$ is in $C$. Let $\omega(f,\cdot)$ be the modulus of continuity of $f$ in $C$, and $\varrho(f,\cdot)$ and $\varrho(f,\cdot)$ be the moduli of monotonicity, decreasing and increasing respectively, of $f$ in $C$.

(For definitions of these moduli, see the author's article "Moduli of Monotonicity with Applications to Monotone Polynomial Approximation", SIAM, J. Math. Anal. 7 (1976), 117-130.) Some Main Results:

In the following, assume $f$ is in $C$ but is not nondecreasing. Let $E_n(f) = (1/2) \omega(f,\cdot)$, $a = \lambda t/A$, where $\lambda = \sup \{\delta \in [0,1]: \omega(f,\delta) = \varrho(f,\delta-t\lambda)\}$.

Theorem 1: If $n > 2a$, then $0 < E_n(f) \leq (8\pi^2/n)(||f||+(1/2)\varrho(f,\cdot))n^{-1}$.

Theorem 2: If $n > 2k$, then $0 < E_n(f) \leq (1+4/n)k^{n+1}(n+1)^{-1} \omega(f,

Then $G(f,n) = \sup_{n>0} \{E_n(f)/A(f,\cdot,n)\}(1+4/n)a(n+1)^{-2}\sqrt{2\pi}$. (Received June 10, 1976.)

42 Fourier Analysis

737-42-1


Abstract. It is shown how an integral representation for the product of Jacobi polynomials can be used to derive a certain integral Lipschitz type condition for the Cesàro kernel for Jacobi expansions. This result is then used to give criteria of Marcinkiewicz type for a sequence to be a multiplier of type $(p,p)$, $1 < p < \infty$, for Jacobi expansions. (Received May 17, 1976.)

737-42-2


If a function $g$ is not necessarily integrable but $xg$ is, we define the generalized sine coefficients of $g$ by $b_n = 2\pi^{-1} \int_0^\pi g(x) \sin nx \, dx$. There is the following known theorem due to Zygmund. THEOREM A. If $g$ is monotonic decreasing in $(0,\delta)$ and bounded on $(\delta,\pi)$, $xg$ is Lebesgue integrable, then $g$ is Lebesgue
integrable if and only if \( \sum_{n=1}^{\infty} |b_n| < \infty \). Boas asked whether the hypothesis of monotonic decreasing function in Theorem A may be replaced by the hypothesis of positive function in the direction \( \int \). We have the following function
\[
g^*(t) = \sum_{n=1}^{\infty} \frac{\sin n t + \sin(2n+1)t}{\log n} + M \]
where \( M \) is an absolute constant. This function \( g^* \) is positive and for this function, Theorem A fails in the direction \( \int \). Hence, this answers the question of Boas in negative. Now question arises whether the hypothesis of decreasing function may be replaced by a weaker hypothesis other than a positive function. The main aim of this paper is to give the answer of second question in affirmative by showing that the hypothesis of monotonic decreasing may be replaced by a weaker hypothesis other than a positive function. (Received May 10, 1976.)


A univariate probability distribution is unimodal with vertex at the point \( a \) if its cumulative distribution function is convex on the interval \((-\infty,a)\) and concave on the interval \((a,\infty)\). (E. Lukacs, Characteristic Functions, Hafner, 1970, pp 91-98.) The following result is proved by making use of the mixture nature of a unimodal distribution and Khintchine's representation theorem of its characteristic function. Theorem. The Fourier integral of an absolutely continuous unimodal distribution converges except for a countable set of points. Conversely, if the Fourier integral of a unimodal distribution converges almost everywhere then it is absolutely continuous. (Received June 14, 1976.)

ROBERT FEINERMAN, Lehman College, CUNY, Bronx, New York 10468 and ROBERT KELMAN, Department of Computer Science, Colorado State University, Fort Collins, Colorado 80521. Dual orthogonal series: An abstract approach. II. Preliminary report.

In a previous note, we announced the development of an abstract approach to dual orthogonal series. One of the main theorems proven was Theorem 1. Let \( P \) and \( Q \) be real Hilbert spaces whose orthogonal sum is \( H \) with \( p \) and \( q \) being the projection operators onto \( P \) and \( Q \) from \( H \). Let \( \{\phi_n\} \) be a complete orthonormal sequence in \( H \) and let \( \{a_n\} \) be a positive sequence (of constants) converging to a positive limit. Then \( \{\phi_n + a_n\phi_0\} \) is an \( L^2 \) basis in \( H \); i.e., given \( f \in H \), there is a unique sequence \( \{j_n\} \in L^2 \) such that
\[
f = \sum_n (\phi_n + a_n\phi_0).\]

This theorem was extended and strengthened as follows: Theorem 2. With the same hypotheses as in Theorem 1, except that now \( \{a_n\} \) is merely assumed to be bounded and bounded away from zero (i.e., \( 0 < m \leq a_n \leq M \)), \( \{\phi_n + a_n\phi_0\} \) is still an \( L^2 \) basis in \( H \). Theorem 3. With the same hypotheses as in Theorem 1, except that now \( \{a_n\} \) converges to zero, \( \{\phi_n + a_n\phi_0\} \) is not an \( L^2 \) basis in \( H \). (Received June 14, 1976.)

43 ▶ Abstract Harmonic Analysis


The problem considered by this paper arises from the existence of the Haar Integral on a locally compact group. This paper considered the more general problem of the existence of a nonzero, additive, and homogeneous real valued transformation on an abstract cone and invariant under a group of operators defined on the cone.

The approach taken to this problem is to first embed a homomorphic image of the cone into a real linear space and then consider the problem of the existence of nonzero additive, and homogeneous real valued transformations on the linear space such that the embedded homomorphic image of the cone is mapped into the nonnegative real numbers.

Two conditions are derived for the existence of nonzero invariant, nonzero, additive, and homogeneous real valued transformations. Also, a necessary and sufficient condition is derived for the uniqueness of a nonzero invariant, additive, and homogeneous real valued transformations on an abstract cone. (Received April 29, 1976.)

H. Kharaghani, Pahlavi University, Shiraz, IRAN. Weakly Almost Periodic Measures on Locally Compact Semigroups. Preliminary report.

Let \( S \) be a locally compact semigroup, \( w \) in \( M(S) \) is called right weakly almost periodic
if \( \{u \in S : s \in S \} \) is relatively \( \sigma (M(S), M(S)^* \) compact. Such semigroups are characterized. Among the other results we have the followings.

Theorem 1: Let \( G \) be a locally compact group. Then there is a right weakly almost periodic measure on \( G \) iff \( G \) is compact.

Theorem 2: Let \( S \) be a locally compact semigroup with jointly continuous multiplication. If there exists a right almost periodic measure on \( S \), then \( \mathcal{W}(S) \) has a right invariant mean iff \( \mathcal{W}(G) \) has one. (Received June 1, 1976.)

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44 ▶ Integral Transforms, Operational Calculus

*737-44-1 Harris S. Shultz, California State University, Fullerton, Ca. 92634.

Operational Calculus for Functions of Two Variables.

Let \( L \) be the space of locally integrable complex-valued functions of two variables defined on the first quadrant. We can inject \( L \) into a commutative algebra of operators on a space of testing functions. Since this injection maps convolution into multiplication it serves as a generalization (there are no growth restrictions) of the two-dimensional Laplace transform. Some useful operational formulas are developed. (Received May 5, 1976.)

*737-44-2 R.S. Pathak and J.N. Pandey, Dept. of Mathematics, Carleton University, Ottawa, Ont., Canada. The Kontorovich-Lebedev transformation of distributions. Preliminary report. Professor Zemanian has extended Kontorovich-Lebedev transformation to distributions of compact supports. The aim of the present paper is to extend this transformation to a larger space of generalized functions and to apply the theory thus developed to a Dirichlet's problem for a wedge. (Received May 17, 1976.)

46 ▶ Functional Analysis


Let \( \{T(t)\} \) be a \( C_0 \)-contraction semigroup on the Hilbert space \( X \), with generator \( A \). Let \( Y \) be another Hilbert space, \( B \in L(X,Y) \) and \( \{S(t)\} \) the \( C_0 \)-contraction semigroup generated by \( A-BB^* \) on \( X \), and take \( \mathcal{C} \) to be the class of strongly measurable, \( Y \)-valued functions on \( X \). Using results of S. Foguel concerning the unitary decomposition of \( X \) corresponding to \( \{S(t)\} \), and the Uniform Ergodic Theorem, the following are established:

**Proposition I:** If the system \( (E)x=Ax+Bu \) with \( u \in \mathcal{C} \) is controllable, then for all \( x \in X \)

\[ \lim_{t \to \infty} S(t)x = 0. \]

**Corollary:** If \( (E) \) is controllable, \( \{T(t)\} \) quasi-compact, and \( B \) compact, then for all \( x \in X \)

\[ \mathcal{M}^{-1} \mathcal{L}(S(t)) \leq M e^{-\varepsilon t} \]

such that \( \|S(t)x\| \leq M e^{-\varepsilon t} \). These results, and a geometric condition equivalent to controllability, are used to give a proof of a theorem of W. Hahn. (Received March 17, 1976.)


In a Hilbert space \( H \), let \( A \) be a closed linear operator with domain \( D(A) \) dense in \( A-534 \).
H. Consider abstract differential operators \( L = - \frac{id}{dt} - A, \ L^* = - \frac{id}{dt} - A^* \). 
\( u \in L^2_{loc} (H) \) is said to be a weak solution of \( L u = f \in L^2_{loc} (H) \) if \( \int_R u(t), L^* \varphi(t) \ dt = \int_R \{ f(t), \varphi(t) \} \ dt, \) for all \( \varphi \in E_A^* \) - space of test functions.

Suppose for some pair \( f \) and \( u \) there holds \( |f(t)| \leq |u(t)| \) and also supp \( u \subset (-\infty, T) \) for \( T < \infty \) then \( u \equiv 0 \) under the hypothesis: the resolvent \( R (\xi, A) \) is bounded by a constant \( M \) on sequence of lines \( \text{Im} \ \xi = a_n \), \( n = 1, 2, \ldots \) with \( a_n \to \infty \) and \( \varphi(t) \leq c \) where \( c M < 1 \). (Received April 12, 1976.)


\( X \) is a real Banach space with dual \( X^* \). \( B(z, r) = \{ x \in X: ||x - z|| \leq r \} \) and \( B^* = B(0, 1) \).

\( K \subseteq X, K \uplus \phi \) is smoothable, if for each \( \epsilon > 0 \) there is an \( f \in X^* \), \( ||f|| = 1 \), and \( B(z, r) \) such that \( \sup f(B(z, r)) = \sup f(K) = s \) and \( \{ x \in K: f(x) \leq s - \epsilon \} \subseteq B(z, r) \).

D. Kemp [Math. Am. 218 (1975), 211-217] shows that \( B^* \) dentable implies \( B^* \) is smoothable.

We show by examples that, in general, \( B_x(B_x^*) \) smoothable does not imply \( B_x^*(B_x) \) dentable. Also, if \( X = WCG, B^*_x \) dentable implies that \( B^*_x \) is smoothable.

Smoothability is not, in general, isomorphically invariant. \( X \) is strongly smoothable (SS), if every bounded subset of \( X \) is smoothable with respect to each equivalent renorming of \( X \). It is shown that if \( X^* \) is separable, \( X \) reflexive, or more generally \( X^* \) WCG, then \( X \) is SS. (Received June 8, 1976.)

737-46-4 JOHN W. BRACE, University of Maryland, College Park, Maryland 20742 and JOEL D. THOMSON, Ithaca College, Ithaca, New York 14850. Function space completions.

We observe that properties usually restricted to \( L_p \) are exhibited in other function spaces. Recall that every convergent sequence in \( L_p \) has a subsequence converging almost everywhere. This can also be achieved by having a filter in the domain and requiring a convergent sequence to have a subsequence converging pointwise on some member of the filter. The \( L_p \) spaces are a specific case. The filter from the domain can sometimes be chosen to determine the linear topological structure of the function space.

Further variations are found by replacing pointwise convergence by uniform convergence and referring either to a subsequence or to the original sequence. The space \( L_\infty (0, 1) \) is the case where a sequence converges if and only if it converges uniformly on a member of the filter. The filter is all subsets of measure one. The four classes of spaces defined in this manner have natural functional completions. (Received June 10, 1976.)

737-46-5 M. M. Rao, University of California, Riverside, California 92502.

Bistochastic operators.

If \( (\Omega, \Sigma, \mu) \) is a measure space, \( T: L^p(\mu) \rightarrow L^p(\mu) \) a linear operator which is contractive for \( p = 1 \) and \( p = \infty \), then it is called a Dunford-Schwartz (or D-S) operator. If \( f, g \in L^1(\mu), \mu(\Omega) = \infty \), then an ordering \( f < g \) is defined through the decreasing rearrangements à la Hardy-Littlewood-Pólya. A linear operator \( T: L^1(\mu) \rightarrow L^1(\mu) \) is called bistochastic if for each \( f \in L^1(\mu), \) \( Tf < f \). Thm. Let \( (\Omega, \Sigma, \mu) \) be a measure space, \( \mu(\Omega) = \infty \). Then \( T: L^1(\mu) \rightarrow L^1(\mu) \) is bistochastic iff it is a positive D-S operator with \( T1 = 1 \). Such an operator can also be given an integral representation. [If \( (\Omega, \Sigma, \mu) \) is the Lebesgue unit interval, this was shown by J. V. Ryff (Pac. J. Math. 13(1963), 1379-1386), but the extension to the abstract case is nontrivial.]

If \( V_n = T_n \cdots T_1, L^p(\mu) \rightarrow L^p(\mu), p \geq 1, \) \( T_n = T_n \cdots T_1, \) where each \( T_i \) is a D-S operator, then conditions for the a.e. convergence of \( V_n f, f \in L^1(\mu), \) generalizing a result of N. Starr, TAMS, 121(1966), 90-115, are obtained. (Received June 10, 1976.)


Let \( (T, Z, m) \) be a finite measure space, with \( m \) a positive, non-atomic measure on \( E \). Let \( X \) be a separable Banach space. If \( F \) is a set-
valued function from $T$ to the collection of compact subsets of $X$, is measurable
\( \{ \{ t \in T \cap A \neq \emptyset \} \} \) is in $E$ for every closed $A$ in $X$, and integrably bounded, and
$P^*$ is the function with values $P^*(t) = \text{closed convex hull } P(t)$, then
$\text{cl} \int P = \int P^*$, where the integral is the Aumann Integral. If the values of $P$
are only weakly compact, the same result holds, provided a range of $P$ is
"nearly" weakly compact. The first result extends the theorem of Debreu,
given for the case of finite dimensional $X$, to the effect that $\int P = \int P^*$.
The second result generalizes to non-reflexive spaces a result of Datko.
(Received June 11, 1976.)

737-46-7 Alan Shuchat, Wellesley College. Wellesley, Massachusetts 02181. Vector measures and
scalar operators in locally convex spaces.

The adjoint of an operator $u$ on a Banach space $E$ with an operational calculus from $C[\phi(u)]$ to $L(E)$ is
a scalar operator whose spectral measure $\nu$ takes values in $L(E^*)$. P.G. Spain (Proc. Camb. Phil. Soc.
69 (1971), 409-410) has shown that $u$ is a scalar operator if and only if the values of $\nu$ are weak*
continuous. In this paper, we point out some connections with the theory of vector measures in locally
convex spaces and generalize Spain's result to such spaces, including barreled spaces that are distin-
guished (i.e., whose strong duals are barreled). (Received June 14, 1976.)

737-46-8 J.K. BROOKS, University of Florida, Gainesville, Florida 32601
and P.W. LEWIS, North Texas State University, Denton, Texas 76203.
Weak Coverage of Operators. Preliminary report.

Let $B = B(C(S,E),F)$ denote the Banach space of bounded operators from the continuous $E$-valued functions defined on the compact
space $S$ to the space $F$, where $E$ and $F$ are Banach spaces. Sufficient
conditions are given in order that certain subsets of $B$ are compact in the
weak topology of $B$. The technique involves embedding the operators
isometrically in a Lebesgue space of bilinear integrals, where weak
compactness is established. (Received June 14, 1976.)


L* = (H* + C) + \{Blaschke products\}.

Let $L^\infty$ (respectively $C$) denote the set of bounded complex valued measurable
(respectively continuous) functions on the unit circle. Let $H^\infty$ denote the usual
subalgebra of $L^\infty$ consisting of boundary values of bounded analytic functions in
the unit disk.

**THEOREM**: Let $f \in L^\infty$. Then there exists a Blaschke produce $b$ and a function
$h \in H^\infty + C$ such that $f = bh$.

**COROLLARY**: Let $E$ be a measurable subset of the unit circle $\partial D$ such that
$0$ is the only function in $H^\infty + C$ which vanishes almost everywhere on $E$. Then
$\partial D \sim E$ has measure zero.

The above corollary answers the question of determining the sets of
uniqueness for $H^\infty + C$; i.e., the only sets of uniqueness for $H^\infty + C$ are the sets
of full measure. (Received June 14, 1976.)

737-46-10 A. K. Snyder, Lehigh University, Bethlehem, Pennsylvania 18015. Duality theory and the
Nevanlinna-Pick property in functional Banach spaces.

Let $E$ be a Banach space of functions on a set $S$, $W \subset S$, and let $M(E)$ be the multiplier algebra of
$E$. Consider the restriction space $E|W$ as a quotient of $E$. The space $E$ has the Nevanlinna-Pick
property if $M(E|W) = M(E)|W$ isometrically; $E$ has the factorization property if there exists
$u \in M(E)$ such that $u$ is an isometry of $E|W$ onto $(S \setminus W)^\perp$ in $E$. We consider the problem of
characterizing those spaces with the Nevanlinna-Pick property. **Theorem 1**: Let $E$ be a Banach
sequence space such that the standard biorthogonal sequence is isometrically series summable. Then
$E$ has the Nevanlinna-Pick property if and only if the series space $\mathcal{S}(E)$ of $E$ satisfies
\[ |x| = \inf \{ s^n \| t^n \| : x = s^n t^n, s^n \in \mathfrak{S}, t^n \in (S \setminus W)^\perp \text{ in } E^f \} \text{ for all } x \in (S \setminus W)^\perp \text{ in } \mathfrak{S}(E). \]

Theorem 2: Let \( E \) be as in Theorem 1 and assume the functional dual \( E^f \) has the factorization property. Then \( E \) has the Nevanlinna-Pick property if and only if \( \mathfrak{S}(E) \) has the factorization property. The classical Nevanlinna-Pick theorem and its failure for the Bergman spaces follow easily from these ideas. Applications are given to problems involving zero-sets and universal interpolating sequences. (Received June 14, 1976.)

47 ▶ Operator Theory

*737-47-1 V. M. Sehgal; University of Wyoming, Laramie, Wyoming and S. P. Singh, Memorial University, St. John's, Newfoundland. Some Fixed Point Theorems for Multivalued Mappings. Preliminary report.

Theorem 1. Let \( C \) be a closed and starshaped subset of a Banach space \( X \) and let \( F: C \rightarrow \mathcal{R}(X) \) (family of non empty compact subsets of \( X \)) be a non expansive function such that \( F(\mathcal{S}C) \subseteq C \). If either (1) \( C \) is compact, or

(2) \( F(C) \) is bounded and \( (1 - F)C \) is closed, then \( F \) has a fixed point.

Theorem 2: Let \( C \) be a star shaped subset of a Hilbert space \( X \) and let \( F: C \rightarrow \mathcal{R}(X) \) be non expansive mapping such that \( F(\mathcal{S}C) \subseteq C \) and either

(1) \( C \) is weakly compact, or

(2) \( C \) is weakly closed and for some weakly compact \( K \subseteq X, F(C) \subseteq K \)

Then \( F \) has a fixed point. (Received May 24, 1976.)


In [Math Z. 125 (1972), 17-31, Theorem 1.7] we presented a set-valued version of Ky Fan's fixed point theorem for inward single-valued mappings [Math Z. 81 (1969), 234-240, Theorem 3]. We assumed there that the set-valued mapping in question was continuous. Using a different approach we are now able to show that the same result is true for upper semi-continuous mappings. (Received June 1, 1976.)


Arising from rather unpretentious origins, this theory has become a focal point from several mathematical perspectives. The relationship of these perspectives and their generalizations will be described. A report on some current interests and some open questions in the theory will also be given. (Received June 10, 1976.)

*737-47-4 Professor Gilbert Strang, Department of Mathematics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139. Uniqueness in the theory of variational inequalities.

For a given convex set \( K \) in \( \mathbb{R}^n \), we ask for the conditions on the matrix \( A \) which ensure uniqueness of the solution \( x \) in \( K \) to the standard variational inequality: \( (Ax - b, y - x) \geq 0 \) for all \( y \) in \( K \). If \( K \) is the unit ball, or any compact strictly convex set, then \( x \) is unique for every \( b \) if and only if \( A \) is both invertible and semidefinite: \( x^T A x \geq 0 \) for all real \( x \). On the unit cube, the condition on \( A \) is the same as the one discovered by Samelson, Thrall, and Wesler for \( K = \mathbb{R}^n_+ \): all principal minors must have positive determinants. We discuss the generalizations to infinite dimensions. (Received June 11, 1976.)

737-47-5 James R. Brown, Oregon State University, Corvallis, Oregon 97331. A brief history of Birkhoff's Problem 111.

In his AMS Colloquium Publication Lattice Theory (Revised Edition), published in 1948, Garrett Birkhoff presented 111 unsolved problems. Problem 111 (p. 266) asks for an extension, under suitable hypotheses, to the infinite-dimensional case of a 1946...
result of Birkhoff's - that any doubly stochastic (row and column sums equal to one) nxn-matrix is a convex combination of permutation matrices. Here we recount the various solutions to this problem that have been offered for infinite matrices, for stochastic matrices with a given invariant probability measure, and for doubly stochastic measures or operators. We also discuss the relationship between these various solutions and with other classical problems in analysis. (Received June 14, 1976.)

A differential operator with constant complex coefficients,

\[ A = \sum_{i,j=1}^{n} a_{ij} \xi_j \xi_i \xi_j, \]

is elliptic if \( 0 \leq V = \sum_{i,j=1}^{n} a_{ij} \xi_i \xi_j \xi_i \xi_j \in \mathbb{R}^n, \ |\xi| = 1 \).

In relating conditions of ellipticity and strong ellipticity (ReV > 0), it is interesting to know that V is a convex subset of for n ≥ 3. (If n = 2 then V is an ellipse.) This can be proved by a simple geometric argument. As a consequence we find that if n ≥ 2, then the numerical range of \( \{a_{ij}\} \), namely

\[ \left\{ \sum_{i,j=1}^{n} a_{ij} \xi_j \xi_i \xi_j | \xi \in \mathbb{C}^n, |\xi| = 1 \right\} \]

is convex. Corresponding results about linear operators in Hilbert spaces follow. The Toeplitz-Hausdorff theorem on the convexity of the numerical range is thus proved in a more transparent way than usual. (Received June 14, 1976.)

Let \( f \in L_p(X,P) \) for some \( p \in [1,\infty) \), \( P \) a probability measure. If \( \int|f|\log(1+|f|) \, dP < \infty \) and each operator \( S_k \) is doubly stochastic, Rota proved that \( \{S_1 S_2 \ldots S_n \} \) converges a.e. and boundedly in \( L_p(X,P) \) (Bull. A.M.S. 68, 95-102). Let \( (X,\mu) \) be a totally sigma-finite positive measure space. Given \( t > 0, \) let \( T(t,s) \) be an operator on each \( L_p(X,\mu), 1 \leq p < \infty, \) having norm at most one and mapping non-negative functions into functions non-negative a.e. Also assume

\[ T(s,t) = T(t,s) = T(t,0)T(s,t) \]

for all \( t, s \geq 0 \). We discuss some ideas of the proofs of limit and maximal behavior theorems for \( T(t,0)T(t,s) \). These results were announced without proof in Trans. A.M.S. 121, 90-115. (Received June 14, 1976.)

A spectral decomposition for continuous operators on Banach spaces is defined in general terms. It is then strengthened by requesting that such spectral decomposition be transferable to the restriction of the given operator to any invariant subspace. The operators endowed by this spectral decomposition property are shown to possess the spectral inclusion property, the single-valued extension property, and the decomposable spectrum property. Furthermore, if 0 is not a limit point of the spectrum, the given operator is the sum of an invertible and a quasinilpotent operator. (Received June 14, 1976.)

50 ▶ Geometry

A space-filling polyhedron is one whose replications can be packed to fill three-space completely. The space-filling tetrahedra, pentahedra and hexahedra have been previously
The search is here extended to the convex space-filling heptahedra. These are obtained by the division of known space-fillers into congruent parts, and by the combination of two or more space-fillers. (Received May 17, 1976.)

**Theorem.** There exist 91 types of isogonal tilings of the plane. (Received June 11, 1976.)

J. Steiner proved in 1826 that \( n \) planes in general position partition \( \mathbb{E}^3 \) into \( C = \sum_{k=0}^{3} \binom{n}{k} \) cells. This formula was generalized to \( \mathbb{E}^d \) by Schlafli (1852) and to \( r \)-faces in \( \mathbb{E}^d \) by Buck (1943). Steiner also showed that if the \( n \) planes of a simple arrangement in \( \mathbb{E}^3 \) fall into \( s \) parallel families with \( n_i \geq 1 \) planes in the \( i \)th family, then \( C = \sum_{k=0}^{3} \sigma_k \), where \( \sigma_k \) is the \( k \)th elementary symmetric function on the \( n_i \). We use the sweep-hyperplane method of Alfred Brousseau to count the \( r \)-faces formed by any simple arrangement in \( \mathbb{E}^d \), generalizing Steiner's formula. Brousseau's method also yields additive formulas when degeneracies beyond parallels occur; subtractive formulas were given in 1889 by S. Roberts (in \( \mathbb{E}^3 \)). We discuss some of the combinatorial identities that interrelate these various formulas and some additional identities that arise through a geometric process we call "lamination." Finally, we find the maximum number of cells formed in \( \mathbb{E}^3 \) by the \( n = \binom{m}{3} \) planes determined by \( m \) points, a problem recently posed by Alfred Brousseau. (Received June 15, 1976.)

52 ▶ **Convex Sets and Geometric Inequalities**

One considers the collection \( C(Y) \) of all nonempty, closed, bounded and convex sets of a reflexive Banach space \( Y \). Using operations of algebraic addition of sets and algebraic multiplication of a set by a scalar the collection forms a semilinear space. Given any neighborhood of zero in a locally convex topology of the space \( Y \), then the family

\[
N = \left\{ B: \text{AB} + V \text{ and } \text{BE} + V \right\}
\]

constitutes a base of neighborhoods for the set \( A \) of \( C(Y) \). This topology is the strong (weak) topology of \( C(Y) \) if it is generated by the strong (weak) topology of the Banach space \( Y \).
Conditions for a series of convex sets presented are essential to determine the structure of additive set valued functions defined on base cones in Banach spaces. (Received May 20, 1976.)


An $m$-set $X$ in $R^d$ is $(r,k)$-divisible if it can be partitioned into $r$ pairwise disjoint subsets whose convex hulls intersect in a set of dimension at least $k$. Thus Radon's theorem says each $(d+2)$-set in $R^d$ is $(2,0)$-divisible. Jurgenecoff recently showed that each $(2d+2)$-set in $R^d$ is $(2,1)$-divisible and asked what analogous results hold for $(r,1)$-divisible sets. (1) Examples of $[2d(r-1)+1]$-sets not $(r,1)$-divisible exist in $R^d$. (2) Upper bounds on $m$ are given for each $m$-set in $R^d$ to be $(r,1)$-divisible. (3) Each $[2d(r-1)+2]$-set in $R^d$ admits two $r$-partitions for which $z_i \in \cap_{j=1}^{r} \text{conv}X_{ij}$ for distinct points $z_i$ and $z_2$. Further, some subset of at most $(d+1)(r-1)+2$ points has this property. (4) Specifically, each $[4(r-1)+2]$-set in $R^2$ is $(r,1)$-divisible. (Received June 14, 1976.)

53 Differential Geometry


Let $M$ be a surface isometrically immersed in the complex plane $\mathbb{C}^2$. If the complex structure of $\mathbb{C}^2$ maps the tangent planes of $M$ into the normal planes of $M$ in $\mathbb{C}^2$, then $M$ is called a totally real surfaces in $\mathbb{C}^2$. Theorem. Up to rigid motions in $\mathbb{C}^2$, locally, totally real isometric analytic immersions of a flat surface in $\mathbb{C}^2$ are one-to-one correspondent to triples $(\nu(t), \sigma(t), \gamma(t))$ of analytic functions of one variable $t$ around $t=0$. (Received June 10, 1976.)

JURAJ VIRSIK, Monash University, Clayton, Australia 3168 and University of California, Berkeley, California 94720. Lifts and prolongations of connections. Preliminary report.

A connection on $M$ can be lifted (in the sense of Kobayashi and Yano) to a connection in the bundle $TTM \rightarrow TM$; on the other hand, it can be prolonged (in the sense of Ehresmann) to a connection in $TTM \rightarrow M$, i.e., in the Lie groupoid $\tilde{\mathbb{M}}^2(M)$ of invertible nonholonomic 2-jets on $M$. This picture can be applied to "higher order situations" by replacing $\tilde{\mathbb{M}}^2(M)$ with $\tilde{\mathbb{F}}^r$ — the $r$th order (nonholonomic) prolongation of a Lie groupoid $\Phi$ acting on a fibred manifold $E \rightarrow M$ and the functor $T$ with $\tilde{\mathbb{r}}^r(V,\star)_G$. Some results of J. Vilm (J. Differential Geometry 1(1967), 233–243) on lifts are generalised to this case. On the other hand, some known results on higher order connections can be applied to the geometry of the iterated tangent bundle $T^r(M) \rightarrow M$. (Received June 14, 1976.)


Let $F$ and $F'$ be two vector bundles over the same manifold $M$. A Lie group $G$ acting by vector bundle automorphisms on $F$ and $F'$ also acts on the vector bundle morphisms from $F$ to $F'$. We characterize the associated infinitesimal action. In particular we find the analogue of the Lie algebra for the group of all automorphisms (not necessarily base-preserving) of a vector bundle. We can deal in the same way with the non-linear case and with the case of a pseudo-group $\pi$ (we consider fiber bundles of "$\pi$-geometric objects”). The motivation for these remarks comes from the study of the covariance of differential operators. (Received June 14, 1976.)


Let $M$ be a complete, $n$-dimensional Riemannian manifold; given any point $p \in M$, let $r_p$ be any positive real number such that the geodesic ball $U_0$ with center $p$ and radius $r_p$ satisfies the following two conditions: (1) for each $q \in U_0$, $r_p$ is less than the convexity radius of $q$; (2) the supremum of the absolute value of all sectional curvatures at any $q \in U_0$, $\sup_{[k]} |k|$, satisfies $\sup_{U_0} |k| \leq \frac{2}{3} r_p^{-2}$. Applying a technique developed independently by J. Cheeger and K. Grove, for any given $k+1$ points $p_0, p_1, \ldots, p_k \in U_0$, one defines the singular
geodesic k-simplex \( \sigma(p_0', \ldots, p_k') \) as a differentiable map of the standard euclidean k-simplex into \( U_0' \), with the given points as vertices, geodesic segments as its \( L \)-edges, etc. If \( k \leq n \) and under suitable, but reasonable, additional conditions, the resulting k-simplex is smoothly imbedded, and its closure is a compact subdomain of a complete, k-dimensional submanifold \( P(p_0', \ldots, p_k') \subset U_0' \). In terms of \( r_0 \) and the piecewise geodesic distances between \( p_0', \ldots, p_k' \), one can estimate: (a) a uniform upper bound for the second fundamental form of \( P(p_0', \ldots, p_k') \); (b) an upper and a lower bound for the k-dimensional volume of \( \sigma(p_0', \ldots, p_k') \). Under the same conditions, if \( r_0 \) is chosen successively smaller, the bound (a) approaches zero, and both bounds (b) are asymptotic to the volume of the linear k-simplex whose \( L \)-edges have the same length as the corresponding ones of \( \sigma(p_0', \ldots, p_k') \). This local construction of "nearly flat" simplices in \( M \) is applied, in a global construction, to obtain a nearly flat triangulation of \( M \), together with a positive, diagonal (i.e., support preserving) duality operator on its real cochain complex, that approximates the Hodge duality operator for exterior forms in \( M \). The resulting combinatorial Laplace operator on the triangulation complex approximates the Hodge-de Rham differential operator in \( M \), as in J. Dodziuk's earlier construction, but furnishes also an approximation to the Green and the heat diffusion operators for k-forms. (Received June 14, 1976.)

## General Topology

### A Note on Preparacompactness. Preliminary report. (Received June 3, 1976.)

Elsayed A. Abo-Zeid, University of Saskatchewan, Saskatoon, Saskatchewan, Canada. On \( \sigma \)-connected spaces.

A topological space \( X \) is said to be \( \sigma \)-connected, provided, it is connected and cannot be decomposed into countably many mutually separated non-empty subsets. It is known that if \( X \) is a connected space and \( A \) is a connected subset of \( X \) such that \( X \setminus A = U \cup V \) where \( U \) is separated from \( V \), then \( A \cup U \) and \( A \cup V \) are connected. We prove this and some related theorems for \( \sigma \)-connected spaces. In particular we answer in the affirmative the following two questions. **Question 1.** (J. Grispolakis, A. Iailek and E. D. Tymchatyn) Suppose \( X \) is a continuum such that each \( \sigma \)-connected subset of \( X \) is a semi-continuum. Is every connected subset of \( X \) arcwise connected? **Question 2.** (J. Mycielski) Does every infinite \( \sigma \)-connected set contain an infinite proper \( \sigma \)-connected subset?

### Hereditarily indecomposable tree-like continua. (Received June 3, 1976.)

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There exists in the plane an uncountable collection \( H \) of mutually exclusive hereditarily indecomposable tree-like continua such that if \( M \) is a compact metric continuum then some member of \( H \) is not a continuous image of \( M \). (Received May 17, 1976.)

### A Note on Preparacompactness. Preliminary report. (Received June 10, 1976.)

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In a paper entitled, "Preparacompactness and \( \mathcal{W} \)-preparacompactness in q-spaces" (Colloq. Math. 1973, Vol. 27, 227-235) R. C. Briggs introduced two generalizations of paracompactness and obtained a number of results relating paracompact and preparacompact spaces.

**Definition:** A \( T_2 \) space \( X \) is called preparacompact (ppc) (resp. \( \mathcal{W} \)-preparacompact) if each open cover of \( X \) has an open refinement \( \mathcal{W} = \{ U_a : a \in A \} \) such that, if \( B \) is any infinite (resp. uncountable) subset of \( A \) and if \( p_B = U \) for each \( B \in B \) with \( p_B \neq p_\beta \) and \( p_\beta \neq p_\delta \) for \( \sigma \neq \beta \), then the set \( Q = \{ q_\beta : \beta \in B \} \) has a limit point whenever \( P = \{ p_B : B \in B \} \) has a limit point. **Definition:** A space \( X \) is called a q-space if each point \( x \in X \) has a sequence of neighborhoods \( \{ N_i \}_{i=1}^\infty \) such that if \( y_i \in N_i \) for each \( i \) and \( y_i \neq y_j \) for \( i \neq j \), then the set \( \{ y_i \}_{i=1}^\infty \) has a limit point.

In this paper we generalize Briggs' results using the notion of irreducible spaces. An example of the results obtained is the following theorem.

**Theorem:** Let \( X \) be a regular q-space. Then \( X \) is paracompact iff \( X \) is \( \mathcal{W} \)-ppc and irreducible.

**Corollary:** Let \( X \) be a regular q-space. Then the following are equivalent: (1) \( X \) is paracompact. (2) \( X \) is \( \mathcal{W} \)-ppc and \( \delta \)-refinable. (3) \( X \) is \( \mathcal{W} \)-ppc and weak \( \delta \)-refinable. (Received June 10, 1976.)

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Let \((X, \mathcal{U})\) be a uniform space and let \(P(X) = \{A: A \subseteq X\}\). In this paper we consider three different uniformities on \(P(X)\). Let \(U \in \mathcal{U}\) and define \(H(U) = \{(A,B): A \subseteq U[B] \text{ and } B \in \mathcal{U}[A]\}\), \(S(U) = \{(A,B): A = B \text{ or } A \times X \subseteq U[B]\}\), and \(G(U) = \{(A,B): A = B \text{ or } A \times X \subseteq U[B]\}\). Then \(H(U) \subseteq \mathcal{U}\), \(S(U) \subseteq \mathcal{U}\), and \(G(U) \subseteq \mathcal{U}\) are respectively bases for uniformities \(\mathcal{U}\) (the usual Hausdorff uniformity), \(\mathcal{U}^s\), and \(\mathcal{U}\) on \(P(X)\). It is proved that \(\mathcal{U}^s \subseteq \mathcal{U}\) The inclusion map from \(X\) into \(P(X)\) is a uniform isomorphism with respect to both \(\mathcal{U}\) and \(\mathcal{U}^s\) (what E. Michael calls admissible uniformities), but not with respect to \(\mathcal{U}\). An attempt is made to determine when there is a uniform isomorphism from \((X, \mathcal{U})\) into \((P(X), \mathcal{U}^s)\).

For a uniformity \(\mathcal{U}\) is generated by a filter \(\mathcal{F}\) iff \(\mathcal{U}\) has a base of the form \(\mathcal{F} \cap \mathcal{F}'\) where \(F \in \mathcal{F}\). Theorem: If \(\mathcal{U}\) has a smallest element, then there exists a uniform isomorphism from \((X, \mathcal{U})\) into \((P(X), \mathcal{U})\) iff \(\mathcal{U}\) is generated by a principal filter. Theorem: If the topology generated by \(\mathcal{U}\) is not discrete, then there exists a uniform isomorphism from \((X, \mathcal{U})\) into \((P(X), \mathcal{U}^s)\) iff \(\mathcal{U}\) is generated by a filter. (Received June 14, 1976.)

By modifying an example of E.A. Michael, we get a stratifiable \(\kappa^-\)-space which is not \(k^-\)-space (i.e. \(X\) is Tychonoff and \(f: X \rightarrow \mathbb{R}\) is continuous on compact subsets implies \(f\) continuous on \(X\)) but not a \(k\)-space (i.e. \(X\) has weak topology over compact subsets). (Received June 14, 1976.)

By modifying an example of E.A. Michael, we get a stratifiable \(\aleph_0\)-space \(X\) which is a \(k^-\)-space but not a \(k\)-space (i.e. \(X\) is Tychonoff and \(f: X \rightarrow \mathbb{R}\) is continuous on compact subsets implies \(f\) continuous on \(X\)).

A function \(f: X \rightarrow Y\) is said to be perfect if it is a continuous closed surjection such that for every \(y \in Y\), \(f^{-1}(y)\) is compact in \(X\).

1. Juhász has proposed that a closed surjection \(f: X \rightarrow Y\) is perfect iff:
   (i) For each \(x \in X\), \(f(\mu(x)) \subseteq \mu(f(x))\).
   (ii) For each remote \(z \in X\), \(f(z)\) is remote in \(Y\).

We have proven this with the additional condition that \(X\) is regular. Also, if \(X\) and \(Y\) are any topological spaces and \(f: X \rightarrow Y\) is perfect, then for each \(x \in X\), \(x\) is near-standard iff \(f(x)\) is near-standard. These two results are used to prove classical results elegantly. (Received June 14, 1976.)

The symbol \(P = Z[Q, R]\) will mean that a space having property \(Q\) is \(Z\)-embedded in every space containing it having property \(R\) if the space satisfies property \(P\). If \(Q = R\) we will say \(P = Z[Q]\). For instance, Blair and Hager have shown that \(P = Z[Q]\) where \(Q\) is Tychonoff and \(P\) is Lindelöf or almost compact. We prove that \(P = Z[Q]\) where \(Q\) is functionally Hausdorff and \(P\) the property that the weak topology is Lindelöf. We also simplify and extend proofs on \(C\)-embedding of various types of pseudocompact subsets using \(Z\)-embedding. See Blair and Hager, Extensions of zero-sets and of real valued functions, Math. Z., 131 (1974), 41-52, for a background on \(Z\)-embedding. (Received June 14, 1976.)

Continuing Abstract 731-54-24. B. Hutton [J. Math. Anal. Appl., 50 (1975), 74-79] defined the fuzzy unit interval and established some of its properties. We prove that the fuzzy unit interval is a-compact for certain \(a\) and, under more general conditions than Hutton's, show that it is compact. We define the fuzzy real line and show that it is not compact and for some \(a\) that it is not a-compact. (Received June 14, 1976.)
A theorem basic to the treatment of open mappings in base of countable order theory (equivalently, in monotone space theory) and in primitive space theory is proved. The proof is direct and involves neither the axiom of choice nor transfinite recursion. With its use, unified and simple proofs can be given for a number of propositions such as preservation of base of countable order by open continuous uniformly complete mappings with (para)regular domain. Theorem. Suppose X is a space and \( f: X \to Y \) is an open mapping. Suppose \( \langle \mathcal{U}_n : n \in \mathbb{N} \rangle \) is an open primitive sequence of \( M \) in \( X \). Then there exists an open primitive sequence \( \langle \mathcal{V}_n : n \in \mathbb{N} \rangle \) of \( f(M) \) in \( Y \) such that for all \( i \in \mathbb{N} \):

1. There is a subcollection \( A_1 \) of \( \mathcal{U}_1 \) such that the mapping \( f_1 \) induced by \( f \) on \( A_1 \) is one-to-one.
2. The well ordering on \( H_1 \) is that induced by \( f_1 \).
3. For all \( y \in f(M) \), if \( H \) is the first element of \( H_1 \) containing \( y \), then \( f^{-1}(y) = f^{-1}(y) \cap N \) is the first element of \( \mathcal{V}_1 \) that contains \( x \). Moreover, if \( \langle H_n : n \in \mathbb{N} \rangle \) is a primitive representative of \( \langle H_n : n \in \mathbb{N} \rangle \), then \( f^{-1}(H_n) : n \in \mathbb{N} \) is a primitive representative of \( \langle \mathcal{V}_n : n \in \mathbb{N} \rangle \). For terminology see the paper by Wicke and Worrell, *A characterization of primitive bases*, Proc. Amer. Math. Soc. 50 (1975) 443–450. (Received June 15, 1976.)

55 ▲ Algebraic Topology

**57-55-1**  VINCENT O. MCBRIDE, College of the Holy Cross, Worcester, MA 01610

The Homology of Literary Schema.

The recent developments by Thom and popularized by Zeeman, relating singularities to manifolds have been accompanied by the literary efforts of Barth and others for a deeper understanding of metre. These efforts have chiefly been engaged in replacing the complex algebra of the iambic metre \((\lambda , \lambda )\), developed since the time of Chaucer, by simple algebraic schema representing the rise and fall of dramatic actions. Actually these schema are more topological in nature and can be extended from simple graph theory to higher dimensional cases when the Euler characteristic is more meaningful. For example, in the case of iambic pentameter \( P \) with a closed subset drama, \( \chi (\cdot) = 1 \). In general the homology of the poem would seem to be a far simpler approach than the list of complex rules permitted for the various substitutions tolerated in any algebraic approach now used in the analysis of metre. (Received March 9, 1976.)


The concept of a linking function is introduced in this paper. This function is of importance in the study of linking numbers between branch curves in branched covering spaces of a knot in \( S^3 \), here called covering linkage invariants. With this function, one can compute covering linkage invariants from a set of linear equations. Apart from this computational application, linking functions allow us many results about covering linkage, including the demonstration of a close connection between the covering linkage invariants and an invariant of Burde, the proof of conjectures of Riley about \( PSL(2, p) \)-covering, a connection with an invariant considered by Reyner and a proof of his conjecture. A certain class of 2-bridged knot is also shown to have property \( P \). (Received May 24, 1976.)


I will begin by considering the problem of group actions on spheres, as an appealing geometrical problem and a source of topological problems. I will show that it leads to problems of the following form: given compact

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Lie groups $G$ and $G'$, with classifying spaces $BG$ and $BG'$, what can we say about the homotopy classification of maps $f: BG \to BG'$? I will start with the case in which $G$ and $G'$ are connected, and here the results are not so recent, though not yet published (to appear September 1976). I will continue with more recent results for the case in which $G$ is finite. I hope to conclude by commenting on work in progress for the general case.

(Received June 1, 1976.)

737-55-4 Stavros G. Papastavridis, University of Athens, Athens, GREECE. Relations among characteristic classes II. Preliminary report.

Let $B=SO$ or $SU$ and $M$ be an $n$-dimensional compact manifold and let us assume that there is a map $\omega: M \to B$ classifying the stable normal bundle of $M$. Let $p$ be a prime number so that the universal bundle over $B$ is $\mathbb{Z}_p$ orientable. All cohomology groups will have $\mathbb{Z}_p$ coefficients. Let $q, k$ be natural numbers so that $q+k \leq n$, $q, k$ and $n$ are fixed throughout. Let $A$ be the mod-$p$ Steenrod algebra.

Let $z \in (H^n(B) \otimes A)^q$ we define a map $f_z : H^k(M) \to H^{k+q}(M)$ as follows: if $z=\sum x_i A_i, x_i \in H^n(B), A_i \in A$ then $f_z(x) = \sum x_i \cdot (A_i x)$ for $x \in H^k(M)$. Let $I^q_n(B, p, k) = \{ z \in (H^n(B) \otimes A)^q : f_z = 0 \}$ for all $n$-dimensional, compact, $\mathbb{C}^\infty$ manifolds and all maps $\omega: M \to B$. In this paper we compute $I^q_n(B, p, k)$.

(Received May 17, 1976.)

*737-55-5 Jonathan Simon, University of Iowa, Iowa City, Iowa 52242. Fibered knots in homotopy 3-spheres.

Using the result, recently obtained by F. Gonzalez-Acuña (1974) and by R. Myers (these Notices, 737-094, Oct. 1975, p. A-651), that each closed, orientable 3-manifold contains a fibered knot, and Theorem 9.2.3 of Neuwirth's Knot Groups, we show that the 3-dimensional Poincaré conjecture is equivalent to the following. Conjecture: If $G$ is a group with the properties listed below then there exists a knot in $S^3$ whose group is isomorphic to $G$; (1) $G/G'$ is infinite cyclic, (2) $G'$ has an element $t$ whose normal closure is all of $G$, and (3) $G'$ is free of rank $2n$ with a free basis $a_1, \ldots, a_n, b_1, \ldots, b_n$ such that $t$ commutes with $[a_i, b_i]$. (The result announced here will appear in Proc. A.M.S. under the above title.)

(Received June 7, 1976.)

*737-55-6 WILBUR WHITTEN, University of Southwestern Louisiana, Lafayette, La. 70504. Prime factors of knot manifolds.

Because of the torus-theorem's proof [C.D. Feustel, "The torus theorem and its applications," Trans. Amer. Math. Soc., in press], the "complements conjecture" (that two tame, prime knots (in $S^3$) with isomorphic groups have homeomorphic complements) holds except perhaps when the knots are cables. Assuming that such exceptions never occur and given an arbitrary knot complement, J. Simon ["On the problems of determining knots by their complements and knot complements by their groups," Proc. Amer. Math. Soc., in press] recently showed that there are at most three knot types whose knots' complements are all homeomorphic to the given one. In this paper, we apply this result to obtain a bound for the number of knot types with the same group. We also prove, modulo the complements conjecture, that a knot's group determines the knot's genus.

(Received June 10, 1976.)

737-55-7 James M. Van Buskirk, University of Oregon, Eugene, Oregon 97403. Alexander polynomials of amphicheiral knots.

Beginning with a knot $K$ and its mirror image $K'$, constructions generalizing the formation of the product of $K$ and $K'$ are used to produce amphicheiral knots. One such construction yields an amphicheiral knot having Alexander
polynomial

$$\Delta^2_K(x) - xz^2L(x,x)$$

where $\Delta_L(x,x)$ is the Alexander polynomial of a 2-component link $L$ determined from $K$. The infinite class of amphicheiral knots constructed in this manner includes a prime amphicheiral knot with trivial Alexander polynomial.

(Received June 14, 1976.)


If $M$ is a compact, orientable, irreducible 3-manifold in which every torus is either compressible or boundary parallel, and $C$ is a finite collection of curves on the boundary of $M$, then there is a finite collection of surfaces $S_i$ in $M$ such that if $T$ is any properly embedded incompressible surface in $M$ with the boundary of $T$ in $C$, then there is an orientation preserving autohomeomorphism $h$ of $M$, which is the identity map on the boundary of $M$, such that $h(T) = S_i$ for some $i$.

As a consequence of this result we show that given a knot $k$ in $S^3$, there is a finite collection of minimal spanning surfaces $S_i$ for $k$ such that if $T$ is any minimal spanning surface of $k$, then there is an orientation preserving autohomeomorphism $h$ of $S^3$ such that $h(T) = S_i$ for some $i$. (Received June 14, 1976.)

*737-55-9 MARK E. KIDWELL, Yale University, New Haven, Connecticut 06511. Relations between the Alexander polynomial and summit power of a closed braid. Preliminary report.

Let $\beta$ be a closed $n$-braid inside a standardly-embedded solid torus $V$ in $S^3$, and let $M$ be a meridian of $\beta V$. Consider the $(n+1)$-string link $\beta U M$ with Alexander polynomial $h(t_1, \ldots, t_n, w)$, and reduce this polynomial to $h(t, w) = \sum_j f_j(t)w^j$.

Theorem 1: Suppose $\beta$ has $p$ positive crossings and $q$ negative crossings, $p + q = m$. Then $h(t, w) = t^p + f_1(t)w + \cdots + f_{m-2}(t)w^{m-2} + t^m w^{m-1}$.

Corollary 1: If $n = 3$, then $f_1(t) = (1 + t + t^2)A_1(t) - t^q - t^p$, where $A_1(t)$ is the Alexander polynomial (suitably reduced and normalized) of $\beta$.

Theorem 2: If $\beta$ is a positive 3-braid prime to $\Delta = \sigma_1^2 \sigma_1$, then $$(1 + t + t^2)A_1(t) = 1 + b_1 t + \cdots + b_1 t^{3n+1} + t^{3n+2},$$ where $\Delta_1(t) = 1 + b_1 t^{3n+1} + \cdots + b_1 t^{3n+1} + t^{3n+2}$. (Received June 14, 1976.)

57 ▲ Manifolds and Cell Complexes

737-57-1 DENNIS McGAVRAN, University of Connecticut, Waterbury, Connecticut 06710. $T^n$-actions on simply connected $(n + 2)$-manifolds. Preliminary report.

In this paper we show that, for each $n \geq 2$, there is a unique, closed, compact, connected, simply connected $(n + 2)$-manifold, $M_{n+2}$, admitting an action of $T^n$ satisfying the following condition: there are exactly $n$ $T^1$-stability groups $T_{i1}, \ldots, T_{in}$ with each $F(T_{ij}, M_{n+2})$ connected. In this case we have $T^n = T_{i1} \times \cdots \times T_{in}$. Any other action $(T^n, M_{n+2})$, $\n_0 \leq n \leq 2$ simply connected, can be obtained from an action $(T^n, M_{n+2})$ by equivariantly replacing copies of $D^4 \times T^{n-2}$ with copies of $S^3 \times D^2 \times T^{n-2}$. As an application, we classify all actions of $T^n$ on simply connected $(n + 2)$-manifolds for $n = 3, 4$. (Received May 11, 1976.)

*737-57-2 E. J. Mayland, Jr., YORK University, Downsview, Ontario M3J 1P3. Inductive arguments on rational (two-bridge) knots.

This paper studies the class of two-bridge knots in terms of their alternating normal forms as Viergeflechte, as described by Bankwitz and Schumann (Hamburg Abh., 10, 1934). Using Conway's notations for these normal forms as rational knots (in Computational problems in abstract algebra, Perga-
The knot types are enumerated and then constructed inductively by iterating a geometrical "twisting" operation, described, e.g., by Fox (Osaka Math. J., 10, 1958). The inductive modification produces minimal spanning surfaces of a particularly simple type, for which result we give the following two alternative formulations:

**Theorem.** A rational knot of genus \( g > 0 \) may be algorithmically constructed from a chain of \( 2g \) factors \( L_n \), each of torus link type \( (2n_i, 2) \), for some \( n_i \neq 0 \), and with each factor non-fibred when \( |n_i| > 1 \).

**Corollary.** A rational knot of genus \( g > 0 \) may be algorithmically constructed from a chain of \( g \) factors, each a two-bridge knot of genus one. (Received June 3, 1976.)

*737-57-3 WITHDRAWN*

*737-57-4 Deborah L. Goldsmith, University of Michigan, Ann Arbor, Michigan, 48109. A new invariant of link concordance.*

Let \( L = K_1 \cup K_2 \cup \ldots \cup K_n \subset S^3 \) be an oriented link. Let \( p:M_a(K) \to S^3 \) be the \( a \)-fold cyclic branched covering of \( S^3 \) branched along \( K \), and let \( \Pi = \mathbb{Z}_a \) be the group of covering translations of \( M_a(K) \). Choose oriented components \( K_i \) and let \( \lambda(x,y) \in \mathbb{Q} \) denote the linking number of \( 1 \)-cycles \( x, y \subset M_a(K) \) if it is defined, and let \( \lambda(x,x) = 0 \). Define matrices \( \Lambda_0, \Lambda_1 \) with entries in \( \mathbb{Q}(\Pi) \) to be equivalent if \( \Lambda' = \Lambda_0 \Lambda_1^{-1} \Lambda \) for some diagonal matrix \( \Lambda \) with entries in \( \Pi \). **Theorem.** Suppose \( a = p^r \) for some prime \( p \). Then the class \([\Lambda_a(K)]\) of the \( nxn \) matrix \( \Lambda_a(K) \) with entries

\[ \lambda_{ij} = \sum_{\sigma \in \Pi} \lambda(K_i, \sigma(K_j)) \sigma \]

is an invariant of concordance and isotopy of \( L \). (Received June 8, 1976.)

*737-57-5 John Hempel, Rice University, Houston, Texas 77001. HNN groups, residual finiteness, and 3-manifolds. Preliminary report.*

Corresponding to a hierarchy for a sufficiently large 3-manifold, \( M \), there is a sequence of HNN groups terminating with \( \pi_1(M) \). As an approach to the question whether \( \pi_1(M) \) need be residually finite we establish some theorems giving positional properties of the bonding subgroups \( C_0, C_1 \) of \( G=\text{HNN}(B;C_0,C_1) \) which will assure that \( G \) be residually finite. Emphasis is given to the case \( \pi_1 \) in which the base group, \( B \), is free and applications are given to the groups of certain classes of knots which span unknotted, incompressible surfaces. (Received June 11, 1976.)

*737-57-6 Daniel R. McMillan, Jr., University of Wisconsin, Madison, Wisconsin 53706. I-equivalent graphs in \( S^3 \).*

Compact sets \( X, Y \) in the \( n \)-sphere \( S^3 \) are I-equivalent if for some \( Z \subset S^3 \times [0,1] \), \( \mathbb{Z} \times x \) to \( X \times 0 \) and \( \mathbb{Z} \times x \) to \( X \times 1 \), and some homeomorphism of \( Z \) with \( X \times [0,1] \) carries \( X \times 0 \) to \( X \times 0 \) and \( Y \times 1 \) to \( X \times 1 \). J. Stallings (J. Algebra 2 (1965), 170-181) has shown that if \( X \) and \( Y \) are I-equivalent and \( A = \pi_1(S^3-X) \), \( B = \pi_1(S^3-Y) \), then for each \( k \) the lower central quotient groups \( A/A_k \), \( B/B_k \) are isomorphic. This is used to show that none of a certain infinite sequence of graphs \( G_1 \subset S^3 \) is I-equivalent to the standard embedding of \( G_1 \) in \( S^3 \) (\( G_1 \) is the disjoint union of two "figure 8's"). This result is used to construct 2-dimensional cell-like continua \( C_1, C_2 \) in a PL 4-manifold \( M^4 \) such that \( C_1 \cap C_2 \) is a point, each \( C_1 \) has a neighborhood in \( M^4 \) embeddable in \( S^4 \), yet the cell-like continuum \( C_1 \cup C_2 \) has no such neighborhood in \( M^4 \). (R.J. Daverman has conjectured that there is such an example with each \( C_1 \) an arc.) (Received June 11, 1976.)
Draw a picture of disjoint curves in 3-space and assign to each a rational number, or \( \infty \). Then you have described a 3-manifold, according to the surgery technique of Max Dehn and the notation introduced in this expository talk. The notation facilitates proving some interesting things about 3-manifolds with only the ability to draw pictures and to add fractions. (Received June 14, 1976.)

**W. JACO and P. B. SHALEN, Rice University. Centralizers, Roots and Relations in 3-manifold groups.**

A complete description is given for the centralizer, and the set of roots, of an element of the fundamental group \( G \) of a sufficiently large, compact, irreducible, orientable 3-manifold. For the case of a knot group this answers a question of L. P. Neuwirth's. The results are stated in a forthcoming BAMS article, "Seifert fibered spaces in irreducible, sufficiently-large 3-manifolds." As an application we show that a relation \( ab^p a^{-1} = b^q \) cannot hold in \( G \) unless \( p = \pm q \) or \( b = 1 \). (Received June 15, 1976.)

### Global Analysis, Analysis on Manifolds

**Chung-wu Ho, Southern Illinois University, Edwardsville, IL 62026.** On certain problems connected with the homeomorphisms which satisfy the Poincaré recurrence theorem.

Let \( X \) be a closed manifold with a nonzero Euler characteristic, and \( H(X) \) be the space of all the homeomorphisms of \( X \) onto \( X \) under the compact open topology. Suppose a Borel measure \( \mu \) is given on \( X \) which assigns a positive value to each nonempty open subset of \( X \). Then an element \( f \in H(X) \) is said to be recurrent if the set of all the points of \( X \) which are nonrecurrent under \( f \) is of measure zero with respect to \( \mu \). The author has established the following Theorem: The set of all the recurrent homeomorphisms of \( X \) is nowhere dense in \( H(X) \). Some possible extension of this theorem will also be discussed. (Received May 10, 1976.)

**LOUIS BLOCK, University of Florida, Gainesville, Florida 32611.** Topological entropy at an \( \Omega \)-explosion. Preliminary report.

An example is given of a function \( f \in \mathbb{C}^2(\mathbb{S}^1, \mathbb{S}^1) \) at which an \( \Omega \)-explosion occurs. It is shown that topological entropy (considered as a map from \( \mathbb{C}^2(\mathbb{S}^1, \mathbb{S}^1) \) to the non-negative real numbers) is continuous at \( f \). (Received June 1, 1976.)

**Robert L. Devaney, Northwestern University, Evanston, Illinois 60201.** Transversal homoclinic orbits in an integrable system.

We construct a mechanical system \( X \) on \( \mathbb{R}^n \) satisfying each of the following:
1. \( X \) admits a unique hyperbolic equilibrium point \( p \).
2. There are 2\( n \) transversal homoclinic orbits to \( p \) within the energy surface.
3. All other orbits in \( \mathbb{W}^s(p) \) and \( \mathbb{W}^u(p) \) are eventually homoclinic.
4. There are no invariant sets on which \( X \) is conjugate to the suspension of a Bernoulli shift.

Thus, transversal intersection of the stable and unstable manifolds together with minimality of the flow on an invariant set do not imply the existence of Smale "horseshoes" nearby. (Received June 1, 1976.)

**R. C. Churchill, Hunter College, New York, New York 10021.**


Let \( f \) be a \( \mathbb{C}^1 \) flow on a compact manifold \( M \).

**Definition:** A closed invariant set \( A \subset M \) is quasi-hyperbolic if

(a) the span \( E \) of the vector field of \( f|_A \) is a subbundle of \( T_A M \); and

(b) the flow on \( T_A M \) induced by the tangent flow has no nontrivial bounded orbits.

This is a weaker and more geometric condition than hyperbolicity. However, coupled with chain recurrence we obtain

**Theorem A:** If \( A \) is chain recurrent, then \( A \) is hyperbolic if and only if \( A \) is quasi-hyperbolic.

Using this result, the hyperbolicity hypothesis can be weakened in several theorems. For example, if \( R \) denotes the maximal chain recurrent set in \( M \) then
Corollary B. If $R$ is quasi-hyperbolic then $f$ satisfies Axiom A and the no cycle property.

Theorem A leads to an easy proof that the geodesic flow on a compact Riemannian manifold of negative curvature is Anosov. Also, it can be used with a computer to verify that certain periodic orbits are hyperbolic. (Received June 11, 1976.)


The usual definition of a cycle for a dynamical system $f$ on a compact manifold $M$ requires that it satisfies Axiom A. The definition eliminates the Axiom A assumption.

Definition: A finite sequence of points $x_1, x_2, \ldots, x_n$ with $x_1 = x_n$ is a cycle if the $\omega$-limit set of $x_i$ intersects the $\omega$-limit set of $x_{i+1}$. A set $A \subset M$ is said to have the no cycle property if every cycle in $M$ is actually in $A$. If $f$ satisfies Axiom A then the nonwandering set $\Omega$ satisfying this no cycle property is equivalent to the standard no cycle property. C. Conley et al. have studied a set which exhibits some recurrence properties and contains $\Omega$. This is called the chain recurrent set $R$. Let $L = \sigma(\bigcup_{x \in \Omega} a(x) \cup \omega(x))$.

Theorem A. If $L$ is hyperbolic then every chain recurrent point is on a cycle.

Theorem B. If $\Omega$ is hyperbolic and has the no cycle property then

1) $\Omega = R$, 2) $\Omega$ satisfies Axiom A, and 3) $f$ is $\Omega$-stable.

A theorem of S. Newhouse follows from these results.

Theorem C. If $L$ is hyperbolic and has the no cycle property then $f$ satisfies Axiom A.

(Received June 11, 1976.)


Fix a smooth mapping $g = (g_1, \ldots, g_c)$ from a given manifold $M^n$ into $R^c$. We assume that $g$ satisfies the generic condition, for any $1 \leq i_1 < \ldots < i_s \leq c$, $g_{i_1}(x) = \ldots = g_{i_s}(x) = 0$ implies the differentials $dg_{i_1}(x), \ldots, dg_{i_s}(x)$ are linearly independent. For simplicity, we also assume the set $W_g = \{g_1(x) > 0, \ldots, g_c(x) > 0\}$ is compact. Now, let $f = (f_1, \ldots, f_p)$ be any smooth mapping from $M^n$ into $R^p$. A point $x$ in $W_g$ is said to be a local Pareto optimum of $f$ with constraints $g \geq 0$ iff there exists a neighborhood $N$ of $x$ such that for any $y \in N \cap W_g$, $f_1(y) \geq f_1(x), \ldots, f_p(y) \geq f_p(x)$ implies that $f_1(y) = f_1(x), \ldots, f_p(y) = f_p(x)$. Under the above assumptions on the constraints $g$, we have the following result:

Theorem. There exists an open dense set of smooth mappings $f = (f_1, \ldots, f_p): M^n \rightarrow R^p$ such that: a) the set $0$ of local Pareto optima of $f$ admits a Whitney (pre)stratification $S$ of dimension $\min(n, p-1)$; b) the set $\Theta$ of all nondegenerate local Pareto optima of $f$ is a finite union of strata in $S$, and $\Theta$ is dense in $\Theta$; c) the set $\Theta$ and $\Theta$ are stable under small perturbation. The proof requires the characterization theorem for local Pareto optima with constraints and the use of stratification and transversality theory. (Received June 11, 1976.)


Theorem: Let $M_i$, $i = 1, \ldots, k$ be pairwise disjoint, closed $(n-1)$-manifolds without boundary embedded in $R^n$. Then one can construct a vector field on $R^n$ for which each $M_i$ is an attractor of the corresponding flow, and there are no other $(n-1)$-dimensional invariant sets.

A similar result holds for codimension 1 submanifolds of compact manifolds. (Received June 14, 1976.)


Consider a parameterized differential equation $\dot{x} = f(x, \alpha)$, $\alpha \in R$ where $f(0, \alpha) \equiv 0$, and such that the associated flow possesses a generic Hopf bifurcation (i.e., a pair of non-zero eigenvalues of the linearized equation transversally crossing the imaginary axis) from the origin $x = 0$ for a sequence $\{\alpha_n\}$.

A global bifurcation theorem of Alexander and Yorke describes what happens if one follows the bifurcating periodic solution from some $x = 0$, $\alpha = \alpha_n$. Specifi-
cally, either the amplitude of the solution tends to infinity, the parameter $\alpha$ tends to plus or minus infinity, the period of the solution tends to infinity or the periodic solution connects to some other Hopf bifurcation (i.e. at some $\alpha_m \neq \alpha_n$).

We describe how this theorem can be proved using transversality techniques, and an index for periodic orbits defined by F. B. Fuller. We also show how the theorem applies to specific equations. (Received June 15, 1976.)

60 ▸ Probability Theory and Stochastic Processes


Let $b_n$ denote the queue size at discrete times $n$. The process considered satisfies

$$b_{n+1} = (b_n - 1)^+ + \sum_{i=0}^{k} \alpha_i x_{n-i},$$

where the non-negative integer valued random variables $x_j$ are i.i.d., and the constant coefficients $\alpha_i$ are non-negative integers. Such a process has been investigated by H. G. Herbert [J. Appl. Prob. 9 (1972), 404-413] under the additional assumption $\alpha_i \neq 0$, $i = 0, \ldots, k$.

Under the stability condition $\left( \sum_{i=0}^{k} \alpha_i \mathbb{E}(x_j^i) \right) < 1$, an expression is derived for the steady state generating function of a joint process involving $(b_{n+1}, x_n, \ldots, x_{n-k+1})$, which is a Markov process. This expression involves the conditional generating function corresponding to zero queue size, and it is shown how an equation for this function, which is a multi-nomial, may be obtained. An alternate procedure for calculating the coefficients in this multi-nomial, which involves a finite number of equations for the steady state probabilities derived from the Markov chain corresponding to the joint process, is also discussed. The case $\alpha_0 = 1 = \alpha_k$, $\alpha_i = 0$ otherwise, is of particular interest, and explicit expressions for the multi-nomial have been derived in this case for $k = 2, 3$ and 4.


A necessary and sufficient condition in order that a measure $\mu$ on a Hilbert space be Gaussian is provided. In this connection is derived a functional equation satisfied by the densities (Radon-Nikodym derivatives), and this equation is similar to the functional equations satisfied by the characteristic functionals. (Received May 21, 1976.)


Let $A_1, A_2, \ldots, A_n$ be a sequence of events on a probability space. Let $S_k$ be the $k$-th binomial moment of the number $t_n$ of those $A_i$'s which occur. Let $B_r$ be the event that exactly $r$, and $E_r$ that at least $r$, of the $A_i$'s occur. An estimate on $F(B_r)$ or $F(E_r)$ by a linear combination of $S_1, S_2, \ldots$ is called a Bonferroni inequality. In the paper, two methods are presented for proving Bonferroni inequalities. One method extends those obtained by the author (J. London Math. Soc. (2) 9 (1975), 561-64) to cases when the coefficients in the linear bounds can vary with $n$. Another method exploits the fact that the sequence $F(E_r) = y_r$ is decreasing and that $S_k$ is a linear combination of the $y_r$. This method will show that Bonferroni inequalities are special cases of non-probabilistic inequalities. The methods will be applied to inequalities, reproducing the sharpest known ones by a unified and simple proof. (Received June 8, 1976.)

Robert Sine, University of Rhode Island, Kingston, Rhode Island 02881. Unique ergodicity for random maps.

Unique ergodicity (at most one invariant probability) is established for a class of Markov operators on the line. The transitions take the form $f \rightarrow p_1 f(x+a_1)$ where each $p_1$ is strictly positive and continuous, the step sizes $\{a_1\}$ generate a dense subgroup of the line, and the hitting probability for
half spaces is monotone. This result is an extension of a theorem of Frank Norman and J. W. Pickands III (Markov Processes and Learning Models, Academic Press 1972). A previous extension (Notices 22 (1975), 6727-C2) obtained by the author and Benton Jamison was much more general as to conditions imposed on the coefficient functions but was limited to two translations. (Received June 9, 1976.)

T. L. Seethoff, Northern Michigan University, Marquette, Michigan 49855 and R. C. Shiflett, California State University Fullerton, Fullerton, California 92634. Doubly stochastic measures with prescribed supports.

Let L and H be measurable maps of the unit interval X into itself such that \( L \leq H \). We investigate the collection \( \mathcal{M}(L, H) \) of all doubly stochastic measures that have their support contained in the union of the graphs of L and H, that is, the doubly stochastic measures that arise from MTF's of the form \( P(x, B) = A(x) \mathbb{I}_B(Lx) + (1 - A(x)) \mathbb{I}_B(Hx) \), where \( A : X \rightarrow X \) is measurable. We show: (i) \( \mathcal{M}(L, H) \) is an extremal subset of the set of all doubly stochastic measures; (ii) \( \mathcal{M}(L, H) \) contains at most one element under appropriate conditions on L and H, and so if \( \mathcal{M}(L, H) \neq \emptyset \) under these conditions, then the corresponding measure is extreme; (iii) \( \mathcal{M}(L, H) \neq \emptyset \) under conditions on L and H compatible with those in (ii); and (iv) there are measurable L and H (namely \( L(x) = x^2 = H^{-1}(x) \)) such that \( \mathcal{M}(L, H) = \emptyset \). (Received June 10, 1976.)


A doubly stochastic measure \( \mu \) can be viewed as the joint distribution of two random variables each of which is uniform on \([0,1]\). Covariance of these random variables is defined with respect to \( \mu \). Bounds for the covariance are obtained, particular examples are considered, and a decomposition result for the set of doubly stochastic measures is found. Also the covariance matrix of a process associated with \( \mu \) is studied. (Received June 11, 1976.)


Let \( 1 \leq k \leq N \) be integers. Consider real random variables \( X_1, \ldots, X_N \) such that each \( k \)-tuple is uniformly distributed in \((0,1)^k\). We will discuss optimal bounds on \( P = \Pr(X_1 \leq c_1, \ldots, X_k \leq c_k) \), where \( 0 \leq c_j \leq 1 \) are given numbers. For instance, if \( k = 1 \) then \( P \geq (1 + \cdots + N - N + 1) \).

(Received June 14, 1976.)


The role of asymptotic distribution theory in the derivation and interpretation of statistical models for paired comparison experiments involving two stimuli is considered. A brief survey of the literature in the general area, along with a description of some unsolved problems is presented. Finally, a method for extending the ideas to the case of experiments involving comparison of three stimuli is described. (Received June 14, 1976.)

Irwin D. Nahinsky, Department of Psychology, University of Louisville, Louisville, Kentucky 40208. Identifiability of theories with sequences of nonobservable underlying states.

The problem of identifying probabilities for sequences involving \( k \) underlying states is investigated for experiments in which there are \( k \) observed states with the conditional distribution for these states specified for each underlying state. No restrictions upon the nature of underlying transitions are assumed for a large number of trials. Applications to some known experimental situations are suggested. The relationships to finite state automata are considered. (Received June 14, 1976.) (Author introduced by Professor A. Rapoport.)


Let \( X_1, X_2, \ldots \) be a sequence of independent identically distributed random variables
in the domain of attraction of a stable law. Define a sequence $Y_n(t)$ of random walks from the partial sums of the $X_i$'s, with appropriate normalization. Using a characterization of weak convergence from Non-standard Analysis, we derive new sufficient conditions for the weak convergence of the conditioned random walks $(Y_n(t)|Y_n(1) \in E_n')$, where the $E_n$'s are subsets of $\mathbb{R}$ such that the conditional probabilities $P(Y_n(1) \in A|Y_n(1) \in E_n')$ converge weakly. (Received June 15, 1976.)

65 Numerical Analysis

*737-65-1 Martin Braun and Joseph Horshenov, Queens College, Flushing, N.Y. 11367. Periodic solutions of finite difference approximations.

Let $x = \phi(t)$ be a periodic solution of the system of equations

(1) $\dot{x} = f(x)$

with characteristic multipliers $\lambda_1, \ldots, \lambda_{n-1}, \lambda_n = 1$, and let

(2) $x_{k+1} = F(x_k, h)$

be a finite difference approximation of (1) which is at least as accurate as Euler's method. Assume that $|\lambda_j| < 1$, $j = 1, \ldots, n-1$, or $|\lambda_j| > 1$, $j = 1, \ldots, n-1$. Then, the difference equation (2) has a closed invariant curve near the periodic solution $x = \phi(t)$ of (1), for $h$ sufficiently small. As a corollary to this result, we show that any finite difference approximation of the Van der Pol equation

$$\ddot{x} + \epsilon(x^2 - 1) \dot{x} + x = 0$$

which is at least as accurate as Euler's method has a closed invariant curve near the circle of radius 2, for $\epsilon > 0$ and $h$ sufficiently small. (Received March 8, 1976.)

737-65-2 HERBERT E. SALZER, 941 Washington Avenue, Brooklyn, NY 11225.

A New Form of Trigonometric Orthogonality and Gaussian-Type Quadrature

An "orthogonal" trigonometric sum $S_{2r+1}(x) = a_0 + \sum_{j=1}^{r} (a_j \cos jx + b_j \sin jx)$ is one that satisfies $\int_{a}^{b} S_{2r+1}(x) S_{2r'+1}(x) dx = 0, r' < r$, and two other arbitrarily imposable conditions needed to make $S_{2r+1}(x)$ unique. Two proofs are given of a fundamental factor theorem for any $S_{2n+1}(x)$ from which we derive 2r-point Gaussian-type quadrature formulas, $r = \lfloor n/2 \rfloor + 1$, which are exact for any $S_{4r-1}(x)$. We have $\int_{a}^{b} S_{4r-1}(x) dx = \sum_{j=1}^{2r} A_j S_{4r-1}(x_j)$, where the nodes $x_j$,

$j = 1(1)2r$, are the zeros of the orthogonal $S_{2r+1}(x)$. It is proven that 2r-1

of the nodes must lie within the interval $[a,b]$, and the remaining node (which may or may not be in $[a,b]$) must be real. Gaussian-type quadrature formulas are applicable to the numerical integration of the Gauss (2n+1)-point interpolation formulas, with extra efficiency when the latter are expressed in barycentric form. $S_{2r+1}(x)$, $x_j$ and $A_j$, $j = 1(1)2r$, were calculated for $[a,b] = [0, \pi/4]$, $2r = 2$ and 4, to single-precision accuracy. (Received May 7, 1976)

*737-65-3 Riaz A. Usmani, The University of Manitoba, Winnipeg, Canada R3T 2N2. An $O(h^4)$ Finite difference analogue for the numerical solution of some differential equations occurring in plate deflection theory.

The bending of a uniformly loaded rectangular plate supported over the entire surface by an elastic foundation satisfies a differential system of the form $y^{(4)} + fy = g$, $y(a), y(b), y''(b)$ being prescribed, $a < b$. An approximate solution of the preceding differential system is obtained by finite difference method. The convergence of the proposed finite difference scheme is established by deriving priori bound on the discretization error. It turns out that our numerical procedure is a fourth order convergent process. The matrix associated with the system of linear equations in the unknowns $y_1, y_2, \ldots, y_n$ is not even assumed to be monotone, as is often the case with two-point boundary value
problems. The only requirement is that the function f(x) be nonnegative on [a,b] and the exact solution $y(x) \in C^8$. The validity of our error analysis is demonstrated in two typical numerical illustrations. (Received May 17, 1976.) (Author introduced by F. M. Arscott.)

Howes & Thrall (1973, Nav. Res. Log. Q.) propose a method for generating the relative values of weapons which are part of a heterogeneous combat force. The method involves solving for eigenvalues and eigenvectors of non-negative matrices, and uses the Perron-Frobenius theory which guarantees the existence of an eigenvalue of maximum norm and an associated positive eigenvector.


Consider the perturbed eigenanalysis problem, $(A-ki)X = E$, $E \neq 0$ and the least squares solution, $X^*(k,E) = (A-ki)^\theta E$, where $(A-ki)^\theta$ is the generalized inverse of $(A-ki)$. We consider the behavior of $X^*(k,E)$ in neighborhoods of $E = 0$ and $k = k_0$. Of particular interest is the problem of (continuously) extending $X^*$ to the points $k = k_0$, $E = 0$. The main result asserts the ability to make this continuation. Moreover, examples are presented showing improved results over previous work. (Received June 11, 1976.) (Author introduced by Dr. Robert M. Thrall.)

70 ▶ Mechanics of Particles and Systems

Consider the following central force problem: $\dot{x} = -\nabla V(x)$. Here $x \in \mathbb{R}^2$ is the position of a particle moving in the potential field $V(x) = ||x||^a$, $a \geq 0$. We show that the singularity at $x = 0$ can be regularized in the sense of Easton if and only if $a = 2(1-n^{-2})$, where $n = 1, 2, 3, \ldots$. (Received June 15, 1976.)

76 ▶ Fluid Mechanics

Several numerical techniques are currently used to study the characteristics of fluid motion by computer experiments. The one used most often is an explicit numerical scheme where the time derivative is approximated by a forward difference formula. Although many researchers claimed computational stability of their solutions for large time steps, mathematically it can be proved that the stability of numerical solutions requires time step $\Delta t$ to be of the order of square of the mesh size $\Delta x$. In this work implicit numerical techniques obtained by approximating the time derivative by a backward difference formula were used to solve (i) Three dimensional heat conduction equation, (ii) Motion of a suddenly accelerated flat plate in a viscous fluid which is otherwise at rest, (iii) One dimensional gas dynamics equation and (iv) Burger's equation on turbulence model. For all these cases analytical solutions exist. Both numerical and computational analysis show that for all time steps, independent of mesh size, implicit numerical solutions were in excellent agreement with those obtained analytically. (Received May 10, 1976.)

A family of steady, two-dimensional irrotational flows of an inviscid and incompressible fluid are considered. The flow in each case involves an unknown free streamline on which the pressure is assumed constant. Using the usual conformal mapping techniques the unknown flows are mapped onto a flow which is determined by Milne-Thomson's Circle Theorem. Utilizing the hodograph plane it is shown that the unknown flow can be found by solving an ordinary differential equation. For particular problems, solutions are obtained. (Received June 11, 1976.)
A maximum principle for compressible flows on manifolds.

An incompressible fluid flow on a Riemannian manifold is described by a harmonic one-form, and it can be shown (Bochner) that there is a relationship between points where local maximum speeds occur and the curvature of the manifold. The author shows that similar results hold for compressible flows which are described by one-forms satisfying a nonlinear elliptic system. More precisely, it is shown that in a neighborhood of a point where the Ricci curvature is positive, the speed is a subelliptic scalar function and, hence, cannot attain a local maximum.

(Received June 15, 1976.)

80 ▶ Classical Thermodynamics, Heat Transfer


Let \( \rho, c, \) and \( \kappa \) be the density, specific heat, and conductivity of an infinite medium subdivided by planes \( z = z_i \), where \( i \in \{0, 1, \ldots, n\} \) and \( z_0 < z_1 < \ldots < z_n \). Suppose that collimated beam of radiation travelling in a line perpendicular to the planes \( z = z_i \) is subject to reflections by the planes and exponential absorptions in the regions between the planes. If the rate of energy deposition per unit volume is \( \xi \), which is the product of a function of time, a polynomial in \( x \) and \( y \) multiplied by \( \exp(-Ax^2 - By^2) \), and a function \( Z(z) \) of \( z \) expressed by

\[
Z(z) = \gamma_1 [P_L(t) \exp(-\gamma_1 (z-z_{1-1})) + P_R(t) \exp(-\gamma_1 (z_1-z))] \quad \text{for} \quad z_{1-1} < z < z_1 \quad \text{and} \quad Z(z) = 0 \quad \text{for} \quad z < z_0 \quad \text{or} \quad z > z_n,
\]

then the four dimensional convolution integral \( u = \Gamma^* S \) represents the increase in temperature in the medium, where \( \Gamma \) is the fundamental solution of the heat operator

\[
H = \rho c (\partial / \partial t) - \text{div}(\kappa \text{grad}).
\]

We show that in fact \( u(x,y,z,t) \) may be expressed explicitly as a one dimensional integral \( \int_0^T G(x,y,z,t,\tau) d\tau \). Moreover, we determine the values of \( \tau \) for which \( G(x,y,z,t,\tau) \) is significant and, thus, optimize the quadrature scheme for evaluating \( u \) on a computer.

(Received June 14, 1976.)

81 ▶ Quantum Mechanics


We discuss a general procedure for approximating Wiener-Feynman integrals which is based upon the Kac-Siegert representation of Brownian motion. This representation allows us to express the integral as an expectation of a self-adjoint operator. Suitable approximation procedures can then be used. (Received June 9, 1976.)


Solving an old problem, we use asymptotic expansions to calculate a perturbation expansion, which yields a convergence proof, for the path integral representation of the Green's function for the Schroedinger equation, where the path integral is defined as the limit of the sequence of iterated improper integrals given by Feynman[Rev. Mod. Phys.20(1948)367-387]. For a single, 1-dimensional particle, the potential is assumed to be a polynomial of degree \( \leq 2 \) with time continuous coefficients plus a perturbation term which has 6 time continuous \( L^2, L^\infty \) space derivatives with \( L^2, L^\infty \) norms uniformly bounded on a finite time interval. This generalizes directly for a system of \( n \) particles in \( 3n \)-dimensional space. Similar results hold when the iterated integrals include an initial \( \psi \) function, verifying previous research, e.g., Nelson[J. Math. Phys.
The limit of these iterated integrals satisfies the Schrödinger equation but the Green's function itself does not in general, having a factor which satisfies a different but related partial differential equation. For either kind of iterated integral, the integrations may be performed in any order. (Received June 10, 1976.)


The author has previously shown (Arch. Rational Mech. Anal., 37(1970), 192-221) that the dynamically natural random variable representing angular momentum in the stochastic mechanics of the Bopp-Haag spin model has the expectation values predicted by quantum mechanics for spin \( \frac{1}{2} \). This result is generalized to all higher values of the spin. (Received June 14, 1976.)

83 ▶ Relativity

John K. Beem, University of Missouri, Columbia, Missouri 65201. Some examples of incomplete space-times.

An example is given of a space-time which is timelike and spacelike complete but null incomplete. An example is also given of a space-time which is geodesically complete but contains an inextendible timelike curve of bounded acceleration and finite length. These two examples may be modified so that in each case they become globally hyperbolic and retain the stated properties. All of the examples are conformally equivalent to open subsets of the two dimensional Minkowski space. (Received May 21, 1976.)

KISHORE B. MARATHE, Brooklyn College, CUNY, Brooklyn, New York 11210. Mean curvature of gravitational fields.

The mean curvature of a gravitational field is defined as a generalization of the average curvature in a given direction which was used by Ricci to give a geometrical interpretation of the Ricci tensor. We find that the mean curvature of gravitational field is independent of the direction as determined by a unit vector. The converse of this result is also true and provides a new characterization of spaces admitting gravitational fields. (Received June 1, 1976.)


It is well-known that several authors, including recently M. Michalski and J. Wainwright [GRG Vol. 6, No. 3 (1975), 289-318], have assumed one or more Killing vector fields when studying the Einstein-Maxwell equations in general relativity. In particular, the existence of a Killing vector field is interpreted as describing some sort of symmetry property of the space-time. In this paper, the technique of differential geometry is used to prove the existence of two Killing vector fields on space-time. Furthermore, we find their explicit relation (locally) with two of the eigenvectors of non-null electromagnetic field tensor. (Received June 15, 1976.)

90 ▶ Economics, Operations Research, Programming, Games

I. STEPHAN SIMS, Memphis State University, Memphis, Tenn. 38152. The Exponential Smoothing Model, Theory and Application. Preliminary report.

The exponentially weighted moving average model, since being introduced 20 years ago, has been found to be an effective predictor.
The exponential smoothing model will be evaluated, setting forth the conditions under which it is an effective predictor. The problem of a simultaneous solution will be explored and a solution will be offered to the problem of deciding between the multiplicative or additive consideration of trend and seasonality. (Received June 14, 1976.) (Author introduced by Ralph J. Faudree.)

92 ▶ Biology and Behavioral Sciences

737-92-1 Anatol Rapoport, University of Toronto, Toronto, Ontario M5S 1A1. A shift of focus from predictive to structural models.

Extensions of mathematical methods to psychology were initially motivated, at least in part, by a hope that these methods would confer on psychology predictive power comparable to that conferred by mathematics on the physical sciences. Although occasionally mathematical models of psychological phenomena are corroborated by experiment, these successes have been, for the most part, sporadic and have contributed little to the sort of integration of psychology that could be seriously compared with the integration achieved in the physical sciences. In investigations now subsumed under mathematical psychology, there has been a shift of focus from predictive to structural models. The role of the latter is not so much that of building blocks in the construction of a science with predictive power as that of developing a repertoire of concepts that can be manipulated with logical rigor and at the same time be appropriate for phenomena investigated. (Received May 17, 1976.)


Retrieval of information stored in memory requires the presence of appropriate retrieval cues. The interaction of cues can be studied and measured when an item is tested for recall with different cues used in succession. In this way it is possible to obtain information, not only of each cue separately, but also of their interaction. In the present investigation, three different cues were used in the six possible orders under immediate and delayed conditions of recall. The model proposed is six independent multinomials having parameters in common for each word. The maximum likelihood estimates of cue effectiveness and cue interactions are presented and the usefulness of model is discussed. (Received May 28, 1976.) (Authors introduced by A. Rapoport.)


The INDSCAL (for INdividual Differences SCALing) model assumes that $d_{jk}^{(i)} = \sqrt{\sum_{t=1}^{T} w_{it} (x_{jt} - x_{kt})^2}$ where $d_{jk}^{(i)}$ is the (subjective) distance between stimuli (or other objects) $j$ and $k$ for subject (or other "data source") $i$. Thus it assumes a direct weighted generalization of the Euclidean metric for the three-way array of subjective distances. Conversion to a scalar products form yields the symmetric trilinear model $b_{jk}^{(i)} = \sum_{t=1}^{T} w_{it} x_{jt} x_{kt}$ where $b_{jk}^{(i)}$ is the scalar product between vectors representing objects $j$ and $k$ for data source $i$. This scalar product form of the model is a special case of the more general CANDECOMP (for CANonical DECOMposition of N-way tables) model of the form $z_{ijk} = \sum_{t=1}^{T} a_{it} b_{jt} c_{kt}$. Fitting the CANDECOMP model to the derived scalar products data via an alternating least squares (ALS) iterative numerical procedure (followed by application of some normalizing steps) yields estimates of parameters (the x's and w's) for INDSCAL. Details of the model and algorithm will be discussed, and applications given. (Received June 10, 1976.) (Authors introduced by Professor Anatol Rapoport.)


This paper reports new results on the continuing long-term development of a mathematical theory of...
learning. It is based on the idea that, as a result of feedback from a changing environment, weights assigned to a set of stored hypotheses, the set of stored hypotheses and the logical system of variables, predicates, etc., with which hypotheses are formed, all change with time so as to enable the learner to recognize and seize (avoid) an increasing variety of opportunities (traps). We study methods for forming and confirming or refuting hypotheses involving such predicates as the convexity, openness, monotonicity of curves. We postulate the existence of a "beetle", capable of moving along plane curves \((x(t),y(t))\). It is provided with information about the presence of absence, and far-to-near or near-to-far discontinuities of projections of the curve along rays to the location of the "beetle". We report an algorithm and its properties, that enables the "beetle" to use its input in recognizing certain topological properties of a discrete representation of a continuous curve. We show that certain global predicates become irrefutably confirmable by local information, that non-fuzzy topological information can be derived, and that fuzzy inputs can lead to the "beetle" being able to locate its position on a map up to "fuzzy isomorphism". (Received June 14, 1976.)

93 ▶ Systems, Control


If \(E\) is a Frechet space and \(F\) a locally convex complete Hausdorff space, a dense subspace \(U\) of \(V = L(D; L(E; F))\) is called a convolution-regular space if (i) \(U\) is locally convex Hausdorff and the injection of \(U\) into \(V\) is continuous and (ii) \(S \in V, \phi \in D, S \not\in \phi \in U, S \in U\). The bijection between \(V\) and \(W = L(D(E); F)\) is used to define convolution-regularity for dense subspaces of \(W\).

Theorem: If \(V\) is a convolution-regular subspace of \(W\) and if \(L: D(E) \rightarrow V\) is a continuous linear time-invariant mapping, then there exists a \(T \in V\) such that \(L \phi = T \ast \phi\) for all \(\phi \in D(E)\). The domain of \(L\) can be extended by means of this convolution representation if additional restrictions are imposed on \(V\). Some examples are given. (Received June 14, 1976.)

94 ▶ Information and Communication, Circuits, Automata

737-94-1 J. K. KARLOF and CHARLES P. DOWNEY, University of Nebraska at Omaha, Omaha, Nebraska 68101. The Existence of odd Group Codes for the Gaussian Channel. Preliminary Report

An \((M,n)\) group code for the Gaussian channel is said to be odd if \(M\) and \(n\) are both odd.

THEOREM: There exist odd \((M,n)\) group codes if and only if \(n\) does not equal 3.
(Received June 3, 1976.)

98 ▶ Mathematical Education, Collegiate


It is possible for and desirable that first-year college students learn something about the language of mathematics (= sets, relations, and functions), something about the principles of modern mathematics, and something about categories. The principles approach has already been sketched out by Hartnett (The CP/PMM Approach to Learning Mathematics, Educ. Studies in Math. 2 (1973), 1-22) and the injection of categories in undergraduate mathematics has been raised by Long, Meltzer, and Hilton (Research in Mathematics Education, Educ. Studies in Math. 2 (1970) 446-468). In this paper we report on a current project. Its aim is to have our serious beginning learners acquire a working
99 ▶ Miscellaneous


A study done under a National Science Foundation grant offers some clues as to why there are relatively few women mathematicians. With the cooperation of the Association for Women in Mathematics (AWM), a detailed questionnaire was sent to its one thousand members in Spring 1975. Responses were received in time for qualitative and quantitative analysis from 40% : 350 women, 52 men. Women reported more discouragement by teachers and advisors. That they were treated differently as mathematics students or professionals, because of being females or males, was reported by 80% of the women and 9% of the men, with the differences increasing as their training progressed. There were no significant differences in career advancement between single and married women. Based on the questionnaires and interviews, recommendations are offered for attracting more women to the mathematical sciences. (Received June 15, 1976.)
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