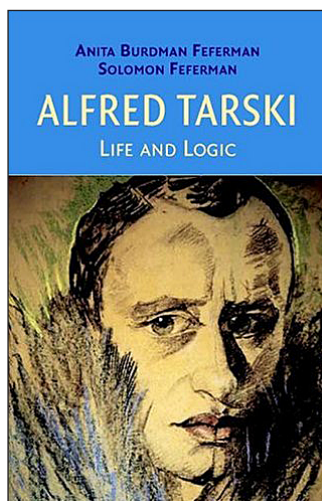


## Book Review



# Alfred Tarski. Life and Logic

*Reviewed by Hourya Benis Sinaceur*

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### **Alfred Tarski. Life and Logic**

*Anita Burdman Feferman and Solomon Feferman*

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Alfred Tarski (1901–1983) is one of the two greatest logicians of the twentieth century, the other being Kurt Gödel (1906–1978). Each began his career in Europe, respectively in Warsaw and Vienna, and came to America shortly before the Second World War. In contrast to the otherworldly Gödel, Tarski was ambitious and practical. He strove for, and succeeded at, building a school of logic at the University of California, Berkeley, that attracted students and distinguished researchers from all over the world.

Tarski was the leader of the “semantic turn” in mathematical logic. This means that he achieved a shift from a view focused on formal systems, axioms, and rules of deduction to a view focusing on the relations between formal systems and their possible interpretations by usual mathematical theories such as real numbers or Cartesian geometry. Hence he gave precise definitions of semantic concepts that had been used informally before. The most important of those concepts are truth, satisfiability, and definability of a formula; logical consequence; and model. Tarski was also the champion of the trend towards reconstructing logi-

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cal notions by mathematical means. For instance, his decision method for elementary algebra and geometry is indeed a generalization of a Sturm’s algorithm for counting the real roots of a polynomial. Tarski was eager to bring to the fore new connections between mathematics and logic and to show how mathematized concepts of logic can help to solve mathematical problems. Thus, Tarski initiated the shift from the foundational aims for which various branches of modern logic were originally developed to heuristic aims for which a new branch would turn out to be especially efficient. Namely, Tarski is the father of model theory, the results and tools of which are nowadays commonly used in various mathematical disciplines (algebra, analysis, geometry, computer science, etc.).

Tarski’s views and achievements have also changed the way we think about the nature, scope, and aims of logic. The rich and detailed biography of Tarski written by Anita and Solomon Feferman show us the roots and the full extent of this change.

The Fefermans’ biography is an enthralling success story of a self-confident, enterprising, untiring, and entrepreneurial scientist, and a rich and scrupulous account of the numerous achievements accomplished by this powerful logician and his colleagues in philosophy of logic, semantics, set theory, decision procedures, universal algebra, algebraic logic, axiomatic geometry, topology, and model theory.

It is particularly remarkable that the Fefermans were able to reconstruct so vividly Tarski’s childhood in Warsaw; the brilliant *gymnasium* and university years; the change of name from Teitelbaum to Tarski (in 1924) and the conversion

to Christianity (motivated by the desire to apply for an academic position); the leftist and socialist bent; the opposition to Zionism and the search for assimilation despite the prejudices against Jews; the trips to Zacopane, a meeting place for Polish intellectuals in the Tatra mountains—in short, the scientific, political, cultural, and artistic atmosphere that favored the development of Tarski's multifaceted personality and help account for his wide interests and capacities. No less remarkable is the description of Tarski's years in America, from the time he arrived in 1939 to give invited lectures at Harvard University until his death in 1983, i.e., from his first uncertain steps in his profession to the heydays crowning the steadfast efforts to build a prestigious logic group at UC Berkeley. All of the events are described in the same meticulous way. The reader follows month by month or year by year the irresistible, though treacherous, ascension of a strong and passionate character.

"Life and Logic" is the meaningful subtitle of the Fefermans' book. As with the biography of Jean van Heijenoort, which Anita published in 1993, the whole picture, or better the whole colorful and stirring movie about Tarski's great adventure, is composed by intertwining scenes of private life, socio-political data, cultural customs and events, academic affairs, and advances in logic. Thus, fifteen chapters deal with the multidimensional context of Tarski's scientific research and achievements, while six technical "Interludes", spliced between the chapters, give a synthetic account of the main results Tarski and his group obtained. A bibliography of selected works by and on Tarski completes the book.

The benefits of reading this fascinating biography are at least threefold. First, one gets a concrete image and a vivid portrait of one of the two logicians who changed the face of logic in the twentieth century. We learn how high were his scientific ambitions and how large was the realm of his passions: for logic and mathematics naturally, but also for philosophy (with a particular interest in classical Greek philosophy), for poetry, for the arts, and also for climbing mountains and visiting botanical gardens. Behind the demanding logician one discovers the "bon vivant" who smoked immoderately; drank heavily; used drugs for keeping awake and working until dawn; womanized all throughout his life; liked parties; and was always ready for hiking, traveling, meeting new people, and exploring new sights of the world.

The second important benefit is to become acquainted with the many people who played a role in Tarski's life and career and built a large community devoted to various subfields of mathematical logic (mainly model theory, set theory, and algebra of logic). As in a Balzac novel, one is introduced to a whole world, the world of logic and set theory from around 1900 to the 1980s, and from Warsaw,

Lvov, and Vienna to Harvard, Princeton, Berkeley, and Stanford, to cite only the main internationally known scientific centers. One meets "mythical" logicians such as Gödel, with whom Tarski shared an interest in completeness, definability, decidability, and recursivity, and many outstanding scholars, such as: Stanisław Lesniewski, Jan Łukasiewicz, Kazimierz Kuratowski, Waclaw Sierpiński, Tadeusz Kotarbiński, Léon Chwistek, Adolf Lindenbaum, Andrzej Mostowski, Wanda Szmielew, Samuel Eilenberg, Bertrand Russell, Rudolf Carnap, Willard Van Orman Quine, Evert Beth, Karl Popper, John McKinsey, Paul Cohen, Alonzo Church, Barkley Rosser, Stephen Kleene, Bjarni Jónsson, Louise Chin, Julia Robinson, Leon Henkin, Robert Vaught, Richard Montague, Dana Scott, Solomon Feferman, Jerome Keisler, Patrick Suppes, Georg Kreisel, John Addison, Robert Solovay, Saul Kripke, and others who have left a mark on the fields of logic or mathematics or philosophy. Everyone is more or less briefly described according to the closeness and depth of his connection to Tarski and in view of his more salient traits.

In addition, Tarski's life story is linked with important intellectual movements. First of all, the extraordinary flourishing from about 1900 onwards of mathematical, logical, and foundational studies in Poland at the hands of the masters of the Warsaw-Lvov School (a thorough account of this school and its emergence is to be found in Jan Woleński's *Logic and Philosophy in the Lvov-Warsaw School*, Kluwer, Dordrecht, 1989). The specific humus provided by the Polish in the interplay of mathematics, logic, and philosophy constituted the fertile ground upon which Tarski's logical masterpieces flourished. In writing them Tarski was concerned on the one hand with showing the importance of logic for mathematics and, on the other hand, with articulating the methodology of deductive theories and the conceptual analysis of logical notions which would give instruments for rigorous philosophical work in this area.

The second major movement that affected, and was affected by, Tarski was the Unity of Science movement, launched in 1934 by Otto Neurath, the coauthor (along with Rudolf Carnap and Hans Hahn) of the manifesto of the Vienna Circle. As is well known, the basic assumption of the manifesto was the division of knowledge into empirical and logical statements, any other statement outside of those categories being considered meaningless. The goal was then to obtain a scientific world conception by applying logical analysis to the empirical material. The brilliant Viennese mathematician Karl Menger, who belonged to the Circle, was invited to lecture in Warsaw in the autumn of 1929. He was so impressed by the precise work of the Polish logicians that he invited Tarski to Vienna. There were encounters that would become important for Tarski's influence outside Poland.

Most notably, Carnap attended Tarski's lectures and in the years following their first meeting in 1930, quickly grasped the intrinsic value of the semantical approach, the importance of the theory of truth and of the Tarskian analysis of the concept of logical consequence, and the possible instrumental contribution of semantics to the development of "scientific philosophy". The 1935 Paris Congress of the Unity of Science, where Tarski presented his concepts of truth and consequence, marked the beginning of his international reputation.

The fifth International Unity of Science Congress was held in 1939 at Harvard University. After a long hesitation, Tarski eventually accepted Quine's invitation and left Poland in August without thinking war was imminent. However, before the opening of the Unity of Science Congress on September 3 Warsaw was already under the Germans' bombs. After the end of the congress, Tarski stayed in America, anxious for his family in Poland. He gave a number of lectures at various universities in the Northeast but had to search for regular teaching positions. No permanent appointment could be found for "the great logician" until his move in 1942 to the University of California at Berkeley. His family could not come before January 1946. Then, he began to organize "a systematic study of logic and foundations," as he wrote in 1948 to Heinrich Scholz, who had helped logicians in Poland during the war. That announced the new age, the American period of the spread of Tarski's semantical methods and their algebraic representation. As the Fefermans put it (p. 185): "Although it became common knowledge that doing graduate work and writing a Ph.D. thesis with Tarski was a harrowing experience, his personal magnetism and brilliance, and his passionate belief in the supreme importance of logic, were so strong that he attracted a steady stream of students eager to take up the challenge."

One has to take account of the social and cultural aspects that Tarski infused into his scientific research to understand the scope of his enterprise and the unending energy he expended for realizing it. For him, science, and first of all logic, was a training in right thinking and a means to promote a kind of philosophy freed from metaphysical views. He created in 1956 at UC Berkeley a new Ph.D. program in Logic and Methodology of Science, which is still thriving today. In some sense, it was nothing but pursuing or reviving in a specific way the ideal of the Warsaw-Lvov School of logic and replacing the comprehensive "scientific world conception" of the Vienna Circle by more objective and realistic goals. According to the Fefermans (p. 252), Tarski was developing the research program of the Polish School "along the lines of Carnap's project for *the logical investigation of scientific language and theories as formal objects of study*" (the italics are mine). He was developing

a scientific and intellectual movement of his own. He was building "a school of thought" (p. 3). International conferences organized in this perspective were not exclusively restricted to logicians; rather they included mathematicians, physicists, and philosophers of science. After the International Symposium on the Axiomatic Method, with Special Reference to Geometry and Physics, held at Berkeley in December 1957/January 1958, Tarski planned the First International Congress for Logic, Methodology and Philosophy of Science, which was held at Stanford University in August 1960, and inaugurated the series, still continuing, of one such international congress every four years.

Thus Tarski created in Berkeley a "home" for a specific way of doing mathematical logic, with its methods, its standards of rigor, its connections to and its impact upon other disciplines, its own meetings and conferences, its own stories, and its own legend.

The third benefit of the Fefermans' biography consists in offering an introduction to the main problems and results that Tarski pushed to the forefront of logical research. Written with a minimum of technicalities, the six Interludes meet the demanding standards of the exceptional teacher that Tarski was and serve perfectly the purpose of pedagogical presentation. Simple but self-contained, explaining step-by-step everything that is needed and leaving nothing fuzzy nor obscure, they are easily understandable even by those not previously acquainted with the subject matters. They furnish a beautiful, though partial, survey of a whole century of logic in Europe and America.

Indeed, these Interludes, none of which exceeds fifteen pages, explain very clearly sophisticated results such as the Banach-Tarski paradox; the completeness and decidability of elementary algebra and geometry, which was considered by Tarski and many others as "one of the two most important research contributions in his entire career;" and the definition of truth, the other most important contribution, which has been "recognized as one of the most important examples of conceptual analysis in twentieth-century logic." The Interludes also give a brief but precise account of Tarski's other endeavors. Thus, in Interlude IV we learn in just a few words (p. 191) what the aim was of Tarski's *Cardinal Algebras* (published in 1949): "to isolate in algebraic form a number of results about finite and infinite cardinal numbers that could be proved *without* using the Axiom of Choice." The content of *Ordinal Algebras*, published in 1956, is described in a similarly concise way. Interlude V is dedicated to model theory, which is "an *informal* mathematical theory whose subject matter is *formal theories and their models*." One of the main novelties is the introduction by the Polish logician Jerzy Łoś of the mathematical notion of "ultrapower". Tarski, together with Anne Morel and Dana Scott, used

this notion to give (in 1958) a mathematical proof of the compactness theorem for first-order logic, according to which if each finite subset of a set of first-order sentences has a model, then the whole set has a model. The compactness theorem is a consequence of Gödel's completeness theorem, the proof of which was carried out by metamathematical means; i.e., it involved reference to the syntactic notion of sentences of a formal language and to the semantic notion of truth in a model. The new proof of the compactness theorem constituted an important milestone on the road Tarski was constructing for logic and ushered in extensive applications of the ultrapower construction in model theory. Indeed, through such applications Tarski hoped to eventually obtain what he aimed at from the very beginnings of his work: to reach a wider mathematical audience beyond those working in closely aligned areas of logic. Interlude VI is devoted partly to relation algebras and one application, which led Tarski and Steven Givant to conceive of a theory of sets with no use of variables or quantifiers, and partly to *Cylindric Algebras*, whose basic aim is the algebraization of logic. Leon Henkin and Donald Monk collaborated in writing down the two thick volumes published respectively in 1971 and 1985. What is striking is the persistence in these last works of two main components of Tarski's initial global project: one had been, and remained, the elimination of quantifiers, which, at the time of the Warsaw Seminar (1926–28), meant the exclusion of set-theoretical methods; and the other had been, and still remained, the replacement of logical notions by corresponding purely algebraic methods. This is only one illustration, but a very significant one, of the steadiness and coherence of Tarski's scientific views throughout his life.

Even a quick perusal of the authors' Notes and Acknowledgments (pages 381–4) will be enough to provide the reader of this fascinating biography with a sense of the vast amount of archival material, documents, articles, and books that were consulted in writing it. One also learns that about 150 people were interviewed or consulted about details of Tarski's personal, and even intimate, life or about different aspects of the scientific work and events in which Tarski had been involved: teaching, supervising Ph.D. students, giving lectures in American universities and abroad, conducting seminars, writing papers and books, heading international congresses and meetings, promoting wherever he was logic, methodology, and philosophy of science. The Notes (pages 393–408) indicate that the first interviews with people who were close to Tarski took place in 1993, that is to say, eleven years before the book came out. The Fefermans started the job while people who knew Tarski in his early days were still alive. They sought even the slightest information coming from relatives; friends; former students or assis-

tants; secretaries; and scholars and professors of mathematics, logic, and philosophy from all over the world. Moreover, they had been personally well acquainted with Tarski since the beginning of the 1950s. Solomon was his Ph.D. student and his teaching and research assistant until 1956. After moving to Stanford, the Fefermans continued to see Tarski often at meetings and at the regular Stanford-Berkeley logic colloquium, which was started in the 1960s. They were among the friends who celebrated Tarski's eightieth birthday (in 1981) at the Henkins' house.

One can imagine how much patience, stubborn determination, and attention to detail and to the complexity of the facts have been invested in this systematic, substantial, and thorough inquiry. Many facts reported in the biography have been double-, triple- or even multichecked. Reports by different persons on how Tarski behaved and thought might have been very dissimilar or even contradictory. To ensure completeness, the Fefermans offer at times many versions of the same fact or event, with the aim of showing the different—and most of the time not separate—roles Tarski played, including the severe and unbending professor who could be a considerate father figure to his students and possibly a lover for those who were female. The complex and varied portrait the Fefermans have painted is a rigorous attempt at capturing with greatest objectivity the complex socio-psychological facts that can help us understand Tarski's personal leanings, his high professional conscientiousness, his “unending concern for clarity, precision, and rigor,” and, last but not least, his strong will to put his mark on his time.