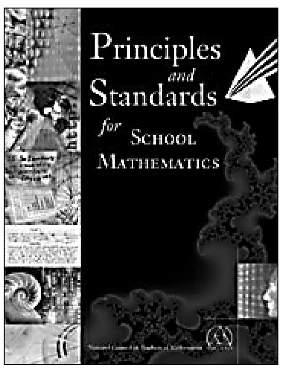
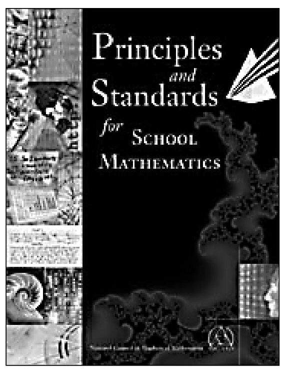
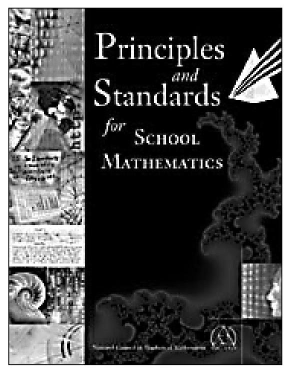
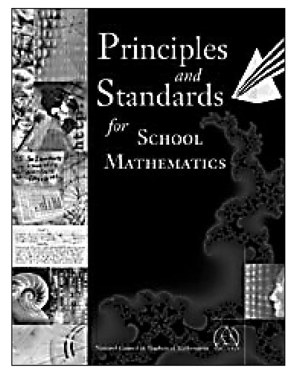
Principles and Standards for School Mathematics: A Guide for Mathematicians

Joan Ferrini-Mundy









In April 2000 the National Council of Teachers of Mathematics (NCTM) released *Principles and Standards for School Mathematics*—the culmination of a multifaceted, three-year effort to update NCTM's earlier standards documents and to set forth goals and recommendations for mathematics education in the prekindergarten-through-grade-twelve years. As the chair of the Writing Group, I had the privilege to interact with all aspects of the development and review of this document and with the committed groups of people, including the members of the Writing Group, who contributed immeasurably to this process. This article provides some background about NCTM and the standards, the process of development, efforts to gather input and feedback, and ways in which feedback from the mathematics community influenced the document. The article concludes with a section that provides some suggestions for mathematicians who are interested in using *Principles and Standards*.

The National Council of Teachers of Mathematics and Standards

Founded in 1920 as an outgrowth of the Chicago Mathematics Club, NCTM is a 100,000-plus member professional organization serving the U.S. and Canada. The NCTM's mission is "to provide the vision and leadership necessary to ensure a mathematics education of the highest quality for all students." The membership includes teachers of K–12 mathematics; university faculty in mathematics, mathematics education, and teacher education; state, provincial, district, and national policymakers; administrators; parents; educational researchers; and mathematicians.

Over the years NCTM has produced various documents intended to guide K–12 mathematics

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education. The 1980 *Agenda for Action* [1] called for the inclusion of problem solving, understanding, and application in school mathematics and was a precursor to the standards documents that followed. NCTM's 1989 *Curriculum and Evaluation Standards for School Mathematics* [2] was the first contemporary set of subject matter standards in the United States. It was followed in 1991 by the *Professional Standards for Teaching Mathematics* [3] and in 1995 by the *Assessment Standards for School Mathematics* [4].

These standards documents have been widely used in various ways. For instance, a total of 49 states now have subject matter standards or frameworks, many of them influenced by the 1989 standards (see, e.g., [5], [6]). Instructional materials have been developed, many with National Science Foundation support, to align with the 1989 standards [7], and recommendations for the education of teachers of mathematics have also reflected ideas from the NCTM recommendations [8]. In recent years the 1989 standards have also generated considerable controversy. Part of the controversy centered on some of the ideas proposed in the standards themselves, but most of it has focused on specific instructional materials developed in alignment with the standards (see, e.g., [9]).

Research efforts to determine the breadth of influence of the standards in U.S. classrooms, the nature of implementation of the ideas, and their effectiveness have provided mixed insights. The Third International Mathematics and Science Study [10] found that about 95 percent of U.S. teachers are aware of current reform directions in mathematics education and that "U.S. teachers believe that their lessons are already implementing the reform recommendations" ([10], p. 46). Examples of incomplete attempts to implement the ideas of standards, focused more on pedagogical changes rather than on mathematics content, also have been described (see, e.g., [11]).

Despite lack of agreement with the particulars of the ideas of standards (whether national, state, or local), about the impact of standards on practice and policy, or about how the ideas of standards should be interpreted and implemented, one general area of agreement has emerged: The nature and quality of K–12 mathematics education needs improvement, and this improvement requires concerted efforts by a wide variety of individuals and organizations.

NCTM has viewed its role in setting and proposing directions for school mathematics as an ongoing one and in the mid-1980s established a first Standards Coordinating Committee, which would evolve into the 1995 Commission on the Future of the Standards. That group recommended to the NCTM Board of Directors in 1996 that a process be established for revising and updating the original NCTM standards documents.

Brief Overview of *Principles and Standards for School Mathematics*

Principles and Standards for School Mathematics [12] is a 402-page document, organized into eight chapters. The preface and first chapter introduce the purpose and overall intention of the effort, and Chapter 1 portrays "A Vision for School Mathematics". In Chapter 2 a set of six principles is provided, expressing the assumptions that serve as a basis for the recommendations in the document. There are ten standards that describe in a general way the mathematics that students should know and be able to do across the prekindergarten-through-grade-twelve years, and each of these is discussed in Chapter 3. Five of these are considered content standards in that they address mathematical topic areas such as algebra and geometry; five are about mathematical processes, such as problem solving, and reasoning and proof. The following four chapters take up each of the four grade bands: prekindergarten to grade 2, grades 3–5, grades 6–8, and grades 9–12. There is a section in each chapter on each of the standards, elaborating how the recommendations might be enacted in classrooms. For the content standards, more specific grade-band "expectations" are offered. The final chapter discusses how various groups might best work together for the continued improvement of mathematics education. Additional examples are provided in the electronic version of *Principles and Standards.*

Developing *Principles and Standards*

By spring 1997 the Standards 2000 Writing Group had been appointed. It included teachers, teacher educators, administrators, researchers, and mathematicians with a wide range of expertise. The Writing Group was charged to establish standards that

- build on the foundation of the original standards documents;
- integrate the classroom-related portions of *Curriculum and Evaluation Standards for School Mathematics*, *Professional Standards for Teaching Mathematics*, and *Assessment Standards for School Mathematics*;
- use four grade bands: prekindergarten through grade 2, grades 3–5, grades 6–8, and grades 9–12.

The 26-person Writing Group met for two- or three-week periods in each of the summers of 1997, 1998, and 1999. Occasional two-day meetings were held during the academic years, and the leaders of the grade-band teams and I met intermittently. The development process was designed to be consultative and open, and we were expected to produce a draft document that would be circulated widely for comment in the 1998–99 academic year.

How To Obtain *Principles and Standards*

Principles and Standards for School Mathematics is available on the World Wide Web, on paper, and on CD-ROM.

The Web version is available at `http://standards.nctm.org/`. A PDF version may be purchased online at `http://www.nctm.org/standards/`. Paper and CD-ROM versions may be ordered on the Web at `http://www.nctm.org/standards/buyonline/`, or by telephone at 800-253-7566 (from overseas, 1-703-620-9840, extension 2601). The postal address for orders is:

National Council of Teachers of Mathematics
1906 Association Drive
Reston, VA 20191-9988

The book (#719) is $45 (NCTM members $36), the CD-ROM (#736) is $30 (members $24), and the PDF version is $30 (members $24). Those purchasing paper copies of the document can also request a free CD-ROM. For orders under $50 there is a shipping and handling charge of $7. Discounts are available on larger orders; consult the NCTM for further details.

—*Allyn Jackson*

Gathering Input and Feedback

In February 1997 all member organizations of the Conference Board of the Mathematical Sciences were invited by NCTM either to form a special new committee or to designate an appropriate existing committee to respond to periodic NCTM questions in the process of planning and writing the new document. Fourteen Association Review Groups (ARGs)[1] were ultimately formed as a result. The collective ARG membership included individuals with widely diverse backgrounds; critics as well as advocates of the NCTM-based reforms were well represented.

Most ARGs operated via e-mail and submitted group responses to a series of letters and questions from NCTM. In almost all cases they were able to produce a consensus report, noting points of disagreement. A November 1996 letter[2] from NCTM Commission chair Mary Lindquist asked ARG members to respond to four questions relating to NCTM's 1989 *Curriculum and Evaluation Standards for School Mathematics* and the 1991 and 1995 teaching and assessment documents. Consider, for example, this response from the AMS ARG, submitted in reply to this first set of questions in January 1997 (for further detail see [13, 14]).

> While there was consensus that the Standards should be made more specific, there was enormous disagreement as to how thoroughly they should delineate the curriculum. This was the subject of the most heated discussion in the group. On the one hand, a number of members argued persuasively for the advantages of a national curriculum…. The idea was brought forward that a national curriculum gives a basis for national discussion, so that the entire teaching profession can receive the benefit of research by small groups of teachers into specific classroom techniques…. On the other hand, other members argued against a national curriculum. Examples were given where specific curriculum items led to a mechanical or rote mastery of the topics and encouraged assessment procedures which searched only for surface-level understanding of what mathematics looks like…. We did agree that a call in the Standards for the development of detailed curricula at appropriate levels—whether school, district, state, or national—would be very helpful…. One role of the new Standards could be to guide the development of such detailed curricula. ([14], pp. 271–2)

A second round of questions, focused on algorithms, was sent to the ARGs in April 1997. The AMS response appears in [14].

All of the ARG responses were distributed to the writers in the summer 1997 meeting, along with a compilation of responses to various issues. The writers read this material carefully and discussed it in detail. The feedback helped shape a number of our early decisions. For example, we recognized that a stronger stand needed to be taken on the importance of proof, both within the standard addressing reasoning and elsewhere. Indeed, in the 1989 document this standard was called "Mathematics as Reasoning", and the name was eventually revised for the 2000 version to be "Reasoning

[1] *The following organizations formed ARGs: the American Mathematical Association of Two-Year Colleges, the American Mathematical Society, the American Statistical Association, the Association for Symbolic Logic, the Association of State Supervisors of Mathematics, the Association for Women in Mathematics, the Benjamin Banneker Association, Inc., the Institute for Operations Research and the Management Sciences, Mathematicians and Education Reform Forum, the Mathematical Association of America, the National Council of Supervisors of Mathematics, the Research in Undergraduate Mathematics Education Community, the Society for Industrial and Applied Mathematics, and the Society of Actuaries.*

[2] *This letter, plus discussion of the positions of the AMS ARG, are described by Roger Howe in [13].*

and Proof". Another topic of discussion that emerged from early ARG responses was the challenge of ensuring that students who plan to go into mathematics or mathematically based fields are not shortchanged by NCTM's strong commitment to mathematics for all. Even recommendations that were not taken up served the important function of highlighting issues that needed to be addressed. For example, a number of ARGs recommended producing grade-by-grade benchmarks rather than more general expectations for a band of grades. After extended discussion the Writing Group decided to maintain the style of the 1989 Standards of offering more general guidance and leaving states and local districts the role of developing specific curricular guidelines. Discussions such as these helped us to clarify and articulate our positions and to develop rationales for all major decisions.

Throughout the fall of 1997 and the spring of 1998, two additional rounds of questions were circulated to the ARGs. These questions asked for more-detailed input on issues raised in previous rounds and requested advice on new issues that arose as the writers continued to plan their work. The third round included a question about how changes in contemporary mathematics might affect the mathematics that is taught in schools. In the fourth round we asked the ARGs about discrete mathematics and about what areas of geometry are most important for school mathematics.

The complete sets of responses to rounds of questions were circulated to the Writing Group and influenced our work in the summer of 1998. *Principles and Standards for School Mathematics: Discussion Draft* [15] was released in October 1998. The ARGs were again asked for comment. The request included suggestions for how the ARGs might organize their review process, referred them to a set of "Reader Reaction" questions embedded within the draft, and posed general questions about the structure and choices of examples. While reviewing a 350-page document by committee was clearly a difficult task, as several ARG chairs confirmed, nearly all the ARGs provided responses. The magnitude of the task made the preparation of consensus reports difficult, and so some ARG responses were compilations of detailed individual reactions, often including line-by-line critique and suggestions. These responses had a significant impact on the synthesis of feedback to the discussion draft and the subsequent revision to produce *Principles and Standards for School Mathematics*.

An elaborate process was developed[3] for gathering and synthesizing feedback to the October 1998 draft document. That process will be described in detail in [16]. The discussion draft was widely distributed in order to attract a range of responses from all groups with an interest in the teaching and learning of precollege mathematics. Over 30,000 copies of the draft were sent out, and tens of thousands of people downloaded the draft from the Web. Open-ended responses were encouraged. Twenty-five reviews of the entire document or parts of it were invited from commissioned reviewers, several of whom were mathematicians. Presentations about the document, inviting feedback, were arranged at more than forty professional meetings (including the Joint Mathematics Meetings in San Antonio in January 1999). We received a total of about 600 responses to the draft; many of these came from groups (an ARG, teachers within a school or a project, students in a university course, etc.). Many responses were identified as coming from mathematicians, both through solicited and unsolicited sources.

Combining and analyzing this broad variety of comments presented a challenge. Using qualitative data analysis methods, a system for categorizing the responses was developed, including use of a software package specially designed to handle narrative data. Once the responses were categorized and entered, the software could then search for all the responses in a particular category or combination of categories. For example, reports could be prepared containing all comments from mathematicians about basic facts, about a particular standard at a particular grade band, or even about the writing style.

Using the categorized feedback, members of the Commission on the Future of the Standards identified nineteen major issues (available in [16]) that needed to be addressed by the writers as they revised the draft. These issues fell into the following clusters:

- Overarching Issues: audience and purpose, specificity of the document's recommendations, view of mathematics, relation to previous standards documents, vision of the document, and support of teachers
- Structure of Document: holistic view of classroom instruction, role of the standards overview chapter, use of research, role of examples, readability, and terminology
- Content Issues: connections within mathematics, articulation across grades, appropriateness of content for grade bands, and influence of technology
- Issues Related to Learning: use of particular theories and balance of skills and understanding
- Issues Related to Equity: special student populations and equity.

[3] *W. Gary Martin, NCTM chief education officer, and Mary Lindquist, chair of the Commission on the Future of the Standards, conceptualized and oversaw this process.*

For each issue the Commission prepared a summary of typical responses for the writers, outlining the representative (and often contrasting) viewpoints expressed in the feedback. The Writing Group convened in May 1999 to plan how they would address each of the nineteen issues and developed draft rationales for each decision that was made.

In the summer of 1999 members of the Writing Group revised the October 1998 draft document, largely in response to the input and critique. Printouts of feedback, organized in various ways, were made available to the writers and were used heavily. For instance, the writers revising the discussion of the Number and Operations Standard for grades 6–8 worked with more than 100 pages of comments. Using the comments, the writers could identify passages in which their messages were not clearly expressed, examples that needed to be fixed, and places in the document that resonated with readers and needed to be preserved.

Using Responses from the Mathematics Community

Responses from all of the ARGs were considered carefully by the Writing Group. Given the diversity both between and within ARGs, we were faced with a wide array of perspectives on each of the issues. The 46-page document provided by the AMS ARG comprised a set of individual, rather than synthesized, responses to issues in the draft. Many of these were quite detailed. Individual opinions, even within this one ARG, varied greatly. Of the nineteen issues that the NCTM Commission identified, ARG responses to the following five are discussed below. In addition, an example is provided of how ARG response to one of the classroom episodes in the October 1998 draft resulted in revision in the final version.

Responses to Selected Issues

- **Issue 2: In what ways should the document be more specific in its recommendations?**

Overall, in the reaction to the draft, responses on this point were mixed. Several of the AMS ARG members' responses called for more specificity, typified by comments such as the following, from the AMS ARG report:

> The nation needs grade-level standards that are realistic yet challenging, but the leadership of NCTM is philosophically opposed to providing such.

As noted earlier, *Principles and Standards* does not provide grade-by-grade specific benchmarks, except in a very few cases where the writers felt that very specific guidance was crucial, such as the following:

> *It is absolutely essential that students develop a solid understanding of the base-ten numeration system and place-value concepts by the end of grade 2.* ([12], p. 81)

> *If by the end of the fourth grade, students are not able to use multiplication and division strategies efficiently, then they must either develop strategies so that they are fluent with these combinations or memorize the remaining "harder" combinations.* ([12], p. 153)

By working in four narrower grade bands rather than the K–4, 5–8, and 9–12 bands of the 1989 Standards, we were able to be quite a bit more specific, through both the examples in the text and the expectations. Those who are designing district and state frameworks will do the work of specifying what is expected at each grade, ideally with *Principles and Standards* as a key reference.

- **Issue 6: Are the needs of teachers addressed?… Has teachers' knowledge of content been taken into consideration?**

Several AMS ARG members commented on their concerns about the inadequacy of teacher knowledge, typified by the following:

> …my first recommendation for PSSM is that it should start with a prominent disclaimer, saying that the Standards require a higher order of teaching than the practices they are supposed to replace, and that no teacher should be asked to implement them without extensive training.

There were a number of comments referring to practices that support teachers in Asian countries.

Principles and Standards was designed to provide curricular goals and guidance for pre-K–12 mathematics, and our charge did not include providing detailed recommendations about the preparation of teachers. However, because of concerns raised by the AMS ARG members, as well as others, in the final version of *Principles and Standards* our message about the importance of teacher knowledge was strengthened in several places. For example, the following language was added to the Teaching Principle:

> *Teachers need several different kinds of mathematical knowledge—knowledge about the whole domain; deep, flexible knowledge about curriculum goals and about the important ideas that are central to their grade level; knowledge about the challenges students are likely to encounter in learning these ideas; knowledge about how the ideas can be*

represented to teach them effectively; and knowledge about how students' understanding can be assessed. This knowledge helps teachers make curricular judgments, respond to students' questions, and look ahead to where concepts are leading and plan accordingly. Pedagogical knowledge, much of which is acquired and shaped through the practice of teaching, helps teachers understand how students learn mathematics, become facile with a range of different teaching techniques and instructional materials, and organize and manage the classroom. Teachers need to understand the big ideas of mathematics and be able to represent mathematics as a coherent and connected enterprise (Schifter 1999; Ma 1999). Their decisions and their actions in the classroom—all of which affect how well their students learn mathematics—should be based on this knowledge.

> *This kind of knowledge is beyond what most teachers experience in standard preservice mathematics courses in the United States.* ([12], p. 17)

Also, there is a section in Chapter 8 titled "How can teachers learn what they need to know?" ([12], pp. 370–371)

- **Issue 15: Is the role of technology sufficiently addressed?**

The role of technology in school mathematics continues to generate strong views in many directions. The AMS ARG responses to the October 1998 draft included the following comments:

> — Surprisingly little is said about the role of technology. What is said seems OK.

> — Reading this standard refutes the idea that the standards overemphasize early use of technology.

> — The Technology Principle continues to endorse this familiar excuse for not learning fundamental algorithms and developing appropriate number sense.

> — I find the advocacy of the use of calculators even in the early grades simply maddening.

After reading the array of input and after much difficult discussion within the Writing Group, we decided ultimately to take a clearer and stronger stand on the importance of technology in mathematics teaching and learning. This was done in part to advance the ideas of the 1989 standards and to support teachers in their efforts to use technology wisely to help students learn mathematics. This resulted in the Technology Principle: *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning* ([12], p. 24). To help clarify the message about technology use, we included more examples, especially in the electronic format, of using technology to support student learning. For instance, in the grades 9–12 chapter, an extended example shows how technology might be an option for analyzing the function $f(x) = \frac{(2x^2+11x+6)}{(x-2)}$ ([12], p. 302). We also added an episode about using technology to understand functions in the grades 9–12 section on problem solving (see [12], pp. 338–339).

- **Issue 17: Has the correct balance been struck between skills and understanding?**

This issue was of particular interest to some ARGs, especially those representing organizations of teachers and supervisors. In general, the AMS ARG comments did not address this area in great detail. The following comments came from other individuals and groups that reviewed the draft:

> — The 3–5 standards focus too much on lists of skills rather than discussion of conceptual issues children confront and work through.

> — Mastery of skills is aggressively downplayed…. Learning arithmetic facts and practice drill is discussed but always in a negative light.

> — The term "computational fluency" may be interpreted in several different ways. For number facts it might suggest recall, or it could include facts that can be derived very quickly from related facts.

> — The words "retrieve the basic facts" are used throughout the whole report, and they automatically cast us back into the last century in their assumptions about teaching and learning basic facts: children that just rotely memorize them randomly without a great deal of thinking about them….

To clarify this issue, we used the same definition of computational fluency consistently in the final version:

> *Equally essential is computational fluency—having and using efficient and accurate methods for computing…. Computational fluency should develop*

in tandem with understanding of the role and meaning of arithmetic operations in number systems. ([12], p. 32)

- **Issue 13: How well is articulation across the grades handled?**

ARG responses included extremely helpful advice about how the writers could do a better job in building the argument for a coherent and connected treatment of ideas across the grades. Here is a comment of this type, from the AMS ARG:

> If students are to succeed in proving and disproving basic statements about numbers and geometry in high school, the discussion in the preceding sections of PSSM should indicate how the thought processes necessary to achieve this goal are to be built up over the years.

To address this, a subset of the Writing Group read the discussion draft sections on Reasoning and Proof in each of the grade-band chapters and tried to trace the development of students' capacity to reason and prove more explicitly over the grades. This is reflected, for instance, in the Chapter 3 overview of Reasoning and Proof, with statements such as: "From children's earliest experiences with mathematics, it is important to help them understand that assertions should always have reasons" ([12], p. 56) or "Children in the lower grades will tend to justify general claims using specific cases…. By the upper elementary grades, justifications should be more general and can draw on other mathematical results…. In high school, students should be expected to construct relatively complex chains of reasoning and provide mathematical reasons" ([12], p. 58).

Influencing the Choices of Classroom Episodes

In addition to commenting on issues of the type just described, ARG members commented critically on some of the examples and classroom episodes that appeared in the October 1998 draft, in many cases leading to revision or replacement. For instance, the draft contained the following example in the grades 9–12 chapter:

> *At the beginning of a tenth-grade unit on circles, for example, a teacher collected the following responses to the request, "Tell me everything you know about circles."*
>
> *Juan: It's made up a a series of arcs that are all connected.*
>
> *Monique: A circle is a shape that has no points.*
>
> *Louis: The outside of the circle is called the circumference.*
>
> *Ramona: A circle is round.*
>
> *William: A circle measures 360 degrees.*
>
> *Eric: Radius is half the distance of the diameter of the circle.*
>
> *Cathy: 1/2 = 180 degrees.*
>
> *Rebecca:* $A = \pi r^2$ *and* $\pi = 3.1415927$.
>
> *Michael: It has two sides (inside and outside).*
>
> *Charles: The center is the radius.* ([15], pp. 321–322)

An ARG member commented:

> I hope the conversation had an end, and that students ended up able to *define* a circle. A definition is not something that can be discovered in a bull session, any more than can the meaning of "consubstantiation". Conversation doesn't yield much about "conventions", except maybe to reveal that the disputants don't agree on the definitions. People do all too much arguing about definitions, which of course have nothing to do with truth. They should merely be *learned,* and then used.

With this comment in mind, here is how the grade-band authors revised the example for the final text:

> *At the beginning of a tenth-grade unit on circles, for example, a teacher might ask students to tell her everything they know about circles. The teacher could then compile the responses, distribute copies of them the next day, and ask the students to agree or disagree with each of the statements. Students should justify the stance they took. The teacher could look at students' incorrect observations and design a lesson to address those misconceptions. In this way, the students' knowledge becomes a starting point for instruction, and the teacher can establish the idea that the students are expected to have reasons for their mathematical opinions.* ([12], p. 351)

No doubt this revision will not completely satisfy the reviewer, because the definition is not stated. The authors wanted to make a point here about the importance of understanding what students are thinking and then designing a lesson that will both build from, or correct, the students' ideas. Obviously such a strategy could lead nicely into a discussion of the definition of a circle and of the role of definitions in mathematics.

Mathematicians' Guide to Working with Principles and Standards

Principles and Standards is intended to serve as a resource to those who make decisions about the nature and direction of mathematics education in prekindergarten through grade 12. Thus for the mathematician who might not be familiar with that arena but who is interested, this document (and some of the references it cites) might be a good place to begin to learn about the issues that are crucial. For such people I suggest first reading Chapter 3, which presents overviews of the ten standards; the content standards (Number and Operations, Algebra, Geometry, Measurement, Data Analysis and Probability); and the process standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation). The chapter presents ten essays intended to help sketch a developmental trajectory across the grades for each of these standards and to delineate which ideas are most essential at which grade levels. Some of the narrative represents current practice; in other cases, it is intended to push practice in new directions. To see in detail what is proposed for a particular grade band in one of these ten areas, one may consult Chapters 4 through 7.

Another way of reading the document is to read "by standard": for example, to see what is recommended about how the ideas of algebra should be developed over the prekindergarten-through-grade-12 years, one can read the section on algebra in each chapter. Some mathematicians have found it interesting to simply read (and do) the mathematical problems and tasks that are included. One might try to predict how students at a particular grade level might approach and solve such problems and what teachers would need to know to teach them.

The document is also intended as a resource for those who are teaching mathematics content courses for prospective or in-service teachers. The classroom episodes and examples of student work, which appear in shaded sections of the text, can help provide a sense of the mathematics that teachers would need to know to teach well and of the nature of the pre-K–12 classrooms we could be aiming for. For instance, one can read the account of a classroom episode about fifth graders' analysis and comparison of computational procedures for division (NCTM, 2000, p. 153). The sections in each grade-band chapter on the process standards are all organized under two headers, "What should *process* X look like in *grades* Y?" and "What should be the teacher's role in developing *process* X in *grades* Y?", to further emphasize what might happen in classrooms. Conversations among pre-K–12 teachers and mathematicians, using *Principles and Standards* as a basis, might be quite productive.

States are working continuously to update their standards, and many state mathematics leaders have been closely following, and involved in, the revision of these NCTM standards. Therefore, it is likely that *Principles and Standards* will be used as a resource as state standards are revised. In many cases the state standards are central in the process of textbook adoption, assessment, and even teacher certification. One kind of comparison on which some states are already embarking is to look at their own standards or grade-specific benchmarks and compare them to the NCTM document. Several groups have been using the Table of Standards and Expectations that appears in Appendix A (see [12], pp. 392–402) as a basis for this. Looking also at some of the specific material in the narrative of *Principles and Standards* could help with this process.

Commentary on and reaction to *Principles and Standards* (via the NCTM Web site at `http://www.nctm.org/`) are welcome. If *Principles and Standards* can be used by mathematicians as a tool for focused, constructive efforts to improve pre-K–12 mathematics education, we will have accomplished a major goal in the Standards 2000 effort. Language from the document itself summarizes our hopes for its use:

> Principles and Standards *offers common language, examples, and recommendations to engage many groups of people in productive dialogue. Although there will never be complete consensus within the mathematics education profession or among the general public about the ideas advanced in any standards document, the Standards provide a guide for focused, sustained efforts to improve students' school mathematics education.* Principles and Standards *supplies guidance and vision while leaving specific curriculum decisions to the local level.* ([12], p. 5)

Further:

> *Any vision of school mathematics teaching and learning needs ongoing examination; it needs to be refined continually in light of the greater understanding achieved through practice, research, and evidence-based critiques. The process that NCTM put in place for developing* Principles and Standards *reflects a commitment to ongoing discussion and reflection. This document, therefore, should be seen as part of a*

> *work in progress that can help guide decision makers in developing excellent mathematics programs, not as a prescription to be rigidly imposed on others.* ([12], p. 380)

Developing and writing *Principles and Standards* was, for those of us involved, a remarkably challenging professional endeavor. Having as part of our work, over several years, the carefully crafted ideas and responses of the ARG members enriched our effort immeasurably. Our hope is that this process and its product, *Principles and Standards*, can make a difference in the effectiveness of school mathematics education.

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