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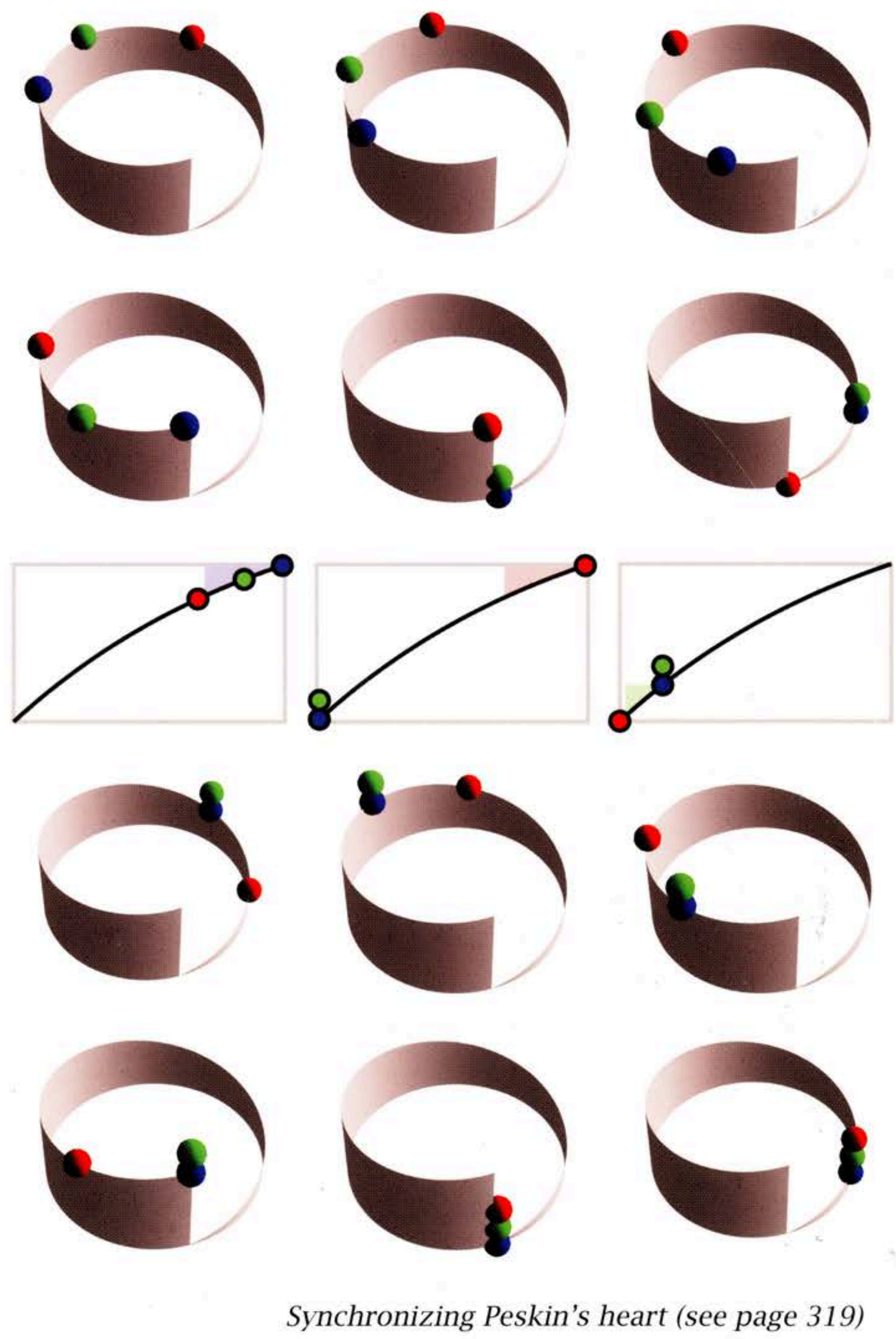

Synchronizing Peskin's heart (see page 319)

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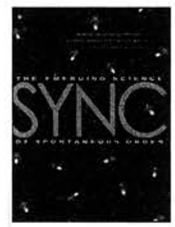
Feature Articles



Communications

Commentary

Notices of the American Mathematical Society

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Departments

From the AMS Secretary

Opinion

A Modest Proposal: Copyright and Scholarly Journals

I am a reactionary—at least when it comes to copyright. The first copyright law in 1709 provided protection for up to 28 years. In most countries today, copyright extends for 70 years beyond an author's life, which means copyright can extend for more than 150 years! The period of 28 years was a compromise between anarchy (zero) and perpetual monopoly (infinity). It balanced the rights and interests of publishers, authors, and the public. The balance worked.

Modern copyright law seems to work for novels, music, and movies, but it doesn't work well for scholarly journals. Journals record the knowledge of one generation for the next; they are long-term affairs. Recently, scholars discovered that copyright is a major impediment to making the older literature available online because obtaining permission decades after a journal was published was often impossible. And scholars realize that these difficulties will increase as we migrate to new formats in the future. For scholarly journals, copyright protection is an obstacle, not a safeguard.

We cannot change the laws that protect novels, movies, and music for the sake of scholarly journals. Publishers have persuaded both themselves and lawmakers that our present copyright laws provide the right balance for these creative works, and we are unlikely to change their minds. We should look for practical solutions, not ideological jousts.

What's a practical solution? Be reactionary—revert to the older traditions of copyright, without changing the law. We should urge scholars when publishing journal articles to dedicate their work to the public domain after 28 years.[1] Until then, authors and publishers control their work as at present (perhaps giving free access much earlier). After 28 years the work belongs to the world, in keeping with the historical traditions of copyright.

Why not merely insist that authors retain copyright? Author-held copyright sounds convincing at first because it sounds "fair". But think what happens in practice. A publisher produces a journal with thousands of articles by thousands of authors. If all the authors retain the copyright, but give the publisher exclusive rights to publish for a period of time, anyone who wants to make the journal available to the public after that period must contact thousands of authors to obtain permission. Some authors will decline permission; some will be hard to find; some will be missing altogether. And since the publisher already has permission, it's the publisher, not the authors or the public, who retains control.

The problem of copyright is not author rights or fairness—the problem is balance.

Publishers are usually horrified when they hear the 28-year proposal, because they misunderstand copyright's purpose. Publishers point out that they will lose revenue from sales of older issues. They will. But copyright is not meant to guarantee publishers every possible penny from a publication. Copyright is meant to provide financial incentives, not guarantees that no one else can make money.

Some publishers object that they will have no incentive to archive legacy material. That is only partially true. In any case, most scholars do not trust publishers to archive their material, and many publishers have little desire to archive. As the older literature falls into the public domain, many different groups will be able to set up archives with many different motives. That is the essence of good archiving, multiplicity.

Some authors object that they will lose control as their work moves into the public domain. And it is certainly true that someone in the future can appropriate a work, either in whole or in part, without attribution. But the legal system has seldom played a role in addressing these problems. No one sues for copyright infringement in such cases—the academic community maintains high standards, and those standards come into play when scholars exhibit poor scholarship.

Finally, publishers *and* authors object that an enterprising entrepreneur may rise up in the future to repackage journal articles as they fall into the public domain, creating new ways to make them available to the public and thereby making a profit. Is that bad? Surely authors, who shared none of the profit initially, should not object to sharing none in the future.

This is a practical proposal. Because journal publishers make most of their money in the first few years after publication, a shorter copyright still provides financial incentives. Because most authors recognize that scholarship derives from the work of countless scholars who came before, placing scholarly work in the public domain appears reasonable. And because the action is simple (unlike complicated licensing arrangements), dedicating work to the public domain is easy to understand.

Why should publishers give up their exclusive right to the material after 28 years? Why should they lose even small amounts of revenue? Why should they let anyone else profit? Because restoring balance to copyright serves the public interest. But also because it is in the publishers' self-interest—our journals are under attack by people who have little understanding of scholarly publishing, but who use the evident flaws in copyright to advance their cause.

We ought to leave a better copyright legacy for the next generation of scholars than the legacy left for us.

—John Ewing, Executive Director
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[1] *Some countries declare certain rights inalienable, making it difficult to devise simple statements that place a work in the public domain everywhere. One scheme for dedicating work to the public domain can be found at* `http://creativecommons.org`, *although this scheme is not directly relevant to the proposal here.*

Letters to the Editor

Long Division by Hand

In the November 2003 *Notices*, Anthony Ralston reviewed *California Dreaming*. In his review, Ralston said about doing long division by hand "...the skill itself has ceased to be useful."

I'm a mathematics undergraduate, and the following day (after reading the article) my teacher was going over the integration of rational functions. One of the methods used when the numerator is of higher degree than the denominator is division. In that case, knowing how to do long division by hand is very useful in dividing the functions.

I think that K–12 students should be taught to do long division (and other arithmetic) by hand simply to teach the basics and get a good foundation.

—*Patrick Baker*
West Virginia State College

(Received November 11, 2003)

California Dreaming Reviewer Partisan

Anthony Ralston's review of *California Dreaming* [November 2003] approvingly describes the author's perspective of the "California Math Wars": roughly paraphrased, *there are extremists on both sides; the solution to the controversy lies squarely in the middle*. If only it were that simple!

I have no first-hand knowledge of the "California Math Wars", but I experienced a similar controversy in Massachusetts quite directly. Together with a high school teacher (Bethe McBride), I carried out the final round of revisions of the Massachusetts Mathematics Curriculum Framework. It was adopted in July 2000 by the Massachusetts Board of Education over the passionate protest of the Massachusetts chapter of the NCTM [National Council of Teachers of Mathematics] and the NCTM president. *The Framework mandates a drill-and-kill approach*, the critics charged; *it is hostile to technology, violates the NCTM's Equity Principle, and deprives teachers of proper guidance*—in short, an extremist document!

What enraged the critics so much? The Massachusetts Framework asks for "...understanding of and the ability to use the conventional algorithms for addition and subtraction...and multiplication...," as well as "...division ...with a single-digit divisor." The Framework encourages the use of technology, including computers and calculators, but warns that "...appropriate use of calculators is essential; calculators should not be used as a replacement for basic understanding and skills....Elementary students should learn how to perform thoroughly the basic arithmetic operations independent of the use of a calculator." At the high school level, the Framework specifies learning standards separately for a sequence of integrated courses and for the standard single-subject sequence (Algebra I, Geometry, Algebra II, Precalculus); the standards for the two sequences are almost equivalent in content but are arranged in different chronological order. To the uninitiated, this requires explanation: the reformers are eager to abolish the single subject sequence—what better way to do that than to have the state frameworks effectively impose an integrated sequence on all students? The Massachusetts Framework is studiously neutral about teaching styles. It does not advocate exclusive use of discovery learning and group learning—thereby depriving teachers of guidance, in the view of its critics—but neither does it discourage a progressive approach in the classroom.

Standard algorithms, limits on the use of calculators, neutrality on the choice of an integrated curriculum over a single-subject sequence, and neutrality on teaching practices are seen as extremist positions by many—but far from all!—mathematics educators. Most academic mathematicians, on the other hand, regard these as unremarkable choices. Extreme or not, who is to decide? Parents, teachers, and politicians are the arbiters, whether or not the educators like it. In California and Massachusetts educators recently lost exclusive control of the framework adoption process, a control they had come to regard as rightfully theirs. The charges of extremism reflect this loss of control as much as they reflect issues of substance.

According to the reviewer, the author of *California Dreaming* occupies the middle ground in the mathematics education debate, where the reviewer presumably also places himself. The final section of the review elaborates his position. Ralston asks the reader to contemplate the recommendation of the Cockroft Report (Great Britain, 1983) to abandon teaching the long division algorithm now that calculators are widely available—a proposal that I personally do not dismiss out of hand. He goes on to question the wisdom of "teaching children to become adept at" using the algorithms for addition, subtraction, and multiplication.

When the reviewer writes for an audience of educators, his language becomes less guarded. In a 1999 paper in the *Journal of Computers in Mathematics and Science Teaching*, Ralston "proposes that paper-and-pencil arithmetic no longer be taught ...," to be replaced by an emphasis on mental computation, with "calculators...used for instructional purposes in all grades including kindergarten". Does Ralston believe this is a middle-ground position? Maybe not; in any case he knows who stands in the way of his grand vision of mathematics education: "politicians, parents, mathematicians—all those antediluvian groups." After an on-the-record comment like that—even if it was made in jest—Ralston should have had the good sense to disqualify himself from reviewing *California Dreaming* for the *Notices*. At the very least, he should have identified himself as a partisan in this debate.

—*Wilfried Schmid*
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Response

Prof. Schmid suggests that I should have recused myself as a reviewer of *California Dreaming* because I am a "partisan" in the Math Wars. But I reject the implicit assumption that only someone with no prior views on a controversial subject is fit to review a book on that subject.

Yes, I have strong opinions on issues related to the Math Wars, as evidenced by the excerpts from a paper of mine that Prof. Schmid so kindly quoted in the last paragraph of his letter. But my language was more "guarded" in my review than in my paper, not because the *Notices* audience is different, but because the opinions expressed in the paper would have been inappropriate in a review of this book anywhere.

Indeed, I am not really a partisan in the Math Wars. While I deplore almost all the pronouncements on one side of these wars, I am not sympathetic to much of what is attributed to the other side. Most of what I have said or written about school mathematics education has few supporters among mathematicians or mathematics educators.

—*Anthony Ralston*
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More Examples of Fuzzy Math

I would like to dispute two points made by Anthony Ralston in his lengthy, fair, and balanced review of Suzanne Wilson's fair and balanced book on the California math wars. First, Ralston states that mathematical errors in the *NCTM Standards* are minor and trivial. Possibly so, but they show that the drafters of the standards were unable to distinguish mathematical right from wrong. The New York City math standards claim that "it is common to approximate $\sqrt{a+b}$ by $\sqrt{a}+\sqrt{b}$." It is unsettling that the author of this (unfortunately true) statement is helping to guide the mathematical education of hundreds of thousands of kids. The present California math standards might be misguided, but at least the authors knew enough math to have an informed opinion.

Then there is the nature of research supporting fuzzy math. Anyone who has opposed fuzzy math in a public forum has been frustrated by claims that "research" supports this or that. For example, there is the "research-based" claim that a kid's ability to learn mathematics is lessened if the kid is exposed to the standard algorithms of arithmetic. This chestnut was repeated, for example, in a letter to New York City parents from school officials. Ralston thinks the pile of fuzzy math research is so high that not all of it can be wrong, but he does not cite any specific research that he feels is useful. The fuzzy math studies I've read have at least one of the following traits: (i) evidence is anecdotal rather than statistical; (ii) proper controls are not applied; (iii) the authors have a conflict of interest; (iv) differences in performance, while statistically significant (small probability of being zero, assuming all kids are independent), are not educationally significant (small percentage difference); (v) do not publish enough raw data for independent evaluation.

—*Jonathan Goodman*
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New Math

Anthony Ralston's book review (November 2003) "California Dreaming: Reforming Mathematics Education" pointed out that the New Math curricula "were mainly the brainchild of university mathematicians." It is equally true, however, that many mathematicians criticized the excessive formalism and abstraction of the New Math. This is demonstrated by the memorandum, signed by 75 leading mathematicians, that was published in the March 1962 issue of *The American Mathematical Monthly* ("On the mathematics curriculum of the high school", pp.189–193).

Ralston also claimed that the new math failed "most notably because teachers were not prepared to teach this mathematics nor were they given adequate support." This major canard continues to be repeated by those who are unable to accept responsibility for having demolished the traditional college preparatory mathematics curriculum in the United States.

Ralston's comment that the New Math's "trajectory spann[ed] little more than the decade of the 1960s" is patently absurd. In fact, the New Math strand developed by E. G. Begle's School Mathematics Study Group (SMSG) had become completely institutionalized by 1970. This was accomplished through the Houghton Mifflin series of books, which were copied by other publishers, coauthored by Mary P. Dolciani. A priceless article describing how this occurred was published in the *Everett School News*, Vol. 3, No. 2, Everett, Massachusetts (Winter 1970). This article is posted at:

http://mathforum.org/epigone/math-teach/blerdzhiclon/

and was reprinted as a filler item in *The American Mathematical Monthly*, January 2002, page 12.

The impeccable SMSG credentials of Dolciani and her coauthors are posted at:

http://mathforum.org/epigone/math-teach/skypablimp/tj3dd0o2xmip@forum.mathforum.com/.

—*Domenico Rosa*
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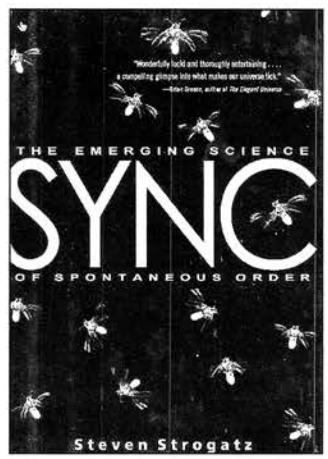
Review of *Sync*: The Emerging Science of Spontaneous Order

Reviewed by G. Bard Ermentrout

Sync: The Emerging Science of Spontaneous Order
Steve Strogatz
Hyperion Press, 2003
338 pages, $24.95
ISBN 0-786-86844-9

Is the universe a chaotic mess? Or, is it possible that there is much more order than we realize? The obvious answer to the second question is a resounding yes. Otherwise, we would not be here to report on a wonderful new book by Steve Strogatz, *Sync: The Emerging Science of Spontaneous Order*. The recent popularity of books on chaos and complexity might lead their readers to assume that chaos and complexity are the rule. Instead, what Strogatz presents, through his personal scientific experiences, is a contrasting point of view. Interactions between individuals—be they fireflies, pendula, or people—can often lead to the emergence of coherent actions. The goal of this readable and intuitive book is to convince us that this is so through dozens of examples ranging from the spread of rumors to Josephson junctions. The book is autobiographical in tone, and Strogatz plays a central role in much of the science that is described. Part of the fun in reading this book is the sense of excitement that ensues from the mere act of doing science and the discovery of a solution to a long-pondered problem. Phrases like "my hand was sweating as I wrote each new line of the calculation" or "I unleashed the computer and stared at the screen" convey the passionate appeal of mathematical creativity.



Strogatz has condensed and simplified many complex and technical subjects into easily understood stories which, remarkably, are completely equation-free. He achieves this through clever analogies such as runners on a circular track (for synchronization of phase-oscillators) or flushing toilets (for excitable media). This is good for the scientifically literate but mathematically naive reader. For me, it was occasionally a source of frustration, since I wanted more details of the underlying phenomena. Of course, for the interested reader, Strogatz provides thirty-three pages of endnotes which include references, sources, and, importantly, the many caveats. One of the major strengths of the book is the way in which Strogatz finds so many connections between seemingly disparate phenomena. But this is also a weakness, as important differences between the mechanisms underlying these phenomena are sometimes glossed over.

Sync is organized into three areas, each divided into several chapters: sync in living systems (cells, animals, people), sync in inanimate objects (pendula, lasers, electrons), and the "frontiers" of sync (chaos, small worlds, vortex rings). The first three

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chapters of the book cover Strogatz's (and others's) work on pulse-coupled oscillators, globally coupled phase oscillators ("Kuramoto" model), and sleep rhythms. The middle three chapters cover pendula and planets; Josephson junctions and their connections to the Kuramoto model; and my favorite of this section, the instability of the Millennium Bridge in London. The last four chapters describe results on the synchronization of chaotic attractors, Strogatz's and Art Winfree's work on the topology of singular filaments in excitable media, his work with Duncan Watts on "small-world" networks, and finally a summation which makes a few allusions to synchrony and cognition.

In the remainder of this review I will attempt to outline some of the mathematics that is hidden in the pages of this book. I hope to also point out places where the conclusions based on the simple models hold firmly with some generality or fall to pieces with the simplest modifications. The first two sections of the book deal with coupled oscillators in the strict sense; each individual generates a stable periodic solution. At issue is whether there is any collective organization once these individuals are coupled together. Almost all of Strogatz's theoretical work deals with the case of "all-to-all" coupling: the effects of any one unit on another are the same for all possible pairs. In the final section of the book he explores systems which either are not intrinsically periodic or in which the connectivity pattern has additional structure.

The term "sync" is short for "synchrony", by which Strogatz means the emergence of order in time. Depending on whom you are talking to, "synchrony" can have different meanings even within the context of periodic phenomena. In the strictest sense it means that each unit oscillator follows an identical trajectory, $x_j(t) = X(t)$ for all units j. I will call this strict synchrony. Alternatively, "synchrony" is used interchangeably with "phase-locked"; each unit is firing with the same period but there are phase-shifts. Finally, there is a notion of synchrony from statistical physics. Consider

$$X(t) = \frac{1}{N} \sum_j x_j(t)$$

in the limit as $N \to \infty$. If $X(t) = C$ a constant, then the oscillators are said to be asynchronous. The emergence of large temporal fluctuations of $X(t)$ is often defined to be the onset of synchrony. Thus, it is the appearance of temporal order where there was none before. I will make this latter definition more precise when I need it. For systems in which the individuals are, say, chaotic, synchrony is defined as having identical trajectories.

An astonishing example of spontaneous order occurs in Southeast Asia, when thousands of male fireflies congregate in trees and synchronously flash at about once per second. Strogatz begins his book with the following quote from Philip Laurent writing in *Science* in 1917 [1]:

> Some twenty years ago I saw, or thought I saw, a synchronal or simultaneous flashing of fireflies. I could hardly believe my eyes, for such a thing to occur among insects is certainly contrary to all natural laws.

This intriguing phenomena was rigorously addressed by John Buck and his colleagues over a period from the 1930s to the 1980s [2], but the theoretical mechanisms remained unknown. In 1975 Charlie Peskin suggested a simple model for the synchronization of two oscillators, each of which obeys the equation

$$\frac{dx_j}{dt} = I - x_j, \quad I > 1 \tag{1}$$

and such that each time $x_j(t)$ crosses 1 it is reset to 0 and the other oscillator is incremented by an amount ϵ. Peskin analyzed the $N = 2$ oscillator case by explicitly solving the ODEs. Strogatz and his colleague Rennie Mirollo attacked the analogue to this problem for arbitrary N and proved that except for a set of initial data with measure zero, all solutions will synchronize [3]. Thus, they were among the first to rigorously analyze a system of pulse-coupled oscillators. In the introduction of their subsequent paper, they hinted that this global pulse-like coupling provided the answer to the enigma of firefly synchronization. In the discussion section of said paper and here in the endnotes of *Sync*, they explain that the actual mechanism for fireflies is very much different from the simple generalization of the Peskin model.

Instead of restricting their system to the above equation, they assume that each oscillator follows a proscribed temporal profile, $x_j(t) = f(t)$ with $f(0) = 0$ and $f(1) = 1$. (I assume that the period is 1 without loss of generality.) Thus, if an oscillator fires (hits 1), it is reset and all others are given a boost of size ϵ. If the kick is enough to push x_j past 1, then this oscillator is "absorbed" into the pool of synchronized oscillators. (That is, $x_j(t) = \max(1, \epsilon + f(t))$.) We now come to some of the "difficulties" with this model. First, should the effects of m oscillators firing together add $m\epsilon$ to all the others? If the impulse of a firing event pushes an oscillator over 1, then does that instantly increment all others? (In their simulations, they don't allow the oscillators that were pushed over to fire until the next iteration.) The method of proof is very clever and can be understood by considering the case $N = 2$. The idea is to create a map for the phase of oscillator B when oscillator A crosses 1. Let $\phi = g(x_B + \epsilon) \equiv f^{-1}(x_B + \epsilon)$ be the phase of B after A fires. In the time it takes B to fire, A moves to $x_A = f(1 - \phi)$ and then jumps to $\epsilon + x_A$

which has phase $g(\epsilon + f(1-\phi)) \equiv h(\phi)$. Since all we have done is interchange the roles of A and B, the return map is just $R(\phi) = h(h(\phi))$. Strogatz and Mirollo show that there is a unique fixed point and that it is a repellor. To show the latter, they note that

$$h'(\phi) = -\frac{g'(u+\epsilon)}{g'(u)}$$

for some $u \in (0,1)$. They now invoke the first of their hypotheses: $f(t)$ is concave down. This means that $g'' > 0$ so that $h' < -1$ and thus $R' > 1$ for all ϕ. Since there is a unique fixed point and it is a repellor, iterations of the map must go to 0 or 1, which is why the hypothesis of absorption is crucial. Then, with technical difficulties aside, they prove the general N case; any fixed points to the resulting map are unstable. The synchronous state is not really even a fixed point of the map. If one starts with A at 0 and B arbitrarily close to but less than 1, then the phases of A and B are arbitrarily close to each other on the circle. However, they will not stay close; B will fire and A will jump, and they will be nearly ϵ apart until A fires again. Small changes in the model such as heterogeneity mean that the system can synchronize only in the sense of absorption. Since synchrony is not a fixed point of the system, there is no notion of local stability, so that small changes in the model can have dramatic effects. If the coupling is not "all-to-all", another problem occurs: not all oscillators will be advanced equally when there is a firing of some group. For example, with nearest neighbors, is synchrony inevitable in this model?

How does this simple model compare to real fireflies or other systems coupled in a pulsatile fashion? Little is known about the transduction of the visual signal to the alteration of the oscillator. However, it is possible to quantify this effect by looking at phase transition curves (PTC). If a pulse stimulus is given to an oscillator at phase ϕ, then the phase will often be shifted to a new phase $F(\phi)$; this function is called the phase transition curve. Strogatz's mentor, Art Winfree, tabulated and measured such curves for a variety of biological oscillators. Frank Hanson (who worked with Buck) measured $F(\phi)$ for several firefly species [4]. In the Strogatz-Mirollo (SM) model, the phase is *always advanced* from a stimulus no matter what the phase. Furthermore, $F(\phi)$ is not one-to-one, since a nonzero interval of phases is mapped to 1. However, in the insect *Pteroptyx malaccae* the PTC is qualitatively like

$$F(\phi) = \phi - \epsilon \sin 2\pi\phi$$

where ϵ is a small positive number. This PTC obviates some of the difficulties that are inherent in the SM model. As long as ϵ is small enough, F is monotone, so no single stimulus can ever make the oscillator fire instantly. Furthermore, the analogous return map for this PTC has 0 as an asymptotically stable fixed point. In the all-to-all case, it can also be shown that synchrony is asymptotically stable for this PTC [5]. Whether sync is inevitable in the all-to-all coupled case with the sinusoidal PTC remains an open mathematical question.

In my opinion, Strogatz's best work is his incisive analysis of the Kuramoto model described in Chapter 2. (This was the model which led to his aforementioned sweaty palms.) The Kuramoto model is a simple coupled system of phase oscillators:

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} \sin(\theta_j - \theta_i). \tag{2}$$

Such models arise through averaging of systems of weakly coupled nonlinear oscillators. Depending on the specifics of the model, the function $\sin(\theta)$ is replaced by an arbitrary periodic function. In a later chapter in *Sync*, Strogatz shows that the Kuramoto model is exact when the physical system is an array of Josephson junctions. When Kuramoto first proposed the model, he considered it to be a toy model for the interactions of biological oscillators. All-to-all coupling has many advantages over interactions with a more specific topology, where details of the boundary conditions and heterogeneities make a general analysis impossible. Kuramoto [6] was interested in the behavior as a function of the coupling parameter K when ω_i are drawn from a symmetric probability density function $g(\omega)$ with zero mean. Numerical solutions for large N revealed that as K increased, there was a transition from complete disorder to order. Key to his subsequent analysis was the fact that the sum in (2) can be written as

$$-R\sin(\theta_i) + Q\cos(\theta_i)$$

where

$$R = \frac{1}{N}\sum_j \cos\theta_j, \qquad Q = \frac{1}{N}\sum_j \sin\theta_j.$$

Kuramoto argued that since the frequency distribution was symmetric about 0, then Q should be zero, for then the equations are symmetric under the transformation $\theta \to -\theta$. Since the mean frequency of the oscillators is zero, he needed to solve:

$$0 = \omega_i - KR\sin\theta_i \tag{3}$$

along with the self-consistency condition

$$R = \frac{1}{N}\sum_j \cos\theta_j. \tag{4}$$

Kuramoto's insight was to divide the oscillators into two groups: those for which equation (3) has a solution ($|\omega| < KR$) and those for which it does not. The latter oscillators drift around the circle, and Kuramoto intuited that their contribution to (4) should be zero over long time. The sum is just the average of $\cos\theta$ over the frequency distribution defined by $g(\omega)$, so that we can replace it by the integral

$$R = \int_{-KR}^{KR} \cos(\theta(\omega))g(\omega)d\omega$$

where $\sin(\theta(\omega)) = \omega/KR$. A change of variables leads to the following equation for R:

$$R = KR \int_{-1}^{1} g(KR\sigma)\sqrt{1-\sigma^2}\, d\sigma. \tag{5}$$

Clearly, $R = 0$ is always a solution. However, a nonzero solution bifurcates at the critical value

$$K_c = \frac{2}{\pi g(0)}.$$

This is a clever argument, but it is completely heuristic. Strogatz relates how he and Nancy Kopell worked on trying to make this rigorous with no results and how he was plagued by it for many years. He then describes in breathless prose how it came to him in a near-dream state: the oscillators are not runners on a track, but like a fluid. Thus, he was led to write an equation for the *density* of oscillators at phase θ and with natural frequency ω. The density evolves as

$$\frac{\partial \rho}{\partial t} = -\frac{\partial(v\rho)}{\partial \theta} \tag{6}$$

where v is the phase velocity given by

$$v(\theta,t) = \omega + K\int_{-\infty}^{\infty} g(\sigma)d\sigma \times \int_0^{2\pi} d\phi \sin(\phi-\theta)\rho(\phi,\sigma,t).$$

This approach has many advantages over the Kuramoto approach. It does not depend on the fact that the interaction is sinusoidal. Furthermore, additive noise in the original model simply adds an additional flux term, $D\partial^2\rho/\partial\theta^2$, to the density equation. The key point is to notice that the asynchronous state is the state in which the phases are uniformly distributed around the circle: $\rho(\theta,\omega,t) = 1/2\pi$. Clearly, this is a stationary state for (6). Thus, the question that Strogatz asked was, how does the stability depend on the strength of the coupling K? Linearizing about this stationary solution leads to an eigenvalue problem whose solution is

$$y(\theta,\omega,t) = e^{\lambda t}e^{i\theta}b(\omega)$$

and

$$\lambda b(\omega) = -i\omega + \frac{K}{2}\int_{-\infty}^{\infty} g(\omega)b(\omega)\, d\omega.$$

Solving this for the discrete eigenvalue λ yields

$$1 = \frac{K}{2}\int_{-\infty}^{\infty} \frac{\lambda}{\lambda^2+\omega^2} g(\omega)\, d\omega.$$

Rescaling $\omega = \lambda\sigma$ results in

$$1 = \frac{K}{2}\int_{-\infty}^{\infty} \frac{g(\lambda\sigma)}{1+\sigma^2}\, d\sigma.$$

Bifurcation occurs in the limit as $\lambda \to 0$, yielding the Kuramoto result:

$$1 = \frac{\pi K_c}{2} g(0).$$

There are many subtleties to this full eigenvalue problem (the essential spectrum), and the reader is urged to consult the lovely review article [7]. But the bottom line is that the phase density approach finally resulted in a rigorous solution to Kuramoto's model. (I should point out that the large but finite N case is still the subject of current research.) This approach remains the method of choice when trying to understand the stability of the asynchronous state. This elegant calculation is the mathematics which underlies Strogatz's statement that "sync" is inevitable. How general is this statement? J. D. Crawford answered this question in a series of papers on the model

$$\frac{d\theta_j}{dt} = \omega_j + \frac{K}{N}\sum_k H(\theta_k - \theta_j)$$

where H is a more general interaction function. For example, if $H(\theta) = \sum_n c_n \sin n\theta$, then for each n there is a critical coupling strength,

$$K_n = \frac{2n}{g(0)\pi c_n}.$$

If K_n is the minimal for $n = n_0$, then the probability distribution which bifurcates will have n_0 peaks. The Kuramoto case corresponds to $n_0 = 1$ and so there is "sync". However, if, e.g., $n_0 = 2$, then a two-cluster state will bifurcate. Thus, "sync" is not really inevitable; it was an accident of the model choice.

The daily 24-hour rhythms which govern our everyday behavior and which are the subject of Chapter 3 are controlled by a small set of neurons buried deep in a part of the brain called the suprachiasmatic nucleus (SCN). In the last twenty years, biologists have uncovered the mechanisms of the genesis of these rhythms. However, the coupling between them remains mysterious. It has been suggested that coupling is through the release of a chemical transmitter (gamma aminobutyric acid, or GABA) that has been shown to affect the phase of individual oscillators. It is possible that this transmitter is pooled so that the cells are effectively coupled in an all-to-all manner. But coupled oscillators play little role in this chapter of *Sync*. Rather, Strogatz focuses on the rather extreme experiments on human subjects kept in isolation from all time cues. In early experiments, volunteers were also isolated from other people and exhibited a variety of psychological problems. Most interestingly, after a long period with no temporal cues, the sleep-wake cycle "separates" from the daily body temperature fluctuations that are controlled by the SCN. This spontaneous desynchronization is what captured the attention of theoreticians. The main idea of the theory is that there is a sleep-wake oscillator and a circadian oscillator that are coupled together but have different intrinsic frequencies. Both are driven by the day-night cycle that keeps them entrained at a 24-hour rhythm. In the absence of the temporal cues, the coupling between the two oscillators is not enough to maintain locking, and they drift apart. Strogatz spends a majority of the chapter describing his work with Dick Kronauer, a modeler of circadian rhythms, and how their data revealed that during certain times of the day it is very difficult to stay awake (around 2:00 p.m., siesta time in many reasonable societies) or go to sleep (around 10:00 p.m., dinner time in those same societies).

The next three chapters deal with physical systems; the most notable (at least to me) is Huygens's clocks. This is a classic example of synchrony in which two pendulum clocks mounted on a beam phase-lock so that they operate a half a cycle out of phase. Strogatz refers to a recent paper by Matthew Bennett and others [8] in which the mystery of this locking is finally solved. Bennett's model is two pendula attached to a rigid beam that is allowed to move vertically. This movement is crucial, since without it the two pendula could not be coupled. It is easy to write down a Lagrangian for this three degree-of-freedom model and solve it numerically. By a simple linear approximation, they reduce the behavior to a map. With this map, Bennett et al. show that Huygens was incredibly lucky: had his pendula been heavier, they would have been too weakly coupled to phase lock; and had they been much lighter, then a solution in which one pendulum rocks and the other is silenced would have occurred. Strogatz uses the example of Huygens's clocks as a perfect illustration of how inanimate objects can sync. The remainder of Chapter 4 gives more examples, such as the electrical grid and the famous Kirkwood gaps in the asteroid belt between Mars and Jupiter. Kirkwood noticed that these occur at radii in which the orbital period is resonant with that of Jupiter. It took another hundred years before a theory was developed that showed this resonance could produce gaps [9].

Chapter 5 describes, mostly through analogies, sync at the quantum level. Strogatz has a nice description of how lasers work, an example of quantum phase coherence. The idea is that occasional photons will join up in sync, and these will recruit more and more of them, and thus there is a positive feedback to produce phase-coherent light. He closes this chapter with a biography of one of the most interesting characters in quantum mechanics, Brian Josephson. The final chapter in this part of the book contains the majority of Strogatz's contributions to "inanimate sync". He makes connections between quantum sync and the Kuramoto model through his prolific work on coupled Josephson junctions which obey the following differential equations:

$$\begin{aligned} \beta_j \phi_j'' + \phi_j' + \sin\phi_j + Q' &= I_j \\ LQ'' + RQ' + C^{-1}Q &= \frac{1}{N}\sum_k \phi_k'. \end{aligned}$$

He first describes observations on the existence of an invariant 2-torus when $C^{-1} = L = 0$. This is a difficult concept to explain to mathematicians, let alone a general audience. But he neatly trumps this difficulty by describing the torus as a sequence of nested Russian dolls. In the limit of small Q' ("weak coupling"), it is possible to average the junction equations, and this is how he ends up with Kuramoto's original model of sinusoidal all-to-all coupling. Thus, while it may be hard to find a biological system that reduces to the exact form of Kuramoto's famous equations, Strogatz has shown that at least one physical system is equivalent.

The middle section of the book concludes with a great description of the Millennium Bridge in London. This bridge was built to celebrate the new millennium, and at its inaugural with all the television cameras running, hundreds of people began to walk across it. As Strogatz notes, "Within minutes it began to wobble, 690 tons of steel and aluminum swaying in a lateral S-shaped vibration like a snake slithering on the ground." He presents a mechanism based on a combination of resonance and phase-locking. This is not like the famous Tacoma Narrows Bridge, which broke apart when wind of a critical velocity blew across it. Rather, when people walk they tend to swing from side to side at roughly two strides per second. Engineers found that the bridge itself had a resonant instability of about one cycle per second. If through purely random chance a number of people transiently synchronized their gaits, this net force could begin to destabilize the bridge so that it starts rocking slightly. If you try to walk on a swaying platform, you tend to compensate by stepping in sync with it. Thus this initially small kernel of synchronous steppers recruited more and more walkers, which further destabilized the bridge, leading to the moral equivalent of the Kuramoto model! As a last word on inanimate sync, Strogatz notes (with an implied wink) that three days after the fiasco (and several months before the engineers analyzed the problem), a letter appeared in *The Guardian* giving essentially the above analysis of the phenomena. The author: Brian Josephson.

Part three of the book covers what is called the "frontiers of sync". I am not exactly sure what this means; Strogatz's work on singular filaments in excitable media (Chapter 8) was done nearly twenty years ago. Nevertheless, the other chapters in this section describe more recent work on chaotic synchronization, small worlds, and a very small bit on neuronal synchrony.

Lou Pecora's work occupies a good part of the text in Chapter 7. He has written a number of papers on synchronization of chaotic systems [10]. Chaotic synchronization asks the simple question as to whether two or more coupled chaotic systems will stably synchronize. Let me start with a very general model that will work for periodic oscillations as well as synchronous rhythms. Consider:

$$\frac{dX_j}{dt} = F(X_j) + \sum_k c_{jk} M(X_k, X_j), \quad j = 1, \dots, N,$$

where $X \in R^m$ and $M(X, X) = 0$. The coupling strength is encoded in the scalars c_{jk}. Suppose that $X' = F(X)$ has a solution $U(t)$ that may be chaotic or periodic but which is time-varying. Clearly $X_j(t) = U(t)$ is a synchronous solution. Stability is determined from the linearized equation $X_j = U + Y_j$ where

$$\frac{dY_j}{dt} = A(t)Y_j + \sum_k d_{jk} B(t) Y_k.$$

where A, B are time-varying $m \times m$ matrices and d_{jk} is the same as c_{jk} except for $j = k$ where $d_{jj} = c_{jj} - \sum_k c_{jk}$. Let (ν, Ψ) be an eigenvalue-eigenvector pair for the $N \times N$ matrix $D = (d_{jk})$. Write $Y_j = \Psi_j Z$ where $Z \in R^m$ and Ψ_j is the j^{th} component of Ψ. This has the effect of putting the system into block diagonal form, leaving us with only the set of problems:

$$\frac{dY}{dt} = A(t)Y(t) + \nu B(t)Y(t) \tag{7}$$

where ν is an eigenvalue of D. Suppose the solution $U(t)$ is chaotic. Then stability is determined by looking at the maximal Liapunov exponents of (7). Since 0 is an eigenvalue of D corresponding to homogeneous perturbations, this means that for a chaotic system there is always at least one positive Liapunov exponent. But, *it is in the homogeneous eigenspace*; thus this instability cannot break the symmetry of synchrony. Pecora's idea for chaotic synchronization is very easy. Find regions in the complex ν-plane for which the Liapunov exponents are all negative. This will characterize the types of coupling that can stabilize the synchronous chaotic state. Suppose, for example, $B(t) = bI$ where b is a scalar and the maximal Liapunov exponent of the uncoupled system is λ. If the nonzero eigenvalues of D are negative, then for b large enough, the synchronous system is stable. If $B(t)$ is a constant nonscalar matrix, then there can be limitations on the strength of the coupling in order to get chaotic synchronization. The methods described here work as well for coupled oscillators. Stability of the synchronous chaotic state is an exercise in linear algebra coupled with numerics.

In the first two-thirds of *Sync* the interactions between individuals is "all-to-all", so that the behavior of the ensemble is pretty simple: there is synchrony or asynchrony, but, of course, no spatial structure. Not until Chapter 8, "Sync in Three Dimensions", does Strogatz break away from this simplifying constraint. Here he first describes how he came to know one of his heroes, Art Winfree, who tragically died last year and to whom the book is dedicated. Winfree was studying active media in the form of the Belousov-Zhabotinsky (BZ) reagent, a mixture of chemicals which generates spontaneous oscillations and spiral waves in a thin layer. (When I first met Winfree at a conference, he passed out specially treated millipore filters and small vials of clear liquid. When you poured the liquid on the filter paper, the BZ reaction took place and spiral waves appeared miraculously on the paper.) Winfree was interested in what could happen in three dimensions and recruited Strogatz to work with him one summer. Imagine a stack of spiral waves exactly lined up; the centers or cores of the spirals produce a straight line (called a singular filament). This produces a scroll wave. Now suppose that we could take such a filament and join the ends to form a ring. The resulting object is called a scroll ring, and associated with each point on the ring is a two-dimensional spiral wave. It is easy to imagine more complex objects such as a trefoil knotted ring or linked rings. In a series of papers published between 1983 and 1984, Winfree and Strogatz [11], [12] described the kinds of patterns that are consistent with chemical and biological excitable media. First, consider a simple planar spiral wave. The isophase lines of this spiral converge in a phase-singularity at the center of the spiral wave. Thus, our filament represents a curve of phase singularities. Consider the simple twisted scroll ring which is obtained as follows. Take a scroll (spiral waves stacked up with the arms aligned) and give it a 360-degree twist and then join the ends. Now imagine a torus that encloses the twisted ring. A surface of the wave at a fixed phase intersects the torus in a closed ring. At the equator of the torus, the isophase points wind around exactly once. Now plug the hole in the torus with a disk; the isophase lines must come together at *another* singularity. Thus the twisted scroll ring cannot exist in isolation. Using clever arguments like this, Winfree and Strogatz show that more exotic possibilities are realizable: for example, a pair of linked twisted scroll rings. The above-mentioned trefoil knot *untwisted* cannot exist; however, with a twist it is realizable. In later work (not with Strogatz) Winfree simulated a simple excitable medium and found apparently stable (or at least long-lasting) knotted twisted scroll rings.

In my mind this chapter is not so well connected to the rest of the book. Let me try to smooth the seams a bit by returning to the coupled phase-models that play a role in Chapter 2 of *Sync*:

$$\frac{d\theta_j}{dt} = \omega_j + \sum_k c_{jk} H(\theta_k - \theta_j), \qquad j = 1, \ldots, N. \tag{8}$$

The Kuramoto model is a special case of this general coupling, where $H(\theta) = \sin\theta$ and $c_{jk} = 1/N$. We have already seen that for the all-to-all case the Kuramoto story is more complex if the coupling is not a pure sinusoid. The zoo of exotic waves described in this chapter is far more complex than sync. Sync for these media, which are essentially homogeneous, would represent either a stable resting state or a bulk oscillation. For chemical and other media these simple states are stable. So where do these exotic creatures originate? The answer lies in the interactions in the medium. Unlike all of the previous models in *Sync*, interactions between individual elements are local in space; coupling is not "all-to-all". Stirring the chemical bath destroys any hope of finding spirals and scroll waves. So, let's make the question really simple. Is sync inevitable in equation (8) when coupling is local and the medium is homogeneous? Consider first a ring of nearest neighbor sinusoidal phase oscillators:

$$\frac{d\theta_j}{dt} = \omega + \sin(\theta_{j+1} - \theta_j) + \sin(\theta_{j-1} - \theta_j), \qquad j = 1, \ldots, N.$$

We identify $j = 0$ with $j = N$ and $j = N + 1$ with $j = 1$. One possible solution to this model is a rotating wave, $\theta_j = \omega t + 2\pi j/N$. It is asymptotically stable as long as $N > 4$. However, if we increase the range of the coupling sufficiently far beyond the nearest neighbor coupling, all that can occur is sync. One might object that the ring is special compared to a simple linear array. Indeed, for a linear nearest-neighbor array of sine models, the following can be proved: suppose that $-\pi/2 < \theta_{j+1}(0) - \theta_j(0) < \pi/2$. Then all solutions go to synchrony. So, based on this, one could conjecture that for *nearest-neighbor linear arrays of sine oscillators*, synchrony is inevitable.

However, what may be surprising is the behavior when we go to two dimensions. Sync is not inevitable for a nearest-neighbor array of sine oscillators on a square grid:

$$\frac{\theta_{ij}}{dt} = \omega + \sum_{(i',j') \in N(i,j)} \sin(\theta_{i'j'} - \theta_{ij}),$$

where $N(i, j)$ is the set of nearest neighbors of (i, j). Thus $(1, 1)$ only has two neighbors, $(1, 2)$ and $(2, 1)$. For a nearest-neighbor sine model on an $2m \times 2m$ grid $(m > 1)$, there are stable rotating patterns that are the discrete analogues of spiral waves in the two-dimensional excitable medium [13]. For m small the basin of attraction of these oscillating but nonsynchronous patterns is small, but for m large the basin for synchrony shrinks. Thus spirals and other patterns are consequences of having a well-defined notion of space such as in locally coupled media. Stacking these sine-model rotating spirals up in a three-dimensional array results in the analogue of a simple scroll wave. Whether exotic structures like scroll rings exist in three-dimensional arrays of locally coupled oscillators remains an open question.

The penultimate chapter in *Sync* covers Strogatz's work with Duncan Watts on the "six degrees of separation" problem. There is a famous game called the "Kevin Bacon" game that started in a Pennsylvania college. The idea is to pick any Hollywood actor and connect him to Kevin Bacon in as few steps as possible. For example, to connect Kevin Bacon to Woody Allen, we note that Bacon was in *Footloose* with Dianne Wiest, who was in *Hannah and Her Sisters* with Woody Allen. Mathematicians play this game with Paul Erdős. You can imagine a network with each actor as a node and links drawn between each actor that appeared in a movie together. Such a network is somewhere between a completely local one and one which is completely random. Many realistic networks have a similar structure to the Erdős and Bacon networks, such as the Internet, the national power grid, the neurons of the nematode *C. elegans*, and many social networks. There are many local links and a few nodes that have links to many other nodes, some quite distant. This type of network is called a "small world network". The material in this chapter is mainly taken from Watts's Ph.D. thesis and subsequent book [15], reviewed in the *Notices* [14]. I should point out that Barabási and his collaborators have developed an alternate theory for scale-free networks [16].

The final chapter is a miscellany of results on the "human side of sync", covering such things as the spread of fads and an interesting phenomena regarding hand clapping. In Eastern Europe, after a good concert the audience often claps rhythmically. This rhythmic clapping can suddenly become asynchronous and then switch back to synchrony. This was recently quantified in [17] by looking at the maximal correlation between a ceiling recording of the audience, $c(t)$, and a sinusoidal function of t. The mechanism is rather interesting. In order to synchronize, the audience must slow down their clapping. But this results in a diminution of the sound intensity of the clapping, contradicting their desire to make the applause as loud as possible. Thus, they speed up and lose synchrony. This leads to a waxing and waning of the synchronous behavior. At high frequencies the spread of individual clapping rates is much greater than at low frequencies, so that the authors suggest that the onset of sync at low frequencies is through the Kuramoto mechanism. There is another possible reason for the loss of synchrony: coupled neural oscillators can often switch from synchrony to asynchrony as the frequency changes, even when the oscillators have identical frequencies [18], [19].

This leads me to the last part of *Sync*, in which Strogatz briefly dives into the neural synchrony tar pit. He touches on work by Charlie Gray, Wolf Singer, and others in which it was suggested that synchronous oscillations may have some importance in cognition; this remains a major controversy in the field. He also cites the famous story in which hundreds of Japanese got seizures from watching an episode of *Pokemon* that featured flashing red lights at a particularly sensitive frequency. The highlight of this chapter, however, is his very funny story about a lunch with Alan Alda in the MIT lunchroom.

Like his textbook on nonlinear dynamics, the present book is a model of clarity. Reading *Sync* has been fun and intellectually stimulating. I can think of about a dozen mathematical questions that came to mind while reading different parts of the book. While it is written for the layperson, there is plenty of interesting mathematics behind the scenes; the endnotes provide many references. I hope that *Sync* will be read by many nonmathematicians so that they can appreciate both the usefulness of mathematics and other forms of "pure" science. And the next time someone asks what I do for a living, I cannot think of a better place to point her.

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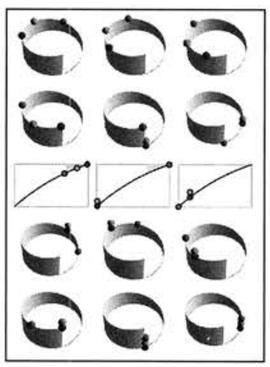
About the Cover

Synchronizing Peskin's heart

The cover this month illustrates how beat synchronization occurs in a heart model proposed around 1975 by Charles Peskin (in the Courant Institute notes "Mathematical aspects of heart physiology").



The heart contains a large number of virtually identical cells that must fire more or less simultaneously (in a healthy heart), and if disturbed, they must return to the synchronized state. In Peskin's model the state of each cell is characterized by its phase, and the system itself is also characterized by a function of phase whose graph is concave (as in the middle row of the cover). At the end of every cycle, each cell fires and adjusts the phase of every other cell, translating it into synchronization with itself or adjusting its phase so as to raise the value of the function by a fixed amount.

Renato Mirollo and Steven Strogatz were the first to prove that Peskin's model incorporates synchronization of an arbitrary population (*SIAM Journal of Applied Mathematics* **50** (1990)). An informal explanation is given in the book by Strogatz reviewed here.

The cover shows how this process works with three cells. It demonstrates roughly how concavity of the graph plays a role.

—*Bill Casselman*
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William T. Tutte (1917–2002)

Arthur M. Hobbs and James G. Oxley

William Thomas Tutte (rhymes with "hut") was the leading graph and matroid theorist of his generation. His first mathematical paper appeared in 1940 and his last in 2000. According to a list published in 1969, between 1940 and 1949 there were fifty-five papers published in graph theory, eleven of them by Tutte; in 2000, more than 1,500 books and papers appeared that were classified under 05C (graph theory) on MathSciNet. Like graph theory, matroid theory has grown dramatically from its introduction in 1935 by Hassler Whitney: MathSciNet shows in the period 1990–2000 over 1,000 items were published having the word "matroid" in the title or review. Despite early contributions by Garrett Birkhoff, S. Mac Lane, and B. L. van der Waerden, the first major advances in matroid theory were made by Tutte in his 1948 Cambridge Ph.D. thesis [3], which formed the basis of an important sequence of papers published over the next two decades. Tutte's work in graph theory and matroid theory has been profoundly influential on the development of both the content and the direction of these two fields.

To summarize Tutte's notable contributions, he broke the German Army High Command's code [10] during World War II, he advanced graph theory from a subject with one text (D. König's) toward its present extremely active state, and he developed Whitney's definitions of a matroid into a substantial theory.

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Tutte's contributions to graph and matroid theory were immense, but his terminology was idiosyncratic, frequently at variance with most other researchers. Hardest of all for a novice approaching Tutte's work is the fact that he often used standard terms in graph and matroid theory in ways that differ from their conventional usage. While Tutte's many theorems are frequently cited today, much of his terminology has been discarded. In the descriptions of his matroid results that appear below, the terminology used follows J. Oxley [2] and, with minor variations, is fairly universally accepted. For his graph theory results, we follow the terminology of D. West [9]. To describe Tutte's results and their significance, we shall need to introduce some of this terminology together with some well-known results.

Background

A *graph* may be viewed as consisting of a set of points in space, called *vertices*, and segments of curves joining pairs of vertices; these curves are called *edges* and share only their ends with other edges and with vertices of the graph. For example, the *complete graph* K_n consists of n vertices and $\binom{n}{2}$ edges, with one edge joining every two distinct vertices. If an edge of a graph joins a vertex to itself, the edge is called a *loop*. If two or more edges join the same pair of vertices, the edges are said to be *parallel*. A graph is *simple* if it has no loops and no parallel edges. The *degree* of a vertex in a graph is the number of ends of edges meeting that vertex. Thus a loop contributes 2 to the degree of the vertex it meets. Since each edge has two ends, the sum of the degrees of a graph is always even. A graph is *cubic* if every vertex has degree three. A *path* is an alternating sequence of vertices and edges, each edge joining the vertex before it to the vertex after it in the sequence, with no repeated vertices. A *cycle* is formed from a path by adding an edge joining its end vertices. A graph is *connected* if between every two distinct vertices there is a path. A *forest* is a graph with no cycles, and a *tree* is a connected forest. A graph is said to be *Hamiltonian* if it has a cycle that includes all of the vertices of the graph, and such a cycle is called a *Hamiltonian cycle*.

Abstracting from the behavior of linearly independent sets of columns of a matrix, in 1935, Whitney defined a *matroid* M to consist of a finite set E (or $E(M)$) and a collection $\mathcal{I}$ (or $\mathcal{I}(M)$) of subsets of E called *independent sets* with the properties that the empty set is independent; every subset of an independent set is independent; and if one independent set has more elements than another, then an element can be chosen from the larger set to adjoin to the smaller set to produce another independent set. A set that is not independent is called *dependent*, and minimal such sets are called *circuits*. For a matrix A over a field $\mathbb{F}$, if E is the set of column labels on A and $\mathcal{I}$ is the set of subsets of E that label linearly independent sets of columns, then $(E, \mathcal{I})$ is a matroid, $M[A]$. Such a matroid is said to be *representable over the field* $\mathbb{F}$. If G is a graph, E is its set of edges, and $\mathcal{I}$ is the collection of the edge sets of the forests in G, then $(E, \mathcal{I})$ is a matroid, $M(G)$, called the *cycle matroid* of G. Its circuits are the edge sets of the cycles in G. A matroid that is isomorphic to the matroid $M(G)$ for some graph G is called *graphic*. Representable and graphic matroids were introduced by Whitney, and these classes have motivated much of the development of matroid theory ever since.

A graph is *planar* if it can be drawn in the plane without edges crossing, and it is a *plane graph* if it is so drawn in the plane. The drawing separates the rest of the plane into regions called *faces*. Every plane graph G has a *dual graph* G^*, formed by introducing a vertex of G^* for each face of G and joining two vertices of G^* by k edges if and only if the corresponding faces of G share k edges in their boundaries. Whitney defined the dual of a graph abstractly and showed that a graph has such a dual if and only if it is a planar graph.

For arbitrary matroids there is a natural notion of a dual matroid that extends the notion of duality for planar graphs. For a matroid M on E, let $\mathcal{B}$ be the collection of *bases*, that is, maximal independent sets, of M. Clearly $\mathcal{B}$ uniquely determines M. The collection $\{E - B : B \in \mathcal{B}\}$ is the set of bases of another matroid on E, namely, the *dual* M^* of M. Evidently,

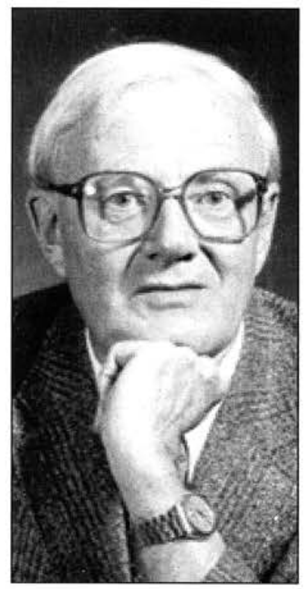
$$(M^*)^* = M. \tag{1}$$

If G is a planar graph and G^* is a dual graph of G, then $(M(G))^* = M(G^*)$. If G is not planar, then $(M(G))^*$ is still defined, but one can show that it is not graphic. So the class of graphic matroids is *not closed under duality*, since the dual of a member of the class need not be in the class. It is easy to show that all bases of a matroid M have the same cardinality; we call this the *rank* $r(M)$ of M.

We will define further concepts relating to graphs and matroids as we need them.

Tutte's Rise to Prominence

William Thomas Tutte was born at Fitzroy House in Newmarket, Suffolk, England, on May 14, 1917, and died on May 2, 2002, in Waterloo, Ontario. He attended Trinity College, Cambridge, from 1935 until January 1941, majoring in chemistry and earning his bachelor's degree in 1939 and his master's in 1941. He spent the rest of World War II working at Bletchley Park.



William T. Tutte

At the end of the war, because of his war work, Tutte was given a Fellowship at Trinity College. The decision to so honor him was justified: With little help from his advisor, Shaun Wylie, he completed a 417-page thesis in three years, graduating in 1948. The thesis is remarkable both for its contents and for its omission of his work on Hamiltonian cycles, which he published while he was writing the thesis.

For the thesis Tutte invented a class of algebraic structures that he called "nets" and that later he learned were equivalent to representable matroids. He developed a theory of "cleavages" of a 2-connected graph into 3-connected parts, thus deepening a result of Whitney. He introduced the dichromatic polynomial, now called the "Tutte polynomial", one of the more important functions in graph and matroid theory. In addition, his thesis introduced matroid minors, now the most important and widely studied of matroid substructures. Finally, he developed this theory to the point that he was able to characterize graphic matroids by giving a list of matroids, which he called "gnarls", such that a matroid is graphic if and only if it has no gnarl as a minor.

In 1948 H. S. M. Coxeter helped Tutte get a position at the University of Toronto. Tutte served there until 1962, when the University of Waterloo (founded in 1958) recruited him. Ralph Stanton, head of the mathematics department at that time,

Tutte's Successful Ph.D. Students

Will Brown (1963)
Ron Mullin (1964)
John Wilson (1966)
Neil Robertson (1969)
Arthur Hobbs (1971)
Stephan Foldes (1977)
Richard Steinberg (1979)
Ken Berman (1979)

went to Toronto with Gerald Berman to talk to Tutte and his wife, Dorothea (married 1949). The opportunity appealed to Tutte, both because it offered the possibility of advancement and because he and Dorothea enjoyed nature and liked the relatively rural setting of Waterloo. Tutte accepted the position, and he and Dorothea bought a house in the small nearby town of West Montrose, next to a covered bridge. They enjoyed showing their lovely garden and nearby scenery, and they could name every bird and plant in the garden. They liked hiking. Bill often organized hiking trips and even bought wilderness properties for them to hike in as birthday presents for Dorothea.

Mathematical Beginnings and Blanche Descartes

Not long after he started his undergraduate studies at Cambridge, Tutte was introduced by his chess-playing friend R. Leonard Brooks to two of Brooks's fellow mathematics students, Cedric A. B. Smith and Arthur Stone. The four became fast friends, and Tutte came to refer to the group as "the Gang of Four" or "the Four Horsemen" [8]. The Four joined the Trinity Mathematical Society and devoted many hours to studying unsolved mathematics problems together.

They were most interested in the problem of squaring a rectangle or square, that is, of finding squares of integer side-lengths that exactly cover, without overlaps, a rectangle or square of integer side-lengths. If the squares are all of different sizes, the squaring is called *perfect.* While still undergraduates at Cambridge, the Four found an ingenious solution involving currents in the wires of an electrical network. Solving this network gives the required sizes of the squares as the currents in the wires. Their first perfect squared square and the accompanying theory were published in 1940 [4, pp. 10–38]; it was anticipated by one year by a perfect squared square found empirically by R. P. Sprague.

About this work Tutte said, "This [research] soon called for much graph theory. It was linked, through a 'Smith diagram,' with the study of 3-connected planar graphs..., and with Kirchhoff's Laws for electrical circuits.... It was linked through rotor theory...with graph symmetry.... It was linked through the tree-number...with the theory of graph functions satisfying simple recursion formulae..." [5, p. xix].

The Gang of Four were typically lively undergraduates. They decided to create a very special mathematician, Blanche Descartes, a mathematical poetess. She published at least three papers, a number of problems and solutions, and several poems. Each member of the Four could add to Blanche's works at any time, but it is believed that Tutte was her most prolific contributor.

The Four carefully refused to admit that Blanche was their creation. Visiting Tutte's office in 1968, Hobbs remarked, "Sir, I notice you have two copies of that proceedings. I wonder if I could buy your extra copy?" Tutte replied, "Oh, no, I couldn't sell that. It belongs to Blanche Descartes."

Bletchley Park

At about the time Tutte completed his master's in chemistry in 1941, his tutor, Patrick Duff, recommended him to the famous code-breaking group at Bletchley Park. After training at the Code and Cypher School in London, Tutte was assigned to the research section and there to a group trying to break the German Army High Command's code that the British called "FISH". It was produced by the Lorenz SZ 40/42 teleprinter cypher attachment, which generated code in the 5-bit 32-letter Baudot Teleprinter Code alphabet [10]. Unlike the case of the 3- or 4-wheel Enigma machine, the British did not have a Lorenz code machine, but the clear 12-word preamble in the early FISH transmissions suggested that it had 12 wheels [10], [6]. Its code was produced letter-by-letter by a mod 2 addition of a letter from the message and a letter from a key produced by the machine. Thus, the message letter R (code 01010) might be added to the key letter A (11000), producing the code letter D (10010).

On August 30, 1941, a clerk in Athens, Greece, sent a Lorenz-coded message of about 4,000 characters. Apparently the receiving office in Vienna didn't quite get it, for they asked for a repeat. Proper protocol would require that the message be sent either exactly as before or with a new setting on the wheels of the machine, but the clerk recoded it with the same initial setting as before and sent it again with some variations in abbreviations, spacing, and punctuation. Even in such circumstances breaking the code is not easy, but eventually John Tiltman found 3,976 characters of the key produced by the machine with that wheel setting. Tutte sought patterns within this key, and after a few months' work he discovered a repetitive pattern of length $574 = 14 \cdot 41$. This indicated that one of the wheels had 41 teeth. It took still more time to fully find the design of the Lorenz machine, but he and the others in his section worked out the placement and number of teeth of the 12 wheels within the machine. Their decoding efforts were paralleled by German efforts to improve the code, so there was a constant struggle between the German encrypters and the British codebreakers. The rest of the story is too long for this article. Suffice it to say that Tutte's work led to the development of the first working electronic computer, Colossus, to run decoding algorithms. By the end of the war, ten Colossus machines were in use [10].

Fragments and Hamiltonian Graphs

A summary of Tutte's work is simplified by the fact that, in the commentaries on his selected works [4] and in other published lectures [7], [8], he gave not only many useful historical facts about the context in which his work was done but also numerous insights into his thinking when he was developing his many theorems. We draw heavily from these works in what follows.

A graph with more than k vertices is *k-connected* if it cannot be disconnected by removing $k-1$ vertices. A cycle with exactly three edges is a *triangle*. If every face of a plane graph is a triangle, the graph is a *plane triangulation*. A cycle C in a plane graph G is *separating* if G has vertices both inside and outside C.

Given a cycle C in a graph G, a *fragment of C*, or a *C-fragment*, consists either of an edge (and its ends) not in C but joining two vertices of C, or of a component Q of $G - V(C)$ together with all edges (and their ends) from vertices of Q to vertices of C. The vertices a C-fragment shares with C are called its *vertices of attachment*.

Tutte introduced fragments in his thesis. He used these tools in several of his graph theory papers, including his 1960 paper "Convex representations of graphs" and his important paper "How to draw a graph" [4, pp. 364–88]. He used a generalization of fragments to matroids in proving his characterization of graphic matroids (1959).

The question of whether a graph has a Hamiltonian cycle is one of the prime exemplars of an NP-complete problem and is a special case of the well-known traveling salesman problem. Consequently, finding significant classes of graphs that certainly are or are not Hamiltonian is important. In 1884 P. G. Tait conjectured that every 3-connected planar cubic graph has a Hamiltonian cycle. If true, a proof of this conjecture would have also proved the famous Four Color Conjecture.

In his occasional free time at Bletchley Park, Tutte thought about Tait's conjecture. He discovered a Hamiltonian graph with the remarkable quality that it had an edge through which all Hamiltonian cycles passed. He then used three copies of that graph to construct a counterexample to Tait's conjecture [4, pp. 47–50].

In 1931 H. Whitney had shown that a plane triangulation with no separating triangle is Hamiltonian. In 1956 Tutte extended Whitney's theorem by showing that every connected planar graph G with at least one cycle has a cycle C, all of whose fragments have at most three vertices of attachment. A nontrivial consequence of this is the following theorem of Tutte.

Theorem 1. *Every 4-connected planar graph is Hamiltonian.*

Excluded-Minor Theorems

The notion of a graph minor was introduced in the 1930s and early 1940s by K. Wagner and H. Hadwiger. Tutte's thesis generalized this notion to matroids. If M is a matroid on a set E and $\mathcal{I}$ is the collection of independent sets, then for an element e of E, there are two natural matroids on $E - \{e\}$: the *deletion* $M\backslash e$ of e and the *contraction* M/e of e. The first has as its independent sets all members of $\mathcal{I}$ that are subsets of $E - \{e\}$. To specify the second, let B_e be $\{e\}$ when $\{e\}$ is independent, and let B_e be $\emptyset$ otherwise. Then M/e has as its independent sets all subsets I of $E - \{e\}$ for which $I \cup B_e \in \mathcal{I}$. These operations of deletion and contraction are natural extensions of their namesakes in graph theory. Since an element e of the cycle matroid $M(G)$ of a graph G is just an edge of G, the matroids $M(G)\backslash e$ and $M(G)/e$ are the cycle matroids of, respectively, the graph $G\backslash e$ that is obtained from G by removing the edge e and the graph G/e that is obtained from G by identifying the ends of e and then deleting e. Both the graph and the matroid operations of deletion and contraction commute with themselves and with each other, so for a matroid M or a graph G, it makes sense to talk about a matroid or a graph of the form $M\backslash X/Y$ or $G\backslash X/Y$, where X and Y are disjoint subsets of E. Such a matroid or graph is called a *minor* of M or of G. (In the case of graphs, we also allow the deletion of vertices of degree zero in constructing a minor.) Several basic classes of matroids and graphs have the property that every minor of a matroid or graph in the class is also in the class. Indeed, the remarks above establish that every minor of a graphic matroid is graphic. We say that the class of graphic matroids is *minor-closed*. For another example, the class of planar graphs is minor-closed.

For any class $\mathcal{M}$ of matroids or graphs that is minor-closed, there is a list of *excluded minors*, that is, those matroids or graphs not in $\mathcal{M}$ such that every deletion and every contraction is in $\mathcal{M}$.

A matroid is *binary* if it is representable over the 2-element field $GF(2)$. A matroid is *regular* if it is representable over $\mathbb{R}$ by a *totally unimodular*

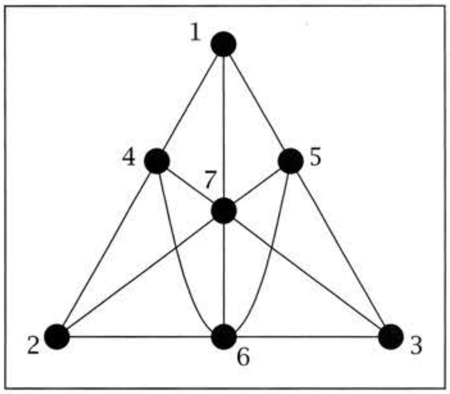


Figure 1. The Fano matroid F_7.

matrix A, that is, by a matrix all of whose subdeterminants are in $\{0, 1, -1\}$. In fact, regular matroids are precisely the matroids that are representable over all fields (see [2, Theorem 6.6.3]), so every regular matroid is binary.

The class of graphic matroids is not closed under duality. By contrast, the classes of regular matroids and of binary matroids are both closed under duality. Indeed, if $[I_r \mid D]$ is an $r \times n$ matrix representing M over a field $\mathbb{F}$, then M^* is represented over $\mathbb{F}$ by the $(n-r) \times n$ matrix $[-D^T \mid I_{n-r}]$, where the order of the labels on the columns of the last matrix matches the order on the original matrix. We observe that duality is a natural generalization of orthogonality, since every row of $[I_r \mid D]$ is orthogonal to every row of $[-D^T \mid I_{n-r}]$.

Every graphic matroid is regular. To see this, let G be a graph, orient its edges arbitrarily, and then take the vertex-edge incidence matrix of the resulting directed graph. It is not difficult to show (see [2, Proposition 5.1.3]) that the resulting matrix A is totally unimodular and that $M[A] = M(G)$. After Tutte recognized the link between the structures he studied in his thesis and Whitney's matroids, his "main preoccupation...was the question: 'When is a matroid graphic?'" [7, p. 100]. Using the observations above, he divided this problem into three subproblems:

(i) When is a matroid binary?
(ii) When is a binary matroid regular?
(iii) When is a regular matroid graphic?

The first of these questions is the simplest and has numerous answers. One of those given by Tutte involves excluded minors. If A is a matrix over a field $\mathbb{F}$ and e labels a column of A, then $M[A]\backslash e$ is represented over $\mathbb{F}$ by the matrix that is obtained by deleting the column labelled e. If e labels a zero column, then $M[A]/e = M[A]\backslash e$; otherwise, by elementary row operations, we can transform A into a matrix A' in which e labels a standard basis vector. It follows from basic linear algebra that $M[A'] = M[A]$; that is, these elementary row operations do not alter the matroid. Now $M[A]/e$ is the matroid that is represented by the matrix that is obtained from A' by deleting both the column labelled by e and the row containing the unique nonzero entry of e. We deduce from these observations that the class of $\mathbb{F}$-representable matroids is minor-closed. In particular, the class of binary matroids is minor-closed. By observing that if the matrix A above is totally unimodular, then so is the matrix A', we deduce that the class of regular matroids is also minor-closed.

Tutte's excluded-minor answer to (i) is contained in the next theorem. It involves a particular uniform matroid where, for $0 \le r \le n$, the *uniform matroid* $U_{r,n}$ is the matroid on $\{1, 2, \dots, n\}$ whose independent sets consist of all subsets of $\{1, 2, \dots, n\}$ with at most r elements.

Theorem 2. *A matroid is binary if and only if it has no minor isomorphic to $U_{2,4}$.*

The answers to Tutte's subproblems (ii) and (iii) were again given in terms of excluded minors but were considerably more difficult to derive. The *Fano matroid*, F_7, is the matroid that is represented over $GF(2)$ by the matrix consisting of the seven nonzero vectors of length 3, that is, by the matrix

$$A = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array}.$$

Geometrically, F_7 can be represented as in Figure 1. There the independent sets consist of all sets of at most three points such that no three are collinear, where the curved line through 4, 5, and 6 indicates that $\{4, 5, 6\}$ is dependent.

It is straightforward to show that the Fano matroid is not regular but that all of its proper minors are, so F_7 is an excluded minor for the class of regular matroids. There is another excluded minor for the class of regular matroids that is easily derived from F_7, namely, its dual F_7^*.

Tutte noted that deletion, contraction, and duality are related by the following beautiful identity:

$$(M\backslash e)^* = M^*/e. \qquad (2)$$

Combining this with (1), we get

$$(M/e)^* = M^*\backslash e. \qquad (3)$$

We conclude, since the class of regular matroids is closed under duality, that F_7^* is another excluded minor for this class. Tutte answered subproblem (ii) by proving the following.

Theorem 3. *A binary matroid is regular if and only if it has no minor isomorphic to F_7 or F_7^*.*

On combining this with the answer to subproblem (i), one obtains the following.

Corollary 4. *A matroid is regular if and only if it has no minor isomorphic to $U_{2,4}$, F_7, or F_7^*.*

Tutte developed a geometry in which the circuits of the matroid were "points", and he used this in the development of a homotopy theorem for matroids by which he proved Theorem 3. He commented [8, p. 7]: "I do not find the homotopy theorem in the later literature. Perhaps it is mentioned with a warning that it is terribly long and then the author tells of some shorter, slicker proof of the excluded minor conditions. That is the way of Mathematics." Theorem 3 is a brilliant result and was always recognized as such, but, as Tutte suggests, most readers were frightened off by the proof technique. The best currently known proof was given by A. M. H. Gerards in 1989 and relies only on some elementary ideas of linear algebra. But the latter proof did not appear till some thirty years after Tutte's landmark result.

Tutte attacked subproblem (iii) by analogy with the proof by fragments of K. Kuratowski's theorem that a graph can be drawn in the plane if and only if it does not have K_5 or $K_{3,3}$ as a minor. Here $K_{3,3}$ is the three-houses, three-utilities graph in which every house is joined to every utility by an edge. Tutte's work on (iii) culminated in the following very pleasing generalization of Kuratowski's theorem.

Theorem 5. *A regular matroid is graphic if and only if it has no minor isomorphic to* $M^*(K_5)$ *or* $M^*(K_{3,3})$.

Theorems 2, 3, and 5, done in the context of nets, first appeared in Tutte's Ph.D. thesis [3] a decade before their publication in a journal. These theorems were the cornerstones of Tutte's "Lectures on Matroids" presented in 1964 at the first matroid theory conference, which was organized by Jack Edmonds and was held at the National Bureau of Standards. They are among Tutte's best-known contributions to matroid theory.

Tutte's work with graph minors was eventually taken up by his former student Neil Robertson in collaboration with P. D. Seymour. Beginning in the 1980s, they have produced an extraordinary sequence of papers describing the structure of graphs. Perhaps their most powerful result so far is the Graph-Minor Theorem.

Theorem 6. *In every infinite set of graphs, there are two such that one is a minor of the other.*

To illustrate the usefulness of this theorem, consider the very old question of characterizing graphs that can be drawn in a surface of genus g without crossing edges. Kuratowski's theorem established that there are exactly two excluded minors for the plane. Around 1980 D. Archdeacon, H. H. Glover, J. P. Huneke, and C. S. Wang found that there are thirty-five excluded minors for the graphs that can be drawn in the projective plane. But the number of excluded minors grows quickly with the genus, and it is not obvious that the list is even finite for other values of g. To see that the list is finite, suppose we have an infinite list $\mathcal{L}$ of excluded minors for a surface. Then by the Graph-Minor Theorem one member of $\mathcal{L}$ is a minor of another. Since this is impossible by the minimality of excluded minors, $\mathcal{L}$ must be finite.

A matroidal conjecture related to the Graph-Minor Theorem is given at the end of the section below on connectivity in matroids.

Colorings, Flows, and the Tutte Polynomial

Perhaps Tutte's most long-lasting legacy will be the polynomial that bears his name. Tutte's work in this area began with Chapter 5 of his thesis. It was published in terms of graphs in his 1947 paper "A ring in graph theory" [4, pp. 55–69]. In a connected graph G, a *spanning tree* is a tree that contains all of the vertices of G. The spanning trees of G coincide with the edge-sets of bases of the matroid $M(G)$. Let $b(G)$ denote the number of spanning trees of G. In their 1940 paper "The dissection of rectangles into squares" [4, pp. 10–38], the Gang of Four had noted a formula that implies that, for all nonloop edges e of G,

$$b(G) = b(G\backslash e) + b(G/e). \tag{4}$$

Recall that $G\backslash e$ is obtained from G by deleting e, while G/e is obtained from G by identifying the ends of e and then deleting e. Tutte wrote [7, p. 53]: "When I was doing my Ph.D. research I began to collect other functions of graphs that satisfied similar recursions." For a graph G and a positive integer λ, let $P(G;\lambda)$ be the number of colorings of the vertices of G using $\{1, 2, \dots, \lambda\}$ such that if two vertices are joined by an edge, then they receive different colors. The smallest positive integer n for which $P(G;n)$ is nonzero is called the *chromatic number* of G. George D. Birkhoff introduced $P(G;\lambda)$ in 1912–13. R. M. Foster made the elementary observation that if e is an edge of G, then

$$P(G;\lambda) = P(G\backslash e;\lambda) - P(G/e;\lambda). \tag{5}$$

Using this, it is easily shown that the function P is a polynomial in λ; it is called the *chromatic polynomial* of G.

Now orient the edges of G arbitrarily and let H be an additive abelian group. An H-*flow* on G is an assignment of nonzero members of H to the edges of G such that at every vertex the total flow into the vertex equals the total flow out from the vertex, where all calculations are done in H. Tutte showed [4, pp. 55–69] that the number of such H-flows depends not on the specific group H but only on the order of H. In particular, if $|H| = m$, then the number of H-flows equals the number of $\mathbb{Z}_m$-flows. We call the latter flows m-*flows* and let $F(G;m)$ be the number of such flows. Tutte showed that

$$F(G;m) = F(G/e;m) - F(G\backslash e;m). \tag{6}$$

The polynomial F is called the *flow polynomial* of G.

These observations led Tutte in [4, pp. 55–69] to discuss functions f on graphs that are invariant under isomorphism:

$$(7) \qquad f(G_1) = f(G_2) \quad \text{if } G_1 \cong G_2,$$

and that satisfy the rule

$$(8) \quad f(G) = f(G\backslash e) + f(G/e) \quad \text{for all nonloop edges } e \text{ of } G.$$

He also considered functions satisfying the additional condition that

$$(9) \qquad f(H + K) = f(H)f(K),$$

where $H + K$ is the graph that is the union of the disjoint graphs H and K. Tutte proved that a graph function satisfying (7)–(9) is uniquely determined once its value is known for all k on the graphs consisting of a single vertex and k loops.

A *cut edge* of a connected graph is an edge whose removal disconnects the graph. Tutte's paper [4, pp. 55–69] also introduced a 2-variable polynomial, which Tutte called the *dichromatic polynomial* and which is now known as the *Tutte polynomial*. This polynomial satisfies (8) provided e is not a cut edge of G. It also satisfies (7) and (9), and, indeed, it is the universal graph invariant satisfying these three conditions in a sense that will shortly be made precise.

Much of the theory described above extends to matroids and was developed for representable matroids in Tutte's Ph.D. thesis. However, Tutte never published his work for matroids, and it was not until 1969 that H. H. Crapo published a fully general matroid form of this theory.

In a matroid M on a set $E(M)$, if $X \subseteq E(M)$, then $M\backslash(E(M) - X)$ is a matroid on the set X; its rank is $r(X)$. The *Tutte polynomial* $T(M; x, y)$ of M is

$$\sum_{A \subseteq E} (x-1)^{r(E)-r(A)}(y-1)^{|A|-r(A)}.$$

This polynomial has the attractive property that

$$T(M; x, y) = T(M^*; y, x),$$

where M^* is the dual of the matroid M. Moreover, for a graph G with $k(G)$ components, the chromatic polynomial and the flow polynomial are related to the Tutte polynomial via

$$P(G; \lambda) = \lambda^{k(G)}(-1)^{|V(G)|-k(G)}T(M(G); 1-\lambda, 0)$$

and

$$F(G; m) = (-1)^{|E(G)|-|V(G)|+k(G)}T(M(G); 0, 1-m).$$

An element e of a matroid is a *loop* if $\{e\}$ is a circuit of the matroid. The Tutte polynomial has the properties that

$$(10) \qquad f(M_1) = f(M_2) \quad \text{if } M_1 \cong M_2,$$

$$(11) \quad f(M) = f(M\backslash e) + f(M/e) \quad \text{if } e \text{ is not a loop in } M \text{ or } M^*,$$

and

$$(12) \quad f(M) = f(M\backslash e)f(M\backslash(E(M) - \{e\})) \quad \text{if } e \text{ is a loop of } M \text{ or } M^*.$$

In his thesis, Tutte showed that the Tutte polynomial is a universal matroid invariant satisfying (10)–(12) in the sense that if the values of f on the two one-element matroids $U_{1,1}$ and $U_{0,1}$ are x and y respectively, then $f(M) = T(M; x, y)$ for all matroids M. Tutte never published this result, and it was rediscovered by T. H. Brylawski and published in 1972.

The applications of the Tutte polynomial extend well beyond graphs into such diverse areas as coding theory, percolation theory, electrical network theory, statistical mechanics, and knot theory. Tutte essentially confined his interest to spanning trees and chromatic and flow polynomials, but in view of the importance of polynomial invariants for knots, the link between one such invariant and the Tutte polynomial is worth describing. Consider a link diagram, as illustrated in Figure 2(a), where the crossings alternate between under and over as the link is traversed. Such a link diagram is called *alternating*. Now the faces in such a link diagram can be colored black and white in such a way that adjacent faces receive different colors and the infinite face is colored white (see Figure 2(b)). This coloring is called the *Tait coloring* of the diagram D. Let the graph $S(D)$ have vertices corresponding to the black faces and an edge joining two such vertices when they are the opposite faces of a crossing (see Figure 2(c)). In 1985 Vaughn Jones introduced a single-variable polynomial for links that now bears his name. For this and other contributions to knot theory, Jones was awarded a Fields Medal in 1990. If L is an alternating link, D is the corresponding link diagram, and $S(D)$ is the graph constructed as above, then as M. B. Thistlethwaite showed, the Jones polynomial of L is given, up to an easily derived factor, by an evaluation of the Tutte polynomial of $S(D)$ along the hyperbola $xy = 1$.

Tutte further developed the ideas introduced in [4, pp. 55–69] in his paper "A contribution to the theory of chromatic polynomials" [4, pp. 157–68]. This paper contained two interesting conjectures. The first, which was settled by F. Jaeger in 1975, asserted that there is a fixed integer n such that every graph without a cut edge has an n-flow. Jaeger proved this with $n = 8$, and subsequently Seymour proved it for $n = 6$. Tutte's second conjecture, the 5-Flow Conjecture, is that every graph without a cut edge has a 5-flow. The *Petersen graph* P_{10} is obtained from the dodecahedron by identifying antipodal vertices and then replacing each pair of parallel edges by a single edge. It has no 4-flow, so 5 is the best possible number in Tutte's conjecture. The 5-Flow Conjecture has now been open for almost fifty years.

The Four Color Problem, that every loopless planar graph has chromatic number at most four, was a dominating influence on graph theory for more than a century following its introduction by F. Guthrie in 1852. In his 1966 paper "On the algebraic theory of graph colorings", following a suggestion of O. Veblen, Tutte developed what he called a "geometrical version of the Four Color Problem." The ideas he introduced were further developed by Crapo and G.-C. Rota in their 1970 consideration of the critical problem for matroids. Let G be a graph. Since we are interested in coloring vertices so that every two adjacent vertices are colored differently, we may assume that G is simple. Now suppose that $M(G)$ has rank r. Take a binary representation of $M(G)$, that is, a matrix A over $GF(2)$ such that $M[A] \cong M(G)$. Then we are effectively viewing $M(G)$ as being embedded in the r-dimensional vector space $V(r,2)$. In a 1966 paper Tutte showed that the graph G has a 4-coloring if and only if there are $(r-1)$-dimensional subspaces H_1 and H_2 of the embedding space $V(r,2)$ such that $H_1 \cap H_2$ avoids the edges of G. He defined a *k-block* to be a set F of nonzero vectors in $V(r,2)$ of rank exceeding k such that F meets every subspace of $V(r,2)$ of dimension $r-k$. Hence a k-block is a special type of embedded matroid. A k-block is *minimal* if no proper subset of it is also a k-block. For instance, Tutte showed that the minimal 1-blocks are the cycle matroids of odd cycles.

A *tangential k-block* is a k-block for which no proper minor without loops is also a k-block. Because the Petersen graph P_{10} has no 4-flow, $M^*(P_{10})$ is a 2-block. Indeed, it is a tangential 2-block. Evidently, K_5 is not 4-colorable, and $M(K_5)$ is a tangential 2-block. Moreover, the Fano matroid is a tangential 2-block. Tutte conjectured that the only tangential 2-blocks are F_7, $M(K_5)$, and $M^*(P_{10})$. He proved this conjecture for tangential 2-blocks of rank at most 6, and B. T. Datta, in 1976 and 1981, proved the cases of rank 7 and 8. The most significant advance toward the resolution of this conjecture was made in 1981 by Seymour, who proved that a tangential 2-block that is not isomorphic to F_7 or $M(K_5)$ has a graphic dual. He used the Four Color Theorem and his 1980 decomposition of regular matroids in his proof of this result. A consequence of this theorem is that Tutte's Tangential 2-Block Conjecture is equivalent to the following 1966 variant of the 5-Flow Conjecture known as Tutte's 4-Flow Conjecture.

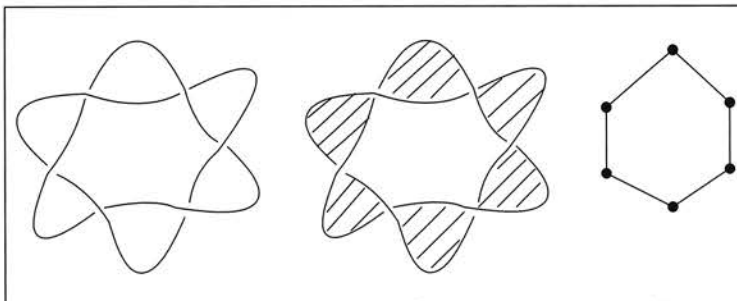


Figure 2.

Conjecture 7. *Suppose that a graph G without cut edges has no 4-flow. Then G has a subgraph contractible to P_{10}.*

Although this conjecture remains open in general, in 1999 N. Robertson, D. Sanders, P. Seymour, and R. Thomas settled it in the important special case when G is a cubic graph.

Connectivity in Matroids

Tutte developed a theory of connection in graphs [4, pp. 229–43] that he subsequently generalized to matroids. This theory has had very important implications in the burgeoning field of matroid structure theory. Concerning the definition of k-connection given earlier, Tutte writes in [4, p. 226] that he now refers to this property as "vertical k-connection [where, in this context, 'vertical' is the adjective from the noun 'vertex'] having come to think of another kind of connection as more natural." Tutte's theory of connection, which is defined in the next paragraph, is preserved under duality. By contrast, the dual of a vertically 3-connected graph need not be vertically 3-connected. For example, adding an edge in parallel to one of the edges of the complete graph K_4 produces a vertically 3-connected graph whose planar dual has a degree-2 vertex and so is certainly not vertically 3-connected.

If m is a positive integer, Tutte [4, pp. 499–522] defined $\{X, E(M)-X\}$ to be an *m-separation* of M if both $|X|$ and $|E(M)-X|$ are at least m and $\xi(X, E(M)-X) \le m$, where

$$\xi(X, E(M)-X) = r(X) + r(E(M)-X) - r(M) + 1.$$

Thus, for example, if A is a matrix in block form $\begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}$, and X_1 and X_2 label the columns of $\begin{bmatrix} A_1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ A_2 \end{bmatrix}$, respectively, then $\{X_1, X_2\}$ is a 1-separation of $M[A]$. This last kind of separation was considered by Whitney. But Tutte went beyond 1-separations, defining a matroid to be *n-connected* if, for all m with $1 \le m \le n-1$, the matroid has no m-separation. In particular, for a graph G with at least four vertices, $M(G)$ is 3-connected if and only if G is both (vertically)

3-connected and simple. In his 1961 paper "A theory of 3-connected graphs" [4, pp. 229–43], Tutte proved a graph result which, for us, is most usefully stated in the matroid language above. An *n-wheel* W_n is the graph that is obtained from a cycle C_n with n edges by adding a new vertex v and then joining v to every vertex of C_n. The cycle C_n is called the *rim* of the wheel. Note that, in particular, $W_3 \cong K_4$.

Theorem 8. *Let G be a graph with at least four vertices and no isolated vertices. Suppose that $M(G)$ is 3-connected but that, for all edges e of G, neither $M(G)\backslash e$ nor $M(G)/e$ is 3-connected. Then G is isomorphic to a wheel.*

Some five years later Tutte extended the last result from graphic matroids to matroids in general. The extension required the introduction of a new class of matroids. This class is constructed from the wheels using the following general method. If C is a circuit of a matroid M such that $E(M) - C$ is a circuit of M^*, then $|C| = r(M)$ and there is a new matroid M' that has $\mathcal{B}(M) \cup \{C\}$ as its set of bases. We say that M' has been obtained from M by *relaxing* C. In particular, the rim C_n of the wheel W_n is a circuit of $M(W_n)$ whose complement is a circuit of $M^*(W_n)$. The matroid $M(W_n)$ is also called an *n-wheel*. Relaxing C_n in $M(W_n)$ gives the *n-whirl* W^n. In his very important paper "Connectivity in matroids" [4, pp. 499–522] Tutte proved the following:

Theorem 9. *Let M be a 3-connected matroid with at least four elements. If, for all elements e of M, neither $M\backslash e$ nor M/e is 3-connected, then M is a wheel or a whirl.*

A consequence of the last two theorems is that every nontrivial 3-connected matroid or graph can be built up one element at a time from a wheel or a whirl so that all intermediate matroids or graphs are 3-connected, where the two allowable building-up operations are the reverses of deletion and contraction.

An important result developed by Tutte during his consideration of separation and connection in matroids appears in the 1965 paper "Menger's Theorem for matroids". The significance of this result has only recently been realized more than thirty years after its publication. We follow J. Geelen and G. Whittle (2002) in describing how this result can be used and its link to Menger's Theorem. Let G be a graph with edge set E and suppose $X \subseteq E$. The *vertex boundary* of X is the set of vertices that meet both X and $E - X$. Now let X and Y be disjoint subsets of E, each having vertex boundary of size k. We can *contract X onto Y* if it is possible to find a minor of G with edge set $X \cup Y$ such that the vertex boundary of X has size k in the minor. Menger's Theorem for graphs implies that X can be contracted onto Y if and only if G has no vertex cut of size less than k that separates X and Y. Rather than describe Tutte's matroid result in full generality, we consider it in the special case of a rank-r matroid M represented over a field $\mathbb{F}$. Effectively, we can view $E(M)$ as a multiset of elements of the vector space $V(r, \mathbb{F})$. For a subset X of $E(M)$, the *subspace boundary* is the intersection of the spans of X and $E(M) - X$. The rank of this subspace boundary is $\xi(X, E(M) - X) - 1$. Now suppose that X and Y are disjoint subsets of $E(M)$ such that both $\xi(X, E(M) - X)$ and $\xi(Y, E(M) - Y)$ equal k. Tutte's theorem asserts that the subspace boundary of X can be contracted onto the subspace boundary of Y provided that M has no smaller separations separating X from Y, that is, provided $\xi(X', Y') \geq k$ for all partitions $\{X', Y'\}$ of $E(M)$ with $X' \supseteq X$ and $Y' \supseteq Y$. This result has played an important role in two 2002 papers of Geelen and Whittle, and of Geelen, Gerards, and Whittle which attack what are currently the two central problems in matroid theory:

(i) Rota's 1971 conjecture that for all finite fields $\mathbb{F}$, the set of excluded minors for representability over $\mathbb{F}$ is finite; and

(ii) the conjecture that for all finite fields $\mathbb{F}$, every infinite sequence of matroids representable over $\mathbb{F}$ contains two matroids, one of which is a minor of another. This conjecture is inspired by Robertson and Seymour's Theorem 6 above.

Further Contributions to Graph Theory

Space prevents our thoroughly explaining Tutte's many other contributions to graph theory. Here we summarize some of these other results.

Factors of Graphs

While he was working on his thesis, Tutte also studied the problem of characterizing all graphs G that have a subgraph having exactly one edge at each of the vertices of G. Such a subgraph is called a *1-factor*. Tutte successfully solved this problem [4, pp. 93–7], thus generalizing J. Petersen's 1891 result for cubic graphs.

Theorem 10. *Given a graph G and a subgraph H of G, let $o(H)$ be the number of components of H having an odd number of vertices. Then G has a 1-factor if and only if $o(G\backslash X) \leq |X|$ for every subset $X \subseteq V(G)$.*

In 1952 Tutte generalized the 1-factor theorem by characterizing for each function f from the vertices of graph G to the set of nonnegative integers, those graphs G which have a subgraph whose degree at each vertex v of G is $f(v)$. These subgraphs are now called "f-factors".

Tree Packing

For a given integer k, which graphs G have k edge-disjoint spanning trees? One obvious condition is that, given any partition of the vertices into m cells, each spanning tree must include enough edges between cells to connect them together; this minimum number is $m - 1$. So for k trees there must be $k(m - 1)$ such edges. In 1961 Tutte [4, pp. 216–25] and independently and simultaneously C. St. J. A. Nash-Williams proved that this condition also suffices. The resulting theory of edge-disjoint packings of graphs with trees has found applications in matroid theory, rigidity of frameworks, electrical circuit theory, and survivability of networks.

Chromatic Polynomials and the Golden Number

During the decade 1879–1890, the Four Color Problem was thought to have been settled by A. B. Kempe (rhymes with "hemp"), using an ingenious device now called a "Kempe chain". In 1890 P. J. Heawood found a flaw in Kempe's proof and showed that Kempe's idea proved the five color theorem. He extended the work to higher genus surfaces. With the problem wide open again, G. D. Birkhoff introduced the function $P(G; \lambda)$ as a tool for attacking it, and in 1946 he and D. C. Lewis published a very long paper that has formed the basis for much subsequent work.

The Four Color Problem was finally settled by K. Appel and W. Haken in 1976, using Kempe chains and a discharging method developed from Euler's polyhedron formula. But the proof is not fully satisfying, because it relies on the treatment of more than 10,000,000 cases by a computer. A later proof by Robertson, Sanders, Seymour, and Thomas is easier to follow but still relies in an essential manner on the use of the computer. Thus work on the Four Color Theorem continues today, with the hope of finding a humanly readable proof.

Birkhoff's chromatic polynomials represented a different approach to the Four Color Theorem. Sometime around the beginning of 1968, Ruth Bari gave Tutte a copy of her thesis, which contained the chromatic polynomials of about 100 planar graphs. Later, Dick Wick Hall sent his collection of 900 chromatic polynomials of planar graphs to Tutte. Tutte asked Gerald Berman to use his newly developed computer program for finding zeros of polynomials to find the zeros of these functions. Examining the resulting stack of 11×15 fanfold printouts, which was many inches thick, Tutte noticed that almost all of the polynomials had a zero near 1.618. Moreover, the chromatic polynomials of the more complicated plane triangulations had zeros closely approximating 2.618. Recognizing that this number is approximately 1 plus the golden mean $\tau = (1 + \sqrt{5})/2$, a zero of $x^2 - x - 1$, he initiated a study of chromatic polynomials evaluated at this number. He discovered that $|P(T; \tau + 1)| \leq \tau^{5-k}$ for any 2-connected plane triangulation T with k vertices [4, pp. 571–8]. In [4, pp. 581–95] he went further, showing that $\tau + 1$ is never a zero of a chromatic polynomial of a plane 2-connected triangulation and that

$$P_T(\tau + 2) = \sqrt{5} \cdot \tau^{3(k-3)} \cdot P_T^2(\tau + 1).$$

To find a number $(\tau + 2)$ close to 4 at which chromatic polynomials of plane triangulations are positive was particularly exciting, since the Four Color Theorem can be stated as "$P(G; 4) > 0$ for every plane graph G."

Enumeration of Graphs

A *near-triangulation* of the plane is a plane graph in which every face but one is a triangle. The remaining face may or may not be a triangle and, in Tutte's research, is taken as the infinite face. A triangulation is *strict* if it has no separating cycle with exactly two edges, and a strict triangulation is *simple* if it has no separating triangle.

Joining the enumerationists, Tutte said, "It occurred to me once that it might be possible to get results of interest in the theory of map-colourings without actually solving the [Four Colour] Problem. For example, it might be possible to find the average number of 4-colourings, on vertices, for planar triangulations of a given size.

"One would determine the number of triangulations of $2n$ faces, and then the number of 4-coloured triangulations of $2n$ faces. Then one would divide the second number by the first to get the required average" [7, p. 114].

To attack this problem, he eliminated all symmetry of a plane triangulation or near-triangulation by rooting the graph. He chose a root vertex v, a root edge A incident with v, and a root face F incident with A. He then set up generating functions for the numbers of rooted plane triangulations and related graphs, with the coefficients depending on the number of vertices in the graph. He found recursions satisfied by these functions and solved the recursions, obtaining some very nice results. For example, if a_n is the number of distinct rooted triangulations or near-triangulations on $2n$ vertices and if

$$g(x) = \sum_{n=0}^{\infty} a_n x^n,$$

then

$$g(x) = 2 \sum_{n=0}^{\infty} \frac{(4n + 1)!\, x^n}{(n + 1)!\,(3n + 2)!}.$$

Similarly, Tutte found that the number of rooted 2-connected plane cubic graphs with $2n$ vertices is

$$p_n = \frac{2^n (3n)!}{(n + 1)!\,(2n + 1)!}.$$

Moreover, he found that the average number of Hamiltonian cycles in a rooted 2-connected plane cubic graph is

$$\frac{3(2n)!\,[(2n+2)!]^2}{2^{n+2}[(n+1)!]^2(n+2)!\,(3n)!} \sim \frac{8\sqrt{3}}{\sqrt{\pi n}}\left(\frac{32}{27}\right)^n.$$

Turning to colorings, he said [7, p. 125], "I now knew the number of general rooted triangulations with $2n$ faces. Why not also find the number of λ-colored ones, for an arbitrary positive integer λ?" Letting N denote a general rooted plane near-triangulation, he considered

$$f(z,\lambda) = \sum_N z^{r(N)} P(N;\lambda),$$

where $r(N)$ is the number of nonroot triangular faces of N. He found a differential equation satisfied by a related function H in a variable $t = z^2$. He said [7, pp. 127–8], "The coefficients of successive powers of t in H can be calculated from this equation. I am tempted to say that the problem of finding the number of 4-coloured rooted triangulations with $2n$ faces is now solved; it is the appropriate coefficient in the power series H defined as that solution of [the differential equation] in which the coefficients of t^0 and t^1 are zero. But we still need an asymptotic approximation. For that we can perhaps now look to the theory of differential equations."

What we have said here only touches the surface of his work in this area, represented by more than twenty-five papers written over a period of more than thirty years.

Reconstruction

Let G be an unlabelled graph without loops and with at least three vertices. For each vertex v of G we make a card showing $G - v$, formed by erasing v and every edge incident with v. Given only the resulting deck of $|V(G)|$ cards, can we reconstruct G? For example, if G is a path of length two, then two of the three cards show two vertices joined by an edge, and the third card shows two isolated vertices. Since each edge appears in all but two of the cards (and so exactly once in this deck of three cards), we conclude that there are two edges, and the vertex removed to form the third card meets both edges. Hence G is a path. In 1941 P. J. Kelly and S. M. Ulam conjectured that any loopless graph with at least three vertices can be reconstructed from the deck of vertex-deleted subgraphs; this is known as the *Reconstruction Conjecture*.

Label the vertices of a graph G with the integers $1, 2, \ldots, n$. The *adjacency matrix* $A(G)$ is an $n \times n$ matrix in which entry (i, j) is 1 if the vertices labelled i and j are adjacent and is 0 otherwise. For a loopless graph, the main diagonal of $A(G)$ is all zeros. For such a graph, the *characteristic polynomial* of G is $\det(A(G) - \lambda I_n)$, and its *spectrum* is the set of zeros, including multiplicities, of the characteristic polynomial. These are independent of the labelling of the vertices of G.

Since the Reconstruction Conjecture has proved to be extremely intractable, the question has arisen, "Which graph properties are reconstructible?" Even this problem remained open for the chromatic polynomial, the chromatic number, the characteristic polynomial, the spectrum, and the number of Hamiltonian cycles of a graph until, in [4, pp. 528–47] and in a 1979 paper entitled "All the King's Horses", Tutte showed that all five of these properties are reconstructible by showing that the Tutte polynomial is reconstructible.

Conclusion

As a result of William Thomas Tutte's contributions to graph theory and matroid theory, both subjects have many results bearing his name. With his code-breaking efforts, he made what has been called the "greatest intellectual feat of the whole war" [1]. Among his many honors, he was awarded the Tory Medal in 1975, the Killam Prize in 1982, and the CRM-Fields Prize in 2001. He was elected a Fellow of the Royal Society of Canada (1958), a Fellow of the Royal Society of London (1987), and an Officer of the Order of Canada (2001).

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Abel's Proof: An Essay on the Sources and Meaning of Mathematical Unsolvability

Reviewed by Lars Gårding

Abel's Proof: An Essay on the Sources and Meaning of Mathematical Unsolvability
Peter Pesic
MIT Press, May 2003
216 pages, $24.95
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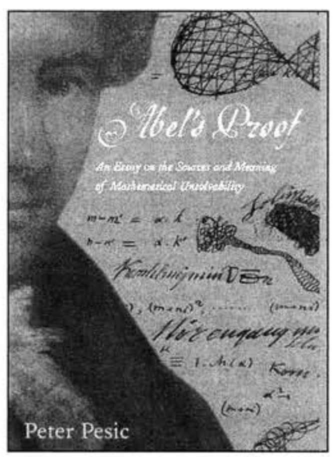
The general polynomial equation of degree 2 was solved in antiquity. Much later, around 1500, it was discovered in Italy that the zeros of general equations of degrees 3 and 4 can be expressed in terms of the coefficients by repeated rational operations and root extractions. One difficulty of this method in more general cases is that an nth root has n values so that simple expressions obtained by rational operations and root extractions from the coefficients easily take on more values than there are zeros of the equation. To give simple examples, the third root of $2 + \sqrt{3}$ has six values, and adding to it the third root of $3 + \sqrt{2}$ gives a root expression with thirty-six values. It is only artful combinations of similar expressions that produce the zeros of the equations of degree less than 5 and nothing else. For instance, in the formula for the zeros of the general equation of degree 3, two third roots are added whose product is required to be one of the coefficients. In spite of the many values of complex root expressions, it turned out in the end that they are still not flexible enough to represent the five zeros of the quintic, i.e., the general equation of degree five. But in the beginning of the nineteenth century past successes made it perhaps natural to be optimistic about the equation-solving power of roots or, with their Latin name, radicals. It may even have appeared that the only way of solving equations was by radicals.



That the quintic is not solvable by repeated root extractions and rational operations on the coefficients is in fact one of the most widely known results in mathematics. The discovery has a long prehistory and a dramatic finale in the first half of the nineteenth century with two heroes, Abel and Galois. Both died in their twenties, and their fame came afterwards. Abel was first with a proof that the quintic is not solvable by repeated rational operations and root extractions starting with the coefficients, and very soon both Abel and Galois gave what is essentially the end result, a beautiful theory of equations in general and solvable equations in particular.

The story of this development has been told in many ways: as history, as the lives of young geniuses; and as the terse, five-page account of the entire theory in van der Waerden's *Algebra*.

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Peter Pesic's book, entitled simply *Abel's Proof*, turns it into a broad, lively, and rather personal account of Abel's proof and its prehistory. The author even uses three appendices to give accounts of Abel's proof and other related mathematical truths: for instance, that the alternating group of permutations of more than four elements has no invariant subgroups. Here his essayistic style and frequent appeals to basics makes his text very difficult for anyone trying to achieve a solid grasp of the material. But this is just a mathematician's remark. The ordinary reader looks for readability rather than a text requiring tough thinking.

The first eight chapters give a fairly straightforward account of the prehistory of unsolvability in radicals of the general quintic. Under the heading "The Scandal of the Irrational", a reader starts the journey in antiquity with Euclid and the questionable existence of irrationals. In the next chapter, "Controversy and Coefficients", the reader passes on to the fifteenth and sixteenth centuries with emerging algebra and equations and the solutions of the equations of the third and fourth degrees by Ferrari and Cardano. In the chapter "Impossibilities and Imaginaries" the reader meets the imaginary numbers; and in the next one, with the enigmatic title of "Spirals and Seashores", he is introduced to preludes to the quintic, including Gauss's proof that every equation has a root—in the reviewer's mind perhaps a more important result than the unsolvability of the quintic.

After yet a chapter on preludes to the quintic, "Premonitions and Permutations", that includes Lagrange's resolvents, the reader reaches a nine-page chapter called "Abel's Proof". It is followed by a historical comparison between the works of Abel and Galois.

When the mathematics gets serious, as in the chapter on Abel's proof, the author is soft on the reader, separating a number of fact boxes of formulas from his running account. In the long-winded fourteen pages of Appendix A, Abel's proof is treated as an object to be seen from all sides. The author's choice to make Abel's proof the high point of the story is historically motivated, but I think that a reader would have been better informed by appendices also on what followed afterwards, i.e., the theory of equations solvable by rational operations and root extractions beginning with the coefficients. Anyway, trying to read Appendix A made the reviewer long for an arrangement that avoids Abel's somewhat awkward proof. This could be done in an appendix explaining and exemplifying the current theory of solvability by radicals in the usual terminology. The climax would then be a brief and lucid argument showing why a solvable quintic cannot be general.

After the preceding chapters the historical theme that kept the book together gives way to more loosely connected, general material. In "Seeing Symmetries" the author sets about to combine explanations of symmetry groups with those of the classical regular bodies where mappings are illustrated by dancers changing places. In the mind of the reviewer this approach creates more confusion than order in a reader wanting to get an idea of what a group is. Like many other places in the book, the trouble here is a style that prevents the author from presenting tight (but highly informative) material.

Of the two last chapters, "Seeing the Order of Things" and "Solving the Unsolvable", the first one deals with the general implications of noncommutativity, including quantum mechanics and relativity theory. But here the line of reasoning ranges over centuries, and a multitude of noncommittal remarks and predictions makes this chapter more obscure than interesting. The implications of Abel's and Galois's result are extended to many cases having only a slender connection with their supposed origin, as when trying to sum up Abel's importance: "The end of the old assumption that all equations have a finite solution revealed a new mathematics of infinite series and noncommutativity." The same is true of the last chapter, which among other things tries to attach some higher philosophical sense, or perhaps, just a pun, to the notion of unsolvability: "By transcending this limitation (unsolvability by radicals) Abel solved the unsolvable."

However, these two quotes do not give a fair idea of the book. The author has written a good historical account of how to solve and not to solve equations by radical extraction that will give a future reader many hours of pleasure and general historical knowledge. I should also add that the book has a very extensive, useful, and reasoned list of references.

Gödel's Proof

Reviewed by Timothy McCarthy

Gödel's Proof
Ernest Nagel and James R. Newman
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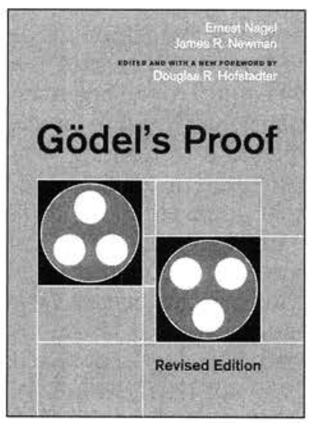
New York University Press, 2001
125 pages, $17.95
ISBN 0-1847-5816-9



In *A Mathematician's Apology*, G. H. Hardy wrote that no book is wholly bad that fires the imagination of clever young people. When *Gödel's Proof* first appeared forty-four years ago, it was the first introductory treatment of Gödel's incompleteness theorems in print and as such had an enormous formative influence on a whole generation of students. One of these was Douglas Hofstadter, the editor of the present revised edition of the book and author of the kaleidoscopic *Gödel, Escher, Bach: An Eternal Golden Braid*; another, a few years later, was the author of this review. This welcome republication of Nagel and Newman's book, with a helpful new preface and a number of substantive corrections, provides a chance to look back at a modern classic in light of the subsequent development of its subject.

The first six chapters of the book canvass the background to Gödel's paper of 1931. This paper establishes, for each formal axiomatic system of a certain not-very-special sort, the existence of sentences which are *undecidable* in the system (sentences in the language of the system which are neither provable nor refutable in the system). The paper also outlines an argument showing the unprovability in each such system of a certain sentence that may be interpreted as expressing its consistency. After a brief introductory chapter, in Chapter 2 the authors sketch the history of the consistency problem as it arose in the nineteenth century. The distinction between relative and absolute consistency proofs is introduced: a relative consistency proof is essentially a structure-preserving interpretation of one theory in another; an absolute consistency proof for a theory is an outright demonstration that no contradiction is derivable in the theory. An interpretation of Riemannian in Euclidean geometry is given, providing a historically important example of a relative consistency proof. The erroneous claim in the original edition that a model of Riemannian geometry results by taking *points* to be points on the surface of a Euclidean sphere and *lines* to be geodesics is corrected by construing the Riemannian points to be pairs of antipodal points (thus ensuring that two points determine a line). The chapter concludes with Russell's paradox, concerning the set of all non-self-membered sets,

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which dramatizes the consistency problem for naïve set theory.

Chapter 3 explores Hilbert's program for obtaining consistency proofs, which sought to provide absolute consistency proofs for fragments of classical mathematics in *finitistic* terms. For Hilbert "finitistic" meant, very roughly, *strictly constructive*. A finitistic consistency proof for a theory T containing arithmetic will specify an effective procedure for showing, for any proof σ in T, that σ is *not* a proof of the sentence $0 = 1$. To make this into a well-defined mathematical problem, T is to be construed as a purely formal object, characterized by a finite formal lexicon and by effective procedures (algorithms) for determining whether a finite sequence of symbols from the lexicon constitutes a formula and whether a finite sequence of formulas constitutes a proof. The systematic codification of formal logic underlying such a construal is described in Chapter 4, and Chapter 5 presents an example of a successful absolute consistency proof which is "finitistic" in the relevant sense: the authors show that a formulation of sentential logic is consistent by showing that every provable formula is a tautology (essentially by induction on the length of proofs). In general, the exposition in the first five chapters of the book is clear, concise, and correct, and especially helpful to the beginner.

In Chapter 6 Nagel and Newman begin to lay the groundwork for their exposition of Gödel's 1931 paper (the bulk of the exposition occurs in Chapter 7). The authors rightly point to a similarity between Gödel's proof and certain semantical paradoxes, pursuing a somewhat strained analogy between Gödel's construction and the Richard Paradox.[1] The idea behind Gödel's proof is also aptly compared to the Liar Paradox. A *Liar sentence* is a sentence of the form "p is not true", where "p" designates the sentence itself; such a sentence thus expresses its own falsity. Gödel showed, somewhat analogously, how to produce a sentence that expresses its own *unprovability*. This chapter goes on to usefully explore ways in which one structure may encode information about another under a mapping. It is this sort of mapping—whereby talk about the arithmetical properties of integers covertly becomes talk about the syntactic properties of a formal theory—that crucially underlies Gödel's construction of the sentence referred to above. It was in Chapter 7, which contains the exposition of the fine structure of Gödel's argument, where the original edition of the book ran into the most trouble.[2] Some of these shortcomings have been corrected in the present edition, but some substantial expository problems remain. In what follows I shall touch on what I take to be the most important of these.

Nagel and Newman clearly describe the process now called "arithmetization of syntax", whereby the syntactic objects of a formal system are coded by positive integers (Gödel numbers) and syntactic relations between them are represented by number-theoretic relations of a sort Gödel called "recursive" [*rekursiv*], now called "primitive recursive". Gödel's original target was a theory he called "PM", a somewhat simplified version of *Principia Mathematica*. PM is essentially a simple type theory, built over elementary arithmetic; that is to say, the theory introduces disjoint types of objects, sets of objects, sets of sets of objects, etc., beginning with the natural numbers as the initial type. The authors state a result they call "The Correspondence Lemma" (p. 73): that each "primitive recursive truth" can be expressed by a sentence of a certain form that is provable in PM and that each provable sentence of this form is true. There is a slight misstatement here: the term "primitive recursive" does not properly apply to propositions or sentences, but rather to certain relations (or functions). It would have considerably clarified the subsequent exposition to simply state the basic result as Gödel does: that each, say, binary primitive recursive relation R can be associated with an open sentence $\psi(x, y)$ in the language of PM such that if a pair $< k_1, k_2 >$ of integers is related by R, then the assertion $\psi(k_1, k_2)$ is provable in PM, and otherwise this assertion is refutable in PM. (A parallel result may be given for relations of any arity.) This is the property that is now called "numeralwise expressibility"; it is a readily understandable rendering of the authors' assertion that primitive recursive notions can be "mirrored" inside PM.

We can now arrive at the heart of the argument. Gödel showed how to define a primitive recursive binary relation R such that for any natural numbers n and m, nRm iff (i) there is an open sentence ϕ (containing just the variable y, say) such that m codes the sentence resulting by substitution of the Gödel number of ϕ itself for y in ϕ, and (ii) n is *not* the Gödel number of a proof in PM of the sentence with code m. Let $\mathbf{R}(x, y)$ be the formula of PM that expresses the relation R in the above sense, and let $B(y)$ be the formula $(\forall x)\mathbf{R}(x, y)$. Finally, let k be the Gödel number of $B(y)$. The sentence G that Gödel showed to be undecidable in PM is just the assertion $B(k)$, which, as the authors observe,

[1] *The analogy was suggested by Gödel himself. First presented by Jules Richard in 1905, this paradox considers an injective enumeration $C_1, C_2, \ldots$ of conditions expressible in English. Now define a condition, R, that applies to exactly the conditions C_n such that C_n does not apply to n. Let R be C_p. Then obviously R applies to* $\mathbf{p}$ *if and only if it does not.*

[2] *For a discussion of the difficulties in the original edition, see the review by Hilary Putnam in* Philosophy of Science **29** *(April 1960).*

expresses a truth of arithmetic if and only if that sentence itself is not a theorem of PM.[3] At this point, however, matters become somewhat muddled. First, the authors attribute to Gödel a proof of the equivalence "G is provable in PM iff $\neg G$ is provable in PM" (p. 99), but then immediately add a footnote asserting, correctly, that this is not what Gödel actually proved. The equivalence is rather attributed to Rosser, who first established the incompleteness of axiomatic theories extending arithmetic on the hypothesis of simple consistency (the condition that if a sentence A is derivable in the theory, then $\neg A$ is not). The authors say that they are using the equivalence attributed to Rosser in order to simplify their exposition; the difficulty is that Rosser did not in fact prove this equivalence! Rosser's undecidable sentence is not the statement G but another statement based on a somewhat more complicated diagonal construction. Rosser's argument is, if anything, *less* simple than Gödel's, not more so; the point of Rosser's theorem is that it gets along with a *weaker hypothesis* than the one assumed by Gödel's.

What is frustrating about this expository situation is that, given the admirable job of stage-setting the authors have done, a sketch of Gödel's actual argument would have been very easy to give. Gödel did in fact establish that the sentence G is not derivable in PM on the assumption that PM is simply consistent, and the authors correctly outline this part of the argument. The argument that $\neg G$ is not derivable in PM is based on the somewhat stronger hypothesis that PM satisfies a condition Gödel called "ω-consistency", saying that for no open sentence $A(x)$ is each sentence of the form $A(n)$ provable in PM while the sentence $(\forall x)A(x)$ is refutable in PM. This is equivalent to the condition that if PM proves each individual natural number falls under a property, then it cannot prove that there are natural numbers that do not. Gödel's argument is quite simple. The hypothesis of ω-consistency ensures that of simple consistency, and so the first half of the argument shows that G is not derivable in PM. So no number codes a proof of it; whence for each n we have nRk. Thus, since the formula $\mathbf{R}(x, y)$ expresses the relation R, for each n the assertion that $\mathbf{R}(n, k)$ holds is provable in PM. By ω-consistency, then, the sentence $\neg G$ cannot be a theorem of PM, since G is the assertion that $(\forall x)\mathbf{R}(x, k)$.

In the concluding part of Chapter 7, Nagel and Newman turn to Gödel's second incompleteness theorem, concerning the unprovability of consistency, and its significance for Hilbert's program. With the exception of some misstatements coming toward the end, this material is clear and helpful. The essential observation here is that the argument of the first half of the first incompleteness theorem, that the sentence G is not provable in PM if in fact PM is consistent, can easily be transcribed into PM by means of a suitable arithmetization of the notion of proof. Gödel represents the consistency claim by the sentence $(\exists x)\neg P(x)$, where $P(x)$ is a certain formula of arithmetic which is true of precisely the codes of the sentences provable in PM. This sentence is thus an arithmetization of the assertion that there is some unprovable sentence, which is easily seen to be equivalent to simple consistency. Let 'CON' abbreviate this arithmetical consistency statement. By mimicking the argument of Gödel's first theorem, we can derive the sentence "If CON, then G holds" *in PM*. Thus, since G is not derivable in PM (assuming PM consistent), neither is CON.

Nagel and Newman subscribe to the standard interpretation of the significance of this result for Hilbert's program but are rather more careful about expressing that interpretation than a number of subsequent writers. Nagel and Newman say that the result excludes the possibility of a finitistic consistency proof for PM *that can be mirrored in PM*, but add that "no one today appears to have a clear idea of what a finitistic proof would be like that is *not* capable of being mirrored inside *Principia Mathematica*" (p. 109, n. 39). Let us say that a formal theory is *finitistically complete* if every finitistically provable arithmetical statement is derivable in the theory. So the authors' claim is that Gödel's second theorem shows that no finitistic consistency proof is available for PM *if in fact PM is finitistically complete*; they then provisionally adopt the hypothesis that PM *is* finitistically complete, and detach the conclusion that no finitistic consistency proof for PM is available. They are surely on strong ground in making this completeness claim for PM; on the most influential analysis of finitism, the finitistically provable number-theoretic statements are identified with the theorems of primitive recursive arithmetic, a very weak fragment of PM. The difficulty lies rather with the first claim: Gödel's second theorem does not, unfortunately, by itself show quite what the authors take it to show. The difficulty is that Gödel shows explicitly only that *one* arithmetic transcription of the consistency claim for PM is underivable in PM (assuming PM consistent). But even if PM is finitistically complete, the question arises whether the syntactic assertion that PM is consistent, which Gödel showed to be unprovable in PM under one arithmetical transcription of its content, may yet be provable under another.

The authors do not discuss this question, but it is interesting to observe that their stance toward

[3] *Here and in what follows, if $P(x)$ is a condition and n an integer, by $P(n)$ we mean the sentence resulting from $P(x)$ by replacing x by a standard numeral for n; the sentence thus says that n satisfies $P(x)$.*

Gödel's second theorem makes possible a short answer to it, and one favorable to their desires. The question is whether there could be an alternative arithmetic transcription of the consistency assertion for PM that *is* derivable in PM. Suppose we had such an alternative candidate, say χ. Although it is not clear in general what χ must do in order to constitute an acceptable representation of the consistency claim, from the standpoint of finitistic consistency proofs, we clearly want to require χ to be a finitistically meaningful number-theoretic statement that can be recognized to be equivalent to the unarithmetized consistency claim by finitistic means: let us say in this case that the two statements are *finitistically equivalent.* Gödel's canonical arithmetization of the consistency claim for PM certainly satisfies this condition. Both Gödel's consistency sentence and the deviant sentence χ, then, are finitistically equivalent to the unarithmetized syntactic claim of consistency for PM. They are thus finitistically equivalent as number-theoretic statements. But the authors' assumption is that PM is finitistically complete. Thus this equivalence must already be demonstrable in PM. It follows that χ is not derivable in PM; else, Gödel's consistency statement would also be derivable in PM. In short, on the authors' hypothesis about PM, *if one finitistically acceptable arithmetic transcription of the consistency claim for PM is unprovable in PM, then they are all unprovable in PM.*

Nagel and Newman recognize that the impossibility of a finitistic consistency proof for a theory does not exclude the possibility of a more general sort of meta-mathematical consistency proof for it. Thus, for example, Gentzen's consistency proof for number theory (1936) is constructive in a reasonably strong sense, exploiting, in addition to finististic devices, transfinite induction over Cantor's ordinal ϵ_0. The authors cite Gentzen's work, but rather oddly construe its significance:

> Meta-mathematical arguments establishing the consistency of formal systems such as PM have, in fact, been devised, notably by Gerhard Gentzen, a member of the Hilbert school, in 1936, and by others since then. These proofs are of great logical significance, because they propose new forms of meta-mathematical constructions, and thereby help make clear how the class of rules of inference has to be enlarged if the consistency of PM and related systems is to be established. (pp. 107–108)

But Gentzen's proof did not target PM—nor, indeed, anything approximating PM—but only elementary number theory, and although Gentzen-style consistency proofs have subsequently been provided for stronger systems, using induction on larger and larger proof-theoretic ordinals, these proofs are conducted in standard set theory and are easily transcribed into systems such as PM. They cannot, therefore, *apply* to systems such as PM.

This misunderstanding of Gentzen-type consistency proofs appears to be connected to some of the reflections with which the book concludes. In the boldest of these, the authors stand at the beginning of a series of writers who have claimed that Gödel's incompleteness theorem bears negatively on the thesis of mechanism in the philosophy of mind. Gödel's results, the authors say, show that "the resources of the human intellect have not been, and cannot be, fully formalized, and that new principles of demonstration forever await invention and discovery" (p. 112). The end-product of formalization is a *formal system*, but a formal system is essentially an algorithm for generating its theorems. We could thus express this negative claim as the thesis that *the theorem-proving capacity of the human mind cannot be represented by a Turing machine.* The argument for this surprising contention is presumably something like the following: suppose that T is a Turing machine that can prove just the number-theoretic statements I can prove. Then via Gödel's procedure I can construct a true number-theoretic proposition (on the model of the sentence G above) that T cannot prove *but that I can recognize to be true.* But then T cannot, after all, prove all the number-theoretic statements I can prove!

In his review of the original edition of the book, Hilary Putnam provided the following rather decisive response to this argument:

> Given an arbitrary machine T, all I can do is find a proposition U such that *I* can prove
>
> (*) If T is consistent, then U is true,
>
> where U is undecidable in T if in fact T is consistent. However, T can perfectly well prove (*) too! And the statement U, which T *cannot* prove (assuming consistency), *I* cannot prove either (unless I can prove that T is consistent, which is unlikely if T is very complicated)!

It follows that if there is a Turing machine T that "represents" me in the present sense, then I cannot recognize that T is *sound* (i.e., proves only true sentences). But of course there is no reason to suppose that the soundness of my entire battery of theorem-proving strategies should be transparent to *me*! I suspect that Nagel and Newman may have been led to an insouciantly optimistic attitude toward the justification of consistency claims by their interpretation of Gentzen's result discussed above; it may have been their view that suitable extensions of Gentzen's procedure would facilitate consistency proofs for (consistent) theories of arbitrary strength.

However it arises, the authors' idea that there are nondemonstrative sources of mathematical justification has antecedents in Gödel's work and has surfaced repeatedly in the subsequent literature. But I want to emphasize that this idea by itself is entirely compatible with mechanism. At one point Gödel speculated that "it is conceivable that every proposition expressible in set theory is decidable from the present axioms plus some true assertion about the largeness of the universe of sets," and in his Gibbs Lecture he considered a strategy for introducing new axioms of the latter sort.[4] This strategy is *ampliative* (in the philosophical sense of *not deductive*; in particular, the combination of Zermelo-Fraenkel set theory with such an axiom implies the consistency statement for each previous system of this sort), but Gödel believed that the axioms in question are implicit in the general concept of set. However, such a strategy may be ampliative and yet be *effective* if in fact the choice of new axiom is effectively determined by the theory to which it is added.

Ampliative inferences are now modeled in what are called "nonmonotonic reasoning systems". In such a system there is a dynamic notion of *projection* for sentences: a sentence projected in one state of the system need not be projected in later states. The notion of *proof* is replaced by an appropriate stability property, which in the simplest of these models consists in a sentence's being projected in all descendents of a given state. When such a model is applied to number-theoretic problems, one can ask whether an analogue of Gödel's incompleteness theorem holds. Rather surprisingly, the answer is that it depends on the precise form of the theorem considered. The theorem of Rosser mentioned above turns out *not* to generalize to this context without restriction: there are nonmonotonic systems which are *effective* (in the sense that the evolution of the states of the system is described by a Turing machine), which satisfy analogues of both the consistency *and completeness* conditions, and in which each theorem of Peano Arithmetic is "provable" (in the relevant sense). But a form of Gödel's theorem itself, using a nonmonotonic analogue of Gödel's ω-consistency condition, holds for a wide variety of such systems. Stronger results are obtainable by reference to nonmonotonic analogues of what are called *reflection principles*, a condition requiring that a sentence of a certain form is provable only if it is true.

The significance of these (relatively recent) developments for Nagel and Newman's position is somewhat mixed. On the one hand, the nonmonotonic models formally implement a coherent conception of nondeductive justification in mathematics and show how it may be that the number-theoretic truths that can be justified in this way need not be effectively enumerable (the range of a computable function). On the other hand, in doing so they exhibit a computational architecture for such a conception and so undermine the idea that there is any tension between these observations and a "computationalist" understanding of mechanism. Nagel and Newman were only the first to allege a connection between Gödel's theorems and the thesis of mechanism.[5] The idea that there is such a connection was circulating at the time the original edition of the book appeared, perhaps inspired by some occasional remarks of Gödel himself, and it has come up repeatedly in the subsequent literature. But no convincing philosophical argument establishing such a connection has ever been given.

Despite its shortcomings, Nagel and Newman's book should be recognized as a classic piece of expository literature. It can still be recommended as an excellent introduction to the background of Gödel's incompleteness theorem and to the philosophical issues to which Gödel's result is connected. Some of its technical exposition is flawed; some of its philosophical claims are suspect. But it is notoriously difficult to say something philosophically sensible in this area, and there are now a number of excellent introductory presentations of the incompleteness theorems, written for people with only a modicum of logical background, containing reasonably complete proofs. Nagel and Newman's book can most profitably be read preparatory to, or in conjunction with, one of these. Its greatest merits are that it conveys the intellectual landscape in which Gödel's paper appeared, the general nature of the ideas involved in that paper, and something of the significance of the conclusions he reached. The book is an excellent point of departure. After reading it, one will want to learn more.

[4] *"Remarks before the Princeton bicentennial conference on problems in mathematics" and "Some basic theorems on the foundations of mathematics and their implications" (the Gibbs Lecture),* Collected Works, *vol. 3, Oxford University Press, 1995.*

[5] *In his foreword to the book, Douglas Hofstadter also finds the supposed connection to be unsupportable for reasons broadly similar to those given here. He has not, however, made any changes in the philosophical parts of Nagel and Newman's text.*

Why Is Mathematical Biology So Hard?

Michael C. Reed

Although there is a long history of the applications of mathematics to biology, only recently has mathematical biology become an accepted branch of applied mathematics. Undergraduates are doing research projects and graduate students are writing Ph.D. dissertations in mathematical biology, and departments are trying to hire them. But what should the Ph.D. training consist of? How should departments judge work in mathematical biology? Such policy questions are always important and controversial, but they are particularly difficult here because mathematical biology is very different from the traditional applications of mathematics in physics. I'll begin by discussing the nature of the field itself and then return to the policy questions.

Where's Newton's Law? The phenomena that mathematical biology seeks to understand and predict are very rich and diverse and not derived from a few simple principles. Consider, in comparison, classical mechanics and continuum mechanics. Newton's Law of Motion is not just a central explanatory principle; it also gives an immediate way to write down equations governing the important variables in a real or hypothetical physical situation. Since the Navier-Stokes equations express Newton's Law for fluids, they are fundamental and have embedded in them both the fundamental principle and the complexity of the fluid phenomena that we see. Thus a pure mathematician who proves a theorem about the Navier-Stokes equations and an applied mathematician who develops new numerical tools knows that he or she has really contributed something. Alas, there are no such central fundamental principles in biology. There are principles of course—some would say dogmas—such as "evolution by natural selection", "no inheritance of acquired characteristics", or "DNA → RNA → proteins". But these are not translatable into mathematical equations or other structures without hosts of additional facts and assumptions that are context-dependent. This means that mathematical biology is very unsatisfying for pure mathematicians, who usually are interested in discovering fundamental and universal structural relationships. It also means that there is no "mathematics of biology" in the same way that ordinary differential equations is the mathematics of classical mechanics and partial differential equations is the mathematics of continuum mechanics.

Diverse, yet special. Because of evolution, biological systems are exceptionally diverse, complex, and special at the same time, and this presents several difficulties to a mathematician. The first is choosing what to work on. There's too much biology! How do changes in the physics or chemistry of a particular environment affect the species that live in the environment (ecology)? How do diseases spread within a population (epidemiology)? How do the organ systems of the human body work (physiology)? How do the neurons in our brain work together to allow us to think and feel and calculate and read (neurobiology)? How does our immune system protect us, and what are the dynamical changes that occur when we are under attack by pathogens (immunology)? How do cells use physics and chemistry to accomplish fundamental tasks (cell biology, biochemistry)? How does the genetic code, inscribed in a cell's DNA, give rise to a cell's biochemical functioning (molecular biology and biochemistry)? How do DNA sequences evolve due to environmental pressures and random events (genomics and genetics)?

The second difficulty is that a priori reasoning is frequently misleading. By "a priori reasoning" I

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mean thinking how we would design a mechanism to accomplish a particular task. As a simple example, both birds and planes burn something to create energy that can be turned into potential energy, and both have to use the properties of fluids that are implicit in the Navier-Stokes equations. But that doesn't mean that one understands birds if one understands planes. To understand how a bird flies, one has to study the bird. Modelers are sometimes satisfied that they have created a mathematical model that "captures" the biological behavior. But that is not enough. Our purpose is to understand how the *biological* mechanisms give rise to the biological behavior. Since these biological mechanisms have been "designed" by evolution, they are often complicated, subtle, and very special or unusual. To understand them, one must immerse oneself in the messy, complex details of the biology; that is, you must work directly with biologists.

Thirdly, different species (or different tissues or different cells) may accomplish the same task by different mechanisms. An astounding array of special mechanisms allows animals to exploit special niches in their environments. For example, a diverse set of locomotory mechanisms are used at different size scales. Thus, when you have understood bird flight completely, you have not even started on the butterfly or the fruit fly. So, even when one is successful, one may have provided understanding only in particular cases.

We can already draw some conclusions. Don't do mathematical biology to satisfy a desire to find universal structural relationships; you'll be disappointed. Don't waste time developing "methods of mathematical biology"; the problems are too diverse for central methods. What's left is the biology. You should do mathematical biology only if you are deeply interested in the science itself. If you are, there's lots of good news. We mathematicians are experts at thinking through complex relationships and formulating scientific questions as mathematical questions. Some of these mathematical questions are deep and interesting problems in pure mathematics. And most biologists know that the scientific questions are difficult and complicated, so they want our help. There's some bad news too; there are three more reasons why the field is so hard.

The problem of levels. In many biological problems one is trying to understand how the behavior of the system at one level arises from structures and mechanisms at lower levels. How does the coordinated firing of neurons give rise to the graceful motion of an arm? How does the genetic code in DNA create, maintain, and adjust a cell's biochemistry? How does the biochemistry of a cell allow it to receive signals, process them, and send signals to other cells? How does the behavior of groups of cells in the immune system give rise to the overall immune response? How do the properties of individual bees give rise to the behavior of the hive? How do the cells in a leaf "cooperate" to turn the leaf towards the sun? How does the varied behavior of individuals contribute to the spread of epidemics?

We are familiar with these types of questions from physics. What are the right variables to describe the behavior of a gas, and how do the values of these variables arise from the classical mechanics of the molecules making up the gas? The behavior at the higher level is relatively simple, and Newton's law suggests the few important variables at the lower level; even so, the proofs are not easy. In the case of biological systems these questions are even more difficult, because the objects at the lower level have been designed by evolution (or trained by feedback control; see below) to have just the right special properties to give rise to the (often complicated) behavior at the higher level. And it is usually not easy to decide what the important variables are at the lower level. If your model has too few, you will not be studying the "real" biological mechanism. If your model has too many, it may be so complicated that a lifetime of computer simulations will not give new biological understanding. You need ideas, guesswork, experience, and luck. You need to be able to deduce the consequences from the assumptions. That is what mathematicians are good at.

The difficulty of experimentation. We mathematicians often have an overly simple view of experiments and the role they play, probably because we don't conduct them ourselves. A theory is tested by deciding on a few crucial variables and designing the right experimental setup. For example, one measures how fast metal beads and feathers fall in a vacuum or the angle subtended by two stars. However, the complicated histories of interaction between theory and experiment in quantum mechanics, nuclear physics, and elementary particle physics in the twentieth century show that this simple view is naive. And for several reasons the experimental situation is even more difficult in biology.

First, one is often interested in how the behavior at one level arises from lower levels. Typically, this "emergent" behavior cannot be seen in any of the parts at the lower level but arises because of complex interactions among the parts. Unfortunately, it can be misleading to study the parts in isolation. For example, I try to understand how certain biochemical networks in mammalian cells function. The networks give rise to systems of ODEs in which the nonlinear terms depend on the enzyme kinetics for each separate enzyme. The enzymes can be isolated and their reaction kinetics studied "in vitro" in experiments that combine pure enzyme with pure substrate. But in the soup, in the real cell, the enzymes and substrates are binding to or being affected by other chemicals too, so one is unsure whether the "in vitro" experiments reflect the true "in vivo" kinetics. That is, each of the parts at the lower level behaves differently in isolation than when it is connected to the other parts.

Second, chance plays a role, not only in experimentation, but perhaps in explanation. Why are two neighboring fields dominated by black-eyed Susans and poppies respectively? Are the soils different? Are the local plant and animal species different? Or perhaps the "explanation" is a chance event in the past (or all of the above).

Third, individuals (whether cells or flowers or people) are both similar and different. How does one know whether data collected is "special" or "typical"? How does one assure oneself that rat data tells us something about humans or, indeed, something about other rats?

Finally, it is characteristic of living systems that the parts themselves are not fixed but ever changing, sometimes even affected by the behavior of the whole (see feedback control, below). A simple true story illustrates this point. An experimenter (#1) who used rats in his experiments was getting very unusual results, and the results were not repeatable week to week. After six months of this he had to stop his experiments and investigate the rats. His rats were housed in his university's vivarium. It turned out that another experimenter (#2) had a mean technician, and when the other experimenter's technician came to get #2's rats, #2's rats would cry out, upsetting the rats belonging to #1. The behavior of #1's rats in experiments depended on whether #2 was doing an experiment the same day! Whew, I'm glad I became a mathematician.

For all these reasons, biological data must be approached cautiously and critically. Since biological systems are so diverse and everything seems to interact with everything else, there are many possible measurements, and enormous amounts of data can be produced. But data itself is not understanding. Understanding requires a conceptual framework (that is, a theory) that identifies the fundamental variables and their causative influences on each other. In messy biological problems, without simple fundamental principles like Newton's Law, useful conceptual frameworks are not easy to propose or validate. One must have ideas about how the structure of (or behavior of) the whole is related to the assumptions about the parts. Thinking through such ideas and proving the consequences of the assumptions are important ways that we mathematicians can make contributions.

The problem of feedback control. It is common to think of biological systems as fragile. However, most are very stable, and it is almost a tautology to say so, because they must all operate in the face of changing and fluctuating environmental parameters; so if they weren't stable, they wouldn't be here. We are familiar from engineering with the concept of feedback control, whereby variables are sensed and parameters are then reset to change the behavior of the system. The nephrons in the kidney sense NaCl concentration in the blood and adjust filtration rate to regulate salt and water balance in the body. The baroreceptor loop regulates blood pressure, heart rate, and peripheral resistance to adjust the circulation to different challenges. Numerous such control systems are known and studied in animal and plant physiology.

There is another kind of feedback that operates between levels that poses special problems. Here are two examples. In the auditory system sensory information is transformed in the cochlea to electrical information that proceeds up the VIIIth nerve to the cochlear nucleus and from there to various other nuclei in the brain stem (a nucleus is a large anatomically distinct group of cells) and on to the midbrain and the cortex. Surprisingly, there are also neural projections from the cortex that influence the sensitivity of the cochlea. Second, the dogma DNA → RNA → proteins → function has turned out to be a naive fiction. Genes (pieces of DNA) don't turn themselves on or off but are activated or inhibited by proteins. That is, proteins affect the genes, adding a reverse loop to the simple picture, implying of course that the genes affect each other through the proteins. The "fundamental" objects, that is, the objects most closely related to "function", may not be genes or proteins but small networks involving both genes and proteins that respond in certain ways to changes in the cell's environment. These kinds of examples show that the nineteenth-century picture of a machine with parts is a very inappropriate metaphor for (at least some) biological systems. When there is feedback between levels, it is hard to say which are the parts! In fact, it may be hard to say which are the levels, and therefore our traditional scientific research paradigm of breaking things into smaller parts (lower levels) may not be successful.

This is not just a philosophical point but a fundamental research issue that deepens the impact of the previous four difficulties. Take, for example, the question of dendritic geometry. It's been one hundred years since Ramon y Cajal made beautiful drawings of complicated dendritic arbors on nerve cells. Is the geometry important? Surely it must be, we feel, since cells in the same brain nucleus in different individuals seem to have roughly similar dendritic arborization. And, indeed, there are examples where it is understood how specific dendritic geometry creates specific neuron-firing properties and presumably specific cell function, though it is not always clear what "function" means for a single cell embedded in layers and layers of a large neural network. On the other hand, suppose a cell is part of a large neural network whose job it is to transform its pattern of inputs into a corresponding pattern of outputs. This neural network may have been trained to do this job by feedback control from a higher level, in which case the details of the dendritic geometry (and even the details of the neural connections) may not be important at all. The details arose from the training, and they are whatever they need to be to give the behavior at the higher level. Furthermore, for large networks there may be many choices of details that give the same network behavior, in which case it will be hard to infer the behavior of the whole by studying the properties of the parts.

I now want to turn to the policy questions that I mentioned at the beginning. I have been using the term "mathematical biology" to refer in the broadest way to quantitative methods in the biological and medical sciences. Physicists, chemists, computer scientists, and biological and medical researchers with some mathematical training can and do contribute to the field I have been referring to as "mathematical biology". But let us now narrow the focus to mathematics education, both undergraduate and graduate, and the mathematics job market.

Undergraduate education. Mathematical biology is an extremely appealing subject to undergraduate students with good training in freshman and sophomore mathematics. Many are naturally interested in biology, and all know that we are in the midst of a revolution in the biological sciences. They are usually amazed and delighted that the mathematical techniques that they have learned can be used to help understand how biological systems work. Further, mathematical biology is a perfect subject for undergraduate research projects. Biology is so diverse and so little quantitative modeling has been done that it is relatively easy to find projects that use undergraduate mathematics in new biological applications. The students find such projects to be very rewarding. They know that the undergraduate major consists mostly of nineteenth-century mathematics; of course they are excited by twenty-first-century applications. Here at Duke we have found that the availability of projects in mathematical biology has attracted many students to the mathematics major. Of course, it helps to have a mathematical biologist on the faculty, but it is not necessary. Any mathematician can create and supervise such projects by working cooperatively with local biologists. It requires only the effort to make the connections and tolerance for appearing "nonexpert" to the students (something we are not used to!).

Graduate education. There is quite a bit of disagreement about the proper mathematics graduate training of a student who wants to be a mathematical biologist. I'll simplify the discussion into two (extreme) positions. The first position emphasizes maximal contact with biology and biologists as part of graduate training. Graduate students should take biology courses (including labs) and should participate in or even initiate collaborative modeling projects. This way they learn a lot of biology, and, even more importantly, they learn modeling and how to communicate with biologists. By doing this they study less mathematics of course, but they can learn what they need later when they need it. The second position emphasizes training as a mathematician first. Graduate students should receive the traditional training in analysis or applied mathematics (or other subjects), and (ideally) the thesis should contain some applications to biological problems. But the graduate student should not spend too much time slogging around in the biological details or working on collaborative projects. It is the job of the thesis advisor to be the interface between the graduate student and the biology. Later, after the Ph.D., when the mathematician is established, he or she can choose to become involved in collaborations and learn more biology.

I guess that most mathematical biologists support the first position. I support the second, perhaps because that is the route that I followed myself. Mathematical biology is really a very hard subject (I hope I have convinced you of that), and a great many ideas and techniques from different branches of mathematics have proven useful. So mathematical biologists need broad training in mathematics. Secondly, I believe that only deep and rigorous graduate training creates mathematicians who can not only learn new mathematics when they need it, but who can also recognize what they need to learn.

Hiring issues. Hiring a mathematical biologist poses special challenges for departments. Most mathematicians have no idea how large the field "biology" is or how large the research communities are. Two examples illustrate this. Here at Duke, Arts and Sciences has 469 tenure-track faculty members, and the Medical Center has 767 (not counting clinical faculty). My colleague Harold Layton is a mathematician who works on the kidney, so of course he goes to the annual meeting of the American Society for Nephrology, where the typical registration number, 12,000, completely dwarfs the registration at the Joint Mathematics Meetings. And those constitute just a subset of the researchers who work on the kidney! So, the first issue is thinking about what kind of mathematical biologist you want. It is a good idea to involve local biologists and medical researchers in preliminary discussions, both to educate department faculty and to understand the local context.

The question of how best to judge job candidates is particularly difficult in mathematical biology. First, most mathematicians know less biology than their tenth-grade daughters. That's just the way it is; biology was not a mathematically related discipline when we were growing up. But, more importantly, mathematical biology really isn't a "field" of mathematics with a coherent community that can testify meaningfully about young people. Mathematical biology is fragmented because biology itself is so diverse. A mathematician working on the lung might want to talk to pulmonary physiologists or geometric analysts who are experts on fractals, but why would he or she want to talk to mathematical biologists working on the kidney, the neurobiology of hearing, or the epidemiology of AIDS? In each area of specialization there is a tremendous amount of biology to learn, and if one doesn't have the background, it's hard to judge the strength of an individual's contributions. This is true even for us, the mathematical biologists, the "experts" you expect to consult. So hiring in mathematical biology necessarily involves intuition and high risks as well as high potential payoffs for the department and the college.

The first step. The best way for departments to overcome these difficulties is to encourage senior faculty to become involved in bringing biological applications and student projects into the undergraduate curriculum. This should be done by working cooperatively with local biologists to create examples and projects related to their own specialties. All mathematicians can do this, and they do not have to give up their own research agendas or become mathematical biologists; it only requires effort.

This strategy for engagement with biology has great benefits both for departments and individuals. The faculty as a whole will become educated in biology and thus better able to judge job candidates in mathematical biology, and the undergraduate curriculum will be more attractive. More importantly, departments and individuals will be participating intellectually in the biological revolution, the greatest scientific revolution of our times, perhaps of all times. The task is to understand how life, in all its diversity and detail, works. This includes how we act, think, and feel, and how we influence and are influenced by other forms of life. We mathematicians have the technical and intellectual tools to make enormous contributions. So, surely, this is our responsibility and our opportunity.

Mathematics People

Kohn Receives 2004 Bergman Prize

JOSEPH J. KOHN of Princeton University has been awarded the 2004 Stefan Bergman Prize. Established in 1988, the prize recognizes mathematical accomplishments in the areas of research in which Stefan Bergman worked. Kohn will receive one year's income from the prize fund, approximately $22,000.

The previous Bergman Prize winners are: David W. Catlin (1989), Steven R. Bell and Ewa Ligocka (1991), Charles Fefferman (1992), Yum Tong Siu (1993), John Erik Fornæss (1994), Harold P. Boas and Emil J. Straube (1995), David E. Barrett and Michael Christ (1997), John P. D'Angelo (1999), Masatake Kuranishi (2000), László Lempert and Sidney Webster (2001), and M. Salah Baouendi and Linda Preiss Rothschild (2003). On the selection committee for the 2003 prize were John P. D'Angelo, John Erik Fornæss, and Yum Tong Siu (chair).

Citation

Joseph J. Kohn's work in partial differential equations (PDE) and several complex variables (SCV) has influenced countless researchers and has fostered an intense interaction between these subjects. Kohn is best known for his solution of the $\overline{\partial}$-Neumann problem and the many subsequent developments for PDE and SCV. The $\overline{\partial}$-Neumann problem was proposed by D. C. Spencer in the 1950s as a way of extending Hodge theory to open domains in complex manifolds. Although the system of partial differential equations (the Cauchy-Riemann equations) is elliptic, the boundary conditions are not; Kohn's solution for strongly pseudoconvex domains, given in 1962, introduced new and deep methods, including the so-called $\frac{1}{2}$ estimate.

Kohn's work soon led to the Kohn-Nirenberg theory of pseudodifferential operators and also to the study of the tangential Cauchy-Riemann equations initiated by Kohn and Rossi. Kohn also proved a global regularity result for the solution of the Cauchy-Riemann equations. In the early 1970s Kohn introduced subelliptic estimates for $\overline{\partial}$; the search for the right geometric conditions for these estimates led to the study of finite-type conditions for weakly pseudoconvex domains and eventually to deep connections between partial differential equations and the algebraic method of multiplier ideals.

Joseph J. Kohn

Kohn's paper in 1979 motivated Nadel's work on the application of multiplier ideal sheaves to the existence problem of Kähler-Einstein metrics on Fano manifolds, which led to the application of multiplier ideal sheaves to many problems in algebraic geometry, such as the Fujita conjecture, the invariance of the plurigenera, and the effective Nullstellensatz.

Kohn's work has provided an impetus to the study of the Bergman projection P and its many applications in regularity for biholomorphic and proper mappings. Kohn's formula $P = I - \overline{\partial}^* N \overline{\partial}$ links the Bergman projection and the Neumann operator N, and this link has been instrumental in the work of several previous winners of the Bergman Prize.

Kohn has also made important contributions to microlocal analysis, to the complex Monge-Ampère equation, and to the study of hypoellipticity for second order operators.

Biographical Sketch

Joseph J. Kohn was born on May 18, 1932, in Prague. He received his B.A. from the Massachusetts Institute of Technology (1953) and his M.A. (1954) and Ph.D. (1956) from Princeton University. His thesis advisor was Donald C. Spencer.

Kohn was an instructor at Princeton University in 1956–57 before moving to Brandeis University, where he was on the faculty for ten years. In 1968 he assumed his present position as a professor at Princeton University. He has held visiting positions at the Courant Institute of Mathematical Sciences of New York University, the Institute for Advanced Study in Princeton, Harvard University, the Institut des Hautes Études Scientifiques, Charles University of Prague, the University of Florence, the University of Buenos Aires, and the University of Mexico. His many professional activities include serving on the AMS Board of Trustees (1972–82) and on the U.S. National Committee for Mathematics (2000–04).

Kohn is the recipient of a Sloan Fellowship and a Guggenheim Fellowship. He was elected a fellow of the American Academy of Arts and Sciences and a member of the U. S. National Academy of Sciences. In 1966 he was an invited speaker at the International Congress of Mathematicians. He was awarded the 1979 AMS Steele Prize for a Seminal Contribution to Research. He received an honorary doctorate from the University of Bologna and the Bolzano Medal of the Czechoslovak Mathematics and Physics Society in 1990. In 1993 he was awarded the First Order Prize of the Union of Czech Mathematicians and Physicists.

About the Prize

The Bergman Prize honors the memory of Stefan Bergman, best known for his research in several complex variables, as well as the Bergman projection and the Bergman kernel function that bear his name. A native of Poland, he taught at Stanford University for many years and died in 1977 at the age of eighty-two. He was an AMS member for thirty-five years. When his wife died, the terms of her will stipulated that funds should go toward a special prize in her husband's honor.

The AMS was asked by Wells Fargo Bank of California, the managers of the Bergman Trust, to assemble a committee to select recipients of the prize. In addition, the Society assisted Wells Fargo in interpreting the terms of the will to assure sufficient breadth in the mathematical areas in which the prize may be given. Awards are made every one or two years in the following areas: (1) the theory of the kernel function and its applications in real and complex analysis, and (2) function-theoretic methods in the theory of partial differential equations of elliptic type with attention to Bergman's operator method.

—Allyn Jackson

Nemirovski and Todd Awarded von Neumann Prize

The 2003 John von Neumann Theory Prize, the highest prize given in the field of operations research and management science, has been awarded to Arkadi Nemirovski of the Technion-Israel Institute of Technology and Michael J. Todd of Cornell University for their "seminal and profound" contributions to continuous optimization. The award, presented by the Institute for Operations Research and the Management Sciences (INFORMS), carries a cash award of $5,000.

The citation for Nemirovski reads in part, "Arkadi Nemirovski has made fundamental contributions in continuous optimization in the last thirty years that have significantly shaped the field. He developed (with D. Yudin) the theory of information-based complexity for convex optimization underlying the majority of modern results on efficient solvability of well-structured convex problems." He has also done "groundbreaking work in the theory and algorithmic implementation of interior-point polynomial-time methods for convex optimization."

The citation for Todd states that he "has made fundamental contributions in a variety of different theoretical domains in continuous optimization" and "developed new triangulations for fixed-point algorithms and developed the critical measure of efficiency of triangulations (average directional density); in addition he made important contributions to combinatorial and pivot theory for fixed-point methods and related mathematical structures." It further states that his work "defined new and critical ways of thinking about the underlying theory of optimization, established the important issues in the field, and explored a variety of mathematical themes and algorithmic concepts."

—From an INFORMS announcement

Prizes and Elections of the French Academy of Sciences

The French Academy of Sciences has announced the awarding of several prizes in mathematics for 2003. Claire Voisin, Université Pierre et Marie Curie, has been awarded the Prix Sophie Germain. Louis Boutet de Monvel, Université Pierre et Marie Curie, received the Prix Fondé par l'État. The Prix Jacques Herbrand was awarded to Wendelin Werner, Université Paris-Sud. Claude Bardos, Université Paris VII, was awarded the Prix Marcel Dassault. Gilles Lebeau, Université de Nice Sophia-Antipolis, received the Prix Ampère de l'Électricité de France. The Prix Gabrielle Sand et M. Guido Triossi was awarded to Damien Gaboriau, Centre National de la Recherche Scientifique. Jean-Marc Delort, Université Paris Nord, received the Prix Langevin.

Three mathematicians were elected to the French Academy of Sciences in 2003. They are Thierry Aubin, Université de Paris 6; Laurent Lafforgue, Institut des Hautes Études Scientifiques; and Marc Yor, Université de Paris 6.

—From French Academy of Sciences announcements

Rhodes Scholarships Awarded

Six students of mathematics are among the thirty-two American men and women chosen as Rhodes Scholars by the Rhodes Scholarship Trust. The Rhodes Scholars were chosen from 963 applicants who were endorsed by 366 colleges and universities in a nationwide competition. The names and brief biographies of the mathematics scholars follow.

Olivia Rissland of Belmont, Massachusetts, is a senior at Brown University majoring in biology, mathematics, and Latin. She is the editor of the Brown classics journal and has gained distinction in the classics, as well as in mathematics and science. She is a radio disc jockey, coxswain in a boat club, and a black belt in karate. She plans a career in medicine and will work on a doctorate in biology at Oxford.

Delavane Diaz of Tampa, Florida, is a senior astronautical engineering major at the United States Air Force Academy with a minor in Spanish. She is wing commander with responsibility for 4,000 cadets, the second woman ever to hold this top position at the academy. She was named the most valuable player for two years on the academy's varsity volleyball team. She plans to work on an M.Sc. in mathematical modeling and scientific computing at Oxford.

Robin M. Rotman of Lake Bluff, Illinois, is a senior at the University of the South, where she majors in geology with minors in environmental studies and mathematics. She has won research internships at both the National Science Foundation and the Environmental Protection Agency. She is also treasurer of the Sewanee student body. At Oxford she plans to work toward an M.Sc. in geology.

Allison Gilmore of Eagan, Minnesota, is a senior at Washington University in St. Louis, where she will receive both her B.A. and M.A. in mathematics. A Byrd Scholar and a Compton Scholar, she has particular interest in algebraic topology. Allison is also president of Washington University Students for a Sensible Drug Policy and is a leader of her campus Stop-the-War coalition. She plans to work toward an M.Phil. in sociology at Oxford.

Decker Walker of Lafayette, Indiana, is a senior at St. Olaf College, where he majors in mathematics and economics. As a Goldwater Scholar, he worked in Zambia on African development, and his work is to be published. Decker plays varsity football and indoor and outdoor track and field. He spent his junior year at Oxford University, where he won blues in both track and field and basketball. He plans to work on a doctorate in economics at Oxford.

Lara B. Anderson of North Logan, Utah, was a double major in physics and mathematics at Utah State University. She graduated first in her class and is currently working on a master's degree. She is a violinist and a black belt in aikido. Lara will work on a doctorate in mathematical physics at Oxford.

Rhodes Scholarships provide two or three years of study at the University of Oxford in England. The value of the Rhodes Scholarship varies depending on the academic field, the degree (B.A., master's, doctoral), and the Oxford college chosen. The Rhodes Trust pays all college and university fees and provides a stipend to cover students' necessary expenses while they are in residence in Oxford, as well as during vacations, and transportation to and from England. The total value averages approximately $30,000 per year.

—From a Rhodes Scholarship Trust announcement

Young Mathematicians Honored in Siemens Westinghouse Competition

Four young mathematics scholars have been awarded prizes in the 2003–2004 Siemens Westinghouse Competition in Math, Science, and Technology. Linda Westrick of Maggie Walker Governor's School in Richmond, Virginia, won a $40,000 scholarship for her project "Investigations of the Number Derivative". The number derivative is closely linked with the prime numbers and calculus. Westrick discovered many "surprising and beautiful" patterns produced by the derivative.

The team of Araceli Fernandez of Harlandale Senior High School, San Antonio, Texas; Yiduo "David" Wang of Lincoln High School, Portland, Oregon; and Hannah Chung of Lyndon B. Johnson High School, Austin, Texas, were honored for their joint project, titled "Eccentric Graphs of Block Graphs and Trees". The project consists of mathematical proofs that explore the structure and properties of eccentric graphs that can be constructed from block graphs and trees. The prize carries a $10,000 scholarship award.

The annual competition, administered by the College Board and funded by the Siemens Foundation, recognizes outstanding talent among high school students in science, mathematics, and technology.

—From a Siemens Foundation announcement

European Mathematical Society Article Competition 2003

The European Mathematical Society (EMS), through its Committee for Raising Public Awareness of Mathematics (RPA), has announced the winners of its first competition for articles published in the general press that seek to present mathematics in interesting ways for the general public. The articles were published in the authors' home countries. They could be in any language that could be read by more than one member of the committee.

First prize was awarded to Nuno Crato, Universidade Técnica de Lisboa, Portugal, for a three-part article, "Cibersegredos inviolàveis" ("Unbreakable cybersecrets"), published in the Portuguese weekly newspaper *Expresso*, September 8, 22, and 29, 2001.

Second prize was given to F. Thomas Bruss, Université Libre de Bruxelles, Belgium, for his article "Der Ungewissheit ein Schnippchen schlagen" ("Playing a Trick on Uncertainty"), published in the magazine *Spektrum der Wissenschaft*, June 6, 2000, and for a similar article in the daily German newspaper *Die Welt*, May 17, 2001.

Third prize went to Sava Grozdev, Ivan Derzhanski, and Evgenia Sendova, Union of Bulgarian Mathematicians, for the article titled "For Those Who Think Mathematics Dreary", published in the Bulgarian daily newspaper *Dnevnik*, December 27, 2001.

The RPA committee which awarded the prizes consisted of Chris J. Budd, Mireille Chaleyat-Maurel, Michele Emmer, Andreas Frommer, Vagn Lundsgaard Hansen (chair), Osmo Pekonen, José Francisco Rodrigues, and Marta Sanz-Solé. For further information, including links to the prizewinning articles, see the website `http://www.mat.dtu.dk/people/V.L.Hansen/rpa/resultartcomp.html`.

—From an EMS announcement

Mathematics Opportunities

Mathematics Awareness Month: April 2004

The theme for Mathematics Awareness Month 2004 is "Mathematics and Connections". This theme concerns the mathematics of networks, an area that has been popularized by several books that have come out in the past couple of years, including:

Small Worlds: The Dynamics of Networks between Order and Randomness, by Duncan J. Watts (Princeton University Press, 1999) (reviewed in the *Notices*, September 2000);

Nexus: Small Worlds and the Groundbreaking Science of Networks, by Mark Buchanan (W. W. Norton & Company, 2002);

Linked: The New Science of Networks, by Albert-Lászl6 Barabási (Perseus Publishing, 2002) (reviewed in the *Notices*, February 2004);

Sync: The Emerging Science of Spontaneous Order, by Steven Strogatz (Hyperion, 2003) (reviewed in this issue of the *Notices*);

Six Degrees: The Science of a Connected Age, by Duncan J. Watts (W. W. Norton & Company, 2003) (reviewed in the *Notices*, February 2004).

As in previous years, there will be a Mathematics Awareness Month poster and a theme essay highlighting some applications of mathematical networks. Mathematics Awareness Month provides a timely occasion for mathematics departments and other organizations to celebrate mathematics and undertake activities to raise awareness among the general public. Refer to the AMS home page, `http://www.ams.org`, for further information about Mathematics Awareness Month.

—Allyn Jackson

Project NExT: New Experiences in Teaching

Project NExT (New Experiences in Teaching) is a professional development program for new and recent Ph.D.'s in the mathematical sciences (including pure and applied mathematics, statistics, operations research, and mathematics education). It addresses all aspects of an academic career: improving the teaching and learning of mathematics, engaging in research and scholarship, and participating in professional activities. It also provides the participants with a network of peers and mentors as they assume these responsibilities. Each year about sixty faculty members from colleges and universities throughout the country are selected to participate in a workshop preceding the Mathematical Association of America (MAA) summer meeting, in activities during the summer MAA meetings and the Joint Mathematics Meetings in January, and in an electronic discussion network. Faculty for whom the 2004–2005 academic year will be the first or second year of full-time teaching (post-Ph.D.) at the college or university level are invited to apply to become Project NExT Fellows.

The application deadline is **April 16, 2004.** For more information see the Project NExT website, `http://archives.math.utk.edu/projnext/`. Project NExT receives major funding from the ExxonMobil Foundation, with additional funding from the Dolciani-Halloran Foundation, the American Mathematical Society, the Educational Advancement Foundation, the American Statistical Association, the National Council of Teachers of Mathematics, the Association of Mathematics Teacher Educators, the Association for Symbolic Logic, and the Greater MAA Fund.

—Christine Stevens, Project NExT

News from the IMA

The Institute for Mathematics and its Applications (IMA) at the University of Minnesota announces the following short course and visiting professorship program.

IMA New Directions Short Course: From July 6 to 16, 2004, the IMA will host an intensive short course designed to efficiently provide mathematicians the basic knowledge prerequisite to understand *Computational Topology*. The course will be taught by Herbert Edelsbrunner, professor of computer science and mathematics at Duke University, and John L. Harer, professor and vice provost for academic affairs, Department of Mathematics, Duke University. Participants will receive full travel and lodging support during the workshop.

Content and philosophy. We understand computational topology as the development of algorithmic tools implementing topological concepts for use in the sciences and engineering. This is different from using the computer to study topological questions, although there is the potential for a beneficial symbiosis between the two efforts. The history of computational topology is short. It grew out of computational geometry as researchers expanded into applications where significant topological issues arise. The two such areas discussed in this course are structural molecular biology and geometric modeling. Both have connections to industries of substantial economical size.

A primary goal in this course is to develop a broad picture in which algorithmic tools connect pure mathematics with scientific applications. Our utilitarian view is that the application should drive the mathematics, the algorithms, and the software development.

Organization. A typical day during the two-week course consists of two general lectures by the principal speakers in the morning, each one-and-a-half hours in duration. There will be a more specialized one-hour topical lecture after lunch. The speakers will vary, and we will occasionally have introductions to topic-related software packages. In the late afternoon there will be a loosely organized two-hour brainstorming session.

Necessary background. The main requirements are mathematical maturity and an open mind toward connecting mathematics to the world around it in new ways. No specialized knowledge in the two application areas will be assumed. Some background in algorithmic thinking and using computers will be helpful.

The application deadline for the New Directions Short Course is **April 1, 2004.**

IMA New Directions Visiting Professorships: The IMA invites applications by established mathematicians for two Visiting Professorships to branch into new directions and increase the impact of their research by spending a year immersed in the 2004–2005 thematic program at the IMA, *Mathematics of Materials and Macromolecules: Multiple Scales, Disorder and Singularities.* Visiting Professors will enjoy an excellent research environment and stimulating scientific program with broad mathematical connections, including differential and integral equations, dynamical systems, statistical mechanics, probability, scientific computing and numerical analysis, and geometry. New Directions Visiting Professors are expected to be resident and active participants in the program but are not assigned formal duties. The New Directions program will supply 50 percent of faculty salary up to $45,000 maximum. The Visiting Professor's home institution must commit to providing a minimum of 50 percent of academic-year salary and all health and other relevant fringe benefits.

The application deadline for New Directions Visiting Professorships is **April 1, 2004.**

For further information and application procedures, please go to `http://www.ima.umn.edu/new-directions/`.

—IMA announcement

Inside the AMS

Math in Moscow Scholarships Awarded

The American Mathematical Society (AMS) has made awards to eight undergraduate students to attend the Math in Moscow program.

The names of the students and their institutions are: OWEN BAKER, North Carolina State University; AMY BIGELOW, Middlebury College; ROBERT EDMAN, Michigan Technical University; JENNIFER HOM, Columbia University; BENJAMIN MERMELSTEIN, Pennsylvania State University; EMMA SMITH, Massachusetts Institute of Technology; MURRAY STOKELY, California State University, Hayward; and JEFFERSON TAFT, University of Arizona.

Math in Moscow is a program of the Independent University of Moscow that offers foreign students (undergraduate or graduate students specializing in mathematics and/or computer science) the opportunity to spend a semester in Moscow studying mathematics. The fifteen-week program is similar to the Research Experiences for Undergraduates programs that are held each summer across the United States. Math in Moscow draws on the Russian tradition of teaching mathematics, which emphasizes creative approaches to problem solving and in-depth understanding. All instruction is in English.

Each semester since 2001 the AMS has awarded several scholarships of approximately $5,000 each for U.S. students to attend the Math in Moscow program. The scholarships are made possible through a grant from the National Science Foundation. Pending renewed funding, the AMS plans to continue awarding the scholarships in the future. Information about how to apply will appear in a future issue of the *Notices* and on the web page `http://www.ams.org/careers-edu/mimoscow.html`. The anticipated deadline is **April 15, 2004**. More information about Math in Moscow may be found at `http://www.mccme.ru/mathinmoscow` and in the article "Bringing Eastern European Mathematical Traditions to North American Students", *Notices*, November 2003, pages 1250–4.

—*Allyn Jackson*

Deaths of AMS Members

A. BOLIBRUKH, of the Steklov Institute of Mathematics, Moscow, died on November 11, 2003. Born on January 30, 1950, he was a member of the Society for 12 years.

JOE ELICH, professor emeritus, Utah State University, Logan, died on November 19, 2003. Born on September 28, 1918, he was a member of the Society for 43 years.

J. H. GOLDSTEIN, retired, of Atlanta, GA, died on October 9, 2003. Born on December 18, 1915, he was a member of the Society for 2 years.

ERNEST L. GRIFFIN JR., retired, Louisiana State University, Baton Rouge, died on August 22, 2003. Born on May 25, 1921, he was a member of the Society for 52 years.

SEMEN Y. KHAVINSON, chair, Moscow State Civil Engineering University, died on January 30, 2003. Born on May 17, 1927, he was a member of the Society for 9 years.

PAUL KINGSTON, retired, from East Syracuse, NY, died on October 15, 2003. Born on February 12, 1932, he was a member of the Society for 47 years.

WILLI LINDEMANN, professor emeritus, Universität Würzburg, Germany, died on October 26, 2003. Born on January 15, 1921, he was a member of the Society for 21 years.

EUGENE A. PFLUMM, retired, of Upper Montclair, NJ, died on September 7, 2003. Born on August 9, 1927, he was a member of the Society for 43 years.

NELSON RICH, emeritus, Nazareth College of Rochester, NY, died on November 13, 2003. Born on September 25, 1942, he was a member of the Society for 9 years.

JORGE D. SAMUR, of the Universidad Nacional de La Plata, Argentina, died on July 21, 2003. Born on August 24, 1948, he was a member of the Society for 9 years.

JOHN P. SHANAHAN, Boston College, died on September 11, 2003. Born on June 29, 1935, he was a member of the Society for 45 years.

PAUL WAUTERS, of the Limburgs Universitair Centrum, Belgium, died on October 26, 2003. Born on December 25, 1958, he was a member of the Society for 4 years.

EVERETT T. WELMERS, retired, from Los Angeles, CA, died on December 5, 2003. Born in October 1912, he was a member of the Society for 68 years.

EDWIN WESTBROOK, retired, from Oxon Hill, MD, died on November 28, 2003. Born on December 22, 1921, he was a member of the Society for 41 years.

D. W. WESTERN, professor emeritus, from Lancaster, PA, died on September 17, 2003. Born on May 7, 1915, he was a member of the Society for 61 years.

DOROTHY J. WILLIAMS, professor emeritus, Portland State University, OR, died on September 25, 2003. Born on February 12, 1930, she was a member of the Society for 45 years.

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.ou.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 405-325-7484 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

February 27, 2004: Nominations for Clay Mathematics Institute Liftoff Fellowships. See http://www.claymath.org/Research/Liftoff; telephone: 617-995-2600; email: researchers@claymath.org.

March 1, 2004: Applications for the summer program of the Christine Mirzayan Science and Technology Policy Internship Program of the National Academies. See http://www7.nationalacademies.org/internship/index.html, or contact The National Academies Christine Mirzayan Science and Technology Policy Internship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667.

March 1, 2004: Applications for the Summer Program for Women Undergraduates. Contact Murli M. Gupta, mmg@gwu.edu, telephone 202-994-4857, or see http://www.gwu.edu/~math/spwm.html.

Where to Find It

A brief index to information that appears in this and previous issues.

March 4, 2004: Applications for EDGE Summer Program. See http://www.edgeforwomen.org/index.html.

March 31, 2004: Nominations for Third World Academy of Sciences prizes. See http://www.ictp.trieste.it/~twas/twas_prizes.html.

March 31, 2004: Nominations for the Prize for Achievement in Information-Based Complexity. Contact Joseph F. Traub, email: traub@cs.columbia.edu.

April 1, 2004: New Directions program at the IMA. See "Mathematics Opportunities" in this issue.

April 16, 2004: Applications for Project NExT. See "Mathematics Opportunities" in this issue.

May 1, 2004: Applications for National Research Council Research Associateships. See http://www4.nationalacademies.org/pga/rap.nsf, or contact Research Associateship Programs, Keck Center of the National Academies, 500 Fifth Street, NW, GR322A, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

May 1, 2004: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html, or contact Association for Women in Mathematics, 4114 Computer and Space Sciences Building, University of Maryland, College Park, MD 20742-2461; telephone 301-405-7892; email: awm@math.umd.edu.

June 1, 2004: Applications for the fall program of the Christine Mirzayan Science and Technology Policy Internship Program of the National Academies. See http://www7.nationalacademies.org/internship/index.html, or contact The National Academies Christine Mirzayan Science and Technology Policy Internship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667.

June 30, 2004: Proposals for DMS/NIGMS Program in Mathematical Biology. See http://www.nsf.gov/pubs/2002/nsf02125/nsf02125.htm.

August 1, 2004: Applications for National Research Council Research Associateships. See http://www4.nationalacademies.org/pga/rap.nsf, or contact Research Associateship Programs, Keck Center of the National Academies, 500 Fifth Street, NW, GR322A, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

September 30, 2004: Nominations for Information-Based Complexity Young Researcher Award. Contact Joseph F. Traub at traub@cs.columbia.edu.

January 1, 2005: Entries for *Cryptologia* undergraduate paper competitions. See http://www.dean.usma.edu/math/pubs/cryptologia/, or contact *Cryptologia*, Department of Mathematical Sciences, United States Military Academy, West Point, NY 10996; email: Cryptologia@usma.edu.

Board on Mathematical Sciences and Their Applications, National Research Council

Peter J. Bickel, University of California, Berkeley

Dimitris Bertsimas, MIT Sloan School of Management

Spencer Bloch, University of Chicago

Avarinda Chakravarti, Institute of Genetic Medicine

Philip Colella, Lawrence Berkeley National Laboratory

John E. Hopcroft, Cornell University

Robert E. Kass, Carnegie Mellon University

Kathryn B. Laskey, George Mason University

Arjen K. Lenstra, Citibank, North America

Robert Lipshutz, Affymetrix, Inc.

Charles M. Lucas, American International Companies

David McLaughlin (chair), New York University

George C. Papanicolaou, Stanford University

Linda R. Petzold, University of California, Santa Barbara

Prabhakar Raghavan, Stanford University

Douglas Ravenel, University of Rochester

Stephen M. Robinson, University of Wisconsin, Madison

Edward J. Wegman, George Washington University

Detlof von Winterfeldt, University of Southern California

The postal address for BMSA is: Board on Mathematical Sciences and Their Applications, National Academy of Sciences, Room K974, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2421; fax 202-334-2422; World Wide Web, http://www7.nationalacademies.org/bms/BMSA_Members.html.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to Book List since the list's last appearance.

1089 and All That: A Journey into Mathematics, by David Acheson. Oxford University Press, July 2002. ISBN 0-19-851623-1.

Abel's Proof: An Essay on the Sources and Meaning of Mathematical Unsolvability, by Peter Pesic. MIT Press, May 2003. ISBN 0-262-16216-4. (Reviewed in this issue.)

After Math, by Miriam Webster. Zinka Press, June 1997. ISBN 0-9647-1711-5. (Reviewed October 2003.)

All the Mathematics You Missed (But Need to Know for Graduate School), by Thomas A. Garrity. Cambridge University Press, December 2001. ISBN 0-521-79707-1.

Beyond the Limit: The Dream of Sofya Kovalevskaya, by Joan Spicci. Forge, August 2002. ISBN 0-765-30233-0. (Reviewed January 2004.)

The Book of My Life, by Girolamo Cardano. The New York Review of Books Classics Series/Granta, 2002. ISBN 1-590-17016-4.

Calculated Risks: How to Know When Numbers Deceive You, by Gerd Gigerenzer. Simon & Schuster, March 2003. ISBN 0-743-25423-6.

California Dreaming: Reforming Mathematics Education, by Suzanne M. Wilson. Yale University Press, January 2003. ISBN 0-300-09432-9. (Reviewed November 2003.)

Codebreakers: Arne Beurling and the Swedish Crypto Program during World War II, by Bengt Beckman, translated by Kjell-Ove Widman. AMS, February 2003. ISBN 0-8218-2889-4. (Reviewed September 2003.)

The Constants of Nature: From Alpha to Omega—The Numbers That Encode the Deepest Secrets of the Universe, by John D. Barrow. Jonathan Cape, September 2002; Pantheon Books, January 2003. ISBN 0-375-42221-8.

Corréspondance Grothendieck-Serre, Pierre Colmez and Jean-Pierre Serre, editors. Société Mathématique de France, 2001. ISBN 2-85629-104-X. (Reviewed October 2003.)

Doing Mathematics: Convention, Subject, Calculation, Analogy, by Martin H. Krieger. World Scientific, April 2003. ISBN 9-812-38200-3.

Einstein's Clocks, Poincaré's Maps: Empires of Time, by Peter Galison. W. W. Norton & Co., August 2003. ISBN 0-393-02001-0.

Emergence of the Theory of Lie Groups. An Essay in the History of Mathematics, 1869–1926, by Thomas Hawkins. Springer-Verlag, 2000. ISBN 0-387-98963-3. (Reviewed June/July 2003.)

Everything and More: A Compact History of Infinity, by David Foster Wallace. W. W. Norton & Co., October 2003. ISBN 0-393-00338-8.

Four Colors Suffice: How the Map Problem Was Solved, by Robin Wilson. Princeton University Press, March 2003. ISBN 0-691-11533-8. (Reviewed February 2004.)

The Fractal Murders, by Mark Cohen. Muddy Gap Press, May 2002. 0-9718986-0-X. (Reviewed October 2003.)

Gamma: Exploring Euler's Constant, by Julian Havil. Princeton University Press, May 2003. ISBN 0-691-09983-9.

Geometrical Landscapes: The Voyages of Discovery and the Transformation of Mathematical Practice, by Amir R. Alexander. Stanford University Press, September 2002. ISBN 0-804-73260-4.

Geometry: Our Cultural History, by Audun Holme. Springer, April 2002. ISBN 3-540-41949-7.

Gödel's Proof, by Ernest Nagel and James R. Newman. New York University Press, revised edition, October 2001. ISBN 0-8147-5816-9. (Reviewed in this issue.)

The Golden Ratio: The Story of Phi, the World's Most Astonishing Number, by Mario Livio. Broadway Books, October 2002. ISBN 0-767-90815-5.

How Economics Became a Mathematical Science, by E. Roy Weintraub. Duke University Press, June 2002. ISBN 0-822-32856-9.

Imagining Numbers (particularly the square root of minus fifteen), by Barry Mazur. Farrar, Straus and Giroux, February 2003. ISBN 0-374-17469-5. (Reviewed November 2003.)

In Code: A Mathematical Journey, by Sarah Flannery and David Flannery. Workman Publishing, May 2001. ISBN 0-761-12384-9. (Reviewed April 2003.)

Infinity: The Quest to Think the Unthinkable, by Brian Clegg. Carroll & Graf, December 2003. ISBN 0-786-71285-6.

Information: The New Language of Science, by Hans Christian von Baeyer. Weidenfeld & Nicolson, October 2003. ISBN 0-297-60725-1 (hardcover), 0-753-81782-9 (paperback).

Isaac Newton, by James Gleick. Pantheon Books, May 2003. ISBN 0-375-42233-1. (Reviewed December 2003.)

Janos Bolyai, Euclid, and the Nature of Space, by Jeremy J. Gray. MIT Press, May 2003. ISBN 0-262-57174-9.

Kepler's Conjecture: How Some of the Greatest Minds in History Helped Solve One of the Oldest Math Problems in the World, by George G. Szpiro. John Wiley & Sons, January 2003. ISBN 0-471-08601-0.

Linked: The New Science of Networks, by Albert-László Barabási. Perseus Publishing, May 2002. ISBN 0-738-20667-9. (Reviewed February 2004.)

M. C. Escher's Legacy: A Centennial Celebration, edited by Doris Schattschneider and Michele Emmer. Springer, January 2003. ISBN 3-540-42458-X. (Reviewed April 2003.)

Math through the Ages: A Gentle History for Teachers and Others, by William P. Berlinghoff and Fernando Q. Gouvêa. Oxton House, 2002. ISBN 1-881929-21-3.

Mathematical Apocrypha: Stories and Anecdotes of Mathematicians and the Mathematical, by Steven G. Krantz. Mathematical Association of America, July 2002. ISBN 0-883-85539-9.

Mathematical Constants, by Steven R. Finch. Cambridge University Press, August 2003. ISBN 0-521-81805-2.

Mathematicians under the Nazis, by Sanford L. Segal. Princeton University Press, July 2003. ISBN 0-691-00451-X.

Mathematics: A Very Short Introduction, by Timothy Gowers. Oxford University Press, October 2002. ISBN 0-192-85361-9.

**Mathematics and Culture I*, edited by Michele Emmer. Springer, January 2004. ISBN 3-540-01770-4.

Mathematics and the Roots of Postmodern Thought, by Vladimir Tasić. Oxford University Press, 2001. ISBN 0-195-13967-4. (Reviewed August 2003.)

**Mathematics, Art, Technology, and Cinema*, edited by Michele Emmer and Mirella Manaresi. Springer, 2003. ISBN 3-540-00601-X.

Mathematics by Experiment: Plausible Reasoning in the 21st Century, by David Bailey and Jonathan Borwein. A K Peters, September 2003. ISBN 1-568-81136-5.

Mathematics Elsewhere: An Exploration of Ideas across Cultures, by Marcia Ascher. Princeton University Press, September 2002. ISBN 0-691-07020-2. (Reviewed May 2003.)

Mathematics for the Imagination, by Peter M. Higgins. Oxford University Press, November 2002. ISBN 0-198-60460-2.

The Mathematics of Juggling, by Burkard Polster. Springer, November 2002. ISBN 0-387-95513-5. (Reviewed January 2004.)

Memoirs of a Proof Theorist: Gödel and Other Logicians, by Gaisi Takeuti, translated by Mariko Yasugi and

Nicholas Passell. World Scientific, February 2003. ISBN 981-238-279-8.

The Millennium Problems: The Seven Greatest Unsolved Mathematical Puzzles of Our Time, by Keith J. Devlin. Basic Books, October 2002. ISBN 0-465-01729-0. (Reviewed September 2003.)

More Mathematical Astronomy Morsels, by Jean Meeus. Willmann-Bell Inc., 2002. ISBN 0-943396-743.

The Music of the Primes: Searching to Solve the Greatest Mystery in Mathematics, by Marcus Du Sautoy. HarperCollins, April 2003. ISBN 0-066-21070-4.

**Newton's Apple: Isaac Newton and the English Scientific Renaissance*, by Peter Aughton. Weidenfeld & Nicolson, October 2003. ISBN 0-297-84321-4.

On the Nature of Human Romantic Interaction, by Karl Iagnemma. Dial Press, April 2003. ISBN 0-385-33593-8.

The One True Platonic Heaven: A Scientific Fiction of the Limits of Knowledge, by John L. Casti. Joseph Henry Press, May 2003. ISBN 0-309-08547-0.

Origami^3, edited by Thomas Hull. A K Peters, July 2002. ISBN 1-568-81181-0.

Predicting Presidential Elections and Other Things, by Ray C. Fair. Stanford University Press, August 2002. ISBN 0-804-74509-9.

Prime Obsession: Bernhard Riemann and the Greatest Unsolved Problem, by John Derbyshire. Joseph Henry Press, March 2003. ISBN 0-309-08549-7.

Proofs from the Book, by Martin Aigner and Günter M. Ziegler. Springer-Verlag, third edition, December 2003. ISBN 3-540-40460-0.

Remarkable Mathematicians, by Ioan James. Cambridge University Press, February 2003. ISBN 0-521-52094-0.

The Riemann Hypothesis: The Greatest Unsolved Problem in Mathematics, by Karl Sabbagh. Farrar Straus & Giroux, April 2003. ISBN 0-374-25007-3.

Science in the Looking Glass, by E. Brian Davies. Oxford University Press, August 2003. ISBN 0-19-852543-5.

The Search for Certainty: A Philosophical Account of Foundations of Mathematics, by Marcus Giaquinto. Oxford University Press, October 2002. ISBN 0-198-75244-X.

Six Degrees: The Science of a Connected Age, by Duncan J. Watts. W. W. Norton & Co., February 2003. ISBN 0-393-04142-5. (Reviewed February 2004.)

Sync: The Emerging Science of Spontaneous Order, by Steven Strogatz. Hyperion, February 2003. ISBN 0-786-86844-9. (Reviewed in this issue.)

Travels in Four Dimensions: The Enigmas of Space and Time, by Robin Le Poidevin. Oxford University Press, February 2003. ISBN 0-19-875254-7.

What the Numbers Say: A Field Guide to Mastering Our Numerical World, by Derrick Niederman and David Boyum. Broadway Books, April 2003. ISBN 0-767-90998-4.

Leroy P. Steele Prizes

Call for Nominations

The selection committee for these prizes requests nominations for consideration for the 2005 awards. Further information about the prizes can be found in the November 2003 *Notices*, pp. 1288-1302 (also available at http://www.ams.org/prizes-awards).

Three Leroy P. Steele Prizes are awarded each year in the following categories: (1) the Steele Prize for Lifetime Achievement: for the cumulative influence of the total mathematical work of the recipient, high level of research over a period of time, particular influence on the development of a field, and influence on mathematics through Ph.D. students; (2) the Steele Prize for Mathematical Exposition: for a book or substantial survey or expository-research paper; and (3) the Steele Prize for Seminal Contribution to Research: for a paper, whether recent or not, that has proved to be of fundamental or lasting importance in its field, or a model of important research. In 2005 the prize for Seminal Contribution to Research will be awarded for a paper in Algebra.

Nominations with supporting information should be submitted to the Secretary, Robert J. Daverman, American Mathematical Society, 312D Ayres Hall, University of Tennessee, Knoxville, TN 37996-1330. Include a short description on the work that is the basis of the nomination, including complete bibliographic citations. A curriculum vitae should be included. The nominations will be forwarded by the Secretary to the prize selection committee, which will, as in the past, make final decisions on the awarding of prizes.

Deadline for nominations is March 31, 2004.

Mathematics Calendar

The most comprehensive and up-to-date Mathematics Calendar information is available on e-MATH at http://www.ams.org/mathcal/.

March 2004

1-26 **Markov Chain Monte Carlo: Innovations and Applications in Statistics, Physics, and Bioinformatics,** Institute for Mathematical Sciences, National University of Singapore, Singapore. (June/July 2003, p. 725)

4-6 **Workshop on Spectral Geometry,** Centre de Recherches Mathématiques, Montréal, Québec, Canada. (Aug. 2003, p. 847)

7 **IMA Tutorial: Control and Pricing in Communication and Power Networks,** University of Minnesota, Minneapolis, Minnesota. (Apr. 2003, p. 499)

8-13 **IMA Workshop 6: Control and Pricing in Communication and Power Networks,** University of Minnesota, Minneapolis, Minnesota. (Apr. 2003, p. 499)

8-June 11 **Proteomics,** Institute for Pure and Applied Mathematics, UCLA, Los Angeles, California. (Sept. 2003, p. 1003)

*11-12 **Workshop on Algebraic Methods in Cryptography,** University of Dortmund, Germany.

Invited Speakers: P. Bellingeri (Univ. of Grenoble), P. Dehornoy (Univ. of Caen), B. Fine (Fairfield Univ.), V. Gebhardt (Univ. of Sydney), D. Goldfeld (Columbia Univ.), D. Grigoriev (Univ. of Rennes), A. G. Myasnikov (City College of New York), V. Shpilrain (City College of New York), M. Teicher (Bar-Ilan Univ.)

Sponsor: The Graduate College "Methods of Mathematics and Engineering for Secure Data Transmission and Information Transfer".

Information: http://www.mathematik.uni-dortmund.de/tagungen/cryptoworkshop2004.html.

12-13 **AMS Southeastern Section Meeting,** Florida State Univ., Tallahassee, Florida. (Dec. 2002, p. 1001)

15-19 **Workshop on Nonlinear Wave Equations,** The Fields Institute, Toronto, Ontario, Canada. (Oct. 2003, p. 1128)

15-30 **Singularities Phenomena in Elliptic and Parabolic Equations,** Technion, Haifa, Israel. (Jan. 2004, p. 61)

17-21 **International Workshop on Hysteresis and Multi-Scale Asymptotics,** University College Cork, Cork, Ireland. (Dec. 2003, p. 1439)

*25-27 **Midwest Geometry Conference, 2004,** University of Arkansas, Fayetteville, Arkansas.

Plenary Speakers: K. Bromberg, R. Bryant, J. Cannon, A. Chang, M. Eastwood, H. Moscovici.

Local Organizing Committee: J. Ryan, C. Goodman-Strauss, Y. Rieck, D. Vassilev, and J. Woodland.

Information: http://www.uark.edu/depts/mathinfo/activities/MWGMeeting/mwg2004.HTM. All information including registration and information for contributing speakers can be found at this Web address.

26-27 **AMS Central Section Meeting,** Ohio University, Athens, Ohio. (Dec. 2002, p. 1001)

29-April 2 **IMA Short Course: Tools for Modeling and Data Analysis in Finance/Asset Pricing,** University of Minnesota, Minneapolis, Minnesota. (Apr. 2003, p. 499)

29-April 2 **Workshop on Kinetic Theory,** The Fields Institute, Toronto, Ontario, Canada. (Oct. 2003, p. 1128)

This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences held in North America carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. Meetings held outside the North American area may carry more detailed information. In any case, if there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to notices@ams.org or mathcal@ams.org.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: http://www.ams.org/.

April 2004

1–May 15 **Econometric Forecasting and High-Frequency Data Analysis**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Aug. 2003, p. 847)

2–4 **Midwest Several Complex Variables Meeting**, University of Western Ontario, London, Ontario, Canada. (Oct. 2003, p. 1128)

3–4 **AMS Western Section Meeting**, University of Southern California, Los Angeles, California. (May 2003, p. 604)

4–7 **Fractal 2004**, Vancouver, Canada. (June/July 2003, p. 725)

5–7 **South African Symposium on Numerical and Applied Mathematics (SANUM 2004)**, University of Stellenbosch, Stellenbosch, South Africa. (Jan. 2004, p. 62)

5–8 **Joint Meeting of the 56th British Mathematical Colloquium and the 17th Annual Meeting of the Irish Mathematical Society (BMC2004)**, Queen's University Belfast, Belfast, Northern Ireland. (Oct. 2003, p. 1128)

9–10 **CombinaTexas 2004: Combinatorics in the South-Central U.S.**, Texas A&M University, College Station, Texas. (Feb. 2004, p. 277)

12–16 **IMA Workshop 7: Risk Management and Model Specifications Issues in Finance**, University of Minnesota, Minneapolis, Minnesota. (Apr. 2003, p. 499)

14–16 **Multi-Objective Programming and Goal Programming (MOPGP'04)**, Hammamet, Tunisia. (Nov. 2003, p. 1314)

15–17 **Aspects of Large Quantum Systems Related to Bose-Einstein Condensation**, Department of Mathematical Sciences, University of Aarhus, Denmark. (Feb. 2004, p. 277)

17–18 **AMS Eastern Section Meeting**, Rider University, Lawrenceville, New Jersey. (Sept. 2002, p. 1001)

19–24 **CHT-04: Advances in Computational Heat Transfer**, Cruise ship between Kirkenes and Bergen, Norway. (June/July 2003, p. 725)

22–24 **SIAM International Conference on Data Mining (SDM04)**, Hyatt Orlando, Orlando, Florida. (Dec. 2003, p. 1439)

22–25 **2004 ASL Spring Meeting (with APA)**, Chicago, Illinois. (June/July 2003, p. 725)

23–26 **Theory of Motives, Homotopy Theory of Varieties, and Dessins d'Enfants**, AIM Research Conference Center, Palo Alto, California. (Feb. 2003, p. 1440)

May 2004

3–5 **SIAM Conference on Imaging Science (IS04)**, Marriott City Center, Salt Lake City, Utah. (Oct. 2003, p. 1128)

3–7 **IMA Workshop 8: Model Implementation, Algorithms and Software Issues**, University of Minnesota, Minneapolis, Minnesota. (Apr. 2003, p. 499)

3–8 **AARMS-CRM—Workshop on Singular Integrals and Analysis on CR Manifolds**, Halifax, Nova Scotia, Canada. (Aug. 2003, p. 847)

3–8 **Lie and Jordan Algebras, Their Representations and Applications**, Guarujá, SP, Brazil. (Jan. 2004, p. 62)

3–June 26 **2004 Geometric Partial Differential Equations**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Aug. 2003, p. 847)

4–7 **Workshop on Spectral Theory and Automorphic Forms**, Centre de Recherches Mathématiques, Montréal, Québec, Canada. (Aug. 2003, p. 848)

9–12 **International Conference on Numerical Combustion (NC04)**, Hilton Sedona Resort & Spa, Sedona, Arizona. (Nov. 2003, p. 1314)

*10–13 **Workshop on Nonlinear and Convex Analysis**, University of Isfahan, Isfahan, Iran.
Main Speakers: Prof. Goeble, Prof. Hadjisavvas, Prof. Lin, and Prof. Thera.
Organizing Committee: B. Djafari Rouhani, M. Fakhar, S. Nobakhtian and J. Zafarani.
Information: Email: `analysis@ui.ac.ir`.

11–15 **IMA Hot Topics Workshop: Compatible Spatial Discretizations for Partial Differential Equations**, Institute for Mathematics & its Applications (IMA), University of Minnesota, Minneapolis, Minnesota. (Oct. 2003, p. 1129)

11–15 **Recent Trends in Kinetic Theory and Its Applications**, Institute of Mathematics, Kyiv, Ukraine. (Feb. 2004, p. 278)

13–15 **Sixth International Joint Meeting of the AMS and the Sociedad Matematica Mexicana (SMM)**, Hyatt Regency Houston, Houston, Texas. (Feb. 2004, p. 278)

*13–15 **Tenth International Conference Devoted to the Memory of Academician Mykhailo Kravchuk (1892–1942)**, Kyiv, Ukraine.
Organizers: National Tech. Univ. of Ukraine (KPI), Inst. of Math. of NASU, National Shevchenko Univ., National Drahomanov Pedagogical Univ.
Topics: (1) Differential and integral equations, its applications; (2) Algebra, geometry: mathematical and numerical analysis; (3) Theory of probability and mathematical statistics; (4) history, methods of teaching of mathematics.
Deadlines: Abstracts of one page by March 1, 2004.
Languages: English, Ukrainian, Russian.
Information: Ukraine, 03056, Kyiv-56, Peremohy Ave. 37, National Technical University (KPI), Phys.-Math. Department, Corpus 7, Room 437, M. Kravchuk Conference, N. Virchenko; tel. (380) 44 441 1 441; email: `conf@ptf.ntu-kpi.kiev.ua`.

*15–25 **Second Annual Spring Institute on Noncommutative Geometry and Operator Algebras**, Vanderbilt University, Nashville, Tennessee.
Description: The meeting is a combined spring school and international conference, directed by A. Connes (College de France, IHES & Vanderbilt Univ.). Graduate students and postdocs are strongly encouraged to attend.
Mini-courses: A. Connes, N. Higson*, and J. Roe* (Penn State); M. Khalkhali (Western Ontario); G. Landi (Trieste), M. Marcolli (Bonn); and H. Moscovici* (Ohio State) (* to be confirmed).
Organizing Committee: D. Bisch, B. Hughes, G. Kasparov, and G. Yu (all at Vanderbilt Univ.).
Deadline: For financial support is April 15, 2004.
Information: `http://www.math.vanderbilt.edu/~ncgoa`.

*18–22 **Conference on Infinite-Dimensional Aspects of Representation Theory and Applications**, University of Virginia, Charlottesville, Virginia.
Description: The week will begin with a three-day workshop that will include three minicourses geared towards those who may have little knowledge about the particular area. It will also prepare attendees for the Invited Talks during the following days.
Minicourse Lecturers: S. Ariki (RIMS, Kyoto Univ., Japan), W. Crawley-Boevey (Univ. of Leeds, UK), A. Okounkov (Princeton Univ., USA).
Invited Speakers: D. Ben Zvi (Univ. of Texas at Austin, USA), R. Bezrukavnikov (Northwestern Univ., USA), V. Chari (Univ. of California, Riverside, USA), M. Geck (Univ. of Lyon, France), M. Kashiwara (RIMS, Kyoto, Japan), S. Kumar (Univ. of North Carolina, USA), H. Li (Rutgers Univ., USA), K. Liu (UCLA, USA), A. Malkin (MIT, USA), T. Miwa (Kyoto Univ., Japan), E. Mukhin (Indiana Univ.-Purdue Univ. Indianapolis, USA), Z. Qin (Univ. of Missouri, Columbia, USA), R. Rouquier (Univ. Paris VII, France), R. Vakil (Stanford Univ., USA), M. Varagnolo (Univ. de Cergy-Pontoise, France).
Organizers: S. Berman (Univ. of Saskatchewan, Canada), B. Parshall (Univ. of Virginia), L. Scott (Univ. of Virginia), W. Wang (Univ. of Virginia).

Sponsors: Dept. of Math., Univ. of Virginia; Inst. of Math. Sci., Univ. of Virginia; National Science Foundation (pending).
Financial Support: It is anticipated that there will be partial support available for some graduate students and junior researchers. (for details and application, see `http://www.math.virginia.edu/LieConf/`).
Information: `http://www.math.virginia.edu/LieConf/`; email: `lieconf@weyl.math.virginia.edu`.

19–24 **2004 ASL Annual Meeting**, Carnegie Mellon University, Pittsburgh, Pennsylvania. (Dec. 2002, p. 1422)

19–25 **The Decidable and the Undecidable in Mathematics Education**, Brno University of Technology, Brno, Czech Republic.

24–28 **IMA Workshop 9: Financial Data Analysis and Applications**, University of Minnesota, Minneapolis, Minnesota. (Apr. 2003, p. 499)

24–28 **Workshop on Hamiltonian Dynamical Systems (jointly with The Fields Institute)**, Centre de Recherches Mathématiques, Montréal, Québec, Canada. (Aug. 2003, p. 848)

24–June 11 **School on Commutative Algebra and Interactions with Algebraic Geometry and Combinatorics**, ICTP, Trieste, Italy. (Feb. 2004, p. 278)

28–31 **International Conference on Mathematics and Its Applications**, City University of Hong Kong. (Sept. 2003, p. 1005)

30–June 3 **Fifth European Conference on Elliptic and Parabolic Problems: A Special Tribute to the Work of Haim Brezis**, Gaeta, Italy. (Jan. 2004, p. 62)

30–June 5 **Commutative Rings and Their Modules**, Cortona, Italy. (Feb. 2004, p. 278)

June 2004

2004 **Fifth Edition of the International Conference on Functional Analysis and Approximation Theory**, Acquafredda di Maratea, Potenza, Italy. (Feb. 2004, p. 278)

2004 **Mathematical Foundations of Learning Theory**, Barcelona, Spain. (Apr. 2003, p. 499)

2004 **WSEAS Conferences**, Corfu Island, Greece. (Apr. 2003, p. 500)

1–4 **International Workshop on Nonlinear Waves**, The Chinese University of Hong Kong, Hong Kong. (Dec. 2003, p. 1440)

1–11 **Workshop on Semi-classical Theory of Eigenfunctions and PDEs**, Centre de Recherches Mathématiques, Montréal, Québec, Canada. (Aug. 2003, p. 848)

2–4 **ICNPAA 2004: Mathematical Problems in Engineering and Aerospace Sciences**, The West University of Timisoara, Romania. (May 2003, p. 604)

* 3–10 **Sixth International Conference on Geometry, Integrability and Quantization**, Sts. Constantine and Elena Resort (near Varna), Bulgaria.
Goal: This sixth edition of the conference aims, like the previous ones, to bring together experts in classical and modern differential geometry, complex analysis, mathematical physics, and related fields in order to assess recent developments in these areas and to stimulate future research.
Organizers: I. M. Mladenov (Sofia) and A. Hirshfeld (Dortmund).
Information: I. M. Mladenov. email: `mladenov@obzor.bio21.bas.bg`; A. C. Hirshfeld, email: `hirsh@physik.uni-dortmund.de`; or visit the conference Web page at `http://www.bio21.bas.bg/conference/`.

3–25 **MRI Spring School 2004: Lie Groups in Analysis, Geometry and Physics**, Utrecht University, Utrecht, The Netherlands. (Jan. 2004, p. 62)

6–10 **Joint Summer Research Conference: String Geometry**, Snowbird Resort, Snowbird, Utah. (Feb. 2004, p. 278)

7–10 **Sixth International Conference on Monte Carlo and Quasi-Monte Carlo Methods in Scientific Computing and Second International Conference on Monte Carlo and Probabilistic Methods for Partial Differential Equations**, Juan-les-Pins, France. (Dec. 2003, p. 1440)

* 7–10 **Symposium on Analysis and PDEs**, Purdue University, West Lafayette, Indiana.
Short Description: The symposium will focus on recent developments in analysis and partial differential equations. It will consist of two 5-hour minicourses presented by the principal lecturers, and thirteen 1-hour lectures given by the invited speakers. There will also be time allocated for contributed talks. The aim of this initiative is, on the one hand, to introduce graduate students and young researchers to a larger mathematical community and help them to establish professional connections with key figures in their areas of interest. On the other hand, it will provide an opportunity to summarize some of the most recent progress in the fields, exchange ideas toward the solution of open questions, and formulate new problems and avenues of research.
Principal Lecturers: L. C. Evans (Univ. of California at Berkeley): Kinetic Formulations of Nonlinear PDEs; C. E. Kenig (Univ. of Chicago): Unique Continuation for Evolution Equations.
Invited Speakers: P. Daskalopoulos (Columbia Univ.); G. David (Univ. Paris 11, France); R. Jensen (Loyola Univ. Chicago); D. Jerison (MIT); F.-H. Lin (Courant Inst. of Math. Sci.); B. Perthame (École Normale Superieure, Paris, France); G. Ponce (Univ. of California at Santa Barbara); S. Salsa (Polytechnic of Milan, Italy); H. Shahgholian (Royal Inst. of Tech., Stockholm, Sweden); G. Staffilani (MIT); T. Toro (Univ. of Washington); G. Uhlmann (Univ. of Washington); L. Vega (Univ. del Pais Vasco, Spain).
Support: The symposium is sponsored by the National Science Foundation and the Department of Mathematics at Purdue University. Some funds are available to help support the participation of graduate students, postdoctoral faculty, and active senior researchers who do not have grant support. We especially encourage people who belong to currently underrepresented groups (women and minorities) to apply. Application forms are available at `http://www.math.purdue.edu/~danielli/support.htm` and should be received by April 15, 2004.
Contributed Talks: Abstracts for 20-minute contributed talks are welcome and should be received by April 15, 2004. Please use the submission form available at `http://www.math.purdue.edu/~danielli/contributed.html`.
Organizer: D. Danielli (`danielli@math.purdue.edu`). Conference secretary: J. Morris (`jmorris@math.purdue.edu`).
Information: `http://www.math.purdue.edu/~danielli/symposium.html`.

7–11 **4th Conference on Poisson Geometry**, University of Luxembourg, Luxembourg City, Grand-Duchy of Luxembourg. (Dec. 2003, p. 1441)

* 7–12 **Application of Mathematics in Engineering and Economics**, Sozopol, Bulgaria.
Description: The Faculty of Applied Mathematics and Informatics in the Technical University of Sofia is organizing the 30th Jubilee International Conference "Application of Mathematics in Engineering and Economics", dedicated to the outstanding mathematician-academician G. D. Bradistilov (1904–1977) on the occasion of his 100th birthday. For the first time the conference will address recent trends in nonlinear differential equations and mathematical physics that have the potential for future developments in the subject, especially in the areas of engineering and economics.
Main Topics: Concentrated around the scientific tradition of Prof. Bradistilov: Potential theory and partial differential equations, mathematical analysis and applications, differential equations and differential geometry, theoretic and applied mechanics, operation research and statistics, numerical methods and mathematical modeling, computer science.

Information: email: gvenkov@tu-sofia.bg, email: andru@tu-sofia.bg.

9–12 **Call for Papers/Abstracts/Submissions: Hawaii International Conference on Statistics, Mathematics and Related Fields**, Sheraton Waikiki Hotel, Honolulu, Hawaii. (Dec. 2003, p. 1441)

13–16 **SIAM Conference on Discrete Mathematics (DM04)**, Loews Vanderbilt Plaza Hotel, Nashville, Tennessee. (Oct. 2003, p. 1129)

13–17 **Joint Summer Research Conference: Complex Dynamics: Twenty-Five Years after the Appearance of the Mandelbrot Set**, Snowbird Resort, Snowbird, Utah. (Feb. 2004, p. 278)

13–18 **Algorithmic Number Theory Symposium VI (ANTS-VI)**, United States Naval Academy, Annapolis, Maryland. (Oct. 2003, p. 1129)

16–19 **AIMS' Fifth International Conference on Dynamical Systems and Differential Equations**, California State Polytechnic University, Pomona, California. (Aug. 2003, p. 848)

*16–23 **5th International Conference on Functional Analysis and Approximation Theory (FAAT 2004)**, Hotel Villa del Mare, Acquafredda di Maratea, Potenza, Italy.
Aim: The meeting is devoted to some significant aspects of contemporary mathematical research in functional analysis, operator theory, and approximation theory, including the applications of these fields in other areas such as partial differential equations, integral equations, numerical analysis, and stochastic analysis. One of the major aims of the conference is to bring together mathematicians working in the above topics in order to spur interdisciplinary collaborations and exchanges of results and techniques.
Main Topics: Banach spaces, Banach lattices, function spaces, (positive) linear operators, semigroups of (positive) linear operators, evolution equations and stochastic analysis, approximate quadratures and integral equations, approximation processes in abstract spaces and in function spaces, approximation by (positive) operators, interpolation, polynomial approximation, constructive approximation, orthogonal polynomials.
Confirmed Invited Speakers: E. Behrends (Berlin), B. Bojanov (Sofia), A. L. Brown (London), N. Jacob (Swansea), N. Kalton (Columbia), A. Krò (Budapest), W. Luxemburg (Pasadena), F. Marcellan (Madrid), G. Milovanovic (Nis), G. Monegato (Torino), B. Silbermann (Chemnitz), V. Totik (Szeged), G. Vinti (Perugia), L. Weis (Karlsruhe), Y. Xu (Eugene, USA).
Scientific Program: Plenary lectures (50 min.), selected section lectures (30 min.), and short communications (15–20 min.).
Organizing Committee: F. Altomare (altomare@pascal.dm.uniba.it), A. Attalienti (attalienti@matfin.uniba.it), M. Campiti (michele.campiti@unile.it), L. D'Ambrosio (dambros@dm.uniba.it), S. Diomede (s.diomede@dse.uniba.it), G. Mastroianni (mastroianni@unibas.it), D. Occorsio (occorsio@unibas.it), M. G. Russo (russo@unibas.it).
Information: http://www.dm.uniba.it/faat2004, http://www.dm.unile.it/faat2004.

18–20 **High Performance Software for Nonlinear Optimization: Status and Perspectives**, Ischia, Italy. (Feb. 2004, p. 278)

18–23 **Mathematical Foundations of Learning Theory**, Barcelona, Spain. (Dec. 2003, p. 1441)

20–25 **Canadian Number Theory Association VIII Meeting**, University of Toronto, Toronto, Ontario, Canada. (Dec. 2003, p. 1441)

20–25 **8th Symposium on Probability and Stochastic Processes**, Universidad de las Americas, Cholula, Puebla, Mexico. (Nov. 2003, p. 1314)

20–27 **42nd International Symposium on Functional Equations**, Opava, Czech Republic. (Jan. 2004, p. 62)

*21–23 **V Italian-Spanish Conference on General Topology and Its Applications**, Almeria, Spain.
Main Speakers: F. Balibrea (Univ. de Murcia, Spain), A. Le Donne (Univ. di Roma "La Sapienza", Italy), V. Gregori (Univ. Politecnica de Valencia, Spain), H. P. A. Kunzi (Cape Town Univ., South Africa).
Organizers: M. A. Sanchez-Granero (Univ. de Almeria), F. G. Arenas (Univ. de Almeria), J. Rodriguez-Lopez (Univ. Politecnica de Valencia).
Information: http://www.upv.es/VItes.

21–25 **Conference in Nonlinear Analysis, in Honor of Haïm Brezis, on the Occasion of His 60th Birthday**, Paris, France. (Jan. 2004, p. 62)

21–25 **Conference on Surface Water Waves**, The Fields Institute, Toronto, Ontario, Canada. (Oct. 2003, p. 1129)

*21–26 **Nonlinear Modelling and Control, An International Seminar**, Nayanova University, Samara, Russia.
Purpose: The seminar's aim is the exchange of information about recent trends in mathematical modeling and control theory and their applications to various problems in physics, chemistry, biology, medicine, economy, and industrial concerns.
Call for Papers: Original papers related to the aim of the seminar are solicited. Potential speakers should submit an abstract before April 30. The cover page should contain title, affiliation, and email address of each author. Electronic submissions in LaTeX are encouraged.
Sponsors: Samara Municipal Nayanova Univ., Samara State Univ., Russian Academy of Natural Sciences, International Federation of Nonlinear Analysts, Univ. College Cork (Ireland).
Organizers: A. Pokrovskii (Cork, Ireland), V. Sobolev (Samara, Russia).
Languages: English and Russian.
Information and Submission: V. Sobolev (sable@ssu.samara.ru) or He. Gorelova (seminar coordinator, email: gorhel@ssu.samara.ru); Nayanova Univ., Molodogvardeiskaya 196, Samara, 443001, Russia.

21–July 2 **Artificial Neural Networks**, University of Wyoming, Laramie, Wyoming. (Feb. 2004, p. 278)

21–July 2 **SMS-NATO Advanced Summer Institute : Morse Theoretic Methods in Nonlinear Analysis and Symplectic Topology**, Université de Montréal, Canada. (Feb. 2004, p. 278)

*22–27 **Representation Theory and Its Applications**, Uppsala University, Uppsala, Sweden.
Description: This is a satellite conference to the Fourth European Congress of Mathematics (Stockholm, Sweden, June 27–July 2, 2004).
Information: email: mazor@math.uu.se.

26–July 1 **The Future of Mathematics Education**, Ciechocinek, Poland. (Jan. 2004, p. 63)

27–July 2 **2004 USENIX Annual Technical Conference**, Boston, Massachusetts. (Dec. 2003, p. 1441)

28–July 1 **Applications and Advances of Plausible and Paradoxical Reasoning for Data Fusion (DSmT)**, Stockholm, Sweden. (Feb. 2004, p. 278)

28–July 2 **16th Annual International Conference on Formal Power Series and Algebraic Combinatorics**, University of British Columbia, Vancouver, BC, Canada. (Dec. 2003, p. 63)

28–July 3 **International Association for Statistical Education (IASE) 2004 Roundtable**, Lund University, Lund, Sweden. (Sept. 2003, p. 1006)

29–July 2 **Days on Diffraction–2004**, St. Petersburg Division of Steklov's Math. Inst. and St. Petersburg Univ., St. Petersburg, Russia. (Dec. 2003, p. 1441)

*30–July 5 **20th International Conference on Operator Theory**, University of the West, Timisoara, Romania.

Topic: Operator theory, operator algebras and their applications (differential operators, complex functions, mathematical physics, matrix analysis, system theory, etc.).
Steering Committee: W. B. Arveson, K. R. Davidson, N. K. Nikolskii, S. Stratila, and F.-H. Vasilescu.
Information: OT20, Inst. of Math., P. O. Box 1-764, 014700 Bucharest, Romania; email: ot@imar.ro; http://www.imar.ro/~ot.

30–July 7 **Fourth World Congress of Nonlinear Analysts (WCNA2004)**, Hyatt Orlando, Orlando, Florida. (Aug. 2003, p. 849)

July 2004

1–December 31 **Wall Bounded and Free-Surface Turbulence and Its Computation**, Institute for Mathematical Sciences, National University of Singapore, Singapore. (Aug. 2003, p. 849)

* 4–7 **ISSAC-2004: International Symposium on Symbolic and Algebraic Computation**, University of Cantabria, Santander, Spain.
Description: ISSAC is the yearly premier international symposium in symbolic and algebraic computation. It provides an opportunity to learn of new developments and to present original research results in all areas of symbolic mathematical computation. Planned activities include invited presentations by P. Parrilo, F. Santos, and J. Verschelde; submitted research papers; poster sessions; tutorial courses; vendor exhibits; and software demonstrations.
Information: http://www.risc.uni-linz.ac.at/issac2004.

4–8 **Joint Summer Research Conference: Algebraic Geometry: Presentations by Young Researchers**, Snowbird Resort, Snowbird, Utah. (Feb. 2004, p. 279)

4–11 **The 10th International Congress on Mathematical Education**, Copenhagen, Denmark. (Oct. 2003, p. 1129)

* 5–8 **International Workshop on Orthogonal Polynomials: Orthogonal Polynomials and Mathematical Physics**, Escuela Politecnica Superior of the Universidad Carlos III de Madrid, Leganes, Madrid, Spain.
Aim: A limited number of invited mathematicians and physicists will discuss and review recent progress of the theory of orthogonal polynomials in mathematical physics.
Topics: Information entropy, imaging theory, integrable systems, special functions, Padé approximation, Riemann-Hilbert problem, orthogonal polynomials.
Invited Speakers: N. Atakishiyev (Inst. de Fisica y Mate., UNAM, Cuernavaca, Mexico), P. A. Clarkson (Inst. of Math., Stat. and Actuarial Science, Univ. of Kent, UK), A. Fokas (Univ. of Cambridge, UK), F. A. Grunbaum (Univ. of California, Berkeley, USA), J. S. Dehesa (Univ. de Granada, Spain), J. A. Tirao (Centro de Invest. y Estud. de Mate., Univ. Nacional de Cordoba, Argentina), and A. Zhedanov (Donestk Inst. for Phys. and Tech., Ukraine).
Deadline: Abstracts for short communications and posters: April 30, 2004. Please visit the Web page of the IWOP04 (http://merlin.us.es/~renato/iwop/) for further information.
Organizing Committee: R. Alvarez-Nodarse (Univ. de Sevilla), J. Arvesu (Univ. Carlos III), and F. Marcellan (Univ. Carlos III de Madrid).
Scientific Committee: J. S. Dehesa (Univ. de Granada), A. Durán (Univ. de Sevilla), G. L. Lagomasino (Univ. Carlos III de Madrid), F. Marcellón (Univ. Carlos III de Madrid), and W. Van Assche (Katholieke Univ. Leuven).
Information: http://merlin.us.es/~renato/iwop/.

5–9 **8th International Conference on p-Adic Functional Analysis**, University Blaise Pascal, Clermont-Ferrand, France. (Dec. 2003, p. 1441)

5–9 **Eleventh International Conference on Fibonacci Numbers and Their Applications**, Braunschweig, Germany. (Dec. 2003, p. 1441)

5–9 **Graphes et Combinatoire, un Colloque a la Memoire de Claude Berge**, Université Paris 6, Paris, France. (Mar. 2003, p. 409)

5–9 **19th "Summer" Conference on Topology and Its Applications**, Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch, South Africa. (Sept. 2003, p. 1006)

* 5–10 **International Conference: 2004—Dynamical Systems and Applications**, Antalya-Pamukkale (Hierapolis), Turkey.
Scope: The conference covers all aspects of dynamical systems, both theoretical and applied.
Information: http://users.kfupm.edu.sa/math/akca/Conference\%202004.htm.

5–10 **Non-commutative Geometry and Representation Theory in Mathematical Physics**, Karlstad University, Karlstad, Sweden. (Dec. 2003, p. 1442)

* 5–11 **Nonstandard Mathematics**, Aveiro, Portugal.
Description: The conference shall be held in honour of Abraham Robinson on the occasion of the 30th anniversary of his death.
Organizing Committee: F. Diener (Univ. of Nice, France); I. van den Berg, A. A. J. Franco de Oliveira (Univ. of Évora, Portugal); J. Teixeira (Technical Univ. of Lisbon, Portugal); K. Stroyan (Univ. of Iowa, USA); V. Neves (Univ. of Aveiro, Portugal).
Topics: The meeting is intended for a broad public, consisting mainly of researchers in any area where nonstandard analysis has been relevant, such as foundations, analysis and functional analysis, control theory, stochastics, differential equations, perturbation theory, economics and quantum physics, amongst others. The conference will consist of plenary talks and contributed talks; the latter will have a duration of 25 minutes. Provision is made for special sessions, such as short courses and debates about past experiences, addressed to university students and teachers of university- or preuniversity-level courses in mathematics.
Speakers: Plenary talks (confirmed): I. van den Berg, N. Cutland, F. Diener, R. Jin, H. J. Keisler, P. Loeb, R. Lutz (special session speaker), E. Nelson, T. Sari, K. Stroyan (special session speaker), M. Wolff.
Deadline: For submitting abstracts of contributed talks is March 31, 2004.
Information: Send abstracts to NSM04@mat.ua.pt or NSM2004@mat.ua.pt. See http://www.mat.ua.pt/eventos/nsmath2004 for detailed information on the event.

5–14 **Workshop on the Moonshine Conjecture, Vertex Algebras, Hyperbolic Lie Algebras and Automorphic Forms**, International Centre for Mathematical Sciences, Edinburgh, UK. (Jan. 2004, p. 63)

5–16 (REVISED) **Advanced Course on Automata Groups**, Barcelona, Spain. (Apr. 2003, p. 500)

* 8–10 **From Arithmetic to Cryptology: Conference on the Occasion of Gerhard Frey's 60th Birthday**, University of Duisburg-Essen, Essen Campus, Essen, Germany.
Topics: Diophantine equations, curves and fundamental groups, abelian varieties, modular forms and modular curves, application of the above to cryptology.
Information: email: diem@exp-math.uni-essen.de; http://www.exp-math.uni-essen.de/~birthday.

11–14 **SIAM Conference on the Life Sciences (LS04)**, Oregon Convention Center, Portland, Oregon. (Dec. 2003, p. 1442)

11–15 **Joint Summer Research Conference: Representations of Algebraic Groups, Quantum Groups, and Lie Algebras**, Snowbird Resort, Snowbird, Utah. (Feb. 2004, p. 279)

12–15 **8th WSEAS CSCC**, Vouliagmeni, Athens, Greece. (Feb. 2004, p. 279)

18–23 **Joint Summer Research Conference: Gaussian Measure and Geometric Convexity**, Snowbird Resort, Snowbird, Utah. (Feb. 2004, p. 279)

18–24 **Workshop on Mathematical Ideas in Nonlinear Optics: Guided Waves in Inhomogenous Media**, International Centre for

Mathematical Sciences, Edinburgh, UK. (Jan. 2004, p. 63)

19-23 **XVIII Escola de Algebra (Eighteenth Algebra School)**, State University of Campinas (UNICAMP), Campinas, SP/Brazil. (Oct. 2003, p. 1129)

19-August 6 **Summer School and Conference on Dynamical Systems**, ICTP, Trieste, Italy. (Feb. 2004, p. 279)

24-28 **European Congress on Computational Methods in Applied Sciences and Engineering**, Jyväskylä, Finland. (Feb. 2003, p. 295)

25-31 **2004 ASL European Summer Meeting (Logic Colloquium '04)**, Torino, Italy. (June/July 2003, p. 725)

* 26-30 **Algebraic Groups, Arithmetic Groups, Automorphic Forms and Representation Theory: An International Conference in Memory of Armand Borel**, Center of Mathematical Sciences, Zhejiang University, Hangzhou, China.
Organizers: J. Coates (co-chair), L-Z. Ji (co-chair), J-S. Li, K-F. Liu, G. Prasad, R. Taylor (co-chair), H-W. Xu, S. T. Yau (chair), S-W. Zhang.
Invited Speakers: W. Casselman, C.-L. Chai, D. Gross, R. Howe, J. P. Labesse, G. Lusztig, W. Li, J. Milne, N. Mok, M. S. Ragunathan, D. Ramakrishnan, L. Saper, W. Schmidt, F. Shahidi, Y. T. Siu, R. Taylor, D. Vogan, C. Voisin, N. Wallach, D. Wan, S-W. Zhang, S. Zucker.
Information: L. Fang, `fangli@zju.edu.cn`; `http://cms.zju.edu.cn`.

26-30 **IMS Annual Meeting/6th Bernoulli World Congress**, Barcelona, Spain. (Aug. 2003, p. 849)

26-30 **Workshop on Spectral Theory of Schrödinger Operators**, Centre de Recherches Mathématiques, Montréal, Québec, Canada. (Aug. 2003, p. 849)

26-30 **XIV Brazilian Topology Meeting**, Campinas, São Paulo, Brazil. (Sept. 2003, p. 1007)

30-August 1 **The Seventh Annual International Conference of Bridges: Mathematical Connections in Art, Music, and Science**, Southwestern College, Winfield, Kansas. (Jan. 2004, p. 63)

August 2004

2-6 **First Announcement: The 9th International Conference on Difference Equations and Applications ICDEA-9**, University of Southern California, Los Angeles, California. (Dec. 2003, p. 1442)

2-6 **Workshop on Dynamics in Statistical Mechanics**, Centre de Recherches Mathématiques, Montréal, Québec, Canada. (Aug. 2003, p. 850)

2-6 **Workshop on Derived Categories, Quivers and Strings**, International Centre for Mathematical Sciences, Edinburgh, UK. (Jan. 2004, p. 63)

2-27 **Magnetic Reconnection Theory**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Apr. 2003, p. 500)

4-6 **2004 IMCC (International Math Conference at Chonbuk National University)**, Chonbuk National University, Chonju, Chonbuk, Korea. (Jan. 2004, p. 63)

4-6 **The Seventh North American New Researchers Conference**, York University, Toronto, Canada. (Jan. 2004, p. 64 (Jan. 2004, p. 63))

6-7 **New Directions in Probability Theory**, The Fields Institute, Toronto, Canada. (Aug. 2003, p. 850)

9-13 **13th USENIX Security Symposium**, San Diego, California. (Dec. 2003, p. 1442)

16-December 17 **Quantum Information Science**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Apr. 2003, p. 500)

9-27 **Advanced School & Conference on Non-commutative Geometry**, ICTP, Trieste, Italy. (Feb. 2004, p. 279)

24-27 **International Conference on Nonlinear Operators, Differential Equations and Applications (ICNODEA-2004)**, Babes-Bolyai University, Cluj-Napoca, Romania. (Sept. 2003, p. 1008)

* 29-September 4 **Seventh International Workshop on Complex Structures and Vector Fields**, Plovdiv, Bulgaria.
Topics: This workshop covers interdisciplinary topics from complex analysis, differential geometry, and mathematical physics.
Description: The participation of students and young researchers is encouraged. Lecturers will give introductory lectures on different topics.
Organizing Committee: S. Dimiev (chair, Sofia-Plovdiv-Smolyan), M. Manev (Plovdiv), R. Lazov (Sofia), Iv. Badev (Plovdiv).
Information: Seventh International Workshop on Complex Structures and Vector Fields, S. Dimiev (chair), Plovdiv Univ., Filial-Smolyan; email: `sdimiev@math.bas.bg`; M. Manev, Plovdiv Univ., Faculty of Math. and Informatics; email: `manevm@pu.acad.bg`; R. Lazov, Inst. of Math. and Informatics, Bulgarian Acad. Sci., Sofia; email: `lazovr@math.bas.bg`; I. Badev, Tech. Univ. of Plovdiv; email: `ivanbadev@abv.bg`.

30-September 3 **9th Conference on Differential Geometry and Its Applications**, Prague, Czech Republic. (Apr. 2003, p. 500)

September 2004

1-6 (REVISED) **Sixth Pan-African Congress of Mathematicians**, Institute National des Sciences Appliquees et del la Technologie (INSAT), Université 7 Novembre à Carthage, Tunis, Tunisia. (May 2003, p. 604)

2-4 **2nd International Conference on Soft Methods in Probability and Statistics**, Edificio Historico de la Universidad, Oviedo, Spain. (Jan. 2004, p. 64)

* 10-14 **International Conference of Numerical Analysis and Applied Mathematics 2004 (ICNAAM 2004)**, Chalkis, Greece.
Aim: The aim of ICNAAM 2004 is to bring together leading scientists of the international numerical and applied mathematics community and to attract original research papers of very high quality. The topics to be covered include (but are not limited to): all the research areas of numerical analysis and computational mathematics, and all the research areas of applied mathematics (see `http://www.uop.gr/~icnaam/res8/aimscope.htm`).
Chairmen and Organizers: T. E. Simos, Department of Comput. Sci. and Tech., Faculty of Sci. and Tech., Univ. of Peloponnese, Greece; and Ch. Tsitouras, Technological Educational Inst. of Chalkis, Greece. Vice-Chairman: G. Psihoyios, Anglia Polytechnic University, Cambridge, UK.
Scientific Committee: G. van den Berghe, Belgium; P. E. Bjorstad, Norway; J. Cash, UK; R. Cools, Belgium; A. Cuyt, Belgium; B. Fischer, Germany; R. W. Freund, USA; I. Gladwell, USA; B. Hendrickson, USA; A. Klar, Germany; W. F. Mitchell, USA; T. E. Simos, Greece; W. Sproessig, Germany; Ch. Tsitouras, Greece; G. Alistair Watson, UK.
Information: Secretary ICNAAM, email: `icnaam@uop.gr`; 26 Menelaou Street, Amfithea Paleon Faliron, GR-175 64, Athens, Greece; fax: +30210 94 20 091; `http://www.uop.gr/~icnaam/`.

14-18 (REVISED) **Third International Conference on Boundary Integral Methods: Theory and Applications**, University of Reading, United Kingdom. (Nov. 2002, p. 1287)

19-22 **The First International Conference on Complex Systems CSIMTA 2004 (Complex Systems Intelligence and Modern Technology Applications)**, Cherbourg, France. (Oct. 2003, p. 1129)

* 20-22 **Workshop on Elliptic Curve Cryptograpy**, Ruhr University, Bochum, Germany.
Description: ECC 2004 is the eighth in a series of annual workshops dedicated to the study of elliptic curve cryptography and related areas.
Main Themes: The discrete logarithm problem, efficient parameter generation and point counting, provably secure cryptographic

protocols, efficient software and hardware implementation, side-channel attacks, deployment of elliptic curve cryptography.
Goal: It is hoped that the meeting will continue to encourage and stimulate further research on the security and implementation of elliptic curve cryptosystems and related areas, and encourage collaboration between mathematicians, computer scientists, and engineers in the academic, industry, and government sectors. There will be approximately 15 invited lectures (and no contributed talks), with the remaining time used for informal discussions. There will be both survey lectures as well as lectures on the latest research developments.
Information: http://www.cacr.math.uwaterloo.ca/conferences/2004/ecc2004/announcement.html.

*20–24 **2004 IEEE/WIC/ACM International Conference on Web Intelligence (WI'04)**, King Wing Hot Spring Hotel, Beijing, China.
Sponsors: IEEE Computer Society, Web Intelligence Consortium (WIC), Association for Computing Machinery (ACM).
Information: http://www.maebashi-it.org/WI04; http://www.comp.hkbu.edu.hk/WI04.

20–24 **12th French-German-Spanish Conference on Optimization**, University of Avignon, Avignon, France. (Jan. 2004, p. 64)

*20–30 **Stochastic Finance 2004 (StochFin2004)**, Coimbra and Lisbon, Portugal.
Description: StochFin2004 is an abbreviation for the Autumn School & International Conference on Stochastic Finance. The **Autumn School** will take place in Coimbra (Portugal) at the Observatório Astronómico de Coimbra, September 20–24, 2004. It is expected that its audience will consist of graduate students, young researchers, and people related to finance enterprises. The **International Conference** will take place in Lisbon (Portugal) at Instituto Superior de Economia e Gestão (ISEG), September 26–30, 2004.
Goals: (1) To present instances of interaction of finance and mathematics by means of a coherent combination of several courses, delivered by specialists in order to stimulate and reinforce the understanding of the subject; (2) to provide an opportunity for graduate students to develop some competence in financial mathematics; (3) to promote the establishment and development of interdisciplinary collaborations between researchers from different areas; (4) to bring together graduate students and specialists either from the academic or business sector, stimulating some interaction between university and business people.
Information: http://pascal.iseg.utl.pt/~stochfin2004/about.html.

October 2004

16–17 **AMS Southeastern Section Meeting**, Vanderbilt University, Nashville, Tennessee. (May 2003, p. 604)

16–17 **AMS Western Section Meeting**, University of New Mexico, Albuquerque, New Mexico. (May 2003, p. 604)

23–24 **AMS Central Section Meeting**, Northwestern University, Evanston, Illinois. (Feb. 2004, p. 279)

*24–31 **The Tenth International Conference in Modern Group Analysis (MOGRAN X)**, Larnaca, Cyprus.
Description: The aim of the meeting is to bring together leading scientists in group analysis and mathematical modelling. The main emphasis of the conference will be on applications of group methods in investigating nonlinear wave and diffusion phenomena; mathematical models in biology; integrable systems; as well as the classical heritage, historical aspects and new theoretical developments in group analysis. The conference will also highlight educational aspects.
Organizing Committee: N. H. Ibragimov, C. Sophocleous, P. A. Damianou.
Information: http://www.ucy.ac.cy/~mogran10.

November 2004

*1–4 **ICDM '04: The Fourth IEEE International Conference on Data Mining**, Brighton, UK.
Topics: Topics related to the design, analysis, and implementation of data mining theory, systems, and applications are of interest.
Sponsor: IEEE Computer Society.
Deadline: June 1, 2004.
Information: R. Rastogi, Room 2B-301, 700 Mountain Avenue, Murray Hill, NJ 07974; phone: +1-908-582-3728; fax: +1-908-582-1239; email: rastogi@research.bell-labs.com; http://icdm04.cs.uni-dortmund.de.

6–7 **AMS Eastern Section Meeting**, University of Pittsburgh, Pittsburgh, Pennsylvania. (Sept. 2003, p. 1009)

7–9 **Constructive Functions Tech-04**, Georgia Institute of Technology, Atlanta, Georgia. (Feb. 2004, p. 1443)

December 2004

5–16 **International Workshop on Nonlinear Partial Differential Equations**, IPM, Tehran, Iran. (Aug. 2003, p. 850)

17–19 **International Conference on Smarandache Algebraic Structures**, Indian Institute of Technology, IIT Madras, Chennai-600 036 Tamil Nadu, India. (Aug. 2003, p. 850)

17–22 **The Third International Congress of Chinese Mathematicians**, The Chinese University of Hong Kong, Shatin, Hong Kong, P. R. China. (Dec. 2003, p. 1443)

January 2005

5–8 **Joint Mathematics Meetings**, Hyatt Regency Atlanta & Atlanta Marriott Marquis, Atlanta, Georgia. (Sept. 2002, p. 1001)

17–July 15 **Model Theory and Applications to Algebra and Analysis**, Isaac Newton Institute for Mathematical Sciences, Cambridge, England. (Apr. 2003, p. 500)

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

March 2005

*19–25 **2005 ASL Annual Meeting**, Stanford University, Stanford, California.
Committees: The members of the Local Organizing Committee include: A. Arana, S. Feferman, G. Mints, J. Mitchell, and R. Sommer. The members of the Program Committee include: J. Mitchell, M. Rathjen, S. Shapiro, R. Solomon, P. Speissegger, and J. Steel (chair).
Information: email: asl@vassar.edu.

July 2006

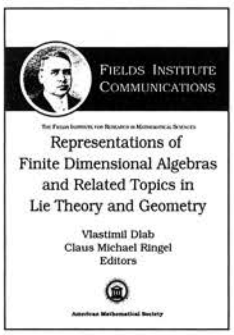
*2–7 **ICOTS 7, Working Cooperatively in Statistics Education**, Salvador (Bahia), Brazil.
Topics: (1) Working cooperatively in statistics education: L. Cordani (Brazil), lisbethk@terra.com.br; M. Shaughnessy (USA), mike@mth.pdx.edu; (2) Statistics Education at the School Level: D. Ben-Zvi (Israel), dbenzvi@univ.haifa.ac.il; L. Pereira (Singapore), lpereira@nie.edu.sg; (3) Statistics Education at the Post Secondary Level: M. Aliaga (USA), aliaga@umich.edu; E. Svensson (Sweden), elisabeth.svensson@esi.oru.se; (4) Statistics Education/Training and the Workplace: P. Silva (Brazil), pedrosilva@ibge.gov.br; P. Martĩn (Spain), pilar.guzman@uam.es; (5) Statistics Education and the Wider Society: B. Phillips (Australia), bphillips@groupwise.swin.edu.au; P. Boland (Ireland), Philip.J.Boland@ucd.ie; (6) Research in Statistics Education: C. Reading (Australia), creading@metz.une.edu.au; M. Pfannkuch (New Zealand), pfannkuc@scitec.auckland.ac.nz; (7) Technology in

Statistics Education: A. Blejec (Slovenia), andrej.blejec@uni-lj.si; C. Konold (USA), konold@srri.umass.edu; (8) Other Determinants and Developments in Statistics Education: T. Chadjipadelis (Greece), chadji@polsci.auth.gr; B. Carlson (USA), bcarlson@eclac.cl; (9) An International Perspective on Statistics Education: D. North (South Africa), delian@icon.co.za; A. S. Haedo (Argentina), haedo@qb.fcen.uba.ar; (10) Contributed Papers: J. Engel (Germany), Engel_Joachim@ph-ludwigsburg.de; A. Mc Lean (Australia), alan.mclean@buseco.monash.edu.au; (11) Posters: C. E. Lopez (Brazil), celilopes@directnet.com.br.

Information: C. Batanero, batanero@ugr.es; http://www.maths.otago.ac.nz/icots7.

New Publications Offered by the AMS

Algebra and Algebraic Geometry



Representations of Finite Dimensional Algebras and Related Topics in Lie Theory and Geometry

Vlastimil Dlab, *Carleton University, Ottawa, ON, Canada*, and **Claus Michael Ringel**, *Universität Bielefeld, Germany*, Editors

These proceedings are from the Tenth International Conference on Representations of Algebras and Related Topics (ICRA X) held at The Fields Institute. In addition to the traditional "instructional" workshop preceding the conference, there were also workshops on "Commutative Algebra, Algebraic Geometry and Representation Theory", "Finite Dimensional Algebras, Algebraic Groups and Lie Theory", and "Quantum Groups and Hall Algebras". These workshops reflect the latest developments and the increasing interest in areas that are closely related to the representation theory of finite dimensional associative algebras. Although these workshops were organized separately, their topics are strongly interrelated.

The workshop on Commutative Algebra, Algebraic Geometry and Representation Theory surveyed various recently established connections, such as those pertaining to the classification of vector bundles or Cohen-Macaulay modules over Noetherian rings, coherent sheaves on curves, or ideals in Weyl algebras. In addition, methods from algebraic geometry or commutative algebra relating to quiver representations and varieties of modules were presented.

The workshop on Finite Dimensional Algebras, Algebraic Groups and Lie Theory surveyed developments in finite dimensional algebras and infinite dimensional Lie theory, especially as the two areas interact and may have future interactions.

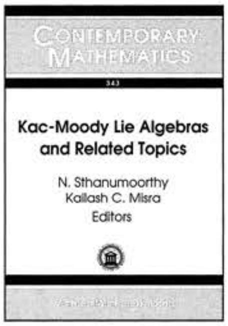
The workshop on Quantum Groups and Hall Algebras dealt with the different approaches of using the representation theory of quivers (and species) in order to construct quantum groups, working either over finite fields or over the complex numbers. In particular, these proceedings contain a quite detailed outline of the use of perverse sheaves in order to obtain canonical bases.

The book is recommended for graduate students and researchers in algebra and geometry.

Contents: *Instructional Workshop:* **K. S. Brown**, Semigroup and ring theoretical methods in probability; **T. Bruestle**, Typical examples of tame algebras; **O. Iyama**, Representation dimension and Solomon zeta function; **S. Koenig**, Filtrations, stratifications and applications; **M. S. Putcha**, Bruhat-Renner decomposition and Hecke algebras of reductive monoids; **L. E. Renner**, Representations and blocks of algebraic monoids; **M. Schocker**, The descent algebra of the symmetric group; *Specialized Workshops. Commutative Algebra, Algebraic Geometry and Representation Theory*: **Y. Berest**, A remark on Letzter-Makar-Limanov invariants; **I. Burban**, Derived categories of coherent sheaves on rational singular curves; **Y. A. Drozd**, Vector bundles and Cohen-Macaulay modules; *Finite Dimensional Algebras, Algebraic Groups and Lie Theory:* **J. Du**, Finite dimensional algebras, quantum groups and finite groups of Lie type; **V. Mazorchuk**, Stratified algebras arising in Lie theory; **T. Tanisaki**, Character formulas of Kazhdan-Lusztig type; **P. Webb**, Weight theory in the context of arbitrary finite groups; *Quantum Groups and Hall Algebras:* **G. Benkart** and **S. Witherspoon**, Restricted two-parameter quantum groups; **B. Deng** and **J. Xiao**, On Ringel-Hall algebras; **Z. Lin**, Lusztig's geometric approach to Hall algebras; **M. Reineke**, The use of geometric and quantum group techniques for wild quivers; **K. Rietsch**, An introduction to perverse sheaves; **Y. Saito**, An introduction to canonical bases; **O. Schiffmann**, Quivers of type A, flag varieties and representation theory.

Fields Institute Communications, Volume 40

March 2004, approximately 504 pages, Hardcover, ISBN 0-8218-3416-9, LC 2003063645, 2000 *Mathematics Subject Classification*: 16-XX, 17-XX, 14-XX; 05-XX, 13-XX, 15-XX, 18-XX, 20-XX, 51-XX, 81-XX, **All AMS members $95**, List $119, Order code FIC/40N



Kac-Moody Lie Algebras and Related Topics

N. Sthanumoorthy, *University of Madras, Chennai, India*, and **Kailash C. Misra**, *North Carolina State University, Raleigh*, Editors

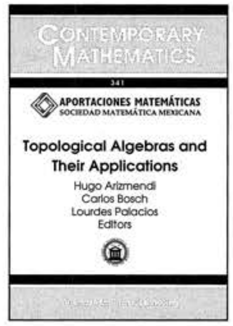
This volume is the proceedings of the Ramanujan International Symposium on Kac-Moody Lie algebras and their applications. The symposium provided researchers in mathematics and physics with the opportunity to discuss new developments in this rapidly-growing area of research. The book contains several excellent articles with new and significant results. It is suitable for graduate students and researchers working in Kac-Moody Lie algebras, their applications, and related areas of research.

Contents: **D. Adamović**, Regularity of certain vertex operator superalgebras; **V. Bekkert**, **G. Benkart**, and **V. Futorny**, Weight modules for Weyl algebras; **E. Date** and **K. Usami**, On an analog of the Onsager algebra of type $D_n^{(1)}$; **A. J. Feingold**, Fusion rules for affine Kac-Moody algebras; **A. J. Feingold** and **H. Nicolai**, Subalgebras of hyperbolic Kac-Moody algebras; **J. Fuchs**, **I. Runkel**, and **C. Schweigert**, Lie algebras, Fuchsian differential equations and CFT correlation functions; **Y.-Z. Huang**, Conformal-field-theoretic analogues of codes and lattices; **H. P. Jakobsen** and **C.-W. H. Lee**, Matrix chain models and Kac-Moody algebras; **K. C. Misra** and **V. Williams**, Combinatorics of quantum affine Lie algebra representations; **H. Nicolai** and **T. Fischbacher**, Low level representations for E_{10} and E_{11}; **M. Primc**, Generators of relations for annihilating fields; **S. E. Rao** and **K. Zhao**, On integrable representations for toroidal Lie superalgebras; **N. Sthanumoorthy** and **C. K. Bagirathi**, Principal vertex operators and super Hirota bilinear equations for $B_3^{(1)}$; **N. Sthanumoorthy** and **P. L. Lilly**, On some classes of root systems of generalized Kac-Moody algebras; **N. Sthanumoorthy**, **P. L. Lilly**, and **A. U. Maheswari**, Root multiplicities of some classes of extended-hyperbolic Kac-Moody and extended-hyperbolic generalized Kac-Moody algebras; **V. N. Tolstoy**, From quantum affine Kac-Moody algebras to Drinfeldians and Yangians.

Contemporary Mathematics, Volume 343

April 2004, approximately 384 pages, Softcover, ISBN 0-8218-3337-5, 2000 *Mathematics Subject Classification*: 16D60, 17B10, 17B37, 17B65, 17B67, 17B69, 22E65, 81R10, 81R50, 81T40; 20F05, 94B05, **All AMS members $71**, List $89, Order code CONM/343N

Analysis



Topological Algebras and Their Applications

Hugo Arizmendi, **Carlos Bosch**, and **Lourdes Palacios**, *Universidad Nacional Autónoma de México*, Editors

The Fourth International Conference on Topological Algebras and Their Applications was held in Oaxaca, Mexico. This meeting brought together international specialists and Mexican specialists in topological algebras, locally convex and Banach spaces, spectral theory, and operator theory and related topics.

This volume contains talks presented at the conference as well as articles received in response to a call for papers; some are expository and provide new insights, while others contain new research.

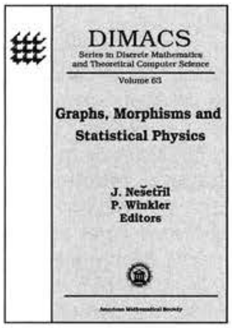
The book is suitable for graduate students and research mathematicians working in topological vector spaces, topological algebras, and their applications.

Contents: **M. Abel**, Description of all closed maximal regular ideals in subalgebras of the algebra $C(X;A;\sigma)$; **M. Abel**, Galbed Gelfand-Mazur algebras; **J. Arhippainen**, On Gelfand representation of topological algebras; **T. Chryssakis**, Relations between numerical range and spectrum–The set of strongly positive elements; **H. Fetter** and **B. Gamboa de Buen**, Some considerations about two properties related to measures of noncompactness in Banach spaces; **A. García**, Regular inductive limits of locally complete spaces; **R. Hadjigeorgiou**, On some more characterizations of Q-algebras; **M. Haralampidou**, Matrix representations of Ambrose algebras; **J. Kakol**, **S. A. Saxon**, and **A. R. Todd**, Docile locally convex spaces; **A. Mallios**, On localizing topological algebras; **A. Martínez Meléndez**, Topological algebras and α-spectrum; **M. Oudadess**, On some nonconvex topological algebras; **F. H. Szafraniec**, Bounded vectors for subnormality via a group of unbounded operators; **Y. Tsertos**, On the C^*-structures of an algebra; **A. Velázquez González** and **A. Wawrzyńczyk**, Spectral mapping formula for Waelbroeck algebras and their subalgebras; **W. Żelazko**, When a commutative unital F-algebra has a dense principal ideal.

Contemporary Mathematics, Volume 341

February 2004, 137 pages, Softcover, ISBN 0-8218-3556-4, LC 2003063641, 2000 *Mathematics Subject Classification*: 18F20, 46A08, 46A13, 46B20, 46H05, 46H10, 46H20, 47A20, 47B20, 55N30, **All AMS members $39**, List $49, Order code CONM/341N

Discrete Mathematics and Combinatorics



Graphs, Morphisms and Statistical Physics

J. Nešetřil, *Charles University, Praha, Czech Republic*, and **P. Winkler**, *Bell Labs, Murray Hill, NJ*, Editors

The intersection of combinatorics and statistical physics has experienced great activity in recent years. This flurry of activity has been fertilized by an exchange not only of techniques, but also of objectives. Computer scientists interested in approximation algorithms have helped statistical physicists and discrete mathematicians overcome language problems. They have found a wealth of common ground in probabilistic combinatorics.

Close connections between percolation and random graphs, graph morphisms and hard-constraint models, and slow mixing and phase transition have led to new results and perspectives. These connections can help in understanding typical behavior of combinatorial phenomena such as graph coloring and homomorphisms.

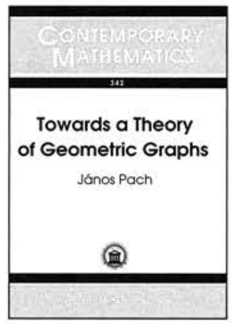
Inspired by issues and intriguing new questions surrounding the interplay of combinatorics and statistical physics, a DIMACS/DIMATIA workshop was held at Rutgers University. These proceedings are the outgrowth of that meeting. This volume is intended for graduate students and research mathematicians interested in probabilistic graph theory and its applications.

This item will also be of interest to those working in mathematical physics and probability.

Contents: S. Boettcher, Efficient local search near phase transitions in combinatorial optimization; **G. R. Brightwell** and **P. Winkler**, Graph homomorphisms and long range action; **C. Borgs, J. T. Chayes, M. Dyer**, and **P. Tetali**, On the sampling problem for H-colorings on the hypercubic lattice; **A. Daneshgar** and **H. Hajiabolhassan**, Random walks and graph homomorphisms; **J. Díaz, M. Serna**, and **D. M. Thilikos**, Recent results on parameterized H-colorings; **M. Dyer, M. Jerrum**, and **E. Vigoda**, Rapidly mixing Markov chains for dismantleable constraint graphs; **D. Galvin** and **P. Tetali**, On weighted graph homomorphisms; **P. Hell** and **J. Nešetřil**, Counting list homomorphisms for graphs with bounded degrees; **G. Istrate**, On the satisfiability of random k-horn formulae; **J. Katriel**, The exchange interaction, spin hamiltonians, and the symmetric group; **M. Loebl**, A discrete non-Pfaffian approach to the Ising problem; **E. Mossel**, Survey: Information flow on trees; **C. Tardif**, Chromatic numbers of products of tournaments: Fractional aspects of Hedetniemi's conjecture; **X. Zhu**, Perfect graphs for generalized colouring-circular perfect graphs.

DIMACS: Series in Discrete Mathematics and Theoretical Computer Science, Volume 63

May 2004, approximately 248 pages, Hardcover, ISBN 0-8218-3551-3, 2000 *Mathematics Subject Classification*: 05-06, 05A16, 05C15, 05C60, 60-06, 60J10, 60K35, 68R10, 82B20, **All AMS members $63**, List $79, Order code DIMACS/63N



Towards a Theory of Geometric Graphs

János Pach, *City College, City University of New York, NY, and Hungarian Academy of Sciences, Budapest*, Editor

The early development of graph theory was heavily motivated and influenced by topological and geometric themes, such as the Königsberg Bridge Problem, Euler's Polyhedral Formula, or Kuratowski's characterization of planar graphs. In 1936, when Dénes König published his classical *Theory of Finite and Infinite Graphs*, the first book ever written on the subject, he stressed this connection by adding the subtitle *Combinatorial Topology of Systems of Segments*. He wanted to emphasize that the subject of his investigations was very *concrete*: planar figures consisting of points connected by straight-line segments. However, in the second half of the twentieth century, graph theoretical research took an interesting turn. In the most popular and most rapidly growing areas (the theory of random graphs, Ramsey theory, extremal graph theory, algebraic graph theory, etc.), graphs were considered as *abstract* binary relations rather than geometric objects. Many of the powerful techniques developed in these fields have been successfully applied in other areas of mathematics. However, the same methods were often incapable of providing satisfactory answers to questions arising in geometric applications.

In the spirit of König, *geometric graph theory* focuses on combinatorial and geometric properties of graphs drawn in the plane by straight-line edges (or more generally, by edges represented by simple Jordan arcs). It is an emerging discipline that abounds in open problems, but it has already yielded some striking results which have proved instrumental in the solution of several basic problems in combinatorial and computational geometry. The present volume is a careful selection of 25 invited and thoroughly refereed papers, reporting about important recent discoveries on the way *Towards a Theory of Geometric Graphs*.

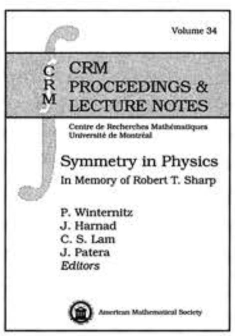
Contents: H. Alt, C. Knauer, G. Rote, and **S. Whitesides**, On the complexity of the linkage reconfiguration problem; **G. Arutyunyants** and **A. Iosevich**, Falconer conjecture, spherical averages and discrete analogs; **P. Brass**, Turán-type extremal problems for convex geometric hypergraphs; **G. Cairns, M. McIntyre**, and **Y. Nikolayevsky**, The thrackle conjecture for K_5 and $K_{3,3}$; **V. Dujmović** and **D. R. Wood**, Three-dimensional grid drawings with sub-quadratic volume; **A. Dumitrescu** and **R. Radoičić**, On a coloring problem for the integer grid; **D. Eppstein**, Separating thickness from geometric thickness; **R. E. Jamison**, Direction trees in centered polygons; **A. Kaneko, M. Kano**, and **K. Suzuki**, Path coverings of two sets of points in the plane; **G. O. H. Katona, R. Mayer**, and **W. A. Woyczynski**, Length of sums in a Minkowski space; **N. H. Katz** and **G. Tardos**, A new entropy inequality for the Erdős distance problem; **A. Kostochka**, Coloring intersection graphs of geometric figures with a given clique number; **L. Lovász, K. Vesztergombi, U. Wagner**, and **E. Welzl**, Convex quadrilaterals and k-sets;

H. Maehara, Distance graphs and rigidity; **J. Nešetřil**, **J. Solymosi**, and **P. Valtr**, A Ramsey property of planar graphs; **J. Pach**, **R. Radoičić**, and **G. Tóth**, A generalization of quasi-planarity; **J. Pach** and **M. Sharir**, Geometric incidences; **M. A. Perles** and **R. Pinchasi**, Large sets must have either a k-edge or a $(k+2)$-edge; **R. Pinchasi** and **R. Radoičić**, Topological graphs with no self-intersecting cycle of length 4; **I. Z. Ruzsa**, A problem on restricted sumsets; **F. Shahrokhi**, **O. Sýkora**, **L. A. Székely**, and **I. Vrťo**, The gap between crossing numbers and convex crossing numbers; **J. Solymosi** and **V. Vu**, Distinct distances in high dimensional homogeneous sets; **J. Spencer**, The biplanar crossing number of the random graph; **K. J. Swanepoel** and **P. Valtr**, The unit distance problem on spheres; **L. Székely**, Short proof for a theorem of Pach, Spencer, and Tóth.

Contemporary Mathematics, Volume 342

April 2004, approximately 312 pages, Softcover, ISBN 0-8218-3484-3, 2000 *Mathematics Subject Classification*: 05C62; 57M50, 90C27, 05C35, 68W35, **All AMS members $63**, List $79, Order code CONM/342N

Mathematical Physics



Symmetry in Physics

P. Winternitz, *University of Montreal, Quebec, Canada*, **J. Harnad**, *Concordia University, Montreal, Quebec, Canada*, **C. S. Lam**, *McGill University, Montreal, Quebec, Canada*, and **J. Patera**, *University of Montreal, Quebec, Canada*, Editors

Papers in this volume are based on the Workshop on Symmetries in Physics held at the Centre de recherches mathématiques (University of Montreal) in memory of Robert T. Sharp. Contributed articles are on a variety of topics revolving around the theme of symmetry in physics.

The preface presents a biographical and scientific retrospect of the life and work of Robert Sharp. Other articles in the volume represent his diverse range of interests, including representation theoretic methods for Lie algebras, quantization techniques and foundational considerations, modular group invariants and applications to conformal models, various physical models and equations, geometric calculations with symmetries, and pedagogical methods for developing spatio-temporal intuition.

The book is suitable for graduate students and researchers interested in group theoretic methods, symmetries, and mathematical physics.

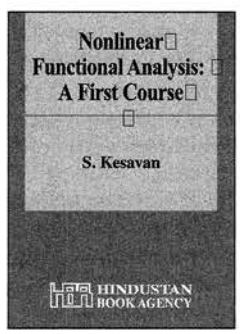
This item will also be of interest to those working in algebra and algebraic geometry.

Contents: S. C. Anco and **J. Pohjanpelto**, Symmetries and currents of massless neutrino fields, electromagnetic and graviton fields; **C. P. Burgess**, Naturalness and quintessence; **C. J. Cummins** and **S. Pauli**, Congruence subgroups of PSL(2,$\mathbb{Z}$); **H. de Guise**, **D. J. Rowe**, and **B. C. Sanders**, Asymptotic SU(2) and SU(3) Wigner functions from the weight diagram; **M. de Montigny**, **F. C. Khanna**, and **A. E. Santana**, Physical applications of a five-dimensional metric formulation of Galilean invariance; **T. Gannon**, Variations on Dedekind's eta; **J.-P. Gazeau**, **T. Garidi**, **E. Huguet**, **M. Lachièze-Rey**, and **J. Renaud**, Examples of Berezin-Toeplitz quantization: Finite sets and unit interval; **A. M. Grundland** and **W. J. Zakrzewski**, A modified Weierstrass representation for CMC-surfaces in multi-dimensional Euclidean spaces; **Č. Burdík** and **M. Havlíček**, Boson realizations of semi-simple Lie algebras; **R. C. King**, **C. Tollu**, and **F. Toumazet**, Stretched Littlewood-Richardson and Kostka coefficients; **P. Kramer**, Group actions on compact hyperbolic manifolds and closed geodesics; **C. S. Lam**, Is there an ultimate symmetry in physics?; **D. Britten** and **F. Lemire**, Formal characters and resolution of infinite-dimensional simple A_r-modules of finite degree; **L. Bégin**, **C. Cummins**, **P. Mathieu**, and **M. A. Walton**, Fusion rules and the Patera-Sharp generating-function method; **M. Moshinsky**, Transient effects in Wigner distribution phase space of a scattering problem; **J. Patera**, R. T. Sharp and generating functions in group theory; **G. Álvarez**, **F. Finkel**, **A. González-López**, and **M. A. Rodríguez**, Quasi-exact solvability in nonlinear optics; **D. J. Rowe**, Coherent states, induced representations, geometric quantization, and their vector coherent state extensions; **M. Bodner** and **G. L. Shaw**, Symmetry math video game used to train profound spatial-temporal reasoning abilities equivalent to dynamical knot theory; **M. A. Walton**, Polytope sums and Lie characters; **P. Winternitz**, Subalgebras of Lie algebras. Example of sl(3,$\mathbb{R}$).

CRM Proceedings & Lecture Notes, Volume 34

April 2004, 227 pages, Softcover, ISBN 0-8218-3409-6, 2000 *Mathematics Subject Classification*: 22-06, 81-06, 81Rxx, **All AMS members $63**, List $79, Order code CRMP/34N

More Publications Available from the AMS



Nonlinear Functional Analysis: A First Course

S. Kesavan, *Institute of Mathematical Sciences, Chennai, India*

Nonlinear functional analysis studies the properties of (continuous) mappings between normed linear spaces and describes methods for solving nonlinear equations involving such mappings. Two major approaches to the solution of nonlinear equations could be described as *topological* and *variational*. Topological methods are derived from fixed point theorems and are usually based on the notion of the topological degree. Variational methods describe the solutions as critical points of a suitable functional and ways of locating them.

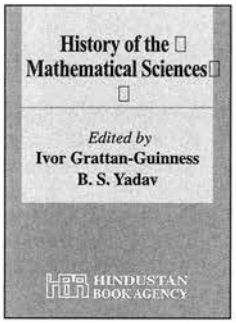
This book presents the basic theory of these methods. It is meant to be a primer of nonlinear analysis and used as a text or reference book by advanced students. Prerequisites are knowledge of functional analysis and topology.

A publication of Hindustan Book Agency; distributed worldwide except in India by the AMS.

Contents: Differential calculus on normed linear spaces; The Brouwer degree; The Leray-Schauder degree; Bifurcation theory; Critical points of functionals.

Hindustan Book Agency

January 2004, 186 pages, Hardcover, ISBN 81-85931-46-1, 2000 *Mathematics Subject Classification*: 46-01; 46T99, 47H11, 47J15, 47J30, 58E99, **All AMS members $24**, List $30, Order code HIN/20N



History of the Mathematical Sciences

Ivor Grattan-Guinness and **B. S. Yadav**, Editors

This book contains selected papers of the proceedings of the International Conference on History of the Mathematical Sciences held in New Delhi. The conference was organized jointly by the Indian Society for History of Mathematics and Ramjas College, University of Delhi, in collaboration with the Indira Gandhi National Open University and the Indian Institute of Advanced Study, Simla. It was the first of its kind ever held in India and covered all aspects of the history of the mathematical sciences. About 200 delegates participated in the deliberations, including leading experts from around the globe and eminent scholars from India.

A publication of Hindustan Book Agency; distributed worldwide except in India by the AMS.

Contents: *Part A: General and Philosophical Issues:* **K. S. Chaudhuri**, Evolution of history of mathematics: Some trends; **I. Grattan-Guinness**, History or heritage? A central question in the historiography of mathematics; **R. S. Kaushal**, Abstraction and structural analogies in mathematical sciences; **V. Lakshmikantham**, The invented and propagated theories in the origin of mathematical sciences; **N. H. Schlomiuk**, Andre Weil: The man and the historian of mathematics; *Part B: History of Mathematics in India:* **G. S. Pandey**, The Vedic concept of infinity and infinitesimal system; **J. Hoyrup**, Mahavira's geometrical problems: Traces of unknown links between Jaina and Mediterranean mathematics in the classical ages; **V. Peckhaus**, Dignaga's logic of invention; **R. G. N. Prasad**, Concept of perception in Vedanta Darshna and modern mathematical sciences; *Part C: History of Mathematics in the Near East:* **M. Gardner**, The Egyptian mathematical leather roll, attested short term and long term; **K. V. Mardia**, Omar Khayyam, Rene Descartes and solutions to algebraic equations; **G. De Young**, A new source of evidence for the lost Arabic translations of Euclid's elements; *Part D: History of Mathematics in Europe and America:* **J. S. Chahal**, Pell's equation and the unity of mathematics; **E. Von Collani**, History, state of the art and future of the science of stochastics; **B. S. Yadav**, $C[a,b]$ as the backbone of evolution of functional analysis; **J.-P. Pier**, Harmonic analysis: A historical manifold during the XXth century; **V. Milman**, From functional analysis to asymptotic geometric analysis.

Hindustan Book Agency

December 2003, 244 pages, Hardcover, ISBN 81-85931-45-3, 2000 *Mathematics Subject Classification*: 00B20, 01-06, 01A75, **All AMS members $32**, List $40, Order code HIN/19N

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Department of Mathematics
Chair

The Howard University College of Arts & Sciences invites applications for the position of chairperson, Department of Mathematics, to begin July 2004. The successful candidate will have excellent interpersonal and leadership skills and will have the vision and ability to bring the department to the next level of national recognition and achievement.

The candidate will hold a Ph.D. in mathematics; will have achieved an outstanding record of teaching experience, scholarly productivity, and professional and community service; will have an easily understood vision for the department; will have exhibited leadership in planning, budgeting, and other fiscal affairs; and will exhibit an understanding of and a commitment to the mission of Howard University.

In addition, the candidate should meet the following criteria:

- Ability to build an effective team and inspire teamwork in accomplishing the goals of the Department of Mathematics.
- Ability to recruit and retain high-quality students and faculty members for the Department of Mathematics.
- Ability to manage a budget and administer it in the best interests of the Department of Mathematics.
- Ability to obtain financial support from government, foundations, agencies, and private sources.
- A keen sensitivity to the needs of staff members in the area of professional development.
- An awareness of curricular and pedagogical designs necessary for the production of educators who must meet the demands of their disciplines.
- An understanding of the connectedness between the Department of Mathematics and other departments, schools, and colleges of the university and its constituent community.

Each candidate should submit a CV; a letter of application that addresses his/her teaching, research, service, and administrative experience; and contact information for at least three references to: ppeart@howard.edu and/or to Professor Paul B. Peart, Chair, Search Committee, Department of Mathematics, Howard University, Washington, DC 20059. Applications should be submitted by March 21, 2004.

A statement of the mission of Howard University is available at http://www.founders.howard.edu/presidentReports/Mission.htm. More information about the position is available at http://www.coas.howard.edu/positions/.

Howard University is an Equal Opportunity/Affirmative Action Employer.

MASSACHUSETTS

THE WORCESTER POLYTECHNIC INSTITUTE
Department of Mathematical Sciences
Harold J. Gay Professorship

The Worcester Polytechnic Institute (WPI) Department of Mathematical Sciences invites applications and nominations for the Harold J. Gay endowed professorship. Candidates are sought in all areas of current departmental research strengths

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2004 rate is \$100 per inch or fraction thereof on a single column (one-inch minimum), calculated from top of headline. Any fractional text of 1/2 inch or more will be charged at the next inch rate. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified advertising.

Upcoming deadlines for classified advertising are as follows: April 2004 issue–January 28, 2004; May 2004 issue–February 27, 2004; June/July 2004 issue–April 29, 2004; August 2004 issue–May 28, 2004; September 2004 issue–June 28, 2004; October 2004 issue–July 28, 2004.

U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P. O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to classads@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

supportive of the strategic development of the department and the university. It is expected that the holder of the professorship will have a distinguished research record, a firm commitment to education, and the ability to exercise strong leadership in advancing the academic programs of the department and the university. The mathematical sciences department has 24 tenured/tenure-track faculty and awards B.S. degrees in mathematical sciences and in actuarial mathematics; M.S. degrees in applied mathematics, applied statistics, financial mathematics, and industrial mathematics; and Ph.D.'s in mathematical sciences. Current research strengths include applied analysis and differential equations, computational mathematics and numerical analysis, discrete mathematics, and statistics. For additional information, go to `http://www.wpi.edu/+math`.

WPI is a private and highly selective technological university with an enrollment of 2,700 undergraduates and about 1,100 full- and part-time graduate students. Worcester, New England's third largest city, offers ready access to the diverse economic, cultural, and recreational resources of the region.

Nominations and applications should be sent to: H. J. Gay Professorship Search Committee, Mathematical Sciences Department, WPI, 100 Institute Rd., Worcester, MA 01609-2280, USA. To be complete, applications must include a detailed curriculum vitae, a brief statement of specific teaching and research objectives, and the names of four references with mail/email addresses and telephone/fax numbers. Applications will be considered on a continuing basis beginning December 1, 2003, until the position is filled.

To enrich education through diversity, WPI is an Affirmative Action/Equal Opportunity Employer. Women and minorities are encouraged to apply.

MICHIGAN

MICHIGAN STATE UNIVERSITY
East Lansing, MI 48824
proMSc Program in Industrial Mathematics

Direct your students toward one of the professional M.Sc. programs. Industry needs business-savvy mathematicians. See `http://www.sciencemasters.com/`.

OHIO

THE OHIO STATE UNIVERSITY AT MARION
Department of Mathematics

Applications are invited for a tenure-track faculty position at the assistant professor level, effective autumn quarter 2004. The successful candidate will hold rank in The Ohio State University in the Department of Mathematics. Candidates must have a Ph.D. degree and a strong commitment to both teaching and research. Candidates in all areas of pure and applied mathematics are invited to apply. Evidence of accomplishment in mathematical research and excellent teaching ability are required. Ability to teach mathematics courses for education students and basic courses in statistics and/or computer science is desired. Teaching responsibilities will be at the undergraduate level. The cover letter should include a brief description of teaching experience and research interests.

Applicants should send a current vita and three letters of reference (at least one of which addresses the candidate's teaching qualifications) to: Math Search, The Ohio State University, Marion Campus, Human Resource Office, 124 Morrill Hall, 1465 Mt. Vernon Ave., Marion, OH 43302. Applications will be screened beginning April 1, 2004, but will be considered until the position is filled.

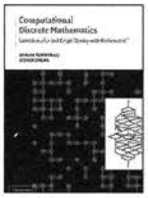
We seek applicants committed to undergraduate education and interested in teaching in a multidisciplinary, small college atmosphere within one of the nation's premier land grant universities. The Ohio State University at Marion provides approximately 1,600 commuter students with courses at the undergraduate and graduate levels. The campus has direct connections to main campus computers and is one hour from Columbus. Information about the Marion campus is at `http://www.marion.ohio-state.edu`. The Department of Mathematics is at `http://www.math.ohio-state.edu`.

The Ohio State University is an Equal Opportunity/Affirmative Action Employer. Women, minorities, veterans, and individuals with disabilities are encouraged to apply.

CLASSIFIEDS

CAMBRIDGE

Exciting New Cambridge Titles in Mathematics



Computational Discrete Mathematics
Combinatorics and Graph Theory with *Mathematica*®
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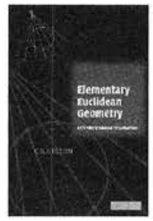
Three interesting classes of exercises are provided—theorem/proof, programming exercises, and experimental explorations—ensuring great flexibility in teaching and learning the material.

$60.00: Hardback: 0-521-80686-0: 512pp



Elementary Euclidean Geometry
An Undergraduate Introduction
C.G. Gibson

This introduction to the geometry of lines and conics in the Euclidean plane is example-based and self-contained, assuming only a basic grounding in linear algebra.

$50.00: Hardback: 0-521-83448-1: 190pp

Financial Derivatives
Pricing, Applications, and Mathematics
Jamil Baz and George Chacko

"The book is fundamentally strong because it is both well-informed technically and also focused on the actual matters that matter in the markets."

–Martin Baxter, Nomura International, London

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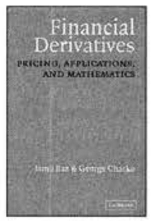


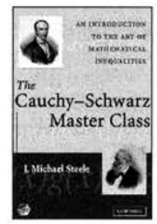
The Cauchy-Schwarz Master Class
A User's Guide to Inequalities
J. Michael Steele

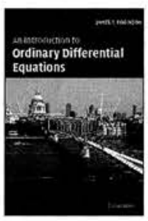
Using the Cauchy-Schwarz inequality as a guide, Steele presents a fascinating collection of problems related to inequalities and coaches readers through solutions.

MAA Problem Books

Copublished with the Mathematical Association of America

$80.00*: Hardback: 0-521-83775-8: 320pp
$29.00*: Paperback: 0-521-54677-X





Introduction to Ordinary Differential Equations
James Robinson

This introduction to ordinary differential and difference equations is suited not only for mathematicians but for scientists and engineers as well. Exact solutions methods and qualitative approaches are covered, and many illustrative examples are included.

$100.00: Hardback: 0-521-82650-0: 414pp
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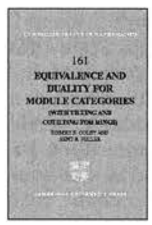


Equivalence and Duality for Module Categories (with Tilting and Cotilting for Rings)
Robert R. Colby and Kent R. Fuller

Collecting and unifying the basic results of investigations with innovative and easily understandable proofs, the authors' provide an aid to further research on this central topic in abstract algebra.

Cambridge Tracts in Mathematics 161

$50.00*: Hardback: 0-521-83821-5: 160pp

Modern Signal Processing
Daniel N. Rockmore and Dennis M. Healy Jr.

The volume emphasizes current challenges, new techniques adapted to new technologies, and certain recent advances in algorithms and theory.

Mathematical Sciences Research Institute Publications

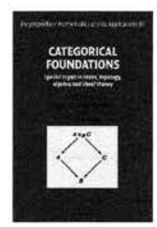
$55.00*: Hardback: 0-521-82706-X: 350pp

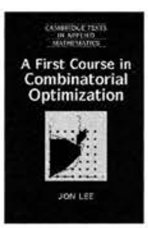
Categorical Foundations
Special Topics in Order, Topology, Algebra, and Sheaf Theory
Edited by Maria Cristina Pedicchio and Walter Tholen

Each of the eight largely independent chapters analyzes a particular subject, revealing the power and applicability of the categorical foundations in each case.

Encyclopedia of Mathematics and its Applications 97

$90.00: Hardback: 0-521-83414-7: 440pp





A First Course in Combinatorial Optimization
Jon Lee

Jon Lee focuses on key mathematical ideas leading to useful models and algorithms, rather than on data structures and implementation details, in this introductory text.

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Representation Theory of Finite Reductive Groups
Marc Cabanes and Michel Enguehard

This book brings together many of the main concerns of modern algebra, synthesizing the past twenty-five years of research, by including some of the most remarkable achievements in the field.

New Mathematical Monographs 1

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Introductory Algebraic Number Theory
Saban Alaca and Kenneth S. Williams

Suitable for senior undergraduates and beginning graduate students in mathematics, this book is an introduction to algebraic number theory at an elementary level.

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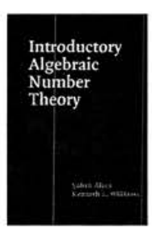

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Use this list to indicate (on the application form) your fields of interest. This information will enable the AMS to keep you informed of new publications or special sales in these fields.

00 General
01 History and biography
03 Mathematical logic and foundations
05 Combinatorics
06 Order, lattices, ordered algebraic structures
08 General algebraic systems
11 Number theory
12 Field theory and polynomials
13 Commutative rings and algebras
14 Algebraic geometry
15 Linear and multilinear algebra; matrix theory
16 Associative rings and algebras
17 Nonassociative rings and algebras
18 Category theory, homological algebra
19 *K*-theory
20 Group theory and generalizations
22 Topological groups, Lie groups
26 Real functions
28 Measure and integration
30 Functions of a complex variable
31 Potential theory
32 Several complex variables and analytic spaces
33 Special functions
34 Ordinary differential equations
35 Partial differential equations
37 Dynamical systems and ergodic theory
39 Difference and functional equations
40 Sequences, series, summability
41 Approximations and expansions
42 Fourier analysis
43 Abstract harmonic analysis
44 Integral transforms, operational calculus
45 Integral equations
46 Functional analysis
47 Operator theory
49 Calculus of variations and optimal control; optimization
51 Geometry
52 Convex and discrete geometry
53 Differential geometry
54 General topology
55 Algebraic topology
57 Manifolds and cell complexes
58 Global analysis, analysis on manifolds
60 Probability theory and stochastic processes
62 Statistics
65 Numerical analysis
68 Computer science
70 Mechanics of particles and systems
74 Mechanics of deformable solids
76 Fluid mechanics
78 Optics, electromagnetic theory
80 Classical thermodynamics, heat transfer
81 Quantum theory
82 Statistical mechanics, structure of matter
83 Relativity and gravitational theory
85 Astronomy and astrophysics
86 Geophysics
90 Operations research, mathematical programming,
91 Game theory, economics, social and behavioral sciences
92 Biology and other natural sciences
93 Systems theory; control
94 Information and communication, circuits
97 Mathematics education

American Mathematical Society

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Please mark the society in which you currently hold membership.

☐ Allahabad Mathematical Society
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☐ Berliner Mathematische Gesellschaft
☐ Calcutta Mathematical Society
☐ Canadian Mathematical Society
☐ Croatian Mathematical Society
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☐ Danish Mathematical Society
☐ Deutsche Mathematiker-Vereinigung
☐ Edinburgh Mathematical Society
☐ Egyptian Mathematical Society
☐ European Mathematical Society
☐ Gesellschaft für Angewandte Mathematik und Mechanik
☐ Glasgow Mathematical Association
☐ Hellenic Mathematical Society
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☐ Palestine Society for Mathematical Sciences
☐ Polskie Towarzystwo Matematyczne
☐ Punjab Mathematical Society
☐ Ramanujan Mathematical Society
☐ Real Sociedad Matemática Española
☐ Saudi Association for Mathematical Sciences
☐ Singapore Mathematical Society
☐ Sociedad Colombiana de Matemáticas
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American Mathematical Society

Application for Membership

January–December
2004

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Day *Month* *Year*

Primary Fields of Interest (choose up to five from the list on reverse)

_______ _______ _______ _______ _______

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☐ Pay by check, money order, UNESCO coupons. Mail to American Mathematical Society, P.O. Box 845904, Boston, MA 02284-5904 USA.

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Choose a Membership Category

Introductory ordinary member **$55** ☐

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With annual professional income below $80,000 **$111** ☐

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Minimum $222. The amount paid which exceeds the higher Ordinary dues level and is purely voluntary may be treated as a charitable contribution.

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One family member pays ordinary dues, based on his or her income. Only this member receives the *Notices* and *Bulletin*.

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With annual professional income at or above $80,000 **$148** ☐

The other member pays ordinary dues, based on his or her income, minus $20. This member does not receive *Notices* or *Bulletin* but will be accorded all other privileges of membership.

With annual professional income below $80,000 **$91** ☐

With annual professional income at or above $80,000 **$128** ☐

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To be eligible for this classification, members must belong to one of those foreign societies with which the AMS has established a reciprocity agreement (see reverse). Annual verification is required.

Reciprocity members who reside outside the U.S. **$74** ☐

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With annual professional income at or above $80,000 **$148** ☐

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M04NO

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See `http://www.ams.org/meetings/`. Programs and abstracts will continue to be displayed on the AMS website in the Meetings and Conferences section until about three weeks after the meeting is over. Final programs for Sectional Meetings will be archived on the AMS website in an electronic issue of the *Notices* as noted below for each meeting.

Tallahassee, Florida

Florida State University

March 12–13, 2004

Friday – Saturday

Meeting #994

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: January 2004
Program first available on AMS website: January 29, 2004
Program issue of electronic *Notices*: March 2004
Issue of *Abstracts*: Volume 25, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Fern Y. Hunt, National Institute of Standards, *Markov decision processes and a potential application to biological sequence alignment.*

William H. Jaco, Oklahoma State University, *On the homeomorphism problem.*

Yair Minsky, Yale University, *Hyperbolic manifolds and their deformations.*

Glenn F. Webb, Vanderbilt University, *Logistic nonlinearities in population dynamics.*

Special Sessions

Algebraic Geometry and Topology, **Eriko Hironaka**, **Paolo Aluffi**, and **Ettore Aldrovandi**, Florida State University.

Applications of Mathematics to Problems in Biology, **Richard Bertram** and **Jack Quine**, Florida State University.

Financial Mathematics, **Alec N. Kercheval**, **Warren D. Nichols**, and **Craig A. Nolder**, Florida State University.

Geometric Topology in Honor of John Bryant, **Washington Mio**, Florida State University, and **Erik K. Pedersen**, Binghamton University (SUNY).

Harmonic Analysis, **Daniel M. Oberlin**, Florida State University, and **Laura de Carli**, Florida International University.

Knot Theory and Applications, **Yuanan Diao**, University of North Carolina at Charlotte.

Modeling and Simulation of Complex Fluid Systems, **Qi Wang** and **Mark Sussman**, Florida State University, and **Xiaoming Wang**, Iowa State University.

PDE's and Turbulence, **Xiaoming Wang**, Iowa State University.

Results in 3-Manifolds and Related Topics, **Wolfgang H. Heil** and **Sergio R. Fenley**, Florida State University.

Robert Gilmer and Joe Mott: Forty Years of Commutative Ring Theory at Florida State University, **William J. Heinzer**, Purdue University, and **James W. Brewer**, Florida Atlantic University.

Syzygies and Hilbert Functions, **Irena Peeva** and **Christopher A. Francisco**, Cornell University.

Athens, Ohio

Ohio University

March 26–27, 2004

Friday – Saturday

Meeting #995

Central Section
Associate secretary: Susan J. Friedlander

Announcement issue of *Notices*: January 2004
Program first available on AMS website: February 12, 2004
Program issue of electronic *Notices*: March 2004
Issue of *Abstracts*: Volume 25, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Mario Bonk, University of Michigan, *Title to be announced.*

Irene M. Gamba, University of Texas at Austin, *Title to be announced.*

Rostislav I. Grigorchuk, Texas A&M University, *Title to be announced.*

Eric G. Zaslow, Northwestern University, *Title to be announced.*

Special Sessions

Algebraic Coding Theory, **Marcus Greferath**, San Diego State University, and **Sergio R. López-Permouth**, Ohio University.

Differential Equations and Control Theory, **Sergiu Aizicovici** and **Nicolai Pavel**, Ohio University.

Discrete Structures and Complex Dynamics, **Mario Bonk** and **Lukas Geyer**, University of Michigan.

Dynamical Systems, **Patrick D. McSwiggen**, University of Cincinnati, and **Todd Young**, Ohio University.

Fast Algorithms in Numerical Analysis, **George Fann**, Oak Ridge National Laboratory, and **Marin J. Mohlenkamp**, Ohio University.

Groups, Representations, and Characters, **Mark Lewis**, Kent State University, and **Thomas R. Wolf**, Ohio University.

Integrable Systems in Mathematics and Physics, **Michael Gekhtman**, University of Notre Dame, and **Luen Chau Li**, Pennsylvania State University.

Linear Algebra and Its Applications, **S. K. Jain**, Ohio University, and **Michael Neumann**, University of Connecticut.

Probabilistic and Asymptotic Aspects of Group Theory, **Rostislav Grigorchuk**, Texas A&M University, **Mark Sapir**, Vanderbilt University, and **Zoran Sunik**, Texas A&M University.

Recent Trends in Infinite-Dimensional Banach Space Theory, **Beata Randrianantoanina** and **Narcisse Randrianantoanina**, Miami University.

Statistics and Probability, **Maria Rizzo** and **Vladimir Vinogradov**, Ohio University.

Theory of Rings and Modules, **Nguyen Viet Dung**, **Franco Guerriero**, **Dinh Van Huynh**, and **Pramod Kanwar**, Ohio University.

Wavelets, Other Multiscale Methods and Their Applications, **En-Bing Lin**, University of Toledo, and **Xiaoping Annie Shen**, Ohio University.

Other Activities

TA Development Using Case Studies: A Workshop for Faculty, organized by Diane Herrmann. Diane Herrmann, University of Chicago, will guide workshop participants in the effective use of the case studies method as a tool in preparing teaching assistants for their important role as classroom instructors. The faculty edition of the publication *Teaching Mathematics in Colleges and Universities: Case Studies for Today's Classroom*, from the CBMS series Issues in Mathematics Education, will be provided to workshop participants at no charge, compliments of the AMS. The workshop will be held on Saturday, March 27. Time and location information is available at `http://www.ams.org/amsmtgs/2090_other.html`.

Travel support awards of $200 per department are available for up to ten departments. Travel support is provided by the Calculus Consortium for Higher Education. To be eligible for this support, faculty representatives must have their department chair send a letter to the AMS stating the department's serious intent to use the case studies materials in their TA training or development program.

Please contact the Membership and Programs Department at the AMS (email: `khb@ams.org` or phone: 401-455-4060) to sign up for the workshop and to request consideration for travel support. For questions about the workshop program, please contact Professor Herrmann (email: `diane@math.uchicago.edu` or phone: 773-702-7332).

Los Angeles, California

University of Southern California

April 3–4, 2004

Saturday – Sunday

Meeting #996

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2004
Program first available on AMS website: February 19, 2004
Program issue of electronic *Notices*: April 2004
Issue of *Abstracts*: Volume 25, Issue 2

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

Invited Addresses

Dan Boneh, Stanford University, *Title to be announced.*

Maria E. Schonbek, University of California Santa Cruz, *Fluid equations and their asymptotic behavior.*

Paul Smith, University of Washington, *Noncommutative algebraic geometry.*

Christopher Martin Thiele, University of California Los Angeles, *Title to be announced.*

Special Sessions

Arithmetic Geometry and K-Theory, **Thomas Geisser** and **Wayne Raskind**, University of Southern California.

Complex and Hyperbolic Geometry, **Francis Bonahon** and **Dragomir Saric**, University of Southern California.

Contact and Symplectic Geometry, **Dragomir Dragnev**, **Ko Honda**, and **Sang Seon Kim**, University of Southern California.

Dynamic Equations on Time Scales: Theory and Applications, **John M. Davis** and **Johnny Henderson**, Baylor University, and **Qin Sheng**, University of Dayton.

Financial Mathematics, **Jaksa Cvitanic** and **Janfeng Zhang**, University of Southern California.

Fluid Problems and Related Questions, **Maria Schonbek**, University of California Santa Cruz, and **Yuxi Zheng**, Pennsylvania State University.

Modern Problems of Integration: Theory and Applications, **Mark Burgin**, University of California Los Angeles.

Noncommutative Algebra and Algebraic Geometry, **Lance W. Small**, University of California San Diego, and **Paul Smith**, University of Washington.

Nonlinear and Harmonic Analysis, **Rowan Killip** and **Christopher Thiele**, University of California Los Angeles.

Partial Differential Equations, **Igor Kukavica**, University of Southern California, and **Qi S. Zhang**, University of California Riverside.

Recent Advances in the Mathematical Analysis of Geophysical and Hydrodynamical Models, **Mohammed Ziane**, University of Southern California.

Smooth Ergodic Theory and Related Topics, **Nicolai Haydn**, University of Southern California, and **Huyi Hu**, Michigan State University.

Other Activities

TA Development Using Case Studies: A Workshop for Faculty, organized by Solomon Friedberg. Solomon Friedberg, Boston College, will guide workshop participants in the effective use of the case studies method as a tool in preparing teaching assistants for their important role as classroom instructors. The faculty edition of the publication *Teaching Mathematics in Colleges and Universities: Case Studies for Today's Classroom*, from the CBMS series Issues in Mathematics Education, will be provided to workshop participants at no charge, compliments of the AMS. The workshop will be held on Sunday, April 4. Time and location information is available at `http://www.ams.org/amsmtgs/2106_other.html`.

Travel support awards of $200 per department are available for up to ten departments. Travel support is provided by the Calculus Consortium for Higher Education. To be eligible for this support, faculty representatives must have their department chair send a letter to the AMS stating the department's serious intent to use the case studies materials in their TA training or development program.

Please contact the Membership and Programs Department at the AMS (email: `khb@ams.org` or phone: 401-455-4060) to sign up for the workshop and to request consideration for travel support. For questions about the workshop program, please contact Professor Freidberg (email: `friedber@bc.edu` or phone: 617-552-3002).

Lawrenceville, New Jersey

Rider University

April 17–18, 2004

Saturday – Sunday

Meeting #997

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: February 2004
Program first available on AMS website: March 4, 2004
Program issue of electronic *Notices*: April 2004
Issue of *Abstracts*: Volume 25, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 24, 2004

Invited Addresses

Sylvia Serfaty, New York University-Courant Institute, *Analysis of vortices in the magnetic Ginzburg-Landau model.*

Dennis P. Sullivan, City College (CUNY), *Title to be announced.*

Wim F. Sweldens, Bell Laboratories, *Title to be announced.*

Gaoyong Zhang, Polytechnic University, *Affine isoperimetric inequalities.*

Special Sessions

Algebraic Geometry and Mirror Symmetry (Code: SS 4A), **Ciprian Borcea**, Rider University.

Analytic Convex Geometry (Code: SS 15A), **Alina Stancu**, Polytechnic University, and **Elisabeth Werner**, Case Western Reserve University.

Automorphic Forms and Analytic Number Theory (Code: SS 1A), **Stephen Miller**, Rutgers University, and **Ramin Takloo-Bighash**, Princeton University.

Commutative Algebra and Algebraic Geometry (Code: SS 14A), **Alberto Corso**, University of Kentucky, **Claudia Polini**, University of Notre Dame, and **Wolmer V. Vasconcelos**, Rutgers University.

Convergence of Riemannian Manifolds (Code: SS 12A), **Christina Sormani**, Herbert H. Lehman College (CUNY), **Xiaochun Rong**, Rutgers State University, and **Guofang Wei**, University of California Santa Barbara.

Convexity and Combinatorics (Code: SS 18A), **James F. Lawrence** and **Valeriu Soltan**, George Mason University.

CR Geometry and Singularities (Code: SS 10A), **Joseph J. Kohn**, Princeton University, **John P. D'Angelo**, University of Illinois, **Xiaojun Huang**, Rutgers University, and **Andreea Nicoara**, Harvard University.

Elliptic Surfaces and Elliptic Fibrations (Code: SS 9A), **William L. Hoyt**, Rutgers University, **Joseph H. Silverman**, Brown University, and **Charles F. Schwartz**, Rider University.

Geometry and Arithmetic of Lattices (Code: SS 13A), **John H. Conway**, Princeton University, and **Derek A. Smith**, Lafayette College.

Geometry of Protein Modelling (Code: SS 5A), **Ileana Streinu**, Smith College, and **Jack Snoeyink**, University of North Carolina at Chapel Hill.

Group Cohomology and Related Topics (in Honor of William Browder's 70th Birthday) (Code: SS 16A), **Alejandro Adem**, University of Wisconsin, and **Jonathan Pakianathan**, University of Rochester.

Homotopical Physics (Code: SS 7A), **James Stasheff**, University of North Carolina, **Thomas J. Lada**, North Carolina State University, and **Alexander A. Voronov**, University of Minnesota.

Homotopy Theory (in Honor of William Browder's 70th Birthday) (Code: SS 3A), **Martin Bendersky**, Hunter College, and **Donald Davis**, Lehigh University.

Strings and Branes (Code: SS 6A), **Thomas P. Branson**, University of Iowa, and **S. James Gates**, University of Maryland.

Surgery (in Honor of William Browder's 70th Birthday) (Code: SS 8A), **Frank S. Quinn**, Virginia Polytechnic Institute & State University.

Theory of Initial- and Boundary-Value Problems for Some Nonlinear Partial Differential Equations (Code: SS 17A), **Laihan Luo**, Richard Stockton College of New Jersey.

Tomography and Integral Geometry (Code: SS 2A), **Andrew Markoe**, Rider University, and **Eric Todd Quinto**, Tufts University.

Variational Methods in Classical Mechanics (Code: SS 11A), **John N. Mather**, Princeton University, and **Vadim Y. Kaloshin**, Institute for Advanced Study.

Houston, Texas

Hyatt Regency Houston

May 13–15, 2004

Thursday – Saturday

Meeting #998

Sixth International Joint Meeting of the AMS and the Sociedad Matemática Mexicana (SMM).
Associate secretary: John L. Bryant
Announcement issue of *Notices*: February 2004
Program first available on AMS website: March 11, 2004
Program issue of electronic *Notices*: April 2004
Issue of *Abstracts*: Volume 25, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: March 2, 2004

Invited Addresses

Luchezar Avramov, University of Nebraska, *Homomorphisms of commutative rings.*

Persi W. Diaconis, Stanford University, *Title to be announced.*

Samuel Gitler, CINVESTAV, *Title to be announced.*

Adolfo Sanchez-Valenzuela, Centro de Investigacion en Matematicas, *Title to be announced.*

Jose Seade-Kuri, UNAM, *Title to be announced.*

Bernd Sturmfels, University of California Berkeley, *Tropical geometry* (Erdős Memorial Lecture).

Special Sessions

Algebraic Geometry (Code: SS 19A), **Pedro Luis Del Angel R.**, CIMAT; **Emma Previato**, Boston University; and **Frank Sottile**, University of Massachusetts, Amherst.

Algebraic Topology (Code: SS 11A), **Miguel A. Xicotencatl**, CINVESTAV, and **Frederick R. Cohen**, University of Rochester.

Associative Rings (Code: SS 5A), **Jose Rios Montes**, UNAM, **Maria-Jose Arroyo**, UAM-Iztapalapa, and **Sergio R. Lopez-Permouth**, Ohio University.

Coding Theory and Cryptography (Code: SS 17A), **Horacio Tapia-Recillas**, UAM-Iztapalapa, and **Neal I. Koblitz**, University of Washington.

Complex Analysis and Operator Theory (Code: SS 10A), **Enrique Ramirez de Arellano**, CINVESTAV, **John E. Fornaess**, University of Michigan, Ann Arbor, and **Norberto Salinas**, University of Kansas.

Continua Theory and General Topology (Code: SS 6A), **Janusz J. Charatonik**, UNAM, **Charles L. Hagopian**, California State University, Sacramento, and **Sergio Macias**, UNAM.

Continuous Distributed Parameters Models in Mathematical Biology (Code: SS 13A), **William E. Fitzgibbon**, University of Houston, and **Jorge X. Velasco Hernandez**, Instituto Mexicana del Petroleo.

Curvature and Geodesics (Code: SS 23A), **David D. W. Bao**, University of Houston, and **Lilia Del Riego**, Universidad Autònoma San Luis Potosi.

Designing Frames and Wavelets: From Theory to Digitization (Code: SS 18A), **Peter R. Massopust**, Tuboscope Pipeline Services, and **Manos I. Papadakis**, University of Houston.

Differential Geometry (Code: SS 20A), **Raul Quiroga Barranco**, CINVESTAV, and **Alberto Candel**, California State University, Northridge.

Dynamical Systems (Code: SS 8A), **Renato Iturriaga**, CIMAT, and **Rafael de la Llave**, University of Texas at Austin.

Geometric Variational Problems (Code: SS 15A), **Clara Garza-Hume**, IIMAS, UNAM, and **Michael Wolf** and **Robert M. Hardt**, Rice University.

Graph Theory and Combinatorics (Code: SS 1A), **Gelacio Salazar**, IICO, UASLP, **Isidoro Gitler**, CINVESTAV, and **Nathaniel Dean**, Texas Southern University.

Harmonic and Functional Analysis (Code: SS 3A), **Salvador Perez-Esteva**, UNAM-Cuernavaca, **Carlos Bosh-Giral**, ITAM, and **Josefina Alvarez**, University of New Mexico.

Homological Algebra of Commutative Rings (Code: SS 21A), **Srikanth Iyengar**, University of Nebraska, and **Graham J. Leuschke**, University of Toronto.

Low Dimensional Topology (Code: SS 7A), **Victor Nuñez**, CIMAT, and **Luis G. Valdez**, University of Texas, El Paso.

Mathematical Physics (Code: SS 16A), **Carlos Villegas-Blas**, UNAM, and **Alejandro Uribe**, University of Michigan, Ann Arbor.

Mathematical Problems in the Analysis of Synchronous States in Networks (Code: SS 22A), **Kresimir Josic**, University of Houston, and **Valentin Afraimovich**, IICO-UASLP.

Nonlinear Analysis (Code: SS 4A), **Monica Clapp**, UNAM, and **Alfonso Castro**, University of Texas at San Antonio.

Problems and Issues in Electronic Publishing (Code: SS 12A), **Klaus Kaiser**, University of Houston, **Bernd Wegner**, Technische Universität Berlin, and **Enrique Ramirez de Arellano**, CINVESTAV.

Representations of Algebras (Code: SS 2A), **Rita Esther Zuazua Vega**, UNAM, and **Gordana G. Todorov**, Northeastern University.

Space and Time Decomposition Methods in Computational and Applied Mathematics (Code: SS 14A), **Roland Glowinski** and **Tsorng-Whay Pan**, University of Houston, and **L. Hector Juarez V.**, UAM-Iztapalapa.

Stochastical Processes and Probability (Code: SS 9A), **Daniel Hernandez Hernandez**, CIMAT, and **Christian Houdre**, Georgia Institute of Technology.

Nashville, Tennessee

Vanderbilt University

October 16–17, 2004

Saturday – Sunday

Meeting #999

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: August 2004
Program first available on AMS website: September 2, 2004
Program issue of electronic *Notices*: October 2004
Issue of *Abstracts*: Volume 25, Issue 4

Deadlines

For organizers: March 16, 2004
For consideration of contributed papers in Special Sessions: June 29, 2004
For abstracts: August 24, 2004

Albuquerque, New Mexico

University of New Mexico

October 16–17, 2004

Saturday – Sunday

Meeting #1000

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: August 2004
Program first available on AMS website: September 3, 2004
Program issue of electronic *Notices*: October 2004
Issue of *Abstracts*: Volume 25, Issue 4

Deadlines

For organizers: March 16, 2004
For consideration of contributed papers in Special Sessions: June 29, 2004
For abstracts: August 24, 2004

Invited Addresses

Sara C. Billey, University of Washington, Seattle, *Title to be announced.*

Peter Ebenfelt, University of California San Diego, *Title to be announced.*

Theodore Stanford, New Mexico State University, *Title to be announced.*

Craig A. Tracy, University of California Davis, *Title to be announced.*

Special Sessions

Algebraic Geometry (Code: SS 3A), **Hirotachi Abo** and **Chris Peterson**, Colorado State University.

Categories and Operads in Topology, Geometry, Physics and Other Applications (Code: SS 5A), **Hanna Ewa Makaruk** and **Robert Michal Owczarek**, Los Alamos National Laboratory, and **Zbigniew Oziewicz**, Universidad Nacional Autónoma de México.

Financial Mathematics: The Mathematics of Derivative Securities (Code: SS 4A), **Maria Cristina Mariani**, New Mexico State University, and **Osvaldo Mendez**, University of Texas at El Paso.

Random Matrix Theory and Growth Processes (Code: SS 1A), **Craig A. Tracy**, University of California Davis.

Several Complex Variables and CR Geometry (Code: SS 2A), **Peter Ebenfelt**, University of California San Diego, and **Marshall A. Whittlesey**, California State University, San Marcos.

Evanston, Illinois

Northwestern University

October 23–24, 2004

Saturday – Sunday

Meeting #1001

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2004
Program first available on AMS website: September 9, 2004
Program issue of electronic *Notices*: October 2004
Issue of *Abstracts*: Volume 25, Issue 4

Deadlines

For organizers: March 23, 2004
For consideration of contributed papers in Special Sessions: July 7, 2004
For abstracts: August 31, 2004

Invited Addresses

Ian Agol, University of Illinois at Chicago, *Title to be announced.*

Robert W. Ghrist, University of Illinois, *Title to be announced.*

Yuri Manin, Northwestern University, *Title to be announced.*

Paul Seidel, Imperial College-London and University of Chicago, *Title to be announced.*

Special Sessions

Algebraic Topology: Interactions with Representation Theory and Algebraic Geometry (Code: SS 13A), **Paul G. Goerss**, Northwestern University, and **Jesper Kragh Grodal**, University of Chicago.

Codes and Applications (Code: SS 5A), **William C. Huffman**, Loyola University of Chicago, and **Vera S. Pless**, University of Illinois at Chicago.

Computability Theory and Applications (Code: SS 8A), **Robert I. Soare** and **Denis R. Hirschfeldt**, University of Chicago.

Differential Geometry (Code: SS 10A), **Anders Ingemar Linner** and **Hongyou Wu**, Northern Illinois University.

Extremal Combinatorics (Code: SS 2A), **Dhruv Mubayi** and **Yi Zhao**, University of Illinois at Chicago.

Fluid Dynamics, Diffusion and Reaction (Code: SS 4A), **Peter S. Constantin** and **Leonid V. Ryzhik**, University of Chicago.

Geometric Partial Differential Equations (Code: SS 7A), **Gui-Qiang Chen** and **Jared Wunsch**, Northwestern University.

Index Theory, Morse Theory, and the Witten Deformation Method (Code: SS 3A), **Igor Prokhorenkov** and **Ken Richardson**, Texas Christian University.

Iterated Function Systems and Analysis on Fractals (Code: SS 12A), **Ka-Sing Lau**, Chinese University of Hong Kong, and **Stephen S.-T. Yau**, University of Illinois at Chicago.

Modern Schubert Calculus (Code: SS 1A), **Ezra Miller**, University of Minnesota, and **Frank Sottile**, University of Massachusetts.

Nonlinear Partial Differential Equations and Applications (Code: SS 6A), **Gui-Qiang Chen**, Northwestern University, and **Mikhail Feldman**, University of Wisconsin at Madison.

Solving Polynomial Systems (Code: SS 9A), **Anton Leykin** and **Jan Verschelde**, University of Illinois at Chicago.

Stability Issues in Fluid Dynamics (Code: SS 11A), **Susan J. Friedlander** and **Roman Shvydkoy**, University of Illinois at Chicago.

Pittsburgh, Pennsylvania

University of Pittsburgh

November 6–7, 2004

Saturday – Sunday

Meeting #1002

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: September 2004
Program first available on AMS website: September 23, 2004
Program issue of electronic *Notices*: November 2004
Issue of *Abstracts*: Volume 25, Issue 4

Deadlines

For organizers: April 7, 2004
For consideration of contributed papers in Special Sessions: July 20, 2004
For abstracts: September 14, 2004

Invited Addresses

Jeffrey F. Brock, Brown University, *Title to be announced.*

Der-Chen Chang, Georgetown University, *Title to be announced.*

Robert Schapire, Princeton University, *Title to be announced.*

Ofer Zeitouni, University of Minnesota, Minneapolis, *Title to be announced.*

Special Sessions

Convexity and Combinatorics (Code: SS 2A), **James F. Lawrence** and **Valeriu Soltan**, George Mason University.

Invariants of Knots and 3-Manifolds (Code: SS 1A), **Marta M. Asaeda**, University of Maryland, **Jozef H. Przytycki**, George Washington University, and **Adam S. Sikora**, SUNY at Buffalo.

Atlanta, Georgia

Atlanta Marriott Marquis and Hyatt Regency Atlanta

January 5–8, 2005

Wednesday – Saturday

Joint Mathematics Meetings, including the 111th Annual Meeting of the AMS, 88th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association of Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL).

Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: October 2004
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2005
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 5, 2004
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Request for Proposals from AMS Special Session Organizers

Lesley M. Sibner, Associate Secretary responsible for the AMS program in Atlanta, solicits proposals for Special Sessions for this meeting. Each proposal must include the full names, email addresses, and institutions of all members of the organizing committee, and identify the one organizer who will serve as contact for all communications about the session; the title and a brief (two or three paragraphs) description of the proposed session; and a *sample* list of speakers whom the proposed organizers plan to invite (please note that it is not at all necessary to have confirmed commitments from these speakers).

It is expected that each Special Session will be allotted 10 hours over two days of the meeting in which to schedule speakers. In order to allow the maximum movement between sessions for all participants, Special Session speakers will be scheduled for either a 20-minute talk, 5-minute discussion, and 5-minute break; or a 40-minute talk, 10-minute discussion, and 10-minute break. All talks must begin on the hour or half-hour. Any combination of 20-minute and 40-minute talks is allowed, provided the schedule conforms to beginning on the hour and half-hour (except on the first afternoon, for technical reasons).

Proposals of AMS Special Sessions must be received by the deadline for organizers, **April 5, 2004**, and submitted (by email) to the AMS Associate Secretary, Lesley M. Sibner (`lsibner@duke.poly.edu`). Late proposals will not be considered.

There is limited space available for Special Sessions on the AMS program, so it is likely that not all proposals will be accepted. Please be sure to submit as detailed a proposal as possible for review by the Program Committee. All proposed organizers will be notified of acceptance or rejection no later than May 1, 2004.

Bowling Green, Kentucky

Western Kentucky University

March 18–19, 2005

Friday – Saturday

Meeting #1003

Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: July 19, 2004
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Newark, Delaware

University of Delaware

April 2–3, 2005

Saturday – Sunday

Meeting #1004

Eastern Section
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 2, 2004
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lubbock, Texas

Texas Tech University

April 8–10, 2005

Friday – Sunday

Meeting #1005

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Santa Barbara, California

University of California Santa Barbara

April 16–17, 2005

Saturday – Sunday

Meeting #1006

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Mainz, Germany

June 16–19, 2005

Thursday – Sunday

Meeting #1007

Second Joint International Meeting with the Deutsche Mathematiker-Vereinigung (DMV) and the Oesterreichische Mathematische Gesellschaft (OMG)
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Invited Addresses

Helene Esnault, University of Essen, *Title to be announced.*

Richard Hamilton, Columbia University, *Title to be announced.*

Michael J. Hopkins, Massachusetts Institute of Technology, *Title to be announced.*

Christian Krattenthaler, University of Lyon, *Title to be announced.*

Frank Natterer, University of Muenster, *Title to be announced.*

Horng-Tzer Yau, New York University and Stanford University, *Title to be announced.*

Special Sessions

Algebraic Geometry, **Yuri Tschinkel**, Georg-August-Universität Göttingen, and **Brendan E. Hassett**, Rice University.

Functional Analytic and Complex Analytic Methods in Linear Partial Differential Equations, **R. Meise**, University of Dusseldorf, **B. A. Taylor**, University of Michigan, and **Dietmar Vogt**, University of Wuppertal.

Mathematics Education, **Gunter Torner**, Universität Duisburg-Essen.

Nonlinear Waves, **Herbert Koch**, University of Dortmund, and **Daniel I. Tataru**, University of California Berkeley.

Stochastic Analysis on Metric Spaces, **Laurent Saloff-Coste**, Cornell University, **Karl-Theodor Sturm**, University of Bonn, and **Wolfgang Woess**, Graz Technical University.

Johnson City, Tennessee

East Tennessee State University

October 15–16, 2005

Saturday – Sunday
Southeastern Section
Associate secretary: John L. Bryant
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 15, 2005
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Lincoln, Nebraska

University of Nebraska in Lincoln

October 21–22, 2005

Friday – Saturday
Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2005
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center

January 12–15, 2006

Thursday – Sunday
Joint Mathematics Meetings, including the 112th Annual Meeting of the AMS, 89th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
Associate secretary: John L. Bryant
Announcement issue of *Notices*: October 2005
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2006
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 12, 2005
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 4–7, 2007

Thursday – Sunday
Joint Mathematics Meetings, including the 113th Annual Meeting of the AMS, 90th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: October 2006
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2007
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 4, 2006
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

San Diego, California

San Diego Convention Center

January 6–9, 2008

Sunday – Wednesday
Joint Mathematics Meetings, including the 114th Annual Meeting of the AMS, 91st Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Associaiton for Symbolic Logic (ASL).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2007
Program first available on AMS website: November 1, 2007
Program issue of electronic *Notices*: January 2008
Issue of *Abstracts*: Volume 29, Issue 1

Deadlines

For organizers: April 6, 2007

For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Washington, District of Columbia

Marriott Wardman Park Hotel and Omni Shoreham Hotel

January 7–10, 2009

Wednesday – Saturday
Joint Mathematics Meetings, including the 115th Annual Meeting of the AMS, 92nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL).
Associate secretary: Lesley M. Sibner
Announcement issue of *Notices*: October 2008
Program first available on AMS website: November 1, 2008
Program issue of electronic *Notices*: January 2009
Issue of *Abstracts*: Volume 30, Issue 1

Deadlines

For organizers: April 7, 2008
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
For summaries of papers to MAA organizers: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Sproul Hall, Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 909-787-3113.

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The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/.

Meetings:

Important Information regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 84 in the January 2004 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Several options are available for speakers submitting abstracts, including an easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. To see descriptions of the forms available, visit http://www.ams.org/abstracts/instructions.html, or send mail to abs-submit@ams.org, typing help as the subject line; descriptions and instructions on how to get the template of your choice will be e-mailed to you.

Completed email abstracts should be sent to abs-submit@ams.org, typing submission as the subject line. Questions about abstracts may be sent to abs-info@ams.org.

Paper abstract forms may be sent to Meetings & Conferences Department, AMS, P.O. Box 6887, Providence, RI 02940. There is a $20 processing fee for each paper abstract. There is no charge for electronic abstracts. Note that all abstract deadlines are strictly enforced.

Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (See http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

June 5–July 24, 2004: Joint Summer Research Conferences in the Mathematical Sciences, Snowbird, Utah. (See November 2003 *Notices*, page 1363.)

MATHEMATICAL MOMENTS

The ***Mathematical Moments*** program is a series of illustrated "snapshots" designed to promote appreciation and understanding of the role mathematics plays in science, nature, technology, and human culture.

Download these and other ***Mathematical Moments*** pdf files at www.ams.org/mathmoments.

MATHEMATICAL MOMENTS

Folding for Fun and Function

Origami—paper-folding—may not seem like a subject for mathematical investigation or one with sophisticated applications, yet anyone who has tried to fold a road map or wrap a present knows that origami is no trivial matter. Mathematicians, computer scientists, and engineers have recently discovered that this centuries-old subject can be used to solve many modern problems. The methods of origami are now used to fold objects such as automobile air bags and huge space telescopes efficiently, and may be related to how proteins fold.

Model designed by Thomas Hull (Merrimack College) and Francis Ow, folded by Pipajoe (Joe Gilardi)

AMS American Mathematical Society

*The **Mathematical Moments** program promotes appreciation and understanding of the role mathematics plays in science, nature, technology, and human culture.*

MM/22.5

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- Getting Results on the Web
- Designing Aircraft
- Deciphering DNA
- Eye-dentifying Yourself
- Making Connections
- Enhancing Your Image
- Simulating Galaxies
- Revealing Nature's Secrets
- Securing Internet Communication
- Making Movies Come Alive
- Listening to Music
- Making Votes Count
- Tracking Products
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