

Remembering Lipman Bers

There are many of us in the mathematical community whom Lipman Bers profoundly affected. These four articles will refresh our memories and introduce this great figure to another generation: his unusual background, his turning to mathematics, his important scientific contributions, his role in human rights, and the powerful support he lent his women students and colleagues. I myself worked with Lipa long ago in the early 1950s during a period when, as C.C. Lin once told me, Lipa was “masquerading as an applied mathematician”. In Abikoff’s biography and mathematical story I followed the thread of

Lipa’s transformation as he left steady fluid dynamics and attacked quasiconformal mappings. Both areas profited from his deep understanding of complex analysis.

Lipa’s life was never far from just causes, and in Corillon and Kra’s article we see just how deeply involved he was and how he drew his colleagues in to worry and do something about political wrongs.

Tilla Weinstein was a Bers student at the Courant Institute when I was a young faculty member. In her article she brought back to me those times when Lipa was sharing with me his “exuberant love of mathematics”, especially trying to teach me a mathematical vocabulary. Gilman’s article reminded me in turn not only of how savvy Lipa could be on the de-

partmental political front but also of how supportive Lipa and Mary were as I struggled to handle mathematics and children.

One memory of my own. When Stalin died, we heard about it around midnight on our car radio. We rushed over to the Bers house. The lights were on, so Mary, Lipa, Herbert, and I were soon drinking toasts to a new era. It was a long time coming, but, especially since his father had been imprisoned for many years in the Gulag, I am very happy that Lipa lived to see the fall of the Soviet Union.

—Cathleen S. Morawetz

*His work
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and beauty*

Lipman Bers

William Abikoff

Lipman Bers was born in Riga, Latvia, on May 22, 1914. He died in New Rochelle, New York, on October 29, 1993, at age seventy-nine. Here I present the story of his life, both personal and mathematical; often it is delivered in his own words. Bers was a mathematician whose work possessed power, grace, and beauty; it has continuing relevance in both mathematics and physics. He also set the highest of standards as

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leader in our community, as human rights advocate, teacher, and mentor.

No person who was close to him can write an article about someone named Lipman Bers (pronounced *bears*). To colleagues he was Lipa or, inevitably, Papa. Within his family, Lipa or Flip were common. Lipa was a gentleman in the traditional European style—he usually wore a tie. Formality, even in the form of a tie, was forsaken in deference either to hot weather or intense activity. When he first arrived in the United States, he had to learn that we do not shake the hand of a colleague each time we pass in the hall—and we even address each other, independent of rank, by first names. Linda Keen relates the story of the time when she received her doctorate. Lipa made clear to her she was free to continue to use an honorific title in conversation with him (to her it was a sign of respect), but he would respond in kind. Henceforth he would refer to her using every title she had ever earned.¹ Lipa possessed a grace and charm that convinced rather than bludgeoned. His linguistic skills—he was fluent in at least four languages—and his sense of humor enhanced his ability to present his ideas.

His humor was often self-effacing; it was delivered with the confidence that captured the attention and affection of his audience. He said, when asked to give an autobiographical lecture,

[Dennis] Sullivan asked me what are the thoughts underlying my research. This is already a compliment, because it assumes that there is a thought underlying the research.

The Early Years

Lipa was born into a family that emphasized learning and insight. His mother was principal of the secular Yiddish language elementary school in Riga; his father was the principal of the corresponding gymnasium. His mother later became a psychoanalyst. His earliest years were spent in Petrograd amidst the turmoil of the Russian revolution, yet he recalled those years as peaceful, a luxury open only to the very young. His youth was spent in Riga and Berlin, where his mother trained at the Berlin Psychoanalytic Institute. In the chaos that was Europe between the world wars, Bers learned the love of mathematics and ideas and began his life-long love affair with Mary Kagan Bers.

He studied for a short time at the University of Zurich, then returned to Riga and its university. There he led the life of a politically involved

¹ *I'm exaggerating here, ever so slightly, to make a point. I learned that from Lipa.*

social democrat in a Europe self-destructing due to the extremism and violent tactics both of the Bolsheviks and a collection of right-wing tyrants and killers.

In 1934 Latvia contributed its own local dictator to the general atmosphere. Lipa wrote for an underground newspaper and spoke publicly against the new regime. Riga soon became too dangerous a place to remain—he left as a warrant for his arrest was issued—and he fled to Prague, where he continued his studies at Charles University under the direction of Karl Löwner. Mary joined him in Prague and they married.

Lipa spoke of feeling neglected, perhaps even not encouraged, by Löwner and said that only in retrospect did he understand Löwner's teaching method. He gave to each of his students the amount of support needed—Lipa used the same technique with the nearly fifty students he advised and the multitude of others who, like me, fell under his spell. It is obvious that Lipa did not appear too needy to Löwner. In 1938 he received the degree of Doctor of Natural Sciences. His doctoral dissertation was never published; indeed Löwner urged him to “submit my thesis without trying to add more results ‘or it will be too late’”. The thesis concerned potential theory.

The year 1938 was cataclysmic for all of Europe but especially for Czechoslovakia. The French and British gave the country to the Nazis in the forlorn hope of achieving “peace in our times”. Among the most endangered people in Nazi Europe were the stateless, the leftists, and the Jews. Bers satisfied all of these conditions. Lipa and Mary crossed a Europe fraught with danger and lived in Paris until the Nazi occupation. While waiting for an American visa,² and living under the stressful condition of being a stateless Jew in France at the start of World War II, Bers demonstrated his devotion to mathematics by continuing his work. He wrote two short notes on Green's functions and integral representations, ideas to which he later returned in the context of Teichmüller theory and Kleinian groups.

² *Functionaries in the American State Department, whether simply afraid of a flood of Europeans of far greater intellect than they possessed or a simple lack of humanitarian concern, created artificial obstacles to the granting of American visas to people fleeing Hitler's New World Order. This additional level of obstruction added to a common American xenophobic reaction to a flood of European (especially Jewish) refugees.*

The official reaction is in stark contrast to the welcome offered by the American mathematical community. Bers pointed out that it was only in countries like the United States, already possessing a strong mathematical environment, that European refugee mathematicians could be appreciated. The refugees found assistance and a collegiality that was graciously offered and gratefully received.

Those representations were then in vogue due to Stefan Bergman's work on kernel functions.

While the Berses lived in Paris, their daughter Ruth, now a psychoanalyst in New York City and professor of psychology at John Jay College of the City University of New York, was born.

American Mathematics Goes to War

In 1940, ten days before Paris fell to the Nazis, the Berses moved into unoccupied France. They finally obtained American visas—10,000 were issued for political refugees after the personal intervention of Eleanor Roosevelt—and the family sailed to New York. Lipa's mother and stepfather, Beno Tumarin—later an actor, theatrical director, and teacher at the Juilliard School—were already in New York. Lipa, like so many of the young mathematicians of our time, sought employment in his specialty only to find that no positions were available. The Berses lived in the circle of unemployed refugees in New York until 1942; he received some support from YIVO, a Yiddish research organization; the result was a paper in Yiddish about Yiddish mathematics textbooks.

It was roughly in this period that he started joint work with Abe Gelbart on Σ -monogenic functions, which later developed into the theory of pseudoanalytic functions.

In 1942 the Berses moved to Providence, where their son Victor, now professor of classics at Yale, was born. Bers had accepted the invitation to participate in the Brown University program Advanced Research and Instruction in Applied Mathematics. Here he was soon joined by Löwner who, in recognition of the haven offered by the United States, had anglicized his name and thenceforth was known as Charles Loewner.

Lipa wrote most eloquently of Loewner, in words that describe their author as well: He

... was a man whom everybody liked, perhaps because he was a man at peace with himself. He conducted a life-long passionate love affair with mathematics, but was neither competitive nor vain. His kindness and generosity in scientific matters, to students and colleagues alike, were proverbial. He seemed to be incapable of malice. ... Without being religious he strongly felt his Jewish identity. ... Without having any illusions about Soviet Russia he was a man of the left. He was a good storyteller, with a sense of humor which was at once Jewish and [humanistic].

It was at Brown that Bers had the first three of his forty-eight doctoral students.

The American universities had become both the training grounds for the war effort and the centers for war-related scientific research. It was there that Bers started his studies of two-dimensional subsonic fluid flow. This was particularly relevant in the late war years as aircraft wings (airfoils) were being designed with sharp edges, jet engines were introduced, and flight speeds nearing that of sound became conceivable. The potential function for the planar flow shared many of the properties of harmonic functions, which are the potentials of an incompressible flow. The techniques used by Bers are particular to the plane. With respect to suitably chosen coordinates the complexified potential is an analytic function. The choice of such coordinates, in the form of isothermal coordinates, is a subject to which we will later return in terms of solving the Beltrami equation—it is central to Bers's major work.

Pseudoanalytic Functions and Subsonic Flow

Bers's work on pseudoanalytic functions—a subject developed independently by I. Vekua in the Soviet Union at a time when East-West scientific communication was often considered treasonable—was one of the generalized complex function theories hinted at in earlier eras by Beltrami, Picard, and Carleman. It was brought to fruition only much later. Quasiconformal mappings, of which we will have more to say, generalized conformal maps, while pseudoanalytic functions were based on generalized Cauchy-Riemann equations. We would like all complex-valued functions in plane domains to have complex derivatives, but, of course, that is impossible. Pseudoanalytic functions come as close as possible to having complex derivatives. Experts will recognize them, after a bit of thought, as nonsingular quasiregular functions.

In some domain D in \mathbb{C} , choose two Hölder continuous functions $F, G: D \rightarrow \mathbb{C}$ with $\Im(\bar{F}G) > 0$. The class $A_D(F, G)$ of pseudoanalytic functions with generators F and G consists of the functions

$$w = \phi F + G$$

with the properties that ϕ and G are real-valued and the "dot" derivative

$$\dot{w}(z) := \lim_{h \rightarrow 0} \left([\phi(z+h) - \phi(z)]F(z+h) + [G(z+h) - G(z)] \right) / h$$

exists and is finite for all $z \in D$. Notice that the $(1, i)$ -pseudoanalytic class defines the class of complex analytic functions.

Some basic properties of pseudoanalytic functions are:

- the notion of pseudoanalytic function is conformally invariant;
- with respect to the “dot” derivative, the functions are infinitely differentiable and there is a Cauchy theory; and
- the zeroes of functions in the class are isolated and each such function admits unique continuation.

On closed subsets of D , w can be written as $w = e^{s(z)}f(z)$ where f is analytic and s is Hölder continuous. Bers and Nirenberg later showed that this factorization of w remains valid when w is a solution to an elliptic system having only measurable coefficients.

To the (F, G) -pseudoanalytic function w we may associate $\omega := \phi + i$ which is pseudoanalytic with respect to $F \equiv 1$ and $G \equiv i$. The functions ω have strong geometric properties, e.g., they are open, orientation-preserving, and locally quasiconformal.

Bers’s work in the period from the early 1940s until the mid-1950s concerned an amalgam of this generalization of single variable complex analysis, together with linear or mildly nonlinear elliptic partial differential equations in the plane and their applications. The early development of the theory of pseudoanalytic functions coincided with Bers’s work on subsonic flow.

The system describing the steady-state planar flow of a compressible fluid is given by the velocity components u, v of the fluid which satisfy the system

$$u_y - v_x = 0$$

and

$$(c^2 - u^2)u_x - uv(u_y + v_x) + (c^2 - v^2)v_y = 0$$

where c is the speed of sound in the fluid. Using components of the velocity vector as independent variables, the equation can be linearized as

$$(c^2 - u^2)\phi_{vv} + 2uv\phi_{uv} + (c^2 - v^2)\phi_{uu} = 0.$$

In 1945 Gelbart was instrumental in bringing both Bers and Loewner to Syracuse University. Led by William Ted Martin and later Stewart Cairns, the department prospered for a time; its faculty included Paul Erdős, Paul Halmos, Arthur Milgram, Dan Mostow, Murray Protter, Paul Rosenbloom, Hans Samelson, and Atle Selberg. Bers left Syracuse in 1949.

It was during his stay at Syracuse that he first worked on the problem of removability of singularities of nonlinear elliptic equations. Earlier, Sergei Bernstein had proved that any solution ϕ of the minimal surface equation

$$(1 + \phi_y^2)\phi_{xx} - 2\phi_x\phi_y\phi_{xy} + (1 + \phi_x^2)\phi_{yy} = 0$$



Lipman Bers at Columbia University in 1984.

which is defined in the whole complex plane must be linear. There are standard techniques that reduce the study of the minimal surface equation to the linear equation

$$(1 + \eta^2)\omega_{\eta\eta} + 2\xi\eta\omega_{\xi\eta} + (1 + \xi^2)\omega_{\xi\xi} = 0.$$

The latter equation may be studied using the methods of pseudoanalytic functions. Bers proved that any finite isolated singularity of a single-valued parametrized minimal surface is removable.³ This profound extension of Riemann’s theorem on removability of singularities of analytic functions shows that nonlinear elliptic equations can have significantly more rigid behavior than their linear counterparts. This removability theorem was both announced and discussed by Bers at the International Congress of Mathematicians in 1950. The *Annals* paper containing the proof is a magnificent synthesis of complex analytic techniques which relate the different parametrizations of minimal surfaces to the representations of the potential function for subsonic flow and thereby achieves the extension across the singularity.⁴ Bers, along

³ I like to think of this theorem as the physically obvious statement that a pinprick will burst a soap bubble.

⁴ Shmuel Agmon, among others, has noted that Lipa was always doing complex analysis—no matter the field in which he was working.

with several others, later extended the theorem to a wide class of nonlinear elliptic equations using generalized complex analytic techniques.

The Transition to Quasiconformal Mappings and Their Applications

Bers then spent two years at the Institute for Advanced Study. It was at this time that he began a ten-year odyssey that took him from pseudo-analytic functions and elliptic equations to quasiconformal mappings, Teichmüller theory, and Kleinian groups. He said,⁵

I was looking for an a priori estimate for an inequality which would say that if a solution to a certain equation exists, then it is less than a fixed number.... I thought there might be an inequality involving quasiconformal mappings which would help me to prove an existence theorem. ... I was trying to prove the existence of a flow that no sane physicist doubted exists. I found the inequality in a paper of Lavrentiev. [He then wrote on the blackboard] If $z \mapsto f(z)$ — [he then turned to the audience and said] This arrow wasn't invented yet— maps the disk onto itself homeomorphically and

$$|\text{dilatation}(f)| \leq K$$

and $f(0) = 0$, then

$$|f(z_1) - f(z_2)| \leq 16|z_1 - z_2|^{1/K}.$$

... The interesting thing is [that] this is called Theorem (Ahlfors-Lavrentiev). This was in a paper by Lavrentiev and I looked in the literature but didn't find the proof anywhere. I was in Princeton at that time. Ahlfors came to Princeton and announced a talk on quasiconformal mappings. He spoke at the University so I went there. And sure enough, he proved this theorem. So I came up to him after his talk and asked him, "Where did you publish it" and he said, "I didn't yet." "So why did Lavrentiev credit you with it?" He [Ahlfors] said, "He probably thought I must know it and was too lazy to look it up in the literature." Well, as a matter of fact, three years later I met Lavrentiev in

Stockholm. I asked him, "Why did you credit Ahlfors with this theorem?" Lavrentiev said, literally, "I thought he must know it and I was too lazy to search the literature." I immediately decided that, first of all, if quasiconformal mappings lead to such powerful and beautiful results and, secondly, if it is done in this gentlemanly spirit—where you don't fight over priority—this is something that I should spend the rest of my life studying.

...

I read or tried to read the papers by Teichmüller. At that time his reputation was not yet established; people doubted whether his main theorem was proved correctly. Ahlfors published a new proof, but a very difficult one, in the *Journal d'Analyse*. Some of you may know that Teichmüller was a Nazi....⁶

Teichmüller's theorem(s) assert the existences and uniqueness of the extremal quasiconformal map between (for simplicity's sake) two compact Riemann surfaces, of the same genus, modulo an equivalence relation. The equivalence relation is given by quasiconformal maps f_i from a base surface Σ to a member S_i of a family of surfaces. $S_1 \sim S_2$ if there is a map $g: S_1 \rightarrow S_2$ so that $g \circ f_1 \circ f_2^{-1}$ is homotopic to a conformal self-map of S_2 . The equivalence classes form the Teichmüller space T_p of compact Riemann surfaces of genus p . Teichmüller also showed how to identify T_p with the unit ball in the space of quadratic differentials on the base surface Σ . The Teichmüller space T_p is the setting for the solution to Riemann's problem of moduli. It was the initial triumph of the Ahlfors-Bers collaboration to give a solution of the moduli problem.

The problem asks for a holomorphic parametrization of the compact Riemann surfaces in a fixed homeomorphism class. The complex structure of this space must be chosen so that the periods of abelian differentials of the first

⁶ *It is still a source of controversy that Teichmüller was a mathematician of the highest order and a human being of the lowest. Some ten years ago, I wrote a biography of Teichmüller and collected recollections of him. Werner Fenchel once told me that Teichmüller lacked the opportunism of a Bieberbach. His life offers a vivid demonstration that a person could simultaneously ascend the heights of insights scientific and descend the depths of human experience as typified by blind obedience to the Nazi credo.*

⁵ *In 1986, he gave an impromptu lecture on his work on quasiconformal mappings in Dennis Sullivan's seminar. As was common there, the talk was videotaped.*

kind are holomorphic functions of the parameters. The idea can be stated far more simply. Riemann worked with algebraic equations like

$$P(z, w) := \sum_{ij} a_{ij} z^i w^j = 0.$$

If we assume that a_{ij} is a holomorphic function of a parameter $t \in \mathbf{C}^N$, then we get a holomorphic family of Riemann surfaces S_t . If we fill in the points that sit over ∞ and desingularize, each S_t becomes a compact Riemann surface. These compact surfaces are topologically equivalent to spheres with $p \geq 0$ handles. Riemann's moduli space R_p is the space of holomorphic equivalence classes $[S]$ of Riemann surfaces of fixed genus p . Riemann wanted to parametrize R_p so that maps of the form $t \mapsto [S_t]$ are holomorphic. Ahlfors and Bers proved that R_p has such structure. The structure projects from the orbifold (or branched) covering by the Teichmüller space T_p of genus p . Ahlfors was first to give a complete description of the complex analytic structure; his work is a tour de force. But there is an easier way, given by Bers, which we will soon describe at length.

In 1951 Bers moved to the Courant Institute—before it had that name—of New York University, where he remained until 1964. At Courant, Bers chaired the graduate program. His teaching and personal style were merged in the lunches that he held each week. Linda Keen recalls that during her graduate years, “Bers taught his class every Friday after lunch. Before class, he would always have lunch with ‘his children’; these included Bernie Maskit, Ronny Wells, Bob and Lesley Sibner, and a number of others. He called it the ‘children’s lunch’. Of course others often came, including students like Jerry Kazdan and Joan Dyer and NYU’s ‘young faculty’, like Ehrenpreis, Garabedian, Morawetz, Moser, Nirenberg, and Lax. I also remember Serge Lang coming. These lunches were both social and mathematical, certainly initiating us into the mathematical culture.” Bernie Maskit adds, “One of the rules of the gathering was that the bill was always presented to the youngest person present—this usually meant Linda—who had to add the tip and divide by the appropriate number. We once turned the tables and insisted that Lipa do it; he got flustered and didn’t get the arithmetic right. We then went on to have some discussion about whether mathematicians could do arithmetic (Lipa, of course, said that they couldn’t; I, who had kept the books in my family’s business for a few years, did not add that I could do arithmetic).”

After the move to the New York City area, the Berses lived in the enclave of European-born mathematicians in New Rochelle, New York, that

had formed around Richard Courant. Members of that community included, at various times, Fritz John, Kurt Friedrichs, Cathleen Morawetz, Harold Grad, Jürgen Moser, Wilhelm Magnus, and Joan Birman. Lipa and Mary created an environment of warmth and excitement in their house on Hunter Avenue. Mathematicians passing through New York, dissidents just expelled from the Soviet Union, friends, and students found a home with the Berses. This fostered both interest in mathematics—the excitement shared between generations—and in human rights—which was hard to ignore after sharing coffee and ideas with incredibly brave people like Chalidze and Litvinov. It was also the place we brought all aspects of our lives; the Berses shared our joys and our sorrows.

One of Lipa’s great skills was to coin phrases. In a filmed lecture, Marc Kac told of describing to Lipa his work on the eigenvalues of the Laplacian for plane domains. Lipa said that he was asking whether one can *hear the shape of a drum*. The answer is no. The proof was given by Gordon, Webb, and Wolpert—Scott Wolpert is Bers’s mathematical grandson.

By 1954 quasiconformal mappings had begun appearing in Bers’s papers, and his emphasis moved into new areas. He signalled his departure from a field by writing up what he knew in the form of a book or survey article.⁷ Between 1954 and 1958 there appeared books or major surveys on sub- and trans-sonic flows, pseudoanalytic functions, and influential sets of lecture notes on algebraic topology and Riemann surfaces. The notes on topology served as an introduction to topological methods for a generation of hard analysts. In particular, it was one of the earliest treatments of fixed point methods and topological obstructions which was accessible to the working analyst. The Riemann surfaces volume describes the state of the field just before the Ahlfors-Bers Theorem revolutionized the field.

⁷ He felt that one should write a book in order to leave a field. After going through the process of writing a book and preparing it for publication, he asserted that it was easy to leave because by then one had come to hate the field. I should note that the planned book on Teichmüller space, which was to be a joint undertaking with Fred Gardiner and Irwin Kra, was never written, although several drafts were produced. I presume that Bers was never quite ready to leave the field.

*...a lifelong,
passionate
love affair
with
mathematics*

A later set of lectures on several complex variables was also quite influential.

Moduli Theory and Kleinian Groups

It was during this period of extensive expository writing that Bers's research blossomed as well. In his 1958 address to the International Congress of Mathematicians, Bers announced a new proof of the so-called measurable Riemann mapping theorem. He then essentially listed the theorems that followed directly from this method—including the solution to Riemann's problem of moduli. He was outlining the work of several years, much of which was either joint with or paralleled by work of Lars Ahlfors. Their professional collaboration was spiritually very close; they were in constant contact although they wrote only one paper together, the paper on the measurable Riemann mapping theorem.⁸ Nonetheless, their joint efforts, usually independent and often simultaneous, and their personal generosity inspired a camaraderie in the vaguely defined group which formed around them. The group has often been referred to as the "Ahlfors-Bers family" or "Bers Mafia"; c.f. the recent article by Kra [O-1]. It is gratifying to see that the spirit of cooperation they fostered lives on among many members of several generations of mathematicians.

First let us set some notation. With ∂_x and (\cdot) , denoting partial derivative with respect to \bullet , define the partial differential operators

$$\partial_z := (1/2)(\partial_x - i\partial_y) \text{ and } \partial_{\bar{z}} := (1/2)(\partial_x + i\partial_y).$$

The Ahlfors-Bers theorem asserts the following: Let $\mu : \mathbb{C} \rightarrow \mathbb{C}$ be measurable with $|\mu(z)| < k < 1$ almost everywhere. Then the Beltrami equation

$$(1) \quad w_{\bar{z}} = \mu w_z$$

has a unique solution w^μ fixing 0, 1 and ∞ which is a homeomorphism of \mathbb{C} onto itself. Given any point z_0 , the map $\mu \mapsto w^\mu(z_0)$ is a holomorphic mapping from the unit ball in $L_\infty(\mathbb{C})$ to \mathbb{C} . The most important consequence of the Ahlfors-Bers approach is that, if μ depends on a parameter t either to some degree of smoothness or real or complex analytically, then w depends on t as well as it possibly can.

⁸ They called it the Riemann mapping theorem for variable metrics. The theorem should properly be called either that or the Ahlfors-Bers theorem or the measurable Riemann mapping theorem with dependence on parameters. Without the dependence on parameters the theorem is due to Morrey. It is precisely the dependence on parameters that displays the complex analytic structure of the Teichmüller space.

A solution w to (1) is conformal with respect to the Riemannian metric

$$ds^2 := \lambda(z)|dz + \mu(z)d\bar{z}|^2 \text{ with } \lambda > 0$$

and gives an isothermal local coordinate for that metric.

In his International Congress paper, Bers asked whether there is an embedding (holomorphic, of course) of the Teichmüller space as a bounded domain in \mathbb{C}^N . In a paper entitled "Spaces of Riemann Surfaces as Bounded Domains", Bers gave an argument which should have proved the theorem. The argument was not correct. In fact, Bers referred to the correction as his most important paper—it is one and one-half pages long, including the references. In the latter work, he described the object now called the Bers embedding of the Teichmüller space.⁹ The embedding brings the idea of moduli (parameters) for Riemann surfaces back to their roots in the work of Gauss, Riemann, Schwarz, and Poincaré on parametrizations of solutions of ordinary differential equations and their monodromy groups. Bers embedded the Teichmüller space of a (say, compact, hyperbolic) Riemann surface S in a space of Schwarzian derivatives $[f]$ of univalent functions f . $[f]$ is a quadratic differential for a Fuchsian group G_0 covering S and f conjugates G_0 into a quasi-Fuchsian group G_f , that is, a group of complex Möbius transformations quasiconformally conjugate to G_0 . This gives an elegant geometric formulation to deformations of S in terms of hyperbolic 3-geometry while holomorphically embedding the Teichmüller space as a cell in the complex vector space of holomorphic quadratic differentials.

The connection between complex Möbius transformations and hyperbolic 3-dimensional geometry had already been discovered by Poincaré. The action of any Möbius transformation on the extended plane can be represented as the composition of two or three reflections in circles or lines C . Poincaré extended the action to the upper halfspace by using reflection in the spheres or planes S meeting the complex plane orthogonally. These reflections on the upper halfspace give the full group of isometries of hyperbolic space.

Using the Ahlfors-Bers Theorem, Bers gave what remains the most intuitive proof of Teichmüller's existence theorem. Teichmüller's proof of his uniqueness theorem remains the standard proof.

⁹ In the correction, Bers states that the same result was obtained, simultaneously and independently, by Ahlfors. So Ahlfors must have named the embedding.

By 1964-65 both Ahlfors and Bers had changed research direction. Also in 1964, Bers moved uptown from the Courant Institute to Columbia University. He remained there until his retirement in 1984 after having served as chairman from 1972 to 1975.

Bers's embedding of the Teichmüller space gives it a boundary, which was first described by Bers and Maskit in separate, but closely coordinated, papers.¹⁰ The boundary points correspond to degenerate Riemann surfaces and also to a class of Kleinian groups whose existence had not previously been suspected. At the time they were called totally degenerate groups; now, following further exploration initiated by Troels Jørgensen and Bill Thurston, they are called singly degenerate groups.

While Bers was exploring the boundary of Teichmüller space, Ahlfors had gone off in a related direction and had proved his finiteness theorem for Kleinian groups.¹¹ In filling a minor gap in Ahlfors's argument, Bers gave numeric bounds for the hyperbolic areas of the Riemann surfaces under discussion.¹² Within the span of ten years, Ahlfors and Bers had led a major portion of the complex analytic community from the solution to Riemann's moduli problem, hence a close interaction with the algebraic geometry community, to the study of groups of motions of hyperbolic 3-space.

In his earliest papers, Bers had examined integral representation theorems. In the 1960s he simply wrote down an operator which generalized the reflection operation $\phi(z) \mapsto \bar{\phi}(\bar{z})$ to reflection across quasicircles. Convergence is only guaranteed if ϕ is a holomorphic q -differential with $q \geq 2$. Since the operator is antianalytic, it gives global holomorphic sections of the bundle of q -differentials. He also introduced a fiber space whose natural quotient space attaches, in

a holomorphic fashion, to a point S in the Teichmüller space, the surface S . Topologically, the quotient space is $T_p \times S$, but using the Bers embedding we actually obtain a holomorphic bundle structure.



Lipman and wife Mary Bers in Hawaii, 1979, attending the U.S./Japan conference on Riemann surfaces.

Yet another major stream of Bers's work concerned Eichler cohomology. First studied by Eichler in number-theoretic investigations, the cohomology classes were later used by André Weil to parametrize the infinitesimal deformations of discrete subgroups of Lie groups. Soon thereafter, Bers used the Eichler theory, in the context of Fuchsian groups, to prove the finiteness theorem that Ahlfors refers to as the model for his finiteness theorem for Kleinian

groups. This represents yet another view of how quadratic differentials parametrize deformations. Bers's Area Theorem is still another use of this cohomology theory. Later work of Ahlfors, Kra, and Bers uses the theory to get a corresponding deRham theory, to find spanning sets for automorphic forms represented as Poincaré series, and later to find bases for these sets and to determine when the Poincaré series of a rational function vanishes identically.¹³

In the late 1970s and beyond, hyperbolic 2- and 3-dimensional geometry became a subject of intense interest to topologists and dynamical systems people. The revolutionary insights of Bill Thurston lead to a classification of surface diffeomorphisms in a fashion that complemented the one previously obtained by Nielsen, but Thurston's has a structure and unity all its own. Bers gave a proof of Thurston's classification using quadratic differentials and Teichmüller theory. In a sense, Bers's proof is easier because the flexibility obtained by working with classes of diffeomorphisms is replaced by the rigidity of working with a unique element, namely the Teichmüller map, in each diffeotopy class. Bers's proof is simply the classification of diffeomorphisms of surfaces via the existence or not of the solution to a new extremal problem on Teichmüller space. The same philosophy of using the rigid map in a class of homeomorphisms underlies the Bers-Royden proof of the Sullivan-

¹⁰ Of course, in the spirit of the Bers-Ahlfors school, it was Ahlfors whose 1962 International Congress address asked what can be said about the boundary.

¹¹ Ahlfors tells us that the proof is an extension of a proof, due to Bers, of the finiteness theorem for Fuchsian groups.

¹² At the same time, Leon Greenberg used a completely different and likewise powerful method to fill the same gap.

¹³ Hejhal had previously solved this last problem in a more restricted case.



Lipman and Ruth Bers.

Thurston improvement of the λ -lemma of Mañé, Sad, and Sullivan, as well as Bers's proof of Sullivan's unified approach to both Ahlfors's finiteness theorem and Sullivan's eventual periodicity theorem (wandering domains cannot exist for iterated rational maps of the Riemann sphere).¹⁴ Bers's paper on the Sullivan unification was proved after he had moved to the (New York) City University Graduate Center following his retirement from Columbia in 1984. He remained at CUNY as visiting professor until 1989.

Professional and Personal Recognition

Bers received many honors throughout his career. Here it is particularly appropriate to mention that he served as vice-president of the AMS from 1963 to 1965 and as president from 1975 to 1977. In 1975 he received the Steele Prize from the Society. He was elected to the National Academy of Sciences in 1964 and served as chair of the Mathematics Section from 1967-70.¹⁵ In one of the other articles in this issue of the *Notices*, you will find an exposition of Bers's work as the inaugural chairperson of the Academy's human rights committee starting in 1979. In 1961 he became a Fellow of the American Academy of Arts and Sciences. He was elected Fellow of the American Association for the Advance-

¹⁴ *The editor, on reading Bers's approach to these theorems, recalled Eli Stein's approach to finding a needle in a haystack. Eli advised us to "pick a small haystack".*

¹⁵ *He once commented in jest that the Academy had two reasons to exist: to make jealous those who did not belong and to bore those who did.*

ment of Sciences in 1965 and chaired its Section A in 1973 and in 1983-84. He was a member of the Council of the Association from 1969-73 and from 1984-85. He was particularly proud to have been elected to the American Philosophical Society, because it is the oldest learned society in the United States, founded by Ben Franklin, whom he particularly admired. He was the first G. H.

Hardy lecturer for the London Mathematical Society in 1967.

When New York City began giving awards for Science and Technology, Bers was one of the group of five people first chosen. The citation said that he was chosen for "his influential and creative contribution to modern mathematics, his inspiring guidance to generations of students, and his tireless campaign in support of the human rights of persecuted scientists throughout the world."

Lipa, ever the optimist, did not possess the power of prophecy. During the years 1966 through 1968, he chaired COSRIMS, the Committee on Support of Research in the Mathematical Sciences, which was a joint endeavor of the National Academy of Sciences and the National Research Council. He took responsibility for the conclusions about the future of the profession which appear in the final report of that committee. It concluded that the future looked incredibly bright for employment in the mathematical sciences. Two years later the job market crashed, leaving a much chagrined Lipman Bers.

During the period from 1969 to 1971, he chaired the Division of Mathematical Sciences of the National Research Council. That was an interesting time for Lipa, an antiwar activist, to be an official of a quasigovernmental agency. Lipa gave heartfelt, impassioned speeches at Columbia against American military involvement in Southeast Asia. The FBI started asking questions about him—they said he was being considered for a high government position. He checked and none was open. He was proud. Later, Lipa felt deflated when his name did not

appear on Nixon's "enemies list", an honor role, as it were, of American dissent. It was not that dissent, in the American political landscape, was his pose but rather that America had given him so much that his expectations were loftier than that of the leadership demonstrated by Nixon and his coterie.

A Personal Note

I worked with Lipa for over twenty years, but I was not his student. I was welcomed to his house by both Lipa and Mary and shared in the warmth of that meeting ground with generations of mathematicians—perhaps two senior to me, my own, and a generation junior to me only in age. I know the mathematics that we shared, the joy, and the enthusiasm that so many people shared with him.

Lipa Bers was a man who had a tremendous influence on me. From the time I was a graduate student, trying to decide what would be a good thesis topic, to the time that, as a (hopefully) less immature member of the community, I was choosing a name for a series of books, I looked to Lipa for counsel, for advice, for a sense of who I am as a mathematician and, more important, of who I would like to be both as a man and as a mathematician.

I first met Lipa at a conference at Stony Brook in 1969. The first lecture was given by Lars Ahlfors. The graduate student in me saw that the theorems that I thought were the end result of mathematics were simply the tools when placed in the right hands. I learned from Lars Ahlfors that mathematics grows on itself. But I learned something else from Lipa. He gave a simple seminar talk, and there I learned about his embedding of Teichmüller space and its boundary. He lectured in typical fashion about quasi-Fuchsian groups; he drew a wiggling simple arc on the blackboard. That was the limit set of a quasi-Fuchsian group—Picasso would have been proud. Almost nothing else was written on the board. He spoke to the audience and created a vision of a set and made us feel that we not only understood it, but that we were seeing it. In reality it is a horribly complicated fractal.

I felt that I was a child and he was an older man—he always had the image of an older, wiser man—who was taking me through a garden and showing me the flowers. The flowers were the theorems, the beauty. I did not know the field, did not even know the definitions; I could see, only through Bers's eyes, the beauty of his ideas. When I left the room, I knew I needed to better understand that garden. That was twenty-five years ago, and I am still finding delight in the freshness of this most classical of mathematical disciplines.

In a way we were children, and Lipa and Mary Bers taught us, by their example, to be thinking and caring people.

A mutual friend, Bill Harvey, first introduced us. I knew of Bers, both because of his lecture notes and because his son's father-in-law was president of (as it is now called) the Polytechnic University, where I was a student. I used a Yiddish word, and he corrected me by pointing out the one I should have used. We did not meet again for the better part of a year. At Bill's urging—actually he twisted my arm, and my wife pushed me out the door—I went to show Lipa some examples of Kleinian groups; I was a nervous wreck. After all, why should a really important and respected mathematician be interested in the silly examples of a graduate student from an engineering school? Along with other students, faculty, visitors, and whoever else wanted to see him, I stood on line outside his office. When I finally entered, he was on the telephone but bade me start to talk. At a certain point in my discourse, he smiled and said into the phone, "Let me call you back." At that moment I knew I had become a mathematician.¹⁶

Lipa's highest compliment to the discoverer of a new idea was the comment, "You sly dog." He attributed to others the observations that, "Mathematics is a collection of bad jokes and cheap tricks" and that "Mathematicians work for the grudging admiration of a few close friends"—but it was acceptable to steal jokes.¹⁷ Lipa was a master storyteller and jokester, but, for all his joking, he knew the good fortune that was his. In the year of the centenary celebration of the founding of the American Mathematical Society, he gave a moving talk in which he thanked the collective body of American mathematicians, both for himself and for his generation of European immigrants, for saving his life and for providing him the opportunity to continue to work at the mathematics he loved.

Lipa loved the beauty and clarity of mathematics yet, as an old man, he remarked,

In mathematics you have complete freedom with a complete lack of arbitrariness. ... Certainly the concept of what is beautiful has changed. And in general when an older man

¹⁶ *It was only in preparing this article that I became aware of Bers's comments on his becoming a mathematician. He said, "When I went to Löwner to ask him for a thesis topic, I expected him to grab me by the neck and say 'What makes you think that you can write a thesis on mathematics? OUT!!!"*

¹⁷ "Lesser artists borrow, great artists steal" can be found in the \LaTeX manual. It is attributed to Stravinsky.

said something was not beautiful, he was wrong.

As a committed social democrat, he was somewhat self-conscious of having devoted his life to the mathematics from which he derived so much pleasure. He said,

Here I am, a grownup man, worrying about whether the limit set of a Kleinian group has positive measure and willing to invest a great deal of effort to find the answer.¹⁸

In his power as a mathematician, his dignity, his enthusiasm, and his caring for others, he set a standard for the people who knew him. In the circle of people who surrounded Lipa and Mary, devotion to mathematics and people, respect for one another's work, public service, and political activism were neither demanded nor expected—they were just done. The personal models set by Lipa and Lars Ahlfors placed no demands on us. We were simply offered a personal ideal, a personal standard, a goal which we could not expect to attain but found pleasure in trying.

Lipa possessed a joy of life and an optimism that is difficult to find at this time and that is sorely missed. Those of us who experienced it directly have felt an obligation to pass it on. That, in addition to the beauty of his own work, is Lipa's enduring gift to us.

Bibliography:¹⁹

There is original autobiographic material. The following are videotaped lectures:

(V-1) *The European mathematicians' migration to America*, AMS-MAA Joint Lecture series, Amer. Math. Soc., Providence, RI, 1988.

(V-2) *My life with QC mappings*, unpublished video of an impromptu lecture in Dennis Sullivan's seminar.

The following are printed interviews:

(P-1) D. J. Albers, G. L. Alexanderson, and C. Reid (eds.), *More mathematical people*, Harcourt Brace Jovanovich, Orlando, 1990, pp. 3–21.

(P-2) Allen L. Hammond, *Mathematics—Our invisible culture in Mathematics Today—Twelve informal essays*, Springer-Verlag, 1978, pp. 15–34.

Bers's autobiographical writings:

(A-1) An as-yet unpublished autobiography of his early years.

¹⁸ This is Ahlfors's measure problem, and, for general finitely generated groups, it remains open to this day.

¹⁹ The bibliographic references given here are only a guide to Lipman Bers's autobiographical and biographical material, together with sufficient guides to his mathematical work and exposition, to permit further study.

(A-2) Charles Loewner, *Collected papers* (L. Bers, ed.), Birkhäuser, Boston, 1988 (see the introduction).
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(O-1) I. Kra, *The Ahlfors-Bers family* (to appear).

(O-2) L. Keen, *Lipman Bers (1914–1993)*, AWM Newsletter **24** (1994), 5–7.

His selected works, which are being edited by Kra and Maskit, will contain lists of his papers and all his students.

Related commentary may be found in Ahlfors's comments on his work with and paralleling Bers's in (A-1). L. V. Ahlfors, *Collected works*, Birkhäuser, Boston, 1982.

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(W-1) *An outline of the theory of pseudoanalytic functions*, Bull. Amer. Math. Soc. **62** (1956), 291–331.

(W-2) *Uniformization, moduli and Kleinian groups*, Bull. London Math. Soc. **4** (1972), 257–300.

(W-3) *What is a Kleinian group*, Crash Course on Kleinian Groups, Lecture Notes in Mathematics, vol. 400, Springer-Verlag, 1974, 1–14.

(W-4) *Finite dimensional Teichmüller spaces and generalizations*, Bull. Amer. Math. Soc. (N.S.) **5** (1981), 131–172.

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On the Social Activism of Lipman Bers

Carol Corillon and Irwin Kra

Throughout his life, Lipman Bers created exciting mathematics and courageously strove to further his social and political ideals. Bers was a mathematician who charted new research paths; he was a humanitarian who set new standards in social activism and responsibility. He stimulated and challenged generations of mathematicians; he cajoled, coaxed, inspired, and oc-

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asionally browbeat colleagues and associates into joining together in the cause of human rights. And in that cause, as in all the other aspects of their life together, his wife Mary was at his side. With Erna and Lars Ahlfors, Lipa and Mary created an international extended family, a *mishpoch*¹ bound by common mathematical and humanitarian principles. Because of the force of their personalities, Mary and Lipa Bers occupy a very special place in the hearts of the members of that family.

One may wonder whether Bers's social activism was an outgrowth of his enthusiasm for mathematical research and teaching or whether his mathematical achievements were a springboard to a career as a social activist.² He was probably born an activist—a social conscience was in his genes—and he developed a taste for mathematics. Lipman attributed his talents and popularity as a teacher to his youthful experiences as a soapbox orator; he credited his research accomplishments to his interactions with brilliant colleagues. While enjoying an active and warm family life, Lipman devoted himself full time to politics and to mathematics, and he excelled in each of these seemingly disparate human endeavors.

Bers devoted the greater part of his active scientific career to the study of moduli of Riemann surfaces. In an irony not lost on Lipa, one of the most outstanding contributors to this field was the well-known function theorist, Oswald Teichmüller, who was also an ardent Nazi. In one of Lipa's earliest papers on the subject, he quoted Plutarch to describe his strong feelings: "...It does not of necessity follow that, if the work delights you with its grace, the one who wrought it is worthy of your esteem."

Two major themes, at times interrelated, dominated Bers's social agenda: the need for structured reform of the AMS³ and the constant need to promote and protect human rights worldwide. The example set by Bers in his professional behavior was, in effect, an integral part of

this agenda. Lipman was an activist long before he became part of the American (United States) mathematics establishment. Even his choice of a place (Prague)⁴ to pursue graduate study had a political component: the need to escape from those who put him on a "wanted list" in his native Riga, Latvia. His education and political activities in Europe presaged the main themes of his career in the United States: as a superb researcher/scholar/teacher, social reformer, and human rights advocate. His service to the AMS on the Council (1957–1959), on its Executive Committee (1960–1961 and 1974–1977), as one of its vice-presidents (1963–1965), and finally as its president (1975–1976) came at critical times for the organization.

Bers was active in AMS affairs during the 1960s and was president of the AMS during the 1970s, a crucial time in American and AMS history. The unrest and social activism of that period strongly affected the lives and careers of AMS members. Nevertheless, throughout that decade the Society continued to be run along more or less traditional, if not authoritarian, lines. Council meetings, for example, were not open to the general membership, and there was little regard for the wishes of the great majority of mathematicians.⁵

Mathematical research is, almost by definition, an elitist activity. The AMS has two faces: it is both an elitist organization serving the highest demands of what most of us, perhaps parochially, consider the queen of the sciences,

³ Meant in the broadest sense. Bers was not concerned only with the reorganization of the Council and improving its committee structure, although he delighted in such activities, he was interested in broadening and humanizing the scope of the AMS mandate and indeed the outlook of the entire mathematical community.

⁴ Prague because it was in a democratic country, because they let him in, because he had an aunt there (hence he could manage without working—a condition of entry for most students to most countries), and perhaps because Charles Loewner was there.

⁵ H. Hasse was invited by the Mathematical Association of America (MAA) to give a lecture during the August 1963 Boulder meeting in a session on the life and work of E. Artin. Bers, M. Kac, and later E. Moise, among others, denounced the invitation to a former Nazi party member to speak at a session honoring a mathematician who left Germany because of Nazi policies and tried to get the AMS to react appropriately to this invitation. The foremost aim of most, though not all, officers of the AMS and MAA was to sweep the whole affair under the rug. Proposed letters to the Notices by Bers and Moise were eventually withdrawn, and how to describe the discussion of the issue in the Council minutes was the subject of much negotiation. In the end, Hasse spoke, and the general mathematical community knew nothing about the controversy.

¹ "Family" in Yiddish, a language dear to Lipa. Fluent and eloquent in English, Russian, and German, among others, Lipman had a special love for this language. When he and Mary arrived in the United States in 1940, Lipa's familiarity with Yiddish and its rich literature served him well and facilitated his entry into American-Yiddish cultural life. Among Lipa's large number of mathematics papers is one in Yiddish on Yiddish arithmetic books.

² One reader of a preliminary version of these articles remarked, "After reading Abikoff's article, it is clear that Bers was a political activist whose taste for mathematics developed after his escape from Latvia." This remark is indeed correct if mathematical life starts with graduate education.

and a professional society serving the interests of its members. There was in the past, and there still is today, an ongoing debate within the AMS between those who would narrow and those who would broaden the scope or definition of the Society's mandate. Lipman Bers was among those who succeeded in broadening the Society's hori-

and he was the key player in the founding of the Committee on the Human Rights of Mathematicians (CHRM). In fact, the Council of the AMS authorized the establishment of CHRM in 1976 only after President Bers agreed to prepare a charge for CHRM for the Council's consideration. Politician Bers decided that the initial charge for

this new line of activities for the AMS should be narrow (restricted to the activities of foreign mathematicians, for example) since optimist Bers was certain that its scope would surely expand with its successes (as indeed it did).

In the mid-1970s Lipman and several other members of the National Academy of Sciences (NAS) initiated a move to create a human rights committee at the Academy. It was to be a committee that would use the prestige of the institution and its members to pressure governments to resolve the cases of scientific colleagues who had disappeared or who had been imprisoned or threatened. By 1976 the committee was functioning and had already publicly denounced the treatment of eight colleagues in three countries: Argentina, Uruguay, and the

Soviet Union. Always active on its behalf, Bers became chair of the committee in 1979 and served as a member until 1984. Today, after almost twenty years, more than 1,000 cases have been championed by the committee and over half have been successfully resolved. The Human Rights Committee is now an integral part of the NAS; it is regarded as the conscience of the Academy and its sister institutions, the National Academy of Engineering and the Institute of Medicine.

Lipman was not merely brilliant and endowed with superb political instincts. Few could express so simply, so strongly, and yet so very eloquently, people's obligations to other human beings. Lipman was an unabashed, unrepenting, and unfailingly articulate fighter for the human rights of all people. He communicated his convictions directly and forcefully, with little regard to personal cost. Those convictions were charmingly expressed in what he himself called "the international language of science: heavily accented English".

He never took freedom for granted, not for others, not for himself. In his successful plea to

zons to include social and political concerns. Before the Bers presidency, and because of inertia for some period after it, most of those governing the AMS defined the scope of the Society's activities very narrowly. At the same time, a group of radical mathematicians was pushing the AMS towards more openness, more involvement in a social agenda, and reform. Bers understood these tensions, and because of his mathematical stature and political past, he was able to communicate with all sides. He was charming; he was political.

Lipman's style imposed civility. He was probably the person most responsible for saving the AMS from a possibly nasty civil war. He respected allies and opponents, and he took both sides seriously. As president of the Society, he once responded to strong accusations of inaction and implied threats by a young radical with, "Relax, xxx, we won."

Lipman was proud of his role in creating the Committee on Committees (in part because of ironic delight in its bureaucratic name) to advise the AMS president on committee assignments,



Bers with Andrei Sakharov, 1988.

the Academy's Council to make a public statement on behalf of Andrei Sakharov, Lipman said: "When Sakharov began speaking out about victims of injustice, he risked everything, and he never knew whether his intervention might help." Lipa then asked the Council members, "Should we, living in a free country, do less?"

He was steadfast in his defense of freedom of expression, whether or not he agreed with the opinion expressed. Of course, it was well known that Lipa was not one inclined to allow those with whom he disagreed to remain blissfully unenlightened for very long. Those who expressed no opinion fared even worse. During the 1978 International Congress, Lipa was heard, sotto voce, in response to the refusal by a colleague to sign a human rights petition on behalf of Shcharansky and Massera, that "The hottest spot in hell [the humanist secular hell, of course] is reserved for those who are eternally neutral."

Bers held fast to the strength of his personal convictions, devoid of moralism or self-righteousness. His concern for human rights was not an abstract policy decision. He was a *mensch*⁶ whose heart was open to all; he always sympathized with the oppressed and the underdog. Yet, in his social actions or political campaigns, as in mathematics, his head always ruled. He did, however, approach one subject on a slightly less-than-rational basis. He harbored a passionate disregard for the high esteem that society accorded to Henry Kissinger.

In his commencement address, on the occasion of receiving an honorary degree at the State University of New York at Stony Brook in 1984, Lipman turned to one of his favorite nonmathematical topics and said:

...By becoming a human rights activist, as I urge you to do, you do take upon yourself certain difficult obligations...I believe that only a truly even-handed approach can lead to an honest, morally convincing, and effective human rights policy. A human rights activist who hates and fears communism must also care about the human rights of Latin American leftists. A human rights activist who sympathizes with the revolutionary movement in Latin America must also be concerned about human rights abuses in Cuba and Nicaragua. A devout Muslim must also care about the human rights of the Bahai in Iran and of the small Jewish community in Syria, while a Jew devoted to Israel

⁶ In Yiddish, "human being", in the most positive sense.

must also worry about the human rights of Palestinian Arabs. And we American citizens must be particularly sensitive to human rights violations for which our government is directly or indirectly responsible, as well as to the human rights violations that occur in our own country, as they do.

At a human rights symposium held during the Academy's Annual Meeting in 1987, Lipman explained why he so strongly believed that the Committee on Human Rights should focus on political and civil rights, which he called "negative" rights, rather than put its energy behind a "positive" economic, social, and cultural agenda.⁷ He began:

As an old social democrat—I would say an old Marxist, if the word had not been vulgarized—I certainly recognize the importance of positive rights. Yet, I think there is a good reason why the international human rights movement, of which our committee is a small part, concentrates on negative rights. It makes sense to tell a government, 'Stop torturing people.' An order by the prime minister or the president to whoever is in charge could make it happen.

It makes sense to tell a foreign ambassador that, 'The American scientific community is outraged that you keep Dr. X in jail. Let him out and let him do his work.' It requires no planning, no political philosophy, and it can unite people with very different opinions.

Lipman, who was never accused of mincing words, added:

...The idea that people of the Third World are somehow less appalled by torture or by government-sponsored murder than citizens of developed nations [is, to me] rank racism.

He then added:

⁷ Negative rights involve restraints by a government against an individual citizen doing something; for example, restricting his or her right to associate freely with colleagues from abroad. Positive rights require a government to fulfill a need, such as providing medical care, food, education, employment, or housing.



Bers with student Tilla Weinstein, 1984.

It is quite a different matter to tell a foreign government of a developing country, 'You really should give this or that positive right to your people.' If we make such a demand in good faith, it must be accompanied by some plan for implementing this right and by some indication of the cost and of who will pay it and how it will be paid....I think that the basic emphasis on negative rights by the international human rights movement is a reasonable thing.

Lipman, ever the politician, continued:

Now, if we want to do things beyond this and participate in organizing a social democratic party in America, I will gladly discuss this later.

At that point, catching that provocative twinkle in Lipman's eye, the symposium leader, Gilbert White, broke in and, in an unsuccessful effort to keep a straight face, said he really did not want to give the panel even a chance to respond to Lipman's challenge.

Over the years Lipman sent scores of appeals to foreign heads of state on behalf of scientific colleagues around the world: Leonid Plyushch in the Ukraine, Jose Luis Massera in Uruguay, Yuri Orlov and Anatoliy Shcharansky in Russia, Ibrahima Ly in Mali, Ales Machacek in Czechoslovakia, Jose Westerkamp in Argentina, Samuel

Greene in Liberia, Sion Assidon in Morocco, Carlos Armando Vargas in El Salvador, and Ismail Mohamed in South Africa are among the names. Bers organized drives that were fully supported by his colleagues—in effect, he forged a consensus that human rights activities on behalf of, and public protests of mistreatment of, scientists are respectable activities for scientists. He also regularly, and without hesitation, enlisted the help of others. In a 1984 telegram, Lipman asked Willy Brandt to help on behalf of Andrei Sakharov to "prevent a tragedy in Gorky". He wrote, in part: "We feel that Sakharov's life is in grave danger. The death of Sakharov would be a grievous blow to the cause of peace and to the hope of im-

proved relations between the West and the USSR. I feel that each of us must do not all we can but more than we can. I appeal to you because of your unique moral authority."

In summary, did Lipman Bers make a difference? Yes, he did. He did what he could, and then he did more; he expected each of us to do no less.

Lipman Bers as Mentor

Tilla Weinstein

Lipman Bers died on October 29, 1993, after a long and debilitating illness which slowly took from us a most extraordinary human being. Others will speak to his mathematical accomplishments, his many contributions to the profession, and his passionate efforts to secure basic human rights for scientists throughout the world. I have the privilege of commenting upon Lipa's legendary talent as a teacher and thesis advisor. People often ask how he managed to produce so many devoted mathematical offspring (forty-

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eight in all). The impressions which follow were formed during the mid- to late 1950s when I was his student at the Courant Institute. But Lipa's inimitable style held steady through the years, as did his unwavering faith in the talent of young people.

Lipa did not wait for students to introduce themselves. He generally approached them early in their graduate careers, sometimes in the hallway or at tea. He would engage them in lively conversation, showing keen interest in whatever they had to say. Then, having quickly ascertained their likely interests and aspirations, he would tell them which courses to take, which books to read, which seminars to attend, and so on. In short, he often took on the role of advisor before being asked for advice.

Lipa's courses were irresistible. He laced his lectures with humorous asides and tasty tidbits of mathematical gossip. He presented intricate proofs with impeccable clarity, pausing dramatically at the few most crucial steps, giving us a chance to think for ourselves and to worry that he might not know what to do next. Then, just before the silence got uncomfortable, he would describe the single most elegant way to complete the argument. A student who asked questions in class was rewarded with more than helpful answers. Lipa welcomed questions as an opportunity to digress or to assign some extra problem or related theorem for the student to report on outside of class. Lipa liked to listen to students talking about mathematics. Then he got to ask the questions, gently prodding us to notice what we needed to learn.

Lipa never seemed concerned about what students could not do. Instead, he concentrated entirely upon what each student could do. He showed instant delight at any sign of progress. But he greeted each mathematical accomplishment with the immediate assignment of a more formidable task, due next Tuesday. Then he sent us on our way with the clear impression that he expected us to succeed.

Lipa generally taught one course or seminar close enough to his research interests so that he could use it to bare the day-to-day details of his mathematical life. He let us see his delight with the lemma he had proved the night before, and he talked openly about any difficulty he was having in establishing a subsequent result or in framing just the right conjecture. Most notably, he would explain the context for his current work, describing the results which lay the groundwork for current progress and pointing to theorems just over the horizon which might be established using the results he was trying to obtain.

When a student was ready to do research, Lipa would suggest not just the open questions he could see to the end of, but the tougher problems which might not be solvable in any routine manner. He was honest about likely difficulties and shared every idea he had on ways to get started. While he took enormous pleasure in the success of students who grappled with particularly challenging questions (some quite distant from his own research interests), he was equally available and respectful to students working on less demanding topics.

Lipa took mathematical parenthood seriously, showing pride and pleasure in our accomplishments, no matter how remote from mathematics. He wanted to know everything about us: our backgrounds, our health, our hobbies, the books we read, our love lives, our politics, our hopes and ambitions. He encouraged a sense of cohesion among us, creating a mutually supportive atmosphere in which professional friendships flourished. Still, it was probably Lipa's wife Mary who most made us feel like a mathematical family. When Lipa invited us home, it was Mary who made us feel at home. Her interest in each one of us was spontaneous and unconditional. Her warmth and loving support were perfect antidotes to the doubts and fears which so easily beset graduate students.

It would be somewhat disingenuous to avoid the question I am most often asked about Lipman Bers. Why did he have so many women graduate students? What did he do (starting way back before it was fashionable) to produce such a disproportionate share of the women earning doctorates in mathematics? I think some of my male colleagues over the years suspected that Lipa exuded some secret Latvian musklike fragrance which drew us mindlessly to his door. On behalf of Lipa's sixteen women Ph.D. students, I am happy to dispel such myths. There was of course a special reason why Lipa succeeded so well in producing women students. He succeeded because he wanted to! Because he made our progress part of his own agenda, he went out of his way to seek us out and to give us the same extraordinary care and attention which he lavished upon his thirty-two male Ph.D. students. The results speak for themselves.

Lipman Bers was a superb mathematician who shared his exuberant love of the subject with several generations of mathematical newcomers. For all he did to expand and enrich our lives, for his faith and his patience, we will forever be grateful.

Lipman Bers: A Personal Remembrance

Jane Gilman

I was Lipman Bers's student from 1968 to 1971. When Bers students write about what he was like as a teacher and a mentor, the result is often criticized for being too personal. But this is precisely the point about Bers, that he affected people on a very personal level. It was not an uncommon experience for me to meet mathematicians, who, when I mentioned that I was a student of Bers, proceeded to tell how Bers had made a difference in their life (e.g., Bers helped them get one more year of a graduate fellowship just when they had given up all hope of continuing graduate school, or Bers recognized and became enthusiastic about their work when no one else seemed to care or notice).

These were common occurrences for people who were not Bers students. His impact upon his students was far greater. Since he treated each student as an individual, the details of our stories vary, but the essence is always the same. Here are some parts of my story.

I entered Columbia in the fall of 1966 an algebraist at heart. That year Bers taught the first year complex variable course. Since he made hard analysis seem as simple and elegant as algebra, I became his student. I was not the only student so influenced by Bers. There were sixteen students in my

year at Columbia. As a result of his course, eight of them became his students. Six of his eight students eventually received Ph.D.s from Columbia, while only three of the other eight did.

A big fuss is always made about the number of female students Bers had. There were very few women in mathematics when I was finishing college and entering graduate school. Some institutions such as Princeton simply did not allow women to apply to graduate school; and even though my undergraduate professors treated me well, my undergraduate advisor at the University of Chicago told me that women were not welcome in the graduate program there. For me the big question was, Could a woman be a mathematician? The question was not just, Could a woman actually get a Ph.D.?; it was, How would the world treat a woman with a Ph.D.?; Could such a woman have the other things in life that a woman might want (family, love, children), or did a woman have to give up her femininity to become a mathematician?

Thus I entered graduate school with a great deal of ambivalence, and this was reflected by an uneven performance. When I took the qualifying exam, I crossed out several correct pages of mathematics on the complex analysis section, instructing the readers to ignore those pages. Bers read the pages I had crossed out and in a typical move, used those pages to justify passing me. Later he asked me why I had crossed out correct work. I replied, "I don't know."

It was with this understanding of my strengths and weaknesses and my ambivalence that Bers took me on as a student. He not only was my mentor during graduate school but also continued to play a significant role shaping my professional life afterwards.

In the early years I wondered whether he ever felt his time with me had been wasted because I was not as mathematically active as others who were not raising small children. But he never said anything and was simply very supportive of what I did do professionally.

If one person suggested to him that I give a seminar talk on a recent result of mine, the suggestion became translated by the time Bers spoke to me into his telling me, "The masses are clamoring for you to speak."

When my children were small and I had a heavy teaching load, it was difficult for me to get to Columbia on a regular basis for Bers's seminar. I recall that when I did make it in one Friday, Bers overheard someone saying to me that

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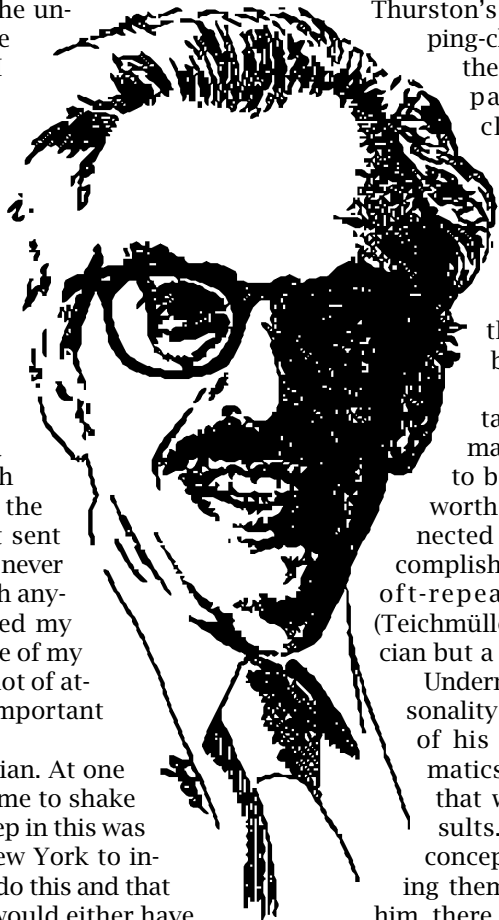
I had come in to socialize and catch up on the gossip. He quickly stepped into the conversation and rebuked the person by saying I had come in to feed my “mathematical soul”, not to gossip. I was grateful to know that he understood how important the seminar was to me even if I came irregularly.

Bers did have some gentle (devious) ways of prodding students into action. Once he called up to ask that I help rewrite a paper of a foreign mathematician whose English was poor. After I ghost edited the paper and it was on its way to publication, I mentioned that using different methods I had previously proved a much stronger theorem which implied the main result of the revised paper. If he had not sent me that paper to edit, I would never have shared my theorem with anyone. I subsequently published my theorem. It was the first piece of my postthesis work to receive a lot of attention, and it played an important role in my getting tenure.

Bers was an astute politician. At one point I decided that it was time to shake up my department. A first step in this was making the pilgrimage to New York to inform Bers that I was going to do this and that it meant that in two years I would either have become chair of my department or be in total disgrace and in desperate need of another job elsewhere, at which point I would need his help. His only reaction was to ask, “Do you have any allies in this venture?” In what was a typical interaction with him, I replied that I had no allies, that was, except for the dean of the college and Danny Gorenstein (who at the time was already university wide an extremely influential Rutgers faculty member). He then changed the subject, and we talked about his most recent reprint from the *Annals of Mathematics*. It was not until much later that I understood that he did not discuss further what he would do when my undertaking failed because he had already figured out that I would succeed. (It actually took ten years to transform my department, and most people thought until the very end that it could not be done.)

Bers refused to suggest problems for me to work on and instead encouraged me to have the confidence to follow my nose. While at the time I might have welcomed more help from him, in the long run it made my successes all the more

valuable to me, a fact for which I am deeply grateful. It also made interactions with him more fun. One day it clicked in my mind that one of the Nielsen papers I had read before



Thurston's classification of the mapping-class group was about one of the (four) classes. (Bers had expanded Thurston's three classes to four by distinguishing two types of infinite reducible mapping-classes, parabolic and pseudohyperbolic.) It was very exciting to call Bers up and say, “Did you know that Nielsen studied parabolic mapping classes?”

All Bers's students were taught very early that while mathematics was king and was to be loved and respected, our worth as a person was not connected with our mathematical accomplishments. This was what the oft-repeated Teichmüller litany (Teichmüller was a great mathematician but a Nazi) was all about.

Underneath the force of Bers's personality and vivacity was the force of his mathematics. His mathematics had a clarity and beauty that went beyond the actual results. He had a special gift for conceptualizing things and placing them in the larger context. For him, there were seldom *technical* lemmas. The λ -lemma is an example. If I close my eyes now and listen, I can visualize him in front of a lecture hall with a gleam in his eye, and I can hear his beautiful and richly accented voice saying the words “the λ -lemma” in a manner that gave a dramatic persona to what another mathematician might have passed off as a mere technical lemma. The lemma took on a life of its own. While I may not be able to give a precise formulation of the lemma, the idea of the lemma and its role in Teichmüller theory remain etched in my mind.

There are a few people in my life whose voice I always hear whatever I am doing. Even during the times when my contact with him was not that extensive, his voice was constantly with me. I still hear his voice, but I miss him.