Racial Equity Requires Teaching Elementary School Teachers More Mathematics

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I strongly believe that the most crucial step for promoting racial equality in this country is to educate all elementary teachers mathematically. This conviction began after a survey I did in the mid-1980s of black mathematicians in New Jersey. Seventy-five black people with at least one degree in mathematics responded to a variety of questions, including, “What can be done to bring more blacks into mathematics?”

The second most common answer to this question was, “Publicize role models.” I might have been planting that answer because I was clearly collecting role models. However, the most common answer (by far) I definitely did not plant; it came as a total surprise to me. It was, “Teach mathematics better to all American children. The way it is now, if children don’t learn mathematics at home, they don’t learn it at all, so any ethnic group that is underrepresented in mathematics will remain so until children are taught mathematics better in elementary school.”

That answer caused me to seek opportunities to work in elementary schools. Much of what follows will be evidence corroborating the statement that was so frequent among the black mathematicians of New Jersey—and the great need to teach mathematics to elementary school teachers. Like most Americans, I found it difficult to believe how poorly prepared mathematically they are. They are well chosen. They are kind, diligent, and smart, qualities that nobody can teach. They have been failed mathematically by our system. They need to be taught. I have found them eager and quick to learn—and appallingly ignorant of the most basic mathematics.

“Teach us math! Teach us math! Teach us math!” chanted dozens of elementary school teachers during one after-school workshop. There was an amazed silence while we all absorbed what had just happened. Then one of them said, “If you taught us math the way you did just now, we could teach it to the children.” They all nodded emphatically. This incident followed my statement that those of us who thrive mathematically have had some good mathematical experience early, typically at home. Someone had asked for an example out of my own childhood, and I had explained how my father had described the meaning of $\pi$ to me several months before I started kindergarten. Their response was the chanting, “Teach us math!”

One study of nine hundred Texas school districts revealed that the large disparities in achievement between black and white students were almost entirely accounted for by socioeconomic status and differences in the measurable qualifications of their teachers.1 It is no secret that minority students across the country have less mathematically educated teachers than whites. However, even in integrated districts, the lack of home preparation of

minority students means that they are more dependent on their teachers for their mathematical knowledge. It has been my observation that the reason that scores are higher in white districts is that some parents teach their children mathematics at home, and these children teach many of the others. It has appeared to me that the teachers are no better prepared in the high-scoring districts.

The teachers are eager and able to learn. I vividly remember one summer class when I taught why the multiplication algorithm works for two-digit numbers using base ten blocks. I have no difficulty doing this with third graders, but this particular class was all elementary school teachers. At the end of the half hour, one third-grade teacher raised her hand. “Why wasn’t I told this secret before?” she demanded. It was one of those rare speechless moments for Pat Kenschaft. In the quiet that ensued, the teacher stood up.

“Did you know this secret before?” she asked the person nearest her. She shook her head. “Did you know this secret before?” the inquirer persisted, walking around the class. “Did you know this secret before?” she kept asking. Everyone shook her or his head. She whirled around and looked at me with fury in her eyes. “Why wasn’t I taught this secret before? I’ve been teaching third grade for thirty years. If I had been taught this thirty years ago, I could have been such a better teacher!!!”

Indeed she could have been. The understanding of the area of a rectangle and its relationship to multiplication underlies an understanding not only of the multiplication algorithm but also of the commutative law of multiplication, the distributive law, and the many more complicated area formulas.

Yet in my first visit in 1986 to a K-6 elementary school, I discovered that not a single teacher knew how to find the area of a rectangle.

In those innocent days, I thought that the teachers might be interested in the geometric interpretation of \((x + y)^2\). I drew a square with \((x + y)\) on a side and showed the squares of size \(x^2\) and \(y^2\). Then I pointed to one of the remaining rectangles. “What is the area of a rectangle that is \(x\) high and \(y\) wide?” I asked.

There was no response, so I asked the question again. “What is the area of a rectangle that is \(x\) by \(y\)?”

The teachers were very friendly people, and they know how frustrating it can be when no student answers a question. “\(x\) plus \(y\)” said two in the front simultaneously.

“What?!!” I said, horrified.

Then all fifty of them shouted together, “\(x\) plus \(y\).” Apparently my nonverbal reaction had not been a sufficient clue that the original answer was wrong. How can children in such a school attain a profound understanding of fundamental mathematics? I am now convinced, after visiting many schools, that this one was not unusual. Perhaps it was above average in the enthusiasm of the teachers and their loyalty to the school.

Its principal invited me to consider that school “my school”. He and the teachers really wanted to help the students. Its students had a median achievement in mathematics of about the 25th percentile on the “Iowas”, one of the lowest levels in Newark. I am now convinced that its rank was due to the fact that the principal did not pressure the teachers to cheat in any way on standardized tests.

When I told him this years later, his eyes widened. He was president of the principals’ union. “What? You are saying…” I nodded. Since then I have read numerous reports of systemic cheating on standardized tests and other forms of deception by school administrators, most notably the recent articles in The New York Times about Houston, while Secretary of Education Roderick Paige was superintendent.

The following year Montclair State facilitated my going once a week after school to that school to talk mathematics with whomever showed up. At least one teacher always did show up, and sometimes six or seven. It’s not an effective way to make change, but we did get acquainted.

A year later I won one of the first K-3 grants from Exxon Education Foundation. This enabled me to spend twelve days on campus in the summer with five teachers from that school and to visit the school two mornings a week during the following school year. I spent those mornings teaching math to one to three first-grade classes, one to three third-grade classes, and one fifth-grade class.

During my first class teaching elementary school children, a fifth grader raised his hand and asked, “What is that word you keep using instead of take away?” Enter “minus”—for fifth graders!

The best first-grade teacher told me she never bothered to teach subtraction during the first half of the year because the children couldn’t learn everything at once. I started visiting the school in October, and it seemed to me natural to teach addition and subtraction together. She told me she would not reinforce my teaching of subtraction between my weekly visits, and I said that was no problem.

One of the games I played with the children was holding five unifix blocks in front of me, putting them behind my back, and bringing forward three.

“How many are behind my back?” I asked. The children could answer correctly. Then I told them that one way of writing this was “\(5 - 3 = 2\)”.

“Oh, no!” said the teacher.

“Why not?” I asked.

“Because subtraction means “take away” and you took away two blocks. So it should be written ‘\(5 - 2 = 3\)’.” I explained that subtraction could mean “take away”, but it could also mean “missing
addend”. It seemed to me that since the children could see three blocks, “5 - 3 = 2” was preferable, but “5 - 2 = 3” is not wrong. The next week we explored the “difference” meaning of subtraction and the “motion” meaning. (I walk five steps toward the window and three steps away. How many steps am I from where I began?)

She was startled when half the children passed the subtraction part of the November standardized test—without any reinforcement from her. She had never had a child pass it before. The crucial role of mathematical knowledge on the part of the teacher was becoming obvious to me.

The following year I led a team that won an Eisenhower grant and began working in an urban-suburban coalition, going to both all-black schools and all-white schools. My first time in a fifth grade in one of New Jersey’s most affluent districts (white, of course), I asked where one-third was on the number line. After a moment of quiet, the teacher called out, “Near three, isn’t it?” The children, however, soon figured out the correct answer; they came from homes where such things were discussed. Flitting back and forth from the richest to the poorest districts in the state convinced me that the mathematical knowledge of the teachers was pathetic in both. It appears that the higher scores in the affluent districts are not due to superior teaching in school but to the supplementary informal “home schooling” of children.

Tests encourage systemic cheating, but there is no way to deceive an educated observer about student and teacher enthusiasm. In the spring of my first year in the Newark school, Exxon sent Pat Hess to observe what was happening. The principal said, “We haven’t had enough standardized tests yet to be sure of a measurable difference, but I can assure you I hear far more conversation about mathematics than I ever did before. I hear the children talking about math in the hallway. When I walk into the teachers’ lunchroom, I hear them talking about math!”

The Eisenhower grant paid me to visit each school only once a month. During the spring of the third year that I had been visiting the original school, one of the third-grade teachers said at the beginning of the class, “Could we put aside the lesson you and I planned and just have you answer the questions of the children that I can’t answer?” Think about what that question indicates about her eagerness to learn and about our relationship—and then what it indicates about the need for third-grade teachers to learn more mathematics.

The children were all African American. The school is in one of the worst neighborhoods of our country’s poorest city. There were no greens growing within a block of the school except an occasional dandelion that would poke up between the sidewalk cracks. When the wind blew as I approached the school, I would feel the flying glass sting my legs. But the next hour was one of the most intellectually exciting of my life. I just answered one question after another and made sure that every child was following. An hour! With eight-year-olds! Totally focused.

At the end, the teacher, who had been on the edge of her seat the entire time, asked, “What do you call this kind of mathematics, Dr. Kenschaft?” Suddenly I began to meta-think—no longer focused on the here and now.

“This is the beginning lesson in calculus that I do with my college students!” I had considered the limit of 60/x as x goes to zero. The children had never heard of division before, but they learned it in that hour—all of them. I used six Cuisenaire ten-rods and asked first how many sixties there are in sixty, then how many thirties, then how many tens, and then how many ones. Then I told them about tenths. Then I asked how many tenths there are in sixty, telling them I didn’t want anyone to shout out. Slowly hands were raised—and they had the right answer. I had everyone whose hand had been raised explain the reason for the answer to the entire class, and then I asked how many hundredths there were in sixty. Of course, I had to explain what a hundredth was first. This time almost half the class raised their hands fairly quickly. The concept of infinity and how it might arise had been constructed in their minds, and they were excited.

Later that spring the Iowa scores were revealed for the three third grade-classes with whom I had been working intermittently for three years. Two classes had median scores at the 60th percentile, a great increase from the 25th percentile only three years earlier. The third class had a median at the 70th percentile, with only one child below the 50th and that child in the 40s. This dramatic increase in Iowa scores was accomplished by the same teachers and a mathematician with no elementary school background whatsoever. I did have high school certification; I had one year of high school teaching experience and the background of having raised two children of my own, but no official professional preparation except a doctorate in mathematics with a specialty in functional analysis. The teachers and I shared a concern for the students. They were good teachers, and I had access to the national materials of the late 1980s. We talked with each other. I certainly was not teaching “to” the lowas. I was trying to share my understanding of fundamental mathematics—and it seems that that was what was needed for the children to do well on the highly computational old-fashioned standardized tests.

A couple of years later I was in another city in another all-black class that was much more unruly. The teacher continually complained to me aloud about the children’s misbehavior. They too were
thinking out of bounds. I watched her try to squelch the children’s impudent mathematical questions, and then hesitantly asked if I could try to answer them. I told her it would take some time, and she agreed. I went through the same explanation of the limit of \(60/x\) as \(x\) goes to zero, not sure if this class could pay attention well enough. But they did. The teacher was amazed not just that they all seemed to understand division lickety-split, but that they paid attention to this obviously very difficult topic.

How much are our social problems due to our not challenging children enough? Life can be boring if you just tread water intellectually. How much do humans need intellectual challenge? How much would providing an excellent mathematics education for our elementary school teachers help mitigate our drug and crime problems?

A couple of years ago I discovered that the problems are even more basic than I had realized earlier; teachers’ understanding of addition is murky. Montclair State certifies teachers without providing a special course in either mathematics or mathematics education for them, so they are scattered in our general education courses. I had one pleasant, diligent young woman in such a class who intended to be an elementary school teacher. On the last day of her formal mathematics education she responded to my offer to answer questions before the exam by saying there was something wrong with exercise 11 on page 69 of the text (my book Mathematics for Human Survival).

“In 1999 U.S. cars achieved an average of 28.11 mpg, but light trucks were rated a mere 20.3 mpg. Their mileage was 23.8 mpg altogether. What proportion of American vehicles were light trucks in 1999?”

“What’s wrong?” I asked.

“Altogether means add, so the mileage altogether must be 48.41 miles per gallon.” I tried to explain but to no avail. Some of the other students gave fine explanations. She is a cooperative person and realized she was outvoted, but it was clear she did not understand.

One of the other students noticed my frustration and anger—not at her, but at a system that will send people so poorly prepared into the elementary school classroom. “You know, Dr. Kenschaft. Key words. We’ve all been taught that ‘altogether’ means add.” The rest of the class nodded as I sighed.

“And ‘left’ always means subtract,” said another with a wry smile.

The student who will be an elementary school teacher earned a legitimate “B” in that course. Without a course about the fundamentals of elementary mathematics, she is woefully unprepared to face young exploring mathematical minds such as I enjoyed so much in the poorest city in our country. I wonder if she realized when she was eight that the average height of the children in her class altogether was between the average height of the boys and the average height of the girls. Was that understanding taught out of her by teachers who insist upon memorization because they themselves don’t understand? Will she, kindly, well-meaning person that she is, do the same to classes of innocent children over her lifetime?

My own interest in elementary school mathematics education grew out of my equity concerns. Ever since my great-great-grandfather came north from a slave-holding family to fight on the Northern side of the Civil War, my family has been active in race relations. The men have encouraged the women to be “real people,” and the women have tried to live up to the advantages we were given.

My survey of black mathematicians in New Jersey, like my earlier survey of black women with doctorates in mathematics, was done by networking. I started with some former Montclair State students, and each time I reached another black mathematician, I asked for names of others. With only two (nonconsecutive) semesters of released time, while teaching three classes during those semesters, I located one hundred fifty black mathematicians in New Jersey. My subjects were surprised there were so many by the time I told them I had located thirty, but at the rate I was going when I ran out of time, I suspect there were three hundred. In my limited time I was able to get responses from only seventy-five—twenty-six written responses to my mailed inquiry, and forty-nine successful follow-up phone calls. I will be glad to send a packet of my writing about blacks in mathematics and/or women in mathematics to anyone who requests them by emailing me at kenschaft@pegasus.montclair.edu. An outside indicator that others appreciate my concern with minorities’ participation in mathematics is indicated by the fact that in 2003 I was chosen to lead the Task Force on Equity and Diversity Integration of the National Council of Teachers of Mathematics, an organization of about 100,000 members.

I support improving pedagogy and helping minorities feel better about themselves. Indeed, my avid collecting of role models has been a major factor in providing young African Americans a mathematical heritage. I strongly approve of programs that involve families. However, none of these activities will begin to close the racial mathematics achievement gap until American elementary school students enjoy so much in the poorest city in our country.

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teachers know mathematics much better than they do now.

What can mathematicians do about this situation? There are many possibilities, but they seem to fall into three categories:

1. **Structural Change**: The mathematical communities need to collaborate with anyone else who will join the effort to lobby strenuously for the need for radically improved teacher knowledge. The major argument is that while once only a few people (white men?) needed to know mathematics, now a large segment (a majority) of the population need to know significant mathematics for career, citizenship, and personal reasons, and it is exceedingly wasteful to have a primary-level teaching corps with such limited knowledge; remedial mathematics learning later is more difficult and, therefore, expensive. That the current situation is also unfair to minorities may have less political clout but should also be emphasized. This change of understanding on the part of decision-makers and the public will not be easy, especially since many harbor deep math anxiety due to their own poor education and are threatened by the thought that others might learn it easily—and/or are reluctant to “inflict” on innocent children the “burden” of learning mathematics well.

The AMS can play a leadership role, but it will need collaboration from all member organizations of the Joint Policy Board for Mathematics. The Institute of Electrical and Electronic Engineers (IEEE) has expressed similar concerns, and there are doubtless other technical societies that could be recruited. The mathematical education of elementary school teachers is basic to the health of all these disciplines, as well as the economic and political health of our country.

2. **Individual Actions**: Those who teach in institutions that certify elementary school teachers can work to make sure adequate specific courses are provided for them and volunteer to do a conscientious job when teaching such courses. Such teaching requires patience, and a determination to direct one’s anger at the system, not the victims of it. Teaching a “profound understanding of fundamental mathematics” is very different from teaching traditional collegiate mathematics, but for the next few decades, some mathematically knowledgeable people must do it if all university mathematicians are to be able to teach university-level mathematics some day.

What mathematics is appropriate to teach aspiring elementary school teachers? What math do they need to know and how do they use it? There already are some adequate programs available, but further insight and improvement is needed. Hyman Bass of the University of Michigan has been working with math educator Deborah Ball, also of Michigan, to investigate these questions.

Roger Howe of Yale University has been working with current math educators to help them clarify their own mathematical knowledge and extend that of their students. Jerome Dancis of the University of Maryland at College Park has been monitoring state tests and finding appalling errors in questions that reflect either lack of mathematical knowledge or careless proofreading among those who compose high-stake tests.

3. **Remedial Work**: Until the current cohort of elementary school teachers retire, the mathematical competence of today’s children will require that their teachers receive continual remedial programs. Hung-Hsi Wu of the University of California, Berkeley, has written about his leadership at Berkeley in summer programs jointly sponsored with a math educator. Paul Sally of the University of Chicago has done extensive work with both teachers and high school students evenings and weekends. He reports working fifty hours a week as a mathematician and another fifty as a mathematics educator. Most of us don’t have that level of energy and/or commitment, but some of us will be needed for teacher remediation until the system is healed.

The above is far from an exhaustive list of either people or activities; it merely indicates examples of good beginnings. Significant efforts at remediation for teachers have taken place throughout the country. However, remediation is far from enough. All aspiring elementary school teachers must be taught appropriate mathematics before they begin teaching children.

Children who have been mathematically abused are much less able to benefit from mathematically competent teachers when they finally reach them. One lesson our current elementary school teachers convey powerfully is that math is too difficult to understand. Because knowledge of mathematics correlates strongly with economic and political achievement, the mathematical education of all elementary school teachers is the paramount equity issue. As Will Rogers said long ago, “You can’t teach what you don’t know any more than you can come back from where you ain’t been.”

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4 This phrase was coined by Liping Ma in her important book, Knowing and Teaching Elementary Mathematics: Teachers’ Understanding of Fundamental Mathematics in China and the United States, Lawrence Erlbaum, Mahwah, NJ, 1999.