

Notices

of the American Mathematical Society

May 2003

Volume 50, Number 5

The Mathematics
of Learning:
Dealing with Data
page 537

PRIMES Is in P:
A Breakthrough
for "Everyman"
page 545

rum tum recentiorum industriam ac sa
n occupauisse, tam notum est, vt de ha
ose loqui superfluum foret. Nihilom
i oportet, omnes methodos hucusque p
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Input: integer $n > 1$

1. If $n = a^b$ for $a \in \mathbb{N}$ and $b > 1$
output COMPOSITE.
2. Find the smallest r
such that $o_r(n) > 4 \log^2 n$.
% $o_r(n)$ is the order of n modulo r
3. If $1 < (a, n) < n$ for some $a \leq r$
output COMPOSITE.
4. If $n \leq r$, output PRIME.
5. For $a = 1$ to $\lfloor 2\sqrt{\phi(r)} \log n \rfloor$ do
If $(X + a)^n \neq X^n + a \pmod{X^r - 1, n}$
output COMPOSITE.
% $\phi(r)$ is Euler's totient
6. output PRIME.

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res magna emolumenta capiendi, quae t
s dispendium mediocre largiter compens
tereaque scientiae dignitas requirere vide
omnia subsidia ad solutionem problematis
antis ac celebris sedulo excolantur. Pro
rationes non dubitamus, quin duae meth
entes, quarum efficaciam ac breuitatem

Is n prime? (see page 553)

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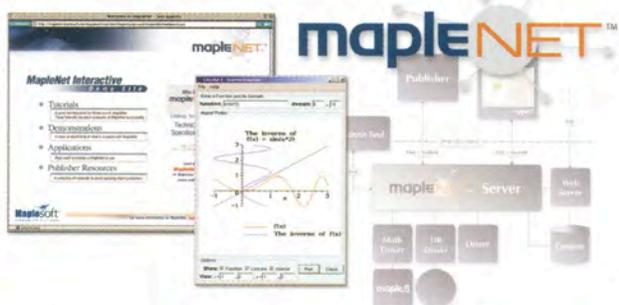
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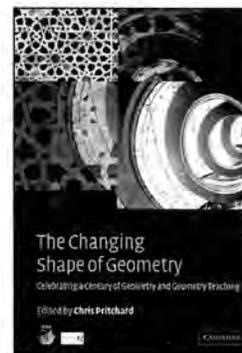
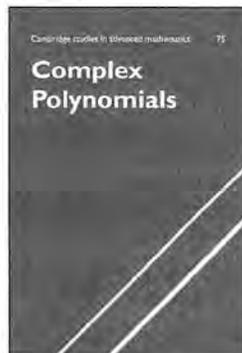
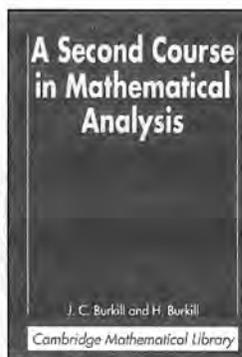
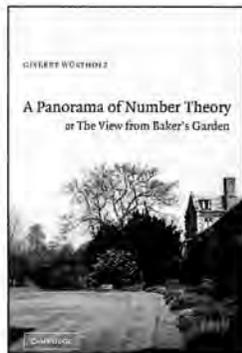
W.K. Nicholson and M.F. Youisif

This book provides an elementary, complete account of quasi-Frobenius rings at a level allowing researchers and graduate students to gain entry to the field.

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G. MAZZOLA, *University of Zürich, Switzerland*
In Collaboration with S. GÖLLER and S. MÜLLER

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Cryptographic Applications of Analytical Number Theory

Complexity Lower Bounds and Pseudorandomness

I. SHPARLINSKI, *Macquarie University, Sydney, Australia*

This book introduces new applications for analytic number theory in cryptography and related areas, such as complexity theory and pseudorandom number generation. Cryptographers will appreciate the new number theoretic techniques, which are invaluable cryptographic tools; number theorists will find challenging new applications for their work.

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PROGRESS IN COMPUTER SCIENCE AND APPLIED LOGIC, VOLUME 22

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M. FALK, *University of Würzburg, Germany*; F. MAROHN, and
B. TEWES, both, *Catholic University of Eichstätt-Ingolstadt,
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This text bridges the gap between theoretical and applied statistics by linking the theory of a selection of statistical procedures used in general practice with their application to real world data sets using the statistical software package SAS (Statistical Analysis System). These applications are intended to illustrate the theory and to help readers use the knowledge effectively and readily in execution. SAS programs and data used in the book can be downloaded from the web page <http://statistics-with-sas.de>.

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Discrete Introduction

P. KOCH MEDINA, *Swiss Re*; and S. MERINO, *UBS*, both,
Zürich, Switzerland

This work gives a self-contained presentation of the theory underlying the valuation of derivative financial instruments, a standard part of the toolbox of finance professionals. The material will be accessible to students and practitioners who have a working knowledge of linear algebra and calculus. The book also offers an introduction to modern probability theory, primarily within the context of finite sample spaces.

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W. BANGERTH, *The University of Texas at Austin, TX*; and
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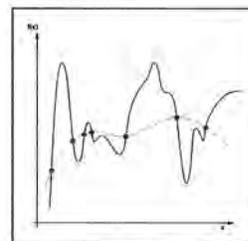
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Tomaso Poggio and Steve Smale

A key to extracting useful information from large quantities of data is to program machines to learn from examples. The authors outline the mathematical foundations of learning theory and describe one important algorithm.



545 PRIMES Is in P: A Breakthrough for “Everyman”

Folkmar Bornemann

Last year saw the discovery of a deterministic algorithm for deciding the primality of integers in polynomial running time. This article tells the story of the new algorithm from the point of view of a nonspecialist.



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Not Business As Usual

Mathematics departments are not taking care of business. Nearly 20 percent fewer mathematics bachelor's degrees were awarded in 1998 than in 1985. How can it be that at a time when higher-quality research is occurring at more and more institutions, interest in mathematics among undergraduates is decreasing?

It should be the department's responsibility to attract undergraduate students to the study of mathematics and to retain them. Departments have failed to accept this responsibility. They have placed an overemphasis on the creation of new mathematics instead of new mathematicians. Faculty fully understand the yardstick by which they are measured. It is not teaching, it is not outreach, it is not mentoring; it is research.

I value the excitement of carrying out a research project, the joy of discovery, the thrill when a proof comes together. Research is vital. Research brings with it the understanding of complex phenomena, it gives us insight into structures, it displays connections that were invisible. But our undergraduates simply never see this aspect of our work. Too often we view research as an esoteric activity, the end result of which is a presentation at a research conference or a publication in a journal. We need to share the excitement with our students and make the grand ideas visible in our undergraduate curriculum.

Departments have failed to understand how critical it is for our undergraduates to obtain a sound and working knowledge of mathematics and how important it is for departments to produce such students. Having more mathematics majors in the work force is beneficial for mathematics. With an increase in the number of mathematics majors, more students will think about advanced degrees. Departments nationwide have difficulty finding qualified U.S. students to enroll in their graduate mathematics programs. Many departments depend on a large cadre of foreign students to keep their graduate programs running. Although the NSF VIGRE program is intended to encourage departments to pay attention to the education of U.S. students, critics have complained about VIGRE funds being restricted to U.S. citizens and permanent residents! Graduate programs want the best students, somehow implying that these students do not exist in the U.S. It makes no sense to ignore the mathematical education of our undergraduates and then to complain about the lack of mathematical preparation of U.S. applicants for graduate school.

The focus on research and the corresponding lack of attention to undergraduate mathematics majors have also served to decrease the number of minorities going into mathematics. Traditionally underrepresented minority groups such as Mexican Americans, Native Americans, and African Americans have been excluded from our profession, and even now these minority groups are almost invisible in our graduate programs. The internationalization of our mathematics departments over the last

twenty years has resulted in very few from these minority groups being hired into faculty positions at our research universities. Since minorities are not part of the mathematical enterprise, minority concerns are not given voice in departmental matters. With so many other pressing concerns in a department, diversity issues are relegated to rhetoric. One doesn't need to have minority faculty in order to reach out to the minority students. However, minority faculty tend to be more concerned about diversity issues. It is just human nature.

I am not saying that our universities should exclude mathematicians trained abroad. I am saying that by depending heavily on this group to fill graduate school and faculty ranks, U.S. colleges and universities have managed to get by without giving proper attention to their undergraduates. Just getting by is a dangerous strategy, since American universities and American research efforts depend on U.S. tax dollars for their funding. Our attitudes toward undergraduates must change for the health of our subject, of our universities, and of our nation.

I would like to close with some recommendations for mathematics departments.

1. Departments should diversify their activities and their hiring. Research and scholarship; effective teaching; outreach to the local community; and recruitment of undergraduate mathematics majors, with special emphasis on integrating these students into the scientific life of the department, the university, and the nation, should be part of the portfolio of a department. Departments should hire and promote faculty who will be helpful in this endeavor.

2. Departments should substantially increase the number of undergraduate mathematics majors. Departments should establish collaborations with those mathematicians who have developed ideas and programs for increasing the number of mathematics majors.

3. Departments should recognize that the bachelor's degree in mathematics is marketable. Departments need to work with employers to ensure that mathematics majors are recruited by more firms.

4. Departments that have graduate programs should look upon their incoming graduate students as a national investment. Every effort should be made to insure that this investment pays off.

5. Departments should recognize that they are part of a neighborhood and that outreach to that neighborhood should be part of their portfolio. There should be faculty in a department that can achieve this goal.

6. Funding agencies like the National Science Foundation should require that all proposals include departmental documentation for the items listed above. The NSF is already asking this from Principal Investigators. An individual's efforts should reflect the activities of the department.

—William Yslas Vélez
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Letters to the Editor

The Lena Image

I was disappointed that Chan et al. chose to use the Lena image in their excellent article about image processing in the January 2003 issue. For those who don't know, the picture is the face of Lena Sjöblom, *Playboy's* 1972 Miss November, and has become a standard test for image processing algorithms since A. Sawchuk and others scanned it at the University of Southern California (USC) in 1973. (See the article by Jamie Hutchinson in the May/June 2001 issue of the IEEE PCS newsletter, http://www.iseeecs.org/may_june01.pdf, for more details.) The use of the image is a kind of joke: in the full picture she is wearing only the hat. It's easy to understand why not everyone finds the joke funny. It also reinforces the perception (reality?) that science is some kind of boys' club, an idea that mathematicians in particular have been trying hard to get away from.

—Christopher Woodward
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(Received January 10, 2003)

Response to Woodward

We are sorry that our inadvertent use of the Lena image has caused negative reactions from some readers. We certainly did not mean it, and we will try our best to refrain from using it again.

—Tony F. Chan
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(Received February 6, 2003)

Leonhard Euler's Blindness

As both an ophthalmologist and mathematics student, I was fascinated by

the article "The World of Blind Mathematicians" in the November 2002 issue. These men are truly remarkable. I would like to add some comments regarding Leonhard Euler's blindness. Although it is not possible to make specific diagnoses so many years later, certain conclusions can be drawn. We are indebted here to René Bernoulli for his comprehensive review of all the existing medical reports, documents, and portraits ("Leonhard Eulers Augenkrankheiten", in *Leonhard Euler 1707-1783: Beiträge zu Leben und Werk*, J. J. Burckhardt, E. A. Fellmann, and W. Habicht, eds., Birkhäuser, 1983, pp. 471-488).

First, the notion that eyestrain causes blindness or leads to serious organic disease is simply an old wives' tale. Although Euler himself made this association, Bernoulli comments correctly that this "legend" is "certainly false." Precisely what did happen to his right eye is not known with certainty: little information is available. It is likely that Euler had a recurrent process in this eye which was inflammatory or infectious in nature. Uveitis, for example, would be a possibility. It is clear that by 1753 there had been significant loss of vision in this eye and that it eventually became phthisical. Phthisis is the end stage of many different catastrophic conditions in which the eye is totally blind and shrunken and its anatomy is completely disorganized. Thus we know that his right eye became totally blind, but the etiology remains speculative.

In terms of the left eye, there is evidence that he had problems as early as 1740. He seems to have had a relapsing inflammatory process. Then in 1766 Euler experienced a sudden, dramatic loss of vision in this eye. His description suggests a vascular event such as a retinal vein occlusion, although numerous other causes of sudden vision loss are considerations as well. He did not, however, become totally blind: he could still distinguish large characters, for example. It is likely that he experienced additional insults to this eye subsequent to this event as well. In 1771 he underwent cataract surgery in this eye. The cataract was very advanced, and one can infer from the operative report

that the eye had suffered an attack of iritis in the past. Nonetheless, the surgery was uncomplicated and proved successful. Although his vision returned to some extent after this surgery, it was ultimately lost again at some point. The details are unclear.

Contrary to what is often repeated, we know that Euler did in fact retain a very small amount of vision in the left eye to the end of his life. Nevertheless, he was profoundly impaired: he was certainly functionally blind and would be considered legally blind today.

One can only stand in awe of an Euler even without knowing of his disability. Apparently some giants can see farther even without the advantage of sight.

—Gary Berman, MD
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(Received January 13, 2003)

Computer Models

In his review of Wolfram's *A New Kind of Science* [February 2003], Lawrence Gray discusses many issues of interest to specialists in the study of cellular automata, but he neglects a more central issue of interest to all mathematicians: that is, the issue of the role of computers in mathematics. In recent years computer models have seen increased use in the sciences, especially in those disciplines where traditional mathematical models based on differential equations have been unsuccessful. (In particular, I am thinking of the computer models used by Robert Axelrod in the study of the social sciences. See <http://www-personal.umich.edu/~axe/homepage.html>.) But computer models and traditional mathematical models are difficult to reconcile. On the one hand, it is difficult to prove general facts about most computer models; they rarely yield to the methods of mathematical analysis. And on the other hand, specific solutions to differential equations are often difficult to calculate, although

general properties of solutions may be quite easy to prove.

Is it possible to reformulate science so that all models are computer models? Wolfram suggests that the answer is "yes" and provides many examples to support his claim. But it is unclear how computer models such as Wolfram's correspond to traditional mathematical models of the same phenomena. Indeed, given the contrast between ease of calculation and ease of proof, it is an open question whether or not the capabilities of computer models and of more traditional models are, in principle or in practice, equivalent. Because computer models will almost certainly play a significant role in twenty-first century science, it is important that we, as a community of mathematicians, begin to try to answer these questions.

—Matthew P. Szudzik
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Mathematics Education

The way we teach it, mathematics would have long gone the way of Greek and Latin if it weren't for the fact that a great number of students are required, much to our financial benefit and very much against their will, to take math courses on the widespread, if questionable, assumption that math courses are the best way to separate the wheat from the chaff.

And how do we teach mathematics, from first grade on to college calculus? To see, one need only look at the textbooks on the market: by show and tell and drill and memorize and... pass/fail the exam and forget it.

The result, as John Holt wrote over thirty years ago in *How Children Fail*, is that students are "answer oriented instead of question oriented," and the consequences are here for all to see, e.g., in the *Trends in International Mathematics and Science Study* (<http://nces.ed.gov/timss>).

It is thus unfortunate that the *Notices* devoted yet more space to Steven Zucker's view that "there is wide agreement over the following statement: The fundamental problem with today's college students is that most arrive thinking that college is a simple continuation of high school" ("Telling the Truth", Opinion, March 2003) when he already said much the same thing in an article in the August 1996 issue ("Teaching at the University Level"). I, for one, am not in agreement.

The real reason that so many students fail in mathematics is that we have turned it into something that is what instant coffee is to espresso. For instance, already in first grade we look at "abstract" numbers such as 2 rather than numerators *cum* denominators such as $2\heartsuit$, thereby preventing our students from distinguishing the *addition* $2\heartsuit + 3\heartsuit = \heartsuit\heartsuit + \heartsuit\heartsuit = \heartsuit\heartsuit\heartsuit = 5\heartsuit$ from the *combination* $2\heartsuit + 3\spadesuit$, where + should be read "and". Students who have seen that if, say, $1\heartsuit = 5*$ and $1\spadesuit = 4*$, then $2\heartsuit + 3\spadesuit = 2(5*) + 3(4*) = 22*$ have no problem dealing with $2/7 + 3/7$ and $2/5 + 3/7$ appropriately. But then, one should not begin with $3/4$ but with $3/4$ of \heartsuit as "3 things, 4 of which can be exchanged for $1\heartsuit$," as with the 25-cent coin. And then, when, much later, we present the few survivors with vector spaces, it is unfortunate that we never gave them a chance to play with combinations of \heartsuit and \spadesuit . We may have had a few more students, both taking the course and passing it.

What is needed is rethinking the *logic* of mathematical exposition. For instance, do signed numbers really come *after* rational numbers? Is the Bolzano-Cauchy-Weierstrass theory of limits truly more appropriate to first-year calculus than polynomial approximations? Does it really simplify the approach to vector spaces not to deal with the dual from the start? etc. And why not appeal to the students' common sense? Students may then realize that things mathematical are the way they are, not because some teacher or some book says so, but because that is the only way they can make sense.

But then, it has always been easier to blame the victims as Steven Zucker does.

—Alain Schremmer
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Response to Schremmer

First, I am in some agreement with Alain Schremmer. There are serious problems that arise from the present state of primary and secondary education in mathematics. If we could fix K-12 education, that would be great. Since we can't, or rather until we can, it behooves us to determine *what we can do* with and for the "victims". Feeling sorry for them is not very practical, nor is simply blaming the system. We are obliged to deal with the high school graduates as they come to us. It is a mistake to equate guiding the victims' recovery with blaming the victims.

Moreover, students have always had to adjust to the higher aspirations and responsibilities of education in college. It's harder for them if they don't even know what these are.

—Steven Zucker
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The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred (notices-letters@ams.org; see the masthead for postal mail addresses). Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.

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The Mathematics of Learning: Dealing with Data

Tomaso Poggio and Steve Smale

The problem of understanding intelligence is said to be the greatest problem in science today and “the” problem for this century—as deciphering the genetic code was for the second half of the last one. Arguably, the problem of learning represents a gateway to understanding intelligence in brains and machines, to discovering how the human brain works, and to making intelligent machines that learn from experience and improve their competences as children do. In engineering, learning techniques would make it possible to develop software that can be quickly customized to deal with the increasing amount of information and the flood of data around us.

Examples abound. During the last decades, experiments in particle physics have produced a very large amount of data. Genome sequencing is doing the same in biology. The Internet is a vast repository of disparate information which changes rapidly and grows at an exponential rate: it is now significantly more than 100 terabytes, while the Library of Congress is about 20 terabytes.

We believe that a set of techniques based on a new area of science and engineering becoming known as “supervised learning” will become a key

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technology to extract information from the ocean of bits around us and to make sense of it.

Supervised learning, or learning from examples, refers to systems that are trained instead of programmed with a set of examples, that is, a set of input-output pairs. Systems that could learn from example to perform a specific task would have many applications. A bank may use a program to screen loan applications and approve the “good” ones. Such a system would be trained with a set of data from previous loan applications and the experience with their defaults. In this example, a loan application is a point in a multidimensional space of variables characterizing its properties; its associated output is a binary “good” or “bad” label.

In another example, a car manufacturer may want to have in its models a system to detect pedestrians about to cross the road to alert drivers to a possible danger while driving in downtown traffic. Such a system could be trained with positive and negative examples: images of pedestrians and images without pedestrians. In fact, software trained in this way with thousands of images has been recently tested in an experimental car from Daimler. It runs on a PC in the trunk and looks at the road in front of the car through a digital camera [1].

Algorithms have been developed that can produce a diagnosis of the type of cancer from a set of measurements of the expression level of many thousands of human genes in a biopsy of the tumor measured with a cDNA microarray containing probes for a number of genes [1]. Again, the software learns the classification rule from a set of examples, that is, from examples of expression patterns in a number

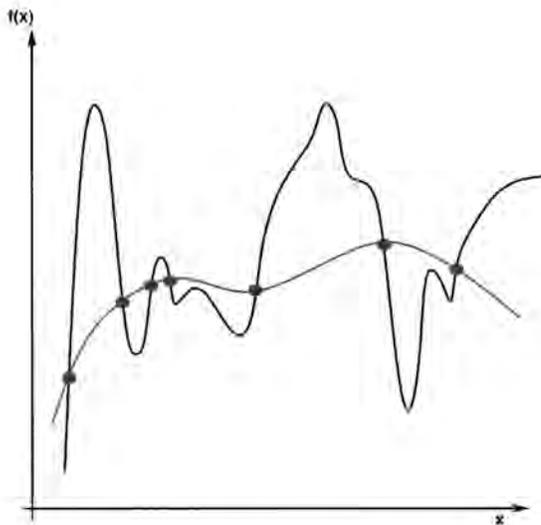


Figure 1. How can we learn a function which is capable of generalization—among the many functions which fit the examples equally well (here $m = 7$)?

of patients with known diagnoses. The challenge in this case is the high dimensionality of the input space—on the order of 20,000 genes—and the small number of examples available for training—around 50. In the future, similar learning techniques may be capable of some learning of a language and, in particular, of translating information from one language to another.

What we assume in the above examples is a machine that is trained instead of programmed to perform a task, given data of the form $(x_i, y_i)_{i=1}^m$. Training means synthesizing a function that best represents the relation between the inputs x_i and the corresponding outputs y_i . The central question of learning theory is how well this function generalizes, that is, how well it estimates the outputs for previously unseen inputs.

As we will see more formally later, learning techniques are similar to fitting a multivariate function to a certain number of measurement data. The key point, as we just mentioned, is that the fitting should be *predictive* in the same way that fitting experimental data (see Figure 1) from an experiment in physics can in principle uncover the underlying physical law, which is then used in a predictive way. In this sense, learning is also a principled method for distilling predictive and therefore scientific “theories” from the data.

We begin by presenting a simple “regularization” algorithm which is important in learning theory and its applications. We then outline briefly some of its applications and its performance. Next we provide a compact derivation of it. We then provide general theoretical foundations of learning theory. In

particular, we outline the key ideas of decomposing the generalization error of a solution of the learning problem into a sample and an approximation error component. Thus both probability theory and approximation theory play key roles in learning theory. We apply the two theoretical bounds to the algorithm and describe for it the tradeoff—which is key in learning theory and its applications—between *number of examples* and *complexity of the hypothesis space*. We conclude with several remarks, both with an eye to history and to open problems for the future.

A Key Algorithm

The Algorithm

How can we fit the “training” set of data $S_m = (x_i, y_i)_{i=1}^m$ with a function $f: X \rightarrow Y$ (where X is a closed subset of \mathbb{R}^n and $Y \subset \mathbb{R}$) that generalizes, i.e., is predictive? Here is an algorithm which does just that and which is almost magical for its simplicity and effectiveness:

1. Start with data $(x_i, y_i)_{i=1}^m$.
2. Choose a symmetric, positive-definite function $K_x(x') = K(x, x')$, continuous on $X \times X$. A kernel $K(t, s)$ is *positive definite* if $\sum_{i,j=1}^n c_i c_j K(t_i, t_j) \geq 0$ for any $n \in \mathbb{N}$ and any choice of $t_1, \dots, t_n \in X$ and $c_1, \dots, c_n \in \mathbb{R}$. An example of such a Mercer kernel is the Gaussian

$$(1) \quad K(x, x') = e^{-\|x-x'\|^2/2\sigma^2}$$

restricted to $X \times X$.

3. Define $f: X \rightarrow Y$ by

$$(2) \quad f(x) = \sum_{i=1}^m c_i K_{x_i}(x)$$

where $\mathbf{c} = (c_1, \dots, c_m)$ and

$$(3) \quad (m\gamma\mathbf{I} + \mathbf{K})\mathbf{c} = \mathbf{y},$$

where \mathbf{I} is the identity matrix, \mathbf{K} is the square positive-definite matrix with elements $K_{i,j} = K(x_i, x_j)$, and \mathbf{y} is the vector with coordinates y_i . The parameter γ is a positive, real number.

The linear system of equations (3) in m variables is well-posed, since \mathbf{K} is positive and $(m\gamma\mathbf{I} + \mathbf{K})$ is strictly positive. The condition number is good if $m\gamma$ is large. This type of system of equations has been studied since Gauss, and the algorithms for solving it efficiently represent one of the most developed areas in numerical and computational analysis.

What does equation (2) say? In the case of a Gaussian kernel, the equation approximates the unknown function by a weighted superposition of Gaussian “blobs”, each centered at the location x_i of one of the m examples. The weight c_i of each Gaussian is such as to minimize a regularized empirical error, that is, the error on the training set.

The σ of the Gaussian (together with y , see later) controls the degree of smoothing, of noise tolerance, and of generalization. Notice that for Gaussians with $\sigma = 0$ the representation of equation (2) effectively becomes a “look-up” table that cannot generalize (it provides the correct $y = y_i$ only when $x = x_i$ and otherwise outputs 0).

Performance and Examples

The algorithm performs well in a number of applications involving regression as well as binary classification. In the latter case the y_i of the training set $(x_i, y_i)_{i=1}^m$ take the values $\{-1, +1\}$; the predicted label is then $\{-1, +1\}$, depending on the sign of the function f of equation (2).

Regression applications are the oldest. Typically they involved fitting data in a small number of dimensions [1]. More recently, they also included typical learning applications, sometimes with a very high dimensionality. One example is the use of an algorithm in computer graphics for synthesizing new images [1]. The inverse problem of estimating facial expression and object pose from an image is another successful application [1]. Still another case is the control of mechanical arms. There are also applications in finance, as, for instance, the estimation of the price of derivative securities, such as stock options. In this case the algorithm replaces the classical Black-Scholes equation (derived from first principles) by learning the map from an input space (volatility, underlying stock price, time to expiration of the option, etc.) to the output space (the price of the option) from historical data [1].

Binary classification applications abound. The algorithm was used to perform binary classification on a number of problems [1]. It was also used to perform visual object recognition in a view-independent way and in particular face recognition and sex categorization from face images [1]. Other applications span bioinformatics for classification of human cancer from microarray data, text summarization, and sound classification.¹

Surprisingly, it has been realized quite recently that the same linear algorithm not only works well but is fully comparable in binary classification problems to the most popular classifiers of today (that turn out to be of the same family; see later).

Derivation

The algorithm described can be derived from Tikhonov regularization. To find the minimizer of the error, we may try to solve the problem—called Empirical Risk Minimization (ERM)—of finding the function in \mathcal{H} that minimizes

$$\frac{1}{m} \sum_{i=1}^m (f(x_i) - y_i)^2,$$

which is in general *ill-posed*, depending on the choice of the hypothesis space \mathcal{H} . Following Tikhonov (see for instance [8]), we minimize instead over the hypothesis space \mathcal{H}_K , for a fixed positive parameter γ , the regularized functional

$$(4) \quad \frac{1}{m} \sum_{i=1}^m (y_i - f(x_i))^2 + \gamma \|f\|_K^2,$$

where $\|f\|_K^2$ is the norm in \mathcal{H}_K , the Reproducing Kernel Hilbert Space (RKHS) defined by the kernel K . The last term in equation (4)—called *regularizer*—forces, as we will see, smoothness and uniqueness of the solution.

Let us first define the norm $\|f\|_K^2$. Consider the space of the linear span of $K_{\bar{x}_j}$. We use \bar{x}_j to emphasize that the elements of X used in this construction do not have anything to do *in general* with the training set $(x_i)_{i=1}^m$. Define an inner product in this space by setting $\langle K_x, K_{\bar{x}_j} \rangle = K(x, \bar{x}_j)$ and extending linearly to $\sum_{j=1}^r a_j K_{\bar{x}_j}$. The completion of the space in the associated norm is the RKHS, that is, a Hilbert space \mathcal{H}_K with the norm $\|f\|_K^2$ (see [4]). Note that $\langle f, K_x \rangle = f(x)$ for $f \in \mathcal{H}_K$ (just let $f = K_{\bar{x}_j}$ and extend linearly).

To minimize the functional in equation (4), we take the functional derivative with respect to f , apply it to an element \bar{f} of the RKHS, and set it equal to 0. We obtain

$$(5) \quad \frac{1}{m} \sum_{i=1}^m (y_i - f(x_i)) \bar{f}(x_i) - \gamma \langle f, \bar{f} \rangle = 0.$$

Equation (5) must be valid for any \bar{f} . In particular, setting $\bar{f} = K_x$ gives

$$(6) \quad f(x) = \sum_{i=1}^m c_i K_{x_i}(x)$$

where

$$(7) \quad c_i = \frac{y_i - f(x_i)}{m\gamma}$$

since $\langle f, K_x \rangle = f(x)$. Equation (3) then follows by substituting equation (6) into equation (7).

Notice also that essentially the same derivation for a generic loss function $V(y, f(x))$, instead of $(f(x) - y)^2$, yields the same equation (6), but equation (3) is now different and, in general, nonlinear, depending on the form of V . In particular, the popular Support Vector Machine (SVM) regression and SVM classification algorithms correspond to special choices of nonquadratic V : one to provide a “robust” measure of error and the other to approximate the ideal loss function corresponding to binary (mis)classification. In both cases the solution is still of the same form as equation (6) for

¹The very closely related Support Vector Machine (SVM) classifier was used for the same family of applications and in particular for bioinformatics and for face recognition and car and pedestrian detection [1].

any choice of the kernel K . The coefficients c_i are no longer given by equation (7) but must be found by solving a quadratic programming problem.

Theory

We give some further justification of the algorithm by sketching very briefly its foundations in some basic ideas of learning theory.

Here the data $(x_i, y_i)_{i=1}^m$ is supposed random so that there is an unknown probability measure ρ on the product space $X \times Y$ from which the data is drawn.

This measure ρ defines a function

$$(8) \quad f_\rho : X \rightarrow Y$$

satisfying $f_\rho(x) = \int y d\rho_x$, where ρ_x is the conditional measure on $X \times Y$.

From this construction f_ρ can be said to be the true input-output function reflecting the environment which produces the data. Thus a measurement of the error of f is

$$(9) \quad \int_X (f - f_\rho)^2 d\rho_x,$$

where ρ_x is the measure on X induced by ρ (sometimes called the *marginal measure*).

The goal of learning theory might be said to "find" f minimizing this error. Now to search for such an f , it is important to have a space \mathcal{H} —the *hypothesis space*—in which to work ("learning does not take place in a vacuum"). Thus consider a convex space of continuous functions $f : X \rightarrow Y$ (remember $Y \subset \mathbb{R}$) that as a subset of $C(X)$ is *compact*, where $C(X)$ is the Banach space of continuous functions with $\|f\| = \max_X |f(x)|$.

A basic example is

$$(10) \quad \mathcal{H} = \overline{I_K(B_R)}$$

where $I_K : \mathcal{H}_K \rightarrow C(X)$ is the inclusion and B_R is the ball of radius R in \mathcal{H}_K .

Starting from the data $(x_i, y_i)_{i=1}^m = z$, one may minimize $\frac{1}{m} \sum_{i=1}^m (f(x_i) - y_i)^2$ over $f \in \mathcal{H}$ to obtain a unique hypothesis $f_z : X \rightarrow Y$. This f_z is called the empirical optimum, and we may focus on the problem of estimating

$$(11) \quad \int_X (f_z - f_\rho)^2 d\rho_x.$$

It is useful towards this end to break the problem into steps by defining a "true optimum" $f_{\mathcal{H}}$ relative to \mathcal{H} by taking the minimum over \mathcal{H} of $\int_X (f - f_\rho)^2$. Thus we may exhibit

$$(12) \quad \begin{aligned} \int_X (f_z - f_\rho)^2 &= S(z, \mathcal{H}) + \int_X (f_{\mathcal{H}} - f_\rho)^2 \\ &= S(z, \mathcal{H}) + A(\mathcal{H}), \end{aligned}$$

where

$$(13) \quad S(z, \mathcal{H}) = \int_X (f_z - f_\rho)^2 - \int_X (f_{\mathcal{H}} - f_\rho)^2.$$

The first term, (S) on the right-hand side in equation (12), must be estimated in probability over z , and the estimate is called the *sample error* (sometimes also the *estimation error*). It is naturally studied in the theory of probability and of empirical processes [7]. The second term, (A) , is dealt with via approximation theory (for a review see [6] and also [10], [11]) and is called the *approximation error*. The decomposition of equation (12) is indirectly related to the well-known bias and variance decomposition in statistics.

Sample Error

First consider an estimate for the sample error, which will have the form

$$(14) \quad S(z, \mathcal{H}) \leq \epsilon$$

with high confidence, this confidence depending on ϵ and on the sample size m .

Let us be more precise. Recall that the covering number $\text{Cov}\#(\mathcal{H}, \eta)$ is the number of balls in \mathcal{H} of radius η needed to cover \mathcal{H} .

Theorem 1. *Suppose $|f(x) - y| \leq M$ for all $f \in \mathcal{H}$ for almost all $(x, y) \in X \times Y$. Then*

$$\text{Prob}_{z \in (X \times Y)^m} \{S(z, \mathcal{H}) \leq \epsilon\} \geq 1 - \delta$$

where $\delta = \text{Cov}\#(\mathcal{H}, \epsilon/24M)e^{-m\epsilon^2/288M^2}$.

The result is Theorem C^* of [4], but earlier versions (usually without a topology on \mathcal{H}) have been proved by others, especially Vapnik, who formulated the notion of VC dimension to measure the complexity of the hypothesis space for the case of $\{0, 1\}$ functions.

In a typical case of Theorem 1 the hypothesis space \mathcal{H} is taken to be as in equation (10), where B_R is the ball of radius R in an RKHS with a smooth K (or in a Sobolev space). In this context R plays an analogous role to VC dimension [16]. Estimates for the covering numbers in these cases were provided by Cucker, Smale, and Zhou [1].

The proof of Theorem 1 starts from the Hoeffding inequality (which can be regarded as an exponential version of Chebyshev's inequality of probability theory). One applies this estimate to the function $X \times Y \rightarrow \mathbb{R}$ which takes (x, y) to $(f(x) - y)^2$. Then extending the estimate to the set of $f \in \mathcal{H}$ introduces the covering number into the picture. With a little more work, Theorem 1 is obtained.

Approximation Error

The approximation error $\int_X (f_{\mathcal{H}} - f_\rho)^2$ may be studied as follows.

Suppose $B : L^2 \rightarrow L^2$ is a compact, strictly positive (selfadjoint) operator. Then let E be the Hilbert space

$$\{g \in L^2, \|B^{-s}g\| < \infty\}$$

with inner product $\langle g, h \rangle_E = \langle B^{-s}g, B^{-s}h \rangle_{L^2}$. Suppose moreover that $E \rightarrow L^2$ factors as $E \rightarrow C(X) \rightarrow L^2$ with the inclusion $J_E : E \hookrightarrow C(X)$ well defined and compact.

Let \mathcal{H} be $J_E(B_R)$ when B_R is the ball of radius R in E . A theorem on the approximation error is

Theorem 2. Let $0 < r < s$ and \mathcal{H} be as above. Then

$$\|f_\rho - f_{\mathcal{H}}\|^2 \leq (1/R)^{\frac{2r}{s-r}} \|B^{-r}f_\rho\|^{\frac{2s}{s-r}}.$$

We now use $\|\cdot\|$ for the norm in the space of square-integrable functions on X , with measure ρ_X . For our main example of RKHS, take $B = L_K^{1/2}$, where K is a Mercer kernel,

$$(15) \quad L_K f(x) = \int_X f(x')K(x, x'),$$

and we have taken the square root of the operator L_K . In this case E is \mathcal{H}_K as above.

Details and proofs may be found in [4] and in [15].

Sample and Approximation Error for the Regularization Algorithm

The previous discussion depends upon a compact hypothesis space \mathcal{H} from which the experimental optimum f_z and the true optimum $f_{\mathcal{H}}$ are taken. In the key algorithm of the preceding section, the optimization is done over all $f \in \mathcal{H}_K$ with a regularized error function. The error analysis of the preceding two subsections must therefore be extended.

Thus let $f_{y,z}$ be the empirical optimum for the regularized problem as exhibited in equation (4):

$$\frac{1}{m} \sum_{i=1}^m (y_i - f(x_i))^2 + \gamma \|f\|_K^2.$$

Then

$$(16) \quad \int (f_{y,z} - f_\rho)^2 \leq S(\gamma) + A(\gamma)$$

where $A(\gamma)$ (the approximation error in this context) is

$$(17) \quad A(\gamma) = \gamma^{1/2} \|L_K^{-1/4} f_\rho\|^2$$

and the sample error is

$$(18) \quad S(\gamma) = \frac{32M^2(\gamma + C)^2}{\gamma^2} v^*(m, \delta)$$

where $v^*(m, \delta)$ is the unique solution of

$$(19) \quad \frac{m}{4} v^3 - \ln\left(\frac{4m}{\delta}\right)v - c_1 = 0.$$

Here $C, c_1 > 0$ depend only on X and K . For the proof one reduces to the case of compact \mathcal{H} and applies Theorems 1 and 2. Thus finding the optimal solution is equivalent to finding the best tradeoff

between A and S for a given m . In our case, this bias-variance problem is to minimize $S(\gamma) + A(\gamma)$ over $\gamma > 0$. There is a unique solution—a best γ —for the choice in equation (4). For this result and its consequences see [5].

Remarks

The Tradeoff between Sample Complexity and Hypothesis Space Complexity

For a given fixed hypothesis space \mathcal{H} , only the sample error component of the error of f_z can be controlled (in equation (12) only $S(z, \mathcal{H})$ depends on the data). In this view, convergence of S to zero as the number of data points increases (Theorem 1) is then the central problem in learning. Vapnik called consistency of ERM (i.e., convergence of the empirical error to the true error) the key problem in learning theory, and in fact much modern work has focused on refining the necessary and sufficient conditions for consistency of ERM (the uniform Glivenko-Cantelli property of \mathcal{H} , finite V_γ dimension for $\gamma > 0$, etc.; see [8]). More generally, however, there is a tradeoff between minimizing the sample error and minimizing the approximation error—what we referred to as the bias-variance problem. Increasing the number m of data points decreases the sample error. The effect of increasing the complexity of the hypothesis space is trickier. Usually the approximation error decreases but the sample error increases. This means that there is an optimal complexity of the hypothesis space for a given number of training data. In the case of the regularization algorithm described in this paper, this tradeoff corresponds to an optimum value for γ as studied in [3], [5], [11]. In empirical work, the optimum value is often found through cross-validation techniques [18].

This tradeoff between approximation error and sample error is probably the most critical issue in determining good performance on a given problem. The class of regularization algorithms, such as equation (4), shows clearly that it is also a tradeoff—quoting Girosi—between the *curse of dimensionality* (not enough examples) and the *blessing of smoothness* (which decreases the effective “dimensionality”, i.e., the complexity of the hypothesis space) through the parameter γ .

The Regularization Algorithm and Support Vector Machines

There is nothing to stop us from using the algorithm we described in this paper—that is, square loss regularization—for *binary classification*. Whereas SVM classification arose from using—with *binary* γ —the loss function

$$V(f(\mathbf{x}, \gamma)) = (1 - \gamma f(\mathbf{x}))_+,$$

we can perform least-squares regularized classification via the loss function

800		250		100	
SVM	RLSC	SVM	RLSC	SVM	RLSC
0.131	0.129	0.167	0.165	0.214	0.211

Table 1. A comparison of SVM and RLSC (Regularized Least Squares Classification) accuracy on a multiclass classification task (the 20newsgroups dataset with 20 classes and high dimensionality, around 50,000), performed using the standard “one versus all” scheme based on the use of binary classifiers. The top row indicates the number of documents/class used for training. Entries in the table are the fraction of misclassified documents. From [14].

52		20		10	
SVM	RLSC	SVM	RLSC	SVM	RLSC
0.072	0.066	0.176	0.169	0.341	0.335

Table 2. A comparison of SVM and RLSC accuracy on another multiclass classification task (the sector105 dataset, consisting of 105 classes with dimensionality about 50,000). The top row indicates the number of documents/class used for training. Entries in the table are the fraction of misclassified documents. From [14].

$$V(f(\mathbf{x}, y)) = (f(\mathbf{x}) - y)^2.$$

This classification scheme was used at least as early as 1989 (for reviews see [13]) and then rediscovered again by many others (see [1]), including Mangasarian (who refers to square loss regularization as “proximal vector machines”) and Suykens (who uses the name “least square SVMs”). Rifkin [14] has confirmed the interesting empirical results by Mangasarian and Suykens: “classical” square loss regularization works well also for binary classification (examples are in Tables 1 and 2).

In references to supervised learning, the Support Vector Machine method is often described (see for instance R. M. Karp’s article in the May 2002 issue of the *Notices*) according to the “traditional” approach, introduced by Vapnik and followed by almost everybody else. In this approach, one starts with the concepts of *separating hyperplanes* and *margin*. Given the data, one searches for the linear hyperplane that separates the positive and the negative examples, assumed to be linearly separable, with the largest margin (the margin is defined as the distance from the hyperplane to the nearest example). Most articles and books follow this approach, go from the separable to the nonseparable case, and use a so-called “kernel trick” (!) to extend it to the nonlinear case. SVM for classification was introduced by Vapnik in the linear, separable case in terms of maximizing the margin. In the nonseparable case, the margin motivation loses most of its meaning. A more general and simpler framework for deriving SVM algorithms

for classification and regression is to regard them as special cases of regularization and follow the treatment of the section above on the key algorithm. In the case of linear functions $f(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x}$ and separable data, maximizing the margin is exactly equivalent to maximizing $1/\|\mathbf{w}\|$, which is in turn equivalent to minimizing $\|\mathbf{w}\|^2$, which corresponds to minimizing the RKHS norm.

The Regularization Algorithm and Learning Theory

The Mercer theorem was introduced in learning theory by Vapnik; RKHS were explicitly introduced in learning theory by Girosi and later by Vapnik [1], [16]. Poggio and Girosi [13], [1] had introduced Tikhonov regularization in learning theory. Earlier, Gaussian Radial Basis Functions were proposed as an alternative to neural networks by Broomhead and Loewe. Of course, RKHS had been pioneered by Parzen and Wahba ([12], [18]; for a review see [18]) for applications closely related to learning, including data smoothing (for image processing and computer vision, see [1]).

A Bayesian Interpretation

The learning algorithm equation (4) has an interesting Bayesian interpretation [18]: the data term—is a model of (Gaussian, additive) noise, and the RKHS norm (the stabilizer) corresponds to a prior probability on the hypothesis space \mathcal{H} .

Let us define $P[f|S_m]$ as the conditional probability of the function f given the training examples $S_m = (\mathbf{x}_i, y_i)_{i=1}^m$, $P[S_m|f]$ as the conditional probability of S_m given f , i.e., a model of the noise, and $P[f]$ as the a priori probability of the random field f . Then Bayes’s theorem provides the posterior distribution as

$$P[f|S_m] = \frac{P[S_m|f] P[f]}{P(S_m)}.$$

If the noise is normally distributed with variance σ , then the probability $P[S_m|f]$ is

$$P[S_m|f] = \frac{1}{Z_L} e^{-\frac{1}{2\sigma^2} \sum_{i=1}^m (y_i - f(\mathbf{x}_i))^2}$$

where Z_L is a normalization constant.

If $P[f] = \frac{1}{Z_r} e^{-\|f\|_k^2}$, where Z_r is another normalization constant, then

$$P[f|S_m] = \frac{1}{Z_D Z_L Z_r} e^{-\left(\frac{1}{2\sigma^2} \sum_{i=1}^m (y_i - f(\mathbf{x}_i))^2 + \|f\|_k^2\right)}.$$

One of the several possible estimates of f from $P[f|S_m]$ is the so-called *Maximum A Posteriori* (MAP) estimate, that is,

$$\max P[f|S_m] = \min \sum_{i=1}^m (y_i - f(\mathbf{x}_i))^2 + 2\sigma^2 \|f\|_k^2,$$

which is the same as the regularization functional if $\lambda = 2\sigma^2/m$ (for details and extensions to models

of non-Gaussian noise and different loss functions, see [8]).

Necessary and Sufficient Conditions for Learnability

Compactness of the hypothesis space \mathcal{H} is *sufficient* for consistency of ERM, that is, for bounds of the type of Theorem 1 on the sample error. The *necessary and sufficient* condition is that \mathcal{H} is a uniform Glivenko-Cantelli class of functions, in which case no specific topology is assumed for \mathcal{H} .² There are several equivalent conditions on \mathcal{H} such as finiteness of the V_γ dimension for all positive γ (which reduces to finiteness of the VC dimension for $\{0, 1\}$ functions).³

We saw earlier that the regularization algorithm equation (4) ensures (through the resulting compactness of the “effective” hypothesis space) well-posedness of the problem. It also yields convergence of the empirical error to the true error (i.e., bounds such as Theorem 1). An open question is whether there is a connection between well-posedness and consistency. For well-posedness the critical condition is usually stability of the solution. In the learning problem, this condition refers to stability of the solution of ERM with respect to small changes of the training set S_m . In a similar way, the condition number characterizes the stability of the solution of equation (3). Is it possible that some specific form of stability may be necessary and sufficient for consistency of ERM? *Such a result would be surprising*, because, a priori, there is no reason why there should be a connection between well-posedness and consistency; they are both important requirements for ERM, but they seem quite different and independent of each other.

Learning Theory, Sample Complexity, and Brains

The theory of supervised learning outlined in this paper and in the references has achieved a

²Definition: A class \mathcal{F} of functions f is a uniform Glivenko-Cantelli class if for every $\varepsilon > 0$

$$\lim_{m \rightarrow \infty} \sup_{\rho} \mathbb{P} \left\{ \sup_{f \in \mathcal{F}} |E_{\rho_m} f - E_{\rho} f| > \varepsilon \right\} = 0,$$

where ρ_n is the empirical measure supported on a set x_1, \dots, x_n .

³In [2], following [17], a necessary and sufficient condition is proved for uniform convergence of $|I_{emp}[f] - I_{exp}[f]|$, in terms of the finiteness for all $\gamma > 0$ of a combinatorial quantity called the V_γ dimension of \mathcal{F} (which is the set $V(x), f(x), f \in \mathcal{H}$), under some assumptions on V . The result is based on a necessary and sufficient (distribution independent) condition which uses the metric entropy of \mathcal{F} defined as $H_m(\varepsilon, \mathcal{F}) = \sup_{x_m \in X^m} \log \mathcal{N}(\varepsilon, \mathcal{F}, x_m)$, where $\mathcal{N}(\varepsilon, \mathcal{F}, x_m)$ is the ε -covering of \mathcal{F} with respect to $l_{x_m}^\infty$ ($l_{x_m}^\infty$ is the l^∞ distance on the points x_m).

Theorem [Dudley, Giné, and Zinn]. \mathcal{F} is a uniform Glivenko-Cantelli class if and only if $\lim_{m \rightarrow \infty} H_m(\varepsilon, \mathcal{F})/m = 0$ for all $\varepsilon > 0$.

remarkable degree of completeness and of practical success in many applications. Within it, many interesting problems remain open and are a fertile ground for interesting and useful mathematics. One may also take a broader view and ask, What next?

One could argue that the most important aspect of intelligence and of the amazing performance of real brains is the ability to learn. How then do the learning machines we have described in the theory compare with brains? There are of course many aspects of biological learning that are not captured by the theory and several difficulties in making any comparison. One of the most obvious differences, however, is the ability of people and animals to learn from very few examples. The algorithms we have described can learn an object recognition task from a few thousand labeled images. This is a small number compared with the apparent dimensionality of the problem (millions of pixels), but a child, or even a monkey, can learn the same task from just a few examples. Of course, evolution has probably done a part of the learning, but so have we, when we choose for any given task an appropriate input representation for our learning machine. From this point of view, as Donald Geman has argued, the interesting limit is not “ m goes to infinity”, but rather “ m goes to zero”. Thus an important area for future theoretical and experimental work is learning from partially labeled examples (and the related area of active learning). In the first case there are only a small number ℓ of labeled pairs $(x_i, y_i)_{i=1}^\ell$ —for instance, with y_i binary—and many unlabeled data $(x_i)_{\ell+1}^m$, $m \gg \ell$. Though interesting work has begun in this direction, a satisfactory theory that provides conditions under which unlabeled data can be used is still lacking.

A comparison with real brains offers another, and probably related, challenge to learning theory. The “learning algorithms” we have described in this paper correspond to one-layer architectures. Are hierarchical architectures with more layers justifiable in terms of learning theory? It seems that the learning theory of the type we have outlined does not offer any general argument in favor of hierarchical learning machines for regression or classification. This is somewhat of a puzzle, since the organization of cortex—for instance, visual cortex—is strongly hierarchical. At the same time, hierarchical learning systems show superior performance in several engineering applications. For instance, a face categorization system in which a single SVM classifier combines the real-valued output of a few classifiers, each trained to a different component of faces (such as eye and nose), outperforms a single classifier trained on full images of faces [1]. The theoretical issues surrounding hierarchical systems of this type are wide open and likely to be of

paramount importance for the next major development of efficient classifiers in several application domains.

Why hierarchies? There may be reasons of *efficiency*—computational speed and use of computational resources. For instance, the lowest levels of the hierarchy may represent a dictionary of features that can be shared across multiple classification tasks (see [9]). Hierarchical systems usually decompose a task in a series of simple computations at each level—often an advantage for fast implementations. There may also be the more fundamental issue of *sample complexity*. We mentioned that an obvious difference between our best classifiers and human learning is the number of examples required in tasks such as object detection. The theory described in this paper shows that the difficulty of a learning task depends on the size of the required hypothesis space. This complexity determines in turn how many training examples are needed to achieve a given level of generalization error. Thus the complexity of the hypothesis space sets the speed limit and the sample complexity for learning. If a task—like a visual recognition task—can be decomposed into low-complexity learning tasks, for each layer of a hierarchical learning machine, then each layer may require only a small number of training examples. Of course, not all classification tasks have a hierarchical representation. Roughly speaking, the issue is under which conditions a function of many variables can be approximated by a function of a small number of functions of subsets of the original variables. Neuroscience suggests that what humans can learn can be represented by hierarchies that are locally simple. Thus our ability of learning from just a few examples, and its limitations, may be related to the hierarchical architecture of cortex. This is just one of several possible connections, still to be characterized, between learning theory and the ultimate problem in natural science—the organization and the principles of higher brain functions.

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PRIMES Is in P: A Breakthrough for “Everyman”

Folkmar Bornemann

“New Method Said to Solve Key Problem in Math” was the headline of a story in the *New York Times* on August 8, 2002, meaning the proof of the statement $\text{PRIMES} \in \mathcal{P}$, hitherto a big open problem in algorithmic number theory and theoretical computer science. Manindra Agrawal, Neeraj Kayal, and Nitin Saxena of the Indian Institute of Technology accomplished the proof through a surprisingly elegant and brilliantly simple algorithm. Convinced of its validity after only a few days, the experts raved about it: “This algorithm is beautiful” (Carl Pomerance); “It’s the best result I’ve heard in over ten years” (Shafi Goldwasser).

Four days before the headline in the *New York Times*, on a Sunday, the three authors had sent a nine-page preprint titled “PRIMES is in P” to fifteen experts. The same evening Jaikumar Radhakrishnan and Vikraman Arvind sent congratulations. Early on Monday one of the deans of the subject, Carl Pomerance, verified the result, and in his enthusiasm he organized an impromptu seminar for that afternoon and informed Sara Robinson of the *New York Times*. On Tuesday the preprint became freely available on the Internet. On Thursday a further authority, Hendrik Lenstra Jr., put an end to some brief carping in the NMBRTHRY email list with the pronouncement:

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This article is a translation by the editor of the Notices of an article by the author that appeared in German in the Mitteilungen der Deutschen Mathematiker-Vereinigung 4-2002, 14-21.

The remarks ... are unfounded and/or inconsequential. ... The proofs in the paper do NOT have too many additional problems to mention. The only true mistake is ..., but that is quite easy to fix. Other mistakes ... are too minor to mention. The paper is in substance completely correct.

And already on Friday, Dan Bernstein posted on the Web an improved proof of the main result, shortened to one page.

This unusually brief—for mathematics—period of checking reflects both the brevity and elegance of the argument and its technical simplicity, “suited for undergraduates”. Two of the authors, Kayal and Saxena, had themselves just earned their bachelor’s degrees in computer science in the spring. Is it then an exception for a breakthrough to be accessible to “Everyman”?

In his speech at the 1998 Berlin International Congress of Mathematicians, Hans-Magnus Enzensberger took the position that mathematics is both “a cultural anathema” and at the same time in the midst of a golden age due to successes of a quality that he saw neither in theater nor in sports. To be sure, some of those successes have many mathematicians themselves pondering the gulf between the priesthood and the laity *within* mathematics. A nonspecialist—cross your heart: how many of us are not such “Everymen”?—can neither truly comprehend nor fully appreciate the proof of Fermat’s Last Theorem by Andrew Wiles, although popularization efforts like the book of Simon Singh help one get an inkling of the connections. Probably no author could be found to help “Everyman”

comprehend all the ramifications and the significance of the successes of last year's recipients of the Fields Medals.

So it is that each one adds bricks to his parapet in the Tower of Babel named Mathematics and deems his constructions there to be fundamental. Rarely is there such a success as at the beginning of August: a foundation stone for the tower that "Everyman" can understand.

Paul Leyland expressed a view that has been in many minds: "Everyone is now wondering what else has been similarly overlooked." Can this explain Agrawal's great astonishment ("I never imagined that our result will be of much interest to traditional mathematicians"): namely, why within the first ten days the dedicated website had over two million hits and three hundred thousand downloads of the preprint?

When a long outstanding problem is finally solved, every mathematician would like to share in the pleasure of discovery by following for himself what has been done. But too often he is stymied by the abstruseness of so much of contemporary mathematics. The recent negative solution to . . . is a happy counterexample. In this article, a complete account of this solution is given; the only knowledge a reader needs to follow the argument is a little number theory: specifically basic information about divisibility of positive integers and linear congruences.

Martin Davis, Hilbert's tenth problem is unsolvable, *American Mathematical Monthly* 80 (1973), 233-69, first paragraph of the introduction.

As a specialist in numerical analysis and not in algorithmic number theory, I wanted to test my mettle as "Everyman", outside of my parapet.

The Problem

Happily the three motivated their work not by the significance of prime numbers for cryptography and e-commerce, but instead at the outset followed the historically aware Don Knuth in reproducing a quotation from the great Carl Friedrich Gauss from article 329 of the *Disquisitiones Arithmeticae* (1801), given here in the 1966 translation by Arthur A. Clarke:

The problem of distinguishing prime numbers from composite numbers and of resolving the latter into their prime factors is known to be one of the most

important and useful in arithmetic. It has engaged the industry and wisdom of ancient and modern geometers to such an extent that it would be superfluous to discuss the problem at length. . . . Further, the dignity of the science itself seems to require that every possible means be explored for the solution of a problem so elegant and so celebrated.

In school one becomes familiar with the sieve of Eratosthenes; unfortunately using it to prove that n is prime requires computation time essentially proportional to n itself. The input length¹ of a number, on the other hand, is proportional to the number of binary digits, thus about $\log_2 n$, so we have before us an algorithm with *exponential* running time $O(2^{\log_2 n})$. To quote Gauss again from article 329 of his *Disquisitiones*:

Nevertheless we must confess that all methods that have been proposed thus far are either restricted to very special cases or are so laborious and prolix that . . . these methods do not apply at all to larger numbers.

Can the primality of very large numbers be decided efficiently *in principle*? This question is rendered mathematical in the framework of modern complexity theory by demanding a *polynomial* running time. Is there a deterministic² algorithm that, with a fixed exponent κ , decides for every natural number n in $O(\log^\kappa n)$ steps whether this number is prime or not; in short, the hitherto open question: is $\text{PRIMES} \in \mathcal{P}$?

The State of Things before August 2002

Ever since the time of Gauss, deciding the primality of a number has been divorced from finding a (partial) factorization in the composite case. In Article 334 of the *Disquisitiones* he wrote:

The second [observation] is superior in that it permits faster calculation, but . . . it does not produce the factors of composite numbers. It does however distinguish them from prime numbers.

The starting point for many such methods is Fermat's Little Theorem. It says that for every *prime*

¹The difference between the size of a number and its length is seen most clearly for such unmistakable giants as the number of atoms in the universe (about 10^{79}) or the totality of all arithmetical operations ever carried out by man and machine (about 10^{24}): 80 (respectively 25) decimal digits can be written out relatively quickly.

²That is, an algorithm that does not require random numbers as opposed to a probabilistic algorithm, which does require such numbers.

number n and every number a coprime to n one has the relation

$$a^n \equiv a \pmod{n}.$$

Unfortunately the converse is false: the prime numbers cannot be characterized this way. On the other hand, “using the Fermat congruence is so simple that it seems a shame to give up on it just because there are a few counterexamples” (Carl Pomerance). It is no wonder, then, that refinements of this criterion are the basis of important algorithms.

An elementary *probabilistic* algorithm of Miller and Rabin from 1976 makes use of a random number generator and shows after k runs either that the number is *certainly* composite or that the number is prime with *high probability*, where the probability of error is less than 4^{-k} . The time complexity is order $O(k \log^2 n)$, where the big-O involves a relatively small constant. In practice the algorithm is very fast, and it finds application in cryptography and e-commerce for the production of “industrial-grade primes” (Henri Cohen). In the language of complexity theory, one says for short $\text{PRIMES} \in \text{co-RP}$.

A *deterministic* algorithm of Adleman, Pomerance, and Rumely from 1983, which uses much more theory and a generalization of Fermat’s Little Theorem to integers in cyclotomic fields, completely characterizes the prime numbers. The best deterministic algorithm prior to August 2002, it has running time of superpolynomial order $(\log n)^{O(\log \log \log n)}$. The triple logarithm in the exponent grows so slowly, however, that concrete versions of the algorithm have had excellent success in the pursuit of record-breaking primality proofs for numbers with more than a thousand decimal digits.³

Another class of modern algorithms uses elliptic curves or abelian varieties of high genus. Thus Adleman and Huang, in a very difficult and technical 1992 monograph, were able to give a *probabilistic* algorithm with polynomial running time that after k iterations either gives a definitive answer (with no possibility of error) or gives no answer, the latter case, however, having probability less than 2^{-k} . In the language of complexity theory, one says for short $\text{PRIMES} \in \text{ZPP}$.

With this background, and in view of the level of difficulty that had been reached and the absence of further successes in over ten years, it was hardly to be expected that there could be a short, elegant resolution of the question that would be understandable by “Everyman”.

³The hero of another story, Preda Mihăilescu, developed essential refinements of this algorithm in his dissertation at ETH Zurich, and with his implementation he was for a long time a player in the prime-number-records game. Recently he proved the Catalan Conjecture.

Enter Manindra Agrawal

The computer scientist and complexity theorist Manindra Agrawal received his doctorate in 1991 from the Department of Computer Science and Engineering of the Indian Institute of Technology in Kanpur (IITK). After a stay as a Humboldt fellow at the University of Ulm in 1995–96 (“I really enjoyed the stay in Ulm. It helped me in my research and career in many ways”), he returned to Kanpur as a professor. Two years ago he gained recognition when he proved a weak form of the isomorphism conjecture in complexity theory.⁴



Manindra Agrawal

Around 1999 he worked with his doctoral supervisor, Somenath Biswas, on the question of deciding the identity of polynomials with a probabilistic algorithm. A new probabilistic primality test appears as a simple application in the publication “Primality and identity testing via Chinese remaindering” [1].

The starting point was a generalization of Fermat’s Little Theorem to *polynomials*, an easy exercise for an introductory course on number theory or algebra. Namely, if the natural numbers a and n are relatively prime, then n is prime *if and only if*

$$(x - a)^n \equiv (x^n - a) \pmod{n}$$

in the ring of polynomials $\mathbb{Z}[x]$. Although this is a very elegant characterization of prime numbers, it is hardly useful. The calculation of $(x - a)^n$ alone requires more computation time than does the sieve of Eratosthenes. But it was precisely for polynomials of this size that Agrawal and Biswas had developed a probabilistic identity test, with bounded error probability, that completely avoided the expansion of the polynomial. Unfortunately the resulting test with polynomial running time was far from competitive with that of Miller and Rabin. A new idea was born, but initially it was interesting only as a footnote in the history of primality testing.

Two years later, with his students at IITK, Agrawal began to examine in detail the potential of the new characterization of prime numbers, in which he had great faith.

⁴The isomorphism conjecture of Berman and Hartmanis implies that $\text{P} \neq \text{NP}$. A proof would therefore solve the first of the seven Millennium Prize Problems of the Clay Mathematics Institute and bring a return of one million dollars.



Neeraj Kayal



Nitin Saxena

Two Bachelor's Projects

The admissions procedure for the Indian Institute of Technology (IIT) is rigorous and selective. There is a two-stage common procedure called the Joint Entrance Examination (JEE) for admission to one of the seven branches of the IIT and two other institutions. Last year 150,000 Indians applied for admission, and after an initial three-hour examination in mathematics, physics, and chemistry, 15,000 were invited to a second test consisting of a two-hour examination in each of the three subjects. Finally 2,900 students were awarded places, of which 45 were for computer science at the very renowned IIT in Kanpur. It is no wonder that good money is earned in India for preparing candidates for the dreaded JEE, and graduates of the IIT are eagerly hired worldwide.

It was with such highly motivated students that Agrawal now worked further on the primality test. With Rajat Bhattacharjee and Prashant Pandey, the idea arose of looking not at the excessively large polynomial power $(x - a)^n$ but instead at its remainder after division by $x^r - 1$. If r stays logarithmic in n , then this very much

smaller remainder can be directly calculated in polynomial time with suitable algorithms.

If n is prime, then certainly⁵

$$(T_{r,a}) \quad (x - a)^n \equiv x^n - a \pmod{(x^r - 1, n)}$$

for all r and n coprime to a . Which a and r permit the converse conclusion that n is prime?

In their joint bachelor's project [5], the two students fixed $a = 1$ and examined the requirements on r . Through analyzing experiments with $r \leq 100$ and $n \leq 10^{10}$, they arrived at the following conjecture. If r is coprime to n and

$$(T_{r,1}) \quad (x - 1)^n \equiv x^n - 1 \pmod{(x^r - 1, n)},$$

then either n is prime or $n^2 \equiv 1 \pmod{r}$. For one of the first $\log_2 n$ prime numbers r , the latter is not

⁵I follow the notation of Agrawal et al. and denote by $p(x) \equiv q(x) \pmod{(x^r - 1, n)}$ the equality of the remainders of the polynomials $p(x)$ and $q(x)$ after division by $x^r - 1$ and division of the coefficients by n .

the case, so one would have a proof of the primality of n in polynomial running time $O(\log^{3+\epsilon} n)$.

Here enter the heroes of our story, off stage until now, the students Neeraj Kayal and Nitin Saxena. Both were members of the Indian team in the 1997 International Mathematical Olympiad. Studying computer science instead of mathematics because of better employment prospects, they found in complexity theory a way to continue working with mathematics on a high level.

In their joint bachelor's project, they examined the relation of the test $(T_{r,1})$ to known primality tests that, like $(T_{r,1})$, in the negative case give a proof that a number is composite and in the positive case give no definitive answer. There was a rich payoff. They were able to show that under the assumption that the Riemann Hypothesis is true, the test $(T_{r,1})$ could be restricted to $r = 2, \dots, 4 \log_2^2 n$ for a primality proof. In this way one would obtain a deterministic algorithm of time complexity $O(\log^{6+\epsilon} n)$. Furthermore, they were able to show that the conjecture formulated by Bhattacharjee and Pandey would follow from a long-standing conjecture of Carl Pomerance. And in connection with one of their investigations of the class of "introspective numbers", they were led to a proof idea that later would turn out to be essential.

The work of the two, submitted in April 2002, bears the title "Towards a deterministic polynomial-time primality test" [9]. A vision, the goal is already clearly in view.

Changing the Viewpoint

That summer they did not go home first but instead directly began doctoral studies. Saxena actually had wanted to go abroad, but—irony of fates—he did not get a scholarship at his university of choice.

Only a small change of viewpoint is still needed. Both bachelor's projects studied the test $(T_{r,a})$ for fixed $a = 1$ and variable r . What happens if one instead fixes r and lets a vary? The breakthrough came on the morning of July 10: through a suitable choice of parameter they obtained nothing less than a characterization of *prime powers*.

The result, as streamlined by Dan Bernstein, is the following.

Theorem . [Agrawal-Kayal-Saxena] Suppose $n \in \mathbb{N}$ and $s \leq n$. Suppose primes q and r are chosen such that $q \mid (r - 1)$, $n^{(r-1)/q} \not\equiv 0, 1 \pmod{r}$, and

$$\binom{q + s - 1}{s} \geq n^{2\lfloor \sqrt{r} \rfloor}.$$

If for all $1 \leq a < s$ we have that

- (i) a is relatively prime to n , and
- (ii) $(x - a)^n \equiv x^n - a \pmod{(x^r - 1, n)}$ in the ring of polynomials $\mathbb{Z}[x]$,

then n is a prime power.

The simple, short, and innovative proof of the theorem is so delightful that I could not resist sketching it in the appendix.

The theorem now leads directly to the so-called **AKS-algorithm**.⁶

1. Decide if n is a power of a natural number. If so, go to step 5.
2. Choose (q, r, s) satisfying the hypotheses of the theorem.
3. For $a = 1, \dots, s - 1$ do the following:
 - (i) If a is a divisor of n , go to step 5.
 - (ii) If $(x - a)^n \not\equiv x^n - a \pmod{(x^r - 1, n)}$, go to step 5.
4. n is prime. Done.
5. n is composite. Done.

Step 1 can be accomplished in polynomial time using a variant of Newton iteration. The running time of the main step 3 using rapid FFT-based arithmetic is $\tilde{O}(sr \log^2 n)$, where the tilde over the big-O incorporates further logarithmic factors in s, r , and $\log_2 n$.

Thus to achieve our goal we must allow s and r to grow at most polynomially in $\log n$. This is the job of step 2. We first show what is possible in principle. Set $s = \theta q$ with a fixed factor θ . Stirling's formula gives the asymptotic relation

$$\log \binom{q+s-1}{s} \sim c_\theta^{-1} q.$$

Accordingly, the conditions of the theorem require the asymptotic estimate

$$q \gtrsim 2c_\theta \lfloor \sqrt{r} \rfloor \log n.$$

Essentially this can happen for large n only if there are infinitely many primes r such that $r - 1$ has a prime factor $q \geq r^{1/2+\delta}$. Now this is related to a much-studied problem of analytic number theory.

Sophie Germain and Fermat's Last Theorem

The optimal cost-benefit ratio q/r is obtained for the primes named after Sophie Germain: these are the odd primes q for which $r = 2q + 1$ is prime too. She had shown in 1823 that for such primes the so-called first case of Fermat's Last Theorem holds: $x^q + y^q = z^q$ has no integer solutions when $q \nmid xyz$. Therefore it became a question of burning interest whether at least there exist infinitely many such friendly primes. Unfortunately one does not know the answer even today. Heuristic considerations, however, led Hardy and Littlewood in 1922 to the following very precise conjecture on the actual density of Germain primes:

$$\#\{q \leq x : q \text{ and } 2q + 1 \text{ are prime}\} \sim \frac{2C_2 x}{\ln^2 x},$$

⁶At <http://www.ma.tum.de/m3/ftp/Bornemann/PARI/aks.txt> there is an executable implementation for the freely available number-theory software package PARI-GP (<http://www.parigp-home.de/>).

where $C_2 = 0.6601618158\dots$ is the twin-primes constant.

If this conjecture were correct, then one could find prime numbers q and $r = 2q + 1$ of size $O(\log^2 n)$ satisfying the hypotheses of the theorem. The AKS-algorithm would then have polynomial running time $\tilde{O}(\log^6 n)$. Since the conjecture impressively has been confirmed up to $x = 10^{10}$, the AKS-algorithm behaves like one of complexity $\tilde{O}(\log^6 n)$ for numbers n up to 100,000 digits.

In 1985, nearly ten years before Andrew Wiles finally proved Fermat's Last Theorem, Adleman, Fouvry, and Heath-Brown proved what one had not been able to accomplish with the aid of the Germain primes: namely, that the first case of Fermat's Last Theorem holds for infinitely many primes [8]. In fact, Adleman and Heath-Brown studied, as a generalization of Germain primes, exactly those pairs (q, r) that also play a key role in the AKS-algorithm.

A Fields Medal

What they required precisely is that the estimate

$$\#\{r \leq x : q, r \text{ prime}; q \mid (r - 1); q \geq x^{1/2+\delta}\} \geq c_\delta \frac{x}{\ln x}$$

hold for a suitable exponent $\delta > 1/6$. The hunt for the largest δ began in 1969 with Morris Goldfeld [7], who obtained $\delta \approx 1/12$, and concluded for the time being in 1985 with Étienne Fouvry [6], whose value was $\delta = 0.1687 > 1/6$. All of these works use very deep methods from analytic number theory that expand on the *large sieve* of Enrico Bombieri. He published this sieve in 1965 at the age of twenty-five, and in 1974 he received the Fields Medal. Thus a heavy task falls on "Everyman" who wishes to understand the proof of this estimate in detail. In answer to my question about whether one of the three undertook this task, Manindra Agrawal wrote:

We tried! But Sieve theory was too dense for us—we have no background in analytical number theory. So after a while we just gave up.

Also they did not need to do it, for "the result was stated there in precisely the form we needed", and they could count on its validity by trusting in the referee and a certain interval of time—the more so since Fouvry's result related to the hot topic of Fermat's Last Theorem appeared in *Inventiones*.

Or maybe not? Fouvry forgot to take into account an additional condition in citing a lemma of Bombieri, Friedlander, and Iwaniec. This additional condition *reduced* the value of δ to $\delta = 0.1683 > 1/6$. It also might have been below the critical threshold. Fouvry later told Roger Baker about this correction, and he and Glyn Harman published it in a survey article [3] in 1996.

Incidentally, it was in an Internet search with Google that Agrawal, Kayal, and Saxena ran across Fouvry's article in the bibliography of an article by Pomerance and Shparlinski. When they inquired about the best-known value for δ , Pomerance referred them to the article of Baker and Harman.

Regardless of the optimal value, $\delta > 0$ suffices to guarantee an allowable triple (q, r, s) for the AKS-algorithm of the necessary polynomial size,

$$r = O(\log^{1/\delta} n), \quad q, s = O(\log^{1+1/2\delta} n).$$

Thus the AKS-algorithm has, all told, a guaranteed running time of $\tilde{O}(\log^{3+3/2\delta} n)$. Hence the statement $\text{PRIMES} \in \mathcal{P}$ is proved; the breakthrough is achieved. Kudos! Fouvry's corrected value for δ gives $\tilde{O}(\log^{11.913} n)$, or, simpler to remember and also without the tilde, $O(\log^{12} n)$.⁷

The director of the IIT in Kanpur, Sanjay Dhande, was so enthusiastic about the headline in the *New York Times* that he declared Agrawal would be nominated for the highest honors in mathematics.⁸ In 2006 Agrawal will be forty years old.

How Practical!?

In Internet newsgroups and in newspapers the question quickly arose of practical applications, since large prime numbers are these days an important component of cryptography and e-commerce. We firmly believe that first of all an important *theoretical* problem was solved that for several decades had eluded the experts. Agrawal himself emphasizes that the problem interested him as an intellectual challenge and that presently the AKS-algorithm is much slower than those algorithms that have raised the record in primality proofs to 5,020 decimal digits.⁹ Finally, one should

⁷On January 22, 2003, Dan Bernstein posted on the Web a new version of his draft paper [4]. There, a small variation of the Agrawal-Kayal-Saxena theorem, which he had learned from Lenstra, allows one to complete the proof of $\text{PRIMES} \in \mathcal{P}$ without referring to any deep analytic number theory. A well-known theorem of Chebyshev, asserting that the primes $\leq 2k$ have product at least 2^k , is enough to guarantee the existence of suitable numbers $r, s = O(\log^5 n)$ for which the algorithm works. This removes the last bulwark of difficult mathematics that might have prevented "Everyman" from completely understanding the result. Probably Paulo Ribenboim is right in writing me: "Our specialists should reflect about their convoluted reasoning."

⁸Already on October 30, 2002, he received the Clay Research Award. Previous winners were Andrew Wiles, the probabilists Smirnov and Schramm, and Fields Medalists Connes, Lafforgue, and Witten.

⁹Please do not confuse this with the record for the largest known prime number, which is at this time $2^{13,466,917} - 1$, a Mersenne prime with 4,053,946 decimal places. These numbers have a lot of structure that allows a customized algorithm to be used.

not forget that the definition of complexity classes like \mathcal{P} is a purely theoretical question of an asymptotic statement as $n \rightarrow \infty$. In a particular case, therefore, the advantage in running time of a polynomial algorithm as opposed to a super-polynomial algorithm very possibly could become manifest only for n so large that neither of the two algorithms would produce an answer within our lifetime on current hardware. In practice the constants in the big-O in the complexity estimate also come into play.

Lower-quality "industrial-grade primes" with 512 binary digits can be produced in a fraction of a second using the Miller-Rabin test on an off-the-shelf 2GHz PC. If required, their primality can actually be *proved* in a couple of seconds with the ECPP-method of Atkin-Morain based on elliptic curves.¹⁰ The running-time complexity of this *probabilistic* algorithm is, to be sure, a "cloudy issue" (Carl Pomerance), but heuristic considerations suggest that the likely value lies right around $\tilde{O}(\log^6 n)$.

On the other hand, because of the high cost of the polynomial congruence in the third step of the AKS-algorithm, the constant in the conjectured $\tilde{O}(\log^6 n)$ running-time bound is so large that the algorithm is estimated to take a couple of days on a 512-bit prime number, although Dan Bernstein, Hendrik Lenstra, Felipe Voloch, Bjorn Poonen, and Jeff Vaaler have already improved this constant by a factor of at least $2 \cdot 10^6$ relative to the original formulation of the algorithm—the status as of January 25, 2003; cf. [4].

Thus a factor of about 10^5 is missing to reach a competitive level. The ECPP-method too started with a completely impractical but groundbreaking new idea of Goldwasser and Kilian. Since the method that Agrawal, Kayal, and Saxena have now produced is so unexpectedly new and brilliant, we may confidently anticipate improved capabilities after further maturation of the algorithm.

The Media Pipeline

Except for an excellently researched, technically correct, very readable, and detailed report in the Indian weekly *Frontline* of August 17, the reporting in the general media was deplorable. Agrawal passed over my inquiry about his impression with a polite, "Leave aside the general public coverage."

To be sure, the previously cited *New York Times* article celebrated the result as a triumph, but opaquely by choosing to simplify to a ridiculous extent: polynomial running time became "quickly"; deterministic became "definitively". The article thus reads as follows: three Indians obtained a

¹⁰See <http://www.ellipsa.net/pages/primo.html> for the freely available program PRIMO by Marcel Martin, which for the time being holds the record.

breakthrough because the computer could now say “quickly and definitively” if a number is prime. On the other hand, the new algorithm has no immediate application, because the already existing methods are faster and do not err in practice. “Some breakthrough,” readers would say to themselves.

The Associated Press (AP) made the *New York Times* article into a wire report in which “definitively” became “accurately” and the aspect of the running time disappeared into the background. The sad end of this pipeline was the website of the *Tagesschau*. On August 12, under the heading “At last: prime numbers can be exactly calculated!” appeared such rubbish as “The joy at German schools is boundless: finally one can calculate prime numbers without tears!” The report was removed after protests from participants in the newsgroup `de.sci.mathematik`.

Aside from the article in the *New York Times*, the story went virtually unnoticed in the American press. In the UK a story in the *New Scientist* of August 17 at least used the words “polynomial time”, but it went on to speak of “an algorithm that gives a definite answer to the problem in a reasonable time.” A retrospective piece on November 4 in the *Wall Street Journal* bore the misleading title “One beautiful mind from India is putting the Internet on alert”. A year-end column by Clive Thompson in the Sunday *New York Times* of December 15 asserted, “Ever since the time of the ancient Greeks, finding a simple way to prove a number is prime has been the holy grail of mathematics. ... This year, it finally arrived. ... This new algorithm could guarantee primes so massive they would afford almost perfect online security.”

And the large German-language daily newspapers? The *Neue Züricher Zeitung* had its first report on August 30. The article falsely suggested that until now no absolutely certain certificate of primality could be calculated “within reasonable time” for prime numbers used in cryptography and that the three Indians had now achieved precisely this; the result was, however, not so greatly lauded by the news agencies and the media because it could not handle the largest known prime number.

In the August 9 arts section, under the heading “Polynomial gods: Resourceful Indians and their prime numbers”, the *Frankfurter Allgemeine Zeitung* had a cryptic text that first made a connection between Indian mathematics and the Indian pantheon and then let four such deities hold a short discussion of the new result:

“What is it good for?” expostulated Agni, and Lakshmi retorted: “For hacking! One needs prime numbers for encoding data for electronic transmission—there are various so-called cryptographic

algorithms like RSA and the Data Encryption Standard DES; the keys are numbers with prime factorizations, and if that can now be easily done in a time that is polynomial in the input data ...” “But it is already well known, for example by the Miller-Rabin test, that if one iterates enough times, one can find a primality test with as large a probability as desired of being correct even for the biggest numbers,” contradicted Rudra. “And the encoding prime factorization has nothing to do with the test of whether a number is prime, which is a completely different problem; for security people what the guys have done is worthless.” At dawn, the hostess Ushas finally found the magic words of reconciliation: “Let us simply take pleasure in an elegant result that the West also admires and in the continuing inspiration of our great mathematical tradition!”

What reader would get from this the reason for all the fuss?

Future Plans

The three plan to submit their work to *Annals of Mathematics* and have been in contact with Peter Sarnak about this. They want to rewrite the article “in a more ‘mathematical’ way as opposed to ‘computer science’ way, as that would be more suitable in *Annals*.”

As to the emotional state and the future of the two doctoral students Kayal and Saxena, Agrawal says:

They are happy, but at the same time quite cool about it. I would say they are very level-headed boys. As for their Ph.D., yes, I am sure that this work will qualify for their Ph.D. But I have advised them to stay back for a couple of years, since this is the best time they have for learning. They still need to pick up so many things. But they are free to make the decision—they already have an offer from TIFR [Tata Institute of Fundamental Research].

Appendix

The following is the promised sketch of the proof of the Agrawal-Kayal-Saxena theorem. I follow the streamlined presentation of Dan Bernstein [4].

Sketch of proof. We take a prime factor p of n for which already $p^{(r-1)/q} \not\equiv 0, 1 \pmod{r}$, and we

show that if (i) and (ii) hold for all $1 \leq a < s$, then the number n is a power of p .

To do this we consider—as did Agrawal on that morning of July 10 when the theorem was found—products of the form $t = n^i p^j$ with $0 \leq i, j \leq \lfloor \sqrt{r} \rfloor$. The pigeon-hole principle gives two distinct pairs (i_1, j_1) and (i_2, j_2) of such exponents for which $t_1 = n^{i_1} p^{j_1} \equiv n^{i_2} p^{j_2} = t_2 \pmod{r}$. The goal is now to prove that actually $t_1 = t_2$, whence $n = p^\ell$ for some ℓ .

Via Fermat's Little Theorem, it follows from (ii) that

$$(*) \quad (x - a)^{t^\mu} \equiv x^{t^\mu} - a \pmod{(x^r - 1, p)}$$

for all $1 \leq a \leq p$ and $\mu = 1, 2$. In their bachelor's project, Kayal and Saxena called such exponents “introspective”, and for these they showed that the congruence $t_1 \equiv t_2 \pmod{r}$ lifts to a congruence $t_1 \equiv t_2 \pmod{\#G}$ with $\#G \gg r$. For a suitable choice of parameters, $\#G$ becomes so large that $t_1 = t_2$ follows. According to Agrawal this lifting is “the nicest part of the paper.”

How does one do the lifting? Since $t_1 \equiv t_2 \pmod{r}$, we have that $x^r - 1$ divides the difference $x^{t_1} - x^{t_2}$, so from (*) it follows finally that

$$(x - a)^{t_1} \equiv (x - a)^{t_2} \pmod{(x^r - 1, p)}.$$

Therefore $g^{t_1} = g^{t_2}$ for all $g \in G$; here G denotes the multiplicative subgroup generated by the linear factors $(\zeta_r - a)$ inside the cyclotomic field over $\mathbb{Z}/p\mathbb{Z}$ generated by adjunction of the r th roots of unity ζ_r . Taking a primitive element g , that is, one of order $\#G$, shows that $\#G \mid (t_1 - t_2)$.

On the other hand, in view of (i) and because $p^{(r-1)/q} \not\equiv 0, 1 \pmod{n}$, the group G has—by some combinatorics and elementary theory of cyclotomic polynomials—at least $\binom{q+s-1}{s}$ elements. Therefore by the hypothesis on the binomial coefficients

$$|t_1 - t_2| < n^{\lfloor \sqrt{r} \rfloor} p^{\lfloor \sqrt{r} \rfloor} \leq n^{2\lfloor \sqrt{r} \rfloor} \leq \binom{q+s-1}{s} \leq \#G,$$

whence follows the desired equality $t_1 = t_2$.

Note Added in Proof

Early in March 2003, Agrawal, Kayal, and Saxena posted on the Web a revision of their preprint:

http://www.cse.iitk.ac.in/news/primality_v3.pdf

It contains the improvements by Lenstra and culminates in the new time-complexity bound $O(\log^{7.5} n)$, cf. Theorem 5.3.

Acknowledgment

My sincere thanks to Manindra Agrawal for his willingness, despite thousands of congratulatory emails, to answer my inquiries about background information graciously and thoroughly.

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About the Cover

Is n prime?

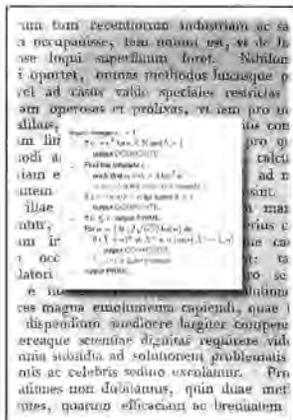
"Problema, numeros primos a compositis dignoscendi ... ad gravissima ac utilissima totius arithmeticae pertinere....," asserted Gauss in §329 of the *Disquisitiones Arithmeticae*. [The problem of distinguishing prime from composite numbers ... is one of the most serious and useful of all that pertain to arithmetic.]

"... scientiae dignitas requirere videtur, ut omnia subsidia ad solutionem problematis tam elegantis ac celebris sedulo excolantur." [The dignity of the science requires that every aid be assiduously cultivated in a search for the solution of such an elegant and celebrated problem.]

Gauss was among the first to begin this search for sophisticated ways to solve the problem, and until very recently there was not even a theoretical understanding of its true complexity. The algorithm displayed on the cover is extraordinarily brief, the most recent version (posted March 4, 2003) of the breakthrough recently found by Agrawal, Kayal, and Saxena of the Indian Institute of Technology at Kanpur. It is the first to show that primality can be tested in polynomial time, as discussed in the article by Folkmar Bornemann.

The image from the *Disquisitiones* was graciously made at extremely short notice by Marcia Tucker, Librarian of the School of Historical Studies at the Institute for Advanced Study. The copy of the book that it was taken from is part of the Lessing J. Rosenwald Collection.

—Bill Casselman
(notices-covers@ams.org)



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A Worm?

Harold P. Boas

My topic is not renegade computer programs, nor Write-Once-Read-Many optical storage devices, nor convoluted plane curves.¹ The “worm” in my title is an important example in multidimensional complex analysis.

A first course on holomorphic (that is, complex analytic) functions of *one* variable introduces two flavors of function theory: namely, functions holomorphic in the entire complex plane \mathbb{C} and functions holomorphic in the unit disc. If one restricts attention to topologically trivial planar domains, then there are no other one-dimensional theories of holomorphic functions. Indeed, the Riemann mapping theorem says that every simply connected planar domain other than the whole plane can be mapped onto the unit disc by a one-to-one holomorphic mapping with a holomorphic inverse, and hence function theory on that domain is equivalent to function theory on the unit disc.

In contrast, the theory of holomorphic functions of two (or more) variables comes in an infinite variety of flavors. Two domains in multidimensional complex space typically are holomorphically inequivalent: each domain supports its own individual theory of holomorphic functions. For example, a bidisc (the product of two one-dimensional discs) cannot be mapped onto a ball in \mathbb{C}^2 by a one-to-one, invertible holomorphic mapping; indeed, Walter Rudin has written both a book titled *Function Theory in Polydiscs* and a book titled *Function Theory in the Unit Ball of \mathbb{C}^n* .

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¹Leo Moser's worm problem asks for a planar convex set of minimal area that contains a congruent copy of every rectifiable arc of length 1. In the November 1973 issue of *Scientific American*, Martin Gardner discussed Paterson's worms, which are certain paths on a planar grid.

For general domains that lack symmetry, there is little hope to understand the function theory in such an explicit way as one can on polydiscs and on balls. One may hope, however, to get some qualitative information. For example, *is function theory stable under perturbations of the domain?*

One of several remarkable discoveries of Fritz Hartogs in his seminal 1905 dissertation was a propagation phenomenon in two-dimensional function theory. An example is furnished by perturbations of the Hartogs triangle T , the set of points (z, w) in \mathbb{C}^2 such that $|z| < |w| < 1$. One may visualize this domain as being fibered over a punctured unit disc in the w -plane, the fiber over a point w being a disc in the z -variable of radius $|w|$. For small positive ϵ , let the perturbation T_ϵ be the union of T and the set of points (z, w) such that $\max(|z|, |w|) < \epsilon$ (the bidisc of radius ϵ). A function that is holomorphic in T_ϵ has a two-variable Maclaurin series that converges in a neighborhood of the origin, and a little trickery with the one-dimensional Cauchy integral formula shows that this series must actually converge in the whole bidisc of radius 1. Thus, *all* holomorphic functions on T_ϵ extend to be holomorphic functions on the unit bidisc, a much larger domain! No such extension phenomenon occurs for holomorphic functions of a single variable.

A domain for which it is *not* the case that all holomorphic functions on the domain extend to a larger domain is called a *domain of holomorphy*. Domains of holomorphy are the natural domains on which to study function theory; they are of interest also in mathematical physics. Some examples of domains of holomorphy are polydiscs, balls, and (more generally) convex domains.

The Hartogs triangle T turns out to be a domain of holomorphy too. Yet every function holomorphic in a neighborhood of the *closure* of T is holomorphic on some T_ϵ and hence extends to be holomorphic on

the unit bidisc. Consequently, every domain of holomorphy that contains the closure of T also contains the unit bidisc. Thus the domain of holomorphy T cannot be approximated from outside by domains of holomorphy.

In 1933, when H. Behnke and P. Thullen raised the general question of when a domain of holomorphy can be approximated from outside by domains of holomorphy, the only negative examples they could offer were variations of the Hartogs triangle T . The boundary of T has a bad singularity at the origin, and one might hope that exterior approximation would be possible for a domain of holomorphy whose boundary is, say, a C^∞ -smooth manifold. In 1976 Klas Diederich and John Erik Fornæss constructed a counterexample in their paper "A strange bounded smooth domain of holomorphy". Their example came to be known as "the worm domain" (actually they constructed a family of domains), because it winds in a way reminiscent of a spiral staircase.

For simplicity I specialize the real parameter in their example to be equal to 25 and discuss one specific worm. A preliminary nonsmooth version is the set of points (z, w) in \mathbb{C}^2 such that

$$\left| z + e^{i \log |w|^2} \right| < 1 \quad \text{and} \quad 1 < |w| < 25.$$

This domain is fibered over an annulus in the w -plane, each fiber in the z -variable being a disc of radius 1 with center at a point of modulus 1. As $|w|$ varies, the center of the fiber moves along a unit circle, and so these disc fibers wind around a central axis.

Since $\log 25^2 > 2\pi$, the fibers wind all the way around a circle, and the projection of this domain into the z -plane is a punctured disc of radius 2. The projection of a neighborhood of the closure of the domain, however, covers a full disc of radius 2 in the z -plane. This filling in of the puncture gives a hint why, as in the case of the Hartogs triangle, one cannot approximate the domain from outside by domains of holomorphy.

The domain just described, in which $|w|$ is chopped off flat at the ends, does not have smooth boundary, but Diederich and Fornæss showed how to add suitable caps to the ends to create counterexample domains of holomorphy with smooth boundary. The smooth domains constructed in this way are the worm domains.

Since these domains depend on the variable w only through the modulus $|w|$, one can represent the worms in ordinary three-dimensional space by collapsing together all the points with equal values of $|w|$. The illustration, with the central axis fattened for clarity, was created by James T. Hoffman for the 1995–96 program in several complex variables at the Mathematical Sciences Research Institute in Berkeley.

The worm is a counterexample to another stability property: Diederich and Fornæss showed that there exist holomorphic functions on the worm that are continuous on the closure of the domain yet cannot be approximated by functions holomorphic in a neighborhood of the closure. Two decades later, the worm was discovered to be a counterexample also to a regularity property in partial differential equations.



Holomorphic functions are the solutions of the homogeneous Cauchy-Riemann equations. When solving the inhomogeneous Cauchy-Riemann equations, one often is interested in the "canonical" solution, the solution whose modulus squared has minimal integral. If the inhomogeneous Cauchy-Riemann equations on a domain of holomorphy with smooth boundary have data whose derivatives of all orders extend continuously to the boundary, must the canonical solution have the same regularity? In 1980 Steve Bell and Ewa Ligocka showed that an affirmative answer has important consequences for determining whether two domains are holomorphically equivalent.

The class of domains for which the answer to the question is yes includes, for example, the convex domains (as Emil J. Straube and I proved). Work of Christer O. Kiselman for the chopped-off worms and of David E. Barrett for the smooth worms suggested that the answer might be negative in general. Finally, a 1996 paper of Michael Christ proved that the smooth worms, originally constructed for a different purpose, are indeed counterexamples to this regularity property in partial differential equations.

For Further Reading

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The "WHAT IS...?" column carries short (one- or two-page) nontechnical articles aimed at graduate students. Each article focuses on a single mathematical object rather than a whole theory. The *Notices* welcomes feedback and suggestions for topics for future columns. Messages may be sent to notices-what-is@ams.org.

Book Review

Mathematics Elsewhere: An Exploration of Ideas Across Cultures

Reviewed by Victor J. Katz

Mathematics Elsewhere: An Exploration of Ideas Across Cultures

Marcia Ascher

Princeton University Press, 2002

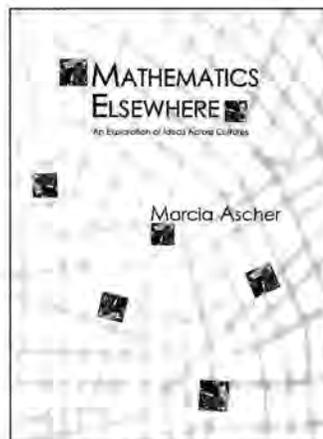
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Marcia Ascher's new book, *Mathematics Elsewhere*, is certain to stimulate discussion of the nature of mathematical ideas and how they find their expression. For many Westerners the term "mathematics" means the intellectual exercise begun by the Greeks of stating theorems and then proving them by beginning with explicitly stated axioms and using logical arguments. Of course, in ancient Greece the axioms were about either geometrical or arithmetical objects. Today the axioms can be about numerous other kinds of objects, but mathematicians are still proving theorems about objects, be they C^* -algebras or regular local rings. So to look at mathematics *elsewhere* might be interpreted to mean the finding of logical arguments from axioms about certain kinds of objects in cultures other than "Western".

Marcia Ascher, however, is not looking for mathematics in that sense. In fact, most of us use a working definition of mathematics much broader than the one implied above. It was "mathematics" when the Babylonians figured out how to solve what we

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know as quadratic equations. It was "mathematics" when Egyptian scribes determined the volume of the frustum of a pyramid. It was "mathematics" when the Chinese figured out how to solve simultaneous congruences by the Chinese remainder theorem. It was "mathematics" when Indian scholars solved the

so-called Pell equation. It was "mathematics" when people from many cultures figured out that the sum of the squares on the legs of a right triangle equals the square on the hypotenuse, or that there was a regular method of determining the number of ways you could choose k objects out of a set of n . In none of these occurrences, at least originally, was there any notion of logical proof from explicit axioms.

These are examples of what Marcia Ascher calls "mathematical ideas"—ideas that she has sought in her research to find *elsewhere*. In particular, Ascher defines mathematical ideas to be those "involving number, logic, spatial configuration, and, more significant, the combination or organization of these into systems and structures." Today the contributions to mathematical ideas of the major literate cultures, including the Babylonian, Egyptian, Chinese, Indian, Islamic, and what we normally

call “Western”, are being studied in more and more detail by numerous scholars. Ascher takes a different path, exploring such ideas in “traditional” or “small-scale” cultures, generally nonliterate in our sense. In her book *Ethnomathematics: A Multicultural View of Mathematical Ideas* [Ascher, 1991], she explored mathematical ideas in certain cultures, and in the current book—really a sequel to the earlier one—she continues her study with other ideas in other cultures.

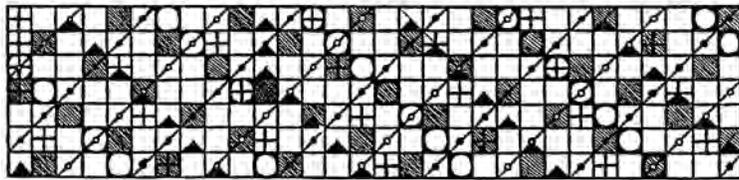
Ascher’s works are examples of what has become known as ethnomathematics, a discipline that grew up beginning in the 1980s, although, of course, its roots go back much further. Initially, the impetus came from mathematics education in traditional societies. The colonial powers ruling many of these societies introduced Western mathematical education in the schools they established, frequently driving out indigenous educational practices. And even after the colonial powers departed, the educational systems frequently remained. But with independence, inhabitants of these countries began to realize that their own heritage was being ignored. All too frequently, the disconnect between the indigenous culture and the Western educational system was too broad for students to bridge. So projects began in some of these places to ferret out the mathematical ideas inherent in the societies, with the hope that these ideas would make mathematics, even “Western” mathematics, more palatable to students. Even in the West, students who traced their heritage to these traditional societies clamored for information about mathematics of their ancestors. Thus researchers began to search out this information. Among the first works of ethnomathematics was Claudia Zaslavsky’s *Africa Counts: Number and Pattern in African Culture* [Zaslavsky, 1973], which grew out of the author’s desire to satisfy the request of her African-American students in New York for information about mathematical ideas in Africa.

It can certainly be debated whether students learn mathematics better by knowing of the mathematical ideas of their own ancestors. Nevertheless, modern theories of education recognize that students need to construct their own knowledge based on what they already know. And if there are mathematical practices in the students’ cultural heritage, it certainly behooves their teachers to be aware of them and to build on that knowledge rather than to tear it down. Thus, after the Brazilian mathematician and mathematics educator Ubiratan D’Ambrosio began to understand how attention to cultural background can improve mathematics education, he formulated his ethnomathematical program, introducing it publicly in a plenary address at the Fifth International Congress on Mathematical Education in Adelaide, Australia, in 1984 and elaborating on it in many writings thereafter [D’Ambrosio, 1985a, 1985b, 1986]. As he put it,

not only are there mathematical ideas and practices in “small-scale” cultures, but certain ideas and practices are part of the culture of identifiable groups within Western societies, such as, for example, carpenters, well-diggers, musicians, and even engineers. Much of this more general ethnomathematics is not taught in schools but is certainly understood and used by practitioners. If one looks at the history of mathematics, it is probably true that much of “Western” mathematics originated in the ad hoc practices and solutions to problems developed by small groups in particular societies. So the theoretical questions posed by D’Ambrosio include: (1) how are these practices developed into methods? (2) how are the methods developed into theories? and (3) how are theories developed into scientific invention? Mainstream history of mathematics considers these questions for topics that have now become part of modern mathematics. However, many mathematical methods that arose in traditional societies never developed into theories but simply remained methods. Despite the lack of theoretical development, the methods work for the purposes for which they were intended and are therefore worthy of study. It is this study that Marcia Ascher has carried out in numerous instances in her earlier *Ethnomathematics* and in the book under review.

Marcia Ascher’s own motivation for her studies of mathematics in “small-scale” societies was not primarily educational. She had a very traditional mathematics background but was married to an anthropologist. Wanting to embark on joint research, they decided to study the *quipus* of the Inca civilization in Peru. Quipus are knotted strings used by the Incas as a data collecting and recording device. The question Ascher wanted to answer was whether the ideas expressed on the quipus had mathematical significance. As her work and the parallel work of her husband on Inca culture progressed, she began to see that in fact many things could be “read” from these artifacts. But she also found that the artifacts alone were not enough to gain an understanding of the meaning of the quipus. As she wrote, “We were constantly amazed that the structural characteristics I was coming up with had resonance in other parts of the culture” [Ascher and D’Ambrosio, 1994, p. 36]. She came to believe that in other cases too the “implicit mathematics” in a society could be understood only by studying the ambient culture, not just the mathematical ideas or artifacts themselves. So, since her original study of the quipus [Ascher and Ascher, 1981], Ascher has studied the implicit mathematics and the cultures in which it is imbedded in societies around the globe.

In *Ethnomathematics* Ascher discussed number words and symbols in several societies, including the Inca. She looked at graph tracing among the Bushoong people of the Congo, the Tshokwe people of Angola,



Paper tika (Figure 3.6, page 78, *Mathematics Elsewhere*).

and the inhabitants of Malekula in the Republic of Vanuatu in the South Pacific. She considered kin relationships in Malekula and among the Warlpiri in Australia's Northern Territory, discussing how these people described these relationships logically. She wrote about games of chance in numerous societies, including a detailed study of the Maori game of *mu torere*. She looked at how various Native American groups described aspects of the space in which they lived. And she considered the symmetric strip decorations of both the Inca and the Maori, analyzing them through the seventeen-fold classification of those two-dimensional patterns.

In *Mathematics Elsewhere* Ascher discusses different mathematical concepts in other societies. In every case, the originators of the particular practice had to solve a certain problem. They did so by developing some mathematical idea. And often, as is so frequent in the history of Western mathematics, they did not stop at the answer to the problem they were solving. They followed the mathematical ideas in new directions, often led by aesthetic considerations. For each mathematical idea she discusses, Ascher describes the particular concept both in modern mathematical terms and, to the extent possible, in the terms understood by the culture itself. She also shows how the idea was imbedded in the ambient culture.

The book begins with a discussion of the logic of divination in three different cultures. Divination is "a decision-making process, utilizing, as part of the process, a randomizing mechanism. The decisions coming out of the process sometimes involve the determination of the cause of an event or, more often, how, when, or whether to carry out some future action" (p. 5). Divination has been a part of virtually every recorded culture, including modern Western ones. For Ascher the interesting part of the subject is the logic behind the randomizing mechanism, that is, the procedures by which a diviner puts together a particular arrangement from which he or she can make a decision.

The most fascinating divination system is that of *sikidy*, a system with a long history that is still used in Madagascar. In this system, the *ombiasy* (expert in *sikidy*) uses a randomizing procedure to lay out four columns of four entries each, where each entry contains either one or two dried seeds of a fano tree. This array is called the *mother-sikidy*.

From the four columns, which we will call C_1 through C_4 , and their associated rows, which we will call C_5 through C_8 , Ascher shows how the *ombiasy* uses what we recognize as Boolean algebra to create eight more columns. (For the purposes of this algebra, we can think of one seed as representing "odd" or "1" and two seeds as representing "even" or "0".) For example, column 11, C_{11} , is created as $C_4 \oplus C_3$, where \oplus represents the XOR (exclusive or) operation on the corresponding elements in each column, namely the operation that gives "even" when combining two elements of the same parity and "odd" when combining two elements of opposite parity. The final arrangement of the columns, together with additional manipulations of them, are the basis for the *ombiasy's* predictions or answers to his client's questions.

Of course, in performing his algorithm the *ombiasy* does not think he is doing Boolean algebra. But when he is done creating his 16 columns, he does several checks to see that he has not made any errors. In other words, he knows that certain relationships must be present in his final arrangement, assuming he carried out the algorithm correctly. For example, the final arrangement must always produce at least two identical columns, and Ascher gives a modern proof of this result. How did the originators of *sikidy* discover this result, and did they develop some sort of proof of it? Ascher does not address this question. But she does note the interest of some of the *ombiasy* in certain special final arrangements and therefore in determining what original layouts lead to these arrangements. That is, the *ombiasy* acted as "mathematicians".

Ascher next has two chapters dealing with time. The first shows how various peoples have organized the physical cycles of the day, the phases of the moon, and the yearly motion of the sun to create calendars, while the second highlights the use of more arbitrary cycles in marking time in the civilization. Although Ascher deals briefly with the luni-solar calendars of the Trobriand islanders and the Kodi people of the South Pacific, she gives the most detail on the development of the Jewish calendar. In particular, she discusses the rules for determining the first day, Tishri 1, of the Jewish year. In general, it is the day of the new moon of Tishri, with four exceptions, detailed in a diagram (which has an unfortunate typographical error). However, Ascher gives a reason for the first exception, that Tishri 1 cannot fall on a Wednesday, Friday, or Sunday. That would mean that either the Day of Atonement (Tishri 10) would fall on the day before or the day after the Sabbath—thus forcing a double Sabbath—or that Tishri 21 would fall on a Sabbath, forcing a contradiction to that day's observances, with activities forbidden on the Sabbath. The reasons for the other exceptions are that

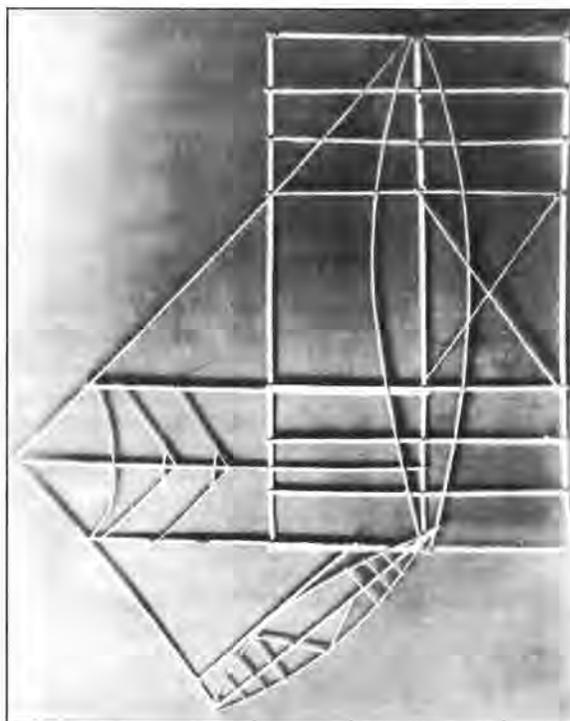
without them either the current year would be too long or the previous year would be too short, since a regular year must have either 353, 354, or 355 days, while a leap year must have 383, 384, or 385 days. Furthermore, all the exceptions together, along with the known inequalities in the Moon's motion, bring Tishri 1 and the first sighting of the crescent moon as close together as possible in a system based on the mean length of a lunation.

In her second chapter on time, Ascher deals primarily with the calendrical system of the Maya. This is based on a 260-day cycle, the Sacred Round, itself based on two independent 13- and 20-day cycles, and a 365-day cycle, the Vague Year, based on 18 months of 20 days each as well as 5 extra days at the end. These two cycles are joined in a larger cycle, the Calendar Round, equivalent to 52 Vague Years or 78 Sacred Rounds, as well as in a Great Cycle of 1,872,000 days, anchored at a fixed historical starting point. An important function of the Mayan priests was to do several types of elapsed time calculations. For example, they needed to find the number of days between two particular dates in the Calendar Round. Since the Calendar Round was made up of several cycles, this amounts to determining the solution of several simultaneous congruences, that is, solving a Chinese Remainder problem. We unfortunately have no records of the methods that the Mayan priests used to solve this problem. But we do know that they calculated correctly. Thus somehow they developed techniques for solving the Chinese Remainder problem. Whether their techniques resembled those originally used in China or those used today to solve such problems is unknown.

Another fascinating calendar is that of the island of Bali, in Indonesia. This calendar is based on 10 arbitrary cycles of 10 days, 9 days, 8 days, and so on, down to 1 day. A year in this calendar is 210 days, a number evenly divisible by all the cycle lengths except 4, 8, and 9. Special adjustments are made for those cycles so they fit reasonably into this calendar. As in the Mayan calendar, to do calculations in this calendar requires the solution of simultaneous congruences. In this case, we know that the Balinese use a wooden board known as the *tika*, on which an array of seven rows and thirty columns is carved or painted. Various symbols are placed in many of the boxes of the array representing important days in the calendar. Manipulations on the *tika* then enable one to find answers to typical calendrical questions. An unanswered question about the Balinese calendar is how the *tika* was originally constructed.

The most impressive chapter of *Mathematics Elsewhere* is the chapter on models and maps, which discusses the stick charts of the Marshall Islanders of the South Pacific. There are two types of these charts, made of palm ribs tied with coconut

fibers, often with a few shells attached, and ranging in size from about 60 cm square to 120 cm square. The first type, known as *mattang*, was designed to introduce prospective Marshall Islands navigators to the interplay of oceanographic phenomena and land masses necessary for navigation through the islands. Thus, these charts represent wave fronts as they approach land and help navigators understand refraction, reflection, and diffraction of the wave fronts. After using the *mattang* under the guidance of an expert, an apprentice can move on to the second type of stick chart, the *rebbelith* and the *meddo*. These are maps of either the entire Marshall Islands archipelago or some smaller region within it, designed to help navigators wend their way among the islands through a knowledge of the typical wave patterns



Meddo (Figure 4.17, page 119, *Mathematics Elsewhere*).

in the ocean and certain physical features of the islands. Interestingly, these stick charts are all intended for study on land. A navigator does not carry one with him on his boat but relies on his memory. Ascher gives many details of both kinds of stick charts, with numerous examples, and shows how they fit into the culture of the Marshall Islanders. As before, we do not know exactly how the islanders organized their sea-faring knowledge into these charts, but we do know that they were able to create these models of a most important aspect of their culture.

Ascher deals with “systems of relationships” in her fifth chapter, considering these among the Basque people of Sainte-Engrâce, France, near the Spanish border; among the Tonga of Polynesia; and among the Borana of Ethiopia, near the Kenyan border. Her discussion revolves around the formal relationships that enable the people to understand their roles in the society. Ascher is careful to note that the people of these communities have “articulated the properties of the relations” in a way easily translatable into our formal terms. Thus she feels comfortable in making that translation, even though the people themselves did not do so. Nevertheless, certain formal consequences of the initial form of the relationships were discovered by the peoples involved, consequences that we can derive “logically”. For example, the Borana historian who discussed the father-son classes in Borana society noted that the consecutive appearances of particular class names occurred in alternate son-to-father lines, while a modern proof of this result requires the algebra of congruences.

Finally, Ascher considers the *kolam* designs of southern India. The women of Tamil Nadu draw these designs each morning at the threshold of the house using a thin stream of rice flour. These designs, which serve both to welcome guests and to avert misfortunes and illness, are mathematically interesting because their creation involves transforming and superimposing several basic units. There are different types of designs, some for daily use and some for special occasions, but the tradition has been passed on from mother to daughter for centuries. Sometimes the kolam are constructed around and within an initial grid of dots, but others are made up of one or a few continuous curves. Throughout the centuries, the women of Tamil Nadu have elaborated these designs well beyond any practical necessity, evidently impressed by the beauty of their creations. Interestingly, the creation of these kolam designs has now become part of the literature in computer science, inspiring those working in formal language theory and with picture languages. Thus, the mathematical ideas inherent in this traditional culture have influenced the development of modern computer science theory.

In *Mathematics Elsewhere*, as in her earlier book, Marcia Ascher has made a significant contribution toward a global perspective on the history of mathematics. Her work has made it increasingly clear that mathematization—that is, the combination and organization of ideas about number, logic, and space into systems and structures—is not the province of a limited number of cultures but has in fact been accomplished by numerous cultures around the world. People have always “mathematized” and have even acted as “mathematicians” by following their ideas well beyond the solution of

their initial problems. Anyone who enjoys this process of mathematization will enjoy seeing how it was accomplished in the diverse societies about which Ascher writes, and those who teach mathematics will be tempted to use her examples in relevant classes. These examples will help us demonstrate to our students the universality of mathematical thinking. And since our students, both at the undergraduate and graduate levels, are increasingly coming from countries outside the West, Ascher’s ideas will prove useful in helping us meet their needs as we teach them the modern mathematics so important to understanding our world.

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Mathematics in a Small Place: Notes on the Mathematics of Romania and Bulgaria

Mark Saul

The Balkan states are proverbially unhappy. Buffeted by history, torn by ethnic and religious strife, mismanaged by unrepresentative or inefficient governments, they sit astride one of the fault lines of history, the division between Eastern and Western Christianity, between Asian and European empires, at the outer extremes of both continents. And yet the people of the Balkans flourish, making signal contributions to European and world culture. On two recent trips to the Balkans, I found much that we might learn from their view of mathematics as part of their cultural heritage.

It has been said that Greece is a poor country where there are many wealthy people, while Romania is a rich country where there are many poor people. For Romania does not prosper. Its economy proceeds on a microscopic scale. With 30,000 lei to the dollar, there is little a traveler would want to buy that would cost as much as \$20, except a day's food and lodging.

I came in August 2001 to work with Balkan mathematicians on the interface between research mathematics and the K-12 curriculum, an area much better worked out in that part of the world than it is in the United States. The group met in Sinaia, a resort town stuffed into a mountain pass between Wallachia and Transylvania, two of the three medieval kingdoms that make up the modern state of Romania. (The third, Moldavia, we did not visit.)

Upon first arriving in Romania, I found myself sitting in a circle with a number of mathematicians. Titu

Andreescu played the role of host. A gifted problem solver, he fled Romania during the 1989 revolution. In the United States he first taught high school and now serves as the coach of the U.S. International Mathematical Olympiad team.

Around the circle the talk was about old friends and new problems. It flowed back and forth from Romanian to English. Romanian, as the name suggests, is a Romance language, derived from Latin, and from time to time I could catch a few familiar words. And I could easily read the many problem books and elementary expositions that are a part of the tradition of Eastern European education. The circle included high school teachers, university professors, and researchers, and the talk flowed uninterrupted, without the issues of turf or rank that would inevitably come up in such a group in America.

Later, one of our sessions was devoted to quadratic equations, and I began discussing these with Ivan Tonov, a Bulgarian mathematician. Among the most tedious topics I have to teach in precalculus is the solution of simultaneous quadratic equations. The students need fluency in solving these so that they can find the intersection points of two conics in a calculus problem. Such questions are a staple of the Advanced Placement examination, which these students will take in about a year. The work is tedious, because there are few new concepts to be mastered, only accuracy in algebraic computation. Or so I thought until my conversation with Tonov. We looked at the problem and immediately saw that it could not be solved in general by elementary means. Eliminating one unknown, in the general case, leads to a quartic equation in the

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other unknown, whose solution is beyond the scope of what we teach in high school. So we must content ourselves with special cases. One special case that the texts unfailingly provide is when both equations lack linear terms. Then we can solve for x^2 and y^2 as variables themselves. So what are some other special cases? With this question the problem immediately became interesting, and I started looking forward to getting to the topic in the classroom.

Another special case occurs when one of the equations is homogeneous in the two variables. Then we can solve this equation for the ratio of the unknowns, express one as a multiple of the other, and substitute in the other equation to obtain a quadratic. We explored this a bit. A new question immediately arises. Suppose for such a system of equations that we eliminated one of the unknowns to get a quartic? What would be special about this quartic that would allow us to solve it? This general question launches us well into a discussion of Galois theory, which quickly goes beyond the usual high school curriculum. The interesting questions then become: What slices of Galois theory can we serve up to high school students? What, in the general situation, might be interesting to show high school teachers? How can we use all this to prepare some students, even a very few, for more advanced work? Or even lure them into more advanced work, using their natural curiosity? We didn't have time for all this, but now I look forward to getting to this topic in the classroom and also to discussing it with interested mathematicians.

The point of our voyage to Romania, for me, was that in Eastern Europe I could find such interested mathematicians. In the United States such dialogue is very rare, and tends to be dominated by one side or the other. In a country like Tonov's Bulgaria, with everyone in the mathematical community crowded together, there is less room for specialization of interests, for carving out one's own territory, for building bridges between those working in different branches of the mathematical professions.

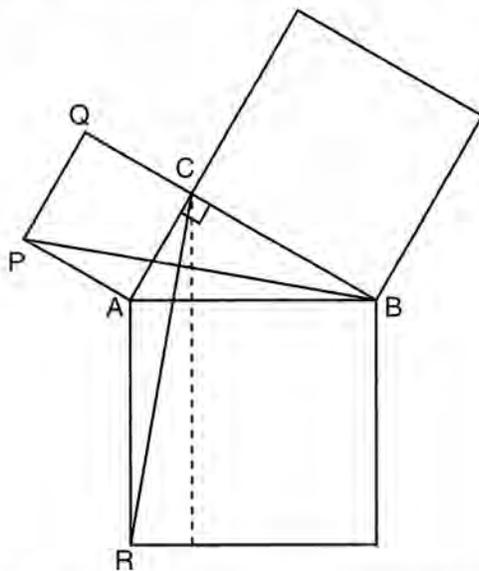
Szeged is the last big town in Hungary before you cross the Romanian border, for years one of the hottest in Europe. There is a sizeable Hungarian minority in Romania, and Transylvania, the largest of the three kingdoms that united to form present-day Romania, was Hungarian territory until 1918. So Szeged is almost in the Balkans, and Jozsef Kosztolanyi teaches there. He told me how he teaches high school, undergraduate courses, and graduate courses for teachers, and I immediately reacted with envy. Kosztolanyi has in one week the experiences of three American careers. In the United States we are too often limited by the institutions we work in.

"I got interested in mathematics very early. The Hungarian tradition in education differs from the Romanian in that we go deeper, not further. We

don't place an emphasis on calculus in the high school years, nor on earlier algebra. Rather, we look for ways to get students to make better use of what they already know. We spend time developing their cognitive abilities."

Curious about this last point, I pressed him. Kosztolanyi replied with a reminiscence of his own education: "I had very good teachers in high school. It was a special school for mathematics. We were encouraged to reinvent for ourselves some of the classic results. For example, one teacher had us discover Euclid's proof of the Pythagorean Theorem."

Again I pressed. This beautiful proof is not at all intuitive. I had often taught it. I knew several ways to make the argument easy to follow. A close reading of Euclid himself tells us how to imbed the proof in a study of a more general method, that of the transformation of areas. But I had never succeeded in getting students to discover it for themselves. How is this done? "Well," explained Kosztolanyi, "we crept up on it. Aside from the various area formulas, which we knew well, the proof depends on the congruence of two particular triangles, which in turn can only be distinguished if we draw in certain lines." Kosztolanyi and I were enjoying a stroll through the park outside a palace of the former Romanian ruling family. But both of us had wrestled long enough with this proof to be able to speak without drawing a diagram. If we had drawn one, Kosztolanyi would be pointing to lines BP and CR in the figure. The congruent triangles in question are PAB and CAR.



"It's not easy to motivate the drawing of those lines. What my teacher did was to ask us to describe the diagram and to find equal elements. We could easily see that the sides of the various squares were equal, but the key insight—which some of us got—was to see that angles PAB and CAR are equal.

Once we saw that, we were able to distinguish the congruent triangles." And so I learned not just about Hungarian mathematics but about something I could use in my own teaching.

Florica Banu is a gifted teacher of junior high school students, so gifted that she has been working for the past year with the Romanian Ministry of Education to construct examinations in mathematics. I asked her how she first discovered mathematics. "I enjoyed mathematics very early," she replied. "But my mother discouraged me from studying it in college. 'Mathematics is not for silly girls,' she said. So I started studying physical education. Only I somehow always wanted to get back to mathematics. And after several years, I did. I enjoyed learning it, and now I enjoy teaching it."

No country has an exemplary record of involving girls in the study of mathematics. But the problem is coming to be recognized, and in Bulgaria I found some indication that the situation is improving. Jordan Tabov is a Bulgarian mathematician who has long been involved in various competitions, including the International Mathematical Olympiad. He told me, "We often have girls on our Olympiad team. Not only do the girls compete, but they often score very high. Once we asked a number of girls about the possibility of having a separate training camp for them, as we do for various sports. They did not want this, and some were even offended."

Petar Kenderov told me a bit more about this subject. Kenderov is the chair of the Department of Operations Research at the Bulgarian Mathematical Institute and has chaired numerous prestigious international committees. "Our special mathematical schools help a lot," Kenderov added. "They begin at age thirteen, a time when girls often show higher achievement than boys. Once they enter these schools, they are safe. Their talent is valued and nurtured."

Back in Romania, I missed one important encounter. It was related to me by Gail Richardson. Richardson works with Best Practices in Education, a not-for-profit group that explores practices abroad that we might use in the United States. The meeting I attended was one of a series arranged by this group, intended to bring teachers and mathematicians together, both American and Balkan. One of the Romanians at the meeting, a small man in a print shirt and suspenders, spoke no English. It was explained that he was the principal of the local Michail Cantacuzino High School. One day he invited the Americans to visit his school. Later, Richardson reported to me on the visit.

"Adrian spoke through a translator," she explained, "who was a former student. Adrian Ghioca has spent his whole working life in this school, first as a teacher, then as its principal. It was originally

attached to a local factory, which closed down shortly after the Ceaușescu regime was toppled. At that point it was not clear what role the school ought to play. Adrian built it into a strong school, with an emphasis on mathematics and the sciences. Now the education it provides is highly coveted, and the school takes in boarding students from nearby regions.

"Adrian showed us the classrooms. Our lack of a common language did not prevent us from perceiving his passion for education and his pride in its achievements. Our translator, his former student, had tears in her eyes as she repeated his words. Pretty soon many of us did too. And this was even before he showed us the twenty-three books he had written on various areas of education."

We knew Adrian Ghioca only as a quiet chain-smoking man who always wore suspenders and who spoke no English. But now we were privileged to have seen the miracles this man had wrought in a tiny crack in the mountains of Transylvania.

One day Mircea Becheanu and I took a hike up one of the mountains girding the town of Sinaia. The exercise was not strenuous, and we talked as we climbed. The conversation ran to teaching. Becheanu is a professor of mathematics (algebra) at the University of Bucharest. He is deputy chair of the Romanian Mathematical Society and has done significant mathematical research. Like many of his colleagues, he is also active in education, editing the society's journal for teachers. Becheanu has taught all his life, including a stint building a mathematics department in Zaire. "Sometimes we lead with intuition," said Becheanu, "but sometimes a formal setting is clearer to the student. I studied tensor analysis for two years at the university, and it wasn't until the concept was made formal that I understood it." We had been talking not about tensor analysis, but about algebra and geometry in high school and the role of intuition in learning. Becheanu saw nothing remarkable in comparing high school algebra to tensor analysis: he had learned from teaching both.

Becheanu continued his remarks: "The formal tradition is very strong in Romania. It is the more or less conscious influence of Bourbaki. Sometimes our texts are too formal, but we also have had the Russian influence, after World War II, and the two schools of mathematics have blended here."

Romania has always felt an affinity for the French. Travelers to Bucharest remark on the resemblance of its architecture to that of Paris. Wedged in between the Turkish and the Russian empires, with their backs to the Germans and Austrians, the Romanians are used to the grindstones of history. France was the nearest European power that seemed not to threaten the country's sovereignty. And so Romanian culture has often followed the French. The Bourbaki school made a lasting impression

here, both on the research community and on the schools. "After the [Second World] War, we had the Russian model," explained Becheanu. "The country was turned eastward. And so the formalism of Bourbaki seemed at that time to be something patriotic, an act of resistance to the satellite status of the country. Now that the political climate is looser, we are more free to choose the influence we think best."

Geography is destiny, some say, and both Romania and Bulgaria have had similar destinies—and similar geographies. But the countries are really very different. Romania is more than twice as large as Bulgaria, both in area and in population, and has always been wealthier. The Bulgarians, however, have had a stronger sense of nationality. They have had this since the Middle Ages, when the first Bulgarian kingdom won its independence from Byzantium. Bulgaria was also the strongest national state in the Balkans during the period just after the Turkish occupation. Romanian national feeling is much newer: there was no strong medieval kingdom, and the country coalesced out of three provinces as the Turkish Empire crumbled.

Bulgarian history came alive on a visit one spring. Jenny Sendova, a researcher at the Bulgarian Academy of Sciences, had invited me to help construct a summer program for high school students. It was to be modeled on the Research Science Institute, a summer program run by the Center for Excellence in Education and the Massachusetts Institute of Technology. Students are provided internships with working scientists and mathematicians. Bulgarian mathematicians and scientists are used to working with young people, so an internship program is a natural development for the country. I was there to help develop internship sites, and I had the delightful job of listening to various researchers describe their work.

"We have a great national treasure," Milena Dobрева told me. "Our libraries and monasteries are filled with medieval manuscripts. I am creating a searchable database so that scholars can work with them more easily. This will be more than just a list of the manuscripts. The database will contain searchable, digital copies."

Being closest to Constantinople, the Bulgarians were among the first Slavs to be Christianized and the first to have a written language. Bulgarians are often taken aback when the Cyrillic alphabet is referred to as the "Russian" alphabet, for it was invented by Bulgarian monks. For centuries, Old Church Slavonic, an archaic variant of Bulgarian, was the literary language of the Slavs. And it was in this language that the earliest Slavic traditions of Christianity developed. One of the difficulties Dobрева faces is that the script is not linear: letters occur over, under, or inside other letters, and the letters themselves assume different forms. All

this information will be important to researchers and will have to be preserved in digital form.

"I am looking at tools to provide for researchers who use these manuscripts," explained Dobрева. "They often have to trace the source of the works. The scribes would make errors or create variants as they transcribed the texts. Sometimes they would insert a letter or a word, or even a whole passage. Sometimes they would delete a word or passage. And sometimes they would replace one whole section with another. In tracing the origin of a manuscript, we have to compare the sequence in one text with the sequence in another."

"So," I commented, "it sounds like what the molecular biologists are doing lately with sequences of nucleotides."

"Exactly," said Dobрева. "The algorithms used in nucleotide sequencing could be applied to the study of some aspects of medieval manuscripts." And so the generality of mathematical methods asserts itself, over thousands of years and thousands of miles.

"'Bulgaria? I think I know where that is. The capital is Bucharest, right?' That's what people sometimes ask me." Jenny Sendova and I were walking around the large church in the downtown section of the Bulgarian capital, which is, of course, Sofia, not Bucharest. The church memorializes the Russian army, which liberated the country from the Turks. Throughout my visit I found this fear of oblivion. Living in a small country, tucked away in a side pocket of Europe, it is not so much the fear of being stigmatized or stereotyped that one feels, as the fear of being forgotten, of being treated as inconsequential. And so every Bulgarian connection seems important: the heritage of Christianity and literacy, the wonderfully inventive folk music and folk dance, the individual mathematicians or artists who have from time to time contributed to Western culture.

I often think that Americans, for all their prosperity, and their robust culture, now emulated by the rest of the world, have oddly parallel fears. Ours is a new country. We don't have our own language or a recognizable national costume. Our country's name might refer to a dozen other entities if it weren't familiar as our own. We are stereotyped, not as quaint, but as callow. And we are too prominent, too "vanilla", on the world stage. We fear being young and inexperienced.

Petar Kenderov told me a bit about the traditions of mathematics in Bulgaria. "Kyril Popov (the name has various transliterations) was an early Bulgarian mathematician. A student of Poincaré, he worked in analysis. He was active and highly respected well into the 1950s. But more important than the French influence in Bulgaria was the Russian. After the 1917 revolution, many Russian intellectuals settled in Bulgaria, including several

mathematicians. In a small country like ours, even two or three hundred remarkable people can influence the intellectual life. But young people, at that time, had to find their own way to mathematics.

"More recently, Ljubomir Iliev encouraged young people more directly to seek careers in mathematics. He started the Balkan Mathematical Society. While his own research was in complex analysis, Iliev worked to enhance development of all branches of the field, including computer science as a branch of mathematics. He encouraged and recruited younger people to work on these branches throughout the 1970s and 1980s. At this time a critical mass was formed of people interested in mathematics in this country. The International Mathematical Olympiad played a large role in this development.

"It was Iliev, on a visit to Russia, who first got a vision of how computers could play a role in the development of Bulgaria's economy. Ours was the first country in the Eastern Bloc to produce CDs and personal computers. It was no coincidence that the first olympiad in computer science was held in Bulgaria. Iliev even obtained exemptions from service in the army for young people, so that they could work with him. The selection process was quite rigorous."

Kenderov adds, a bit too modestly, "I was one of those chosen. At that time, the minister of education, Gancho Ganev, had a background in mathematics. He sent many students to Moscow. Bulgarian students formed a large contingent at Moscow State University. And almost no one in mathematics ever defected to the West. Ironically, it is now that mathematicians are leaving, especially in computer science.

"In Bulgaria, we have always recognized the role of teachers in identifying talent. Their job is not just to teach mathematics. The identification and support of talented students is their more difficult task. And this is the most vulnerable place in our system just now. The Cyril and Methodius Foundation [a charitable international organization named after the two brothers who brought literacy to Bulgaria, of which Kenderov is president] gives prizes to teachers with success in identifying and developing talented students."

While the cultures of the Balkans are old, the nations themselves are young, and political traditions are still evolving. I met one day with Blagovest Sendov at his office in the Central Laboratory for Parallel Computing. He comes here every morning before going off to assume his duties in the Bulgarian parliament. Sendov is an imposing figure, both physically and intellectually. Having enjoyed a robust career as a mathematician, he took up politics after the fall of Communism. He was elected to the parliament, then to its leadership as chairman. He is as deeply concerned

about the fate of his country as he is about that of the mathematics it has produced. Concern marks his wide brow as he speaks. "We have trouble here. Our government is chaotic. Money is flowing out of the country at an alarming rate, and we have lost a large part of our irrigated cropland. I have tried to speak out about this situation, but some mistake my words for a call to return to the old system."

Sendov speaks with the intensity of a man used to solving hard problems, problems that one must wrestle with for long periods of time before subduing them. But even he seems weighed down by the problems of his country. It may take more than one lifetime to solve them. Sendov might have rested content with his achievements in mathematics. Or he could have emigrated to a Western university. But he stayed and is trying to use his problem-solving abilities to help his country. In fact, it is not that uncommon in small countries for people prominent in the sciences to engage also in political activity. With fewer human resources, sometimes active minds must do double duty.

I have written elsewhere¹ about Russian mathematics and how it played the role of a refuge from the totalitarian state for intellectually active people. The same situation held all over Eastern Europe. Perhaps now that these societies have burst open, mathematical minds are free to range over a larger domain of activity. On the other hand, it may be useful to think of the field of education as less differentiated than that of research. We need people who are skilled in introspection in a close examination of mathematical worlds. Sometimes these same people are also gifted in stimulating the thoughts of others—the quintessential act of teaching. And even if these talents reside in different people, it is important that these different people achieve a synergy of effort. I observed such a synergy in the Balkans.

That is, I found the Balkan mathematics community less balkanized than the American.

Acknowledgment

In addition to the people quoted in this article, I would like to acknowledge the help of Christinel Mortici, Bogdan Enescu, Bozhidar Sendov, Michael Tachev, Svetoslav Markov, and Ivan Derzhanski.

¹M. Saul, *Love Among the Ruins: The Education of High Ability Mathematics Students in the USSR*, Focus 12 (February 1992).

Carnegie Initiative on the Doctorate

In 2001 The Carnegie Foundation for the Advancement of Teaching launched a multiyear project to examine the doctoral degree in the United States. Entitled the Carnegie Initiative on the Doctorate (CID), the project aims to stimulate rethinking and renewal of doctoral programs in six disciplines, one of which is mathematics. During 2003 the CID will publish a set of twelve essays, two in each discipline, about doctoral education. And earlier this year, thirty-two "partner departments", including eight mathematics departments, were chosen as participants in the initiative.

Founded by Andrew Carnegie in 1905, the Carnegie Foundation has a long tradition of carrying out research and policy studies in education. One of its most famous and influential projects was the 1910 "Flexner report", a tough indictment of medical schools in the United States that led to widespread reform. It was partly because of his fame as author of that report that Abraham Flexner became the founding director of the Institute for Advanced Study. Although the Flexner report is sometimes mentioned in connection with the CID, the similarity is not strong. "Of course we would like [the CID] to be influential," remarked Chris Golde, research director for the CID. The difference is that Flexner clearly had an agenda in mind when he wrote his report, while the CID does not. "We are committed to that [agenda] needing to arise from the disciplines," Golde said.

In fact, the CID does not even take it for granted that doctoral education needs a huge overhaul. "I don't see [the CID] as necessarily dramatically changing what we do," said Kevin Corlette, chair of

the University of Chicago mathematics department, one of the CID participating departments. "The idea is to go back to basic questions about what we are trying to accomplish in doctoral education and to think about what we could do differently or better." A sign of the excellence of U.S. doctoral programs is that they attract students from all over the globe. Still, evolution in the societal and academic contexts in which these programs exist has sometimes caused mismatches between the goals of doctoral programs and what is expected of the programs' graduates. The idea of the CID is to ask anew the question, What is the purpose of doctoral education today? George Walker, a theoretical physicist at Indiana University who serves as the CID project director, said the main goal is to "convince people to think carefully and deeply about PhD programs, and then to act on their thoughts, just as they do in their disciplinary research."

An organizing notion for the initiative is that doctoral programs should produce "stewards of the discipline". In addition to having made an original contribution to research in the discipline, a steward possesses perspective on the history of the discipline, on the "big questions" and ideas that drive the field, and on its relations to other areas. A steward should also be a communicator in the broadest sense: someone who not only can teach students effectively but also can communicate in such a way that the tools and ideas of the discipline are available to those outside it. The notion of a steward of the discipline "calls in an ethical dimension," Golde remarked. "It's about integrity.

It's about, What are your responsibilities to the discipline?"

The CID is "conceptually based," Golde explained. "Its heart is in ideas, not in very specific practices." The purpose is not, say, to promote the use of interdisciplinary team projects in doctoral programs. Rather, the goal would be to get departments to think carefully about the communication skills that stewards of the discipline should possess. Departments would then consider what kinds of experiences—team projects may be one of them—would lead to the development of those skills. As Walker put it, "What experiences should future stewards of the field have in graduate school to allow them to evolve and be effective in a future we can't imagine, a future thirty or fifty years from now?"

Six fields were chosen for the CID: chemistry, education, English, history, mathematics, and neuroscience. There were several reasons for choosing these particular disciplines. One was the aim of including disciplines that span a wide area of academe. Carnegie also looked for central, fundamental fields where there are a large number of doctoral programs and students. The inclusion of neuroscience reflected the desire to bring in a multidisciplinary field.

In addition, Carnegie sought areas in which discussions of the doctorate were already taking place, and in that regard mathematics was a natural choice. Over the past decade or so the mathematical community has engaged in intensive discussions about the doctoral program. Among the major stimulants of these discussions were the reports *Educating Mathematical Scientists: Doctoral Study and the Postdoctoral Experience in the United States* (National Academy Press, 1992) and *Towards Excellence: Leading a Doctoral Mathematics Department in the 21st Century* (AMS, 1999). These discussions influenced the establishment of the VIGRE (Vertical Integration of Research and Education in the Mathematical Sciences) program of the National Science Foundation, which has stimulated much innovation and change in doctoral programs.

To begin the CID, Carnegie commissioned the writing of twelve essays, two in each of the six disciplines, that explore the purpose of doctoral education and the notion of "stewards" in the context of each discipline. The essays are not "how-to" manuals for structuring doctoral programs; their purpose is not to tell departments what to do but rather to provoke discussions within disciplines and across academe. The essays in mathematics were written by Hyman Bass of the University of Michigan, who is now past president of the AMS, and Tony Chan of the University of California, Los Angeles (the essays by Bass and Chan will be published in future issues of the *Notices*). All of the essays are scheduled to be

CID Participating Mathematics Departments

Partner Departments

Duke University
Ohio State University
State University of New York, Stony Brook
University of Chicago
University of Illinois, Urbana-Champaign
University of Michigan, Ann Arbor
University of Nebraska, Lincoln
University of Southern California

Allied Departments

Howard University
Kent State University
University of North Carolina, Chapel Hill
University of Utah

completed in spring 2003 and will eventually be published together as a book.

The second part of the CID began in early 2003 with the selection of thirty-two "partner departments". These departments have made a commitment to undertake a thorough examination of their doctoral programs. The intention is that the commissioned essays will stimulate the departments' discussions. CID staff will visit the partner departments, not for the purpose of evaluating them, but rather to encourage discussions and bring in new ideas. There will also be periodic meetings of partner department representatives, the first one taking place in July 2003 at the Carnegie headquarters on the campus of Stanford University. After the initial conceptual phase, the departments will experiment with new ways of designing doctoral programs. In the "research and dissemination" stage, departments will evaluate the outcomes of their experiments and share them with colleagues at other institutions; the Carnegie Foundation will also help to publicize ideas and results growing out of the CID.

The departments were chosen after a call for applications was issued last fall. Because there were more worthy applications than it could accommodate, Carnegie also selected some "allied departments" that can participate in discussions and receive materials but which will not have site visits or support for attending CID meetings. (See the sidebar for a listing of partner and allied mathematics departments.)

The VIGRE program has had an enormous effect on mathematics departments across the country, spawning innovations in education at the undergraduate and graduate levels. One reason for the large impact is that the goals of VIGRE are backed by grant dollars. By contrast, the CID provides essentially no funding to its partner departments

(staffing for the CID is funded jointly by the Carnegie Foundation and Atlantic Philanthropies). So what do departments get out of the CID? "It seems to be a very serious look at the doctorate and an opportunity to be there at the table when a lot of people are putting thought into the nature of the doctorate and how it might be protected and changed and improved," explained Jim Lewis, chair of the mathematics department at the University of Nebraska. "I believe this is likely to be an effort that has an impact on the profession," he added. Also, there has been great pressure on mathematics departments to get and keep VIGRE grants, and the competition for money has been fierce. By contrast, the focus of the CID is entirely on enduring ideals and values, which is "refreshing," Corlette remarked.

The now-famous crisis at the University of Rochester in the mid-1990s, in which the university administration planned to close down the mathematics doctoral program, was a wake-up call for many departments that had become insular. Breaking out of that insularity is another motivation for departments to participate in the CID. "We are a typical department," said Francis Bonahon, chair of the mathematics department at the University of Southern California. "For many years we were living in our little corner, complaining that we were not funded at the right level and not given positions. But we were not doing much about it. Now we want to reach out." The department has introduced some new features in its doctoral program in recent years, such as a seminar in which faculty talk about their research areas as a way of helping students choose an advisor. The students also work on problems that are devised by the faculty to relate to the research.

Indeed, one of the criteria Carnegie used in choosing the partner departments was evidence that the departments had already begun to examine seriously their doctoral programs. Some of the departments have VIGRE grants and others are planning to apply for them, but all have made innovations in their doctoral programs. For example, at the University of Chicago the mathematics department has begun offering three-to-five week modules to introduce students to basic tools used in mathematical research. In contrast to regular courses, which usually begin with foundations and work up to big results, these modules provide a "user's guide to ideas", so that students can begin to use the tools without having to work through all the prerequisites. A few years ago the University of Illinois at Urbana-Champaign revamped its comprehensive examinations, which are taken during the first two years of doctoral study. Originally there were two exams, one in algebra and one in analysis. Now there is a more flexible system in which students choose four out of

thirteen exams covering a much wider variety of areas. At the University of Nebraska the mathematics department has given a high priority to educating women in its doctoral program. In February 2003 the department held its fifth annual conference for women in mathematics, which drew 175 women students from eighty colleges and universities. The goal of the conference is to encourage women to pursue graduate education in mathematics.

The list of CID partner and allied mathematics departments is not dominated by the elite doctoral programs, which have historically been seen as the leaders in doctoral education in the U.S. Is this a problem? Not necessarily. For one thing, the CID list is not lacking in representation of the very best doctoral programs in the nation. Also, the diversity among the CID departments may mean that the ideas and experiences that come out of the project will be useful to a wide variety of departments. The received wisdom that all mathematics departments should strive to be like the elite departments has been questioned in recent years. One of the main conclusions of the *Towards Excellence* report was that it is better for most departments to create their own niches and build on their individual strengths rather than pursue an unreachable goal of being an elite department.

The purpose of the CID is not to define, once and forever, what a doctoral program should be. "It's not something you do once and never again, because you have it right for all time," Walker remarked. Rather, the purpose is to foster a culture of continual discussion and renewal, so that improving doctoral education becomes an ongoing mission. This is a worthwhile endeavor for even the best doctoral programs. Phillip Griffith, the director of graduate studies in the mathematics department at the University of Illinois and a CID departmental representative, put it this way: "If you are complacent, you will go backwards."

—Allyn Jackson

Sato and Tate Receive 2002-2003 Wolf Prize

The 2002–2003 Wolf Prize in Mathematics has been awarded to MIKIO SATO, of the Research Institute for Mathematical Sciences, Kyoto University, Kyoto, Japan; and to JOHN T. TATE, Department of Mathematics, University of Texas, Austin. Sato was honored “for his creation of ‘algebraic analysis’, including hyperfunction and microfunction theory, holonomic quantum field theory, and a unified theory of soliton equations.” Tate was honored “for his creation of fundamental concepts in algebraic number theory.” The two share the \$100,000 prize.



Mikio Sato



John T. Tate

Mikio Sato

Mikio Sato’s vision of “algebraic analysis” and mathematical physics initiated several fundamental branches of mathematics. He created the theory of hyperfunctions and invented microlocal analysis, which allowed for a description of the structure of singularities of (hyper)functions on cotangent bundles. Hyperfunctions, together with integral Fourier operators, have become a major tool in linear partial differential equations. Along with his students, Sato developed holonomic quantum field theory, providing a far-reaching extension of the mathematical formalism underlying the two-dimensional Ising model, and introduced along the way the famous tau functions. Sato provided a unified geometric description of soliton equations in the context of tau functions and infinite-dimensional Grassmann manifolds. This was extended by his followers to other classes of equations, including self-dual Yang-Mills and Einstein equations. Sato has generously shared his ideas

with young mathematicians and has created a flourishing school of algebraic analysis in Japan.

Mikio Sato was born in 1928 in Tokyo. He received his B.Sc. (1952) and his Ph.D. (1963) from the University of Tokyo. He was a professor at Osaka University and at the University of Tokyo before moving to the Research Institute for Mathematical Sciences at Kyoto University in 1970. He served as director of that institute from 1987 to 1991. He is now a professor emeritus at Kyoto University. He received the Asahi Prize of Science (1969), the Japan Academy Prize (1976), the Person of Cultural Merits award of the Japanese Education Ministry (1984), the Fujiwara Prize (1987), and the Schock Prize of the Royal Swedish Academy of Sciences (1997). In 1993 he was elected to foreign membership in the U.S. National Academy of Sciences.

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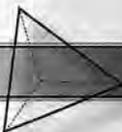
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John T. Tate

For over a quarter of a century, John Tate's ideas have dominated the development of arithmetic algebraic geometry. Tate has introduced path-breaking techniques and concepts that initiated many theories that are very much alive today. These include Fourier analysis on local fields and adèle rings, Galois cohomology, the theory of rigid analytic varieties, and p -divisible groups and p -adic Hodge decompositions, to name but a few. Tate has been an inspiration to all those working in number theory. Numerous notions bear his name: Tate cohomology of a finite group, Tate module of an abelian variety, Tate-Shafarevich group, Lubin-Tate groups, Neron-Tate heights, Tate motives, the Sato-Tate conjecture, Tate twist, Tate elliptic curve, and others. John Tate is a revered name in algebraic number theory.

John Tate was born in 1925 in Minneapolis. He received his A.B. from Harvard College (1946) and his Ph.D. from Princeton University (1950). He was a research assistant and instructor at Princeton (1950-53) and a visiting professor at Columbia University (1953-54) before moving to Harvard University. He was a professor at Harvard until 1990, when he accepted his present position as professor and Sid W. Richardson Chair in Mathematics at the University of Texas at Austin. Tate received the AMS Cole Prize (1956), a Sloan Fellowship (1959-61), and a Guggenheim Fellowship (1965-66). He was elected to the U.S. National Academy of Sciences (1969) and was named a foreign member of the French Academy of Sciences (1992) and an honorary member of the London Mathematical Society (1999).

About the Wolf Prize

The Israel-based Wolf Foundation was established by the late German-born inventor, diplomat, and philanthropist Ricardo Wolf. A resident of Cuba for many years, Wolf became Fidel Castro's ambassador to Israel, where Wolf lived until his death in 1981. The Wolf Prizes have been awarded since 1978 to outstanding scientists and artists "for achievements in the interest of mankind and friendly relations among peoples, irrespective of nationality, race, color, religion, sex, or political view." The prizes of \$100,000 are given each year in four out of five scientific fields, in rotation: agriculture, chemistry, mathematics, medicine, and physics. In the arts the prize rotates among architecture, music, painting, and sculpture. The 2002-2003 prizes will be conferred by the president of Israel at a ceremony at the Knesset (the Israeli parliament) in Jerusalem on May 11, 2003.

—Allyn Jackson

2003 JPBM Communications Award

The 2003 Communications Award of the Joint Policy Board for Mathematics (JPBM) was presented at the 108th Annual Meeting of the AMS in Baltimore in January 2003.

The JPBM Communications Award is presented annually to reward and encourage journalists and other communicators who, on a sustained basis, bring accurate mathematical information to non-mathematical audiences. The award carries a cash prize of \$1,000.

Previous recipients of the JPBM Communications Award are: James Gleick (1988), Hugh Whitmore (1990), Ivars Peterson (1991), Joel Schneider (1993), Martin Gardner (1994), Gina Kolata (1996), Philip J. Davis (1997), Constance Reid (1998), Ian Stewart (1999), John Lynch and Simon Singh (special award, 1999), Sylvia Nasar (2000), Keith J. Devlin (2001), and Claire and Helaman Ferguson (2002).

The 2003 JPBM Communications Award was presented to ROBERT OSSERMAN. The text that follows presents the selection committee's citation, a brief biographical sketch, and the recipient's response upon receiving the award.

Citation

The 2003 JPBM Communications Award is given to Robert Osserman, professor emeritus at Stanford University and Special Projects Director at the Mathematical Sciences Research Institute in Berkeley.

For many years, Bob Osserman has been an erudite spokesman for mathematics, communicating its charm and excitement to thousands of people from all walks of life.

His slim volume *Poetry of the Universe* has been described as "artful and beguiling", introducing readers to the inherent beauty and power of mathematical thinking. It has appeared in more than ten languages. But he has communicated with the public in a more unconventional style as well, through his open conversations and dialogues with playwrights and writers from Tom Stoppard to Steve Martin. These informal and relaxed interviews give mathematical and lay audiences alike an understanding of mathematics through its connections to media and literature. The interviews make mathematics part of our modern culture.

Bob Osserman believes in making mathematics accessible to the general public. He has done more than explain mathematics, however. He has made "mathematics appreciation" more than the title of a course—Bob Osserman has changed people's attitudes towards the subject.

Biographical Sketch

Robert Osserman was born and raised in New York City. He attended the Bronx High School of Science and New York University before being drafted into the army. He received his M.A. and Ph.D. from Harvard, with breaks to study in Zurich and Paris.

His research work has had a geometric slant, starting with geometric function theory and Rie-



Robert Osserman

mann surfaces, then to differential geometry, the complex variable and PDE approaches to minimal surfaces, isoperimetric inequalities, and a brief foray into ergodic theory. He has had a broad array of coauthors in this work, including former students Blaine Lawson, Robert Gulliver, and David Hoffman, as well as Henry Landau, S.-S. Chern, Halsey Royden, Max Schiffer, Robert Finn, Richard Schoen, Peter Sarnak, and Min Ru.

Osserman taught at Stanford University from 1955 to 1994, with years off as a visitor to Harvard University, a Fulbright Lecturer at Paris, a Guggenheim Fellow at the University of Warwick, the head of the Mathematics Branch of the Office of Naval Research, and a visiting member of the Courant Institute of Mathematical Sciences, New York University. At Stanford he received the Dean's Award for Teaching and the Mellon Professorship for Interdisciplinary Studies. He also received the Lester R. Ford Award from the Mathematical Association of America for excellence in expository writing. Since 1990 he has been associated with the Mathematical Sciences Research Institute (MSRI), first as deputy director and then as special projects director.

Response

My main concerns throughout most of my career were teaching and research, and along with the usual related duties of academic life, these pretty well filled up the available time. However, the urge to expose a broader public to some of the most beautiful and interesting parts of mathematics was clearly always there. Already as a graduate student I succeeded in attracting an audience of some 300 to a talk on Gödel's undecidability theorem by pairing it with a performance by fellow student Tom Lehrer.

Over the years I made occasional forays in a similar direction, talking to high school students, alumni groups, and others. A course on mathematics, science, and technology designed for a non-technical (and even technophobic) audience led to my writing a book on geometry and cosmology in which I tried to offer something of interest to everyone, from those with no mathematical background all the way to the professional mathematician. One of my main goals was to make the presentation not only accessible but also accurate, since I had found so much misinformation in many "popular" presentations of science and mathematics.

After retiring from teaching in 1994 and trading in my position as deputy director of MSRI for that of special projects director in 1995, I finally had the freedom to think more deeply about how to reach those parts of the general public who would normally stay far away from anything billed as "mathematics".

The time and place could not have been more propitious. Bill Thurston, who was MSRI director

at the time, and David Eisenbud, who took over in 1997, were both fully supportive of this goal, as have been the relevant MSRI governing bodies. I owe them all great thanks, as I do the many staff members at MSRI during these years, who brought enormous talent and energy to our public events.

I further owe a debt to the mysterious zeitgeist that just at this time was turning the interest of the general public toward mathematics through a series of books, plays, and movies. They provided the perfect vehicle to attract an audience whose main interest may have been in theater, film, or literature.

Most of all I am grateful to those authors who wrote the books, plays, and screenplays, then agreed to participate in our public events and engage in a broad-ranging dialogue, including the mathematical angles about which they often felt not very sure: Tom Stoppard (*Arcadia*), David Auburn (*Proof*), Michael Frayn (*Copenhagen*), Sylvia Nasar (*A Beautiful Mind*), and Steve Martin (*The Pleasure of My Company*) in particular.

MAA Awards Presented in Baltimore

The Mathematical Association of America (MAA) presented several awards at the Joint Mathematics Meetings in Baltimore in January 2003.

Chauvenet Prize

The Chauvenet Prize, first awarded to Gilbert Bliss in 1925, is presented for an outstanding expository article on a mathematical topic by a member of the MAA. The prize is named in honor of William Chauvenet (1820–70), who was a professor of mathematics at the U.S. Naval Academy.

The 2003 Chauvenet Prize was awarded to THOMAS C. HALES of the University of Pittsburgh for his article “Cannonballs and Honeycombs”, *Notices Amer. Math. Soc.* **47** (2000), 440–9.

The prize citation states: “The classical sphere packing conjecture, also known as the Kepler Conjecture, asserts that the natural cannonball arrangement gives the maximum density packing of the Euclidean 3-dimensional space with congruent solid balls. The problem evaded solution for almost 400 years, until Thomas C. Hales, the author of this article, gave a difficult, computer-aided, yet ingenious proof. Another old problem tackled by Hales and described in the article, the Honeycomb Conjecture, is of equally appealing geometric character: Any partition of the plane into regions of equal area has perimeter at least that of the regular hexagonal honeycomb tiling.

“‘Cannonballs and Honeycombs’ is an extremely worthy recipient of the Chauvenet Prize. It has humor, history, talks about real people, presents significant mathematics, and has handholds throughout the article so you can keep finding

good things even if you choose not to follow all the details as you go. The writing is delightful. It connects us to famous scientists of the past and to nature, it talks about the resolution of a centuries-old conjecture, it points out philosophical issues about mathematics and rigor, and it describes intriguing, understandable open questions that have an interesting history, thereby situating us in the flow of history and the challenges of the future.”

Haimo Award

The Deborah and Franklin Tepper Haimo Award for Distinguished College or University Teaching of Mathematics, established in 1991, honors college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has had influence beyond their own institutions.

The 2003 Haimo Award was presented to JUDITH V. GRABINER of Pitzer College, RANJAN ROY of Beloit College, and PAUL ZEITZ of the University of San Francisco.

Grabiner was honored “[f]or her extraordinary scholarship in the history of mathematics, her remarkable teaching, and her compelling exposition to every audience.” The award citation states: “Professor Grabiner enjoys an international reputation as a scholar of the history of mathematics. Her teaching career spans 35 years, with most of that at California State University, Dominguez Hills and (since 1985) Pitzer College. She is universally praised for the depth and range of her knowledge of mathematical history and is famous for giving talks that are knowledgeable, witty, charming, and

beautifully organized and that hold the interest of both the trained mathematician and the 'I hate math' undergraduate. She is a sought-after speaker." She has won three Allendoerfer Awards and two Ford Awards for outstanding writing.

The citation for Roy reads in part: "Professor Roy teaches mathematics as a body of ideas of great depth and beauty, and as a way of thinking which can improve the lives of all who study it. He has read systematically the original works of Newton, Euler, Gauss, Jacobi, and Ramanujan, and uses his deep familiarity with their creative methods to show students that mathematics can be lived. He has an uncanny ability to find ways to connect mathematics to individual students' lives. He teaches, using mathematics as his example, that the key to successful thinking in any discipline is to master a few important ideas deeply and reason from those ideas to solve new problems. 'Ranjan is the kind of teacher who changes your life,' say many students. Professor Roy was Beloit College's Teacher of the Year in 1986 and again in 2000. Professor Roy is also a creative mathematician and a nationally-known expositor of mathematics."

The citation for Zeitz reads in part: "Paul Zeitz's passion for problem solving permeates his teaching. 'Charismatic' is the best descriptor of his teaching style. A teacher at the University of San Francisco since completing his Ph.D. at U.C. Berkeley in 1992, he has been teaching and participating in mathematical contests since he was captain of the Math Team at Stuyvesant High School. In 1974 he took first place in the USAMO (USA Mathematical Olympiad) and was a member of the first U.S. team to compete in the IMO (International Mathematical Olympiad). Although he did not major in mathematics at Harvard, Zeitz taught high school mathematics for six years after graduation. This experience, as well as his talent and enthusiasm for mathematical competitions, led him to be recruited to write problems for the Committee on the American Mathematics Competitions."

Gung and Hu Award

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the MAA. The 2003 award was presented to CLARENCE STEPHENS of the State University of New York, Potsdam.

Stephens, born in 1917, received his Ph.D. from the University of Michigan in 1943, becoming the ninth African American to receive a Ph.D. in mathematics in the U.S. From 1969 until his retirement in 1987, he was chair of the Department of Mathematics at the State University of New York at Potsdam. Stephens is honored "for his role in achieving the 'Potsdam Miracle' in the production of undergraduate mathematics majors at SUNY Potsdam in the 1980's, which led to a model for

creating a welcoming atmosphere for undergraduate mathematics majors at many other institutions." He had already received accolades for a long and distinguished career in undergraduate mathematics education by the time he came to SUNY Potsdam in 1969.

The citation states: "Though SUNY Potsdam is a relatively small regional state college with a total enrollment of just over 4,000 students during Stephens' time there, in 1985 the college "graduated" 184 mathematics majors, the third largest number of any institution in the U.S. that year (exceeded only by two University of California campuses). This represented about a quarter of the degrees given by SUNY Potsdam that year, and over 40 percent of the institution's honor students were mathematics majors. The 'Potsdam Miracle' was not in any sense accomplished by lowering standards, but rather by raising the standards for teaching the students and providing a supportive environment for them...For his pioneering accomplishments in undergraduate mathematics education, and the provision of a national model for institutions that wish to replicate the 'Potsdam Miracle', the MAA Gung-Hu Award Committee is pleased to recommend Clarence Stephens for this award."

Certificates of Meritorious Service

Each year the MAA presents Certificates of Meritorious Service for service at the national level or for service to a section of the MAA. Those honored in 2003 are: KARIN CHESS of Owensboro Community College, Kentucky Section; LESTER H. LANGE of San Jose State University, Northern California Section; LUISE-CHARLOTTE KAPPE of the State University of New York at Binghamton, Seaway Section; LARRY J. MORLEY of Western Illinois University, Illinois Section; ALVIN R. TINSLEY of Central Missouri State University, Missouri Section; and FREDRIC TUFTE of the University of Wisconsin-Platteville, Wisconsin Section.

AWM Awards Presented in Baltimore

The Association for Women in Mathematics (AWM) presented two awards during the Joint Mathematics Meetings in Baltimore in January 2003.

Louise Hay Award

The Louise Hay Award for Contributions to Mathematics Education was established in 1990 to honor the memory of Louise Hay, who was widely recognized for her contributions to mathematical logic and for her devotion to students.

The 2003 Hay Award was presented to KATHERINE PUCKETT LAYTON of Beverly Hills High School “[i]n recognition of her significant contributions to mathematics education, her outstanding achievements as a teacher and scholar, and her role in bridging mathematics education communities.”

Layton began her teaching career in 1960 after receiving a bachelor's degree in mathematics from the University of California, Los Angeles (UCLA). She taught mathematics for forty years at Beverly Hills High School. She spent one year studying for an M.Ed. in mathematics at Harvard University and was a visiting lecturer at Clemson University and in the UCLA mathematics department. In 1990 she received the California Presidential Award for Teaching Excellence. After her retirement in 1999 she spent two years in the Graduate School of Education at UCLA, where she was a field supervisor for a teaching intern program for mathematics majors. She has served on the Mathematical Sciences Education Board of the National Research Council, the National Board for Professional Teaching Standards, and the College Entrance Examination Board.

Schafer Prize

The Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman was established in 1990. The prize is named in honor of Alice T. Schafer, one of the founders of AWM and one of its past presidents.

The 2003 Schafer Prize was awarded to KATE GRUHER of the University of Chicago. Two runners-up were also honored: WEI HO of Harvard University and JOSEPHINE T. YU of the University of California, Davis.

Gruher, a senior at the University of Chicago, excelled in the honors calculus, honors algebra, and honors analysis sequences. During the summer after her sophomore year, she participated in the ergodic theory group of the Research Experiences for Undergraduates program at Williams College. A paper she coauthored on power weak mixing will appear in the *New York Journal of Mathematics*. In the summer of 2002 she participated in the highly exclusive Director's Summer Program at the National Security Agency (NSA). In addition to her classes and research, Kate has graded and run problem sessions for calculus, assisted with new student orientation, and worked as a counselor with the University of Chicago's middle school Young Scholars Program.

Mathematics People

National Academy of Sciences Awards Announced

Two mathematicians have been honored with National Academy of Sciences (NAS) Awards for 2003. DAVID A. FREEDMAN, of the University of California, Berkeley, received the John J. Carty Award for the Advancement of Science, given this year for achievement in statistics. Freedman was selected for "his profound contributions to the theory and practice of statistics, including rigorous foundations for Bayesian inference and trenchant analysis of census adjustment." The award, which carries a cash prize of \$25,000, is given annually for distinguished accomplishment in various fields of science. DAVID R. KARGER, of the Massachusetts Institute of Technology, received the NAS Award for Initiatives in Research, given annually in a field supporting information technology; this year the prize was awarded in algorithms and computation. Karger was selected "for the elegant use of randomness to design improved algorithms for classically studied problems such as network flow, graph coloring, finding minimum trees, and finding minimum cuts." The prize carries a cash award of \$15,000.

—From an NAS announcement

Chern Receives 2002 Lobachevskii Medal

On December 1, 2002, Kazan State University awarded the Lobachevskii Medal for Distinguished Works in Geometry to SHIING-SHEN CHERN, honorary director of the Nankai Institute of Mathematics, Tianjin, China.

The Lobachevskii Medal, established in 1991 by the government of the Soviet Union, is awarded every five years on December 1, the birthday of N. I. Lobachevskii. The statutes of the Lobachevskii Medal also permit awarding of honorary diplomas from Kazan State University to some of the nominees for the medal.

The first Lobachevskii Medal was awarded in 1992 to Aleksandr P. Norden (1904–1993), Kazan State University, for development of the normalization method (called

Norden's method of normalization) in the theory of surfaces in projective spaces, for applications of this method to the theory of non-Euclidean spaces, and for the development and popularization of Lobachevskii's ideas. Three honorary diplomas were also awarded.

In 1997 ten scientists were nominated for the Lobachevskii Medal. On the advice of an international jury, the Council of the Kazan State University awarded two Lobachevskii Medals. The medalists were Mikhael Gromov, Institut des Hautes Études Scientifiques, Paris; and Boris P. Komrakov, International Sophus Lie Center, Minsk. Gromov was honored for a series of papers (1967–1996) in geometry and topology which were devoted to the theory of hyperbolic groups and to the development of the theory of embeddings for various classes of spaces. Komrakov was honored for contributions to the theory of Lie groups and homogeneous spaces presented in the monographs *Structures on Manifolds and Homogeneous Spaces* and *Primitive Actions and the Sophus Lie Problem*. Three nominees received honorary diplomas.

For the competition for the 2002 Lobachevskii Medal, four scientists were nominated. On the advice of an international jury, the Council of Kazan State University awarded the Lobachevskii Medal to Shiing-Shen Chern for his fundamental contributions to differential geometry, integral geometry, web geometry, complex analysis, and characteristic classes. IDZHAD KH. SABITOV, Moscow State University, was awarded an honorary diploma for a series of papers on metric geometry in the large and polyhedra solving.

—Boris Shapukov, Kazan State University

Develin Awarded AIM Five-Year Fellowship

The American Institute of Mathematics (AIM) has awarded its Five-Year Fellowship for 2003 to MIKE DEVELIN, of the University of California, Berkeley. He is currently studying discrete geometry and combinatorics under the supervision of Bernd Sturmfels.

The AIM five-year fellowships are awarded each year to outstanding new Ph.D. students in an area of pure

mathematics. The fellowships cover sixty months of full-time research, as well as funds for travel and equipment. Each fellowship carries a stipend of \$4,000 per month, with an additional \$4,000 per year allocated for travel and equipment.

—From an AIM announcement

Chudnovsky and Lindenstrauss Awarded CMI Long-Term Prize Fellowships

The Clay Mathematics Institute (CMI) has announced its selection of two long-term prize fellows for 2003. They are MARIA CHUDNOVSKY, of Princeton University, and ELON LINDENSTRAUSS, of Stanford University. Chudnovsky “has made significant contributions to the field of combinatorics and graph theory,” including helping to solve the Strong Perfect Graph Conjecture, “one of the best known open problems in combinatorics.” She is working on a related problem for her Ph.D. dissertation. Lindenstrauss was chosen “for his novel work in ergodic theory and dynamical systems,” most notably “his research on the problem of arithmetic quantum unique ergodicity, which is a problem at the interface between the theory of automorphic forms and mathematical physics.”

The prize fellowships are awarded to mathematicians who are thirty years old or younger and who have contributed profound ideas and major achievements to the discipline of mathematics. The long-term prize fellows are employed by CMI for terms ranging from one to five years and are paid a salary to conduct research at institutions of their choice. Additional research funding can be requested.

The Clay Mathematics Institute is a private, nonprofit foundation dedicated to increasing and disseminating mathematical knowledge. It sponsors a series of programs that includes creating new mathematical knowledge, disseminating mathematical insights, inspiring talented students, and recognizing extraordinary mathematical achievement and solutions of specific mathematical problems.

—From a CMI announcement

Klein Awarded Leibniz Prize

RUPERT KLEIN, of the Free University of Berlin and the Potsdam Institute for Climate Impact Research, has been awarded the Gottfried Wilhelm Leibniz Prize for 2003 by the Deutsche Forschungsgemeinschaft (DFG). Klein works in both mathematics and climate research. The DFG prize consists of 1.55 million euros (about US\$1.7 million) to support research over a period of five years.

Klein studied theoretical engineering at RWTH Aachen, from which he received a Ph.D. in 1988. He did postdoctoral work at Princeton University and held a University Professorship at Bergische Universität before joining the

Free University and the Potsdam Institute. His current research interests are in integrating applied mathematics and computer sciences with climate impact research and in the mathematical modeling of multiscale interactions in natural and social systems. His goal is to further develop both mathematical concepts and concepts relating to the natural and social sciences and thus to solidify interdisciplinary research.

The aim of the Leibniz Prize Program, which was instituted by the DFG in 1985, is to improve the working conditions of outstanding scientists and scholars, to broaden their opportunities for research, to relieve them of administrative burdens, and to allow them to hire especially highly qualified young academics. The prizewinners are permitted the greatest possible freedom in the way they use the prize funds. The DFG is the main scientific research funding agency of the German government.

—From a Potsdam Institute announcement

AWM Essay Contest Winners Announced

The Association for Women in Mathematics (AWM) has announced the winners of its 2002 essay contest, “Biographies of Contemporary Women in Mathematics”. The grand prize winner was ALYSSA CHASE, of Townsend Harris High School in Flushing, New York, for her essay “Peggy Tang Strait: A Pioneer in Uncharted Territory”. Chase’s essay will be published in the AWM newsletter. The first-place winner in the graduate school category was JEFFREY B. FARR, of Clemson University, for an essay on Renu Laskar. ALICIA RICHARDSON, of Morgan State University, won first prize in the college division for an essay on Fern Hunt. The winner in the middle school category was ROSS CATON, of Jack Jouett Middle School, Charlottesville, Virginia, who wrote an essay on Lois Williams. A complete list of the winners, as well as copies of their essays, can be found on the AWM website at <http://www.awm-math.org/biographies/contest/2002.html>.

—From an AWM announcement

Mathematics Opportunities

NSF Postdoctoral Research Fellowships

The National Science Foundation (NSF) awards Mathematical Sciences Postdoctoral Research Fellowships for appropriate research in areas of the mathematical sciences, including applications to other disciplines. Awardees are permitted to choose research environments that will have maximal impact on their future scientific development. Stipends provide support for two 9-month academic years and 6 summer months, for a total of 24 months.

The deadline for applications is **October 17, 2003**. For more information and application instructions, see the NSF website at <http://www.fastlane.nsf.gov/d11/D11Menu.htm>.

—From an NSF announcement

Call for VIGRE Proposals

The Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) has announced a new competition for Grants for Vertical Integration of Research and Education in the Mathematical Sciences, known as VIGRE grants.

VIGRE grants are designed to allow departments in the mathematical sciences to carry out innovative educational programs in which research and education are integrated and in which undergraduates, graduate students, postdoctoral fellows, and faculty are mutually supportive. The goals of VIGRE are: (1) to prepare undergraduate students, graduate students, and postdoctoral fellows for the broad range of opportunities available to individuals with training in the mathematical sciences; and (2) to encourage departments in the mathematical sciences to initiate or improve educational activities that lend themselves to integration with research, especially activities that promote the interaction of scholars across boundaries of academic age and departmental standing.

The deadline for proposals for the new competition is **July 28, 2003**. The Division of Mathematical Sciences anticipates making between three and ten awards in the competition this year. For further information consult the DMS website, <http://www.nsf.gov/pubs/2002/nsf02120/nsf02120.txt>.

—Elaine Kehoe

Science Visualization Contest Announced

The National Science Foundation (NSF) and the journal *Science* are now accepting entries for the inaugural 2003 Science and Engineering Visualization Challenge. This new international contest will recognize outstanding achievement by scientists, engineers, and visual information practitioners in the use of visual media to promote understanding of research results.

The contest is open to individual scientists, engineers, visual information practitioners, and scientific teams—including technicians and support team members—who produce or commission photographs, illustrations, animations, interactive media, video sequences, or computer graphics for research.

Winning selections will be featured in a special section of the September 12, 2003, issue of *Science*, and winners will receive an expense-paid trip to the NSF for its “Art of Science Project” exhibit and accompanying lecture.

Entries must have been produced after January 1, 2000. The deadline for entries to be postmarked is **May 31, 2003**. Contest rules and entry submission instructions are available at the website <http://www.nsf.gov/od/1pa/events/sevc/>.

—From an NSF announcement

Correction

The next Pan-African Congress of Mathematicians will be held September 1–6, 2004, at the Université de 7 Novembre à Carthage in Tunis, Tunisia. An announcement about the congress that appeared in the February 2003 issue of the *Notices*, page 257, inadvertently omitted the following information:

Those interested in speaking at or participating in the Pan-African Congress are invited to contact: A. Boukricha, Chairman, Local Organizing Committee of the Pan-African Mathematical Congress, B.P. 63, 1013 EL Menzah 9, Tunis, Tunisia; email aboukricha@fst.rnu.tn.

—Allyn Jackson

For Your Information

Change in Procedures for 2003 AMS Elections

As part of AMS elections for 2003, members will have the option of voting either online or by the traditional paper ballot. All AMS members who are open to receiving email from the AMS will be contacted in May and given the opportunity to choose to receive a paper ballot; otherwise their voting instructions will be emailed to them at the start of elections in early September. Members who receive the traditional paper ballot will still have the option of casting their votes online. It is anticipated that the availability of online voting will provide a much appreciated convenience to AMS members, especially the approximately 7,000 AMS members who live outside of the United States.

—Robert Daverman, AMS secretary

CPST Project on Master's Degrees

The Commission on Professionals in Science and Technology (CPST) has received funding from the Alfred P. Sloan Foundation to conduct a series of activities designed to promote the revitalization of the master's degree in science and mathematics.

In 1997 the Sloan Foundation launched the Professional Science Master's Initiative with the goal of bringing into being new two-year professional master's degree programs that would equip people for work outside academia. The coordinator of the Sloan initiative, Sheila Tobias, coauthored an article entitled "The growth of the professional master's in mathematics", which appeared in the May 2001 issue of the *Notices*, pages 491-7.

The CPST project will conduct a series of activities, including carrying out an in-depth review of the state of master's education and employment in science and mathematics, supporting efforts of professional societies

to inventory master's programs, and developing and maintaining a cross-disciplinary database on master's education. The CPST will convene key policymakers for a conference on professional master's programs, to be held in Washington, DC, in October 2003.

For further information visit the website <http://www.cpst.org/>.

—Allyn Jackson

Film about Mathematicians to Air

A film about mathematicians entitled *The Math Life* will air on public television stations across the U.S. this spring. The film was produced by Daniel Rockmore, a mathematician at Dartmouth College, and filmmakers Wendy Conquest and Bob Drake. In 52 minutes distilled from interviews with seventeen mathematicians, the film examines what it is like to do mathematics. Among the mathematicians interviewed are David Mumford, Ingrid Daubechies, Persi Diaconis, Michael Freedman, Fan Chung Graham, Kate Okikiolu, Jennifer Tour Chayes, Peter Sarnak, and Steven Strogatz.

Further information about the film may be found on the website <http://www.cs.dartmouth.edu/~rockmore/mathlife.html>, which also contains a link to a list of broadcast dates.

—Allyn Jackson

Reference and Book List

The *Reference* section of the Notices is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.

Contacting the Notices

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.tamu.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 979-845-6028 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

April 11, 2003: Applications for Project NExT. See <http://archives.math.utk.edu/projnext/>.

April 15, 2003: Applications for National Research Council Research Associateship Program. See <http://www4.nationalacademies.org/pga/rap.nsf/>, or contact the National Research Council, Associateship Programs (TJ 2114), 2101 Constitution Avenue, NW, Washington, DC 20418;

telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

April 18, 2003: Full proposals for NSF IGERT program. See <http://www.nsf.gov/pubsys/ods/getpub.cfm?nsf02145>.

April 30, 2003: Nominations for Maria Mitchell Women in Science Award. See <http://www.mmo.org/>, or contact the Maria Mitchell Women in Science Award Committee at the Maria Mitchell Association, 2 Vestal

Where to Find It

A brief index to information that appears in this and previous issues.

AMS Bylaws—November 2001, p. 1205

AMS Email Addresses—November 2002, p. 1275

AMS Ethical Guidelines—June/July 2002, p. 706

AMS Officers 2002 and 2003 (Council, Executive Committee, Publications Committees, Board of Trustees)—May 2003, p. 594

AMS Officers and Committee Members—October 2002, p. 1108

Backlog of Mathematics Research Journals—September 2002, p. 963

Conference Board of the Mathematical Sciences—September 2002, p. 955

Information for Notices Authors—June/July 2002, p. 697

Mathematics Research Institutes Contact Information—August 2002, p. 828

National Science Board—January 2003, p. 64

New Journals for 2001—June/July 2002, p. 698

NRC Board on Mathematical Sciences and Their Applications—March 2003, p. 383

NRC Mathematical Sciences Education Board—April 2003, p. 489

NSF Mathematical and Physical Sciences Advisory Committee—February 2003, p. 261

Program Officers for Federal Funding Agencies—October 2002, p. 1103 (DoD, DoE); November 2002, p. 1278 (NSF Education Program Officers); December 2002, p. 1406 (DMS Program Officers)

Street, Nantucket, MA 02554; telephone 508-228-9198.

May 1, 2003: Applications for NSF/AWM Travel Grants for Women. See <http://www.awm-math.org/travelgrants.html>, telephone 301-405-7892, email: awm@math.umd.edu.

May 15, 2003: Applications for fall semester of Math in Moscow and for AMS scholarships. See <http://www.mccme.ru/mathinmoscow>, or contact Math in Moscow, P. O. Box 524, Wynnwood, PA 19096; fax +7095-291-65-01; email: mim@mccme.ru. For information about and application forms for the AMS scholarships, see <http://www.ams.org/careers-edu/mimoscow.html> or contact Math in Moscow Program, Professional Services Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904; email: prof-serv@ams.org.

May 19, 2003: NSF Teacher Professional Continuum Program. See <http://www.ehr.nsf.gov/ehr/esie/>.

May 31, 2003: Entries for 2003 Science and Engineering Visualization Challenge. See "Mathematics Opportunities" in this issue.

May 31, 2003: Nominations for Oberwolfach Prize. See <http://www.oberwolfach.org/>, or contact Gert-Martin Greuel, Director, Mathematisches Forschungsinstitut Oberwolfach, Lorenzenhof, 77709 Oberwolfach-Walke, Germany.

June 1, 2003: Applications for Christine Mirzayan Science and Technology Policy Internship Program. See <http://www7.nationalacademies.org/internship/index.html>, or contact The National Academies Christine Mirzayan Science and Technology Policy Internship Program, 500 5th Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667.

June 30, 2003: Nominations for the Fermat Prize for Mathematics Research. See www.ups-tlse.fr/ACTUALITES/Sciences/Prix_Fermat_2003/Areglement.html.

July 15, 2003: Applications for Women's International Science Collaboration (WISC) Program. See <http://www.aaas.org/international/wiscnew.shtml>, or contact WISC Travel Grant, American Association for the

Advancement of Science, Directorate for International Programs, 1200 New York Avenue, NW, Washington, DC, 20005.

July 28, 2003: Proposals for VIGRE grants. See "Mathematics Opportunities" in this issue.

August 15, 2003: Applications for National Research Council Research Associateship Program. See <http://www4.nationalacademies.org/pga/rap.nsf/>, or contact the National Research Council, Associateship Programs (TJ 2114), 2101 Constitution Avenue, NW, Washington, DC 20418; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

October 15, 2003: Applications for spring semester of Math in Moscow and for AMS scholarships. See <http://www.mccme.ru/mathinmoscow>, or contact Math in Moscow, P. O. Box 524, Wynnwood, PA 19096; fax +7095-291-65-01; email: mim@mccme.ru. For information about and application forms for the AMS scholarships, see <http://www.ams.org/careers-edu/mimoscow.html>, or contact Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence, RI 02904; email: prof-serv@ams.org.

October 17, 2003: Applications for NSF Postdoctoral Research Fellowships. See "Mathematics Opportunities" in this issue.

December 31, 2003: Entries for *Cryptologia* paper competitions. See <http://www.dean.usma.edu/math/pubs/cryptologia/>, or contact *Cryptologia*, Department of Mathematical Sciences, United States Military Academy, West Point, NY 10996; email: Cryptologia@usma.edu.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician,

coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

* *1089 and All That: A Journey into Mathematics*, by David Acheson. Oxford University Press, July 2002. ISBN 0-19-851623-1.

The Algorithmic Beauty of Seaweeds, Sponges and Corals, by Jap Kaandorp and Janet Kübler. Springer-Verlag, January 2001. ISBN 3-540-67700-3.

The Annotated Flatland: A Romance of Many Dimensions, Edwin A. Abbott; introduction and notes by Ian Stewart. Perseus Publishing, November 2001. ISBN 0-7382-0541-9. (Reviewed November 2002.)

The Art of the Infinite: The Pleasures of Mathematics, by Robert Kaplan and Ellen Kaplan. Oxford University Press, March 2003. ISBN 0-195-14743-X.

Behind Deep Blue: Building the Computer That Defeated the World Chess Champion, by Feng-hsiung Hsu. Princeton University Press, November 2002. ISBN 0-691-09065-3.

The Bit and the Pendulum: How the New Physics of Information Is Revolutionizing Science, by Tom Siegfried. John Wiley & Sons, February 2000. ISBN 0-47132-174-5. (Reviewed August 2002.)

The Book of Nothing: Vacuums, Voids, and the Latest Ideas about the Origins of the Universe, by John D. Barrow. Pantheon Books, April 2001. ISBN 0-375-42099-1. (Reviewed June/July 2002.)

* *Codebreakers: Arne Beurling and the Swedish Crypto Program during World War II*, by Bengt Beckman; translated by Kjell-Ove Widman. AMS, February 2003. ISBN 0-8218-2889-4.

Codes and Ciphers: Julius Caesar, the Enigma, and the Internet, by Robert Churchhouse. Cambridge University Press, January 2002. ISBN 0-521-81054-X.

The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems, by Martin Gardner. W. W. Norton & Company, August 2001.

ISBN 0-393-02023-1. (Reviewed October 2002.)

Conned Again, Watson! Cautionary Tales of Logic, Math, and Probability, by Colin Bruce. Perseus Publishing, January 2001. ISBN 0-7382-0345-9. (Reviewed November 2002.)

The Constants of Nature: From Alpha to Omega—The Numbers That Encode the Deepest Secrets of the Universe, by John D. Barrow. Jonathan Cape, September 2002. Pantheon Books, January 2003. ISBN 0-375-42221-8.

Correspondance Grothendieck-Serre, Pierre Colmez and Jean-Pierre Serre, editors. Société Mathématique de France, 2001. ISBN 2-85629-104-X.

Curve Ball: Baseball, Statistics, and the Rules of Chance in the Game, by Jim Albert and Jay Bennett. Copernicus-Springer Verlag, July 2001. ISBN 0-387-98816-5.

Damned Lies and Statistics: Untangling Numbers from the Media, Politicians, and Activists, by Joel Best. University of California Press, May 2001. ISBN 0-520-21978-3. (Reviewed February 2003.)

Does God Play Dice? The New Mathematics of Chaos, by Ian Stewart. Blackwell, revised second edition, January 2002. ISBN 0-631-23251-6. (Reviewed December 2002.)

Entanglement: The Greatest Mystery in Physics, by Amir D. Aczel. Four Walls Eight Windows, October 2002. ISBN 1-56858-232-3.

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From the AMS Secretary

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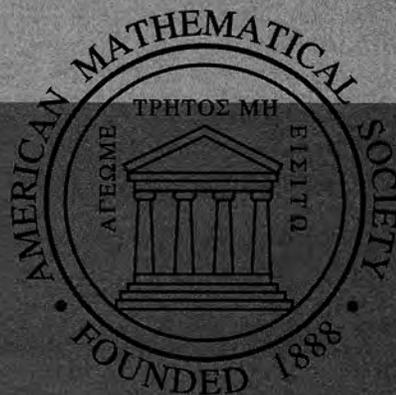
2003 Frank and Brennie Morgan AMS-MAA-SIAM Prize for Outstanding Research in Mathematics by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students having submitted joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, or Canada or Mexico, is eligible to be considered for this prize.

The prize recipient's research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be submitted while the student is an undergraduate; they cannot be submitted after the student's graduation. The research paper (or papers) may be submitted for consideration by the student or a nominator. All submissions for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student's research. Publication of research is not required.



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The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. The 2003 prize will be awarded for papers submitted for consideration no later than **June 30, 2003**, by (or on behalf of) students who were undergraduates in December 2002.

Nominations and submissions should be sent to:

Morgan Prize Committee
c/o Robert J. Daverman, Secretary
American Mathematical Society
312D Ayres Hall
University of Tennessee
Knoxville, TN 37996-1330

Questions may be directed to the chairperson of the Morgan Prize Committee:

Dr. Martha J. Siegel, Chair
Department of Mathematics
Towson University
Towson, MD 21252-0001
telephone: 410-704-4379
e-mail: siegel@towson.edu

*Call for Nominations***E. H. MOORE**
Research Article Prize

At its meeting in January 2002, the AMS Council approved the establishment of a new award called the E. H. Moore Research Article Prize. It is to be awarded every three years for an outstanding research article that has appeared in one of the AMS primary research journals (namely, the *Journal of the AMS*, *Proceedings of the AMS*, *Transactions of the AMS*, *Memoirs of the AMS*, *Mathematics of Computation*, *Electronic Journal of Conformal Geometry and Dynamics*, and the *Electronic Journal of Representation Theory*) during the six calendar years ending a full year before the meeting in which the prize is awarded.

Among other activities, E. H. Moore founded the Chicago branch of the AMS, served as the Society's sixth president (1901–2), delivered the Colloquium Lectures in 1906, and founded and nurtured the *Transactions of the American Mathematical Society*. The name of the prize honors his extensive contributions to the discipline and to the Society.

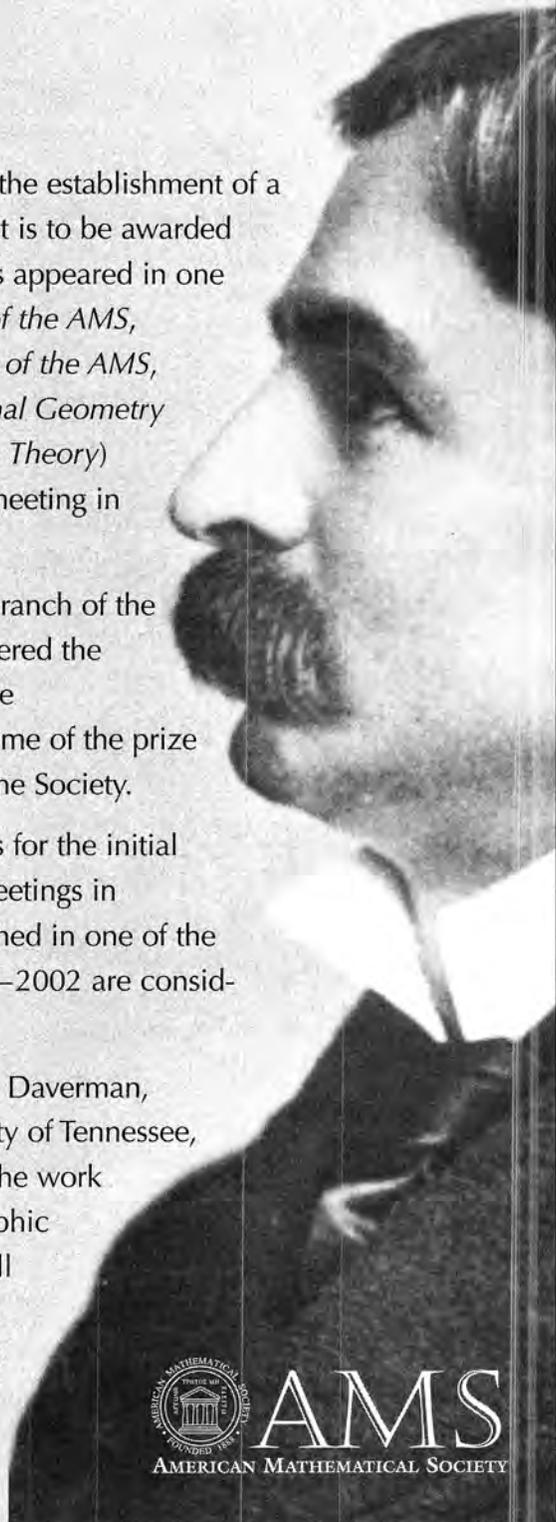
The Moore Prize Selection Committee requests nominations for the initial award, which will be presented at the Joint Mathematics Meetings in Phoenix, AZ, in January 2004. To be specific, papers published in one of the journals named in the first paragraph during the years 1997–2002 are considered eligible for the 2004 award.

Nominations should be submitted to the secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville, TN 37996-1330. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations. A brief curriculum vitae should be included for all nominees. The nominations will be forwarded by the secretary to the prize selection committee, which will make final decisions on the awarding of this prize.

Deadline for nominations is June 30, 2003.



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Call for
NOMINATIONS

The selection committees for these prizes request nominations for consideration for the 2004 awards, which will be presented at the Joint Mathematics Meetings in Phoenix, AZ, in January 2004.

Information about these prizes may be found in the November 2001 *Notices*, pp. 1211–1223. (Also available at <http://www.ams.org/prizes-awards>.)

Levi L. Conant

PRIZE

The Levi L. Conant Prize, first awarded in January 2001, is presented annually for an outstanding expository paper published in either the *Notices* or the *Bulletin of the American Mathematical Society* during the preceding five years.

DISTINGUISHED

Public Service

AWARD

The Award for Distinguished Public Service is presented every two years to a research mathematician who has made a distinguished contribution to the mathematics profession during the preceding five years.

Oswald Veblen

PRIZE

The Oswald Veblen Prize is now presented every three years in recognition of a notable research memoir in geometry or topology published in the preceding six years. To be considered, either the nominee should be a member of the Society or the memoir should have been published in a recognized North American journal.

Norbert Wiener

PRIZE

The Norbert Wiener Prize is awarded jointly by the AMS and SIAM for an outstanding contribution to “applied mathematics in its highest and broadest sense.” The award was first made in 1968 and usually has been presented every fifth year since then. Beginning in 2004 future awards will be made on a three-year cycle.

Nominations should be submitted to the secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville, TN 37996-1330. Include a short description of the work that is the basis of the nomination, with complete bibliographic citations when appropriate. A brief curriculum vitae should be included for the nominee. The nominations will be forwarded by the secretary to the appropriate prize selection committee, which, as in the past, will make final decisions on the awarding of these prizes.

Deadline for nominations is June 30, 2003.

Add this Cover Sheet to all of your Academic Job Applications

How to use this form

1. Using the facing page or a photocopy, (or visit the AMS web site for a choice of electronic versions at www.ams.org/cover-sheet/), fill in the answers which apply to *all* of your academic applications. Make photocopies.
2. As you mail each application, fill in the remaining questions neatly on one cover sheet and include it *on top of* your application materials.

The purpose of the cover form is to aid department staff in tracking and responding to each application for employment. Mathematics departments in Bachelor's-, Master's-, and Doctorate-granting institutions are expecting to receive the form from each applicant, along with the other application materials they require.

The AMS suggests that applicants and employers visit the Job Application Database for Mathematicians (www.mathjobs.org), a new electronic resource being offered by the AMS (in partnership with Duke University) for the second year in 2002-03. The system provides a way for applicants to produce printed coversheet forms, apply for jobs, or publicize themselves in the "Job Wanted" list. Employers can post a job listing, and once applications are made, search and sort among their applicants. Note-taking, rating, e-mail, data downloading and customizable EOE functions are available to

employers. Also, reference writers can submit their letters online. A paperless application process is possible with this system, however; employers can choose to use any portion of the service. There will be annual employer fees beginning this year. This system was developed at the Duke University Department of Mathematics.

Please direct all questions and comments to: emp-info@ams.org.

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Ph.D. Thesis Title (optional) _____

Indicate the mathematical subject area(s) in which you have done research using the Mathematics Subject Classification printed on the back of this form or on the AMS website. Use the two-digit classification which best fits your interests in the Primary Interest line and additional two-digit numbers in the Secondary Interest line.

Primary Interest _____

Secondary Interests optional _____

Give a brief synopsis of your current research interests (e.g. finite group actions on four-manifolds). Avoid special mathematical symbols and please do not write outside of the boxed area.

Most recent, if any, position held post Ph.D.

University or Company _____

Position Title _____

Indicate the position for which you are applying and position posting code, if applicable

If unsuccessful for this position, would you like to be considered for a temporary position?

Yes No If yes, please check the appropriate boxes.

Postdoctoral Position 2+ Year Position 1 Year Position

List the names, affiliations, and e-mail addresses of up to four individuals who will provide letters of recommendation if asked. Mark the box provided for each individual whom you have already asked to send a letter.

This form is provided courtesy of the American Mathematical Society.

This cover sheet is provided as an aid to departments in processing job applications. It should be included with your application material.

Please print or type. Do not send this form to the AMS.



2000 Mathematics Subject Classification

- 00 General
- 01 History and biography
- 03 Mathematical logic and foundations
- 05 Combinatorics
- 06 Order, lattices, ordered algebraic structures
- 08 General algebraic systems
- 11 Number theory
- 12 Field theory and polynomials
- 13 Commutative rings and algebras
- 14 Algebraic geometry
- 15 Linear and multilinear algebra, matrix theory
- 16 Associative rings and algebras
- 17 Nonassociative rings and algebras
- 18 Category theory, homological algebra
- 19 K -theory
- 20 Group theory and generalizations
- 22 Topological groups, Lie groups
- 26 Real functions
- 28 Measure and integration
- 30 Functions of a complex variable
- 31 Potential theory
- 32 Several complex variables and analytic spaces
- 33 Special functions
- 34 Ordinary differential equations
- 35 Partial differential equations
- 37 Dynamical systems and ergodic theory
- 39 Difference and functional equations
- 40 Sequences, series, summability
- 41 Approximations and expansions
- 42 Fourier analysis
- 43 Abstract harmonic analysis
- 44 Integral transforms, operational calculus
- 45 Integral equations
- 46 Functional analysis
- 47 Operator theory
- 49 Calculus of variations and optimal control, optimization
- 51 Geometry
- 52 Convex and discrete geometry
- 53 Differential geometry
- 54 General topology
- 55 Algebraic topology
- 57 Manifolds and cell complexes
- 58 Global analysis, analysis on manifolds
- 60 Probability theory and stochastic processes
- 62 Statistics
- 65 Numerical analysis
- 68 Computer science
- 70 Mechanics of particles and systems
- 74 Mechanics of deformable solids
- 76 Fluid mechanics
- 78 Optics, electromagnetic theory
- 80 Classical thermodynamics, heat transfer
- 81 Quantum theory
- 82 Statistical mechanics, structure of matter
- 83 Relativity and gravitational theory
- 85 Astronomy and astrophysics
- 86 Geophysics
- 90 Operations research, mathematical programming
- 91 Game theory, economics, social and behavioral sciences
- 92 Biology and other natural sciences
- 93 Systems theory, control
- 94 Information and communication, circuits
- 97 Mathematics education

Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on e-MATH at <http://www.ams.org/mathcal/>.

May 2003

*14–16 **DIMACS Workshop on Data Depth: Robust Multivariate Analysis, Computational Geometry and Applications**, DIMACS Center, Rutgers University, Piscataway, New Jersey.

Short Description: The concept of data depth provides new perspectives to probabilistic as well as computational geometries. In particular, the development of implementable computing algorithms for depth-based statistics has brought about many new challenges in computational geometry. This workshop would create a unique environment for multidisciplinary collaboration among computer scientists, theoretical and applied statisticians, and data analysts. It would bring together active researchers in these fields to discuss significant open issues, establish perspective on applications, and set directions for further research.

Organizers: R. Liu, Rutgers Univ., rliu@stat.rutgers.edu; R. Serfling, Univ. of Texas at Dallas, serfling@utdallas.edu; D. Souvaine, Tufts Univ., dls@eecs.tufts.edu; Y. Vardi, Rutgers Univ., vardi@stat.rutgers.edu.

Contact: R. Liu, Rutgers Univ., rliu@stat.rutgers.edu.

Local Arrangements: M. Mercado, DIMACS Center, mercado@dimacs.rutgers.edu, 732-445-5928.

Deadline: Participants interested in presenting talks/posters at the workshop please submit abstracts to R. Liu (rliu@stat.rutgers.edu) by March 25, 2003.

Information: <http://dimacs.rutgers.edu/Workshops/Depth/>.

*20–24 **Conference and Workshop on Coding Theory and Quantum Computing**, University of Virginia, Charlottesville, Virginia.

Topics: Coding Theory and Quantum Computing, including Kerdock/Preparata Codes, Orthogonal Geometry and Quantum Computing, Quantum Cryptography, and Quantum Entanglement.

Description: The week will begin with a three-day workshop that will include three minicourses geared towards those who may have little knowledge about the particular areas and that will prepare attendees for the invited talks during the following days.

Minicourse Lecturers: R. Calderbank (AT&T Labs Research), S. Lomonaco, Jr. (Univ. of Maryland, Baltimore County), D. Meyer (Univ. of California, San Diego).

Invited Speakers: S. van Enk (Bell Labs), S. Gao (Clemson), M. Hillery (Hunter College of CUNY), G. Matthews (Clemson), B. Terhal (IBM Watson Research Center), C. van der Wal (Harvard), L. Viola (Los Alamos National Laboratory), J. Walker (Univ. of Nebraska), Q. Xiang (Univ. of Delaware).

Contributed Talks: Abstracts for contributed talks are welcome and should be sent to the email address below.

Organizers: D. Evans (Univ. of Virginia), J. Holt (Univ. of Virginia), J. Howland (Univ. of Virginia), C. Jones (Washington and Lee), B. Parshall (Univ. of Virginia), O. Pfister (Univ. of Virginia), H. Ward (Univ. of Virginia).

Sponsors: Department of Mathematics, University of Virginia; Institute of Mathematical Sciences, University of Virginia; Dean, Faculty of Arts and Sciences, University of Virginia; National Science Foundation.

Information: <http://www.cs.virginia.edu/~evans/quantum/>; email: conf2003@weyl.math.virginia.edu.

June 2003

*2–6 **DIMACS Workshop on Complexity and Inference**, DIMACS Center, Rutgers University, Piscataway, New Jersey.

Short Description: In this workshop we will explore both the foundational aspects of complexity-based inference as well as

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with

respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence eight months prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: <http://www.ams.org/>.

applications of these ideas to challenging modeling problems. Participants will be drawn from the fields of statistics, information and coding theory, machine learning, and complexity theory. Application areas include biology, information technologies, physics and psychology. The following specific topics will be covered by the workshop: Kolmogorov complexity and inference, MDL (MML) theory and applications, Lossy compression and complexity theory, Complexity and Bayesian methods, Individual sequence/online prediction and predictive complexity, Compression methods for clustering, Machine learning and computational complexity, Complexity and cognitive science, Applications.

Sponsor: DIMACS Center.

Organizers: M. Hansen, Bell Labs., email: cocteau@research.bell-labs.com; P. Vitanyi, CWI and the Univ. of Amsterdam, email: Paul.Vitanyi@cwi.nl; B. Yu, UC Berkeley, email: binyu@stat.berkeley.edu.

Local Arrangements: M. Mercado, DIMACS Center; email: mercado@dimacs.rutgers.edu, 732-445-5928.

Deadline: Submission of contributed papers: March 1, 2003.

Information: Visit <http://dimacs.rutgers.edu/Workshops/Inference/>.

*9-14 **Variational Methods in Celestial Mechanics**, AIM Research Conference Center, Palo Alto, California.

Organizers: R. Montgomery and A. Chenciner.

Workshop Topics: The focus of this workshop is variational methods in celestial mechanics, more precisely, minimization of the action in the presence of symmetries. Using the direct method, a number of new orbits have been established over the last three years. In the workshop we will aim to summarize the state of knowledge to date and pose problems within the N-body problem which may be accessible to variational methods.

Deadline: February 22, 2003.

Information: Visit <http://aimath.org/ARCC/workshops/varcelest.html>.

*11-12 **DIMACS Workshop: Algorithms for Multidimensional Scaling II**, Doubletree Hotel, Tallahassee, Florida.

Sponsors: DIMACS Center.

Organizers: J. Douglas Carroll (chair), Rutgers University, email: dcarroll@rci.rutgers.edu; P. Arabie, Rutgers Univ., email: arabie@andromeda.rutgers.edu; L. Hubert, Univ. of Illinois, email: lhubert@s.psych.uiuc.edu; M. Trosset, The College of William & Mary, email: trosset@math.wm.edu; M. Brusco, Florida State Univ., email: mbrusco@garnet.acns.fsu.edu; M. Janowitz, DIMACS liaison, email: melj@dimacs.rutgers.edu.

Local Arrangements: M. Mercado, DIMACS Center, mercado@dimacs.rutgers.edu, 732-445-5928.

Information: Visit <http://dimacs.rutgers.edu/Workshops/Scaling2/>.

*11-14 (REVISED) **Curvature in Geometry, in Honour of Prof. L. Vanhecke**, Grand Hotel Tiziano e dei Congressi, Lecce, Italy.

Topics: The program will concern topics in differential geometry and will focus on the role of curvature in geometry, a research field in which Prof. L. Vanhecke gave relevant contributions.

Main Speakers: J. Berndt (Univ. of Hull, UK), D. Blair (Michigan State Univ., USA), A. Borisenko (Kharkov Nat. Univ., Ukraine), V. Cortes (Univ. of Nancy, France), P. De Bartolomeis (Univ. di Firenze, Italy), P. Gauduchon (Ecole Polytechnique, France), K. Grove (Univ. of Maryland, College Park, USA), V. Muñoz (Univ. Autónoma de Madrid, Spain), A.M. Naveira (Univ. de Valencia, Spain), S. Nishikawa (Tohoku Univ., Japan), M. Pontecorvo (Univ. di Roma Tre, Italy), C.-L. Terng (Northeastern Univ., Boston, USA), J. A. Wolf (Univ. of California, Berkeley, USA).

Organizers: R. A. Marinosci, rosanna@ilenic.unile.it; G. De Cecco, giuseppe.dececco@unile.it; G. Calvaruso, giovanni.calvaruso@unile.it; E. Boeckx, eric.boeckx@wis.kuleuven.ac.be; L. Nicolodi, lorenzo.nicolodi@unipr.it.

Information: <http://www.diffgeo.unile.it>.

*15-July 4 **The Arithmetic, Geometry and Topology of Algebraic Cycles**, Morelia, Mexico.

Description: The first two weeks (June 15-28) will be in a Summer School format, devoted to short courses and surveys. The third week (June 29-July 4) will be a research-level conference.

Organizers: J. Elizondo, P. Luis del Angel, J. D. Lewis, V. Srinivas, C. Weibel.

Funding: Limited funding is available to cover the expenses of advanced graduate students, postdoctoral fellows and young researchers. We encourage those people to let us know they need funding.

Information: Details of the application process and program can be found at <http://www.math.unam.mx/cycles/>.

*27-29 **NKS 2003**, Boston, Massachusetts.

Conference Outline: Since its release in May 2002, Stephen Wolfram's *A New Kind of Science* has generated immense interest across many areas of science and beyond.

Description: NKS 2003 will be the first conference devoted to the ideas and implications of *A New Kind of Science*. The conference will bring together individuals from a broad range of fields to learn and interact and to get involved with NKS. Major conference features will include: A series of in-depth lectures by Stephen Wolfram on key aspects of *A New Kind of Science*; specialized sessions focusing on implications and applications for: computer science, biological sciences, social sciences, physical sciences, fundamental physics, mathematics, foundations of mathematics, philosophy, future technology and more...; workshops and case studies on: modeling, computer experimentation, defining NKS problems, NKS Explorer, NKS-based education and more...; group discussions on selected topics of mutual interest; poster/demo sessions providing opportunities to showcase NKS-based work; gallery of NKS-based art pieces; special preconference Mathematica course.

NKS 2003 will be appropriate for anyone interested in learning about or pursuing the ideas of *A New Kind of Science*. Dr. Wolfram's core lectures will assume a background of general scientific knowledge at a basic college level. Specialized sessions will assume graduate-level knowledge of relevant fields.

Call for Materials: Being held only a year after the publication of *A New Kind of Science*, the focus of NKS 2003 will be on education about the ideas and implications of the book. NKS 2003 will nevertheless provide poster sessions, demo stations, and an art gallery to showcase projects that are under development based on the book. Please see <http://www.wolframscience.com/conference/2003/materials.html>.

Deadline: The deadline is April 15, 2003 for new submissions.

Contact: email: nks2003@wolframscience.com, 1-800-WOLFRAM (965-3726).

Information: <http://www.wolframscience.com/conference/2003/>.

July 2003

*6-10 **New Connections between Dynamical Systems and PDEs**, AIM Research Conference Center, Palo Alto, California.

Topics: This workshop will explore new connections between dynamical systems and PDEs. Specific topics include emerging connections between Mather sets and viscosity solutions of nonlinear PDEs, recent progress on PDE versions of Aubrey-Mather theory, and KAM theory for dynamical systems and its PDE analogues.

Organizers: L. C. Evans and P. Rabinowitz.

Deadline: March 21, 2003.

Information: <http://aimath.org/ARCC/workshops/dynpde.html>.

*6-19 **Computational Noncommutative Algebra and Applications**, NATO Advanced Study Institute, Il Ciocco Resort, Tuscany, Italy.

Description: Computational algebra has emerged as a key tool to handle the most demanding applications of signal and image

processing in remote sensing, computer vision, medical image processing, and biological signal processing. This ASI will allow a new generation of mathematicians and engineers to share the insights of some of the outstanding contributors to the state of the art.

Organizer: J. Byrnes, asi@prometheus-inc.com.

Principal Speakers: Y. Aloimonos, Univ. of Maryland, Action representations and harmonic computational geometry; W. Baylis, Univ. of Windsor, Canada, The geometry of paravector space, with applications to relativistic physics and The quantum/classical interface: a geometric approach from the classical side; T. Beth, Univ. Karlsruhe, Germany, Group algebraic approach to algorithm engineering; J. Byrnes, Prometheus Inc., USA, Algebraic structures of certain complementary sequences and their wireless communications ramifications; P. Cannarsa, Univ. di Roma 'Tor Vergata', Italy, Algebraic methods in control theory: an overview; M. Clausen, Univ. Bonn, Germany, A group theoretical approach to content-based multimedia information retrieval; A. Dress, Univ. Bielefeld, Germany, In a vector space, all bases are equal, but some are more equal; T. Havel, MIT, USA, Representations of quantum operations in geometric algebra; V. Labunets, Urals State Technical Univ., Russia, Toward realization of the 'Erlangen Program' for classical and quantum theory of signals/systems on groups and hypergroups and Applications of Clifford algebras to multispectral image processing and recognition; J. Lasenby, Cambridge Univ., UK, Using Clifford/geometric algebra in robotics; W. Moran, Prometheus Inc., USA, Group theory in radar and signal processing; N. Nikolski, V. A. Steklov Mathematical Institute, Russia, Harmonic analysis methods for infinite dimensional dynamic systems; B. Saffari, Univ. Paris-Sud, France, Complex-valued Golay pairs of complementary sequences; J. Selig, South Bank Univ., UK, Lie Groups and Lie algebras in robotics; G. Sobczyk, Univ. de las Americas, Mexico, Clifford geometric algebras in multilinear algebra and non-Euclidean geometries; G. Sommer, Christian-Albrechts-Univ. zu Kiel, Germany, The Clifford algebra approach to robotic vision; R. Tolimieri, Prometheus Inc., USA, Group theoretic methods in image processing.

Deadline: Applications and abstracts for poster sessions are due May 1, 2003.

Funding: Limited funding is available.

Information: Visit <http://www.prometheus-inc.com/asi/algebra2003/>.

* 6-23 **33rd Probability Summer School**, Saint-Flour, France.

Program: Three 15-hour courses will be given. The speakers this year will be A. Dembo, "Multiscale occupation analysis: Favourite points, cover times and fractals"; T. Funaki, "Stochastic interface models"; P. Massart, "Concentration inequalities and model selection".

Deadline: For registration: April 4. Registration can be made online on our website.

Organizer: J. Picard, Laboratoire de Mathématiques Appliquées, Univ. Blaise Pascal, 63177 Aubiere, France.

Information: Further information can be obtained at <http://www.lma.univ-bpclermont.fr/stflour/>; email: stflour@math.univ-bpclermont.fr.

* 7-18 **Advanced School in Basic Algebraic Geometry**, The Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy.

Organizer: L. Göttsche.

Directors: L. Göttsche (ICTP); C. S. Seshadri (Chennai Mathematical Institute, Chennai, India); A. Vistoli (Univ. di Bologna, Italy).

Information: <http://agenda.ictp.trieste.it/smr.php?1487/>.

August 2003

* 4-9 **Holomorphic Curves in Contact Geometry**, AIM Research Conference Center, Palo Alto, California.

Topics: This workshop, sponsored by AIM and the NSF, will be devoted to the development of holomorphic curve techniques in contact geometry and topology. The advent of holomorphic curve

techniques in contact topology, as exemplified in Symplectic Field Theory (SFT), and asymptotically holomorphic curve techniques, in the spirit of Donaldson, has allowed one to use a diverse set of geometric, analytic, and topological tools when studying contact structures. The goal of this workshop is to expose, develop, and apply these tools.

Organizers: Y. Eliashberg and J. Etnyre.

Deadline: March 28, 2003.

Information: Visit <http://aimath.org/ARCC/workshops/contactgeom2.html>.

* 4-22 **Summer School and Conference on Real Algebraic Geometry and Its Applications**, The Abdus Salam International Centre for Theoretical Physics (ICTP), Trieste, Italy.

Directors: F. Broglia (Pisa, Italy); K. Kurdyka (Chambery, France); M. F. Roy (Rennes, France); C. Traverso (Pisa, Italy).

Deadline: March 15, 2003.

Information: <http://agenda.ictp.trieste.it/smr.php?1537/>.

* 6-9 **Workshop, Spectral Geometry of Manifolds with Boundary and Decomposition of Manifolds**, Field Laboratory Søminestation at Holbæk Bay near Copenhagen, Roskilde University, Denmark.

Topics: Well-posed boundary value problems for operators of Dirac type, spectral invariants in global analysis, gluing formulas.

Organizers: B. Booss-Bavnbek (Roskilde), G. Grubb (Copenhagen), K. P. Wojciechowski (IUPUI, Indianapolis).

Participants: B. Booss-Bavnbek, G. Grubb, Y. Lee, M. Lesch, P. Loya, R. Nest, V. Nistor, J. Park, P. Piazza, A. Savin, E. Schrohe, D. Vassilevich, K. P. Wojciechowski, C. Zhu.

Contact: email: workshop@mmf.ruc.dk.

Information: <http://mmf.ruc.dk/conf/Workshop/>.

* 12-16 **Conformal Structure in Geometry, Analysis, and Physics**, AIM Research Conference Center, Palo Alto, California.

Topics: This workshop, sponsored by AIM and the NSF, will be devoted to differential invariants of conformal and analogous structures and their applications in geometric analysis and physics. The main questions to be addressed concern solidifying the link between invariant differential operators on the one hand and curvature prescription problems, sharp inequalities, scattering theory, overdetermined systems, and other important analytic problems on the other. We especially hope to advance the understanding of the Q-curvature and to set an agenda for communication among geometers, analysts, and physicists.

Organizers: T. Branson, M. Eastwood, A. R. Gover, and M. Wang.

Deadline: April 4, 2003.

Information: <http://aimath.org/ARCC/workshops/confstruct.html>.

September 2003

* 15-21 **International Conference on Nonlinear Partial Differential Equations**, Alushta, the Crimea, Ukraine.

Topics: 1. Qualitative properties of solutions of nonlinear elliptic and parabolic equations. 2. Degenerate nonlinear elliptic and parabolic equations and their applications in mathematical physics. 3. Blow-up and singularities for quasilinear elliptic and parabolic equations. 4. Homogenization problems for nonlinear PDE. 5. Free boundary problems

Visas: All foreign nationals coming to the Ukraine should have a valid passport and most will need a visa. If you need a visa, we advise you to apply for one at a Ukrainian consular office in your country.

Information: Contacts: A. A. Kovalevsky, Institute of Applied Mathematics & Mechanics of NAS of Ukraine, R. Luxemburg St. 74 83114 Donetsk, Ukraine Phone: 38(0622)552394 Fax: 38(0622)552265; email: NPDE2003@iamm.ac.donetsk.ua; <http://www.iamm.ac.donetsk.ua/mmmain.html>.

* 30-October 3 **International Silk Road Conference: Quantum Theory, Partial Differential Equations of Mathematical Physics and Their Applications**, Tashkent, Uzbekistan.

Organizer: Inst. of Nuclear Physics, Uzbekistan Acad. of Sciences, Tashkent; The INTAS-Network Project Nr.15,2000, entitled "Partial Differential Equations Modelling Semiconductors"; The Wolfgang Pauli Institute, Vienna, <http://www.wpi.ac.at/>; The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy.

Topics: (1) Nonlinear Differential Equations of Mathematical Physics, (2) Inverse problems, (3) Kinetic equations and Problems of Statistical Physics, (4) Mathematical Modelling of Semiconductors, (5) Fluid Dynamics, (6) Spectral properties of Hamiltonians, (7) Quantum Semiconductor Device Models and Nanotechnology.

Language: English at plenary sessions and both English and Russian at parallel sessions.

Deadlines: Applications for financial support: April 15, 2003. Registration form: May 1, 2003. Abstract: June 1, 2003. The second information bulletin: May 1, 2003. The conference fee: September 1, 2003.

Information: Principal Contacts: Further information about the conference may be obtained from: M. Yu. Rasulova, Institute of Nuclear Physics, Ulugbek, Tashkent, 702132, Uzbekistan; tel.: +998712/(3712) 60 67 53; fax: +998712/(3712) 64 25 90; email: qtpm2003@suninp.tashkent.su or wittgenstein.mathematik@univie.ac.at.

October 2003

* 2-4 **AMS Joint Central and Western Section Meeting**, University of Colorado, Boulder, Colorado

Information: <http://www.ams.org/amsmtgs/sectional.html>.

April 2004

* 3-4 **AMS Western Section Meeting**, University of Southern California, Los Angeles, California.

Information: <http://www.ams.org/amsmtgs/sectional.html>.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

June 2004

* 2-4 **ICNPAA 2004: Mathematical Problems in Engineering and Aerospace Sciences**, The West University of Timisoara, Romania.

Scope: Includes mathematical problems in all areas of engineering and aerospace sciences.

Organizers: S. Sivasundaram (general chair); S. Balint (local organizing chair).

Sponsors: IFNA, IFIP, IEEE, AIAA.

Deadlines: 1. Organizing Special Session (the title of the session, name of the organizers): June 30, 2003; to send the title of the talks and speakers: November 30, 2003. 2. For abstracts of the talks: January 30, 2004; full papers for the proceedings: July 15, 2004.

Contact: ICNPAA 2004, 104, Snow Goose Ct., Daytona Beach, FL 32119; email: SeenithI@aol.com.

Information: <http://www.icnpaa.com/>.

September 2004

* 1-6 (REVISED) **Sixth Pan-African Congress of Mathematicians**, Institute National des Sciences Appliquees et del la Technologie (INSAT), Université 7 Novembre à Carthage, Tunis, Tunisia.

Theme: Mathematical Sciences and the Development of Africa—Challenges for Building a Knowledge Society in Africa. The scientific program will include plenary lectures, invited lectures, contributed research papers, a symposium, and exhibitions.

Contact: Those interested in speaking at or participating in the congress are invited to contact: A. Boukricha, local organizing committee, Université de Tunis EL Manar Departement de Mathématiques, Faculté des Sciences De Tunis, 1060 Tunis, Tunisia; email: aboukricha@fst.rnu.tn.

Information: Please submit curriculum vitae and abstract to: J. Persens, Pres., African Mathematical Union, Univ. of the Western Cape, Private Bag X17, Belville 7535, South Africa; jpersens@uwc.ac.za; and copies to: J.-P. Ezin, Sec. General, African Mathematical Union, Institut de Mathematiques et de Sciences Physiques, BP613, Porto Novo, Benin; jpezin@syfed.bj.refer.org.

October 2004

* 16-17 **AMS Southeastern Section Meeting**, Vanderbilt University, Nashville, Tennessee.

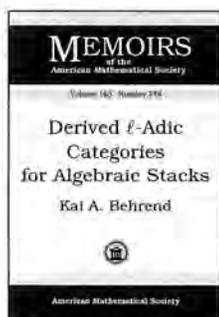
Information: <http://www.ams.org/amsmtgs/sectional.html>.

* 16-17 **AMS Western Section Meeting**, University of New Mexico, Albuquerque, New Mexico.

Information: <http://www.ams.org/amsmtgs/sectional.html>.

New Publications Offered by the AMS

Algebra and Algebraic Geometry



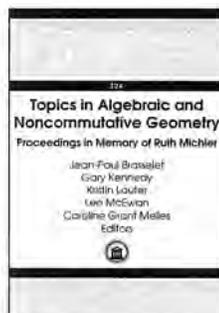
Derived ℓ -Adic Categories for Algebraic Stacks

Kai A. Behrend, *University of British Columbia, Vancouver, Canada*

Contents: Introduction; The ℓ -adic formalism; Stratifications; Topoi; Algebraic stacks; Convergent complexes; Bibliography.

Memoirs of the American Mathematical Society, Volume 163, Number 774

May 2003, 93 pages, Softcover, ISBN 0-8218-2929-7, LC 2003040432, 2000 *Mathematics Subject Classification*: 14D20, 18Gxx, **Individual member \$31**, List \$51, Institutional member \$41, Order code MEMO/163/774N



Topics in Algebraic and Noncommutative Geometry

Proceedings in Memory of Ruth Michler

Caroline Grant Melles, *U.S. Naval Academy, Annapolis, MD*, **Jean-Paul Brasselet**, *CNRS, Marseille, France*,

Gary Kennedy, *Ohio State University, Mansfield*, **Kristin Lauter**, *Microsoft Corporation, Redmond, WA*, and **Lee McEwan**, *Ohio State University, Mansfield*, Editors

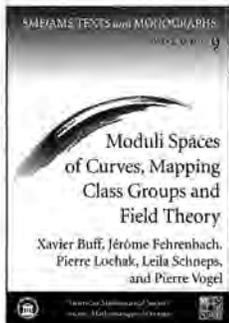
This book presents the proceedings of two conferences, Résolution des singularités et géométrie non commutative and the Annapolis algebraic geometry conference. Research articles in

the volume cover various topics of algebraic geometry, including the theory of Jacobians, singularities, applications to cryptography, and more. The book is suitable for graduate students and research mathematicians interested in algebraic geometry.

Contents: **A. Iarrobino, Jr.**, Dr. Ruth I. Michler's research; **S. S. Abhyankar** and **A. Assi**, Jacobian pairs; **P. Aluffi**, Inclusion-exclusion and Segre classes, II; **G. Bérczi**, **L. M. Fehér**, and **R. Rimányi**, Expressions for resultants coming from the global theory of singularities; **D. Boneh** and **A. Silverberg**, Applications of multilinear forms to cryptography; **A. Campillo** and **J. Castellanos**, On Puiseux exponents for higher dimensional singularities; **V. Cossart**, Desingularization: A few bad examples in dim. 3, characteristic $p > 0$; **P. D. González Pérez**, **L. J. McEwan**, and **A. Némethi**, The zeta-function of a quasi-ordinary singularity II; **E. Hironaka**, Lehmer's problem, McKay's correspondence, and 2,3,7; **V. P. Kostov** and **O. Gabber**, On the weak Deligne-Simpson problem for index of rigidity 2; **A. E. Ksir** and **S. G. Naculich**, Elliptic fibrations and elliptic models; **K. E. Lauter**, The equivalence of the geometric and algebraic group laws for Jacobians of genus 2 curves; **D. B. Massey**, Invariant subspaces of the monodromy; **L. J. McEwan** and **A. Némethi**, Some conjectures about quasi-ordinary singularities; **A. Simis**, Two differential themes in characteristic zero; **M. A. Vitulli**, Some normal monomial ideals; **J. F. Voloch**, Surfaces in P^3 over finite fields; **A. Yekutieli**, The continuous Hochschild cochain complex of a scheme (Survey).

Contemporary Mathematics, Volume 324

June 2003, approximately 257 pages, Softcover, ISBN 0-8218-3209-3, LC 2003043689, 2000 *Mathematics Subject Classification*: 14-06; 14B05, **All AMS members \$55**, List \$69, Order code CONM/324N



Moduli Spaces of Curves, Mapping Class Groups and Field Theory

Xavier Buff, *Université Paul Sabatier, Toulouse, France*,
 Jérôme Fehrenbach, *University of Nice Sophia-Antipolis, Valbonne, France*,

Pierre Lochak, *Centre de Mathématiques de Jussieu, Université Paris VI*, Leila Schneps, *Université Paris VI*, and Pierre Vogel, *Université Paris VII*

From a review of the French edition:

A collective monograph dedicated to the new and profound relations between various theories previously considered as unrelated ... A specific feature of the book, which distinguishes it from many other monographs and textbooks on the same subjects, is its nature of a "guide for the non-specialist" ... it also contains full proofs of some results difficult to find elsewhere ... Examples are studied in great detail ... Recommended as a first reading for a non-specialist who wants to get acquainted with the subject but who does not want to get lost in its many intricacies and ramifications.

—*Mathematical Reviews*

This is a collection of articles that grew out of a workshop organized to discuss deep links among various topics that were previously considered unrelated. Rather than a typical workshop, this gathering was unique as it was structured more like a course for advanced graduate students and research mathematicians.

In the book, the authors present applications of moduli spaces of Riemann surfaces in theoretical physics and number theory and on Grothendieck's dessins d'enfants and their generalizations. Chapter 1 gives an introduction to Teichmüller space that is more concise than the popular textbooks, yet contains full proofs of many useful results which are often difficult to find in the literature. This chapter also contains an introduction to moduli spaces of curves, with a detailed description of the genus zero case, and in particular of the part at infinity. Chapter 2 takes up the subject of the genus zero moduli spaces and gives a complete description of their fundamental groupoids, based at tangential base points neighboring the part at infinity; the description relies on an identification of the structure of these groupoids with that of certain canonical subgroupoids of a free braided tensor category. It concludes with a study of the canonical Galois action on the fundamental groupoids, computed using Grothendieck-Teichmüller theory. Finally, Chapter 3 studies strict ribbon categories, which are closely related to braided tensor categories: Here they are used to construct invariants of 3-manifolds which in turn give rise to quantum field theories. The material is suitable for advanced graduate students and researchers interested in algebra, algebraic geometry, number theory, and geometry and topology.

This item will also be of interest to those working in number theory and geometry and topology.

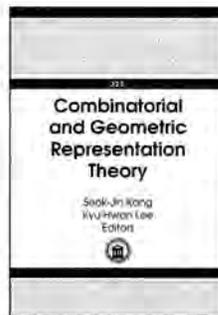
SMF members are entitled to AMS member discounts.

Contents: X. Buff, J. Fehrenbach, and P. Lochak, Elements of the geometry of moduli spaces of curves; L. Schneps, Funda-

mental groupoids of genus zero moduli spaces and braided tensor categories; P. Vogel, Witten-Reshetikhin-Turaev invariants and quantum field theories.

SMF/AMS Texts and Monographs, Volume 9

June 2003, 131 pages, Softcover, ISBN 0-8218-3167-4, 2000 *Mathematics Subject Classification:* 32G15, 20F34, 11R32, 20F36, All AMS members \$35, List \$44, Order code SMFAMS/9N



Combinatorial and Geometric Representation Theory

Seok-Jin Kang, *Korea Institute for Advanced Study, Seoul*, and Kyu-Hwan Lee, *University of Toronto, ON, Canada*, Editors

This volume presents the proceedings of the international conference on Combinatorial and Geometric Representation Theory. In the field of representation theory, a wide variety of mathematical ideas are providing new insights, giving powerful methods for understanding the theory, and presenting various applications to other branches of mathematics. Over the past two decades, there have been remarkable developments. This book explains the strong connections between combinatorics, geometry, and representation theory. It is suitable for graduate students and researchers interested in representation theory.

Contents: H. H. Andersen, Twisted Verma modules and their quantized analogues; S. Ariki, On tameness of the Hecke algebras of type B ; G. Benkart and D. Moon, Tensor product representations of Temperley-Lieb algebras and their centralizer algebras; J. F. Carlson, Z. Lin, D. K. Nakano, and B. J. Parshall, The restricted nullcone; W. J. Haboush, Projective embeddings of varieties of special lattices; G. James, Representations of general linear groups; S.-J. Kang and J.-H. Kwon, Fock space representations for the quantum affine algebra $U_q(C_2^{(1)})$; M. Kashiwara, Realizations of crystals; H. Nakajima, t -analogs of q -characters of quantum affine algebras of type A_n, D_n ; A. Ram, Skew shape representations are irreducible.

Contemporary Mathematics, Volume 325

May 2003, 189 pages, Softcover, ISBN 0-8218-3212-3, LC 2002041753, 2000 *Mathematics Subject Classification:* 05Exx, 14Lxx, 16Gxx, 17Bxx, 20Cxx, 20Gxx, 81Rxx, All AMS members \$39, List \$49, Order code CONM/325N



Bases cristallines des groupes quantiques

Masaki Kashiwara, *Research Institute for the Mathematical Sciences, Kyoto University, Japan*

Since their introduction by Drinfeld and Jimbo in 1985 in the studies of exactly solvable models, quantum enveloping algebras have been one of the most important tools to describe

new symmetries.

For $q = 0$, there is a good base (the so-called *crystal base*) of the representation of a quantum enveloping algebra $U_q(\mathfrak{g})$ of a semi-simple Lie algebra \mathfrak{g} . A modified action of root vectors sends the crystal base to itself, thus providing a rich combinatorial structure. Therefore one can reduce many properties of representation to the combinatorics of crystal bases.

In this book, the author presents crystal bases and their applications to multiplicities and weights of the tensor products of two representations.

A publication of the Société Mathématique de France. Distributed by the AMS in North America. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: Représentations de l'algèbre quantique $U_q(\mathfrak{sl}_2)$; Bases cristallines des $U_q(\mathfrak{sl}_2)$ -modules; L'algèbre enveloppante quantique $U_q(\mathfrak{g})$; Bases cristallines des $U_q(\mathfrak{g})$ -modules; Cas de \mathfrak{gl}_n ; Bases globales des $U_q(\mathfrak{g})$ -modules; Base cristalline $B(\infty)$ de l'algèbre $U_q^-(\mathfrak{g})$; Réalisation des bases cristallines par des chemins; Cristaux et groupe de Weyl; Bibliographie; Index des notations; Index terminologique.

Cours Spécialisés—Collection SMF, Number 9

January 2003, 115 pages, Softcover, ISBN 2-85629-126-0, 2000 *Mathematics Subject Classification:* 17B37, **Individual member \$30, List \$33, Order code COSP/9N**

Recommended Text

Independent Study

Algebraic Geometry 3 Expansion of Scheme Theory

Kenji Ueno, *Kyoto University, Japan*

Algebraic geometry plays an important role in several branches of science and technology. This is the third of three volumes by Kenji Ueno on scheme theory, the most natural form of algebraic

geometry. This, in addition to *Algebraic Geometry 1* and *Algebraic Geometry 2*, makes an excellent textbook for a second course in algebraic geometry.

In this volume, the author goes beyond introductory notions and presents the theory of schemes and sheaves with the goal of studying the properties necessary for the full development of modern algebraic geometry. The main topics discussed in the book include dimension theory, flat and proper morphisms, regular schemes, smooth morphisms, completion, and Zariski's

main theorem. Ueno also presents the theory of algebraic curves and their Jacobians, and the relation between algebraic and analytic geometry, including Kodaira's Vanishing Theorem.

The book contains numerous exercises and problems with solutions. It is suitable for a graduate course on algebraic geometry or for independent study.

Contents: Fundamental properties of scheme theory; Algebraic curves and Jacobi varieties; Algebraic geometry and analytic geometry; Overview and references; Solutions to problems; Solutions to exercises; Index.

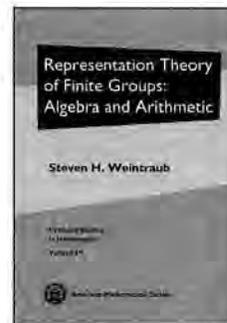
Translations of Mathematical Monographs (Iwanami Series in Modern Mathematics), Volume 218

July 2003, approximately 240 pages, Softcover, ISBN 0-8218-1358-7, LC 99-22304, 2000 *Mathematics Subject Classification:* 14-01, **All AMS members \$31, List \$39, Order code MMONO/218N**

Recommended Text

Representation Theory of Finite Groups: Algebra and Arithmetic

Steven H. Weintraub, *Lehigh University, Bethlehem, PA*



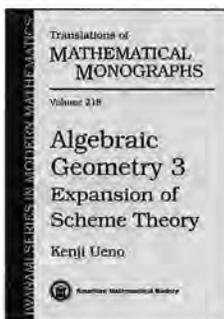
We explore widely in the valley of ordinary representations, and we take the reader over the mountain pass leading to the valley of modular representations, to a point from which (s)he can survey this valley, but we do not attempt to widely explore it. We hope the reader will be sufficiently fascinated by the scenery to further explore both valleys on his/her own.

—from the Preface

Representation theory plays important roles in geometry, algebra, analysis, and mathematical physics. In particular, representation theory has been one of the great tools in the study and classification of finite groups. There are some beautiful results that come from representation theory: Frobenius's Theorem, Burnside's Theorem, Artin's Theorem, Brauer's Theorem—all of which are covered in this textbook. Some seem uninspiring at first, but prove to be quite useful. Others are clearly deep from the outset. And when a group (finite or otherwise) acts on something else (as a set of symmetries, for example), one ends up with a natural representation of the group.

This book is an introduction to the representation theory of finite groups from an algebraic point of view, regarding representations as modules over the group algebra. The approach is to develop the requisite algebra in reasonable generality and then to specialize it to the case of group representations. Methods and results particular to group representations, such as characters and induced representations, are developed in depth. Arithmetic comes into play when considering the field of definition of a representation, especially for subfields of the complex numbers. The book has an extensive development of the semisimple case, where the characteristic of the field is zero or is prime to the order of the group, and builds the foundations of the modular case, where the characteristic of the field divides the order of the group.

The book assumes only the material of a standard graduate course in algebra. It is suitable as a text for a year-long graduate course. The subject is of interest to students of algebra, number



theory and algebraic geometry. The systematic treatment presented here makes the book also valuable as a reference.

Contents: Introduction; Semisimple rings and modules; Semi-simple group representations; Induced representations and applications; Introduction to modular representations; General rings and modules; Modular group representations; Some useful results; Bibliography; Index.

Graduate Studies in Mathematics, Volume 59

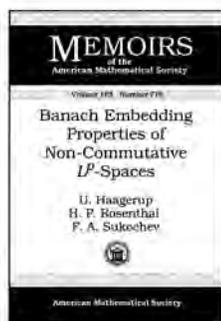
July 2003, approximately 232 pages, Hardcover, ISBN 0-8218-3222-0, 2000 *Mathematics Subject Classification*: 20C05, 20C15, 20C20; 20-01, **All AMS members \$36**, List \$45, Order code GSM/59N

for 0-pseudodifferential operators; *Part 2. Algebras of 0-pseudodifferential operators of order 0: C^* -algebras of 0-pseudodifferential operators; Ψ^* -algebras of 0-pseudodifferential operators; Appendix A. Spaces of conormal functions; Bibliography; Notations; Index.*

Memoirs of the American Mathematical Society, Volume 163, Number 777

May 2003, 92 pages, Softcover, ISBN 0-8218-3272-7, LC 2003040426, 2000 *Mathematics Subject Classification*: 58J40, 58J05, 58J35, 47G30, 46K10, 46L45, **Individual member \$31**, List \$51, Institutional member \$41, Order code MEMO/163/777N

Analysis



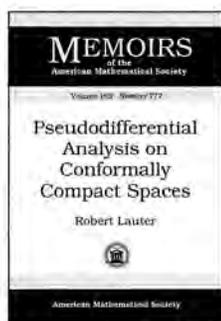
Banach Embedding Properties of Non-Commutative L^p -Spaces

U. Haagerup, *SDU Odense University, Denmark*,
H. P. Rosenthal, *University of Texas, Austin*, and
F. A. Sukochev, *Flinders University of South Australia, Adelaide*

Contents: Introduction; The modulus of uniform integrability and weak compactness in $L^1(\mathcal{N})$; Improvements to the main theorem; Complements on the Banach/operator space structure of $L^p(\mathcal{N})$ -spaces; The Banach isomorphic classification of the spaces $L^p(\mathcal{N})$ for \mathcal{N} hyperfinite semi-finite; $L^p(\mathcal{N})$ -isomorphism results for \mathcal{N} a type III hyperfinite or a free group von Neumann algebra; Bibliography.

Memoirs of the American Mathematical Society, Volume 163, Number 776

May 2003, 68 pages, Softcover, ISBN 0-8218-3271-9, LC 2003040431, 2000 *Mathematics Subject Classification*: 46B20, 46L10, 46L52, 47L25, **Individual member \$27**, List \$45, Institutional member \$36, Order code MEMO/163/776N

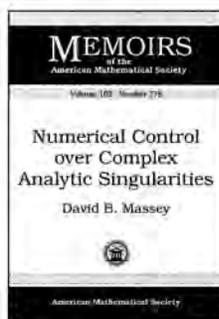


Pseudodifferential Analysis on Conformally Compact Spaces

Robert Lauter, *University of Mainz, Germany*

Contents: *Part 1. Fredholm theory for 0-pseudodifferential operators:* Review of basic objects of 0-geometry; The small 0-calculus and the 0-calculus

with bounds; The b - c -calculus on an interval; The reduced normal operator; Weighted 0-Sobolev spaces; Fredholm theory



Numerical Control over Complex Analytic Singularities

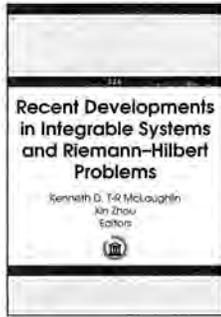
David B. Massey,
Northeastern University, Boston

This item will also be of interest to those working in geometry and topology.

Contents: Overview; *Part I. Algebraic Preliminaries: Gap Sheaves and Vogel Cycles:* Introduction; Gap sheaves; Gap cycles and Vogel cycles; The Lê-Iomdine-Vogel formulas; Summary of Part I; *Part II. Lê Cycles and Hypersurface Singularities:* Introduction; Definitions and basic properties; Elementary examples; A handle decomposition of the Milnor fibre; Generalized Lê-Iomdine formulas; Lê numbers and hyperplane arrangements; Thom's a_f condition; Aligned singularities; Suspending singularities; Constancy of the Milnor fibrations; Another characterization of the Lê cycles; *Part III. Isolated Critical Points of Functions on Singular Spaces:* Introduction; Critical avatars; The relative polar curve; The link between the algebraic and topological points of view; The special case of perverse sheaves; Thom's a_f condition; Continuous families of constructible complexes; *Part IV. Non-Isolated Critical Points of Functions on Singular Spaces:* Introduction; Lê-Vogel cycles; Lê-Iomdine formulas and Thom's condition; Lê-Vogel cycles and the Euler characteristic; Appendix A. Analytic cycles and intersections; Appendix B. The derived category; Appendix C. Privileged neighborhoods and lifting Milnor fibrations; References; Index.

Memoirs of the American Mathematical Society, Volume 163, Number 778

May 2003, 268 pages, Softcover, ISBN 0-8218-3280-8, LC 2003040369, 2000 *Mathematics Subject Classification*: 32B15, 32C35, 32C18, 32B10, **Individual member \$43**, List \$72, Institutional member \$58, Order code MEMO/163/778N



Recent Developments in Integrable Systems and Riemann-Hilbert Problems

Kenneth D. T-R McLaughlin,
*University of North Carolina,
Chapel Hill and University of
Arizona, Tucson*, and
Xin Zhou, *Duke University,
Durham, NC*, Editors

This volume is a collection of papers presented at a special session on integrable systems and Riemann-Hilbert problems. The goal of the meeting was to foster new research by bringing together experts from different areas. Their contributions to the volume provide a useful portrait of the breadth and depth of integrable systems.

Topics covered include discrete Painlevé equations, integrable nonlinear partial differential equations, random matrix theory, Bose-Einstein condensation, spectral and inverse spectral theory, and last passage percolation models. In most of these articles, the Riemann-Hilbert problem approach plays a central role, which is powerful both analytically and algebraically.

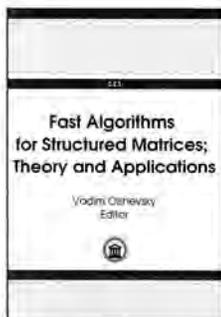
The book is intended for graduate students and researchers interested in integrable systems and its applications.

Contents: **J. Baik**, Riemann-Hilbert problems for last passage percolation; **R. Beals**, **D. H. Sattinger**, and **J. Szmigielski**, Inverse scattering and some finite-dimensional integrable systems; **D. J. Kaup** and **H. Steudel**, Recent results on second harmonic generation; **M. Kovalyov** and **A. H. Vartanian**, On long-distance intensity asymptotics of solutions to the Cauchy problem for the modified nonlinear Schrödinger equation for vanishing initial data; **W. M. Liu** and **S. T. Chui**, Integrable models in Bose-Einstein condensates; **A. H. Vartanian**, Long-time asymptotics of solutions to the Cauchy problem for the defocusing non-linear Schrödinger equation with finite-density initial data. I. Solitonless sector.

Contemporary Mathematics, Volume 326

May 2003, approximately 198 pages, Softcover, ISBN 0-8218-3203-4, 2000 *Mathematics Subject Classification*: 35Q15, 35Q51, 35Q53, 35Q55, 35P25, 15A52, 05E10, 05E15, 34E05, All AMS members \$39, List \$49, Order code CONM/326N

Applications



Fast Algorithms for Structured Matrices; Theory and Applications

Vadim Olshevsky, *University of Connecticut, Storrs*, Editor

One of the best known fast computational algorithms is the fast Fourier transform method. Its efficiency is

based mainly on the special structure of the discrete Fourier transform matrix. Recently, many other algorithms of this

type were discovered, and the theory of structured matrices emerged.

This volume contains 22 survey and research papers devoted to a variety of theoretical and practical aspects of the design of fast algorithms for structured matrices and related issues. Included are several papers containing various affirmative and negative results in this direction. The theory of rational interpolation is one of the excellent sources providing intuition and methods to design fast algorithms. The volume contains several computational and theoretical papers on the topic. There are several papers on new applications of structured matrices, e.g., to the design of fast decoding algorithms, computing state-space realizations, relations to Lie algebras, unconstrained optimization, solving matrix equations, etc.

The book is suitable for mathematicians, engineers, and numerical analysts who design, study, and use fast computational algorithms based on the theory of structured matrices.

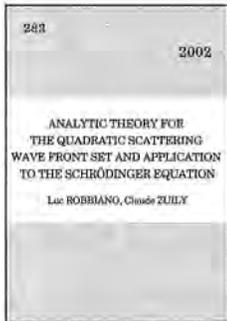
This volume is a joint publication of the American Mathematical Society and the Society for Industrial and Applied Mathematics.

Contents: **V. Olshevsky**, Pivoting for structured matrices and rational tangential interpolation; **G. Heinig**, Inversion of Toeplitz-plus-Hankel matrices with arbitrary rank profile; **D. Fasino** and **L. Gemignani**, A Lanczos-type algorithm for the QR factorization of Cauchy-like matrices; **D. Fasino**, **N. Mastronardi**, and **M. Van Barel**, Fast and stable algorithms for reducing diagonal plus semiseparable matrices to tridiagonal and bidiagonal form; **A. Olshevsky**, **V. Olshevsky**, and **J. Wang**, A comrade-matrix-based derivation of the eight versions of fast cosine and sine transforms; **D. A. Bini**, **L. Gemignani**, and **B. Meini**, Solving certain matrix equations by means of Toeplitz computations: algorithms and applications; **F. T. Luk** and **S. Qiao**, A fast singular value algorithm for Hankel matrices; **D. Calvetti**, **L. Reichel**, and **F. Sgallari**, A modified companion matrix method based on Newton polynomials; **J. Hendrickx**, **R. Vandebril**, and **M. Van Barel**, A fast direct method for solving the two-dimensional Helmholtz equation, with Robbins boundary conditions; **C. Di Fiore**, Structured matrices in unconstrained minimization methods; **N. Ito**, **W. Schmale**, and **H. K. Wimmer**, Computation of minimal state space realizations in Jacobson normal form; **A. Mayo**, High order accurate particular solutions of the biharmonic equation on general regions; **T. Wen**, **A. Edelman**, and **D. Gorsich**, A fast projected conjugate gradient algorithm for training support vector machines; **V. Olshevsky** and **M. A. Shokrollahi**, A displacement approach to decoding algebraic codes; **M. Bollhöfer** and **V. Mehrmann**, Some convergence estimates for algebraic multilevel preconditioners; **D. Noutsos**, **S. S. Capizzano**, and **P. Vassalos**, Spectral equivalence and matrix algebra preconditioners for multilevel Toeplitz systems: a negative result; **W. F. Trench**, Spectral distribution of Hermitian Toeplitz matrices formally generated by rational functions; **D. Fasino** and **S. S. Capizzano**, From Toeplitz matrix sequences to zero distribution of orthogonal polynomials; **K. R. Driessel**, On Lie algebras, submanifolds and structured matrices; **H. Dym**, Riccati equations and bitangential interpolation problems with singular Pick matrices; **V. Bolotnikov**, **A. Kheifets**, and **L. Rodman**, Functions with Pick matrices having bounded number of negative eigenvalues; **Yu. M. Arlinskiĭ**, **S. Hassi**, **H. S. V. de Snoo**, and **E. R. Tsekanovskiĭ**, One-dimensional perturbations of selfadjoint operators with finite or discrete spectrum.

Contemporary Mathematics, Volume 323

June 2003, approximately 448 pages, Softcover, ISBN 0-8218-3177-1, 2000 *Mathematics Subject Classification*: 68Q25, 65Y20, 65F05, 65F10, 65G50, 65M12, 15A57, 15A18, 47N70, 47N40, All AMS members \$69, List \$99, Order code CONM/323N

Differential Equations



Analytic Theory for the Quadratic Scattering Wave Front Set and Application to the Schrödinger Equation

Luc Robbiano, *CNRS, Université de Versailles, France*, and Claude Zuily, *Université de Paris-Sud, Orsay*

In this book, the authors consider the microlocal propagation of analytic singularities for the solutions of the Schrödinger equation with variable coefficients. Following R. Melrose and J. Wunsch, they introduce a \mathbb{R}^n compactification and a cotangent compactification. They define by FBI transform an analytic wave front set on this cotangent bundle. The main part of this paper is to prove the propagation of microlocal analytic singularities in this wave front set.

A publication of the Société Mathématique de France. Distributed by the AMS in North America. Orders from other countries should be sent to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: Introduction; The geometrical context; The analytic *qsc* wave front set; The Laplacian and its flow; Statements of the main results and reductions; Proof of Theorem 4.6; Proof of Theorem 4.7; Proof of Theorem 4.8; Proof of Theorem 4.9; Appendix; Bibliography.

Astérisque, Number 283

January 2003, 128 pages, Softcover, ISBN 2-85629-131-7, 2000 *Mathematics Subject Classification:* 35J10, 35A20, 35A27, 35A18, 35A21, **Individual member \$30**, List \$33, Order code AST/283N

much more current view of important algorithmic developments in intersection graph classes than is currently available and includes a large number of new open problems.

It deals with the questions that arise from storing a graph in a computer. Different classes of graphs admit different forms of computer representations, and focusing on the representations gives a new perspective on a number of problems. For a variety of classes of graphs, the book considers such questions as existence of good representations, algorithms for finding representations, questions of characterizations in terms of representation, and how the representation affects the complexity of optimization problems. General models of efficient computer representations are also considered.

The book is designed to be used both as a text for a graduate course on topics related to graph representation and as a monograph for anyone interested in research in the field of graph representation. The material is of interest both to those focusing purely on graph theory and to those working in the area of graph algorithms.

This item will also be of interest to those working in applications.

Contents: Explanatory remarks; Introduction; Implicit representation; Intersection and containment representations; Real numbers in graph representations; Classes which use global information; Visibility graphs; Intersection of graph classes; Graph classes defined by forbidden subgraphs; Chordal bipartite graphs; Matrices; Decomposition; Elimination schemes; Recognition algorithms; Robust algorithms for optimization problems; Characterization and construction; Applications; Glossary; Survey of results on graph classes; Bibliography; Index.

Fields Institute Monographs, Volume 19

April 2003, 342 pages, Hardcover, ISBN 0-8218-2815-0, 2000 *Mathematics Subject Classification:* 05C62, 05C17, 05C50, 05C85, 05-00, 05-02, 68R10, 68W01, 68P05, 68Q30, 68-01, **All AMS members \$76**, List \$95, Order code FIM/19N

Discrete Mathematics and Combinatorics

Supplementary Reading

Efficient Graph Representations

Jeremy P. Spinrad, *Vanderbilt University, Nashville, TN*

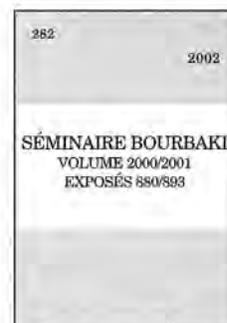
This monograph the first to deal with graph representation as a field of study. It is written from both a mathematical and computer science perspective. Synthesizing the two traditions opens a number of interesting new research areas. Some

individual classes of graphs are important, but are not adequately covered in any current text. This book gives a

General and Interdisciplinary

Séminaire Bourbaki

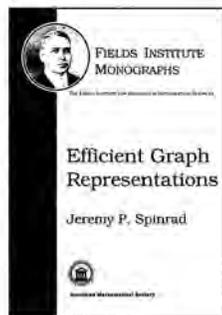
Volume 2000/2001 Exposés 880/893



As in the preceding volumes of this seminar, one finds here fourteen survey lectures on topics of current interest; four lectures on algebraic geometry, two on probability, classical or free, one on Riemannian geometry, one on non-commutative geometry, one on (non-)integrability of Hamiltonian systems, one on *L*-functions

and random matrices, one on Langlands functoriality, one on polylogarithms, one on geometric quantization and one on equations of hydrodynamics. Among the authors are leading French mathematicians P. Cartier, M. Vergne, M. Audin, G. Henniart, and others.

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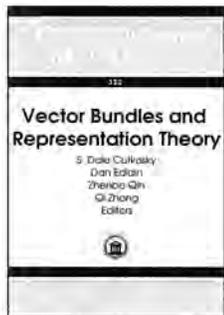
to the SMF, Maison de la SMF, B.P. 67, 13274 Marseille cedex 09, France, or to Institut Henri Poincaré, 11 rue Pierre et Marie Curie, 75231 Paris cedex 05, France. Members of the SMF receive a 30% discount from list.

Contents: *Novembre 2000:* L. Bonavero, Factorisation faible des applications birationnelles; M. Brunella, Courbes entières dans les surfaces algébriques complexes; M. Émery, Espaces probabilisés filtrés: de la théorie de Vershik au mouvement brownien, via des idées de Tsirelson; M. Herzlich, L'inégalité de Penrose; *Mars 2001:* M. Audin, Intégrabilité et non-intégrabilité de systèmes hamiltoniens; P. Cartier, Fonctions polylogarithmes, nombres polyzêtas et groupes pro-unipotents; A. Chambert-Loir, Théorèmes d'algébricité en géométrie diophantienne; P. Michel, Répartition des zéros des fonctions L et matrices aléatoires; M. Vergne, Quantification géométrique et réduction symplectique; *Juin 2001:* P. Biane, Entropie libre et algèbres d'opérateurs; G. Henniart, Progrès récents en fonctorialité de Langlands; E. Peyre, Points de hauteur bornée et géométrie des variétés; G. Skandalis, Géométrie non commutative, opérateur de signature transverse et algèbres de Hopf; C. Villani, Limites hydrodynamiques de l'équation de Boltzmann.

Astérisque, Number 282

December 2002, 443 pages, Softcover, ISBN 2-85629-130-9, 2000 *Mathematics Subject Classification:* 14Exx, 14J29, 32Q45, 37F75, 60G05, 60G25, 60G42, 60G44, 53C21, 58J35, 58J60, 83C30, 83C57, 34-XX, 37Jxx, 37K10, 53Dxx, 70G45, 11J82, 40B05, 17B01, 33E20, 34M35, 14G40, 11Gxx, 11F66, 11M36, 15A52, 22-XX, 53-XX, 46L54, 46L10, 22E55, 22E50, 14G05, 11G35, 16S38, 16W30, 57T05, 76D05, 76P05, **Individual member \$62, List \$69, Order code AST/282N**

Geometry and Topology



Vector Bundles and Representation Theory

S. Dale Cutkosky, Dan Edidin, Zhenbo Qin, and Qi Zhang, *University of Missouri, Columbia*, Editors

This volume contains 13 papers from the conference on "Hilbert Schemes, Vector Bundles and Their Interplay

with Representation Theory". The papers are written by leading mathematicians in algebraic geometry and representation theory and present the latest developments in the field.

Among other contributions, the volume includes several very impressive and elegant theorems in representation theory by R. Friedman and J. W. Morgan, convolution on homology groups of moduli spaces of sheaves on K3 surfaces by H. Nakajima, and computation of the S^1 fixed points in Quot-schemes and mirror principle computations for Grassmanians by S.-T. Yau, et al.

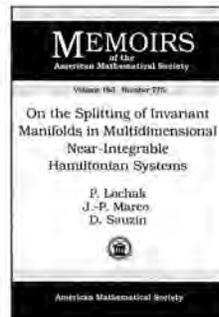
The book is of interest to graduate students and researchers in algebraic geometry, representation theory, topology and their applications to high energy physics.

This item will also be of interest to those working in algebra and algebraic geometry.

Contents: R. Friedman and J. W. Morgan, Minuscule representations, invariant polynomials, and spectral covers; S. Hosono, B. H. Lian, K. Oguiso, and S.-T. Yau, Fourier-Mukai partners of a K3 surface of Picard number one; J. Li, Moduli spaces associated to a singular variety and the moduli of bundles over universal curves; H. Nakajima, Convolution on homology groups of moduli spaces of sheaves on K3 surfaces; W.-P. Li, Z. Qin, and Q. Zhang, Curves in the Hilbert schemes of points on surfaces; X. Wu, Limiting linear subspaces on non-reduced schemes; B. P. Purnaprajna, Geometry of canonical covers of varieties of minimal degree with applications to Calabi-Yau threefolds; W. Wang, Universal rings arising in geometry and group theory; D. Burns, Y. Hu, and T. Luo, HyperKähler manifolds and birational transformations in dimension 4; N. M. Kumar, C. Peterson, and A. P. Rao, Standard vector bundle deformations on \mathbb{P}^n ; B. H. Lian, C.-H. Liu, K. Liu, and S.-T. Yau, The S^1 fixed points in Quot-schemes and mirror principle computations; W. Li, The semi-infinity of Floer (co)homologies; R. Friedman and J. W. Morgan, Automorphism sheaves, spectral covers, and the Kostant and Steinberg sections.

Contemporary Mathematics, Volume 322

May 2003, approximately 256 pages, Softcover, ISBN 0-8218-3264-6, 2000 *Mathematics Subject Classification:* 14C05, 14D20, 14F05, 14J28, 17B10, 20C05, 32L05, 57R99, **All AMS members \$55, List \$69, Order code CONM/322N**



On the Splitting of Invariant Manifolds in Multidimensional Near-Integrable Hamiltonian Systems

P. Lochak and J.-P. Marco, *University of Paris*, and D. Sauzin, *Astronomie et Systems Dynamiques, Paris*

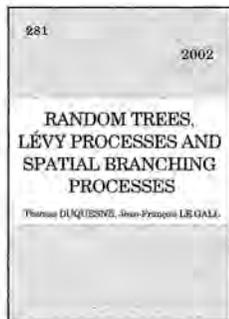
This item will also be of interest to those working in analysis.

Contents: Introduction and some salient features of the model Hamiltonian; Symplectic geometry and the splitting of invariant manifolds; Estimating the splitting matrix using normal forms; The Hamilton-Jacobi method for a simple resonance; Appendix. Invariant tori with vanishing or zero torsion; Bibliography.

Memoirs of the American Mathematical Society, Volume 163, Number 775

May 2003, 145 pages, Softcover, ISBN 0-8218-3268-9, LC 2003040368, 2000 *Mathematics Subject Classification:* 70H08, 70H09, 37J40, 37D10, 34C37, **Individual member \$33, List \$55, Institutional member \$44, Order code MEMO/163/775N**

Probability



Random Trees, Lévy Processes and Spatial Branching Processes

Thomas Duquesne, *Université de Paris-Sud, Orsay*, and **Jean-François Le Gall**, *École Normale Supérieure, Paris*

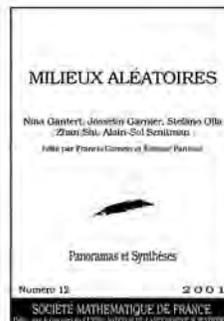
In the book, the authors investigate the genealogical structure of general critical or subcritical continuous-state branching processes. Analogously to the coding of a discrete tree by its contour function, this genealogical structure is coded by a real-valued stochastic process called the height process, which is itself constructed as a local time functional of a Lévy process with no negative jumps. They present a detailed study of the height process and of an associated measure-valued process called the exploration process, which plays a key role in most applications. Under suitable assumptions, it is proven that whenever a sequence of rescaled Galton-Watson processes converges in distribution, their genealogies also converge to the continuous branching structure coded by the appropriate height process. The authors apply this invariance principle to various asymptotics for Galton-Watson trees and then use the duality properties of the exploration process to compute explicitly the distribution of the reduced tree associated with Poissonian marks in the height process and the finite-dimensional marginals of the so-called stable continuous tree. This last calculation generalizes to the stable case a result of Aldous for the Brownian continuum random tree. Finally, they combine the genealogical structure with an independent spatial motion to develop a new approach to superprocesses with a general branching mechanism. In this setting, they derive certain explicit distributions, such as the law of the spatial reduced tree in a domain, consisting of the collection of all historical paths that hit the boundary.

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Contents: Introduction; The height process; Convergence of Galton-Watson trees; Marginals of continuous trees; The Lévy snake; Bibliography; Notation index; Index.

Astérisque, Number 281

October 2002, 146 pages, Softcover, ISBN 2-85629-128-7, 2000 *Mathematics Subject Classification*: 60J80, 60J25, 60G57, 60F17, 60G52, **Individual member \$30**, List \$33, Order code AST/281N



Milieux aléatoires

Nina Gantert, *Universität Karlsruhe, Germany*, **Josselin Garnier**, *Université Paul Sabatier, Toulouse, France*, **Stefano Olla**, *Université de Paris 9-Dauphine, France*, **Zhan Shi**, *Université de Paris VI, France*, and **Alain-Sol Sznitman**, *Mathematisches ETH-Zentrum, Zürich, Switzerland*

Random media are natural models for nonhomogeneous materials which possess some kind of statistical regularity. The study of stochastic processes in random media is currently an active field of research, and new techniques have recently been developed, including mathematical forms of renormalization. Those techniques apply to models which are much more delicate than exactly soluble ones or even reversible ones.

The session, "États de la Recherche", presented the state of the art in the field and brought it to a large portion of the scientific community. Based on the notes of the courses delivered during the session, this volume is composed of five articles and a general introduction, where all basic notions from probability theory are defined. The introduction and the style of the articles make the volume readable by nonspecialists.

The article by Alain Sznitman studies the survival of Brownian motion moving among randomly located obstacles, and the ballistic behavior of the random walk in random media on the $d \geq 2$ -dimensional lattice. This illustrates the role of atypical pockets in the medium and of abnormally small eigenvalues. The second article, by Zhan Shi, presents the analysis via stochastic calculus of Sinai's random walk and of the one dimensional diffusion in a Brownian potential. Nina Gantert studies the random walk on a random Galton-Watson tree, in particular, the probability of rare events. Stefano Olla studies random homogenization, taking the point of view of the environment seen from the particle, as well as applications to interacting particle systems. In the last article, Josselin Garnier studies wave propagation in random media, the competition between nonlinear and random effects, and solitons in this framework.

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Contents: Introduction générale; A.-S. Sznitman, Milieux aléatoires et petites valeurs propres; N. Gantert, Galton-Watson trees as random environments; Z. Shi, Sinai's walk via stochastic calculus; S. Olla, Central limit theorems for tagged particles and for diffusions in random environment; J. Garnier, Wave propagation in one-dimensional random media.

Panoramas et Synthèses, Number 12

December 2002, 159 pages, Softcover, ISBN 2-85620-127-9, 2000 *Mathematics Subject Classification*: 60K37, 82D30, 60F10, 82C44, **Individual member \$30**, List \$33, Order code PASY/12N

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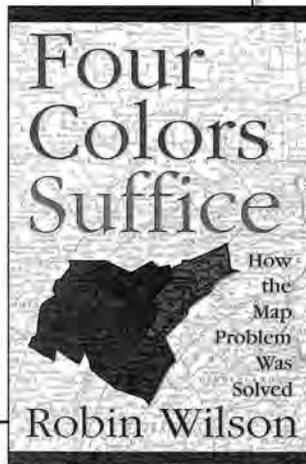
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Reciprocity member (please verify) ³	<input type="checkbox"/> \$72 <input type="checkbox"/> \$108 <input type="checkbox"/> \$144
Category-S member ⁴	<input type="checkbox"/> \$16
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³ Reciprocity Membership Verification (sign below) I am currently a member of the society indicated on the right and am therefore eligible for reciprocity membership.

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| <input type="checkbox"/> European Mathematical Society | <input type="checkbox"/> Société Mathématique Suisse |
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| <input type="checkbox"/> Hellenic Mathematical Society | <input type="checkbox"/> Society of Mathematicians, Physicists, and Astronomers of Slovenia |
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Members of the Society

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Journal mailing lists must be printed four to six weeks before the issue date.

Therefore, in order to avoid disruption of service, members are requested to provide the required notice well in advance.

Besides mailing addresses for members, the Society's records contain information about members' positions and their employers (for publication in the Combined Membership List). In addition, the AMS maintains records of members' honors, awards, and information on Society service.

When changing their addresses, members are urged to cooperate by supplying the requested information. The Society's records are of value only to the extent that they are current and accurate.

If your address has changed or will change within the next two or three months, please fill out this form, supply any other information appropriate for the AMS records, and mail it to:

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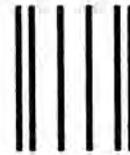
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Recent honors and awards

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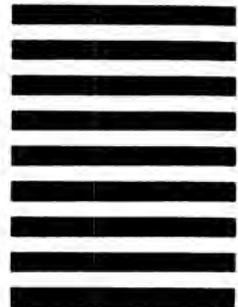


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Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See <http://www.ams.org/meetings/>. Programs and abstracts will continue to be displayed on the AMS website in the Meetings and Conferences section until about three weeks after the meeting is over. Final programs for Sectional Meetings will be archived on the AMS website in an electronic issue of the *Notices* as noted below for each meeting.

New York, New York

Courant Institute

April 12–13, 2003

Meeting #986

Eastern Section

Associate secretary: Lesley M. Sibner

Announcement issue of *Notices*: February 2003

Program first available on AMS website: February 27, 2003

Program issue of electronic *Notices*: April 2003

Issue of *Abstracts*: Volume 24, Issue 3

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions:
Expired

For abstracts: Expired

Invited Addresses

Matthias Aschenbrenner, University of California Berkeley, *Asymptotic differential algebra*.

John Etnyre, University of Pennsylvania, *Legendrian knots*.

Hans Foellmer, Humboldt University Berlin, *Mathematical aspects of financial risk*.

Wilfrid Gangbo, Georgia Institute of Technology, *The Riemannian geometry of the 2-Wasserstein metric and the nonlinear kinetic Fokker-Planck equation*.

Special Sessions

Algebraic and Topological Combinatorics, **Eva-Maria Feichtner**, ETH, Zurich, Switzerland, and **Dmitry N. Kozlov**, University of Bern, Switzerland, and KTH, Stockholm, Sweden.

Algebraic Geometry, Integrable Systems, and Gauge Theory, **Marcos Jardim** and **Eyal Markman**, University of Massachusetts, Amherst.

Analytical and Computational Methods in Electromagnetics, **Alexander P. Stone**, University of New Mexico, and **Peter A. McCoy**, U. S. Naval Academy.

Combinatorial and Statistical Group Theory, **Alexei Myasnikov** and **Vladimir Shpilrain**, City College, New York.

Contact and Symplectic Geometry, **John B. Etnyre** and **Joshua M. Sabloff**, University of Pennsylvania.

Galois Module Theory and Hopf Algebras, **Daniel R. Replegle**, College of Saint Elizabeth, and **Robert G. Underwood**, Auburn University.

The History of Mathematics, **Patricia R. Allaire**, Queensborough Community College, CUNY, and **Robert E. Bradley**, Adelphi University.

Hopf Algebras and Quantum Groups, **M. Susan Montgomery**, University of Southern California, **Earl J. Taft**, Rutgers University, and **Sarah J. Witherspoon**, Amherst College.

Low-Dimensional Topology, **James Conant**, Cornell University, **Slava Krushkal**, University of Virginia, and **Rob Schneiderman**, NYU-Courant Institute.

Nonlinear Partial Differential Equations in Differential Geometry, **John C. Loftin** and **Mu-Tao Wang**, Columbia University.

Rigidity in Dynamics, Geometry, and Group Theory, **David Fisher**, CUNY, Herbert H. Lehman College, and **Steven E. Hurder** and **Kevin M. Whyte**, University of Illinois at Chicago.

Topological Aspects of Complex Singularities, **Sylvain E. Cappell**, NYU-Courant Institute, and **Walter D. Neumann** and **Agnes Szilard**, Barnard College, Columbia University.

San Francisco, California

San Francisco State University

May 3–4, 2003

Meeting #987

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: March 2003

Program first available on AMS website: March 20, 2003

Program issue of electronic *Notices*: May 2003

Issue of *Abstracts*: Volume 24, Issue 3

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions:
Expired

For abstracts: Expired

Invited Addresses

Joe P. Buhler, Reed College, *A problem in symmetric functions arising from phase determination in crystallography*.

Raymond C. Heitmann, University of Texas at Austin, *The direct summand conjecture in dimension three*.

Alexei Y. Kitaev, California Institute of Technology, *Quantum media and topological quantum computation*.

Arkady Vaintrob, University of Oregon, *Higher spin curves and Gromov-Witten theory*.

Special Sessions

Beyond Classical Boundaries of Computability, **Mark Burgin**, University of California Los Angeles, and **Peter Wegner**, Brown University.

Combinatorial Commutative Algebra and Algebraic Geometry, **Serkan Hosten**, San Francisco State University, and **Ezra Miller**, Mathematical Sciences Research Institute.

Commutative Algebra, **Raymond C. Heitmann**, University of Texas at Austin, and **Irena Swanson**, New Mexico State University.

Efficient Arrangements of Convex Bodies, **Dan P. Ismailescu**, Hofstra University, and **Włodzimierz Kuperberg**, Auburn University.

Geometry and Arithmetic over Finite Fields, **Bjorn Poonen**, University of California Berkeley, and **Joe P. Buhler**, Reed College.

Gromov-Witten Theory of Spin Curves and Orbifolds, **Tyler Jarvis**, Brigham Young University, **Takashi Kimura**, Boston University, and **Arkady Vaintrob**, University of Oregon.

The History of Nineteenth and Twentieth Century Mathematics, **Shawnee McMurrin**, California State University, San Bernardino, and **James A. Tattersall**, Providence College.

Numerical Methods, Calculations and Simulations in Knot Theory and Its Applications, **Jorge Alberto Calvo**, North Dakota State University, **Kenneth C. Millett**, University of California Santa Barbara, and **Eric J. Rawdon**, Duquesne University.

PDEs and Applications in Geometry, **Qi S. Zhang**, University of California Riverside.

Q-Series and Partitions, **Neville Robbins**, San Francisco State University.

Qualitative Properties and Applications of Functional Equations, **Theodore A. Burton**, Southern Illinois University at Carbondale.

Topological Quantum Computation, **Alexei Kitaev**, California Institute of Technology, and **Samuel J. Lomonaco**, University of Maryland, Baltimore County.

Seville, Spain

June 18–21, 2003

Meeting #988

First Joint International Meeting between the AMS and the Real Sociedad Matemática Española (RSME).

Associate secretary: Susan J. Friedlander

Announcement issue of *Notices*: February 2003

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions:
Expired

For abstracts: Expired

Invited Addresses

Xavier Cabre, Universidad Politècnica de Catalunya, Barcelona, *Title to be announced*.

Charles Fefferman, Princeton University, *Title to be announced*.

Michael Hopkins, Massachusetts Institute of Technology, *Title to be announced*.

Ignacio Sols, Universidad Complutense, Madrid, *Title to be announced*.

Luis Vega, Universidad del País Vasco, Bilbao, *Title to be announced*.

Efim Zelmanov, Yale University, *Title to be announced.*

Special Sessions

Affine Algebraic Geometry, **Jaime Gutierrez**, University of Cantabria, **Vladimir Shpilrain**, City College of New York, and **Jie-Tai Yu**, University of Hong Kong.

Algebraic Geometry, **Felix Delgado**, Universidad de Valladolid, and **Andrey N. Todorov**, University of California Santa Cruz.

Algebraic Topology, **Alejandro Adem**, University of Wisconsin, **J. Aguade**, Universitat Autònoma de Barcelona, and **Eric M. Friedlander**, Northwestern University.

Banach Spaces of Analytic Functions, **Daniel Girela**, University of Malaga, and **Michael Stessin**, SUNY at Albany.

Biomolecular Mathematics, **Thomas J. Head** and **Fernando Guzman**, SUNY at Binghamton, **Mario Perez**, Universidad de Sevilla, and **Carlos Martín-Vide**, Rovira i Virgili University.

Classical and Harmonic Analysis, **Nets Katz**, Washington University, **Carlos Perez**, Universidad de Sevilla, and **Ana Vargas**, Universidad Autònoma de Madrid.

Combinatorics, **Joseph E. Bonin**, George Washington University, and **Marc Noy**, Universitat Politècnica de Catalunya.

Commutative Algebra: Geometric, Homological, Combinatorial and Computational Aspects, **Alberto Corso**, University of Kentucky, **Philippe Gimenez**, Universidad de Valladolid, and **Santiago Zarzuela**, Universitat de Barcelona.

Computational Methods in Algebra and Analysis, **Eduardo Cattani**, University of Massachusetts, Amherst, and **Francisco Jesus Castro-Jimenez**, Universidad de Sevilla.

Constructive Approximation Theory, **Antonio Duran**, Universidad de Sevilla, and **Edward B. Saff**, Vanderbilt University.

Control and Geometric Mechanics, **Manuel de Leon**, Instituto de Matemáticas y Física Fundamental, **Alberto Ibort**, Universidad Carlos III, and **Francesco Bullo**, University of Illinois, Urbana-Champaign.

Differential Galois Theory, **Teresa Crespo** and **Zbigniew Hajto**, Universitat de Barcelona, and **Andy R. Magid**, University of Oklahoma.

Differential Structures and Homological Methods in Commutative Algebra and Algebraic Geometry, **Gennady Lyubeznik**, University of Minnesota, and **Luis Narvaez-Macarro**, Universidad de Sevilla.

Discrete and Computational Geometry, **Ferran Hertado**, Universitat Politècnica de Catalunya, and **William Steiger**, Rutgers University.

Dynamical Systems, **George Haller**, Massachusetts Institute of Technology, **Zbigniew H. Nitecki**, Tufts University, **Enrique Ponce**, Universidad de Sevilla, **Tere M. Seara**, Universitat Politècnica de Catalunya, and **Xavier Jarque**, Universitat Autònoma de Barcelona.

Effective Analytic Geometry over Complete Fields, **Luis-Miguel Pardos**, Universidad de Cantabria, and **J. Maurice Rojas**, Texas A&M University.

Geometric Methods in Group Theory, **José Burillo**, Universitat Politècnica de Catalunya, **Jennifer Tayback**, University of Albany, and **Enric Ventura**, Universitat Politècnica de Catalunya.

History of Modern Mathematics—Gauss to Wiles, **Jose Ferreiros**, Universidad de Sevilla, and **David Rowe**, Universität Mainz.

Homological Methods in Banach Space Theory, **Jesus M. F. Castillo**, Universidad de Extremadura, and **N. J. Kalton**, University of Missouri.

Homotopy Algebras, **Pedro Real**, Universidad de Sevilla, **Thomas J. Lada**, North Carolina State University, and **James Stasheff**, University of North Carolina.

Interpolation Theory, Function Spaces and Applications, **Fernando Cobos**, Universidad Complutense de Madrid, and **Pencho Petrushev**, University of South Carolina.

Lorentzian Geometry and Mathematical Relativity, **Luis J. Alias**, Universidad de Murcia, and **Gregory James Galloway**, University of Miami.

Mathematical Aspects of Semiconductor Modeling and Nano-technology, **Irene Martínez Gamba**, University of Texas at Austin, and **Jose Antonio Carrillo**, Universidad de Granada.

Mathematical Fluid Dynamics, **Diego Cordoba**, CSIC, Madrid, and Princeton University, **Susan Friedlander**, University of Illinois, Chicago, and **Marcos Antonio Fontelos**, Universidad Rey Juan Carlos.

Mathematical Methods in Finance and Risk Management, **Santiago Carrillo Menendez**, Universidad Autònoma de Madrid, **Antonio Falcos Montesinos**, Universidad Cardinal Herrera CEU, **Antonio Sanchez-Calle**, Universidad Autònoma de Madrid, and **Luis A. Seco**, University of Toronto at Mississauga.

Mathematics of Electronmicroscopic Imaging, **Jose-Maria Carazo**, Centro Nacional de Biotecnología-CSIC, and **Gabor T. Herman**, City University of New York.

Moduli Spaces in Geometry and Physics, **Steven B. Bradlow**, University of Illinois, Urbana-Champaign, and **Oscar Garcia-Prada**, Universidad Autònoma de Madrid.

Nonassociative Algebras and Their Applications, **Efim I. Zelmanov**, Yale University, **Santos Gonzalez**, Universidad de Oviedo, and **Alberto Elduque**, Universidad de Zaragoza.

Nonlinear Dispersive Equations, **Gustavo Ponce**, University of California Santa Barbara, and **Luis Vega**, Universidad del País Vasco.

Numerical Linear Algebra, **Lothar Reichel**, Kent State University, and **Francisco Marcellan**, Universidad Carlos III de Madrid.

Operator Theory and Spaces of Analytic Functions, **Jose Bonet**, Universitat Politècnica de Valencia, **Pedro Paul**, Universidad de Sevilla, and **Cora S. Sadosky**, Howard University.

PDE Methods in Continuum Mechanics, **Juan L. Vazquez**, Universidad Autònoma de Madrid, and **J. W. Neuberger**, University of North Texas.

Polynomials and Multilinear Analysis in Infinite Dimensions, **Richard M. Aron**, Kent State University, **J. A. Jaramillo** and **Jose G. Llavona**, Universidad Complutense de Madrid, and **Andrew M. Tonge**, Kent State University.

Quantitative Results in Real Algebra and Geometry, **Carlos Andradas** and **Antonio Diaz-Cano**, Universidad Complutense, **Victoria Powers**, Emory University, and **Frank Sottile**, University of Massachusetts, Amherst.

Recent Developments in the Mathematical Theory of Inverse Problems, **Russell Brown**, University of Kentucky, **Alberto Ruiz**, Universidad Autónoma de Madrid, and **Gunther Uhlmann**, University of Washington.

Riemannian Foliations, **Jesus Antonio Alvarez Lopez**, Universidade de Santiago de Compostela, and **Efton L. Park**, Texas Christian University.

Ring Theory and Related Topics, **Jose Gomez-Torrecillas**, University of Granada, **Pedro Antonio Guil Asensio**, University of Murcia, **Sergio R. Lopez-Permouth**, Ohio University, and **Blas Torrecillas**, University of Almeria.

Variational Problems for Submanifolds, **Frank Morgan**, Williams College, and **Antonio Ros**, Universidad de Granada.

Boulder, Colorado

University of Colorado

October 2-4, 2003

Meeting #989

Joint Central/Western Sections

Associate secretaries: Susan J. Friedlander and Michel L. Lapidus

Announcement issue of *Notices*: August 2003

Program first available on AMS website: August 21, 2003

Program issue of electronic *Notices*: October 2003

Issue of *Abstracts*: Volume 24, Issue 4

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions:
June 6, 2003

For abstracts: August 12, 2003

Invited Addresses

J. Brian Conrey, American Institute of Mathematics, *Random matrix theory and the Riemann zeta-function*.

Giovanni Forni, Northwestern University, *Title to be announced*.

Juha M. Heinonen, University of Michigan, *Title to be announced*.

Joseph D. Lakey, New Mexico State University, *Recent progress in time-frequency analysis*.

Albert Schwarz, University of California Davis, *Maximally supersymmetric gauge theories*.

Brooke E. Shipley, Purdue University, *Title to be announced*.

Avi Wigderson, Institute for Advanced Study, *Title to be announced* (Erdős Memorial Lecture).

Special Sessions

Algebraic Geometry (Code: AMS SS M1), **Holger Kley**, **Rick Miranda**, and **Chris Peterson**, Colorado State University.

Algebras, Lattices and Varieties (Code: AMS SS A1), **Keith A. Kearnes**, University of Colorado, Boulder, **Agnes Szendrei**, Bolyai Institute, and **Walter Taylor**, University of Colorado, Boulder.

Analysis on Singular Spaces (Code: AMS SS L1), **Mario Bonk**, University of Michigan, and **Juha Heinonen**, Mathematical Sciences Research Institute.

Applications of Number Theory and Algebraic Geometry to Coding (Code: AMS SS B1), **David R. Grant**, University of Colorado, Boulder, **Jose Felipe Voloch**, University of Texas at Austin, and **Judy Leavitt Walker**, University of Nebraska, Lincoln.

Associative Rings and Their Modules (Code: AMS SS J1), **Gene Abrams**, University of Colorado, Colorado Springs, and **Kent Fuller**, University of Iowa.

Computational Number Theory (Code: AMS SS R1), **Brian Conrey** and **Michael Rubinstein**, American Institute of Mathematics.

Dynamics of Rational Polygonal Billiards and Related Systems (Code: AMS SS K1), **Giovanni Forni**, Northwestern University.

Finite Geometries (Code: AMS SS N1), **Stanley E. Payne**, University of Colorado, Denver, and **Robert Allen Liebler**, Colorado State University.

Geometric Methods in Partial Differential Equations (Code: AMS SS C1), **Jeanne N. Clelland**, University of Colorado, Boulder, and **George R. Wilkins**, University of Hawaii.

Graphs and Diagraphs (Code: AMS SS H1), **Michael Jacobson**, University of Colorado, Denver, and **Richard J. Lundgren**, University of Colorado, Denver.

Groupoids in Analysis and Geometry (Code: AMS SS D1), **Lawrence Baggett**, University of Colorado, Boulder, **Jerry Kaminker**, Indiana University-Purdue University Indianapolis, and **Judith Packer**, University of Colorado, Boulder.

Homotopy Theory (Code: AMS SS F1), **Daniel Dugger**, University of Oregon, and **Brooke E. Shipley**, Purdue University.

Noncommutative Geometry and Geometric Analysis (Code: AMS SS E1), **Carla Farsi**, **Alexander Gorokhovskiy**, and **Siye Wu**, University of Colorado.

Nonlinear Waves (Code: AMS SS P1), **Bernard Deconinck**, Colorado State University, and **Harvey Segur**, University of Colorado, Boulder.

Structured Population and Epidemic Models: Periodicity, Chaos, and Extinction (Code: AMS SS G1), **Linda J. S. Allen**, Texas Technical University, and **Sophia R.-J. Jang**, University of Louisiana at Lafayette.

Ubiquitous Heat Kernel (Code: AMS SS Q1), **Jay Jorgenson**, City College of New York, and **Lynne Walling**, University of Colorado, Boulder.

Binghamton, New York

Binghamton University

October 11–12, 2003

Meeting #990

Eastern Section

Associate secretary: Lesley M. Sibner

Announcement issue of *Notices*: August 2003

Program first available on AMS website: August 28, 2003

Program issue of electronic *Notices*: October 2003

Issue of *Abstracts*: Volume 24, Issue 4

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions:
June 24, 2003

For abstracts: August 19, 2003

Invited Addresses

Peter Kuchment, Texas A&M University, *Title to be announced.*

Zlil Sela, Einstein Institute of Mathematics, *Title to be announced.*

Zoltan Szabo, Princeton University, *Title to be announced.*

Jeb F. Willenbring, Yale University, *Title to be announced.*

Special Sessions

Biomolecular Mathematics (Code: AMS SS A1), **Thomas J. Head** and **Dennis G. Pixton**, Binghamton University, **Mitsunori Ogihara**, University of Rochester, and **Carlos Martin-Vide**, Universitat Rovira i Virgili.

Boundary Value Problems on Singular Domains (Code: AMS SS C1), **Juan B. Gil**, Temple University, and **Paul A. Loya**, Binghamton University.

Character Theory of Finite Groups and Algebraic Combinatorics (Code: AMS SS P1), **Kenneth W. Johnson**, Pennsylvania State University, and **Eirini Poimenidou**, New College of Florida.

Dowling Lattices: The 30th Anniversary (Code: AMS SS N1), **Thomas Zaslavsky**, Binghamton University.

Finite Solvable Groups and Their Representations (Code: AMS SS K1), **Ben Brewster**, Binghamton University, and **Arnold Feldman**, Franklin & Marshall College.

Geometric Group Theory (Code: AMS SS B1), **Zlil Sela**, Einstein Institute of Mathematics, and **Ross Geoghegan**, Binghamton University.

Homotopy Theory: Honoring Peter Hilton on His Eightieth Birthday (Code: AMS SS J1), **Martin Bendersky** and **Joseph Roitberg**, Hunter College (CUNY).

Infinite Groups and Group Rings (Code: AMS SS D1), **Luise-Charlotte Kappe**, Binghamton University, and **Derek J. S. Robinson**, University of Illinois, Urbana-Champaign.

Inverse Problems and Tomography (Code: AMS SS H1), **Peter Kuchment**, Texas A&M University, **Leonid A. Kunyansky**, University of Arizona, and **Eric Todd Quinto**, Tufts University.

Lie Algebras, Conformal Field Theory, and Related Topics (Code: AMS SS E1), **Chongying Dong**, University of California Santa Cruz, and **Alex J. Feingold** and **Gaywalee Yamskulna**, Binghamton University.

Manifold Theory (Code: AMS SS L1), **Erik K. Pedersen**, Binghamton University, and **Ian Hambleton**, McMaster University.

Noncommutative Ring Theory (Code: AMS SS M1), **Howard E. Bell** and **Yuanlin Li**, Brock University.

Probability Theory (Code: AMS SS F1), **Miguel A. Arcones**, Binghamton University, and **Evarist Gine**, University of Connecticut.

Statistics (Code: AMS SS G1), **Miguel A. Arcones**, **Anton Schick**, and **Qiqing Yu**, Binghamton University.

Chapel Hill, North Carolina

University of North Carolina at Chapel Hill

October 24–25, 2003

Meeting #991

Southeastern Section

Associate secretary: John L. Bryant

Announcement issue of *Notices*: August 2003

Program first available on AMS website: September 11, 2003

Program issue of electronic *Notices*: October 2003

Issue of *Abstracts*: Volume 24, Issue 4

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions:
July 19, 2003

For abstracts: September 3, 2003

Invited Addresses

James N. Damon, University of North Carolina, *Title to be announced.*

Erica L. Flapan, Pomona College, *Title to be announced.*

Mary Ann Horn, Vanderbilt University, *Title to be announced.*

Helmut Voelklein, University of Florida, *Title to be announced.*

Special Sessions

Association Schemes: 1973–2003 (Code: AMS SS F1), **William J. Martin**, Worcester Polytechnic Institute, and **Dijen K. Ray-Chaudhuri**, Ohio State University.

Ergodic Theory (Code: AMS SS B1), **Idris Assani**, University of North Carolina.

Group Actions on Curves (Code: AMS SS A1), **Kay Magaard**, Wayne State University and University of Florida, and **Helmut Voelklein**, University of Florida.

Group Cohomology in Algebra and Geometry (Code: AMS SS E1), **Richard M. Hain**, Duke University, and **Kevin P. Knudson**, Mississippi State University.

Knots, Links, and Embedded Graphs (Code: AMS SS C1), **Joel S. Foisy**, SUNY at Potsdam, and **Erica L. Flapan**, Pomona College.

Linear Operators on Function Spaces (Code: AMS SS G1), **Nathan S. Feldman**, Washington and Lee University, and **William T. Ross**, University of Richmond.

Measurable, Complex, and Symbolic Dynamics (Code: AMS SS D1), **Jane M. Hawkins** and **Karl E. Petersen**, University of North Carolina at Chapel Hill.

Bangalore, India

Indian Institute of Science

December 17–20, 2003

Meeting #992

First Joint AMS-India Mathematics Meeting

Associate secretary: Susan J. Friedlander

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions:
Not applicable

For abstracts: September 1, 2003

Invited Addresses

R. Balasubramanian, Institute for Mathematical Sciences, *Title to be announced.*

George C. Papanicolaou, Stanford University, *Title to be announced.*

M. S. Raghunathan, Tata Institute of Fundamental Research, *Title to be announced.*

Peter Sarnak, Princeton University and NYU-Courant Institute, *Title to be announced.*

K. B. Sinha, Indian Statistical Institute, *Title to be announced.*

Vladimir Voevodsky, Institute for Advanced Study, *Title to be announced.*

Special Sessions

Algebraic and Geometric Methods in Multivariable Operator Theory, **Ronald G. Douglas**, Texas A&M University, and **Gadadhar Misra**, Indian Statistical Institute.

Algebraic and Geometric Topology, **Parameswaren Sankaran**, Institute of Mathematical Sciences, and **P. B. Shalen**, University of Illinois.

Automorphic Forms and Functoriality, **James Cogdell**, Oklahoma State University, and **T. N. Venkataramana**, Tata Institute of Fundamental Research.

Buildings and Group Theory, **N. S. Narasimha Sastry**, Indian Statistical Institute, and **Richard M. Weiss**, Tufts University.

Commutative Algebra and Algebraic Geometry, **Sudhir Ghorpade**, Indian Institute of Technology, Bombay, **Hema Srinivasan**, University of Missouri, and **Jugal K. Verma**, Indian Institute of Technology, Bombay.

Cycles, K-Theory, and Motives, **Eric M. Friedlander**, Northwestern University, **Steven Lichtenbaum**, Brown University, **Kapil Paranjape**, Institute of Mathematical Sciences, and **Vasudevan Srinivas**, Tata Institute of Fundamental Research.

Differential Equations and Applications to Population Dynamics, Epidemiology, Genetics and Microbiology, **Bindhyachal Rai**, University of Allahabad, **Sanjay Rai**, Jacksonville University, **Terrance Quinn**, Ohio University Southern, and **Sunil Tiwari**, Sonoma State University.

History of Indian Mathematics, **Gerard G. Emch**, University of Florida, and **R. Sridharan**, Chennai Mathematical Institute.

L-Functions, Automorphic Forms and Cryptography, **R. Balasubramanian**, Institute of Mathematical Sciences, and **K. Soundararajan**, University of Michigan.

The Many Facets of Linear Algebra and Matrix Theory, **Richard Brualdi**, University of Wisconsin, and **Rajendra Bhatia**, Indian Statistical Institute.

PDE and Applications, **Susan B. Friedlander**, University of Illinois, and **P. N. Srikanth**, Tata Institute of Fundamental Research.

Probability Theory, **Rajeeva Karandikar**, Indian Statistical Institute, and **Srinivasa R. S. Varadhan**, NYU-Courant Institute.

Quantum Dynamics, **William Arveson**, University of California Berkeley, and **B. V. Rajarama Bhat**, Indian Statistical Institute.

Reductive Groups: Arithmetic, Geometry and Representation Theory, **Vikram Mehta** and **R. Parimala**, Tata Institute of Fundamental Research, and **Gopal Prasad**, University of Michigan, Ann Arbor.

Spectral and Inverse Spectral Theories of Schrödinger Operators, **Peter David Hislop**, University of Kentucky, and **Krishna Maddaly**, Institute of Mathematical Sciences.

Phoenix, Arizona

Phoenix Civic Plaza

January 7–10, 2004

Wednesday – Saturday

Meeting #993

Joint Mathematics Meetings, including the 110th Annual Meeting of the AMS, 87th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: October 2003

Program first available on AMS website: November 1, 2003

Program issue of electronic *Notices*: January 2004

Issue of *Abstracts*: Volume 25, Issue 1

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions:
August 6, 2003

For abstracts: October 1, 2003

For summaries of papers to MAA organizers: To be announced

Tallahassee, Florida

Florida State University

March 12–13, 2004

Meeting #994

Southeastern Section

Associate secretary: John L. Bryant

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 13, 2003

For consideration of contributed papers in Special Sessions:
To be announced

For abstracts: To be announced

Athens, Ohio

Ohio University

March 26–27, 2004

Meeting #995

Central Section

Associate secretary: Susan J. Friedlander

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 26, 2003

For consideration of contributed papers in Special Sessions:
To be announced

For abstracts: To be announced

Los Angeles, California

University of Southern California

April 3–4, 2004

Meeting #996

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced

For consideration of contributed papers in Special Sessions:
To be announced

For abstracts: To be announced

Special Sessions

Contact and Symplectic Geometry (Code: AMS SS A1),
Dragomir Dragnev, **Ko Honda**, and **Sang Seon Kim**, University of Southern California.

Lawrenceville, New Jersey

Rider University

April 17–18, 2004

Meeting #997

Eastern Section

Associate secretary: Lesley M. Sibner

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 17, 2003
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Special Sessions

Algebraic Geometry and Mirror Symmetry (Code: AMS SS D1), **Ciprian Borcea**, Rider University.

Automorphic Forms and Analytic Number Theory (Code: AMS SS A1), **Stephen Miller**, Rutgers University, and **Ramin Takloo-Bighash**, Princeton University.

Geometry of Protein Modelling (Code: AMS SS E1), **Ileana Streinu**, Smith College, and **Jack Snoeyink**, University of North Carolina at Chapel Hill.

Homotopy (Code: AMS SS C1), **Martin Bendersky**, Hunter College, and **Donald Davis**, Lehigh University.

Tomography and Integral Geometry (Code: AMS SS B1), **Andrew Markoe**, Rider University, and **Eric Todd Quinto**, Tufts University.

Houston, Texas

University of Houston

May 13–15, 2004

Sixth International Joint Meeting of the AMS and the Sociedad Matemática Mexicana (SMM).

Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Nashville, Tennessee

Vanderbilt University

October 16–17, 2004

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 16, 2004
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Albuquerque, New Mexico

University of New Mexico

October 16–17, 2004

Southeastern Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 16, 2004
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Pittsburgh, Pennsylvania

University of Pittsburgh

November 6–7, 2004

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 7, 2004
For consideration of contributed papers in Special Sessions:
To be announced
For abstracts: To be announced

Atlanta, Georgia

*Atlanta Marriott Marquis and Hyatt Regency
Atlanta*

January 5–8, 2005

*Wednesday – Saturday
Joint Mathematics Meetings, including the 111th Annual Meeting of the AMS, 88th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of*

the Association of Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL). Associate secretary: Lesley M. Sibner

Announcement issue of *Notices*: October 2004
 Program first available on AMS website: To be announced
 Program issue of electronic *Notices*: January 2005
 Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 5, 2004
 For consideration of contributed papers in Special Sessions:
 To be announced
 For abstracts: To be announced
 For summaries of papers to MAA organizers: To be announced

Newark, Delaware

University of Delaware

April 2–3, 2005

Eastern Section
 Associate secretary: Lesley M. Sibner
 Announcement issue of *Notices*: To be announced
 Program first available on AMS website: To be announced
 Program issue of electronic *Notices*: To be announced
 Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 2, 2004
 For consideration of contributed papers in Special Sessions:
 To be announced
 For abstracts: To be announced

Mainz, Germany

Deutsche Mathematiker-Vereinigung (DMV) and the Osterreichische Mathematische Gesellschaft (OMG)

June 16–19, 2005

Second Joint AMS-Deutsche Mathematiker-Vereinigung (DMV) Meeting
 Associate secretary: Susan J. Friedlander
 Announcement issue of *Notices*: To be announced
 Program first available on AMS website: To be announced
 Program issue of electronic *Notices*: To be announced
 Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
 For consideration of contributed papers in Special Sessions:
 To be announced
 For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center

January 12–15, 2006

Thursday – Sunday
Joint Mathematics Meetings, including the 112th Annual Meeting of the AMS, 89th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
 Associate secretary: John L. Bryant
 Announcement issue of *Notices*: October 2005
 Program first available on AMS website: To be announced
 Program issue of electronic *Notices*: January 2006
 Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 12, 2005
 For consideration of contributed papers in Special Sessions:
 To be announced
 For abstracts: To be announced
 For summaries of papers to MAA organizers: To be announced

New Orleans, Louisiana

*New Orleans Marriott and Sheraton
 New Orleans Hotel*

January 4–7, 2007

Thursday – Sunday
Joint Mathematics Meetings, including the 113th Annual Meeting of the AMS, 90th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
 Associate secretary: Susan J. Friedlander
 Announcement issue of *Notices*: October 2006
 Program first available on AMS website: To be announced
 Program issue of electronic *Notices*: January 2007
 Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 4, 2006
 For consideration of contributed papers in Special Sessions:
 To be announced
 For abstracts: To be announced
 For summaries of papers to MAA organizers: To be announced

San Diego, California

San Diego Convention Center

January 6–9, 2008

Sunday – Wednesday

Joint Mathematics Meetings, including the 114th Annual Meeting of the AMS, 91st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: October 2007

Program first available on AMS website: November 1, 2007

Program issue of electronic *Notices*: January 2008

Issue of *Abstracts*: Volume 29, Issue 1

Deadlines

For organizers: April 6, 2007

For consideration of contributed papers in Special Sessions:

To be announced

For abstracts: To be announced

For summaries of papers to MAA organizers: To be announced

Washington, District of Columbia

*Marriott Wardman Park Hotel and
Omni Shoreham Hotel*

January 7–10, 2009

Wednesday – Saturday

Joint Mathematics Meetings, including the 115th Annual Meeting of the AMS, 92nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).

Associate secretary: Lesley M. Sibner

Announcement issue of *Notices*: October 2008

Program first available on AMS website: November 1, 2008

Program issue of electronic *Notices*: January 2009

Issue of *Abstracts*: Volume 30, Issue 1

Deadlines

For organizers: April 7, 2008

For consideration of contributed papers in Special Sessions:

To be announced

For abstracts: To be announced

For summaries of papers to MAA organizers: To be announced



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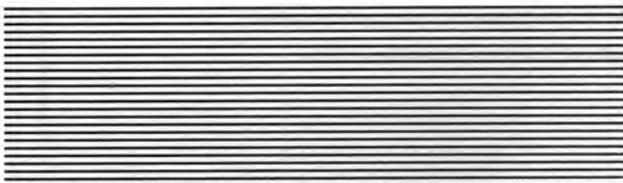
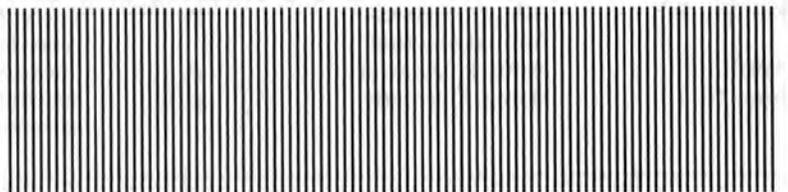
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Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Sproul Hall, Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 909-787-3113.

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Eastern Section: Lesley M. Sibner, Department of Mathematics, Polytechnic University, Brooklyn, NY 11201-2990; e-mail: lsibner@duke.poly.edu; telephone: 718-260-3505.

Southeastern Section: John L. Bryant, Department of Mathematics, Florida State University, Tallahassee, FL 32306-4510; e-mail: bryant@math.fsu.edu; telephone: 850-644-5805.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated.** **Up-to-date meeting and conference information at www.ams.org/meetings/.**

Meetings:

2003

April 12-13	New York, New York	p. 621
May 3-4	San Francisco, California	p. 622
June 18-21	Seville, Spain	p. 622
October 2-4	Boulder, Colorado	p. 624
October 11-12	Binghamton, New York	p. 625
October 24-25	Chapel Hill, North Carolina	p. 625
December 17-20	Bangalore, India	p. 626

2004

January 7-10	Phoenix, Arizona Annual Meeting	p. 627
March 12-13	Tallahassee, Florida	p. 627
March 26-27	Athens, Ohio	p. 627
April 3-4	Los Angeles, California	p. 627
April 17-18	Lawrenceville, New Jersey	p. 627
May 13-15	Houston, Texas	p. 628
October 16-17	Nashville, Tennessee	p. 628
October 16-17	Albuquerque, New Mexico	p. 628
November 6-7	Pittsburgh, Pennsylvania	p. 628

2005

January 5-8	Atlanta, Georgia Annual Meeting	p. 628
April 2-3	Newark, Delaware	p. 629
June 16-19	Mainz, Germany	p. 629

2006

January 12-15	San Antonio, Texas Annual Meeting	p. 629
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2007

January 4-7	New Orleans, Louisiana Annual Meeting	p. 629
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2008

January 6-9	San Diego, California Annual Meeting	p. 630
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2009

January 7-10	Washington, D.C. Annual Meeting	p. 630
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Important Information regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 108 in the January 2003 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Several options are available for speakers submitting abstracts, including an easy-to-use interactive Web form. No knowledge of \LaTeX is necessary to submit an electronic form, although those who use \LaTeX may submit abstracts with such coding, and all math displays and similarly coded material (such as accent marks in text) must be typeset in \LaTeX . To see descriptions of the forms available, visit <http://www.ams.org/abstracts/instructions.html>, or send mail to abs-submit@ams.org, typing `help` as the subject line; descriptions and instructions on how to get the template of your choice will be e-mailed to you.

Completed abstracts should be sent to abs-submit@ams.org, typing `submission` as the subject line. Questions about abstracts may be sent to abs-info@ams.org.

Paper abstract forms may be sent to Meetings & Conferences Department, AMS, P.O. Box 6887, Providence, RI 02940. There is a \$20 processing fee for each paper abstract. There is no charge for electronic abstracts. Note that all abstract deadlines are strictly enforced. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

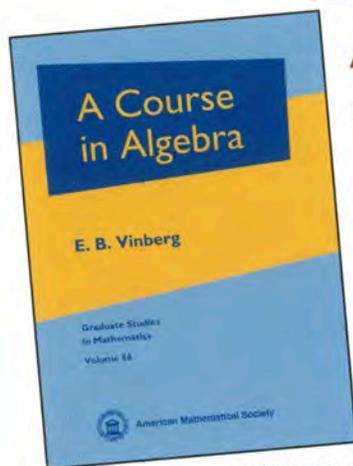
Conferences: (See <http://www.ams.org/meetings/> for the most up-to-date information on these conferences.)

June 8 - July 24, 2003: Joint Summer Research Conferences in the Mathematical Sciences, Snowbird, Utah.

New and Noteworthy Algebra Titles from the AMS



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E. B. Vinberg, *Moscow State University*

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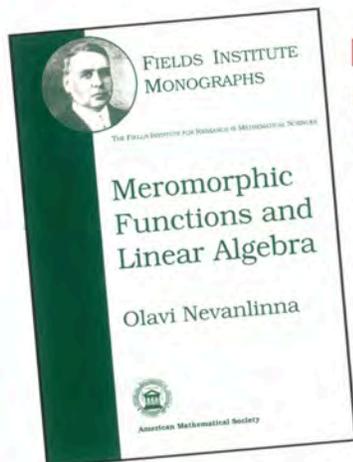
—Irving Kaplansky, MSRI

This is a comprehensive textbook on modern algebra written by an internationally renowned specialist. It covers material traditionally found in advanced undergraduate and basic graduate courses and presents it in a lucid style. It is written with extreme care and contains over 200 exercises and 70 figures. It is an ideal textbook or suitable for independent study for advanced undergraduates and graduate students.

Graduate Studies in Mathematics, Volume 56; 2003; 511 pages

Hardcover: ISBN 0-8218-3318-9; List \$89; All AMS members \$71; Order code GSM/56BK305

Softcover: ISBN 0-8218-3413-4; List \$59; All AMS members \$47; Order code GSM/56.SBK305



Meromorphic Functions and Linear Algebra

Olavi Nevanlinna, *Helsinki University of Technology*

This volume describes for the first time in monograph form important applications in numerical methods of linear algebra. The author presents new material and extended results from recent papers in a very readable style.

The book is intended for researchers in mathematics in general and especially for those working in numerical linear algebra. Much of the book is understandable if the reader has a good background in linear algebra and a first course in complex analysis.

Fields Institute Monographs, Volume 18; 2003; 136 pages; Hardcover;

ISBN 0-8218-3247-6; List \$49; All AMS members \$39; Order code FIM/18BK305

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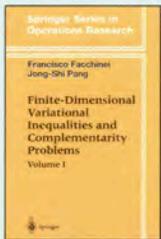
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This comprehensive book presents a rigorous and state-of-the-art treatment of variational inequalities and complementarity problems in finite dimensions. This class of mathematical programming problems provides a powerful framework for the unified analysis and development of efficient solution algorithms for a wide range of equilibrium problems in economics, engineering, finance, and applied sciences. The book is published in two volumes, with the first volume concentrating on the basic theory and the second on iterative algorithms. Both volumes contain abundant exercises and feature extensive bibliographies.

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G. GRAS, University of Franche-Comte, Besancon, France

Global class field theory is a major achievement of algebraic number theory, based on the functorial properties of the reciprocity map and the existence theorem. In this book, the author works out the consequences and the practical use of these results by giving detailed studies and illustrations of classical subjects (classes, idèles, ray class fields, symbols, reciprocity laws, Hasse's principles, the Grunwald-Wang theorem, Hilbert's towers, etc.). This book, intermediary between the classical literature published in the sixties and recent computational one, gives much material in an elementary way, and is suitable for students, researchers, and all those who are fascinated by this theory.

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Numerical Methods and Diffpack Programming

SECOND EDITION

H.P. LANGTANGEN, University of Oslo, Norway

The target audience of this book is students and researchers in computational sciences who need to develop computer codes for solving partial differential equations. The exposition is focused on numerics and software related to mathematical models in solid and fluid mechanics. The book teaches finite element methods, and basic finite difference methods from a computational point of view. The main emphasis regards development of flexible computer programs, using the numerical library Diffpack. The second edition contains several new applications and projects; improved explanations; and correction of errors, and is up to date with Diffpack version 4.0.

2003/864 PP./HARDCOVER/\$69.95
ISBN 3-540-43416-X
TEXTS IN COMPUTATIONAL SCIENCE AND ENGINEERING, VOL. 1

NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS

P. KNABNER, Friedrich-Alexander-Universität, Erlangen, Germany; and L. ANGERMAN, Otto-von-Guericke-Universität, Magdeburg, Germany

This text provides an application-oriented introduction to the numerical methods for partial differential equations. It covers finite difference, finite element and finite volume methods, interweaving theory and applications throughout. Extensive exercises are provided throughout the text. Graduate students in mathematics, engineering and physics will find this book useful.

2003/440 PP., 50 ILLUS./HARDCOVER/\$69.95
ISBN 0-387-95449-X
TEXTS IN APPLIED MATHEMATICS, VOL. 44

NUMERICAL OPTIMIZATION

Theoretical and Practical Aspects

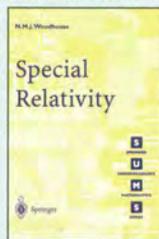
J.F. BONNANS and J.C. GILBERT, both, INRIA Rocquencourt, Le Chesnay, France; C. LEMARÉCHAL, INRIA Rhône-Alpes, Montbonnot, France; and C.A. SAGASTIZÁBAL, IMPA, Rio de Janeiro, Brazil

Numerical Optimization has numerous applications in engineering sciences, operations research, economics, finance, etc. Starting with illustrations of this ubiquitous character, this book is essentially devoted to numerical algorithms for optimization, which are exposed in a tutorial way. It covers fundamental algorithms as well as more specialized and advanced topics for unconstrained and constrained problems. This level of detail is intended to familiarize the reader with some of the crucial questions of numerical optimization: how algorithms operate, why they converge, difficulties that may be encountered and their possible remedies. Theoretical aspects of the chosen approaches are also addressed with care, often using minimal assumptions.

2003/437 PP./SOFTCOVER/\$54.95
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UNIVERSITEXT

SPECIAL RELATIVITY

N.M.J. WOODHOUSE, Oxford University, Oxford, UK



This book provides mathematics students with the tools they need to understand the physical basis of special relativity and leaves them with a confident mathematical understanding of Minkowski's picture of space-time. *Special*

Relativity is loosely based on the tried and tested course at Oxford, where extensive tutorials and problem classes support the lecture course. This is reflected in the book in the large number of examples and exercises, ranging from the rather simple through to the more involved and challenging. Written with the second year undergraduate in mind, the book will appeal to those studying the Special Relativity option in their Mathematics or Mathematics and Physics course. However, a graduate or lecturer wanting a rapid introduction to special relativity would benefit from the concise and precise nature of the book.

2002/192 PP., 17 ILLUS./SOFTCOVER/\$34.95
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