THE CALENDAR BELOW lists all of the meetings which have been approved by the Council up to the date this issue of the Notice was sent to press. The summer and annual meetings are joint meetings of the Mathematical Association of America and the American Mathematical Society. The meeting dates which fall rather far in the future are subject to change; this is particularly true of meetings to which no numbers have yet been assigned.

ABSTRACTS SHOULD BE SUBMITTED ON SPECIAL FORMS which are available in most departments of mathematics; forms can also be obtained by writing to the headquarters of the Society. Abstracts to be presented at the meeting in person must be received at the headquarters of the Society in Providence, Rhode Island, or before the deadline for the meeting.

--- CALENDAR OF MEETINGS ---

<table>
<thead>
<tr>
<th>MEETING NUMBER</th>
<th>DATE</th>
<th>PLACE</th>
<th>DEADLINE for ABSTRACTS * and NEWS ITEMS</th>
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</thead>
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<tr>
<td>753</td>
<td>March 20-25, 1978</td>
<td>Columbus, Ohio</td>
<td>JANUARY 17</td>
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<tr>
<td>755</td>
<td>April 7-8, 1978</td>
<td>Houston, Texas</td>
<td>FEBRUARY 15</td>
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<td>756</td>
<td>April 14-15, 1978</td>
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<tr>
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<td>(82nd Summer Meeting)</td>
<td>October 20-21, 1978</td>
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<td>November 3-4, 1978</td>
<td>Charleston, South Carolina</td>
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<td>November 12, 1978</td>
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<td>January 24-28, 1979</td>
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<td>April 6-8, 1979</td>
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<td>January 3-7, 1980</td>
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<td></td>
<td></td>
<td>January 8-12, 1981</td>
<td>San Francisco, California</td>
</tr>
<tr>
<td></td>
<td>(87th Annual Meeting)</td>
<td>(86th Annual Meeting)</td>
<td></td>
</tr>
</tbody>
</table>

*Deadline for abstracts NOT presented at a meeting (by title)

--- OTHER EVENTS ---

March 20-23, 1978  Symposium on Relations Between Combinatorics and Other Parts of Mathematics  Columbus, Ohio
June 12-23, 1978  Summer Seminar on Nonlinear Oscillations in Biology  University of Utah, Salt Lake City, Utah
July 10-28, 1978  1978 Summer Research Institute on Harmonic Analysis in Euclidean Spaces and Related Topics  Williams College, Williamstown, Massachusetts
August 15-23, 1978 International Congress of Mathematicians  Helsinki, Finland

PLEASE AFFIX THE PEEL-OFF LABEL on these Notices to correspondence with the Society concerning fiscal matters, changes of address, promotions, or when placing orders for books and journals.

The Notices of the American Mathematical Society is published by the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, in January, February, April, June, August, October, November, and December. Subscription prices for the 1978 volume (Volume 26) are list $19.00, member $9.50. The subscription price for members is included in the annual dues. Back issues of the Notices are available for a two year period only and cost $6.00 per issue list price, $4.50 per issue member price for Volume 23 (1976) and Volume 24 (1977). Orders for subscriptions or back issues must be accompanied by payment and should be sent to the Society at P. O. Box 1571, Annex Station, Providence, Rhode Island 02901. Other correspondence should be addressed to P. O. Box 6248, Providence, Rhode Island 02940. Second class postage paid at Providence, Rhode Island, and additional mailing offices. U. S. Postal Service Publication Number: 398-820.

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ROOM RESERVATION FORM (New York Meeting) ...................... A-218
The eighty-fourth annual meeting of the American Mathematical Society will be held in Atlanta, Georgia, from Tuesday, January 3, through Saturday, January 7, 1978. Sessions will be held in the Hyatt Regency Atlanta Hotel, located at 265 Peachtree Street, N.E.

The fifty-first Josiah Willard Gibbs Lecture will be presented by DONALD E. KNUTH of Stanford University at 8:30 p.m. on Wednesday, January 4. The title of the lecture is "Mathematical typography."

There will be one series of Colloquium Lectures to be delivered by HYMAN BASS of Columbia University. The title of the series is "Algebraic K-theory." The four lectures in the series are scheduled for 1:00 p.m. on Wednesday, Thursday, Friday, and Saturday, January 4, 5, 6, and 7.

The George David Birkhoff Prize in Applied Mathematics will be awarded at a session at 3:15 p.m. on Thursday, January 5.

By invitation of the Program Committee, there will be nine invited one-hour addresses as follows: 9:00 a.m., Wednesday, JOEL E. COHEN, Rockefeller University, "Ergodic theorems in demography"; 10:30 a.m., Wednesday, YUM TONG SIU, Yale University, "Pseudoconvexity and the problem of Levi"; 3:30 p.m., Wednesday, THOMAS SPENCER, Rockefeller University, "Classical and quantum field theory"; 9:00 a.m., Thursday, JOAN S. BIRMAN, Columbia University, "Mapping class groups of surfaces"; 10:30 a.m., Thursday, ROBERT I. SOARE, University of Chicago, "Recursively enumerable sets and degrees"; 2:15 p.m., Thursday, JEFF CHEEGER, State University of New York at Stony Brook, "Aspects of the geometry and topology of the spectrum"; 2:30 p.m., Friday, CHARLES W. CURTIS, University of Oregon, "representations of finite groups of Lie type"; 4:00 p.m., Friday, MICHAEL E. TAYLOR, Rice University, "Propagation, reflection, and diffraction of singularities of solutions to wave equations"; 3:30 p.m., Saturday, ROBERT D. EDWARDS, University of California, Los Angeles, "Images of manifolds under cell-like maps."

Also by invitation of the Program Committee, there will be thirteen special sessions of selected twenty-minute papers. The titles of these special sessions, the names of the mathematicians arranging them, and the times of their first meetings are as follows: Harmonic analysis on nilpotent and solvable groups, LOUIS AUSLANDER, Thursday morning; Representations of finite dimensional algebras and finite groups, MAURICE AUSLANDER, Wednesday morning; Approximate solutions of random equations, A. T. BHARUCHA-REID, Friday afternoon; Ill-posed problems for partial differential and integrodifferential equations, FREDERICK BLOOM, Thursday morning; Ramsey theory and its ramifications, STEFAN A. BURR, Wednesday morning; Foliations, LAWRENCE W. CONLON, Wednesday morning; Mathematics in neurobiology, JACK D. COWAN, Friday afternoon; Operator theory, RONALD G. DOUGLAS, Wednesday morning; Direct sum representations of rings and modules, CARL FAITH, Thursday morning; History of mathematics, UTA C. MERZBACH, Thursday afternoon; Nonacademic mathematical research, ROBERT J. THOMPSON, Friday afternoon; Capacity in several complex variables, PIT-MANN WONG, Thursday afternoon; Number theory, KENNETH ROSEN, Friday afternoon.

There will be sessions for contributed ten-minute papers on Tuesday evening, all day and evening on Wednesday and Thursday, Friday afternoon and evening, and Saturday afternoon.

There will be a poster session of contributed papers in a variety of fields from 2:15 p.m. to 4:15 p.m. on Wednesday, January 4, in the Flemish Room at the Hyatt. At the session participants will display their papers on posters and remain nearby to expand on the material and answer questions during the two-hour session. Those attending the meeting are urged to visit the poster session and participate in the discussions.

The AMS Committee on Employment and Educational Policy (CEEP) will sponsor a panel discussion at 8:30 p.m. on Thursday, on "An employer's viewpoint on nonacademic employment." Panel members are Daniel H. Wagner (moderator), Daniel H. Wagner, Associates; Brockway McMillan, Bell Laboratories; Shmuel Winograd, International Business Machines; and James A. Dewar, Dynamic Sciences, Inc. In addition to this panel discussion on nonacademic employment, a special session on Nonacademic mathematical research has been arranged by Robert J. Thompson at the suggestion of CEEP. (See list of special sessions above.)

COUNCIL AND BUSINESS MEETING

The Council of the Society will meet in the Stuart Room of the Hyatt at 2:00 p.m. on Tuesday. The Business Meeting of the Society will be held in the Regency Ballroom at the Hyatt at 4:30 p.m. on Thursday. The secretary notes the following resolution of the Council: "Each person who attends a business meeting of the Society shall be willing and able to identify himself as a member of the Society." In further explanation, it is noted that "Each person who is to vote at a meeting is thereby identifying himself as and claiming to be a member of the American Mathematical Society."
American Mathematical Society Short Course Series

Numerical Analysis
January 3–4, 1978

The American Mathematical Society will present a one and one-half day short course on "Numerical Analysis" at the Hyatt Regency Hotel in Atlanta, Georgia on January 3 and 4, 1978. This course has been designed to provide a survey of topics in numerical analysis for the non-specialist. The speakers will survey subareas, present applications and discuss current research directions and problems. The topics have been chosen to emphasize active research areas. The talks will be directed to a mathematically mature audience without assuming knowledge of the topics to be discussed. The six seventy-five-minute lectures will cover computational linear algebra, optimization theory, approximation theory and quadrature, and methods for solving ordinary and partial differential equations.

The program is under the direction of Gene H. Golub and Joseph E. Oliger of the Computer Science Department of Stanford University. The short course was recommended by the Society's Committee on Employment and Educational Policy: Lida K. Barrett, David Blackwell, Wendell H. Fleming (chairman), Hugo Rossi, Martha K. Smith, and Robert J. Thompson.

The program will consist of six seventy-five-minute lectures as follows: Cleve B. Moler (Department of Mathematics, University of New Mexico), will speak on "Numerical linear algebra"; J. E. Dennis (Department of Computer Science, Cornell University) will speak on "Nonlinear optimization"; Carl de Boor (Mathematics Research Center, University of Wisconsin) will speak on "The approximation of functions and linear functionals: Best vs. good approximation"; James M. Varah (Department of Computer Science, Stanford University) will speak on "Numerical methods for the solution of ordinary differential equations"; Joseph E. Oliger (Department of Computer Science, Stanford University) will speak on "Methods for time dependent partial differential equations"; and George J. Fix (Department of Mathematics, Carnegie-Mellon University) will speak on "Variational methods for elliptic boundary value problems."

In addition Herbert B. Keller (Department of Mathematics, California Institute of Technology) will present some concluding remarks.

Summaries of these talks and accompanying reading lists appear on pages A-583 through A-586 of the October 1977 issue of these Notices.

The short course is open to all who wish to participate upon payment of the registration fee. There are reduced fees for students and unemployed individuals. Please refer to the section entitled MEETING REGISTRATION for details.

OTHER ORGANIZATIONS

The Mathematical Association of America (MAA) will hold its annual meeting on January 6-8 in conjunction with this meeting of the Society. At the Business Meeting of the Association at 10:00 a.m. on Saturday, the seventeenth Award for Distinguished Service to Mathematics and the 1978 Chauvenet Prize will be presented. A more detailed listing of the program of the Association appears in the timetable beginning on page 8 of these Notices.

Mark P. Hale, Jr., of the University of Florida has scheduled a session for persons interested in computer-assisted test construction and other forms of technical support for instruction. This meeting will be held in the Grecian Room of the Hyatt on Friday, January 6, from 7:30 p.m. to 9:30 p.m. The program will consist of several short presentations followed by an open discussion. Persons wishing additional information should write to Professor Hale at the Department of Mathematics, University of Florida, Gainesville, Florida 32611.

The Association for Women in Mathematics (AWM) will hold an open meeting of its Council at 5:00 p.m. on Saturday, and will sponsor a panel discussion on "Black women mathematicians" at 7:30 p.m. the same day.

The Conference Board of the Mathematical Sciences (CBMS) will sponsor a panel discussion on Friday at 2:00 p.m. The topic of the discussion is "The growing role of applications in mathematical higher education"; Clayton V. Aucoin will moderate. The CBMS Council will meet at 2:15 p.m. on Saturday.

The Mathematicians Action Group (MAG) will hold an open meeting of its Steering Committee at 9:00 a.m. on Tuesday, and its Business Meeting will be held at 4:30 p.m. on Wednesday. MAG will sponsor a panel discussion at 5:00 p.m. on Friday.

National Science Foundation (NSF) staff members will be available in the Embassy Room at the Atlanta Hilton Hotel to provide counsel and information on NSF programs of interest to mathematicians from 9:00 a.m. to 5:00 p.m. on January 5, 6, and 7.

The Board of Directors of the Rocky Mountain Mathematics Consortium will meet at 2:15 p.m. on Saturday in the Vienna Room at the Hilton.

MATHEMATICAL SCIENCES
EMPLOYMENT REGISTER

The Employment Register will be in session on Thursday, Friday, and Saturday, January 5–7, in the Crystal Ballrooms at the Hilton. A short (optional) orientation session will be held by the AMS-MAA-SIAM Joint Committee on Employment Opportunities at 9:00 a.m. on Thursday, January 5. The purpose of this session is to familiarize participants with the operation of the Employment Register and with registration procedures.

Registration for the Register will begin at 9:30 a.m. on Thursday and interviews will begin...
at 9:30 a.m. on Friday, January 6, and 9:30 a.m. on Saturday, January 7. Interview request cards must be turned in to the code clerk before 4:00 p.m. on the day prior to the interview. There will be no interviews scheduled for Thursday.

This year arrangements have again been made for computer scheduling of interviews. In order to allow applicants and employers to participate in as many interviews as possible, interviews are scheduled by the computer on the basis of preferences expressed by the participants. Provision has been made for scheduling of interviews in half-day modules. This allows for four half-days of interviews; Friday, A.M. and P.M., and Saturday, A.M. and P.M. This year the AMS-MAA-SIAM Joint Committee on Employment Opportunities has suggested that the Saturday, January 7, afternoon interview session allow the employer the opportunity of requesting interviews with applicants exclusively. Requests for interviews must be submitted by the employer on Friday, January 6, prior to the deadline of 4:00 p.m. in order to receive a schedule for Saturday afternoon. Applicants may not submit interview request forms for this session.

Applicants and employers should be sure to indicate exactly what times they will be available for interviews in the appropriate place on the forms. Applicants and employers are asked not to duplicate their interview requests for both morning and afternoon schedules on the same day; applicants and employers should also be advised that the program will NOT automatically reschedule a morning appointment to an afternoon session, if it could not be scheduled when requested for the morning. Interview requests should not be submitted, of course, unless the other individual requested has indicated availability during the time period desired. Morning schedules will be distributed on Friday and Saturday at 8:45 a.m.; the afternoon schedules will be distributed at 9:00 a.m. on the same days.

All participants in the Employment Register are required to register for the Joint Mathematics Meetings. For applicants there is no additional fee for participation in the Employment Register. For employers the additional fee for participation in the Employment Register is $15 when paid at the meeting.

EXHIBITS

The book and educational media exhibits will be located in Ivy Hall of the Hyatt from Wednesday through Saturday, January 4-7. The exhibits will be open from 1:00 p.m. to 5:00 p.m. on Wednesday, from 9:00 a.m. to 5:00 p.m. on Thursday and Friday, and from 9:00 a.m. to noon on Saturday. All participants are encouraged to visit the exhibits during the meeting.

BOOK AND AUDIO TAPES SALE

Books published by the Society and the Association, and audio tapes of AMS invited addresses, will be sold for cash prices somewhat below the usual prices when these same books and tapes are sold by mail. The book sales will be located in Ivy Hall of the Hyatt.

MEETING REGISTRATION

Participants who wished to preregister for the meetings should have done so before the deadline of December 2.

Registration for the short course only will begin on Tuesday, January 3, at 10:00 a.m. outside the Phoenix Room at the Hyatt. Participants who are not attending the short course are advised that no general meeting information (or registration material) will be available prior to the time listed below for the Joint Mathematics Meetings registration.

The Joint Mathematics Meetings registration desk will be located in Ivy Hall of the Hyatt. The desks will be open during the hours listed below:

**Numerical Analysis Short Course**

(outside Phoenix Room)

- **Tuesday, January 3** 10:00 a.m. - 8:00 p.m.
- **Wednesday, January 4** 8:30 a.m. - 9:30 a.m.

**Joint Mathematics Meetings** (Ivy Hall)

- **Tuesday, January 3** 2:00 p.m. - 8:00 p.m.
- **Wednesday, January 4** 8:00 a.m. - 5:00 p.m.
- **Thursday, January 5**
  - **Friday, January 6** 8:00 a.m. - 4:00 p.m.
  - **Saturday, January 7**
  - **Sunday, January 8** 8:30 a.m. - 2:30 p.m.

Meeting preregistration and registration fees partially cover expenses of holding the meetings. The preregistration fee does not represent an advance deposit for lodgings. Please note that separate registration fees are required for the Short Course and the Joint Meetings. These fees are as follows:

**Numerical Analysis Short Course**

<table>
<thead>
<tr>
<th>At Meeting</th>
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<tbody>
<tr>
<td>Members/Nonmembers</td>
<td>$20</td>
</tr>
<tr>
<td>Student/Unemployed</td>
<td>5</td>
</tr>
<tr>
<td>One-day fee for second day only</td>
<td>10</td>
</tr>
</tbody>
</table>

**Joint Mathematics Meetings**

| Members of AMS/MAA | $25 |
| Nonmembers         | 35  |
| Student/Unemployed | 3   |

There will be no extra charge for members of the families of registered participants, except that all professional mathematicians who wish to attend sessions must register independently.

Students are considered to be only those currently working toward a degree who do not receive compensation totaling more than $7,000 from employment, fellowships, and scholarships.

The unemployed status refers to any person currently unemployed, actively seeking employment, and who is not a student. It is not intended to include persons who have voluntarily resigned or retired from their latest position.

A fifty percent refund of the preregistration fee will be made for all cancellations received in Providence prior to January 2. There will be no
refunds granted for cancellations received after that date, or to persons who do not attend the meetings.

HOTEL ACCOMMODATIONS

Participants should have preregistered and secured room reservations before December 2; after that date participants should contact the hotels listed on an individual basis. Participants who have already preregistered for the meetings, and obtained hotel accommodations through the Housing Bureau, are asked to notify the hotel directly of any change in plans. The rates quoted below are subject to a 7 percent sales tax. Walking time from the Hyatt is given in parentheses.

HYATT REGENCY ATLANTA (headquarters)
265 Peachtree Street, N.E., 30303
Telephone: (404) 577-1234
Single $29 Double $41 Twin Double $41
Triple $32 Quadruple $63
Suite (1 bedroom) $95, 110, 125, 135, 165

ATLANTA AMERICAN MOTOR HOTEL
(10-15 minutes)
Spring Street at Carnegie Way, 30301
Telephone: (404) 688-8600
Single $26 Double $30 Twin $30
Triple $35 Quadruple $40
Suite (1 bedroom) $65 (2 bedrooms) $250

ATLANTA CENTRAL-TRAVELodge
(10-15 minutes)
311 Courtland Street, N.E., 30303
Telephone: (404) 659-4545
Single $21 Double $24 Twin Double $27
Triple $30 Quadruple $33

ATLANTA HILTON (5 minutes)
Courtland and Harris Streets, N.E., 30303
Telephone: (404) 659-2000
Single $29 Twin Double* $41 Twin* $41
*Extra person in room $4

BEST WESTERN WHITE HOUSE MOTOR HOTEL
(10-15 minutes)
70 Houston Street, N.E., 30303
Telephone: (404) 659-2660
Single $24 and $32 Double $28 Triple $36
Twin Double $32 Quadruple $40
Suite (1 bedroom) $95 (2 bedrooms) $125

HOLIDAY INN DOWNTOWN (10-15 minutes)
175 Piedmont Avenue, N.E., 30303
Telephone: (404) 659-2727
Single $26 Double $29 Twin $34
Triple $38 Quadruple $42
Suite (1 bedroom) $80 (2 bedrooms $110)

PASCHAL'S MOTOR HOTEL (20-30 minutes)
330 Martin Luther King, Jr. Drive, S.W., 30314
Telephone: (404) 577-3150
Single $18 Twin Double $21
Triple $24 Quadruple $27

PASSPORT CENTER INN (5 minutes)
231 Ivy Street, N.E., 30303
Telephone: (404) 577-1510
Single $24 Double* $28 Twin* $28
*Extra person in room $4

INEXPENSIVE ACCOMMODATIONS

On October 24, 1977, we were advised by the management of the Metropolitan YMCA in Atlanta that they would be unable to abide by their original commitment to provide housing for some of our student and unemployed participants during the Joint Mathematics Meetings in January. We regret that this information was not received prior to the printing of the November issue of these Notices.

ENTERTAINMENT AND LOCAL INFORMATION

A no host cash bar cocktail party will take place in the lobby of the Hyatt at 6:00 p.m. on Friday, January 6. All participants are invited to attend.

The Hyatt is located on Atlanta’s Peachtree Street, often regarded as the Main Street of the South. The city has a full complement of museums, theater and concert groups, movie houses, serious drinking establishments, amusement parks and a zoo. The Atlanta Symphony performs at the Memorial Arts Center on the northern edge of the business district, while many sporting events are held at the Omni complex in the heart of the city.

Much of the city’s center can be seen from a circular trip on the Downtown Loop bus, with service every 10 minutes between 8:00 a.m. and 6:00 p.m. on weekdays. MARTA, the Atlanta bus system, is one of the nation’s best and most economical, taking riders almost anywhere in the city or suburbs for 15¢ (exact change). A MARTA information booth is but a few steps from the Hyatt.

Sightseeing tours of the area are available, appealing to a wide range of interests, from the older (“plantation tours,” though not to the nonexistent Tara), to the recent (the tomb and the restored birthplace of Dr. M. L. King, Jr.), to the present (the skyline and Peachtree Center), and to the future (a subway system under construction). An all-day tour to Plains by bus is available any day, following advance registration by a group (minimum twenty-five).

Atlanta has a large number of restaurants catering to a wide spectrum of tastes, including continental, southern and ethnic. Restaurant guides and city information will be available daily at the Local Information section of the Joint Mathematics Meetings registration desk.

CHILD CARE

A number of commercial child care agencies, offering hotel care at hourly rates, are available in the metropolitan Atlanta area. A list of agencies, telephone numbers, and rates will also be available at the Local Information section of the Joint Mathematics Meetings registration desk.
1. Hyatt Regency Atlanta
2. Atlanta Hilton
3. Best Western White House Motor Hotel
4. Holiday Inn Downtown
5. Passport Center Inn
6. Atlanta American Motor Hotel
7. Metropolitan YMCA
8. Phoenix Halls of Atlanta
9. Atlanta Central-Travelodge
10. Paschal's Motor Hotel
MAIL AND MESSAGE CENTER

All mail and telegrams for persons attending the meetings should be addressed in care of Mathematics Meetings, Hyatt Regency Atlanta Hotel, 265 Peachtree Street, N.E., Atlanta, Georgia 30303. Mail and telegrams so addressed may be picked up at the meeting registration area in Ivy Hall.

A telephone message center will be located in the same area to receive incoming calls for all participants. The center will be open from January 4 through January 8 during the same hours as the Joint Meetings registration desk. Messages will be taken down and the name of any individual for whom a message has been received will be posted until the message has been picked up at the message center. Individuals planning on attending the meetings are advised to leave the following number with anyone who might want to reach them at the meeting: (404) 525-8240.

TRAVEL

Those planning to attend the AMS short course or the early sessions of the Joint Mathematics Meetings should have made reservations for travel accommodations early. It should be kept in mind that Monday, January 2, will be the legal New Year's holiday, so that more people than usual will be traveling on both Monday, January 2 and Tuesday, January 3.

In winter, Atlanta is on Eastern Standard Time. Hartsfield International Airport, the second busiest in the world, is served by Braniff, Delta, Eastern, Northwest, Piedmont, Southern, TWA and United Airlines. Four commuter airlines serve nearby cities. The airport is about eight miles from the city center. The airport limousine to all downtown hotels costs $3.50. The trip by taxi takes about 30 minutes from the Hyatt and costs about $6.50 for one person and $3 each for three or more persons sharing the cab.

American International, Avis, Budget, Dollar, Hertz and National maintain car rental desks in the airport terminal, with a number of additional companies located adjacent to the terminal.

The Southern Railroad serves Peachtree Station, two miles from the Hyatt, as follows: thrice weekly service on the New York-Atlanta-New Orleans-Los Angeles route; one train daily on the New York-Washington-Atlanta route.

Three interstate highways intersect in downtown Atlanta a few blocks from the Hyatt, whose blue dome is visible for many miles: I-75 leads north to Cincinnati and Detroit and south to Tampa; I-85 leads northeast to Charlotte and Richmond and southwest to Montgomery; I-20 leads west to Birmingham and Jackson and east to Columbia.

WEATHER

Atlanta is located in the foothills of the Blue Ridge Mountains. With an elevation of over 328 meters (1050 feet), it enjoys a relatively mild yearly climate. Normal mean temperature during the month of January is 6°C (43°F). The average daily maximum temperature is 11°C (52°F), and the average daily minimum temperature is 1°C (34°F). The average January rainfall is 11 cm. (4.34 inches or 548 microfurlongs). Early morning fog, restricting visibility to less than one-quarter mile, occurs about five days each January. Below-freezing temperatures occur, on the average, about sixteen days in the month.

LOCAL ARRANGEMENTS COMMITTEE

All sessions are at Hyatt Regency, unless noted as follows:
Hi - Atlanta Hilton

TIMETABLE
(Eastern Standard Time)

AMERICAN MATHEMATICAL SOCIETY

<table>
<thead>
<tr>
<th>TUESDAY, January 3</th>
<th>SHORT COURSE ON NUMERICAL ANALYSIS</th>
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</thead>
<tbody>
<tr>
<td>10:00 a.m. - 8:00 p.m.</td>
<td>REGISTRATION (Short Course Only) - Outside Phoenix Room</td>
</tr>
<tr>
<td>1:30 p.m. - 2:45 p.m.</td>
<td>Numerical linear algebra</td>
</tr>
<tr>
<td>3:00 p.m. - 4:15 p.m.</td>
<td>Nonlinear optimization</td>
</tr>
<tr>
<td>4:30 p.m. - 5:45 p.m.</td>
<td>The approximation of functions and linear functionals: Best vs. good approximation</td>
</tr>
<tr>
<td>7:30 p.m. - 8:45 p.m.</td>
<td>Numerical methods for the solution of ordinary differential equations</td>
</tr>
<tr>
<td>9:00 p.m. - 9:40 p.m.</td>
<td>Methods for time dependent partial differential equations I</td>
</tr>
</tbody>
</table>

WEDNESDAY, January 4

| 8:30 a.m. - 9:30 a.m. | REGISTRATION (Short Course Only) - Outside Phoenix Room |
| 9:00 a.m. - 9:40 a.m. | Methods for time dependent partial differential equations II |
| 10:00 a.m. - 11:15 a.m. | Variational methods for elliptic boundary value problems |
| 11:15 a.m. - 11:45 a.m. | Concluding remarks |

JOINT MATHEMATICS MEETINGS

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<th>TUESDAY, January 3</th>
<th>American Mathematical Society</th>
<th>Other Organizations</th>
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<td>9:00 a.m. - noon</td>
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<td>Mathematicians Action Group</td>
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<td>STEERING COMMITTEE - Open Meeting</td>
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<td>Goya Room</td>
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<tr>
<td>2:00 p.m. - 8:00 p.m.</td>
<td>COUNCIL MEETING</td>
<td>REGISTRATION - Ivy Hall</td>
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<tr>
<td>2:00 p.m.</td>
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<td>Stuart Room</td>
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<td>SPECIAL SESSION</td>
<td>Essex Room</td>
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<tr>
<td>7:00 p.m. - 9:50 p.m.</td>
<td>Direct Sum Representations of Rings and Modules</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS</td>
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<tr>
<td>7:00 p.m. - 9:40 p.m.</td>
<td>Logic and Foundations</td>
<td>Lancaster Room D</td>
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<tr>
<td>7:00 p.m. - 9:10 p.m.</td>
<td>Algebraic Number Theory, Field Theory and Polynomials</td>
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<tr>
<td>8:00 a.m. - 12:10 p.m.</td>
<td>Functions of a Complex Variable</td>
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<tr>
<td>8:00 a.m. - 11:25 a.m.</td>
<td>Ordinary Differential Equations I</td>
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<tr>
<td>8:00 a.m. - 11:55 a.m.</td>
<td>General Topology I</td>
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<tr>
<td>8:00 a.m. - 11:55 a.m.</td>
<td>Functional Analysis I</td>
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<td>Time</td>
<td>Special Sessions</td>
<td>Other Organizations</td>
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<tr>
<td>8:30 a.m. - noon</td>
<td>Representations of Finite Dimensional Algebras and Finite Groups I, Expository Session</td>
<td>Lancaster Rooms A and B</td>
</tr>
<tr>
<td>8:30 a.m. - 11:50 a.m.</td>
<td>Ramsey Theory and its Ramifications I</td>
<td>Lancaster Room E</td>
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<tr>
<td>8:30 a.m. - 11:30 a.m.</td>
<td>Foliations</td>
<td>Lancaster Room D</td>
</tr>
<tr>
<td>9:00 a.m. - 10:50 a.m.</td>
<td>Operator Theory I</td>
<td>Essex Room</td>
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<tr>
<td>9:00 a.m. - 10:00 a.m.</td>
<td>INVITED ADDRESS: Ergodic theorems in demography</td>
<td>Joel E. Cohen, Regency Ballroom</td>
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<tr>
<td>9:00 a.m. - 11:40 a.m.</td>
<td>INVITED ADDRESS: Pseudoconvexity and the problem of Levi</td>
<td>Yum Tong Siu, Regency Ballroom</td>
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<tr>
<td>9:00 a.m. - 11:25 a.m.</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS</td>
<td>Statistics</td>
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<tr>
<td>9:00 a.m. - 11:30 a.m.</td>
<td>Numerical Analysis</td>
<td>Austrian Room</td>
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<tr>
<td>10:30 a.m. - 11:30 a.m.</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS</td>
<td>Statistics</td>
</tr>
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<td>1:00 p.m. - 2:00 p.m.</td>
<td>COLLOQUIUM LECTURES: Algebraic K-theory, Lecture I</td>
<td>Hyman Bass, Regency Ballroom</td>
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<td>1:00 p.m. - 5:00 p.m.</td>
<td>EXHIBITS - Ivy Hall</td>
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<tr>
<td>2:15 p.m. - 5:10 p.m.</td>
<td>Special Sessions</td>
<td>Representations of Finite Dimensional Algebras and Finite Groups II, Problems</td>
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<td>2:15 p.m. - 5:05 p.m.</td>
<td>Operator Theory II</td>
<td>Essex Room</td>
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<tr>
<td>2:15 p.m. - 6:00 p.m.</td>
<td>Ramsey Theory and its Ramifications II</td>
<td>Lancaster Room E</td>
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<tr>
<td>2:15 p.m. - 4:55 p.m.</td>
<td>Ordinary Differential Equations II</td>
<td>English Room</td>
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<td>2:15 p.m. - 4:55 p.m.</td>
<td>General Topology II</td>
<td>Lancaster Room C</td>
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<tr>
<td>2:15 p.m. - 5:10 p.m.</td>
<td>Algebraic Topology</td>
<td>Tudor Room</td>
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<td>2:15 p.m. - 7:25 p.m.</td>
<td>Integral Equations and Transforms</td>
<td>Dutch Room</td>
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<td>2:15 p.m. - 5:10 p.m.</td>
<td>Functional Analysis II</td>
<td>York Room</td>
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<tr>
<td>2:15 p.m. - 3:10 p.m.</td>
<td>Finite Differences, Functional Equations and Computer Science</td>
<td>Grecian Room</td>
</tr>
<tr>
<td>2:15 p.m. - 3:10 p.m.</td>
<td>Geometry, Convex Sets and Geometric Inequalities</td>
<td>French Room</td>
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<tr>
<td>2:15 p.m. - 4:15 p.m.</td>
<td>POSTER SESSION</td>
<td>Flemish Room</td>
</tr>
<tr>
<td>3:30 p.m. - 4:30 p.m.</td>
<td>INVITED ADDRESS: Classical and quantum field theory</td>
<td>Thomas Spencer, Regency Ballroom</td>
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<tr>
<td>4:30 p.m. - 6:30 p.m.</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS</td>
<td>French Room</td>
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<tr>
<td>7:00 p.m. - 7:40 p.m.</td>
<td>Group Theory and Generalizations I</td>
<td>Austrian Room</td>
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<tr>
<td>7:00 p.m. - 7:40 p.m.</td>
<td>Algebraic Geometry</td>
<td>Grecian Room</td>
</tr>
<tr>
<td>7:00 p.m. - 7:55 p.m.</td>
<td>Calculus of Variations and Optimal Control</td>
<td>York Room</td>
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<tr>
<td>7:00 p.m. - 8:10 p.m.</td>
<td>Several Complex Variables and Analytic Spaces</td>
<td>English Room</td>
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<tr>
<td>7:00 p.m. - 8:10 p.m.</td>
<td>Fourier Analysis</td>
<td>Italian Room</td>
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<td>Time</td>
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<tr>
<td>7:00 p.m. - 7:55 p.m.</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS: Global Analysis, Analysis on Manifolds, French Room</td>
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<tr>
<td>7:00 p.m. - 7:40 p.m.</td>
<td>Sequences, Series Summability, Dutch Room</td>
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<tr>
<td>7:00 p.m. - 7:55 p.m.</td>
<td>Fluid Mechanics, Flemish Room</td>
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<tr>
<td>8:30 p.m. - 9:30 p.m.</td>
<td>JOSEPH WILLARD GIBBS LECTURE: Mathematical typography, Donald E. Knuth, Regency Ballroom</td>
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<td>8:00 a.m. - 4:00 p.m.</td>
<td>SPECIAL SESSION: Registration - Ivy Hall</td>
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<tr>
<td>8:00 a.m. - 11:50 a.m.</td>
<td>Harmonic Analysis on Nilpotent and Solvable Groups, Essex Room</td>
<td></td>
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<tr>
<td>8:00 a.m. - 11:55 a.m.</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS: Associative Rings and Algebras I, Austrian Room, York Room</td>
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<tr>
<td>8:00 a.m. - 10:55 a.m.</td>
<td>Number Theory, York Room</td>
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<td>8:00 a.m. - 11:40 a.m.</td>
<td>Combinatorics I, French Room</td>
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<tr>
<td>8:00 a.m. - 11:40 a.m.</td>
<td>Probability Theory and Stochastic Processes I, Italian Room</td>
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<tr>
<td>8:00 a.m. - 11:40 a.m.</td>
<td>Partial Differential Equations I, Lancaster Room D</td>
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<tr>
<td>8:30 a.m. - 11:50 a.m.</td>
<td>Ill-posed Problems for Partial Differential and Integrodifferential Equations I, Lancaster Room E</td>
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<tr>
<td>8:30 a.m. - 11:20 a.m.</td>
<td>Capacity in Several Complex Variables, Lancaster Room C</td>
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<tr>
<td>8:30 a.m. - 11:55 a.m.</td>
<td>SESSIONS FOR CONTRIBUTED PAPERS: Group Theory and Generalizations II, Tudor Room, English Room</td>
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<tr>
<td>9:00 a.m. - 10:55 a.m.</td>
<td>Manifolds and Cell Complexes, English Room</td>
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<tr>
<td>9:00 a.m. - 9:30 a.m.</td>
<td>EMPLOYMENT REGISTER ORIENTATION SESSION - Crystal Ballrooms (HI)</td>
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<tr>
<td>9:00 a.m. - 10:00 a.m.</td>
<td>INVITED ADDRESS: Mapping class groups of surfaces, Joan S. Birman, Regency Ballroom</td>
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<tr>
<td>9:00 a.m. - 11:30 a.m.</td>
<td>SPECIAL SESSION: History of Mathematics I, Lancaster Rooms A and B</td>
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<tr>
<td>9:00 a.m. - 4:00 p.m.</td>
<td>Mathematical Association of America, BOARD OF GOVERNORS MEETING, Strasbourg-Vienna Rooms (III)</td>
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</tr>
<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td>EXHIBITS - Ivy Hall</td>
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<tr>
<td>9:30 a.m. - 4:00 p.m.</td>
<td>EMPLOYMENT REGISTER REGISTRATION - Crystal Ballrooms (HI)</td>
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<tr>
<td>10:30 a.m. - 11:30 a.m.</td>
<td>INVITED ADDRESS: Recursively enumerable sets and degrees, Robert L Soare, Regency Ballroom</td>
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<tr>
<td>1:00 p.m. - 2:00 p.m.</td>
<td>COLLOQUIUM LECTURE II, Hyman Bass, Regency Ballroom</td>
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<tr>
<td>2:15 p.m. - 3:15 p.m.</td>
<td>INVITED ADDRESS: Aspects of the geometry and topology of the spectrum, Jeff Cheeger, Regency Ballroom</td>
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<tr>
<td>2:15 p.m. - 2:55 p.m.</td>
<td>LINEAR AND MULTILINEAR ALGEBRA, MATRIX THEORY, English Room</td>
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<tr>
<td>2:15 p.m. - 3:10 p.m.</td>
<td>GENERAL MATHEMATICAL SYSTEMS, Dutch Room</td>
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## TIMETABLE

### THURSDAY, January 5

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<td>2:15 p.m. - 2:55 p.m.</td>
<td><strong>SESSIONS FOR CONTRIBUTED PAPERS</strong>&lt;br&gt;Category Theory, Homological Algebra&lt;br&gt;Ballroom</td>
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<tr>
<td>2:15 p.m. - 3:10 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Real Functions&lt;br&gt;Grecian Room</td>
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<tr>
<td>2:15 p.m. - 3:10 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;General Topology III&lt;br&gt;French Room</td>
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<td>2:15 p.m. - 3:10 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Partial Differential Equations II&lt;br&gt;Lancaster Room D</td>
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<tr>
<td>2:15 p.m. - 3:10 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Biology and Behavioral Sciences&lt;br&gt;Flemish Room</td>
</tr>
<tr>
<td>3:15 p.m. - 4:15 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;GEORGE DAVID BIRKHOFF PRIZE&lt;br&gt;IN APPLIED MATHEMATICS&lt;br&gt;Regency Ballroom</td>
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<tr>
<td>4:30 p.m. - 5:30 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;BUSINESS MEETING&lt;br&gt;Regency Ballroom</td>
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<td>7:30 p.m. - 10:20 p.m.</td>
<td><strong>SPECIAL SESSIONS</strong>&lt;br&gt;Il-posed Problems for Partial Differential and&lt;br&gt;Integrodifferential Equations II&lt;br&gt;Lancaster Room E</td>
</tr>
<tr>
<td>7:30 p.m. - 9:50 p.m.</td>
<td><strong>SESSIONS FOR CONTRIBUTED PAPERS</strong>&lt;br&gt;Real Functions and Generalizations II&lt;br&gt;Tudor Room</td>
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<tr>
<td>7:30 p.m. - 9:25 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Ordinary Differential Equations III&lt;br&gt;English Room</td>
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<td>7:30 p.m. - 8:40 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Topological Groups, Lie Groups&lt;br&gt;French Room</td>
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<td>7:30 p.m. - 9:25 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Nonassociative Rings and Algebras&lt;br&gt;Lancaster Room C</td>
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<tr>
<td>7:30 p.m. - 9:40 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Functional Analysis III&lt;br&gt;York Room</td>
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<tr>
<td>7:30 p.m. - 9:10 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Probability Theory and Stochastic Processes II&lt;br&gt;Ballan Room</td>
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<tr>
<td>7:30 p.m. - 8:55 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Associative Rings and Algebras II&lt;br&gt;Austrian Room</td>
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<tr>
<td>8:00 p.m. - 10:30 p.m.</td>
<td><strong>SPECIAL SESSION</strong>&lt;br&gt;History of Mathematics II&lt;br&gt;Lancaster Rooms A and B</td>
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<tr>
<td>8:30 p.m. - 10:00 p.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;Committee on Employment and Educational Policy&lt;br&gt;PANEL DISCUSSION: An employer’s viewpoint on nonacademic employment&lt;br&gt;James A. Dewar&lt;br&gt;Brockway McMillan&lt;br&gt;Daniel H. Wagner (moderator)&lt;br&gt;Shmuel Winograd&lt;br&gt;Regency Ballroom</td>
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<td><strong>REGISTRATION - Ivy Hall</strong></td>
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<tr>
<td>9:00 a.m. - 10:50 a.m.</td>
<td><strong>Other Organizations</strong>&lt;br&gt;MAA - Chauvenet Symposium&lt;br&gt;James C. Abbott (moderator)&lt;br&gt;Regency Ballroom&lt;br&gt;Boolean-valued models in set theory, analysis, and quantum mechanics&lt;br&gt;Martin D. Davis&lt;br&gt;Offbeat integral geometry&lt;br&gt;Lawrence A. Zalcman</td>
</tr>
<tr>
<td>9:00 a.m. - 5:00 p.m.</td>
<td><strong>EXHIBITS - Ivy Hall</strong></td>
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<td>9:00 a.m. - 5:30 p.m.</td>
<td><strong>OTHER ORGANIZATIONS</strong>&lt;br&gt;EMPLOYMENT REGISTER INTERVIEWS - Crystal Ballrooms (III)</td>
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<tr>
<td>11:00 a.m. - 11:50 a.m.</td>
<td>**MAA - PANEL DISCUSSION: Numerical analysis in the undergraduate curriculum&lt;br&gt;William S. Dorn&lt;br&gt;Gunter H. Meyer (moderator)&lt;br&gt;Thomas A. Porsching&lt;br&gt;James S. Vandergraft&lt;br&gt;Regency Ballroom</td>
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**TIMETABLE**

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<td>1:00 p.m. - 2:00 p.m.</td>
<td><strong>COLLOQUIUM LECTURE III</strong>&lt;br&gt;Hyman Bass, Regency Ballroom</td>
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<td>2:00 p.m. - 4:00 p.m.</td>
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**SPECIAL SESSIONS**

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<td>2:15 p.m. - 4:35 p.m.</td>
<td><strong>Number Theory I</strong>&lt;br&gt;Essex Room</td>
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<tr>
<td>2:15 p.m. - 5:05 p.m.</td>
<td><strong>Mathematics in Neurobiology</strong>&lt;br&gt;Lancaster Room D</td>
</tr>
<tr>
<td>2:15 p.m. - 5:05 p.m.</td>
<td><strong>Nonacademic Mathematical Research</strong>&lt;br&gt;Lancaster Rooms A and B</td>
</tr>
<tr>
<td>2:15 p.m. - 5:05 p.m.</td>
<td><strong>Approximate Solutions of Random Equations I</strong>&lt;br&gt;Lancaster Room E</td>
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<td>2:15 p.m. - 5:10 p.m.</td>
<td><strong>Measure and Integration</strong>&lt;br&gt;English Room</td>
</tr>
<tr>
<td>2:15 p.m. - 5:55 p.m.</td>
<td><strong>Commutative Rings and Algebras</strong>&lt;br&gt;Italian Room</td>
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<tr>
<td>2:15 p.m. - 5:40 p.m.</td>
<td><strong>Combinatorics II</strong>&lt;br&gt;French Room</td>
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<tr>
<td>2:15 p.m. - 5:55 p.m.</td>
<td><strong>Operator Theory I</strong>&lt;br&gt;Austrian Room</td>
</tr>
<tr>
<td>2:15 p.m. - 5:55 p.m.</td>
<td><strong>Order, Lattices, Ordered Algebraic Structures</strong>&lt;br&gt;Tudor Room</td>
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<tr>
<td>2:30 p.m. - 3:30 p.m.</td>
<td><strong>INVITED ADDRESS:</strong>&lt;br&gt;Representations of finite groups of Lie type&lt;br&gt;Charles W. Curtis, Regency Ballroom</td>
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<td>4:00 p.m. - 5:00 p.m.</td>
<td><strong>INVITED ADDRESS:</strong>&lt;br&gt;Propagation, reflection, and diffraction of singularities of solutions to wave equations&lt;br&gt;Michael E. Taylor, Regency Ballroom</td>
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<tr>
<td>5:00 p.m. - 6:30 p.m.</td>
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<tr>
<td>6:00 p.m. - 8:00 p.m.</td>
<td><strong>NO-Host COCKTAIL PARTY</strong>&lt;br&gt;Hyatt Regency Lobby</td>
</tr>
<tr>
<td>7:00 p.m.</td>
<td><strong>MAA - FILM PROGRAM</strong>&lt;br&gt;Phoenix Room</td>
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<tr>
<td>7:00 p.m. - 7:14 p.m.</td>
<td>Regular homotopies in the plane: Part I&lt;br&gt;(Nelson Max, Mark Thiel)</td>
</tr>
<tr>
<td>7:20 p.m. - 7:25 p.m.</td>
<td>Butterfly catastrophe</td>
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<td>7:30 p.m. - 7:40 p.m.</td>
<td>Geometric introduction to partial differential equations (Roy E. Myers)</td>
</tr>
<tr>
<td>7:45 p.m. - 8:15 p.m.</td>
<td>Weather by the numbers</td>
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<tr>
<td>8:20 p.m. - 8:33 p.m.</td>
<td>Orthogonal projection (Daniel Pedoe)</td>
</tr>
<tr>
<td>8:35 p.m. - 9:35 p.m.</td>
<td>What is an integral? (Edwin Hewitt) (b&amp;w)</td>
</tr>
<tr>
<td>7:30 p.m. - 9:30 p.m.</td>
<td><strong>MAA - Session on computer-assisted test construction</strong>&lt;br&gt;Grecian Room</td>
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**SESSIONS FOR CONTRIBUTED PAPERS**

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<tr>
<td>8:15 p.m. - 9:55 p.m.</td>
<td><strong>Applied Mathematics</strong>&lt;br&gt;English Room</td>
</tr>
<tr>
<td>8:15 p.m. - 9:40 p.m.</td>
<td><strong>Combinatorics III</strong>&lt;br&gt;French Room</td>
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<tr>
<td>8:15 p.m. - 9:40 p.m.</td>
<td><strong>Operator Theory II</strong>&lt;br&gt;Austrian Room</td>
</tr>
<tr>
<td>8:15 p.m. - 9:25 p.m.</td>
<td><strong>Special Functions</strong>&lt;br&gt;Italian Room</td>
</tr>
<tr>
<td>8:15 p.m. - 9:25 p.m.</td>
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**Conference Board of the Mathematical Sciences**

**PANEL DISCUSSION:** The growing role of applications in mathematical higher education<br>Clayton V. Aucoin (moderator)<br>Phoenix Room
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<tr>
<th>Time</th>
<th>American Mathematical Society</th>
<th>Other Organizations</th>
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<tr>
<td>8:00 a.m. - 4:00 p.m.</td>
<td>REGISTRATION - Ivy Hall</td>
<td>MAA - PANEL DISCUSSION: A course in applied mathematics based on problems from regional industries Jeanne L. Agnew (moderator) Regency Ballroom Faculty presentation: Marvin S. Keener Student presentations: Jeri Ezell Bob Hayes Kathy Stewart</td>
</tr>
<tr>
<td>9:00 a.m. - 9:50 a.m.</td>
<td>MAA - PANEL DISCUSSION: A course in applied mathematics based on problems from regional industries Jeanne L. Agnew (moderator) Regency Ballroom Faculty presentation: Marvin S. Keener Student presentations: Jeri Ezell Bob Hayes Kathy Stewart</td>
<td></td>
</tr>
<tr>
<td>9:00 a.m. - noon</td>
<td>Employment Register Interviews - Crystal Ballrooms (HI)</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom MAA - POSTER SESSION Ivy Hall</td>
</tr>
<tr>
<td>9:00 a.m. - 5:30 p.m.</td>
<td>MAA - BUSINESS MEETING Regency Ballroom</td>
<td>MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>10:00 a.m. - 10:50 a.m.</td>
<td>MAA - BUSINESS MEETING Regency Ballroom</td>
<td>MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
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<tr>
<td>11:00 a.m. - 11:50 a.m.</td>
<td>MAA - BUSINESS MEETING Regency Ballroom</td>
<td>MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>noon - 1:30 p.m.</td>
<td>MAA - BUSINESS MEETING Regency Ballroom</td>
<td>MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>1:00 p.m. - 2:00 p.m.</td>
<td>COLLOQUIUM LECTURE IV Hyman Bass, Regency Ballroom</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>2:15 p.m. - 4:35 p.m.</td>
<td>Number Theory II Essex Room</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>2:15 p.m. - 4:35 p.m.</td>
<td>Approximate Solutions of Random Equations II Lancaster Room E</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>2:15 p.m. - 4:55 p.m.</td>
<td>Semigroups Tudor Room</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>2:15 p.m. - 4:10 p.m.</td>
<td>Abstract Harmonic Analysis Lancaster Room C</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
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<td>2:15 p.m. - 4:25 p.m.</td>
<td>Differential Geometry English Room</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>2:15 p.m. - 3:40 p.m.</td>
<td>Approximations and Expansions French Room</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
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<tr>
<td>2:15 p.m. - 3:10 p.m.</td>
<td>Mathematics Education Italian Room</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>2:15 p.m. - 4:40 p.m.</td>
<td>Operations Research, Systems Control and Information Theory Austrian Room</td>
<td>MAA - BUSINESS MEETING Regency Ballroom MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>2:15 p.m. - 5:00 p.m.</td>
<td>MAA - BUSINESS MEETING Regency Ballroom</td>
<td>MAA - RETIRING PRESIDENTIAL ADDRESS An inside outsider looks at mathematics education Henry O. Pollak, Regency Ballroom</td>
</tr>
<tr>
<td>2:15 p.m. - 6:00 p.m.</td>
<td>Rocky Mountain Mathematics Consortium BOARD OF DIRECTORS MEETING Vienna Room (HI)</td>
<td>CBMS - COUNCIL MEETING Strasbourg Room (HI)</td>
</tr>
<tr>
<td>3:30 p.m. - 4:30 p.m.</td>
<td>INVITED ADDRESS: Images of manifolds under cell-like maps Robert D. Edwards, Regency Ballroom</td>
<td>MAA - FILM PROGRAM Phoenix Room Unless otherwise noted, all films are in color</td>
</tr>
<tr>
<td>5:00 p.m. - 6:00 p.m.</td>
<td>Association for Women in Mathematics COUNCIL - Open Meeting Grecian Room</td>
<td>MAA - FILM PROGRAM Phoenix Room Unless otherwise noted, all films are in color</td>
</tr>
<tr>
<td>5:30 p.m. - 7:00 p.m.</td>
<td>MAA - Committee on Two-Year Colleges INFORMAL MEETING Stuart Room</td>
<td>MAA - FILM PROGRAM Phoenix Room Unless otherwise noted, all films are in color</td>
</tr>
<tr>
<td>7:00 p.m.</td>
<td>MAA - FILM PROGRAM Phoenix Room Unless otherwise noted, all films are in color</td>
<td>MAA - FILM PROGRAM Phoenix Room Unless otherwise noted, all films are in color</td>
</tr>
<tr>
<td>7:00 p.m. - 7:22 p.m.</td>
<td>Shapes of the future: Some unsolved problems in geometry—two dimensions (Victor L. Klee, Jr.)</td>
<td>MAA - FILM PROGRAM Phoenix Room Unless otherwise noted, all films are in color</td>
</tr>
<tr>
<td>7:25 p.m. - 7:33 p.m.</td>
<td>Powers of ten (Charles Eames)</td>
<td>MAA - FILM PROGRAM Phoenix Room Unless otherwise noted, all films are in color</td>
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<tr>
<td>SATURDAY, January 7</td>
<td>American Mathematical Society</td>
<td>Other Organizations</td>
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<tr>
<td>7:35 p.m. - 8:00 p.m.</td>
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<td>MAA - FILM PROGRAM (Continued)</td>
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<tr>
<td></td>
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<td>Numbers now and then</td>
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<td></td>
<td>(B. L. van der Waerden) – A BBC production for the Open University’s History of Mathematics course</td>
</tr>
<tr>
<td>8:05 p.m. - 8:07 p.m.</td>
<td></td>
<td>Similar triangles</td>
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<td>(Bruce and Katharine Cornwell)</td>
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<tr>
<td>8:10 p.m. - 8:36 p.m.</td>
<td></td>
<td>Isometries</td>
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<td>(W. O. J. Moser, Seymour Schuster)</td>
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<tr>
<td>8:40 p.m. - 8:48 p.m.</td>
<td></td>
<td>Accidental nuclear war (David S. Gillman)</td>
</tr>
<tr>
<td>8:50 p.m. - 9:12 p.m.</td>
<td></td>
<td>Adventures in perception (Maurits Escher)</td>
</tr>
<tr>
<td>7:30 p.m. - 8:30 p.m.</td>
<td></td>
<td>AWM - PANEL DISCUSSION: Black women mathematicians</td>
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<tr>
<td>8:00 p.m. - 10:00 p.m.</td>
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<td>Regency Ballroom</td>
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<tr>
<td>8:00 p.m. - 10:00 p.m.</td>
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<td>CBMS - COUNCIL MEETING</td>
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<td>Strasbourg Room (III)</td>
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<td>8:30 a.m. - 2:30 p.m.</td>
<td>REGISTRATION - Ivy Hall</td>
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<tr>
<td>9:00 a.m. - 9:50 a.m.</td>
<td>PANEL DISCUSSION: Using the history of mathematics to teach mathematics</td>
</tr>
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<td></td>
<td>Arthur Schlissel (moderator)</td>
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<td>Regency Ballroom</td>
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<td></td>
<td>Using contemporary sources</td>
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<td></td>
<td>Edward J. Barbeau</td>
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<td>Pedagogical values of the history of mathematics</td>
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<td>Morris Kline</td>
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<td>Two case histories</td>
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<td></td>
<td>Frederick V. Pohle</td>
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<tr>
<td>10:00 a.m. - 10:50 a.m.</td>
<td>INVITED ADDRESS</td>
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<tr>
<td></td>
<td>The Serre problem concerning polynomials over a field</td>
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<td>Barbara L. Osofsky, Regency Ballroom</td>
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<tr>
<td>10:00 a.m. - 10:50 a.m.</td>
<td>PANEL DISCUSSION: Remediation</td>
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<td></td>
<td>Martin L. Bittinger</td>
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<td>Joseph E. Cicero (moderator)</td>
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<td>Leonard Feldman</td>
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<td>Jack E. Forbes</td>
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<td>Gail S. Young</td>
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<td>Hanover Hall</td>
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<tr>
<td>11:00 a.m. - 11:50 a.m.</td>
<td>INVITED ADDRESS</td>
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<tr>
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<td>Some old and new ideas on partial differential equations</td>
</tr>
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<td>Charles L. Fefferman, Regency Ballroom</td>
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<tr>
<td>11:00 a.m. - 11:50 a.m.</td>
<td>INVITED ADDRESS</td>
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<tr>
<td></td>
<td>Combinatorics really does count</td>
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<td>Ivan Niven, Hanover Hall</td>
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<tr>
<td>1:30 p.m. - 2:20 p.m.</td>
<td>PANEL DISCUSSION: Credit and placement by examination</td>
</tr>
<tr>
<td></td>
<td>Beverly L. Brechner</td>
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<td>Betty J. Himman</td>
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<td>Donald L. Kreider (moderator)</td>
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<td>Alfred L. Putnam</td>
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<td>Regency Ballroom</td>
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<tr>
<td>2:30 p.m. - 3:20 p.m.</td>
<td>PANEL DISCUSSION: Interim report of the NRC Committee on Applied Mathematics Training</td>
</tr>
<tr>
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<td>Regency Ballroom</td>
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<td></td>
<td>The committee’s activities</td>
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<tr>
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<td>Peter J. Hilton (moderator)</td>
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<tr>
<td></td>
<td>The formation and charge to the committee</td>
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<td>George D. Mostow</td>
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SUNDAY, January 8

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<td>PANEL DISCUSSION (Continued)</td>
<td>A draft proposal for a mathematics major</td>
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<td>Stephen Smale</td>
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<tr>
<td>Noncurricular programs</td>
<td>Shmuel Winograd</td>
</tr>
<tr>
<td>INVITED ADDRESS</td>
<td>Combinatorial problems in elementary geometry</td>
</tr>
<tr>
<td>Paul Erdős, Regency Ballroom</td>
<td></td>
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3:30 p.m. - 4:20 p.m.

PRESENTERS OF PAPERS

Following each name is the number corresponding to the speaker's position on the program

- Invited one-hour lecturers
- Special session speakers

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Al-Amri, H. S. #51
Alexander, C. C. #501
*Alexander, H. S. #555
Allan, R. #507
*Alperin, J. L. #92
Anacker, S. E. #547
Anantharaman, R. #472
Andrus, G. F. #517
Arkowitz, M. #157
Arnold, E. J., Jr. #382
*Asinov, D. J. #103
*Aul, C. F. #466
*Auslander, M. #51, #201
Aznaria, N. #522
Aznaria, N. #123
Azoff, E. #514
Aspetia, A. G. #34
*Bailey, P. B. #531
Baird, B. #591
Baker, J. M. #435
Bakke, V. L. #228
Bakker, W. C. #44
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Bang, C. M. #221
Bardwell, M. A. #525
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Barth, K. F. #480
*Baum, H. #590
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*Bècgs, G. A. #555
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*Berndt, B. C. #535
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*Bhurucha-Reid, A. T. #554
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*Birman, J. S. #572
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*Bloom, F. #547
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*Ewing, R. E. #445
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*Fischer, D. R. #625
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PROGRAM OF THE SESSIONS

SHORT COURSE ON NUMERICAL ANALYSIS

All Sessions in Phoenix Room

TUESDAY, 1:30 P.M. - 9:40 P.M.

1:30-2:45 p.m. Numerical linear algebra. CLEVE MOLER, University of New Mexico, Albuquerque, New Mexico (SC 78-1)

3:00-4:15 p.m. Nonlinear optimization. J. E. DENNIS, Cornell University, Ithaca, New York (SC 78-2)

4:30-5:45 p.m. The approximation of functions and linear functionals: Best vs. good approximation. CARL de BOOR, University of Wisconsin, Madison (SC 78-3)

7:30-8:45 p.m. Numerical methods for the solution of ordinary differential equations. JAMES M. VARAH, University of British Columbia, Vancouver, British Columbia (SC 78-4)

9:00-9:40 p.m. Methods for time dependent partial differential equations. I. JOSEPH OLIGER, Stanford University, Stanford, California (SC 78-5)

WEDNESDAY, 9:00 A.M. - 11:45 A.M.

9:00-9:40 a.m. Methods for time dependent partial differential equations. II. JOSEPH OLIGER, Stanford University, Stanford, California (SC 78-5)

10:00-11:15 a.m. Variational methods for elliptic boundary value problems. GEORGE J. FIX, Carnegie-Mellon University, Pittsburgh, Pennsylvania (SC 78-6)

11:15-11:45 a.m. Concluding remarks. HERBERT B. KELLER, California Institute of Technology, Pasadena, California

THE EIGHTY-FOURTH ANNUAL MEETING

The time limit for each contributed paper in the general sessions is ten minutes. In the special sessions the time varies from session to session and within sessions. To maintain the schedule, the time limits will be strictly enforced.

TUESDAY, 7:00 P.M.

Session on Logic and Foundations, Lancaster Room D

7:00-7:10 (1) Topological dimension as a first order theory. Professor LUDVIK JANOS, Mississippi State University (752-02-1)

7:15-7:25 (2) The logic of contradiction. Preliminary report. Professor NICOLAS D. GOODMAN, State University of New York at Buffalo (752-02-2)

7:30-7:40 (3) Non-generable RE sets. Professor DOUGLAS CENZER, University of Florida (752-02-3)

7:45-7:55 (4) Some theorems extended to higher-level analytic hierarchy, Preliminary report. Professor HISAO TANAKA, Hosei University, Koganei, Tokyo and University of California, Los Angeles (752-02-4)

8:00-8:10 (5) A topology-like approach to completeness theorems for logics with restricted substitution. Dr. THOMAS M. LESCHINE, Woods Hole Oceanographic Institution (752-02-5)

8:15-8:25 (6) Straight-line computation in abstract algebra. Preliminary report. Dr. PETER M. WINKLER, Emory University (752-02-6)

8:30-8:40 (7) A Burnside-style theorem for equational classes. Preliminary report. GEORGE McNULTY, University of South Carolina (752-02-8)

8:45-8:55 (8) Robinson's consistency theorem for game quantifier logics. Dr. FRED HALPERN, Bishop College (752-02-9)

9:00-9:10 (9) Automatic construction of finite models. Professor RICHARD STARK, San Jose State University (752-02-10)

9:15-9:25 (10) Ultrahomogeneous structures. Dr. BRUCE I. ROSE* and Dr. ROBERT E. WOODROW, University of Notre Dame (752-02-11)

9:30-9:40 (11) Fuzzy sets and cardinals. Preliminary report. Professor E. WILLIAM CHAPIN, University of Maryland, Eastern Shore (752-02-12)

*For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
TUESDAY, 7:00 P. M.

Session on Algebraic Number Theory, Field Theory and Polynomials, Lancaster Room E

7:00- 7:10 (12) Real quadratic function fields of genus $g$ with regulator $g + 1$. Preliminary report. Professor JAMES R. C. LEITZEL, Ohio State University (752-12-4)

7:15- 7:25 (13) Hensel's lemma and the $p$-adic inverse function theorem. Professor JOHN W. SHUCK, Ursinus College (752-12-5)

7:30- 7:40 (14) Matrix field extensions. Professor JACOB T. B. BEARD, Jr. *, University of Texas at Arlington and Professor ROBERT M. McCONNEL, University of Tennessee (752-12-6)

7:45- 7:55 (15) On the invariants of some $Z_2$-extensions. Dr. JOHN R. BLOOM, Texas A & M University (752-12-8)

8:00- 8:10 (16) Decomposition of primes in biquadratic extensions of $Q$. Professor JOSEPH B. DENNIN, Jr., Fairfield University (752-12-9)

8:15- 8:25 (17) Capitulation patterns in class field extensions of type (p,p). Preliminary report. Dr. SHIAN-MING CHANG, University of Toronto and Dr. RICHARD FOOTE*, California Institute of Technology (752-12-10) (Introduced by Professor Marshall Hall, Jr.)

8:30- 8:40 (18) A divisibility property of ideals in rings of integral-valued polynomials. Dr. DEMETRIOS BRIZOLIS, California State Polytechnic University (752-12-11)

8:45- 8:55 (19) Some remarks on an algorithm of Minkowski. Dr. THERESA P. VAUGHAN, University of North Carolina at Greensboro (752-12-12)

9:00- 9:10 (20) Imaginary quadratic number fields with special class groups. Preliminary report. Dr. JAMES SOLDERITSCH, Lehigh University (752-12-13)

TUESDAY, 7:00 P. M.

Special Session on Direct Sum Representations of Rings and Modules, Essex Room

7:00- 7:20 (21) Unique decomposition and isomorphic refinement theorems in additive categories. Professor CAROL L. WALKER* and Professor ROBERT B. WARFIELD, Jr., New Mexico State University (752-13-6)

7:30- 7:50 (22) On a generalization of serial rings. Professor KENT R. FULLER, University of Iowa (752-16-19)

8:00- 8:20 (23) Recent developments in the representation theory. Professor V. DLAB, Carleton University (752-16-18)

8:30- 8:50 (24) Commutative rings whose finitely generated modules decompose. Professor WILLY BRANDAL, University of Tennessee (752-13-3)

9:00- 9:20 (25) Algebras whose finitely generated modules are direct sums of cyclics. Preliminary report. Professor ROGER WIEGAND, University of Nebraska (752-16-12)

9:30- 9:50 (26) The basis theorem for modules over noncommutative rings. Preliminary report. Professor CARL FAITH, Rutgers University and Institute for Advanced Study (752-16-11)

WEDNESDAY, 8:00 A. M.

Session on Functions of a Complex Variable, Tudor Room

8:00- 8:10 (27) Holomorphic functions on the group algebra of the symmetric group. Professor RALPH W. WILKerson, Southern Illinois University (752-30-1)

8:15- 8:25 (28) Convolution conditions for convexity, starlikeness, and spiral-likeness. HERB SILVERMAN, College of Charleston, EVELYN M. SILVIA*, University of California, Davis, and DON N. TELAGE, University of Kentucky (752-30-2)

8:30- 8:40 (29) Functions defined by gap power series, Dr. S. M. SHAH, University of Kentucky at Lexington and University of Texas at Arlington (752-30-3)

8:45- 8:55 (30) Imaginary values of meromorphic functions in the disk. Dr. DOUGLAS W. TOWNSEND, Indiana University-Purdue University at Fort Wayne (752-30-4)

9:00- 9:10 (31) New classes of close-to-convex functions. H. S. AL-AMMII, Bowling Green State University (752-30-5)

9:15- 9:25 (32) Boundary distortion and the uniform convergence of quasiconformal mappings. Preliminary report. R. NÄKKI, University of Jyväskylä, Jyväskylä, Finland and B. PALKA*, University of Texas at Austin (752-30-6)

9:30- 9:40 (33) Generalized regularity on linear associative algebras in the sense of Fueter. Professor HERBERT H. SNYDER, Southern Illinois University (752-30-7)
9:45- 9:55 (34) On the growth of entire functions defined by Dirichlet series. Professor ALFONSO G. AZPETITIA, University of Massachusetts at Boston (752–30–8)

10:00-10:10 (35) On moduli of continuity of analytic and harmonic functions. Professor THAD DANKEI, Jr., University of North Carolina at Wilmington (752–30–9)


11:15-11:25 (40) \( \Phi \)-like polynomial approximation. Dr. KENNETH R. GURGANUS, University of North Carolina at Wilmington (752–30–14)

11:30-11:40 (41) On the relative growth of area for subordinate functions. Preliminary report. Professor PAUL EENIGENBURG* and Professor DON NELSON, Western Michigan University (752–30–15)

11:45-11:55 (42) On Robinson's \( \frac{1}{n} \) conjecture. ROGER W. BARNARD, University of Kentucky (752–30–16)

12:00-12:10 (43) Locally univalent functions and coefficient distortions. HERB SILVERMAN*, College of Charleston, E. M. SILVIA, University of California, Davis, and D. N. TELAGE, University of Kentucky (752–30–17)

WEDNESDAY, 8:00 A. M.

Session on Ordinary Differential Equations I, English Room

8:00- 8:10 (44) A method of iteration for solving ordinary differential equations with advanced and retarded arguments. Mr. WILLIAM C. BAKKER, Spelman College and Emory University (752–34–1)

8:15- 8:25 (45) An expansion theorem for an eigenvalue problem with eigenvalue parameter in the boundary condition. Professor DON HINTON, University of Tennessee (752–34–3)

8:30- 8:40 (46) Forced non-oscillations in second order functional equations. Professor BHAGAT SINGH, University of Wisconsin Center, Manitowoc (752–34–4)


9:00- 9:10 (48) Positivity properties of singular linear differential operators. Preliminary report. Dr. J. K. SHAW, Virginia Polytechnic Institute and State University (752–34–6)

9:15- 9:25 (49) An existence theorem for nonlinear ordinary differential equations at an irregular type singularity. Professor PO–FANG HSIEH, Western Michigan University (752–34–7)

9:30- 9:40 (50) The functional differential equation \( \psi' (g(x)) = m(g(x)) \psi(x) \) with \( g(g(x)) = x \). Professor WILLIAM R. DERRICK and Professor KEITH YALE*, University of Montana (752–34–8)

9:45- 9:55 (51) Green's functions for focal type boundary value problems. Professor ALLAN C. PETERSON, University of Nebraska, Lincoln (752–34–9)

10:00-10:10 (52) A bounded solution of \( x^s+1 y' = F(x, y, z) x z' = G(x, y, z) \) when \( G_z(0, 0, 0) = 0 \). Preliminary report. Mr. JEROME J. PRZYBYLSKI (752–34–10)

10:15-10:25 (53) Solution in the large of a certain linear homogeneous differential equation of order \( n \). Professor T. K. PUTTASWAMY, Ball State University (752–34–11)

10:30-10:40 (54) On the existence of periodic boundary condition for certain nonlinear vector differential equation. Dr. G. G. HAMEDANI, Arya–Mehr University of Technology, Tehran, Iran (752–34–12)

10:45-10:55 (55) Further results on the nonoscillation of perturbed nonlinear differential equations. Professor JOHN R. GRAEF* and Professor PAUL W. SPIKES, Mississippi State University (752–34–13)

11:00-11:10 (56) Periodic solutions to a nonlinear Volterra integro–differential equation. Professor HARLAN W. STECH, Virginia Polytechnic Institute and State University (752–34–14)
11:15-11:25 (57) Oscillation theorems for $y'' + pf(y) = 0$. Dr. G. J. BUTLER, University of Alberta (752-34-15)

WEDNESDAY, 8:00 A. M.

Session on General Topology I, Lancaster Room C

8:00-8:10 (58) GO spaces and quasi-uniformities. Preliminary report. PETER FLETCHER*, Virginia Polytechnic Institute and State University and WILLIAM LINDGREN, Slippery Rock State College (752-54-1)

8:15-8:25 (59) The number of closed subsets of a topological space. H. R. E. HODEL, Duke University (752-54-2)

8:30-8:40 (60) Generalized closedness. Professor ROBERT A. HERRMANN, U. S. Naval Academy (752-54-3)

8:45-8:55 (61) Neighborhoods of the diagonal and strong normality properties. Professor H. L. SHAPIRO*, Northern Illinois University and Professor F. A. SMITH, Kent State University (752-54-4)

9:00-9:10 (62) Discrete and countably discrete maps. Dr. RICHARD H. WARREN, University of Nebraska at Omaha (752-54-5)

9:15-9:25 (63) Kuratowski closure operators without the expansive axiom. Dr. J. GLENN BROOKSHEAR, Marquette University (752-54-6)

9:30-9:40 (64) Metrizability of certain ordered spaces. Professor H. R. BENNETT and Professor D. J. LUTZER*, Texas Tech University (752-54-7)

9:45-9:55 (65) Irreducibly essential maps from inverse limits. Preliminary report. Dr. GARY A. FEUERBACHER, Bellaire Research Laboratory, Texaco, Inc., Bellaire, Texas (752-54-8)

10:00-10:10 (66) Homoclinic points of mappings of the interval. Dr. LOUIS BLOCK, University of Florida (752-54-9)

10:15-10:25 (67) Applications of shrinkable covers. J. C. SMITH, Virginia Polytechnic Institute and State University (752-54-10)

10:30-10:40 (68) The topology of $\theta$-convergence is the semiregular topology associated with $(X, T)$. Preliminary report. Professor T. R. HAMLETT, Arkansas Tech University (752-54-11)

10:45-10:55 (69) Bitopological spaces with a unique quasi-proximity. Preliminary report. Professor WORTHEN HUNSAKER*, Southern Illinois University and Professor WILLIAM LINDGREN, Slippery Rock State College (752-54-12)

11:00-11:10 (70) On the Lindelöf degree. Preliminary report. Ms. MARLENE E. GEWAND, State University of New York at Buffalo (752-54-13)

11:15-11:25 (71) The effectiveness of nets of cardinal $\mathfrak{K}_1$. Preliminary report. Professor SCOTT W. WILLIAMS, State University of New York at Buffalo (752-54-14)

11:30-11:40 (72) Perfect functions. Dr. VICTOR SAKS, Brooklyn, New York (752-54-15)

11:45-11:55 (73) On pseudo-open mappings of paracompact spaces. Mr. HEIKKI JUNNILA, Virginia Polytechnic Institute and State University (752-54-16)

WEDNESDAY, 8:00 A. M.

Session on Functional Analysis I, York Room

8:00-8:10 (74) A generalization of infinite tree structure. Preliminary report. Mr. PHILIP W. McCARTNEY, Harvey Mudd College and Claremont Graduate School (752-46-29)

8:15-8:25 (75) A nonlinear theorem of ergodic type II. GORDON R. FEATHERS* and W. G. DOTSON, Jr., North Carolina State University (752-46-2)

8:30-8:40 (76) Weak compactness in the space of Pettis-integrable functions. Preliminary report. Professor JAMES K. BROOKS and Professor NICOLAE DINCULEANU*, University of Florida, Gainesville (752-46-5)

8:45-8:55 (77) Convex Fréchet differentiable map of the unit ball of $C(X)$ into $C(X)$. Professor LARRY F. HEATH*, University of Texas at Arlington and Professor TED J. SUFFRIDGE, University of Kentucky (752-46-4)

9:00-9:10 (78) Functions which operate on the real parts of functions in a uniform algebra. Preliminary report. Professor S. J. SIDNEY, University of Connecticut (752-46-5)


9:30-9:40 (80) Non-standard representations. Mr. MICHAEL A. FREEDMAN, Georgia Institute of Technology (752-46-7)
9:45- 9:55 (81) A measure derived Banach algebra topology. Professor SIMON COHEN, New Jersey Institute of Technology (752-46-8)

10:00-10:10 (82) Some geometry in Lebesgue–Bochner function spaces. Preliminary report. Dr. MARK A. SMITH*, Miami University and Dr. BARRY TURETT, Texas Tech University (752-46-9)

10:15-10:25 (83) Rotundity in Lebesgue–Bochner function spaces. Preliminary report, Dr. MARK A. SMITH, Miami University and Dr. BARRY TURETT*, Texas Tech University (752-46-10)

10:30-10:40 (84) Normal structure and the Banach–Mazur distance coefficient. Preliminary report. Mr. W. L. BYNUM, College of William and Mary (752-46-11)

10:45-10:55 (85) A Baire property in linear topological spaces. Professor AARON R. TODD, City University of New York, Brooklyn College (752-46-12)

11:00-11:10 (86) Function spaces with trivial duals. Preliminary report. Mr. DAVID BETOUNES, University of Southern Mississipi (752-46-13)


11:30-11:40 (88) Nonexpansive mappings, asymptotic regularity and successive approximations. Professor M. EDELSTEIN and Dr. R. C. O'BRIEN*, Dalhousie University (752-46-15)

11:45-11:55 (89) Duality for C*-algebras. MAURICE J. DUPRE, Tulane University (752-46-16)

WEDNESDAY, 8:30 A. M.

Special Session on Representations of Finite Dimensional Algebras and Finite Groups I, Expository Session, Lancaster Rooms A & B

8:30- 9:15 (90) Groups of integral representation type. Professor HYMAN BASS, Columbia University (752-16-9)

9:25-10:10 (91) Representation theory of Artin algebras. MAURICE AUSLANDER, Brandeis University (752-16-30)

10:20-11:05 (92) A survey of block theory from the point of view of representations of algebras. Professor J. L. ALPERIN, University of Chicago (752-20-36)

11:15-12:00 (93) Representations of semisimple Lie algebras. Professor JAMES E. HUMPHREYS, University of Massachusetts (752-17-1)

WEDNESDAY, 8:30 A. M.

Special Session on Ramsey Theory and its Ramifications I, Lancaster Room E

8:30– 8:50 (94) Isomorphic Ramsey numbers. Professor FRANK HARRY* and Professor ROBERT W. ROBINSON, University of Michigan (752-05-40)

9:00– 9:20 (95) A generalization of Ramsey theory for graphs. Preliminary report. Mr. C. L. LIU* and Mr. K. M. CHUNG, University of Illinois (752-05-14) (Introduced by Dr. S. A. Burr)


10:00–10:20 (97) Size Ramsey numbers for bipartite graphs. Dr. J. SHEEHAN* and Dr. C. C. ROUSSEAU, Memphis State University (752-05-36)

10:30–10:50 (98) Survey on Ramsey-multiplicity of graphs. VERA ROSTA, University of Waterloo (752-05-51)

11:00–11:20 (99) Ramsey numbers involving tree-like hypergraphs. Dr. STEFAN A. BURR, AT&T Long Lines, Bedminster, New Jersey and Dr. RICHARD A. DUKE*, Georgia Institute of Technology (752-05-17)

11:30–11:50 (100) An improved bound for the classical Ramsey number R(4,4;3). Professor ALLEN J. SCHWENK, United States Naval Academy (752-05-8)

WEDNESDAY, 8:30 A. M.

Special Session on Foliations, Lancaster Room D

8:30– 8:00 (101) Foliated 3-manifolds with solvable fundamental group. Preliminary report. Professor J. F. PLANTE, University of North Carolina (752-57-5)

9:10– 9:40 (102) Homological uniqueness of invariant measures. Professor WILLIAM THURSTON, Princeton University (752-57-14)
9:50-10:20 (103) On the average Gaussian curvature of leaves of foliations. DANIEL ASIMOV, Haverford College (752-53-10)

10:30-11:00 (104) Codimension one foliations of closed manifolds. PAUL R. DIPPOLITO, Institute for Advanced Study (752-57-7)

11:10-11:30 (105) Leaves of finite class. Preliminary report. Professor JOHN CANTWELL, St. Louis University and Professor LAWRENCE CONLON*, Fordham University (752-57-8)

WEDNESDAY, 9:00 A. M.

Session on Statistics, Italian Room

9:00- 9:10 (106) On the behavior of grouping sets from exponential distributions. Preliminary report. Dr. CURTIS K. CHURCH*, University of North Florida and Dr. CHRIS P. TSOKOS, University of South Florida (752-62-1)

9:15- 9:25 (107) Deriving parameter bounds for a stochastic ecological model. Preliminary report. Dr. ROBERT JERNIGAN* and Dr. JOHN C. TURNER, University of South Florida (752-62-2) (Introduced by Dr. Chris P. Tsokos)

9:30- 9:40 (108) Bayesian analysis of the alpha failure model. Preliminary report. Dr. D. C. KOUTRAS*, Polytechnic Institute of New York and Dr. CHRIS P. TSOKOS, University of South Florida (752-62-3)

9:45- 9:55 (109) A sequential rank test for a small change in distribution. Preliminary report. Dr. DARGAN FRIERSON, Jr., University of North Carolina at Wilmington (752-62-4)

10:00-10:10 (110) Optimal acceptance sampling schemes using the alpha failure model. Preliminary report. Dr. D. C. KOUTRAS, Polytechnic Institute of New York and Dr. A. E. RUST*, University of South Florida (752-62-5) (Introduced by Dr. Chris P. Tsokos)

10:15-10:25 (111) Estimation of monotone failure rate in multivariate failure models. Preliminary report. Dr. E. CHARATSIS*, Athens Graduate School of Economics, Greece and Dr. A. N. V. RAO, University of South Florida (752-62-6) (Introduced by Dr. Chris P. Tsokos)

10:30-10:40 (112) Multivariate non-Gaussian Bayes discriminant analysis. Preliminary report. Dr. R. L. W. WELCH*, Federal Reserve Board, Washington, D.C. and Dr. C. P. TSOKOS, University of South Florida (752-62-7)

10:45-10:55 (113) A stochastic model for BOD and DO in streams. Dr. A. S. PAPADOPOULOS*, College of Charleston and Dr. W. J. PADGETT, University of South Carolina (752-62-8)

11:00-11:10 (114) Approximating moment sequences to obtain consistent estimates of distribution functions. PATRICK L. BROCKETT, University of Texas (752-62-9)

11:15-11:25 (115) N-dimensional measures of dependence derived from copulas. Preliminary report. Dr. EDWARD F. WOLFF, Beaver College (752-62-10)


WEDNESDAY, 9:00 A. M.

Session on Numerical Analysis, Austrian Room

9:00- 9:10 (117) An iterative solution of nonlinear equations in several variables. Preliminary report. Professor DAVID A. FIELD, College of the Holy Cross (752-65-1)

9:15- 9:25 (118) An advance in machine estimation of numbers of fixed points. Preliminary report. Dr. BAKER KEARFOTT, University of Southwestern Louisiana (752-65-2)

9:30- 9:40 (119) Numerical eigenvalues for second order differential equations. Professor JOHN GREGORY* and Professor RALPH WILKERSON, Southern Illinois University (752-65-4) (Introduced by Professor Ronald Kirk)

9:45- 9:55 (120) Computation of eigenvalues for operator equations. Preliminary report. Mr. STEVE HENNAGIN, University of California, Davis (752-65-6)

10:00-10:10 (121) Finding eigenfunction expansions for PDE’s using Taylor series. Dr. GEORGE CORLISS*, Dr. Y. F. CHANG, and Dr. GREGORY KRIEGSMAN, University of Nebraska, Lincoln (752-65-8)

10:15-10:25 (122) An iterative variant of collocation. Dr. LUIS KRAMARZ, Emory University (752-65-10)

10:30-10:40 (123) The extrapolation of Galerkin methods for parabolic equation. Preliminary report. Mr. NOZAR AZARNIA, University of Cincinnati (752-65-11)
10:45-10:55 (124) General moment methods for a class of nonlinear models. Professor JEROME EISENFELD and Mr. STEPHEN W. CHENG, University of Texas at Arlington (752-65-12)

11:00-11:10 (125) The constant strain condition, Preliminary report. Mr. ADDISON E. FREY, University of Pittsburgh (752-65-13)

11:15-11:25 (126) Approximation of zeros of functions. Preliminary report. Dr. J. JONES, Jr., Air Force Institute of Technology, Wright-Patterson AFB (752-65-14)

WEDNESDAY, 9:00 A. M.

Special Session on Operator Theory I, Essex Room

9:00-9:20 (127) Bounded integral operators. Professor P. R. HALMOS, University of California, Santa Barbara (752-47-6)

9:30-9:50 (128) The transitive algebra problem: recent results and possible approaches. Professor PETER ROSENTHAL, University of Toronto (752-47-10)

10:00-10:20 (129) Recent developments in the theory of hyponormal operators. Dr. JOSEPH G. STAMPFLI, Indiana University (752-47-19)

WEDNESDAY, 10:30 A. M.

Invited Address, Regency Ballroom

(131) Ergodic theorems in demography. Professor JOEL E. COHEN, Rockefeller University (752-15-4)

WEDNESDAY, 1:00 P. M.

Colloquium Lectures, Lecture I, Regency Ballroom

(133) Algebraic K-theory. Professor HYMAN BASS, Columbia University

WEDNESDAY, 2:15 P. M.

Session on Ordinary Differential Equations II, English Room


2:30-2:40 (135) Existence of weak solutions to a nonlinear boundary value problem. Preliminary report. Dr. ALAN V. LAIR, University of South Dakota (752-34-18)

2:45-2:55 (136) Further extensions of Lettenmeyer's theorem. Preliminary report. Professor LEON M. HALL, University of Nebraska, Lincoln (752-34-19)

3:00-3:10 (137) An analysis of the sum of closed operators. Professor PHILIP W. WALKER, University of Houston (752-34-20)


3:30-3:40 (139) Periodic solutions of perturbed conservative systems. Professor JAMES R. WARD, Pan American University (752-34-22)

3:45-3:55 (140) Hilbert space methods for nonlinear systems of differential equations. Dr. PETER W. BATES, Pan American University (652-34-23)

4:00-4:10 (141) Flow invariant sets for second order differential equations. JAMES H. LIGHTBOURNE III, Pan American University (752-34-24)

4:15-4:25 (142) Monotone methods for nonlinear boundary value problems in Banach space. Dr. S. R. BERNFELD* and Dr. V. LAKSHMIKANTHAM, University of Texas at Arlington (752-34-25)

4:30-4:40 (143) Persistence of positive solutions of differential equations. Preliminary report. Professor THOMAS C. GARD*, University of Georgia and Professor THOMAS G. HALLAM, University of Tennessee (752-34-26)

4:45-4:55 (144) Hille–Wintner type oscillation criteria. Professor G. J. ETGEN*, University of Houston and Professor R. T. LEWIS, University of Alabama (752-34-27)
WEDNESDAY, 2:15 P. M.

Session on General Topology II, Lancaster Room C

2:15– 2:25 (145) A functional characterization of primitive base. Professor HOWARD H. WICKE, Ohio University (752-54-17)

2:30– 2:40 (146) An improvement of the Poincaré–Birkhoff fixed point theorem, Preliminary report. Ms. PATRICIA H. CARTER, University of Florida (752-54-18)

2:45– 2:55 (147) $G_δ$-sets in symmetrizable and related spaces. Preliminary report. Dr. S. W. DAVIS*, Dr. G. GRIENHAGE, and Dr. P. J. NYIKOS, Auburn University (752-54-19)

3:00– 3:10 (148) Metric decompositions. Professor E. van DOUWEN and Professor G. M. REED*, Ohio University, Professor D. J. LUTZER, Texas Tech University, and Professor J. PELANT, Institute of Mathematics, ČSAV, Czechoslovakia (752-54-20)

3:15– 3:25 (149) Remote points and $Z$-ultrafilters on metric spaces. Professor SOO BONG CHAE* and Mr. JEFF H. SMITH, University of South Florida, New College (752-54-21)

3:30– 3:40 (150) A compact nonmetrizable space $X$ such that $X^2$ is hereditarily normal, Preliminary report. Professor PETER J. NYIKOS, Auburn University (752-54-22)

3:45– 3:55 (151) A fixed point theorem for tree-like continua. Preliminary report. Professor J. B. FUGATE*, University of Kentucky and Professor T. BRUCE McLEAN, James Madison University (752-54-23)

4:00– 4:10 (152) Some topological properties of function spaces. Dr. ROBERT A. MCCOY, Virginia Polytechnic Institute and State University (752-54-24)


4:30– 4:40 (154) Embeddings and concordances of embeddings. Professor L. S. HUSCH*, University of Tennessee and Professor I. IVANŠIĆ, University of Zagreb, Yugoslavia (752-54-26)

WEDNESDAY, 2:15 P. M.

Session on Algebraic Topology, Tudor Room

2:15– 2:25 (156) A spectrum realization of a finite chain complex over the cohomology ring of the stable integral Eilenberg–MacLane space at the prime two. Preliminary report. Dr. PEDRO A. SUAREZ, Northwestern University (752-55-1)


2:45– 2:55 (158) Two results concerning nilpotency and cofibrations. Preliminary report. Professor ROBERT H. LEWIS, Lander College (752-55-3)

3:00– 3:10 (159) The homotopy groups of knots, I, II. Professor SAMUEL J. LOMONACO, Jr., State University of New York at Albany (752-55-4)

3:15– 3:25 (160) Orbits and attractors. Preliminary report. Professor MARGARET M. LASALLE, University of Southwestern Louisiana (752-55-5)

3:30– 3:40 (161) Simplicial sets from categories. Professor DANA MAY LATCH*, North Carolina State University, Professor W. STEPHEN WILSON, Institute for Advanced Study, and Dr. ROBERT W. THOMASON, Massachusetts Institute of Technology (752-55-6)


4:00– 4:10 (163) Obstructions to free product decompositions of rational loop space homology of polyhedra. Preliminary report. Professor JOHN L. CUADRADO, Wright State University (752-55-8)


4:30– 4:40 (165) Homology with models. Preliminary report. DARYL GEORGE, East Carolina University (752-55-10)

4:45– 4:55 (166) On Brown–Peterson (co)homology theories of the Eilenberg–MacLane space $K(Z,3)$. Preliminary report. Professor JACK UCCI, Syracuse University (752-55-11)
5:00–5:10 (167) Free and projective crossed modules. Dr. JOHN G. RATCLIFFE, Massachusetts Institute of Technology (752–55–12)

WEDNESDAY, 2:15 P. M.

Session on Functional Analysis II, York Room


2:30–2:40 (169) Sufficient conditions for the relative uniform topology to be a vector topology. Preliminary report. Ms. NANCY J. FORDYCE, Florida State University (752–46–18)

2:45–2:55 (170) Inner products on groups with transition probability structure and a theorem of Kakutani–Mackey. Preliminary report. Professor Č. V. STANOJEVIĆ and Dr. S. J. GUCCIONE, Jr.*, University of Missouri, Rolla (752–46–19)

3:00–3:10 (171) Bohr compactifications of products. Professor R. RAO CHIVUKULA, University of Nebraska (752–46–20)


4:00–4:10 (175) Fractional orders of periodic distributions. Dr. WILLIAM TADD FRANKE, Emory University (752–46–24)

4:15–4:25 (176) Weakly null sequences equivalent to the unit vector basis of $c_0$. Preliminary report. Dr. E. ODELL*, University of Texas and Dr. M. WAGE, Yale University (752–46–25)

4:30–4:40 (177) Compact abelian groups of automorphism of von Neumann algebras. Preliminary report. Dr. JON E. KRAUS, State University of New York at Buffalo (752–46–26)

4:45–4:55 (178) Smoothness properties of functions in $R^2(X)$ at certain boundary points. Preliminary report. Dr. EDWIN M. WOLF, East Carolina University (752–46–27)

5:00–5:10 (179) Conditional basic sequences in Fréchet spaces. Preliminary report. Professor S. F. BELLENOT, Florida State University (752–46–28)

WEDNESDAY, 2:15 P. M.

Session on Integral Equations and Transforms, Dutch Room

2:15–2:25 (180) Arithmetical Laplace transforms. Professor RAIMOND A. STRUBLE, North Carolina State University (752–44–1)

2:30–2:40 (181) On the locality of generalized Radon transforms. Preliminary report. Mr. ERIC TODD QUINTO, Massachusetts Institute of Technology (752–44–3)

2:45–2:55 (182) Eigenvalues of a Stieltjes–Volterra integral equation. Dr. JOHN A. CHATFIELD, Southwest Texas State University (752–45–1)

3:00–3:10 (183) Determinantal and product integral solutions of nonlinear Volterra integral equations. Preliminary report. Professor ALVIN J. KAY, Texas A & I University (752–45–2)


WEDNESDAY, 2:15 P. M.

Session on Finite Differences, Functional Equations and Computer Science, Grecian Room

2:15–2:25 (185) Remarks on summation representation functional equations. Professor PL. KANNAPPAN, University of Waterloo and Professor P. N. RATHIE*, Universidade Estadual de Campinas, Brazil (752–39–1)

2:30–2:40 (186) Distributional solutions in information theory. P.L. KANNAPPAN*, A. KAMIŃSKI, and J. MIKUSIŃSKI, University of Waterloo and Silesian University, Katowice (752–39–2)


3:00–3:10 (188) A vector (ordinal) switching algebra as conservative extension of present scalar (typological) switching algebra. Preliminary report. Mr. JOHN HAYS, ITERACTICS, Reston, Virginia (752–68–2)
WEDNESDAY, 2:15 P. M.

Session on Geometry, Convex Sets and Geometric Inequalities, French Room

2:15- 2:25 (189) A purely geometric proof. Mrs. JANET D. THOMAS, Jenkintown, Pennsylvania (752-50-1)

2:30- 2:40 (190) The theorems of Euler and Eberhard for tilings of the plane. Professor BRANKO GRÜNBAUM*, University of Washington and Professor G. C. SHEPHERD, University of East Anglia, Norwich, England (752-50-2)

2:45- 2:55 (191) Lopsided sets and collections of orthants intersected by convex sets. Dr. JAMES F. LAWRENCE, National Bureau of Standards (752-52-1)

3:00- 3:10 (192) Remarks on billiards. ROBERT SINE*, University of Rhode Island and VLADISLAV KREINOVIC, Academy of Sciences of the USSR (752-52-2)

WEDNESDAY, 2:15 P. M.

Poster Session, Flemish Room

2:15- 4:15 (193) Upper radicals and essential ideals. Preliminary report. Professor DWIGHT M. OLSON, Cameron University and Professor TERRY L. JENKINS*, University of Wyoming (752-16-21)

2:15- 4:15 (194) Oscillation of first-order, linear, Hamiltonian systems of ordinary differential equations. Preliminary report. Dr. CARL H. RASMUSSEN, University of Michigan, Dearborn (752-34-16)


2:15- 4:15 (196) An $L_p$ analytic Fourier-Feynman transform. Professor GERALD JOHNSON and Professor DAVID SKOUG*, University of Nebraska (752-44-2)

2:15- 4:15 (197) Functors on compact pairs. Professor DONALD G. HARTIG, United States Naval Academy (752-46-32)

2:15- 4:15 (198) Error bounds for approximations to the generalized inverse. Preliminary report. Professor C. W. GROETSCHEL, University of Cincinnati (752-47-3)

2:15- 4:15 (199) On one sided Chebyshev approximation. Professor CHRIS CORAY* and Professor E. R. HEAL, Utah State University (752-65-3) (Introduced by Duane Loveland)

2:15- 4:15 (200) A method for computing directional drilling surveys. Dr. ROBERT D. SIDMAN* and Ms. BRENDA F. YOUNGBLOOD, University of Southwestern Louisiana (752-99-2)

2:15- 4:15 (200A) Geometric convergence of Chebyshev rational approximations; factorization of the derivative. Preliminary report. Professor M. S. HENRY*, Montana State University and Professor J. A. ROULIER, North Carolina State University (752-41-5)

WEDNESDAY, 2:15 P. M.

Special Session on Representations of Finite Dimensional Algebras and Finite Groups II, Problems, Lancaster Rooms A & B

2:15- 3:00 (201) Representation theory of Artin algebras. Professor MAURICE AUSLANDER, Brandeis University (752-16-30)

3:10- 3:55 (202) Some problems in modular representation theory. WALTER FEIT, Yale University (752-20-22)

4:05- 4:50 (203) Problems in integral representation theory. Professor Dr. K. W. ROGENKAMP, University of Stuttgart, West Germany (752-16-15)

5:00- 5:45 (204) Representations of semisimple Lie algebras. Professor JAMES E. HUMPHREYS, University of Massachusetts, Amherst)

WEDNESDAY, 2:15 P. M.

Special Session on Operator Theory II, Essex Room

2:15- 2:35 (205) Generalizations of pseudo differential operators. I. M. SINGER, University of California, Berkeley (752-57-15)

2:45- 3:05 (206) Topics related to extension of $C^*$-algebras. Preliminary report. LAWRENCE G. BROWN, Purdue University (752-47-11)


3:45- 4:05 (208) Concrete model theory. Professor DOUGLAS N. CLARK, University of Georgia (752-47-12)
4:15– 4:35 (209) Aspects of cyclic subnormal operators. THOMAS KRIETE, University of Virginia
(752-47-31)

4:45– 5:05 (210) Operator theory and electrical engineering. J. WILLIAM HELTON, University of
California, San Diego (752-47-20)

WEDNESDAY, 2:15 P. M.

Special Session on Ramsey Theory and its Ramifications II, Lancaster Room E

2:15– 2:35 (211) The Nešetřil and Rödl-theorem on Ramsey's theorem for set systems. B. L.
ROTHSCHILD, University of California, Los Angeles (752-05-49)

2:45– 3:05 (212) On a problem of subsets and Ramsey's theorem. Professor PAUL ERDOS, Univer-
sity of Colorado and Hungarian Academy of Sciences (752-05-16)

3:15– 3:35 (213) Intersection theorems for subsets of integers, Dr. MIKLÓS SIMONOVITS* and
Dr. VERA T, SÓS, Eötvös Loránd University, Budapest, Hungary (752-05-24)
(Introduced by Stephan Burr)

3:45– 4:05 (214) A general Ramsey product theorem. R. L. GRAHAM, Bell Laboratories, Murray
Hill, New Jersey (752-05-43)

4:15– 4:35 (215) Euclidean Ramsey theory. Professor E. G. STRAUS, University of California,
Los Angeles (752-05-30)

4:45– 5:05 (216) All finite configurations are almost Ramsey. JOEL H. SPENCER, State University
of New York at Stony Brook (752-05-15)

5:15– 6:00 (217) Problem session

WEDNESDAY, 3:30 P. M.

Invited Address, Regency Ballroom

(218) Classical and quantum field theory. Professor THOMAS SPENCER, Princeton Uni-
versity (752-81-4)

WEDNESDAY, 7:00 P. M.

Session on Group Theory and Generalizations I, Austrian Room

7:00– 7:10 (219) Infinite groups with finitely many automorphisms. THOMAS FOURNELLE, Univer-
sity of Illinois (752-20-33)

7:15– 7:25 (220) Commutator equations over free nilpotent class 2 groups. Preliminary report.
Professor KENNETH W. WESTON, University of Wisconsin–Parkside (752-20-34)

7:30– 7:40 (221) Valuation and height function of countable p-groups. Dr. CHANG M. BANG, Emory
University (752-20-35)

WEDNESDAY, 7:00 P. M.

Session on Algebraic Geometry, Grecian Room

7:00– 7:10 (222) A two variable criterion for extrema with related effective procedure. Dr. PAUL
CHERENACK, University of Cape Town, South Africa (752-14-1)

7:15– 7:25 (223) Endomorphisms of Abelian varieties. Dr. ROBERT J. FISHER, Illinois State Uni-
versity (752-14-2)

7:30– 7:40 (224) Interference patterns and algebraic geometry. Preliminary report. Professor
KEITH KENDIG, Cleveland State University (752-14-3) (Introduced by Professor
Allen J. Silberger)

WEDNESDAY, 7:00 P. M.

Session on Calculus of Variations and Optimal Control, York Room

SURYANARAYANA, Eastern Michigan State University and RUSSELL D. RUPP*,
Rutgers University, Camden (752-49-1)

Professor DENNIS D. BERKEY* and Professor MARVIN I. FREEDMAN, Boston
University (752-49-2)

Professor F. A. MASSEY* and Professor R. E. WORTH, Georgia State Univer-
sity (752-49-3)

7:45– 7:55 (228) Optimal control problems with multiple delays. Preliminary report. Professor
VERNON L. BAKKE, University of Arkansas (752-49-4) (Introduced by Dr. James
F. Porter)
Session on Several Complex Variables and Analytic Spaces, English Room

7:00- 7:10 (229) A two-sided H. Lewy extension phenomenon, I, Professor L. R. Hunt, Texas Tech University and Dr. M. Kazlow*, Rice University (752-32-8)

7:15- 7:25 (230) Higher order analogues to T. C. R. equations for smooth real submanifolds of \( \mathbb{C}^n \) with C.R. singularity. Preliminary report. Dr. Gary A. Harris, Texas Tech University (752-32-2)

7:30- 7:40 (231) Analytic structures for \( H^\infty \) of certain domains in \( \mathbb{C}^n \). Professor Eric P. Kronstadt, University of Michigan (752-32-4)

7:45- 7:55 (232) A two-sided H. Lewy extension phenomenon. II. Preliminary report. Professor L. R. Hunt*, Texas Tech University and Professor Mike Kazlow, Rice University (752-32-1)

8:00- 8:10 (233) Simplified descriptions of the exceptional bounded symmetric domains. Professor Daniel Drucker, Wayne State University (752-32-9)

WEDNESDAY, 7:00 P. M.

Session on Fourier Analysis, Italian Room

7:00- 7:10 (234) Jacobi and Hankel multipliers of type \((p,q), 1 < p < q < \infty\). Professor George Gasper*, Northwestern University and Professor Walter Trebels, Technische Hochschule, Darmstadt, West Germany (752-42-1)

7:15- 7:25 (235) Weighted norm inequalities for the \( \gamma \)-function. Preliminary report. Mr. Douglas S. Kurtz, Rutgers University (752-42-2)

7:30- 7:40 (236) Multipliers for Hardy spaces and BMO. Professor Günther W. Goes, Illinois Institute of Technology (752-42-3)

7:45- 7:55 (237) The Littlewood-Paley theory for Jacobi expansions. Dr. William C. Connett* and Dr. Alan L. Schwartz, University of Missouri-St. Louis (752-42-4)

8:00- 8:10 (238) Helsen curves and multiplicity. Preliminary report. Professor O. Carruth McGehee, Louisiana State University and Professor Gordon S. Woodward*, University of Nebraska (752-42-5)

WEDNESDAY, 7:00 P. M.

Session on Global Analysis, Analysis on Manifolds, French Room

7:00- 7:10 (239) Timelike homotopy groups and a characteristic class for Lorentz manifolds. Dr. William F. Rich, Sam Houston State University (752-58-2)

7:15- 7:25 (240) Function groups associated with constraint submanifolds. Professor R. O. Fulp* and Professor J. A. Marlin, North Carolina State University (752-58-3)

7:30- 7:40 (241) Axiom A dynamical systems, cycles, and stability. Preliminary report. Alan Dankner, Institute for Advanced Study (752-58-4)

7:45- 7:55 (242) Mathematical analysis of a cellular control process with positive feedback. James P. Selgrade, North Carolina State University (752-58-5)

WEDNESDAY, 7:00 P. M.

Session on Sequences, Series Summability, Dutch Room

7:00- 7:10 (243) Summability of matrix transforms of stretchings and subsequences. Professor David F. Dawson, North Texas State University (752-40-1)

7:15- 7:25 (244) Consistency theory in semiconservative spaces. Professor A. K. Snyder, Lehigh University (752-40-2)

7:30- 7:40 (245) Conullity of a pair of conservative matrices. Preliminary report. Dr. Robert DeVos, Villanova University (752-40-3)

WEDNESDAY, 7:00 P. M.

Session on Fluid Mechanics, Flemish Room

7:00- 7:10 (246) Propagation of magnetohydrodynamic dispersive waves on a running stream. Dr. Lokenath Debnath*, East Carolina University and Dr. Uma Basu, University of Calcutta, India (752-76-1)

7:15- 7:25 (247) Symmetry principles for integrodifferential equations. Dr. Mayer Humi, Worcester Polytechnic Institute (752-76-2)

7:30- 7:40 (248) Two and three dimensional periodic waves in water of infinite depth. Preliminary report. Professor John Reeder, University of Missouri and Professor Marvin Shinbrot*, University of Victoria (752-76-3)

7:45- 7:55 (249) Flows and continued fractions over a vector space. Preliminary report. Professor F. A. Roach, University of Houston (752-76-4) (Introduced by W. T. Ingram)
Josiah Willard Gibbs Lecture, Regency Ballroom  
(250) Mathematical Typography. Professor DONALD E. KNUTH, Stanford University

THURSDAY, 8:00 A. M.

Session on Associative Rings and Algebras I, Austrian Room
8:00- 8:10 (251) Algebraic extensions of skew fields. Preliminary report. JOHN DAUNS, Tulane University (752-16-1)
8:15- 8:25 (252) Indecomposable decompositions and the minimal direct summand containing the nilpotents. Professor G. F. BIRKENMEIER, Floyd Junior College (752-16-2)
8:30- 8:40 (253) Negative d. g. near rings. Preliminary report. Professor HENRY E. HEATHERLY, University of Southwestern Louisiana (752-16-3)
8:45- 8:55 (254) Fixed rings of simple rings. Mr. JAMES OSTERBURG, University of Cincinnati (752-16-4)
9:00- 9:10 (255) The descending chain condition relative to a torsion theory. Preliminary report. Professor MARK L. TEPLY* and Professor ROBERT W. MILLER, University of Florida (752-16-5)
9:15- 9:25 (256) C-commutativity in R[x] and in R[[x]]. Dr. TOM CHEATHAM*, Samford University and Dr. ED ENOCHS, University of Kentucky (752-16-6) (Introduced by Dr. W. D. Peeples)
9:30- 9:40 (257) Hopf algebras with one-sided antipodes. Professor JAMES A. GREEN, University of Warwick, Coventry, England and Professor EARL J. TAFT*, Rutgers University (752-16-7)
9:45- 9:55 (258) Reduced rank in rings with Krull dimension. Preliminary report. Professor T. H. LENAGAN, University of Southern California (752-16-8) (Introduced by Charles P. Lanski)
10:00-10:10 (259) Finiteness conditions for modular lattices with finite group actions. Professor JOE W. FISHER, University of Cincinnati (752-16-10)
10:15-10:25 (260) Unitary invariance in algebraic algebras. CHARLES LANSKI, University of Southern California (752-16-13)
10:30-10:40 (261) The centralizer of a group automorphism. Professor CARLTON J. MAXSON* and Professor KIRBY C. SMITH, Texas A & M University (752-16-14)
10:45-10:55 (262) On homogeneous components of socle. Professor JITENDRA MANOCHA, Kent State University (752-16-17) (Introduced by Dr. Richard Shoop)
11:00-11:10 (263) Structure of q-rings. K. A. BYRD, University of North Carolina at Greensboro (752-16-16)
11:15-11:25 (264) Note on separable extensions of rings. Preliminary report. Professor GEORGE SZETO, Bradley University (752-16-20)
11:30-11:40 (265) State spaces of Grothendieck groups of rings. Preliminary report. Dr. K. R. GOODEARL*, University of Utah and Dr. R. B. WARFIELD, Jr., University of Washington (752-16-22)
11:45-11:55 (266) Direct sums of semirings. Preliminary report. Dr. H. E. STONE, University of Southwestern Louisiana (752-16-23)

Session on Number Theory, York Room
8:00- 8:10 (267) Large sieve inequality for algebraic number fields. Preliminary report. Professor DONALD G. HAZLEWOOD, Southwest Texas State University (752-10-1)
8:15- 8:25 (268) Equidistribution of linear recurring sequences in finite fields. Professor HARALD NIEDERREITER, University of Illinois and Professor JAU-SHYONG SHIUE*, University of Illinois and National Cheng-Chi University, Taipei, Taiwan, Republic of China (752-10-6)
8:30- 8:40 (269) The generating function for Gordon's identity. Dr. DAVID M. BRESSOUD, Pennsylvania State University (752-10-7)
8:45- 8:55 (270) The six smallest prime factors of an odd perfect number. Dr. MASAO KISHORE, University of Toledo (752-10-9)
9:00- 9:10 (271) The distribution of the zeroes of the Lerch zeta function. Dr. GEORGES GRINESTEIN, Auburn University at Montgomery (752-10-10)
9:30–9:40 (273) Relationship of spinor genus definitions for quadratic forms. Preliminary report. Dr. ANDREW G. EARNEST, University of Southern California (752–10–12)


10:00–10:10 (275) On the minimum of zero indefinite binary quadratic forms. MARY E. GBUR, Texas A & M University (752–10–15)


10:30–10:40 (277) Smoothing slow sequences of distributions modulo one. J. H. B. KEMPERMANN, University of Texas and University of Rochester (752–10–19)


THURSDAY, 8:00 A. M.

Session on Combinatorics I, French Room

8:00–8:10 (279) Graphical regular representations of non-Abelian groups of order 24. Preliminary report. Ms. PAULA A. RUSSO*, Indiana University and Professor MARK E. WATKINS, Syracuse University (752–05–1)

8:15–8:25 (280) The enumeration of labelled and unlabelled line graphs. PHIL HANLON, Dartmouth College (752–05–2) (Introduced by Robert Z. Norman)

8:30–8:40 (281) Circulant quasi-Hadamard matrices. Professor JOHN KONVALINA*, University of Nebraska at Omaha and Professor RODNEY KOSLOSKI, Indiana University Northwest (752–05–4)

8:45–8:55 (282) Four-discordant permutations. Dr. EARL GLEN WHITEHEAD, Jr., University of Pittsburgh (752–05–25)

9:00–9:10 (283) On maximal antichains containing no set and its complement. Professor G. F. CLEMENTS, University of Colorado (752–05–7)

9:15–9:25 (284) Reducible partitions. Preliminary report. Mr. STEVEN E. ANACKER, Ohio State University (752–05–9)


10:00–10:10 (287) A double circulant presentation for quadratic residue codes. Mr. RICHARD A. JENSON, Oberlin College (752–05–12)

10:15–10:25 (288) Integral and rational combinatorial matrix completions. Mr. ERIC VERHEIDEN, California Institute of Technology (752–05–13)

10:30–10:40 (289) Toroidal graphs and the strong perfect graph conjecture. Preliminary report. Mr. CHARLES M. GRINSTEAD, University of California, Los Angeles (752–05–18) (Introduced by Bruce Rothschild)

10:45–10:55 (290) An additivity theorem for maximum genus of a graph. Dr. CHARLES H. C. LITTLE, Royal Melbourne Institute of Technology, Australia and Dr. RICHARD D. RINGEISEN*, Indiana University–Purdue University at Fort Wayne (752–05–19)

11:00–11:10 (291) Graphs with given girth and circumference. Preliminary report. Dr. JOHN ROBERTS, University of Louisville (752–05–20)

11:15–11:25 (292) Detours in digraphs. Professor R. C. ENTRINGER*, University of New Mexico, Dr. D. E. JACKSON, Los Alamos Scientific Laboratories, Los Alamos, New Mexico, and Dr. HENDA SWART, University of Natal, Durban, South Africa (752–05–21)

11:30–11:40 (293) The standard formulas for combinations with various repetitions obtained by functional equations. Dr. DONALD R. SNOW, Brigham Young University (752–05–22)

THURSDAY, 8:00 A. M.

Session on Probability Theory and Stochastic Processes I, Italian Room

8:00–8:10 (294) Positive frequency and a zero-one law. Preliminary report. THEODORE P. HILL, University of California, Berkeley (752–60–1)

8:15–8:25 (295) The distribution of the likelihood ratio for additive processes. Professor P. L. BROCKETT, University of Texas at Austin, Professor W. N. HUDSON, Tulane University, and Professor H. G. TUCKER*, University of California, Irvine (752–60–2)

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8:30- 8:40 (296) Three discrete-time queues in tandem. Preliminary report. Dr. JOHN A. MORRISON, Bell Laboratories, Murray Hill, New Jersey (752-60-3)

8:45- 8:55 (297) Equiconvergence and the entropy of a transformation. Professor LOUIS H. BLAKE, City University of New York, College of Staten Island (752-60-4)

9:00- 9:10 (298) Doubly stochastic measures supported by measurable functions. Professor PAUL N. DELAND* and Professor RAY C. SHIFLETT, California State University, Fullerton (752-60-6)


9:30- 9:40 (300) A continuous-time analogue of random walk in a random environment. Professor GRANT A. RITTER, University of Florida (752-60-10)

9:45- 9:55 (301) Markov solutions of stochastic differential equations. Professor PHILIP PROTTER, Institute for Advanced Study (752-60-12)

10:00-10:10 (302) Asymptotic regular almost compactness and convergence of nets of probability measures. Preliminary report. Professor HERMAN RUBIN, Purdue University (752-60-13)

10:15-10:25 (303) A strong law of large numbers for stationary point processes. Dr. ROBERT M. CRANWELL*, University of Wisconsin-Parkside and Dr. N. A. WEISS, Arizona State University (752-60-15)

10:30-10:40 (304) Compactifications of dual processes. Preliminary report. Mr. JOSEPH GLOVER, University of California, San Diego (752-60-16)

10:45-10:55 (305) Matrix-stable distributions and limit theorems. Preliminary report. Mr. RICHARD SALTER, Indiana University (752-60-17)

11:00-11:10 (306) An application of the random version of Schauder's fixed point theorem to experience theory. Preliminary report. Mr. PATRICK SUTHERLAND, University of Texas at Arlington (752-60-18)

11:15-11:25 (307) Stability of McShane systems. Preliminary report. Dr. A. N. V. RAO* and Dr. C. P. TSOKOS, University of South Florida (752-60-19)

11:30-11:40 (308) A strong Markoff property for multiparameter processes. Dr. R. WOLPERT, Duke University (752-60-23)

THURSDAY, 8:00 A.M.

Session on Partial Differential Equations I, Lancaster Room D

8:00- 8:10 (309) Uniqueness and norm convexity for products of solvable operators. Preliminary report. Professor MONTY J. STRAUSS, Texas Tech University (752-35-3)

8:15- 8:25 (310) Polynomial approximation of generalized biaxisymmetric potentials. Professor PETER A. MCCOY, U. S. Naval Academy (752-35-4)

8:30- 8:40 (311) A uniqueness theorem for a generalized initial value problem. Dr. JOHN F. SCHMEELK, Virginia Commonwealth University (752-35-5)

8:45- 8:55 (312) Potentials for symmetric hyperbolic systems. Preliminary report. Dr. DAVID GILLIAM*, Texas Tech University and Dr. JOHN SCHULEMBERGER, Tucson, Arizona (752-35-6)

9:00- 9:10 (313) A nonlinear Dirichlet problem. Preliminary report. Dr. ALFONSO CASTRO, Centro de Investigacion del IPN, Apartado Postal 14740, Mexico 14, D. F., Mexico (752-35-7)

9:15- 9:25 (314) Weighted translation semigroups and a time dependent Cauchy problem. Preliminary report. Dr. LARRY GEARHART, Wright State University (752-35-8)

9:30- 9:40 (315) A sharp maximum principle for elliptic operators of second order with discontinuous coefficients, Dr. R. E. WHITE, North Carolina State University (752-35-9)


10:00-10:10 (317) A new maximum principle for systems of partial differential equations. JERRY M. FEINBERG, Brandeis University (752-35-12)

10:15-10:25 (318) A regularity theorem for linear elliptic equations. Preliminary report. JAMES E. ROSS, San Diego State University (752-35-14)

10:30-10:40 (319) Fredholm theory of partial differential equations on complete Riemannian manifolds. Preliminary report. Mr. ROBERT C. McOWEN, University of California at Berkeley (752-35-18)
10:45-10:55 (320) On the biharmonic Green's function for plane regions with a corner. CHARLES V. COFFMAN, Carnegie-Mellon University (752-35-19)

11:00-11:10 (321) Parabolic differential inequalities in cones. Dr. V. LAKSHMIKANTHAM and Mr. R. L. VAUGHN*, University of Texas at Arlington (752-35-20)

11:15-11:25 (322) Commutator conditions for local solvability of partial differential operators with multiple characteristics. Professor CAROLE GROVER, University of Pittsburgh (752-35-21)

11:30-11:40 (323) A nonlinear heat equation. Preliminary report. Professor B. C. BURCH*, University of Texas at El Paso and Professor JEROME A. GOLDSTEIN, Tulane University (752-35-22)

THURSDAY, 8:00 A. M.

Special Session on Harmonic Analysis on Nilpotent and Solvable Groups, Essex Room

8:00- 8:20 (324) Nilpotent groups of Heisenberg type. Preliminary report. Professor THOMAS A. FARMER, Miami University (752-22-7)

8:30- 8:50 (325) On the transience of random walks on nilpotent Lie groups. Preliminary report. Mr. DAVID R. FISCHER, Louisiana State University (752-22-2)

9:00- 9:20 (326) Distinguished subspace theory of $L^2$ of Heisenberg nilmanifolds. Ms. SHARON GOODMAN, City University of New York, Graduate School (752-33-2)

9:30- 9:50 (327) The Fourier transform of a Schwartz class function on a nilpotent Lie group. Preliminary report. Professor LAWRENCE CORWIN*, Rutgers University and Professor FREDERICK P. GREENLEAF, Courant Institute of Mathematical Sciences, New York University (752-22-10)

10:00-10:20 (328) Poisson summation on Kiriílov orbits. Preliminary report. Professor LEONARD RICHARDSON, Louisiana State University (752-22-4)

11:00-11:20 (330) Nilmanifolds in algebraic number theory. Preliminary report. Professor JONATHAN BREZIN, University of North Carolina (752-12-1)

11:30-11:50 (331) Asymptotic behavior of matrix coefficients of unitary representations. Professor ROGER E. HOWE, Yale University and Professor CALVIN C. MOORE*, University of California, Berkeley (752-22-8)

THURSDAY, 8:30 A. M.

Session on Group Theory and Generalizations II, Tudor Room

8:30- 8:40 (332) A lower bound for the number of conjugacy classes in a finite nilpotent group. Professor GARY J. SHERMAN, Rose-Hulman Institute of Technology (752-20-1)

8:45- 8:55 (333) Wreath products with faithful projective representations. Professor K. BOLLING FARMER, University of Florida (752-20-3)

9:00- 9:10 (334) On normnilpotent inseparable groups of order $p^nq^2$. Professor HOMER BECHTELL, University of New Hampshire, Durham (752-20-4)

9:15- 9:25 (335) On Bieberbach's decomposition of discrete Euclidean groups. Dr. R. K. OLIVER, Pittsburgh, Pennsylvania (752-20-6)

9:30- 9:40 (336) The linear index of a group representation. Preliminary report. Dr. MARC J. LIPMAN, Indiana University-Purdue University at Fort Wayne (752-20-8)

9:45- 9:55 (337) Simple groups with a Sylow normalizer of order $3p$. Dr. LEO J. ALEX, State University of New York, College at Oneonta and Dr. D. C. MORROW*, Ferrum College (752-20-9)

10:00-10:10 (338) Four-dimensional symplectic groups. ROBERT E. SOLAZZI, Virginia Polytechnic Institute and State University (752-20-10)

10:15-10:25 (339) On the automorphism group of the semigroup of complexes of a finite abelian group. Dr. RICHARD D. BYRD, Dr. JUSTIN T. LLOYD, and Dr. JAMES W. STEPP*, University of Houston (752-20-11)

10:30-10:40 (340) A classification of indecomposable S-sets. WILLIAM R. NICO, Tulane University (752-20-13)

10:45-10:55 (341) Finite slightly injective groups. Dr. DENNIS BERTHOLF, Oklahoma State University and Dr. GARY WALLS*, University of Southern Mississippi (752-20-14)

11:00-11:10 (342) Direct summands of simply presented groups. Dr. ROBERT O. STANTON, St. John's University (752-20-15)
11:15-11:25 (343) The groups of order 128. EUGENE RODEMICH, Jet Propulsion Laboratory, Pasadena, California (752-20-16)

11:30-11:40 (344) On presentations of certain unipotent groups. Preliminary report. Mr. JOHN F. KIRCHMEYER, Northwestern University (752-20-18)

11:45-11:55 (345) L \text{KK} -admissible sets. Dr. STANLEY H. STAHL, Smith College (752-04-1)

SPECIAL SESSION ON ILL-POSED PROBLEMS FOR PARTIAL DIFFERENTIAL AND INTEGRODIFFERENTIAL EQUATIONS I, Lancaster Room E

THURSDAY, 8:30 A.M.

8:30-8:50 (346) The exact amount of nonuniqueness for the Euler–Poisson–Darboux equation. Professor JEROME A. GOLDSTEIN, Tulane University (752–35–15)

9:00-9:20 (347) Some new upper bounds for lengths of existence intervals for solutions of a class of nonlinear evolutionary equations, (with R. J. Knops). Professor H. A. LEVINE, University of Rhode Island (752–35–23)


THURSDAY, 8:30 A.M.

SPECIAL SESSION ON CAPACITY IN SEVERAL COMPLEX VARIABLES, Lancaster Room C

8:30-8:50 (353) Extremal length in several complex variables. Preliminary report. H. ALEXANDER, University of Illinois at Chicago Circle (752–32–7)

9:00-9:20 (354) The infinite dimensionality of complex structures. Preliminary report. Professor R. E. GREENE, University of California, Los Angeles (752–32–6)

9:30-9:50 (355) Strictly parabolic manifolds. Preliminary report. Professor WILHELM STOLL, University of Notre Dame (752–32–5)

THURSDAY, 9:00 A.M.

SESSION ON MANIFOLDS AND CELL COMPLEXES, English Room

9:00-9:10 (359) Reduction of standard spines. Preliminary report. DAVID GILLMAN, University of California, Los Angeles (752–57–1)


9:45-9:55 (362) Factoring free actions. Preliminary report. Professor BRADD CLARK, University of Southwestern Louisiana (752–57–4)

10:00-10:10 (363) Complements of codimension-two submanifolds, III: Cobordism theory. Mr. JUSTIN SMITH, Rice University (752–57–6)

10:15-10:25 (364) Embeddings of balls into contractible manifolds. KENNETH C. MILLETT, University of California, Santa Barbara (752–57–9)

10:30-10:40 (365) Book decompositions of manifolds. Professor FRANK QUINN, Virginia Polytechnic Institute and State University (752–57–11)

10:45-10:55 (366) Involutions on Klein spaces \(M(p,q)\). Dr. PAIK K. KIM, University of Kansas (752–57–12)
THURSDAY, 9:00 A. M.

Special Session on History of Mathematics I, Lancaster Rooms A & B
9:00–9:20 (367) American women in mathematics—The first Ph.D.'s. JUDY GREEN, Rutgers University, Camden (752-01-1)

9:30–9:50 (368) On the history of manifolds. Professor MORRIS W. HIRSCH, University of California, Berkeley (752-01-4)

10:00–10:20 (369) Asymptotic and normal series solutions of ordinary differential equations. Professor A. SCHLISSEL, City University of New York, John Jay College of Criminal Justice (752-01-3)

10:30–10:50 (370) The R. L. Moore school as a subject of historical research. Dr. ALBERT C. LEWIS, University of Texas (752-01-5) (Introduced by Uta C. Merzbach)

11:00–11:30 (371) Discussion session

THURSDAY, 9:00 A. M.

Invited Address, Regency Ballroom
(372) Mapping class groups of surfaces. Professor JOAN S. BIRMAN, Columbia University (752-57-13)

THURSDAY, 10:30 A. M.

Invited Address, Regency Ballroom
(373) Recursively enumerable sets and degrees. Professor ROBERT I. SOARE, University of Chicago (752-02-7)

THURSDAY, 1:00 P. M.

Colloquium Lectures. Lecture II, Regency Ballroom
(374) Algebraic K-theory. Professor HYMAN BASS, Columbia University

THURSDAY, 2:15 P. M.

Session on Linear and Multilinear Algebra, Matrix Theory, English Room

2:30–2:40 (376) Partitions of a symmetric matrix over a finite field. Preliminary report. NICK MOUSOURIS* and A. DUANE PORTER, Humboldt State University (752-15-2)

2:45–2:55 (377) Complex eigenvalues of a non-negative matrix with a specified graph. II. Dr. R. B. KELLOGG, University of Maryland and Dr. A. B. STEPHENS*, Mount St. Mary's College (752-15-3)

THURSDAY, 2:15 P. M.

Session on General Mathematical Systems, Dutch Room
2:15–2:25 (378) Non-Euclidean real numbers. Preliminary report. Professor LINO GUTIERREZ-NOVOA, University of Alabama (752-08-1) (Introduced by Dr. Joseph Hornback)

2:30–2:40 (379) A nondesarguesian finite projective plane of order nine. Preliminary report. Professor W. M. SANDERS, James Madison University (752-08-2)

2:45–2:55 (380) n-associative groupoids. Preliminary report. Professor WILLIAM P. WARDLAW, U. S. Naval Academy (752-08-3)

3:00–3:10 (381) On algebraic structures with three operations. Preliminary report. Professor M. E. COHEN and Professor H. S. SUN*, California State University, Fresno (752-08-4)

THURSDAY, 2:15 P. M.

Session on Category Theory, Homological Algebra, Italian Room
2:15–2:25 (382) A generalized Cartan isomorphism for the Grothendieck group of a finite group. Professor JAMES E. ARNOLD, Jr., University of Wisconsin–Milwaukee (752-18-1)

2:30–2:40 (383) On proper subobjects in a topos. Preliminary report. Mr. HARRY J. PORTA* and Professor OSWALD WYLER, Carnegie–Mellon University (752-18-2)

2:45–2:55 (384) Obstruction theory for Lie algebras. Professor CHARLES CHING-AN CHENG and Professor YEL-CHIANG WU*, Oakland University (752-18-3)

THURSDAY, 2:15 P. M.

Session on Real Functions, Grecian Room
2:15–2:25 (385) Borel parametrizations. Preliminary report. DAN MAULDIN, North Texas State University (752-26-1)

2:30–2:40 (386) Positive linear maps of Baire functions. Professor C. T. TUCKER, University of Houston (752-26-2)
2:45- 2:55 (387) Symmetric and ordinary differentiation. Preliminary report. Professor C. L. BELNA, Pennsylvania State University, Professor M. J. EVANS*, and Professor P. D. HUMKE, Western Illinois University (752-26-3)

3:00- 3:10 (388) Classification of degenerate critical points. Dr. THEODORE S. BOLIS, State University of New York, College at Oneonta (752-26-4)

THURSDAY, 2:15 P. M.

Session on General Topology III, French Room
2:15- 2:25 (389) Stable homeomorphisms of the pseudo-arc. Dr. WAYNE LEWIS, Texas Tech University (752-54-28)

2:30- 2:40 (390) Initially compact fuzzy topological spaces. Dr. HAROLD MARTIN, College of St. Scholastica (752-54-29)

2:45- 2:55 (391) A selector for equivalence with $G_δ$ orbits. Professor DOUGLAS E. MILLER, University of Illinois at Chicago Circle (752-54-30)

3:00- 3:10 (392) Stratifiable $σ$-discrete spaces are $M_1$. Dr. GARY GRUENHAGE, Auburn University (752-54-31)

THURSDAY, 2:15 P. M.

Session on Partial Differential Equations II, Lancaster Room D
2:15- 2:25 (393) The oscillation of elliptic systems. Dr. R. T. LEWIS*, University of Alabama and Dr. G. J. ETGEN, University of Houston (752-35-24)

2:30- 2:40 (394) Dirichlet eigenvalues. Dr. JAMES R. KUTTLER, Johns Hopkins University, Applied Physics Laboratory (752-35-25) (Introduced by D. W. Fox)

2:45- 2:55 (395) Asymptotic behavior for a single hyperbolic conservation law with periodic initial data, Preliminary report. Dr. JOSEPH G. CONLON, Courant Institute, New York University (752-35-26) (Introduced by Jerry Feinberg)

3:00- 3:10 (396) The principal eigenvalue for second order linear elliptic equations with Neumann boundary conditions. Professor CHARLES HOLLAND, Courant Institute, New York University (752-35-27) (Introduced by Jerry Feinberg)

THURSDAY, 2:15 P. M.

Session on Biology and Behavioral Sciences, Flemish Room

2:30- 2:40 (398) A convergence theorem for clinical cardiology. Preliminary report. Dr. J. N. REED* and Professor J. M. WORRELL, Jr., Ohio University (752-92-2)

2:45- 2:55 (399) Nonparametric classification scheme for the detection of myocardial infarction. Preliminary report. Dr. JOHN C. TURNER* and Dr. CHRIS P. TSOKOS, University of South Florida, and Dr. LARRY H. BERNSTEIN, University of South Alabama (752-92-3)

3:00- 3:10 (400) Inverse problems in neurobiology. Preliminary report. Professor W. T. KYNER* and Dr. G. A. ROSENBERG, University of New Mexico (752-92-6)

THURSDAY, 2:15 P. M.

Invited Address, Regency Ballroom
(401) Aspects of the geometry and topology of the spectrum. Professor JEFF CHEEGER, State University of New York at Stony Brook (752-53-11)

THURSDAY, 3:15 P. M.

The George David Birkhoff Prize in Applied Mathematics, Regency Ballroom

THURSDAY, 4:30 P. M.

Business Meeting, Regency Ballroom

THURSDAY, 7:30 P. M.

Session on Group Theory and Generalizations III, Tudor Room
7:30- 7:40 (402) Wreath products and converse of Lagrange's theorem. Dr. ROBERT C. MERS, North Carolina A & T State University (752-20-21)

7:45- 7:55 (403) Linear groups generated by elements containing an eigenspace of codimension two. Preliminary report. Professor W. CARY HUFFMAN, Union College (752-20-23)

8:00- 8:10 (404) $fσ$-union of a countable number of pure, completely decomposable subgroups. Preliminary report. Dr. CHARLES MEGIBBEN, Vanderbilt University and Dr. ANGELA B. SHIFLET*, Lander College (752-20-27)
8:15-8:25 (405) Small critical groups having central monoliths of a nilpotent by abelian product variety of groups. Dr. JAMES J. WOEPEL, Indiana University Southeast (752-20-28)

8:30-8:40 (406) A characterization of J1. Dr. MANLEY PERKEL, Western Illinois University (752-20-30)

8:45-8:55 (407) On the cardinality of set products in groups. Professor RICHARD D. BYRD, Professor JUSTIN T. LLOYD*, and Professor JAMES W. STEPP, University of Houston (752-20-31)

9:00-9:10 (408) Pushing up Lq(2N). Preliminary report. Mr. NEVILLE CAMPBELL, California Institute of Technology (752-20-32)

THURSDAY, 7:30 P.M.

Session on Ordinary-Differential Equations III, English Room
7:30-7:40 (409) The forced equation oscillates if and only if the unforced equation oscillates. Order n = even. Professor ATHANASSIOS G. KARTSATOS, University of South Florida (752-34-28) (Introduced by Professor M. N. Manougian)

7:45-7:55 (410) A technique in stability theory of delay differential equations. Dr. S. LEE LA*, State University of New York, College at Geneseo and Dr. V. LAKSHMIKANTHAM, University of Texas at Arlington (752-34-29)

8:00-8:10 (411) On Poisson stability. Dr. ROGER C. McCANN, Mississippi State University (752-34-30)

8:15-8:25 (412) A boundary value problem arising in the flow of a viscous fluid. Preliminary report. Dr. TAI-CHI LEE, Louisiana State University (752-34-31)

8:30-8:40 (413) On the angular variation of solutions of second order linear systems. Dr. STEVEN D. TALIAFERRO, Texas A&M University (752-34-32)

8:45-8:55 (414) Characterization of oscillation of solutions of the delay equation x(n)(t) + a(t)f(x(q(t))) = 0. Dr. W. E. MAHFOUD, Murray State University (752-34-33)

9:00-9:10 (415) Singular eigenvalue problems with eigenvalue parameter contained in the boundary conditions. Dr. CHARLES T. FULTON, Pennsylvania State University (752-34-34)


THURSDAY, 7:30 P.M.

Session on Topological Groups, Lie Groups, French Room
7:30-7:40 (417) Analytic H-spaces and alternative algebras. Dr. A. A. SAGLE*, University of Hawaii at Hilo and Dr. J. P. HOLMES, Auburn University (752-22-1)

7:45-7:55 (418) Continuous homomorphic images of compact 0-dimensional semigroups. Dr. AUGUST LAU, North Texas State University (752-22-3)

8:00-8:10 (419) Dense subgroups of Lie groups. II. Dr. DAVID ZERLING, Philadelphia College of Textiles and Science (752-22-5)

8:15-8:25 (420) SL2-orbits in flag domains. Professor EDUARDO CATTANI and Professor AROLDO KAPLAN*, University of Massachusetts, Amherst (752-22-6)

8:30-8:40 (421) Spectra of discrete uniform subgroups of semisimple Lie groups. Preliminary report. Dr. S. CHEN, University of Florida (752-22-9)

THURSDAY, 7:30 P.M.

Session on Nonassociative Rings and Algebras, Lancaster Room C
7:30-7:40 (422) Some radical properties of Jordan matrix rings. Preliminary report. Professor MICHAEL RICH, Temple University (752-17-2)

7:45-7:55 (423) Alternators of a right alternative algebra. Professor IRVIN R. HENTZEL, Iowa State University (752-17-3)

8:00-8:10 (424) Derivation alternator rings with idempotent. Preliminary report. Professor IRVIN HENTZEL and Professor HARRY SMITH*, Iowa State University (752-17-4)

8:15-8:25 (425) Right nilpotent generalized alternative rings. Professor DAVID J. POKRASS, Emory University (752-17-5)

8:30-8:40 (426) The classification of algebras. Preliminary report. Dr. DAVID T. PRICE, Wheaton College, Illinois (752-17-6)

8:45-8:55 (427) Lie algebra of class D. BYOUNG-SONG CHWE, University of Alabama-Tuscaloosa (752-17-7)
Orders in simple Lie algebras of Chevalley type. Professor JAMES F. HURLEY, University of Connecticut (752-17-8)

On the subclasses of antiflexible rings. Preliminary report. Professor HASAN A. CELIK, California State Polytechnic University (752-17-9)

THURSDAY, 7:30 P. M.

Session on Functional Analysis III, York Room

7:30-7:40 (430) Gelfand representation of tensor products of Banach modules. Professor J. W. KITCHEN, Duke University and Professor D. A. ROBBINS*, Trinity College (752-46-1)

7:45-7:55 (431) A representation for normed vector lattices. Preliminary report. Professor WILLIAM A. FELDMAN* and Professor JAMES F. PORTER, University of Arkansas (752-46-30)

8:00-8:10 (432) Relative uniform convergence for $\phi$-algebras. Preliminary report. Professor JAMES F. PORTER* and Professor WILLIAM A. FELDMAN, University of Arkansas (752-46-31)

8:15-8:25 (433) A characterization of inner product spaces. Preliminary report. Dr. ROY RAKESTRAW, Wheaton College (752-46-6)

8:30-8:40 (434) On supremum and infimums of function seminorms. Dr. SZE-KAI TSUI, Oakland University (752-46-34)

8:45-8:55 (435) A note on compact operators which attain their norm. Preliminary report. Dr. JOHN M. BAKER, Western Carolina University (752-46-35)

9:00-9:10 (436) Linear isometries of a certain metric space. Preliminary report. R. J. FLEMING and J. E. JAMISON*, Memphis State University (752-46-36)

9:15-9:25 (437) Commutants of triangular matrix operators. Preliminary report. Professor RICHARD J. FLEMIN* and Professor JAMES E. JAMISON, Memphis State University (752-46-37)

9:30-9:40 (438) $M$-ideals in function algebras. Dr. ROGER R. SMITH, Texas A & M University (752-46-38)

THURSDAY, 7:30 P. M.

Session on Probability Theory and Stochastic Processes II, Italian Room

7:30-7:40 (439) Distribution estimates of barrier-crossing probabilities of the Yeh-Wiener process. Professor CHULL PARK*, Miami University and Professor DAVID L. SKOUG, University of Nebraska (752-60-24)

7:45-7:55 (440) Fixed-points of random set-valued maps. Dr. GARY F. ANDRUS, University of Prince Edward Island and Dr. TOGO NISHIURA*, Wayne State University (752-60-25)

8:00-8:10 (441) Dependence with fixed marginals from a density viewpoint. Preliminary report. Mr. JOSHUA P. SEEGER, Temple University (752-60-26)

8:15-8:25 (442) Results concerning the characteristic operators associated with a class of pieced-together Markov processes. Preliminary report. Mr. KYLE SIEGRIST, Georgia Institute of Technology (752-60-27)

8:30-8:40 (443) Concomitants of extreme order statistics. Professor JANOS GALAMBOS, Temple University (752-60-28)

8:45-8:55 (444) Infinite dimensional moment problems, Preliminary report. Dr. RICHARD M. PHELPS, Sperry Systems Management (752-60-30) (Introduced by Kevin T. Phelps)

9:00-9:10 (445) On the behavior of characteristic functions. Professor STEPHEN JAMES WOLFE, University of Delaware (752-60-31)

THURSDAY, 7:30 P. M.

Session on Associative Rings and Algebras II, Austrian Room

7:30-7:40 (446) Stable equivalence and rings whose modules are a direct sum of finitely generated modules. Dr. HARLAN HULLINGER, University of Kansas (752-16-24)

7:45-7:55 (447) On projective modules. Preliminary report. JAMES WHITEHEAD, Kent State University (752-16-25)

8:00-8:10 (448) Determining homomorphisms to skew fields. Preliminary report. PETER MALCOLMSON, Wayne State University (752-16-26)

8:15-8:25 (449) Hereditary module-finite algebras. Professor JOHN FUELBERTH, University of Northern Colorado, Professor ELLEN KIRKMAN, and Professor JAMES KUZMANOVICH*, Wake Forest University (752-16-27)

8:45–8:55 (451) Polynomial rings of weakly primitive rings. Professor JULIUS ZELMANOWITZ, University of California, Santa Barbara (752–16–29)

THURSDAY, 7:30 P. M.

Special Session on Ill-posed Problems for Partial Differential and Integrodifferential Equations II, Lancaster Room E


8:00–8:20 (453) Ill-posed problems for integral equations with emphasis on the Radon equation. Professor M. ZUHAIR NASHED, University of Delaware (752–45–3)

8:30–8:50 (454) On a remarkable system of nonlinear partial differential equations describing cylindrical plasma collapse. Professor A. CARASSO, University of New Mexico (752–35–17)

9:00–9:20 (455) Development of momentary singularities in solutions of the Korteweg-deVries equation. Professor AMY C. MURRAY, Rutgers University (752–35–16)

9:30–9:50 (456) Solution techniques for first kind Abel integral equations under inequality constraints. Professor R. GORENFLO, Freie Universität Berlin, Berlin, West Germany (752–45–6) (Introduced by Professor Frederick Bloom)

10:00–10:20 (457) Bounds for solutions to a class of damped integrodifferential equations in Hilbert space with applications to the theory of nonconducting material dielectrics. Professor FREDERICK BLOOM, University of South Carolina, Columbia (752–45–5)

THURSDAY, 7:30 P. M.

Special Session on Ramsey Theory and its Ramifications III, Lancaster Room D

7:30–7:50 (458) Simple combinatorial problems without reasonable solutions, Professor J. PARIS, Manchester University, England and Professor L. HARRINGTON*, University of California, Berkeley (752–05–52)

8:00–8:20 (459) A quantitative version of the infinite Ramsey theorem, Preliminary report, Professor PAUL ERDÖS, University of Colorado and Professor FRED GALVIN*, University of Kansas (752–05–6)

8:30–8:50 (460) Partitions and sums and products of integers. II. Professor NEIL HINDMAN, California State University, Los Angeles (752–05–3)

9:00–9:20 (461) Some recent results in the study of the partition calculus. Preliminary report, Dr. JEAN A. LARSON, University of Florida (752–04–2)

9:30–9:50 (462) Partition theorems and ideals. Preliminary report. Professor ALAN D. TAYLOR, Union College (752–05–42)

THURSDAY, 8:00 P. M.

Special Session on History of Mathematics II, Lancaster Rooms A & B

8:00–8:20 (463) Blacks in mathematics. Professor KENNETH R. MANNING, Massachusetts Institute of Technology (752–01–6) (Introduced by Professor Uta C. Merzbach)

8:30–8:50 (464) The development of the theory of \( \theta \)-functions: Unpublished work of Riemann. Professor H. L. RESNIKOFF, University of California, Irvine (752–01–7)

9:00–9:20 (465) An American in Göttingen 1926–1927: Letters from Kline to R. L. Moore. Professor LUCILLE WHYBURN, University of Texas (752–01–5)

9:30–9:50 (466) On the early history of general topology. C. E. AULL, Virginia Polytechnic Institute and State University (752–01–2)

10:00–10:30 (467) Discussion session

FRIDAY, 1:00 P. M.

Colloquium Lectures, Lecture III, Regency Ballroom

(468) Algebraic K-theory. Professor HYMAN BASS, Columbia University

FRIDAY, 2:15 P. M.

Session on Measure and Integration, English Room

2:15–2:25 (469) Convolution iterates of Banach algebra valued measures. Professor G. D. FAULKNER and Professor J. E. HUNEYCU TT, Jr.*, North Carolina State University (752–28–1)

2:30–2:40 (470) Extending the product of two regular Borel measures. Professor ROY A JOHN-SON, Washington State University (752–28–2)

3:00–3:10 (472) On a theorem of Nikodym on vector-valued measures. Dr. R. ANANTHARAMAN, State University of New York, College at Old Westbury (752–28–4)

3:15–3:25 (473) The weak integral convergence theorems and countable additivity. Mr. WILLIAM V. SMITH and Professor DON H. TUCKER*, University of Utah (752–28–5)

3:30–3:40 (474) Convergence in measure of integrands. Preliminary report. Mr. WILLIAM V. SMITH, University of Utah (752–28–6)

3:45–3:55 (475) On integral representation of members of ba(S,Σ)*. Dr. MICHAEL KEISLER, Arkansas Tech University (752–28–7)

4:00–4:10 (476) A decomposition theorem for vector measures on a Boolean algebra. Professor WILLIAM H. GRAVES, University of North Carolina (752–28–8)

4:15–4:25 (477) N-functions and integration by substitution. Professor GERALD S. GOODMAN, University of Florence, Italy (752–26–5)

4:30–4:40 (478) On the zeros of the derivatives of balanced trigonometric polynomials. Preliminary report. CARL PRATHER, Northwestern University (77T–B188)

4:45–4:55 (479) Harmonic mapping in two and three dimensions. Professor C. WAYNE MASTIN, Mississippi State University (752–31–1)

5:00–5:10 (480) The growth of plane harmonic functions on asymptotic paths. II. Professor K. F. BARTH*, Syracuse University and Dr. D. A. BRANNAN, Queen Elizabeth College, London (752–31–2)

FRIDAY, 2:15 P. M.

Session on Commutative Rings and Algebras, Italian Room

2:15–2:25 (481) On R-homomorphisms of power series rings. Professor ROBERT GILMER, Florida State University and Professor MATTHEW O’MALLEY*, University of Houston (752–13–1)

2:30–2:40 (482) Commutative rings with homomorphic mth power functions. Professor JOHN O. KILTINEN, Northern Michigan University (752–13–2)


3:00–3:10 (484) Complete intersections and Gorenstein ideals. Professor E. L. GREEN, Virginia Polytechnic Institute and State University (752–13–5)

3:15–3:25 (485) The spectrum of two-dimensional Noetherian rings. Professor SYLVIA WIEGAND, University of Nebraska (752–13–7)

3:30–3:40 (486) On a duality of modules over valuation rings. Professor LASZLO FUCHS, Tulane University (752–13–8)

3:45–3:55 (487) Differential criteria for flatness. Dr. SARAH GLAZ, Rutgers University (752–13–9) (Introduced by Professor W. V. Vasconcelos)

4:00–4:10 (488) Weierstrass families of ideals in commutative rings. Professor JOHN W. PETRO*, Western Michigan University and Professor M. EDWARD PETTIT, Jr., Augusta College (752–13–10)

4:15–4:25 (489) Regular elements in one-dimensional rings. Preliminary report. Professor JAMES E. CARRIG*, George Mason University and Professor WOLMER VASCONCELOS, Rutgers University (752–13–11)

4:30–4:40 (490) The number of generators of a colength N ideal in a power series ring. Mr. DAVID BERMAN, University of Texas (752–13–12) (Introduced by Professor Anthony Iarrobino)

4:45–4:55 (491) Arithmetical semigroup rings. Dr. LEO CHOIJNARD, Dr. BONNIE HARDY, and Dr. THOMAS SHORES*, University of Nebraska (752–13–13)

5:00–5:10 (492) A connection between Amitsur and group cohomology for some non-Galois extensions, Preliminary report. Mr. MARK TRZASKA, Northwestern University (752–13–14)

5:15–5:25 (493) Factoring ideals into semiprime ideals, Professor N. H. VAUGHAN*, North Texas State University and Professor R. W. YEAGY, Stephen F. Austin State University (752–13–15)

5:30–5:40 (494) Unique factorization of matrices and Towber rings, Preliminary report. Professor D. R. ESTES and Professor J. R. MATLEVIC*, University of Southern California (752–13–16)
5:45- 5:55 (495) Pseudo-hereditary rings of continuous functions. Mr. MARVIN N. COHEN and Professor J. D. McKNIGHT, Jr.*, University of Miami (752-13-17)

FRIDAY, 2:15 P. M.

Session on Combinatorics II, French Room

2:15- 2:25 (496) Postage stamp problem. Dr. WILLIAM T. SPEARS and Dr. BARBARA JEFF-COTT*, University of Waterloo (752-05-5)

2:30- 2:40 (497) Extremal values of the interval number of a graph. Preliminary report. Dr. JERROLD GRIGGS*, California Institute of Technology and Mr. DOUGLAS B. WEST, Massachusetts Institute of Technology (752-05-26)


3:00- 3:10 (499) A forbidden subgraph characterization of Robert's inequality for boxicity. Preliminary report. Dr. WILLIAM T. TROTTER, Jr., University of South Carolina (752-05-28)

3:15- 3:25 (500) Uniformly deep families and cyclic designs. WAYNE M. DYMAZEC* and D. P. ROSELLE, Virginia Polytechnic Institute and State University (752-05-29)

3:30- 3:40 (501) Connectivity and traceability. Preliminary report. Dr. JAMES KINNEY, George C. Wallace Community College and Dr. CHARLES C. ALEXANDER*, University of Mississippi (752-05-31)


4:00- 4:10 (503) Biplanes with k = 6 and the (16,11) extended Hamming code. Professor E. F. ASSMUS, Jr., Lehigh University and Professor CHESTER J. SALWACH*, Lafayette College (752-05-33)

4:15- 4:25 (504) Combinatorics, Preliminary report. Mr. ROBERT CALDERBANK, California Institute of Technology (752-05-34) (Introduced by Professor Marshall Hall, Jr.)

4:30- 4:40 (505) GRR's for simple groups. Mr. C. GODSIL, Syracuse University (752-05-37)

4:45- 4:55 (506) On domination and independent domination numbers of a graph. Professor RENU LASKAR* and Mr. ROBERT B. ALLAN, Clemson University (752-05-38)

5:00- 5:10 (507) On the domatic number of a graph and the domination number of its complement. Mr. ROBERT B. ALLAN* and Professor RENU LASKAR, Clemson University (752-05-39)

5:15- 5:25 (508) Enumeration of d-nary trees. Preliminary report. Professor FRANK W. OWENS, Ball State University (752-05-41)

5:30- 5:40 (509) Skew chain orders and sets of rectangles. Preliminary report. Mr. DOUGLAS B. WEST* and Professor DANIEL J. KLEITMAN, Massachusetts Institute of Technology (752-05-44)

FRIDAY, 2:15 P. M.

Session on Operator Theory I, Austrian Room

2:15- 2:25 (510) Translation semigroups. Preliminary report. Professor RALPH GELLAR, North Carolina State University (752-47-1)

2:30- 2:40 (511) Characterization of Hilbert space operators with unitary cross sections. DON DECKARD and L. FIALKOW*, Western Michigan University (752-47-2)

2:45- 2:55 (512) A characterization of doubly stochastic operators induced by measure-preserving transformations. Professor RAY C. SHIFLETT, California State University, Fullerton (752-47-4)

3:00- 3:10 (513) Lifting the commutant of an analytic Toeplitz operator. Dr. CARL C. COWEN, University of Illinois (752-47-5)

3:15- 3:25 (514) A von Neumann algebra is determined by its logic of projections. Professor EDWARD AZOFF*, and Professor DAVID EDWARDS, University of Georgia (752-47-7)

3:30- 3:40 (515) The spectrum of a Toeplitz operator with a multiplicatively periodic symbol. M. B. ABRAHAMSE, University of Virginia (752-47-8)

3:45- 3:55 (516) The canonical form of a scalar operator in a Banach space. Professor G. D. FAULKNER* and Professor JAMES E. HUNNEYCUTT, Jr., North Carolina State University (752-47-9)

4:00- 4:10 (517) Fixed point theorems in locally convex spaces. Professor TROY L. HICKS, University of Missouri–Rolla (752-47-14)
4:15- 4:25 (518) Spectral mapping theorem for essential spectra. Professor KIRTI K. OBERAI, Queen's University (752-47-15)

4:30- 4:40 (519) Cyclic vectors and seminormal operators. Professor KEVIN F. CLANCEY, University of Georgia and Professor DONALD D. ROGERS*, U. S. Naval Academy (752-47-16)

4:45- 4:55 (520) A decomposition theorem for Riesz elements in a C*-algebra. Professor DAVID LEGG, Indiana University-Purdue University at Fort Wayne (752-47-17)

5:00- 5:10 (521) Symmetry principles for homogeneous and nonhomogeneous equations, Preliminary report. Ms. KATHERINE YERION, Gonzaga University (752-47-18)

5:15- 5:25 (522) On the dense operators in a KH-module. Preliminary report. Dr. NAZANIN AZARNIA, Miami University (752-47-21)

5:30- 5:40 (523) Contraction operator lemma from the study of wide-sense Markov processes. Dr. MILTON ROSENBERG, University of Montana (752-47-22)

5:45- 5:55 (524) Self-adjointness of certain second order differential operators on a Riemannian manifold. Preliminary report. Mr. PHILIP UNEW, Northwestern University (752-47-23)

FRIDAY, 2:15 P. M.

Session on Lattices, Ordered Algebraic Structures, Tudor Room
2:15- 2:25 (525) Lattice-ordered groups of order automorphisms of partially ordered sets. Ms. MAUREEN A. BARDWELL, Bowling Green State University (752-06-1)

2:30- 2:40 (526) Continuous cluster methods. Mr. M. F. JANOWITZ, University of Massachusetts, Amherst (752-06-2)

2:45- 2:55 (527) Path connected posets. RUSSELL BELDING, United States Naval Academy (752-06-3)

3:00- 3:10 (528) A variety of semilattices which generalize orthomodular lattices. Professor GARY M. HARDEGREE, University of Massachusetts, Amherst (752-06-4) (Introduced by David J. Foulis)

3:15- 3:25 (529) Traditional morphisms on manuals of operations. Preliminary report. Professor W. ROBERT COLLINS, Christopher Newport College (752-06-5)

3:30- 3:40 (530) Partitions into chains of a class of partially ordered sets. Dr. N. METROPOLIS, Los Alamos Scientific Laboratory, Los Alamos, New Mexico, Professor GIAN-CARLO ROTA, Massachusetts Institute of Technology, and Professor NEIL L. WHITE*, University of Florida (752-06-6)

3:45- 3:55 (531) $\omega$-automorphisms of the Hahn group. Preliminary report. Mr. TODD FEIL, Bowling Green State University (752-06-7)

4:00- 4:10 (532) An example of a manifold group. Preliminary report. Dr. JANET D. BLAIR, Ohio Northern University (752-06-8)

4:15- 4:25 (533) Free p-rings and the two countable chain conditions. Professor ALEXANDER ABIAN, Iowa State University (752-06-9)

4:30- 4:40 (534) On Conrad's partial order relation on semiprime rings and on semigroups. Professor R. RAPHAEL, Concordia University (752-06-10)

4:45- 4:55 (535) Homomorphisms of bounded lattices. Dr. M. E. ADAMS* and Dr. J. SICHLER, University of Manitoba (752-06-11)

5:00- 5:10 (536) A new family of 1-group varieties. Dr. J. E. H. SMITH, Boise State University (752-06-12)


5:30- 5:40 (538) Sheaf sectional representation of some partially ordered rings. Dr. CHEN-JYI SU, University of Miami (752-06-14)

5:45- 5:55 (539) $\ell$-convergence and $\ell$-Cauchy structures on lattice ordered groups. Preliminary report. Professor RICHARD N. BALL, Boise State University (752-06-15)

FRIDAY, 2:15 P. M.

Special Session on Number Theory I, Essex Room
2:15- 2:35 (540) Another minimal property associated with Markoff forms. Preliminary report. Professor HARVEY COHN, College of the City of New York (752-10-3)

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2:45- 3:05 (541) Signatures of units and congruences (mod 4) in certain real quadratic fields. Dr. J. C. Lagarias, Bell Laboratories, Murray Hill, New Jersey (752-12-2)

3:15- 3:35 (542) A study of the asymptotic oscillations of a certain sequence. Andrew M. Odlyzko, Bell Laboratories, Murray Hill, New Jersey (752-10-5)

3:45- 4:05 (543) Units arising from Kronecker's first limit formula. Professor Kenneth H. Rosen, University of Colorado (752-12-7)

4:15- 4:35 (544) Biotic Gauss sums and sixteenth power residue difference sets. Dr. Ronald J. Evans, University of California, San Diego (752-10-2)

FRIDAY, 2:15 P. M.

Special Session on Mathematics in Neurobiology, Lancaster Room D
2:15- 3:05 (545) Development of topological mappings between neural sheets with evolution-type differential equations. Preliminary report. Dr. Christoph Von Der Malsburg*, Max-Planck-Institut für biophysik, Göttingen, West Germany and Dr. David J. Willsshaw, NIMR, London, England (752-93-4) (Introduced by Professor Jack D. Cowan)

3:15- 4:05 (546) Flies and synapses: A Volterra-like functional representation of algorithms and interactions. T. Poggio, Max-Planck-Institut für biologische Kybernetik, Tübingen, West Germany (752-92-7)

4:15- 5:05 (547) A mathematical model for nonlinear waves of spreading cortical depression. Preliminary report, Professor Robert M. Miura, University of British Columbia (752-92-5)

FRIDAY, 2:15 P. M.

Special Session on Nonacademic Mathematical Research, Lancaster Rooms A & B
2:15- 2:35 (548) Elementary excitations in disordered alloys. Dr. Leonard J. Gray* and Dr. Theodore Kaplan, Oak Ridge National Laboratory, Oak Ridge, Tennessee (752-81-1)

2:45- 3:05 (549) Crystal growing. Dr. Lynn O. Wilson, Bell Laboratories, Murray Hill, New Jersey (752-92-1) (Introduced by Henry O. Pollak)


3:45- 4:05 (551) The problem of thermal instability of explosive materials. Paul B. Bailey, Sandia Laboratories, Albuquerque, New Mexico (752-80-2) (Introduced by Professor Robert J. Thompson)


4:45- 5:05 (553) Queueing models for data communications. Dr. Alan G. Konheim, IBM T. J. Watson Research Center (752-68-3)

FRIDAY, 2:15 P. M.

Special Session on Approximate Solutions of Random Equations I, Lancaster Room E
2:15- 2:35 (554) A survey of methods for solving random equations. Professor A. T. Bharucha-Reid, Wayne State University (752-60-14)

2:45- 3:05 (555) Successive approximation solutions of a class of random equations. Dr. George A. Bécus, University of Cincinnati (752-60-8)


3:45- 4:05 (557) Iterative and projectional methods for random operator equations. Preliminary report. Professor M. Zuhaire Nashed, University of Delaware (752-65-7)

4:15- 4:35 (558) Stochastic approximation and random nonlinear operator equations. R. Kannan, University of Texas at Arlington and University of Missouri, St. Louis (752-60-20)

4:45- 5:05 (559) Approximate solution of stochastic evolution equations. Preliminary report. Professor P. L. Chow, Wayne State University (752-60-7)

FRIDAY, 2:30 P. M.

Invited Address, Regency Ballroom

(560) Representations of finite groups of Lie type. Professor Charles W. Curtis, University of Oregon (752-20-17)
FRIDAY, 4:00 P. M.

Invited Address, Regency Ballroom

(561) Propagation, reflection, and diffraction of singularities of solutions to wave equations. Professor MICHAEL E. TAYLOR, Rice University (752-35-1)

FRIDAY, 8:15 P. M.

Session on Applied Mathematics, English Room
8:15–8:25 (562) Local interactions defined by boundary conditions. Preliminary report. E. C. SVENSEN, University of Illinois (752-81-2)

8:30–8:40 (563) A rigorous development of Feynman’s path integral and perturbation expansion. Preliminary report. Professor RICHARD H. BURKHART, University of North Carolina at Wilmington (752-81-3)

8:45–8:55 (564) Caratheodory’s temperature equations revisited. Preliminary report. Dr. TERRY D. LENKER, Central Michigan University (752-80-1)

9:00–9:10 (565) On the accuracy of Megerlin's method for phase change problems. Dr. ALAN SOLOMON, Union Carbide Corporation, Nuclear Division, Oak Ridge, Tennessee (752-80-3)

9:15–9:25 (566) Quantum potentials and the microcanonical entropy. Professor WILLIAM GREENBERG, Virginia Polytechnic Institute and State University (752-82-2)


9:45–9:55 (568) Specification of scalar curvature on $\mathbb{R}^3$ and solutions of the scalar constraint equation of general relativity. Dr. MURRAY CANTOR, University of Texas (752-83-2)

FRIDAY, 8:15 P. M.

Session on Combinatorics III, French Room
8:15–8:25 (569) On the classification of generalized quadrangles. Dr. CLIFTON E. EALY, Jr., Northern Michigan University (752-05-45)

8:30–8:40 (570) The orientable genus of a 2-connected graph. RICHARD W. DECKER*, HENRY H. GLOVER, and JOHN PHILIP HUNEKE, Ohio State University (752-05-46)

8:45–8:55 (571) Characteristic and pseudocharacteristic polynomials of permutation representations of groups. Professor KENNETH P. BOGART*, Dartmouth College and Professor JEAN GORDON, Williams College (752-05-47)

9:00–9:10 (572) Tournament regular representations of infinite groups. Preliminary report. Dr. DEREK A. HOLTON, University of Melbourne, Parkville, Australia and Professor MARK E. WATKINS*, Syracuse University (752-05-48)

9:15–9:25 (573) On graphs with equal edge connectivity and minimum degree. Professor DONALD L. GOLDSMITH* and Professor ARTHUR T. WHITE, Western Michigan University (752-05-50)


FRIDAY, 8:15 P. M.

Session on Operator Theory II, Austrian Room
8:15–8:25 (575) Two-point inequalities, the Hermite semigroup, and the Gauss–Weierstrass semigroup. FRED B. WEISSLER, University of Texas (752-47-24)

8:30–8:40 (576) Equations with unbounded nonlinearities. Preliminary report. R. KENT NAGLE and KAREN SINGKOFER*, University of South Florida (752-47-27)

8:45–8:55 (577) Strong convergence of the farthest closest point algorithm. Preliminary report. Professor RONALD E. BRUCK, University of Southern California (752-47-28)

9:00–9:10 (578) Purely contractive analytic functions and characteristic functions of noncontractions. Dr. BRIAN W. MCENNIS, University of Missouri–Rolla (752-47-29)

9:15–9:25 (579) An application of the minimum principle to the study of essential self-adjointness of Dirac operators. Preliminary report. Professor JOHN LANDGREN, Georgia Institute of Technology (752-47-30)

9:30–9:40 (580) A fixed point theorem and attractors. LUDVIK JANOS and J. L. SOLOMON*, Mississippi State University (752-47-32)

FRIDAY, 8:15 P. M.

Session on Special Functions, Italian Room
8:15–8:25 (581) From the zero–transform to combinatorial identities. Preliminary report. Dr. GLORIA OLIVE, University of Otago, New Zealand (752-33-1)

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8:30–8:40 (582) Computing elliptic integrals by duplication. Professor B. C. CARLSON, Ames Laboratory-ERDA, Iowa State University (752-33-3)

8:45–8:55 (583) Growth and complete sequences of generalized biaxially symmetric potentials. Professor ALLAN J. FRYANT, U. S. Naval Academy (752-33-4)

9:00–9:10 (584) An analytic proof of the product formula for Hahn polynomials. Preliminary report. Mr. THOMAS P. Laine, Northwestern University (752-33-5)

9:15–9:25 (585) Removable singularities for generalized Prüfer transformations. Dr. KURT CREITH, University of California, Davis (752-34-2)

SATURDAY, 1:00 P. M.

Colloquium Lectures. Lecture IV, Regency Ballroom,

(586) Algebraic K-theory. Professor HYMAN BASS, Columbia University

SATURDAY, 2:15 P. M.

Session on Semigroups, Tudor Room

2:15–2:25 (587) The structure of idempotent relations. Professor EUGENE M. NORRIS, University of South Carolina (752-20-2)

2:30–2:40 (588) On maximal inverse subsemigroups of \( T_X \). Dr. NATHANIEL KNOX, Morgan State University (752-20-5)

2:45–2:55 (589) Compactifications of finite-dimensional cones. Professor MICHAEL FRIEDBERG, University of Houston (752-20-7)

3:00–3:10 (590) The minimum group congruence on an unipotent semigroup. Preliminary report. Dr. C. C. EDWARDS, Indiana University-Purdue University at Fort Wayne (752-20-12)

3:15–3:25 (591) Inverse semigroups of homeomorphisms are hopfian. Preliminary report. Dr. BRIDGET BAIRD, University of Florida (752-29-19)

3:30–3:40 (592) The closure of the extended bicyclic semigroup. Preliminary report. Dr. ANNE ALEXANDER SELDEN* and Dr. JOHN SELDEN, Jr., Boğaziçi University, Istanbul, Turkey (752-20-20)

3:45–3:55 (593) Quasi ring-semigroups. Professor PATRICIA JONES and Professor STEVE LIGH*, University of Southwestern Louisiana (752-20-24)

4:00–4:10 (594) Green's relations for semigroups of continuous selfmaps. Preliminary report. Professor K. D. MAGILL, Jr., State University of New York at Buffalo (752-20-25)

4:15–4:25 (595) Elementary orthodox semigroups. Professor CARL EBERHART, University of Kentucky and Professor W. WILEY WILLIAMS*, University of Louisville (752-20-26)

4:30–4:40 (596) Quotient group topologies for topological semigroups. Professor JOHN C. HIGGINS, Brigham Young University (752-20-29)

4:45–4:55 (597) Local permutation polynomials in three variables over GF(p). Preliminary report. Professor GARY L. MULLEN, Pennsylvania State University, Sharon (752-12-3)

SATURDAY, 2:15 P. M.

Session on Abstract Harmonic Analysis, Lancaster Room C

2:15–2:25 (598) Multipliers of tensor products of CMA's and Radon–Nikodym derivatives, Dr. DAVID L. JOHNSON and Dr. CHARLES D. LAHR*, Dartmouth College (752-43-1)

2:30–2:40 (599) A kernel representation theorem. Dr. DAVID L. JOHNSON, Dartmouth College (752-43-2)

2:45–2:55 (600) Multipliers of Banach algebras and unbounded approximate identities. Mr. CHARLES A. JONES* and Dr. CHARLES D. LAHR, Dartmouth College (752-43-3)

3:00–3:10 (601) Entropy on discrete abelian groups. Professor JUSTIN R. PETERS, Iowa State University (752-43-4)

3:15–3:25 (602) A characterization of \( L^1(G) \) among its subalgebras. Preliminary report. Mr. SUNG-WOO SU, University of Connecticut (752-43-5)

3:30–3:40 (603) On existence of discontinuous almost periodic functions on topological groups. Preliminary report. Professor TER-JENQ HUANG, State University of New York, College at Cortland (752-43-6)

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<tr>
<th>Time</th>
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<th>Location</th>
<th>Speaker(s)</th>
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<td>4:00-4:10</td>
<td>605 Maximal functions and the harmonic structure of Jacobi series.</td>
<td>English Room</td>
<td>Dr. WILLIAM C. CONNETT and Dr. ALAN L. SCHWARTZ*</td>
<td>University of Missouri, St. Louis (752-43-8)</td>
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<td><strong>SATURDAY, 2:15 P. M.</strong></td>
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<td>2:15-2:25</td>
<td>606 The space-time distance function and singularities. Preliminary report.</td>
<td>English Room</td>
<td>Professor JOHN K. BEEM* and Professor PAUL E. EHRlich</td>
<td>University of Missouri, Columbia (752-53-1)</td>
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<td>2:30-2:40</td>
<td>607 An integral transform proof of the ham sandwich theorem.</td>
<td>English Room</td>
<td>Dr. JAMES V. PETERS, St. Bonaventure University</td>
<td>(752-53-2)</td>
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<td>2:45-2:55</td>
<td>608 Exterior recurrent forms. Preliminary report.</td>
<td>English Room</td>
<td>DILIP K. DATTA, University of Rhode Island</td>
<td>(752-53-3)</td>
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<td>3:00-3:10</td>
<td>609 Focal sets, taut embeddings and the cyclides of Dupin. Preliminary report.</td>
<td>English Room</td>
<td>Professor THOMAS E. CECIL, Vassar College and Professor PATRICK J. RYAN*</td>
<td>Indiana University at South Bend (752-53-4)</td>
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<td>3:30-3:40</td>
<td>611 On minimal submanifolds of the sphere. Preliminary report.</td>
<td>English Room</td>
<td>DORIS H. FISCHER-COLBRIE, University of California, Berkeley</td>
<td>(752-53-6)</td>
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<td>3:45-3:55</td>
<td>612 Almost contact submersions. Preliminary report.</td>
<td>English Room</td>
<td>Professor BILL WATSON, Case Western Reserve University</td>
<td>(752-53-7)</td>
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<td>4:00-4:10</td>
<td>613 An application of numerical analysis to differential geometry. Preliminary report.</td>
<td>English Room</td>
<td>Professor GERALD R. CHACHE, Howard University</td>
<td>(752-53-8)</td>
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<td><strong>SATURDAY, 2:15 P. M.</strong></td>
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<td>2:15-2:25</td>
<td>615 An asymptotic theorem for a class of positive linear convolution operators.</td>
<td>French Room</td>
<td>Professor S. EISENBERG*, University of Hartford and Professor B. WOOD, University of Arizona</td>
<td>(752-41-1)</td>
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<td>2:30-2:40</td>
<td>616 Quantitative estimates for L_p approximation with positive linear operators.</td>
<td>French Room</td>
<td>Professor BRUCE WOOD, University of Arizona and Professor JOHN SWETITS*, Old Dominion University</td>
<td>(752-41-2)</td>
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<td>2:45-2:55</td>
<td>617 Approximation of generalized random functions.</td>
<td>French Room</td>
<td>Professor GARY F. ANDRUS, University of Prince Edward Island</td>
<td>(752-41-3)</td>
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<td>3:00-3:10</td>
<td>618 Best L_p approximate solutions on nonlinear integro-differential equations.</td>
<td>French Room</td>
<td>Professor KENNETH L. WIGGINS, College of Charleston</td>
<td>(752-41-6)</td>
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<td>3:15-3:25</td>
<td>619 Korovkin sets and subspaces with property U.</td>
<td>French Room</td>
<td>Dr. RALPH L. JAMES, California State College</td>
<td>(752-41-7)</td>
</tr>
<tr>
<td>3:30-3:40</td>
<td>620 Bounds for incomplete polynomials.</td>
<td>French Room</td>
<td>Professor E. B. SAFF, University of South Florida and Professor R. S. VARGA*</td>
<td>Kent State University (752-41-9)</td>
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<td><strong>SATURDAY, 2:15 P. M.</strong></td>
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<td>2:15-2:25</td>
<td>621 Sex differences in the visualization of number.</td>
<td>Italian Room</td>
<td>Professor JAMES H. JORDAN, Washington State University</td>
<td>(752-96-1)</td>
</tr>
<tr>
<td>2:30-2:40</td>
<td>622 On the equality of the mixed partial derivatives.</td>
<td>Italian Room</td>
<td>Professor A. L. ANDREW, Dr. SIDNEY A. MORRIS, Professor P. J. STACEY, La Trobe University, Bundoora, Victoria, Australia, and Dr. GERARD P. PROTONASTRO*, Saint Peter's College</td>
<td>(752-98-1)</td>
</tr>
<tr>
<td>2:45-2:55</td>
<td>623 An elementary proof of the fundamental theorem of algebra.</td>
<td>Italian Room</td>
<td>Professor LAIRD E. TAYLOR, California State College</td>
<td>(752-98-2)</td>
</tr>
<tr>
<td>3:00-3:10</td>
<td>624 A consumer mathematics course for liberal arts students.</td>
<td>Italian Room</td>
<td>ROLAND E. LARSON, Pennsylvania State University, Erie</td>
<td>(752-98-3)</td>
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<td><strong>SATURDAY, 2:15 P. M.</strong></td>
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<tr>
<td>2:15-2:25</td>
<td>625 Asymptotic equilibria in a class of N-person symmetric games.</td>
<td>Austrian Room</td>
<td>Professor HARVEY R. DIAMOND, West Virginia University</td>
<td>(752-90-2) (Introduced by S. M. Rankin III)</td>
</tr>
</tbody>
</table>
2:30- 2:40 (626) Geometric games. Preliminary report. Dr. WILLIAM H. RUCKLE, Clemson University (752-90-3)

2:45- 2:55 (627) Generic necessary conditions for a family of nonlinear programming problems. JONATHAN SPINGARN*, Georgia Institute of Technology and R. TYRRELL ROCKAFELLAR, University of Washington (752-90-4)

3:00- 3:10 (628) Regular piecewise smooth economies. Preliminary report. Dr. HSUEH-CHENG CHENG, University of Southern California (752-90-5)

3:15- 3:25 (629) A generalized weight for linear codes and a Witt-MacWilliams theorem. Mr. DONALD Y. GOLDBERG, Dartmouth College (752-94-1) (Introduced by Professor Kenneth P. Bogart)

3:30- 3:40 (630) Information measures depending directly on events. Preliminary report. Dr. BRUCE EBANKS, Texas Tech University (752-94-2)

3:45- 3:55 (631) Two inertia theorems for Hessenberg matrices and their applications to stability problems. Dr. B. N. DATTA, Universidade Estadual de Campinas, Campinas, Brasil (752-93-1)

4:00- 4:10 (632) Controllability of linear neutral systems. Dr. HERMAN RIVERA, Universidad Mayor de San Andreas, La Paz, Bolivia and Professor C. E. LANGENHOP*, Southern Illinois University (752-93-2)

4:15- 4:25 (633) On a connexion between Thom's 'catastrophe theory' and Wiener's 'cybernetics'. Preliminary report. Dr. G. ARTHUR MIHRAM, P.G.Box 234, Haverford, Pennsylvania (752-90-1)

4:30- 4:40 (634) Applications of catastrophe theory to semantics: Ambiguity, jokes, and scientific revolutions. Professor JOHN PAULOS, Temple University (752-99-1)

SATURDAY, 2:15 P. M.

Special Session on Number Theory II, Essex Room

2:15- 2:35 (635) Gauss and Jacobsthal sums over GF(p^2). Professor BRUCE C. BERNDT*, University of Illinois and Professor RONALD J. EVANS, University of California, San Diego (752-10-4)

2:45- 3:05 (636) \( \delta \) -values for cusp forms on \( \Gamma(1) \). Preliminary report, Professor MARK SHEINGORN, City University of New York, Baruch College (752-10-17)

3:15- 3:35 (637) Eisenstein series for \( GL(n) \) and the Minkowski-Hlawka theorem. AUDREY A. TERRAS, University of California, San Diego (752-10-14)

3:45- 4:05 (638) A note on the first prime in an arithmetic progression. Professor CARL POMERANCE, University of Georgia (752-10-16)

4:15- 4:35 (639) Limit theorems for uniformly distributed p-adic sequences. Preliminary report. Dr. JEFFREY D. VAALER, University of Texas at Austin (752-10-8)

SATURDAY, 2:15 P. M.

Special Session on Approximate Solutions of Random Equations II, Lancaster Room E


2:45- 3:05 (641) Applications of the Liouville equation. Preliminary report, Professor WILLIAM E. BOYCE, Rensselaer Polytechnic Institute (752-60-22)

3:15- 3:35 (642) Obtaining approximate solutions of random equations by means of the method of moments. Preliminary report. Professor MELVIN D. LAX, California State University, Long Beach (752-60-21)

3:45- 4:05 (643) Approximate solution of fixed point equations for random operators. I. P. S. CHANDrasekharan* and A. T. BHIARUCHA-REID, Wayne State University (752-60-11)

4:15- 4:35 (644) Approximate solution of random nonlinear equations by random contractors. Professor W. J. PADGETT, University of South Carolina (752-60-5) (Introduced by R. M. Stephenson, Jr.)

SATURDAY, 3:30 P. M.

Invited Address, Regency Ballroom

(645) Images of manifolds under cell-like maps. Professor ROBERT D. EDWARDS, University of California, Los Angeles (752-57-10)

New Orleans, Louisiana

Frank T. Birtel
Associate Secretary
Contingent upon the receipt of financial support from a Federal agency, the twelfth annual symposium on Some Mathematical Questions in Biology will be held on February 14, 1978, in the Board Room of the Shoreham Americana Hotel in Washington, D.C. The Symposium is being held in conjunction with the annual meeting of the American Association for the Advancement of Science. It will be cosponsored by the American Mathematical Society, the Society for Industrial and Applied Mathematics, and Section A of the American Association for the Advancement of Science. Registration and local arrangements were announced in the November 1977 issue of Science.

The program has been arranged by the AMS-SIAM Committee on Mathematics in the Life Sciences, whose members are Hans J. Bremermann, Jack D. Cowan, Murray Gerstenhaber, Stuart Kauffman, Simon A. Levin (chairman), Robert M. May, George F. Oster, and Sol I. Rubinow.

**PROGRAM**

**FEBRUARY 14, 1978, 9:00 A.M.**

Chairman: George F. Oster, University of California, Berkeley, California

9:00 a.m. Generality and uniqueness in the history of life: an exploration using random models, STEPHEN J. GOULD, The Agassiz Museum of Comparative Zoology, Harvard University, Cambridge, Massachusetts

10:00 a.m. Mathematical optimization models and sociobiology, GEORGE F. OSTER, University of California, Berkeley, California

11:00 a.m. Towards a computational theory of musical perception, CHRISTOPHER LONGUET-HIGGINS, The University of Sussex, England

**FEBRUARY 14, 1978, 3:00 P.M.**

Chairman: Jack D. Cowan, University of Chicago, Illinois

3:00 p.m. Some stochastic problems in biology, JOSEPH B. KELLER, Courant Institute of Mathematical Sciences, New York, and Stanford University, Stanford, California

4:00 p.m. Immunological pattern formation, PETER H. RICHTER, Max-Planck-Institut für Biophysikalische Chemie, Göttingen, Federal Republic of Germany

5:00 p.m. Optimal strategies for an immune response, ALAN S. PERELSON, University of California, Los Alamos Scientific Laboratory, New Mexico

As of the date this program was prepared, abstracts for the contributed paper session had been accepted from the following: Richard A. Alo* and William Titus (BIO 78-4); Martha O. Bertman* and John R. Jungck (BIO 78-3); John A. Feroe (BIO 78-1); Robert N. Miller* and John Rinzel (BIO 78-2).

Simon A. Levin, Chairman, Organizing Committee
Twelfth Annual AMS-SIAM Symposium
on Some Mathematical Questions in Biology

*Ithaca, New York

* For papers with more than one author, an asterisk follows the name of the author who plans to present the paper at the meeting.
PRELIMINARY ANNOUNCEMENTS OF MEETINGS

753RD MEETING
Ohio State University
Columbus, Ohio
March 20–25, 1978

The seven hundred fifty-third meeting of the American Mathematical Society will be held at the Ohio State University, Columbus, Ohio, from Monday, March 20, through Saturday, March 25, 1978. The sessions of the meeting will be held in Hagerty Hall, Page Hall, and Sullivant Hall, all three of which are near the Faculty Club and the Ohio Union on the Ohio State Campus.

The period March 20–23 will be devoted to a symposium on Relations between Combinatorics and Other Parts of Mathematics. The topic of the symposium was selected by the 1976 Committee to Select Hour Speakers for Western Sectional Meetings, which consisted of Richard A. Askey, Paul T. Bateman (chairman), and Richard G. Swan. Further information is given below under the section SYMPOSIUM.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be four invited one-hour addresses. PETER J. HILTON of Battelle Memorial Institute and Case Western Reserve University, will speak at 11:00 a.m. on Friday, March 24; his subject is "Free maps and free homotopies." STEVEN OREY, University of Minnesota, will address the Society at 1:45 p.m. on Friday on the topic "Old and new questions on limit theorems for sums of independent random variables." JOSEPH LIPMAN, Purdue University, will give an hour talk at 11:00 a.m. on Saturday; his topic is "Some aspects of equisingularity on analytic varieties." PAUL J. SALLY, Jr., University of Chicago, will speak at 1:45 p.m. on Saturday; his topic is "The Fourier transform of orbital integrals."

By invitation of the same committee, there will be ten special sessions of selected twenty-minute papers on Friday and Saturday. EARL R. BERKSON is organizing a special session on Hardy spaces and related topics; the tentative list of speakers includes Sheldon Axler, Donald L. Burkholder, Sun-Yung Alice Chang, Joseph A. Cima, Peter L. Duren, Frank Forelli, Dattatraya J. Patil, Lee A. Rubel, Joel H. Shapiro, and Guido L. Weiss. WILLIAM J. DAVIS is organizing a special session on Banach spaces; the tentative list of speakers includes Dale E. Alspach, Z. Altschuler, Simon J. Bernau, Leonard E. Dor, Gerald A. Edgar, James N. Hagler, Nigel J. Kalton, Edward W. Odell, Jr., G. Schechtman, Thomas W. Sari, J. Serrin, and John E. Wolfe.

JOE W. FISHER is organizing a special session on Rings and modules; the tentative list of speakers includes Maurice Chacon, John H. Cozzens, Vlastimil Dlab, Robert Gordon, D. Handelman, Charles P. Lanski, Wallace S. Martindale III, Bernard R. McDonald, Donald S. Passman, Richard D. Rosco, and Sudarshan K. Sehgal.

GEZA FREUD is organizing a special session on Orthogonal polynomials; the tentative list of speakers includes Richard A. Askey, Willard Miller, Jr., Paul G. Nevai, and Joseph L. Ullman. (Professor Freud would welcome suggestions for further speakers.) J. S. HSIA is organizing a special session on Quadratic forms; the tentative list of speakers includes Ronald Brown, Andrew G. Earnest, Richard S. Elman, Dennis Estes, Larry J. Gerstein, Alexander J. Hahn, Donald G. James, O. Timothy O'Meara, Takashi Ono, Meinhard Peters, Arnold K. Pizer, Barth Pollak, Cari R. Siegmund, Alex Rosenberg, Adrian R. Wadsworth, and Roger P. Ware.

PHILIP KUTZKO is organizing a special session on The representation theory of p-adic linear groups; the tentative list of speakers includes Charles A. C. M. Azumaya, Radha G. Laha, Courtney H. Moen, Alexandre Nobs, Joseph S. Repka, Allan J. Silberger, Bham A. Srivastava, and Gregg J. Zuckerman.

HENRY B. LAUPF is organizing a special session on Analytic and algebraic singularities; the tentative list of speakers includes Robert M. Ephraim, Melvin Hochster, Pusparaj Kanungo, Philip D. Wagreich, and Stephen S. T. Yau. V. S. MANDREKAR is organizing a special session on Limit theorems and the geometry of Banach spaces; the tentative list of speakers includes Alejandro de Acosta, A. P. DeArnao, Naresh C. Jain, James D. Kuelbs, Louis Sucheston, Wojbor A. Woyczynski, and Joel Zinn.

SURINDER K. SEHGAL is organizing a special session on Group theory; the tentative list of speakers includes James C. Beidleman, Wilbur E. Deskins, Wolfgang P. Kappe, Luise-Charlottte Kappe, L. Patel, A. H. Rhetmulla, Michio Suzuki, and Pavel Winternitz.

HANS J. ZASSENHAUS is organizing a special session on Constructive number theory; the tentative list of speakers includes David A. Ford, J. K. S. McKoy, Andrew M. Odlyzko, M. Pohst, Hans J. Zassenhaus, and Horst G. Zimmer.

There will be sessions for contributed papers as needed. Abstracts should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive by the abstract deadline of January 17, 1978. Contributed papers on Combinatorics and related subjects will be scheduled during the period March 20–23. Authors of such papers are requested to send copies of their abstracts to D. K. Ray-Chaubhuri, Department of Mathematics, The Ohio State University, 231 West
18th Avenue, Columbus, Ohio 43210 as early as possible. Other contributed paper sessions will be scheduled on Friday and Saturday, March 24-25.

The Council of the Society will meet at 5:00 p.m. on Thursday, March 23, at the Holiday Inn, SYMPOSIUM

With the anticipated support of the Army Research Office, National Science Foundation, and Office of Naval Research, a symposium on Relations between Combinatorics and Other Parts of Mathematics will be held from March 20-23, 1978.

Mathematical problems of a combinatorial nature arise in various fields of application; namely, statistics, computer sciences, communication (coding theory), physics, graph theory, and so forth. In recent years there has been much progress in combinatorics; researchers have made deep studies and solved many outstanding classical problems. These new results in combinatorics are making impacts in the fields of application. The symposium will bring together internationally reputed experts in combinatorics and other areas of mathematics, facilitate communication and exchange of ideas, and stimulate research in both pure and applied combinatorics.

The Organizing Committee for the symposium, responsible for selecting the speakers and arranging the program, consists of D. K. Ray-Chaudhuri (chairman), Marshall Hall, Jr., Peter J. Hilton, Gian-Carlo Rota, W. T. Tutte, and Richard M. Wilson. The speakers, and the titles of their lectures, are: George E. Andrews (Pennsylvania State University), "Connection coefficient problems and partitions"; David W. Barnette (University of California, Davis), "Path problems and extremal problems for convex polytopes"; James E. Baumgartner (Dartmouth College), "Independence proofs and combinatorics"; Raj C. Bose (Colorado State University), "Combinatorial problems of experimental design"; F. Buekenhout (University of Brussels), "The geometry of diagrams"; Peter J. Cameron (Merton College, Oxford, England), "A combinatorial tool kit for permutation groups"; Joel E. Cohen (Rockefeller University), "The probability of an interval graph, and why it matters"; H. S. M. Coxeter (University of Toronto), "Trivalent graphs of large girth"; Charles F. Dunkl (University of Virginia and Georgia Institute of Technology), "Orthogonal functions on some permutation groups"; Paul Erdős (Hungarian Academy of Sciences and University of Colorado), "Combinatorial problems in geometry and number theory"; Dominique Foata (University of California, San Diego), "A combinatorial study of Meherl type identities for Hermite polynomials"; Adriano M. Garcia (University of California, San Diego), "Some applications of combinatorics to analysis"; Branko Grünbaum (University of Washington), "Patterns in geometry and combinatorics"; Alan J. Hoffman (IBM, T. J. Watson Research Center), "Linear programming and combinatorics"; Victor Klee (University of Washington), "The d-step conjecture and its relatives"; Joel Koplik (École Normale Supérieure), "The combinatorics of graphs in field theory"; Johan J. Seidel (Eindhoven University of Technology, Netherlands), "Spherical designs";

Neil J. A. Sloane (Bell Laboratories), "On the classification of geometric lattices"; Louis Solomon (University of Wisconsin, Madison), "Partially ordered sets with colors"; Alan P. Sprague (Ohio State University), "Incidence structures arising from projective spaces"; and Richard P. Stanley (Massachusetts Institute of Technology), "Combinatorics and invariant theory." Other lecturers, the title of whose talks are not yet available, are: John Conway (University of Cambridge, England), John P. Hunke (Ohio State University), R. M. Karp (University of California, Berkeley), and Saharon Shelah (University of Wisconsin, Madison).

REGISTRATION

The registration desk will be located in Sullivan Hall, and will be open Monday through Saturday from 8:00 a.m. until 4:00 p.m. The registration fees for the meeting and symposium are:

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<th>Meeting Only</th>
<th>Symposium Only</th>
<th>Meeting and Symposium</th>
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<tr>
<td>Nonmember</td>
<td>$5</td>
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<td>Member AMS</td>
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<tr>
<td>Student/Un-</td>
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ACCOMMODATIONS

Blocks of rooms have been reserved at the motels listed below, and will be held for participants until March 6. Individuals should make their own reservations directly with the motels, and should identify themselves as participants in the AMS meeting. Rates listed do not include the 7% sales tax. The Ohio Stater Inn is located just northeast of campus, 0.4 miles from the meeting area. The Holiday Inn is just northwest of the campus, 0.9 miles from the meeting area. Stouffer's University Inn is 2.4 miles northwest of the meeting area. All motels have parking facilities for guests.

OHIO STATER INN MOTOR HOTEL

(614) 294-5381
2060 N. High Street, Columbus, Ohio 43201
Single (double bed, 1 person) $14
Twin (twin beds, 2 people) 16
Double/Double (2 double beds, 2-4 people) 20

HOLIDAY INN

(614) 294-4848
328 W. Lane Avenue, Columbus, Ohio 43201
Single (1 person) $21
Double (2 people) 26
Extra person in room 4

STOUFFER'S UNIVERSITY INN

(614) 287-9291
3025 Olentangy River Road, Columbus, Ohio 43202
Single (double bed, 1 person) $24
Single (double bed, 2 people) 28

FOOD SERVICE AND ENTERTAINMENT

Since the meeting will take place during the spring quarter break, many university dining facilities will be closed. The meeting area is adjacent to High Street, on which numerous campus eating establishments are located. A list of recommended restaurants and open uni-
Participants staying at the Ohio State Inn or the Holiday Inn will be within 15 minutes walking distance of the meeting area. Visitor’s parking permits will be issued at the Ohio Union Parking Ramp near the meeting area for a $0.75 daily fee.

TRAVEL AND LOCAL INFORMATION

Columbus is located in central Ohio, midway between Cincinnati and Cleveland. The city is served by Greyhound and Continental bus lines, Amtrak, and the following airlines: Allegheny, American, Delta, Eastern, North Central, TWA, and United.

The average temperature in March is 40°F. Precipitation averages 3.5 inches during March.

Paul T. Bateman
Associate Secretary

Urbana, Illinois

754TH MEETING

Biltmore Hotel
New York, New York
March 28–31, 1978

The seven hundred fifty-fourth meeting of the American Mathematical Society will be held at the Biltmore Hotel, Madison Avenue at 43rd Street, New York City, on Thursday and Friday, March 30 and 31, 1978.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings, there will be five invited one-hour addresses and seven special sessions. Two invited addresses and a special session devoted to the theme "Functional analysis in mathematical physics" will be presented on Thursday, March 30. ARTHUR JAFFE, Harvard University, will speak on some aspect of the above theme; and BARRY SIMON, Princeton University, will lecture on "Spectral analysis of Schroedinger operators." JAMES GLIMM and ARTHUR WIGHTMAN are organizing the related special session on Applications of functional analysis to mathematical physics; the speakers include David Brydges, Bertram Kostant, Elliot Lieb, Joel Liebowitz, Vladimir Scheffer, Erhard Seiler, and Eugene Trubowitz. RONALD DOUGLAS, SUNY Center at Stony Brook, will lecture on "Operators on Hilbert space" and, in conjunction with this lecture, LEWIS COBURN is organizing a special session on Operator theory and several complex variables; the names of the speakers will be announced later. ALAN DURFEE, University of Washington and Columbia University, will give an invited address entitled "Topological algebraic singularities." To accompany this address, a special session is being organized by RICHARD RANDELL on Topology of varieties and singularities. OVED SHISHA, University of Delaware, will give an invited address on "Dominant and simple integrability: Advances in the theory and numerical analysis of improper integrals." HARRY MC LAUGHLIN is organizing a related special session on Approximation of functions and integrals; the list of speakers in this session includes Alex Babopoulos, Seymour Haber, Charles Osgood, Jack Roulier, and Arthur Sroou.

Three additional special sessions are being organized; the titles of these special sessions, and the names of the mathematicians arranging them, are as follows: Scattering theory, JEFFREY COOPER; Analytic and computational number theory, LOWELL SCHOENFELD; and Ergodic theory, CHOY-TAK TAAM.

There will be sessions for contributed ten-minute papers. Abstracts should be sent to the American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940, so as to arrive by the abstract deadline of January 17, 1978. If necessary, late papers will be accepted for presentation at the meeting, but it will not be possible to list these in the printed program.

SYMPOSIUM ON MATHEMATICAL PROBLEMS IN FRACTURE MECHANICS

With the anticipated support of the Energy Research and Development Administration and the National Science Foundation, a symposium on Mathematical Problems in Fracture Mechanics will be held on Tuesday and Wednesday, March 28 and 29, 1978. This topic was selected by the AMS–SIAM Committee on Applied Mathematics, whose members are Donald S. Cohen (chairman), D. J. Benney, Edward L. Reiss, Martin Schultz, David Siegmund, and Stephen Smale.

Fracture mechanics is a respected and long-established engineering discipline of obvious and undisputed importance, which has given rise to some of the most challenging mathematical problems in continuum mechanics. These fall into several categories, each possessing a distinctive feature which provides significant mathematical interest. Thus, brittle fracture and seismic-source theory lead to free boundary problems for the wave equation. The crack-tip region is usually treated as a singularity in brittle fracture theory, but may be treated on a microscale as an inner nonlinear, possibly plastic region leading to an associated problem of matching to the outer solution. The study of the initiation of fracture is essentially a bifurcation from a state of continuous deformation to a discontinuous state.

The Organizing Committee, comprised of Keiti Aki, Massachusetts Institute of Technology; Robert Burridge, Courant Institute of Mathematical Sciences (chairman); James K. Knowles, California Institute of Technology; and James R. Rice, Brown University, has organized the symposium into four sessions. The lecturers, and titles of their talks, are: J. D. Achenbach (Northwestern University), "Elastodynamic fracture mechanics"; Keiti Aki, "Evolution of quantitative

REGISTRATION

The registration desk will be located in the Vanderbilt Suite (N, O, P) on the first floor of the Biltmore Hotel. The desk will be open from 8:00 a.m. to 4:30 p.m. on Tuesday and Wednesday, from 8:30 a.m. to 4:30 p.m. on Thursday, and from 8:00 a.m. to 3:30 p.m. on Friday.

Registration fees for the meeting and symposium are as follows:

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<th>Fee Type</th>
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Meeting and Symposium fees include three meals and coffee breaks.

ACCOMMODATIONS

A block of rooms has been set aside for use by participants at the Biltmore Hotel. Persons planning to stay at the Biltmore should make their own room reservations with the hotel. A reservation form and listing of the room rates will be found on page A-218 of these Notices. The deadline for receipt of reservations is March 7, 1978.

TRAVEL

The Biltmore Hotel is located on Madison Avenue at 43rd Street, on the east side of New York City. Walkways from Grand Central Station are located under the hotel, and signs are posted directing persons to the hotel lobby.

Those arriving by bus at the Port Authority Bus Terminal may take the Independent Subway System, taxi, or bus to the hotel. There is shuttle bus service from LaGuardia, Kennedy, and Newark airports directly to Grand Central Station, and starters will direct passengers to the correct bus.

Persons arriving by car will find several parking garages in the area, in addition to the garage at the hotel. Parking service can be arranged through the hotel doorman. The present rate in the hotel garage is $10 for each 24-hour period, and there is an additional charge for extra pickup and delivery service if it is required. The parking fee is subject to New York City taxes.

MAIL ADDRESS

Registrants at the meeting may receive mail addressed to them in care of the American Mathematical Society, Biltmore Hotel, Madison Avenue at 43rd Street, New York, New York 10017.

Raymond G. Ayoub
Associate Secretary
University Park, Pennsylvania

755TH MEETING

University of Houston
Houston, Texas
April 7–8, 1978

The seven hundred fifty-fifth meeting of the American Mathematical Society will be held at the University of Houston, Houston, Texas, from 11:00 a.m., Friday, April 7, 1978 to 6:00 p.m., Saturday, April 8, 1978. The meeting will be held in conjunction with the 1978 spring meeting of the Association for Symbolic Logic, which will be held from 1:00 p.m., April 6, to 11:00 a.m., April 7. All sessions will be held in the Continuing Education Center of the University of Houston.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings, there will be four invited one-hour addresses as follows: FRED GALVIN, University of Kansas, 11:00 a.m., Friday; ANDY R. MAGID, University of Oklahoma, 1:45 p.m., Friday; TODD DUPONT, University of Chicago, 11:00 a.m., Saturday; and JOHN P. HEMPEL, Rice University, 1:45 p.m., Saturday.

By invitation of the same committee, there will be five special sessions of selected twenty-minute papers. The topics of these special sessions and the names of the mathematicians organizing them are as follows: Approximation theory, CHARLES K. CHUI, Texas A & M University; Topological and generalized manifolds, WILLIAM T. EATON, University of Texas at Austin; Commutative algebra, ROBERT M. FOSSUM, University of Illinois at Urbana-Champaign; Finite element approximations for partial differential equations, PETER PERCELL, University of Houston, and MARY PANETT WHEELER, Rice University; Recursion theoretic aspects of model theory and algebra, ANIL NERODE, Cornell University. Most of the papers to be presented at these special sessions will be by invitation; however, anyone contributing an abstract for the meeting who feels that his or her paper would be particularly appropriate for one of these special sessions should indicate this.
clearly in the abstract and submit it by January 25, 1978, three weeks before the normal deadline for contributed papers.

There will be sessions for contributed ten-minute papers as needed. Abstracts should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive by the abstract deadline of February 15, 1978.

The registration fee for those attending the meeting will be $5 for nonmembers, $3 for members of either the AMS or the ASL, and $1 for students and unemployed persons.

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**756th Meeting**

**San Francisco State University**

**San Francisco, California**

**April 14–15, 1978**

The seven hundred fifty-sixth meeting of the American Mathematical Society will be held at San Francisco State University, San Francisco, California, on Friday and Saturday, April 14 and 15, 1978. The meeting will be held jointly with the Northern California Section of the Society for Industrial and Applied Mathematics.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, there will be two invited one-hour addresses. These addresses will be given by WILLIAM CASSELMAN, University of British Columbia, and ALEXANDRE JOEL CHORIN, University of California, Berkeley.

By invitation of the same committee, there will be at least three special sessions of invited papers. VINCENT J. BRUNO of San Francisco State University is organizing a special session on Nonlinear analysis; ROLF JELTSCH of Stanford University is organizing a special session on Numerical solutions of initial value problems of ODE’s; and GARO KIREMIDJIAN of Stanford University is organizing a special session on Partial differential equations and geometry on complex manifolds. Most of the papers to be presented at special sessions will be by invitation, however, anyone contributing an abstract for the meeting who wishes the paper to be considered for a special session should indicate this clearly on the abstract and submit it by January 25, 1978, three weeks prior to the normal deadline for contributed papers.

There will be sessions for contributed ten-minute papers. Abstracts should be sent to the American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, so as to arrive by the deadline of February 15, 1978. Late papers will be accepted for presentation at the meeting, but will not be listed in the printed program of the meeting.

Information about registration, travel, and accommodations will appear in the February issue of the Notices and the final program of the meeting will appear in the April Notices.

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**1978 Summer Seminar**

**Applied Mathematics**

**University of Utah**

**Salt Lake City, Utah**

**June 12–23, 1978**

The tenth AMS Summer Seminar in Applied Mathematics will be held at the University of Utah, Salt Lake City, Utah, June 12–23, 1978. The seminar will be sponsored jointly by the American Mathematical Society and the Society for Industrial and Applied Mathematics, and it is anticipated that the seminar will be supported by a grant from a Federal agency. The topic, "Nonlinear oscillations in biology," was selected by the AMS-SIAM Committee on Applied Mathematics whose members include D. J. Benney, Donald S. Cohen (chairman), Edward L. Reiss, Martin Schultz, David Siegmund, and Stephen Smale. The members of the Organizing Committee are W. S. Childress, Courant Institute of Mathematical Sciences; Donald S. Cohen, California Institute of Technology; F. C. Hoppensteadt, University of Utah (chairman); Paul Waltman, University of Iowa; and A. S. Winfree, Purdue University.

The theory and methods of nonlinear oscillations are widely used in the analysis of oscillatory phenomena observed in the life sciences. The seminar will focus on these applications, Oscillatory problems of current and potential interest in population biology, physiology, and developmental biology will be introduced and discussed. The theory and applications of nonlinear oscillations methods will be developed.
from an introduction to descriptions of recent results and methods.

Individuals may apply for admission to the seminar. Application blanks for admission and/or financial assistance can be obtained from the Meeting Arrangements Department, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940. The application deadline is March 15, 1978. An applicant will be asked to indicate his or her scientific background and interest, and should have completed at least one year of graduate school. A graduate student's application must be accompanied by a letter from his or her faculty advisor concerning his or her ability and promise. Those who wish to apply for a grant-in-aid should so indicate; however, funds available to the seminar are limited and so individuals who can obtain support from other sources should do so.

1978  
Williams College  
Williamstown, Massachusetts  
July 10–28, 1978

Harmonic Analysis in Euclidean Spaces and Related Topics

The twenty-sixth Summer Research Institute sponsored by the American Mathematical Society will be devoted to the above subject, and will take place at Williams College, Williamstown, Massachusetts, for a period of three weeks from July 10–28, 1978. The Organizing Committee consists of Donald Burkholder, Alberto P. Calderon, Y. Meyer, Elias M. Stein, Stephen Wainger and Guido Weiss (co-chairmen), and Antoni Zygmund. It is anticipated that the institute will be supported by a grant from the National Science Foundation.

A main objective of the institute will be to discuss the considerable development in harmonic analysis in Euclidean spaces, and related areas, that has taken place during the last decade. Much of this development was due to interactions between harmonic analysts and research workers in the fields of probability, partial differential equations, and several complex variables. It is hoped that this conference will serve as a vehicle to bring experts in all these fields together and, hence, will be a stimulus for new progress in this area. Emphasis will be placed primarily on instruction at a very high level with seminars, study sessions, and lectures by distinguished mathematicians in these fields.

Housing accommodations for participants will be provided primarily in a complex of residence houses on the campus, and meals will be served in the central dining hall of the complex. A brochure of information containing details about the program, room and board rates, physical facilities, registration fees, etc., will be available in the early spring. The brochure will be sent to all persons invited to attend the institute.

Funds for participant support will be limited, and it is hoped that a number of participants who wish to attend will obtain their own sources for support. Those wishing to take part in the institute and/or be considered for financial assistance should write to Dr. William J. LeVeque, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940, before March 10, 1978. The committee will consider these requests, and applicants will be informed of its decision early in the spring.

NEWS ITEMS AND ANNOUNCEMENTS

INTERNATIONAL CONGRESS OF MATHEMATICIANS
Helsinki, Finland
August 15–23, 1978

The first announcement of the Congress appeared on pp. 274–275 of the August 1977 issue of these Notices. Those who requested the second announcement should have received this information from Helsinki by now. Those who have not should be aware that there are eight categories of hotel rates available for the Congress, ranging from $12.37 to $48.26 single, and $17.32 to $64.35 double. In addition, group accommodation rates are available from $2.72 to $8.66 per person per night. Information on travel arrangements for the Congress appears on pp. A-181 to A-187 of this issue of these Notices.

VISITING MATHEMATICIANS AVAILABLE FOR LECTURES

Among the visiting Fulbright-Hays scholars from abroad each year, there are many who welcome opportunities to participate in programs and meet colleagues on campuses other than those where they are officially located. Each year the Council for International Exchange of Scholars prepares a list of these scholars who are available for occasional lectures. The list, available in November, contains details on specializations and interests and faculty associates at host institutions. A copy of the list is available from Mrs. Mary Ernst, Faculty Fulbright Advisor, CIES, 11 Dupont Circle, N.W., Washington, D. C. 20036.
ORGANIZERS AND TOPICS OF SPECIAL SESSIONS

Abstracts of contributed papers to be considered for possible inclusion in special sessions should be submitted to the Providence office by the deadlines given below. The latest abstract form has a section for indicating special sessions. Lacking this, be sure your abstract form is clearly marked “For consideration for special session (title of special session).” Those papers not selected for special sessions will automatically be considered for regular sessions unless the author gives specific instructions to the contrary.

Columbus, Ohio, March 1978

Deadline: December 27

Earl R. Berkson, Hardy spaces and related topics
William J. Davis, Banach spaces
Joe W. Fisher, Rings and modules
Geza Freud, Orthogonal polynomials
J. S. Hsia, Quadratic forms
Philip Kutzko, The representation theory of p-adic linear groups
Henry B. Laufer, Analytic and algebraic singularities
V. S. Mandrekar, Limit theorems and the geometry of Banach spaces
Surinder K. Sehgal, Group theory
Hans J. Zassenhaus, Constructive number theory

New York, New York, March 1978

Deadline: December 27

Lewis Coburn, Operator theory and several complex variables
Jeffrey Cooper, Scattering theory
James Glimm and Arthur Wightman, Applications of functional analysis to mathematical physics
Harry McLaughlin, Approximation of functions and integrals
Richard Randell, Topology of varieties and singularities
Lowell Schoenfeld, Analytic and computational number theory
Choy-Tak Taam, Ergodic theory

Houston, Texas, April 1978

Deadline: January 25

Charles K. Chui, Approximation theory
William T. Eaton, Topological and generalized manifolds
Robert M. Fossum, Commutative algebra
Anil Nerode, Recursion theoretic aspects of moral theory and algebra
Peter Percell and Mary Fanett Wheeler, Finite element approximations for partial differential equations

San Francisco, California, April 1978

Deadline: January 25

Vincent J. Bruno, Nonlinear analysis
Rolf Jeltsch, Numerical solutions of initial value problems of ODE’s
Garo Kiremidjian, Partial differential equations and geometry on complex manifolds

Eugene, Oregon, June 1978

Deadline: March 28

Kenneth A. Ross, Commutative harmonic analysis

INVITED SPEAKERS AT AMS MEETINGS

This section of these Notices lists regularly the individuals who have agreed to address the Society at the times and places listed below. For some future meetings, the lists of speakers are incomplete.

Columbus, Ohio, March 1978

Peter J. Hilton
Joseph Lipman

New York, New York, March 1978

Ronald Douglas
Alan Durfee
Arthur Jaffe

Houston, Texas, April 1978

Todd Dupont
Fred Galvin
John P. Hempel
Andy R. Magid

San Francisco, California, April 1978

William Casselman
Alexandre J. Chorin
DOCTORATES CONFERRED IN 1976–1977 — Supplementary List

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during the interval July 1, 1976–June 30, 1977. This is a supplement to the list printed in the October 1977 issue of these Noticeis. The numbers appearing in parentheses after each university indicate the following: the first number is the number of additional degrees listed for that institution; the next seven numbers are the number of degrees in the categories of 1. Pure Mathematics (i.e., algebra, number theory, geometry, topology, analysis, functional analysis, logic, or probability), 2. Statistics, 3. Operations Research, 4. Computer Science, 5. Applied Mathematics, 6. Mathematics Education, 7. Other. Each entry contains the dissertation title. There are seven universities listed with a total of 36 individual names. This total, added to the previous list, includes doctorates from 153 universities with a total of 1,008 individual names; 235 departments granting doctorates.

CALIFORNIA

STANFORD UNIVERSITY (25;0, 0, 9, 16, 0, 0, 0)

Department of Computer Science
Bolles, Robert Coy
Verification vision within a programmable assembly system
Brown, Mark Robbin
The analysis of a practical and nearly optimal priority queue
Cartwright, Robert, Jr.
A practical formal semantics and verification system for TYPED LISP
Finkel, Raphael Ari
A manipulator language for automation
Ginsparg, Jerrold M.
A parser for English and its application in an automatic programming system
Lenat, Douglas Bruce
The automated mathematician
Lewis, John Gregg
Algorithms for sparse matrix eigenvalue problems
Paxton, William Hamilton
A framework for speech understanding
Plaisted, David Alan
Theorem proving and semantic trees
Reiser, John Fredrick
Analysis of additive random number generators
Simpson, Irv
On some subrecursive reducibilities
Simon, Charles
Metaprogramming—a software production technique
Sprull, Robert Fletcher
Strategy construction using a synthesis of heuristic and decision-theoretic methods
Stirratt, Edward Preble
The control and analysis of online file systems
Sweet, Richard Eric
Empirical estimates of program entropy
Taylor, Russell Highsmith
The synthesis of manipulator control programming from task-level specifications

Department of Operations Research
Avila, David M.
Some polyhedral cones related to metric spaces
Durrett, Richard
Some general conditioned limit theorems
Gheorghe, Mark Stewart
A special class of large quadratic programs
Hatoyama, Yukio
Markov maintenance models with repair
Kao, Peichuen
Conditioned limit theorems in queueing theory
Kochman, Gary
Decomposition in integer programming
Orkenyi, Peter
Optimal control of intermittent M/G/1 queueing and inventory systems
Pang, Jong-Shi
Least element complementarity theory
Shipley, Robert Scott
Stochastic capacity expansion models

ILLINOIS

ILLINOIS INSTITUTE OF TECHNOLOGY (1;0, 0, 0, 1, 0, 0, 0)

Department of Computer Science
Buroff, Steven J.
ALGOL 68 implementation techniques

LOUISIANA

LOUISIANA STATE UNIVERSITY (2;2, 0, 0, 0, 0, 0, 0)

Department of Mathematics
Gills, Johnny
Totally factorable operators
Smith, Brent
Helson sets are uniform Fatou-Zygmund sets

NEW YORK

STATE UNIVERSITY OF NEW YORK AT BUFFALO (5;3, 0, 0, 0, 2, 0, 0)

Department of Mathematics
Chan, Wai Kit
A rate of approach to the steady state for solutions of 2nd order hyperbolic equations with time dependent variable coefficients
Jakel, David W.
Contributions to the theory of 0-dimensional Hausdorff spaces
Marvin, Bertram K.
Closest lattice points to a plane in \( \mathbb{R}^3 \)
Wolfe, Robert J.
The existence of periodic solutions in a detailed kinetic model of the Belousov-Zhabotinskii reaction
Yang, Shi Nine
Biholomorphic mappings and \( H^p \) spaces on bounded symmetric domains

TENNESSEE

GEORGE PEABODY COLLEGE (1;0, 0, 0, 0, 0, 0, 1)

Department of Mathematics
Lowman, Bertha Pauline
F. Lynwood Wren: His contributions to the field of mathematics education

TEXAS

UNIVERSITY OF TEXAS AT AUSTIN (1;0, 0, 0, 1, 0, 0, 0)

Department of Computer Science
McGlothin, John Joseph, Jr.
An investigation of the projection requirements for programming language objects

VIRGINIA

VIRGINIA POLYTECHNIC INSTITUTE AND STATE UNIVERSITY (1;0, 1, 0, 0, 0, 0, 0)

Department of Statistics
Morris, Max D.
Designs for the detection of inadequacy in factional models
1976 MEMBERSHIP SURVEY

A questionnaire, reproduced below, appeared on the AMS member dues notice distributed in the summer and fall of 1976. It was accompanied by an explanatory paragraph (also reproduced below) which appeared on the reverse of the notice. At the summer meeting in Seattle, August 1977, Professor Lida K. Barrett, a member of the Data Subcommittee of the Society's Committee on Employment and Educational Policy (CEEP), summarized results from the membership survey at an open session sponsored by CEEP on Monday afternoon, August 15, in Meany Hall on the campus of the University of Washington. In her summary, Professor Barrett presented several graphs depicting aspects of the Society's membership. Annotated versions of the items selected by Professor Barrett appear in this report.

In the summer of 1976 there were approximately 16,400 members of the Society, including 13,250 with addresses in the U.S. Usable responses were received from 7,061 members who indicated that they were citizens, permanent residents, or temporarily in the United States. The Society has no records of the number of members who are U.S. citizens residing abroad, but some of those responding were in this category. The response rate cannot be estimated accurately, but was apparently on the order of 50 percent. Readers should be warned that although this is a rather large sample, it cannot be considered a random one since its members were self-selected. An independent check, based on a random sample of male and female first names of AMS members listed in the 1976-1977 Combined Membership List, gave no evidence of a biased response rate according to sex in the Membership Survey. On the other hand, there may be an age related bias in the data as regards the youngest and oldest age groups. Among the 16,400 AMS members who received the questionnaire are included 694 members emeriti and 3,773 institutional nominees. The response rates among these groups (25% and 11%, respectively) were significantly lower than for the membership as a whole.

The various Tables and Figures below give a profile of the respondents according to sex, ethnic group, employment status, age, etc. A striking feature of the profile is the very uneven age distributions shown in Figures 1 and 2. About 45% of the respondents are between ages 31 and 40, and this figure rises to 50% if only those holding doctorates are included. Most AMS members included in this profile hold academic positions. Over 60% of those employed full-time are in universities; nearly 20% in four-year colleges; less than 5% in two-year colleges or high schools; and approximately 15% in government, non-profit organizations, business or industry. About 2.4% of respondents reported that they were unemployed, and 2.7% reported that they had part-time employment (Table 2). About 11% of all respondents are women, and 9% of those holding doctorates are women. About 80% of respondents hold doctorates. The median age at receipt of doctorate was slightly over 27, and 66% obtained their doctorate before age 30 (Figure 6). Among unemployed respondents, the age at receipt of doctorate tended to be slightly higher (Figure 7).

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**MEMBERSHIP SURVEY (OPTIONAL), FOR MEMBERS IN OR FROM THE UNITED STATES.** See reverse for instructions.

<table>
<thead>
<tr>
<th>1</th>
<th>Year of birth: (check one)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>U.S. Citizen or Permanent Resident</td>
</tr>
<tr>
<td>3</td>
<td>Year highest degree conferred:</td>
</tr>
<tr>
<td>4</td>
<td>Primary Employment Status, Fall 1976</td>
</tr>
<tr>
<td>5</td>
<td>If Employed (check one)</td>
</tr>
<tr>
<td>6</td>
<td>With Tenure</td>
</tr>
<tr>
<td>---</td>
<td>-------------</td>
</tr>
<tr>
<td>Student</td>
<td>Retired or not seeking employment</td>
</tr>
<tr>
<td>Two-year college or High School</td>
<td>Federal government</td>
</tr>
<tr>
<td>Four-year college</td>
<td>Other government or nonprofit</td>
</tr>
<tr>
<td>University</td>
<td>Business/Industry</td>
</tr>
</tbody>
</table>

---

The Society's Committee on Employment and Educational Policy and its Data Subcommittee have prepared the brief questionnaire which appears on the face of this dues notice. The purpose of the questionnaire is to make it possible to construct profiles of the membership as a part of the continuing efforts of these Committees to survey the mathematical profession and to monitor the job market in the United States. If you are willing to participate, you are requested to supply six digits (first indicating the year of your birth, and then the year in which you received your highest degree). You are also requested to check the one box which most accurately describes your situation for each of the six groups listed (five groups if you are not employed for fall 1976). The Committee on Employment and Educational Policy expects to report the results of this survey in the NOTICE if the response rate is high enough to make it possible to draw any conclusions from the data. The Committee also plans to survey mathematicians with nonacademic positions, and expects to use information gathered in the present survey to identify them. The information requested in group 4 is sought so that the Society may respond to requests received from department heads and other members in the United States for data required in connection with affirmative action programs. The categories listed are: government agencies.

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57
In Table 1, the ethnic group of each of the respondents is shown. Table 2 shows the breakdown of the employment status of the respondents.

<table>
<thead>
<tr>
<th>Ethnic Groups</th>
<th>Women</th>
<th>Men</th>
<th>Unspecified</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>American Indian, Eskimo</td>
<td>2</td>
<td>11</td>
<td>-</td>
<td>13</td>
</tr>
<tr>
<td>Oriental, Pacific Islander</td>
<td>34</td>
<td>219</td>
<td>12</td>
<td>265</td>
</tr>
<tr>
<td>Black, Afro-American</td>
<td>11</td>
<td>59</td>
<td>3</td>
<td>73</td>
</tr>
<tr>
<td>Mexican American, Puerto Rican</td>
<td>1</td>
<td>27</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>None of the above</td>
<td>683</td>
<td>4,768</td>
<td>202</td>
<td>5,653</td>
</tr>
<tr>
<td>Decline to state</td>
<td>32</td>
<td>525</td>
<td>42</td>
<td>599</td>
</tr>
<tr>
<td>Blank</td>
<td>54</td>
<td>340</td>
<td>34</td>
<td>428</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>817</strong></td>
<td><strong>5,949</strong></td>
<td><strong>295</strong></td>
<td><strong>7,061</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Employment Status</th>
<th>Women</th>
<th>Men</th>
<th>Unspecified</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student</td>
<td>96</td>
<td>385</td>
<td>8</td>
<td>489</td>
</tr>
<tr>
<td>Retired or not seeking employment</td>
<td>52</td>
<td>163</td>
<td>5</td>
<td>220</td>
</tr>
<tr>
<td>Unemployed</td>
<td>43</td>
<td>119</td>
<td>7</td>
<td>169</td>
</tr>
<tr>
<td>Employed part-time</td>
<td>62</td>
<td>122</td>
<td>5</td>
<td>189</td>
</tr>
<tr>
<td>Employed full-time</td>
<td>544</td>
<td>5,127</td>
<td>268</td>
<td>5,939</td>
</tr>
<tr>
<td>Not yet known</td>
<td>17</td>
<td>27</td>
<td>2</td>
<td>46</td>
</tr>
<tr>
<td>Employment status unspecified</td>
<td>3</td>
<td>6</td>
<td>-</td>
<td>9</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>817</strong></td>
<td><strong>5,949</strong></td>
<td><strong>295</strong></td>
<td><strong>7,061</strong></td>
</tr>
</tbody>
</table>

Of the 7,061 respondents, 817 indicated that they were female, 5,949 that they were males and 295 did not specify either sex. Of the 7,061, 5,576 reported that their highest degree was a doctorate, 1,051 a masters, 251 a bachelors, and 219 did not specify any degree.

The total number of individuals included in each of the following graphs varies considerably. This is the case because some forms could not be used in certain graphs for lack of essential information. Thus only 6,877 of the 7,061 respondents (those who gave the year of their birth) are included in Figure 1. Figure 2 includes only 5,431, the number who both reported that they had doctorates and gave the year of their birth. It should also be noted that in those graphs where women are singled out (shaded areas) for comparison with the total, the unshaded area does not represent men only, but includes all those who checked "male" as well as all those who did not specify either sex on the questionnaire. Finally, on each of the graphs, numbers over the boxes indicate the number of individuals in each category; numbers in parentheses indicate those included in the shaded region of the graph.
Figure 1 shows the age distribution of those respondents who supplied the necessary information. Figure 2 shows the age distribution of the subset of all those who reported they have a doctorate. In these graphs, the shaded areas represent the women included in each category. It is interesting to note that over half of the respondents are under 40 years of age and nearly 80 percent have doctoral degrees. Also, the proportion of women in both graphs is similar.

Figure 4 gives the age distribution of members who reported that they were unemployed; the shaded area indicates the subset who reported that they were women. 5.3 percent of the women reported they were unemployed and 2.0 percent of the men reported they were unemployed. Also, while eleven percent of all respondents were female, twenty-five percent of the respondents who reported that they were unemployed were female. Note that the peak in unemployment coincides with the age of the shift from untenured to tenured academic employment.

189 respondents reported part-time employment; this included 107 holding doctoral degrees, 33 of whom were women.

Figure 3 provides age distributions for members who reported that they were employed full-time by (a) universities (63 percent of those employed full-time), (b) four-year colleges (18 percent), (c) governments, non-profit organizations, business or industry (16 percent). (Shaded areas in the first two parts indicate the fraction of the totals without tenure.) The remaining 3 percent are those who reported they were employed full-time in a two-year college or high school.

Figure 5. Years since receipt of doctoral degree for members employed full-time by (a) universities, (b) four-year colleges, and (c) non-academic organizations
In Figure 5, doctorate holding respondents who provided the year the degree was conferred are distributed according to the number of years elapsed since receipt of the doctorate for the three groups employed full-time by (a) universities, (b) four-year colleges, (c) governments, non-profit organizations, business, and industry. Shaded areas indicate those without tenure at academic institutions.

**FIGURE 6. Age at receipt of doctorate**

Figures 6 and 7 give distribution of the age at which the doctorate was received for four groups: all respondents who provided the necessary information, the subset of unemployed members, and the women in each of these classes (shaded regions).

**FIGURE 7. Age at receipt of doctorate: Unemployed members**

The Committee on Employment and Educational Policy has devised a questionnaire which it plans to distribute in the fall of 1977 to mathematical scientists with doctoral degrees employed in the nonacademic sector. Information requested includes salaries, professional experience, and type of work. The Committee hopes to report its findings in these Notices.

The data used to compile each of the preceding tables and graphs were taken from a set of computer printed tables.

**PERSONAL ITEMS**

LEE R. ABRAMSON of the Energy Research and Development Administration has been appointed statistical adviser in the Applied Statistics Branch of the Nuclear Regulatory Commission, Washington, D. C.

FRED C. ANDREWS has returned to the University of Oregon from University College, Cork, Ireland. He is now starting a second three-year appointment as head of the Department of Mathematics.

KENNETH ASTBURY of Ohio State University has been appointed to an assistant professorship at Wayne State University.

GREGORY T. BACHELIS of Wayne State University has been appointed to a visiting associate professorship at the University of Texas at Austin.

HYMAN BASS of Columbia University has been appointed to a visiting professorship at the University of California, Berkeley, for the 1978 winter and spring quarters.

ROBERT CAHN of the University of Miami has been appointed to a visiting associate professorship at the University of California, Berkeley, for the 1978 spring quarter.

MURRAY CANTOR of Duke University has been appointed to an assistant professorship at the University of Texas at Austin.

JIA-ARNG CHAO of the University of Texas, Austin, has been appointed to a visiting assistant professorship at New Mexico State University.

RUTH M. CHARNEY of Princeton University has been appointed a lecturer at the University of California, Berkeley.

CAMERON M. GORDON of the Institute for Advanced Study has been appointed to an assistant professorship at the University of Texas at Austin.

WILLIAM F. HILL of East Texas State University retired July 6, 1977.

JOSEPH HINTZ of Gonzaga University has been appointed to an assistant professorship at the University of Akron.

NEAL HULKOWER of Northwestern University has been appointed to an assistant professorship at Vanderbilt University.

J. H. B. KEMPERMAN of the University of Rochester has been appointed to a visiting professorship at the University of Texas at Austin.

PRESTON KOHN has been appointed to a visiting assistant professorship at Villanova University.

DAVID I. LIEBERMAN of Brandeis University has been appointed deputy director of the
Communications Research Division of the Institute for Defense Analyses.

PETER MALCOLMSON of the University of California, Berkeley, has been appointed to an assistant professorship at Wayne State University.

R. DANIEL MAULDIN of the University of Florida has been appointed to an associate professorship at North Texas State University.

PAUL MUHLY of the University of Iowa has been appointed to a visiting professorship at the University of California, Berkeley.

JOHN W. NEUBERGER of Emory University has been appointed to a professorship at North Texas State University.

R. DANIEL MAULDIN of the University of Florida has been appointed to an associate professorship at North Texas State University.

PAUL MUHLY of the University of Iowa has been appointed to a visiting professorship at the University of California, Berkeley.

EVERETT PITCHER of Lehigh University has announced his intention to retire at the end of the current academic year.

SAMUEL M. RANKIN of Murray State University has been appointed to an assistant professorship at West Virginia University.

KENNETH RIBET has been appointed to an associate professorship at the University of California, Berkeley.

ERVIN RODIN of Washington University, St. Louis, will be cited for his contributions in physics and engineering in teaching at the University's Founders Day Banquet.

DONALD ROSE of Harvard University has been appointed to the Chairmanship of the Department of Systems and Information Science and to a professorship at Vanderbilt University.

MILTON ROSENBERG has been appointed to a visiting associate professorship at the University of Montana.

STANLEY SAWYER of Yeshiva University has been appointed to a visiting professorship at the University of Washington.

STEVEN SHREVE of the University of Delaware has been appointed to a visiting assistant professorship at the University of California, Berkeley.

I. M. SINGER of the Massachusetts Institute of Technology has been appointed to a visiting professorship at the University of California, Berkeley.

KONDAGUNTA SUNDARESAN of the University of Wyoming has been appointed to a professorship at Cleveland State University.

GARRETT S. SYLVESTER of Rockefeller University has been appointed to a visiting assistant professorship at New Mexico State University.

ANGUS E. TAYLOR has retired as Chancellor of the University of California, Santa Cruz. He is now Professor Emeritus with an office at the University of California, Berkeley.

CLARENCE WILKERSON of the University of Pennsylvania has been appointed to an associate professorship at Wayne State University.

E. DON WILLIAMS has been appointed R. C. Fish Professor of Mathematics at Austin College.

WOJIBOR A. WOYCZYNKSI of Northwestern University and Wrocław University has been appointed to a professorship at Cleveland State University.

WOLFGANG ZILLER of the University of Bonn has been appointed a lecturer at the University of California, Berkeley.

**PROMOTIONS**

To Director, Center for Pure and Applied Mathematics, University of California, Berkeley: CALVIN C. MOORE; Communications Research Division, Institute for Defense Analyses: LEE P. NEUWIRTH.

To Chairman, Department of Mathematics, University of Texas at Austin: JAMES W. DANIEL.

To Vice Chairman, Faculty Appointments of the Department of Mathematics, University of California, Berkeley: P. EMERY THOMAS.

To Professor, East Texas State University: BILL D. ANDERSON; University of Texas at Austin: WILLIAM T. EATON; Wayne State University: PAO-LIU CHOW.

To Associate Professor, University of Akron: DAVID C. BUCHTHAL; University of California, Berkeley: LEO A. HARRINGTON, ARTHUR OGUS; Villanova University: ROBERT DEVOS.

To Assistant Professor, New Mexico State University: ROGER H. HUNTER.

**INSTRUCTORSHIPS**

New Mexico State University: MICHAEL H. FRESE, SHERRY D. FRESE.

University of Texas at Austin: FREDRIC D. ANCEL, VASILIOS ALEXIADES, RONALD M. DOTZEL, JOSEPH O. HOWELL, JOHN R. MYERS.

**DEATHS**

Dr. JAMES T. DAY of Pennsylvania State University died on May 20, 1977, at the age of 44. He was a member of the Society for 15 years. Professor FLOYD S. HARPER of Lincoln, Nebraska, died on March 11, 1977, at the age of 81. He was a member of the Society for 52 years.

Professor Emeritus WILLIAM T. REID of the University of Oklahoma and a visiting scholar at the University of Texas at Austin died on October 14, 1977, at the age of 70. He was a member of the Society for 49 years.

Professor CLARENCE A. E. SWANSON of the University of Southern Colorado died on June 29, 1977, at the age of 59. He was a member of the Society for 10 years.

Professor Emeritus FRANK M. WEIDA of George Washington University died on September 13, 1977, at the age of 85. He was a member of the Society for 58 years.
SPECIAL MEETINGS INFORMATION CENTER

THIS CENTER maintains a file on prospective symposia, colloquia, institutes, seminars, special years, and meetings of other associations, helping the organizers become aware of possible conflicts in subject matter, dates, or geographical area.

AN ANNOUNCEMENT will be published in these (NOTICES) if it contains a call for papers, place, date, subject (when applicable), and speakers; a second full announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in each issue until it has been held and a reference will be given in parentheses to the volume and page of the issue in which the complete information appeared.

IN GENERAL, SMIC announcements of meetings held in the United States and Canada carry only date, title of meeting, place of meeting, speakers (or sometimes general statement on the program), deadline dates for abstracts or contributed papers, and name of person to write for further information. Meetings held outside the North American area may carry slightly more detailed information. Information on the pre-­preliminary planning will be stored in the files, and will be available to anyone desiring information on prospective conferences. All communications on special meetings should be sent to the Special Meetings Information Center of the American Mathematical Society.

DEADLINES are the same as the deadlines for abstracts. They appear on the inside front cover of each issue.


barg, Pasadena; J. Mikusinski, Katowice.

9-15. Funktionenräume und Funktionenalgebren
   **Chairmen:** H. Bauer, Erlangen; H. König, Saarbrücken.

16-22. Arbeitsgemeinschaft Algebra: Schiefkörper
   **Chairmen:** P. M. Cohn, London; G. Michler, Essen.

23-29. Endliche Gruppen und Permutationsgruppen
   **Chairmen:** Ch. Hering, Tübingen; B. Huppert, Mainz.

30-August 5. Allgemeine Ungleichungen

6-12. Konstruktive Verfahren in der komplexen Analysis
   **Chairmen:** D. Gaier, Giessen; P. Henrici, Zurich.

13-19. Himmelsmechanik
   **Chairmen:** E. Steifel, Zürich; V. Szebehely, Austin.

20-26. Formale Sprachen
   **Chairmen:** R. V. Book, Santa Barbara; G. Hotz, Saarbrücken; H. Walter, Darmstadt.

27-September 2. Komplexe Analysis
   **Chairmen:** H. Grauert, Göttingen; R. Remmert, Münster; K. Stein, München.

September

3-9. Methoden der algebraischen Geometrie in der algebraischen Topologie
   **Chairmen:** E. Friedlander, Evanston; G. Harder, Wuppertal.

10-16. Topologie
   **Chairmen:** T. tom Dieck, Göttingen; K. Lamotke, Köln; C. B. Thomas, London.

17-23. Geometrie
   **Chairman:** K. Leichtweiss, Stuttgart.

24-30. Funktionalanalysis
   **Chairmen:** K.-D. Bierstedt, Paderborn; H. König, Saarbrücken; G. Köthe, Frankfurt; H. H. Schaefer, Tübingen.

October

1-7. Numerische Integration
   **Chairman:** G. Hämerlin, München.

8-14. Arbeitsgemeinschaft Geyer-Harder

   **Chairmen:** R. Henn, Karlsruhe; H. P. Künzi, Zürich; H. Schubert, Düsseldorf.

22-28. Grundlagen der Geometrie
   **Chairman:** R. Lingenberg, Karlsruhe.

29-November 4. Zahlentheorie (insbesondere elementare und analytische Zahlentheorie)
   **Chairmen:** H. E. Richter, Ulm; W. Schwarz, Frankfurt; E. Wirsing, Ulm.

November

12-18. Fortbildungslehrgang für Studienräte

19-25. Konstruktive Methoden bei nichtlinearen Randwertaufgaben und nichtlinearen Schwingungen
   **Chairmen:** J. Albrecht, Clausthal-Zellerfeld; L. Collatz, Hamburg; K. Kirchgässner, Stuttgart.

26-December 2. Multivariante Statistical Analysis
   **Chairmen:** D. Plachky, Münster; S. Schuch, Dortmund.

December

3-9. Operator-Ungleichungen
   **Chairmen:** N. Bazley, Köln; J. Schröder, Köln.


JANUARY 1978


30-February 2. Ninth Southeastern Conference on Combinatorics, Graph Theory and Computing, Florida Atlantic University, Boca Raton, Florida.

**Purpose:** The purpose of the program is to bring together mathematicians interested in Combinatorics, Graph Theory and Computing and their interactions and to promote better understanding of modern applied mathematics, combinatorics and computer science, emphasizing the interplay of combinatorics with other areas.

**Invited Speakers:** J. Adrian Bondy (University of Waterloo); Paul Erdős (Hungarian Academy of Sciences); Gian-Carlo Rota (Massachusetts Institute of Technology); Hugh C. Williams (University of Manitoba).

**Call for Papers:** There will be sessions for fifteen-minute contributed papers. Deadline for abstracts is January 18, 1978.

**Information:** Frederick Hoffman, Director, Ninth Southeastern Conference, Department of Mathematics, Florida Atlantic University, Boca Raton, Florida 33432.

FEBRUARY 1978

5-8. Applied Mathematics Conference, Broadbeach Hotel, Gold Coast, Queensland, Australia. (24, p. 282)

**Invited Speakers:** The principal invited speaker will be J. Keller of the Courant Institute, New York. There will also be a series of invited addresses in the nature of review or general interest lectures.

12-17. National Meeting of the American Association for the Advancement of Science, Sheraton-Park Hotel, Washington, D.C.

**Program:** The meeting will focus not only on advances in research in various areas of science, engineering, and medicine, but also on science and technology uses and their social and ethical implications. Topics will include sociobiology; advances in knowledge using recombinant DNA techniques; space exploration and the search for extraterrestrial intelligence; recent extremes in weather and long term climatic patterns; stress; aging; the high school class of 1972; nuclear and alternative energy systems; whistle-blowing and scientific responsibility; desertification; minorities, women, and the handicapped in science; and others. Ten public lectures and the AAAS Science Film Festival also will be included in the meeting program. (A preliminary program for the meeting appeared in the 4 November 1977 issue of *Science*.)

**Information:** AAAS Meetings Office. 1776 Massachusetts Avenue, N.W., Washington, D.C. 20036.


MARCH 1978

6-9. International Conference on Developing Mathematics in Third World Countries, University of Khartoum, Khartoum, Sudan. (24, p. 282)

15-17. Eleventh Annual Simulation Symposium, Tampa, Florida. (24, p. 221)


**Program:** Cameron Gordon, Kenneth Kumen, Sibe Mardesic, and Richard Schori have accepted invitations to speak. Other invited speakers will be announced.

**Call for Papers:** Talks of 15-20 minutes are desired. Title and abstract should be sent to John W. Green at the address below.

**Information:** Leonard R. Rubin, Department of Mathematics, University of Oklahoma, 601 Elm Avenue, Room 423, Norman, Oklahoma 73019.


APRIL 1978

4-7. Second International Conference on Combinatorial Mathematics, Barbizon Plaza Hotel, New York, New York. (24, p. 375; 440)


6-14. Eighth Conference on Stochastic Processes and their Applications, Australasian National University, Canberra ACT, Australia.


14-17. Working Conference on Codes for Boundary-value Problems for O.D.E.s, University of Houston, Houston, Texas.


JUNE 1978


8. Joint Regional Mathematics Conference, Oregon State University, Corvallis, Oregon.

JULY 1978


Information: E. N. Cramer, Southern California Edison Company, P. O. Box 800, 2244 Walnut Grove Avenue, Rosemead, California 91770.

14-17. Working Conference on Codes for Boundary-value Problems for O.D.E.s, University of Houston, Houston, Texas.


LETTERS TO THE EDITOR

For early consideration Letters to the Editor should be mailed directly to the Executive Director, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940.

Editor, the Notices

I was shocked when I saw the author index of MR 53. This change means a considerable loss of information. To give an example: as it happened I was interested in the review of a paper by Davis or Davies whose initials I did not remember, but I remembered the approximate title of the paper. Please have a look at the index and you know what I mean. My suggestion: please
1) produce an author index old style of MR 53; 2) keep the old style.

Of course, the new style is cheaper I am sure.

G. J. Rieger

Editor, the Notices

There is no doubt that the reduction in the author index for Volume 53 of the Mathematical Reviews, referred to in Professor Rieger's letter, does involve some loss of information and some inconvenience. The question is whether this inconvenience is so great that it negates the saving of roughly $20,000 in production costs.

I estimate that this index contains about 8,600 authors for 15,600 reviews. Since all of the authors of joint papers are listed in the index, there are fewer than two reviews per author, on the average. It strikes me that looking up two entries for the average author is not a vast increase in effort.

He is, of course, correct that in the case of prolific authors and forgotten names the work is increased. However, if one has a vague recollection of both the author's name and the subject area of the paper, it is probably quicker and easier to refer to the subject index rather than the author index. For example, if one recalls that the article is in functional analysis, a quick scan of the subject index would lead one to W. J. Davis #6287.

Ultimately, the volume indexes should be considered to be temporary indexes, designed to tide one over until the IMP (= Index of Mathematical Papers) comes along with full bibliographic information. We hope that the IMP for the year 1977 will appear in the early part of 1978 and definitely plan that the IMP for 1978 (which will be a privilege of subscription) will appear in January 1979. When the IMP has appeared, the utility of the volume indexes is greatly reduced.

Finally, I regret that financial considerations have forced a curtailing of some of our former features. Perhaps, once the computerization at MR is completed, it will be financially possible to restore somewhat fuller indexing.

Robert G. Bartle
Executive Editor
Mathematical Reviews

Editor, the Notices

Professor Halmos (Notices, August 1977, p. 283) suggests that the ideal book review is a "chatty expository essay." While I agree that a review should contain material of this type, it seems to me that it should also be designed to help the reader decide whether the book is likely to be worth buying or recommending to students. I therefore feel that the central point of a review should be a discussion of such questions as how the book fits in to the general development of the subject, how comprehensive it is, at what level it is aimed, whether it is clearly written, whether the exposition has any unusual or noteworthy features, etc.

P. E. Newstead
VISITING MATHEMATICIANS — Supplementary List

The list of visiting mathematicians includes both foreign mathematicians visiting in the United States and Canada, and Americans visiting abroad. Note that there are two separate lists. These are a supplement to the listings in the October and November 1977 issues of these Notices.

### American and Canadian Mathematicians Visiting Abroad

<table>
<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Interest</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Andreotti, Aldo (U. S. A.)</td>
<td>Université de Strasbourg, France</td>
<td>Algebraic Geometry, Complex Analysis, Partial Differential Equations</td>
<td>9/76 – 6/78</td>
</tr>
<tr>
<td>Anselone, Philip M. (U. S. A.)</td>
<td>University of Hamburg, West Germany</td>
<td>Approximation Theory, Integral Equations</td>
<td>9/77 – 8/78</td>
</tr>
<tr>
<td>Carlson, David H. (U. S. A.)</td>
<td>Universidade de Coimbra, Portugal</td>
<td>Matrix Theory</td>
<td>9/77 – 6/78</td>
</tr>
<tr>
<td>Hager, Anthony W. (U. S. A.)</td>
<td>University of Padua, Italy</td>
<td>Topological Algebra</td>
<td>1/78 – 7/78</td>
</tr>
<tr>
<td>Hironaka, Heisuke (U. S. A.)</td>
<td>Kyoto University</td>
<td>Algebraic Geometry</td>
<td>6/77 – 8/78</td>
</tr>
</tbody>
</table>

### Visiting Foreign Mathematicians

<table>
<thead>
<tr>
<th>Name and Home Country</th>
<th>Host Institution</th>
<th>Field of Interest</th>
<th>Period of Visit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Catanese, Fabrizio (Italy)</td>
<td>Harvard University</td>
<td>Geometry</td>
<td>9/77 – 6/78</td>
</tr>
<tr>
<td>Edwards, C. Martin (England)</td>
<td>Queen's University</td>
<td>Functional Analysis</td>
<td>1/78 – 8/78</td>
</tr>
<tr>
<td>Gallone, Franco (Italy)</td>
<td>University of Texas at Austin</td>
<td>Quantum Mechanics</td>
<td>Spring 1978</td>
</tr>
<tr>
<td>Govaerts, Willy (Belgium)</td>
<td>SUNY at Buffalo</td>
<td>Theory of Spaces of Continuous Functions</td>
<td>1/78 – 6/78</td>
</tr>
<tr>
<td>Grinblat, Leon (Israel)</td>
<td>Harvard University</td>
<td>Functional Analysis, General Topology</td>
<td>7/77 – 6/78</td>
</tr>
<tr>
<td>Hoffmann, Karl-Heinz (Germany)</td>
<td>Oregon State University</td>
<td>Approximation Theory, Optimization, Control Theory</td>
<td>9/77 – 12/77</td>
</tr>
<tr>
<td>Hughes, Anthony (England)</td>
<td>Harvard University</td>
<td>Group Theory</td>
<td>9/77 – 6/78</td>
</tr>
<tr>
<td>Khosrovshahi, Gholamreza (Iran)</td>
<td>University of Illinois, Chicago Circle</td>
<td>Positive Spectrum of Schrödinger Operators</td>
<td>8/77 – 8/78</td>
</tr>
<tr>
<td>Morris, Hedley C. (Ireland)</td>
<td>Oregon State University</td>
<td>Theoretical Physics, Math Biology</td>
<td>9/77 – 6/78</td>
</tr>
<tr>
<td>Natterer, Frank (Germany)</td>
<td>Oregon State University</td>
<td>Applied Mathematics</td>
<td>4/78 – 6/78</td>
</tr>
<tr>
<td>Procesi, Claudio (Italy)</td>
<td>Harvard University</td>
<td>Algebra</td>
<td>2/78 – 6/78</td>
</tr>
<tr>
<td>Segal, Graeme (England)</td>
<td>Harvard University</td>
<td>Topology</td>
<td>9/77 – 1/78</td>
</tr>
<tr>
<td>Sprekels, Jürgen (Germany)</td>
<td>Oregon State University</td>
<td>Differential and Integral Equations, Fixed Point Theorems</td>
<td>9/77 – 6/78</td>
</tr>
<tr>
<td>Zirilli, Francesco (Italy)</td>
<td>Harvard University</td>
<td>Mathematical Physics</td>
<td>8/77 – 6/78</td>
</tr>
</tbody>
</table>
135. J. McCormack 12 Conyngham Road, Dublin 8, Ireland). The 24 coloured squares problem. There are 24 squares the edges of which are coloured with colours from the set (red, black, yellow). It is required to fit the 24 squares into a $6 \times 4$ rectangle so that (1) Problem 1. (a) Each pair of touching edges is of the same colour; (b) The border of the rectangle is of one colour and (2) Problem 2. (a) Each pair of touching edges is of the same colour; (b) The border of the rectangle is equally divided between any two colours. The 24 coloured squares are realisations of the 24 equivalence classes defined by the cyclic group of order four (of rotations of the square) on the set of 81 ordered quadruples $(x, y, z, u); x, y, z, u \in \{0, 1, 2\}$. Is there a theory and/or literature on problems of this sort?


137. Andreas Zachariou (Mathematical Institute, University of Athens, Athens 143, Greece). I recently saw an advertisement for a rather recent bibliography on finite algebraic systems or something like this which I cannot relocate. Would someone provide an exact reference (author, title, publishing company)?

138. Ludvik Janos (Department of Mathematics, Mississippi State University, Mississippi State, Mississippi 39762). The well-known Schreier refinement theorem says that two given normal series in a group have refinements which are isomorphic. We ask whether the analogous statement holds concerning three given normal series. More precisely: Given normal series $\alpha_1$, $\alpha_2$, $\alpha_3$, do there exist normal series $\beta_1$, $\beta_2$, $\beta_3$ such that (1) $\beta_i$ is a refinement of $\alpha_i$ for $i = 1, 2, 3$ and (2) $\beta_i$ is isomorphic to $\beta_j$ for $i, j = 1, 2, 3$?
NEWS ITEMS AND ANNOUNCEMENTS

AMS RESEARCH FELLOWSHIP FUND
Request for contributions

The AMS Research Fellowship Fund was established in 1973 because of the scarcity of funds for postdoctoral fellowships. From this fund AMS Research Fellowships are awarded annually to individuals who have received the Ph.D. degree, who show unusual promise in mathematical research, and who are citizens or permanent residents of a country in North America. Currently each fellowship carries a partially tax-exempt stipend of approximately $11,000.

Two Research Fellowships were awarded in 1974-1975, three in 1975-1976, and two in 1976-1977. In 1977-1978 it was possible to award four Fellowships, with eleven persons receiving Honorable Mention. The Society hopes that the number of fellowships to be awarded for 1978-1979 can again be increased. This number, of course, depends on the contributions the Society receives. The Society itself contributes a minimum of $9,000 to the Fund each year, matching one-half the funds in excess of $18,000 raised from other sources, up to a total contribution by the Society of $20,000. It is hoped that every member of the Society will contribute to the Fund.

Contributions to the AMS Research Fellowship Fund are tax deductible. Checks should be made payable to the American Mathematical Society, clearly marked "AMS Research Fellowship Fund," and sent to the American Mathematical Society, P. O. Box 1571, Annex Station, Providence, Rhode Island 02901.

DECLINE IN NSF GRADUATE FELLOWSHIPS

The National Science Foundation has announced that it plans to award 420 new graduate fellowships in 1978. Truman Botts, in an editorial in the October-November CBMS Newsletter, points out that this figure represents a sharp decline from the 550 new awards which have been made in each of the past three years. As the mathematical sciences have usually received between 10 and 11 percent of these fellowships, the number of new mathematical awards for 1978 is expected to decrease from 60 to about 45. Dr. Botts, referring to a recent article by Smith and Karlesky (The State of Academic Science, May-June CBMS Newsletter), calls attention to the long-term effect such an erosion in the number of fellowships could have on the national research effort.

The NSF at present estimates that in 1979 it will award between 525 and 550 new fellowships. In view of the Smith-Karlesky report, Dr. Botts feels that the member societies of CBMS can justifiably support the NSF Science Education Directorate in calling for the awarding of at least 550 new fellowships in 1979.

COMPUTERIZED REGISTRY OF WOMEN SCIENTISTS

The Association for Women in Science (AWIS) announces the expansion of its computerized Registry of Women in Science and Engineering. The Registry contains brief biographies of women in all fields of science and engineering and provides job candidates for recruiters from business, government, and academia. The Registry also serves as a resource list for advisory panels and other projects involving women in science. For more information write: AWIS Registry, Suite 1122, 1346 Connecticut Avenue, N.W., Washington, D. C. 20036.

J. E. LITTLEWOOD

John Edensor Littlewood of Trinity College, Cambridge, England, died on September 7, 1977, at the age of 92. He had been a Fellow of Trinity College since 1908. Professor Littlewood was a scholar at Trinity College, bracketed Senior Wrangler in 1905, and subsequently received an M.A. from the University of Cambridge. He held several honorary doctorates. He was Lecturer of Trinity College from 1910 to 1928; Cayley Lecturer in the University of Cambridge from 1920 to 1928; and Rouse Ball Professor of Mathematics in the University of Cambridge from 1928 to 1950. He was a Fellow of the Royal Society and of the Cambridge Philosophical Society. His awards and prizes include the Royal Medal of the Royal Society (1929) and the De Morgan Medal of the London Mathematical Society (1939), among others. Professor Littlewood was a corresponding member of the French and Göttingen Academies and a Foreign Member of the Royal Dutch, Royal Danish and Royal Swedish Academies.

NEW NSF ASSISTANT DIRECTOR

James A. Krumhansl has been nominated by the President as Assistant Director for Mathematical and Physical Sciences and Engineering of the National Science Foundation. He will replace Edward C. Creutz who has resigned. Dr. Krumhansl received his B.S. in electrical engineering from the University of Dayton, the M.S. from Case Institute of Technology, and the Ph.D. in physics from Cornell University. He has been an active researcher in academic and industrial circles since 1943, with emphasis in the fields of theoretical solid state physics, materials science, applied mathematics and electrical engineering.

COLLOQUIUM LECTURES

Copies of the lecture notes prepared by Professor Herbert Federer for the set of Colloquium Lectures presented at the summer meeting in Seattle, Washington, are no longer available for sale.
RECENT APPOINTMENTS

President R. H. Bing has appointed Peter D. Lax to the Committee on National Awards and Public Representation. Continuing members of the committee are President Bing and Daniel Mostow.

Murray Gerstenhaber and Mary W. Gray have been appointed by President R. H. Bing to the Committee on Human Rights of Mathematicians. The continuing members of the committee are Lipman Bers, Charles Herbert Clemens, Morris W. Hirsch, Nathan Jacobson (chairman), and John A. Nohel.

New members of the Committee on Prizes, appointed by President R. H. Bing, are: Louis Auslander, James C. Cantrell, and Mary E. Rudin. Other members of the committee are Walter Feit, Victor L. Klee, Jr., Peter D. Lax, and John W. Milnor.

President R. H. Bing has appointed several new members to the Committee on Steele Prizes. They are Irving Kaplansky, H. Blaine Lawson, Jr., Joseph L. Taylor, and Raymond O. Wells, Jr. Continuing members of the committee are Edward B. Curtis, Paul R. Halmos, Joseph R. Shoenfield, Frank Spitzer, and Hans F. Weinberger (chairman). This is a selection committee. The definition of the prizes was reconstituted by the Council in April, 1977, as recorded on page 309 of the August 1977 issue of these Notices.

President R. H. Bing has appointed Barbara Osofsky and Donald S. Ornstein to the Program Committee. Other members of the committee are C. Edmund Burgess, Edwin E. Floyd (chairman), James G. Glimm, Everett Pitcher, and Harold M. Stark.

James M. Greenberg and Jack C. Kiefer have been appointed by President R. H. Bing to the Committee to Select Hour Speakers for Eastern Sectional Meetings. Elias M. Stein has been appointed chairman of the committee, and Raymond G. Ayoub (Associate Secretary) and Hyman Bass are continuing members.

Robert Osserman and Rimhak Ree have been appointed by President R. H. Bing to the Committee to Select Hour Speakers for Far Western Sectional Meetings. Paul C. Fife has been appointed chairman of the committee, and David M. Goldschmidt and Kenneth A. Ross (Associate Secretary) are continuing members.

William K. Allard and Thomas A. Chapman have been appointed by President R. H. Bing to the Committee to Select Hour Speakers for South-eastern Sectional Meetings. Robert B. Gardner has been appointed chairman of the committee, and Frank T. Birtel (Associate Secretary) and James D. Stasheff are continuing members.

Melvin Hochster and Karen Uhlenbeck have been appointed by President R. H. Bing to the Committee to Select Hour Speakers for Western Sectional Meetings. Raymond O. Wells, Jr., has been appointed chairman of the committee, and Paul T. Bateman (Associate Secretary) and Mark Mahowald are continuing members.

President R. H. Bing has appointed Mary W. Gray to continue as representative of the Society on the joint AMS-MAA-SIAM Committee on Women in Mathematics. Other members of the Committee are Dorothy L. Bernstein, Jane K. Cullum, Etta Z. Falconer, I. N. Herstein, Linda C. Kaufman, Victor L. Klee, Jr., Margaret S. Menzin, Cathleen S. Morawetz, Jacqueline C. Moss, and Alice T. Schafer (chairman).

Bruce W. Mielke and James T. Sedlock have been appointed as Tellers in the election of 1977 by President R. H. Bing.

President R. H. Bing has appointed Arthur Mattuck and Alan J. Goldman to the Committee on Employment and Educational Policy. Continuing members of this committee are Lida K. Barrett, Hugo Rossi, Martha Kathleen Smith, and Robert J. Thompson.

NEW SUBCOMMITTEE

The following have been appointed members of the Employment Concerns Subcommittee of the AMS Committee on Employment and Educational Policy: Hugo Rossi (chairman), Audrey Terras, Robert J. Thompson and Barnet Weinstock.

COMMITTEE DISCHARGED

The Committee on Publication Problems has been discharged with the thanks of the Council.
THE OCTOBER MEETING IN WELLESLEY

The seven hundred forty-eighth meeting of the American Mathematical Society was held at Wellesley College, Wellesley, Massachusetts, on Saturday, October 29, 1977. There were 118 registrants including 103 members of the Society.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings there were two invited one-hour addresses. GIAN-CARLO ROTA of the Massachusetts Institute of Technology spoke on "Recent progress in combinatorics," and JEAN E. TAYLOR of Rutgers University spoke on "The geometry of soap films and crystals."

There were three special sessions of selected twenty-minute papers. GIAN-CARLO ROTA organized a special session on Combinatorics; the speakers were Steve Fisk, Stephen Milne, Steven Roman, and Gregory Wulczyn. JEAN E. TAYLOR organized a special session on Geometric problems in the calculus of variations; the speakers were F. J. Almgren, Jr., John B. Baillieul, Enrico Bombieri, Frank Morgan, and Jon T. Pitts. CARL S. LEDBETTER of Wellesley College arranged a special session on Recent advances in the theory of sheaves and topoi; the speakers were Julian C. Cole, Michael Fourman, Carl S. Ledbetter, and George Reynolds. Michael Fourman moderated a discussion on Analysis in Topoi during this special session.

There were three sessions on contributed ten-minute papers.

University Park, Pennsylvania
Raymond G. Ayoub
Associate Secretary

THE OCTOBER MEETING IN WEST LAFAYETTE

The seven hundred forty-ninth meeting of the American Mathematical Society was held at Purdue University, West Lafayette, Indiana, on Saturday, October 29, 1977. There were 179 registrants including 152 members of the Society.

By invitation of the Committee to Select Hour Speakers for Western Sectional Meetings there were two invited one-hour addresses. ALBERT BAERNSTEIN II of Washington University gave an hour talk on "Maximal functions in complex analysis"; he was introduced by Casper Goffman, KAREN K. UHLENBECK of the University of Illinois at Chicago Circle addressed the Society on the subject "Variational problems on manifolds with a conformal structure"; Felix Haas presided at her lecture.

By invitation of the same committee there were four special sessions of selected twenty-minute papers. GEORGIA M. BENKART of the University of Wisconsin arranged a special session on Nonassociative algebras and their connections with physics; the speakers were Harry P. Allen, Yu-chte Chow, Gabor Domokos, Pierre M. Ramond, Robert Lee Wilson, and Hans J. Zassenhaus. JOHN B. CONWAY of Indiana University arranged a special session on Subnormal operators; the speakers were James E. Brennan, Kevin Clancey, James A. Deddens, Ronald G. Douglas, William W. Hastings, Thomas L. Kriete III, Robert F. Olin, James E. Thompson, and Warren R. Wogen. JOSEPH B. MILES of the University of Illinois at Urbana-Champaign arranged a special session on Functions of one complex variable; the speakers were Joseph A. Cima, David Drasin, Peter L. Duren, Mats R. Essén, Lowell J. Hansen, Larry J. Kotman, John L. Lewis, Lee A. Rubel, Glenn E. Schober, and Allen W. Weissman. JOHANNES C. C. NITSCHKE of the University of Minnesota arranged a special session on Methods of the calculus of variations and partial differential equations applied to geometrical or physical problems; the speakers were Giles Auchmuty, Kenneth A. Brakke, Luis A. Caffarelli, Robert Finn, Avner Friedman, Robert D. Gulliver II, Stefan Hildebrandt, Jerry L. Kazdan, David S. Kinderlehrer, Frank Morgan, and Henry C. Wente.

There was also one session of contributed ten-minute papers, for which Jean E. Rubin served as presiding officer.

Urbana, Illinois
Paul T. Bateman
Associate Secretary

NEWS ITEMS AND ANNOUNCEMENTS

AAAS DIRECTORY
OF HANDICAPPED SCIENTISTS

The Project on the Handicapped in Science of the American Association for the Advancement of Science (AAAS), under a grant from the National Science Foundation, is developing a directory of handicapped scientists for distribution to public agencies, consumer organizations, and other handicapped scientists. The directory is intended to identify scientists in specific areas of specialization who are willing to share their experiences, to educate the scientific community as to their problems, to organize necessary action, and possibly to act as advisors, consultants and/or evaluators in areas concerning science and the handicapped.

The directory will contain basic biographical data in each entry and will include all those with training in all areas of natural and social sciences, including mathematics, engineering, and medicine, whether employed, working in non-scientific areas, or unemployed. All handicapped scientists and graduate students are urged to participate by contacting Martha Ross Redden, Project Director, or Janette Alsford Owens, Project Assistant, Project on the Handicapped in Science, Office of Opportunities in Science, AAAS, 1776 Massachusetts Avenue N.W., Washington, D. C. 20036.
ABSTRACTS

The abstracts are grouped according to subjects chosen by the author from categories listed on the abstract form and are based on the AMS (MOS) Subject Classification Scheme (1970). Abstracts for which the author did not indicate a category are listed under miscellaneous.

Abstracts for which the author did not indicate a category are listed under miscellaneous. Authors are urged to submit blank copy preferably using a carbon ribbon or reasonably new black copy, in order to improve the typography of the abstract section; when copy is gray rather than black, the printing is light and some characters may print only in part or not at all.

* Indicates that preprints are available from author. • Indicates invited addresses.

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Abstracts Presented to the Society

The abstracts printed in this section were accepted by the American Mathematical Society for written presentation. An individual may present only one abstract by title in any one issue of the Notices, but joint authors are treated as a separate category. Thus, in addition to abstracts from two individual authors, one joint abstract by them may also be accepted for the same issue.

Algebra and Theory of Numbers (05, 06, 08, 10, 12-18, 20)


Let $E$ be a semilattice $Y$ of rectangular bands $(E_\alpha : \alpha \in Y)$ and let $V$ be an inverse semigroup with semilattice $Y$. Let $\alpha \to \theta_\alpha$ and $\alpha \to \Phi_\alpha$ be mappings of $V$ into $T_E$, the full transformation semigroup on $E$. Let $I_\alpha (J_\alpha)$ denote a fixed $\alpha$-class ($\alpha$-class) of $E_\alpha (\alpha, \beta$ are Green's relations). Let $f(g)$ be a function of $V^2$ into $I = \bigcup \{I_\alpha : \alpha \in Y\} (J = \bigcup \{J_\alpha : \alpha \in Y\})$. Furthermore, assume $\theta_\gamma (\Phi_\gamma)$ is a homomorphism of $E_\gamma$ into $e\gamma \gamma^{-1}(E_\gamma \gamma^{-1} e)$ for some $e \in E$; if $x \in Ef$ and $y \in gE$ for some $f, g \in E_e (E_e^{-1} e) \in E_E$, then $(xy)e = x\Phi_\beta y\Theta_\beta (xy)\Phi_\beta = x\Phi_\beta y\Theta_\beta$ for some $e \in E_\beta \beta^{-1} (E_\beta^{-1} e)$; for $j \in J$, $j\Phi_\beta \alpha e \in (f(\beta, \gamma))\Phi_\beta \gamma (\gamma) (\in E)$; for $z \in I$, $z\Phi_\beta \alpha e \in f(\gamma, \beta)(\gamma, \beta) z\Theta_\beta (\gamma) (\in E)$. Let $(Y, E, V, \theta, \phi, f, g)$ denote $((\alpha, \beta, \gamma), \alpha \in V, i \in I, \alpha \gamma^{-1}(E_\gamma \gamma^{-1} e)$ under the multiplication $(i, \alpha, j); p, \beta, \gamma = ((p, \theta_\alpha)\Phi_\beta \alpha \Theta_\beta \gamma (E_\beta \beta^{-1} e). Theorem. S is an orthodox semigroup if and only if $S$ is isomorphic to some $(Y, E, V, \theta, \phi, f, g)$.

(Received February 14, 1977.)

78T-A2 RONSON J. WARNE, University of Alabama, Birmingham, Alabama 35294 and JANE LA FRANCE, 729 Jones Street, Apartment 116, San Francisco, California 94109. Split extensions and a class of simple semigroups. II.

A simple semigroup $S$ ($S$ has no proper ideals) such that $T$, the union of the maximal subgroups of $S$, is an $\omega$-chain of completely simple semigroups $(T_n : n \in \mathbb{N})$, the nonnegative integers) is termed a natural simple semigroup of type $\omega$. Let $d$ be a positive integer and let $H = \{0, 1, 2, \ldots, d-1\}$. Let $C_d = N \times N \times H$ under the multiplication $(a, b, c) \to (a, b, c) = (n + r - \min(k, r), k + s - \min(k, r), f(k, r))$, where $f(k, r) = 1, j$, or $\max(1, j)$ according to whether $k > r$, $r > k$, or $r = k$. Denote $N \times H$ under the order $(a, i) < (m, j)$ if $n > m$ or $n = m$ and $i > j$ by $\omega d$. Theorem. $(\omega d, T, C_d, \phi, A)$ (notation of R. J. Warne, "Split extensions and a class of simple semigroups"), I, these Notices 24(1977), A-607 is a natural simple semigroup of type $\omega$, and, conversely, every such semigroup is isomorphic to some $(\omega d, T, C_d, \phi, A)$ (valid with "simple" replaced by "bisimple" and $d = 1$). The theorem is specialized to the semigroups considered in R. J. Warne, "Natural orthodox semigroups". I, II, these Notices 24(1977), A-423, A-523. (Received March 30, 1977.)

A-1
Let $S$ (respectively $S_0$) denote the category of all join-semilattices (resp. join-semilattices with 0) with (0-preserving) semilattice homomorphisms. For $A \in S$ let $A_0$ represent the object of $S_0$ obtained by adjoining a new 0-element. In either category the tensor product of two objects may be constructed in such a manner that the tensor product functor is left adjoint to the hom functor. An object $A \in S$ ($S_0$) is called flat if the functor $- \otimes_S A$ ($- \otimes_{S_0} A$) preserves monomorphisms in $S$ ($S_0$).

**Theorem:** For $A \in S$ ($S_0$) the following conditions are equivalent: (1) $A$ is flat in $S$ ($S_0$), (2) $A_0$ ($A$) is distributive (see Grätzer, Lattice Theory, p. 117), (3) $A$ is a directed colimit of a system of f. g. free algebras in $S$ ($S_0$). The equivalence of (1) and (2) in $S$ was previously known to James A. Anderson. (1) $\iff$ (3) is an analogue of Lazard's well-known result for R-modules. (Received August 23, 1977.)

Robert B. Allan and Renu Laskar, Clemson University, Clemson, S.C. 29631. On the domatic number of a graph and the domination number of its complement.

Let $G = (V,E)$ be a graph where $V = \{v_1,v_2,\ldots,v_p\}$. A set $A \subseteq V$ is a dominating set for $G$ if for $u \notin A$, there exists a $v \in A$, such that $uv \in E$. If $A$ is a dominating set and no proper subset of $A$ is dominating set, then $A$ is called a minimal dominating set. A $\mathcal{D}$-partition of $G$ is a partition of $V(G)$ into dominating sets of $G$. Let $\gamma(G)$ denote the smallest number of vertices in a minimal dominating set and let $d(G)$, the domatic number of $G$, denote the maximum order of a $\mathcal{D}$-partition of $G$. A constructive proof of the result $\gamma(G) \leq d(G)$ [due to Jaeger, F. and Payan, C. "Relations du Type Nordhaus-Gaddum Pour le Nombre d'Absorption d'un Graphes Simple", C. R. Acad. Sc. Paris, Series A, t 274, 1972.] is given here. (Received September 19, 1977.)

JONATHAN SAMIT, University of Massachusetts, Amherst, Massachusetts 01003. An intersection theorem for subdirect sums.

Let $A$ be the subdirect sum of finitely generated algebras $A_j$ over a field $F$ such that all the identities of the full matrix algebra of some order hold in each $A_j$. Then the intersection of all the powers of the quasi-regular radical of $A$ is zero. (Received September 20, 1977.)

PHILIP DWINGER, University of Illinois at Chicago Circle, Chicago, IL 60680. Completeness of Boolean powers of Boolean algebras.

Various results are derived concerning the completeness of a Boolean power $B_2(B_1)$, where $B_1$ and $B_2$ are complete Boolean algebras. Typical results: 1. If $B_2(B_1)$ is complete then $B_2(B_1)$ is the normal completion of the free product of $B_1$ and $B_2$; 2. $B_2(B_1)$ is not complete iff there exists a (set) function $f : B_2 \rightarrow B_1$ and $a \in B_1$, $a \neq 0$, such that $\sum_{x \in z} f(u) \leq \sum_{y \in x} f(y)$ for each $x \in B_2$; 3. If $B_2(B_1)$ is $\kappa$-distributive then $B_2(B_1)$ is complete; 4. If $B_1$ is not $\kappa$-distributive then $2\kappa(B_1)$ is not complete. (Received September 22, 1977.)
Theorem 1. If \( \text{cf} \kappa > \aleph_0 \) (in particular, if \( \kappa \) is uncountable and regular), \( A \) is a free BA, and \( B \) is a subset of \( A \) with \( |B| = \kappa \), then there is an independent subset \( C \) of \( B \) with \( |C| = \kappa \). The theorem does not extend to \( \kappa \) with \( \text{cf} \kappa = \aleph_0 \). Corollary. Any uncountable subalgebra of a free BA has an uncountable free subalgebra. Hence Conjecture (C) of Rotman, Fund. Math. 75, 187-197, is true. This was first noticed by M. Rubin, by an argument essentially proving the corollary.

Theorem 2. If \( \kappa = (2^{\aleph_0})^+ \), \( \lambda \) is minimum such that \( \lambda \geq \kappa \), \( \lambda \) is regular, \( A \) is a free BA, \( B \) is a subset of the completion \( \hat{A} \) of \( A \), and \( |B| = \kappa \), then there is a \( C \subset B \), \( |C| = \lambda \), with \( C \) independent. Thus the conclusion holds with \( \kappa = \lambda = (\omega_1)^+ \), for any \( \mu \). Theorem 3. If \( \kappa \) is singular, \( \text{cf} \kappa = (2^{\aleph_0})^+ \), \( \mu < \kappa \) whenever \( \mu < \kappa \), \( \nu < \text{cf} \kappa \) whenever \( \nu < \text{cf} \kappa \), \( A \) is a free BA, \( B \subset \hat{A} \), and \( |B| = \kappa \), then there is an independent \( C \subset B \) with \( |C| = \kappa \). Thus the conclusion holds under GCH for \( \kappa = \omega_2 \). (Received October 6, 1977.)

R. E. OSTEEN, University of Florida, Gainesville, Florida 32601.

A note on maximal rectangles in digraphs and cliques in graphs.

The problem of finding the maximal rectangles of a directed graph can be reduced to that of identifying the cliques of an undirected graph.

Let \( D \) be a given (finite) digraph, consisting of a finite nonempty set \( P \) of points and a set of directed lines, \( L \subset P \times P \). A rectangle in \( D \) is a nonempty subset \( A \times B \) of \( L \); a rectangle is maximal if it is not properly contained in a rectangle in \( D \).

The undirected graph, \( G=(V, E) \), associated with \( D=(P, L) \) is defined as follows: \( V=L \); if \( (a,b) \in L \) and \( (c,d) \in L \) then \( \{(a,b),(c,d)\} \in E \) in case \( (a,b),(c,d) \in L \), \( (a,d) \in L \), and \( (c,b) \in L \).

Theorem. The family of maximal rectangles of \( D \) coincides precisely with the family of cliques of \( G \). (Received September 23, 1977.) (Author introduced by A. R. Bednarek).

James C. Owings, Jr., University of Maryland, College Park, Maryland 20742. Solution of planar Diophantine equations.

We find all integral solutions of any equation of the form \( x^2+y^2+z^2+yz+zx+xy+gx+hy+kz+m=0 \) where \( g,h,k,m \) are integers. Every solution to such an equation generates a "plane" of solutions - an infinite triangular lattice of integers in which each atomic triad is a solution. It is shown that an equation of the above type has finitely many such planes. (Received September 26, 1977.)


Let \( A \) be a central simple algebra over an arithmetic field \( K \). Let \( R \) be a Dedekind domain with quotient field \( K, R \not= K \). Suppose further that \( A \) satisfies Eichler condition relative to \( R \). If \( K \) satisfies certain conditions given by Queen and Weinberger, then any maximal principal \( R \)-order \( \Lambda \) in \( A \) is two-sided Euclidean for some algorithm, suited to take care of the divisors of zero. (Received September 30, 1977.)
We introduce the duals of the notions of finitely generated modules, finitely related modules and noetherian rings and study their properties. Then, as dual to the notion of pure submodules, we introduce the notion of copure submodules. A submodule $A$ of an $R$-module $B$ is said to be copure if for every cofinitely related $R$-module $M$, every homomorphism from $A$ into $M$ has an extension to $B$. Our main theorem asserts that the class of all copure short exact sequences of $R$-modules is a 'proper class' in the sense of Mac Lane. We are able to settle definitely that copurity does not imply purity, when $R$ is arbitrary. It is reasonable to expect also that purity does not imply copurity. However, we prove that over a Dedekind domain these notions are equivalent.

(Received October 6, 1977.) (Author introduced by Professor M. Rajagopalan).

Let $V$ be an inner product $n$ dimensional vector space over the field $\mathcal{F}$. Let $G$ be a subgroup of the symmetric group $S_n$ and $\lambda:G\to\mathcal{F}$ an irreducible character of $G$. By $o(G)$ we denote the order of $G$. For each $\sigma \in G$ define a linear map $P(\sigma): V \to V$ by $P(\sigma)v_{1}\theta \ldots v_{m} = v_{\sigma(1)}^{-1} \theta \ldots v_{\sigma(m)}^{-1}$. Put $T(\sigma) = \sum_{\sigma \in G} \lambda(\sigma)P(\sigma)$. If $x_{1}, \ldots, x_{m} \in V$ we define the star product $x_{1}^{*} \ldots x_{m}^{*}$ by $x_{1}^{*} \ldots x_{m}^{*} = T(\sigma)x_{1}\theta \ldots x_{m}^{*}$.

Theorem. If $x_{1}, \ldots, x_{m} \in V$ are linearly independent and $y_{1}, \ldots, y_{m} \in V$, then $x_{1}^{*} \ldots x_{m}^{*}$ if and only if there exists an $m \times m$ matrix $B = [b_{ij}]$ such that $y_{1} = \sum_{j=1}^{m} b_{ij}x_{j}^{*}$ for $i = 1, \ldots, m$ and $d_{G}^{\lambda}(BB^{*}) \leq d_{G}^{\lambda}(AA^{*})$. (Received October 3, 1977.) (Author introduced by Russell Merris).

A definition of the conceptual structure of abstract mathematics (as delineated in the subject classification index of Mathematical Reviews) is sought. The following design features were desired for the structure: (1) an integrated picture of the subject including interdependence of geometry, algebra, analysis and topology; (2) relationship of abstract mathematics to applications and their joint relationship to computer algorithms and implementations, and their separation from non-mathematical disciplines; (3) one-to-one mapping between abstract and applied mathematics; (4) outline of the historical growth pattern; (5) directions for future growth; (6) a framework for the history of the past century; (7) ability to be reproduced by an automaton; (8) the subsets of the structure to be recognized by an automaton; (9) the objects and morphisms of category theory can be embedded within the diagram. (Received September 28, 1977.)
if the set of \( \tau_1 \)-elements form a subgroup of \( G \). \( G \) is weakly \( \tau_1 \)-closed if for every subgroup \( U \) of \( G \), the number of \( \tau_1 \)-elements of \( U \) is exactly \( |U| \). Frobenius posed the following problem: Let \( G \) be a weakly \( \tau_1 \)-closed group. Is \( G \), \( \tau_1 \)-closed? In Theorem 1, we show the answer is affirmative. **Theorem 1:** Assume \( \tau_1 \) is a set of primes. If \( G \) is a weakly \( \tau_1 \)-closed group, then \( G \) is \( \tau_1 \)-closed. (Received September 29, 1977.)

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On the relations between the genus and the cardinality of the maximum matchings. Preliminary report.

Some relations between the genus and the cardinality of the maximum matchings of graphs are established. Let \( M(G) \) denote a maximum matching of a simple finite undirected graph \( G \) and \( y \) the genus of \( G \). Let \( G \) contain \( p \) vertices, and let \( p^{(1)} \) and \( p^{(2)} \) be the number of vertices of degree 1 or 2, respectively. \( [x] \) means the greatest integer \( \leq x \), and \( \lceil x \rceil \) the least integer \( \geq x \).

**Theorem 1.** Let \( G \) be a connected graph with \( p \) vertices. \[ |M(G)| \geq \left\lceil \frac{(p-p^{(1)}-p^{(2)}+2-4y)}{3} \right\rceil \] if \( p+2p^{(1)}+2p^{(2)}+8y \geq 10 \); and \( |M(G)| = \left\lfloor \frac{p}{2} \right\rfloor \) otherwise.

**Theorem 2.** Let \( G \) be a 2-connected graph. \[ |M(G)| \geq \left\lceil \frac{(p-p^{(2)}+4-4y)}{3} \right\rceil \] if \( p+2p^{(2)}+8y \geq 14 \); and \( |M(G)| = \left\lfloor \frac{p}{2} \right\rfloor \) otherwise.

**Theorem 3.** Let \( G \) be a connected nonplanar graph with minimum degree \( \geq 2 \). \[ |M(G)| \geq \left\lceil \frac{(p-p^{(2)}+4-4y)}{3} \right\rceil \], i.e., \( y \geq \left\lceil \frac{(p-p^{(2)}-3|M(G)|)}{4} \right\rceil + 1 \).

**Theorem 4.** Let \( G \) be a tree with \( p \) (\( \geq 2 \)) vertices. \[ |M(G)| \geq \left\lceil \frac{(p-p^{(1)})}{2} \right\rceil + 1 \]. (Received September 29, 1977.) (Author introduced by Professor R. J. Duffin).

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Regular sequences of minors. Preliminary report.

We give an answer to a particular case of a question posed by D. Lazard in suites régulières dans les ideaux déterminantiels, Comm. in Algebra vol. 4 (1976), 327-340.

A sequence of minors of a \( 2 \times s \) (\( s \geq 2 \)) matrix \( M \) is called regularizable if the corresponding sequence of a \( 2 \times s \) generic matrix is regular. We give a description of all regularizable sequences of \( M \). The main theorem reads: Let \( R \) be a commutative noetherian Macaulay ring containing an infinite field \( K \). Let \( M \) be a \( 2 \times s \) (\( s \geq 2 \)) matrix with entries in \( R \) such that the ideal generated by the \( 2 \times 2 \) minors has grade \( r \). Then we can find an elementary matrix \( E \in SL_s(K) \) such that every regularizable sequence of \( 2 \times 2 \) minors of \( ME \) having \( t \leq r \) elements is actually regular. (Received September 28, 1977.) (Author introduced by T.M. Viswanathan).

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Coproducts in the categories of Kleene and Three-valued Lukasiewicz algebras.

In a recent paper, W. H. Cornish and P. R. Fowler (Bull. Austral. Math. Soc., 16(1977), 1-14) have shown that in general the coproduct of a family of Kleene algebras in the category of De Morgan algebras does not coincide with the coproduct in the subcategory of Kleene algebras. The aim of this paper is to give an explicit description of coproducts in the category of Kleene algebras in terms of the dual topological spaces. As an application, a description of dual spaces of free Kleene algebras is given. It is also shown that the coproduct of a family of three-valued Lukasiewicz algebras in the category of Kleene algebras is the same as the coproduct in the subcategory of three-valued Lukasiewicz algebras. This result generalizes a construction of free three-valued Lukasiewicz algebras due to A. Monteiro and this author (Rev. Unión Mat. Argentina, 22(1965), 152-153). (Received October 11, 1977.)
An exact crossed complex of finite length, generalizing the notion of a crossed module, is interpreted as a "crossed n-fold extension" of a group Q by a Q-module A. We show that crossed n-fold extensions are classified by the cohomology group $H^{n+1}(Q,A)$ which may thus be thought of as group of crossed n-fold operator extensions. To this end we construct certain projective crossed resolutions of groups and introduce a calculus of crossed n-fold extensions which enables us to avoid cocycle calculations completely. With a group extension we associate a long exact sequence of certain groups of crossed n-fold operator extensions.

Examples of crossed n-fold extensions arise from abstract kernels, central simple algebras normal over their center, CW-complexes aspherical in a certain range of dimensions, fibrations of certain topological spaces, and in Algebraic K-theory. We represent each of the cohomology classes, associated with either example, by a unique crossed n-fold extension. This provides conceptual proofs of the old classification theorems. (Received October 11, 1977.)

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On Ford-spheres.

Let $p = \frac{1+\sqrt{3}}{2}$, $L := \{mnp : m \in \mathbb{Z}, n \in \mathbb{Z}, \text{gcd}(m,n) = 1\}$, $H := \mathbb{Q}(p)$. Write $z \in H$ in the form $z = a/b$, $a, b \in L$, $0 \neq b \in L$, $(a, b) = 1$. For $z \in C$, $0 < d \in \mathbb{R}$ denote by $B(z, d)$ the closed sphere with center $(z, d/2)$ and diameter $d$ in euclidean 3-space $\mathbb{C} \times \mathbb{R}$.

For $z \in H$, $F(z) := B(z, |b|^{-2})$ is called Ford-sphere. Let $V := \bigcup_{z \in H} F(z)$.

Spheres of the form $B(z, d)$ with $z \in C$, $0 < d \in \mathbb{R}$ are called admissible. Let $W_1 := \bigcup_{z \in L} F(z)$.

To $W_k$ we add all admissible spheres of maximal diameter and having no interior point in common with $W_k$; this gives $W_{k+1}$, say $(k \in \mathbb{N})$. Theorem 1. $V = \bigcup_{k \geq 1} W_k$.

Let $h \in \mathbb{R}$, $0 < h \leq 1$; denote by $M(h)$ the parallelogram with corners $(0, h)$, $(1, h)$, $(\rho, h)$, $(\rho^2, h)$ in $\mathbb{C} \times \mathbb{R}$; denote by $r(h)$ the area of $M(h) \cap V$. For primes $p \neq 2$, let $\chi(p) := 1$ if $p \equiv 1 \mod 3$ and $\chi(p) := -1$ for $p \equiv 2 \mod 3$. For $1 < s \in \mathbb{R}$, let $L(s) := \prod_{p \neq 2} (1-\chi(p)p^{-s})^{-1}$.

Theorem 2. For $0 < h \leq 1$, $r(h) = \left(\frac{L(2)}{\sqrt{3}}\right)^2 + O(1/h)$. (Received October 11, 1977.)

Matrices simultaneously similar to block triangular matrices and matrices unitarily similar to block diagonal matrices.

Let $V$ be an $n$ dimensional vector space over an algebraically closed field $F$ and let $\mathcal{A}$ be an algebra of $n \times n$ matrices over $F$. Regard $V$ as a left $\mathcal{A}$-module and let $n_1, \ldots, n_t$ be the dimensions of the composition factors of $V$. Applying a similarity, we may assume $\mathcal{A}$ consists of block upper triangular matrices with blocks of sizes $n_1$. Gaines (1968) used commutator relations to study such block triangular matrices. Theorem: Let $P$ be a polynomial in $r$ noncommuting variables with coefficients in $F$. Suppose $P = 0$ is satisfied by every $r$-tuple of $k \times k$ matrices but not by every $r$-tuple of $(k+1) \times (k+1)$ matrices over $F$.

Then if $P = 0$ is satisfied by every $r$-tuple of matrices of $\mathcal{A}$, max \{n_i\} $\leq k$. Note that polynomials satisfying the hypothesis do exist. Theorem: Let $A$ be an $n \times n$ complex matrix. $A$ is unitarily similar to a block diagonal matrix with blocks of sizes $n_1, \ldots, n_t$ if and only if $A$ and $A^*$ are simultaneously similar to block upper triangular matrices with blocks of sizes $n_1$. If $S$ is a set of complex matrices and $\mathcal{A}$ is the algebra generated by $S \cup \{1\}$, a similar result holds for $\mathcal{A}$, provided $\mathcal{A} = \mathcal{A}^*$. These results yield a new proof of a theorem of Watters (1974): If $S$ is a family of $n \times n$ normal matrices and $\mathcal{A}$ is the algebra generated by $S$ over the complex numbers, then the matrices in $S$ are simultaneously unitarily similar to quasi-diagonal matrices if and only if $[A_iB]Q = QA_iB_i^{-1}$ for all $A_i$ and $B_i$ in $\mathcal{A}$, and $Q$ in $S$. (Received October 11, 1977.)
By generalizing the Liapunov-Yoshizawa techniques, we give necessary and sufficient conditions for uniform boundedness and uniform ultimate boundedness of a rather general class of nonlinear differential equations of neutral type. This is then applied to several linear and nonlinear systems of equations including the generalized Lienard equation of neutral type. The calculations seem less cumbersome than is the case when Liapunov-Razamikhin techniques are adopted as suggested by Lopes in SIAM J. Appl. Math., Vol. 29, No. 1 (1975), pp. 196-207. Only explicit Liapunov-Yoshizawa functionals are utilized in the applications. (Received July 25, 1977.)

Let K be a compact convex subset of a Hausdorff l.c.t.v.s. Consider the property (SS) every continuous affine map of K onto any other compact convex set is open.

THEOREM. If K has property (SS), then K is finite dimensional.

THEOREM. If K ∩ R^d, then K has property (SS) if and only if for each extremepoint b of K and each sequence e_n of extremepoints of K which converge -- say, s = lim e_n -- there is a sequence of scalars t_n converging to 1 such that e_n + t_n(b-s) is in K for each n.

Thus polytopes and finite dimensional strictly convex sets, as well as products of such, have property (SS). (Received August 2, 1977.)

In this paper we continue our investigation of the hyperinvariant subspace lattice, Hyperlat T, for a completely non-unitary C^1_1 contraction T with defect indices n < ω. Consider T being acting on H ≅ [H^2_n] ⊕ [λ^2_n] ⊕ [H^2_w : w ∈ H^2] by T(f ⊗ g) = P(e^{it}f ⊗ e^{it}g) for f ⊗ g ∈ H, where ΩT denotes the characteristic function of T, ∆ = (1 - Ω_T Ω_T*)^½ and P denotes the (orthogonal) projection onto H. We show that Hyperlat T consists of subspaces of the form K_T = {f ⊗ g ∈ H : -Δ_g f + Ω_g g = 0} on f^½, where Δ_g = (1 - Ω_T Ω_T*)^½ and F is some Borel subset of the unit circle. As an application, we show that for the operator T considered every element in Hyperlat T is the kernel and the range of operators in [f^½]. We also show that Lat T = Hyperlat T if and only if T is multiplicity-free and Ω_T(t) is isometric on a set of positive Lebesgue measure, where Lat T denotes the invariant subspace lattice of T. (Received September 7, 1977.)
ALBERT A. MULLIN, USA BMD Advanced Technology Center, Data Processing, P.O. Box 1500, Huntsville, AL 35807. Combinatorial fixed-point theory.

Lemma. Let F be a non-empty index set whose cardinality |F| ≥ n+1. Let \( \{S_f, f \in F\} \) be a class of compact convex subsets of \( \mathbb{R}^n \). Let \( \{G_f, f \in F\} \) be a class of non-empty commuting families of non-expansive mappings of \( S_f \) into \( S_f \), \( f \in F \). Let \( \{G_f \subseteq S_f, f \in F\} \) be the class of sets of all common fixed-points over \( G_f, f \in F \). Suppose that for each subclass of \( (n+1) \) members of \( \{G_f\} \) there exists a point of \( \mathbb{R}^n \) whose distance from each of the \( (n+1) \) members of \( \{G_f\} \) does not exceed a positive real number \( d \). Then there exists a point of \( \mathbb{R}^n \) whose distance from all members of \( \{G_f, f \in F\} \) does not exceed \( d \). Note 1. Analogous combinatorial results hold in any Minkowski space for sets of common fixed points over commuting families of affine continuous mappings of \( S_f \) into \( S_f \), \( f \in F \).

As a corollary, one has a new kind of \( \epsilon \)-fixed-point theorems, viz., common \( \epsilon \)-fixed-points over certain families of mappings can be used to establish the existence of a common \( \epsilon \)-fixed-point over a significantly broader family of mappings. Dimension-free analogues of the Lemma are discussed informally. (Received September 12, 1977.)


We give some sufficient conditions for the validity of equalities \( S_n(B)S_n^{-1}(A) \rightarrow A \), \( \lambda_n(B)\lambda_n^{-1}(A) \rightarrow \lambda(A) \), where \( S_n(A) \), \( \lambda_n(A) \) are the singular and eigenvalues of \( A \), \( A \), \( B \) are linear operators. Thm 1. Let \( A \) be a closed operator in the Hilbert space \( H \), spectrum of \( A \) being discrete, \( A^{-1} \) being bounded, \( T \) be a linear operator \( \mathcal{D}(A) \rightarrow \mathcal{D}(A) \), \( A^*T \) be compact operator, \( B = AT, D(B) = D(A) \). Then the spectrum of \( B \) is discrete and \( SN(B)S_n^{-1}(A) \rightarrow A, n \rightarrow \infty \). If, in addition, \( A = A^* \), \( B = B^* \), then \( \lambda_n(B)\lambda_n^{-1}(A) \rightarrow A, n \rightarrow \infty \). Thm 2. Let \( Q, S \) be compact operators in \( H \), \( \dim R(Q) = \infty \), \( N(I - S) = 0 \). Then \( S = (Q+S)S_n^{-1}(A), S = (Q+S)S_n^{-1}(A) \rightarrow A, n \rightarrow \infty \). Here \( \omega(Q) \) is the range of \( Q \), \( N(I - S) = 0 \) is the null space of \( S \), \( S_n \) is the set of all common fixed-points over \( \epsilon \)-fixed-points of \( B \) and \( A \). Let \( A \) be a countably cofinal if \( A \) is a direct sum of \( S \) where \( S \), \( \epsilon \), \( \omega \), \( A \), \( A \), \( B \) are compact operators in \( H \), \( \epsilon \), \( \omega \), \( A \), \( A \), \( B \) are countably cofinal. Thm 3. Suppose \( A \) is countably cofinal and \( \omega(A) \) is the spectrum of \( A \). Then \( \omega(A) \) is countably cofinal if and only if \( \omega(A) \) is a spectrum of \( A \). A-8


Let \( \pi, \rho \) be unitary representations from a separable C*-algebra \( A \) into the set \( B(H) \) of operators on a Hilbert space \( H \). Write \( \pi \sim \rho \) if there is a sequence \( \{U_n\} \) of unitary operators such that \( ||U_n(\pi(a)U_n^* - \rho(a))|| \rightarrow 0 \) for all \( a \) in \( A \). Let \( \alpha \) be an infinite cardinal and \( J_\alpha \) be the closure of the set of operators with rank less than \( \alpha \). If \( \pi \sim \rho \) and the preceding \( U_n \)'s can be chosen so that \( U_n(\pi(a)U_n^* - \rho(a)) \in J_\alpha \) for all \( a \) and \( n \), then write \( \pi \sim_{J_\alpha} \rho \). Let \( \text{Irr}(A) \) be the set of irreducible representations of \( A \). If \( \sigma \in \text{Irr}(A) \), write \( \text{mult}(\sigma; \rho) = \sup(\beta: B \text{ a cardinal, a direct sum of } \beta \text{ copies of } \sigma \text{ is a direct summand of } \rho \text{ with } \rho \sim \sigma) \). Call \( \alpha \) a countably cofinal if \( \alpha \) is a l.u.b. of a countable set of cardinals less than \( \alpha \). Thm 1. Each implies the others: (1) \( \pi \sim_{J_\alpha} \rho \), (2) \( \text{rank } \pi(a) = \text{rank } \rho(a) \) for all \( a \) in \( A \), (3) \( \text{mult}(\sigma; \rho) = \text{mult}(\sigma; \rho) \) for all \( \sigma \) in \( \text{Irr}(A) \).

Thm 2. If \( H = \alpha \) is countably cofinal and \( \pi \sim \rho \), then \( \pi \sim_{J_\alpha} \rho \). Thm 3. Suppose \( \alpha \leq \text{dim } H \). Then \( \alpha \) is countably cofinal if and only if closed, unitarily separable subalgebra of \( B(H)/J_\alpha \) is reflexive. These results extend and utilize the results of D. Voiculescu [Rev. Rom. Math. P. et Appl. 21 (1976) 97-113]. (Received September 19, 1977.)

SANFORD S. MILLER, State University of New York, Brockport, New York 14420, MAXWELL O. READE, The University of Michigan, Ann Arbor, Michigan 48104 and PETER T. MOCANU, Babes-Bolyai University, Cluj-Napoca, Romania. The order of starlikeness of alpha-convex functions. Let \( f(z) \) be an alpha-convex function and define the Koebe alpha-convex function.
\[ k_\alpha(z) = \left[ \frac{1}{\alpha} \int_0^z w^{1/\alpha - 1} (1 + w)^{-2/\alpha} \, dw \right]^\alpha. \]

The authors show that \( z f'(z)/f(z) \) is subordinate to \( z k'_\alpha(z)/k_\alpha(z) \) and use this result to determine the order of starlikeness of alpha-convex functions.

(Received September 21, 1977.)

*78T-B9  ALFONSO G. AZPEITIA, University of Massachusetts, Boston, Massachusetts 02125. On the lower type of entire functions defined by Dirichlet series.

Let \( f(z) = \sum_{n=0}^{\infty} a_n \exp(\lambda_n z) \) be absolutely convergent for all \( z = x + iy \). The Ritt order of \( f(z) \) is \( \rho = \lim \sup_{x \to \infty} \log M(x)/x \), with \( M(x) = \sup \{|f(z)| : |y| < \infty \} \). The lower type is \( w = \lim \inf_{x \to \infty} \exp(p \log M(x)/x) \), if \( p \) is any sequence of indices, then: Theorem 1. (i) \( w \geq \rho/\lambda_{n_0} \), and (ii) \( w \leq \rho/\lambda_{n_1} \), \( n_0 \leq n \leq n_1 \). Theorem 2. If \( \lim \inf_{x \to \infty} \exp(p \log M(x)/x) > 0 \), then \( w = (\rho/\lambda_{n_0})^{-1} \lim \inf_{x \to \infty} \exp(p \log M(x)/x) \). A lower bound for \( w \) previously shown by A. V. Bočurk and A. E. Ėremenko ["The growth of entire functions that are representable by Dirichlet series", Izv. Vyss. Učebn. Zaved. Matematika 1975, no. 5 (156), 93-95, Theorem 2; English transl., Soviet Math. (Iz. VUZ) 19 (1975), no. 5, 73-75. MR 53 #3302] is shown to be incorrect. (Received September 28, 1977.)


For terminology, see Abstract 747-02-6, these Notices, 24(1977), p. A-447. Let \( K \) be a TODF and let \( C \) be the algebraic closure of \( K \). Theorem 1. The Cantor completion of \( C \) is a copy of \( C \). Theorem 2. A necessary and sufficient condition for a commutative Hausdorff topological algebra \( X \) to be a commutative Hausdorff topological division algebra with continuous inversion is that \( X \) be a commutative division algebra.

Theorem 3. A commutative Hausdorff division algebra over \( C \) is a copy of \( C \). Theorem 4. A commutative Hausdorff topological division algebra over \( K \) is a copy of either \( K \) or \( C \). Definition. A neocomplex number is an element of \( C \). Remark. There exists a counterexample to Zelazko's conjecture (8.10. Conjecture, W. Zelazko, Rozprawy Matematyczne, XLVII, Warszawa, 1965, Metric generalizations of Banach algebras, p. 28). There is a commutative Hausdorff topological algebra which has no generalized topological divisors of zero and which is not a copy of \( K \) or \( C \). (Received September 23, 1977.)

*78T-B11  DAVID ZEITLIN, 1650 Vincent Ave., North, Minneapolis, Minnesota, 55411. An inequality for a class of polynomials.

Theorem. Let \( \{C_k\} \) be a non-decreasing sequence with \( C_0 = 0 \) and \( C_k - B k_{k-1} \leq 1 \), \( k = 1, 2, \ldots \), where \( B \) is a constant, \( 0 \leq B \leq 1 \). Then, for \( x \geq 1 \) and \( p = 2, 3, \ldots \), we have the polynomial inequality

\[ \sum_{k=1}^{n} 2^{p-1} C_k x^k \leq p \left( \sum_{k=1}^{n} 2^{p-1} x^k \right)^2, \quad n = 1, 2, \ldots. \] REMARK. For \( x = B = 1 \), (1) gives (18), p. 29, in the paper by Klaasen and Newman, Amer. Math. Monthly, 83(1976)26-30. Double induction is used to prove (1). For integer sequences, we have the following CONJECTURE. Let \( U_k \), with \( U_0 = 0 \), \( U_1 = 1 \), and \( V_k \), with \( V_0 = 2 \), \( V_1 = 3 \), be two solutions of \( W_{k+2} = P W_{k+1} + Q W_k \), \( k = 0, 1, \ldots \), where \( P \) and \( Q \) are integers with \( P \geq 2 \) and \( P \geq 2 \). We now claim that

\[ \sum_{k=1}^{n} U_k \leq V_n \left( \sum_{k=1}^{n} U_k \right)^2, \quad n = 1, 2, \ldots. \] REMARK. For \( P = 2 \) and \( Q = 1 \), \( U_k = k \), \( V_n = 2 \), and (1) gives the well-known identity, \( \sum_{k=1}^{n} k^3 = \left( \sum_{k=1}^{n} k \right)^2 \). Using double induction, one can prove the conjecture for \( P \geq 3 \), which leaves the two cases \( P = 2 \) and \( P = 1 \) open. (Received September 30, 1977.)

*78T-B12  JENG-ENG LIN, National Tsing Hua University, Hsinchu, Taiwan 300, Republic of China. Asymptotic behavior in time of the solutions of three nonlinear partial differential equations.

Let \( p \geq 3 \) be an odd integer and \( s \) be a positive integer. Consider (1) \( u_t + u_{xx} - A-9 \)
\[ u^p - u_x = 0, \quad (2) \quad u_{tt} - u_{xx} + (g(x) + 1)u^p + g(x)u = 0, \quad \text{where} \quad g_x < 0, \quad g > 0, \quad \text{and} \quad g \text{ and all its derivatives up to order two are bounded}, \quad (3) \quad iu_{t} - u_{xx} + |u|^2s + u + k(x)u = 0, \quad \text{where} \quad k(x) = 1/(1 + a^2 x^2), \quad \text{with} \quad 0 < a < (2/3)^{1/2}. \quad \text{Then the local} \quad L^2 \text{ norms of the smooth solutions of the above three equations with nice initial data decay to zero for large time. (Received September 28, 1977.)} \]

**An Expansion Theorem in Certain B*-algebras.**

Let \( A \) be an \( H \)-algebra, that is a \( B^* \)-algebra for which there exists a positive linear functional \( \pi \) satisfying \( c \| x \| ^2 \leq \pi (x^*x) \) for all \( x \in A \) for some real positive constant \( c \). Then the following result holds: Let \( A \) be a countably compact \( H \)-algebra, \([a,b]\) a finite interval and \( g(x) \) an \( A \)-valued, continuous function on \([a,b]\) with spectrum \( \sigma = \sigma (g(x)) \) such that \( \inf \{ \sigma (g(x)) : x \in [a,b] \} \) is finite. Then the problem \( y'' + (\lambda - g(x)) y = 0, y(a,\lambda) \cos \mu - y'(a,\lambda) \sin \mu = 0, 0 < \mu < \pi/2, y(b,\lambda) = 0 \), has a non-trivial solution for only a countable number of real values \( \lambda \) of \( \lambda \), and if \( z_1 \) is the solution corresponding to \( \lambda_1 \), then for any \( A \)-valued, twice continuously differentiable function \( q(x) \) on \([a,b]\) satisfying the above boundary conditions, we have \( g(x) = \sum \frac{1}{2} g(z_1 z_1^*) z_1 (x) \).

An approximation theorem in the singular case for functions on \((-\infty, \infty)\) is also obtained. (Received October 3, 1977.) (Author introduced by Dr. C.E. Billigheimer.)

**Some Generalizations of Markovian Decision Processes.** Preliminary report.

Consider the transformation \( x = My \), where \( M \) is a Markov matrix. This transformation preserves the region \( x_i \geq 0, \sum x_i = 1 \). Conversely, if we ask that this region be preserved then \( M \) must be a matrix. If we consider the transformation \( x = M(g) y \), where \( g \) is a decision variable, we obtain Markovian decision processes. One generalization of Markov processes is the matrix transformation \( x_i = \sum A_i X_i A_i^t \). R. Bellman, Introduction to Matrix Analysis, McGraw-Hill Book Company, New York, 1960; 2nd Edition, 1970. This prime indicates the transpose. This transformation preserves the region \( \text{tr}(\sum X_i = 1), x_i \geq 0 \). Here the order relation is that of non-negative definite matrices.

If we allow the \( A_i \) to depend upon a decision variable we obtain a generalization of Markovian decision processes.

If we consider the transformation \( x = Ty \), where \( T \) is an orthogonal matrix, it preserves the region \( (x, x) = 1 \). The \( x_i^2 \) may be interpreted as probabilities. Conversely, if we ask that this region be preserved, then \( T \) must be orthogonal. If we allow \( T \) to depend upon a decision variable, we obtain another generalization of Markovian decision processes. The continuous version of these processes plays a role in quantum mechanics in the design of experiments. There is a corresponding matrix generalization. (Received October 6, 1977.)

**Maximizing functions for analytic diameter calculations.**

Notation will be as in the author's Analytic Centers and Analytic Diameters of Planar Continua, Trans. Amer. Math. Soc. 191(1974), 83-93. In Theorem 4 of that paper, the analytic diameter function \( \beta(K, z) \) was calculated for the case \( K \) a planar continuum, but the corresponding maximizing functions were not exhibited. We announce here the following

**Theorem:** Let \( K, g, \omega \) be as usual, and let \( z \in \mathbb{C} \).

1. If \( |z - \omega| \leq 2 \gamma(K) \), let \( f(z) = g(\xi)(g(\xi) - \lambda)/(1 - g(\xi)), \quad \lambda = (z - \omega)/2 \gamma(K) \).
2. If \( |z - \omega| > 2 \gamma(K) \), let \( f(z) = -(\bar{z} - \omega)g(z)/|z - \omega| \).

Then \( f \) is the unique element of \( A(K) \) such that \( \beta(f, z) = \beta(K, z) \gamma(K) \).

This result will not be published elsewhere, but further details are available upon request. (Received October 6, 1977.)
We consider the problem of determining the region of variability of \((a_p, a_q)\) for the trinomial 
\[ z + a_p z^p + a_q z^q, \quad p < q \]
to be univalent in \(|z| < 1\). It is shown that for \(0 < t = a_q < \frac{1}{q}\),
\[ a_p \in \bigcap_{0 \leq \theta < \pi/2} \left\{ \frac{\sin \theta}{\sin \theta} \right\} \cap G_q, \]
where \(G_q\) is the region determined by the curve \(w(\phi) = e^{-(p-1)i\phi + t} \sin \phi \sin \theta(q-p)i\phi, \quad \phi \in [0, 2\pi]\)
and containing the origin.

In general it is difficult to simplify this intersection but in certain special cases it turns out to be \(\frac{1}{p} \cos \phi\). If \(t = \frac{1}{q}\) then \(a_p = 0\) except when \(p = \frac{q+1}{2}\) in which case \(a_p\) must be real and 
\[ |a_p| \leq \frac{2p-1}{2q} \sin \frac{\pi}{2p}. \]
The information about the coefficient region \((a_p, a_q)\) helps us to prove that if 
\[ f(z) = z + a_q z^q \]
is univalent in \(|z| < 1\) then so are
\[ f(z) = z + a_p z^p + a_q z^q, \quad p < q. \]
(Received October 11, 1977.) (Author introduced by P. M. Gauthier).

**Applied Mathematics**

(65, 68, 70, 73, 76, 78, 80–83, 85, 86, 90, 92–94)

#78T-C1


**Nonlinear Mathematical Model of the Flow Past an Island in a Tidal Estuary.**

A nonlinear mathematical model of the flow past an island in a tidal estuary is developed with the aid of the explicit finite difference scheme with leap-frog operator. The present model requires no additional approximations than what are usually adopted in the case of a single channel without any branching or island in it. The results of the model are analyzed and then compared with the case of the Nayachara Island in the river Hooghly. A detailed comparative study between the computed and observed results is presented in order to prove the effectiveness of the model. (Received August 24, 1977.)

#78T-C2

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**A mixed theory of information I: Recursive, symmetric and measurable entropies of randomized systems of events.**

In classical, probabilistic information theory, measures of information depend only upon probabilities of events (messages, outcomes of an experiment, market situation, etc.). In the theory of information without probability, such measures do not depend on probabilities at all, only upon the events themselves. We propose a mixed theory of information where measures of information depend both on the events and their probabilities. For instance, in the entropies
\[ H_n(x_1, \ldots, x_n) \]
of sets, \(x_k = \emptyset (k \neq 1, 2, \ldots, n)\), \(p_k > 0\), \(\sum p_k = 1\). We state here the first characterization theorem on inset entropies.

**Theorem.** Inset entropies are 3-symmetric
\[ I_3(x_1, x_2, x_3) = I_3(x_1, x_3, x_2), \]
recursive
\[ I_n(x_1, x_2, x_3, \ldots) = I_{n-1}(x_1, x_2) + (p_1 + p_2)I_2((p_1 + p_2), p_2 / (p_1 + p_2)) \]
\(0.1. \quad x_1, x_2 \geq 0\) and \(\forall x_1, x_2 \) is measurable for all fixed \(x_1, x_2\), iff there exists a constant \(b\) and a function \(g: [0, 1] \to \mathbb{R}\) such that
\[ I_n(x_1, \ldots, x_n) = g(x_k) - b\sum k \log p_k, \]
\(0. \log 0 = 0\), \((n=2, 3, \ldots)\). (Received September 15, 1977.)
Here is a second characterization theorem in the mixed theory of information (cf. the above abstract).

**Theorem:** Inset entropies are \((2,3)\)-additive

\[
I(x_1 \oplus \cdots \oplus x_n) = \sum_{k=1}^{n} \left( x_k \log P_k \right) \quad (g: B \rightarrow [0,1] \text{ measurable}) \text{ iff there exist constants } a, b \text{ such that } I(x_1 \oplus \cdots \oplus x_n) = a(n+1) - b \log P_k \quad (0.0 \log 0 := 0), \quad (n=2,3,\ldots) \]

So, in this case, the inset entropies turn out to depend only on the probabilities.

(Received September 15, 1977.)

Linear multistep methods for the numerical solution of

\[ y'' = f(x,y) \]

are considered. A method is said to have an interval of periodicity if the numerical solution of \( y'' = -\lambda^2 y, \lambda > 0 \), \( \lambda \) fixed, neither decays nor explodes for all stepsizes \( h \in (0, h_0) \), \( h_0 \) sufficiently small. It is shown that a convergent multistep method has an interval of periodicity if and only if it is a symmetric method and all growth parameters are positive. Moreover a multistep method with optimal error order which has -1 as an essential root cannot have an interval of periodicity.

(Received October 3, 1977.)


Let \( FG \) be the group ring of a finite cyclic extension group \( G \) over the field \( F \) of two elements. The left ideals in \( FG \) are called non-abelian codes. They are generalizations of cyclic codes, i.e., ideals in \( F \mathbb{Z} \) (where \( \mathbb{Z} \) denotes the cyclic group of order \( n \)). The structure of such codes is studied. Distance bounds are found. The implementation of such codes in terms of linear sequential circuits is briefly given. (Received October 6, 1977.)

Herman Rubin, Department of Statistics, Purdue University, W. Lafayette, IN 47907. Computational complexity of the generation of non-uniform random variables. Preliminary report.

Let \( F \) be a distribution depending on parameters \( \lambda_1, \ldots, \lambda_n \) which are computable real numbers, i.e., \( a_{ij} \leq \lambda_j \leq b_{ij}, \quad a_{ij}, b_{ij} \text{ rational, and } b_{ij} - a_{ij} \rightarrow 0 \) at a computable rate. Then under reasonable regularity conditions, there exists an infinite precision procedure (see these Notices, February 1977, Abstract 77T-C21) for obtaining a random variable \( X \) whose distribution is exactly \( F \). Under suitable additional restrictions the expected cost can be shown to be finite. These results remain true if some of the \( \lambda \)'s are themselves the results of infinite precision procedures. (Received October 11, 1977.)
A set $A$ of $n \times n$ complex matrices is stable if for every neighborhood of the origin $U$, there exists another neighborhood of the origin $V$ such that for each $M \in A'$ (the set of finite products of matrices in $A$), $MV \subseteq U$. Matrix and Liapunov stability are related:

**Theorem** A set of matrices $A$ is stable iff there exists a Liapunov function $w(z)$ such that for all $M \in A$, $w(Mz) \leq w(z)$, where $w$ is convex, continuous, and absolutely homogeneous of degree 1.

The Liapunov function need not be smooth; using smooth functions to prove stability can be inadequate. The totally novel central result is a constructive algorithm which can determine the stability of $A$ based on the following:

**Theorem** $A = \{M_0, M_1, \ldots, M_{m-1}\}$ is a set of $m$ distinct complex matrices. Let $W_0$ be a bounded neighborhood of the origin. Define for $k > 0$, $W_k = \text{convex hull}[U_{w_0^k} = (M_k)W_{k-1}]$, where $k' = (k-1) \mod m$.

Then $A$ is stable iff $W_\infty = \bigcup_0 \infty W_k$ is bounded.

The constructive algorithm represents a convex set by its extreme points and uses linear programming to construct the successive $W_k$. Sufficient conditions for finiteness and for stopping the algorithm are presented. $A$ is generalized to be any convex set. A dynamical system of differential equations is stable if a corresponding set of matrices - associated with the logarithmic norms of the matrices of the linearized equations - is stable. (Received October 11, 1977.) (Author introduced by Mar..)

**Geometry (50, 52, 53)**

*78T-D1* Bang-yen Chen, Michigan State University, East Lansing, Michigan 48824. **Cobordism, A-genus and Einstein-Kaehler metrics.**

Let $A(M)$ and $T(M)$ be the A-genus and Todd genus of a complex 4-dimensional complex manifold $M$ and let $(M) = a(M)\gamma_4(C) + b(M)\gamma_2(C)\cdot\gamma_2(C)$ be the cobordisme decomposition of $M$ with cobordisme coefficients $a(M)$, $b(M)$. The following results are obtained. **Theorem 1.** If $M$ admits an Einstein-Kaehler metric, then $A(M) \leq 256T(M)$. If $A(M) = 256T(M)$, every Einstein-Kaehler metric on $M$ is Ricci-flat. Conversely, if $M$ admits a Ricci-flat Kaehler metric, $A(M) = 256(M)$. **Theorem 2.** If the cobordisme coefficients of $M$ satisfy $3a(M) + 2b(M) > 128T(M)$, then $M$ admits no Einstein-Kaehler metric. Some applications are also obtained. Two typical examples are as follows. **Corollary 1.** Let $P^2[k]$ be the rational surface obtained from the complex projective plane by blowing up $k$ points. Then $P^2[k+1] \ni P^2[m+1]$ admits no Einstein-Kaehler metric if $km > 64$. **Corollary 2.** If a complex manifold $N$ is obtained from a given 4-dimensional complex manifold by blowing down sufficient many points, then $N$ admits no Einstein-Kaehler metric. (Received September 12, 1977.)

*78T-D2* Martin A. Magid, Brown University, Providence, RI 02912. **Isometric Immersions Between Indefinite Flat Spaces with V*a=0, Examples.** Prel. Report

Let $\mathbb{R}^n$ denote the $n$-dim. affine space equipped with the standard metric of signature $(p,n-p)$, $\mathbb{R}^n_+(r) = \{x \in \mathbb{R}^n : <x,x> = r > 0\}$, $\mathbb{R}^n_-(r) = \{x \in \mathbb{R}^n : <x,x> = r < 0\}$ and $\mathbb{R}^n_{\pm}=\{x \in \mathbb{R}^n : <x,x> = 0\}$.

The following examples are obtained. 1. Umbilic immersions with light-like mean curvature vector. $f: \mathbb{R}^n \rightarrow \mathbb{R}^{n+2}$, $(x_1, \ldots, x_n) \rightarrow (x_1, \ldots, x_n, a_1x_1^2 + \cdots + a_nx_n^2)$ where the metric on $\mathbb{R}^n$ is $(\sigma_1, \ldots, \sigma_n)$ and that on $\mathbb{R}^n_{\pm}$ is $(\sigma_1, \ldots, \sigma_n, -\sigma_k)$. $\sigma_k = +$ or $-1$.

2. Complex circle of radius $\kappa \in \mathbb{C}$. $f: \mathbb{R}^2 \rightarrow \mathbb{H}^3 \subset \mathbb{R}^4$. $(x,v) \rightarrow x+iv = z = \kappa(\cos\phi, \sin\phi)$ where $c^2$ is identified with $\mathbb{R}^4$ by sending $(u, v, r, s)$ to $(u, v, r, s)$. The metric on $\mathbb{R}^4$ is $(u, v, r, s) = u^2 - v^2 + r^2 - s^2$.

3. Following L. Graves (Notices, Aug. 1977) take the null curve $x(v)$ in $\mathbb{R}^2$ with frame $A(v), B(v)$ light-like and $C(v), D(v), E(v)$ Lorentz orthonormal, $x' = A, A' = B, B' = 0, C' = -D, D' = -E, F' = -B - D$. Let $f(u, v, w) = wB(u) + vD(u) + x(v)$. (Received October 11, 1977.)
Logic and Foundations (02, 04)

1978-T-01 PAUL BANKSTON, University of Kansas, Lawrence, Kansas, 66045. On elementary categories.

Let C be a category, k an infinite cardinal. C is k-elementary if there is a first order theory T in a predicate language of power at most k such that C is naturally equivalent to the category of models of T plus all homomorphisms. C is dual k-elementary if C^op is k-elementary. The category of rings of continuous functions on zero-dimensional compact Hausdorff spaces, plus homomorphisms, is ω-elementary (T = the theory of Boolean algebras). The category of compact Abelian groups, plus continuous homomorphisms, is dual ω-elementary (T = the theory of Abelian groups). Let C, C denote respectively the categories of complete and complete atomic Boolean algebras, plus homomorphisms. We use ultraproduct techniques in a category-theoretic setting to prove that neither category can be (dual) k-elementary for any k. (Received August 9, 1977.)


If A is an admissible set, let HC(A) = {x|A^x is hereditarily countable}. Theorem 1. HC(A) is admissible. If X is a set of reals, write X=Ext I1^1-AC iff (∗) \( \forall n \exists R \in XQ(n,R) \land \exists s \in X \forall n \forall (s)_n \) where Q is \( \Pi^1_1 \) (in \( \forall \)) with parameter from X. Theorem 2. X is the set of reals in an admissible set iff X=Ext I1^1-AC. X is \( \Pi^1_1 \)-Strong if WO\( \times \)X is \( \Pi^1_1 \) over X (WO = the set of well-ordered reals). Theorem 3. X is the set of reals in an admissible set iff (i) X is \( \Pi^1_1 \)-Strong, X/= Ext I1^1-AC, or (ii) X is not \( \Pi^1_1 \)-Strong, X/= Ext I1^1-AC. Theorem 5. X is the set of reals in an admissible set satisfying \( \Sigma^1_1 \)-DC iff (i) X is a \( \beta \)-model of \( \Sigma^1_2 \)-AC, or (ii) X is not a \( \beta \)-model, X/= \( \Sigma^1_1 \)-DC. Theorem 6. If \( \phi \) is a sentence of analysis true in some \( \omega \)-model of \( \Sigma^1_1 \)-AC(\( \Sigma^1_1 \)-DC), then \( \phi \) is true in some admissible set (some admissible set satisfying \( \Sigma^1_1 \)-DC). The first part of Theorem 6 is due to Leo Harrington. (Received September 26, 1977.)


Let \( \mathcal{M} \) be the Fraenkel-Mostowski model of ZFU obtained from an uncountable set of urelements U, the group of all permutations of U, and the filter of subgroups obtained by allowing supports to be countable. (See [1]).

Theorem. \( \mathcal{M} \models \) (Every two cardinals have a greatest lower bound) \( \& \rightarrow \) AC.

This shows that AC is independent of "Every two cardinals have a greatest lower bound" in ZFU, answering a question posed in [2]. This result does not seem to be transferable to ZF.


A-14
Let $k$ be a finite or infinite cardinal. Define: A class of structures $K$ is of $k$-character whenever: $\forall \in K$ iff every substructure of $\forall$ with $< k$ generators belongs to $K$. (As generators, we use but do not count the constants of $\forall$.) Note: $\omega$-character is called finite character.

Define: A sentence of $L(V)$ is $\omega$-universal if it is of the form $\forall \bar{x} \exists \bar{y} \Phi(\bar{x}, \bar{y})$ where $\Phi(\bar{x})$ is a quantifier-free sentence and $\bar{x}$ has length $< \omega$. A sentence of $L(V)$ is $\in$-finite if it is of the form $\forall \exists \Phi(\bar{x}, \bar{y})$ where $\Phi(\bar{x}, \bar{y})$ is a quantifier-free sentence and $\bar{x}$ has length $<\in$. Theorem 1 Let $\kappa < \omega$, let $A$ be a countable admissible set and let $\Phi \subseteq L_{\omega}$. Then the models of $\Phi$ form a class of $\kappa$-character iff there is an $A$-finite set of $L_{\omega}$-sentences $\forall_1, \ldots, \forall_n$ $(n < \kappa)$ such that $\forall_\kappa$ is $(\in\kappa)$-universal and $\forall \Leftrightarrow \bigwedge \forall_i$. Theorem 2 Let $\kappa > \omega$, let $L$ be a language, let $L_\kappa$ be an abstract logic (see J. Barwise, Annals of Math. Logic 7 (1974), 221-265) and let $T$ be a theory in $L^\kappa$. Then the models of $T$ form a class of $\kappa$-character iff there is a set of $\kappa$-universal sentences $\Phi \subseteq L_{\omega}(\kappa, L(T))$ such that $\bigwedge \Phi \Leftrightarrow \bigwedge T$ is valid. (Received September 19, 1977.)

*87T-E5* ALAN H. MEKLER, Carleton University, Ottawa, Canada K1S 5B6. Model Completions with a distinguished subset. Preliminary report.

For a language $L$, unary predicate $U$ not in $L$, and $F \in L$, $F^U$ is the relativization of $F$ to $U$. If $K \subseteq L$ then $K^U = [F]^U [F \in K]$. Suppose $T \subseteq L$ and $S \subseteq L(U)$. $S$ is an enriched model completion of $T$ if: (1) $S \cap L$ is the model completion of $T$ (2) $S \geq T$ (3) for all $M \models T$, $S \cup \{M \in L(U)\}$ is complete and has a model where the interpretation of $U$ is $M$. Theorem 1 (analogue of elimination of quantifiers): Assume $S$ is an enriched model completion of $T$ then for any formula $F(x_0, \ldots, x_n) \in L(U)$ there exists $G(x_0, \ldots, x_n) \in L$ such that $S \models (\forall x_0 \ldots x_n) \bigwedge F(x_0, \ldots, x_n) \leftrightarrow G(x_0, \ldots, x_n)^U$. Theorem 2 Suppose $T'$, a countable theory, is the model completion of a universal theory, $T$, and for all countable $M \models T'$, $T' \cup \text{Diag}(M)$ is $\omega_1$-categorical. Then $T$ has an enriched model completion, namely $T' \cup \{U(x_0, \ldots, x_n) \mid x_0 \neq x_1 \neq \cdots \neq x_n \}$. Theorem 3 There is a universal theory with an $\omega$-stable model completion and no enriched model completion. (Received September 30, 1977.)


We establish a relation between higher order logic and definability in the Levy-hierarchy. It is well known that the finite order logic $L^{(\omega)}$ coincides with the second order logic $L^{II}$ as far as decision and spectrum problems are concerned. We prove: A model class is $\Delta$-definable in $L^{II}$ iff it is $\Delta$-definable. $\Delta$-definability with parameters corresponds to in definability in $L^{II}$ is $\Delta$-definable. This is a special case of a general theorem about abstract logics. The decision problem of $L^{II}$ is the complete $\Pi^2_1$-definable set of integers. The Löwenheim number $\hat{\theta}^{II}$ of $L^{II}$ is the supremum of all $\Delta^2_2$-definable cardinals, and the Hanf number $\hat{\aleph}^{II}$ of $L^{II}$ is the supremum of all $\Delta^2_2$-definable cardinals. The proof of this uses the following characterizations: we say a cardinal $\kappa$ is (weakly) $\text{largest}$ order describable if there is a sentence $\phi$ of set theory such that $\phi$ is true in $R_{\alpha}$ for $\lambda > \kappa$ and false for (arbitrary large) $\lambda < \kappa$. $\hat{\theta}^{II} = \sup\{\kappa \mid \kappa$ is largest order describable $\}$ $= \text{the least } \kappa$ such that $R_{\kappa}^{\text{ZFC}}$. $\hat{\aleph}^{II} = \sup\{\kappa \mid \kappa$ is weakly largest order describable $\}$. (Received October 4, 1977.) (Author introduced by K. J. Devlin).

*87T-E7* MATATYAHU RUBIN, University of Colorado, Boulder, Colorado 80309, U.S.A. An almost Johnson Boolean algebra.

This result is a variant of our construction which yielded the results in: "Some results on Boolean algebras", These Notices, Nov., 1977. Define: An algebra $A$ of power $\kappa$ is almost Johnson (AJ), if for every subalgebra $C$ of $A$ of power $\kappa$, there is a subset $D$ of $A$,
In $\mathbb{K}$, s.t. $C \cup D$ generates $A$. $A$ has $K$ intersection property (is $K$-IP), if the intersection of any two subalgebras of power $K$ is of power $K$. Theorem: (1) There is a BA $B$ of power $\aleph_1$ such that: (1) $B$ is AJ; (2) $B$ is $\aleph_1$-IP; (3) $B$ has just $\aleph_1$ sublattices. Let $I \subseteq B$ be the ideal of all elements $b$ in $B$ that have just $\leq \aleph_0$ elements $\preceq b$. So $I$ is a prime ideal; (4) $I$ is an AJ lattice. (5) $I$ is an $\aleph_1$-IP lower-semi-lattice (LSL).
(6) $I$ has just $\aleph_1$ sub-LSL's. (7) Every uncountable subset of $I$ contains a chain of order type $\omega_1$, and three distinct elements $a, b, c$ s.t. $a \land b = c$. Remark: Using Shelah's omitting type theorem this result transfers to $\lambda^+$ whenever $\diamondsuit$ and $\diamondsuit^+$ hold. Remark: The undecidability of the theory of BA's in Magidor-Malitz's language follows from $\mathsf{GH}$. One can use the BA's constructed by R. Bonnet, and Berney, Nyikos. So $\diamondsuit^+$ is not needed. (Received September 23, 1977.) (Introduced by J. D. Monk.)

Robert P. Daley, Computer Science Department, University of Pittsburgh, Pittsburgh, PA 15260. Busy beaver sets and the degrees of unsolvability.

This paper, which is a greatly expanded version of the preliminary report announced in these Notices (June 1977, p A-392), uses the primitives of axiomatic computational complexity theory to simplify and/or eliminate a number of finite injury and infinite injury priority constructions in the degrees of unsolvability. In addition to the construction of a solution to Post's problem without the finite injury priority method it is shown that Sacks' Jump Theorem can be proven using a zero injury priority argument and Sacks' Density Theorem can be proven using only a finite injury priority argument. The latter two theorems heretofore have required the use of infinite injury priority arguments. (Received October 11, 1977.)

Topology (22, 54, 55, 57, 58)

Matthew Witten, State University of New York at Buffalo, Buffalo, New York 14226. Conjugacy classes in the "Simplest Dynamical System". The conjugacy classes of the simplest dynamical system $f_b(x) = bx(1-x)$ are discussed. It is shown that the conjugacy classes of $f_b$ can be parameterized by $b$ in the following manner: $b = 0$, $b \in (0,1]$, $b \in (1,2)$, $b = 2$, $b \in (2,3)$, $b = 3$, $b = 4$, and at least countably many conjugacy classes $\mathfrak{R}_j$ in the interval $(3,4)$. In fact, it can be shown that there are countably many conjugacy classes in the interval $(3,3.58)$, and at least countably many more conjugacy classes $\mathfrak{R}_j$ in the interval $(3,3.58,4)$. A method for construction of these conjugacy classes is exhibited. It is shown that there are some conjugacy classes $\mathfrak{R}_j$ in the interval $(3,3.58)$ which are the union of two or more conjugacy classes $\mathfrak{R}_j$. An example of this property is also illustrated. Some general conjugacy theorems which are natural extensions of this work are presented are discussed. (Received July 5, 1977.)

J. GRISPOLAKIS and E. D. TYMCHATYN, University of Saskatchewan, Saskatoon, Sask. S7N 0W0, Canada. Embedding smooth dendroids in hyperspaces. Preliminary report.

A dendroid is an arcwise connected hereditarily unicoherent metric continuum. A dendroid $D$ is said to be smooth provided there exists some point $p \in D$ such that $\lim_{n \to \infty} a_n = a$ implies that $\lim_{n \to \infty} [p, a_n] = [p, a]$, where $[x, y]$ denotes the unique arc in $D$ with end-points $x$ and $y$. Let $C(X)$ denote the hyperspace of all subcontinua of the metric continuum $X$ with the Vietoris topology. The following theorem answers in the affirmative a question of S.B. Nadler, Jr. Theorem. Let $X$ be a hereditarily indecomposable continuum.
Then every smooth dendroid is embeddable in \( C(X) \). Combining this theorem with a result of Nadler we obtain the following: Corollary. An arcwise connected one-dimensional continuum is a smooth dendroid if and only if it is embeddable in \( C(X) \) for some hereditarily indecomposable continuum \( X \).

(Received August 23, 1977.)

\*78T-G3 CARLOS R. BORGES, University of California, Davis, California 95616. Extension properties of CW-complexes: answer to a question of O. Hanner.

In 1952 O. Hanner (Ark. Mat., 2(1952), 315-360) gave necessary and sufficient conditions for a simplicial complex with the Lefschetz metric topology to be an ANR (collectionwise normal; normal; Tychonoff), respectively. He also remarked that "The corresponding problems in the case of the weak topology are unsolved". Our primary purpose is to answer Hanner's questions. **Theorem.** Let \( K \) be a simplicial complex with the weak topology. Then (a) \( K \) is an ANE (collectionwise normal; paracompact) if and only if \( K \) is a locally finite polyhedron. (b) \( K \) is an ANE (normal) if and only if \( K \) is a locally finite polyhedron with a countable number of vertices, if and only if \( K \) is an ANE (Tychonoff). (Received September 19, 1977.)

\*78T-G4 Jonathan A. Hillman, Pure Mathematics, S.G.S., Australian National University, P.O. Box 4, Canberra, 2600. Alexander Ideals of Classical Links.

**Theorem** If \( G \) is a finitely presentable group with \( G/G' = \mathbb{Z}^\mu \) then the following two conditions are equivalent
1) \( \epsilon_{\mu-1}(G) = 0 \)
2) For each integer \( q \geq 1 \), the Chen group \( Q(G;q) = G/G'/G_{q+1}^\mu \) is isomorphic to \( Q(F;q) \), where \( F \) is the free group of rank \( \mu \).

Further, if \( G \) is the group of a \( \mu \)-component link \( L : \bigcup_{i=1}^{\mu} s^1_s \), then 1) and 2) are equivalent to:
3) The longitudes of \( L \) are in \( G(\infty) \).

(Received September 19, 1977.)

\*78T-G5 David A. Singer and Robert J. Wolfe, Case Western Reserve University, Cleveland, OH 44106. Structurally Stable Endomorphisms of the Unit Interval. Preliminary Report.

For \( k \geq 0 \) the \( C^k \)-endomorphisms of the closed unit interval form a topological space \( g^k \) in the uniform \( C^0 \)-topology. Two endomorphisms \( f \) and \( g \) are said to be **topologically conjugate** if there is a homeomorphism \( h \) of the interval for which \( hf = gh \). An endomorphism is **structurally stable** provided it is an interior point of its equivalence class under topological conjugacy. Our main result gives sufficient conditions for structural stability for \( C^k \)-endomorphisms \( (k \geq 3) \) having a single critical point.

Let \( \mathcal{E} \) consist of those endomorphisms \( F \) that satisfy the following conditions:
(1) \( F(0) = F(1) = 0 \); (2) \( F \) has one critical point; (3) the Schwarzian derivative of \( F \) is negative on the interval. The Schwarzian derivative of \( F \) at \( x \) is defined to be the number \( (F_3(x))^2 (F_3(x) F''(x) - \frac{3}{2} [F''(x)]^2) \); it originated in the work of H. A. Schwarz on conformal mappings.

**Theorem.** There exists an open dense subset \( \mathcal{W} \subseteq \mathcal{E} \) consisting of structurally stable endomorphisms. (Received September 22, 1977.)
A new property of pseudo-open functions is presented in this paper. Pseudo-open functions are monotonic decreasing with respect to ordinal and cardinal invariants defined by compact and sequential closures. A weak form of continuity, called k-continuous, is defined, characterized and used in the proof of the monotonicity properties of pseudo-open mappings. The relationships between the classes of k-continuous and sequentially continuous mappings in the category of all topological spaces and the continuous mappings in the subcategories of k-spaces and sequential spaces are presented. (Received September 26, 1977.)

Birman, Gonzalez-Acuna and Montesinos (Mich. Math. J. 23 (1976), 97-103) have shown that there are prime 3-manifolds which can be represented as 2-fold covering spaces of \( S^3 \) branched over two inequivalent knots. We prove that for a large class of irreducible 3-manifolds with finite fundamental group, this phenomenon cannot happen.

**Theorem 1.** Suppose \( M \) is a Seifert manifold with \( S^2 \) as orbit surface and 3 exceptional fibres of multiplicity \( (2, 2, n) \), \( n > 1 \). Then any two orientation-preserving PL involutions on \( M \) with fixed-points are equivalent.

**Theorem 2.** Suppose \( M \) is a lens space of the type \( L(4k, 2k - 1) \) or \( L(2k, q) \), \( k \geq 1, q \) arbitrary odd. Then any orientation-preserving PL involution on \( M \) with fixed-points is fibre-preserving for some Seifert fibering of \( M \).

**Corollary.** Assume \( M \) is a 3-manifold as in Theorems 1 or 2. Then there is a unique representation (up to equivalence) of \( M \) as a 2-fold covering of \( S^3 \) branched over a knot or link. (Received September 30, 1977.)

**Theorem 1:** Let \( n > 0, q \geq 2, v_{2n+1} : S^{2n+1} \rightarrow \Omega^q_{2n+1} \) be the projection onto the standard Lens space. There is a map \( \gamma_{2n+1} : \mathbb{Z}/q\mathbb{Z} \rightarrow \Omega^q_{2n+1} \) such that \( \mathbb{Z}/q\mathbb{Z} \times \mathbb{Z}^{2n+1} \rightarrow \Omega^q_{2n+1} \times \mathbb{Z}^{2n+1} \rightarrow \mathbb{Z}/q\mathbb{Z} \) is an H-equivalence, where \( \mathbb{Z}/q\mathbb{Z} \times \mathbb{Z}^{2n+1} \) and \( \mathbb{Z}^{2n+1} \) are given the obvious H-group structure.

**Corollary 2:** \( v_q \geq 2 \Omega^3_q = \Omega^7_q \) are homotopy abelian. Remark: For \( q > 2 \), \( L^3_q \) and \( L^7_q \) are not H-spaces. Let \( X \) be a connected CW complex and for \( m \geq 1 \), let \( v_m(X) \) be the nilpotency class of the action of \( \pi_1(X) \) on \( \pi_m(X) \); \( \text{W-long}(X) \) be the Whitehead product length. **Theorem 2:** For all \( m \geq 1 \), \( v_m(X) \leq \text{W-long}(X) \). **Theorem 3:** Now let \( X \) also be nilpotent and let \( X_n \) denote the \( n \)-th stage in the Postnikov system of \( X \). If \( X_n \neq X_{n-1} \times K(n, n) \) for \( s \) values of \( n, n = n_1, n_2, \ldots \), \( n_s \) say, where \( 2 \leq n_1 < \ldots < n_s \) and \( s < \infty \), then a) if \( s = 0 \), \( \text{cokat} X \leq \text{max} \{ v_1(X), 1 \} \), b) if \( s \geq 1 \), \( \text{cokat} X \leq \sum_{m=1}^{s} v_{n_m}(X) + \text{max} \{ v_1(X), 1 \} \). Theorem 4 readily yields the upper bound conjectured in [Bernstein and Ganea, Ill. J. Math. 5 (1961), 99-130, Remark 4.14]. (Received September 27, 1977.)

**WITHDRAWN**
are regular and is irreducible, in the case when \( X \) is a regular \( T_\text{s} \)-space, iff all the preimages \( f^{-1}y, y \in Y \) are cores. If each point \( x \in X \) is semi-regular, the map \( f \) is called half-open. A topological space is called isocompact, if every of its closed countably compact subset is compact and is said to be a \( \varepsilon \)-space, if for every countable discrete family of its points there exists a discrete family of their neighborhoods. Let \( f : X \rightarrow Y \) be a closed, continuous map of an isocompact \( \varepsilon \)-space \( X \) onto a \( \varepsilon \)-space (in the sense of E. Michael) \( Y \), then \( \text{Th.1: The map } f \text{ is peripherally compact and so there exists a closed set } P \subset X \text{ such that } fP=Y \text{ and } f|_P \text{ is irreducible.} \)

\( \text{Th.2: Each point } y \in Y \text{ has a core. } \text{Th.3: If } X \text{ is normal with a point-countable base and } y \in Y \text{ is not isolated in } Y, \text{ then a closed set } P \subset f^{-1}y \text{ is the trace on } f^{-1}y \text{ of some closed set } P \subset X \text{ such that } fP=Y \text{ and } f|_P \text{ is irreducible iff } P \text{ is regular and all of its points are semi-regular. } \text{Th.4: If } X \text{ is normal with a point-countable base, then each point of any core is semi-regular and there exists a maximal closed set } H \subset X \text{ such that } f|_H \text{ is half-open and } TH=X. \text{(Received September 28, 1977.)} \)

\[ \text{WITHDRAWN} \]

\*78T-G12

ROSS GEOGHEGAN, State University of New York Center at Binghamton, Binghamton, N.Y. 13901. Stable fibered compacta have finite homotopy type.

A compactum (\( \varepsilon \) compact metric space) is fibered if it is homeomorphic to an inverse limit

\[ \lim_{\rightarrow} \cdots X_n \leftarrow f_n \]

in which each \( X_n \) is a compactum homotopy equivalent to a finite complex, and each \( f_n \) is a Hurewicz fibration. \( \text{Theorem 1: A fibered compactum which is homotopy equivalent to a complex is homotopy equivalent to a finite complex. } \text{Theorem 2: A fibered compactum which is shape equivalent to a complex is homotopy equivalent to that complex. (Compacts shape equivalent to complexes are sometimes said to be stable, so the title summarizes Theorems 1 and 2. Pointed ANR's are stable; a compactum is stable iff it admits a Siebenmann I-regular neighborhood.) } \text{Theorem 3: Let } X \text{ be the limit of an inverse sequence of compact ANR's and fibrations. Then } X \text{ is stable iff } X \text{ is a compact ANR.} \text{(Received October 6, 1977.)} \]

\*78T-G13

F. T. FARRELL, Penn. State University, University Park, Pa. 16802 and L. E. JONES, SUNY at Stony Brook, Stony Brook, N.Y. 11794. Examples of dynamical systems on an exotic torus.

Let \( T^n = S^1 \times S^1 \cdots \times S^1 \) \((n \text{-factors})\) denote the n-torus with its standard differential structure.

\[ \text{Theorem. For some integer } n, \text{ there exists a smooth manifold } M^n \text{ homeomorphic to } T^n \text{ but not diffeomorphic to } T^n \text{ such that } M^n \text{ supports both an expanding endomorphism and an Avosov diffeomorphism; e.g., } n \text{ can be chosen to be } 9. \text{(Received October 6, 1977.)} \]

78T-G14

H.E. WINKELNKEMPER, University of Maryland, College Park, Maryland, 20742. On twist maps of the annulus. Preliminary report.

Let \( A = \{(r, \theta) \in \mathbb{R}^2 \mid 0 < a \leq r \leq b\} \) denote the planar annulus and \( h : A \rightarrow A \) a twist homeomorphism (in the sense of Birkhoff-Poincaré) with fixed point set \( \text{Fix } h \); let \( A_0 = A - \text{Fix } h \) and \( h_0 : A_0 \rightarrow A_0 \) be the restriction of \( h \) to \( A_0 \). Assume that \( \text{Fix } h \) does not divide \( A \) and let \( \hat{A}_0 \) be the universal cover of \( A_0 \).

\[ \text{Theorem: If } h_0 : A_0 \rightarrow A_0 \text{ lifts to a homeomorphism } \hat{h}_0 : \hat{A}_0 \rightarrow \hat{A}_0 \text{ which sends each component of the boundary of } \hat{A}_0 \text{ into itself, then } h \text{ cannot be topologically transitive.} \]
Corollary: There exists a non-empty open subset of the space of area preserving $C^1$-diffeomorphisms of $A$, with the $C^1$ topology, consisting entirely of diffeomorphisms which are not ergodic.

Hence despite Takens' counterexample (Indag. Math. 33) to the $C^1$ Moser Twist Theorem, the Oxtoby-Ulam Theorem is false in the $C^1$ case. (Received October 7, 1977.)

781-G15 Birgit Speh, University of Chicago, Chicago, IL 60637. Classification of unitary representations of $GL(4, \mathbb{R})$, Preliminary report.

Using Langland's classification of irreducible representations, a theorem of T. Sherman, which states that the coefficients of a unitary (not one-dimensional) representation vanish at infinity, and reducibility results for generalized principal series representations of $GL(n, \mathbb{R})$, a classification of unitary representations of $GL(4, \mathbb{R})$ was obtained. (Received October 11, 1977.)

750TH MEETING
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750-B10 JOHN C. POLKING, Rice University, Houston, Texas 77001. Fundamental solutions for the Cauchy Riemann equations.

- A fundamental solution for the Cauchy Riemann equations in an open set $\Omega \subset \mathbb{C}^n$ is an $n, n-1$ current $K$ in the product space $\Omega \times \Omega$ which satisfies the differential equation $\delta K = [\Delta]$, where $[\Delta]$ is the current of integration over the diagonal $\Delta = \{(z, z) | z \in \Omega\}$. If $K$ is a fundamental solution, we let $K$ also denote the operator with kernel $K$, and then we have the homotopy formula $\phi = \delta K\phi + K \delta \phi$ for a smooth p, q form $\phi$ with compact support in $\Omega$. Explicit fundamental solutions can be constructed using a formalism that goes back to Fantappie and Leray. Specific fundamental solutions of this type have support properties which are useful in showing the existence of solutions to the inhomogeneous Cauchy Riemann equations and to the inhomogeneous induced Cauchy Riemann equations in a large number of situations. (Received September 30, 1977.)

750-G15 MARK FESHBACH, Northwestern University, Evanston, Illinois 60201. On the cohomology of compact Lie groups, Preliminary report.

The following theorem is proved by a simple transfer argument. Let $h$ be any cohomology theory. Let $G$ be a compact Lie group and let $N$ be the normalizer of a maximal torus in $G$. Let $\rho : BN \to BG$ be the natural projection of the classifying spaces.

Theorem. $\rho^* : h(BG) \to h(BN)^s$ is an isomorphism where $h(BN)^s$ is the summand of stable elements in $h(BN)$ over $BG$. For a definition of stable elements see Cartan and Eilenberg p.257.

Other theorems about the cohomology of compact Lie groups also are derived by transfer methods. This talk is a continuation of one given in April at an AMS Regional Conference at Northwestern. (Received September 29, 1977.)

A-20
Commutative Nil Semigroups

A commutative nil semigroup is a semigroup satisfying the following condition. S has a zero and some power of every element is 0. A commutative nil semigroup is called fundamental if the ordering defined by divisibility satisfies the ascending chain condition.

Theorem 1 A commutative nil semigroup S is fundamental if and only if IS I for all nonzero ideals I of S.

A commutative nil semigroup is called globally idempotent if S^2 = S. If every nonzero ideal I of a commutative nil semigroup S satisfies I^2 I, then S is called globally idempotent-free.

Theorem 2 Every commutative nil semigroup S is the ideal extension of a globally idempotent nil semigroup J by a globally idempotent-free nil semigroup G. Moreover J and G are uniquely determined by S.

Theorem 3 A commutative nil semigroup S is globally idempotent-free if and only if S is a tower of fundamental nil semigroups. (Received November 4, 1977.)


Let G be a commutative semigroup with a compatible quasi-order π. R = [−∞, +∞] is the additive (partial) semigroup of real numbers equipped with −∞ and +∞. Let δ (ω) be an R-valued sub (super)-additive function of G. An R-valued function f of G is called monotone if x π y means f(x) ≤ f(y). As a generalization of the so-called Sandwich theorem, we get

Theorem 1. There exists a monotone additive function f of G satisfying w f for all w such that the following hold:

- x π y implies w(x) ≤ δ(y).

From Theorem 1, we get an extension theorem of additive functions:

Theorem 2. Let H be a subsemigroup of G and f be an additive function of H. Suppose there exist h_0, h'_0 ∈ H, x_0 ∈ G, such that f(h_0), δ(h'_0), δ(x_0) I and ω(h_0), f(h'_0), ω(x_0) I. Then, if f is extendible to a monotone additive function f of G satisfying w < f < δ if and only if (**) x_1 ∈ H, x_2 ∈ G, h_1, h_2 ∈ H implies w(x_1) + f(h_1) ≤ δ(x_2) + f(h_2).

(Received November 4, 1977.)

Existence and Uniqueness of the Periodic Solution to a Nonlinear Wave Equation with Boundary Conditions.

Abstract results in Hilbert space are established which apply to many nonlinear P.D.E's with boundary conditions which satisfy a nonresonance condition. For example, consider the nonlinear system

\[ (1) \frac{\partial^2 y}{\partial t^2} + \text{grad}_y G(t,x,y) = f(t,x), \text{ f in } L^2, \text{ with conditions } (2) y(0,x) = y(2\pi,x), y_t(0,x) = y_t(2\pi,x) \]

and \( y(t,x) = 0 \) for \( x \) in the boundary of \( (0,\pi)^m \). Suppose that there exist \( q < p \) such that

\[ q I \leq (\frac{\partial^2 G/\partial y_j \partial y_j}{\partial y_j \partial y_j}) \leq p I \text{ on } (0,2\pi) \times (0,\pi)^m \times \mathbb{R}^k. \]
THEOREM: If $f$ and $G$ are as above, then (1), (2) has a unique weak solution provided

$$\left\{ \begin{array}{l}
\sum_{k_0}^2 - \sum_{k_1}^2 : k_0 \in \mathbb{Z}, k_1 \in \mathbb{N}, 1 \leq i \leq m \right\} 
\end{array}$$

Remark: The boundary conditions, in this case, (2), are reformulated in an $L_2$ setting to make the linear part of the equation a self-adjoint operator in $L_2$. (Received October 21, 1977.)

751-F7 JOHN WALSH, University of British Columbia, Vancouver, B. C., Canada V6T 1W5. Two-parameter martingales.

We look at several situations in probability and analysis which give rise to two-parameter martingales—classical differentiation theory, the convergence of multiple Fourier series, and the boundary behavior of biharmonic functions, for instance—and see how some of the elementary martingale convergence theorems apply. The emphasis is less on the theory of multiparameter martingales than on some of the examples which have motivated the theory. (Received October 3, 1977.)

84TH ANNUAL MEETING
Hyatt Regency Atlanta
Atlanta, Georgia
January 3–7, 1978

01 ▶ History and Biography

752-01-1 JUDY GREEN, Rutgers University Camden College of Arts and Sciences, Camden, New Jersey. American Women in Mathematics—The First Ph.D’s.

The first Ph.D in mathematics granted to an American woman was earned Summa Laude by Winifred Edgerton from Columbia University in 1886. During the next 25 years at least 34 more American women received this degree. Their careers show the effects of attitudes and patterns which still influence the position of women in the academic profession. (Received October 6, 1977.)


The work of E. H. Moore and his early topology students and their interaction with Fréchet is discussed. Special attention is given to their contributions to the development of early metrization theory with emphasis on the work of E. W. Chittenden (deceased June 16, 1977). (Received October 12, 1977.)


In 1886 H. Poincaré introduced the concept of asymptotic series solutions for ordinary differential equations. By the use of Laplace transforms he obtained asymptotic series solutions for an equation of the type

$$P_n(x) \omega^{(n)} + P_{n-1}(x) \omega^{(n-1)} + \cdots + P_0(x) \omega = 0$$

A-22
where the $Z^m_{(n)} \in \mathbb{C}$ are polynomials of the same degree, which he identified with the equation's normal series solutions (formal solutions) previously obtained by R. Thomé. However, no substantiating argument is given by Poincaré. The same identification for the case of other equations was made by J. Horn and later by G. Birkhoff, again without any supporting argument. We give a proof for this identification and also indicate a historical significance of this oversight by Poincaré, Horn and Birkhoff. (Received October 18, 1977.)

752-01-4 MORRIS W. HIRSCH, University of California, Berkeley, CA 94720. On the history of manifolds.

Manifolds are global geometrical objects which can be locally parameterized by $n$-tuples of real numbers. Such concepts were first used in nineteenth century function theory. Riemann's use of surfaces and his connectivity number focused attention on global topological questions. Attempts at the classification of surfaces were published in the 1860s by Möbius and Jordan. Today it is difficult to understand even the statements of their theorems. Their work will be discussed, and as time permits, also that of Betti, Poincaré, and more recent writers. (Received October 18, 1977.)

752-01-5 ALBERT C. LEWIS, University of Texas, Austin, Texas 78712. The R. L. Moore School as a subject of historical research.

The Texas topologist Robert Lee Moore (1882-1974), with his school of mathematics and method of teaching, has probably been the subject of more treatises than any other modern American mathematician. The reasons for this, which distinguishes him in this respect from comparable mathematicians such as G.D. Birkhoff, L.E. Dickson, and H.H. Moore (his teacher at Chicago), appear to range from the force of his character to the richness of the personal papers he left behind. Some historical work has been initiated by mathematicians and some by historians, but most significant has been a co-operative effort of historians, mathematicians, and archivists. Thus, the activities that have centered on Moore or grown from such activities may help to point up many of the opportunities and difficulties in tackling the largely unexplored area of the development of modern American mathematics. (Received October 18, 1977.) (Author introduced by Professor Uta C. Merzbach).


The paper treats, in general, the various factors affecting the decisions of blacks to enter the field of mathematics, and examines, in particular, the opportunity for training blacks in mathematics in American institutions before 1940. In addition, it studies the mathematics curricula at black institutions and the reception of black graduate students in white institutions. Finally, it analyzes the place of black mathematicians in the larger professional world. (Received October 18, 1877.) (Author introduced by Professor Uta C. Merzbach).

752-01-7 H.L. RESNIKOFF, University of California, Irvine, CA 92717. The Development of the theory of $\Theta$-functions: Unpublished work of Riemann.

In the years just before his death, Riemann attempted to generalize Jacobi's differential equation $\frac{\partial^3 u}{\partial z^3} = \Pi \theta_\phi \theta_\psi \Theta_{\bar{z}}$ to $\Theta$-functions of several variables. He anticipated work of Thome and Frobenius. His remarkable formulae were given without
proof and those for genus 4 through 7 have still not been verified. The problem will be presented from a modern viewpoint, and photographs of Riemann’s working papers will be shown. (Received October 18, 1977.)

\*752-01-8 LUCILLE WHYBURN, University of Texas, Austin, Texas 78712. An American in Göttingen 1926-1927: Letters from Kline to R.L. Moore

J.R. Kline, secretary of the AMS 1941-1951, was the only mathematician among the first group of Guggenheim fellows. He chose to spend the year February 1926 to February 1927 in Göttingen. This paper presents revealing letters from Kline to R.L. Moore written from Göttingen about such mathematicians as Hilbert, Courant, and Emmy Noether. He also gives an account of G.D. Birkhoff’s visit that year, on behalf of the Rockefeller Foundation in connection with the large grant Courant was seeking for their Institute. In addition, he tells of sharing his interest in topology with Alexandroff. (Received October 18, 1977.)

02 ▶ Logic and Foundations

\*752-02-1 Ludvik Janos, Mississippi State University, Mississippi State, Mississippi 39762. Topological dimension as a first order theory.

Let \( \mathcal{U} \) be a nonempty family of subsets of the Hilbert space \( l^2 \). For every integer \( n \geq 0 \) we define on \( \mathcal{U} \) two unary relations \( C_n \) and \( I_n \) as follows: For \( U \in \mathcal{U} \), \( C_n(U) \) is true if there is some affine subspace of dimension \( 2n + 1 \) containing \( U \); and \( I_n(U) \) is true if every affine subspace of dimension \( n + 1 \) intersects \( U \) in a set of dimension \( \leq 0 \). We denote by \( \langle \mathcal{U}; C_0^*, C_1^*, \ldots; I_0^*, I_1^*, \ldots \rangle \) the relational structure thus defined and by \( L \) the corresponding first order language based upon the unary predicate letters \( C_0^*, C_1^*, \ldots; I_0^*, I_1^*, \ldots \). For \( n = 0, 1, \ldots \) let \( S_n^* \) denote the sentence \( \exists x (C_n^* x \land I_n^* x) \) and let \( S_n^* \) be defined by \( S_0^* = S_0^* \), \( S_1^* = S_1^* \lor \neg S_2^* \), \( S_3^* = S_3^* \lor \neg S_4^* \lor \ldots \lor \neg S_5^* \). If \( Y \) is a separable metric space and \( S \) a sentence of \( L \) we say that \( Y \) satisfies \( S \) iff the above structure is a model for \( S \) where \( \mathcal{U} \) is the family of all subsets of \( l^2 \) which are homeomorphic to \( Y \). Applying a theorem of J. H. Roberts and one recent result of H. W. Martin we obtain Theorem: A separable metric space \( Y \) has dimension \( n \) \((n = 0, 1, \ldots)\) if and only if \( Y \) satisfies the sentence \( S_n^* \). (Received September 22, 1977.)


By anti-intuitionistic logic we mean the predicate calculus which is dual to the usual intuitionistic predicate calculus. More specifically, anti-intuitionistic logic is to the Brouwerian algebras of McKinsey and Tarski [Ann. Math. 47(1946), 122-162] as intuitionistic logic is to Heyting algebras or as classical logic is to Boolean algebras. All cases of excluded middle are anti-intuitionistically valid, but not all contradictions are anti-intuitionistically rejected. Let \( S \) be the anti-intuitionistic set theory having full comprehension as its sole non-logical axiom scheme. In \( S \) one can derive Russell’s paradox. Nevertheless, Theorem: \( S \) is consistent in the sense that not every sentence is derivable. (Received October 3, 1977.)

\*752-02-3 DOUGLAS CENZER, University of Florida, Gainesville, Florida 32611. Non-generable RE sets.

An RE set of natural numbers is said to be generable if it is the closure of a recursive inductive operator. In this paper, a non-generable RE set is constructed by means of a finite injury priority argument. It is shown that all generable sets are mitotic. The construction is refined to obtain, for any degree \( d \), a non-generable set of degree \( \leq d \), to obtain a non-generable set of degree \( 0^* \) and, finally, to obtain a non-generable RE set which is strongly mitotic (and therefore non-maximal). (Received October 11, 1977.)
We have higher-level analogues of some theorems in the first-level analytic hierarchy, under the assumption of the projective determinacy (PD). Here we state a main theorem. Let \( \Gamma \) be a pointclass. A set of natural numbers \( A \) is called a \( \Gamma \)-maximal set if \( A \in \Gamma \), \( \mathbb{A} (= \omega - \mathbb{A}) \) is infinite and for every set \( R \in \Gamma \) either \( \mathbb{A} \cap R \) or \( \mathbb{A} \cap \bar{R} \) is finite. Let \( E_k^1 = \Sigma_k^1 \) for \( k \) even and \( E_k^1 = \Pi_k^1 \) for \( k \) odd. **Theorem.** Assume PD for the case of \( k > 2 \). For each \( k \) there is an \( E_k^1 \)-maximal set. (For \( k = 0 \) this theorem is due to Friedberg, Journal of Symbolic Logic, 23(1958) and for \( k = 1 \) to Kreisel-Sacks, Ibid 30(1965).) Proof uses Addison-Moschovakis-Martin's Prewellordering Theorem which is a consequence of PD. (Received October 14, 1977.)

**A Topology-like Approach to Completeness Theorems for Logics with Restricted Substitution.**

The topological semantics first described by Tarski embodies a number of connections between properties of topological significance and semantic properties of the corresponding propositional calculi. In this presentation a class of "topology-like" logical matrices is defined. We show that a generalization of the usual notion of validity, motivated by a desire to mirror topology-like behavior in logic, produces a completeness theorem with respect to a subclass of these matrices, for a calculus intermediate by virtue of having restricted substitution rules. While such a completeness theorem is of course impossible with respect to the usual notion of validity, it is shown that the generalized notion is coincident with the usual notion on both the intuitionistic and classical propositional calculi. See Abstract 76T-E66, Notices, October, 1976, p. A-597. (Received October 17, 1977.)

**Straight-line computation in abstract algebra.** Preliminary report.

The one-pass (resp., read-only) rank of a straight-line computation is defined to be the minimum number of registers needed to perform the computation when the inputs are handled in one-pass (read-only) fashion. An abstract algebra or class of algebras is one-pass (resp., read-only) if for some \( k \) all expressions can be formulated so as to have one-pass (read-only) rank less than \( k \).

**Theorem.** The above notions depend only on the equational identities holding in an algebra or class of algebras, are invariant under equivalence of equational class, and are preserved in subclasses.

Some one-pass classes: abelian groups, modules, structures with pairing functions. Read-only but not one-pass: groups, rings, finite primal algebras, the real numbers with continuous functions (cf. Kolmogorov's solution to Hilbert's Thirteenth Problem). Not even read-only: groupoids, lattices, non-associative algebras. In practice, "read-only" turns out to be a sort of wild generalization of the associative law, "one-pass" a generalization of commutativity. (Received October 17, 1977.)

**Recursively enumerable sets and degrees.**

A set of nonnegative integers is recursively enumerable (r.e.) if there is an algorithm for enumerating its members. Such sets have played a major role in undecidability results such as Hilbert's 10th problem on Diophantine equations and the word problem for finitely presented groups. We survey the major results on the lattice \( \mathcal{E} \) of r.e. sets, and on the r.e. degrees \( \mathbb{R} \) from the time of Post's famous paper (Bull A.M.S. 1944) up to the present state of the subject. We consider three main areas: (1) the generalized Post program and degrees of classes invariant under automorphisms of \( \mathcal{E} \); (2) the structure of \( \mathcal{E} \), its
ideals, filters, automorphises, and the partial decidability of its elementary theory, (3) the same questions for the upper semi-lattice $\mathbb{R}$ of r.e. degrees. An expanded session of this talk will appear in the Bulletin A.M.S.

(Received October 17, 1977.)

752-02-8 GEORGE F. McNUlTY, University of South Carolina, Columbia, South Carolina 29208

A Burnside-style theorem for equational classes. Preliminary report.

For any term $\sigma$ in some language, $t(\sigma)$ is the number of occurrences of variables and constant symbols in $\sigma$, while $v(\sigma)$ is the number of distinct variables and constant symbols which occur in $\sigma$.

**THEOREM:** Let $L$ be a language with some operation symbol of rank at least two. If $\Sigma$ is a set of $L$-equations such that

(i) only finitely many distinct variables and constant symbols occur in $\Sigma$, and

(ii) $t(\sigma) \geq 2^{v(\sigma)}$ and $t(\tau) \geq 2^{v(\tau)}$ for all $\sigma, \tau \in \Sigma$,

then some finitely generated model of $\Sigma$ is infinite.

An upper bound on the number of generators necessary to obtain an infinite model can be computed from the number of variables and constant symbols occurring in $\Sigma$. The theorem can also be relativized to certain classes of algebras (e.g. semigroups). The proof employs avoidable words.

(Received October 18, 1977.)

752-02-9 FRED HALPERN, Bishop College, Dallas, Texas 75241. Robinson's consistency theorem for game quantifier logic.

Let $L^G_A$ be the logic with game quantifiers based on the (possibly uncountable) $\Sigma$ compact admissible set $A$. Robinson's consistency theorem holds in $L^G_A$. (Received October 18, 1977.)

752-02-10 RICHARD STARK, San Jose State University, San Jose, California 95192.

Automatic construction of finite models.

A computer program, based on the Henkin constant construction, will be described which is capable of constructing arbitrary finite models. The use of this algorithm in finitary mathematical research will be discussed. (Received October 18, 1977.)

752-02-11 Bruce I. Rose and Robert E. Woodrow, University of Notre Dame, Notre Dame, Indiana 46556. Ultrahomogeneous Structures.

A structure is ultrahomogeneous if any isomorphism of substructures of smaller cardinality can be extended to an automorphism. The relationship between the various notions of homogeneity that have appeared in the literature is quite subtle. The table relates ultrahomogeneity (u) to homogeneity (h) and saturatedness (s). The utility of this concept is seen by using u > h > it to obtain easy proofs that algebraically closed fields admit elimination of quantifiers and that the existence of homogeneous (cf. Sacks) models of a theory of cardinality $\mathcal{L}_1$ is independent of ZFC. It can also be used to show that the reduced power by the cofinite filter on $\omega$ of a finite structure which admits elimination of quantifiers does not necessarily admit elimination of quantifiers. Although a seemingly algebraic definition ultrahomogeneity imposes many restrictions on the theory of a structure. The existence and properties of ultrahomogeneous structures are explored and results are obtained. (Received October 18, 1977.)
Gödel-Bernays- and Zermelo-Fraenkel-type axiomatizations of Zadeh's theory of fuzzy sets give rise to models similar to those obtained for classical set theory using Boolean values or forcing techniques. Transfer techniques permitting the construction of models of classical ZF and GB from models of the fuzzy theory allow simple demonstrations of the possible relationships between cardinals in the classical case. A study of statements not transferring from the fuzzy models to classical models in any simple way helps to elucidate the fine structure of the classical theory, giving 'intermediate' models between some classical possibilities. (Received October 18, 1977.)

**04 ▶ Set Theory**

Stanley H. Stahl, Smith College, Northampton, Massachusetts 01063. L(K) - Admissible Sets.

Given a regular cardinal κ, the property of a set being κ-admissible and/or κ-power admissible is defined. A characterization of when H(α) is κ-admissible is provided and it is shown that when A is κ-power admissible and α is the least ordinal not in A, then C^κ_α, the αth level of the sets constructible from L^K_κ, is also κ-admissible and C^κ_α ⊆ A. (Received October 12, 1977.)


P. Erdos and R. Rado (in Bull. Am. Math. Soc. 62(1956)) have defined various partition relations, including \( \varphi(\varphi, \theta)^2 \), where \( \varphi, \psi, \theta \) are order types.

Theorem 1: If K and \( \lambda \) are Ramsey cardinals K\( > \) and K\( ^\lambda \) is obtained from K and \( \lambda \) by ordinal exponentiation, then K\( ^\lambda \) \( \not\rightarrow (\lambda^+, 3)^2 \).

E. Specker introduced a relation between ordinals called pinning which is denoted \( \alpha \leftrightarrow \beta \). (See Fund. Math. 83(1975) for the definition.) Pinning is useful for transferring results in the partition calculus from one ordinal to another. The following theorem gives an example of a pair of ordinals where the relation holding is consistent and independent of the axioms of set theory.

Theorem 2: If ZF is consistent, then so are ZF together with \( w_1^{w+2} < w^2 \) and ZF together with \( w_1^{w+2} = w^2 \). (Received October 14, 1977.)

**05 ▶ Combinatorics**


A group G is in Class I if it admits a graphical regular representation, i.e., a graph X whose automorphism group is isomorphic to G and acts regularly on the vertex set of X. G is in Class II if for each subset H \( \subseteq G \) with H = H^-1, there exists \( \phi \in \text{Aut}(G) \), \( \phi \neq 1_G \), such that \( \phi[H] = H \).

Classes I and II are disjoint, and all "small groups" have been classified in this way except some non-Abelian groups of order 24 [M.E. Watkins, Aequationes Math. 11 (1974), 40-50]. We announce that of the twelve non-Abelian groups of order 24, nine are in Class I and three are in Class II. (Received August 9, 1977.)

Phil Hanlon, Dartmouth College, Hanover, N.H. \( \Theta3755 \). The Enumeration of Labelled and Unlabelled Line Graphs.

Line graphs with p points are enumerated (both labelled and unlabelled). The methods employed are cycle index sums utilizing basic theorems of R.W. Robinson, The Enumeration of Non-Separable Graphs.
Let \( L(\delta) \) denote the line graph of a graph \( \delta \), \( \Gamma(\delta) \) the vertex automorphism group of \( \delta \), \( \Gamma_{\ell}(\delta) \) its group of vertex-induced line automorphisms, and \( Z_{p,L}(\delta) \) its mixed point-line cycle index (where \( a_{i} \) mark cycles of vertices and \( b_{i} \) mark cycles of lines). Then \( Z_{p,L}(\delta)(a_{i}+1) = Z(\Gamma(\delta)) \).

For a set \( S \) of graphs, let \( Z_{p,L}(S) = \sum_{\delta \in S} Z_{p,L}(\delta) \) and \( Z(\Gamma_{\ell}(S)) = \sum_{\delta \in S} Z(\Gamma_{\ell}(\delta)) \). With three exceptions, connected graphs \( \delta \) satisfy \( Z(\Gamma(\delta)) = Z(\Gamma_{\ell}(\delta)) \), so that if \( C \) is the set of all connected graphs and \( L_{C} \) is the set of connected line graphs, then \( Z(L_{C}) \) is easily computed from \( Z(\Gamma(\delta)) \). Let \( G \) be the set of all graphs. Then

\[
Z_{p,L}(G) = \sum_{i=1}^{\infty} a_{i} Z_{p,L}(S(2))^{i} Z_{p,L}(G) = \sum_{i=1}^{\infty} b_{i} Z_{p,L}(G) \]

Applying the familiar relation between graphs and connected graphs, \( Z_{p,L}(G) = \sum_{i=1}^{\infty} \lambda_{i}[Z_{p,L}(C)] \), and inverting we get

\[
Z_{p,L}(C) = \sum_{i=1}^{\infty} \lambda_{i}[Z_{p,L}(C)] \]

Given an unlabelled graph \( \delta \), the number of ways to label \( \delta \) is \( p^{\lambda(\delta)} \). Hence the cycle index sums just computed serve to enumerate the labelled case as well.

(Received August 2, 1977.) (Author introduced by Robert Z. Norman).
Problem is to determine positive integer stamp denominations $d_1, \ldots, d_d$, which yield the largest possible integer $M$ satisfying the condition that for each $m \leq M$ it is possible to frank a letter with $m$ cents in postage.

We study the case $d = 3$. We construct an algebraic framework which provides a classification of all solutions, and use this both to approximate solutions and obtain a new, improved lower bound on $M$. These methods yield the optimal solution in all but one of the known examples and appear to provide good approximations for all values of $t$. (Received September 23, 1977.)

752-05-6 PAUL ERDŐS, University of Colorado, Boulder, Colorado 80309, and FRED GALVIN, The University of Kansas, Lawrence, Kansas 66045. A quantitative version of the infinite Ramsey theorem. Preliminary report.

Thm. Given positive integers $r$ and $k$, there is a constant $c > 0$ such that, for any function $f : [\omega]^r \rightarrow k$, there exists $A \subseteq \omega$ such that:

1. $|\{f(X) : X \in [A]^r\}| \leq 2^{r-1}$;

2. $|A \cap \omega| > c \log^{r-1} n$ for infinitely many $n$. (Received September 26, 1977.)


Let $1 \leq k_1 \leq k_2 \leq \ldots \leq k_n$ be integers and let $F$ denote the set of all vectors $x = (x_1, x_2, \ldots, x_n)$ with integral coordinates $x_i$ satisfying $1 \leq x_i \leq k_i$, $i = 1, 2, \ldots, n$.

The complement of $x$ is $(k_1 - x_1, k_2 - x_2, \ldots, k_n - x_n)$ and a subset $X$ of $F$ is an antichain provided that for any two elements $x, y$ of $X$, the inequalities $x_i < y_i$, $i = 1, 2, \ldots, n$ do not all hold. We determine the maximal cardinality of an antichain containing no vector and its complement, thus generalizing a result of Hans-Dietrich Gronau who recently solved the $k_n = 1$ case. (Received September 26, 1977.)

752-05-8 ALLEN J. SCHWENK, United States Naval Academy, Annapolis, Maryland 21402. An Improved Bound for the Classical Ramsey Number $R(4,4;3)$.

We demonstrate that the first nontrivial face-coloring classical ramsey number is at most 15. In other words, any 2-coloring of the faces of the complete 2-plex on 15 vertices must contain a monochromatic tetrahedron. Sobczyk has reported that $R(4,4;3) \geq 14$, so the value is nearly determined. (Received September 29, 1977.)

752-05-9 STEVEN E. ANACKER, The Ohio State University, Columbus, Ohio 43210. Reducible Partitions. Preliminary Report.

Let $G$ be a permutation group on a set $X$. Let $X_1: 0 \leq i \leq t$ be an ordered partition of $X$. A partition is reducible if $t \geq 1$, $X_t \neq \emptyset$, and for every $y \in X_1$, $i \geq 1$, there exists a $g \in G$ such that $g(y) \in X_{i-1}$ and $g$ respects the partition $X = X_1 \cup \ldots \cup X_{i-1} \cup y \cup X_{i-1} \cup \ldots \cup X_t$. Let $t(G,X)$ be the largest positive integer such that a reducible partition of $X$ exists wrsp. to $G$. Announced in the October Notices was Theorem 1: If $X_1: 0 \leq i \leq t$ is a reducible partition
of X and t ≠ 3, then o(Gx) ≤ t!. The restriction t ≠ 3 can be removed. **Theorem 2:** Let G be transitive and imprimitive on a set X. Let \( \mathcal{B} \) be a complete block system of A on X, then

\[ t(G, X) \leq t(G, \mathcal{B}) + t(G, \mathcal{B}) \]

where \( \mathcal{B} \neq \emptyset \). (Received September 30, 1977.)

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N colored beads are threaded on a string, and the interstices between the beads are labeled as shown:

\[
\cdots a_1 a_2 \cdots a_b \cdots b_1 b_2 \cdots b_{N-1} \cdots c_1 c_2 \cdots c_b \cdots d_1 d_2 \cdots d_{N-1} \cdots e_1 e_2 \cdots e_b \cdots f_1 f_2 \cdots f_{N-1} \cdots g_1 g_2 \cdots g_b \cdots h_1 h_2 \cdots h_{N-1} \cdots
\]

There is a \( b \)-symmetry about interstice \( i \) if each of the \( b \) beads immediately to the left of \( i \) has the same color as the bead in the corresponding position to the right of \( i \). For \( i = 1, \ldots, N - 1 \), let \( A_i \) be the event that there is \( b \)-symmetry at axis \( i \). The general problem is to compute \( \text{Prob}(A_1 \cap A_2 \cap \cdots \cap A_{i-1} \cap A_{i+1} \cap \cdots \cap A_{N-1} ) \) if there are \( w \) possible colors and their distribution is uniform. It is easy to show that \( \text{Prob}(A_i \cap A_j) = \text{Prob}(A_i) \text{Prob}(A_j) \) for \( i \neq j \). **Theorem:** If \( i < j < k \), if \( r = \min(j - i, k - j) \), if \( s = \max(j - i, k - j) \), then, assuming \( b \leq i \) and \( k \leq N - b \),

\[
\text{Prob}(A_1 \cap A_j \cap A_k) = w^{-a},
\]

where

\[
a = \begin{cases} 
3b, & 2b \leq s \\
2b + r + s - \max((r + s - b), [r, s]), & s < 2b 
\end{cases}
\]

where \([r, s]\) is the greatest common divisor of \( r \) and \( s \). (Received September 30, 1977.)

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The contractibility number (also known as the Hadwiger number) of a connected graph \( G \), \( Z(G) \), is defined as the maximum order of a complete graph onto which \( G \) is contractible. An elementary proof is given of a theorem of Ore about this invariant: If the minimum degree of \( G \) is at least 3, then \( Z(G) \geq 4 \). Also, we solve the problem of finding \( Q(p, k) \), the maximum \( Z(G) \) over all \( G \) having \( p \) points that are regular of degree \( k \). It is found that for each \( x \),

\[
Q(p, k) = \max \{ \lambda \in \mathbb{Z} : \lambda (\lambda - 3)/(k-2) \leq p \}
\]

for all but finitely many \( p \) where \( \lfloor x \rfloor \) denotes the least integer greater than or equal to the real number \( x \). (Received October 3, 1977.) (Author introduced by Professor F. Harary).

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*752-05-12 RICHARD A. JENSON, Oberlin College, Oberlin, Ohio 44074. *A Double Circulant Presentation for Quadratic Residue Codes.*

Let \( Q(p) \) denote the \((p+1, (p+1)/2)\) extended binary quadratic residue code where \( p \) is a prime congruent to \( \pm 1 \) modulo 8. It is known that when \( p = 7 \) and \( p = 23 \) the code \( Q(p) \) has a generator matrix which is a double circulant (i.e. two juxtaposed \((p+1)/2\) - square circulants).

Query: is this the case for all \( Q(p) \)? This paper proves that there are two families of primes, one each possibly infinite, for which \( Q(p) \) must have a double circulant presentation. Furthermore, not every \( Q(p) \) enjoys this property. Two counterexamples verify this last assertion.

(Received October 6, 1977.)

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*752-05-13 Eric Verheiden, California Institute of Technology, Pasadena, California 91125. *Integral and Rational Combinatorial Matrix Completions.*

Rational normal completions of partial integral matrices satisfying \( AA^T = \mu I \) (including Hadamard matrices) and partial incidence matrices of symmetric block designs are investigated. Known results on rational lattices are used to show that completions in which all denominators are powers of two are always possible in the former case and usually possible in the latter. Bounds are given when small numbers of rows are to be added.

As a related result, it is shown computationally that \((1, -1)\) completions of partial Hadamard matrices and \((0, 1)\) completions of partial incidence matrices are possible when no more than seven rows remain to be added, extending results by Marshall Hall. (Received October 6, 1977.)
A generalization of Ramsey theory for graphs. Preliminary report.

Let \( c, d_1, d_2, \ldots, d_k \) be integers such that \( 1 \leq d_1 < d_2 < \ldots < d_k < c \). Let \( t \) denote the sum
\[
(c) + \binom{c}{2} + \cdots + \binom{c}{k}.
\]
Given \( c \) distinct colors, we order the \( \binom{c}{1} \) subsets of \( d_1 \) colors, \( \binom{c}{2} \) subsets of \( d_2 \) colors, \( \ldots \), \( \binom{c}{k} \) subsets of \( d_k \) colors in some arbitrary manner. Let \( G_1, G_2, \ldots, G_t \) be graphs. The \( (d_1, d_2, \ldots, d_k) \)-chromatic Ramsey number, denoted \( R^c_{d_1, d_2, \ldots, d_k}(G_1, G_2, \ldots, G_t) \), is defined to be the least number \( p \) such that if the edges of the complete graph \( K_p \) are colored in any fashion with \( c \) colors, then for some \( i \) the subgraph whose edges are colored with \( i \)th subset of colors contains a \( G_i \). The Ramsey numbers \( R^c_{d_1, d_2, \ldots, d_k}(G_1, G_2, \ldots, G_t) \) were studied previously in the literature. In this paper, we shall report some recent results on the numbers \( R^3_{d_1, d_2, \ldots, d_6}(G_1, G_2, \ldots, G_t) \) and \( R^3_{d_1, d_2, \ldots, d_3}(G_1, G_2, G_3) \). (Received October 7, 1977.) (Author introduced by Dr. S. A. Burr).

Let \( C = \{C_1, \ldots, C_k\} \) be a subset of Euclidean space. \( C \) is called Ramsey if for all \( k \) there exists \( n \) so that if \( R^n \) is \( k \)-colored there exists a monochromatic \( B \subset C \). The determination of the Ramsey configurations remains an open problem.

Call \( C \) Almost Ramsey if for all \( k, \epsilon > 0 \) there exists \( n \) so that if \( R^n \) is \( k \)-colored there exists a monochromatic \( A = \{a_1, \ldots, a_k\} \) and a set \( B = \{b_1, \ldots, b_k\} \subset C \) so that \( |a_i - b_i| < \epsilon \) for \( 1 \leq i \leq k \).

Theorem: All Finite Configurations are Almost Ramsey. (Received October 7, 1977.)

Let \( \{S_i\} = n, A_k \subset S, 1 \leq k \leq m \), \( |A_k| \geq cn \). Denote \( f(n, m, c, \epsilon) \) the largest integer so that there always is a subfamily \( A_{k_1}, \ldots, A_{k_r} \) \( r \geq f(n, m, c, \epsilon) \) so that \( |A_{k_1} \cap A_{k_2}| > \epsilon n \) for every \( 1 \leq i_1 \leq i_2 \leq r \). I am particularly interested in case \( m = n, \epsilon \) small. Ramsey’s theorem easily implies \( f(n, n, c, \epsilon) > n(1/c)^{-1} \) (i.e., if \( 1/3 \leq c > 1/3 \) then \( f(n, n, c, \epsilon) > n^{1/2} \). Perhaps this holds for every \( c \) if \( \epsilon = \epsilon(c) \) is sufficiently small and \( n > n_0(\epsilon, c) \) is large enough. I have no counterexample to \( f(n, n, c, \epsilon) > \eta n \).

Several related problems will also be discussed. (Received October 7, 1977.)

Let \( P_{k_0} \) denote the complete 3-graph on \( n \) vertices and let \( T_m \) be a "tree-like" 3-graph on \( m \) vertices, where the latter is defined as follows: \( T \) is tree-like if it consists of a single triple or it has \( 2k + 1 \) vertices and can be formed from a tree-like 3-graph \( T' \) on \( 2k-1 \) vertices by adjoining a single triple which shares exactly one vertex with \( T' \). Extending a result of Chvátal (J. Graph Theory 1 (1977) 93) we have that the Ramsey number for such a pair of 3-graphs is given by
$r(P_{kn},T_m) = \frac{1}{2}(n-1)(m-1) + 1$ when $n$ is odd. For even $n$ the corresponding values are given by $\frac{1}{2}(n-2)(m-1) + 2$ when $m \leq 7$ and when $T_m$ is a "path," but the general problem seems to be much more difficult. The higher dimensional analogues are also considered. (Received October 11, 1977.)

Let $G$ be a graph such that neither $G$ nor $\overline{G}$ contains an odd irreducible cycle of length $\geq 5$. Then the following two theorems hold. **Theorem 1:** If $G$ is of genus 1, then $\gamma(G) = \omega(G)$. **Theorem 2:** If $G$ has maximum degree $= 6$, then $\gamma(G) = \omega(G)$. (Received October 11, 1977.) (Author introduced by Bruce Rothschild).

Let $G$ be a finite, connected graph with no loops or multiple edges. **Theorem:** If $G$ is the union of two blocks, then a necessary and sufficient condition for the maximum genus of $G$ to be the sum of the maximum genera of its blocks is that at most one of the blocks has a maximum embedding in which the cutpoint belongs to the boundary of more than one face. The relationship of this characterization theorem to the resolution of some questions concerning upper embeddable graphs is also explored. (Received October 11, 1977.)

The **girth** $g(G)$ of a graph $G$ is the length of a smallest cycle in $G$ while the **circumference** $c(G)$ is the length of a longest cycle in $G$. A graph $G$ is an $<m,n>$ graph if $m = g(G)$ and $n = c(G)$.

Adopting the convention that a graph $G$ is smaller than graph $H$ if $|V(G)| < |V(H)|$ or $|V(G)| = |V(H)|$ and $|E(G)| < |E(H)|$, the following theorem is true.

**Theorem 1.** For integers $m$ and $n$ where $3 \leq m \leq n$, there is a unique smallest $<m,n>$ graph. This graph is 2-connected and has order $n$ if $m = n$ or $m \leq \lfloor (n+1)/2 \rfloor$ and order $m + \lfloor (n-1)/2 \rfloor$ otherwise.

**Theorem 2.** If $m$ and $n$ are integers such that $3 \leq m \leq \lfloor (n+1)/2 \rfloor$, then there exists a 2-connected $<m,n>$ graph of order $p$ for each $p \geq n$. (Received October 13, 1977.)

A **detour** for a path $P = (p_0, ..., p_k)$ in a digraph $D$ is any path from $p_0$ to $p_k$ having no other points or arcs in common with $P$. $D$ is a **detour digraph** iff $D$ is weakly connected and each path of $D$ has a detour. A simple characterization of strong detour digraphs is obtained and for arbitrary detour digraphs $D$ it is shown that after the deletion of certain arcs of $D$, in the condensation of the remaining subgraph every path of length at least 2 has a detour. (Received October 13, 1977.)

This paper will describe how certain standard combinatorial functions, namely combinations with no or various allowable repetitions, can be obtained, studied, and categorized by a
simple functional equation system. The approach is by first obtaining a functional equation based on the definition of the desired function and then solving this to determine the functions, relationships they satisfy, and even their generating function. The functions discussed in this paper satisfy the same functional equation system and are thereby shown to have many properties in common and generating functions of the same general form. The initial conditions for the required allowable repetitions distinguish between the various functions satisfying the functional equation system and specialize the generating function from the general class in each case. The author has also successfully applied this method to several other types of combinatorial functions and it provides a natural approach showing how such functions may be found, studied and classified. (Received October 13, 1977.)


For a random walk on a tree with the endpoints as absorbing states, bounds are obtained on the probability that a walk originating at v passes through u after departure from v (Notices, Jan. 1977, Vol. 24, No. 1, Abstract No. 76T-A12).

**Theorem.** If $T$ is a tree with $q$ edges and $k = \text{d}(v,u)$, then $P(v \to u) \leq \frac{q-k}{q}$.

**Theorem.** If $T$ is a tree with $q$ edges and $u \in T^0$ is of degree $\delta$, then $P(u \to u) \leq \frac{q-\delta}{q}$.

Further bounds on the expected duration of a walk originating at $v$, $E(v)$, are obtained.

**Theorem.** If $T$ is a tree with $q$ edges and $e$ is an endpoint of $T$ with $d(v,e) = k$, then $E(v) \leq (q-k)k$.

Theorem. Let $T$ be a tree with diameter $d$. Then $E(v) \leq \frac{d^2}{4}$.

Effects of edge subdivision, edge deletion and edge removal are investigated for their effect on absorption probabilities and expectations. (Received October 17, 1977.)

752-05-24 Miklóš Simonovits, Eötvös Loránd University, Budapest, and Vera T. Sós, Eötvös Loránd University, Budapest. Intersection theorems for subsets of integers.

Let $A_1, \ldots, A_n$ be a family of subsets of $\{1,2,\ldots,n\}$, and for a fixed integer $k$ we assume that $A_i \cap A_j$ is an arithmetic progression of $\geq k$ elements for $1 \leq i < j \leq n$. We would like to determine the maximum value of $N$ under this condition. For $k=0$ R.L. Graham and the authors have proved that the maximum value of $N$ is $\left(\binom{n}{2} + \binom{n}{3}\right) + n + 1$ obtained iff $A_1, \ldots, A_n$ is the family of all the subsets of at most $3$ elements in $\{1,2,\ldots,n\}$. The main result of this communication is that for $k=1$ one extremal system, that is, for which the maximum is attained is the following one: for a fixed $c \in \mathbb{N}$, take all the triplets of form $\{c, x, y\}$, $x, y \in \mathbb{N}$, further all the sets $\{c, x\}$ and $\{c\}$ (The other extremal systems are also completely known.) For $k=2$ the extremal systems are "asymptotically the same", independently of $k$: take all the finite arithmetic progressions of form $\{c+jd: j=0,1,2,\ldots,k_2\}$, where $c$ is fixed, $d \in \mathbb{N}^{1/4}$, $k_1 \geq 0$ and $k_2 \leq n^{3/4}$. Clearly, any two of these arithmetic progressions intersect in an arithmetic progression of at least $n^{1/4}$ elements, their number $N$ is asymptotically $\frac{n^2}{24}$ if $c = \frac{n}{2} + o(n)$ and we claim that even for $k=2$ $N$ cannot be larger than that. (Received October 17, 1977.) (Author introduced by Stephan Bure.)


Let $p_1$ and $p_2$ be two permutations on the $n$-set $\{1,2,\ldots,n\}$. Permutations $p_1$ and $p_2$ are discardant if $p_1(i) \neq p_2(i)$ for all integers $i$ such that $1 \leq i \leq n$. John Riordan [Scripta Math. 20 (1954), 14-23] enumerated the permutations discardant with three given permutations. Here the permutations discardant with four given permut-
tations are enumerated. In the terminology of W. O. J. Moser [Canad. J. Math. 19 (1967), 1011-1017], this is the enumeration of the very reduced $5 \times n$ Latin rectangles. This enumeration involves rook polynomials, hit polynomials, and generating functions. (Received October 17, 1977.)

The interval number $i(G)$ of a simple graph $G$ is defined to be the smallest number $t$ such that to each vertex $v$ in $G$ there can be assigned a collection of at most $t$ closed intervals on the real line, such that there is an edge in $G$ between distinct vertices $v$ and $w$ if and only if some interval for $v$ intersects some interval for $w$. This is a natural generalization of the well-known interval graphs, which are precisely those graphs $G$ with $i(G) = 1$.

We prove the following external results about interval numbers:

**Theorem 1.** If $G$ has maximum degree $d$, then $i(G) \leq \lceil (d+1)/2 \rceil$, and this bound is best possible. This bound is attained by every regular graph of degree $d$ with no triangles, e.g., $K_d$, and the cube graph, $Q_4$.

**Theorem 2.** If $G$ has $n$ vertices, then $i(G) \leq \lceil n/3 \rceil$. This bound is not best possible, even for $n$ as low as 7. We conjecture that the correct bound should be $\lceil (n+1)/4 \rceil$, which is attained by $K_{\lceil n/2 \rceil, \lfloor n/2 \rfloor}$. (Received October 17, 1977.)

A graph $G$ is the line or intersection graph of a hypergraph $H = (X, \mathcal{E})$ if the vertices of $G$ are the edges of $H$ with two vertices adjacent in $G$ if and only if the corresponding edges of $H$ have a nonempty intersection. We prove that all hypergraphs with the same line graph, $G$, can be constructed by first constructing a canonical hypergraph and then applying the two operations of "node duplication" and "contraction by $K_n$'s."

This result has Whitney's theorem that if $n > 3$ there is a unique graph with a given line graph as a corollary. The proof is constructive. (Received October 17, 1977.)

The boxicity of a graph $G$, denoted $Box(G)$, was defined by Fred Roberts as the smallest integer $n$ for which there exists a function $f$ assigning to each vertex $u \in G$ a sequence $f(u)(1), f(u)(2), \ldots, f(u)(n)$ of closed intervals of the real line so that distinct vertices $u$ and $v$ are adjacent in $G$ if and only if $f(u)(i) \cap f(v)(i) \neq \emptyset$ for $i = 1, 2, 3, \ldots, n$. By convention, a complete graph has boxicity zero. Roberts proved that $Box(G) \leq \lceil |G|/2 \rceil$. We construct three families of graphs $\{G_n: n \geq 1\}$, $\{H_n: n \geq 2\}$, and $\{W_n: n \geq 3\}$ so that if $|G| = 2n + 1$, then $Box(G) < n$ unless $G$ contains $G_n, H_n$ or $W_n$ as an induced subgraph. A graph with $Box(G) \leq 2$ is called a rectangle graph. We also provide a forbidden subgraph characterization for rectangle graphs among the class of graphs with clique covering number two. (Received October 17, 1977.)

Let $n$ and $k$ be nonnegative integers with $0 \leq k \leq n$. Let $Z_n = \{0, 1, \ldots, n-1\}$, and let $\mathcal{F}$ be an arbitrary subfamily of $\{A | A \subseteq Z_n \text{ and } |A| = k\}$. For each $x \in Z_n$, let $S_{\mathcal{F}}(x)$ denote $|\{A | x \in A \in \mathcal{F}\}|$.

Following, Silberger, J. of Comb. Theory (A), 31-37, (1977), we say that $\mathcal{F}$ is uniformly deep provided $S_{\mathcal{F}}(x) = S_{\mathcal{F}}(y)$ for all pairs $x, y \in Z_n$. As observed by Silberger, $|\mathcal{F}| = \frac{tn}{n,k}$ for some $t \leq \frac{(n,k)^2}{n}$. We answer a question posed by Silberger with the following: **Theorem:** There exists a uniformly deep subfamily of $\{A | A \subseteq Z_n \text{ and } |A| = k\}$ with $|\mathcal{F}| = \frac{tn}{n,k}$ for all $t \leq \frac{(n,k)^2}{n}$. **Corollary:** If $(n,k)$ is a practical number, $t = \frac{kn}{(n,k)}$ - cyclic design exists for all $t \leq \frac{(n,k)^2}{n}$. (Received October 17, 1977.)
A set $S$ in $E^n$ (infinite dimensional Euclidean space, that is points in separable Hilbert space with only finitely many nonzero coordinates) has the Euclidean Ramsey (ER) property if in every finite coloring of $E^n$ there is a monochromatic congruent copy of $S$. This concept and related questions have been treated in 3 papers by P. Erdős, R. L. Graham, P. Montgomery, B. L. Rothschild, J. Spencer and E. G. Straus (J. Combinatorial Theory - A 14 (1973), 341-363, Colloquia Math. Soc. János Bolyai 10 (1973), 550-558 and 559-583), L. Schader (J. Combinatorial Theory - A 20 (1976), 385-389) and R. Juhász (to appear in J. Combinatorial Theory - A).

Since $E^n$ is isometric to $E^m \times E^n$ it follows that the Cartesian product of two ER sets is ER. Since $E^n$ contains regular simplices of arbitrary finite dimension it follows that every point pair — and hence by direct products every vertex set of a rectangular parallelepiped — is ER. These sets and their subsets are the only ER sets known so far. On the other hand it is known that every ER set $S$ is a subset of a sphere and bounds depending only on $|S|$ were obtained for the number of colors in a coloring of $E^n$ which contains no monochromatic set isometric to a given non-spherical set $S$ (E. G. Straus, A combinatorial theorem in group theory, Math. Comp. 29(1975), 303-309).

(Rceived October 18, 1977.)


For any natural number $n$, a graph is $n$-traceable if and only if every set of $n$ distinct vertices of the graph lie on a common path. The largest integer $n$ not exceeding $|V|$ such that a graph $G$ is $n$-traceable is called the traceability of $G$, denoted $\tau(G)$. In a manner similar to the work of Mesner and Watkins (Canadian Journal of Mathematics, 19(1967), 1319-1328), this paper shows that $\tau(G) \geq K(G) + 1$, where $K(G)$ denotes the vertex connectivity of $G$, for each non-hamiltonian graph with $K(G) \geq 2$. For a graph $G$ having $K(G) \geq 2$, $\tau(G) = K(G) + 1$ if and only if there exists a separating set $S$ such that $|S| = K(G)$ and $G - S$ has at least $K(G) + 2$ components. A graph having connectivity one has traceability 2 iff the block cut-vertex tree of $G$ has a vertex with degree exceeding two.

(Received October 17, 1977.)


By using the notion of normalized shift basis from [1] and ideas from [2], it is shown that the sequences of Eulerian binomial type of [3] have a simple expression as the renormalization of shift bases from the 0-evaluation functional to the 1-evaluation functional.

Main Theorem. Let $T$ be a shift on a vector space with $L$-normalized shift basis $q_n$. Then the $M$-normalized shift basis for $T$ is

$$p_n = \left\{ \sum_{L} <L, p_k > T^k/|k|! \right\} q_n = \sum_{L} \left( \frac{n}{L} \right) <L, p_k > q_{n-k},$$

$$\sum_{L} <L, p_n > t^n/n! = 1/\left( \sum_{M} <M, q_n > t^n/n! \right)$$
determines the numbers $<L, p_k >$.


Elementary techniques of algebraic coding theory are here used to discuss the three biplanes with $k=6$ (i.e., the three projective $(16,6,2)$ designs). These three designs are intimately related to the $(16,11)$ extended binary Hamming code and to one another; we systematically investigate these relationships.

(Received October 17, 1977.)

A-35
THEOREM 1: Let $C$ be a linear code of length $n$ over an arbitrary field. If the permutation automorphism group of $C$ contains $A_n$ then it contains $S_n$ and $C$ is known to within isomorphism.

THEOREM 2: Let $C$ be a linear code of length $(p+1)$ ($p$, an odd prime) over the finite field $GF(q)$. If $C$ admits a permutation automorphism group strictly between $PSL_2(p)$ and $A_{p+1}$ then $C$ is isomorphic to the extended quadratic residue code of length $(p+1)$, provided that $q$ is a quadratic residue modulo $p$. (Received October 17, 1977.) (Author introduced by Professor Marshall Hall, Jr.)

Let $F$, $G$ and $H$ be finite, undirected graphs without loops or multiple edges. Write $F \rightarrow (G,H)$ to mean that if the edges of $F$ are colored with two colors, say red and blue, then either the red subgraph of $F$ contains a copy of $G$ or the blue subgraph contains a copy of $H$. The class of all graphs $F$ such that $F \rightarrow (G,H)$ will be denoted by $R(G,H)$. The size Ramsey number $r(G,H)$ is the minimum number of edges of a graph in $R(G,H)$. The following theorem is proved: Theorem: The size Ramsey number $\hat{r}(mK_1, nK_{1,t}) = (m+n-1)(k+t-1)$, where $sK_{1,t}$ denotes $s$ disjoint copies of a star with $t+1$ vertices. Also some results are obtained on the following question. If $F \rightarrow (mG, nH)$, how many disjoint copies of $G$ (or $H$) must $F$ contain? (Received October 17, 1977.)

The statement $G \rightarrow (G_1, G_2)$ signifies that for every partition $(E_1, E_2)$ of $E(G)$, either $E_1 \supseteq G_1$ or $E_2 \supseteq G_2$. The size Ramsey number $\hat{r}(G_1, G_2)$ is defined to be $\min(|E(G)|)$ such that $G \rightarrow (G_1, G_2)$. For connected bipartite graphs $G_1$, $G_2$, we define $\hat{r}_B(G_1, G_2)$ to be $\min(ab)$ such that $K_{a,b} \rightarrow (G_1, G_2)$. We have determined $\hat{r}_B(G_1, G_2)$ in various cases, in particular where $G_1 \cong K_{k,m}$ or $C_{2m}$ and $G_2 \cong K_{1,n}$. Also, in a few of these cases we have determined $\hat{r}(G_1, G_2)$. The following theorems describe cases in which both $\hat{r}_B$ and $\hat{r}$ are known. THEOREM 1. $\hat{r}_B(C_4) = 21$ and $\hat{r}(C_4) = 15$. THEOREM 2. For $n \geq 5$, $\hat{r}(K_{2,2,1,n}) = \hat{r}(K_{2,2,1,n}) = 4n$. If $m \geq 9$ and $n$ is sufficiently large, then $\hat{r}(K_{2,2,m,1,n}) = \hat{r}(K_{2,2,m,1,n}) = 2m + 4(n-1)$. (Received October 17, 1977.)

Let $X$ be a vertex-transitive graph. We show that if both $N$, the neighbourhood of a vertex in $X$, and $\overline{N}$, the neighbourhood of a vertex in the complement of $X$, are disconnected then either $X$ is isomorphic to $K_2 \times K_2$ or both $N$ and $\overline{N}$ contain isolated vertices.
After characterizing the vertex-transitive graphs satisfying the latter condition we are able to show that if \( X \) is a GRR (graphical regular representation) then \( N \) and \( \overline{N} \) are not both disconnected. Using this we are able to determine those groups which are generated by their involutions and do not admit a GRR. The largest such group has order 18, and it follows in consequence that all non-abelian simple groups have GRR's. (Received October 17, 1977.) (Author introduced by Dr. M. E. Watkins).

*752-05-38 ROBERT B. ALLAN and RENU LASKAR, Clemson University, Clemson, S.C. 29631. On domination and independent domination numbers of a graph.

Let \( G = (V, E) \) be a graph where \( V = \{v_1, v_2, \ldots, v_p\} \). A set \( A \subseteq V \) is a dominating set for \( G \) if for \( u \in A \), there exists a \( v \in A \), such that \( uv \in E \). If \( A \) is a dominating set and no proper subset of \( A \) is a dominating set, then \( A \) is called a minimal dominating set. A subset \( B \subseteq V \) is independent if no two vertices of \( B \) are adjacent. \( B \) is a maximal independent set if \( B \) is independent and is not a proper subset of an independent set. Let \( i(G) \) denote the smallest number of vertices in a maximal independent set and \( \gamma(G) \) denote the smallest number of vertices in a minimal dominating set. **Theorem.** If \( G \) does not contain an induced subgraph isomorphic to \( K_{1,3} \), then \( i(G) = \gamma(G) \). **Cor 1.** \( \gamma(L(G)) = i(L(G)) \), where \( L(G) \) is the line-graph of \( G \). (This extends the result \( \gamma(L(T)) = i(L(T)) \). where \( T \) is a tree. Mitchell and Hedetniemi, S. E. Conf Baton Rouge, 1977). **Cor 2.** \( \gamma(M(G)) = i(M(G)) \), where \( M \) is the middle graph of \( G \). The middle graph of \( G \) is an intersection graph \( \overline{G(F)} \) of \( V \) of \( F = V' \cup E \), where \( V' = \{\{v_1\}, \{v_2\}, \ldots, \{v_p\}\} \). (Received October 14, 1977.)

*752-05-39 ROBERT B. ALLAN and RENU LASKAR, Clemson University, Clemson, S.C. 29631. On the domatic number of a graph and the domination number of its complement.

Let \( G = (V, E) \) be a graph where \( V = \{v_1, v_2, \ldots, v_p\} \). A set \( A \subseteq V \) is a dominating set for \( G \) if for \( u \notin A \), there exists a \( v \in A \), such that \( uv \in E \). If \( A \) is a dominating set and no proper subset of \( A \) is a dominating set, then \( A \) is called a minimal dominating set. A \( D \)-partition of \( G \) is a partition of \( V(G) \) into dominating sets of \( G \). Let \( \gamma(G) \) denote the smallest number of vertices in a minimal dominating set and let \( d(G) \), the domatic number of \( G \), denote the maximum order of a \( D \)-partition of \( G \). A constructive proof of the result \( \gamma(G) \leq d(G) \) [due to Jaeger, F. and Payan, C. "Relations du Type Nordhans - Gaddum Pour Le Nombre d'Absorption d'un Graphpe Simple," C. R. Acad. Sc. Paris, Series A, t 274, 1972] is given here. (Received October 14, 1977.)

*752-05-40 Frank Harary, University of Michigan, Ann Arbor, Michigan 48109 and Robert W. Robinson, University of Newcastle, New South Wales 2308, Australia. Isomorphic Ramsey Numbers.

The Ramsey number of a graph \( G \) with no isolates has been defined as the minimum \( p \) such that every 2-coloring of (the lines of) the complete graph \( K_p \) contains a monochromatic \( G \). An isomorphic factorization of \( K_p \) is a partition of its lines into isomorphic subgraphs. Combining these concepts, the isomorphic Ramsey number of \( G \) is the minimum \( p \) such that for all \( n > p \), every 2-coloring of \( K_n \) which induces an isomorphic factorization contains a monochromatic \( G \). The isomorphic Ramsey numbers of all the small graphs (with at most four points) are determined. The extension to \( c > 2 \) colors is also studied. (Received October 14, 1977.)

*752-05-41 Frank W. Owens, Ball State University, Muncie, Indiana 47306. Enumeration of d-nary trees. Preliminary report.

Functional equations are obtained for the generating functions which enumerate d-nary trees for \( d \leq 6 \). The generating functions are then expanded using their functional equations. (Received October 14, 1977.)
THEOREM: Let $I$ be a countably complete proper ideal on $\omega_1$ and let $I^+$ denote $\mathcal{P}(\omega_1) - I$. Then the following are equivalent conditions on $I$:

1. $I$ is weakly selective (i.e. if $A \in I^+$ and $f: A \to \omega_1$ then there exists a set $B \in I^+$ such that $f|B$ is either constant or 1 to 1.)

2. $I^+ \cdot (I_1^+ \cdot \omega_1^+ + 1)^2$ (i.e. if $A \in I^+$ and $f: [A]^2 \to 2$ then either $f([X]^2) = \{0\}$ for some $X \in I^+$ or $f([Y]^2) = \{1\}$ for some $Y \subseteq \omega_1$ of order type $\omega + 1$.)

3. $I^+ \cdot (I_1^+ \cdot \omega_1^+)^2$.

4. $I$ is a weak $P$-point (i.e. if $A \in I^+$ and $f: A \to \omega_1$ then there exists a set $B \in I^+$ such that $f|B$ is either constant or $\omega_1$ to 1.)

(Received October 14, 1977.)

We will also briefly discuss the (topological) complexity of certain ideals on $\omega$ that arise naturally from some (finite and infinite) Ramsey type theorems. (Received October 14, 1977.)

A General Ramsey Product Theorem

We prove the following theorem which extends a number of recent results in Ramsey theory. Let us call a family $F$ of finite subsets of an abelian group $(G, +)$ Ramsey if for any partition of $G$ into finitely many classes, there exists $F \subseteq F$ and $g \in G$ such that $F + g = \{f + g: f \in F\}$ is contained entirely in one of the classes.

Theorem: Let $\{F_a\}_{a \in A}$ be an arbitrary collection of Ramsey families of $(G, +)$. Then for any partition of $G$ into finitely many classes, say $G = C_1 \cup \cdots \cup C_k$, there are elements $F_a \subseteq F_a$ and $g_a \in G$ such that for some $k$, $P_a + E_a \subseteq C_k$ for all $a \in A$. (Received October 17, 1977.)

Consider a rectangle with integer corners. We seek the maximum collection of integer-cornered subrectangles such that no pair of them has projections equal in one coordinate and ordered by inclusion in the other. To deal with this problem we introduce a concept of skew chain orders (as opposed to symmetric chain orders), which are partial orders which can be partitioned into saturated chains starting at the same rank. The rectangles, ordered by inclusion, are the direct product of two skew chain orders, and the desired set is the maximum semi-antichain. In general, the maximum semi-antichain in such a direct product consists of the two sets whose rank in the two components is equal. The arguments used parallel those of Sperner-type theorems on symmetric chain orders. The answer to the original question is the collection of all subrectangles which are squares. Some other questions are raised. (Received October 17, 1977.)

On the classification of generalized quadrangles.

Let $Q$ be a generalized quadrangle. Let $\text{St}(x) = \{y | y$ is a point of $Q$ and $y$ is collinear with $x$\}$. For each point $x$ of $Q$, let $E(x) = \{a \in \text{Aut}(Q) | a$ is the identity on $\text{St}(x)\}$. Let $U(Q) = \langle E(u) | u$ is a point of $Q\rangle$. We have the following result. THEOREM. The order of $E(u)$ is even for every point $u$ of $Q$ if and only if $Q$ is a hermitian or symplectic generalized quadrangle. If either condition holds, there is a power of 2,
q, such that \((U(Q_x), Q)\) is permutation isomorphic to one of the following:
\((\text{PSp}(4,q), \text{W}(3,q)), (\text{PSU}(4,q^2), \text{H}(3,q^2)), \) or \((\text{PSU}(5,q^2), \text{H}(4,q^2))\). This resolves a conjecture of E. Shult (Mathematical Centre Tracts 57, 1974). We also obtain the following result. **THEOREM.** Let \(p\) be a prime. Suppose that \(p\) divides the order of \(E(u)\) for each point \(u\) of \(Q\). Then \(U(Q_x)\) acts primitively on the points of \(Q\). (Received October 17, 1977.)

752-05-46 Richard W. Decker, Henry H. Glover and John Philip Huneke, The Ohio State University, Columbus, OH 43210. The Orientable Genus of a 2-connected Graph.

**Theorem.** Let \(G\) be a graph such that \(G = K \cup L\), \(K \cap L = \{\alpha, \beta\}\). Then the orientable genus of \(G\),
\[\gamma(G) = \gamma(K) + \gamma(L) + \epsilon, \quad \epsilon = 0, 1.\]
Further, \(\epsilon\) can be computed in terms of the genus of \(K, L\) and two particular completions of each.
This theorem generalizes the result of Battle, et al. giving \(\gamma(G)\) as the sum \(\Sigma \gamma(B_i)\) where \(B_i\) are the blocks of \(G\). (Received October 17, 1977.)


Given a permutation group \(P\) acting on a set \(X\), Snapper has shown that the coefficient \(\gamma_1\) of \(x^{n-1}\) in the characteristic polynomial of the permutation matrix representing an element \(\sigma\) of \(P\) is a difference of two permutation characters, one of which, \(\Delta_1\), is the permutation character of \(P\) acting on the 1 element subsets of \(X\). In the case where \(P\) has odd order, \(\gamma_1\) is \(\Delta_1\). One consequence of this is that the number of orbits of \(P\) acting on the 1-element subsets of \(X\) is equal to the inner product of \(\gamma_1\) and the all 1's character in this case. We present an analogous result using the characteristic polynomial of another matrix representation of elements \(\sigma\) of \(P\). Our polynomial turns out to have \(\pm \gamma_1\) as coefficients, which enables us to generalize most of Snapper's results to the even order case. We also show how to compute \(\Delta_1(\sigma)\) and \(\gamma_1(\sigma)\) for any group element \(\sigma\). We give a polynomial which may be computed by a substitution into the cycle index of \(P\) on \(X\) in which the coefficient of \(x^K\) is plus or minus the number of orbits of \(P\) acting on the \(k\) element subsets of \(X\). (Received October 17, 1977.)

752-05-48 Derek A. Holton, University of Melbourne, Parkville, Victoria 3052, Australia and Mark E. Watkins, Syracuse University, Syracuse, New York 13210. Tournament regular representations of infinite groups. Preliminary report.

A tournament \(T\) is a tournament regular representation (TRR) of an abstract group \(G\) if the automorphism group of \(T\) is isomorphic to \(G\) and acts as a regular permutation group on the set of vertices of \(T\). **Theorem 1:** Let \(G\) be a group containing an infinite, finitely generated, normal subgroup \(N\) such that \(G/N \neq Z\); if \(N\) admits a TRR, then \(G\) admits a TRR in which, for each vertex \(x\), some vertex dominated by \(x\) is dominated by finitely many other vertices dominated by \(x\). In particular, every finitely generated free abelian group admits a TRR having this property.

**Theorem 2:** Let \(G\) be a group containing an infinite normal subgroup such that \(G/N \neq Z_n\) for some odd \(n\); if \(N\) admits a TRR with the above dominance property, then \(G\) does also.

L. Babai and W. Imrich (in preparation) have shown that every finite abelian group of odd order except for \(Z_3 \times Z_3\) admits a TRR. We combine this result with special cases of the foregoing theorems to prove **Theorem 3:** Except for \(Z_3 \times Z_3\), every finitely generated abelian group without elements of order 2 admits a TRR. (Received October 17, 1977.)

A-39
The recent work of Nešetřil and Rodl on Ramsey Theory for set systems with forbidden subconfigurations will be discussed. The work is very general, and an attempt will be made here to present more intuitive form of the argument for special cases which will still preserve the basic ideas in the proof. (Received October 18, 1977.)

DONALD L. GOLDSMITH and ARTHUR T. WHITE, Western Michigan University, Kalamazoo, Michigan 49008. On graphs with equal edge connectivity and minimum degree.

It was proved by Chartrand in connection with a problem concerning communication networks that if G is a graph of order p for which the minimum degree is at least \([p/2]\), then the edge connectivity of G equals the minimum degree of G. We prove here the following theorem: Let G be a graph of order p, minimum degree \(\delta(G)\), and edge connectivity \(\kappa_1(G)\). If the vertex set \(V(G)\) can be partitioned into \([p/2]\) pairs of vertices \(v_i, v'_i\) \((i = 1, 2, \ldots, [p/2])\) (and, if p is odd, one "unpaired" vertex u) such that \(\deg v_i + \deg v'_i \geq p (i = 1, 2, \ldots, [p/2])\), then \(\kappa_1(G) = \delta(G)\). (Received October 18, 1977.)

VERA ROSTA, University of Waterloo, Waterloo, Ontario, Canada. Survey on Ramsey-multiplicity of graphs. Preliminary report.

The Ramsey number, \(r(F)\), of a graph F is the smallest number \(p\) such that in any 2-colouring of the edges of the complete graph \(K_p\) there occurs a monochromatic F. The Ramsey-multiplicity \(R(F)\) is the minimum number of monochromatic F in any 2-colouring of \(K_{r(F)}\). As the Ramsey number for some graphs is already known it was possible to investigate the Ramsey-multiplicity for some of these graphs (mainly for small graphs, paths, stars and cycles). There also exist some interesting results on the minimum number of monochromatic F in any 2-colouring of \(K_n\) if \(n \geq r(F)\).

(Received October 18, 1977.) (Author introduced by Dr. S. A. Burr).


For \(X\) a finite subset of \(\mathbb{N}\) = the natural numbers, \(X\) is \(l\)-dense if \(l, \min X \leq |X|\); \(X\) is \(n\)-dense if for all \(f: [X]^3 \rightarrow \mathbb{N}\), \(\exists y \subseteq X (y\ is\ n\-dense\ and\ homogeneous\ for\ f)\). For \(a,b,c \in \mathbb{N}\), \(X \varphi (a,b)^c\) means: for all \(f: [X]^b \rightarrow [0,c-1]\), \(\exists Y \subseteq X (\min Y, a \leq |Y|\ and\ Y\ is\ homogeneous\ for\ f)\). Let \(\eta(n) = \text{the minimum}\ m\ \text{such}\ \{0,m\}\ \text{is}\ n\-dense\.\)\ Let \(\lambda(n) = \text{the minimum}\ m\ \text{such}\ \{0,m\}\ \text{is}\ n\-dense\.\)\ Let \(\mathbb{N}\) be the natural numbers.

The following combinatorial statements are true but not provable in Peano's first order axioms:

1) \(\forall n \exists m (\{0,m\} \text{ is } n\-dense)\)
2) \(\forall a,b,c \exists m (\{0,m\} \varphi (a,b)^c)\).

In fact, the functions \(\eta\) and \(\lambda\) eventually dominate any computable function which is provably total in Peano's first order axioms (this includes, in particular, the primitive recursive functions).

(Received October 18, 1977.)

06 Order, Lattices, Ordered Algebraic Structures

MAUREEN A. BARDWELL, Bowling Green State University, Bowling Green, Ohio 43403. Lattice-ordered groups of order automorphisms of partially ordered sets.

Let \(\mathcal{O}\) be a partially ordered set and let \(A(\mathcal{O})\) denote the group of all order preserving automorphisms of \(\mathcal{O}\). We may define a partial order of the set \(A(\mathcal{O})\) by the following rule: \(f \leq g\) means that for all \(a \in \mathcal{O}\), \(af \leq ag\). When \(\mathcal{O}\) is a totally ordered set, it is well-known that \(A(\mathcal{O})\) inherits a lattice order from \(\mathcal{O}\) and that positive elements of \(A(\mathcal{O})\) are algebraically disjoint if and only if they have disjoint supports. In this report, we present a theorem which leads to the
classification of all the partially ordered sets \( \mathcal{A} \) and the groups \( \mathcal{A}(\mathcal{A}) \) which enjoy the two properties listed. We also present examples of the \( \ell \)-groups \( \mathcal{A}(\mathcal{A}) \) which arise in this fashion. (Received September 30, 1977.)

*752-06-2  M. F. Janowitz, University of Massachusetts, Amherst, MA 01003. Continuous cluster methods.

The notation and terminology of Abstracts 76T-C19 and 76T-C36 (vol. 23 (1976), these Notices) will be followed. Equip \( LC(P) \) with the metric topology determined by the sup norm. Call a cluster method \( F \) left order-continuous if \( d \) in the pointwise ordering of \( LC(P) \) implies the existence of an index \( b \) such that \( Fd = \bigvee \{Fd_a : a \geq b\} \).

Call \( F \) 0-isotone if \( d_1 \leq d_2 \) in \( LC(P) \) with \( Td_1(0) = Td_2(0) \) implies \( Fd_1 \leq Fd_2 \). Theorem 1. For a cluster method \( F \), TAE: (1) \( F \) is monotone equivariant (ME) and left continuous, (2) \( F \) is ME and left order-continuous. (3) \( F \) is ME and \( \wedge \)-isotone. (4) \( F \) is semiflat. Theorem 2. TAE: (1) \( F \) is ME and continuous. (2) \( F \) is ME and right continuous. (3) \( F \) is ME and right order-continuous. (4) \( F \) is flat (in the sense of 76T-C19). (Received October 4, 1977.)

*752-06-3  RUSSELL BELDING, United States Naval Academy, Annapolis, Maryland 21402. Path Connected Posets.

The graph (or Hasse diagram) of a partially ordered set \( (X, \leq) \) has \( X \) as its set of vertices and \( (x, y) \) is an edge if and only if \( x \) covers \( y \) or \( y \) covers \( x \). The poset is path connected if its graph is connected. Two integer valued metrics, zig-zag and distance, are defined for path connected posets. Together the values of these metrics characterize path connected posets to within isomorphism and duality. (Received October 11, 1977.)

*752-06-4  GARY M. HARDEGREE, University of Massachusetts, Amherst, Massachusetts 01003. A Variety of Semilattices which generalize Orthomodular Lattices.

The relevant algebraic structures are called quasi-implication algebras (QIA's), which are intended to generalize orthomodular lattices (OML's) in the same way that implication algebras (J.C. Abbott) generalize Boolean lattices. A QIA is a set \( Q \) together with a binary operation \( \rightarrow \) (called quasi-implication) satisfying the following axioms (\( a \rightarrow b \) is denoted \( ab \)): 1. \( (ab)a = a \) 2. \( (ab)(ac) = (ba)(bc) \) 3. \( ((ab)(ba))a = ((ba)(ab))b \). A bounded QIA is a QIA with a distinguished element \( 0 \) additionally satisfying: 4. \( 0a = aa \). Every QIA induces an upper bounded join semilattice, where the following identifications are made: \( 1 = aa \); \( a \leq b \Rightarrow ab = 1 \); \( a \wedge b = (ab)(ba)a \). Furthermore, every pair of elements \( a, b \) that have a lower bound \( c \) have a greatest lower bound \( (ab)(ac)c \).

Consequently, every bounded QIA induces a lattice; indeed, every bounded QIA induces an orthomodular lattice, where the following identifications are made: \( a \wedge b = ((ab)(a0))a \); \( a \sqcap = a0 \). Conversely, every OML induces a bounded QIA, where \( ab = a \rightarrow \top \wedge (a \wedge b) \), and \( 0 \) is the OML zero element. Finally, every QIA can be embedded into a bounded QIA (OML) preserving the quasi-implication operation. Thus, every QIA is isomorphic to a quasi-implication subalgebra of an OML, and the theory of QIA's completely characterizes the quasi-implicational fragment of OML theory. (Received October 11, 1977.) (Author introduced by David J. Foulis).


A morphism from one manual \( A \) to a second manual \( B \) is a map \( \lambda \) from the events of \( A \) to the events of \( B \) which induces an isotope map from the logic \( \Pi(A) \) of \( A \) to the logic \( \Pi(B) \) of \( B \) and which satisfies \( \lambda(D) = \top \wedge \lambda([x]) \) for the \( A \)-event \( D \). A traditional morphism is a morphism which can be written as a composition of some or all of these pure morphisms: a phenomenological identification, a collapse, an outcome refinement, a truncation, an amnesiation, and an immersion. Pure morphisms correspond to modifications a scientist might make in a laboratory to a manual of operations.
because of theoretical or practical considerations. This note will deal with the kinds of morphisms which are traditional, and the kinds of manuals which admit only traditional morphisms. [For basic definitions, see "The Operational Approach to Quantum Mechanics" by Randall and Foulis in The Logico-Algebraic Approach to Quantum Mechanics, Volume III, D. Reidel Publishing Co.] (Received October 13, 1977.)

Let a cube of side $k$ in $\mathbb{R}^n$ be dissected into $k^n$ unit cubes. The collection of all affine subspaces of $\mathbb{R}^n$ determined by the faces of the unit cubes forms a lattice $L(n,k)$ when ordered by inclusion. We explicitly construct a Dilworth partition into chains of $L(n,k)$. (Received October 14, 1977.)

A generalization of the concept of a cyclically ordered group is given; namely, a manifold group is defined to be a group $G$ together with a collection $\mathcal{O}$ of partially ordered subsets satisfying certain compatibility conditions. Conditions are given for a system of partially ordered subsets of a group to generate a manifold structure. Finally it is shown that the group of orientation preserving permutations of the oriented unit circle is, in fact, a manifold group. (Received October 14, 1977.)

A method is given for constructing a free $p$-ring $F$ on any set of preassigned (finite or infinite) cardinality of free generators. It is shown that any set $D$ of pairwise disjoint elements (i.e., $xy = 0$ for every two distinct elements $x$ and $y$ of $D$) of $F$ is countable. Similarly, it is shown that any simply ordered subset (with respect to the partial order $\leq$ in $F$ defined by $x \leq y$ if and only if $xy = x^2$) of $F$ is countable.

The above results are extended to a second method for constructing free $p$-rings and to the case of free $p^e$-rings. (Received October 17, 1977.)

For a semigroup $S$ the relation $a \equiv b \equiv asb = bas = asa$ for all $s \in S$, is called Conrad's relation. The author presents joint work with W. D. Burgess in which the notion of a weakly separative semigroup is defined. This notion generalizes separative semigroups to the noncommutative case, and is a necessary and sufficient condition.
condition for Conrad's relation to give a partial ordering of a semigroup. The authors study several notions of completeness with respect to this order, continuing their study of reduced and semiprime rings. One interesting consequence of the work is that an inverse semigroup admits a partial ordering which is, in general, different from the 'natural' one, which is well known. The work has been submitted to the Semigroup Forum. (Received October 17, 1977.)

752-06-11 M.E. Adams and J. Sichler, University of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2

*Homomorphisms of bounded lattices.*

Let \( \mathcal{L} \) denote the category of nontrivial bounded lattices and all \((0,1)\)-preserving homomorphisms. **Theorem:** Let \( \mathcal{C} \) be a small category and let \( \mathcal{F} \) be a functor such that \( \mathcal{F} : \mathcal{C} \to \mathcal{L} \). Then there exists a full embedding \( \mathcal{Y} : \mathcal{C} \to \mathcal{L} \) such that \( \mathcal{F} \) is a subfunctor of \( \mathcal{Y} \).

(Received October 17, 1977.)

752-06-12 Jo E. Smith, Boise State University, Boise, Idaho 83725.

*A new family of \( J \)-group varieties.*

The family of Scrimger varieties, \( J_n \), for any natural number \( n \), has been shown to be of great interest in the study of the lattice \( \mathcal{L} \) of all \( J \)-group varieties. It is well-known that \( J_n \) is contained in the variety of all \( J \)-groups in which \( n \)th powers commute, and it was conjectured that this containment is proper. In this report, the conjecture is proved true for \( n \) not prime. For the proof, however, it is useful to consider a new family of varieties \( J_{n,m} \) generated by a type of lex extension of the \( J \)-groups which generate the \( J_n \)'s. In fact, it is possible to continue this process of extending \( J \)-groups and thus produce a family of new varieties subscripted over sequences of natural numbers. The role that this family plays in the structure of \( \mathcal{L} \) will be discussed, as well as the uses for this family in answering some of the other standing questions concerning \( J \)-group varieties. (Received October 18, 1977.)

752-06-13 R.L. Wilson, University of Tennessee, Knoxville, Tennessee 37916.

*Semigroups Supporting Unique Compact Hausdorff Topologies.* Preliminary report.

Let \( \mathcal{C} \) denote the class of all pairs \((S,T)\) such that \( S \) is a normal, \( T \)-trivial semigroup with \( S^2 = S \) and \( T \) is a compact Hausdorff topology on \( S \) for which multiplication is continuous. Let \( \mathcal{A} \) denote the subclass of \( \mathcal{C} \) whose objects have the property that if \((S,T_1) \in \mathcal{C} \) and \((S,T_2) \in \mathcal{C} \) then \( T_1 = T_2 \).

Order theoretic techniques are used to obtain a sufficient algebraic condition on members of \( \mathcal{C} \) which will imply that they must also be members of \( \mathcal{A} \). It is shown that this condition is preserved by continuous homomorphic images and is satisfied, for example, if \( S \) is a normal, \( T \)-trivial semigroup with \( S^2 = S \) on a uniquely arcwise connected continuum with maximal arcs having idempotent endpoints. This encompasses semigroups on generalized topological trees with idempotent endpoints.

These facts along with the result that \( \mathcal{A} \) is closed under products and the result that each compact semilattice is a member of \( \mathcal{A} \), give an indication of the extensiveness of the class \( \mathcal{A} \).

(Received October 17, 1977.) (Author introduced by J. H. Carruth).

752-06-14 Chen-jyi Su, University of Miami, Coral Gables, Florida 33124.

*Sheaf Sectional Representation of Some Partially Ordered Rings.*

A PBI ring is directed partially ordered ring which satisfies: \( a^{-1} \exists \) exists and \( a^{-1} \geq 0 \) whenever \( a \geq 1 \). It is called Archimedean of \( a \leq 0 \) whenever \( \inf\{n/m : ma < n, m \in \mathbb{N}\} = 0 \). An ideal \( I \) in a partially ordered ring is called an \( 0 \)-ideal if \( x \in I \) whenever \( 0 < x \leq y \) and \( y \in I \). The order prime spectrum \( O(A) \) of a partially ordered ring \( A \) is the subspace of the prime spectrum...
of $A$ consisting of ordered prime ideals. A sheaf of partially ordered rings is a sheaf of rings in which every stalk is a partially ordered ring.

**Theorem** Let $A$ be an Archimedean PBI ring. Then there is a sheaf of partially ordered rings over $O(A)$ such that $A$ is isomorphic to the ring of all continuous sections. (Received October 18, 1977.)

Let $G$ be a lattice ordered group and $\tau$ a convergence structure on $G$ making group and lattice operation continuous (an $\varepsilon$-convergence structure). Define the two-sided $\varepsilon$-Cauchy structure $C$ of $\tau$ by declaring $F \in C$ if $F^{-1}, F^{-1} F \in (1)\tau$. Let $G^C$ be the set of equivalence classes of $C$ under the usual equivalence relation. **Thm:** The obvious definitions make $G^C$ a lattice ordered group which contains $G$ if $\tau$ is Hausdorff. **Thm:** For any formula $\psi$ which is a combination of $\varepsilon$-equations by disjunction and conjunction, $G^C \models \psi$ iff $G \models \psi$. In particular, $G$ and $G^C$ generate the same variety, and one is totally ordered iff the other is. **Thm:** If $\tau$ is an order closed convergence structure ($F \in (1)\tau$ implies the order closure of $F \in (1)\tau$) then $G$ is order dense in $G^C$. **Thm:** If $\tau$ is the order convergence structure on $G$, $G^C$ is the ($\varepsilon$-group of units of) the Dedekind-McNeill completion of $G$. **Thm:** If $\tau$ is the polar convergence structure $((1)\tau$ generated by filters of polars which intersect $1)$, then $G^C$ is laterally complete.

The last result provides an alternative proof of the existence of the lateral completion due to Bernau. (Received October 18, 1977.)

08 | General Mathematical Systems


Among the locally distributive systems, i.e., systems with two operations and a topology and such that \[ \lim_{h \to 0} (ha+hb)(h(a+b))^{-1} = 1 \]
are related to the real field in a way entirely similar to the positions of non-Euclidean geometries with respect to the Euclidean one. Twelve axioms are presented which characterize the hyperbolic real numbers. Proofs of consistency and categoricalness are sketched. Elliptic real numbers are introduced and applications to elliptic geometry are discussed. (Received October 12, 1977.) (Author introduced by Dr. Joseph Hornback).

752-08-2 W. M. SANDERS, James Madison University, Harrisonburg, Virginia 22801. A non-desarguesian finite projective plane of order nine. Preliminary report.

L. J. Paige and C. Wexler used digraph-complete latin squares to obtain an incidence matrix for a finite projective plane. Here a set of nine permutation matrices is used to form the kernel of a canonical incidence matrix for a non-desarguesian plane of order nine. From the incidence structure two binary operations are obtained. Under one operation the system yields an abelian group; under the other, the system is noncommutative, nonassociative, and at least one of the distributive laws fails. (Received October 17, 1977.)

*752-08-3 WILLIAM P. WARDLAW, U. S. Naval Academy, Annapolis, Maryland 21402. n-Associative Groupoids. Preliminary report.

A groupoid $G$ (set with binary operation) is $n$-associative if the product of any $n$ elements of $G$ is independent of the way in which the factors are associated.

**Theorem** For $n > 3$, $n$-associativity implies $(n + 1)$-associativity.

This natural generalization of the generalized associative law motivates an investigation of groupoids which are $n$-associative but not $(n - 1)$-associative. Some results are obtained on the structure and possible cardinalities of such groupoids. (Received October 17, 1977.)

A nonempty set $S$, with 3 binary operations $+$, $\times$, and $\ast$, is known as a Sring if $(S,+)$ and $(S,\times)$ are monoids, $(S,\ast)$ is a groupoid, and for all $a,b,c \in S$ we have:

- $(a \times (b + c)) = ((a \times b) + (a \times c))$,
- $(b \times a + (c \times b)) = (a \ast (b + c))$,
- $s(b \times c) = (s \ast b) \times c$.

A nonempty set $T$ with 3 binary operations $+$, $\times$, and $\ast$, is known as a Tring if $(T, +, \times)$ is a ring, and for all $a,b,c \in S$ we have:

- $(a \ast (b + c)) = (a \ast b) \times (a \ast c)$,
- $(a \times (b + c)) = (a \times b + (a \times c))$,
- $(a \ast b) \times (b \ast c) = (a \ast (b \times c))$.

In this paper, we study the ideals, homomorphisms, and other properties of srings and trings. (Received October 18, 1977.)

10 ▶ Number Theory

Large Sieve Inequality for Algebraic Number Fields. Preliminary report.

The author will present an application of a weighted large sieve inequality for algebraic number fields with a fixed integral basis. The inequality is derived after the manner of H. L. Montgomery and R. D. Vaughan [Mathematika 20(1973), 119-134] for rational integers and contains the formulation of the large sieve for algebraic number fields with a fixed integral basis given by M. N. Huxley [Mathematika 15 (1968), 178-187]. (Received September 23, 1977.)

Biotic Gauss sums and sixteenth power residue difference sets.

The evaluation of the Gauss sum $G_{16} = \sum_{n=0}^{p-1} e^{2\pi in^2/p}$ (for $p$ prime, $p \equiv 1 \pmod{16}$) is used to prove the nonexistence of sixteenth power residue difference sets. This completes the work of Whiteman [Trans. American Math. Soc. 86(1957), 401-413]. (Received September 26, 1977.) (Author introduced by Bruce Berndt.)

Another minimal property associated with Markoff forms. Preliminary report.

If $A$ and $B$ lie in a group, define $S$, the "step-power", as $(A,B)^u,v = \prod_{i=1}^{t_j} (A,B)^{j}$

where $t_j = \lfloor jv/u \rfloor - \lfloor (j-1)v/u \rfloor$ if $u > 0$, $S=B^v$ if $u=0$, $S^{-1}=(A,B)^{-u,-v}$ if $u < 0$.

Let $A$ and $B$ be matrices in $SL_2(\mathbb{R})$ with trace $[A,B]=-2$. (For instance, the modular group has its commutator subgroup generated by $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$; and the fixed points of the step-powers are the roots of the Markoff forms). Now consider all words $W = A^u B^v$ with preassigned coset modulo the commutators, i.e., with preassigned $u \equiv \Sigma_{a_j}$ and $v \equiv \Sigma_{b_j}$. We assert that minimum $\min \{[A,B]^u,v}\}$, and this minimum is strict to within equivalence. This is seen by geodesics in the upper half plane. [See author, Math. Ann. 196 (1972) and A. Schmidt, Crelle 286/7 (1976)]. The half plane is further tessalated by an elliptic function. There surely must be a direct way to prove this matrix theorem! (Research supported by NSF Grant MPS 76-06714). (Received September 29, 1977.)
Let $p$ be an odd prime, $k$ and $r$ positive integers, $\alpha \in \text{GF}(p^r)$, and $\text{tr}(\alpha) = \alpha + \alpha^p + \alpha^{p^2} + \ldots + \alpha^{p^{r-1}}$. We define the Gauss sum $G_{k,r}$ over $\text{GF}(p^r)$ by

$$G_{k,r} = \sum_{\alpha \in \text{GF}(p^r)} e^{2\pi i \frac{\text{tr}(\alpha^k)}{p^r}}.$$

Furthermore, let $n$ be a positive integer, $\phi$ a quadratic character on $\text{GF}(p^r)$, and $\beta \in \text{GF}(p^r)^*$. We define the Jacobsthal sum $\Phi_{n,r}(\beta)$ over $\text{GF}(p^r)$ by

$$\Phi_{n,r}(\beta) = \sum_{\alpha \in \text{GF}(p^r)} \phi(\alpha)\phi(\alpha^n + \beta).$$

For $r = 1$, several Gauss and Jacobsthal sums have been evaluated in the literature. For $r > 1$, no such determinations are found in the literature. By developing the theory of Jacobi and Eisenstein sums over $\text{GF}(p^2)$, the authors evaluate several Gauss and Jacobsthal sums when $r = 2$. (Received October 3, 1977.)

ANDREW M. ODLYZKO, Bell Laboratories, 600 Mountain Avenue, Murray Hill, N.J. 07974. A study of the asymptotic oscillations of a certain sequence.

The sequence $\{t_n\}$ in question enumerates so-called 2,3-trees. It will be shown through analytic methods that this sequence exhibits very regular oscillations for large $n$;

$$t_n \sim n^{\frac{1}{2}} \psi(n \log n) \text{ as } n \to \infty.$$  

Here $\psi = (1+\sqrt{5})/2$ is the golden ratio, and $\psi$ is a nonconstant positive function which is periodic with period $\log(3-\phi^{-1})$. (Received October 3, 1977.)


According to a general principle established by the first author [On the cycle structure of linear recurring sequences, Math. Scand. 38, 53-77 (1976).] any linear recurring sequence in a finite field which has a sufficiently long period is almost equidistributed, in the sense that each element of the field occurs about equally often in the full period of the sequence. This raises the question of characterizing those linear recurring sequences for which we have exact equidistribution. We settle this problem for linear recurrences of low order. Although the pattern of a general method emerges clearly from the ensuing investigation, a detailed discussion becomes increasingly complex for higher-order linear recurring sequences. Therefore we restrict the attention to linear recurrences of order at most 4. (Received October 6, 1977.)

David M. Bressoud, Pennsylvania State University, University Park, Pa. 16802. The generating function for Gordon's identity.

Let $(a)_n = (1-a)(1-aq)(1-aq^2)\ldots(1-aq^{n-1})$. We demonstrate that

$$\sum_{n_1,\ldots,n_{k-1},n_k \geq 0} \frac{q^{N_1+N_2^2+\ldots+N_{k-1}^2+N_k^r+N_k^{r+1}+\ldots+N_k^{r+k-1}}}{(q)_n(q^2)_n^2(\ldots)(q^{r+k-1})_{n_{k-1}}} \quad (N_i = n_1+n_{i+1}+\ldots+n_{k-1})$$

generates $B_{k,r}(n)$, the number of partitions of $n$ of the form $(d_1 + d_2 + \ldots + d_s)$ where $d_1 \geq d_{i+1}$, $d_1 - d_{i+k-1} \geq 2$, and at most $r-1$ of the $d_i$ equal 1. (Received October 7, 1977.)
Let $\xi = a_0 + a_1 p + a_2 p^2 + \ldots$ be the canonical representation of the $p$-adic integer $\xi$.

Define $T: \mathbb{Z}_p \to \mathbb{Z}_p$ by $T(\xi) = a_1 + a_2 p + a_3 p^2 + \ldots$. If $f: \mathbb{Z}_p \to \mathbb{C}$ has $p$-adic total variation $V(f)$ and $s_f(\xi) = \int f(\zeta) d\lambda(\zeta) = 0$, where $\lambda$ is Haar measure on $\mathbb{Z}_p$, we show that

$$\lim \sup_{N \to \infty} \frac{1}{N \log \log N} \left| \int f(T^k(\xi)) + f(T^{k+1}(\xi)) + \ldots + f(T^N(\xi)) \right| \leq \frac{A_p}{p} V(f) \|f\|_2$$

for $\lambda$-almost all $\xi \in \mathbb{Z}_p$. Here $A_p$ is a positive constant depending only on $p$. We also prove a law of the iterated logarithm for $s_f(\xi)$, namely:

$$\lim \sup_{N \to \infty} \frac{1}{N \log \log N} \left| \int s_f(T^n(\xi)) \right| < B$$

for $\lambda$-almost all $\xi \in \mathbb{Z}_p$. Again $B > 0$ depends only on $p$. (Received October 11, 1977.)

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Suppose $p_i$ is the $i$-th smallest prime factor of an odd perfect number with $r$ distinct prime factors. Grün [Math. Z. 55 (1952), 353-354] proved that $p_1 < 2r/3 + 2$, and Pomerance [Math. Ann. 226 (1977), 195-206] proved that $p_i < (4r)^{2^{i+1}/2}$

Using computer, we prove that for $2 \leq i \leq 6$, $p_i < 2^{i-1}(r - i + 1)$. (Received October 11, 1977.)

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Let $\zeta(s,a,x) = \sum_{n=0}^{\infty} \frac{\sin nx}{(na)^s}$, the Lerch zeta function, valid for $a,x \in (0,1]$ and $s > 1$ if $x = 1$ and $0$ otherwise. We analyze the distribution of zeroes. Let $x = \min\{x,1-x\}$ and $\log' u = \log u$ if $0 < u < 1$. The critical strip in question is contained in the strip $0 < \sigma < 1+a$, there being no zeroes for $\sigma > 1+a$ and only 

"trivial" ones for $0 < \sigma < 1$ given asymptotically (as $|s| \to \infty$) by the formulae $\sigma = \frac{-2\log^2(1-x) - 4a - 2k + \frac{1}{x} + o(1)}{2 \log^2(1-x) + \frac{1}{x}}$

and $t = \frac{-1}{2 \pi^2} \log^2\left(\frac{1-x}{x}\right) \log'(\frac{1-x}{x})$ with $k \in \mathbb{R}$ and $k > \frac{1}{2 \pi^2} \log^2\left(\frac{1-x}{x}\right) - 2a$. If we let

$N_+(T,a,x)$ (respectively $N_-(T,a,x)$) denote the number of zeroes in $[-x,1+x]$ and $0 < t < T$ (respectively $-T < t < 0$) then as $T \to \infty$ we have $N_+(T,a,x) = \frac{T}{2\pi} \log T - \frac{x}{2\pi} \log 2 + \log a + \log x + o(1)$

and $N_-(T,a,x) = \frac{T}{2\pi} \log T - \frac{x}{2\pi} \log 2 + \log a + \log'(1-x) + o(1)$. We then propose a reasonable conjecture on the growth of $\arg \zeta(s,a,x)$ and consider some of its consequences one of which is the unsolved Tchebyshov conjecture: $\limsup_{s \to 0} \frac{1}{p} \sum_{p \leq x} e^{-\frac{1}{p}} = +\infty$ and $\liminf_{s \to 0} \frac{1}{p} \sum_{p \leq x} e^{-\frac{1}{p}} = -\infty$, (p an odd prime).

(Received October 14, 1977.)

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The Cassini oval $0(a,b,k^2) = \{u \in \mathbb{C} ; |a - u|,|b - u| \leq k^2\}$, defined for $a, b$ in the complex numbers $\mathbb{C}$ and $k > 0$, and the function $m$ defined for $c > 0$ by $m(c) = \max \min |a - u|,|b - u|$, where the maximum is over all complex numbers $a$ and $b$ for which $|a - b| = 2c$ and the minimum is over all Gaussian integers $u$, are related by the equivalence $0(-c,c,k^2)$ contains a lattice point no matter how it is rotated and translated in the plane iff $k^2 > m(c)$. The value of $m(c)$ for $0 \leq c^2 \leq 7/8$ has been known (these Notices Vol. 16 (1969) p. 298, abstract #663-723). We evaluate $m(c)$ for $7/8 < c^2 \leq 25/16$.

For $c^2 = 25/16$, we obtain the smallest lemniscate of Bernoulli $0(-5/4,5/4,25/16)$ which always contains a lattice point no matter how it is moved in the plane. (Received October 14, 1977.)
We compare three notions of equivalence recently introduced into the theory of quadratic forms: the spinor genera of quadratic lattices, the spinor equivalence of quadratic forms over $\mathbb{Z}$ and the quasi-genera of quadratic forms over $\mathbb{Z}$, as introduced by M. Eichler, G. L. Watson and B. W. Jones, respectively. We first show that forms over $\mathbb{Z}$ are spinor equivalent if and only if the corresponding $\mathbb{Z}$-lattices are in the same spinor genus. The concept of the quasi-genus is compared with that of the spinor genus with respect to a certain set of rational primes. Finally, it is noted that while Eichler's definition requires the existence of $p$-adic completions of the coefficient ring, a generalization of Watson's can be given applying to matrices over arbitrary commutative rings.

(Received October 17, 1977.)

Suppose $r_1, r_2, \ldots, r_m$ are positive irrational numbers such that $r_i/r_j$ is irrational for $1 \leq i < j \leq n$. Let $r_0 = 1$, and let

$$S_j = \left\{ \frac{n r_i}{r_j} : n = 1, 2, \ldots \right\} \quad \text{for} \quad j = 0, 1, \ldots, m.$$  

These are complementary sets; that is, each positive integer lies in one and only one $S_j$. This result is a generalization of Beatty's Theorem, which is obtained in case $n = 1$. (Received October 17, 1977.)

The Eisenstein series for $GL(n)$ are functions of 2 variables $s\in\mathbb{C}$ and $P\in\mathbb{P}_n^+$, the symmetric space of all positive definite $n\times n$ real matrices. Such functions are important for the harmonic analysis of $GL(n)$ (cf. the book of Kubota or the proceedings of the AMS Summer Institute on Automorphic Forms, Corvallis, Oregon, 1977). These Eisenstein series are simultaneously zeta functions, i.e., Mellin transforms in higher dimensions of Siegel modular forms. Thus the analytic continuation in the variable $s$ is also important for the understanding of the higher dimensional version of the Hecke correspondence between automorphic forms and Dirichlet series. We use Siegel's integral formulas which appear in his proof of the Minkowski-Hlawka theorem (Ges. Abh. III,p.46) to obtain some simple results about Eisenstein series. For example, one can show that for any $s$ in the interval $(0,n/2)$ there is a positive definite $P\in\mathbb{P}_n^+$ such that the Epstein zeta function of $P$ is positive at $s$. Or we can find a $P$ to make the Epstein zeta function negative at $s$. In fact we can show that for $ResE(0,n/2)$ the integral of the Epstein zeta function over the determinant 1 surface in Minkowski's fundamental domain for $P_n^+$ modulo $GL(n,\mathbb{Z})$ is zero. (Received October 17, 1977.)

As initiated by T. H. Jackson [J. London Math. Soc. (2) 14 (1976), 178-182], we consider a Markoff spectrum for the set of indefinite binary quadratic forms which represent zero non-trivially. As was done for the classical Markoff spectrum, we show that $1/3$ is the largest accumulation point for the set and explicitly determine the countably infinite number of elements greater than $1/3$. Unlike the situation for the classical Markoff spectrum, there are a countably infinite number of limit points greater than $1/3$. (Received October 17, 1977.)
Let \( k > 0 \) and say \((\ell, k) = 1\). Denote by \( p(k, \ell) \) the least prime \( p \equiv \ell \pmod{k} \). Let \( P(k) \) denote the largest of the \( p(k, \ell) \) for all \( \ell \). It follows from the work of Linnik and others that \( P(k) \ll k^c \) for some constant \( c \). We are here concerned with lower bounds for \( P(k) \). Since \( P(k) \) is at least as big as the \( \varphi(k) \)-th prime, it follows from the prime number theorem that

\[
\alpha \overset{\text{def}}{=} \lim \inf P(k)/\varphi(k) \log k \geq 1.
\]

Using a result of Erdős (Math. Scand. 10(1962), 163-170) and an idea of Hensley and Richards (Acta Arith. 25(1974), 375-391) we show \( \alpha \geq e^{\gamma} = 1.78107 \ldots \) where \( \gamma \) is Euler's constant. Moreover we show that \( P(k)/\varphi(k) \log k \) tends to infinity on a set of density 1. (Received October 17, 1977.)

A method is introduced which may be used to prove theorems of a type exemplified by the following. Theorem: Let \( F \) be a non-vanishing cusp form of negative dimension \(-2q\) on \( \Gamma(1) \), the modular group. Let \( \xi \) be an irrational number with continued fraction representation \( \xi = (a_0; a_1, a_2, \ldots) \) having convergents \( p_n/q_n \). Then

(i) if \( a_{n+1} > C_p  \log q_n \) infinitely often, \( |F(\xi + iy)| \) has both 0 and \( \infty \) as cluster points as \( y \to 0 \).

(ii) if \( C_p  \log q_n - a_{n+1} \to \infty \), \( |F(\xi + iy)| \to \infty \) as \( y \to 0 \).


The method also has application to questions of transitivity for \( \Gamma(1) \). (Received October 18, 1977.)

Let \( d \) be a discriminant of a quadratic field and let \( \chi \) be a real primitive character \( \pmod{d} \). We prove the following variation on Siegel's and Tatazawa's theorems: If \( \varepsilon > 0 \) and \( |d| > e^{\tau} \) then

\[
L(1, \chi) > \min \left( \frac{1}{(15.24)\log d}, \frac{\varepsilon}{(1 + \varepsilon) \log d} \right)^2, \quad \text{with one possible exception. (Received October 18, 1977.)} \]

(Author introduced by Howard Hiller)

A function \( h \) on \( \mathbb{R}/2 \pi \) is also interpreted as a 1-periodic function on \( \mathbb{R} \). Let \( (x_k, k = 1, 2, \ldots) \) be a fixed sequence of real numbers and let \( F \) consist of all nonnegative and regular summation matrices \( A = (a_{nk}) \) to which there corresponds a probability measure \( P'(A) \) on \( \mathbb{R} \) such that, for all \( h \in C(\mathbb{R}/2\pi), \lim_{n} \int_{\mathbb{R}/2\pi} h(x_n) \, dP'(A)(dx) = 0 \). The associated measure \( P(A) \) on \( \mathbb{R}/2\pi \) is unique. Let \( h_{nk} = h(x_n) \) and consider the smoothing process \( h_{rn} = E_{k}^{\infty} h_{r-1, k} = E_{k}^{\infty} h_{k} \). Here, \( A^{r} \in F \) and further \( B^{r} = A^{r} A^{r-1} \ldots A^{1}, \quad (r = 1, 2, \ldots) \). Theorem. One has that \( B^{r} \in F \) and \( P(B^{r}) = P(A^{r}) \cdot \ldots \cdot P(A^{1}) \). (convolution). Moreover, as soon as either \( h \in C(\mathbb{R}/2\pi) \) or \( P(B^{r}) \) is absolutely continuous and \( h \) is
Riemann integrable, one has \( \lim_{n \to \infty} (h_n - h(x_n)) = 0 \), where \( h(x) = \int_{-t}^{t} h(x - t)P'(B^r)(dt) \). It follows that \( \lim_{n \to \infty} h_n(x) = \max h(x) \), provided \((x_k, \ k = 1, 2, \ldots)\) is dense (mod 1). We further present rather precise error bounds for the important case that \( x_k = u \log k \) while \( A^r = (C, 1) \), for all \( r \). Then \( P'(A^r) \) may be taken as the exponential distribution with parameter \( 1/u \) and \( P'(B^r) \) as the \( \Gamma(r, 1/u) \) distribution. Clearly, \( P(B^r) \) converges (weakly) to the uniform distribution on \( \mathbb{R}/\mathbb{Z} \). (Received October 10, 1977.)

**12 ▶ Algebraic Number Theory, Field Theory and Polynomials**


Using harmonic analysis on the usual compact Heisenberg manifold, one can compute classical Gaussian sums explicitly, including the determination of the sign (work independently done by Tolimieri).

Thus one can prove classical quadratic reciprocity over \( \mathbb{Q}_2 \) via such analysis. One would like to generalize to other symbols. For quadratic reciprocity over more general fields, it appears that only slight changes are needed. For higher reciprocity laws the situation is, as of this writing, unclear. Some computations have been done, however, and the current state of this problem will be the main topic. (Received September 19, 1977.)

*752-12-2* JEFFREY C. LAGARIAS, Bell Telephone Laboratories, Murray Hill, New Jersey 07974. Signatures of units and congruences (mod 4) in certain real quadratic fields.

The following curious result is proved using genus theory. **Theorem.** Let \( d \equiv 1 \pmod{8} \) and \( d = x^2 + y^2. \) Let \( \epsilon \) be any unit of \( \mathbb{Q}(\sqrt{d}) \), \( \overline{\epsilon} \) its conjugate. (i) \( \epsilon \equiv 1 \pmod{4} \) iff \( \epsilon > 0, \overline{\epsilon} > 0. \)

(ii) \( \epsilon \equiv -1 \pmod{4} \) iff \( \epsilon < 0, \overline{\epsilon} < 0. \) (iii) \( \epsilon \equiv \sqrt{d} \pmod{4} \) iff \( \epsilon > 0, \overline{\epsilon} < 0. \) (iv) \( \epsilon \equiv -\sqrt{d} \pmod{4} \) iff \( \epsilon < 0, \overline{\epsilon} > 0. \) The theorem does not guarantee the existence of units of each signature type, however. The theorem holds in fact for all integer \( a \) in \( \mathbb{Q}(\sqrt{d}) \) such that \( a \) is squarefree as an integer but \( (a) \) is the square of an ideal, provided \( (a) \) is relatively prime to \( (2) \). (Received September 29, 1977.)

*752-12-3* Gary L. Mullen, The Pennsylvania State University, Sharon, Pa. 16146. Local Permutation Polynomials in Three Variables over \( \text{GF}(p) \). Preliminary report.

A polynomial \( f(x_1, x_2, x_3) \) with coefficients in \( \text{GF}(p) \), \( p \) an odd prime, is a local permutation polynomial (LPP) over \( \text{GF}(p) \) if \( f(x_1, a, b), f(c, x_2, d), \) and \( f(e, f, x_3) \) are permutations in \( x_1, x_2, \) and \( x_3 \) respectively for all \( a, b, c, d, e, f \in \text{GF}(p) \). The number of LPP's in three variables over \( \text{GF}(p) \) equals the number of Latin cubes of order \( p \). A set of necessary and sufficient conditions is obtained for the coefficients of a polynomial \( f(x_1, x_2, x_3) \) in order that it be a LPP. These conditions extend to three variables, the corresponding results in Abstract 720-12-12, these Notices 22(1975). The conditions however are too complicated to state here. (Received October 13, 1977.)
Let $K$ be a finite field and $F$ be a real quadratic extension of the rational function field $K(x)$. In this case the order, $h_F$, of the ideal class group of the integral closure of $K[x]$ in $F$ divides the order, $h_K$, of the group of divisor classes of degree zero of $F$. The quotient is $r$, the regulator of $F$, i.e., $h_F = rh_K$. If $F$ is a field of genus $g$, it is known that $r \geq g + 1$. This paper first studies some properties of Lucas and Fibonacci polynomials when their coefficients are restricted to finite fields. Using these properties and some results from continued fraction expansions of algebraic functions it is shown that for a given field $K$, with char $K \neq 2$ there exist infinitely many real quadratic extensions $F/K(x)$ of genus $g$ for which the regulator $r$ is exactly $g + 1$. (Received October 13, 1977.)

*752-12-5 JOHN W. SHUCK, Ursinus College, Collegeville, Pennsylvania 19426. Hensel's lemma and the p-adic inverse function theorem.

Generalized versions of Hensel's lemma enable one to deduce information (e.g. existence) about p-adic zeros of a polynomial (or a system of polynomials) from information about zeros modulo powers of $p$. There are also theorems which deduce information about zeros modulo powers of $p$ from information about p-adic zeros. Here we show that results in both directions follow from precise, quantitative versions of the p-adic inverse and implicit function theorems. (Received October 13, 1977.)

752-12-6 JACOB T. B. BEARD, JR., University of Texas at Arlington, Arlington, TX 76019 and Robert M. McConnel, University of Tennessee, Knoxville, TN 37916. Matrix field extensions.

In the cases that $R$ is the ring of integers modulo $m$ or the Galois field $GF(p)$, the distinct field extensions $M'$ in the complete matrix ring $(R_n)$ of a subfield $M$ of $(R_n)$ are enumerated according to the degree $[M':M]$ of $M'$ over $M$. This number is shown to depend on $[M':M]$ and the order and rank of $M$, but not on $M$ itself. A matrix representation is obtained for the Galois group of $M'$ over $M$, and the numbers of non-singular matrices $F \in (R_n)$ and similarity transformations $\phi_p$ on $(R_n)$ which induce $M$-automorphisms of $M'$ are determined. Generalizations of these results are shown to hold for subfields $M$ of $(R_n)$ when $R = GF(p^d)$, $d > 1$, provided the canonical projection of the normal form of $M$ contains all $r \times r$ scalar matrices over $R$ where $r = rank M$. The techniques used are similar to those employed earlier by the author(s) [Duke Math J., 39(1972), 313-321; 475-484], [Acta Arith., 25(1974), 315-329; 331-335], [Lin. Alg. & App., 14(1976), 95-105]. (Received October 14, 1977.)

752-12-7 KENNETH H. ROSEN, University of Colorado, Boulder, Colorado 80309. Units arising from Kronecker's First Limit Formula.

Let $L(s, \chi_K, \zeta) = \frac{1}{2} \sum \chi_K((x,y))Q(x,y)^{-s}$ where $Q$ is a positive definite quadratic form of discriminant $d < 0$. Stark has shown that $L(1, \chi_K, \zeta) = \frac{2\pi}{24|\zeta|\sqrt{|d|}} \log \epsilon_0$ where $\epsilon_0$ is a unit in $K^* = \mathbb{Q} + \sqrt{d}$; $\epsilon_0 \epsilon^d_{2a}$; his conjecture that $\epsilon_0$ is actually the 24th power of a unit in $K^*$ has been proved by Schertz. Here, the square root of Schertz's unit will be investigated. By the use of Kronecker's First Limit Formula and a theorem of Sohngen it will be shown that this square root is in a certain class field. The relationship between the location of this unit and the value at $s = 1$ of certain L-series for genera will be given. (Received October 17, 1977.)

752-12-8 JOHN R. BLOOM, Texas A&M University, College Station, Texas 77843. On the invariants of some $\mathbb{Z}_p$-extensions. Preliminary report.

Various results will be presented which give relationships among the invariants of related $\mathbb{Z}_p$-extensions. In particular, congruence relations will be given for the $\lambda$-invariant when the base field is extended by a cyclic $\mathbb{Z}_p$-extension. If the base field of the extension $K/k$ is extended
by composing with $\mu_i$, the $i$-th layer of another $Z_k$-extension $M/k$, the above results can be strengthened, under the hypothesis $\mu(K/k) = 0$. This gives an asymptotic formula $\lambda(K_m/k_m) = r_k + c$, for $i > i_k$, as well as information about the constant $c$. Finally, the invariants for all $Z_k$-extensions contained in the composite $MK$ are studied, and one finds that the $\mu$-invariant is nonzero for at most one extension. Certain results are also obtained for the $\lambda$-invariant in this case. (Received October 17, 1977.)

*752-12-9* Joseph B. Dennin, Jr., Fairfield University, Fairfield, Connecticut 06430. Decomposition of Primes in Biquadratic Extensions of $\mathbb{Q}$.

Let $L = \mathbb{Q}(m^{\frac{1}{2}}, n^{\frac{1}{2}})$ be a biquadratic extension of the rationals $\mathbb{Q}$ where $m$ and $n$ are distinct square free rational integers. Let $p$ be a rational prime.

We describe how the ideal $(p)$ factors in the ring of integers of $\mathbb{Q}(m^{\frac{1}{2}}, n^{\frac{1}{2}})$. (Received October 17, 1977.)

752-12-10 SHIAN-MING CHANG, University of Toronto, Toronto, Ontario, M5S 1A1 and RICHARD FOOTE, California Institute of Technology, Pasadena, California 91125. Capitulation patterns in class field extensions of type $(p,p)$. Preliminary report.

Let $p$ be a prime $\geq 5$ and for a number field $K$ let $K^{(1)}$ denote the $1$-th member of the Hilbert $p$ class field tower of $K$. For each $n \in \{0,1,\ldots, p+1\}$ the authors construct a family $\mathcal{M}(n)$ of metabelian $p$-groups with commutator quotients of type $(p,p)$ such that for any $G \in \mathcal{M}(n)$ if $\mathfrak{z} \in K$ a number field $K$ with $G = \mathcal{M}(K^{(2)}/K)$, then $K$ has the following property: for exactly $n$ of the subfields of $K^{(2)}$ of degree $p$ over $K$, say $K_1$, $K_2$, ..., $K_n$, every ideal class of $K$ of order $p$ becomes principal (capitulates) in $K_i$ for $1 \leq i \leq n$. Moreover, if $n \geq 3$, for any $k \in \{0,1,\ldots, p+1\}$ such that $G \in \mathcal{M}(n)$ such that if $K,K_1$ are as above, then, in addition, $K^{(2)}$ has exactly $k$ subfields, say $K_{n+1}, \ldots, K_{n+k}$ which are of degree $p$ over $K$ and are of type $(B)$, i.e. the subgroup $\mathfrak{S}_1$ of order $p$ in the class group of $K$ corresponding to the field $K_1$ does not become principal in $K_i$, $n+1 \leq i \leq n+k$. Thus, unlike for primes 2 and 3, group theory puts no restriction on possible capitulation patterns (values of $n$). The authors further show that if $K$ is a number field with $\mathcal{M}(K^{(1)}/K)$ of type $(p,p)$, then certain capitulation patterns force $[K^{(2)}:K]$ to be "large" and, for some $K$, force $[K^{(\infty)}:K] = \infty$. (Received October 17, 1977.) (Author introduced by Professor Marshall Hall, Jr.)

*752-12-11* DEMETRIOS BRIZOLIS, California State Polytechnic University, Pomona, California, 91768. A Divisibility Property of Ideals in Rings of Integral-Valued Polynomials.

Let $A$ be an integral domain with quotient field $K$.

Define $\text{Int}(A) = \{f \in K[x] \mid f(a) \in A \text{ for all } a \in A\}$, the ring of integral-valued polynomials associated with $A$.

**Theorem** Let $A$ be a Dedekind domain satisfying

i) the quotient field of $A$ is perfect

ii) $A/P$ is either finite or algebraically closed for each prime ideal $P \subset A$.

iii) For each non-constant polynomial $f \in A[x]$ the set of all prime ideals $P \subset A$ such that $f(x) = 0$ has a solution in $A$ is not empty.

Then the following are true:

a) $\text{Int}(A)$ is a Prüfer domain

b) Suppose $I,J$ are finitely generated ideals of $\text{Int}(A)$ satisfying $\{f(a) \mid f \in I\} = \{g(a) \mid g \in J\}$.

Then $I \cap J = \{0\}$.

**Remark:** This generalizes a theorem proved by the author in the American Mathematical Monthly, Vol. 81, No. 9, Nov., 1974, pp. 997-999. Namely, if $A$ is a ring of algebraic integers and if $f, g \in \text{Int}(A)$ satisfy $g(a)|f(a)$ for every $a \in A$ then $f/g \in \text{Int}(A)$. (Received October 17, 1977.)
Minkowski has given a criterion for algebraic numbers, which may be summarized as follows: Given a vector \( v = (1, a, a^2 \cdots a^{n-1}) \). For each integer \( R = 1, 2, 3, \cdots \) find \( n \) linearly independent vectors \( u_1, u_2, \cdots, u_n \) satisfying (i) the entries of \( u_i \) are integers between \(- R \) and \( R \), (ii) the quantities \( |u_1 \cdot v| \) are as small as possible, and (iii) \( |u_1 \cdot v| \leq |u_2 \cdot v| \leq \cdots \leq |u_n \cdot v| \). The matrix \( P(R) = \text{col}(u_1, u_2, \cdots, u_n) \) is called a matrix belonging to \( R \). Set \( W(R) = P(R) \cdot v \), and suppose that the vectors \( W(R) \) are normalized in some fashion (such as dividing through by the largest entry.) Then \( \alpha \) is an algebraic number of degree \( \leq n \) if and only if the set of these normalized vectors is finite. With the aid of the computer, we have carried out many steps of this procedure, for \( n = 3 \), for various cubic irrationals. So far we have found that (a) all the matrices \( P(R) \) are unimodular; (b) all the matrices \( (P(R))^{-1} \), after a few steps, have every column of the form \( (a, \langle ax \rangle, \langle ax^2 \rangle) \) (where \( \langle x \rangle \) is the nearest integer to \( x \)); (c) a "great many" of the quantities \( u_i \cdot v \) are units in the field \( \mathbb{Q}(\alpha) \). (Received October 17, 1977.)

For any prime \( p \geq 3 \), a relationship between two Diophantine equations of degree \( p \) is used to produce a homogeneous polynomial of degree \( 2p \) in two variables. This polynomial is used to generate discriminants of imaginary quadratic number fields whose ideal class groups may contain subgroups of type \( C(p) \times C(p) \).

Sufficient conditions for this behavior are investigated. For each \( p \), infinitely many fields with such class groups are constructed. The class groups for many small discriminants produced in this manner are analyzed by a computer program developed from a published report by D. Shanks.

Several examples of fields with class groups containing \( C(5) \times C(5) \times C(5) \) and \( C(3) \times C(3) \times C(3) \times C(3) \) found by the program among these discriminants are discussed in detail. The present computer implementation of Shanks' ideas is described. (Received October 18, 1977.)

13 ▶ Commutative Rings and Algebras

Let \( R \) be a commutative ring with identity, let \( \{X_i\}_{i=1}^n \) and \( \{Y_j\}_{j=1}^m \) be sets of indeterminates over \( R \), and let \( R^{(n)} \) and \( R^{(m)} \) denote the formal power series rings in \( \{X_i\}_{i=1}^n \) and \( \{Y_j\}_{j=1}^m \), respectively. Several papers have recently dealt with the structure of the sets of \( R \)-endomorphisms and \( R \)-automorphisms of \( R^{(n)} \), with the case \( n = 1 \) having drawn special attention. Here we answer some previously open questions, ask some new questions, and extend some of the known results to the general case of an \( R \)-homomorphism of \( R^{(n)} \) into \( R^{(m)} \). For example, we prove the following result.

**Theorem.** Let \( \phi \) and \( \psi \) be \( R \)-homomorphisms of \( R^{(n)} \) into \( R^{(m)} \). Then

(i) If \( \phi(X_i) = \psi(X_i) \) for each \( i \), then \( \phi = \psi \).

(ii) If \( \{\phi(X_i)^n\}_{i=1}^n = \{\psi(X_i)^n\}_{i=1}^n \) and \( \phi \) is surjective, then \( \psi \) is surjective if and only if \( \phi \) is surjective. However, \( \psi \) need not be injective if \( \phi \) is injective. (Received September 29, 1977.)

A ring \( A \) will be called \( m \)-linear for \( m \) a fixed positive integer if the function \( \phi_m: x + x^m \) is a ring homomorphism on \( A \). All rings considered are commutative.
Earlier work by the author established relationships between \( m \) and \( \text{char}(A) \), the characteristic of an \( m \)-linear \( A \). This paper presents more detailed structure theory for \( m \)-linear rings. It is shown that \( m \)-linear rings decompose into \( m \)-linear rings of prime characteristic, so attention is given to the case where \( \text{char}(A) = p \) and \( p \) is prime. From well known facts, one sees that the finite field \( GF(p^k) \) is \( m \)-linear if and only if for some \( i \), \( 0 \leq i < k-1 \), \( (p^k-1)|(m-p^i) \), or equivalently, \( a^m = a^i \) for every \( a \) in \( GF(p^k) \). More generally, it is shown that for any ring \( A \) with identity such that \( \text{char}(A) = p \), \( A \) is \( m \)-linear if and only if there is some integer \( s \) such that \( a^m = a^s \) for every \( a \) in \( A \). Furthermore, a decomposition is obtained for such rings which is necessary and sufficient for \( m \)-linearity. Equivalent conditions are obtained for an \( m \)-linear ring to be embeddable in an \( m \)-linear ring with identity, and non-embeddable examples are given. (Received September 30, 1977.)

752-13-3  WILLY BRANDAL, University of Tennessee, Knoxville, Tennessee 37916. Commutative rings whose finitely generated modules decompose.

The following is the culmination of recent work by several people including R. Wiegand, T. Shores, P. Vamos, S. Wiegand, and myself. Theorem: Let \( R \) be a commutative ring with identity. Then every finitely generated \( R \)-module decomposes into a direct sum of cyclic submodules if and only if \( R \) is a finite direct product of rings of the following three types: (i) (almost) maximal valuation rings, (ii) almost maximal Bezout domains, and (iii) toric rings.

Besides the well known ones, examples of such rings have been given by B. Osofsky, S. Wiegand, P. Vamos, and T. Shores and R. Wiegand. (Received October 11, 1977.)

752-13-4  STUART S. WANG, Texas Tech University, Lubbock, Texas 79409. Some partial results on the Jacobian problem. Preliminary report.

Let \( X_1, \ldots, X_n \) be indeterminates over a field \( k \), \( Y_1, \ldots, Y_n \) be polynomials in \( X_1, \ldots, X_n \) with coefficients in \( k \) such that the determinant of the Jacobian matrix \( \frac{\partial(Y_1, \ldots, Y_n)}{\partial(X_1, \ldots, X_n)} \) is a non-zero element of \( k \). Then \( k[X_1, \ldots, X_n] \triangleleft k[Y_1, \ldots, Y_n] = k[Y_1, \ldots, Y_n] \), and the \( k[Y_1, \ldots, Y_n] \)-module \( k[X_1, \ldots, X_n] \) has a projective resolution of length 1. If, furthermore, characteristic \( k \neq 2 \) and the degrees of \( Y_1, \ldots, Y_n \) are less than or equal to 2, then \( X_1, \ldots, X_n \) are polynomials of \( Y_1, \ldots, Y_n \) with coefficients in \( k \). (Received October 12, 1977.)

752-13-5  Edward L. Green, Virginia Polytechnic Institute and State University, Blacksburg, Va. 24061. Complete intersections and Gorenstein ideals.

A criterion is given for determining when a zero dimensional Gorenstein ideal in a polynomial ring is a complete intersection. The criterion is algorithmic in that it only involves ranks of matrices. (Received October 12, 1977.)

752-13-6  CAROL L. WALKER, New Mexico State University, Las Cruces, New Mexico 88003 and ROBERT B. WARFIELD, JR. Unique decomposition and isomorphic refinement theorems in additive categories.

Under suitable hypotheses, if an object in an additive category is a direct sum of subobjects with local endomorphism rings, then any two such decompositions have the summands isomorphic in pairs, and any other decomposition has a refinement into a decomposition where the summands are of this sort. These theorems are applied in a quotient category of abelian groups to get quasi-isomorphic refinement theorems for direct sums of certain abelian groups of torsion free rank one. (Received October 13, 1977.)
752-13-7 Sylvia Wiegand, University of Nebraska, Lincoln, Nebraska 68588. The spectrum of two dimensional Noetherian rings. Preliminary report.

We will discuss the problem of determining those partially ordered sets that arise as the set of prime ideals in a two-dimensional noetherian ring. (Received October 17, 1977.)

752-13-8 LASZLO FUCHS, Tulane University, New Orleans, Louisiana 70118. On a duality of modules over valuation rings.

Let $E$ be a unital module over a commutative ring $R$. Let $D(E)$ denote class of submodules of powers of $E$, and $C(E)$ the class of closed submodules of topological powers of $E$, with $E$ discrete. Taking continuous homomorphisms into $E$, every $M \in D(E)$ [$C(E)$] defines a character module $M^* \in C(E)$ [$D(E)$]. A. Orsatti (Annali di Mat., to appear) investigates the cases when this yields a duality. A question raised by Orsatti is answered by pointing out that the choice $E = R$ yields a duality for complete discrete valuation rings $R$. It is also shown that these are the only noetherian domains for which $E = R$ defines a duality. (Received October 17, 1977.)


Let $A$ be a commutative ring, $d: A \to M$ a derivation; a flat ideal $I$ of $A$ has the property that $d(f, g) = fd(g) - gd(f)$ lies in $I^2M$ for every two elements $f$ and $g$ of $I$. Our purpose is to investigate to what extent the above and related properties are characteristic of flat ideals. Two positive results are: (1) A fairly complete treatment for regular (geometric) rings, and (2) Reduction, in Cohen-Macauley cases, to one dimensional rings. (Received October 17, 1977.) (Author introduced by Professor W. V. Vasconcelos).

*752-13-10 JOHN W. PETRO, Western Michigan University, Kalamazoo, Michigan 49008 and M. EDWARD PETTIT, JR., Augusta College, Augusta, Georgia 30904. Weierstrass families of ideals in commutative rings.

The ideas in this paper are motivated by classical results concerning entire functions. Let $R$ be a commutative ring with 1. Definition. A non-empty family $W = \{b_\lambda | \lambda \in \Lambda\}$ of proper $R$-ideals is a Weierstrass family associated with the $R$-ideal $a$ in case for all $\lambda, \mu$ (i) $\lambda + \mu = b_\lambda + b_\mu = R$, (ii) $a \subseteq b_\lambda$, and (iii) $b_\lambda + a : b_\lambda = R$. We prove an algebraic analog of the Weierstrass Factorization Theorem which is valid in general. In case Spec$(R)$ is tree'd (for example, a Prüfer domain) we prove that the following conditions are equivalent: (1) there exists an $R$-ideal with infinitely many minimal primes, (2) there exists a countably infinite Weierstrass family in $R$ consisting of finitely generated $R$-ideals, and (3) there exists an infinite Weierstrass family in $R$. (Received October 17, 1977.)

752-13-11 James E. Carrig, George Mason University, Fairfax, Virginia, 22030 and Wolmer Vasconcelos, Rutgers University, New Brunswick, New Jersey 08903 Regular Elements in One-Dimensional Rings. Preliminary Report.

For a reduced ring $R$ of valuative dimension one - that is Krull dim $R[T] = 2$ finding regular elements in $R$ is aided by the following. Theorem: Every finitely generated faithful ideal $I$ contains a projective ideal $J$ with the same radical. As consequences we have: (1) A finitely generated ideal $I$ in a ring of valuative dimension one, contained in the Jacobson radical, contains an element with the same radical, (2) If, in addition, $R$ is reduced and contains a field of cardinality greater than that of its minimal prime spectrum, then any finitely generated faithful ideal contains a regular element. (Received October 17, 1977.)
752-13-12 DAVID BERMAN, University of Texas, Austin, Texas 78712. The number of generators of a colength $N$ ideal in a power series ring.

In the main Theorem, it is shown that if $I$ is a colength $N$ graded ideal in the ring $R = k[[x_1, \ldots, x_t]]$, then the number of generators of $I \subseteq$ the number of generators of $I_N$, where $I_N$ is intuitively the ideal of colength $N$ closest to an ideal of the form $m^j = (x_1, \ldots, x_t)^j$ for an appropriate $j$. As a consequence, a sharp upper bound on the number of generators of an ideal of colength $N$ in $R$ is given: $(\# \text{gens } I) \leq (r/r_{j-1} + \varepsilon)N(1-1/r)$ when $N \equiv N(r, \varepsilon)$. (Received October 17, 1977.) (Author introduced by Professor Anthony Iarrobino).

752-13-13 Leo Chouinard, Bonnie Hardy and Thomas Shores, Department of Mathematics, University of Nebraska, Lincoln, NE 68588. Arithmetical semigroup rings.

Let $R$ be a commutative ring with identity and $S$ a commutative monoid. Necessary and sufficient conditions on $R$ and $S$ for the ring $R[S]$ to be reduced arithmetical (equivalently, have weak global dimension $\leq 1$) are given. Likewise, those rings $R[S]$ which are semihereditary are determined in terms of conditions on $R$ and $S$. (Received October 17, 1977.)


Let $S$ be a commutative $G$-Galois extension of $R$. We generalize the result that Amitsur cohomology, $H^0(S/R, \text{Pic})$, and group cohomology, $H^0(G, \text{Pic}(S))$, are isomorphic with the following. Theorem: Let $R \subseteq S$ be commutative rings and $G$ a finite group of automorphisms of $S$ with $R \subseteq S^G$. Denote by $h$ the homological different of $S$ with respect to $R$. If $S/h$ is Artin, $S$ is $R$-flat and the usual map from $S^G$ to $C^1(G, S)$ is an inclusion, then there exists an exact sequence of groups: $1 \rightarrow U_n \rightarrow H^0(S/R, \text{Pic}) \rightarrow H^0(G, \text{Pic}(S)) \rightarrow U_n \rightarrow \cdots$. As a corollary we describe the $U_n$ for $R$ a Noetherian domain of Krull dimension less than two and $S$ a quadratic extension of $R$. The key idea is the inclusion of $S^G \rightarrow S = S^n$ into $C^1(G, S)$ as a subdirect sum of copies of $S$. The usual cartesian square involving $S^n$, $C^1(G, S)$, the conductor between them and the resulting Mayer-Vietoris sequence are investigated to produce the aforementioned results. (Received October 17, 1977.)

752-13-15 N. H. Vaughan, North Texas State University, Denton, Texas 76203 and R. W. Yeagy, Stephen F. Austin State University, Nacogdoches, Texas 75962. Factoring ideals into semiprime ideals.

Let $D$ be an integral domain with $1 \neq 0$ and quotient field $K$. An ideal $A$ of $D$ is said to be semiprime if $A = \sqrt{A}$. We prove Theorem 1. If every ideal of $D$ is a finite product of semiprime ideals, then $D$ is an almost Dedekind domain. We give an example of a non-Noetherian almost Dedekind domain in which every ideal is a finite product of semiprime ideals. We also give an example of a non-Noetherian almost Dedekind domain which has an ideal which is not a finite product of semiprime ideals. (Received October 17, 1977.)

752-13-16 D.R. ESTES and J.R. MATIJEVIC, University of Southern California, Los Angeles, California 90007, Unique factorization of matrices and Towber rings. Preliminary report.

Let $R$ be a commutative ring with identity. We say $R_n$, the ring of $n \times n$ matrices with entries in $R$, has a UF if the following two conditions hold: (1) Each $A$ in $R_n$ with determinant not a zero divisor in $R$ may be factored into irreducibles, where a matrix $B$ of determinant not a zero divisor, not a unit in $R$ is irreducible if $B = CD$ means $C$ or $D$ is invertible; and (2) If $A = B_1 \cdots B_r = C_1 \cdots C_s$ are two factorizations of $A$ into irreducibles then $r = s$ and upon a rearrangement, $\text{det } B_i$ and $\text{det } C_0(i)$ are associates in $R$. (Received October 17, 1977.)
Theorem: The following are equivalent for a Noetherian ring \( R \) with no nonzero nilpotent elements:

1. \( R_n \) has \( \delta \) - UF for each \( n > 0 \),
2. \( R_2 \) has \( \delta \) - UF, and
3. \( R \) is a finite direct sum of factorial Towber domains.

A stronger condition for the uniqueness of factorization of matrices is also investigated, and shown for the same Noetherian rings as above to be equivalent to the condition that \( R \) is a finite direct sum of principal ideal domains. (Received October 18, 1977.)

Marvin N. Cohen and J. D. McKnight, Jr., University of Miami, Coral Gables, Florida 33124. Pseudo-hereditary rings of continuous functions.

To ask whether \( C(X) \) is hereditary is uninteresting because \( X \) must be finite for this to be so. However, let \( m(I) \) be the smallest ideal of \( C(X) \) having the same \( m \)-closure as \( I \) (i.e., \( m(I) \) consists of all \( f \) satisfying \( f \in I \)). Then every ideal of the form \( m(I) \) is projective if and only if \( X \) is compact and hereditarily paracompact. (Received October 18, 1977.)

14  Algebraic Geometry

Paul Cheranack, University of Cape Town, Cape Town, South Africa. A two variable criterion for extrema with related effective procedure.

Let \( f : \mathbb{R}^2 \to \mathbb{R} \) be a mapping. Suppose that \( f(x,y) \) has a Taylor's series expansion in a neighbourhood \( N \) of \((0,0)\) agreeing with \( f(x,y) \) on \( N \).

Necessary and sufficient conditions guaranteeing the existence of an extremum for \( f(x,y) \) at \((0,0)\) are given. These conditions differ from previously stated conditions in that they are of an algebraic rather than analytic nature and are such that an effective procedure provided by the author can be implemented to decide whether \( f(x,y) \) has a local minimum or maximum at \((0,0)\). Many of the notions used derive from algebraic geometry. (Received May 2, 1977.)


Endomorphisms of Abelian varieties.

The endomorphism algebra of a polarized abelian scheme is a semi-simple \( \mathbb{Q} \)-algebra endowed with a positive involution. Given such an algebra \( L \) and an order \( R \) in \( L \) we construct moduli families parametrizing polarized abelian schemes \( X \) of a given relative dimension \( g \) together with homomorphisms of \( R \) into the endomorphism ring of \( X \) such that the Rosati involution on \( \text{End}^0(X) \) agrees with the given involution on \( L \). If we specify the degree \( d \) of the polarization and require a level \( n \)-structure on \( X \) then for \( n \) sufficiently large the moduli family \( E_{g,d,n} \) is a quasi-projective over \( \text{Spec}(\mathbb{Z}) \). The "forgetful" morphism of \( E_{g,d,n} \) into the moduli family \( A_{g,d,n} \) of abelian schemes constructed by Mumford is a finite morphism. Comparisons are made with similar families defined over the complex numbers by Shimura. (Received October 17, 1977.)

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Interference Patterns and Algebraic Geometry. Preliminary report.

The theory of "interference fringes" generated by superimposing two diffraction gratings is generalized to finding the Moiré patterns which arise from superimposing two families of algebraic varieties. A general density function describing these patterns will be presented. (Received October 17, 1977.) (Author introduced by Professor Allen J. Silberger).
Let $V$ be a non-singular quadratic space over a field $F$ of characteristic $\neq 2$. The map $m:F*/F^2 \rightarrow \mathbb{Z}$ is defined by: $m(a)$ is the maximum number of mutually orthogonal vectors $A$ in $V$ such that $A^2 = a$. Trivially, $M = \{ m(a): a \in F*/F^2 \}$ is a complete set of invariants of $V$ if $F$ has only one square class. It is also easy to see that if the dimension of $V \leq 2$, then $M$ together with the dimension and discriminant of $V$ completely determine $V$ up to isometry (for any field). **Theorem.** If $F$ is a finite field or a superpythagorean field with a finite number of square classes, then $M$ completely determines $V$ up to isometry. **Theorem.** If $F$ is a Pythagorean field with $2^k$ square classes and $k$ orders, then $M$ and the dimension of $V$ completely determine $V$ up to isometry. **Theorem.** If $F$ is either a non-formally real field with four square classes (e.g. non-dyadic $p$-adic fields) or $F = \mathbb{Q}_2$, then $M$ and the discriminant of $V$ completely determine $V$ up to isometry. (Received October 13, 1977.)

Let $GF(q)$ denote the finite field with $q = p^w$ elements, $p$ odd. Let $A$, $B$, and $X_i$, $i = 1, \ldots, n$, be matrices with elements from $GF(q)$. Let $X^t$ denote the transpose of the matrix $X$ and take $B$ to be a symmetric matrix. In this paper we discuss the partition $A'X_1\ldots X_n + X_n'\ldots X_1A = B$. Subject to certain conditions on $A$ and $B$, we find the number of solutions $X_1, \ldots, X_n$ to the above matrix equation both for the unranked case, no ranks specified for $X_1, \ldots, X_n$, and for the ranked case, rank $X_i = k_i$, $i = 1, \ldots, n$. (Received October 17, 1977.)

Let $G$ be a digraph with nodes $\{1, \ldots, n\}$. Let $M(G)$ be the collection of row stochastic matrices of order $n$ such that $a_{ij} > 0$ implies that $(i,j)$ is a link of $G$. Let $M(G)$ be the set of all complex numbers $\lambda$ such that $\lambda$ is an eigenvalue of an $A \in M(G)$. $M(G)$ is determined for several graphs; in particular, when $G$ is the cycle graph with directed links $(1,2), (2,3), \ldots, (n-1,n), (n,1)$. Some other results are also obtained. For example, $M(G)$ is the union of the closure of an open set and a line segment on the real axis. If $\lambda \in \partial M(G)$, then either (i), $\lambda$ is the eigenvalue of an $A \in M(G)$ with at most two non-zero entries per row and per column, or, (ii), for some $\epsilon > 0$ and some subgraph $H$ of $G$ with fewer than $n$ nodes, $(1 + \epsilon)\lambda \in M(H)$. (Received October 17, 1977.)

The ergodic theorems of demography describe the properties of a product of certain nonnegative matrices in the limit as the number of matrix factors in the product becomes large. We review the historical motivation and applications of these theorems. We present some new, perhaps surprising, properties of these products when successive factors are chosen from a set of possible matrices by a Markov chain. We assume elementary knowledge of linear algebra and stochastic processes, but no previous exposure to demography. We indicate some unanswered questions which may require more mathematical power. (Received October 19, 1977.)

**16 ▶ Associative Rings and Algebras**

For a skew field $F$, let $L = F(t;\delta)$ be the right Ore quotient division ring, where $(ab)\delta = a\delta(b) + a(b)\delta$ and $ct = t(c)\delta + c\delta$ for $a, b, c \in F$. For $0 < m$, let $K \subseteq L$ be the skew subfield generated by $F$ and $t^m$. **THEOREM I.** If $\delta^{m-1} \neq 0$, $\delta^m = 0$, then $(i)K \subseteq L = K[t;S,D]$ is
binomial, where $t^{mS} = t^m$, $cS = c8$, and $BD = \beta t - t(\beta S)$ for $\beta \in K$. (ii) $S^{mD} = DS^m$ on $L$. (iii) $\mathcal{D}[t; 8, 0] = 0$. Results of [T.H.M. Smits, Indag. Math. 30 (1968), p. 72] are used to prove I and to extend P. M. Cohn's Theorem (Illinois J. Math. 10 (1966), p. 420) from $m = p$ prime to arbitrary $m$, and for $w \in \text{center } F \Omega$ instead of $w \in \text{center } F$. **THEOREM II.** Let $w$ be a primitive $m$-th root of unity, $w_{\phi} = w$, $w_{\phi} = 0$, $\delta \theta = \delta \phi$, and $w \in \text{center } F \Omega$. Then

(i) $K \subseteq L = K[t; S, D]$ is binomial, where $S, D : L \to L$, $tS = tw$, and $yD = yt - t(yS)$

(ii) $\delta \phi S = S \delta \phi$; $\delta \phi t = t \delta \phi$; $\delta \phi$ is inner by $t^m$. (Received October 18, 1977.)

*752-16-2* G. F. Birkenmeier, Floyd Junior College, Rome, Ga. 30161. **Indecomposable Decompositions and the Minimal Direct Summand Containing the Nilpotents.**

The **MDSN** is the unique minimal direct summand (i.e. idempotent generated right ideal) containing the nilpotent elements of a ring. We note that the MDSN is a semicompletely prime (two-sided) ideal which contains the right singular ideal. A reduced ring is one with no nonzero nilpotents. It is well known that an indecomposable right ideal decomposition of a ring is not necessarily unique. We show that the reduced right ideals of such a decomposition are unique up to isomorphism and the remainder of the decomposition forms the unique MDSN. In the main theorem we use triangular matrices to prove that a ring with an indecomposable decomposition is basically composed of a nilpotent ring, a ring (containing a unity) with an indecomposable decomposition which equals its MDSN, and a direct sum of indecomposable reduced rings with unity. Finally, if a ring $R$ has a MDSN and $x \in R$ such that $x^2 = x$, then $xR = A \oplus B$ where $A$ is reduced and $B \subseteq \text{MDSN}$. (Received October 26, 1977.)

*752-16-3* HENRY E. HEATHERLY, University of Southwestern Louisiana, Lafayette, Louisiana 70504. **Negative D.G. Near Rings, Preliminary Report.**

A (left) near-ring $(N, +, \cdot)$ is defined to be **negative distributively generated** (n.d.g.) if there is a set $D$ of right distributive elements such that (1) $D$ generates $N$ additively and (2) $d \in D$ implies $-d$ is right distributive. This class lies between d.g. and distributive. **Theorem.** If $N$ is n.d.g., then the commutator subgroup $N'$ of $(N, +)$ is an ideal of $(N, +, \cdot)$ and $N' \cdot N = \{an : a \in N', n \in N\} = (0)$. **Corollary 1.** An n.d.g. near-ring with right identity is a ring. **Corollary 2.** An n.d.g. near-ring with no non-zero nilpotent ideals is a ring. The results of imposing conditions on the ring $N/N'$ are investigated. (Received October 6, 1977.)

*752-16-4* James Osterburg, University of Cincinnati, Cincinnati, Ohio 45221. **Fixed Rings of Simple Rings.**

Let $R$ be a simple ring with an identity and $G$ a finite group of outer automorphisms of $R$. Then the fixed ring of $R$ is primitive with the intersection of all the nonzero two-sided ideals of the fixed ring being a nonzero simple ring. The fixed ring of $R$ is simple if and only if there is an element of trace 1. Finally, we give an example based on one due to Zalesskii-Nerovlovavskii to show that the fixed ring is not simple. (Received October 11, 1977.)


Let $R$ be a ring with identity element. Let $\sigma$ be an idempotent kernel functor, and let $(T_\sigma, F_\sigma)$ be its associated hereditary torsion theory of left $R$-modules. A left ideal $L$ of $R$ is called $\sigma$-closed if $R/L \in F_\sigma$. **Theorem 1.** If $R$ has dcc on $\sigma$-closed left ideals, then $R$ also has acc on $\sigma$-closed left ideals.
Let $L_\sigma$ be the localization functor associated with $\sigma$. Theorem 2. If $L_\sigma$ is exact and if each module in $F_\sigma$ is contained in a direct sum of finitely generated modules, then $R$ has dcc on $\sigma$-closed left ideals. A module $W$ is called a $F_\sigma$-cogenerator if every module in $F_\sigma$ is contained in a product of copies of $W$. Theorem 3. If $L_\sigma$ is exact, then the following statements are equivalent: (1) $R$ has dcc on $\sigma$-closed left ideals, and $R/\sigma(R)$ is an $F_\sigma$-cogenerator; (2) $R$ has acc on $\sigma$-closed left ideals, and $R/\sigma(R)$ is an $F_\sigma$-cogenerator; (3) every injective module in $F_\sigma$ is projective as an $R/\sigma(R)$-module. (Received October 11, 1977.)

Taft has shown (cf. Amer. Math. Monthly 77 (1970), p. 315) that if $R$ is c-commutative (i.e. if $a, b \in R$ and $ab = c$ then $ba = c$ where $c$ is a central, non-zero divisor of $R$ then $R[x]$ is c-commutative). He raises the question of whether either condition on $c$ (i.e. central or non-zero divisor) can be omitted. We give examples to show that even for $R[x]$ neither condition can be omitted. We show that if $R[x]$ is $h(x)$-commutative for any $h(x) \in R[x]$ then so is $R$ with any finite number of (commuting) indeterminates adjoined. Examples are given to show that $R[x]$ need not be c-commutative even if $R[x]$ is. Finally, examples are given to answer Taft's question for the special case of a zero-commutative ring. (Received October 11, 1977.) (Author introduced by Dr. W. D. Peeples).

Let $B$ be a bialgebra over a field $F$ with multiplication $m$, comultiplication $\Delta$, unit $\mu$ and counit $\epsilon$. $\text{Hom}_B(B, B)$ is an $F$-algebra with unit element $\mu e$ under the convolution product $f * g = m(\text{Reg})\Delta$. Let $S$ be a left inverse of the identity mapping $I$ in $\text{Hom}_B(B, B)$. Question: Is $S$ a right inverse of $I$, i.e., is $B$ a Hopf algebra with antipode $S$? The answer is affirmative if either (1) $B$ is finite-dimensional, (2) $B$ is a commutative algebra, or (3) $B$ is an irreducible coalgebra. (Received October 11, 1977.)

The concept of reduced rank, introduced by Goldie for Noetherian rings, is shown to be meaningful for rings with Krull dimension and the concept is then used to prove results in this case. (Received October 11, 1977.) (Author introduced by Charles F. Lanski).

These are groups $\Gamma$ such that, for any representation $\rho: \Gamma \to \text{GL}_n(K)$, $K$ a field, the character $\chi^\rho$ of $\rho$ takes values which are integral over $\mathbb{Z}$. This implies the following strong finiteness properties. (1) For any finite subset $X$ of $\Gamma$, the subring of $M_n(K)$ generated by $\rho(X)$ is a finitely generated $\mathbb{Z}$-module; (2) If $\Gamma$ is finitely generated there is a constant $c_n$ such that, in any characteristic, $\Gamma$ has $\leq c_n$ irreducible representations of dimension $\leq n$, each of which is defined over a finite extension of the prime field. The methods used to derive these results,
Theorem. Let \( \tau \) be a finitely generated subgroup of \( \text{GL}_2(\mathbb{C}) \) with finite commutator quotient, which is not a nontrivial amalgamated free product, and which acts irreducibly on \( \mathbb{C}^2 \). Then \( \tau \) is conjugate to a subgroup of \( \text{GL}_2(A) \) where \( A \) is a ring of algebraic integers. (Received October 11, 1977.)

Let \( \Lambda \) be a modular lattice and let \( G \) be a finite group of automorphisms acting on \( \Lambda \). We prove that if the sublattice of fixed points \( \Lambda^G \) satisfies ACC(DCC), then \( \Lambda \) satisfies ACC(DCC). We apply this to rings with finite group actions and to skew group rings (crossed products with trivial factor sets). In particular, it results that if a skew group ring of a finite group is Noetherian (Artinian), then the coefficient ring is Noetherian (Artinian). Also, we show that if a ring with a finite group \( G \) acting by automorphisms satisfies the ACC(DCC) on \( G \)-invariant semiprime ideals, then it satisfies ACC(DCC) on all semiprime ideals. See also Notices AMS 24 June, 1977, p. A-377. (Received October 11, 1977.)

A ring \( A \) is an FGC-ring provided every finitely generated left \( A \)-module is a direct sum of cyclic modules. A complete classification for the commutative FGC-rings has recently been obtained. Here we will discuss the situation where \( A \) is a finitely generated module over its center. (Received October 12, 1977.)

Let \( R \) be a prime algebra with involution over an infinite field \( F \) with char \( F \neq 2 \). A structure theorem is obtained for subspaces invariant under conjugation by the unitary group \( U \). For invariant subalgebras \( W \), either \( W \) is central, \( W \) contains an ideal of \( R \), or \( R \) satisfies the standard identity of degree eight. (Received October 14, 1977.)
Let $A$ be an automorphism of the finite group $G$. The set $C(A)$ of identity preserving functions $f: G \to G$ which commute with $A$ forms a near-ring under the usual operations. The structure of $C(A)$ is investigated. In particular it is determined when $C(A)$ is simple, semi-simple, and a description of the radical is given. These results are then applied to the situation in which $G$ is a finite vector space and $A$ is an invertible linear transformation. (Received October 14, 1977.)

The following problems will be discussed, in each case giving the historical background, the present state of knowledge and pointing out some directions in which further progress can be expected. 1.) Conditions on orders to have finitely many indecomposable representations 2.) Picard groups and class groups 3.) Extension categories of finite groups and the isomorphism problems. (Received October 14, 1977.)

A right $q$-ring $R$ is an associative ring with identity such that every right ideal of $R$ is quasi-injective as an $R$-module. The structure of an arbitrary right $q$-ring is described in terms of division rings, local rings and right $q$-rings with no primitive idempotents. A right $q$-ring need not be a left $q$-ring. A right $q$-ring has no infinite set of orthogonal non-central idempotents. (Received October 14, 1977.)

For a ring $R$, it is proved (Comm. Algebra 4 (11), 1077-1086 (1976) ) that if the right socle $S$ of $R$ is essential as a right ideal then (i) $R/S^2$ is flat as a left $R$-module and (ii) $S^2$ is projective as a right ideal. As an application of the notion of generalized singular submodules (Notices, January 1977) the above result is improved as: If $I$ is a sum of homogeneous components of the right socle of $R$, then $R/I^2$ is flat as a left $R$-module and $I^2$ is projective as a right ideal. Some immediate corollaries are stated. (Received October 14, 1977.) (Author introduced by Dr. Richard Shoop).

A brief report on some of the latest results in the representation theory of algebras with emphasis on applications. (Received October 14, 1977.)
The rings of the hypothesis are said to be of \textbf{left distributive module type (DMT)} and those of the conclusion are called \textbf{biserial}. The theorem has been used to prove that a left serial ring is of left (equivalently, right) DMT iff it is biserial; an hereditary artinian ring is of left (right) DMT iff it is biserial of finite module type; a split symmetric algebra is of DMT iff it is given by a Brauer tree with no exceptional vertex. (Received October 17, 1977.)

Let $R$ be a ring with identity $1$, and $S$ a subring containing $1$. Then $R$ is called a \textbf{ring extension of $S$}. The ring $R$ is a \textbf{separable extension of $S$} if there exists an element $\sum a_i b_i$ in $R \otimes S$ such that $\sum R a_i b_i = \sum a_i b_i R$ for each $r$ in $R$ and $\sum a_i b_i = 1$, (Definition 2, Section 2, K. Hirata and K. Sugano, On semisimple extensions and separable extensions over non-commutative rings, J. Japan Math. Soc. 18(1966),360-373). Then the group ring extension $RG$ of a finite group $G$ over $R$ is separable if and only if the order of $G$ is invertible in $R$. Thus $Z_{(p)} Q$ is separable over $Z_{(p)}$ if and only if $p \neq 2$, where $Q$ is the group of quaternions and $Z_{(p)}$ is the local ring of integers at a prime $p$, (this corrects an error in Example 4.1 in an author's paper: The structure of semiperfect rings, Communications in Algebra, 5(3), 219-229 (1977)). The above error was pointed out to the author by G. J. Janusz. (Received October 14, 1977.)

A \textbf{state} on the Grothendieck group $K_0(R)$ of a ring $R$ is an additive, real-valued function $f$ on $K_0(R)$ such that $f([R]) = 1$ and $f([A]) \leq 0$ for all finitely generated projective $R$-modules $A$. The set $S$ (the \textbf{state space}) of all states on $K_0(R)$ is a compact convex subset of the topological vector space of all real-valued functions on $K_0(R)$. In case $R$ is a von Neumann regular ring, then $S$ is a simplex (possibly infinite-dimensional), but for other rings, $S$ need not be a simplex. For instance, there exist HNP (hereditary noetherian prime) rings $R$ for which $S$ is a square, a trapezoid, or an octahedron. For an HNP ring $R$ with only finitely many idempotent ideals, $S$ is isomorphic to a certain amalgamation of simplexes associated with cycles of idempotent maximal ideals of $R$. (Received October 17, 1977.)

We say that a semiring $H$ is a \textbf{sum} of a family of subsemirings $\{S_i\}$ if for each $h \in H$ there is at least one selection $s_i \in S_i$, all but finitely many 0, with $h = \sum s_i$. The following analogues of the \textbf{direct sum concept} are analysed: (1) 0 has unique representation; (2) $S_j \cap \Sigma S_i = 0$ for all $j$; (3) for each subset $A$ of indices, $\Sigma S_i : i \in A \cap \Sigma S_j : j \notin A = 0$;

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(4) $S_j \cap \langle S \setminus \{ j \} \rangle = 0$ for all $j$ (where $\langle S \rangle$ denotes the subtractive subsemiring generated by $S$); (5) every $h \in H$ has unique representation. We show that these analogues are distinct and are arranged in increasing order of strength. Theorems relating the concepts and applying them are given. (Received October 17, 1977.)

**752-16-24 HAPLAN L. HULLINGER, University of Kansas, Lawrence, Kansas 66045. Stable equivalence and rings whose modules are a direct sum of finitely generated modules.**

The representation theory of a ring $\Delta$ has been studied by examining the category of contravariant (additive) functors from the category of finitely generated left $\Delta$-modules to the category of abelian groups. Closely connected with the representation theory of rings is the study of stable equivalence, which is a relaxing of the notion of Morita equivalence. Here we relate two stably equivalent rings via their respective functor categories and examine left artinian rings with the property that every left $\Delta$-module is a direct sum of finitely generated modules. (Received October 17, 1977.)

**752-16-25 JAMES WHITEHEAD, Kent State University, Kent, Ohio 44242. On Projective Modules, Preliminary report.**

Let $R$ be a ring and $\{ a_i \}_{i=1}^\infty$ a collection of elements of $R$. In his paper on finitistic dimension, Bass showed that if $\lim (R_\lambda r_a \lambda \to R_\lambda)$ is projective, the descending chain of principal left ideals $R_\lambda \supseteq R_{\lambda+1} \lambda \to \cdots$ terminates. This result is strengthened and the converse is proved. It is also shown that for any ring $R$, each idempotent ideal $I$ which is finitely generated as a left $R$-module is the trace of a countably generated projective right $R$-module. (Received October 17, 1977.)

**752-16-26 Peter Malcolmson, Wayne State University, Detroit, Michigan 48202. Determining homomorphisms to skew fields. Preliminary report.**

Given an associative ring $R$ with $1$, an $R$-field is a homomorphism $R \to K$, where $K$ is a skew field (division ring) which is generated as a skew field by the image of $R$. Then each $R$-field is determined by a rank function $r$ (with integer values) on matrices of $R$ which satisfies: $r(0) = 0$; $r(1) = 1$; $r(ab) \leq \min( r(a), r(b) )$; $r( \begin{pmatrix} 0 & a \\ b & 0 \end{pmatrix} ) = r(a) + r(b)$; and $r( \begin{pmatrix} a & c \\ 0 & b \end{pmatrix} ) \geq r(a) + r(b)$ (for matrices $a,b,c$ over $R$).

Thus such a rank function is equivalent to a prime matrix ideal of $R$ as defined by F. M. Cohn (Free Rings and their Relations, Academic Press, 1971). Other equivalent forms are given in terms of closure operators or dependence relations on subsets of finitely generated free $R$-modules. (Received October 17, 1977.)

**752-16-27 John Fuelberth, University of Northern Colorado, Greeley, CO 80639, Ellen Kirkman and James Kuzmanovich, Wake Forest University, Winston-Salem, NC 27109. Hereditary Module-Finite Algebras**

Let $R$ be a commutative hereditary ring with total quotient ring $K$ and let $\Sigma$ be a central separable $K$-algebra. A subring $\Lambda$ of $\Sigma$ is called an $R$-order if $AK = \Sigma$ and $\Lambda$ is a finitely generated $R$-module. Structure theorems for hereditary $R$-orders can be given which show that such orders are very similar in structure to classical hereditary orders over Dedekind domains, for example it can be shown that every order is contained in a maximal order, maximal orders are hereditary, and every hereditary order is an idealizer of a maximal order.

**THEOREM.** Every hereditary module-finite algebra is the ring direct product of an Artinian ring and a hereditary order over a commutative hereditary ring. (Received October 17, 1977.)
Let $R$ be a complete, 2-dimensional regular local ring. By an order, we mean a 2-dimensional (global dimension), $R$-free, $R$-order. Earlier results of the author for the reduced (self-basic) case are used to determine the structure of all non-reduced orders, and, in particular, to obtain an explicit matrix representation for all orders which contain a reduced order. Moreover, the generating function (in the combinatorial sense) for the number of orders of varying types, containing a fixed reduced order, is determined. Extensions of these results to higher dimensions will also be discussed. (Received October 17, 1977.)

Julius Zelmanowitz, University of California, Santa Barbara, California 93106. Polynomial rings of weakly primitive rings.

An $R$-module $M$ is called critically compressible if for all nonzero submodules $N$ of $M$, $\text{Hom}_R(M,N)$ contains a monomorphism, while $\text{Hom}_R(M,M/N)$ contains no monomorphisms. Rings which possess a faithfully critically compressible module are called weakly primitive, and these rings are characterized as weakly dense subrings of full linear rings (see these Notices, January 1976, abstract 731-16-27). It can be shown that when $M$ is critically compressible, so is $R[x] \otimes_R M$. Consequently, one learns that, unlike primitivity, weak primitivity of rings is preserved under formation of polynomials. In particular, a polynomial ring over a primitive ring is weakly primitive. (Received October 17, 1977.)

Maurice Auslander, Brandeis University, Waltham, Massachusetts 02154. Representation theory of Artin algebras.

These talks are expository in nature. The main purpose is to discuss recent results in the representation theory of nonsemisimple artin algebras which includes the special case of finite dimensional algebras over fields. Topics to be included are the first and second Brauer-Thrall conjectures, representation theory of hereditary algebras and Brauer trees. The second talk will be devoted to a discussion of some problems in the area of the representation theory of artin algebras, mainly centered around the notions of almost split sequences and irreducible maps. (Received October 17, 1977.)

Nonassociative Rings and Algebras

James E. Humphreys, University of Massachusetts, Amherst, Massachusetts 01003. Representations of semisimple Lie algebras.

Two related topics will be surveyed, with some discussion of open questions and conjectures:

1. Modules, both finite and infinite dimensional, for a complex semisimple Lie algebra, with emphasis on the category $\mathcal{O}$ of Bernstein-Gel'fand-Gel'fand. (2) Finite dimensional modules for the analogous Lie algebra in prime characteristic (or more generally, for the associated hyperalgebra).

In each case there are natural objects to consider, related by a Brauer-type reciprocity; irreducible modules with a highest weight, projective (or injective) modules, and modules of an intermediate type. (Received September 21, 1977.)


Let $A$ be a ring with 1 in which $2x \cdot a$ is uniquely solvable for each $a \in A$. Let $J$ be an involution on $A$, $J_a$ the induced involution on $A_n$, $n \geq 3$, determined by a diagonal matrix $a$ of
A\n all of whose entries are in the nucleus of A, and H(A\n,J\n) the set of symmetric elements of A\n. We show that for certain radicals there is a natural connection between the radical of A and that of H(A\n,J\n). In particular if R denotes the prime or Levitzki radical then 
R(H(A\n,J\n)) = H(A\n,J\n) \cap R(A) \n. Also, if A is 3-torsion free and J\n is the standard involution, then the same result holds for the semiprime radical. (Received October 3, 1977).

Irvin R. Hentzel, Iowa State University, Ames, Iowa 50011. Alternators of a right 
alternative algebra.

We show that in any right alternative algebra, the additive span of the alternators is nearly an ideal. We give an easy test to use to determine if a given set of additional identities will imply that the span of the alternators is an ideal. We apply our technique to the class of right alternative algebras satisfying the condition (a,a,b) = 1\{a,[a,b]\}. We show that any semi-prime algebra over a field of characteristic \neq 2, 3 which satisfies the right alternative law and the above identity with \lambda \neq 0 is a subdirect sum of (associative and commutative) integral domains. (Received October 7, 1977.)

Irvin Hentzel, Iowa State University, Ames, Iowa 50011 and 
HARRY SMITH, Iowa State University, Ames, Iowa 50010. 
Derivation alternator rings with idempotent. Preliminary report.

A nonassociative ring R is called a derivation alternator ring if (yz,x,x) = 
y(z,x,x) + (y,x,x)x, (x,y,xz) = y(x,x,z) + (x,x,y)z, and (x,x,x) = 0 for all 
x, y, z in R. Simple derivation alternator rings with characteristic \neq 2, 3 and idempotent e \neq 1 are alternative. (Received October 7, 1977.)

David J. Pokrass, Emory University, Atlanta, Georgia 30322. Right nilpotent 
generalized alternative rings.

A generalized alternative ring is one which satisfies the identities (i) 0 = (x,y,x) and 
(ii) 0 = (xy,z,w) + (x,y,[z,w]) - x(y,z,w) - (x,z,w)y, where (a,b,c) = (ab)c - a(bc) and 
[a,b] = ab - ba.

Theorem: A right nilpotent generalized alternative ring is nilpotent. This generalizes the known result for alternative rings. (Received October 11, 1977.)

David T. Price, Wheaton College, Wheaton, Illinois 60187. The classification of 
algebras. Preliminary report.

Let R be a commutative ring with unity. Let A be an R-algebra (not necessarily associative).
Set Ann A = {a \in A | ab = ba = 0, all b \in A}. Then the multiplication of A is given by an R-bilinear balanced map from A/Ann A \times A/Ann A to A^2. Two algebras A and B lie in the same tribe if there is a pair of R-isomorphisms \phi:A/Ann A \rightarrow B/Ann B and \psi:A^2 \rightarrow B^2; the set of all such pairs is Trib\n(A,B). For 
a, b in A, set (a+bAnn A)\times(b+Ann A) = ab. Two algebras A and B in the same tribe lie in the same clan if there is (\phi,\psi) \in Trib\n(A,B) such that \phi(ab) = \psi(ab) for all a, b \in A; the set of all such pairs is Clan\n(A,B). Two algebras A and B in the same clan lie in the same family if there is (\phi,\psi) \in Clan\n(A,B) such that \phi and \psi are R-algebra isomorphisms; the set of all such pairs is Fam\n(A,B). If one algebra in a family is associative then all are.

Let M be an R-module. Procedures are given for constructing the clans of algebras with module M, the families in a clan, and the isomorphism classes of algebras in a family. The procedure uses the groups Clan\n_\n(A,A), Fam\n_\n(A,A), and Auto\n_\n(M,M).

Robert L. Kruse and the author have used this procedure to calculate the associative rings of order p^4. (Received October 17, 1977.)
Let $D$ be the family of Lie algebras (infinite dimensional) such that each subalgebra is a sub ideal.

Theorem. If $L \in D$ and $[L, L]$ is nilpotent then any two nilpotent subalgebras generate nilpotent subalgebra.

Corollary. If $L \in D$ and $[L, L]$ is nilpotent then there is a series $N_1 \leq N_2 \leq N_3 \leq \ldots$ of nilpotent subalgebra of $L$ such that $L = \bigcup N_i$.

It is left to study how to improve the condition $[L, L]$ is nilpotent. For instance if we change it to "$[L, L]$ is T-nilpotent" (in the similar sense as of H. Bass), then we get analogous results. (Received October 17, 1977.)


The integral structure of a simple Lie algebra $L$ of Chevalley type over the field of fractions of an integral domain $D$ is studied. Sandwich relations for sufficiently large orders are obtained using techniques of the author (Trans. Amer. Math. Soc. 137, 245-258) and I. Stewart (Compositio Math. 26, 111-118). These involve orders naturally associated with a Chevalley basis, and generalize the principal results of the (unpublished) Ph.D. dissertation of W. H. Hymann. In addition, a new sandwich relation is obtained in the case when $D$ is an integrally closed Noetherian domain. (Received October 17, 1977.)


Nonassociative rings satisfying the identities $(x,y,z) = (z,y,x)$ and $(x_1 x_2) x_3 = (x_1 x_3) x_2$ or $(x_1 x_2) x_3 = x_1 (x_3 x_2)$, $(i,j,k) \in S_3$ are generalized antiflexible rings. Commutativity and associativity of such rings are characterized. Structure theorems for prime and semi-prime rings are proved. (Received October 17, 1977.)

18  Category Theory, Homological Algebra


Given a ring $R$, $G_0(R)$ is the Grothendieck group of f.g. $R$ modules, and $K_0(R)$ the Grothendieck group of f.g. projective $R$ modules. The natural map $K_0(R) \to G_0(R)$ is called the Cartan map and is an isomorphism when $R$ is Noetherian and every f.g. $R$ module has finite projective dimension. If $G$ is a finite group, however, a $\mathbb{Z}$-free $\mathbb{Z}[G]$ module $M$ has finite projective dimension if and only if $M$ is projective. As a result, the Cartan map for $R = \mathbb{Z}[G]$ is in general neither injective nor surjective. We consider resolutions of $\mathbb{Z}[G]$ modules by summands of permutation modules in place of projective modules. With more stringent exactness conditions, the usual theorems of homological algebra hold, and in particular the dimension of a module is well defined. We then generalize the Cartan map to the corresponding Grothendieck group and obtain an isomorphism whenever the generalized global dimension is finite. In particular, f.g. modules over cyclic groups have generalized dimension $\leq 1$ and consequently, the generalized Cartan map is an isomorphism for $G$ cyclic. (Received October 17, 1977.)


In an elementary topos $E$, a subobject $m$ of an object $A$ is proper (in the sense of being "nonempty") iff $\exists k_A \cdot m^n = true$. The subobject of $PA$ with $k_A$ as
characteristic morphism is labeled \( q_A: QA \rightarrow PA \) and called the object of proper subobjects. \( Q \) is made a functor from the category of relations in \( E \) to the category of partial morphisms in \( E \), right adjoint to the inclusion functor. This adjunction is composed with an adjunction \( \text{Rel} E \rightarrow E^{\text{op}} \) to obtain an adjunction on the right of contravariant functors between \( E \) and \( \text{Part} E \). The category of algebras for the resulting monad on \( E \) is equivalent to \( (\text{Part} E)^{\text{op}} \). (Received October 17, 1977.)

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**20 ▶ Group Theory and Generalizations**

**752-20-1** Gary J. Sherman, Rose-Hulman Institute of Technology, Terre Haute, Indiana 47803. A lower bound for the number of conjugacy classes in a finite nilpotent group.

Let \( G \) be a finite nilpotent group with \( k(G) \) conjugacy classes. It is known that \( k(G) \geq \log_2 |G| \).

In this paper we prove the following Theorem. If \( G \) is a finite nilpotent group of nilpotency class \( m \), then \( k(G) \geq m|G|^{1/m} - m + 1 \). (Received July 27, 1977.)

**752-20-2** Eugene M. Norris, University of South Carolina, Columbia, SC 29208. The Structure of Idempotent Relations.

Theorem 1. Let \( R \) be a relation on \( X \), let \( E \) be the domain of \( R \cap R^{-1} \), \( F = X/E \), and define: \( R_1 = R \cap E \times E, R_2 = R \cap F \times E, R_3 = R \cap E \times F, R_4 = R \cap F \times F \). Then \( R \) is idempotent if and only if \( R_1 \) is a quasiorder on \( E \) and the following equations hold: \( R_1 = R_1 \circ R_1 \cup R_2 \circ R_2 \), \( R_2 = R_2 \circ R_1 \cup R_4 \circ R_4 \), \( R_3 = R_1 \circ R_3 \cup R_3 \circ R_4 \), \( R_4 = R_2 \circ R_3 \cup R_4 \circ R_4 \). It devolves that \( R_1 \) is the maximal quasiorder contained in \( R \) and \( R_4 \) is the maximal strict partial order contained in \( R \). Given a quasiorder \( R_1 \) on a subset \( E \) of \( X \) and a strict partial order \( R_4 \) on \( X/E \), Theorem 2 tells how to construct all idempotents \( R \) having \( R_1 \) and \( R_4 \) respectively as its maximal quasiorder and maximal strict order parts. We give an internal and an external construction of neat idempotents, introduced in our paper [Union Preserving Homomorphisms of Semigroups of Closed Relations, Semigroup Forum, to appear]. (Received September 6, 1977.)

**752-20-3** K. BOLLING FARMER, University of Florida, Gainesville, Florida 32611. Wreath products with faithful projective representations.

If \( A \) and \( B \) are finite groups with \( B \) a permutation group on \( \{1, 2, \ldots, n\} \), then the wreath product \( A \curlywedge B \) is the semidirect product of \( k \), the direct sum of \( n \) copies of \( A \), by \( B \). We investigate conditions under which \( A \curlywedge B \) has a faithful projective representation. The main result is:

Theorem. If \( \omega \) is a factor set of \( A \curlywedge B \) and \( K \) has a faithful, irreducible \( \omega \)-representation, then \( A \curlywedge B \) does. (Received September 22, 1977.)
A finite group is called inseparable if the only proper normal subgroup over which it splits has order one. The residual $G_E$, for the formation $E$ of groups in which all Sylow subgroups are elementary abelian, appears to control the action of splitting. In this paper the structure is identified for the nonnilpotent inseparable groups of order $p^a q^b$ that have $G_E$ as an $A$-group and in which the Frattini subgroups of the Sylow subgroups are cyclic. (Received September 28, 1977.)

For any set $X$, let $T_X$ and $I_X$ denote, respectively, the full transformation semigroup on $X$ and the symmetric inverse semigroup on $X$.

E. S. Ljapin has shown that any inverse semigroup of partial transformations on a set $X$ is isomorphic to an inverse subsemigroup of $I_X$. It follows then that any inverse subsemigroup of $T_X$ is isomorphic to an inverse subsemigroup of $I_X$.

J. W. Nichols recently characterized a class of maximal inverse subsemigroups of $T_X$ as follows: Let $a \in X$ and define $I_a = \{a \in T_X : aa = a$ and $|ya^{-1}| = 1$ for $ya = a\}$. Then, for each $a \in X$, $I_a$ is a maximal inverse subsemigroup of $T_X$.

In this note it is shown that $I_a = I_X - \{a\}$. (Received October 3, 1977.)

This is an elementary proof of Bieberbach's result that a discrete Euclidean group is a subdirect product of a point group and a space group. (Here a point group is a Euclidean group whose elements have a common invariant point and a space group is one whose translations span the underlying space.) The main feature is an inductive argument based on the two main steps of Frobenius' proof of the slightly weaker result that a discrete Euclidean group without an invariant subspace is a space group. (Received October 7, 1977.)

Given a topological semigroup $S$, the universal compactification of $S$ is a (unique) pair $(U,u)$ where $U$ is a compact semigroup, $u : S \rightarrow U$ is a continuous homomorphism with $u(S)$ dense in $U$ and with the following property: for each continuous homomorphism $\alpha : S \rightarrow T$, with $T$ a compact semigroup, there is a continuous homomorphism $\bar{\alpha} : U \rightarrow T$ satisfying $\bar{\alpha} \circ u = \alpha$. In a previous work to appear in the Rocky Mountain Journal this author described the universal compactification of a closed cone in a finite-dimensional real vector space supplied with vector addition and the discrete topology. In the present work, the universal compactification of such a cone supplied instead with the relative Euclidean topology is described. (Received October 7, 1977.)
In the case that $V$ is a module arising from a permutation representation $(G,S)$, $L(G,V)$ and $P(G,S)$, the cycle index, are recoverable from each other by variable substitutions.

Among other relationships discussed is the following theorem: If $W$ is a $\mathbb{C}$-vector space then

$$L(G, \text{Hom}(W,V)) = L(G, \text{Hom}(V,W)) = L(G,V) \sum_{i=1}^{\dim W} x_i \mapsto x_i^i , \ldots , n.$$ (Received October 11, 1977.)

**#752-20-9** LEO J. ALEX, State University College, Oneonta, New York 13820 and DEAN C. MORROW, Ferrum College, Ferrum Virginia 24088, *Simple groups with a Sylow normalizer of order $3p$.*

In this paper the following main theorem is proved. **Theorem.** Let $G$ be a finite simple group with a Sylow $p$-normalizer of order $3p$. If there is a non-principal ordinary irreducible character of degree at most 29 in the principal $p$-block, then $G$ is isomorphic to one of the groups $L(3,2)$, $A_7$, $U(3,3)$, $L(3,3)$, $U(3,4)$, $L(3,4)$, $U(3,5)$ or $A_8$.

(Received October 12, 1977.)

**#752-20-10** ROBERT E. SOLAZZI, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061. *Four-dimensional symplectic groups.*

In a previous paper (J. Alg. 21, 1972, 91-102), the automorphisms of the symplectic congruence groups and their corresponding projective groups were found over any integral domain, in dimensions greater than four. The purpose here is to determine these automorphisms in dimension four. In characteristic not 2 the automorphisms of the projective congruence groups have the same characterization: they are determined by transformation by a symplectic semi-similitude. This is the main Theorem. The remainder deals with exceptional automorphisms of the congruence groups that do not preserve transvections and which can occur with four-dimensional congruence groups in characteristic 2. (Received October 11, 1977.)

**#752-20-11** Richard D. Byrd, Justin T. Lloyd and James W. Stepp, University of Houston, Houston, Texas 77004. *On the automorphism group of the semigroup of complexes of a finite abelian group.*

Let $G$ be a finite abelian group and let $F(G)$ be the semigroup of nonempty subsets of $G$. In this note we discuss when the automorphism group of $G$ is the same as the automorphism group of $F(G)$. (Received October 13, 1977.)

**#752-20-12** Constance C. Edwards, Indiana University-Purdue University at Fort Wayne, Fort Wayne, Indiana 46805. *The Minimum Group Congruence on an $\delta$-Unipotent Semigroup.* Preliminary Report.

Let $S$ be an $\delta$-unipotent semigroup, i.e., a semigroup in which each principal left ideal is generated by a unique idempotent. Let $E$ denote the subset of idempotents of $S$, and for each $a \in S$, let $V(a)$ denote the subset of inverses of $a$. Define

$$E_n = \{x | x^\epsilon x = x \text{ and } \forall e \in E, \forall x \in V(x)\};$$

$$E_w = \{x | x \in \bigcap E \# \};$$ and $$(E_n)w = \{x | x \in \bigcap E \#\} .$$

The greatest idempotent separating congruence $\mu$ on $S$ is given by $(a,b)\in \mu \iff a\epsilon b$ and $a\in E_n \forall b \in E(b)$. The minimum inverse semigroup congruence $\rho$ on $S$ is given by $(a,b)\in \rho \iff a\epsilon (b$ and $a\epsilon b \in E \forall a \epsilon (a)$). And the minimum group congruence $\sigma$ on $S$ is given by $(a,b)\in \sigma \iff a\epsilon b \epsilon E$. Furthermore, $\rho \subseteq \mu \subseteq \sigma$ and $\mu \epsilon (\bigcap E \# = E$ and $(a,b)\in \sigma \iff a\epsilon (E_n)w \forall b \epsilon E(b)$.

(Received October 13, 1977.)
A classification of indecomposable S-sets.

It is possible to define an equivalence relation on the class of indecomposable left S-sets over a monoid S. Call the resulting equivalence classes indecomposability types. Let $C$ be the set of left congruences on $S$, and let $S$ act on $C$ by left translation, i.e., $S \times C \rightarrow C$ by $(s,\sigma) \mapsto \sigma s$ where $a \sigma s b$ iff $a \sigma b$. Then the indecomposability types of left S-sets are in one to one correspondence with the indecomposable components of the left S-set $C$. In the case in which $S$ is a group, this specializes to the fact that isomorphism classes of transitive permutation representations of a group are in one-to-one correspondence with conjugacy classes of subgroups. (Received October 14, 1977.)

Finite Slightly Injective Groups.

A group is said to be slightly injective provided endomorphisms of subgroups can be extended to endomorphisms of the whole group. Necessary and sufficient conditions are determined for a group to be slightly injective. (Received October 14, 1977.)

Direct Summands of Simply Presented Groups.

Theorem. Let $G$ be a direct summand of a simply presented Abelian group. Then there is a torsion group $T$ such that $G \oplus T$ is simply presented.

As a consequence of this theorem, the classification theorem due to the author extends to summands of simply presented groups. (Received October 14, 1977.)

The Groups of Order 128.

For any positive integer $N$, there are a finite number of non isomorphic groups of order $N$. This paper deals with the 2358 groups of order 128. Their classification into families is presented with the number of groups in each family and some of their properties. (Received October 17, 1977.)

Representations of finite groups of Lie type.

A survey will be given of recent progress towards the construction of the irreducible representations, in the field of complex numbers, of finite groups of Lie type. The groups will be considered both from the point of view of finite group theory, and as groups of rational points on connected reductive algebraic groups defined over finite fields. The groups occur in infinite families $\{G_q\}$, with each family associated with a fixed root system, and parametrized by finite fields $\{F_q\}$. For example $\{\text{GL}_n(F_q)\}$ is a family, with $n$ fixed, associated with the root system $A_{n-1}$, and parametrized by all finite fields $\{F_q\}$. The problem is to construct the representations for all the groups in a given family in such a way that the representation-theoretic information such as character degrees, other character values, etc. is parametrized by the finite fields $\{F_q\}$. Three general methods will be described: Harish-Chandra's "philosophy of cusp forms"; the use of Hecke algebras for decomposing induced representations into their irreducible components; and the Deligne-Lusztig construction of virtual representations parametrized by the tori, defined over a given finite field, of the algebraic group. The application of these methods involves a mixture of finite group theoretic techniques and ideas from algebraic geometry of varieties defined over finite fields. (Received October 12, 1977.)
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We discuss presentations of maximal unipotent subgroups of certain Chevalley groups over finite fields of characteristic \( p > 2 \). Analogous presentations for \( p = 2 \) give the right groups if and only if the rank is 1 or 2. (Received October 17, 1977.)

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Inverse semigroups of homeomorphisms are hopfian. Preliminary Report.

Let \( X \) be a topological \( T_2 \) space and let \( I_p(X) \) be the inverse semigroup (under partial composition of functions) of all homeomorphisms whose domains and ranges are closed subsets of \( X \). Epimorphisms from \( I_p(X) \) onto \( I_p(Y) \) are examined where \( Y \) is any \( T_1 \) space. The main theorem states that for many spaces \( X \) (including all \( T^n \) and \( S^n \)) every such epimorphism must be an isomorphism. This then yields the result that for these spaces \( X \), the semi-group \( I_p(X) \) is hopfian (every surjective endomorphism is an isomorphism). (Received October 17, 1977.)


The extended bicyclic semigroup \( B \) is the union of a countably infinite chain of bicyclic subsemigroups \( B_p \) [Warne, R. J., Trans. Amer. Math. Soc. 130 (1968), 367-386]. Let \( B \) be contained in \( T \) a locally compact topological inverse semigroup. Let \( A^* \) denote the closure of \( A \) in \( T \). If \( H = B^* - \bigcup B_p^* \neq \emptyset \), then \( H \) is the group of units and \( H \subseteq B \). \( H \) is either a single point or is isomorphic to the integers. If \( B^* \neq B \), then \( B \) has two proper ideals \( U, B^* \) and \( B^* - B \), which is known to be isomorphic to the integers [Eberhart, Carl and John Selden, Jr., Trans. Amer. Math. Soc. 144 (1969), 115-126]. \( B^* \) is shown to be algebraically and topologically one of a collection of constructed examples. The algebraic structure of one of these examples is as follows: Let \( Z \) be the integers and \( S = \{1, 2, \ldots, m\} \). The multiplication is 2\( (m,n) = (m+2r,n) \); \( (m,n)2r = (m,n-2r) \); \( (m,n)(r,s) = (m+r-q,n+s-q) \) where \( q = \min (n,r) \). Here \( H = 2Z, B = Z \times Z, \) and \( B^* - B_p = \emptyset \). (Received October 20, 1977.)

ROBERT C. MERS, North Carolina A. & T. State University, Greensboro, N. C. 27411. Wreath Products and Converse of Lagrange's Theorem

Let \( W = G \wr H, G \) and \( H \) finite groups, \( F = \oplus C_p, C_n \) the cyclic group of order \( n, p \) prime, \( (p,m) = 1 \), and \( d_m \) exponent of \( p \mod m \). We look at conditions for \( W \) to be CLT (satisfy converse of Lagrange's Theorem). Theorem 1. If \((|G|, |H|) = 1, W \) is CLT if and only if \( H \) is CLT and there are \( D \leq F \) of every prime power index such that \( H \subseteq N(D) \) (\( N(D) \) the normalizer of \( D \) in \( W \)). As a consequence we have Theorem 2. If \( W = C_p \wr C_m \), then \( W \) is CLT if and only if for each \( \alpha \in \{1, 2, \ldots, m\} \), \( \lambda^m-1 \) has distinct irreducible factors \( m_1(\lambda), \ldots, m_k(\lambda) \) such that \( \alpha = \sum_{i=1}^{k} \deg m_i(\lambda) \).

Theorem 3: Let \( n = mp^a, F = \oplus C_p, \langle x \rangle = C_n, Tf = x^{-1}fx \) for all \( f \in F \), \( C = \{\ell|F \) has a \( T \) invariant subspace of dimension \( \ell\}, D = \{\ell|0 < \ell < \min 2^a + a, \ell \notin C\}, M \) the maximum number of consecutive integers in \( D \). Then \( C_p \wr C_n \) is CLT if and only if \( M < \alpha \). Theorem 4: Let \( W = C_p \wr C_p^a, e_1, e_2, \ldots, e_l \) be the divisors of \( m \) chosen so that \( d_{e_1} < d_{e_i+1} \) for all \( 1 \leq i < \ell - 1 \). \( W \) is CLT if and only if \( d_{ek} \leq 1 + a + p^a \sum_{i=1}^{k-1} \phi(e_i) \) for all \( 2 \leq k \leq \ell \). (Received October 17, 1977.)
There are many open questions concerning projective modules of finite groups in characteristic $p$ and their characters. In particular little is known even for groups of Lie type in characteristic $p$. Some of these questions will be discussed, together with what little is known about them. Aside from the group $\text{PSL}_2(q)$ the only known results for any infinite clan of groups seems to be for some very special classes in characteristic 2. (Received October 12, 1977.)

W. Cary Huffman, Union College, Schenectady, New York 12308. Linear groups generated by elements containing an eigenspace of codimension two. Preliminary report.

Let $G$ be a finite linear group of degree $n$ with corresponding representation $X$. An element $g \in G$ is called special if $X(g)$ has exactly $n - 2$ equal eigenvalues. The author along with D. B. Wales has handled the case where $G$ contains a special element and $X$ is quasiprimitive. In this paper the case where $X$ is irreducible but not quasiprimitive is discussed. (Received October 17, 1977.)

PATRICIA JONES and STEVEN LIGH, University of Southwestern Louisiana, Lafayette, La. 70504. Quasi ring-semigroups.

A multiplicative semigroup $(S, \cdot)$ is said to be a ring-semigroup if addition, $+$, can be defined on $S$ so that $(S, +, \cdot)$ is a ring. A semigroup that is not a ring-semigroup but every proper subsemigroup containing 0 does support a ring is called a quasi ring-semigroup. The purpose of this paper to give a complete classification of all quasi ring-semigroups. (Received October 17, 1977.)


$S(X)$ denotes the semigroup under composition of all continuous selfmaps of the topological space $X$. Results are obtained about Green's relations which do not require the elements involved to be regular. Special attention is given to the polynomial functions in $S(\mathbb{R})$ where $\mathbb{R}$ denotes the space of real numbers. In fact, necessary and sufficient conditions are given for two odd degree polynomials to be equivalent for each of Green's relations and in each case the conditions concern properties of the local maxima and minima. For example, let $P$ and $Q$ be two odd degree polynomials (not homeomorphisms) with local maxima and minima (in increasing order) at $\{a_i\}_{i=0}^M$ and $\{b_i\}_{i=0}^N$ respectively. Then $P$ and $Q$ are $R$-equivalent if and only if $M = N$ and either $P(a_i) = Q(b_i)$ for all $i$ or $P(a_i) = Q(b_{N-i})$ for all $i$. Similar but somewhat more complicated results are obtained for each of the other relations as well. Finally, $\mathcal{L}(S(X))$ the partially ordered family of all $\Sigma$-classes of $S(X)$ is investigated and is shown in various instances to be a complete lattice. (Received October 17, 1977.)

Carl Eberhart, University of Kentucky, Lexington, Kentucky 40506 and W. Wiley Williams, University of Louisville, Louisville, Kentucky 40208. Elementary Orthodox Semigroups.

The free elementary orthodox semigroup $\mathcal{E}$, generated by an element $p$ and an inverse $q$, will be presented and coordinatized. The lattice of congruences $\Lambda(\mathcal{E})$ will be described, and some interesting examples of elementary orthodox semigroups will be given. (Received October 17, 1977.)
Hill proved that if a torsion-free abelian group \( G \) is the \( \varphi \)-union of a countable number of pure, completely decomposable subgroups that are completely decomposable and homogeneous of type \((0,0,\ldots,0,\ldots)\), then \( G \) is itself completely decomposable and homogeneous of this type (P. Hill, "On the freeness of abelian groups," Bull. Amer. Math. Soc., Vol. 76 (1970), 1118-1120). This result may be generalized, using Hill and Megibben's back-and-forth technique, to the case where the pure subgroups are completely decomposable and homogeneous of any type. Homogeneity, however, cannot be removed from the hypothesis and conclusion of this theorem. Let \( \{q, p_2, p_3, \ldots\} \) be a set of distinct primes, and let \( \{e_i\}_{i=1}^{a} \) be a set of independent elements. Define rank one groups \( E_1 = \langle e_1 \rangle \) and \( E_i = \langle q^{-\infty} e_i \rangle \) for \( i \geq 2 \). Then, define groups \( C_1 = E_1 \) and for \( n \geq 2 \) \( C_n = \langle \prod E_i, p_2^{-1}(e_1+e_2), \ldots, p_n^{-1}(e_1+e_n) \rangle \). Even though each \( C_n \) is completely decomposable and pure in \( G = \bigcup_{n=1}^{\infty} C_n \), \( G \) is not completely decomposable. (Received October 17, 1977.)

Let \( \mathfrak{H} \) be a variety of groups that has nilpotency class two and finite odd exponent, let \( \mathfrak{A} \) be an abelian variety of groups with finite exponent relatively prime to the exponent of \( \mathfrak{H} \), let \( \mathfrak{S} \) be the variety of groups whose third term in the derived series is central. The structure of the nonnilpotent critical groups in the product variety \( \mathfrak{H} \times \mathfrak{A} \times \mathfrak{S} \) having central monoliths is characterized in terms of an invariant \( k \). A critical group \( C \) is said to be a small critical group of \( \mathfrak{H} \times \mathfrak{A} \times \mathfrak{S} \) if \( \text{Fit} C \) is generated by \( k \) elements. Necessary and sufficient conditions are obtained for the intersection \( \mathfrak{H} \times \mathfrak{A} \) to be join-irreducible. For clarification of the notation and terminology see Hanna Neumann's "Varieties of Groups," Springer-Verlag, 1967. (Received October 17, 1977.)

The conditions under which a semigroup \( S \) can be embedded in a group are well known. We refer to the group which allows such an embedding as the quotient group \( Q \) of \( S \). If \( S \) is a topological semigroup one can in an obvious way construct a topology for \( Q \) using the topology of \( S \). This is termed the quotient topology for \( Q \). If \( Q \) is a topological group then \( S \) is automatically a topological semigroup with the relative topology. In this paper we treat the question of finding conditions such that the quotient topology of the relative topology is the original group topology on \( Q \). We call such topologies on \( Q \) recoverable. (Received October 18, 1977.)

It is known that \( G = J_1 \), Janko's first simple group, has a primitive, rank 5, permutation representation on a set \( \Omega \) of degree 266, with point stabilizer \( \text{PSL}(2,11) \) (see for example, D. G. Higman, J. Alg. 6 (1967), p. 37). The graph \( \mathcal{K} \) on \( \Omega \) defined with respect to the suborbit of length 11 has girth 5 and satisfies the following properties: (a) the valency of \( \mathcal{K} \) is 11; (b) for any \( x \in \Omega \), \( \Delta(x) = \text{PSL}(2,11) \); and (c) for some path \( (x,y,z) \) of length 2, \( x,y,z \in \Omega \) \( x \neq z \), \( \Delta_{xyz} \) fixes a pentagon (circuit of length 5) containing \( (x,y,z) \). (Here \( \Delta(x) \) denotes the vertices of \( \mathcal{K} \) adjacent to \( x \).) The following is proved. **Theorem:** Let \( \mathcal{K} \) be a connected, regular graph (undirected, no loops or multiple edges) of girth 5, and \( G \leq \text{Aut}(\mathcal{K}) \) such that \( G \) and \( \mathcal{K} \) satisfy (a), (b) and (c) above. Then \( G = J_1 \). The characterization is in terms of Janko's original characterization of his group (Z. Janko, J. Alg. 3 (1966)). (Received October 18, 1977.)
Let $G$ be a group and let $F(G)$ be the collection of finite nonempty complexes of $G$. Then $F(G)$ is a semigroup with respect to the usual set product. For $A \in F(G)$, let $|A|$ denote the cardinality of the set $A$. **Theorem 1.** If $|AB| = |BA|$ for all $A, B \in F(G)$ such that $|A| = |B| = 3$, then $G$ is abelian. **Theorem 2.** If $|AB| - |BA| \leq 1$ for all $A, B \in F(G)$, then $G$ is either abelian or isomorphic to the quaternion group $Q$ of order eight. (Received October 18, 1977.)

**Neville Campbell,** California Institute of Technology, Pasadena, California 91125. **Pushing Up $L_3(2^k)$.** Preliminary report.

Let $G$ be a finite group such that $F^*(G) = O_2(G)$ and $G/O_2(G) \cong L_3(q)$ or $SL_3(q)$ where $q = 2^N$.

Now let $T \in \text{Syl}_2(G)$ and $B \leq \text{Aut} T$. Sufficient conditions are given for the existence of $1 \neq H < T$ such that $H \triangleleft G$ and $H$ is $B$-invariant. (Received October 18, 1977.)

**Thomas Fournelle,** University of Illinois, Urbana, Illinois 61801. **Infinite groups with finitely many automorphisms.**

A finite group $A$ is representable if there exists an infinite nonabelian group $G$ such that $\text{Aut} G = A$. We prove that neither a finite simple group nor a generalized quaternion group is representable. The symmetric group $S_n$ and the dihedral group $D_n$ of order $2n$ are representable if and only if $n = 4$ and $n = 6$, respectively. An elementary abelian $p$-group of rank $n$ is representable

(i) for $p = 2$ if and only if $n \equiv 3$,

(ii) for $p = 3$ if and only if $n \equiv 8$,

(iii) for $p > 3$ if and only if $n = 8$, or $n \equiv 10$ and $n$ is composite.

(Received October 18, 1977.)

**Kenneth W. Weston,** University of Wisconsin-Parkside, Kenosha, Wisconsin. **Commutator equations over free nilpotent class 2 groups.** Preliminary report.

Let $N$ designate the variety of nilpotent groups of class 2 and $F_m(N)$ the relatively free group of $N$ in $m$ generators. The author discusses the equation problem for systems of commutator equations over $F_2(N)$. It is shown that $F_2(N)$ contains 2 constants such that no algorithm exists for solving systems of commutator equations in only these constants. However there is an algorithm for solving the set of single commutator equations over $F_2(N)$. (Received October 18, 1977.)

**Chang M. Bang,** Emory University, Atlanta, Georgia 30322. **Valuation and height function of countable $p$-groups.**

Any valuation $v$ of any countable $p$-group $A$ can be realized as a height function of $A$ in a larger group $B$, that is, $v(a) = h_B(a)$ for all $a \in A$. (Received October 18, 1977.)

**J. L. Alperin,** University of Chicago, Chicago, Illinois 60637. **A survey of block theory from the point of view of representations of algebras.**

The role of modular representations in group theory and the main problem of block theory. Structure of the modular group algebra. Vertices, sources and the Green correspondence. The three main theorems of block theory. (Received September 20, 1977.)
As in the theory of Lie groups and Lie algebras, the theory of a power-associative analytic H-space \((M,m)\) is developed where \(m: M \times M \to M\) is the analytic multiplication function with two-sided identity element \(e\). A canonical coordinate system is constructed in a neighborhood of \(e\) similar to the canonical coordinate system given by the exponential map for Lie groups. Thus if \(V\) is the representation of \(m\) in a neighborhood \(O\) of \(a\) in \(\mathbb{R}^n\) in this canonical coordinate system, then \(V: O \times O \to O\) has a Taylor's series representation \(V(x,y) = \sum k! V^k(x,y)\) where \(V^k\) is the \(k\)th derivative of \(V\) at \((0,0)\). Furthermore \(V(sx,tx) = sx + tx\) and \(V^k(sx,tx) = 0\) for \(k \geq 2; s, t \in \mathbb{R}\) (reals). This Taylor's series is analogous to the Campbell-Hausdorff formula for Lie groups and the anti-commutative algebra \((\mathbb{R}^n, +, y^2)\) is analogous to the Lie algebra. Investigating when the algebra \((\mathbb{R}^n, +, y^2)\) determines \(V\) gives the Theorem: Let \(A\) be an \(n\)-dimensional algebra containing the \(n\)-dimensional manifold \(M\) such that the multiplication \(m\) is that of \(A\). If \(A\) is an alternative algebra, then the Taylor's series for \(V\) is given by the Campbell-Hausdorff formula. Conversely, if the Taylor's series for \(V\) is such that \(V^3(x,y)^3\) is a homogeneous polynomial in terms of \(V^2(x,y)^2\) and \(A\) is semi-simple, then \(A\) (or its complexification \(A_C\)) is quasi-equivalent to a semi-simple alternative algebra. The H-space \(S^7\) obtained from the Cayley numbers of norm 1 is discussed in this context. (Received August 2, 1977.)

A class of random walks on simply connected nilpotent Lie groups is considered. It is shown that for the random walks considered that certain series associated with the walks converge more rapidly than do the series associated with corresponding random walks on the Lie algebras. The rate of convergence of these series is associated with the rate at which the random walks wander to infinity. (Received September 21, 1977.)

It is well-known that every continuous homomorphic image of a compact \(O\)-dimensional group is \(O\)-dimensional. The interesting question is: what is a continuous homomorphic image of a compact \(O\)-dimensional semigroup? Concepts like finitely neighborable and ultrametrizable semigroups are discussed and they will be used to characterize continuous homomorphic images of compact \(O\)-dimensional semigroups. (Received September 21, 1977.)
**M·A, M/A = Z(G), \tilde{M} = M·Z(G), and L = M·\tilde{A}.** Theorem: Let \(L_1\) and \(L_2\) be (CA) analytic groups and let \(G_1\) and \(G_2\) be isomorphic proper dense analytic subgroups of \(L_1\) and \(L_2\), respectively. (i) If \(Z(L_1)\) and \(Z(L_2)\) are both finite, then \(L_1\) and \(L_2\) are diffeomorphic and locally isomorphic. (ii) If \(Z(L_1)\) and \(Z(L_2)\) are both discrete, then \(L_1\) and \(L_2\) are locally isomorphic. (Received October 13, 1977.)

**752-22-6** Eduardo Cattani and Araldo Kaplan, University of Massachusetts, Amherst, Mass. 01003. SL_2-orbits in Flag Domains.

A classification is given for the "horizontal" orbits of \(SL(2,\mathbb{R})\) in complex manifolds of the form \(G/V\), where \(G\) is a non-compact real form of a complex semisimple Lie group \(G_C\) and \(V\) is the compact intersection of \(G\) with a maximal parabolic subgroup of \(G_C\). Such orbits and their classification are relevant in the study of variations of Hodge structures. (Received October 17, 1977.)


Let \(W = \mathbb{R}^n \times \mathbb{R}^m\) and \(Z = \{t \in \mathbb{R}^n \times \mathbb{R}^n : t = t'\}\). Suppose \(B: W \times W \to Z\) is given by \(B(w,v) = w \bar{B} v' + v \bar{B}' w\), where \(\bar{B} \in \mathbb{R}^{m \times n}\). Define the 2-step, nilpotent Lie group \(N(B) = W \times Z\), with \((w,t)(v,s) = (w + v, t + s + B(w,v))\). Let \(A(B) = \frac{1}{2}(B - B^T)\) be the alternating part of \(B\). Theorem. Let \(B_i\) \((i = 1,2)\) be as above. Then \(N(B_1) \cong N(B_2) \iff \dim \ker A(B_1) = \dim \ker A(B_2)\).

Now suppose \(\ker A(B) = 0\). The resulting group \(N(B)\), which can be represented as a subgroup of the symplectic group, is a natural generalization of the \(2n + 1\) dimensional Heisenberg group. We can exhibit the dual object of \(N(B)\) in a very explicit way. It is likely that some recent results concerning the Heisenberg groups can be extended to the groups \(N(B)\) of Heisenberg type. (Received October 17, 1977.)

**752-22-8** Roger E. Howe, Yale University, New Haven, Connecticut 06520 and Calvin C. Moore, University of California, Berkeley, California 94720. Asymptotic behavior of matrix coefficients of unitary representations.

Let \(\rho\) be an irreducible unitary representation of the group \(G\) of points of a connected linear algebraic group in a local field \(F\), and let \(P\) be the projective kernel of \(\rho\). We prove that the absolute value of all matrix coefficients of \(\rho\) tend to zero at \(\infty\) on \(G/P\). For general \(G, F\) should be characteristic zero, but for reductive \(G, F\) may be arbitrary. For reductive \(G\) we can give a simple, short, and direct argument that is perhaps of interest even though Wallach and others have obtained stronger results by employing more powerful techniques. Our results are deduced from a stronger theorem which asserts that for some integer \(k\), the \(k\)-th tensor power \(\rho \otimes k\) of \(\rho\) has a dense set of matrix coefficients whose absolute values are square integrable on \(G/P\), or, equivalently, that \(\rho \otimes k\) is a subrepresentation of a multiple of the \(\lambda\)-regular representation of \(G\) on \(G/P\) for some central character \(\lambda\) of \(P\). For reductive \(G\) we appeal to the work of Wallach and others noted above. For general \(G\) we employ an inductive argument using the Mackey machine, which involves at a key point a close analysis of the matrix coefficients of the oscillator representation. We also establish the same result on tensor powers for all exponential solvable groups. (Received October 17, 1977.)


Let \(G\) be a connected semisimple Lie group with finite center and without compact factor such that \(\text{rank}(G/K) = 1\). If \(\{G'_{\alpha}\}\) is an infinite sequence of nonisomorphic discrete uniform torsion free subgroups of \(G\) with the same spectrum, then this sequence is finite. (Received October 18, 1977.)
Let $N$ be a nilpotent Lie group, and let $\phi$ be a Schwartz class function on $N$. The Fourier transform of $\phi$, $\hat{\phi}$, is the operator-valued function on $N$, the unitary dual of $N$, defined by $\hat{\phi}(\omega) = \int \phi(n) \sigma(n) \, dn$. We give some results and examples concerning (a) formulas for $\hat{\phi}(\omega)$ and (b) characterizations of the space of Fourier transforms of Schwartz class functions. (Received October 18, 1977.)

**Theorem.** There is a left-commutative (xyz = yzx), associative, nilpotent algebra $\mathcal{G}$ and a Lie ideal $\mathcal{I}$ of $\mathcal{G}$ such that, as a Lie algebra, $N = \mathcal{G}/\mathcal{I}$

**Corollary.** $N$ is meta-abelian.

**Corollary.** Every unitary irreducible representation of $N$ is square integrable modulo its kernel. (Received October 18, 1977.) (Author introduced by Professor Louis Auslander).

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**Real Functions**

Let $X$ and $Y$ be uncountable Polish spaces and $B$ a Borel subset of $X \times Y$ such that for each $X$, $B_X$ is uncountable. A Borel parameterization of $B$ is a Borel isomorphism, $g$, of $X \times Y$ onto $B$ such that for each $x$, $g(x, \cdot)$ maps $Y$ onto $B_x = \{ y : (x, y) \in B \}$. It is shown that $B$ has a Borel parameterization if and only if $B$ contains a Borel set $M$ such that for each $X$, $M_X$ is a nonempty compact perfect set, or, equivalently, there is an atomless conditional probability distribution, $\mu$, so that for each $X$, $\mu(X, B_X) > 0$. It is also shown that if $B_X$ is not meager, for each $x$, then $B$ has a Borel parameterization. (Received July 25, 1977.)

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If $K$ is a set of real valued functions denote by $B_1(K)$ the set of point wise limits of sequences of functions in $K$ and by $B_2(K)$ the set $B_1(B_1(K))$. Suppose $L$ is a lattice ordered vector space of real valued functions containing the constant functions.

**Theorem 1.** If $\phi$ is a positive linear functional on $B_1(L)$ then $\phi$ is the sum of a finite number of Riesz homomorphisms.

**Theorem 2.** If $K$ is a vector lattice of real valued functions, and $\phi$ is a positive linear map of $B_1(L)$ to $K$, then $\phi$ can be extended to a positive linear map from $B_2(L)$ to $B_1(K)$. (Received October 17, 1977.)
It is known that if a measurable function \( f: \mathbb{R} \rightarrow \mathbb{R} \) is symmetrically differentiable at every point, then \( f \) has an ordinary derivative except on a set of measure zero. If, in addition, \( f \) is continuous, then the exceptional set is known to be of first category. Here, these results are sharpened by showing that if \( f \) is measurable and symmetrically differentiable at every point, then it has an ordinary derivative except on a \( \sigma \)-porous set. (Received October 18, 1977.)

**THEODORE S. BOLLIS, State University College, Oneonta, New York 13820.**

**Classification of degenerate critical points.**

We suppose that \( f \) is a sufficiently smooth function defined on an open set in \( \mathbb{R}^n \) and that \( a \) is a critical point of \( f \). Let \( F_1 \) be the homogeneous form on \( \mathbb{R}^n \) defined by \( F_1(x) = \sum \frac{\partial^2 f}{\partial x_i \partial x_j} (a) x_i x_j \), \( i \geq 2 \), and let \( F_k \) be the first non-zero such form. If \( F_k \) is positive definite (resp. negative definite, resp. non-semi-definite), then \( a \) is a local minimum (resp. local maximum, resp. saddle point) of \( f \). If \( F_k \) is positive semi-definite but not positive definite, we consider the next non-zero form \( F_{k+1} \). If \( F_{k+1} \) is positive definite on the cone \( C_k \) of \( F_k \), then \( a \) is a local min. If \( F_{k+1} \) is non-semi-definite or negative semi-definite but not zero, then \( a \) is a saddle point of \( f \). The test fails if \( F_{k+1} \) vanishes on \( C_k \). Examples are provided. (Received October 18, 1977.)

**GERALD S. GOODMAN, University of Florence, Florence, Italy.**

**N-functions and integration by substitution.**

By use of a new result on the chain rule for the derivative of a composite function, the author is able to extend the classical change of variable formula for Lebesgue and Denjoy/Perron integrals to the case in which the change is effected by continuous N-functions of unbounded variation. In the resulting formula the integration of the chain rule derivative need only be carried out over the set where the N-function is differentiable, and that set may fail to have full measure, as an example shows. (Received October 18, 1977.)

28 ▶ Measure and Integration

**G. D. Faulkner and J. E. Huneycutt, Jr.*, North Carolina State University, Raleigh, North Carolina 27607.**

**Convolution iterates of Banach algebra valued measures.**

Let \( B(K) \) be the Borel subsets of a compact \( T_2 \) semigroup \( K \) with unity and let \( E \) be a Banach algebra. The collection \( M(K,E) \) of all \( E \)-valued measures on \( B(K) \) with finite variation \( \| \cdot \| \) is itself a Banach algebra where multiplication is taken to be convolution. A sequence \( \mu_n \) in \( M(K,E) \) is said to converge to \( \mu \) in the \( C(K) \) topology if \( \int f dm_n + \int f dm \) for each continuous real-valued function defined on \( K \). If \( \| \mu \| \leq 1 \) and the range of each \( \mu^i = \mu^* \ldots \mu^* \) is contained in a given weakly compact subset of \( E \), then the sequence \( \frac{1}{n} \sum \mu^i \) converges in the \( C(K) \) topology to an idempotent of \( M(K,E) \). In particular, if \( G \) is a compact group with left-invariant Haar measure \( m \) and \( E \) is a reflexive Banach algebra, then for \( f \in L^1(G,E) \) with \( \| f \| \leq 1 \), the sequence \( \frac{1}{n} \sum f^i \) converges in the \( C(K) \) topology to \( g \in L^1(G,E) \) which is idempotent. (Received September 29, 1977.)

**Roy A. Johnson, Washington State University, Pullman, Washington 99164.**

**Extending the product of two regular Borel measures.**

Suppose \( \mu \) and \( \nu \) are regular Borel measures on the Borel sets \( \mathcal{B}(X) \) and \( \mathcal{B}(Y) \) of compact (Hausdorff) spaces \( X \) and \( Y \). It is shown that the product measure \( \mu \times \nu = \mathcal{B}(X) \mathcal{B}(Y) \) has only one extension to a Borel measure on \( \mathcal{B}(X \times Y) \), the Borel sets of \( X \times Y \). (Received October 17, 1977.)
Let \((X, \Sigma, \mu)\) be a \(\sigma\)-finite measure space and \(T\) a positive linear operator on \(L_1\).
Suppose \(\sup_n \|T_n^*\|_1 < \infty\) and \(\sup_n \|T_n \|_\infty < \infty\), where \(T_n = \sum_{k=0}^{n-1} \mu(\mathcal{E}_k), n = 1, 2, \ldots\).

In \("A pointwise ergodic theorem for positive bounded operators, Proc. Japan Acad. 48 (1972), 458-60\), Y. Kubokawa showed that the pointwise ergodic theorem holds for \(T\) on \(L_1\) provided \(\mu(X) < \infty\) and

\[
(1) \quad \lim_{n \to \infty} T_n f = f \quad \text{for} \quad f \in L_1, \quad \lim_{n \to \infty} \|T_n f - f\|_1 = 0.
\]

Kubokawa's result may be extended by allowing \(\mu(X) = \infty\), dropping \((1)\), and weakening \((2)\) to:

\[
(2) \quad \lim_{n \to \infty} \sup_{\mathcal{E} \in \Sigma} |\mu(\mathcal{E})| = 0 \implies f = 0 \text{ a.e.}
\]

Also, pointwise and mean convergence may be established in \(L_p, 1 < p < \infty\). Apparently the question of mean convergence in \(L_1\) cannot be resolved by applying the techniques used to obtain the above results. (Received October 17, 1977.)

We present a simple proof of the above "uniform boundedness principle" for vector measures, and pose some problems. (Received October 17, 1977.)

An integration theory for vector functions and operator valued measures is outlined and it is shown that in the setting of locally convex topological vector spaces, the dominated and bounded convergence theorems are almost equivalent to the countable additivity of the integrating measure. The measures studied are those representing the continuous linear operators on a space of continuous functions. When certain restrictions are imposed on the spaces involved, actual equivalence of countable additivity and the above theorems obtains. An example is given which shows that in general spaces, convergence in measure no longer implies the almost everywhere convergence of a subsequence. (Received October 18, 1977.)

In this note we investigate properties which imply the convergence in measure of a collection of vector-valued functions when their indefinite integrals (with respect to an operator-valued measure) converge. The functions for which such a theorem is true turn out to be those which, roughly speaking, satisfy the usual maximal inequality of the scalar case: that is, \(\int \mu(E) \leq \int \mu(E)\) for \(E \in \Sigma\). What this means is that the "essential range" of \(f\) stays away from "zero" on the proper kinds of sets; this will be made precise in the paper. Our setting is that of (not necessarily locally convex) topological vector spaces, but we also indicate the situation for Banach spaces. (Received October 18, 1977.)

Horn and Tarski (Trans. Amer. Math. Soc. 64 (1948), pp. 467-497) showed that any \(\mu' \in \mathfrak{ba}(S, \Sigma')\), where \(\Sigma'\) is a subfield of the field \(\Sigma\), can be extended to a \(\mu \in \mathfrak{ba}(S, \Sigma)\). \(\mathfrak{ba}(S, \Sigma)\) is said to be continuous if for \(\varepsilon > 0\) there is a subdivision \(D\) of \(S\) such that if \(E\) refines \(D\) and \(I \in E\), then \(|\mu(I)| < \varepsilon\). \(\mu \in \mathfrak{ba}(S, \Sigma)\) is said to be discrete if there is no non-zero continuous \(\lambda \in \mathfrak{ba}(S, \Sigma)\) such that \(|\lambda(I)| \leq |\mu(I)|\), for \(I \in \Sigma\). By proving a result similar to that cited above, a theorem by the author (these Notices, abstract 77T, Bill, June, 1977) can be extended as follows (\(\Sigma^*\) is as defined in abstract): Theorem. The following are equivalent. (1) Each member of \(\mathfrak{ba}(S, \Sigma)^*\) has an integral represen-


**A decomposition theorem for vector measures on a Boolean algebra.**

Let \( \mathfrak{e} \) be the unit in a Boolean algebra \( \mathfrak{A} \). For \( a \in \mathfrak{A} \), let \( (a) \) denote the principal ideal generated by \( a \) and let \( e \setminus a \) be the complement of \( a \). \( W \in \text{lcs} \) means that \( W \) is a locally convex space, and \( W(0) \) is a base for its neighborhoods of zero. \( \phi \in \text{sb}(\mathfrak{A}, W) \) means that \( \phi : \mathfrak{A} \to W \) is strongly bounded (\( \phi(a \lor b) = \phi(a) + \phi(b) \) when \( a \land b = 0 \) and \( \phi(a_j) = 0 \) for every sequence \( (a_j) \) with \( a_j \land a_j = 0 \) for \( i \neq j \)).

Theorem. Let \( \mathfrak{D} \) be any non-empty family of ideals of \( \mathfrak{A} \) directed by \( \supseteq \). For any complete \( W \in \text{lcs} \) and \( \phi \in \text{sb}(\mathfrak{A}, W) \), \( \exists \phi_\mathfrak{D}, \phi_\mathfrak{D} \in \text{sb}(\mathfrak{A}, W) \) such that \( \phi = \phi_\mathfrak{D} + \phi_\mathfrak{D} \) and such that for all \( \phi \in W(0) \), \( \exists \phi_\mathfrak{D} \in \mathfrak{D} \) and \( \forall \lambda \in \mathfrak{A}, \exists \phi(\mathfrak{D}) \subseteq \mathfrak{N} \) while \( \forall \lambda \in \mathfrak{D}, \exists \phi(\mathfrak{D}) \subseteq \mathfrak{N} \) with \( \phi(\mathfrak{D}) \subseteq \mathfrak{N} \).

Applications of this decomposition theorem include Lebesgue decompositions, the Yosida-Hewitt decomposition, and other classical decompositions. (Received October 18, 1977.)


**Functions of a Complex Variable**

We consider the six-dimensional algebra \( \mathbb{R}S_3 \) of all elements \( z = x_0 + x_1 h + x_2 h^2 + x_3 i + x_4 j + x_5 k \) over the reals, and functions \( f(z) \) from \( \mathbb{R}S_3 \) to itself, \( f(z) = f_0(x_0, \ldots, x_5) + \cdots + f_5(x_0, \ldots, x_5)k \). Let the \( f_j \) be \( C^1(\mathbb{D}) \) on a domain \( \mathcal{D} \subseteq \mathbb{R}S_3 \) (for definitions, cf. Ward, Duke Math. J. 7(1940)). We obtain the following theorems:

I. \( f(z) \) is left (right) differentiable on \( \mathcal{D} \) iff the \( f_j \) satisfy a system of first-order linear PDEs. In particular, there exist non-trivial left (right) component-wise polynomial functions of arbitrary degree. (Note: \( z^2 \) is differentiable in neither sense.) II. The Hausdorff derivative of \( f(z) \) lies in an algebra \( \mathfrak{A} \supseteq \mathbb{R}S_3 \), \( \dim(\mathfrak{A}) = 18 \). Polynomials are regular in this sense. The regular representation matrices of \( \mathfrak{A} \) are given explicitly (with essential use of a computer program). III. Let \( f(z) \) be regular in the sense of II. Then there exists a linear map \( A \) and component functions \( g_j \) with \( A[f_0 \ldots f_5] = [g_0 \ldots g_5] \) such that \( g(z) \) is two-sidedly regular in the sense of Fueter (cf. also, Comm. Math. Helv. 8(1936)). (Received September 29, 1977.)

We prove necessary and sufficient conditions in terms of convolutions for functions to be convex of order \( \alpha \), starlike of order \( \alpha \), and spiral-like. Applications to radius of starlikeness and radius of convexity problems are given. (Received September 30, 1977.)

Functions \( g \) defined by the gap power series \( g(z) = \sum_{n=0}^{\infty} a_n z^{\lambda_n} \) are considered. Assuming suitable conditions on \( \{\lambda_n\} \) and \( \{a_n\} \) it is shown that there exist functions \( f \) of bounded index and functions \( F \) of bounded value distribution which are asymptotically equal to the given function, that is, \( \log M(r,g) \sim \log M(r,f) \sim \log M(r,F) \). These functions \( g \) do not belong to the class \( B \) of entire functions of exponential type and bounded on the real axis. Some results in this paper are joint work with C. G. Frick. (Received October 5, 1977.)

A-81
Imaginary values of meromorphic functions in the disk.

For a meromorphic function $f$ let $\phi(r,f)$ be the number of points on $|z| = r$ at which $f$ is purely imaginary. If $f$ is entire, then A. Gel'fond, H. S. Wilf, and J. Korevaar and S. Hellerstein have given bounds for $\phi(r,f)$ in terms of the order of $f$. For meromorphic functions in the plane, J. Miles and D. Townsend have given bounds in terms of the Nevanlinna characteristic function $T(r,f)$ for $\phi(r,f)$, with an exceptional set of $r$-values, and for $\phi(r,f) = \int_0^r \phi(t,f)(1-t)^{-1}dt$ for all sufficiently large values of $r$.

Bounds are now obtained for $\phi(r,f)$ and $\phi(r,f) = \int_0^r \phi(t,f)(1-t)^{-1}dt$ for meromorphic functions in the disk. The bound for $\phi(r,f)$ holds off an exceptional set of $r$-values. Methods of proof are significantly different from those used to prove similar results in the plane. (Received October 6, 1977.)

New classes of close-to-convex functions.

Let $D^k f(z) = (z/(1-z)^k+1)^t f(z)$, $k > -1$ where $(*)$ is the Hadamard product of power series. The classes $K_n$ of functions $f(z)$ regular in the unit disc $E$ with $f(0) = f'(0) - 1 = 0$ and satisfying $\text{Re}(D^n+1 f(z)/D^n f(z)) > 1/2$, $z \in E$, $n \in N_0 = \{0,1,2,\ldots,\}$, have been introduced by St. Ruscheweyh. The author introduces the classes $G$ of functions $f(z)$ regular in $E$ with $f(0) = f'(0) - 0$ and satisfying $\text{Re}(D^n+1 f(z)/D^n g(z)) > 1/2$ for some $g \in K_{n+1}$ and all $z \in E$, $n \in N_0$. The following results are obtained:

1. $K_{n+1} \subset G_n$, $n \in N_0$.
2. Let $g \in K_n$ relative to $g \in K_{n+1}$.

Then $f(z) = \left[ \sum_{j=1}^{m} \frac{Y+1}{Y+j} z^j \right] * f(z) \in G_n$ relative to $G(z) = \left[ \sum_{j=1}^{m} \frac{Y+1}{Y+j} z^j \right] * g(z) \in K_{n+1}$ with $n \in N_0$ and $\text{Re} \gamma > n/2$. (Received October 6, 1977.)

Boundary distortion and the uniform convergence of quasiconformal mappings. Preliminary report.

Let $D$ denote a proper subdomain of $\mathbb{R}^n = \mathbb{R}^n \cup \{\infty\}$ and let $\{f_j\}$ be a sequence of K-quasiconformal mappings of $D$ into $\mathbb{R}^n$ which converges uniformly on compact subsets of $D$ to a K-quasiconformal mapping $f$. Under what circumstances can it be inferred that $f_j \to f$ uniformly on all of $D$? This paper describes a number of criteria which can be used to test $\{f_j\}$ for uniform convergence. These criteria typically involve extension properties of the mappings $f_j$ and/or geometric conditions on either $D$ or the sequence of image domains $f_j(D)$. The following is an example of our results.

Theorem. Suppose that each of the mappings $f_j$ has a continuous extension $\overline{f}_j$ to the closure $\overline{D}$ of $D$. Then $f_j \to f$ uniformly on $D$ if and only if $f$ has a continuous extension $\overline{f}$ to $\overline{D}$ and $\overline{f}_j \to \overline{f}$ uniformly on the boundary of $D$. (Received October 7, 1977.)

Generalized regularity on linear associative algebras in the sense of Fueter.

Let $\mathcal{A}$ be a linear associative algebra of dimension $n$ over $\mathbb{R}$; $\mathcal{L}$ an $m$-dimensional subspace of $\mathcal{A}$; $\mathcal{M}$ an $m$-dimensional manifold with boundary $\partial M$; do a $(p-1)$-form (with coefficients in $\mathcal{A}$), $2 \leq p \leq m \leq n$; $\omega = x_1 e_1 + \ldots + x_m e_m \in \mathcal{L}$; $f(\omega) = \int f_1(x_\omega) e_\omega \in \mathcal{A}$, with the $f_\omega$ real $C(\Omega)$-functions on $\mathcal{L}$. DEF.: Say that $f(\omega)$ is $(n,m,p,d)\text{-left}$ $(right)$ regular on $\mathcal{L}$ if $\int_{\partial M} f(\omega) d\omega = 0$ ($\int_{\partial M} d f(\omega) d\omega = 0$) for every $M \subset \mathcal{L}$. Then we prove that:

1. $f(\omega)$ is left regular in the sense of R. Fueter iff $f(\omega)$ is $(n,m,m,d)\text{-left}$ regular with $d\omega = e_1 dx_1 \ldots dx_m + \ldots + e_m dx_m \ldots dx_1$; similarly for right regular.

2. $f(\omega)$ is regular in the sense of G. Scheffers (=classically differentiable) on the
commutative algebra \( \mathfrak{A} \) iff \( f(z) \) is \( \{n,n,2,dz\} \)-regular on \( \mathfrak{A} \). (3) Let \( \mathfrak{A} \) be the complex field. There exists a 1-form \( \Omega = a(z) dx + b(z) dy \) such that \( f(z) \) is pseudo-analytic iff \( f(z) \) is \( \{2,2,2,d\} \)-regular. Apart from bringing several different versions of generalized holomorphy within a single definition, we now also notice the existence of infinitely many entirely new regularity classes. (Received October 11, 1977.)

*752-30-8 ALFONSO G. AZPEITIA, University of Massachusetts at Boston, Boston, Massachusetts 02125. On
the growth of entire functions defined by Dirichlet series.
\[
\sum_{n=1}^\infty a_n \exp(\lambda_n z)
\]
where the series is absolutely convergent for all \( z \) and \( D = \lim \sup_{n \to \infty} \log n/\lambda_n < \infty \). The question of whether finite Ritt order, i.e.,
\[
\rho = \lim \sup_{x \to \infty} \log M(x)/x < \infty \quad (M(x) = \sup \{ |f(z)| : -\infty < y < \infty \}),
\]
with \( D = 0 \) imply: (1) \( \log M(x) \to \log \mu(x) \) as \( M(x) = \max \{ |a_n|n^{\lambda_n} : n = 0, 1, 2, \ldots \} \), is answered negatively by using an example of
Sugimura [Math. Z. 29 (1928-1929), Satz 6, P. 266 and pp. 275-276]. The additional condition
\[
0 < \lim_{x \to \infty} \log M(x)/\exp(\rho x) < \infty
\]
is shown to be sufficient for (1). The proof is based on results of Yu [Ann. Sci. Ecole, Norm. Sup. 68 (1951), 65-104] and Azpeitia [Trans. Amer. Math. Soc. 104 (1962), 495-501]. It is
also proved that if \( D = 0 \) then for any \( \alpha \neq 0 \),
\[
\lim \sup_{x \to \infty} \log M(x)/\exp(\alpha x) = \lim \sup_{x \to \infty} \log \mu(x)/\exp(\alpha x).
\]
(Received October 11, 1977.)

*752-30-9 THAD DANKEL, JR., University of North Carolina at Wilmington, Wilmington, North Carolina
28401. On Moduli of Continuity of Analytic and Harmonic Functions.

ABSTRACT: We consider inequalities relating the modulus of continuity of an analytic or harmonic
function in a planar region to its modulus of continuity on the boundary of the region. Using har-
monic measure, we give a new proof of such a result for harmonic functions in the unit disc. We also
generalize results for both analytic and harmonic functions in the unit disc to such functions defined
on a Jordan region \( G \) such that \( \partial G \) satisfies certain smoothness assumptions. (Received October 14, 1977.)

752-30-10 A. W. Goodman, University of South Florida, Tampa, Florida 33620.
Koebe domains for certain classes of functions. Preliminary report.

We determine the Koebe domain for the class of all normalized bounded functions in the unit disk. From this result it is easy to derive the Koebe domains for certain other related classes. (Received October 14, 1977.)

752-30-11 J.W. NOONAN, College of the Holy Cross, Worcester, Massachusetts
01610. A length-area principle, Preliminary report.

Let \( n \geq 1 \), set \( D = \{ z \in \mathbb{C}^n : ||z||<1 \} \), and let \( f:D \to \mathbb{C}^n \) be locally biholomorphic. Let \( B(R) = \{ w : ||w||=R \} \), \( \tilde{f}(R) = B(R) \cap f(D) \), and \( D(R) = f^{-1}(\tilde{f}(R)) \). Let \( f^{-1} \) be
a local inverse for \( f \), and let \( Jf^{-1} \) denote the Jacobian of \( f^{-1} \). Set \( \|(f^{-1})'\| \)
equal to the operator norm of \( (f^{-1})' \). Let \( \lambda(R) \) be the (real) 2n-1 dimensional measure of the manifold \( D(R) \), and put \( p(R) = \int |Jf^{-1}(w)| \, ||(f^{-1})'(w)||^{-2n} dv \),
where the integral is over the manifold \( \tilde{f}(R) \). Theorem. If \( 1/(2n) + 1/(2m) = 1 \), and if \( p(D) \) is the (complex) \( n \) dimensional measure of \( D \), then
\[
\int_0^\infty \lambda(R) \frac{2m}{(2n-1)(2m-1)} \frac{1}{p(R)^{m/n}} dR \leq V(D).
\]
If \( n = 1 \), this reduces to a well-known
length principle. Since the integrand in the definition of \( p(R) \) reduces to \( 1 \) if and only if \( f \) is conformal, \( p(R) \) can be interpreted as a measure of the
average extent to which \( f \) fails to be conformal. (Received October 17, 1977.)
Let \( p(z) \) be regular in the unit disc \( U \) and let \( \psi(r,s,t) \) be a complex function defined in \( \mathbb{C}^3 \).

For the univalent functions \( h(z) \) and \( q(z) \) the authors determine conditions on \( \psi \) such that if \( \psi(p(z),zp'(z),z^2p''(z)) \) is subordinate to \( h(z) \) then \( p(z) \) will be subordinate to \( q(z) \). Applications of this result in the field of univalent functions and differential equations are given. (Received October 17, 1977.)


For \( p > 0 \), let \( \mathcal{P}(p) \) be the space of locally integrable functions \( f \) on the complex plane \( \mathbb{C} \) such that \( \int |f|^2 e^{-|z|^{p+\epsilon}} < \infty \) for every \( \epsilon > 0 \). Also, let \( \mathcal{L}(p) \) denote the space of such functions \( f \) such that \( \int |f|^2 e^{-|z|^p} < \infty \) for some \( C > 0 \). Theorem: As an operator on \( \mathcal{L}(p) \), \( \frac{3}{2} \frac{\partial}{\partial z} \) has a continuous, linear cross section. As an operator on \( \mathcal{P}(p) \), \( \frac{3}{2} \frac{\partial}{\partial z} \) has no continuous linear cross section. (Received October 17, 1977.)

752-30-14 KENNETH R. GURGANUS, University of North Carolina at Wilmington, Wilmington, N. C. \( \phi \)-like Polynomial Approximation.

Let \( || \cdot || \) be a norm on \( \mathbb{C}^n \) and \( f \) be a univalent holomorphic function defined on \( B = \{ z \in \mathbb{C}^n : ||z|| < 1 \} \) such that \( f(0) = 0 \) and \( Df(0) = I \). Theorem 1. There exists a sequence of univalent polynomial mappings \( \{ p_k(z) : k = 1, 2, \ldots \} \) where \( p_k(z) \) has degree \( k \), \( p_k(B) \subseteq p_{k+1}(B) \), and \( p_k(z) + f(z) \) uniformly on compact subsets of \( B \). This extends the results of MacGregor (Trans. Amer. Math. Soc., 148 (1970), 199-209) and Cima (Trans. Amer. Math. Soc., 175 (1973), 491-497). We have shown earlier (Trans. Amer. Math. Soc., 205 (1975), 389-406) that \( f \) is univalent if and only if \( f \) is \( \phi \)-like for some \( \phi \). Theorem 2. If \( || \cdot || \) is either the Euclidean norm or the sup norm on \( \mathbb{C}^n \), and \( f \) is \( \phi \)-like, then the subordination sequence of polynomial mappings in Theorem 1 may be chosen so that each \( p_k \) is \( \phi \)-like. This latter result can be extended to an arbitrary norm on \( \mathbb{C}^n \) with appropriate conditions on \( \phi \). (Received October 17, 1977.)


Let \( f \) and \( F \) be analytic in \( |z| < 1 \), \( f \) subordinate to \( F \), and let \( A(r,f) \) denote the area of the region of the Riemann surface onto which \( |z| < r \) is mapped by \( f \). Finally, let \( B(r,f,F) = A(r,f)/A(r,F) \). Reich ["An inequality for subordinate analytic functions", Pacific J. Math 4(1954), 259-274] has shown that \( B(r,f,F) < \alpha(n-k^2-2) \), \( 0 < r < 1 \), with equality holding for each \( r \), for \( k=1,2,\ldots \) certain pair \( (f,F) \), dependent on \( r \). Asymptotically, \( B(r,f,F) = O((1-r)^{-1}) \).

We show that this order result improves if one specifies that \( F \) map \( |z| < 1 \) onto certain wedge shaped regions. For example, \( B(r,f,F) \) is bounded if \( F(z) = [(1 + z)/(1 - z)]^\alpha, \alpha > 1/2 \).

Finally, if \( F(z) = z \), then \( B(r,f,F) \), (or, \( A(r,f) \)), \( = O((1-r)^{-1}) \). (Received October 17, 1977.)
ROGER W. BARNARD, University of Kentucky, Lexington, KY 40506. On Robinson's 1/2 Conjecture.

In 1947 R. Robinson conjectured that if \( f \) is in \( S \), i.e. a normalized univalent function on the unit disk, then the radius of univalence of \( [zf(z)]'/2 \) is at least 1/2. He proved in that paper that it was at least 0.38. The conjecture has been shown to be true for most of the known subclasses of \( S \). As to the entire class \( S \) the exact lower bound is still open. This author shows by use of the Grunski inequalities that the lower bound is at least 0.49. (Received October 18, 1977.)

H. Silverman, College of Charleston, Charleston, South Carolina 29401, E. M. Silvia, University of California, Davis, California 95616, and D. N. Telage, University of Kentucky, Lexington, Kentucky 40506. Locally Univalent Functions and Coefficient Distortions.

We look at functions \( f(z) = z + \sum_{n=2}^{\infty} a_n z^n \) satisfying \( \sum_{n=2}^{\infty} \left| a_n \right| > 1 \) and determine conditions for which the arguments of the coefficients may vary without affecting the univalence of the function. A bound on the radius of starlikeness for the convolution of functions taken from the closed convex hull of convex functions and a special subclass of starlike functions is also obtained. (Received October 18, 1977.)

Potential Theory

C. WAYNE MASTIN, Mississippi State University, Mississippi State, Mississippi 39762. Harmonic mappings in two and three dimensions.

Some basic properties of harmonic mappings of simply-connected regions onto rectangular regions are discussed. Harmonic mappings are easy to construct, the boundary correspondence may be arbitrarily assigned, and the mapping has a nonvanishing Jacobian and is one-to-one whenever the boundary correspondence is one-to-one. Applications to mesh generation problems and the solution of partial differential equations on arbitrary regions are included. (Received October 11, 1977.)

Karl F. Barth, Syracuse University, Syracuse, NY 13210 and David A. Brannan, Queen Elizabeth College, London W87AH, England. The growth of plane harmonic functions on asymptotic paths II.

Continuing the research begun in these Notices, 24(1977), p.A-84, we prove the following theorem. Let \( u(z) \) be harmonic in the plane and suppose that \( T \) is a component of \( \{ z: u(z) > 0 \} \). Let \( s(r) \) be the length of \( T \cap \{ |z| = r \} \) and \( \theta(r) = s(r)/2\pi \). Suppose that \( \theta(r) < \beta \) \( (0 < \beta < 2) \) and that \( \phi(r) \) is a positive, increasing function of \( r \) for \( r > r_0 \) and that

\[
\int_{r_0}^{\infty} \frac{\phi(t)}{1 + \frac{1}{\beta}} dt < \infty.
\]

Then there exists a positive constant \( a \) and a path \( \Gamma \subset T \) going to \( \infty \) such that

\[
u(z) > a\phi(|z|) \quad \text{on} \quad T
\]

for \( |z| > r_0 \). The proof consists of combining the techniques used in the previous work with a theorem of Tsuji, Tohoku Math. J., 2nd Ser., 3 (1951), 13-23. (Received October 17, 1977.)

Several Complex Variables and Analytic Spaces

L.R. HUNT, Texas Tech University, Lubbock, Texas 79409, and MIKE KAZLOW, Rice University, Houston, Texas 77001. A two-sided H. Lewy extension phenomenon II. Preliminary report.

Suppose \( M \) is a connected real \( k \)-dimensional \( C^\infty \) CR-submanifold of \( \mathbb{R}^n \), \( k, n \geq 2 \). Sufficient conditions on \( M \) are found which imply that there is a connected open set \( U \) in \( \mathbb{R}^n \) which contains \( M \) in its closure.
so that all \( C^\infty \) CR-functions on \( M \) extend to holomorphic functions on \( U \). If certain stronger conditions are satisfied, it is shown that the open set \( U \) is actually a neighborhood of \( M \) in \( \mathbb{C}^n \), and all CR-functions on \( M \) are the restrictions to \( M \) of holomorphic functions on \( U \). (Received October 11, 1977.)

*752-32-2 GARY A. HARRIS, Texas Tech University, Lubbock, Texas 79409. Higher order analogues to T.C.R. equations for smooth real submanifolds of \( \mathbb{C}^n \) with C.R. singularity. Preliminary report.
A possibly infinite succession of higher order differential operators are defined which, for a large class of real submanifolds, reduce to the first-order induced Tangential Cauchy-Riemann equations in the event the submanifold is C.R. Local approximation and extension results are obtained for functions defined on certain smooth submanifolds. These results recover known results for C.R. submanifolds. (Received October 11, 1977.)

*752-32-3 JOSEPH A. BECKER, Purdue University, West Lafayette, Indiana 47907 and WILLIAM R. ZAME*, Tulane University, New Orleans, Louisiana 70118. Applications of functional analysis to the solution of power series equations.
It is well-known that any homomorphism between complete local rings (e.g., rings of formal power series) is both open and closed in the respective Krull topologies, and that the corresponding statement for analytic rings (e.g., rings of convergent power series) is false. The main result of this paper is that every homomorphism between analytic rings which is closed is necessarily open (the converse is not true); this can be interpreted as a statement about the formal and convergent solutions of certain power series equations. This result is proved by deriving certain connections between the Krull topology and the simple and inductive topologies on the respective analytic rings, and applying some functional-analytic results about the automatic continuity of linear operators and the uniqueness of topologies. For example, it is shown that the algebra of formal power series modulo any ideal carries a unique Frechet algebra topology (namely the simple topology). (Received October 11, 1977.)

*752-32-4 Eric P. Kronstadt, University of Michigan, Ann Arbor, Michigan 48109. Analytic structures for \( H^\omega \) of certain domains in \( \mathbb{C}^n \).
A necessary and sufficient condition for the existence of certain types of \( n \)-dimensional analytic structures in the maximal ideal space of \( H^\infty(D) \), where \( D \) is a bounded homogeneous domain in \( \mathbb{C}^n \), is given. It is shown that points in the closure of interpolating sequences are contained in such structures. A partial converse for the case of the polydisk is also presented. (Received October 14, 1977.)

*752-32-5 WILHELM STOLL, University of Notre Dame, Notre Dame, Indiana 46556. Strictly parabolic manifolds. Preliminary report.
Let \( \tau \geq 0 \) be a \( C^\infty \)-function on a connected, complex manifold \( M \) of dimension \( m \) such that \( \{ x \in M \mid \tau(x) \leq r^2 \} \) is compact for all \( r \geq 0 \). Then \( (M,\tau) \) is said to be strictly parabolic if \( dd^C \log \tau \geq 0 \), if \( dd^C \tau > 0 \), and if \( (dd^C \log \tau)^m \equiv 0 \). Example: \( (\mathbb{C}^m,\tau_0) \) with \( \tau_0(z) = |z|^2 \). Let \( (M,\tau) \) be strictly parabolic. Then \( M(0) \) consists of exactly one point \( 0_M \). Let \( u: \hat{M} \to M \) be the blow up of \( M \) at \( 0_M \) by a \( \sigma \)-process. Theorem 1: "The manifold \( M \) carries the structure of a differentiable linebundle \( \tau: \hat{M} \to \mathbb{E}^m+1 \) whose zero section and fibers are complex submanifolds of \( \hat{M} \). A differentiable bundle isomorphism \( h: \mathbb{C}^m \to \mathbb{E}^m \) exists such that \( \tau_0 \circ h = \tau \circ \sigma \)." The dualism defined by the Kaehler metric \( dd^C \tau \) maps \( \mathcal{F} \) onto a vector field \( f \) on \( M \). Theorem 2: "If \( f \) is holomorphic, \( h \) is biholomorphic and a biholomorphic map \( g: \mathbb{C}^m \to M \) exists such that \( \tau_0 = \tau \circ g \)." Let \( \rho \) be the Ricci form of \( (dd^C \tau)^m \). Then \( f \) is holomorphic, iff \( \rho(f,\overline{f}) = 0 \). (Received October 17, 1977.)
The space of biholomorphically distinct complex structures on $\mathbb{R}^{2n}$, $n \geq 2$, is known to be large in the sense, for instance, that there are infinite dimensional families of deformations of the unit ball structure, no member of the family being equivalent to any other. The usual method of establishing such conclusions is to use the Fefferman extension-to-the-boundary theorem (for biholomorphic mappings of strictly pseudoconvex domains) to reduce the problem to a boundary question which is analyzed with Chern-Moser invariant theory. By considering domains with special symmetries, the existence of infinite dimensional families of distinct structures can be established by a method which uses neither the Fefferman theorem nor the theory of Chern-Moser invariants. (Received October 17, 1977.)

We consider a natural extension of the notion of extremal length to several complex variables where invariance under biholomorphism is a basic requirement. Application is made to a mapping problem considered by Chern-Levine-Nirenberg via their intrinsic norm and by Eisenman (Pelles) via hyperbolic analysis. (Received October 17, 1977.)

If $M$ is a real hypersurface of $\mathbb{C}^n$, then it is known that if there exists a point $p \in M$ such that the Levi form at $p$ has eigenvalues of opposite sign then the $C^\omega$ solutions of the tangential Cauchy-Riemann equations on $M$ near $p$ are restrictions of holomorphic functions. We generalize the above theorem to the case where $M$ is a submanifold of arbitrary codimension and assume that the image of the Levi form of $M$ at $p$ is the normal space to $M$ at $p$. Furthermore, if $M$ has real codimension 2 and the Levi form of $M$ at $p$ has eigenvalues of opposite sign in every direction, then the image of the Levi form of $M$ at $p$ is the normal space to $M$ at $p$. (Received October 17, 1977.)

Let $\mathbb{C}$ be the complex Cayley numbers with Cayley conjugation $x \rightarrow \overline{x}$, complex conjugation $x \rightarrow \overline{x}$, and unit element $e$. Define $(x:y) \in \mathbb{C}$ by $(x:y)e = \frac{1}{2}(xy + yx)$; put $N(x) = (x:x) \in \mathbb{C}$ and $B(x) = (x:x) \neq 0$. Let $\mathcal{J}$ be the Jordan algebra of Cayley-hermitian $3 \times 3$ matrices over $\mathbb{C}$ with multiplication $U \circ V = \frac{1}{2}(UV + VU)$. For $Z = (z_{ij}) \in \mathcal{J}$, set $Z = (z_{ij})$. Theorem. The 16-dimensional exceptional domain can be described as $\{(x,y) \in \mathbb{C} \oplus \mathbb{C}: 2[B(x) + B(y)] < \min(2, 1 + |N(x)|^2 + |N(y)|^2 + B(xy))\}$. The 27-dimensional domain equals $\{(z,\overline{z}) \in \mathcal{J}: \operatorname{tr}(z + \overline{z}) < 3 \text{ and } \frac{1}{2}[\operatorname{tr}(z + \overline{z})]^2 + \operatorname{tr}(z^2 + \overline{z}^2) - 2[\operatorname{tr}(z + \overline{z})^2] > \max(2 \operatorname{tr}(z + \overline{z}) - 3, \operatorname{tr}(z + \overline{z}) + |\det z|^2 - 1)\}$. Previous descriptions were hard to use because it was necessary to test operators on $\mathbb{C} \oplus \mathbb{C}$ or $\mathcal{J}$ for positive definiteness. (Received October 17, 1977.)

The Chern-Levine-Nirenberg semi-norm $N(\Gamma)$ of the homology class of the outer boundary $\Gamma$ of a domain $\Omega \subset \mathbb{C}^n$ is sup $|\int_{\partial \Omega} (dd^c v)^{p-1}|$, sup taken over $\mathcal{J} = \{\text{plurisubharmonic } v \in C^2(\Omega), 0 < v < 1, (dd^c v)^p = 0\}$. Let $\mu \in \mathcal{J}$ be the function with boundary value $\equiv 1$ on $\Gamma$, $\equiv 0$ on all other components of $\partial \Omega$. We show that if $\mu$ is sufficiently regular, then $\mu$ is the unique function realizing the sup above. This follows from uniqueness in the Cauchy problem for $(dd^c v) = 0$, and a study of the foliation of $\Omega$ by integral manifolds of $dd^c u$. Mapping theorems are derived from this untiqqeness, generalizing the one variable technique of Landau and Osserman.
The CLN semi-norm leads to a notion of "negligible" set in $\mathbb{C}^n$. We show that sets polar for p.s.h. functions are negligible, and that negligible sets are sets of removable singularity for p.s.h. solutions of $(dd^c \varphi)^n = 0$. Applied to mappings this gives (typically): if $n \geq 2$, and $\mathbb{C}^n - \Omega$ has a finite number $\geq 2$ of components with interior, then a holomorphic $f: \Omega \to \Omega$ is biholomorphic iff $f_{\Omega}: H_{2n-1}(\mathbb{C}) \to H_{2n-1}(\mathbb{C})$ is an isomorphism. Examples are given of sets not removable for singularities of p.s.h. functions. (Received October 15, 1977.)


* This is a survey of the developments in the theory of several complex variables which arose from the Levi problem. The Levi problem in its original form was solved long ago. However, over the years various extensions and generalizations of the Levi problem were proposed and investigated. Some of the more general forms of the Levi problem still remain unsolved. In this survey we will discuss known results, counterexamples, and unsolved problems concerning locally Stein sets, increasing unions of Stein sets, the Serre problem, weakly pseudoconvex boundaries, and curvature conditions. (Received October 18, 1977.)

*752-32-12 R. O. WELLS, JR., Rice University, Houston, TX 77001. Holomorphic Representation of Solutions of the Zero-Rest-Mass Field Equations.

R. Penrose established a geometric correspondence between $\mathbb{P}_3 = \mathbb{P}_3(\mathbb{C})$ and complexified compactified Minkowski space $M_\mathbb{C}$. The self-dual solutions of the zero-rest-mass field equations of spin $s$, $s = 1/2, 1, 3/2, \ldots$, on the Hermitian symmetric space $M_\mathbb{C}^+$ of $2 \times 2$ complex matrices with imaginary part positive definite are isomorphic to the cohomology vector space $H^1(\mathbb{P}_3^+, H^{2s-2}(\mathbb{H}))$, where $H \to \mathbb{P}_3$ is the hyperplane section bundle, and $\mathbb{P}_3^+$ is the open strictly 1-convex subset of $\mathbb{P}_3$ defined by means of $|z_0|^2 + |z_1|^2 - |z_2|^2 - |z_3|^2 > 0$, where $[z_0, \ldots, z_3]$ are homogenous coordinates for $\mathbb{P}_3$. An SU(2,2) invariant proof of this isomorphism, first described by Penrose, is presented. (Received October 18, 1977.)

33 ▲ Special Functions

752-33-1 GLORIA OLIVE, University of Otago, Dunedin, New Zealand. From the zero-transform to combinatorial identities. Preliminary report.

A variety of combinatorial identities are easily obtained from identities for $b$-transforms and their related functions by letting $b = 0$. The following identities can be obtained by this conversion.

(*) \[ \sum_{m+j} k^m j^j s^m_j = \sum_{m+j} k^{m+j} s^m_j j^j k^j \]

where $s^m_j$ is the Stirling number of the first kind defined by $k^m = \sum_{j=0}^{k} s^m_j j^m/k!$.

(**) \[ \sum_{k=0}^{\infty} \alpha^{2k} \binom{x}{2k} = (1 - \alpha)^x \cosh \frac{x}{2} \log \left( \frac{1 + \alpha}{1 - \alpha} \right) \] when $|\alpha| < 1$.

We do not claim that any of the identities are new, nor do we deny this possibility. (Received June 15, 1977.)

*752-33-2 SHARON GOODMAN, GRADUATE SCHOOL, THE CITY UNIVERSITY OF NEW YORK, N.Y., 10036. DISTINGUISHED SUBSPACE THEORY OF $L^\infty$ OF HEISENBERG NILMANIFOLDS.

Let $N$ denote the $n$-dimensional Heisenberg group. A lifted subgroup of $N$ is a discrete subgroup $\Gamma$ of $N$ such that $\Gamma \backslash N$ is compact and such that $[\Gamma, \Gamma] = \Gamma \cap Z$, where $Z$ denotes the center of $N$, and $[\Gamma, \Gamma]$ denotes the commutator subgroup of $\Gamma$. The right
regular representation $R$ of $N$ on $L^2(\Gamma \setminus N)$ decomposes this space into a direct sum of $R$ invariant, closed subspaces: $L^2(\Gamma \setminus N) = \sum \mathbb{H}_m(\Gamma)$. Although each space $H_m(\Gamma)$ for $m \neq 0$ can be decomposed into the direct sum of $R$ invariant, irreducible, closed subspaces, in an infinite number of ways, there are a finite number of decompositions into "distinguished subspaces" originally discussed by L. Auslander and J. Brezin, that are in some way nicer than others.

In this paper, we discuss two ways in which these decompositions are particularly nice. In the first section, we prove that two lifted subgroups $\Gamma_1$ and $\Gamma_2$ of $N$ belong to the same rational form of $N$ if and only if $L^2(\Gamma_1 \setminus N)$ and $L^2(\Gamma_2 \setminus N)$ have a distinguished subspace in common. In the second section we give a necessary and sufficient condition for an $R$ invariant, closed, irreducible subspace of $L^2(\Gamma \setminus N)$ to be a distinguished subspace in terms of the zero sets of the "nil-theta" function that lies in that subspace. (Received October 5, 1977.)


Elliptic integrals of all three kinds (including complete and degenerate cases) are evaluated numerically by successive applications of the duplication theorem. When the convergence is improved by including a correction term obtained from series expansion, the error ultimately decreases by a factor of 256 in each cycle of iteration. No serious cancellations occur. Square roots are extracted in each cycle, but the computation can be done to arbitrary precision without transcendental operations. Circular and hyperbolic cases of the third integral do not need to be distinguished, and there is no separation of cases according to the values of the variables except the principal-value case of the third integral. (Received October 6, 1977.)

752-33-4 Allan J. Fryant, United States Naval Academy, Annapolis, Maryland 21402. Growth and complete sequences of generalized bi-axially symmetric potentials.

The growth of entire symmetric solutions of the equation $u_{xx} + u_{yy} + (2\beta + 1)x^{-1}u_x + (2\alpha + 1)y^{-1}u_y = 0$ is characterized in terms of coefficients in the expansion $u(x,y) = \sum_{n=0}^{\infty} \sqrt{n!}P_n(\alpha, \beta) \cos 2\beta$. Expression for order and type are given explicitly in terms of the coefficients $a_n$. Using the Braaksma-Muelenbeld Laplace type integral for Jacobi polynomials, application is made to generating complete sequences of solutions from a single entire solution $u$ by application of a sequence of dilations: $u_n(x,y) = u(\lambda_n x, \lambda_n y)$, $\lambda_n$ real, $n = 1, 2, \ldots$. Completeness is with respect to the uniform norm, and is obtained on a disk of any radius by appropriate choice of the sequence of reals $\{\lambda_n\}$. (Received October 11, 1977.)


**Ordinary Differential Equations**

**752-34-1** William C. Bakker, Spelman College, Atlanta, Georgia 30314 and Emory University, Atlanta, Georgia 30322. A method of iteration for solving ordinary differential equations with advanced and retarded arguments.

An iterative procedure is used to get Theorem: Suppose $B$ is a Banach space, $p > 2$ is an integer, and $g$ is a continuous function from $[1,p+1]$ to $B$. Suppose also that each of $a$ and $b$ is a Lipschitz function on $B$ with the property that $\|b(1+|b|)|a| < 1$ where $|a|$ and $|b|$ are the Lipschitz constants for $a$ and $b$ respectively. If $y$ is a bounded integrable function from $[0,1]U(p+1,p+2]$ to $B$ then there is an extension $z$ of $y$ which is continuous on $[1,p+1]$, differentiable on $(2,p)$ and satisfies

(1) $z'(t) + b(z(t-l)) + a(z(t+l)) = g(t)$

for each $t$ in $(2,p)$. Furthermore, if $y$ is continuous then $z$ is continuous on $[0,p+1]$, differentiable on $(1,p+1)$ and satisfies equation (1) for each $t$ in $(1,p+1)$. A corollary extends the theorem to problems with advanced and retarded argument $r > 0$ as long as $p$ is an integer multiple of $r$. (Received August 17, 1977.)

**752-34-2** Kurt Kreith, University of California, Davis, California 95616. Removable singularities for generalized Prüfer transformations.

The classical Prüfer transformation allows a natural generalization to selfadjoint 2n-th order differential equations of the form

$$
\sum_{k=0}^{n} (-1)^k \left( p_k(x) u^{(k)} \right)^{(k)} = 0
$$

by setting

$$
\sum_{j=k}^{n} (-1)^j \frac{p_j(x)^{(j-k)}}{1/r(t)dt} = r_k \cos \theta_k, k=1,2,...,n.
$$

For $n = 2$ such a transformation has been studied by Banks and Kurowski [Trans. Amer. Math. Soc. 199(1974), 203-222] subject to additional conditions which assure the nonvanishing of the corresponding Jacobian in the interval under consideration.

However, a simple sign convention makes these singularities removable and enables one to extend such generalized Prüfer transformations to a larger class of problems and equations. (Received September 2, 1977.)

**752-34-3** DON HINTON, University of Tennessee, Knoxville, Tennessee 37916. An expansion theorem for an eigenvalue problem with eigenvalue parameter in the boundary condition.

The boundary value problem considered is

$$
u u = r^{-1} \{-r u' + qu\} = \lambda u$$

with boundary conditions

$$
cos \alpha u(a) + \sin \alpha (pu')(a) = 0, \quad a \in [0,r), \quad -[p_1 u(b) - p_2 (pu')(b)] = \lambda [\beta_1 u(b) - \beta_2 (pu')(b)].
$$

The functions $p$, $q$, and $r$ are real continuous functions on $[a,b]$ with $r$ and $p$ positive. The real numbers $\beta_1$, $\beta_2$, $\beta_1'$, and $\beta_2'$ satisfy

$$
\beta_1^2 - \beta_1 \beta_2 < 0.
$$

Using an operator theoretic formulation, J. Walter [Math. Z. 133, 301-312] and C.T. Fulton [Proc. R.S.E., to appear] have associated a self-adjoint operator $A$ with this eigenvalue problem. For $f$ in a certain class $\mathcal{J}$, it is proved that $f$ can be expanded in a uniformly and absolutely convergent series of eigenfunctions of $A$.

If $A$ has a non-negative spectrum, the class $\mathcal{J}$ is the domain of $A^{1/2}$. (Received September 29, 1977.)

**752-34-4** Dr. Bhagat Singh, University of Wisconsin Center, 705 Viebahn St., Manitowoc, Wisconsin 54220. Forced Non-oscillations in second order functional equations.

The retarded differential equation

$$
(A) \quad \left[r(t)y'(t)\right]' + p(t)f(y(g(t))) = q(t)
$$

is studied for its oscillatory behaviour subject the regularity assumptions:

The main results state:

**Lemma (4.1)** In addition to assumed regularity conditions suppose $q(t) \geq 0$, $\int p^+(t)dt \leq \infty$ and $\int 1/r(t)dt \leq \infty$ then all oscillatory solutions of (A) are bounded above.
Theorem (4.1) Suppose conditions of Lennna (4.1) hold. Further suppose \( p \equiv 0 \) and \( \int q \, dt = \infty \); then all solutions of (A) are nonoscillatory.

**Remark** A case study of bounded \( r(t) \) is made in the last section. It is shown that for bounded \( r(t) \) the oscillatory solutions of (A) are precisely those which are "slowly oscillating."

(Received September 30, 1977.)

752-34-5 Harry Hochstadt, Polytechnic Institute of N.Y. 333 Jay St. Brooklyn, N.Y. 11201

An Inverse Problem for a Hill's Equation. Preliminary report.

Let \( q(x) \) be an \( L_1 \), periodic real function, so that (*) \( y'' + (\lambda - q(x))y = 0 \) is a Hill's equation. With such an equation one can associate an entire function \( \Delta(\lambda) \). The zeros of \( \Delta(\lambda)^{-2} \) represent the eigenvalues corresponding to which (*) has solutions satisfying \( y(1) = y(0), \ y'(1) = y'(0) \).

The first eigenvalue must be simple, but the remainder may be simple or double.

**Theorem:** All zeros of \( \Delta(\lambda)^{-2} \), except the first are double iff \( q(x) = \lambda x + I'(x) + I''(x) \), where \( I(x + \frac{t}{2}) = -I(x) \) and \( \lambda \) is the first eigenvalue. (Received October 7, 1977.)

752-34-6 John K. Shaw, Virginia Polytechnic Institute and State University, Blacksburg, Va. 24061.


Let the differential operator \( Ly = -(py')' + qy \) be defined on the interval \([0,b)\), regular there but singular at \( b \), \( 0 < b < \infty \). Suppose \( L \) has a positive spectrum, let \( \mu < 0 \), and define the class \( L_\mu^+ \) of \( L_\mu \)-positive functions by \( L_\mu^+ = \{ f \in C^m[0,b) : (L - \mu)^k f(t) \geq 0, k = 0, 1, \ldots \} \). Let \( X(t,\lambda) \) and \( \psi(t,\lambda) \) denote functions of \((L - \lambda)y = 0\) such that \( X(0,\lambda) = 1, \psi(0,\lambda) = 0 \) and \( \psi'(0,\lambda) = 1 \) for all \( \lambda \), and \( X(t,\lambda) \in C^2(0,b) \) for \( \lambda < 0 \). It is shown that each \( f \in L_\mu^+ \) admits a unique representation \( f(t) = \sum c_k P_k(t) \), where \( P_0(t) = X(t,\mu), P_1(t) = \psi(t,\mu) \), and \( (L - \mu)P_k = P_{k-2}, k = 2, 3, \ldots \). Explicit formulas for the \( c_k \) are given. Among the examples studied are the harmonic oscillator \( Ly = -y'' \) on \([0,\infty) \), and the Legendre operator \( Ly = -(1 - x^2)y'' \) on \([0,1) \). (Received October 11, 1977.)

*752-34-7 Po-Fang Hsieh, Western Michigan University, Kalamazoo, MI 49008. An existence theorem for nonlinear ordinary differential equations at an irregular type singularity.

Consider a system of equations \((1) x^s y' = f(x, v, w, y, z), xx' = g(x, v, w, y, z)\), where \( x \) is a complex variable, \( s \) is a positive integer, \( f \) and \( g \) are holomorphic in a neighborhood of the origin and vanish there, \( f_y(0,0,0,0,0) = a, (\text{Re } a > 0) \), and \( g(0,0,0,0,z) = z^{m+1}(b + cz^m) \), \( m: \text{positive integer, } b \neq 0 \). Furthermore, \( v \) and \( w \) satisfy the following respective equations: \((2) xv' = v^{m+1}(b + cv^m), (3) xx' = w^2(b + cvw). \) Let \( V(x) \) be a general solution of (2) which tends to \( 0 \) as \( x \to 0 \).

**Theorem.** If (1) possesses a formal solution \( y \sim \sum_{n=0}^\infty V(x)^n P_n(x,w), z \sim \sum_{n=0}^\infty V(x)^n q_n(x,w) \) with \( P_n \) and \( q_n \) holomorphic in certain sectors of \( x- \) and \( w- \) planes with power series asymptotic expansions. Then, these formal series converge uniformly in a neighborhood of \( V = 0 \) to an analytic solution of (1). (Received October 11, 1977.)

*752-34-8 William R. Derrick and Keith Yale, University of Montana, Missoula, Montana 59812. The functional differential equation \( \psi'(g) = m(g)\psi \) with \( q(g(x)) = x \). Closed form solutions to the FDE \( \psi'(g) = m(g)\psi \) where \( g(g(x)) = x, g'(x) < 0 \) are constructed for various classes of non-constant coefficients \( m \). The techniques
are elementary and involve transforming the FDE to an ordinary Riccati DE. The constant coefficient case was examined by R. G. Kuller in 1969. (Received October 11, 1977.)

*752-34-9 Allan C. Peterson, University of Nebraska, Lincoln, Nebraska 68588. Green's Functions for Focal Type Boundary Value Problems.

We are mainly concerned with the differential equations \( y^{(n)} = \lambda p(x)y \) where \( \lambda = \pm 1 \) and \( p(x) \) is a positive continuous function on \([a,b]\). Following Nehari, we say that a differential equation of this form is disfocal on \([a,b]\) provided there is no nontrivial solution \( y(x) \) such that each of \( y^{(i)}(x) \) vanishes at least once in \([a,b]\).

Our main result is that if a differential equation of this form is disfocal and \( k \in \{1, \ldots, n-1\} \) then the Green's function \( G_k(x,s) \) for the focal boundary value problem \( y^{(n)} - \lambda p(x)y = h(x), y^{(i)}(a) = 0, i = 0, \ldots, k-1, y^{(j)}(b) = 0, j = k, \ldots, n-1, \)
\( h \in C[a,b], \) satisfies \((-1)^{n-k}G_k^{(i)}(x,s) > 0 \) on \((a,b) \times (a,b) \) for \( i = 0, \ldots, k-1. \)

(Received October 11, 1977.)

752-34-10 JEROME J. PRZYZYLSKI, Western Michigan University, Kalamazoo, Michigan 49008. A bounded solution of \( x^{s+2}y' = \mathcal{P}(x,y,z)xz' = \mathcal{G}(x,y,z) \) when \( g(0,0,0) = 0. \) Preliminary report.

A function \( f(x,y) \) has Property-U with respect to \( y \) in \( ||y|| < b, A < \arg x < B, 0 < |x| < a, \) if in the domain it admits a uniformly convergent expansion in powers of \( y \) whose coefficients have asymptotic expansions in powers of \( x \) as \( x \to 0 \) through the sector. Consider a system of \( m+n \) nonlinear differential equations \( x^{s+1}y' = \mathcal{F}(x,y,z), xz' = \mathcal{G}(x,y,z), s \) a positive integer, where \( \mathcal{F}(0,0,0) = 0, \mathcal{G}(0,0,0) = 0, \mathcal{F}_y(0,0,0) = \text{diag}(\mu_1, \ldots, \mu_m), \text{Re} \mu_i > 0, \mathcal{G}(0,0,0) = 0. \) Further suppose that (i) \( \mathcal{G}(0,y,z) = z_n^q(g(z))z \) where \( q \) is an \( n \) row vector of non-negative integers, not all zero; and (ii) \( g(0) = t \) where all components of \( t/q \) have positive real parts. Then the system has a solution of the form \( y = j(x;v,w) + x^{s+1}J(x,v,w), z = k(x;v,w) + x^{s+1}K(x,v,w) \) where \( J \) and \( K \) are polynomials of degree \( s \) in \( x \) with coefficients having Property-U with respect to \( v \) in \( ||v|| < b, C < \arg w < D, 0 < |w| < a, \) and \( J \) and \( K \) have Property-U with respect to \( v \) in \( ||v|| < b, C < \arg w < D, 0 < |w| < a, A < |x| < B, 0 < |x| < a. \) Here \( V(x) \) is a solution of \( xz' = z_n^q(h(z))z \) for a suitable polynomial \( h(z), h(0) = t, \) and \( W(x) \) is determined by \( 2\omega dx/dw = x/qh(v), wdv/dw = 1_n(h(v)/qh(v))v. \) (Received October 11, 1977.)

*752-34-11 T. K. PUTTASWAMY, Ball State University, Muncie, Indiana 47304. Solution in the large of a certain linear homogeneous differential equation of order \( n. \)

In order to introduce the investigations of this paper, let us consider the linear homogeneous differential equation of order \( n \)
\[
2 \sum_{j=0}^{n-1} \left( \begin{array}{c} n \\ j \end{array} \right) a_j + b_j z + c_j z^2 \left( \begin{array}{c} n \\ j \end{array} \right) \frac{d^2 y}{dz^2} = 0
\]
(1)

Here, the variable \( z \) is regarded complex as likewise the constants \( a_i, b_i, c_i \)
\((i=0, 1, 2, \ldots, n-1)\) with \( i \neq 0 \) and \( a_{n-1} + b_{n-1} z + c_{n-1} z^2 = 0. \) Then (1) will have three regular singular points, namely \( z=0, z=\mu \) and \( z=\infty. \) The indicial equation about \( z=0 \) is found to be
\[
a_0 + \sum_{j=0}^{n-1} a_j \sum_{j=0}^{n-1} b_j (h-j) = 0
\]
(2)

It is also assumed that no two roots of (2) differ by an integer. (Received October 13, 1977.)

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Various existence and uniqueness theorems for the solutions of the following vector boundary value problem are established: 
\[ x'' + Ax = f(t, x, x'), \quad x(0) - x(\omega) = x_1(0) - x_1(\omega) = 0, \]
where \( x = (x_1, \ldots, x_n) \) is an n-dimensional vector; \( A \) is a constant diagonal \( nxn \) matrix; and 
\[ f(t, x, y) = (f_1(t, x_1, \ldots, x_n, y_1, \ldots, y_n), \ldots, f_n(t, x_1, \ldots, x_n, y_1, \ldots, y_n)) \]
is a vector valued function, defined for \( (t, x_1, y_1, \ldots, y_n) \in [0, \omega] \times \mathbb{R}^n \times \mathbb{R}^n \).

We consider the equations (1) \( (a_1(t)x')' + q_1(t)f_1(x) = r(t, x, x') \) and (2) \( (a_2(t)y')' + q_2(t)f_2(y) = 0 \)
where \( a_1, q_1, f_1, a_2, q_2, f_2, r \) are continuous and \( a_i(t) > 0 \), \( i = 1, 2 \). We further assume that
\[ a_1(t) \geq a_2(t), \quad u f_1(u) \geq 0, \quad f_2'(u) > 0, \text{ and there exists } k > 0 \text{ such that} \]
\[ 0 \leq f_1'(u)/f_2'(v) \leq k \quad \text{and} \quad q_2(t) \geq k q_1(t). \]

**Theorem 1.** Suppose that \( q_1(t) \geq 0 \) and either \( r(t, u, u') \geq 0 \) or \( r(t, u, u') < 0 \). If equation (2) has a nonoscillatory solution, then all solutions of (1) are nonoscillatory.

**Theorem 2.** If \( ur(t, u, u') \geq 0 \) and (2) has a nonoscillatory solution, then all solutions of (1) are nonoscillatory. Separation theorems are obtained under slightly less restrictive hypotheses. (Received October 14, 1977.)

A nonlinear Volterra integrodifferential equation arising from the theory of population dynamics is shown to have a nonconstant periodic solution. The equation is treated as an autonomous functional differential equation with infinite delay. Existence of the periodic solution is proved by a fixed point argument. (Received October 14, 1977.)

Some of the better known criteria for the oscillation of the second order linear differential equation
\[ y'' + p(t)y = 0 \]
are the Fite-Wintner condition \( \int_0^\infty p(s)ds = \infty \), Wintner's condition \( \lim_{t \to \infty} t^{-1}\int_0^t p(r)drds = \infty \), and Hartman's condition \( -\infty < \lim_{t \to \infty} t^{-1}\int_0^t p(r)drds < \lim_{t \to \infty} t^{-1}\int_0^t p(r)drds < \infty \). Waltman showed that the Fite-Wintner condition is also valid for the oscillation of all (extendable) solutions of \( y'' + p(t)y^{2n+1} = 0 \).

We establish that the Wintner and Hartman conditions are also valid for the equation \( y'' + p(t)f(y) = 0 \) for a large class of functions \( f \) including \( f(y) = |y|^\alpha \text{sgn } y, \alpha > 0 \), thereby answering a question of Wong in the case \( \alpha > 1 \). (Received October 14, 1977.)

Let \( A, B, \) and \( C \) be \( n \times n \) matrices whose entries are (i) complex valued functions on a real \( t \)-interval \( J = (a, \infty) \), and (ii) Lebesgue integrable on compact subintervals of \( J \). Let \( B(t) \) and \( C(t) \) be hermitian on \( J \) (i.e., \( B*(t) = B(t) \) and \( C(t) = C*(t) \)). Then the system
\[ X' = A(t)X + B(t)Y, \quad Y' = -C(t)X - A*(t)Y \]
is called a Hamiltonian system.

**Theorem.** Let \( B(t) \) and \( C(t) \) be positive semidefinite on \( J \). Then system \((H)\) is nonoscillatory (at \( \infty \)) if and only if there exists a \( 2n \times n \) self-conjugate solution matrix \((X^*, Y^*)^*\) (one where \( X^*(t)Y(t) \) is identically hermitian on \( J \)) such that both \( X(t) \) and \( Y(t) \) are nonsingular on some terminal subinterval \((b, \infty)\) of \( J \). (Received October 17, 1977.)
By employing the variation of parameters method, a sufficient condition is given for the oscillatory behavior of first order linear functional integro-differential equations. Furthermore, the non-oscillatory behavior of such equations is also investigated. (Received October 17, 1977.)

The existence of a solution in $H^1_0(I)$ ($I = (0,1)$) to the nonlinear boundary value problem $(h[t,x]|x|')' + f[t,x] = 0$ with $x(0) = x(1) = 0$ is developed. The existence proof is based primarily on the following assumption: For each positive integer $n$, there exist numbers $\lambda_n > 0$, $M_n > 0$, $K_n$ such that $\lambda_n \leq |h[t,x]| \leq M_n$ and $x \cdot f[t,x] \leq -\beta_n x^2 + K_n$ whenever $|x| \leq n$, and there exists a positive integer $N$ such that either 1) $K_N \leq \beta_n N^2$ if $\beta_n > 0$ for all $n$, or 2) $K_N \leq (\lambda_n \pi^2 + \beta_n N^2)$ if $-\pi^2 \lambda_n < \beta_n < 0$ for all $n$. Examples are included to show that uniqueness fails under the above assumption. (Received October 17, 1977.)

Let $D$ be an $n \times n$ diagonal matrix with nonnegative integers on the diagonal, $B(z)$ an $n \times n$ matrix of functions analytic at $z = 0$, and $y(z)$ an $n$-vector function. The classical theorem of Lettenmeyer states that the system of differential equations in the complex plane given by $z^D y'(z) + B(z)y(z) = 0$ has at least $n - \text{tr}(D)$ solutions analytic at $z = 0$. In this report we treat two cases for which Lettenmeyer's theorem is inappropriate, viz. when $D = I$ and when $D = 2I$. By working in a Banach space setting and using the alternative problem technique developed by L. Cesari, J. K. Hale, and others, we were able to find a procedure which yields the exact number of solutions analytic at $z = 0$ in each case. (Received October 17, 1977.)

An iterative method is given for determining solvability of and for constructing solutions to linear operator equations involving sums and compositions of closed operators. (Received October 17, 1977.)

Existence and asymptotic behavior of solutions for the equation $u' = -A(t)u(t) + F(t,u(t))$ for $t > 0$, $u_0 \in X$ is given. $X$ is a Banach space; the family of unbounded linear operators $\{A(t)\}$ generates an evolution system of linear operators, and $F$ is continuous with respect to a fractional power of $A(t_0)$ for some $t_0 \in [0,T]$. This problem has been considered by C. Travis and G. Webb when $A(t_0) = A$ and $-A$ generates an analytic semigroup of linear operators. (Received October 17, 1977.)

Consider the differential equation (*) $x'' + \nabla G(x) = p(t,x)$. Here $p: \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous, uniformly bounded, $2\pi$-periodic in $t$ for each $x \in \mathbb{R}^n$, and $G \in \mathcal{C}^2(\mathbb{R}^n, \mathbb{R})$. Theorem: Suppose there
exist an integer \( n \) and positive numbers \( p, q, \) and \( r \) such that whenever \( a \in \mathbb{R}^n \) and \( |a| \geq r \) the inequality (**) \( n^2 I < pI \leq (a_i^2 G(a)/a_i^2 x_i^2) \leq qI < (n+1)^2 I \) holds. Then there is at least one \( 2n \)-periodic solution of (***). This result improves a result of A.C. Lazer and D.A. Sánchez ["On periodically perturbed conservative systems", Michigan Math. J. 16 (1969), 193-200]. (Received October 17, 1977.)

#752-34-23 Peter W. Bates, Pan American University, Edinburg, Texas 78539. Hilbert space methods for nonlinear systems of differential equations.

Let \( H_1 \subseteq H \) be Hilbert spaces and let \( \text{dom} L = H_1 \longrightarrow H \) be a linear selfadjoint operator with spectrum \( s \) and compact resolvent \((L - cI)^{-1}\) for \( c \) not in \( s \). Suppose that \( h \) maps \( H_1 \) continuously into the symmetric bounded linear operators on \( H \). Further, suppose that there exist real numbers \( q \leq p \) such that \( qI \leq h(w) \leq pI \) for all \( w \) in \( H_1 \).

Theorem: If \( L \) and \( h \) satisfy the nonresonance condition: \([q,p] \cap s = \emptyset\) and if \( f: H \longrightarrow H \) is continuous, mapping bounded sets into bounded sets and satisfies the condition: (F) For some \( R > 0 \) we have \( |f(w)| < R \) dist \( ([q,p],s) \) for \( w \) in \( H_1 \), \( |w|_1 = R \). Then the equation \( Lu - h(u)u = f(u) \) has at least one solution \( u \) in \( \text{dom} L \) with \( |u|_1 < R \).

Remark: The condition (F) is satisfied if \( f \) exhibits either sublinear growth outside, or superlinear growth inside some ball in \( H_1 \). The theorem is applied to obtain existence of solutions to systems of O.D.E.'s and systems of elliptic P.D.E.'s. (Received October 17, 1977.)


Let \( \mathcal{U} \subseteq \mathbb{R}^n \) be closed and convex, \( f: [0,\infty) \times \mathcal{D} \times \mathbb{R}^n \rightarrow \mathbb{R}^n \) be continuous, and consider the second order differential equation: (***) \( x''(t) = f(t, x(t), x'(t)) \) with initial conditions \( x(0) = x_0 \in \mathcal{D} \) and \( x'(0) = x'_0 \in \mathbb{R}^n \). We give criteria ensuring existence of solution \( x \) to (**) with \( x(t) \in \mathcal{D} \) for \( t \geq 0 \). Existence is established via approximate solutions to (**). We also give conditions for global existence and apply these results to the periodic problem (**) with \( x(0) = x(1) \). (Received October 17, 1977.)


A generalized comparison principle for second order differential inequalities is established in a Banach space where the inequalities are given relative to an arbitrary cone. We use the comparison result to develop a monotone method that generates two sided bounds on the solutions of the nonlinear boundary value problem (**) \( u'' = f(t, u, u'), \quad a_i u(t) + b_i u(t) = c_i, \quad i = 0, 1 \). Here \( f \in C \left((0,1] \times \mathbb{B} \times \mathbb{B}, \mathbb{B}\right) \), \( \mathbb{B} \) any Banach space, and \( a_i > 0, \quad b_i > 0, \quad i = 0, 1 \). With given upper and lower solutions of (**) we generate monotone sequences of functions which converge to the maximal and minimal solutions of (**) within the segment bounded by the upper and lower solutions. (Received October 17, 1977.)

#752-34-26 Thomas C. Gard, University of Georgia, Athens, Georgia 30602, and Thomas G. Hallan, University of Tennessee, Knoxville, Tennessee 37916. Persistence of positive solutions of differential equations. Preliminary report.

Let \( \varphi(t,x_0) \) be the solution of the initial value problem (**) \( x' = f(x), \quad x(0) = x_0 \in \text{int} \mathbb{R}^n \), where \( f: \mathbb{R}^n = \{x \in \mathbb{R}^n: x_i > 0, \quad 0 < i < n\} \rightarrow \mathbb{R}^n \). \( \varphi = \{\varphi_i\} \) is persistent if for each \( i, 1 \leq i \leq n \) and each \( \tau, 0 < \tau < =, \prod_{i=1}^n \varphi_i(t,x_0) > 0 \). Let \( \rho(x) = \prod_{i=1}^n \rho_i(x_i) \), where for
each \( i, \rho_i : R_+ \to R_+ \) is \( C^t \) on \( \text{int } R_+ \) and \( \rho_i(u) \to 0 \) if and only if \( u \to 0 \).

**Theorem.** Assume either \( \rho_i \) is bounded, each \( i \), or that solutions of (*) that fail to persist are bounded. If \( \sum_{i=1}^n (\ln \rho_i(x_j))'f_1(x) \geq 0 \) on a set \( S = \bigcup_{j=1}^n \{ x : \forall x_j \leq \varepsilon \} \), some \( \varepsilon > 0 \), then all solutions of (*) persist. If \( \sum_{i=1}^n (\ln \rho_i(x_j))'f_1(x) \leq -\alpha \), some \( \alpha > 0 \) on \( S \), any solution of (*) eventually in \( S \) fails to persist. The location of recurrent solutions in the boundary of \( R_+^n \) is used with this result to obtain more general persistence criteria. Applications have been made to Lotka-Volterra food chain models. (Received October 17, 1977.)

*752-34-27 G. J. ETGEN, University of Houston, Houston, Texas 77004 and R. T. Lewis, University of Alabama, Birmingham, Alabama 35294. Hille-Wintner type oscillation criteria. Let \( X \) be a Hilbert space, \( A = Q(K,K) \) the \( B^*\)-algebra of bounded linear operators from \( K \) to \( K \) with the standard operator topology, and \( S \) the subset of selfadjoint elements of \( A \). Let \( \phi \) be the set of positive linear functionals on \( A \). Consider the differential equation (1) \( Y'' + Q(x)Y = 0 \) on \( R_+ = [0, \infty) \), where \( Q : R_+ \to S \) is continuous. Let \( Y \) be a nontrivial, conjoined solution of (1). Then: (i) \( Y \) is nonsingular at \( x = a \), \( a \in R_+ \), if the range of \( Y(a) \) is \( K \) and \( Y(a) \) has a bounded inverse, otherwise \( Y \) is singular at \( x = a \); and (ii) \( Y \) is oscillatory if for each \( a \in R_+ \) there exists \( b \geq a \) such that \( Y(b) \) is singular. Equation (1) is oscillatory if all nontrivial, conjoined solutions are oscillatory. Oscillation criteria for (1) in the case where \( X = R_n \), Euclidean \( n \)-space, and \( A \) is the \( B^*\)-algebra of \( n \times n \) matrices, have been developed by a wide variety of authors. Almost all such criteria involve assumptions of the form \( \int_0^\infty Q(t)dt = \sigma \). In this paper we develop so-called Hille-Wintner oscillation criteria for (1) in which it is assumed that \( \int_0^\infty Q(t)dt < \sigma \). For example, **Theorem:** If there is a \( g \in \phi \) such that \( g(I) = 1 \), \( \int_0^\infty g(Q(t))dt \) converges (possibly only conditionally), and \( \lim \inf \int_x^\infty g(Q(t))dt > 1/4 \), then (1) is oscillatory. Extensions to equations of the form \([P(x)Y']' + Q(x)Y = 0\) are also considered. (Received October 17, 1977.)

*ATHANASSILOS G. KARTSATOS, University of South Florida, Tampa, Florida 33620. The forced equation oscillates if and only if the unforced equation oscillates. Order \( n \)-even. Consider the equation: (1) \( x(n) + H(t,x) = Q(t), n \geq 2 \), where \( H \) is increasing in its second variable and such that \( \forall u \neq 0 \), \( H(t,u) > 0 \). **Theorem:** Assume the existence of a function \( S(t), t \in [0, \infty) \) such that \( S(0, \infty) = Q(t), t \in [0, \infty) \) and \( S(t) \) is oscillatory with \( \lim S(t) = 0 \) as \( t \to \infty \). Then (1) is oscillatory if and only if the unforced (1) is oscillatory. The sufficiency part of this result was shown by the author in [J. Math. Anal. Appl., 52 (1975), 1 - 9]. (Received October 17, 1977.) (Author introduced by Professor M. N. Manougian).

*V. LAKSHMIKANTHAM, University of Texas at Arlington, Arlington, Texas 76019 and S. LEELA, SUNY College at Genesco, Genesco, New York 14454. A technique in stability theory of delay differential equations. A new comparison theorem which simultaneously yields upper and lower bounds is proved. It is then applied to obtain stability criteria of delay differential equations. It is shown that this method gives sharper results compared to the usual Lyapunov method when applied to certain reactor equations. (Received October 17, 1977.)

*ROGER C. MCCANN, Mississippi State University, Mississippi State, Mississippi 39762. On Poisson stability. Let a point \( x \) be an element of its own limit set and be neither critical nor periodic. Then the closure of the trajectory through \( x \) contains uncountably many trajectories. (Received October 17, 1977.)
A boundary value problem of the form \( x''' + 2xx'' + 2B(1-x'^2) = 0 \)
where \( x(a) = x'(a) = 0 \), \( x'(b) = 1 \), is known
to arise in the study of solutions to the boundary layer equations. Although
various conclusions were drawn by many authors. We establish the existence of
solutions to the above problem by using a slightly different approach which is
applicable to either a finite or an infinite interval. Therefore the consideration
are given to a larger class of equations. Moreover, the true solution obtained
lies between the constructed upper and lower solutions. (Received October 18, 1977.)

We study the angular variation of solutions of the second order linear system
\( x'' + P(t)x = 0, \ x(a) = x(b) = 0 \) (la,b)
where \( P(t) \) is a real, positive definite, symmetric, \( n \times n \) matrix. We assume \( (la) \) is disconjugat-
on the interval \( (a,b) \) and \( (la,b) \) has a nontrivial solution. By the angular variation of a
nontrivial solution, \( x(t) \), of \( (la,b) \) we mean \( \frac{\int_a^b \|x'(t)\| dt}{\|x(a)\|} \) where \( \theta(t) = x(t)/\|x(t)\| \). Let
\[ \gamma(t) = \begin{cases} (b-a)^{-1}(a+b-2t)(b-t), & \text{if } a \leq t \leq \frac{1}{2}(a+b) \\ (b-a)^{-1}(2t-(a+b))(t-a), & \text{if } \frac{1}{2}(a+b) \leq t \leq b \end{cases} \]
Let \( \lambda_1(t) \geq \lambda_2(t) \geq \ldots \geq \lambda_n(t) \) be the eigenvalues of \( P(t) \).

Theorem 1. The angular variation of every solution of \( (la,b) \) is less than
\[ \frac{1}{2} \int_a^b \gamma(t)(\lambda_1(t) - \lambda_n(t)) dt. \]

Theorem 2. The angular variation of every solution of \( (la,b) \) is less than
\[ [(b-a)^{-1} \int_a^b \lambda_1(t) dt - 4]^1/2. \] (Received October 18, 1977.)
where \( B_1^-(\cdot), B_2^-(\cdot) \) and \( B_1^+(\cdot), B_2^+(\cdot) \) are linearly independent boundary values for \( \gamma \) at \( a \) and \( c \) respectively (cf. Dunford and Schwartz, Linear Operators, II, p. 1238), and establish the associated eigenfunction expansion theory. This work represents an extension of the author's previous paper, 'Two-point boundary value problems with eigenvalue parameter contained in the boundary conditions' (Proc. R.S.E., to appear), in which the regular problem on a finite closed interval was considered. As in the regular case, \( (\cdot) \) admits interpretation as a Self-adjoint operator in the direct sum Hilbert space, \( H = L_2^2((a,c);k|x|;\mathcal{H}, \mathcal{E}) \), where \( \mathcal{E} = \text{complex numbers} \).

(Received October 18, 1977.)

**752-35-3** LARRY KURTZ and DAVID WEINMAN, Hollins College, Virginia 24020. **Eigenvalue Estimates for a Sturm-Liouville Problem.**

This paper presents a method for computing eigenvalue estimates of the Sturm-Liouville problem

\[-y'' + q(x)y = \lambda y, \quad y(0) = 0 = y(1),\]

where \( q(x) \) has a bounded third derivative on \([0,1]\). The estimates are obtained by making quadratic approximations over \( n \) subintervals of \([0,1]\), of length \( h \). The error is \( O(h^3) \). (Received October 18, 1977.)

**35 ▶ Partial Differential Equations**

**752-35-1** Michael E. Taylor, Rice University, Houston, Texas 77001. **Propagation, reflection, and diffraction of singularities of solutions to wave equations.**

The classical rules of geometrical optics have been justified in applications to mixed initial boundary value problems for hyperbolic equations, granted convexity of the boundary with respect to the bicharacteristic flow. The analysis reveals a corrected form of the Kirchoff approximation, which enables one to give rigorous solutions to many classical problems of scattering theory. These results extend the results of Hörmander on propagation of singularities and use a generalized type of Fourier integral operators. (Received August 18, 1977.)

**752-35-2** R. E. Showalter, The University of Texas, Austin, Texas, 78712. **Quasi-Reversibility of Degenerate Parabolic Evolution Equations.**

Consider the ill-posed final-value problem

\[
\frac{d}{dt} \mathcal{M}u(t) + \mathcal{L}u(t) = 0, \quad 0 < t < T, \quad \mathcal{M}u(T) = \mathcal{M}\xi
\]

where the continuous linear and coercive \( \mathcal{L} : V \rightarrow V' \) is \( \pi/4 \)-sectorial on the complex Hilbert space \( V \) and the continuous linear \( \mathcal{M} : V \rightarrow V' \) is symmetric and non-negative. We discuss uniqueness and a constructive existence result for this problem and present applications to parabolic partial differential equations whose coefficients may contain singularities or may degenerate. (Received August 22, 1977.)

**752-35-3** MONTY J. STRAUSS, Texas Tech University, Lubbock, Texas 79409. **Uniqueness and norm convexity for products of solvable operators. Preliminary report.**

Norm convexity estimates are obtained for the forward and backward Cauchy problems for partial and pseudo differential operators that are products of first order operators for which this is possible. Uniqueness results then follow.

(Received September 29, 1977.)
Let the real valued GBASP $F^{(a,\beta)}$ be regular in the open unit hypersphere $E(a,\beta)$ about the origin and continuous on $\overline{E}(a,\beta)$. Let the minimum error in the approximation of $F^{(a,\beta)}$ over $H(a,\beta)_n$ the set of all real biaxisymmetric harmonic polynomials of degree at most $2n$, be defined by

$$e_n(F^{(a,\beta)}) = \inf \{ \sup_{(x,y) \in E(a,\beta)} |F^{(a,\beta)}(x,y) - P^{(a,\beta)}_n(x,y)| : x^2 + y^2 = 1 \}; F^{(a,\beta)} \in H(a,\beta)_n,$$

$n = 0, 1, 2, \ldots$. Then we shall prove that $F^{(a,\beta)}$ continues analytically as an entire function GBASP of index $k$, order $\rho(k)$ and type $\gamma(k)$ or of logarithmic order and type are specified as limits of functions of the form $e_n$. That is, certain local approximation properties of solutions to the GBASP equation,

$$(\partial^2/\partial x^2 + \partial^2/\partial y^2 + (2a+1)/x \partial/\partial x + (2\beta+1)/y \partial/\partial y)F^{(a,\beta)} = 0,$$

characterize global existence and growth. These results are established via the Bergman and Gilbert Integral Operator Method. (Received September 29, 1977.)

**John F. Schmeelk, Virginia Commonwealth University, Richmond, Virginia 23284. A uniqueness theorem for a generalized initial value problem.**

The author previously derived solutions to an initial value problem. (Siam Review Vol. 19 #2 April 1977). Therein the author discussed the set of test surfuctions which were equipped with appropriate norms and were then shown to form a scale of Fréchet Spaces. The initial value functions for this initial value problem were members of the space of test surfuctions. We then established an existence theorem which provided us with a solution to the problem. In this recent development the author will discuss the uniqueness of the solution. The proof of the uniqueness will employ techniques used in the proof of the Ovchinnikov Theorem. (Received October 5, 1977.)


A technique is exhibited whereby a large class of symmetric hyperbolic systems of partial differential equations can be factored unitarily into a sum of wave equations via a potential decomposition.

This class of equations contains many of the common first order symmetric hyperbolic systems which arise in classical physics. (Received October 11, 1977.)


Let $\Omega$ be a bounded region in $\mathbb{R}^n$. We study the problem

$$(I) \quad \Delta u(x) + g(x, u(x)) + f(x, \nabla u(x)) = 0 \quad \forall x \in \Omega, \quad u(x) = 0 \quad \forall x \in \partial \Omega. \quad \text{We assume that } \frac{\partial g}{\partial u} (x, u) \text{ exists and is bounded away from the eigenvalues of } \Delta u(x) + \lambda u(x) = 0 \quad \forall x \in \Omega, \quad u(x) = 0 \quad \forall x \in \partial \Omega, \text{ moreover we assume that } f \text{ is sublinear. By relating the growth of } g \text{ and the growth of } f \text{ we obtain an existence theorem for the problem (I). The proof uses a combination of a maxmin principle with the Schauder fixed point theorem.} (Received October 11, 1977.)
We consider the Cauchy problem \( \frac{\partial u}{\partial t} + \sum_{k=1}^{n} a_k(x,t) u + b(x,t) u = f(x,t) \) with initial condition \( u(\cdot, 0) = \varphi(\cdot) \). We require \( a_k, b \in C(\mathbb{R}^n \times [0, T]) \) for \( k = 1, \ldots, n \), \( \frac{\partial a_k}{\partial x_k}, \frac{\partial b}{\partial x_m} \in L^\infty(\mathbb{R}^n \times [0, T]) \) for \( m, k = 1, \ldots, n \) and the map \( t \mapsto f(\cdot, t) \) to be continuous from \( [0, T] \) to \( H^2(\mathbb{R}^n) \). Given these conditions we prove the existence of a unique solution \( u \) such that \( t \mapsto u(\cdot, t) \) is continuous in \( H^1(\mathbb{R}^n) \) and strongly continuously differentiable in \( L^2(\mathbb{R}^n) \). (Received October 13, 1977.)

**752-35-9**


We develop a sharp maximum principle for elliptic operators of the form studied by G. Stampacchia in "Equations elliptiques du second ordre a coefficients discontinus", Montreal Press, 1966, whose solutions are continuous. We give two applications. First, we show uniqueness to the third boundary value problem. Second, we construct via monotone methods a solution to the semilinear problem with the third boundary value problem's data on the boundary. (Received October 17, 1977.)

**752-35-10**


A boundary-integral representation is obtained for nonnegative solutions of the parabolic problem

\[
\sum_{i,j=1}^{n} a_{ij}(x,t) \frac{\partial u}{\partial x_i} - \frac{\partial u}{\partial t} = 0
\]

in a finite cylinder \( Q = \mathbb{R}^n \times (0, T) \). Boundary trace methods are employed. (Received October 17, 1977.)

**752-35-11**

RICHARD E. EWING, The Ohio State University, Columbus, Ohio 43210. The Cauchy problem for a linear parabolic partial differential equation. Preliminary report.

The Cauchy problem for the linear parabolic partial differential equation (p(x)U)_x - q(x)U = \rho(x) U_t, \quad 0 < x < 1, \quad 0 < t \leq T, \quad U(0,t) = f(t), \quad 0 < t \leq T, \quad U(1,t) = g(t), \quad 0 < t \leq T, \quad p(0)U_x(0,t) = h(t), \quad 0 < 0 \leq T, \quad, \quad is ill-posed in the sense of Hadamard. Complex variable techniques are used to establish Holder continuous dependence upon the data under the additional assumption of a uniform bound for \( |U(x,0)| \), the initial temperature distribution in the physical heat flow problem. Numerical results are obtained for the problem where the data \( f, g, \) and \( h \) are known only approximately. Applications of the techniques to some other ill-posed problems are also mentioned. (Received October 17, 1977.)

**752-35-12**


Let \( \tilde{r}_j \) and \( \tilde{r}_j \) each satisfy its own system of nonlinear PDE's and assume that for each \( j, \quad j = 1, \ldots, n, \)

1. \( \tilde{r}_j \geq \tilde{r}_j \) in a neighborhood of \( p \in \mathbb{R}^n \) and

2. \( \tilde{r}_j(\tilde{p}) = \tilde{r}_j(p) \).

If the differences \( u_j = \tilde{r}_j - \tilde{r}_j \) satisfy a system of the form

\[
L^k_k + \sum_{j=1}^{n} \sum_{k=1}^{n} b_{jk} u_j = 0
\]

where the \( L^k_k \) are linear second order elliptic operators, then \( u_j = 0 \), or \( \tilde{r}_j = \tilde{r}_j \), \( j = 1, \ldots, n \).

In particular, the second order quasi-linear systems defining a nonparametric m-dimensional
minimal submanifold of $\mathbb{R}^{n+m}$ are of this type. The maximum principle provides a new characterization of the way minimal submanifolds may touch.

The proof uses an expansion theorem of Bers and a unique continuation theorem of Aronszajn.

(Received October 18, 1977.)

*752-35-13 J. R. CANNON, The University of Texas at Austin, Texas, 78712. A class of inverse problems.

A survey of results and open questions regarding the determination of coefficients in parabolic and elliptic equations from overspecified boundary and/or initial data will be presented.

(Received October 14, 1977.)


Let $\Omega$ be a bounded open set in $\mathbb{R}^n$ and $u$ be a real valued function contained in the Sobolev space $W^{1,p}(\Omega)$, for some $p > 1$. Suppose that $u$ satisfies the equation $\Delta (a_{ij}u) = 0$ in the sense of distributions in $\Omega$. The real valued functions $a_{ij}$ satisfy the ellipticity condition, $a_{ij} \xi_i \xi_j \geq \lambda \xi^2$ for all $\xi \in \mathbb{R}^n$ and for all $x \in \Omega$. We also assume that

$$
\frac{1}{n} \int_{\Omega} \left| a(x + t) - a(x) \right|^p \, dx \leq n \|t\|^p,
$$

for all $t \in \mathbb{R}^n$ for some $\alpha, p$ satisfying $mp > n, \alpha < 1$. (Here we have set $a_{ij} \equiv 0$ outside $\Omega$.) We prove that $u_x$, the gradient of $u$, is H"older continuous in $\Omega$ with any exponent $\beta < \alpha$. Note that the coefficients $a_{ij}$ are, by our assumptions, H"older continuous with exponent $\alpha_0 = \alpha - \frac{n}{p}$ and therefore by well known results $u_x$ is H"older continuous with exponent $\alpha_0$. The methods apply to higher order equations and systems. (Received October 17, 1977.)

*752-35-15 JEROME A. GOLDSTEIN, Tulane University, New Orleans, Louisiana 70118. The Exact Amount of Nonuniqueness for the Euler-Poisson-Darboux Equation.

Let $A$ be a self-adjoint operator on a Hilbert space $H$. (E.g. $A^2 = -d^2/dx^2$ in $L^2(-\infty,\infty)$.) Consider the E-P-D problem (*) $\frac{d^2 v}{dt^2} + (\rho(t) \frac{dv}{dt} + A^2 v = 0 \quad (t > 0), \quad v^{(k)}(0) = v_k, \quad k = 0, 1, \ldots, n+1 (n \geq 0)$. The EPD equation is singular when $\rho < 0$, and $\rho$ is a measure of the amount of singularity. Theorem 1: If $\rho > -n$, (*) has at most one solution. Theorem 2: (*) has infinitely many linearly independent solutions with $v_k = 0 \quad (0 \leq k \leq n+1)$ if $\rho$ is not an integer and $\rho < -n$. Theorem 1 is based on a new (sharp) uniqueness theorem for nonlinear ODEs in Banach spaces. (Received October 14, 1977.)

*752-35-16 AMY C. MURRAY, Rutgers University, New Brunswick, New Jersey 08803. Development of momentary singularities in solutions of the Korteweg-deVries equation.

The regularity of solutions of the Korteweg-deVries equation (KdV) evolving from initial data in $C^{K}$ is controlled by the decay rate of the data. For $n \geq 0$, there are solutions $u$ of KdV such that $u$ is $C^\infty$ both before and after time $T$, but at time $T$, $u$ is $C^0$ and not $C^{n+1}$. Indeed a solution may be $C^\infty$ until time $T$, be a step function at time $T$, and then smooth out again. If such a solution is considered as a family of potentials in the Schrödinger equation (s), $-\psi'' + a k^2 \psi$, parametrized by $t$, then the scattering data evolve in a simple way. In appropriate topologies, this shows discontinuous dependence of the potential on scattering data. The explanation of such "blow-up in finite time" for KdV involves the relation between the decay and regularity properties of the reflection coefficient and of the potential in (s). (Received October 14, 1977.)

We consider the snow-plough model for cylindrical plasma collapse in the case of a specified constant driving term. This is a coupled nonlinear system consisting of five partial differential equations in two independent variables, one of which is the time. The initial value problem for similar systems is improperly posed in general. We show that this is not the case here by direct construction of the unique solution, explicitly in terms of the initial data. The solution exists for all positive times and is generally an infinitely differentiable function of the independent variables. Nevertheless, the solution always develops a non-physical singularity after a certain positive time, and thereafter ceases to be relevant to the underlying physical problem. Our theory leads to an a-priori bound, in terms of the initial data, for the time interval during which the snow-plough model is physically realistic. We discuss several examples which illustrate the pathologies exhibited by the solution. (Received October 14, 1977.)


Let $\Omega$ be a complete Riemannian manifold, and $H^N_0$ ($N=0,1,\ldots$) the $N$th Sobolev space. Let $L$ be an $N$th-order differential operator obtained by taking sums of products of bounded functions and vector fields whose covariant derivatives vanish at infinity. We can associate to $L$ an $N$th-order symbol $\sigma^N_L$ which is a continuous function on $\mathbb{R}^n \times \mathbb{R}^n$ where $\mathbb{R}^n$ denotes a certain compactification of the cotangent bundle $T^*\Omega$. The following holds under a few restrictions on $\Omega$.

Theorem. The operator $L$ is Fredholm as a map $H^N_0 \to H^0_0$ if $\sigma^N_L$ is never zero on $\mathbb{R}^n \times \mathbb{R}^n$.

The proof involves a $C^*$-algebra $\mathcal{A}$ of bounded (pseudo-differential) operators over $L^2(\Omega)$, and the maximal ideal space of the commutative algebra obtained by quotienting $\mathcal{A}$ by the compact ideal. The theorem generalizes the corresponding results for compact $\Omega$ (Seeley, 1965) and $\Omega = \mathbb{R}^n$ (Cordes, 1968). (Received October 17, 1977.)

Charles V. Coffman, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213. On the Biharmonic Green's function for plane regions with a corner.

Let $\Omega$ be a smoothly bounded convex region in the plane whose boundary contains a $\left(90^\circ\right)$ corner. We show that the Green's function for the biharmonic boundary value problem

$$\Delta^2 u = f \quad \text{in} \quad \Omega, \quad u = \frac{\partial u}{\partial n} = 0 \quad \text{on} \quad \partial \Omega,$$

is not of one sign. (Received October 17, 1977.)


Introducing the notions of ellipticity, parabolicity and quasimonotonicity relative to a cone $K$ in $\mathbb{R}^n$, the theory of weakly coupled systems of parabolic differential inequalities is developed which in turn is used to derive bounds on the solutions of diffusion-reaction equations. Flow-invariance of solutions in a closed set of such equations is also considered. (Received October 17, 1977.)


Sufficient conditions are found for local solvability of a linear partial differential operator $P$ of order $m$ with smooth coefficients and with characteristics of order $k$. The conditions are on the commutators $[P^\alpha, P^{\alpha+\gamma}]$ for $|\alpha| < k$ and $|\gamma| = 0$ or $1$. In the case of simple characteristics,
these conditions reduce to Hormander's condition of principal normality. Under the additional assumption that $P_m = \ldots + P_{m-k+2}$ has constant coefficients, regularity of the local solutions is shown. Energy inequalities are also obtained for certain compositions of operators satisfying the commutator conditions. Applications are given, including examples of solvable operators with variable multiplicities. (Received October 17, 1977.)


We consider the nonlinear heat equation

$$\begin{cases}
\frac{du}{dt} = q(x)u_{xx} & 0 < x < 1, \ 0 < t < T \\
u(t,0) = au(t,0) & 0 < t < T \\
u(t,1) = -bu(t,1) & 0 < t < T \\
u(0,x) = u_0(x) & 0 < x < 1.
\end{cases}$$

For $a, b > 0$, $q \in C^1(\mathbb{R})$, and $q > 0$, we show (*) is governed by a contraction semigroup via the Crandall-Liggett Theorem in the spaces $L^p[0,1]$ for $1 \leq p < \infty$ and $C[0,1]$ with the sup-norm.

For $q \in C^3(\mathbb{R})$, (*) is governed by a semigroup of type $\omega = \max(2^{\frac{1}{a} + b})$ via the Crandall-Liggett Theorem in the space $C^1[0,1]$ with norm, $||v|| = \max(||v||, ||v_x||)$. Intermediate to these results, are a priori estimates and existence results for the stationary equation $u - q(u)u_{xx} = h$.

(Received October 17, 1977.)


We consider the evolutionary equation $\frac{\partial u}{\partial t} = -Au + f(u)$ where $u = (0,T) \subset H$ is a Hilbert space valued function taking values in $D$, subdomain of $H$, contained in the domains of $A$, and the gradient operator $f$. Using so called weighted energy arguments, we can obtain improved estimates for the size of the (finite) existence interval, both for weak and strong solutions. Similar techniques also give improved estimates for equations of 'parabolic' type $(\frac{\partial u}{\partial t} = -Au + f(u))$ and for evolutionary equations with nonlinearities in the boundary condition. (Here $P$, $A$ are symmetric and positive definite and semidefinite on $D$ and $D_A$ respectively. Neither need be bounded.)

(Received October 14, 1977.)

752-35-24 Roger T. Lewis, University of Alabama, Birmingham, Alabama, 35294 and G. J. Etgen, University of Houston, Houston, Texas, 77004. The oscillation of elliptic systems.

A unification and extension of much of the known criteria for the oscillation of elliptic systems of differential equations is presented. Using positive functionals, a theorem relating the oscillation of a scalar differential equation to the oscillation of systems of partial differential equations is given. Similarly, the relation between the oscillation of systems of partial differential equations to the oscillation of a related system of ordinary differential equations is examined. (Received October 17, 1977.)


The lowest Dirichlet eigenvalue $q_1$ satisfying $\Delta^2 w = 0$ in $\Omega, \Delta w - q_1 \Delta w = 0$ on $\partial \Omega$, is the optimal constant in the a priori inequality used to approximate solutions to the Dirichlet problem for Poisson's equation. Tight bounds are given on $q_1$ for rectangles by the method of a posteriori-a priori inequalities. (Received October 17, 1977.) (Author introduced by D. W. Fox)
Consider the conservation law \( (*) \ u_t + f(u)_x = 0 \), with initial data of period \( p \). The average value, \( \omega(u) \), of \( u \) over a period is conserved under the motion. If \( f(u) \) is a convex or concave function it is known that \( u(x,t) + \omega(u) \) uniformly as \( t \to \infty \) and that \( u(x,t) \) still tends to \( \omega(u) \) uniformly. It seems therefore that the only case where a different type of asymptotic behavior might occur is when \( \omega(u) \) is a point of inflection of \( f(u) \). One considers equation \( (*) \) when \( f(u) = u^3 \) and \( \omega(u) = 0 \). It is shown that equation \( (*) \) is then equivalent to a coupled system of differential delay equations. Using these delay equations one obtains an asymptotic shape different to that which occurs in the convex case. (Received October 18, 1977.)

**39 ▶ Finite Differences and Functional Equations**

**752-39-1** PL. Kannappan, Univ. of Waterloo, Waterloo and P.N. Rathie, Universidade Estadual de Campinas, Campinas. **Remarks on sum representation functional equations.**

The object of this paper is to solve the following functional equations under the measurability of the functions in each variable by direct and simpler methods.

\[
\begin{align*}
2 & \sum_{i=1}^{3} \sum_{j=1}^{3} F(x_i u_j, y_i v_j) = 2 \sum_{i=1}^{3} F(x_i, y_i) + 3 \sum_{j=1}^{3} F(u_j, v_j) \\
2 & \sum_{i=1}^{3} \sum_{j=1}^{3} G(x_i u_j, y_i v_j, z_i w_j) = 2 \sum_{i=1}^{3} G(x_i, y_i, z_i) + 3 \sum_{j=1}^{3} G(u_j, v_j, w_j).
\end{align*}
\]

(Received August 9, 1977.)

**752-39-2** PL. KANNAPPAN, A. KAMINSKI and J. MIKUSINSKI, University of Waterloo, Waterloo, and Silesian University, Katowice. **Distributional Solutions in Information Theory.**

The measures directed divergence and inaccuracy are characterized with the help of the functional equation \( K(p_1^q, p_2^q, p_3^q) = K(p_1^q, p_2^q, p_3^q) + K(p_1^q, p_2^q, p_3^q) \), which is solved by means of distributions. (Received October 3, 1977.)


A representation theorem is obtained for solutions of the nonlinear functional differential equation of infinite delay \( (1) \ u'(t) + Bu(t) = F(u_t), \ t \geq 0, u(t) = \phi(t), \ t \leq 0, \) as a semigroup of
nonlinear operators on a space of initial data $X$ of "fading memory type." Equation (1) is studied in the abstract setting of a Banach space $E$. The nonlinear functional $F$ is a uniformly Lipschitz continuous mapping from $X$ to $E$. The operator $B$ has the property that $B - cI$ is accretive for some $c \geq 0$. The semigroup is constructed by transforming (1) to an abstract Cauchy problem $(CP)$ $w'(t) + Aw(t) = 0$, $w(0) = \phi$, in the space $X$ and applying a generation theorem of M. Cranell and T. Liggett to the operator $A$ in $X$. It is shown that under certain conditions on $c$, $F$, and the memory in $X$, $A$ generates a contraction semigroup on $X$ which represents solutions of (1). This implies the stability of solutions of (1) with respect to initial data. (Received October 14, 1977.)

40 ▶ Sequences, Series, Summability

752-40-1 David F. Dawson, North Texas State University, Denton, Texas 76203. Summability of matrix transforms of stretchings and subsequences.

R. C. Buck has shown that if $A$ is a regular matrix, $I$ is the identity matrix, and $x$ is a sequence such that $Ay$ is $I$-summable for every subsequence $y$ of $x$, then $x$ is convergent. In the present paper, several results of this type are obtained by either (1) changing conditions on $A$, or (2) using matrices other than $I$, or (3) using stretchings instead of subsequences, or (4) using absolute convergence instead of convergence. The following is a typical result. Theorem. Suppose $B$ is any regular matrix summability method. If $A$ is a regular matrix and $x$ is a complex sequence (bounded or not) such that $Ay$ is absolutely $B$-summable for every stretching $y$ of $x$, then $x$ is absolutely convergent. (Received October 6, 1977.)


A sequence space $V$ will be called pseudoconull if every matrix convergence domain containing $V$ is conull. $V$ has the gliding humps property if for every increasing sequence $\{p_n\}$ of positive integers and each bounded sequence $\{v^n\}$ in $V$ satisfying $v^n_1 = 0$ unless $p_n \leq i \leq p_{n+1}$, there exists a subsequence $\{v^{q_n}\}$ such that $\sum v^{q_n} \in V$. The following theorems are established for suitable spaces $K_0$ and $V$: 1. If $E$ is conull, $K_0 \subseteq E$, and $V$ has the gliding humps property, then $W_E \cap V$ is pseudoconull; 2. If $A$ is a matrix, $A: c_0 \rightarrow c_0$, and $Ae \in V$, if the multiplier algebra $M(V)$ of $V$ has the gliding humps property, then $V_A \cap m$ is pseudoconull. A semiconservative consistency theory can be based on these theorems. We obtain: 3. If $E$ is an FK space, $K_0 \subseteq E$, and $M(V)$ has the gliding humps property, then $K_0^\theta$ is $\sigma(K_0, M(V))$ sequentially complete; 4. If $A: c_0 \rightarrow c_0$ and $M(V)$ has the gliding humps property, then $l_1^\phi$ is $\sigma(l_1, V_A \cap m)$ sequentially complete. As an application we observe that $M(ac_0)$ has the gliding humps property. Therefore, (3) and (4) are generalizations of the consistency theorems for almost convergence of G. Bennett and N. Kalton. (Received October 18, 1977.)


In a preprint, "The $u$ property of FK spaces," A. Wilansky defined the property of conullity of a pair of conservative matrices. For $D$ & $A$ conservative matrices, $A$ $\mu$-unique, $D$ is conull w.r.t. $A$ providing $c_D \supseteq c_A$ and $\mu_A(\lim_D) = 0$. Thus conull in the classical sense means conull w.r.t. 1. We prove for $D$ & $A$ conservative matrices with $c_D \supseteq c_A$ and $B_A \neq W_A$ the following are equivalent: (a) $D$ is conull w.r.t. $A$, (b) $B_A \subseteq W_D$, (c) For some $z \in B_A \setminus W_A$, $z \in W_D$. For matrices $A$ s.t. $B_A \neq W_A$ this solves the naming problem. Additional problems in the aforementioned paper will be considered. (Received October 18, 1977.)

A-105
A theorem of Müller ("Approximation Theory", edited by A. Talbot, Academic Press, New York, 1970, pp. 315-320) is applied to obtain an asymptotic result for positive linear convolution operators of P.P. Korovkin ("Linear Operators and Approximation Theory", Hindustan Publ., Delhi, 1960, Chapter 1). (Received August 16, 1977.)

We establish $L_p$ analogues to the well known uniform norm estimates of O. Shisha - B. Mond and G. Freud. Our estimates are in the spirit of recent results by H. Berens and R. DeVore (Approximation Theory II, Academic Press (1976), 289-298). Let $e_i(t) = t^i$ for $i = 0, 1, 2$. Theorem: Let $\{L_n\}$ be a sequence of uniformly bounded, positive linear operators from $L_p[a,b]$ to $L_p[c,d]$ ($a \leq c < d \leq b$). Assume that, for each $n$, $L_n$ can be expressed as a singular integral with non-negative kernel and that $||L_n(e_i) - e_i||_p \to 0$ ($n \to \infty$) for $i = 0, 1, 2$. Then for $1 \leq p < \infty$, $f \in L_p[a,b]$ and call $n$ sufficiently large, $||L_n(f) - f||_p \leq C_p t_n \|f\|_p + w_2,p(f, t_n/2p+1)$. (Received September 26, 1977.)

Let $\Omega$ be a $\sigma$-finite measure space and let $Y$ be a Souslin subset of a Polish space $(X,d)$. Suppose that $\Phi: \Omega \to X$ and $c: \Omega \to (0,\infty)$ are both measurable functions. Define a relation $F: \Omega \to Y$ by $F(\omega) = \{x \in Y : d(\Phi(\omega), x) < c(\omega)\}$, for $\omega \in \Omega$. It is proved that there exists a measurable map $\sigma: \Omega \to Y$, satisfying $d(\Phi(\omega), \sigma(\omega)) < c(\omega)$ a.e., whenever $F(\omega) \neq \emptyset$ a.e. This result provides a uniform approach to probabilistic analogues of deterministic approximation theorems. Conditions are established, under which a given random function can be approximated by a random polynomial or a random rational function. We generalize the theorems of Hune and Mergety in that the functions being approximated depend not only on the complex variable $z$ but are also allowed to depend on a second variable $\omega$, from a measure space $\Omega$. In addition, further generalizations are established by allowing the domain of definition of the functions to depend on $\omega$. (Received October 5, 1977.)

Much of the elegant univariate theory of best approximation does not extend to classical multivariate approximation theory. However, product approximation in several variables does preserve some of the elegant univariate features and appears computationally efficient. Most of the results for product approximation have been established in $C(D)$, where $D$ is a rectangle or a discrete set in two space. More complicated domains in two space have been considered, but the admissibility of these domains has been based on fairly technical conditions. In the present paper product approximation is shown to be a special case of a general theory developed in a continuous field of Banach spaces. This setting reveals basic product approximation properties on more complicated domains. Other examples of approximation in a continuous field of Banach spaces are also given. (Received October 5, 1977.)
Let $f \in \mathcal{C}[0, \infty)$ and denote by $\mathbb{P}_n$ the set of algebraic polynomials of degree at most $n$ with real coefficients. Suppose there is a number $q > 1$ and a sequence of polynomials $\{p_n\}_{n=0}^\infty$ such that
\[
\limsup_{n \to \infty} \left( \| (1/f) - (1/p_n) \| \right)^{1/n} \leq 1/q.
\]
Then $f$ is said to have geometric convergence. In this paper two new theorems that insure geometric convergence are proven. The proofs are based on appropriate factorizations of the derivative of $f$. One of these theorems is closely related to Theorem 6 of Meinardus et al., Trans. Amer. Math. Society, 170 (1972), 171-185. Examples demonstrating the utility of the theorems are given. (Received October 7, 1977.)

Define $\mathcal{L}\subset \mathcal{C}[0, \infty)$ by $\mathcal{L}(X)(t) = A(t, X) + \int_0^t F(t, s, X(s))ds$ for $t \in I = [0, \sigma]$ where $\sigma > 0$ and $A, F$ are continuous. Consider the initial value problem (IVP) $X'(t) = \mathcal{L}(X)(t), t \in I, X(0) = X_0$. Let $\mathbb{P}_n$ denote the set of $n$-vectors whose components are polynomials of degree at most $k$ and set $P_k = \{ p \in \mathbb{P}_n : p(0) = X_0 \}$. For each positive integer $k$ and for $1 < p \leq \infty$ there exists $p_k \in P_k$ satisfying $\| p_k - \mathcal{L}(p_k) \|_p = \inf_{Q \in P_k} \| Q - \mathcal{L}(Q) \|_p$. If $1 < p \leq \infty$ then there is a number $\sigma^* > 0$ such that $\sigma \leq \sigma^*$ implies that there is a function $W$ and a subsequence $\{ p_k(\xi) \}_{k=1}^\infty$ of the sequence $\{ p_k \}_{k=1}^\infty$ satisfying $\lim_{\xi \to \infty} \| p_k(\xi) - W \|_\infty = 0$ and $\lim_{\xi \to \infty} \| p_k(\xi) - W' \|_p = 0$. Moreover $W$ must be a solution of the IVP. This analysis extends some of the work of Kartsatos and Saff. (Received October 14, 1977.)

Let $X$ and $Y$ be Banach spaces, $M$ a subspace of $X$ and $S(Y^*)$ the unit sphere in the dual space $Y^*$. Assume $M$ has property $U$, i.e. each continuous linear functional on $M$ has a unique Hahn-Banach extension to $X$. If $P$ is a norm-one linear operator mapping $X$ into $Y$, let $K(M,P) = \{ f \in S(Y^*) : \| f \|_M = 1 \}$.

Theorem: Suppose $\{ P_n \}$ is a sequence of linear contractions mapping $X$ into $Y$. If $\| P_n(x) - P(x) \| \to 0$ for each $x \in M$, then for each $y \in X$, $foP_n(y) \to foP(y)$ for all $f \in K(M,P)$ and the convergence is uniform on weak* closed subsets of $K(M,P)$.

This theorem is similar to a result of Wulbert's [Convergence of Operators and Korovkin's Theorem, J. of Approximation Theory 1 (1968), 381-390] with $P$ replacing the identity operator. (Received October 17, 1977.)

WITHDRAWN

For each $\theta$, $0 < \theta \leq 1$, and real $t$ set $\xi = \xi(t, \theta) = \{ t + \theta - \sqrt{(1-t)(2\theta^2 - 1-t)}/(1+\theta) \}$ and define $G(t, \theta) = |(\xi^2 - 1)/(\xi - t)|^{1-\theta}(1+\xi^2)^{1/2}$, for $t \leq 2\theta^2 - 1$, $t \neq -1$, and $G(-1, \theta) = 0$. For any...
(not identically zero) incomplete polynomial of the form \( p_n(t) = \sum_{k=s}^{n} a_k (1+t)^k \), with \( 0 < s \leq n \), we prove that

\[(*) \quad |p_n(t)| \leq ||p_n||_{[-1,1]} \{G(t, s/n)\}^n < ||p_n||_{[-1,1]} \]

for any \( t \) with \(-1 \leq t < 2(s/n)^2 - 1\), where \( ||\cdot|| \) denotes the uniform norm. Inequality \((*)\) is best possible in the sense that for each \( 0 < \theta \leq 1 \) there exists a sequence of polynomials \( q_i(t) \) with the following properties: \( (1) \ q_i(t) = \sum_{k=0}^{n} a_k (1+t)^k \) with \( n + 1 \) and \( \lim_{s \to s} s-n = 0 \); \( (2) \ \lim_{s \to s} \sup_{|t| \leq 1} \{q_i(t)\}^{1/n_l} \leq 1 \); and \( (3) \ \lim_{s \to s} |q_i(t)|^{1/n_l} = c(t, \theta) \) for each \( t < 2\theta^2 - 1 \). The results obtained were inspired by some open problems of G. G. Lorentz. (Received October 18, 1977.)

42 Fourier Analysis

George Gasper, Northwestern University, Evanston, Illinois, 60201 and Walter Trebels, TH. Darmstadt, D-6100 Darmstadt, West Germany. Jacobi and Hankel multipliers of type \((p,q)\), \( 1 < p < q < \infty \).

It is shown how the multiplier criteria of type \((q,q)\) for Jacobi expansions and Hankel transforms in Gasper and Trebels [A characterization of localized Bessel potential spaces and applications to Jacobi and Hankel multipliers, Studia Math., to appear.] can be used to derive multiplier criteria of type \((p,q)\). In particular, it is proved that if \( \alpha = \beta - 1/2, 1 < p < q < \infty \), \( \gamma = 2(\alpha+\gamma)/|\alpha - 1/2| + 1/2 \), \( 1/q = 1/p - \delta/(2\alpha+2) \) and if \( m = \{m_k\}_{k=0}^\infty \) is in the sequence space \( wb_{1,\gamma,\delta} \), i.e.,

\[ \|\{k^\mu m_k\}_\infty + \|m_k\|_{2^n-1}^{2^n-1} k^{-1} k^{-\mu+\delta} \Delta^\gamma m_k \| < \infty , \]

where \( \Delta^\gamma \) is a certain fractional difference operator, then \( m \) is a multiplier of type \((p,q)\) for expansions in Jacobi polynomials \( \{p_n(\alpha,\gamma)(x)\} \). (Received October 5, 1977.)

Douglas S. Kurtz, Rutgers University, New Brunswick, New Jersey 08903. Weighted norm inequalities for generalized \( \gamma \)-function. Preliminary report.

Let \( \Delta = \{\rho\} \) be a lacunary decomposition of \( \mathbb{R}^d \). For \( \rho \in \Delta \), set \( \sigma_{\rho} f(x) = (x \cdot \rho)^\gamma(x) \), and define \( \gamma(x) = \gamma(f,\Delta)(x) = (1 \in \{\sigma_{\rho} f(x)\}^{1/2} \). Suppose \( w(x) \) is a positive function, locally in \( L^p \) and satisfies

\[ \sup_{\rho \in \Delta} \left( \frac{1}{|R|} \int_R w(x) dx \right) \left( \frac{1}{|R|} \int_R \frac{1}{|f|} \left| \int_R f(x) dx \right|^{p-1} dx \right)^{p-1} \leq C, \]

the sup taken over all rectangles, \( R, \) in \( \mathbb{R}^d \).

Theorem: Let \( \Delta = \{\rho\} \) be a lacunary decomposition of \( \mathbb{R}^d \), \( 1 < p < \infty \), and \( w \) satisfy Ap. Then, there exist constants \( A \) and \( B \), depending on \( p, w, \) and \( \Delta \), such that \( A ||f||_{L^p} \leq ||\gamma(f)||_{L^p} \leq B ||f||_{L^p} \). A partial converse is proved in the case where \( \Delta \) is the dyadic decomposition of \( \mathbb{R}^d \).

The theorem is shown to be equivalent to a weighted version of the Marcinkiewicz Multiplier Theorem. The proof relies on a generalization of the Hörmander Multiplier Theorem. (Received October 14, 1977.)

Günter W. Goes, Illinois Institute of Technology, Chicago, Illinois 60616. Multipliers for Hardy spaces and BMO.

Let \( H^1 = H^1(T) \) be the classical Hardy space on \( T = \mathbb{R}/(2\pi \mathbb{Z}) \), i.e., \( f \in H^1 \) if and only if \( f \in L^1(T) \) and \( f(t) \sim \sum_{k} a_k e^{ikt} \). The multipliers \( \lambda \in (H^1 \to H^1) \) from \( H^1 \) into \( H^1 \) can be characterized as follows: \( \lambda = (\lambda_k) \in (H^1 \to H^1) \), i.e., for every \( f \in H^1 \), the transformed Fourier series \( \sum_{k} \lambda_k a_k e^{ikt} \) belongs to a function \( g \in H^1 \) if and only if \( \sum_{k} (\lambda_k / ik) e^{ikt} \sim h(t) \), where \( h \in \mathcal{V}_{BMO} \), the space of functions in \( H^1 \) which have generalized bounded variation in the norm of the space BMO (bounded mean oscillation à la John and Nirenberg). This and related facts will be discussed. (Received October 17, 1977.)
The convolution kernel of Gasper is used to define the various functionals \( g, \tilde{g}, \gamma, \tilde{\gamma} \), and \( S \) for Jacobi expansions \( P(a, \beta)_{n}(X) \). For each functional, the range of \( a, \beta \) which yields the standard inequalities is found. This leads to a very general multiplier theorem for Jacobi expansions. (Received October 18, 1977.)

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Let \( \Gamma \) be a continuous graph in \( \mathbb{R}^2 \). Then there is a line \( L = \{ r(\cos \theta, \sin \theta) : r \in \mathbb{R} \} \) and a probability measure \( \mu \) supported on \( \Gamma \) such that the Fourier-Stieltjes transform \( \hat{\mu} \in C_0(\mathbb{C}) \) for any closed cone \( C \) not containing \( L \). (Received October 18, 1977.)

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**43 ▶ Abstract Harmonic Analysis**

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Let \( A_1, A_2 \) be commutative semisimple CMA's with approximate identities of norm one and structure semigroups \( \Gamma_1, \Gamma_2 \), and let \( A_1 \hat{\otimes} A_2 \) be their projective tensor product CMA. Further, let \( \mathcal{M}(A_1), \mathcal{M}(A_2), \mathcal{M}(A_1 \hat{\otimes} A_2) \) be their respective multiplier algebras, and assume that \( \mathcal{M}(A_1) \) and \( \mathcal{M}(A_2) \) are CMA's. This paper is devoted to characterizing those elements of \( \mathcal{M}(A_1 \hat{\otimes} A_2) \) that belong to \( \mathcal{M}(A_1) \hat{\otimes} \mathcal{M}(A_2) \). For example, it is shown that, regarding \( m \in M(\Gamma_1 \times \Gamma_2) \cap (\mathcal{M}(A_1) \hat{\otimes} \mathcal{M}(A_2)) \) as a vector-valued measure in \( M(\Gamma_1, M(\Gamma_2)) \), \( m \) automatically has range in \( \mathcal{M}(A_2) \) and its variation measure \( \mu^m \) is in \( \mathcal{M}(A_1) \). Consequently, such a multiplier measure \( m \) is in \( \mathcal{M}(A_1) \hat{\otimes} \mathcal{M}(A_2) \) if and only if it possesses a Bochner-integrable Radon-Nikodym derivative with respect to \( \mu^m \). Most results are stated in the context of determining properties of measures in \( M(X_1 \times X_2) \) that cause them to belong to \( L_1 \hat{\otimes} L_2 \), where \( L_1 \) is an \( L \)-subspace of \( M(X_1) \), for \( X_1 \) a compact Hausdorff space, \( i = 1, 2 \). (Received August 25, 1977.)

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Let \( G \) be a locally compact group and let \( D(G) \) be the space of test functions on \( G \) having compact support as defined by Bruhat [Bull. Soc. Math. France 89 (1961), 3-75]. Then \( D'(G) \), the topological dual of \( D(G) \), is the space of distributions on \( G \). For \( \varphi \) in \( D(G) \) and \( T \) in \( D'(G) \), let \( \langle \varphi, T \rangle \) denote the value of \( T \) on \( \varphi \). By an adjunctive kernel on \( D(G) \), we mean a separately continuous sesquilinear form \( b \) on \( D(G) \times D(G) \) such that \( b(\rho \varphi, \psi) = b(\varphi, \rho^* \psi) \), for all \( \varphi, \psi, \rho \) in \( D(G) \). Let \( K_G \) denote the set of all adjunctive kernels on \( D(G) \). Then we have proved that the following representation theorem for \( K_G \) in terms of \( D'(G) \), which generalizes a result of Goodman [Pacific J. Math. 45 (1973), 83-91] for second-countable Lie groups, is valid for arbitrary locally compact groups.

**Theorem.** If \( G \) is a locally compact group, then there is a bijective correspondence between the set \( K_G \) of all adjunctive kernels on \( D(G) \) and the space \( D'(G) \) of distributions on \( G \). This correspondence \( b \leftrightarrow T \), where \( b \in K_G \) and \( T \in D'(G) \), is implemented by the identity: \( b(\varphi, \psi) = \langle \psi^* \varphi, T \rangle \), valid for all \( \varphi, \psi \) in \( D(G) \). (Received September 22, 1977.)
Let $B$ be a commutative semi-simple Banach algebra, $AB$ its maximal ideal space, and $\Lambda$ the closed linear span of $AB$ in $B^*$. Further, let $f*XB^*$ denote the pre-Arens product of $fCB^*$ and $XCB$. Then $B$ is said to have property Q2 if $\sum_{k=1}^{\infty} f_kx_k=0$ implies $\sum_{k=1}^{\infty} f_k(x_k)=0$ whenever $f_k\in \Lambda$, $x_k\in B$ and $\sum_{k=1}^{\infty} |f_k| |x_k|<\infty$.

Under the assumption that $B$ has property Q2, it is proved that the multiplier algebra $M(B)$ of $B$ is isomorphic to a subalgebra of $((B\Lambda)/K)^*$ (with an Arens product), where $K$ is the kernel of the map from $B\Lambda$ to $\Lambda$ defined on simple tensors by $x\otimes \ast f\mapsto fx$, for $x\in B$ and $f\in \Lambda$. These theorems generalize the work of L. Mâr (who defined property P2) and others. It is shown that a necessary condition for $B$ to have property Q2 (P2) is that $B$ have a $\Lambda$-approximate ($B^*$-approximate) identity. Furthermore, it is proved that $B$ has a $B^*$-approximate identity if and only if $B$ has an approximate identity. A sufficient condition for $B$ to have property Q2 (P2) is that $B$ have an operator bounded $\Lambda$-approximate ($B^*$-approximate) identity. Finally, when $B$ has a bounded $\Lambda$-approximate identity, the Q2 multiplier theory yields the same representation of $M(B)$ considered in various instances by F.T. Birtel, C.D. Lahr and others. (Received October 7, 1977.)

Let $G$ be a countable discrete abelian group and $\alpha$ an automorphism of $G$. The adjoint automorphism $^\alpha$ is defined on the dual group $\hat{G}$ by $(^\alpha)(x)(\tau) = \tau(\alpha^{-1}x)$, $\tau \in \hat{G}$, $x \in G$. Let $h(^\alpha)$ denote the Kolmogorov-Sinai entropy of $^\alpha$ (with respect to haar measure on $\hat{G}$), or equivalently, the topological entropy of $^\alpha$. Now define $h(\alpha)$ as follows: let $E \subseteq G$ be a finite set and $E_{a,n} = E + a^{-1}E + \ldots + a^{-(n-1)}E$. Let $h(\alpha) = \lim \sup_{E \subseteq G} 1/\log m(E_{a,n})$, where $m$ denotes haar measure (i.e., cardinality) and the limit is taken over the net of all finite subsets $E$ of $G$, directed by inclusion. Then $h(\alpha) = h(^\alpha)$. (Received October 17, 1977.)

Let $G$ be a LCA group, $\Gamma$ be its dual group, $A(\Gamma)$ be the set of all Fourier transforms of $L^1(G)$. It was shown by Rudin and Katznelson that every separating self-adjoint subalgebra $\hat{A}$ of $A(\Gamma)$ with $Z(A)=\Phi$ is dense in $A(\Gamma)$ if $\Gamma$ is totally disconnected. (Pacific J. Math. 11, 253-265, '61)

In this direction, the following general result is obtained:

**Theorem 1.** Let $A$ be a commutative Banach algebra and $B$ be a closed subalgebra of $A$ such that $R\hat{A}$ is dense in $C_0(\Sigma A)$, then every idempotent element in $C_0(\Sigma A)$ is contained in $\hat{A}$.

As a direct consequence of Th.1, the following Wermer type characterization is obtained:

**Theorem 2.** Let $A$ be a closed subalgebra of $L^1(G)$ such that $\hat{A}$ separates points on $\Gamma$, and suppose that not all members of $\hat{A}$ vanish at any point of $\Gamma$. If $\Gamma$ is totally disconnected, then ($*)R\hat{A} \subset C_0(\Sigma A)$ implies $A=L^1(G)$. Since self-adjointness of $\hat{A}$ always implies ($*$) and ($*$) does not imply self-adjointness in general, Theorem 2 is an extension of the Rudin-Katznelson Th. (Received October 13, 1977.)

Using a recent result of the author concerning topologies of maximally almost periodic groups [On topologies of maximally almost periodic groups, Proc. of AMS, to appear], it is proved:

**Theorem.** Every non-discrete compact group admits complex-valued discontinuous almost periodic functions. (Received October 17, 1977.)
Let $G$ be a locally compact abelian group with nondiscrete dual group $H$. Let $A(H)$ denote the Banach algebra of Fourier transforms on $H$ with induced norm from $L(G)$. For a closed ideal $J$ of $A(H)$ and $f$ in $C(H)$ define the sets $L(f,J) = \{x \in H : f \text{ belongs locally to } J \text{ at } x \}$ and $I(f,J) = \{g \in J : gf \text{ belongs to } J \}$. In a recent paper the authors completely characterized these sets in the special case that $J = A(H)$. In this report various results are obtained under such assumptions as:

- $f$ belongs to $A(H)$, the spectrum of $J$ is an S-set, $H$ is metric, etc. (Received October 17, 1977.)

In the course of developing a Littlewood-Paley theory for Jacobi series, three maximal functions arise (for notation and definitions see the authors' A Multiplier Theorem For Jacobi Series. Studia Math. LII (1975) 243-261):

$$Mf(\phi) = \sup_{0 < \varepsilon < 1} \int_{\phi - \varepsilon}^{\phi + \varepsilon} |f(\mu)| d\mu (\phi - \varepsilon, \phi + \varepsilon) \quad (\varepsilon > 0)$$

$$\mathcal{M} f(\phi) = \sup_{0 < \varepsilon < 1} X_{\varepsilon} * |f| (\phi) / \mu (0, \varepsilon) \quad (\varepsilon > 0)$$

$$f * (\phi) = \sup_{0 < r < 1} |f(r, \phi)| \quad (0 < r < 1)$$

where $X_{\varepsilon}$ is the characteristic function of $[0, \varepsilon]$ and $f(r, \phi)$ is the (Jacobi) Poisson integral of $f$. In the Fourier case the analogs of the first two coincide.

Theorem. Let $a > a > -1$. The three maximal functions are of weak type $(1,1)$ and strong type $(p,p)$ for $1 < p < \infty$ provided $a + a > -1$ for $Mf$, and $a > -1/2, a + a > 0$ for $f^*$. (Received October 18, 1977.)

### 44 ▶ Integral Transforms, Operational Calculus

**752-44-1** Raimond A. Struble, North Carolina State University, Raleigh, North Carolina 27607. 

**Arithmetical Laplace Transforms.**

A field of convolution quotients of $C^\infty$-functions in $L^2$ is constructed for which an extension of the classical and distributional Laplace transformation becomes an isomorphism onto the field of all meromorphic functions in (various) open, vertical strips about the imaginary axis. (Received September 12, 1977.)

**752-44-2** Gerald Johnson and David Skoug, University of Nebraska, Lincoln, Nebraska 68588. An $L_p$ Analytic Fourier-Feynman Transform.

Let $C[a,b]$ be the set of real continuous functions $x(t)$ vanishing when $t = a$. In 1972, Brue (Thesis, Univ. of Minn.) introduced an $L_1$ analytic Fourier-Feynman transform. Recently Cameron and Storvick (Michigan Math. J. 23(1976), 1-30) introduced an $L_2$ analytic Fourier-Feynman transform. In this paper we study an $L_p$ analytic Fourier-Feynman transform for $1 \leq p \leq 2$. Fix real $q > 0$. For a functional $F(x)$, $x \in C[a,b]$, the $L_p$ analytic Fourier-Feynman transform $\mathcal{T}_p F(y)$ is defined to be the scale-invariant limit in the mean of order $p'(\frac{1}{p} + \frac{1}{p'} = 1)$ of the analytic extension in $\lambda$ of the Wiener integral $\int F(\lambda^{-\frac{1}{2}}x + y) dx$ as $\lambda \to -iq$. (In case $p = 1$ we take a scale-invariant pointwise limit.) We show that $\mathcal{T}_p F$ exists for various classes of functionals $F$. In addition we indicate various relationships between the $L_1$ and $L_2$ theories and extend the previous results in the cases $p = 1$ and $p = 2$. (Received October 14, 1977.)
Let \( Z \times X \times Y \) be a double fibering of manifolds. Under certain conditions on \( Z \) and the projections \( \pi \) and \( \rho \), a generalized Radon transform, \( R:C^\infty(Z)\rightarrow C^\infty(X) \), and its dual, \( R^t:C^\infty(X)\rightarrow C^\infty(Z) \), can be defined as Fourier integral operators. More specifically, for \( y \in Y \) and \( f \in C^\infty(X) \), \( Rf(y) \) is the integral of \( \pi f \) over the manifold \( \rho^{-1}(y) \), and similarly for \( R^t \).

It is known that \( R^tR \) is an elliptic pseudodifferential operator. Both \( R^t \) and \( R \) depend on a choice of measures on \( X \), \( Y \), and \( Z \), and we give a necessary condition on the measures for \( R^tR \) to be invertible (mod smoothing operators) by a differential operator. To discover this condition, we calculate the top order symbol of \( R^tR \) as a pseudodifferential operator. For the classical Radon transforms on points and hyperplanes in \( \mathbb{R}^n \), we show that a similar criterion on the measures is sufficient for \( R^tR \) to have a local inverse. (Received October 17, 1977.)

### Integral Equations

#### 752-45-1 John A. Chatfield, Southwest Texas State University, San Marcos, Texas 78666. Eigenvalues of a Stieltjes-Volterra Integral Equation.

If each of \( f, g, \) and \( h \) is a function from the real numbers \( \mathbb{R} \) to a ring \( N \), \( K \) is a bounded function from \( \mathbb{R} \times \mathbb{R} \) to \( N \), \( g \) is of bounded variation, the function \( y \) defined by
\[
y(x) = \lfloor 1 - K(x,x) \rfloor (g(x) - g(x^-))^{-1},
\]
\( g(x^-) \neq g(x^-) \) is bounded, then the following two statements are equivalent:

1. \( (R)(f,K,g) \) is in \( OA^* \) on \( [a,b] \) and if \( x \) is in \( [a,b] \), then
   \[
f(x) = h(x) + (R) \int_0^x K(x,t)dg(t);
   \]
2. \( (R)(h,K,g) \) is in \( OM^* \) on \( [a,b] \) and if \( x \) is in \( [a,b] \), then
   \[
f(x) = (R)V(a,x,h,K,dg).
   \]

Further, necessary and sufficient conditions are determined for the operator \( \mu \) defined by
\[
\mu f(x) = (R) \int_0^x K(x,t)dg(t)
\]
to have non-zero eigenvalues of non-trivial eigenfunctions of \( \mu \).

(Received October 3, 1977.)


Let \( E \) denote a number interval \( [a,b] \), \( G,*,0,\| \|, H \) the set of functions from \( G \) to \( H \), and \( K \) a function from \( SXSXS \) to \( H \) with the property that there is a super function \( k \) from \( S \) to the numbers such that for each \( \{x,u,v\} \) in \( SXSXS \) and \( \{p,q\} \) in \( GXS \)
\[
\| K(x,u,v)P - K(x,u,v)Q \| < k(v) - k(u) \| P - Q \|.
\]
Let \( \{t_j\}_0 \) be a subdivision of \( S \); then for \( 0 < m \leq n \)
\[
V(\{t_j\}_0,h,K) = h(x_0) + K(x_m,x_0,x_1) + \sum_{p=2}^m K(x_m,x_{p-1},x_p)V(\{t_j\}_0^{-1};h,K);
\]
\( V(a,b;h,K) \) denotes the subdivision-refinement-type limit of the set \( \{V(\{t_j\}_0,h,K)\} \).

The pair \( \{h,K\} \in OA^*[OM^*] \) means if \( c > 0 \), then there is a subdivision \( s \) of \( S \) such that if \( \{t_j\}_0 \) is a refinement of \( s \) and \( 0 < m \leq n \), then
\[
\| V(a,x;h,K) - V(\{t_j\}_0,h,K) \| < c.
\]

#### 752-45-3 M. Zuhair Nashed, University of Delaware, Newark, Delaware 19771. Ill-posed problems for integral equations with emphasis on the Radon equation.

The aim of this talk is to examine the scope and limitations of several approximation and regularization methods for finding numerically minimum weighted norm least-squares solutions of first kind integral equations. A concept of a criticality index (c.i.) is introduced which quantifies in a single number information involving smoothness of the kernel and solution, degree of approximation,
tolerable and data error, etc. Each method, whether regularized or nonregularized, involves a c.i. whose determination is crucial to the numerical realization and amenability of the method. In RKHS methods or in a suitable Sobolev space the c.i. reflects the number of derivatives that the integral operator imparts to the $L_2$-functions; in Tikhonov regularization it is the regularization parameter; in projection and finite element methods it is the optimal dimension of the approximating subspaces; etc.

It is proposed that numerical methods for ill-posed problems should be assessed and compared on the basis of the c.i. A modulus of convergence can be defined for the methods as a function of the c.i. and the tolerance error. The ramifications are illustrated with the Radon equation. (Received October 14, 1977.)


Qualitative properties of the solutions of the nonlinear Volterra integrodifferential equation of nonconvolution type 

$$-u'y'(t) = \int_0^t b(t,s) F(y(t),y(s))ds \quad (t > 0), \quad y(t) = g(t) \quad (\infty < t \leq 0),$$

and of the associated reduced equation, obtained by setting $y = 0$, are discussed. The relations between those solutions as $u \to 0^+$, both for large $t$ and for $t$ near zero where a boundary layer occurs, are also treated. In these equations $\mu$ is a small positive parameter, $b(t,s)$ is a given real kernel, and $F, g$ are given real functions. The results are extensions of recent theorems of Lodge, McLeod, and Nohel for the convolution case $b(t,s) = a(t-s)$ which, for particular choices of $a, F,$ and $g$, models the elongation ratio of a homogeneous polyethylene filament which is stretched on the time interval $(\infty, 0]$ and then released. (Received October 17, 1977.)

FREDERICK BLOOM, University of South Carolina, Columbia, South Carolina 29208. Bounds for Solutions to a Class of Damped Integrodifferential Equations in Hilbert Space with Applications to the Theory of Nonconducting Material Dielectrics.

Let $T > 0$ be an arbitrary real number and $H, H_+,$ real Hilbert spaces with $H_- \subseteq H$ algebraically and topologically dense in $H$. Let $H_-$ be the dual of $H_+$ via the inner product of $H$ and denote by $L_0(H_+, H_-)$ the space of symmetric bounded linear operators from $H_+$ into $H_-$. We prove that the evolution of the electric displacement field in a simple class of holohedral isotropic dielectrics can be modeled by an abstract initial-value problem of the form

$$u_{tt} - {a_\alpha} u_t - Lu + \int_0^T M(t-\tau)u(\tau) d\tau = \frac{\beta(t)}{\alpha}, 0 \leq t < T,$$

$$u(0) = u_0, \quad u_t(0) = u_1 (u_0, u_1 \in H_+),$$

where $L \in L_0(H_+, H_-)$, $M(t) \in L^2(0,T)$, $L_0(H_+, H_-)$, $\beta(t) \in C^1((0,T))$, and $\alpha$ is an arbitrary (non-zero) real number. By employing a logarithmic convexity argument we derive growth estimates for solutions of the above system which lie in uniformly bounded classes of the form ($N > 0$ a real number)

$$N = \{u \in C^2([0,T); H_+) \mid \sup_{[0,T]} ||u||_{H_+} \leq N\}$$

(Received October 14, 1977.)

Prof. R. Gorenflo, Freie Universität Berlin, Berlin, West Germany. Solution techniques for first kind Abel integral equations under inequality constraints.

The integral equation

$$I(x) = \frac{1}{x} \int_0^x \frac{2i(t) r \, dr}{\sqrt{r^2 - x^2}}, \quad 0 \leq x \leq 1,$$

is equivalent to

$$G(t) = \int_0^t (T-s)^{-1/2} g(s) \, ds, \quad 0 \leq t \leq 1,$$

which occurs in the evaluation of side on measurements of light intensities of rotationally symmetric gas discharges. For physical reasons we should have $i(r) \geq 0$; because of measurement errors, however,
this may not be the case everywhere for the computed $i(r)$. Some methods are discussed for making best use of extra inequality conditions and of the available information on $I(x)$ (usually a finite set of inaccurately determined values of linear functionals). (Received October 14, 1977.)

(Author introduced by Professor Frederick Bloom)

46  Functional Analysis


Let $\Pi: E \rightarrow S$ and $\varrho: F \rightarrow T$ be full bundles of Banach spaces.

A bundle $\pi \otimes \varrho: G \rightarrow S \times T$ is constructed. The product bundle $\pi \otimes \varrho$ is then used to determine the Gelfand representation of a large class of Banach modules of form $(M \otimes A, N \otimes B)$, where $A$ and $B$ are commutative Banach algebras and $(M, A)$ and $(N, B)$ are Banach modules, and of form $(M \otimes A, N, A)$, where $A$ is a commutative Banach algebra and $(M, A)$ and $(N, A)$ are Banach modules.

(For terminology, see these Notices, 24 (1977), A 115-116.) (Received September 19, 1977.)

*752-46-2  Gordon R. Feathers and W. G. Dotson, North Carolina State University, Raleigh, North Carolina 27607. A Nonlinear Theorem of Ergodic Type II.

Let $B$ be a uniformly convex Banach space with weak duality mapping $[2]$ and $C$ a closed convex subset of $B$. Suppose $G = \{S_m\}$ is a collection of self-mappings of $C$ such that $I - S_m$ is demi-closed for $m = 1, 2, \ldots$ Suppose further that there is a collection of nonexpansive self-mappings of $C$, $\{C_n\}$ satisfying

\begin{enumerate}
  \item $C_n(x) \in \{S_m(x): S_m \in G\}$ for all $x \in C$
  \item there exist $x_0$ such that $C_n(x_0) = C_n(x_0) + 0$ as $n \rightarrow \infty$
  \item suppose there exists a subsequence $\{C_m(x_0)\}$ of $\{C_n(x_0)\}$ such that $C_m(x_0) \rightarrow y \in C$ [since $C$ is weakly closed]
\end{enumerate}

then

1) $S_m(y) = y$ for all $S_m \in G$

2) $C_n(x_0) \rightarrow y$. (Received October 3, 1977.)


Let $(X, \tau, \mu)$ be a measure space, $E$ a Banach space and $L^1_E(\mu)$ the space of Pettis-integrable functions $f: X \rightarrow E$, endowed with the Pettis norm $\|f\| = \sup\{|\langle f, x' \rangle| \mid x' \in E', \|x'\| \leq 1\}$. For each finite partition $\pi = \{A_i\}_{i=1}^n$ with $A_i \in \mathcal{F}$, $0 < \mu(A_i) < \infty$, and each $f \in L^1_E(\mu)$ consider the conditional expectation $f_\pi = \frac{1}{\mu(A_i)} \int_{u(A_i)} f d\mu$. Theorem. A set $K \subset L^1_E(\mu)$ is conditionally weakly compact (i.e. each sequence of $K$ contains a weak Cauchy subsequence) iff: 1) For every $A \in \mathcal{F}$ with $\mu(A) < \infty$, the set $\{f \in \mathcal{F} | f \in K\}$ is conditionally weakly compact in $E$; 2) for every countable subset $K \subset K$, there exists a sequence $\{f_n\}$ of finite partitions such that $f_n \rightarrow f$ weakly in $L^1_E(\mu)$, uniformly for $f \in K$. Using this theorem, a theorem by Dan Lewis can be deduced (which has the same statement, except that condition 2) is replaced by: 2') For every $x' \in E'$, the set $\{\langle f, x' \rangle | f \in K\}$ is relatively weakly compact in $L^1_E(\mu)$. (Received October 11, 1977.)
Let \( X \) be a compact \( T_2 \)-space such that each point of \( X \) is a \( G_δ \) point. Let \( B \) be the unit ball in \( C(X) \), the space of continuous functions on \( X \). A function \( F: B \to C(X) \) is said to be biholomorphic in \( B \) if \( F \) and \( F^{-1} \) are Fréchet differentiable. If \( P(\mathbb{B}) \) is a convex set, then \( P \) is said to be a convex function. A function \( F: \mathbb{B} \to C(X) \) is said to be Lorch analytic in \( \mathbb{B} \) such that \( DF(0) = I_\mathbb{B} \), then \( F \) is Lorch analytic in \( B \). (Received October 11, 1977.)

Some variants are also obtained. J. Wermer had done the case \( h(t) = t^2 \) (and so by induction on degree, \( h \) any polynomial of degree greater than 1), and A. Bernard had done the case \( h(t) = ||t|| \) (which is not included in the present result). The problem remains open for general non-affine \( h \), and it is hoped that current methods will soon yield a general proof. (Received October 11, 1977.)


The following theorem is proved: Theorem. Let \( X \) be a normed linear space over a field which contains the real numbers. Then \( X \) is an inner product space if, and only if, the following implication is true in \( X \). If \( n \geq 3, x_1, x_2, \ldots, x_n \in X \) and \( \alpha_1, \alpha_2, \ldots, \alpha_n \) are real numbers such that \( \sum_{1 \leq i < j \leq n} \alpha_i \alpha_j ||x_i - x_j||^2 = 0 \), then \( \sum_{i=1}^{n} \alpha_i x_i = 0 \). It is then shown that this characterization encompasses characterizations previously given by Jordan and von Neumann, Ficken and Lorch in the sense that the necessity of each of the earlier implications is easily derived from the present one without employing the inner product. (Received October 11, 1977.)

Non-Standard Representations.

Let \( A \) be an arbitrary self-adjoint operator acting in a separable \( L^2 \) space \( H \). It is shown that up to infinitesimals \( A \) is strongly unitarily equivalent to multiplication by some function in \( H \). As well, if \( V \) is an arbitrary Hermitian form acting in \( H \) then \( V(\cdot, \cdot) \cong < U \cdot, T_F U \cdot > \) for some unitary map \( U \) and multiplication operator \( T_F \) in \( \hat{H} \). (Received October 12, 1977.)

A measure derived Banach algebra topology. Preliminary report.

Let \( X \) be a complex commutative semisimple Banach algebra with identity of norm one. Let \( \mathcal{M} \) be the maximal ideal space of \( X \) with the Gelfond topology and let \( \mu \) be a probability measure on \( \mathcal{M} \) which is positive on nonempty open sets. The subspace of linear functionals \( \{ f_y \} \), where \( f_y(x) = \int x \hat{y} dm \) (\( \hat{x} \) and \( \hat{y} \) are the Gelfond functions associated with \( x \) and \( y \) resp.), is total and hence generates a topology on \( X \). We are concerned with the properties of \( \mathcal{G} \). For example, we show that \( \mathcal{G} \) is separable if \( X \) is separable and \( \mathcal{G} \) is metrizable iff \( X \) is finite dimensional. (Received October 12, 1977.)
Let $\lambda$ denote Lebesgue measure on $[0,1]$, let $1 < p < \infty$ be fixed and let $X$ be a Banach space.

A Banach space $B$ is said to have property (H) if norm and weak convergence of norm-1 sequences coincide. Theorem 1. If $L^p(\lambda, X)$ has property (H), then $X$ is rotund. Applying this result with $X$ equal the two-dimensional $l^1$ space, a space with property (H), it follows that $L^p(\lambda, X)$ does not have property (H). This answers a question raised by E. Leonard [J. Math. Anal. and Appl. 54 (1976), 245-265].

Theorem 2. If $X$ is uniformly non-$l^p(n)$, then $L^p(\lambda, X)$ is uniformly non-$l^p(n)$. As a corollary, a known result (due to T. Figiel) follows, namely, if $X$ is $B$-convex then $L^p(\lambda, X)$ is $B$-convex.

Other related properties are discussed. (Received October 14, 1977.)
Suppose \((\Omega, \mathcal{B}, \mu)\) is a \(\sigma\)-finite measure space and \(S\) the space of real-valued, measurable functions on \(\Omega\). Let 
\[ L_F = \{ x \in S : \|x\| = \int_{\Omega} F_\omega(x(\omega))d\mu < \infty \}. \]
Here 
\[ \{ F_\omega : \omega \in \Omega \} \]

is a family of left-continuous maps \(\mathbb{R} \rightarrow \mathbb{R}\) such that 
1. \(\forall \omega \in \Omega, F_\omega(0) = 0 \) iff \(\|F_\omega(0)\| = 0 \), 
2. \(\forall \omega \in \Omega, F_\omega(s) \leq F_\omega(t) \) if \(|s| \leq |t|\), 
3. \(\forall t > 0, \|F_\omega(t)\| < \infty \) if \(\mu(E) < \infty \). 

A class of \(F\)-ideals of \(S\) whose duals are represented by their Köthe duals is described. The Köthe dual \(L_F^+ = \{ y : xy \text{ is integrable} \forall x \in L_F \} \) can be more precisely identified via the function \(\theta(\omega) = \inf\{F_\omega(t)/t : t > 0\}\).

**Theorem** If \((\Omega, \mathcal{B}, \mu)\) is non-atomic, then 
\[ L_F^+ = \{ \alpha \theta : \alpha \in L_\infty \}. \]
Therefore \(L_F^+\) is isomorphic to \(L_\infty(\text{Support}(\theta))\). 

**Corollary** \(L_F^+ = \{ 0 \}\) iff \(\mu(\text{Support}(\theta)) = 0\).

A similar description of \(L_F^+\) for atomic measure spaces is presented. The above generalizes several results of B. Gramsch [Math. Annalen 171 (1967), 61-78].

(Received October 14, 1977.)

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Differentiability of the norm in an abstract \(L\)-space is examined independently of the Kakutani representation of such spaces as \(L^1(\mu)\). A central result is the following equivalence of uniform differentiability and uniform absolute continuity: **Theorem** If \(X\) is an \(L\)-space, \(f \in X\), and \(K \subset X\) is bounded, then the derivative of the norm at \(f\) in the direction \(g\) exists uniformly for \(g \in K\) iff 
\[ |nf| \wedge |g| \rightarrow |g| \text{ uniformly for } g \in K. \]

The failure of uniform differentiability to characterize weak compactness in arbitrary Banach spaces, or even in the space \(L^1(\mu, E)\) of Bochner integrable functions is demonstrated. A uniform differentiability result for compact sets is used to give a new proof of the Vitali-Hahn-Saks Theorem. A contemporary version of the Vitali-Hahn-Saks Theorem is extended. 

(Received October 14, 1977.)

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Suppose that \(f : C \rightarrow C\) is a nonexpansive mapping of a convex subset of a normed linear space into itself. For any \(\lambda \in (0,1)\) define the nonexpansive mapping \(F_\lambda : C \rightarrow C\) by 
\[ F_\lambda(x) = \lambda x + (1-\lambda)f(x). \]

We show that if \(C\) is a bounded set, then for any \(\lambda \in (0,1)\), \(F_\lambda\) is uniformly asymptotically regular; i.e. given \(\varepsilon > 0\) there exists an \(N\) such that for all \(x \in C\) and for all \(n \geq N\), 
\[ \|F_\lambda^{n+1}(x) - F_\lambda^n(x)\| < \varepsilon. \]

A number of corollaries are derived concerning the convergence of the sequence of iterates \(\{F_\lambda^n(x)\}\) to a fixed point of \(f\) under certain additional assumptions on \(f\) and \(C\). The more important ones include:

**Corollary** If \(C\) is compact, then \(\{F_\lambda^n(x)\}\) converges to a fixed point of \(C\) for any \(x \in C\). (This generalizes a result originally due to Krasnoselski for uniformly convex spaces and extended by the first named author to strictly convex spaces.)

**Corollary** If \(f\) is affine and \(C\) is weakly compact, then \(\{F_\lambda^n(x)\}\) converges (strongly) to a fixed point of \(f\) for any \(x \in C\). (This generalizes a result due to Dotson for uniformly convex spaces.)

(Received October 17, 1977.)
We consider non-commutative generalizations of the Gelfand theory characterizing abelian C*-algebras in terms of locally compact Hausdorff spaces. The result is a type of representation of a C*-algebra as an algebra of sections of a C*-bundle, a type of fibre space each of whose fibres is a C*-algebra. A C*-bundle can be very non-locally trivial unlike ordinary fibre bundles, in fact the fibres can even vary in dimension from point to point over a connected base space. The representation of a C*-algebra as an algebra of sections of a C*-bundle is difficult to deal with when the primitive ideal space of the C*-algebra is not Hausdorff. In an attempt to circumvent this difficulty, we introduce the notion of a C*-bundle semigroup, a type of C*-bundle which in particular has a topological semigroup as its base space. Each C*-algebra will have a representation as the C*-algebra of compatible sections of a C*-bundle semigroup, and for C*-algebras of certain types, the representations expose a good deal of their structure. (Received October 17, 1977.)

Every pseudonormed function space $G(S)$, composed of scalar valued functions with domain $S$, has associated with it natural set functions defined on subsets of $S$. The capacities of potential theory are prime examples. These set functions are closely related to the filters, $F$, which give the topology on $G$, [Duke Math. J. 36(1969) pp. 709-714], and are actually restrictions of more basic set functions, $\mu_F$, which are constructed directly from $F$. Each $F$ is a filter of subsets of a linear space $X$ which has $S$ as a Hamel basis. For a subset $A$ of $X$, $\mu_F$ is simply defined as follows: $\mu_F(A) = \inf\{r \geq 0 \mid r(A) \in F\}$. The immediate importance of $\mu_F$ is that it locates singularities of certain classes of functions or determines those sets of the domain which can be ignored. This parallels the most important use of the classical capacities. However, $\mu_F$ may also arise from considerations completely apart from potential theory. For example, all measures, $\mu$, are but restrictions of a $\mu_F$ to subsets of $S$, where $F$ is a filter giving the topology on $L'(\mu, S)$. (Received October 17, 1977.)

Let $X$ be a real vector space ordered by a generating cone $K$, with $K$ giving an Archimedean order. In the case that $(X,K)$ has the diagonal property for relative uniform convergence, we give an explicit description of a neighborhood basis at 0 for the relative uniform topology $T_{ru}$. We show that if there is a countable neighborhood basis at 0 for $T_{ru}$, then $(X,K)$ has the diagonal property. Thus one can find a countable collection of sets of the type mentioned above which form a neighborhood basis at 0 for $T_{ru}$. The special properties of these neighborhoods, together with the fact that the countability of the family insures that addition will be continuous for $T_{ru}$, enables us to conclude the main result: If there exists a countable neighborhood basis at 0 for the relative uniform topology, then it is a vector topology. (Received October 17, 1977.)

The authors concern themselves with the following basic questions: (1) What is the minimal internal structure on the set of quantum states $S$ and what are reasonable measuring devices on $S$ which allow $S$ to be embedded in a Hilbert space model? (2) What is the minimal internal structure on $S$ for which a Kakutani-Mackey theorem will hold? The following answers are obtained: Theorem 1. Let $(G,*)$ be an abelian group admitting square roots. If $(G,(e),p)$ is a Mielnik probability space of dimension 2 with all bases of the form $B = \{b, b^{-1}\}$, $b \in G$ where $p(x,y) = q(x-y)$, and $q(x) = q(x^{-1})$, then $(G,\ast)$ can be embedded densely in a real Hilbert space $G$. Theorem 2: Let $(G,\ast)$ be an abelian group with square roots. If there is a sublattice $L(G)$ of subgroups of $G$ admitting an involution "$\ast$"
satisfying (1) the Kakutani-Mackey conditions and (2) $<x^*>$ is a maximal subgroup, $\forall x \in G$, and if there exists an isomorphism $J: (G, e) \rightarrow (\hat{G}, \ast)$ such that $J(<x>) = G_{<x^*>}$, then $(G, e)$ can be completed into a generalized real inner product space $G$. Here, $\hat{G} = \{ h \mid h$, homomorphism from $(G, e)$ to $(0, +)$ (the dyadic rationals)$\}$ and $G_{<x^*>} = \{ he \mid \ker h = <x^*>\}$. (Received October 17, 1977.)

*752-46-20 R. Rao Chivukula, University of Nebraska, Lincoln, Nebraska 68588. Bohr compactifications of products.

It is proved that the Bohr compactification of the product of a family of Hausdorff topological groups is (isomorphic to) the product of the Bohr compactifications. (Received October 17, 1977.)

*752-46-21 C.J. Seifert, University of Kansas, Lawrence, Kansas 66045. The Banach-Saks property in $L_p(\sum)$.

The Banach space is said to have the weak Banach-Saks property if every weakly null sequence admits a subsequence which has arithmetic means converging in norm.

Let $(\Omega, \Sigma, \mu)$ be a finite measure space and let $\sum$ be a Banach space. For $L_p(\Omega, \Sigma, \mu, \sum)$ (or $L_p(\sum)$) denotes the Banach space of all $\sum$-valued, $p^{th}$ power Bochner integrable functions. We study the conditions on $\sum$ which guarantee that $L_p(\sum)$ has the weak Banach-Saks property. (Received October 17, 1977.)


We prove the following characterization of infrabarreled spaces, with an application to the theory of spectral operators in locally convex spaces. Theorem. A locally convex space $E$ is infrabarreled if and only if the adjoint map $T \rightarrow T'$ is continuous for $L_b(E) \rightarrow L_b(E')$.

Corollary. Let $T$ be a bounded linear operator on a sequentially complete infrabarreled space $E$. If $T$ has a continuous operational calculus $F: C(\text{spec } T) \rightarrow L_b(E)$ then the operational calculus $F'(f) = (Ff)'$ for $T'$ is continuous for $C(\text{spec } T') \rightarrow L_b(E')$. [For general locally convex spaces, the properties of $F$, $F'$ and their associated spectral measures are developed in the authors Vector measures and scalar operators in locally convex spaces, Mich. Math. J., to appear.] (Received October 17, 1977.)

*752-46-23 STEVE WRIGHT, Dept. of Mathematical Sciences, Oakland University, Rochester, Michigan, 48063. Derivations on $C^*$-algebras. Preliminary report.

Definitions. Let $A$ be a unital $C^*$-algebra, $X$ a unital Banach $A$-module, $D: A \rightarrow X(A \rightarrow X^*)$ a derivation. $D$ is inner if $\exists x \in X(f \in X^*) \ni D(a) = xa - ax(fa - af)$, $\forall a \in A$. $D$ is strongly inner if this $x$ (this $f$) can be chosen in $\text{co}(D(u)u^* : u \in U(A))$, where $U(A)$ = unitary group of $A$ and $\sum S = o(X(X^*) \cup (X(X^*) - \text{closed convex hull of } S \subseteq X(X^*) \subseteq X^*)$. The set $\text{co}(D(u)u^* : u \in U(A))$ is called the hull of $D$, denoted hull $D$.

Theorem 1. Let $A$ and $X$ be as above. Let $H_s$ denote the set of all derivations $D: A \rightarrow X^*$ for which hull $D$ is norm-separable. Then $H_s$ is norm-closed and every element is strongly inner.

Theorem 2. Let $A$ and $X$ be as above. Let $H_{ws}$ denote the set of derivations $D: A \rightarrow X$ for which hull $D$ is weakly compact. Then $H_{ws}$ is norm-closed and every element is strongly inner.

Corollary. Let $A$ and $X$ be as above. Suppose $D: A \rightarrow X$ is the norm-limit of finite-rank derivations. Then $D$ is a strongly inner derivation. (Received October 17, 1977.)
The fractional derivative of $C^r$-periodic functions, the action of fractional derivatives of periodic distributions on test functions, and the fractional order of periodic distributions are defined.

Several theorems clarifying the properties of the fractional derivative are stated and examples exhibiting the not quite perfect relationship between fractional order and derivatives of measures are given. (Received October 17, 1977.)

E. Odell and M. Wage, University of Texas, Austin, Texas 78712 and Yale University, New Haven, Connecticut 06520. Weakly null sequences equivalent to the unit vector basis of $c_0$. Preliminary report.

Let $(x_n)$ be a normalized weakly null sequence in a Banach space. Then there is a subsequence $(x'_{n_k})$ of $(x_n)$ so that either (1) $(x'_{n_k})$ is equivalent to the unit vector basis of $c_0$ or (2) for all further subsequences $(x''_{n_k})$ and all choices of sign $\varepsilon_j = \pm 1$, $\sup_{1 \leq k \leq n} \| \varepsilon_j x''_{n_k} \| = \infty$. (Received October 17, 1977.)


We give necessary and sufficient conditions for an ultraweakly continuous compact abelian group of automorphisms of a von Neumann algebra to be inner. These conditions depend on the structure of the lattice of projections of the center of the fixed-point algebra of the automorphism group.

Arveson has shown (J. Functional Analysis 15(1974), 217-243) that a one-parameter group of automorphisms of a von Neumann algebra is inner, with the implementing unitary group having positive spectrum, if and only if a spectrum condition holds. As an application of our theorem, we show that an analogous result holds for compact abelian groups of automorphisms when positive is suitably interpreted. Some of the results to be discussed in this talk were originally announced in these Notices (24(1977): A-238), where more detailed statements of them can be found. (Received October 17, 1977.)

Edwin M. Wolf, East Carolina University, Greenville, North Carolina 27834. Smoothness properties of functions in $R^2(X)$ at certain boundary points. Preliminary report.

Denote by $R^2(X)$ the closure in the $L^2(X)$ norm of the rational functions with poles off the compact set $X$ in the complex plane. Suppose that $L$ is a line segment with an end point $x \in \partial X$ such that $L \setminus \{x\} \subset \text{Int} X$. We prove the following result which is related to our earlier result (Abstract 76T-B39, these Notices 23(1976), A-279)

**Theorem.** Suppose that $x \in \partial X$ is a bounded point evaluation for $R^2(X)$ which is represented by the function $g \in L^2(X)$. Let $\varphi$ be an admissible function such that $g(z)\varphi(|z-x|^{-1}) \in L^2(X)$. Then for any $\varepsilon > 0$ there is a $\delta > 0$ such that if $f \in H^2(X)$ and $y \in L \cap (0, \delta)$, $|f(y) - f(x)| \leq \varepsilon \varphi(|y-x|) \| f \|_2$. We use theorems of Fernström and Polking in their work on bounded point evaluations and approximation in $L^P$ by solutions of elliptic partial differential equations. (Received October 14, 1977.)

Steven F. Bell, Florida State University, Tallahassee, Florida 32306. Conditional basic sequences in Fréchet spaces. Preliminary report.

Let space mean locally convex non-nuclear Fréchet space and let CBS stand for conditional basic sequences. **Theorems:** 1. Non-Montel spaces have CBS. 2. Non-Schwartz spaces have CBS if each Montel space with a basis has CBS. 3. There is a technical condition which, if satisfied by the non-Schwartz space $E$, implies that $E$ has CBS. In particular, this condition is satisfied if $E$ is a $X$-Köthe sequence space, where $X$ is some Banach space with an unconditional basis; or if $E$ is a subspace of a $L^p$-Köthe space, where $p$ is finite. (Received October 18, 1977.)
If \( \epsilon > 0 \) and \( 0 < \delta < \epsilon/4 \) then a Banach space \( X \) is said to have the \((\epsilon,\delta)\)-neighborly tree property if and only if its unit ball contains a sequence of \((\epsilon,\delta)\) trees \( \{T_n\} \), such that if \( x_{n,k,j} \) denotes the \( j \)th entry of the \( k \)th column of \( T_n \), then for all \( k \) and \( j \), there is a ball of radius \( \delta \) which contains \( \{x_{n,k,j} : n \in \mathbb{N}\} \).

There is a Banach space which has the \((2,\delta)\)-neighborly tree property for all \( 0 < \delta < \frac{1}{2} \), but which contains no infinite tree. For dual spaces, having the \((\epsilon,\delta)\)-neighborly tree property for some \( \epsilon > 0 \) is equivalent to not having the Radon-Nikodým property, but this result is not known to be true for Banach spaces in general. (Received October 18, 1977.)

A representation for normed vector lattices as spaces of extended real-valued continuous functions is considered. By generalizing H. H. Schaefer's concept of a topological orthogonal system (relaxing the orthogonality condition) one obtains carrier spaces other than those which are direct sums of compact spaces. This representation, while retaining many features of Schaefer's representation theorem, applies to a broader class of spaces. (Received October 18, 1977.)

Let \( V \) be a \( \Phi \)-subalgebra of \( D(K) \), the extended real-valued continuous functions \( f \) on a compact set \( K \) which are finite on a dense subset \( A_f \) of \( K \). It is shown that a net converges relatively uniformly in \( V \) if and only if it is order bounded and converges uniformly on the compact subsets of \( A_f \) for some \( f \) in \( V \). (Thus relative uniform convergence on \( V \) is realized as a convergence space inductive limit of factors related to \( C(A_f) \).

Relative uniform convergence for \( \Phi \)-algebras. Preliminary report.

(Received October 18, 1977.)
(\mathcal{W}_i^n)_{i \in I} is taken as the usual supremum of \{\rho_i \mid i \in I\}, and \rho' is the associate norm of \rho. But (\mathcal{W}_i^n)_{i \in I} = (\mathcal{W}_i^n)_{i \in I} when all \rho_i have the Fatou property. The collection of all function norms having the Fatou property form a lattice, whereas the lattice distributive law proves to be false in general. A direct proof for the fact that \rho_1 A \rho_2 has the Fatou property whenever both \rho_1 and \rho_2 have is provided. Finally an integral of a measurable family of seminorms \{\rho_y \mid y \in Y\} is studied to provide function seminorm on the product measurable space X \times Y. (Received October 18, 1977.)


The compact operators from the Banach space X into a Banach space Y attain their norm on the unit cell S_{X**} of X** when S_{X**} is w*-sequentially compact. Consequently, every operator with reflexive domain X is the sum of two operators which attain their norm in X. For Banach spaces X such that the quotient space X**/X is separable and Y the space of absolutely summable sequences, a proper subset \mathcal{P}_0 of the finite rank operators from X into Y is exhibited. The set \mathcal{P}_0 is shown to consist of operators which attain their norm and to be norm-dense in the operator space. (Received October 18, 1977.)


Let \((\Omega, \Sigma, \mu)\) be a finite measure space, E a Banach space, and M be the space of all strongly measurable E valued functions on \(\Omega\). For f and g in M, define:

\[
\rho(f, g) = \int_\Omega \frac{||f - g||}{1 + ||f - g||} \, d\mu.
\]

If the functions which are equal almost everywhere are identified, then (M, \rho) is a metric space.

In this talk we will characterize the linear isometries of the metric space (M, \rho). (Received October 18, 1977.)


Let A be a strictly lower (upper) triangular n by n matrix whose principal sub (super)-diagonal entries are nonzero. Every matrix which commutes with A is a polynomial in A. We extend this result to infinite matrices of the same type by showing that the commutant of such a matrix is algebraically isomorphic to a space of power series. It follows that operators on locally convex spaces which possess strictly triangular matrix representations will have abelian commutants. This last statement applies to weighted shift operators, backward shifts, and generalized integration operators. (Received October 18, 1977.)

*752-46-38 ROGER R. SMITH, Texas A&M University, College Station, Texas 77843. M-ideals in function algebras.

A characterization of the M-ideals in a function algebra is given. The main result is that a closed subspace I is an M-ideal if and only if it is an algebraic ideal with a bounded approximate identity. (Received October, 18, 1977.)

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**47 ▶ Operator Theory**


Studied here: $C^0$ semigroups $R_t$, which can be represented as right translations on $B$-spaces $\mathcal{I}$ of distributional 1st derivatives of continuous functions on $[0, \infty)$ with values in $B$-space $C$. (1) The weighted translations of Embry and Lambert, and their higher dimensional analogues are examples. (II) So is any semigroup with a left inverse semigroup but which is nowhere invertible. The infinitesimal generator is described completely. $[R_t]'$ is represented as a $B$-algebra of 2nd derivatives of continuous functions with values in $\mathcal{I}(C)$, acting on $L^2$ by convolution. Here $R_t$ is a point mass at $t$. When $\dim C = 1$, $[R_t]'$ is Abelian, its members have zero kernel and connected spectrum. Growth rates are studied and if exponential (yes in case II) the Laplace transform represents $\mathcal{I}$ as a space of analytic functions on a right $1/2$-plane, $R_t$ as multiplication by $e^{-tz}$, elements of $[R_t]'$ as multiplications by bounded and analytic functions. Large lattices of $[R_t]'$-invariant and $[R_t]$-invariant subspaces appear naturally. (Received August 4, 1977.) (Author introduced by Stephen Campbell).


The following results supplement those in [Abstract *77T-B13, these Notices, 24(1977), A-423]. Let $U(A)$ denote the unitary group of a C*-algebra $A$ (with identity). Let $U(X)$ denote the unitary orbit of an element $X$ in $A$, and let $\pi: U(A) \rightarrow U(X)$ be given by $\pi(U) = U^*XU$. A (local unitary) cross section for $X$ is a pair $(B, \varphi)$, where $B \subset U(X)$ is relatively open, $\varphi: B \rightarrow U(A)$ is norm continuous, $\varphi(X) = 1$, and $\pi \varphi = 1_B$. If $X$ has a cross section, then $X$ satisfies property (P): if $(U_n) \subset U(A)$ and $\lim \|U_n^*XU_n - X\| = 0$, then there exists $(W_n) \subset U(A)$ such that $\lim \|W_n - 1\| = 0$ and for each $n$, $W_n^*W_n = U_n^*XU_n^*$. 

**Theorem 1.** If $X$ is in $A$ and $C^*(X)$ is finite dimensional, then $X$ has a cross section.

Although the converse of Theorem 1 is false, the following result holds for the case $A = L(H)$, the algebra of all bounded linear operators on a separable Hilbert space $H$.

**Theorem 2.** The following are equivalent for $T$ in $L(H)$: (i) $T$ satisfies property (P); (ii) $U(T)$ is closed; (iii) $C^*(T)$ is finite dimensional; (iv) $T \approx R \oplus S \oplus S \oplus \ldots \oplus S$ where $R$ and $S$ are operators on finite dimensional spaces; (v) $T$ has a cross section.

(The equivalence of (ii), (iii), and (iv) is a result of D. Voiculescu [Rev. Roum. Math. Pures et Appl., 21(1976), 97-113, Proposition 2.4].) (Received September 23, 1977.)

*752-47-3* C.W. Groetsch, University of Cincinnati, Cincinnati, Ohio 45221. Error bounds for approximations to the generalized inverse. Preliminary report.

An error bound for a general class of approximations (see Chapter III of the author's monograph "Generalized Inverses of Linear Operators: Representation and Approximation", Dekker, 1977) to the generalized inverse is provided. (Received September 12, 1977.)

*752-47-4* Ray C. Shiflett, California State University, Fullerton, Fullerton, California 92634. A Characterization of Doubly Stochastic Operators Induced by Measure-Preserving Transformations.

Let $(X, \mathcal{A}, \mu)$ be the unit interval with the Lebesgue structure. An operator $T$ is doubly stochastic if it is a positive contraction on $L^2(X)$, with $T^* = T^*$. We say $T$ is induced by a measure-preserving map $\phi$ of $X$ to $X$ if $Tf = f \circ \phi$ for all $f$ in $L^2(X)$.

**Theorem:** $T$ is induced by a measure-preserving map $\phi$ if and only if $\|Tf\|^2 = \|f\|^2$ for some non-negative one to one $f$ in $L^2(X)$.

Corollaries:
1. $T$ is an isometry if and only if $\|Tf\|^2 = \|f\|^2$ for some non-negative one to one $f$ in $L^2(X)$.
2. If $f$ and $Tf$ are increasing, then $Tg = g$ for all $g$ in $L^2(X)$.
3. Let $f(x) = x$ and $\phi$ be measure-preserving. $Tf = \phi$ if and only if $T$ is an isometry.

**Theorem:** An invertible measure-preserving map $\phi$ is the fixed point for no non-trivial doubly stochastic operator.

**Theorem:** If $f$ is a non-negative $L^2(X)$ function and if $Tf = f$ for exactly one operator $T$, then that $T$ is an extreme operator. (Received September 19, 1977.)
Lifting the Commutant of an Analytic Toeplitz Operator.

For \( f \) a bounded analytic function on the unit disk \( D \), let \( T_f \) and \( M_f \) denote the operators, on \( H^2(D) \) and \( L^2(D) \) respectively, of multiplication by \( f \). It is well known that \( T_f \) is subnormal and \( M_f \) is its minimal normal extension. We will say that the commutant of \( T_f \), denoted \( \{T_f\}' \), lifts if each \( A \) in \( \{T_f\}' \) has an extension \( \tilde{A} \) in \( \{M_f\}' \). Choose a complex number \( \lambda \) so that \( f - \lambda = \sigma g \) where \( g \) is outer and \( \sigma \) is a non-constant inner function.

Theorem: \( \{T_f\}' \) lifts if and only if there is a constant \( C \) so that \( \|A\| \leq C\|AT_g^N\| \) for all \( A \) in \( \{T_f\}' \) and all positive integers \( n \). Moreover, in this case, we have \( \|\tilde{A}\| \leq C\|A\| \).

Corollary: If \( \{T_f\}' \) lifts for some non-constant function \( f \), then \( T_f \) does not commute with any non-zero compact operators.

Corollary: If \( f \) is a covering map of the disk onto a bounded domain, then \( \{T_f\}' \) lifts isometrically. (Received September 19, 1977.)

Bounded integral operators.

This is a report on bounded operators \( A \), between \( L^2 \) spaces, induced by kernels \( k \) via equations such as \( f(x) = \int k(x,y)g(y)dy \). The theory has been growing in the last ten years; among the principal contributors to it are J. Weidmann and V. B. Korotkov. Several old questions have recently received definitive and satisfactory answers. Sample (V. B. Korotkov): over non-atomic measure spaces every operator unitarily equivalent to \( A \) is an integral operator if and only if \( A \) is a Hilbert-Schmidt operator. Sample (V. S. Sunder): over non-atomic measure spaces some operator unitarily equivalent to \( A \) is an integral operator if and only if \( 0 \) belongs to the right essential spectrum of \( A \). Application: a unilateral shift of finite multiplicity is not unitarily equivalent to an integral operator, but one of infinite multiplicity is. Hope: the analytic techniques will permit concrete calculations, such as, for instance, the determination of all invariant subspaces of some operators. Several interesting problems are still open. Sample: does every bounded operator have an absolutely bounded matrix? (Received September 21, 1977.)

A von Neumann algebra is determined by its logic of projections.

There are two basic approaches to the foundations of quantum mechanics: the first is based upon the logic \( \mathcal{L} \) of questions associated to a physical system \( \mathcal{J} \); and the second is based upon the partial algebra \( \mathcal{O} \) of continuous real valued observables associated to \( \mathcal{J} \). It is usual to assume either that \( \mathcal{L} \) is isomorphic to the logic \( \text{Proj}(\mathcal{S}) \) of projections in a von Neumann algebra \( \mathcal{S} \), or that \( \mathcal{O} \) is isomorphic to the partial algebra \( \text{Herm}(\mathcal{D}) \) of Hermitian elements of a \( C^* \)-algebra \( \mathcal{D} \). By combining together work of Lodkin and Kadison, we can provide a satisfactory resolution of the question of determining to what extent \( \text{Proj}(\mathcal{S}) \) determines \( \mathcal{S} \). The analogous question for \( C^* \)-algebras remains open.

Theorem. If \( \text{Proj}(\mathcal{S}_1) \cong \text{Proj}(\mathcal{S}_2) \), then \( \mathcal{S}_1 \cong \mathcal{S}_1' \oplus \mathcal{S}_1'' \) and \( \mathcal{S}_2 \cong \mathcal{S}_2' \oplus \mathcal{S}_2'' \) and \( \mathcal{S}_1' \cong \mathcal{S}_2' \) and \( \mathcal{S}_1'' \) is anti-isomorphic to \( \mathcal{S}_2'' \). (Received September 23, 1977.)

The spectrum of a Toeplitz operator with a multiplicatively periodic symbol.

Let \( \phi \) be a continuous complex function on \( (-\infty,0) \cup (0,\infty) \) which satisfies the equation \( \phi(2x) = \phi(x) \) and let \( T_\phi \) be the Toeplitz operator on \( H^2 \) of the upper half plane with...
The function $\psi$ is determined by the two closed curves $\psi_- = \psi|[-2,-1]$ and $\psi_+ = \psi|[1,2]$.

Theorem. A point $\lambda$ is in the spectrum of $T_{\psi}$ if and only if one of the following three conditions holds: (1) the point $\lambda$ is in the range of $\psi$; (2) the point $\lambda$ is not in the range of $\psi$ and the sum of the winding numbers of $\psi_+$ and $\psi_-$ about $\lambda$ is non-zero; (3) the point $\lambda$ is not in the range of $\psi$ and the difference of the average arguments of $\psi_+$ and $\psi_-$ about $\lambda$ with respect to the measure $dx/|x|$ is equal to $\pi$ modulo integral multiples of $2\pi$. (Received September 23, 1977.)

Gary D. Faulkner* and James E. Huneycutt, Jr., North Carolina State University, Raleigh, North Carolina 27607. The canonical form of a scalar operator in a Banach space.

For a finite scalar measure $\mu$, the operator $Q:L^m(\mu) \to L^m(\mu)$ defined by $Qf(s) = sf(s)$ is a scalar operator with resolution $E(A)g = l_A^{-1}g$. For $g = 1$ the vector valued measure $\mu(A) = E(A)1 = l_A^{-1}1$ has the property that $\overline{co}(R(\mu))$ has nonvoid interior. We show that if $A = \int \lambda dE(\lambda)$ is a scalar operator for which there exists some $g$ for which $\overline{co}(R(E_g))$ has nonvoid interior, then $A$ is similar to $Q$ on a quotient space of a suitably constructed $L^m$ space. (Received September 26, 1977.)

Peter Rosenthal, University of Toronto, Toronto, Canada. The transitive algebra problem; recent results and possible approaches.

After a quick summary of the history of the problem, a few recent results (of several authors) will be described in detail. Then an outline will be given of some approaches to the problem which Nordgren, Radjavi and the speaker have attempted. These approaches have been unsuccessful so far; perhaps one of the listeners will be able to carry them further. (Received September 28, 1977.)

Lawrence G. Brown, Purdue University, W. Lafayette, Indiana 47907. Topics related to extensions of C*-algebras. Preliminary report.

The study of operators with compact self-commutators can be reformulated as the study of C*-algebra extensions of the ideal of compact operators on Hilbert space by $C(X)$, where $X$, the essential spectrum of the operator involved is a compact subset of the plane. For this reason extensions of the compacts by a general separable C*-algebra, $A$, have also been studied. This leads to an abelian semigroup, $\text{Ext}(A)$. $\text{Ext}(A)$ is often, but not always, a group. When $A = C(X)$, $\text{Ext}(A)$ can be identified as the $K$-homology of $X$; and in general $\text{Ext}(A)$ is closely related to the $K$-theory of $A$. Because of the fact, proved by C. Olsen and W. Zame, that many tensor product C*-algebras are singly generated, $\text{Ext}(A)$ is relevant to operator theory for general $A$. In addition to $K$-theory, $\text{Ext}$ is related to index theory, quasidiagonality, commutator theory, and the structure of certain group C*-algebras. The talk will be a survey, emphasizing open questions. (Received September 28, 1977.)

Douglas N. Clark, University of Georgia, Athens, Georgia 30602. Concrete model theory.

Concrete model theory consists of the explicit computation of the Sz.-Nagy-Foias characteristic function and hence the canonical model of specific contraction operators. Actually, "concrete" refers to computing, in addition, the unitary operator that makes the contraction unitarily equivalent to its model. Several operators have yielded to this attack and they give rise to interesting theories, but each example is constructed in such a way as to make the concrete model theory easily obtainable. In this talk, we discuss the possibility of computing the model theory for operators chosen because they are interesting in their own right. As an example, the author's recent work on Toeplitz operators of the form $T_{\phi/\psi}$, where $\phi$ and $\psi$ are finite Blaschke products, is discussed. (Received October 5, 1977.)
The author will discuss a new, short proof (by himself, Apostol, and Foiaş) of the theorem that every quasinilpotent operator on a separable, infinite dimensional, complex Hilbert space is a norm-limit of nilpotent operators. He will also make some remarks about the hypertransitive operator problem. (Received October 6, 1977.)

Let C be a convex subset of a nuclear locally convex space that is also an F-space. Suppose T: C → C is non-expansive and \( \{v_n\} \) is given by the Mann iteration process. It is shown that if \( \{v_n\} \) is bounded, T has a fixed point. Also, a sequence \( \{y_n\} \) can be constructed such that \( y_n \rightarrow y \) weakly where \( Ty = y \). If C is a linear subspace and T is linear, then \( \lim y_n = y \). (Received October 11, 1977.)

Let T be a bounded linear operator on a Banach space and let \( f \) be a holomorphic function defined on a neighbourhood of the spectrum, \( \sigma(T) \), of T. Let \( \sigma_2(T) = \{ \lambda: \lambda I - T \) is not a semi-Fredholm operator\} and \( \sigma_5(T) = \) Browder's limit point spectrum. We show that the mappings \( T \rightarrow \sigma_2(T) \) and \( T \rightarrow \sigma_5(T) \) are upper semicontinuous while the mapping \( T \rightarrow \sigma_1(T) \) may not be so. We use these results to show that \( f(\sigma_5(T)) = \sigma_2(f(T)) \) and that \( f(\sigma_2(T)) \subseteq \sigma_2(f(T)) \) and this inclusion may be proper even when \( f \) is a polynomial function. We give some conditions under which \( f(\sigma_2(T)) = \sigma_2(f(T)) \). We also give some results on the stability of essential spectra of T when T is perturbed by a commuting quasi-nilpotent operator or by a compact operator. (\( \sigma_1(T) = \{ \lambda: \lambda I - T \) does not have closed range\}. (Received October 11, 1977.)

By using a result of G.R. Allan and a result due to the first named author, it is shown that a completely non-normal cohyponormal operator whose right spectrum has planar Lebesgue measure zero has a dense set of cyclic vectors. In case T is an irreducible cohyponormal operator with \( TT^* - T^*T = (,f) \) if a result of N. Aronszajn is used to show that the vector f is not cyclic for both T and T* if \( TT^* \) has singular spectrum or if \( T^* \) has singular spectrum other than a zero eigenvalue. An example is given to show f can be cyclic for both T and T* if T has a non-zero kernel. (Received October 13, 1977.)

Recently, Ward et. al. (Proc. of A.M.S., 60, (1976) p. 92) showed that every Riesz operator R on a Hilbert space has a decomposition \( R = C + Q \), where C is compact and both Q and CQ - QC are quasinilpotent. We use a Riesz theory in Banach algebras as developed by Smyth (Math. Z, 145, (1975), p. 145) to extend this result to C*-algebras. (Received October 13, 1977.)
The operator equations (1)_a: \( F_a(\lambda, w, z) = c \cdot T(\lambda, z) \), \( a = \theta \) or \( \alpha \), are considered, where \( w \) and \( z \) are elements of a real Banach space \( \mathcal{B} \), \( \lambda \) and \( c \) are real numbers, the operator \( T(\cdot, z) : \mathbb{R} \rightarrow \mathcal{B} \) is arbitrary, and the operators \( F_a(\cdot, \cdot, z) : \mathbb{R} \times \mathcal{D}_a \rightarrow \mathcal{B} \) satisfy the identity \( F_0(\lambda, w, z) = ||w||^2 F_\alpha(\lambda, w/||w||^2, z) \) for \( w \neq \theta \) and \( F_0(\lambda, \theta, z) = \theta \) where \( \mathcal{D}_0 \) is an open neighborhood of \( \alpha \) in \( \mathcal{B} \).

The results of Rabinowitz [J. Diff. Eqns. 14 (1973), 462-475] and Toland [Proc Roy Soc Edin. 73A (1975), 137-147] for a special class of homogeneous equations are generalized showing that (i) homogeneous equations \( (1)_\alpha \) (\( \alpha = 0 \)) are symmetric with respect to the existence, continuity, and positivity of solutions bifurcating from \( a \) at \( \lambda = \lambda_0 \); and (ii) nonhomogeneous equations \( (1)_a \) for \( \lambda \) fixed, are symmetric with respect to existence, continuity, and positivity of solutions on an interval \( [\alpha, c_0] \).

The results also hold for multiparametered equations where \( \lambda \) and \( c \) are vectors. These principles allow sublinear and singular equations to be converted to superlinear equations. A generalization of a result by Sather [Arch Ratl Mech Anal 36 (1970), 47-64] can be applied to obtain continuous solutions. (Received October 14, 1977.)

752-47-19 Joseph G. Stampffli, Indiana University, Bloomington, IN 47401

Recent developments in the theory of hyponormal operators.

A survey on recent developments in the theory of hyponormal operators with particular emphasis on the invariant subspace problem. (Received October 14, 1977.)

752-47-20 J. William Helton, University of California, San Diego, La Jolla, California 92093

Operator theory and electrical engineering.

The canonical models branch and indeed much operator theory involving complex variables has strong connections with theoretical engineering. First of all there are existing theorems in operator theory which correspond closely to existing theorems in systems theory. Secondly, the techniques mathematicians use are appropriate for studying many theoretical engineering problems. The talk addresses the second assertion.

I will try to give some feel for types of mathematical questions which come up and then concentrate on one particular topic. Probably the topic will be gain equalization (passive) in amplifiers. It is an area which gives rise to a heirarchy of mathematical problems. They are nonlinear problems in \( H^p(\mathbb{C}^n) \). The best tools to use on these problems are the linear operator theoretic ones which deal with matrix valued analytic functions (e.g., various factorizations, the commutant lifting theorem). The talk will describe existing theory and the type of (hard) mathematical problems which are left to be solved before one can get a good theory of gain equalization. (Received October 14, 1977.)

752-47-21 Nazanin Azarnia, Miami University, Hamilton, Ohio 45011

On the dense operators in a KH-Module. Preliminary report.

H. Takemoto proves (Tohoko Math. J., 27(1975), 413-435) that an unbounded operator \( A \) on a KH-Module \( H \) is decomposable if \( D(A) \), the domain of \( A \), is dense in \( H \). He then obtains results analogous to those known for unbounded operators in a Hilbert space. In this paper we prove that the density condition on \( D(A) \) reduces the operator to a very special type. We then apply this result to the modular operator associated with a faithful normal semi-finite center valued weight on a von Neumann algebra. We prove that with density condition imposed on the modular operator the von Neumann algebra has to be the product of a semi-finite algebra and at most a finite number of properly infinite factors. (Received October 14, 1977.)

752-47-22 Milton Rosenberg, University of Montana, Missoula, Montana 59812

Contraction operator lemma from the study of wide-sense Markov processes.

Let \( W \) be a Hilbert space with an o.n. basis \( (z_i)_{i=\mathbb{Q}} \). Contraction Lemma. Let \( (A_k)_{k=1}^\infty \) be an arbitrary sequence of contractions on \( W \) to \( W \) (\( |A_k|_B \leq 1 \)), \( (\lambda_k)_{k=1}^\infty \) be a
sequence of orthogonal projections on $\mathcal{K}$ such that for $k \geq 1 \quad J_{k+1} = A_k = A_k \mathcal{J}_k$ (e.g., take $\mathcal{J}_k = I$). Then $\mathbb{V}_n \geq 1$, the hermitian operator $H : \mathfrak{h}^{n+1} \to \mathfrak{h}^{n+1}$ is $\geq 0$, bounded, where $H_n = [H_n]_{i,j=1}^{n+1}$, $H_{i,j} = J_i$, $H_{i,j} = A_{i-1}A_i \ldots A_j$ for $n+1 \geq i > j \geq 1$. Cor. 1. Let $(B_t)_{t \in \mathbb{S}}$ be a family of contractions such that for $t \geq s \geq r \quad B_t B_s = B_r$. Then $\mathbb{V}(t,s) = B_{t-s}$ for $t \geq s$, $= B_{s-t}$ for $t < s$. Then $\mathbb{V}$ is a positive kernel:

$$
\mathbb{V}(t,s) = \mathbb{V}(s,t) = 0, \quad \text{if } t \neq \pm s \quad \text{and } \quad \mathbb{V}(t,s) = 1, \quad \text{if } t = s.
$$

Cor. 2. Let $\mathbb{V}(t,s)$ be the matrix of $\mathbb{V}(t,s)$ wrt $(\mathcal{J}_k)$. Then $\mathbb{V}(\mathcal{J}_k)$ is a positive kernel:

$$
\mathbb{V}(\mathcal{J}_1, \mathcal{J}_n) = 0, \quad \text{if } 1 < \mathcal{J}_n \leq \mathbb{K}.
$$

Cor. 3. Let $\mathbb{V}(\mathcal{J}_1)_{t \to 0}$ be a semigroup (discrete or cont.) on $\mathcal{K}$.

$\text{Self-adjointness of certain second order differential operators on a Riemannian manifold.}$

Preliminary report.

Let $M$ be a complete, connected, non-compact $n$-dimensional Riemannian manifold whose sectional curvatures satisfy: $0 < K(\sigma) \leq \lambda$. Let $T$ be a 2nd order differential operator on $M$ which is given locally by

$$
(g(x))^{-1/2} \sum_{j,k=1}^n (a_j - i b_j(x)) a_{jk}(x) (g(x))^{1/2} \cdot (a_k - i b_k(x)),
$$

where $a_j = \frac{\partial}{\partial x_j}$ and $(g(x))^{1/2} dx$ is the local volume element derived from the metric. This note gives conditions which guarantee that $L = -T + q$ is essentially self adjoint in $L^2(M)$ — here $q : M \to \mathbb{R}$ is bounded below and in $L^2_{\text{loc}}(M)$.

To prove a similar result in $\mathbb{R}^n$ Devinatz employed a distributional inequality of T. Kato and a maximum principle of W. Littman. For the manifold result much of the proof is local and uses these two results also. However, two additional results are needed: one is a global version of the maximum principle and the other provides uniform bounds on the local components of the metric valid on any normal coordinate ball of radius $\pi^{1/2}$. (Received October 14, 1977.)

Fred R. Weissler, The University of Texas, Austin, Texas 78712. Two-point inequalities, The Hermite semigroup, and the Gauss-Weierstrass semigroup.

Let $e^{-2H}$ be the Hermite semigroup and $\mu$ Gauss measure on $\mathbb{R}$. If $1 \leq p \leq q < \infty$, then $e^{-2H}$ is a bounded map from $L^p(\mu)$ into $L^q(\mu)$ if and only if

$$
|p-2-(q-2)e^{-2x}| \leq |p-q|e^{-2x}.
$$

In this case, except for the values $3/2 < p < q < 2$ and $2 < p < q < 3$, it is proved that $e^{-2H}$ is indeed a contraction from $L^p(\mu)$ into $L^q(\mu)$. This information and a formula relating the Hermite semigroup and Gauss-Weierstrass semigroup $e^{-2M}$ are used to calculate precise norms for $e^{-2M} : L^p(dx) \to L^q(dx)$, $1 < p \leq q < \infty$ and $\text{Re } x > 0$, but excluding the same values of $p$ and $q$ as above. (Received October 17, 1977.)

Joel D. Pincus, State University of New York, Stony Brook, N.Y. 11794. Symmetric singular integral operators with arbitrary deficiency indices. One dimensional singular integral operators with discontinuous coefficients furnish natural examples where symbols alone do not suffice for index determination. This fact is related to the study of unbounded Weiner-Hopf or Toeplitz operators with real square integrable symbols which define symmetric operators on $L^2(\mathbb{R})$ and to the study of singular integral operators of the form $L \xi(\lambda) = A(\lambda) \mathcal{X}(\lambda) + \frac{1}{\pi i} \int \frac{K(\lambda)K(t)}{t - \lambda} \mathcal{X}(\lambda) dt$ with $A(\lambda)$ real and measurable and $\int |K(\lambda)|^2 (1 + \lambda^2)^{-1} d\lambda < \infty$. The deficiency spaces of the associated symmetric operators are determined explicitly through the construction of scattering problems in a large Hilbert space which contains $L^2(\mathbb{R})$. Although the Cayley transforms of these symmetric operators do not have locally sectorial symbols...
the phase shifts of such scattering problems furnish arguments for the symbols whose variation (properly understood) gives the index. All of these results emerge from a study of certain associated almost commuting algebras of operators defined on the large Hilbert space. (Received October 17, 1977.)

*752-47-26  Thomas I. Seidman, University of Maryland, Baltimore County, Baltimore, Md. 21228.

A Class of Approximation Schemes for Ill-Posed Operator Equations.

Let $X$ be a uniformly convex Banach space, $Y$ a Hausdorff space and $F$ a mapping from a domain $D$ in $X$ to $Y$ whose range need not be closed in $Y$. Suppose $b$ in $Y$ is such that there is a unique minimal-norm solution $x_*$ of

$$F(x) = b.$$  

We consider subsets $B_k$ in $Y$ and approximations $F_k$ to $F$ and find appropriate interpretations of: $B_k \rightarrow \{b\}$, $F_k \rightarrow F$ such that any sequence $x_k$ given by

$$F_k(x_k) \in B_k \text{ with } ||x_k|| \leq \min$$

must then converge to $x_*$. Some applications are noted in which the $F_k$ represent computational approximations to $F$ for which (2) will be finitary. (Received October 14, 1977.)

*752-47-27  R. KENT NAGLE and KAREN SINGKOFER, University of South Florida, Tampa, Florida 33620.


We are concerned with the equation $Lx + N(x) = 0$ where $x$ is an element of a real Hilbert space, $H$, $L$ is a densely defined linear operator generated by a coercive bilinear form, and $N$ is a nonlinear operator such that $D(L) \subseteq D(N)$. Alternative methods are used to study this equation where $N$ is hemicontinuous on a dense subspace, $H_1$, of $H$ and satisfies a condition somewhat weaker than monotonicity, namely

$$(N(x) - N(y), x - y) \geq -a||x-y||^2 - b||x-y||^2$$

for $a, b$ positive constants, $x, y \in H_1$ with $||\cdot||_1$ and $||\cdot||$ the norms in $H_1$ and $H$, respectively. It is shown that there always exists an equivalent finite-dimensional alternative problem. The conditions imposed on $N$ allow for certain nonlinearities involving the derivatives of $x$ to be considered as well as nonlinearities in $x$ itself. These results are applied to boundary value problems for ordinary and partial differential equations. (Received October 17, 1977.)

*752-47-28  RONALD E. BRUCK, University of Southern California, Los Angeles, California 90007.

Strong convergence of the farthest closest point algorithm. Preliminary report.

Let $\Gamma$ be a finite family of nonempty closed convex subsets of a real Hilbert space $H$ with the property that $-C \cap \Gamma$ whenever $C \in \Gamma$, and let $P_C$ denote the proximity map of $H$ on $C$. Define a sequence

$$\{x_n\} \text{ by: } x_1 = x \text{ arbitrary;} \text{ given } x_n, \text{ let } x_{n+1} = P_{C'_n}(x_n), \text{ where } C'_n \text{ is any set in } \Gamma \text{ such that }$$

$$|x_n - P_{C'_n}(x_n)| = \max \{|x_n - P_{C}(x_n)|: C \in \Gamma\}. \text{ Theorem. } \{x_n\} \text{ converges strongly to a point of } \bigcap \Gamma.$$

Idea of the proof: put $\phi(x) = \max \text{ dist}(x, C)^2$, note that $x_{n+1} \in x_n - \delta \phi(x_n)$, $\phi$ is even and convex, $C \in \Gamma$, and appeal to a general theorem on convergence of the method of steepest descent for even convex functions. Some of the difficulties in extending this result to Banach spaces are discussed. (Received October 17, 1977.)

*752-47-29  Brian W. McEnnis, University of Missouri, Rolla, Missouri 65401.

Purely Contractive Analytic Functions and Characteristic Functions of Non-Contractions.

The characteristic function of a bounded operator $T$ is an analytic function $\Theta_T$ which takes values that are continuous operators between two Krein spaces $D_T$ and $D_{T^*}$. (A Krein space is a generalization of a Hilbert space, with an inner product $[\cdot, \cdot]$ that need not be positive definite.)

Suppose $\Theta$ is a function that is analytic in the open unit disk $D$ and which takes values that are continuous operators between Krein spaces $D$ and $D_{a*}$. $\Theta$ is said to be purely contractive if $[\Theta(\lambda)a, \Theta(\lambda)a] < [a, a]$ (where $a \in D$, $a \neq 0$) and if

$$[\Theta(\lambda)^*a, \Theta(\lambda)^*a] < [a^*, a^*]$$

(where $a \in D_{a*}$, $a \neq 0$) for all $\lambda \in D$.

Theorem $\Theta$ coincides with the characteristic function $\Theta_T$ of some operator $T$ if and only if $\Theta$ is purely contractive.

This generalizes a result of Sz.-Nagy and Foia for contractions. The proof is based on a theorem of J. Ball (J. Math. Anal. Appl., 52(1975), 235-254) which gives more complicated necessary and sufficient conditions for $\Theta$ to be of the form $\Theta_T$. (Received October 18, 1977.)
An application of the minimum principle to the study of essential self-adjointness of Dirac operators. Preliminary report.

Let $H_0$ be the free particle Dirac operator defined, at least formally, in the usual manner. Next define, again formally, the interaction between the electron and the nucleus in a one-electron ion with atomic number $e$ by

$$V_0(e) = -\frac{e}{r}, \quad r = r(x) = \sqrt{x_1^2 + x_2^2 + x_3^2}.$$  

It is shown that $V_0(e)$ has relative bound zero with respect to $H_0$ in all but a finite number of reducing subspaces of $H_0$. These estimates are obtained by an application of the minimum principle to a family of related Laplace-Beltrami operators.

(Received October 18, 1977.)

Aspects of cyclic subnormal operators.

Let $\mu$ be a finite Borel measure having compact support in the complex plane. Some recent results concerning the $L^2(\mu)$ closure of the complex polynomials, under various hypotheses on $\mu$, will be discussed. (Received October 18, 1977.)

A fixed point theorem and attractors.

We investigate attractors for compact sets by considering a certain quotient space. The following theorem is included. Let $f : G \to G$, $G$ a closed convex subset of a Banach space, $f$ a mapping satisfying (i) there exists $M \subseteq G$ which is an attractor for compact sets under $f$; (ii) the family $\{f^n\}_{n=1}^\infty$ is equicontinuous. Then $f$ has a fixed point. (Received October 18, 1977.)

49 ▲ Calculus of Variations and Optimal Control

Existence of Nash Equilibria for N-person Games.

We consider a payoff functional of the form $J(u_1, \ldots, u_N)$ subject to an operator-theoretic equation of the form $Lx = Ex + \sum_{i=1}^N B_i u_i$. The controls $u_i$ lie in convex weakly compact subsets of a collection of Hilbert spaces. The operator-theoretic equation is such that $(L - E)^{-1}$ is linear and continuous from a Hilbert space into a Banach space. The existence of a Nash equilibrium point is established. Applications are made to games governed by appropriate systems of differential equations, particularly of the hyperbolic and linear Volterra-type, and to a stochastic problem. (Received September 6, 1977.)

A Perturbation Method for Systems from Control with Multiple Switch Points.

We consider a system of ordinary differential equations of the form

$$x' = f(x, u, \epsilon)$$

$$\lambda' = g(x, \lambda, u, \epsilon)$$

$$x(0) = \lambda(T) = 0$$

arising from an application of the Maximal Principle to a free endpoint trajectory optimization problem in control. We assume the presence of a control constraint of the type $|u| \leq 1$, and we assume that the solution of the problem corresponding to $\epsilon = 0$
has the property that the control variable \( u \) moves on and off the boundary of the control region a finite number of times. We develop a procedure for obtaining uniformly valid parameter expansions for the solution in the case \( \delta_{\text{ese}} \), and we establish conditions on the Hamiltonian for the system under which the procedure can be carried out. (Received October 17, 1977.)

752-49-3 F. A. MASSEY and R. E. WORTH, Georgia State University, Atlanta, Georgia 30303. Interior equilibrium solutions of optimal control problems. Preliminary report.

Many authors consider the following special optimal control problem (see, e.g., Silvert and Smith, Math. Biosci. 33(1977), 121-134): find a control function \( u \) and a state function \( x \) which maximizes \( \int_0^\infty e^{-\rho t} R(x,u) \, dt \) subject to \( \frac{dx}{dt} = g(x,u) \), with \( x \) and \( u \) constrained to certain sets \( X \) and \( U \), and such that \( x \) and \( u \) are equilibrium (constant) solutions which are interior to \( X \) and \( U \). We show that \((x_0,u_0)\) is an interior equilibrium solution of the problem if and only if \((x_0,u_0)\) is an interior global maximum for \( R(x,u) \) which satisfies \( g(x_0,u_0) = 0 \). (Received October 17, 1977.)


Necessary conditions are derived for optimal control problem where the state variables are solutions of integral equations containing delayed arguments. Multiple fixed delays and variable delays are considered and a maximum principle is derived by considering an equivalent problem which involves no delays. The results for the special case in which the states are solutions of delay differential equations are used to obtain necessary conditions for a problem in which the states are governed by integral equations. (Received October 18, 1977.) (Author introduced by Professor James F. Roter).

50 ▲ Geometry


No surveyor ever constructed, nor ever will construct, the abstract object known as a Jordan arc. Instead, to approximate an arc he places a finite sequence of straight segments so that no two meet and so that the end of each equals the start of the next. If the first segment follows the last, the Jordan curve is approximated by the loop bounding a face in a geographic map. Let \( A \) be an arbitrary point not in a loop \( L \). Let \( B \) be a point not equal to \( A \) and not in \( L \). Then \( B \) is inside or outside \( L \) according as segment \( AB \) meets \( L \) at an even or an odd number of points. Usually map \( M \) is defined by bonding across a face of \( M_{n-1} \). Here the bonding is limited so that the sides introduced are chords (arcs of segments) inside or outside the original \( L \) of \( M_n \). There are thus two sequences of faces, those inside are colored 45 alternately and those outside 67 so that simultaneous construction and coloring is possible and trivial. If a map already constructed is proposed, an \( L \) containing all the vertices, called a ridge, must be found, unless the map is a special type, reducible by a time-honored method. Two other reductions are used in the inductive proof. The algebra used previously is eliminated. If the Jordan curve theorem is assumed, both proofs hold without further appeal to topology, for the abstract, physically unrealizable maps. Neither graph theory nor machine computation is needed. As pointed out by Cayley in the early days, an algorithm was needed and has sufficed. (Received October 18, 1977.)


For convex polyhedra in 3-space (or maps on the sphere), Euler's theorem relates the numbers of vertices, edges and faces (countries). Its corollaries include relations between the numbers \( p_k \) of \( k \)-sided faces and \( v_n \) of \( n \)-valent vertices. Eberhard-type theorems investigate to what extent are those necessary conditions also sufficient for the existence of polyhedra or maps with prescribed numbers \( p_k \) and \( v_n \). We consider analogues of these theorems for the plane. While Euler's theorem for tilings has been mentioned in the literature many times (frequently in incorrect formulations or with invalid proofs), the Eberhard-type theorems we establish appear to be completely new. (Received October 18, 1977.)

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52 ▶ Convex Sets and Geometric Inequalities

752-52-1 JAMES F. LAWRENCE, Applied Mathematics Division, National Bureau of Standards, Washington, D. C. 20234. **Lopsided sets and collections of orthants intersected by convex sets.**

Consider the set of the \(2^d\) closed orthants in \(d\)-dimensional Euclidean space. Given a subset \(L\) of this set of orthants, is there a convex set \(K\) which intersects those closed orthants in \(L\), while missing those not in \(L\)? There is a strong combinatorial condition on the set \(L\) which is necessary for the existence of such a convex set. The sets \(L\) satisfying this condition—the "lopsided" sets—have a rich combinatorial structure which can be exploited in the study of convex sets and systems of linear inequalities. (Received October 17, 1977.)

*752-52-2 Robert Sine, University of Rhode Island, Kingston, Rhode Island 02881 and Vladislav Kreinovic, Institute of Mathematics, Academy of Sciences of USSR, 630090 Novosibirsk 90, USSR. Remarks on Billiards.

Theorem 1. Let \(K\) be a smooth convex body in \(\mathbb{R}^n\), \(n \geq 3\). Then \(K\) is a ball if and only if each billiard orbit is contained in a 2-dim affine subset. Theorem 2. characterizes smooth convex bodies in \(\mathbb{R}^2\) in terms of billiard properties. Corollary. Billiards in a smooth convex body of constant breadth in \(\mathbb{R}^2\) is not ergodic. (Received October 18, 1977.)

53 ▶ Differential Geometry

*752-53-1 John K. Beem and Paul E. Ehrlich, University of Missouri, Columbia, Missouri 65201. **The space-time distance function and singularities.** Preliminary report.

Let \(M\) be a time oriented Lorentz manifold of dimension \(\geq 3\). Let \(d: M \times M \to \mathbb{R}\) denote the space-time distance function defined by \(d(p, q) = 0\) for \(q \in M - J^+(p)\) and for \(q \in J^+(p)\) by the sup of lengths of all future directed causal curves from \(p\) to \(q\). We say that \(M\) satisfies the finite distance condition iff \(d(p, q)\) is finite for all \(p, q \in M\). A compact subset \(K\) of \(M\) causally disconnects a future directed inextendible timelike curve \(c: (-\infty, 0) \to M\) iff \(\exists a < b\) such that every future directed nonspacelike curve from \(c(t_1)\) to \(c(t_2)\) meets \(K\) for all \(t_1 \leq a\) and \(t_2 \geq b\).

Theorem: Let \(M\) satisfy the strong energy condition and generic condition of Hawking-Penrose and the finite distance condition. Suppose \(M\) has a past and future inextendible timelike curve which is causally disconnected by some compact set. Then \(M\) contains an incomplete nonspacelike geodesic which is inextendible. (Received October 3, 1977.)

*752-53-2 JAMES V. PETERS, St. Bonaventure University, St. Bonaventure, N.Y. 14778. **An Integral Transform Proof of The Ham Sandwich Theorem.**

We establish a quite general form of the Ham Sandwich theorem on the basis of the following proposition: Given \(n\) continuous functions \(f_1(\theta), f_2(\theta), \ldots, f_n(\theta)\) defined on \(S^n\) and satisfying \(f_j(\theta) = -f_j(-\theta)\) for all \(\theta \in S^n\) and \(1 \leq j \leq n\), there exists a zero common to all the \(f_j'\)s. The method of proof requires the construction of an appropriate integral transform. By developing the inverse transform theory it is also shown that the Ham Sandwich theorem implies the above proposition. In the process we obtain a weak \(L^1\) inversion theorem for the Radon transform. The precise relationship between the Borsuk-Ulam theorem and the above results is also given. (Received October 7, 1977.)

*752-53-3 Dilip K. Datta, University of Rhode Island, Kingston, Rhode Island 02881. **Exterior recurrent forms.** Preliminary report.

A differential \(\sigma\)-form \(\omega\) on a \(\sigma\)-manifold \(M\) of dimension \(n\) is exterior recurrent if \(\omega\) is not a closed form and if there exists a 1-form \(\lambda\) on \(M\) such that \(d\omega = \lambda \cdot \omega\), where \(d\) denotes
exterior derivative and $\wedge$ denotes exterior product of forms. The 1-form $\lambda$ is called the recurrence 1-form. A tensor $S$ of type $(\alpha, \rho)$ on $M$ is exterior recurrent if $\text{Alt}(S)$ is exterior recurrent, where $\text{Alt}(S)$ is the alternation of $S$.

Exterior recurrent forms appear in conditions of integrability while exterior recurrent tensors may be obtained from certain recurrent tensors on a linearly connected manifold, discussed by Wong (Trans. Amer. Math. Soc. 99 (1961), 325-341) and others. Several results will be proved for exterior recurrent forms. It will be shown that these results are useful in obtaining properties of recurrent tensors. (Received October 12, 1977.)

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THOMAS E. CECIL, Vassar College, Poughkeepsie, New York 12601 and PATRICK J. RYAN, Indiana University at South Bend, South Bend, Indiana 46615. Focal sets, taut embeddings and the cyclides of Dupin.

The compact cyclides, discovered by Dupin in 1822, are the images under stereographic projection of a product torus in a Euclidean 3-sphere. Classically, these were characterized as the only surfaces in $E^3$ with the property that both sheets of the focal set are curves. The cyclides reappeared when Banchoff (J. Differential Geometry 4(1970), 193-205) proved that the only taut compact surfaces in $E^3$ are spheres and cyclides. In this paper, the above characterizations are extended to the higher dimensional cyclides which are those hypersurfaces in Euclidean $(n+1)$-space obtained as the image under stereographic projection of a standard product of spheres in an $(n+1)$-sphere. Theorem 1. Let $M$ be a connected, compact hypersurface embedded in $E^{n+1}$. If the focal set consists of two distinct submanifolds of codimension greater than one, then $M$ is a cyclide. Theorem 2. Let $M$ be a compact, connected manifold with the same integral homology as the product of a $k$-sphere with an $(n-k)$-sphere, $M$ is tautly embedded as a hypersurface in Euclidean $(n+1)$-space, then $M$ is a cyclide. The proof of Theorem 1 is different than the classical proofs which relied on special properties of so-called focal conics. The crux of the proof of Theorem 2 is to demonstrate that there are only two distinct principal curvatures on $M$, a complication which, of course, did not arise in Banchoff's proof for surfaces. (Received October 13, 1977.)

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THOMAS E. CECIL, Vassar College, Poughkeepsie, NY 12601 and PATRICK J. RYAN, Indiana University at South Bend, South Bend, Ind. 46615. Tight and Taut Immersions into Hyperbolic Space.

Three natural classes of distance functions on hyperbolic space $H^m$ are defined. The level sets of the three types of functions are hyperspheres, equidistant hypersurfaces, and horospheres, respectively. If $M$ is a submanifold of $H^m$, one shows that almost all of the functions of each of the three types restrict to Morse functions on $M$. We first prove the following characterization of umbilical submanifolds of $H^m$ which generalizes a similar theorem in Euclidean space of Nomizu and Rodriguez (Nagoya Math. J. 48 1972, 197-201).

Theorem: Let $M$ be a connected, complete Riemannian $n$-manifold ($n \geq 2$) which is isometrically immersed in $H^m$. If every Morse function of each of the three types has index 0 or $n$ at any of its critical points, then $M$ is embedded as an umbilical hypersurface of a totally geodesic $(n+1)$-dimensional submanifold of $H^m$.

A submanifold $M$ is called taut in $H^m$ if every Morse function of the first type has the minimum number of critical points on $M$, and $M$ is called tight in $H^m$ if every Morse function of the second type has the minimum number of critical points on $M$. As in the Euclidean case, tightness is related to minimal total absolute curvature. We characterize taut and tight immersions for certain particular manifolds. These examples demonstrate that taut and tight immersions into hyperbolic space have many properties with no Euclidean counterparts. (Received October 13, 1977.)

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Let $M^k$ be an oriented compact immersed minimal submanifold of $S^{n+k} \subset R^{n+k+1}$. For $x \in M^k$, let $N_x$ denote the $k$-plane normal to $M^k$ in $S^{n+k}$. Identify $N_x$ with the corresponding $k$-plane through the origin in $R^{n+k+1}$, and let $A$ be a fixed $k$-plane through the origin. Define $P = \langle N_x, A \rangle$. A-133
Theorem: For \( n=2 \) (when \( M_2 \) is a minimal surface in \( \mathbb{S}^k+2 \)) if \( P > 0 \), then \( M_2 \) is totally geodesic. This theorem was proved by Barbosa for \( M_2 \) homeomorphic to \( S^2 \).

Theorem: For general \( n \) and \( k \), let \( r = \min(n,k) \). If \( P \geq \cos^2 \frac{\pi}{2n} \), then \( M \) is totally geodesic. This improves previous constants of Simons and Reilly. The technique involved can also be used to solve the Dirichlet problem for the minimal surface system on a convex domain for boundary data having a sufficiently small computable Lipschitz constant. (Received October 17, 1977.)

752-53-7 BILL WATSON, Case Western Reserve University, Cleveland, Ohio 44106. Almost contact submersions. Preliminary report.

There are three classes of almost contact submersions, Riemannian submersions which transfer almost contact and almost complex structures. Type I, of the form \( f: \mathcal{M}^{2m+1} \to \mathcal{N}^{2n+1} \), has almost contact total and base spaces and almost complex fibres. Type II, \( f: \mathcal{M}^{2m} \to \mathcal{N}^{2n+1} \), has almost contact base space and fibres and almost complex total space. Type III, \( f: \mathcal{M}^{2m+1} \to \mathcal{N}^{2n} \), has almost contact total space and fibres and almost complex base space. The fundamental properties of these submersions are presented and examples given. For instance, an example of a Type III almost contact submersion is given by the Hopf map, \( \pi: S^3 \to S^2 \). (Received October 17, 1977.)


A Zoll surface is a surface of revolution diffeomorphic to \( S^2 \), the 2-sphere, with the property that all geodesics are closed with period \( 2\pi \). Eigenvalues of the (positive) Laplacian on \( S^2 \) are \( k(k+1) \) with multiplicity \( 2k+1 \) (\( k=0,1,2, \ldots \)). For Zoll surfaces other than \( S^2 \), each of the multiple eigenvalues \( k(k+1) \) splits into a "cluster" of eigenvalues near \( k(k+1) \); specifically, there is a number \( M > 0 \), independent of \( k \), such that the eigenvalues in the \( k \)-th cluster are contained in the interval \( [k(k+1)-M, k(k+1)+M] \). The eigenvalues of the first 20 clusters of selected Zoll surfaces were calculated by numerical methods. The results of these calculations led to two conjectures. (The conjectures are now results of Alan Weinstein.) Conjecture 1. The arithmetic mean of the eigenvalues in the \( k \)-th cluster approaches \( k(k+1) \) as \( k \) approaches \( \infty \). Conjecture 2. The eigenvalues of the \( k \)-th cluster approach a limiting distribution as \( k \) goes to \( \infty \). (Received October 18, 1977.)


For a generic set of smooth immersions of a closed surface into 3-space, the Gauss spherical image mapping is known to be excellent (i.e. with only folds and cusps as singularities). We characterize cusps of the Gauss map by any one of a number of geometric properties involving the relation of parabolic curves to lines of curvature, asymptotic curves, and ridges (corresponding to curves of cusps on focal surfaces). We indicate how certain embeddings with non-excellent Gauss maps (e.g. the torus of revolution and the monkey saddle) can be perturbed so that their Gauss mappings become excellent. (The presentation will be accompanied by computer graphics illustrations by Banchoff and Charles M. Strauss.) (Received October 18, 1977.)


Let \( W^{n+1} \) be a smooth compact Riemannian manifold of constant sectional curvature \( = c \). If \( M^n \subset W^{n+1} \) is a hypersurface then there is a Gaussian curvature function \( K: M \to \mathbb{R} \), (defined only up to sign if \( n \) is odd). Let \( \mathcal{F} \) be a smooth codimension-one transversely-oriented foliation of \( W \). Then we have a continuous function \( K: W \to \mathbb{R} \) via \( K(x) = \text{Gaussian curvature at } x \) of the leaf through \( x \). Let \( \text{vol} \) be the canonical volume form on \( W \), and let \( v = \int_W \text{vol} \). Let \( \overline{K} \) denote \( (1/v) \int_W K \text{vol} \). Theorem. \( \overline{K} = 2^n c^{n/2} / (\pi^{n/2}) \). In particular, \( \overline{K} \) is independent of \( \mathcal{F} \). (Received October 18, 1977.)
Let $M^n$ be a compact Riemannian manifold and $\{\lambda_j\}$ the spectrum of the Laplace operator on $i$-forms. The series $\zeta(s) = \sum_{j=1}^{\infty} \lambda_j^{-s}$ converges for $\Re s > n/2$ and may be analytically continued to a meromorphic function which is regular at $s = 0$. The analytic torsion $T(M)$ is defined by $T(M) = \pi \sum_{j=1}^{\infty} (-1)^{j+1} \lambda_j^{j/2}$. Let $\Omega$ denote the Reidemeister torsion of $M$. It is a combinatorial invariant which depends on the metric via volume element in cohomology induced by the identification of $H^*(M, \mathbb{R})$ with the space of harmonic $i$-forms. Then we have the following theorem which was conjectured by Ray and Singer. Theorem. $T(M) = \Omega(M)$. This result is one example of recent progress on the old question of understanding the relation between the shape of an object and its spectrum. The method of proof provides some new directions and techniques. (Received October 18, 1977.)

54  General Topology

*752-54-1 Peter Fletcher, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061 and William Lindgren, Slippery Rock State College, Slippery Rock, Pennsylvania 15067. GO spaces and quasi-uniformities. Preliminary report.

Let $(X, \mathcal{J}, \mathcal{K})$ be a topological space and let $\preceq$ be a linear order on $X$. Then $(X, \mathcal{J}, \mathcal{K})$ is a GO space if, and only if, there is a quasi-uniformity $U$ on $X$ such that $\cap U = \mathcal{G}(\mathcal{K})$ and $T(U \cup U^{-1}) = \mathcal{J}$. There follows a theory of uniformities for GO spaces, which parallels the usual theory of uniformities for completely regular spaces. (Received April 27, 1977.)

*752-54-2 R. E. Hodel, Duke University, Durham, N. C. 27706. The number of closed subsets of a topological space II.

The reader is referred to A-653 for the abstract on I. Let $X$ be an infinite topological space. Results are obtained which give the number of denumerable closed sets in $X$ and the number of closed sets in $X$ of cardinality $|X|$. Theorem 1. Let $X$ be an infinite, regular space such that every point is a $G_0$. Then the number of denumerable closed sets in $X$ is the maximum possible, namely $\aleph_0$. Example. There is a compact, Hausdorff space $X$ such that the number of denumerable closed sets in $X$ is neither $\aleph_0$ nor $|X|$. Theorem 2. Let $X$ be an infinite Hausdorff space. Then there are at least $|X|$ closed sets in $X$ of cardinality $|X|$. Theorem 3. Assume CH, and let $X$ be an infinite Hausdorff space. Then $X$ has $\mathcal{O}(X)$ closed sets of cardinality $|X|$. ($\mathcal{O}(X)$ is the number of open subsets of $X$.) (Received August 1, 1977.)

*752-54-3 ROBERT A. HERRMANN, U. S. Naval Academy, Annapolis, MD 21402. Generalized closedness.

In this long paper, nonstandard topology with respect to a kappa-saturated enlargement is used to generalize the concept of the closed map and closed graph, among others. The generalized perfect map and its relation to the generalized closed map with thick fibers is studied in detail. In the process, more than 25 published propositions in general topology are either extended or improved upon. These include results of Halfar, Dickman and Porter, Herrington, Herrington and Long, Joseph, Kasahara, Powsa, Noiri, Stephenson, Thompson and Viglino, among others, relative to such diverse concepts as closed graphs, strongly-closed graphs, closed maps, regular-closed maps, star-closed maps, absolutely closed maps, almost continuity, weak $\Theta$-continuity, $\Theta$-continuity, continuity, subcontinuity, (w, $L_X$)-perfect maps, $Z$-closed projections; completely Hausdorff-closed, nearly-compact, quasi-H-closed and $S$-closed spaces as well as other less well known concepts. From the 58 propositions which appear in this paper, we give the following example:

Corollary 5.5.1. Let $X, Y$ be completely Hausdorff-closed. Then $X \times Y$ is completely Hausdorff-closed iff the first projection $P_1 : X \times Y \to X$ is $(w, I_X)$-perfect iff $P_1 : X \times Y \to X$ is $Z$-closed. (Received August 17, 1977.)
We say that X is strongly collectionwise normal if the universal uniformity on X equals the collection of all neighborhoods of the diagonal. A family $G = (G_a)_{a \in I}$ is even-screenable (resp. even-expandable) if there exist an open (resp. locally finite) family $F = (F_a)_{a \in I}$ (resp. $H = (H_a)_{a \in I}$) and a neighborhood $W$ of the diagonal of X such that $uG = uF$ (resp. $uG = uH$) and $W(F_a) \subseteq G_a$ (resp. $W(G_a) \subseteq H_a$) for all $a \in I$.

**Theorem.** If $G = (G_a)_{a \in I}$ (resp. $F = (F_a)_{a \in I}$) is a locally finite family of open (resp. closed) subsets of X such that $F_a \subseteq G_a$ for all $a \in I$ and $uF = uG$ then $G$ is even-screenable and $F$ is even-expandable. Theorem. X is normal iff every binary (resp. finite, locally finite, star-finite) open cover of X is even-screenable. Theorem. X is collectionwise normal iff whenever $(F_a)_{a \in I}$ is a discrete family of closed subsets of X there exists a neighborhood $W$ of the diagonal such that $(W(F_a))_{a \in I}$ is a pairwise disjoint family. (Received September 22, 1977.)

**Discrete and countably discrete maps.** A subset $S$ of a topological space is called discrete iff each point in the space has a neighborhood that contains at most one point of $S$. A map is called discrete (resp. countably discrete) iff the image of any discrete subset (resp. countable discrete subset) is a closed subset. Discrete and countably discrete maps are characterized in $T_1$-spaces. Theorems due to Vašinštein and Engelking are extended to discrete and countably discrete maps. (Received October 7, 1977.)

**Kuratowski Closure Operators Without the Expansive Axiom.** A pseudo-closure operator is the same as a Kuratowski closure operator except that a set may or may not be contained in its closure. This paper investigates the basic properties of such operators. For example, those families of subsets that constitute the closed sets for some pseudo-closure operator are characterized. Moreover, it is shown that knowing the family of closed sets allows one to establish bounds for the closure of a set. (The possible closures form a join semilattice.) A representation theorem for the pseudo-closure operators of a finite set is presented and used to motivate a natural ordering of the pseudo-closure operators definable on a given set. This order induces a lattice structure and the properties of this lattice are investigated. Finally, the concept of a continuous function is explored. Several examples are also given. (Received October 7, 1977.)

**Metrizability of certain ordered spaces.** In this paper we use a metrization theorem of Bennett [A note on the metrizability of $M$-spaces, Proc. Japan Acad. 45(1969), 6-9] to study those generalized ordered spaces which are hereditarily $p$-spaces in the sense of Arhangelskii [On hereditary properties, Gen. Top. & Appl. 3(1973), 39-46] and we prove Theorem: Let X be a $G_δ$ space. The following are equivalent: (1) X is metrizable; (2) every subspace of X is a $p$-space; (3) every subspace of X is an $M$-space; (4) every subspace of X is a $\Delta$-space; (5) every subspace of X is quasi-complete in the sense of Creede. In proving that theorem, we establish the following results which may be of some independent interest: (a) if every subspace of the $G_δ$ space X is a $\beta$-space in Hodel's sense, then X is paracompact; (b) for $G_δ$ spaces, $M$-space = quasi-complete. (Received October 11, 1977.)
Irreducibly essential maps from inverse limits. Preliminary report.

It is well known that if $f$ is an essential map from a compact metric continuum $M$ into the unit circle, then there is a subcontinuum $H$ of $M$ such that the restriction of $f$ to $H$ is irreducibly essential. In this paper it is shown that in case $M$ is one-dimensional, $H$ may be characterized by the properties of its inverse limit representation. Letting $H$ be the inverse limit of an inverse system of one-dimensional polyhedra, then in a sense $H$ is "almost" circle-like. In general terms, there is an infinite sequence of simple closed curves, each curve contained in the corresponding factor space of the inverse system, such that each term of the sequence is contained in the bonding map image of its successor; the bonding maps, restricted to the simple closed curves, are essential; and the image of such a restricted bonding map is "close" to every point in its range space. This result is used to show that one-dimensional, hereditarily unicoherent continua may be characterized by the properties of their inverse limit representations. (Received October 11, 1977.)

Homoclinic points of mappings of the interval.

Let $f$ be a continuous map of a closed interval $I$ into itself. A point $x$ in $I$ is called a homoclinic point of $f$ if there is a periodic point $p$ of $f$ such that $x \neq p$, $x$ is in the unstable manifold of $p$, and $p$ is in the orbit of $x$ under $f^n$, where $n$ is the period of $p$. It is shown that $f$ has a homoclinic point if and only if $f$ has a periodic point whose period is not a power of 2. (Received October 14, 1977.)

Applications of Shrinkable Covers.

An open cover $Q = \{G_\alpha : \alpha \in A\}$ of a topological space $X$ is shrinkable if there exists a closed cover $\mathcal{F} = \{F_\alpha : \alpha \in A\}$ of $X$ such that $F_\alpha \subseteq G_\alpha$ for each $\alpha \in A$.

In this paper the author determines the conditions necessary for a variety of general covers to be shrinkable. In certain cases shrinkability of these type covers are characterizations for these properties. The types of covers investigated are, weak $\Theta$-covers, weak $\theta$-covers, point countable covers, $\theta$-covers and weak $\theta$-covers. Applications of these results are answers of unsolved problems and new results for irreducible spaces. (Received October 14, 1977.)

The topology of $\Theta$-convergence is the semi-regular topology associated with $(X, T)$. Preliminary report.

A filterbase is said to $\Theta$-converge to a point $x \in (X, T)$ if for every open set $U$ containing $x$ there exists an element $F$ of the filterbase such that $F \subseteq \text{Cl}(U)$. It is shown that this is the convergence of the semi-regular topology associated with $(X, T)$. This result leads to characterizations of $C$-compact and $H$-closed spaces, as well as a characterization of $\Theta$-continuous functions. This result also yields straightforward proofs of some known results, and is used to strengthen others. (Received October 17, 1977.)


$(X, P, Q)$ will always denote a pairwise completely regular bitopological space. $B_X$ denotes the largest pair compactification of $(X, P, Q)$ and $S_X$ denotes the Stone-Cech compactification of $(X, P, Q)$. Consider the following statements: 1. $(X, P \lor Q)$ admits a unique proximity. 2. $|S_X - X| \leq 1$. 3. $|B_X - X| \leq 1$. 4. If $E$ is $Q$-closed, $F$ is $P$-closed and $(E, F)$ is completely separated then either $E$ or $F$ is $P \lor Q$-compact. 5. If $E$ is $Q$-closed, $F$ is $P$-closed and $(E, F)$ is completely separated, then either $E$ is $P$-compact or $F$ is $Q$-compact. 6. $(X, P, Q)$ admits a unique pair compactification.
7. \((X, P, Q)\) admits a unique quasi-proximity. 8. If \(E\) is \(P\)-closed and \(F\) is \(Q\)-closed and \((E, F)\) is completely separated, then either \(E\) is \(P\)-compact or \(F\) is \(Q\)-compact.

We prove \(1 \Rightarrow 2 \Rightarrow 3 \Rightarrow 4 \Rightarrow 5 \Rightarrow 6 \Rightarrow 7 \Rightarrow 8\). We give examples which show that \(5 \neq 4\) and \(8 \neq 7\). Certain applications to the theory of ordered spaces are presented. (Received October 17, 1977.)


We discuss recent results on the Lindelof degree of the Tychonoff product of spaces with various properties. The relationship of the Lindelof degree with other cardinal functions will also be considered. (Received October 17, 1977.)

*752-54-14 SCOTT W. WILLIAMS, SUNY at Buffalo, Buffalo, N.Y. 14214. The Effectiveness of Nets of Cardinal \(\aleph_1\). Preliminary report.

Given two directed sets \(D\) and \(E\), J. W. Tukey defines \(D\) to be as effective for convergence as \(E\) (\(D \leq E\)), if for each topological space \(X\) and net \(\Phi : E \to X\) converging to \(x \in X\), there exists a net \(\Phi : D \to X\) converging to \(x\) with \(\Phi(D) \subseteq \Phi(E)\). "\(\equiv\)" induces an equivalence relation and a "direction" to the class of directed sets. In answer to questions of J. R. Isbell, we show

1. CH implies there are three (up to equivalence) ultra-filters on \(\aleph_1\) and
2. the existence of an \(\omega_1\)-scale in \(\omega\) implies the existence of a directed set \(D\) of cardinal \(\aleph_1\), equivalent to \(\omega\) and \(\omega_1\), and lying above \(\omega \times \omega_1\) and below the set of finite subsets of \(\omega_1\). Therefore, the existence of six non-equivalent directed sets of cardinal \(\aleph_1\) is independent of CH. (Received October 17, 1977.)


In these (Mathematical Reviews 22(1975), A-218), we defined, for \(m \in \aleph_0\) and \(B \in \beta m\), the concepts \(\beta\)-limit point and \(\beta\)-closed. Definition. A function \(f : X \to Y\) is \(\beta\)-perfect if whenever \(y = \beta\)-lim \(x \in X\) \(f(x) = y\), then there exists \(x \in X\) such that \(x = \beta\)-lim \(x \in X\) and \(f(x) = y\). Theorem. Let \(f\) be continuous, onto and \(Y\) Hausdorff. Then \(f\) is perfect if and only if \(f\) is \(\beta\)-perfect for every \(B\). Examples. There exists \(f\) which is \(\beta\)-closed for every \(B\) but \(f\) is not \(\beta\)-perfect for any \(B \in \beta \omega \setminus \omega\). Let \(B \in \beta \omega \setminus \omega\). Then there exists \(f : X \to Y\) which is closed but not \(\beta\)-closed. (These results correct the assertion made in the previous abstract that if \(f\) is \(\beta\)-closed for every \(B\), then \(f\) is perfect.) (Received October 18, 1977.)

*752-54-16 Heikki J. K. Jumnila, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061. On pseudo-open mappings of paracompact spaces.

We show that a topological space \(X\) is metacompact if and only if every open covering of \(X\) has a point-finite semi-open refinement (a cover \(L\) of \(X\) is semi-open if \(St(x, L)\) is a neighborhood of \(x\) for every \(x \in X\)).

It follows that the image of a paracompact space under a pseudo-open, compact and continuous mapping is metacompact; this answers a question of A. V. Arhangel'skii The intersection of topologies, and pseudo-open compact mappings, Soviet Math. Dokl. 17 (1976) 160-163. (Received October 17, 1977.)

*752-54-17 HOWARD H. WICKE, Ohio University, Athens, Ohio 45701. A functional characterization of primitive base.

All formulations of the concept of primitive base in the published literature involve the concept of well ordering in a prominent way. One of the most felicitous of these formulations is that a space \((X, T)\) has a primitive base if and only if there is a sequence \(W_1, W_2, W_3, \ldots\) of well ordered open covers of \(X\) such that for each \(p \in X\) the collection \(\{V : \text{for some } n \in \aleph_0, V \text{ is the first element of } W_n \text{ that contains } p\}\) is a local base at \(p\). For others see [Wicke and Worrell, Proc. Amer. Math. Soc. 50 (1975) 443-450]. The present paper gives various conditions not involving well ordering on a function \(f : X \times \aleph_0 \to T\) such that if the space has a function satisfying one of these conditions, then it may be proved (using the Axiom of Choice) that the space has a primitive base. One such condition is the
following: Suppose $(X,T)$ is a space. Then $h:X \times \mathbb{N} \rightarrow T$ is called an initial function if and only if for all $j \in \mathbb{N}$ and $A \subseteq X$, if $A \neq \emptyset$, then there exists $a \in A$ such that for all $b \in A$ either $h(a,j) = h(b,j)$ or $b \neq h(a,j)$. Theorem. A space has an initial first countable function if and only if it has a primitive base. Conditions equivalent to initiality are also given. Although having a primitive base clearly implies the existence of an initial function, examples show that without AC the concepts of first countable initial function and primitive base are not equivalent. (Received October 17, 1977.)


If $h$ is a twist homeomorphism of the annulus $\mathbb{S}^1 \times [0,1]$ onto itself then either (1) there is a simple closed curve $C$ in the interior of the annulus with $r=1$ in its interior so that the annulus bounded by $r=1$ and $C$ is mapped onto a proper subset of itself by either $h$ or $h^{-1}$, or else (2) $h$ has at least two distinct fixed points. In case $h$ is area-preserving (1) cannot occur and so $h$ has at least two distinct fixed points. (Received October 17, 1977.)


In this paper we answer questions of Arhangel'skii and Michael by providing an example of a regular symmetrizable space which is not subparacompact and has a closed subset which is not a $G_\delta$-set.

We also use the idea of sequential order to obtain some positive results, and several examples are provided which show that these results are in some sense the best possible. (Received October 17, 1977.)

752-54-20 E. van Douwen and G. M. Reed, Institute for Medicine and Mathematics, Ohio University, Athens, Ohio 45701, D. J. Lutzer, Texas Tech University, Lubbock, Texas 79409, and J. Pelant, Institute of Mathematics, CAS AV, Prague, Czechoslovakia. Metric Decompositions.

In [Gen. Top. and Its Appl. 1 (1971), 102-103], B. Fitzpatrick gave an example of a $\sigma$-space which is not the union of countably many closed metrizable subsets. In this paper, the following two results are obtained: Theorem: Each $\sigma$-space is the union of a $\sigma$-c closed metrizable subsets. Example: There exists a paracompact, linearly ordered, perfect space which is not the union of $\sigma$-c closed metrizable subsets. (Received October 17, 1977.)

752-54-21 SOO BONG CHAE, JEFF H. SMITH, New College/Univ. of South Florida, Sarasota, FL 33580. Remote points and Z-ultrafilters on metric spaces.

N. J. Fine and L. Gillman, Proc. AMS, Vol. 13, pp-29-36 proved the existence of remote points for $\mathbb{R}$ with the help of the Continuum Hypothesis. In this paper we show that the existence of such a point in any noncompact metric space does not depend on the Continuum Hypothesis. (Received October 17, 1977.)

752-54-22 Peter J. Nyikos, Auburn University, Auburn, Alabama 36830. A compact nonmetrizable space $X$ such that $X^2$ is hereditarily normal. Preliminary Report.

In 1948, M. Katětov showed [Fund. Math. 36, 271-274] that if $X$ is a compact space such that $X \times X$ is hereditarily normal, then $X$ is metrizable, leaving open the question of whether hereditarily normality of $X \times X$ is enough. I now show it consistent that the answer be No. Example. Let $A$ be a subset of $I = [0,1]$ of cardinality $\mathfrak{c}$. Replace each $a \in A$ by two points, $a^-$ and $a^+$, setting $a^- < a^+$ and otherwise obeying the real-line order. Then $X = (I - A) \cup A^+ \cup A^-$ with the order topology is compact, perfectly normal, and nonmetrizable.
Theorem [MA + CH] $X$ is hereditarily normal. On the other hand: Theorem [2 $^{\aleph_0} < 2^{\aleph_1}$]

If $Y$ is a compact nonmetrizable space such that $Y^2$ is hereditarily normal, then either $Y$ is an L-space, or $Y^2$ is an S-space, or $Y^2$ contains both an S-space and an L-space.

(Received October 17, 1977.)

J. B. FUGATE, University of Kentucky, Lexington, KY 40506 and T. BRUCE McLEAN, James Madison University, Harrisonburg, VA 22801. A fixed point theorem for tree-like continua. Preliminary report.

A tree-like continuum is a compact connected metric space having finite open covers, of arbitrarily small mesh, whose nerves are trees. Whether tree-like continua have the fixed point property for continuous functions is unsettled. At the 1977 Louisiana State University Topology Conference, Fugate and Mohler announced that this question reduces to the case that the function is a homeomorphism. We extend a theorem of P. A. Smith (Amer. J. Math. 63 (1941), 1-8) and obtain the following result.

Theorem A periodic homeomorphism on a tree-like continuum has a fixed point. (Received October 17, 1977.)

Robert A. McCoy, Virginia Polytechnic Institute and State University, Blacksburg, Va., 24061. Some topological properties of function spaces.

Necessary and sufficient conditions are given for function spaces to have certain topological properties, including: first and second countability, separability, metrizability, submetrizability, and almost $\sigma$-compactness. The two topologies of primary interest are the compact-open topology and the topology of pointwise convergence. (Received October 17, 1977.)

J.E. VAUGHAN, University of North Carolina at Greensboro, N.C. 27412.

A countably compact space and its products.

For an ultrafilter $x$ on the natural numbers $\omega$, let $t(x)$ denote the type of $x$ (i.e., the set of all ultrafilters on $\omega$ to which $x$ can be mapped by a permutation of $\omega$). The Rudin-Keisler order on $\omega$ is defined as follows: Write $t(x) \leq t(y)$ provided there exists a function $f: \omega \to \omega$ such that the extension $\overline{f}: \beta(\omega) \to \beta(\omega)$ maps $y$ to $x$. An ultrafilter $p$ in $\beta(\omega)$ is called a $\omega$-point provided $p$ is not in the boundary of any countable subset of $\beta(\omega) \backslash \omega$. Theorem. Let $X = \beta(\omega) \setminus t(p)$ where $p$ is a $\omega$-point for which there exists an ultrafilter $r$ such that $t(p) \leq t(r)$. Then (1) $X$ is r-compact, and hence the product space $X^k$ is countably compact for all cardinals $k$. (2) For every countably compact space $Y$, $X \times Y$ is countably compact. (3) The set $\omega \times X$ is a countable set of isolated points in $X$, and no infinite subset of $\omega$ has compact closure in $X$. This last property implies that $X$ is not totally countably compact, and (by a theorem of Z. Frolík) that there exists a pseudocompact space $Y$ such that $X \times Y$ is not pseudocompact. The pair $(p,r)$ of ultrafilters required by the theorem exists under CH, MA, P(c), and other set-theoretic assumptions. (Received October 17, 1977.)

L. S. HUSCH, University of Tennessee, Knoxville, Tennessee 37916 and I. IVANŠIĆ, University of Zagreb, Zagreb, Yugoslavia. Embeddings and Concordances of Embeddings.

The following is the generalization of a result of J. Stallings in the shape category:

THEOREM. Let $X$ be a continuum of fundamental dimension $k \geq 3$ which is pointed $(2k-q+1)$-movable. If there exists a shape $(2k-q+1)$-connected shape map $f:X \to M$, of $X$ into a $q$-dimensional PL manifold which is either closed or is open and dominated by a finite complex and if $q-k \geq 3$, then there exists a $k$-dimensional continuum $Y \subseteq M$ and a shape equivalence $g:X \to Y$ such that $gf=i$ where $i$ is the shape map induced by the inclusion of $y$ into $M$. An example is given to show that the condition pointed $(2k-q+1)$-movable cannot be removed. (Received October 18, 1977.)
Let $X$ be a Banach space and let $X^*$ be its dual. By a lemma of Kato if $X^*$ is uniformly convex there is a duality map $J : X \to X^*$. **THEOREM 2.** Let $X$ be a Banach space with a uniformly convex dual. Let $F : X \to 2^{X^*}$ be a set-valued function satisfying the following three conditions: (A1) $F$ has a closed graph. (A2) $F(x)$ is a nonempty compact convex set for each $x \in X$. (A3) $F$ maps bounded sets into relatively compact sets. Suppose there is a positive number $a < 1$, a bounded set $\{x_i : i \in I\}$ in $X$ and bounded positive numbers $\{d_i : i \in I\}$ such that for each $x \in X$ there is an $i \in I$ so that at least one of the following three conditions holds: (B1) $\|x - x_i\| < d_i$. (B2) $\|y - x_i\| < d_i$ for each $y \in F(x)$. (B3) For each $y \in F(x)$, 
$$\langle \text{Re} \, y - x_i, J(x - x_i) \rangle \leq a \|y - x_i\| \|x - x_i\|. $$
Then $F$ has a fixed point, i.e., there is an $x_0 \in X$ such that $x_0 \in F(x_0)$. Furthermore for each fixed point $x_0$ of $F$ there is an index $i \in I$ such that $\|x_0 - x_i\| < d_i$.

This theorem extends a theorem of Van de Vel (1975 Dissertation). (Received October 18, 1977.)

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**Stable homeomorphisms of the pseudo-arc.**

Noting that certain restrictions are placed on homeomorphisms of the pseudo-arc, since it is hereditarily indecomposable, in 1955 R.H. Bing asked if the identity is the only stable homeomorphism of the pseudo-arc. We prove the following theorem.

**Theorem.** Let $U$ be a non-empty open subset of the pseudo-arc $P$. Let $p$ and $q$ be distinct points of $P$ so that the subcontinuum $M$ irreducible between $p$ and $q$ does not intersect $c(U)$. The there exists a homeomorphism $h : P \to P$ with $h(p) = q$ and $h|_U = 1_U$. (Received October 18, 1977.)

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**Initially compact fuzzy topological spaces.**

Let $F$ be a fuzzy topology for a set $X$. Following R. Lowen, let $i(F)$ denote the smallest topology for $X$ such that if $f$ is in $F$, then $f$ is lower semi-continuous with respect to $i(F)$. We define the fuzzy topological space $(X,F)$ to be **initially compact** iff $(X,i(F))$ is compact. **Theorem.** If $(X,i(F))$ is a Tychonoff space, then there exists a fuzzy topology $F'$ for $\beta X$ such that $(\beta X,F')$ is initially compact and such that $(X,F)$ may be densely embedded in $(\beta X,F')$.

Moreover, if $h:(X,F) \to (Y,W)$ is a fuzzy continuous map from $X$ into an initially compact fuzzy space $(Y,W)$ where $(Y,i(W))$ is Hausdorff, then $h$ has a unique fuzzy continuous extension $g:(\beta X,F') \to (Y,W)$. (Received October 18, 1977.)

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**A selector for equivalence with $G_5$ orbits.**

Assume that $X$ is a separable, completely metrizable (i.e., Polish) space, $\mathcal{N}$ is a countable base for the topology of $X$, and $E$ is an equivalence on $X$ such that each equivalence class (orbit) is a $G_5$ subset of $X$. Given $O \in \mathcal{N}$, $O^+$ is the $E$-saturation of $O$.

**Theorem 1.** The collection $\{O^+ : O \in \mathcal{N}\}$ separates orbits.

**Theorem 2.** Assume further that for each $O \in \mathcal{N}$, $O^+$ is both $F_\sigma$ and $G_6$. Then the Borel space $X/E$ is standard and there is a Borel measurable cross-section $s:X/E \to X$.

**Corollary.** Let $A$ be a separable $C^*$-algebra. There is a Borel measurable right inverse for the canonical map $\text{Irr}(A) \to \text{Prim}(A)$. This establishes a conjecture of R. Kallman and R. D. Mauldin. (Received October 18, 1977.)
In proving that stratifiable \sigma-discrete spaces are \( M_1 \), we generalize and greatly simplify the author's published proof that countable stratifiable spaces are \( M_1 \). As a corollary to this result and a result of P. Nyikos, we have that scattered stratifiable spaces are \( M_1 \). (Received October 18, 1977.)

\[ \text{Let } A \text{ be the Steenrod algebra modulo 2, and } \hat{A} = A/\langle Sq^1 \rangle = H^*\left(\mathbb{Z}, \mathbb{Z}_2\right). \text{ For every integer } i \geq 0 \text{ let } A_i = \hat{A} \text{ and } d_i = Sq^1 \cdot Sq^2 \cdot \ldots \cdot Sq^i. \text{ Theorem 1: } A_1 \leftarrow A_2 \leftarrow A_3 \leftarrow \ldots \text{ is a chain complex.} \]

\[ \text{Theorem 2: There exists a spectrum } X \text{ such that: } P1) X \text{ realizes } \mathbb{Z} \text{ up to } A_5; P2) \pi_k(X) = \mathbb{Z} \text{ if } k = 2^m - 2 \text{ (for } m \geq 6) \text{ and 0 otherwise. The proof of Theorem 1 involves relations in } \hat{A}. \text{ The proof of Theorem 2 is by construction of a tower of fibrations, induced over a Postnikov tower with bottom space } \mathbb{Z}^3 \text{ and } k-\text{invariant the nonzero bottom class } x_j \in H^3(\mathbb{Z}^3; \mathbb{Z}). \text{ (Received July 5, 1977.)} \]

Let \( \lambda: J \rightarrow \text{DGLA} \) be the Quillen functor from the category of 1-connected spaces to the category of connected differential graded Lie algebras over \( \mathbb{Q} \). (Quillen, Ann. of Math. 90(1969), p. 210). Define a functor \( \omega: J \rightarrow \text{DGLA} \) as follows: for a space \( X \), \( \omega(X) \) is the Lie algebra of primitives of the cobar construction applied to \( \mathbb{M}_X \), the dual of the Sullivan minimal algebra \( \mathbb{M}_X \). (Sullivan, Proc. Conf. Manifolds, Tokyo 1973). Let \( \lambda \) be the functor which assigns to an object in DGLA its minimal Lie algebra [Baues-Lemaire, Math. Ann. 225(1977), §2]. Let \( L_X = \text{min } \lambda(X) \) and \( L^X = \text{min } \omega(X) \). The conjecture [ibid., p. 240] is that \( L_X \) and \( L^X \) are isomorphic in DGLA. We prove this from a homology decomposition of \( X \). For the inductive step we establish a dual Hirsch lemma for \( \lambda \) and \( \omega \). The former follows from Quillen's work and the latter is found in a preprint by Neisendorfer [Lie Algebras, Co-algebras and Rational Homotopy Theory for Nilpotent Spaces, §8]. (Received August 3, 1977.)

**THEOREM 1:** (A Blakers-Massey Theorem) Given a pair of n-connected nilpotent CW complexes \( (X,A) \), if \( H_1 \otimes H_{n+1} X/A = 0 \) then \( (X,A) \rightarrow (X/A,*) \) induces an isomorphism on \( \pi_{n+1} \) and an epimorphism on \( \pi_{n+2} \). If in addition \( H_2 \otimes H_{n+1} X/A = 0 \), then the induced map is an isomorphism in dimension \( n+2 \) and an epimorphism in dimension \( n+3 \).

**THEOREM 2:** Let \( (X,A) \) have cofiber \( C \). If \( X \) is nilpotent and \( Z(\pi_1 C) \otimes H_1 A \) is a nilpotent \( \pi_1 C \) module for \( i \geq 1 \), then \( C \) is nilpotent. This is the dual of a well known result relating nilpotency and fibrations. It can be used to construct many examples of nilpotent complexes. (Received August 29, 1977.)

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Let $X$ be the complement of a locally flat PL $(4,2)$-knot (or link). A method is given for computing from hyperplane cross sections the second and third homotopy groups of $X$, thus solving problem 36 of R.H. Fox in his 1962 paper, "Some problems in knot theory." (Received September 9, 1977.)

Theorems of Algebraic Topology are necessary to generalize and explain conditions of structural stability, instability and closed orbits as generated by a differential equation with conditions on eigenvalues. A vector field is constructed on a manifold and a flow is defined which leaves a fixed point. From the above observations one is led to questions of turbulence. Quasi-periodic functions were accepted as the basic definition but these functions do not have the properties of a Baire set. Since turbulent solutions are now considered as those near an attractor, the purpose of this presentation is to construct an attractor with conditions placed on points of hypersurfaces orthogonal to orbit. (Received September 15, 1977.)

The Categorical Homotopy Theorem of Latch ["The uniqueness of homology for the category of small categories," J. Pure App. Alg. 9(1977), 221-237] proves that the respective homotopic categories of $\text{Cat}$, the category of small categories, and $K$, the category of simplicial sets, are equivalent. The equivalence is given by the nerve functor $N: \text{Cat} \to K$ and the "category of simplices" functor $F: \Gamma:K \to \text{Cat}$. We give the following characterization of all "reasonable" homotopy inverses for

\[ N \sim S \text{ for all } A \in \text{Cat}. \]

This theorem implies that up to homotopy, the classifying space $B_\sim = |NA|$ is the only "reasonable" way of assigning a topological space to a small category $A$. (Received October 11, 1977.)

We consider maps $f: V \to E(n)$ where $E = \mathbb{R}^q$, $E(n)$ is the $n$-fold symmetric product of $E$, $V$ is open in $E$, and the fixed point set $K_f$ is compact ($K_f = \{ x \in V \mid x \text{ is a coordinate of } f(x) \}$), $n > 0$, $q > 0$. For such a map, a rational number $I(f)$ is defined which reduces to the usual integer-valued fixed-point-index if $n = 1$. It is shown that the basic properties of the ordinary index have generalizations for such maps: the index is preserved under homotopy, the index is additive, and $I(f) = 0$ whenever $f$ is fixed point free. A join $f \circ g: V \to E(n+m)$ of such maps $f: V \to E(n)$ and $g: V \to E(m)$ is defined by concatenation, and it is proven that $(n + m)I(f \circ g) = nI(f) + mI(g)$. (Received October 14, 1977.)

Let the simply connected simplicial complex $K$ be the union of two non-empty simply connected subcomplexes $K_1$ and $K_2$. Using the method of formal power series connections, obstructions to the decomposition of the rational loop space homology of $K$ as a free product of the rational loop space homologies of $K_1$ and $K_2$ are
obtained. These obstructions are given in terms of the multiplicative structure
of the Sullivan-de Rham algebra of K. Obstructions for the rational loop space
homology of K to be a free algebra will also be presented. This last result
represents an extension to the Bott-Samelson theorem. (Received October 14, 1977.)

DONALD M. DAVIS, Lehigh University, Bethlehem, Pennsylvania 18015. Stable
It is determined which stunted complex projective spaces with at most \(3p-2 + (-1)^0\) cells
are stably p-equivalent, where \(p\) is any prime. Examples are given of stunted complex
projective spaces of distinct stable homotopy type which are stably p-equivalent for all
p. The methods include localization, BP-operations, and Chern character. (Received October 17, 1977.)

DONALD M. DAVIS, Lehigh University, Bethlehem, Pennsylvania 18015. Stable

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752-55-10 DARYL GEORGE, East Carolina University, Greenville, North Carolina 27834. Homology
The derived functors of the Kan extension are used to define a topological homology theory based
on a subcategory \(M\) (called the model category) of \(\text{Top}\) and a coefficient functor \(F\) from \(M\) to
abelian groups. The homology theory takes values in the category of pro-abelian groups. It is
shown to satisfy the Eilenberg-Steenrod axioms when \(F\) is homotopy invariant and \(\pi_0[0,1]\) is
an endofunctor on \(M\). Singular and Vietoris homology theories provide examples. (Received
October 17, 1977.)

752-55-11 JACK UCCI, Syracuse University, Syracuse, New York 13210. On Brown-Peterson
(co) homology theories of the Eilenberg-MacLane space \(K(\mathbb{Z},3)\). Preliminary report.
The Atiyah-Hirzebruch spectral sequences \(H^* \Rightarrow K^*\), \(H_* = K_*\) (\(K\) denotes complex K-theory with
coefficients \(Z/p\) or \(Z(p)\)) are determined for \(X = K(\mathbb{Z},3)\). As a consequence the analogous sequences
with \(BP\langle 1\rangle\) replacing \(K\) are determined, from which the calculation of \(BP\langle 1\rangle^*(X)\), \(BP\langle n\rangle^*(X)\)
follows. Implications for \(BP\langle n\rangle^*(X)\), \(BP\langle n\rangle^*(X)\) are considered.
The calculation exhibits a decomposition of the \(E_2\) term into submodules stable with respect to
the differentials. In particular, (1) \(\{v_0, v_{i+1}\}_{i \geq 0}\), \(v_0 = \text{fundamental 3-dimensional class and}
v_{i+1} = pP^1 p^1 \ldots p^1 u_0\), generates one such module, and (2) each remaining submodule is contained
in \(H^*(SP^3, SP^3, SP^3; Z(p))\) for some \(r\). Here \(SP^3\) is the usual k-fold symmetric product. The
suggestion of a Steenrod-type decomposition of \(h^*(SP^3, X)\) for \(h\) a p-typical theory is very strong.
(Received October 17, 1977.)

752-55-12 JOHN G. RATCLIFFE, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139.
Free and Projective Crossed Modules.
The concept of a free crossed module is due to J.H.C. Whitehead. He proved that if \(A\) is a
path connected space and \(X\) is obtained from \(A\) by attaching 2-cells, then \(\pi_2(X,A)\) is a free
\(\pi_1(A)\)-crossed module with basis corresponding to the attached 2-cells. His proof is highly geo-
metric. A more algebraic proof can be given as an application of the following homological charac-
terization of free crossed modules. Theorem: Suppose \(C\) is a G-crossed module with boundary
\(\partial; C \rightarrow G\). Let \(N = \text{Im } \partial\) and \(Q = G/N\), then \(C\) is free with basis \(\{c_\alpha\}\) if and only if
(i) \(H_1(C)\) is a free \(Q\)-module with basis the image of \(\{c_\alpha\}\),
(ii) the normal closure of \(\{c_\alpha\}\) in \(G\) is \(N\),
(iii) \(\partial_*; H_2(C) \rightarrow H_2(N)\) is trivial. (Received October 18, 1977.)
57 ▶ Manifolds and Cell Complexes

752-57-1 DAVID GILLMAN, University of California, Los Angeles, CA 90024. Reduction of Standard Spines. Preliminary report.

**Theorem.** If $P$ is a counterexample to the 3-dimensional Poincare conjecture, $K$ is a standard spine of $P$ with $n$ vertices, and there exists a 2-cell $C$ in $K$ with less than four vertices on its boundary, then there exists a counterexample to the 3-dimensional poincare conjecture having a standard spine with fewer than $n$ vertices.

This theorem may be useful in a computer search for a counterexample.

*Question.* Is there a standard spine for the 3-ball in which no 2-cell has less than four vertices on its boundary? (Received October 6, 1977.)


Let $M$ be a closed smooth $n$-manifold. **Theorem.** $M$ is cobordant to an $n$-manifold admitting a smooth $(Z_2 \times Z_2)$-action whose set of stationary points has codimension two. **Theorem.** $M$ is cobordant to an $n$-manifold appearing as the set of stationary points of a closed $(n+2)$-dimensional manifold with smooth $(Z_2 \times Z_2)$-action. (Received October 6, 1977.)


A 2-manifold is a separable metric space such that each point has a neighborhood homeomorphic to a 2-cell. Let $\pi_o(X)$ denote the space of ends of $X$.

**Theorem:** The homeomorphism type of a connected 2-manifold $M$ is determined by:

1. the map $\pi_o(\partial M) \to \pi_o(M)$ induced by inclusion,
2. the 2-to-1 function $\pi_o(\partial M) \to \pi_o(\partial M)$,
3. the subsets of $\pi_o(M)$ of non-planar ends, non-orientable ends, and ends that are limits of compact components of $\partial M$,
4. a subset of $\pi_o(\partial M)$ that indicates how orientations of neighborhoods of orientable ends in $\pi_o(M)$ agree with orientations of neighborhoods of ends in $\pi_o(\partial M)$,
5. the genus, orientability type, and number of compact boundary components of $M$.

An example of each homeomorphism type is constructed. (Received October 11, 1977.)

752-57-4 Bradd Clark, University of Southwestern Louisiana, Lafayette, Louisiana 70504. Factoring Free Actions. Preliminary report.

If $f$ is a free action of period $p$ on $X = N \cup M$, we say that $f$ factors over $N$ and $M$ if the restriction of $f$ to $N$ is a free action on $N$. Using the definitions given by Jaco and Myers in "An algebraic characterization of closed 3-manifolds," one can write $X = X' \cup T'$ where $X$ is a cabled knot-manifold, $X'$ is a knot manifold, $T$ is a homotopy solid torus and $T$ and $X'$ have been attached along an essential annulus $A$. **Theorem:** If $X$ is a prime, irreducible, cabled knot manifold then either $X$ admits the structure of a Seifert fiber space with decomposition surface a disk and having three singular fibers, or there exists a homeomorphism between $X$ and $T$ which takes $A \subset \partial X'$ to $A \subset \partial T$, or every free action of finite period on $X$ is conjugate to a free action which factors over $X'$ and $T$. **Theorem:** If $K_1$ and $K_2$ are nonhomeomorphic prime cubes-with-knotted hole and $K = K_1 \cup K_2$ is the composite cube-with-knotted hole, then every free action of finite period is conjugate to a free action which factors over $K_1$ and $K_2$. (Received October 12, 1977.)

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Let $M$ be a compact 3-manifold and $F$ a codimension one foliation of $M$. We say that a group is almost solvable if it has a solvable subgroup of finite index. The point of the present work is that the only obstruction to classifying sufficiently smooth $F$ (up to topological conjugacy) when $\pi_1(M)$ is almost solvable is the possible existence of Reeb components. Along the way the following generalization of a theorem of S. Novikov is also obtained. **Theorem.** If $\pi_1(M)$ is almost solvable and $H^1(M;\mathbb{Z}) = 0$ then any codimension one foliation $F$ of $M$ (of class $C^1$) has a compact leaf. (Received October 13, 1977.)
embeddings of $D^k$ into $N$ which are fixed on $S^{k-1}$, i.e., we have selected one such embedding and all others are to agree with it on $S^{k-1}$. It is well known that $E(D^k, N)$ is contractible if $n - k \geq 3$. The purpose of this note is to show that if $N$ is contractible and $n - k \geq 3$ then $E(D^k, N)$ is contractible. (Received October 17, 1977.)

752-57-10 ROBERT D. EDWARDS, University of California, Los Angeles, California 90024. Images of manifolds under cell-like maps.

The objects of study in this subject are cell-like maps $f: M \to X$ whose domains are manifolds and whose images are absolute neighborhood retracts (ANR's). A manifold here will mean a compact connected metric space which is locally homeomorphic either to some Euclidean half-space $R^m_+ = R^{m-1} \times [0, \infty)$, or to the Hilbert cube $[-1,1]^\infty$. A map defined on a manifold is cell-like if each point-inverse has the property that it is null homotopic inside any given neighborhood of itself. As an example, any map $f: M \to X$ which is a limit of disc-bundle projections from $M$ to $X$ is a cell-like map. Here we are assuming that the boundary-sphere-bundle subsets of the disc-bundle structures on $M$ correspond precisely to $\partial M$. Hence $\partial M = \emptyset$ if and only if the discs are 0-dimensional, i.e., the projections are homeomorphisms. It is not assumed in this example that $X$ is a manifold, but necessarily $X$ is a manifold factor, i.e., $X \times B^k$ is a manifold for some cell $B^k$. Various questions regarding this situation will be discussed, among them: (1) Is every ANR the image of some manifold under a cell-like map? (Answer - yes.) (2) Is every cell-like map $f: M \to X$ which looks homotopically like a limit of disc-bundle projections from $M$ to $X$, in fact a limit of disc-bundle projections from $M$ to $X$, after stabilizing to $f \circ \text{proj}: M \times B^k \to X$, for some cell $B^k$? (Answer - unknown, although some special cases are known.) The relevance of this question is that it is equivalent to the following question: Is every ENR homology manifold $X$ a manifold factor? The 0-disc-bundle (i.e., $\partial M = \emptyset$) version of question (2) is itself interesting and worthy of attention, for it amounts to the study of cell-like decompositions of manifolds-without-boundary. This subject, largely pioneered by R. H. Bing, has led to the broader problems mentioned above, the partial solutions of which are due to too many people to mention in this abstract. (Received October 21, 1977.)

*752-57-11 Frank Quinn, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061. Book decompositions of manifolds.

A book decomposition is a representation as a relative mapping torus $t(h, \alpha, h)$ of an automorphism $h: P \to P$ which is the identity on $\partial P \subset \partial P$.

Suppose $M$ is a compact $m$-manifold with a book decomposition of $\partial M$. Theorem: if $m$ is odd $\geq 5$ the decomposition extends to $M$. If $m$ is even $\geq 6$ the decomposition extends iff $i(M) = 0$. $i(M)$ takes values in a Witt group of bilinear forms over $\mathbb{Z}[\pi_1 M]$ with no symmetry requirements. This extends the closed simply connected theorem of E. Winkelnkemper [BAMS 79(1973) 45-51]. (Received October 18, 1977.)

*752-57-12 Paik K. Kim, University of Kansas, Lawrence, Kansas 66045. Involutions on Klein spaces $K(p,q)$. Preliminary report.

A Klein space $K = K(p,q)$ is a closed, orientable, irreducible 3-manifold with $|\pi_1(K)| < \infty$, in which a Klein bottle can be embedded. The class of Klein spaces $K(p,q)$ is completely classified by their fundamental groups $<\alpha, \beta | \alpha^2 \beta^{-1} \beta = 1, \alpha^q = \beta^p>$, where $p$ and $q$ are relatively prime (see the author, Some 3-manifolds which admit Klein bottles, preprint). We characterize PL involutions on Klein spaces $K$. The result can be applied to transformation group actions on the 3-sphere $S^3$. (Received October 18, 1977.)


The topological classification of closed 2-manifolds is one of the famous solved problems of mathematics. On a deeper level, however, there is rich structure to study in the group of isotopy classes of homeomorphisms.
of 2-manifolds. We will attempt to review what is known about these groups and their subgroups, and to show some ways in which they play a role in 3 and even 4-dimensional topology. This will be an expository talk, aimed at an audience of non-specialists. (Received October 18, 1977.)

752-57-14 WILLIAM THURSTON, Princeton University, Princeton, New Jersey 08540.

Homological uniqueness of invariant measures.

Let $M$ be a closed manifold having fundamental group with polynomial growth. Any transverse invariant measure for a codimension one foliation of $M$ defines a homology class. This class will be shown to be unique up to a scalar multiple. Applications will be given to analytic foliations. (Received October 18, 1977.)

752-57-15 I. M. Singer, University of California, Berkeley, California 94720.

Generalizations of pseudo differential operators.

We will discuss various generalizations of pseudo differential operators and various problems which they suggest. (Received October 18, 1977.)

58 Global Analysis, Analysis on Manifolds

752-58-1 WITHDRAWN

752-58-2 William F. Rich, Sam Houston State University, Huntsville, Texas 77341.

Timelike homotopy groups and a characteristic class for Lorentz Manifolds. Preliminary report.

Let $M$ be a manifold of dimension $m$ with a Lorentz tensor field. A restricted homotopy theory on $M$ is defined in which the homotopy is always timelike with respect to the Lorentz tensor field. The homotopy so restricted is called timelike homotopy, and the timelike homotopy groups are denoted $T(M)$. The construction of the timelike homotopy groups is then applied to fibre bundles over $M$, the bundle space $E$ and the fibre $F$ both admitting Lorentz structures. The bundle of timelike homotopy groups $E(T)$ is then used as the coefficient group for a cohomology theory. Obstructions to the construction of sections of $E$ are defined, and it is shown that the obstructions determine an element of the cohomology group $H^{m+1}(M,E(T))$. It is then shown that if $E$ is the subbundle of the tangent bundle consisting of unit timelike vectors, then this cohomology class in dimension one vanishes if and only if $M$ is time orientable. Finally, the non-vanishing of the n-dimensional cohomology class is shown to be a sufficient condition for the existence of a singularity in the tensor field. (Received October 17, 1977.)


Function groups associated with constraint submanifolds.

Let $Q$ be a manifold and $\omega$ a constant rank closed two-form on $Q$. If $M$ is a submanifold of $Q$ then any integral $f$ of $\ker \omega$ is said to be a first-class function if and only if whenever $g$ is an integral of $\ker \omega$ and $g|M$ is constant, then $(f,g)|M = 0$. A first-class function which is constant on $M$ is called a first-class constraint and a submanifold $M$ which is given locally by first-class constraints is called a first-class submanifold of $Q$. It turns out that when the image of a constant rank Legendre transformation is a submanifold, then it is a first-class submanifold. The set of all first-class functions on $Q$ is a function group and the set of all first-class constraints is a subgroup. When $M$ is a first-class submanifold of $M$ it can be realized as a leaf of the singular foliation induced by the function group of first-class functions. (Received October 17, 1977.)
Let $\Omega(f)$ denote the nonwandering set of a diffeomorphism $f$ of a compact manifold $M$. We construct an Axiom A diffeomorphism $f$ of a 3-manifold that cannot be approximated by an $\Omega$-stable diffeomorphism $g$ with $\Omega(g) = \Omega(f)$. $f$ and all nearby $g$ with $\Omega(g) = \Omega(f)$ have cycles, and their instability results from the $\Omega$-explosion construction of Palis. This contrasts sharply with the situation in dimension 2, which has been studied by Newhouse and Palis. They have shown that any Axiom A diffeomorphism $f$ of a 2-manifold can always be approximated by a diffeomorphism $g$ such that $g$ has no cycles and $\Omega(g) = \Omega(f)$. Using Smale’s $\Omega$-stability theorem, they conclude that such $g$ are $\Omega$-stable. (Received October 17, 1977.)

J. S. Griffith proposed the following system of equations to model a cellular process for control of gene expression by positive feedback:

$$
\begin{align*}
\dot{x}_1 &= x_3^m(1 + x_3^m)^{-1} - ax_1, \\
\dot{x}_2 &= x_1 - 8x_2, \\
\dot{x}_3 &= x_2 - \gamma x_3
\end{align*}
$$

where $a, S, \gamma$ are positive real parameters and $m$ is a positive integer. The origin is a critical point. For $m = 1$, if the positive octant $O$ contains another critical point, then all solutions in $O$ except the origin are positively asymptotic to this critical point. For $m > 1$, $O$ may contain two nonzero critical points on a half-line from the origin. We show that there is a 2-dimensional surface of solutions positively asymptotic to the smaller of these two critical points; all solutions in $O$ on one side of this surface are asymptotic to the origin and all solutions in $O$ on the other side are asymptotic to the larger critical point, provided $a \geq 1$ and $\gamma > \frac{1}{8}$. (Received October 18, 1977.)

**60 ▶ Probability Theory and Stochastic Processes**

DEFINITION: For a sequence of sets $A_1, A_2, \ldots$, let "$A_n$ positive sup frequency" be the set $A_n \text{ p.s.f.} = \{x \in O A_n : \limsup_{k} (1/k) \sum_{j=1}^{k} \chi_{A_j}(x) > 0\}$. Define the sets "$A_n$ positive inf frequency" and "$A_n$ positive frequency" similarly.

REMARKS: 1. $\limsup A_n \leq \limsup A_n \text{ p.s.f.} \leq \liminf A_n \text{ p.f.}$

2. If $A_1, A_2, \ldots$ are measurable, so are $A_n \text{ p.s.f.}$, $A_n \text{ p.i.f.}$, and $A_n \text{ p.f.}$

3. If $T$ is a measure preserving transformation on a probability space $(X,A,P)$, and if $A \in A$, then $P(A \cap \{T^{j}(A) \text{ p.f.}\}) = P(A)$.

PROPOSITION: Let $A_1, A_2, \ldots$ be measurable sets in a probability space $(X,A,P)$, and let $p_k$ be the conditional probability of $A_{k+1}$ given (the field generated by) $A_1, \ldots, A_k$. Then

$$
\limsup_{k} (1/k) \sum_{j=1}^{k} \chi_{A_j} \text{ is positive almost surely where } \limsup_{k} (1/k) \sum_{j=1}^{k} p_j \text{ is positive.}
$$

COROLLARY: (A Zero-One Law). If the events $A_1, A_2, \ldots$ are independent, then $P(A_n \text{ p.s.f.}) = 0$ or $1$ according as $\limsup_{k} (1/k) \sum_{j=1}^{k} p_j = 0$ or $> 0$.

Unlike the Borel-Cantelli Lemma, both parts of the conclusion may fail without independence. The analogous results hold for $A_n \text{ p.i.f.}$ and $A_n \text{ p.f.}$. (Received August 15, 1977.)

Let $[X(t), 0 \leq t \leq T]$ and $[Y(t), 0 \leq t \leq T]$ be two stochastically continuous stochastic processes with independent increments over the interval $[0,T]$ which, as measures over $D[0,T]$, are absolutely continuous with respect
to each other. Let \( \mu_X \) and \( \mu_Y \) be the measures over \( D[0,T] \) determined by the two processes. The characteristic function of \( \mathbb{E}(d\mu_X/d\mu_Y) \) with respect to \( \mu_Y \) is obtained in terms of the determining parameters of the two processes.

(Received September 12, 1977.)


A discrete-time queueing problem involving three queues in tandem, with unit service times, is considered. Arrivals to the queues occur from separate sources, and it is assumed that the corresponding arrival processes are mutually independent and that, for each queue, the number of arrivals is independently and identically distributed in each unit time interval. In addition, departures from the first queue arrive instantaneously at the second queue, and departures from the second queue arrive instantaneously at the third queue. The three queues are assumed to have unlimited size, and the generating function of the joint equilibrium distribution of the three queue lengths is calculated, under the assumption that the sum of the mean arrival rates from the three sources is less than unity. (Received September 22, 1977.)

Louis H. Blake, College of Staten Island-CUNY, Staten Island, N.Y. 10301. Equiconvergence and the entropy of a transformation.

It is proved that if \( \mathcal{A}_n \) is an increasing sequence of finite partitions which are getting close at a certain rate then \( h(\mathcal{A}_n, T) - h(\mathcal{A}_p, T) \to 0 \) as \( n,p \to \infty \) at a rate independent of \( T \) where \( T \) is a measure preserving transformation and \( h(\mathcal{A}, T) \) is the entropy of \( \mathcal{A} \) relative to \( T \). (Received October 3, 1977.)


Let \((\Omega, \mathcal{A}, P)\) be a complete probability space, let \( \omega \in \Omega \), and suppose that \( X \) and \( Y \) are separable Banach spaces. The concept of a random contractor of a nonlinear random operator \( U(\omega): X \to Y \) and the usefulness of this concept in obtaining existence and uniqueness of random solutions to random nonlinear operator equations are discussed. For example, the random nonlinear operator equation \( U(\omega)x = y(\omega) \) has a unique random solution \( x^*(\omega) \) in \( X \) for an arbitrary \( Y \)-valued random variable \( y(\omega) \) if \( U(\omega) \) has a random contractor \( \Gamma(x;\omega) \) which satisfies certain conditions. The random solution \( x^*(\omega) \) is approximated by the iteration procedure \( x_{n+1}(\omega) = x_n(\omega) - \Gamma(x_n(\omega);\omega)[U(\omega)x_n(\omega) - y(\omega)] \) a.s., \( n = 0, 1, 2, \ldots \).

As an application of these results, the solutions of random nonlinear Volterra integral equations and random nonlinear discrete systems are considered. (Received October 6, 1977.) (Author introduced by Professor R. M. Stephenson, Jr.)

Paul N. DeLand and Ray C. Shiflett, California State University, Fullerton, Fullerton, California 92634. Doubly stochastic measures supported by measurable functions. Preliminary report.

Measures defined on the unit square whose marginals are Lebesgue measure are called doubly stochastic. There is a one to one correspondence between such measures and temporally discrete Markov processes which have Lebesgue measure as a stationary distribution. Among such processes are those whose transition probabilities \( P(x,*) \) are purely atomic on one or two atoms. Conditions are given under which a measurable function \( f \) can be paired to a measurable function \( g \) so that \( \{f(x)\} \) and \( \{g(x)\} \) are the atoms for such a doubly stochastic measure. (Received October 7, 1977.)
Stochastic evolution equations include the partial differential equations of parabolic and hyperbolic types with random coefficients. These equations are often used to model the evolution of physical or biological systems in a random or turbulent environment. Under such circumstances, one is commonly interested in determining certain average functionals of a random solution, such as its moments. We shall discuss some modes of approximating these functionals and their computational feasibility. Even though it is sometimes possible to show the convergence of successive approximations, by iteration or perturbation, the convergence could be slow and nonuniform. Consequently a truncation of the approximating sequence after a small number of terms, as usually done for practical reasons, may lead to erroneous or inaccurate results of limited validity. To circumvent this kind of difficulty, various methods of approximations have been developed in the physical context. Some of these methods will be presented from the viewpoint of stochastic modeling and statistical estimation theory. (Received October 11, 1977.)

GEORGES A BÉCUS, Engineering Science Department, University of Cincinnati, Cincinnati, OH 45221. Successive approximation solutions of a class of random equations.

Weak solutions for random equations of the form \( u = u_0(w) + T q(w)u \) where \( T \) is a deterministic operator and \( w \in (\Omega, \mathcal{A}, P) \) a suitable probability space, are defined and constructed by the method of successive approximations. Sufficient conditions on \( T \) and \( q \) ensuring the convergence of the approximations are obtained. The method allows for an easy computation of the expected value and correlation function of the solution. Applications to stochastic versions of classical problems in mathematical physics and engineering are considered. (Received October 11, 1977.)


Let \( X_t \) be standard Brownian motion in \( \mathbb{R}^k \) with nonrandom starting point \( X_0 \). Explicit formulas for the joint distribution of the time \( T \) and place \( X_T \) where \( X_t \) first hits a sphere or exits from a spherical shell are obtained, in terms of a natural transform. Let \( |X_0| = x \) and consider a pair of spheres with centers at \( 0 \) and radii \( a < b \) ; for \( x \neq 0 \) let \( \theta_t \) be the angle \( \angle X_0 O X_t \). By rotational symmetry only the joint distribution of \( T, \theta_t \) is needed. Set \( h = (k-2)/2 \), let \( C_n^h \) be the Gegenbauer polynomial of order \( h \), degree \( n \), and let \( I \) and \( K \) be the Bessel functions "with imaginary argument". Now for \( s > 0 \) define \( Y_n(s,t) = \exp(-st)C_n^h(\cos \theta_t)/C_n^h(1) \). The results are as follows. For \( x < a \), \( E(Y_n(s,T)) = (a/x)^{n+h}(x/2s)/I_{n+h}(a/2s) \). Replacing \( a \) by \( b \) and \( I \) by \( K \) gives the formula for the case \( x > b \). When \( a < x < b \), \( E(Y_n(s,T); |X_T| = a) = (a/x)^{h}(I_{n+h}(x/2s)/I_{n+h}(x/2s)) \). The expectation on the set where \( |X_T| = b \) is obtained by interchanging \( a \) and \( b \) throughout. The problem, for the case \( k = 2, x < a \), was suggested by C.T. Shih; the method of solution goes back to Darling-Siegert [Ann. Math. Stat. 24(1953) 634-639]. (Received October 11, 1977.)


Let \( N_t \) be a birth and death process on the integers with time parameters \( \{\lambda_i, \mu_i\}_{i=-\infty}^{\infty} \). In making these parameters random variables, we create a continuous-time analogue of random walk in a random environment. Criteria for recurrence or transience is discussed and an a.s. convergence law is determined. (Received October 11, 1977.)
In this paper we consider approximate solutions of the fixed point equation (*) \( T(w)x = x \) where \( T(w) \) is a compact random operator. Let \( \{ T_n(w), n \geq 1 \} \) be an a.s. collectively compact family; and assume that \( T_n(w) \) converges pointwise to \( T(w) \). The approximate solutions of (*) are given by the solutions \( x_n(w) \) of the family of random operator equations (**) \( T_n(c.u)x = x \), \( n \geq 1 \).

We establish the existence, uniqueness, measurability and convergence of the solutions of (**) in a neighborhood of an isolated solution of (*). (Received October 13, 1977.)

Let \( X \) be the unique solution of (1) \( X_t = X_0 + \int_0^t f(s,X_s)dz + \int_0^t g(s,X_s)da \), where \( f \) and \( g \) are (say) jointly continuous and Lipschitz in the space variable. If \( Z \) is Brownian motion and \( \Lambda_t = t \) then it is classical that \( X \) is a diffusion; a strong Markov process with continuous paths. If instead one lets \( Z \) be a strong Markov process and a semimartingale and \( \Lambda \) be an additive functional of \( Z \), then it is shown that \( X \) need not be Markov, but the vector process \( (X,Z) \) is strong Markov. If \( Z \) is a Hunt process and \( \Lambda \) is quasi-left-continuous then \( (X,Z) \) is shown to be a Hunt process and its Lévy system is computed. (Received October 13, 1977.)

A (net) family \( \{ P_a : a \in \Omega \} \) is (asymptotically) regularly almost compact if for every open \( \Omega \) and every \( \epsilon > 0 \) there is a finite closed refinement \( J \) such that for every (sufficiently large) \( a, P_a (\omega \cup J) < \epsilon \). On a regular space, a net \( P_a \) has the property that every subnet clusters at an almost Lindelöf measure if and only if it is asymptotically regularly almost compact. As a corollary, a set \( \{ P_a : a \in \Omega \} \) has a compact closure in the class of almost Lindelöf measures if and only if it is regularly almost compact. This answers the question of characterization of families with compact closure for metric spaces which are not complete. (Received October 13, 1977.)

This paper is intended as a general introduction to the study of random equations. After giving some basic definitions from the theory of random equations, a brief survey is given of various methods which have been used to solve random equations. (Received October 13, 1977.)

Suppose that at time zero \( A_0(x) \) particles are placed at \( x \in \mathbb{Z}^d \) (\( \mathbb{Z}^d \) denotes the space of \( d \)-dimensional integers), where the process \( \{ A_n(x) \} \) is covariance stationary with \( \sum_{x \in \mathbb{Z}^d} \left| C_0(x) \right| < \infty \) and \( \mathbb{E}(A_0(x)) = \lambda \) for each \( x \in \mathbb{Z}^d \). Here \( C_0(x) \) denotes the covariance function of the \( A_0 \) process. The particles are then assumed to move independently according to the transition function \( P(x,y) \) of a \( d \)-dimensional random walk \( \{ Y_n \} \).

Let \( B \) denote a finite nonempty subset of \( \mathbb{Z}^d \) and \( A_n(B) \) the number of particles in \( B \) at time \( n \). Then \( S_n(B) = \sum_{k=1}^n A_k(B) \) is the total occupation time of \( B \) by time \( n \) of all the particles. It is shown that if \( P(0,0) \neq 1 \) then \( \lim_{n \to \infty} \frac{S_n(B)}{n} = \lambda |B| \), where \( |B| \) is the cardinality of \( B \). (Received October 13, 1977.)
Let $X$ be a Borel right Markov process on a Lusin state space $(E, \mathcal{F})$. Assume $X$ satisfies Meyer's Hypothesis (H) and a condition to assure the existence of a sigma-finite excessive reference measure. Under these conditions Smythe & Walsh construct a process in duality with $X$ which, in general, has only the moderate Markov property. A strong Markov version of the dual may not exist on $E$. We discuss a method of completing $E$ in a new metric (via a modification of the Ray-Knight compactification) so that $X$, considered as a process on the new metric space, has a strong Markov dual $Y$. $Y$ may have branch points. (Received October 14, 1977.)


We shall say that an $\mathbb{R}^n$-valued random variable $X$ has a matrix-stable distribution if for each $n$ there exists a matrix $B_n$ and vector $\beta_n \in \mathbb{R}^n$ such that

$$X - B_n(X_1 + \cdots + X_n) \sim \beta_n,$$

where the $X_i$ represent independent copies of $X$, and $\sim$ denotes equidistribution. We investigate these distributions, which generalize ordinary stable laws (which correspond to $B_n = c_n I$ for $c_n \in \mathbb{R}$) and analyze their corresponding role in limits of the form

$$\lim_{n \to \infty} A_n(Y_1 + \cdots + Y_n) - \alpha_n,$$

where $A_n$ is an $n \times n$ matrix and $\alpha_n \in \mathbb{R}^n$. These results are then extended to the analogous problems involving $\mathbb{R}^n$-valued processes with stationary independent increments. (Received October 14, 1977.)

Patrick Sutherland, University of Texas at Arlington, Arlington, Texas 76019. An application of the random version of Schauder's fixed point theorem to experience theory. Preliminary report.

Let $X$ be a Banach space and $T(\omega, s, x)$ be a stochastic process mapping $\Omega \times [0, \infty) \times X$ into $X$. Sufficient conditions are given to assure that the solution of the random operator equation

$$x_0(\cdot) = T(\cdot, 0, x_0(\cdot))$$
$$x_t(\cdot) = t^{-1} \int_0^t T(\cdot, s, x_s(\cdot)) ds$$

reaches the solution of the regression operator equation. (Received October 17, 1977.)


A stochastic McShane system of the form

$$x(t) = x(0) + \int_0^t f(s, x(s)) ds$$
$$+ \int_0^t \sigma(s, x(s)) d\beta(s)$$

is considered. Sufficient conditions are given under which the system has stochastically stable solutions. (Received October 17, 1977.)

R. Kannan, University of Texas at Arlington, Arlington, Texas 76019 and University of Missouri, St. Louis, Missouri 63121. Stochastic approximation and random nonlinear operator equations.

Convergence properties of stochastic approximation schemes are studied. These results are then applied to random nonlinear equations with applications to random integral equations and nonlinear stochastic programming. (Received October 17, 1977.)

The validity of using the method of moments to generate approximate solutions of certain types of random equations is established. Special attention is given to the application of this method to linear ordinary random differential equations and linear random integral equations. Several examples are provided to illustrate the implementation, effectiveness, and limitations of this approach. (Received October 17, 1977.)


The Liouville equation can be viewed as a probabilistic analogue of the continuity equation in fluid dynamics. In the past it has been used to determine the marginal density function for the solution of an initial value problem involving a finite number of random variables. Here we discuss extensions to joint density functions, to initial value problems involving stochastic processes, and to random boundary value problems. (Received October 17, 1977.)


We provide a coordinate-free definition of optional time for a multiparameter process; special cases include optional and cooptional times for ordinary stochastic processes and the stopping times of Wong and Zakai for the two-parameter Wiener process. We describe a strong Markoff property which is invariant under time reversal and other Euclidean transformations on parameter space, and prove sufficient conditions for a Markoff process to have the strong Markoff property. These conditions are satisfied by the multiparameter Wiener process. (Received October 17, 1977.)

752-60-24 Chull Park, Miami University, Oxford, Ohio 45056 and David L. Skoug, University of Nebraska 68588. Distribution estimates of barrier-crossing probabilities of the Yeh-Wiener process.

Let Q = [0, S] x [0, T] be a rectangle and \( \{X(s,t) : s,t \geq 0\} \) be the two-parameter Yeh-Wiener process. This paper finds probabilities of \( X(s,t) \) and \( |X(s,t)| \) crossing barriers of the type \( ast+bs+ct+d \) on the boundary of the rectangle \( Q \). These probabilities give lower bounds for the yet unknown probabilities of \( X(s,t) \) and \( |X(s,t)| \) crossing the barrier \( ast+bs+ct+d \) over \( Q \). The paper also gives sharper bounds for the latter probabilities. Also included are similar results for the two-parameter Brownian bridge \( \{Y(s,t) : 0 \leq s,t \leq 1\} \neq \{X(s,t) : 0 \leq s,t \leq 1 \mid X(1,1)=0\} \). (Received October 17, 1977.)

752-60-25 GARY F. ANDRUS, University of Prince Edwards Island, Charlottetown, P.E.I., C0, CLA4P3 and TOGO NISHIURA, Wayne State University, Detroit, Michigan, 48202. Fixed-points of random set-valued maps.

Let \( (\Omega, \mathcal{F}) \) be a measurable space and \( (\text{CB}(X), D) \) be the space of closed bounded nonvoid subsets of a complete metric space \( (X,d) \) with the Hausdorff metric. A function \( k: \Omega \times [0,\infty) \rightarrow [0,\infty) \) is said to be measurably contractive if it is measurable and for each \( \omega \in \Omega \), \( k(\omega, t) < t \) for \( t > 0 \) and \( k(\omega, \cdot) \) is usc from the right. An g-function \( \sigma: [0,\infty) \rightarrow [0,1] \) is a decreasing function with \( \sigma(0) = 1 \) and \( \sigma(t) \neq 1 \) for \( t > 0 \). Consider a map \( F: \Omega \times X \rightarrow \text{CB}(X) \) for which \( F(\cdot, x) \) is weakly measurable in the sense of Himmelberg,
Fund. Math., 87 (1975), 53-72. F is said to have shrinking diameter at w if there is \( \theta_w : [0, \infty) \rightarrow [0, \infty) \) such that \( \theta_w (0) = 0 = \theta_w (0^+) \) and \( \text{diam} \ F(w,x) \leq \theta_w (d(x,F(w,x))) \). The following generalizes results of S. Itoh, A random fixed point theorem for multivalued contraction mappings (to appear) and A. Nowak, Random fixed-point theorems (to appear). Assume \( \Omega \) is separable.

**Theorem.** Let \( F \) be as above and \( \Omega_A, \Omega_B \) be a measurable partition of \( \Omega \). Suppose for \( w \in \Omega_A \) either (1) \( \text{D}(F(w,x),F(w,x')) = k_w d(x,x') \) where \( 0 \leq k_w < 1 \) or (2) \( F(w, \cdot) \) has shrinking diameter and \( \text{D}(F(w,x),F(w,x')) = k_w d(x,x') \) where \( \alpha_w \) is an \( \alpha \)-function. Suppose for \( w \in \Omega_B \), \( F(w, \cdot) \) has shrinking diameter and \( \text{D}(F(w,x),F(w,x')) = k_w d(x,x') \) where \( k: \Omega \times \{0, \infty) \rightarrow [0, \infty) \) is measurable contractive. Then there is a measurable map \( x:\Omega \rightarrow \mathbb{R} \) such that \( x(w) \in F(w,x(w)) \) for \( w \in \Omega \).

(Received October 17, 1977.)


For purposes of parametrizing dependence in bivariate (multivariate) distributions, a novel representation of a certain kind of subclass, \( \{H_\theta \} \), of the class \( \{F_1, F_2\} \), bivariate c.d.f.s with common univariate marginals \( F_1 \) and \( F_2 \), is derived. The representation of the bivariate density, \( h_\theta \), of \( H_\theta \) is given in the form (in obvious notation) \( f_1(x)f_2(y) \phi(x,y) \) where \( \phi(x,y) \) is called the density weighting function. Properties of \( \phi(x,y) \) are studied for various known bivariate classes and some striking similarities are discovered. These results explain some previously obscured dependence properties of the distributions investigated. The approach taken facilitates an improvement and a natural multivariate extension of the Farlie–Gumbel–Morgenstern distributions. (Received October 17, 1977.)

### 752-60-27 Kyle Siegrist, Georgia Institute of Technology, Atlanta, Georgia 30332. Results concerning the characteristic operators associated with a class of pieced-together Markov processes. Preliminary report.

For each \( i=1,2,\ldots,n \) let \( X_i \) be a continuous Markov process with state space \( E \) and with characteristic operator \( a_i \). A discontinuous Markov process \( Y \) is constructed by taking the process formed from 'killing' \( X_i \) at a rate \( q_i : E \rightarrow [0, \infty) \) and 'piecing-together' these processes through jump probabilities \( n_{ij} : E \rightarrow [0,1] \) satisfying \( \sum_{j \neq i} n_{ij}(x) = 1 \). Under suitable conditions on the processes \( X_i, i=1,2,\ldots,n \) and on the functions \( q_i \) and \( \pi_{ij}, i,j=1,2,\ldots,n \), the characteristic operator of the process \( Y \) is shown to be an extension of the operator \( a_Y(x,i) = a_i f_i(x) - q_i(x) f(x,i) + \sum_{j=1, j \neq i} n_{ij}(x) \pi_{ij}(x) f(x,j) \) where \( f_i(x) = f(x,i) \) and is used to characterize the harmonic functions of \( Y \). These results and methods of proof are placed within the literature of random evolutions, as surveyed in the article of R. Hersh (Rocky Mountain J. of Math., 4 (1974) pp.443-477) and in particular are compared to the work of D. Heath (Doctoral dissertation, University of Illinois, 1969). (Received October 17, 1977.)


Let \( (X_j, Y_j), j = 1,2,\ldots,n, \) be independent and identically distributed random vectors. We consider the order statistics \( X_{1:n} \leq X_{2:n} \leq \cdots \leq X_{n:n} \) of the first component and we denote the corresponding \( Y \)’s by \( Y_{(1:n)}, Y_{(2:n)}, \ldots, Y_{(n:n)} \). This latter sequence is called the concomitants of the order statistics of the first component. The foundations of the asymptotic theory of these concomitants is laid down in a joint paper of H.A. David and the present author (J. Appl. Prob. 11 (1974), 762-770). This, and all subsequent papers, however, concentrate on the quantile case. In the present paper, a general limit theorem is established for the concomitant \( Y_{(n:n)} \) of the extreme \( X_{n:n} \) for a large class of bivariate distributions. The interesting feature of the result is that \( Y_{(n:n)} \) and \( Y_{n:n} \)

(Received October 17, 1977.)

752-60-29 WENDELL H. FLEMING, Brown University, Providence, Rhode Island 02912. Exit probabilities and optimal stochastic control. Preliminary report.

The estimates of Ventcel-Freidlin give an asymptotic formula for the probability that the state of a nearly deterministic Markov diffusion process exits from a given bounded region during a given time interval. See A. Friedman, Stochastic Differential Equations and Applications, vol 2, Chap. 14; Academic Press, 1976. We give a new proof, using stochastic control methods. (Received October 18, 1977.)


Algebraic moment problems associated with Random Processes $x(t), t \in T$ are defined, and existence and uniqueness results derived using M. Riesz's functional approach. The $(\mathcal{G}_c)$ Moment Problem: For an indexed set of numbers $\{A_{c_1}, \ldots, A_{c_n}\}$ $\forall c_1, c_2 \in T$ does there exist a random process $x(t), t \in T$ with spectrum in the closed set $\mathcal{G}_c$, which generates this indexed set as its moments, and if so is the process unique in law? It is proved that a solution exists iff the linear functional $\mu$ induced by the indexed set on the space of polynomials $\left[\lambda_{c_1}^{n_1}, \ldots, \lambda_{c_m}^{n_m}\right]$ is positive (i.e. maps polynomials positive on $\mathcal{G}_c$ into positive numbers). Consequently the $(\mathcal{G}_c)$ moment problem has a solution iff every sub-moment problem has a solution. Nec. and suff. conditions for uniqueness are derived in terms of the functional $\mu$.

These definitions and results are an extension of the work done by C.K. Haviland in 1935 and 1936 on the n-dimensional moment problem. (Received October 18, 1977.)

*752-60-31 STEPHEN JAMES WOLFE, University of Delaware, Newark, Delaware 19711, On the Behavior of Characteristic Functions

This paper is concerned with the following question: If a characteristic function satisfies a certain property at the origin, what can be said about its behavior on the entire real line? If $k$ is an even integer and $f(u)$ is a characteristic function, then the existence of $f^{(k)}(0)$ implies the existence of $f^{(k)}(u)$, for all $u$. If $k$ is an odd integer, then it is possible to construct a characteristic function $f(u)$ such that $f^{(k)}(0)$ exists but $f^{(k)}(u)$ fails to exist for almost all $u$. However the existence of $f^{(k)}(0)$, when $k$ is odd, implies that $f(u)$ satisfies a $k$th order smoothness condition uniformly on the real line and thus $f(u)$ has many of the properties of a characteristic function with a continuous $k$th derivative. Several other results are obtained that show that if a characteristic function has a property $P$ at 0 then it either has the same property everywhere on the real line or comes close to having the property everywhere. (Received October 18, 1977.)


A finite element method is derived for solving differential equations with random non-homogeneous terms and proper boundary conditions. The properties of solutions are discussed and the rate of convergence is estimated. The case of differential equations with random coefficients is also discussed. (Received October 18, 1977.)
Let $X_1(1), X_2(2), \ldots, X_i(r_i), i = 1, 2, \ldots, k$ be the first $r_i$ ordered observation of a sample of size $n$ from a two parameter exponential distribution. A multistage procedure is developed to determine $t < k$ grouping so that in each group the distribution has $\mu_i$'s that are not appreciably different. The method extends the approach of Kumar and Patel, Technometrics, Volume 13, 1971, pages 183-189. The emphasis is on the development of a procedure based on the null sampling distribution of the maximum gap of the ordered first order observation from the exponential distribution. The distribution of the test statistic under the null hypothesis, $\mu_1 = \mu_2 = \ldots = \mu_k$, is derived. The significance points for $k = 2(1)5$ are tabulated for various $r_i$'s. The probabilities of correct rankings are investigated and sample sizes are determined for certain important cases. (Received October 17, 1977.)

A differential equation model of a marine ecosystem is formulated as a stochastic process. The ecosystem is modeled by considering the random exchange of a chemical nutrient between three components of the ecosystem. The Chapman-Kolmogorov equations and the moment or cumulant generating functions for the process are derived to examine analytically the behavior of the moments of the process. Through the use of differential inequalities, bounds on the exchange rate parameters are derived to reflect component extinction. Bounds on the moments of the process are also obtained. (Received October 17, 1977.) (Authors introduced by Professor Chris P. Tsokos).

In the present study it is assumed that the underlying failure model in the usual life-testing procedures is the recently introduced alpha probability distribution. Bayesian estimates of the reliability function and the shape parameter of the failure model are developed. The uniform truncated normal and gamma probability density functions were utilized to characterize the random behavior of the parameter. A sensitivity analysis of the assumed priors is also investigated using the mean square error criteria. A Monte Carlo simulation is used to illustrate the usefulness of the Bayesian estimates. (Received October 17, 1977.)

Let $X_1, \ldots, X_N$, $X_{[N]} + 1, \ldots, X_N$ be a long sequence of independent r.v.'s, the first $[N]$ with continuous cdf $F$, the remaining with continuous cdf $G$, where $\int_0^1 F(x) \, dx = \frac{1}{2} + \delta + o(1), \delta > 0$, $\delta \in (0,1)$. Let $R_1 = \text{sequential rank of } X_1 \text{ among } X_1, \ldots, X_N$ and $Z_1 = \frac{1}{1} (R_1 - \frac{1}{2}(1 + 1))$. If $S_N(t)$ is the random function on $[0,1]$ obtained by linearly interpolating between the points $(0,0)$.
and \[ \left\{ \begin{array}{l} \left( \frac{K}{N}, \frac{12}{2}, \frac{K}{N} \sum_{i=1}^{N} Z_i \right) \end{array} \right\}, K \geq 1, \] then a stopping rule based on \[ S_N(t) \] is defined.

Asymptotic properties of the rule are studied by showing that the weak limit of \[ S_N(t) \] is a standard Brownian motion with logarithmic drift \[ \int_0^t \theta \log(1 + \theta) \] starting at \( t = 0 \).

Applications to quality control are given. (Received October 17, 1977.)


An acceptance sampling scheme for lots of items having failure times subject to an alpha distribution with unknown shape parameter \( \alpha \) is developed. The algorithm presented is based on an acceptance criterion involving the order statistic \( t_{n,m} \), which represents the \( n \)th failure time from a subsample of \( m \) items drawn from the lot. A lot is accepted if \( t_{n,m} > t_c \). The algorithm chooses \( t_{c,n} \), and \( m \) to minimize a linear function of \( m \) and of the posterior consumer risk \( \pi_c \) and produces risk \( \pi_p \) under an assumed probability distribution for the parameter \( \alpha \). The sensitivity of the optimal \( n, m, \) and \( t_c \) to the distribution of \( \alpha \) is investigated. (Received October 17, 1977.) (Author introduced by Dr. Chris P. Tsokos).


Multivariate failure models with monotone failure rate are considered. The problem of the maximum likelihood estimation of the failure rate and the asymptotic distribution of the estimator are investigated. (Received October 17, 1977.) (Author introduced by Dr. Chris P. Tsokos).


This paper studies the robustness of the Bayes discriminant procedure with respect to multivariate data which does not follow a Gaussian (normal) probability distribution. The authors present discriminant criteria based on the Dirichlet (multivariate beta), gamma, and lognormal distribution functions as alternatives to the usual Gaussian rule. These procedures are compared on both actual and simulated data. We find that Bayesian discrimination is sensitive to the incorrect use of the normal distribution. (Received October 17, 1977.)

\#752-62-8 A. S. Papadopoulos, The College of Charleston, Charleston, South Carolina 29401 and W. J. Padgett, University of South Carolina, Columbia, South Carolina 29208. A stochastic model for BOD and DO in streams.

In this study a stochastic model for stream pollution will be presented. The model involves a random differential equation of the form \[ \dot{X}(t) = AX(t) + Y(t), \] where \( X(t) \) is a two dimensional vector-valued stochastic process giving the biochemical oxygen demand (BOD) and dissolved oxygen (DO) at distance \( t \) downstream from a source of pollution. The random inhomogeneous term \( Y(t) \) is a function which describes a discharge of pollutant along a continuous stretch of stream. The fundamental Liouville's theorem is utilized to obtain the solution \( X(t) \) of the random differential equation, at each \( t \). Computer simulated trajectories of the BOD and DO processes are presented which illustrate the theoretical results. (Received October 17, 1977.)

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Suppose \( \mathbf{a} = (a_1, a_2, \ldots) \) is a moment sequence for a distribution function \( F \) which is uniquely determined by its moments. We prove a general theorem stating that if a sequence of (not necessarily moment) sequences \( \mathbf{a} = (a_1, a_2, \ldots) \) converges coordinate wise, then we can obtain from \( \mathbf{a} \) a distribution function \( F \) with \( F \to F \). This result is applied to obtain consistent estimates of the mixing distribution of a compound or mixed Poisson distribution, and to obtain consistent estimates of the mixing distribution in scale mixtures of exponential distributions and in scale mixtures of symmetric stable distributions. (Received October 17, 1977.)

Let \( X_1, \ldots, X_n \) be random variables with continuous distribution functions \( F_1, \ldots, F_n \), joint distribution \( F \), and unique connecting copula \( C \), so that \( F(t_1, \ldots, t_n) = C(F_1(t_1), \ldots, F_n(t_n)) \). We introduce the quantities \( \sigma_n(X_1, \ldots, X_n) = \frac{k}{n} \int \left[ C(u_1, \ldots, u_n) - u_1 \cdots u_n \right] du_1 \cdots du_n \) and \( \rho_n(X_1, \ldots, X_n) = \frac{k}{n} \int \left[ C(u_1, \ldots, u_n) - u_1 \cdots u_n \right] du_1 \cdots du_n \) as measures of dependence of \( X_1, \ldots, X_n \). \( \sigma_n \) is the n-dimensional generalization of the quantity introduced in [C.R. Acad. Sci. de Paris, 283A(1976), 659-661], whereas \( \rho_n \) generalizes Spearman's \( \rho \). For any sequence of random variables \( \{X_n\} \), \( \lim \sigma_n(X_1, \ldots, X_n) = \lim \rho_n(X_1, \ldots, X_n) \). Furthermore, for an important class of random variables, including the trivariate normal, \( \rho_3(X,Y,Z) = \frac{1}{3} (\rho(X,Y) + \rho(X,Z) + \rho(Y,Z)) \). Similarly, under certain reasonable additional restrictions, \( \sigma_3(X,Y,Z) \) satisfies the same averaging relation. Various reduction formulae of this type also hold in higher dimensions. (Received October 18, 1977.)

65 ▶ Numerical Analysis

Rational functions are used to derive an iterative solution of nonlinear equations in several variables. The classic Newton's method is a special case and for \( n = 1 \), Halley's method can be derived. Orders of convergence are given as well as adjustments to interpolation methods. (Received August 21, 1977.)
simplex search methods. The algorithm is related to previous schemes for finding the topological
degree of maps except that computation proceeds recursively by dimension reduction. Generalized
bisects are used to subdivide the boundary of D. (Received September 2, 1977.)

*752-65-3 CHRIS CORAY and E.R. HEAL, Utah State University, Logan, Utah 84321
On One Sided Chebyshev Approximation

Let \( X \) be a compact space \& \( C(X) \) the space of continuous real-valued functions on \( X \). If
\( S \) is a Haar subspace of \( C(X) \) the problem of best one-sided approximation is to minimize
\[ \| \alpha - f \|, \text{ subject to } f(x) - \alpha(x) \leq 0 \quad \forall \ x \in X. \] In this paper we establish the existence of a
positive function in Haar spaces on certain domain spaces, and using this theorem, we establish
a characterization of best one sided approximation to \( f \in C(X) \) by a Haar subspace which is a
Markoff system. Theorem 1. If \( f(x) \in C[a,b] \& S_n \) a Haar space on \([a,b] \) with a Markoff basis
of dimension \( n \), then \( q(x) \in S_n \) is a best one sided approximation to \( f \) iff \( \exists \ n + 1 \) points
\( a = x_1 < x_2 < \ldots < x_{n+1} = b \) where \( E(x) = q(x) - f(x) \) alternately assume the values \( ||q - f|| \) and 0.
(Received October 11, 1977.) (Author introduced by Duane Loveland).

*752-65-4 JOHN GREGORY and RALPH WILKERSON, Southern Illinois University, Carbondale,

A new algorithm for computing eigenvalues of second order differential equations
of the form \( (p(t) x'(t))' + q(t) x(t) = \lambda r(t) x(t), x(a) = x(b) = 0, p(t) > 0 \) has
been developed. The algorithm is very fast, accurate, and easy to implement. The
ideas follow from the first author's approximation theory of quadratic forms on
Hilbert Spaces and are applicable to a wide variety of Raleigh-Ritz type problems.
The algorithm is derived as follows: The above differential equation is con­
verted to its equivalent quadratic form which is in turn approximated by a finite
dimensional quadratic form using Spline "hat" functions. The "Euler-Lagrange"
equation of the finite dimension quadratic form is constructed and is actually the
numerical solution we seek. (Received October 11, 1977.) (Author introduced by Professor
Ronald Kirk).

752-65-5 JAMES C. CAVENDISH, General Motors Research Laboratories, Warren, Michigan 48090.
The mathematics of catalytic converter modelling

Mathematical models of automotive catalytic converters have been developed in order to analyze
converter performance, to identify important control variables and to study design improvements
and modifications. These models are characterized by coupled nonlinear transient boundary value
problems which describe the diffusion and reaction of mass and energy in a catalytic converter.
The existence of multiple steady-state solutions, possible discontinuities in the dependent
variables, and rapid variations in system control parameters are characteristics of these
boundary value problems which lead to computational difficulties. In this presentation we will
discuss the formulation and analysis of a typical mathematical model, the numerical methods
developed and implemented to efficiently handle the computational complexities, and the subsequent
analysis of these numerical methods. (Received October 14, 1977.) (Author introduced by Allen V.
Butterworth).

752-65-6 Stephen C. kennagin, University of California, Davis, California 95616. Computation of

Let \( \sigma(L)= \{ \lambda_1, \lambda_2, \ldots, \lambda_n, \ldots \} \) be the eigenvalues of the self-adjoint operator \( L \). \( \sigma(L_n)= \)
\( \{A_1, \ldots, A_n \} \wedge \) will be the eigenvalues of \( L_n \), an approximation to \( L \). By using computable error

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bounds we are able to infer properties of $\sigma(L)$ from similar properties of $\sigma(L_n)$. By "computable" is meant an error bound which comes out of the computational process.

Specifically, we prove the following:

1. If $\sigma(L_n) \cap [A_1 + e_1, A_2 - e_2] = \emptyset$ then $\sigma(L) \cap [A_1 + e'_1, A_2 - e'_2] = \emptyset$ where $e_1, e'_1$ are computable.

2. If $A_1$ is simple, the corresponding $a_j$ is simple.

3. If $A_1$ is of multiplicity $m$, the corresponding $a_j$ is of multiplicity $m' \geq m$.

Examples are given from integral equations, ordinary differential equations, and integro-(ordinary) differential equations. (Received October 14, 1977.)


This talk will focus on general principles of applicability and convergence of iterative and projectional methods to random operator equations given the convergence of these methods in a corresponding deterministic setting. Thus, the main questions which need to be addressed are the measurability of the solution and the measurability of the approximates, in order that the methods produce random approximations which converge to a random solution.

Several results validating such general principles will be given, and some open problems will be stated. (Received October 17, 1977.)

*752-65-8 George Corliss, Y. F. Chang, and Gregory Kriegsman, University of Nebraska, Lincoln, NE 68588. Finding Eigenfunction Expansions for PDE's using Taylor Series.

Given a linear, separable partial differential equation defined on a rectangular region, it is well known that the solution can be expressed as a linear combination of certain functions. These are the eigenfunctions of appropriate Sturm-Liouville problems. We illustrate our method on an elliptic problem. We generate a Taylor series approximation to the solution of the separated ordinary differential equations in each dimension. If necessary, the solution is extended to the far boundary by analytic continuation. An extended precision arithmetic package is used to find the eigenvalues. The solution of the original problem is computed as a linear combination of the two families of eigenfunctions. A numerical example is given. (Received October 17, 1977.)


We consider the important problem of reconstructing a radiographic density function $f(x,y)$ from approximate data $g_k(t)$ for its parallel beam X-ray projections $A_k f(t)$ at $P$ equally spaced angles. The author's Method of Partial Eigenfunction Expansion (see Arch. Rat. Mech. Anal., 16 (1964) pp. 126-154, or for a more concise exposition see J. Math. Phys., Vol. 14, No. 8 (1973) pp. 1037-1048) can be applied to give a solution which is "optimal" within a certain very natural setting. We assume the data accuracy (i) $\| A f - g_k \| \leq \epsilon$, and the stabilizing constraint (ii) $\| f \| \leq 1$. (The composite $L^2$ norm in (i) is weighted by $\exp(t^2)$ and the $L^2$ norm in (ii) by $\exp(x^2 + y^2)$) Furthermore, we seek to approximate the pointwise values not of $f$ itself, but of its $\delta$-mollifier $J_\delta f$, with a small "blur-
ring radius \( \delta \). Fortunately, the magical symmetries and identities associated with these exponential weight functions have allowed us to track down the complete eigenfunction decomposition of \( A^*A \) in explicit form. One can thus apply MPEE, prove convergence (with a best-possible error bound), etc. The method can be written in an extremely efficient computational form. (Received October 14, 1977.) (Introduced by J. R. Cannon).

*752-65-10 Luis Kramarz, Emory University, Atlanta, Georgia 30322. An iterative variant of collocation.

To approximate the solution of a two-point boundary value problem by collocation, two partitions \( \Delta_n \) and \( \Delta_q \) are chosen, with \( \Delta_n \) containing more points than \( \Delta_q \). A sequence of collocation problems are solved on \( \Delta_q \), obtaining a sequence of functions which converge to the collocation solution of the original problem on \( \Delta_n \). (Received October 17, 1977.)


The use of Richardson extrapolation in conjunction with general single step discrete time Galerkin method for the approximate solution of the parabolic initial-boundary value problem

\[
\begin{align*}
    u_{k+1} - \Delta u &= f(x,t) & \text{in } Q = (0,T) \times \Omega, \\
    u(x,t) &= g(x,t) & \text{on } \Sigma = (0,T) \times \Gamma, \text{the lateral surface of } Q, \\
    u(x,0) &= u_0(x) & \text{in } \Omega,
\end{align*}
\]

is studied. It is shown that, for \( j=1,2 \), the extrapolation of Galerkin approximations which are \( j \)-th order correct in time yields an improvement of \( j \) orders of accuracy in time per extrapolation. (Received October 17, 1977.)

*752-65-12 JEROME EISENFELD and STEPHEN W. CHENG, University of Texas at Arlington, Arlington, Texas 76019. General moment methods for a class of nonlinear models.

This paper deals with the following problems: (i) the development of a general theory which incorporates several methods as special cases; (ii) the applicability of moment methods to a class of nonlinear problems; (iii) the specification of the class of admissible weighting functions; (iv) the estimation of the number of parameters; (v) the elimination of the cut-off error in the analysis of fluorescence decay data. (Received October 17, 1977.)


For second order elliptic boundary value problems a finite element is said to satisfy the constant strain condition if the associated interpolation scheme reproduces linear behavior of the dependent variable. While numerical integration may introduce sizeable errors in the computation of the entries of the stiffness matrix, if it is shown that the validity of the constant strain condition need not be destroyed by numerical integration. In addition a new \( C' \) curved element is described which, assuming exact integration, satisfies the constant strain condition for fourth order problems. (Received October 17, 1977.)


Given any real-valued function \( f(x) \) for \( x \in [a, b] \) let \( f(x) \) be an L-subsolution, i.e., \( L[f] \leq 0 \) for \( x \in [a, b] \) and \( L[f] = (d/dx)(A(x)df/dx) + 2B(x)df/dx + C(x)f(x) \), where \( A(x), B(x), C(x), G(x) \) are continuous real-valued functions of \( x \) for \( x \in [a, b] \) and the matrix \( Q(x) = \begin{pmatrix} A(x) & B(x) \\ B(x) & G(x) \end{pmatrix} \) is positive-definite for \( a \leq x \leq b \). Let

\[
    I[f] = \int_a^{a+h} \left[ A(x) (u(x, h/dx))^2 + B(x)u(x, h/dx) [du(x, h/dx) + G(x)] - C(x) \right] u^2(x, h) dx, \ a < a + h \leq b, \ h > 0, \ \text{where} \ u(x, h), [A(x)du(x, h)/dx] \in C'(a, b), \ u(x, h) \neq 0, u(a, h) = u(a + h, h) = 0. \ \text{Theorem.} \ \text{If} \ f(x) \ \text{is} \ \text{any} \ \text{real-valued} \ \text{function of} \ x \ \text{for} \ a \leq x \leq b \ \text{such that} \ I[f] < 0, Q(x) \ \text{is} \ \text{a} \ \text{positive-definite} \ \text{matrix for} \ a \leq x \leq b \ \text{and} \ f(x) \ \text{is} \ \text{an} \ L\text{-subsolution such that} \ f(x)L[f] \leq 0 \ \text{for} \ a \leq x \leq b, \ \text{then} \ f(x) \ \text{has at least one zero on the interval} \ a \leq x \leq b. \ \text{(Received October 18, 1977.)}}
Computer Science


A program of research and subsequent software development is outlined to illustrate the conduct of mathematics activity within the laboratories of the Department of Defense. The illustrative example is the implementation into robust and portable computer codes of numerical algorithms to solve large, sparse eigenproblems which arise typically in the dynamic analysis of aerospace vehicle structures. Frequently the matrices which obtain are better considered as diagonalizable rather than merely symmetric so that the problem at hand is the numerical approximation of a selection of eigenvalues and corresponding eigenvectors of $C\mathbf{u} = \lambda\mathbf{u}$ where $C$ is of large order, has many zero entries and is similar to a diagonal matrix. The role of the research mathematician as program manager is discussed and how the efforts of mathematicians and computer scientists both in academic research and in industry are focused on the resolution of this problem. (Received October 17, 1977.)


A. N. Whitehead (Universal Algebra, 1900) noted nonidempotency in numerical algebra $(xx=x, xy=xy)$, idempotency in boolean algebra $(xx=x)$. Def. 1. Given structure $\mathbf{A}=(\mathbf{U}, \mathbf{L}, \mathbf{O}, \mathbf{O}_2)$ of type $\langle 2,2,2,0,0 \rangle$, with (for all $x \in \mathbf{A}$) $x_0L=L$, $x_0U=U$, $x_0x=x$, $x_0x=x$, put (for some $x \in \mathbf{A}$) $x_0x=xy$; $\mathbf{A}$ is a panalgebra iff $x_0\gamma=xy$ whenever $x_0\gamma=L$. Def. 2. $\mathbf{D}=(\mathbf{D}_1, \tau)$ is a supplemented lattice if $\mathbf{D}$ is distributive lattice and $\tau$ recognizes as the commutative, associative, well-defined (cancellative, nonidempotent) operation generating chains of proper (nonatomic) join-irreducibles in $\mathbf{D}$. Th. 1. A factor algebra $\mathbf{F}^n$ (n, natural or integral) is a panalgebra, as is $\mathbf{F}$. Proof: $\mathbf{F}=(gcf, lcm, \cdot, n, \pi)$. Def. 3. 0 is field of set-theory in Hays, Notices 18,4,666(1971); $G$, generalized combination system in H., N. 18,5,806 (1971), V its variously restricted occupancy theory; $L$, lattice logic of H., N. 22,6,647(1975); M its matrix algebra; S, switching algebra of H., N. 24,7,619(1977). Th. 2. $O$, $G$, $V$, $L$, $M$ are (contain) panalgebras. Corollary. Each is isomorph of some $\mathbf{F}^n(F^n)$. Def. 4. A lattice is a cancellative distributive monoid (e.g. natural number system $N$). Def. 5. A structure $\langle 0, \mathbf{O}_2, \mathbf{O}_3, \mathbf{O}_4, \mathbf{O}_5, \mathbf{L} \rangle$ is a spanalgebra iff $\mathbf{O}$ is a set countable in type and tokenage (e.g. prime set of $N$); $\langle 0, \mathbf{O}_2, \mathbf{O}_3, \mathbf{O}_4, \mathbf{L} \rangle$, a panalgebra; $\langle 0, \mathbf{O}_4, \mathbf{L} \rangle$, a larutan. Def. 6. $H$ denotes hyperboolean system of $H., N. 14, 6, 827(1967)$; $H_i$ any monotypal subsystem. Th. 3. $H$ is a spanalgebra; $H_1$ is isomorph of $N$. Corollary. H is imbeddable in an integral domain. (Received October 18, 1977.)

752-68-3 ALAN G. KONHEIM, IBM Research Center, P. O. Box 218, Yorktown Heights, New York 10598. Queueing Models For Data Communications.

The field of computer communications has witnessed rapid growth and technological innovation in recent years. By computer communications we usually mean the user-to-computer or computer-to-computer interfaces realized by communication links. These range from various forms of teleprocessing and time-sharing to computer networks like the ARPANET. The principal problem is the movement of data between users and processors and between processors. Computer communications has defined a class of mathematical problems which deal with the behavior of large service systems. In this talk we will describe some of the underlying mathematical problems and work in this area. (Received October 18, 1977.)

Fluid Mechanics

752-76-1 Lokenath Debnath, Mathematics Department, East Carolina University, Greenville, North Carolina 27834 and Uma Basu, Applied Mathematics Dept., University of Calcutta, Calcutta, India. Propagation of Magnetohydrodynamic Dispersive Waves on a Running Stream.

A study is made of the propagation and generation of the Alfven-gravity waves on a running stream of inviscid electrically conducting liquid of finite depth by an oscillating pressure distribution on the free surface of the liquid. An asymptotic analysis of the integral solution is carried out to determine the transient and the ultimate steady state wave motions. It is shown that there is a critical value of the basic stream velocity $U$, above and below which the ultimate wave systems have a different
character, and that when $U$ assumes its critical value the solution is singular. It is shown that the ultimate steady state solution consists of either two or four Alfvén-gravity waves depending on the relative value of $U$ and its critical value. The parametric equations of the critical curve which separates these two possible states are obtained. For an infinitely deep fluid, the critical velocity is $U = U^* = \frac{g}{a} + \frac{U^2}{g}$. Several limiting cases of physical interest are discussed. (Received August 24, 1977.)


We show how the covariance of an integrodifferential equation with respect to a discrete transformation group can be used to generate new solutions from a given one. These results can be applied to integrodifferential equations that appear in various application. (Received October 6, 1977.)

752-76-3 JOHN REEDER, University of Missouri, Columbia, Mo., 65201 and MARVIN SHINBROT, University of Victoria, Victoria, B.C., Canada V8W 2Y2. Two and three dimensional periodic waves in water of infinite depth. Preliminary report.

We begin with a new, constructive, and very simple proof of Levi-Civita’s famous theorem on the existence, in water of infinite depth, of nonlinear, two dimensional, periodic, progressive waves with small amplitude. Then, we turn to various generalizations to three dimensional waves, the simplest of which is the following. At any time, the wave forms appear two dimensional, in that all curves of constant phase are horizontal, parallel straight lines, but which are actually three dimensional, since the waves move at a given angle $\theta$ to their crests. These waves reduce to the Levi-Civita waves when $\theta = 0$. We also present some other, more complicated, doubly periodic wave forms that move progressively without change in shape, but these are too complicated to describe here. We emphasize that all proofs are constructive, and, in all cases, we present explicit, approximate formulas (valid for small amplitudes) for the flows and the waveforms. (Received October 14, 1977.)

#752-76-4 F. A. Roach, University of Houston, Houston, Texas 77004. Flows and continued fractions over a vector space. Preliminary report.

Suppose that $S$ is a real inner product space and that $u$ is a unit vector in $S$. For each nonzero point $P$ of $S$, let $1/P$ denote $[(2u, P)u - P] / ||P||^2$. By making use of this reciprocal, transformations analogous to those of complex analysis can be defined. For example, transformations of the form $b_0 + 1/b_1 + \ldots + 1/b_n + z$ are quite similar to linear fractional transformations and $1/1/z + 1/z + 1/1/z - z$ is analogous to $z^2$. Connections between such transformations and flows on $S$ are investigated. Generally, these transformations do not give rise to potential flows unless $S$ is $E^2$. However, with certain modifications, functions which do so can be obtained from them. Several examples in $E^3$ are given. (Received October 18, 1977.) (Author introduced by W. T. Ingram).

80 ▲ Classical Thermodynamics, Heat Transfer


In 1909 C. Caratheodory (Math. Ann. 67 (1909), 355-386) claimed that a temperature scale can be constructed if one assumes the existence of a certain set of relations defined on pairs of state spaces of physical systems. G. Whaples (J. Rational Mech.
Anal. 1 (1952), 301-307) constructed a counter-example to Caratheodory's assertion. Sufficient conditions are stated that establish Caratheodory's statement. The major theorem is a generalization of S. Eilenberg's work (Amer. J. of Math 63 (1941), 39-45). Theorem. Let \((X, \sim)\) be a topological space with an equivalence relation \(\sim\) which satisfies the following properties: i) the equivalence class of \(x\) is closed and connected in \(X\) for all \(x\) in \(X\), ii) \((X, \sim)\) is connected and locally connected, iii) \(P(X) = \{(x,y): x \neq y\}\) is not connected. Then \((X, \sim)\) possesses exactly two partial orders which agree with the equivalence relation \(\sim\) and the topology on \(X\).

(Received September 12, 1977.)

*752-80-2 PAUL B. BAILEY, Sandia Laboratories, Applied Mathematics Division, Albuquerque, New Mexico 87115. The problem of thermal instability of explosive materials.

When a material undergoes an exothermic chemical reaction with heat loss by conduction through the material to its boundary, thermal instability will occur if the energy liberated during the reaction is greater than that lost through the surface. So the steady-state heat conduction equation obeyed by explosive materials is studied in order to be able to obtain information about the onset of thermal instability. (Received October 3, 1977.) (Author introduced by Professor Robert J. Thompson).

*752-80-3 ALAN D. SOLOMON, Union Carbide Corporation Nuclear Division, P.O. Box Y, Oak Ridge, Tennessee 37830. ON THE ACCURACY OF MEGERLIN'S METHOD FOR PHASE CHANGE PROBLEMS.

The Megerlin method for the solution of the Stefan problem is based on the solution of an ordinary differential equation for the phase boundary, arising from the fitting of an appropriately chosen temperature approximation to the conditions of the problem. In practice one finds that the phase boundary locations predicted by the method are extremely accurate, in spite of the at-times poor fit of the temperature approximation. In this discussion we derive an estimate of the error in the phase front location for a one-phase problem with prescribed, variable, boundary flux. We find that this error is of the order of the Stefan number (the ratio of sensible to latent heats of the material), implying its high accuracy for materials with small Stefan number. Examples are given in the context of specific materials and flux functions. (Received October 18, 1977.)

81 ▶ Quantum Mechanics

*752-81-1 LEONARD J. GRAY and THEODORE KAPLAN, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830. Elementary excitations in disordered alloys.

A disordered alloy is a solid in which two types of atoms are arranged in the solid according to some probability distribution. The Hamiltonian matrix \(H\) which describes the elementary excitation (electron, magnon, or phonon) is therefore a random matrix, and one seeks to average the resolvent \(G(z) = (zI - H)^{-1}\) over all possible configurations of the alloy. The widely used Coherent Potential Approximation is only valid for very special disordered alloys. A completely general approach to this problem, which relies on ideas and techniques from operator theory and functional integration, is presented. Applications of this method to a problem in photosynthesis and to the computation of functional integrals are also mentioned. (Received October 3, 1977.)


Let \(L\) be a translation-invariant self-adjoint operator in \(L^2(\mathbb{R}^n, \mathbb{C}^m)\) with domain \(D\) and order \(p\). Let \(X\) be a \(d\)-dimensional linear subspace of \(\mathbb{R}^n\), \(D_X\) be those functions in \(D\) that are zero in a neighborhood of \(X\), and \(L_X\) be the restriction of \(L\) to \(D_X\). In this paper we find all the self-adjoint extensions of \(L_X\) and interpret some of them as local interactions of pairs of quantum mechanical particles. One result, similar to a result of W. Littman, is that \(L_X\) is essentially
self-adjoint if \((n-d)\geq 2p\). Define a local interaction of two particles in \(\mathbb{R}^n\) to be a self-adjoint extension of \(L_\Delta\), where \(L\) is the non-interacting Hamiltonian (an operator in \(L^2(\mathbb{R}^{2n},\mu)\) for some \(\mu\)) and \(\Delta\) is the diagonal in \(\mathbb{R}^{2n}\). There are non-trivial local interactions of relativistic particles only if \(n=1\) and of Schrödinger particles only if \(n\leq 3\). The last result is similar to a result of C. Friedman. Let \(G\) be the inhomogeneous Lorentz group in one spatial dimension and \(\mathfrak{g}\) be its Lie algebra. We use exact representations of \(\mathfrak{g}\) defined by boundary conditions at \(\Delta\) to show that some of the non-trivial local interactions of relativistic particles in \(\mathbb{R}^n\) come from unitary representations of \(G\). (Received October 17, 1977.)


We announced previously (AMS Notices 23, 5(1976), p. A-533) the rigorous computation of the Feynman path integral, using Feynman's original definition as the propagator or kernel for the Schrödinger equation, calculated as the limit of certain iterated improper integrals. This work derived from repeated asymptotic expansions and gave a perturbation expansion, or time-ordered exponential, but not the usual one (see Feynman & Hibbs, Quantum Mechanics and Path Integrals (1965), p. 120). Now we give a parallel development, replacing asymptotic by Fourier transform techniques, which yields Feynman's expansion. It works for any \(L^2\) potential or potential which is the Fourier transform of a measure of bounded variation, or for a potential which is one of these plus a linear or quadratic part. (Received October 17, 1977.)

752-81-4 THOMAS SPENCER, Princeton University, Princeton, New Jersey 08540 and Rockefeller University, New York, New York 10021. Classical and quantum field theory.

We shall discuss some aspects of the relation between classical field equations which are typically nonlinear partial differential equations and the more complicated quantum field models. Examples will include the anharmonic oscillator, some special two dimensional models and the Yang Mills equation. (Received October 17, 1977.)

82 Statistical Physics, Structure of Matter

752-82-1 LYNN O. WILSON, Bell Laboratories, Murray Hill, New Jersey 07974. Crystal growing.

Integrated circuits are fabricated on thin semiconductor wafers sliced from large single crystals. A semiconductor crystal consists primarily of a pure material, such as silicon, but also intentionally contains a small amount of a dopant or impurity which substantially affects its electrical properties. The amount of impurity in a crystal depends in part upon the rate at which it was grown. Experimentally, the microscopic crystal growth rate is found to fluctuate considerably with time. In order to understand how time varying growth rates affect the dopant distribution, we model the crystal growth process mathematically. A hierarchy of interesting mathematical research problems emerges. (Received October 11, 1977.) (Author introduced by Henry O. Rollak).

752-82-2 William Greenberg, Virginia Polytechnic Institute & State University, Blacksburg, Va. 24061. Quantum Potentials and the Microcanonical Entropy.

The microcanonical entropy for quantum (operator valued) potentials on a lattice is studied. We show that existence of an infinite volume limit is implied by a spectral estimate for the perturbation of normal matrices; such an estimate has been an open conjecture for many years. Conditions for continuity of the entropy as a function of density and the conditional variables are derived. (Received October 18, 1977.)
Relativity

752-83-1 J. G. MILLER, Texas A&M University, College Station, Texas 77843. Bifurcate Killing horizons. Preliminary report.

Canonical coordinate systems are constructed where a Killing vector field on a Lorentz manifold vanishes. This result is used to prove the following theorem: If $K$ is a Killing horizon of an analytic Lorentz manifold with respect to a Killing vector field $\xi$ and the null orbits of $\xi$ on $K$ are incomplete, then the gradient of $\xi^2$ on the horizon equals a nonvanishing function $\kappa$ on $K$ times the Killing vector field $\xi$. If $\kappa$ is a constant function, then there exists a unique local analytic prolongation of the Lorentz metric and the Killing vector field $\xi$ that contains a bifurcate Killing horizon. (Received October 17, 1977.) (Author introduced by William L. Perry).

752-83-2 Murray Cantor, The University of Texas at Austin, Austin, Texas 78712. Specification of Scalar Curvature on $\mathbb{R}^3$ and Solutions of the Scalar Constraint Equation of General Relativity.

Let $\sigma(x) = (1 + |x|^2)^{\frac{1}{2}}$ and $|x|_{p,s,5} = \sum |a|^{s+p'|a|^{2p}}$. The following theorems are proved:

Theorem 1. Let $p > 3$, $s > 3/p$, and $\delta$ be a Riemannian metric on $\mathbb{R}^3$ such that $|\delta_{ij}|_{p,s,0} < \infty$ and scalar curvature $R(g) \leq 0$. Let $\bar{g}$ be any function such that $|\bar{g}|_{p,s,2,2} < \infty$ and $C_1(x) \leq R(g)(x) \leq \bar{R}(x) \leq 0$ for all $x \in \mathbb{R}^3$ and some $C_1 > 0$. Then there is a unique positive function $\delta$ such that $|\delta^{-1}|_{p,s,0} < \infty$ and $\bar{g} = \delta^4 g$ has scalar curvature $\bar{R}$. Furthermore $\delta$ depends smoothly on $g$. This theorem is used to establish the following: Theorem 2. Suppose $p,s,5$ and $R(g)$ are as in Theorem 1. Also suppose $\delta$ is a nonnegative function with $|\delta|_{p,s,2,2}$ sufficiently small. Then there is a unique positive $\delta$ with $|\delta^{-1}|_{p,s,0} < \infty$ satisfying $8\Delta g = R(g)\delta + M\delta^2 = 0$. These results are contained in the paper "The Existence of Non-trivial Asymptotically Flat Initial Data for Vacuum Spacetimes", Comm. Math. Phys. To appear. (Received October 18, 1977.)

Economics, Operations Research, Programming, Games

752-90-1 G. ARTHUR RHEINGOLD, Ph. D.; P. O. Box 234; Haverford, Pennsylvania 19041. On "catastrophe theory" and Wiener's 'cybernetics'. Preliminary report.

The paper reviews the author's 'A Closer Look at Thom's Catastrophe Theory' [PROCEEDINGS OF THE FIRST INTERNATIONAL CONFERENCE ON MATHEMATICAL MODELING, St. Louis (1977)], placing Zeeman's notions [SCIENTIFIC AMERICAN, April 1976] regarding the applicability of Thom's catastrophe theory (to the description of social and political systems) in two contexts: Norbert Wiener's introduction of the term, 'cybernetics', to mean other than its proper etymological definition: "political science" [cf. his HUMAN USE OF HUMAN BEINGS, 1954]; and Wiener's self-acknowledged political orientation [cf. his I AM A MATHEMATICIAN, pp. 175-176]. Political systems are thus shown to be ideally suited for modelling in combined discrete/continuous programming languages [CASP IV, e.g.] rather than in terms of either Thom's catastrophic surfaces or Wiener's cybernetic time-series [cf. AN EPSILITE TO DR. BENJAMIN FRANKLIN, Exposition-University Press, 1975 (1974)]. The dynamics of human social systems are shown to be representable in terms of a set of algorithms, a single algorithm for each decision-maker (elected or reigning politician) within the social system. Hence we may resolve the complexities of socio-politico-economic systems by means of (not strictly mathematical) simulations, avoiding thereby both Thom's 'catastrophe theory' and Wiener's ambiguous 'cybernetics'. (Received October 11, 1977.)

752-90-2 Harvey R. Diamond, West Virginia University, Morgantown, West Virginia 26506. Asymptotic Equilibria in a Class of $n$-Person Symmetric Games

We consider the following class of games: Let $n$ players independently choose one of $N$ alternatives. The payoff to each of $n$ players who choose alternative $i$ is $S_i(n)$, a non-increasing function of $n$. We construct a unique symmetric equilibrium, the probability distribution $p_i(n)$, $i = 1, \ldots, N$ over the $N$ alternatives, along with the equilibrium payoff $C(n)$. If for each $i$, $S_i(n) \to 0$ then $C(n) \to 0$ and $np_i(n) \to \infty$. We treat the problem of determining the asymptotic
behavior of $C(N)$ and $p_i(N)$ for $i$ large when the $S_i(n) \sim n^{-\alpha_i}$. A transform technique converts the expected payoffs to an integral form which is evaluated asymptotically. To first order the equilibrium satisfies $C(N) = S_i(N)p_i(N)$, $i = 1, \ldots, M$. Explicit two term expansions for $p_i(N)$ and $C(N)$ are calculated for cases $\alpha_i = \alpha \forall i$ and $\alpha_i \neq \alpha_j$ if $i \neq j$.

We solve (asymptotically) a related sequential game and discuss variations of the model treated. (Received October 17, 1977.) (Author introduced by S. M. Rankin, III).

752-90-3 WILLIAM H. RUCKLE, Clemson University, Clemson, South Carolina 29631. Geometric Games. Preliminary report.

Two antagonists RED and BLUE occupy closed subintervals $R$ and $B$ respectively of the unit interval $[0,1]$. The interval $R$ may have length at most $r$; the interval $B$ must have length at least $b$. The payoff to RED is the length of the intersection $R \cap B$. This game is similar but not identical to one studied by Arnold in 1962 (ORC, IRM-10, AD277843). If $r \leq b$ the value of this game to RED is $r/|1/b|$ for $b-r \geq 1- b/|1/r|$ and $(r/|1/b| + b/|1/r| + b-1)/(1+b)^2$ for $b-r \leq 1-b/|1/r|$. Here $[ \cdot ]$ denotes the greatest integer function. If $r > b$ the value of this game to RED is $(b-[1/r] + [1/r]^2 r)/(1/r)^2 + [1/r])$ for $1-[1/r] r \leq b$ and $b/(1 + [1/r])$ for $1-[1/r] r > b$. We also solve this game played on the circumference of a unit circle and played with rectangles within a unit square. (Received October 17, 1977.)

*752-90-4 JONATHAN SPINGARN, Georgia Institute of Technology, Atlanta, Georgia 30332 and R. T. ROCKAFELLAR, University of Washington, Seattle, Washington 98195. Generic Necessary Conditions for a Family of Nonlinear Programming Problems. For each $v \in \mathbb{R}^n$ and $u = (u_1, \ldots, u_m) \in \mathbb{R}^m$, let $(Q_{vu})$ denote the nonlinear programming problem in which the function $f(x) - x \cdot v$ is to be minimized over all $x \in \mathbb{R}^n$ satisfying the constraints $g_i(x) < u_i (i = 1, \ldots, m)$.

Theorem: If $f$ is of class $C^2$ and $g_1, \ldots, g_m$ are of class $C^{n+1}$, then for all $(v,u) \in \mathbb{R}^{n+m}$ not belonging to a certain set of measure zero, if $x$ is a local minimizer for $(Q_{vu})$ then there exists a multiplier vector $y = (y_1, \ldots, y_m) \in \mathbb{R}^m$ such that (i) $\nabla f(x) + \sum_{i=1}^m y_i \nabla g_i(x) = 0$, (ii) for all $i$, $y_i \geq 0$ and ($y_i > 0$ if and only if $g_i(x)=0$), (iii) the gradients of the active constraints are linearly independent, and (iv) if $H = \nabla^2 f(x) + \sum_{i=1}^m y_i \nabla^2 g_i(x)$, then $\zeta^T H \zeta > 0$ for all nonzero $\zeta$ such that $\zeta \cdot \nabla g_i(x) = 0$ for all $i$ such that $g_i(x) = 0$. (Received October 17, 1977.)


An economy consists of a finite numbers of traders. Each trader has a utility function, which is piecewise smooth in the interior and on the boundary of the consumption set. Regular economies are defined and it is proved that outside a closed set of measure 0 in the parameter space, economies are regular, have odd numbers of equilibria, and the equilibria vary smoothly with the parameters. The proof is an application of Transversal Density Theorem. Genericity of the utility functions are discussed. This involves mutually transversal smooth functions and Jet Transversality Theorem. We also discuss the connection between "corners" of piecewise smooth functions and "singularities" of smooth functions. (Received October 18, 1977.)

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Let S and I be the classes of susceptible and infective individuals in a population of fixed size N. F. Wang [Asymptotic behavior of some deterministic epidemic models, SIAM J. on Math. Analysis (to appear)] proposed a model in which susceptibles become infective in the time interval \( [t, t + dt] \) at a rate \( a(I(t))dt \) and remain infective with probability \( F(t) \) for at least a length of time \( t \). Here \( a \in C' \) is nondecreasing. The model yeilds the system\( S'(t) = -a(I(t))S(t), \)
\[ I(t) = r(t) = r(t) + \int_0^t \alpha(I(u))S(u)F(t-u)du, \]
where \( r(t) \) is the proportion of initial infectives still infective at time \( t \). Assume \( F(t) = 1 \) for \( [0, T] \) and \( \exp A(T-t) \) for \( t > T \), and \( r(t) = 0 \). The system has a unique solution \( (I(t), S(t)) \neq (0, 0) \) as \( t \to \infty \). \textbf{Theorem.} If \( I' \leq 0 \) on \( [t-T, t] \) then \( I' \leq 0 \) on \( [t, \infty) \). \textbf{Theorem.} If \( \inf[I(t); t_0 - T \leq t \leq t_0] = I(t_1), t_1 > t_0 \) then \( I' \leq 0 \) on \( [t_1, \infty) \). \textbf{Corollary.} \( I \) does not oscillate indefinitely. (Received October 11, 1977.)

With the supposition that the excretion of digitoxin and certain other glycosides from the body is exponential, it can be shown that a digitalizing average body level \( B \) can be accomplished by administering a constant maintenance dose \( d < B \). This has been emphasized by R.W. Jelliffe [c.f. Math Biosci 1 (1967) 305, 325]. In certain clinical situations, the excretion is not exponential. A theorem is obtained which applies to certain of these clinical situations in which the hypothesis that the excretion of the pharmaceutical is exponential is relaxed to a general condition on the curvature of the decay functions. Certain decay functions evidently of clinical bearing may be then shown on inspection by differentiation to satisfy the hypothesis of the theorem. (Received October 17, 1977.)

The detection of myocardial infarction in patients is an especially important medical problem. Various diagnostic techniques have been suggested based on the analysis and evaluation of blood enzymes. The present paper is concerned with the development of a statistical classification procedure for implementation of such techniques utilizing nonparametric density estimators and a recently developed penalized maximum likelihood estimator are used. The problems involved in applying these estimators to the problem of classification are discussed. In addition, the results for the two estimators are compared and contrasted with the results obtained using classical parametric methods. The nonparametric methods promise to yield more accurate classification schemes and illustrate some problems involved in using classical parametric approaches to the problem. (Received October 17, 1977.)

The development of topological eye-brain mappings may be regarded as a model system for self-organi-
zation of ordered structures in the brain. Early label theories have been shown insufficient by expe-
riments revealing a high degree of adaptability to variations in eye and brain. We present a novel the-
ory that could explain most of the experimental results. It postulates a small number of marker mole-
cule types which are synthesized in isolated regions of the retina and which diffuse and decay; their
stationary distribution is then in the form of smooth gradients. The markers are transported to the
target structure (tectum) by the connecting fibres themselves. Differential equations are given for
the developing contacts and are discussed in terms of minimal requirements. The equations are analogous
to those for population dynamics in evolution with self-reproduction and selection according to fitness
(computed on the basis of marker concentrations) and limited resources (conservation of axonal arbri-
zation). Examples of numerical solutions of the differential equations are discussed. (Received
October 18, 1977.) (Authors introduced by Professor Jack D. Cowan).

752-92-5 ROBERT M. MIURA, University of British Columbia, Vancouver, B.C., Canada V6T 1W5.
A mathematical model for nonlinear waves of spreading cortical depression. Preliminary
report.

Physiologists have studied waves of spreading cortical depression for the past 33 years but they have
not yet proposed an adequate explanation of this phenomenon. The waves are characterized by a slowly
propagating (2-5 mm/min.) negative d.c. surface potential and a depression of the electroencephalo-
gram. Physiological mechanisms are proposed to explain the instigation and propagation of these waves
and a mathematical model consisting of a system of reaction-diffusion type equations coupled with a
system of ordinary differential equations is constructed. A simplified version of this model which
retains the important mechanisms has been studied numerically and the solutions exhibit qualitative
agreement with the principal phenomena observed experimentally. Some of the mathematical problems
will be discussed. (Received October 17, 1977.)

752-92-6 WALTER T. KYNER and GARY A. ROSENBERG, University of New Mexico, Albuquerque, N.M. 87131.

An example of an inverse problem in neurobiology is the determination of blood-brain transfer
constants for the capillaries at the cortical surface of a cat by supracollosial-cisternal perfusion
with an extracellular marker and a permeable test molecule. The corresponding mathematical problem
is the determination of parameters in a partial differential equation modeling the perfusion by an
optimization algorithm. The algorithm and its application to several experiments is given.
(Received October 18, 1977.)

752-92-7 T. Poggio, Max Planck Institut für biologische Kybernetik, Tübingen, W.Germ
Flies and synapses: A Volterra-like functional representation of algorithms
and interactions. Preliminary report.

The control of orientation behavior in the fly relies on a few powerful and fast
computations performed on the visual input. A phenomenological theory, based on
quantitative behavioral data, implies that a few specific computations are performed
on the visual input. A general representation of "smooth" nonlinear systems (func-
tionals) can be used to analyze the algorithms and the underlying neural interactions
that implement these computations in the fly's visual system. The approach, which is
suitable for a large class of "simple" nonlinear operations, rests on a few theorems
that characterize rigorously the class of systems admitting a series representation
of the Volterra type. The possible "hardware" implementation of such nonlinear
interactions in the nervous system will be discussed in terms of a new approach to
synaptic interactions. The classical approach to (passive) neural integration,
mainly due to W. Rall, can be extended to the nonlinear case in which conductance
changes are inputs to a cable-like dendritic tree. A simple theorem ensures that
the membrane potential V is always an analytic functional of bounded, localized
conductance inputs and provides explicitly the various kernels of the polynomial
expansion of V. Finally, several implications of this approach for linking speci-
fic synaptic organizations with elementary information processing will be briefly
outlined. (Received October 18, 1977.) (Author introduced by Professor Jack D. Cowan).
**Systems, Control**

*752-93-1  B. N. DATTA, Universidade Estadual de Campinas, Campinas, Brasil. Two Inertia theorems for Hessenberg matrices and their applications to stability problems."

Theorems relating to the inertia of a matrix and those involving results about the distribution of eigenvalues with respect to the unit circle are called inertia theorems. In this paper we present two inertia theorems for Hessenberg matrices. The importance of these theorems lie in the fact that the procedure of solving stability problems of linear control systems, both continuous and discrete time systems, via Lyapunov matrix equations, become much simplified. We also show that the well-known result of Sowarz on the distribution of eigenvalues with respect to the imaginary axis, a recent result of the author on the stability of the Routh matrix and the classical results of Fujiwara on the solution of the Routh-Hurwitz and the Schur-Cohn problems, all follow as special cases of these inertia theorems. (Received September 26, 1977.)

*752-93-2  HERNAN RIVERA, Universidad Mayor de San Andres, La Paz, Bolivia, and C. E. LANGENHOP, Southern Illinois University, Carbondale, Illinois 62901. Controllability of Linear Neutral Systems."

The n-dimensional real system (1) \( \dot{x}(t) = A_1 \dot{x}(t-h) + A_0 x(t) + A_1 x(t-h) + b u(t) \) (with \( h > 0 \) and scalar control functions \( u \) which are locally \( L_2 \)) is said to be controllable on \([0, T]\) if for any \( \psi: [-h, 0] \to \mathbb{R}^n \) which is continuous with \( L_2 \) derivative a.e. there is a control \( u \) such that the solution of (1) satisfying \( x_0 = 0 \) also satisfies \( x_T = \psi \). Treating \( D \) (for derivative) and \( S \) (for shift) as scalar indeterminants, form the transposed matrix of cofactors \( P(D,S) = \text{adj}(ID - A_1 DS - A_0 - A_1 S) \) and define the polynomial matrix \( K(D) \) by \( K(D) \zeta(S) = P(D,S)b \) where \( \zeta(S) = [1, S, \ldots, S^{n-1}]^T \). Theorem: Let \( T > nh \). In order for (1) to be controllable on \([0, T]\) it is necessary and sufficient that rank \([b, A_1 b, \ldots, A_1^n b]\) = \( n \) and \( K(\lambda) \zeta(e^{-\lambda h}) \neq 0 \) for every complex \( \lambda \). Necessity was proved and sufficiency conjectured in Criteria for function space controllability of linear neutral systems by M. Q. Jacobs and C. E. Langenhop, SIAM J. Control and Optimization, 14 (1976), 1009-1048. (Received October 11, 1977.)

**Information and Communication, Circuits, Automata**

*752-94-1 Donald Y. Goldberg, Dartmouth College, Hanover NH 03755. A generalized weight for linear codes and a Witt-MacWilliams theorem."

Let \( F \) be a finite field and \( H \) a subgroup of \( F^* \). For each \( n \)-tuple \( u = (u_1, \ldots, u_n) \in F^n \) define \( w_H(u) = \#\{u_i : u_i \in H\} \). An \( H \)-monomial map on \( F^n \) is an automorphism of \( F^n \) whose matrix with respect to the co-ordinate basis is of the form \( P \cdot D \), where \( P \) is a permutation matrix and \( D \) is a diagonal matrix with non-zero entries from \( H \). Suppose \( C \) is an \( (n,k) \) code over \( F \) (that is, a \( k \)-dimensional subspace of \( F^n \)) and that \( \varphi : C \to F^n \) is an injective homomorphism which preserves \( w_H \) in the sense that \( w_H(\varphi(u)) = w_H(u) \) for all \( u \in C \). We prove that \( \varphi \) may be extended to an \( H \)-monomial map on \( F^n \). This generalization of a theorem of MacWilliams on the (Hamming) equivalence of codes may be considered to be an analogue of the Witt Theorem of metric vector spaces. (Received October 14, 1977.)

*Author introduced by Professor Kenneth Bogart.*

752-94-2 BRUCE R. EBANKS, Texas Tech University, Lubbock, Texas 79409. Information measures depending directly on events. Preliminary report.

Classical measures of information (or generalized information) depend on the outcomes.
of an experiment only through the probabilities (or information measures) of the outcomes. We consider measures of information which may depend directly on the events (outcomes). The classical branching property is generalized to

\[ I_n(x_1, x_2, \ldots, x_n; p(x_1), p(x_2), \ldots, p(x_n)) = I_{n-1}(x_1 \cup x_2, x_3, \ldots, x_n; p(x_1 \cup x_2), p(x_3), \ldots, p(x_n)) + \varphi(x_1, x_2; p(x_1), p(x_2), p(x_1 \cup x_2)) \]

for events \( x_i \) with probabilities \( p(x_i) \). It is shown that all such branching measures which are expansible and symmetric (in the pairs \( (x_1, p(x_1)) \)) are of the form

\[ \sum_{i=1}^{n} \varphi(x_i, p(x_i)) + \varphi_0(i \cup_1 x_i). \]

Among these measures, the continuous ones which satisfy the natural generalization of classical recursivity are of the form

\[ A \sum_{i=1}^{n} p(x_i) \log p(x_i) + \sum_{i=1}^{n} p(x_i) B(x_i) - B(i \cup_1 x_i), \]

for some function \( B \) of the events and some constant \( A \). (Received October 17, 1977.)

96 ▶ Mathematical Education, Elementary

*752-96-1 James H. Jordan, Washington State University, Pullman, Washington 99164.

Sex differences in the visualization of number.

A person is shown six pennies. A visual barrier then is placed so the person cannot see the pennies. One penny is brought from the hiding place and set in view of the person being tested. A simple problem involving the hidden pennies is posed. By the responses we conclude that girls, much more often than boys, lose track of the number of objects still hidden. (Received October 12, 1977.)

98 ▶ Mathematical Education, Collegiate

752-98-1 A.L. ANDREW, SIDNEY A. MORRIS, P.J. STACEY, La Trobe University, Bundoora, Victoria 3083, Australia, and GERARD P. PROTONASTRO, Saint Peter's College, Jersey City, New Jersey 07306. On the equality of the mixed partial derivatives.

In a standard calculus text, after discussing higher-order partial derivatives, the natural question which arises is the following: what is a set of conditions for which the mixed partial derivatives \( f_{xy} \) and \( f_{yx} \) are equal? The usual answer to this question is essentially that the continuity of the mixed partials implies their equality. In this paper we discuss some lesser-known but interesting conditions which imply that \( f_{xy} = f_{yx} \). A consideration of such results has been found to be useful in a junior-senior level student participation seminar, in which topics not usually found in textbooks are discussed. (Received October 17, 1977.)

*752-98-2 LAIRD E. TAYLOR, California State College, Bakersfield, CA 93309

An elementary proof of the fundamental theorem of algebra.

We give a proof of the fundamental theorem which seems quite suitable for students. No complex integration is involved: in fact, the deepest analytic result needed is the

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convergence of subsequences on compacta. To a point we follow H. W. Kuhn (Mathematical Programming Study 1 (1974) p148-158) but we avoid computations by relying on an (easily proved) strengthened form of Sperner's Lemma. (Received October 17, 1977.)

752-98-3 ROLAND E. LARSON, Pennsylvania State University, Erie, Pennsylvania 16510. A consumer mathematics course for liberal arts students.

One of the Baccalaureate Degree Requirements at Penn State is that each graduate acquire at least six credits (two courses) of quantification. One way a student can partially fulfill this requirement is by taking a "mathematics appreciation" course. Typically, such courses attempt to introduce the student to the beauty of mathematics by giving him a bird's eye view of several topics such as set theory, geometry, probability, topology, statistics, number theory, logic, etc. Since many of the clientele for these courses are suffering from math anxiety, fear of fractions, decimal distress, numerical neurosis, or some other type of figuring phobia, the courses are often organized so that the student can learn about mathematics without doing mathematics. Current texts for such courses are heavy on pictures and cartoons and short on computation. After several unpleasant attempts at teaching a mathematics appreciation course, I decided to try a different approach. From my students' reaction, as well as my own, I believe the attempt was successful. At any rate, this paper describes this alternate approach. (Received October 18, 1977.)

99 ▶ None of the above


The notion of an ambiguous string of expressions is modeled using Thom's cusp catastrophe. In a similar way the structure of simple jokes is explicated, the punch line bringing about the catastrophic interpretation switch. More complicated jokes as well as Kuhn's notion of a scientific revolution are conceived in terms of Thom's butterfly catastrophe. The 3 control factors in a Kuhnian revolution are the extent of observational support for the alternative theories and time. (Received August 4, 1977.)


A directionally drilled oil or gas well is one whose direction is altered to achieve certain objectives, such as relieving blowouts and reaching reservoirs that would otherwise be inaccessible. Information such as the direction and inclination angles and depth of such a well are available only at selected points along the wellbore, since such data are expensive and difficult to obtain.

Since it is important for the directional driller to locate the bottom of the wellbore several models have been developed to simulate the wellbore by a curve from which this location can be inferred. We present here a model that invokes only reasonable physical assumptions about the actual wellbore, and, when applied to existing wells, is at least as accurate as methods currently in use. (Received October 17, 1977.)
**Bio 78-1** JOHN A. FEROE, Vassar College, Poughkeepsie, N. Y. 12601. *Multiple impulse solutions of a nerve equation.*

A piecewise linear partial differential equation which models the propagation of a nerve impulse along an axon is considered. As well as two previously investigated solutions, one representing a slow and the other a fast velocity solitary impulse traveling with fixed form, the existence of slow and fast multiple impulse solutions is exhibited. The local stability of these solutions is analyzed using a technique which identifies the number of unstable modes. In general fast impulses are stable and slow impulses unstable, however, both fast and slow multiple impulse solutions may exhibit a mode of instability which indicates a change in the spacing between two pulses. (Received October 17, 1977.)


Nerve model equations typically possess periodic traveling wave solutions corresponding to repetitive firing in nerves. We have calculated a family of such solutions and determined the dependence of the propagation speed on firing frequency for the Hodgkin-Huxley nerve conduction equations at 6.3°C. For each frequency below the maximum frequency of 147Hz, we find two solutions, one fast and one slow. As frequency approaches zero, these solutions approach the fast and slow solitary pulse solutions (as previously calculated by other investigators). The speed of the fast wave is observed to drop from 12.3 meters/second for the solitary pulse to 4.6 m/sec at maximum frequency.

Our calculations were done using a finite difference method rather than a shooting scheme. The wave profile, evaluated on a discrete grid over the period, along with the speed are determined as the solution to a set of simultaneous nonlinear equations. We believe our method to be widely applicable, and have used it to compute analogous solutions to the FitzHugh-Nagumo equations. Numerical experiments with the full set of partial differential equations have been consistent with the conjecture that the fast wavetrains are stable and the slow ones are not. (Received October 17, 1977.)


While David Berlinski ("On Systems Analysis," MIT Press, Cambridge, 1977) has contended that "biochemists have taken pleasure in the thought that decisive progress in the unraveling of the code was achieved in a way that owed nothing to mathematical methods," we will argue that the work of Gamow and others not only laid the basis for the biochemical solution of genetic coding, but that it, furthermore, led to other contributions of combinatorics, information theory, graph theory, group theory, and coding theory, as well as substantial contributions of statistics, which deepened our understanding of genetic coding and presented unresolved problems. We will present a tesseract graph of a $4 \times 4$ Klein group representation of the systematically degenerate genetic code (Bertman and Jungck, J. Bio. Ed., in press), a combinatoric analysis of possible genetic codes given certain evolutionary assumptions and a statistical justification for why certain amino acids are coded for by particular anticodon oligonucleotide sequences (Jungck, unpublished), notions on overlapping code limitations on synchronizability of coded sequences (e.g., 0X174), application of Shannon's first and second theorems of information theory to efficient coding, and finally to suggest some problems in error-detection and error-correction properties of the genetic code. (Received October 17, 1977.)
This paper analyzes several mathematical models of tumor growth. These models consist of groups of equations that represent the rules governing the movement of tumor cells through various stages of the cell cycle. The models allow one to predict the tumor population after a chemotherapy regimen. Thus, these models make it possible to predict the optimal doses, schedules and combinations of drugs for any tumor. (Received October 17, 1977.)

**ERRATA—Volume 24**

The original list of doctorates awarded in June 1977 appeared in the October 1977 issue of these Notices. On page 345 under the Mathematics Department of the University of California, Los Angeles, the dissertation title of Robert Lowell Miller was listed incorrectly. It should read "Totally positive matrices, semi-simple algebras, and error-correcting codes."

**SITUATIONS WANTED**

UNEMPLOYED MATHEMATICIANS, or those under notice of involuntary unemployment, are allowed two free advertisements during the calendar year; retired mathematicians, one advertisement. The service is not available to professionals in other disciplines, nor to graduate students seeking their first postdoctoral positions; however, veterans recently released from service will qualify. Applicants must provide:

1. name of institution where last employed;
2. date of termination of service;
3. highest degree;
4. field.

APPLICATIONS FROM NONMEMBERS must carry the signature of a member. Free advertisements may not exceed fifty words (not more than six 65-space lines), including address of advertiser; excess words are charged at the rate of $0.15 per word (minimum charge $1). Anonymous listings are carried for an additional fee of $5; correspondence for such applicants will be forwarded to them.

EMPLOYED MEMBERS of the Society may advertise at the rate of $0.15 per word; nonmembers, currently employed, $0.50 per word (minimum charge $15).

DEADLINE for receipt of advertisements is the same as that for abstracts; date appears inside front cover of each issue of the Notices.

APPLICATION FORMS may be obtained from, and all correspondence should be directed to, the Editorial Department, American Mathematical Society, P. O. Box 6248, Providence, Rhode Island 02940.

CORRESPONDENCE TO APPLICANTS LISTED ANONYMOUSLY should be directed to the Editorial Department; the code which is printed at the end of the listing should appear on an inside envelope in order that correspondence can be forwarded.

MATHEMATICS PROFESSOR, TEACHING AND/OR RESEARCH. Ph. D. 1969. Age 36. Specialty: numerical analysis—partial D, E. Seven published articles in numerical analysis and computational fluid dynamics, Five fellowships, Superb teacher, seven years experience, Excellent references and résumé upon request. Area of at least 100,000 preferred. Larry Kurtz, Cloverdale Farms, #14, Cloverdale, Virginia 24077.


ANONYMOUS

AUSTRALIAN MATHEMATICIAN, Ph. D. 1965. Have done work in Class Field Theory. Considerable teaching experience at universities. At present teaching Mathematics at an extremely low level. Regular position or visiting position for any period may be considered. SW 59
SUGGESTED USES for classified advertising are books or lecture notes for sale, books being sought, positions available, summer or semester exchange or rental of houses, mathematical typing services and special announcements of meetings. The rate is $3.00 per line. Ads will be typed in the AMS office and will be typed solid. If centering and spacing of lines is requested, the charge will be per line with the same rate for open space as for solid type.

TO CALCULATE the length of an ad assume that one line will accommodate 52 characters and spaces.

DEADLINES for the next few issues are: for February, January 24; for April, February 21; for June, April 25; for August, June 12.

SEND AD AND CHECK TO: Advertising Department, AMS, P. O. Box 6248, Providence, Rhode Island 02940. Individuals are requested to pay in advance, institutions are not required to do so.

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**EXCHANGE POSITION**

EXCHANGE POSITION, acad. yr. 77-78 with someone from school offering Ph. D. in Statistics, preferably mountain west or west. If interested write to Dr. Oskar Feichtinger, assoc. prof., Dept. Math., Univ. Maine, Orono, Maine 04473.

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**FOR SALE**

1978 MATHEMATICAL CALENDAR

An informative and attractive calendar for the desk of the mathematician. Each month displays a different, colorful tessellation of the plane. In the centerfold is an original composite portrait of Gauss with a full-page biography. Birthdates and biographical sketches of 168 famous mathematicians are included. Other features are meeting dates, quotations, humor and problems, 28 pages, $1/2 by 11, $4.50 plus 50¢ postage. N. C. residents add 18¢ tax. Rome Press, Dept. N, Box 21451, Raleigh, North Carolina 27612.

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**UPCOMING SYMPOSIUM**

CONSTRUCTIVE APPROACHES TO MATHEMATICAL MODELS: A Symposium in Honor of R. J. Duffin will be held at the Mellon Institute of Carnegie-Mellon University, July 10-14, 1978.

The Symposium is an appropriate forum for papers in the following general topics: Graphs and Network Models; Mathematical Programming and Optimization; Mathematical Models in Engineering, Management and Science; Constructive Theory of Differential Equations and Inequalities.


You are cordially invited to submit a paper in any of the four general topics. Please send your abstract, not to exceed 200 words, to: George J. Fix, Department of Mathematics, Carnegie-Mellon University, Pittsburgh, PA 15213. Abstracts should indicate the topic for which they are intended. Deadline for abstracts is March 1, 1978; however, due to space limitations, it is advised to send abstracts in at your earliest convenience.

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**POSITIONS AVAILABLE**

University of Wisconsin-Madison, Instructorships and Assistant Professorships in Mathematics. We invite applications from outstanding young mathematicians who are recent recipients of a doctorate. We are looking for people who will interact well with members of our department, who care about teaching, and who can contribute to our research and instructional programs. The regular teaching load is two courses per semester, with at least one in your specialty every other year. There is a high probability of additional income through research or teaching during summers between consecutive years of appointment. The salary will be dependent on experience, with a minimum of $14,500 per academic year. All positions are for specified two- or three-year terms, and are not renewable. Those holding instructorships are eligible for consideration for Assistant Professorships on an equal basis with other applications. Application blanks may be obtained by writing Professor Joshua Chover, Chairman, Department of Mathematics, 213 Van Vleck Hall, University of Wisconsin, Madison, Wis. 53706, University of Wisconsin-Madison is an Equal Opportunity Employer.

Hunter College of the City University of New York invites applications for faculty positions in computer science beginning in September 1978. Candidates with a Ph. D. in computer science specializing in theoretical computer science, applied software or information systems will be preferred. Rank and salary will be commensurate with credentials and experience. The candidates will be expected to have a firm commitment to research and undergraduate teaching. Applications, including a curriculum vita and references, should be addressed to: Professor Barry M. Cherkas, Chairman, Department of Mathematics, Hunter College, 695 Park Avenue, New York, New York 10021, an Equal Opportunity/Affirmative Action Employer.

Chairman-Mathematics Department, Georgia State University. Applications are invited for the position of Chairman, Department of Mathematics. The chairman should have leadership ability, administrative experience, demonstrated research ability and an appreciation of the role of pure and applied mathematics in a modern university. Anticipated position available September 1978. Application deadline is February 15, 1978. Applications should include a vita and three names of references. Applications or inquiries should be sent to: Dr. Charles S. Frady-Chairman of Mathematics Search Committee, Department of Mathematics, Georgia State University, University Plaza, Atlanta, GA 30303. Georgia State University is an Affirmative Action/Equal Opportunity Employer.

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POSITIONS AVAILABLE

FACULTY, all ranks, Computer Sci/Stat/Ops Res/Math in Math Dept with strong CS & Stat OR programs. Outstanding research required; rank, salary depend on qualifications. Send vita, direct three references to H. A. Smith, Chair, Dept of Math, Arizona State Univ, Tempe, Arizona 85281. Closing dates 12/15/77, 1/15/78 and biweekly thereafter, ASU is an equal opportunity, affirmative action employer and complies with Title IX of the Educational Amendments of 1972.

The College of Industrial Management at the Georgia Institute of Technology will fill one or more faculty positions in Management Science. The College seeks candidates with prospects for an outstanding scholarly career and with a commitment to effective teaching. For further information contact Professor Matthew J. Sobel, College of Industrial Management, Georgia Institute of Technology, Atlanta, Georgia 30332.

The Mathematics Department of Winthrop College invites applications for a tenure-track position as ASSISTANT PROFESSOR OF MATHEMATICS beginning August 15, 1978. The Ph.D. degree is required along with evidence of strong research potential, commitment to scholarly achievement and related public service, and ability to teach well on the college level. The applicant should have training in numerical analysis, statistics, operations research, and computing. Send vita and names and addresses of at least three persons who know the candidate professionally to Edward P. Guettler, Chairman, Department of Mathematics, Winthrop College, Rock Hill, South Carolina 29733. Application deadline is January 15, 1978, Equal Opportunity/Affirmative Action Employer.

The Mathematics Department of Winthrop College invites applications for a position as INSTRUCTOR OF MATHEMATICS beginning August 15, 1978. A graduate degree in mathematics is required along with evidence of commitment to scholarly achievement and related public service, and ability to teach courses in diverse areas of applied mathematics. It is also expected that some visiting positions will be given to candidates who can work on problems that are of interest in engineering and physical sciences, or the management and social sciences. Carnegie-Mellon University is an Equal Opportunity Employer. Write to Professor Daniel Gorenstein, Chairman, Department of Mathematics, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213.

The Department of Mathematics at Rutgers University is seeking a senior applied mathematician with research interests in mathematical physics. The candidate should have outstanding research credentials and concern for teaching.

Inquiries should be directed to:
Professor Daniel Gorenstein, Chairman
Department of Mathematics
at New Brunswick
Rutgers University, The State University of New Jersey
New Brunswick, New Jersey 08903
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RUTGERS UNIVERSITY
THE STATE UNIVERSITY OF NEW JERSEY
Applications are invited in the fields of applied mathematics or topology. The candidate must have a Ph.D., show outstanding promise in research and a concern for teaching.

Inquiries should be directed to:
Professor Daniel Gorenstein, Chairman
Department of Mathematics
at New Brunswick
Rutgers University, The State University of New Jersey
New Brunswick, New Jersey 08903
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RUTGERS UNIVERSITY
THE STATE UNIVERSITY OF NEW JERSEY
Applications are invited from mathematicians for junior level positions in all specialties who have outstanding research ability in pure or applied mathematics and concern for teaching. These are temporary positions of one or two years' duration.

Applications should be sent to:
Professor Daniel Gorenstein, Chairman
Department of Mathematics
at New Brunswick
Rutgers University, The State University of New Jersey
New Brunswick, New Jersey 08903
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The Department of Mathematics of the UNIVERSITY OF GEORGIA invites applications for positions in mathematics education and in areas of modern applied mathematics. It is also expected that some visiting positions for one to three years will be available. Tenure track and tenure level positions are possible for persons of exceptional ability in any of the traditional areas of mathematics. The UNIVERSITY OF GEORGIA is an equal opportunity/affirmative action employer. Write to James C. Cantrell, Head, Department of Mathematics, University of Georgia, Athens, GA 30602.
HEAD—DEPARTMENT OF MATHEMATICS, University of Connecticut. Nominations and applications are invited for the position of Head of the Department of Mathematics. The department has 35 full-time faculty members at the main campus at Storrs and 13 full-time faculty members at the University's branch campuses. The University has an enrollment of approximately 12,500 undergraduate students and 3,500 graduate students at its Storrs campus. Qualifications include strong administrative and leadership ability, an established record of distinguished research and a commitment to excellent teaching. This position will be available September 1, 1978. Send résumé and names and addresses of at least three references to: Frank D. Vasington, Search Committee Chairman, Box A-178, University of Connecticut, Storrs, Connecticut 06268, An Equal Opportunity Employer.

Two vacancies in tenure-track positions are anticipated at the Southern Technical Institute for Fall, 1978, to teach lower-division mathematics courses. M. S. in Mathematics in required, Ph. D. strongly preferred. Preference will be given to applicants with some of all of the following qualifications: (a) Experience in teaching lower-division courses; (b) Nonacademic engineering or technical experience; (c) Evidence of continued interest and work in mathematics. Rank and salary commensurate with qualifications. Southern Technical Institute is a division of Georgia Institute of Technology, with a separate campus in Marietta, GA, a suburb of Atlanta. Two- and four-year degrees are offered in Engineering Technology. The Mathematics Department is a service department with nine members. Faculty have access to the facilities of Georgia Institute of Technology. Apply to: Dr. Simon A. Stricklen, Jr., Head Mathematics Department Southern Technical Institute 534 Clay Street Marietta, Georgia 30060 Deadline for applications is 1 February 1978, An equal-opportunity/affirmative action educational institution.

DEPARTMENT OF MATHEMATICAL SCIENCES CLEMSON UNIVERSITY invites applications for the position of DEPARTMENT HEAD Qualifications include a Ph. D. degree, teaching experience, proven research ability, and leadership capacity. Administrative experience is highly desirable, but not required. This is a 12-month position. The Department has approximately 50 full-time faculty members and offers a broad program in the mathematical sciences including B. S. and M. S. options in mathematics, statistics, operations research, and computer science. The Ph. D. degree is also offered, as well as a Ph. D. in Management Science administered jointly with the Department of Industrial Management in the College of Industrial Management and Textile Science. Position available July 1, 1978; application deadline is January 15, 1978. Applications or inquiries should be sent to: Dr. Paul T. Holmes, Chairman, Search Committee, Department of Mathematical Sciences, Clemson University, Clemson, South Carolina 29631. Applicants should request 3 letters of recommendation to be sent directly to the Chairman of the Search Committee, Clemson University is an Equal Opportunity/Affirmative Action Employer.

The Department of Mathematics and Statistics of the University of Massachusetts at Amherst expects to have positions available for the academic year 1978-1979. Possible openings include a senior level one-year visiting position in Statistics, a junior level tenure track position in Statistics, Applied Mathematics or Numerical Analysis, and a junior level two-year visiting position in Statistics, Applied Mathematics or Numerical Analysis. Curriculum vitae should be sent to Professor Edward A. Connors, Head, Department of Mathematics and Statistics University of Massachusetts, Amherst, MA 01003. The University of Massachusetts is an Affirmative Action/Equal Opportunity Employer.

CHAIRMAN, Department of Mathematics, Wright State University, at the rank of professor. Fall 1978. Qualifications include leadership ability, demonstrated scholarly achievement, administrative potential and an appreciation of the roles of pure and applied mathematics. The Department offers Bachelors and Masters degrees with concentrations in mathematics and statistics. The University has recently received planning approval from the Ohio Board of Regents for an interdisciplinary Ph. D. program in the biomedical sciences, Salary open. Usual benefits. Contact Carl C. Maneri, Department of Mathematics, Wright State University, Dayton, Ohio 45435. Equal Opportunity, Affirmative Action Employer.

DEPARTMENT OF MATHEMATICS INDIANA STATE UNIVERSITY Applications are invited for the position of Chairperson, Department of Mathematics. Applicants should have a doctorate, an established record of research, a commitment to quality teaching, and administrative ability. The department has 19 full-time members and offers programs of study leading to the traditional liberal arts B. S. and M. S. degrees in Mathematics and the B. S. and M. S. in Mathematics for Teacher Certification. The department offers a minor in computer science and is developing a major. The rank and salary of the chairperson are open. The application deadline is March 1, 1978. The starting date for the position could be either July 17, 1978, or August 29, 1978. Applications should include a vita, four letters of recommendation, and should be addressed to: Search Committee Department of Mathematics Indiana State University Terre Haute, Indiana 47809 Indiana State University is an equal opportunity employer.

ASSISTANT/ASSOCIATE PROFESSOR -- College of Charleston. Two regular faculty positions beginning September, 1978, Teaching in an undergraduate mathematics and computer science program (12 hrs/wk). Ph. D. in mathematics or computer science and a sincere interest in teaching undergraduates required. One position requires a strong background in operations research, applied mathematics, or computer science. The second position requires outstanding teaching credentials and proven research ability, with preference given to persons with research interests in classical complex analysis, approximation theory, probability or statistics. Salary and rank are dependent on credentials. Min. salary: $12,500. Three letters of recommendation and a résumé should be sent to W. Hugh Haynsworth, Chairman, Department of Mathematics, College of Charleston, Charleston, South Carolina 29401. Equal Opportunity/Affirmative Action Employer.

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Application Deadlines for Grants and Assistantships

The following schedule gives upcoming application deadlines for grants and assistantships described in the special December issue of these Notices. For further information about the various programs the reader is referred to the appropriate section of the December 1977 issue as follows: [GS] = Graduate Support Section; [PS] = Postdoctoral Support Section; [TSA] = Travel and Study Abroad Section.

**January 1**
- Courant Institute, Instructorships in Mathematics and Computer Science [PS]
- Courant Institute, Postdoctoral Visiting Memberships [PS]
- Jacob David Tamarkin Instructorships [PS]
- Lady Davis Fellowship Trust [TSA]
- Václav Hlavatý Research Assistant Professorships [PS]
- Zonta International [GS]

**January 3**
- T. H. Hildebrandt Research Assistant Professorships [PS]

**January 6**
- L. E. Dickson Instructorships in Mathematics [PS]

**January 9**
- Benjamin Peirce Lectureships [PS]

**January 15**
- E. R. Hedrick Assistant Professorships in Mathematics [PS]
- G. C. Evans Instructorships [PS]
- IBM Thomas J. Watson Research Center [PS]
- Institute for Advanced Study Memberships [PS]
- Kosciuszko Foundation [GS] [TSA]
- National Research Council [PS]
- National Research Council of Canada, Visiting Fellowships [TSA]
- North Atlantic Treaty Organization [TSA]
- Smithsonian Institution [GS]
- Smithsonian Institution, Postdoctoral Fellowships [PS]

**January 20**
- State University of New York at Buffalo [PS]

**January 30**
- Solomon Lefschetz Research Instructorship [TSA]

**January 31**
- American Mathematical Society Research Fellowships [PS]

"Before February"

- J. Willard Gibbs Instructorships [PS]

**February 1**
- American Association for the Advancement of Science [GS]
- American Society for Engineering Education [PS]
- Hughes Aircraft Company Fellowships [GS]
- Sigma Delta Epsilon, Grants-in-Aid [GS]

**February 6**
- California State Graduate Fellowships

**February 10**
- American Philosophical Society [PS]

**February 15**
- AMS-MAA-SIAM Congressional Science Fellowship [PS]
- Kappa Kappa Gamma Fraternity [GS]

**March 1**
- Rotary Foundation

**March 15**
- United States-India Exchange [TSA]

**April 1**
- Air Force Office of Scientific Research [PS]

**April 7**
- American Philosophical Society [PS]

**April 30**
- North Atlantic Treaty Organization [TSA]

**May 15**
- Weizmann Institute of Science, Feinberg Graduate School Postdoctoral Fellowships [TSA]

**June 1**
- Fulbright-Hays Program (Americas, Australia, New Zealand) [TSA]

**July 1**
- Fulbright-Hays Program (Africa, Asia, Europe) [TSA]

**August 11**
- American Philosophical Society [PS]
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STATISTICS AND ACTUARIAL SCIENCE GROUP

The Statistics and Actuarial Science Group of the Department of Mathematics, The University of Western Ontario, invites applications for an anticipated position. Preference will be given to applicants with demonstrated records of scholarly achievement in the area of probability theory. Duties shall include teaching graduate and undergraduate courses, direction of Ph. D. theses and conducting research. Salary and rank are negotiable. Current salary minima are: Assistant Professor $17,000; Associate Professor $20,300; Professor $26,500. Subject to funds being available the effective date of the appointment will be July 1, 1978.
Applications should be addressed to:

THE DIRECTOR
Statistics and Actuarial Science Group
Department of Mathematics
The University of Western Ontario
London, Ontario, Canada N6A 5B9

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Program I
HELSINKI
August 13-28, 1978

Program II
HELSINKI/ RUSSIA
August 13-September 1, 1978

Program III
SCANDINAVIA
August 8-29, 1978

The International Congress will commence on Tuesday, August 15, 1978. This follows immediately after the conclusion on Saturday, August 12 of the Joint Mathematics Meeting in Providence, Rhode Island.

Programs have been planned to enable participants to depart from Boston on their choice of origin cities in order to arrive in Helsinki Monday, August 14, rested and relaxed for the sessions which will commence on Tuesday, August 15.

Programs afford a choice of 15 day, 18 day and 21 day schedules.

PROGRAM I
• HELSINKI •
August 13 – 28, 1978

YOUR DAY-BY-DAY ITINERARY

SUNDAY, AUGUST 13
TO HELSINKI
Depart Boston (or other gateway city) by evening flight to Helsinki via Finnair or other IATA carrier.

MONDAY, AUGUST 14
HELSEINKI
Arrive Helsinki. Transfer from the airport to the hotel.

TUESDAY, AUGUST 15 through WEDNESDAY, AUGUST 23
Attend sessions of the International Congress of Mathematicians.

THURSDAY, AUGUST 24 through SUNDAY, AUGUST 27
Opportunity to participate in any of the official post-congress tours (four days/three nights):
OPTION A: Lapland
OPTION B: Lake District
OPTION C: Leningrad by motorcoach (Escorted tours, including transportation, accommodations, sightseeing and all meals!)
OPTION D: "Visa-free" cruise to Leningrad
N.B. To enable participation in this very popular four-day/three-night cruise, space is being reserved on August 18, 22, and 25.

Itineraries follow for all optional tours. Prices are approximate, based on current rate of exchange which may fluctuate slightly.

MONDAY, AUGUST 28
HOMEWARD
Depart Helsinki via return flight to Boston (or other gateway city).

AIR TRANSPORTATION – PROGRAM I

The least expensive applicable airfare which can be used is the APEX fare. This fare (Advance Purchase Excursion Fare) represents a savings of approximately $300.00 per person over a regular excursion fare. Certain restrictions, however, do apply and must be strictly adhered to.

RESERVATIONS
Space is subject to availability at time of booking. Reservations must be secured at least 50 days prior to departure and regulations require a minimum of 14 days and a maximum of 45 days in Europe.

Reservations may not be changed after ticket has been issued. Tickets must be issued no later than 7 days from date of confirmation of reservations.
Stopovers are not allowed although regulations do allow travel from the United States to one city and return travel from a different city. Transportation between cities is additional.

Cancellation or failure to use confirmed space as ticketed prior to departure will result in a forfeiture of 10% of the APEX fare paid, or $50.00, whichever is higher.

Special conditions do allow full refund and upgrading of ticket to higher alternate fares. Detailed information will be furnished.

For flight departure Sunday, August 13, 1978 and flight return Monday, August 28, 1978 the following are examples of applicable APEX fares. Fares from other gateway cities will be furnished on request.

- Boston/Helsinki/Boston: $548.00
- New York/Helsinki/New York: $553.00
- Chicago/Helsinki/Chicago: $616.00
- San Francisco/Helsinki/San Francisco: $728.00
- Dallas/Helsinki/Dallas: $744.00

All fares include U.S. departure tax of $3.00 and peak season westbound surcharge of $20.00.

**HOTEL ACCOMMODATIONS IN HELSINKI**

Garber Travel has secured accommodations at the Inter-Continental Hotel (deluxe) and the Vaakuna Hotel (superior first class) in Helsinki, August 14—28. Daily rates, including service charge, tax and continental breakfast, are (per person):

- Hotel Inter-Continental: $31.50 basis double, $47.25 basis single
- Hotel Vaakuna: $25.15 basis double, $38.25 basis single

Hotel rates are based on 1977 tariff and rate of currency exchange in effect at time of printing. Information on other accommodations available may be found on page 54 of these NOTICES.

**PROGRAM I – OPTION A**

**KUUSAMO**

THURSDAY, AUGUST 24

Morning flight from Helsinki to Rovaniemi, the capital of Finnish Lapland. The tour continues from Rovaniemi Airport to Posio village and Anu Pentik’s leather and ceramics studio. Lunch in Posio. Further on to Kuusamo. The Kuusamo district borders the Soviet Union in the east and includes large tracts of wilderness. Dinner and overnight in Kuusamo.

FRIDAY, AUGUST 25

SUOMUTUNTURI FELL

After morningswim and breakfast at hotel, the breath-taking experience of shooting the Kayla rapids in Kitka awaits the group. Lunch is served at the lakeside Kitkapiirri Motel, an old lumberjack cottage turned into a restaurant. The tour then continues north to Suomutunturi Fell and Hotel Suommu. Dinner of Lapp specialties is served in the unique restaurant built to resemble a giant “kota”, a Lapp tepee. After dinner the participants are admitted as members of the Polar Circle Club on the top of the Suomutunturi Fell. Overnight at Hotel Suommu.

SATURDAY, AUGUST 26

ROVANIEMI

After breakfast there is an excursion by water launch to Lehtoniemi old farm house where typical Lapp lunch will be served. A sauna-bath in a genuine smoke sauna is optional. In the afternoon the drive continues to Pyhatunturi Fell which gives a majestic view over the surrounding National Park with its numerous wild animals and birds, and via Sodankyla village to Rovaniemi. Dinner and overnight at the Hotel Pohjanhovi.

SUNDAY, AUGUST 27

HELSINKI

After breakfast there is a sightseeing tour of Rovaniemi which is located at the confluence of the Kemi and Ounasjoki Rivers, just south of the Arctic Circle. Rovaniemi was completely rebuilt after the Second World War and the city plan was designed by the famous Finnish architect Alvar Aalto. During the tour visits are made to the Library and Rovaniemi Parish Church with its unusual altar piece. Lunch is served at the Polar-Ounasvaara at the toe of Ounasvaara Fell. Transfer to Rovaniemi airport for afternoon flight to Helsinki. Transfer to hotel for overnight.

**COST:** $300.00 per person basis double occupancy
PROGRAM I – OPTION B

• Lakeland Tour •

THURSDAY, AUGUST 24
Departure from the hotel between 9:00 and 10:00 AM for Hameenlinna, the birthplace of Jean Sibelius, the famous Finnish composer. Lunch is taken in town and then the tour continues to Aulanko, a popular holiday resort. During the afternoon there is a sightseeing tour, including a visit to Aulanko National Park the church of Hattula, which dates back to the 14th century. After a sauna bath and a swim in the indoor pool of Aulanko there is a dinner. This hotel with a pleasant, cosmopolitan atmosphere also has a nightclub with dancing and roulette. Overnight at the Aulanko Hotel.

FRIDAY, AUGUST 25
After breakfast the participants board a Silverline boat for a wonderful 2-hour cruise to Visavuori, the studio of the late artist Emil Wikstrom. From there the drive continues to Tampere with lunch, followed by a sightseeing tour of Tampere. Visits are made to Kaleva Church, the revolving theater of Pyynikki, and the Planetarium-Aquarium. In the late afternoon a bus takes the group northwards to Jyvaskyla, a town in central Finland, on the northern shore of Lake Paijanne. Dinner is served at the Laajavuori Sandpiper Hotel on the outskirts of the town. Overnight is at this same hotel, which is an attraction in itself owing to its unusual architecture.

SATURDAY, AUGUST 26
Breakfast is followed by a sightseeing tour of the town. Jyvaskyla is sometimes called Finland’s Athens, since it was the seat of learning during the 19th century and the site of the country’s first Finnish-language secondary school and teacher’s college. This tradition is carried on today by the University of Jyvaskyla. After lunch the tour continues by bus to Kuopio, the heart of of the Finnish lake district. Visit to the revolving Puijo Tower, from which a magnificent view opens over thousands of lakes. Dinner and overnight in Kuopio.

SUNDAY, AUGUST 27
After breakfast there is a sightseeing tour of the town. A visit is made to the Orthodox Church Museum, the only one of its kind in entire western Europe. A few hours of the trip between Kuopio and Heinavesi will be on picturesque inland boat. Lunch is taken en route. From Heinavesi the tour continues by bus to Savonlinna, a town whose center is built on islands surrounded by the water of Lake Saimaa. The main sight of Savonlinna is the 500-year-old castle, Olavinlinna, where an opera festival is held each summer. Transfer to Savonlinna Airport for late afternoon flight to Helsinki. Transfer to Helsinki hotel for overnight.

COST: $300.00 per person basis double occupancy

PROGRAM I – OPTION C

• Leningrad by Motorcoach •

THURSDAY, AUGUST 24
Departure from Helsinki at 8:30 AM. The route goes through the southern coastal district via Porvoo, Loviisa and Hamina. Lunch is taken en route. After lunch the tour continues to Leningrad, arriving at about 8:00 PM. Dinner and overnight.

FRIDAY, AUGUST 25 and SATURDAY, AUGUST 26
Leningrad, with more than 600 bridges, numerous parks and treelined boulevards, is considered by many to be one of the most beautiful cities in the world. The city was founded by Czar Peter the Great in the 18th century and was called St. Petersburg until 1914, when the name was changed to Petrograd. In 1924 the city received its present name. The construction of the city was a great undertaking. The site is on low ground, only a few meters above sea level and subject to flooding. But these obstacles were overcome and a magnificent city was erected. Leningrad is rich in history. It was the setting for the major events of the Russian Revolution in 1917 and endured a 900-day siege during the Second World War, sustaining tremendous human and material losses. Despite the destruction, historical monuments still stand in Leningrad. The sightseeing tour takes you to the most famous places of interest, including the Nevsky Prospect, which is Leningrad’s main street; St. Isaac’s Cathedral; the fortress of Peter and Paul, around which Leningrad originally grew up. Excursions are arranged to the Hermitage Art Museum adjacent to the Winter Palace, the former residence of the Russian Czars, and to Peterhof, the summer residence of Peter the Great, with its hundreds of fountains and beautifully laid-out park. A visit is made to the Leningrad Metro.

SUNDAY, AUGUST 27
There is free time for shopping, etc. in the morning. The tour leaves Leningrad after lunch and dinner is taken en route. Arrival in Helsinki at about 11:00 PM. Transfer to hotel for overnight.

COST: $225.00 per person basis double occupancy
PROGRAM I – OPTION D

• Visa-Free Leningrad Cruise •

Friday, August 18; Tuesday, August 22; or Friday, August 25

FIRST DAY
Departure from Helsinki South Harbour at 6:00 PM. Dinner and overnight on board.

SECOND DAY
Breakfast on board. Arrival in Leningrad at 9:00 AM. Sightseeing tour of town. After lunch (taken in one of the Leningrad restaurants) visit to the Hermitage. In the evening either Russian dinner in town or visit to ballet/circus. Overnight on board.

THIRD DAY
Breakfast on board. Excursion to Peterhof, the summer residence of Peter the Great, famous for its 140 fountains and gilded statues. Lunch in town. Departure from Leningrad at 7:00 PM. Dinner on board.

FOURTH DAY
Breakfast on board. Arrival in Helsinki at 8:00 AM. Full day at leisure. Overnight at hotel in Helsinki.

COST: $320.00 per person basis double occupancy in Category A cabin

PROGRAM II

• HELSINKI • RUSSIA •

August 13 – September 1, 1978

SUNDAY, AUGUST 13
Depart New York in the evening on your overnight flight to Helsinki.

MONDAY, AUGUST 14
Arrive Helsinki. Transfer to VAAKUNA HOTEL or similar. (Accommodations during Congress period are included.)

TUESDAY, AUGUST 15 through SUNDAY, AUGUST 21
Attend sessions of the International Congress of Mathematicians.

MONDAY, AUGUST 22
Transfer to the airport for your flight to Moscow. Transfer upon arrival from the airport to the hotel.

TUESDAY, AUGUST 23
Sightseeing tour of the city. USSR Exhibition of Economic Achievements.

WEDNESDAY, AUGUST 24
Visit the Moscow Kremlin and Tretyakov Art Gallery.

THURSDAY, AUGUST 25
Day at leisure in Moscow.

FRIDAY, AUGUST 26
Museum of Revolution. Time at leisure for exploring Moscow on your own, resting, relaxing, shopping, etc. Depart for Kiev.

SATURDAY, AUGUST 27
Sightseeing tour of the city. Time at leisure.

SUNDAY, AUGUST 28
Museum of Art of the Ukrainian SSR. Depart for Leningrad.

MONDAY, AUGUST 29
Sightseeing tour of the city. The Hermitage museum.

TUESDAY, AUGUST 30
Excursion to Petrodvorets.

WEDNESDAY, AUGUST 31
Day at leisure in Leningrad.

THURSDAY, SEPTEMBER 1
Transfer from the hotel to the airport for return flight to New York.

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PROGRAM III
• SCANDINAVIA •
August 8 – 29, 1978

TUESDAY, AUGUST 8
Depart New York via regularly scheduled flight to Copenhagen.

WEDNESDAY, AUGUST 9
Arrive Copenhagen. Transfer from the airport to the SHERATON HOTEL. Balance of the day at leisure.

THURSDAY, AUGUST 10 and FRIDAY, AUGUST 11
In Copenhagen. A tour of the city and the harbor, plus a full-day excursion to Hans Christian Anderson country is included.

SATURDAY, AUGUST 12
Transfer to the airport for morning flight to Stockholm. Upon arrival transfer to SHERATON HOTEL. This afternoon – a “get-acquainted” tour of Sweden’s capital.

SUNDAY, AUGUST 13
Today’s sightseeing will include Drottningholm Palace, the Royal Palace and Millesgarden, with its sculptures and art treasures.

MONDAY, AUGUST 14
Morning at leisure. Transfer to the airport for afternoon flight to Helsinki. Transfer on arrival from the airport to VAAKUNA HOTEL or similar. (Accommodations during Congress period are included.)

TUESDAY, AUGUST 15 through MONDAY, AUGUST 22
Attend sessions of the International Congress of Mathematicians.

TUESDAY, AUGUST 23
TO HELSINKI
Attend Closing Session of the Congress. Transfer to the airport for evening flight to Oslo. Transfer upon arrival from the airport to the hotel.

WEDNESDAY, AUGUST 24
TO OSLO
Full day Grand Tour of Oslo by motorcoach and motorboat, including a cruise on the Oslofjord, and visits to the Viking Ships, Holmenkollen Ski Jump, and Vigeland Sculpture Park.

THURSDAY, AUGUST 25
Depart Oslo on three-day Norwegian Fjord Line Tour. Overnight at Tyin.

FRIDAY, AUGUST 26
Continue on Fjord Line Tour, cruise on the majestic Sognefjord and overnight at Stalheim Hotel, in magnificent surroundings.

SATURDAY, AUGUST 27
Continue on Fjord Line Tour, through exciting mountain passes and past roaring waterfalls to Bergen, arriving in late afternoon. Transfer to the hotel.

SUNDAY, AUGUST 28
In the morning, visit the colorful marketplace (fish . . . fruit . . . flowers) and drive up to the suburban hillsides for a sweeping panoramic view. In the afternoon, enjoy an excursion visiting Trollhaugen (home of Edvard Grieg) and the Fantoft Stave Church.

MONDAY, AUGUST 29
Transfer from the hotel to the airport for return flight to New York.

TOUR CONDITIONS

COST:

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PROGRAM III
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TRANSFERS: Round-trip transfer service between airport(s) and hotel(s) by private motocar, including porterage of luggage.

ACCOMMODATIONS: In twin-bedded rooms with private bath in first class hotels. The right is reserved to substitute hotels of equal category if necessary.

MEALS: Program II – Continental breakfast in Helsinki; all meals in Russia. Program III – Continental breakfast daily in Copenhagen, Helsinki and Stockholm; breakfast and dinner (MAP) daily in Bergen and Oslo; all meals on Fjord Line Tour from Oslo to Bergen. Meals served afloat when flying at normal mealtime hours.

SIGHTSEEING: A complete sightseeing program by private deluxe motorcar including the services of an English-speaking guide. All entrance fees and tips are included. No sightseeing while in Helsinki.

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§§0 and 1 have preliminary character. §§2 and 3 are devoted to studying properties of $p$-absolutely summing operators from the disc algebra. §§4 and 5 should convince the reader that the disc algebra is a pathological space from the point of view of Banach space theory, or at least that it is very different from $C(S)$-spaces, and that the same is true for a large class of uniform algebras. The Havin lemma is presented in §6. In §§7 and 8 the author shows that the disc algebra shares various properties of $C(S)$-spaces like the Dunford-Pettis property, characterizations of weakly compact operators, weak sequential completeness of the dual, etc. In §9 the author applies the main result of §2 to study asymptotic behavior of norms of projections from the disc algebra onto its finite dimensional subspaces as the dimensions of these subspaces tend to infinity. §10 is loosely related to other sections. It is a survey (without proofs) of results concerning bases and various approximation properties in the classical spaces of analytic functions. In §11 the author considers the problem of isomorphic classification of spaces of analytic functions of different numbers of variables.

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