

The 1989 Curriculum and Evaluation Standards of the National Council of Teachers of Mathematics have become the centerpiece of most efforts to reform school mathematics. Indeed they have come to be seen by other disciplines as a model for “standards”-based national curricula. Happily, an increasing number of research mathematicians have become interested in the problems of school mathematics. Some of these people have been strong supporters of the NCTM Standards, others vocal critics. The following two articles present both sides. While important for what they say, these articles are published foremost to stimulate further discussion about school mathematics among AMS members. NCTM is undertaking a review of the Standards and has encouraged and actively engaged input from the research mathematics community. (Please see “NCTM Updating Standards Documents” on page 444 of this issue of the *Notices*.)

## The NCTM Standards: Helping Shape a Mathematically Literate Society

Jack Price

We educators must continually ask ourselves, “Are the skills we provide our students those that they will be using in their jobs and in their adult lives?” Hopefully the answer is a resounding yes; if not, then we are not living up to our obligation as educators.

The harsh reality we faced in mathematics education in the 1970s was that we were not doing enough. School mathematics programs did not adequately challenge the mathematically talented, and they did not adequately entice the mathematically disinterested. As educators we did not show students the real-life applications of mathematics and therefore did not answer the question on many students’ minds, “When will I ever use this stuff?”

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The need to change was both evident and well documented. Publications like *Agenda for Action* (1980) and *A Nation at Risk* (1983) charged that in order for students to achieve mathematics had to be placed in a familiar context and that change had to be a well-thought-out process sustained over time.

The National Council of Teachers of Mathematics recognized this need and put in motion plans to develop a focused, coherent approach to mathematics education in schools that sets high standards of achievement for every student. NCTM’s *Curriculum and Evaluation Standards for School Mathematics* (1989), *Professional Standards for Teaching Mathematics* (1991), and *Assessment Standards for School Mathematics* (1995) present guidelines that can be used to shape such a mathematics curriculum in every school, in every state.

The effort to develop the NCTM Standards was a broad-based, concerted effort articulated over time. Many of those with an interest in mathematics education—teachers, researchers, mathematicians, parent groups, and business and political leaders—shared in the development process.

The underlying philosophy driving the NCTM Standards is the belief that all students can—and must—learn mathematics. Two complementary philosophies guided the development of the NCTM Standards: an educational philosophy on the need to change, and a conceptual philosophy to articulate the individual standards.

## **The Need to Change**

Changes in the world necessitate changes in the way we prepare our students to be successful. These changes have a profound impact on the way we evaluate the mathematics education we provide our students and determine the education we believe our students need to succeed in the twenty-first century.

Changes in the need for mathematics, in mathematics and its uses, in the role of technology both in today's society and in the future, in society, in what we know about how students learn, and in international competitiveness necessitate a change in mathematics education.

### **Changes in the Need for Mathematics**

Each recent generation has needed more mathematics than the previous generation, with no exception for today's students. Problem solving and reasoning skills rank high on employers' lists of skills, as do competence in higher levels of mathematics such as algebra, probability, and statistics. Similarly, we cannot open a newspaper or make an informed decision about a candidate for election without seeing mathematics in many forms. Citizens need to be more mathematically literate to participate fully in society.

### **Changes in Mathematics and Its Uses**

In the past quarter century we have seen the development of new branches of mathematics and an unprecedented increase in the types and variety of problems requiring mathematical applications. Some of the credit must be given to the development of computers and the subsequent development of computer applications. For example, with technology, two high school students in Connecticut have discovered a new way to carry out a Fibonacci sequence. And we have also seen such complex theorems as Fermat's Last Theorem being proved.

### **Changes in the Role of Technology**

What mathematics is important, as well as how mathematics is done, has been affected by technology. Without mathematics there would be no computers; yet computers extend the reach and power of mathematics. It is now possible to execute almost all of the algorithmic techniques taught from kindergarten through the first two years of college on hand-held calculators. The impact of technology—calculators and computers—is so profound that all aspects of school mathematics had to be revisited.

### **Changes in Society**

When today's children enter the workforce, they will find that most jobs require greater mathematical skills than in the past. At the same time, white males—once the base of U.S. mathematically trained workers—will represent a significantly smaller portion of new workers. These

changes forced us to rethink our approach to mathematics education and how diverse audiences can be successful.

### **Changes in What We Know about How Students Learn**

We have increasing evidence that most students learn best when ideas are introduced in context—a known, real-world situation, problem, or structure into which the mathematics falls. Against this background, students can begin to think abstractly and see the value of proof. A mathematically literate person knows much more than a series of techniques acquired through repetitive drill.

### **Changes in International Competitiveness**

International studies have shown that U.S. students are not high achievers in mathematics. Although international data are useful, they often lead to international rankings, with inadequate consideration given to the varying social contexts and educational practices in which schooling takes place. Still, the information we are able to garner from international studies—of both education and business practices—provides us with much needed information to prepare our students to compete in the global marketplace.

Although these six components helped form the foundation supporting the need to change mathematics education, they did not provide answers to the question, What do we change?

## **What Do We Change**

The process of changing school mathematics is far reaching and time consuming, encompassing all aspects of mathematics education—content, teaching, and assessment. Although the NCTM Standards were released in three separate documents, the message forms a cohesive whole. To improve learning, teaching must improve; and to improve content, assessment must improve.

The changes in educational practices recommended in the NCTM Standards include:

- In curriculum, a shift toward a more extensive study of mathematical ideas and concepts and their uses in today's world.
- In learning, a shift toward more active student involvement with mathematics, including mathematical problems that relate to their world, and the use of a variety of mathematical tools for solving these problems.
- In teaching, a shift toward creating classrooms that are stimulating learning environments in which all students have the opportunity to reach their full mathematical potential.

- In assessment practices, a shift toward student evaluations that are continuous and based on many sources of evidence.

Each set of Standards is built around five curricular goals for students: become a mathematical problem solver, learn to communicate mathematically, learn to reason mathematically, learn to value mathematics, and become confident in one's own ability.

One look at any of the NCTM Standards documents and you will see the interrelation between the mathematics content, teaching, and assessment, as well as opportunities to reach the five curricular goals. For example, the NCTM *Assessment Standards for School Mathematics* contends that assessments are learning opportunities as well as opportunities for students to demonstrate what they know and can do. Specifically, the document contends that assessment that enhances mathematics learning becomes a routine part of ongoing classroom activity rather than an interruption. It does not simply mark the end of a learning cycle; it is an integral part of instruction that encourages and supports further learning. To a visitor entering a classroom, instruction and assessment might be indistinguishable.

The *Assessment Standards* also acknowledge that teachers must have high expectations for themselves as well as for all students, provide for different learning styles, and encourage the active involvement of all students in learning mathematics. Assessment must give each student, including those with special needs or talents, the opportunity to demonstrate his or her understanding in a variety of ways. Results of the assessment must be used to ensure that each student is given not only the opportunity but also the necessary support to reach the highest possible levels of achievement.

The NCTM Standards are guidelines to help shape the mathematics curriculum—they are not a national mandate. Recognizing the need for autonomy among the states, the Standards enable each state, textbook company, and test publisher to interpret the recommendations and develop appropriate curricula. They are meant to stimulate policymakers, parents, teachers, administrators, and local communities and school boards to improve mathematics programs at all levels.

The question is no longer what needs to change, nor is it how do we change. The recommendations noted in the NCTM Standards reflect the changes that must take place in mathematics education if we are to ensure that our students know—and understand—the mathematics they do.

It is time to move forward, time to raise the standards of mathematics education.

## Next Steps to Ensuring Mathematics Excellence for All

We have two choices: either submit to the problem or take responsibility for fixing it. As a mathematics educator, I am committed to improving mathematics education. We have existence proofs that point the way.

Indications that positive change is taking place in mathematics classrooms nationwide are now available. Where NCTM recommendations are being systematically implemented, the achievement data are encouraging.

The Third International Mathematics and Science Study (TIMSS) is a wonderful opportunity for all of us to learn, to refine our approach to improving the mathematics education provided to all students. What is unique about the TIMSS data is the abundance of information that will be available. For example, videotapes of classroom instruction in Japan, Germany, and the United States will be released in the near future, offering us opportunities to understand better the teaching and learning that is taking place in the classroom. Preliminary reports note that “it seems clear that Japanese teachers, on average, come closer to implementing the spirit of current ideas advanced by American reformers than do American teachers.”

The findings illustrate what mathematics teachers have been saying for years—that an intellectual, coherent, focused approach to mathematics education, addressing content, teaching practices, and assessment is needed to ensure our students' success.

We must provide all of our students with the skills needed to be competitive on a global scale. Yet we can neither mimic the education system of one country nor provide a national curriculum that fails to capitalize on the diversity in our society and on the states' responsibility for education. Improving mathematics education is a multiyear journey that demands continual rethinking. Just as our country is predicated on constant improvement, so must our education system be.

As we move forward in our efforts to improve the mathematics we provide to all students, we must view the NCTM Standards as the means to raise our student achievement. And we must all work together toward the common goal of mathematics excellence for all students.

## Resources

- [1] *Reshaping School Mathematics: A Philosophy and Framework for Curriculum*, Mathematical Sciences Education Board, Washington, DC, 1990.
- [2] *Pursuing Excellence*, National Center for Education Statistics, Washington, DC, 1996.

- [3] *Assessment Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 1995.
- [4] *Curriculum and Evaluation Standards for School Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 1989.
- [5] *Professional Standards for Teaching Mathematics*, National Council of Teachers of Mathematics, Reston, VA, 1991.

# Commentary on Assessment Standards for School Mathematics<sup>1</sup>

*George E. Andrews*

Everyone recognizes that mathematics education is in some difficulty in the USA. In one way or another, with more or less evidence, many of us feel that standards have fallen. Obviously assessment is not working well currently. Something must be done to ensure that our students learn more before they are assessed as passing.

There are numerous assessment problems that could be addressed, so let me base my commentary on three of the most obvious.

## Grade Inflation

In a recent study carried out at the University of California at San Diego, J. R. Betts and S. Boedeker made a careful study of the relationship between learning and grade inflation. Their central conclusion is: "Students who attend schools with more lax grading standards learn less during the school year than do students at schools with more stringent grading standards, even after controlling for a wide variety of measures of family background and school resources." [1]

Yet according to the College Board, the past decade has witnessed substantial increases in the number of As given in high school, with corresponding decreases in lower letter grades. It might (and undoubtedly will) be argued that grade inflation is a problem of evaluation, not

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<sup>1</sup>Assessment Standards for School Mathematics, *Assessment Standards Working Groups of the National Council of Teachers of Mathematics*, 112 pp., NCTM, 1995.

of assessment, surely a distinction without a difference. Grades, whatever their merits, are perceived by students and public alike as substantial rewards or punishment for academic work. Given this perception, they will have enormous impact on any assessment process, and this impact merits serious consideration in any discussion of assessment.

## Homework

*The Economist* of May 6, 1995, ran an article summarizing various studies on homework:

Timothy Keith of the University of Iowa and several colleagues find that homework's power to influence success ranks second only to ability, and ahead of race and family background. "Graded homework," Mr. Walberg and his colleagues (at the University of Illinois-Chicago) found, "produces an effect...three times larger than social class."

One of the most careful comparisons, by Chuansheng Chen and Harold Stevenson of the University of Michigan, found that eleven-year-olds in Taiwan do twice as much homework as do their counterparts in Japan, who in turn do twice as much as American fifth-graders. [2]

Of course, those anxious to contradict the obvious will say that the recent TIMSS (Third International Mathematics and Science Study) shows that the Chen-Stevenson report is wrong. This faulty view is based on the TIMSS assertion that lack of homework is not a problem:

It found that American math and science teachers assign much more than teachers in Germany or Japan. Eighty-six percent of American teachers surveyed in the study said they assigned homework three to five times a week. In Japan, 21 percent of teachers said they did, yet their students rank much higher in an overall assessment of math and science skills. Most Japanese eighth-graders do, however, attend after-school classes in math for a few hours each week to prepare for high-school entrance exams, the study notes. [3]

Obviously the number of assignments is hardly a measure of how much work is done outside of class. Are the assignments all collected? Are they corrected? How much do they count for the class grade? Is anyone seriously suggesting

that our students work nearly enough on studies outside of class?

Clearly the homework question is a difficult one. How can homework best be utilized in the assessment process? Homework is done outside the supervision of the teacher. How much should it count towards a grade, given this lack of supervision? How can a teacher use assessment incentives to encourage more and better homework?

### The Role of Calculators in Assessment

I have been unable to unearth controlled studies on the effects of technology in assessment. To illustrate some of the problems, I shall rely on four vignettes.

The *NCTM Assessment Standards* uses many vignettes to illustrate its points. There is, however, an important difference between my vignettes and those used by NCTM. My stories actually happened. The NCTM notes that "Except for those identified as adapted from research, the vignettes are fictional illustrations and, although drawn from experience, are not factual accounts." My stories are too ridiculous to make credible fiction.

**Vignette 1.** The scene is an oral, one-on-one exam being given to a nonmath major in a freshman course at a Big Ten university.

*Professor:* Please give me an example of a sequence decreasing geometrically.

*Student:*  $1/3, 1/6, 1/9, 1/12\dots$

*Professor:* Actually the rate of decrease must be much faster. Suppose that we start with  $1/3$ . Then, for example, to form each new term we might divide by 3. What is  $1/3$  divided by 3?

*Student:* I don't know! I don't have my calculator.

*Professor:* "Really, you don't need a calculator to figure this out. Dividing  $1/3$  by 3 is the same as multiplying  $1/3$  times  $1/3$ . What is  $1/3$  times  $1/3$ ?"

*Student* (after a long pause): Two sixths?,

**Vignette 2.** Five students are preparing a psychology lab report in a major East Coast state university.

*Student A:* What is the average of  $.12$  and  $.12$ ? Student A pulls out his sophisticated graphics calculator and enters  $.12 + .12/2$ . The calculator responds  $.18$ .

*Student A:* The average of  $.12$  and  $.12$  is  $.18$ . Three other students write this down.

*Student B:* Wait a minute! The average of  $.12$  and  $.12$  is obviously  $.12$ .

*Student A:* That's what I thought, but my calculator says it's  $.18$ .

Student B asks to use the calculator. He enters  $(.12 + .12)/2$ . The calculator responds  $.12$ .

Seeing this, the other four are immediately convinced that the average of  $.12$  and  $.12$  is  $.12$ .

**Vignette 3.** The scene is a professor's office in a major East Coast state university. The professor is talking with a student from an elementary statistics class.

*Student:* I can't get the right answer to Problem 10. The book says it is  $0.5$ .

*Professor:* What answer did you get?

*Student:*  $1/2$ .

*Professor:* (Unprintable).

Student pulls calculator from his bag and enters  $1/2$ . The calculator responds  $.5$ .

*Student:* Wow, I got it after all.

**Vignette 4.** The scene is a large, elementary physics-for-engineers lecture in the flagship state university in a large state.

*Instructor:* How many of you know what  $968$  divided by  $10$  is without using a calculator?

Fewer than 10 percent knew the answer.

In subsequent discussion with one of these students, the instructor found that the student could calculate  $1000/2$  by hand but not  $1000/10$ .

It would be a terrible misreading of these stories to infer that today's students are just stupid. Rather, many are ignorant due to a miseducation which involves heavy use of calculators. So with this introduction I would hope that even the most skeptical reader will at least concede that the issues raised above merit some consideration as assessment issues.

And how does *NCTM Assessment Standards for School Mathematics* address these problems?

Concerning grade inflation they have NOTHING to say. I would argue that this is one of the most important problems we face today. Yet NOTHING is said by the NCTM. Well, not exactly. The Standards wishes to shift assessment practice "TOWARD assessing students' full mathematical power (and) AWAY FROM assessing only students' knowledge of specific facts and isolated skills." [4]

What does this imply? The impression I get from the Standards is that students should know a lot and that the main fault of current assessment is its failure to sample the vast array of student skills. Is this your view of what has happened to the students in your classes? It is obvious that university students who have no grasp of basic arithmetic have been given a high school diploma under false pretenses.

Concerning homework, the *NCTM Assessment Standards* has ALMOST NOTHING to say. The word *homework* never appears, even though guidance on the role of homework in the assessment process would be welcomed by countless math teachers, including me. They do once refer to "work at home".

A threat to the validity of inferences comes from potential bias in the evidence. New forms of assessment, such as portfolios or extended projects, may create new sources of bias. Extended projects may allow students to complete some of the work at home [God forbid! (GEA)], with the result that differences in home resources (including assistance from the parents [No! No! Not the parents! Anything but the parents! (GEA)]) may bias the results.[5]

This quote highlights one of the problems with the NCTM *Assessment Standards*. This is obviously a document written by people who truly desire a more decent and equitable society. Time after time we are warned that bias may creep into assessment.

“Another source of potential bias lies in assessment activities that rely heavily on students’ ability to use the English language to communicate mathematical knowledge.” [6]

“How is the role of students’ background and experiences recognized in judging their responses to the assessment?” [7]

“Assessments have too often ignored differences in students’ experience, physical condition, gender, and ethnic, cultural, and social backgrounds in an effort to be fair.” [8]

Now, there is nothing wrong with each of these three statements per se, but admitting their validity does not ensure greater equity in mathematics achievement, a goal clearly desired by all. Indeed, excessive attention to these statements at the expense of proven indices for achieving the desired equity may have negative implications on both equity and high standards. Note, for example, this further excerpt from *The Economist* article:

Indeed, homework turns out to be a powerful educational leveler—and it levels upwards. Studying British grammar school boys, Michael Holmes and Paul Croll, both British researchers, found that working-class children benefited more from homework than did their wealthier schoolmates. Working-class boys who spent an hour or more at night on homework achieved just as much as middle-class boys who did the same—whereas among low-homework boys, class differences were pronounced. [2]

Thus an emphasis on homework and standards increased equity as an indirect result.

In all the NCTM vignettes with group learning, peanut-eating elephants, postal-rate histo-

ries, areas of salt marshes, and the appropriate use of dynamic geometry software, one nagging question continues to plague the reader: What is the path of least resistance through all this stuff?

Will all students participating in groups with appropriate software work like little beavers to achieve “mathematical power”? Or will a number realize that if they sort of get the idea, then the group can carry them to a passing grade, especially if they have Energizer batteries in their calculators?

If students know they can pass with little effort, if they know they can cry “Bias!” when homework is assigned, then some, perhaps many, will work the system to their advantage.

Of course something bad like this will never happen provided all teachers are the highly motivated, single-minded dynamos that populate the admitted fiction of the NCTM *Assessment Standards*. Unfortunately, out in the real world a variety of educationist “leaders” leave many math teachers uncertain of what is really important in math education. Out of this confusion emerge many students who are poorly educated in arithmetic, algebra, and trigonometry.

It is impossible in this commentary to cover the multitude of sins in this book; fortunately, there are numerous teachers and school boards who will just ignore it.

Recently I had a long conversation with an eleven-year-old boy who goes to our local elementary school. I was then in the process of writing this commentary, and when the subject of arithmetic came up, I decided to ask a few questions. “What’s  $8 \times 7$ ?” “56,” he responded immediately. “What’s  $11 \times 12$ ?” “132,” he responded almost immediately. “That’s easy because the middle digit is the sum of the outer two when you multiply by 11.” “OK, what’s  $8 \times 12$ ?” There was a pause. “96,” he said eventually. “I didn’t know it, but I did know  $3 \times 12$  and  $5 \times 12$ , so I added 36 to 60.” He has obviously learned some “specific facts” from his teacher. Will all these specific facts prevent him from achieving mathematical power?

Mathematics is almost tactile: it is like playing the piano. You can’t do it holistically. If you can’t play a scale, it is highly unlikely that you can play “The Moonlight Sonata” (highly unlikely, not impossible). Of course, you can always put a CD of Rubinstein into a CD player, and if you are a big enough fool, you can believe that you are playing the piano with some technological assistance. If anyone really believes that a typical student can become mathematically powerful while remaining arithmetically, algebraically, and trigonometrically ignorant, then he has an obligation to provide some compelling evidence before this hypothesis becomes an un-

intended consequence of efforts to implement national recommendations.

Reviewing the NCTM *Assessment Standards* left me with the depressing feeling that they can only add to the deplorable mathematical ignorance that we struggle against daily. Does anyone have any evidence that allows a different conclusion?

#### References

- [1] J. R. BETTS and S. BOEDEKER, Research Report, UCSD, January 1995, p. 15.
- [2] *The Economist*, May 6, 1995.
- [3] *The Washington Post*, November 21, 1996.
- [4] *Assessment Standards for School Mathematics*, NCTM, Reston, 1995, pp. 56, 58, 83.
- [5] *Ibid*, p. 19.
- [6] *Ibid*, p. 19.
- [7] *Ibid*, p. 16.
- [8] *Ibid*, p. 15.

## Response of George Andrews

OK, class! Our assessment activity for the day is the following quiz on Jack Price's essay, *The NCTM Standards: Helping Shape a Mathematically Literate Society*.

1. How many times does the word "change" (singular or plural) occur in Price's piece?

- a. 5
- b. 23
- c. 432
- d. 11317.6

**Answer: b**

2. Which century is it that has so badly frightened us into all this change?

- a. the 12th
- b. the 16th
- c. the 19th
- d. the 21st

**Answer: d**

3. Price clearly identifies the culprits in our current educational turmoil with the following sentence: "Preliminary reports note that it seems clear that Japanese teachers, on average, come closer to implementing the spirit of current ideas advanced by American reformers than do American \_\_\_\_\_." The word that belongs in the blank is:

- a. educationists
- b. celebrities
- c. capitalists
- d. teachers

**Answer: d**

4. In his next sentence, Price clearly identifies the heroes in our current educational turmoil with the following assertion: "The findings illustrate what mathematics \_\_\_\_\_ have been saying for years—that an intellectual, coherent, focused approach to mathematics education, addressing content, teaching practices, and assessment is needed to ensure our students' success." The word that belongs in the blank is:

- a. educationists
- b. celebrities
- c. capitalists
- d. teachers

**Answer: d**

5. Price tells us that "improving mathematics education" is:

- a. an impossible job
- b. a lucrative business
- c. an ongoing process
- d. a multiyear journey

**Answer: d**

6. What is demanded by the multiyear journey mentioned in Question 5?

- a. a car
- b. funding
- c. a calendar & map
- d. continual rethinking

**Answer: d**

7. What does a mathematically literate person know?

- a. 17th century algebra
- b. 15th century arithmetic
- c. geometry from ancient Greece
- d. much more than a series of techniques acquired through repetitive drill

**Answer: d**

8. What do international data often lead to?

- a. a clear understanding of what the NCTM has already done for us
- b. funding
- c. the smoking gun
- d. international rankings

**Answer: d**

9. Do international rankings often give inadequate consideration to the varying social contexts and educational practices in which schooling takes place?

- a. yes

**Answer: a**

10. According to Price, almost all algorithmic techniques taught from kindergarten through the first two years of college have been \_\_\_\_\_ by hand-held calculators. The word or phrase that belongs in the blank is:

- a. turned into incomprehensible magic
- b. destroyed
- c. assassinated
- d. executed

**Answer: d**

11. According to Price's essay, grade inflation is:

- a. a major assessment problem
- b. not a major assessment problem
- c. irrelevant to assessment
- d. ?

**Answer: d**

12. According to Price's essay, the "calculator as crutch" phenomenon is:

- a. a major assessment problem
- b. not a major assessment problem
- c. irrelevant to assessment
- d. ?

**Answer: d**

13. According to Price's essay, study outside of class is:

- a. overemphasized in the U.S.
- b. underemphasized in the U.S.
- c. a source of bias
- d. ?

**Answer: d**

14. We discern directly from Price's article that the shape of a mathematically literate society is:

- a. hexagonal
- b. scalene
- c. rather indeterminate but its surface is warm and fuzzy
- d. ?

**Answer: d**

Count the number you got right, and get out your calculators. Multiply the number you got right by 2. Add 200. Divide the result by 2. Finally subtract the number you got right. The result is your grade for the quiz. Happy? Me too!

## Response of Jack Price

Mathematics education is currently suffering under a burden of what John Gardner called "unloving critics" and "uncritical lovers". There is a huge chasm between these two groups—nostalgic and reactionary on the one hand and well meaning but overreactive on the other. In the meantime, most of us are trying our best to keep a balance.

We all have the same goal: to heighten the mathematics achievement of U.S. students. But

it is time to do more than just point out the problem; we must take responsibility for fixing it.

Although I agree with Dr. Andrews on both the counterproductive impact of grade inflation and the importance of homework, I do not agree with his inference that current reform efforts dilute grades and homework. Clearly, if we are to improve mathematics education, then we must focus on what is most important: high student achievement in high-quality mathematics.

The Standards emphasize high expectations and high standards for teacher and student alike without specific recommendations for homework, grades, or any one part of the educational enterprise. Would it help if these issues were dealt with overtly? Undoubtedly. However, their omission from the *Assessment Standards* hardly invalidates the document. The Standards provide a philosophy to guide decision making.

Although I share Dr. Andrews' discouragement at the actions noted in his vignettes, I take exception to his remarks that "many [students] are ignorant due to a miseducation which involves heavy use of calculators." We all hope that there will be few such stories in the future, but I cannot help but notice that each vignette involves a college-aged student whose education predates widespread implementation of a Standards-based mathematics education. His choice of vignettes validates the need to change an education process that has failed our students.

We have seen a shift from longhand to typewriters to word processors. Just as the word processor has not replaced our ability to compose, calculators and computers have not replaced our ability to think—quite the opposite. The appropriate use of technology enables us to reach higher levels of achievement. Miseducation is the culprit, not technology. And it is miseducation that the Standards address.

So, in response to Dr. Andrews's "nagging question", I would like to point out that since the dawn of time, there have been those who choose the path of least resistance. As educators we are compelled to do what we can to encourage every student to reach his or her full potential. Will we succeed with each student? Probably not. Have we, in the past, succeeded with every student? Clearly not. We have ample evidence that what we have done in the past does not work. Today we have a growing number of skilled professionals who can effect change, and we have the technology and curriculum materials needed to support their efforts. It is time to stop complaining that the cup is half empty and time to start working together to make the cup full.