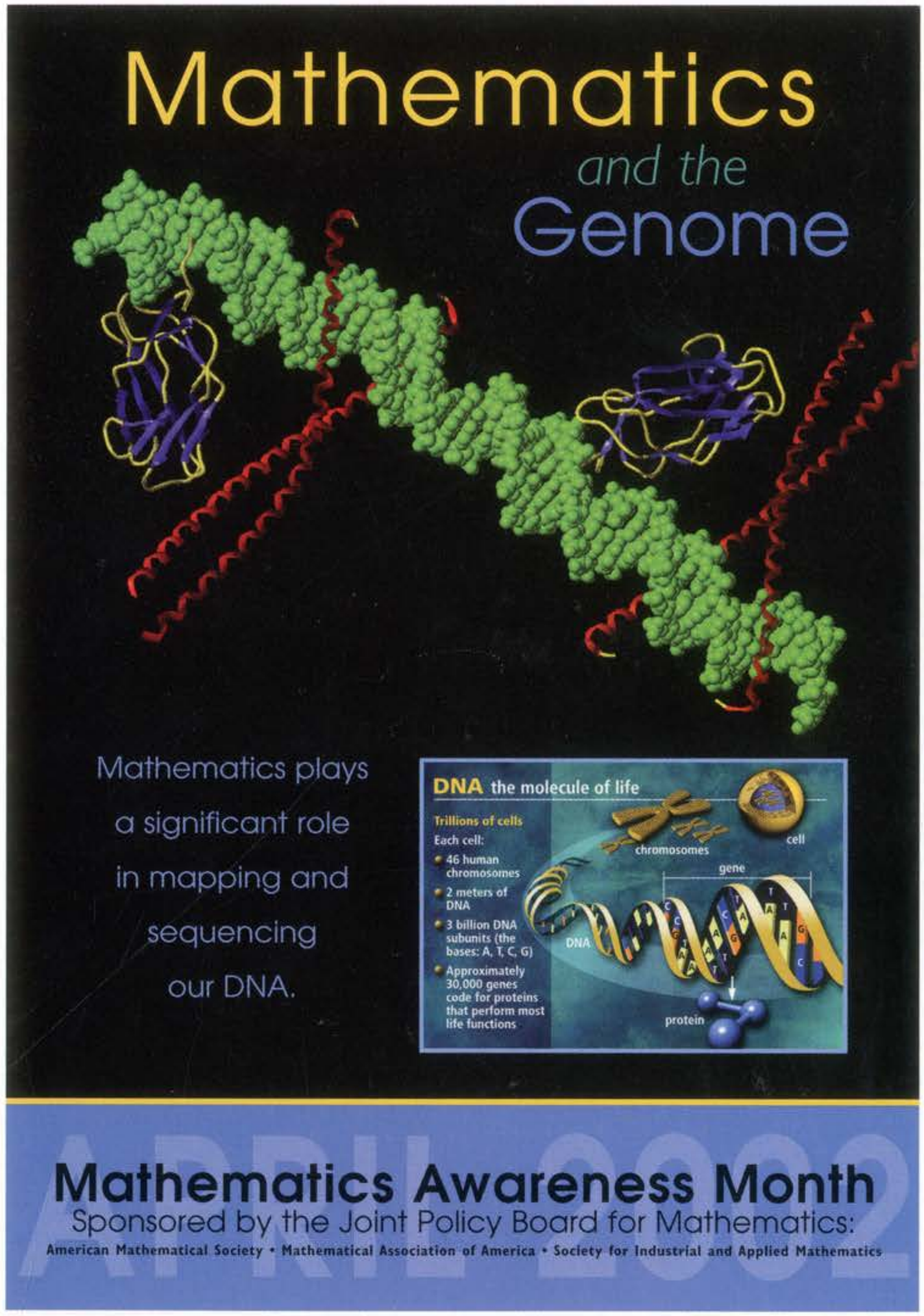
ISSN 0002-9920

Notices

of the American Mathematical Society

April 2002

Volume 49, Number 4



Mathematics Awareness Month 2002 (see page 429)

Notices
of the American Mathematical Society

April 2002

Feature Articles

Communications

Commentary

Notices
of the American Mathematical Society

EDITOR: Harold P. Boas

ASSOCIATE EDITORS:
Susanne C. Brenner, Bill Casselman (*Covers Editor*), Robert J. Daverman, Nathaniel Dean, Rick Durrett, Susan Friedlander, Robion Kirby, Elliott H. Lieb, Andy Magid, Judith Roitman, Mark Saul, Karen E. Smith, Audrey Terras, Lisa Traynor

SENIOR WRITER and DEPUTY EDITOR:
Allyn Jackson

MANAGING EDITOR: Sandra Frost

CONTRIBUTING WRITER: Elaine Kehoe

PRODUCTION ASSISTANT: Muriel Toupin

PRODUCTION: Marcia Almeida, Kyle Antonevich, Siulam Fernandes, Thomas J. Kacvinsky, Lori Nero, Donna Salter, Deborah Smith, Peter Sykes

ADVERTISING SALES: Anne Newcomb

SUBSCRIPTION INFORMATION: Subscription prices for Volume 49 (2002) are $360 list; $288 institutional member; $216 individual member. (The subscription price for members is included in the annual dues.) A late charge of 10% of the subscription price will be imposed upon orders received from nonmembers after January 1 of the subscription year. Add for postage: Surface delivery outside the United States and India—$17; in India—$38; expedited delivery to destinations in North America—$35; elsewhere—$75. Subscriptions and orders for AMS publications should be addressed to the American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904. All orders must be prepaid.

ADVERTISING: *Notices* publishes situations wanted and classified advertising, and display advertising for publishers and academic or scientific organizations. Advertising material or questions may be faxed to 401-331-3842 (indicate "*Notices* advertising" on fax cover sheet).

SUBMISSIONS: Articles and letters may be sent to the editor by e-mail at `notices@math.tamu.edu`, by fax at 979-845-6028, or by postal mail at Department of Mathematics, Texas A&M University, College Station, TX 77843-3368. E-mail is preferred. Correspondence with the managing editor may be sent to `notices@ams.org`. For more information, see the section "Reference and Book List".

NOTICES ON THE AMS WEBSITE: Most of this publication is available electronically through the AMS website, the Society's resource for delivering electronic products and services. Use the URL `http://www.ams.org/notices/` to access the *Notices* on the website.

[*Notices of the American Mathematical Society* is published monthly except bimonthly in June/July by the American Mathematical Society at 201 Charles Street, Providence, RI 02904-2294. Periodicals postage paid at Providence, RI, and additional mailing offices. POSTMASTER: Send address change notices to *Notices of the American Mathematical Society*, P.O. Box 6248, Providence, RI 02940-6248.] Publication here of the Society's street address and the other information in brackets above is a technical requirement of the U.S. Postal Service. Tel: 401-455-4000, e-mail: `notices@ams.org`.

Departments

From the AMS Secretary

Mathematics and the Human Genome

The genetic material in our chromosomes is our genome. Chromosomes are long DNA molecules, that is, double-helices consisting of long paired polymers of the bases Adenine, Cytosine, Guanine, and Thymine bound together in sequence, attached to sugar-phosphate backbones. The twenty-two regular human chromosomes and the X and Y chromosomes that are a human genome comprise about three billion base pairs of DNA. Genes are segments of chromosomes of differing lengths, with the typical human gene being a few thousand base pairs in length. There are believed to be thirty to forty thousand genes in the human genome, making up about 3 percent of the total base pairs. Identifying all the genes and understanding their function is the major task of genomics.

For most of the twentieth century mathematics played a minor role in genomics, but its role has expanded greatly over the last twenty years. With the ability to sequence complete genomes, the expansion accelerated. Why? What kinds of mathematics arise here? Why might mathematicians find the field interesting? What are the opportunities? Are there barriers to the involvement of mathematicians in genomics?

Why mathematics and the human genome? The answer is: data. Early genetic data were either counts, typically of genotypes at a few loci or discrete phenotypes, or measurements of continuously varying phenotypes. We now have very large quantities of genotypic data on a genome-wide scale: many loci and many phenotypes on many people. There is a vast and growing amount of DNA, RNA, and protein sequence data in public databases, much of it on a genome-wide scale (e.g., billions of bases of human DNA sequence). Human sequence variation data is now accumulating at the level of the individual. The generation of massive amounts of messenger RNA expression data on a genome-wide scale has just begun; similar data at the protein level is the next great growth area. The amount can hardly be imagined: In principle we could generate data on the expression of all genes in all cell types under all conditions for individuals of all genotypes.

What kinds of mathematics? This is a hard question to answer briefly, but let's consider the recent human genome sequencing and the initial follow-up. Several different human genomes were copied many times and broken into millions of small fragments. Fairly accurate but imperfect DNA sequence was obtained for each of these fragments. The algorithms which turned laser-scanned electrophoretic traces into sequences of A's, C's, G's, and T's, along with quality scores, involved dynamic programming, machine learning, Fourier analysis, and other signal processing techniques. Then came the massive assembling of these millions of fragments into large pieces. There a wide range of combinatorial, graph-theoretic, probabilistic, and statistical tools were combined into algorithms which ran for thousands of hours on some of the world's largest computers.

After the genome was assembled—finished data in one sense, raw data in another sense—the task of interpretation began. Identifying the 3 percent comprising the genes was the first priority. Protein-coding genes have a modular structure, including several highly but not completely conserved signals, and DNA sequences that have small but consistent compositional differences from nongenic DNA. The task was to use this information, together with that on known genes in sequence databases, to identify new genes. The best current methods use hidden Markov models. Stochastic context-free grammars play an analogous role in identifying RNA genes. Comparing new DNA sequences to those in databases involves string search algorithms (e.g., finite-state machines), sequence alignment algorithms (more dynamic programming), and statistical scoring. When we have predicted genes, we want to learn about the proteins they encode. Alignment, hidden-Markov models, clustering, and phylogenetic tree algorithms all play roles here. Having predicted proteins we next want to learn their structure and function. How do they fold? When and where are they expressed? With which other proteins do they interact? In which pathways are they involved? More questions, more data, more mathematics.

If your interest is in algorithms, combinatorics, machine learning, optimization, probability, statistics, knot theory, to mention the obvious areas, there is a good chance that there are problems in genomics to interest and challenge you. If you like biology as well, the opportunities are endless. What about barriers? They are the usual ones when boundaries are crossed: language and culture. For mathematicians getting into this area there is much to learn. And as in any application of mathematics, the problems mathematicians want to study may not be those biologists think important. The genome community may not embrace your contribution in the way you wish. But none of this is new or unique to genomics. The challenges and satisfaction are there for anyone wishing to get involved; the benefits far outweigh the risk of investing time and effort to find out what is needed.

—Terry Speed
University of California at Berkeley

About the Cover

April is Mathematics Awareness Month (MAM). The theme for MAM 2002 is "Mathematics and the Genome". Visit `http://www.mathforum.org/mam/02/` for more information and to download the poster. Images: *Protein bound to DNA*, courtesy of Akinori Sarai, Ryken Bioinformatics Database, `www.rtc.riken.go.jp/jouhou/image/dna-protein/all/N1hjb.gif`; *DNA, the molecule of life*, courtesy of U.S. Department of Energy Human Genome Program, `www.ornl.gov/hgmis/`.

—Annette Emerson, AMS Public Awareness Officer

Expansions That Grow on Trees

Arieh Iserles

Linear Ordinary Differential Equations

How to solve linear ordinary differential equations? Like many outstanding mysteries of mathematics, this question has the virtue of simplicity. Linear differential equations are the staple of every mathematical syllabus, familiar to all and sundry, and their investigation has informed much of the development of mathematical analysis in the last three hundred years. Needless to say, we can all provide *partial* answers to this question. Thus, the solution of the scalar equation $y' = a(t)y$, $y(0) = y_0$, is

$$y(t) = \mathrm{e}^{\int_0^t a(\xi)\mathrm{d}\xi} y_0, \quad t \ge 0,$$

while the solution of the vector equation $\mathbf{y}' = A\mathbf{y}$, $\mathbf{y}(0) = \mathbf{y}_0 \in \mathbb{R}^N$, where A is a *constant* $N \times N$ matrix, is

$$\mathbf{y}(t) = \mathrm{e}^{tA}\mathbf{y}_0, \quad t \ge 0.$$

The matrix exponential above is defined by Taylor expansion,

$$\mathrm{e}^{\Omega} = \sum_{m=0}^{\infty} \frac{1}{m!}\Omega^m.$$

It is natural, as the next step in our investigation, to attempt "mental interpolation" between the last two explicit formulae and to check whether the solution of the vector system with variable coefficients, $\mathbf{y}' = A(t)\mathbf{y}$, $\mathbf{y}(0) = \mathbf{y}_0$, might perhaps be

$$(1) \qquad \mathbf{y}(t) = \mathrm{e}^{\int_0^t A(\xi)\mathrm{d}\xi}\mathbf{y}_0, \quad t \ge 0,$$

where the integral of the matrix function A is the matrix whose entries are the integrals of the components of A. Unfortunately, this mental leap is in general false.

The general form of $\mathbf{y}$, and, indeed, of the fundamental solution Y, where

$$(2) \qquad Y' = A(t)Y, \quad t \ge 0, \qquad Y(0) = I,$$

is considerably more intriguing. Its exploration will take us on a tour spanning a hundred years of mathematics and ranging from differential geometry, Lie algebra theory, and graph theory, all the way to contemporary numerical analysis. The basic idea is to represent the solution of (2) in the form $Y(t) = \mathrm{e}^{\Omega}$ and to rephrase the differential equation in terms of the *logarithm* of Y: the matrix function Ω. The equation for Ω is originally due to Felix Hausdorff [10] and is given by

$$(3) \qquad \Omega' = \sum_{m=0}^{\infty} \frac{\mathrm{B}_m}{m!}\mathrm{ad}_{\Omega}^m A, \quad t \ge 0, \qquad \Omega(0) = O.$$

Here $\{\mathrm{B}_m\}_{m\in\mathbb{Z}_+}$ are the *Bernoulli numbers,* defined by

$$\sum_{m=0}^{\infty} \frac{\mathrm{B}_m}{m!} z^m = \frac{z}{\mathrm{e}^z - 1} = 1 - \tfrac{1}{2}z + \tfrac{1}{12}z^2 - \tfrac{1}{720}z^4 + \cdots,$$

while ad_{Ω} is a shorthand for an iterated commutator,

$$\mathrm{ad}_{\Omega}^0 A = A, \quad \mathrm{ad}_{\Omega}^1 A = [\Omega, A], \quad \mathrm{ad}_{\Omega}^2 A = [\Omega, [\Omega, A]],$$

Arieh Iserles is professor in the Department of Applied Mathematics and Theoretical Physics, University of Cambridge, UK. His e-mail address is `ai@damtp.cam.ac.uk`.

and, in general, $\mathrm{ad}_\Omega^{m+1}A = [\Omega, \mathrm{ad}_\Omega^m A]$, where $[B,C] = BC - CB$.

On the face of it, we have just taken a large step in rendering the simple equation (2) in a more complex, indeed obscure, manner, as an infinite series of nonlinear terms. Yet, as so often in mathematics, this is just the price of an ultimate simplification, new insight, and superior computational algorithms.

We commence by noting that if $A(t)$ and $\Omega(t)$ commute (which, as we will see soon, is the case whenever $A(t)$ and $\int_0^t A(\xi)\mathrm{d}\xi$ commute), all the terms on the right of (3), except for the first, vanish. We have $\Omega' = A$ and the solution is (1). In other words, the $m \geq 1$ terms and the nonlinearity of (3) are the cost of noncommutativity.

Wilhelm Magnus

The main insight into the equation (3) was provided in a remarkable paper by a remarkable mathematician, Wilhelm Magnus. A topologist, an algebraist, an authority on differential equations and on special functions, a mathematical physicist, Magnus exercised great (and often not sufficiently acknowledged) influence on twentieth century mathematics [1]. Magnus observed in [17] that Ω can be written as a linear combination of multiple integrals. Specifically, employing Picard's iteration,

$$\Omega_0(t) \equiv O,$$

$$\Omega_{s+1}(t) = \int_0^t \sum_{m=0}^{\infty} \frac{\mathrm{B}_m}{m!} \mathrm{ad}^m_{\Omega_s(\xi)} A(\xi)\mathrm{d}\xi, \quad s \in \mathbb{Z}_+,$$

it is possible to show that

(4)
$$\begin{aligned}\Omega(t) = &\int_0^t A(\xi)\mathrm{d}\xi \\ &- \tfrac{1}{2}\int_0^t\int_0^{\xi_1}[A(\xi_2),A(\xi_1)]\mathrm{d}\xi_2\mathrm{d}\xi_1 \\ &+ \tfrac{1}{12}\int_0^t\int_0^{\xi_1}\int_0^{\xi_1}[A(\xi_3),[A(\xi_2),A(\xi_1)]]\mathrm{d}\xi_3\mathrm{d}\xi_2\mathrm{d}\xi_1 \\ &+ \tfrac{1}{4}\int_0^t\int_0^{\xi_1}\int_0^{\xi_2}[[A(\xi_3),A(\xi_2)],A(\xi_1)]\mathrm{d}\xi_3\mathrm{d}\xi_2\mathrm{d}\xi_1 \\ &+ \cdots.\end{aligned}$$

Next there are four fourfold integrals, each with three nested commutators, and in general the number of terms in each "generation" grows exponentially. So does the complexity, which rapidly gets out of hand. As a matter of fact, the situation is even worse, since naive Picard iteration generates a multitude of higher-order terms that need to be excised in each step. Nonetheless, the above *Magnus expansion* became an important tool in numerous application areas, in particular in quantum chemistry and theoretical physics. Many attempts have been made to derive the expansion in explicit form, the most remarkable due to Fomenko and Chakon [8]. This, however, did not eliminate the obvious combinatorial complexity inherent in dealing with multiple integrals and "multilayered" iterated commutators and, arguably, did not lead to a viable means to generate recursively and analyse the function Ω.

On Expansions That Grow on Trees

Several years ago, Syvert Nørsett and I came across the Magnus expansion in the context of *geometric integration*, the emerging discipline concerned with discretising differential equations while respecting their qualitative properties [5]. Blissfully unaware of the substantial research to which Magnus's creation has been subjected, we developed an alternative approach, using *rooted trees* as a shorthand for expansion terms. This has led to a framework that elucidates the structure of individual terms and their relationship, thus allowing for convenient construction of the expansion (4) by recursion [14].

Examine (4) for a moment: Each term is made out of integrals and commutators of the matrix function A. Specifically, each term on the right, except for the first, is of the form

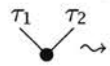
$$\alpha \int_0^t [H_1(\xi), H_2'(\xi)]\mathrm{d}\xi, \tag{5}$$

where both H_1 and H_2 have already featured earlier in the expansion, while α is a scalar constant. We thus require terminology to denote repeated integration and commutation in a clear, transparent fashion.

We commence by assigning to A a trivial tree, consisting of single vertex,

$$\bullet \rightsquigarrow A.$$

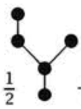
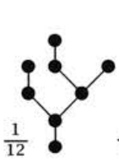
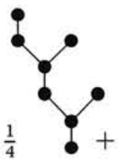
Moreover, suppose that we have already constructed two expansion terms, H_{τ_1} and H_{τ_2}, say, such that $\tau_1 \rightsquigarrow H_{\tau_1}$ and $\tau_2 \rightsquigarrow H_{\tau_2}$. Then



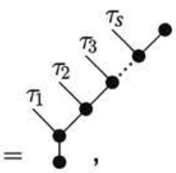
$$\rightsquigarrow [H_{\tau_1}, H_{\tau_2}],$$


$$\rightsquigarrow \int_0^t H_{\tau_1}(\xi)\mathrm{d}\xi.$$

Given an $N \times N$ matrix function A with integrable entries, and commencing from $\bullet$, we have just defined a map $\tau \to H_\tau$ from $\mathbb{F}$, a subset of binary rooted trees, into $N \times N$ matrix functions. In particular, (4) can be denoted in the shorthand

(6)  $-\frac{1}{2}$  $+\frac{1}{12}$  $+\frac{1}{4}$  $+\cdots.$

Each tree in (6), except for the first, is of the form on the right side of (7),

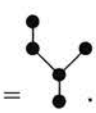
(7) $\tau =$ 

,

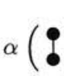
where $s \in \mathbb{Z}_+$ and the trees $\tau_1, \tau_2, \ldots, \tau_s$ have already featured earlier in the expansion. For example, in the third tree in (6)

$$s = 2, \qquad \tau_1 = \tau_2 =$$  ,

while the fourth tree is consistent with

$$s = 1, \qquad \tau_1 =$$  .

It is possible to deduce from (5) that the form (7) remains valid for subsequent expansion terms, indeed that it characterises all the trees in $\mathbb{F}$. With a little more effort it is also possible to deduce from (7) the explicit form of the constant α in (5) [14]. Set

$\alpha\Big($  $\Big) = 1$

and suppose that $\alpha(\tau_j)$ is known for $j = 1, 2, \ldots, s$. Then

$$\alpha(\tau) = \frac{\mathrm{B}_s}{s!}\prod_{j=1}^{s}\alpha(\tau_j), \qquad s \in \mathbb{N}, \tag{8}$$

where B_s is the sth Bernoulli number.

We have now all the ingredients to describe a general recursive rule for the generation of a Magnus expansion, except that, in our quest for tidiness, we are reluctant to lump all expansion coefficients into a hotchpotch of unclassified terms. In our earlier discussion of (4) we have aggregated terms according to the number of integrals therein: In the shorthand of rooted trees, this corresponds to the number of "vertical" edges. This, however, is not the best way to display the expansion in the most economical fashion, without including terms which might be redundant once the expansion has been truncated for practical purposes. Instead, we say that a tree τ is of *power* $r \in \mathbb{N}$ if r is the greatest integer such that

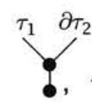
$$H_\tau(t) = \mathcal{O}(t^r), \qquad t \to 0,$$

for all sufficiently smooth matrix functions A. Let $\mathbb{F}_r$ be the set of all trees of power r. The Magnus expansion can be written in the form

$$\Omega(t) = \sum_{r=1}^{\infty}\sum_{\tau\in\mathbb{F}_r}\alpha(\tau)H_\tau(t) \tag{9}$$

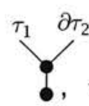
according to the following rules:

1. Let $\mathbb{F}_1 = \Big\{$  $\Big\}$ and $\alpha\Big($  $\Big) = 1$.
2. Given $r \geq 2$, assemble $\mathbb{F}_r$ by taking all the trees of the form

   $, \quad \tau_1 \in \mathbb{F}_{k_1},\ \tau_2 \in \mathbb{F}_{k_2},\ k_1 + k_2 = r-1,\ \tau_1 \neq \tau_2,$

   where $\partial\tau$ stands for the tree τ with its root excised, i.e., $\partial\tau \rightsquigarrow \mathrm{d}H_\tau/\mathrm{d}t$. Moreover, if r is even, then add to $\mathbb{F}_r$ all trees of the form

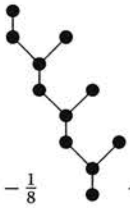
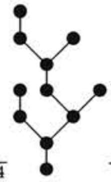
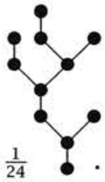
   

   $, \quad \tau_1 \in \mathbb{F}_{k_1},\ k_1 = r/2 - 1.$
3. Identify each tree τ with the representation (7) and use (8) to evaluate its coefficient $\alpha(\tau)$.

Note that $\mathbb{F}_2 = \emptyset$, since (as follows from the algorithm above but can be easily verified directly) the power of

is three.

All trees of power up to four are displayed in (6). The next contribution to Ω, corresponding to trees in $\mathbb{F}_5$, is

(10) $-\frac{1}{8}$  $-\frac{1}{24}$  $-\frac{1}{24}$  .

As we go along, the number of terms in $\mathbb{F}_r$ increases exponentially: It was proved in [15] that $\limsup_{r\to\infty}[\#\mathbb{F}_r]^{1/r} = 3.1167\ldots$. In comparison, the number of terms with r integrals increases as $c(r)4^r$, where c is a slowly-varying function [14]. Even for moderate r the difference is highly significant.

By construction, once we truncate (9),

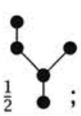
$$\Omega^{[p]}(t) = \sum_{r=1}^{p} \sum_{\tau\in\mathbb{F}_r} \alpha(\tau)H_\tau(t),$$

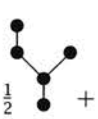
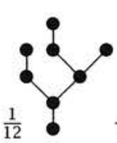
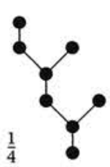
we can expect $\Omega^{[p]}(t) = \Omega(t) + \mathcal{O}(t^{p+1})$, an order-$p$ approximation. However, the situation is better than this!

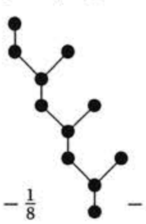
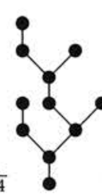
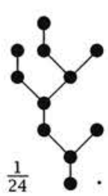
An operator Ψ_t, which for every t maps (for example) $N \times N$ matrix functions to themselves, is said to be *time symmetric* if $\Psi_{-t} \circ \Psi_t = \mathrm{Id}$. In particular, every solution operator of a differential equation, $\Psi_h Y(t) = Y(t+h)$, is time symmetric, but this is not necessarily the case with approximations to the solution—and the truncation $\Omega^{[p]}(t)$ can be envisaged as a numerical method to compute Ω with step size t. (Much more later about Magnus expansions as numerical methods.) Yet, it is possible to prove that the map $\Psi_t X(t) = \mathrm{e}^{\Omega^{[p]}(t)}X(t)$ is time symmetric [15].

Time symmetry is important because it implies that for odd p, the tail $\Omega(t) - \Omega^{[p]}(t)$ is $\mathcal{O}(t^{p+2})$ and we gain an extra unit of order [15]. The first few such truncated Magnus expansions are

order 2:  ;

order 4:  $-\frac{1}{2}$  ;

order 6:  $-\frac{1}{2}$  $+\frac{1}{12}$  $+\frac{1}{4}$ 

$-\frac{1}{8}$  $-\frac{1}{24}$  $-\frac{1}{24}$  .

As a simple example, which the reader (perhaps with the help of a symbolic algebra program) is encouraged to verify, let us consider the *Airy equation*

(11) $y'' + ty = 0, \quad t \ge 0, \qquad y(0) = 1, \ y'(0) = \frac{1}{2},$

which we rewrite in a vector form as

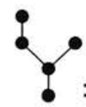
$$\mathbf{y}' = \begin{bmatrix} 0 & 1 \\ -t & 0 \end{bmatrix}\mathbf{y}, \quad t \ge 0, \quad \mathbf{y}(0) = \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix}.$$

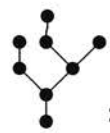
Although the exact solution of (11) can be written explicitly in terms of Airy functions, it is more illustrative to express it as a series,

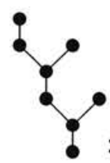
$$y = 1 + \tfrac{1}{2}t - \tfrac{1}{6}t^3 - \tfrac{1}{24}t^4 + \tfrac{1}{180}t^6 + \tfrac{1}{1008}t^7 + \mathcal{O}(t^9).$$

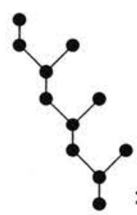
The leading Magnus trees and corresponding matrix functions are

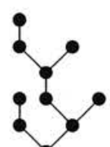
 : $\begin{bmatrix} 0 & t \\ -\frac{1}{2}t^2 & 0 \end{bmatrix}$;

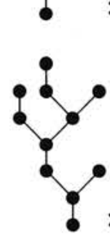
 : $\begin{bmatrix} -\frac{1}{6}t^3 & 0 \\ 0 & \frac{1}{6}t^3 \end{bmatrix}$;

 : $\begin{bmatrix} 0 & \frac{1}{4}t^4 \\ \frac{1}{10}t^5 & 0 \end{bmatrix}$;

 : $\begin{bmatrix} 0 & -\frac{1}{12}t^4 \\ -\frac{1}{15}t^5 & 0 \end{bmatrix}$;

 : $\begin{bmatrix} \frac{1}{40}t^6 & 0 \\ 0 & -\frac{1}{40}t^6 \end{bmatrix}$;

 : $\begin{bmatrix} -\frac{1}{12}t^6 & 0 \\ 0 & \frac{1}{12}t^6 \end{bmatrix}$;

 : $\begin{bmatrix} -\frac{7}{120}t^6 & 0 \\ 0 & \frac{7}{120}t^6 \end{bmatrix}$.

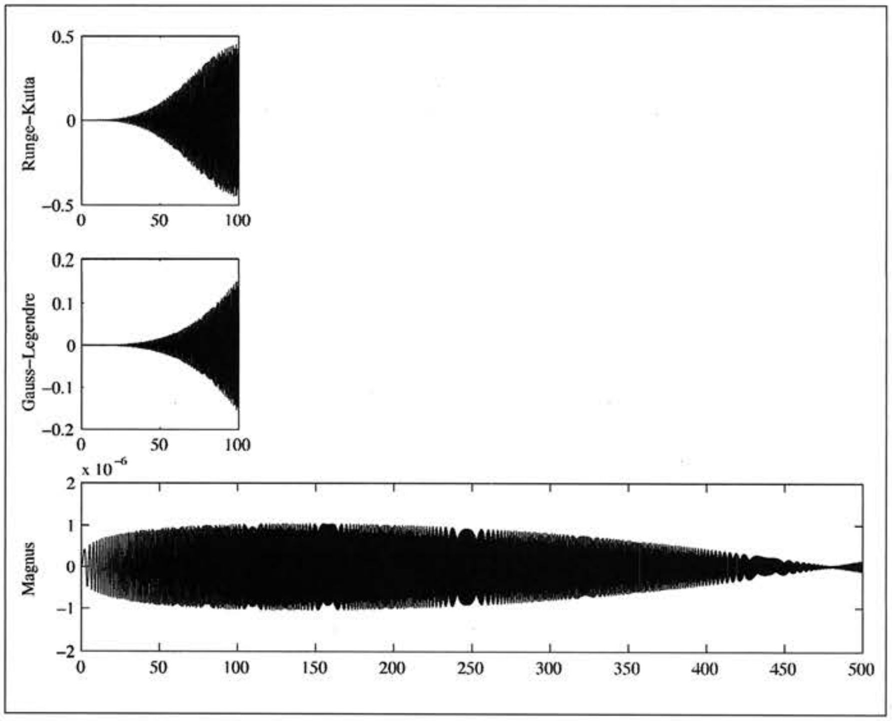


Figure 1: The error in the solution of the Airy equation by three numerical methods.

We thus deduce that

$$\Omega^{[1]} = \begin{bmatrix} 0 & t \\ -\frac{1}{2}t^2 & 0 \end{bmatrix};$$
$$\Omega^{[3]} = \begin{bmatrix} \frac{1}{12}t^3 & t \\ -\frac{1}{2}t^2 & -\frac{1}{12}t^3 \end{bmatrix};$$
$$\Omega^{[5]} = \begin{bmatrix} \frac{1}{12}t^3 + \frac{1}{360}t^6 & t \\ -\frac{1}{2}t^2 - \frac{1}{120}t^5 & -\frac{1}{12}t^3 - \frac{1}{360}t^6 \end{bmatrix}.$$

Letting $\mathbf{x}_k = e^{\Omega^{[k]}}\mathbf{y}(0)$, we obtain

$$\mathbf{x}_1 = 1 + \tfrac{1}{2}t^2 - \tfrac{1}{4}t^3 + \mathcal{O}(t^4),$$
$$\mathbf{x}_3 = 1 + \tfrac{1}{2}t^2 - \tfrac{1}{6}t^3 - \tfrac{1}{24}t^4 + \tfrac{1}{144}t^6 + \mathcal{O}(t^7),$$
$$\mathbf{x}_5 = 1 + \tfrac{1}{2}t^2 - \tfrac{1}{6}t^3 - \tfrac{1}{24}t^4 + \tfrac{1}{180}t^6 + \tfrac{1}{1080}t^7 + \mathcal{O}(t^9).$$

Comparison with the Taylor expansion of y confirms that this is consistent with order $k+1$.

Figure 1 displays the pointwise error in the solution of the Airy equation (11) by three fourth-order methods: the classical Runge–Kutta method, the (implicit, A-stable) Gauss–Legendre Runge–Kutta method, and a truncation of the Magnus expansion, all time-stepped with constant step size of $\frac{1}{10}$. The exact solution of (11) oscillates with increasing frequency, and it behaves asymptotically like $t^{-1/4}\sin\frac{2}{3}t^{3/2}$ as $t \to \infty$. This is sufficient to defeat even the hardiest "classical" numerical schemes, while Magnus, less impressed by rapid oscillation, delivers significantly superior performance: Note that the vertical scale on the bottom diagram in Figure 1 is six orders of magnitude smaller than in the other two diagrams.

For the record, to obtain similar accuracy of six significant digits in $[0, 500]$ with a Runge–Kutta method would have required a step size of $\approx \frac{1}{1200}$, at a relative cost (in computer time) of more than sixty times that of the Magnus method. More sophisticated methods need not be necessarily more expensive!

Lie Groups and Geometric Integration

Like Molière's Monsieur Jourdain, who had been speaking prose all his life and did not know it, we have just been, perhaps unknowingly, speaking the language of Lie groups.

Since their introduction by Sophus Lie in the late nineteenth century, Lie groups have been among our most powerful tools for understanding structure, symmetry, and geometry, in particular within the context of differential equations [22].

A *Lie group* $\mathcal{G}$ is a manifold endowed with group structure. Being a manifold, it can be covered by an atlas of overlapping local charts, which are homeomorphic to open subsets of Euclidean space. Being a group, $\mathcal{G}$ is closed under inversion and multiplication. Moreover, we stipulate that these two operations are continuous with respect to the underlying topology of the manifold. In case this sounds mysterious and abstract, let us consider several examples. First, consider the *orthogonal group* $\mathrm{O}(N)$ of real $N \times N$ orthogonal matrices. Clearly, it is a group: The identity is an orthogonal matrix, and if $Q_1, Q_2 \in \mathrm{O}(N)$, then also $Q_1^{-1} = Q_1^\top, Q_1Q_2 \in \mathrm{O}(N)$. It is also a $\frac{1}{2}(N-1)N$-dimensional manifold (can you prove it?) which can be embedded in the N^2-dimensional space of all real $N \times N$ matrices, while both matrix multiplication and inversion are smooth operations. Another example is the *special linear group* $\mathrm{SL}(N)$ of real $N \times N$ matrices with unit determinant. Yet another is the *symplectic group* $\mathrm{Sp}(N)$, comprised of $(2N) \times (2N)$ real matrices X such that $XJX^\top = J$, where

$$J = \begin{bmatrix} O & I \\ -I & O \end{bmatrix},$$

with $N \times N$ zero and identity matrices O and I respectively. Finally, consider the set of all *rigid motions* (rotations, reflections, and translations) of the space $\mathbb{R}^N$, namely $\mathbf{x} \mapsto A\mathbf{x} + \mathbf{b}$, where $A \in \mathrm{O}(N)$ and $\mathbf{b} \in \mathbb{R}^N$. Each rigid motion is thus characterised by the pair $(A, \mathbf{b})$, and we can endow such pairs with group structure. The outcome is the *Euclidean group* $\mathrm{E}(N)$, with the operations

$$(A_2, \mathbf{b}_2) \cdot (A_1, \mathbf{b}_1) = (A_2 A_1, A_2 \mathbf{b}_1 + \mathbf{b}_2),$$
$$(A, \mathbf{b})^{-1} = (A^\top, -A^\top \mathbf{b})$$

(verify!).

All our examples are subgroups of matrices, so we should perhaps comment that Lie groups need be neither multiplicative subgroups of matrices nor, indeed, finite dimensional. Glossing over some technical details, an important example of an infinite-dimensional group is SDiff $\mathcal{M}$, the set of all volume-preserving diffeomorphisms from a manifold $\mathcal{M}$ to itself.

Lie groups feature in multitudes of applications. Orthogonal and Euclidean groups are crucial in mechanics, computer vision, and numerical algebra, the special linear group in volume conservation, the symplectic group in Hamiltonian mechanics, and SDiff $\mathcal{M}$ in geometric theories of fluid flow. Lie groups are increasingly used in diverse areas of applied mathematics where it is important to identify and preserve geometric structure, e.g., in control theory, robotics, Kalman filtering, subdivision schemes in computer graphics, geometric mechanics....

Typically, a mathematical model is phrased as a solution of a differential system. Whenever such a system is known to evolve in a Lie group, this represents valuable information since, more often than not, Lie group structure embodies qualitative features which are central to the underlying natural phenomenon that we are endeavouring to understand.

Once we consider differential equations evolving on a manifold $\mathcal{M}$, we are concerned with *tangents* to points on $\mathcal{M}$, since these are all the possible values that a vector field may attain. Let $\mathrm{T}\mathcal{M}|_x$ be the linear space of all vectors tangent to $\mathcal{M}$ at some $x \in \mathcal{M}$. It is easy to verify that any differential equation evolving in $\mathcal{M}$ can be written in the form

$$y' = f(t, y), \quad t \geq 0, \qquad y(0) \in \mathcal{M},$$

where the (sufficiently smooth) function $f(t, y)$ maps $\mathbb{R}_+ \times \mathcal{M}$ to $\mathrm{T}\mathcal{M}|_y$. Typically, the structure of the *tangent bundle* $\mathrm{T}\mathcal{M}$ of all tangent spaces $\mathrm{T}\mathcal{M}|_x$, $x \in \mathcal{M}$, is complicated. One of the great virtues of Lie groups is that their tangency relations are relatively transparent and endowed with beautiful features. We first note that the group structure implies that $\mathrm{T}\mathcal{G}|_X = \mathrm{T}\mathcal{G}|_I X$ for all $X \in \mathcal{G}$. Therefore, all ordinary differential equations evolving on $\mathcal{G}$ can be brought to the form

$$(12) \qquad Y' = A(t, Y)Y, \quad t \geq 0, \qquad Y(0) \in \mathcal{G},$$

where the (suitably smooth) function A maps $\mathbb{R}_+ \times \mathcal{G}$ to $\mathfrak{g} = \mathrm{T}\mathcal{G}|_I$. Secondly, the set $\mathfrak{g}$ is not just a humble linear space but a *Lie algebra:* It is closed under a bilinear, skew-symmetric operation $[\![\cdot, \cdot]\!]$, say. For every $x_1, x_2, x_3 \in \mathfrak{g}$ and $\alpha_1, \alpha_2 \in \mathbb{R}$ (or, for example, in $\mathbb{C}$, but we do not aim here for the greatest generality) it is true that

$$[\![x_1, x_2]\!] + [\![x_2, x_1]\!] = 0,$$
$$[\![\alpha_1 x_1 + \alpha_2 x_2, x_3]\!] = \alpha_1 [\![x_1, x_3]\!] + \alpha_2 [\![x_2, x_3]\!],$$

and

$$[\![x_1, [\![x_2, x_3]\!]\,]\!] + [\![x_2, [\![x_3, x_1]\!]\,]\!] + [\![x_3, [\![x_1, x_2]\!]\,]\!] = 0$$

(the *Jacobi identity*). Thirdly, it is possible to construct a smooth map $e : \mathfrak{g} \to \mathcal{G}$ which takes zero to the identity and affords us a way back from the algebra to the group.

Recalling our examples of Lie groups, we just list the corresponding Lie algebras. Thus, the Lie algebra of $\mathrm{O}(N)$ is $\mathfrak{so}(N)$, the linear space of $N \times N$ skew-symmetric matrices; to $\mathrm{SL}(N)$ there corresponds the algebra $\mathfrak{sl}(N)$ of zero-trace matrices; the Lie algebraic counterpart of the symplectic group $\mathrm{Sp}(N)$ is $\mathfrak{sp}(N)$, such that $X \in \mathfrak{sp}(N)$ implies $XJ + JX^\top = O$, and, finally, the Euclidean algebra $\mathfrak{se}(N)$ comprises all pairs $(X, \mathbf{b})$ such that $X \in \mathfrak{so}(N)$ and $\mathbf{b} \in \mathbb{R}^N$.

All this may sound unrelated to linear equations. Yet, a powerful connection becomes apparent once we restrict our attention to finite-dimensional groups and algebras. According to a beautiful theorem of Ado, every finite-dimensional Lie algebra is isomorphic to a Lie subalgebra of matrices [22] and, without loss of generality, we may assume that $\mathfrak{g}$ is indeed a subalgebra of $N \times N$ real matrices. In that case $[\![X_1, X_2]\!] = [X_1, X_2]$ and $e(X) = \mathrm{e}^X$, the familiar commutator and exponential functions. Thus, the entire discourse of the previous section can be rendered in the language of Lie: The matrix function $A(t)$ assumes values in a matrix Lie algebra $\mathfrak{g}$, hence the solution of (2) evolves in "its" Lie group $\mathcal{G}$. (In the most unrestricted case, $\mathfrak{g} = \mathfrak{gl}(N)$, the Lie algebra of all $N \times N$ matrices, and $\mathcal{G} = \mathrm{GL}(N)$, the *general linear group* of all nonsingular $N \times N$ matrices.) Replacing Y with Ω and equation (2) with (3) corresponds to translating the problem from a Lie group to a Lie algebra. Since the latter is closed with respect to linear combinations, it is also closed with respect to integration, which is a limit of Riemann sums. Therefore, both the Magnus expansion (9) and its truncation $\Omega^{[p]}$ evolve in $\mathfrak{g}$ and the exponential restores the solution to the Lie group.

The great virtue of working in a Lie algebra, rather than a Lie group, becomes clear once we consider approximation, whether numerical or perturbative. General Lie groups are nonlinear creatures and classical numerical methods are often useless when the retention of Lie group structure is at issue. For example, no familiar discretisation method, e.g., Runge–Kutta or multistep or a truncated Taylor expansion, can be assured to evolve in $\mathrm{SL}(N)$ for $N \geq 3$ [11, 16]. On the other hand, $\mathfrak{g}$ is a linear space: As long as we restrict our arsenal to linear combinations and commutation, we cannot go wrong! Thus, Magnus expansions are a prime example of *geometric integrators*, which are guaranteed to respect the underlying geometry of the differential equation [13].

An obvious alternative to Lie group methods, like the Magnus expansion, is *projection:* Use an arbitrary discretisation method and ensure that geometry is modelled correctly by projecting the numerical solution on $\mathcal{G}$. Although naive projection may run foul of instabilities and rapid error accumulation, a more sophisticated approach can be effective [9].

Computing Magnus

In principle, the Magnus expansion presents a powerful means to compute numerically the solution of a linear system (2). However, nothing is ever simple or straightforward with computation. The first stumbling block is that the expansion (9) has only a limited radius of convergence: To converge in a norm $\|\cdot\|$, we require

$$\int_0^t \|A(\xi)\| \mathrm{d}\xi \le \int_0^{2\pi} \frac{\mathrm{d}\xi}{4+\xi[1-\cot(\xi/2)]} \approx 1.08687$$

[2, 18]. If t is so large that this inequality is violated, then we can use time-stepping with, say, constant step $h > 0$. A more substantial obstacle is that the evaluation of $\Omega^{[p]}$ requires the computation of several multivariate integrals, each over a different polytope, at each time step. Now, it is very easy to approximate a univariate integral but most challenging and labour intensive to do so in a multivariate setting [7].

Before we can run, though, we need to learn how to walk. Or, more specifically, how to evaluate univariate integrals. Even if you are familiar with numerical quadrature, do bear with me, for, miraculously, the univariate case extends in our setting to multiple integrals. Let $c_1, c_2, \ldots, c_\nu$ be the zeros of the ν-degree Legendre polynomial, shifted to the interval $[0, 1]$. It is well known since Gauss's time that there exist weights $b_1, b_2, \ldots, b_\nu > 0$ such that the *quadrature*

$$\int_0^h A(\xi)\mathrm{d}\xi \approx h \sum_{k=1}^{\nu} b_k A(c_k h) \tag{13}$$

is of order 2ν: For sufficiently smooth functions A it carries an error of $\mathcal{O}(h^{2\nu+1})$. Moreover, no other choice of quadrature nodes $c_1, c_2, \ldots, c_\nu$ and weights can improve (or, indeed, match) this order [7]. It is helpful to rephrase (13). Thus, let the matrices $B_1, B_2, \ldots, B_\nu$ be the solution of the Vandermonde linear system

$$\sum_{j=1}^{\nu} (c_k - \tfrac{1}{2})^{j-1} B_j = hA(c_k h), \quad k = 1, 2, \ldots, \nu.$$

It is possible to show that

$$B_j = d_j h^j A^{(j-1)}(\tfrac{1}{2}h) + \mathcal{O}(h^{j+1}),$$

where the d_j's are nonzero constants, and that the order-2ν quadrature (13) is equivalent to

$$\int_0^h A(\xi)\mathrm{d}\xi \approx \sum_{j=0}^{\lfloor(\nu-1)/2\rfloor} \frac{1}{4^j(2j+1)} B_{2j+1}.$$

We will soon see that, when generalisation to "Magnus integrals" is at issue, the currency of the B_j's is superior to that of the $A(c_k h)$'s.

Taking a leaf from [14], we next consider the double integral from (4),

$$\int_0^h \int_0^{\xi_1} [A(\xi_2), A(\xi_1)] \mathrm{d}\xi_2 \mathrm{d}\xi_1.$$

Being thrifty by nature, we are loath to abandon the matrices B_j. Instead, in the spirit of environmental awareness, we recycle them. The integrand being a commutator of two values of A, we may attempt to approximate the integral by a linear combination of commutators of the form $[B_k, B_j]$. This works surprisingly well and it is possible to prove that

$$\sum_{k=1}^{\nu-1} \sum_{\substack{j=k+1 \\ k+j \text{ odd}}}^{\nu} \frac{(j-k)-(-1)^k(j+k)}{2^{j+k-1} jk(j+k)} [B_k, B_j]$$
$$= -\tfrac{1}{6}[B_1, B_2] + \tfrac{1}{120}[B_2, B_3] - \tfrac{1}{40}[B_1, B_4] + \cdots$$

approximates the double integral to order 2ν, identical to the univariate Gaussian quadrature (13) [13].

It is not just the bivariate integral. *All* the integrals in the Magnus expansion can be evaluated to order 2ν using and reusing again the same ν matrices B_j. The general pattern for an s-fold integral is

$$\int_{hS} \mathcal{L}(A(\xi_s), A(\xi_{s-1}), \ldots, A(\xi_1)) \mathrm{d}\xi_s \mathrm{d}\xi_{s-1} \cdots \mathrm{d}\xi_1 \approx \sum_{\mathbf{j} \in C_s^\nu} \tilde{b}_{\mathbf{j}} \mathcal{L}(B_{j_1}, B_{j_2}, \ldots, B_{j_s}). \tag{14}$$

Here $hS \in \mathbb{R}^s$ is the domain of integration, $\mathcal{L}$ is a nested commutator, and C_s^ν is the set of all combinations of length s from $\{1, 2, \ldots, \nu\}$. The weights $\tilde{b}_{\mathbf{j}}$ are given explicitly by

$$\tilde{b}_{\mathbf{j}} = \int_S \prod_{i=1}^{s} (\xi_i - \tfrac{1}{2})^{j_i-1} \mathrm{d}\xi_s \mathrm{d}\xi_{s-1} \cdots \mathrm{d}\xi_1.$$

Regardless of the number of terms in the order-(2ν) approximation $\Omega^{[2\nu-1]}$, their computation requires just ν evaluations of the matrix function A, the same number as we would have needed were the single-integral formula (1) right. Great news? Not exactly, since the *quid pro quo* of this amazing economy in function evaluations is a stupendous price tag in linear algebra. The number of terms in C_s^ν is very large, increasing exponentially with s, and each such term requires the computation of nested commutators. Fortunately, further insight, combined with the magic wand of Lie algebra theory, has led Hans Munthe-Kaas and Brynjulf Owren to an approach which reduces the cost of algebra down to a significantly more modest size and allows the computation of high-order Magnus approximations relatively cheaply [20].

Graded Lie Algebras and Optimised Magnus

There are three mechanisms at play that allow us to reduce the number of computations. First, each matrix B_j is $\mathcal{O}(h^j)$, hence

$$\mathcal{L}(B_{j_1}, B_{j_2}, \ldots, B_{j_s}) = \mathcal{O}(h^{j_1+j_2+\cdots+j_s}).$$

We say that $\mathcal{L}(B_{j_1}, B_{j_2}, \ldots, B_{j_s})$ is of *grade* $|\mathbf{j}| = \sum_{i=1}^{s} j_i$ and note that, whenever $|\mathbf{j}| > 2\nu$, we can throw away the underlying nested commutator without affecting the order. Secondly, it is possible to prove that the act of replacing all integrals in $\Omega^{[2\nu-1]}$ with quadratures (14) retains time symmetry [20]. The consequence is that we know in advance that roughly half of all terms come with zero coefficients, hence need not be computed. Specifically, whenever the grade of a term is even, it makes no contribution to the quadrature of $\Omega^{[2\nu-1]}$. It is, however, a third mechanism, that of *graded Lie algebras,* which brings down computational expense to the greatest extent.

It is implicit in our construction that, no matter how complicated the terms $\mathcal{L}(B_{j_1}, B_{j_2}, \ldots, B_{j_s})$, they are all "words" written in an "alphabet" that consists of just ν separate "letters", $B_1, B_2, \ldots, B_\nu$. Formally, we say that every such term resides in a *free Lie algebra* $\mathcal{B}$, generated by $\mathbf{B} = \{B_1, B_2, \ldots, B_\nu\}$: the linear combinations of all possible "words" that can be created from the elements of $\mathbf{B}$ by repeated commutation. We have already seen that each such "word" has an important attribute, its grade. Formally, we endow each *generator* B_j with the grade $w(B_j) = j$ and propagate this quantity in the recursive generation of nested commutators by

$$Z = [X_1, X_2] \quad \Rightarrow \quad w(Z) = w(X_1) + w(X_2).$$

This formula is clearly consistent with our definition of the grade of $\mathcal{L}(B_{j_1}, B_{j_2}, \ldots, B_{j_s})$. It is easy to verify that the span, in $\mathcal{B}$, of all the terms of grade m forms a linear space, which we denote by $\mathcal{B}_m$. (In fact, $\mathcal{B}$ is a direct sum of $\mathcal{B}_m$, $m \in \mathbb{N}$.) Remarkably, the *dimension* of $\mathcal{B}_m$ is very small. Let $\lambda_1, \lambda_2, \ldots, \lambda_\nu$ be all the zeros of the polynomial

$$1 - \sum_{i=1}^{\nu} z^i = \frac{1 - 2z + z^{\nu+1}}{1 - z}.$$

It is possible to prove that

$$\dim \mathcal{B}_m = \frac{1}{m} \sum_{d|m} \left(\sum_{i=1}^{\nu} \lambda_i^{m/d} \right) \mu(d), \tag{15}$$

where μ is the *Möbius function,*

$$\mu(n) = \begin{cases} 1, & n = 1, \\ (-1)^k, & j_i \equiv 1 \text{ for all } i = 1, 2, \ldots, k, \\ 0, & \text{otherwise,} \end{cases}$$

where $n = p_1^{j_1} p_2^{j_2} \cdots p_k^{j_k}$ is the *prime factorization* of the natural number n and the summation in (15) is carried out over all the integer divisors of m [20]. Moreover, it is also possible to construct (by recursion, how else?) a basis of $\mathcal{B}_m$ and express all the elements in the linear space as linear combinations from the basis. The reason for this truly amazing economy is that a Lie algebra is replete with redundancy, originating in the skew-symmetry of the commutator and in the Jacobi identity. The language of graded free Lie algebras allows us to take advantage of this feature.

To appreciate the savings inherent in this approach, we note that the number of commutators required for an order-6 method drops from 80 to 7 in each step and for an order-8 method goes down from 3,304 to 22. For order 10 the number of commutators is reduced from 1,256,567 in a naive implementation all the way down to 73.

As an example, we herewith present a sixth-order Magnus method, written in a variable-step time-stepping terminology, using quadrature (14) and taking advantage of economies inherent in graded free Lie algebras. In each step n we commence by evaluating $A_k = A(t_n + c_k h_n)$, $k = 1, 2, 3$, where $h_n = t_{n+1} - t_n$ and

$$c_1 = \tfrac{1}{2} - \tfrac{\sqrt{15}}{10}, \quad c_2 = \tfrac{1}{2}, \quad c_3 = \tfrac{1}{2} + \tfrac{\sqrt{15}}{10}.$$

Next, we set

$$B_1 = hA_2, \qquad B_2 = \frac{\sqrt{15}h}{3}(A_3 - A_1),$$

$$B_3 = \frac{10h}{3}(A_3 - 2A_2 + A_1).$$

The method now reads

$$\begin{aligned} \tilde{\Omega}^{[5]} = B_1 &+ \tfrac{1}{12} B_3 - \tfrac{1}{12}[B_1, B_2] + \tfrac{1}{240}[B_2, B_3] \\ &+ \tfrac{1}{360}[B_1, [B_1, B_3]] - \tfrac{1}{240}[B_2, [B_1, B_2]] \\ &+ \tfrac{1}{720}[B_1, [B_1, [B_1, B_2]]] \end{aligned}$$

(note that all the elements are of odd grade!). We conclude each Lie-algebraic step with

$$Y_{n+1} = \mathrm{e}^{\tilde{\Omega}^{[5]}} Y_n,$$

where $Y_n \approx Y(t_n)$. Just three function evaluations, seven commutators, and a single exponential, an expense perfectly affordable in a single time step.

But have we extracted every single ounce of information to drive down the computational cost? No, not yet! Aggregating terms and computing commutators in a clever manner, Sergio Blanes, Fernando Casas, and Pepe Ros have recently presented what is probably the best possible implementation of the numerical Magnus expansion [3]. Revisiting the order-6 case, we compute

$$\begin{aligned} C_1 &= [B_1, B_2], \\ C_2 &= [B_1, 2B_3 + C_1], \\ C_3 &= [-20B_1 - B_3 + C_1, B_2 - \tfrac{1}{60}C_2], \\ \hat{\Omega}^{[5]} &= B_1 + \tfrac{1}{12}B_3 + \tfrac{1}{240}C_3, \\ Y_{n+1} &= \mathrm{e}^{\hat{\Omega}^{[5]}} Y_n, \end{aligned}$$

with just three commutators. Note that $\tilde{\Omega}^{[5]}$ and $\hat{\Omega}^{[5]}$ are not the same, but they differ only in $\mathcal{O}(h_n^7)$ terms, that have no effect on the order.

Beyond Magnus

One of main reasons why linear equations are interesting is because they might tell us something about their nonlinear brethren. Indeed, the idea of a Magnus expansion has been generalised by Antonella Zanna to nonlinear Lie group equations (12) [23]. This, however, leads to implicit methods and nonlinear algebraic equations in every step: Other things being equal, it is probably easier to employ explicit Runge–Kutta methods in the Lie algebra, an approach pioneered by Hans Munthe-Kaas [19]. A considerably more significant extension of the scope of Magnus (and other Lie group) methods is due to Hans Munthe-Kaas and Antonella Zanna, who have applied them to equations evolving in homogeneous manifolds [21].

We say that a Lie group $\mathcal{G}$ *acts* on a manifold $\mathcal{M}$ if there exists a map $\lambda : \mathcal{G} \times \mathcal{M} \to \mathcal{M}$ which is consistent with the group multiplication, $\lambda(x_1, \lambda(x_2, y)) = \lambda(x_1 x_2, y)$ for all $x_1, x_2 \in \mathcal{G}$, $y \in \mathcal{M}$. A group action is said to be *transitive* if for every $y_1, y_2 \in \mathcal{M}$ there exists $x \in \mathcal{G}$ such that $\lambda(x, y_1) = y_2$. If there exists a transitive group action for $\mathcal{M}$, we say that the latter is a *homogeneous manifold* (or a homogeneous space).

An example of a homogeneous manifold is the Euclidean unit sphere $\mathbb{S}_{N-1} \subset \mathbb{R}^N$, which is acted upon transitively by orthogonal matrices $\mathrm{O}(N)$,

$$\|\mathbf{x}\|_2 = 1, \quad Q \in \mathrm{O}(N) \quad \Rightarrow \quad \|Q\mathbf{x}\|_2 = 1.$$

The set $\mathrm{Sym}(N)$ of real symmetric $N \times N$ matrices is subject to an action of $\mathrm{GL}(N)$ by congruence, $S \to VSV^\top$. If we restrict the latter action from $\mathrm{GL}(N)$ to the *special orthogonal group* $\mathrm{SO}(N) = \mathrm{O}(N) \cap \mathrm{SL}(N)$, it acquires an important attribute: The eigenvalues of S and of $VSV^\top$ are the same.

Suppose that a differential equation evolves in a homogeneous manifold $\mathcal{M}$ and we desire to solve it numerically. The conventional objective is thus, given an approximation $\mathcal{M} \ni \mathbf{y}_n \approx \mathbf{y}(t_n)$, to seek $\mathbf{y}_{n+1}$ that approximates $\mathbf{y}(t_{n+1})$ to sufficient order of accuracy. Unfortunately, if we employ a "traditional" numerical method, it is very unlikely that $\mathbf{y}_{n+1}$ remains in $\mathcal{M}$. We can instead set a goal more consistent with the underlying geometry of the manifold: Given an approximation $\mathcal{M} \ni \mathbf{y}_n \approx \mathbf{y}(t_n)$, find an element $V_n \in \mathcal{G}$ such that $\mathbf{y}_{n+1} = \lambda(V_n, \mathbf{y}_n)$ approximates $\mathbf{y}(t_{n+1})$ to requisite order of accuracy. Note that this guarantees that we stay in $\mathcal{M}$.

As an example, consider the *isospectral flow*

(16)
$$Y' = [A(t, Y), Y], \quad t \geq 0, \quad Y(0) = Y_0 \in \mathrm{Sym}(N),$$

where $A : \mathbb{R}_+ \times \mathrm{Sym}(N) \to \mathfrak{so}(N)$. It is easy to show that the eigenvalues of $Y(t)$ remain constant as the flow evolves by demonstrating that it can be subjected to the (congruent) similarity action by $\mathrm{SO}(N)$, $Y(t) = Q(t) Y_0 Q^\top(t)$, where

$$Q' = A(t, QY_0Q^\top)Q, \quad t \geq 0, \quad Q(0) = I \tag{17}$$

(compare with (12)). This feature is important in many applications, not least in numerical linear algebra [6], but it cannot be retained for $N \geq 3$ by classical numerical methods like Runge–Kutta or multistep. The remedy is to solve (17) by a Lie group method. Of course, (17) is typically nonlinear, outside the scope of "plain vanilla" Magnus methods. One remedy is nonlinear Magnus, another is a Runge–Kutta method in the algebra. However, in special cases the logic underlying the Magnus expansion can be extended to nonlinear equations. For example, consider the *double bracket equation*

(18)
$$Y' = [[M, Y], Y], \quad t \geq 0, \quad Y(0) = Y_0 \in \mathrm{Sym}(N),$$

where $M \in \mathrm{Sym}(N)$ is given. Equation (18) has many interesting applications, which are underpinned by an exciting feature. Suppose that $[M, Y_0] \neq O$. Then $\hat{Y} = \lim_{t \to \infty} Y(t)$ exists, is unique, and minimizes the distance from M in the *Frobenius norm*, $\|B\|_\mathrm{F} = \left(\sum_{k,j=1}^N B_{k,j}^2\right)^{1/2}$, among all symmetric matrices that share the eigenvalues of Y_0 [4].

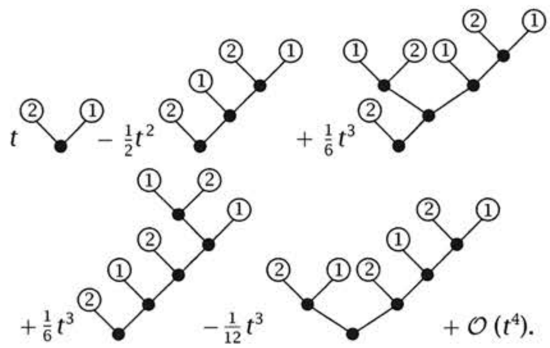
Have a look at (18). No matter how complicated the solution (whether the function Y itself or its Lie group and Lie algebraic counterparts), it must be expressible in an "alphabet" comprising just two "letters", the matrices Y_0 and M. Since we are keen to stay on the *isospectral orbit*, we express the solution in the form

$$Y(t) = e^{\Omega(t)} Y_0 e^{-\Omega(t)}, \qquad t \geq 0,$$

where Ω evolves in $\mathfrak{so}(N)$. Although the equation for Ω is highly nonlinear, it is possible to express this function using our terminology of binary rooted trees, except that we paint their leaves in two colours: We let

$$① \rightsquigarrow Y_0, \qquad ② \rightsquigarrow M.$$

Then the Taylor expansion of Ω can be represented in an arboreal fashion [12],



As before, there exist recursive rules that allow us to approximate Ω to arbitrarily high order. Double-bracket equations (18) are just one example from a large (and largely unexplored) menagerie of Lie group and homogeneous manifold equations that can be written in a finite "alphabet", thereby lending themselves to similar treatment.

The tale of Magnus expansions and geometric integrators is by no means complete. And the moral of this tale? Two, really: first, the importance of reconciling qualitative information—geometry, structure, invariants—with computation and approximation. It is not a zero-sum game where you must opt for either quality or quantity, but a complicated interactive procedure with huge scope for synergy. Second, modern mathematical computation is not about discretising everything in sight by elementary means and massive number crunching. It is about employing every suitable pure mathematical tool, in our case rooted trees and graded Lie algebras, to render our computations more accurate and affordable. Mathematics can lead us to better computational algorithms, and expansions can indeed grow on trees.

Acknowledgments

Local thanks to all my colleagues who have read various versions of this paper, offered their comments, and saved me from errors, misprints, and self-inflicted embarrassment. In particular, my gratitude is due to Brad Baxter, Harold Boas, Chris Budd, Martin Buhmann, Fernando Casas, Dganit Iserles, Joe Keller, and Peter Olver.

No mathematical research, least of all *this* mathematical research, is a creation of a single mind or of several isolated minds. It evolves in the shared intellectual space spanned by the expertise, excitement, and sense of fun and discovery of many individuals. Thus, global thanks to the "Lie group" of colleagues, friends, and collaborators who have created in the last few years the new subject area of geometric integration and of computational methods in a Lie group setting.

References

[1] W. Abikoff, J. S. Birman, and K. Kuiken, editors, *Wilhelm Magnus: Groups, Geometry and Special Functions*, vol. 169 of *Contemp. Math.*, Providence, RI, 1994, American Mathematical Society.

[2] S. Blanes, F. Casas, J. A. Oteo, and J. Ros, Magnus and Fer expansions for matrix differential equations: The convergence problem, *J. Phys. A*, **31** (1998) 259–268.

[3] S. Blanes, F. Casas, and J. Ros, High order optimized geometric integrators for linear differential equations, Technical Report NA07, DAMTP, University of Cambridge, 2000.

[4] R. W. Brockett, Dynamical systems that sort lists, diagonalize matrices and solve linear programming problems, *Linear Algebra Appl.*, **146** (1991), 79–91.

[5] C. J. Budd and M. D. Piggott, Geometric integration and its applications. In P. G. Ciarlet and F. Cucker, editors, *Handbook of Numerical Analysis*, North Holland, Amsterdam, 2002, to appear.

[6] M. T. Chu, A list of matrix flows with applications. In *Hamiltonian and Gradient Flows, Algorithms and Control*, vol. 3 of *Fields Inst. Commun.*, pp. 87–97, Amer. Math. Soc., Providence, RI, 1994.

[7] P. J. Davis and P. Rabinowitz, *Methods of Numerical Integration*, Academic Press, Orlando, FL, 2nd edition, 1984.

[8] A. T. Fomenko and R. V. Chakon, Recurrence formulas for homogeneous terms of a convergent series that represents a logarithm of a multiplicative integral on Lie groups, *Funct. Anal. Applcs.*, **1** (1990), 41–49.

[9] E. Hairer, Symmetric projection methods for differential equations on manifolds, *BIT*, **40** (2000), 726–734.

[10] F. Hausdorff, Die symbolische Exponentialformel in der Gruppentheorie, *Berichte der Sächsischen Akademie der Wißenschaften, Math. Phys. Klasse*, **58** (1906), 19–48.

[11] A. Iserles, Multistep methods on manifolds, *IMA J. Num. Anal.*, **21** (2001), 407–419.

[12] ———, On the discretization of double-bracket flows, Technical Report NA08, DAMTP, University of Cambridge, 2001, to appear in *Found. Comp. Maths.*

[13] A. Iserles, H. Z. Munthe-Kaas, S. P. Nørsett, and A. Zanna, Lie-group methods, *Acta Numerica*, **9** (2000), 215–365.

[14] A. Iserles and S. P. Nørsett, On the solution of linear differential equations in Lie groups, *R. Soc. Lond.*

Philos. Trans. Ser. A Math. Phys. Eng. Sci., **357** (1999), 983–1020.

[15] A. Iserles, S. P. Nørsett, and A. F. Rasmussen, Time symmetry and high-order Magnus methods, *Appld. Num. Maths*, **39** (2001), 379–401.

[16] A. Iserles and A. Zanna, Preserving algebraic invariants with Runge-Kutta methods, *J. Comp. Appld. Maths*, **125** (2000), 69–81.

[17] W. Magnus, On the exponential solution of differential equations for a linear operator, *Comm. Pure Appl. Math.*, **7** (1954), 649–673.

[18] P. C. Moan, Efficient approximation of Sturm-Liouville problems using Lie-group methods, Technical Report NA11, DAMTP, University of Cambridge, 1998.

[19] H. Munthe-Kaas, Runge-Kutta methods on Lie groups, *BIT*, **38** (1998), 92–111.

[20] H. Munthe-Kaas and B. Owren, Computations in a free Lie algebra, *R. Soc. Lond. Philos. Trans. Ser. A Math. Phys. Eng. Sci.*, **357** (1999), 957–982.

[21] H. Munthe-Kaas and A. Zanna, Numerical integration of differential equations on homogeneous manifolds. In F. Cucker and M. Shub, editors, *Foundations of Computational Mathematics*, pp. 305–315, Springer Verlag, 1997.

[22] V. S. Varadarajan, *Lie Groups, Lie Algebras, and Their Representations*, GTM 102, Springer-Verlag, 1984.

[23] A. Zanna, Collocation and relaxed collocation for the Fer and the Magnus expansions, *SIAM J. Num. Anal.*, **36** (1999), 1145–1182.

Interview with Louis Nirenberg

Louis Nirenberg is one of the outstanding analysts of the twentieth century. He has made fundamental contributions to the understanding of linear and nonlinear partial differential equations and their application to complex analysis and geometry.

He was born on February 28, 1925, in Hamilton, Ontario, Canada. After receiving his bachelor's degree from McGill University in 1945, he went to New York University as a graduate student, obtaining his M.S. in 1947 and his Ph.D. in 1949, under the direction of James Stoker. Nirenberg then joined the faculty of NYU and was an original member of the Courant Institute of Mathematical Sciences. After spending his entire academic career at Courant, he retired in 1999.

Nirenberg received the AMS Bôcher Prize in 1959 for his work on partial differential equations. In 1982 he was the first recipient in mathematics of the Crafoord Prize, established by the Royal Swedish Academy of Sciences in areas not covered by the Nobel Prizes. In 1995 he received the National Medal of Science, the United States' highest honor for contributions to science.

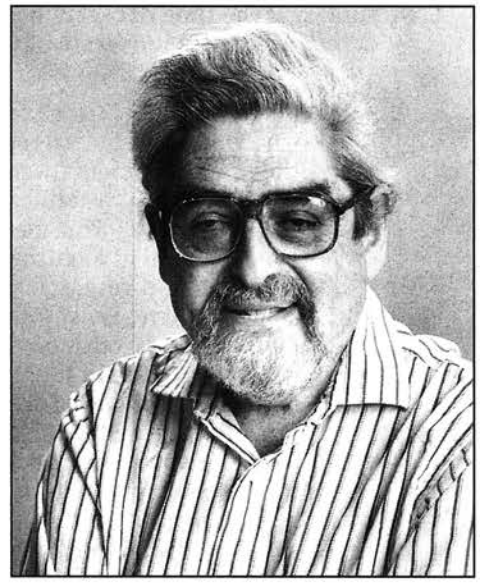
The following is the edited text of an interview with Nirenberg, conducted on December 8, 2001, by *Notices* senior writer and deputy editor Allyn Jackson. The assistance of Dieter Kotschick, Ludwig-Maximilians-Universität München, is gratefully acknowledged.

—*A. J.*

Allyn Jackson is senior writer and deputy editor of the Notices. *Her e-mail address is* axj@ams.org.

Early Experiences

Notices: What were your early experiences with mathematics?



Louis Nirenberg

Nirenberg: I always liked mathematics in school. My father was a Hebrew teacher, and he wanted me to learn Hebrew. But I foolishly resisted. I went to Hebrew school for a while, and that didn't take, and he tried to give me lessons, but that didn't take either. Then a friend of his gave me private lessons, and that man liked mathematical puzzles. So half of the so-called Hebrew lessons were spent on mathematical puzzles.

I went to a very good high school in Montreal called Baron Byng High School. It was full of bright students. It was during the Depression, and to be a high school teacher was considered a very good job, so there were good teachers who were very devoted. I especially liked the physics teacher, who actually had a Ph.D. in physics. His courses made me think that I might want to be a physicist. I didn't even know that there was such a career as "mathematician". I knew you could be a math teacher, but I didn't know you could be a mathematician.

When I finished high school, I decided I would do mathematics and physics. At that time one could do a major in both, which I did at McGill

L. Nirenberg at the time of receiving his B.Sc. from McGill University, 1945.

University. I graduated in 1945, just when the war ended.

Notices: *How was the mathematics at McGill at that time?*

Nirenberg: The training was pretty good. I guess the most prominent mathematician there at the time was Gordon Pall, who was in number theory. He was really an inspiration to the students around him. But I had planned to do theoretical physics. I'll tell you the story of how I went into mathematics. When I graduated, the war had finished in Europe but was still on in Japan. In Canada, the science students were not drafted, and that's why I wasn't in the armed services. In the summer of 1945, I got a job at the National Research Council of Canada in Montreal. They were working on atomic bomb research. Richard Courant's older son was there, Ernst Courant. Ernst had recently married a girl from Montreal whom I knew, and she was working there too. One day she said, "We're going down to New York for the weekend, to see Ernst's father." I had read part of Courant-Hilbert, so I knew about Courant. I said, "Could you ask him to recommend a place where I might study theoretical physics?" I knew nothing about where to apply or what to do. She came back and said she'd talked to Courant about me, and he had suggested I get a master's in mathematics at New York University, where he was, and then maybe go on to do physics. So I went down for an interview, and I met him and Friedrichs. They were very kind and offered me an assistantship. Then I stayed in mathematics.

Notices: *Was it the Courant Institute at that time?*

Nirenberg: No, it was just the Graduate Mathematics Department of New York University. The department was tiny, but there were several very good fellow students. Some of us who got Ph.D.'s there stayed, like Harold Grad, Joe Keller, Peter Lax, and Cathleen Morawetz. Peter Lax's wife, Anneli, was a student there when I arrived. She was the first person I met when I came there as a student. It was a remarkable group of people.

Richard Courant, A Complicated Man

Notices: *What are your impressions of Courant?*

Nirenberg: I remember him very well. He was a complicated man; he isn't easy to summarize. He was enormously intelligent and terrific with young people. He loved to be with young people and was very encouraging. As a teacher he was good when he prepared, which was seldom, but I enjoyed his classes. Very often on the weekend he would invite some graduate students to his home, which was in New Rochelle. I discovered that one of the reasons was to weed his garden.

Courant was a great lover of music, as was his whole family. They often played chamber music at home, and sometimes I attended concerts. The story went that when Courant hired somebody, he would ask if the person played an instrument. If so, the person had a better chance of being hired. But if the instrument was piano—no, because Courant played the piano. Probably the story isn't true, but that was the story at the time.

Some of the other students were closer to Courant than I was. Kurt Friedrichs was a big influence—I would say the major influence—on me in mathematics. His view of mathematics very much formed my view. I started with Friedrichs as an adviser, and he gave me a problem in operator theory. I thought about it for a while, but I didn't get anywhere. Some months later Jim Stoker suggested a problem in geometry. Stoker was my official adviser, and he was a very kind man. But I actually talked more with Friedrichs than I did with Stoker during the time I was working on the thesis. So I was really closer to Friedrichs.

Notices: *What was Friedrichs's view of mathematics that influenced you?*

Nirenberg: I'd have trouble saying. When you were trying to resolve something, it didn't matter so much whether you would prove it was true or false. The thing was to understand the problem. Also, Friedrichs was a great lover of inequalities, and that affected me very much. The point of view was that the inequalities are more interesting than the equalities, the identities. I also liked the things he did in partial differential equations, which I followed very closely. But he did other things, in quantum theory, operator theory, shock wave theory. When I was a graduate student, I felt that what Friedrichs was doing was where the action was. So I went to him to do a thesis, but in the end I didn't do the thesis with him.

Notices: *What was the problem you worked on in your thesis?*

Nirenberg: It was a problem that Hermann Weyl had worked on, a problem in geometry. Weyl had solved it partly, and what I did was complete the proof. Hans Lewy solved it in the analytic case. You're given a Riemannian metric on the 2-sphere, having positive Gauss curvature, and the question is, can you embed this 2-sphere isometrically into 3-space as a convex surface? Weyl worked on it, around 1916 I think, and had made some crucial estimates. One needed some more estimates before one could finish the problem. What I did was to get the additional estimates, essentially using ideas of C. B. Morrey. Morrey's work was a very big influence on me, and later I got to know him. He was a very nice man. He didn't have many joint papers, but we did write one paper together. Morrey understood a lot of things, but he was hard to understand. I remember a story I heard. He ran a seminar every year at Berkeley. One year, the semester started, and the seminar met for the first time. He said, "Well, I'll use the same notation as last year."

Notices: *So if you weren't in the seminar last year, too bad?*

Nirenberg: You'd have to catch up. I once attended a conference in Pisa, and Morrey was there. A number of people gave a series of talks, and of course we spoke English; we didn't know Italian. When Morrey spoke he had a strong Ohioan accent, and the Italians found him very hard to understand. And he would use expressions like, "Well, if you try this kind of technique, you'll never get to second base." They had no idea what this referred to. They called him "The Sheriff". During the meeting the local newspaper published some photos from the lectures. There was a photo of Morrey standing at a blackboard lecturing, and the caption read, "Professor Nirenberg from New York University." Morrey saw this and said, "That's not Nirenberg! Those are my formulas!"

Notices: *In your later work did you follow up on your thesis on the embedding problem in greater generality?*

Nirenberg: No. The work on the embedding problem involved nonlinear partial differential equations. That's how I got into partial differential equations. After that I worked essentially in partial differential equations connected to other things.

There is still a local embedding problem that has been open for maybe 150 years. If you're given a Riemannian metric in a neighborhood of the origin in the plane, can you embed that isometrically as a piece of surface in R^3? The general case is still open. If the metric is analytic, the answer is yes; you use the Cauchy-Kowalevsky theorem. If the curvature of the metric is strictly positive, the equations you get are elliptic, and again the answer is yes. If the curvature is strictly negative, the equations are hyperbolic; again the answer is yes. But if the curvature can change sign or might have a zero, then the problem is more difficult. Years later I gave a case of the problem to a graduate student, who did a beautiful thesis. He solved the problem in case the curvature vanishes at some point, but where its gradient is not zero. He solved that case in a beautiful paper. His name is Chang-Shou Lin. He also worked on the case where the curvature is nonnegative but might have a zero. Other people have worked on this too.

Notices: *Going back to Morrey, what did he work on?*

Nirenberg: One of his famous papers, which he did around 1932 or 1933, was to solve one of Hilbert's problems, which Hilbert had formulated in two dimensions. Morrey proved the analyticity of the solution of the nonlinear variational problem. The n-dimensional problem was done in 1957 independently by De Giorgi and Nash. De Giorgi did it first.

Notices: *You knew [John] Nash. There was one year when he hung around the Courant Institute.*

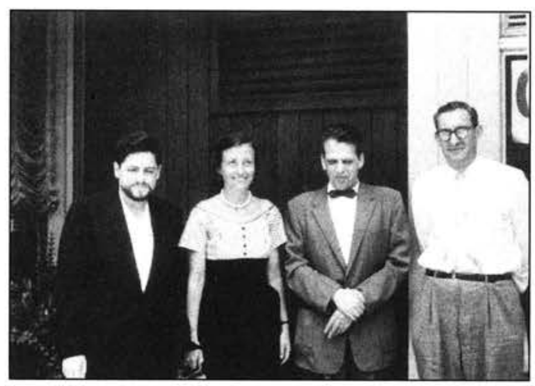
Nirenberg: He was officially visiting Princeton, but his girlfriend—I think they were not yet married at the time, but I don't quite remember—lived in New York. So he spent a lot of time in New York, and he hung around the Courant Institute a lot. I knew him pretty well that year. That's the year when he did the paper connected with the De Giorgi paper.

Notices: *Were you the one who suggested that problem to Nash?*

Nirenberg: I think Sylvia Nasar writes that in her biography of Nash, but I don't remember. I would say it's likely, because it was a problem that I was

New York City, ca. 1953. Left to right: K. O. Friedrichs, J. Stoker, M. Nagumo, F. John, L. Nirenberg.

Left to right: L. Nirenberg, P. Lax, O. Oleinik, F. Treves.



Left to right: L. Nirenberg, C. Morawetz, L. Bers, M. Protter, 1960s.

Nirenberg with wife Susan and children Lisa and Marc, Spain, 1962.

With Joe Kohn around 1965.

interested in and had tried to solve. I knew lots of people were interested in this problem, so I might have suggested it to him, but I'm not absolutely sure.

Notices: *What did you make of him at the time?*

Nirenberg: About twenty years ago somebody asked me, "Were there any mathematicians you would consider as geniuses?" I said, "I can think of one, and that's John Nash." I first heard of him when he did his paper on the isometric embedding problem, and I studied that paper. I found that to be a remarkable paper. I met him after he had done it, and I heard him speak on it at a meeting in Seattle. When he was hanging around Courant and working on the other problem, he would come around and ask questions like, "Do you think such-and-such inequality might be true?" Sometimes the inequalities weren't true. I wasn't sure he was getting anywhere. But then in the end, he did it. He had a remarkable mind. He thought about things differently from other people.

Pure versus Applied?

Notices: *How do you see the relationship between so-called "pure" and "applied" mathematics?*

Nirenberg: That was one of the nice things about the Courant Institute—and very much due to Courant and Friedrichs—that there was hardly any difference between pure and applied. There was just mathematics, and people were interested in both pure problems and applied problems and didn't distinguish so much. There was a period when I was a graduate student when a number of people—Friedrichs, Stoker, Hans Lewy, Fritz John—worked on the theory of water waves. But the work was analysis, meaning partial differential equations or complex analysis.

Courant also was a great believer that you must not only do research, you must also teach. That's very different from, say, the Russian or Soviet system, where there were many institutes where people just did research and didn't teach. Courant always thought that was very bad. In fact, the young people at Courant taught less than the older people. In general the atmosphere is terrific at Courant. The graduate students are very fond of the place. There's a very warm relationship between the faculty and the students.

Notices: *You spent 1951–52 in Europe. Where did you go?*

Nirenberg: I went to Zurich, to see Heinz Hopf, and I also went to Göttingen. This was arranged by Courant. I went first to Zurich, and then my wife and I went to Göttingen. She was very unhappy to be there, and we stayed only a month in Göttingen and then went back to Zurich.

Notices: *Why was she so unhappy?*

Nirenberg: Just the idea of being in Germany. Both of us were not so happy with the idea, but Courant had arranged it, and we thought we should do it. In fact, she went back to Zurich a little before I did.

When I was in Göttingen, Carl Ludwig Siegel invited me to dinner, and he served white asparagus. I had never had white asparagus before, which I found delicious. I was told the next day he complained, "Nirenberg ate all the asparagus!" But I didn't have much contact with Siegel. Jürgen Moser was a student at the time I was there. He was a student of Franz Rellich, and, in fact, when I was there I spoke more with Rellich than with other people.

In Zürich I attended Hopf's lectures. He was my favorite lecturer for many years. He spoke absolutely gorgeous, musical German. He gave a wonderful course in geometry, and I later attended a course he gave at Courant. I kept up my interest in geometry, although I didn't work so much in geometry. In Zurich I also attended lectures by Nevanlinna and van der Waerden, who were at the University of Zurich. van der Waerden was giving a course in Riemann surface theory, which he then made into a book. At that time I could speak a little German, but since then I haven't spoken German, so I've forgotten. Yiddish was my first language, so it wasn't hard to pick up some German, and I still speak Yiddish a little and can understand it almost perfectly. I also met Hermann Weyl, but he wasn't teaching, he was just living in Zurich. I attended some lectures of Pauli in relativity theory. I found him hard to understand.

Newlander-Nirenberg Theorem

Notices: *Around 1957 you worked on the integrability problem with your student Newlander. How did that paper come about?*

Nirenberg: That happened in an interesting way. I heard of the problem from two people, first from André Weil. He said, "Ah, you people in partial differential equations! You're not working on the important problems! Here is an important problem that we need in complex analysis. Why aren't you working on that?" And later Chern brought the problem to my attention. So I thought, okay, let's have a stab at that. So I suggested to Newlander that we work on it.

The problem, a local one, is this: In R^{2n}, can one recognize the Cauchy-Riemann operators if they are given in some arbitrary coordinate system? For $n = 1$, the long-known answer is yes. For $n > 1$, there are necessary integrability conditions, and these turn out to be sufficient. In the problem you have, sort of, Cauchy-Riemann equations with some extra terms. The idea was to get rid of those terms one at a time. In the simplest case we first looked at, there was just one extra term, and Newlander had the idea of how to get rid of that term. Then we worked together on the more general case, first in two complex dimensions. I was sure that everything would work in higher dimensions, but then when it came to writing it down, we discovered that the idea didn't work in higher dimensions. We then came up with a different proof, which got rid of everything at the same time, but by making a fully nonlinear transformation of the whole problem. But I was drawn to the problem because of André Weil and Chern.

Notices: *They were right, because this turned out to be an important result.*

Nirenberg: Yes, it's a very natural question, and the result has been used. I find it interesting that in recent years Gromov and others have done fantastic things with nonintegrable structures.

Notices: *Where is Newlander now?*

Nirenberg: He got a job in Seattle, but he had some psychological problems and he couldn't teach. After a few months he gave up his job and gave up mathematics. He moved to his hometown, Denver, Colorado, and we were in touch every New Year's for a number of years, but now I've lost touch with him. I don't know where he is. He stopped mathematics shortly after the Ph.D. thesis. That was the only paper he ever wrote. He was a very bright guy, but he had problems.

Notices: *Was he your first student?*

Nirenberg: No, my first student was Walter Littman. He's at the University of Minnesota and works in partial differential equations. I've had quite a number of students, about forty-five. I had a colleague at Courant, Wilhelm Magnus, and he was marvelous with students. Once he said to me, "You know, I don't mind writing a student's thesis. I object when they come to check on my rate of progress."

Notices: *You had a couple of especially influential papers with Agmon and Douglis. Can you say a little bit about these papers and why they have been influential?*

International Congress in Stockholm, 1962. Front row: M. Stone, J. Moore, N. Wiener. Second row: Nirenberg center (with glasses). To his (physical) left, H. Hopf.

Nirenberg: In the theory of second-order elliptic equations, there was famous work done by Schauder. Douglis and I decided we would like to extend the Schauder theory to higher-order equations. So we did that first for so-called interior estimates, meaning away from the boundary. Then we started working on the case on boundaries, and we discovered that Agmon was also working on that. So we thought the three of us could collaborate. The collaboration with Agmon was mainly by mail, because he wasn't at Courant then. This was shortly after the work of Calderón-Zygmund on L^p theory, and we knew that the Calderón-Zygmund theory could extend easily to interior estimates in L^p, so we thought we should do that up to the boundary also. And that was a lot of work. We decided to do it in general because we figured these estimates would be useful to people working in partial differential equations. Indeed, that's been the case; they have been useful. Douglis was also a student at Courant, but he left shortly after he got his Ph.D. We were good friends for many years.

Mathematical Taste

Notices: *When you were at Courant, people would come and talk to you about the problems they were working on.*

Nirenberg: One reason was that I was very good at catching mistakes. I'm no longer good at that. I don't catch my own mistakes anymore! I have to be very careful and check everything I do. But I was very good at catching mistakes, so people came for that reason. They would show me a proof to have it checked because I had a good nose for mistakes.

Notices: *Then out of these conversations also came collaborations.*

Nirenberg: Sometimes, yes.

Joint Soviet-American Conference in Partial Differential Equations in Novosibirsk, USSR, August 1963. L. Nirenberg provided partial identification of those pictured in photo above. Front row (left to right): J. Moser, A. Zygmund, L. Ahlfors, N. Brunswick, S. L. Sobolev, C. B. Morrey, C. Loewner, R. Courant, M. M. Lavrent'ev, N. Vekua, S. Bergman, A. N. Tikonov, G. I. Marchuk, D. Spencer, A. Dynin. Second row: 2) M. Krein, 3) O. A. Oleinik, 4) H. Weinberger, 5) H. Grad, 6) M. Schechter, 7) J. Douglas, 8) F. Browder, 9) M. H. Protter, 11) Ju. M. Berezanskii, 13) A. P. Calderón, 14) P. D. Lax, 15) E. B. Dynkin. Third row: 1) A. Ja. Povzner, 3) P. E. Sobolevskii, 6) B. V. Shabat, 11) S. Krein, 13) R. Richtmeyer. Fourth row: 3) R. Finn, 6) L. Nirenberg, 13) M. Vishik, 16) Yu. V. Egorov. Fifth row: 1) M. Agranovich, 3) N. Vvedenskaya, 4) L.. Volevich, 5) T. Venttsel. Sixth row: 1) V. A. Solonnikov, 2) M. A. Lavrent'ev, 14) L. Faddeev, 16) M. S. Birman.

Notices: *What would guide you in choosing what to work on? What kinds of things would interest you about problems you would hear about?*

Nirenberg: I myself don't understand so very well. I remember meeting a young Frenchman years ago, and he had been trying to do research for several years. He asked me, "How do you do research? How do you start on a problem?" I said, "Well, sometimes it happened to me that I read a paper and I didn't like the proof. So I started to think about something that might be more natural, and very often this led to some new work." Then I asked him, "What about your case?" He said, "I never found a proof I didn't like." I thought, "This is hopeless!"

Notices: *Did you then give him a few proofs you thought were especially bad?*

Nirenberg: No.

Notices: *There is a question of taste there.*

Nirenberg: Yes, taste plays a very important role in mathematics. Some mathematicians I think have very good taste, others I am not so drawn to the kind of problems they work on. Taste is very important, and it's very hard to define or even to describe.

Notices: *But what usually appeals to you? A general theoretical question? Or a specific problem?*

Nirenberg: I greatly admire people who develop theories in mathematics, but I am not one of those. I am more of a problem solver. I hear a problem, and if it appeals to me, I work on it.

I remember the work I did with Joe Kohn on pseudo-differential operators. We were trying to extend his work on regularity of the so-called $\bar{\partial}$-Neumann problem to other degenerate problems. We were trying to work with singular integral operators, and we seemed to need facts about products and commutators of singular integral operators, which were then not in the literature. We said, "Well, we'll try and develop what we need." That's how we did the work that we then called pseudo-differential operators—by the way, the name "pseudo-differential operators" is due to Friedrichs. We needed this for a specific problem. But in the case of the work with Agmon and Douglis—there we felt we should develop the general estimates for the general systems under general boundary conditions because we thought those estimates would be useful.

Notices: *You said you are problem oriented and you choose problems that appeal to you. Can you say what the appeal is, or which type of problem appeals to you?*

Nirenberg: That's hard. Inequalities, certainly. I love inequalities. So if somebody shows me a new inequality, I say, "Oh, that's beautiful, let me think about it," and I may have some ideas connected with it.

Let me say a word about the paper with Luis Caffarelli and Bob Kohn on Navier-Stokes equations. There was a paper of I. M. Sheffer, a mathematician at Rutgers who had very interesting results on the dimension of possible singularities. One day I was walking through Chinatown with Caffarelli and Bob Kohn, and I said, "You know, we should study that paper. Why don't we study it together?" So that's how that came about—we decided to study Sheffer's paper.

Notices: *The Millennium Prize Problem about the Navier-Stokes equations asks whether or not the solutions have singularities. How do you see this problem?*

Nirenberg: It's a great problem. I think it will be settled in the not-too-distant future, one way or another. I won't bet which way it will go. Maybe twenty years ago—before I had done the work on the Navier-Stokes problem—I asked Jean Leray which way he thought it would go. He didn't predict. He was a great mathematician and greatly influenced me. I first met him at the International Congress at Harvard in 1950. He had also worked on the isometric embedding problem, and I couldn't understand his paper, so I made an appointment to talk about it with him. We met in his office at Harvard for one or two hours. He was extremely kind, but I never understood that paper.

Moscow, 1988, at a conference organized by "refuseniks".

Receiving the Crafoord Prize, Stockholm 1962, with Mrs. Crafoord and the King of Sweden.

Notices: *With the Navier-Stokes problem, do you think that the existing methods in PDE are enough to eventually crack it, or is some new idea needed?*

Nirenberg: My feeling is one needs more harmonic analysis. But I haven't worked on it since the work we did in our paper.

Notices: *But your result is about the best that's been done.*

Nirenberg: About the nature of the singularities, yes.

Notices: *It's strange that the problem is so open, that one isn't leaning one way or the other.*

Nirenberg: When I was working on it, I sometimes felt one way, sometimes I felt the other way. At the moment I don't have any particular feeling about which way it should go.

Mathematical Vision

Nirenberg: In 1963 there was a joint Soviet-American meeting in Novosibirsk on partial differential equations. That was one of the best meetings I ever went to. I met many Soviet colleagues and made friends, and we've remained friends to this day. Afterwards several of us went to Moscow for a few days and attended Gelfand's famous seminar. It goes on for hours, with different speakers, and Gelfand interrupts the speakers to make comments and ask questions. When we returned to New York, Friedrichs said, "You know, we should run a seminar like that, and we could take turns playing Gelfand."

Gelfand is still active, doing research, running a seminar, and working with different people. It's just incredible—he is now 88 years old. I saw him just a few weeks ago. He's always been the sort of person who sparks ideas, and then other people carry them out. Whenever I saw him in Moscow, he would ask me, "What do you consider important now in mathematics? What are the future directions?" These

Photograph courtesy of Chang-Shou Lin.

September 11, 2000, the International Conference on Partial Differential Equations in Celebration of the Seventy-fifth Birthday of Louis Nirenberg (Nirenberg in center, front row).

were questions I could never answer. It was always embarrassing to me, because I never think in those terms. But he does think in those terms.

Notices: *You would have things you were working on then. But you didn't necessarily think that that would be the future direction?*

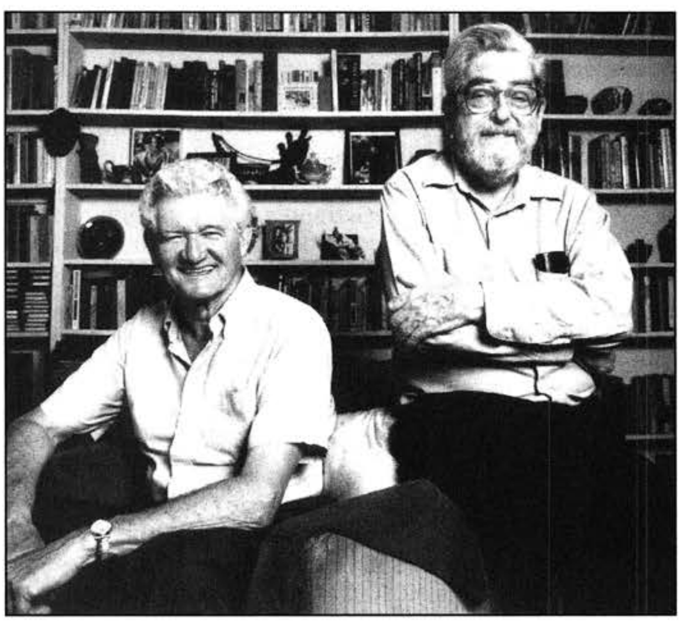
Nirenberg: No, one doesn't know. And I don't have such a vision of mathematics as he does.

Notices: *But do you think his vision has been good and accurate?*

Nirenberg: Oh yes, I think he has remarkable vision. He's worked on so many different things, and he helped develop so many different fields.

Notices: *Do you know of other mathematicians who have that kind of vision?*

Nirenberg: Maybe André Weil had that kind of vision. Perhaps Hirzebruch, Atiyah, Milnor, Smale.



Nirenberg with colleague Peter Lax, 1999.

This reminds me of a story I heard about von Neumann. Somebody once asked him, "Today, how much of all of mathematics can a mathematician know?" He replied, "Uh—two-thirds."

The Joy of Collaboration

Notices: *You wrote one paper with Fritz John.*

Nirenberg: I think I was the first person he wrote a paper with. He then wrote several with Klainerman, towards the end of his life, but mostly he worked by himself. He never followed any fashion. In fact, he always would apologize, "Oh, I haven't read this, I haven't read that." But he created fashions, because he had such wonderful ideas, and people then followed his ideas and developed what he did. He was very independent, at the same time very modest.

The paper I wrote with Fritz John started when he came to me and said, "I believe such-and-such inequality should be true" and that something should be in L^p. I was able to prove that, and then he improved what I had done. So we wrote a joint paper.

Notices: *This was the paper in which you defined BMO [bounded mean oscillation].*

Nirenberg: Yes. That was the only paper I had with him. He was a wonderful mathematician—extremely deep and original.

Notices: *How did he come upon the problem you worked on together?*

Nirenberg: He did several papers in elasticity theory, and it was one problem in particular that came up.

Notices: *How have the BMO spaces been used subsequently?*

With Haim Brezis and Joe Kohn, Taiwan, September 2000.

Nirenberg: They have been used in harmonic analysis and in martingale theory. More and more in analysis people are working on something called VMO—which I call the son or daughter of BMO—vanishing mean oscillation. That's due to Donald Sarason at Berkeley, and it's turned out to be an extremely useful tool. A few years ago I did a paper with Haim Brezis in which we extended degree theory to mappings belonging to VMO.

I've worked with several French mathematicians and written lots of papers with Brezis and Henri Berestycki. And a lot of papers with Luis Caffarelli.

***Notices:** What is Caffarelli like as a mathematician?*

Nirenberg: Fantastic intuition, just remarkable. We haven't worked together for several years now, but when we worked together, I had a hard time keeping up with him. He somehow immediately sees things that other people don't see, but he has trouble explaining them. He says things and writes very little, so when we were working at the board, I would always say, "Luis, please write more, write down more." Once I said to him, "Luis, to use a Biblical expression, 'Where is it written?'" Somebody said he once heard a talk in which Luis proved something in partial differential equations—using nothing! Just somehow out of thin air, he can come up with ideas. He's really fantastic—and a very nice person.

I must say all the people I've worked with have been extremely nice. It's one of the joys of working with colleagues. For instance, Peter Lax—although we wrote only one paper together, he seems like a brother to me. He was a big influence on me. He always knew more mathematics than I did. I learned a lot from him.

Speaking of collaborating, let me just mention a few other people I enjoyed working with very much. I wrote several papers with François Treves, and they were a great pleasure. We worked on certain classes of equations that came out of work of Hans Lewy—equations that have no solutions at all, even locally. We wrote several papers on this problem. That was fun. I would like to mention also David Kinderlehrer, Joel Spruck, and Yan Yan Li. I wrote one paper with Philip Hartman that was elementary but enormous fun to do. That's the thing I try to get across to people who don't know anything about mathematics, what fun it is! One of the wonders of mathematics is you go somewhere in the world and you meet other mathematicians, and it's like one big family. This large family is a wonderful joy.

Photograph courtesy of Yan Yan Li.

Above, photos left to right: Nirenberg with Jürgen Moser; with Yan Yan Li and Chang-Shou Lin in Taiwan, Sept. 2000; with Irene and Luis Cafarelli, 2000; with Yan Yan Li, October 2001.

Except where otherwise noted, all photographs in this article are courtesy of Louis Nirenberg.

Photograph by Rachel S. Brezis.

A mathematical discussion—Haim Brezis on left, Louis Nirenberg on right, Haifa, January 2002.

September 11th Did Not Change Cryptography Policy

Whitfield Diffie and Susan Landau

At the turn of the twentieth century, cryptography was a labor-intensive, error-prone process incapable of more than transforming a small amount of written material into an encoded *ciphertext* form. At the turn of the twenty-first century, cryptography can be done quickly, reliably, and inexpensively by computers at rates approaching a billion bits a second. As telecommunication has improved in quality and gained in importance, police and intelligence organizations have made ever more extensive use of the possibilities for electronic eavesdropping. These same agencies expect that the growth of cryptography in the commercial world will deprive them of sources of information on which they have come to rely. The result has been a struggle between the business community, which needs cryptography to protect electronic commerce, and elements of government that fear the loss of their surveillance capabilities. Export control emerged as an important battleground in this struggle.

On January 14, 2000, the Bureau of Export Administration issued long-awaited revisions to the rules on exporting cryptographic hardware and software. The new regulations, which grew out of a protracted tug of war between the computer industry and the U.S. government, were seen by industry as a victory. These changes allowed the export of cryptography in retail products, without limit on the strength of the system. On September 11, 2001, the United States was attacked by Al-Qaeda, a terrorist organization. Although there was no evidence to indicate that encryption played a role in the intelligence lapses that allowed the terrible events of September 11th to occur, New Hampshire Senator Judd Gregg argued for controls on encryption. However, to the surprise of many who had not been following the encryption debate closely, Senator Gregg's call was not supported by the Bush administration or by other members of Congress, and, after several weeks of urging controls, the senator quietly dropped his efforts. In this communication we explain what caused the reversal of U.S. export policy on encryption and why the events of September 11th have not led to cryptographic controls.

In the 1970s, after many years as the virtually exclusive property of the military, cryptography appeared in public with a dual thrust. First came the work of Horst Feistel and others at IBM that produced the U.S. Data Encryption Standard (DES). Adopted in 1977 as Federal Information Processing Standard 46, DES was mandated for the protection of all government information legally requiring protection but not covered under the provisions for protecting classified information—a category later called "unclassified sensitive." The second development was the work of several academics that was to lead to *public-key cryptography*, the technology underlying the security of Internet commerce today.

The government response was to try to acquire the same sort of "born classified" legal control

Whitfield Diffie and Susan Landau are at Sun Microsystems, Inc. Susan Landau's e-mail address is `susan.landau@sun.com`. *Whitfield Diffie's e-mail address is* `whitfield.diffie@sun.com`. *Diffie and Landau are co-authors of* Privacy on the Line: The Politics of Wiretapping and Encryption, *MIT Press, 1998.*

over cryptography that the Department of Energy claimed in the area of atomic energy. The effort was a dramatic failure. The National Security Agency (NSA) hoped an American Council on Education committee set up to study the problem would recommend legal restraints on cryptographic research and publication. Instead, it proposed only that authors voluntary submit papers to NSA for its opinion on the possible national security implications of their publication [7, p. 10].

It did not take the government long to realize that even if control of research and publication were beyond its grasp, control of deployment was not. Although laws directly regulating the use of cryptography in the U.S. appeared out of reach—and no serious effort was ever made to get Congress to adopt any—adroit use of export control proved effective in diminishing the use of cryptography, not only outside the U.S. but inside as well. The export controls even had an impact on research [1].

Current export controls are rooted in the growth of the Cold War that followed World War II. In the immediate post-war years the U.S. accounted for a little more than half of the world's economy. The country was coming off a war footing, with its machinery of production controls, rationing, censorship, and economic warfare. The U.S. not only had the economic power to make export control an effective element of foreign policy but the inclination and the regulatory machinery to do so.

Primary legal authority for regulating exports was given to the Department of State, with the objective of protecting national security. Although the goods to be regulated are described as *munitions*, the law does not limit itself to the common meaning of that word. The affected items are determined by the Department of State acting—through the *Munitions Control Board*—on the advice of other elements of the executive branch. In the case of cryptography, this was primarily NSA.

Exports that are deemed to have civilian as well as military uses are regulated by the Department of Commerce. Such items are termed *dual-use* and present a wholly different problem from "munitions". A broad range of goods—vehicles, aircraft, clothing, copying machines—are vital to military functioning just as they are to civilian. If the sale of such goods was routinely blocked merely because they might benefit the military of an unfriendly country, there would be little left of international trade. Control of the export of dual-use articles therefore balances considerations of military application with considerations of foreign availability. Munitions controls are far more severe than the dual-use controls.

Application of export controls naturally depends heavily on the destination for which goods are bound. Clearly the effectiveness of export controls will be vastly magnified by coordination of the export policies of allied nations. During the Cold War, the major vehicle for such cooperation among the U.S. and its allies was *COCOM*, the *Coordinating Committee on Multilateral Export Controls*, whose membership combined Australia, New Zealand, and Japan with the U.S. and most western European countries. The end of the Cold War realigned the world and made the "east versus west" structure of COCOM inappropriate. The organization was replaced by a new coalition, the Wassenaar Arrangement, that included former enemies from the Soviet Union and the Warsaw Pact.

In the post-WWII period, cryptography was an almost entirely military technology. As the information revolution progressed—particularly as computers began to "talk" more and more to other computers—the argument for dual-use status slowly improved. To achieve high security in communication between computers without human intervention, using cryptography to achieve authentication is indispensable. Nonetheless, cryptography remained in the "munition" category long after this seemed reasonable to most observers. As munitions, cryptographic devices required individually approved export licenses, which proved quite a burden for industry.

The problem of distinguishing military from civilian cryptosystems remained elusive. Some cases—such as the MK XII IFF[1] devices that identify aircraft to military radars—were straightforward, but many—such as cryptosystems running in ordinary commercial computing equipment in ordinary office environments—were not. The challenge of export control is to develop a policy that interferes as little as possible with international trade while limiting the ability of other countries to develop military capabilities that threaten U.S. interests. A cryptographic system adequate to protect a billion dollar electronic funds transfer is indistinguishable from one adequate to protect a top-secret message.

As the U.S. share of the world's economy has declined over the past five decades, export controls have become less effective as a mechanism of U.S. foreign policy. In 1950, it cost U.S. companies little to be prevented from exporting something for which there were few foreign customers. Today, with a majority of potential customers outside the U.S., a product's exportability can make the difference between success and failure. This change in impact of export controls has changed their role, and export controls on cryptography have come to be used at least as much for their effect on the domestic market as on the foreign one. Three factors made this possible:

- The typical American computer company makes more than half its sales abroad and must manufacture exportable products to be competitive.

[1] *Identification Friend or Foe.*

- To be usable and effective, security must be integrated from scratch with the features it supports. Even when it is feasible, adding cryptography to a finished system is undesirable.
- Making two versions of a product is complicated and expensive. Making a more secure product for domestic use, furthermore, points out to foreign customers that you have given them less than your best.

The result of U.S. export controls has been to limit the availability of strong cryptography, not merely abroad but at home.

These policies, which put the interests of intelligence and law enforcement agencies ahead of other national concerns, were made possible by the dominant position of U.S. companies in the world market for computer hardware and software. But as the fast-growing computer industry in both Europe and Asia began to challenge the U.S. position, and the growth of the World Wide Web and electronic commerce made the commercial importance of cryptography more obvious, the U.S. government came under more and more pressure to amend its regulations.

The end of the Cold War at the beginning of the 1990s set the stage for a change in export policy. The first step, a deal struck in 1992 between the National Security Agency, the Department of Commerce, and RSA Data Security (a leading maker of cryptographic software), was not encouraging. It provided for streamlined export approval for products using approved algorithms with keys no longer than 40 bits.[2] In 1992, a message encrypted using a 40-bit key could be cracked by a personal computer using the crudest techniques in a month or so, yet at the same time, any encryption applied to even a few percent of the world's communications would have created a formidable barrier to signals intelligence, which must determine in a fraction of a second whether a message is worth recording.

A few months into the Clinton administration, the government proposed *Clipper* as a compromise. Clipper was an exportable encryption system for "publicly switched telephones" in which the encryption keys would be *escrowed* with agencies of the federal government. The system, which was strongly opposed by industry and civil liberties groups, was eventually approved as a Federal Information Processing Standard, but never did well in the marketplace.

In response to the key-escrow concerns raised by Clipper, the National Research Council (NRC) released *Cryptography's Role in Securing the Information Society* (the *CRISIS* report) in the summer of 1996. Acting on a mandate from Congress, the NRC convened a panel of sixteen experts from government, industry, and science, thirteen of whom received security clearances, for an eighteen-month study. The panel was heavily weighted towards former members of the government—the chair, Kenneth Dam, for example, had been Under Secretary of State during the Reagan administration—and many opponents of the government's policies anticipated that the NRC report would support the Clinton administration's cryptography policy. It did not.

The report concluded that "on balance, the advantages of more widespread use of cryptography outweigh the disadvantages," and that current U.S. policy was inadequate for the security requirements of an information society [4, pp. 300–1]. Observing that existing export policy hampered the domestic use of strong cryptosystems, the panel recommended loosening export controls and said that products containing DES "should be easily exportable" [4, p. 312]. This was not a message the Clinton administration wanted to hear, and no immediate effect on policy was discernible.

The year 1996 also saw the start of congressional interest in cryptography export. The absurdity of U.S. export controls and the danger that they would have a devastating impact on the growing electronic economy led various members of Congress to introduce bills that would have diminished executive discretion in controlling cryptographic exports. None of the bills—which in their later forms were called SAFE for Security and Freedom through Encryption—was close to having enough votes to override a promised presidential veto. Nonetheless, congressional support for the liberalization of cryptographic export policy was to grow over the next few years.

In behind-the-scenes negotiations in 1998 at the Wassenaar Arrangement the Clinton administration scored a coup: Wassenaar agreed that "mass market" cryptography using a key length not exceeding 64 bits would not be controlled.[3] The implication was that anything else would be. The Wassenaar Arrangement is subject to "national discretion," and various nations in the agreement had not previously restricted the export of cryptography. The Clinton administration believed that these nations would now begin to restrict cryptographic exports. Then evidence surfaced suggesting that the U.S. might be using Cold War intelligence agreements for commercial spying.

A U.S. signals intelligence network called *ECHELON* that had been in existence for at least

[2] *If the encryption algorithm is properly designed, then the difficulty of unauthorized decryption is determined by the number of bits in the key; an increase of one bit doubles the cost to the intruder. A good encryption algorithm with a 56-bit key is thus* 2^{16} *or 65,000 times more difficult to crack than one with a 40-bit key.*

[3] *The 64-bit limit was for symmetric, or private-key, cryptography. This translates to approximately 650 bits for public-key cryptography.*

twenty years came embarrassingly to light. The Echelon system is a product of the UK-USA agreement, an intelligence association of the English speaking nations dominated by Britain and the United States. According to a report prepared for the European Parliament [3], Echelon targets major commercial communication channels, particularly satellite systems. Many in Europe drew the inference that the purpose of the system was commercial espionage, and indeed, former Central Intelligence Agency Director James Woolsey acknowledged that was at least part of the system's purpose [11]. The potential targets of such spying could hardly be expected to regard U.S. policy as adequate protection under the circumstances. Consternation replaced cooperation in the European community. Nations whose policies had previously ranged from the no controls stance of Denmark to the relatively strict internal controls of France were now united on the need to protect their communications from the uninvited ear of U.S. intelligence, and cryptography was key to any solution.

In 1999, a SAFE bill passed the five House committees with jurisdiction and was headed to the floor, when the White House announced that the regulations would be revised to similar effect. By giving in, the administration avoided the loss of control that would have resulted from a change in the law.

On September 16, 1999, U.S. Vice President and presidential candidate Albert Gore Jr. announced that the government would capitulate.[4] Beginning with regulations announced for December—and actually promulgated on January 14, 2000—key length would no longer be a major factor in determining the exportability of cryptographic products.

The new rules split the market based on the type of buyer. *Retail* products could be freely exported. (An item is retail if it is sold widely in large volume, made freely available, not customized for each individual user, not extensively supported after sale, and not explicitly intended for communications infrastructure protection.) Windows NT with strong encryption would not be subject to export controls; custom-designed telephone switches would be. For nonretail items, export was freely permitted to commercial customers but restricted to government ones. Special provision was also made for software distributed in source code.

The new rules are a clever compromise between the needs of business and the needs of the intelligence community. Products employed by individual users, small groups, or small companies are fairly freely exportable. Products intended for protecting large communications infrastructures—and it is national communication systems that are the primary target of American communications intelligence—are explicitly exempted from retail status.

In June 2000 the European Council of Ministers announced the end of cryptographic export controls within the European Union (EU) and its "close trading and security partners," which include the Czech Republic, Hungary, Japan, Poland, Switzerland, and the U.S. The liberalized export regulations of January 14, 2000, will no longer provide the level playing field the U.S. administration has sought.

On July 17, 2000, in response to the European liberalizations, the U.S. adopted similar ones: Export licenses would no longer be required for export of cryptographic products to the fifteen EU members and the same additional countries. Furthermore, although companies would have to provide one-time technical reviews to the U.S. government prior to export, they would be able to export products immediately.

What forces drove the U.S. government from complete intransigence to virtually complete capitulation in under a decade? Most conspicuous is the Internet, which created a demand for cryptography that could not be ignored and which at the same time made it more difficult than ever to control the movement of information. More subtle forces were also at play; one of these was the *open-source* movement.

Ever since software became a big business, most software companies have distributed object code and treated the source code as a trade secret. For many years, the open-source approach to software development—freely sharing the source code with the users—was limited to hobbyists, some researchers, and a small movement of true believers. That changed in the mid-1990s, as some businesses found that an open-source operating system gave them more confidence and better reliability due to rapid bug fixes and the convenience of customization.

Open-source software has taken its place as a major element in the software marketplace. The consequence is a general decrease in the controllability of software and, in particular, a serious threat to effectiveness of the government efforts to stop the export of software containing strong cryptography. A policy predicated on the concept of software as a finished, packaged product, one that was developed and controlled by an identifiable and accountable manufacturer, foundered when confronted with programs produced by loose associations of programmers/users scattered around the world.

Open-source software was widely distributed—arguably published—on websites. If a program, such as an operating system, leaves the U.S. without cryptography, foreign programmers can add cryptographic components immeasurably more

[4]*The administration's anticryptography policy was inimical to Silicon Valley, whose support was seen as crucial for the vice president's bid for president.*

easily than they could with a proprietary source operating system. U.S. export controls have little influence on this process.

To make that matter more arcane, the government has stopped short of claiming that source code published on paper lacks First Amendment protection, maintaining that only source code in electronic form is subject to export control.

In 1996, Daniel Bernstein, a mathematics graduate student at the University of California, Berkeley, decided that rather than ignore the law, as most researchers had, he would assert a free speech right to publish the code of a new cryptographic algorithm electronically. Bernstein did not apply for an export license, maintaining that export control was a constitutionally impermissible infringement of his First Amendment rights. Instead, he sought injunctive relief from the federal courts. Bernstein won in both the district court [1] and the Appeals Court for the Ninth Circuit [2]. Unfortunately for the free speech viewpoint, the opinion of the appeals court was withdrawn in preparation for an *en banc* review by a larger panel of Ninth Circuit judges, a review that never took place. The appearance of new regulations provided the government with an opportunity to ask the court to declare the case moot. To the government's delight, the court obliged, indefinitely postponing what the government perceived as the danger that the Supreme Court would strike down export controls on cryptographic source code as an illegal prior restraint of speech.

A final adverse influence on export control came from the government's role as a major software customer and the military's desire to stretch its budget by using more *commercial off-the-shelf* software and hardware. If export regulations discouraged the computer industry from producing products that met the government's security needs, the government would have to continue the expensive practice of producing custom products for its own use. This was uneconomical to the point of being infeasible; the only way to induce the manufacturers to include sufficiently strong encryption in domestic products was to loosen export controls.

The decision in 2000 to change the export controls on cryptography was not made lightly. For fifty years the U.S. used export controls to prevent the widespread deployment of cryptography. This policy succeeded for forty of those years, but changes in computing and communications in the last decade of the twentieth century increased the private sector need for security and reduced the policy to a Cold War relic. Although the particular actions of September 11th were unanticipated, the fact that the changed export controls would lead to encrypted traffic being unreadable by U.S. intelligence was not. Nonetheless the National Security Agency signed off on the January and July 2000 liberalizations of cryptographic export controls. September 11th did not change the facts that led to the reversal of export control regulations governing cryptography, and it is not expected that controls will be reinstated.

References

[1] Daniel Bernstein v U.S. Department of State, 922 F. Supp. 1426, 1428–30 (N.D. Cal. 1996).

[2] Bernstein v U.S. Department of State, 176 F. 3d 1132, 1141, rehearing en banc granted, opinion withdrawn, 192 F. 3d 1308 (9th Cir. 1999).

[3] Duncan Campbell, Interception 2000: Development of surveillance technology and risk of abuse of economic information, Report to the Director General for Research of the European Parliament, Luxembourg, April 1999.

[4] Kenneth Dam and Herbert Lin, *Cryptography's Role in Securing the Information Society*, National Academy Press, 1996.

[5] Whitfield Diffie and Susan Landau, *Privacy on the Line: The Politics of Wiretapping and Encryption*, MIT Press, 1998.

[6] David Kahn, *The Codebreakers*, Scribners, 1996.

[7] Susan Landau, Primes, codes and the National Security Agency, *Notices of the Amer. Math. Soc.* [Special Article series], **30** (1983), 7–10.

[8] United States Department of Commerce, National Bureau of Standards (1977), Data Encryption Standard, *Federal Information Processing Standard Publication* 46.

[9] Department of Commerce, Bureau of Export Administration: 15 CFR Parts 734, 740, 742, 770, 772, and 774, Docket No. RIN: 0694-AC11, Revisions to Encryption Items. Effective January 14, 2000.

[10] U.S. House of Representatives, Select Committee on U.S. National Security, *Final Report of the Select Committee on U.S. National Security and Military/Commercial Concerns with the People's Republic of China*, 1999.

[11] James R. Woolsey, Why we spy on our allies, *The Wall Street Journal*, March 17, 2000.

A Beautiful Mind

Reviewed by Lynne M. Butler

A Beautiful Mind
Movie directed by Ron Howard

John Nash's Life

West Virginian John Nash earned a Ph.D. in mathematics from Princeton for foundational work on the theory of noncooperative games, published in 1950. He accepted a position at MIT, where he met Alicia Larde, a student to whom he taught multivariable calculus. They married and conceived a son before Nash was involuntarily committed to a psychiatric hospital. In the next few decades, Nash experienced both remission and relapse of his paranoid schizophrenia. Cared for by Alicia at their home near Princeton, he gradually rejoined the academic community and learned to reject paranoid thoughts. His genius is diminished, but he is valued by his family and honored by his colleagues. He was awarded the 1994 Nobel Prize in Economics for his early work in game theory.

The movie *A Beautiful Mind* incorporates these biographical details, but omits others to tell its story using invented characters and plot. In 1951 John Nash was hired at MIT as a Moore Instructor. In 1953 he was promoted to assistant professor for his work on the embedding problem for Riemannian manifolds. The mathematics faculty voted to grant him tenure just before his fifty-day hospitalization at McLean in 1959. In the next thirty-five years, he was involuntarily hospitalized three more times. In 1961 at Trenton State he was aggressively treated to achieve a remission, but he later relapsed and Alicia sued for divorce. In 1963 at the Carrier Clinic he responded quickly to Thorazine but was not released until well after his divorce was finalized. Although Alicia and John did not remarry until 2001, he has lived at her house near Princeton since 1970. Their son also suffers from schizophrenia.

John Nash was awarded the 1978 von Neumann Theory Prize for his foundational work on noncooperative games and a 1999 Steele Prize in recognition of his embedding theorem for Riemannian manifolds. See [1] for a discussion of his mathematical work.

Ron Howard's Movie

A Beautiful Mind is at heart a love story between John, played by Russell Crowe, and Alicia, played by Jennifer Connelly. Nash is reported in Sylvia Nasar's biography [2] as an arrogant and egotistical anomaly, admired for his brilliance and pitied for his illness. Crowe's character John is not anomaly but outsider. As we come to understand him, we see not arrogance but confidence, not egotism but self-awareness. Alicia hears not rudeness but honesty in his suggestion that they bypass platonic activities on the way to lovemaking, and she recognizes sincerity in his proposal to marry her if she can provide proof that their love will last.

Lynne Butler's e-mail address is `lbutler@haverford.edu`. *She is professor of mathematics at Haverford College. Before joining its faculty in 1991, she taught for four years at Princeton University and married a codebreaker who earned his first Ph.D. at the age of twenty and his mathematics Ph.D. at Princeton. They live with her brother, who has survived schizophrenia for thirty years.*

The Noncooperative Game in *A Beautiful Mind*

The movie suggests that a motivating example for the discovery of Nash equilibria might have been the strategies of five suitors most attracted to the same woman in a group of five. As suggested by the movie's visuals, positive outcomes occur only when each woman is approached by one suitor. In the two-person version of this game, each of two suitors, say John and Martin, decides with what probability, say x and y respectively, he will approach the more attractive of two women. The expected payoff to John is $xa(1-y)+(1-x)by$, where $a > b > 0$ since John prefers the more attractive woman. Likewise the expected payoff to Martin is $(1-x)cy+xd(1-y)$, where $c > d > 0$. Two Nash equilibria for this game, as suggested in the movie, occur when $x = 1$ and $y = 0$ (with payoffs a and d) and when $x = 0$ and $y = 1$ (with payoffs b and c). The only other Nash equilibrium is when $x = c/(c+d)$ and $y = a/(a+b)$ (with payoffs $ba/(a+b) < b$ and $dc/(c+d) < d$). At a Nash equilibrium, neither player can improve his expected payoff by unilaterally changing his strategy. Visit `http://www.haverford.edu/math/lbutler/maths-illustrated.html` for a complete explanation of a similar example and of the wit in John's courtship line "I believe in assigning value to things."

Alongside Alicia, we love him for his desire to make a valued contribution and sympathize with him as we realize how his mind betrays him and us. The love story connects John's two stories of personal accomplishment: Thought is fueled by emotion in the first; emotion is directed by thought in the second. To succeed in graduate school—to be recognized for one original idea—required a brilliant mind fueled for years by personal ambition; to survive schizophrenia—to work again in his "art form", to help care for his son, and to share intimacy with his wife—required a loving heart directed for decades by honest self-assessment.

This review looks beyond what some view as flaws in Akiva Goldsman's screenplay to gain an intelligent appreciation of a movie that transcends stereotypes of mathematical genius and mental illness. Those who have read Nasar's biography of Nash might fault the movie for inaccurately portraying his life: He did not participate in Cold War codebreaking efforts. Those who have studied a reference book like [3] will find the symptoms suffered by the movie's protagonist fantastic: Auditory, not visual, hallucinations are characteristic of this brain disease. Finally, those who have seen David Auburn's play *Proof* [4] know a story inspired by Nash's life that is suspenseful without a car chase or a homicide. However, to dismiss *A Beautiful Mind* based on any of these well-informed observations is to fail to appreciate the creative choices that enable this movie to tell the most compelling, truthful, and important story about mental illness staged or screened since President Kennedy championed deinstitutionalization in the 1960s.

A story is compelling if those told it feel they have shared the experience of its characters. Nash, his colleagues, and his wife did not know he was going mad. They tried to understand his unusual perspective and strange behavior in light of his unique mathematical mind. It was thrilling, then agonizing, to recognize first his genius, then his madness. Is it thrilling to believe in a paranoid or grandiose delusion? Is it agonizing to realize that experiences are not real but delusional? Audiences of Howard's *A Beautiful Mind* feel the thrill of that belief and the agony of that realization because Goldsman invented delusions for John that the audience experiences as credible (though far-fetched) and exciting (though formulaic). The audience and Alicia share John's desperation to understand what is happening to him. We sympathize with his self-mutilation in the hospital, submit to the doctor who orders insulin coma therapy, laugh at the light-hearted joke he plays on an old friend who visits him at home, feel Alicia's loss of companionship and anxiety for her child, suffer with John when he chooses far-fetched and formulaic delusions over reality where he feels worthless, appreciate his reawakening as a personal accomplishment made possible by Alicia's understanding care, and are grateful for the appreciation offered him by mathematics students and colleagues at Princeton.

A story is truthful if it is based on understanding. *A Beautiful Mind* tells a truthful story about an academic subculture that values genius. In that subculture, creativity is used to solve hard problems, and competition is a way to negotiate friendships. At Princeton, John's refusal to attend classes frees him to search for a truly original idea, and his decision not to romance women saves him from those who can see only his physical appeal. His talent and true appeal lie in his ability to see mathematical beauty everywhere. He seeks the mathematical principles that govern everything from the movement of pigeons in a grounded flock to the selection of strategies in noncooperative games. His Ph.D. thesis on game theory wins him the instant celebrity he craves and the placement he wants most. At MIT, John notices a beautiful woman in his advanced calculus class, who boldly challenges the self-absorption of her handsome and celebrated professor. Their love grows from a shared appreciation of the beauty to be found in pattern and color. Together they appreciate the patterns high above made by the stars at night and the colors deep inside a glass prism that refracts light. Alicia's efforts to understand John's genius transform her admiration to love, and her efforts to understand his madness transform her fear and pity to sympathy. Their meeting of minds enables them together to solve the problem of surviving John's schizophrenia. The mathematics community is the extended family to which they turn for support.

A story is important if it sheds light on some aspect of human experience. The reality of deinstitutionalization of the mentally ill in the United States is darkly oppressive; as explained in [3], it will lighten only when the experience of schizophrenics is understood by families, employers, and neighbors. *A Beautiful Mind* treats seriously and sensitively the issues they face, in sharp contrast to movies such as *Birdy* (1984) and *Benny and Joon* (1993). Unlike *Shine* (1996) and *Pi* (1998), this movie does not assume a questionable relationship between schizophrenia and either abuse in childhood or genius in adulthood. *A Beautiful Mind* shows us the reality of this brain disease: Its onset is not necessarily rapid or apparent and its causes are unknown, effective treatments can be torturous, side effects of antipsychotics include sexual dysfunction and tardive dyskinesia, delusional thinking can result in refusal of needed hospitalization or medication, and the stigma of the disease exacerbates suffering. Many schizophrenics are periodically imprisoned or homeless, and some resort to self-mutilation or suicide. The emotional impact of *A Beautiful Mind* breaks down the barrier of intimidation that blocks understanding of individuals endowed with genius or afflicted with schizophrenia. True appreciation and sympathy are impossible without understanding. This movie offers unobscured understanding to an audience much wider than that reached by Nasar's biography or Auburn's play.

Mathematicians and Moviemakers

The movie *A Beautiful Mind* is as concise and unexpected as an elegant proof. Its logic is tight and its acting is precise. Facial expressions and subtle movements reveal John's thoughts: Standing behind Alicia, he smiles to himself as she studies a painting; seated at his desk, he reaches for her but she has turned away to go alone to bed. I wonder at Crowe's visually informed intelligence. What inspired him in the fall of 2000 as he watched a Rademacher lecture at Penn alone at the back of the hall or as he studied photographs of the young John Nash supplied by Princeton consultant Harold Kuhn? Is asking how Crowe created his character like questioning what led Nash to the concept of equilibrium in noncooperative games? *A Beautiful Mind's* answer to the latter question is elaborated in the shaded box.

Moviemakers found Nash an incomprehensible expositor of his work. Howard's interview at `http://www.countingdown.com/beautifulmind/ronhoward.html` reads, "I tried to get Nash to lecture us and explain some of his important breakthroughs...it was pretty hopeless. But...we copied some of what he wrote on the board." So they hired a mathematics consultant more attuned to their needs, Dave Bayer of Barnard College. During filming Goldsman described him as "an academic who is also movie savvy" whom "we were and continue to be lucky to have around." In addition to designing visuals for the movie, like the blackboards on the Riemann hypothesis and de Rham cohomology, Bayer served as Crowe's hand double.

First week of filming at Princeton: Nash (left) with Crowe (center) and Howard.

Nash watched from a distance while Bayer placed stones on a go board before each take, but his visits to the set did not go unnoticed. The red knit cap worn by Crowe late in the movie is like the one Nash wore to the set the first week of filming. The last week of March 2001 was still cold, so Crowe offered him a hot cup of tea. Nash responded with detailed mutterings about his palate subsequently used in the movie. The last week of June was uncomfortably hot, so my brother offered Crowe a cold bottle of beer after the rooftop scene shot on the last day of filming in Princeton. (For details, visit `http://www.murphsplace.com/crowe/mind/fanlast.html`.) Crowe responded with genuine friendliness rarely shown to strangers who appear mentally ill. My brother, who suffers the stigma of schizophrenia, told me he would remember the encounter for a long time.

References

[1] *The Essential John Nash*, edited by Harold W. Kuhn and Sylvia Nasar, Princeton University Press, 2001.

[2] *A Beautiful Mind: A Biography of John Forbes Nash Jr., Winner of the Nobel Prize in Economics 1994*, by Sylvia Nasar, Simon & Schuster, 1998. Reviewed by John Milnor in the *Notices* **45** (1998), pp. 1329–32.

[3] *Surviving Schizophrenia: A Manual for Families, Consumers, and Providers*, by E. Fuller Torrey, HarperCollins, 2001.

[4] Reviews of the play *Proof*, Dave Bayer, *Notices* **47** (2000), pp. 1082–4; Mark Saul, *Notices* **48** (2001), pp. 596–7.

Book Review

The Story of Mathematics

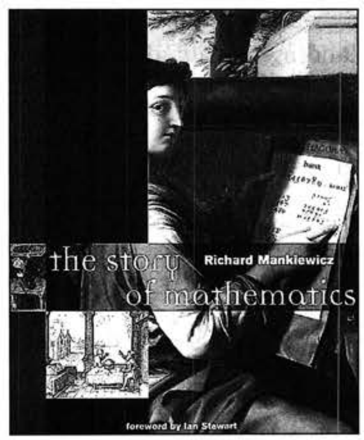
Reviewed by Karen Hunger Parshall

The Story of Mathematics
Richard Mankiewicz
Princeton University Press, 2001
ISBN 0-691-08808-X
$24.95, hardcover

Richard Mankiewicz has done the unimaginable: he has written a "coffee-table book" on mathematics. From its richly colored dust jacket to its handsomely reproduced color plates and illustrations to its sumptuously thick paper, *The Story of Mathematics* is a beautiful example of the publisher's art that should adorn the coffee table of anyone who has ever wondered about the interrelations between mathematics and the broader culture.

Mankiewicz's book is not, strictly speaking, a book for mathematicians or even, one might argue, for undergraduate students of mathematics. Thumbing through its pages, one finds virtually no definitions of terms or mathematical notation—one exception occurs in the discussion of Sir William Rowan Hamilton's discovery in 1843 of the quaternions—for, as Mankiewicz explains in his preface, his intention was not to "tak[e] the reader through a sequence of 'great theorems'" but rather "to illustrate how the mathematical sciences were intimately linked to the interests and aspirations of the civilizations in which they flourished" [p. 8]. The story of mathematics that

Karen Hunger Parshall is professor in the departments of history and mathematics at the University of Virginia, Charlottesville. Her e-mail address is khp3k@virginia.edu.



Mankiewicz wants to tell, then, is not the history of mathematics viewed as a body of knowledge. It is the story of how mathematics developed in tandem with and in response to diverse social and cultural concerns. As Ian Stewart confesses in his foreword, this "is the sort of book I would have loved to have read when I was a teenager" [p. 7], but he sees the book's real target audience as something other than that small group of teenaged mathematics enthusiasts that searches the shelves of the high school library for something, anything, to read on mathematics. This book, in Stewart's view, is for that (large) segment of the general public that "still associates math[ematic]s with school, and *with nothing else*" [p. 6; his emphasis]. In particular, it is for those who remain unaware of the field's "unbroken history of involvement with the mainstream of human culture, a history that has been going on for at least five thousand years" [p. 7]. How does Mankiewicz attempt to reach and educate this readership?

First consider the book's physical presentation. Mankiewicz has produced an approachable book. Under two hundred pages in length, it could hardly be considered intimidating by one who might want to read something—but not too much—about mathematics. Moreover, it is divided up into twenty-four chapters, the majority of which are five rather sparsely typeset pages long and include three or more illustrations, some full-page. Given this format, it is a book that can easily be read in thoroughly digestible, fifteen-minute chunks.

The volume is also visually seductive with over a hundred images, most in full color, illustrating what one might term the material culture of mathematics. These range widely. In the chapters from the prehistory of mathematics, what Mankiewicz calls "year zero," through the Middle Ages in the Latin West, we find, among many others, a photograph of a Mesopotamian clay tablet inscribed in cuneiform with a table of accounts that dates from the third millennium BCE [p. 9], pages from a medieval Islamic text [p. 25] and from a thirteenth-century Chinese manuscript [p. 35] dealing with the Pythagorean theorem, a page from a sixteenth-century chronicle of Moghul India [p. 41], and a photograph of an astrolabe from ninth-century Iraq [p. 45]. The book's middle third includes, for example, a reproduction of *The Flagellation of Christ* by the fifteenth-century Italian painter Piero della Francesca [p. 62], a sixteenth-century French illustration on how to use a cross staff [p. 70], a photograph of a calculating device based on the principle of Napier's rods [p. 76], and an early sixteenth-century map of North Africa [p. 113]. Images in the book's final third range from a drawing by Moritz Escher inspired by the Möbius band [p. 129], to a photograph of a positive electrical charge taken in 1892 [p. 146], to Marcel Duchamps's 1912 *Nude Descending a Staircase* [p. 169], to computer-generated snapshots of cellular automata [p. 187]. Even those who simply flip idly through this book cannot fail to be impressed at how deeply mathematics is and has been embedded in cultures around the globe.

Once readers have been drawn into the book through its presentation, what do they encounter in its text? To a large extent, they find much of what historians of mathematics would recognize as the now-standard textbook account, except that Mankiewicz *has* placed more emphasis than some textbook authors on the cultural aspects of mathematics.

Still, the now-standard account is not necessarily what one would find in popularizations from, say, fifty years ago. In his book, *Mathematics: Queen and Servant of Science* (1951), Eric Temple Bell took a thoroughly Eurocentric approach to the history of the subject. He saw modern, Western mathematics as springing from classical Greek antiquity and the works of men like Plato, Euclid, and Archimedes. In his view, mathematics lay fallow until the period of the Scientific Revolution brought geniuses like Galileo and Isaac Newton into the picture. Mathematics then developed explosively in the eighteenth—but especially the nineteenth—century to result in the twentieth-century pinnacle of mathematical achievement. Although Bell mentions in passing the Babylonians and their presumably empirical understanding of the Pythagorean Theorem, as well as the Chinese and their possible proof of Fermat's "little" theorem, neither medieval Islamic mathematics nor mathematics in the Latin West figure into Bell's historical account. These civilizations and these time periods were simply not deemed as having contributed to the mathematical progress that had resulted in the mathematics of Bell's own day.

In some sense, Bell must not be criticized for his Eurocentric approach, which was, after all, the norm, especially in 1931 and 1937 when he first wrote the two pieces that he later revised and combined to form the 1951 text. Even so, Florian Cajori, writing his *A History of Mathematics* in 1893 for an audience of teachers and students of mathematics, spent almost a dozen of his 400 pages on the Babylonians and Egyptians and fifty pages on the Middle Ages including some sixteen pages each on "The Hindoos" and "The Arabs" but omitting the Chinese. By the 1960s, these "other cultures" began to figure prominently both in popularizations like Lancelot Hogben's *Mathematics in the Making* (1960) and in textbooks on the history of mathematics such as Carl Boyer's *A History of Mathematics* (1968). Serious historical scholarship on mathematics in Mesopotamia, in (to a lesser extent) Egypt, in medieval Islam, and in South and East Asia has subsequently been incorporated into the standard histories, culminating in texts like Victor Katz's *A History of Mathematics* (1993, 2nd ed. 1998).

If Mankiewicz draws from this now-standard textbook presentation, he also gives it a more cultural, less technically mathematical spin. Consider the chapter "Mathematics for the Common Wealth" [pp. 68–76], in which he gives an overview of sixteenth- and early seventeenth-century Europe. This was a time when the possibilities of the printing press came to be appreciated fully. This was a time, following the Hundred Years War, of increasing economic prosperity. This was a time of myriad voyages of discovery and strange and wonderful stories of distant lands, peoples, creatures, and plants. All of these broader cultural phenomena had implications for and reverberations in mathematics.

The printing press allowed mathematical knowledge to spread more easily, and the economies of the new medium gradually forced the development of a kind of shorthand for what had traditionally been a purely rhetorical, word-for-word way of expressing mathematical notions. Not a true

Mesopotamian cuneiform · medieval Islamic · Escher

Piero della Francesca · printing press · Francis Bacon

notation, this so-called "syncopated" style of mathematical expression began to appear in printed works like Luca Pacioli's 1494 text *Summa*. The *Summa* also contained detailed explanations of how to use Hindu-Arabic numerals and discussions of, for example, the principles of accounting. Texts like Pacioli's—and so mathematics—thus contributed to Europe's economic development by permitting the training of numerate workers hired specifically to keep financial records.

The voyages of discovery were also intertwined with mathematics in significant ways. In particular, sailors needed to determine accurately their position on the globe. This problem, one that often involved calculations in terms of large numerical values, was made much simpler by the application of the logarithms that John Napier developed in the early seventeenth century. Napier's technique similarly made the calculational aspects of astronomers' work easier.

The complex interplay between mathematics and society illustrated in these examples was, moreover, not lost on contemporaries. In works like *The Advancement of Learning* (1605), *The Great Instauration* (1620), and the *Novum Organon* (1620), Francis Bacon argued that science, properly done, would be crucial to the prosperity of the commonwealth. In particular, Mankiewicz writes that "[t]he use of mathematics by merchants, navigators and scientists was seen as contributing to the creation of greater wealth for the nation. The promotion of mathematics was no longer the concern of a few scholars, but a full-blown call to arms" [p. 75]. Mathematics and science were fully embedded in the culture.

Another point of departure between Mankiewicz's book and the now-standard textbook accounts of the history of mathematics is its engagement with trendier contemporary topics like game theory, computing, fractals, and chaos. For example, chapter twenty-one, entitled "War Games", treats game theory, the mathematical analysis of "games" involving pure strategy. Mankiewicz opens with a brief description of the nineteenth-century Prussian game of *Kriegspiel*, a war simulation game that helped train the mighty Prussian army prior to World War I. Germany's defeat in that war may have marked the end of what Mankiewicz terms *Kriegspiel*'s "mythical status" [p. 160], but another lesson of the war was that the whole notion of military strategy needed rethinking. "The military thus needed mathematicians and scientists not only for developing military hardware, but also for strategic advice—hitherto the domain of generals steeped in military history" [p. 160]. Once again, the import of extrascientific and extramathematical factors on the development of mathematics becomes manifest in Mankiewicz's rendition of the story of mathematics.

A brief history of game theory follows. It starts with Émile Borel's series of papers in the 1920s, in which the mathematics of games is applied not only to situations like bluffing at cards but also to the realms of economics and politics. Next, it mentions the ground-breaking work of John von Neumann and Oskar Morgenstern on the *Theory of Games and Economic Behavior* (1944) and discusses their analysis of "the two-player two-strategy zero-sum game—a game in which two perfectly rational players are each intent on winning, and in which the total utility is zero, i.e. one player's gain is the other's loss" [p. 161]. The presentation here is not mathematical but rather analogical; the situation is likened to "a scenario replayed in many a household...the division of a cake between two children so that neither feels that the other has the larger piece" [p. 161]. Biographical information on von Neumann adds color and a human dimension to the discussion as it does in the glimpse that follows of the Nobel Prize-winning work of John Nash from the 1940s and early 1950s on nonzero-sum games and optimal strategies. Finally, after raising the question "[w]as there an optimal strategy for nuclear weapons?", the chapter concludes with a description of the RAND Corporation, a group founded in 1945 as a think tank to "think the unthinkable" and to devise national strategies in a nuclear world [p. 163]. Mankiewicz takes yet another opportunity to emphasize the intimate relationship between mathematics and society in his chapter's closing line: "[t]he whole global marketplace is a shifting scene between collaborations and competition—a world of game theory" [p. 164].

For all of its strengths as a popular work on mathematics in culture, *The Story of Mathematics* also has some weaknesses. First, despite its efforts to discuss in an integrated way the development of mathematics by many different peoples and cultures, the book fails to interweave the role of women in mathematics (with the exception of Hypatia) into its narrative. There are also some—but not too many—misleading statements and outright errors, typographical or otherwise. For example, the Babylonians are described as "highly proficient in algebra, although questions and methods of solution were stated rhetorically in words rather than symbols" [p. 11]. To label what the Babylonians did as "algebra"—even with the qualification about their lack of symbols—is to create the misperception that they approached a certain range of questions in ways reminiscent of high school algebra. A better way to finesse the linguistic problem of applying the word "algebra" ahistorically would have been to provide the briefest example of a Babylonian algorithmic problem solution and to have described it as "translatable" into what we would call algebraic terms. The allusion to "algebraic thinking" without further qualification or explanation again in the context of the medieval Islamic mathematicians perpetuates this misperception [p. 46].

One of the book's actual errors occurs in chapter sixteen on "New Geometries", where Euclid's fifth postulate is said to state that given a point not on a line "through this point there is one and only one line which is parallel to the first line" [p. 129]. While this is one of the many logically equivalent statements of the postulate in the context of Euclidean geometry, it is not Euclid's formulation of it; a correct statement appears earlier in the chapter [p. 126]. Finally, what must have been some sort of typographical error in interpreting 20^2 and 20^3 has resulted in this nonsensical rendering of the Mayan, partly vigesimal number system: "A true vigesimal system would have place values in the sequence 1, 20, 202, 203, and so on, but the Mayan system uses the sequence 1, 20, 18×20, 18×202, and so on" [p. 16]. Another unfortunate typographical error occurs in the title of chapter three on the so-called "Pythagorean theorum" [p. 21], and not enough attention was paid in the proofreading stages to the diacritical marks on words from foreign languages.

Admittedly, these final criticisms smack of the school marm, but one wishes that a book—especially one as beautifully produced as this—could be totally error-free. Mankiewicz's *The Story of Mathematics* is not a book for everyone, but, then, it is not meant to be. It is not a book for historians of mathematics. It is not a book for mathematicians (but perhaps I am being overly optimistic about the extent to which the modern mathematical community appreciates its cultural roots) or for those with technical expertise who want to read deeply into the history of mathematics. It is not a book that defines all of its terms, or covers every possible mathematical topic, or treats every conceivable geographical region or constituency. It *is*, however, an unintimidating point of departure into the world of mathematics. It is a book for all of those who never managed to see the point of mathematics. In short, it is a book for the vast majority of the English-reading public, who could and should read it with great benefit.

Book Review

Flatterland: Like Flatland, Only More So

Reviewed by Jody Trout

Flatterland: Like Flatland, Only More So
Ian Stewart
Perseus Press, April 2001,
ISBN 0-7382-0675-X
320 pages, $25.00

Geometry, the Final Frontier. These are the mathematical voyages of Vikki Line of Flatland....Wait a minute. Does this sound like a review of a mathematics book or a science fiction novel? Mathematics and science fiction? For several generations, the reading public has assumed that the main focus of science fiction (SF) was mainly, well, science and its technological toys. Lasers, spaceships, robots, time machines, atomic reactors, intelligent computers, warp drives, and genetic engineering are just some of the familiar literary devices of mainstream SF. But, why not curved surfaces, hyperspheres, fractals, Hamming metrics, projective lines, and non-Euclidean geometries? Couldn't mathematics also be the queen and servant of science fiction, to corrupt that famous saying?

Of course, as many sci-fi fans and readers of the *Notices* know, mathematical concepts have appeared in several science fiction stories over the past century or so. The main examples, from a literary viewpoint, are the fourth dimension of time in the classic novel *The Time Machine* (1895) by H. G. Wells and a hypercubical home in Robert A. Heinlein's timeless tale "—And He Built a Crooked House" (1940).

Jody Trout is associate professor of mathematics at Dartmouth College. His e-mail address is jody.trout@dartmouth.edu.

Because of its special relation to Einstein's theories of relativity, the geometry of the fourth dimension—along with its resident hypercubes and such twisted topological beasties as the Klein bottle and the Möbius strip—has provided the most popular mathematical morsel. But, with apologies to Euclid, no element of geometric literature could be more famous or enjoyable than that satirical Victorian romance of many dimensions, *Flatland.*

Written in 1884 by the school headmaster, clergyman, Shakespearean (and decidedly non-mathematical) scholar Edwin Abbott Abbott, that delightful little book has charmed generations of readers and tempted many of them to become mathematicians, including me. Many of you already know the plot by heart. *Flatland* tells the tale of how the lowly A. Square, a four-sided inhabitant of a two-dimensional Euclidean universe, receives heretical knowledge of higher dimensions from a visit by that most symmetric of Solids, The Sphere. Armed with the Theory of the Third Dimension, our planar hero sets out on a crusade to convert the narrow-minded and sexist polygonal citizens of Flatland to a more enlightened higher-dimensional view of the mysteries of space and time. However, like Galileo, A. Square discovers the timeless truth that those who put the prevailing cosmic paradigm on trial are all too often the subject of a trial themselves.

Since it first appeared, *Flatland* has been in continuous print in numerous editions and in many foreign languages. And, as many good books do, it has spawned several sequels. The main examples are the story *An Episode of Flatland* (1907) written by the colorful logician Charles Howard Hinton, the novel exposition of curved spaces *Sphereland* (1965) crafted by the Dutch physicist Dionys Berger, and *The Planiverse* (1984) by the computer scientist A. K. Dewdney, which develops the physics, astronomy, and biology of a 2D universe in a more rigorous and consistent manner. By the way, it is rumored that C. H. Hinton is the person to whom Abbott obliquely refers in the dedication of *Flatland* when he writes, "To the Inhabitants of Space In General And H. C. In Particular...." Hinton was influential in getting the public at the turn of the twentieth century interested in the fourth dimension by writing popular science articles and books on the mysterious topic. (He even claimed he could see four-dimensionally and, by the way, also invented the baseball throwing machine!)

There have also been several short stories involving Flatland or discussing it in some detail, such as a Flatland spoof by A. G. Birch called "An Adventure in the Fourth Dimension", which appeared in the October 1923 edition of the famous pulpzine *Weird Tales*. Nelson Bond wrote "The Monster from Nowhere" (1974), a creepy story about a 4D being captured by a human, and there is Rudy Rucker's dark tale "Message Found in a Copy of Flatland", which appeared in *Mathenauts* (1997), the anthology he edited (but which, sadly, is now out of print). In this story, Rucker places the physical location of Flatland in the basement of a questionable Indian restaurant in the city of London!

In fact, over the past several decades there have been so many science fiction tales involving ideas streaming from *Flatland*, the fourth dimension, and mathematics in general, that "mathematical science fiction" should be treated now as its own subgenre of SF. Indeed, three years ago I collaborated with my Dartmouth colleague Laurence Davies, who is in the comparative literature and English departments, in designing a course to study this emerging literary form. The course, which we called *Mathematics and SF: The Fire in the Equations*, was developed under the auspices of the Mathematics Across the Curriculum (MATC) project. The MATC grant was part of a multi-institutional effort by the National Science Foundation to foster interdisciplinary courses involving mathematics. We taught the course for the second time during the spring 2001 term and concentrated mainly on sci-fi stories involving geometric ideas, such as the fourth dimension, relativistic spacetime, parallel and curved universes, projective geometry, other non-Euclidean geometries, and topology. And, of course, *Flatland* was the perfect starting point for the course, as well as a useful source of metaphors and analogies for more advanced geometric concepts.

Had Ian Stewart's novel been published sooner, we would have surely considered it for our course syllabus since it discusses all of the topics we covered and then some! *Flatterland: Like Flatland, Only More So* is the latest sequel to the tri-dimensional journey of A. Square. Stewart is a mathematician at the University of Warwick, where he also directs the Mathematics Awareness Center. A well-known popular writer about science and mathematics and the author of over sixty books, Stewart was awarded the prestigious Michael Faraday Medal from the Royal Society for his contributions to furthering the public's understanding of science and mathematics. In 1999 he received the Communications Award from the Joint Policy Board for Mathematics. Stewart has also written the "Mathematical Recreations" column in *Scientific American*.

Flatterland begins with the discovery of an old family copy of A. Square's original testimony one hundred years later by his lineal great-great-granddaughter Victoria Line. Upset by her father's pigheaded insistence to make sure that the embarrassing memories of the imprisonment of crazy old Albert and the suppression of his subversive 3D Theory no longer cause the family any shame, the precocious Vikki secretly scans the ancient scroll into her personal computer before handing it over to her father to be burned. Studying the files, she finds a secret message from her ancestor on how to contact Those from the Third Dimension.

Vikki is soon visited by the Space Hopper, a tame horned sphere homeomorphic to the original spatial sage. (Alexander's wild horned sphere makes a brief appearance later when they visit Topologica.) The Space Hopper promises to take her on a fantastic voyage of the Mathiverse (short for the Mathematical Universe), that Platonic realm where mathematical objects, geometries, and spaces have their own Alice-in-Wonderland existence. To help his one-dimensional charge understand and visualize the multidimensional marvels of the Mathiverse, the Space Hopper equips the "Flatty" Vikki with a Virtual Unreality Engine (VUE) and then whisks her away from her planar home, without her even saying goodbye.

First they explore Spaceland, the idealized three-dimensional world of Euclid, which is separate from the "Planiturthian" Universe of the earth-bound humans, whose mathematical mindsets give rise to the quasi-independent existence of the Mathiverse. Running throughout the story is a philosophical chicken-and-egg conundrum about the exact nature of the relationship between mathematics and the

For more information concerning Edwin Abbott and *Flatland*, as well as the mathematics and applications of higher dimensions, visit the "Math Spans All Dimensions" website

`http://mam2000.mathforum.com/765/index.html`

designed by Thomas Banchoff of Brown University and David Cervone of Union College. A math-sf bibliography and related links can be found at our course website

`http://math.dartmouth.edu/~c18s01/.`

Other useful bibliographic references are Alex Kasman's "Mathematical Fiction" website

`http://math.cofc.edu/faculty/MATHFICT/`

(discussed in the August 2000 issue of the *Notices)* and Appendix B of Clifford Pickover's delightfully X-File-ish book *Surfing through Hyperspace.*

—*J. T.*

true geometry and structure of the universe. Is the universe constructed out of mathematics or is mathematics the construct of human minds, which are part of the universe? Stay tuned.

After stacking circles and spheres to highlight the differences between 2D and 3D, the duo then proceeds to a brief tour of higher dimensions. Various uses of "dimension" are investigated, such as treating time as a dimension and using parametric dimensions, for example, when they visit the discrete geometry of the Double- and Triple-Digit Districts, where Vikki learns about error-correcting codes and the Hamming metric. She then has a weird dream about painting herself into the interior of a closed hole by painting the outside of a ball, thus discovering the boundaryless nature of a 3-sphere. There then ensues a discussion of the kissing number for hyperspheres. (It is mentioned that the kissing numbers are known only in dimensions 1, 2, 3, 8, and 24, but they are known now also in dimensions 5, 6, and 7.) They then finish with how dimensions affect knots: There are no knots in 2D, there are oodles in 3D, and every knot can be undone in 4D.

Interspersed throughout her travels are disconnected vignettes of the plight of her Flatland family at the sudden disappearance of their beloved yet high-strung daughter and Vikki's feeble whining to her dear diary about how much she supposedly misses her family. These scenes are a little flat (pardon the pun) and the tension they are supposed to create is not very believable.

Through the next several chapters, Vikki and her smiling horned companion take a fantastic voyage through various geometries and spaces of the Mathiverse. They take a hike through the Fractal Forest where they meet the infinitely-crinkly Helge the Snowflake on their way to the infinitely-paved Quadratic City, which has streets and avenues for each (x, y) in the plane. A simple recursive rule for taxi drivers leads to mass confusion if one tries to leave this complex city! So, of course, the perfect fractal for the job of Taxi Controller is none other than the "Mandelblot" himself! (Groan...) Next, they warp themselves to the continuously deforming landscape of Topologica, the Rubber-Sheet Continent, where they meet such Carrollian characters as the Doughmouse, who can turn himself into a saucer, and Moobius the Cow, that half-twisted strip of beef that keeps her milk in...you guessed it...a Klein bottle. (Double groan...The word play is cute at first but then becomes a bit tiresome.) Afterwards, they take a safari out to infinity in the Projective Plain to capture Projective Lions, which are always polite when meeting each other and never run in parallel paths. Taking a much-needed wine break (not suitable for underage readers) at the Running Turtle Bar, where they are served by the Chicken Mock Nugget, the tired dimensional explorers help the bartender solve his vineyard planting problem using the geometry of finite projective planes.

With all these conflicting notions of what constitutes a "geometry", the Space Hopper then helps Vikki understand that geometry is nothing more than a space equipped with a transformation group of symmetries. This helps her to digest the distastefully non-Euclidean geometry of Platterland, the politely curved cousin of her own 2D world where curves are lines and lines are curves and the circle at infinity is always just beyond one's next ever-shrinking step. Thus ends the purely mathematical voyage of Victoria Line two-thirds of the way into the tale.

The rest of the story concentrates on Vikki's trying to understand the geometry and structure of the Planiturthian Universe. The best way is to start out small—really, really small—by going into the subatomic litter box of Cat Country, which is the domain of Superpaws, that mortally confused pet of the master of quantum superposition himself, Erwinschrödinger. (No, I did not make a typo. For some obscure reason, Ianstewart concatenates all—and only—human names in his story. I am sure Edwinabbottabbott would not have been amused.) By studying the photoelectric effect, Vikki learns of the slippery dual nature of subatomic quanta that can discreetly change their clothes from solid particles to pastel waves. (What Stewart actually describes here is not the photoelectric effect, where high frequency photons kick off electrons from certain reactive metals, but rather the rarer inverse process of electron-impact photon emission.)

Next, they enter the cosmological arena of Alberteinstein where they meet the likes of the Paradox Twins: Twindledumb and Twindledumber. Guess who made the wrong travel plans and ended up forty years older? To spice things up a bit, they then make the acquaintance of the very causal Space Girls: Curvy, Bendy, Pushy, Squarey, and Minny Space. (Outer Space must have left the band before the book was written....) Minny instructs Vikki in the subtle nature of relativity using her light-coney diagram by Hermannminkowski. Then, they dash off to the spatial engineering domain of the capitalist Hawk King. Vikki desperately wants his majesty to grant them a temporal trip through one of his exotic matter wormholes, but they are instead duped into falling through the looking glass event horizon of a common black hole. Thanks to space-time gymnastics that would challenge even Mr. Spock's temporal lobes and make the Cheshire cat frown, they arrange to bootstrap themselves out of the hungry maw of the naked singularity.

To finish up her Mathiversian instruction, Vikki is led to the forefront of mathematical physics: cosmic strings, supersymmetry, quantum gravity, p-branes, and M-theory. And, according to her diary logs, she experiences the wondrous totality of her entire voyage through the Mathiverse in only a semester's worth of time! (If only our students could absorb even a fraction of this material in the same amount of time.) But, alas, she finally misses her family too much, so her horned conductor takes Vikki back to her pentagonal flat just in time to have leftover Crisp Moose dinner and celebrate New Year's with her grateful family.

However, using her enhanced VUE of things, she discovers a secret that will set her on a revolutionary social crusade in the spirit of her rectangular ancestor. The one-dimensional women of polygonal-dominated Flatland are, in fact, the intersection of Flatland with orthogonal polygons when viewed from the larger, supersymmetric world of Shadow Matter! Vikki then begins to spread her mathematical feminism through the cyberworld of the Flatland Interline. Grrl-power goes interdimensional.

Personally, I enjoyed reading this book. It presents advanced mathematical and physical topics in a fun and whimsical manner. But, of course, with so many topics, several are not discussed in any significant detail. And, regrettably, hypercubes were shorted the most, despite being the higher-dimensional topic that is the most fascinating and accessible to the general public. Some may complain that Stewart attempted to cover too much mathematics in one book; after all, analogies and metaphors can only go so far before the need for rigor sets in. Some might also find the book too long, especially when compared to the original tale, which gives a brief exposition of just one topic. *Flatterland* can be far too cute at times, and, as already mentioned, the word play can grate on the ears. Also, some of the British in-jokes, relating to railways and the tawdry soap Eastenders, will not tickle American readers. My biggest complaint has to do with, of all things, the title! There is so little to do with Flatland itself, except for the handful of miniscenes where Vikki's family mopes about her absence, and Vikki so quickly begins to talk and act like a 3D human (seeming to know several obscure facts of human science and history) that there really was no need to set it in Flatland to begin with. One might be confused at times as to whether *Flatterland* is supposed to be a sequel to Abbott's book or to Lewis Carroll's *Through the Looking Glass*, and in fact I think it would have been better to make Vikki a descendant of Alice rather than of Albert.

Acknowledgments

Thanks to Allyn Jackson and the referee for their helpful comments. Thanks also to Laurence Davies, Katie Lynch, Thomas Banchoff for his extensive research on Abbott, Rudy Rucker for his copy of *Mathenauts*, and Alex Kasman for his informative website.

References

[1] Nelson Bond, The monster from nowhere, *As Tomorrow Becomes Today* (Charles W. Sullivan, ed.), New York: Prentice-Hall, 1974.

[2] Clifford Pickover, *Surfing through Hyperspace: Understanding Higher Universes in Six Easy Lessons*, Oxford University Press, 1999.

2002 Steele Prizes

The 2002 Leroy P. Steele Prizes were awarded at the 108th Annual Meeting of the AMS in San Diego in January 2002.

The Steele Prizes were established in 1970 in honor of George David Birkhoff, William Fogg Osgood, and William Caspar Graustein. Osgood was president of the AMS during 1905–06, and Birkhoff served in that capacity during 1925–26. The prizes are endowed under the terms of a bequest from Leroy P. Steele. Up to three prizes are awarded each year in the following categories: (1) Mathematical Exposition: for a book or substantial survey or expository-research paper; (2) Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research; and (3) Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students. Each Steele Prize carries a cash award of $5,000.

The Steele Prizes are awarded by the AMS Council acting on the recommendation of a selection committee. For the 2002 prizes, the members of the selection committee were: M. S. Baouendi, Sun-Yung A. Chang, Michael G. Crandall, Constantine M. Dafermos, Daniel J. Kleitman, Hugh L. Montgomery, Barry Simon, S. R. S. Varadhan (chair), and Herbert S. Wilf.

The list of previous recipients of the Steele Prize may be found in the November 2001 issue of the *Notices*, pages 1216–20, or on the AMS website at `http://www.ams.org/prizes-awards/`.

The 2002 Steele Prizes were awarded to YITZHAK KATZNELSON for Mathematical Exposition, to MARK GORESKY and ROBERT MACPHERSON for a Seminal Contribution to Research, and to MICHAEL ARTIN and ELIAS STEIN for Lifetime Achievement. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Mathematical Exposition: Yitzhak Katznelson

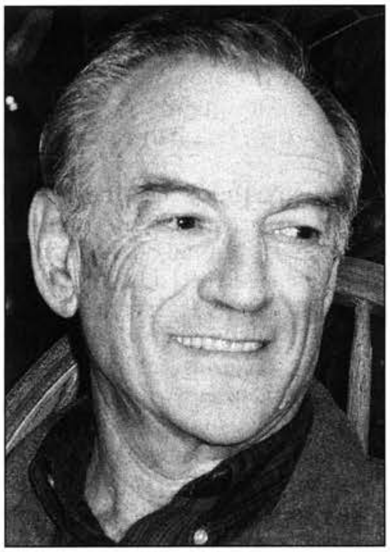
Citation

Although the subject of harmonic analysis has gone through great advances since the sixties, Fourier analysis is still its heart and soul. Yitzhak Katznelson's book on harmonic analysis has withstood the test of time. Written in the sixties and revised later in the seventies, it is one of those "classic" Dover paperbacks that has made the subject of harmonic analysis accessible to generations of mathematicians at all levels.

The book strikes the right balance between the concrete and the abstract, and the author has wisely chosen the most appropriate topics for inclusion. The clear and concise exposition and the presence of a large number of exercises make it an ideal source for anyone who wants to learn the basics of the subject.

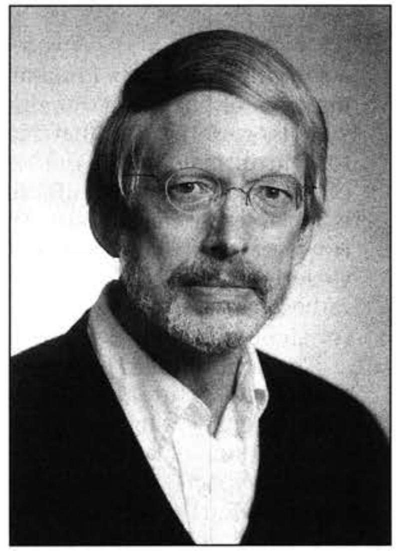
Biographical Sketch

Yitzhak Katznelson was born in Jerusalem in 1934. He graduated from the Hebrew University with a



Yitzhak Katznelson

Mark Goresky



Robert MacPherson

master's degree in 1956 and obtained the Dr. ès Sci. degree from the University of Paris in 1959.

After a year as a lecturer at the University of California, Berkeley, and a few more at the Hebrew University, Yale University, and Stanford University, he settled in Jerusalem in 1966. Until 1988 he taught at the Hebrew University, while making extended visits to Stanford and Paris. He is now a professor of mathematics at Stanford University.

Katznelson's mathematical interests include harmonic analysis, ergodic theory (and in particular its applications to combinatorics), and differentiable dynamics.

Response

What a pleasant surprise!

I am especially gratified by the committee's approval of "the balance between the concrete and the abstract," which was one of my main concerns while teaching the course and while developing the notes into a book.

How should one look at things, and in what generality? If a statement and its proof apply equally in an abstract setup, should it be introduced in the most general or the most familiar terms?

When I came to Paris in 1956 I heard a rumor that the old way of doing mathematics was being replaced by a new, "abstract" fashion which was the only proper way of doing things. The rumor was spread mostly by younger students—typically hugging a freshly-purchased volume of Bourbaki—but seemed confirmed also by the way some courses were taught.

As late as 1962, Kahane and Salem found the need to apologize (undoubtedly tongue-in-cheek) in the preface to their exquisite book *Ensembles Parfaits et Séries Trigonométriques* for dealing with subject matter that might be considered too concrete.

The balance I tried to strike in the book—and I believe that I was strongly influenced by Kahane and Salem—was to set up the subject matter in the most concrete terms and allow as much generality and abstraction as needed for development, methods, and solutions.

Seminal Contribution to Research: Mark Goresky and Robert MacPherson

Citation

In two closely related papers, "Intersection homology theory", *Topology* 19 (1980), no. 2, 135–62 (IH1) and "Intersection homology. II", *Invent. Math.* 72 (1983), no. 1, 77–129 (IH2), Mark Goresky and Robert MacPherson made a great breakthrough by discovering how Poincaré duality, which had been regarded as a quintessentially manifold phenomenon, could be effectively extended to many singular spaces. Viewed topologically, the key difficulty had been that Poincaré duality reflects the transversality property that holds within a manifold but which fails in more general spaces. IH1 introduced "intersection chain complexes", which are the subcomplexes of usual chain complexes consisting of those chains which satisfy a transversality condition with respect to the natural strata of a space. More precisely, by introducing a kind of measure, called a "perversity", of the amount of variation from transversality a chain would be allowed, Goresky and MacPherson actually introduced a parametrized family of intersection chain complexes. Each of these yielded a corresponding sequence of intersection homology groups, and these theories intermediated between homology and cohomology. Starting with methods of local piecewise-linear transversality that had been developed by investigations of M. Cohen, E. Akin, D. Stone, and C. McCrory, IH1 showed that its intersection homology theories were related to each other by a version of Poincaré duality; in particular, the intersection homology theory which was positioned midway between homology and cohomology satisfied, when defined, a self-duality, as was familiar for manifolds. This immediately yielded a signature invariant for many singular varieties, and that, in turn, was used in IH1 to yield, in analogy with the Thom-Milnor treatment of piecewise linear manifolds, rational characteristic classes for many triangulated singular varieties. However, these characteristic classes of singular varieties naturally were elements in homology rather than cohomology groups, a distinction which for singular varieties was significant.

The continuation paper, IH2, reformulated this theory in a natural and powerful sheaf language. This language, suggested by Deligne, gave local formulations of a version of Poincaré duality for singular spaces in terms of a Verdier duality of sheaves. Furthermore, IH2 presented beautiful axiomatic characterizations of its intersection chain sheaves. These were all the more valuable as the achievement of duality for nonsingular spaces came at the cost of giving up the familiar functorial and homotopy properties that characterized usual homology theories; in particular, intersection homology theory is not a "homology theory" in the sense of homotopy theory.

IH1 and IH2 made possible investigations across a great spectrum of mathematics which further extended key classical manifold phenomena and methods to singular varieties and used these to solve well-known problems. While it is impossible to list all of these, a few important ones in 1) differential geometry, 2) algebraic geometry and representation theory, 3) geometrical topology, and 4) geometrical combinatorics will be indicated.

1) An immediate question was the relation of intersection homology theory to an analytic theory of L^2 differential forms and L^2 cohomology on suitable singular varieties with metrics that J. Cheeger had concurrently developed. In fact, for many metrics the resulting groups were seen to be isomorphic by a generalization of the classical de Rham isomorphism of manifold theory. Questions about when and how this can be generalized to various natural metrics have since occupied many investigators.

2) The work of IH2 led to the discovery of the important category $P(X)$ of perverse sheaves on an algebraic variety X. In the case when X is a smooth algebraic variety over a field of characteristic zero the [generalized] Riemann-Hilbert theorem says that the category $P(X)$ is equivalent to the category of D-modules on X. This equivalence made possible the applications of Grothendieck's yoga to the theory of D-modules and, in particular, to the formulation and proof of the Kazhdan-Lusztig conjecture, which gives a formula for characters of reducible representations of Lie groups in terms of intersection homology of the closures of Schubert cells. In the case when X is an algebraic variety over a finite field F, $P(X)$ is used in investigating "good" functions on the points $X(F)$ of X over F. This is the basic ingredient in the geometrization of representation theory which has had remarkable successes in recent years.

3) Paul Segal used the methods of Goresky and MacPherson and a cobordism theory of singular varieties to show that their rational characteristic classes could in many cases be lifted, after inverting 2, to a KO-homology class. Intersection chain sheaves were extensively used in various collaborations of Cappell, Shaneson, and Weinberger which extended results of classical Browder-Novikov-Sullivan-Wall surgery theory of manifolds to yield topological classifications of many singular varieties, which developed new invariants for singular varieties and their transformation groups, which gave methods of computing the characteristic classes of singular varieties, and which related these to knot invariants.

4) In investigations of the geometrical combinatorics of convex polytopes, the intersection homology groups of their associated toric varieties have become a fundamental tool. This began with R. Stanley's investigations of the face vectors of polytopes. A calculation of the Goresky-MacPherson characteristic classes of toric varieties was used by Cappell and Shaneson in obtaining an Euler-MacLaurin formula with remainder for lattice sums in polytopes. Recent works of MacPherson and T. Braden on flags of faces of polytopes used results on the intersection chain sheaves of toric varieties. The already astonishing range of research areas influenced by this seminal work continues to grow.

Biographical Sketch: Mark Goresky

Mark Goresky received his B.Sc. from the University of British Columbia in 1971 and attended graduate school at Brown University. He spent the 1974–75 academic year at the Institut des Hautes Études Scientifiques and received his Ph.D. in 1976. He was a C. L. E. Moore Instructor at the Massachusetts Institute of Technology (1976-78) and an assistant professor at UBC (1978-81). In 1981 he moved to Northeastern University, where he eventually attained the rank of professor with a joint appointment in mathematics and computer science. Since 1995 he has lived in Princeton, New Jersey, where he is currently a member at the Institute for Advanced Study. He has held other visiting positions at the University of Chicago, the Max-Planck-Institut für Mathematik, the IHÉS, and the University of Rome.

Goresky received a Sloan Fellowship in 1981. He is a fellow of the Royal Society of Canada, and he received the Coxeter-James Award (1984) and the Jeffrey-Williams Prize (1996) from the Canadian Mathematical Society.

Biographical Sketch: Robert MacPherson

Robert MacPherson received a B.A. from Swarthmore College and a Ph.D. from Harvard University.

He held faculty positions at Brown University from 1970 to 1987, at MIT from 1987 to 1994, and at the Institute for Advanced Study since then. Over the years, he has held visiting positions at the Institut des Hautes Études Scientifiques in Paris, Université de Paris VII, Steklov Institute in Moscow, IAS in Princeton, Università di Roma I, University of Chicago, Max-Planck-Institut für Mathematik in Bonn, and Universiteit Utrecht. He received the National Academy of Sciences Award in Mathematics and honorary doctorates from Brown University and Université de Lille. He served as chair of the National Research Council's Board on Mathematical Sciences from 1997 to 2000. He is a member of the American Academy of Arts and Sciences, the National Academy of Sciences, and the American Philosophical Society.

Response

We are very grateful to the American Mathematical Society for awarding us the Steele Prize. We are particularly pleased to receive a joint prize for our joint research. We know of no other mathematical prize that is awarded jointly to the participants of a collaboration. Given the increasing role of collaborative research in mathematics, this policy on the part of the AMS seems particularly enlightened to us.

In September 1974 we began a year at the Institut des Hautes Études Scientifiques with a pact to try to understand what intersection theory should mean for singular spaces. We thought the question might have importance for several areas of mathematics, given the ubiquity with which singular spaces naturally arise. By late autumn, we had found intersection homology and Poincaré duality. Jeff Cheeger, Pierre Deligne, Clint McCrory, John Morgan, and Dennis Sullivan played significant roles in the early stages of this research.

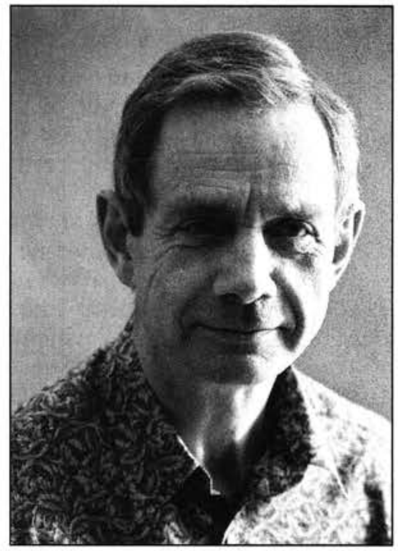
Starting around 1980, an explosion of activity surrounding intersection homology occurred. Our dream that the subject would find applications suddenly became true. Many mathematicians contributed a remarkable collection of ideas to this activity, and our collaboration was swept along with this flow into new fields such as combinatorics and automorphic forms.

Today, extensions and applications of the theory are pursued by a new generation of highly talented mathematicians, some of whom have already received mathematical awards in Europe (where prizes for younger mathematicians are more common). It is gratifying to see that these ideas, in whose discovery we participated, are now in such capable hands.

Lifetime Achievement: Michael Artin

Citation

Michael Artin has helped to weave the fabric of modern algebraic geometry. His notion of an algebraic space extends Grothendieck's notion of scheme. The point of the extension is that Artin's theorem on approximating formal power series solutions allows one to show that many moduli spaces are actually algebraic spaces and so can be studied by the methods of algebraic geometry. He showed also how to apply the same ideas to the algebraic stacks of Deligne and Mumford. Algebraic stacks and algebraic spaces appear everywhere in modern algebraic geometry, and Artin's methods are used constantly in studying them.



Michael Artin

He has contributed spectacular results in classical algebraic geometry, such as his resolution (with Swinnerton-Dyer in 1973) of the Shafarevich-Tate conjecture for elliptic $K3$ surfaces. With Mazur, he applied ideas from algebraic geometry (and the Nash approximation theorem) to the study of diffeomorphisms of compact manifolds having periodic points of a specified behavior.

For the last twenty years he has worked to create and define the new field of noncommutative algebraic geometry.

Artin has supervised thirty doctoral students and influenced a great many more. His undergraduate algebra course was for many years one of the special features of an MIT education; now some of that insight is available to the rest of the world through his textbook.

Biographical Sketch

I have departed from the usual format here to write a bit about my early life and the origins of my interest in mathematics.

When I was nearly forty years old I had a revelation: A recurring dream that I'd had since age twelve was an allegory of my birth! In the dream, I was stuck in a secret passage in our house but eventually worked my way out and emerged into a sunlit cupola. After my revelation, the dream went away.

My mother says that I was a big baby and it was a difficult birth, although I don't know what I weighed. The conversion from German to English pounds adds ten percent, and I suspect that my mother added another ten percent every few years. She denies this, of course. Anyway, I'm convinced that a birth injury caused my left-handedness and some seizures, which, fortunately, are under control.

The name Artin comes from my great-grandfather, an Armenian rug merchant who moved to Vienna in the nineteenth century.

Armenians were declared "Aryan" by the Nazis, but one side of my mother's Russian family was Jewish, and because of this, my father Emil was fired from the university in Hamburg. We came to America in 1937, when I was three years old.

My father loved teaching as much as I do, and he taught me many things: sometimes mathematics, but also the names of wild flowers. We played music and examined pond water. If there was a direction in which he pointed me, it was toward chemistry. He never suggested that I should follow in his footsteps, and I never made a conscious decision to become a mathematician.

I had decided to study science when I began college, but fields such as chemistry and physics gradually fell away, until biology and mathematics were the only ones left. I loved them both, but decided to major in mathematics. I told myself that changing out of mathematics might be easier, since it was at the theoretical end of the science spectrum, and I planned to switch to biology at age thirty when, as everyone knew, mathematicians were washed up. By then I was too involved with algebraic geometry. My adviser Oscar Zariski had seen to that.

Response

I thank the AMS and the prize committee for choosing to award me the Steele Prize for Lifetime Achievement, and I congratulate my fellow recipient Eli Stein. This award gives me great pleasure.

I also want to thank the many inspiring people who have surrounded me throughout my career. It has been a privilege to teach at MIT, where the students are gifted and motivated, and where my colleagues are as deserving of an award for lifetime achievement as I am. My thesis students there have been a constant source of inspiration. The financial support provided by the National Science Foundation for my work has been invaluable.

Alexander Grothendieck, Barry Mazur, John Tate, and of course my thesis adviser Oscar Zariski, are among the people who influenced me the most during the 1950s and 1960s. Those were exciting times for algebraic geometry. The crowning achievement of the Italian school, the classification of algebraic surfaces, was just entering the mainstream of mathematics. The sheaf theoretic methods introduced by Jean-Pierre Serre were being absorbed, and Grothendieck's language of schemes was being developed. Zariski's dynamic personality, and the explosion of activity in the field, persuaded me to work there. I became his student along with Peter Falb, Heisuke Hironaka, and David Mumford. Later, in the 1960s, I visited the Institut des Hautes Études Scientifiques several times to work with Grothendieck and Jean-Louis Verdier.

My interest in noncommutative algebra began with a talk by Shimshon Amitsur and a visit to Chicago, where I met Claudio Procesi and Lance Small. They prompted my first foray into ring theory, and in subsequent years noncommutative algebra gradually attracted more of my attention. I changed fields for good in the mid-1980s, when Bill Schelter and I did experimental work on quantum planes using his algebra package, Affine.

My early training has led me to concentrate on dimension two, or noncommutative surfaces. They display many interesting phenomena which remain to be explained, and I've come to understand that two is a critical dimension. Thanks to recent work of people such as Johan de Jong, Toby Stafford, and Michel Van den Bergh, the methods of algebraic geometry are playing a central role in this area too, and I hope to see it absorbed into the mainstream in the near future.

Lifetime Achievement: Elias Stein

Citation

During a scientific career that spans nearly half a century, Eli Stein has made fundamental contributions to different branches of analysis.

In harmonic analysis, his Interpolation Theorem is a ubiquitous tool. His result about the relation between the Fourier transform and curvature revealed a deep and unsuspected property and has far reaching consequences. His work on Hardy spaces has transformed the subject. He has made important contributions to the representation theory of Lie groups as well.

Elias Stein

His work on several complex variables is equally striking. His explicit approximate solutions for the $\bar{\partial}$-problems made it possible to prove sharp regularity results for solutions in strongly pseudoconvex domains. In this connection he also obtained subelliptic estimates which sharpened and quantified Hörmander's hypoellipticity theorem for second order operators.

Besides his contributions through his own research and excellent monographs, Stein has worked with and influenced many students, who have gone on to make profound contributions of their own.

Biographical Sketch

Elias M. Stein was born in Belgium in 1931 and came to the U.S. at the age of ten. He received his Ph.D. from the University of Chicago in 1955. Since 1963 he has taught at Princeton University, where he has served twice as chair of the mathematics department (1968–71 and 1985–87).

Stein's many fellowships and awards include a National Science Foundation Postdoctoral Fellowship (1955–56), an Alfred P. Sloan Foundation Fellowship (1961–63), Guggenheim Fellowships (1976–77 and 1984–85), membership in the National Academy of Sciences (1974) and the American Academy of Arts and Sciences (1982), the von Humboldt Award (1989–90), the Schock Prize from the Swedish Academy of Sciences (1993), and the Wolf Prize (1999). He was awarded the AMS Steele Prize in 1984 for his book *Singular Integrals and the Differentiability Properties of Functions* published in 1970 by Princeton University Press. Stein received an honorary Ph.D. from Peking University and an honorary D.Sc. from the University of Chicago.

Response

I want to express my deep appreciation to the American Mathematical Society for the honor represented by this award. At this occasion I am mindful of the great debt I owe others for my present good fortune. Beginning with my teachers and mentors and continuing with my peers, colleagues, and students, I have had the advantage of their warm support and encouragement and the indispensable benefit of their inspiration and help. To all of them I am very grateful.

I would like also to say something about the area of mathematics of which I am a representative. For more than a century there has been a significant and fruitful interaction between Fourier analysis, complex function theory, partial differential equations, real analysis, as well as ideas from other disciplines such as geometry and analytic number theory, etc. That this is the case has become increasingly clear, and the efforts and developments involved have, if anything, accelerated in the last twenty or thirty years. Having reached this stage, we can be confident that we are far from the end of this enterprise and that many exciting and wonderful theorems still await our discovery.

AMS PRIZES PRIZES

2002 Bôcher Prize

The 2002 Maxime Bôcher Memorial Prize was awarded at the 108th Annual Meeting of the AMS in San Diego in January 2002.

The Bôcher Prize is awarded every three years for a notable research memoir in analysis that has appeared during the previous five years in a recognized North American journal (until 2001, the prize was usually awarded every five years). Established in 1923, the prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer in 1896 and who served as AMS president during 1909–10. Bôcher was also one of the founding editors of *Transactions of the AMS*. The prize carries a cash award of $5,000.

The Bôcher Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2002 prize, the members of the selection committee were: Luis Caffarelli, Sergiu Klainerman (chair), and Linda Preiss Rothschild.

Previous recipients of the Bôcher Prize are: G. D. Birkhoff (1923), E. T. Bell (1924), Solomon Lefschetz (1924), J. W. Alexander (1928), Marston Morse (1933), Norbert Wiener (1933), John von Neumann (1938), Jesse Douglas (1943), A. C. Schaeffer and D. C. Spencer (1948), Norman Levinson (1953), Louis Nirenberg (1959), Paul J. Cohen (1964), I. M. Singer (1969), Donald S. Ornstein (1974), Alberto P. Calderón (1979), Luis A. Caffarelli (1984), Richard B. Melrose (1984), Richard M. Schoen (1989), Leon Simon (1994), Demetrios Christodoulou (1999), Sergiu Klainerman (1999), and Thomas Wolff (1999).

The 2002 Bôcher Prize was awarded to Daniel Tataru, Terence Tao, and Fanghua Lin. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Daniel Tataru

Citation

The Bôcher Memorial Prize in 2002 is awarded to Daniel Tataru for his fundamental paper "On global existence and scattering for the wave maps equations", *Amer. Jour. of Math.* 123 (2001) no. 1, 37–77. The paper introduces a remarkable functional framework which has played an important role in the recent breakthrough of T. Tao on the critical regularity for wave maps in two and three dimensions. The work of Tataru and Tao opens up exciting new possibilities in the study of nonlinear wave equations.

The prize also recognizes Tataru's important work on Strichartz estimates for wave equations with rough coefficients and applications to quasilinear wave equations, as well as his many deep contributions to unique continuation problems.

Biographical Sketch

Daniel Tataru was born on May 6, 1967, in a small city in the northeast of Romania. He received his undergraduate degree in 1990 from the University of Iasi, Romania, and his Ph.D. in 1992 from the University of Virginia. He was assistant, associate, and then full professor at Northwestern University (1992–2001) with a two year interruption when he visited the Institute for Advanced Study and Princeton University (1995–97). Since 2001 he has been a professor of mathematics at the University of California at Berkeley.

Response

I feel very honored to receive the 2002 Bôcher Prize, for which I am grateful to the selection committee and the American Mathematical Society. I would like to take this opportunity to acknowledge several people who have significantly influenced my work. My undergraduate mentor, Viorel Barbu, provided a model and an inspiration for me on what it means to be a mathematician. Later, my thesis advisors, Irena Lasiecka and Roberto Triggiani, through their professional support as well as their warmth, helped me grow, move on confidently, and adjust successfully here in the U.S. From them I learned control theory, which subsequently served both as a motivation and as a source of good problems in unique continuation. Sergiu Klainerman is the one who introduced me to nonlinear hyperbolic equations. I thank him for his constant support and for a fruitful collaboration during my years in Princeton. I am also grateful for the help and the encouragement that I received earlier in my career from M. G. Crandall, J. L. Lions, and P. L. Lions, as well as for the support of my friends and former colleagues at Northwestern. In addition, I continue to learn from my collaborators Herbert Koch and Hart Smith.

The wave maps equation is a semilinear second order hyperbolic equation which models the evolution of "waves" which take values into a Riemannian manifold. The starting point of the work in the citation was an earlier article of Klainerman and Machedon. At the time it was clear that bridging the gap between their result and the main wave maps conjecture required two distinct improvements of their argument. One was the so-called "division problem", which is related to controlling the bilinear interaction of waves at a fixed size of the frequency; the second was to control the interaction of low and high frequency waves in order to prevent the migration of the energy toward high frequency (which could lead to blow-up). These two problems correspond to two separate logarithmic divergencies in the work of Klainerman and Machedon, but, more importantly, they also correspond to the difference between a local and a global (in time) result. In the article cited I solved the first of these two problems; the second one was later solved by Tao. This was not an easy problem. Part of the difficulty lies in the construction of an appropriate functional framework. However, one does not have a good starting point for this, since the main condition this framework has to satisfy is a self-consistency condition. The solution was to start with a "reasonable" framework, proceed with the proof, and then backtrack and readjust the initial set-up whenever the argument did not work. While fairly intuitive, my approach is quite technical and I hope it can be simplified in the future. After the recent work of Tao there are still some finishing touches to be put on the study of the wave-maps equation. However, the more interesting problems which are still open are the other unsolved critical problems for the Yang-Mills equation, the nonlinear Schrödinger equation, and others.

My work on second order nonlinear hyperbolic equations was initially a byproduct of my attempt to use a phase space localization technique called the FBI transform for the analysis of partial differential operators with rough coefficients. Originally the FBI transform had been employed by Sjöstrand for the study of partial differential operators with analytic coefficients; as I learned later, it has also been used by physicists under the name of the Bergman transform. This approach produced sharp Strichartz type (dispersive) estimates for linear second order hyperbolic equations with rough coefficients, which in turn led to considerable progress in the local theory for second order nonlinear hyperbolic equations. At the same time an alternate approach for the nonlinear equations was pursued by Bahouri and Chemin, with comparable success. Later on, using Klainerman's vector fields method, Klainerman and Rodnianskii were able to improve my results. Around that time it became clear that the FBI transform is not robust enough for the study of nonlinear hyperbolic equations. My recent joint work with Hart Smith is based on another way of constructing a parametrix for the wave equation, using wave packets (which are highly localized solutions that stay coherent on a given time scale). The idea of constructing approximate solutions as superpositions of wave packets goes back to Fefferman, but its first effective use for second order hyperbolic equations is due to Smith. The joint work of myself and Smith largely completes the local theory for general second order nonlinear hyperbolic equations. The main open problem that remains is to understand whether the results can be improved for equations which have a special structure such as the Einstein equations, nonlinear elasticity, and other related problems.

Daniel Tataru

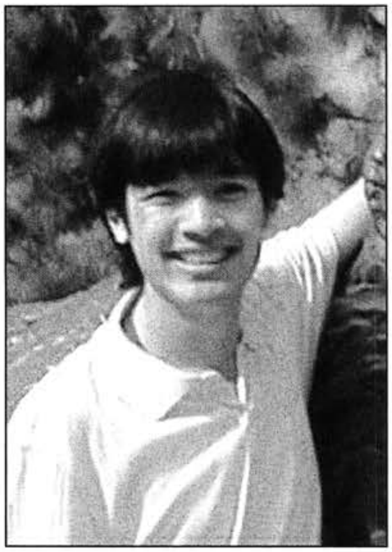
Unique continuation problems for PDEs have long been on my list of favorite topics. Originally my interest was motivated by problems in control theory, but later it took a life of its own. My view of the subject was influenced by the work of several mathematicians: L. Hörmander, G. Lebeau, L. Robbiano, C. Zuily, and others. Initially I worked on unique continuation problems which, up to that time, had received little or no attention: for boundary value problems, for operators with partially analytic coefficients, for anisotropic operators. Later on, in an ongoing joint project with Herbert Koch, I have returned to some of the more classical problems, but with a new twist: rough coefficients and/or unbounded potentials. The starting point for us was the seminal work of D. Jerison, C. Kenig, and T. Wolff.

Terence Tao

Citation

The Bôcher Memorial Prize in 2002 is awarded to Terence Tao for his recent fundamental breakthrough on the problem of critical regularity in Sobolev spaces of the wave maps equations, "Global regularity of wave maps I. Small critical Sobolev norm in high dimensions", *Int. Math. Res. Notices* (2001), no. 6, 299–328 and "Global regularity of wave maps II. Small energy in two dimensions", to appear in *Comm. Math. Phys.* (2001 or early 2002).

The committee also recognizes his remarkable series of papers, written in collaboration with J. Colliander, M. Keel, G. Staffilani, and H. Takaoka, on global regularity in optimal Sobolev spaces for KdV and other equations, as well as his many deep



Terence Tao

contributions to Strichartz and bilinear estimates.

Biographical Sketch

Terence Tao was born in Adelaide, Australia, in 1975. He received his Ph.D. in mathematics from Princeton University in 1996 under the advisorship of Elias Stein. He has been at the University of California, Los Angeles, as a Hedrick assistant professor (1996–1998), assistant professor (1999– 2000), and professor (2000–). He has also held visiting positions at the Mathematical Sciences Research Institute (1997), the University of New South Wales (1999–2000), and Australian National University (2001). He is currently on leave from UCLA as a Clay Prize Fellow.

Tao has been supported by grants from the National Science Foundation and fellowships from the Sloan Foundation, Packard Foundation, and the Clay Mathematics Institute. He received the Salem Prize in 2000.

Tao's research is divided into three areas: real-variable harmonic analysis (especially estimates for rough operators and connections with geometric combinatorics); nonlinear evolution equations (especially the global behavior of rough solutions); and algebraic combinatorics (specifically the understanding of the Littlewood-Richardson rule and its generalizations, and its applications to linear algebra, algebraic geometry, and representation theory).

Response

I am deeply flattered and honored to be nominated for the Bôcher Prize, and I am grateful to the prize committee for their recognition of this research. I have been extremely fortunate to have been supported, encouraged, and taught by many wonderful people and collaborated with many more. For the papers cited above I was particularly influenced by many invaluable conversations with Elias Stein, Tom Wolff, Jean Bourgain, Sergiu Klainerman, Chris Sogge, Daniel Tataru, Michael Christ, and my collaborators Mark Keel, Jim Colliander, Gigliola Staffilani, Hideo Takaoka, Ana Vargas, and Luis Vega. I am particularly grateful to Mark Keel and Sergiu Klainerman for giving me a thorough and expert introduction to the field of nonlinear wave and dispersive equations.

In the analysis of nonlinear dispersive equations, the tools used can be roughly divided into "analytical" tools and "algebraic" ones. By analytic tools I mean the use of function spaces such as Sobolev or Lebesgue spaces, coupled with linear, bilinear, multilinear, or nonlinear estimates in these spaces (which are often proven by harmonic analysis techniques). These estimates can allow one to apply perturbation theory and approximate a nonlinear equation by the linear analogue, at least for short times. By algebraic tools I refer to the use of conservation laws, symmetries, monotonicity formulae, special transformations, integrability, and explicit solutions. These algebraic identities give some partial control on the global development of solutions to the nonlinear PDE. To obtain satisfactory global control of solutions, one often combines the partial global control coming from the algebraic identities, with the more detailed but local control coming from the analytic techniques. For instance, perturbation theory might show that smooth solutions exist as long as the energy remains finite, while algebraic identities (i.e., integration by parts) might show that the energy remains constant. Combining the two one would then be able to show that smooth solutions exist globally in time, so that no singularities can ever form if the initial energy is finite.

Both the algebraic and analytic tools have been under development for many decades, the groundwork being laid by many excellent mathematicians. In the last ten years there has been immense progress, particularly in the analytic side of things, thanks to the efforts of Bourgain, Klainerman-Machedon, Kenig-Ponce-Vega, and many, many other authors. Indeed, our understanding of the local theory of nonlinear wave and dispersive equations has become quite satisfactory. Unfortunately, even when this local theory is completely understood, it does not always match up with the algebraic tools needed to create good global results; for instance, the local theory may need control of the solution in the Sobolev space H^2, but the conserved quantities might only control the solution in H^1.

One interesting development in recent years is that hybrid techniques, combining both analytical and algebraic ideas, have started to bridge some of the above gaps. In particular, the use of cutoff functions (in space, or in frequency, or in both), together with the latest linear and multilinear estimates, have been used to obtain "localized" conservation laws, "localized" evolution equations, "localized" gauge transforms, etc., which are more flexible than their global algebraic counterparts, and have had some recent successes. Notable applications of this type of philosophy include Bourgain's series of papers on nonlinear Schrödinger equations; the work by Martel and Merle on the stability of solitons for the generalized Korteweg de Vries equations; the many papers on global solutions below the energy norm (starting with work of Bourgain, and also including the papers by Colliander, Keel, Staffilani, Takaoka, and myself); the recent breakthroughs on quasilinear wave equations by Bahouri-Chemin, Tataru-Smith, and Klainerman-Rodnianski; and the recent series of papers on wave maps. For instance, one effective technique for constructing global solutions when the energy is infinite is to construct a smoothing operator, define the associated smoothed out energy, and show an approximate conservation law for the smoothed out energy. For wave maps, one new technique has been to localize the wave map to different frequency modes, and then gauge transform each frequency mode independently. The work on quasilinear wave equations is also interesting in that it seems to bring geometric optics back into the cutting edge of the theory. (Intriguingly, this has also occurred in the theory of oscillatory integrals, thanks to the work of Bourgain, Wolff, and others.)

In the future I believe we will see a more systematic synthesis of the analytic and algebraic techniques, perhaps ultimately leading to a unified theory for treating the development of nonlinear PDE; at present there are only tantalizing hints of such a theory. The end result should be a more powerful and flexible theory, allowing for much more detailed control on the global behavior of nonlinear PDE. (In particular, I hope to see finer control on the possible cascade of energy between frequencies, and on tracking particle-like behavior of solutions.)

Fanghua Lin

Citation

The Bôcher Memorial Prize in 2002 is awarded to Fanghua Lin for his fundamental contributions to our understanding of the Ginzburg-Landau (GL) equations with a small parameter. In a remarkable series of papers, among which we single out his pioneering work "Some dynamical properties of the GL vortices", *Comm. Pure Appl. Math.* (1996), 323–59, he has established, both in the stationary and evolutionary cases of GL equations, that the limiting phenomenon is governed by a finite dimensional system associated to the BBH renormalized energy.

The prize also recognizes his many deep contributions to harmonic maps and liquid crystals. Of particular note is his paper "Gradient estimates and blow-up analysis for stationary harmonic maps", *Annals of Math.* (2), 149 (1999), no. 3, 785–829.

Biographical Sketch

Fanghua Lin was born in China in 1959. He received a Ph.D. from the University of Minnesota (1985). He has held faculty positions at the Courant Institute (1985–88) and the University of Chicago (1988–89; 1996–97). Since 1989 he has been a professor of mathematics at New York University. Over the years, he has held numerous visiting positions, including those at MSRI, IAS, University of Paris-Sud, the Max Planck Institute, Academia Sinica, Hong Kong University of Science and Technology, and the University of Minnesota.

Fanghua Lin

Lin's awards and honors include an Alfred P. Sloan Fellowship (1989–91), a Presidential Young Investigator Award (1989–94), and the Chang Jiang Professorship (1999). He has served on the editorial committees of twelve mathematics journals, published over 110 research articles, and given numerous invited addresses.

Response

It is a great honor to be awarded the Bôcher Prize, and I am grateful to the members of the Bôcher Prize selection committee and the American Mathematical Society for their recognition of my work and this kind citation.

The Ginzburg-Landau (GL) equations with a small parameter were used to model superconductivity. They are also among the standard equations used to describe various phase transition phenomena. Many people have contributed to the study of these equations, and I interpret my receipt of this prize as a tribute to all of them.

My first paper on this subject established the connection between the critical points of the Bethuel, Brezis, Hélein (BBH) renormalized energy and the static solutions of the GL equations, one of many open problems posed in the seminal work of BBH. Soon after that, I learned the importance of various dynamical issues, particularly those concerned with vortex dynamics, and the scientific views of several of my colleagues at the Courant Institute (Weinan E and Andy Majda) and also of my collaborator Jack Xin have greatly influenced me. I had a lot of fun learning and solving some of these problems related to the vortex dynamics in 2-D.

The study of the GL equations in high dimensions is more subtle and involved. The analytical method that I developed jointly with T. Rivière can also be applied to study similar problems for the Seiberg-Witten and Yang-Mills equations. Nevertheless, many difficulties remain, especially those concerned with dynamical issues.

Much of my work relies on ideas from geometric measure theory, and I take this opportunity to thank my Ph.D. advisor, my long-time friend and collaborator, R. Hardt, for introducing me to this fascinating subject. I have been extremely lucky to have been at the Courant Institute, and I thank my colleagues there for their advice, support, and friendship during these past years. I also want to thank friends at the University of Chicago who have been very kind and offered a great deal of help during my career.

2002 Cole Prize in Number Theory

The 2002 Frank Nelson Cole Prize in Number Theory was awarded at the 108th Annual Meeting of the AMS in San Diego in January 2002.

The Cole Prize in Number Theory is awarded every three years for a notable research memoir in number theory that has appeared during the previous five years (until 2001, the prize was usually awarded every five years). The awarding of this prize alternates with the awarding of the Cole Prize in Algebra, also given every three years. These prizes were established in 1928 to honor Frank Nelson Cole on the occasion of his retirement as secretary of the AMS after twenty-five years of service. He also served as editor-in-chief of the *Bulletin* for twenty-one years. The Cole Prize carries a cash award of $5,000.

The Cole Prize in Number Theory is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2002 prize, the members of the selection committee were: Benedict Gross, Carl Pomerance, and Paul Vojta (chair).

Previous recipients of the Cole Prize in Number Theory are: H. S. Vandiver (1931), Claude Chevalley (1941), H. B. Mann (1946), Paul Erdős (1951), John T. Tate (1956), Kenkichi Iwasawa (1962), Bernard M. Dwork (1962), James B. Ax and Simon B. Kochen (1967), Wolfgang M. Schmidt (1972), Goro Shimura (1977), Robert P. Langlands (1982), Barry Mazur (1982), Dorian M. Goldfeld (1987), Benedict H. Gross and Don B. Zagier (1987), Karl Rubin (1992), Paul Vojta (1992), and Andrew J. Wiles (1997).

The 2002 Cole Prize in Number Theory was awarded to Henryk Iwaniec and Richard Taylor. The text that follows presents, for each awardee, the selection committee's citation, a brief biographical sketch, and the awardee's response upon receiving the prize.

Henryk Iwaniec

Citation

The Frank Nelson Cole Prize in Number Theory is awarded to Henryk Iwaniec of Rutgers University for his fundamental contributions to analytic number theory. In particular, the prize is awarded for his paper (with J. Friedlander) "The polynomial $X^2 + Y^4$ captures its primes", in *Ann. Math.*, which is the first paper ever to show that an integer polynomial with "sparse" range takes on infinitely many prime values. The method is robust, and already D. R. Heath-Brown has extended the method to certain cubic polynomials. In addition, the prize is awarded for the series of papers (with W. Duke and J. Friedlander) "Bounds for automorphic L-functions, I, II, III", in *Invent. Math.*, and the paper (with B. Conrey) "The cubic moment of central values of automorphic L-functions", in *Ann. Math.* In these papers, critical-line bounds for L-functions associated to certain modular forms were greatly improved by novel methods, including an amplification technique that provided the starting point for J. W. Cogdell, I. Piatetskii-Shapiro, and P. Sarnak to finally resolve Hilbert's eleventh problem (on representation by quadratic forms in a number field). And, the prize is awarded for the paper (with P. Sarnak) "The nonvanishing of central values of automorphic L-functions and Landau-Siegel zeros", *Israel J. Math.*, for the introduction of far-reaching averaging and mollification techniques for families of automorphic L-functions.

Biographical Sketch

Henryk Iwaniec was born in Elblag, Poland, on October 9, 1947. He graduated from Warsaw University in 1971, and he received his Ph.D. there in 1972. From 1971 until 1983 he held various positions in the Institute of Mathematics of the Polish Academy of Sciences. In 1976 he defended his *habilitation* thesis. In the year 1976–77 he enjoyed a fellowship of the Accademia Nazionale dei Lincei at the Scuola Normale Superiore di Pisa. In 1979–80 he visited the University of Bordeaux. In 1983 he was promoted to professor. The same year he became member correspondent of the Polish Academy of Sciences.

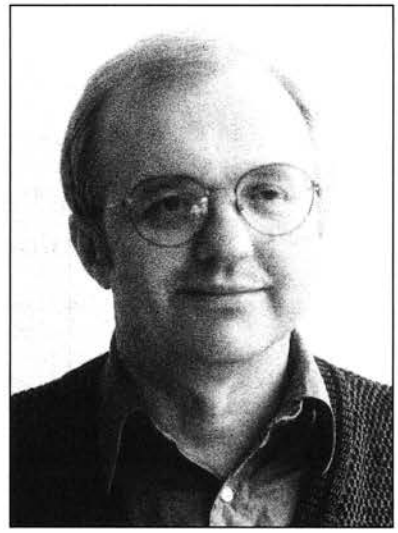
Iwaniec left Poland in 1983 to take visiting positions in the USA at the Institute for Advanced Study in Princeton (1983–84 and 1985–86) and the University of Michigan at Ann Arbor (summer 1984), and he was the Ulam Distinguished Visiting Professor at Boulder (fall 1984). In January 1987 he assumed his present position as New Jersey State Professor of Mathematics at Rutgers University. He was elected to the American Academy of Arts and Sciences in 1995. He spent the year 1999–2000 as a distinguished visiting professor at IAS. Recently he became a citizen of the USA.

Iwaniec received first prizes in the Marcinkiewicz contests for student works in the academic years 1968–69 and 1969–70. In 1978 he received the State Prize from the Polish Government, in 1991 he received the Jurzykowski Award from the Alfred Jurzykowski Foundation in New York, and in 1996 he received the Sierpinski Medal. Iwaniec was an invited speaker at the International Congress of Mathematicians in Helsinki (1978) and in Berkeley (1986).

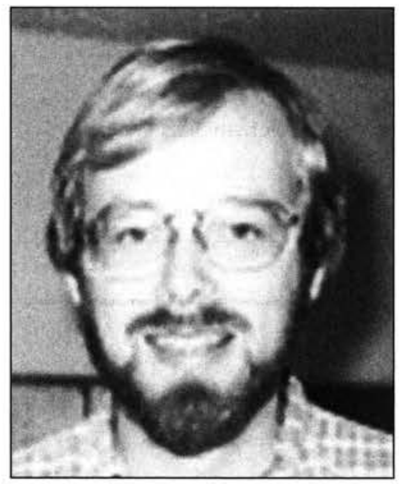
Response from Professor Iwaniec

I thank from my heart the American Mathematical Society and the committee of the Cole Prize for selecting me for this award. My joy is even greater when I think that this is a significant award for professional accomplishments from beyond my native country, and in particular that this is coming from my new homeland in the USA. Less emotional, nevertheless important for me, is also the feeling of larger recognition of analytic number theory which the Cole Prize manifests in this case. Indeed, all the works cited for the prize are joint with many of my colleagues. Without their collaboration I cannot imagine how could I get that far. Yes, working together offers an immediate satisfaction from sharing ideas, but above all it is the only way we can cultivate in depth the modern analytic number theory.

Analytic number theory pursues hard classical problems of an arithmetical nature by means of best available technologies from any branch of mathematics, and that is its beauty and strength. Analytic number theory is not driven by one concept; consequently it has no unique identity. Fourier analysis was always present, but in the last two decades it has been expanded to nonabelian harmonic analysis by employing the spectral theory of automorphic forms. For example, applying this analysis implicitly we have established the asymptotic distribution of primes in residue classes in the range beyond the capability of the Grand Riemann Hypothesis. Moreover, along these lines, we were able to produce primes in polynomial sequences. To this end one needs to enhance the Dirichlet characters by more powerful cusp forms on congruence groups. In a different direction we performed amplified spectral averaging from which to deduce important estimates for individual values of L-functions and to apply the latter to questions of equidistribution of many arithmetical objects. Other fruitful resources for solving problems in analytic number theory were uncovered by exploiting the Riemann hypothesis for varieties. Connections of these problems with the profound theory of Deligne are by no means straightforward. Perhaps these brief words may give some idea of what the trends are in the subject today, or at least what we are doing there.



Henryk Iwaniec



Richard Taylor

There are many colleagues to whom I owe my gratitude for inspiration and joint research over the last years; among them I would like to mention Enrico Bombieri, Brian Conrey, Jean-Marc Deshouillers, William Duke, John Friedlander, Etienne Fouvry, Philippe Michel, and Peter Sarnak.

Richard Taylor

Citation

The Frank Nelson Cole Prize in Number Theory is awarded to Richard Taylor of Harvard University for several outstanding advances in algebraic number theory. He led an effort to extend his earlier work with Wiles, to show that all elliptic curves over Q are modular, i.e., are factors of the Jacobians of modular curves. In his book with M. Harris, he established the local Langlands conjecture, giving a complete parametrization of the n-dimensional representations of a

Galois group of a local field. He has also made important progress on 2-dimensional Galois representations, establishing the Artin conjecture for an infinite class of nonsolvable cases, and increasing our understanding of the conjectures of Fontaine-Mazur and Serre.

Biographical Sketch

Richard Taylor was born on May 19, 1962, in Cambridge, England. At the age of two he moved to Oxford, where he grew up. In 1980 he went back to Cambridge for his undergraduate studies. In 1984 he moved to Princeton University for his graduate studies, receiving his Ph.D. in 1988 for a thesis on congruences between modular forms. His advisor was Andrew Wiles, who had a very great influence on Taylor's mathematical development.

After graduating Richard Taylor spent a postdoctoral year at the Institut des Hautes Études Scientifiques outside Paris. Encouraged and supported by John Coates, he then moved back to Cambridge University for six years. Following his marriage in 1995, he left Cambridge first for the Savilian chair of geometry at Oxford University, and a year later moved to Harvard University, where he is still employed. In 1990 he was awarded a junior Whitehead Prize by the London Mathematical Society, in 1992 he was awarded the Prix Franco-Brittanique by the French Académie des Sciences, and in 1995 he was elected a fellow of the Royal Society.

Richard Taylor is an algebraic number theorist working on the interconnections between automorphic forms and representations of Galois groups. In 1994 he collaborated with Andrew Wiles to repair the gap in Wiles' proof of Fermat's last theorem.

Response from Professor Taylor

It is a great honour and pleasure for me to receive the Frank Nelson Cole Prize. It is also an honour to share the prize with a mathematician I admire as much as Henryk Iwaniec. The citation mentions three papers and one book, on which I have worked with a total of seven collaborators. I would like to thank them all, above all for the enjoyment I have had working on these various projects.

2002 Award for Distinguished Public Service

Margaret H. Wright

The 2002 Award for Distinguished Public Service was presented at the 108th Annual Meeting of the AMS in San Diego in January 2002.

The Award for Distinguished Public Service is presented every two years to a research mathematician who has made a distinguished contribution to the mathematics profession during the preceding five years. The purpose of the award is to encourage and recognize those individuals who contribute their time to public service activities in support of mathematics. The award carries a cash prize of $4,000.

The Award for Distinguished Public Service is made by the AMS Council, acting on the recommendation of a selection committee. For the 2002 award, the members of the selection committee were: Frederick W. Gehring (chair), Peter D. Lax, D. J. Lewis, Calvin C. Moore, and William Y. Velez.

Previous recipients of the award are: Kenneth M. Hoffman (1990), Harvey B. Keynes (1992), I. M. Singer (1993), D. J. Lewis (1995), Kenneth C. Millett (1998), and Paul J. Sally Jr. (2000).

The 2002 Award for Distinguished Public Service was presented to Margaret H. Wright. The text that follows presents the selection committee's citation, a brief biographical sketch, and the recipient's response upon receiving the award.

Citation

The 2002 American Mathematical Society Award for Distinguished Public Service is presented to Professor Margaret H. Wright, newly appointed chair of computer science at New York University after fourteen years with the Computing Sciences Research Center at Bell Laboratories.

Professor Wright was elected to the National Academy of Engineering in 1997 and was chosen Emmy Noether Lecturer by the Association for Women in Mathematics and Forsythe Lecturer by the Computer Science Department at Stanford University in 2000.

Among her notable contributions to the federal government are service as chair of the Advisory Committee for the Directorate of Mathematical and Physical Sciences at the National Science Foundation, as current chair of the Advanced Scientific Computing Advisory Committee for the Department of Energy, and recently as a member of committees of the National Research Council.

Professor Wright's contributions to the scientific community include service as president of SIAM in 1995–96, as cochair of the Scientific Advisory Committee of the MSRI at Berkeley, California, as the current editor-in-chief of the *SIAM Review* and as an associate editor of the *SIAM Journal on Scientific Computation*, the *SIAM Journal on Optimization*, and the IEEE/AIP journal *Computation in Science and Engineering*.

Finally, Professor Wright has been active for many years in encouraging women and minority students, for example, by means of programs that brought them together with leaders and researchers from industry to discuss opportunities outside academia.

Biographical Sketch

Margaret H. Wright is professor of computer science and mathematics and chair of the Computer Science Department in the Courant Institute, New York University. From 1988–2001 she was with the Computing Sciences Research Center at Bell Laboratories, Lucent Technologies (formerly AT&T Bell Laboratories), where she was named a Distinguished Member of Technical Staff in 1993 and a Bell Labs Fellow in 1999. She served as head of the Scientific Computing Research Department from 1997–2000. From 1976–1988 she was a research staff member in the Systems Optimization Laboratory, Department of Operations Research, Stanford University.

She received her B.S. in mathematics and her M.S. and Ph.D. in computer science from Stanford University. Her research interests include optimization, linear algebra, numerical and scientific computing, and scientific and engineering applications.

She was elected to the National Academy of Engineering in 1997 and to the American Academy of Arts and Sciences in 2001. During 1995–1996 she served as president of the Society for Industrial and Applied Mathematics (SIAM), and she is now a member of the Board of Trustees; she was previously a member of the SIAM Council and Vice-President at Large. She is chair of the Advisory Committee on Advanced Scientific Computing for the Department of Energy's Office of Science and is currently chair of the peer committee in computer science and engineering at the National Academy of Engineering. She is also a member of the National Science Foundation Blue Ribbon Panel on Cyberinfrastructure. From 1996–2001 she served on the Scientific Advisory Committee of the Mathematical Sciences Research Institute (MSRI) and was cochair during 1999–2001.

In 2000 she was chosen as the Noether Lecturer by the Association for Women in Mathematics and as the Forsythe Lecturer by the Computer Science Department, Stanford University; she also received the Award for Distinguished Service to the Profession from SIAM.

Wright is editor-in-chief of *SIAM Review*, as well as an associate editor of the *SIAM Journal on Scientific Computing*, the *SIAM Journal on Optimization*, *Mathematical Programming*, and *Computing in Science and Engineering*.

Response

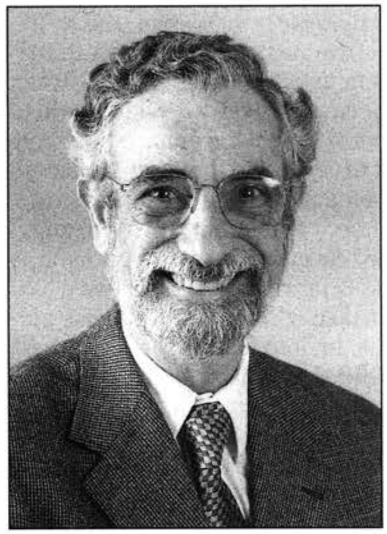
It is a great privilege for me to receive the 2002 Award for Distinguished Public Service, and I am deeply grateful to the selection committee and the American Mathematical Society.

Thinking about public service, I would like to echo some thoughts of Don Lewis, the 1995 recipient of this award and one of my heroes. In his response, Don stressed a point that deserves frequent repetition: Mathematical sciences research will thrive only if constant attention is paid to the multiple environments in which we work and live. Because mathematical scientists function in many different contexts, some broad, some narrow, it follows that public service takes many forms—improving education, encouraging students to pursue careers in mathematics, supporting young people in the mathematical sciences, arguing for funding, sustaining the vitality of scientific societies, and conveying the excitement and importance of scientific research.

Some of the activities mentioned in my citation involve service on committees, and I want to offer a plug for the joys of committee service. Despite the stereotype (undeniably true at times!) that the way not to get something done is to form a committee, being in the room when decisions are made—and they are often made by a committee—does matter. Since our community needs to be involved in discussions at all levels about science policy and education, we also need to be on committees at all levels. Happily, the best committees provide an opportunity to meet fascinating people and to appreciate and understand other points of view.

In everything that I have done, it has been a privilege to work with many outstanding, dedicated individuals. I thank them for providing irrefutable proof that public service can make a difference.

2002 Conant Prize



Elliott Lieb

Jakob Yngvason

The 2002 Levi L. Conant Prize was awarded at the 108th Annual Meeting of the AMS in San Diego in January 2002.

The Conant Prize is awarded annually to recognize the best expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years. Established in 2000, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic University. The prize carries a cash award of $1,000.

The Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2002 prize, the members of the selection committee were: Brian J. Parshall, Anthony V. Phillips (chair), and Joseph H. Silverman.

The previous recipient of the Conant Prize is Carl Pomerance (2001).

The 2002 Conant Prize was awarded to Elliott H. Lieb and Jakob Yngvason. The text that follows presents the committee's citation, a brief biographical sketch, and the awardees' response upon receiving the prize.

Citation

The Levi L. Conant Prize in 2002 is granted to Elliott H. Lieb and Jakob Yngvason for their appealing and thought-provoking article "A Guide to Entropy and the Second Law of Thermodynamics", *Notices of the AMS* **45**, no. 5 (1998), 571–581.

"This article is intended for readers who, like us, were told that the second law of thermodynamics is one of the major achievements of the nineteenth century...but who were unsatisfied with the 'derivations' of the entropy principle as found in textbooks and in popular writings." Thus do Lieb and Yngvason begin their article. They proceed to take the reader on a tour of the second law of thermodynamics as seen through an axiomatic-mathematical lens, without ever losing the friendly and conversational tone of the start.

Abstractly, there is only a set G and a preorder $<$ on G. Interpreted physically, the elements of G represent states of a system, and the preorder $<$ is required to satisfy certain natural axioms that characterize when one state can "lead to" another state (specifically, when the second is adiabatically accessible from the first, in a precise sense that the authors make clear). The second law of thermodynamics is then formulated in terms of an entropy function on $(G, <)$, that is, a real-valued function S on G that characterizes $<$ and has certain additivity and scaling properties. The authors detail the search for simple, elegant, and mathematically precise axiom systems that allow the construction of an entropy function and, thus, that capture the powerful predictive capabilities of thermodynamics. In doing so, they illuminate a fascinating trail between the "pure" world of mathematical abstraction and the "real" world of physics, chemistry, and engineering.

Biographical Sketch: Elliott H. Lieb

Elliott H. Lieb was born in Boston, Massachusetts, in 1932. He received his B.Sc. degree from MIT in 1953 and his Ph.D. degree in mathematical physics from the University of Birmingham (UK) in 1956 under the direction of S. F. Edwards. He holds honorary doctorates from Copenhagen University and the École Polytechnique Fédérale de Lausanne. After a Fulbright postdoc in Kyoto, he held positions in Illinois, Cornell, IBM, Sierra Leone, Yeshiva, Northeastern, and MIT. From 1975 he has been a professor in the mathematics and physics departments of Princeton University.

He has received a number of prizes for his work in mathematics and mathematical physics, including the Birkhoff Prize of the AMS and the Society for Industrial and Applied Mathematics, the Rolf Schock Prize in mathematics of the Swedish Academy, the Heinemann Prize in mathematical physics of the American Physical Society, the Boltzmann Prize in statistical mechanics of the International Union of Pure and Applied Physics, and the Max-Planck Medal of the German Physical Society. He is a member of the U.S., Austrian, and Danish Academies of Science, and the American Academy of Arts and Sciences. He served twice as president of the International Association of Mathematical Physics. Invited lectures include the AMS Gibbs Lecture and the Hedrick Lecture of the Mathematical Association of America.

Biographical Sketch: Jakob Yngvason

Jakob Yngvason was born in Reykjavik, Iceland, in 1945. He studied physics at the University of Göttingen, Germany, receiving his Ph.D. there in 1973 under the direction of H. J. Borchers. He was assistant professor at the University of Göttingen from 1973 to 1978, and from 1978 to 1985 he was senior research scientist at the Science Institute of the University of Iceland in Reykjavik. From 1985 to 1996 he was professor of theoretical physics at the University of Iceland. Since 1996 he has been professor of mathematical physics at the University of Vienna, Austria. He is also president of the Erwin Schrödinger Institute for Mathematical Physics in Vienna and vice president of the International Association of Mathematical Physics. He has held visiting positions at many research institutions, including the Universities of Göttingen and Leipzig, Rutgers University, the Institut des Hautes Études Scientifiques in Bures-sur-Yvette, DESY in Hamburg, NORDITA in Copenhagen, and the Max Planck Institute for Physics in Munich.

His main research interests are in quantum field theory and rigorous quantum many-body theory. He was plenary speaker at the 13th International Congress on Mathematical Physics in Paris, 1994. He received the Olafur Danielsson Prize for Mathematics in 1993.

Response from Lieb and Yngvason

This award was a pleasant surprise to us. We had worked for many years to try to formulate the second law of thermodynamics—the law of increasing entropy—in a mathematically precise, yet accessible, way and were not sure to what extent we had succeeded in communicating our enthusiasm for the subject to our colleagues. It is a very much appreciated honor to have our *Notices* article counted as "the best expository paper published in the *Notices* or the *Bulletin* in the preceding five years."

Our article is based on a long and detailed analysis (in *Physics Reports* **310** (1999), 1–96) of one of the most precise laws of physics. It was discovered in the first half of the nineteenth century and by the beginning of the twentieth century had attracted the attention of mathematicians, notably Carathéodory. To this day many schools of thought continue this interest.

The twentieth century, however, tended to see the law as an "easy" consequence and "incomplete expression" of statistical mechanics (Gibbs). This is an overstatement since the "derivation" from statistical mechanics is, after more than a century, still in a rudimentary phase, and because the law itself makes no reference to statistical mechanics. That is to say, the second law could well hold even if the world were made of vortices in a seamless fluid instead of being made of atoms. Statistical mechanics is a beautiful and important subject, but it is essential to understand the second law in its own right if we are ever going to derive it from statistical mechanics. Beginning in the fifties some people (e.g., P. Landsberg, H. Buchdahl, G. Falk and H. Jung, and, most notably, R. Giles) advocated an approach to the law based on an order relation among equilibrium states. We built on this structure. The earlier work introduced a basic new axiom which we call "comparison"; one of our main contributions was to convert this from an axiom to a theorem.

The subject is not, and may never be, finished. Also, the logical structure may have use in other fields, such as economics. We would be delighted if our article motivated other mathematicians to take up the thread.

Our sincere thanks go to Beth Ruskai for urging us to write this article; to the editor, Tony Knapp, for patience and much helpful criticism; and to Sandy Frost for essential help with editing.

Mathematics People

Prizes of the Mathematical Society of Japan

Several annual prizes are awarded to mathematicians by the Mathematical Society of Japan (MSJ) at the autumn meeting of the Society.

The Autumn Prize of the MSJ is awarded for outstanding contributions to mathematics, in the highest and broadest sense, in the past five years. The Autumn Prize in the year 2001 is awarded to ATSUSHI MORIWAKI of Kyoto University for his distinguished contributions to Arakelov geometry.

The Geometry Prize is awarded to a maximum of two geometers in recognition of major fundamental research in geometry. This prize was established with funds donated to the MSJ. The Geometry Prize in the year 2001 is awarded to REIKO MIYAOKA of Sophia University for her outstanding contributions to the theories of Dupin hypersurfaces and minimal surfaces.

The Takebe Prize for outstanding research was established to encourage young mathematicians. The Takebe Senior Prize is awarded to recipients chosen from nominations by members of the MSJ. The Takebe Junior Prize is awarded to self-nominated recipients. The Takebe Prize in the year 2001 is awarded to the recipients listed below.

Takebe Senior Prize: YUKARI ITO of Tokyo Metropolitan University for the study of crepant resolutions and the McKay correspondence; YOSHIHIDE KAKIZAWA of Hokkaido University for the study of the asymptotic theory of statistics in time series analysis; MASANORI HINO of Kyoto University for the study of stochastic analysis in infinite dimensional spaces; and HIDEO TAKAOKA of Hokkaido University for the study of nonlinear dispersive equations by the high and low frequency method.

Takebe Junior Prize: OSAMU IYAMA of Kyoto University for the study of representation theory of orders; AKIRA USHIJIMA of the Tokyo Institute of Technology for the study of standard division of hyperbolic manifolds; KEN-ICHI KAWARABAYASHI of Keio University for the study of circuits and chromatic numbers in graph theory; and KANETOMO SATO of Nagoya University for the study of cycle maps for varieties over arithmetic fields.

—MSJ announcement

Siemens Westinghouse Competition Winners Announced

Six high school mathematics students were among the winners in the Siemens Westinghouse Science and Technology National Competition. Individual prizes were awarded to the following students. ALEXANDRA OVETSKY (Central High School, Philadelphia, Pennsylvania) won second place overall in the competition for her project "Surreal Dimensions and their Applications". She was awarded a \$50,000 scholarship. JACOB LICHT (William H. Hall High School, West Hartford, Connecticut) won fourth place overall with his project "Rainbow Ramsey Theory: Rainbow Arithmetic Progressions and Anti-Ramsey Results". He received a \$30,000 scholarship. PETER BEHROOZI (Malcolm Price Laboratory School, Cedar Falls, Iowa) was awarded fifth place and a \$20,000 scholarship for his project "A Proof of the Collatz Conjecture for Rational Patterns". Fourth place in the team competition went to REBECCA WILLIAMS (North Lamar High School, Paris, Texas), CYNTHIA CHI (William P. Clements High School, Sugar Land, Texas), and CHARLES HALLFORD (Texas Academy of Mathematics and Science, Denton, Texas) for their joint project "The Generalization of the deBruijn Edge Sums". They will receive scholarships worth \$30,000.

The annual competition, administered by the College Board and funded by the Siemens Foundation, recognizes outstanding talent among high school students in science, mathematics, and technology.

—From a College Board announcement

Mathematics Opportunities

2002 NSF-CBMS Regional Conferences

With funding from the National Science Foundation (NSF), the Conference Board of the Mathematical Sciences (CBMS) will hold seven NSF-CBMS Regional Research Conferences during the summer of 2002. These conferences are intended to stimulate interest and activity in mathematical research.

Each five-day conference features a distinguished lecturer who will deliver ten lectures on a topic of importance in current research. Support for about thirty participants will be provided for each conference. Established researchers and interested newcomers, including postdoctoral fellows and graduate students, are invited to attend. The title of each conference follows, along with the name of the principal speaker, the date and location of the conference, and the names of the organizers, as well as contact information. More information about the conferences can also be found at the website `http://www.maa.org/cbms/nsf/2002_conf.htm` or by contacting the Conference Board of the Mathematical Sciences, 1529 18th Street, NW, Washington, DC 20036-1385, telephone: 202-293-1170, fax: 202-293-3412, e-mail: `kolbe@math.georgetown.edu` or `rosier@math.georgetown.edu`.

Nonhomogeneous Harmonic Analysis, Weights, and Applications to Problems in Complex Analysis and Operator Theory, Alexander Volberg, May 13–17, University of North Carolina. Organizers: Joseph Cima and Alec Matheson. Telephone: 919-962-1050, e-mail: `cima@email.unc.edu`, World Wide Web: `http://www.math.unc.edu/`.

Solving Polynomial Equations, Bernd Sturmfels, May 20–24, Texas A&M University. Organizers: Paulo Lima-Filho, J. Maurice Rojas, and Henry Schenck. Telephone: 979-845-5298, e-mail: `cbms@math.tamu.edu`, World Wide Web: `http://www.math.tamu.edu/conferences/cbms/`.

Structure and Decomposition of Graphs, Robin Thomas, May 20–24, East Tennessee State University. Organizers: Debra Knisley and Anant Godbole. Telephone: 423-439-6979, e-mail: `knisleyd@etsu.edu`, World Wide Web: `http://www.etsu.edu/math/cbms/`.

Mathematical Methods in Nonlinear Wave Propagation, J. Kenneth Shaw, May 20–24, North Carolina A&T State University. Organizers: Dominic P. Clemence and Guoqing Tang. Telephone: 336-334-7822, e-mail: `clemence@ncat.edu`, World Wide Web: `http://www.ncat.edu/~math/`.

Geometric Graph Theory, Janos Pach, May 28–June 1, University of North Texas. Organizer: Farhad Shahrokhi. Telephone: 940-565-2805, e-mail: `farhad@cs.unt.edu`, World Wide Web: `http://www.cs.unt.edu/~farhad/CBMSNSF/cbms.html`.

Numerical Methods in Forward and Inverse Electromagnetic Scattering, Peter Monk, June 3–7, Colorado School of Mines. Organizers: Paul Martin and Graeme Fairweather. Telephone: 303-273-3895, e-mail: `pamartin@mines.edu`, World Wide Web: `http://www.mines.edu/academic/macs/nsfcbms/`.

The Dynamical Behavior of the Newtonian N-Body Problem, Donald Saari, June 9–15, Eastern Illinois University. Organizers: Patrick Coulton, Peter Andrews, Gregory Galperin, and Hillel Gauchman. Telephone: 217-581-6276, e-mail: `cfprc@eiu.edu`, World Wide Web: `http://www.eiu.edu/~math/news/cbmsconf.html`.

—From a CBMS announcement

Request for Proposals for 2003 NSF-CBMS Regional Conferences

The National Science Foundation (NSF), with the sponsorship of the Conference Board of the Mathematical Sciences (CBMS), intends to support up to five NSF-CBMS Regional Research Conferences in 2003.

Each five-day conference features a distinguished lecturer who delivers ten lectures on a topic of important current research in one sharply focused area of the mathematical sciences. The lecturer subsequently prepares an expository monograph based on these lectures, which, depending on the topic, is published either by the AMS, by the Society for Industrial and Applied Mathematics (SIAM), or jointly by the American Statistical Association and the Institute of Mathematical Statistics. Support is provided for about thirty participants at each conference, including postdoctoral researchers and graduate students.

Colleges and universities with at least some research competence in the field of the proposal are eligible to apply. Because a major goal of these conferences is to attract new researchers into the field of the conference and to stimulate new research activity, institutions that are interested in upgrading or improving their research efforts are especially encouraged to apply.

Proposals should reach the NSF by **April 8, 2002**. For further information on the NSF-CBMS Regional Conferences and guidelines for preparing proposals, contact: Conference Board of the Mathematical Sciences, 1529 18th Street, NW, Washington, DC 20036-1385; telephone: 202-293-1170; fax: 202-293-3412; World Wide Web: `http://www.maa.org/cbms/nsf/2003_call.htm`; e-mail: `kolbe@math.georgetown.edu` or `rosier@math.georgetown.edu`.

—From a CBMS announcement

Career Awards for Research Addressing Biological Questions

In recognition of the vital role mathematical and physical scientists will play in furthering biomedical research, the Burroughs Wellcome Fund has developed the Career Awards at the Scientific Interface program. These awards are intended to foster the early career development of researchers with backgrounds in the physical, mathematical, and computational sciences whose work addresses biological questions and who are dedicated to pursuing a career in academic research.

Applicants are expected to draw from their training in a scientific field other than biology to propose innovative approaches to answer important questions in the biological sciences. Examples of approaches include, but are not limited to, physical measurement of biological phenomena, computer simulation of complex processes in physiological systems, mathematical modeling of self-organizing behavior, building probabilistic tools for medical diagnosis, developing novel imaging tools or biosensors, applying nanotechnology to manipulate cellular systems, predicting cellular responses to topological clues and mechanical forces, and developing a new conceptual understanding of the complexity of living organisms. Proposals that include experimental validation of theoretical models are particularly encouraged.

The awards provide $500,000 over five years to support up to two years of advanced postdoctoral training and the first three years of a faculty appointment. During both the postdoctoral and the faculty periods, awards must be taken at degree-granting institutions in the United States or Canada.

Candidates must hold a Ph.D. degree in the fields of mathematics, physics, chemistry (physical, theoretical, or computational), computer science, statistics, or engineering. Exceptions will be made only if the applicant can demonstrate significant expertise in one of these areas, evidenced by publications or advanced course work. Candidates must have completed at least six months but not more than forty-eight months of postdoctoral training at the time of application. Candidates must not hold or have accepted a faculty appointment as a tenure-track assistant professor at the time of application.

Candidates who are not citizens of the United States or Canada must provide documentation of their visa status at the time of application. Permanent residents must provide a copy of their Alien Registration card (green card) for the United States or their Landed Immigrant Status form for Canada.

The application deadline is **May 1, 2002**. For more information and complete application materials, visit the website `http://www.bwfund.org/interfaces_in_science_career_awards.htm`.

—Burroughs Wellcome Fund announcement

Project NExT

Project NExT (New Experiences in Teaching) is a program for new or recent Ph.D.'s in the mathematical sciences who are interested in improving the teaching and learning of undergraduate mathematics. It addresses the full range of faculty responsibilities in teaching, research, and service, and it provides professional support for new faculty as they undertake these activities. Faculty for whom the 2002–03 academic year will be the first or second year of full-time employment with significant teaching responsibilities at the college/university level are invited to apply to become Project NExT Fellows.

The first event for the 2002–03 Project NExT Fellows will be a workshop, July 29–31, 2002, just prior to the summer MAA meeting (the Mathfest) in Burlington, Vermont, August 1–3, 2002. At this workshop and at Project NExT sessions during the Mathfest, Fellows will explore and discuss issues that are of special relevance to beginning faculty. The Fellows will also have an opportunity to meet with Fellows who began the program in previous years.

Following the workshop, Project NExT Fellows will attend the Mathfest, participating in all the opportunities of that meeting, and will choose among special short courses organized by Project NExT. During the following year, Project NExT Fellows will participate in an electronic network that links Fellows with one another and with distinguished teachers of mathematics. Fellows will also attend special events at the Joint Mathematics Meetings in Baltimore, Maryland, January 15–18, 2003; the MAA Mathfest in Boulder, Colorado, July 30–August 2, 2003; and a workshop preceding the Mathfest.

There is no fee for participation in Project NExT itself, and Fellows will be provided with room and board at the Project NExT workshop in Burlington. Fellows also do not have to pay for the special short courses at the summer Mathfest that are organized by Project NExT. Institutions employing the Project NExT Fellows are expected to provide all other expenses associated with the meetings, and assurances of institutional support are of critical importance in the application process.

Application forms are available on the Project NExT Web page (`http://archives.math.utk.edu/projnext/`). The application deadline is **April 12, 2002**. For more information, contact one of the following:

T. Christine Stevens, director of Project NExT, Department of Mathematics and Mathematical Computer Science, Ritter Hall 104, Saint Louis University, 220 N. Grand Blvd., St. Louis, MO 63103; telephone: 314-977-2436; e-mail: `stevensc@slu.edu`.

Joseph Gallian, co-director, Department of Mathematics and Statistics, University of Minnesota, Duluth, MN 55812; telephone: 218-726-7576; e-mail: `jgallian@d.umn.edu`.

Aparna Higgins, co-director, Department of Mathematics, University of Dayton, Dayton, OH 45469; telephone: 937-229-2103; e-mail: `higgins@saber.udayton.edu`.

Project NExT is a program of the Mathematical Association of America. Major funding is provided by the ExxonMobil Foundation, with additional funding from the American Mathematical Society, the Dolciani-Halloran Foundation, and the Educational Advancement Foundation.

—From a Project NExT announcement

AMS Scholarships for "Math in Moscow"

The Independent University of Moscow has created a program called "Math in Moscow," which offers foreign students (undergraduate or graduate students specializing in mathematics and/or computer science) the chance to spend a semester in Moscow studying mathematics.

Math in Moscow provides students with a fifteen-week program similar to the Research Experiences for Undergraduates programs that are held each summer across the United States. Math in Moscow draws on the Russian tradition of teaching mathematics, which emphasizes creative approaches to problem solving rather than memorizing theorems. The focus is on developing in-depth understanding of carefully selected material rather than broad surveys of large quantities of material. Discovering mathematics under the guidance of an experienced teacher is the central principle of Math in Moscow. Most of the program's teachers are internationally recognized research mathematicians, and all of them have considerable teaching experience in English, typically in the United States or Canada. (All instruction is in English.)

Each semester, five $5,000 scholarships will be granted to U.S. students to attend the Math in Moscow program. Funding is provided by the National Science Foundation, and the scholarships are administered by the AMS. To be eligible for the scholarships, students must submit applications to *both* the Math in Moscow program and the AMS. An applicant should be an undergraduate mathematics or computer science major enrolled at a U.S. institution. **May 15** is the deadline for applications to enroll in Math in Moscow for the following fall semester; **October 15** is the deadline for the spring semester. The same deadlines apply for the AMS scholarships.

Information and application forms for Math in Moscow are available on the Web at `http://www.mccme.ru/mathinmoscow/`, or by writing to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax +7095-291-65-01; e-mail: `mim@mccme.ru`. Information and application forms for the AMS scholarship are available on the Web at `http://www.ams.org/careers-edu/mimoscow.html`, or by writing to: Math in Moscow Program, Professional Services Department, American Mathematical Society, 201 Charles Street, Providence RI 02904; e-mail: `prof-serv@ams.org`.

—Allyn Jackson

AP Calculus Readers Sought

The Educational Testing Service and the College Board invite interested college faculty to apply to be readers for the Advanced Placement Calculus Exam. The AP Calculus exams (AB and BC) were taken by approximately 185,000 high school students last year. The six free-response problems on the exam are graded during seven days in June by more than 600 high school and college mathematics teachers at Colorado State University in Ft. Collins, Colorado. This is an excellent opportunity for teachers, especially those just starting their professional careers, to enhance their knowledge of the AP Calculus Program and of teaching and to meet with other faculty from around the country. To learn more about this opportunity or to apply for a position as a reader, see the website `http://apcentral.collegeboard.com/` and follow the links to "Colleges & Universities" and then "Faculty Involvement", or send e-mail to `apreader@ets.org`. Questions about the reading may be sent to Larry Riddle, Chief Faculty Consultant for the AP Calculus Program, at `LRiddle@agnesscott.edu`.

—Larry Riddle, Agnes Scott College

For Your Information

EMS Issues Statement on Money-Making Schemes

In December 2001, the European Mathematical Society (EMS) issued a statement warning mathematicians about schemes designed to take advantage of scholarly communication in order to make money. The advent of electronic communications seems to have caused an increase in such schemes.

In some cases, individuals have organized conferences with very high registration fees, and the main purpose of the conferences did not appear to be the advancement of scholarship and research. There are also instances of journals charging authors very high page charges that appeared to be difficult to justify. Further, individuals' reputations sometimes appeared to be exploited when they were asked to sit on boards of such journals.

The EMS statement does not condemn all financial gain from scholarly communications. Academic and scholarly organizations often make modest amounts of money on conferences and publications and use that money to support other activities. Thus the EMS statement mainly serves as a warning to mathematicians to exercise caution. The full text of the statement follows.

"**A common sense tip**

"From time to time active mathematicians receive invitations to submit papers to research journals or conferences, to have their name included as a member of a journal editorial board, to speak at a conference, to participate in a conference, to have their name included as a member of the organising committee of a conference, and so on. Sometimes these invitations include a request for payment of some sort (e.g. conference charges, page charges, or whatever).

"The vast majority of such invitations are of course entirely genuine and welcome to the recipient! However a small minority of such invitations represent money-making schemes of a type that might not be immediately obvious to the recipient, and that might not be at all welcome to them if they understood what was going on. The internet is often used for such invitations, just as it is for a number of other doubtful financial schemes.

"If you receive an invitation to be involved in a journal or conference whose organiser's reputability you do not already know, it is wise therefore to check out the integrity of what is proposed before sending any money or agreeing to let your name be used in what might be a purely money making scheme.

"David A. Brannan, EMS Secretary"

—*Allyn Jackson*

Correction

In the article "Doctoral Degrees Conferred" in the February 2002 issue of the *Notices*, page 241, Cecilia Fosser and Anupama Rao were listed as receiving degrees from Arizona State University. Their correct affiliation is the University of Arizona.

Correction

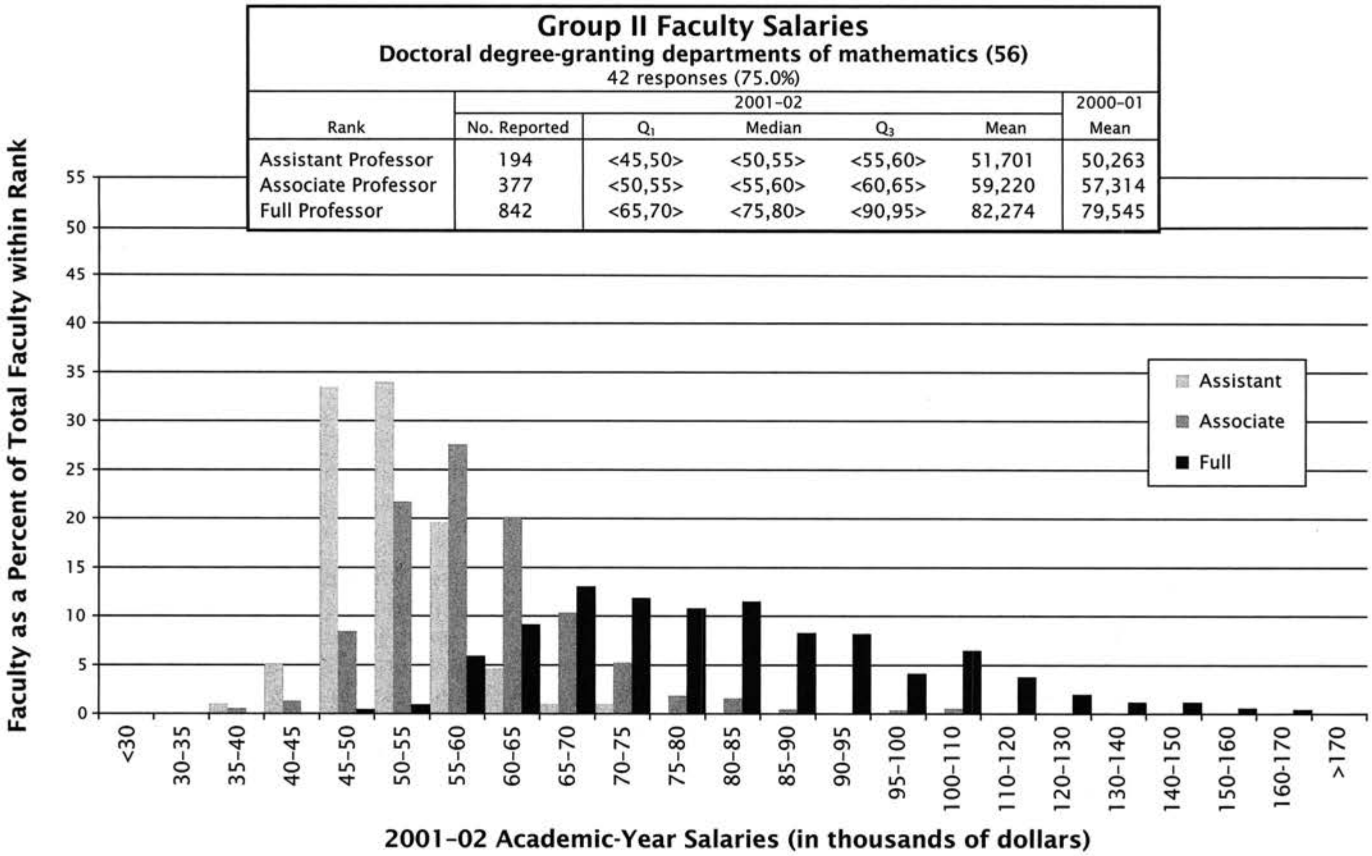
Reprinted on the following page are tables depicting Group II Faculty Salaries (corrected) and Group B Faculty Salaries. When the tables appeared in the "2001 Annual Survey of the Mathematical Sciences" article in the February 2002 issue, the data from Group B was erroneously printed in the table of Group II. Data for both groups is shown correctly here.

Group II Faculty Salaries

Doctoral degree-granting departments of mathematics (56)

42 responses (75.0%)

Rank	2001-02 No. Reported	2001-02 Q1	2001-02 Median	2001-02 Q3	2001-02 Mean	2000-01 Mean
Assistant Professor	194	<45,50>	<50,55>	<55,60>	51,701	50,263
Associate Professor	377	<50,55>	<55,60>	<60,65>	59,220	57,314
Full Professor	842	<65,70>	<75,80>	<90,95>	82,274	79,545

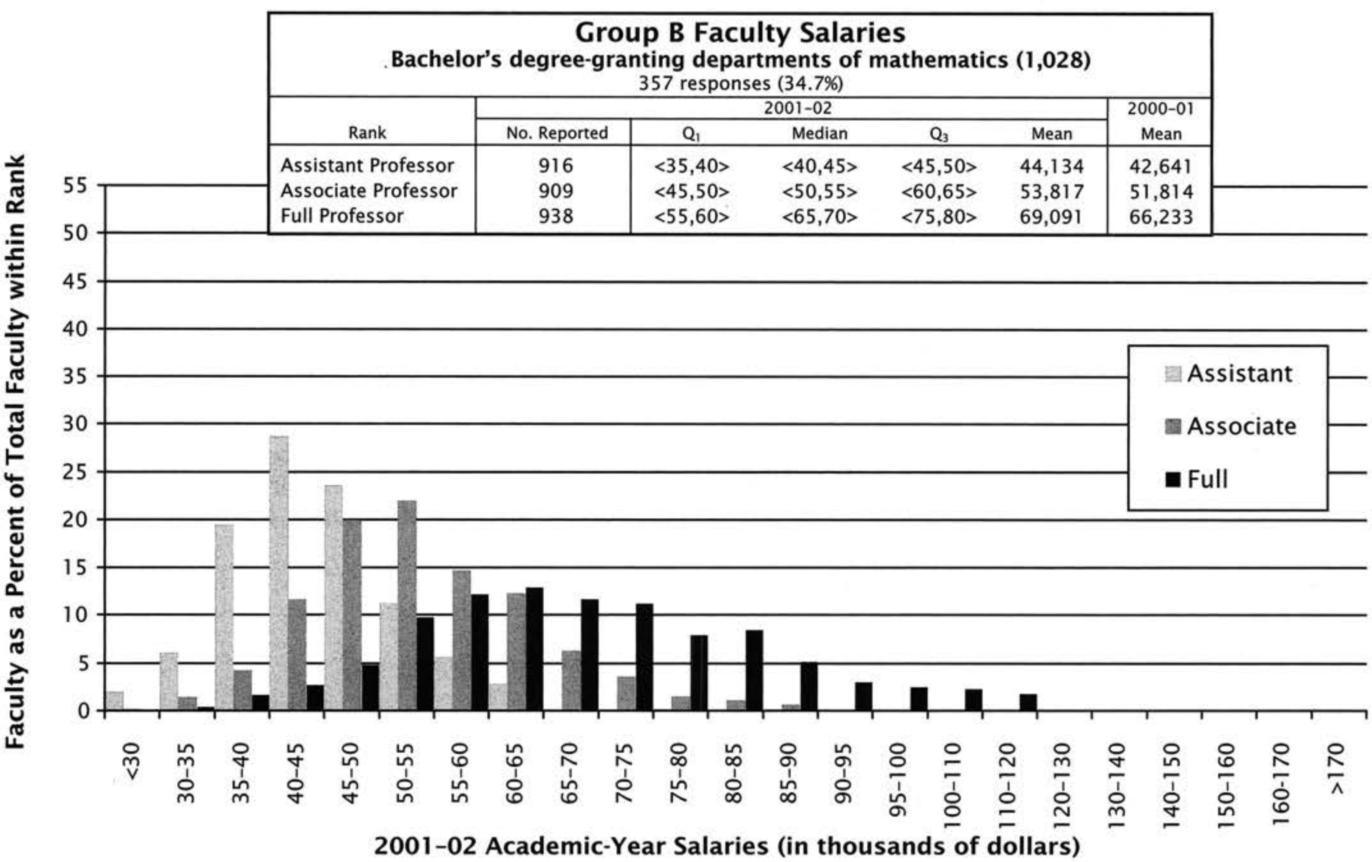


Group B Faculty Salaries

Bachelor's degree-granting departments of mathematics (1,028)

357 responses (34.7%)

Rank	2001-02 No. Reported	2001-02 Q1	2001-02 Median	2001-02 Q3	2001-02 Mean	2000-01 Mean
Assistant Professor	916	<35,40>	<40,45>	<45,50>	44,134	42,641
Associate Professor	909	<45,50>	<50,55>	<60,65>	53,817	51,814
Full Professor	938	<55,60>	<65,70>	<75,80>	69,091	66,233



Inside the AMS

AMS Participates in Planning for 2004 NAEP

Every four years, the National Assessment of Educational Progress (NAEP) gathers information about student achievement in several academic areas, including mathematics. The results of the assessment are published in *The Nation's Report Card*, which is used by citizens, teachers, curriculum specialists, school systems, and policymakers as a barometer of student achievement. Begun in 1973, NAEP has grown into a major enterprise. It has both a national and a state component, each of which assesses students in the fourth, eighth, and twelfth grades. In the mathematics part of the 2000 NAEP, 47,000 students were tested for the national component, and another 212,000 were tested by individual states.

The AMS Committee on Education (COE) has had substantial involvement in the development of the mathematics framework for the 2004 NAEP. The Council of Chief State School Officers (CCSSO) is overseeing the NAEP Mathematics Consensus Project, which is designed to bring views of different groups to bear on the preparation of the framework. CCSSO appointed four mathematicians to committees working on this project. Herb Clemens of the University of Utah and Carl Cowen of Purdue University are on the planning committee, and Wilfried Schmid of Harvard University and H. H. Wu of the University of California, Berkeley, are on the steering committee. Clemens and Cowen are also COE members (though they do not officially represent the COE in the CCSSO project).

In August 2001 a draft of the 2004 NAEP mathematics framework was issued for public comment. The framework identifies five main content areas to be assessed at the different grade levels: Number, Measurement, Geometry, Data Analysis and Probability, and Algebra. Clemens and Samuel M. Rankin III, director of the AMS Washington Office, brought the draft before the COE and asked the committee members to comment. COE Chair Roger Howe assembled an ad hoc subcommittee of COE to study and discuss the draft framework via e-mail.

As part of their commentary, the subcommittee provided a proposal for revising the introduction to the section of the framework describing the Number content area. The COE commentary was presented by Clemens to the National Assessment Governing Board, which oversees NAEP, during a public hearing devoted to testimony concerning the draft framework. The sample revision for the introduction to Number was received enthusiastically by the board, and subsequently a decision was made to solicit introductions to each subject area from subject matter experts.

The COE was commissioned to write the introductions for Algebra and Geometry, as well as the one for Number. Introductions for Measurement and for Data Analysis and Probability were solicited from other outside experts. The COE received a commission for its writing work and donated the commission to the AMS Epsilon Fund. The COE will continue to provide advice as the draft framework evolves.

The COE's work on the 2004 NAEP mathematics framework is a substantive contribution to an assessment that has a large impact on perceptions about student achievement in mathematics and, in turn, on efforts to improve mathematics education. Appropriate input from mathematicians can greatly aid these efforts. "I was delighted that the [National Assessment Governing Board] session for public comment enabled CoE to contribute positively to the NAEP framework," Howe said. "I believe that, with appropriate mechanisms, research mathematicians could do a lot to benefit mathematics education."

—*Allyn Jackson*

Babbitt Retires as AMS Publisher

At the end of June 2002, Donald G. Babbitt will retire as AMS publisher. His eight years in that position have been a time of fundamental changes in scholarly publishing and in the AMS publishing enterprise.

Babbitt first joined the staff of the AMS in July 1992 as executive editor of *Mathematical Reviews* (MR) in Ann Arbor, Michigan. He took that position while on a leave of absence from the University of California, Los Angeles. At that time, electronic versions of MR were available on compact disks and tape, but the paper version was still dominant. Babbitt recalls the fall 1993 meeting of the AMS Executive Committee and Board of Trustees (ECBT), in which two board members, John Franks of Northwestern University and John Polking of Rice University, insisted that the AMS had to leave the disks and tapes behind and move MR to the Web. As a result, one of Babbitt's last acts as executive editor was to call upon the staff to create a Web prototype. Over the next two years the work of many people throughout the AMS developed that prototype into MathSciNet.

In 1994, Babbitt moved to the Providence staff of the AMS, taking the newly created position of publisher. Some of his duties had previously been handled by Samuel M. Rankin III, who served as director of publications from 1991 to 1993 (Rankin now heads the AMS Washington Office). In addition to those duties, Babbitt took over the production parts of the AMS publication operation, as well as electronic product development and the warehouse and distribution facilities.

One of Babbitt's main achievements was the creation in 1997 of a consortium pricing model for MR products based on the "data access fee" subscription model, which had just been introduced for subscribers to MR products. Consortium pricing arrangements are designed to provide groups of institutions greater access to MR products at lower prices. With this model of consortium pricing, "There are now hundreds of institutions around the world that have access to MathSciNet that wouldn't have been able to otherwise," Babbitt says. "The AMS was one of the pioneers in this kind of pricing."

Babbitt's tenure as publisher was a time of upheaval and uncertainty for scholarly publishers, as new modes of scholarly communication, such as free electronic journals and preprint servers, called into question the traditional subscription model for journals. Many professional societies including the AMS rely on journals as a main source of revenue. One of the strategies the AMS pursued in order to prepare for a possible decline in journal revenue was to build up the book program. Since Babbitt became AMS publisher the number of books the Society produces annually has grown by about 35 percent, and the unit sales have increased significantly as well. Babbitt says, however, that the book program must develop further before its income could replace a significant drop in income in the journal program.

Babbitt officially stepped down as publisher in January 2002 and will remain on the staff as "publisher emeritus" for six months, to help with planning for the future of the AMS book program. AMS executive director John H. Ewing is serving as interim publisher. After he leaves the AMS, Babbitt will move to southern California, where he lived for thirty years while on the UCLA faculty. Going to the AMS "was a great opportunity," he says. "It has been very interesting and very rewarding."

—*Allyn Jackson*

E. H. Moore Prize Established

At its meeting in January 2002, the AMS Council approved the establishment of the E. H. Moore Research Article Prize. The prize will recognize an outstanding research paper that has appeared in one of the primary AMS journals (*Journal of the AMS, Proceedings of the AMS, Transactions of the AMS, AMS Memoirs, Mathematics of Computation, Electronic Journal of Conformal Geometry and Dynamics*, and *Electronic Journal of Representation Theory*).

The prize will be given every three years. The amount of the prize will be the same as for other AMS prizes honoring research achievements; this amount is at present $5,000. To be eligible, an article must have appeared in one of these journals during a time window of six calendar years; the window ends one year before the meeting at which the prize is to be given.

The prize honors the memory of Eliakim Hastings Moore (1862–1932), a University of Chicago mathematician who had considerable influence on the development of mathematics research in the United States. He founded and nurtured the *Transactions of the AMS* and served as vice-president (1898–1900) and as president (1901–02) of the Society.

The first awarding of the E. H. Moore Prize will take place at the Joint Mathematics Meetings in Baltimore in January 2003.

—*Allyn Jackson*

Journal Backfiles to be Made Freely Available

At its meeting in September 2001, the AMS Committee on Publications considered a proposal to make available for free electronic backfiles of AMS journal articles that were published more than five years ago. In its deliberations, the committee had to weigh two factors: the benefit to the mathematical community and the possibility that making the files available for free might erode the subscription base of AMS journals. The committee decided that a good balance could be achieved by making available only material that had been out for more than five years.

The Executive Committee and Board of Trustees, as well as the AMS Council, have approved this plan. Current plans call for the backfiles to be made available in the summer of 2002

—*Allyn Jackson*

Death of AMS Member

Edmund J. Pinney, professor emeritus, University of California, Berkeley, died on December 19, 2000. Born in August 1917, he was a member of the Society for 58 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.tamu.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 979-845-6028 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

March 31, 2002: Nominations for 2002 Prize for Achievement in Information-Based Complexity. Send nominations to Joseph Traub, `traub@santafe.edu`.

April 8, 2002: Proposals for 2003 NSF-CBMS Regional Conferences. See "Mathematics Opportunities" in this issue.

April 12, 2002: Applications for Project NExT. See "Mathematics Opportunities" in this issue.

April 15, 2002: Applications for National Research Council Research Associateship Program. See `http://www4.nationalacademies.org/pga/rap.nsf/` or contact the National Research Council, Associateship Programs (TJ 2114), 2101 Constitution Avenue, NW, Washington, DC 20418; telephone 202-334-2760; fax 202-334-2759; e-mail: `rap@nas.edu`.

April 15, 2002: Proposals for COBASE collaborative grants. For application forms and instructions, visit the website `http://www.`

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices.*

nationalacademies.org/oia/. For more information, telephone 202-334-2644, send a fax to 202-334-2614, or e-mail: ocee@nas.edu.

April 30, 2002: Nominations for the Maria Mitchell Women in Science Award. See http://www.mmo.org/, or contact the Maria Mitchell Women in Science Award Committee at the Maria Mitchell Association, 2 Vestal Street, Nantucket, MA 02554; telephone 508-228-9198.

May 1, 2002: Applications for Career Awards for Research Addressing Biological Questions. See "Mathematics Opportunities" in this issue.

May 1, 2002: Nominations for Lobachevskii Medal. See http://www.ksu.ru/lobmed/.

May 1, October 1, 2002: Applications for NSF/AWM Travel Grants for Women. See http://www.awm-math.org/travelgrants.html; telephone 301-405-7892; e-mail: awm@math.umd.edu.

May 15, 2002: Applications for fall semester of Math in Moscow and for AMS scholarships. See http://www.mccme.ru/mathinmoscow/ or contact Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax +7095-291-65-01; e-mail: mim@mccme.ru. For information about and application forms for the AMS scholarships, see http://www.ams.org/careers-edu/mimoscow.html or contact Math in Moscow Program, Professional Services Department, American Mathematical Society, 201 Charles Street, Providence RI 02904; e-mail: prof-serv@ams.org.

July 15, 2002: Applications for the AAAS Women's International Science Collaboration Program (WISC). See http://www.aaas.org/international/wiscnew.shtml, or contact WISC Travel Grant, American Association for the Advancement of Science, Directorate for International Programs, 1200 New York Avenue, NW, Washington, D.C., 20005.

August 15, 2002: Applications for National Research Council Research Associateship Program. See http://www4.nationalacademies.org/pga/rap.nsf/ or contact the National Research Council, Associateship Programs (TJ 2114), 2101 Constitution Avenue, NW, Washington, DC 20418; telephone 202-334-2760; fax 202-334-2759; e-mail: rap@nas.edu.

October 15, 2002: Applications for spring semester of Math in Moscow and for AMS scholarships. See http://www.mccme.ru/mathinmoscow/ or contact Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax +7095-291-65-01; e-mail: mim@mccme.ru. For information about and application forms for the AMS scholarships, see http://www.ams.org/careers-edu/mimoscow.html or contact Math in Moscow Program, Professional Services Department, American Mathematical Society, 201 Charles Street, Providence RI 02904; e-mail: prof-serv@ams.org.

Board on Mathematical Sciences and Their Applications, National Research Council

Peter J. Bickel, (chair), University of California, Berkeley
Dimitris Bertsimas, MIT Sloan School of Management
George Casella, University of Florida
Jennifer Chayes, Microsoft Research
David Eisenbud, Mathematical Sciences Research Institute
Ciprian Foias, Texas A&M University
Raymond L. Johnson, University of Maryland
Iain M. Johnstone, Stanford University
Arjen K. Lenstra, Citibank, NA
Robert Lipshutz, Affymetrix, Inc.
George C. Papanicolaou, Stanford University
Alan S. Perelson, Los Alamos National Laboratory
Linda R. Petzold, University of California, Santa Barbara
Douglas Ravenel, University of Rochester
Stephen M. Robinson, University of Wisconsin, Madison
S. R. Srinivasa Varadhan, New York University
Sallie Keller-McNulty (ex officio), Los Alamos National Laboratory
Scott Weidman, (Director, BMSA), National Academy of Sciences

The postal address for BMSA is: Board on Mathematical Sciences and Their Applications, National Research Council, Room HA476U, 2101 Constitution Avenue, NW, Washington, DC 20418; World Wide Web, http://www4.nationalacademies.org/cpsma/bms.nsf/web/bmsa_members?OpenDocument/.

U.S. National Committee for Mathematics

The U.S. National Committee for Mathematics (USNCM) is the adhering organization representing the United States in the International Mathematical Union (IMU). The USNCM is convened under the auspices of the National Academy of Sciences. The members of the USNCM are listed below.

Donald G. Saari, (chair), University of California, Irvine
Richard Askey, University of Wisconsin at Madison
Augustin Banyaga, Pennsylvania State University
Lynne Billard, University of Georgia
David Eisenbud, Mathematical Sciences Research Institute
Barbara L. Keyfitz, University of Houston
Joseph Kohn, Princeton University
Donald J. Lewis, University of Michigan
Gilbert Strang, Massachusetts Institute of Technology
Ruth J. Williams, University of California, San Diego

Ex-Officio Members

Secretary, IMU
Phillip A. Griffiths, Institute for Advanced Study

Past President, IMU
David Mumford, Brown University

Chair, Board on Mathematical Sciences
Peter Bickel, University of California, Berkeley

President, International Commission on Mathematical Instruction
Hyman Bass, University of Michigan

Secretary, Commission on Development and Exchanges

Herbert Clemens, University of Utah

Foreign Secretary, NAS
F. Sherwood Rowland (NAS, IOM), National Academy of Sciences

Program Director, USNC/Math
Tamae Maeda Wong, National Research Council

Book List

The Book List highlights books that have mathematical themes and hold appeal for a wide audience, including mathematicians, students, and a significant portion of the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to the managing editor,* e-mail: notices@ams.org.

Battle of Wits: The Complete Story of Codebreaking in World War II, by Stephen Budiansky. Free Press, October 2000. ISBN 0-684-85932-7.

The Bit and the Pendulum: How the New Physics of Information Is Revolutionizing Science, by Tom Siegfried. John Wiley & Sons, February 2000. ISBN 0-47132-174-5.

The Book of Nothing: Vacuums, Voids, and the Latest Ideas about the Origins of the Universe, by John D. Barrow. Pantheon Books, April 2001. ISBN 0-375-42099-1.

Calculated Bets: Computers, Gambling, and Mathematical Modeling to Win, by Steven S. Skiena. Cambridge University Press, September 2001. ISBN 0-521-00962-6.

Chaotic Elections! A Mathematician Looks at Voting, by Donald G. Saari. AMS, April 2001. ISBN 0-8218-2847-9.

The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems, by Martin Gardner. W.W. Norton & Company, August 2001. ISBN 0-393-02023-1.

Computers Ltd.: What They Really Can't Do, by David Harel. Oxford University Press, November 2000. ISBN 0-198-50555-8.

A Concise History of Mathematics, by Dirk J. Struik. Dover Publications, 1987. ISBN 0-486-60255-9. (Reviewed June/July 2001.)

Conned Again, Watson! Cautionary Tales of Logic, Math, and Probability, by Colin Bruce. Perseus Publishing, January 2001. ISBN 0-7382-0345-9.

Conquering Statistics: Numbers without the Crunch, by Jefferson Hane Weaver. Perseus Publishing, paperback edition, August 2001. ISBN 0-732-820495-1.

Conversations with a Mathematician: Math, Art, Science, and the Limits of Reason, by Gregory J. Chaitin. Springer, November 2001. ISBN 1-85233-549-1.

Creators of Mathematics: The Irish Connection, by Ken Houston. University College Dublin Press, September 2000. ISBN 1-900-62149-5.

Damned Lies and Statistics: Untangling Numbers from the Media, Politicians, and Activists, by Joel Best. University of California Press, May 2001. ISBN 0-520-21978-3.

The Difference Engine: Charles Babbage and the Quest to Build the First Computer, by Doron Swade. Viking Press, September 2001. ISBN 0-670-91020-1.

The Dream Machine: J. C. R. Licklider and the Revolution That Made Computing Personal, by M. Mitchell Waldrop. Viking Press, 2001. ISBN 0-670-89976-3.

The Essential John Nash, Harold Kuhn and Sylvia Nasar, editors. Princeton University Press, December 2001. ISBN 0-691-09527-2.

Euclid's Window: The Story of Geometry from Parallel Lines to Hyperspace, by Leonard Mlodinow. Free Press, April 2001. ISBN 0-684-86523-8.

Exploring Randomness, by Gregory J. Chaitin. Springer, December 2000. ISBN 1-852-33-417-7. (Reviewed October 2001.)

Finite vs. Infinite, Contributions to an Eternal Dilemma, Cristian S. Calude and Gheorghe Paun, editors. Springer, March 2000. ISBN 1-852-33251-4.

Flatterland: Like Flatland, Only More So, by Ian Stewart. Perseus Publishing, May 2001. ISBN 0-7382-0442-0. (Reviewed in this issue.)

Fooled by Randomness: The Hidden Role of Chance in the Markets and Life, by Nassim Nicholas Taleb. Texere, October 2001. ISBN 1-587-99071-7.

The Fractal Murders, by Mark Cohen. E-book published by Southern Cross Review, 2001. World Wide Web: www.southerncrossreview.org/.

Fragments of Infinity: A Kaleidoscope of Math and Art, by Ivars Peterson. John Wiley & Sons, October 2001. ISBN 0-471-16558-1.

Geometry: Our Cultural History, by Audun Holme. Springer, to appear March 2002. ISBN 3-540-41949-7.

Gödel: A Life of Logic, by John L. Casti and Werner DePauli. Perseus Publishing, August 2000. ISBN 0-7382-0274-6. (Reviewed September 2001.)

The Hilbert Challenge, by Jeremy J. Gray. Oxford University Press, December 2000. ISBN 0-198-50651-1.

The Hole in the Universe: How Scientists Peered over the Edge of Emptiness and Found Everything, by K. C. Cole. Harcourt Brace, January 2001. ISBN 0-151-00398-X.

How the Other Half Thinks: Adventures in Mathematical Reasoning, by Sherman Stein. McGraw-Hill, July 2001. ISBN 0-071-37339-X.

How to Solve It: Modern Heuristics, by Zbigniew Michalewicz and David B. Fogel. Springer, December 1999. ISBN 3-540-66061-5.

In Code: A Mathematical Journey, by Sarah Flannery and David Flannery. Workman Publishing, May 2001. ISBN 0-761-12384-9.

Lebesgue's Theory of Integration: Its Origins and Development, by Thomas Hawkins. AMS, September 2001. ISBN 0-8218-2963-7.

Logical Dilemmas: The Life and Work of Kurt Gödel, by John Dawson. A K Peters, December 1997. ISBN 1-56881-025-3. (Reviewed September 2001.)

Mathematical Mountaintops: The Five Most Famous Problems of All Time, by John Casti. Oxford University Press, October 2001. ISBN 0-195-14171-7.

The Mathematician Sophus Lie: It Was the Audacity of My Thinking, by Arild Stubhaug. Springer, 2002. ISBN 3-540-42137-8.

Mathematics and the Roots of Postmodern Thought, by Vladimir Tasic. Oxford University Press, 2001. ISBN 0-195-13967-4.

Mathematics Galore: Masterclasses, Workshops, and Team Projects in Mathematics and Its Applications, by C. J. Budd and C. J. Sangwin. Oxford University Press, June 2001. ISBN 0-198-50769-0 (hardcover), 0-198-50770-4 (paperback).

The Measure of the World, by Denis Guedj. University of Chicago Press, October 2001. ISBN 0-226-31030-2.

A New Kind of Science, by Stephen Wolfram. Wolfram Media, Inc., (to appear March 2002). ISBN 1-579-55008-8.

Newton's Tyranny: The Suppressed Scientific Discoveries of John Flamsteed and Stephen Gray, by David H. Clark and Stephen P. H. Clark. W. H. Freeman, October 2000. ISBN 0-716-74215-2.

Number: From Ahmes to Cantor, by Midhat Gazalé. Princeton University Press, March 2000. ISBN 0-691-00515-X. (Reviewed August 2001.)

Proofs from THE BOOK, by M. Aigner and G. M. Ziegler. Revised and expanded second edition, Springer, January 2001. ISBN 3-540-67865-4. (First edition reviewed August 1999.)

Ptolemy's Geography, translated by J. Lennart Berggren and Alexander Jones. Princeton University Press, November 2000. ISBN 0-691-01042-0.

The Quest for the Quantum Computer, by Julian Brown. Touchstone Books, August 2001. ISBN 0-684-87004-5.

Radical Equations: Math Literacy and Civil Rights, by Robert P. Moses and Charles E. Cobb Jr. Beacon Press, February 2001. ISBN 0-807-03126-7. (Reviewed March 2002.)

The Riddle of the Compass, by Amir Aczel. Harcourt Brace, August 2001. ISBN 0-151-00506-0.

Sacred Geometry, by Miranda Lundy. Walker & Company, April 2001. ISBN 0-802-71382-3.

The Science of Conjecture: Evidence and Probability before Pascal, by James Franklin. Johns Hopkins University Press, June 2001. ISBN 0-8018-6569-7.

The Search for Mathematical Roots, 1870–1940: Logics, Set Theories, and the Foundations of Mathematics from Cantor through Russell to Gödel, by I. Grattan-Guinness. Princeton University Press, February 2001. ISBN 0-691-0587-1.

* *Signs of Life: How Complexity Pervades Biology*, by Richard Solé and Brian Goodwin. Basic Books, January 2001. ISBN 0-465-01927-7.

Statisticians of the Centuries, edited by C. C. Heyde and E. Seneta. Springer, September 2001. ISBN 0-387-953283-7.

The Story of Mathematics, by Richard Mankiewicz. Princeton University Press, February 2001. ISBN 0-691-08808-X. (Reviewed in this issue.)

Things a Computer Scientist Rarely Talks About, by Donald Knuth. Center for the Study of Language and Information, July 2001. ISBN 1-57586-327-8.

Thinks, by David Lodge. Viking Press, May 2001. ISBN 0-670-89984-4.

Triangle of Thoughts, by Alain Connes, André Lichnerowicz, and Marcel Paul Schützenberger. AMS, July 2001. ISBN 0-8218-2614-X. (Reviewed March 2002.)

Turing and the Universal Machine: The Making of the Modern Computer, by Jon Agar. June 2001, Totem Books. ISBN 1-840-46250-7.

Understanding Mathematics for Aircraft Navigation, by James S. Wolper. McGraw-Hill, May 2001. ISBN 0-07-137572-4.

The Universal Computer: The Road from Leibniz to Turing, by Martin Davis. W.W. Norton & Company, October 2000. ISBN 0-393-04785-7. (Reviewed May 2001.)

The Universal History of Computing: From the Abacus to the Quantum Computer, by Georges Ifrah; translated from the French and with notes by E. F. Harding, assisted by Sophie Wood, Ian Monk, Elizabeth Clegg, and Guido Waldman. John Wiley & Sons, November 2000. ISBN 0-471-39671-0. (Reviewed in two parts, January 2002 and February 2002.)

The Universal History of Numbers: From Prehistory to the Invention of the Computer, by Georges Ifrah; translated from the French by David Bellos, E. F. Harding, Sophie Wood, and Ian Monk. John Wiley & Sons, December 1999. ISBN 0-471-37568-3. (Reviewed in two parts, January 2002 and February 2002.)

The Universe in a Nutshell, by Stephen Hawking. Bantam Doubleday Dell, November 2001. ISBN 0-553-80202-X.

The Unknowable, by Gregory J. Chaitin. Springer, August 1999. ISBN 9-814-02172-5. (Reviewed October 2001.)

What Is Mathematics? An Elementary Approach to Ideas and Methods, by Richard Courant and Herbert Robbins; second edition, revised by Ian Stewart. Oxford University Press, August 1996. ISBN 0-195-10519-2. (Reviewed December 2001.)

What Shape is a Snowflake?, by Ian Stewart. W. H. Freeman & Co, November 2001. ISBN 0-716-74794-4.

Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being, by George Lakoff and Rafael Núñez. Basic Books, October 2000. ISBN 0-465-03770-4. (Reviewed November 2001.)

White Light, by Rudy Rucker. Four Walls Eight Windows, April 2001. ISBN 1-56858-198-X.

Women Becoming Mathematicians: Creating a Professional Identity in Post-World War II America, by Margaret A. M. Murray. MIT Press, September 2000. ISBN 0-262-13369-5. (Reviewed August 2001.)

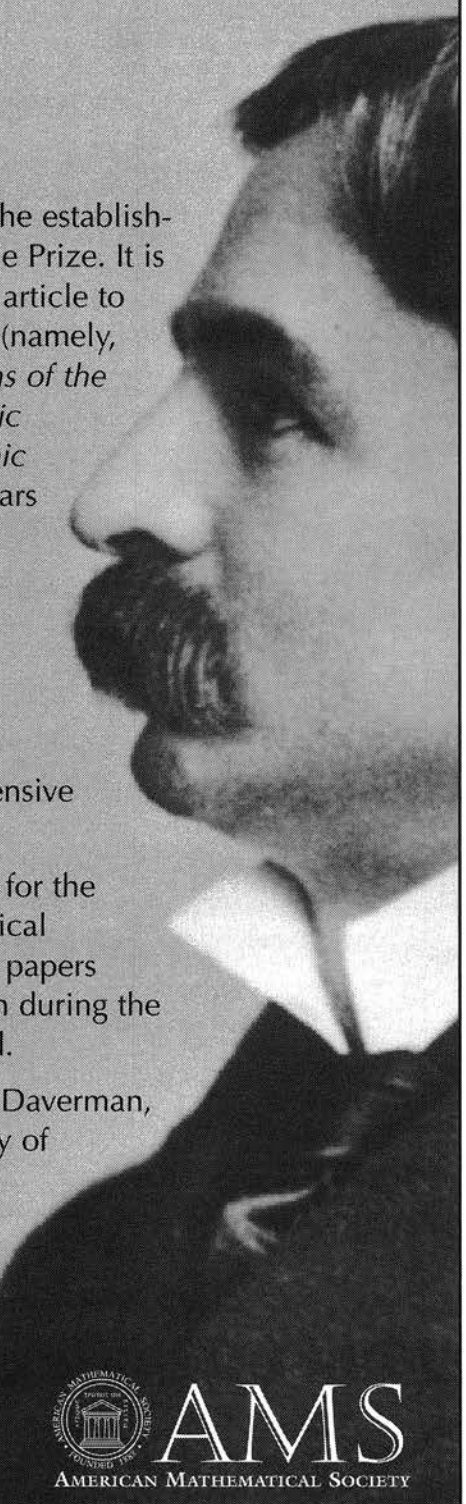
The Zen of Magic Squares, Circles, and Stars: An Exhibition of Surprising Structures across Dimensions, by Clifford A. Pickover. Princeton University Press, January 2001. ISBN 0-691-07041-5.

**Added to "Book List" since the list's last appearance.*

AMERICAN MATHEMATICAL SOCIETY

Call for Nominations

E. H. MOORE
Research Article Prize



At its meeting of January 2002, the AMS Council approved the establishment of a new award called the E. H. Moore Research Article Prize. It is to be awarded every three years for an outstanding research article to have appeared in one of the AMS primary research journals (namely, the *Journal of the AMS*, *Proceedings of the AMS*, *Transactions of the AMS*, *AMS Memoirs*, *Mathematics of Computation*, *Electronic Journal of Conformal Geometry and Dynamics*, and *Electronic Journal of Representation Theory*) during the six calendar years ending a full year before the meeting in which the prize is awarded.

Among other activities, E. H. Moore founded the Chicago branch of the AMS, served as the Society's sixth president (1901–2), delivered the Colloquium Lectures in 1906, and founded and nurtured the *Transactions of the American Mathematical Society*. The name of the prize honors his extensive contributions to the discipline and to the Society.

The Moore Prize Selection Committee requests nominations for the initial award, which will be presented at the Joint Mathematical Meetings in Baltimore, MD, in January 2003. To be specific, papers published in one of the journals named in the first paragraph during the years 1996–2001 are considered eligible for the 2003 award.

Nominations should be submitted to the secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville TN 37996-1330. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations. A brief curriculum vita should be included for all nominees. The nominations will be forwarded by the Secretary to the prize selection committee, which will make final decisions on the awarding of this prize.

Deadline for nominations is June 30, 2002.

AMS
AMERICAN MATHEMATICAL SOCIETY

Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on e-MATH at http://www.ams.org/mathcal/.

May 2002

* 6 **International Workshop on Knowledge Discovery in Multimedia and Complex Data (KDMCD'02)**, Taipei, Taiwan.
Note: In conjunction with the Sixth Pacific-Asia Conference on Knowledge Discovery and Data Mining (PAKDD-02), Taipei, Taiwan, May 6-8 2002; http://arbor.ee.ntu.edu.tw/pakdd02/.
Information: http://db.cs.ualberta.ca/kdmcd02/.

* 8-10 **Source Coding and Harmonic Analysis**, DIMACS Center, Rutgers University, Piscataway, New Jersey.
Short Description: The aim of this workshop is to continue and to accelerate the exchange of ideas between the various groups that contribute to compression. Including the following overlapping fields: source coding theory (information theorists); compression practice (signal processors); statistical modeling and inference (statisticians); harmonic analysis (applied mathematics). Talks that make progress in one field accessible to the spectrum of participants and those that are speculative are particularly encouraged.
Sponsors: DIMACS Center.
Organizers: V. Goyal, Digital Fountain and J. Kovacevic, Bell Labs .
Contacts: J. Kovacevic, Bell Labs, jelena@bell-labs.com.
Local Arrangements: J. Thiemann, DIMACS Center, jennifer@dimacs.rutgers.edu, 732-445-5928.
Information: http://dimacs.rutgers.edu/Workshops/index.html.

* 13-July 31 **TOPCON02: International Workshop and Seminar on Topology in Condensed Matter Physics**, Max Planck Institute for the Physics of Complex Systems, Dresden, Germany.
Program: The central scientific theme of our program is the recent development of applications of topology to several areas in condensed matter physics and related fields in biology. The seminar will provide a unique opportunity to bring together researchers from the three fields of mathematics, biology, and physics, experimentalists as well as theoreticians. The two one-week workshops will consist of a program of invited talks and are devoted to applications of topology to physics and to biology. The first one will be held from June 17-21 and the second one from July 1-5. The topics which we expect to discuss are liquid crystals and superfluid liquids, Fermi surfaces, membranes, quasicrystals, the general theory of defects, biopolymers and polymers, DNA, proteins, apoptosis and molecular recognition. This is not a limiting list of topics, and the seminar will largely be the result of the actual activity of the participants. From May 13 to June 7 we organize minicourses on topology for physicists. The seminar program will be organized in a way to optimize interactions between participants of different communities.
Preliminary List of Invited Participants: J. Avron, J. Bellisard, M. Berger, Joan Birman, Joe Birman, R. Brooks, Y. Bouligand, N. Cazzarelli, D. Chilingworth, B. Coleman, I. Darcy, S. Fraden, M. Frank-Kamenetsky, A. Gabibov, S. Golovin, A. Grosberg, R. Holyst, T. Janssen, A. Joets, B. Jilinsky, L. Kauffman, R. Kerner, A. Kister, A. Kosevich, R. Kusner, S. Lambropolou, M. Lapidus, S. Levine, F. Livolant, J. Maddocks, M. Magnasco, D. R. Nelson, A. Nepomnyashy, L. Pitaevski, Y. Pomeau, R. Ricca, N. Rivier, D. Roux, L. Sadun, U. Seifert, I. M. Sigal, E. Siggia, T. Sluckin, A. Stasiak, H.-R. Trebin, A. Vologodski
Information: http://www.mpipks-dresden.mpg.de/~topcon02/.

* 14-19 **4th Conference on Function Spaces**, Southern Illinois University, Edwardsville, Illinois.
Topics: Function algebras, Banach algebras, spaces and algebras

---

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.
An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.
In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.
In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **six months** prior to the scheduled date of the meeting.
The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.
The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

of analytic functions, Lp spaces, geometry of Banach spaces, isometries of function spaces.
Information: For more information visit the conference web page at: http://www.siue.edu/MATH/conference/ or contact the organizer: Krzysztof Jarosz, Department of Mathematics & Statatistics, Southern Illinois University, Edwardsville, IL 62026-1653; kjarosz@siue.edu; tel: (618) 650-2354; fax: (618) 692-0095.

* 16–18 **NMMATYC-TexMATYC**, El Paso, Texas.
Description: The New Mexico and Texas Mathematical Associations of Two Year Colleges will hold a joint annual meeting.
Conference Chair: J. Peeples may be reached at Joannep@epcc.edu.
Information: Details on the conference may be found at the NMMATYC website: http://www.unm.edu/~nmmatyc/.

* 20–24 **Solving Systems of Polynomial Equations, NSF-CBMS Regional Research Conference**, Texas A & M University, College Station, Texas.
Speaker: B. Sturmfels, UC Berkeley.
Topics: A series of ten lectures on solving systems of polynomial equations. Lecture 1: Polynomials in One Variable, Lecture 2: Hypergeometric Functions, Lecture 3: Groebner Bases and Numerical Linear Algebra, Lecture 4: Rational Univariate Representation, Lecture 5: Resultants, Lecture 6: Homotopy Methods, Lecture 7: The Real Nullstellensatz, Lecture 8: Polynomial Optimization and Semidefinite Programming, Lecture 9: Primary Decomposition, Lecture 10: The Palamodov-Ehrenpreis Theorem.
Organizers: P. Lima-Filho, J. M. Rojas, H. Schenck.
Information: http://www.math.tamu.edu/conferences/cbms/.

* 22–25 **Visualization and Mathematics 2002**, Berlin–Dahlem, Germany.
Description: This workshop provides an active forum for mathematicians and computer graphics researchers on the fundamental problems of visualization techniques, on applications in mathematics, and on mathematical concepts in visualization. It is the third symposium in a series of workshops bringing together mathematicians and experts from computer graphics.
Sponsor: Under sponsorship of the Deutsche Forschungsgemeinschaft (DFG).
Themes: Visualization in differential geometry and partial differential equations, algorithmic representation of mathematical structures, computational aspects of topology, discrete geometry of meshes, compression of large and time-dependent geometric models, virtual laboratories for mathematics and applications, online visualization and computational web services.
Information: See http://www.math.tu-berlin.de/vismath/.

* 23–25 **International Conference on Asymptotic Methods in Stochastics**, Carleton University, Ottawa, Ontario, Canada.
Desciption: The conference is in honour of the work of Miklos Csorgo.
Co-Sponsor: The meeting will be cosponsored by the Fields Institute. It will be a smaller version of an earlier conference, ICAMPS'97 (International Conference on Asymptotic Methods in Probability and Statistics), that was held at Carleton University in July 1997. We are planning to revisit major topics of ICAMPS'97.
Organizers: The official organizers of the conference are L. Horvath of the Univ. of Utah and B. Szyszkowicz, Carleton Univ. Both are colleagues and collaborators of Professor Csorgo and both are currently active doing research in asymptotic stochastic analysis. The conference will cover topics related to Professor Csorgo's research interests, which are wide-ranging and cover a broad spectrum of probability theory and stochastic processes.

* 23–25 **Progress in Partial Differential Equations and Applications**, Dept. of Math., Washington State Univ., Pullman, Washington.
Description: The conference will focus on recent progress in partial differential equations and applications. May 24 will be in honor of John R. Cannon on the occasion of his 65th birthday and his research achievement. May 25 will be the Pacific Northwest PDE seminar series.
Invited Speakers: W. Allegretto (Univ. of Alberta), S. Avdonin (Univ. of Alaska), John R. Cannon (Univ. of Central Florida), J. M. Chadam (Univ. of Pittsburgh), X. Chen (Univ. of Pittsburgh), E. DiBenedetto (Univ. of Vanderbilt), P. DuChateau (Colorado State Univ.), R. Ewing (Texas A&M Univ.), Gary Lieberman (Iowa State Univ.), Y. Lin (Univ. of Alberta), Zhilin Li, (North Carolina State Univ.), T. Seidman (Univ. of Maryland at Baltimore), R. Showalter (Univ. of Texas at Austin), W. Rundall (Texas A&M Univ.), D. Wollkind (Washington State Univ.), G. Uhlmann (Univ. of Washington). Jun Zou (Univ. of Hong Kong).
Information: For accommodation and travel information, please visit the web page http://www.math.wsu.edu.

* 25–31 **A Workshop on Homological Methods in Commutative Algebra**, Institute for Studies in Theoretical Physics and Mathematics (IPM), Tehran, Iran.
Organizing Committee: L. L. Avramov, E. E. Enochs, H. B. Foxby, and S. Yassemi.
Tentative List of Speakers and Topics: L. L. Avramov, Cohomology theories for modules of finite *G*-dimension, Growth of (co)homology over complete intersection rings, Vanishing of (co)homology over Gorenstein rings; W. Bruns, The discretization of homological properties, Invariants of the Knuth-Robinson-Schensted correspondence, Algebras defined by minors; E. E. Enochs, Gorenstein flat modules, Towards a coGalois theory of covers, Cotorsion theories; H. B. Foxby, Homological dimensions of modules and complexes, Relative homological dimensions and restricted homological dimensions, Local homomorphisms and homological dimensions; J. Herzog, Koszul algebras and modules, On the asymptotic behavior of regularity, Discrete polymatroids; O. M. G. Jenda, Gorenstein injective modules, Omega-Gorenstein modules.
Information: The announcement http://www.ipm.ac.ir/IPM/news/homological/announcement.jsp; registration form: http://www.ipm.ac.ir/IPM/news/homological/register.jsp. There will be no registration fee for this workshop. The local committee can provide hotel reservations for the participants. Transportation services will be provided by IPM.

June 2002

* 2–6 **7th Valencia International Meeting on Bayesian Statistics**, Tenerife, Canary Islands, Spain.
Description: Residential conference held at a conference center at the seaside (Playa de las Americas). World Conference on Bayesian Statistics, held every four years. Fully refereed, selected proceedings published by Oxford University Press.
Deadline: The deadline for abstracts for contributed papers is April 10, 2002.
Organizer: Universidad de Valencia/ISBA.
Information: Futher information and registration forms at the conference websites: http://www.uv.es/valencia7/ (USA mirror: http://www.stat.duke.edu/valencia7/); e-mail: valencia7@uv.es.

* 2–7 **Foundations of Probability and Physics-2**, International Center for Mathematical Modelling in Physics, Engineering and Cognitive Sciences, Växjö University, Sweden.
Invited Speakers: L. Accardi* (Univ. of Rome-2, Italy), V. Belavkin (Nottingham Univ.), K.F. Berggren (Linkopnig Univ.), C. Fuchs (Bell Lab., Murray Hill), R. Gill (Univ. of Eindhoven), K. Gustafsson (Univ. of Boulder), B. Hellsings (Chalmers, Gothenburg), B. Hiley (Univ. of London), A. Khrennikov (Växjö Univ.), S. Kozyrev (Steklov Mathematical Inst., Moscow), B. Nilsson (Växjö Univ.), M. Ohya (Science Univ. of Tokyo), A. Plotnitsky (Purdue Univ.), A. Segienko (Boston Univ.), H. Rauch (Atominstitute, Vienna), E. Sjöqvist (Uppsala Univ.), J. Summhammer (Atominstitut, Vienna), I. Volovich (Steklov Mathematical Inst., Moscow). (*)- to be confirmed
Information: See http://www.msi.vxu.se/aktuellt/konferens/foundations2.pdf/. Andrei Khrennikov, Prof. of Applied Math-

ematics, The Director of International Center in Mathematical Modelling in Physics, Engineering and Cognitive Sciences.

* 5–8 **Variational Methods: Open Problems, Recent Progress, and Numerical Algorithms**, Northern Arizona University, Flagstaff, Arizona.
Principal Speakers: A. Castro, G. Chen, W.-M. Ni.
Organizer: J. M. Neuberger, NAU.
Contact: John.Neuberger@NAU.Edu.
Information: http://odin.math.nau.edu/~vari/.

* 6–8 **Conference on Zero-dimensional Schemes and Related Topics**, Acireale, Italy (Sicily).
Topics: This conference, which honors Tony Geramita's 60th birthday, includes topics in algebraic geometry and commutative algebra related to Geramita's work in these areas. Talks will be given by the invited speakers listed below; shorter talks are also solicited. To register or to submit a request to speak visit the website listed below.
Organizers: E. Campbell, Queen's University (eddy@mast.QueensU.CA); B. Harbourne, Univ. of Nebraska (bharbourne@unl.edu); J. Migliore, Univ. of Notre Dame (migliore.1@nd.edu); F. Orecchia, Univ. of Napoli (orecchia@matna2.dma.unina.it); A. Ragusa, Univ. of Catania (ragusa@dmi.unict.it); L. Robbiano, Univ. of Genova (robbiano@dima.unige.it).
Invited Speakers: K. Chandler, Univ. of Notre Dame; L. Chiantini, Univ. of Siena; A. Conca, Univ. of Genova; A. Gimigliano, Univ. of Bologna; M. Kreuzer, Univ. of Regensburg; R. M. Miró-Roig, Univ. of Barcelona; U. Nagel, Univ. of Paderborn; L. Roberts, Queens Univ.; R. Strano, Univ. of Catania; B. Ulrich, Purdue Univ.
Information: Contact one of the organizers above or visit http://cocoa.dima.unige.it/conference/acireale/first.html.

* 21–26 **Symmetries and Integrability of Difference Equations–EuroConference on Discrete Painleveé Equations and the Solvability of Difference Equations**, VVF La Badine, Giens (near Toulon), France.
Scope: The topics covered by this conference will include: analytic and algebraic aspects of difference equations, difference Galois theory, the Painleve property and singularity analysis, growth and branching phenomena in rational mappings, difference analogues of the Painleve equations, isomonodromic deformation theory, asymptotics of orthogonal polynomials, symmetries of difference equations and applications to numerical analysis. The conference is open to researchers worldwide, whether from industry or academia. Participation will be limited to 100. The emphasis will be on discussion about new developments. The conference fee covers registration, full board, and lodging. Grants will be available, in particular for nationals under 35 from EU or Associated States.
Speakers will include: M. J. Ablowitz (Boulder, US), C. Brezinski (Lille, France), P. Clarkson (Canterbury, UK), R. Conte (Saclay, France), C. Favre (Paris, France)*; R. Halburd (Loughborough, UK), P. E. Hydon (Surrey, UK), G. Immink (Groningen, Netherlands), A. Iserles (Cambridge, UK), A. Its (Indianapolis, US), N. Joshi (Adelaide, Australia), M. Kruskal (Rutgers, US), I. Laine (Joensuu, Finland), D. Levi (Rome, Italy), J.-M. Maillard (Paris, France), F. Nijhoff (Leeds, UK), V. Papageorgiou (Patras, Greece), R. Quispel (Bundoora, Australia), A. Ramani (Paris, France), K. Rerikh (Dubna, Russia), J. Sauloy (Toulouse, France), J. Satsuma (Tokyo, Japan), T. Takenawa (Tokyo, Japan)*; W. van Assche (Louvain, Belgium), A. Veselov (Loughborough, UK), P. Winternitz (Montreal, Canada) (* means to be confirmed).
Information: Mr. Rachid Adghoughi, Conference Organiser, European Science Foundation, 1 quai Lezay-Marnési, 67080 Strasbourg, France; tel +33 388 76 71 35; fax +33 388 36 69 87; e-mail: radghoughi@esf.org; http://www.esf.org/euresco/02/pc02185/

* 23–28 **Recent Advances and New Directions in Mechanics, Continuum Thermodynamics, and Kinetic Theory: Symposium in Memory of Clifford Ambrose Truesdell III**, Virginia Polytechnic Institute and State University, Blacksburg, Virginia.
Description: This Symposium will be held in conjunction with the 14th U.S. National Congress of Theoretical and Applied Mechanics. Online information on the National Congress is available at the website http://www.esm.vt.edu/usncam14/.
Information: Updated information on the present Symposium is posted at the URL http://www.ms.uky.edu/~mclxyh/ctsymp.htm.

* 24–27 **International Conference on "Information and Knowledge Engineering" (IKE'02)**, Las Vegas, Nevada.
Information: http://www.gsw.edu/~bhanu/Conferences.html.

* 24–27 **International Workshop on Orthogonal Polynomials: Orthogonal Polynomials and Approximation Theory (IWOP'02)**, Leganés, Madrid, Spain.
Invited Speakers: J. S. Geronimo (Georgia Inst. of Technology, USA), P. González Vera (Univ. of La Laguna, Spain) and L. Jodar (Polythecnical Univ. of Valencia, Spain) A. B. J. Kuijlaars (Katholieke Univ. Leuven, Belgium), A. Martínez-Filkenshtein (Univ. of Almería, Spain), H. Stahl (Technical Univ. Berlin, Germany).
Information: R. Álvarez-Nodarse (Univ. of Sevilla, ran@us.es), J. Arvesú (Univ. of Carlos III de Madrid, jarvesu@math.uc3m.es), and P. Marcellán (Univ. of Carlos III de Madrid, pacomarc@ing.uc3m.es). There will be six two-hour invited lectures and 20 research seminars (20+5 min). Further information will appear on the official URL site of the IWOP'02, http://merlin.us.es/~renato/iwop/.

* 24–27 **Special Session on "Knowledge Representation and Reasoning" in 2002 International Conference on Artificial Intelligence (IC-AI)**, Las Vegas, Nevada.
Information: http://www.gsw.edu/~bhanu/Conferences.html.

* 24–28 **Workshop on "Bone Mechanics: Mathematical and Mechanical Models for Analysis and Synthesis"**, IST, Lisbon, Portugal.
Workshop topics: Homogenization models for cellular materials; mechanosensation system in bone; adaptive elasticity; material models in topology optimization of structures; optimization and biological designs; bone remodeling: analytical and computational models; bone prostheses and implants; material optimization models applied to bone remodeling simulation; computational assessment of bone mechanical quality; biological versus topological optimization models of bone.
Speakers: M. P. Bendsoe, Mathematical Inst., Technical Univ. of Denmark; A. Cherkaev, Dept. of Mathematics, Univ. of Utah; S. C. Cowin, Dept. of Mechanical Engineering at City College, City Univ. of New York; M. Doblaré, Univ. of Zaragoza; R. Huiskes, Dept. of Biomedical Engineering, Eindhoven Univ. of Technology; H. Weinans, Erasmus Orthopaedic Research Lab, Erasmus Univ. Rotterdam; J. M. Guedes, Dept. of Mechanical Engineering, Instituto Superior Técnico, Portugal; H. Rodrigues, Dept. of Mechanical Engineering, Instituto Superior Técnico, Portugal.
Information: http://www.dem.ist.utl.pt/~bonemec/.

* 24–29 **Topology, Geometry and Quantum Field Theory**, Oxford, England.
Description: Geometry and quantum physics developed in parallel since the recognition of the central role of nonabelian gauge theory in elementary particle physics in the late seventies and the emerging study of supersymmetry and string theory. The topics of this symposium will be centered around string theory, M-theory, and quantum gravity on the one hand, and K-theory, elliptic cohomology, quantum cohomology, and string topology on the other. Its purpose is to bring experts in topology, geometry, and theoretical physics together.
Invited Speakers: Who have provisionally accepted: Sir M. Atiyah (Edinburgh), R. Cohen (Stanford), R. Dijkgraaf (Amsterdam), M. Hopkins (MIT), M. Kontsevich (IHES), D. McDuff (SUNY), N. Nekrasov (IHES), D. Sullivan (CUNY), S. Stolz (Notre Dame), C. Teleman

(Cambridge), G. Wilson (King's College London), E. Witten (IAS Princeton).
Symposium organizer: U. Tillmann (Oxford).
Information: Please register your interest with L. Mildenhall (mildenha@maths.ox.ac.uk). Further details will appear at http://www.maths.ox.ac.uk/notices/events/. The symposium is supported by the LMS.

* 27–July 2 **19th International Conference on Operator Theory,** West University, Timisoara, Romania.
Organizers: Institute of Mathematics of the Romanian Academy and the Faculty of Mathematics of the West University of Timisoara.
Topics: Operator theory, operator algebras and their applications (differential operators, complex functions, mathematical physics, matrix analysis, systems theory, etc.).
Steering Committee: W. B. Arveson, N. K. Nikolskii, N. Salinas, S. Stratila, F.-H. Vasilescu.
Information: http://www.imar.ro/~ot/.

* 30–July 20 **2002 Summer Session: IAS/Park City Mathematics Institute (PCMI)**, Park City, Utah.
Description: PCMI is a three-week residential Summer Session for research mathematicians, graduate students, undergraduate students, high school teachers, college teaching faculty, mathematics education researchers.
Sponsor: PCMI is sponsored by the Institute for Advanced Study, Princeton, New Jersey.
Research topic: Automorphic forms and their applications.
Organizers: P. Sarnak, Princeton Univ., and F. Shahidi, Purdue Univ.
Information: http://www.ias.edu/parkcity/.

July 2002

* 1–6 **2002: Workshop on "Wavelets and Applications"**, Barcelona, Spain.
Organizer: Institute of Mathematics of the University of Barcelona (IMUB).
Organizing Committee: M. J. Carro, J. Cerda, J. Martin, and J. Soria.
Information: IMUB, Univ. of Barcelona; phone: (34)-93 402 1385, fax: (34)-93 403 5963, http://www.imub.ub.es/wavelets/.

* 10–13 **VISIT-ME 2002: Vienna International Symposium on Integrating Technology into Mathematics Education**, Vienna, Austria.
Contact: B. Kutzler at kutzler@eunet.at.
Information: http://www.acdca.ac.at/visit-me-2002/.

* 12–15 **XIth Oporto Meeting on Geometry, Topology and Physics,** Porto, Portugal.
Description: The main topic of the XIth Oporto meeting will be: "Geometry of Differential Equations". The confirmed invited main speakers are: I. M. Anderson (Dept. of Mathematics and Statistics, Utah State Univ.), P. A. Griffiths (Institute for Advanced Study, Princeton), N. Kamran (Dept. of Mathematics and Statistics, McGill Univ., Montreal, Canada), J. Krasil'shchik (Independent Univ. of Moscow), A. Verbovetsky (Independent Univ. of Moscow), A. M. Vinogradov (Diffiety Inst. and Salerno Univ., Italy). The main courses will be delivered by the following invited speakers: Prof. Anderson, Prof. Kamran, Prof. Griffiths, and Prof. Vinogradov.
Further Information and Registration: http://www.fc.up.pt/mp/actividades/om.html.

* 27–August 9 **Banach Algebras and Their Applications: Banach Algebras 2003**, Edmonton, Alberta, Canada.
Description: This conference is the sixteenth in a series of conferences on Banach algebras that started in 1974 in Los Angeles. We expect that most specialists in Banach algebras as well as leading mathematicians from related areas will attend this conference. In the past, these conferences have always led to fruitful interaction between the participants, and we expect this tradition to continue.

In addition to the regular conference program consisting of one-hour and half-hour talks by the participants, we also plan to hold five workshops on the following topics, each of which will be chaired by an internationally recognized specialist in the respective area: Banach algebras in harmonic analysis (to be held in the honor of Eberhard Kaniuth on the occasion of his retirement), Chair: A. T.-M. Lau (Edmonton); Banach algebras in operator theory, Chair: M. M. Neumann (Starkville; USA); Banach algebras and operator spaces, Chair: Z.-J. Ruan (Urbana-Champaign; USA); K-theory of Banach algebras, Chair: J. Cuntz (Muenster, Germany); Topological homology, Chair: A. Ya. Helemskii (Moscow, Russia). Each workshop will occupy two afternoons. The chairs are completely free to decide on the format of their workshops.
Information: For more detailed information, including a list of invited speakers, see the conference website at http://www.math.ualberta.ca/~ba03/.

* 29–August 2 **10th International Conference on Finite or Infinite Dimensional Complex Analysis and Applications,** Silla University in Pusane, Korea.
Deadlines: Registration form by April 15, 2002; abstract by May 31, 2002.
Topics: Finite or infinite dimensional complex analysis and applications international advisory board: J. Kajiwara, Kyushu Univ. (Chairman); H. Begehr, Free Univ. of Berlin; H. Kazama, Kyushu Univ.; M. Nishihara, Fukuoka Institute of Technology; K. H. Shon, Pusan National Univ.; L. H. Son, Hanoi Univ. of Technology; S. Wu, Peking Univ.; C. C. Yang, Hongkong Univ. of Science and Technology.
Speakers: H. Begehr, K. S. Chang, S. H. Cho, S. Dineen, J. K. Han, H. O. Kim, K. T. Kim, M. Morimoto, M. Nishihara, F. Ronning, H. M. Srivastava, L. H. Son, T. Sugawa, S. Tajima, C. C. Yang.
Information:http://lotus.silla.ac.kr/~mathema/conference/ .

* 30–August 2 **Mysticism, Reason, Art and Literature: East West Perspectives,** The Society of Indian Philosophy & Religion, Calcutta, India.
Topics: The theme can be addressed critically, reflectively and creatively by the philosophical, religious and scientific traditions of the world's great civilizations. The program will include plenary addresses, volunteered papers, invited papers and panel discussions. Registered participants who are members of professional associations or societies are encouraged to submit proposals for holding meetings in the conference on behalf of their associations or societies. The organizers are committed to upholding the highest academic standards with emphasis on the exchange of ideas and dialogues among thinkers drawn from a wide range of the world's cultural traditions and movements.

Possible topics include: art, literature and religion; mysticism and romanticism; art and creativity; art and imitation; myth, metaphor and reality; fact, fiction and reason; language, thought and reality; deductivism and inductivism; holism and atomism; ineffable and unknowable; esoteric traditions of the world; God and absolute; transcendence and immanence; scepticism and agnosticism; realism and antirealism; reason, revelation and faith; self and absolute; Tao and Brahman; emptiness and nothingness; theories of deconstruction, existentialism and essentialism. This list is illustrative and not exhaustive.
Information: For information or to contribute a paper, contact: Dr. Chandana Chakrabarti, Elon Univ., Elon, NC 27244; e-mail: chakraba@elon.edu; phone: (336) 278-5713; fax: (336) 278-5627; webpage:http://www.elon.edu/chakraba/conference_2002.htm.

August 2002

* 4–9 **Modeling and Simulating Biocomplexity for Mathematicians and Physicists**, Santa Fe Institute (SFI), Santa Fe, New Mexico.
Participants: Ph.D. students, post-doctoral students and young faculty members in mathematics, and physicists who are interested in applying their knowledge in the biological sciences.

Prerequisites: This workshop is dedicated to assisting theoreticians who wish to switch their interests into biology. Thus, no biological background will be assumed (though general background reading is of course helpful). Preference will be given to those for whom this experience is their first, or one of their first, in theoretical biology. Mathematicians should have had some experience in applying mathematics to some other area; please indicate this experience in your statement of research interests.

Lecturers (Partial List): Note that the following are authors or coauthors of major textbooks in mathematical/theoretical biology: Edelstein, Keener and Sneyd, Murray, Segel. L. Edelstein (Univ. of British Columbia), J. Keener (Univ. of Utah), B. Korber (Los Alamos National Lab.), H. Markram (Weizmann Inst.), M. Laessig (Univ. of Cologne), J. Murray (Univ. of Washington), L. Peliti (Univ. of Naples), L. Segel (Weizmann Inst.), J. Sneyd (Massey Univ.).

Information: Available at `http://www.santafe.edu/mathmodel/` or `mathmodel@santafe.edu`.

* 8–September 3 **First Sino-German Meeting on Stochastic Analysis (Satellite Conference to the ICM 2002)**, Beijing, China.

Sections: 1) Geometry on path space (organizer: R. Léandre, e-mail: `leandre@iecn.u-nancy.fr`), 2) Infinite dimensional analysis, measure-valued processes and Dirichlet forms (organizer: K.-T. Sturm, e-mail: `sturm@uni-bonn.de`), 3) Noncommutative and quantum probability (organizer: M. Schuermann, e-mail: `schurman@uni-greifswald.de`), 4) Pseudodifferential operators and jump processes (organizer: N. Jacob, e-mail: `N.Jacob@swansea.ac.uk`), 5) Random media (organizer: A. Bovier, e-mail: `bovier@wias-berlin.de`), 6) Statistical mechanics and particle systems (organizer: Yu. Kondratiev, e-mail: `kondrat@mathematik.uni-bielefeld.de`), 7) Stochastic finance (organizer: M. Schweizer, e-mail: `martin.schweizer@mathematik.uni-muenchen.de`), 8) Stochastic methods in quantum field theory and hydrodynamics (organizers: P. Blanchard, e-mail: `blanchard@physik.uni-bielefeld.de` and L. Streit, e-mail: `streit@physik.uni-bielefeld.de`), 9) Stochastic partial differential equations (organizer: L. Tubaro, e-mail: `tubaro@science.unitn.it`).

Information: If you are interested in participating, please contact the organizer of the section closest to your interest and send a copy to the following address: M. L. Wang (`wang@uni-bonn.de`).

* 9–11 **Colloquium Logicum 2002**, Münster, Germany.

Description: Biannual Meeting of the DVMLG (Deutsche Vereinigung für Mathematische Logik und für Grundlagen der Exakten Wissenschaften).

Invited Speakers: T. Arai (Hiroshima), J. Bagaria (Barcelona), A. Nies (Chicago, IL), M. Otto (Swansea), C. Parsons (Cambridge, MA), A. Pillay (Urbana-Champaign, IL), M. Rathjen (Leeds), J. van Benthem (Amsterdam/Stanford, CA).

Information: Satellite conference to `http://wwwmath.uni-muenster.de/LC2002/CL2002.html`.

* 12–16 **Geometric Topology—A Satellite Conference of ICM 2002, Beijing**, Shaanxi Normal University, Xi'an, China.

Confirmed Plenary Speakers: D. Calegari (Harvard Univ.), M. Bestvina (Univ. of Utah), S. Bigelow (Univ. of Melbourne), M. Boileau (Universite de Toulouse III), F. Bonahon (Univ. of Southern California), M. Bridson (Univ. of Oxford), Y. Eliashberg (Standford Univ.), M. Furuta (Univ. of Tokyo), D. Gabai (Princeton Univ.), C. Gordon (Univ. of Texas at Austin), W.-C. Hsiang (Princeton Univ.), J. Jezierski (Univ. of Agriculture, Warsaw), S. Kamada (Osaka City Univ.), J. Roberts (Univ. of California, San Diego), H. Rubinstein (Univ. of Melborne), V. Turaev (IRMA, Universite de Strasbourg).

Organizing Committee: Chair: B. Jiang (Peking Univ.); Vice-Chair: S. Zhao (President, Shaanxi Normal Univ.); F. Fang (Nankai Inst. of Math., Nankai Univ.), B. Li (Inst. of Systems Science, Chinese Acad. of Sci.), X.-S. Lin (Univ. of California, Riverside), F. Luo (Rutgers Univ.), G. Wang (Shaanxi Normal Univ.), S. Wang (Peking Univ.), Y.-Q. Wu (Univ. of Iowa), B. Zhao (Shaanxi Normal Univ.).

Information and registration: Participants should register at the website `http://www.math.uiowa.edu/~wu/gtc/gtc.html`. Every effort will be made to provide support for local expenses to all registered participants.

* 19–23 **24th European Meeting of Statisticians, 14th Prague Conference on Information Theory, Statistical Decision Functions and Random Processes**, Prague, The Czech Republic.

Information: `http://siprint.utia.cas.cz/24_ems/`.

September 2002

* 9–13 **Axiomatic and Enriched Homotopy Theory**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England.

Workshop Organizers: P. G. Goerss and J. P. C. Greenlees.

Information: Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge, CB3 0EH; tel: +44 (0) 1223 335999; fax: +44 (0) 1223 330508; e-mail: `info@newton.cam.ac.uk`. Please refer to our website: `http://www.newton.cam.ac.uk/programs/NST/` for full details of how to apply for these workshops.

* 16–18 **KES'2002 Sixth International Conference on Knowledge-Based Intelligent Information & Engineering Systems**, Podere d'Ombriano, Crema, Italy.

Information: Special session: Machine learning in bioinformatics, `http://www.dsi.unifi.it/~paolo/kes02.html`. This special session aims at presenting and discussing state-of-the-art algorithms and methodologies in computational molecular biology where machine learning plays a key role. Topics of interest include (but are not limited to) applications of machine learning to: protein folding and protein structure prediction; gene expression data and DNA micro-arrays; protein-protein interaction; finding signals and motifs in DNA sequences; inference of genetic networks; analysis of gene structure and regulation; phylogenetic analysis; QSAR and QSPR. Paolo Frasconi, Dept. of Systems and Computer Science, University of Florence, Via di Santa Marta 3, I-50139 Firenze, Italy, Phone: +39 055 4796 362, Fax: +39 055 4796 363, `http://www.dsi.unifi.it/~paolo/`.

* 16–20 **Homotopy Theory of Geometric Categories**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England.

Workshop Organizers: J. F. Jardine and F. Morel.

Information: Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge, CB3 0EH; tel: +44 (0) 1223 335999; fax: +44 (0) 1223 330508; e-mail: `info@newton.cam.ac.uk`. Please refer to our website: `http://www.newton.cam.ac.uk/programs/NST/` for full details of how to apply for these workshops.

* 17–23 **International Algebra Conference Dedicated to the Memory of Zenon Borewicz (1922–1995)**, Saint Petersburg, Russia.

Topics: The conference will cover all areas of algebra, algebraic number theory and algebraic geometry, with special emphasis in topics close to the research interests of Borewicz, viz. local and global fields, representation theory, homological algebra and algebraic K-theory, linear and algebraic groups. We plan both plenary talks and short communications in smaller thematic sections.

Information: If you are interested in participating in the conference and would like to receive further information, please respond to `Borevich.conf@norths.spb.su`.

* 20–22 **Yamabe Memorial Symposium**, University of Minnesota, School of Mathematics, Minneapolis, Minnesota.

Description: The Yamabe Memorial Symposium is held in memory of Hidehiko Yamabe (1923-1960) whose significant work on topological groups and geometry were outstanding contributions to modern mathematics.

Main Speakers: P. Li, R. Hamilton, F.-H. Lin, and B. Chow.

Organizing Committee: R. Gulliver, D. Kahn, N.-C. C. Leung, J. Wang.

Financial support: Interested younger participants are invited to apply for partial support for travel and local expenses. Please supply a brief research summary or C.V. and a letter of reference.
Information and Contacts: E-mail: yamabe@math.umn.edu, tel: (612) 625-5591. The final list of speakers and other conference details will be posted at http://www.math.umn.edu/~gulliver/confs/yamabe.html.

* 30–October 4 **K-theory and Arithmetic**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England.
Workshop Organizers: S. Lichtenbaum and V. P. Snaith.
Information: Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge, CB3 0EH; tel: +44 (0) 1223 335999; fax: +44 (0) 1223 330508; e-mail: info@newton.cam.ac.uk. Please refer to our website: http://www.newton.cam.ac.uk/programs/NST/ for full details of how to apply for these workshops.

November 2002

* 18–22 **Twenty Years of Tilting Theory: An Interdisciplinary Symposium**, Fraueninsel, Germany.
Description: Tilting modules were born about twenty years ago in the context of finite dimensional algebras. Since then, tilting theory has spread in many different directions, and nowadays it plays an important role in various branches of modern algebra, ranging from Lie theory and algebraic geometry to homotopical algebra. The aim of this meeting is to bring together for the first time experts from different fields where tilting is relevant or even of central importance. There will be several lecture series and survey talks on the use of tilting theory in different contexts, as well as a number of additional talks contributed by the participants.
Tentative List of Invited Speakers: M. van den Bergh (Univ. of Limburg), S. Brenner (Univ. of Liverpool), T. Brüstle (Univ. of Bielefeld), M. Butler (Univ. of Liverpool), S. Donkin (Univ. of London), K. Erdmann (Univ. of Oxford), K. Fuller (Univ. of Iowa), B. Keller (Univ. of Paris VII), S. König (Univ. of Leicester), H. Lenzing (Univ. of Paderborn), O. Mathieu (Univ. of Lyon), J. Miyachi (Tokyo Gakugei Univ.), I. Reiten (NTNU Trondheim), J. Rickard (Univ. of Bristol), R. Rouquier (Univ. of Paris VII), J. Trlifaj (Charles Univ. Prague).
Information: See http://www.mathematik.uni-muenchen.de/~tilting/.

* 25–28 **Fifth International Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing (MCQMC 2002)**, Singapore, Republic of Singapore.
Description: This conference will bring together experts in the fields of mathematics, computer science, statistics, operations research, physics, engineering, and finance to discuss the latest developments in Monte Carlo and quasi-Monte Carlo methods and their applications. MCQMC 2002 is the fifth in a series of international meetings. The program will consist of invited plenary talks, several special thematic sessions, and contributed talks.
Program Committee: K.-T. Fang (Hong Kong), P. Glasserman (USA), S. Heinrich (Germany), F. J. Hickernell (Hong Kong), P. L'Ecuyer (Canada), H. Niederreiter (Singapore, chair), E. Novak (Germany), A. Owen (USA), I. H. Sloan (Australia), J. Spanier (USA), D. Talay (France), S. Tavare (USA), J.-S. Wang (Singapore), H. Wozniakowski (USA/Poland).
Invited Speakers: P. Boyle (Canada), S. Heinrich (Germany), P. L'Ecuyer (Canada), J.S. Liu (USA), D. Talay (France), W. Wagner (Germany), G. Wasilkowski (USA), C. P. Xing (Singapore).
Call for Papers: Abstracts of contributed talks should be submitted to H. Niederreiter by July 31, 2002. The abstract should fit on one page and include the title of the talk, the name, affiliation, full postal address, and e-mail address of the speaker, and a summary of the talk which provides sufficient information to assess the relevance and novelty of the results. The preferred mode is electronic submission in LaTeX or Postscript format. Notification of the acceptance of the talk will be given about one month after the above deadline.
Contact: H. Niederreiter, Department of Mathematics, National University of Singapore, 2 Science Drive 2, Singapore 117543, Republic of Singapore; e-mail: nied@math.nus.edu.sg.
Information: Regularly updated information can be obtained from the web page http://www.mcqmc2002.math.nus.edu.sg/.

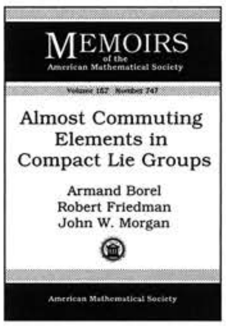
December 2002

* 9–13 **Elliptic Cohomology and Chromatic Phenomena**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England.
EuroWorkshop Organizers: D. C. Ravenel and H. R. Miller.
Information: Isaac Newton Institute for Mathematical Sciences, 20 Clarkson Road, Cambridge, CB3 0EH; tel: +44 (0) 1223 335999; fax: +44 (0) 1223 330508; e-mail: info@newton.cam.ac.uk. Please refer to our website: http://www.newton.cam.ac.uk/programs/NST/ for full details of how to apply for these workshops.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

New Publications Offered by the AMS

Algebra and Algebraic Geometry



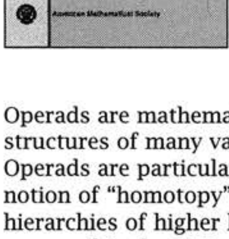
Almost Commuting Elements in Compact Lie Groups

Armand Borel, *Institute for Advanced Study, Princeton, NJ*, and **Robert Friedman** and **John W. Morgan**, *Columbia University, New York City, NY*

Contents: Introduction; Almost commuting N-tuples; Some characterizations of groups of type A; c-pairs; Commuting triples; Some results on diagram automorphisms and associated root systems; The fixed subgroup of an automorphism; C-triples; The tori $\overline{S}(k)$ and $\overline{S}^{W_c}(\overline{\mathfrak{g}}, k)$ and their Weyl groups; The Chern-Simons invariant; The case when $\langle C \rangle$ is not cyclic; Bibliography; Diagrams and tables.

Memoirs of the American Mathematical Society, Volume 157, Number 747

May 2002, 136 pages, Softcover, ISBN 0-8218-2792-8, LC 2002018237, 2000 *Mathematics Subject Classification*: 22C05, 17B20; 57R20, 17B40, **Individual member $31**, List $52, Institutional member $42, Order code MEMO/157/747N



Operads in Algebra, Topology and Physics

Martin Markl, *Czech Academy of Sciences, Prague, Czech Republic*, **Steve Shnider**, *Bar-Ilan University, Ramat-Gan, Israel*, and **Jim Stasheff**, *University of North Carolina, Chapel Hill*

Operads are mathematical devices which describe algebraic structures of many varieties and in various categories. Operads are particularly important in categories with a good notion of "homotopy" where they play a key role in organizing hierarchies of higher homotopies. Significant examples first appeared in the sixties though the formal definition and appropriate generality waited for the seventies. These early occurrences were in algebraic topology in the study of (iterated) loop spaces and their chain algebras. In the nineties, there was a renaissance and further development of the theory inspired by the discovery of new relationships with graph cohomology, representation theory, algebraic geometry, derived categories, Morse theory, symplectic and contact geometry, combinatorics, knot theory, moduli spaces, cyclic cohomology, and, not least, theoretical physics, especially string field theory and deformation quantization. The generalization of quadratic duality (e.g., Lie algebras as dual to commutative algebras) together with the property of Koszulness in an essentially operadic context provided an additional computational tool for studying homotopy properties outside of the topological setting.

The book contains a detailed and comprehensive historical introduction describing the development of operad theory from the initial period when it was a rather specialized tool in homotopy theory to the present when operads have a wide range of applications in algebra, topology, and mathematical physics. Many results and applications currently scattered in the literature are brought together here along with new results and insights. The basic definitions and constructions are carefully explained and include many details not found in any of the standard literature.

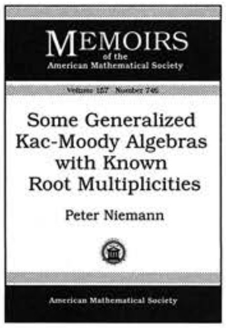
There is a chapter on topology, reviewing classical results with the emphasis on the W-construction and homotopy invariance. Another chapter describes the (co)homology of operad algebras, minimal models, and homotopy algebras. A chapter on geometry focuses on the configuration spaces and their compactifications. A final chapter deals with cyclic and modular operads and applications to graph complexes and moduli spaces of surfaces of arbitrary genus.

This item will also be of interest to those working in geometry and topology.

Contents: *Part I:* Introduction and history; *Part II:* Operads in a symmetric monoidal category; Topology–review of classical results; Algebra; Geometry; Generalization of operads; Epilog; Bibliography; Glossary of notations; Index.

Mathematical Surveys and Monographs, Volume 96

June 2002, approximately 360 pages, Hardcover, ISBN 0-8218-2134-2, LC 2002016342, 2000 *Mathematics Subject Classification*: 18D50, 55P48, **Individual member $53**, List $89, Institutional member $71, Order code SURV/96N



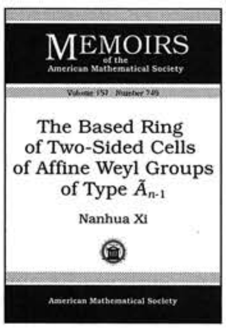
Some Generalized Kac-Moody Algebras with Known Root Multiplicities

Peter Niemann, *Logica UK Ltd, London, UK*

Contents: Introduction; Generalized Kac-Moody algebras; Modular forms; Lattices and their Theta-functions; The proof of Theorem 1.7; The real simple roots; Hyperbolic Lie algebras; Appendix A; Appendix B; Bibliography; Notation.

Memoirs of the American Mathematical Society, Volume 157, Number 746

May 2002, 119 pages, Softcover, ISBN 0-8218-2888-6, LC 2002018236, 2000 *Mathematics Subject Classification*: 17B65, **Individual member $30**, List $50, Institutional member $40, Order code MEMO/157/746N



The Based Ring of Two-Sided Cells of Affine Weyl Groups of Type $\widetilde{A}_{n-1}$

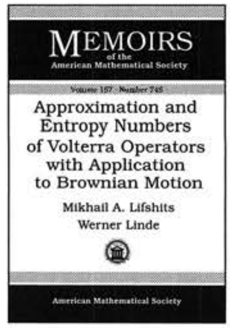
Nanhua Xi, *University of Sydney, NSW, Australia*

Contents: Cells in affine Weyl groups; Type $\widetilde{A}_{n-1}$; Canonical left cells; The group F_λ and its representation; A bijection between $\Gamma_\lambda \cap \Gamma_\lambda^{-1}$ and Irr F_λ; A factorization formula in $J_{\Gamma_\lambda \cap \Gamma_\lambda^{-1}}$; A multiplication formula in $J_{\Gamma_\lambda \cap \Gamma_\lambda^{-1}}$; The based rings $J_{\Gamma_\lambda \cap \Gamma_\lambda^{-1}}$ and J_c; Bibliography; Index; Notation.

Memoirs of the American Mathematical Society, Volume 157, Number 749

May 2002, 95 pages, Softcover, ISBN 0-8218-2891-6, LC 2002018239, 2000 *Mathematics Subject Classification*: 20G05, 18F25; 16S80, 20C07, **Individual member $29**, List $48, Institutional member $38, Order code MEMO/157/749N

Analysis



Approximation and Entropy Numbers of Volterra Operators with Application to Brownian Motion

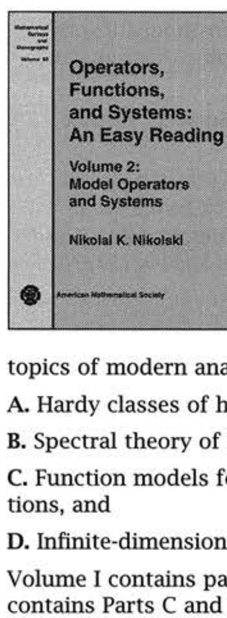
Mikhail A. Lifshits, *Saint Petersburg State University, St. Petersburg, Russia*, and **Werner Linde**, *Friedrich-Schiller University, Jena, Germany*

This item will also be of interest to those working in probability.

Contents: Introduction; Main results; Scale transformations; Upper estimates for entropy numbers; Lower estimates for entropy numbers; Approximation numbers; Small ball behaviour of weighted Wiener processes; Appendix; Bibliography.

Memoirs of the American Mathematical Society, Volume 157, Number 745

May 2002, 87 pages, Softcover, ISBN 0-8218-2791-X, LC 2002018235, 2000 *Mathematics Subject Classification*: 47G10; 47B06, 60G15, 47B38, **Individual member $29**, List $48, Institutional member $38, Order code MEMO/157/745N



Operators, Functions, and Systems: An Easy Reading

Volume 2: Model Operators and Systems

Nikolai K. Nikolski, *University of Bordeaux I, Talence, France*

This unique work combines together in two volumes four formally distinct topics of modern analysis and its applications:

A. Hardy classes of holomorphic functions

B. Spectral theory of Hankel and Toeplitz operators

C. Function models for linear operators and free interpolations, and

D. Infinite-dimensional system theory and signal processing

Volume I contains parts A and B; this volume, Volume II, contains Parts C and D.

Hardy classes of holomorphic functions: This topic is known to be the most powerful tool of complex analysis for a variety of applications, starting with Fourier series, through the Riemann zeta-function, all the way to Wiener's theory of signal processing.

Spectral theory of Hankel and Toeplitz operators: These now become the supporting pillars for a large part of harmonic and complex analysis and for many of their applications. In this book, moment problems, Nevanlinna-Pick and

Carathéodory interpolation, and best rational approximations are considered to illustrate the power of Hankel and Toeplitz operators.

Function models for linear operators and free interpolation: This is a universal topic and, indeed, is the most influential operator theory technique in the post-spectral-theorem era. In this book, its capacity is tested by solving generalized Carleson-type interpolation problems.

Infinite-dimensional system theory and signal processing: This topic is the touchstone of the three previously developed techniques. The presence of this applied topic in a pure mathematics environment reflects important changes in the mathematical landscape of the last 20 years, in that the role of the main consumer and customer of harmonic, complex, and operator analysis has more and more passed from differential equations, scattering theory, and probability, to control theory and signal processing.

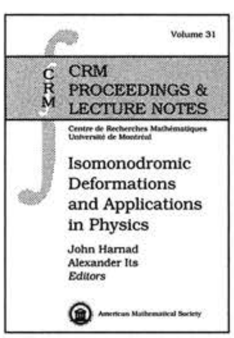
These volumes are geared toward a wide audience of readers, from graduate students to professional mathematicians. They develop an elementary approach while retaining an expert level that can be applied in advanced analysis and selected applications.

Contents: *Model operators and free interpolation:* Foreword to part C; The basic function model; Elements of spectral theory in the language of the characteristic function; Decompositions into invariant subspaces and free interpolation; *Analytic problems in linear system theory:* Foreword to part D; Basic theory; First optimizations: Multiplicity of the spectrum and the DISC; Eigenvector decompositions, vector valued exponentials, and squared optimization; A glance at bases of exponentials and of reproducing kernels; A brief introduction to H^∞ control; Bibliography; Author index; Subject index; Symbol index; Errata to volume 1.

Mathematical Surveys and Monographs, Volume 93

May 2002, 439 pages, Hardcover, ISBN 0-8218-2876-2, LC 2001053556, 2000 *Mathematics Subject Classification*: 47-02, 30-02, 93-02, 30D55, 47B35, 47A45, 93B05, 93C05, **Individual member $59**, List $98, Institutional member $78, Order code SURV/93N

Differential Equations



Isomonodromic Deformations and Applications in Physics

John Harnad, *University of Montreal, QC, Canada*, and **Alexander Its**, *Indiana University - Purdue University, Indianapolis*, Editors

The area of inverse scattering transform method or soliton theory has evolved over the past two decades in a vast variety of exciting new algebraic and analytic directions and has found numerous new applications. Methods and applications range from quantum group theory and exactly solvable statistical models to random matrices, random permutations, and number theory. The theory of isomonodromic deformations of systems of differential equations with rational coefficents, and most notably, the related apparatus of the Riemann-Hilbert problem, underlie the analytic side of this striking development.

The contributions in this volume are based on lectures given by leading experts at the CRM workshop (Montreal, Canada). Included are both survey articles and more detailed expositions relating to the theory of isomonodromic deformations, the Riemann-Hilbert problem, and modern applications.

The first part of the book represents the mathematical aspects of isomonodromic deformations; the second part deals mostly with the various appearances of isomonodromic deformations and Riemann-Hilbert methods in the theory of exactly solvable quantum field theory and statistical mechanical models, and related issues. The book elucidates for the first time in the current literature the important role that isomonodromic deformations play in the theory of integrable systems and their applications to physics.

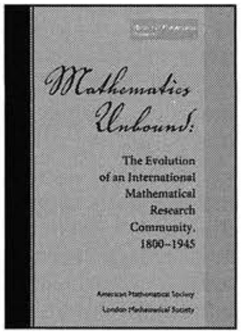
This item will also be of interest to those working in mathematical physics.

Contents: *Isomonodromic deformations:* **A. Bolibruch**, Inverse problems for linear differential equations with meromorphic coefficients; **J. Harnad**, Virasoro generators and bilinear equations for isomonodromic tau functions; **A. A. Kapaev**, Lax pairs for Painlevé equations; **D. A. Korotkin**, Isomonodromic deformations and Hurwitz spaces; **Y. Ohyama**, Classical solutions of Schlesinger equations and twistor theory; **M. A. Olshanetsky**, *W*-geometry and isomonodromic deformations; **C. A. Tracy** and **H. Widom**, Airy kernel and Painlevé II; *Applications in physics and related topics:* **M. Bertola**, Jacobi groups, Jacobi forms and their applications; **P. A. Clarkson** and **C. M. Cosgrove**, Symmetry, the Chazy equation and Chazy hierarchies; **F. Göhmann**, Universal correlations of one-dimensional electrons at low density; **F. Göhmann** and **V. E. Korepin**, A quantum version of the inverse scattering transformation; **Y. Nakamura**, Continued fractions and integrable systems; **A. Yu. Orlov** and **D. M. Scherbin**, Hypergeometric functions related to Schur functions and integrable systems; **J. Palmer**, Ising model scaling functions at short distance; **N. A. Slavnov**, The partition function of the six-vertex model as a Fredholm determinant.

CRM Proceedings & Lecture Notes, Volume 31

March 2002, 218 pages, Softcover, ISBN 0-8218-2804-5, LC 2002016306, 2000 *Mathematics Subject Classification*: 34Exx, 34Mxx, 35Qxx, 58Cxx, 81Txx, 30-XX, 33-XX, **Individual member $38**, List $64, Institutional member $51, Order code CRMP/31N

General and Interdisciplinary



Mathematics Unbound: The Evolution of an International Mathematical Research Community, 1800–1945

Karen Hunger Parshall, *University of Virginia, Charlottesville*, and **Adrian C. Rice**, *Randolph-Macon College, Ashland, VA*, Editors

Although today's mathematical research community takes its international character very much for granted, this "global nature" is relatively recent, having evolved over a period of roughly 150 years—from the beginning of the nineteenth century to the middle of the twentieth century. During this time, the practice of mathematics changed from being centered on a collection of disparate national communities to being characterized by an international group of scholars for whom the goal of mathematical research and cooperation transcended national boundaries. Yet, the development of an international community was far from smooth and involved obstacles such as war, political upheaval, and national rivalries. Until now, this evolution has been largely overlooked by historians and mathematicians alike.

This book addresses the issue by bringing together essays by twenty experts in the history of mathematics who have investigated the genesis of today's international mathematical community. This includes not only developments within component national mathematical communities, such as the growth of societies and journals, but also more wide-ranging political, philosophical, linguistic, and pedagogical issues.

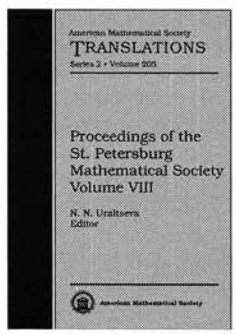
The resulting volume is essential reading for anyone interested in the history of modern mathematics. It will be of interest to mathematicians, historians of mathematics, and historians of science in general.

Copublished with the London Mathematical Society. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

Contents: **K. H. Parshall** and **A. C. Rice**, The evolution of an international mathematical research community, 1800–1945: An overview and an agenda; **I. Grattan-Guinness**, The end of dominance: The diffusion of French mathematics elsewhere, 1820–1870; **E. Ausejo** and **M. Hormigón**, Spanish initiatives to bring mathematics in Spain into the international mainstream; **S. E. Despeaux**, International mathematical contributions to British scientific journals, 1800–1900; **J. Lützen**, International participation in Liouville's *Journal de mathématiques pures et appliquées*; **H. Gispert**, The effects of war on France's international role in mathematics, 1870–1914; **T. Archibald**, Charles Hermite and German mathematics in France; **J. E. Barrow-Green**, Gösta Mittag-Leffler and the foundation and administration of *Acta Mathematica*; **L. Martini**, An episode in the evolution of a mathematical community: The case of Cesare Arzelà at Bologna; **A. Brigaglia**, The first international mathematical community: The *Circolo matematico di Palermo*; **J. J. Gray**, Languages for mathematics and the language of mathematics in a world of nations; **C. Sasaki**, The emergence of the Japanese mathematical community in the modern western style, 1855–1945; **J. W. Dauben**, Internationalizing mathematics east and west: Individuals and institutions in the emergence of a modern mathematical community in China; **X. Yibao**, Chinese–U. S. mathematical relations, 1859–1949; **D. D. Fenster**, American initiatives toward internationalization: The case of Leonard Dickson; **R. Siegmund-Schultze**, The effects of Nazi rule on the international participation of German mathematicians: An overview and two case studies; **S. L. Segal**, War, refugees, and the creation of an international mathematical community; **O. Lehto**, The formation of the international mathematical union; Index.

History of Mathematics, Volume 23

June 2002, approximately 416 pages, Hardcover, ISBN 0-8218-2124-5, 2000 *Mathematics Subject Classification*: 01A55, 01A60, 01A70, 01A72, 01A73, 01A74, 01A80, **All AMS members $68**, List $85, Order code HMATH/23N



Proceedings of the St. Petersburg Mathematical Society, Volume VIII

N. N. Uraltseva, *St. Petersburg State University, Russia*, Editor

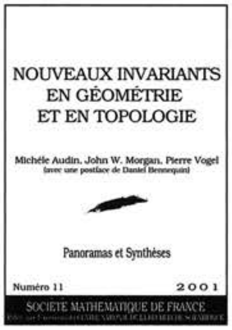
The articles in this collection present new results in partial differential equations, numerical analysis, probability theory, and geometry. The results, ideas, and methods given in the book will be of interest to a broad range of specialists.

Contents: **A. N. Borodin**, Distribution of the supremum for linear combinations of local times of random processes; **O. L. Vinogradov** and **V. V. Zhuk**, Sharp Jackson type inequalities for differentiable functions and minimization of the step of the moduli of continuity; **Yu. K. Dem'yanovich**, Biorthogonal system of minimal splines on a nonuniform grid; **V. V. Zhuk** and **G. I. Natanson**, S. N. Bernstein and direct and converse theorems of contructive function theory; **I. A. Ibragimov**, On S. N. Bernstein's work in probability; **N. M. Ivochkina**, Some trends in the development of the theory of fully nonlinear second order evolution equations; **R. Lauter**, **B. A. Plamenevskii**, and **O. V. Sarafanov**, Coefficients in asymptotics of solutions to pseudodifferential equations on manifolds with conical points; **G. A. Leonov**, The Brockett problem in the theory of nonstationary stabilization of linear differential equations; **V. N. Malozemov** and **A. N. Sergeev**, Discrete nonperiodic splines on a uniform grid; **A. I. Nazarov** and **Ya. Yu. Nikitin**, Some extremal problems for Gaussian and empirical random fields; **F. V. Petrov** and **S. E. Rukshin**, Two theorems about convex polyhedra.

American Mathematical Society Translations—Series 2, Volume 205

May 2002, 208 pages, Hardcover, ISBN 0-8218-2941-6, 2000 *Mathematics Subject Classification*: 01A60, 35K60, 35S15, 41-03, 41A25, 52B45, 58J40, 60-03, 60G55, 60G60, 60J65, 65D07, 93C10; 41A15, 47G30, 60Fxx, **Individual member $57**, List $95, Institutional member $76, Order code TRANS2/205N

Geometry and Topology



Nouveaux Invariants en Geometrie et en Topologie

Michéle Audin, *Université Louis Pasteur et CNRS, Strasbourg, France*, **John W Morgan**, *Columbia University, New York*, and **Pierre Vogel** and **Daniel Bennequin**, *Université Paris, France*

A publication of the Société Mathématique de France.

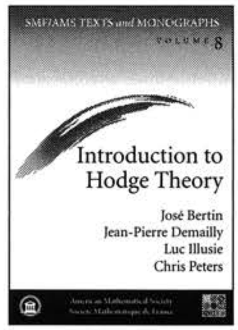
This volume offers a presentation of recent developments of three types of geometric invariants: symplectic invariants, including Gromov-Witten invariants, invariants of four-manifolds and Seiberg-Witten theory, and finite type invariants for three-manifolds. The book concludes with a description of the links between these three types of invariants and contemporary quantum field theory. The text is in French and English.

Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: M. Audin, Invariants en géométrie symplectique via les courbes holomorphes; **J. W. Morgan**, Seiberg-Witten invariants; **P. Vogel**, Invariants de type fini; **D. Bennequin**, Invariants contemporains.

Panoramas et Synthèses, Number 11

December 2001, 159 pages, Softcover, ISBN 2-85629-111-2, 2000 *Mathematics Subject Classification*: 53Dxx, 53C23, 57M25, 57M27, 57R57, 58Jxx, 81-XX, **Individual member $30**, List $33, Order code PASY/11N



Introduction to Hodge Theory

José Bertin, *University of Grenoble I, St. Martin D'Heres, France*, **Jean-Pierre Demailly**, *University of Grenoble I, St. Martin d'Heres, France*, **Luc Illusie**, *University of Paris-Sud, Orsay, France*, and **Chris Peters**, *University of Grenoble I, St. Martin d'Heres, France*

From reviews of the French Edition:

The present book ... may be regarded as a masterly introduction to Hodge theory in its classical and very recent, analytic and algebraic aspects ... it is by far much more than only an introduction to the subject. The material leads the reader to the forefront of research in many areas related to Hodge theory, and that in a detailed highly self-contained manner ... this text is also a valuable source for active researchers and teachers in the field ...

—Zentralblatt MATH

The book under review is a collection of three articles about Hodge theory and related developments, which are all aimed at non-experts and fulfill, in an extremely satisfactory manner, two functions. First, the basic methods used in the theories are discussed and developed in great detail; second, some newer developments are described, giving the reader a good overview of the more important applications. Furthermore, the style makes these articles a joy to work through, even for the mathematician not encountering these subjects for the first time.

—Mathematical Reviews

Hodge theory originated as an application of harmonic theory to the study of the geometry of compact complex manifolds. The ideas have proved to be quite powerful, leading to fundamentally important results throughout algebraic geometry. This book consists of expositions of various aspects of modern Hodge theory. Its purpose is to provide the nonexpert reader with a precise idea of the current status of the subject. The three chapters develop distinct but closely related subjects: L^2 Hodge theory and vanishing theorems; Frobenius and Hodge degeneration; variations of Hodge structures and mirror symmetry. The techniques employed cover a wide range of methods borrowed from the heart of mathematics: elliptic PDE theory, complex differential geometry, algebraic geometry in characteristic p, cohomological and sheaf-theoretic methods, deformation theory of complex varieties, Calabi-Yau manifolds, singularity theory, etc. A special effort has been made to approach the various themes from their most natural starting points. Each of the three chapters is supplemented with a detailed introduction and numerous references. The reader will find precise statements of quite a number of open problems that have been the subject of active research in recent years.

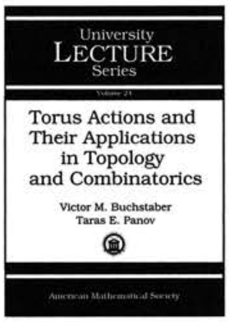
The reader should have some familiarity with differential and algebraic geometry, with other prerequisites varying by chapter. The book is suitable as an accompaniment to a second course in algebraic geometry.

SMF members are entitled to AMS member discounts.

Contents: J.-P. Demailly, L^2 Hodge theory and vanishing theorems; **L. Illusie**, Frobenius and Hodge degeneration; **J. Bertin** and **C. Peters**, Variations of Hodge structure, Calabi-Yau manifolds and mirror symmetry.

SMF/AMS Texts and Monographs, Volume 8

May 2002, 232 pages, Softcover, ISBN 0-8218-2040-0, LC 2002019611, 2000 *Mathematics Subject Classification*: 14C30, 14D07, 14F17, 13A35, 58A14, 14-02, 32-02; 81-02, **All AMS members $52**, List $65, Order code SMFAMS/8N



Torus Actions and Their Applications in Topology and Combinatorics

Victor M. Buchstaber and **Taras E. Panov**, *Moscow State University, Russia*

Here, the study of torus actions on topological spaces is presented as a bridge connecting combinatorial and convex geometry with commutative and homological algebra, algebraic geometry, and topology. This established link helps in understanding the geometry and topology of a space with torus action by studying the combinatorics of the space of orbits. Conversely, subtle properties of a combinatorial object can be realized by interpreting it as the orbit structure for a proper manifold or as a complex acted on by a torus. The latter can be a symplectic manifold with Hamiltonian torus action, a toric variety or manifold, a subspace arrangement complement, etc., while the combinatorial objects include simplicial and cubical complexes, polytopes, and arrangements. This approach also provides a natural topological interpretation in terms of torus actions of many constructions from commutative and homological algebra used in combinatorics.

The exposition centers around the theory of moment-angle complexes, providing an effective way to study invariants of triangulations by methods of equivariant topology. The book includes many new and well-known open problems and would be suitable as a textbook. It will be useful for specialists both in topology and in combinatorics and will help to establish even tighter connections between the subjects involved.

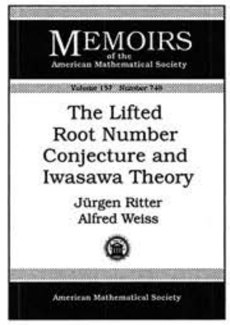
This item will also be of interest to those working in discrete mathematics and combinatorics.

Contents: Introduction; Polytopes; Topology and combinatorics of simplicial complexes; Commutative and homological algebra of simplicial complexes; Cubical complexes; Toric and quasitoric manifolds; Moment-angle complexes; Cohomology of moment-angle complexes and combinatorics of triangulated manifolds; Cohomology rings of subspace arrangement complements; Bibliography; Index.

University Lecture Series, Volume 24

May 2002, approximately 152 pages, Softcover, ISBN 0-8218-3186-0, LC 2002018243, 2000 *Mathematics Subject Classification*: 52B70, 57Q15, 57R19, 14M25, 52B05, 13F55, 05B35, **All AMS members $23**, List $29, Order code ULECT/24N

Number Theory



The Lifted Root Number Conjecture and Iwasawa Theory

Jürgen Ritter, *University of Augsburg, Germany*, and **Alfred Weiss**, *University of Alberta, Edmonton, AB, Canada*

Contents: Introduction; The Tripod; Restriction, deflation; change of maps, and variance with S; Definition of $\mho_S$; Ω_Φ as a shadow of $\mho_S$; $\mho_S$ over the maximal order in the case when G is abelian; Local considerations; Towards a representing homomorphism for Ω_{φ_L}; Real cyclotomic extensions tame over l; References.

Memoirs of the American Mathematical Society, Volume 157, Number 748

May 2002, 90 pages, Softcover, ISBN 0-8218-2928-9, LC 2002018238, 2000 *Mathematics Subject Classification*: 11R23, 11R27, 11R32, 11R33, 11R37, 11R42, 11S20, 11S23, 11S31, 11S40; 11-02, 11R18, 11R65, **Individual member $29**, List $48, Institutional member $38, Order code MEMO/157/748N

Previously Announced Publications

Pick Interpolation and Hilbert Function Spaces

Jim Agler, *University of California at San Diego*, and **John E. McCarthy**, *Washington University, St. Louis, MO*

The book first rigorously develops the theory of reproducing kernel Hilbert spaces. The authors then discuss the Pick problem of finding the function of smallest H^∞ norm that has specified values at a finite number of points in the disk. Their viewpoint is to consider H^∞ as the multiplier algebra of the Hardy space and to use Hilbert space techniques to solve the problem. This approach generalizes to a wide collection of spaces.

The authors then consider the interpolation problem in the space of bounded analytic functions on the bidisk and give a complete description of the solution. They then consider very general interpolation problems. The book includes developments of all the theory that is needed, including operator model theory, the Arveson extension theorem, and the hereditary functional calculus.

Graduate Studies in Mathematics, Volume 44

April 2002, 308 pages, Hardcover, ISBN 0-8218-2898-3, LC 2001056501, 2000 *Mathematics Subject Classification*: 47A57, 30E05, 46E20, 32A70, **All AMS members $39**, List $49, Order code GSM/44RT204

Celestial Mechanics

Dedicated to Donald Saari for his 60th Birthday

A. Chenciner, *Institute de Mécanique Céleste, Paris, France*, **R. Cushman**, *University of Utrecht, Netherlands*, and **C. Robinson** and **Z. Xia**, *Northwestern University, Evanston, IL*, Editors

This volume reflects the proceedings from an international conference on celestial mechanics held at Northwestern University (Evanston, IL) in celebration of Donald Saari's sixtieth birthday. Many leading experts and researchers presented their recent results.

Don Saari's significant contribution to the field came in the late 1960s through a series of important works. His work revived the singularity theory in the n-body problem which was started by Poincaré and Painlevé. Saari's solution of the Littlewood conjecture, his work on singularities, collision and noncollision, on central configurations, his decompositions of configurational velocities, etc., are still much studied today and were reflected throughout the conference.

This volume covers various topics of current research, from central configurations to stability of periodic orbits, from variational methods to diffusion mechanisms, from the dynamics of secular systems to global dynamics of the solar systems via frequency analysis, from Hill's problem to the low energy transfer orbits and mission design in space travel, and more. This classic field of study is very much alive today and this volume offers a comprehensive representation of the latest research results.

Contributors include: A. Albouy, J. Llibre, E. Belbruno, F. Beukers, R. Cushman, A. Chenciner, M. Corbera, W. S. Koon, J. E. Marsden, S. D. Ross, M. W. Lo, E. A. Lacomba, E. Perez-Chavela, C. Marchal, R. Moeckel, R. Montgomery, P. H. Rabinowitz, E. W. Stredulinsky, C. Robinson, C. Simó, Y.-S. Sun, J.-L. Zhou, J.-Q. Zheng, M. Valtonen, Q. Wang, and Z. Xia.

Contemporary Mathematics, Volume 292

February 2002, approximately 280 pages, Softcover, ISBN 0-8218-2902-5, LC 2001055364, 2000 *Mathematics Subject Classification*: 70Fxx, 70Hxx, 37N05, 37Jxx, **Individual member $41**, List $69, Institutional member $55, Order code CONM/292RT204

Wavelet Analysis and Applications

Donggao Deng, *Zhongshan University, Guangzhou, People's Republic of China*, **Daren Huang**, *Zhejiang University, Hangzhou, People's Republic of China*, **Rong-Qing Jia**, *University of Alberta, Edmonton, AB, Canada*, **Wei Lin**, *Zhongshan University, Guangzhou, People's Republic of China*, and **Jianzhong Wang**, *Sam Houston State University, Huntsville, TX*, Editors

Wavelet analysis has been one of the major research directions in science in the last decade. More and more mathematicians and scientists join this exciting research area. Certainly, wavelet analysis has had a great impact in areas such as approximation theory, harmonic analysis, and scientific computation. More importantly, wavelet analysis has shown great potential in applications to information technology such as signal processing, image processing, and computer graphics.

China has played a significant role in this development of wavelet analysis as evidenced by many fruitful theoretical results and practical applications. A conference on wavelet analysis and its applications was organized to exchange ideas and results with international research groups at Zhongshan University (Guangzhou, China). This volume contains the proceedings from that conference.

Comprised here are selected papers from the conference, covering a wide range of research topics of current interest. Many significant results are included in the study of refinement equations and refinable functions, properties and construction of wavelets, spline wavelets, multi-wavelets, wavelet packets, shift-invariant spaces, approximation schemes and subdivision algorithms, and tilings. Several papers also focus on applications of wavelets to numerical solutions of partial differential equations and integral equations, image processing and facial recognition, computer vision, and feature extraction from data.

Titles in this series are copublished with International Press, Cambridge, MA.

Contributors include: A. Aldroubi, Q. Sun, W.-S. Tang, R. Ashino, C. Heil, M. Nagase, R. Vaillancourt, S. Basu, C. A. Micchelli, P. Olsen, O. Bratteli, P. E. T. Jorgensen, G. J. Chae, H. O. Kim, R. Y. Kim, W. Chen, W. Lin, D.-Q. Dai, T. N. T. Goodman, S. L. Lee, L. Gori, F. Pitolli, B. Han, S. D. Riemenschneider, D. P. Hardin, T. A. Hogan, D. Huang, Z. Wang, Z. Zhang, R.-Q. Jia, Q.-T. Jiang, J. Ning, J. K. Lim, I. Kirat, K.-S. Lau, X.-z. Liang, M.-c. Liu, S. Li, I. Ya. Novikov, J. Tang, S. Kawato, J. Ohya, J. Wang, M. V. Wickerhauser, L. Yang, Y. Y. Tang, P. C. Yuen, G. C. Feng, J. H. Lai, D. Q. Dai, and D.-X. Zhou.

AMS/IP Studies in Advanced Mathematics, Volume 25

March 2002, 326 pages, Softcover, ISBN 0-8218-2991-2, LC 2001056568, 2000 *Mathematics Subject Classification*: 42C40, **All AMS members $44**, List $55, Order code AMSIP/25RT204

Diffusions, Superdiffusions and Partial Differential Equations

E. B. Dynkin, *Cornell University, Ithaca, NY*

Interactions between the theory of partial differential equations of elliptic and parabolic types and the theory of stochastic processes are beneficial for both probability theory and analysis. At the beginning, mostly analytic results were used by probabilists. More recently, analysts (and physicists) took inspiration from the probabilistic approach. Of course, the development of analysis in general and of the theory of partial differential equations in particular, was motivated to a great extent by problems in physics. A difference between physics and probability is that the latter provides not only an intuition, but also rigorous mathematical tools for proving theorems.

The subject of this book is connections between linear and semilinear differential equations and the corresponding Markov processes called diffusions and superdiffusions. Most of the book is devoted to a systematic presentation (in a more general setting, with simplified proofs) of the results obtained since 1988 in a series of papers of Dynkin and Dynkin and Kuznetsov. Many results obtained originally by using superdiffusions are extended in the book to more general equations by applying a combination of diffusions with purely analytic methods. Almost all chapters involve a mixture of probability and analysis.

Similar to the other books by Dynkin, *Markov Processes* (Springer-Verlag), *Controlled Markov Processes* (Springer-Verlag), and *An Introduction to Branching Measure-Valued Processes* (American Mathematical Society), this book can become a classical account of the presented topics.

This item will also be of interest to those working in differential equations.

Colloquium Publications, Volume 50

March 2002, 236 pages, Hardcover, ISBN 0-8218-3174-7, LC 2001058957, 2000 *Mathematics Subject Classification*: 60J60, 35Jxx; 35K55, 60J65, **All AMS members $39**, List $49, Order code COLL/50RT204

Supplementary Reading

Ruled Varieties

An Introduction to Algebraic Differential Geometry

Gerd Fischer and **Jens Piontkowski**

A publication of the Vieweg Verlag.

The simplest surfaces, aside from planes, are the traces of a line moving in ambient space or, more precisely, the unions of one-parameter families of lines. The fact that these lines can be produced using a ruler explains their name, "ruled surfaces". The mechanical production of ruled surfaces is relatively easy, and they can be visualized by means of wire models. These models are not only of practical use, but also provide artistic inspiration.

Mathematically, ruled surfaces are the subject of several branches of geometry, especially differential geometry and algebraic geometry. In classical geometry, especially differential geometry and algebraic geometry. In classical geometry, we know that surfaces of vanishing Gaussian curvature have a ruling that is even developable. Analytically, developable means that the tangent plane is the same for all points of the ruling line, which is equivalent to saying that the surface can be covered by pieces of paper. A classical result from algebraic geometry states that rulings are very rare for complex algebraic surfaces in three-space: Quadrics have two rulings, smooth cubics contain precisely 27 lines, and in general, a surface of degree at least four contains no line at all. There are exceptions, such as cones or tangent surfaces of curves. It is also well-known that these two kinds of surfaces are the only developable ruled algebraic surfaces in projective three-space.

The natural generalization of a ruled surface is a ruled variety, i.e., a variety of arbitrary dimension that is "swept out" by a moving linear subspace of ambient space. It should be noted that a ruling is not an intrinsic but an extrinsic property of a variety, which only makes sense relative to an ambient affine or projective space. This book considers ruled varieties mainly from the point of view of complex projective algebraic geometry, where the strongest tools are available. Some local techniques could be generalized to complex analytic varieties, but in the real analytic or even differentiable case there is little hope for generalization: The reason being that rulings, and especially developable rulings, have the tendency to produce severe singularities.

As in the classical case of surfaces, there is a strong relationship between the subject of this book, ruled varieties, and differential geometry. For the purpose of this book, however, the Hermitian Fubini-Study metric and the related concepts of curvature are not necessary. In order to detect developable rulings, it suffices to consider a bilinear second fundamental form that is the differential of the Gauss map. This method does not give curvature as a number, but rather measures the degree of vanishing of curvature; this point of view has been used in a fundamental paper of Griffiths and Harris. One of the purposes of this book is to make parts of this paper more accessible, to give detailed and more elementary proofs, and to report on recent progress in this area.

The AMS is exclusive distributor in North America, and non-exclusive distributor worldwide except in Germany, Switzerland, Austria, and Japan.

Vieweg Advanced Lectures in Mathematics

May 2001, 141 pages, Softcover, ISBN 3-528-03138-7, 2000 *Mathematics Subject Classification*: 14M99, 53A20, **All AMS members $41**, List $45, Order code VWALM/8RT204

Supplementary Reading

Conformal Field Theory and Topology

Toshitaki Kohno, *University of Tokyo, Japan*

Geometry and physics have been developed with a strong influence on each other. One of the most remarkable interactions between geometry and physics since 1980 has been an application of quantum field theory to topology and differential geometry. This book focuses on a relationship between two-dimensional quantum field theory and three-dimensional topology which has been studied intensively since the discovery of the Jones polynomial in the middle of the 1980s and Witten's invariant for 3-manifolds derived from Chern-Simons gauge theory. An essential difficulty in quantum field theory comes from infinite-dimensional freedom of a system. Techniques dealing with such infinite-dimensional objects developed in the framework of quantum field theory have been influential in geometry as well. This book gives an accessible treatment for a rigorous construction of topological invariants originally defined as partition functions of fields on manifolds.

The book is organized as follows: The Introduction starts from classical mechanics and explains basic background materials in quantum field theory and geometry. Chapter 1 presents conformal field theory based on the geometry of loop groups. Chapter 2 deals with the holonomy of conformal field theory. Chapter 3 treats Chern-Simons perturbation theory. The final chapter discusses topological invariants for 3-manifolds derived from Chern-Simons perturbation theory.

This item will also be of interest to those working in mathematical physics.

Translations of Mathematical Monographs (*Iwanami Series in Modern Mathematics*), Volume 210

May 2002, approximately 184 pages, Softcover, ISBN 0-8218-2130-X, 2000 *Mathematics Subject Classification*: 54C40, 14E20; 46E25, 20C20, **All AMS members $28**, List $35, Order code MMONO/210RT204

Recommended Text

Introduction to the Theory of Random Processes

N. V. Krylov, *University of Minnesota, Minneapolis*

This book concentrates on some general facts and ideas of the theory of stochastic processes. The topics include the Wiener process, stationary processes, infinitely divisible processes, and Itô stochastic equations.

Basics of discrete time martingales are also presented and then used in one way or another throughout the book. Another common feature of the main body of the book is using stochastic integration with respect to random orthogonal measures. In particular, it is used for spectral representation of trajectories of stationary processes and for proving that Gaussian stationary processes with rational spectral densities are components of solutions to stochastic equations. In the case of infinitely divisible processes, stochastic integration allows for obtaining a representation of trajectories through jump measures. The Itô stochastic integral is also introduced as a particular case of stochastic integrals with respect to random orthogonal measures.

Although it is not possible to cover even a noticeable portion of the topics listed above in a short book, it is hoped that after having followed the material presented here, the reader will have acquired a good understanding of what kind of results are available and what kind of techniques are used to obtain them.

With more than 100 problems included, the book can serve as a text for an introductory course on stochastic processes or for independent study.

Other works by this author published by the AMS include, *Lectures on Elliptic and Parabolic Equations in Hölder Spaces* and *Introduction to the Theory of Diffusion Processes.*

Graduate Studies in Mathematics, Volume 43

April 2002, approximately 240 pages, Hardcover, ISBN 0-8218-2985-8, 2000 *Mathematics Subject Classification*: 60-01; 60G99, **All AMS members $28**, List $35, Order code GSM/43RT204

Quantum Computation

A Grand Mathematical Challenge for the Twenty-First Century and the Millennium

Samuel J. Lomonaco, Jr., Editor

This book presents written versions of the eight lectures given during the AMS Short Course held at the Joint Mathematics Meetings in Washington, D.C. The objective of this course was to share with the scientific community the many exciting mathematical challenges arising from the new field of quantum computation and quantum information science. The course was geared toward demonstrating the great breadth and depth of this mathematically rich research field. Interrelationships with existing mathematical research areas were emphasized as much as possible. Moreover, the course was designed so that participants with little background in quantum mechanics would, upon completion, be prepared to begin reading the research literature on quantum computation and quantum information science.

Based on audience feedback and questions, the written versions of the lectures have been greatly expanded, and supplementary material has been added. The book features an overview of relevant parts of quantum mechanics with an introduction to quantum computation, including many potential quantum mechanical computing devices; introduction to quantum algorithms and quantum complexity theory; in-depth discussion on quantum error correcting codes and quantum cryptography; and finally, exploration into diverse connections between quantum computation and various areas of mathematics and physics.

Contributors include: S. J. Lomonaco, Jr., H. E. Brandt, P. W. Shor, U. V. Vazirani, D. Gottesman, A. Kitaev, and L. H. Kauffman.

Proceedings of Symposia in Applied Mathematics, Volume 58

March 2002, approximately 360 pages, Hardcover, ISBN 0-8218-2084-2, LC 2001055366, 2000 *Mathematics Subject Classification*: 81-01, 81-02, 81P68, 68Q05, 94A60; 22E70, 57M99, 81V80, 94A15, **Individual member $41**, List $69, Institutional member $55, Order code PSAPM/58RT204

Far-from-Equilibrium Dynamics

Yasumasa Nishiura, *Hokkaido University, Sapporo, Japan*

This book is devoted to the study of evolution of nonequilibrium systems. Such a system usually consists of regions with different dominant scales, which coexist in the space-time where the system lives. In the case of high nonuniformity in special direction, one can see patterns separated by clearly distinguishable boundaries or interfaces.

The author considers several examples of nonequilibrium systems. One of the examples describes the invasion of the solid phase into the liquid phase during the crystallization process. Another example is the transition from oxidized to reduced states in certain chemical reactions. An easily understandable example of the transition in the temporal direction is a sound beat, and the author describes typical patterns associated with this phenomenon.

The main goal of the book is to present a mathematical approach to the study of highly nonuniform systems and to illustrate it with examples from physics and chemistry. The two main theories discussed are the theory of singular perturbations and the theory of dissipative systems. A set of carefully selected examples of physical and chemical systems nicely illustrates the general methods described in the book.

This item will also be of interest to those working in differential equations.

Translations of Mathematical Monographs (*Iwanami Series in Modern Mathematics*), Volume 209

March 2002, approximately 336 pages, Softcover, ISBN 0-8218-2625-5, 2000 *Mathematics Subject Classification*: 34D15, 35B25, 35B32, 35B40, 35K57, 37D10, 37L10, 74N20, **All AMS members $47**, List $59, Order code MMONO/209RT204

Classified Advertisements

Positions available, items for sale, services available, and more

MASSACHUSETTS

HARVARD UNIVERSITY
Bauer Center for Genomics Research (CGR)

Applications are invited for Fellow positions from exceptional researchers at the post-doctoral level who are interested in interdisciplinary genomics research. CGR funds each Fellow, along with a team of 1-2 post-docs or technicians, for 3-5 years to work in one of three areas: applying genomics techniques to biological problems, developing new experimental methods for collecting genome-wide information, or building new analytical, computational or theoretical tools for analyzing genome-wide data. We welcome any proposal directed at a system-wide approach to biology.

For more information, visit the website: `http://cgr.harvard.edu/fellows/fellows_program.html`. To apply, please send your c.v. and a few pages describing your proposed research to Laura Garwin, Director of Research Affairs, Bauer Center for Genomics Research, 16 Divinity Avenue, Cambridge, MA 02138, USA, or email them to `lgarwin@cgr.harvard.edu`. Please also arrange for three letters of recommendation to be sent. There is no deadline for applications, but for fullest consideration, all materials should arrive by April 30, 2002.

Harvard University is an Affirmative Action, Equal Opportunity Employer.

MICHIGAN

MICHIGAN STATE UNIVERSITY
proMSc Program in Industrial Mathematics
East Lansing, MI 48824

Direct your students toward one of the professional M.Sc. programs. Industry needs business-savvy mathematicians. See `http://www.sciencemasters.com/`.

NEVADA

UNIVERSITY OF NEVADA, RENO
Department of Mathematics

The Department of Mathematics anticipates the possibility of one or more full-time three-year visiting lecturer positions (with year by year continuation dependent on satisfactory teaching evaluations) in mathematics beginning in the fall of 2002. Minimum qualifications are a master's degree in a mathematical science and demonstrated excellence in teaching mathematics at the college level. The usual teaching load is 12 credit hours per semester at the level of calculus and below. Evidence of excellence in teaching may include extent and variety of classroom teaching experience, student evaluation of instruction (if available), letters from responsible faculty, and results of oral interviews, including communication skills. Candidates are sought whose interests are compatible with those of the current faculty. See `http://www.unr.edu/math/faculty/`. For the full advertisement with instructions see `http://www.unr.edu/math/` or `http://jobs.unr.edu/`. To guarantee full consideration, applications should be received by April 19, 2002. EEO/AA

UNIVERSITY OF NEVADA, RENO
Department of Mathematics

The Department of Mathematics anticipates the possibility of one or more full-time three-year teaching postdoctoral appointments (with year by year continuation dependent on satisfactory teaching evaluations) in mathematics beginning in July 2002. The usual teaching load for this position is 9-10 credit hours per semester (usually with just one preparation) at the level of calculus and below. The candidate is also expected to maintain a vigorous program of research and professional development. The department is committed to mentoring new graduates in this process. Candidates are sought whose interests are compatible with those of the current faculty. See `http://www.unr.edu/math/faculty/`. For the full advertisement with instructions see `http://www.unr.edu/math/` or `http://jobs.unr.edu/`. Applications will be accepted until the positions are filled. To guarantee full consideration, applications should be received by April 19, 2002. EEO/AA

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2002 rate is \$100 per inch or fraction thereof on a single column (one-inch minimum), calculated from top of headline. Any fractional text of 1/2 inch or more will be charged at the next inch rate. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified advertising.

Upcoming deadlines for classified advertising are as follows: May 2002 issue–February 26, 2002; June/July 2002 issue–April 24, 2002; August 2002 issue–May 24, 2002; September 2002 issue–June 26, 2002, October 2002 issue–July 25, 2002; November 2002 issue–August 27, 2002.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send e-mail to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

VIRGINIA

NATIONAL SCIENCE FOUNDATION
Director, Division of Mathematical Sciences
Arlington, VA

NSF's Directorate for Mathematical and Physical Sciences (MPS) seeks candidates for Director of its Division of Mathematical Sciences. The division supports research and education in mathematics and closely related areas. Information about the division's activities may be found at `http://www.nsf.gov/mps/divisions/dms/start.htm`.

Appointment to this Senior Executive Service position may be on a career basis, or on a 2-to-3 year limited term basis, with a salary range of $125,972 to $138,200. Alternatively, the incumbent may be assigned under Intergovernmental Personnel Act provisions. Announcement S20020068 with position requirements and application procedures is located on the NSF home page at `http://www.nsf.gov/home/chart/work.htm#hrm`. Applicants may also obtain the announcements by contacting the Executive Personnel and Development Branch at 703-292-8755 (hearing impaired individuals may call TDD 703-292-8044). Applications must be received by May 15, 2002.

NSF is an Equal Opportunity Employer committed to employing a highly qualified staff reflecting the diversity of our nation.

FRANCE

THE INSTITUT DES HAUTES ETUDES SCIENTIFIQUES (IHES)
Post-doctoral Fellowships

The Institut des Hautes Etudes Scientifiques (IHES), an international research institute based near Paris (France), whose activity has been so far focused on mathematics and theoretical physics, started a new research activity at the interface of these disciplines with genetics and molecular biology. One of its aims is to contribute to the analysis and design of biological experiments by developing new tools to represent and study biological data.

The Institute offers two two-year postdoctoral positions for doctors having a background in mathematics, computer science, biophysics, biology and/or biochemistry, with special interests in one of the following topics: biophysics of macromolecules, macromolecular networks, sequence analysis, molecular evolution and phylogenetic trees, databases and datamining, architecture of high-throughput experiments, pattern recognition, as well as related areas. Applications should be sent before April 30, 2002, to:

Mme Helga Dernois
Institut des Hautes
Etudes Scientifiques
35, route de Chartres
F 91440 - Bures-sur-Yvette, France
tel: +33 1 6092 6667
fax: +33 1 6092 6669
website: www.ihes.fr
e-mail: `dernois@ihes.fr`

The decision will be made by May 30, 2002.

PUERTO RICO

UNIVERSITY OF PUERTO RICO
Department of Mathematics and Computer Science
Río Piedras Campus
San Juan, Puerto Rico

We announce the opening of two tenure-track positions, beginning in August 2002. The candidates should have a Ph.D. in the mathematical sciences and should be willing to teach and supervise thesis at undergraduate and graduate levels. Conversational knowledge of Spanish is a plus.

One position requires a strong commitment for teaching and continuing and directing research in the areas of statistics, probability and its applications, and/or actuarial science.

The second position requires specific interest in the improvement of teaching at the introductory levels and research in any area of mathematical sciences. At least one of the recommendation letters for this position should address teaching.

Salary will depend on qualifications, according to the scale of the University of Puerto Rico, Río Piedras Campus. Please send a curriculum vitae, a letter describing the interest and projects of the candidate, and three letters of recommendation by mail to the address below. It would be useful to have a copy of the academic transcripts and diplomas. The evaluation of candidates will start on March 1, 2002, and will continue until the positions are filled.

UPR is an Equal Opportunity Employer.
University of Puerto Rico, Río Piedras
Department of Mathematics
and Computer Science
Luis Raúl Pericchi, Chairman
PO Box 23355,
San Juan, PR 00931-3355
e-mail:
`pericchi@goliath.cnnet.clu.edu`
fax: 787-281-0651

International Mathematics Research Notices

Web Site: http://imrn.hindawi.com

AIMS AND SCOPE

IMRN provides very fast publication of research articles of high current interest in all areas of mathematics. All articles are fully refereed and are judged by their contribution to advancing the state of the science of mathematics. Issues are published as frequently as necessary. IMRN is expected to publish 36+ issues in 2002. The articles of the IMRN are reviewed/indexed in COMPUMATH Citation Index, INSPEC, ISI Alerting Services, Mathematical Reviews, PubSCIENCE, Science Citation Index, SciSearch, and Zentralblatt für Mathematik.

INSTRUCTIONS FOR AUTHORS

IMRN is devoted to advancing the state of the science of mathematics by publishing research articles of high current interest in all fields of mathematics. Articles of any length are welcome and all articles are refereed and judged for correctness, interest, originality, depth, and applicability. There are no page charges. Submissions are made by email to submit@imrn.hindawi.com. An abstract for each article should be included. A copy may also be sent to an editor. Only an acknowledgment from the editorial office officially establishes the date of receipt. Submissions not prepared using $\TeX$ should be typed or printed on one side of the page, be double-spaced (including references), have ample margins, and be accompanied by a page that lists all potentially ambiguous notations. Check contact information for submission by fax or post. When articles are accepted, production begins immediately; authors should be available to assist the editorial staff.

FORTHCOMING ARTICLES

- A Counterexample to the Hodge Conjecture Extended to Kähler Varieties, *Claire Voisin*
- Asymptotic Properties of a Family of Solutions of the Painlevé Equation P_{VI}, *Ovidiu Costin and Rodica D. Costin*
- Formulas of Ramanujan for the Power Series Coefficients of Certain Quotients of Eisenstein Series, *Bruce C. Berndt, Paul R. Bialek, and Ae Ja Yee*
- G-Bundles, Isomonodromy, and Quantum Weyl Groups, *P. P. Boalch*
- Generating Functions for Intersection Numbers on Moduli Spaces of Curves, *Andrei Okounkov*
- Invariant Stein Domains in Stein Symmetric Spaces and a Nonlinear Complex Convexity Theorem, *Simon Gindikin and Bernhard Krötz*
- Multivalued Minimal Graphs and Properness of Disks, *Serguei Barannikov*
- On a Sharp Estimate for Oscillatory Integrals Associated with the Schrödinger Equation, *Giacomo Gigante and Fernando Soria*
- Singular Principal Bundles over Higher-Dimensional Manifolds and Their Moduli Spaces, *Alexander H. W. Schmitt*
- Spectral Deformations and Eisenstein Series Associated with Modular Symbols, *Yiannis N. Petridis*
- Uniform Approximate Functional Equation for Principal L-Functions, *Gergely Harcos*

SUBSCRIPTION INFORMATION

Institutional subscription rates for year 2002 (36+ issues) are $1190.00 for print and online editions, $952.00 for online edition only. Please contact orders@hindawi.com for personal subscription rates, multiple year subscriptions, or purchase of back volumes. Back volumes are available starting from the volume of 1991. IMRN is also available as part of "Hindawi Mathematics Bundle" and "Hindawi Journals Bundle." Subscription information for these bundles is available at http://www.hindawi.com/subs.html.

HINDAWI Hindawi Publishing Corporation, P.O. Box 3079, Cuyahoga Falls, OH 44223, USA
Fax 1-866-446-3294 (USA, toll-free), 31-20-5248282 (Amsterdam, Netherlands)

AMS APPLICATION FOR MEMBERSHIP 2002

AMERICAN MATHEMATICAL SOCIETY

JANUARY—DECEMBER

WWW.AMS.ORG/MEMBERSHIP

Please read the "Membership Categories" section of this form to determine the membership category for which you are eligible. Then fill out this application and return it as soon as possible.

Date ______________ **20** ____

____

Family Name First Middle

Place of Birth: _______________________________________

City State Country

Date of Birth: _______________________________________

Day Month Year

If formerly a member of AMS, please indicate dates ____________________

Check here if you are now a member of either ☐ MAA or ☐ SIAM

Degrees, with institutions and dates ________________________________

____

____

Present position _______________________________________

Firm or institution _____________________________________

____

____

City State Zip/Country

Primary Fields of Interest (choose five from the list at right)

_________ _________ _________ _________ _________

Secondary Fields of Interest (choose from the list at right)

_________ _________ _________ _________ _________

_________ _________ _________ _________ _________

Address for all mail

____

____

____

____

Telephone: home: (____)____________ office: (____)____________

Fax: (____)________________

E-mail: _______________________________________

____

Signature

☐ ***Check here if you would like to receive your dues renewal notices electronically.***

☐ ***Check here to receive free quarterly* PUBLICATIONS UPDATE *by mail.***

Fields of Interest

If you wish to be on the mailing lists to receive information about publications in fields of mathematics in which you have an interest, please consult the list of major headings below. These categories will be added to your computer record so that you will be informed of new publications or special sales in the fields you have indicated.

00 General
01 History and biography
03 Mathematical logic and foundations
05 Combinatorics
06 Order, lattices, ordered algebraic structures
08 General algebraic systems
11 Number theory
12 Field theory and polynomials
13 Commutative rings and algebras
14 Algebraic geometry
15 Linear and multilinear algebra; matrix theory
16 Associative rings and algebras
17 Nonassociative rings and algebras
18 Category theory, homological algebra
19 *K*-theory
20 Group theory and generalizations
22 Topological groups, Lie groups
26 Real functions
28 Measure and integration
30 Functions of a complex variable
31 Potential theory
32 Several complex variables and analytic spaces
33 Special functions
34 Ordinary differential equations
35 Partial differential equations
37 Dynamical systems and ergodic theory
39 Difference and functional equations
40 Sequences, series, summability
41 Approximations and expansions
42 Fourier analysis
43 Abstract harmonic analysis
44 Integral transforms, operational calculus
45 Integral equations
46 Functional analysis
47 Operator theory
49 Calculus of variations and optimal control; optimization
51 Geometry
52 Convex and discrete geometry
53 Differential geometry
54 General topology
55 Algebraic topology
57 Manifolds and cell complexes
58 Global analysis, analysis on manifolds
60 Probability theory and stochastic processes
62 Statistics
65 Numerical analysis
68 Computer science
70 Mechanics of particles and systems
74 Mechanics of deformable solids
76 Fluid mechanics
78 Optics, electromagnetic theory
80 Classical thermodynamics, heat transfer
81 Quantum theory
82 Statistical mechanics, structure of matter
83 Relativity and gravitational theory
85 Astronomy and astrophysics
86 Geophysics
90 Operations research, mathematical programming,
91 Game theory, economics, social and behavioral sciences
92 Biology and other natural sciences
93 Systems theory; control
94 Information and communication, circuits
97 Mathematics Education

Prepayment Methods and Mailing Addresses

All prices quoted in U.S. dollars.
Payment by check must be drawn on U.S. bank if paid in U.S. dollars.

Send checks, money orders, UNESCO coupons to American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904.

To use credit cards, fill in information requested and mail to American Mathematical Society, P.O. Box 6248, Providence, RI 02940-6248 or call (401) 455-4000 or 1-800-321-4AMS.

For Foreign Bank Transfers: Citizens Bank, 1 Citizens Drive, Riverside, RI 02915, ABA #011-500-120, for American Mathematical Society, Acct #1107802021.

☐ American Express ☐ Discover ☐ VISA ☐ MasterCard

____

Account number **Expiration date**

M02NO

Membership Categories

Please read the following to determine what membership category you are eligible for, and then indicate below the category for which you are applying.

Members can purchase a ***multi-year membership*** by prepaying their current dues rate for either two, three, four or five years. This option is not available to category-S, unemployed, or student members.

Introductory ordinary member rate applies to the first five ***consecutive*** years of ordinary membership. Eligibility begins with the first year of membership in any category other than student and nominee. Dues are $52.

For ***ordinary members*** whose annual professional income is below $75,000, the dues are $105; for those whose annual professional income is $75,000 or more, the dues are $140.

Minimum dues for ***contributing members*** are $210. The amount paid which exceeds the higher ordinary dues level and is purely voluntary may be treated as a charitable contribution.

For a ***joint family membership,*** one member pays ordinary dues, based on his or her income; the other pays ordinary dues based on his or her income, less $20. (Only the member paying full dues will receive the *Notices* and the *Bulletin* as a privilege of membership, but both members will be accorded all other privileges of membership.)

The annual dues for ***reciprocity members*** who reside outside the U.S. are $70. To be eligible for this classification, members must belong to one of those foreign societies with which the AMS has established a reciprocity agreement. Annual verification is required. Reciprocity members who reside in the U.S. must pay ordinary member dues ($105 or $140).

The annual dues for ***category-S members,*** those who reside in developing countries, are $16. Members can chose only one privilege journal. Please indicate your choice below.

For either ***students*** or ***unemployed individuals,*** dues are $35, and annual verification

2002 Dues Schedule (January through December)

Ordinary member, introductory rate			☐$52
Ordinary member		☐$105	☐$140
Joint family member (full rate)		☐$105	☐$140
Joint family member (reduced rate)		☐$85	☐$120
Contributing member (minimum $210)			☐
Student member (please verify)[1]			☐$35
Unemployed member (please verify)[2]			☐$35
Reciprocity member (please verify)[3]	☐$70	☐$105	☐$140
Category-S member [4]			☐$16
Multi-year membership	$..................	for...............	years

[1] ***Student Verification*** (sign below)
I am a full-time student at ________________________________

________________________________ currently working toward a degree.

[2] ***Unemployed Verification*** (sign below) I am currently unemployed and actively seeking employment.

[3] ***Reciprocity Membership Verification*** (sign below) I am currently a member of the society indicated on the right and am therefore eligible for reciprocity membership.

____

Signature

[4]☐ ***send*** **NOTICES** ☐ ***send*** **BULLETIN**

Reciprocating Societies

- ☐ Allahabad Mathematical Society
- ☐ Australian Mathematical Society
- ☐ Azerbaijan Mathematical Society
- ☐ Balkan Society of Geometers
- ☐ Berliner Mathematische Gessellschaft
- ☐ Calcutta Mathematical Society
- ☐ Canadian Mathematical Society
- ☐ Croatian Mathematical Society
- ☐ Cyprus Mathematical Society
- ☐ Dansk Matematisk Forening
- ☐ Deutsche Mathematiker-Vereinigung
- ☐ Edinburgh Mathematical Society
- ☐ Egyptian Mathematical Society
- ☐ European Mathematical Society
- ☐ Gesellschaft für Angewandte Mathematik und Mechanik
- ☐ Glasgow Mathematical Association
- ☐ Hellenic Mathematical Society
- ☐ Icelandic Mathematical Society
- ☐ Indian Mathematical Society
- ☐ Iranian Mathematical Society
- ☐ Irish Mathematical Society
- ☐ Israel Mathematical Union
- ☐ János Bolyai Mathematical Society
- ☐ The Korean Mathematical Society
- ☐ London Mathematical Society
- ☐ Malaysian Mathematical Society
- ☐ Mathematical Society of Japan
- ☐ Mathematical Society of Serbia
- ☐ Mathematical Society of the Philippines
- ☐ Mathematical Society of the Republic of China
- ☐ Mongolian Mathematical Society
- ☐ Nepal Mathematical Society
- ☐ New Zealand Mathematical Society
- ☐ Nigerian Mathematical Society
- ☐ Norsk Matematisk Forening
- ☐ Österreichische Mathematische Gesellschaft
- ☐ Palestine Society for Mathematical Sciences
- ☐ Polskie Towarzystwo Matematyczne
- ☐ Punjab Mathematical Society
- ☐ Ramanujan Mathematical Society
- ☐ Real Sociedad Matemática Española
- ☐ Saudi Association for Mathematical Sciences
- ☐ Singapore Mathematical Society
- ☐ Sociedad Colombiana de Matemáticas
- ☐ Sociedad Española de Matemática Aplicada
- ☐ Sociedad de Matemática de Chile
- ☐ Sociedad Matemática de la Republica Dominicana
- ☐ Sociedad Matemática Mexicana
- ☐ Sociedad Uruguaya de Matemática y Estadística
- ☐ Sociedade Brasileira Matemática
- ☐ Sociedade Brasileira de Matemática Aplicada e Computacional
- ☐ Sociedade Paranaense de Matemática
- ☐ Sociedade Portuguesa de Matemática
- ☐ Societat Catalana de Matemàtiques
- ☐ Societatea de Ştiinţe Matematice din România
- ☐ Societatea Matematicienilor din Romania
- ☐ Société Mathématique de Belgique
- ☐ Société Mathématique de France
- ☐ Société Mathématique du Luxembourg
- ☐ Société Mathématique Suisse
- ☐ Société Mathématiques Appliquées et Industrielles
- ☐ Society of Associations of Mathematicians & Computer Science of Macedonia
- ☐ Society of Mathematicians, Physicists, and Astronomers of Slovenia
- ☐ South African Mathematical Society
- ☐ Southeast Asian Mathematical Society
- ☐ Suomen Matemaattinen Yhdistys
- ☐ Svenska Matematikersamfundet
- ☐ Ukrainian Mathematical Society
- ☐ Union Mathemática Argentina
- ☐ Union of Bulgarian Mathematicians
- ☐ Union of Czech Mathematicians and Physicists
- ☐ Union of Slovak Mathematicians and Physicists
- ☐ Unione Matematica Italiana
- ☐ Vijnana Parishad of India
- ☐ Wiskundig Genootschap

Change of Address

Members of the Society who move or change positions are urged to notify the Providence Office as soon as possible.

Journal mailing lists must be printed four to six weeks before the issue date. Therefore, in order to avoid disruption of service, members are requested to provide the required notice well in advance.

Besides mailing addresses for members, the Society's records contain information about members' positions and their employers (for publication in the Combined Membership List). In addition, the AMS maintains records of members' honors, awards, and information on Society service.

When changing their addresses, members are urged to cooperate by supplying the requested information. The Society's records are of value only to the extent that they are current and accurate.

If your address has changed or will change within the next two or three months, please fill out this form, supply any other information appropriate for the AMS records, and mail it to:

Customer Services
AMS
P.O. Box 6248
Providence, RI 02940

or send the information on the form by e-mail to:

amsmem@ams.org or
cust-serv@ams.org

Name ______________________________

Customer code ______________________________

Change effective as of ______________________________

Old mailing address

____

____

____

New mailing address

____

____

____

New position ______________________________

If mailing address is not that of your employer,

please supply the following information:

New employer ______________________________

Location of employer (city, state, zip code, country)

____

____

____

Telephone/home: ________________ **office:** ________________

Fax: ______________________________

e-mail ______________________________

Recent honors and awards

____

Fold here

NO POSTAGE
NECESSARY
IF MAILED
IN THE
UNITED STATES

BUSINESS REPLY MAIL

FIRST-CLASS MAIL PERMIT NO. 554B PROVIDENCE, RI

POSTAGE WILL BE PAID BY ADDRESSEE

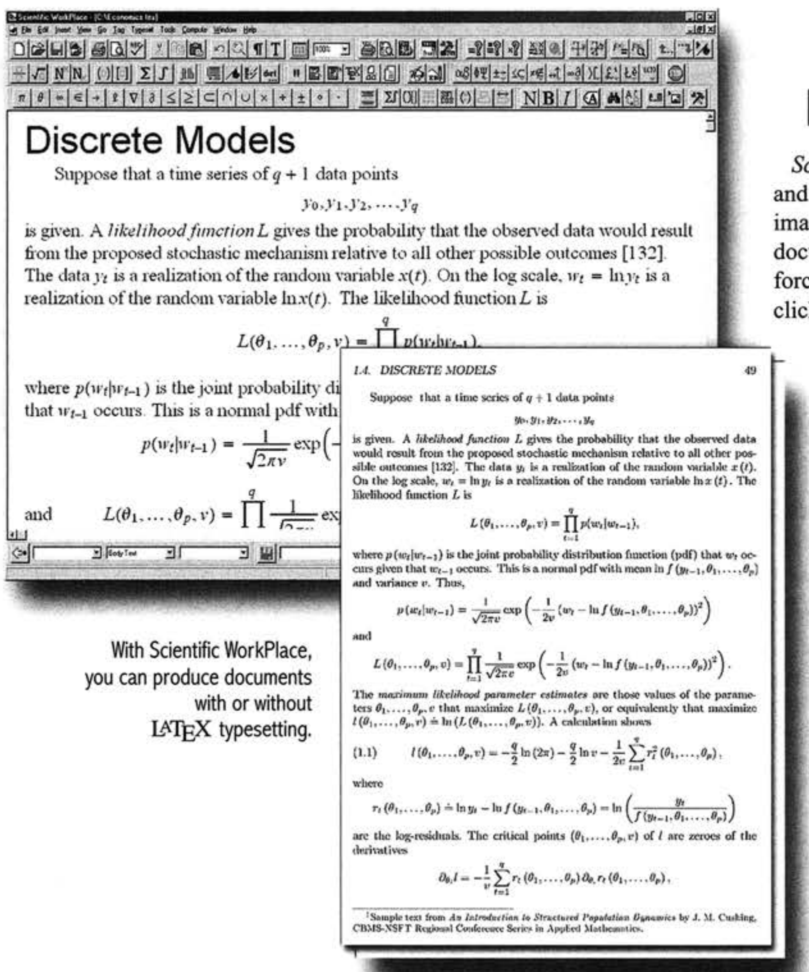
Membership and Customer Services
AMERICAN MATHEMATICAL SOCIETY
P.O. Box 6248
Providence, RI 02940-4248

Fold here

Introducing Version 4.0

Scientific WorkPlace®

Mathematical Word Processing • LaTeX Typesetting • Computer Algebra



Discrete Models

1.4. DISCRETE MODELS 49

Suppose that a time series of $q+1$ data points

$$y_0, y_1, y_2, \ldots, y_q$$

is given. A *likelihood function* L gives the probability that the observed data would result from the proposed stochastic mechanism relative to all other possible outcomes [132]. The data y_t is a realization of the random variable $x(t)$. On the log scale, $w_t = \ln y_t$ is a realization of the random variable $\ln x(t)$. The likelihood function L is

$$L(\theta_1, \ldots, \theta_p, v) = \prod_{t=1}^{q} p(w_t|w_{t-1}),$$

where $p(w_t|w_{t-1})$ is the joint probability distribution function (pdf) that w_t occurs given that w_{t-1} occurs. This is a normal pdf with mean $\ln f(y_{t-1}, \theta_1, \ldots, \theta_p)$ and variance v. Thus,

$$p(w_t|w_{t-1}) = \frac{1}{\sqrt{2\pi v}} \exp\left(-\frac{1}{2v}(w_t - \ln f(y_{t-1}, \theta_1, \ldots, \theta_p))^2\right)$$

and

$$L(\theta_1, \ldots, \theta_p, v) = \prod_{t=1}^{q} \frac{1}{\sqrt{2\pi v}} \exp\left(-\frac{1}{2v}(w_t - \ln f(y_{t-1}, \theta_1, \ldots, \theta_p))^2\right).$$

The *maximum likelihood parameter estimates* are those values of the parameters $\theta_1, \ldots, \theta_p, v$ that maximize $L(\theta_1, \ldots, \theta_p, v)$, or equivalently that maximize $l(\theta_1, \ldots, \theta_p, v) \doteq \ln(L(\theta_1, \ldots, \theta_p, v))$. A calculation shows

$$l(\theta_1, \ldots, \theta_p, v) = -\frac{q}{2}\ln(2\pi) - \frac{q}{2}\ln v - \frac{1}{2v}\sum_{t=1}^{q} r_t^2(\theta_1, \ldots, \theta_p), \tag{1.1}$$

where

$$r_t(\theta_1, \ldots, \theta_p) \doteq \ln y_t - \ln f(y_{t-1}, \theta_1, \ldots, \theta_p) = \ln\left(\frac{y_t}{f(y_{t-1}, \theta_1, \ldots, \theta_p)}\right)$$

are the log-residuals. The critical points $(\theta_1, \ldots, \theta_p, v)$ of l are zeroes of the derivatives

$$\partial_{\theta_i} l = -\frac{1}{v}\sum_{t=1}^{q} r_t(\theta_1, \ldots, \theta_p)\,\partial_{\theta_i} r_t(\theta_1, \ldots, \theta_p),$$

[1] Sample text from *An Introduction to Structured Population Dynamics* by J. M. Cushing, CBMS-NSF Regional Conference Series in Applied Mathematics.

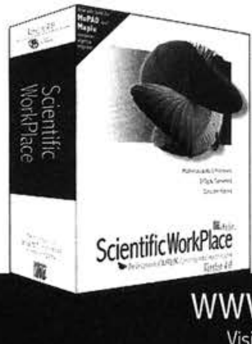
With Scientific WorkPlace, you can produce documents with or without LaTeX typesetting.

The Gold Standard for Mathematical Publishing

Scientific WorkPlace makes writing, publishing, and doing mathematics easier than you ever imagined possible. You compose and edit your documents directly on the screen without being forced to think in a programming language. A click of a button allows you to typeset your document in LaTeX. You can also compute and plot solutions with the integrated computer algebra system.

A few of the new features included in Version 4.0 are:

- Two computer algebra engines, MuPAD® and Maple®, so you can take advantage of the strengths of each
- HTML document export, with mathematics exported as graphics or MathML
- Configurable Symbol Cache toolbar
- Two new manuals:
  - *Typesetting Documents in Scientific WorkPlace® & Scientific Word® Version 4.0 (printed)*
  - *A Gallery of Document Shells for Scientific WorkPlace® & Scientific Word® Version 4.0 (on CD)*
- And more



MacKichan SOFTWARE, INC.

Tools for Scientific Creativity since 1981

www.mackichan.com/notices

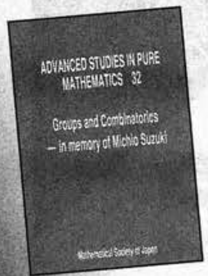
Visit our website for free trial versions of all our products.

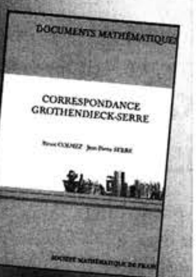
Toll Free: 877-724-9673 • Email: info@mackichan.com

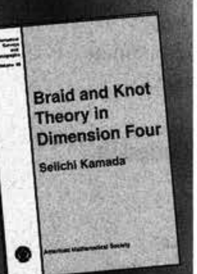
AMERICAN MATHEMATICAL SOCIETY

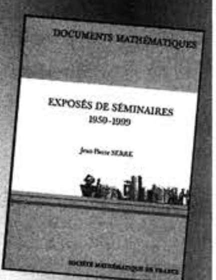
For many more publications of interest, visit the AMS Bookstore:

www.ams.org/bookstore









New Titles from the AMS

Groups and Combinatorics—in Memory of Michio Suzuki

Eiichi Bannai, *Kyushu University, Fukuoka, Japan*, **Hiroshi Suzuki**, *International Christian University, Tokyo, Japan*, **Hiroyoshi Yamaki**, *Kumamoto University, Japan*, and **Tomoyuki Yoshida**, *Hokkaido University, Sapporo, Japan*, Editors

A publication of the Mathematical Society of Japan.

Professor Suzuki had a profound influence on the development of group theory over the last 50 years. It's generally believed that his work in the 1950s ignited work on the classification of finite simple groups, and in the 1960s and 1970s, he was a leader in its development.

Just prior to his death in 1998, Professor Suzuki completed a 150-page manuscript containing his most recent contribution to group theory. This paper, "On the Prime Graph of a Finite Simple Group—an Application of the Method of Feit-Thompson-Bender-Glauberman", is included in this volume. Here, the editors have been meticulous in making minimal corrections to the work in order to honor the writing style and original flow of Professor Suzuki's thoughts. The book also includes contributions from the speakers at the conference, as well as papers from researchers who shared close ties with Professor Suzuki.

Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

Advanced Studies in Pure Mathematics, Volume 32; 2001; 474 pages; Hardcover; ISBN 4-931469-15-9; List $105; Individual member $74; Order code ASPM/32NT204

Correspondance Grothendieck-Serre

Pierre Colmez and **Jean-Pierre Serre**, Editors

A publication of the Société Mathématique de France.

This remarkable volume contains a large part of the mathematical correspondence between A. Grothendieck and J.-P. Serre. This correspondence forms a vivid introduction to the study of algebraic geometry during the years 1955–1965. For example, readers will discover the genesis of some of Grothendieck's ideas on Sheaf cohomology, schemes, Riemann-Roch, the fundamental group, motives, and more. They also will get an idea of the mathematical athomsphere of this time (Bourbaki, seminars in Paris, Harvard, Princeton, war in Algeria, etc.). This is a remarkable book.

Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Number 2; 2001; 288 pages; Hardcover; ISBN 2-85629-104-X; List $55; Individual member $50; Order code SMFDM/2NT204

Supplementary Reading

Braid and Knot Theory in Dimension Four

Seiichi Kamada, *Osaka City University, Japan*

Braid theory and knot theory are related via two famous results due to Alexander and Markov. Alexander's theorem states that any knot or link can be put into braid form. Markov's theorem gives necessary and sufficient conditions to conclude that two braids represent the same knot or link. Thus, one can use braid theory to study knot theory and vice versa.

In this book, the author generalizes braid theory to dimension four. He develops the theory of surface braids and applies it to study surface links. In particular, the generalized Alexander and Markov theorems in dimension four are given. This book is the first to contain a complete proof of the generalized Markov theorem.

Included is a table of knotted surfaces with a computation of Alexander polynomials. Braid techniques are extended to represent link homotopy classes. The book is geared toward a wide audience, from graduate students to specialists. It would make a suitable text for a graduate course and a valuable resource for researchers.

Mathematical Surveys and Monographs, Volume 95; 2002; 305 pages; Hardcover; ISBN 0-8218-2969-6; List $79; Individual member $47; Order code SURV/95NT204

Exposés de Séminaires 1950–1999

Jean-Pierre Serre

A publication of the Société Mathématique de France.

Jean-Pierre Serre has made significant contributions to several areas of mathematics, in particular to topology, algebraic geometry, and number theory. He is also renowned for his remarkable expository skill.

This volume gathers seminar talks he gave between 1950 and 1999 in various seminars: Bourbaki, Cartan, Chevalley, and Delange-Pisot-Poitou. The themes extend from algebraic topology to number theory, covering also Lie group theory, algebraic geometry and modular forms. It gives both a presentation of works by other mathematicians (Borel, Dwork, etc.) and personal works, such as his talk at the Chevalley seminar on algebraic fibre spaces, which inspired Grothendieck in his construction of étale cohomology.

Distributed by the AMS in the United States, Canada, and Mexico. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Number 1; 2001; 259 pages; Hardcover; ISBN 2-85629-103-1; List $55; Individual member $50; Order code SMFDM/1NT204

Cohomology of Arithmetic Groups, L-Functions and Automorphic Forms

T. N. Venkataramana, *Tata Institute of Fundamental Research, Mumbai, India*

A publication of the Tata Institute of Fundamental Research.

This collection of papers is based on lectures delivered at the Tata Institute of Fundamental Research (TIFR) as part of a special year on arithmetic groups, L-functions and automorphic forms. The volume opens with an article by Cogdell and Piatetski-Shapiro on Converse Theorems for GL_n and applications to liftings. It ends with some remarks on the Riemann Hypothesis by Ram Murty. Other talks cover topics such as Hecke theory for Jacobi forms, restriction maps and L-values, congruences for Hilbert modular forms, Whittaker models for p-adic $GL(4)$, the Seigel formula, newforms for the Maaß Spezialchar, an algebraic Chebotarev density theorem, a converse theorem for Dirichlet series with poles, Kirillov theory for $GL_2(\mathcal{D})$, and the L^2 Euler characteristic of arithmetic quotients. The present volume is the latest in the Tata Institute's tradition of recognized contributions to number theory.

Distributed worldwide except in India, Bangladesh, Bhutan, Maldavis, Nepal, Pakistan, and Sri Lanka.

Tata Institute of Fundamental Research; 2001; 251 pages; Softcover; ISBN

To order, call: 1-800-321-4AMS (4267), in the U.S. and Canada, or 1-401-455-4000; fax: 1-401-455-4046; email: cust-serv@ams.org. Visit the AMS Bookstore and order online at **www.ams.org/bookstore**. Or write to: American Mathematical Society, 201 Charles Street, Providence, RI 02904-2294. Prices subject to change without notice.

AMS
AMERICAN MATHEMATICAL SOCIETY

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Programs and abstracts will continue to be displayed on the AMS website in the Meetings and Conferences section until about three weeks after the meeting is over. Final programs for Sectional Meetings will be archived on the AMS website in an electronic issue of the *Notices* as noted below for each meeting.

Atlanta, Georgia

Georgia Institute of Technology

March 8-10, 2002

Meeting held in conjunction with the Mathematical Association of America.

Meeting #975

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: January 2002
Program first available on AMS website: January 31, 2002
Program issue of electronic *Notices*: May 2002
Issue of *Abstracts*: Volume 23, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Georgia Benkart, University of Wisconsin, Madison, *Going up and down.*

Robert L. Bryant, Duke University, *De-prescribing the shape of surfaces.*

Johnny Henderson, Auburn University, *Uniqueness implies existence for solutions of boundary value problems for dynamic equations on time scales.*

Nigel J. Kalton, University of Missouri, Columbia, *Banach space theory, sectorial operators and partial differential equations.*

James G. Oxley, Louisiana State University, *The interplay between graphs and matroids.*

Special Sessions

Algebraic Combinatorics, **Mihai A. Ciucu**, Georgia Institute of Technology.

Automated Reasoning in Mathematics and Logic, **Johan G. F. Belinfante**, Georgia Institute of Technology.

Banach Spaces and Their Applications, **Peter G. Casazza** and **N. J. Kalton**, University of Missouri-Columbia.

Bridges from 'Applied' to 'Mathematics', **Peter Mucha**, **John A. Pelesko**, **John E. McCuan**, and **Guillermo H. Goldsztein**, Georgia Institute of Technology.

Collaborative Learning Classroom Activities, **Sabrina A. Hessinger**, Armstrong Atlantic State University.

Combinatorics and Graph Theory, **John M. Harris**, Furman University.

Computation in the Mathematical Sciences, **Sabrina A. Hessinger**, Armstrong Atlantic State University, and **Mark D. Cawood**, Clemson University.

Dynamic Equations on Time Scales, **Martin J. Bohner**, Florida Institute of Technology, and **Billur Kaymakcalan**, Georgia Southern University.

Elementary Mathematical Modeling, **Mary Ellen Davis**, Georgia Perimeter College.

Frames, Wavelets, and Operator Theory, **Christopher E. Heil** and **Yang Wang**, Georgia Institute of Technology.

Graphs and Matroids, **James G. Oxley** and **Bogdan Oporowski**, Louisiana State University, and **Robin Thomas**, Georgia Institute of Technology.

Harmonic Analysis, **Gerd Mockenhaupt** and **Michael T. Lacy**, Georgia Institute of Technology, and **Akos Magyar**, University of Georgia.

Introductory/Elementary Statistics, **Patricia G. Monroe**, Greenville Technical College.

Knot Theory, 3-Manifolds, 4-Manifolds, and Geometric Group Theory, **Wolfgang H. Heil**, Florida State University, and **Jose Carlos Gómez-Larrañaga**, CIMAT, Mexico.

Linear Algebra and Matrix Theory, **Frank J. Hall** and **Zhongshan Li**, Georgia State University.

Mathematical Models in Biology, **Robert D. Fray**, Furman University.

Number Theory, **David Penniston**, Furman University.

Numerical Linear Algebra and Its Applications, **Michele Benzi**, Emory University, **Steven B. Damelin**, Georgia Southern University, and **James Nagy**, Emory University.

Probability and Combinatorics, **Russell D. Lyons** and **Prasad V. Tetali**, Georgia Institute of Technology.

Quantum Structures, **Alexander G. Wilce**, Juniata College, **Richard J. Greechie**, Louisiana Technical University, and **Franklin E. Schroeck**, Florida Atlantic University.

Real World Applications of Mathematics, **Mark C. Ginn**, Appalachian State University.

Research on the Mathematical Education of Undergraduates, **Joe Wimbish**, Huntingdon College.

Symplectic and Contact Topology, **Margaret Symington**, Georgia Institute of Technology, and **Gordana Matic**, University of Georgia.

Technology and Distance Learning, **Tom Morley**, Georgia Institute of Technology, and **Martha Abel**, Georgia Southern University.

Montréal, Quebec Canada

Centre de Recherches Mathématiques, Université de Montréal

May 3-5, 2002

Meeting #976

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: March 2002
Program first available on AMS website: March 21, 2002
Program issue of electronic *Notices*: July 2002
Issue of *Abstracts*: Volume 23, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: March 12, 2002

Invited Addresses

Nicholas M. Ercolani, University of Arizona, *Title to be announced.*

Lars Hesselholt, Massachusetts Institute of Technology, *Title to be announced.*

Niky Kamran, McGill University, *Title to be announced.*

Rafael de la Llave, University of Texas at Austin, *Title to be announced.*

Special Sessions

Asymptotics for Random Matrix Models and Their Applications (Code: AMS SS J1), **Nicholas M. Ercolani**, University of Arizona, and **Kenneth T.-R. McLaughlin**, University of North Carolina at Chapel Hill and University of Arizona.

Combinatorial Hopf Algebras (Code: AMS SS C1), **Marcelo Aguiar**, Texas A&M University, and **François Bergeron** and **Christophe Reutenauer**, Université du Québec à Montréal.

Combinatorial and Geometric Group Theory (Code: AMS SS A1), **Olga G. Kharlampovich**, McGill University, **Alexei Myasnikov** and **Vladimir Shpilrain**, City College, New York, and **Daniel Wise**, McGill University.

Commutative Algebra and Algebraic Geometry (Code: AMS SS G1), **Irena Peeva**, Cornell University, and **Hema Srinivasan**, University of Missouri-Columbia.

Curvature and Topology (Code: AMS SS E1), **Regina Rotman**, Courant Institute, New York University, **Christina Sormani**, Lehman College, CUNY, and **Kristopher R. Tapp**, SUNY at Stony Brook.

Function Spaces in Harmonic Analysis and PDEs (Code: AMS SS D1), **Galia D. Dafni** and **Jie Xiao**, Concordia University.

Potential Theory (Code: AMS SS B1), **Paul M. Gauthier**, Université de Montréal, **K. Gowri Sankaran**, McGill University, and **David H. Singman**, George Mason University.

Shape Theory in Dynamics (Code: AMS SS F1), **Alex Clark**, University of North Texas, and **Krystyna M. Kuperberg**, Auburn University.

Spectral Geometry (Code: AMS SS H1), **Dmitry Jakobson**, McGill University, and **Yiannis Petridis**, McGill University and Centre de Recherches Mathématiques.

Pisa, Italy

June 12-16, 2002

Meeting #977

First Joint International Meeting between the AMS and the Unione Matematica Italiana.
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: March 2002
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: March 15, 2002

Invited Addresses

Luigi Ambrosio, Scuola Normale Superiore, *Title to be announced.*

Luis A. Caffarelli, University of Texas at Austin, *Title to be announced.*

Claudio Canuto, Politecnico of Torino, *Title to be announced.*

L. Craig Evans, University of California Berkeley, *Title to be announced.*

Giovanni Gallavotti, University of Rome I, *Title to be announced.*

Sergiu Klainerman, Princeton University, *Title to be announced.*

Rahul V. Pandharipande, California Institute of Technology, *Title to be announced.*

Claudio Procesi, University of Roma, *Title to be announced.*

Special Sessions

Advances in Complex, Contact and Symplectic Geometry, **Paolo De Bartolomeis**, University of Firenze, **Yakov Eliashberg**, Stanford University, **Gang Tian**, MIT, and **Giuseppe Tomassini**, Scuola Normale Superiore, Pisa.

Advances in Differential Geometry of PDEs and Applications, **Valentin Lychagin**, New Jersey Institute of Technology, and **Agostino Prastaro**, University of Roma, La Sapienza.

Algebraic Logic and Universal Algebra, **Paolo Agliano**, University of Siena, **Keith A. Kearnes**, University of Colorado, **Franco Montagna**, University of Siena, **Don Pigozzi**, Iowa State University, and **Aldo Ursini**, University of Siena.

Algebraic Vector Bundles, **Vincenzo Ancona**, University of Firenze, **Mohan Kumar**, Washington University, **Giorgio Maria Ottaviani**, University of Firenze, **Christopher Peterson**, Colorado State University, and **Prabhakar Rao**, University of Missouri.

Analytic Aspects of Convex Geometry, **Stefano Campi**, University of Modena, **Richard Gardner**, Western Washington University, **Erwin Lutwak**, Polytechnic University Brooklyn, and **Alijosa Volcic**, University of Trieste.

Classification Theory and Topology of Algebraic Varieties, **Fabrizio Catanese**, University of Gottingen, **Janos Kollar**, Princeton University, and **Shing-Tung Yau**, Harvard University.

Commutative Algebra and the Geometry of Projective Varieties, **Ciro Ciliberto**, University of Roma II, **Anthony Geramita**, University of Genova, **Rick Miranda**, Colorado State University, and **Ferruccio Orecchia**, University of Napoli.

Commutative Algebra: Hilbert Functions, Homological Methods and Combinatorial Aspects, **Aldo Conca**, University of Genova, **Anna Guerrieri**, University of L'Aquila, **Claudia Polini**, University of Oregon, and **Bernd Ulrich**, Michigan State University.

Commutative Rings and Integer-valued Polynomials, **Stefania Gabelli**, University of Roma III, and **Thomas G. Lucas**, University of North Carolina Charlotte.

Complex, Contact and Quaternionic Geometry, **David E. Blair**, Michigan State University, and **Stefano Marchiafava**, University of Roma, La Sapienza.

Contemporary Developments in Partial Differential Equations and in the Calculus of Variations, **Irene Fonseca**, Carnegie Mellon University, and **Paolo Marcellini**, University of Firenze.

Didattica della Dimostrazione, **Ferdinando Arzarello**, University of Torino, **Guershon Harel**, Purdue University, and **Vinicio Villani**, University of Pisa.

Dynamical Systems, **Antonio Giorgilli**, University of Milano-Bicocca, **Stefano Marmi**, Scuola Normale Superiore, Pisa, and **John Norman Mather**, Princeton University.

Elliptic Partial Differential Equations, **Angelo Alvino**, University of Napoli, **Luis Caffarelli**, University of Texas, **Giorgio Talenti**, University of Firenze, and **Vladimir Oliker**, Emory University.

Equazioni di Evoluzione Nonlineari, **Alberto Tesei**, University of Roma, La Sapienza, and **Wei-Ming Ni**, University of Minnesota, Minneapolis.

Free Boundary Problems, **Ricardo Horacio Nochetto**, University of Maryland, College Park, and **Augusto Visintin**, University of Trento.

Geometric Properties of Solutions to PDEs, **Donatella Danielli**, Purdue University, and **Sandro Salsa**, Politecnico of Milano.

Harmonic Analysis, **Fulvio Ricci**, Scuola Normale Superiore, Pisa, and **Elias M. Stein**, Princeton University.

Higher Dimensional Algebra, **John Baez**, University of California, Riverside, and **Giuseppe Rosolini**, University of Genova.

History of Mathematics, **Piers Bursil-Hall**, Cambridge University, **Enrico Giusti**, University of Firenze, and **James J. Tattersall**, Providence College.

Hyperbolic Equations, **Sergiu Klainerman**, Princeton University, and **Sergio Spagnolo**, University of Pisa.

Hyperbolic Systems of Conservation Laws, **Alberto Bressan**, SISSA, Trieste, and **Shi Jin**, University of Wisconsin.

Inverse Boundary Problems and Applications, **Giovanni Alessandrini**, University of Trieste, and **Gunther Uhlmann**, University of Washington.

Jump Processes in Option Pricing Theory, **Claudio Albanese**, University of Toronto, and **Marco Isopi**, University of Bari.

Kolmogorov Equations, **Giuseppe Da Prato**, Scuola Normale Superiore, Pisa, and **Nicolai V. Krylov**, University of Minnesota.

Logarithmic De Rham Cohomology and Dwork Cohomology, **Alan Adolphson**, Oklahoma State University, Stillwater, **Francesco Baldassarri**, University of Padova, **Arthur Ogus**, University of California Berkeley, and **Steven Sperber**, University of Minnesota, Minneapolis.

Mathematical Problems in Soft Matter Modelling, **Eugene C. Gartland**, Kent State University, and **Epifanio Virga**, University of Pavia.

Mathematical Problems in Transport Theory, **Carlo Cercignani**, Politecnico of Milano, and **Irene Gamba**, University of Texas.

Mathematical Schools: Italy and the United States at the Turn of the Twentieth Century, **Umberto Bottazzini**, University of Palermo, and **Karen Hunger Parshall**, University of Virginia.

Mathematics in Polymer Science, **Antonio Fasano**, University of Firenze, and **Kumbakonam R. Rajagopal**, Texas A&M University.

Microlocal Analysis and Applications to PDE, **Daniele Del Santo**, University of Trieste, **M. K. Venkatesha Murthy**, University of Pisa, and **Daniel Tataru**, Northwestern University.

Nonlinear Analysis, **Antonio Ambrosetti**, SISSA, Trieste, **Vieri Benci**, University of Pisa, **Haim Brezis**, Rutgers University, and **Paul Rabinowitz**, University of Wisconsin.

Nonlinear Elliptic and Parabolic Equations and Systems, **Gary Lieberman**, Iowa State University, and **Antonio Maugeri**, University of Catania.

Nonstandard Methods and Applications in Mathematics, **Alessandro Berarducci**, University of Pisa, **Nigel Cutland**, University of Hull, **Mauro Di Nasso**, University of Pisa, and **David Ross**, University of Hawaii.

Operator Algebras, **Sergio Doplicher**, University of Roma, La Sapienza, and **Edward George Effros**, University of California Los Angeles.

Optimization and Control, **Roberto Triggiani**, University of Virginia, and **Tullio Zolezzi**, University of Genova.

Partial Differential Equations of Mixed Elliptic-Hyperbolic Type and Applications, **Daniela Lupo**, Politecnico of Milano, **Cathleen S. Morawetz**, Courant Institute, and **Kevin R. Payne**, University of Milano.

Periodic Solutions of Differential and Difference Equations, **Massimo Furi**, University of Firenze, and **Mario Umberto Martelli**, Claremont McKenna College.

Poisson Geometry and Integrable Systems, **Franco Magri**, University of Milano, and **Ping Xu**, Pennsylvania State University.

Quantum Cohomology and Moduli Spaces, **Angelo Vistoli**, University of Bologna, and **Aaron Bertram**, University of Utah.

Scaling Limits and Homogenization Problems in Physics and Applied Sciences, **Mario Pulvirenti**, University of Roma, and **George Papanicolaou**, Stanford University.

Semigroups of Operators and Applications, **Francesco Altomare**, University of Bari, and **Frank Neubrander**, Louisiana State University.

Semigroups, Automata and Formal Languages, **Alessandra Cherubini**, Politecnico of Milano, and **John Meakin**, University of Nebraska-Lincoln.

Some Mathematics Around Composites, **Robert V. Kohn**, Courant Institute, and **Vincenzo Nesi**, University of Roma, La Sapienza.

Structured Matrix Analysis with Applications, **Dario Andrea Bini**, University of Pisa, and **Thomas Kailath**, Stanford University.

The Topology of 3-manifolds, **Ricardo Benedetti** and **Carlo Petronio**, University of Pisa, **Dale Rolfsen**, University of British Columbia, Vancouver, and **Jeffrey Weeks**, Canton, New York.

Variational Analysis and Applications, **Franco Giannessi**, University of Pisa, **Boris S. Mordukhovich**, Wayne State University, Detroit, **Biagio Ricceri**, University of Catania, and **R. Tyrrell Rockafellar**, University of Washington.

Viscosity Methods in PDEs and Applications, **Piermarco Cannarsa**, University of Roma II, **Italo Capuzzo Dolcetta**, University of Roma, La Sapienza, and **Panagiotis Souganidis**, University of Texas, Austin.

White Noise Theory and Quantum Probability, **Luigi Accardi**, University of Roma, Tor Vergata, and **Hui-Hsiung Kuo**, Louisiana State University.

Portland, Oregon

Portland State University

June 20–22, 2002

Meeting #978

Meeting held in conjunction with the Pacific Northwest Section of the Mathematical Association of America.
Western Section
Associate secretary: Bernard Russo
Announcement issue of *Notices*: April 2002
Program first available on AMS website: May 9, 2002
Program issue of electronic *Notices*: August 2002
Issue of *Abstracts*: Volume 23, Issue 2

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions: Expired
For abstracts: April 30, 2002
For summaries of papers to MAA organizers: Various

Joint Invited Addresses

Kenneth A. Ribet, University of California Berkeley, *Title to be announced* (AMS-MAA Invited Address).

Invited Addresses

Edward B. Burger, Williams College, *Innovative Experiments...and how I survived them*; Friday evening at the MAA Banquet (MAA Polya Lecturer)

Richard W. Montgomery, University of California Santa Cruz, *Variational methods for the N-body problem.* (AMS)

Kenneth A. Ribet, University of California, Berkeley, Title to be announced; Friday, 1:30 p.m. (AMS-MAA)

Tina H. Straley, Executive Director of the MAA, *The MAA's role in the future of undergraduate mathematics*; Friday, 11:00 a.m. (MAA)

Edriss S. Titi, University of California Irvine, Title to be announced. (AMS)

Jim Valerio, Intel Desktop Architectural Lab, Improving pc graphics; Saturday, 1:30 p.m. (MAA)

Michael Wolf, Rice University, *Minimal surfaces, flat structures, and moduli spaces.* (AMS)

AMS Special Sessions

Algebraic Geometry and Combinatorics (Code: AMS SS B1), **Eric Babson** and **Rekha Thomas**, University of Washington, and **Sergey Yuzvinsky**, University of Oregon.

Association Schemes and Distance-Regular Graphs (Code: AMS SS J1), **John S. Caughman**, Portland State University, and **Paul M. Terwilliger**, University of Wisconsin.

Flat Structures, Moduli Spaces, and Minimal Surfaces (Code: AMS SS F1), **Matthias Weber**, Indiana University, and **Michael Wolf**, Rice University.

Low Dimensional Homotopy and Combinatorial Group Theory (Code: AMS SS H1), **F. Rudolf Beyl**, Portland State University, **Paul Latiolais**, Portland State University, **William A. Bogley**, Oregon State University, and **Micheal N. Dyer**, University of Oregon.

Mathematical Biology (Code: AMS SS D1), **Richard S. Gomulkiewicz**, Washington State University, and **Sebastian Schreiber**, Western Washington University.

Matroid Theory (Code: AMS SS E1), **Jennifer M. McNulty**, University of Montana, and **Nancy Ann Neudauer**, Pacific University.

Qualitative Properties and Applications of Functional Equations (Code: AMS SS A1), **Theodore A. Burton**, Southern Illinois University.

Quantum Topology (Code: AMS SS G1), **Douglas G. Bullock**, Boise State University, **Joanna M. Kania-Bartoszynska**, Boise State University, and **Uwe Kaiser**, Boise State University.

The Quintic Equation: Algebra and Geometry (Code: AMS SS C1), **Jerry Shurman**, Reed College, and **Scott Crass**, California State University, Long Beach.

AMS Contributed Paper Sessions

There will be sessions for ten-minute contributed talks, grouped by similar topic insofar as possible. See the electronic abstract submission instructions at `http://www.ams.org/abstracts/instructions.html`. Submitting via the website form is the easiest method. Please select AMS CP 1 as the event code for this session. The deadline for receipt of abstracts is April 30, 2002; this deadline will be strictly enforced.

MAA Minicourses

Four half-day minicourses will be given on Thursday. There is a registration fee of $20 for each course, which is in addition to the general meeting fee. Please see detailed descriptions and instructions for how to register in advance (no later than June 1) at `http://www.mth.pdx.edu/pnwmaa-ams`.

What to Teach and How Not to Teach It, **Edward B. Burger**, Williams College.

Using Geometer's Sketchpad (Version 4) in Undergraduate Mathematics Courses, **Keith Leatham**, Portland State University.

Visualization in Mathematics: Using Proofs Without Words in Your Classroom, **Roger B. Nelsen**, Lewis & Clark College, and **Jeanette R. Palmiter**, Portland State University.

Senior Capstones: Meaningful Closure to the Undergraduate Experience, **Dusty E. Sabo** and **Kemble R. Yates**, Southern Oregon University.

MAA Sessions

General Sessions of Contributed Papers. Any participant may present fifteen-minute contributed talks. Proposals must be submitted no later than June 1. Undergraduates should submit proposals to **Curtis J. Feist**, Southern Oregon University, `feistc@sou.edu`. Other proposals should be submitted to **Ken Ross**, University of Oregon, `ross@math.uoregon.edu`.

Mathematically Inspired, Computer Generated Poster Art. **Jeffrey Ely**, Dept. of Mathematical Sciences, Lewis and Clark College, `jeff@lclark.edu`. The advent of inexpensive large format color printing affords another opportunity to explore the resonance between computer graphics and mathematics. Several such pieces, one of which won a "best poster of the day award" at SIGGRAPH 2000, will be displayed and the mathematical issues in their creation will be discussed. The talk is scheduled for 9:00–9:45 a.m. on Friday, but the gallery will be open from 8:00 to 10:50 a.m.

Innovations in Teaching Undergraduates. This session organized by **Monte Boisen**, University of Idaho, `boisen@uidaho.edu`, welcomes presentations that report on what institutions are doing to improve the teaching of undergraduate courses. Proposals, with a title and brief abstract, should be submitted to the organizer by April 15.

Mathematics Education Research, organizers **Barbara Edwards**, Oregon State University, `edwards@math.orst.edu`, and **Sam Hall**, Willamette University, `shall@willamette.edu`, welcome proposals, with a title and brief abstract, for

20-minute presentations on education research in undergraduate mathematics.

On Undergraduate Research: Programs That Work, organized by **Daniel Kim**, Southern Oregon University, Kimd@sou.edu. All proposals with short abstract should be sent to the organizer by April 15.

The Use of Technology in Community College Mathematics Courses (pre-algebra through linear algebra and differential equations). There will be several individual presentations, and a panel discussion with a question and answer session and audience discussion as well. Typical questions that will be addressed are: What do MAA and AMATYC have to say about the use of technology at this level? What is the guiding philosophy that calls for, or prohibits, the use of technology in various courses? At what point in the curriculum is it appropriate to start using technology and why? What special assessment issues are raised by using technology and how do people deal with these issues? What do departments do when their faculty members hold differing views on the use of technology? What do instructors and departments do to insure that all students have access to the required technology? This session will not focus on specific uses of technology in the classroom. Anyone interested in giving a brief (10- to 15- minute) presentation or in serving as a panel member should contact **Peter Haberman**, Portland Community College, phaberma@pcc.edu, by March 29.

Junior Faculty on their Research, **Jennifer Ann Firkins**, Linfield College, jfirkins@linfield.edu, is organizing this session. Proposals, with a title and brief abstract, should be submitted to the organizer by April 15.

Special Presentation on Assessment, **Bonnie Gold**, Monmouth University, bgold@monmouth.edu, will represent Supporting Assessment in Undergraduate Mathematics (SAUM), which is a three-year NSF-supported project to provide college math departments with reasons why they should integrate assessment of student learning into their instructional programs and how to do that. This 90-minute session will consist of three parts: 1) A PowerPoint presentation with discussion of assessment and the SAUM project, 2) Mini-minicourse on assessment with some take-home tools, 3) Open discussion about what is going on locally (in the PNW section of MAA) in assessment.

Accommodations

Participants should make their own arrangements directly with a hotel of their choice. Because of Portland's popularity as a tourist attraction, hotels may sell out early. Blocks of rooms have been reserved at special rates at the properties below for the nights of Thursday, Friday, and Saturday, June 20–22, 2002 (most properties will honor the special rate for Wednesday, June 19). Room rates do not include the tax of 11.5%. Please cite the group name **PSU Math** when making a reservation. Hotels have varying cancellation or early checkout penalties; be sure to ask details when making your reservation. The AMS and MAA are not responsible for rate changes or for the quality of the accommodations.

Days Inn City Center, 1414 S.W. Sixth, 503-221-1611 or 800-899-0248; $89/one queen bed or two double beds; complimentary parking, a pass for a nearby Fitness Center; full service restaurant; within walking distance to the PSU campus, about one quarter mile to the meeting site. Please cite reservation code PSU Math 06/02.

Doubletree Hotel-Portland Downtown, 310 S.W. Lincoln, 503-221-0450; $74/two double beds or one king bed, additional person is $15; restaurant on premises; heated swimming pool and exercise center; overnight parking is $10/night; within easy walking distance to the PSU campus, less than one half mile to the meeting site. Please cite reservation code PSU Math.

Mallory Hotel, 729 S.W. 15th Avenue, 800-223-6311; $75/one double bed, $90/two twin beds or one queen, $105/two double beds, $120/two queen beds; $135/king-sized bed; historic hotel featuring complimentary parking, in-room refrigerators (most rooms); about 1.2 miles from the meeting site on campus. Please cite reservation code PSU Math.

Portland State Campus: A very limited number of dormitory-style rooms are available for students (undergraduate or graduate) in **Ondine Hall** about one block from the meeting site. Rates for one or two persons per room are $38/standard and $46/deluxe. Each room has two twin beds, private bath, telephone, coffee maker, linens, and clock. Deluxe rooms include a television, refrigerator, and microwave. Parking is $7/day. Please contact Katura Smith at 503-725-4336 or ksmith@chnw.pdx.edu and state you are with the PNW MAA/AMS Conference.

Downtown Value Inn, 415 S.W. Montgomery, 503-253-0578, $45/night. Budget-minded participants may find it convenient. Although no special rates were negotiated, this is a small motel very close to campus with complimentary parking (see http://www.downtownvalueinn.com).

Food Service

Information will be available on site, in the program, and at the website maintained by the local organizers.

Local Information

See the websites maintained by the local organizers at http://www.mth.pdx.edu/pnwams/ (this includes links to Portland state and maps) and the Portland Oregon Visitors Association at http://www.pova.org/.

Other Activities

Banquet: All are invited to this banquet at the Great China Seafood Restaurant on Friday at 7:00 p.m. The after-dinner speaker is Edward B. Burger, Williams College, *Innovative Experiments...and how I survived them.* The cost is $18/person. Reservations must be made through advance registration (see below) no later than **June 1**.

Book Sales: Examine the newest titles from the AMS and the MAA! Several other publishers have been invited to participate in the exhibits. Many of the AMS books will be available at a special 50% discount available only at the meeting. Complimentary coffee will be served courtesy of AMS Membership Services. Exhibits will be located adjacent to on-site registration in Neuberger Hall.

Editorial Activity: An acquisitions editor from the AMS Book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Reception: All registered participants are invited to a reception on Thursday evening from 5:30 p.m. to 7:00 p.m. at the Simon Benson House on the PSU campus.

Project NExT (New Experiences in Teaching) is a professional development program for new college-level faculty interested in "improving the teaching and learning of undergraduate mathematics." The PNW Project NExT is an extension of the MAA sponsored national program to the sectional level, and is scheduling a program for Wednesday. More information can be found at: `http://www.math.umt.edu/pnwnext/`.

TOTOM: The Oregon Teachers of Teachers of Mathematics (TOTOM) will hold its annual meeting on Wednesday, June 19, at PSU. Proposals for workshops and 20-minute presentations on the education of public school mathematics teachers may be sent to Sam Hall (`shall@willamette.edu`) by April 1. A joint session with Project NExT Fellows is being considered for the afternoon.

Parking

Parking is available across from Neuberger Hall in Parking Structure I. Enter on 6th St. between Hall and Harrison. The cost is $4/day for those who state they are with the PNW MAA/AMS conference (fees are higher otherwise). As parking is limited, participants are advised to carpool or take public transportation where possible.

Registration and Session Information

Registration fees (either in advance or on site) are $35/AMS or MAA members; $50/nonmembers; $5/Students/unemployed/emeritus members. Students making a presentation and first-year members of the MAA section's Project NExT receive complimentary registration. In addition those celebrating exactly 25 years as a member of MAA receive special complimentary registration.

Participants may register in advance through the PSU website or on site at the meeting. Either way, the funds will be shared between the AMS and MAA using the same equitable formula. Advance registration will help the organizing committee in assigning session and event space appropriately. Advance registration payable by check or money order is available at `http://www.mth.pdx.edu/pnwmaa-ams`; the advance registration deadline is **June 1**. Note that if you plan to attend the banquet and/or register for a minicourse, **you must register in advance**.

On-site registration will take place on Thursday and Friday from 8:00 a.m. to 4:00 p.m. in the Atrium on the third floor of Neuberger Hall. On-site registration fees are payable by cash, check, VISA, MasterCard, Discover, or American Express.

Sessions will take place in Neuberger Hall and Smith Memorial Center.

Travel

The City of Portland is served by Portland International Airport (PDX) located about 20 miles from downtown. Service to the downtown area is provide by the MAX light-rail Red Line service every 15 minutes from about 4:30 a.m. to 11:35 p.m. The trip takes about 38 minutes and costs $1.55 one-way. Scheduled shuttle service via Greyline Express provides service to most downtown hotels, departing every 45 minutes, 5:00 a.m. to midnight. The trip takes from 20 to 40 minutes and costs $15 (one-way) or $22 (round-trip); discounts are available for seniors and children. Metered taxis provide door-to-door service 24 hours per day; the fare per vehicle is $22 to $28 one-way and takes about 20–40 minutes.

Travel to campus: For an area map, campus map, and driving directions to PSU see the Web page maintained by the local organizers.

Car rental: Special rates have been negotiated with **Avis Rent A Car** for the period June 13 to June 29, 2002, beginning at $23.99/day for a subcompact car at the weekend rate (or beginning at $33.99 weekday). All rates include unlimited free mileage; the weekend rates quoted are available from noon Thursday until Monday at 11:59 p.m. Rates do not include state or local surcharges, tax, optional coverages, or gas refueling charges. Renter must meet Avis' age, driver, and credit requirements, and return to the same renting location. Make reservations by calling 800-331-1600 or online at `http://www.avis.com`. Higher nonweekend and weekly rates are also available. Please quote **Avis Discount Number B159266** when making reservations.

Weather

June weather in Portland is delightful and temperatures usually range from a low of 53° to a high of 74°. Expect sunny skies with an occasional shower. A light jacket is needed in the evenings.

Boston, Massachusetts

Northeastern University

October 5–6, 2002

Meeting #979

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: August 2002
Program first available on AMS website: August 22, 2002
Program issue of electronic *Notices*: December 2002
Issue of *Abstracts*: Volume 23, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 18, 2002
For abstracts: August 13, 2002

Invited Addresses

Lou P. van den Dries, University of Illinois, Urbana-Champaign, *Title to be announced.*

Diane Henderson, Pennsylvania State University, *Title to be announced.*

Christopher K. King, Northeastern University, *Title to be announced.*

Xiaobo Liu, University of Notre Dame, *Title to be announced.*

Special Sessions

Developments and Applications in Differential Geometry (Code: AMS SS C1), **Chuu-Lian Terng**, Northeastern University, and **Xiaobo Liu**, University of Notre Dame.

Ergodic Theory and Dynamical Systems (Code: AMS SS B1), **Stanley J. Eigen**, Northeastern University, and **Vidhu S. Prasad**, University of Massachusetts, Lowell.

Hilbert Schemes (Code: AMS SS G1), **Mark De Cataldo**, SUNY at Stony Brook, and **Anthony A. Iarrobino**, Northeastern University.

Modern Schubert Calculus (Code: AMS SS A1), **Frank Sottile**, University of Massachusetts, Amherst, and **Christopher T. Woodward**, Rutgers University.

Number Theory and Arithmetic Geometry (Code: AMS SS D1), **Matthew A. Papanikolas**, Brown University, and **Siman Wong**, University of Massachusetts, Amherst.

Quantum Information Theory (Code: AMS SS J1), **Christopher K. King**, Northeastern University, and **Mary Beth Ruskai**, University of Massachusetts Lowell.

Quivers and Their Generalizations (Code: AMS SS E1), **Alex Martsinkovsky**, **Gordana G. Todorov**, **Jerzy M. Weyman**, and **Andrei V. Zelevinsky**, Northeastern University.

Recent Developments in the Orbit Method for Real and p-adic Groups (Code: AMS SS F1), **Donald R. King**, Northeastern University, and **Alfred G. Noel**, University of Massachusetts, Boston.

Singularities in Algebraic and Analytic Geometry (Code: AMS SS H1), **Terence Gaffney** and **David B. Massey**, Northeastern University, and **Caroline Grant Melles**, U. S. Naval Academy.

The Mathematics of Water Waves (Code: AMS SS K1), **Diane Henderson**, Pennsylvania State University, and **Gene Wayne**, Boston University.

the History of Mathematics (Code: AMS SS L1), **Adrian C. Rice**, Randolph-Macon College, and **Amy E. Shell**, U. S. Military Academy.

Madison, Wisconsin

University of Wisconsin-Madison

October 12-13, 2002

Meeting #980

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2002
Program first available on AMS website: August 29, 2002
Program issue of electronic *Notices*: December 2002
Issue of *Abstracts*: Volume 23, Issue 4

Deadlines

For organizers: March 12, 2002
For consideration of contributed papers in Special Sessions: June 25, 2002
For abstracts: August 20, 2002

Invited Addresses

Lawrence Ein, University of Illinois at Chicago, *Title to be announced.*

Eleny Ionel, University of Wisconsin, *Title to be announced.*

Mikhail Safonov, University of Minnesota, *Title to be announced.*

John Sullivan, University of Illinois, Urbana-Champaign, *Title to be announced.*

Special Sessions

Arithmetic Algebraic Geometry (Code: AMS SS A1), **Ken Ono** and **Tonghai Yang**, University of Wisconsin-Madison.

Arrangements of Hyperplanes (Code: AMS SS E1), **Daniel C. Cohen**, Louisiana State University, **Peter Orlik**, University of Wisconsin-Madison, and **Anne Shepler**, University of California Santa Cruz.

Biological Computation and Learning in Intelligent Systems (Code: AMS SS S1), **Shun-ichi Amari**, RIKEN, **Amir Assadi**, University of Wisconsin-Madison, and **Tomaso Poggio**, Massachusetts Institute of Technology.

Characters and Representations of Finite Groups (Code: AMS SS U1), **Martin Isaacs**, University of Wisconsin, Madison, and **Mark Lewis**, Kent State University.

Combinatorics and Special Functions (Code: AMS SS T1), **Richard Askey** and **Paul Terwilliger**, University of Wisconsin-Madison.

Dynamical Systems (Code: AMS SS P1), **Sergey Bolotin** and **Paul Rabinowitz**, University of Wisconsin-Madison.

Effectiveness Questions in Model Theory (Code: AMS SS J1), **Charles McCoy**, **Reed Solomon**, and **Patrick Speissegger**, University of Wisconsin-Madison.

Geometric Methods in Differential Equations (Code: AMS SS H1), **Gloria Mari Beffa**, University of Wisconsin-Madison, and **Peter Olver**, University of Minnesota.

Geophysical Waves and Turbulence (Code: AMS SS M1), **Paul Milewski**, **Leslie Smith**, and **Fabian Waleffe**, University of Wisconsin-Madison.

Group Cohomology and Homotopy Theory (Code: AMS SS G1), **Alejandro Adem**, University of Wisconsin-Madison, and **Jesper Grodal**, Institute for Advanced Study.

Harmonic Analysis (Code: AMS SS C1), **Alex Ionescu** and **Andreas Seeger**, University of Wisconsin-Madison.

Hyperbolic Differential Equations and Kinetic Theory (Code: AMS SS K1), **Shi Jin**, **Marshall Slemrod**, and **Athanassios Tzavaras**, University of Wisconsin-Madison.

Lie Algebras and Related Topics (Code: AMS SS N1), **Georgia Benkart** and **Arun Ram**, University of Wisconsin-Madison.

Multiresolution Analysis and Data Presentation (Code: AMS SS F1), **Amos Ron**, University of Wisconsin-Madison.

Partial Differential Equations and Geometry (Code: AMS SS D1), **Sigurd Angenent** and **Mikhail Feldman**, University of Wisconsin-Madison.

Probability (Code: AMS SS R1), **David Griffeath**, University of Wisconsin-Madison, and **Timo Seppalainen**, Iowa State University.

Ring Theory and Related Topics (Code: AMS SS L1), **Don Passman**, University of Wisconsin-Madison.

Several Complex Variables (Code: AMS SS B1), **Pat Ahern**, **Xianghong Gong**, **Alex Nagel**, and **Jean-Pierre Rosay**, University of Wisconsin-Madison.

Salt Lake City, Utah

University of Utah

October 26–27, 2002

Meeting #981

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: September 2002
Program first available on AMS website: September 16, 2002
Program issue of electronic *Notices*: January 2003
Issue of *Abstracts*: Volume 23, Issue 4

Deadlines

For organizers: March 26, 2002
For consideration of contributed papers in Special Sessions: July 10, 2002
For abstracts: September 4, 2002

Invited Addresses

Yakov Eliashberg, Stanford University, *Title to be announced.*

Hart F. Smith, University of Washington, *Title to be announced.*

Michael Ward, University of British Columbia, *Title to be announced.*

Amie Wilkinson, Northwestern University, *Title to be announced.*

Orlando, Florida

University of Central Florida

November 9–10, 2002

Meeting #982

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: September 2002
Program first available on AMS website: September 26, 2002
Program issue of electronic *Notices*: January 2003
Issue of *Abstracts*: Volume 23, Issue 4

Deadlines

For organizers: April 10, 2002
For consideration of contributed papers in Special Sessions: July 23, 2002
For abstracts: September 17, 2002

Baltimore, Maryland

Baltimore Convention Center

January 15–18, 2003

Joint Mathematics Meetings, including the 109th Annual Meeting of the AMS, 86th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: October 2002
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 15, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Baton Rouge, Louisiana

Louisiana State University

March 14–16, 2003

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 14, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Bloomington, Indiana

Indiana University

April 4–6, 2003

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 4, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

New York, New York

Courant Institute

April 12–13, 2003

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 12, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Seville, Spain

June 18–21, 2003

First Joint International Meeting between the AMS and the Real Sociedad Matematica Española (RSME).
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: May 15, 2002
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses

Xavier Cabre, Universidad Politécnica de Cataluña, Barcelona, *Title to be announced.*

Charles Fefferman, Princeton University, *Title to be announced.*

Michael Hopkins, Massachusetts Institute of Technology, *Title to be announced.*

Ignacio Sols, Universidad Complutense, Madrid, *Title to be announced.*

Luis Vega, Universidad del Pais Vasco, Bilbao, *Title to be announced.*

Efim Zelmanov, Yale University, *Title to be announced.*

Special Sessions

Banach Spaces of Analytic Functions, **Daniel Girela**, University of Malaga, and **Michael Stessin**, SUNY at Albany.

Classical and Harmonic Analysis, **Nets Katz**, Washington University, **Carlos Perez**, Universidad de Sevilla, and **Ana Vargas**, Universidad Autonoma de Madrid.

Computational Methods in Algebra and Analysis, **Eduardo Cattani**, University of Massachusetts, Amherst, and **Francisco Jesus Castro-Jimenez**, Universidad de Sevilla.

History of Modern Mathematics–Gauss to Wiles, **Jose Ferreiros**, Universidad de Sevilla, and **David Rowe**, Universitat Mainz.

Interpolation Theory, Function Spaces and Applications, **Fernando Cobos**, University Complutense de Madrid, and **Pencho Petrushev**, University of South Carolina.

Mathematical Fluid Dynamics, **Diego Cordoba**, CSIC, Madrid, and Princeton University, and **Susan Friedlander**, University of Illinois, Chicago.

Nonlinear Dispersive Equations, **Gustavo Ponce**, University of California Santa Barbara, and **Luis Vega**, Universidad del Pais Vascos.

Operator Theory and Spaces of Analytic Functions, **Jose Bonet**, Universidad Politecnica de Valencia, **Pedro Paul**, Universidad de Sevilla, and **Cora S. Sadosky**, Howard University.

Variational Problems for Submanifolds, **Frank Morgan**, Williams College, and **Antonio Ros**, Universidad de Granada.

Binghamton, New York

SUNY-Binghamton

October 10–12, 2003

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 10, 2003
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Phoenix, Arizona

Phoenix Civic Plaza

January 7–10, 2004

Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 2, 2003
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Athens, Ohio

Ohio University

March 26–27, 2004

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 26, 2003
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Atlanta, Georgia

Atlanta Marriott Marquis and Hyatt Regency Atlanta

January 5–8, 2005

Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 5, 2004
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Sproul Hall, Riverside, CA 92521-0135; e-mail: `lapidus@math.ucr.edu`; telephone: 909-787-3113.

Central Section: Susan J. Friedlander, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: `susan@math.nwu.edu`; telephone: 312-996-3041.

Eastern Section: Lesley M. Sibner, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: `lsibner@duke.poly.edu`; telephone: 718-260-3505.

Southeastern Section: John L. Bryant, Department of Mathematics, Florida State University, Tallahassee, FL 32306-4510; e-mail: `bryant@math.fsu.edu`; telephone: 850-644-5805.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information** at `www.ams.org/meetings/`.

Meetings:

Important Information regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 175 in the January 2002 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Several options are available for speakers submitting abstracts, including an easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding. To see descriptions of the forms available, visit `http://www.ams.org/abstracts/instructions.html`, or send mail to `abs-submit@ams.org`, typing `help` as the subject line; descriptions and instructions on how to get the template of your choice will be e-mailed to you.

Completed abstracts should be sent to `abs-submit@ams.org`, typing `submission` as the subject line. Questions about abstracts may be sent to `abs-info@ams.org`.

Paper abstract forms may be sent to Meetings & Conferences Department, AMS, P.O. Box 6887, Providence, RI 02940. There is a $20 processing fee for each paper abstract. There is no charge for electronic abstracts. Note that all abstract deadlines are strictly enforced. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (See `http://www.ams.org/meetings/` for the most up-to-date information on these conferences.)

May 20–25, 2002: 6th International Conference on Clifford Algebras and Their Applications to Mathematical Physics, Cookeville, TN.

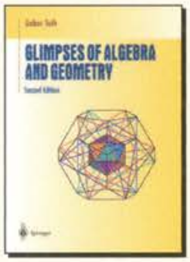
June 3–8, 2002: Abel Bicentennial Conference 2002, University of Oslo, Norway.

June 7–August 1, 2002: Joint Summer Research Conferences in the Mathematical Sciences, Mount Holyoke College, South Hadley, MA. See pages 1289-1291, November 2001 issue, for details.

SPRINGER FOR MATHEMATICS

Second Edition

GLIMPSES OF ALGEBRA AND GEOMETRY



G. TOTH, Rutgers University, Camden, NJ

The purpose of this book is to fill a gap between undergraduate and graduate mathematics studies. It is one of the few undergraduate texts to explore the subtle and sometimes puzzling connections between number theory, classical geometry and modern algebra in a clear and easily understandable style. Over 160 computer-generated images, accessible to readers via the World Wide Web, facilitate an understanding of mathematical concepts and proofs even further. This new edition includes innumerable improvements throughout the text, including an in-depth treatment of root formulas, a detailed and complete classification of finite Möbius groups a la Klein, and a quick, direct, and modern approach to Felix Klein's "Normalformsatzs."

2002/484 PP., 117 ILLUS., 18 IN COLOR
HARDCOVER/$64.95
ISBN 0-387-95345-0
UNDERGRADUATE TEXTS IN MATHEMATICS

FROM ELEMENTARY PROBABILITY TO STOCHASTIC DIFFERENTIAL EQUATIONS WITH MAPLE®

S. CYGANOWSKI, Tipperay Institute, Tipperary, Ireland; **P. KLOEDEN,** Johann Wolfgang Goethe Universität, Frankfurt/Main, Germany; and **J. OMBACH,** Jagiellonian University, Krakow, Poland

The authors provide a fast introduction to probabilistic and statistical concepts necessary to understand the basic ideas and methods of stochastic differential equations. The book is based on measure theory, which is introduced as smoothly as possible. It is intended for advanced undergraduate students or graduates, not necessarily in mathematics, providing an overview and intuitive background for more advanced studies as well as some practical skills in the use of MAPLE in the context of probability and its applications.

2002/310 PP./SOFTCOVER/$44.95
ISBN 3-540-42666-3
UNIVERSITEXT

SPECTRAL METHODS WITH APPLICATIONS TO INCOMPRESSIBLE VISCOUS FLOW

R. PEYRET, University of Nice-Sophia Antipolis, Nice, France

This well-written book explains the theory of spectral methods and their application to incompressible fluid flow in clear and elementary terms. It begins with an introduction to the fundamentals of spectral methods and then covers the Fourier and Chebyshev methods. Further chapters handle streamfunction-vorticity and velocity-pressure for Navier-Stokes equations, self- adaptive coordinate transform, domain decomposition, treatment of singularities, and free-surface flow.

2002/366 PP., 61 ILLUS./HARDCOVER/$59.95
ISBN 0-387-95221-7
APPLIED MATHEMATICAL SCIENCES, VOL. # 148

Third Edition

RIEMANNIAN GEOMETRY AND GEOMETRIC ANALYSIS

J. JOST, Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany

From the reviews of the first and second editions:

"... a very readable introduction to Riemannian geometry and geometric analysis ... With the vast development of the mathematical subject of geometric analysis, the present textbook is most welcome."

—MATHEMATICAL REVIEWS

This third edition gives a new presentation of Morse theory and Floer homology that emphasizes the geometric aspects and integrates it into the context of Riemannian geometry and geometric analysis. It also gives a new presentation of the geometric aspects of harmonic maps.

2002/532 PP./SOFTCOVER/$54.95
ISBN 3-540-42627-2
UNIVERSITEXT

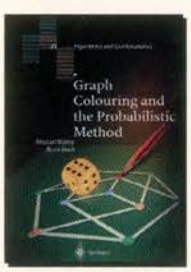
CREDIT RISK: MODELING, VALUATION AND HEDGING

T.R. BIELECKI, Northeastern Illinois University, Chicago, IL; and **M. RUTKOWSKI,** Warsaw University of Technology, Warsaw, Poland

Mathematical finance and financial engineering have been rapidly expanding fields of science over the past three decades. Here the author presents a comprehensive survey of past and recent developments in the area of credit risk research. One of this book's novel features is that it bridges the gap between the mathematical theory of credit risk and the financial practice, which serves as the motivation for the mathematical modeling studied in the book.

2002/500 PP./HARDCOVER/$64.95
ISBN 3-540-67593-0
SPRINGER FINANCE

GRAPH COLOURING AND THE PROBABILISTIC METHOD



M. MOLLOY, University of Toronto, ON, Canada; and **B. A. REED,** University of Paris VI, France

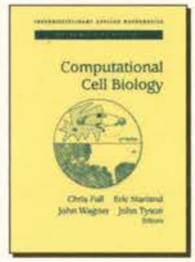
Over the past decade, many major advances have been made in the field of graph coloring via the probabilistic method. This monograph provides an accessible and unified treatment of these results, using the Lovasz Local Lemma and Concentration Inequalities developed by Talagrand. This gentle introduction to the probabilistic method will be useful to researchers and graduate students in graph theory, discrete mathematics, theoretical computer science and probabilists.

2002/326 PP., 19 ILLUS./HARDCOVER/$79.95
ISBN 3-540-42139-4
ALGORITHMS AND COMBINATORICS, VOL. # 23

Springer

www.springer-ny.com

COMPUTATIONAL CELL BIOLOGY



C. FALL, New York University, New York, NY; **J. WAGNER,** University of Connecticut, Storrs, CT; **E. MARLAND,** Appalachian State University, Boone, NC; and **J. TYSON,** Virginia Polytechnic Institute & State University, Blacksburg, VA (Eds.)

This textbook provides an introduction to dynamic modeling in molecular cell biology, taking a computational and intuitive approach. Selected biological examples are used to motivate concepts and techniques used in computational cell biology through a progression of increasingly more complex cellular functions modeled with increasingly complex mathematical and computational techniques. Advanced undergraduate and graduate theoretical biologists, and mathematics students and researchers, who wish to learn about modeling in cell biology, will find this book useful. The core of this book was the beginning of a textbook written by Joel Keizer before his death in 1999. Keizer founded and directed the Institute of Theoretical Dynamics at the University of California, Davis.

2002/APPROX. 445 PP., 210 ILLUS./HARDCOVER/$59.95
ISBN 0-387-95369-8
INTERDISCIPLINARY APPLIED MATHEMATICS, VOL. # 20

COMPUTATIONS IN ALGEBRAIC GEOMETRY WITH MACAULAY 2

D. EISENBUD, Mathematical Sciences Research Institute, Berkeley, CA; **D.R. GRAYSON,** University of Illinois at Urbana-Champaign, Urbana, IL; **M. STILLMAN,** Cornell University, Ithaca, NY; and **B. STURMFELS,** University of California, Berkeley, CA (Eds.)

This book presents algorithmic tools for algebraic geometry and experimental applications of them. It also introduces a software system in which the tools have been implemented and with which the experiments can be carried out. Macaulay 2 is a computer algebra system devoted to supporting research in algebraic geometry, commutative algebra, and their applications. The reader of this book will encounter Macaulay 2 in the context of concrete applications and practical computations in algebraic geometry. The expositions of the algorithmic tools presented here are designed to serve as a useful guide for those wishing to bring such tools to bear on their own problems.

2002/345 PP./HARDCOVER/$44.95
ISBN 3-540-42230-7
ALGORITHMS AND COMPUTATION IN MATHEMATICS, VOL. # 8

ORDER TODAY!

CALL: Toll-Free 1-800-SPRINGER

WEB: www.springer-ny.com

E-MAIL: orders@springer-ny.com

WRITE: Springer-Verlag New York, Inc., Dept. S357, PO Box 2485, Secaucus, NJ 07096-2485

VISIT: Your local scientific bookstore or urge your librarian to order for your department.

Prices subject to change without notice

4/02 *Promotion #S357*

0002-9920(200204)49:4;1-U