2010 Chern Medal Awarded

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Louis Nirenberg

On August 19, 2010, the first Chern Medal Award was presented at the opening ceremonies of the International Congress of Mathematicians (ICM) in Hyderabad, India. The awardee is Louis Nirenberg of the Courant Institute of Mathematical Sciences, New York University. He was honored "for his role in the formulation of the modern theory of nonlinear elliptic partial differential equations and for mentoring numerous students and postdocs in this area."

The Chern Medal Award was instituted for 2010 in memory of the outstanding mathematician Shiing-Shen Chern. It will be awarded jointly with the Chern Medal Foundation (CMF) to an individual whose lifelong outstanding achievements in the field of mathematics warrant the highest level of recognition.

Shiing-Shen Chern devoted his life to mathematics, both in active research and education, and to nurturing the field whenever the opportunity arose. He obtained fundamental results in all the major aspects of modern geometry and founded the area of global differential geometry. Chern exhibited keen aesthetic tastes in his selection of problems, and the breadth of his work deepened the connections of modern geometry with different areas of mathematics.

The award consists of a medal and a monetary prize of US$500,000. It is required that half of the award be donated to organizations of the recipient's choice to support research, education, outreach, or other activities to promote mathematics. Chern was generous during his lifetime in his personal support of the field, and it is hoped that this philanthropy requirement for the promotion of mathematics will set the stage and the standard for mathematicians to continue this generosity on a personal level.

Louis Nirenberg is undoubtedly one of the outstanding analysts of the twentieth century. His work has had major influence in the development of different areas of mathematics and their applications. In particular, he has been a leader in most of the developments in the theory of linear and nonlinear partial differential equations (PDEs) and related areas of analysis, the basic mathematical tools of modern science. PDEs arise in physics and geometry when systems depend on several variables simultaneously, and the most interesting ones are nonlinear. The importance of PDEs is clear in the fact that, of the seven million-dollar Millennium Problems posed by the Clay Mathematics Institute, three are in or are related to PDEs. Nirenberg's work in PDEs is deep and fundamental. He developed intricate connections between analysis and differential geometry and applied them to the theory of fluid phenomena and other physical processes.

Nirenberg's thesis concerned a fundamental issue in geometry: the solution to an embedding problem in differential geometry that Hermann Weyl had posed around 1916. *Given a Riemannian metric on a unit sphere with positive Gauss curvature, can you embed this two-sphere isometrically in three-space as a convex surface?* The thesis itself was a window to Nirenberg's main interests—PDEs, and especially elliptic PDEs. In giving a positive answer to Weyl's question, Nirenberg used ideas due to Charles Morrey. He solved the problem by reducing it to a problem in nonlinear PDE. This equation is also an elliptic PDEs. (In elliptic PDEs, the coefficients satisfy a positivity condition, and these have applications in almost all areas of mathematics, as well as numerous applications in physics. As with a general PDE, an elliptic PDE may have nonconstant coefficients and be nonlinear. The basic example of an elliptic PDE is Laplace's equation.)

As mathematician J. Mawhin has pointed out in his recent tribute to Nirenberg, ellipticity is a key word in Nirenberg's mathematical work. More than one-third of Nirenberg's articles contain the word "elliptic" in their titles, and a much larger fraction of them deal with elliptic equations or systems. "There is hardly any aspect of those equations he has not considered," Mawhin said. Despite their variety, elliptic PDEs have a well-developed theory to which Nirenberg contributed in large measure. In 1953, the first year in which he published, he published three further papers, two of which involved nonlinear elliptic PDEs, to which he returned in later years.

Through the next twenty years, with several collaborators, he developed the theory of elliptic equations satisfying certain regularity criteria that he had formulated. Although proofs for the existence and uniqueness of weak solutions to elliptic problems were well known, Nirenberg addressed the much more difficult question of how regular (or well behaved) the weak solution is. (A weak solution to a PDE is one for which the derivatives appearing in the equation may not all exist but which nevertheless is deemed to satisfy the equation in a certain dual sense.) Today, Nirenberg's method of differences for proving the interior and boundary regularity is part of a graduate student's education in PDEs.

A high point in his research in this area is the extension of this work, which was done in collaboration with Shmuel Agmon and Avron Douglis on a priori estimates for general linear elliptic systems. This is one of the most widely quoted results in analysis. As Nirenberg has said, the objective was to obtain "general estimates for general systems under general boundary conditions". The citation for the Steele Prize for Lifetime Achievement, which he received from the American Mathematical Society (1994), said: "Nirenberg is a master of the art and science of obtaining and applying a priori estimates in all fields of analysis."

Essential questions about regularity for Navier-Stokes equations, whose solutions determine fluid motion, are still open, and it is one of the Clay Institute's Millennium Problems. Among the best results today is the Caffarelli-Kohn-Nirenberg estimate of the measure of the set of singularities. Nirenberg feels that the problem will be solved soon but that it will require more input from harmonic analysis.

Building on earlier estimates of Alberto Calderón and Antoni Zygmund, he and Joseph Kohn introduced the concept of a pseudodifferential operator, a generalization of the idea of parametrix for a partial differential operator, which is useful in addressing the question of regularity of solutions to elliptic boundary value problems. They were addressing a problem that involved singular integral operators (integral operators that are not mathematically defined at a point), and they needed facts about certain properties of singular integral operators, but these were not available in the literature. So they developed what they needed, and what resulted was the extremely useful notion of a pseudodifferential operator. This has helped generate copious developments. Another important work was the one with François Treves on the solvability of general linear PDEs. Some other highlights are his research on the regularity of free boundary problems with David Kinderlehrer and Joel Spruck. Such problems have found a wide range of applications, including flame propagation.

As mentioned earlier, Nirenberg, a pioneer in nonlinear PDEs, has turned his attention again at various stages to fully nonlinear equations and made striking breakthroughs. An example is the series of papers on the existence of smooth solutions of equations of Monge-Ampère type, which are nonlinear second-order PDEs with special symmetries, with Luis Caffarelli and Spruck. Nirenberg's study of symmetric solutions of nonlinear elliptic equations using moving plane methods, with Basilis Gidas and Wei Ming Ni, and later with Henri Berestycki, is an ingenious application of the maximum principle. Mawhin has called Nirenberg a "Paganini of the maximum principle". Thanks to Nirenberg's ideas and methods, this has developed into a beautiful theory and has led to applications in combustion theory.

The following famous quotations by Nirenberg are indicative of his approach to handling nonlinearity; in Nirenberg's view, the problem determines the method. In his invited lecture at the Stockholm International Congress of Mathematicians (ICM 1962), he said: "Most results for nonlinear problems are still obtained via linear ones, despite the fact that the problems are nonlinear and not because of it." In the same lecture he also said, commenting on someone's result, "The nonlinear character of the equation is used in an essential way; indeed, he obtains results because of the nonlinearity and not despite it."

Nirenberg's wide range of interests also includes differential geometry and topology, in which he has made significant contributions. In harmonic analysis a function of bounded mean oscillation (BMO) is a real-valued function whose mean oscillation is bounded (finite). Motivated by Fritz John's earlier work in elasticity, Nirenberg, in association with John, investigated for the first time the topology of the space of such functions and gave a precise definition for it. The space is also sometimes called the John-Nirenberg space. The results were crucial for later work of Charles Fefferman on Hardy spaces. This function space has subsequently been used in many parts of analysis and in martingale theory. More recently, motivated by some nonlinear problems in physics, Nirenberg returned to this field and, in collaboration with Haim Brezis, investigated the space of functions with vanishing mean oscillation (VMO). The VMO functions form a predual for the first Hardy space. This function space is actually a subspace of BMO. Nirenberg and Brezis extended the "degree theory" in topology to mappings belonging to VMO, a result that took topologists by surprise. A fundamental question in the study of complex manifolds is: *When is an almost complex structure given by a complex structure?* A manifold is a topological space that can be locally described in terms of simpler, well-understood spaces such as Euclidean spaces. For a manifold to be a complex manifold (in which operations with imaginary numbers can be defined), the existence of an almost complex structure is necessary but not sufficient. (An almost complex manifold is a smooth manifold equipped with a structure that, roughly speaking, defines operations with imaginary numbers on each tangent space, the differentiable manifold attached to every point.) That is, every complex manifold is an almost complex manifold, but not vice versa.

In 1957 Nirenberg, with his student August Newlander, proved a fundamental result that answered this question, which had been open for many years. According to Nirenberg, André Weil and Shiing-Shen Chern had drawn his attention to the problem of proving the integrability condition for almost complex structures. Its resolution paved the way for its use in the study of many aspects of complex manifolds, particularly deformation theory. Although the problem is linear, Newlander and Nirenberg's proof reduces the problem to a system of nonlinear PDEs such that each equation involves derivatives with respect to only one complex variable. Nirenberg's tremendous insight into the properties of PDEs and his unique ability to connect PDEs, analysis, and geometry runs through all of his work.

Inequalities have had a special attraction for Nirenberg, and there are several important inequalities associated with his name. A very important result is the set of Gagliardo-Nirenberg inequalities. The AMS citation called it a "minor gem". Nirenberg's love for inequalities comes from his long association with Kurt Friedrichs at the Courant Institute of Mathematical Sciences at New York University, which Nirenberg joined after receiving his bachelor's degree in physics and mathematics from McGill University in 1945. Though he wanted to go into physics, thanks to an excellent physics teacher at McGill, he turned to mathematics on the advice of Richard Courant. Nirenberg has said that Friedrichs was a major influence on him and that Friedrichs's views of mathematics very much formed his view.

"Friedrichs was a great lover of inequalities, and that affected me very much. The point of view was that inequalities are more interesting than equalities, the identities," Nirenberg said in an interview with AMS. Elsewhere he has said, "I love inequalities. So if somebody shows me a new inequality, I say: Oh! that is beautiful, let me think about it." He had wanted to study for his Ph.D. under Friedrichs but instead studied under Jim Stoker, finishing in 1949. But Friedrichs's influence is clearly visible in Nirenberg's choice of problems, which are often drawn from physics. He does not distinguish between "pure" and "applied" mathematics, an attitude that has resulted from spending his entire career at Courant, "where there is just mathematics and people are interested in pure and applied problems," a heritage of Courant and Friedrichs. Though Nirenberg has said that he is more of a problem solver than one who develops theories, his approach to problems has resulted in the formulation of theories in different areas of mathematics.

Nirenberg is recognized for his excellent lectures and lucid expository writing. He has published over 185 papers and has had 46 students and 245 descendants, according to the Mathematics Genealogy Project. In each of the past ten years, the top fifteen cited papers in mathematics have included at least two by Nirenberg, according to MathSciNet. The fact that nearly 90 percent of Nirenberg's papers are written collaboratively shows how Nirenberg, for over six decades, has shared his knowledge and mathematical insight with mathematicians from all over the world. He is also known to be very generous and enthusiastically presents the results of other mathematicians in his lectures and survey papers.

Kohn has said this of him: "Nirenberg's career has been an inspiration; his numerous students, collaborators, and colleagues have learned a great deal from him. Aside from mathematics, Nirenberg has taught all of us the enjoyment of travel, movies, and gastronomy. An appreciation of Nirenberg also must include his ever-present sense of humor. His humor is irrepressible, so that on occasion it makes its way to the printed page."

In his AMS interview, Nirenberg said: "I wrote one paper with Philip Hartman that was elementary but enormous fun to do. That's the thing I try to get across to people who don't know anything about mathematics: what fun it is! One of the wonders of mathematics is you go somewhere in the world and you meet other mathematicians, and it's like one big family. This large family is a wonderful joy."

Louis Nirenberg was born in 1925 in Hamilton, Ontario, Canada. After receiving his bachelor's degree from McGill University in 1945, he obtained his M.S. (1947) and Ph.D. (1949) degrees from New York University. Nirenberg then joined the faculty of NYU. He was one of the original members of the Courant Institute of Mathematical Sciences. After spending his entire academic career at Courant, he retired in 1999 and is now emeritus professor. His distinctions include the 1959 Bôcher Prize of the AMS, the Jeffrey-Williams Prize of the Canadian Mathematical Society in 1987, and the AMS Steele Prize for lifetime achievement in 1994. In 1982 he (along with the late V. I. Arnold) was the first recipient in mathematics of the Crafoord Prize, established by the Royal Swedish Academy of Sciences in areas not covered by the Nobel Prizes. In 1995 he received the National Medal of Science, the highest honor in the United States for contributions to science.

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