Scientific WorkPlace®
Scientific Word®

- Mathematical Word Processing
- \LaTeX Typesetting
- Computer Algebra

Includes the Beamer Package for slide presentations

The Gold Standard for Mathematical Publishing

Scientific WorkPlace and Scientific Word Version 5.5 make writing, sharing, and doing mathematics easier. You compose and edit your documents directly on the screen, without having to think in a programming language. A click of a button allows you to typeset your documents in \LaTeX. You choose to print with or without \LaTeX typesetting, or publish on the web. Scientific WorkPlace and Scientific Word enable both professionals and support staff to produce stunning books and articles. Also, the integrated computer algebra system in Scientific WorkPlace enables you to solve and plot equations, animate 2D and 3D plots, rotate, move, and fly through 3D plots, create 3D implicit plots, and more.

Visit our website for free trial versions of all our products.

www.mackichan.com/notices  Email: info@mackichan.com  Toll free: 877-724-9673
Mathematical Moments
A series of over 70 posters that describe the role mathematics plays in science, nature, technology, and human culture. Some have been translated into other languages, and some feature podcast interviews with experts in the field.

Listen Up! — Podcast Interviews with Experts

Diandra L. Leslie-Pelecky, University of Texas at Dallas, on the mathematics of NASCAR
Ken Golden, University of Utah, on the mathematics of sea ice
Kevin Short, University of New Hampshire, on digitizing a wire recording of Woodie Guthrie (and his resulting Grammy Award)
Tim Chartier, Davidson College, on the connections between mathematics and soccer
and more...

Mathematical Moments in Other Languages
Read translated versions in Chinese, Spanish, French, German, Portuguese, Polish, Japanese, and Russian.

Using Mathematical Moments

• Display in your office
• Use in the classroom
• Distribute at special events
• Promote math to colleagues
• Post on bulletin boards
• Launch a math club activity

www.ams.org/mathmoments
New and Forthcoming

Nonlinear Partial Differential Equations
Asymptotic Behavior of Solutions and Self-Similar Solutions
Mi-Ho Giga, University of Tokyo, Japan; Yoshikazu Giga, University of Tokyo, Japan; Jürgen Saal, University of Konstanz, Germany
The main focus of the textbook, in two parts, is on showing how self-similar solutions are useful in studying the behavior of solutions of nonlinear partial differential equations, especially those of parabolic type. The exposition moves systematically from the basic to more sophisticated concepts with recent developments and several open problems. With challenging exercises, examples, and illustrations to help explain the rigorous analytic basis for the Navier—Stokes equations, mean curvature flow equations, and other important equations describing real phenomena, this book is written for graduate students and researchers, not only in mathematics but also in other disciplines.
2010. 350 P. 20 ILLUS., HARDCOVER
PROGRESS IN NONLINEAR DIFFERENTIAL EQUATIONS AND THEIR APPLICATIONS, VOL. 79
APPROX.$99.00

Now Available in Softcover
Discrete Groups, Expanding Graphs and Invariant Measures
Alexander Lubotzky, Hebrew University, Jerusalem, Israel; Jonathan D. Rogawski, University of California, Los Angeles, CA, USA
1ST ED. 1994, 2ND PRINTING, 2010. 208 P., SOFTCOVER
ISBN 978-3-0346-0331-7
ORIGINALLY PUBLISHED IN THE SERIES: PROGRESS IN MATHEMATICS VOL 125
MODERN BIRKHAUSER CLASSICS
$39.95

New Journal for 2010
Journal of Pseudo-Differential Operators and Applications
The Journal of Pseudo-Differential Operators and Applications is a forum for high quality papers in the mathematics, applications and numerical analysis of pseudo-differential operators. Pseudo-differential operators are understood in a very broad sense, embracing but not limited to, harmonic analysis, functional analysis, operator theory and algebras, partial differential equations, geometry, mathematical physics, and novel applications in engineering, geophysics, and medical sciences.
ISSN: 1662-9981 (PRINT VERSION)
ISSN: 1662-999X (ELECTRONIC VERSION)
JOURNAL NO. 11868

Arrangements, Local Systems and Singularities
CIMPA Summer School, Galatasaray University, Istanbul, 2007
Fouad El Zien, American University of Beirut, Lebanon; Alexander I. Suciu, Northeastern University, Boston, MA, USA; Meral Tosun, Galatasaray University, Istanbul, Turkey; Muhammed Uludag, Galatasaray University, Istanbul, Turkey; Sergey Yuzvinsky, University of Oregon, Eugene, OR, USA (Eds).
This volume comprises the Lecture Notes of the CIMPA/TUBITAK Summer School Arrangements, Local systems and Singularities held at Galatasaray University, Istanbul during June 2007. The reader will find a variety of open problems involving arrangements, local systems and singularities proposed by the lecturers at the end of the school.
SERIES: PROGRESS IN MATHEMATICS, VOL. 283
2010. 305 P., HARDCOVER
ISBN 978-3-0346-0208-2
$99.00

Now Available in Softcover
Cardinal Invariants on Boolean Algebras
Donald J. Monk, University of Colorado, Boulder, CO, USA
ORIGINALLY PUBLISHED IN THE SERIES: PROGRESS IN MATHEMATICS VOL. 142
1ST ED. 1996. 2ND PRINTING, 2010.
308 P., SOFTCOVER
ISBN 978-3-0346-0333-1
MODERN BIRKHAUSER CLASSICS
$39.95

The Theory of the Top
Volume 2: Development of the Theory for the Heavy Symmetric Top
Felix Klein, Arnold Sommerfeld,
Translators: Raymond J. Nagel, Guido Sandri, Boston University, Boston, MA, USA
The Theory of the Top was originally presented by Felix Klein as an 1895 lecture at Göttingen University that was broadened in scope and clarified as a result of collaboration with Arnold Sommerfeld. The Theory of the Top: Volume II. Development of the Theory for the Heavy Symmetric Top is the second installment in a series of four self-contained English translations that provide insights into kinetic theory and kinematics.
APPROX. 300 P. 25 ILLUS., HARDCOVER
ISBN 978-0-8176-4824-4
APPROX. $79.95

Also Available:
The Theory of the Top
Volume 1: Introduction to the Kinematics and Kinetics of the Top
2008. XVIII, 279 P. 43 ILLUS., HARDCOVER
$79.95
Ideas of differential geometry have permeated all parts of mathematics. Partial differential equations, topology, analysis, and many other branches of our subject have taken on a geometric patina. The recent solution of the Poincaré conjecture used geometry in surprising ways. This issue of the Notices showcases some recent developments in the geometry of Riemann.

—Steven G. Krantz
Editor

Features

208 An Invitation to Cauchy-Riemann and Sub-Riemannian Geometries
John P. D’Angelo and Jeremy T. Tyson

220 Mathematics of the Gateway Arch
Robert Osserman

230 Model Theory and Complex Geometry
Rahim Moosa
Departments

About the Cover .......................... 266
Mathematics People ........................ 260
MacPherson Awarded Hopf Prize, Lindenstrauss and Villani Awarded Fermat Prize, Pujals Awarded TWAS Prize in Mathematics, Dereich Receives 2009 Information-Based Complexity Young Researcher Award, NSF CAREER Awards Made, Memories of Eddie Nussbaum.

Mathematics Opportunities .................. 264

Inside the AMS ........................... 268
From the AMS Public Awareness Office, AMS Email Support for Frequently Asked Questions, Deaths of AMS Members.

Reference and Book List ...................... 271
Mathematics Calendar ......................... 303
New Publications Offered by the AMS .............. 308
Classified Advertisements ...................... 312
Meetings and Conferences of the AMS ............. 314
Meetings and Conferences Table of Contents ........ 327

From the AMS Secretary

2009 Election Results ...................... 300
2010 AMS Election—Nominations by Petition ........ 301

I thank Randi D. Ruden for her splendid editorial work, and for helping to assemble this issue. She is essential to everything that I do.

—Steven G. Krantz
Editor
Collaborating on Research with Mathematicians from Less Developed Countries

For a decade, I have collaborated on research with mathematicians from Jordan, Iran, Pakistan, India, and South Africa. Hardly any of my collaborators have high mathematical prestige or have positions at institutions that have it. By and large, these mathematicians make up for deficiencies that include high teaching loads and very poor libraries with enthusiasm, patience, goodwill, and hard work. Most, though not all, such efforts have led to publication in respectable journals published in western countries.

Common sense told me to expect that collaborating with mathematicians from other cultures using different technologies would bring me problems, but these problems were far more difficult than I expected.

Since the end of the Second World War, most published mathematics is written in English, which is a second language for most people. Even if your foreign co-author’s knowledge of English is very good, it is often not idiomatic and needs to be corrected in a way that is effective without being pedantic or patronizing. Treat your co-author with respect and make it easy for your co-author to be critical of what you have written. A co-author’s English may be less than perfect, a co-author from a poor country may know less mathematics than you do, but still may be extremely gifted. Once I overcame their reluctance to criticize my writing or mathematics because of their fear of being discourteous, my Third World co-authors could be very good at detecting errors.

Avoid political discussions or comparison of governments. Ignoring this, especially in writing, can get you into trouble. Stick to mathematics.

Keep in mind two differences in how mathematicians are “rewarded for publishing papers” in some countries. The author whose name appears first gets more of a reward (sometimes financial) than the remaining authors. There is also an Impact Factor assigned to journals that is ill understood. Publishing in a journal with a high impact factor gets a bigger reward.

Communication, electronic or otherwise, is difficult for a number of reasons. Email works most of the time, but power failures are much more frequent than in North America or Europe. Regular mail tends to be slow, and fax machines are usually turned off at night. Acknowledge all emails and encourage your co-authors to do the same. If possible, get a list of PDF files of your own publications to potential co-authors. Even governments you may not like encourage their citizens to study and earn advanced degrees abroad. Should you visit your co-author, there are many books named “Culture Shock (name of country)” written to describe local customs with a view to warn you of things that might insult your hosts. For example, in many countries your host will feel obligated to urge you to accept as a gift something that you admire—and resent it when you accept.

One communication difficulty pertaining to mathematics is that few of these foreign co-authors have access to good library facilities or even an effective interlibrary loan system. References mailed to potential collaborators often result in a polite reply saying that some book or journal is unavailable. This is a serious problem everywhere because the number of distinct mathematics journals and books has grown substantially and gotten both more specialized and more expensive.

The best substitute for a poor library is a subscription to MathSciNet, which is an electronic version of Math Reviews. Its reviews and papers can be read and printed mostly, but not always, sometimes for an additional charge. Were it up to me, I would provide open access to MathSciNet to all Third World AMS members even if some dues had to be increased. The main problem for these mathematicians is cost.

I am most fearful at this prospect: The quantity of research in the less developed countries is growing rapidly, while many journals’ articles are becoming available only in electronic form. In some countries ideology, economics, or technological limitations restrict a mathematician’s access to Web portals. Commercial publisher-maintained sites are only available to subscribers at costs that some institutions cannot afford. Ignoring this trend may result in serious negative consequences for Third World authors, indeed for all of us.

We want a worldwide community of research mathematicians. Quality of published research in mathematics that is independent of geographic location or financial wealth should be our common goal. Perhaps there are less expensive methods for making mathematical research and thoughts readily available. What is most required is a desire to reach out to all mathematicians in all countries as a way to make our combined research efforts easier to implement.

Collaborating with international mathematicians is both important and rewarding. It is difficult to exaggerate the enjoyment I feel after a successful collaboration with someone from a different culture. I hope that a committee of the AMS will study this important problem.

—Melvin Henriksen

Editor’s Note: Professor Henriksen died after a long illness on October 14, 2009.
GOOD MATH.

IT’S WEBASSIGN, ONLY BETTER.

Only WebAssign is supported by every publisher and supports every course in the curriculum from developmental math to calculus and beyond. That’s good. Now we’ve added the latest homework tools to bring you the ultimate learning environment for math. That’s better.

Welcome to a better WebAssign. There are new tools — including calcPad, that lets students enter higher level mathematical functions, then automatically assesses their work. There are more books, over 170 titles from every major publisher. And there’s more server capacity with an expanded support team just waiting to serve you and over half a million loyal WebAssign users each term. It’s the culmination of a team-wide effort to build the best WebAssign ever.

The ingenuity of WebAssign for math. It’s available now and it’s free to faculty at WebAssign.net. Now what could be better than that?

Visit [www.webassign.net](http://www.webassign.net) to sign up for your free faculty account today.

calcPad lets students easily enter higher-level mathematical functions, then automatically assesses them.
Andrew Gleason

Many readers of the Notices are, I am sure, grateful for the fine articles about the late Andrew Gleason and his accomplishments, in the November 2009 issue. I myself was particularly interested in Paul Chernoff’s article, for I once spent much effort to completely master Gleason’s profound theorem characterizing abstractly defined “states” in quantum mechanics, and I even hoped to be able to shorten the proof, something in which, however, I failed. Because Gleason is probably most widely known for his path-breaking contribution to solving Hilbert’s Fifth Problem, I thought that a personal experience of mine may be of interest.

Gleason once visited Indiana University in Bloomington, and I met him at a post-lecture party. I mentioned to him my admiration for “Gleason’s Theorem”, “Which one?” he asked. Upon my replying that I meant the one about states in quantum theory, his face darkened, he thought for a few moments, and then said “You know, that was the most difficult thing I ever did in my life.”

—Andrew Lenard
Indiana University
lenard@indiana.edu

(Received October 2009)

As a former physics major, now physician and neuroscientist, and one of Professor Gleason’s “last students” in a couple of different ways, I would just like to add to the recent recollections in the Notices.

I was the only nonmathematician taking Gleason’s graduate real analysis class. Gleason passed out a set of notes from a prior time he had taught the class and requested that we bring any errors to his attention—there were less than a handful. I mentioned that he must be pleased that when he taught the class again, the notes would be error free! He responded that he would never teach the class again, but he continued to collect errors with interest and concern. The final for the class was half true/false questions, scored +1 for a correct answer, -3 for a wrong answer, zero for no answer. One question was so tricky that no one among the crew of past, present, and future deep-thinking and Putnam Prize-winning students dared answer! The professor still knew best!

Later I wanted to enter a team in the COMAP Mathematical Contest in Modeling. Knowing that Gleason was a three-year Putnam champion and literally wrote the book on this pure math contest, I approached him about being our faculty advisor with some trepidation, but he immediately agreed.

We solved the discrete problem for the specific given set of conditions, and also showed that with general conditions the problem was NP-complete. On Monday morning after Gleason mailed our sealed write-up, we proudly told him our results. He listened with a knowing look. He too had found the same, but didn’t look nearly as tired as we were! We were awarded an Outstanding Paper. Gleason’s legacy is that Harvard now regularly competes in this applied math contest.

I have done some work on J. J. Thomson’s century-old problem of the minimum energy configuration of charges on the surface of a sphere. I discussed with Gleason one paper in which we were studying the difference in energies of configurations for certain numbers of charges that we had found admitted more than one nice lattice configuration. He asked why we hadn’t given the general formula for such N—his legendary ability to generalize your results on the spot! My co-author Richard Stong was then able to produce the formula. I have never seen reviewers so impressed! I continued to have the pleasure and good fortune of discussing work on other problems in physics and neuroscience with Andrew Gleason.

From his lectures and comments, it seems to me that Andrew Gleason was never fully comfortable with the notion of completeness of the real numbers. When last we talked, Gleason mentioned to me that he was studying Hilbert’s axioms of geometry, in particular the axioms on order.

Our teacher and friend is gone. But the example he set for us lives on for us to follow.

—Eric Altschuler, M.D., Ph.D.
New Jersey Medical School–University of Medicine & Dentistry of New Jersey

(Received October 2009)
An Invitation to Cauchy-Riemann and Sub-Riemannian Geometries

John P. D’Angelo and Jeremy T. Tyson

A certain amount of complex analysis, in both one and several variables, combines with some differential geometry to form the basis of research in diverse parts of mathematics. We first describe that foundation and then invite the reader to follow specific geometric paths based upon it. We discuss such topics as the failure of the Riemann mapping theorem in several variables, the geometry of real hypersurfaces in complex Euclidean space, the Heisenberg group, sub-Riemannian manifolds and their metric space structure, the Hopf fibration, sub-Riemannian geodesics on the three-sphere, CR mappings invariant under a finite group, and finite type conditions. All of these topics fall within the branch of mathematics described by the title, but of course they form only a small part of it. We hope that the chosen topics are representative and appealing. The connections among them provide a fertile ground for future research. We modestly hope that this article inspires others to develop these connections further. To this end, our reference list includes a diverse collection of books and accessible articles.

Given Riemann’s contributions to geometry, it is not surprising that his name occurs twice in the title. It is more surprising that the uses of his name here come from different parts of mathematics. We find it delightful that contemporary mathematics is forging connections that even Riemann could not have anticipated.

John P. D’Angelo is professor of mathematics at the University of Illinois. His email address is jpd@math.uiuc.edu.

Jeremy T. Tyson is associate professor of mathematics at the University of Illinois. His email address is tyson@math.uiuc.edu.

There Is No Riemann Mapping Theorem in Higher Dimensions

In an attempt to understand complex analysis in several variables, especially questions revolving around the failure of the Riemann mapping theorem, many mathematicians have focused attention on geometric properties of the boundaries of domains and how those geometric properties influence the complex analysis on the domain. Such investigations have led to the fields of CR geometry and sub-Riemannian geometry.

We begin with the Riemann mapping theorem. Let Ω be an open, connected, and simply
We define and discuss the Levi form on page 211. We say that \( \Omega \) we need only mention the classical text by Ahlfors in the boundary. There is no Levi form in one dimension, different boundary curves are biholomorphic mapping can exist. By contrast, the boundary geometries are so different that no example, the subset of \( \mathbb{C} \) indistinguishable from this point of view. At least 2 the Levi form is identically zero. For acts does not exist. For a half plane in dimension real dimension and the part on which the Levi form plays a crucial role. The Levi form for a real hypersurface in this case, the \( \mathbb{C} \) Levi form for a real hypersurface in \( \mathbb{C} \). Let us now consider complex Euclidean space \( \mathbb{C}^n \) of arbitrary dimension. We equip \( \mathbb{C}^n \) with the usual Hermitian inner product given by \( \langle z, w \rangle = \sum z_j w_j \) and the corresponding norm given by \( |z| = \langle z, z \rangle^{1/2} \). The unitary group \( U(n) \) consists of linear mappings preserving the inner product. The unit ball \( B_n \) is the set of \( z \) for which \( |z| < 1 \).

In dimension at least two the unit ball is not biholomorphically equivalent to a half space. There are many possible proofs. One approach takes into account the geometry of the boundaries. When the boundaries are smooth manifolds (as they are in this case), the Levi form plays a crucial role. The Levi form for a real hypersurface in \( \mathbb{C}^n \) (for \( n \geq 2 \)) is the complex variable analogue of the second fundamental form for a real hypersurface in \( \mathbb{R}^n \). We define and discuss the Levi form on page 211.

For now we simply note that the Levi form is a Hermitian form defined on part of the tangent spaces of the boundary. There is no Levi form in one dimension, because the tangent space is one real dimension and the part on which the Levi form acts does not exist. For a half plane in dimension at least 2 the Levi form is identically zero. For the unit sphere it is positive definite. As a result the boundary geometries are so different that no biholomorphic mapping can exist. By contrast, in one dimension, different boundary curves are indistinguishable from this point of view.

A polydisc is a Cartesian product of discs. For example, the subset of \( \mathbb{C}^2 \) defined by \( |z_1| < 1 \) and \( |z_2| < 1 \) is a polydisc. It is also not biholomorphically equivalent to \( B_2 \). In fact, for \( n \geq 2 \) there is no biholomorphic map from any polydisc to any ball. One classical proof of this statement involves showing that the groups of biholomorphic automorphisms of the ball and of the polydisc have different dimensions.

A second proof (see [D1, page 14]) proceeds in the following manner. First consider a biholomorphic (or even proper) image \( \Omega \) of the Cartesian product of bounded domains; one shows that \( b\Omega \) must contain complex analytic sets. Then an easy computation, given in Lemma 6, shows that the definiteness of the Levi form precludes the existence of positive-dimensional complex analytic subsets of \( b\Omega \). This computation helps unify several of the ideas in this article. For example, higher order commutators of complex vector fields can determine obstructions to the existence of complex analytic sets in a real hypersurface. Such obstructions lead to geometric finite type conditions for subelliptic estimates. In two complex dimensions a subelliptic estimate (see page 217) holds if and only if the bracket-generating property (see page 212) holds.

Consider a domain \( \Omega \) with smooth boundary \( b\Omega \). In order that geometric results about \( b\Omega \) be meaningful for complex analysis on \( \Omega \), one requires results concerning the boundary smoothness of biholomorphic maps. Two early references in this area are [Bel] and [Fef]; we do not attempt to list any of the many newer references. In general, smoothly bounded domains, topologically equivalent to the unit ball \( B_n \), are seldom biholomorphically equivalent to it. On the other hand, Fridman [Fri] has established a wonderful approximate Riemann mapping theorem: given any topological ball and a positive \( \epsilon \), there is a biholomorphic mapping to a domain whose boundary is always within distance \( \epsilon \) of the unit sphere.

The Unit Sphere and the Heisenberg Group

The quintessential principle connecting CR geometry and sub-Riemannian geometry arises from the correspondence between the unit sphere \( bB_n \) and the Heisenberg group, which bounds the unbounded Siegel upper half space \( H_n \). We will obtain a useful identification of \( bH_n \) with \( \mathbb{C}^{n-1} \times \mathbb{R} \), on which we will impose a nonabelian group law arising from the biholomorphic automorphisms of \( H_n \). The equivalence between \( H_n \) and \( B_n \) generalizes the one-dimensional Cayley transformation.

The Siegel upper half space \( H_n \) is the set of \( w \) in \( \mathbb{C}^n \) such that

\[
\text{Im}(w_n) > \sum_{j=1}^{n-1} |w_j|^2.
\]

The following elementary but useful identity is the key to discovering the biholomorphic map:

\[
\text{Im}(\zeta) = \left| \frac{i + \zeta}{2} \right|^2 - \left| \frac{i - \zeta}{2} \right|^2.
\]
With \( \zeta = w_n \) we plug (3) into (2) and rewrite the result to obtain

\[
\left| \frac{i + w_n}{2} \right|^2 > \sum_{j=1}^{n-1} |w_j|^2 + \left| \frac{i - w_n}{2} \right|^2.
\]

After dividing by \( |\frac{i + w_n}{2}|^2 \) and changing notation, inequality (4) becomes

\[1 > \sum_{j=1}^{n-1} |Z_j|^2.\]

Here \( Z_j = \frac{2w_j}{i + w_n} \) for \( j < n \) and \( Z_n = \frac{i - w_n}{i + w_n} \). The reader should check that this transformation \( f : w \to z \) is biholomorphic from \( H_n \) to \( B_n \) and also compare it with the transformation in (1).

**The automorphism group of \( H_n \)**

We alluded earlier to the biholomorphic automorphism groups of the ball and the polydisc. For any complex manifold the automorphism group is a fundamental algebraic invariant. For example, the automorphism group of the upper half plane in \( \mathbb{C} \) is \( \text{PSL}(2, \mathbb{R}) \). Its elements act on the (one-point compactification of the) boundary of the upper half plane as linear fractional transformations \( x \to \frac{ax + b}{cx + d} \), for \( ad - bc = 1 \). Any such transformation can be represented as the composition of dilations \( x \to rx \), translations \( x \to x + x_0 \), and possibly the inversion \( x \to \frac{1}{x} \). Automorphisms fixing the point at infinity do not require the inversion.

We now describe the analogous situation on \( bH_n \), defined by equality in (2). The last variable in (2) plays a different role, and hence for \( w \in \mathbb{C}^n \) we write \( w = (w', w_n) \). Two natural families of biholomorphic self-maps of \( H_n \) are the dilations \( \delta_r : H_n \to H_n \), for \( r > 0 \), given by \( \delta_r (w', w_n) = (rw', r^2w_n) \), and the rotations \( R_{\alpha} : H_n \to H_n \), for \( \alpha \in \mathcal{U}(n - 1) \), given by \( R_{\alpha} (w', w_n) = (\alpha (w'), w_n) \).

To introduce an analogue of translation we consider \( \bar{w} \) variables. Finally we observe that the group of biholomorphic automorphisms of \( H_n \) which fix the point at infinity is generated by dilations, rotations, and translations. See Stein’s textbook in harmonic analysis [Ste, Chapters XII and XIII] for the complete story.

**CR Structures**

We next discuss the geometry of a general real hypersurface \( M \) in \( \mathbb{C}^n \). The beautiful interplay between real and complex geometry dominates the discussion. See [BER], [D1], [DT], [Jac], [Tre] and their references for more on CR geometry.

We wish to consider complex vector fields and therefore start by considering the complexified tangent bundle

\[\mathcal{C}T(\mathbb{C}^n) = T(\mathbb{C}^n) \otimes \mathbb{C}.\]

As usual in complex analysis we define the complex partial derivative operators

\[
\frac{\partial}{\partial z_j} = \frac{1}{2} \left( \frac{\partial}{\partial x_j} - i \frac{\partial}{\partial y_j} \right)
\]

and

\[
\frac{\partial}{\partial \bar{z}_j} = \frac{1}{2} \left( \frac{\partial}{\partial x_j} + i \frac{\partial}{\partial y_j} \right).
\]

Using these operators we decompose the exterior derivative \( d \) as

\[
df = \partial f + \bar{\partial} f = \sum_{j=1}^{n} \frac{\partial f}{\partial z_j} dz_j + \sum_{j=1}^{n} \frac{\partial f}{\partial \bar{z}_j} d\bar{z}_j.
\]

Since \( d^2 = 0 \), it follows that \( \partial^2 = \bar{\partial}^2 = \partial \bar{\partial} + \bar{\partial} \partial = 0 \).

A section of the bundle (6) is a complex vector field. Each such vector field is a combination of both \( z \) and \( \bar{z} \) derivatives with smooth coefficients:

\[L = \sum_{j=1}^{n} a_j(z) \frac{\partial}{\partial z_j} + \sum_{j=1}^{n} b_j(z) \frac{\partial}{\partial \bar{z}_j}.
\]

We obtain two naturally defined integrable subbundles of \( \mathcal{C}T(\mathbb{C}^n) \). Let \( T^{10}(\mathbb{C}^n) \) denote the bundle whose sections are vector fields \( L \) of the form

\[
L = \sum_{j=1}^{n} a_j(z) \frac{\partial}{\partial \bar{z}_j},
\]

where the \( a_j \) are smooth complex-valued functions. We note that this bundle is integrable in the sense of Frobenius: if \( K, L \) are sections of \( T^{10}(\mathbb{C}^n) \), then so is their commutator, or Lie bracket, given by \( [K, L] = KL -LK \). The complex conjugate bundle, denoted \( T^{01}(\mathbb{C}^n) \), is also integrable. Since the zero section is the only section of both bundles, we have \( T^{10}(\mathbb{C}^n) \cap T^{01}(\mathbb{C}^n) = 0 \). We obtain a splitting

\[\mathcal{C}T(\mathbb{C}^n) = T^{10}(\mathbb{C}^n) \oplus T^{01}(\mathbb{C}^n) \]

This splitting of the tangent bundle plays a crucial role in all aspects of complex geometry. CR geometry studies the extent to which this splitting holds on real manifolds.

Let \( M \) be a smooth real hypersurface of \( \mathbb{C}^n \), or more generally, of a complex manifold. The tangent spaces of \( M \) then inherit some of the ambient complex structure. We write \( \mathcal{C}TM \) for \( T(M) \otimes \mathbb{C} \). We define \( T^{10}(M) = T^{10}(\mathbb{C}^n) \cap CTM \) and denote its complex conjugate bundle by \( T^{01}(M) \). Again we have

\[T^{10}(M) \cap T^{01}(M) = 0.\]
Let $M$ be an abstract real manifold with complexified tangent bundle $CTM$. We say that a subbundle $T^{10}(M)$ of $CTM$ defines a CR structure on $M$ if it is integrable and its intersection with its conjugate bundle satisfies $(9)$. We then call $M$ a CR manifold. Its horizontal bundle is the direct sum
\[ \mathcal{H}(M) = T^{10}(M) \oplus T^{01}(M). \]
We say that $M$ is of hypersurface type if the fibers of $\mathcal{H}(M)$ have codimension one in $CTM$.

In general, the CR codimension of $M$ is the codimension of $\mathcal{H}(M)$ in $CTM$. Real submanifolds of $\mathbb{C}^n$ of arbitrary dimension define CR manifolds of various CR codimensions. Consider a $\eta$ type. On such manifolds there is a nonvanishing real hypersurfaces in $\mathbb{C}^n$ to order two at $r$. For a compact hypersurface second fundamental form is defined only modulo the sign of the horizontal space. In general, the second fundamental form can be any integer codimension can be any integer.

If it is complex. Any complex manifold can be regarded as a CR manifold of CR codimension zero. Such manifolds are precisely the integrable almost complex manifolds.

We focus on CR manifolds of hypersurface type. On such manifolds there is a nonvanishing differential one-form $\eta$, defined up to a multiple, annihilating $\mathcal{H}(M)$. We may assume that $\eta$ is purely imaginary. Each of the summands in $(9)$ is integrable, but something new happens; their sum is not integrable. The Levi form measures the failure of integrability of the sum and leads to both CR geometry and sub-Riemannian geometry.

**Definition 1.** Let $M$ be a CR manifold of hypersurface type. The Levi form $\lambda$ is the Hermitean form on $T^{10}(M)$ defined by
\[ \lambda(L, K) = \langle \eta, [L, K] \rangle. \]

Any vector field $T$ for which $\langle \eta, T \rangle \neq 0$ has a component in the missing direction. The Levi form $\lambda(L, T)$ gives the component of $[L, T]$ in the missing direction. In the following definition we identify $\lambda$ with a Hermitean linear transformation of $T^{10}(M)$.

**Definition 2.** A CR manifold of hypersurface type is pseudoconvex if all nonzero eigenvalues of $\lambda$ have the same sign. It is called strongly pseudoconvex if $\lambda$ is definite, that is, all eigenvalues have the same nonzero sign.

Observe that there is an ambiguity of sign in the definition of the Levi form. Even for a real hypersurface in real Euclidean space the sign of the second fundamental form is defined only modulo the choice of normal. For a compact hypersurface in $\mathbb{R}^n$ consider a point $p$ farthest from the origin. Since the sphere centered at the origin osculates $M$ to order two at $p$, the second fundamental form at $p$ agrees with that of the sphere, thus determining its sign. The story is the same for real hypersurfaces in $\mathbb{C}^n$ and the Levi form. On the other hand, there is no natural way to resolve the ambiguity of sign for an abstract CR manifold.

Next we express the Levi form on a hypersurface in terms of partial derivatives. In a neighborhood of a given point we suppose that $M$ is the zero set of $r$, where $r$ is smooth and $dr \neq 0$ where $r = 0$. A complex vector field $L$ is tangent to $M$ if and only if $L(r) = \langle dr, L \rangle = 0$ on $M$. For the one-form $\eta$ we may use $\frac{1}{2} (\partial - \overline{\partial}) r$.

The Cartan formula for the exterior derivative of a 1-form $\eta$ states that
\[ d\eta = A(\eta, B) - B(\eta, A) - \langle \eta, [A, B] \rangle. \]
The commutator term ensures that $d\eta$ is linear over the module of smooth functions. Let $L$ and $K$ be local sections of $T^{10}(M)$. Then $\langle \eta, L \rangle = \langle \eta, K \rangle = 0$. By the Cartan formula,
\[ \lambda(L, K) = -\langle d\eta, L \wedge K \rangle = \langle \partial \overline{\partial} \rangle r, L \wedge K \rangle. \]
We have interpreted the Levi form as the restriction of the complex Hessian of $r$ to sections of $T^{10}(M)$. It is therefore analogous to the real Hessian, which governs Euclidean convexity. Euclidean convexity implies pseudoconvexity, but the converse fails. See [Hör] and [Kra] for lengthy discussion, especially for the meaning of pseudoconvexity on a domain whose boundary is not smooth.

**Example 3.** The zero set of $r(z) = \text{Im}(z_n)$ is a half-space $\Sigma$. Its horizontal space $\mathcal{H}_\Sigma(\Sigma)$ decomposes into holomorphic and conjugate holomorphic subspaces, spanned respectively by the vector fields $L_j = \partial / \partial z_j$, $j = 1, \ldots, n - 1$, and their conjugates. In this case, the complex Hessian of the defining function is identically equal to zero, and hence the Levi form also vanishes identically.

**Example 4.** The zero set of $r(z) = \sum_{j=1}^n |z_j|^2 - 1$ is the sphere $S^{2n-1}$. The horizontal space $\mathcal{H}_\Sigma(S^{2n-1})$ decomposes into the holomorphic subspace $T^{10}_\Sigma(S^{2n-1}) = \text{span}[L_1, \ldots, L_{n-1}]$ and its conjugate $T^{01}_\Sigma(S^{2n-1})$, where
\[ L_j = z_j \frac{\partial}{\partial z_n} - z_n \frac{\partial}{\partial z_j}. \]
The annihilating one-form can be taken to be
\[ \eta = \frac{1}{2} \sum_{j=1}^n z_j dz_j - z_j d\bar{z}_j. \]
The Levi form, after dividing out a nonzero factor, satisfies $\lambda(L_j, \bar{L}_k) = \delta_{jk} + z_j \bar{z}_k$. Hence the unit sphere is strongly pseudoconvex. The Heisenberg group is also strongly pseudoconvex. Its Levi form, computed similarly, is $\lambda(L_j, \bar{L}_k) = \delta_{jk}$. See [Tre] for an elegant and readable
account of developments in PDE and CR geometry based on this operator.

Positive definiteness of the second fundamental form for a real hypersurface $M$ in $\mathbb{R}^n$ is a curvature condition: it precludes the existence of straight line segments in $M$. The next lemma provides a subtle analogue for the Levi form.

**Lemma 6.** Let $M$ be a strongly pseudoconvex real hypersurface in $\mathbb{C}^n$. Then $M$ contains no complex analytic sets of positive dimension.

**Proof.** We prove the contrapositive statement: if $M$ contains such sets, then $M$ cannot be strongly pseudoconvex. Assume $M$ contains a complex analytic set $V$ of positive dimension. Let $p$ be a nonsingular point of $V$. We can then find a one-dimensional nonsingular holomorphic curve $t \mapsto z(t) \in \mathbb{C}^n$ with $z(0) = p$, with tangent vector $z'(0) \neq 0$, and with $z(t) \in M$ for $|t| < 1$. For each local defining function $r$ for $M$ near $p$ we have $r(z(t)) = 0$ for $|t|$ small.

Taking $\frac{\partial}{\partial t}$ of the identity $r(z(t)) = 0$ and evaluating at $t = 0$ gives

\[ (\frac{\partial}{\partial t} r(p), z'(0)) = 0. \tag{15} \]

Taking the Laplacian $\Delta$ of the same identity and evaluating at $t = 0$ gives

\[ (\Delta r(p), z'(0) \wedge \overline{z'(0)}) = 0. \tag{16} \]

Equation (15) says that $z'(0) \in T_p^{1,0}(M)$ and equation (16) says that $z'(0)$ is in the null space of the Levi form. Since $z'(0) \neq 0$, $M$ is not strongly pseudoconvex at $p$. \hfill $\square$

**Sub-Riemannian Geometry**

It has been evident so far that the stratification of the complexified tangent spaces of a CR manifold given by the horizontal bundle together with the missing direction is at the heart of CR geometry. In this section we will consider real manifolds equipped with a similar horizontal subbundle of the (uncomplexified) tangent bundle. In both cases there are horizontal directions which are infinitesimally accessible and missing directions. When the Levi form is nonzero we can recover the missing direction by commutators. The ability to recover missing directions via commutators of horizontal vector fields is the defining property of a sub-Riemannian manifold. This nonintegrability condition translates to a connectivity condition whereby such manifolds are equipped with a singular metric. See Theorem 9.

CR manifolds provide a natural class of examples, but the general theory covers a much wider class of spaces arising in PDE, control theory, geometric group theory, and many other settings. Example 8 puts a sub-Riemannian structure on spaces of $k$th order Taylor polynomials (jets). This example formalizes the well-known procedure for reducing a differential equation of high order to a first-order system. The references [Mon], [Str], [Gro], [Bel], [FS], [CCG], [CDPT], and [CC] provide many examples and discussion of other sub-Riemannian manifolds.

**Differential geometric aspects**

Let $M$ be a smooth real manifold and let $\mathcal{H}(M)$ be a distribution (subbundle) in the tangent bundle $T(M)$. The classical Frobenius theorem deals with the case when $\mathcal{H}(M)$ is integrable, that is, closed under the Lie bracket. In this case $M$ is foliated by $\mathcal{H}(M)$-integral submanifolds. We consider the opposite extreme, when $\mathcal{H}(M)$ is completely nonintegrable: the Lie bracket span of $\mathcal{H}(M)$, at any point $p \in M$, coincides with $T_p(M)$. In this case we say that the pair $(M, \mathcal{H}(M))$ satisfies the bracket-generating property, also known as Hörmander’s condition. Put another way, for each $p \in M$ there exists an integer $s = s(p) < \infty$ so that the values at $p$ of all $s$-fold iterated Lie brackets of vector fields valued in $\mathcal{H}(M)$ fill out the entire tangent space $T_p(M)$. We call $s(p)$ the step of $M$ at $p$. We emphasize that $s(p)$ may depend on the point $p$. In the case when $s$ is uniformly bounded on $M$, we call sup$_{p \in M} s(p)$ the step of $M$.

We elaborate with some examples. If $M$ is a CR manifold of hypersurface type, we define $\mathcal{H}(M)$ as in (10). The missing direction might or might not be obtained via iterated Lie brackets of horizontal vector fields. Example 3 shows that Levi flat hypersurfaces do not satisfy the bracket-generating property. The opposite extreme is the strongly pseudoconvex case. If $L$ is a $(1,0)$ vector field and not zero at $p$, then the bracket $[L, \overline{L}]$ has a component in the missing direction. In this case the missing direction arises upon taking a single Lie bracket of horizontal vector fields, and the induced sub-Riemannian structure is of step two. To be step two we need only one such vector field; we recover the missing direction at $p$ with a single Lie bracket whenever the Levi form is not zero at $p$. The simplest example of a higher step structure is the pseudoconvex hypersurface defined by $\text{Re}(z_2) = |z_1|^{2m}$ in $\mathbb{C}^2$ near the origin. One requires an iterated commutator

$$[\ldots [L, \overline{L}], \ldots, \overline{L}]$$

with $2m$ total brackets to obtain the missing direction. In this case the origin is called a point of type $2m$, and one says that the vector field $L$ is of type $2m$ there. Notice in this case that the step at most points is 2. See page 217 for related information.

**Example 7.** The Heisenberg group $bH_n$ provides the canonical example of a sub-Riemannian manifold. The group law was discussed in the paragraph following (5). As usual, the left invariant vector fields define the Heisenberg Lie algebra. As a basis
We say that a pair \((X, Y)\) of vector fields satisfies the Lie bracket identities
\[
[X_j, Y_j] = -2\frac{\partial}{\partial t^j} T_{\delta j},
\]
where \(w_j = x_j + iy_j\) and \(t = Re(w_j)\). Let \(T = \frac{\partial}{\partial t}\). Then \([X_j, Y_j] = -4T\delta_{jk}\) for \(j, k = 1, \ldots, n-1\). By putting \(L_j = \frac{1}{2}(X_j - iY_j)\) and \(L^*_j = \frac{1}{2}(X_j + iY_j)\) and regarding them as sections of \(T^*(bH_n)\), we return precisely to the CR setting.

**Example 8.** Jet spaces provide a geometric interpretation for Taylor polynomials, by viewing the equivalence classes of \(C^m\) functions modulo \(m\)-jets \((m\text{th order Taylor approximations})\) as points in an abstract space. We illustrate in the one-dimensional case. Consider \(C^m\) functions \(f: \mathbb{R} \rightarrow \mathbb{R}\). The underlying space \(M\) is \(\mathbb{R}^{m+2}\), with coordinates \(x\) (corresponding to the base point \(x_0\) for the Taylor approximation of \(f\)) and \(u_0, \ldots, u_m\) (corresponding to \(f^{(m)}(x_0), \ldots, f(x_0)\)). The sub-Riemannian structure is defined by a family of smooth 1-forms
\[
\eta_j = du_j - u_{j+1}dx, \quad j = 0, \ldots, m-1,
\]
corresponding to the differential equalities \(df^{(j)}(x) = f^{(j)}(x)dx\). These forms, together with \(dx\) and \(du_m\), frame the cotangent bundle \(T^*(M)\). Introduce the dual frame \(X, U_m, \ldots, U_0\) for \(T(M)\). Let \(\mathcal{H}(M)\) be spanned by \(X\) and \(U_m\). The Cartan formula (11) gives
\[
\langle \eta_{j-1}, [U_k, X] \rangle = -\langle d\eta_{j-1}, X \wedge U_k \rangle = \langle du_j \wedge dx, U_k \wedge X \rangle = \delta_{jk},
\]
which yields the Lie bracket identities \([U_j, X] = U_{j-1}, \quad j = 1, \ldots, m\). We observe that \(\mathcal{H}(M)\) is a bracket-generating horizontal distribution of rank two inducing a sub-Riemannian structure on \(M = \mathbb{R}^{m+2}\) of step \(m + 1\), called the \(m\text{th order jet space}\) (on the real line). The first-order jet space is isomorphic (as a Lie group) to the Heisenberg group \(bH_2\). For additional details see [CDPT, Chapter 3.1] or [Mon, Chapter 6].

**The fundamental theorem of sub-Riemannian geometry**
Recall that a Riemannian metric on a smooth manifold \(M\) is a smoothly varying family of inner products defined on the tangent spaces. We say that a pair \((M, \mathcal{H}(M))\) satisfying the Hörmander condition is a sub-Riemannian manifold if it is equipped with a smoothly varying family of inner products \(\langle \cdot, \cdot \rangle_p\) (the sub-Riemannian metric) defined on the horizontal tangent spaces \(H_p(M)\). Such a metric permits us to define notions of length, volume, angle and other geometric concepts for objects taking values in the horizontal subbundle. For instance, a smooth curve \(y: \{a, b\} \rightarrow M\) is termed horizontal if \(y'(t) \in \mathcal{H}_{y(t)}M\) for all \(t\).

As in the Riemannian case, the sub-Riemannian metric induces a distance function \(d: M \times M \rightarrow \mathbb{R}\), in the sense of Carnot-Carathéodory (the so-called Carnot-Carathéodory distance):
\[
d(p, q) = \inf \int_a^b (y'(t), y'(t))^{1/2} dt
\]
over all smooth horizontal curves \(y: [a, b] \rightarrow M\) with \(y(a) = p\) and \(y(b) = q\). The fundamental result is Theorem 9 below, the Chow-Rashevsky theorem. See the preface in [Mon] for the history leading to this theorem.

**Theorem 9.** Assume that Hörmander’s condition is satisfied for the pair \((M, \mathcal{H}(M))\). Then all pairs of points in \(M\) are horizontally connected. Consequently, the Carnot-Carathéodory distance function on a sub-Riemannian manifold \(M\) endows \(M\) with the structure of a metric space.

An adapted Riemannian metric on \(M\) is an extension of the sub-Riemannian metric to the full tangent bundle. If the step of \(M\) is finite, then the topologies defined by the sub-Riemannian metric and any adapted Riemannian metric coincide. However, the geometric and analytic properties of these two metrics differ substantially, unless the step is equal to one. For instance, if \(M\) has step at least two, then these two metrics are never bi-Lipschitz equivalent. (They are bi-Hölder equivalent with Hölder exponent \(\frac{1}{s}\), where \(s\) is the step of \(M\).)

Sub-Riemannian geodesics are smooth horizontal curves \(y\) in \(M\) which locally minimize the length \(\ell(y)\) defined in (18). The horizontality condition is a family of nonlinear constraints which can be understood in the language of control theory. The path planning problem for wheeled motion (including such concrete subproblems as parallel parking an automobile) can be reinterpreted as the problem of finding geodesics joining prespecified points on certain sub-Riemannian manifolds modeled locally on the first Heisenberg group. Again, we reference [Mon] for more details.

Sub-Riemannian geodesics behave rather differently from their Riemannian counterparts. Abnormal geodesics are locally length minimizing curves which fail to satisfy the geodesic equations. Such curves cannot exist in Riemannian geometry. We refer to [Mon] for a thorough discussion of the geodesics in sub-Riemannian spaces and especially the issue of abnormal geodesics. We remark, however, that all geodesics in the sub-Riemannian geometries arising from CR structures on the boundaries of strictly pseudoconvex domains are necessarily normal. Additional information on the structure of sub-Riemannian geodesics can be found in [CCG] and the recent publication [CC].

A rich vein of current research activity in sub-Riemannian geometry that we do not address is the study of the Carnot-Carathéodory geometry of.
submanifolds of sub-Riemannian manifolds. For more information see [CDPT, Chapter 4].

**Example 10.** We equip the Heisenberg group $bH_n$ with a Carnot-Carathéodory metric as follows. The left invariant vector fields from (17) define the horizontal distribution. We declare them to be an orthonormal basis. The geodesics for this metric have a beautiful geometric description. For simplicity, we restrict to the case $n = 2$. In view of the group structure it suffices to describe the geodesics from the identity element $o = (0,0,0)$ to an arbitrary point $p = (x_1,y_1,t_1)$ with $t_1 \leq 0$. Each geodesic is the horizontal lift of a circular arc in the $(x,y)$ plane with endpoints $(0,0)$ and $(x_1,y_1)$ so that the area of the planar region bounded by the arc together with the line segment joining these two points is equal to $-4t_1$. The sub-Riemannian distance from $o$ to $p$ is just the Euclidean length of the projected arc. When $t_1 = 0$ the geodesic is just a ray in the $t = 0$ plane from $o$ to $p$. When $(x_1,y_1) = (0,0)$ geodesics are not unique; the family of geodesics from $o$ to $(0,0,t_1)$ is rotationally symmetric about the $t$-axis.

The Unit 3 Sphere

*Just like the circle and the two-sphere, the three-sphere is very round. But there are some beautiful, classical aspects to its roundness that are not easy to guess from its lower-dimensional sisters.*

William Thurston [Thurston]

We illustrate the preceding circle of ideas by describing the sub-Riemannian geodesics in the three-sphere $S^3$ equipped with the Carnot-Carathéodory metric arising from its canonical CR structure. These geodesics have recently been computed by several authors using different techniques. See [HR], [BR], and [CC, Chapter 10]. We loosely follow the presentation by Hurtado and Rosales [HR]. The geodesics admit a simple and elegant description in terms of the Hopf fibration.

$S^3$ as a Lie group and the Hopf fibration

Hamilton’s search for an “algebra of vectors in $\mathbb{R}^3$”, parallel to the identification of plane vectors with complex numbers, is mathematical folklore. His search led him to develop the theory of the *quaternions*, a noncommutative algebraic structure for vectors in $\mathbb{R}^4$. An arbitrary quaternion takes the form $a + bi + cj + dk$, where $a, b, c, d \in \mathbb{R}$ and $i, j, k$ are indeterminates satisfying $i^2 = j^2 = k^2 = ijk = -1$. We will identify quaternions with pairs of complex numbers by the rule $p = z_1 + z_2 j \leftrightarrow (z_1, z_2) \in \mathbb{C}^2$. Thus we identify $\mathbb{C}$ with the subspace of the quaternions $\mathbb{H}$ obtained by setting the coefficients of $j$ and $k$ equal to zero. Note that the quaternionic conjugate $\overline{p}$ is equal to $\overline{z_1 - z_2 \cdot j}$ in this presentation. We point out the elementary identities $z \cdot j = j \cdot z$ and $z \cdot k = k \cdot z$ for $z \in \mathbb{C}$.

We view $S^3$ as the set of unit quaternions $p = z_1 + z_2 j$, where $|z_1|^2 + |z_2|^2 = 1$. With the preceding conventions in place, $S^3$ is equipped with the Lie group law

$$p \cdot p' = (z_1 + z_2 j) \cdot (z_1' + z_2' j) = (z_1 z_1' - z_2 z_2') + (z_1 z_2' + z_2 z_1') j.$$  

It is well known that $S^3$ is parallelizable: its tangent bundle admits a smoothly varying orthonormal frame. The quaternionic presentation provides an elegant description for such a frame: just consider the vector fields $U$, $V$, and $W$, where $U(p) = i \cdot p$, $V(p) = j \cdot p$, and $W(p) = k \cdot p$. These vector fields are linked by the bracket relations


In particular, setting $\mathcal{H}(S^3) = \text{span}\{V, W\}$ defines a bracket-generating distribution on $S^3$. If we set $p = z_1 + z_2 \cdot j$, then $V(p) = j \cdot (z_1 + z_2 j) =$
We begin by observing that the annihilating one-form $\eta$ from (14) and the corresponding horizontal distribution $\mathcal{H}(S^3)$ are invariant under the $S^1$ action. This remark suggests that the structure of the sub-Riemannian geodesics should be related to the Hopf fibration. We now confirm this suggestion.

Extend the inner product to a Riemannian metric $g$ on $S^3$ by declaring $U$, $V$ and $W$ to be orthonormal. Let $D$ be the Levi-Civita connection for this metric $g$. Define a bundle isomorphism $J : \mathcal{H}(S^3) \to \mathcal{H}(S^3)$ by setting $J(V) = W$ and $J(W) = -V$. The data $(S^3, \mathcal{H}(S^3), \eta, J)$ defines a pseudo-Hermitian structure on $S^3$, and the ensuing discussion can be framed in the language of pseudo-Hermitian geometry. See, for instance, [CHMY] and [DT].

The sub-Riemannian geodesics are critical points for the first variation of (horizontal) length by horizontal curves with fixed endpoints. Let $y : I \to S^3$ be a $C^2$ horizontal curve defined on a compact interval $I$. An admissible variation of $y$ is a $C^2$ map $G : I \times (-\epsilon_0, \epsilon_0) \to S^3$ satisfying

1. $G(s, 0) = y(s)$ for all $s \in I$,
2. $G(s, \epsilon) = G(s, 0)$ for all $\epsilon$ and all $s \in \partial I$,
3. For each $\epsilon$, $y_\epsilon(s) := G(s, \epsilon)$ is horizontal.

Let $X$ be the vector field along $y$ whose value at $y(s)$ is equal to $(\partial G/\partial \epsilon)(s, \epsilon)|_{\epsilon=0}$. Admissibility of the variation can be characterized in terms of $X$: $G$ is admissible if and only if $X$ vanishes at the endpoints of $y$ and

$$y'(\langle X, U \rangle) = 2(\pi_H X, J(y')),$$

where $\pi_H$ denotes projection into the horizontal space.

The classical formula for variation of length in $(S^3, g)$, specialized to admissible variations, reads

$$\frac{\partial}{\partial \epsilon} \ell(y_\epsilon)|_{\epsilon=0} = -\int_I \langle D_y y', U \rangle.$$

Considering variations of the form $X = f J(y')$ for $C^1$ functions $f : I \to \mathbb{R}$ with $f|_{\partial I} = 0$ and $\int_I f = 0$, and taking into account (20), one finds that the local minimizers for the sub-Riemannian length functional (18) are characterized by the Euler–Lagrange equation

$$D_y y' + 2\kappa J(y') = 0,$$

where $\kappa$ is a real constant. This constant $\kappa$ can be interpreted as a curvature of the geodesic $y$. Geodesics of curvature zero are just great circles on $S^3$. Geodesics of nonzero curvature are horizontal lifts of non-great circles on $S^3$ through the Hopf fibration. Indeed, consider a smooth curve $y$ taking values in $S^3$ and parameterized by arc length. The tangent space to $C^2$ at $p = y(t)$ splits as $T_p(C^2) = T_p(S^3) \oplus T_p(S^3)^\perp$, which yields the following decomposition for the acceleration vector:

$$y''(t) = D_y y'(t) - y'(t).$$

Then (21) reads $y'' + 2\kappa J(y') + y = 0$. Viewing $S^3$ as a subset of $C^2$, this equation takes the form

$$y'' + 2i\kappa y' + y = 0.$$

Equation (23) is a system of constant coefficient second-order ODE's whose explicit solution is easily obtained. Rather than presenting this solution, however, we focus on its geometric content.

**Theorem 11** (Hurtado–Rosales). Let $y$ be a $C^2$ horizontal curve on $S^3$ parameterized by arc length. Then the following are equivalent:

1. $y$ is a geodesic of curvature $\kappa$ in the sub-Riemannian metric on $S^3$,
2. $\langle y'', J(y') \rangle = -2\kappa,$
(3) $\gamma$ is the horizontal lift under the Hopf fibration $S^3 \rightarrow S^2$ of a piece of a circle in $S^2$ of constant geodesic curvature $\kappa$.

We recall that the geodesic curvature of a space curve $\gamma$ contained in $S^2$ is its curvature computed relative to the embedding in $S^2$. For a curve $\gamma$ parameterized at arbitrary speed, this curvature is given explicitly by the formula $\kappa = \langle y' \times y'' \rangle / |y'|^3$. For $|h| < 1$, the geodesic curvature of the circle $\gamma = S^2 \cap \{ z = h \}$ is $h/\sqrt{1 - h^2}$.

We sketch the proof of Theorem 11. To see that (1) and (2) are equivalent, note that the triple $\{ y', J(y'), U(y) \}$ forms an orthonormal frame for $TS^3$ along $\gamma$. The fact that $\gamma$ is horizontal and arc length parameterized immediately gives that the components of $D_{y'}y'$ in the directions $y'$ and $U(y')$ are equal to zero. Thus $D_{y'}y'$ is a multiple of $J(y')$ and (21) can be rewritten $\langle D_{y'}y', J(y') \rangle = -2\kappa$. Equation (22) shows that $\langle y'', J(y'') \rangle = \langle D_{y'}y', J(y') \rangle$, since $y'(t)$ is normal to the sphere and $J(y'(t))$ is tangent to the sphere. To see the equivalence of (1) and (3), one calculates the binormal of the Hopf projection of $\gamma$, which turns out to be a constant. Thus this projection is a circle on $S^2$. Another calculation shows that its geodesic curvature agrees with the curvature $\kappa$ from (21).

The topological structure of these geodesics is quite interesting.

Proposition 12 (Hurtado–Rosales). If $h := \frac{\kappa}{\sqrt{1 + \kappa^2}}$ is a rational number, then $\gamma$ is a closed curve, isotopic to a torus knot inside a group translate of the Clifford torus $T_h$. If $h$ is irrational, then $\gamma$ is diffeomorphic to $\mathbb{R}$ and is dense in some group translate of $T_h$.

Figure 3. The Hopf fibration and sub-Riemannian geodesics.

Curves of the latter type are the well-known skew lines on the torus. The Clifford tori in $S^3$ are tori of the form $T_{\rho} := \{(z_1, z_2) \in S^3 : |z_1|^2 = (1 + \rho)/2, |z_2|^2 = (1 - \rho)/2\}$, where $|\rho| < 1$. Figure 3 illustrates the Hopf fibration, sub-Riemannian geodesics, and the Clifford tori.

Group-invariant Mappings between Spheres in Complex Spaces

The unit sphere $S^3$ has dominated our discussion, because of its symmetry and its role in both CR and sub-Riemannian geometry. Earlier we observed an action of the circle on $S^3$ defined by $(z_1, z_2) \rightarrow (e^{i\theta}z_1, e^{i\theta}z_2)$. In this section we study symmetries of $S^3$ via actions of finite subgroups of $U(2)$. The simplest case is when the group is cyclic; even in the cyclic case there is a remarkable depth and breadth of ideas. Given a positive integer $p$ and an integer $q$ with $1 \leq q < p$, we choose a primitive $p$-th root of unity $\alpha$, and consider the group action $(24)$

$$\alpha^q \cdot (z_1, z_2) = (\alpha^q z_1, \alpha^q z_2).$$

The quotient of the sphere by this group is called a lens space, written $L(p, q)$. Lens spaces, introduced by Tietze in 1908, are important in topology. The fundamental group of $L(p, q)$ is cyclic of order $p$ for all $q$, and thus lens spaces with different $p$ are not homeomorphic equivalent. Famous work of Alexander in 1919 showed that the lens spaces $L(5, 1)$ and $L(5, 2)$ are not homeomorphic even though they have isomorphic fundamental groups and the same homology. Necessary and sufficient conditions on $p$ and $q$ for being homeomorphic known; similarly, necessary and sufficient conditions on $p$ and $q$ for being homotopy equivalent are also known. In Figure 4, the colors represent successive iterations of the group action (24). See http://lukyanenko.net/math/heisenberg09/lens.html for animation and additional details.

Figure 4. A provocative view of the lens space $L(5, 2)$.

Lens spaces also arise in CR geometry. We naturally ask the following question. Given a finite subgroup $\Gamma$ of $U(n)$, is there an integer $N$ and a nonconstant rational mapping, invariant under $\Gamma$, from $S^{2n-1}$ to $\mathbb{R}^{2N}$? The answer is, in general, no. In fact, according to a theorem of Lichtblau [Lic], such a map can exist only when $\Gamma$ is cyclic. Furthermore, (see [D1]), most representations of cyclic groups are also ruled out. We describe the results in the

216
NOTICES OF THE AMS
VOLUME 57, NUMBER 2
special case when \( n = 2 \). Roughly speaking, the only possibilities for nonconstant invariant rational maps to spheres are the special cases when the quotient space is \( L(p, 1) \) or \( L(p, 2) \). One amazing consequence is that there are homeomorphic lens spaces, for example \( L(4, 1) \) and \( L(4, 3) \), only one of which (in this case \( L(4, 1) \)) admits a nonconstant rational CR mapping to a sphere.

We do not give up. In order to get more groups involved, we can proceed in two ways. First, we can weaken the assumptions on the mapping. Smooth CR mappings in this context must be rational. Second, we could allow continuous functions satisfying the tangential Cauchy-Riemann equations in the sense distributions. We then obtain spheres as targets of group-invariant maps for any fixed-point free subgroup of \( \mathcal{U}(n) \). Such results require considerable machinery in function theory of several complex variables. If we weaken the assumptions on the target by allowing hyperquadrics, then more elementary but fascinating ideas appear. Given a finite subgroup \( \Gamma \) of \( \mathcal{U}(n) \), by [D1] we can always find an \( N \) and a nonconstant polynomial mapping \( p : \mathbb{C}^n \to \mathbb{C}^N \) such that \( p \) is \( \Gamma \)-invariant, and \( p \) maps the unit sphere to a hyperquadric. The polynomial \( p \) arises from the following construction. First, one defines the Hermitian symmetric polynomial \( \Phi \) as follows:

\[
\Phi(z, \overline{w}) = 1 - \prod_{y \in \Gamma} (1 - \langle yz, w \rangle).
\]

By elementary linear algebra, there exist holomorphic vector-valued \( \Gamma \)-invariant polynomial mappings \( g, h \) such that

\[
\Phi(z, \overline{z}) = \sum_{j=1}^{N_2} |g_j(z)|^2 - \sum_{j=1}^{N_1} |h_j(z)|^2.
\]

Then the polynomial map \( p = (g, h) \) has the desired properties. Even for the cyclic subgroups \( \Gamma(p, q) \) used to define lens spaces, the computation of \( \Phi_{(p,q)} \) is interesting. For \( \Gamma(p, 1) \) the result is \( (|z_1|^2 + |z_2|^2)^p \). For \( \Gamma(p, 2) \) and \( \Gamma(p, p - 1) \) explicit formulas exist. For all \( q \), the formula for \( \Gamma(p, q) \) is a polynomial \( f_{p,q} \) in the variables \( |z_1|^2 \) and \( |z_2|^2 \) with integer coefficients. In general these polynomials have many interesting properties; we give here a few examples. By [D2], for each \( q \), the congruence

\[
f_{p,q}(|z_1|^2, |z_2|^2) \equiv |z_1|^{2p} + |z_2|^{2p} \quad (\text{mod } p)
\]

holds if and only if \( p \) is prime. A combinatorial interpretation of the integer coefficients appears in [LWW]. See [Mus] for an application to counting points on elliptic curves. See [D2] for many additional properties of these polynomials. See [BR] for a study of sub-Riemannian geometries on the lens spaces \( L(p, q) \); these geometries are obtained by projecting the horizontal distribution from \( S^5 \) to \( L(p, q) \) via the natural quotient map.

### Finite Type and Higher Brackets

Iterated commutators of vector fields provide a systematic method for dealing with higher order conditions, and their role in sub-Riemannian geometry has been evident in this article. Such conditions are also significant in partial differential equations. Given a finite set of vector fields \( X_1, \ldots, X_k \) in a neighborhood of a point in \( \mathbb{R}^d \), consider the operator \( T = \sum_j x_j^2 \). Such *sums of squares* of vector fields arise throughout partial differential equations and probability. When the \( X_j \) form a basis for the tangent space, \( T \) is elliptic, and hence hypoelliptic. Hypoellipticity means that solutions \( u \) to the equation \( Tu = f \) are smooth when \( f \) is smooth. Subellipticity is almost as useful as ellipticity, because it also implies hypoellipticity. An elliptic operator of order two gains two derivatives in the Sobolev scale; a subelliptic operator gains fewer derivatives than its order, but the gain suffices to establish regularity results. The Hörmander condition, applied to the span of the \( X_j \)'s, provides the necessary and sufficient condition for subellipticity of \( T \).

On the other hand, subellipticity for systems is much more difficult. In two dimensions, subellipticity for the Cauchy-Riemann equations reduces to estimates for a scalar sum of squares operator. The following theorem precisely relates the gain in a subelliptic estimate with the step of the horizontal distribution on the boundary. We omit the technical definition of a subelliptic estimate with gain \( \epsilon \). Kohn established the first subelliptic estimate for weakly pseudoconvex domains in \( \mathbb{C}^2 \) under the condition of finite step, and Greiner established the necessity of this condition. The sharp result in Theorem 13 relies on a sophisticated theory of estimates developed by Rothschild and Stein [RS] based on the sub-Riemannian geometry of nilpotent Lie groups; their theory made it possible to relate the geometry and the estimates in a precise fashion.

**Theorem 13.** Let \( \Omega \) be a smoothly bounded pseudoconvex domain in \( \mathbb{C}^2 \), and suppose \( p \in b \Omega \). The following are equivalent:

- There is a subelliptic estimate at \( p \) with gain \( \epsilon \), but no larger value of \( \epsilon \).
- For an \( (0,1) \) vector field \( L \) on \( b \Omega \) with \( L(p) \neq 0 \), we have \( \text{type}(L, p) = 2m \).
- The bundle \( \mathcal{H}(M) = T^{10}(M) \oplus T^{01}(M) \) has step \( 2m \) at \( p \).
- The maximum order of contact at \( p \) of a complex analytic curve with \( b \Omega \) equals \( 2m \).

In higher dimensions things are more subtle; algebraic geometry enters the story. Consider the hypersurface \( M \) in \( \mathbb{C}^3 \) defined by

\[
\text{Re}(z_3) = |z_1| - |z_2|^2.
\]

The step at the origin is 4, and each \( (1,0) \) vector field has type either 4 or 6 there. Thus each such
vector field is of finite type at the origin. On the other hand, \( M \) contains the complex analytic variety \( V \) defined by \( z_1^2 - z_2^2 = z_3 = 0 \), which has a cusp at the origin. (See Figure 5.) At all nonsingular points of \( V \), the step is 2, but there is a \((1, 0)\) vector field (tangent to \( V \)) of infinite type. In particular the condition that each \((1, 0)\) vector field be of finite type holds at the origin but fails at nearby points. This failure of openness has consequences in the study of subelliptic estimates. The condition for subellipticity must be an open finite-type condition. The answer turns out to be that there is a bound on the order of contact of (possibly singular) complex analytic one-dimensional varieties with \( M \) at \( p \). See [Cat], [CD], [D1], [D3], [DK], and [Koh1].

**Figure 5. A singular variety in the hypersurface \( M = \{ z \in \mathbb{C}^3 : \text{Re}(z_3) = |z_1^2 - z_2^2|^2 \} \).**

**Concluding Remarks**

Math 499, a required course for first-year graduate students at the University of Illinois, exposes students to a wide variety of mathematical points of view through engaging invitations to the research areas of the speakers. This note arose from discussing the background common to talks we have given in this course but includes material well beyond what we actually presented. We hope that we have provided sufficient insights and references to motivate further study.

Much of the material here is only tangentially related to differential geometry, especially if one appeals to the hilarious definition “Differential geometry is the study of those properties invariant under change of notation” sometimes attributed to Calabi. Regardless of the definition, differential geometry encompasses a diverse collection of subjects, including both CR geometry and sub-Riemannian geometry. While both these subjects have their roots in mapping theorems from complex analysis, they have diverged considerably over the past forty years. Perhaps this article is a small step toward their eventual convergence.

**Acknowledgements**

We would like to thank Harold Parks for encouraging us to write an article for the *Notices* and for his useful comments on a preliminary draft. We also acknowledge valuable suggestions from two referees. The Graduate Program at UIUC provided us the opportunity to speak in the Math 499 series. Luca Capogna, Joe Kohn, Chris Leininger, and Tom Nevins have invariably shared useful geometric insights with us over the years and thus indirectly aided the preparation of this article. The pictures of lens spaces, the Hopf fibration, and the sub-Riemannian geodesics on \( S^3 \) were produced by Anton Lukyanenko using MATHICA; we gratefully recognize his contribution. We acknowledge the National Science Foundation for research funding under NSF Grants DMS-0500765 and DMS-0753978 (JPD) and Grants DMS-0555869 (JTT).

**References**


[CNS] D.-C. Chang, A. Nagel, and E. M. Stein, Estimates for the \( \bar{\partial} \)-Neumann problem in...


One of the oldest problems to have been solved using the calculus of variations was to find the equation for the shape formed by a hanging chain or flexible cord. Most often it is said that Galileo was the first to pose the problem and that he claimed that the resulting curve was a parabola. (See, for example, Goldstine [5], p. 32, footnote 42, and Truesdell [18], pp. 43–44.) Both of these statements are at best half true. Galileo’s exact words (in a classic English translation of his book Two New Sciences [4], p. 290) are

…a cord stretched more or less tightly assumes a curve which closely approximates the parabola… the coincidence is more exact in proportion as the parabola is drawn with less curvature or, so to speak, more stretched; so that using parabolas described with elevations less than 45° the chain fits its parabola almost perfectly…

Nowhere, to my knowledge, does Galileo address the question of finding the exact shape of the curve formed by the hanging chain. And he is exactly right that for a parabola with elevation less than 45° the curve formed by the chain is an extremely close approximation to the parabola. (See Figure 1.)

This passage follows immediately after an extended discussion during which Galileo presents one of his most famous discoveries: that the path of a projectile in a vacuum under the influence of gravity is a parabola ([4], pp. 244–290). In the course of the discussion, he points out that for a fixed initial velocity, the range will be maximized by an initial elevation of 45°. Finally he addresses the question of how to quickly draw a number of parabolas, leading to the passage cited above.

The reason that one might mistake Galileo’s intentions and conclusions is that elsewhere in Two New Sciences ([4], p. 149) Galileo addresses the same problem and says simply that the “chain will assume the form of a parabola.” The reason that virtually everyone refers to this passage rather than the one in which Galileo spells out his conclusions in far greater detail and complete accuracy is not as clear.

Some of Galileo’s earliest experiments with hanging chains apparently date from the summer of 1602, when he was working with Guidobaldo del Monte. Had he thought to pose and try to answer the question of the exact equation for the shape of a hanging chain, he would have encountered two serious obstacles. First, the essential tool needed—the calculus—had not yet been invented. Second, the equation for the curve involves logarithms, which had also not yet been invented.

After the problem was finally explicitly stated, it was treated frequently by a number of authors between 1690 and 1720 and the correct answer derived, although some of the initial reasoning used was decidedly suspect. The curve became known as a *catenary*. Its equation is $y = \cosh x$, the hyperbolic cosine, up to change of coordinates.

One method used to arrive at this result was the calculus of variations, finding the shape of a curve whose center of gravity is the lowest among all curves having a prescribed length and prescribed endpoints.

The Gateway Arch

It was Robert Hooke who in 1675 made the connection between the ideal shape of an arch and that of a hanging chain in an aphorism that says, in abbreviated form, “As hangs the chain, so stands the arch.” In other words, the geometry of a standing arch should mirror that of a hanging chain. The horizontal and vertical forces in a hanging chain must add to a force directed along the chain, since any component perpendicular to the chain would cause it to move in that direction to gain equilibrium. Similarly, one wants the combined forces at each point of an arch to add up to a vector tangent to the arch. In both cases, the horizontal component of the force is constant and simply transmitted along the arch or chain, while the vertical forces are mirror images.
At least in part for these reasons, the shape of the Gateway Arch is often described mistakenly as a catenary (when not even more mistakenly as a parabola). In fact, the equation on which the arch is based is

$$y = A \cosh Bx + C,$$

which is a catenary only if $A = 1/B$. For the Gateway Arch, $A$, $B$, and $C$ are numerical constants, with $A$ approximately equal to .69(1/B). As a result, those who wish to be more accurate describe the shape of the Arch as a “modified catenary” or, more often, a “weighted catenary”. The idea of the weighted catenary is to use either a chain with literal weights attached at various points or else a continuous version, such as a cord of varying density. (Both of those methods, incidentally, were used by Saarinen to find a shape for the Gateway Arch that appealed to him esthetically.)

Some natural questions to ask are:

1. Is there a density function that yields the curve defined in equation (1) above, and if so, what is that function?
2. How descriptive is the term “weighted catenary”? In other words, what are the curves that fall in this category?

In order to answer these questions, we start with some precise definitions.

**Definitions.**

1. A weighted chain $C$ is a pair $(f(x), \rho(s))$ where $f(x)$ is a suitably smooth function on an interval $x_1 \leq x \leq x_2$, $s$ is arclength of the curve $y = f(x)$, and $\rho(s)$ is a positive continuous function of $s$.

   - The function $\rho(s)$ is called the density function of $C$.

2. A weighted catenary is a weighted chain $C$ in which
   
   (i) $-\infty < f'(x_1) < 0 < f'(x_2) < \infty$,
   
   (ii) $f''(x)$ is continuous and positive,
   
   (iii) for $x_1 \leq x \leq x_2$,

   $$f'(x) = V(X)/H,$$

   with

   $$V(X) = \int_{x_0}^{X} \rho(s(x)) \frac{ds}{dx} dx$$

   where $f'(x_0) = 0$, and $H$ is a constant that depends on both $f$ and $\rho$:

   $$H = V(x_2)/f'(x_2).$$

There is a certain amount of arbitrariness in these definitions. The smoothness of the function $f(x)$ is deliberately left vague, since one might well want to allow different classes of functions in different situations. Similarly, one could allow the density function to have point masses or to be even more general.

Nonetheless, conditions (i)–(iii) are natural ones based on the physics underlying the problem. As noted earlier, for equilibrium we want the sum of the horizontal and vertical forces acting on the chain to be a vector tangent to the curve. The horizontal force is constant and simply transmitted along the curve, while the vertical force is due to gravity acting on the part of the chain from the lowest point to the point that we are considering on the curve and is equal to the total weight of that part of the chain.

Note that we do not require that the endpoints be at the same height—in other words, that $f(x_1) = f(x_2)$—although that will be true in all the examples of interest to us. In that case, condition (i) would follow from the physics of the situation. However, condition (i) does guarantee the existence of an interior point $x_0$ of the interval where $f' = 0$, while condition (ii) implies that there is a unique such point.

Note also that the fact that there is a constant nonzero horizontal component to the force acting on the curve implies that the slope cannot be infinite at the endpoints. For example, a weighted catenary cannot be in the form of a semicircle.

Note finally that if $\rho(s)$ is multiplied by a positive constant, then both $H$ and $V(x)$ are multiplied by the same constant. It follows that if the pair $(f(x), \rho(s))$ defines a weighted catenary, then so does the pair $(f(x), c \rho(s))$ for any positive constant $c$.

We are now able to state our first result.

**Theorem 1.** If $f(x)$ satisfies conditions (i) and (ii) above, then there is a function $\rho(s)$, unique up to a multiplicative constant, such that the pair $(f(x), \rho(s))$ is a weighted catenary. Namely

$$\rho(s(x)) = \frac{Hf''(x)}{\sqrt{1 + f''(x)^2}},$$

where $H$ is an arbitrary positive constant. The value of $H$ is then given by

$$H = \frac{W}{|f'(x_1)| + |f'(x_2)|},$$
with
\[ W = \int_{x_1}^{x_2} \rho(s(x)) \frac{ds}{dx} dx \]
equal to the total weight of the chain.

Proof. Suppose first that there exists a function \( \rho(s) \) such that the pair \((f(x), \rho(s))\) is a weighted catenary. Then by equations (2) and (3), for \( X \geq x_0 \),
\[ Hf''(X) = V'(X) = \rho(s(X)) (ds/dx)|_{x=X}. \]

Since \( ds/dx = \sqrt{1 + f'(x)^2} \), it follows that \( \rho(s) \) must have the form given in equation (5). For \( X < x_0 \),
\[ V(X) = \int_{x_0}^{X} \rho(s(x)) (ds/dx) dx \]
and
\[ Hf''(X) = V'(X) = \rho(s(X)) (ds/dx)|_{x=X} \]
as before. Finally, the total weight \( W \) is
\[ W = \int_{x_1}^{x_2} \rho(s(x)) ds = \int_{x_1}^{x_0} + \int_{x_0}^{x_2} = V(x_2) - V(x_1). \]
But \( V(x_2) = Hf''(x_2) > 0 \), and \( V(x_1) = Hf''(x_1) < 0 \), so that \( H \) takes the form indicated.

This proves that the density function \( \rho \) is uniquely determined by the function \( f(x) \) up to the choice of the constant \( H \).

For the existence of a suitable density function \( \rho(s) \), given \( f(x) \), define \( \rho(s) \) by equation (5). Then
\[ V(X) = \int_{x_0}^{X} \rho(s(x)) \frac{ds}{dx} dx = \int_{x_0}^{X} Hf''(x) dx \]
\[ = Hf'(X) - Hf'(x_0) = Hf'(X), \]
so that the pair \((f(x), \rho(s))\) form a weighted catenary, and then the constant \( H \) must have the form indicated in equation (6). \( \square \)

Example 1 (catenary),
\[ f(x) = \frac{1}{B} \cosh Bx + C. \]
Then
\[ f'(x) = \sinh Bx, \quad \sqrt{1 + f'(x)^2} = \cosh Bx \]
and \( f''(x) = B \cosh Bx \), so
\[ \rho(s(x)) = HB, \quad \text{constant}. \]

Comment: This is a kind of inverse to the original catenary problem of asking for the shape taken by a uniformly weighted hanging chain. It tells us that only a uniform chain will result in a catenary.

Example 2 (parabola),
\[ f(x) = ax^2 + b, \quad f'(x) = 2ax, \quad f''(x) = 2a. \]
Then
\[ \rho(s(x)) = 2aH/\sqrt{ds/dx} \]
and
\[ \int \rho(s) ds = \int \rho(s(x)) \frac{ds}{dx} dx = 2aH \int dx, \]
and
\[ \int_{s(x_1)}^{s(x_2)} \rho(s) ds = 2aH(x_2 - x_1). \]

Comment: This is in a sense the answer to Galileo's original question of finding a mechanical method for drawing a parabola. It tells us exactly how we have to weight a chain so that it will hang in the form of a parabola. The key is the weight distribution has to be uniform in the horizontal direction. For this reason, it is usually stated that the cables on a suspension bridge will hang in the shape of a parabola, since the weight of the cables themselves, together with that of the vertical support cables holding up the roadway, is small compared to the weight of the roadway, which has (in essence) a uniform horizontal weight distribution.

A curious by-product of the equation for the density \( \rho \) in the case of a parabola is that, since \( ds/dx \geq 1 \), with equality only where \( f'(x) = 0 \), it follows that \( \rho(s(x)) \) is maximum at the vertex and then decreases monotonically as \( |f'(x)| \) increases. In other words, the chain has to be weighted the most near the vertex and then decrease as the steepness of the curve increases. As a result, if Saarinen had decided that he found a parabolic arch most pleasing esthetically, he would have been faced with the paradox that in order to have the line of thrust be everywhere directed along the arch, the arch would have to be thickest at the top and taper down toward the bottom, which would be both ungainly esthetically and potentially disastrous structurally.

Example 3 (circular arc),
\[ x = R \cos \theta, \quad y = R \sin \theta, \quad \pi < \theta_1 \leq \theta \leq \theta_2 < 2\pi. \]
Then
\[ f'(x) = \frac{dy}{d\theta} \frac{dx}{d\theta} = -\cot \theta, \quad f''(x) = -\frac{1}{R} \csc^3 \theta, \]
while
\[ s = R \theta, \quad ds/d\theta = -\csc \theta > 0. \]
So
\[ \rho(s(x)) = \frac{H}{R} \csc^2 \theta = HR/y^2. \]

Comment: As noted earlier, a full semicircle cannot be a weighted catenary. We see explicitly that the density function would tend to infinity. However, any circular arc that is short of a semicircle at both ends can be realized (at least in theory) as a weighted catenary. The weighting is simply
\[ \rho(s) = \frac{H}{R \cos^2(s/R)} \quad \pi R < s_1 \leq s = R \theta \leq s_2 < 2\pi R. \]

Although this example may seem a typical "textbook example", of no practical interest but merely a case that can be worked out explicitly, the opposite is true. A monumental "Gateway Arch" had been proposed as an entryway to a planned 1942 International Exposition in Rome, but it was derailed by the war. The arch was to be in the form
of a semicircle, and the question of exactly how much it should be tapered was potentially critical. (See [12] for more on this subject.)

Note that this theorem answers the first part of Question 1 above about the existence of a density function yielding the particular “weighted catenary” shape of the Gateway Arch. It also answers Question 2 concerning the term “weighted catenary” itself, which turns out to have essentially no content beyond “convex curve”. Before turning to the second part of Question 1 regarding the exact form of the density function that yields the curve underlying the Gateway Arch, we prove a strong form of the converse to the theorem.

Converse of Theorem 1. Let C be a curve with endpoints \( P_1 = (x_1, y_1, 0) \) and \( P_2 = (x_2, y_2, 0) \), where \( x_1 < x_2 \). Assume that C is parameterized by arc length s in the form

\[
X(s) = (x(s), y(s), z(s)), \quad P_1 = X(s_1), \quad P_2 = X(s_2).
\]

Let \( \rho(s) > 0 \) be a continuous function along C, taken as a density function. Assume gravity acts in the negative y-direction, and the pair \((C, \rho(s))\) is in equilibrium. Then:

1. C lies entirely in the \( x, y \)-plane.
2. C is in the form of a graph \( y = f(x), \quad x_1 \leq x \leq x_2 \).
3. \( f'(x) \) is finite and monotone increasing for \( x_1 \leq x \leq x_2 \).

The idea underlying the proof is that if we know that C is a graph and \((C, \rho(s))\) is in equilibrium, so that equation (2) above holds, where \( V(X) \) is defined by (3), then

\[
-f'(x_1) = \frac{1}{H} \int_{x_1}^{x_0} \rho(s(x))(ds/dx)dx < \infty,
\]

\[
f'(x_2) = \frac{1}{H} \int_{x_0}^{x_2} \rho(s(x))(ds/dx)dx < \infty,
\]

and for \( x_1 \leq x_3 < x_4 \leq x_2 \),

\[
f'(x_4) - f'(x_3) = \frac{1}{H} \int_{x_3}^{x_4} \rho(s(x))(ds/dx)dx > 0.
\]

Hence condition 3 in the statement of the converse must hold.

In terms of the representation above for C as X(s), the unit tangent vector to C will be given by \( T = dx/ds \). We denote the tension at the point X(s) by \( \tau \), and for the case considered here, the tension vector will be

\[
\tau T = (H, V, W),
\]

where \( H \) is constant and \( W = 0 \), while \( V \) is given by (3). Then

\[
\frac{d}{ds} \tau T = \left( \frac{dV}{ds}, 0, \frac{dV}{ds} / \frac{dx}{ds}, 0 \right) = (0, \rho(s), 0)
\]

by (3), or

\[
\frac{d}{ds} \tau T = \rho(s)j,
\]

where \( j \) is the unit vector directed along the positive y-axis.

The proof of the above converse depends on the fact that equation (9) is the equation of equilibrium for an arbitrary curve joining the points \( P_1, P_2 \) in the \( x, y \)-plane, with no special assumptions about its shape. (See [2], Chapter III, sections 1–3, for a detailed discussion of these questions in maximum generality. I would like to thank Joe Keller for providing this reference, as well as further helpful information. Note that in contrast to [2], we take the density \( \rho \) to mean weight, rather than mass, per unit length, so that the acceleration of gravity is absorbed into it.)

Proof of the converse. Let C be defined by

\[
X(s) = (x(s), y(s), z(s)), \quad s_1 \leq s \leq s_2,
\]

where \( X(s_1) = (x_1, y_1, 0), \quad X(s_2) = (x_2, y_2, 0), \quad x_1 < x_2 \), and let \( \rho(s) > 0 \) be a continuous density function along C. Then the pair \((C, \rho(s))\) will be in equilibrium if and only if (9) holds; that is,

\[
\frac{d}{ds} \tau T = \rho(s)j,
\]

where \( j \) is the unit vector directed along the positive y-axis.

The proof of the above converse depends on the fact that equation (9) is the equation of equilibrium for an arbitrary curve joining the points \( P_1, P_2 \) in the \( x, y \)-plane, with no special assumptions about its shape. (See [2], Chapter III, sections 1–3, for a detailed discussion of these questions in maximum generality. I would like to thank Joe Keller for providing this reference, as well as further helpful information. Note that in contrast to [2], we take the density \( \rho \) to mean weight, rather than mass, per unit length, so that the acceleration of gravity is absorbed into it.)

Proof of the converse. Let \( C \) be defined by

\[
X(s) = (x(s), y(s), z(s)), \quad s_1 \leq s \leq s_2,
\]

where \( X(s_1) = (x_1, y_1, 0), \quad X(s_2) = (x_2, y_2, 0), \quad x_1 < x_2 \), and let \( \rho(s) > 0 \) be a continuous density function along \( C \). Then the pair \((C, \rho(s))\) will be in equilibrium if and only if (9) holds; that is,

\[
\frac{d}{ds} \tau T = \rho(s)j,
\]

where \( j \) is the unit vector directed along the positive y-axis.

The proof of the above converse depends on the fact that equation (9) is the equation of equilibrium for an arbitrary curve joining the points \( P_1, P_2 \) in the \( x, y \)-plane, with no special assumptions about its shape. (See [2], Chapter III, sections 1–3, for a detailed discussion of these questions in maximum generality. I would like to thank Joe Keller for providing this reference, as well as further helpful information. Note that in contrast to [2], we take the density \( \rho \) to mean weight, rather than mass, per unit length, so that the acceleration of gravity is absorbed into it.)

Proof of the converse. Let \( C \) be defined by

\[
X(s) = (x(s), y(s), z(s)), \quad s_1 \leq s \leq s_2,
\]

where \( X(s_1) = (x_1, y_1, 0), \quad X(s_2) = (x_2, y_2, 0), \quad x_1 < x_2 \), and let \( \rho(s) > 0 \) be a continuous density function along \( C \). Then the pair \((C, \rho(s))\) will be in equilibrium if and only if (9) holds; that is,

\[
\frac{d}{ds} \tau T = \rho(s)j,
\]

where \( j \) is the unit vector directed along the positive y-axis.
defining $C$ as a graph. It follows from equations (11) and (12) that
\begin{equation}
\frac{df}{ds} = \frac{1}{c} \tau(s) \left( \frac{dy}{ds} + \frac{1}{c} \int_s^y \rho(\sigma) d\sigma \right).
\end{equation}
Hence $f'(x)$ is a monotone increasing function of $s$ and therefore of $x$. This completes the proof of the converse.

Note that it follows from the above that
\begin{equation}
\frac{d}{ds} f'(x) = \frac{1}{c} \rho(s(x)) \frac{ds}{dx} > 0.
\end{equation}
Since $c$ is the horizontal component of the tension vector, it is the quantity we denoted earlier by $H$, and equation (15) is the same as equation (5) above.

We now come to our principal example of the theorem.

**Definition 1.** A flattened catenary is a curve of the form $y = f(x) = A \cosh Bx + C$, or
\begin{equation}
y = Dg(x) + C, \quad D = AB,
\end{equation}
where $0 < D < 1$, and
\begin{equation}
g(x) = \frac{1}{B} \cosh Bx
\end{equation}
is a catenary.

**Example 4** (flattened catenary). We examine this case in the following theorem.

**Theorem 2.** Let $C$ be a weighted catenary defined by the pair $(f(x), \rho(s))$, and let $p(x) = \rho(s(x))(ds/dx)$. Assume that coordinates are chosen so that $f'(0) = 0$. Then $p(x)$ will be of the form
\begin{equation}
p(x) = ay + b = a f(x) + b
\end{equation}
for some constants $a > 0$ and $b$ if and only if $f(x)$ is a flattened catenary:
\begin{equation}
f(x) = A \cosh Bx + C,
\end{equation}
for constants $A, B, C$.

**Proof.** We have for all $X \in [x_1, x_2]$ that $f''(x) = V(X)/H$, where
\begin{equation}
V(X) = \int_{x_0}^X p(x) dx, \quad H = \int_{x_1}^{x_2} \frac{p(x)}{|f'(x_1)| + |f'(x_2)|} dx.
\end{equation}
Then
\begin{equation}
f''(X) = V'(X)/H = p(X)/H.
\end{equation}
Suppose first that
\begin{equation}
y = f(x) = A \cosh Bx + C.
\end{equation}
Then $f''(x) = AB \sinh Bx$ and
\begin{equation}
f''(x) = AB^2 \cosh Bx = B^2 (f(x) - C).
\end{equation}
Then
\begin{equation}
p(x) = H f''(x) = af(x) + b,
\end{equation}
where
\begin{equation}
a = HB^2 > 0, \quad b = -HB^2 C.
\end{equation}
Conversely, suppose that $p(x) = af(x) + b$. Then $f''(x) = (af(x) + b)/H$. Let
\begin{equation}
g(x) = f(x) + b/a.
\end{equation}
Then $g'(0) = f'(0) = 0$, and
\begin{equation}
g''(x) = f''(x) = ag(x)/H.
\end{equation}
It follows that $g(x) = A \cosh Bx$ where $B^2 = a/H$. Hence
\begin{equation}
f(x) = g(x) - b/a = A \cosh Bx + C,
\end{equation}
where
\begin{equation}
A = f(0) + b/a, \quad B = \sqrt{a/H}, \quad C = -b/a.
\end{equation}

**Historical Note**

The flattened catenary was studied in the nineteenth century by a number of authors in France, England, Ireland, and Scotland. The first may have been Villarceau [19]; see Heyman [6], p. 48. Others include W. J. M. Rankine [13] and A. M. Howe [9]. (I thank William Thayer for these latter references and for alerting me to the connection between the equation for the Gateway Arch and the nineteenth-century application discussed here.) In those works, the flattened catenary is often referred to as a “transformed catenary”, and the term “two-nosed catenary” is used for a highly flattened catenary, for reasons explained below. The context in which it arises is in answer to the question: What is the ideal shape of an arch that supports a horizontal roadway held up by evenly packed dirt on top of the arch? The vertical load applied to each point on the arch is then proportional to the distance from that point up to the level of the roadway. If we write the equation of the arch as $y = f(x)$, where $y$ is the distance from the roadway down to a given point on the arch, then the total vertical thrust acting on the part of the arch from the apex to an arbitrary point of the arch will correspond to the weight of a hanging chain from the minimum point to a given point, and it will be of the form $\int p(x) dx$, where $p(x)$ is a linear function of $y$. The same reasoning as in the above theorem then shows that $f(x)$ should be of the form (16): a flattened catenary. (See, for example, Heyman [6], pp. 48-49.)

We now come to the Gateway Arch itself. At first sight it would appear to be a very different situation from that of an arch supporting a bridge, since it supports nothing except its own weight. On closer look, however, the fact that the Arch is tapered means that the weight supported at each level is not determined just by the length of the arch above it but by an integral with respect to arclength of a function exactly analogous to the density function for a weighted chain. The physical Gateway Arch is reasonably well modeled as one of a class of geometric shapes that are of interest in their own right.
Tube-like Domains
Two chance meetings in 1939 led to one of the major advances in differential geometry, the intrinsic proof by Chern of an \( n \)-dimensional Gauss–Bonnet theorem. The first occurred when mathematician Hermann Weyl attended a lecture by statistician Harold Hotelling in which Hotelling described a new result of his and posed an open question. Hotelling had been led by a statistical problem to pose the question: Given \( r > 0 \), what is the probability that a point chosen at random on the \( n \)-sphere would lie within a distance \( r \) of a given curve \( C \)? The answer amounts to computing the volume of the domain consisting of all points whose distance to \( C \) is less than \( r \) and comparing that to the total volume of the sphere. Hotelling was able to answer the question in both Euclidean \( n \)-space and the \( n \)-sphere for \( r \) sufficiently small [8]. The answer in Euclidean space was that the volume is equal to the length of \( C \) times \( n - 1 \), with an analogous result for the \( n \)-sphere. The answer was perhaps a surprise, since one might think that the volume of the “tube” around the curve \( C \) would depend somehow on the twists and turns in the curve, and not simply on its total length.

What Hotelling wanted to know, and was unable to derive, was a formula for the case that the curve \( C \) was replaced by a surface of two or more dimensions. Hermann Weyl [22] provided the answer in spectacular fashion: for a compact smooth submanifold \( S \) of Euclidean space, the volume of the domain consisting of all points lying within a fixed distance \( r \) of \( S \) is given, for \( r \) sufficiently small, by a polynomial in \( r \) whose coefficients are intrinsic quantities associated with \( S \), and do not depend on the way that \( S \) is immersed in space. Most strikingly, the top coefficient does not even depend on the geometry of \( S \), but only its topology; it is just a constant times the Euler characteristic \( \chi \) of \( S \).

The other chance meeting in 1939 took place between André Weil and Lars Ahlfors in Finland. Weil first learned about the classical Gauss–Bonnet theorem from Ahlfors, who expressed an interest in having a generalization to higher dimensions in order to develop a higher-dimensional Nevanlinna theory of value distribution ([20], p. 558, [21]. When Weil came to America several years later he met Carl Allendoerfer and learned that both Allendoerfer and Werner Fenchel had independently proved a Gauss–Bonnet theorem for manifolds embedded in Euclidean space, using Weyl’s formulas. Remembering Ahlfors’s desire for a general Gauss–Bonnet theorem, Weil was able, together with Allendoerfer, to prove the Gauss–Bonnet theorem for all Riemannian manifolds ([1], [20], p. 299). Their proof went via Weyl’s formulas together with local embedding theorems. Shortly after, using the appropriate form of the integrand from these earlier approaches, Chern was able to provide an intrinsic proof of the Gauss–Bonnet theorem without the detour through embeddings.

Our interest here is in a generalization of Hotelling’s result in a different direction. We go back to the case of a curve \( C \) in Euclidean space, but are interested in much more general domains.

**Notation 1.** A curve \( C \) in \( \mathbb{R}^n \) will be denoted by \( X(s) = (x_1(s), \ldots, x_n(s)), 0 \leq s \leq L \), \( s = \text{arc length parameter}, \)
\[ e_1(s) = dX/ds \]
the unit tangent vector,
\[ e_1(s), e_2(s), \ldots, e_n(s) \]
a positively oriented orthonormal frame field along \( C \).

**Definition 2.** A tube-like domain \( T \) with centerline \( C \) is the image under a diffeomorphism \( F \) of a domain \( D \) in \( \mathbb{R}^n \), with coordinates \( u_1, \ldots, u_n \), such that \( 0 < u_1 < L \), and for any \( t, 0 < t < L \), the intersection \( D_t \) of \( D \) with the hyperplane \( u_1 = t \) contains the point \( (t, 0, \ldots, 0) \). The map \( F \) satisfies \( F(u_1, u_2, \ldots, u_n) = X(u_1) + u_2 e_2(u_1) + \cdots + u_n e_n(u_1) \), with \( u_1 = s = \text{arc length parameter along} C \).

For \( 0 < s < L \), let \( T_s \) be the intersection of \( T \) with the normal hyperplane to \( C \) at the point \( X(s) \). Then \( F : D_s - T_s \) is a linear transformation mapping \( D_s \) isometrically to \( T_s \).

**Theorem 3.** Let \( T \) be a tube-like domain in \( \mathbb{R}^n \) with centerline \( C \). Then the volume \( V \) of \( T \) is given by
\[
V = \int_0^L \left[ \int_{D_t} \tilde{k}(s) \cdot (F(s, u_2, \ldots, u_n) - X(s)) du_2 \ldots du_n \right] ds,
\]
where \( \tilde{k}(s) = d^2X/ds^2 \) is the curvature vector, and for each fixed value of \( s \), the vector
\[ F(s, u_2, \ldots, u_n) - X(s) = u_2 e_2(s) + \cdots + u_n e_n(s) \]
traces out the normal section \( T_s \) of \( T \) as the point \( (s, u_2, \ldots, u_n) \) ranges over the parameter domain \( D_s \).

**Proof.** Let \( J = \det dF \) be the Jacobian of the map \( F, Y = (y_1, \ldots, y_n) \). Then
\[
J = \frac{\partial(y_1, \ldots, y_n)}{\partial(u_1, \ldots, u_n)}, \quad V(T) = \int_{D_s} J du_2 \ldots du_n
\]
\[ = \int_0^L \left[ \int_{D_s} J du_2 \ldots du_n \right] ds.
\]
Now write
\[ Y = X(u_1) + u_2 e_2 + \cdots + u_n e_n, \]
with \( e_j = e_j(u_1), \ 0 \leq u_1 \leq L \). Then
\[
\frac{\partial Y}{\partial u_1} = e_1 + u_1 \frac{de_2}{du_1} + \cdots + u_n \frac{de_n}{du_1} \quad \text{and} \quad \frac{\partial Y}{\partial u_i} = e_i, \quad i = 2, \ldots, n,
\]
and
\[
\frac{\partial Y}{\partial u_1} \wedge \cdots \wedge \frac{\partial Y}{\partial u_n} = \frac{\partial Y}{\partial u_1} \wedge e_2 \wedge \cdots \wedge e_n = fe_1 \wedge \cdots \wedge e_n.
\]
Write
\[
\frac{\partial e_i}{\partial u_1} = \sum_{j=1}^n a_{ij} e_j = a_{i1} e_1 + \cdots ,
\]
where the coefficients are given by
\[
a_{i1} = \frac{de_1}{du_1} \cdot e_1 = - \frac{de_1}{du_1} \cdot e_1 = - \frac{d}{ds} \frac{\partial Y}{\partial u_1},
\]
since the curvature vector \( \hat{k}(s) = \frac{d^2 X}{ds^2} \) equals \( \frac{de_1}{ds} \).

Note. Different versions of this theorem have been known at least since 1978, when Michael Raugh [14] studied the case \( n = 3 \) in the context of the growth and structure of tendrils. Another discussion of the case \( n = 3 \) appears in Chapter 3 (Cálculo de várias variáveis) of [3], pp. 202–210. Both of those discussions assume the existence of a Frenet frame field for the centerline of the domain. A recent paper of Raugh [15] also treats the \( n \)-dimensional case.

Note. For the proof to be valid, one needs the Jacobian to be positive. If it were to change sign, then there would be cancellations, and the integral would not represent the actual volume. Since our definition of a tube-like domain includes the assumption that the defining map is a diffeomorphism, the Jacobian cannot change sign. However, if one starts with a domain \( D \) and tries to represent it as a tube domain, then one must check the positivity of the Jacobian. We shall return to this point later on.

**Corollary 1.** If the point \( X_s \) is the centroid of \( T_s \) for each \( s \in [0, L] \), then
\[
(21) \quad V(T) = \int_0^L V(T_s) \ ds.
\]

Note that it suffices to consider points where \( k(s) \neq 0 \).

**Proof.** For each value of \( s \), the inner integral in the second term of the formula for \( V(T) \) is a sum of terms consisting of integrals over \( D_s \) of terms of the form \( c_j u_i \), where \( c_j \) is a constant. But if \( X(s) \) is the centroid of \( T_s \), then the origin is the centroid of \( D_s \), and each of those integrals must vanish. \( \square \)

**Definition 3.** Under the hypotheses of Corollary 1, the centerline \( C \) is called a centroid curve of \( T \).

A given domain may have more than one centroid curve; for example, a Euclidean ball can be viewed as a tube-like domain with respect to any diameter as centerline, and that diameter will be a centroid curve for the ball.

**Corollary 2 (Hotelling’s theorem).** If \( T \) is a tube domain in \( \mathbb{R}^n \) of radius \( r \) around a curve \( C \) of length \( L \), then
\[
V(T) = \omega_{n-1} r^{n-1} L,
\]
where \( \omega_k \) is the volume of the ball of radius \( r \) in \( \mathbb{R}^k \).

**Proof.** In this case \( D \) is a cylinder of radius \( r \) and height \( L \). For \( 0 < s < L \), the normal section \( T_s \) of \( T \) is an \((n - 1)\)-ball of radius \( r \) centered at the point \( X(s) \) of \( C \). \( \square \)

A number of statisticians returned to questions related to Hotelling’s theorem after a lapse of many years. See the paper [11] and further references given there.

**Special Cases**

\( n = 2 \). Then \( e_2 \) is uniquely determined by \( e_1 = dX(ds) \). The orthogonal section \( T_s \) is an interval \( I_s \), and the second term vanishes if \( X(s) \) is the midpoint of \( I_s \) at all points where the curvature is nonzero.

**Note.** For \( n = 2 \), \( \hat{k}(s) = \kappa(s)e_2(s) \), where the curvature \( \kappa \) can be positive, negative, or zero.

\( n = 3 \). If \( \hat{k}(s) \neq 0 \) at a point, then the same holds in an interval around the point, and we may define \( e_2 = \hat{k}(s)/\kappa(s) \), where \( \kappa(s) = |\hat{k}(s)| \) is the curvature. Then \( e_2(s) \) is a smooth unit vector field on the interval, and \( e_1 \), \( e_2 \), together determine \( e_3 \). Since \( de_1/ds = \hat{k}e_2 \), we have \( a_{21} = -\kappa, \ a_{31} = 0 \), and \( J = 1 - \kappa(s)u_1 \). As already noted, this equation also holds whenever \( \kappa = 0 \). Therefore, for \( n = 3 \),
\[
V(T) = \int_0^L \left[ \int_{T_s} (1 - \kappa(s)u_2) \ du_2 \ du_3 \right] \ ds
\]
\[
= \int_0^L V(T_s) \ ds - \int_0^L \kappa(s)M_s \ ds,
\]
where \( M_i \) is the moment of \( T_s \) about the line through \( X(s) \) in the \( e_i \) (binormal) direction. It follows that for (21) to hold, one does not need \( X(s) \) to be the centroid of the orthogonal section, but only that the moment of that section be zero with respect to the line through \( X(s) \) in the binormal direction, at all points where \( \kappa \neq 0 \). (See Raugh [15].)

An analogous statement holds for arbitrary \( n \).

Another formulation is
\[
\int_{D_s} J \, du_2 \ldots du_n = \int_{D_s} (1 + \sum_{i=2}^{n} a_{1i} u_i) \, du_2 \ldots du_n = V(T_s) + \sum_{i=2}^{n} a_{1i} M_i,
\]
where \( M_i \) is the \( i \)-th moment of \( D_s \), and \( a_{1i} = a_{1i}(s) \).

**Gateway Arch Specifications**

The *shape of the Gateway Arch* is a polyhedral surface, the piecewise linear approximation by quadrilaterals to the surface \( S \) of the theoretical Gateway Arch, with two of the sides of each quadrilateral lying on designated straight-line segments of \( S \).

**Note.** The surface just described is an abstract mathematical representation of the surface of the physical Gateway Arch, that surface consisting of the exterior of a set of stainless steel plates subject to a variety of distortions. Among those are the (uneven) expansion and contraction with changes of temperature, deflection due to wind forces, and deformation under the moving weight distribution of the internal transportation system carrying passengers to the top.

The *theoretical Gateway Arch* is a tube-like domain \( T \) whose centerline \( C \) is the reflection in a horizontal plane of a flattened catenary. The curve \( C \) is given by the equation
\[(22) \quad y = 693.8597 - A \cosh Bx,\]
where \( A \) and \( B \) are the numerical constants
\[(23) \quad A = 68.7672, B = 0.100033,\]
\[(24) \quad -299.2239 \leq x \leq 299.2239,\]
and \( x, y \) represent distances measured in feet. We may also write the equation of \( C \) as
\[(25) \quad y = 625.0925 - Y\]
where
\[(26) \quad Y = A \cosh Bx + C = A(\cosh Bx - 1)\]
represents the vertical distance down from the vertex of \( C \).

The orthogonal section \( T(x,y) \) at the point \((x, y)\) of \( C \) is in the form of an equilateral triangle of area \( Q(x, y) \), where the sides of \( T(x, y) \) when \( x = 0 \) (at the vertex of \( C \)) have length 17 feet and when \( y = 0 \) (at the base of the arch) have length 54 feet. The area \( Q(x, y) \) is defined by interpolating linearly in \( Y \) between the corresponding areas so defined:
\[Q(x, y) = Q_v + (Q_b - Q_v)Y/Y_b,\]
where \( Q_v \) is the cross-sectional area at the vertex, and \( Q_b \) is the corresponding area at the base. Since the area \( Q \) of an equilateral triangle is equal to
\[Q = \frac{\sqrt{3}}{4}d^2,\]
where \( d \) is the length of a side, and since \( d = 17 \) at the vertex \( v \) and \( d = 54 \) at the base, we find
\[Q_v = 125.1406, \quad Q_b = 1262.6651,\]
while \( Y_b = Y|_{y=0} = 625.0925 \). It follows that
\[Q(x, y) = 125.1406 + 1.81977y = 1262.6651 - 1.81977y.\]

Using the relation above between the area \( Q \) and the side \( d \) of an equilateral triangle, we obtain the exact dimensions of the cross-sections at each height \( y \). It remains to specify that the positions of the triangles in the planes orthogonal to the curve \( C \) are symmetric with respect to the \( x, y \)-plane, with one vertex lying in that plane and interior to the curve \( C \), and that the centroid of each of those triangles lies on \( C \), so that \( C \) is a centroid curve of the Arch.

It follows from these specifications that the surface of the theoretical Gateway Arch is the union of three surfaces: a pair of ruled surfaces that are symmetric images with respect to the \( x, y \)-plane and meet in a curve lying in the \( x, y \)-plane, called the *intrados* of the Arch, together with a cylindrical surface orthogonal to that plane: the *extrados*. The maximum height of the extrados occurs at the point directly above the vertex of \( C \) where \( x = 0 \) and \( y = y_v = 625.0925 \). At this point the orthogonal plane is vertical, and the distance from the vertex of \( C \) to the base of the triangle is one third of the length of the median, or \( 17\sqrt{3}/6 = 4.90748 \). Adding this to \( y_v \), gives for the maximum height of the Arch \( h = 629.99998 \).

The width of the Arch is a bit more subtle. We take the points where \( y = 0 \) on the centroid curve \( C \) to represent ground level. At those points, \( x = 299.2239 \) and \( -299.2239 \) so that the width of the curve at ground level is 598.4478. The equilateral triangle representing the cross-section at those points has side 54, and hence the distance to the outer curve of the Arch is
\[54\sqrt{3}/6 = 9\sqrt{3} = 15.58846.\]

Adding this on each side to the width of \( C \) at ground level gives a total width for the Arch of 629.6247. However, since the curve \( C \), although steep, is not vertical at ground level, the cross-section of the Arch is not horizontal, and the actual outer width is slightly larger. However, one sees that the dimensions of the centroid curve together with the size and shape of the cross-sections produce an arch
that for all practical purposes has exactly the same
total height as width. It may be worth noting, how-
ever, since it is sometimes a source of confusion,
that the centroid curve is distinctly taller than wide,
and the same is even more true of the inner curve
of the Arch, whose height is 615.3 feet, and width
is 536.1. For more on this and related matters, see
[12].

Curvature Computations

Let \( y = f(x) = A \cosh Bx = D(1/2 \cosh Bx) \) define
a flattened catenary, where \( A, B \) are positive
constants, and \( D = AB \). Then \( f''(x) = B^2 y > 0 \) so that
\( f \) is convex and the curvature \( \kappa > 0 \). Specifically,
\[
\kappa = \frac{B^2 y}{(B^2 y^2 + 1 - D^2)^{3/2}}.
\]

For a catenary, \( D = 1 \) and \( \kappa = 1/By^2 \), with a
maximum of \( B \) at the vertex \((0, 1/B)\) and decreasing
monotonically to zero as \( y \to \infty \).

In general, let \( g(y) = (B^2/\kappa)^2 = B^4 R^2, R = 1/\kappa. \)
Then \( dR/dy \geq 0 \iff g'(y) \geq 0 \). But one finds
\[
g'(y) = \frac{2}{y^3} (B^2 y^2 + 1 - D^2)^2 (2B^2 y^2 - 1 + D^2)
\]
and since \( y \geq A, B^2 y^2 + 1 - D^2 \geq 1, \) and
\[
g'(y) > 0 \iff 2B^2 y^2 > 1 - D^2.
\]

Case 1: \( D > 1 \). An “elongated” catenary, obtained
by stretching a catenary uniformly in the vertical
direction. Then the right-hand side is negative,
g' \( (y) > 0 \) along the whole curve, so that \( R \) has a
minimum and \( \kappa \) a maximum at the vertex, where
\( x = 0, y = D/B, \) and \( \kappa = BD \).

Case 2: \( D = 1 \). A catenary, for which, as we have
seen, \( R \) has a minimum value of \( 1/B \) at the vertex
and \( R \to \infty \) as \( y \to \infty \).

Case 3: \( 0 < D < 1 \). Then there are two possibilities.

Case 3a: \( D^2 \geq 1/3 \). Since \( y \) takes its minimum
value \( A \) at the vertex, we have
\[
2B^2 y^2 + D^2 \geq 2B^2 A^2 + D^2 = 3D^2 \geq 1
\]
so that \( g'(y) \geq 0 \) along the whole curve, and as
in cases 1 and 2, the curvature has its maximum
value of \( BD \) at the vertex.

Case 3b: \( D^2 < 1/3 \). Then \( 1 - D^2 > 2D^2 = 2B^2 A^2 \),
and we have \( g'(y) < 0 \) in the interval
\[
y|_{y=0} = A \leq y < \left( \frac{1 - D^2}{2B^2} \right)^{1/2}.
\]

In this case, the vertex of the curve will be a local
minimum of the curvature, which will increase
monotonically until \( y \) reaches the value indicated
on the right, where the curvature will reach its max-
imum. The curve is in this case sometimes referred
to as a two-nosed catenary.

Figure 2 shows a two-nosed catenary for which
\( D = 1/3 \), obtained by stretching a catenary uni-
formly in the horizontal direction by a factor of
3, which is equivalent, up to a similarity, to com-
pressing it vertically by a factor of 3.

It may be worth observing that when illustrating
geometric graphs, one often sees different scales
used on the two axes. For a parabola, the resulting
curve will still be a parabola, but for a catenary, the
result will be a flattened or elongated catenary. As
the figure, as well as the above calculation, makes
clear, the shape may be very different geometrically
from a true catenary. Such differences are obvious
when different scales in the horizontal and vertical
directions turn a circle into an ellipse, but they are
often overlooked for noncompact curves such as
the catenary.

Finally, we note that for the centroid curve of
the Gateway Arch, we have \( D = AB = .69 \), well
above the critical value \( D = \sqrt{3}/3 = .577 \ldots \), so
that the curvature will take its maximum value at
the vertex, where it is equal to \( DB = .0069 \) so that
the radius of curvature at that point is \( R = 145 \) feet,
and it grows monotonically from there. Since all
points of the cross-sectional equilateral triangles
that define the Gateway Arch are less than 54 feet
(and in fact, \( < 54\sqrt{3}/3 < 32 \) feet) from the centroid
curve, it follows that the Jacobians entering into
the computations leading up to equation (21) are
all positive, where the Gateway Arch is considered
as a tube-like domain.

Note. This paper is an expanded version of the
Roever Lecture given by the author at Washington
University in St. Louis on March 24, 2009. William H.
Roever was born in St. Louis, and after receiving
his Ph.D. at Harvard University and teaching at MIT,
he returned to Washington University, where he
served as chairman of the mathematics department.
He wrote two papers on the curves of light formed
by reflection under particular circumstances [16],
[17]. He would undoubtedly have been fascinated
by the light curve that forms on the Gateway Arch
when the sun is at a certain angle.

Final Note

Although the essence of Hooke’s dictum relating
the standing arch to the hanging chain seems clear,
there are subtle issues that arise, some involving
the fact that a standing arch is a three-dimensional
body, whereas the hanging chain is modeled just
as a curve. In fact, a careful analysis of the stability
of an arch and its relation to the geometry of the
arch is a subject about which whole books could
be (and have been) written. (See, for example, [6].)

Here is an example of a seemingly paradoxical
fact that does not seem to have been mentioned
anywhere. As we noted at the outset, the fact that
the shape of a hanging chain is a catenary is often
derived by using the calculus of variations, finding
the curve of given length that has the lowest center
of gravity. It follows then, according to Hooke, that
the ideal shape of an arch would be an upside-down
catenary, which would have the highest center of gravity among all arches of the same length. But that would appear to make it maximally unstable.

Some of these and related issues are discussed in the paper [12]. We hope to return to the subject for a fuller discussion at a later date.

Acknowledgment. My thanks to Silvio Levy and Paul Shields for valuable technical assistance.

References
Model Theory and Complex Geometry

Rahim Moosa

Model theory is a branch of mathematical logic whose techniques have proven to be useful in several disciplines, including algebra, algebraic geometry, and number theory. The last fifteen years have also seen the application of model theory to bimeromorphic geometry, which is the study of compact complex manifolds up to bimeromorphic equivalence. In this article I will try to explain why logic should have anything to say about compact complex manifolds. My primary focus will be on the results in bimeromorphic geometry obtained by model-theoretic methods and the questions about compact complex manifolds that model theory poses.

Structures and Definable Sets

Besides being a discipline in its own right, model theory is also a way of doing mathematics. Given a mathematical object, such as a ring or a manifold, we begin by stating explicitly what structure on that object we wish to investigate. We then study those sets that can be described using formal expressions that refer only to the declared structure and whose syntax is dictated by first-order logic. Let me give a few details.

A structure consists of an underlying set $M$ together with a set of distinguished subsets of various cartesian powers of $M$ called the basic relations. It is assumed that equality is a basic (binary) relation in every structure. One could also allow basic functions from various cartesian powers of $M$ to $M$, but by replacing them with their graphs I will, without loss of generality, restrict myself to relational structures. For example, a ring can be viewed as a structure where the underlying set is the set of elements of the ring and there are, besides equality, two basic relations: the ternary relations given by the graphs of addition and multiplication. If the ring also admits an ordering that we are interested in, then we can consider the new structure where we add the ordering as another basic binary relation. The definable sets of a structure are those subsets of cartesian powers of $M$ that are obtained from the basic relations in finitely many steps using the following operations:

- intersection,
- union,
- complement,
- cartesian product,
- image under a coordinate projection, and
- fibre of a coordinate projection.

I am avoiding talking about the logical syntax here, but the reader familiar with some first-order logic will recognize that the basic relations form the language of the structure and the various operations correspond to logical symbols such as conjunction, disjunction, and negation. The operation of taking the image under a coordinate projection corresponds to existential quantification, while that of taking a fibre of a coordinate projection corresponds to substituting parameters for variables (that is, specialisation). The definable sets are then the sets described by first-order formulae. This way of viewing definable sets is an essential feature of model theory, even though many expositions (including this one) avoid formulae by introducing definability set-theoretically, just as I have done here.

Rahim Moosa is professor of mathematics at the University of Waterloo, Canada. His email address is rmoosa@math.uwaterloo.ca.

The author was supported by an NSERC Discovery Grant.
In any case, given a structure we have an associated collection of definable sets. When \((R, +, \times)\) is a commutative unitary ring, for example, it is not hard to see that if \(f_1, \ldots, f_r\) are polynomials in \(R[x_1, \ldots, x_n]\), then the \(\text{algebraic set}\) they define, namely their set of common zeros in \(R^n\), is definable. Hence the finite boolean combinations of such sets, that is, the \(\text{Zariski constructible sets}\), are all definable. It is an important fact that if \(R\) is an algebraically closed field, then these are the only definable. Hence the finite boolean combinations of such sets, that is, the \(\text{Zariski constructible sets}\), are all definable. It is an important fact that if \(R\) is an algebraically closed field, then these are the only definable. Hence the finite boolean combinations of such sets, that is, the \(\text{Zariski constructible sets}\), are all definable. It is an important fact that if \(R\) is an algebraically closed field, then these are the only definable.

But we are interested in a somewhat different sort of example. Fix a compact complex manifold \(X\) and consider the structure \(\mathcal{A}(X)\) where the basic relations are the complex analytic subsets of \(X^n\), for all \(n > 0\). By a \textit{complex analytic subset}, or just \textit{analytic} subset for short, I mean a subset \(A\) such that for all \(p \in X^n\) there is a neighbourhood \(U\) of \(p\) and finitely many holomorphic functions \(f_1, \ldots, f_r\) on \(U\) such that \(A \cap U\) is the common zero set of \(\{f_1, \ldots, f_r\}\). Note that the local data of \(U\) and \(f_1, \ldots, f_r\) are not part of our structure; only the global set \(A\) is named as a basic relation.

The model theory of compact complex manifolds was begun by Zilber’s [14] observation in the early 1990s that \(\mathcal{A}(X)\) is “tame”. In particular, as a consequence of Remmert’s proper mapping theorem, Zilber shows that every definable set in \(\mathcal{A}(X)\) is a finite boolean combination of analytic subsets. But the tameness goes much further, making the geometry of analytic sets susceptible to a vast array of model-theoretic techniques.

Zilber’s analysis of individual compact complex manifolds extends to the many-sorted structure \(\mathcal{A}\), which includes all compact complex manifolds at once, and where all complex analytic subsets are named as basic relations. Note that algebraic geometry is part of this structure; amongst the compact complex manifolds in \(\mathcal{A}\) are the complex projective spaces, and so all quasi-projective algebraic varieties are definable in \(\mathcal{A}\). But there are also nonalgebraic compact complex manifolds, and in some sense the model-theoretic analysis focuses on those. Let me discuss two examples that we will see again later.

Suppose \(\{\alpha_1, \ldots, \alpha_{2n}\}\) is an \(\mathbb{R}\)-basis for \(\mathbb{C}^n\), and \(\Lambda = \mathbb{Z}\alpha_1 + \cdots + \mathbb{Z}\alpha_{2n}\) is the real 2\(n\)-dimensional lattice it generates. Then the quotient \(T = \mathbb{C}^n/\Lambda\) is a compact complex manifold of dimension \(n\), called a \textit{complex torus}, that inherits a holomorphic group structure from \((\mathbb{C}^n, +)\). While some complex tori can be embedded into projective space, if \(\Lambda\) is chosen sufficiently generally, then the resulting torus is nonalgebraic. For example, if the real and imaginary parts of \(\{\alpha_1, \ldots, \alpha_{2n}\}\) form an algebraically independent set over \(\mathbb{Q}\), then \(T\) will not contain any proper infinite analytic subsets. In particular, such tori, which are called \textit{generic complex tori}, cannot be projective varieties if \(n > 1\). Another example of a nonalgebraic compact complex manifold is the following \textit{Hopf surface}: fix a pair of complex numbers \(\alpha = (\alpha_1, \alpha_2)\) with \(0 < |\alpha_1| \leq |\alpha_2| < 1\), and consider the infinite cyclic group \(\Gamma\) of automorphisms of \(\mathbb{C}^2 \setminus \{(0, 0)\}\) generated by \((u, v) \mapsto (\alpha_1 u, \alpha_2 v)\). The quotient \(H_\alpha = \mathbb{C}^2 \setminus \{(0, 0)\}/\Gamma\) is a compact complex surface that is never algebraic. Indeed, this can already be deduced from its underlying differentiable structure; \(H_\alpha\) is diffeomorphic to \(S^3 \times S^3\), something that is never the case for a projective surface. Unlike tori, Hopf surfaces always contain proper infinite analytic subsets: the images of the punctured axes in \(\mathbb{C}^2 \setminus \{(0, 0)\}\) give us at least two irreducible curves on \(H_\alpha\). If \(\alpha\) is chosen sufficiently generally, then these will be the only curves. In that case, like generic complex tori, \(H_\alpha\) will have no nonconstant meromorphic functions.

In model theory there are several notions, of varying resolution, that describe the possible interaction between two definable sets. These notions, specialised to the structure \(\mathcal{A}\), have relevant counterparts in bimeromorphic geometry. Suppose that \(X\) and \(Y\) are irreducible complex analytic sets in \(\mathcal{A}\). The strongest possible interaction is if \(X\) and \(Y\) are biholomorphic. This can be weakened to \textit{bimeromorphic equivalence}: there exists an irreducible analytic subset \(G \subset X \times Y\) such that both projections \(G \to X\) and \(G \to Y\) are generically bijective. By “generically bijective” I mean that off a countable union of proper analytic subsets of \(X\) the fibres of \(G \to X\) are singletons, and similarly for \(G \to Y\). If \(G \to X\) and \(G \to Y\) are only assumed to be generically finite-to-one, then we say that \(G\) is a \textit{generically finite-to-finite correspondence} between \(X\) and \(Y\). Finally, we can weaken the condition much further by merely asking that there exist some proper analytic subset of \(X \times Y\) that projects onto both \(X\) and \(Y\). In that case we say that \(X\) and \(Y\) are \textit{nonorthogonal}. For example, any two projective varieties are nonorthogonal. On the other hand, any compact complex manifold without nonconstant meromorphic functions—such as a generic complex torus or a generic Hopf surface—is \textit{orthogonal} to any projective variety. Indeed, nonorthogonality to some projective variety implies nonorthogonality to the projective line \(\mathbb{P}_1\), and if \(G \subset X \times \mathbb{P}_1\) witnesses this, then \(G\) has nonconstant meromorphic functions, and as \(G \to X\) will necessarily be generically finite-to-one, so does \(X\).
Classifying Simple Compact Complex Manifolds

One way for a structure to be considered “tame” is if the definable sets have a dimension theory, that is, if a certain intrinsic model-theoretically defined dimension function, called the rank, takes on ordinal values on all definable sets. Zilber’s initial analysis of $\mathcal{A}$ showed that every analytic subset of a compact complex manifold is of finite rank. In fact, this rank is bounded by, but typically not equal to, the complex dimension. I will not define rank here, but I will in a moment explain what it means for a compact complex manifold to have rank 1. This reticence is partially justified by the fact that there is general model-theoretic machinery available, due mostly to Shelah, that analyses arbitrary finite-rank definable sets in terms of rank 1 sets. In particular, and this is only the starting point of such an analysis in $\mathcal{A}$, every analytic set will be nonorthogonal to one of rank 1. It follows that the study of rank 1 sets is central to understanding compact complex manifolds in general. Note that this is not true of complex dimension: understanding compact complex curves tells us essentially nothing about compact complex surfaces that contain no curves.

So what does it mean for a compact complex manifold $X$ to be of “rank 1”? Here is a geometric characterisation: $X$ is not covered by a definable family of proper infinite analytic subsets. Pillay [10] observed that such manifolds were already of interest to complex geometers and were called simple. More precisely:

**Definition.** A compact complex manifold $X$ is said to be simple if $\dim X > 0$, and whenever $Y$ is a compact complex manifold, $A \subset Y \times X$ is an analytic subset, and $E \subset Y$ is a proper analytic subset such that the fibres of $A$ above $Y \setminus E$ are proper infinite subsets of $X$, then the union of all the fibres of $A$ above $Y \setminus E$ is contained in a proper analytic subset of $X$.

Projective curves are simple; they are the only simple projective varieties. But so is a generic complex torus, or indeed any compact complex manifold without proper infinite analytic subsets. The generic Hopf surfaces, having precisely two curves on them, are also simple. It is not hard to see that simplicity is a bimeromorphic invariant; in fact, it is preserved by generically finite-to-finite correspondences. In a sense that I have hinted at above and will return to again later, simple manifolds are the building blocks for all compact complex manifolds.

The contribution of model theory to bimeromorphic geometry begins with the following dichotomy for simple compact complex manifolds, which is a consequence of the deep results of Hrushovski and Zilber from [5]. It says that a simple compact complex manifold is either algebraic or its cartesian powers have no “rich” definable families of analytic subsets. More precisely, if $X$ is a simple compact complex manifold, then exactly one of the following holds:

I. $X$ is a projective curve, or
II. $X$ is modular: whenever $Y$ is a compact complex manifold with $\dim Y > 0$, and $A \subset Y \times X$ is an analytic subset whose generic fibres over $Y$ are distinct proper infinite irreducible analytic subsets of $X^2$ that project onto each coordinate, then $Y$ is simple.

While the condition of modularity only explicitly mentions $X^2$, it actually restricts the rank of families of analytic subsets of $X^n$, for all $n$. All projective curves are nonmodular. For example, the family of lines $y = ax + b$ is a two-parameter algebraic family; it gives rise to a family of subvarieties of $\mathbb{P}_1 \times \mathbb{P}_1$ that is parameterised by a two-dimensional projective variety, and two-dimensional projective varieties are not simple.

So, by the above dichotomy, every simple manifold of dimension at least two is modular. As we have seen, examples can be found among the complex tori and the Hopf surfaces. In fact the model-theoretic analysis provides us with a further dichotomy which distinguishes sharply between these two examples. Very roughly speaking, a modular manifold that is not a complex torus admits only binary definable relations. More precisely, if $X$ is a simple modular compact complex manifold, then exactly one of the following holds:

I. $X$ is in generically finite-to-finite correspondence with a complex torus, or
II. $X$ is relationally trivial: if $A \subset X^n$ is an irreducible analytic subset that projects onto $X$ in each coordinate, and if $\{\pi_1, \ldots, \pi_\ell\}$ is an enumeration of all the coordinate projections from $X^n$ to $X^\ell$, then $A$ is an irreducible component of $\bigcap_{i=1}^{\ell} \pi_i^{-1}(\pi_i(A))$.

Note that complex tori are not relationally trivial: being complex Lie groups they admit a truly ternary analytic relation, namely the graph of the group law. Simple Hopf surfaces are relationally trivial. Actually one can be more precise in case II: there exists a generically finite-to-one meromorphic surjection from $X$ to a **Kummer manifold**, a manifold of the form $T/\Gamma$ where $T$ is a complex torus and $\Gamma$ is a finite group of holomorphic automorphisms of $T$.

The above characterisation of simple modular compact complex manifolds was suggested by Hrushovski in his 1998 address to the International Congress of Mathematicians [4]. It was proved by Pillay and Scanlon in [12], building on some unpublished work of Scanlon. The result is obtained as a corollary to the main theorem in...
that paper which I would at least like to state in
brief. Meromorphic groups are a natural generali-
sation of algebraic groups to the complex analytic
category; they are complex Lie groups that are
“compactifiable” in an appropriate sense. What
Pillay and Scanlon actually prove, using model-
theoretic methods and building on work of Fujiki,
is that every meromorphic group is the extension
of a complex torus by a linear algebraic group.

In any case, putting the two dichotomies to-
gether, we get:

**Theorem 1.** If $X$ is a simple compact complex
manifold, then

- $X$ is a projective curve, or
- $X$ is in generically finite-to-finite correspondence
  with a simple complex torus of dimension $> 1$, or
- $X$ is relationally trivial.

It remains then to understand the relationally
trivial complex compact manifolds. In dimension
one there are none because all compact curves
are algebraic (this is by the Riemann existence
theorem), and it is not hard to see that projective
curves are not relationally trivial. Besides Hopf
surfaces, examples of relationally trivial surfaces
can also be found among the $K3$ and Inoue sur-
faces. Relationally trivial manifolds remain quite
evasive, and, except for the case of surfaces, there
are only conjectural results. These conjectures are
restricted to Kähler manifolds, which also play
a special role from the model-theoretic point of
view, and to which I now turn.

**Compact Cycle Spaces and Kähler Manifolds**

One of the obstacles to the full application of
model-theoretic techniques to compact complex
manifolds is the size of the language, the fact that
every analytic set is a basic relation. Let me point
out that in the algebraic case there is a more eco-
nomical choice of structure. Consider, for example,
the complex compact manifold $X = \mathbb{P}_m$, projective
$m$-space over the complex numbers. Instead of
$\mathcal{A}(X)$ we can consider the structure $\mathcal{A}_Q(X)$, where
we only include as basic relations the algebraic
subsets of $X^g$ that are defined by polynomials with
rational coefficients. Then $\mathcal{A}_Q(X)$ has only count-
ably many basic relations. But all analytic subsets
of $X^g$ are still definable in $\mathcal{A}_Q(X)$. This is because
every complex analytic subset of projective space
is algebraic (Chow’s theorem) and every algebraic
set is obtained by specialisation from an algebraic
set over $\mathbb{Q}$. Hence $\mathcal{A}(X)$ and $\mathcal{A}_Q(X)$ have the same
definable sets, even though the latter is equipped
with a much reduced collection of basic relations.

This motivates the following definition: A com-
 pact complex manifold $X$ is called essentially
saturated if there exists a countable collection $\mathcal{L}_0$
of analytic subsets of cartesian powers of $X$ such
that every analytic set is definable in the reduct
where only the sets in $\mathcal{L}_0$ are named as basic
relations. Essentially saturated compact complex
manifolds are significantly more amenable to a
model-theoretic analysis. Exactly why is described
in [7] and is somewhat beyond the scope of this
article. Instead, I would like to focus on a very
suggestive geometric characterisation of essential
saturation.

Recall that a $k$-cycle of a compact complex man-
ifold $X$ is a finite linear combination $Z = \sum n_i Z_i$
where the $Z_i$ are distinct $k$-dimensional irreduc-
ible complex analytic subsets of $X$ and each $n_i$ is
a positive integer. In particular, every irreducible
analytic subset is itself a cycle. The set of all $k$-cycles
is denoted by $\mathcal{B}_k(X)$, and $\mathcal{B}(X)$ denotes the disjoint
union of all the $\mathcal{B}_k(X)$. In the 1970s Barlet endowed
$\mathcal{B}(X)$ with a natural structure of a complex ana-
lytic space. If $X$ is algebraic, then $\mathcal{B}(X)$ coincides
with the Chow scheme. With this terminology in
place, we can characterise essential saturation as
follows: $X$ is essentially saturated if and only if all
the irreducible components of $\mathcal{B}(X^n)$ are compact,
for all $n \geq 0$. This follows from [7], in which the
universal family of complex analytic subspaces
(the Douady spaces) were used instead of Barlet’s
cycle spaces. In any case, it is important here that
the condition is on all the cartesian powers; if $H$
is a Hopf surface, then $\mathcal{B}(H)$ has only compact
components, but $\mathcal{B}(H \times H)$ has a noncompact
component. In particular, Hopf surfaces are not
essentially saturated. While the property of $\mathcal{B}(X)$
having only compact components is one that ap-
pears often in bimeromorphic geometry, it seems
that the extension of this condition to all cartesian
powers of $X$ has not been studied. For example,
is essential saturation preserved under cartesian
products and bimeromorphic equivalence?

How does one check if a component of the
Barlet space is compact? In the late 1970s Lieber-
man [6] gave the following differential geometric
criterion: a set of cycles is relatively compact in
the cycle space if and only if the volume of the
cycles in that set (with respect to some hermitian
metric) is bounded. This gives us many examples
of essentially saturated manifolds. Recall that a
Kähler manifold is one that admits a (hermitian)
metric which locally approximates to order 2 the
standard euclidean metric on $\mathbb{C}^n$. With respect to
such a metric, the volume of the cycles in any given
component of the Barlet space is constant, and
this remains true of cartesian powers because käh-
lerianity is preserved under cartesian products. It
follows from Lieberman’s theorem that all com-
 pact Kähler manifolds are essentially saturated.
Moreover, any compact complex analytic space
that is bimeromorphic to a Kähler manifold—this
is Fujiki’s class $C$—is also essentially saturated.
While $C$ does not include all essentially saturated manifolds (Inoue surfaces of type $S_d$ are counterexamples, see [8]), it is a large class of manifolds that contains all projective varieties and complex tori and that is preserved under various operations including meromorphic images and generically finite-to-finite correspondences. Compact Kähler manifolds play a prominent role in bimeromorphic geometry largely because they are susceptible to many of the techniques of algebraic geometry. Because of essential saturation, they are also important to the model-theoretic approach.

Let us return to the classification problem for simple compact complex manifolds. In the previous section I explained how this reduces to understanding the relationally trivial manifolds. We can restrict the problem further and ask: What are the relationally trivial simple Kähler manifolds? We have already seen that there are none in dimension one. In dimension two, inspecting the Enriques-Kodaira classification, we see that all relationally trivial simple Kähler surfaces are bimeromorphic to $K3$ surfaces. These surfaces (introduced by Weil and named in honour of Kummer, Kodaira, Kähler, and the mountain K2) are by definition simply-connected compact surfaces with trivial canonical bundle. In higher dimensions the correct generalisation of $K3$ seems to be irreducible hyperkähler: simply connected Kähler manifolds with the property that their space of holomorphic 2-forms is spanned by an everywhere nondegenerate form. Pillay has conjectured that all relationally trivial simple Kähler manifolds are in generically finite-to-finite correspondence with an irreducible hyperkähler manifold. Since irreducible hyperkähler manifolds are always even-dimensional, Pillay’s conjecture, coupled with Theorem 1 above, would imply that every odd-dimensional simple Kähler manifold is in generically finite-to-finite correspondence with a complex torus. In dimension three this is essentially a conjecture of Campana and Peternell, namely that every simple Kähler threefold is Kummer.

**Variation in Families**

In justifying our focus on simple manifolds I have already mentioned the fact that every compact complex manifold is nonorthogonal to a simple one. From this one can deduce that for every compact complex manifold $X$ there exists a meromorphic surjection $f : X \to Y$, where $\dim Y > 0$ and $Y$ is in generically finite-to-finite correspondence with some cartesian power of a simple manifold. The same is then also true of each generic fibre $X_y$ of $f$. At least in the Kähler case, using essential saturation, one can show that the corresponding meromorphic surjections on $X_y$ vary uniformly in the parameter $y$. Since the dimension of $X_y$ is strictly less than that of $X$, such an analysis must stop after finitely many iterations. It is in this sense, via sequences of meromorphic fibrations, that simple manifolds control the structure $\mathcal{A}$. Of course this does not reduce the classification problem to the case of simple manifolds; at the very least one needs to also understand how such manifolds fit into meromorphic fibrations and how they interact with each other. I want to state one conjecture that is central to this question.

First of all, from the definition of simplicity it follows that two simple manifolds are nonorthogonal to each other if and only if they are in generically finite-to-finite correspondence. The three classes of compact complex manifolds appearing in Theorem 1—projective curves, simple complex tori of dimension $> 1$, and simple relationally trivial manifolds—are all mutually orthogonal in the sense that any two manifolds coming from different classes will be orthogonal. Moreover, while all curves are nonorthogonal to each other, there exist orthogonal pairs within each of the other two classes.

One fundamental question is whether or not there exist entire definable families of simple manifolds any two of which are orthogonal. Actually, it follows from observations of Pillay and Scanlon that such families do exist, but not, conjecturally, among the Kähler manifolds. More precisely, the conjecture is that if $f : X \to Y$ is a meromorphic surjection between compact Kähler manifolds with simple generic fibres, then any two generic fibres are in generically finite-to-finite correspondence. In 2005 Campana [2] proved an isotriziality result which proves the conjecture, with the stronger conclusion of isomorphism rather than correspondence, in the following cases: when the generic fibres are surfaces, when the generic fibres are “general” complex tori, and when the generic fibres are irreducible hyperkähler.

**Analogies**

By way of conclusion, I want to discuss the special role of model theory as a medium between different geometric contexts. Certain results from bimeromorphic geometry have informed advances in abstract model theory which can then be reapplied in other areas. In this way model theory acts as a conduit between bimeromorphic geometry and, in the case I want to discuss, differential-algebraic geometry. I need to say a few words about differential-algebraic geometry.

Like bimeromorphic geometry, differential-algebraic geometry is an “expansion” of algebraic geometry. A differential field is a field $K$ equipped with a derivation; an additive map $\delta : K \to K$ such that $\delta(xy) = x\delta(y) + y\delta(x)$. For example, $(\mathbb{C}(t), \frac{d}{dt})$ is a differential field. Differential algebra is then commutative algebra in the presence of...
this derivation. The role of polynomials is played by differential polynomials: functions of the form \( P(x, \delta(x), \ldots, \delta''(x)) \), where \( P \in K[X_0, \ldots, X_t] \) is an ordinary polynomial. A differential field \((K, \delta)\) is differentially closed if any system of differential polynomial equations and inequations having a solution in some differential field extension already has a solution in \( K \). The model theory of differentially closed fields of characteristic zero is also tame. In particular, every definable set in \((K, +, \times, \delta)\) is a finite boolean combination of differential-algebraic sets: zero sets of systems of differential polynomial equations. These are the objects of study in differential-algebraic geometry.

Many of the model-theoretic techniques that apply to the structure \( \mathcal{A} \) also apply to \((K, +, \times, \delta)\). In fact, there is a fruitful analogy between these structures whereby complex analytic sets correspond to finite-rank differential-algebraic sets (see [10]). This analogy can lead to transferring results between the two disciplines that these structures represent. The example I have in mind is based on the following theorem of Campana [1], due also independently to Fujiki [3], from the early 1980s. Suppose \( X \) is a compact complex manifold and \( C \) is a compact analytic subset of the cycle space \( B(X) \). Then for any \( a \in X \), the set of cycles in \( C \) that pass through \( a \) is (up to bimeromorphism) an algebraic set. Moreover this happens uniformly as \( a \) varies. In 2001 Pillay [11] observed that this theorem could be used to give a more direct proof of the first dichotomy; the fact that every simple manifold is either a curve or modular. Pillay’s argument involves formulating an abstract model-theoretic counterpart to Campana’s theorem, which replaces the difficult and much more general results of Hrushovski and Zilber [5]. Pillay and Ziegler [13] then show that this model-theoretic counterpart also holds in differentially closed fields. As a result one has an analogue of Campana’s theorem in differential-algebraic geometry, as well as a direct proof of the corresponding dichotomy for rank 1 differential-algebraic sets. Another related example of this phenomenon can be found in [9], in which Campana’s “algebraic connectedness” is abstracted from bimeromorphic geometry to model theory and then applied to differentially closed fields. The outcome in that case is a criterion for when a finite-rank differential-algebraic set is in generically finite-to-finite correspondence with the \( C_K \)-points of an algebraic variety, where \( C_K = \{ x \in K : \delta(x) = 0 \} \).

The kind of transfer of ideas that we see in the above examples, and the role that model theory plays here of recognising, formalising, and facilitating analogies between different geometric settings, is not something new or unique to its interaction with bimeromorphic geometry. This has been a defining feature of model theory since the 1970s and continues to fuel the internal development of the subject.

References

What is a Halting Probability?

Cristian S. Calude and G. J. Chaitin

Turing’s famous 1936 paper “On computable numbers, with an application to the Entscheidungsproblem” defines a computable real number and uses Cantor’s diagonal argument to exhibit an uncomputable real.

Roughly speaking, a computable real is one that can be calculated digit by digit, one for which there is an algorithm for approximating as closely as one may wish. All the reals one normally encounters in analysis are computable, like $\pi$, $\sqrt{2}$, and $e$. But they are much scarcer than the uncomputable reals because, as Turing points out, the computable reals are countable, whilst the uncomputable reals have the power of the continuum. Furthermore, any countable set of reals has measure zero, so the computable reals have measure zero. In other words, if one picks a real at random in the unit interval with uniform probability distribution, the probability of obtaining an uncomputable real is unity. One may obtain a computable real, but that is infinitely improbable.

But how about individual examples of uncomputable reals? We will show two: $H$ and the halting probability $\Omega$, both contained in the unit interval. Their construction was anticipated in 1927 by Émile Borel, who “defined” a real number $0 < B < 1$ in the following way. The $N$th digit $b_N$ of $B$ answers the $N$th question in an enumeration of all possible yes/no questions that one can write in French. Borel argued that knowledge of every point of the continuum would have strange consequences, and $B$ is a case in point. The number $B$ is an oracle that “would give answers to all past, present, and future enigmas of science, history, and curiosity.”

Let’s change this into something more mathematical, more realistic: an oracle for the halting problem $H$.

Systematically enumerate all programs with no input (say, lexicographically). The $N$th bit $h_N$ of the real number $H$ tells us whether or not the $N$th computer program ever halts. $H$ is an oracle for the halting problem, which would be extremely useful to have, as we will show later.

However, this oracle $H$ for the halting problem is extremely wasteful, because $N$ instances of the halting problem are not $N$ bits of mathematical information, they are only $\log_2 N$ bits of mathematical information. One only needs to know how many of these $N$ programs halt—a number expressed in less than or equal to $1 + \log_2 N$ bits—to be able to determine which ones halt. Indeed, one just runs in parallel all programs till exactly the known number of programs that halt have stopped: all the remaining programs won’t halt.

Using a slightly more sophisticated version of this idea, we finally arrive at the halting probability $\Omega$, which is defined by the following formula:

$$0 < \Omega = \sum_{\text{program } p \text{ halts}} 2^{-\text{(size of } p\text{ in bits)}} < 1.$$ 

This is the probability that a computer program whose bits are generated one by one by independent tosses of a fair coin will eventually halt. There are actually many halting probabilities, not one, because the precise numerical value of $\Omega = \Omega_L$ depends on the choice $L$ of programming language for the programs $p$. 

Cristian S. Calude is professor of computer science at the University of Auckland. His email address is cscalude@gmail.com.

G. J. Chaitin is emeritus researcher at the IBM Watson Research Center and honorary professor at the University of Buenos Aires. His email address is gjchaitin@gmail.com.
Nota Bene: For this definition of a halting probability to work properly, the computer programming language \( L \) that the programs \( p \) are written in has to satisfy certain requirements:

- The programs \( p \) have to be self-delimiting (no extension of a valid program is a valid program) or the sum won’t converge. This enables one to construct large programs by concatenating (abutting) subroutines.
- The language \( L \) has to be universal and concise. More precisely, if any other language \( L' \) can program something in \( K \) bits, then \( L \) can do it in \( \leq K + c_L \) bits.

For historical reasons, such a programming language \( L \) is often referred to as a universal self-delimiting Turing machine, and we shall do so below.

What are the key properties of a halting probability \( \Omega \)?

By writing \( \Omega \) in binary one can show that given the first \( N \) bits \( \omega_1 \ldots \omega_N \) of

\[
\Omega = \sum_{k=1,2,3,\ldots} \omega_k / 2^k,
\]

one can solve the halting problem for all programs up to \( N \) bits in size. This implies that the bits of \( \Omega \) are algorithmically and logically irreducible:

- The smallest program for computing the first \( N \) bits of \( \Omega \) has \( \geq N - O(1) \) bits. (This algorithmic irreducibility property is normally called (algorithmic) randomness.)
- Any formal axiomatic theory, like ZFC, that enables one to prove what are the values of the first \( N \) bits of \( \Omega \) must have \( \geq N - O(1) \) bits of axioms. (This is logical irreducibility.)

In other words, the bits of \( \Omega \) are the most concise oracle for solving the halting problem, the best possible compression of all the answers to individual instances of the halting problem.

The philosophical and epistemological significance of \( \Omega \) is primarily due to the following surprising fact:

*The bits \( \omega_1 \omega_2 \omega_3 \ldots \) of the halting probability \( \Omega \) provide a perfect simulation within pure mathematics, where all truths are necessary, of an infinite stream of contingent, accidental yes/no facts.*

For instance, \( \Omega \) provides us with a natural example of what É. Borel termed an absolutely normal real number. This means that if \( \Omega \) is written in any base \( b \geq 2 \), then all blocks of \( K \) base-\( b \) digits will occur with equal limiting relative frequency \( b^{-K} \).

It is an immediate corollary of \( \Omega \)'s algorithmic and logical irreducibility that \( \Omega \) is uncomputable and that, in fact, no algorithm can compute more than finitely many scattered bits of \( \Omega \). Some of these bits have actually been determined. For a natural choice of the programming language \( L \), the first 40 bits of \( \Omega = \Omega_L \) are

\[
000100000010001010011101100011111010.
\]

If we knew the first 7,780 bits,\(^2\) which is less than one quarter of this note’s size in bits, we would know whether the Riemann hypothesis is correct.

Now for some more advanced results.

Although uncomputable, \( \Omega \) is the most "computable" among all algorithmically random reals: any \( \Omega_L \) is left-computable, i.e., the least upper bound of a computable sequence of rationals. The set \( \{ \Omega_2 \} \) of all halting probabilities of universal self-delimiting Turing machines \( L \) coincides with the set of all left-computable algorithmically random reals.

Can a halting probability be formally proved algorithmically random in Peano arithmetic (PA)? The answer depends on the representation: if \( \Omega = \Omega_L \) is given by a self-delimiting Turing machine \( L \) whose universality (and conciseness) is proved in PA, then PA proves that \( \Omega \) is algorithmically random. Every \( \Omega \) can be written as \( \Omega = \Omega_L \), for some \( L \) satisfying the above requirements, so it is provably random in PA.\(^3\)

**Further Reading.** For the intellectual history leading up to \( \Omega \), including the role played by Leibniz, see the second author’s *Meta Math!*. For a technical treatment, see the first author’s *Information and Randomness*, or *An Introduction to Kolmogorov Complexity and Its Applications* by M. Li and P. Vitányi. For halting probabilities for versions of a real programming language, LISP, and a plethora of other nonstandard halting probabilities and their applications, see the second author’s *Information-Theoretic Incompleteness*. For some of the latest results, see *Computability and Randomness* by A. Nies and *Algorithmic Randomness and Complexity* by R. G. Downey and D. Hirschfeldt (forthcoming). For discussions of the philosophical impact and additional historical material, see the second author’s *Thinking about Gödel and Turing*.


\(^3\)C. S. Calude, N. J. Hay, Every computably enumerable random real is provably computably enumerable random, Logic Jnl. IGPL 17 (2009), 325–350.
Headlines & Deadlines for Students, a service from the AMS Public Awareness Office, provides email notification of mathematics news and of upcoming deadlines. These email notifications are issued about once a month, and when there’s special news. Imminent deadlines are included in these emails, which link to a web page that’s a centralized source for information relevant to students and faculty advisors, at:

[www.ams.org/news-for-students/](http://www.ams.org/news-for-students/)

Sign up for the email service at [www.ams.org/news-for-students/signup](http://www.ams.org/news-for-students/signup)

**Check out the latest Headlines & Deadlines for Students for news and due dates for applications, registrations, proposals...**

- **Pi Mu Epsilon Student Paper Presentation Awards**
- **Fellowships and Grants**
- **New Mathematical Moments**
- **Stipends for Study and Travel**
- **Marshall Scholarships**
- **Trjitzinsky Awards**
- **Math in Moscow Semester - Call for Applications**
- **AWM Essay Contest**
- **Special Book Sales on AMS Bookstore**
- **Poster Session Proposals**
- **Putnam Exam Results**
- **Employment Center Registration**
- **Clay Research Fellowships**

[www.ams.org/news-for-students](http://www.ams.org/news-for-students)
What Then?
Plato’s Ghost: The Modernist Transformation of Mathematics

Reviewed by Yuri Manin

Plato’s Ghost: The Modernist Transformation of Mathematics
Jeremy Gray
Princeton University Press, 2008
US$45.00, 526 pages

The new book of the renowned historian of mathematics Jeremy Gray can be succinctly described as a rich and thorough study of Western ethnomathematics in the period of five decades or so (1880–1930) preceding and following World War I. What distinguishes such a project from more conventional Bourbaki–style enterprises is the stress on extra–mathematical background: social and political structures of society, economic practices, forms of professional self–organization, and culture.

In fact, culture (represented by visual arts, music, philosophy) dominates the discourse at several key points, and culture furnished the basic metaphor for the book. The time span in question is characterized as the period of modernist transformation in mathematics, and the first quotation in the Introduction is taken from Guillaume Apollinaire’s “The Beginning of Cubism” (1912).

The scope of the book is already ambitious. Chronologically, it starts well before the arrival of “modernism”: There are brief essays on Monge, Poncelet, and projective geometry, followed by a discussion of Lobachevsky, Bolyai, and non-Euclidean geometry, including a description of Gauss’s role in its formation and the subsequent contributions of Riemann and Poincaré that led to a new vision of what geometry is and what it should be. Similar essays on algebra, analysis, “British Algebra and Logic”, and philosophy follow, all of this interspersed with information about professional organizations, teaching, journals, and quotations from contemporary articles and letters. (I cannot resist the temptation to reproduce a delightfully funny sentence from a letter by C. G. J. Jacobi to A. von Humboldt, although in the book it appears only considerably later, on page 268:

If Gauss says he has proved something, it seems very probable to me; if Cauchy says so, it is about as likely as not; if Dirichlet says so, it is certain.)

Chapter 2, containing this wealth of information, is called “Before Modernism”, and ends with a brief synopsis “Consensus in 1880”.

The arrival of mathematical modernism is addressed in Chapter 3. Again, the discussion is consecutively focused on geometry (in particular, reemerging connections to physics and Schwarzschild’s paper of 1900); then analysis, algebra, and logic. With hindsight, the central modernizing figure appears in the person of Georg Cantor. Especially interesting for me were pages discussing religious overtones of Cantor’s transcendental flights in the infinity of infinities and the related subsection on “Catholic Modernism”. (As I have written elsewhere, I discern a subtle mockery in the famous and often quoted sentence of Hilbert

Yuri Manin holds the Trustee Chair and is professor of mathematics at Northwestern University and professor emeritus at the Max-Planck Institut für Mathematik in Bonn, Germany. His email address is manin@mpim-bonn.mpg.de.
about “Cantor’s Paradise”, usually perceived only as unconditional support for set theory).

The subsequent chapters are called “Modernism Avowed”, “Faces of Mathematics”, “Mathematics, Language and Psychology”, and “After the War” and are bursting with information, insight, and scholarship. As far as I know, nobody before Jeremy Gray discussed in this context “Popularizing Mathematics around 1900” (section 5.4), or “Languages Natural and Artificial” (section 6.1).

Constrained by the restrictions of space, I will end this brief survey of the detailed contents of this remarkable book and turn to the discussion of several of its grand themes, sometimes underlying the exposition and sometimes explicitly invoked in it.

Mathematics is a product of civilization: it is discovered or created (see discussion below) by a very limited number of human beings in each generation, then encoded in a mixture of words, formulas, and pictures (nowadays software and hardware might be added to this list), and in this way transmitted to the next generation, which continues this process. To estimate the number of persons creating mathematics these days, one can refer to the attendance of International Congresses of Mathematicians that are held every four years: recently it was about three to five thousand. Undoubtedly, this should be viewed as a great explosion in comparison with previous centuries. In Newton’s time the respective number probably did not exceed a few dozen.

One remarkable feature of mathematical knowledge is this: we learn more and more about the same objects that ancient mathematicians already started to see with their mental eyes: integers and prime numbers, real numbers, polynomial equations in one or many variables, space and various space forms... To illustrate this statement, I will mention just one string of events. Some time around 300 BC, the Hellenistic scholar Pappus discovered a remarkable and, in the Euclidean context, quite unusual theorem (see the modernized statement and the picture on page 463 of Gray’s book, and beware of a small misprint: $B \ A$ must be $BA'$). Pappus’s Theorem does not conform to the spirit of Euclidean geometry, because lengths, angles, and rigid plane motions are quite irrelevant here: only points, lines, and incidence relations “a point $x$ lies on a line $X$” play a role. In the nineteenth century it was understood that Pappus's Theorem characterizes projective geometry of the real plane. When an abstract projective plane—as a set of points $P$, with a set of subsets $I \subset P$ called lines—was introduced at the beginning of the twentieth century, it turned out that Pappus’s Theorem, now taken as an axiom, is a necessary and sufficient condition for the possibility of introducing projective coordinates in $P$ with values in an abstract field (at least, if $P$ is infinite). Several more decades elapsed, and it was discovered that this latter statement can be vastly generalized in the theory of models of formal languages: “Zariski geometries” of Zilber and Hrushovski describe when abstract sets with subsets satisfying certain combinatorial requirements can be conceived as sets of solutions of arbitrary polynomial equations over a field.

A sine qua non condition of such permanency, continuity of preoccupation, is the creation and maintenance of a very safe, reliable, intergenerational flow of information. Linguistic resources of mathematics by which this information is encoded and transferred are arguably much more variable, fickle, subject to sweeping winds of history, than the content of the information itself.

The whole bulk of discoveries of one generation, or one of its active but geographically restricted subgroups, might in a few decades become almost incomprehensible to another generation, unless new expressive means, with their new intuitions and hygienic rules, come to the rescue. The old arguments are translated into a new language, rewritten, purged of perceived obscurities and errors, in the process of mastering old results and moving to new discoveries.

My professional youth was spent in this exhilarating process of assimilation of glorious Italian algebraic geometry and recasting it using schemes, coherent sheaves, and homological algebra of the emerging "new age" algebraic geometry, which in two to three decades was to become one of the powerful vehicles not only in pure mathematics but also in theoretical physics and quantum field theory. Perhaps for this personal reason, I became perceptive of similar events that took place in other times and locations.

From this perspective, “the modernist transformation of mathematics”, which is the subtitle of Jeremy Gray’s book, was one such periodically occurring refurbishing of basic vocabulary, grammar, and, yes, aesthetic requirements for mathematical thought and mathematical texts. It took about thirty years, and its outcomes dominated the next thirty years: all in all, just about the span of three generations. The fact that during the lives of these generations two world wars swept Europe (or, according to some accounts, one Thirty Years’ War 1914–1945) influenced mathematics and the wider culture in many ways.
The Bourbaki group formed itself in the 1930s in a conscious attempt to fill the void left by the “lost generation”: not in the somewhat egocentric sense of the phrase coined by Gertrude Stein, who referred to bohemian American émigrés in Paris, but a real and painful void. About 40 percent of French mathematicians died in the first world war.

Although my argument posits “the modernist transformation” as one of the series of structurally similar transformations that occurred in other times and places, both the choice of the time span and the use of the adjective “modernist” for it seem to me very felicitous. After formulating what he sees as characteristic of modernist aesthetics, creative psychology, the relationship with society, and self-perception, shared by, say, painting and mathematics of that period, Jeremy Gray himself warns that this looks like a case of “convergent evolution” (page 8) rather than an effect of “common genes”.

In his own sober words: “I am not sure there is more than a resonance that links mathematical modernism to the arrival of modernity, modern capitalism, and its horrific opponent, the Nazi state. We are far from knowing much about the societies we inhabit.”

In a book largely dedicated to the history and philosophy of mathematics, Plato’s name might have been invoked in many contexts, but the first that comes to mind, that of “Mathematical Platonism”, is relegated to the very end of the discussion, section 7.3, pages 440–452. Besides this section 7.3, Jeremy Gray, according to the Index, mentions Plato or Platonism on twelve more pages scattered throughout the text. By contrast, more than fifty entries in the Index invoke Poincaré and only slightly fewer, Bertrand Russell.

Why, then, is this monograph called “Plato’s Ghost”? The explanation is on the very first page, which quotes in its entirety Yeats’s “What Then?”, a poem of four five-line stanzas about an accomplished life of an accomplished human being:

Everything he wrote was read,
After certain years he won
Sufficient money for his need;
Friends that have been friends indeed;
“What then?” sang Plato’s ghost, “What then?”

The most direct reading of these stanzas suggests to me a simple and universal human emotion, existential anxiety, as beautifully expressed by Yeats as by many others before and after him. Jeremy Gray himself hears in Plato’s voice denial of a claim to perfection (page 14). This interpretation is readily accommodated in a chronicle of (para)philosophical controversies around mathematics. But, significantly, the word “anxious” appears already in the first paragraph of the Introduction, and “anxiety” pops up on page 4, where it is first introduced as “a well established theme in writing about modernism”. Later this theme is developed in the section 4.8, “Anxiety”, already fully in the context of the mathematics of the first half of the twentieth century, against the background of the “crisis of foundations”.

Is it conceivable that Gray’s whole discourse on modernism in mathematics is informed by existential anxiety and that the last section, 7.5, “The Work Is Done”, is as self-referential as it sounds?

Of course it is, as the present author willingly testifies, relying upon his own experience, and as Jeremy Gray himself hints in the last lines of page 4.

But before bidding farewell to the reader, let’s brace ourselves and try to face the Ghost question “What then?”, which I will interpret in the most prosaic way, as a question about the direction in which self-consciousness of mathematicians moved during the last few decades, separated by more than a half-century from the 1930s and 1940s where the detailed analysis of Gray stops.

So far as I can judge, “Platonism” of working mathematicians is based on a feeling that important mathematical facts are discoveries rather than inventions.

The Bering Strait was named after Vitus Bering who discovered it, whereas the diesel engine was named after Rudolf Diesel who invented the engine. What about Galois groups? If you feel that they were discovered by Évariste Galois, rather than invented by him, you are in a sense a Platonist.

I will call such an attitude emotional Platonism in order to stress that (in my view) it is intellectually indefensible, but not to the least degree invalidated by this fact, since our emotions happily resist rational arguments.

Being such an emotional Platonist myself, I do not want to say that all mathematics is a discovery of a Platonic world, whatever that could mean. Certainly, the history of mathematics is also marked by inventions of marvelous intellectual telescopes and efficient vehicles, allowing people to travel from one discovery to another. Moreover, there are mathematicians whose oeuvre deserves comparison to the Odyssey rather than to Columbus’s travelogue, insofar as mathematics is exteriorized in texts forming a part of the much vaster general culture of the written word.

Elaborating the latter metaphor, one can represent the history of the “modernist transformation of mathematics” simply as a tale about the birth and development of a certain style of thought, expression, and teaching, starting with Georg Cantor and culminating with Bourbaki. Much will have to be left out of such a tale, but it could serve as a good starting point for guessing “What then?”

This style is formed around a system of pretty explicit prescriptions: how a mathematician is supposed to introduce his or her object of study (“what
I am talking about”), how his or her results should be stated (“what I am saying”), and finally how one should write arguments convincing her/himself and potential readers that the stated results are correct (“proofs”).

Briefly, the object of study (group, space with measure, topological space…) is introduced by a definition, presenting it as a Bourbaki-style structure, which in turn is a collection of Cantorian sets satisfying certain relationships stated in terms of the basic relation “being an element of” and standard logical means. The results are stated as theorems, statements of the type “if a structure has a certain property \( P \), then it has another property \( Q \) as well” etc. Finally, proofs are texts written in a mixture of natural language, formulas, and sometimes pictures (although the latter are considered “bad taste” in the Bourbaki aesthetics). Such a text can be considered as a valid proof only if in principle it can be replaced by a sequence of statements such that each term of this sequence is valid either by definition, or can be obtained by an elementary logical step from previously obtained statements.

There are several more or less hidden or explicit sources of self–referentiality in this picture, of which I will mention two.

One is that the notion of mathematical reasoning invoked in the previous paragraphs can itself be rigidified to become a mathematical structure. This was of course the main discovery of the formalist program; as soon as it had crystallized, Gödel’s and Tarski’s theorems formalizing the “liar’s paradox” or Cantor’s diagonal argument became inevitable.

Another source of self–referentiality is less formal. Namely, a “proof” presented in a mathematical paper must convince the reader not just that the stated theorem is true, but that it by itself is a PROOF, the incarnation of an ideal object residing in the Platonic territory of formalized mathematics. Severe rules of hygiene are imposed upon mathematical exposition in order to ensure this. The reader must be alerted even to the occurrence of an “empty proof” (empty in the set–theoretical sense), as Edmund Landau used to practice with his inimitable “Beweis: Klar. (Proof: Clear.)”.

The imposition of the hygienic restrictions was a part of the reaction to the perceived crisis of foundations, but in the real life of mathematics of the twentieth century it played another, and probably unexpected, role: that of unification of many diverse fields of mathematical studies. Algebraists, analysts, geometers, probabilists, number theorists, logicians could now speak in the same language, even when speaking about different structures.

Logicists stubbornly struggled in the losing battle against this loss of their dominance as Keepers of Foundations. Nicolas Bourbaki did not really understand the structures emerging in new mathematical logic and botched the respective chapters of his Treatise. As a result, the basic new discoveries in logic, the Turing–Church Thesis and the theories of computability and of complexity, suffered because their ties to the mainstream mathematics were loosened. Fortunately, such researchers as A. N. Kolmogorov helped tighten these ties.

What we see now in the flow of mathematical research can be termed loosely as a “post-modern” period, but the term can be taken literally only if we adopt Jeremy Gray’s concept of “modernism”. Contemporary mathematics bears no traces or connotations of Lyotard’s description of the postmodern condition, whose main characteristic is the loss of credibility of all grand narratives. The grand narrative of mathematics is steadily developing, reaching new depths and new sophistication.

One trend that visibly changes the face of set theory as the foundational language of mathematics is the current popularity of categories (functors, enriched categories, polycategories…) as the dialect in which basic definitions of mathematical objects are formulated. Another related, but not identical, trend is the evolution of homotopy theory, which has gradually become a kind of new language for postmodern mathematics (for an initiated reader, I can mention “brave new rings”).

Briefly, these two trends together revolutionize our collective vision of both semantics and syntax of the language(s) of mathematics.

First, basic new objects “category” (up to equivalence) and “homotopy type” are not Bourbaki structures: they are formed by a class of Bourbaki structures that are related by certain equivalence relations. Both “class” and “relation” in this statement can be represented set–theoretically by “infinitely large” collections of sets.

Accepting this, a working mathematician not only sheds the primeval horror of large infinities but opens his/her imagination to a radically new vision even of common objects. For example, natural numbers “simply” count things (or cardinalities of finite sets), but the history of mathematics teaches us how late zero and negative integers were in–introduced and accepted. External references such as the notion of “debt” in trade were necessary in order to legitimate negatives. Nowadays integers are homotopy classes of pointed loops in a real plane with a deleted point: this is a germ of the idea of “brave new rings”. Even more generally, “small sets” that in the Cantor/Bourbaki paradigms could be turned into topological spaces by imposing an additional structure now become secondary/derived objects: say \( \pi_0 \) of a homotopy type. The traditional view “continuous from discrete” gives way to the inverted paradigm: “discrete from continuous”.

Second, the notion of formal language that crystallized in the 1930s as a purified written
form of natural languages, after it became treated as a Bourbaki structure, could be vastly extended. Seemingly, there are other Bourbaki structures (or better, categories) that have “language-like properties”, such as categories of graphs. The intuition behind considering, say, a directed graph as a potential linguistic entity is that of “flowcharts”. Generally, computer science, that no-nonsense child of mathematical logic, will exert growing influence on our thinking about the languages by which we express our vision of mathematics.

Finally, I want to discuss briefly the collaboration of mathematicians and physicists, which became dormant during the several decades of “mathematical modernism” but renewed with new vigour after 1950s and 1960s. I will primarily stress the benefits of this collaboration for mathematics and will describe these benefits as the emergence of a vast research program that could be called “quantization of classical mathematics”.

This program is historically related to the fact that when physicists started to see quantum phenomena as the basic natural causes underlying observable classical behavior of matter and fields, they had to gradually discover what kind of changes must be made in order to proceed from a known mathematical description of a classical system to a new, quantum description (“quantization”), and back (“classical approximation”). Some of the discovered prescriptions, such as “deformation quantization” involving Planck’s constant as a small parameter, turned out to be much more universal mathematically than suggested just by their initial uses.

Another prescription stressing quantum observables as operators in (generally infinite-dimensional) Hilbert spaces led to “non-commutative geometry”, one germ of which was the Heisenberg commutation relation, and later to “quantum groups”.

Philosophical problems related to the changed role of observation and measurement led to discussions of “quantum logic” and later, in a more applied vein, “quantum computation”.

In the 1940s a development started that produced some mathematical miracles. Richard Feynman developed path integrals, a notion that is highly intuitively appealing though mathematically vague, as well as the powerful machinery of Feynman diagrams, which are well understood mathematically but are motivated only by the idea that they somehow capture path integrals. It was a great success in quantum field theory and elementary particles, but nobody could foresee how, in mathematics, it would backfire.

This became reality after many physics papers of Witten and his collaborators, which mathematicians could interpret as rich and powerful heuristic tools to guess precise and striking new mathematical facts, such as “quantum invariants of knots”, from the physical intuition related to, say, “topological quantum field theories” and the respective formalism of path integrals. These facts could afterwards be investigated and put on firm ground by mathematicians: we got “quantum topology”, “quantum cohomology” (from string theory), and much more. If you search in the arXiv, MathSciNet, and Google for the terms I put in quotation marks above, you will find oceans of information about this stuff.

At this point, Plato’s apparition intervenes again and sings “What then?”

And I respond: “For us, there is only the trying. The rest is not our business.”

References

\[ T. S. Eliot, Four Quartets. \]
Solving Mathematical Problems: A Personal Perspective

Reviewed by Loren Larson

In 1980 Paul Halmos concluded his Monthly article “The Heart of Mathematics” [1], with this thought:

I do believe that problems are the heart of mathematics, and I hope that as teachers, in the classroom, in seminars, and in the books and articles we write, we will emphasize them more and more, and that we will train our students to be better problem posers and problem solvers than we are.

Now, thirty years later, I think he would be pleased with what has transpired. Problem solving is central in current mathematics education, from kindergarten through middle school, from high school through college and beyond. Learning mathematics means doing mathematics, a mix of practice exercises to develop skills and problems for deeper understanding. The proportion of each depends on the context, anything from 100/0 for crash courses where the problems will come later (in more than one sense?) to 0/100 for Moore-method courses where mastering techniques will come later. Both are necessary, but from a pedagogical point of view, appropriately chosen problems under the right conditions are more fun and offer more satisfaction, at least for mathematically inclined students. To paraphrase Albrecht Dürer,

“love and delight are better teachers than compulsion.”

One means of promoting problem solving is by organizing math contests, and there are now a variety of contests and plenty of opportunities for participation. These contests are supported by a vast literature of preparatory supplements: anthologies, compilations, how-to-solve books, special topics, problem-solving strategies, online classes, forums, summer math camps, and videos (e.g., see artofproblemsolving.com). Contests have a proven record for fostering interest in mathematics, but they aren’t for everyone. Some students are turned off by competition, or have mathematical interests that aren’t particularly amenable to a contest format, or feel that contests favor speed over power (not that they can’t coexist). But there are other ways of promoting problem solving and personal involvement, such as problem seminars, independent research, and interdisciplinary projects.

Finding suitable problems is a time-intensive task. Problem books can provide ideas and starting points, for example nontrivial tweaks or special cases of published results. But this approach can lead to stagnation and irrelevance. A better source for keeping ideas meaningful and up-to-date is the

Solving Mathematical Problems: A Personal Perspective
Terence Tao
Oxford University Press, September 2006
US$34.99, 128 pages

Reviewed by Loren Larson
Loren Larson is professor of mathematics at St. Olaf College. His email address is lllarsson@earthlink.net.
Algebra: Algebra is the basic foundation of a large part of applied mathematics. Problems of mechanics, economics, chemistry, electronics, optimization, and so on are answered by algebra and differential calculus, which is an advanced form of algebra. In fact, algebra is so important that most of its secrets have been discovered—so it can be safely put into a high school curriculum. However, a few gems can still be found here and there.

Number Theory: Unlike algebra, which has as its backbone the laws of manipulating equations, number theory seems to derive its results from a source unknown. Take, for example, Lagrange's Theorem, … . Number theory is a fundamental cornerstone which supports a sizeable chunk of mathematics.

Analysis: Analysis is the study of functions and their properties. … Functional equations form a sort of “pocket mathematics”, where instead of the three dozen or so axioms and countless thousands of theorems, one has only a handful of “axioms” (i.e., data) to use and there is a clear direction in which to go. And yet, it still has surprises.

Geometry: The true beauty of geometry is in how a non-obvious-looking fact can be shown to be undeniably true by repeated application of obvious facts. As an example: the midpoints of the four sides of a quadrilateral always make up a parallelogram. These facts—they have a certain something about them.

Statements like these have to be put into context. Remember that his target reader is someone like himself at age nine, intensely eager and capable and focused on learning as much as possible about Olympiad-level math problems. A certain panache and dash, even some exaggeration and eccentricity, is expected at this age. I was all ears, a teenager again, fired-up and flattered to be treated as an equal. His intentions for the book are laid out in the Preface:

I will try to demonstrate some tricks of the trade when problem solving. Two of the main weapons—experience and knowledge—are not easy to put into a book: they have to be acquired over time. But there are many simpler tricks that take less time to learn. There are ways of looking at a problem that make it easier to find a feasible attack plan. There are systematic ways of reducing a problem into successively easier subproblems.

Math “how-to” books usually focus on specific applications of methods given by George Pólya [3], and this book is no exception—the intention is to motivate the solution. But the manner in which Tao
does it is the book’s most distinguishing feature. For not only does Tao motivate the solution but he also walks through his entire thinking process, including rejected ideas and false starts. He seems to be thinking aloud, explaining as he goes—each journey an apparently effortless flow of ideas. It’s a slim book, about 100 pages, but the problems are well chosen, only twenty-six of them, with about the same number of instructive exercises, without solutions. The solutions to the problems are almost incidental to Tao’s observations and reformulations along the way. He shows us how to think like mathematicians. For example, the actual solution to Problem 2.2 (Is there a power of 2 whose digits could be rearranged and made into another power of 2?) takes only six lines, but it is preceded by five pages of discussion.

The defining characteristic of Tao’s perspective on problem solving is based on Pólya’s dictum, nicely stated by Halmos [1]:

Make it easier. In slightly greater detail: if you cannot solve a problem, then there is an easier problem that you cannot solve, and your first job is to find it! Make it sharp. By that I mean: do not insist immediately on asking the natural question (“what is...?”), “when is...?”, “how much is...?”), but focus first on an easy (but nontrivial) yes-or-no question (“is it...?”).

Essentially, this approach is analogous to the Bolzano-Weierstrass Method for catching a lion in the desert [2]: Repeatedly bisect the area with alternating vertical and horizontal lines, choosing the half that contains the lion. At each stage, build a fence. The lion is ultimately enclosed by a fence of arbitrarily small perimeter.

In other words, first you have to survey the desert, that is, understand the problem and write down everything you can think of that might be relevant. This is especially important in problems at this level because the most obvious beginnings probably won’t work. Then, with this list in hand, you start bisecting, eliminating ideas that appear to make things more difficult. Iterate on this process, systematically asking questions and reducing it into easier subproblems until the tricky parts are no longer tricky. “Make it easy” is the theme of the book, which also makes it fun to read. Expressed more flamboyantly by Tao:

So that is it. We keep reducing the equation into simpler and simpler formulas, until it just collapses into nothing. A bit of a long haul, but sometimes it is the only way to resolve these very complicated questions: step by step reduction.

Simplify repeatedly until the more unusable and unfriendly parts of the problem are exchanged with more natural, flexible, and cooperative methods.

It is best to try elementary techniques first, as it may save a lot of dashing about in circles later.

As long as one always tries to simplify and connect, chances are that the solution will soon fall into place. (Assuming, of course, that there is one—and most problems are not trying to pull your leg.)

Here’s an example of how it plays out: One of the geometry problems asks you to prove that either the given triangle is isosceles or a particular angle is 60°. Tao quickly rules out coordinate geometry: “Hack-and-slash coordinate geometry is one long and boring way that is prone to abysmal complications and huge errors. Let us try that as a last resort.” Realizing that the conclusion involves angles and the given data involves equal line segments, he needs results that relate length and angle. So he writes down relevant facts about right triangles, isosceles triangles, the law of cosines, the law of sines. There aren’t a lot of right triangles in the figure so let’s not create any yet. The equal line segments aren’t part of an isosceles triangle that directly connects to the conclusion, so we’ll pass on that for now. He continues: “The law of cosines usually complicates rather than simplifies, and it just creates more unknown lengths. This leaves only the law of sines as a feasible alternative.” This leads to equations, then to further manipulations and reformulations and simplifications driven by the overall objective, and finally to the relevant angles.

Granted, this step-by-step reduction is particularly effective for Olympiad-like problems, and Tao acknowledges this in the original Preface:

[Olympiad-like] mathematics problems are “sanitized” mathematics, where an elegant solution has already been found, the question is stripped of all superfluousness and posed in an interesting and (hopefully) thought-provoking way. If mathematics is likened to prospecting for gold, solving a good mathematics problem is akin to a “hide-and-seek” course in gold-prospecting: you are given a nugget to find, and you know what it looks like, that it is out there somewhere, that it is not too hard to reach, that its unearthing is within your capabilities, and you have conveniently been given the right equipment (i.e., data) to get it. It may be hidden in a cunning place,
but it will require ingenuity rather than digging to get it.

Contest problems provide a setting in which to acquire the habit of thinking mathematically. Here the psychological advantage makes it ideally suited for developing a way of thinking that will transfer to "unsanitized" mathematics, the habit of doing what you can and thinking ahead about how the complicated parts might be simplified.

The book assumes that the reader has a strong background in basic arithmetic and algebra (modular arithmetic, factorization formulas, elementary trig identities), standard Euclidean geometry, including elementary vector methods, and properties of polynomials and functions, as well as mathematical induction. If Tao were fifteen today his book would probably have included more topics from discrete math, such as the pigeonhole principle, elementary graph theory (Euler's formula \( V - E + F = 2 \), Euler and Hamiltonian circuits), and combinatorics (counting principles, recurrences). Nevertheless, the overall problem-solving approach would be the same, and it is largely this feature, along with the thorough, caring, and energizing delivery, that makes the book a noteworthy addition to the literature on problem solving and how to teach it.

References

Simons Foundation Launches US$40-Million Program for Theoretical Research

Allyn Jackson

The Simons Foundation has launched a program that is now spending an estimated US$40 million a year to support research in mathematics and theoretical aspects of areas related to mathematics. The program began in fall 2009, creating seventy postdocs intended to ease the effects of the tight academic job market.

In mathematics, US$40 million is a serious chunk of change: For comparison, the yearly budget of the Division of Mathematical Sciences at the National Science Foundation (NSF) was US$226 million in fiscal year 2009. The Simons Foundation program will not be solely devoted to mathematics but will also fund “theoretical science radiating from mathematics”, as David Eisenbud put it. Eisenbud, a professor at the University of California Berkeley and former director of the Mathematical Sciences Research Institute (MSRI), is heading up the program as the Simons Foundation Vice President for Mathematics and the Physical Sciences.

James Simons, of Chern-Simons invariant fame, has made billions through his investment company, Renaissance Technologies. In 1994 he and his wife Marilyn established the Simons Foundation, which supports basic science and mathematics. The Simons fortune also funds Math for America, a separate entity from the Simons Foundation that focuses on attracting and retaining outstanding individuals to teach mathematics in public secondary schools. In 2009 James Simons, seventy-one years old, announced he is retiring from day-to-day management of the firm he has run for thirty-one years and will spend more time on his philanthropic work, which also includes support for autism research.

“The mission of the Simons Foundation is to support basic scientific research,” said James Simons in an email message. “There is already an active program in the life sciences, and several important but isolated activities in mathematics and the physical sciences, such as the newly founded Center for Geometry and Physics, but there is no coherently articulated program in the latter area. David Eisenbud was brought on to establish such a program, and over the next several months he will be gathering ideas and advice to guide him in carrying out this mission.” (For more information on the center, see “Major Gift to Stony Brook for Simons Center in Geometry and Physics”, Notices, June/July 2008.)

The new program does not have a name yet—nor has it been decided exactly how the US$40 million will be used. But the driving idea is to “strengthen basic research internationally and across science”, Eisenbud said. The plan is to structure the program so that it complements rather than duplicates modes of support already available through existing funders of theoretical research, such as the NSF. Private sources of support for mathematics include the Clay Mathematics Institute, which provides funding for individual mathematicians and specific projects like conferences, and the electronics-chain-store magnate John Fry, who is the main funder of the American Institute of Mathematics in Palo Alto, California. The Simons Foundation program will be “substantially larger” than what these other private funders provide, Eisenbud said.

As a first step in establishing the program, the foundation decided to fund about seventy

Allyn Jackson is senior writer and deputy editor of the Notices. Her email address is axj@ams.org.
postdoctoral positions, called Simons Postdoctoral Fellowships. Fifteen three-year positions in mathematics and ten three-year positions in theoretical physics will be funded this year and next; in theoretical computer science, nine two-year positions will be offered for three consecutive years. According to Eisenbud, the breakdown of positions was dictated by a sense of the “size of the enterprise” in each area, partly measured by the number of new doctorates in each. With about 1,400 new doctorates last year, mathematics had by far the largest number. The funding will be given to university departments, each of which will be able to hire one or two postdocs recruited from anywhere in the world. Recruitment began in fall 2009, and the positions will start in fall 2010.

The Simons Foundation has decided not to announce publicly the full list of departments receiving the postdoc funds. The selection of mathematics departments was made by a five-member committee of “distinguished mathematicians”, Eisenbud said; parallel committees in theoretical physics and theoretical computer science made the selections in those areas. The awards were made to departments that can nurture postdocs well; they were “not seen as rewards to great departments,” Eisenbud noted. The departments were chosen on the basis of “whether the committee felt that people in a reasonably broad range of subjects would advise their graduate students to go to those departments,” he explained.

These postdoctoral positions are sorely needed in the job market facing new and recent Ph.D.s, a market that Eisenbud described as “gruesome”. “The number of postdoctoral positions is down at research universities” by as much as one-half, he said. “This is something we will feel for years.” A survey conducted in early 2009 by an AMS task force on employment estimated that the number of academic positions open to new doctorates had declined by 39 percent from summer 2008 to summer 2009 (see http://www.ams.org/prof-services/employtaskforce/ETF.html for the task force report). The NSF pitched in to help in 2009 with a set of temporary postdoctoral positions managed jointly by the mathematical sciences research institutes. These postdocs will probably not be repeated in 2010, absent the flexibility provided by funds from the Obama administration’s one-time stimulus package.

The Simons Foundation decided to give the funds for the postdocs to departments, rather than to individuals, because there was not time to set up a mechanism for reviewing individual applications. In the future, if it does fund more postdocs, it will have a mechanism for direct application by individuals. Will the program in fact continue to fund postdocs? That is not clear yet—and indeed only a few aspects of the program have been decided upon. One of these is the areas to be supported: mathematics and theoretical topics in areas connected to it, such as theoretical physics and theoretical computer science (but not the biological sciences, which are the focus of a different Simons Foundation program). Support for large experimental facilities is not within the scope of the program, but Eisenbud said that it might be possible to fund some experimental projects “if the right project comes along”. Researchers from anywhere in the world are eligible for support. Unlike grants from the NSF, which pay overhead rates set by the institutions receiving the grants, the grants from the Simons Foundation program will pay only the foundation’s standard overhead rate of 20 percent of certain direct costs.

Exactly what kinds of activities the program will fund has yet to be determined. Asked whether the money might be used for a new institute, Eisenbud said that he thought it unlikely, given that the foundation just recently launched the Simons Center. However, the foundation has been giving funds to enhance activities at some institutes, and this might continue. How about a small grants program, which many mathematicians in the United States have said is what they really need? That is a possibility. “Nothing is settled yet,” Eisenbud said. “Figuring out something really useful and really effective to do, even with a lot of money, is, I think, not so easy,” Eisenbud remarked. To this end the Simons Foundation will in coming months hold several roundtable events in which mathematicians and scientists “will offer us, we hope, sage advice about how to spend this money,” Eisenbud said. “We will listen hard before we make any decisions.” The foundation will also work closely with organizations such as the NSF and the Clay Mathematics Institute to ensure that the new program complements rather than duplicates existing ones.

Eisenbud said he would be interested in hearing from researchers who have ideas for how best to use the funds (his email address is de@simonsfoundation.org). It is not appropriate at this stage to send proposals for specific research projects; what is needed at this time are ideas about what kind of grant mechanisms would be most effective. Input from a broad segment of the mathematical community can help ensure that this unusual program has a lasting and positive impact on the field.
2009 Annual Survey of the Mathematical Sciences
(First Report)


Polly Phipps, James W. Maxwell, and Colleen Rose

This report presents a statistical profile of recipients of doctoral degrees awarded by departments in the mathematical sciences at universities in the United States during the period July 1, 2008, through June 30, 2009. All information in the report was provided over the summer and early fall of 2009 by the departments that awarded the degrees. The report includes a preliminary analysis of the fall 2009 employment plans of 2008-09 doctoral recipients and a demographic profile summarizing characteristics of citizenship status, gender, and racial/ethnic group. This preliminary report will be updated in the Second Report of the 2009 Annual Survey to reflect subsequent reports of additional 2008-2009 doctoral recipients from the departments that did not respond in time for this report. No adjustments have been made to the numbers in this report for the non-responding departments. The Second Report, to appear in the August 2010 issue of Notices, will also reflect additional information provided by the new doctoral recipients themselves, including their starting salaries.

Table 1 provides the number of departments responding to the 2009 Survey of New Doctoral Recipients in time for this report. This year’s response rates were above 80% for all groups except Group IV which was 74% (up from 67% last year.) Overall, eighteen more departments responded in time for the First Report this year than responded in time for last year’s First Report. Efforts continue to obtain data from as many of the non-responding departments as possible.

Table 1: Number of Departments Responding to Doctorates Granted Survey

| Group I (Pu) | 23 of 25 including 0 with no degrees |
| Group I (Pr) | 21 of 23 including 0 with no degrees |
| Group II | 50 of 56 including 3 with no degrees |
| Group III | 72 of 81 including 18 with no degrees |
| Group IV Statistics | 68 of 92 including 4 with no degrees |
| Group IV Biostatistics | 45 of 57 including 2 with no degrees |
| Group Va | 18 of 21 including 0 with no degrees |

Table 2: New Doctoral Degrees Awarded by Group, Preliminary Count

<table>
<thead>
<tr>
<th>Group</th>
<th>I (Pu)</th>
<th>I (Pr)</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Va</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999-00</td>
<td>256</td>
<td>157</td>
<td>223</td>
<td>132</td>
<td>284</td>
<td>67</td>
<td>1119</td>
</tr>
<tr>
<td>2000-01</td>
<td>233</td>
<td>129</td>
<td>203</td>
<td>125</td>
<td>237</td>
<td>81</td>
<td>1008</td>
</tr>
<tr>
<td>2001-02</td>
<td>218</td>
<td>139</td>
<td>164</td>
<td>124</td>
<td>222</td>
<td>81</td>
<td>948</td>
</tr>
<tr>
<td>2002-03</td>
<td>158</td>
<td>138</td>
<td>170</td>
<td>121</td>
<td>239</td>
<td>91</td>
<td>1017</td>
</tr>
<tr>
<td>2003-04</td>
<td>195</td>
<td>187</td>
<td>215</td>
<td>111</td>
<td>243</td>
<td>90</td>
<td>1041</td>
</tr>
<tr>
<td>2004-05</td>
<td>243</td>
<td>146</td>
<td>203</td>
<td>153</td>
<td>285</td>
<td>86</td>
<td>1116</td>
</tr>
<tr>
<td>2005-06</td>
<td>307</td>
<td>184</td>
<td>216</td>
<td>140</td>
<td>287</td>
<td>111</td>
<td>1245</td>
</tr>
<tr>
<td>2006-07</td>
<td>300</td>
<td>119</td>
<td>234</td>
<td>138</td>
<td>279</td>
<td>87</td>
<td>1157</td>
</tr>
<tr>
<td>2007-08</td>
<td>234</td>
<td>172</td>
<td>290</td>
<td>142</td>
<td>291</td>
<td>106</td>
<td>1235</td>
</tr>
<tr>
<td>2008-09</td>
<td>326</td>
<td>211</td>
<td>282</td>
<td>171</td>
<td>352</td>
<td>88</td>
<td>1430</td>
</tr>
</tbody>
</table>

Table 2: New Doctoral Degrees Awarded by Group, Preliminary Count

<table>
<thead>
<tr>
<th>Group</th>
<th>I (Pu)</th>
<th>I (Pr)</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Va</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999-00</td>
<td>256</td>
<td>157</td>
<td>223</td>
<td>132</td>
<td>284</td>
<td>67</td>
<td>1119</td>
</tr>
<tr>
<td>2000-01</td>
<td>233</td>
<td>129</td>
<td>203</td>
<td>125</td>
<td>237</td>
<td>81</td>
<td>1008</td>
</tr>
<tr>
<td>2001-02</td>
<td>218</td>
<td>139</td>
<td>164</td>
<td>124</td>
<td>222</td>
<td>81</td>
<td>948</td>
</tr>
<tr>
<td>2002-03</td>
<td>158</td>
<td>138</td>
<td>170</td>
<td>121</td>
<td>239</td>
<td>91</td>
<td>1017</td>
</tr>
<tr>
<td>2003-04</td>
<td>195</td>
<td>187</td>
<td>215</td>
<td>111</td>
<td>243</td>
<td>90</td>
<td>1041</td>
</tr>
<tr>
<td>2004-05</td>
<td>243</td>
<td>146</td>
<td>203</td>
<td>153</td>
<td>285</td>
<td>86</td>
<td>1116</td>
</tr>
<tr>
<td>2005-06</td>
<td>307</td>
<td>184</td>
<td>216</td>
<td>140</td>
<td>287</td>
<td>111</td>
<td>1245</td>
</tr>
<tr>
<td>2006-07</td>
<td>300</td>
<td>119</td>
<td>234</td>
<td>138</td>
<td>279</td>
<td>87</td>
<td>1157</td>
</tr>
<tr>
<td>2007-08</td>
<td>234</td>
<td>172</td>
<td>290</td>
<td>142</td>
<td>291</td>
<td>106</td>
<td>1235</td>
</tr>
<tr>
<td>2008-09</td>
<td>326</td>
<td>211</td>
<td>282</td>
<td>171</td>
<td>352</td>
<td>88</td>
<td>1430</td>
</tr>
</tbody>
</table>

Doctoral Degrees Granted in 2008–09

Table 2 shows the number of new doctoral degrees granted by the different doctoral groups surveyed in the Annual Survey for the past ten years. The preliminary count of 1,430 new doctorates granted by these departments in 2008–09 is an increase of 195 from the preliminary count for 2007–08.

From Table 2 we see that all groups except Groups II and Va reported an increase in the number of doctoral recipients from the previous year. The

Polly Phipps is a senior research statistician with the Bureau of Labor Statistics. James W. Maxwell is AMS associate executive director for special projects. Colleen A. Rose is AMS survey analyst.
were new citizens to doctoral academic this year's U.S. U.S. doctoral all new are in U.S. IV, 303 of Group positions doctoral the recipients, 4 among the 1,430 new doctoral recipients are found on pages 276–99 of this issue of the Notices.

By way of background, additional information about various types of full-time graduate students is available in the Third Report of the 2008 Annual Survey (Notices, November 2008), Table 6B, page 1299.

Employment Plans of 2008–09 New Doctoral Recipients

Tables 4A and 4B each provide a cross-tabulation of the 1,430 new doctoral recipients in the mathematical sciences. These tables contain a wealth of information about these new doctoral recipients, some of which will be discussed in this report. Note that these tables give a breakdown by gender for type of employer and type of degree-granting department. Additional information is available on the AMS website at employment/surveyreports.html

The preliminary unemployment rate for these data is 7.9%. This preliminary rate will be updated later with information gathered from the

Figure 1: New Doctoral Degrees Awarded by Combined Groups, Preliminary Count

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Degrees</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999-00</td>
<td>1200</td>
</tr>
<tr>
<td>2000-01</td>
<td>1100</td>
</tr>
<tr>
<td>2001-02</td>
<td>1000</td>
</tr>
<tr>
<td>2002-03</td>
<td>900</td>
</tr>
<tr>
<td>2003-04</td>
<td>800</td>
</tr>
<tr>
<td>2004-05</td>
<td>700</td>
</tr>
<tr>
<td>2005-06</td>
<td>600</td>
</tr>
<tr>
<td>2006-07</td>
<td>500</td>
</tr>
<tr>
<td>2007-08</td>
<td>400</td>
</tr>
<tr>
<td>2008-09</td>
<td>300</td>
</tr>
</tbody>
</table>

Highlights

There were 1,430 new doctoral recipients reported for 2008-09 by departments responding in time for the 2009 First Report. The number of departments responding in time for this year’s report increased by 8 and in every group except Group IV. When one considers only the 208 departments that responded in time for the First Report in both years, the 2009 figure reported by these departments is up 9.7% over that reported for 2008.

There were 669 U.S. citizens reported among this year’s new doctoral recipients, 47% of the total. Last year’s figure was 44%. The fall preliminary 2009 unemployment rate for the 1,231 new doctoral recipients whose employment status is known is 7.9%, up from 5.4% for fall 2008.

Seventy-two new doctoral recipients hold positions at the institution that granted their degree, although not necessarily in the same department. This is 6% of the new doctoral recipients who are currently known to have jobs and 10% of those who have academic positions in the U.S. Seventeen new doctoral recipients have part-time positions.

The number of new doctoral recipients employed in the U.S. is 987, up 101 from last year. The number of new doctoral recipients employed in academic positions in the U.S. has increased to 741, compared to 650 last year.

Of the 987 new doctoral recipients taking positions in the U.S., 184 (19%) have jobs in business and industry, a decrease of 11% from last year’s figure of 207. The fall 2009 number remains up 69 (60%) from fall 2005. The number of new doctoral recipients taking jobs in government is up 30 (107%) over fall 2008.

Among the 987 new doctoral recipients having employment in the U.S., 501 (51%) are U.S. citizens (up from 426 (48%) last year). The number of non-U.S. citizens having employment in the U.S. is 486; last year it was 460.

Among the 333 new doctoral recipients hired by U.S. degree-granting departments, 47% are U.S. citizens (down from 49% last year). Among the 408 having other academic positions in the U.S., 62% are U.S. citizens (up from 57% last year).

Of the 1,430 new doctoral recipients, 32% (462) are female, the same percentage reported in fall 2008. Of the 669 U.S. citizen new doctoral recipients, 30% (202) are female, down from 31% in fall 2008.

Among the 669 U.S. citizen new doctoral recipients, 4 are American Indian or Alaska Native, 44 are Asian, 19 are Black or African American, 19 are Hispanic or Latino, 1 is Native Hawaiian or Other Pacific Islander, 575 are White, and 7 are of unknown race/ethnicity.

Group IV produced 352 new doctorates, of which 159 (45%) are female, compared to all other groups combined, where 303 (28%) are female. In Group IV, 92 (26%) of the new doctoral recipients are U.S. citizens (while in the other groups 54% are U.S. citizens).

Twenty-nine percent of the new doctorates had a dissertation in statistics/biostatistics (410). The next highest percentage was in algebra and number theory with 14% (203).

The Faculty Salary Survey report, will appear in the March issue of Notices.
individual new doctoral recipients. The additional information from prior years is reflected in the final unemployment rates displayed in Figure 2. The preliminary rate for fall 2008 was 6.0%. The unemployment rates shown in Figure 2 differ from those given in previous Annual Survey reports. The rates shown are now based on only those 1,072 individuals in the U.S. labor market.

For further details, see the explanatory note on unemployment rates at the end of the report. The unemployment rates, calculated by type of doctoral degree-granting department using Table 4A, vary from group to group, with a high of 12.8% for Group III and a low of 3.3% for Group IV.

There are 987 new doctoral recipients employed in the U.S. Table 5A gives a breakdown of type of employer by type of degree-granting department for these 987 new doctoral recipients. Of these, 741 (75%) hold academic positions, 62 (6%) are employed by government, and 184 (19%) hold positions in business and industry. In the First Report for 2007–08, there were 886 new doctoral recipients employed in the U.S., of which 649 (73%) held academic positions, 30 (3%) were in government, and 207 (23%) were in business and industry. The number of new doctoral recipients employed in the U.S. increased in all categories except "Business and Industry" which decreased 11%. "Government" showed the largest percentage increase, 107%.

Table 5B shows the number of new doctoral recipients who took positions in business and industry by the type of department granting their degree for fall 2005 to fall 2009. The number of new doctoral recipients taking jobs in business and industry, which had been steadily increasing, decreased from 207 to 184, an 11%
Table 4B: Field of Thesis of 2008–09 New Doctoral Recipients by Type of Degree-Granting Department

<table>
<thead>
<tr>
<th>TYPE OF DOCTORAL DEGREE-GRANTING DEPARTMENT</th>
<th>FIELD OF THESIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group I (Public)</td>
<td>77</td>
</tr>
<tr>
<td>Group I (Private)</td>
<td>52</td>
</tr>
<tr>
<td>Group II</td>
<td>57</td>
</tr>
<tr>
<td>Group III</td>
<td>16</td>
</tr>
<tr>
<td>Group IV</td>
<td>0</td>
</tr>
<tr>
<td>Group Va</td>
<td>1</td>
</tr>
<tr>
<td>Column Total</td>
<td>203</td>
</tr>
<tr>
<td>Column Subtotals</td>
<td>Male</td>
</tr>
<tr>
<td></td>
<td>Female</td>
</tr>
</tbody>
</table>

Table 5A: 2008–09 New Doctoral Recipients Employed in the U.S. by Type of Degree-Granting Department

<table>
<thead>
<tr>
<th>Type of Employer in U.S.</th>
<th>I (Pu)</th>
<th>I (Pr)</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Va</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groups I, II, III, IV, and Va</td>
<td>101</td>
<td>70</td>
<td>59</td>
<td>29</td>
<td>59</td>
<td>15</td>
<td>333</td>
</tr>
<tr>
<td>Master’s, Bachelor’s, and 2-Year Colleges</td>
<td>48</td>
<td>20</td>
<td>82</td>
<td>52</td>
<td>21</td>
<td>9</td>
<td>232</td>
</tr>
<tr>
<td>Other Academic and Research Institutes</td>
<td>32</td>
<td>14</td>
<td>17</td>
<td>16</td>
<td>75</td>
<td>22</td>
<td>176</td>
</tr>
<tr>
<td>Government</td>
<td>7</td>
<td>1</td>
<td>10</td>
<td>13</td>
<td>27</td>
<td>4</td>
<td>62</td>
</tr>
<tr>
<td>Business and Industry</td>
<td>31</td>
<td>17</td>
<td>23</td>
<td>13</td>
<td>85</td>
<td>15</td>
<td>184</td>
</tr>
<tr>
<td>TOTAL</td>
<td>219</td>
<td>122</td>
<td>191</td>
<td>123</td>
<td>267</td>
<td>65</td>
<td>987</td>
</tr>
</tbody>
</table>

Table 5B: Number of New Doctoral Recipients Taking Positions in Business and Industry in the U.S. by Type of Degree-Granting Department, Fall 2005 to Fall 2009

<table>
<thead>
<tr>
<th>Year</th>
<th>I (Pu)</th>
<th>I (Pr)</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Va</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall 2005</td>
<td>6</td>
<td>9</td>
<td>14</td>
<td>15</td>
<td>64</td>
<td>8</td>
<td>115</td>
</tr>
<tr>
<td>Fall 2006</td>
<td>27</td>
<td>14</td>
<td>19</td>
<td>9</td>
<td>80</td>
<td>18</td>
<td>167</td>
</tr>
<tr>
<td>Fall 2007</td>
<td>39</td>
<td>10</td>
<td>16</td>
<td>19</td>
<td>88</td>
<td>15</td>
<td>187</td>
</tr>
<tr>
<td>Fall 2008</td>
<td>24</td>
<td>19</td>
<td>32</td>
<td>22</td>
<td>87</td>
<td>23</td>
<td>207</td>
</tr>
<tr>
<td>Fall 2009</td>
<td>31</td>
<td>17</td>
<td>23</td>
<td>13</td>
<td>85</td>
<td>15</td>
<td>184</td>
</tr>
</tbody>
</table>

Among the 987 new doctoral recipients known to have employment in the U.S. in fall 2009, Group III has the smallest percentage taking jobs in business and industry at 11% and Group IV the highest at 32%.

Table 5C shows the number of new doctoral recipients who took academic positions in the U.S. by type of department granting their degree for fall 2005 to fall 2009. The total number of new doctoral recipients taking academic employment in fall 2009 increased 14% to 741 from 649 last year. Among the 987 new doctoral recipients employed in the U.S. in fall 2009, 75% have academic positions. This percentage is highest for Group I (Pr) at 85% and lowest for Groups IV at 58%.

Table 5D shows the number of positions filled with new doctoral recipients for each type of academic employer. Increases in positions filled by new doctoral recipients were realized by all groups. The biggest increase in hires of new doctorates into academic positions was in Groups IV and Va with 43% and 44%, respectively. Hires of new doctorates into positions at research institutes increased from 29 in fall 2008 to 42 in fall 2009.

In fall 2009, 72 new doctoral recipients held positions in the institution that granted their degree, although not necessarily in the same department. This represents 6% of new doctoral recipients who are currently employed in the U.S. and 10% of the U.S. academic positions held by new doctoral recipients. In fall 2008 there were 69 such individuals making up 7% of the new doctoral recipients who were employed at the time of the First Report. Seventeen new doctoral recipients have taken part-time positions in fall 2009 compared with 18 in fall 2008.

Information about 2008–09 Female New Doctoral Recipients

Tables 4A and 4B give the breakdown of the new doctoral recipients in 2008–09 by Field of Thesis, by Type of Degree-Granting Department and by Type of Employer.

Overall, 462 (32%) of the 1,430 new doctoral recipients in 2008–09 are female. In 2007–08, 388 (31%) of the new doctoral recipients were female. This percentage varies over the different groups, and these percentages are given in the first row of Table 5E. This year the percentage of females produced is highest again for Group IV at 45%, compared with 52% last year. The second row of Table 5E gives the percentage of the new doctoral recipients hired who are female for each of the Groups I, II, III, IV, and Va. In addition, 40% of the
new doctoral recipients hired in Group M, Master’s departments, are female; 43% of the new doctoral recipients hired in Group B, Bachelor’s departments, are female, up from 40% last year. This year, Group IV hired the highest percentage of women with 38%, while Groups I, II, III ranged from 16% to 31%.

The unemployment rate for female new doctoral recipients is 5.8%, compared to 9.0% for males and 7.9% overall.

The percentage of female new doctoral recipients within fields of thesis ranged from 19% in linear, nonlinear optimization/control, to 69% in mathematics education and 45% in statistics.

Later sections in this First Report give more information about the female new doctoral recipients by citizenship and the female new doctoral recipients in Group IV.

### Employment Information about 2008–09 New Doctoral Recipients by Citizenship and Type of Employer

Table 5F shows the pattern of employment within employer categories broken down by citizenship status of the new doctoral recipients. The unemployment rate for the U.S. citizens is 8.6% compared to 5.8% in fall 2008. The unemployment rate for non-U.S. citizens is 7.3%. This varies by type of visa. The unemployment rate for non-U.S. citizens with a permanent visa is 8.3%, while that for non-U.S. citizens with a temporary visa is 7.2%. Among U.S. citizens whose employment status is known, 84% are employed in the U.S. 

### Table 5C: Number of New Doctoral Recipients Taking U.S. Academic Positions by Type of Degree-Granting Department, Fall 2005 to Fall 2009

<table>
<thead>
<tr>
<th>Year</th>
<th>I (Pu)</th>
<th>I (Pr)</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>Va</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall 2005</td>
<td>131</td>
<td>88</td>
<td>130</td>
<td>83</td>
<td>131</td>
<td>39</td>
<td>602</td>
</tr>
<tr>
<td>Fall 2006</td>
<td>167</td>
<td>108</td>
<td>123</td>
<td>86</td>
<td>137</td>
<td>50</td>
<td>671</td>
</tr>
<tr>
<td>Fall 2007</td>
<td>178</td>
<td>76</td>
<td>146</td>
<td>87</td>
<td>120</td>
<td>43</td>
<td>650</td>
</tr>
<tr>
<td>Fall 2008</td>
<td>126</td>
<td>90</td>
<td>174</td>
<td>77</td>
<td>133</td>
<td>49</td>
<td>649</td>
</tr>
<tr>
<td>Fall 2009</td>
<td>181</td>
<td>104</td>
<td>158</td>
<td>97</td>
<td>155</td>
<td>46</td>
<td>741</td>
</tr>
</tbody>
</table>

### Table 5D: Academic Positions in U.S. Filled by New Doctoral Recipients by Type of Hiring Department, Fall 2005 to Fall 2009

<table>
<thead>
<tr>
<th>Year</th>
<th>I–III</th>
<th>IV</th>
<th>Va</th>
<th>M&amp;B</th>
<th>Other*</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fall 2005</td>
<td>231</td>
<td>45</td>
<td>12</td>
<td>188</td>
<td>126</td>
<td>602</td>
</tr>
<tr>
<td>Fall 2006</td>
<td>262</td>
<td>69</td>
<td>12</td>
<td>185</td>
<td>143</td>
<td>671</td>
</tr>
<tr>
<td>Fall 2007</td>
<td>264</td>
<td>42</td>
<td>9</td>
<td>180</td>
<td>162</td>
<td>649</td>
</tr>
<tr>
<td>Fall 2008</td>
<td>256</td>
<td>42</td>
<td>9</td>
<td>180</td>
<td>162</td>
<td>649</td>
</tr>
<tr>
<td>Fall 2009</td>
<td>260</td>
<td>60</td>
<td>13</td>
<td>210</td>
<td>198</td>
<td>741</td>
</tr>
</tbody>
</table>

*Includes other academic and research institutes/nonprofit.

### Table 5E: Females as a Percentage of 2008–09 New Doctoral Recipients Produced by and Hired by Doctoral-Granting Groups

<table>
<thead>
<tr>
<th>Group</th>
<th>NON-U.S. CITIZENS</th>
<th>Permanent Visa</th>
<th>Temporary Visa</th>
<th>Unknown Visa</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. Employers</td>
<td>501</td>
<td>66</td>
<td>414</td>
<td>6</td>
</tr>
<tr>
<td>U.S. Academic</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Groups I, II, III, and Va</td>
<td>412</td>
<td>45</td>
<td>280</td>
<td>4</td>
</tr>
<tr>
<td>Group IV</td>
<td>135</td>
<td>14</td>
<td>122</td>
<td>2</td>
</tr>
<tr>
<td>Non-Ph.D. Department</td>
<td>238</td>
<td>20</td>
<td>107</td>
<td>1</td>
</tr>
<tr>
<td>Research Institute/Other Nonprofit</td>
<td>16</td>
<td>5</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>U.S. Nonacademic</td>
<td>89</td>
<td>21</td>
<td>134</td>
<td>2</td>
</tr>
<tr>
<td>Non-U.S. Employer</td>
<td>39</td>
<td>3</td>
<td>99</td>
<td>3</td>
</tr>
<tr>
<td>Non-U.S. Academic</td>
<td>36</td>
<td>3</td>
<td>87</td>
<td>3</td>
</tr>
<tr>
<td>Non-U.S. Nonacademic</td>
<td>3</td>
<td>0</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>Not Seeking Employment</td>
<td>8</td>
<td>0</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>Still Seeking Employment</td>
<td>47</td>
<td>6</td>
<td>32</td>
<td>0</td>
</tr>
<tr>
<td>SUBTOTAL</td>
<td>595</td>
<td>75</td>
<td>552</td>
<td>9</td>
</tr>
<tr>
<td>Unknown (U.S.)</td>
<td>72</td>
<td>8</td>
<td>29</td>
<td>0</td>
</tr>
<tr>
<td>Unknown (non-U.S.)*</td>
<td>2</td>
<td>1</td>
<td>80</td>
<td>7</td>
</tr>
</tbody>
</table>

*Includes those who left the U.S. and whose employment status is reported as "unknown" or "still seeking employment".

### Table 5F: Employment Status of 2008–09 New Doctoral Recipients by Citizenship Status

<table>
<thead>
<tr>
<th>CITIZENSHIP</th>
<th>U.S. CITIZENS</th>
<th>NON-U.S. CITIZENS</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent Visa</td>
<td>Temporary Visa</td>
<td>Unknown Visa</td>
<td></td>
</tr>
<tr>
<td>U.S. Employer</td>
<td>501</td>
<td>66</td>
<td>414</td>
</tr>
<tr>
<td>U.S. Academic</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Groups I, II, III, and Va</td>
<td>412</td>
<td>45</td>
<td>280</td>
</tr>
<tr>
<td>Group IV</td>
<td>135</td>
<td>14</td>
<td>122</td>
</tr>
<tr>
<td>Non-Ph.D. Department</td>
<td>238</td>
<td>20</td>
<td>107</td>
</tr>
<tr>
<td>Research Institute/Other Nonprofit</td>
<td>16</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>U.S. Nonacademic</td>
<td>89</td>
<td>21</td>
<td>134</td>
</tr>
<tr>
<td>Non-U.S. Employer</td>
<td>39</td>
<td>3</td>
<td>99</td>
</tr>
<tr>
<td>Non-U.S. Academic</td>
<td>36</td>
<td>3</td>
<td>87</td>
</tr>
<tr>
<td>Non-U.S. Nonacademic</td>
<td>3</td>
<td>0</td>
<td>12</td>
</tr>
<tr>
<td>Not Seeking Employment</td>
<td>8</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>Still Seeking Employment</td>
<td>47</td>
<td>6</td>
<td>32</td>
</tr>
<tr>
<td>SUBTOTAL</td>
<td>595</td>
<td>75</td>
<td>552</td>
</tr>
<tr>
<td>Unknown (U.S.)</td>
<td>72</td>
<td>8</td>
<td>29</td>
</tr>
<tr>
<td>Unknown (non-U.S.)*</td>
<td>2</td>
<td>1</td>
<td>80</td>
</tr>
</tbody>
</table>

TOTAL | 669 | 84 | 661 | 16 | 1430 |
Among non-U.S. citizens with a permanent visa whose employment status is known, 88% have jobs in the U.S. (last year this percentage was 90%), while the similar percentage for non-U.S. citizens with a temporary visa is 75% (last year the percentage was 76%). The number of non-U.S. citizens having employment in the U.S. is 486, up from 459 last year.

Table 5G is a cross-tabulation of the 987 new doctoral recipients who have employment in the U.S. by type of employer and citizenship. Using numbers from Table 5F. Of the 987 new doctoral recipients having jobs in the U.S., 51% are U.S. citizens (up from 48% last year). Of the 333 new doctoral recipients who took jobs in U.S. doctoral-granting departments, 47% are U.S. citizens (down from 49% last year). Of the 486 non-U.S. citizens employed in the U.S., 36% have jobs in doctoral-granting departments, 32% are in other academic positions, and 32% are in nonacademic positions. For the 486 non-U.S. citizens employed in the U.S., the analogous percentages are 36%, 32%, and 32% respectively.

Table 6 presents a breakdown of new doctoral recipients according to gender, racial/ethnic group, and citizenship status. The information reported in this table was obtained in summary form from the departments granting the degrees. Additional reports on gender, race/ethnicity, and citizenship is available on the Web at www.ams.org/employment/specialreports.html.

There were 669 (47%) U.S. citizens among the 1,430 new doctoral recipients in 2008–09. Among U.S. citizens, 4 are American Indian or Alaska Native, 44 are Asian, 19 are Black or African American, 19 are Hispanic or Latino, 1 is Native Hawaiian or Other Pacific Islander, 575 are White, and 7 are of unknown race/ethnicity. Among non-U.S. citizens, there are no American Indians or Alaska Natives, 471 Asians, 18 Blacks or African Americans, 49 Hispanics or Latinos, 217 Whites, and 6 of unknown race/ethnicity.

Table 7 gives the number of new U.S. doctoral recipients and the number of U.S. citizens back to 1998–99. The 669 U.S. citizen new doctoral recipients is up by 236 (55%) from its low in 2004–05.

Females make up 30% of the 669 U.S. citizens receiving doctoral degrees in the mathematical sciences in 2008–09. Last year this percentage was 31%. Among the 761 non-U.S. citizen new doctoral recipients, 34% (260) are female, up from last year’s 32%. Figure 3 gives the historical record of U.S. citizen female new doctoral recipients and the percentage of females among U.S. citizen (full-time) graduate students beginning in fall 1989. The number of female U.S. citizen new doctoral recipients is up 15 (8%) from 187 in 1998–99; reaching an all time new high. Additional historical information on U.S. citizen doctoral recipients is available on the Web.

### Table 5G: 2008–09 New Doctoral Recipients Having Employment in the U.S. by Type of Employer and Citizenship

<table>
<thead>
<tr>
<th>U.S. EMPLOYER</th>
<th>CITIZENSHIP</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S.</td>
<td>Non-U.S.</td>
<td>TOTAL</td>
<td></td>
</tr>
<tr>
<td>Academic</td>
<td>412</td>
<td>329</td>
<td>741</td>
<td></td>
</tr>
<tr>
<td>Groups I–Va</td>
<td>158</td>
<td>175</td>
<td>333</td>
<td></td>
</tr>
<tr>
<td>M, B, &amp; 2-Year</td>
<td>166</td>
<td>66</td>
<td>232</td>
<td></td>
</tr>
<tr>
<td>Other Acad. &amp; Research Inst.</td>
<td>88</td>
<td>88</td>
<td>176</td>
<td></td>
</tr>
<tr>
<td>Government, Business &amp; Industry</td>
<td>89</td>
<td>157</td>
<td>246</td>
<td></td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>501</strong></td>
<td><strong>486</strong></td>
<td><strong>987</strong></td>
<td></td>
</tr>
</tbody>
</table>

### Table 6: Gender, Race/Ethnicity, and Citizenship of 2008–09 New Doctoral Recipients

<table>
<thead>
<tr>
<th>RACIAL/ETHNIC GROUP</th>
<th>MALE</th>
<th>FEMALE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>U.S. CITIZENS</td>
<td>Permanent</td>
</tr>
<tr>
<td>American Indian or Alaska Native</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Asian</td>
<td>31</td>
<td>14</td>
</tr>
<tr>
<td>Black or African American</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>Hispanic or Latino</td>
<td>10</td>
<td>4</td>
</tr>
<tr>
<td>Native Hawaiian or Other Pacific Islander</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>White</td>
<td>413</td>
<td>24</td>
</tr>
<tr>
<td>Unknown</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>467</strong></td>
<td><strong>45</strong></td>
</tr>
</tbody>
</table>


2008–09 New Doctoral Recipients with Dissertations in Statistics/Biostatistics and Probability

Group IV contains U.S. departments (or programs) of statistics, biostatistics, and biometrics reporting a doctoral program. In the Annual Survey Reports, Group IV is referred to as the Statistics Group. In addition, other groups in the Annual Survey produce new doctoral recipients with dissertations in statistics/biostatistics or probability. The other groups produced 140 new doctoral recipients with dissertations in statistics/biostatistics or probability in 2008–09 and have averaged 90 per year over the ten-year period reported in Table 8. Information about these 140 new doctoral recipients and the 347 new doctoral recipients in Group IV is found in this section of the report.

Table 8 contains information about new doctoral recipients in Group IV as well as those with dissertations in statistics/biostatistics and probability in other groups for this year as well as for the past nine years. In addition, the last two rows of Table 8 give a split of the 2008–09 results between the 57 statistics departments and the 35 biostatistics and biometrics departments in Group IV. This year 487 new doctorates had a dissertation in statistics/biostatistics (410) or probability (77), a 27% increase from last year’s number. Those with dissertations in statistics/biostatistics and probability accounted for 34% of new doctorates in 2008–09. Quite a bit of the year-to-year variation in these numbers is due to the changes made in the departments included in Group IV over the ten years and to the response rate variation in this group.

Group IV has 92 departments for 2008–09, 11 more than the next largest doctoral group. It contains 31% of all doctoral departments surveyed, and the 68 Group IV departments responding to the Annual Survey reported 352 new doctoral recipients, 25% of all new doctoral recipients in 2008–09. The number of new doctoral recipients in Group IV is up 61 from the number reported at this time last year, while the number of departments responding is up 8 from the number responding by this time last year.

Because of its size, the data from Group IV have a large effect on the results when all doctoral groups are combined. Furthermore, Group IV results are often quite different from those for Groups I (Pu), I (Pr), II, III, and Va. Group IV results can mask important changes in the other doctoral groups. In the following paragraphs some of these differences are presented.

Group IV is producing a larger percentage of female doctorates than the other doctoral groups. Females accounted for 45% of the 352 new doctoral recipients in Group IV, while 28% of 1,078 are female in the other doctoral groups. Among U.S. citizens, females accounted for 59% of the 92 Group IV new doctoral recipients, while for the other groups 26% of 577 were female. Overall, 30% of the 669 U.S. citizen new doctoral recipients were female.

Group IV is producing a smaller percentage of U.S. citizen new doctorates than the other doctoral groups. In Group IV, 36% of the 352 new doctoral recipients are U.S. citizens, while in other groups 50% of the 1,078 are U.S. citizens. In Group IV, 66% of the 159 females were not U.S. citizens.

Group IV doctorates are more likely to take jobs in business and industry than those in other doctoral
groups. Of the 267 new doctoral recipients from Group IV who found employment in the U.S., 85 (32%) took jobs in business or industry. From the other groups, 720 new doctoral recipients found employment in the U.S., of which 99 (14%) took jobs in business or industry.

Group IV doctorates have a lower unemployment rate than the other doctoral groups. The employment status for 301 Group IV new doctoral recipients is known, and 9 (3.3%) are unemployed. For the other groups, the employment status of 930 is known, and 66 (9.5%) are unemployed. Group IV is hiring a bigger percentage of females than the other doctoral groups. Twenty-three of 60 (38%) new doctoral recipients hired by Group IV departments were female, down from last year's 50%. The other doctoral groups reported that 63 of 273 (23%) new doctoral recipients hired were female, down from last year's 50%. The other doctoral departments had 140 (5.0%) new doctoral recipients hired by Group IV departments, of which 57 (39%) were female, up from last year's 22%.

Group IV had 347 new doctoral recipients with fields of thesis in statistics/biostatistics (343) and the other doctoral departments had 140 with fields of thesis in statistics/biostatistics (67) and probability (73) (last year the other doctoral departments had 57 new doctorates in statistics and 44 in probability). The distribution of these degrees among the various groups can be found in Table 4B. The number of new doctoral recipients with theses in statistics/biostatistics and probability (487) is substantially larger than any other field, with algebra and number theory next with 203.

Changes in Reporting of Unemployment Rate

In the unemployment calculations provided in this report the individuals employed outside the U.S. have been removed from the denominator used in the calculation of the rate, in addition to the routine removal of all individuals whose employment status is unknown. This is a change from prior Annual Survey Reports. As a consequence, the unemployment rate now being reported more accurately reflects the U.S. labor market experienced by the new doctoral recipients. This change tends to increase the rate of unemployment over that produced in prior years.

In a further small change from prior years, those individuals reported as not seeking employment have also been removed from the denominator. The number of individuals so designated is small each year, and the impact of this change is to produce a slight increase in the rate over that reported in prior years.

The unemployment rates for years prior to 2009 shown in this report have been recalculated using this new method. One can view a comparison of the unemployment rates using the traditional method and the new method by visiting the AMS website at [www.ams.org/employment/surveyreports.html].

Previous Annual Survey Reports

The 2008 First, Second, and Third Annual Survey Reports were published in the Notices of the AMS in the February, August, and November 2009 issues respectively. These reports and earlier reports, as well as a wealth of other information from these surveys, are available on the AMS website at [www.ams.org/employment/surveyreports.html].

### Table 8: New Doctoral Recipients with Dissertations in Statistics/Biostatistics and Probability

<table>
<thead>
<tr>
<th>Year</th>
<th>Group IV Depts Surveyed</th>
<th>Group IV Depts Responding (percent)</th>
<th>New Doctoral Recipients in Group IV only</th>
<th>New Doctoral Recipients in Statistics/Biostatistics and Probability, Group IV and Other* Groups</th>
<th>New Doctoral Recipients Hired by Group IV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Total (percent)</td>
<td>Total</td>
<td>Total (percent)</td>
<td>Male (percent)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Male (percent)</td>
<td>Female (percent)</td>
<td>Group IV</td>
<td>Female (percent)</td>
</tr>
<tr>
<td>1999-00</td>
<td>89</td>
<td>284 (110) (39%)</td>
<td>79 (5.0%)</td>
<td>351 (278) (73)</td>
<td>2.2% (24)</td>
</tr>
<tr>
<td>2000-01</td>
<td>86</td>
<td>237 (98) (41%)</td>
<td>59 (5.9%)</td>
<td>289 (221) (68)</td>
<td>6.1% (27)</td>
</tr>
<tr>
<td>2001-02</td>
<td>86</td>
<td>222 (92) (41%)</td>
<td>56 (6.6%)</td>
<td>288 (221) (67)</td>
<td>6.1% (31)</td>
</tr>
<tr>
<td>2002-03</td>
<td>86</td>
<td>239 (98) (41%)</td>
<td>45 (2.3%)</td>
<td>302 (234) (68)</td>
<td>3.6% (20)</td>
</tr>
<tr>
<td>2003-04</td>
<td>87</td>
<td>243 (97) (40%)</td>
<td>50 (3.4%)</td>
<td>318 (241) (77)</td>
<td>4.2% (48)</td>
</tr>
<tr>
<td>2004-05</td>
<td>87</td>
<td>285 (126) (44%)</td>
<td>64 (5.1%)</td>
<td>374 (283) (91)</td>
<td>6.1% (26)</td>
</tr>
<tr>
<td>2005-06</td>
<td>88</td>
<td>287 (134) (47%)</td>
<td>80 (1.8%)</td>
<td>396 (278) (118)</td>
<td>3.2% (41)</td>
</tr>
<tr>
<td>2006-07</td>
<td>86</td>
<td>279 (127) (46%)</td>
<td>88 (3.1%)</td>
<td>380 (279) (101)</td>
<td>4.2% (24)</td>
</tr>
<tr>
<td>2007-08</td>
<td>89</td>
<td>291 (151) (52%)</td>
<td>87 (1.3%)</td>
<td>382 (281) (101)</td>
<td>3.2% (21)</td>
</tr>
<tr>
<td>2008-09</td>
<td>92</td>
<td>352 (159) (45%)</td>
<td>85 (3.3%)</td>
<td>487 (347) (140)</td>
<td>5.0% (37)</td>
</tr>
<tr>
<td>Statistics</td>
<td>57</td>
<td>245 (103) (42%)</td>
<td>74 (3.7%)</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>Biostatistics</td>
<td>35</td>
<td>107 (56) (52%)</td>
<td>11 (1.3%)</td>
<td>-------------------------------------------------------------------------------------------------</td>
<td></td>
</tr>
</tbody>
</table>

* Includes other academic departments and research institutes/other nonprofits.

** Of 347, there were 343 in statistics/biostatistics and 4 in probability. For complete details, see Table 4B.

*** Of 140, there were 67 in statistics/biostatistics and 73 in probability. For complete details, see Table 4B.
Acknowledgments
The Annual Survey attempts to provide an accurate appraisal and analysis of various aspects of the academic mathematical sciences scene for the use and benefit of the community and for filling the information needs of the professional organizations. Every year, college and university departments in the United States are invited to respond. The Annual Survey relies heavily on the conscientious efforts of the dedicated staff members of these departments for the quality of its information. On behalf of the Annual Survey Data Committee and the Annual Survey Staff, we thank the many secretarial and administrative staff members in the mathematical sciences departments for their cooperation and assistance in responding to the survey questionnaires.

Other Sources of Data
Visit the AMS website at www.ams.org/employment/specialreports.html for a listing of additional sources of data on the Mathematical Sciences.

The Annual Survey series begun in 1957 by the American Mathematical Society is currently under the direction of the Data Committee, a joint committee of the American Mathematical Society, the American Statistical Association, the Institute of Mathematical Statistics, the Mathematical Association of America, and the Society of Industrial and Applied Mathematics. The current members of this committee are Richard Cleary, Richard M. Dudley, Susan Geller, John W. Hagood, Abbe H. Herzig, Ellen Kirkman, Joanna Mitro, James W. Maxwell (ex officio), Bart Ng, Polly Phipps (chair), Douglas Ravelan, Jianguo (Tony) Sun, and Marie Vitulli. The committee is assisted by AMS survey analyst Colleen A. Rose. Comments or suggestions regarding this Survey Report may be directed to the committee.

Definitions of the Groups
As has been the case for a number of years, much of the data in these reports is presented for departments divided into groups according to several characteristics, the principal one being the highest degree offered in the mathematical sciences. Doctoral-granting departments of mathematics are further subdivided according to their ranking of “scholarly quality of program faculty” as reported in the 1995 publication Research-Doctorate Programs in the United States: Continuity and Change. These rankings update those reported in a previous study published in 1982. Consequently, the departments which now compose Groups I, II, and III differ significantly from those used prior to the 1996 survey.

The subdivision of the Group I institutions into Group I Public and Group I Private was new for the 1996 survey. With the increase in number of the Group I departments from 39 to 48, the Annual Survey Data Committee judged that a further subdivision of public and private would provide more meaningful reporting of the data for these departments.

Brief descriptions of the groupings are as follows:
Group I is composed of 48 departments with scores in the 3.00–5.00 range. Group I Public and Group I Private are Group I departments at public institutions and private institutions respectively.
Group II is composed of 56 departments with scores in the 2.00–2.99 range.
Group III contains the remaining U.S. departments reporting a doctoral program, including a number of departments not included in the 1995 ranking of program faculty.
Group IV contains U.S. departments (or programs) of statistics, biostatistics, and biometrics reporting a doctoral program.
Group V contains U.S. departments (or programs) in applied mathematics/applied science, operations research, and management science which report a doctoral program.
Group Va is applied mathematics/applied science; Group Vb, which was no longer surveyed as of 1998–99, was operations research and management science.
Group M contains U.S. departments granting a master’s degree as the highest graduate degree.
Group B contains U.S. departments granting a baccalaureate degree only.

Listings of the actual departments which compose these groups are available on the AMS website at www.ams.org/employment/groups_des.htm.

2These findings were published in An Assessment of Research-Doctorate Programs in the United States: Mathematical and Physical Sciences, edited by Lyle V. Jones, Gardner Lindzey, and Porter E. Coggeshall, National Academy Press, Washington, DC, 1982. The information on mathematics, statistics, and computer science was presented in digest form in the April 1983 issue of the Notices of the AMS, pages 257–67, and an analysis of the classifications was given in the June 1983 Notices of the AMS, pages 392–3.
ADVANCED FINANCIAL MODELLING
Ed. by
Hansjörg Albrecher / Wolfgang J. Runggaldier / Walter Schachermayer

RRP € [D] 109.95/*US$ 170.00
ISBN 978-3-11-021313-3
(Radon Series on Computational and Applied Mathematics)

This book is a collection of state-of-the-art surveys on various topics in mathematical finance, with an emphasis on recent modelling and computational approaches.

Gerrit van Dijk
INTRODUCTION TO HARMONIC ANALYSIS AND GENERALIZED GELFAND PAIRS

RRP € [D] 69.95/*US$ 108.00
ISBN 978-3-11-022019-3
(de Gruyter Studies in Mathematics 36)

From the contents:
• Fourier series
• Fourier integrals
• Locally compact groups
• Haar measures
• Harmonic analysis on locally compact abelian groups
• Theory and examples of Gelfand pairs
• Theory and examples of generalized Gelfand pairs

Ivanka Stamova
STABILITY ANALYSIS OF IMPULSIVE FUNCTIONAL DIFFERENTIAL EQUATIONS

RRP € [D] 99.95/*US$ 155.00
ISBN 978-3-11-022181-7
(de Gruyter Expositions in Mathematics 52)

This book is devoted to impulsive functional differential equations which are a natural generalization of impulsive ordinary differential equations (without delay) and of functional differential equations (without impulses).

Andrei Khrennikov
INTERPRETATIONS OF PROBABILITY

RRP € [D] 129.95/*US$ 201.00
ISBN 978-3-11-020748-4

This is the first fundamental book devoted to non-Kolmogorov probability models. It provides a mathematical theory of negative probabilities, with numerous applications to quantum physics, information theory, complexity, biology and psychology.

*For orders placed in North America. eBooks are currently only available for Libraries/Institutions. Prices are subject to change. Prices do not include postage and handling.
MacPherson Awarded Hopf Prize

ROBERT MACPHERSON of the Institute for Advanced Study has been chosen the first winner of the Heinz Hopf Prize given by ETH Zurich for outstanding scientific work in the field of pure mathematics. MacPherson, a leading expert in singularities, delivered the Heinz Hopf Lectures, titled “How Nature Tiles Space”, in October 2009. The prize also carries a cash award of 30,000 Swiss francs, approximately equal to US$30,000.

The following quotation was taken from a tribute to MacPherson by Gisbert Wüstholz of ETH Zurich: “Singularities can be studied in different ways using analysis, or you can regard them as geometric phenomena. For the latter, their study demands a deep geometric intuition and profound geometric insight; this is what MacPherson masters in a most striking and extremely artful way. He combines geometric visions with algebraic rigidity. If you study his work you see that it is glowing with elegance and profound in its depth.

“He consistently was ahead of his time, developing new ideas and new approaches—one often not shaped by the main streams of mathematical thought of the day, but rather characterized by great vision. Repeatedly the mathematical community came to embrace, extend, and apply his ideas and results as they caught up with that vision.

“MacPherson’s connections with the Swiss mathematical community date back to 1983, when he participated in and significantly contributed to the famous Borel seminar, a joint seminar organized by several Swiss universities, including ETH Zurich, Lausanne, Geneva, Bern, and Basel. The seminar was initiated by Armand Borel, one of the most distinguished Swiss mathematicians of the last century. The topic of the seminar was the Goresky-MacPherson intersection homology and its use for the cohomology of arithmetic groups, one of the main research areas of Armand Borel.

“This illustrates only a small part of the work of MacPherson, for it spans a wide spectrum of contributions in many very different areas: algebraic geometry and topology; algebraic groups, group actions, and representation theory; enumerative geometry and combinatorics; locally symmetric spaces, $L^2$-cohomology, arithmetic groups, and the Langlands program. The list is by far not complete, and we try only to give a representative selection of his contribution to mathematics. He influenced a whole generation of mathematicians by giving them new tools to attack difficult problems and teaching them novel geometric, topological, and algebraic ways of thinking.”

Robert MacPherson was born in 1944 in Lakewood, Ohio. He received his B.A. from Swarthmore College in 1966 and his Ph.D. from Harvard University in 1970. He taught at Brown University from 1970 to 1987 and at the Massachusetts Institute of Technology from 1987 to 1994. He has been at the Institute for Advanced Study in Princeton since 1994. His work has introduced radically new approaches to the topology of singular spaces and promoted investigations across a great spectrum of mathematics. MacPherson works in several fields of geometry-topology, algebraic geometry, differential geometry, and singularity theory, and he is especially interested in aspects of geometry that interact with other areas of mathematics. In 1992 he received the National Academy of Sciences Award in Mathematics, and in 2002 he received the Leroy P. Steele Prize of the AMS.

The Heinz Hopf Prize at ETH Zurich was established through a donation made by Dorothee and Alfred Aeppli and will be awarded every two years for outstanding scientific work in the field of pure mathematics.

—From an ETH announcement

Lindenstrauss and Villani Awarded Fermat Prize

ELON LINDENSTRAUSS of Princeton University and CÉDRIC VILLANI of École Normale Supérieure de Lyon have been awarded the 2009 Fermat Prize by the Institut de Mathématiques de Toulouse. Lindenstrauss was honored for his contributions to ergodic theory and their applications in number theory. Villani was selected for his contributions to the theory of optimal transport and his studies of non-linear evolution equations.
The Fermat Prize, given every two years, recognizes outstanding research in the fields in which Pierre de Fermat made significant contributions: statements of variational principles, foundations of probability and analytical geometry, and number theory. The prize is intended to reward research that is accessible to the greatest number of professional mathematicians within these fields.

The prize carries a cash award of €20,000 (approximately US$29,700).

—Elaine Kehoe

Pujals Awarded TWAS Prize in Mathematics

ENRIQUE PUJALS of the Institute of Pure and Applied Mathematics (IMPA) in Rio de Janeiro, Brazil, has been named the winner of the 2009 TWAS Prize in mathematics, awarded by the Academy of Sciences for the Developing World (TWAS). He was honored for his contributions toward developing “a theory about robust dynamics and about the role of homoclinic bifurcation as a universal mechanism to describe the way to produce very rich and complex dynamics.” Pujals will receive a cash prize of US$15,000 and will deliver a lecture at the academy’s general meeting in 2010.

—From a TWAS announcement

Dereich Receives 2009 Information-Based Complexity Young Researcher Award

STEFFEN DEREICH of the Technische Universität Berlin, Germany, has been awarded the Information-Based Complexity Award for 2009. The award is given every year for significant contributions to information-based complexity by a young researcher who has not reached his or her thirty-fifth birthday by September 30 of the year of the award. The prize consists of US$1,000 and a plaque.

The award committee this year consisted of Dirk Nuyens, Katholieke Universiteit, Leuven; Andreas Neuenkirch, University of Frankfurt; Jan Vybiral, University of Jena; Joseph F. Traub, Columbia University; and Henryk Wozniakowski, Columbia University and University of Warsaw.

—Joseph Traub, Columbia University

NSF CAREER Awards Made

The Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) has honored twenty-nine mathematicians in fiscal year 2009 with Faculty Early Career Development (CAREER) awards. The NSF established the awards to support promising scientists, mathematicians, and engineers who are committed to the integration of research and education. The grants provide funding of at least US$400,000 over a five-year period. The 2009 CAREER grant awardees and the titles of their grant projects follow.

Miklos Abert, University of Chicago, Asymptotic Invariants of Residually Finite Groups; Rafail Abramov, University of Illinois, Chicago, Predicting Global Climate Change through Fluctuation-Dissipation: A Practical Computational Strategy for Complex Multiscale Dynamics; Orly Alter, University of Texas, Austin, Integrative and Comparative Tensor Algebra Models of DNA Microarray Data from Different Studies of the Cell Cycle; Benjamin Brubaker, Massachusetts Institute of Technology, Multiple Dirichlet Series, Automorphic Forms, and Combinatorial Representation Theory; Francesco Calegari, Northwestern University, Arithmetic of Cohomological Automorphic Forms; Gautam Chinta, City College, City University of New York, Multiple Dirichlet Series and Metaplectic Groups; Tommaso de Fernex, University of Utah, Singularities in the Minimal Model Program and Birational Geometry; Ioana Dumitriu, University of Washington, Synergistic Interactions between Numerical Linear Algebra and Stochastic Eigenanalysis (Random Matrix Theory); Noureddine El Karoui, University of California Berkeley, Random Matrices and High-Dimensional Statistics; Yongtao Guan, Yale University, New Statistical Methods for Massive Spatial, Temporal, and Spatial-Temporal Processes; Jeffrey Humpherys, Brigham Young University, Interdisciplinary Mentoring Program in Analysis, Computation, and Theory (IMPACT); Marta Lewicka, University of Minnesota, Twin Cities, Thin Shells: Problems in Nonlinear Elasticity and Fluid Dynamics; Fengyan Li, Rensselaer Polytechnic Institute, Development and Applications of Discontinuous Galerkin Methods; Di Liu, Michigan State University, Modeling, Analysis, and Computation of Stochastic Intracellular Reactions; Gregory Lyng, University of Wyoming, Behavior of Solutions of Nonlinear Partial Differential Equations; Mauro Maggioni, Duke University, Multiscale Methods for High-Dimensional Data, Graphs, and Dynamical Systems; Dionisios Margetis, University of Maryland, College Park, Thermodynamic and Kinetic Approaches for Epitaxial Material Systems; Lenhard Ng, Duke University, Symplectic Field Theory and Low-Dimensional Topology; JiaWang Nie, University of California San Diego, Linear Matrix Inequality Representations in Optimization; Duane Nykamp, University of Minnesota, Twin Cities, Toward a Second-Order Description of Neuronal Networks; Jeffery Schenker, Michigan State University, Analysis of Disordered Systems; Jian Song, Rutgers University, Canonical Metrics, Complex Monge-Ampére Equations, and Geometric Flows; Jason Starr, State University of New York, Stony Brook, Higher Rational Connectedness, Higher Fano Manifolds, and Applications; Katrin Wehrheim, Massachusetts Institute of Technology, The Symplectic Category, Floer Field Theory, and Relations to Gauge Theory and Topology; Anna Wienhard, Princeton University, Higher Teichmüller Theory; Lexing Ying, University of Texas, Austin, Fast Algorithms for Oscillatory Integrals; Ming Yuan, Georgia...
Institute of Technology, Sparse Modeling and Estimation with High-Dimensional Data; ALEKSEY ZINGER, State University of New York, Stony Brook, Holomorphic Geometry and Symplectic Topology; and HUI ZOU, University of Minnesota, Twin Cities, New Statistical Methodology and Theory for Mining High-Dimensional Data.

—Elaine Kehoe

Memories of Eddie Nussbaum

A. Edward (Eddie) Nussbaum died of congestive heart failure on October 31, 2009, at the age of eighty-four. He was a faculty member in the Department of Mathematics at Washington University in St. Louis for thirty-seven years.

Eddie Nussbaum was born in 1925 in the region Moenchen-Gladbach-Rheydt (three adjacent towns), Germany, where his parents operated a department store. His elder brother was arrested on Kristallnacht in 1938. Soon thereafter, Eddie and his sister were sent by the Kindertransport train to live in Belgium. However, conditions in Belgium were not safe, and Eddie and his sister were soon separated. Eddie fled to southern France, and when conditions there also became unsafe, he crossed into Switzerland with the help of two local women and their woodsman father. When he was quickly put in jail by the Swiss authorities, he invented a story that led to his release, and he lived for several years with a spinster and her nephew in Switzerland while studying mathematics at the University of Zurich. Although Eddie’s sister also survived the Holocaust, sadly both his parents and his elder brother died in the Nazi death camp at Auschwitz.

In 1947 Eddie arrived penniless in New York and began taking courses at Brooklyn College while supporting himself by rolling clay tennis courts. Soon thereafter he was admitted to Columbia University for graduate studies in mathematics, receiving his M.A. from Columbia in 1950. The high regard in which he was held by the Columbia mathematics department is attested to by his appointment as a lecturer for the academic year 1951–52. For 1952–53 he was a staff member of the electronic computer project headed by John von Neumann at the Institute for Advanced Study in Princeton. After serving as an instructor at the University of Connecticut (1953–55) and an instructor at Rensselaer Polytechnic Institute (1955–57), he received his Ph.D. from Columbia in 1957 with a dissertation titled “The Hausdorff-Bernstein-Widder Theorem for Semigroups in Locally Compact Abelian Groups”. Following a year of service as an assistant professor at RPI, he was appointed assistant professor of mathematics at Washington University in 1958; he was quickly promoted to associate professor in 1961 and to full professor in 1965.

In 1955 Eddie, with Allen Devinatz and John von Neumann, coauthored a paper published by the Annals of Mathematics titled “On the permutability of self-adjoint operators”. This led to a distinguished career in functional analysis, with numerous important papers on unbounded operators on Hilbert spaces and on a variety of related topics. His lectures were regarded by students and faculty as a model of mathematical clarity and precision. Both within the mathematics department and among his circle of friends and relatives, he was considered to be a kind, gentle, and compassionate man. Although he suffered a serious heart attack in 1979, he made a remarkable recovery, aided by an intensive rehabilitation and exercise program that led to his becoming an avid runner; indeed, he won a medal in a 1983 race. He retired from Washington University in 1995.

In 1989 Eddie and his wife visited his hometown of Rheydt in the Moenchen-Gladbach-Rheydt region and were very well received there; remarkably, he said he harbored no bitterness over his family’s experiences. He is survived by Anne, his wife of fifty-two years, and their children Karl, who teaches film studies at Montclair State University and has produced a number of films, and Franziska, who works with photographers and serves as a stylist for advertising agencies in the St. Louis area.

—Guido Weiss and Edward Wilson, Washington University
Grants to support collaborations between Chinese and U.S./Canadian researchers are made possible through the generosity of Ky and Yu-Fen Fan.

The Fan China Exchange Program is intended to send eminent mathematicians from the U.S. and Canada to make a positive impact on the mathematical research community in China and to bring Chinese scientists in the early stages of their research to the U.S. and Canada to help further their careers. The program encourages host institutions to provide some type of additional support for the travel or living expenses of the visitor and to ensure a suitable length of stay.

Applications received before March 15 will be considered for the following academic year.

For more information on the Fan China Exchange Program and application process see [www.ams.org/employment/chinaexchange.html](http://www.ams.org/employment/chinaexchange.html) or contact the AMS Membership and Programs Department by telephone at 800-321-4267, ext. 4170 (U.S. and Canada), or 401-455-4170 (worldwide), or by email at chinaexchange@ams.org.
Mathematics Opportunities

Summer Program for Women Undergraduates

The 2010 Summer Program for Women in Mathematics (SPWM2010) will take place at George Washington University in Washington, DC, from June 26 to July 31, 2010. This is a five-week intensive program for mathematically talented undergraduate women who are completing their junior years and may be contemplating graduate study in mathematical sciences. The goals of this program are to communicate an enthusiasm for mathematics, to develop research skills, to cultivate mathematical self-confidence and independence, and to promote success in graduate school.

Applicants must be U.S. citizens or permanent residents studying at a U.S. university or college who are completing their junior years or the equivalent and have mathematical experience beyond the typical first courses in calculus and linear algebra. Sixteen women will be selected. Each will receive a travel allowance, campus room and board, and a stipend of US$1,750. The deadline for applications is March 1, 2010. Early applications are encouraged. Applications are accepted only by mail. For further information, please contact the director, Murli M. Gupta, email: mmg@gwu.edu; telephone: 202-994-4857; or visit the program’s website at http://www.gwu.edu/~spwm/. Application material is available on the website.

—From an SPWM announcement

NSF Support for Undergraduate Training in Biological and Mathematical Sciences

The National Science Foundation (NSF) offers opportunities for support through its Undergraduate Biology and Mathematics (UBM) program. The goal of the program is to enhance undergraduate education and training at the intersection of the biological and mathematical sciences and to better prepare undergraduate biology or mathematics students to pursue graduate study and careers in fields that integrate the mathematical and biological sciences.

The program will provide support for jointly conducted long-term research experiences for interdisciplinary teams of at least two undergraduates from departments in the biological and mathematical sciences. Projects should focus on research at the intersection of the mathematical and biological sciences and should provide students exposure to contemporary mathematics and biology addressed with modern research tools and methods. Projects must involve students from both areas in collaborative research experiences and include joint mentorship by faculty in both fields.

Between six and nine awards are expected to be made in 2010. The deadline for full proposals is February 11, 2010. For more information, see http://www.nsf.gov/pubs/2008/nsf08510/nsf08510.htm#awd_info. The UBM program is a joint effort of the Education and Human Resources (EHR), Biological Sciences (BIO), and Mathematical and Physical Sciences (MPS) directorates of the NSF.

—From an NSF announcement

Monroe H. Martin Prize

The Institute for Physical Science and Technology at the University of Maryland, College Park, is seeking applications and nominations for the eighth Monroe H. Martin Prize. The prize is awarded for an outstanding paper in applied mathematics (including numerical analysis) by a young researcher who is a resident of North America and who has not reached his or her thirty-sixth birthday by July 31, 2010. Submitted papers must be written by single authors and must have been published or accepted for publication. The work must not have been performed in connection with the completion of requirements for an
Clay Mathematics Institute 2010 Summer School

The 2010 Clay Mathematics Institute (CMI) Summer School on Probability and Statistical Physics in Two (and More) Dimensions will be held in Buzios, Brazil, from July 11 to August 7, 2010.

In the past ten to fifteen years, various areas of probability theory related to rigorous statistical mechanics, disordered systems, and combinatorics have enjoyed an intensive development. A number of these developments deal with two-dimensional random structures. The questions related to critical systems are twofold: understanding large-scale properties of lattice-based models (on a periodic deterministic lattice or in the case where the lattice is itself random) and, on the other hand, being able to construct and manipulate a continuous object that describes directly their scaling limits. In the case of a fixed planar lattice, a number of conjectures originating in the physics literature have now been proved, but many questions remain open. In the case of statistical physics on random planar graphs, sometimes referred to as quantum gravity, many results have been recently understood, and a relation between discrete and continuous structures is now emerging. The aim of the Summer School is to provide a complete picture of the current state of the art in these and related topics.

Organization: During the first two weeks of the school, three foundational courses will be combined with afternoon activities in thematic working groups and evening seminars. Weeks three and four will be dedicated to shorter advanced courses (the fourth week will run in parallel with the Fourteenth Brazilian School of Probability).

Foundational Courses: The foundational courses include "Large random planar maps and their scaling limits" (Jean-Francois LeGall and Gregory Miermont), "SLE and other conformally invariant objects" (Vincent Beffara), and "Noise sensitivity and percolation" (Jeffrey Steif and Christophe Garban).

Minicourses: These courses include "Random geometry and Gaussian free field" (Scott Sheffield), "Conformal invariance of lattice models" (Stanislav Smirnov), "Integrable combinatorics" (Philippe Di Francesco), "Fractal and multifractal properties of SLE" (Gregory Lawler), "The double dimer model" (Rick Kenyon), "Random polymers" (Frank den Hollander), and "Self-avoiding walks" (Gordon Slade). The latter two will be held jointly with the Brazilian School on Probability. Another possible minicourse, "Supersymmetric methods in disordered systems and two-dimensional critical behavior" (John Cardy), is awaiting confirmation. See the website [http://www.claymath.org/summerschool] for updated information.

Financial Support: Graduate students and postdoctoral fellows who are within five years of receipt of the Ph.D. degree can apply for financial support. Support is decided on a competitive basis and may include accommodations plus funds toward the cost of economy travel.

The scientific committee consists of David Alexandre Ellwood, Charles Newman, Vladas Sidoravicius, and Wendelin Werner.

The deadline for applications is March 1, 2010. More information and online application forms are available at [http://www.claymath.org/summerschool] or by sending email to summerschool@claymath.org.

—From a CMI announcement

News from CRM

The Centro di Ricerca Matematica Ennio De Giorgi (CRM) invites applications for four one-year junior visiting positions for the academic year 2010–11. Applicants should be new or recent Ph.D. recipients in mathematics and should have exceptional potential in research. The annual stipend is 25,000 euros (approximately US$36,800), along with a research allowance of 2,500 euros (approximately US$3,600) intended for support of other researchers invited to CRM by the junior visitors. Junior visitors will participate in a variety of scientific activities, including intensive research periods, workshops, and seminars, and will interact with prominent scientists who participate in the senior visiting program at CRM.

The application deadline is January 28, 2010, and junior visitors are expected to begin their research activities at CRM no later than October 2010. For further information and application details, see the website [http://www.crm.sns.it/hpp/grants.html?year=2010].

The 2010 scientific program includes the following workshops:

January 25–February 5, 2010: Periodic Approximation in Dynamics
Mathematics Opportunities

February 9–12, 2010: Linear and Nonlinear Hyperbolic Equations
March 8–14, 2010: Italy-India Conference on Diophantine and Analytic Number Theory
March 1–April 30, 2010: Euclidean Harmonic Analysis, Nilpotent Lie Groups and PDEs
May 2–June 30, 2010: Configuration Spaces: Geometry, Combinatorics and Topology
September 6–10, 2010: Geometric Evolutions and Minimal Surfaces in Lorentzian Manifolds
September 13–17, 2010: On the Contested Expanding Role of Applied Mathematics from the Renaissance to the Enlightenment
October 12–16, 2010: Optimal Transportation and Applications

For further information see the website http://www.crm.sns.it.

—From a CRM announcement

Everett Pitcher Lectures

The next series of Everett Pitcher Lectures will be held March 22–25, 2010, on the campus of Lehigh University in Bethlehem, Pennsylvania. The speaker will be Rick Durrett of Cornell University, who will speak on “Three Faces of Probability”, with three lectures on improbabilities, life on a random graph, and cancer models. Durrett’s research focuses on probability problems that come from ecology and genetics. He is a member of the National Academy of Sciences.

The lectures, which are free and open to the public, are held in honor of Everett Pitcher, who was secretary of the AMS from 1967 until 1988. Pitcher served in the mathematics department at Lehigh University from 1938 until 1978, when he retired as Distinguished Professor of Mathematics. He passed away in December 2006 at the age of ninety-four. For further information, contact the Everett Pitcher Lecture Series, Department of Mathematics, Lehigh University, Bethlehem, PA, 18015; telephone 610-758-3731; or see the website http://www.lehigh.edu/-math/pitcher.html.

—From a Lehigh University announcement

About the Cover

Differential geometry issue
In 2009 geometer Robert Osserman visited St. Louis to observe and to lecture at Washington University on the mathematical properties of the Gateway Arch. An article based on Osserman’s lecture is included in this issue. The cover photograph by Geir Arne Hjelle attests to the awesomeness of a marriage between mathematics and architecture on a grand scale.

The cover photograph was taken on November 23, 2008, from the south leg of the St. Louis Arch, looking north and slightly up. For this magnificent wide angle, unusual shot, the photographer used a Canon EOS 40D camera fitted with a Canon EF-S 17-85mm f4-5.6 IS USM lens.

“I was down at the arch,” Geir Arne comments, “looking for a cover picture for my St. Louis photo book, and I guess this was one of the candidates.” The photographer later favored a different shot. You can see the other shot at http://www.flickr.com/photos/hjelle/3439055921/.

The cover image was composed using GIMP (GNU Image Manipulation Program) freeware. Find it at http://www.gimp.org/.

—Steven G. Krantz
See the AMS website for the most up-to-date lists of Invited Addresses and Special Sessions.

www.ams.org/amsmtgs/sectional.html

Come to hear presentations, connect with researchers in your field, and meet colleagues, mentors, and students. While at the meetings visit the AMS exhibit to see the book sale and receive information on AMS membership and programs.
Inside the AMS

From the AMS Public Awareness Office

• Mathematical Moment. Recent topics include “Knowing How to Fold Them” (how math is involved in understanding the complex behavior of proteins) and “Resisting the Spread of Disease”. Listen to a podcast interview with Mac Hyman talking about his research on H1N1 (the swine flu) and view more than seventy-five topics at http://www.ams.org/mathmoments/.

• Feature Column. Recent essays include “Marian Rejewski and the First Break into Enigma”, by Bill Casselman; “A Non-commutative Marriage System in the South Pacific”, by Tony Phillips; “School Choice”, by Joe Malkevitch; and “We Recommend a Singular Value Decomposition”, by David Austin. Read the current essay and explore the archive at http://www.ams.org/featurecolumn.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
paoffice@ams.org

AMS Email Support for Frequently Asked Questions

A number of email addresses have been established for contacting the AMS staff regarding frequently asked questions. The following is a list of those addresses together with a description of the types of inquiries that should be made through each address.

.abs-info@ams.org for questions regarding a particular abstract.
.abs-submit@ams.org for information on how to submit abstracts for AMS meetings and MAA sessions at January Joint Mathematics Meetings. Type HELP as the subject line.
.acquisitions@ams.org to contact the AMS Acquisitions Department.
.ams@ams.org to contact the Society’s headquarters in Providence, Rhode Island.
.amsdc@ams.org to contact the Society’s office in Washington, DC.
.amsmem@ams.org to request information about membership in the AMS and about dues payments or to ask any general membership questions; may also be used to submit address changes.
.ams-survey@ams.org for information or questions about the Annual Survey of the Mathematical Sciences or to request reprints of survey reports.
.bookstore@ams.org for inquiries related to the online AMS Bookstore.
.classads@ams.org to submit classified advertising for the Notices.
.cust-serv@ams.org for general information about AMS products (including electronic products), to send address changes, place credit card orders for AMS products, or conduct any general correspondence with the Society’s Customer Services Department.
.development@ams.org for information about giving to the AMS, including the Epsilon Fund.
.emp-info@ams.org for information regarding AMS employment and career services.
.eprod-support@ams.org for technical questions regarding AMS electronic products and services.
.mathcal@ams.org for queries about the “Mathematics Calendar” section of the Notices. To submit conference information, see the form at http://www.ams.org/cgi-bin/mathcal/mathcal-submit.pl.
mathrev@ams.org to submit reviews to Mathematical Reviews and to send correspondence related to reviews or other editorial questions.
meet@ams.org to request general information about Society meetings and conferences.
meetreg-request@ams.org to request meeting registration forms be emailed.
meetreg-submit@ams.org to submit completed registration forms.
mmsb@ams.org for information or questions about registration and housing for the Joint Mathematics Meetings (Mathematics Meetings Service Bureau).
msn-support@ams.org for technical questions regarding MathSciNet.
notices@ams.org (or smf@ams.org) to send correspondence to the managing editor of the Notices, including items for the news columns. The editor (notices@math.wustl.edu) is the person to whom to send articles and letters. Requests for permission to reprint from the Notices should be sent to reprint-permission@ams.org (see below).
notices-ads@ams.org to submit electronically paid display ads for the Notices.
notices-booklist@ams.org to submit suggestions for books to be included in the “Book List” in the Notices.
notices-letters@ams.org to submit letters and opinion pieces to the Notices.
notices-whatish@ams.org to comment on or send suggestions for topics for the “WHAT IS…?” column in the Notices.
paooffice@ams.org to contact the AMS Public Awareness Office.
president@ams.org to contact the president of the American Mathematical Society.
prof-serv@ams.org to send correspondence about AMS professional programs and services.
pub@ams.org to send correspondence to the AMS Publication Division.
pub-submit@ams.org to submit accepted electronic manuscripts to AMS publications (other than Abstracts). See [http://www.ams.org/submit-book-journal] to electronically submit accepted manuscripts to the AMS book and journal programs.
reprint-permission@ams.org to request permission to reprint material from Society publications.
sales@ams.org to inquire about reselling or distributing AMS publications or to send correspondence to the AMS Sales Department.
secretary@ams.org to contact the secretary of the Society.
statements@ams.org to correspond regarding a balance due shown on a monthly statement.
technical-support@ams.org to contact the Society’s typesetting Technical Support Group.
textbooks@ams.org to request examination copies or inquire about using AMS publications as course texts.
webmaster@ams.org for general information or for assistance in accessing and using the AMS website.

Deaths of AMS Members

LINDA BARKLEY, Boeing Communications, died on August 15, 2009. Born on December 12, 1951, she was a member of the Society for 31 years.

ENRIQUE BAYO, professor, from San Juan, PR, died on May 28, 2002. Born on April 20, 1922, he was a member of the Society for 51 years.

LEONARD D. BERKOVITZ, professor emeritus, Purdue University, died on October 13, 2009. Born on January 24, 1924, he was a member of the Society for 59 years.

THOMAS DIETMAIR, ERGO Insurance Group, Germany, died on September 6, 2009. Born on January 22, 1961, he was a member of the Society for 17 years.

F. BROCK FULLER, professor emeritus, Caltech, died on November 6, 2009. Born on July 8, 1927, he was a member of the Society for 59 years.

PAUL GERMAIN, professor, Paris, France, died in February 2009. Born on August 28, 1920, he was a member of the Society for 55 years.

ALBERT E. HURT, professor, University of Victoria, Canada, died on October 28, 2009. Born on October 22, 1931, he was a member of the Society for 51 years.

EDWARD S. KENNEDY, professor emeritus, American University of Beirut, died on May 4, 2009. Born on January 3, 1912, he was a member of the Society for 72 years.

ARNAUD I. LAFONTE, from Rio de Janeiro, Brazil, died on October 28, 2009. Born on October 5, 1938, he was a member of the Society for 4 years.

GEORGE J. MALTESE, professor, Wesleyan University, died on October 27, 2009. Born on June 24, 1931, he was a member of the Society for 51 years.

DAVID B. MERONK, retired professor, Bowling Green State University, died on July 30, 2009. Born on August 10, 1934, he was a member of the Society for 17 years.

ALAN A. MEYERHOFF, professor, Rutgers University, New Brunswick, died on May 18, 2009. Born on March 20, 1926, he was a member of the Society for 35 years.

TETSURO MIYAKAWA, professor, Kanazawa University, died on November 6, 2009. Born on March 10, 1948, he was a member of the Society for 21 years.

CHARLES T. MOLLOY, Falls Church, VA, died on May 5, 2009. Born on November 22, 1914, he was a member of the Society for 58 years.

A. EDWARD NUSBAUM, professor emeritus, Washington University at St. Louis, died on October 31, 2009. Born on January 10, 1925, he was a member of the Society for 57 years.

HOWARD L. PENN, professor, U.S. Naval Academy, died in November 2009. Born on November 9, 1946, he was a member of the Society for 37 years.

SAM PERLIS, professor emeritus, Purdue University, died on June 22, 2009. Born on April 18, 1913, he was a member of the Society for 71 years.

MARIAN POUR-EL, professor emeritus, School of Mathematics, University of Minnesota, died on June 10, 2009. Born on April 19, 1928, she was a member of the Society for 56 years.
Inside the AMS

CALVIN R. PUTNAM, professor emeritus, Purdue University, died on April 24, 2008. Born on May 25, 1924, he was a member of the Society for 63 years.

BETTINA RICHMOND, professor, Western Kentucky University, died on November 22, 2009. Born on January 30, 1958, she was a member of the Society for 30 years.

PIERRE SAMUEL, professor emeritus, University of Paris-Sud, died on August 23, 2009. Born on September 12, 1921, he was a member of the Society for 63 years.

ALICE T. SCHAFER, former AWM president, died on September 27, 2009. Born on June 18, 1915, she was a member of the Society for 68 years.

STEFAN SCHWARBIK, professor emeritus, Institute of Mathematics of the Academy of Sciences of the Czech Republic, died on November 4, 2009. Born on March 15, 1941, he was a member of the Society for 16 years.

PETE SEIBERT, from Mexico City, died on August 13, 2009. Born on April 13, 1927, he was a member of the Society for 49 years.

NOBUO SHIMADA, from Nagoya, Japan, died on December 17, 2007. Born on October 13, 1925, he was a member of the Society for 47 years.

EDSEL F. STIEL, professor emeritus, California State University, Fullerton, died on January 18, 2008. Born on December 19, 1933, he was a member of the Society for 48 years.

TAKAYUKI TAMURA, professor emeritus, University of California, Davis, died on June 1, 2009. Born on March 12, 1919, he was a member of the Society for 51 years.

LINCOLN H. TURNER, from Lakewood, CO, died on November 25, 2007. Born on June 30, 1928, he was a member of the Society for 50 years.

JOHN H. URSSELL, professor emeritus, Queen’s University, died on July 30, 2009. Born on June 9, 1938, he was a member of the Society for 24 years.

J. RICHARD VANDEVELDE, professor, Arrupe House, Loyola University of Chicago, died on August 11, 2009. Born on February 21, 1935, he was a member of the Society for 43 years.

ALFRED G. VASSALOTTI, associate professor emeritus, Hofstra University, died on October 21, 2009. Born on January 11, 1929, he was a member of the Society for 45 years.

THERESA P. VAUGHAN, retired professor, University of North Carolina at Greensboro, died on June 13, 2009. Born on October 13, 1941, she was a member of the Society for 37 years.

FREDERICK N. WEBB, from Littleton, MA, died on July 12, 2009. Born on July 22, 1944, he was a member of the Society for 8 years.

ALVIN M. WHITE, professor emeritus, Harvey Mudd College, died on June 2, 2009. Born on June 21, 1925, he was a member of the Society for 49 years.

KATHLEEN B. WHITEHEAD, retired professor, Tufts University, died on April 18, 2009. Born on November 9, 1920, she was a member of the Society for 66 years.

JOSEPH ZELLE, from Euclid, OH, died on July 11, 2009. Born on February 25, 1912, he was a member of the Society for 51 years.
Reference and Book List

The Reference section of the Notices is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.

Contacting the Notices
The preferred method for contacting the Notices is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people’s mathematics research.

The managing editor is the person to whom to send items for “Mathematics People”, “Mathematics Opportunities”, “For Your Information”, “Reference and Book List”, and “Mathematics Calendar”. Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are notices@math.wustl.edu in the case of the editor and notices@ams.org in the case of the managing editor. The fax numbers are 314-935-6839 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines
MassMedia/, or contact Stacey Pasco, Manager, Mass Media Program, AAAS Mass Media Science and Engineering Fellows Program, 1200 New York Avenue, NW, Washington, DC 20005; telephone 202-326-6441; fax 202-371-9849; email: spasco@aaas.org. Also see http://www.ams.org/government/massmediaaann.html or contact the AMS Washington Office, 1527 Eighteenth Street, NW, Washington, DC 20036; telephone 202-588-1100; fax 202-588-1853; email: amsdc@ams.org.

January 15, 2010: Applications for Jefferson Science Fellows Program. See http://sites.nationalacademies.org/PGA/jefferson/ or email: jsf@nas.edu or telephone 202-334-2643.


February 1, 2010: Applications for February review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

February 1, 2010: Applications for AWM Travel Grants and Mentoring Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.


February 11, 2010: Full proposals for NSF Undergraduate Biology and Mathematics (UBM) program. See “Mathematics Opportunities” in this issue.


April 15, 2010: Applications for fall 2010 semester of Math in Moscow. See http://www.mccme.ru/mathinmoscow or write to: Math in Moscow, P.O. Box 524, Wynnewood, PA 19096; fax: +7095-291-65-01; email: mim@mccme.ru. For information on AMS scholarships see http://www.ams.org/outreach/mimoscowl.html or write to: Math in Moscow Program, Membership and Programs Department, American Mathematical Society, 201 Charles Street, Providence RI 02904-2294; email: student-serv@ams.org.

May 1, 2010: Applications for May review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

May 1, 2010: Applications for the fall 2010 program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship program of the National Academies. See http://sites.nationalacademies.org/PGA/policyfellows/index.htm or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: policyfellows@nas.edu.

May 1, 2010: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.


August 1, 2010: Applications for August review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

October 1, 2010: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.org; or contact Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

November 1, 2010: Applications for November review for National Academies Postdoctoral and Senior Research Associateship Program. See http://sites.nationalacademies.org/PGA/RAP/PGA_050491 or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

**MPS Advisory Committee**

Following are the names and affiliations of the members of the Advisory Committee for Mathematical and Physical Sciences (MPS) of the National Science Foundation. The date of the expiration of each member’s term is given after his or her name. The website for the MPS directorate may be found at http://www.nsf.gov/home/mps/. The postal address is Directorate for the Mathematical and Physical Sciences, National Science Foundation, 4201 Wilson Boulevard, Arlington, VA 22230.

Hector D. Abruna (10/10)
Department of Chemistry and Cornell University
Reference and Book List

February 2010

NOTICES OF THE AMS

273

**Taft Armandroff** (10/12)
W. M. Keck Observatory
Kamuela, Hawaii

**James Berger** (10/11)
Department of Statistical Science
Duke University

**Daniela Bortoletto** (10/11)
Department of Physics
Purdue University

**Kevin Corlette** (10/12)
Department of Mathematics
University of Chicago

**Eric A. Cornell** (10/10)
JILA
University of Colorado

**Juan J. de Pablo** (10/12)
Department of Chemical and Biological Engineering
University of Wisconsin-Madison

**Joseph M. DeSimone** (10/12)
Department of Chemistry
University of North Carolina at Chapel Hill

**Barbara J. Finlayson-Pitts** (10/11)
Department of Chemistry
University of California, Irvine

**Irene Fonseca** (10/11)
Department of Mathematical Sciences
Carnegie Mellon University

**Sharon L. Neal** (10/11)
Department of Chemistry and Biochemistry
University of Delaware

**Luis Orozco** (10/12)
Department of Physics
University of Maryland, College Park

**John Peoples Jr.** (10/11)
Fermilab
Batavia, IL

**Elsa Reichmanis** (10/11)
School of Chemical and Biomolecular Engineering
Georgia Institute of Technology

**Fred S. Roberts** (10/12)
DIMACS
Rutgers University

**Joel E. Tohline** (10/10)
Department of Physics and Astronomy
Louisiana State University

**Geoffrey West** (10/11)
Santa Fe Institute
Santa Fe, NM

---

**Book List**

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers’ attention to older books. Suggestions for books to include on the list may be sent to notices-booklist@ams.org.

*Added to "Book List" since the list’s last appearance.


Decoding the Heavens: A 2,000-Year-Old Computer—and the Century-Long Search to Discover Its Secrets, by Jo Marchant. Da Capo
Reference and Book List


Doctoral Degrees Conferred

2008–2009

ALABAMA

Auburn University (10)

DEPARTMENT OF MATHEMATICS AND STATISTICS

Baker, Charla, The intersection problem for Latin squares with holes of size 2 and 3
Bobga, Benkam, Some necessary conditions for list colorability of graphs and a conjecture on completing partial Latin squares
Chidume, Chukwudi, Iteration methods for approximation of solutions of nonlinear equations in Banach spaces
Divavahi, Venkata, Graph decompositions
Gammon, Kevin, Factorwise rigidity involving hereditarily indecomposable spaces
Jin, Kang, On the lattice Boltzmann method: Implementation and applications
Newman, Nicolas, Enclosings of small cycle systems
Nudurupati, Sai, Robust nonparametric discriminant analysis procedures
Peng, Man, Palm measure invariance and exchangeability for marked point processes
Turkmen, Asuman, Robust partial least squares for regression and classification

University of Alabama-Birmingham (11)

DEPARTMENT OF BIOSTATISTICS

Azuero, Andres, Comparisons of sequential testing approaches for detection of association between disease and haplotype blocks
Banerjee, Sampa, Bayesian genome-wide QTL mapping for multiple traits
Chen, Lang, Microarray data analysis for SNPs effects and inferring alternative splicing

Dube, Tina, Assessing and correcting the effects of measurement error among correlated covariates in a proportional hazards setting
Keith, Scott, Free-knot splines and bootstrapping for nonlinear modeling in complex samples
Li, Qing, Interim monitoring efficacy, safety and futility in Phase III clinical trials
Nair, Nittin, Adaptive procedures to detect treatment effects under unexpected covariate interactions
Prucka, William, Wavelet-based regression and classification for longitudinal diffusion tensor imaging data
You, Zhiying, Power and sample size of cluster randomized trials

DEPARTMENT OF MATHEMATICS

Kulkarni, Mandar, Multi-coefficient Dirichlet-Neumann type elliptic inverse problems with application to reflection seismology
Mavinga, Nsoki, Nonlinear second order parabolic and elliptic equations with nonlinear boundary conditions

University of Alabama-Huntsville (2)

DEPARTMENT OF MATHEMATICS

Bowie, Miranda, Liar’s domination and the domination continuum
Puckett, Matthew, Minimum wave speed and uniqueness of monotone traveling waves

University of Alabama-Tuscaloosa (4)

DEPARTMENT OF MATHEMATICS

Fayoumi, Hiba, A study of binary systems on sets and applications to several classes of such systems
Kang, Hyuna, Stochastic and key rate duration for value at risk

Liu, Congxiao, Mathematical study for the switch-initiating field of $2d$ ferroanti-ferromagnet exchange coupled systems
Wu, Leina, Grid refinement method for partial differential equations

ARIZONA

Arizona State University (14)

SCHOOL OF MATHEMATICAL AND STATISTICAL SCIENCES

Alvarado, Alejandra, Arithmetic progressions on curves
Fortney, Jon Pierre, Dirac structures in pseudo-gradient systems with an emphasis on electrical networks
Kang, Yun, The dynamics of plant-herbivore interactions and their implications for spatial expansion
Kelkar, Ashwini, A study of the subgraphs and the conjecture of the middle two layers graph, using modular matchings
Kupresanin, Ana Maria, Topics in functional canonical correlation and regression
La Marca, Michael Benedetto, Control of continuum models of production systems
Park, Russell, Optimal compression and numerical stability for Gegenbauer reconstructions with applications
Saxena, Rishu, High order methods for edge detection and applications
Spiriti, Steven Mark, Random search optimization for free-knot splines and P-splines
Stefan, Wolfgang, Total variation regularization for linear ill-posed inverse problems: Extensions and applications
Unver, Ali Kemal, Observation based PDE models for stochastic production systems
Wallington, Rachel, Number fields with solvable Galois groups and small Galois root discriminants

The above list contains the names and thesis titles of recipients of doctoral degrees in the mathematical sciences (July 1, 2008, to June 30, 2009) reported in the 2009 Annual Survey of the Mathematical Sciences by 223 departments in 138 universities in the United States. Each entry contains the name of the recipient and the thesis title. The number in parentheses following the name of the university is the number of degrees listed for that university. A supplementary list containing names received since compilation of this list will appear in a summer 2010 issue of the Notices.
Yang, Hongling, A study of additive coefficient models
Zhang, Guoyi, Smoothing splines using compactly supported, positive definite, radial basis functions

University of Arizona (9)

School of Mathematical Sciences
Bazan, Carlos, PDE-based image and structure enhancement for electron tomography of mitochondria
Hui, Kwok, Risks analysis of software development using Bayesian belief network and non-linear programming methods
Limón, Alfonso, A multilevel framework for PDEs whose solution exhibits fast transitions

Naval Postgraduate School (1)

Department of Applied Mathematics
Dea, John, High-order non-reflecting boundary conditions for the linearized Euler equations

University of California, Berkeley (41)

Department of Mathematics
Armstrong, Scott, Principal half-eigenvalues of fully nonlinear homogeneous elliptic operators
Boginski, Paul, Stable SO(2)-categorical algebraic structures
Blasiak, Jonath, Cyclage, catabolism, and the affine Hecke algebra
Brown, Jeffrey, Gromov-Witten invariants of toric fibrations
Chen, Qingtao, Some mathematical aspects of quantum field theory

Cotton-Clay, Andrew, Symplectic Floer homology of area-preserving surface diffeomorphisms and sharp fixed point bounds
Dutter, Seth, Logarithmic jet spaces and intersection multiplicities
Flemmer, Joseph, The relative structure of Henselian valued fields
Judson, Zachary, Connectivity in the Higson corona of the positive real access
Long, Yun, Timing of the Swendsen-Wang dynamics in complete graphs and trees
Mkrtychyan, Sevak, Scaling limits of random skew plane partitions
Peters, Emily, A planar algebra construction of the Haagerup subfactor
Prat-Waldron, Arturo, Pfaffian line bundles over loop spaces, spin structures and the index theory
Ramakrishnan, Janak, Type of o-minimal theories
Rothbach, Brian, Borel orbits of $X^2 = 0$ in $GL_n$
Sain, Jeremy, Berezniz quantization from ergodic actions of compact quantum groups and quantum Gromov-Hausdorff distance
Sammis, Ian, Implicit and fourth-order semi-Lagrangian contouring for geometric moving interface problems
Schommer-Pries, Christopher, The classification of two-dimensional extended topological field theories
Snyder, Noah, Quantum groups, tensor categories and knot invariants
Somersille, Stephanie, Biased tug of war, the biased infinity Laplacian and comparison with exponential cones
Stovall, Lindsay, $L^p$ inequalities for certain generalized Radon transforms
Tohaneanu, Mihai, Local energy estimates and structure estimates on Schwarzschild and Kerr black hole backgrounds
Tolland, Andrew, Gromov-Witten gauge theory
Trekhimchouk, Maxim, Regularity for certain parabolic diffusion systems
Varilly-Alvarado, Anthony, Arithmetic of del Pezzo surfaces of degree 1
Wu, Guoliang, Impulse control of multidimensional diffusion and jump diffusion processes
Zhu, Xinwen, Gerbal representations of double loop groups

Department of Statistics
Ahn, Soyeon, Statistical topics in gene regulation
Bae, Chongsoo, Analyzing random forests
Hather, Greg, Statistical analysis of DNA sequence motifs and microarray data
Liang, Richard, Two continuum-sites stepping stone models in population genetics with delayed coalescence
Obozinski, Guillaume, Simultaneous variable selection and simultaneous subspace selection for multitask learning
Ralph, Peter, Most recent common ancestors, genetic inheritance, stochastic gene transcription and Brownian motion on disconnected sets: A probabilistic analysis of a few models
Rosenberg, David, Semi-supervised learning with multiple views
Sly, Allan, Spatial and temporal mixing of Gibbs measures
Vu, Vinc, High dimensional data analysis: Entropy and regression
Xu, Na, Transcriptome detection by multiple RNA tilting array analysis and identifying functional conserved non-coding elements by statistical testing
Yan, Donghui, Some issues with dimensionality in statistical inference

GROUP IN BIOSTATISTICS
Gilbert, Houston, Multiple hypothesis testing: Methodology, software implementation, and applications to genomics

Moore, Kelly, Targeted maximum likelihood estimation of treatment effects in randomized controlled trials and drug safety analysis
Rubin, Daniel, Applications of double robustness

University of California, Davis (20)

DEPARTMENT OF MATHEMATICS
Baba, Chinpei, Decomposition theorems for complex projective structures
Berg, Christopher, Combinatorics of (1,0)-JM partitions, l-cores, the ladder crystal and the finite Hecke algebra
Hallowell, Karl, Higher spin approaches to quantum field theory and (pseudo)-Riemannian geometrics
Haws, David, Matroid polytopes: Algorithms, theory, and applications
Miller, Adam, How disturbance creates the storage effect and promotes maintenance of species diversity
Mulherkar, Jai, Some properties of the XXZ spin chain and their applications to quantum computing
Needell, Deanna, Topics in compressed sensing
Raz, Hillel, Lieb-Robinson bounds in the anharmonic lattice
Sershen, Cheryl, A dynamic model of DNA structure and function
Stevens, Alice, Knots in Heegaard surfaces
Strawbridge, Eva, Mechanics, dynamics, and structures of DNA

DEPARTMENT OF STATISTICS
Barkauskas, Donald, Statistical analysis of matrix-assisted laser desorption/ionization Fourier transform ion cyclotron resonance mass spectrometry data with applications to cancer biomarker detection
Chen, Kun, Functional approaches for high dimensional and gene expression data
Chen, Ying, Statistical approaches for detection of relevant genes and pathway in analysis of gene expression data
Liu, Bitao, Functional data analysis for online auction data
Liu, Jingyi, A shrinkage-average method for fitting high dimensional linear models
Liu, Ziadi, Nonparametric bootstrap method on Stiefel manifold and GPAN algorithm for ASP fit
Norris, Ann (Michelle), Parametric and semiparametric joint modeling for longitudinal diagnostic outcomes
Weng, Qian, Modeling progression of neurodegenerative disease with longitudinal neuroimaging data
Xu, Quyan, Estimation of integrated covariability for asynchronous assets in the presence of microstructure noise

University of California, Irvine (8)

DEPARTMENT OF MATHEMATICS
Bjork, Andrew-David, Criteria for rank-one transformations to be weakly mixing, and the generating property
Cao, Yaping, On the α-regularization of non-linear partial differential equations-analysis and error estimates
Cox, Sean, Covering theorems for the core model, with applications
Hu, Zhensheng, Phase-field modeling of thin film epitaxial growth
Kypriotakis, Kyriakos, Combinatorial principles in Jensen type extender models of the form L[1]
Le, Phong, Coherent decomposition of p-adic Newton polygons for L-functions of exponential sums
Luo, Songting, Numerical methods for static Hamilton-Jacobi equations
Matayoshi, Jeffrey, The zeros of random polynomials

University of California, Los Angeles (22)

DEPARTMENT OF MATHEMATICS
Blunk, Mark, Del Pezzo surfaces of degree 6 over an arbitrary field
Chen, Lin, Chern-Simons theory of knot invariants
Chifan, Ionut, Deformation spectral gap rigidity in von Neumann algebras
Citrin, Craig, L-invariants of adjoint square Galois representations coming from modular forms
Conley, Clinton, Some applications of combinatorics in descriptive set theory
Dong, Bin, Applications of variational models and partial differential equations on medical image and surface processing
Gautam, Sushrut Zhu, Two geometric obstruction results in harmonic analysis
Herman Jr., Paul Edward, Applications of beyond endoscopy
Kim, Yunho, Variational methods: Theory and its applications to image deblurring and denosing problems
Liu, Jiang, Controlling the dynamics of recurrent neural networks with synaptic learning rules
Lu, Steve, New constructions in pairings-based cryptography
Mathews, Bryant, Canonical dimension of projective homogeneous varieties of inner Type A and Type B
Nguyen, Dinh Huu, On P-generic splitting varieties for Milnor K-symbols mod P
Roy, Tristan, Global analysis of the defocusing cubic wave equation in dimension 3
Shao, Shuanglin, Restriction of estimates for paraboloids and cones in the cylindrically symmetric case
Shibver, Dan, Tracial and non-tracial random matrix models in free probability
Thompson, Alexander, A metamathematical condition equivalent to the existence of a complete left invariant metric for a Polish group
Vanvalkenburgh, Michael, Hermite/Laguerre-Gaussian modes and lower bounds for quasimodes of semiclassical operators
Virtanen, Jukka Tapio, Structure of elementary particles in non-Archimedean spacetime

DEPARTMENT OF STATISTICS
Anderson, Ariana, Machine learning for classification and diagnosis of functional magnetic resonance image scans
Zhu, Long, Recursive composition in computer vision: Modeling, inference, and learning

University of California, Riverside (7)

DEPARTMENT OF STATISTICS
Bi, Yingtao, Theoretical analysis of classification under CCC-noise and its application to semi-supervised text mining
Kim, Songsu, Inverse circular regression with possibly asymmetric error distribution
Montes de Oca, Veronica, Nonparametric cusum algorithms with applications to network surveillance
Pal, Rupam Ranjan, Efficient designs for discriminating between two competing linear regression models
Shah, Payal, Sequential sampling methods using generalized linear models with applications to pest density estimation
SUAREZ ESPINOZA, Javier, Inference for the multiparameter skew normal distribution
Yu, Hua, Neutral zone classifiers within a decision-theoretic framework

University of California, San Diego (16)

DEPARTMENT OF MATHEMATICS

Armel, Jonathan, Holomorphic extension of solutions to homogeneous analytic partial differential equations
Averett, Maia Christine, On real Johnson-Wilson theories
Beeson, Amanda, Groups of special units
Comstock, Jana, A finite presentation of knotted trivalent graphs
Eldredge, Nathaniel, Holomorphic extension of solutions to homogeneous analytic partial differential equations
Fuller, Evan, Generating functions for composition/word statistics
Grice, Jon, Discrete quantum control
Gurvich, Michael, Some results on the topology of quasitoric manifolds and their equivalent mapping spaces
Horn, Paul, Random subgraphs of a given graph
Jehring, Kristin, Harmonic functions on Walsh’s Brownian motion
Kinnally, Michael, Stational distributions for stochastic delay differential equations with non-negativity constraints
Lee, Nam Heon, A sufficient condition for stochastic stability of an Internet congestion control model in terms of fluid model stability
Niedermaier, Andrew, Statistics on wreath products
Raleigh, Sean, Ruled Legendrian knots and Lagrangian surfaces
Szypowski, Ryan, Least-squares finite elements and constrained evolution systems
Yacobi, Oded, An analysis of multiplicity spaces in classical symplectic branching

University of California, Santa Cruz (1)

DEPARTMENT OF APPLIED MATHEMATICS

Schuresso, Michael, Controlling global network connectivity of robot swarms with local interactions

University of Southern California (12)

DEPARTMENT OF MATHEMATICS

Aydilek, Harun, Optimal decisions under recursive utility
Glatt-Holtz, Nathan, Stochastic equations in geophysical fluid dynamics
Guest, Simon, A solvable version of the Baer-Suzuki theorem and generalizations
Jedwab, Andrea, Representations of finite dimensional Hopf algebras
Oleksandr, Lytvak, Fully discrete approach for estimating local volatility in the generalized Black-Scholes setting
Piterbarg, Yulya, Population modeling and Bayesian estimation for the deconvolution of blood alcohol concentration from transdermal biosensor data
Psiloyenis, Yannis, Mixing conditions and long return times on Markov towers
Seliger, Philip, Optimal device design
Song, Qian, Optimal and exact control of deterministic, stochastic or parabolic differential equations
Yang, Yan, Mathematical modeling and analysis of representation error and system uncertainties for discretized distributed parameters systems
Zeng, Yu, Large deviations approach to the bi-stable systems
Zhao, Yuegang, Credit risk of a leveraged firm in a controlled optimal stopping framework

University of California, Santa Barbara (12)

DEPARTMENT OF MATHEMATICS

Dai, Dongyun, Convergence of the Yamabe flow on manifolds with boundary
Hendrata, Melissa, A dynamical system model for simulating myxobacteria life cycle
Kessenich, Paul, Global existence with small initial data for three-dimensional incompressible isotropic viscoelastic materials
Levitt, John, Embeddings of Mori dream spaces
Mohler, George, Efficient, non-stiff, and multiscale methods for complex fluids
Qi, Jiawei, Effective dynamics for ferrofluids
Taylor, Scott, Boring split links and unknots

COLORADO

Colorado State University (13)

DEPARTMENT OF MATHEMATICS

Bikowski, Jutta, Electrical impedance tomography reconstructions in two and three dimensions; form Calderon to direct methods
Frederick, Christopher, Persistence homology of sequences of neighborhood complexes for graphs
Jamshidi, Arthur, Modeling spatio-temporal systems with skew radial basis functions: Theory, algorithms and applications

University of Colorado, Boulder (13)

DEPARTMENT OF APPLIED MATHEMATICS

Barker, Andrew, Parallel monolithic fluid-structure interaction algorithms with application to blood flow simulation
Haut, Terry, Analysis and applications of nonlocal spectral formulations of fluids with free interfaces and surfaces
Sanders, Geoffrey, Two extensions to adaptive smooth aggregation (aSA) multigrid Eigenvalue initialization and nonsymmetric problems
Simpson, David, Bifurcations in piecewise-smooth, continuous systems

DEPARTMENT OF MATHEMATICS

D’Andrea, Jonas, Wavelet frames on fractal spaces of Dutkay-Jorgensen type
Denoncourt, Hugh, Some combinatorial models for reduced expressions in Coxeter group
Doctoral Degrees Conferred

Ernst, Dana, A diagrammatic representation of an affine C Temperley-Lieb algebra
Formichella, Marc, Functional equations among Barnes’ integrals and hypergeometric series
Kang, Sooran, The Yang-Mills functional and Laplace’s equation on quantum Heisenberg manifolds
Mishev, Ilia, Coxeter group actions on supplementary pairs of Salschützian $F_4(1)$ hypergeometric series
Pohlmann, Brent, Structural properties of acyclic heaps with applications to Kazhdan-Lusztig theory
Schumacher, Timothy, Removable boundary singularities for the equation $\Delta u = u^p$ in non-smooth domains
Shaw, Jason, Commutator relations and the clones of finite groups

University of Colorado, Denver (5)

Department of Biostatistics and Informatics
Boyd, Adam, Censoring-robust treatment effect estimation in clinical trials with time-to-event outcomes

Department of Mathematical and Statistical Sciences
Beezley, Jonathan, High-dimensional data assimilation and morphing ensemble Kalman filters with applications in wildfire modeling
Fink, Stephen, Truncated quadratics and elliptic curves
Harris, Angela, Cycle structures in graphs
Zakaryan, Armen, Stochastic diffusion model of heterogeneous populations

University of Northern Colorado (2)

School of Mathematical Sciences
Phipps, Marnie, A phenomenological investigation on eighth graders’ number sense of fractions
Toney, Allison, Women with advanced degrees in mathematics in doctoral programs in mathematics education

University of Connecticut, Storrs (13)

Department of Mathematics
Alagic, Gorjan, Uncertainty principles for compact groups
Bella, Thomas, Topics in numerical linear algebra related to quasiregular and other structured matrices
Corluy, Marc, Rates of conveyance in the central limit theorem for Markov chains
Jura, Matthew, Reverse mathematics and the coloring number of graphs
Levin, Oscar, Computability theory, reverse mathematics and ordered fields
Markkanen, Tyler, Separating the degree spectra of structures
Prime, Russell, Averaging quadratic L-functions over function fields
Rong, Zhou, Numerical simulation of cell movement
Sang, Hallin, Asymptotic properties of generalized kernel density estimators
Termine, Lisa, Existence of solutions to semilinear elliptic differential equations using computer verification
Wooster, Robert, Evolution systems of measure for stochastic differential equation with Levy noise

Department of Statistics
Xie, Wangang, Bayesian phylogenetic model selection and applications
Zhao, Yifang, Contributions to microarray data analysis

Wesleyan University (4)

Department of Mathematics and Computer Science
Gould, Franklin, On certain classes of minimally almost periodic groups
Hall, Becky, An improved method for computing group cohomology of congruence subgroups of $SL_3(\mathbb{Z})$
Pan, Weiwei, Group actions on categorized bundles
Shea, Steve, Finitary isomorphisms of r-processes

Yale University (4)

Department of Mathematics
Aryapoor, Masood, The Penrose transform and self-dual Yang-Mills equations in split signature
Le Donne, Enrico, Bi-Lipschitz homogeneous geodesic manifolds
Mohammadi, Amir, Unipotent flows in positive characteristics and applications
Warshall, Andrew, Depth and related properties of infinite finitely generated groups

Florida Atlantic University (4)

Department of Mathematics
Al-Islam, Najja, Pseudo almost periodic solutions to some systems of non-linear hyperbolic second-order partial differential equations
Salama, Lifoma, Combinatorial statistics on phylogenetic trees

University of Delaware (8)

Department of Mathematical Science
Gutekunst, Todd, Subsets of finite groups exhibiting additive regularity
Heryudono, Alfa, Adaptive radial basis function methods for the numerical solution of partial differential equations with application to the simulation of the human tear film
Maki, Kara, Computational solutions of linear systems and models of the human tear film
Wang, Zeying, Skew Hadamard difference sets, strongly regular graphs and p-ary bent functions

Wei, Ang, Random-harmonic functions and multivariate Gaussian estimates
Wu, Junhua, Geometric structures and linear codes related to conics in classical projective planes of odd orders
Xu, Liwei, Computational methods for a class of problems in acoustic, elastic and water waves
Zeev, Noam, Direct and inverse scattering problems for thin dielectric and partially coated objects

DISTRICT OF COLUMBIA

George Washington University (4)

Department of Mathematics
Luse, Terry, Invariants of knots, graphs, and Feynman diagrams
Pingrey, Sarah, Strong degree spectra of relations

Department of Statistics
Hikawa, Hiroyuki, Local linear Peters-Belson regression and its applications to employment discrimination cases
Yan, Lihan, Group sequential robust designs in genetic studies

Howard University (2)

Department of Mathematics
Al-Islam, Najja, Pseudo almost periodic solutions to some systems of non-linear hyperbolic second-order partial differential equations
Salama, Lifoma, Combinatorial statistics on phylogenetic trees

FLORIDA

Florida Atlantic University (4)

Department of Mathematical Sciences
Bozovic, Vladimir, Algebraic and combinatorial aspects of group factorizations
Hopkins, Mary, Weakly integrally closed monoids and forbidden patterns
Kasprikova, Eva, Higher-order commutators in the method of orbits
Wess, Mark, Computing topological dynamics from time series

Florida Institute of Technology (1)

Department of Mathematical Sciences
Huang, Weijun, Sequential stochastic games

CONNECTICUT

University of Connecticut, Storrs (13)

Department of Mathematics
Alagic, Gorjan, Uncertainty principles for compact groups
Bella, Thomas, Topics in numerical linear algebra related to quasiregular and other structured matrices
Corluy, Marc, Rates of conveyance in the central limit theorem for Markov chains
Jura, Matthew, Reverse mathematics and the coloring number of graphs
Florida State University (12)

Department of Mathematics
Lebedev, Yuri, Open math library for computing on Riemann surfaces
Lin, Haomin, An optimal control problem for a time dependent Ginzburg-Landau model of superconductivity
Mavroudis, Konstantinos, Constant proportions portfolio strategies in an evolutionary context under a dividend factor model
Salta, Emmanuel, Variance reduction techniques in pricing financial derivatives
Shah, Manan, Randomized quasi-Monte Carlo methods and the computation of endogenous mortgage rates
Zigantcheva, Milena, Stochastic volatility extensions of the swap market model
Yoo, Eunjoo, Variance gamma pricing of American futures options

Department of Statistics
Balov, Nikolay, Covariance on manifolds
Ivanescu, Andrada, Revealing sparse signals in functional data
Lin, Lanjia, Association models for clustered data with binary and continuous responses
Liu, Yang, Transformation models for survival data analysis and applications

University of Central Florida (8)

Department of Mathematics
Boustique, Hatim, Lattice valued convergence: Quotient maps
Bryant, Donald, Analysis of Kolmogorov’s superposition theorem and its implications with low and high dimensional data
Galifia, Daniel J., The Sheffer B-type 1 orthogonal polynomial sequences
Khosravi, Mehrdad, Pseudoquotients: Construction, applications, and their Fourier transform
Konate, Souleymane, Efficient cone beam reconstruction for the distorted circle and line trajectory
Landon, Benjamin, Degree of approximation of Holder continuous functions
Lopez, Jerry, Optimal dual frames for erasures and discrete Gabor frames
Macun, Lisa Fischer, Almost regular graphs and edge face colorings of plane graphs

University of Florida (6)

Department of Mathematics
Dashti, Seyyed, Effective symbolic dynamics
Nguyen, Hung Nguyen, Representations of finite groups of Lie type

Department of Statistics
Li, Zhen, Bayesian methodologies for genomic data with missing covariates
Roy, Vivekananda, Theoretical and methodological developments for Markov chain Monte Carlo algorithms for Bayesian regression
Wu, Song, A robust approach for genetic mapping of complex traits
Yap, John, Nonparametric covariance estimation in functional mapping of complex dynamic traits

University of Miami (3)

Department of Mathematics
Coburn, Brian, Multi-species influenza models with recombination
Donzelli, Fabrizio, Algebraic density property of homogeneous spaces
Zivanovic, Sanja, Attractors in dynamics with choice

University of South Florida (1)

Department of Mathematics and Statistics
Miladinovic, Branko, Kernel density estimation of reliability with applications to extreme value distributions

Georgia Institute of Technology (13)

School of Mathematics
Bilinski, Mark, Approximating the circumference of 3-connected claw-free graphs
Bird, Csaba, Problems and results in partially ordered sets, graphs and geometry
Greenberg, Samuel, Random sampling of lattice configurations using local Markov chains
Jimenez, David, Analysis of two problems in signal quantizations and A/D conversion
Li, Yongfeng, Nonlinear oscillation and control in the BZ chemical reaction
Litherland, Trevis, On the limiting shape of random young tableaux for Markovian words
Marcus, Adam, New combinatorial techniques for non-linear orders
Pearson, John, The noncommutative geometry of ultrametric Cantor sets
Pugliese, Alessandro, Theoretical and numerical aspects of coalescing of eigenvalues and singular values of parameter dependent matrices
Savinien, Jean, Cohomology and K-theory of aperiodic tilings
Xu, Hua, Aspects of random matrix concentration and subsequence problems
Young, Stephen, Random dot product graphs: A flexible model for complex networks
Yurchenko, Aleksey, Some problems in the theory of open dynamical systems and deterministic walks in random environments

University of Georgia (13)

Department of Mathematics
Bagci, Irfan, Cohomology and support varieties in Lie superalgebras
Hower, Jeremiah, On elliptic curves and arithmetical graphs
Zhuang, Chao, Approximation methods and applications in financial optimization problems

Department of Statistics
Breazel, Ellen Hepfer, Effects of common errors in microsatellite data on estimates of population differentiation and inferring genotypic structure of complex loci using genome-wide expression data
Chen, Jen, Applications of empirical likelihood to quantile estimation and longitudinal data
Le-Rademacher, Jennifer, Principal component analysis for interval-valued and histogram-valued data and likelihood functions and estimators for symbolic data
Liu, Shangbin, Statistical issues on mass spectrometry-based protein identification and quantitation
Qiu, Junshan, Variable selection methods with applications
Sun, Guoying, A genetical genomics to genome scan for complex traits
Tewari, Susanta, Construction of high-resolution likelihood-based integrated genetic and physical map of neurospora crassa
Wang, Qin, Sufficient dimension reduction and sufficient variable selection
Ye, Jun, Geostatistical methods for spatio-temporal analysis of fMRI data
Zhen, Xiaoa, Categorical time series

IDAHO
University of Idaho (3)
Department of Mathematics
Bauer, Ryan, Disjoint cycles and tree packings in graphs
Buzbas, Erkan Ozge, Likelihood methods for balancing selection under the k-allele models
Tran, Luong Dinh, Nonlinear extensions of the Erdős-Ginzburg-Ziv theorem

ILLINOIS
Illinois Institute of Technology (5)
Department of Applied Mathematics
Aljun, Du, Deviation, ergodicity and reduction for stochastic dynamical systems
Chen, Bao Hua, Stochastic modeling of water vapor in the climate system
Vidozzi, Frederico, Two essays on multivariate stochastic processes and applications to credit risk modeling
Vidozzi, Sebastiano, Selected applications of the theory of Markov processes to mathematical finance
Zeng, Xiaoyan, Integration and approximation using lattice designs

Illinois State University (2)
Department of Mathematics
Casey, Stephanie, Subject matter knowledge for teaching statistical association
Williams, Nicole, Interpreting scatterplots: The role of statistical knowledge and personal knowledge

Northern Illinois University (6)
Department of Mathematical Sciences
Bilgili, Devrim, Quantitative trait loci detection using an accelerated failure time cure model
Corley, Todd, Classification methods for primitive table algebras of dimension 4
Lund, Mark, Nevanlinna theory in annuli
Meshes, Jonathan, Functions in class F in the unit disc which have integrals not in F
Racovitan, Michael, Generalizations for the blocks of the symmetric group and Nakayama conjecture
Sokolov, Vadim, Quadratic inverse eigenvalue problems: Theory, methods and applications

Northwestern University (11)
Department of Engineering Sciences and Applied Mathematics
Conway, Jessica, Complex patterns in oscillatory systems
Hansen, David, Axisymmetric Stokes flow solutions and bubble microstreaming
McCallum, Matthew, Eigenfunctions of cyclic integral transforms
Menon, Vihan, Building better models of the C41 pyramidal neuron: A combined computational and experimental approach
Merkey, Brian, Biofilm modeling for wastewater treatment: Multiple species and multiple components
Sample, Christine, Nonlinear dynamics of interacting interfaces in electrochemical and biological systems
Smith, Bryan, The extended finite element method for special problems with moving interfaces

Department of Mathematics
Nanes, Kalman, On the ergodicity of partially hyperbolic flows
Ouyang, Cheng, Asymptotics of implied volatility in local volatility models
Qian, Randy, Diffractive theorems for the wave equation with inverse square potential
Wang, Fang, Minimal measures for Lagrangian systems

Southern Illinois University, Carbondale (5)
Department of Mathematics
A, Min, Solvability of diffusion models of insect movement in complex landscapes
Ghebremicael, Aman, Latin squares and applications
Schwartz, Andrew, Decompositions of graphs and trees
Teweldemedhin, Amanuel, Nonparametric estimators of mean residual life function
Ye, Ping, Tests of symmetry with ordered alternatives in three-dimensional contingency tables

University of Chicago (15)
Department of Mathematics
Beccana, Marius, A critical centre-stable manifold for the semilinear Schrödinger equation
Bennett, Hanna, Volume distortion in groups
Biringer, Ian, Algebra versus geometry in hyperbolic 3-manifolds
Brosh, Michael, Moduli of finite flat group schemes with G-structure
Conidis, Chris, Applications of computability theory
Cudney, Richard, A category of representations at the critical level
Fowler, James, Poincaré duality groups and homology manifolds
Ginensky, Adam, A generalization of the Clifford index and determinantal equations for curves and their secant varieties
Malestein, Justin, Topological properties of a surface and the lower central series
Patel, Deepam, De-Rham epsilon factors
Patterson, Eric, Singularities of configuration and graph hypersurfaces
Shulman, Michael, Double categories and base change in homotopy theory
Smith Zbarsky, Emma, Families of A-infinity algebras and homotopy group actions
Sparling, Jonathan, On the local relative trace formula
Townsend Beazley, Elizabeth, Codimensions of Newton strata for SL(3) in the Igahori case

University of Illinois at Chicago (15)
Division of Epidemiology and Biostatistics
Arval, Subhash, Small sample tests and sample size determination for hierarchical mixed-effects models
Joslin, Charlotte, The association between Acanthamoeba keratitis and domestic water exposure in the Chicago area
Lee, Jungwha, Variance estimation of partial population attributable fraction and its applications
Rankin, Kristin, Applying multifactorial population attributable fractions to the problem of childhood overweight
Ricks, Phillip, Tuberculosis control among substance users: The indigenous leadership outreach model vs. standard care
Department of Mathematics, Statistics, and Computer Science
de los Angeles Ortiz Rodriguez, Rosario, A statistical look at day trading
Fazioli, Christopher Carlo, On analyticity of variations of the Dirichlet-Neumann operator, and computational concerns
Jordan, Richard, Asymptotic methods applied to finance: Equity and volatility derivatives
Kakleas, Maria, Numerical simulation of a weakly nonlinear model for water waves with viscosity
University of Illinois, Urbana-Champaign (25)

**Department of Mathematics**

**Barrus, Michael**, On induced subgraphs, degree sequences, and graph structure

**Choi, Jeong Ok**, Extremal problems in coloring and labeling of graphs and partial orders

**Dong, Han**, Minimal volume lattice d-simplices

**Gunaydin, Ayhan**, Model theory of fields with multiplicative group

**Johnson, Mathew**, On the stability of periodic solutions of nonlinear dispersive equations

**Jung, Jaebum**, Pareto optimization in robotics with acceleration constraints

**Koh, Ngin-Tee**, Approximable quasidisks

**Konwerska, Malgorzata**, The law of the iterated logarithm in noncommutative probability

**Landquist, Eric**, Infrastructure, arithmetic, and class number computations in purely cubic function fields of characteristics at least 5

**Law, Ka Lung**, Laurent polynomial inverse matrices and multidimensional multivariate systems

**Liu, Qi**, Some extremal problems on graphs and partial orders

**Lopes, Vinicius Cifu**, Grothendieck semirings and definable endofunctions

**Markkova, Jana**, 0-minimal fields with standard past map

**Masri, Nadia**, Fourier coefficients of modular forms and their applications

**Mizrak, Ozgur**, Asymptotic stability of the ground states of the nonlinear Schrödinger equation

**Popa, Ana-Maria**, On completely bounded multipliers of the Fourier algebra A(G)

**Rojas, Ricardo**, Loops and finite affine planes

**Sahin, Mehmet**, Moduli space of sheaves on \( P^n \) with Hilbert polynomial 4n + 1

**Santhanam, RKeka**, Units of equivariant ring spectra

**Sheik, Naem N.**, Extremal problems for partitions of edge-sets of graphs

**Stan, Florin**, Trace problems in algebraic number fields and applications to characters of finite groups

**Stodolsky, Burak**, Domination in sparse graphs

**Department of Statistics**

**Hong, Hyokyung**, Prediction of conditional quantiles in ordinal regression with applications to aging research

**Huebner, Alan**, Modeling correlated ordinal data: Marginal and conditional approaches

Indiana University, Bloomington (16)

**Department of Mathematics**

**Alan, Muhammed Ali**, Weighted pluripotential theory

**Altok, Serdar**, Reversibility of simple random walk on Galton-Watson trees

**Bateman, Michael**, Maximal averages over rectangles in the plane

**Chen, Qingshan**, Analysis and computation of the invididual primitive equations and the corner singularities for some nonlinear diffusive equations

**Crawford, Jesse**, Interpretation of eigenvalues in multivariate statistical analysis

**Gash, Justin**, On efficient Grobner basis computation

**Hackett, Michelle**, A new family of screw-motion invariant minimal surfaces

**Hagge, Tobias**, Graphical calculus for fusion categories and quantum invariants for 3-manifolds

**Hong, Seung-Moon**, Classification and applications of tensor categories

**Kuo, Jung-Miao**, The Clifford algebra of a cubic form

**Mazur, Justin**, Motivic zeta functions and the Grothendieck ring for varieties with group actions

**Nguyen, Toan**, Asymptotic stability of noncharacteristic viscous boundary layers

**Van Cott, Cornelia**, Obstructions to slicing generalized doubles

**Yarahmadian, Shantia**, Pointwise Green function bounds and stability of noncharacteristic boundary layers

**Yari, Masoud**, Formation and long-time persistence of patterns outside of equilibrium

**Yasamin, Ahamed**, Maximal invariants over symmetric cones

**Purdue University** (37)

**Department of Mathematics**

**Bakhtary, Parsa**, On the cohomology of a simple normal crossings divisor and its dual complex, with applications to isolated singularities

**Crosby, Charles**, A monoidal structure in the category of cosimplicial chain complexes

**Dean, Noah**, Vertex reinforced jump processes on one dimensional weighted environments

**Fang, Qirong**, Efficient and stable spectral methods for acoustic scattering over bounded obstacles

**Hassapis, George**, Bergman coordinates on finite Riemann surfaces

**Jang, Junmyeon**, Generic \( p \)-ranks for semi-stable fibrations

**Ji, Yu-Juan**, Stochastic analysis and differential equation with respect to fractional Brownian motion

**Joyner, Sheldon**, Zeta functions as iterated integrals

**Kim, Jaehong**, Holomorphic Banach bundles over compact manifolds

**Kjeldgaard Mikkelsen, Carl Christian**, Numerical methods for large Lyapunov equations

**Kwan, Yuen Vicky**, Fast spectral methods for incompressible flows

**Marrazzi Lopez, Leonardo**, Direct and inverse scattering on conformally compact manifolds and asymptotically hyperbolic manifolds with polyhomogeneous metric

**Orfanos, Stefanos**, Crossed products and their finite dimensional approximation properties

**Pham, David**, G-Frobenius algebras, twisting, and the Drinfeld double

**Price, James**, Weighted homogeneous polynomials and the Jacobian

**Shang, Ning**, Low genus algebraic curves in cryptography

**Taylor, Courtney**, Computation of fixed point data from equivariant cohomology

**Wang, Jinyue**, Error bounds for numerical methods for the ROF image smoothing model

**Wang, Yi**, Variational solutions to SPDE perturbed by a general Gaussian noise

**Wu, Qingyu**, Image of transfer from \( GL(2) \times GL(3) \) to \( GL(6) \)

**Xie, Zhenqiu**, Minimizers of the Lawrence-Poniat model under weak coupling and a parallel or slightly tilted field

**Yang, Kai**, The logistic, two-sex, age-structured population model with births outside marriage

**Yao, Song**, Quadratic nonlinear expectations and risk measures

**Yuan, Chunyan**, Gamma-convergence of the Lawrence-Poniat energy for layered superconductors to a three-dimensional anisotropic Ginzburg-Landau energy

**Zhang, Ji**, A solution method for zero dimensional equation systems and a pose-free framework for the robust solution of the structure from motion problem

**Zhang, Songfu**, Smoluchowski-Kramers approximation for stochastic equations with Levy-noise

**Zhao, Ruijun**, Some mathematical models and scientific computations in epidemiology and immunology
Department of Statistics
An, Lingling, Dynamic clustering of time series gene expression
Anderson, David, Multifractal fractional sum-difference models for internet traffic
Daye, Zhongyin John, Some new approaches in high-dimensional variable selection and regression
Jeng, Xinge Jessie, Covariance adaptation and regularization in large-scale hypothesis testing and high-dimensional variable selection
Li, Jingguang Tony, An additive-interactive nonlinear volatility model, estimating and testing
Qi, Xiaoli, Nonparametric calibration of two common susceptibility tests using interval-censored data with measurement error
Shu, Lei, Stepwise model building for mixed-effect models with random scale effects
Tuzov, Nikita, Mutual fund performance evaluation methodology and local false discovery rate approach
Yoo, Suk-Young, Statistical methods for integrating epigenomic results
Zeng, Lingmin, Group variable selection methods in analyses of genomic data

University of Notre Dame (7)
Department of Mathematics
Broad, Steven, Index formulas for higher order Loewner vector fields
Cibotaru, Daniel, Localization formulae in odd $K$-theory
Hauenstein, Jonathan, Regeneration, local dimension, and applications in numerical algebraic geometry
Hoehn, Stacy, The moduli space of compact fiber bundle structures on a fibration
Jensen, Raymond, Integral boundary invariants for conformally compact manifolds of constant curvature and measure-theoretic Einstein condition
Maher, Christina, On embeddings of computable structures, classes of structures and computable isomorphism
Rissler, Matthew, Stochastic models of collective motion in *Myxococcus Xanthus*

IOWA

Iowa State University (23)
Department of Mathematics
Ahmed, Haseena, High-resolution alternating evolution schemes for hyperbolic conservation laws and Hamilton-Jacobi equations
Allen, Aaron, Stability results for damped multilayer composite beams and plates
Bhattacharyya, Gargi, Terwilliger algebras of wreath products of association schemes
Dagli, Mehmet, Lie algebra decompositions with applications to quantum dynamics
Dragnei, Mihaela Cristina, Inverse Sturm-Liouville problems using multiple spectra
Lee, Jangwoon (Leo), Analysis and finite element approximations of stochastic optimal control problems constrained by stochastic elliptic partial differential equations
Mikkelsen, Rana, Minimum rank of graphs that allow loops
Pryporova, Olga, Types of convergence of matrices
Shin, Jaemin, Inverse scattering problems for first-order systems
Xie, Xiaoliang, Large time-stepping methods for higher order time-dependent evolution equations
Xu, Ying, Modeling and direct numerical simulation of particle-laden turbulent flows

Department of Statistics
Bingham, Melissa, Likelihood and Bayes inference for a class of distributions on orientations in 3 dimensions
Chapman, Jessica, Computationally efficient resource allocation for complex system reliability studies
Dancik, Garrett, Exploring host-pathogen relationships through computer simulations of intracellular infection
Diao, Liqia, Robust small area estimation
Lu, Lu, Statistical methods in a high school transcript survey
Ma, Haiming, New developments in planning accelerated life tests
Ramler, Ivan, Improved statistical methods for k-means clustering of noisy and directional data
Reising, Monica, Modeling and discrimination for spectral-temporal data
Tang, Chengyong, Parameter estimation and bias correction for diffusion processes and a nonparametric approach to census population size estimation
Villanueva-Morales, Antonio, Modified pseudo-likelihood estimation for Markov random fields with Winsorized Poisson conditional distributions
Wang, Jianqiang, Estimating the distance distributions of subpopulations and testing observation outlyingness for a large-scale complex survey
You, Lifeng, Cross-validation in model-assisted estimation

University of Iowa (11)
Department of Applied Mathematics and Computational Sciences
Gillispie, Brian, Instabilities of elastic objects in motion
Jiang, Yu, Inference and prediction in a multiple structural break model of economics time series
Kazmi, Kamran, Mathematical theory and numerical analysis of integrated bioluminescence tomography and diffuse optical tomography
Kim, Sujin, The role of bases and wavelets in stochastic processes
Ortega, Omaya, Evaluation of rotavirus vaccination programs

Department of Biostatistics
Bayman, Emine O., Bayesian hierarchical models for multi-center clinical trials: Power and subgroup analyses
Chen, Yi, Optimal adaptive group sequential design for Phase II clinical trials: A Bayesian decision theoretic approach

Department of Mathematics
Chun, Sangmin, The annihilator semi-group ring
Froelich, Jennifer, Universal deformation rings related to the symmetric group $S_3$
Ljumanovich, Leonida, Horn theories
Steinova, Jitka, Theta correspondence for unitary groups

KANSAS

Kansas State University (6)
Department of Mathematics
Cipra, James, Waring’s number in a finite field
Froehlich, Dmytro, Hirzebruch-Riemann-Roch theorem for differential graded algebras

Department of Statistics
Jung, Yoonsung, Effects of unequal treatment variances on the analysis of crossover designs
Ndum, Edwin, Individual treatment effect heterogeneity in multiple time points trials
Tang, Zhongwen, Lack of fit of logistic GEE models and cost efficient Bayesian optimal designs for nonlinear combinations of parameters in nonlinear regression models
Zhang, Ke (Kurt), Inference of nonparametric hypothesis testing on high dimensional longitudinal data and its application in DNA copy number variation and microarray data analysis

University of Kansas (5)
Department of Mathematics
Kastens, Jude, Some new developments on two separate topics: Statistical cross validation and floodplain mapping
Kummini, Manoj, Homological invariants of monomial and binomial ideals
Moen, Kabe, Linear and multilinear fractional operators: Weighted inequalities, sharp bounds, and other properties
LOUISIANA

Louisiana State University, Baton Rouge (6)

DEPARTMENT OF MATHEMATICS

Alali, Bacim, Multiscale analysis of heterogeneous media for local and nonlocal continuum theories

Jara, Patricio, Rational approximation schemes for solutions of abstract Cauchy problems and evolution equations

Meng, Chao, Stochastic and Copula models for credit derivatives

Morris, Karli, Trace forms of Abelian extensions of number fields

Ozer, Koray, Laplace transform inversion and time-discretization methods for evolution equations

Tugurlan, Maria Cristina, Fast marching methods—Parallel implementation and analysis

University of Louisiana at Lafayette (4)

DEPARTMENT OF MATHEMATICS

Marinov, Tchavdar, An inverse problem for the Euler-Bernoulli equation and a new scheme for solving a hierarchical size-structured model with nonlinear growth, mortality, and reproduction rates

Peng, Jie, Sample size calculation and tests for binomial and Poisson distributions

Stanford, Alex, Topological rings and associated topological groups

Trapoonsrisak, Patcharin, Quenching and blow-up phenomena due to concentrated nonlinear sources in $R^n$

MARYLAND

Johns Hopkins University (22)

DEPARTMENT OF APPLIED MATHEMATICS AND STATISTICS

Beer, Elizabeth, Random latent position graphs: Theory, inference, and applications

Hutchinson, David, Stopping times and confidence bounds for small-sample stochastic approximation algorithms

Lin, Yue, Rank-based methods for statistical analysis of gene expression microarray data

Ye, Xugang, Random disambiguation paths: Models, algorithms, and analysis

DEPARTMENT OF BIOSTATISTICS

Carvalho, Benilton, Statistical methods and software for high density oligonucleotide arrays

Chan, Kwan Chuen (Gary), Recurrent marker process before failure event: A backward process approach

Cheng, Yu-Jen, Statistical methods for failure time data with biased sampling and measurement errors

Di, Chongzhi, Likelihood ratio testing under nonidentifiability with applications to biomedical studies

Eckel, Sandra, Quantifying individual and city level modification of the health effects of air pollution in older adults

Ho, Yen-Yi, Statistical methods for failure time data with biased sampling and measurement errors

Li, Qing, Genetic association tests: Trio logic regression and score test

Shum, Kim Fai (Kenny), Estimating the mean medical cost in small samples

DEPARTMENT OF MATHEMATICS

Agarwala, Susama, The geometry underlying renormalization

Banerjee, Abhishek, Nearby cycles, Archimedean complex and periodicity in cyclic homology

Breiner, Christine, Embedded minimal surfaces of finite topology and one end: Conformal structure and asymptotic properties

Chen, Yifei, Strong rational connectedness of toric varieties

Hezari, Hamid, Eigenvalues and eigenfunctions of Schrödinger operators: Inverse spectral problems and zeros of eigenfunctions

Jiang, Jin-Cheng, Bilinear Strichartz estimates in 2 dimensional compact manifolds and cubic nonlinear Schrödinger equations

Kramer, Joel, Examples of embedded stable minimal surfaces of large area

Seyyedali, Reza, Balanced metrics in Kahler geometry

Wright, Thomas, On convergence of singular series for a pair of quadratic forms

Zulkowski, Patrick, The plateau problem of Alexandrov spaces satisfying the Perel’man conjecture

University of Maryland, Baltimore County (5)

DEPARTMENT OF MATHEMATICS AND STATISTICS

Gurtuna, Filiz, Geometric optimization, problems with applications to ellipsoidal approximations and quasi-Newton methods

Klein, Martin, Statistical analysis based on physiologically-based pharmacokinetic models

Newcomer, Justin, Estimation procedures for multinomial models with overdispersion

Wang, Dan, Partial differential equation constrained optimization and its applications to parameter estimation in models of nerve dendrites
University of Maryland, College Park (28)

**Department of Mathematics**

Belding, Juliana, Number theoretic algorithms for elliptic curves
Brody, Justin, On the model theory of random graphs
Chen, Qiwen, Dependence structure for Levy processes and its applications in finance
Delgado, Robert, Density properties of subsets of the Euler characteristic-2 surface group, PSL(2,R) character variety
Draper, Thomas, Non-linear complexity of Boolean permutations
Furnival, Darran, Iterative methods for the stochastic diffusion problem
Galaz-Garcia, Fernando, Non-negatively curved fixed-point homogeneous manifolds in low dimensions
Groer, Christopher, Parallel and serial algorithms for the vehicle routing problem and several variations
Hoban, Ryan, Local rigidity of triangle groups in Sp(4,R)
Johnson, Gregory, Abstract elementary classes with L"owenheim-Skolem number cofinal with \( \omega \)
Kim, Hyejin, Asymptotic problems for stochastic processes and related differential equations
Long, Jane, The cohomology of the affine group over \( F_p^2 \) and of \( PSL(3,F_p) \)
Lott, Paul Aaron, Fast solvers for models of fluid flow with spectral elements
Manon, Christopher, Presentations of semigroup algebras of weighted trees
Nicholls, Stacey, Column generation in predictor-corrector methods for solving linear programs
Olsson, James Scott, Combining evidence from unconstrained spoken term frequency estimation for improved speech retrieval
Pauletti, Miguel, Parametric AFEM for geometric evolution equations and coupled fluid-membrane interaction
Prakash, Samvit, Pricing volatility derivatives using space scaled Levy processes
Presley, Joanna, Response dynamics of integrate-and-fire neuron models
Price, Carter, Applications of operations research models to problems in health care
Rosario, Dalton, Algorithm development for hyperspectral anomaly detection
Sam, Chon (Denise), Variable selection properties of L1 penalized regression in generalized linear models
Shaw, Christopher, Weakly o-minimal structures and Skolem functions
Song, Xin, A new scheme for monitoring multivariate process dispersion
Spiliopoulos, Konstantinos, Asymptotic problems for stochastic processes with reflection and related partial differential equations
Sun, Weiran, Mathematical problems arising when connecting kinetic to fluid regions
Tang, Guojing, Discrete time stochastic volatility model
Wang, Xia, Applications of genetic algorithms, dynamic programming, and linear programming to combinatorial optimization problems

**Massachusetts**

Boston University (7)

**Department of Mathematics and Statistics**

Emmerling, Thomas, Essays on American and game-type options
Hamrick, David, Topics in stationarity, volatility, and contagion
Jiang, Xiaoyu, Network-based information integration for protein function prediction
Robertson, Scott, Applications of large deviations principles to options pricing and portfolio choice
Russell, Elizabeth, The complex dynamics of rational maps
Spurgin, Greg, Dynamics of the four body problem with large and small masses
Uminsky, David, The viscous N-vortex problem: A Helmholtz/Kirchhoff approach

Boston University School of Public Health (6)

**Department of Biostatistics**

Blood, Emily, Performance of mixed effects models in the analysis of longitudinal data generated from a structural equation model
Chibnik, Lori, Development and evaluation of risk prediction models in the presence of correlated markers and non-linear associations between markers and outcome using logistic regression and net-benefit analysis
Nguyen, Anh-Hoa, Family-based association tests and genetic risk scores for survival data
Stedman, Margaret, A comparison of methods for physician randomized trials with binary and survival outcomes
Sukprrut, Suporn, The estimation of the probability of treatment greater than control in clinical trials
Wu, Hong Sheng, Methods for genetic association studies using longitudinal and multivariate phenotypes in families

Harvard University (34)

**Department of Biostatistics, School of Public Health**

Hoffmann, Thomas, Contributions to multivariate association models for nuclear families
Maghsoudi, Kaveh, Transcript mapping with genome tiling microarrays
Manjourides, Justin, Distance based methods for space time modelling of the health of populations
Naylor, Melissa, Novel multivariate and Bayesian approaches to genetic association testing and integrated genomics
Othus, Megan, Survival analysis with complex censoring mechanisms with applications in population-based studies and clinical trials
Torgovitsky, Roman, Resampling methods for high-dimensional data and nonparametric regression with applications to brain imaging, bioinformatics, and sleep/circadian medicine
Wang, Lu, Nonparametric and semiparametric regression with missing data
Wu, Michael, Statistical methods for high-dimensional genomic data
Zhang, Jing Jenny, Novel methodologies for the analysis of complex failure time data and alternative progression-free survival estimators

**Department of Mathematics**

Barnet-Lamb, Thomas James, On potential automorphy, and other topics in number theory
Basu, Samik, A\textsuperscript{\textregistered} structures on Thom spectra
Bullock, Evan Merrill, Subcanonical points on algebraic curves
Chi, Chen Yu, Pseudonorms and theorems of Torelli type for birational equivalence
Choi, Suh Hyun, Local deformation lifting spaces of mod1 Galois representations
Hornet, Stefan Laurentiu, A generalization of a theorem of Ravenel and Wilson
Isacsson, Samuel Baruch, Cubical homotopy theory and monoidal model categories
Kass, Jesse, Good completions of Neron models
Rasmussen, Sarah Dean, A number theoretic result for Berge’s conjecture
Smith, Camilla, Enumeration of the distinct shuffles of permutations
Tarnita, Corina Elena, Evolutionary dynamics in structured populations
Tosatti, Valentino, Geometry of complex Monge-Ampère equations
SCHOOL OF ENGINEERING AND APPLIED SCIENCE
Abbot, Dorian, A high-latitude convective cloud feedback
Constantin, Gabriel, Expressiveness and optimization under incentive compatibility constraints in dynamic auctions
Dechadilok, Panadda, Electrokinetic effects in hindered transport
Dias, Joao, Automatically generating the back end of a computer using declarative machine descriptions
Hendrix, Philip, Learning and decision making with reputation information
Kang, Laura Suh Young, Open mechanism design: Ensuring and verifying the strategyproofness of mechanisms in open environments
Kordab, Imad, Bilevel optimization methods to evaluate intermodal freight transportation investments in Colombia
McClean, Megan, Principles of signal processing and fidelity in biological networks
Nesson, Rebecca, Synchronous and multicomponent tree-adjoining grammars: Complexity, algorithms and linguistic applications
Patel, Ankit, Modeling and inferring cleavage patterns in proliferating epithelia
Redwine, Kevin, Inference rules plus proof-search strategies equals programs
Shneidman, Jeffrey, Rationally motivated failure in distributed systems
Wapinski, Iian, Genome-wide reconstructions of the evolution of genes and regulatory elements in Ascomycota fungi

Massachusetts Institute of Technology (19)

DEPARTMENT OF MATHEMATICS
Aboofateh Beikizefzali, Salman, Quantum proof systems and entanglement theory
Bernstein, Jacob, Conformal and asymptotic properties of genus g minimal surfaces with one end
Chen, Victor, The Gowers norm in the testing of Boolean functions
Davis, Christopher, The overconvergent de Rham-Witt complex
Eu, Ching-Hwa, Hochschild homology/cohomology of preprojective algebras of ADST quivers
Iacovino, Vito, Open strings in the cotangent bundle and Morse homotopy
Jung, Kyomin, Approximate inference: Decomposition methods with applications to networks
Kilic, Mustafa Sabri, Induced-charge electrokinetics at large voltages
Lekili, Yanki, Broken Lefschetz fibrations, Lagrangian matching invariants and Ozsváth-Szabó invariants
Lim, Joungkeun, On the capacity of the erasure channel and the construction of an ε-randomizing map
Liu, Ruochuan, On the slope filtration of φ-modules over the Robba ring
Maydanskiy, Maksim, Exotic symplectic manifolds from Lefschetz fibrations
Ritter, Alexander, The Novikov theory for symplectic cohomology and exact Lagrangian embeddings
Sabbatini, Silvia, The topology of GKM spaces and GKM fibrations
Spiridonov, Alexey, Pattern-avoidance in binary fillings of grid shapes
Tievsky, Aaron, Analogues of Kähler geometry on Sasakian manifolds
Xiao, Liang, Nonarchimedean differential modules and ramification theory
Yick, King Yeung, A sphere settling in a stratified fluid at small Reynolds number
Yin, Jingbin, A q-analogue of spanning trees: Nilpotent transformations over finite fields

Northeastern University (5)

DEPARTMENT OF MATHEMATICS
Brown, Justin, Some geometric properties of certain toric varieties and Schubert varieties
Cutler, Anthony, Regular polyhedra of index 2
Donovan, Elizabeth, Various parameters of subgraphs and supergraphs of the hypercube
Koldan, Nilufer, Semiclassical asymptotics on manifolds with boundary
Mukherjee, Himadri, Singularities of a certain class of toric varieties

Tufts University (4)

DEPARTMENT OF MATHEMATICS
Espanol, Malena Inez, Multilevel methods for discrete ill-posed problems: Application to deblurring
Gonzalez Martinez, Cristian, Some aspects of the geometry and the cohomology of the moduli spaces of vector bundles and coherent systems over algebraic curves
O’Brien, Mark Richard, Right-angled Coxeter groups and CAT(0) spaces
Zhang, Wei, Haar based multi-resolution stochastic processes

University of Massachusetts, Amherst (7)

DEPARTMENT OF BIOSTATISTICS, PUBLIC HEALTH
Li, Xiaohong, Approaches to estimation of haplotype frequencies and haplotype-trait associations
Xu, Bo, Predictors of treatment means for a one factor completely randomized design

DEPARTMENT OF MATHEMATICS AND STATISTICS
Grigoryan, Viktor, Stability of geodesic wave maps
Hall-Seelig, Laura, Asymptotically good towers of global function fields and bounds for the Ihara function
Kucukasakili, Onur, Class numbers of ray class fields of imaginary quadratic fields
Milburn, Brett, Dirac maps and generalized complex geometry on homogeneous spaces
Shieh, Meng-Shiou, Correction methods, approximate biases, and inference for misclassified data

Worcester Polytechnic Institute (1)

DEPARTMENT OF MATHEMATICAL SCIENCES
Huang, Xueying (Shirley), In vivo MRI-based three-dimensional fluid-structure interaction models and mechanical image analysis for human carotid atherosclerotic plaques

MICHIGAN

Michigan State University (13)

DEPARTMENT OF MATHEMATICS
Arikan, Mehmet, Topological invariants of contact structures and planar open books
Condori, Alberto, Thematic indices and superoptimal singular values of matrix functions
Durusoy, Daniel, Heegaard Floer homology of certain 3-manifolds and cobordism invariants
Dyer, Kimberly, Almost dual FF-modules
Enders, Joerg, Generalizations of the reduced distance in the Ricci flow—Monotonicity and applications
Geng, Weihua, Interface method and Green’s function based Poisson Boltzmann equations solver and interface technique based molecular dynamics
Kim, Dong-Joong, Comparison of native-English and native-Korean speaking university students’ discourses on infinity and limit
Knapp, Adam, Computations of Lagrangian Floer homology and gauge theoretic invariants for Montesinos twins
Li, Mingfei, Strategies in repeated games
Tsai, Chih-Hsiung, Algorithms for solving polynomial systems by homotopy continuation method and its parallelization
Zhang, Yinfeng, Mixed volume and total degree
Doctoral Degrees Conferred

DEPARTMENT OF STATISTICS AND PROBABILITY
Aggarwal, Deepa, On some inference problems for current status data
Zhu, Yongfang, Statistical models for assessing the individuality of fingerprints

Oakland University (4)
DEPARTMENT OF MATHEMATICS AND STATISTICS
Coffield, Daniel, A model for single phase flow in horizontally fractured porous media using homogenization techniques
Gan Hewage, Jayantha Lanel, Complex root isolation
Jia, Fentiy, Contributions to Bayesian ranking and selection rules and to randomized response survey analysis
Rathugamage, Sriyani Renuka Menike, Models and analysis for adhesive contact, material damage, and cardiac waves

University of Michigan (25)
DEPARTMENT OF MATHEMATICS
Felgueiras, Oscar, The ample cone for a morphism
Gentry, Sara, Mathematical modeling of mutation acquisition in hierarchical tissues: Quantification of the cancer stem cell hypothesis
Gomez-Guerra, Jose, Models of twisted K-theory
Gong, Jason, Derivations on metric measure spaces
Iwen, Mark, Combinatorial compressive sampling with applications
Jain, Harsh, Quantitative modeling of molecular pathways associated with intratumoral angiogenesis
Kutch, Jason, Signal in human motor unsteadiness: Detecting the action and activity of muscles
Lee, Kyu Yong, On realization of line arrangements as multiplier ideals
Mackay, John, Conformal dimension and the quasisymmetric geometry of metric spaces
More, Yogesh, Arc valuations on smooth varieties
Robbins, Hannah, Finiteness of associated primes of local cohomology modules
Sierra, Susan, The geometry of birationally commutative graded domains
Spencer, Craig, Analytic methods for Diophantine problems
Stubbs, Joe, Potent elements and tight closure in Artinian modules
Urzua, Giancarlo, Arrangements of curves and algebraic surfaces
Vavrineck, Diane, Accessibility and JSJ decompositions of groups
Yang, Bo, Application of perturbation methods to pricing credit and equity derivatives

DEPARTMENT OF STATISTICS
Hamidieh, Kamal, Topics in statistical modeling of extremes and their dependence
Linn, Matthew, Nonlinear filtering of random fields in the presence of long-memory noise and related problems in stochastic analysis
Sen, Bodhisatta, A study of bootstrap and likelihood based methods in non-standard problems
Singhal, Harsh, Statistical inverse problems on graphs with applications to flow volume estimation in computer networks
Sun, Hakeun, Hybrid bootstrap for mapping quantitative trait loci and change point problems
Wagaman, Amy, Topics in high-dimensional inference with applications to Raman spectroscopy
Yang, Lili, Sample based estimation of network traffic flow characteristics
Zhao, Ou, Isotonic regression and stationary random walks

Wayne State University (5)
DEPARTMENT OF MATHEMATICS
Holdai, Veera, Multichannel change point problems
Kan, Shaobai, System identification in stochastic hybrid systems
Li, Wei Yuan, Fundamental solution and $L^p$ estimates for higher order subelliptic Schrödinger operators on stratified groups
Truong, Bao, Variational analysis and applications to multiojective optimization
Zhu, Huiping, Discontinuous Galerkin methods for singularly perturbed problems

Western Michigan University (7)
DEPARTMENT OF MATHEMATICS
Madden, Sandra, High school mathematics teachers’ evolving study of comparing distributions

DEPARTMENT OF STATISTICS
Awosoga, Oluwagbogumbi, Meta analyses of multiple baseline time series designs intervention models for dependent and independent series
Nantz, Eric, Testing equality of competing risks of patient discontinuation across multiple disease states
Nyirenda, Themba, Heteroscedastic linear model estimation based on ranks: An iterated reweighted least squares
Ponnathapura, Srinand, Statistical procedures for bioequivalence analysis
Rosales, Mathew, Robustness of confidence intervals for the mean of delta distribution

Tena, Jezania Kira, Test procedures for equality of two variances in delta distributions

MINNESOTA
University of Minnesota-Twin Cities (10)
DIVISION OF BIOSTATISTICS, SCHOOL OF PUBLIC HEALTH
Li, Meijuan, Statistical models for a censored quantitative trait and candidate genes association mapping in structured populations with multilevel genetic relatedness
Wei, Peng, Network-based mixture models for genomic data

SCHOOL OF STATISTICS
Adragni, Kofi Placid, Principal fitted components on small samples
Deng, Qiai, Developing a distribution-free change-point model for unknown-baseline data
Dong, Hua, Statistical properties of Gaussian reproducing kernel in univariate density estimation
Flegal, James, Monte Carlo standard errors for Markov chain Monte Carlo
Joo, Jonghoon, Curve/surface estimation with possible jumps/roofs/valleys preserved
Rendahl, Aaron, Graphical methods of determining predictor importance and effect
Wang, Huixin, On large margin hierarchical classification
Wu, Seongho, Homotopy, nonconvexity and regularization

MISSISSIPPI
Mississippi State University (1)
DEPARTMENT OF MATHEMATICS AND STATISTICS
Shahul-Hameed, Jaffar Ali, Multiple positive solutions for classes of elliptic systems with combined nonlinear effects

University of Mississippi (4)
DEPARTMENT OF MATHEMATICS
Chism, Lyrall, On independence polynomials and independence equivalence in graphs
Craddock, Michelle, Reflexivity and the Grothendieck property for positive tensor products of Banach lattices
Lai, Wei-Kai (Brian), The Radon-Nikodym property for positive tensor products of Banach lattices
Song, Lanzhen, Independence polynomials of $k$-tree related and well-covered graphs
<table>
<thead>
<tr>
<th>University</th>
<th>Department</th>
<th>Name</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>Missouri University of Science and Technology</td>
<td>Department of Mathematics and Statistics</td>
<td>Cowell, Simon</td>
<td>Spatially adaptive priors for essay on topics related to Riemann surfaces</td>
</tr>
<tr>
<td>Missouri University of Science and Technology</td>
<td>Department of Statistics</td>
<td>Marciniak, Małgorzata</td>
<td>Holomorphic extensions in toric varieties</td>
</tr>
<tr>
<td>Missouri University of Science and Technology</td>
<td>Department of Mathematics</td>
<td>Mukhopadhyay, Parna</td>
<td>A modified approach for obtaining sieve bootstrap prediction intervals for time series</td>
</tr>
<tr>
<td>Missouri University of Science and Technology</td>
<td>Department of Statistics</td>
<td>Supapakorn, Thidaporn</td>
<td>Estimating bounds for nonidentifiable parameters using potential outcomes</td>
</tr>
<tr>
<td>St. Louis University</td>
<td>Department of Mathematics and Computer Science</td>
<td>Manche, Jaime</td>
<td>Best representations and completions in weakened topologies for the integers</td>
</tr>
<tr>
<td>University of Missouri-Columbia</td>
<td>Department of Mathematics</td>
<td>Bihun, Oksana</td>
<td>Approximate isometries and distortion energy functionals</td>
</tr>
<tr>
<td>University of Missouri-Columbia</td>
<td>Department of Mathematics</td>
<td>Cowell, Simon</td>
<td>Asymptotic unconditionality in Banach spaces</td>
</tr>
<tr>
<td>University of Missouri-Columbia</td>
<td>Department of Mathematics</td>
<td>Felts, Kenneth</td>
<td>Harmonics and excitations of oscillators</td>
</tr>
<tr>
<td>University of Missouri-Columbia</td>
<td>Department of Statistics</td>
<td>Koh, Doowon</td>
<td>Extension theorems in vector spaces over finite fields</td>
</tr>
<tr>
<td>University of Missouri-Columbia</td>
<td>Department of Statistics</td>
<td>Shane, Christopher</td>
<td>Uniqueness theorems for non-symmetric convex bodies</td>
</tr>
<tr>
<td>University of Missouri-Columbia</td>
<td>Department of Mathematics</td>
<td>Lin, Xiaoyan</td>
<td>Bayesian hierarchical models for the recognition-memory experiment</td>
</tr>
<tr>
<td>University of Missouri-Columbia</td>
<td>Department of Mathematics</td>
<td>Yue, Yu</td>
<td>Spatially adaptive priors for regression and spatial modeling</td>
</tr>
<tr>
<td>University of Missouri-Columbia</td>
<td>Department of Mathematics</td>
<td>Zhang, Jing</td>
<td>Bayesian spatial analysis with application to the Missouri Ozark Forest Ecosystem Project</td>
</tr>
<tr>
<td>University of Missouri-Columbia</td>
<td>Department of Mathematics</td>
<td>Zhu, Liang</td>
<td>Semiparametric analysis of multivariate longitudinal data</td>
</tr>
<tr>
<td>Washington University in St. Louis</td>
<td>Department of Electrical and System Engineering</td>
<td>Yang, Ruoting</td>
<td>Data-driven two degree of freedom control for robust intelligent life science automation</td>
</tr>
<tr>
<td>Washington University in St. Louis</td>
<td>Department of Mathematics</td>
<td>Abel, Haley</td>
<td>The role of positive selection in molecular evolution: Alternative models for within-locus selective effects</td>
</tr>
<tr>
<td>Washington University in St. Louis</td>
<td>Department of Mathematics</td>
<td>Bohanon, Joseph</td>
<td>Groups in which the normalized of every non-normal sugroup is maximal</td>
</tr>
<tr>
<td>Brieler, Robert</td>
<td>Symmetric and alternating groups as monodromy groups of compact Riemann surfaces: The case of four branch points</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Browder, Jonathan</td>
<td>Proper group actions and the face structure of simplicial complexes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fan, Chunlin</td>
<td>Contributions to the theory of copula</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gill, James</td>
<td>Functions of finite distortion and a lower bound for the weak-type constant of the Beurling-Ahlfors transform</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lin, Yonhow</td>
<td>The interplay between harmonic analysis, functions theory and operators</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lott, Tim</td>
<td>Dehn fillings of hyperbolic punctured-torus bundles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ronshausen, Emily</td>
<td>The Liouville property in the discrete group-action setting</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standeven, Bennett</td>
<td>The role of first order logic in analysis of several complex variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MONTANA</td>
<td>University of Montana, Missoula</td>
<td>Department of Mathematical Sciences</td>
<td>Alonso, Nicomedes, III</td>
</tr>
<tr>
<td>MONTANA</td>
<td>University of Montana, Missoula</td>
<td>Department of Mathematical Sciences</td>
<td>Elias, Joran</td>
</tr>
<tr>
<td>MONTANA</td>
<td>University of Montana, Missoula</td>
<td>Department of Mathematical Sciences</td>
<td>YATES, Rebekah</td>
</tr>
<tr>
<td>NEBRASKA</td>
<td>University of Nebraska-Lincoln</td>
<td>Department of Mathematics</td>
<td>Bockelman, Brian</td>
</tr>
<tr>
<td>NEBRASKA</td>
<td>University of Nebraska-Lincoln</td>
<td>Department of Mathematics</td>
<td>Crabbe, Andrew</td>
</tr>
<tr>
<td>NEBRASKA</td>
<td>University of Nebraska-Lincoln</td>
<td>Department of Mathematics</td>
<td>Einfeld, Duane</td>
</tr>
<tr>
<td>NEBRASKA</td>
<td>University of Nebraska-Lincoln</td>
<td>Department of Mathematics</td>
<td>Ford, Pari</td>
</tr>
<tr>
<td>NEBRASKA</td>
<td>University of Nebraska-Lincoln</td>
<td>Department of Mathematics</td>
<td>Hummel (Miller), Livia</td>
</tr>
<tr>
<td>NEBRASKA</td>
<td>University of Nebraska-Lincoln</td>
<td>Department of Mathematics</td>
<td>Moore, W. Frank</td>
</tr>
<tr>
<td>NEW HAMPSHIRE</td>
<td>University of New Hampshire</td>
<td>Department of Mathematical Sciences</td>
<td>Das, Paramita</td>
</tr>
<tr>
<td>NEW HAMPSHIRE</td>
<td>University of New Hampshire</td>
<td>Department of Mathematical Sciences</td>
<td>Li, Weihua</td>
</tr>
<tr>
<td>NEW HAMPSHIRE</td>
<td>University of New Hampshire</td>
<td>Department of Mathematical Sciences</td>
<td>Paddock, Megan</td>
</tr>
<tr>
<td>NEW JERSEY</td>
<td>New Jersey Institute of Technology</td>
<td>Department of Mathematical Sciences</td>
<td>Espin, Leonardo</td>
</tr>
<tr>
<td>NEW JERSEY</td>
<td>New Jersey Institute of Technology</td>
<td>Department of Mathematical Sciences</td>
<td>Joshi, Yogesh</td>
</tr>
<tr>
<td>NEW JERSEY</td>
<td>New Jersey Institute of Technology</td>
<td>Department of Mathematical Sciences</td>
<td>Matakatu, Kamyar</td>
</tr>
<tr>
<td>Princeton University</td>
<td>Department of Mathematics</td>
<td>Brooks, Shimon</td>
<td>Entropy bounds for quantum limits</td>
</tr>
<tr>
<td>Princeton University</td>
<td>Department of Mathematics</td>
<td>Bukh, Boris</td>
<td>Extremal problems in combinatorial geometry and additive combinatorics</td>
</tr>
<tr>
<td>Princeton University</td>
<td>Department of Mathematics</td>
<td>Greene, Joshua</td>
<td>Donaldson’s theorem, Heegaard Floer homology, and results on knots</td>
</tr>
<tr>
<td>Princeton University</td>
<td>Department of Mathematics</td>
<td>Guttikar, Chattanya</td>
<td>Recovering a variety from its derived category</td>
</tr>
<tr>
<td>Princeton University</td>
<td>Department of Mathematics</td>
<td>Matchett Wood, Melanie</td>
<td>Moduli spaces for rings and ideals</td>
</tr>
<tr>
<td>Princeton University</td>
<td>Department of Mathematics</td>
<td>Michalakiorakia, Mia</td>
<td>On rational blow-downs of smooth 4-manifolds</td>
</tr>
<tr>
<td>Princeton University</td>
<td>Department of Mathematics</td>
<td>Naber, Aaron</td>
<td>Ricci solitons and collapsed spaces</td>
</tr>
<tr>
<td>Princeton University</td>
<td>Department of Mathematics</td>
<td>Pierce, Lillian</td>
<td>Discrete analogues in harmonic analysis</td>
</tr>
<tr>
<td>Princeton University</td>
<td>Department of Mathematics</td>
<td>Sarkar, Sucharit</td>
<td>Topics in Heegaard Floer homology</td>
</tr>
</tbody>
</table>
Snowden, Andrew, The Jacquet-Langlands correspondence for GL(2)
Volpato, Michael, Some counting problems in arithmetic
Wong, Willie, On the uniqueness of Kerr-Newman black holes
Young, Andrew, Complex analytic Néron models for degenerating Abelian varieties over higher dimensional parameter spaces
Yun, Zhiwei, Towards a Springer theory for global function fields

PROGRAM IN APPLIED AND COMPUTATIONAL MATHEMATICS
Eckhoff, Philip, Neuromodulation of decision making: Incorporating single-cell physiology in low-dimensional models
Gevertz, Jana, Multi-scale mathematical modeling of heterogeneous tumor growth
Ward, Rachel, Extracting information from systems with a redundant representation: Three problems in signal and image processing

Rutgers University-New Brunswick (6)

DEPARTMENT OF STATISTICS AND BIOSTATISTICS
Chen, Chuanwen, Admissible, consistent multiple testing with applications
Li, Jiuxin, Saddle point approximations
Makherjee, Somnath, A statistical test spectrum—from robust to powerful
San, Qiankun, Statistical modeling and inference for multiple temporal or spatial cluster detection
Wang, Jue, Some properties of robust statistics under asymmetric models
Zhang, Juan, Higher order conditional inference using parallels with approximate Bayesian techniques

Rutgers University-Newark (1)

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
Puri, Karan, Factorization of isometries of hyperbolic 4-space and a discreteness condition

NEW MEXICO

New Mexico State University, Las Cruces (3)

DEPARTMENT OF MATHEMATICAL SCIENCES
Eydenberg, Michael, Kernel methods on $\Omega$-ultradistributions
Noussi, Hubert, Stabilization of competition models in the chemostat via feedback linearization
Tran, Hien, Statistics of course data with applications to financial risk analysis

University of New Mexico (13)

DEPARTMENT OF MATHEMATICS AND STATISTICS
Collins, Dave, Nonparametric solution to flowgraph models
Cuadros, Jaime, On null Sasakian structures
Deshler, Jessica, Predicting student success in undergraduate mathematics courses
Grundini, Daniele, Quaternionic Kähler reductions of Wolf spaces
Guba, Okysana, The spectra of ordinary differential equations
Meng, Chen, Phylogenetic models for hybridization in a coalescent framework
Panek, Danusz, On sharp extrapolation theorems
Peterson, Kara, Modeling artic sea ice using the material-point method and an elastic-decohesive rheology
Ringler, Adam, Positivity problems in higher dimensional geometry
Srinivasan, Gowri, Uncertainty quantification in stochastic processes
Villaen-Hernandez, Alejandro, Monte Carlo strategies for calibration in climate models
Ying, Wenxia, Bayesian modeling of nonstationary extremal events
Zhu, Min, Functional transformed models: Frequentist on Bayesian approaches

NEW YORK

Clarkson University (2)

DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
Tang, Yuefeng, Interpolation algorithms in program verification
Zhou, Guohua, Fourier analysis and truncation error estimates of multigrid methods and conservative Jacobians on hexagonal grids

Columbia University (18)

DEPARTMENT OF MATHEMATICS
Cohen, Dvora, The existence of asymptotically parallel spinors on manifolds asymptotic to hypersurfaces in Minkowski space
DeLand, Matthew, Geometry of rational curves on algebraic varieties
Goia, Irina, Bessel and volatility-stabilized processes
Li, Zhi, On $\Lambda$-adic Saito-Kurokawa lifting and its application
Maharaj, Yogishwar, Line bundles, curves, and quasi-projective surfaces
Mapes, Sonja, Finite atomic lattices and their relationship to resolutions of monomial ideals
McFeron, Donovan, Remarks on some non-linear heat flows in Kahler geometry
Mosincu, Natalia, Probability on graphs and groups: Theory and applications
Pedersen, Helge, Splice diagrams, singularity links and universal Abelian covers
Ross, Joseph, The Hilbert-Chow morphism and the incidence divisor
Simring, Eric, Representations and combinatorics of Ocneanu’s essential paths
Welij, Shaffiq, On the Heegaard Floer homology of singular knots and their unoriented resolutions
Zhang, Wei, Modularity of generating functions of special cycles on Shimura varieties
Zhou, Yun, A stochastic representation problem with applications in optimal singular control, Dynkin games and obstacle problems

DEPARTMENT OF STATISTICS
Ichiba, Tomoyuki, Topics in multi-dimensional diffusion theory: Attainability, reflection, ergodicity and rankings
Jin, Man, Variable selection in canonical discriminant analysis
Paik, Jane, Contributions to the analysis of survival data subject nonstandard sampling schemes
Pospisil, Libor, Risk measures and maximum drawdown

Cornell University (12)

CENTER FOR APPLIED MATHEMATICS
Diener, Nicolas, Mathematical models for swing options and subprime mortgage derivatives
Huerta-Sanchez, Emilia, An analysis of sequence polymorphism under alternative population genetic models
Owen, Megan, Distance computation in the space of phylogenetic trees
Restrepo, Mateo, Computational methods for static allocation and real-time redeployment of ambulances
Verdugo, Angel, Dynamics of gene networks with time delays

DEPARTMENT OF MATHEMATICS
Adams, Bryant, Representing processes as update automata and transducers
Hawertz, Noam, Free resolutions of monomial ideals
Koch, Sarah, A new link between Teichmüller theory and complex dynamics
Mattucci, Francesco, Algorithms and classification in groups of piecewise-linear homeomorphisms
Minnes, Mor Mia, Computability and complexity properties of automatic structures and their applications
Murgescu, Radu, On the $p$-class groups of the pure number field $\mathbb{Q}(N^{1/p})$ and its Galois closure $\mathbb{Q}(N^{1/p}, \zeta_p)$
Workman, John, End-point estimates and multi-parameter paraproducts on higher dimensional tori
Graduate Center, City University of New York (7)

PH.D. PROGRAM IN MATHEMATICS

Dean, Margaret, FINITELY GENERATED METABELIAN AND SOLVABLE GROUPS

Garbin, Daniel, ASYMPOTICS FOR THE PARABOLIC, HYPERBOLIC, AND ELLIPTIC EISENSTEIN SERIES THROUGH HYPERBOLIC AND ELLIPTIC DEGENERATION

Kim, Youngju, RIGIDITY AND STABILITY FOR ISOMETRY GROUPS IN HYPERBOLIC 4-SPACE

Kojaran, Hana, ON CRITICAL POINTS FOR GAUSSIAN VECTORS WITH INFINITELY_DIVISIBLE SQUARES

Lovell, Jonathan, TOTAL VARIATION OF GAUSSIAN PROCESSES AND LOCAL TIMES OF ASSOCIATED LÉVY PROCESSES

Orosz, Brooke, PROBLEMS IN ADDITIVE NUMBER THEORY

Pepper, Michael, ENDOMORPHISMS OF N-DIMENSIONAL PROJECTIVE SPACE OVER FUNCTION FIELDS

New York University, Courant Institute (17)

COURANT INSTITUTE OF MATHEMATICAL SCIENCES

Albuts, Thomas, DIMENSION AND MEASURE OF SLE ON THE BOUNDARY

Bae, Hantaek, ON THE NAVIER-STOKES EQUATIONS

Bringley, Thomas, ANALYSIS OF THE IMMERSED BOUNDARY METHOD FOR STOKES FLOW

Freire, Juliana, ANALYSIS TECHNIQUES FOR NUDGED RANDOM PROCESSES

Goldberg, Daniel, NUMERICAL K-THEORETICAL TREATMENT OF GROUNDING LINE MOVEMENT AND ICE SHELF BUTTRESSING IN MARINE ICE SHEETS

Hand, Paul, HOMOGENIZATION IN CARDIAC ELECTROPHYSIOLOGY AND BLOW-UP IN BACTERIAL CHEMOTAXIS

Kerzherner, Yakov, EISENSTEIN SERIES, UNIQUENESS PRINCIPLE, AND LIMITS OF HOMOGENIZE

Le, Nam Quang, ANALYSIS OF SEVERAL SHARP-INTERFACE LIMITS IN VARIATIONAL PROBLEMS

Lee, Darren, TWO CONSTRUCTIONS OF COMPLETE MINIMAL SURFACES

Mintchev, Stanislav, SELF-ORGANIZATION PHENOMENA IN NETWORKS OF PULSE-COUPLED PHASE OSCILLATORS

Momin, Al Saeed, INTERSECTON THEORY IN 3-DIMENSIONAL LINEARIZED CONTACT HOMOLOGY

Render, Paris, HARMONIC FUNCTIONS FOR RANDOM WALKS IN RANDOM ENVIRONMENTS

Rodriguez, Ignacio, A MATHEMATICAL MODEL OF TELOMERE LENGTH REGULATION AND CELLULAR SENESCENCE

Schutzer, Emmanuel, MARKING THE (1, 2) POINTS OF THE BROWNIAN WEB AND APPLICATIONS

Tulloch, Ross, GEOSTROPHIC DYNAMICS AT SURFACES IN THE ATMOSPHERE AND OCEAN

Yarmola, Tatiana, INVARIANT MEASURES FOR DEGENERATE RANDOM PERTURBATIONS OF DISCRETE-TIME DYNAMICAL SYSTEMS

Yilmaz, Atilla, LARGE DEVIATIONS FOR RANDOM WALK IN A RANDOM ENVIRONMENT

Yin, Yong, STATIC BORN-INFELD THEORY

Rensselaer Polytechnic Institute (9)

DEPARTMENT OF MATHEMATICAL SCIENCES

Cristobal La Torre, Juan, EFFECTIVE TRANSPLANT PROPERTIES OF STOCHASTICS BIOPHYSICAL MODELS

Ding, Xuexin, STATISTICAL EQUILIBRIA OF THE COUPLED BAROTROPIC FLOW AND SHALLOW WATER FLOW ON A ROTATING SPHERE

Fang, Yi, Imaging from sparse measurements

Frenkel, Yevgeniy, PROPAGATION OF ULTRA-SHORT ELECTROMAGNETIC PULSES IN A MAXWELL-DUFFING MEDIUM

Lin, Kui, ERROR ESTIMATION FOR THE DIRECT INVERSION MODEL AND NUMERICAL SCHEMES FOR THE FULL INVERSION MODEL IN ELASTOGRAPHY

Morales, Jose, SYNTHETIC-APERTURE RADAR IMAGING AND WAVEFORM DESIGN FOR DISPERSIVE MEDIA

Rasmussen, Kathryn, OPTICAL PULSE DYNAMICS IN ACTIVE MEDIA WITH NEGATIVE REFRACTIVE INDEX

Saintval, Wendy, INFLUENCE OF MODAL ATTENUATION ON SHALLOW WATER PROPAGATION

Wang, Chaoxiong, POWER CONTROL FOR MULTIUSER COMMUNICATION SYSTEMS AND COMPUTATION OF GENERALIZED NASH EQUILIBRIA

State University of New York at Buffalo (5)

DEPARTMENT OF BIOSTATISTICS

Cai, Xueya, Generalized linear model approach for estimating and testing equality of control correlations

Miller, Austin, Statistical methods for biodosimetry in the presence of both Berkson and classical measurement error

DEPARTMENT OF MATHEMATICS

Buettgens, Matthew, Property Tao and von Neumann algebras

Ivanov, Miodrag C., THE REPRESENTATION THEORY OF PROFINITE ALGEBRAS—INTERACTIONS WITH CATEGORY THEORY, ALGEBRAIC TOPOLOGY AND COMPACT GROUPS

Li, Bo, REGULARITY ESTIMATES FOR BEREZIN'S OPERATOR CALCULUS

State University of New York at Binghamton (4)

DEPARTMENT OF MATHEMATICS AND SCIENCE

Junes, Leandro, DUALITY OF HIGHER ORDER NON-EUCLIDEAN PROPERTY FOR ORIENTED MATROIDS

Pallekonda, Seshendra, BOUNDED CATEGORY OF AN EXACT CATEGORY

Sullivan, Margaret, DNA COMPUTING WITH CUTTING, PASSING, FILTERING AND WASHING

Tuncer, Nigar, GLOBALIZATION THEOREMS IN TOPOLOGY

State University of New York at Stony Brook (15)

DEPARTMENT OF APPLIED MATHEMATICS AND STATISTICS

Chang, Su-Wei, GROWTH MIXTURE MODELING AS AN EXPLORATORY ANALYSIS TOOL TO IDENTIFY LONGITUDINAL QUANTITATIVE TRAIT LOCUS IN GENOME-WIDE ASSOCIATION STUDIES

Chu, Adrienne, CLASSIFICATION OF GASTROINTESTINAL BLEEDING DATA

Kifle, Hagos, LATTICE BOLTZMANN SIMULATION OF IMMISCIBLE TWO-PHASE FLOW IN THREE DIMENSIONAL POROUS MEDIA

Kim, Daesang, NETWORK FLOW MODEL FOR THE SIMULATION OF CO2 SEQUESTRATION PROCESS

Kim, Dongyoung, CONSERVATIVE DATA COLLECTIONS AND COMPARISON STUDY FOR FRONT-TRACKING

Lee, Jungyeon, EXTENDING CONMINGLING ANALYSIS TO EXPOSURAL DISTRIBUTIONS AND ITS GENETIC APPLICATIONS

Yongzhi, Chen, TASK MAPPING ON SUPER-COMPUTERS WITH CELLULAR NETWORKS

DEPARTMENT OF MATHEMATICS

Chandramouli, Vasu V. M. Sarma, Renormalization and non-rigidity

Clarke, Andrew, RIGIDITY OF MINIMAL SUBMANIFOLDS

Hao, Ning, D-BAR SPARK THEORY AND DELIGNE COHOMOLOGY

Hazard, Peter, HÉNON-LIKE MAPS RENORMALIZATION

Ostrovy, Stanislav, WEIGHTED L2 INTERPOLATION ON NON-UNIFORMLY SEPARATED SEQUENCES

Savelyev, Yakov, QUANTUM CHARACTERISTIC CLASSES

Thind, Jaimal, CATEGORICAL COMBINATORIAL CONSTRUCTIONS OF A, D, E ROOT SYSTEMS

Xu, Dezhen, CONFIGURATIONS OF GRAPHS AND THE MASTER EQUATION

Syracuse University (5)

DEPARTMENT OF MATHEMATICS

Graves, Christina, ON THE STRUCTURE OF RELIABILITY POLYNOMIALS

Graves, Stephen, GROWTH OF TESSELLATIONS

Lu, Yao, FAST MULTISCALE INTEGRAL EQUATION METHODS FOR IMAGE RESTORATION

McCaffrey, Adam, STRUCTURE OF INFINITELY-ENDED, EDGE-TRANSITIVE PLANAR MAPS AND THEIR PETRE WALKS

Zhang, Haizhang, SAMPLING WITH REPRODUCING KERNELS
The University of Albany, SUNY (3)

Department of Mathematics and Statistics

Clark, Timothy, Poset resolutions of monomial ideals
Savopouloou, Anna, Almost everywhere convergence of convolution measures
Wedrychowicz, Christopher, Almost everywhere convergence of weighted averages

University of Rochester (7)

Department of Biostatistics and Computational Biology

Chen, Linlin, The correlation structure of microarray data and its statistical implications
Ma, Yan, Inference for instrument reliability and rater agreement within a multi-rater and longitudinal data setting
Pearson, Alexander, Subset selection for high-dimensional data, with applications to gene array data

Department of Mathematics

Huang, Haijun, Heat equation with noise restricted to fractal set
Li, Huibin, Invariant measure of Burger’s system of equations
Palsu-Andriescu, Emanuel, Fourier integral operators on Colombeau spaces
Salch, Andrew, Moduli and cohomology

North Carolina State University (40)

Department of Mathematics

Andjelkovic, Ivan, Auxiliary signal design for fault detection for nonlinear systems: Direct approach
Benedict, Brandy, Axisymmetric poroleastic boundary element methods for biphasic mechanics of articular cartilage
Davis, Jimena, Uncertainty quantification in the estimation of probability distributions on parameters in size-structured population models
Dickson, Kelly, Condition estimates for numerical continuation with applications to atomic and molecular fluids
Ellwein, Laura, Cardiovascular and respiratory regulation, modeling and parameter estimation
Ernstberger, Jon, High speed parameter estimation for a homogenized energy model
Ernstberger, Stacey, Sensitivity methods for dynamical systems
Govan, Angela, Ranking theory with application to popular sports
Janovitz-Freireich, Itnuit, Computation of the exact and approximate radicals of ideals: Techniques based on matrices of traces, moment matrices and Bezoutians
Jiang, Qunlei, Analysis and computation for a fluid mixture model of tissue deformations
Kalhorn, Rebecca, On the solvability of nonlinear boundary value problems on time scales
Kim, Eun Jung, Computational biomechanical models for the pericellular matrix of articular cartilage
Levy, Louis, Multipliers for the lower central series of strictly upper triangular matrices
Okay, Irfan, The additional dynamics of the least squares completions of linear differential algebraic equations
Olson, Sarah, Mathematical models for analysis of tissue regeneration in articular cartilage
Pasteur, Roger, A multiple inhibin model of the human menstrual cycle
Pope, Scott, Parameter identification in lumped compartment cardiorespiratory models
Porter, Gloria, Multiobjective optimal control methodology for the analysis of certain sociodynamic problems
Qasimov, Heydar, Lacunae based stabilization of PMLs
Samuels, John, Inverse problems and post analysis techniques for a stenosis-driven acoustic wave propagation model
Sele, Teresa, Stochastic matrices: Ergodicity coefficients and applications to ranking
Sköld, Ryan, Model development for shape memory polymers

University of North Carolina at Chapel Hill (19)

Department of Biostatistics

Abraham, Anita, Model selection methods for the linear mixed model for longitudinal data
Andraca Carrera, Eugenio, Approaches to parameter and variance estimation in generalized linear models
Chung, Yeonseung, Nonparametric Bayesian inferences on predictor-dependent response distributions
Clement, Meagan, Analysis techniques for diffusion tensor imaging data
Lu, Tsui-Shan, Statistical inferences for outcome dependent sampling with continuous multivariate outcomes
Park, Ju-Hyun, Bayesian density regression and predictor-dependent clustering
Saville, Benjamin, Bayesian multilevel models and medical applications
Shi, Xiaoyan, Diagnostic measures for missing covariate data and semiparametric models for neuroimaging
Simpson, Sean, Linear models with generalized AR(1) covariance structure for longitudinal and spatial data

DEPARTMENT OF MATHEMATICS
Bouzarth, Elizabeth, Regularized singularities and spectral deferred correction methods: A mathematical study of numerically modeling Stokes fluid flow
Dodson, Benjamin, Caustics and the indefinite signature Schrödinger equation, linear and nonlinear
Newport, Jason, Solutions to the nonlinear Schrödinger equation with Dirac mass initial data
Olli, Jeanette, Tiling systems, division point measures, and endomorphisms

DEPARTMENT OF STATISTICS AND OPERATION RESEARCH
Lim, Changwon, Robust statistical theory and methodology for nonlinear models with applications to toxicology
Samarov, Daniel, Analysis and advanced extensions of canonical correlation analysis
Sen, Suman, Classification on manifolds
Wang, Fangfang, Statistical analysis of some financial time series
Wang, Zhaohui, Capacity investment strategies under operational flexibility
Yoon, Jungeun, Contributions to stochastic volatility models and option pricing

University of North Carolina at Charlotte (4)

DEPARTMENT OF MATHEMATICS AND STATISTICS
Huang, Hongwei, Statistical properties and applications of Turing’s formula
Lee, Jimin, Some statistical methods for failure time data
Li, Jing, Risk minimizing portfolio optimization and hedging with conditional value-at-risk
Wang, Xian, Selection of mixed copulas and finite mixture models with applications in finance

NORTH DAKOTA
North Dakota State University, Fargo (2)

DEPARTMENT OF MATHEMATICS
Lambert, Joshua, Determining the bifilar crossing number of $C_k \times C_l \times C_m \times C_n$
Pile, Angela, The space of regular polygons with a small number of edges

OHIO
Bowling Green State University (3)
DEPARTMENT OF MATHEMATICS AND STATISTICS
Attanayake, Champike, Finite elements and practical error—analysis of Huxley and EFK equations
Dong, Fanglong, Bayesian model checking in multivariate discrete regression problems
Rashid, Mamunur, Inference on logistic regression models

Case Western Reserve University (15)
DEPARTMENT OF EPIDEMIOLOGY AND BIOSTATISTICS
Chang, Shelley, Predicting methicillin-resistant S. aureus carriage and dissemination in a veterans affairs medical center
Conroy, Britt, Racial differences in surgical procedure utilization and cardiac procedure mortality among American Indian and Alaska Native (AI/AN), Black, and White Medicare beneficiaries 65 years of age and older in 15 states, 1997
Hawthorn, Rachael, Finding a role: Health care professional’s perspectives on and responses to role uncertainty in end-of-life care planning
Heaphy, Emily, Evaluation of HIV-risk behaviors of Puerto Rican women with severe mental illness in Cuyahoga County, Ohio
Hixson, Eric, Ambulatory heart failure treatment: Process and outcomes effects of practice and patient adherence
Lu, Qing, Designing and forming predictive genetic test using optimal ROC curve
Onwuzulike, Kaine, The genetics and definition of salt-sensitivity hypertension in African Americans
Ou, Ju-Chi, Evaluation of exposure/treatment effect via spatial propensity score in observation studies
Parrado, Tony, Improved individual ancestry estimates for proper adjustments of ancestral confounding in association analysis
Wang, Yang, Gene discovery for age-related macular degeneration
Won, Sungho, Improving genetic analysis of case-control studies
Zullo, Melissa, Cardiovascular disease management and functional capacity improvement in patients with metabolic syndrome

Ohio State University, Columbus (32)

DEPARTMENT OF MATHEMATICS
Arms, Scott, Minimal heights in number fields
Ault, Shaan, On the symmetric homology of algebras
Griesmer, John, Ergodic averages, correlation sequences, and sunsets
Kar, Aditi, Discrete groups and CAT(0) asymptotic cones
Khouri, Jr., Michael, Multiplicity-one results and explicit formulas for quasi-split p-adic unitary groups
Li, Lingfei, Functoriality of harmonic foliations
Liu, Sheng-Chi, Mass equidistribution of Hecke eigenforms on the Hilbert modular varieties
McSweeney, John, Time to coalescence for a class of nonuniform allocation models
Niu, Liang, The vertex primitive and vertex bi-primitive s-arc regular graphs
Pikula, Rafal, Enveloping semigroups of affine skew products and Sturmian-like systems
Schnell, Christian, The boundary behavior of cohomology classes and singularities of normal function
Shi, Ronggang, Equidistribution of expanding measures with local maximal dimension and Diophantine approximation
Swartz, Eric, 2-arc transitive polygonal graphs of large girth and valency

DEPARTMENT OF MATHEMATICS
Kuceyeski, Amy, Efficient computational and statistical models of hepatic metabolism
Ye, Deping, Topics in convex geometry and phenomena in high dimension

DEPARTMENT OF STATISTICS
Liu, Peng, Adaptive mixture estimation and subsampling PCA

Kent State University (6)
DEPARTMENT OF MATHEMATICAL SCIENCES
Dekock, Mienie, Absolute continuity and on the range of a vector measure
Dugan, Carrie, Solvable groups whose character degree graphs have diameter three
Fenta, Aderaw, Lucunary power sequences and extremal vectors
Ignatyev, Oleksiy, The compact support property for hyperbolic SPDE: Two contrasting equations
Smith, Tabrina, Orbits and ranges of operators on Banach spaces
Wakefield, Thomas, Verifying Huppert’s Conjecture for the nonabelian simple groups of rank two

DEPARTMENT OF STATISTICS
Onwuzulike, Kaine, Designing and forming predictors and predictor-dependent clusterings of continuous multivariate outcomes
Won, Sungho, Gene discovery for age-related macular degeneration

Doctoral Degrees Conferred

FEBRUARY 2010
NOTICES OF THE AMS
Doctoral Degrees Conferred

Xiong, Wei, Brownian motion of a particle immersed in a viscous, incompressible, thermally fluctuating solvent
Yang, Keyan, On orbit equivalent permutation groups
Yeum, Ji-A, Probability of solvability of random systems of 2-linear equations over GF(2)
Young, Justin, The twisted tensor L-function of GSp(4)
Zhao, Peng, Quantum variance of Maass-Hecke cusp forms

DEPARTMENT OF STATISTICS
Bautista, Dianne Carrol, A sequential design for approximating the Pareto front using the expected Pareto improvement function
Ding, Jie, Monte Carlo pedigree disequilibrium test with missing data and population structure
Han, Gang, Modeling the output from computer experiments having quantitative and qualitative input variables and its applications
Kohlschmidt, Jessica, Ranked set sampling: A look at allocation issues and missing data complications
Kumar, Arun, Sequential calibration of computer models
Li, Dongmei, Resampling-based multiple testing with applications to microarray data analysis
Markiewicz, Shannon, Nonparametric inference using order restricted randomized designs
Paul, Rajib, Theoretical and algorithmic developments in Markov chain Monte Carlo
Roy, Soma, Sequential-adaptive design of computer experiments for the estimation of percentiles
Sroka, Christopher, Extending ranked set sampling to survey methodology
Walters, Kimberly, The use of post-intervention data from waitlist controls to improve estimation of treatment effect in longitudinal randomized controlled trials
Wang, Zhen, Semi-parametric Bayesian models extending weighted least squares
Yao, Yonggang, Statistical applications of linear programming for feature selection via regularization methods
Zhang, Xiuyun, Efficient algorithms for fitting Bayesian mixture models

Ohio University, Athens (2)

DEPARTMENT OF MATHEMATICS
Blackwood, Brian, A study of partial orders on nonnegative matrices and von Neumann regular rings
Stover, Derrick, Continuous mappings and some new classes of spaces

University of Cincinnati (5)

DEPARTMENT OF MATHEMATICAL SCIENCES
Baena, John, Fast signature schemes over odd characteristic
Clough, Crystal, Square: A new family of multivariate encryption schemes
Jin, Yan, Bayesian solution to the analysis of data with values below the limit of detection (LOD)
Sauer, John, Theoretical and algorithmic developments in Markov chain Monte Carlo
Young, Justin, The twisted tensor L-function of GSp(4)
Zhao, Peng, Quantum variance of Maass-Hecke cusp forms

University of Cincinnati, Medical College (3)

DEPARTMENT OF ENVIRONMENTAL HEALTH
Liu, Xiaolei, Sample size calculations in matched case-control studies and unmatched case-control studies with controls contaminated
Xi, Hufeng, Allele frequency distribution and its implication in association study
Yu, Tao, A self-building system for ranked set sampling

University of Toledo (2)

DEPARTMENT OF MATHEMATICS
Liu, Nan, Theory and applications of Legendre polynomials and wavelets
Yousef, Abdel, Two problems in the theory of Toeplitz operators on the Bergman space

OKLAHOMA

Oklahoma State University (4)

DEPARTMENT OF MATHEMATICS
Dollarhide, John, On the L-function of multiplicative character sums
Fisher, Brian, Students' conceptualizations of multivariable limits
Mukherjee, Antara, Isoperimetric inequalities using Varopoulos transport
Srinivasan, Varadarajan, On certain towers of extensions by antiderivatives
Tonic, Vera, Bockstein basis and resolution theorems in extension theory

University of Oklahoma (7)

DEPARTMENT OF MATHEMATICS
Cho, Sangbum, A finite presentation of the Goeritz group
Frey, Timothy, Graph reflection groups
Martin, Jason, Expert conceptualizations of the convergence of Taylor series, yesterday, today and tomorrow
Mecham, Talal, Largeness of graphs of Abelian groups
Mukherjee, Antara, Isoperimetric inequalities using Varopoulos transport
Srinivasan, Varadarajan, On certain towers of extensions by antiderivatives
Tonic, Vera, Bockstein basis and resolution theorems in extension theory

Oregon State University (6)

DEPARTMENT OF MATHEMATICS
Cook, Sam, Killing spinors and affine symmetry tensors in Godel's universe
Pierce, Larry, Computing entropy for Z^2-actions
Schmurr, Jason, Triangular billiards surfaces and translation covers
Ulman, Shari, The cohomology rings of seven dimensional primitive cohomogeneity one manifolds
Wills, Dean, Connections between combinatorics of permutations and algorithms and geometry

DEPARTMENT OF STATISTICS
Taylor, Aimee, Statistical enhancement of support vector machines

Portland State University (4)

DEPARTMENT OF MATHEMATICS AND STATISTICS
Ahmed, Aden Omar, Quaternions, octonions, and quantizations of games
Godinez Vazquez, Humberto, Data assimilation, adaptive observations and applications
Khan, Faisal Shah, Quantum multiplexers, Parrando games, and proper quantization
Swinyard, Craig, Students' reasoning about the concept of limit in the context of reinventing the formal definition

University of Oregon (7)

DEPARTMENT OF MATHEMATICS
Brown, Jonathan Scott, Finite w-algebras of classical type
Collins, John Patrick, Gluing Bridgeland stability conditions and Z_2-equivariant sheaves on curves
Leeman, Aaron Christopher, Stabilization of chromatic functors
Phan, Christopher Lee, Koszul and generalized Koszul properties for noncommutative graded algebras
Vanderpool, Ruth, Non-existence of a stable homotopy category for P-complete abelian groups
Wade, Jeremy, Summability of Fourier orthogonal expansions and a discretized Fourier orthogonal expansion involving radon projections for functions on the cylinder
Walsh, Mark G., Metrics of positive scalar curvature and generalised Morse functions

Pennsylvania

Carnegie Mellon University (14)

DEPARTMENT OF MATHEMATICAL SCIENCES
Arama, Danut, On a variational approach for Stokes conjectures in water waves
Brunick, Gerard, A weak existence result with applications to the financial engineer’s calibration problem

Chebolu, Prasad, Topics in random graphs

Melsted, Pall, Algorithms on random graphs

Obermeyer, Fritz, Automated equational reasoning in nondeterministic λ-calculi modulo theorems H^*

Offner, David, Extremal problems on the hypercube

Picolletti, Michael, Extremal problems and random processes on graphs

Rand, Alexander, Delannay refinement algorithms for numerical methods

Sousa, Bernardo, Variational methods for phase transitions

Wimmer, Karl, Fourier methods and combinatorics in learning theory

Department of Statistics

Bertolet, Marianne, To weight or not to weight: Incorporating sampling designs into model-based analyses

Damouras, Sotirios, Nonparametric time series analysis using Gaussian processes

Luca, Diana, Genetic matching by ancestry in genome-wide association studies

Xie, Libo, Long-memory stochastic volatility models: A new method for inference and applications in options pricing

Lehigh University (2)

Department of Mathematics

Mohler, Joel, Residues of Weyl group multiple Dirichlet series

Qian, Huiyu, On data-driven chi square statistics

Pennsylvania State University (21)

Department of Mathematics

Endres, Erik, Shocks and wave interactions for compressible flow

Ethier, John, Strong forms of orthogonality for sets of hypercubes

George, Chris, The Mackey analogy for SL(n,R)

Kursungoz, Kagan, Parity considerations in Andrews-Gordon identities and the k-marked Durfee symbols

Li, Hengguang, Elliptic equations with singularities: A-priori analysis and numerical approaches

Otgonbayar, Uuye, The local index theorem in noncommutative geometry

Pan, Jia, Optimal intermediated investment in a liquidity-driven business cycle

Weaver, Bryce, Growth rate of periodic orbits for geodesic flows on surfaces with regions of positive curvature

Xue, Guangri, Mathematical modeling and computation of multiphysics fuel cells

Zhang, Lei, Phase field model for the nucleation in solid state phase transformations: Theories, algorithms and applications

Zheng, Bin, Finite element approximations of high order partial differential equations

Zhu, Yunrong, Robust preconditioners for H(div), (curl) and H(curl) systems with strongly discontinuous coefficients

Department of Statistics

Benaglia, Tatiana, Nonparametric estimation in multivariate finite mixture models

Dong, Yuxiao, Dimension reduction for non-illiptically distributed predictors

Gaugler, Trent, Nonparametric estimation in multivariate finite mixture models

Kim, Daeyoung, Mixture inference at the edge of identifiability

Recta, Virginia, A model-based analysis of semicontinuous spatial data

Tyukucheva, Svitlana, Augmenting the bootstrap to analyze high-dimensional genomic data

Wang, Yu, Nonlinear dimension reduction in feature space

Zhang, Wei, A general class of agreement-coefficient methods for categorical and continuous responses

Zhu, Junjia, Essays on monitoring profile data

Temple University (2)

Department of Mathematics

Du, Xiaohong, Additive Schwarz preconditioned GMRES, inexact Krylov subspace methods and applications of inexact CG

Wang, Linhong, Noetherian skew power series rings

University of Pennsylvania (18)

Department of Mathematics

Auel, Asher N., Cohomological invariants of line bundle-valued symmetric bilinear forms

Bressler, Andrew E., Quantum random walks on the integer lattice via generating functions

Favero, David, A study of the geometry of the derived category

Herreros, Pilar, Rigidity of magnetic and geodesic flows

Holschbach, Armin, A Chebotarev-like density theorem in algebraic geometry

Jankowski, Christopher, On type II_0 E_8 semigroups induced by q-pure maps on M_n(C)

Obus, Andrew, Ramification of primes in fields of moduli of three-point covers

Rowe, Paul, Policy compliance, confidentiality and complexity in collaborative systems

Shonkwiler, Clayton, Poincaré duality angles on Riemannian manifolds with boundary

Tsay, Joe-Kai, Formal analysis of the Kerberos authentication protocol

Vela-Vick, David Shea, Applications of Oszvath-Szabo invariants to contact geometry

Yoo, Meeuse, Combinatorial formulas connected to diagonal harmonics and Macdonald polynomials

Zhu, Tong, Nonlinear Polya urn models and self-organizing processes

Zong, Ying, On hyper-symmetric abelian varieties

Department of Statistics

Gupta, Abhishek, Statistical methods for high-dimensional data analysis

Levin, David, Causal inference methods for randomized controlled trials with noncompliance

Nagaraja, Chaitra, Statistical methods for modeling house prices and indices

Wang, Lie, Variance function estimation in nonparametric regression

University of Pittsburgh (22)

Department of Biostatistics

Blakesley-Ball, Richard, Parametric control familywise error rates with dependent P-values

Cheng, Chunrong, Enhanced inter-study prediction and biomarker detection in microarray with application to cancer studies

Haile, Sarah, Inference on competing risks in breast cancer data

Jakobsdottir, Johanna, Genetics of age-related maculopathy and score statistics for X-linked quantitative trait loci

Li, Jia, Statistical issues in meta-analysis for identifying signature genes in the integration of multiple genomic studies

Mi, Zhibao, Robust cross-platform disease prediction using gene expression microarrays

Oh, Sunghee, Effects of missing value imputation on down-stream analyses in microarray data

Scott, John, Spatio-temporal mixed models for diffusion tensor magnetic resonance imaging

Tudorascu, Dana, Partial least squares on data with missing covariates: A comparison approach

Zhu, Fang, An index of local sensitivity to nonignorability and a penalized pseudolikelihood method for data with nonignorable nonresponse

Department of Mathematics

Bahwe, Chetan, Geometric motivic integration on Artin n-stacks

Berry, Robert, Lipschitz estimates for geodesics in the Heisenberg group
Doctoral Degrees Conferred

Labovshchii, Alexandr, Mathematical architecture for models of fluid flow phenomena
Manica, Evandro, Effects of coupling and heterogeneity in the pre-Bötzinger complex cells using first return maps
Poerio, Thomas, Topological algebraic structure in the density topology and on Sounis lines
Reynolds, Angela, Mathematical models of acute inflammation and a full lung model of gas exchange under inflammatory stress
Stanculescu, Iuliana, Turbulence modeling as an ill-posed problem

DEPARTMENT OF STATISTICS
Ghebregiorgis, Ghideon, Modeling and analyzing multivariate longitudinal left-censored biomarker data
Ha, Wonho, Applications of the reflected Ornstein-Uhlenbeck process
Han, Sangdae, Comparing spectral densities in replicated time series by smoothing spline ANOVA
Nickleach, Scott, Numerical algorithms for stock option valuation
Wang, Jie, Bathtub failure rates in reliability and dependence in multiple testing

RHODE ISLAND
Brown University (11)

CENTER FOR STATISTICAL SCIENCE
Dunsiger, Shira, Analysis of longitudinal binary data from behavioral medicine
Li, Hong, Statistical methods for monitoring disease recurrence

DIVISION OF APPLIED MATHEMATICS
Carvalho, Luis, Bayesian centroid estimation
Grinberg, Leopold, Topics in ultrascale scientific computing with application in biomedical modeling
Luo, Xian, A spectral element/smoothed profile method for complex-geometry flows

Narayan, Aki, A generalization of the Wiener rational basis functions on infinite intervals
Papamichael, Giorgio, Event-by-event Monte Carlo simulation of radiation transport in vapor and liquid water
Pollard, Thomas, Comprehensive 3-d change detection
Schiemenz, Alan, Advances in the discontinuous Galerkin method: Hybrid schemes and applications to the reactive infiltration instability in an up-welling compacting mantle
Srinivasan, Ravi, Closure and complete integrability in Burgers turbulence
Zhang, Wei, Statistical inference and probabilistic modeling in compositional vision

University of Rhode Island (2)

DEPARTMENT OF MATHEMATICS
Basu, Sukanya, Global behavior of solutions to a class of second-order rational difference equations
Kostrov, Yevgeniy, Global behavior in rational difference equations

SOUTH CAROLINA
University of South Carolina (12)

DEPARTMENT OF EPIDEMIOLOGY AND BIOSTATISTICS
Ogbuanu, Ikachukwu, Do prenatal immunological exposures modify the effect of Interleukin-13 (IL13) gene on childhood atopy?
Parker, David, HIV risk behaviors and knowledge: Associations with HIV prevalence in Estonia
Rasathurai, Samithran, Bayesian identification methods
Wagner, Sara, Spatial assessment of groundwater uranium and cancer in South Carolina
Zhen, Huiling, Multiple imputation of missing data based on prediction of conditional quantiles
Zhou, Li, Quantile regression with ordinal and discrete data

DEPARTMENT OF MATHEMATICS
Sircar, Sarthok, Dynamics and rheology of biaxial liquid crystal polymers
Chen, Peng, Topics in binary regression models with group testing data
McLain, Alex, Accelerated testing with recurrent events and fundamental issues in marginal models
Pritchard, Nick, Geometric group testing
Taylor, Laura, Competing risks in a recurrent event setting
Wu, Meng, Based hypothesis test for unidimensionality

TENNESSEE
University of Memphis (5)

DEPARTMENT OF MATHEMATICAL SCIENCES
Clarke, Teddy J., Asymptotic analysis of the abstract telegraph equation
Garland, Sarah Elizabeth, Factorial crossover designs with randomization restrictions and balanced cross-over design from terraces
Naheed, Naima, Mathematical contributions to spin-polarized Thomas-Fermi theory
Parrish, Anca, On the geometric structure of Lorentz and Marcinkiewicz spaces

Parrish, Andrew, Pointwise convergence of ergodic averages in Orlicz spaces

University of Tennessee, Knoxville (4)

DEPARTMENT OF MATHEMATICS
Clayton, Timothy, Optimal control of epidemic models involving rabies and West Nile viruses
Khader, Masa, The dissipative nonlinear wave equations with space-time dependent potentials
Labuz, Brendon, Generalized uniform covering maps characterized as inverse limits of uniform covering maps
Neilan, Michael, Numerical methods for fully nonlinear second order partial differential equations

Vanderbilt University (2)

DEPARTMENT OF MATHEMATICS
Acosta Reyes, Ernesto, Optimal non-linear signal models and stability of sampling-reconstruction
Kozakova, Iva, Percolation and Ising model on tree-like graphs

TEXAS
Baylor University (7)

DEPARTMENT OF STATISTICAL SCIENCES
Crixell, Jo Anna, Logistic regression with covariate measurement error in an adaptive design: A Bayesian approach
Greer, Brandy, Bayesian and likelihood interval estimation for comparing two Poisson rate parameters using under-reported data
Hetzer, Joel, Statistical considerations for the analysis of multivariate Phase II testing
Miyamoto, Kazutoshi, Bayesian and maximum likelihood methods for some two-segment generalized linear models
Powers, Stephanie, Bayesian approaches to inference and variable selection for misclassified or under-reported response models
Wang, Jie, Sample size determination for EMAX model, equivalence/non-inferiority test and drug combination in fixed dose trials
Zhang, Lin, Semiparametric AUC regression for testing treatment effect in clinical trials

Rice University (16)

DEPARTMENT OF COMPUTATIONAL AND APPLIED MATHEMATICS
Gonzalez del Cueto, Fernando, Filtering random layering effects for imaging and velocity estimation
Gonzalez, Edward, Efficient alternating gradient-type algorithms for the approximate non-negative matrix factorization problem
Papakonstantinou, Joanna, Historical development of the BFGS secant method and its characterization properties
Sun, Kai, Domain decomposition and model reduction for parabolic optimal control problems

DEPARTMENT OF MATHEMATICS
Douglas, Casey, Perturbed genus one Scherk surfaces and their limits
Dunning, Ryan, Asymptotics under self-intersection for minimizers of self-avoiding energies
Hardway, Heather, Pattern formation in systems of reaction diffusion equations modeling genetic networks
Horn, Peter, Higher-order analogues of genus and slice genus of classical knots
McLellan, Matthew, Deformation of symmetric Scherk type minimal surfaces

DEPARTMENT OF STATISTICS
Ahern, Charlotte, Statistical modeling in the optimization of breast cancer screening schedules
Chatman, Jamie, Computing diversity in undergraduate admissions
Christian, James, Incorporating annotation data in quantitative trait loci mapping with mRNA transcripts
Fofanov, Viacheslav, Statistical models in protein structural alignments
Mathaes, Matthias, Statistical tests of neutrality based on SNP and Alu repeat data
Neeley, Eleanor Shannon, Models for the preprocessing of reverse phase protein arrays
Zhu, Hongqiao, Functional data classification and covariance estimation

Southern Methodist University (2)

DEPARTMENT OF MATHEMATICS
Haque, Mohammed Ziaul, An adaptive finite element method for systems of second-order hyperbolic partial differential equations in one space dimension
Taylor, Michael, Epidemic models for partial-temporary immunity with delay

Texas A&M University (18)

DEPARTMENT OF MATHEMATICS
Chen, Xianjin, Analysis and computation of multiple unstable solutions to nonlinear elliptic systems
Fulkerson, Michael, Radial limits of holomorphic functions on the ball
Jiang, Lijian, Multiscale finite element methods using limited global information and applications
Kimball, James, Bounds on codes from smooth toric threefolds with Pic(X) = 2
Muntynan, Yevgen, Automata groups
Nam, Dukjin, Multiscale numerical methods for some types of parabolic equations
Nguyen, Nga, Surgery on frames
Oeding, Luke, G-varieties and the principal minors of symmetric matrices
Sendova, Tsvetanka, A new approach to the modeling and analysis of fracture through an extension of continuum mechanics to the nanoscale
Sethuraman, Swaminathan, Volumes of certain cones of multi homogeneous polynomials
Tucci, Gabriel, Some results in the hyper-invariant subspace problem and free probability
Vega, Maria, Hypergeometric functions over finite fields and their relations to algebraic curves
Yang, Haibo, RO(G)-graded equivariant Bredon cohomology and sheaves

DEPARTMENT OF STATISTICS
Chen, Min, Modeling covariance structure in unbalanced longitudinal data
George, Nysia, Mixture modeling and outlier detection in microarray data analysis
Jeong, Jae Sik, Some applications of wavelets to time series data
Lee, Seokho, Principal components analysis for binary data
Maity, Arnab, Efficient inference in general semiparametric regression models

University of Houston (11)

DEPARTMENT OF MATHEMATICS
Branton, Sheena, Sub-actions for Young towers
Dulock, Matthew, Uniqueness of holomorphic curves encountering divisors
Filiplnski, Natasha, Periodic solutions in systems with finite abelian symmetries
Kashyap, Upasana, Morita equivalence of dual operator algebras
Kouam, Cedvic, Pricing spread options under stochastic volatility
Larson, Craig, Graph theoretic independence and critical independent sets
Pons, Victoria, Numerical methods for non-smooth problems from calculus of variations: Applications
Ragupathi, Mrinal, Constrained Nevanlinna-Pick interpolation
Wang, Ting, Numerical methods for free boundary and particulate flow problems: Applications to biomechanics
Xie, Weiwei, Valuation of spread options by bivariate Edgeworth expansion
Yavich, Nikolay, Multilevel preconditioners for strongly anisotropic problems

University of North Texas (3)

DEPARTMENT OF MATHEMATICS
Kazemi, Parimah, A constructive method for finding critical points of the Ginzburg-Landau energy functional
Shao, Chuang, Urysohn ultrametric spaces and isometry groups

Snyder, Jason, The global structure of iterated function systems

University of Texas at Arlington (1)

DEPARTMENT OF MATHEMATICS
Alonso, Ricardo, The Boltzmann equation: Sharp Povzner inequalities applied to regularity theory
Anthropelos, Michail, Agents’ agreement and partial equilibrium pricing in incomplete markets
Callahan, Jason, The arithmetic and geometry of two-generator Kleinian groups
Callegari, Renato, Existence and persistence of invariant objects in dynamical systems and mathematical physics
Carneiro, Emanuel, Extremality, symmetry and regularity issues in harmonic analysis
Charters, Philippa, Generalizing binary quadratic residue codes to higher power residues over larger fields
DiTanna, Anthony, The optimal control of a Lévy process
Kahle, Alexander, Superconnections and index theory
Mallmann, Katja, The discriminant algebra in cohomology
Ortiz, Michael, Differential equivalent K-theory
Salerno, Adriana, Hypergeometric functions in arithmetic geometry
Schwah, Russell, Random and periodic homogenization for some nonlinear partial differential equations
Stirling, Spencer, Abelian Chern-Simons theory with toral gauge group, modular tensor categories, and group categories
Swenson, Michelle, Phylogenetic superetree methods
Zhou, Ti, Essays on pricing and portfolio choice in incomplete markets

INSTITUTE FOR COMPUTATIONAL ENGINEERING AND SCIENCES
De Basabe, Jonas, High-order finite element methods for seismic wave propagation
Iglesias-Hernandez, Marco Antonio, An iterative representer-based scheme for data inversion in reservoir modeling
Santillana, Mauricio, Analysis and numerical simulation of the diffusive wave approximation of the shallow water equations
Stagner, Ray, Parallel adaptive C1 macro-elements for nonlinear thin film and non-Newtonian flow problems
Doctoral Degrees Conferred

Tharkabhushanam, Sri Harsha, A conservative deterministic spectral method for rarefied gas flows

UTAH

Brigham Young University (2)

Department of Mathematics

Lian, Zeng, Lyapunov exponents and invariant manifold for random dynamical systems in a Banach space

Kunsmach, Gretchen, Complete tropical Bezout’s theorem and its consequences

University of Utah (6)

Department of Mathematics

Centellegh, Tommaso, A conjectural mass formula for Galois representations modp

Copene, Elizabeth, Ephaptic coupling of cardiac cells

Crofts, Scott, Duality of Spin(m + 1, n)

Preszler, Jason, Nilpotent orbits of symplectic p-adic Lie algebras and quadratic forms

Simeonova, Lyubima, Wave propagation in composite materials: Effective properties and optimization

Thompson, Joshua, Grafting real complex projective structures

Utah State University (3)

Department of Mathematics and Statistics

Maslova, Inga, Testing and estimation for functional data with applications to magnetometer records

Morphet, William, Simulation, kriging, and visualization of circular-spatial data

Yurk, Brian, Modeling the evolution of insect phenology with particular reference to mountain pine beetle

 VIRGINIA

George Mason University (1)

Department of Statistics

Markaryan, Tigran, Exact distributional properties of Efron’s biased coin design with application to clinical trials

Old Dominion University (3)

Department of Mathematics and Statistics

Fernanda, Anne, DGM—A finite difference scheme based on the discontinuous Galerkin method

Grant, Terri, Improved constrained global optimization for estimating molecular structure from atomic distance

Tournon, Charles, An adaptive method for calculating blow-up solution

University of Virginia (7)

Department of Mathematics

Drupieski, Christopher, Cohomology of Frobenius-Lusztig kernels of quantized enveloping algebras

Kapp, Brian, On the structure of certain groups associated in division algebras over fields of power series

Khongsap, Totrakool, Spin Hecke algebras

Pitsillides, Achilles, Structure of unitary groups over number fields

Schmitz, Rebecca, Toeplitz-composition $C^*$-algebras with piecewise continuous symbols

Tysses, Jill, The centers of spin symmetric and spin hyperoctahedral group algebras

Wang, Yao, On a stochastic wave equation modeling heat flow

WASHINGTON

University of Washington (17)

Department of Mathematics

Ballard, Matthew, Derived categories of quasi-projective schemes

Koo, Tak-Lun, Change of Selmer group under isogeny, Iwasawa invariants of $A$-adic representation, and coefficients of newforms

Kunkel, Christopher, Quaternionic contact pseudohermitian normal coordinates

Moody, Dustin, The Diffie-Hellman problem and generalization of Verheul’s theorem

Sekar, Anusha, Earthquake source inversion for tsunami runup prediction

Vance, Stephanie, Lattices and sphere packings in Euclidean space

Whitcher, Ursula, Families of polarized K3 hypersurfaces

Department of Statistics

Alkema, Leontine, Uncertainty assessments of demographic estimates and projections

Di, Yanning, Conditional tests for localizing trait genes

Fu, Quiyun, Models and inference of transmission of DNA methylation patterns in mammalian somatic cells

Gile, Krista, Inference from partially-observed network data

Li, Quinhua, Statistical methods for peptide and protein identification using mass spectrometry

Sloughter, James McLean, Probabilistic weather forecasting using Bayesian model averaging

Stanberry, Larissa, Statistical solutions to some problems in medical imaging

White, Toby, Extensions of latent class transition models with application to chronic disability survey data

Youn, Ahrim, Learning transcriptional regulatory networks from the integration of heterogeneous high-throughput data

Zhang, Shengyu, Statistical analysis of portfolio risk and performance measures: The influence function approach

WEST VIRGINIA

West Virginia University (1)

Department of Mathematics

Zhou, Ju, Cycles in graph theory and matroids

WISCONSIN

Marquette University (1)

Department of Mathematics, Statistics and Computer Science

Ratnakumar, Shivani, Protocol for estimation of target registration error in registration of acquired CT to live x-ray left atrial images

Medical College of Wisconsin (2)

Division of Biostatistics

Fan, Xiaolin, Bayesian nonparametric inference for competing risks data

Pajewski, Nicholas, Bayesian semiparametric hierarchical models for genetic association studies in the presence of population structure and multiplicities

University of Wisconsin, Madison (21)

Department of Mathematics

Akers, Benjamin, On model equations or gravity-capillary waves

Andrejko, Erik, Between O and Osztaszewski

Darnall, Matthew, On the discrepancy of random ($t, m, s$)-nets

Galindo, Diego, Self similar solutions of cold ion plasma equations

Hua, Zheng, Derived categories of toric stacks and Calabi-Yau varieties

Kang, Hye-Won, Multiple scaling methods in chemical reaction networks
Kim, Yeon Hyang, Representations of almost periodic functions using $L^2$-frames
McGinn, Daniel, Quantifier elimination in intuitionistic JRS theories
Milovich, David, Order-theoretic invariants in set-theoretic topology
Nam, Sangnam, Construction and analysis of local wavelet-like pyramidal representations in several dimensions
Otto, Benjamin, Supercharacters of algebra groups
Owen, Robert, Outer model theory and the definability of forcing
Petro, Matthew, Moduli spaces of Riemann surfaces
Rault, Patrick, On uniform bounds for rational points on rational curves and thin sets
Rhoades, Robert, The interplay between harmonic weak Maass forms and classical modular forms
Sengun, Mehmet Haluk, Serre's conjecture over imaginary quadratic fields
Shi, Yingzhe, Numerical methods for coupling of multispecies kinetic and hydrodynamic equations
Tang, Yudong, Geodesic rays and test configurations
Thorne, Frank, Extensions of results on the distribution of primes
Tonejc, Jernej, Formal normal forms for almost complex structures
Yang, Xu, Numerical methods for multiscale kinetic transport and high frequency waves

University of Wisconsin, Milwaukee (8)

Chiappetta, Shawn, Non-overlapping domain decomposition parallel algorithms for convection-diffusion equations
Fuhrman, Kseniya, Mathematical modeling and analysis of virus-host interaction in aquatic systems
He, Hao, Utility maximization of a portfolio that includes an illiquid asset
Michael, Martin, Local and global solutions to quasilinear wave equations via Nash-Moser algorithm
Rus, George, Finite element methods for control of singular stochastic processes
Sebert, Florian, Algebraic and geometric properties of generalized wavelet matrices
Shomberg, Joseph, Explicit construction of a robust family of compact inertial manifolds
Zaglauer, Katharina, Fair pricing of participating life insurance contracts in a regime-switching market environment

WYOMING

University of Wyoming (2)

Department of Mathematics
Rahunanthan, Arunasalam, A study of spatial and time discretizations for discontinuous Galerkin methods

Department of Statistics
Li, Yumei, Hidden Markov modeling of earthquake swarms
2009 Election Results

In the elections of 2009 the Society elected a president elect, a vice president, a trustee, five members at large of the Council, three members of the Nominating Committee, and two members of the Editorial Boards Committee.

**President Elect**
Elected as the president elect is **Eric M. Friedlander** from the University of Southern California. Term is one year as president elect (1 February 2010–31 January 2011), two years as president (1 February 2011–31 January 2013), and one year as immediate past president (1 February 2013–31 January 2014).

**Vice President**
Elected as the new vice president is **Sylvain Cappell** from Courant Institute of Mathematical Sciences, New York University. Term is three years (1 February 2010–31 January 2013).

**Trustee**
Elected as new trustee is **Mark L. Green** from the University of California, Los Angeles. Term is five years (1 February 2010–31 January 2015).

**Members at Large of the Council**
Elected as new members at large of the Council are
- **Alejandro Adem** from the University of British Columbia
- **Richard Hain** from Duke University
- **Jennifer Schultens** from the University of California, Davis
- **Janet Talvacchia** from Swarthmore College
- **Christoph Thiele** from the University of California, Los Angeles
Terms are three years (1 February 2010–31 January 2013).

**Nominating Committee**
Elected as new members of the Nominating Committee are
- **William Beckner** from University of Texas at Austin
- **Richard T. Durrett** from Cornell University
- **Carla D. Savage** from North Carolina University
Terms are three years (1 January 2010–31 December 2012).

**Editorial Boards Committee**
Elected as new members of the Editorial Boards Committee are
- **Anatoly Libgober** from the University of Illinois at Chicago
- **Simon Tavener** from Colorado State University
Terms are three years (1 February 2010–31 January 2013).
2010 AMS Election
Nominations by Petition

Vice President or Member at Large

One position of vice president and member of the Council *ex officio* for a term of three years is to be filled in the election of 2010. The Council intends to nominate at least two candidates, among whom may be candidates nominated by petition as described in the rules and procedures.

Five positions of member at large of the Council for a term of three years are to be filled in the same election. The Council intends to nominate at least ten candidates, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

Petitions are presented to the Council, which, according to Section 2 of Article VII of the bylaws, makes the nominations. The Council of 23 January 1979 stated the intent of the Council of nominating all persons on whose behalf there were valid petitions.

Prior to presentation to the Council, petitions in support of a candidate for the position of vice president or of member at large of the Council must have at least fifty valid signatures and must conform to several rules and operational considerations, which are described below.

Editorial Boards Committee

Two places on the Editorial Boards Committee will be filled by election. There will be four continuing members of the Editorial Boards Committee.

The President will name at least four candidates for these two places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate’s assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Nominating Committee

Three places on the Nominating Committee will be filled by election. There will be six continuing members of the Nominating Committee.

The President will name at least six candidates for these three places, among whom may be candidates nominated by petition in the manner described in the rules and procedures.

The candidate’s assent and petitions bearing at least 100 valid signatures are required for a name to be placed on the ballot. In addition, several other rules and operational considerations, described below, should be followed.

Rules and Procedures

Use separate copies of the form for each candidate for vice president, member at large, or member of the Nominating and Editorial Boards Committees.

1. To be considered, petitions must be addressed to Robert J. Daverman, Secretary, American Mathematical Society, Department of Mathematics, 302C Aconda Court, University of Tennessee, 1534 Cumberland Avenue, Knoxville, TN 37996-0614, USA, and must arrive by 24 February 2010.

2. The name of the candidate must be given as it appears in the <Combined Membership List> (www.ams.org/cml). If the name does not appear in the list, as in the case of a new member or by error, it must be as it appears in the mailing lists, for example on the mailing label of the Notices. If the name does not identify the candidate uniquely, append the member code, which may be obtained from the candidate’s mailing label or by the candidate contacting the AMS headquarters in Providence (<amsmem@ams.org>).

3. The petition for a single candidate may consist of several sheets each bearing the statement of the petition, including the name of the position, and signatures. The name of the candidate must be exactly the same on all sheets.

4. On the next page is a sample form for petitions. Petitioners may make and use photocopies or reasonable facsimiles.

5. A signature is valid when it is clearly that of the member whose name and address is given in the left-hand column.

6. The signature may be in the style chosen by the signer. However, the printed name and address will be checked against the <Combined Membership List> and the mailing lists. No attempt will be made to match variants of names with the form of name in the CML. A name neither in the CML nor on the mailing lists is not that of a member. (Example: The name Robert J. Daverman is that of a member. The name R. Daverman appears not to be.)

7. When a petition meeting these various requirements appears, the secretary will ask the candidate to indicate willingness to be included on the ballot. Petitioners can facilitate the procedure by accompanying the petitions with a signed statement from the candidate giving consent.
Nomination Petition for 2010 Election

The undersigned members of the American Mathematical Society propose the name of

______________________________

as a candidate for the position of (check one):

☐ Vice President
☐ Member at Large of the Council
☐ Member of the Nominating Committee
☐ Member of the Editorial Boards Committee

of the American Mathematical Society for a term beginning 1 February, 2011

Return petitions by 24 February 2010 to:
Secretary, AMS, 302C Aconda Court, University of Tennessee, Knoxville, TN 37996-0614 USA

Name and address (printed or typed)

______________________________
Signature

______________________________
Signature

______________________________
Signature

______________________________
Signature

______________________________
Signature

______________________________
Signature

______________________________
Signature
February 2010

* 1–March 6 Math-Info 2010: Towards new interactions between mathematics and computer science, International Center for Mathematical Meetings (C.I.R.M.), Marseille, France.

Description: This thematic month will be split into five weeks:
- * from Feb. 1–5: Lattice Reduction
- * from Feb. 8–12: Dynamics and Computation
- * from Feb. 15–19: Multi-dimensional Subshifts and Tilings
- * from Feb. 22–26: Sage Days

The mornings are devoted to lectures (around three lectures of four hours each for each week), given by renowned researchers. The specificity of this conference is that participants can propose topics and names for the afternoon working sessions in order to speak about their results or discuss some problems with other participants. *You can propose such working sessions* by sending an email or directly on the webpage: http://www.lirmm.fr/arith/wiki/MathInfo2010/CallForWorkshopProposals. If you intend to participate, "you need to register" as soon as possible on the website: http://www.lirmm.fr/arith/wiki/MathInfo2010/Pre-registration.


* 1–12 X Winter Diffiety School, Academic Gymnasium, St. Petersburg State University, St. Petersburg, Russia.

Description: The aim of this permanent School is to introduce undergraduate and Ph.D. students in Mathematics and Physics as well as post-doctoral researchers in a recently emerged area of Mathematics and Theoretical Physics: SECONDARY CALCULUS. A "diffiety" is a new geometrical object that properly formalizes the concept of the solution space of a given system of (nonlinear) PDEs, much as an algebraic variety does with respect to solutions of a given system of algebraic equations. SECONDARY CALCULUS is a natural diffiety analogue of the standard Calculus on smooth manifolds, and as such leads to a very rich general theory of nonlinear PDEs. It appears that it is this only natural language of quantum physics, just as the standard Calculus is for classical physics.

Information: http://www.levi-civita.org.Activities/DiffietySchools/XWDS.

* 14–19 Young Set Theory Workshop 2010, Seminarzentrum Raach, near Vienna, Austria.

Description: The third annual Young Set Theory Workshop will take place between 15-19 February 2010 at Seminarzentrum Raach (http://www.srz.at) located one hour south of Vienna in Raach am Hochgebirge. The aim of this conference is to bring together Ph.D. students and postdocs in Set Theory in order to learn from leading researchers in the field, hear about the latest research, and to discuss research issues in a co-operative environment. The conference format will be similar to previous years, including tutorials, postdoc research talks, and discussion sessions.

Information: http://www.math.uni-bonn.de/people/logic/events/young-set-theory-2010/.
Mathematics Calendar


**Description:** The aims of WINBIS’10 is bring together researchers, engineers, and practitioners interested on wireless information Networks & Business information systems to make international forum WINBIS’10 will not only present the papers, also give the ideas on how to analyze & approach problems by combining wireless information system & Business information system and it’s applications.

**Information:** http://www.win-bis.com.

March 2010

*14–17 2010 Interpore Conference and Annual Meeting, Texas A&M, Mar 2010, Texas A&M University, College Station, Texas.

**Description:** The goal of the conference is to provide a forum for discussing the academic and industrial challenges of porous media, as well as to foster collaboration among theoreticians, modelers and experimentalists working in porous media research. The "International Society for Porous Media" (InterPore) is a non-profit-making independent scientific organization established in 2008. The general aim of the Society is to advance and disseminate knowledge for the understanding, description, and modeling of natural and industrial porous media systems (see http://www.interpore.org/ for more information). The conference will include keynote lectures, invited oral presentations, and poster sessions. No parallel sessions are planned, and oral presentations are by invitation only.

**Deadline:** The abstracts for poster session presentations are to be sent to: interpore2010@gmail.com.


*14–17 2010 Interpore Conference and Annual Meeting, Texas A&M, Mar 2010, Texas A&M University, College Station, Texas.

**Description:** The goal of the conference is to provide a forum for discussing the academic and industrial challenges of porous media, as well as to foster collaboration among theoreticians, modelers and experimentalists working in porous media research. The "International Society for Porous Media" (InterPore) is a non-profit-making independent scientific organization established in 2008. The general aim of the Society is to advance and disseminate knowledge for the understanding, description, and modeling of natural and industrial porous media systems (see http://www.interpore.org/ for more information). The conference will include keynote lectures, invited oral presentations, and poster sessions. No parallel sessions are planned, and oral presentations are by invitation only.

**Deadline:** The abstracts for poster session presentations are to be sent to: interpore2010@gmail.com.


*14–15 A Celebration of Mathematics and the 40th Anniversary of Jeffery Hall, Queen’s University, Kingston, Ontario, Canada.

**Description:** In the post-Sputnik era, Canada and the United States faced an immense challenge to bring the scientific research levels of both nations to a higher standard. In the mathematical field, this challenge was met in Canada by several mathematicians, the most notable being Professor R. L. Jeffery of Queen’s University. By strengthening the Canadian Mathematical Society and by organizing the summer research seminars held at Queen’s University during the 1950s and 1960s, Professor Jeffery was instrumental in raising the level of mathematical research in Canada. Jeffery Hall, which presently houses the Department of Mathematics and Statistics at Queen’s University, is named after him. This building was 40 years old in October, 2009. The world now faces a similar challenge to the one Professor Jeffery faced in the 1950s. To keep the importance of mathematical research in the forefront of our scientific consciousness, as well as to commemorate the 40th anniversary of Jeffery Hall and honour Professor Jeffery, the Department of Mathematics and Statistics at Queen’s University is planning a two-day celebration from May 14-15, 2010. Several presentations are planned. Professor Sir David Cox of the University of Oxford, Professor Hale Trotter of Princeton University and Professor Gerhard Frey of the University of Duisburg, Essen, have already agreed to speak. A lecture surveying the development of research in mathematics in Canada, as well as R. L. Jeffery’s role in the Canadian Mathematical Society, is also planned. It is anticipated that this celebration will galvanize and reinforce the mathematical talent in Canada to meet the present scientific and technological challenges. It is expected that many alumni and others will attend.

**Information:** If you would like to be put on the mailing list to receive more information, please write by mail to: Dr. A. M. Herzberg FRSC, Department of Mathematics, Queen’s University, Kingston, Ontario K7L 3N6; by email: Miss A. Burns; burnsa@queensu.ca.

*16–22 ESF Mathematics Conference in partnership with EMS and ERCOM Algebraic Methods in Dynamical Systems, The Institute of Mathematics Conference Centre, Bedlewo, Poland.

**Description:** The conference is intended to give an account of the new results concerning algebraic methods in dynamical systems. Morales-Ramis theorem establishing the quasi-abelianity of the differential Galois group of the variational equation along a particular solution of a meromorphically completely integrable Hamiltonian system have led during the recent years to a big number of new results on non-integrability of Hamiltonian systems. More recently, the application of differential Galois theory to the study of Hamiltonian systems is no longer limited to Picard-Vessiot theory but is extended to the more general setting based on recent results of Malgrange and Umemura.

**Information:** http://www.esf.org/conferences/10320.

*17–20 The Seventh International Conference on Computational Physics, Fragrant Hill Hotel, Beijing, China.

**Focus:** The conference will focus on key aspects of computational physics, including theory, numerical methods, and applications. Twenty-two invited speakers have confirmed their participation in ICCP7. Papers submitted to ICCP7 will have an opportunity to be published in one of two journals (Communications in Computational Physics, Chinese Journal of Computational Physics). The Scientific Committee will give recommendation for publication of quality papers. For any enquiry you are welcome to send an email to: iccp7@iapcm.ac.cn.

**Organizer:** ICCP7 is organized by the Institute of Applied Physics and Computational Mathematics.

**Information:** For further information on the conference venue, please visit the website: http://www.xsf.com; http://www.iapcm.ac.cn/iccp7/.

*23–28 Conformal maps: from probability to physics, Monte Verita, Ascona, Ticino, Switzerland.

**Description:** The conference centers on random structures in the context of complex analysis, inspired by interactions between math-
rheumatology and physics. The central example is perhaps the Stochastic Loewner Evolution, introduced by Oded Schramm. SLE arises as the scaling limit of interfaces in 2D lattice models at criticality (percolation, Ising model, self-avoiding polymers), and its elegant combination of probability and complex analysis has led to the proofs of many conjectures originating in physics. Other topics include Diffusion Limited Aggregation (a model for electrodeposition and other phenomena), Hele-Shaw flow (describing interfaces between fluids of different viscosities, e.g. oil and water), 2D Quantum Gravity and Random Maps.


*26–29 Workshop on Combinatorial and Additive Number Theory (CANT 2010), CUNY Graduate Center, New York, New York.

Description: This is the eighth in a series of annual spring workshops sponsored by the New York Number Theory Seminar on problems in combinatorial and additive number theory and related parts of mathematics. In addition to long and short talks, each day of the conference will include a problem and discussion session. Previous CANT conferences have attracted many graduate students as well as research mathematicians, and students are encouraged to attend. A list of lecturers and other information will be posted on the conference website. Mathematicians who wish to speak at the meeting are encouraged to submit a title and abstract to: melvyn.nathanson@lehman.cuny.edu.

Information: [http://www.theoryofnumbers.com](http://www.theoryofnumbers.com).

June 2010

*7–11 International Conference on Applied Mathematics (with the first William Benter Prize in Applied Mathematics), City University of Hong Kong, Hong Kong.

Description: In recent years, huge advances have been achieved through the application of mathematical ideas and techniques to a wide variety of fields. The International Conference on Applied Mathematics will cover a wide range of topics including all aspects of applied mathematics. The objective of this conference is to provide a forum for the exchange of expertise, experience, and insights by mathematical scientists, physicists, and young researchers who are active in the area of applied mathematics, and to encourage leading scientists from abroad to further strengthen their cooperation with local scientists. The first William Benter Prize in Applied Mathematics will be awarded and the winner will be presented with the prize ([http://www6.cityu.edu.hk/rcms/WBP/the_prize.html](http://www6.cityu.edu.hk/rcms/WBP/the_prize.html)) during the conference. The aim of the prize is to recognize outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, finance, or engineering applications.


*7–11 International Functional Analysis Meeting in Valencia on the Occasion of the 80th Birthday of Professor Manuel Valdivia, University of Valencia, Valencia, Spain.

Description: This conference will emphasize different areas of functional analysis. It will provide a venue for established scholars to interact with each other and with junior scholars. Partial support for a small number of participants is expected to be available. Recent recipients of doctoral degrees and pre-doc students are encouraged to apply. The meeting will take place in Valencia as a joint venture of the University of Valencia and the University Politècnica de Valencia. It will take place in the Mathematics building (and surrounding large lecture theaters) in Burjasot (Valencia) from June 7–11, 2010 (both days included). The structure of the conference is of plenary and long invited lectures in the morning, and twelve parallel sessions in the afternoon.


*7–11 7th Annual Conference on Theory and Applications of Models of Computation- TAMC 2010, Charles University, Prague, Czech Republic.

Description: TAMC is an international conference series with an interdisciplinary character, bringing together researchers working in computer science, mathematics and the physical sciences. The cross-disciplinary character, together with its focus on algorithms, complexity and computability theory, gives the conference a special flavor and distinction. The TAMC conference series arose naturally in response to important scientific developments affecting how we compute in the twenty-first century. Originally aimed to become a significant contributor to the scientific resurgence seen in East Asia, TAMC is now playing an important international role in contemporary applications of theoretical computer science. After six successful conferences held in China (Beijing 2004 and 2006, Kunming 2005, Shanghai 2007, Xi’an 2008, ChangShao 2009) TAMC is leaving Asia for the first time.


Description: The objective of this conference is to bring together researchers, practitioners, and students with interest in theoretical analysis, numerical simulations, and applications of hyperbolic PDEs and related mathematical models appearing in the area of applied sciences. The conference will keep the traditional balance of the HYP series, blending theory, numerics and applications. In addition, additional new attractive points are also taken into granted.

Information: For more information about the conference list of the Scientific Committee and Invited Speakers, registration fee and the related things including possible support for young mathematicians from developing countries, please visit the HYP2010 conference website at: [http://www.amt.ac.cn/hyp2010/](http://www.amt.ac.cn/hyp2010/).

*16–21 XI International Conference “Current Geometry”, Vietri sul Mare Salerno, Italy.

Description: The power of synthesis of Geometry, which led in the past to the formulation of “grand unification theories”, has got an essential role nowadays, especially because of the growing fragmentation of knowledge due to scientific progress. In order to avoid too big a dispersion, geometers need a constant dialogue. Therefore, a stable experience of personal meetings, apart from telematic interchanges, cannot be renounced. Current Geometry was born to allow a periodic update about actual progresses in Geometry (and its applications) on the international scene.

Information: [http://www.levi-civita.org/Activities/Conferences/xicurrentgeometry.html](http://www.levi-civita.org/Activities/Conferences/xicurrentgeometry.html).

*17–19 [FG60] Computational and Geometric Topology, Bertinoro, Forli, Italy.

Description: The research groups of Geometric Topology and Computational Topology of the Universities of Bologna and Modena-Reggio Emilia, Departments of Mathematics, will be organizing a conference in order to celebrate the 60th birthdays of Massimo Ferri and Carlo Gagliardi.


Information: All the information on the conference and the form to register and/or submit a talk can be found on the conference’s web page [http://fg60.dm.unibo.it/](http://fg60.dm.unibo.it/).

*19–24 International Algebraic Conference dedicated to the 70th birthday of A. V. Yakovlev, St. Petersburg, Russia.


*20–25 The Twelfth National Conference in Algebra (China), Northwest Normal University, Lanzhou, Gansu, China.
Description: The National Conference in Algebra is the largest national conference in China. It is held every two years. All areas of algebra and their applications will be covered. At the last conference, there were several hundred presentations and over 300 participants. It will be a good opportunity for both the experts and beginning researchers to learn the most recent developments in algebra in China. The conference encourages participation from within and outside of China.

Information: http://www.nwu.edu.cn/algebra.


Description: The Summer School and Workshop "Harmonic Analysis and Related Topics" is intended both for Ph.D. students and young researchers and also experts in various topics related to Harmonic Analysis and Function Spaces. Summer School includes three short courses: "Sobolev, capacity and isocapacitary inequalities" (Vladimir Maz’ya), "Weighted problems for operators of Harmonic Analysis in some Banach Function Spaces" (Vakhram Kokilashvili), "Variable Lebesgue Spaces" (David Cruz-Uribe). The workshop is supposed to cover a number of topics in Harmonic Analysis, Function spaces, and related areas.


* 21–26 2nd International Conference for Promoting the Application of Mathematics in Technical and Natural Sciences (AMIiTnS’10), Sozopol, Bulgaria.

Description: The conference will be scheduled in plenary and keynote lectures followed by special and contributed sessions. The accents of the conference will be on Difference and Spectral methods; Applied Analysis, Biomathematics, Continuum Mathematics, which can be complemented by new topics. You are welcomed to announce and organize special sessions that should be within the general topic of the conference. Everybody, who is interested in attending AMiTnS’10 please let him/her prepare a short abstract within 300 words clearly stating the goal, tools and results and submit it at: conference@ea4amitans.org and fill out the online Application form. In case of problems please send its text version as attachment to e-mail: conference@ea4amitans.org.

Deadline: For both submissions is March 31, 2010.


* 22–25 Group Representation Theory and Related Topics, EPFL, Centre Interfacultaire Bernoulli, Lausanne, Switzerland.

Description: This is an international conference that will take place in Lausanne, Switzerland. Group representation theory is a very active area of mathematics which has connections and overlaps with areas such as algebraic topology, K-theory, algebraic geometry and commutative ring theory. Research in the area has been driven by several conjectures and open problems which have pushed the development of new methodology and broader interactions with other areas of mathematics. The aim of the conference is to stimulate activity in and enhance interaction between representation theory and other areas of mathematics, like those presented above. In addition to the lectures, there will be a conference dinner in honor of Professor Jacques Thévenaz.

Information: http://grt.epfl.ch.

* 29-July 4 23rd International Conference on Operator Theory, West University of Timisoara, Timisoara, Romania.

Description: The conference is devoted to operator theory, operator algebras and their applications (differential operators, complex functions, mathematical physics, matrix analysis, system theory, etc.).

Information: http://www.imar.ro/~ot/.

July 2010

* 4–17 40th Probability Summer School, Saint-Flour, France.

Description: Founded in 1971, this school is intended for Ph.D. students, teachers and researchers who are interested in probability theory, statistics, and in applications of these techniques. The three 12-hour courses of this year will be given by Franco Flandoli, Giambattista Giacomin, and Takashi Kumagai.

Information: http://math.univ-bpclermont.fr/stflour/.


Description: TMFCS is an important event in the theoretical, mathematical and logical areas of computer science. The conference will be held at the same time and location where several other major international conferences will be taking place.

Information: For other conferences associated with MULTICONF-10, please visit: http://www.promotoreresearch.org.


Description: A celebration of the long and illustrious careers of Victor Klee and Branko Grunbaum and their fundamental contributions to discrete mathematics and geometry. There will be 18 invited talks, a poster session and an open problems session. Details available at the above website.

Information: http://sites.google.com/a/alaska.edu/klee-grunbaum/.

August 2010


Topics: The conference topics include (but are not limited to) enumeration questions, forbidden pattern questions, study of the permutation pattern order, algorithms for computing with permutation patterns, applications and generalizations of permutation patterns, and others.

Plenary Lectures: There will be two invited plenary lectures by Nik Ruskuc and Richard P. Stanley, as well as contributed talks.

Information: http://math.dart.edu/~pp2010/.

* 27–31 Differential Geometry and its Applications, Masaryk University, Faculty of Science, Brno, Czech Republic, Europe.

Description: The DGA conferences take place regularly at one of the Czech universities every three years; the tenth conference in the series took place in Olomouc in 2007.

Invited plenary speakers: Shun-ichi Amari (Japan), Robert Bryant (MSRI, USA), Andreas Cap (Vienna, Austria), Anna Fino (Torino, Italy), Joseph M. Landsberg (Texas, USA), Franz Pedit (Amsterdam & Tuebingen), Lorenz Schwachhoefer (Dortmund, Germany), Zhongmin Shen (Indianapolis, USA), Jian Song (Rutgers, USA), Vladimir Soucek (Prague, Czech Republic), Guddlaut Thorbergsson (Koeln, Germany). The Editorial Board meeting of the journal of the same name, cf. http://www.elsevier.com/locate/difgeo will take place during the conference.


September 2010

* 7–11 International Conference "Modern Stochastics: Theory and Applications II", Kyiv National Taras Shevchenko University, Kyiv, Ukraine.

Description: Conference is dedicated to the anniversaries of prominent Ukrainian scientists: A. Skorokhod, V. Korolyuk, I. Kovalenko.


Keynote speakers: Yu. Belyaev, Sweden; V. Buldygin, Ukraine; A. Bulinski, Russia; J. M. Corcuera, Spain; P. Doukhan, France; M. Dozzi, France; A. Dudin, Belarus; A. Gushchin, Russia; V. Konakov, Russia;...
Yu. Kondratiev, Germany; K. Kubilius, Lithuania; N. Leonenko, UK; N. Limnios, France; E. Merzbach, Israel; I. Molchanov, Switzerland; E. Orsingher, Italy; G. Peccati, France; D. Silvestrov, Sweden; E. Valkaella, Finland; V. Vatutin, Russia; A. Veretennikov, UK.

Information: http://probability.univ.kiev.ua/msta2conf.

* 7–11 Logic, Algebra and Truth Degrees 2010, Prague, Czech Republic.
Description: Mathematical Fuzzy Logic is a sub-discipline of Mathematical Logic which studies the notion of comparative truth. The assumption that "truth comes in degrees" has proved to be very useful in many, both theoretical and applied, areas of Mathematics, Computer Science, and Philosophy.
Goal: The main goal of this meeting is to foster collaboration between researchers in the area of Mathematical Fuzzy Logic, and to promote communication and cooperation with members of neighbouring fields.
Tutorials by: George Metcalfe and Vilém Novák.
Program committee: Petr Hájek (Chair), Antonio Di Nola, Christian Fernmuller, Siegfried Gottwald, Daniele Mundici, and Carles Noguera.

* 2010 Program committee:
Preliminary Announcement and Call for Papers: The Mathematics Education into the 21st Century Project has just completed its tenth successful international conference in Dresden, Germany, following conferences in Egypt, Jordan, Poland, Australia, Sicily, Czech Republic, Malaysia and the USA. Our project was founded in 1986 and is dedicated to the planning, writing and disseminating of innovative ideas and materials in Mathematics, Statistics, Science and Computer Education.
Organizing Committee: The chairman is Professor Marc Schaefer of Rhodes University.
Information: For further conference details please email Alan Rogerson, Chairman of the International Programme Committee: a1anrogerson.pol.pl.
New Publications Offered by the AMS

To subscribe to email notification of new AMS publications, please go to http://www.ams.org/bookstore-email.

Analysis

**Differential Algebra, Complex Analysis and Orthogonal Polynomials**

Primitivo B. Acosta-Humánez, Universidad Sergio Arboleda, Bogotá, Colombia, and Francisco Marcellán, Universidad Carlos III de Madrid, Leganés, Spain, Editors

This volume represents the 2007–2008 Jairo Charris Seminar in Algebra and Analysis on Differential Algebra, Complex Analysis and Orthogonal Polynomials, which was held at the Universidad Sergio Arboleda in Bogotá, Colombia.

It provides the state of the art in the theory of Integrable Dynamical Systems based on such approaches as Differential Galois Theory and Lie Groups as well as some recent developments in the theory of multivariable and $q$-orthogonal polynomials, weak Hilbert’s 16th Problem, Singularity Theory, Tournaments in flag manifolds, and spaces of bounded analytic functions on the unit circle.

The reader will also find survey presentations, an account of recent developments, and the exposition of new trends in the areas of Differential Galois Theory, Integrable Dynamical Systems, Orthogonal Polynomials and Special Functions, and Bloch–Bergman classes of analytic functions from a theoretical and an applied perspective.

The contributions present new results and methods, as well as applications and open problems, to foster interest in research in these areas.

This item will also be of interest to those working in algebra and algebraic geometry.

A co-publication of the AMS and Instituto de Matemáticas y sus Aplicaciones (IMA).


Contemporary Mathematics, Volume 509


Advances in $p$-adic and Non-Archimedean Analysis

Martin Berz, Michigan State University, East Lansing, MI, and Khodr Shamseddine, University of Manitoba, Winnipeg, Manitoba, Canada, Editors

This volume contains the proceedings of the Tenth International Conference on $p$-adic and Non-Archimedean Analysis, held at Michigan State University in East Lansing, Michigan, on June 30–July 3, 2008.

This volume contains a kaleidoscope of papers based on several of the more important talks presented at the meeting. It provides a cutting-edge connection to some of the most important recent developments in the field. Through a combination of survey papers, research articles, and extensive references to earlier work, this volume allows the reader to quickly gain an overview of current activity in the field and become acquainted with many of the recent sub-branches of its development.

Contents: J. Aguayo, S. Navarro, and M. Nova, Strict topologies on spaces of vector-valued continuous functions over non-Archimedean field; B. Diarra, Some subalgebras of the algebra of bounded linear operators of the one variable Tate algebra; A. Escassut and N. Mainetti, The ultrametric corona problem;

Contemporary Mathematics, Volume 508


Global Aspects of Ergodic Group Actions

Alexander S. Kechris, California Institute of Technology, Pasadena, CA

The subject of this book is the study of ergodic, measure preserving actions of countable discrete groups on standard probability spaces. It explores a direction that emphasizes a global point of view, concentrating on the structure of the space of measure preserving actions of a given group and its associated cocycle spaces. These are equipped with canonical topological actions that give rise to the usual concepts of conjugacy of actions and cohomology of cocycles. Structural properties of discrete groups such as amenability, Kazhdan’s property (T) and the Haagerup Approximation Property play a significant role in this theory as they have important connections to the global structure of these spaces. One of the main topics discussed in this book is the analysis of the complexity of the classification problems of conjugacy and orbit equivalence of actions, as well as of cohomology of cocycles. This involves ideas from topological dynamics, descriptive set theory, harmonic analysis, and the theory of unitary group representations. Also included is a study of properties of the automorphism group of a standard probability space and some of its important subgroups, such as the full and automorphism groups of measure preserving equivalence relations and connections with the theory of costs.

The book contains nine appendices that present necessary background material in functional analysis, measure theory, and group representations, thus making the book accessible to a wider audience.

Contents: Measure preserving automorphisms; The space of actions; Cocycles and cohomology; Realifications and complexifications; Tensor products of Hilbert spaces; Gaussian probability spaces; Wiener chaos decomposition; Extending representations to actions; Unitary representations of abelian groups; Induced representations and actions; The space of unitary representations; Semidirect products of groups; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 160


Differential Equations

Lectures on Elliptic Boundary Value Problems

Shmuel Agmon, Hebrew University of Jerusalem, Israel

This book, which is a new edition of a book originally published in 1965, presents an introduction to the theory of higher-order elliptic boundary value problems. The book contains a detailed study of basic problems of the theory, such as the problem of existence and regularity of solutions of higher-order elliptic boundary value problems. It also contains a study of spectral properties of operators associated with elliptic boundary value problems. Weyl’s law on the asymptotic distribution of eigenvalues is studied in great generality.

Contents: Notations and conventions; Calculus of $L^2$ derivatives—Local properties; Calculus of $L^2$ derivatives—Global properties; Some inequalities; Elliptic operators; Local existence theory; Local regularity of solutions of elliptic systems; Gårding’s inequality; Global existence; Global regularity of solutions of strongly elliptic equations; Coerciveness; Coerciveness results of Aronszajn and Smith; Some results on linear transformations on a Hilbert space; Spectral theory of abstract operators; Eigenvalue problems for elliptic equations; The self-adjoint case; Non-self-adjoint eigenvalue problems; Completeness of the eigenfunctions; Bibliography; Notation index; Index.

AMS Chelsea Publishing, Volume 369

Micro- and nanoelectromechanical systems (MEMS and NEMS), which combine electronics with miniature-size mechanical devices, are essential components of modern technology. It is the mathematical model describing “electrostatically actuated” MEMS that is addressed in this monograph. Even the simplified models that the authors deal with still lead to very interesting second- and fourth-order nonlinear elliptic equations (in the stationary case) and to nonlinear parabolic equations (in the dynamic case). While nonlinear eigenvalue problems—where the stationary MEMS models fit—are a well-developed field of PDEs, the type of inverse square nonlinearity that appears here helps shed a new light on the class of singular supercritical problems and their specific challenges.

Besides the practical considerations, the model is a rich source of interesting mathematical phenomena. Numerics, formal asymptotic analysis, and ODE methods give lots of information and point to many conjectures. However, even in the simplest idealized versions of electrostatic MEMS, one essentially needs the full available arsenal of modern PDE techniques to do the required rigorous mathematical analysis, which is the main objective of this volume.

This monograph could therefore be used as an advanced graduate text for a motivational introduction to many recent methods of nonlinear analysis and PDEs through the analysis of a set of equations that have enormous practical significance.

Probability

Markov processes are among the most important stochastic processes for both theory and applications. This book develops the general theory of these processes, and applies this theory to various special examples. The initial chapter is devoted to the most important classical example—one-dimensional Brownian motion. This, together with a chapter on continuous time Markov chains, provides the motivation for the general setup based on semigroups and generators. Chapters on stochastic calculus and probabilistic potential theory give an introduction to some of the key areas of application of Brownian motion and its relatives. A chapter on interacting particle systems treats a more recently developed class of Markov processes that have as their origin problems in physics and biology.

This is a textbook for a graduate course that can follow one that covers basic probabilistic limit theorems and discrete time processes.

Contents: One-dimensional Brownian motion; Continuous time Markov chains; Feller processes; Interacting particle systems; Stochastic integration; Multidimensional Brownian motion and the Dirichlet problem; Appendix; Bibliography; Index.

Graduate Studies in Mathematics, Volume 113


New AMS-Distributed Publications

Number Theory

**Formes Quadratiques sur Un Corps**

Bruno Kahn, *Institut de Mathématiques de Jussieu, Paris, France*

This book presents the theory of quadratic forms over a field, focusing on the Pfister-Arason-Knebusch technique of extensions to function fields of quadrics.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

**Contents:** La théorie de Witt; La théorie de Pfister; Corps de fonctions de quadriques; La théorie de Knebusch; Formes devenant isotropes sur le corps des fonctions d’une quadric; Invariants élémentaires; Le théorème de réduction d’indice et ses applications; Formes de base dimension; Invariants supersuvers; Descente; Rappels sur le groupe de Brauer; Rappels de cohomologie galoisienne; Courbes algébriques; Un aperçu sur les formes quadratiques en caractéristique 2; Formes quadratiques et cycles algébriques; Solutions de certains exercices; Bibliographie; Glossaire; Index.

**Cours Spécialisés—Collection SMF, Number 15**


---

**Advertise in the Notices of the American Mathematical Society**

Target your message to 33,000 active readers!

Make an impact through **Display Advertising**

Increase your sales and connect with new customers.

The Notices of the AMS has the largest circulation of any publication in the field.

Select premium positions are available.

Target your audience through **Classified Advertising**

An effective and economical way to get the most applicants for your open position.

“We have been a long-time advertiser in the Notices and consider it a highly effective way to reach our customers in the mathematical community.” —Cambridge University Press

Please contact Anne Newcomb to create a program that will address your advertising needs.

Anne Newcomb
Phone: 401-455-4084
Email: aen@ams.org

Please visit our online media kit: http://www.ams.org/notices/adnot.html
Positions available, items for sale, services available, and more

KANSAS

KANSAS STATE UNIVERSITY
Department of Mathematics

Applications are invited for two Visiting Assistant Professorships commencing August 8, 2010. These will be annual appointments with the possibility of two subsequent one-year appointments depending on performance, funding, and need of services. A Ph.D. in mathematics or a Ph.D. dissertation—accepted with only formalities to be completed—is required by the time of appointment. The department seeks candidates whose research interests mesh well with current faculty. The department has research groups in algebra, analysis, differential equations, geometry/topology, and number theory. Successful candidates are expected to participate in the department’s programs integrating undergraduate and graduate research, including mentoring undergraduate students during summer programs. The successful candidate should have strong research credentials as well as strong accomplishments or promise in teaching, and should value working with colleagues and students from diverse backgrounds.

Applicants must submit the following: a letter of application; curriculum vita; outline of teaching philosophy; a statement of research objectives; and four letters of reference, at least one of which addresses the applicant’s teaching ability or potential. All application materials must be submitted electronically via http://www.mathjobs.org. Screening of applications begins December 1, 2009, and continues until positions are closed.

Kansas State University is an Equal Opportunity Employer and actively seeks diversity among its employees and encourages applications from women and minorities. A background check is required.

NEW JERSEY

RAMAPO COLLEGE OF NEW JERSEY
Assistant Professor of Mathematics
Tenure-Track—Fall 2010

JOB DESCRIPTION: Responsibilities encompass a wide range of undergraduate mathematics courses, and the teaching and development of General Education mathematics courses.

REQUIREMENTS: Ph.D. in mathematics by September 1, 2010, is required. College teaching experience preferred. Faculty members are expected to maintain active participation in research, scholarship, college governance, service, academic advisement, and professional development activities.

All applications must be completed online at http://www.ramapojobs.com. Attach resume, cover letter, statement of teaching philosophy, research interests, and a list of three references to your completed application. Since its beginning, Ramapo College has had an intercultural/international mission. Please tell us how your background, interest, and experience can contribute to this mission, as well as to the specific position for which you are applying.

Review of applications will begin immediately and continue until the position is filled. Position offers excellent state benefits. To request accommodations, call (201) 684-7718. Additional supportive materials in nonelectronic format may be sent to Dr. Maxim Goldberg-Kugalev, Search Committee chair, Ramapo College of New Jersey, 505 Ramapo Valley Road, Mahwah, NJ 07430.

Ramapo College is a member of the Council of Public Liberal Arts Colleges (COPLAC), a national alliance of leading liberal arts colleges in the public sector. EEO/AFFIRMATIVE ACTION.

BRAZIL

INSTITUTE FOR PURE AND APPLIED MATHEMATICS (IMPA)
Rio de Janeiro, Brazil

The Institute for Pure and Applied Mathematics (IMPA), invites applications for a two-year postdoctoral position in any field of mathematics, with a monthly take-home salary of R$6,000 (about US$3,500 at the current exchange rate). Candidates must have obtained their Ph.D. degrees after March 31, 2006. IMPA, located in Rio de Janeiro, Brazil, is widely recognized as one of the leading mathematical research centers worldwide. Its main goal is the generation of high-level mathematical

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services.

The 2010 rate is $3.25 per word. No discounts for multiple ads or the same ad in consecutive issues. For an additional $10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the “Positions Available” classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.


U.S. laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. “Positions Available” advertisements from institutions outside the U.S. cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to U.S. laws. Details and specific wording may be found on page 536 (vol. 56).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 world wide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02940; or via fax: 401-331-3842; or send email to classifieds@ams.org. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.
research. It offers also graduate level programs at the Ph.D. and MSc level. Currently, its faculty includes specialists in Real and Complex Dynamical Systems, Analysis, Algebra, Geometry, Probability, Fluid Dynamics, Optimization, Mathematical Economics, and Computer Graphics. Applications should be sent to: opening@impa.br until March 31, 2010. Further inquiries should be addressed to the same email address. For information on application submissions, see http://www.impa.br/openings/en/pesquisa/postdoc/opening.html.

KOREA

POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY (POSTECH)
Department of Mathematics
P-Assistant Professorship Positions

The Department of Mathematics at Pohang University of Science and Technology (POSTECH) solicits applications for non-tenure-track P-Assistant Professorship. All areas of pure and applied mathematics are considered. Applicants are expected to have demonstrated exceptional research potential, including major contributions beyond or through the doctoral dissertation. The applicants should be within the first 4 years of their research activities since gaining their Ph.D. degrees. No linguistic ability in Korean is required for successful candidates who are fluent in English.

The P-Assistant Professorship is a two-year position and is non-renewable. Exceptional research performance during the two years may warrant considerations for possible tenure-track appointment. The annual salary is up to 60,000,000 KRW. An on-campus apartment unit may be provided at a low cost. This position will remain open until it is filled.

Applicants should submit;
1. A signed cover letter
2. A curriculum vitae
3. A description of research
4. Three letters of recommendation sent directly by recommenders
5. Copies of publications including the Ph.D. dissertation

Please have your applications sent to:
Hyungju Park, Chair
Department of Mathematics
POSTECH
Pohang, 790-784 Korea;
tel:+82-54-279-2302;
fax:+82-54-279-2799;
email:alanpark@postech.ac.kr.

Inquiries can be made to:
Yongkyoo Lee (Mr.)
Department Administrative Manager;
email:ykyoo@postech.ac.kr.

POHANG UNIVERSITY OF SCIENCE AND TECHNOLOGY (POSTECH)
Department of Mathematics
Tenure-track Faculty Positions Available

The Department of Mathematics at Pohang University of Science and Technology (POSTECH) invites applications for several tenure-track faculty positions in any areas of Pure and Applied Mathematics including Statistics. The positions are primarily at the Assistant Professor level, although senior candidates with strong credentials will also be considered. The positions are open to mathematicians of any nationality who are fluent in English, and applications are accepted throughout the year. To be considered for Spring & Fall, they must be received by early January & early July, respectively. New faculty members are provided with a competitive benefits package and free use of faculty apartments for ten years.

POSTECH was founded in 1986 by the world-leading steel company, POSCO, and is an internationally renowned academic institution dedicated to education and research in science and engineering. Successful candidates are expected to establish strong research programs and to excel in teaching at both the undergraduate and graduate levels.

1. Qualification: Applicants must hold a Ph.D. in Mathematics or a related discipline with an outstanding research record and should be able to teach in English.
2. Applicants should submit
   a. A signed cover letter
   b. A curriculum vitae
   c. A description of research
   d. Three letters of recommendation sent directly by recommenders
   e. Copies of publications including the Ph.D. dissertation

For more information, please visit our webpage: POSTECH Homepage: http://www.postech.ac.kr.
Mathematics Department Homepage: http://math.postech.ac.kr.

Please have your package sent by email, fax, and regular mail, to
Hyungju Park, Chair
Department of Mathematics
POSTECH
Pohang, 790-784 Korea;
tel:+82-54-279-2302;
fax:+82-54-279-2799;
email:alanpark@postech.ac.kr.

• Email your new address to us: amsmem@ams.org
• or make the change yourself online at: www.ams.org/ml-update
• or send the information to:
  Member and Customer Services
  American Mathematical Society,
  201 Charles Street, Providence, RI
  02904-2294 USA
  Phone: (800) 321-4267
  (US & Canada)
  (401) 455-4000 (Worldwide)
Meetings & Conferences of the AMS

Lexington, Kentucky
University of Kentucky
March 27–28, 2010
Saturday – Sunday
Meeting #1057
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: January 2010
Program first available on AMS website: February 11, 2010
Program issue of electronic Notices: March 2010
Issue of Abstracts: Volume 31, Issue 2

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: January 26, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses
Percy A. Deift, Courant Institute–New York University, Open problems in integrable systems and random matrix theory.
Irina Mitrea, Worcester Polytechnic Institute, Recent progress in the area of elliptic boundary value problems on rough domains.
Bruce Reznick, University of Illinois at Urbana-Champaign, The secret lives of polynomial identities.
Bernd Ulrich, Purdue University, Title to be announced.
Doron Zeilberger, Rutgers University, 3x+1 (Erdős Memorial Lecture).

Special Sessions
Advances in Algebraic Coding Theory (Code: SS 6A), Heide Gluesing-Luerssen, University of Kentucky, and Jon-Lark Kim, University of Louisville.
Advances in Algebraic Statistics (Code: SS 2A), Sonja Petrović, University of Illinois, Chicago, and Ruriko Yoshida, University of Kentucky.
Advances in Algebraic and Geometric Combinatorics (Code: SS 14A), Richard Ehrenborg and Margaret A. Readdy, University of Kentucky.
Analysis and Control of Dispersive Partial Differential Equations (Code: SS 25A), Michael J. Goldberg and Bingyu Zhang, University of Cincinnati.
Combinatorial Algebra (Code: SS 7A), Juan C. Migliore, University of Notre Dame, and Uwe Nagel, University of Kentucky.
Commutative Algebra (Code: SS 1A), Alberto Corso, University of Kentucky, Claudia Polini, University of Notre Dame, and Bernd Ulrich, Purdue University.
Complex Analysis and Potential Theory (Code: SS 4A), James E. Brennan and Vladimir Eiderman, University of Kentucky.
Financial Mathematics and Statistics (Code: SS 22A), Kiseop Lee, University of Louisville, and Jose Figueroa-Lopez, Department of Statistics, Purdue University.
Function Theory, Harmonic Analysis, and Partial Differential Equations (Code: SS 5A), Joel Kilty, Centre College, Irina Mitrea, Worcester Polytechnic Institute, and Katharine Ott, University of Kentucky.
Geometric Function Theory and Analysis on Metric Spaces (Code: SS 3A), John L. Lewis, University of Kentucky, and Nageswari Shanmugalingam, University of Cincinnati.
Homotopy Theory and Geometric Aspects of Algebraic Topology (Code: SS 16A), Serge Ochanine, University of Kentucky, and Marian F. Anton, Centre College.

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the Notices. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See http://www.ams.org/meetings/. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the Notices as noted below for each meeting.
Interactions between Logic, Topology, and Complex Analysis (Code: SS 23A), Matt Insall, Missouri University of Science and Technology, and Malgorzata Marciniak, University of Toledo.
Inverse Problems, Riemann-Hilbert Problems, and Nonlinear Dispersive Equations (Code: SS 10A), Peter A. Perry, University of Kentucky, and Peter Topalov, Northeastern University.
Large Scale Matrix Computation (Code: SS 19A), Qiang Ye, University of Kentucky, and Lothar Reichel, Kent State University.
Mathematical Economics (Code: SS 21A), Adib Bagh and Robert E. Molzon, University of Kentucky.
Mathematical Problems in Mechanics and Materials Science (Code: SS 20A), Michel E. Jabbour and Chi-Sing Man, University of Kentucky, and Kazumi Tanuma, Gunma University.
Mathematical String Theory (Code: SS 18A), Al Sharpe, Department of Physics and Astronomy, University of Kentucky, Eric Sharpe, Physics Department, Virginia Polytechnic Institute and State University, and Mark A. Stern, Duke University.
Mathematics Outreach (Code: SS 26A), Carl W. Lee and David C. Royster, University of Kentucky.
Matroid Theory (Code: SS 9A), Jakayla Robbins, University of Kentucky, and Xiangqian Zhou, Wright State University.
Multivariate and Banach Space Polynomials (Code: SS 15A), Richard A. Aron, Kent State University, and Lawrence A. Harris, University of Kentucky.
Noncommutative Algebraic Geometry (Code: SS 24A), Dennis S. Keeler and Kimberly Retert, Miami University.
Partial Differential Equations in Geometry and Variational Problems (Code: SS 8A), Luca Capogna, University of Arkansas, and Changyou Wang, University of Kentucky.
Recent Progress in Numerical Methods for Partial Differential Equations (Code: SS 12A), Alan Demlow, University of Kentucky, and Xiaobing H. Feng, University of Tennessee at Knoxville.
Relative Homological Algebra (Code: SS 11A), Edgar E. Enochs, University of Kentucky, and Alina C. Iacob, Georgia Southern University.
Sharp Spectral Estimates in Analysis, Geometry, and Probability (Code: SS 17A), Richard S. Laugesen and Bartłomiej Siudeja, University of Illinois.
Spectral and Transport Properties of Schrödinger Operators (Code: SS 13A), Peter D. Hislop, University of Kentucky, and Jeffrey H. Schenker, Michigan State University.

St. Paul, Minnesota
Macalester College
April 10–11, 2010
Saturday - Sunday

Meeting #1058
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: February 2010
Program first available on AMS website: February 25, 2010
Program issue of electronic Notices: April 2010
Issue of Abstracts: Volume 31, Issue 2

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: February 16, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses
Charles Doering, University of Michigan, Title to be announced.
Matthew James Emerton, Northwestern University, Title to be announced.
Vladimir Touraev, Indiana University, Title to be announced.
Peter Webb, University of Minnesota, Title to be announced.

Special Sessions
Applications of a Geometric Approach to Chaotic Dynamics (Code: SS 16A), Evelyn Sander, George Mason University, Judy Kennedy, Lamar University, and James Yorke, University of Maryland.
Cohomology and Representation Theory of Algebraic Groups and Related Structures (Code: SS 7A), Christopher Bendel, University of Wisconsin-Stout, Bobbe Cooper, University of Minnesota, and Terrell Hodge, Western Michigan University.
Combinatorial Representation Theory (Code: SS 3A), Tom Halverson, Macalester College, and Victor Reiner, University of Minnesota.
Commutative Ring Theory (Code: SS 5A), Michael Axtell, University of St. Thomas, and Joe Stickles, Millikin University.
Differential Equations and Applications (Code: SS 15A), Nicolai Tarfulea, Purdue University Calumet, and Catalin Turc, Case Western Reserve University.
Fractals, Convolution Measures, and Frames (Code: SS 13A), Keri Kornelson, University of Oklahoma, and Karen Shuman, Grinnell College.
Meetings & Conferences

**Geometric Flows, Moving Frames and Integrable Systems**
(Code: SS 8A), **Gloria Mari-Beffa**, University of Wisconsin-Madison, and **Peter Olver**, University of Minnesota.

**Hecke Algebras and Deformations in Geometry and Topology**
(Code: SS 11A), **Matthew Douglass** and **Anne Shepler**, University of North Texas.

**Mathematical Developments in Cell and Systems Biology**
(Code: SS 14A), **Anastasios Matzavinos**, Iowa State University, and **Nicoleta Eugenia Tarfulea**, Purdue University Calumet.

**Matrices and Graphs**
(Code: SS 9A), **Luz M. DeAlba**, Drake University, **Adam Berliner**, St. Olaf College, **Leslie Hogben**, Iowa State University, and **In-Jae Kim**, Minnesota State University.

**Partition Theory and the Combinatorics of Symmetric Functions**

**Pattern Formation in Biological Systems**
(Code: SS 12A), **Magdalena Skolarska**, University of St. Thomas, and **Chad Topaz**, Macalester College.

**Physical knotting and Linking and its Applications**
(Code: SS 10A), **Eric Rawden**, University of St. Thomas, **Yuanan Diao**, University of North Carolina at Charlotte, and **Claus Ernst**, Western Kentucky University.

**Probabilistic and Extremal Combinatorics**
(Code: SS 2A), **Ryan Martin** and **Maria Axenovich**, Iowa State University.

**Quantum Invariants of 3-manifolds and Modular Categories**
(Code: SS 1A), **Thang Le**, Georgia Institute of Technology, **Eric Rowell**, Texas A&M University, and **Vladimir Touraev**, Indiana University.

**Universal Algebra and Order**

**Accommodations**
Participants should make their own arrangements directly with the hotel. When making a reservation identify yourself as being with the AMS-Macalester College meeting. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation or early checkout penalties; be sure to ask for details when making your reservation.**

A special hotel rate has been negotiated at the **Crowne Plaza Saint Paul-Riverfront Hotel**. The room rate is US$109 per night, plus occupancy tax. Please be advised to make your reservations as soon as possible. **This special rate is valid while rooms are available up to and including March 15, 2010.** Participants can reserve their room through the direct link below, or by calling 651-292-1900 or 866-422-3185 and mentioning that they will be attending the AMS-Macalester College meeting. A complimentary continental breakfast is included with this room rate.

Link to hotel reservations at the group rate: https://secure.ichotelsgroup.com/h/d/cp/1/en/cwshome/DPRD-7X8K3X/MSP57?secure=true&secure=true&justLoggedln=true&requestid=638587.

The Crowne Plaza is located in downtown Saint Paul, an area with many restaurants and museums. The hotel is 4 miles from Macalester College, where the meeting will take place. The meeting will provide free bus transportation to Macalester College on Saturday and Sunday morning and back to the hotel on Saturday evening. There are also public buses between the hotel and Macalester which run every half hour on Saturday and every hour on Sunday. For those with a car, parking is available at the hotel for US$10 per day, and it is an easy 12-minute drive to Macalester along Saint Paul’s Grand Avenue.

**Food Service**
The Campus Center Dining Hall will be open for lunch and dinner on Saturday and Sunday. All-you-can-eat-meals cost approximately US$8. Macalester is located on Saint Paul’s well-known Grand Avenue, which offers many varied dining opportunities. A list of nearby restaurants is available at http://www.macalester.edu/admissions/campusvisit/restaurant.html. A list of local restaurants will be available at the registration desk.

**Local Information and Campus Map**
For further information please consult the following sites:
- **Campus Map**: http://www.macalester.edu/about/mapbynumber.html
- **Macalester College**: http://www.macalester.edu
- **The Macalester College Math Department**: http://www.macalester.edu/mathcs
- **Grand Avenue**: http://www.grandave.com/
- **Visit Saint Paul**: http://www.visitsaintpaul.com/

**Other Activities**
**AMS Book Sale**: Stop by the on-site AMS Bookstore—review the newest titles from the AMS, enter the FREE book drawing, enjoy up to 25% off all titles or even take home the new AMS T-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

**AMS Editorial Activity**: An acquisitions editor from the AMS Book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

**Parking**
Parking is readily available on campus and is free. We recommend that you park in the lot just south of the Leonard Center accessible via Snelling Avenue or in the lots just west of Olin-Rice or west of the Janet Wallace Fine Arts Center, both of which are accessed via Macalester Drive. See the campus map: http://www.macalester.edu/about/mapbynumber.html.

**Social Event**
The Department of Mathematics will host a reception for all participants. The reception will be held from 6:15 p.m. to 7:45 p.m. in the Small Gallery of Olin-Rice Hall. Hearty appetizers, beer, and wine will be served. The AMS thanks the department for its hospitality.

**Registration and Meeting Information**
Registration will take place in Olin-Rice Hall, from 7:30 a.m. to 4:00 p.m. on Saturday, April 10, and 8:00 a.m. to noon on Sunday, April 11. The invited addresses will
take place in the J. B. Davis Lecture Hall of the Campus Center. Most Special Sessions will take place in Olin-Rice Hall and the others will take place in the Humanities Building, Carnegie Hall, and Old Main.

Registration fees: (payable on-site only) US$40/AMS members; US$60/nonmembers; US$5/emeritus members, students, or unemployed mathematicians. Fees are payable by cash, check, VISA, Mastercard, Discover, or American Express.

Travel
Macalester College is located on Saint Paul’s Grand Avenue in a pleasant neighborhood which is 4 miles west of downtown Saint Paul and 6 miles east of downtown Minneapolis.

By Air:
All major airlines service the Minneapolis-Saint Paul International Airport (MSP). The airport is 6.3 miles from Macalester College and 8.5 miles to the Crowne Plaza Hotel in downtown St. Paul.

Cab fares from the airport to Macalester cost about US$20 and from the airport to the Crowne Plaza cost about US$25.

Super Shuttle (http://www.supershuttle.com) costs about US$14 each way between the airport and the Crowne Plaza.

Metro Transit (http://metrorail.org) provides public bus options. The 54 bus from the airport to the Crowne Plaza takes about 25 minutes. Buses from the airport to Macalester take 35–45 minutes and require at least one transfer.

If you rent a car, it is easy to drive to the Crowne Plaza: Follow the signs for St. Paul/Highway 5. Hwy 5 turns into 7th Street in Saint Paul. Travel along Hwy 5 (7th Street) for 6.7 miles until you reach Kellogg Blvd. Turn right on Kellogg and travel for .5 miles until you reach the hotel at 11 E. Kellogg Blvd.

Driving from the airport to Macalester: Follow the signs for St. Paul/Highway 5. After you cross the river, exit right onto Edgecumbe Road (keep left as you exit) and follow for 1 mile; stay to the left and take Fairview Avenue. Follow Fairview Avenue for 1.5 miles. Turn right (east) on Saint Clair Avenue and go .5 miles to Snelling Avenue. Turn left (north) on Snelling, travel .25 miles to the Leonard Center parking lot.

By Car
Driving from the East or the West: Take interstate I94 to Exit 238 at Snelling Avenue. Travel 1 mile south on Snelling Avenue to the corner of Snelling and Grand (you are now at the corner of campus). Continue .25 miles further south along Snelling to reach the Leonard Center parking lot, just before the football stadium.

Driving from the South: Take interstate I35 to I35E. Take Exit 104A at Randolph Avenue. Turn left (west) and travel 1 mile along Randolph Avenue until you reach Snelling Avenue. Turn right (north) on Snelling and travel .8 miles. The Leonard Center parking lot is accessible from Snelling Avenue just after the football stadium.

Car Rental
Avis is the official car rental company for the sectional meeting in Albuquerque. All rates include unlimited free mileage. Weekend daily rates are available from noon Thursday to Monday at 11:59 p.m. Rates do not include any state or local surcharges, tax, optional coverages, or gas refueling charges. Renters must meet Avis’s age, driver, and credit requirements. For the best available rate and to make a reservation please call Avis at 800-331-1600 or go online at http://www.avis.com. Please use the AMS meeting Avis Discount Number J098887.

Weather
April weather in Saint Paul is unpredictable. The average high for the month is 58 degrees F and the average low is 37 degrees F. It has been known to snow in mid April and it has been known to be warm, bright, and sunny. For up-to-the-minute weather please visit http://www.weather.com/outlook/driving/interstate/local/55105.

Information for International Participants
Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the United States found at http://www7.nationalacademies.org/visas/Traveling_to_US.html and http://travel.state.gov/visa/index.html. If you need a preliminary conference invitation in order to secure a visa, please send your request to wsd@ams.org.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of “binding” or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:
  - family ties in home country or country of legal permanent residence
  - property ownership
  - bank accounts
  - employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor’s home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location, and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

Meeting & Conferences
**Meetings & Conferences**

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

**Albuquerque, New Mexico**

*University of New Mexico*

**April 17–18, 2010**

Saturday – Sunday

**Meeting #1059**

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of Notices: February 2010

Program first available on AMS website: March 4, 2010

Issue of Abstracts: Volume 31, Issue 3

**Deadlines**

For organizers: Expired

For consideration of contributed papers in Special Sessions: Expired

For abstracts: February 23, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

**Invited Addresses**

- **Kenneth Bromberg**, University of Utah, *Title to be announced.*
- **Danny Calegari**, California Institute of Technology, *Title to be announced.*
- **Ioana Dumitriu**, University of Washington, *Title to be announced.*
- **Steffen Rohde**, University of Washington, *Title to be announced.*

**Special Sessions**

- *Dyadic and Non-Dyadic Harmonic Analysis* (Code: SS 2A), **M. Cristina Pereyra**, University of New Mexico, and **Stephanie A. Salomone**, University of Portland.

- **Geometric Structures and PDEs** (Code: SS 8A), **Charles Boyer** and **Dimitor Vassilev**, University of New Mexico.
- **Kleinian Groups and Teichmüller Theory** (Code: SS 15A), **Kasra Rafi**, University of Oklahoma, **Hossein Namazie**, University of Texas, and **Kenneth Bromberg**, University of Utah.
- **Positivity in Noncommutative Settings** (Code: SS 12A), **Roger Roybal**, California State University Channel Islands, and **Terry Loring**, University of New Mexico.
- **Selected Topics in Analysis and Numerics for PDEs** (Code: SS 11A), **Thomas Hagstrom**, Southern Methodist University, and **Stephen Lau** and **Jens Lorenz**, University of New Mexico.
- **Subjects in between Pure and Applied Mathematics** (Code: SS 7A), **Hanna Makaruk** and **Robert Owczarek**, Los Alamos National Laboratory.
- **Topics in Geometric Group Theory** (Code: SS 1A), **Matthew Day**, California Institute of Technology, **Daniel Peter Groves**, University of Illinois at Chicago, **Jason Manning**, SUNY at Buffalo, and **Henry Wilton**, California Institute of Technology.
- **Trends in Commutative Algebra** (Code: SS 3A), **Louiza Fouli**, New Mexico State University, and **Janet Vassilev**, University New Mexico.

**Accommodations**

Participants should make their own arrangements directly with the hotel. When making a reservation identify yourself as being with the UNM Math and Stat group attending the AMS Meeting. The AMS is not responsible for rate changes or for the quality of the accommodations. Hotels have varying cancellation or early checkout penalties; be sure to ask for details when making your reservation.

**Plaza Inn Downtown Albuquerque**

900 Medical Arts Ave NE, Albuquerque, NM 87102; phone 505-243-5693 or 866-925-7881. Rates start at US$56.99 + tax. **Deadline for reservations is March 26, 2010.** Be sure to check the cancellation policy. Amenities include: Shuttle to/from airport and UNM from 6:30 a.m. to 11 p.m., complimentary continental breakfast, complimentary high-speed wireless Internet access in public areas and guest rooms, hot tub, pool, restaurant. The hotel is approximately one mile to campus. For more information please visit http://www.plazainnabq.com/.
Albuquerque Airport Fairfield Inn by Marriott, 2300 Centre Ave. SE, Albuquerque, NM 87106, phone: 505-247-1621. Price: US$59 + tax. Deadline for reservations is March 19, 2010. Be sure to check the cancellation policy. Amenities include: shuttle to/from airport and UNM, continental breakfast, coffee/tea in room, outdoor pool & spa, complimentary breakfast, wired high-speed Internet in guest rooms. The hotel is approximately 1.5 miles to campus.

Food Service
A list of local restaurants will be available at the registration desk.

Local Information and Campus Map
For further information please consult the website maintained by the Department of Mathematics at the University of New Mexico: http://www.math.unm.edu. To view a campus map please visit: http://www.unm.edu/campusmap.html. Dane Smith Hall is building 48 at F-G-3. For travel information please visit: http://www.math.unm.edu/about/index.php.

Other Activities
AMS Book Sale: Stop by the on-site AMS Bookstore—review the newest titles from the AMS, enter the FREE book drawing, enjoy up to 25% off all titles or even take home the new AMS T-shirt! Complimentary coffee will be served courtesy of AMS Membership Services.

AMS Editorial Activity: An acquisitions editor from the AMS Book program will be present to speak with prospective authors. If you have a book project that you would like to discuss with the AMS, please stop by the book exhibit.

Parking
Visitors can park anywhere on campus on weekends (6:00 p.m. Friday until 8:00 a.m. Monday) without a permit. City parking rules still apply. For further information please visit http://pats.unm.edu/visitors.cfm.

Registration and Meeting Information
Registration will take place in the atrium of Dane Smith Hall located on Las Lomas Blvd., across from University House on Yale Avenue, from 7:30 a.m. to 4:00 p.m. on Saturday, April 17, and 8:00 a.m. to noon on Sunday, April 18.

Registration fees: (payable on-site only) US$40/AMS members; US$60/nonmembers; US$5/emeritus members, students, or unemployed mathematicians. Fees are payable by cash, check, VISA, Mastercard, Discover, or American Express.

Travel
By Air:
Albuquerque Sunport International Airport: Albuquerque is served by many of the major commercial carriers and several commuter airlines. The Sunport is located two (2) miles south of UNM.

Traveling to Albuquerque by car: Albuquerque is served by two major interstates, I25 (North-South) and I40 (East-West). The Martin Luther King exit 1 mile south of the I25-I40 interchange allows access to the Plaza Inn Albuquerque (take the service road north from Martin Luther King, on the east side of I25) and UNM (go east to University Blvd., enter campus, turn right on Redondo Rd. and follow it to visitor parking or residence halls). The Albuquerque Airport Fairfield Inn by Marriott is located off of Yale (east of Yale) between the airport and UNM.

Transportation from/to the Sunport: Shuttle and taxi service available after all flights outside at baggage claim level. Albuquerque Cab (505-883-4888, typical charge US$18.00), Sunport provides a shuttle to Rental Car Center. Call 505-247-1621 for the free shuttle to Albuquerque Airport Fairfield Inn by Marriott, and 505-243-5693 for the free shuttle to Plaza Inn Albuquerque.

Getting around Albuquerque is easiest by car, but the city has regular bus service along Central Ave. from downtown to UNM for participants who wish to stay downtown, see http://www.cabq.gov/transit/routes-and-schedules. Taxi service is available but best arranged beforehand.

Car Rental
Avis is the official car rental company for the sectional meeting in Albuquerque. All rates include unlimited free mileage. Weekend daily rates are available from noon Thursday to Monday at 11:59 p.m. Rates do not include any state or local surcharges, tax, optional coverages, or gas refueling charges. Renters must meet Avis’s age, driver, and credit requirements. For the best available rate and to make a reservation please call Avis at 800-331-1600 or go online at http://www.avis.com. Please use the AMS meeting Avis Discount Number J098887.

Weather
April weather is generally pleasant with daytime temperatures in the 60 degree F. range, and nighttime temperatures in the 30–45 degree F. range. For up-to-the-minute weather please visit http://www.weather.com/look/driving/local/USNM0004.

Information for International Participants
Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the U.S. found at http://www7.nationalacademies.org/visas/Traveling_to_US.html and http://travel.state.gov/visa/index.html. If you need a preliminary conference invitation in order to secure a visa, please send your request to dls@ams.org.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of “binding” or sufficient ties to their home country or permanent
residence abroad. This may include documentation of the following:
- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;
- Visa applications are more likely to be successful if done in a visitor’s home country than in a third country;
- Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;
- Include a letter of invitation from the meeting organizer or the U.S. host, specifying the subject, location, and dates of the activity, and how travel and local expenses will be covered;
- If travel plans will depend on early approval of the visa application, specify this at the time of the application;
- Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Newark, New Jersey

New Jersey Institute of Technology

May 22–23, 2010
Saturday – Sunday

Meeting #1060
Eastern Section

Associate secretary: Steven H. Weintraub
Announcement issue of Notices: March 2020
Program first available on AMS website: April 8, 2010
Program issue of electronic Notices: May 2020
Issue of Abstracts: Volume 31, Issue 3

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: February 2, 2010
For abstracts: March 30, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Simon Brendle, Stanford University, Hamilton’s Ricci flow and the sphere theorem in geometry.
Konstantin M. Mishaikov, Rutgers University, Computational topology applied to the global dynamics of nonlinear systems.
Ricardo H. Nochetto, University of Maryland, Curvature driven flows in deformable domains.

Richard E. Schwartz, Brown University, Polygonal outer billiards.

Special Sessions

Automorphic Forms, L-functions, and Applications (Code: SS 6A), Ameya Pitale, American Institute of Mathematics, and Anantharam Raghuram, Oklahoma State University.

Biomembranes: Modeling, Analysis, and Computation (Code: SS 16A), Ricardo H. Nochetto and Dionisis Margetis, University of Maryland.

Elliptic and Parabolic Problems in Geometry (Code: SS 12A), Simon Brendle, Stanford University, and Mu-Tao Wang, Columbia University.

Expandable Computations, Algorithms, Methodologies and Experiments for Engineering Interpretation (Code: SS 1A), Mustapha S. Fofana, Worcester Polytechnic Institute, Marie D. Dahleh, Harvard School of Engineering and Applied Sciences, Harvard University, and Kenji Kawashima, Precision and Intelligence Laboratory, Tokyo Institute of Technology.


Graph Theory (Code: SS 10A), Nathan W. Kahl, Seton Hall University, Michael J. Ferrara, University of Colorado at Denver, and Arthur H. Busch, University of Dayton.

Groups, Computations, and Applications (Code: SS 2A), Delaram Kahrobaei, City University of New York.

Homology Theories for Knots and Skein Modules (Code: SS 3A), Mikhail Khovanov, Columbia University, and Jozef H. Przytycki and Radmila Sazdanovic, George Washington University.

Invariants of Knots, Links, and 3-Manifolds (Code: SS 4A), Abhijit Champanerkar and Ilya S. Kofman, College of Staten Island, CUNY, and Philip J. P. Ording, Medgar Evers College, CUNY.

Lie Algebras and Representation Theory (Code: SS 8A), Gautam Chinta, City College, City University of New York, Andrew Douglas, New York City College of Technology, City University of New York, and Bart Van Steirteghem, Medgar Evers College, City University of New York.

Logic and Groups (Code: SS 17A), Peggy Dean, Claire Wladis, and Marcos Zyman, Borough of Manhattan Community College, City University of New York.


Mathematics and Computations of Fluid Dynamics (Code: SS 15A), Yuan N. Young, New Jersey Institute of Technology.

Mathematics of Optics and Matter Waves (Code: SS 13A), Roy Goodman, New Jersey Institute of Technology.

Nonlinear Waves (Code: SS 19A), A. David Trubatch, Montclair State University.

Recent Trends in Cayley Graphs to Model Interconnection Networks (Code: SS 18A), Daniela Ferrero, Texas State University, and Beth Novick, Clemson University.

Teichmüller Theory, Hyperbolic Geometry, and Complex Dynamics (Code: SS 5A), Zheng Huang, College of Staten Island, CUNY, and Ren Guo, University of Minnesota.
Topological and Computational Dynamics (Code: SS 7A), Jean-Philippe Lessard, Institute for Advanced Study and Rutgers University, and Konstantin M. Mishaikow, Rutgers University.

Vortex Dynamics: Theory and Applications (Code: SS 11A), Denis Blackmore, New Jersey Institute of Technology, Morten Brons, Technical University of Denmark, and Chjran Lim, Rochester Polytechnic Institute.

Berkeley, California
University of California Berkeley

June 2–5, 2010
Wednesday – Saturday

Meeting #1061
Eighth Joint International Meeting of the AMS and the Sociedad Matemática Mexicana.
Associate secretary: Susan J. Friedlander
Announcement issue of Notices: March 2010
Program first available on AMS website: April 22, 2010
Program issue of electronic Notices: June 2010
Issue of Abstracts: Volume 31, Issue 3

Deadlines
For organizers: Expired
For consideration of contributed papers in Special Sessions: February 16, 2010
For abstracts: April 13, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses
Alejandro Adem, University of British Columbia and PIMS, Title to be announced.
Peter W-K Li, University of California Irvine, Title to be announced.
Ernesto Lupercio, CINVESTAV, Title to be announced.
Victor Perez Abreu, CIMAT, Title to be announced.
Alberto Verjovsky, IM-UNAM, Title to be announced.
Maciej Zworski, University of California Berkeley, Title to be announced.

Special Sessions
Algebraic Topology and Related Topics (Code: SS 3A), Alejandro Adem, University of British Columbia, Gunnar E. Carlsson and Ralph L. Cohen, Stanford University, and Ernesto Lupercio, CINVESTAV.
Analytic Aspects of Differential Geometry (Code: SS 2A), Nelia Charalambous, ITAM, Lizhen Ji, University of Michigan, and Jiaping Wang, University of Minnesota.


Dynamical Systems (Code: SS 4A), Alberto Verjovsky, IM-UNAM, and Rodrigo Perez, Indiana University-Purdue University, Indianapolis.

Graph Theory and Combinatorics with Emphasis on Geometric and Topological Aspects (Code: SS 9A), Gelasio Salazar, Instituto de Fisica, Universidad Autonoma de San Luis Potosi, and Dan S. Archdeacon, University of Vermont.

Harmonic Analysis, Microlocal Analysis, and Partial Differential Equations (Code: SS 1A), Gunther Uhlmann, University of Washington, and Salvador Perez Esteava, UNAM.
Low-Dimensional Topology (Code: SS 8A), Kenneth L. Baker, University of Miami, and Enrique Ramirez Losada, CIMAT.

Singularity Theory and Algebraic Geometry (Code: SS 6A), David Eisenbud, University of California, Berkeley, Anatoly S. Libgober, University of Illinois at Chicago, Jose Seade, UNAM, and Xavier Gomez-Mont, CIMAT.

Toeplitz Operators and Discrete Quantum Models (Code: SS 5A), Alejandro Uribe, University of Michigan, and Maciej Zworski, University of California, Berkeley.

Syracuse, New York
Syracuse University

October 2–3, 2010
Saturday – Sunday

Meeting #1062
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: To be announced
Program first available on AMS website: August 19, 2010
Program issue of electronic Notices: October
Issue of Abstracts: Volume 31, Issue 4

Deadlines
For organizers: March 2, 2010
For consideration of contributed papers in Special Sessions: June 15, 2010
For abstracts: August 10, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses
Alan Frieze, Carnegie-Mellon University, Title to be announced.
Yan Guo, Brown University, Title to be announced.
William Minicozzi, Johns Hopkins University, Title to be announced.
Andrei Zelevinsky, Northeastern University, Title to be announced.

Special Sessions

Difference Equations and Applications (Code: SS 2A), Michael Radin, Rochester Institute of Technology.

Graphs Embedded in Surfaces, and Their Symmetries (Code: SS 4A), Jack E. Graver and Mark E. Watkins, Syracuse University.

Mathematical Image Processing, Lixin Shen and Yuesheng Xu, Syracuse University.

Nonlinear Analysis and Geometry (Code: SS 1A), Tadeusz Iwaniec, Leonid V. Kovalev, and Jani Onninen, Syracuse University.

Several Complex Variables (Code: SS 3A), Dan F. Coman and Evgeny A. Poletsky, Syracuse University.

Los Angeles, California

University of California Los Angeles

October 9–10, 2010
Saturday – Sunday

Meeting #1063
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: August 2010
Program first available on AMS website: August 26, 2010
Program issue of electronic Notices: October 2010
Issue of Abstracts: Volume 31, Issue 4

Deadlines
For organizers: March 10, 2010
For consideration of contributed papers in Special Sessions: June 22, 2010
For abstracts: August 17, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtg/sectional.html.

Invited Addresses

Greg Kuperberg, University of California Davis, Title to be announced.

Cris Moore, University of New Mexico, Title to be announced.

Stanley Osher, University of California Los Angeles, Title to be announced.

Terence Tao, University of California Los Angeles, Title to be announced (Einstein Public Lecture in Mathematics).

Melanie Wood, Princeton University, Title to be announced.

Special Sessions

Applications of Nonlinear PDE (Code: SS 5A), Susan J. Friedlander and Igor Kukavica, University of Southern California.

Combinatorics and Probability on Groups (Code: SS 3A), Jason Fulman and Robert Guralnick, University of Southern California, and Igor Pak, University of California Los Angeles.

Extremal and Probabilistic Combinatorics (Code: SS 4A), Benny Sudakov, University of California Los Angeles, and Jacques Verstraete, University of California San Diego.

Large Cardinals and the Continuum (Code: SS 2A), Matthew Foreman, University of California Irvine, Aleks Kechris, California Institute for Technology, Itay Neeman, University of California Los Angeles, and Martin Zeman, University of California Irvine.

Recent Trends in Probability and Related Fields (Code: SS 6A), Marek Biskup, University of California Los Angeles, Yuval Peres, Microsoft Research, and Sebastien Roch, University of California Los Angeles.

Topology and Symplectic Geometry (Code: SS 1A), Robert Brown and Ciprian Manolescu, University of California Los Angeles, and Stefano Vidussi, University of California Riverside.

Notre Dame, Indiana

Notre Dame University

October 29–31, 2010
Friday – Sunday

Meeting #1064
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: August 2010
Program first available on AMS website: September 16, 2010
Program issue of electronic Notices: October 2010
Issue of Abstracts: Volume 31, Issue 4

Deadlines
For organizers: February 19, 2010
For consideration of contributed papers in Special Sessions: July 20, 2010
For abstracts: September 7, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtg/sectional.html.

Invited Addresses

Laura DeMarco, University of Illinois at Chicago, Title to be announced.

Jordan Ellenberg, University of Wisconsin, Title to be announced.

David Fisher, Indiana University, Title to be announced.
Meetings & Conferences

Jared Wunsch, Northwestern University, Title to be announced.

Special Sessions

Commutative Algebra and Its Interactions with Algebraic Geometry (Code: SS 2A), Claudia Polini, University of Notre Dame, Alberto Corso, University of Kentucky, and Bernd Ulrich, Purdue University.

Groups, Representations, and Characters (Code: SS 4A), James P. Cossey, University of Akron, and Mark Lewis, Kent State University.

Hilbert Functions in Commutative Algebra and Algebraic Combinatorics (Code: SS 3A), Fabrizio Zanello, Michigan Technological University, Juan Migliore, University of Notre Dame, and Uwe Nagel, University of Kentucky.

Interdisciplinary Session on Deterministic and Stochastic Partial Differential Equations (Code: SS 5A), Nathan Glatt-Holtz, Indiana University, and Vlad Vicol, University of Southern California.

Quasigroups, Loops, and Nonassociative Division Algebras (Code: SS 6A), Clifton E. Ealy, Western Michigan University, Stephen Gabola, University of Arizona, Julia Knight, University of Notre Dame, J. D. Phillips, Northern Michigan University, and Petr Vojtechovsky, University of Denver.

Singularities in Algebraic Geometry (Code: SS 1A), Nero Budur, University of Notre Dame, and Lawrence Ein, University of Illinois at Chicago.

Richmond, Virginia

University of Richmond

November 6–7, 2010
Saturday – Sunday

Meeting #1065
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: September
Program first available on AMS website: September 23, 2010
Program issue of electronic Notices: November Issue of Abstracts: Volume 31, Issue 4

Deadlines
For organizers: March 8, 2010
For consideration of contributed papers in Special Sessions: July 27, 2010
For abstracts: September 14, 2010

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

AMS Invited Addresses

Ricardo Baeza, Universidad de Talca, Chile, Title to be announced.

Igor Dolgachev, University of Michigan, Title to be announced.

Andres Navas, Universidad de Santiago de Chile, Title to be announced.

Rodolfo Rodriguez, Universidad de Concepcion, Title to be announced.

Gunther Uhlmann, University of Washington, Title to be announced.

S. R. Srinivasa Varadhan, New York University, Title to be announced.

AMS Special Sessions

Arithmetic of Quadratic Forms and Integral Lattices. (Code: SS 6A), Maria Ines Icaza, Universidad de Talca, Chile, Wai Kiu Chan, Wesleyan University, and Ricardo Baeza, Universidad de Talca, Chile.

Automorphic Forms and Dirichlet Series (Code: SS 5A), Yves Martin, Universidad de Chile, Chile, and Solomon Friedberg, Boston College.
Meetings & Conferences

Complex Algebraic Geometry (Code: SS 1A), Giancarlo Urzua and Eduardo Cattani, University of Massachusetts.
Foliations and Dynamics (Code: SS 4A), Andrés Navas, Universidad de Santiago de Chile, and Rostislav Grigorchuk, University of Texas.
Group Actions: Probability and Dynamics (Code: SS 3A), Andrés Navas, Universidad de Santiago de Chile, and Rostislav Grigorchuk, University of Texas.
Non-Associative Algebras (Code: SS 2A), Alicia Labra, Universidad de Chile, and Kevin McCrimmon, University of Virginia.

New Orleans, Louisiana
New Orleans Marriott and Sheraton New Orleans Hotel
January 5–8, 2011
Wednesday – Saturday
Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: October 2010
Program first available on AMS website: November 1, 2010
Program issue of electronic Notices: January 2011
Issue of Abstracts: Volume 32, Issue 1

Deadlines
For organizers: April 1, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Iowa City, Iowa
University of Iowa
March 18–20, 2011
Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: July 16, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Worcester, Massachusetts
College of the Holy Cross
April 9–10, 2011
Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: September 9, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Statesboro, Georgia
Georgia Southern University
March 12–13, 2011
Saturday – Sunday
Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: August 12, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Las Vegas, Nevada
University of Nevada
April 30 – May 1, 2011
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced
Meetings & Conferences

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions
Advances in Modeling, Numerical Analysis and Computations of Fluid Flow Problems (Code: SS 2A), Monika Neda, University of Nevada Las Vegas.

Geometric PDEs (Code: SS 1A), Matthew Gursky, Notre Dame University, and Emmanuel Hebey, Universite de Cergy-Pontoise.

Lincoln, Nebraska
University of Nebraska-Lincoln
October 14–16, 2011
Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: August 2011
Program first available on AMS website: To be announced
Program issue of electronic Notices: October 2011
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Salt Lake City, Utah
University of Utah
October 22–23, 2011
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Program issue of electronic Notices: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boston, Massachusetts
John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel
January 4–7, 2012
Wednesday – Saturday
Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic Notices: January 2012
Issue of Abstracts: Volume 33, Issue 1

Deadlines
For organizers: April 1, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California
San Diego Convention Center and San Diego Marriott Hotel and Marina
January 9–12, 2013
Wednesday – Saturday
Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of Notices: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic Notices: January 2012
Issue of Abstracts: Volume 34, Issue 1

Deadlines
For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced
Baltimore, Maryland

**Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor**

**January 15–18, 2014**

Wednesday – Saturday

Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller

Announcement issue of Notices: October 2013

Program first available on AMS website: November 1, 2013

Program issue of electronic Notices: January 2013

Issue of Abstracts: Volume 35, Issue 1

**Deadlines**

For organizers: April 1, 2013

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

---

Seattle, Washington

**Washington State Convention & Trade Center and the Sheraton Seattle Hotel**

**January 6–9, 2016**

Wednesday – Saturday

Joint Mathematics Meetings, including the 122nd Annual Meeting of the AMS, 99th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Michel L. Lapidus

Announcement issue of Notices: October 2015

Program first available on AMS website: To be announced

Program issue of electronic Notices: January 2016

Issue of Abstracts: Volume 37, Issue 1

**Deadlines**

For organizers: April 1, 2015

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

---

San Antonio, Texas

**Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio**

**January 10–13, 2015**

Saturday – Tuesday

Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub

Announcement issue of Notices: October 2014

Program first available on AMS website: To be announced

Program issue of electronic Notices: January 2015

Issue of Abstracts: Volume 36, Issue 1

**Deadlines**

For organizers: April 1, 2014

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

---

Atlanta, Georgia

**Hyatt Regency Atlanta and Marriott Atlanta Marquis**

**January 4–7, 2017**

Wednesday – Saturday

Joint Mathematics Meetings, including the 123rd Annual Meeting of the AMS, 100th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Georgia Benkart

Announcement issue of Notices: October 2016

Program first available on AMS website: To be announced

Program issue of electronic Notices: January 2017

Issue of Abstracts: Volume 38, Issue 1

**Deadlines**

For organizers: April 1, 2016

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced
Meetings and Conferences of the AMS

### Associate Secretaries of the AMS

**Western Section:** Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

**Central Section:** Georgia Benkart (after January 31, 2010), University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

**Eastern Section:** Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

**Southeastern Section:** Matthew Miller, Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

---

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated. Up-to-date meeting and conference information can be found at [www.ams.org/meetings/](http://www.ams.org/meetings/).

### Meetings:

#### 2010

- **March 27–28**  
  Lexington, Kentucky  
  p. 314
- **April 10–11**  
  St. Paul, Minnesota  
  p. 315
- **April 17–18**  
  Albuquerque, New Mexico  
  p. 318
- **May 22–23**  
  Newark, New Jersey  
  p. 320
- **June 2–5**  
  Berkeley, California  
  p. 321
- **October 2–3**  
  Syracuse, New York  
  p. 321
- **October 9–10**  
  Los Angeles, California  
  p. 322
- **October 29–31**  
  Notre Dame, Indiana  
  p. 322
- **November 6–7**  
  Richmond, Virginia  
  p. 323
- **December 15–18**  
  Pucon, Chile  
  p. 323

#### 2011

- **January 5–8**  
  New Orleans, Louisiana  
  Annual Meeting  
  p. 324
- **March 12–13**  
  Statesboro, Georgia  
  p. 324
- **March 18–20**  
  Iowa City, Iowa  
  p. 324
- **April 9–10**  
  Worcester, Massachusetts  
  p. 324
- **April 30–May 1**  
  Las Vegas, Nevada  
  p. 324
- **October 14–16**  
  Lincoln, Nebraska  
  p. 325
- **October 22–23**  
  Salt Lake City, Utah  
  p. 325

#### 2012

- **January 4–7**  
  Boston, Massachusetts  
  Annual Meeting  
  p. 325

#### 2013

- **January 9–12**  
  San Diego, California  
  Annual Meeting  
  p. 325

#### 2014

- **January 15–18**  
  Baltimore, Maryland  
  Annual Meeting  
  p. 326

#### 2015

- **January 10–13**  
  San Antonio, Texas  
  Annual Meeting  
  p. 326

#### 2016

- **January 6–9**  
  Seattle, Washington  
  Annual Meeting  
  p. 326

#### 2017

- **January 4–7**  
  Atlanta, Georgia  
  Annual Meeting  
  p. 326

### Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 92 in the January 2010 issue of the Notices for general information regarding participation in AMS meetings and conferences.

### Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of \LaTeX is necessary to submit an electronic form, although those who use \LaTeX may submit abstracts with such coding, and all math displays and similarly coded material (such as accent marks in text) must be typeset in \LaTeX. Visit [http://www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl). Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

---

### Conferences:

(see [http://www.ams.org/meetings/](http://www.ams.org/meetings/) for the most up-to-date information on these conferences.)

Co-sponsored conferences:

- **February 18–22, 2010:** AAAS Meeting in San Diego, CA (please see [www.aaas.org/meetings](http://www.aaas.org/meetings) for more information).
- **March 18–21, 2010:** First International Conference on Mathematics and Statistics, AUS-ICMS 10, American University of Sharjah, Sharjah, United Arab Emirates (please see [http://www.aus.edu/conferences/icms10/](http://www.aus.edu/conferences/icms10/) for more information).
- **May 24–29, 2010:** From Carthage to the World, the Isoperimetric Problem of Queen Dido and its Mathematics Ramifications, Carthage, Tunisia (for more information please see [http://math.arizona.edu/~dido/welcome.html](http://math.arizona.edu/~dido/welcome.html)).
- **June 17-19, 2010:** Coimbra Meeting on 0-1 Matrix Theory and Related Topics, University of Coimbra, Portugal (for more information please see [http://www.mat.uc.pt/~cmf/01MatrixTheory](http://www.mat.uc.pt/~cmf/01MatrixTheory)).
Bayesian Nonparametrics
Edited by Nils Lid Hjort, Chris Holmes, Peter Müller, and Stephen G. Walker
Cambridge Series in Statistical and Probabilistic Mathematics

Second Edition!
Smooth Compactifications of Locally Symmetric Varieties
Avner Ash, David Mumford, Michael Rapoport, and Yung-sheng Tai
Cambridge Mathematical Library

Logical Foundations of Proof Complexity
Stephen Cook and Phuong Nguyen

Geometry of Riemann Surfaces
Edited by Frederick P. Gardiner, Gabino González-Diez, and Christos Kourouniotis
London Mathematical Society Lecture Note Series

Zariski Geometries
Geometry from the Logician’s Point of View
Boris Zilber
London Mathematical Society Lecture Note Series

Now in Paperback!
An Algebraic Introduction to K-Theory
Bruce A. Magurn
“This volume is a very useful graduate algebra text with an orientation towards algebraic K-theory....This volume will form an excellent basis for several types of one-and two-semester graduate algebra courses.”
-Mathematical Reviews
Encyclopedia of Mathematics and its Applications

Stochastic Control and Mathematical Modeling
Applications in Economics
Hiroaki Morimoto
Encyclopedia of Mathematics and its Applications

Locally Convex Spaces over Non-Archimedean Valued Fields
C. Perez-García and W. H. Schikhof
Cambridge Studies in Advanced Mathematics

Epidemics and Rumours in Complex Networks
Moez Draief and Laurent Massoulié
London Mathematical Society Lecture Note Series

Prices subject to change.

www.cambridge.org/us/mathematics • 800-872-7423

CAMBRIDGE UNIVERSITY PRESS
A SELECTION OF THE BEST-SELLING AMS TITLES IN 2009

What's Happening in the Mathematical Sciences
Dana Mackenzie
Volume 7; 2009; 127 pages; Softcover; ISBN: 978-0-8218-4478-6; List US$19.95; AMS members US$15.95; Order code HAPPENING/7

Introduction to Analysis
Fifth Edition
Edward D. Gaughan, New Mexico State University, Las Cruces, NM
Pure and Applied Undergraduate Texts, Volume 1; 1998; 240 pages; Hardcover; ISBN: 978-0-8218-4787-9; List US$62; AMS members US$50; Order code AMSTEXT/1

Lectures on Quantum Mechanics for Mathematics Students
L. D. Faddeev, Steklov Mathematical Institute, St. Petersburg, Russia, and O. A. Yakubovskii, St. Petersburg University, Russia

A Decade of the Berkeley Math Circle
The American Experience, Volume 1
Zvezdelina Stankova, Mills College, Oakland, CA, and University of California, Berkeley, CA, and Tom Rike, Oakland High School, CA, Editors
Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).
MSRI Mathematical Circles Library, Volume 1; 2008; 326 pages; Softcover; ISBN: 978-0-8218-4683-4; List US$49; AMS members US$39; Order code MCL/1

Markov Chains and Mixing Times
David A. Levin, University of Oregon, Eugene, OR, Yuval Peres, Microsoft Research, Redmond, WA, and University of California, Berkeley, CA, and Elizabeth L. Wilmer, Oberlin College, OH

Quantum Mechanics for Mathematicians
Leon A. Takhtajan, Stony Brook University, NY

Tools of the Trade
Introduction to Advanced Mathematics
Paul J. Sally, Jr., University of Chicago, IL

Discrete Differential Geometry
Integrable Structure
Alexander I. Bobenko, Technische Universität Berlin, Germany, and Yuri B. Suris, Technische Universität München, Garching bei München, Germany

Mathematics and Music
David Wright, Washington University, St. Louis, MO

The Geometry of Heisenberg Groups
With Applications in Signal Theory, Optics, Quantization, and Field Quantization
Ernst Binz and Sonja Pods, University of Mannheim, Germany

Pioneering Women in American Mathematics
The Pre-1940 PhD’s
Judy Green, Marymount University, Arlington, VA, and Jeanne LaDuke, DePaul University, Chicago, IL
Co-published with the London Mathematical Society beginning with Volume 4. Members of the LMS may order directly from the AMS at the AMS member price. The LMS is registered with the Charity Commissioners.

Class Field Theory
Emil Artin, and John Tate, University of Texas at Austin, TX

www.ams.org/bookstore