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Mathematics in Barcelona: Time Past, Time Future
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Los Angeles Meeting
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Institut d'Estudis Catalans, Barcelona (see page 560)
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Featured Reviews in Mathematical Reviews 1997–1999
With Selected Reviews of Classic Books and Papers from 1940–1969
Donald G. Babbitt, Publisher, American Mathematical Society, Providence, RI, and Jane Kister, Executive Editor, Mathematical Reviews, Ann Arbor, MI, Editors

This second volume of Featured Reviews makes available special detailed reviews of some of the most important mathematical articles and books published from 1997 through 1999. Also included are excellent reviews of several classic books and articles published prior to 1970. Among those reviews, for example, are the following: Homological Algebra by Henri Cartan and Samuel Eilenberg, reviewed by G. Hochschild; Faisceaux algébriques cohérents by Jean-Pierre Serre, reviewed by C. Chevalley; and On the Theory of General Partial Differential Operators by Lars Hörmander, reviewed by J. L. Lions. In particular, these reviews and the background information on current developments outside their own area of expertise will find the volume very useful.

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The Game’s Afoot: Game Theory in Myth and Paradox
Alexander Mehlmann, Vienna University of Technology, Austria
Reviews of the German edition:
The author, well known for various imaginative, entertaining and instructive writings in the area of game theory, and for his game-theoretic excursions into classical literature, has now brought out this delightful little book on the basics of noncooperative games. . . (The book is) rewarding reading for a rather wide variety of reasonably well-educated persons. The reader will gain an appreciation for the mathematical modelling of conflict in economics, the social sciences and biology, and get a glimpse of game-theoretic analysis of conflict in some of the classical literature.
—Zentralblatt für Mathematik

Through the amusing exposition of the material, overflowing with jokes and general culture, the new book by Alexander Mehlmann has become bedtime reading for me. . . . It is a pleasure to see such things as the Dilemmas of the Arms Race, Goethe’s Mephisto, the Chain-Store Paradox, and the Madness of Oedipus brought under one game-theoretic roof.
—Eric Lessing (from a translation of “What I am reading” in Die Presse)

In this book, Mehlmann presents mathematical foundations and concepts illustrated via social quandaries, mock political battles, evolutionary confrontations, economic struggles, and literary conflict. Most of the standard models—the prisoners’ dilemma, the arms race, evolution, duels, the game of chicken, etc.—are here. Many non-standard examples are also here: the Legend of Faust, shootouts in the movies, the Madness of Oedipus, to name a few.

Originally written in German and published by Vieweg-Verlag, this AMS edition is a translation tailored for the English-speaking reader. The Game’s Afoot would make an excellent book for an undergraduate course in game theory. It can also be used for independent study or as supplementary course reading. The connections to literature, films and everyday life also make it highly suitable as a text for a challenging course for non-majors. Its refreshing style and amusing combination of game-theoretic analysis and cultural issues even make it appealing as recreational reading.

Student Mathematical Library, Volume 5; 2000; 159 pages; Softcover; ISBN 0-8218-2121-4; List $26; All AMS members $21; Order code STML/5NT005

Dynamical Systems on Homogeneous Spaces
Alexander N. Starkov, Moscow State University, Russia
A homogenous flow is a dynamical system generated by the action of a closed subgroup $H$ of a Lie group $G$ on a homogeneous space of $G$. The study of such systems is of great significance since they constitute an algebraic model for more general and more complicated systems. Also, there are abundant applications to other fields of mathematics, most notably to number theory.

The present book gives an extensive survey of the subject. In the first chapter the author discusses ergodicity and mixing of homogeneous flows. The second chapter is focused on unipotent flows, for which substantial progress has been made during the last 10–15 years. The culmination of this progress was M. Ratner’s celebrated proof of far-reaching conjectures of Raghunathan and Dani. The third chapter is devoted to the dynamics of nonunipotent flows. The final chapter discusses applications of homogeneous flows to number theory, mainly to the theory of Diophantine approximations. In particular, the author describes in detail the famous proof of the Oppenheim-Davenport conjecture using ergodic properties of homogeneous flows.

Translations of Mathematical Monographs, Volume 190; 2000; 243 pages; Hardcover; ISBN 0-8218-1385-7; List $56; Individual member $57; Order code MMONO/190NT005

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This self-contained introduction to some problems and asymptotic behavior of Toeplitz matrices looks at Toeplitz matrices obtained by working with appropriate stochastic algorithms toward creative solutions and problem-solving techniques. The work is divided into problems clustered in three self-contained sections (algebra & analysis, geometry & trigonometry, number theory & combinatorics) with solutions provided separately. Along with background material, each section includes representative examples, beautiful diagrams, and lists of unconventional problems. Additionally, historical insights and asides are presented to stimulate further inquiry.

Recent Trends in Nonlinear Analysis
Festschrift Dedicated to Alfonso Vignoli on the Occasion of His Sixtieth Birthday
J. Appell, University of Wurzburg, Germany (Ed.)
This collection of 21 original research papers reports on recent developments in various fields of nonlinear analysis covering a wide variety of topics, from abstract fields such as algebra topology, functional analysis, operator theory, spectral theory, analysis on manifolds, partial differential equations, boundary value problems, geometry of Banach spaces, measure theory, variational calculus, and integral equations, to more application-oriented fields like control theory, numerical analysis, mathematical physics, mathematical economics, and financial mathematics.

Units in Skew Fields
E. Kleine, University of Hamburg, Germany
This book is devoted to a study of the unit groups of orders in skew fields, finite dimensional and central over the rational field; it thereby belongs to the field of noncommutative arithmetic. The book gives a synopsis of results and methods, including full proofs of the most important results.

New Horizons in pro-p Groups
M. du Sautoy, Cambridge University, Cambridge, U.K.
D. Segal, All Souls College, Oxford, U.K. & A. Shalev, Hebrew University, Jerusalem, Israel (Eds.)
This book presents a clear picture of the rich universe of pro-p groups in its unity and diversity, with a systematic emphasis on the construction and examination of many classes of examples. A correspondingly diverse range of mathematical techniques is successfully applied, leading to new results and pointing to exciting new directions for research. A large number of open research problems are introduced and discussed, and excellent introductory material and numerous examples are presented throughout.

Mathematical Olympiad Challenges
T. Andreescu, American Mathematics Competition, University of Nebraska, Lincoln, NE & R. Gelca, University of Michigan, Ann Arbor, MI
This comprehensive collection of problems, written by two experienced teachers and coaches of the U.S. International Mathematical Olympiad Team, presents hundreds of beautiful, challenging, and instructive problems from decades of international competitions, encouraging readers to move away from routine exercises and memorized algorithms toward creative solutions and problem-solving techniques. The work is divided into problems clustered in three self-contained sections (algebra & analysis, geometry & trigonometry, number theory & combinatorics) with solutions provided separately. Along with background material, each section includes representative examples, beautiful diagrams, and lists of unconventional problems. Additionally, historical insights and asides are presented to stimulate further inquiry.

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Stochastic Analysis and Mathematical Physics
R. Rebolledo, Pontificia Universidad Católica de Chile, Santiago, Chile (Ed.)
This volume presents refereed papers that highlight emergent research in the area of quantum probability. Several papers present a qualitative analysis of quantum dynamical semigroups and new results on q-deformed oscillator algebras, while others stress the application of classical stochastic processes in quantum modeling.

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Feature Articles

Program Verification
Robert P. Kurshan

Program verification—the formal checking of a computer program for correctness—
involves steps of reduction and decomposition, as well as a check of the resulting
component programs. It provides an economically viable method for testing the design
of integrated circuits.

Invariant Distances and Metrics in Complex Analysis
Alexander V. Isaev and Steven G. Krantz

The authors work with a variety of distance functions that are defined infinitesimally
on complex manifolds, showing how the properties of these distances lead to
restrictions on the automorphism group of the manifold and sometimes to an
identification of the manifold itself.

Mathematics in Barcelona: Time Past, Time Future
Allyn Jackson

Barcelona, the site of the European Congress of Mathematicians in July 2000, boasts
four universities and an international center for mathematics research.

Memorial Article
Ralph Phillips (1913-1998)
Peter Sarnak

Communications

Fragile Dominion—A Book Review
Reviewed by David C. Krakauer and Martin A. Nowak

Browder, Coifman, and Kadanoff Receive
2000 National Medals of Science

Ingrid Daubechies Receives NAS Award
in Mathematics

Bott and Serre Share 2000 Wolf Prize

2000 JPBM Communications Award

AWM Awards Presented in Washington, DC

MAA Prizes Presented in Washington, DC

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AMS in the Twentieth Century

AMS Prizes

In his history book of the AMS from 1888 until 1938, Raymond Archibald makes no mention of any discussion of AMS prizes before 1916. He writes, "In 1916 it became known that, because of the admiration of one of its most loyal members, the Society would one day be able to offer a very large prize.... By the will of Prof. L. L. Conant (1857-1916), who had been a member of the Society from 1892, the sum of $10,000 was left to the Society, subject to Mrs. Conant's life interest." It was Conant's clear intention that the income from the bequest ultimately be used for an AMS prize. Conant's wife was still alive in 1938.

Upon the death of Maxime Böcher (1867-1918), "a committee ... was authorized to receive contributions," Archibald writes, "from members of the Society and others who wished to participate in establishing a suitable memorial." The money amounted to $1,161.79 by 1921 and was used to establish the Böcher Memorial Prize in Analysis. The first award of the prize of $100 went to G. D. Birkhoff in 1923.

When Frank Nelson Cole (1861-1926) retired in 1920 as AMS secretary and Bulletin editor, he was honored by the Society for his distinguished service. Among the tokens of recognition was a sum of money obtained from small contributions, amounting to $472.88. In turn Cole presented this money to the Society in 1921, and the money was the beginning of the "Cole Fund." Archibald writes that a committee "recommended, among other things, that the Fund shall accumulate until, by interest and contributions, it reaches the amount of $1,000; that at the end of every five years thereafter the Council shall award from the available income not more than $200 as a prize, to be called the Frank Nelson Cole Prize in Algebra." By 1923 the amount in the Fun exceeded $1,000, and the first award of the Cole Prize was to L. E. Dickson in 1928. The Cole Prize was soon awarded twice every five years, once in algebra and once in number theory.

Everett Pitcher's history book about the AMS deals with the interval 1939-1988. No further AMS prizes were founded until the 1960s. In rapid succession came the Oswald Veblen Prize in Geometry, first awarded in 1964; the George David Birkhoff Prize in Applied Mathematics, joint with SIAM and first awarded in 1968; the Norbert Wiener Prize in Applied Mathematics, joint with SIAM and first awarded in 1970; and the Leroy P. Steele Prizes, first awarded in 1970. The Steele Prizes were well endowed and were to be given annually, but there were no awards in five of the first nine years. At that point the system was changed, and thereafter Steele Prizes were awarded annually for lifetime achievement, for a memorial paper, and for exposition.

Pitcher says, "The Council has always been reluctant to have too many prizes and to subdivide mathematics too finely. Thus subsequent money was often incorporated into funds or used to supplement existing prizes. There were some exceptions in the 1990s, with the establishment of the Fulkerson Prize in Discrete Mathematics, the Morgan Prize for undergraduate research, the Satter Prize for mathematics research by a woman, and the Award for Distinguished Public Service."

But what about the Conant bequest? Conant's wife lived until 1976, and at that time the AMS received $9,500. That money grew rapidly, and in 1999 the AMS began to address the question of what to do with the Conant Fund. At its January 2000 meeting, the Council voted to establish an annual Conant Prize for Exposition, specifically, for the best article in the Notices or Bulletin in the past five years. Current plans are for the first prize to be given in 2001.

—Anthony W. Knapp

In This Issue

New Department

This issue of the Notices inaugurates a new department called "Inside the AMS". This department will contain information about AMS events and activities of interest to the membership. The name of the "From the AMS Secretary" section has been changed to "From the AMS Secretary" in order to reflect better the contents of that section and to avoid confusion with the new department.

—Allyn Jackson
Commentary

In My Opinion

Imminent Danger—From a Distance

In Gulliver's Travels, Jonathan Swift describes a society in which students learn mathematics by swallowing pieces of paper on which theorems have been transcribed. Recently, California students have learned mathematics by studying Mathland, which has no textbooks, but uses "manipulatives". Soon thousands of university students will learn mathematics either by interacting with a computer or over the Internet via a process called "distance-learning".

These phenomena share one or more of the following three features:

(i) They eschew the method of placing students in a classroom—in a controlled environment—under the supervision of a trained professional or teacher.

(ii) They reject the notion that students should learn from a curriculum or a text that has been written by practicing faculty scholars.

(iii) They value form over substance.

Those promoting distance-learning—i.e., students learning over the Internet without direct, synchronous interaction with a human instructor—want to substitute the act of "logging on" for the productive interaction of first-class minds that takes place in the classroom. Those promoting computer learning want to do much the same. When I think about undergraduate education and when I question one of these methodologies, I am questioning both.

Not to demonize Mathland, but it shares an alarming feature with many other modern educational products: it has no author. It is written by the publisher's paid staff. Likewise, the distance-learning companies are not hawking a curriculum created by you and me. They are packaging and selling materials that were created by their staff or that were hired out as piecework and later made into "learning materials" by staff.

Years ago we deviated from the true path with graphing calculators, which we neither designed nor consciously chose for our classrooms. They were foisted upon us by the manufacturers. We conveniently rationalized the educational value of calculators after the fact. Now, with distance- and computer-learning we have the opportunity to repeat the error on a much larger scale.

At a recent trade show in Atlanta, a major software vendor pledged his company to take over the teaching of lower-division college mathematics in America, because his company's software can do a better job of it than university faculty.

All these observations describe a dangerous trend. In every instance, the proper role of the trained mathematician instructor/scholar is being usurped by a machine or by a device or by the paid staff of a retailer.

Learning from software has merits: (i) a course is self-paced, (ii) there are no "missed classes", (iii) the student can "try things". But the give-and-take of human interaction is central to the learning process; that dynamic is lost when a Pentium chip does the teaching.

Among other qualities, a good teacher

• shows the students how to read the subject matter;
• sets a pace for the students and evaluates their progress;
• adjusts the material to the audience;
• uses voice, style, personality, and knowledge to communicate;
• instills a love for learning;
• helps students to become engaged in the learning process;
• teaches students to reason and to think critically;
• sets a standard for what it means to be educated.

Can a machine perform any of these activities?

Proponents claim that distance- and computer-learning products lower attrition rates and raise scores. The important question is whether students are internalizing and retaining the material. Are they mastering the mathematical method? Can they think critically? Are they attracted to mathematical science? For the distance- and computer-educated, we do not know.

Provosts and deans have dollar signs in their eyes. They envision teaching more students with fewer faculty. But true education is never efficient. Often it is two steps forward and one step back, and it does not come cheap. As Harvard president Derek Bok said, "If you think education is expensive, try ignorance."

Traditional education may not be linear and bullet-like, but it enables students to master the ideas and to retain them for future use. Computer- and distance-learning are slick and quick and high-tech, but their efficacy is unestablished. We should be hesitant to undermine or discard the traditional methods, which have had—and continue to have—considerable success. The vast majority of today's college faculty, for example, were educated with traditional methods. If such caution is not taken, then we are in danger of sacrificing the finest education system in the world on the altar of expediency, austerity, and bottom-line statistics.

If your university is investing in distance- or computer-learning, then apprise yourself of the attendant changes and of how they will affect the quality of learning. Mathematics courses should be designed, taught, and controlled by those who are best qualified—that is, by the mathematics faculty. What is at stake is the education of the next generation of mathematical scientists.

—Steven G. Krantz
Associate Editor

MAY 2000
NOTICES OF THE AMS 533
How can a computer program developer ensure that a program actually implements its intended purpose? This article describes a method for checking the correctness of certain types of computer programs. The method is used commercially in the development of programs implemented as integrated circuits and is applicable to the development of "control-intensive" software programs as well. "Divide-and-conquer" techniques central to this method apply to a broad range of program verification methodologies.

Classical methods for testing and quality control no longer are sufficient to protect us from communication network collapses, fatalities from medical machinery malfunction, rocket guidance failure, or a half-billion dollar commercial loss due to incorrect arithmetic in a popular integrated circuit. These sensational examples are only the headline cases. Behind them are multitudes of mundane programs whose failures merely infuriate their users and cause increased costs to their producers.

A source of such problems is the growth in program complexity. The more a program controls, the more types of interactions it supports. For example, the telephone "call-forwarding" service (forwarding incoming calls to a customer-designated number) interacts with the "billing" program that must determine whether the forwarding number or the calling number gets charged for the additional connection to the customer-designated number. At the same time, call-forwarding interacts with the "connection" program that deals with the issue of what to do in case the called number is busy, but the ultimate forward destination is free. One property to check is that a call is billed to the customer if and only if the connection is completed. If the call connection and billing programs interact incorrectly, a called number that was busy and then became free could appear busy to one program and free to the other, resulting in an unbilled service or an unwarranted charge, depending upon their order of execution.

If a program includes \( n \) interrelated control functions with more than one state, the resulting program may need to support \( 2^n \) distinct combinations of interactions, any of which may harbor a potential unexpected peculiarity. When \( n \) is very small, the developer can visualize all the combinations and deal with them individually. Since the size of a program tends to be proportional to the number of functions it includes (one program block per function), the number of program interactions as a function of program size may grow exponentially. As a result, the developer can use only a very small proportion of the possible program interactions to guide the development and testing of the program. When some unconsidered combination produces an eccentric behavior, the result may be a "bug".

While a computer could be programmed to check a program under development for eccentric behavior by searching exhaustively through all combinations of program interactions, the exponential growth could soon cause the computer to exceed available time or memory. On account of the potential for interactions, adding a few functions to a program can substantially alter its testability and thus viability. From the program developer's perspective, this is unsatisfactory. If the program is developed carefully, however, the correct behavior of each of the \( n \) individual control functions of the program can be checked in a way...
such that the complexity of the checks grows more or less proportionally to the size of the program.

Overview
This article presents some key ideas of “program verification”. It focuses on the “reduction and decomposition” process, which addresses the problem of how to verify a program automatically in a way that the computational complexity scales tractably with increasing program size. It does not attempt to survey the field. A survey of automation-based verification is included in [9]; for a survey of logic-based verification, see [4], [7].

Reduction and Decomposition
Reduction and decomposition circumvent the exponential cost of checking correctness. Checking each control function separately, reduction refers to an algorithm 1 through which the program to be checked is replaced, relative to the respective control function, with a simpler program that it is sufficient to check. For example, to check the operation of call-forwarding, the part of the program that performs billing may be largely ignored, except to the extent that it may interfere with call-forwarding. The reduced program is checked through an algorithm that analyzes every possible program state. In order for a computer to implement this effectively, the reduced program must be sufficiently simple.

Decomposition refers to checking that a given control function is correctly implemented by splitting the function into several simpler-to-check “subfunctions”. Taken together, the subfunctions implement the original function. A subfunction is simpler to check if it gives rise to a simpler reduction than the original function. Decomposition and reduction thus are used together as divide-and-conquer techniques. As an example, call-forwarding may be decomposed into its “setup”, where the customer designates the new destination for incoming calls, and “execution”, where an incoming call gets forwarded to the designated destination. In verifying setup, the reduced program can, in addition, ignore most of the parts of the original program that perform the execution subfunction. Likewise in the verification of the execution subfunction, most parts of the program that deal with setup can be ignored. A computer program can check that these subfunctions collectively implement the original function. In general, it is a manual step to decide how to decompose a function.

Conventional Testing vs. Program Verification
Suppose one wanted to check the C program in Figure 1, whose stated purpose is to read a non-negative binary integer from standard input and convert it to a decimal integer printed to standard output. For example, entering “11001” followed by a carriage return should produce “25”.

```c
#include <stdio.h>
main()
{
    unsigned x=0;
    int input;
    while((input=getchar())!=EOF) {
        if(input=='0')x=x*2;
        else if (input=='1')x=x*2+1;
        else if (input=='\n') {
            printf(“%u
”, x);
            x=0;
        }
    }
}
```

Figure 1. A program for converting binary to decimal.

Program verification could demonstrate that this program fails to fulfill its stated purpose. Readers who are C programmers are invited to stop reading this article now and find the bug in this trivial program.

In fact, the bug is nowhere to be found in the text of the program. Computers commonly truncate “integer” variables to 32 bits (in binary representation), thus performing integer arithmetic modulo 2^32. In the application of program verification, this truncation would be inferred from knowledge of the type of computer on which the program is run. The catastrophic failure of the European Space Agency’s Ariane 5 Flight 501 has been traced to a numerical overflow like the one in this example.

To check the program completely for the stated purpose, one would need to confirm that it is fulfilled at every program state. If the program is tested for billions of “typical” inputs, all of which happen to have fewer than 32 bits, the bug would be missed.

The program above is utterly trivial, giving no inkling of the complexities to be found in routine programs, which could be hundreds of pages long. Nonetheless, if one tried to test this program for the stated purpose by executing the program from the keyboard, entering the binary integers successively, it would take about one hundred years to hit the bug. This time could be reduced to a couple of hours by writing an automated tester. For a program not so trivial, even an automated tester would be unable in any lifetime to execute the program under test to reach all its states. The automated tester would need to make “educated guesses” at input sequences likely to uncover errors. Since programming errors tend to be unintentional, they can hide in hard-to-guess corners of a program.

---

1 An algorithm is a precisely specified succession of steps to obtain a desired result, such as Euclid’s gcd algorithm. In this article the term refers to a procedure that generally is an automatic step in the verification process (a step implemented by a computer program).
Fulfilling Condition
Discharging Condition

Figure 2. In the commercial model-checking tool FormalCheck [11], required program operation is specified in terms of an enabling condition that determines the onset of a requirement specified by a fulfilling condition that must hold unless released by a discharging condition, ad infinitum. Each such specification is represented internally by a \( \omega \)-automaton. Any property expressible with an \( \omega \)-automaton can be expressed by a collection (logical conjunction) of such specifications.

Tailoring a tester to the program under test is a nontrivial programming burden that adds significantly to the cost of program development and delays the onset of testing. This task can be simplified by generating inputs randomly. While easier (and faster) to set up, random testing often is nearly worthless. For the example above, if input strings of 0's, 1's, and carriage returns were randomly generated with respective probabilities .25, .25, and .5 (biased in favor of "typical" shorter input strings), the expected time to hit the bug would be more than the time required to execute the program for each successive input string with value increasing from 0 to \( 2^{32} \). For less trivial programs the probability of hitting a bug in a reasonable amount of time with randomly generated inputs may be close to 0. For each less trivial program to be tested, the "problematic" regions of its state space must somehow be predicted, and a "test bench" must be built to guide the tests to these regions and capture errors.

By contrast, a program verification algorithm can find this bug in a few seconds on a modern computer. No problematic regions need be predicted, and no test bench need be built. Instead, it is necessary only to give a precise definition of what is to be checked. For the example program, this can also be done in a few seconds using a user-interface aid like the one pictured in Figure 2 to specify the property that the decimal output always is equal to the binary input. To express this property, a user of the Figure 2 aid first sets the Type to Always, causing the qualifier of the Fulfilling Condition to change from Eventually (as shown) to Always, and then the user enters the Fulfilling Condition that expresses in program variables the condition decimal (inputbits) = output (leaving the other two fields blank). The user interface aid will automatically generate an "automaton" that defines this property. This automaton is described below. (Hardware verification tools such as [11] operate on hardware description languages instead of C code, but the principle is the same.)

If the purpose of the example program is modified, requiring that it work only for input strings of length 32 or less, then the program can be verified to be correct through an algorithmic check, described below, that takes a few seconds as well.

Conventional testing with either selectively generated input scenarios or random inputs can determine what happens only in selected cases. Especially in case of the presence of unknown bugs, a program's state space may not conform to expectations. A definitive test would need to include every possible input sequence. Since general input sequences are unbounded, there are an infinite number of sequences that should be tested. Executing the program for an infinite number of input sequences is not possible, so testing such a program by executing it is intrinsically incomplete.

Formal Verification
Program verification is also called "formal verification" to contrast it with conventional testing,
which lacks a precise description of what was tested. Formal verification begins with a formal statement of some high-level purpose of the program and determines whether that purpose is supported in every possible execution of the program. This is accomplished by analyzing the logic of the program, not by executing it. Since formal verification can account for every possible program execution, it is more reliable than conventional testing. There are numerous accounts of bugs that could have caused significant commercial damage had they remained but that were found quickly with formal verification after having been missed by conventional testing.

As a program’s purpose evolves, the influence of one part on another grows, so correcting defects becomes increasingly costly. A common observation is that the cost of fixing a bug doubles with each successive stage of development. Finding and correcting program errors early in the design cycle thus can decrease development time and cost significantly. Conventional testing requires the creation of program-specific test benches, so it tends to be deferred until late in the program development cycle, when the program development has stabilized. By contrast, formal verification algorithms are independent of the program to be verified. Therefore, formal verification can be applied early in the program development cycle.

Within the last ten years research in formal verification has found its way to commercial products that implement program verification algorithms. For the present these products are specialized mainly for industries that manufacture integrated circuits. However, the program verification methodologies described here can be applied to significant portions of general software development, and this is the direction of the industry.

Verification Methodologies

Formal verification refers not just to a single methodology but to a spectrum of methodologies. In this spectrum there is a tradeoff between expressiveness—what program properties can be described and analyzed—and degree of automation—the extent to which the verification can be automated with an efficient algorithm.

The most expressive form of formal verification could be said to be mathematical argument. However, the reliability of mathematical argument is based upon peer review [5]. It is unlikely that too many pairs could be muster to check a 100-page proof of the correctness of an integrated circuit. Moreover, it is unlikely that the circuit manufacturer would be willing to wait for the years it might take to produce such a proof in the first place.

This latter problem is shared by automated theorem-proving [2]. The dream of writing a computer program capable of automatically proving marvelous theorems never was completely realized. Instead, the “automatic theorem provers” are in fact proof checkers. While often quite sophisticated, powerful, and useful, automatic theorem provers mainly require the user to come up with the proof, which must be entered into the “theorem-proving” program using the special notation accepted by the program. Theorem provers thus do not at present support a methodology that can keep up broadly with a commercial development cycle.

This disappointment led researchers to investigate the other end of the spectrum: methodologies that sacrifice expressiveness in favor of automation. Among the first to discuss this strategy were E. M. Clarke and E. A. Emerson, who, in 1980, proposed a limited but completely algorithmic form of verification they called “model checking” [3]. Their mode of verification was founded in a logic that supported a very simple model-satisfaction check. Around the same time and independently, J.-P. Queille and J. Sifakis made a similar proposal.

In 1982, ignorant of the above work, I proposed a verification method based on automata. Eventually (ten years later!) the method was implemented in a commercial tool (Figure 2) that is marketed to manufacturers of integrated circuits. Since for the moment at least this is the dominant commercial model-checking tool, I will exercise a certain prerogative and restrict my discussion of model-checking to automata. There is no loss of generality in using automata, as logic-based model checking can be reduced to automaton-based model checking (and vice versa).

Models

There is a substantial literature reporting the impressive benefits of formally verifying systems for everything from communication to safety-critical devices. It may come as a surprise then (cf. [1]) that there is no mathematical sense in which these can be verified. The thinking is that the mathematical model of a computer system can be the exact and entire computer program itself. That is, the computer’s programming language may be given a formal semantics,2 and one may then reason formally about the actual program. So it may seem that if the formal model is the entire system program, then any formal proof about the behavior of the model must be a proof about the actual system, since they are one and the same.

2A computer programming language has the formal attributes of “syntax” and “semantics”. Syntax is a set of rules, such as those checked by a program compiler, governing when a string of characters defines a “well-formed” expression. Semantics is a map of syntax to a well-understood mathematical model, such as an automaton, which gives meaning to a computer program. The semantics masks both “uninteresting” variations in syntax and “irrelevant” details of the computer program execution, such as transient states ignored by the computer operating system. Program semantics gives a basis to determine whether two computer programs are “essentially” equivalent.
The fallacy is that a computer program by itself embodies no physical behavior. Physical behavior is manifest when the program is entered into a computer and executed. The observable physical behavior thus is not of the computer program but of the computer. The computer's behavior is related to its hardware, to whether the computer is plugged in and operated properly, and to the nature of other programs, such as the compiler. Conceptually (but definitely not realistically), mathematical models of all these could be taken into account too. Physical models of even wires and transistors are imperfect predictors, however, depending on uncertainties like model inaccuracies, manufacture process, and defects. The important point here is that there is no absolute value in formally verifying a program, such as a guarantee that its execution on a physical computer will behave correctly.

Moreover, an insurmountable deficiency of formal verification and conventional testing alike is the inability to know that the operational definition of "behave correctly" is "correct". My computer has two implementations of the Unix "word count" utility wc(1). They disagree on whether a final string terminated without white space or a newline counts as a word. In the fall of 1999 another incompatibility in software minutiae led to the destruction of NASA's Mars Climate Orbiter. Even had the derelict NASA software been formally verified module by module, the mismatch in measurement units between modules could have gone unnoticed unless the entire system had been verified as a single entity. To do this may have been impossible within the time available. If the navigation table, erroneously expressed in English units rather than metric units, was preexisting or otherwise deemed "external" to the verification process—and one must draw the boundary somewhere—then the mismatch in any case could have been overlooked.

While all this uncertainty may seem to cast a cloud over the entire enterprise of program verification, the uncertainty should be taken in perspective. There can be no absolute guarantee that a program will behave correctly, even in principle. As long as a verification process improves the reliability of programs or shortens the program design cycle more than other methods, it is worthwhile. Model checking turns out, above all, to be an uncommonly effective debugging tool.

To apply a program verification algorithm to the C program in Figure 1, the first step is to translate the C program to a simple representation with a clear semantics, like automata described below, on which the verification algorithm has been designed to operate. Commercial model checkers use a general automated translator to do this for "suitable" programs. Not all C programs are amenable to algorithmic verification. The C language supports unbounded constructs like character pointers that can be incremented to point to an unspecified range of memory locations. Each location in effect defines (an unnamed) program variable, and the possible extent of this is determined by the memory configuration of the particular computer on which the program is run.

The synergy between programs that define integrated circuits and program verification derives in part from the fact that programming restrictions sufficient to derive an automaton model from a program—a finite number of states, and a well-defined state-transition structure—are necessary to generate an integrated circuit automatically from a program. Unlike most C programmers, circuit designers thus intrinsically write programs that also can be verified. Nonetheless, many portions of C programs like the example in Figure 1 are finite-state and thus can be translated automatically to an automaton model. An infinite-state C program that implements a "control" function like call-forwarding typically has core components that can be formally abstracted to a finite-state program. It is an open problem of software engineering to identify such translatable portions automatically.

For translatable programs the translation process is fraught with pitfalls. A given input program may compute different results on different computers. For example, incrementing an integer variable on a computer with a 32-bit or a 64-bit architecture is implicitly modulo $2^{32}$ or $2^{64}$ respectively. There are myriad unpublished conventions concerning the transparency to the user of transient program states, and yet there is no universal convention concerning what constitutes a stable program state. All such issues must be covered by the translator.

Program State
In the von Neumann stored program model, "instructions" update "data" stored in "memory registers" in a sequential fashion. "Program variables" designate memory locations and their data contents and also define via "assignment expressions" the rules by which the data stored in the variables get modified.

The program state is the vector of simultaneous values of all program variables. It captures the entire instantaneous condition of the program, independent of its history. A program's behavior is captured by its succession of states. However, the computer or integrated circuit that implements the program makes it effectively impossible (on purpose)—or renders it unnecessary—to discern the precise sequential evolution of certain "transient" states. The program model should reflect this and moreover can exploit it to reduce the computational complexity of verification. For example, for variables $x$, $y$, and $z$, suppose $x$ is assigned to take the value of $y+1$ after $y$ is assigned to take the value of $z+1$. An analysis of the program may reveal that with regard to its implementation, $x$ effectively is or can be treated as a "macro" or synonym for $y+1$ and
likewise \( y \) for \( z + 1 \), simplifying the two successive assignments to a simultaneous assignment of the value of \( z + 1 \) to \( y \) and of \( z + 2 \) to \( x \).

Deciding which assignments are to be modeled as simultaneous must be done with great care in order to preserve the semantics of the implemented program. Commercial model checkers perform this task algorithmically. In the program model a **sequential** variable is one that attains its value after (rather than simultaneously with) the value attained by its assignment expression. Typically, if a variable \( x \) is assigned the value of an expression that depends on \( x \)—for example, if \( x \) is assigned the value of \( x + 1 \)—then \( x \) would be designated a sequential variable. The new value of \( x \) would then be modeled as succeeding the prior value of \( x \).

Limiting the number of sequential variables in a program is very important for the performance of an integrated circuit and for the performance of model checking too. There is a multibillion-dollar industry for "synthesis" tools that help create an integrated circuit layout automatically from its defining program, focused in important part on this partitioning problem.

The nonsequential program variables are called **combinational** variables (to use the hardware term). These hold the values of respective "transient" steps in a computation, such as the value of \( y \) used in \( x \) above. They include the primary inputs, which are variables whose values are assigned according to decisions made outside of the program, e.g., which keyboard key a human pushes. Typically, program outputs also are represented by combinational variables.

In the program model the program state is partitioned into the **sequential state**, defined by the values of the sequential variables, and the **combinational state**, defined by the remaining (combinational) variables. The combinational state vector \( C \) is given as a function \( C = f(S) \) of the sequential state vector \( S \). The sequential state \( S \) is initialized, and each successive value

\[
S' = f(S, C)
\]

is expressed in terms of its current value \( S \) and the combinational state.

**Nondeterminism**

In a model of the program the assignment of primary inputs must be abstracted to account for all possible assignments. Abstraction is used more generally in program verification to simplify parts of a program whose detailed function is irrelevant to the verification objective. Abstracting duration in respective asynchronous parallel processes can result in a simpler model that is useful when the required behavior is independent of the relative execution speeds of the processes.

An effective way to abstract a program is through the use of **nondeterminism** in program variable assignments. If the function \( f(S) \) is allowed to be multivalued, at each sequential state \( S \) a combinational variable may assume several values in its range. This gives rise to a set of program behaviors or "lifetimes". Each nondeterministic assignment splits the execution of the program model into separate respective behaviors, ad infinitum.

Without nondeterminism a program would have only one behavior, and its analysis would be simple. Although it is the nondeterminism that makes program analysis difficult, there is no real alternative with regard to the primary inputs. It also may be simpler and sufficient to model with a nondeterministic assignment certain variables that are assigned within the actual program. Imagine a program subroutine that performs a complex computation producing a nonzero result, writing the result to the program variable \( v \). Suppose the program is required to perform some task unrelated to the computation each time the subroutine completes its calculation, the completion indicated by \( v \neq 0 \). In order to verify the property \( P \) that whenever \( v \neq 0 \) the task is correctly performed, the computation that assigns \( v \) is irrelevant. To verify \( P \), it is sufficient to have the program model assign \( v \) nondeterministically to \( 0 \) or \( 1 \), where \( v = 1 \) is an abstraction that stands for all the nonzero values of the variable \( v \) in the original program. In this abstraction, the verification routine can determine, through a localization reduction described below, that the complex computation is irrelevant and exclude it from the analysis leading to the verification of \( P \). (That computation will be relevant to other properties, with respect to which other portions of the program may be irrelevant.) If \( P \) is verified in the abstract model and the computation that assigns \( v \) in the original program is subsequently altered, it is unnecessary to reverify \( P \).

If \( w \neq 0 \) signals the conclusion of another program, then the order in which \( v \) and \( w \) assume nonzero values indicates the relative speeds of the associated programs. Assigning \( v \) nondeterministically allows it to become nonzero both before and after \( w \) does. A property verified with this abstraction will hold for alternative implementations that vary the relative speeds of the programs.

Nondeterministic assignment may introduce additional behaviors not present in the original program (while retaining all original behaviors). In case several variables are assigned nondeterministically, not all combinations of assignments may be possible in the original program. On the other hand, nondeterministic assignment can reduce program complexity by masking relative execution speeds and program structure (as above). This makes it a vital tool in program verification.

**Automata**

The automaton is a fundamental structure in computer science, useful for much analysis and proof
concerning "regular" sequential behavior. In the translation of a program to an automaton, defined next, the program's sequential state becomes the automaton "state," and the program's combinational states define automaton state transition conditions. For the C program in Figure 1, the states of the corresponding automaton are the $2^{32}$ values of the program's sequential variable $x$. The program's combinational variable input defines the state transition conditions as follows. For each value of $x$ there are transitions to the states

$$2x, 2x + 1, 0$$

and back to $x$, conditioned respectively on input $= 0$, input $= 1$, input $= v$ (carriage return), and every other value of input. Program behaviors are modeled by the sequences of combinational states consistent with successive automaton state transitions (cf. a discrete Markov process). The set of all such sequences forms the language of the automaton, which provides the basis for model checking as defined here. The combinational states of the C program in Figure 1 are its respective input/output pairs (input, output), where output is the value imparted by the print statement (namely, the value of $x$, or "nothing", encoded by some distinct designated value).

To facilitate reduction and decomposition, the modeling automaton is "factored" into component automata, in emulation of the program's modularity. This factoring requires a means to correlate factor automata transition conditions that mediate the respective automaton state transitions. For example, if one program component reads the value of a variable set by another program component, the corresponding automata must have a mechanism to reflect a common view of the value of that variable. For reduction there also must be a means to "abstract" transition conditions. If a state transition is conditioned on a variable $v$ being nonzero and the nonzero values of $v$ are abstracted to the value "1," as above, then the corresponding automaton transition condition must be abstracted in a consistent well-defined manner. To meet these needs, the set of valuations of combinational program variables is given the structure of a Boolean algebra.

In this context the correlation of transition conditions is captured by their conjunction in a consistent well-defined manner. To meet these needs, the set of valuations of combinational program variables is given the structure of a Boolean algebra and abstraction is obtained through a Boolean algebra homomorphism. The principal role of the automaton defined next is to serve as a "scaffolding" to carry this Boolean algebra: the factorization (5) needed for the decomposition (6) appears as a matrix tensor product on the automaton state transition matrices, and the simplification needed for reduction is given as a Boolean algebra homomorphism that acts on these matrices elementwise, in (12) below. The details are now explained.

In the context of program verification the most useful type of automaton is the $\omega$-automaton, whose behaviors are (infinite) sequences. The automaton is defined in terms of a directed graph with a finite set of vertices called states, some of which are designated initial; a transition condition for each directed edge or state transition in the graph; and an acceptance condition defined in terms of sets of states and state transitions. Each transition condition is a nonempty set of elements called letters drawn from a fixed set called the alphabet of the automaton. There are several different equivalent definitions of acceptance conditions in use. The acceptance condition of an $\omega$-automaton can capture the concept of something happening "eventually" or "repeatedly" (ad infinitum), "forever" or "finally" (forever after). The set of all subsets of the alphabet forms a Boolean algebra $L$ in which the singleton sets are atoms. The set of atoms of $L$, denoted by $S(L)$, determines $L$ when $L$ is finite.

How are automaton transition conditions associated with the program's combinational states? If $v$ and $w$ are states of the automaton $A$, let $A(v, w)$ denote the transition condition on the directed edge $(v, w)$. Expressing conjunction (set intersection) in $L$ by $\cdot$, we see for a letter $a$ that $a \in A(v, w)$ if and only if the atom $\{a\}$ satisfies $\{a\} \cdot A(v, w) \neq 0$. In the context of $L$, we refer to the atom $\{a\}$ as a "letter" and $S(L)$ as the "alphabet" of $A$. We say the automaton $A$ is over $L$. $L$ is associated with the Boolean algebra generated by all valuations of combinational program variables: terms of the form $x = c$ for a value $c$ in the range of the combinational variable $x$. Thus, $L$ is all Boolean expressions in these terms. Each letter is a conjunction of terms of the form $x = c$, the conjunction over all the combinational variables $x$. This corresponds to the combinational state $C$ with $x$-th component $c$. Thus, we associate combinational states with letters, i.e., atoms of $L$. Automaton transition conditions are nonzero elements of $L$. For example, a transition condition $x = 0$ corresponds to the set of all letters (i.e., disjunction of atoms) of the form $\cdots \cdot (x = 0) \cdot \cdots$. Referring to (1), we have

$$A(v, w) = \sum_{F(v, C) = w} C,$$

the summation expressing disjunction in $L$. 

\begin{equation}
A(v, w) = \sum_{F(v, C) = w} C,
\end{equation}
A sequence \((v_i)\) of states of \(A\) is a run of a sequence of letters \((s_i) \in S(L)^n\) provided \(v_1\) is an initial state and for all \(i, s_i \cdot A(v_i, v_{i+1}) \neq 0\). A run is accepting if its infinitely repeating states and transitions satisfy the automaton acceptance condition.

The set of sequences of letters with respective accepting runs in an automaton \(A\) is the language of \(A\), denoted \(L(A)\).

The stated purpose of the C program in Figure 1 can be expressed by a 2-state automaton \(P\) that uses its state to remember whether the current output is the decimal equivalent of the last binary input. The state of \(P\) can be represented with the program variable \(y\), having initial state \(y = 0\) and transition condition for the state transition from \(y = 0\) to \(y = 1\) represented by the expression in program variables that expresses the failure

\[
(4) \quad \text{decimal(inputbits)} \neq \text{output}
\]

The acceptance condition accepts runs that remain forever in the state \(y = 0\).

Program Factorization

The automaton model of a program is "built" from smaller automaton models of program components. A program component may comprise a single variable or a group of closely related variables. The program consists of its components. For example, the call-forwarding program may be implemented by respective components that define its setup and execution, as described earlier. A program is translated component by component to respective "component" automaton models. The component automata are combined as a "product" to form a single automaton that models the original program. This "factoring" of the program model into components follows the natural program structure and aids in translation as well as in reduction and decomposition. All component automata are over one fixed Boolean algebra \(L\) determined by the program.

The combinational variables of a program component may be interpreted as the "outputs" of that component. The simultaneous evaluations of the outputs of a program component get translated to elements of \(L\) that generate a subalgebra of \(L\), the "output subalgebra" of the corresponding component automaton. (In this context, program primary inputs translate to outputs of trivial automata.) The call-forwarding setup component \(p\) may have as outputs the variables \(m\) and \(n\) that designate the called and forward numbers respectively. If there are no other outputs of \(p\), then the output subalgebra of the automaton that models \(p\) has as its atoms all expressions of the form \((m = m_0) \cdot (n = n_0)\), for \(m_0, n_0\) in the range of \(m\) and \(n\) respectively. The (interior) product of the respective output subalgebras of all the component automata is \(L\). Each letter (atom of \(L\)) is a conjunction of atoms from the respective output subalgebras.

A program component is modeled by an \(L\)-process \(P\): an \(\omega\)-automaton over the Boolean algebra \(L\) with an identified "output" subalgebra \(L_p \subset L\) that models the combinational states of the program component. The atoms \(L_p\) are the "output values" of \(P\). By (3), the output values \(f(v)\) represented above (cf. 1), possible at the state \(v\) of \(P\), are the elements of \(L_p\) that have nonzero conjunction with the transition condition of some state transition leaving \(v\). (The "inputs" to \(P\) are the simultaneous collective output values of the various \(L\)-processes, i.e., the alphabet \(S(L)\). The transitions of \(P\) may be independent of any particular outputs, of course.)

If \(P\) and \(Q\) are \(L\)-processes modeling respective program components \(p\) and \(q\), the product \(P \circ Q\) is an \(L\)-process that models the program component formed by taking \(p\) and \(q\) together. (This sometimes is called the "synchronous product" of \(p\) and \(q\), subsuming their "asynchronous" or "interleaving" product [9].) The set of states of \(P \circ Q\) is the Cartesian product of their respective state spaces, and the output subalgebra of \((P \circ Q)\) is the interior product of the component output subalgebras.

If \(P(v, w)\) is the transition condition for the transition \((v, w)\) from state \(v\) to state \(w\) of \(P\) and \(Q(v', w')\) is likewise for \(Q\), then the transition condition for the transition \((v', v'), (w', w')\) of the product \(P \circ Q\) is \(P(v, w) \cdot Q(v', w')\). Some program variables that define a component automaton's transition conditions may come from program components other than the one modeled by the automaton. This allows automata to share conditions on the sequences they accept and reflect coordination among the program components. For example, the call-forwarding execution component modeled by \(L\)-process \(Q\) refers to the setup component's output variable \(n\) that designates the ultimate call destination number. In this way \(Q\) coordinates with the setup established by the setup component. The products of their respective transition conditions define the coordinated behavior of the two automata. If \(C, D \in L\), then the product of the setup transition condition \(P(v, w) = (n = n_0) \cdot C\) and \(Q(v', w') = (n = n_1) \cdot D\) is nonzero only if \(n_0 = n_1\). Thus, the transition condition \((P \circ Q)((v, v'), (w, w'))\) of the product automaton is nonzero only when the two numbers agree.

The coordination thus defined by automata transition conditions supports the automaton "factoring" mentioned earlier. For any \(L\)-process \(P\) we define its state transition matrix to be the matrix over \(L\) whose \(j, k\)-th element is the transition condition for the state transition from the \(j\)-th state to the \(k\)-th state. By the above definition of process product, the matrix for the product process \(P \circ Q\) is the tensor product of the matrices for \(P\) and \(Q\).
In general, if \( M \) is the state transition matrix for an \( L \)-process that is factored into component \( L \)-processes with respective matrices \( M_i \), then
\[
M = M_1 \otimes \cdots \otimes M_k.
\]
Moreover, if each process is designated by its respective matrix,
\[
L(M) = L(M_1) \cap \cdots \cap L(M_k).
\]
The intuition for (6) is that each component \( M_i \) imposes certain restrictions on the behaviors of the product \( M \), and the behaviors of \( M \) are the "simultaneous solutions" for behaviors of the respective components. If \( M_1 \) restricts the setup of the call-forwarding number \( n \) to numbers in a specified area code and \( M_2 \) permits execution only if the designated number \( n \) is not busy, then the product pertains to numbers \( n \) in the specified area code that are not busy.

**Model Checking**

We model as respective \( L \)-processes the program, its components, and the property to be verified. The program model is the product of its components (5).

It is convenient to use the same symbol to denote a process and its matrix, and henceforth the term program will be used to designate both the syntactic computer program and the \( L \)-process automaton model into which it gets translated.

Our formal definition of verification of property \( P \) for a program \( M \) is the automaton language containment check
\[
L(M) \subseteq L(P).
\]
In words, this says that all behaviors of the program are behaviors "consistent with" (i.e., "of") the property. This is equivalent to checking that
\[
L(M) \cap L(\bar{P}) = \emptyset
\]
where \( \bar{P} \) is the "complementary" automaton satisfying \( L(\bar{P}) = S(L(P))^c \). By (6) verification is transformed into the automaton language emptiness check
\[
L(M \otimes \bar{P}) = \emptyset.
\]

If \( M \) models the C program in Figure 1 according to (2) and \( P \) is the automaton that expresses its desired behavior, defined by (4), then verification consists of analyzing the composite system \( M \otimes \bar{P} \). Here \( \bar{P} \) is \( P \) with a complementary acceptance condition that accepts runs for which eventually \( y = 1 \). The state of this system is the value of the pair \((x, y)\), where \( x \) is the program variable of Figure 1. The verification algorithm checks whether any state \((x, 1)\) can ever be reached through a succession of state transitions that begins at the initial state \((0, 0)\). The analysis concludes when either a violation of the property is found (i.e., \((x, 1)\) is reached) or all reachable pairs \((x, y)\) have been examined. The latter entails an examination of \(2^{32}\) states, which could be costly were it not for a symbolic technique described below.

The foregoing is an automata-theoretic formulation of model checking. There are other formulations of model checking, expressed in terms of "temporal" logic formula satisfiability [7]. Each of these formulations can be transformed into (7) for some class of automata. In some cases the best way known to perform the check is first to construct an automaton corresponding to the temporal logic formula and then to check (7). See [4], [7], [10] for a way to express the original and widely practiced form of model checking for the logic CTL without reverting to automata. The automata-theoretic formulation of model checking was developed with a somewhat different perspective from the one presented here by M. Y. Vardi and P. Wolper [12] in the early 1980s and by this author independently around the same time.

**Algorithms**

A general method to check whether \( L(A) = \emptyset \) for (7) is based on the finite structure of \( A \). \( L(A) \) is non-empty (and the required property fails) if and only if \( A \) has an accepting run. This is equivalent to the graph of \( A \)'s having a cycle "consistent" with the acceptance structure, since \( A \)'s state space is finite. The path from an initial state to such a cycle may be retraced, and this "error track" provides a step-by-step account of how the property can fail in the program.

One way to check for an accepting run involves an explicit enumeration of the states of \( A \) reachable from an initial state. Since the number of states can be exponential in the size of the program description (5), explicitly enumerating the states has severe computational limitations. Fortunately, explicit enumeration is not necessary.

In 1986 R. Bryant made a seminal observation that played a major role in spurring the commercialization of model-checking techniques—a role that resulted in the ACM Kanellakis Prize for Bryant and his colleagues Clarke, Emerson, and McMillan, who showed how to use this beneficially for model checking.

Bryant's observation was that binary decision graphs, long used for planning and programming, could be viewed as automata and thus minimized in a canonical fashion. The minimized structures were dubbed "binary decision diagrams" or BDDs. The idea was to represent the global state as a binary vector and represent a set of states by its characteristic function (having value 1 for each state in the set), encoded symbolically as a Boolean function. The set of reachable states could be represented as a fixed point of the state set transformer that starts with the set of initial states and repeatedly applies the state transition matrix as an operator on the state space, adding the new states
reached in one transition, from the set of states reached thus far. Since the set of states is finite, iterating this transformation has a least fixed point—the set of reachable states—in the lattice of state sets. For the Figure 1 program, with respect to checking (7), \( A = M \otimes \bar{P} \) is expressed as a Boolean function of \( x, y \) (the state variable of \( P \)), \( \text{input} \), and added variables \( x' \) and \( y' \) that encode the “next state” of \( A \), as per (1). In terms of a Boolean function,

\[
M(x, \text{input}, x') = (x' = x \times 2) \cdot (\text{input} = 0) + (x' = x \times 2 + 1) \cdot (\text{input} = 1) + \cdots
\]

and similarly for \( \bar{P} \), where + and \( \cdot \) denote the respective arithmetic operations and also the Boolean operations “disjunction” and “conjunction” in the Boolean algebra \( L \). As Boolean functions, \( A = M \otimes \bar{P} \).

If the system starts from the set of all initial states defined by the Boolean function \( I \)—in this case, the single state that is the solution to \( I(x, y) = (x = 0) \cdot (y = 0) = 1 \) (1 being the universal element of \( L \)), the states reachable through a single state transition from an initial state are the solutions \( I(x', y') \) to \( I(x, y) \cdot A(x, y, \text{input}, x', y') = 1 \). With \( I \) in place of \( I \), the set of states reachable through two transitions is determined. Since the set of all states is finite, this process may be iterated until no new solutions are found. The cumulative set of solutions defines the set of reachable states. Each iteration is expressed symbolically in terms of the Boolean characteristic functions, reduced to their canonical form and represented as a BDD. A very simple Boolean function can express an arbitrarily large set of states. For example, \( x_1 = 0 \) defines the set of all global states whose first bit is 0, representing half of the global states, no matter how large the state space. Thus, there is the potential to manipulate very large sets of states. In practice, the ability to compute reachable state sets with \( 10^{10} \) states for typical commercial programs is considered fairly trivial, \( 10^{20} \) states is routine, and \( 10^{50} \) states and higher is not unusual. For the example of Figure 1, the fixed point of \( 2^{32} \) states is reached very quickly, in 32 iterations. Each iteration the set of states reached is represented by a computationally simple Boolean function. While the worst-case complexity of performing state reachability symbolically is the same as for explicit enumeration of the states, symbolic search lowered the threshold of acceptability for model checking, leading to its commercialization.

**Homomorphic Reduction**

A program modeled by an \( L \)-process \( M \) can be “abstracted” by a “simpler” program (with fewer variables or variables with smaller ranges) that is modeled by an \( L' \)-process \( M' \). The relationship between \( M \) and \( M' \) is given by a Boolean algebra homomorphism \( \phi : L' \rightarrow L \). If \( \phi M' \) is the \( L \)-process with transition matrix obtained by applying \( \phi \) elementwise to the transition matrix of \( M' \) and \( L(M) \subset L(\phi M') \), then we say \( M' \) is a reduction (or “abstraction”) of \( M \) and \( M \) is a refinement of \( M' \).

If this is the case and, moreover,

(8) \[ L(M') \subset L(P') \]

for a property defined by the \( L' \)-process \( P' \), then applying \( \phi \) to both sides gives \( L(\phi M') \subset L(\phi P') \). So for \( P = \phi P' \) it follows that

(9) \[ L(M) \subset L(P), \]

which means that verifying the reduction verifies the refinement.

One type of reduction, localization reduction, described below, is derived by an algorithm. So \( M' \) and \( \phi \) are defined algorithmically, and \( L(M) \subset L(\phi M') \) is guaranteed by construction. Alternatively, a reduction can be “guessed” and then verified as above, where the “guess” consists of a definition of \( M' \) and \( \phi \). As an example of the latter, imagine a component adder used in the context of a program for an integrated circuit that computes the norm of an incoming signal. The correctness of the adder can be established by considering it in isolation from the rest of the circuit. \( M' \) in this case could be just the model of the adder, with \( \phi \) mapping the adder to the full circuit with all nonadder variables assigned nondeterministically.

In this example a failure of the isolated adder does not necessarily imply it would fail in the context of the full circuit. If the otherwise-correct adder had errors in case of negative inputs, but negative inputs were impossible in the context of the full circuit, then the failure for negative inputs would not matter, and this “error” in fact may reflect an intentional optimization. In this case one could constrain the inputs to the adder to be positive and prove it correct in that context. This approach would create a “proof obligation” with respect to the remainder of the circuit: to verify that the full circuit does in fact produce only positive inputs to the adder. In discharging such proof obligations, one must beware of circular reasoning: “the adder is good if the rest of the circuit offers it only positive inputs; the rest of the circuit offers only positive inputs if the adder is good” (perhaps a faulty adder gives feedback to the rest of the circuit that can cause the rest of the circuit to offer negative inputs).

In the earlier example of a “complex computation” all nonzero values of the program variable \( v \) were abstracted by the value “1”, giving another example of a homomorphic reduction. The homomorphism maps \( v = 1 \) in the abstract Boolean algebra to the disjunction of all nonzero assignments to \( v \) in the refined Boolean algebra.

In order to verify (9), the reduction (8) may be simplified by decomposing the property \( P \) into small “subproperties” \( P_1, \ldots, P_k \), where

(10) \[ L(P_1 \otimes \cdots \otimes P_k) \subset L(P) \]
Design by refinement is an application to verification of a general “top-down” program design methodology that has been around for many years [6], [13]. By evolving a design together with its verification, a development methodology that leads to a tractable verification, as above, may be built into the design process. Program details are added incrementally through a succession of program refinements:

\[(11) \quad \mathcal{L}(M_{i+1}) \subset \mathcal{L}(\phi_i M_i),\]

Here the increasing “level” \(i\) indexes progressively more refined program models, leading ultimately to the program with all its details elaborated. Since \(M_i\) is a reduction of \(M_{i+1}\), any property verified for \(M_i\) holds for \(M_{i+1}\), and on. Properties of the program thus may be verified once and for all at the most abstract level possible, and each level may be verified before the next level is defined.

Based on the composition (5) of each level \(M_i = M_1 \otimes \cdots \otimes M_{i-1}\), the check (11) can be decomposed into a set of smaller sufficient checks: for each \(j\),

\[(12) \quad \mathcal{L}(M_{i+1}) \subset \mathcal{L}(\phi_i M_i),\]

where the factors \(M_{i+1}\) in the successive checks (12) are factors of \(M_{i+1}\).

**Localization Reduction**

The variable dependency graph of a program is the directed graph whose vertices are the program’s variables, with an edge \((v, w)\) whenever \(v\) appears in the program’s assignment expression for \(w\). An automatic way to derive a homomorphic reduction involves a traversal of the variable dependency graph to determine which values of which variables are equivalent, with respect to the property being checked. A variable \(v\) is irrelevant if it has no directed path to the variables that implement the automaton \(P\) that defines the property being checked. In this case, if we transform the program’s acceptance conditions to \(P\), the particular values assigned to \(v\) can have no bearing on the check (7). Two values of a variable are equivalent if the assignment expressions of the relevant variables do not distinguish them. A homomorphism may associate together all equivalent values. Localization reduction is an iterative algorithm that starts with a small “active” program component that is topologically close in the variable dependency graph to \(P\) (Figure 3). All other program variables are abstracted with nondeterministic assignments. This renders the values of the variables beyond the boundary of the active variables equivalent, so operationally these variables may be “pruned” out of the model. If the property is thus verified, then by (6) it holds for the full program. On the other hand, if the property fails for this reduction, the algorithm attempts to expand the resulting error track to an error track for the full program. If this

Figure 3. Localization Reduction. In the variable dependency graph, a program component close to the automaton \(P\) that defines an aspect of the program's required operation is designated active. The remaining variables are pruned, removing their effect. If the required operation is verified for this reduction, it holds for the full program. If it neither verifies nor falsifies, the active component is augmented, and the check is repeated.

The required check (9) is replaced by

\[\forall i, \quad \mathcal{L}(M_i) \subset \mathcal{L}(P_i).\]

By (6), this implies (9). Although one check is replaced with \(k\), (9) may entail \(O(|M|)\) computations for a model \(M\) with \(|M|\) sequential states, whereas each of the \(k\) component checks can engender a reduction (8), with \(P_i\) in place of \(P\), that entails only \(O(|M|^{1/2})\) computations. The check (10) can be recursively decomposed into a tree of such checks. At each node of the tree the respective \(k\) is small and the \(P\) is checked against the product of the \(P_i\)'s at the successive nodes [9]. If the total number of nodes is \(O(\log |M|)\), i.e., is proportional to the number of program components, then the complexity of program verification is proportional to the size of the program. In some cases, when \(P\) refers to a “repetitive” structure in \(M\) like an array, \(P\) can be decomposed algorithmically by restricting it to each successive array element. In general, a decomposition is obtained by a manual “guessing” step, and the guess is verified algorithmically as above.

Reduction and decomposition apply more generally to infinite-state program models. The principles behind them are central to most program verification methodologies that scale tractably with increasing program size.
succeeds, it means the error in fact reveals a “bug” in the full program. If, however, the expansion fails, it means the error is an artifact of the localization. In this case the active component is augmented by a heuristically small set of topologically relevant program variables whose assignments are inconsistent with (and thus invalidate) the error track. The verification check is repeated for the expanded program component, and the process is iterated until the algorithm terminates with a verification or a genuine program error.

**Conclusion**

Program verification can increase the reliability of a program by checking it in ways that may be overlooked in conventional testing. Conventional testing intrinsically comes at the end of the design cycle, whereas program verification may be introduced at the beginning, eliminating bugs earlier than otherwise possible. This can accelerate program development.

A principal type of program verification is model checking, which may be expressed in terms of automata. Although the worst-case time complexity for model checking is for all practical purposes exponential in the size of the program to be verified, model checking often can be accomplished in time proportional to the size of the program, as follows. Define the properties to be checked, decompose each property into subproperties that admit of respective tractable reductions, verify each subproperty on its respective reduction. This approach is "bottom-up", since it begins with the full program.

The alternative “top-down” approach starts with the same property decomposition, but before the program exists. Rank the subproperties according to level of abstraction. For each level define an “abstraction” of the program-to-be. This abstraction corresponds to a reduction in the bottom-up approach. Program details are added incrementally, ending with the fully elaborated program. Each increment is verified to be a refinement of the previous level. A property verified at one level thus remains true at all subsequent levels. Since the program is written and checked incrementally, debugging starts earlier in the program development cycle than with conventional testing, which requires the fully elaborated program. With the top-down approach, the program can be designed so that each required check is tractable.

Reductions can be derived algorithmically in the bottom-up approach. In either approach a prospective reduction may be verified to be an actual reduction by checking the refined program against a homomorphic image of the reduced program.

For both the bottom-up and top-down approaches, property decomposition is a fundamental step. In special cases, when a property refers to a repetitive structure, its decomposition can be derived algorithmically. In general, decomposition can be verified—but not derived—by a tractable algorithm. In fact, it is not tractable to determine whether a “good” decomposition—one that gives rise to tractable bottom-up verification—exists. Finding useful heuristics for decomposition is a foremost open problem in model checking.

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**References**


Constituting a distance that is invariant under a given class of mappings is one of the fundamental tools for the geometric approach in mathematics. The idea goes back to Klein and even to Riemann. In this article we will consider distances invariant under biholomorphic mappings of complex manifolds. There will be many such distances. A number of these will come from functions on the tangent spaces, in the way that a Riemannian metric on a manifold yields a distance on the manifold.

Following Riemann’s lead, we think of a suitable function on the tangent spaces as a way to measure “lengths” of tangent vectors, and we can use it to define lengths of curves and ultimately a distance on the manifold. We do not insist that this function be related to an inner product. If the complex manifold is $M$ and its tangent spaces are denoted $T_p(M)$, then we shall work with any nonnegative function $f(p, v)$, for $v \in T_p(M)$, such that $f(p, v)$ suitably respects scalar multiplication (real or complex as appropriate) in $v$. In most cases the function will be continuous in $(p, v)$, but sometimes we allow the continuity to be slightly relaxed. Motivated by the Riemannian case, we shall refer to this function as a metric if $f(p, v)$ vanishes only for $v = 0$, or a pseudometric in general. We do not assume that $f(p, v)$ satisfies the triangle inequality in $v$ if $p$ is fixed; if the triangle inequality is in fact satisfied, the function will be called a norm or pseudonorm.

If the “length” function comes from an inner product, then the metric is said to be Riemannian as usual. If that inner product is the real part of a Hermitian inner product, then we call the metric Hermitian. If the Hermitian inner product behaves almost like the Euclidean metric (in the sense that they agree to order two), then we call the metric Kählerian. If, on the other hand, there is no inner product present at all and only the notion of vector length is defined, then we call the metric Finslerian.

The way that the word “Hermitian” is used here may be unfamiliar to some readers, and we offer a note of clarification by considering the simple example of $\mathbb{C}^n$. If we think of $\mathbb{C}^n$ as a complex space, then it is natural to use the inner product, for $z = (z_1, \ldots, z_n)$ and $w = (w_1, \ldots, w_n)$ in $\mathbb{C}^n$, given by

$$\langle z, w \rangle = \sum_{j=1}^{n} z_j \overline{w}_j.$$

This is the inner product that we call the (standard) Hermitian inner product on $\mathbb{C}^n$. Sometimes, however, we wish to think of $\mathbb{C}^n$ as just $\mathbb{R}^{2n}$. So we write $z = (z_1, \ldots, z_n) \approx (x_1, y_1, \ldots, x_n, y_n)$ and $w = (w_1, \ldots, w_n) \approx (u_1, v_1, \ldots, u_n, v_n)$, where we have made the natural identifications $z_j = x_j + iy_j \approx (x_j, y_j)$ and $w_j = u_j + iv_j \approx (u_j, v_j)$. Then a natural inner product to use is
\[ z \cdot w = \sum_{j=1}^{n} x_j u_j + y_j v_j. \]

Notice that this real inner product is simply the real part of the Hermitian inner product introduced a moment ago.

All manifolds in this article will be complex. For a complex manifold, the real tangent space at a point may be regarded canonically as a complex vector space of half the dimension. We will regard it that way throughout. The differential of a holomorphic mapping is then complex linear.

We intend in this article to demonstrate that different distances are suitable for different applications. In five different contexts, we shall try to indicate what some of these may be.

The first result in this general line of inquiry is due to Poincaré. On the unit disc \( \mathcal{D} := \{z \in \mathbb{C} : |z| < 1\} \) in the complex plane, he constructed the distance

\[
\rho(z, w) := \frac{1}{2} \ln \frac{1 + \frac{z-w}{1-z\bar{w}}}{1 - \frac{z-w}{1-z\bar{w}}},
\]

which is invariant under conformal transformations of \( \mathcal{D} \). Poincaré used this distance in his model for the Lobachevskii geometry in \( \mathcal{D} \).

Consider the Hermitian metric on \( \mathcal{D} \) given in traditional notation by

\[
\mathcal{P}(z) := \frac{dz \, d\bar{z}}{(1 - |z|^2)^2}.
\]

Then, for a vector \( v \) tangent to \( \mathcal{D} \) at a point \( z \), one can calculate the length of \( v \) as

\[
\rho(z, v) := \frac{|v|}{1 - |z|^2}
\]

(here \( | \cdot | \) is just Euclidean length). It turns out that \( \rho \) is an infinitesimal form of the distance \( \rho \). This means that, for given points \( p, q \in \mathcal{D} \), the distance \( \rho(p, q) \) can be obtained from \( \rho \) as follows: for any smooth curve \( y \) (parametrized by the interval \([0, 1]\)) that joins \( p \) and \( q \), we define its length \( |y|_\rho \) as

\[
|y|_\rho := \int_0^1 \rho(y(t), y'(t)) \, dt,
\]

then we take the infimum of the lengths of all such curves to determine the distance from \( p \) to \( q \). Thus the distance \( \rho \) on \( \mathcal{D} \) can be recovered by measuring the lengths of tangent vectors in the metric \( \rho \). This relation will be of importance for us below when we look at various generalizations of \( \rho \) and \( \rho \).

Buried inside the Poincaré distance is another distance. Define

\[
\hat{\rho}(z, w) = \frac{|z-w|}{1 - z\bar{w}}.
\]

Then it can be verified that \( \hat{\rho} \) is a distance function. The classical name for \( \hat{\rho} \) is the pseudohyperbolic distance (see [Gar])—not to be confused with the "pseudodistances" that will be discussed in the sequel. If \( \phi \) is any holomorphic function from the unit disc to itself, then

\[
\hat{\rho}(\phi(z), \phi(w)) \leq \hat{\rho}(z, w).
\]

This inequality comes from the Schwarz-Pick lemma, which in turn is just the conformally invariant version of the classical Schwarz lemma.

So holomorphic functions are distance nonincreasing in the metric \( \hat{\rho} \). If \( \phi \) happens to be a holomorphic automorphism of the disc (i.e., \( \phi : \mathcal{D} \to \mathcal{D} \) is one-to-one and onto, as well as holomorphic), then inequality (3) applies both to \( \phi \) and to \( \phi^{-1} \), so that we obtain

\[
\hat{\rho}(\phi(z), \phi(w)) = \hat{\rho}(z, w).
\]

If the pseudohyperbolic distance has the nice invariance properties that we want, then why bother with the Poincaré metric, which seems to involve an additional level of (computational) complexity? Let us try to answer this question as follows. The group of holomorphic automorphisms of the unit disc consists of certain Möbius transformations. These act transitively on the disc: if \( z, w \in \mathcal{D} \), then there is an automorphism \( \phi \) such that \( \phi(z) = w \). In fact, more is true: if \( \xi, \zeta \) are tangent directions at \( z, w \) respectively, then there is an automorphism \( \psi \) such that \( \psi(z) = w \) and the tangent direction \( \xi \) is taken to a scalar multiple of the tangent direction \( \zeta \). From this observation it follows that there is, up to multiplication by a scalar, just one Riemannian metric on the disc that is invariant under the automorphism group. And of course that metric is the Poincaré metric.

So how does the pseudohyperbolic distance fit in? It is plainly not a constant multiple of the Poincaré distance. The answer is that \( \hat{\rho} \) does not come from a Riemannian metric: it is in fact impossible to find a way to measure the lengths of vectors tangent to \( \mathcal{D} \) to obtain \( \hat{\rho} \) in the same way as \( \rho \) was obtained from \( \rho \). Nevertheless, the pseudohyperbolic distance meshes naturally with the Schwarz-Pick lemma and is therefore a cornerstone of classical geometric function theory (see [Gar]).

In what follows we will give five separate examples of generalizations of the Poincaré distance and metric and provide one application of each. Among a large number of potential illustrative examples we concentrate on just a few results, mostly those relevant to the study of the group \( \text{Aut}(M) \) of biholomorphic automorphisms of a given

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1 Specialists are fond of the pseudohyperbolic distance also because it meshes well with Nevanlinna-Pick interpolation, which is the backbone of the celebrated corona problem.
complex manifold $M$ (all manifolds throughout this paper are assumed to be connected). Here
\[ \text{Aut}(M) = \left\{ \phi : M \to M \bigg| \phi \text{ is holomorphic, one-to-one and onto} \right\}. \]

Observe that Aut($M$), equipped with the binary operation of composition of mappings, is a group (the inverse of a holomorphic, one-to-one mapping is automatically holomorphic). We topologize Aut($M$) with the compact-open topology, which turns Aut($M$) into a topological group. For some classes of manifolds the group Aut($M$) can be given the additional structure of a Lie group; one such class can be defined via an invariant distance (see Theorem 2 below).

**The Carathéodory Pseudodistance**

The "pseudodistance" that now bears his name was introduced by Carathéodory in 1926. Let $M$ be a complex manifold and $p, q \in M$. Set
\[ C_M(p, q) := \sup_{\omega \in \Omega} \rho(f(p), f(q)), \]
where the supremum is taken over all holomorphic mappings from $M$ into the unit disc $\mathbb{D}$ and $\rho$ is the Poincaré distance (1). In effect, $C_M$ is the pullback to $M$ of the distance $\rho$ on $\mathbb{D}$ (Figure 1). It turns out that $C_M$ is a pseudodistance; i.e., it satisfies

(i) $C_M(p, q) \geq 0$,
(ii) $C_M(p, p) = 0$,
(iii) $C_M(p, q) = C_M(q, p)$,
(iv) $C_M(p, q) \leq C_M(p, r) + C_M(r, q),$

for all $p, q, r \in M$. It is in general not a distance; that is to say, it may happen that, for $p \neq q$, one has $C_M(p, q) = 0$. For example, $C_M = 0$. On the other hand, it follows from the Schwarz-Pick lemma that $C_M = \rho$. It can be shown that the Carathéodory pseudodistance $C_M$ is invariant under biholomorphic mappings and, moreover, is nonincreasing under holomorphic mappings. Specifically, if $f : M_1 \to M_2$ is a holomorphic mapping between complex manifolds $M_1$ and $M_2$, then
\[ C_M(f(p), f(q)) \leq C_M(p, q), \]
for all $p, q \in M_1$. It turns out that $C_M$ is the least pseudodistance among all pseudodistances on $M$ that are nonincreasing under holomorphic mappings from $M$ to $\mathbb{D}$, where distances on $\mathbb{D}$ are measured in the Poincaré distance.

A pseudodistance $d$ is said to be inner if $d(p, q) = d'(p, q)$, which is the infimum of the lengths of all curves connecting $p$ to $q$. Here the length of a curve $\gamma : [0, 1] \to M$ is understood as the supremum of $\sum_{k=1}^{K} d(\gamma(\tau_{k-1}), \gamma(\tau_k))$ over all partitions $0 = \tau_0 < \tau_1 < \ldots < \tau_K = 1$ of the interval $[0, 1]$.

The Carathéodory pseudodistance $C_M$ is not in general inner. Although $C_M$ is continuous as a mapping from $M \times M$ into $\mathbb{R}$, it does not in general induce the topology of $M$, even if it is in fact a distance.

For a point $p \in M$ and a tangent vector $\nu \in T_p(M)$ to $M$ at $p$, one can also define the Finslerian pseudometric
\[ c_M(p, \nu) := \sup_{\omega \in \Omega} \rho(p, df(p)\nu), \]
where the supremum is taken over the same set of mappings $f$ as in (4) and $p$ is defined in (2). The function $c_M$ is a pseudonorm on the tangent bundle $T(M)$ to $M$ and is not in general given by a Hermitian pseudometric. It is an infinitesimal form of the inner pseudodistance $C_M$ induced by $C_M$ and is continuous on $T(M)$. The pseudometric $c_M$ is nonincreasing under holomorphic mappings. This last fact generalizes the usual Schwarz-Pick lemma for the unit disc $\mathbb{D}$; indeed, for the unit disc $\mathbb{D}$, one has (cf. (2))
\[ c_{\mathbb{D}}(z, \nu) = p(z, \nu) = \frac{|\nu|}{1 - |z|^2}, \]
and the property that $c_{\mathbb{D}}$ does not increase under a holomorphic mapping $\phi : \mathbb{D} \to \mathbb{D}$ means precisely that
\[ \frac{|\phi'(z)|}{1 - |\phi(z)|^2} \leq \frac{1}{1 - |z|^2}, \]
which is the infinitesimal version of the Schwarz-Pick lemma.

Both $C_M$ and $c_M$ are directed to the study of holomorphic mappings between manifolds rather than to the geometric properties of manifolds. Applications of $C_M$ and $c_M$ to the understanding of holomorphic mappings are numerous, and we mention just one of them.

A complex manifold $M$ is called $C$-hyperbolic if, for its universal cover $M'$, $C_{M'}$ is a genuine distance. Examples of $C$-hyperbolic manifolds are bounded domains in complex space $\mathbb{C}^n$. Also, a compact quotient of a bounded domain $D$ by a properly discontinuous group acting freely on $D$ is $C$-hyperbolic.
**Theorem 1.** There exist only finitely many holomorphic mappings from a compact complex manifold $M_1$ onto a $C$-hyperbolic compact complex manifold $M_2$. In particular, the group of biholomorphic automorphisms of a compact $C$-hyperbolic manifold is finite.

**Sketch of the proof of Theorem 1 (Urata).** Let $\text{Hol}(M_1, M_2)$ denote the set of all holomorphic mappings from $M_1$ into $M_2$. It is known that $\text{Hol}(M_1, M_2)$, equipped with the compact-open topology, admits a complex structure (think of the graphs of the mappings as forming a sort of Teichmüller space). Using the $C$-hyperbolicity of $M_2$, one can show that $\text{Hol}(M_1, M_2)$ is compact.

Further, let $S \subset \text{Hol}(M_1, M_2)$ be the set of all holomorphic surjections in $\text{Hol}(M_1, M_2)$. The set $S$ is a complex subvariety of $\text{Hol}(M_1, M_2)$ because $S$ is given by the equation $f(M_1) = M_2$; i.e.,

$$S = \{ f \in \text{Hol}(M_1, M_2) \mid f(M_1) = M_2 \},$$

and hence it is also compact. The core of the proof is to show that $\dim S = 0$ (which is far too technical to be discussed here). Since $S$ is compact, this gives that $S$ is in fact finite. $\square$

**The Kobayashi Pseudodistance**

The pseudodistance named after Kobayashi was introduced by him in 1967. Let $M$ be a complex manifold and $p, q \in M$. A *chain of discs* from $p$ to $q$ is a collection of points $p = p_0, p_1, \ldots, p_k = q$ of $M$; pairs of points $a_1, b_1, a_2, b_2$ of $\varepsilon$; and holomorphic mappings $f_1, \ldots, f_k$ from $\varepsilon$ to $M$ such that $f_j(a_j) = p_{j-1}$ and $f_j(b_j) = p_j$ for all $j$ (Figure 2).

The length $l(\alpha)$ of a chain $\alpha$ is the sum $\rho(a_1, b_1) + \cdots + \rho(a_k, b_k)$, where $\rho$ is the Poincaré distance (1). The Kobayashi pseudodistance is then defined to be

$$K_M(p, q) := \inf_{\alpha} l(\alpha),$$

where the infimum is taken over all chains of discs from $p$ to $q$. $K_M$ is indeed a pseudodistance, but not a distance in general (e.g., $K_M = 0$); it coincides with the Poincaré distance (1) on $\varepsilon$ and is nonincreasing under holomorphic mappings. The Kobayashi pseudodistance $K_M$ is the greatest pseudodistance among all pseudodistances on $M$ that do not increase under holomorphic mappings from $\varepsilon$ to $M$, where distances on $\varepsilon$ are measured in the Poincaré distance. For any complex manifold $M$, one has $C_M(p, q) \leq K_M(p, q)$.

The Kobayashi pseudodistance is continuous as a mapping from $M \times M$ to $\mathbb{R}$ and, unlike the Carathéodory pseudodistance, is always inner. In particular, if $K_M$ is a distance on $M$, then it induces the topology of $M$. A manifold for which $K_M$ is a genuine distance is called (Kobayashi) hyperbolic. Any $C$-hyperbolic manifold, any bounded domain in complex space in particular, is hyperbolic. The property “hyperbolic” is preserved under biholomorphic mappings. There are unbounded domains that are not biholomorphically equivalent with bounded domains and are nevertheless hyperbolic. For example, one can show that the domain

$$\{ (z_1, z_2) \in \mathbb{C}^2 \mid |z_1| < 1, |z_2| < 1/(1 - |z_1|) \}$$

is hyperbolic but is not biholomorphically equivalent to any bounded domain in $\mathbb{C}^2$, as any bounded holomorphic function on $\varepsilon$ is independent of $z_2$. This last fact also implies that $\varepsilon$ is not $C$-hyperbolic.

An infinitesimal form of the Kobayashi pseudodistance is given by the Finslerian pseudometric

$$t_M(p, q) := \inf_{f, z, u} p(z, u),$$

where the infimum is taken over all holomorphic mappings $f$ from $\varepsilon$ into $M$, points $z \in \varepsilon$, and tangent vectors $u \in T_z(\varepsilon)$ such that $f(z) = p$ and $df(z)u = v$. In general $t_M$ is not even a pseudonorm as $C_M$ is (i.e., $t_M$ does not in general satisfy the triangle inequality). Similar to $C_M$, the infinitesimal pseudometric $t_M$ does not increase under holomorphic mappings.

As with the Carathéodory pseudodistance, $K_M$ and $t_M$ are primarily used for studying holomorphic mappings between manifolds rather than a manifold’s intrinsic geometry. The Kobayashi pseudodistance now has probably many more applications than any other invariant pseudodistance, and we are not even going to try to mention all the areas in complex analysis where it is used. We will give just one example of such an application below.

**Theorem 2.** Let $M$ be a hyperbolic complex manifold of complex dimension $n$. Then the following holds:

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Figure 2. Constructing the Kobayashi pseudodistance.
The group Aut(M) can be given the structure of a real Lie group whose topology agrees with the compact-open topology, and for every \( p \in M \), the stabilizer \( I_p(M) := \{ f \in \text{Aut}(M) \mid f(p) = p \} \) of \( p \) in \( \text{Aut}(M) \) is compact.

(ii) If \( n \geq 2 \) and \( \dim \text{Aut}(M) \geq n^2 + 3 \), then \( M \) is biholomorphically equivalent to the unit ball in \( \mathbb{B}^n \subset \mathbb{C}^n \).

The proof of Theorem 2 involves some interesting arguments of counting dimension, together with some rather subtle Lie group theory. We give an indication of the ideas.

Sketch of the proof of Theorem 2. Since \( K_M \) is an inner distance on \( M \), it induces the topology of \( M \) and thus turns \( M \) into a locally compact metric space. By a classical result of van Dantzig and van der Waerden, the group \( G(M) \) of all isometries of \( M \) is locally compact and the stabilizer \( S_p(M) := \{ f \in G(M) \mid f(p) = p \} \) is compact for every \( p \in M \) with respect to the compact-open topology. Since \( \text{Aut}(M) \) and \( I_p(M) \) are closed in \( G(M) \) and \( S_p(M) \), respectively, it follows that \( \text{Aut}(M) \) is locally compact and \( I_p(M) \) is compact. Further, by a theorem of Böchner and Montgomery, a locally compact group of differentiable transformations of a manifold is a Lie transformation group, and (i) is proved.

For (ii) we need the following theorem due to Greene-Krantz (1985) and Bland-Duchamp-Kalka (1987).

**Theorem 3.** Let \( M \) be a complex manifold of complex dimension \( n \) and \( p \) a point in \( M \). Suppose that there exists a compact group \( K \subset I_p(M) \) such that, for every pair of nonzero vectors \( v, u \in T_p(M) \), there exists \( \phi \in K \) and \( \mu \in \mathbb{C} \) such that \( d\phi(p)u = \mu v \). Then \( M \) is biholomorphically equivalent to either the unit ball \( \mathbb{B}^n \) or the complex space \( \mathbb{C}^n \) or the complex projective space \( \mathbb{CP}^n \).

We will now proceed with the proof of part (ii) of Theorem 2. Since the complex dimension of \( M \) is \( n \), it follows that the real dimension of any orbit of the action of \( \text{Aut}(M) \) on \( M \) does not exceed \( 2n \), and therefore we have \( \dim I_p(M) \geq n^2 - 2n + 3 \) for every \( p \in M \). Consider the isotropy representation \( \alpha_p : I_p(M) \to \text{GL}(I_p(M), \mathbb{C}) \):

\[
\alpha_p(\phi) := d\phi(p), \quad \phi \in I_p(M).
\]

It can be shown that \( \alpha_p \) is faithful (one-to-one). Since \( I_p(M) \) is compact by (i) of Theorem 2, there is a positive definite Hermitian form \( h_p \) on \( T_p(M) \) such that \( \alpha_p(I_p(M)) \subset U_{h_p}(n) \), where \( U_{h_p}(n) \) is the group of complex-linear transformations of \( T_p(M) \) preserving the form \( h_p \). We choose a basis in \( T_p(M) \) such that \( h_p \) in this basis is given by the identity matrix. Since \( \alpha_p \) is faithful, we see that \( \alpha_p(I_p(M)) \) is a compact subgroup of \( U(n) \) of dimension \( \geq n^2 - 2n + 3 \).

To wrap up the proof, it is necessary to apply some fairly technical results from Lie theory to see that a connected closed subgroup of \( U(n) \) of dimension \( \geq n^2 - 2n + 3 \) must in fact be either \( U(n) \) or \( SU(n) \) (this holds for \( n \neq 4 \); for \( n = 4 \) there are additional possibilities). Each of \( U(n) \) and \( SU(n) \) acts transitively on the unit sphere \( S^{2n-1} \subset \mathbb{C}^n \). Then, assuming \( n \neq 4 \), we can fix \( p \in M \) and apply Theorem 3 with \( K = \alpha_p(I_p(M)) \) to conclude that \( M \) is biholomorphically equivalent to either \( \mathbb{B}^n \) or \( \mathbb{C}^n \) or \( \mathbb{CP}^n \). Since \( M \) is hyperbolic, it in fact has to be equivalent to \( \mathbb{B}^n \) (\( \mathbb{C}^n \) and \( \mathbb{CP}^n \) are not hyperbolic). For \( n = 4 \) an additional technical argument is required.

**The Bergman Pseudometric**

The Bergman pseudometric is a Hermitian pseudometric on complex manifolds introduced by Bergman in 1922 for one variable and in 1933 for several variables.

We first describe Bergman's construction for \( M \) a bounded domain in \( \mathbb{C}^n \). Consider \( L^2(M) \) with respect to Lebesgue measure. The subspace of \( L^2(M) \) of holomorphic functions is closed, and we let \( \{ f_j \}_{j=1}^\infty \) be an orthonormal basis. Then the Bergman kernel function is given by

\[
b_M(z, w) = \sum_{j=1}^{\infty} f_j(z) \overline{f_j(w)}.
\]

It is independent of the choice of orthonormal basis. For \( z = w \), we have \( b_M(z, z) > 0 \).

For a general \( M \) of complex dimension \( n \), let \( \omega_1, \omega_2, \ldots \) be a complete orthonormal basis in the space of square-integrable holomorphic \( n \)-forms on \( M \). Then the differential form

\[
B_M(z, w) := \sum_{j=1}^{\infty} \omega_j(z) \wedge \omega_j(w),
\]

for \( z, w \in M \), is independent of the basis and is called the Bergman kernel form. In a local coordinate system \( z_1, \ldots, z_n \) in \( M \), one can write every \( \omega_j(z) \) as \( \omega_j(z) = f_j(z) dz_1 \wedge \cdots \wedge dz_n \) where \( f_j(z) \) is a locally defined holomorphic function, and therefore \( B_M \) can be written locally as

\[
B_M(z, w) = b_M(z, w) dz_1 \wedge \cdots \wedge dz_n
\]

\[
\wedge d\overline{w}_1 \wedge \cdots \wedge d\overline{w}_n,
\]

where \( b_M(z, w) := \sum_{j=1}^{\infty} f_j(z) f_j(w) \).

Assume now that \( b_M(z, z) > 0 \) for every \( z \in M \), and define the Bergman pseudometric as follows:

\[d_B(z, w) = \sup_{\|f\|_\infty \leq 1} |f(z) f(w)|, \quad f \in L^\infty(M), \quad z, w \in M.\]

\[\]
It turns out that $B_M$ is independent of the coordinate system, is positive semidefinite, biholomorphically invariant, smooth, and Kählerian. For the unit disc $\mathbb{D}$ we have $B_M = 1$. However, it is not true in general that $B_M$ is nonincreasing under holomorphic mappings. Those manifolds for which $B_M$ is well defined and positive definite (and thus is a Hermitian metric) are of particular interest, although they are relatively few. Bounded domains in complex space are examples of such manifolds, and for them the Bergman metric has been studied most extensively. If $D$ is a bounded domain in $\mathbb{C}^n$, then one has $c_D(p, v) \leq 4 \langle v, v \rangle B_D(p)$, but no general relation between $t_D$ and $B_D$ is known (in fact, a famous example of Diederich and Fornaess suggests that there is no such relation).

Unlike $t_D$ and $B_D$, the Bergman pseudometric $B_D$ is Hermitian and even Kählerian, so one would expect that it could be used to obtain differential-geometric information about the domain $D$. Below we give an example of one such application.

For any invariant metric on a domain $D \subset \mathbb{C}^n$, an important characteristic is its boundary behavior, in particular, the boundary behavior of its curvature tensor. The best-studied case is that of bounded strongly pseudoconvex domains with $C^\infty$ smooth boundary. Strong pseudoconvexity means that the boundary of the domain can be locally made strongly convex by a biholomorphic change of coordinates. For such domains C. Fefferman (1974) found a remarkable asymptotic formula for the Bergman kernel form. He used it to show that suitable geodesics of the Bergman metric approach the boundary of the domain in a "pseudotransverse" manner, and therefore that biholomorphic mappings of strongly pseudoconvex domains extend smoothly to the boundaries.

Using Fefferman's asymptotic formula, Klembeck showed—for a bounded strongly pseudoconvex domain $D$ with $C^\infty$ smooth boundary—that the holomorphic sectional curvature of $B_D$ near the boundary of $D$ approaches the negative constant $-2/(n + 1)$; that number is the holomorphic sectional curvature of the Bergman metric of the unit ball $\mathbb{B}^n \subset \mathbb{C}^n$. This fact is essential for the following characterization of the unit ball $\mathbb{B}^n$, which is entirely different from that in Theorem 2. We sketch the proof because it is readily appreciated and the techniques have been quite influential.

**Theorem 4 (Bun Wong, Rosay).** Let $D \subset \mathbb{C}^n$ be a bounded, strongly pseudoconvex domain with $C^\infty$ smooth boundary, and suppose that the automorphism group $\text{Aut}(D)$ of $D$ is noncompact in the compact-open topology. Then $D$ is biholomorphically equivalent to $\mathbb{B}^n$.

**Proof of Theorem 4 (Klembeck).** For any bounded domain $D \subset \mathbb{C}^n$, the noncompactness of $\text{Aut}(D)$ is equivalent to the existence of a sequence $\{\phi_j\} \subset \text{Aut}(D)$ such that $\phi_j - \phi$ as $j \to \infty$, uniformly on compact subsets of $D$, with $\phi : D \to \mathbb{C}^n$ holomorphic and $\phi \not\in \text{Aut}(D)$. Clearly, $\phi$ maps $D$ into $\mathbb{D}$, and $\phi$ maps $D$ under each of the automorphisms $\phi_j$, this implies that the holomorphic sectional curvature of $B_D$ approaches $-2/(n + 1)$. Since the Bergman metric is invariant under each of the automorphisms $\phi_j$, it follows that the holomorphic sectional curvature of $B_D$ is constant and equal to $-2/(n + 1)$.

We will now show that $D$ is simply connected. Let $\gamma$ be a closed curve in $D$. Since $\phi_j$ converges to $\phi$ uniformly on compact subsets of $D$, we see that the curve $\phi_j(\gamma)$ for $j$ large enough sits in a prescribed neighborhood of the point $\phi(q)$, and this neighborhood can be chosen (by the boundary smoothness) to be simply connected. Therefore, $\phi_j(\gamma)$ is homotopic to zero, and hence so is $\gamma$, thus proving that $D$ is simply connected.

In summary, $D$ is a simply connected Kähler manifold with negative constant holomorphic sectional curvature. It now follows from standard results of differential geometry (see [KoN]) that $D$ is biholomorphically equivalent to $\mathbb{B}^n$.  

The boundary behavior of the Bergman metric on more general domains is an interesting subject in its own right; it is very important, for example, in problems of extendability of holomorphic mappings between domains to mappings between their boundaries. The literature on this topic is vast.

Figure 3. Proof of Theorem 4.
and we shall not discuss the results here (see, e.g., [Kra]).

The Kähler-Einstein Metric
A Kähler-Einstein metric on a complex manifold is a Hermitian metric for which the Ricci tensor coincides up to multiplication by a real constant with the metric tensor. Thanks to Cheng-Yau (1980) and Mok-Yau (1983), such a metric is known to exist, for example, on any domain $D \subset \mathbb{C}^n$ that is bounded and pseudoconvex (i.e., can be exhausted by an increasing union of strongly pseudoconvex domains). It is given by

$$E_D(z) := \sum_{m,k=1}^n \frac{\partial^2 u(z)}{\partial z_m \partial \bar{z}_k} \, dz_m \, d\bar{z}_k,$$

where $u$ is a solution to the boundary value problem

$$\det \left( \frac{\partial^2 u}{\partial z_m \partial \bar{z}_k} \right) = e^{2u} \quad \text{on } D,$$

$$u = \infty \quad \text{on } \partial D.$$

The solution $u$ is known to be strongly plurisubharmonic; that is, the matrix $(\partial^2 u/\partial z_m \partial \bar{z}_k)$ is required to be positive definite. The metric $E_D$ is Kählerian, complete, and biholomorphically invariant. For the unit disc $\mathcal{C}$ one has $E_\mathcal{C} = \mathcal{P}$. It is, however, not true in general that $E_M$ does not increase under holomorphic mappings.

For a bounded strongly pseudoconvex domain $D$ with $C^\infty$ smooth boundary, it is known from the work of Cheng-Yau that the holomorphic sectional curvature of $E_D$ near the boundary of $D$ approaches the negative constant $-2/(n+1)$. Therefore, $E_D$ can be used in the proof of Theorem 4 above instead of $B_D$. In some cases the boundary behavior of $E_D$ is easier to determine than that of $B_D$, since $E_D$ is found from a solution to the boundary value problem (6) and therefore can be studied by methods of partial differential equations.

More Pseudometrics and Pseudodistances
In 1993 Hung-Hsi Wu proposed a way to define a whole new family of biholomorphically invariant pseudometrics. The actual definitions are rather complicated, and we mention only the following existence theorem related to one of these pseudometrics (see [Wu] for more detail).

Theorem 5. It is possible to construct on every complex manifold $M$ an upper semicontinuous Hermitian (not just Finslerian!) pseudometric $\mathcal{W}_M$ such that:

(i) $\mathcal{W}_\mathcal{C} = \mathcal{P}$.
(ii) If $f : M_1 \to M_2$ is a holomorphic mapping and $\dim M_1 = n$, then $f^* \mathcal{W}_M \leq \sqrt{n} \mathcal{W}_M$.
(iii) If $f : M_1 \to M_2$ is a biholomorphic mapping, then $f^* \mathcal{W}_M = \mathcal{W}_M$.
(iv) If $M$ is hyperbolic, then $\mathcal{W}_M$ is an upper semicontinuous Hermitian metric.

The Wu pseudometric $\mathcal{W}_M$ in fact combines some of the most attractive features of the Finsler pseudometrics (Kobayashi and Carathéodory) and the Hermitian pseudometrics (Bergman and Kähler-Einstein) discussed earlier. For the Wu pseudometric $\mathcal{W}_M$ is (essentially) nonincreasing under holomorphic mappings, yet it is Hermitian. One of the first triumphs of the Wu pseudometrics is the following theorem of Wu:

Theorem 6. A compact $C$-hyperbolic manifold is a projective variety.

It is also conjectured that a similar result will be true for general Kobayashi hyperbolic manifolds.

The pseudometric $\mathcal{W}_M$ becomes a Hermitian metric on hyperbolic manifolds. This fact once again indicates the importance of the concept of hyperbolicity. It turns out that the hyperbolicity of a manifold can be understood by using a pseudometric introduced by Sibony in 1981 via plurisubharmonic functions (see [For, pp. 357–72] for details). Klimek in 1985 and Azukawa in 1986 also used plurisubharmonic functions to define an invariant pseudodistance and an invariant pseudometric different from the one of Sibony. More information about this subject can be found in [Kob2].

In fact there are other invariant pseudodistances and pseudometrics, too numerous to mention here explicitly, that have been created for various special purposes in complex analysis. We mention particularly the pseudometric of Hahn and the Lempert function. The monograph [JAP] is an excellent source of information about the panorama of ideas that is available.

Concluding Remarks
In this short article we have discussed some ideas in the geometric approach to complex analysis—in particular, ideas that use invariant distances and metrics. We would like to stress once again that such distances and metrics have many applications, of which we have indicated just a few. We hope, however, that we have been able to show at least to some extent the beauty of this active and rapidly developing subject. The geometric approach has given rise to powerful new versions of function theory on manifolds, and the problems currently under study should occupy researchers for many years to come. For more information we refer the reader to the monographs and survey articles listed in the reference section below.

References
I must have been born with skeptical and satirical genes, if there are such, and I was aware of it early. Regarding skepticism, the precept “question everything” looks good on a bumper sticker, but if one adhered to it rigorously, one couldn’t get on with one’s life. Part of the mathematizations installed today consists of rating things on scales. “On a scale of one to ten, where do you stand on the issue of race?”; “On a scale of one to ten, how much do you like Norwich Terriers as opposed to Airedales?” I question such ratings. Yet if someone were to ask me “On a scale of one to ten, how skeptical do you consider yourself, rating Pyrrho of Elis as ten?” I would answer “Six.”
Barcelona is known for its great weather, arresting architecture, and excellent seafood. But this city, the site of the European Congress of Mathematics to be held in July of this year, is not just another pretty conference town. With four universities, an international center for mathematics research, hundreds of mathematicians, and thousands of students, Barcelona's mathematical community bustles in tune with the city's famed avenue of shopping and street artists, Las Ramblas. It is a community in transition, one that still bears the marks of its country's turbulent history but is nevertheless looking resolutely to the future.

The Marks of the Past
Mathematically, Spain is in many respects a developing country. Part of the lack of development has to do with Spanish culture, which tends to emphasize arts and humanities over science and mathematics. Other influences can be found in Spain's history, including the Spanish Inquisition and its suppression of scientific ideas. A more recent influence was the Fascist dictatorship of Francisco Franco, which began after the Spanish Civil War (1936-39) and lasted until his death in 1975. Franco controlled the press and many other institutions, including universities. Although Spanish science did not thrive in this climate, the Franco regime was not inimical to science; for example, it was only months after Franco took power that he established the Consejo Superior de Investigaciones Científicas (CSIC, Higher Council for Scientific Research), which today remains an important sponsor of research in Spain. Under CSIC in Franco's time, there was funding for mathematics research. However, it tended to accumulate around those having influential connections rather than around the best researchers.

This is the general problem that has stunted the development of science and mathematics in Spain in this century: connections and the ability to maneuver within the system have been rewarded more than research excellence. A good example of this effect is the career of the Catalan mathematician Ferran Sunyer i Balaguer, who lived from 1912 to 1967. Sunyer suffered from a congenital nervous atrophy, which confined him to a wheelchair and left him unable to write and able to speak only with difficulty. Despite his disability he taught himself mathematics and made a number of original contributions to research, particularly in classical analysis. His work was recognized internationally, and he had contacts with some of the most important mathematicians of his day, including Jacques Hadamard, Szolem Mandelbrojt, and Waclaw Sierpiński.

Although on the international scene Sunyer was one of the most important mathematicians in Spain, he lacked recognition within his own country. He earned a position at the CSIC only late in life, and despite working for a number of Spanish mathematics journals he was never asked to join their editorial boards. His life exemplifies the saying, common in academic circles in Spain, that one must be dead before one is recognized by one's colleagues. Indeed, Sunyer's name may be better known today than when he was alive, in part because of the establishment by the Institut d'Estudis Catalans of the Ferran Sunyer i Balaguer Prize, an annual award of 10,000 euros (about $9,850) recognizing an outstanding mathematical research monograph. Sunyer's difficulties were also compounded by the fact that he was an open Catalan nationalist. Franco

1 An excellent account of his life is "Ferran Sunyer i Balaguer (1912-1967) and Spanish Mathematics after the Civil War", by Antoni Malet (Mathematical Intelligencer 20 (2) (1998)). Malet has also written, in Catalan, a biography of Sunyer, which was published in 1995 by the Catalan Mathematical Society and the Catalan Society for the History of Science.
perceived Catalan nationalism as one of the biggest threats to his regime, and in an effort to root out Catalan culture he forbade any official use of the Catalan language.

Many things have changed in Spain since Franco’s death. The Generalitat de Catalunya was established as an autonomous government of Catalonia, and there followed a great revival of Catalan language and culture. In Catalonia and the rest of Spain, universities are now better funded, have more independence from the government, and are building research strength. Mathematics in particular is flowering in many places in Spain, including Barcelona. Sebastià Xambó, president of the Societat Catalana de Matemàtiques (Catalan Mathematical Society, SCM), noted that in the past twenty-five years mathematical research in Spain has gone from nearly zero to a reasonable international standard today. Holding the European Congress of Mathematics in Barcelona, he said, “gives us a good opportunity to symbolize this stage in a public event.”

A Mathematical Society, a Mathematical Center

One of Catalonia’s most famous native sons is the architect Antoni Gaudi (1852-1926), whose designs have become emblems of Barcelona. The free-form, undulating surfaces of his buildings look almost as if they had been molded into shape by human hands. At first glance his designs appear to have little of the symmetry and geometric form that typically connect mathematics and architecture. However, the mathematics simply lies deeper. For example, in his great unfinished work, the Sagrada Familia (Sacred Family) church, many of the fluid shapes are created using ruled surfaces, particularly the ruled hyperboloid, a shape often found in nature. Gaudi also developed a highly geometric method of using regular polygons to generate the shapes for the church’s columns. Gaudi died in 1926, when only one of the church’s four 100-meter parabolic towers had been finished (eight more spires have yet to be built; the highest will soar 130 meters). During the Spanish Civil War vandals burned the drawings for the church and smashed many of the models. Despite this loss, construction of the church continues, and the mathematical aspects of his designs have facilitated the use of computers to generate new models from which the builders can work. During the European Congress there will be organized tours of the church, emphasizing the mathematical elements of Gaudi’s work.

The Catalan affinity for architecture can also be seen in a remarkable fifteenth-century building in Barcelona’s Gothic quarter, a building that is home to the National Catalan Library, as well as to the Institut d’Estudis Catalans (Institute of Catalan Studies, IEC). Originally a convalescent home, the building features elaborate tiled wainscoting and a sculpture of Saint Paul in its courtyard. The building is currently undergoing massive renovations, which should be complete by the time of the European Congress. Founded in 1907, the IEC is the central custodian of Catalan language, history, and culture. During the Franco regime, the IEC was mostly dormant, though some of its members continued to meet clandestinely in private homes. Funded by the Generalitat, the IEC today is Catalonia’s main scholarly academy for all areas, from the humanities to the sciences. The current institute president is Manuel Castellet, one of the four members of the IEC’s mathematics section. The IEC oversees two important institutions on the mathematical scene in Barcelona: the SCM and the Centre de Recerca Matemàtica (Mathematical Research Center).

SCM president Sebastià Xambó is a busy man: He is a full professor of mathematics at the Universitat Politècnica de Catalunya, vice rector of that institution, and chair of the executive committee of the European Congress. Since Xambó became president in 1995 the SCM has grown from around 400 members to nearly 1,000. Xambó credits the expansion in part to the interest surrounding the Congress, of which the SCM is the sponsoring society. The growth of the SCM came at a time when Spain’s main national mathematical society, the Real Sociedad Matemática Española (Royal Spanish Mathematical Society), was relatively inactive. The SCM has been especially visible on the European scene: it was one of the first organizations to join the European Mathematical Society when the EMS was founded in 1990, and in 1996 it put in its successful bid to hold the 2000 European Congress in Barcelona (bids are made by mathematical societies rather than by nations).

The SCM publishes a research journal, *Butlletí de la Societat Catalana de Matemàtiques*, and a news publication, *Notícies*. The society is in many ways resolutely Catalan: for example, it recently published translations into Catalan of Gauss’s *Disquisitiones Arithmeticae*, beautifully bound and printed on high-quality paper, and of René Descartes’s *La Géométrie*, extensively annotated with explanations in modern terminology. The SCM also holds meetings, the most
The Barcelona Congress

The European Congress of Mathematics (ECM) is held every four
years in a city in Europe. The third ECM will take place July 10
to 14, 2000, in Barcelona. The congress is organized by the
Catalan Mathematical Society, under the auspices of the
European Mathematical Society (EMS). The International
Mathematical Union has designated the Barcelona congress as
a World Mathematical Year 2000 event.

The ECM scientific program features nine 1-hour plenary
lectures and thirty 50-minute parallel-session lectures. Ten
minisymposia, lasting four and a half hours apiece, will focus
on emerging topics that have important interactions with areas
outside of mathematics. As at the previous two ECMs—in Paris
in 1992 and in Budapest in 1996—several “round table” sessions
will explore issues such as education and public awareness of
mathematics. Demonstrations of mathematical software and
video exhibitions are also planned.

At the opening ceremonies up to ten EMS Prizes will be given
to recognize outstanding research by European mathematicians
under the age of thirty-two. In addition, the first Felix Klein Prize
will be awarded to a young researcher or to a small group for
using sophisticated methods to give a solution to a concrete and
difficult industrial problem.

The ECM organizers say they expect about 1,500 participants.
Because of the complications of holding such a large meeting
at a university and because the universities lack an auditorium
large enough for the plenary lectures, the organizers opted to
hold the ECM at the Palau de Congressos, Barcelona’s main
convention center. From there, a short subway ride takes one
to the old center of the city or to the universities. Participants
will be housed in hotels or university dormitories that are
either near the congress site or easily accessible by public transpor-
tation.

Funding for the congress comes from the government of
Catalonia and the central Spanish government, and from the city
of Barcelona, the universities, the Institut d’Estudis Catalans,
the International Mathematical Union, and private companies.
Beverages will be donated by Codorniu, one of the top makers of
cava, the Spanish counterpart of champagne.

—A. J.

ECM Web Sites:
http://www.iec.es/3ecm/
or
http://www.si.upc.es/3ecm/

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recent one being the third Trobada Matemàtica, or
Mathematical Meeting, held in March 2000, which
brought together around fifty Catalan mathematicians
for a program of five lectures.

Xamò is an enthusiastic promoter of all SCM
activities, but what seems closest to his heart are
the SCM’s two programs for young people. The first,
which is in fact the oldest program of the SCM, is
the organization in Catalonia of a phase of the Math-
ematical Olympiad. The second, begun in 1996, is a
less exclusive and more playful rendition of the
Olympiad called Cangur. Originally started in Aus-
tralia and later transplanted to France under the
name Kangaroo (“cangur” means kangaroo in Catal-
lan), the competition spread around Europe, where
it is sometimes also called the Mathematical Feast.
Cangur aims to capture the interest of a wide range
of students, not just those with special mathemat-
ic talent. When the SCM started Cangur, it drew
1,500 participants; this year over 5,000 students are
expected to take part across all of Catalonia. In ad-
dition to promoting mathematics among students,
Cangur has provided a connection between SCM and
teachers in secondary schools.

The Centre de Recerca Matemàtica (CRM) has the
distinction of being Spain’s only international mathe-
ematics research center. Manuel Castellet has served
as the director since founding the center in 1984.
He received his Ph.D. from the Eidgenössische
Technische Hochschule in Zürich in the 1970s as
a student of Beno Eckmann, the founder of the
Forschungsinstut für Mathematik (Mathematics
Research Institute) of the ETH. Castellet returned
to Spain in 1976 to take a position at the Universitat
Autònoma de Barcelona, and in 1982 he organized
the first Barcelona Conference on Algebraic
Topology and invited some of the top international
researchers. Two years later, influenced by the
Zürich Institute and wanting to expand further the
international contacts available to mathematicians
in Barcelona, Castellet founded the CRM as a
center under the IEC.

The CRM is a visitors’ institute, in the sense
that it has no permanent faculty, apart from the
director. Visitor stays range from one month to a
year, or sometimes two years for postdoctoral
researchers. Each year there are eight or nine
visitors. About six months and about sixty who stay for shorter periods. Specific
areas of emphasis are chosen for the fall and the
spring semesters; for the academic year 1999–2000
these were differential geometry and dynamical
systems. Because of Castellet’s own interests and
the strength of the research group in algebraic
topology that has grown at the Autònoma, the
CRM has historically been especially strong in this
area. In coordination with the Barcelona Conference
on Algebraic Topology, which now takes place
every four years, the CRM organizes a semester em-
phasis in that subject. Five years ago the CRM
started a series of advanced courses in mathematics that last one to two weeks and feature lectures by two experts in an area that has seen recent developments. Course topics have included mathematical aspects of image processing and integral geometry (1999) and algebraic aspects of quantum groups (2000).

The CRM is located in the mathematics building of the Autònoma, a thirty-minute commuter train ride from the center of Barcelona. Renovated in 1994, the CRM quarters are elegant and comfortable, with excellent office and lecture facilities. The center is a tremendous asset for mathematicians in Catalonia, and especially for those at the Autònoma, for whom CRM activities and visitors are just down the hall. In addition, the Autònoma mathematics department can rely on the CRM staff for logistical help when inviting visitors. Indeed, a substantial amount of the activity of the CRM has had ties to the Autònoma mathematics department. The CRM, being a part of the IEC, is a resource for all Catalan mathematicians. However, some believe that the lion's share of advantages accrue to mathematicians at the Autònoma. In addition, although there seems to be general agreement about the high quality of the CRM research activities, some would like to see the center distribute its activities more evenly across the full range of mathematical areas.

Such concerns, together with a desire to enhance support for research, have led mathematicians at the Universitat de Barcelona to launch their own mathematics research institute. Early this year they received approval and a small amount of start-up funding from the university to establish the Institut de Matemàtica de la Universitat de Barcelona (IMUB). The main purpose of the IMUB is to increase the research strength of the university's mathematics faculty by bringing in visitors and postdocs. The IMUB will likely bear some similarity to the CRM; for example, plans call for the IMUB to choose areas of emphasis for each semester and to offer advanced courses. Some have wondered whether Barcelona ought to have two mathematics institutes. The CRM receives steady funding from the Generalitat, making up about 30 percent of the center's budget; the rest comes from grants requiring yearly applications. As the IMUB gets under way, the two institutes might find themselves competing for the same funds.

Four Universities, Nine Mathematics Departments

The heart of Barcelona is Las Ramblas, a long avenue that runs from the harbor to the Plaça de Catalunya and has a central pedestrian island suited to strolling. Every hundred meters or so one finds a street performer: a foot-tall puppet convincingly plays Beethoven, fingers flying and arms flailing, on a pint-sized piano; a mime dressed as a cowboy and bathed head to toe in bronze makeup looks like a sculpture with a fine patina. Here one also finds high art, in the form of the Gran Teatre del Liceu, Spain's primary opera house, which reopened last fall after a 1994 fire gutted the auditorium and stage. Another form of entertainment is the Boqueria, Barcelona's central market, featuring an extraordinary variety of sea creatures, from staples like salmon and prawns to local specialties like espardenyes, a kind of sea cucumber.

A left turn at the Plaça de Catalunya and a five-minute walk takes one to the regal main building of the Universitat de Barcelona (UB). The building dates from 1871, but the university was founded in 1450. With 74,000 students, a little less than half the total in Catalonia, UB is the state's largest university and the second largest in all of Spain. A little farther out from the historic center of the city is the Universitat Politècnica de Catalunya (UPC), which is primarily an engineering school. The UPC was founded in 1971 when two engineering schools and an architecture school united, but the mathematics faculty was established only later, in 1992. Today the UPC has around 28,000 students. Also founded in the 1970s was the Universitat Autònoma de Barcelona (UAB), which has around
45,000 students. Situated in the Barcelona suburb of Bellaterra, the UAB is the only one of Barcelona's universities that has an American-style campus. Back in the center of Barcelona one finds the city's newest university, Universitat Pompeu Fabra (UPF); founded in 1990, it has only 7,000 students and six departments. UPF has no mathematics department, but there are some mathematicians in the Department of Economics and Business. The growth of universities in Barcelona is part of a general university expansion across Spain, which began in the 1970s and seems now to be ending.

In Spain professorships in mathematics always come with a specific area designation, such as algebra, analysis, or geometry. In many cases whole departments grow up around these areas. At the UB there are four departments under the Faculty of Mathematics: the departments of applied mathematics and analysis, of probability and statistics, of algebra and geometry, and of logic and the history of science. Each department had responsibility for organizing and teaching all courses within its specialty. At UPC, mathematics teaching is organized in coordination with engineering disciplines. For example, the Applied Mathematics I Department specializes in teaching mathematics for industrial engineering students. Similar teaching specialties are found in the other three departments: Applied Mathematics II, Applied Mathematics III, and Applied Mathematics and Telematics. These departments also have clusters of people who share research specialties, some of which are unrelated to the engineering specialties they serve. The UAB is the only one of the three having a single department that organizes the teaching of mathematics courses across all specialties, so that, for example, a professor of algebra might teach a course in analysis.

The division of mathematics into several departments at the UB and the UPC strikes some, including many mathematicians at those institutions, as fragmented. But changes do not seem to be on the horizon. For one thing, habit and tradition die hard. One observer also noted that this structure tends to create "small parcels of power" that faculty cling to. In addition, the level of teaching specialization provides a measure of security to faculties that have become quite large: The UPC and the UB each have about 100 professors. The UPC mathematicians can argue that they need many professors to provide teaching tailor-made for engineering specialties; their own undergraduate program in mathematics is small and selective, with only 200-250 students. The UB offers about seventy courses, some of them quite specialized, to its 1,000 mathematics students. The mathematics department at the UAB, with about 60 professors and around 400 students, offers around fifty such courses.

All three universities did a lot of hiring in mathematics in recent years and have now reached capacity in numbers of professors. A crisis may be looming on the horizon, for the number of mathematics majors has been declining. The number of first-year undergraduate students in mathematics at the UB fell in recent years from 300 to 200; at the UAB the numbers went from around 100 to around 80. (The UPC mathematics program has been somewhat sheltered from this problem, because since its inception it has accepted only about 50 students per year.) One reason for the decline seems to boil down to job prospects: Spanish companies do not have a tradition of hiring mathematicians. Companies in Catalonia tend to be branches of multinationals whose research laboratories are outside of Spain. Therefore, companies' appreciation for theoretical training is not as high as one finds in, say, Germany.

Barcelona mathematicians are working on ways to address these problems. For instance, Joaquim Bruna, chair of the mathematics department at the UAB, has begun a program of soliciting consulting work from local industry. An example is a project from a casino in Catalonia, which wanted to be sure that its card-shuffling machines were mixing the cards sufficiently; UAB mathematicians concluded they were. The connections to industry have led the mathematics department to establish new degree programs in industrial mathematics and in mathematical finance, the latter jointly organized with the CRM and supported by the Catalan Stock Exchange. At the UPC the curriculum for the mathematics degree program has a distinctly applied flavor and is oriented toward training students for positions in industry. Pere Pascual, head of the UPC mathematics faculty, said UPC mathematicians are working on strengthening ties to industry and are making concerted efforts to see that their students get jobs.

The decline in numbers of undergraduate mathematics students also does not bode well for doctoral programs. For the academic year 1999-2000, the number of first-year mathematics Ph.D. students is 14 at the UB, 9 at the UAB, and 20 at the UPC. These numbers are quite small, considering that there are around 300 mathematics professors across the three universities. The main reason for the low numbers is the weak academic job
market. In Barcelona the normal career path for a mathematician is to get a tenured job at the same institution where he or she received the Ph.D. With the Barcelona mathematics faculties full of young, recently hired professors, this career path looks quite bleak. Lackluster academic job markets in the rest of Spain, in Europe, and in the United States face those who consider taking jobs outside Barcelona. Another problem is that professors in Spain are not well paid, and students find that they can make higher salaries in a different occupation with only a bachelor's degree in mathematics.

Bruna noted that while the level of mathematical research in Barcelona is such that one can do a very good Ph.D. there, the research groups tend to be quite specialized. During the expansion of Spanish universities such a Ph.D. was in demand, but no longer. In industry such a degree is seen as too specialized and insufficiently practical. For this to change, said Bruna, "the mentality must change, in universities and in industry."

The Promise of the Future

Barcelona's hosting of the 1992 Summer Olympics provided impetus for a major revamping of the city's infrastructure. New buildings were erected and others renovated in the construction of the Olympic Ring atop Montjuïc, the highest point in Barcelona, with a commanding view of the city and the sea. One of the creations was a sleek white telecommunications tower overlooking the esplanade of the Olympic Ring. The tower was designed by Santiago Calatrava, a Valencian whose fluidly curved creations have made him one of today's most original and sought-after architects. The main column of the 136-meter tower is set at a precise angle so that on the summer solstice sunlight travels the entire length of the column to hit the pavement below. A silhouette image of Calatrava Tower symbolizes the city in the logo for the European Congress of Mathematics. In choosing such a modern image, the Congress organizers are indicating that the purpose of the meeting is to look resolutely to the future and not to the past.

Calatrava Tower is just barely visible from the office of Andreu Mas-Colell, commissioner for universities and research in the Generalitat. Mas-Colell, one of the world's leading mathematical economists, held a joint appointment in mathematics and economics at the University of California, Berkeley, before moving to Harvard University in the early 1980s. In 1995 he returned to his native Barcelona to lead the Department of Economics and Business at the Universitat Pompeu Fabra, which has gained the reputation of being Barcelona's elite university. During his time at Pompeu Fabra, Mas-Colell built up an economics faculty that today counts among the best in Europe.

This success is due to Mas-Colell's international stature, but also to the fact that he started essentially from scratch. He was thus able to avoid what is probably the biggest obstacle to improving scientific and mathematical research in Spain: the inbred nature of the professoriate. In Barcelona's mathematics departments, not only is it rare to find any non-Spaniards, there are only a few non-Catalans. Many mathematicians in Barcelona hold jobs at the same institutions where they received their Ph.D.'s. This is a common situation across Spain and has much to do with Spaniards' attachment to their hometowns and their reluctance to move, even within Spain. But the real problem lies in the hiring system.

In Spanish universities there are basically two kinds of tenured positions, Titular and Catedrático, which are roughly equivalent to associate and full professor respectively. The fact that these are civil servant positions meant for a long time that one had to be Spanish to apply for them; today one need only be a citizen of a nation in the European Community. When a position becomes vacant, a hiring jury is formed, consisting of two representatives from the university offering the position and three others chosen randomly from among mathematicians at other Spanish universities who are in the same research area as the open position. In principle, the hiring process is open, and there is plenty of external advice to the university doing the hiring. "De facto, it is just the reverse," said Mas-Colell. "When I say the reverse, I mean 100 percent the reverse." University faculty watch for such openings in the hope of placing one of their students. They also have some control over the timing and placement of the advertising: sometimes nontenured assistantships are advertised only in local newspapers and with a week's notice before the deadline. The two university representatives on the jury can usually influence one of the three external members in order to gain a majority.

If one asks Barcelona mathematicians about the hiring system, the descriptions range from "bad" to "terrible". Everyone seems to dislike it, and everyone seems to feel powerless to change it. There are a few individuals who take steps on their own to work against the system—for example, Joaquim Bruna of the UAB refuses to serve on the jury of a position for which one of his students is being considered—but they are the exceptions.

The current system, established in 1986, bears some resemblance to the Franco-era system, which suffered from some of the same problems but which some believe was actually fairer. But perhaps the problems arise not out of the system itself but out of the way in which the system is used: it serves to reward those who can pull the right strings rather than to find the highest-quality candidates.

Still, said Mas-Colell, "slowly, slowly the system is changing." Pompeu Fabra successfully developed a tenure-track system, and Mas-Colell noted
that other universities are moving in this direction. He is also trying to stimulate change through a couple of new government programs. In one, the Generalitat will help universities to fund positions for people whom the universities can demonstrate are chosen in an open, external review. As Mas-Colell described it, the program “gives universities incentives to come up with excellent candidates.” In another program, thirty scholars across all fields will be chosen each year to receive special grants running four to six years. One possible use of the funds is to buy out the scholars’ teaching time, freeing them up to concentrate on research. A program initiated not by the Generalitat but by the Spanish government will create over the next three or four years about 2,000 research-only positions across Spain. These positions will function in a similar way to those of France’s CNRS (Centre National de la Recherche Scientifique), except that the Spanish positions will have limited durations, at most eight to ten years. Mas-Colell said that he expects 300 to 400 of these positions to go to scholars in Catalonia.

While inbreeding has hindered the development of mathematical research in Barcelona, it has not stopped it. Indeed, across the city’s universities one can find many groups doing excellent work. For example, David Nualart and Marta Sanz at the UB lead a group of about fifteen professors and students in stochastic analysis, the only group in Spain actively working in this area. The group’s research is quite theoretical, focusing mainly on the study of stochastic partial differential equations and applications of Malliavin calculus. The group has many ties to researchers in similar areas in other countries, particularly France. It is also a node on the network “Stochastic Analysis and Its Applications”, funded by the European Community, which encourages collaborations mainly through exchanges of postdocs. Last year David Nualart received one of the highest honors in science and technology in Spain, the Iberdrola Prize, which is given by the Iberdrola electrical power company. The prize carries a monetary award of 12 million pesetas (about $71,000).

Nualart and Sanz both got their Ph.D.’s at the UB in the 1970s. At that time there was very little research, and they had to find their own way in building their research group. The situation of their UB colleague Carles Simó is similar: he did a doctorate in celestial mechanics at the UB in the early 1970s. Together with Amadeo Delshams at the UPC, Simó now leads a dynamical systems group of about thirty-five people at the two universities. A prodigious worker, Simó holds a three-hour seminar each week and has been known to give courses that meet for six hours in a single day. His research exemplifies the can-do spirit of Barcelona mathematicians, bringing in tools from all across mathematics and beyond to bear on questions in dynamical systems. If the right tools do not exist, the group tries to build them. Needing serious computing power for a simulation, they built their own supercomputer on a shoestring by networking in parallel eighty-four Pentium 500 processors. Simó has also worked on many applications of dynamical systems, including projects for the European Space Agency and the Jet Propulsion Laboratory.

Three mathematicians from Barcelona will speak at the European Congress of Mathematics: Simó will give a plenary lecture, and Joaquim Bruna of the UAB and Xavier Cabrè of the UPC will speak in parallel sessions. The ECM speakers were chosen by a scientific committee that was appointed by the European Mathematical Society and has representatives from all across the continent. The fact that there are three locals among the invited speakers (out of a total of almost forty) is an indication of how well Barcelona mathematicians hold their own in a major international meeting like the ECM.

For Marta Sanz, who serves as the secretary of the executive committee of the ECM, the Barcelona Congress represents a “normalization” of mathematical relations between Spain and the rest of Europe and the world. Mathematics in Spain still bears the burdens of what the Catalan science historian Antoni Malet called “the peculiar country that kept alive the last dictatorship of Western Europe.” But those burdens are slowly being shed. Today mathematics in Spain is growing, and the European Congress provides a way to celebrate this achievement. In holding the ECM in Barcelona, said Sanz, “There is a general feeling that we are doing something important for the development of mathematics in our country.”

About the Cover

Ceramic tile walls in the “galeria” entrance to the Institut d’Estudis Catalans (Institute of Catalan Studies) in Barcelona. The fifteenth-century building is a former convalescent home. Photograph courtesy of the Institut d’Estudis Catalans. Photographer: Manuel Armengol.

—Sandra Frost
Ralph Phillips died on November 23, 1998, a year after his wife of fifty-five years, Jean, had died. To his colleagues, many friends, and collaborators around the world, Ralph will be remembered as the upbeat, tough-but-fair, straightforward, modest, and delightful person that he was.

Born in Oakland on June 23, 1913, he received his bachelor's degree from the University of California at Los Angeles in 1935 and his Ph.D. from the University of Michigan in 1939. His thesis advisor was T. H. Hildebrandt. There is no doubt that parts of Ralph's personality, particularly his toughness and his ability to work hard steadily, were rooted in his living through the Great Depression.

From 1939 until 1942 he was a member of the Institute for Advanced Study in Princeton, an instructor at the University of Washington (where he met his future wife, Jean), and an instructor at Harvard. In a recent article in *The Mathematical Intelligencer* [8], Ralph described the prevalence of anti-Semitism in the academic culture of that period and how it affected his life. During the war he led a research group at MIT's Radiation Laboratory, the facility where much of the theoretical and practical work on radar technology was done. This work led to his book *Theory of Servomechanisms* [2], which for many years was the standard text in the subject. After the war he returned to mathematics, taking an assistant professorship at the Courant Institute of Mathematical Sciences. He moved to the University of Southern California at Los Angeles in 1958. In 1960 he moved for good to Stanford.

Ralph's research covered many areas, primarily in analysis, establishing him as one of the leading analysts of his time. He wrote over 125 research articles and 3 books over a period spanning more than sixty years. He was actively engaged in research almost to the very end. Moreover, the quality of his work remained consistently at the highest level. He was a prominent counterexample to the common view that mathematicians do their best work in their youth.

I will try to describe briefly the impact of some of Ralph's work. In his response [9] to receiving the 1997 AMS Steele Prize for Lifetime Achievement, Ralph recorded a list of his papers that he considered his most insightful. I have used this list as the basis for choosing which of his works to highlight here. There are three periods in his mathematical works, the first being the period before 1957, in which he was mainly concerned with functional analysis and particularly with semigroups of linear operators. In a series of influential papers on the foundations of semigroup theory, he developed new approaches that led to refinements and extensions of the theory. For example, his paper [7] introduced Banach algebra techniques into the subject. These works led to a coauthored revision of Hille's book [1] *Functional
Analysis and Semi-Groups, which contains many of Ralph's new results and which is still a standard text today. After this he wrote very little on the theory of semigroups, though they play a prominent role throughout his later works.

The second period spans the years 1957 to 1977, during which time Ralph turned to partial differential equations and in particular to dissipative and hyperbolic systems. He also began studying the wave equation, which I think was his greatest mathematical love and which was from then on always at the center of his research. This period also marked the beginning of his long and very fruitful collaboration with Peter Lax. (When I once compared this collaboration with that of Littlewood and Hardy, Ralph's immediate reaction was the question, Which was he—Littlewood or Hardy?) Together Lax and Phillips developed a novel and powerful geometric approach to scattering theory for the wave equation in the exterior of a compact obstacle in $\mathbb{R}^n$. Their method, which is time dependent, allowed them to resolve many problems associated with solutions to the wave equation in such a domain (such as exponential decay of solutions [3]).

One finds in their work the key insights into the relations between poles of the scattering matrix and dynamics of rays, a relation that in related and analogous settings became a very active area of investigation. A key role in their axiomatic theory is played by a semigroup and its infinitesimal generator. Their theory is described in detail in the second edition of their book [4], which also contains a survey of the solution by others of some of the fundamental conjectures by Lax and Phillips about the poles of the scattering matrix.

In the mid 1970s Ralph was diagnosed with cancer, but successful treatment gave him a new lease on life and a new mathematical journey. In this, the third period of his work, he turned to automorphic forms, especially scattering theory on locally symmetric spaces, and related geometry and discrete groups. It is amazing that at age sixty-five his research branched out into completely new areas. He was always open to learning new things. He and Lax pursued their geometric time-dependent approach to scattering theory in the setting of finite- and infinite-volume hyperbolic manifolds. Their work, appearing in the book [5], contained new proofs of the spectral analysis of the Laplacian for these manifolds and, in particular, the analytic continuation of Eisenstein series and a derivation of the Selberg Trace Formula. An important and novel feature in their method is the introduction of a nonselfadjoint operator (naturally enough, the generator of the appropriate semigroup) whose spectrum consists of the eigenvalues of the Laplacian together with the poles of the Eisenstein series. With this setup one can study what happens to the spectrum of this operator when the hyperbolic surface is deformed. In his paper [10] this technique was used to show that the discrete spectrum (that is, the so-called Maass cusp-form spectrum) is very unstable under deformation. In particular, in today's understanding these elusive cusp forms, which are the building blocks in modern automorphic-form theory, exist in abundance only for reasons related to arithmetic. Ralph's interests in spectral problems for the Laplacian broadened, and in the paper [6] it is shown (in connection with the problem of hearing the shape of a drum) that the set of planar drums that sound the same is compact in a suitable $C^0$ topology. During the last years of his life Ralph worked on generalizing his approach to scattering theory to higher-rank locally symmetric spaces. Unfortunately his health failed him, and he could not see this interesting project through.

I would like to end with some personal memories of Ralph. They all come from the period during which I knew him (1977-98) at Stanford. From the point of view of students and young postdocs, he was a model professor. He always welcomed people into his office to discuss mathematics or to do joint readings. Naturally he directed the theses of many students, many of whom are leaders in their fields today (see sidebar). In seminars and colloquia he never hesitated to ask a basic question, even though it might well show some ignorance. His were the questions that many in the audience were wondering about but were afraid to ask. In fact, Ralph was never interested in dazzling; he never put on airs. He was happy and satisfied with what he was doing, and it was
contagious. He much preferred to work together with others, and as is clear from his many successful collaborations, he was very good at it. He would listen to what a person had to say and come back the next day with a critical and insightful analysis of the previous day's discussion. He was a great colleague, and his contributions to the Stanford mathematics department, administratively and otherwise, were substantial. He had many passions besides mathematics. These included playing classical music. (Jean played the violin, and Ralph the clarinet.) He was an avid sailor and was always looking for some young hands to help man his boat "Wave Equation". He was a die-hard 49-er football fan (not unique among Stanford faculty), watching and analyzing each game. He told me not long before he died that he had had a good life and was lucky to enjoy what he did. I also feel very lucky to have enjoyed what he did, and no doubt others feel the same way.

References

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Fragile Dominion: Complexity and the Commons
Reviewed by David C. Krakauer and Martin A. Nowak

In his book *The End of Certainty*, Prigogine notes that "If the world were formed by stable dynamical systems, it would be radically different from the one we observe around us. It would be a static, predictable world, but we would not be here to make the predictions." In *Fragile Dominion*, Simon Levin describes the evolution and dynamics of the world's ecosystems and the loss of biological diversity attendant upon human activities. These losses, we learn, are largely the result of our inability to cope with complex, nonlinear systems. Thus, while life is in essence derived from nonlinearity, we risk the loss of life and biodiversity through the fine sensitivities of these essential processes. It is appropriate therefore to endeavor to understand how populations, communities, and ecosystems are constructed, both in the long term through mutation and selection and in the shorter term through colonization and competition. In this way we might arrive at decisions and policies with fewer adverse consequences for our planet.

Historically, these differences in time scales have translated into evolutionary versus ecological approaches to biological diversity; Hutchinson contrasted the ecological theatre with the evolutionary play. Thus Darwin sought to understand the origins of diversity among individuals and species, while ecologists such as Elton and Hutchinson have tended to concentrate on the maintenance of diversity at the species level and above. As Levin repeatedly informs us, this division of processes is largely expedient and need not reflect what is going on in nature. The ecology of the future will be one that explains selection acting at many scales of organization and therefore is made a partner of evolutionary theory. This will include describing ecological patterns using measures of fitness derived at lower levels of organization. The advantages are obvious: in place of descriptive theories of patterning, one obtains an optimality criterion upon which to base the observed order.

The interweaving of individual evolutionary processes and ecology forms the warp and woof of this book. Good examples of this approach are "individual-based" models of populations and ecosystems, an area of research in which Levin has made considerable contributions. In these models, large scale structure is derived from the
bottom up—as the statistical outcome of the behavior of countless, selfish, Darwinian replicators. Consider Levin's work on the dynamics of herds. Many species spontaneously form social groups in which some spatial structure emerges from the coordination of individual members. Reflecting a "Lagrangian" approach, these models consider the locations \( r_i = (x_i(t), y_i(t)) \) of each individual \( i \) at time \( t \), moving at some velocity towards a target with a velocity component derived from the interaction with other individuals. Hence \( r_i = \sum_{j=1}^{n} w_{ij} \cdot \) where \( w_{i1} \) is the intrinsic velocity vector and \( w_{i2} \) the complement. These interaction components are attractive towards distant neighbors and repulsive towards close neighbors. The tendency of groups to fragment at some threshold density can be characterized by the magnitude of these opposing forces. Furthermore, by allowing the velocities to vary across the population, diverse patterns of motion can be produced: slow speeds tend to produce bands, whereas fast speeds produce columns.

The first two chapters offer an overview of ideas and themes. In the first chapter we are introduced to the scientific study of biodiversity, which examines, for example, how certain "keystone" species play a vital role in preserving species' heterogeneity and the implications this might have on myopic stewardship of the environment. In the second we are introduced to the notion of the self-organizing system. Such systems are typified by patterns that emerge from nonlinear interactions among simple constituents rather than through imposition from above. A common example one finds in biology is Turing's reaction-diffusion formulation for steady-state heterogeneous spatial patterns. Thus, if one has a vector describing the abundance of two species \( c \) which interact nonlinearly according to \( f(c) \) and where each species has a constant rate of diffusion (represented by positive diagonal entries in the 2-square matrix \( D \)), then one can write an equation of the form \( \partial c / \partial t = f(c) + D \partial^2 c \). If the diagonal entries in \( D \) are not equal, Turing showed how this system could give rise to diffusion-driven instability. The earth or biosphere can be viewed as a very elaborate self-organizing system in which the majority of global patterns we observe are derived from local interactions among countless biotic and abiotic components. Complex adaptive systems, by contrast, are modified by selection acting most forcibly at the level of the individual components. In these cases we can refer to the structures as "evolved" for some purpose. Nonadaptive and adaptive systems share some characteristics, such as a diversity of components, localized nonlinear interactions, some form of hierarchical organization, and flows or energy connecting the hierarchies. But only the adaptive system has undergone systematic modification under pressure of selection. Levin goes to some length to clarify these ideas, as so often in biology it is wrongly assumed that pattern implies some form of selection. It is refreshing to read that "What is good for the ecosystem is neither well defined nor relevant evolutionarily." For those of us struggling with strong versions of the Gaia hypothesis (the earth as a complex adaptive system), this is a great source of comfort.

In the third chapter we are presented with six fundamental questions, each of which is covered in one chapter and around which the remainder of the book is structured. The questions are: (1) What patterns exist in nature? We are introduced to the "facts" of ecology relating to species diversity at numerous different scales. (2) Are these patterns determined by the current environment or by historical processes? In other words, can we understand ecological patterns through an appreciation of the environment, or have a large number of unpredictable events throughout history culminated in the patterns that we observe today? (3) How do ecosystems assemble themselves? Can we understand the competitive processes leading to biodiversity, and how do these relate to the evolution of individual characteristics? (4) How does evolution shape these ecological assemblages? We accept that evolution can act on individuals making up the species and thereby influence multispecies patterns, but is there a sense in which these higher-level patterns feed back on selection at the individual level? (5) What is the relationship between ecosystem structure and function? How do the parts contribute to the stability of the whole, and are there components which make a disproportionate contribution to the total assemblage? (6) Does evolution increase the resilience of an ecosystem? Do higher levels of selection lead to communities typified by robustness, or are ecosystems at some critical threshold and prone to dramatic losses of diversity?

The tendency of the human mind to uncover patterns is somewhat bewildering: we have seen weasels in clouds and prurient posturing in spilt ink. The extent to which nature manifests interesting order rather than reflects cognitive biases represents a serious challenge to biological science. This becomes very clear when one considers that just about everything in nature above the species level is an arbitrary classification. Herein lies a problem for ecology, as ecologists seek to understand the details of species numbers and distributions and their aggregate properties as populations, communities, guilds, and ecosystems. Is it possible to have a science for these higher levels of organization when we remain uncertain whether they exist as natural types?

Great progress in describing the abundance of species derived from the work of Wilson, MacArthur, and Simberloff analyzing the diversity...
of islands in the 1960s. The "equilibrium theory" of island biogeography states that the number of species on islands is determined by a balance of immigration and extinction. A previously unoccupied island, or one that has been cleared of most species, such as Java after the eruption of Krakatoa, is rapidly colonized by a large number of different species. As the numbers of resident species increase, the rate of immigration diminishes, reaching zero once the island's biodiversity matches that of the source population (on the mainland). However, on the island, species will be going extinct both through maladaptedness to the new environment and through competitive exclusion. The result is that extinction is proportional to species richness. The equilibrium solution of this dynamical system gives a measure of the biodiversity on an island. One can think about this using an ordinary differential equation in which the number of species \( x \) changes in time according to \( \dot{x} = c(1 - x) - ex \), where \( c \) is a colonization parameter and \( e \) an extinction parameter. The equilibrium solution is given by \( x^* = c/(c + e) \). If one assumes that extinction decreases as island diameter \( S \) increases such that \( e = e_0 \exp(-aS) \), and if colonization decreases as the distance to the mainland \( D \) increases such that \( c = c_0 \exp(-bD) \), then one can derive an approximate expression for the number of species at equilibrium: \( x^* = c_0 \exp(-bD)/(c_0 \exp(-bD) + e_0 \exp(-aS)) \). This assumes of course that all species are equally adept at migrating and equally strong competitors. Simberloff's experiments on mangrove swamps in the Florida Keys, in which he removed species by fumigation and then chartered recolonization, bore out the predictions of the model.

An important question in ecology has been whether there is a true "island effect" or whether the reduced biodiversity on islands is a simple consequence of small areas. Thus the colonization terms in the "equilibrium" model could be replaced with a speciation function or a general dispersal kernel and still produce good approximations simply because competitive processes remain the dominant determinants of species richness. To evaluate this possibility, a general model of species-area relationships is required. The most general finding in ecology has been that the logarithm of the species richness \( R \) is proportional to the logarithm of the area \( A \) plus some constant. Thus the species area relationship is described by a power law of the form \( R = A^P \). One feature of these relationships is that the exponent \( p \) lies in the range 0.15 to 0.4 in different regions around the world. Levin, in collaboration with Durrett, has used simple spatial models to derive the power law relationship from interactions among individuals and has attempted to explain the variation in the parameter \( p \) based on individual mechanisms. These models are stochastic cellular automata. Space is divided into square cells positioned on a two-dimensional integer lattice. At any given time the state of the system is determined by the ensemble of species at each site. The dynamics of the system are determined by speciation/immigration, dispersal, and competition. The model is a multiple contact process with mutation. Speciation/immigration is modeled by occupying a vacant site with a new species at a rate \( \alpha \). Dispersal to another site occurs with a probability proportional to the distance between sites at a rate \( \beta \), and competition is approximated by including a constant death rate \( d \) at which sites become vacant. The timing of events is independent and described by the exponential distribution \( \exp(-\lambda t) \). It can be shown that this system has a unique nontrivial equilibrium when \( \beta/d > \lambda_c \), where \( \lambda_c \) is a critical value of \( \lambda \). Such a model produces a power law relationship for which the parameter \( \alpha \) has the greatest impact on the value of the exponent. Thus these models suggest there is an "island effect", as the source of new biodiversity has the greatest impact on species richness. As speciation is orders of magnitude slower than immigration, the "distance effect" (the rate at which new species reach the island) provides a strong explanation for diversity.

The attraction of these simple models is that we obtain a good insight into qualitative properties of ecosystems. The pioneering work of May on complexity in simple model ecosystems taught us that we need not assume that the mechanisms underlying biodiversity are as elaborate as the product. The canonical example is the discrete logistic map in which the state of a variable \( x_t \) at time \( t \) is mapped onto a new value at \( t + 1 \) according to \( x_{t+1} = rx_t(1 - x_t) \). This is a noninvertible map whose behavior depends critically on the parameter \( r \). Gradually increasing the parameter \( r \) leads to a series of bifurcations, attracting limit cycles, and eventually chaos. In ecology the value of \( r \) can be loosely interpreted as fertility. The disadvantage of simple models is that we cannot incorporate detailed knowledge gleaned from the field in order to generate new quantitative predictions. This has led to two approaches to modeling ecosystems: those based on simple analytical reasoning and those based on simulations parameterized by much of the available data. The disadvantage of many of these simulations was articulated by Janzen, who stated that they got "all the nouns, but none of the verbs." Levin is very much in the "verb" school of modeling and well aware of the risks of not seeing the forest for the trees. Nonetheless, in Chapter 5 we are provided with a fascinating introduction to Botkin and Pacala's individual-based forest simulations, which have to a large extent bridged the gap between these two cultures. Pacala's recent work has been to derive expressions for the moments (mean densities and spatial covariances) of ensembles of
organisms as approximations of stochastic spatial models. The first and second moments are written as an ordinary differential equation for the average density of individuals and an integro-partial differential equation describing the average spatial covariance across all possible interactions.

Chapter 6 grapples with the relationship between ecological diversity (at many scales) and the evolutionary process. If we assume we start with a homogeneous system, how do we arrive at diversity? This is the problem of pattern formation in which symmetry must be broken and subsequent heterogeneity enhanced and maintained. It was Darwin and his successors' great insight that mutation and selection could achieve this result. While Darwin tended to think about the match between the individual and the environment (optimization), we now realize that fitness tends to be frequency dependent. This means that individual fitness is a function of the strategies employed by other individuals in the population. Hence "perpetual adaptation does not necessarily translate into optimization." The theoretical framework for investigating frequency-dependent fitness is evolutionary game theory. Game theory involves determining the existence, accessibility, and stability properties of a large family of equilibria. The most well known of these is the Nash equilibrium. If one considers two individuals A and B who can play strategies p or q, the pair of strategies (p_A, p_B) are Nash equilibria if p_A is the best reply against p_B and vice versa. If p_A is the unique best reply to p_B and p_B is the unique best reply to p_A, then this is a strict Nash equilibrium.

The evolution of altruism provides a test case for such a notion of fitness, as individual success is determined largely by interaction with conspecifics. Altruism presented a serious challenge to Darwin, as nature, "red in tooth and claw," should rarely favor a strategy that enhances the reproductive output of others at the expense of one's own fitness. In other words, how can cooperation reach a Nash equilibrium when selfish behavior increases an individual's fitness? Haldane presaged the answer in his remark that he would lay down his life for two of his siblings or eight of his cousins. This intuition was provided with a full theoretical explanation in William Hamilton’s revolutionary work on kin selection in the 1960s. The essential idea is that fitness is the net contribution of a shared set of genes to the population gene pool. Thus closely related individuals have an overlapping fitness referred to as "inclusive fitness" with respect to their shared genes, expressed as a correlation coefficient of relatedness. The correlation is between individual and group fitness. The greater the correlation, the more frequently altruism evolves. This insight is summarized by Hamilton's rule, which states that altruism can evolve provided that \( rB - C > 0 \), where \( r \) is the relatedness, \( B \) the reproductive increment to the recipient(s) of an action, and \( C \) the cost to the actor of providing the benefit.

Levin demonstrates how limiting dispersal, thereby promoting spatially structured populations, can produce such correlations. One way to think about this is to realize that fitness in a mean-field model (nonspatial or fully mixed) is defined simply as the per capita growth rate of mutants when rare. A mutant is simply an individual adopting a unique strategy in the population, while the growth rate is given by the invasion exponent or dominant eigenvalue of a "jacobian" matrix evaluated at the wildtype population’s steady-state value. In spatially viscous populations, multiple differential equations are required to capture the invasion dynamics. If one represents this system in matrix form, Van Baalen and Rand have shown how the dominant eigenvalue provides information about invasion and the corresponding eigenvector, the structure of clusters of the rare mutants. It turns out that these clusters correspond to the coefficient of relatedness defined locally. Thus spatial structure favors the emergence of altruistic strategies, because as Levin states, "the success of the group feeds back to affect the individual’s fitness on relatively fast time scales."

The importance of short feedback loops for stability is a recurrent theme in Levin’s discussions and in May’s earlier work on stability and complexity. These interactions are responsible for the robustness of populations, ecosystems, and communities. In recent years it has become common to see treatments of complex adaptive systems couched in terms of critical phenomena. Two of the most prominent are Kauffman’s metaphor of “life at the edge of chaos" and Bak’s "self-organizing criticality". While these are very different theories, they share a perspective that life is positioned close to a critical state, which upon perturbation undergoes a phase transition to disorder. Bak and colleagues have interpreted periodicities in the fossil record as evidence of self-organized criticality. This result is reached by observing that fluctuations in species abundance through time can be described by a 1/f power spectrum \( P(f) \) which measures the contribution of each frequency \( f \) to the time series. Self-similarity is observed in these data as \( P(f) \approx f^{-\beta} \) with \( 0 < \beta < 2 \). Such a relationship can be generated by modeling individuals using contact processes as discussed above, and then the system evolves to a critical point with 1/f fluctuations. However, recent work suggests that the results on the fossil record could be an artifact of interpolation which can produce spurious correlations in data. Levin’s position is quite different and closer to the biology. He argues that selection has led to a situation in which individuals obtain a high fitness in a complicated network of interactions. Those species without robust properties...
are rarely observed. By extension, those ecosystems that are volatile have been prone to extinction, and those that we observe are among those that were able to remain stable. We can ask what are features of the ecosystems that we observe, adopting an assumption akin to the “anthropic principle”, which avers that the mere existence of complex interactions argues for some stability mechanisms. The properties we see are hierarchical organization into modular-like components. The importance of modularity is that it partitions variance in such a way that ablation of one module has a minimal impact on the remaining structure. Thus few species are members of keystone functional groups, those groups whose activity has an impact on many other species. Similarly, food webs have characteristic numbers of levels, limiting the range of influence. These properties arise quite naturally out of a consideration of individual-based selection.

The book closes with some thoughts on how we as humans might deal with these insights. The notion of tight feedback is as useful an insight into human perception and behavior as into the behavior of dynamical systems in general. Thus by becoming responsible for our actions locally (tight feedback) we are far more likely to behave with ecological probity. Levin's final thoughts are presented as “eight commandments of environmental management”: (1) reduce uncertainty, (2) expect surprise, (3) maintain heterogeneity, (4) sustain modularity, (5) preserve redundancy, (6) tighten feedback loops, (7) build trust, and (8) Do unto others as you would have them do unto you. All of these are necessary as we now find ourselves the guardians of this fragile dominion. It is reassuring to have Levin as a guide.
Browder, Coifman, and Kadanoff Receive 2000 National Medals of Science

In January 2000 President Clinton announced the names of twelve individuals to receive the National Medal of Science, the nation's highest scientific honor. Among these were three who work in the mathematical sciences: Felix E. Browder, Ronald R. Coifman, and Leo P. Kadanoff.

Felix E. Browder
Felix E. Browder was honored for "pioneering mathematical work in the creation of nonlinear functional analysis that opened up new avenues in nonlinear problems, and for leadership in the scientific community that broadened the range of interactions among disciplines." Browder, University Professor of Mathematics at Rutgers University, is president of the AMS.

Browder's work in nonlinear functional analysis and its applications to nonlinear partial differential equations have had a long-term impact on mathematics. One of his major early achievements was to advance the study of elliptic partial differential equations to treat nonlinear problems that had previously been out of reach. The thrust of his theory was a liberation from the requirements of compactness and convexity, thus opening up a wide range of problems of nonlinear partial differential equations to precise analysis. His seminal work on nonlinear equations of evolution has had a profound influence on the subject.

Browder's progressive international view of science made him a leader for his time. It was through his efforts that the French analysts developed the strong interactions with their American counterparts that characterizes present-day research efforts. His supportive efforts to improve undergraduate and graduate education in the mathematical sciences included bringing about the successful AMOCO project at the University of Chicago, a program to engage inner-city youth in science; as well as the Center for Mathematics, Science and Computer Education; and the Outreach Program in Mathematics at Rutgers University. He has sustained over many years, advocacy of the involvement of women and minorities in science and mathematics.

At a time when it was not popular within mathematical circles, Browder advocated including applied mathematics at the highest levels into mathematics departments and did so successfully at Rutgers and at the University of Chicago, where he is the Max Mason Distinguished Service Award Professor Emeritus.

Ronald R. Coifman
Ronald R. Coifman was honored "for fundamental contributions to the field of harmonic analysis and for adapting that field to the capabilities of the digital computer to produce a family of fast, robust computational tools that have substantially benefited science and technology." He is the Phillips Professor of Mathematics at Yale University.

Coifman is a world leader in harmonic analysis. He introduced tools powerful enough to solve key problems in pure mathematics, yet sufficiently simple and flexible to become the basis for new, fast algorithms to handle the problems of wave propagation, data storage, de-noising, and medical imaging. As Coifman moved to applied mathematics, his work in the development of wavelet analysis had a revolutionary impact.

In collaboration with Yves Meyer, Coifman constructed a huge library of waveforms of various duration, oscillation, and other behavior. Through a clever algorithm developed with Victor Wickerhauser, it became possible to do very rapidly computerized searches through an enormous range of signal representations in order to quickly find the most economical transcription of measured data. This development allowed, for example, the FBI to compress a fingerprint database of 200 terabytes into less than 20 terabytes, saving millions of dollars in transmission time and storage costs.

Coifman also used wavelet analysis to develop tools for processing noisy data. He recognized
that one can essentially remove noise completely, allowing for short time exposure magnetic resonance images that would enable real-time "movies" inside the human body.

Coifman has continued to work on many computational problems in numerical analysis. The Beylkin-Coifman-Rokhlin matrix multiplication algorithm makes it possible to solve certain problems in high-intensity computations that were beyond the capability of any computer one might envision using previous algorithms.

Coifman's intellectual leadership has attracted first-class scientific talent from around the world to come to the United States to work on problems of national importance in signal processing and scientific computing.

Leo P. Kadanoff
Leo P. Kadanoff was honored "for leadership in fundamental theoretical research in statistical, solid state, and nonlinear physics which has led to numerous and important applications in engineering, urban planning, computer science, hydrodynamics, biology, applied mathematics, and geophysics." Kadanoff is the John D. MacArthur Distinguished Service Professor at the James Franck Institute of the University of Chicago.

Kadanoff has been a force in theoretical physics for nearly forty years. His concepts of scaling and universality have been used widely, both in research and in teaching. His textbook with Gordon Baym, Quantum Statistical Mechanics, is considered a classic and has been translated into many languages.

In his most important study, Kadanoff showed that sudden changes in material properties (for example, the magnetization of a magnet or the boiling of a fluid) could be understood in terms of scaling and universality. With his collaborators he showed how all the experimental data then available for the changes, called second-order phase transitions, could be understood in terms of these two ideas. These same ideas have now been extended to apply to a broad range of scientific and engineering problems and have found numerous and important applications in a wide range of areas.

Kadanoff has played a major role in the education of students. At the University of Chicago he was awarded the Quantrell Award for excellence in undergraduate teaching. He has been instrumental in introducing computers into physics laboratories as well as in developing associated instructional material. He has influenced the University of Chicago's entire academic approach by his strongly interdisciplinary techniques that blend experiment, theory, and computation. He creates opportunities for significant participation by students, postdoctoral associates, and young colleagues in a very broad range of research topics.

About the National Medal of Science
Established by Congress in 1959, the National Medal of Science is bestowed annually by the President of the United States on a select group of individuals deserving of special recognition by reason of their outstanding contributions to knowledge in the physical, biological, mathematical, engineering, social, or behavioral sciences. In 1962 President John F. Kennedy awarded the first Medal of Science to the late Theodore Von Karman, professor emeritus of the California Institute of Technology. The National Science Foundation administers the National Medal of Science program for the President. A distinguished, independent, twelve-member, presidential-appointed committee reviews the nominations and sends its list of recommendations to the President for final selection.

—Compiled from NSF news releases
Ingrid Daubechies Receives NAS Award in Mathematics

Ingrid Daubechies has received the 2000 National Academy of Sciences (NAS) Award in Mathematics. The $5,000 award, established by the AMS in commemoration of its centennial in 1988, is presented every four years for excellence in published mathematical research. Daubechies was chosen "for fundamental discoveries on wavelets and wavelet expansions and for her role in making wavelet methods a practical basic tool of applied mathematics."

Ingrid Daubechies received both her bachelor's and Ph.D. degrees (in 1975 and 1980) from the Free University in Brussels, Belgium. She held a research position at the Free University until 1987. From 1987 to 1994 she was a member of the technical staff at AT&T Bell Laboratories, during which time she took leaves to spend six months (in 1990) at the University of Michigan and two years (1991–93) at Rutgers University. She is now a professor in the Mathematics Department and in the Program in Applied and Computational Mathematics at Princeton University. Her research interests focus on the mathematical aspects of time-frequency analysis, in particular wavelets, as well as applications.

In 1993 Daubechies was elected as a member of the American Academy of Arts and Sciences, and in 1998 she was elected as a member of the NAS and as a fellow of the Institute of Electrical and Electronics Engineers. The AMS awarded her the 1994 Steele Prize for mathematical exposition for her book Ten Lectures on Wavelets (Society for Industrial and Applied Mathematics, 1992), as well as the 1997 Ruth Lyttle Satter Prize. From 1992 to 1997 she was a fellow of the John D. and Catherine T. MacArthur Foundation.

The previous recipients of the NAS Award in Mathematics are Robert P. Langlands (1988), Robert D. MacPherson (1992), and Andrew J. Wiles (1996).

—Allyn Jackson

Ingrid Daubechies

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Bott and Serre Share 2000 Wolf Prize

RAOUL BOTT and JEAN-PIERRE SERRE will share the $100,000 Wolf Prize in Mathematics for 2000. Bott is honored “for his deep discoveries in topology and differential geometry and their applications to Lie groups, differential operators, and mathematical physics.” Serre is recognized “for his many fundamental contributions to topology, algebraic geometry, algebra, and number theory and for his inspirational lectures and writing.” The prize is to be conferred by Israeli president Ezer Weizman at a special ceremony in Jerusalem on May 21, 2000.

Raoul Bott

Raoul Bott has been one of the leading figures in differential geometry, particularly in its links with topology and Lie groups. Through his publications, his students, and his personal qualities, he has significantly influenced the mathematics of our times.

Bott’s first major contribution was the application of Morse theory to the topology of Lie groups and homogeneous spaces, culminating in the famous “periodicity theorems” for the stable homotopy of the classical groups. This result provided the foundation for the development of K-theory, to which Bott also contributed greatly, particularly through his joint work with Michael Atiyah on the index theory of differential operators and its applications to equivariant topology. Bott also obtained seminal results in the theory of foliations. His later work on Yang-Mills theory, moduli spaces of vector bundles, and elliptic genera has been marked by a combination of the same geometric insight, coupled with new points of view coming from mathematical physics.

Raoul Bott was born in 1923 in Budapest, Hungary. After receiving bachelor’s and master’s degrees from McGill University, he received his Ph.D. in 1949 from the Carnegie Institute of Technology. After two years at the Institute for Advanced Study (IAS) in Princeton, he moved to the University of Michigan in 1951. Since 1959 he has been at Harvard University. He is now William Casper Graustein Research Professor. He has also held visiting positions at the IAS and at the Institut des Hautes Études Scientifiques in Bures-sur-Yvette, France. Bott was elected to the National Academy of Sciences of the USA in 1965, and he is an honorary member of the Académie des Sciences de Paris. In 1990 he received the AMS Steele Prize for Lifetime Achievement. He received the National Medal of Science in 1987.

Jean-Pierre Serre

Jean-Pierre Serre is a mathematician of enormous versatility who has had a huge influence on an astonishingly wide range of subjects.

Serre’s initial work in algebraic topology and complex geometry made him the youngest recipient ever of the Fields Medal, in 1954. His application of algebraic methods to infinite-dimensional spaces was to become a major theme in all modern geometry. He transformed algebraic
geometry and commutative algebra through use of sheaf-theoretical and homological methods, constructed the first sheaf cohomology theory in characteristic p, created modern geometric class field theory, and made major contributions to Galois cohomology and to the theory of arithmetic groups. In number theory Serre’s influence is inestimable. He introduced the notion of ℓ-adic representations and gave spectacular applications to elliptic curves, abelian varieties, and the theory of modular forms. His conjecture about the modularity of Galois representations was a key step toward the eventual proof of Fermat’s Last Theorem.

Through his lectures, books, and courses, each of which is a gem of mathematical exposition and clarity, Serre has inspired generations of mathematicians.

Jean-Pierre Serre was born in 1926 in Bages, France. He studied at the École Normale Supérieure and received his D.Sc. in 1951 from the Sorbonne. After holding a position through the Centre National de la Recherche Scientifique, he was a professor at the Université de Nancy. In 1956 he assumed the position of professor at the Collège de France; he has been an honorary professor there since 1994. Serre is a member of a number of honorary scientific societies in various nations, including the Académie des Sciences de Paris and the National Academy of Sciences of the USA. In addition to receiving the Fields Medal, Serre also won the Balzan Prize in 1985. In 1995 he received the AMS Steele Prize for Mathematical Exposition.

About the Wolf Prize
The Wolf Foundation was established by the late German-born inventor, diplomat, and philanthropist Ricardo Wolf (1887–1981). A resident of Cuba for many years, he became Fidel Castro’s ambassador to Israel and held this position until 1973, when Cuba severed diplomatic ties. Wolf decided then to stay on in Israel, where he spent his final years.

Five annual Wolf Prizes have been awarded since 1978 to outstanding scientists and artists “for achievements in the interest of mankind and friendly relations among peoples, irrespective of nationality, race, color, religion, sex, or political view.” The prizes of $100,000 in each area are given every year in four out of five scientific fields in rotation: agriculture, chemistry, mathematics, medicine, and physics. In the arts the prize rotates among architecture, music, painting, and sculpture.

—From Wolf Foundation news releases
The Joint Policy Board for Mathematics (JPBM) Communications Award was established in 1988 to reward and encourage those who, on a sustained basis, bring accurate mathematical information to nonmathematical audiences. This lifetime award recognizes a significant contribution or accumulated contributions to public understanding of mathematics.

At the Joint Mathematics Meetings in Washington, DC, in January 2000, the 2000 JPBM Communications Award was presented to SYLVIA NASAR. What follows is the award citation, a biographical sketch, and the recipient's response to receiving the award.

Citation
The Joint Policy Board for Mathematics presents its 2000 Communications Award to Sylvia Nasar for A Beautiful Mind, her biography of John Forbes Nash Jr. Based on extensive research, the vivid, beautifully written account of the life of the troubled mathematical genius provides rare insight into the world of academic research in mathematics. By portraying the mathematical culture at several leading institutions and explaining the significance of John Nash's contributions in game theory, geometry, and analysis, Sylvia Nasar has given the general public a glimpse into the world of mathematical research and an understanding of its impact on society.

Biographical Sketch
Sylvia Nasar, who was born in Germany and grew up in the United States and Turkey, was trained as an economist. She studied under William Baumol and Fritz Machlup at New York University and subsequently worked with Wassily Leontief, the 1973 winner of the Nobel Prize in economics for his invention of input-output analysis. Nasar has been writing about economics for many years, first at Fortune and U.S. News & World Report, and, in the past decade, at the New York Times. She is now working on her second book, about great twentieth-century economic thinkers, which picks up where Robert Heilbroner's classic, The Worldly Philosophers, leaves off and which will be published by Simon & Schuster in 2003.

A Beautiful Mind, the biography of John Nash, was Nasar's first book. It grew out of her New York Times article "The Lost Years of the Nobel Laureate", written right after Nash won the Nobel Prize in 1994. A Beautiful Mind won the 1998 National Book Critic's prize for biography and many other accolades.

Response
When John Forbes Nash Jr. won the Nobel Prize in economics in 1994, "the most remarkable mathematician of the second half century" was known around Princeton as "The Phantom of Fine Hall". He was not affiliated with any university, was not a member of the National Academy, was not listed in Who's Who. The "Nash equilibrium", "Nash bargaining solution", "Nash embedding", "Nash-De Giorgi result", "Nash-Moser theorem", and other of Nash's contributions from the 1950s had become famous in fields as disparate as geometry and game theory, but he himself was shrouded in obscurity. Thirty years of devastating mental illness had not only shattered Nash's life, they had also erased his personal history. Without the loyal support of his colleagues in the mathematical community and his wife, Alicia, Nash could not have survived those lost years, much less recovered from his illness and won worldwide acclaim. And without their recollections, letters, and photographs, I could not have reconstructed Nash's profoundly moving and inspiring story. For these gifts—as well as this wonderful award—I am profoundly grateful.

—From JPBM prize announcement
AWM Awards Presented in Washington, DC

The Association for Women in Mathematics (AWM) presented two prizes during the Joint Mathematics Meetings in Washington, DC, in January 2000.

Louise Hay Award
In 1990 the Executive Committee of the AWM established the annual Louise Hay Award for Contributions to Mathematics Education. The purpose of this award is to recognize outstanding achievements in any area of mathematics education, to be interpreted in the broadest possible sense. While Louise Hay was widely recognized for her contributions to mathematical logic and for her strong leadership as head of the Department of Mathematics, Statistics, and Computer Science at the University of Illinois at Chicago, her devotion to students and her lifelong commitment to nurturing the talent of young women and men secure her reputation as a consummate educator. The annual presentation of this award is intended to highlight the importance of mathematics education and to evoke the memory of all that Hay exemplified as a teacher, scholar, administrator, and human being.

JOAN FERRINI-MUNDY of Michigan State University received the 2000 Hay Award. "Ferrini-Mundy is both a leader and a scholar," the citation states. "She is one of the leading intellectual authorities on the broad landscape of mathematics education in the United States, and a leading researcher in teacher education and development, and reform. Her knowledge, strong organizational skills, and ability to listen to and understand people from different intellectual communities have enabled her to lead, administer, and implement numerous organizational initiatives, at every scale."

Ferrini-Mundy is the chair of the Writing Group for Standards 2000, the update of the National Council of Teachers of Mathematics Standards. Prior to going to Michigan State she served as director of the Mathematical Sciences Education Board and associate executive director of the Center for Science, Mathematics, and Engineering Education at the National Research Council.

Alice T. Schafer Prize
In 1990 the Executive Committee of the AWM established the annual Alice T. Schafer Prize for excellence in mathematics by an undergraduate woman. The prize is named for former AWM president and one of its founding members Alice T. Schafer (emerita of Wellesley College), who has contributed a great deal to women in mathematics throughout her career.

MARIANA E. CAMPBELL, a senior at the University of California, San Diego, received the 2000 Schafer Prize. The citation states: "After distinguishing herself ('best in the class') as a junior in both undergraduate and graduate classes at UCSD, Ms. Campbell participated in the Mount Holyoke REU program, where the faculty described her as 'astonishing.' At the program she wrote a paper entitled "The Igusa local zeta function for the different reduction types of the special fiber of an elliptic curve", which is currently being revised for publication. In addition to her mathematical achievements, Campbell is a talented violinist. The citation concludes, "The consensus is that Ms. Campbell has 'the drive, intellect, and creativity to become a leading mathematician.' She is 'remarkable' and 'someone who will make a difference in the lives of those around her down the line.'"

-From AWM prize announcements
The Mathematical Association of America (MAA) presented a number of prizes during the Joint Mathematics Meetings in Washington, DC, in January 2000.

Gung-Hu Award for Distinguished Service to Mathematics

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the MAA. This award, first given in 1990, is the successor to the Award for Distinguished Service to Mathematics, awarded since 1962, and has been made possible by the late Dr. Charles Y. Hu and his wife, Yuh-Gin Gung. Hu was not a mathematician but a retired professor of geology at the University of Maryland. He had such strong feelings about the basic nature of mathematics and its importance in all human endeavors that he felt impelled to contribute generously to the discipline of mathematics.

The 2000 Gung-Hu Award was presented to Paul R. Halmos. The citation states in part: "No mathematician alive today has influenced American mathematics more than Paul Halmos. He has written important research papers in operator theory, ergodic theory, and logic. He has edited major journals and served on many committees for the AMS and MAA. He has written textbooks and expository articles. His lectures always give us new insights."

Haimo Awards for Distinguished Teaching

The Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching of Mathematics recognize teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions. Deborah Tepper Haimo was president of the MAA during 1991-92.

There are three recipients of the 2000 Haimo Award. Arthur T. Benjamin was cited for "widespread recognition as a truly extraordinary teacher, his passionate dedication to mathematics and its pedagogy, and his nationwide reputation as one of the most successful ambassadors for mathematics." Benjamin is known as a scintillating lecturer as well as a magician and "lightning mental calculator." Benjamin is an associate professor of mathematics at Harvey Mudd College.

The citation for Donald S. Passman calls him "a mathematician of exceptional intelligence, keen insight, extraordinary mathematical ability, and keen sensitivity, who has a deep and natural
understanding of the learning process in mathematics. He brings all these attributes to bear on both his teaching and research." The author of more than 140 research publications and 5 books, Passman is the Richard Brauer Professor of Mathematics at the University of Wisconsin.

According to the citation for Gary W. Towsley, "[t]he hallmarks that set [him] apart from other excellent teachers are versatility, creativity across many disciplines, and personal qualities of integrity, helpfulness, and caring." Truly a "Renaissance man", Towsley has taught a wide range of mathematics courses as well as a medieval studies course for English majors, history and philosophy of science as an interdisciplinary offering, and "Roots of 20th Century Science" for the college honors program. Towsley is the Lockhart Professor of Mathematics at the State University of New York in Geneseo.

Beckenbach Book Prize
The Beckenbach Book Prize is awarded for distinguished, innovative books published by the MAA. David M. Bressoud of Macalester College has received the prize for his book *Proofs and Confirmations, the Story of the Alternating Sign Matrix Conjecture* (MAA Spectrum Series, jointly published with Cambridge University Press, 1999).

The prize citation states: "This book has several outstanding features. First and foremost, it carefully presents a significant chapter of mathematics. Moreover, it demonstrates how mathematics is actually created. It brings out the interplay among several seemingly unrelated branches, and also discusses unsolved problems ... Bressoud's book is a model of how a mathematics book should be written."

Chauvenet Prize
The Chauvenet Prize for expository writing is given for an outstanding expository article on a mathematical topic by a member of the MAA. Don B. Zagier of the Max-Planck-Institut für Mathematik has received the prize for his paper "Newman's short proof of the prime number theorem", which appeared in the *American Mathematical Monthly* 104 (1997).

The article, which Zagier dedicated to the prime number theorem on the occasion of its 100th birthday, gives a completely self-contained yet concise analytic proof of the theorem. The prize citation says that the article "is a masterpiece of excellent mathematical exposition and is accessible to anyone with a minimum background in complex analysis."

Certificates of Meritorious Service
Certificates of Meritorious Service are presented for service to the MAA at the national level or for service to an MAA section. Those recognized for 2000 are: Stanley Eison of University of Oklahoma (Oklahoma-Arkansas Section); Mario Martelli of California State University at Fullerton (Southern California Section); Sister M. Stephanie Sloan, emerita of Georgian Court College (New Jersey Section); Kathleen Taylor of Duquesne University (Allegheny Mountain Section); and Elizabeth J. Teles of Montgomery College and the National Science Foundation (Maryland-District of Columbia-Virginia Section).

—From MAA prize announcements
What is most impressive about this directory is its scope and size. It includes worldwide organizations of every type in the mathematical sciences. This title will be especially useful to academic libraries that support graduate programs in mathematics.

—American Reference Books Annual

This edition of the World Directory of Mathematicians 1998 incorporates updates and corrections to the 1994 edition, and includes nearly 30 percent more names. This valuable reference contains the names and addresses of over 50,000 mathematicians from 69 countries. There is also an increase in the number of fax numbers and email addresses in this edition. Listings for the directory are arranged both alphabetically and geographically and are based on information supplied by National Committees for Mathematics (or corresponding organizations). Libraries, mathematics departments, and individuals will find this most recent edition to be a valuable resource for its extensive coverage of the international mathematical community.

Contents: Preface; Ordering; List of Main Abbreviations; Members of the International Mathematical Union; List of Mathematical Organizations; Alphabetical List of Mathematicians; Geographical List of Mathematicians.

Published by the International Mathematical Union.

1998; 1093 pages; Softcover; List $50; All individuals $25; Order code WRLDIR/11

All prices subject to change. Charges for delivery are $3.00 per order. For optional air delivery outside of the continental U.S., please include $6.50 per item. Prepayment required. Order from: American Mathematical Society, P. O. Box 5904, Boston, MA 02206-5904, USA. For credit card orders, fax 1-401-455-4046 or call toll free 1-800-321-4AMS (4267) in the U. S. and Canada, 1-401-455-4000 worldwide. Or place your order through the AMS bookstore at www.ams.org/bookstore. Residents of Canada, please include 7% GST.
Mathematics People

Prizes of the Académie des Sciences de Paris

The Académie des Sciences de Paris has awarded a number of prizes to mathematicians.

Ampère Prize

The Prix Ampère de l'Electricité de France has been awarded to Yves Colin de Verdière of the Université Joseph Fourier, Grenoble I. The prize of 200,000 FF (about $30,000) is one of the major prizes of the Académie des Sciences.

Yves Colin de Verdière has made fundamental contributions to spectral theory. He established a relation between the spectrum of the Laplace operator on a Riemannian manifold and the lengths of closed geodesics. He has proved that if the geodesic flow is ergodic, almost all eigenfunctions are equidistributed ("quantum chaos"). He also obtained important results on the multiplicities of the first eigenvalues and their links with topology. As a corollary he obtained a planarity criterion for graphs. More recently, he has worked on the semiclassical limit for the Schrödinger equation and electrical networks.

State Prize

The State Prize of 50,000 FF is awarded to Bernard Maurey of Université Denis Diderot, Paris VII. Bernard Maurey has done fundamental work on the geometry and structure of Banach spaces, beginning with his doctoral thesis devoted to operators between $L_p$ spaces. Around 1980 he proved that the Hardy space $H^1$ has an unconditional basis, thus solving a major problem. More recently, his joint work with T. Gowers attracted a great deal of attention, in particular with the construction of a Banach space called the "Gowers-Maurey space") without any infinite unconditional basis. This construction led to a new concept, that of a hereditarily indecomposable Banach space.

Theme Prizes

The following prizes were awarded for achievement in mathematics: Élie Cartan Prize (25,000 FF): Laurent Clozel, Université Paris-Sud, Orsay, for his work on base change in the theory of automorphic forms.

Jacques Herbrand Prize (15,000 FF): Laurent Manivel, Institut Fourier of the Université Joseph Fourier, Saint-Martin d'Hères, for his work on the cohomology of homogeneous fibers and the geometry of projective varieties.

Joint Gabrielle Sand and M. Guido Triossi Prize (15,000 FF): Pierre Colmez, École Normale Supérieure, Paris, for his work on $p$-adic analysis.

Langevin Prize (10,000 FF): Sylvester Gallot, Université Joseph Fourier, Grenoble I, for his pioneering studies on the relationships between topology and integrals of curvature.

Paul Doistau-Emile Blutet Prize (10,000 FF): Wandelin Werner, Université Paris-Sud, Orsay, for his contribution to critical indices for Brownian motion.

—From Académie des Sciences announcements

2000 Leibniz Prize Awarded

The Deutsche Forschungsgemeinschaft (DFG) has selected the recipients of its Gottfried Wilhelm Leibniz Prize for the year 2000. Fourteen scientists and scholars have been awarded the prize. Among these are two who work in the mathematical sciences: Stefan Müller and Dieter Lust will each receive 1.5 million DM (approximately $750,000) to support research over a period of five years.
Stefan Müller

Stefan Müller, 37 years old, studied mathematics in Bonn, Edinburgh, and Paris and earned his Ph.D. in Edinburgh. He was an assistant professor at Carnegie-Mellon University before receiving his habilitation in Bonn. After professorships in Freiburg and Zürich, Müller assumed his present position as a director of the Max Planck Institute for Mathematics in the Natural Sciences in Leipzig. He is also deputy director of the Mathematical Research Institute at Oberwolfach. Müller is the recipient of the Max Planck Research Prize.

Müller's chief interests lie in applied analysis and partial differential equations. His special achievements have been to apply analysis to a multitude of practical problems in mechanics and material science and to have gained deep and sometimes surprising results. In this manner, with pure mathematics and excursions into physics and mechanics as his starting point, he has created a body of work which could be called "mathematical material science".

Dieter Lüst

Dieter Lüst, age 43, studied physics at the Technische Universität in Munich and earned his Ph.D. at Ludwig-Maximilians-Universität in Munich. He held a postdoctoral position at the California Institute of Technology during 1985-86. After receiving his habilitation in Munich, he was employed at CERN in Geneva as a Heisenberg Fellow. In 1993 he took up an appointment as a full professor at Humboldt-Universität in Berlin. He leads a DFG-sponsored postgraduate research group in particle physics and since 1998 has been an external member of the Max Planck Institute for Gravitational Physics in Potsdam-Golm.

Lüst's fields are string and supersymmetrical field theories. His research is directed at basic questions that combine quantum and gravitational physics in order to gain a uniform understanding of all physical forces and to achieve a uniform description of the numerous elementary particles constituting matter.

About the Leibniz Prize

The aim of the Leibniz Prize program, which was instituted by the DFG in 1985, is to improve the working conditions of outstanding scientists and scholars, to broaden their opportunities for research, to relieve them of administrative burdens, and to facilitate their employment of especially highly qualified young academics. The prizewinners are permitted the greatest possible freedom in the way they use the prize funds. Universities, the Max Planck Society, and former prizewinners made over 100 nominations for the 2000 prize. The DFG is the main scientific research funding agency of the German government.

---From a DFG announcement

Sznitman Receives 1999 Loève Prize

The 1999 Line and Michel Loève International Prize in Probability has been awarded to ALAIN-SOL SZNITMAN of the Eidgenössisches Technisches Hochschule, Zürich. The prize, which carries a monetary award of about $30,000, was presented at the University of California, Berkeley, in January 2000.

Biographical Sketch

Alain-Sol Sznitman was born in 1955 in France. He was a student at the École Normale Supérieure in Paris from 1975 to 1979. In 1987 he became associate professor at the Courant Institute for Mathematical Sciences, New York University, and he assumed his present position as professor at the ETH in Zürich in 1991. He was director of the Mathematics Research Institute of the ETH from 1995 to 1999. He received the Rollo Davidson Prize in 1991, delivered a plenary lecture at the European Congress of Mathematicians in 1992 in Paris, and was an invited speaker at the International Congress of Mathematicians 1998 in Berlin.

The Work of Sznitman

The central topics of Sznitman's mathematical research are interacting stochastic systems, mainly in connection with nonlinear partial differential equations, and random media. One of his main research interests in the late 1980s was interacting systems of Boltzmann type and corresponding propagation of chaos results. A central problem in interacting particle systems is to derive equations that govern the macroscopic development of the system from a microscopic description. The macroscopic equations are usually linked to this microscopic description via propagation of chaotic behavior, meaning that on large microscopic pieces of the space the process propagates some purely random part, for instance by exhibiting and maintaining some purely Poisson structure. Sznitman has greatly advanced our understanding of such phenomena in a number of important cases. One is a model of mutually (locally) annihilating Brownian particles in a Boltzmann type regime. He found a surprising link between the nonlinear equation for the macroscopic development and a probabilistic tree construction, where the interplay comes from a local propagation of chaos result. Quite surprisingly, such probabilistic tree constructions also show up in much later work of...
Sznitman (with Yves Le Jan) in a different context, namely, in their treatment of the Navier-Stokes equation.

Since around 1990 Sznitman has mainly worked on random media. He has developed a completely new method for problems of random walks and Brownian motion among random traps and related issues. A typical question is the following: Given random media consisting of a Poisson potential, which is influencing the Brownian, for instance, by local killing, what are the long-time survival probabilities and what are the (probabilistic) properties of the surviving paths? The survival rates can be expressed in properties of low-lying eigenvalues for the corresponding Dirichlet problem in random medium. Sznitman has developed an extremely powerful method for treating such eigenvalue problems in disordered media. A basic difficulty is the high complexity generated by the environment. The problem should "simplify" in the limit, in the sense that simply structured "pockets of low-lying eigenvalues" become dominant. These dominant pockets are responsible for what is sometimes called "intermittency" of the system. In order to treat these questions, Sznitman has developed his method of enlargement of obstacles. The idea is to replace the obstacles by bigger ones, reducing the combinatorial complexity of the system. However, one must make a careful analysis differentiating between different types of obstacles according to their surroundings. In this way, Sznitman has been able to analyze very precisely a number of problems connected with random traps, like giving precise information about pinning of random walks. The method is not restricted to random trap problems, as has become increasingly clear in recent years. It has been applied and extended, for instance, to random saturation problems and to random Schrödinger operators in the presence of a magnetic field (in works by G. Ben Arous, L. Erdős, and others). It is also clear that these problems are closely related to complex problems on disordered Gibbs measures in the regime of the so-called Griffiths singularity.

In his most recent investigations Sznitman applies similar ideas to a non-self-adjoint setting, namely, to multidimensional random walks in random environments, where his research has led to considerable progress in understanding. Based on ideas closely related to the ones described above, this work again stresses the dominant role of pockets of exceptional behavior but is carried out in a more delicate non-self-adjoint setting, which can no longer be described by eigenvalues.

About the Prize

The Line and Michel Loève International Prize in Probability was established in 1992 by Mrs. Line Loève and the Department of Statistics at the University of California, Berkeley. It is meant to recognize outstanding contributions by researchers in probability who are less than 45 years of age. The committee awarding the prize consists of approximately thirty internationally recognized probabilists who are more than 45 years of age. The prize is awarded in alternate years. Past recipients are David Aldous (1993), Michel Talagrand (1995), and Jean-François Le Gall (1997). Michel Loève was professor of mathematics and statistics at the University of California, Berkeley, from 1948 until his unexpected death in 1979. His wife, a psychologist, died very shortly after establishing the prize in 1992.

—Lucien LeCam, University of California, Berkeley, and Erwin Bolthausen, University of Zürich

Lebowitz Receives Poincaré Prize

Joel L. Lebowitz, who has made outstanding contributions to both statistical physics and the fight for human rights for oppressed scientists around the world, has received the Henri Poincaré Prize for mathematical physics from the International Association of Mathematical Physics (IAMP). Lebowitz is the George William Hill Professor of Mathematics and Physics at Rutgers University.

The Poincaré Prize recognizes outstanding contributions that set the foundation for novel developments in mathematical physics. The prize will be awarded at the thirteenth IAMP conference in London in the summer of 2000 and includes a cash award of 5,000 euros (about 5,5000).

Other honors conferred on Lebowitz include the Boltzmann Medal of the International Union of Pure and Applied Physics, the A. Cressy Morrison Award in Natural Sciences from the New York Academy of Sciences, and the Rutgers Board of Trustees Award for Excellence in Research.

—From a Rutgers University news release

Guionnet Awarded Oberwolfach Prize

Alice Guionnet of Université de Paris-Sud has received the 1999 Oberwolfach Prize for outstanding research in the mathematical field of stochastics. She received the prize at a ceremony on November 5, 1999, at the Mathematical Research Institute in Oberwolfach, Germany, and presented a lecture entitled "Large random matrices".

The Oberwolfach Prize is awarded by the Gesellschaft für Mathematische Forschung e.V. to European mathematicians not older than 35 years. The prize recognizes excellent achievements in a specific field of mathematics, which changes each time the prize is given. The prize carries a monetary award of 10,000 DM (approximately $5,000) and was financed by the friends of Oberwolfach (Förderverein für mathematische Forschung e.V.).

Previous recipients of the Oberwolfach Prize are Peter Kronheimer (topology and geometry, 1991), Jörg Brüdern and Jens Franke (number theory and algebra, 1993), and Gero Friesecke and Stefan Sauter (analysis and applied mathematics, 1996).

—Allyn Jackson
AIM Five-Year Fellows Announced

The American Institute of Mathematics (AIM) has announced that the recipients of five-year fellowships for the year 2000 are HENRY COHN of Harvard University and VADIM KALOSHIN of Princeton University. They were chosen from a pool of more than 120 applicants.

Henry Cohn received his B.S. in mathematics from the Massachusetts Institute of Technology in 1995. He will receive his Ph.D. in 2000 with a thesis on “New bounds on sphere packings”. In his thesis he develops new techniques that improve upper estimates on the packing density of spheres in Euclidean spaces of dimensions 4 through 36. He has published several papers in combinatorics and number theory. His interests also include the question of the irrationality of the Riemann zeta function for arguments that are odd integers and that are greater than or equal to 5.

Vadim Kaloshin received his B.S. from Moscow State University in 1994. He will receive his Ph.D. in 2000 in the area of dynamical systems with a thesis on “Growth of number of periodic orbits of generic diffeomorphisms”. His thesis contains a new, more elementary, proof of the Artin-Mazur result that “every smooth invertible self-map of a compact manifold can be approximated by one for which the number of periodic points of period p is less than an exponential function of p” and also shows that the number of periodic points of period p can grow arbitrarily fast with p for a generic set of smooth invertible self-maps of a compact manifold. With B. Hunt, Kaloshin has shown that the Hausdorff dimension of a fractal set in a Banach space is not necessarily preserved under projection to finite-dimensional Euclidean space.

The AIM five-year fellowships are awarded each year to outstanding new Ph.D. recipients to support research in an area of pure mathematics. The fellowships cover sixty months of full-time research, as well as funds for travel and equipment. Each fellowship carries a stipend of $4,000 per month, with an additional $4,000 per year allocated for travel and equipment.

—From an AIM announcement

National Academy of Engineering Elections

The National Academy of Engineering has announced the election of seventy-eight new members and eight foreign associates, including six for work in the mathematical sciences. TAMER BASAR of the University of Illinois, Urbana-Champaign, was elected for work on the development of dynamic game theory and application to robust control of systems with uncertainty. JAMES R. BASSINGTHWAITE of the University of Washington, Seattle, was elected for contributions to integrative physiology and bioengineering using transport theory and computational methods. HOWARD R. BAUM of the National Institute of Standards and Technology was elected for developing and implementing broadly applicable analytical models and numerical tools for understanding and mitigating fire phenomena. JAMES W. COOLEY of the University of Rhode Island was elected for the creation and development of the Fast Fourier Transform (FFT) algorithm for time series analysis. HENRY H. RACHFORD JR. of Stoner Associates, Inc., Houston, Texas, was elected for contributions in the numerical solution of partial differential equations to solve petroleum reservoir and pipeline hydraulics problems. JACOB T. SCHWARTZ of the Courant Institute of Mathematical Sciences of New York University was elected for contributions to the theory and practice of programming language design, compiler technology, and parallel computation.

—From a National Academy of Engineering announcement

Deaths

CHARLES L. BOTTOMS, a teacher at Booker T. Washington High School, Tulsa, OK, died on December 5, 1999. Born on January 14, 1945, he was a member of the Society for 10 years.

ARTHUR B. BROWN, of Floral Park, NY, died on November 8, 1999. Born on February 10, 1905, he was a member of the Society for 71 years.

JOHN A. NOHEL, professor emeritus at the University of Wisconsin, Madison, died on November 1, 1999. Born on October 24, 1924, he was a member of the Society for 49 years.

GEORGE W. PETRIE III, of Port Angeles, WA, died on October 18, 1999. Born on May 6, 1912, he was a member of the Society for 62 years.

RAYMOND W. SOUTHWORTH, of Williamsburg, VA, died on January 22, 2000. Born on October 23, 1920, he was a member of the Society for 31 years.
Mathematics Opportunities

NSF Mathematical Sciences Postdoctoral Research Fellowships

The Mathematical Sciences Postdoctoral Research Fellowship program of the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) awards fellowships each year for research in pure mathematics, applied mathematics and operations research, and statistics. The deadline for this year’s applications is October 16, 2000. Applications must be submitted via FastLane on the World Wide Web. Go to http://www.fastlane.nsf.gov/, and click on "Postdoctoral Fellowship". Information can be found there for the Mathematical Sciences Postdoctoral Research Fellowships as well as for other NSF fellowship opportunities. For more information telephone the DMS at 703-306-1870 or e-mail: msprf@nsf.gov.

-AWM announcement

AWM Workshops for Women Graduate Students and Postdocs

Over the past twelve years the Association for Women in Mathematics (AWM) has held a series of workshops for women graduate students and recent Ph.D.’s in conjunction with major mathematics meetings. The workshops are also supported by the Office of Naval Research and the National Science Foundation.

The next AWM workshop will be held in conjunction with the annual Joint Mathematics Meetings in New Orleans, Louisiana, January 10-13, 2001. The exact date of the workshop has not yet been determined, but it will most likely be held on Saturday, January 13, 2001, with an introductory dinner held on either Thursday (January 11) or Friday (January 12) evening.

Twenty women will be selected in advance of the workshop to present their work; the selected graduate students will present posters, and the postdocs will give 20-minute talks. AWM will offer funding for travel and two days’ subsistence for the selected participants. The workshop will also include a panel discussion on issues of career development, a luncheon, and a dinner with a discussion period. Participants will have the opportunity to meet with other women mathematicians at all stages of their careers. All mathematicians (female and male) are invited to attend the program. Departments are urged to help graduate students and postdocs who do not receive funding to obtain some institutional support to attend the workshop presentations and the associated meetings.

The AWM also seeks volunteers to lead discussion groups and to act as mentors for workshop participants. Anyone interested in volunteering should contact the AWM office.

Applications are welcome from graduate students who have made substantial progress toward their theses and from women who have received their Ph.D.’s within approximately the last five years. (The word “postdoc” refers to recent Ph.D.’s, whether or not they currently hold a postdoctoral or other academic position.) Women with grants or other sources of support are still welcome to apply. All non-U.S. citizens must have a current U.S. address.

All applications should include a curriculum vitae, a concise description (two to three pages) of research, and the title of the proposed talk or poster. All applications should also include at least one letter of recommendation; in particular, graduate students should include a letter of recommendation from their thesis advisors. Nominations by other mathematicians (along with the information described above) are also welcome.

Send five complete copies of the application materials (including the cover letter) to: Workshop Selection Committee, Association for Women in Mathematics, 4114 Computer & Space Sciences Building, University of Maryland, College Park, MD 20742-2461; telephone 301-405-7892; e-mail: awm@math.umd.edu. The AWM Web site is at http://www.awm-math.org/.

Please note that applications via e-mail or fax are not acceptable. The deadline for receipt of applications at the AWM office is September 1, 2000.

-AWM announcement
Inside the AMS

AMS Participates in Free Font Project

The AMS has banded together with a small group of scientific societies and publishers on a project to produce a free font set to meet the needs of scientific and technical publishing. The project also includes an effort to obtain unique, universally standardized computer codes for a large collection of mathematical and technical symbols.

The problem being addressed is a common one, seen in such everyday nuisances as not being able to read a mathematics paper received electronically because one's computer or printer lacks all the necessary fonts. As electronic publishing and information exchange permeate all aspects of the professional lives of scientists, mathematicians, and engineers, the lack of a commonly used set of fonts has arisen as an important problem. The wide variety of symbols used in scientific and technical writing mean that authors and publishers often patch together font sets by mixing proprietary and freely available fonts. Font licensing considerations can restrict the exchange of these font sets and sometimes of the documents created with them.

To address this problem, the STIPUB (Scientific and Technical Information Publishers) group initiated the STIX (Scientific and Technical Information eXchange) project. STIPUB consists of the AMS, the American Chemical Society, the American Institute of Physics, the American Physical Society, Chemical Abstracts, the Institute of Electrical and Electronics Engineers, and Elsevier, Inc. The STIX project is overseen by a team of about ten professionals who are on the staffs of the STIPUB organizations and who have expertise in electronic publishing.

The STIX project has two goals. The first goal is to ensure that all the symbols to be included in the STIX font set have unique, universally standardized computer codes. When the STIX team assembled the collection of symbols to be included in the STIX font set, many symbols were found that do not have Unicode codes. Like the familiar ASCII system, Unicode is a system for encoding written material into a form that can be processed by computer. But where ASCII represents only the (unaccented) Latin alphabet, Unicode has the capacity to provide 16-bit codes for 64,000 characters and symbols. An evolving international standard, Unicode is coming into widespread use and will eventually replace ASCII. The STIX project is currently working on obtaining Unicode codes for all of the symbols to be included in the STIX fonts.

The second goal of the STIX project is the actual creation of the STIX fonts. At the time of this writing the STIPUB group was assessing bids from companies to develop the fonts. The cost of the development will be shared among the STIPUB members. Plans call for development to be under way during 2000, though it is too soon yet to say when the work will be complete. The STIX fonts will be available under license but free of charge.

Free availability of a universal font set will facilitate the flow of scientific and technical communication by simplifying exchange of documents among authors and publishers. In addition, the incorporation of the full STIX symbol collection into Unicode, together with the availability of the MATHML markup language for presenting mathematics in Web documents, will eventually permit development of a new archival format for mathematics in which one can search not just for text but also for symbols and expressions and from which one can lift expressions and feed them directly into symbolic manipulation tools. In this way the STIX project is an important step toward realizing the full potential of mathematics publishing on the World Wide Web.

—Allyn Jackson

MR Celebrates Its Sixtieth

At the Joint Mathematics Meetings in Washington, DC, in January 2000, Mathematical Reviews celebrated its sixtieth birthday. It was in 1940 that Otto Neugebauer fled Europe and wound up at Brown University. There he founded MR, which has become one of the most important tools for accessing the mathematical literature.

The celebration featured presentations offering three views on MR: executive editor Jane Kister spoke on MR today, V. Frederick Rickey of the U.S. Military Academy offered a look back at MR's history, and Andrew Odlyzko of AT&T Research pondered what the future might bring. A reception followed the talks.

Available at the celebration was a special booklet produced to mark MR's sixtieth anniversary, MathSciNet—Mathematical Reviews on the Web: Guiding you through the literature of mathematics. Warning readers on the cover that it contains “Everything you wanted to know...and then some,” the booklet offers interesting historical tidbits, descriptions of the MR production process, an overview of the MR database, and a guide to using MathSciNet. The booklet is available free on request from the AMS (see the AMS Bookstore, www.ams.org/bookstore/) and can be downloaded from www.ams.org/mathscinet/guidebook/.

—Allyn Jackson
The Reference section of the Notices is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.

Contacting the Notices
The preferred method for contacting the Notices is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, book reviews and other communications, columns for "Another Opinion", and "Forum" pieces. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.sunysb.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 631-751-5730 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines
April 18, 2000: Proposals for 2001 NSF-CBMS Regional Conferences. See http://www.maa.org/cbms/nsf/2000_conf.htm or contact the Conference Board of the Mathematical Sciences, 1529 18th St. NW, Washington, DC 20036-1385; telephone: 202-293-1170; fax: 202-293-3412; e-mail: kolbe@math.georgetown.edu or rosier@math.georgetown.edu.

April 28, 2000: Nominations for the Maria Mitchell Women in Science Award. Contact Maria Mitchell Association, 2 Vestal Street, Nantucket, MA 02554; telephone: 508-228-9198; or see http://www.mma.org/.

May 1 and October 1, 2000: Applications for NSF/AWM Travel Grants for Women. See http://www.awm-math.org/travelgrants.html; telephone: 301-405-7892; e-mail: awm@math.umd.edu.

Where to Find It
A brief index to information that appears in this and previous issues of the Notices.

AMS e-mail addresses
November 1999, p. 1269
AMS Ethical Guidelines
June 1995, p. 694
AMS officers and committee members
November 1999, p. 1271
Board on Mathematical Sciences and Staff
April 2000, p. 495
Bylaws of the American Mathematical Society
November 1999, p. 1252
Information for Notices authors
January 2000, p. 69

Mathematical Sciences Education Board and Staff
April 2000, p. 495

Mathematics Research Institutes
May 1999, p. 580; August 1999, p. 804

National Science Board
January 2000, p. 71

National Science Foundation
March 2000, p. 381

Regional Conference (NSF-CBMS)
April 2000, p. 495

References
November 1999, p. 1252

Information for Notices authors
January 2000, p. 69
July 31, 2000: Nominations for the Monroe Martin Prize. Contact J. A. Yorke, Director, Institute for Physical Sciences and Technology, University of Maryland, College Park, MD 20742.

August 15, 2000: Third competition for NRC Research Associateships. See http://www.national-academies.org/rap, or contact the National Research Council, Associateship Programs (TJ 2114/D3), 2101 Constitution Avenue, NW, Washington, DC 20418; telephone: 202-334-2760; fax: 202-334-2759; e-mail: rap@nas.edu.

September 1, 2000: Applications for the AWM 2001 Workshop for Women Graduate Students and Postdocs. See “Mathematics Opportunities” in this issue.


Book List
The Book List highlights books that have mathematical themes and hold appeal for a wide audience, including mathematicians, students, and a significant portion of the general public. When a book has been reviewed in the Notices, a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to the managing editor, e-mail: notices@ams.org.


Noeuds: Genèse d'une théorie mathématique (Knots: Genesis of a Mathematical Theory), by Alexei Sossinsky.


*Added to the "Book List" since the List's last appearance.
How to use this form

1. Using the facing page or a photocopy, (or a TeX version which can be downloaded from the e-math "Employment Information" menu, http://www.ams.org/employment/), fill in the answers which apply to all of your academic applications. Make photocopies.

2. As you mail each application, fill in the remaining questions neatly on one cover sheet and include it on top of your application materials.

The Joint Committee on Employment Opportunities has adopted the cover sheet on the facing page as an aid to job applicants and prospective employers. The form is now available on e-math in a TeX format which can be downloaded and edited. The purpose of the cover form is to aid department staff in tracking and responding to each application.

Mathematics Departments in Bachelor's, Master's and Doctorate granting institutions have been contacted and are expecting to receive the form from each applicant, along with any other application materials they require. Obviously, not all departments will utilize the cover form in the same manner. Please direct all general questions and comments about the form to: emp-info@ams.org or call the Professional Programs and Services Department, AMS, at 800-321-4267 extension 4105.

JCEO Recommendations for Professional Standards in Hiring Practices

The JCEO believes that every applicant is entitled to the courtesy of a prompt and accurate response that provides timely information about his/her status. Specifically, the JCEO urges all institutions to do the following after receiving an application:

(1) Acknowledge receipt of the application—immediately; and
(2) Provide information as to the current status of the application, as soon as possible.

The JCEO recommends a triage-based response, informing the applicant that he/she
(a) is not being considered further;
(b) is not among the top candidates; or
(c) is a strong match for the position.
AMS STANDARD COVER SHEET

Last Name ____________________________
First Name ____________________________
Middle Names ____________________________

Address through next June ____________________________ Home Phone ____________________________

Current Institutional Affiliation ____________________________ Work Phone ____________________________

Highest Degree and Source ____________________________
Year of Ph.D. (optional) ____________________________
Ph.D. Advisor ____________________________

If the Ph.D. is not presently held, date on which you expect to receive ____________________________

Indicate the mathematical subject area(s) in which you have done research using, if applicable, the Mathematics Subject Classification printed on the back of this form or on e-MATH. If listing more than one number, list first the one number which best describes your current primary interest.

Primary Interest ____________________________
Secondary Interests optional ____________________________

Give a brief synopsis of your current research interests (e.g. finite group actions on four-manifolds). Avoid special mathematical symbols and please do not write outside of the boxed area.

Most recent, if any, position held post Ph.D.
University or Company ____________________________
Position Title ____________________________

Indicate the position for which you are applying and position posting code, if applicable ____________________________

If unsuccessful for this position, would you like to be considered for a temporary position?
□ Yes □ No If yes, please check the appropriate boxes.

□ Postdoctoral Position □ 2+ Year Position □ 1 Year Position

List the names, affiliations, and e-mail addresses of up to four individuals who will provide letters of recommendation if asked. Mark the box provided for each individual whom you have already asked to send a letter.

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2000
Mathematics
Subject Classification

00 General
01 History and biography
03 Mathematical logic and foundations
05 Combinatorics
06 Order, lattices, ordered algebraic structures
08 General algebraic systems
11 Number theory
12 Field theory and polynomials
13 Commutative rings and algebras
14 Algebraic geometry
15 Linear and multilinear algebra, matrix theory
16 Associative rings and algebras
17 Nonassociative rings and algebras
18 Category theory, homological algebra
19 K-theory
20 Group theory and generalizations
22 Topological groups, Lie groups
26 Real functions
28 Measure and integration
30 Functions of a complex variable
31 Potential theory
32 Several complex variables and analytic spaces
33 Special functions
34 Ordinary differential equations
35 Partial differential equations
37 Dynamical systems and ergodic theory
39 Difference and functional equations
40 Sequences, series, summability
41 Approximations and expansions
42 Fourier analysis
43 Abstract harmonic analysis
44 Integral transforms, operational calculus
45 Integral equations
46 Functional analysis
47 Operator theory
49 Calculus of variations and optimal control, optimization
51 Geometry
52 Convex and discrete geometry
53 Differential geometry
54 General topology
55 Algebraic topology
57 Manifolds and cell complexes
58 Global analysis, analysis on manifolds
60 Probability theory and stochastic processes
62 Statistics
65 Numerical analysis
68 Computer science
70 Mechanics of particles and systems
74 Mechanics of deformable solids
76 Fluid mechanics
78 Optics, electromagnetic theory
80 Classical thermodynamics, heat transfer
81 Quantum theory
82 Statistical mechanics, structure of matter
83 Relativity and gravitational theory
85 Astronomy and astrophysics
86 Geophysics
90 Operations research, mathematical programming
91 Game theory, economics, social and behavioral sciences
92 Biology and other natural sciences
93 Systems theory, control
94 Information and communication, circuits
97 Mathematics education
From the AMS Secretary

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Editor: A.W. Wickstead, Queen's University of Belfast, N. Ireland

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- Potential theory, harmonic analysis, positive harmonic functions and diffusion.
- Variational analysis and variational inequalities.
- Optimization and optimal control theory.
- Convex functions and convex analysis.
- Nonstandard analysis and Boolean valued models.

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Boris S. Mordukhovich, Yongheng Shao, Qiji Zhu

Branching Random Walk in a Catalytic Medium. I. Basic Equations
Sergio Albeverio, Leonid V. Bogachev

Subscription Information:
2000, Volume 4 (4 issues), ISSN 1385-1292
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Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on e-MATH at http://www.ams.org/mathcal1/.

May 2000

*1–6 Isomonodromic Deformations and Applications in Physics, Centre de Recherches Mathématiques, Université de Montréal, Montréal, Quebec, Canada.

**Workshop Topics:** Matrix Riemann-Hilbert problem, nonlinear WKB and applications; Tau functions, Vertex operators, Darboux transformations; Computation of correlation functions in quantum integrable systems and lattice models of statistical mechanics; The spectral theory of random matrices, with applications to quantum gravity; The theory of Frobenius manifolds, with applications to topological field theory, quantum cohomology, mirror symmetry; Scaling reductions of classical integrable systems; Hamiltonian structure and duality.

**Organizers:** J. Harnad (CRM & Concordia Univ.), A. Its (Indiana Univ. / Purdue Univ. at Indianapolis).


*13 East Coast Computer Algebra Day 2000, Ontario, Canada.

**Description:** The 7th East Coast Computer Algebra Day (ECCAD'2000), a joint meeting with Southern Ontario Numerical Analysis Day 2000 and the 70th Birthday Celebration for Professor Hans J. Stetter, will be held on Saturday, May 13, 2000. It will be hosted at the Ontario Research Centre for Computer Algebra, The University of Western Ontario, London, Ontario, Canada. This meeting will be immediately preceded by the Southern Ontario Numerical Analysis Day (SONAD'2000) on May 12, 2000, at the same location.


**Information:** You can register and get more information by looking at the World Wide Web site http://www.orcca.on.ca/events/. Alternatively, send e-mail to one of the organizers, and registration information will be sent to you. There is no registration fee for either conference, and limited travel support may be available on request.

June 2000

*1–3 Fabes Lectures on Real Analysis & PDEs, International Conference, Firenze, Italy.

**Description:** A conference in memory of Gene Fabes, continuing a series of meetings which took place in Italy with the purpose of gathering friends, colleagues, and former students of Gene together.

**Speakers:** The following mathematicians have agreed to give a talk: G. Alessandrini (Trieste), P. Baumann (Purdue), R. Brown (Lenington), L. Caffarelli (Austin), D. Jerison (MIT), M. Jodeit (Minneapolis), P. Manselli (Florence), C. Pagani (Milan), J. K. Seo (Seoul).

**Information:** Anyone interested in receiving subsequent announcements, registration form, and hotel information should send e-mail to fabes00diaga.fi.cnr.it.


**Information:** V. Shtukar, pletnev@gn1.belpak.bogilev.by.

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the Notices if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the Notices in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the Notices prior to the meeting in question. To achieve this, listings should be received in Providence six months prior to the scheduled date of the meeting. The complete listing of the Mathematics Calendar will be published only in the September issue of the Notices. The March, June, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through e-MATH on the World Wide Web. To access e-MATH, use the URL: http://www.ams.org/ (or http://www.ams.org/). For those with VT100-type terminals or for those without WWW browsing software, connect to e-MATH via Telnet (telnet e-math.ams.org; login and password e-math) and use the Lynx option from the main menu.)
Mathematics Calendar

* 8-9 Fifth International Conference on Mathematical Modeling and Application, Institute of Mathematics, Riga Jurmala, Latvia. Focus: The conference focuses on various aspects of mathematical modeling and usage of difference methods for numerical solution of modern problems of science and engineering. It aims, in particular, to foster cooperation among practitioners and theoreticians in this field.

Topics: Analysis of numerical methods for solving problems of mathematical physics, parallel numerical algorithms and parallel computing, applications of difference methods to engineering problems, analysis of ODE and PDE problems and applications. The scientific program includes invited plenary talks (40 min.) and contributed talks (20 min.). A volume of proceedings will be published after the conference. All papers will be refereed.

Program and Organizing Committee: A. Buiks (vice chair), J. Cepitis (secretary), R. Ceigis (vice chair), O. Jadrups, H. Kalis, A. Reinfields (chair), M. Sapagovas, A. Zemtis.

Call for Papers: Abstracts for contributed papers should be received by April 25, 2000.


July 2000

* 10-15 4th World Multiconference on Circuits, Systems, Communications & Computers (CSCC 2000), Vouliagmeni, Greece. Description: The conference, dedicated to the memory of Sidney Darlington (1906-1997), is being held jointly with the 2nd WorldSES MCP and 2nd WorldSES MCME.


Sponsors: WSES, IEEE, IMACS, IARID.

* 17-21 VI Workshop on Real and Complex Singularities, Departamento de Matematica-ICMC-USP, Sao Carlos, Sao Paulo, Brazil.


Organizer: M. J. Saia.


Financial Support: CNPq, Fapesp, FAPESP, Finep, CCInt, SBM.

Information: http://www.icmc.usp.br/~eventos/.

August 2000

* 2-9 Summer School on Quantum Field Theory—From a Hamiltonian Point of View, Sanktjorg Manor, Denmark.

Organizers: J. P. Solovej (Copenhagen) and S. Fournais (Maphysto, Aarhus).


Deadline: The deadline for applications is June 15, 2000.

Information: More information can be obtained from the course home page, located at http://www.maphysto.dk/events/SummerSP2000/.

* 4-10 Stokes Millennium Summer School, Skreen, Co. Sligo, Ireland.

Speakers: P. Clarkson (Univ. of Kent), P.G. Drazin (Univ. of Bristol), P. Lynch (Trinity College Dublin), G.I. Shishkin (Ekeferburg, Russia), B. Straughan (Univ. of Glasgow).

Topics: For the Year 2000 Summer School, held at his birthplace, we will be concentrating on a single aspect of Stokes’s work, the Navier-Stokes Equations: Computational Methods and Applications. These will include physical theory, engineering applications, numerical and analytical methods, including techniques such as asymptotics, singular perturbations, and similarity solutions. The format will be an instructional course followed by a workshop. Participants may register for either or both.

Information: Available by visiting Web site http://www.dcu.ie/math/stokes2.html, by e-mailing alastair.wood@dcu.ie, or by fax to +353 1 704 5786, or by telephone to +353 1 704 5293.

* 16-19 12th Canadian Conference on Computational Geometry, University of New Brunswick, Fredericton, New Brunswick, Canada.

Program: Computational geometry is a discipline concerned with algorithms, software, and mathematical foundations for the treatment of geometric data by computer. The Canadian Conference on Computational Geometry (CCGG) reflects this diversity of interest, with invited speakers and contributed papers on topics ranging from geometric applications in industry to the frontiers of pure mathematics.

Speakers: G. Kalai, Hebrew Univ. of Jerusalem (to be confirmed), T. Pardon Memorial Lecture on Discrete Geometry; N. Kato, Univ. of Kyoto, Applications of Computational Geometry in Architecture.

Information: http://www.cs.ubc.ca/cont/ccgg/ or e-mail: ccgg@umb.ca.

* 17-September 2 Laboratory of Applied Mathematics, Thirty Third International Probability School, Saint-Flour (Cantal), France.

Speakers: S. Albeverio (Univ. Böchum, Germany), Dirichlet forms and infinite dimensional processes: Theory and applications; W. Schachermayer (Univ. Wien, Austria), Mathematical finance; M. Talagrand (CNRS, France), Spin glasses.

Organizer: P. Bernard, Univ. Blaise Pascal, e-mail: bernard@ucfna.univ-bpclermont.fr. This is the Year 2000 Summer School of the European Mathematical Society.

Registration: D. Courgeot, Laboratoire de Mathématiques Appliquées, Les Cézeaux-63177 Aubiere Cédex; tel: +33 (0)4 73 40 70 50; fax:+33 (0)4 73 40 70 64; e-mail: lama@ucfna.univ-bpclermont.fr. Web: http://www.lima.univ-bpclermont.fr/fftflour/. Deadline for registration is April 15, 2000.

September 2000

* 3-6 EMPC 2000 - 31st European Mathematical Psychology Group Meeting, Graz, Austria.

Information: e-mail: empc2000@psyserver.kfunigraz.ac.at; D. Albert, Dept. of Psychology, Universitätsplatz 2/I, A-8010 Graz, Austria; tel: +43 (3) 16-380-518 (-5104); fax: +43 (3) 16-380-9806; URL: http://www.psyserver.kfunigraz.ac.at/empc2000/.

* 5-8 Brno Colloquium on Differential and Difference Equations, Masaryk University in Brno, Czech Republic.

Topics: Qualitative theory of differential and difference equations and applications.

Format: The scientific program consists of survey lectures, lectures, short communications, posters, and extended abstracts. There will be enough time for individual discussions. Proceedings will be published. Approx. 70 participants will be accepted for this colloquium.

Survey Lectures: O. Dosly (Czech Rep.), I. Gyori (Hungary), T. Kusano (Japan), I. Kiguradze (Georgia), M. Marini (Italy), S. Schwabik (Czech Rep.).

Call for Papers: Participants are invited to present communications, posters, or extended abstracts.


Information: http://www.math.muni.cz/cdde/ or e-mail: cdde@math.muni.cz.

* 13-15 7th DIMACS Implementation Challenge: Semidefinite and Related Optimization Problems, DIMACS Center, Rutgers University, Piscataway, New Jersey.

Short Description: In the past few years much has been learned about the kinds of problem classes that SDP can tackle, the best SDP algorithms for the various classes, and the various limits of the current approaches to solving SDP’s. Similar to, and indeed an extension of, semidefinite programming, a great deal is known
about optimization with convex quadratic constraints as well as the
limitation of current methods. This workshop attempts to distill and
expand upon accumulated knowledge. We have collected a variety
of interesting and challenging SDP instances, whose solution would
expand our knowledge on the applicability of SDP.

Deadline for Submissions: March 30, 2000: Progress reports
due for comment and feedback. This should mostly be helpful to
newcomers to the area. August 15, 2000: Extended abstracts due for
consideration for the workshop. All proposals and abstracts should
be sent to the chair of the conference e-mail: challenge@dimacs.rutgers.edu.

Organizers: F. Alizadeh, Rutgers Univ.; D. Johnson, AT & T Labs
Research; G. Pataki, Columbia Univ.

Contacts: F. Alizadeh, Rutgers University, challenge@dimacs.
rutgers.edu. Local arrangements: Jessica Herold, DIMACS Center,
jessica@dimacs.rutgers.edu. 732-445-5928.

Information: http://www.rutgers.edu/Workshops/
index.html.

*22-27 Geometry, Analysis and Mathematical Physics, San Feliu
de Guixols, Spain.

Speakers: B. Altshuler, Princeton, USA; V. Bach, Mainz, Germany; N.
Burq, Orsay, France; V. Buslaev, St. Petersburg, Russia; J. Dereziński,
Warsaw, Poland; B. Eckhart, Marburg, Germany; M. Esteban, Paris,
France; J. Ginibre, Orsay, France; B. Helffer, Orsay, France; V. Ivrii,
Toronto, Canada; V. Jaksic, Ottawa, Canada; A. Knauf, Erlangen,
Germany; A. Komech, Moscow, Russia; G. Lebeau, Palaiseau, France;
C. A. Pillet, Toulon, France; I. M. Sigal, Toronto, Canada; U. Smilanski,
Rehovot, Israel; J. P. Solovej, Copenhagen, Denmark; T. Tao,
Los Angeles, USA; A. Vanney, Berkeley-Boston, USA; G. Velo, Bologna,
Italy; G. Vodev, Nancy, France; S. Zelditch, Baltimore, USA; M.
Zworski, Berkeley, USA.

Scope: The interaction between physics and mathematics has
come back ever more important in recent years. This meeting aims
to bring together mathematicians and some physicists working
in areas related to quantum physics and mathematical analysis.
Lectures and discussions will focus on recent progress in
non-linear scattering theory and evolution equations, solitons, quantum
many-body problems, resonances (scattering poles), field theory,
quantum chaos, and also some related topics such as control
theory, spectral theory on manifolds, semi-classical problems. The
conference is open to researchers worldwide, whether from industry
or academia. Participation will be limited to 100. The emphasis
will be on discussion about new developments. The conference fee
covers registration, full board, and lodging. Grants will be available
for younger scientists, in particular those from less-favored regions
in Europe.

Information: Deadline for applications: May 2000. For information
& application forms, contact the chair of the EURESCO Unit:
Dr. J. Hendelko, European Science Foundation, Ix Lezay-Marnésia,
67080 Strasbourg Cedex, France; tel: +33 388 76 71 35; fax: +33
388 36 69 87; e-mail: eureesco@esf.org. On-line information and
application on Web site http://www.esf.org/eureesco/.

*26-30 XI Biannual Conference of ECMI - The European Consortium
for Mathematics for Industry, Villaggio Torre Normanna, Altavilla
Milicia (Palermo), Italy.

Organizing Committee: A. M. Anile (Univ. of Catania and ECMI
Council); V. Capasso (president of ECMI, 1999-2001); co-chairman;
A. Donato (Univ. of Messina); A. Fasano (Univ. of Florence); A. Greco
(Univ. of Palermo), chairman; R. M. Mattheij (past president of ECMI,

Information: Please visit the Web site http://www.itdf.pa.cnr.
it/ecmi2k/ or http://www.ecmi.de/. For additional information or
to submit contributed talks or minisymposia, please contact
the conference organizers at ecmi2k@pia.itdf.pa.cnr.it.

October 2000

*19 Fargo Preconference Workshop on BVPs and Oscillation
Theory for Differential Equations on Measure Chains, North
Dakota State University, Fargo, North Dakota.
New Publications Offered by the AMS

Algebra and Algebraic Geometry

Higher Regulators, Algebraic K-Theory, and Zeta Functions of Elliptic Curves
Spencer J. Bloch, University of Chicago, IL

This book is the long-awaited publication of the famous Irvine lectures by Spencer Bloch. Delivered in 1978 at the University of California at Irvine, these lectures turned out to be an entry point to several intimately-connected new branches of arithmetic algebraic geometry, such as: regulators and special values of L-functions of algebraic varieties, explicit formulas for them in terms of polylogarithms, the theory of algebraic cycles, and eventually the general theory of mixed motives which unifies and underlies all of the above (and much more). In the 20 years since then, the importance of Bloch's lectures has not diminished. A lucky group of people working in the above areas had the good fortune to possess a copy of old typewritten notes of these lectures. Now everyone can have their own copy of this classic.

Contents: Introduction; Tamagawa numbers; Tamagawa numbers. Continued; Continuous cohomology; A theorem of Borel and its reformulation; The regulator map. I; The dilogarithm function; The regulator map. II; The regulator map and elliptic curves. I; The regulator map and elliptic curves. II; Elements in $K_2(E)$ of an elliptic curve $E$; A regulator formula; Bibliography; Index.

CRM Monograph Series
Mathematics Subject Classification: 19F27; 14G10, 19D50.
All AMS members $19$, List $24$, Order code CRMM-BLOCHN

Analysis

The Backward Shift on the Hardy Space
Joseph A. Cima, University of North Carolina, Chapel Hill, and William T. Ross, University of Richmond, VA

Shift operators on Hilbert spaces of analytic functions play an important role in the study of bounded linear operators. For example, "parts" of direct sums of the backward shift operator on the classical Hardy space $H^2$ model certain types of contraction operators and potentially have connections to understanding the invariant subspaces of a general linear operator.

This book is a thorough treatment of the characterization of the backward shift invariant subspaces of the well-known Hardy spaces $H^p$. The characterization of the backward shift invariant subspaces of $H^p$ for $1 < p < \infty$ was done in a 1970 paper of R. Douglas, H. S. Shapiro, and A. Shields, and the $0 < p \leq 1$ was done in a 1979 paper of A. B. Aleksandrov which is not well known in the West. This material is pulled together in this single volume and includes all the necessary background material needed to understand (especially for the $0 < p < 1$ case) the proofs of these results.

Several proofs of the Douglas-Shapiro-Shields result are provided so readers can get acquainted with different operator theory and theory techniques: applications of these proofs are also provided for understanding the backward shift operator on various other spaces of analytic functions. The results are thoroughly examined. Other features of the volume include a description of applications to the spectral properties of the backward shift operator and a treatment of some general real-variable techniques that are not taught in standard graduate seminars. The book includes references to works by Duren, Garnett, and Stein for proofs, and a bibliography for further exploration in the areas of operator theory and functional analysis.

Contents: Introduction; Classical boundary value results; The Hardy spaces of the disk; The Hardy spaces of the upper-half
plane; The backward shift on $H^p$ for $p \in [1, \infty)$; The backward shift on $H^p$ for $p \in (0,1)$; Bibliography; Index.

Mathematical Surveys and Monographs
Mathematics Subject Classification: 47B38; 46E10, 46E15.
Individual member $29, List $49, Institutional member $39.
Order code SURV-CIMAN

Operator Theory and Its Applications
A. G. Ramm, Kansas State University, Manhattan;
P. N. Shivakumar, University of Manitoba, Winnipeg, MB, Canada, and A. V. Strauss, Ulyanovsk Pedagogical University, Russia, Editors

This volume contains a selection of papers presented at an international conference on operator theory and its applications held in Winnipeg. The papers chosen for this volume are intended to illustrate that operator theory is the language of modern analysis and its applications. Together with the papers on the abstract operator theory are many papers on the theory of differential operators, boundary value problems, inverse scattering and other inverse problems, and on applications to biology, chemistry, wave propagation, and many other areas.

The volume is dedicated to the late A. V. Strauss, whose principal areas of research were spectral theory of linear operators in Hilbert spaces, extension theory for symmetric linear operators, theory of the characteristic functions and functional models of linear operators, and boundary value problems with boundary conditions depending on spectral parameter. The bibliography of publications by A. V. Strauss combined with the papers from the conference provide both historical perspective and contemporary research on the field of operator theory and its applications.


Fields Institute Communications, Volume 25
Mathematics Subject Classification: 47-02, 35-02, 34-02, 81-02.
Individual member $78, List $130, Institutional member $104.
Order code FIC/25N

Differential Equations

Some Current Topics on Nonlinear Conservation Laws
Lectures at the Morningside Center of Mathematics, 1
Ling Hsiao, Institute of Mathematics, Academia Sinica, Beijing, People's Republic of China, and Zhouching Xin, New York University, Courant Institute, NY, Editors

This volume resulted from a year-long program at the Morningside Center of Mathematics at the Academia Sinica in Beijing. It presents an overview of nonlinear conversation laws and introductions to developments in the expanding field of the subject of nonlinear conservation laws. Zhouping Xin's introductory overview of the subject is followed by lecture notes of leading experts who have made fundamental contributions to this field of research. A. Bressan's theory of $L^1$-well-posedness for entropy weak solutions to systems of...
nonlinear hyperbolic conservation laws in the class of viscosity solutions is one of the most important results in the past two decades; G. Chen discusses weak convergence methods and various applications to many problems; P. Degond details mathematical modelling of semi-conductor devices; B. Perthame describes the theory of asymptotic equivalence between conservation laws and singular kinetic equations; Z. Xin outlines the recent development of the vanishing viscosity problem and nonlinear stability of elementary wave—a major focus of research in the last decade; and the volume concludes with Y. Zheng’s lecture on incompressible fluid dynamics.

This collection of lectures represents previously unpublished expository and research results of experts in nonlinear conservation laws and is an excellent reference for researchers and advanced graduate students in the areas of nonlinear partial differential equations and nonlinear analysis.

Titles in this series are co-published with International Press, Cambridge, MA.


AMS/IP Studies in Advanced Mathematics, Volume 15
May 2000, 226 pages, Softcover, ISBN 0-8218-1965-8, LC 00-025164, 2000 Mathematics Subject Classification: 35-02, 35L65, 35L67, 35L60, 35L80, 76N10, 76P05, 46N20, 35Q30, All AMS members $34, List $42, Order code AMSIP/15N

Differential Equations and Mathematical Physics

Rudi Weikard and Gilbert Weinstein, University of Alabama, Birmingham, Editors

This volume contains the proceedings of the 1999 International Conference on Differential Equations and Mathematical Physics. The contributions selected for this volume represent some of the most important presentations by scholars from around the world on developments in this area of research. The papers cover topics in the general areas of linear and nonlinear differential equations and their relation to mathematical physics, including multiparticle Schrödinger operators, stability of matter, relativity theory, fluid dynamics, spectral and scattering theory including inverse problems.

This item will also be of interest to those working in mathematical physics.

Titles in this series are co-published with International Press, Cambridge, MA.

Contents: A. A. Balinsky and W. D. Evans, On the Brown-Ravenhall relativistic Hamiltonian and the stability of matter; R. Bartsch, Assessing accuracy in a numerical Einstein solver; R. D. Benguria and M. C. Depassier, Variational principle for the limit cycle of Rayleigh’s equation; B. K. Berger, Approach to the singularity in spatially inhomogeneous cosmologies; M. Sh. Birman and T. A. Suslina, On the absolute continuity of the periodic Schrödinger and Dirac operators with magnetic potential; T. Bodineau and B. Helffer, Correlations, spectral gap and log-Sobolev inequalities for unbounded spins systems; R. Brummelhuis, M. B. Ruskai, and E. Werner, One-dimensional regularizations of the Coulomb potential with application to atoms in strong magnetic fields; D. Chae and O. Yu. Imanuvilov, Construction of a solution to the semi-linear elliptic equation in Chern-Simons gauge theory; M. Christ, A. Kiselev, and Y. Last, Approximate eigenvectors and spectral theory; D. Christodoulou, The initial value problem in the large and spacetime singularities; L. Erdös and J. P. Solovej, On the kernel of $\gamma_i^\ast$; Dirac operators on $S^2$ and $\mathbb{R}^2$; R. Froese and I. Herbst, Realizing holonomic constraints in classical and quantum mechanics; F. Gesztesy and H. Holden, A combined sine-Gordon and modified Korteweg-de Vries hierarchy and its algebro-geometric solutions; M. Griesemer, A minimax principle for eigenvalues in spectral gaps; G. A. Hagedorn and A. Joyce, Semiclassical dynamics and exponential asymptotics; R. Hempel and K. Lienau, Genericity of the band-gap structure of periodic media in the large coupling limit; A. M. Hinz, Distribution of eigenvalues in the dense point spectrum of Schrödinger operators; P. D. Hislop, On the distribution of scattering resonances for asymptotically hyperbolic manifolds; T. Hupfer, H. Leschke, and S. Warzel, The multiformity of Lifshits tails caused by random Landau Hamiltonians with repulsive impurity potentials of different decay at infinity; W. Karwowski and V. Koshmanenko, Schrödinger operator perturbed by dynamics of lower dimension; Y. V. Kurylenko and M. Lassas, Hyperbolic inverse problem with data on a part of the boundary; Y. Li, Best Sobolev inequalities on Riemannian manifolds; E. H. Lieb and M. Loss, Self-energy of electrons in non-perturbative QED; E. H. Lieb and J. Yngvason, The ground state energy of a dilute Bose gas; M. Ohnima, Formulæ and completely integrable Hamiltonians; Y. Pinchover, On the maximum and anti-maximum principles; T. C. Sideris, The null condition and global existence of nonlinear elastic waves; H. Siedentop, The Hartree-Fock approximation in quantum electrodynamics—Positivity of the energy; J. A. Smoller and J. B. Temple, Shock-wave cosmology; S. B. Sontz, On some inverse inequalities in the Segal-Bargmann space; G. Teschl, On the initial value problem of the Toda and Kac-van Moerbeke hierarchies; V. Tkachenko, A class of non-selfadjoint Hill’s operators with analytic potentials; M. M. Tom, Regularized long wave-KP models; C. Tretter, Spectral issues for block operator matrices; I. A. Vdovlevsky, Some fully nonlinear equations in conformal geometry; R. Weder, $L^p - L^q$ estimates for the Schrödinger equation and inverse scattering; G. Wolanski, Stationary states of Vlasov system.

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MAY 2000

NOTICES OF THE AMS

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Algebraic Numbers and Functions
Helmut Koch, Humboldt-University, Berlin, Germany

Algebraic number theory is one of the most refined creations in mathematics. It has been developed by some of the leading mathematicians of this and previous centuries. The primary goal of this book is to present the essential elements of algebraic number theory, including the theory of normal extensions up through a glimpse of class field theory. Following the example set for us by Kronecker, Weber, Hilbert and Artin, algebraic functions are handled here on an equal footing with algebraic numbers. This is done on the one hand to demonstrate the analogy between number fields and function fields, which is especially clear in the case where the ground field is a finite field. On the other hand, in this way one obtains an introduction to the theory of 'higher congruences' as an important element of 'arithmetic geometry'.

Early chapters discuss topics in elementary number theory, such as Minkowski's geometry of numbers, public-key cryptography and a short proof of the Prime Number Theorem, following Newman and Zagier. Next, some of the tools of algebraic number theory are introduced, such as ideals, discriminants and valuations. These results are then applied to obtain results about function fields, including a proof of the Riemann-Roch Theorem and, as an application of cyclotomic fields, a proof of the first case of Fermat's Last Theorem. There are a detailed exposition of the theory of Hecke L-series, following Tate, and explicit applications to number theory, such as the Generalized Riemann Hypothesis. Chapter 9 brings together the earlier material through the study of quadratic number fields. Finally, Chapter 10 gives an introduction to class field theory.

The book attempts as much as possible to give simple proofs. It can be used by a beginner in algebraic number theory who wishes to see some of the true power and depth of the subject.

Contents: Introduction; The geometry of numbers; Dedekind's theory of ideals; Valuations; Algebraic functions of one variable; Normal extensions; L-series; Applications to Hecke L-series; Quadratic number fields; What next?; Divisibility theory; Trace, norm, different, and discriminant; Harmonic analysis on locally compact abelian groups; References; Index.

Graduate Studies in Mathematics


Large Deviations
Frank den Hollander, Nijmegen University, Netherlands

This volume offers an introduction to large deviations. It is divided into two parts: theory and applications. Basic large deviation theorems are presented for i.i.d. sequences, Markov sequences, and sequences with moderate dependence. The rate function is computed explicitly. The theory is explained without too much emphasis on technicalities. Also included is an outline of general definitions and theorems. The goal is to expose the unified theme that gives large deviation theory its overall structure, which can be made to work in many concrete cases. The section on applications focuses on recent work in statistical physics and random media.

This book contains 60 exercises (with solutions) that should elucidate the content and engage the reader. Prerequisites for the book are a strong background in probability and analysis and some knowledge of statistical physics. It would make an excellent textbook for a special topics course in large deviations. This item will also be of interest to those working in mathematical physics.

Fields Institute Monographs, Volume 14

An Introduction to the Theory of Local Zeta Functions
Jun-ichi Igusa, Johns Hopkins University, Baltimore, MD

This book is an introductory presentation to the theory of local zeta functions. As distributions, and mostly in the archimedean case, local zeta functions are called complex powers. The volume contains major results on complex powers by Atiyah, Bernstein, I. M. Gelfand, and S. I. Gelfand. Also included are related results by Sato. The section on p-adic local zeta functions presents Serre's structure theorem, a rationality theorem and many examples by the author. It concludes with theorems by Denef and Meuser.

Prerequisites for understanding the text include basic courses in algebra, calculus, complex analysis, and general topology. The book follows the usual pattern of progress in mathematics: examples are given, conjectures follow, conjectures are developed into theorems.

This book is accessible and self-contained. Results illustrate the unity of mathematics by gathering important theorems from algebraic geometry and singularity theory, number theory, algebra, topology, and analysis. The ideas are then employed in essential ways to prove the theorems.

Titles in this series are co-published with International Press, Cambridge, MA.

AMS/IP Studies in Advanced Mathematics, Volume 14
Problems in Mathematical Analysis I
Real Numbers, Sequences and Series

W. J. Kaczor and M. T. Nowak, Marie Curie-Sklodowska University, Lublin, Poland

We learn by doing. We learn mathematics by doing problems. This book is the first volume of a series of books of problems in mathematical analysis. It is mainly intended for students studying the basic principles of analysis. However, given its organization, level, and selection of problems, it would also be an ideal choice for tutorial or problem-solving seminars, particularly those geared toward the Putnam exam. The volume is also suitable for self-study.

Each section of the book begins with relatively simple exercises, yet may also contain quite challenging problems. Very often several consecutive exercises are concerned with different aspects of one mathematical problem or theorem. This presentation of material is designed to help student comprehension and to encourage them to ask their own questions and to start research. The collection of problems in the book is also intended to help teachers who wish to incorporate the problems into lectures. Solutions for all the problems are provided.

The book covers three topics: real numbers, sequences, and series, and is divided into two parts: exercises and/or problems, and solutions. Specific topics covered in this volume include the following: basic properties of real numbers, continued fractions, monotonic sequences, limits of sequences, Stolz's theorem, summation of series, tests for convergence, double series, arrangement of series, Cauchy product, and infinite products.

Student Mathematical Library, Volume 4
April 2000, 380 pages, Softcover, ISBN 0-8218-2050-8, LC 99-087039, 2000 Mathematics Subject Classification: 00A07; 40-01,
All AMS members $31, List $39, Order code STML/4RT005

Dynamical Properties of Diffeomorphisms of the Annulus and of the Torus

Patrice Le Calvez, University of Paris, Villetaneuse, France

The first chapter of this monograph presents a survey of the theory of monotone twist maps of the annulus. First, the author covers the conservative case by presenting a short survey of Aubry-Mather theory and Birkhoff theory, followed by some criteria for existence of periodic orbits without the area-preservation property. These are applied in the area-decreasing case, and the properties of Birkhoff attractors are discussed. A diffeomorphism of the closed annulus which is isotopic to the identity can be written as the composition of monotone twist maps.

The second chapter generalizes some aspects of Aubry-Mather theory to such maps and presents a version of the Poincaré-Birkhoff theorem in which the periodic orbits have the same braid type as in the linear case. A diffeomorphism of the torus isotopic to the identity is also a composition of twist maps, and it is possible to obtain a proof of the Conley-Zehnder theorem with the same kind of conclusions about the braid type, in the case of periodic orbits. This result leads to an equivariant version of the Brouwer translation theorem which permits new proofs of some results about the rotation set of diffeomorphisms of the torus.

This is the English translation of a volume previously published as volume 204 in the Astérisque series.

SMF members are entitled to AMS member discounts.

SMF/AMS Texts and Monographs, Volume 4
All AMS members $17, List $21, Order code SMFAMS/4RT005
Introduction to Hilbert Space
Sterling K. Berberian
Completely self-contained ... All proofs are given in full detail ... recommended for unassisted reading by beginners ... For teaching purposes this book is ideal.
—Proceedings of the Edinburgh Mathematical Society
AMS Chelsea Publishing; 1961; ISBN 0-8218-1912-7; 206 pages; Hardcover; All AMS members $15, List $17, Order Code CHEL/287.HCT005

Introduction to Probability
Second Revised Edition
Charles M. Grinstead, Swarthmore College, PA, and J. Laurie Snell, Dartmouth College, Hanover, NH
The book is a beautiful introduction to probability theory at the beginning level. The book contains a lot of examples and an easy development of theory without any sacrifice of rigor, keeping the abstraction to a minimal level.
—Zentralblatt für Mathematik
1977; ISBN 0-8218-0749-8; 510 pages; Hardcover; All AMS members $39, List $49, Order Code IPROBCT005

Groups and Symmetry: A Guide to Discovering Mathematics
David W. Farmer, Bucknell University, Lewisburg, PA
Nicely produced and concentrates on the informal analysis of geometrical patterns with the emphasis on informality .... could serve as a useful collection of activities to precede a formal course.
—The Mathematical Gazette
1999; ISBN 0-8218-0451-0; 101 pages; Softcover; All AMS members $15, List $19, Order Code MAWRLD/6CT005

Knots and Surfaces: A Guide to Discovering Mathematics
David W. Farmer, Bucknell University, Lewisburg, PA, and Theodore B. Stanford, University of Nevada, Reno
Would serve well as the basis of an independent study course in which the student would work through the tasks in a journal subject to periodic review by the instructor .... the writing is clear and engaging, and the tasks should be effective at setting a reasonable pace.
—American Mathematical Monthly

How to Teach Mathematics
Second Edition
Steven G. Krantz, Washington University, St. Louis, MO
Praise for the First Edition ...
An original contribution to the educational literature on teaching mathematics at the post-secondary level. The book itself is an explicit proof of the author's claim "teaching can be rewarding, useful, and fun".
—Zentralblatt für Mathematik
1999; ISBN 0-8218-1398-6; 307 pages; Softcover; All AMS members $19, List $24, Order Code HTM/2CT005

Techniques of Problem Solving
Steven G. Krantz, Washington University, St. Louis, MO
Krantz has collected a thoroughly engaging arsenal of problems and problem-solving techniques ... After a delightful introductory chapter, the chapters are primarily organized around specific techniques and their applicability in areas such as geometry, logic, recreational math, and counting.
—CHOICE
1997; ISBN 0-8218-0619-X; 465 pages; Softcover; All AMS members $23, List $28, Order Code TPSCT005

Solutions Manual for Techniques of Problem Solving
Luis Fernandez and Haedeh Gooransarab, Washington University, St. Louis, MO
1997; ISBN 0-8218-0639-8; 158 pages; Softcover; All AMS members $10, List $12, Order Code SMTPSCT005

Algebra
Third Edition
Saunders Mac Lane, University of Chicago, IL, and Garrett Birkhoff
The book is clearly written, beautifully organized, and has an excellent and wide-ranging supply of exercises ... contains ample material for a full-year course on modern algebra at the undergraduate level.
—Mathematical Reviews
1988; ISBN 0-8218-1646-2; 626 pages; Hardcover; All AMS members $35, List $49, Order Code CHEL/390.HCT005

For more publications in your subject area visit the AMS Bookstore: www.ams.org/bookstore.
The mathematics department of Loyola Marymount University invites applications for the Clarence J. Wallen, S. J. Endowed Chair in Mathematics. The individual holding the chair will teach two classes per semester, carry out his/her own research agenda, develop programs that involve the undergraduate mathematics majors in research or professional activities, and engage in departmental and university service. Individuals working in any mathematical area, including mathematics education (especially K-12 teacher preparation), are invited to apply.

The appropriate candidate will have an established scholarly and academic record and should be able to demonstrate success at involving undergraduates in research or professional activities. The appointment to the endowed chair will provide a competitive salary at the rank of associate or full professor and budgetary support for program development and research activities. Applications must include a letter of interest that briefly outlines a plan for the development of a program that will involve undergraduates in research or professional activities, a curriculum vitae, and the names of three references. References may be contacted during the initial screening of applications; finalists for the position will be asked to provide three letters of reference.

The position will remain open until filled. The appointment could begin as early as the fall of 2000. Loyola Marymount University is a comprehensive Catholic university whose focus is excellence in undergraduate education. The mathematics department, housed within the university's College of Science and Engineering, is a community of 15 full-time faculty members and 30-40 mathematics majors who work in an atmosphere of mutual respect and collegiality. Additional information about the LMU mathematics department and this position can be found on the Web at http://cse.lemou.edu/natmath/lnumath.html. Please send applications and inquiries to:

Dr. Gerald Jakubowski, Dean
College of Science and Engineering
Loyola Marymount University
7900 Loyola Boulevard
Los Angeles, CA 90045-8135
e-mail: gjakubow@lmu.edu
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NEW YORK

NEW CITY TECHNICAL COLLEGE
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Mathematics Dept./Asst. Professor
Tenure-track assistant professor. Requirements: Ph.D. in mathematics, minimum 2 years' experience in teaching mathematics courses up to and including calculus, demonstrated ability to interact with students in a multicultural environment, excellent communication skills in both written and spoken English, knowledge of computers. Preferred Qualifications: Expertise in use of graphing calculators and computers in instruction, other forms of innovative pedagogy, experience in grant proposal writing and curriculum development. Applications will be accepted until position is filled. Résumés to: Michelle Schlein, Human Resources, New York City Technical College, 300 Jay St., Namm 321, Brooklyn, NY 11201. AA/EEO/ADA/IRCA.

SIENA COLLEGE

The Department of Mathematics invites applications for tenure-track and visiting faculty positions starting in the fall of 2000, subject to administrative approval. A Ph.D. in the mathematical sciences is required, as is a strong commitment to undergraduate teaching and to undergraduate research development. Strong preference will be given to applicants who can teach computer science courses as well as mathematics.

Siena College is a liberal arts college of 2,600 full-time undergraduates located just north of Albany in upstate New York. There are numerous cultural, professional, and recreational opportunities in the area.

Applications should send a curriculum vitae along with a description of their teaching philosophy and research interests to:

Professor Steven Bloom, Chairman
Department of Mathematics

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2000 rate is $100 per inch or fraction thereof on a single column (one-inch minimum), calculated from top of headline. Any fractional text of 1/2 inch or more will be charged at the next inch rate. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted. There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified advertising.

Upcoming deadlines for classified advertising are as follows: June/July issue—April 25, 2000; August issue—May 18, 2000; September issue—June 20, 2000; October issue—July 19, 2000; November issue—August 23, 2000; December issue—September 29, 2000.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send e-mail to classifieds@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.
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New Paltz, New York 12561-2440
The Department of Mathematics and Computer Science invites applications for three 1-year visiting positions in mathematics. These positions are sabbatical replacements for the 2000-01 academic year and require excellent teaching skills.

One position is for a visiting assistant professor to teach a variety of upper-division courses. A Ph.D. in mathematics is required. Two positions are for visiting lecturers to teach a variety of courses beyond the elementary level. A master’s degree in mathematics is required; a Ph.D. is preferred. Send a curriculum vitae and three letters of reference to the address above.

The department offers a bachelor’s degree in mathematics, an M.A. in mathematics, and an M.S. in secondary education in mathematics. For additional information about the department and the college, see http://www.mcs.newpaltz.edu/. New Paltz is located in the mid-Hudson Valley, 75 miles north of New York City.

We will accept applications until we fill the positions. We are an AA/EOE/ADA Employer.

SWITZERLAND

SWISS FEDERAL INSTITUTE OF TECHNOLOGY ZURICH (ETHZ)
Assistant Professor of Mathematics

Duties of this position include, in addition to research, an active participation in the teaching of courses for students of mathematics, natural sciences, and engineering. Candidates should have the doctorate or equivalent and have demonstrated the ability to carry out independent research. Willingness to teach at all university levels and to collaborate with colleagues is expected.

This assistant professorship has been established to promote the careers of younger scientists. Initial appointment is for three years, with the possibility of renewal for an additional three years.

Applications with curriculum vitae and a list of publications should be submitted to the president of ETH Zurich, Prof. Dr. O. Kübler, ETH Zentrum, CH-8092 Zurich, no later than May 20, 2000 (or by e-mail also to the assistant to the president, Dr. Th. Eichenberger, eichenberger@al.ethz.ch). The ETHZ specifically encourages female candidates to apply with a view towards increasing the proportion of female professors.

PUBLICATIONS WANTED

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Pure & appl. adv. & research level, any age, usable cond. Reprints OK. One box to whole libraries sought. Contact: Collier Brown or Kirsten Berg @ Powell’s Technical Bks., Portland, OR. Call 800-225-6911, fax 503-228-0505, or e-mail kirsten@technical.powells.com.

NOTICES OF THE AMS
VOLUME 47, NUMBER 5

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Confidence in knowing. It's important to feel secure about your insurance coverage. Now you can. AMS carefully selects experienced providers with the financial stability to ensure competitive insurance options for its members.

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The catastrophic major medical and high limit accident insurance plans are underwritten by The United States Life Insurance Company in the city of New York, (member American General Financial Group), 5000 Route 9, P.O. Box 1380, Neptune, NJ 07753-1380.
The term life insurance plan is underwritten by Connecticut General Life Insurance Company, a CIGNA Company, Hartford, CT 06152.
Volume 6, 2000 (Most Recent Articles)

S. R. Bullett and W. J Harvey, Mating quadratic maps with Kleinian groups via quasiconformal surgery

A. A. Kirillov, Family algebras

Alejandro Adem and Jeff H. Smith, On spaces with periodic cohomology


ERA-AMS publishes high-quality research announcements of significant advances in all branches of mathematics. Authors may submit manuscripts to any editor. All papers are reviewed, and the entire Editorial Board must approve the acceptance of any paper. Papers are posted as soon as they are accepted and processed by the AMS.

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Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the Notices. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on e-MATH. See http://www.ams.org/meetings/. Programs and abstracts will continue to be displayed on e-MATH in the Meetings and Conferences section until about three weeks after the meeting is over. Final programs for Sectional Meetings will be archived on e-MATH in an electronic issue of the Notices as noted below for each meeting.

Odense, Denmark

Odense University

Note: This is a World Math Year 2000 (WMY2000) event.

June 13–16, 2000

Meeting #955

First AMS-Scandinavian International Mathematics Meeting.
Sponsored by the AMS, Dansk Matematikforening, Suomen matemaattinen yhdistys, Icelandic Mathematical Society, Norsk Matematisk Forening, and Svenska matematikföreningen.

Associate secretary: Robert M. Fossum
Announcement issue of Notices: March 2000
Program first available on e-MATH: Not applicable
Program issue of electronic Notices: None
Issue of Abstracts: None

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: March 15, 2000

Invited Addresses
Tobias Colding, Courant Institute, New York University, Embedded minimal surfaces and applications to 3-manifold topology.

Johan Håstad, Royal Institute of Technology, Stockholm, On efficient approximability of NP-hard optimization problems.
Nigel J. Hitchin, University of Oxford, The geometry of three-forms.
Elliott Lieb, Princeton University, Title to be announced.
Pertti Mattila, University of Jyväskylä, What has Menger curvature given to complex and harmonic analysis?
Curtis T. McMullen, Harvard University, The shape of the moduli space of Riemann surfaces.
Alexei N. Rudakov, Norwegian University of Science & Technology, Representations of $E(3,6)$ and the related structures.
Karen Uhlenbeck, University of Texas, Austin, Integrable systems in geometry.
Dan Voiculescu, University of California, Berkeley, Title to be announced.

Special Sessions
Algebraic Groups and Representation Theory, Henning Haahr Andersen and Niels Lauritzen, Aarhus University.
Complex Analysis in Higher Dimensions, Finnur Larusson, University of Western Ontario, and Ragnar Sigurdsson, University of Iceland.
Differential Geometry, Claude R. LeBrun, State University of New York at Stony Brook, and Peter Petersen, University of California, Los Angeles.
Discrete Mathematics, Jiri S. Honkala, University of Turku, and Carsten Thomassen, Technical University of Denmark.
Dynamical Systems, Michael Benedicks, Royal Institute of Science, Stockholm, and Carsten Lunde Petersen, Roskilde.

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NOTICES OF THE AMS 607
Geometric Analysis/PDE, Gerd Grubb, University of Copenhagen, and Bent Ørsted, Odense University.

Joint EWM and AWM Session, Lisbeth Fajstrup, Aalborg University, Tinne Hoff Kjeldsen, Roskilde, and Christina Wiis Tommesen-Friedman, Aarhus University.


Linear Spaces of Holomorphic Functions, Peter L. Duren, University of Michigan, Ann Arbor, Michael Stessin, State University of New York, Albany, and Harold S. Shapiro, Royal Institute of Technology, Stockholm.

Mathematical Physics, Bergfinnur Durhuus, University of Copenhagen, and Kurt Johansson, Royal Institute of Technology, Stockholm.

Mathematics Education, Claus Michelsen, Odense, and Martti E. Pesonen, University of Joensuu.

Stochastic DE and Financial Mathematics, Tomas Björk, University of Stockholm, and Bernt Øksendal, University of Oslo.

Michael V. Berry, University of Bristol, Wave asymptotics and borderland physics.

Haim Brezis, University of Paris XI and Rutgers University, The interplay between analysis and topology in some non-linear PDEs.

Alain Connes, Collège de France, Noncommutative geometry.

David L. Donoho, Stanford University, will speak on interactions among harmonic analysis, statistical analysis, and information theory.

Charles L. Fefferman, Princeton University, Unsolved problems of fluid mechanics.

Michael H. Freedman, Microsoft Research, The physics of computation.


Helmut H. W. Hofer, Courant Institute, New York University, Dynamical systems at the interface of symplectic geometry and three-dimensional topology.

Richard M. Karp, University of Washington, will speak on Mathematical challenges from genomics and molecular biology.

Sergiu Klainerman, Princeton University, On the analysis of geometric evolution equations.

Maxim Kontsevich, Institut des Hautes Études Scientifiques, will speak on deformations, supermanifolds, and homotopical algebra.

Peter D. Lax, Courant Institute, New York University, Mathematics and computing.

Simon A. Levin, Princeton University, Ecosystems as complex adaptive systems.

László Lovász, Yale University, Algorithms and their analysis: Classical mathematics and new challenges.

David Mumford, Brown University, Modeling perception and inference in intelligent systems.

Peter Sarnak, Princeton University, Some problems in number theory and related analysis.

Saharon Shelah, The Hebrew University and Rutgers University, Logical dreams.

Peter W. Shor, AT&T Labs, will speak on quantum computing/quantum information theory.

Yakov G. Sinai, Princeton University, From renormalization in dynamics to renormalization in statistical physics.

Richard P. Stanley, Massachusetts Institute of Technology, Recent progress in algebraic combinatorics.

Dennis P. Sullivan, The CUNY Graduate School, String topology.

Clifford Taubes, Harvard University, Bliss and Ignorance in 4-dimensions.

Jean E. Taylor, Rutgers University, Applications of geometric analysis.
William P. Thurston, University of California, Davis, will speak on three-dimensional topology and geometry.
Karen Uhlenbeck, University of Texas, Austin, Geometric partial differential equations: From Hilbert's Twenty-third Problem to nonlinear waves.
S. R. S. Varadhan, Courant Institute, New York University, Stochastic analysis and applications.
Edward Witten, Institute for Advanced Study, will speak on the mathematical impact of quantum fields and strings.
Shing-Tung Yau, Harvard University, will speak on geometry and its relation to physics.
Don B. Zagier, Max-Planck-Institut für Mathematik, will speak on number theory: modular forms.

Contributed Papers
Sessions for ten-minute contributed papers will be held over the long lunch hour and perhaps on some evenings. Papers will be grouped by related subject classifications into sessions. Abstracts will be published in the program. To submit an abstract, see www.ams.org/abstracts/instructions.html, or send an empty e-mail message to abs-submit@ams.org and type help as the subject line to receive instructions through e-mail.

Opening Ceremonies and Reception
At the conclusion of the AMS-MAA Presidents' Lecture by Ronald L. Graham, all participants are invited to the Mathematical Challenges Opening Ceremonies on Sunday at 5:00 p.m. in UCLA's spectacular Royce Hall. As we hear from organizers and distinguished guests, we will begin to explore and celebrate the glorious future of mathematics. A reception will be held immediately following remarks by the speakers.

Other Events of Interest
AMS Information Booth: All Math Challenges participants are invited to visit the AMS Information Booth during the meeting. Complimentary coffee and tea will be served. A special gift will be available for participants, compliments of the AMS. The membership manager of the Society will be at the booth to answer questions about membership.

Book Sales and Exhibits: All participants are encouraged to visit the book, education media, and software exhibits from 8:00 a.m. to 4:30 p.m. Monday through Friday, and from 8:00 a.m. to noon on Saturday. Books published by the AMS will be sold at discounted prices somewhat below the cost for the same books purchased by mail. These discounts will be available only to registered participants wearing the official Math Challenges badge. Most major credit cards will be accepted for book sale purchases at the meetings. Also, AMS electronic products and e-MATH will be demonstrated.
Social Events

It is strongly recommended that for any event requiring a ticket, tickets should be purchased through advance registration. Only a very limited number of tickets, if any, will be available for sale on site. If you must cancel your participation in a ticketed event, you may request a 50% refund by returning your ticket(s) to the Mathematics Meetings Service Bureau (MMSB) by July 10, 2000. After that date no refunds can be made. Special meals are available at the banquet upon advance request, but this must be indicated on the Advance Registration/Housing Form.

The Math Challenges Pub, a private cash bar, will be open for meeting participants only on Monday through Friday evenings on the terrace outside the Grand Horizons Ballroom in Covel Commons. The Commons is adjacent to the hotel-style residence halls where meeting participants will stay and serves as the dining facility. The terrace overlooks the beautiful UCLA campus. Join your colleagues for a glass of wine, beer, or soft drinks at the end of the day to discuss mathematical issues (blackboards will be provided) or simply to watch the sunset.

Saturday, August 5

All participants are invited to a reception at the new NSF-funded Institute for Pure and Applied Mathematics (IPAM). See www.ipam.org/ for more information on this new institute, whose main purpose is to encourage the cross-fertilization between mathematics and other scientific disciplines and to broaden the range of mathematical techniques used in science. Time to be announced.

Sunday, August 6

Getty Museum Tour: The Getty offers a vast collection of European paintings, decorative arts, Old Master drawings, and Renaissance manuscripts, and American and European photographs. The beautifully landscaped grounds feature tranquil gardens and panoramic views of the city, mountains, and sea. Buses will leave the UCLA campus at approximately 10:00 a.m. and return at 3:00 p.m. For those on the UCLA meal plan, box lunches will be provided. There are restaurants at the museum. Cost for this trip is $5/person; there is no charge for a child sitting on an adult’s lap. Space is limited, so reserve early during advance registration! See www.getty.edu/ for more information on special exhibitions, programs, and children’s activities.

Tuesday, August 8

Southern California Barbecue: Bring your appetite for a country shindig! Feast on mesquite-grilled chicken, pork spareribs with hickory barbecue sauce, tossed green salad, red potato salad, corn on the cob, and country biscuits. Top it all off with some mouthwatering strawberry shortcake! So kick off your shoes, relax with your colleagues outdoors in Sunset Village, and enjoy this bountiful buffet. Cash bar available. Cost is $27 per person; children 5 to 11 are $12; no charge for children 4 and under.

Thursday, August 10

A Night at the Hollywood Bowl: Here is your chance to experience this historic outdoor amphitheater, the Los Angeles Philharmonc’s summer home. This evening’s theme will be “Thunder and Lightning”, featuring the music of Beethoven and Tchaikovsky. Buses will take us from the UCLA campus at about 7:30 p.m., returning at approximately 10:30 p.m. Cost is $45 per person, including round-trip bus transportation. Reserve early during advance registration! Tickets are limited for this fabulous evening of music under the stars.

Saturday, August 12

Millennium Banquet: At the conclusion of the week’s plenary talks, come join your colleagues to summarize what the mathematical challenges of the 21st century will be. Enjoy a beverage at the cash bar, then dine on grilled fillet of salmon glazed with a caramelized orange sauce, accompanied by herbed orzo and a salad of fresh tomatoes, grilled vegetables, and goat cheese, dressed with a champagne vinaigrette; dessert is blackberry sorbet garnished with fresh berries. Complimentary wine will be served with dinner. This evening’s event will feature a special presentation by Persi Diaconis, probabilist, magician, and expert on the mathematics of card shuffling, in addition to being a very stimulating lecturer. Don’t miss this marvelous event celebrating the achievements of mathematics! Cost is $35 per person.

Registering in Advance and Hotel/Residence Hall Accommodations

How to Register in Advance: The importance of advance registration cannot be overemphasized. Advance registration fees are considerably lower than the fees that will be charged for registration at the meeting. Participants registering by June 8, 2000, will receive their badges, programs, and tickets purchased in advance by mail approximately three weeks before the meeting, unless they check the appropriate box to the contrary on the Advance Registration/Housing Form. Because of delays that occur in U.S. mail to Canada, it is strongly suggested that advance registrants from Canada choose to pick up their materials at the meeting. Because of delays that occur in U.S. mail to overseas, we do not mail materials overseas. There will be a special Registration Assistance Desk at the meetings to assist individuals who either do not receive this mailing or who have a problem with their registration. Please note that a $5 replacement fee will be charged for programs and badges that are mailed but not taken to UCLA. Acknowledgments of registrations will be sent by e-mail to the e-mail addresses given on the Advance Registration/Housing Form. If you do not wish your registration acknowledged by e-mail, please mark the appropriate box on the form.

E-mail Advance Registration: This service is available for advance registration and housing arrangements. Please request the e-mail registration form either by sending e-mail to meetreg-request@ams.org or by going to http://www.ams.org/amsmtgs/mathchall.html and
looking under "Registration". VISA, MasterCard, Discover, and American Express are the only methods of payment which can be accepted for e-mail advance registration, and charges to credit cards will be made in U.S. funds. Completed e-mail forms should be sent to meetreg-submit@ams.org. All advance registrants will receive acknowledgment of payment prior to the meetings.

**Internet Advance Registration:** This service is available for advance registration and housing arrangements at [http://www.ams.org/amstgs/mathchal1_regsh.html](http://www.ams.org/amstgs/mathchal1_regsh.html) or by going directly to [https://www.ams.org/cgi-bin/meetreg/meetings?meetnum=2062/](https://www.ams.org/cgi-bin/meetreg/meetings?meetnum=2062/). VISA, MasterCard, Discover, and American Express are the only methods of payment that can be accepted for Internet advance registration, and charges to credit cards will be made in U.S. funds. All Internet advance registrants will see an on-screen acknowledgment upon the submission of this form.

**Cancellation Policy:** Those who cancel their advance registration for the Mathematical Challenges meeting by July 27, 2000, will receive a 50% refund of fees paid. No refunds will be issued after this date.

### Mathematical Challenges Registration Fees

<table>
<thead>
<tr>
<th>Description</th>
<th>By July 10</th>
<th>At Meeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Member of AMS, CMS, MAA, SIAM</td>
<td>$155</td>
<td>$202</td>
</tr>
<tr>
<td>Temporarily Employed</td>
<td>115</td>
<td>127</td>
</tr>
<tr>
<td>Emeritus Member of AMS, MAA:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graduate Student, Unemployed</td>
<td>35</td>
<td>45</td>
</tr>
<tr>
<td>Librarian, High School Teacher</td>
<td>20</td>
<td>26</td>
</tr>
<tr>
<td>Developing Countries Special Rate</td>
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<td>312</td>
</tr>
<tr>
<td>Undergraduate Student</td>
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<td>5</td>
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<tr>
<td>Nonmember</td>
<td>240</td>
<td>312</td>
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<tr>
<td>High School Student</td>
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<td>5</td>
</tr>
<tr>
<td>One-Day Member</td>
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<tr>
<td>of AMS, CMS, MAA, SIAM</td>
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<td>172</td>
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<tr>
<td>One-Day Nonmember</td>
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</tr>
<tr>
<td>Special: 3-day Member Registration</td>
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<td>132</td>
</tr>
<tr>
<td>3-day Nonmember Registration</td>
<td>132</td>
<td>204</td>
</tr>
<tr>
<td>Nonmathematician Guest</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

**Full-Time Students:** Those currently working toward a degree or diploma. Students are asked to determine whether their status can be described as graduate (working toward a degree beyond the bachelor's), undergraduate (working toward a bachelor's degree), or high school (working toward a high school diploma) and to mark the Advance Registration/Housing Form accordingly.

**Emeritus:** Persons who qualify for emeritus membership in either Society. The emeritus status refers to any person who has been a member of the AMS for twenty years or more and who retired because of age or long-term disability from his or her latest position.

**Librarian:** Any librarian who is not a professional mathematician.

**Unemployed:** Any person currently unemployed, actively seeking employment, and not a student. It is not intended to include any person who has voluntarily resigned or retired from his or her latest position.

**Developing Country Participant:** Any person employed in developing countries where salary levels are radically noncommensurate with those in the U.S.

**Temporarily Employed:** Any person currently employed but who will become unemployed by June 1, 2000, and who is actively seeking employment.

**Nonmathematician Guest:** A registered participant may request a nonmathematician guest badge for a spouse, guest, or child. We request that all mathematicians register in a scientific category.

Advance registration and on-site registration fees only partially cover the expenses of holding meetings. All mathematicians who wish to attend sessions are expected to register and should be prepared to show their badges if so requested. Badges are required to obtain discounts at the AMS Book Sale and to cash a check with the meeting cashier.

Advance registration forms accompanied by insufficient payment either will be returned, thereby delaying the processing of any housing request, or a $5 charge will be assessed if an invoice must be prepared to collect the delinquent amount. Overpayments of less than $5 will not be refunded.

For each invalid check or credit card transaction that results in an insufficient payment for registration or housing, a $5 charge will be assessed. Participants should check with their tax preparers for applicable deductions for education expenses as they pertain to this meeting.

If you wish to be included in a list of individuals sorted by mathematical interest, please provide the one mathematical subject classification number of your major area of interest on the Advance Registration/Housing Form. A list of these numbers is available by sending an empty e-mail message to abs-submit@ams.org; include the number 956 as the subject of the message.

If you do not wish to be included in any mailing list used for promotional purposes, please indicate this in the appropriate box on the Advance Registration/Housing Form.

**Advance Registration Deadlines**

There are two separate advance registration deadlines, each with its own advantages and benefits.

- **Early Advance Registration** (materials mailed and airfare lottery) **June 8**
- **Final Advance Registration** (advance registration, banquets, and events) **July 10**

**Free Airfare!** Early Advance Registration: Those who register by the early deadline of June 8 will be eligible for a free airfare lottery, provided by the Los Angeles Convention & Visitor's Bureau (LACVB). Early registrants may also choose to
receive their registration packets before the meeting. Registration packets will be mailed on July 13, 2000.

Final Advance Registration: Those who register after June 8 and by the final deadline of July 10 may register for the meeting and obtain tickets for social events. Those who register after June 8 must pick up their badges, programs, and any tickets for social events at the Registration Desk in Royce Hall at UCLA. Please note that the July 10 deadline is firm; any forms received after that date will be returned and full refunds issued. Please come to the Registration Desk in Royce Hall at UCLA to register on site.

The deadline to reserve residence hall housing at UCLA is July 3 (see below).

Hotel Reservations

Participants should be aware that the AMS contracts only with facilities who are working toward being in compliance with the public accommodations requirements of the ADA. Participants requiring hotel reservations should call the following hotels directly.

**Doubletree Hotel Brentwood**
(1 mile from UCLA)
Deadline for reservations: July 10, 2000
10740 Wilshire Boulevard
Los Angeles, CA 90024
Telephone: 310-475-8711; 310-475-5220 (fax)
Toll-Free Reservations: 1-800-472-8556
Rooms: $129 single/double
Please note: Credit card guarantee necessary, full service, 24-hour cancellation policy, full amenities, restaurant/lounge, covered parking $16 overnight with in/out privileges (subject to change). Limited complimentary shuttle to and from UCLA.

**Hotel del Capri**
(9 blocks from UCLA, car needed)
Deadline for reservations: July 10, 2000
10586 Wilshire Boulevard
Los Angeles, CA 90024
Telephone: 310-474-3511; 310-470-9999 (fax)
Toll-Free: 1-800-44HOTEL
Rooms: $90 single /$100 double
Guest Suites: $110 single/$120 double
Please note: European hotel; multilingual staff, credit card guarantee necessary, number of rooms is limited. Rates include parking. There is no restaurant, but complimentary continental breakfast is served each morning; a charge of $1.00 per breakfast per day is added to the bill at checkout. Four-week cancellation policy with no penalty; two-week cancellation policy with 50% penalty.

Participants should be aware that most hotels are starting to charge a penalty fee to guests for departure changes made after guests have checked into their rooms. Participants should inquire about this at check-in and make their final plans accordingly.

Participants should also be aware that it is general hotel practice in most cities to hold a nonguaranteed reservation until 6:00 p.m. only. When one guarantees a reservation by paying a deposit or submitting a credit card number as a guarantee in advance, however, the hotel usually will honor this reservation up until checkout time the following day. If the individual holding the reservation has not checked in by that time, the room is then released for sale, and the hotel retains the deposit or applies one night's room charge to the credit card number submitted.

If you hold a guaranteed reservation at a hotel but are informed upon arrival that there is no room for you, there are certain things you can request the hotel do. First, they should provide for a room at another hotel in town for that evening at no charge. (You already paid for the first night when you made your deposit.) They should pay for taxi fares to the other hotel that evening and back to the meeting the following morning. They should also pay for one telephone toll call so that you can let people know you are not at the hotel you expected. They should make every effort to find a room for you in their hotel the following day and, if successful, pay your taxi fares to and from the second hotel so that you can pick up your baggage and bring it to the first hotel. Not all hotels in all cities follow this practice, so your request for these services may bring mixed results or none at all.

**Residence Hall Reservations**
Rooms will be available from August 5 to August 12 at the following halls.

**Deadlines:**

**Housing Reservations**
July 3

**Housing Changes/Cancellations**
(90% refund on cancellations—no refunds after this date)
July 12

**Sunset Village:** Hotel-style air-conditioned rooms with private bathrooms in shared bathrooms, all nonsmoking. Rooms include daily maid service and hotel-type amenities, including cable TV. Room with a private bathroom is $106.50 per person for one, and $60.00 per person for a double. Rates include breakfast and lunch. Room with a shared bathroom is $95.50 per person for a single and $54.50 per person for a double. (Please note: Doors in shared bathrooms do not lock from the bedroom side; however, rubber door stops will be provided free of charge upon request at the Mathematical Challenges Registration Desk.) Check-in will be in Sunset Village lobby.

**Dykstra Hall:** Residence hall with community bathrooms on each floor. Rooms are all nonsmoking, all nonsmoking. The rooms are $74.50 per person for a single and $44.00 per person for a double. Rates include breakfast and lunch. Daily towel exchange at the front desk is included. Check-in will be in Dykstra lobby.

**Residence Suites:** Suites are located in Saxon Hall or Hitch Hall; the walk to these halls is slightly uphill. All suites have two bedrooms (two twin beds in each), a living room, and a private bathroom. All suites are nonsmoking, but are not air-conditioned. Kitchen facilities are not available.
in these suites. They are $168.50 per person for a single, $91.00 per person for a double, $65.17 per person for a triple, and $52.25 per person for a quad. Rates include breakfast and lunch. Daily towel exchange at front desk is included. Check-in will be in Rieber Hall lobby or Hedrick Hall lobby.

All residence hall rates listed include complimentary guest privileges at recreational facilities on campus and a daily meal package of breakfast and lunch, starting with breakfast on the day after arrival and ending with lunch daily meal package of breakfast and lunch, starting with guest privileges at recreational facilities on campus and a maximum number of people in a room in Sunset Village or Dykstra Hall is three. Maximum number of people in a residence suite in Saxon Hall or Hitch Hall is four.

Meals will be served for all residents at Covel Commons in Sunset Village. Operation hours and meal prices (for cash basis only) are (Sunday–Saturday):

- Breakfast: 7:00 a.m. to 10:00 a.m. (door price: $6.50)
- Lunch: 11:00 a.m. to 2:00 p.m. (door price $8.50)
- Dinner: 5:00 p.m. to 9:00 p.m. (door price $12.00)

To reserve university housing, you must complete an Advance Registration/Housing Form. All completed Advance Reservation/Housing Forms must be received by the MMSB by July 3, 2000, for housing. Changes to reservations will be accepted by the MMSB until July 12, 2000. After this date, changes will be taken by the MMSB based on availability. A 10% cancellation fee will be charged for all university housing cancellations made by July 12, 2000. There will be no refunds issued for changes and cancellations after July 12, 2000.

**Miscellaneous Information**

Audio-visual Equipment: Standard equipment is one overhead projector and screen in the session room (plenary speakers are provided with two overhead projectors and screens). No additional equipment is available.

E-mail/Internet Access: An Internet access port is available in every residence hall sleeping room. Ethernet cards are available from the Student Technology Center located in the Sunset Village plaza. The UCLA Department of Mathematics also plans to have terminals available for checking e-mail, as does the new Institute for Pure and Applied Mathematics.

Getting around Campus/Parking: Participants residing in the Sunset Village complex may purchase parking permits during advance registration through the MMSB. Cost for parking permits is $6 per day per vehicle. See the Advance Registration/Housing Form for details. Parking spaces will be available to residents in the PSV parking garage located on DeNeve Drive. You may also park in this garage while you unload your belongings and check in.

Commuters may purchase individual parking permits at the parking kiosk in parking structure #4. The parking structure entrance is located at Sunset Boulevard and Westwood Plaza. Current rate for parking permits is $6 per day per vehicle, subject to change.

Complimentary shuttle service available exclusively for Math Challenges participants will operate Monday through Saturday between Sunset Village and Royce Hall during these times: 7:45 a.m. to 8:45 a.m., 11 a.m. to 2 p.m., and 5:30 p.m. to 6:30 p.m. It is about a 15- to 20-minute walk to Royce from Sunset Village.

Information Distribution: A table is set up in the exhibits area for dissemination of information of possible interest to participants but not promoting a product or program for sale. Those who wish to display information promoting a product or program for sale may do so in the exhibit area at the Joint Books, Journals, and Promotional Materials display for a fee of $50 per item. Contact the exhibits coordinator at 401-455-4143 or send e-mail to meet@ams.org for more information.

Local Information: Please see pages maintained by the UCLA Department of Mathematics at \http://www.math.ucla.edu/news/events/news/ams2000.html\ and by the Los Angeles Convention & Visitors Bureau at \http://www.lacvb.com/\.

Messages: Telephone messages of a routine nature should be left at the participant’s place of accommodation, either the hotel or the Sunset Village reception desk (310-206-9633).

Emergency messages will be taken at the meeting registration desk in Royce Hall during the hours that the desk is open. The telephone number will be published in the program.

Registration Desk: The meeting registration desk in the Founders Room in Royce Hall will be open Monday through Friday, August 7–11, from 8:00 a.m. to 4:30 p.m., and on Saturday from 8:00 a.m. to noon. Participants who register in advance and do not elect to receive their badges and programs in the mail may pick up pertinent materials at the outside ticket booth at Royce Hall on Sunday afternoon between 2:00 p.m. and 5:00 p.m.; otherwise, all badge and program pickup and new registrations will take place in the Founders Room during the hours cited above.

Travel: The nearest airport to campus is the Los Angeles International Airport (LAX).

The following specially negotiated rates on US Airways are available exclusively to mathematicians and their families for the period July 29–August 15, 2000, for travel to Los Angeles. Discounts apply only to travel within the continental U.S. Other restrictions may apply, and seats are limited. Receive a 5% discount off first or Envoy class and any published US Airways promotional round-trip fare. By purchasing your ticket 60 days or more prior to departure, you can receive an additional 5% bonus discount. Or you may receive a 10% discount off unrestricted coach fares with 7-day advance purchase. For reservations call (800) 843-2535 or have your travel agent call US Airways Group and Meeting Reservation Office toll-free at 877-874-
Meetings & Conferences

7687 between 8:00 a.m. and 9:30 p.m. Eastern Time. Refer to Gold File number 18611161.

AVIS is offering special convention car rental rates for the meeting, effective July 26-August 19, 2000. Rentals include unlimited free miles and are available from Los Angeles/Orange County Avis locations. There are no additional drop-off charges at the Los Angeles International Airport or John Wayne Airport.

If at the time of your reservation a lower qualifying rate becomes available, Avis will extend a 5% discount on that rate. The following rates are available to renters 25 years and older:

<table>
<thead>
<tr>
<th>Car Group</th>
<th>Daily</th>
<th>Weekly</th>
<th>Weekend</th>
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<tr>
<td>4-door subcompact</td>
<td>$38.99</td>
<td>$171.99</td>
<td>$27.99</td>
</tr>
<tr>
<td>4-door compact</td>
<td>41.99</td>
<td>181.99</td>
<td>28.99</td>
</tr>
<tr>
<td>2/4-door intermed.</td>
<td>44.99</td>
<td>190.99</td>
<td>29.99</td>
</tr>
<tr>
<td>2-door full size</td>
<td>51.99</td>
<td>204.99</td>
<td>32.99</td>
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<tr>
<td>4-door full size</td>
<td>53.99</td>
<td>208.99</td>
<td>35.99</td>
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<tr>
<td>Premium</td>
<td>57.99</td>
<td>245.99</td>
<td>41.99</td>
</tr>
<tr>
<td>Luxury</td>
<td>67.99</td>
<td>312.99</td>
<td>46.99</td>
</tr>
<tr>
<td>Minivan</td>
<td>72.99</td>
<td>331.99</td>
<td>80.99</td>
</tr>
<tr>
<td>Sport Utility (SUV)</td>
<td>70.99</td>
<td>322.99</td>
<td>77.99</td>
</tr>
</tbody>
</table>

*Weekend rates are available from noon on Thursday to 11:59 p.m. on Monday.

Sales tax, optional LDW, PDI, ALI, PEP, and gas refueling charges are extra. Advance reservations are strongly encouraged and can be made 24 hours a day, seven days a week at 1-800-331-1600; cite rate code 00 for daily and weekly rentals, and rate code E2 for weekend rentals. Also cite Avis Worldwide Discount #AWD K167407.

Participants who may find it convenient are invited to use UCLA’s discounted airline and car rental services. Airfare on United Airlines will be discounted by 15%. Alamo Rent A Car offers the following rates:

<table>
<thead>
<tr>
<th>Size</th>
<th>Daily</th>
<th>Weekly</th>
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<tr>
<td>Compact</td>
<td>$31</td>
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<tr>
<td>Intermediate</td>
<td>33</td>
<td>165</td>
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<td>175</td>
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<tr>
<td>Premium</td>
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Some restrictions may apply to discounted airfares and car rentals. Please contact the UCLA Travel Center by telephone at 1-800-235-UCLA, 1-310-794-8267, or by fax at 1-310-794-2875. When telephoning, please select “summer conferences” during their business hours of 7:30 a.m.-6:00 p.m. (PT), Monday through Friday. Please state that you are part of the AMS conference.

Directions/Transportation from LAX: From the airport take the 405 Freeway north to the Sunset Blvd. exit. Proceed east on Sunset Blvd. to Bellagio Drive. Turn right onto Bellagio and proceed to the stop sign at the top of the hill. Turn left at the stop sign onto DeNeve Drive. Proceed 1/2 mile down the hill through one stop sign to the Sunset Vill-

age parking garage. The campus is well marked with directional signs to other facilities.

Taxi fare is approximately $24. Door-to-door van service is provided by three carriers with various fare schedules. Each operates 24 hours per day; fares quoted are current as of this printing; pickup at LAX is at the blue van spot on the center traffic island. Prime Time Shuttle (800-733-8267) and SuperShuttle (800-BLUEVAN): $14/one person, $28/two persons, $42/three persons; Xpress Shuttle (800-427-7483): $12/one person, $19/two persons, $26/three persons.

Weather: In August Los Angeles is generally sunny with comfortable humidity. For more current information use your favorite Net search engine or try the sites: http://www.usatoday.com/weather/basemaps/nw722950.htm or http://www.weather.com/weather/cities/us_ca_los_angeles.html.

Program Committee

Richard Askey, University of Wisconsin
Spencer Bloch, University of Chicago
Felix Browder (chair), Rutgers University
Charles Fefferman, Princeton University
Peter Lax, Courant Institute
Robert MacPherson, Institute for Advanced Study
David Mumford, Brown University
Gian-Carlo Rota (deceased)
Peter Sarnak, Princeton University
Audrey Terras, University of California, San Diego
Srinivasa Varadhan, Courant Institute

Local Arrangements Committee

Christopher Anderson, UCLA
Tony Chan, UCLA (chair)
Philip C. Curtis, UCLA
Bjorn Engquist, UCLA
John B. Garnett, UCLA
Thomas Liggett, UCLA
Sorin Popa, UCLA
Barry Simon, California Institute of Technology
Ronald J. Stern, University of California, Irvine
Eitan Tadmor, UCLA

Toronto, Ontario
Canada

University of Toronto

September 22-24, 2000

Meeting #957

Central Section

Associate secretary: Susan J. Friedlander
Announcement issue of Notices: August 2000
Program first available on e-MATH: August 10, 2000
Program issue of electronic Notices: November 2000
Issue of Abstracts: Volume 21, Issue 3
**Deadlines**

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 6, 2000
For abstracts: July 14, 2000

**Invited Addresses**

John H. Conway, Princeton University, *Title to be announced* (Erdős Memorial Lecture).

George Elliott, University of Toronto, *Title to be announced.*

Benson Farb, University of Chicago, *Title to be announced.*

Yongbin Ruan, University of Wisconsin, *Title to be announced.*

Boris Tsyagan, Pennsylvania State University, *Title to be announced.*

**Special Sessions**

*Analytic Number Theory* (Code: AMS SS T1), *Man-Duen Choi* and *George Elliott,* University of Toronto.

*Probability* (Code: AMS SS S1), *Neal Madras,* Thomas Salisbury, and Donna Salopek, York University.


*Set Theory and Set-Theoretic Topology* (Code: AMS SS G1), *Franklin D. Tall,* University of Toronto.

**San Francisco, California**

*San Francisco State University*

**October 21-22, 2000**

**Meeting #958**

Western Section

Associate secretary: Bernard Russo

Announcement issue of Notices: August 2000

Program first available on e-MATH: September 11, 2000

Program issue of electronic Notices: December 2000

Issue of Abstracts: Volume 21, Issue 4

**Deadlines**

For organizers: Expired

For consideration of contributed papers in Special Sessions: June 21, 2000

For abstracts: August 29, 2000

**Invited Addresses**

Steven N. Evans, University of California, Berkeley, *Title to be announced.*

Lisa J. Fauci, Tulane University, *Title to be announced.*

Kristin Lauter, Microsoft Corporation, *Title to be announced.*

**Special Sessions**

*Algebraic and Geometric Combinatorics* (Code: AMS SS A1), *Jesus De Loera,* University of California, Davis, and *Frank Sottile,* University of Wisconsin.

*History of Mathematics* (Code: AMS SS B1), Shawnee McMurran, University of Redlands, and James J. Tattersall, Providence College.
Meetings & Conferences

New York, New York
Columbia University
November 4-5, 2000
Meeting #959
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: September 2000
Program first available on e-MATH: September 28, 2000
Program issue of electronic Notices: December 2000
Issue of Abstracts: Volume 21, Issue 4

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: July 18, 2000
For abstracts: September 12, 2000

Invited Addresses
Paula Cohen, Université des Sciences et Technologies de Lille, France, Title to be announced.
Brian Greene, Columbia University, Title to be announced.
Sergey Novikov, University of Maryland, College Park, and Landau Institute for Theoretical Physics, Title to be announced.
Alexander I. Suciu, Northeastern University, Title to be announced.

Special Sessions
Algebraic Geometry (Code: AMS SS H1), Sorin Popescu and Lev A. Borisov, Columbia University.
Arithmetic Geometry and Modular Forms (Code: AMS SS D1), Dorian Goldfeld, Columbia University, and Paula Cohen, University of Lille.
Arrangements of Hyperplanes (Code: AMS SS C1), Michael J. Falk, Northern Arizona University, and Alexander I. Suciu, Northeastern University.
Combinatorial Group Theory (Code: AMS SS A1), Gilbert Baumslag, Alexei Myasnikov, and Vladimir Shpilrain, City College (CUNY).
Commutative Algebra (Code: AMS SS F1), Irena Peeva, Cornell University, and Luchezar Avramov, Purdue University.
Differential Algebra and Related Topics (Code: AMS SS E1), Li Guo and William Keigher, Rutgers University at Newark, and William Sit, City College (CUNY).
Symbolic Computation and Kleinian Groups (Code: AMS SS G1), Jane P. Gilman, Rutgers University, and Mika K. Seppala, Florida State University.

Birmingham,
Alabama
University of Alabama-Birmingham
November 10-12, 2000
Meeting #960
Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of Notices: September 2000
Program first available on e-MATH: October 5, 2000
Program issue of electronic Notices: January 2001
Issue of Abstracts: Volume 21, Issue 4

Deadlines
For organizers: April 10, 2000
For consideration of contributed papers in Special Sessions: July 25, 2000
For abstracts: September 19, 2000

Special Sessions
Analytical Problems in Mathematical Physics (Code: AMS SS E1), Roger T. Lewis, University of Alabama at Birmingham, Michael P. Loss, Georgia Institute of Technology, and Marcel Griesemer, University of Alabama at Birmingham.
Billiards and Related Topics (Code: AMS SS C1), Nikolai I. Chernov and Nandor Simanyi, University of Alabama at Birmingham.
Dynamics and Low-Dimensional Topology (Code: AMS SS G1), Alexander M. Blokh, Lex G. Oversteegen, and John C. Mayer, University of Alabama at Birmingham.
Inverse Problems (Code: AMS SS A1), Ian Walker Knowles and Rudi Weikard, University of Alabama at Birmingham.
Operator Algebras and Their Representations (Code: AMS SS F1), Alan Hopenwasser, University of Alabama, and Justin R. Peters, Iowa State University.
Relations between Spectral Theory and Analytic Number Theory (Code: AMS SS B1), Robert M. Kauffman, University of Alabama at Birmingham, and Martin N. Huxley, Cardiff University, Wales.
Spectral and Transport Problems in Solid State Physics (Code: AMS SS D1), Peter D. Hislop, University of Kentucky, and Yulia Karpeshina and Gunter H. Stolz, University of Alabama at Birmingham.
Hong Kong, People's Republic of China

December 13–17, 2000

Meeting #961
First Joint International Meeting between the AMS and the Hong Kong Mathematical Society.
Associate secretary: Bernard Russo
Announcement issue of Notices: June/July
Program first available on e-MATH: Not applicable
Program issue of electronic Notices: None
Issue of Abstracts: None

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: July 15, 2000
For abstracts: September 15, 2000

Invited Addresses
Thomas Liggett, UCLA, Title to be announced.
Michael Shub, IBM, Title to be announced.
Gang Tian, MIT, Title to be announced.

Special Sessions
Combinatorial and Computational Methods in Commutative Algebra and Algebraic Geometry, Vladimir Shpilrain, CUNY, City College, and Jie-Tai Yu, University of Hong Kong.
Combinatorics and Graph Theory, Beifang Chen, Hong Kong University of Science and Technology, Jeong Han Kim, Microsoft, USA, and Che Bor Lam, Hong Kong Baptist University.
Geometric Analysis, Peter Li, University of California, Irvine, and Luen Fai Tam and Tom Wan, Chinese University of Hong Kong.
Integrable Systems, Jishan Hu, Hong Kong University of Science and Technology, Wen Xiu Ma, City University of Hong Kong, Peter Olver, University of Minnesota, and Min Yan, Hong Kong University of Science and Technology.
Iterative Methods in Scientific Computation, Michael Ng, University of Hong Kong, and Robert Plemmons, Wake Forest University.
Low Dimensional Topology, Iain Aitchison and Hyam Rubinstein, University of Melbourne.
Mathematics of Learning Theory, Felipe Cucker and Stephen Smale, City University of Hong Kong.
Mathematics of Optimization, Kung Fu Ng, Chinese University of Hong Kong, and Jong-shi Pang, Johns Hopkins University.
Nonlinear Elliptic and Parabolic Partial Differential Equations, Kai Seng Chou, Chinese University of Hong Kong, Yanyan Li, Rutgers University, and Juncheng Wei, Chinese University of Hong Kong.

Nonlinear Waves, Zhouping Xin, Courant Institute of Mathematical Sciences and Chinese University of Hong Kong, and Xiaoping Wang, Hong Kong University of Science and Technology.
Numerical Methods for Partial Differential Equations, Susanne Brenner, University of South Carolina, and Jun Zou, Chinese University of Hong Kong.
Optimization and Applications, Kok Lay Teo and X. Q. Yang, Hong Kong Polytechnic University.
Representation Theory, Jian Shu Li and Jinsong Huang, Hong Kong University of Science and Technology.
Theoretical and Numerical Aspects of Nonlinear Conservation Laws, Tao Tang, Hong Kong Baptist University, and Zhouping Xin, Courant Institute of Mathematical Sciences and Chinese University of Hong Kong.
Value Distribution Theory and Complex Dynamics, Chung Chun Yang, Hong Kong University of Science and Technology, and William Cherry, University of North Texas.

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 10–13, 2001

Meeting #962
Joint Mathematics Meetings, including the 107th Annual Meeting of the AMS, 84th Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: October 2000
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: Volume 22, Issue 1

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: August 8, 2000
For abstracts: October 3, 2000
For summaries of papers to MAA organizers: To be announced
Columbia, South Carolina
University of South Carolina
March 16-18, 2001
Meeting #963
Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: August 15, 2000
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Hoboken, New Jersey
Stevens Institute of Technology
April 28-29, 2001
Meeting #966
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: September 28, 2000
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses
Alexander Barvinok, University of Michigan, Ann Arbor, Title to be announced.
Frank Sottile, University of Wisconsin, Madison, Title to be announced.

Lyon, France
July 17-20, 2001
First Joint International Meeting between the AMS and the Société Mathématique de France.
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on e-MATH: Not applicable
Program issue of electronic Notices: None
Issue of Abstracts: None

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lawrence, Kansas
University of Kansas
March 30-31, 2001
Meeting #964
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: June 28, 2000
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Special Sessions
Commutative Algebra (Code: AMS SS A1), Craig Huneke and Daniel Katz, University of Kansas.

Las Vegas, Nevada
University of Nevada
April 21-22, 2001
Meeting #965
Western Section
Associate secretary: Bernard Russo
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lyon, France
July 17-20, 2001
First Joint International Meeting between the AMS and the Société Mathématique de France.
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on e-MATH: Not applicable
Program issue of electronic Notices: None
Issue of Abstracts: None

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses
Alexander Barvinok, University of Michigan, Ann Arbor, Title to be announced.
Frank Sottile, University of Wisconsin, Madison, Title to be announced.

Lyon, France
July 17-20, 2001
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Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on e-MATH: Not applicable
Program issue of electronic Notices: None
Issue of Abstracts: None

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses
Alexander Barvinok, University of Michigan, Ann Arbor, Title to be announced.
Frank Sottile, University of Wisconsin, Madison, Title to be announced.
Chattanooga, Tennessee
University of Tennessee, Chattanooga

October 5-6, 2001
Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: March 5, 2001
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Williamstown, Massachusetts
Williams College

October 13-14, 2001
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: March 11, 2001
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Special Sessions
History of Mathematics (Code: AMS SS A1), Glen R. Van Brummelen, Bennington College, Della D. Fenster, Richmond University, and James J. Tattersall, Providence College.

San Diego, California
San Diego Convention Center

January 6-9, 2002
Joint Mathematics Meetings, including the 108th Annual Meeting of the AMS, 85th Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).

Pisa, Italy
June 16-20, 2002
First Joint International Meeting between the AMS and the Unione Matematica Italiana.
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on e-MATH: Not applicable
Program issue of electronic Notices: None
Issue of Abstracts: None

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boston, Massachusetts
Northeastern University

October 5-6, 2002
Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of Notices: To be announced
Program first available on e-MATH: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: March 5, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
# Math Challenges Timetable

University of California Los Angeles

## Sunday, August 6

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:00 a.m. - 3:00 p.m.</td>
<td><strong>Getty Museum Tour Option</strong></td>
</tr>
<tr>
<td>2:00 p.m. - 5:00 p.m.</td>
<td><strong>Meeting Registration</strong> For pickup of advance registration packets only. Outside Ticket Booth, Royce Hall</td>
</tr>
<tr>
<td>4:00 p.m. - 5:00 p.m.</td>
<td><strong>AMS-MAA Presidents' Address</strong> Mathematics in the 21st century: Problems and prospects. RONALD L. GRAHAM</td>
</tr>
<tr>
<td>5:00 p.m. - 6:00 p.m.</td>
<td><strong>Opening Ceremonies, Royce Hall</strong></td>
</tr>
<tr>
<td>6:00 p.m. - 8:00 p.m.</td>
<td><strong>Opening Reception, Dickson Plaza</strong></td>
</tr>
</tbody>
</table>

## Monday, August 7

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m. - 4:30 p.m.</td>
<td><strong>Meeting Registration</strong>, Founders Room, Royce Hall</td>
</tr>
<tr>
<td>8:00 a.m. - 4:30 p.m.</td>
<td><strong>Exhibits and Book Sales</strong>, West Lobby, Royce Hall</td>
</tr>
<tr>
<td>8:30 a.m. - 9:30 a.m.</td>
<td><strong>Charles L. Fefferman</strong>, Unsolved problems of fluid mechanics.</td>
</tr>
<tr>
<td>10:00 a.m. - 11:00 a.m.</td>
<td><strong>David Mumford</strong>, Modeling perception and inference in intelligent systems.</td>
</tr>
<tr>
<td>11:00 a.m. - 2:00 p.m.</td>
<td><strong>Sessions for Contributed Papers</strong></td>
</tr>
<tr>
<td>2:00 p.m. - 3:00 p.m.</td>
<td><strong>Shing-Tung Yau</strong> will speak on geometry and its relation to physics.</td>
</tr>
<tr>
<td>3:15 p.m. - 4:15 p.m.</td>
<td><strong>Richard P. Stanley</strong>, Recent progress in algebraic combinatorics.</td>
</tr>
<tr>
<td>4:30 p.m. - 5:30 p.m.</td>
<td><strong>Karen Uhlenbeck</strong>, Geometric partial differential equations: From Hilbert's Twenty-third Problem to nonlinear waves.</td>
</tr>
</tbody>
</table>

## Tuesday, August 8

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m. - 4:30 p.m.</td>
<td><strong>Meeting Registration</strong>, Founders Room, Royce Hall</td>
</tr>
<tr>
<td>8:00 a.m. - 4:30 p.m.</td>
<td><strong>Exhibits and Book Sales</strong>, West Lobby, Royce Hall</td>
</tr>
<tr>
<td>8:30 a.m. - 9:30 a.m.</td>
<td><strong>Clifford Taubes</strong>, Bliss and ignorance in 4-dimensions.</td>
</tr>
<tr>
<td>10:00 a.m. - 11:00 a.m.</td>
<td><strong>David L. Donoho</strong> will speak on interactions among harmonic analysis, statistical analysis, and information theory.</td>
</tr>
<tr>
<td>11:00 a.m. - 2:00 p.m.</td>
<td><strong>Sessions for Contributed Papers</strong></td>
</tr>
<tr>
<td>2:00 p.m. - 3:00 p.m.</td>
<td><strong>Haim Brezis</strong>, The interplay between analysis and topology in some nonlinear PDEs.</td>
</tr>
<tr>
<td>3:15 p.m. - 4:15 p.m.</td>
<td><strong>Michael H. Freedman</strong>, The physics of computation.</td>
</tr>
<tr>
<td>4:30 p.m. - 5:30 p.m.</td>
<td><strong>Peter W. Shor</strong> will speak on quantum computing/quantum information theory.</td>
</tr>
<tr>
<td>6:30 p.m. - 9:00 p.m.</td>
<td><strong>Southern California Barbecue Option</strong></td>
</tr>
</tbody>
</table>

## Wednesday, August 9

<table>
<thead>
<tr>
<th>Time</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>8:00 a.m. - 4:30 p.m.</td>
<td><strong>Meeting Registration</strong>, Founders Room, Royce Hall</td>
</tr>
<tr>
<td>8:00 a.m. - 4:30 p.m.</td>
<td><strong>Exhibits and Book Sales</strong>, West Lobby, Royce Hall</td>
</tr>
</tbody>
</table>
WEDNESDAY, AUGUST 9 (cont'd)

8:30 a.m. - 9:30 a.m.  SERGIU KLAINERMAN, On the analysis of geometric evolution equations.
10:00 a.m. - 11:00 a.m.  JEAN E. TAYLOR, Applications of geometric analysis.
11:00 a.m. - 2:00 p.m.  Sessions for Contributed Papers
2:00 p.m. - 3:00 p.m.  RICHARD M. KARP, Mathematical challenges from genomics and molecular biology.
3:15 p.m. - 4:15 p.m.  YAKOV G. SINAI, From renormalization in dynamics to renormalization in statistical physics.
4:30 p.m. - 5:30 p.m.  DENNIS P. SULLIVAN, String topology.

THURSDAY, AUGUST 10

8:00 a.m. - 4:30 p.m.  Meeting Registration, Founders Room, Royce Hall
8:00 a.m. - noon  Exhibits and Book Sales, West Lobby, Royce Hall
8:30 a.m. - 9:30 a.m.  JAMES G. ARTHUR, The principle of functorality.
10:00 a.m. - 11:00 a.m.  EDWARD WITTEN will speak on the mathematical impact of quantum fields and strings.
11:00 a.m. - 2:00 p.m.  Sessions for Contributed Papers
2:00 p.m. - 3:00 p.m.  PETER D. LAX, Mathematics and computing.
3:15 p.m. - 4:15 p.m.  SAHARON SHELAH, Logical dreams.
4:30 p.m. - 5:30 p.m.  WILLIAM P. THURSTON will speak on three-dimensional topology and geometry.
7:30 p.m. - 10:30 p.m.  Concert at the Hollywood Bowl Option

FRIDAY, AUGUST 11

8:00 a.m. - 4:30 p.m.  Meeting Registration, Founders Room, Royce Hall
8:00 a.m. - 4:30 p.m.  Exhibits and Book Sales, West Lobby, Royce Hall
8:30 a.m. - 9:30 a.m.  ALEXANDER A. BEILINSON, Some results on the geometric Langlands conjecture.
10:00 a.m. - 11:00 a.m.  HELMUT H. W. HOFER, Dynamical systems at the interface of symplectic geometry and three-dimensional topology.
11:00 a.m. - 2:00 p.m.  Sessions for Contributed Papers
2:00 p.m. - 3:00 p.m.  PETER SARNAK, Some problems in number theory and related analysis.
3:15 p.m. - 4:15 p.m.  S. R. S. VARADHAN, Stochastic analysis and applications.
4:30 p.m. - 5:30 p.m.  LÁSZLÓ LOVÁSZ, Algorithms and their analysis: Classical mathematics and new challenges.

SATURDAY, AUGUST 12

8:00 a.m. - noon  Meeting Registration, Founders Room, Royce Hall
8:00 a.m. - noon  Exhibits and Book Sales, West Lobby, Royce Hall
8:30 a.m. - 9:30 a.m.  MICHAEL V. BERRY, Wave asymptotics and borderland physics.
10:00 a.m. - 11:00 a.m.  SIMON A. LEVIN, Ecosystems as complex adaptive systems.
11:00 a.m. - 2:00 p.m.  Sessions for Contributed Papers
2:00 p.m. - 3:00 p.m.  DON B. ZAGIER will speak on number theory: modular forms.
3:15 p.m. - 4:15 p.m.  MAXIM KONTSEVICH will speak on deformations, supermanifolds, and homotopical algebra.
4:30 p.m. - 5:30 p.m.  ALAIN CONNES, Noncommutative geometry.
6:00 p.m. - 9:30 p.m.  Millennium Banquet Option
Meetings and Conferences of the AMS

**Associate Secretaries of the AMS**

**Western Section:** Bernard Russo, Department of Mathematics, University of California, Irvine, CA 92697; e-mail: brusso@math.uci.edu; telephone: 949-824-5505.

**Central Section:** Susan J. Friedlander, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: susan@math.nwu.edu; telephone: 312-996-3041.

**Eastern Section:** Lesley M. Sibner, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2900; e-mail: lsibner@magnus.poly.edu; telephone: 718-260-3505.

**Southeastern Section:** John L. Bryant, Department of Mathematics, Florida State University, Tallahassee, FL 32306-4510; e-mail: bryant@math.fsu.edu; telephone: 850-644-5805.

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated.** Up-to-date meeting and conference information is available on the World Wide Web at [www.ams.org/meetings/](http://www.ams.org/meetings/).

### Meetings: 2000

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<th>Date</th>
<th>Location</th>
<th>Pages</th>
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<td>June 13-16</td>
<td>Odense, Denmark</td>
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<tr>
<td>August 7-12</td>
<td>Los Angeles, California</td>
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<td>September 22-24</td>
<td>Toronto, Ontario, Canada</td>
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<tr>
<td>October 21-22</td>
<td>San Francisco, California</td>
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<tr>
<td>November 4-5</td>
<td>New York, New York</td>
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</tr>
<tr>
<td>November 10-12</td>
<td>Birmingham, Alabama</td>
<td>616</td>
</tr>
<tr>
<td>December 13-17</td>
<td>Hong Kong, People's Republic of China</td>
<td>616</td>
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</table>

### Meetings: 2001

<table>
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<tr>
<th>Date</th>
<th>Location</th>
<th>Pages</th>
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</thead>
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<tr>
<td>January 10-13</td>
<td>New Orleans, Louisiana</td>
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<tr>
<td>March 16-18</td>
<td>Columbia, South Carolina</td>
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<tr>
<td>March 30-31</td>
<td>Lawrence, Kansas</td>
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<tr>
<td>April 21-22</td>
<td>Las Vegas, Nevada</td>
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<tr>
<td>April 28-29</td>
<td>Hoboken, New Jersey</td>
<td>618</td>
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<tr>
<td>July 17-20</td>
<td>Lyon, France</td>
<td>618</td>
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<tr>
<td>October 5-6</td>
<td>Chattanooga, Tennessee</td>
<td>618</td>
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<tr>
<td>October 13-14</td>
<td>Williamstown, MA</td>
<td>619</td>
</tr>
</tbody>
</table>

### Conferences: (See [http://www.ams.org/meetings/](http://www.ams.org/meetings/) for the most up-to-date information on these conferences.)

June 11-July 20, 2000: Joint Summer Research Conferences in the Mathematical Sciences, Mt. Holyoke College, South Hadley, MA. (See pages 1325-30, November 1999 issue, for details.)

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2002

<table>
<thead>
<tr>
<th>Date</th>
<th>Location</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 6-9</td>
<td>San Diego, California</td>
<td>619</td>
</tr>
<tr>
<td>June 16-20</td>
<td>Pisa, Italy</td>
<td>619</td>
</tr>
<tr>
<td>October 5-6</td>
<td>Boston, Massachusetts</td>
<td>619</td>
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</tbody>
</table>

**Important Information regarding AMS Meetings**

Potential organizers, speakers, and hosts should refer to page 106 in the January 2000 issue of the Notices for general information regarding participation in AMS meetings and conferences.

**Abstracts**

Several options are available for speakers submitting abstracts, including an easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX or AMS-LaTeX may submit abstracts with such coding. To see descriptions of the forms available, visit [http://www.ams.org/abstracts/instructions.html](http://www.ams.org/abstracts/instructions.html), or send mail to abs-submit@ams.org, typing help as the subject line; descriptions and instructions on how to get the template of your choice will be e-mailed to you.

Completed abstracts should be sent to abs-submit@ams.org, typing submission as the subject line. Questions about abstracts may be sent to abs-info@ams.org.

Paper abstract forms may be sent to Meetings & Conferences Department, AMS, P.O. Box 6887, Providence, RI 02940. There is a $20 processing fee for each paper abstract. There is no charge for electronic abstracts. Note that all abstract deadlines are strictly enforced. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.
### Mathematical Challenges of the 21st Century

**Name**  
**Mailing Address**  
**City**  
**State**  
**Zip**  
**Country**  
**Phone**  
**Fax**  
**Email**  
**Affiliation/Univ/Company (for badge)**  
**Name of nonmathematician guest (if purchasing a badge)**  
**DIDO NOT want my program and badge mailed to the address above on 7/13/00.**  
*(Mathfest materials will be mailed on 7/6/00.)*

**DIDO NOT want acknowledgment of this registration by U.S. Mail, not email.**

---

#### Registration Fees

<table>
<thead>
<tr>
<th>Member</th>
<th>AMS</th>
<th>MAA</th>
<th>CMS</th>
<th>SIAM</th>
<th><strong>by July 10</strong></th>
<th><strong>At mtg</strong></th>
<th><strong>Total</strong></th>
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<td></td>
<td>$5</td>
<td></td>
<td>$7</td>
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**Subtotal for Registration**  

---

**Mathematical Reviews subject classification number**  
**Special needs: I have special needs and require special arrangements or services.**

---

#### Banquets/Tours

<table>
<thead>
<tr>
<th>Social Events</th>
<th># tickets</th>
<th>Price per</th>
<th><strong>Total</strong></th>
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</thead>
<tbody>
<tr>
<td>Getty Museum Tour (8/6)</td>
<td>___</td>
<td>$6</td>
<td>$___</td>
</tr>
<tr>
<td>Barbecue (Adult) (8/6)</td>
<td>___</td>
<td>$27</td>
<td>$___</td>
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<tr>
<td>Barbecue (Child 5-11) (8/6)</td>
<td>___</td>
<td>$12</td>
<td>$___</td>
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<tr>
<td>Thunder and Lightning Hollywood Bowl Concert (8/10)</td>
<td>___</td>
<td>$45</td>
<td>$___</td>
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<tr>
<td>Millennium Banquet (8/12)</td>
<td>___</td>
<td>$35</td>
<td>$___</td>
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</table>

**Subtotal for Banquets & Tours**  

---

#### MAA Mathfest* (8/3-8/5) Special Opportunity!

<table>
<thead>
<tr>
<th></th>
<th># tickets</th>
<th>Price per</th>
<th><strong>Total</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Special One Day (8/5) Mathfest Registration</td>
<td>___</td>
<td>N/A</td>
<td>$___</td>
</tr>
<tr>
<td>Special One Day (8/5) Mathfest Registration for Students</td>
<td>___</td>
<td>N/A</td>
<td>$___</td>
</tr>
<tr>
<td>MAA 25-year Member Banquet (8/5)</td>
<td>___</td>
<td>N/A</td>
<td>$___</td>
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</table>

**Note:** Deadlines for Mathfest apply. For full Mathfest Registration, see form in Focus.

---

#### Payment

<table>
<thead>
<tr>
<th>Payment Details</th>
<th><strong>Subtotal for Registrations</strong></th>
<th><strong>Subtotal for Housing (other side)</strong></th>
<th><strong>Grand Total to be paid</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Purchase order #</td>
<td>____________________________</td>
<td></td>
<td>_______________________</td>
</tr>
<tr>
<td>(please enclose copy)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Check</td>
<td>Make checks payable to AMS. Checks drawn on foreign banks must be in equivalent foreign currency at current exchange rates. A $5 processing fee will be charged for each returned check or invalid credit card.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Credit Card (VISA, MasterCard, AMEX, Discover) Card number</td>
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<td></td>
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</tr>
<tr>
<td>Exp. date</td>
<td>Zipcode of credit card billing address</td>
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<td></td>
</tr>
<tr>
<td>Signature</td>
<td>Name on card</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

**Mail this form to:**

MMSB  
P. O. Box 6897  
Providence, RI 02940  
FAX: 401-455-4004

---

**Deadlines:**

To have your badge and program mailed and to be eligible for the free airfare lottery, return this form by:  
**June 8, 2000**

For residence hall reservations, return form by:  
**July 3, 2000**

For Advance Registration only, return form by:  
**July 10, 2000**

For residence hall changes and cancellations (90% refund on residence hall cancellations) request by:  
**July 12, 2000**

To cancel and receive a 50% refund on registration, request by:  
**July 27, 2000**

To cancel a banquet or tour and receive a 50% refund, request by:  
**July 10, 2000**

* No refunds after this date.

---

**UCLA**

**Residence Hall Reservations**
Residence Hall Reservations

Name ________________________________________________________________

Date and time of arrival __________________________ Date and time of Departure __________________________

Please check all that apply:

☐ I am choosing a double, triple or quad, and I am paying all charges for this room.

☐ Male ☐ Female ☐ Mixed couple or group ☐ Smoking ☐ Nonsmoking

☐ I have disabilities as defined by the ADA that require a sleeping room that is accessible to the physically challenged. My needs are ________________________________________________.

Child under 5 are free of charge, and do not indicate a roommate, you will be charged for the full price of the room.

Roommate's name __________________ Arrival ___________ Depart ________ Child (give age) ______

Parking Permits: ☐ 8/3 ☐ 8/4 ☐ 8/5 ☐ 8/6 ☐ 8/7 ☐ 8/8 ☐ 8/9 ☐ 8/10 ☐ 8/11 ☐ 8/12

University Housing Rates (Rates include breakfast and lunch)

<table>
<thead>
<tr>
<th>UCLA University Housing</th>
<th>Single One Adult</th>
<th>One Adult &amp; 1 child 5-12</th>
<th>Double 2 Adults</th>
<th>Double w/cot 2 adults &amp; 1 child 5-12</th>
<th>Triple w/cot 3 adults</th>
<th>Quad 4 adults</th>
<th># of nights</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunset Village</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>private bath</td>
<td>$106.50</td>
<td>$113.25</td>
<td>$120.00</td>
<td>$146.75</td>
<td>$153.51</td>
<td>$151.17</td>
<td>N/A</td>
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<tr>
<td>shared bath</td>
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<td>$135.75</td>
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<td>Dykstra Hall</td>
<td>$74.50</td>
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<td>$88.00</td>
<td>$114.75</td>
<td>$121.50</td>
<td>$119.00</td>
<td>N/A</td>
<td>x</td>
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</table>

| Suites in Saxon & Hitch Halls |                 |                          |                |                                  |                      |              |             |        |
| Saxon & Hitch Suites      | $168.50         | $175.25                  | $182.00        | $198.75                         | $206.51              | $204.00      | x           |        |

Parking Permits: $6.00 per day x

Total housing charges:

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(see full map in Notices)
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