One in a Million—Remembrances of Laurie Snell (1925–2011)

Dan Rockmore

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Laurie Snell

When I arrived at Dartmouth College in the fall of 1991, at the recommendation of my thesis advisor, Persi Diaconis, one of the first people I looked up was Laurie Snell. Like many people (as I was later to find out), I was surprised when I discovered that in fact "Laurie" was not a woman, but rather, a man—a tall, amiable, and wonderfully friendly man who happened to know a lot about probability besides having coauthored my favorite book on Markov chains. Over twenty years we became the best of friends and from time to time collaborators too on aspects of his *Chance* project, to which he devoted much of his energies during his emeritus years. I miss him deeply.

Below is a stroll through Laurie's life, decorated by the reminiscences of several of Laurie's friends and colleagues and even by some of Laurie's own stories. What comes through is a man of great intelligence, humility, and warmth, a lover of tradition, and a loyal friend, but also a person of endless curiosity and boundless energy, a fine maker of mathematics but perhaps an even better communicator of the subject. At the heart of any great communicator is a great storyteller, and one of Laurie's chief joys was hearing and telling tales about family and friends—some mathematicians, others not—often over a languid evening of dinner, drinks, and always dessert. I like to think that he would have enjoyed reading this—or even better, hearing it, surrounded by laughter, deeply happy in what was his favorite environment, the warm glow of friendship.

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Before Dartmouth

It seems as if Laurie was always at Dartmouth College, but in fact he had a life before that. Born James Laurie Snell on January 15, 1925, he grew up in Wheaton, Illinois, the youngest of the three children (all boys) of Lucille Ziegler Snell and Roy Judson Snell. Lucille was a piano teacher and performer (Laurie had a lifelong love of music). Roy was a very successful writer, best known as the author of mystery and adventure books for boys and girls (seventy-eight in all!) and a scriptwriter for the *Jack Armstrong, The All-American Boy* radio show.

Laurie received his B.A. from the University of Illinois, served for several years in the navy, and then returned to Illinois for graduate studies, receiving his Ph.D. under the direction of the famous probabilist J. L. Doob. Laurie and Doob stayed in close touch over the years, and Laurie forever affectionately referred to Doob as "my teacher". Laurie's thesis [7] focused on extending to submartingales results by Doob related to various martingale convergence questions. A discrete time martingale is a sequence of random variables with finite expectation such that if we view the sequence as the "fortune" of a game (or gamble), then (in Laurie's own words) "a martingale is a fair game, a

supermartingale represents an unfavorable game and a submartingale a favorable game. [8]"

Laurie recalls in his obituary for Doob [8] how he came to work with "the great man": *Doob kept a card file of ideas for theses. When he got a new graduate student he would pull out a card and suggest the problem on the card. If the student could not solve it, Doob put it back in the file and chose the next card…I succeeded on the third card, which proposed extending to submartingales an inequality called the 'upcrossing inequality' that Doob proved for martingales…It turned out to be easy to think of the submartingale as the price of the stock…Actually it turned out to be too easy because Doob said that a thesis had to be at least 30 pages long, so I would have to find something else to pad it with.*

The best-known result in Laurie's thesis is "the Snell Envelope", the smallest supermartingale dominating the given payoff process or, as Laurie's longtime friend and colleague Peter Doyle puts it, "Everyone knows that you should quit when you are ahead. To decide whether you are far enough ahead to quit, you compute the Snell Envelope." The Snell Envelope continues to play an important role in the analysis of many stochastic processes.

From Urbana, at Doob's urging, Laurie took a three-year position at Princeton as a Fine Instructor, mainly to be around legendary probabilist William Feller and a young Al Tucker, who at that time was just inventing linear programming. Life in Princeton was not all mathematics. Laurie was always a very sociable person and chance brought him a lifelong companion, Joan. They were engaged just three weeks after meeting. Joan was a great match for Laurie: a fine musician, fluent in French, and as befits her deep interest in and degree in English literature, an outstanding editor. Joan was often the first and last reader of Laurie's books, and no edition of *Chance News* (more on that later) left Laurie's computer without Joan's final proofreading.

While at Princeton, Laurie learned from his calculus students that in the philosophy department there was a young assistant professor who often posed probability problems for his classes. Laurie recalled that he told his students that "Philosophers did not know mathematics, so their professor's solutions were probably wrong." He goes on to say, "I later learned that their professor was John Kemeny."[1] Even a probabilist loses a bet from time to time.

Early Years at Dartmouth—Life with Kemeny[2]

Kemeny was lured away from Princeton by the challenge and charge to rebuild the Dartmouth mathematics department, and Laurie was one of the first hires. In those days before the interstate highway system, Dartmouth was both isolated and insulated, and this close environment certainly served to quickly cement Laurie and Kemeny's productive collaboration and deep friendship, which expanded to include the families, which for the Snells eventually included a son, John, and a daughter, Mary Paige. Some of the most important mathematical work that Laurie was to do at Dartmouth was related to various generalizations of Markov theory, much of it accomplished with Kemeny. Kemeny and Snell wrote some joint papers in probability in the late 1950s, and their books on "finite mathematics" led them to a special interest in Markov chains with a finite state space, summarized in their 1960 classic, *Finite Markov Chains* [2]. Important examples of Markov chains, however, also involved a countably infinite number of states, and Kemeny and Snell turned their attention to this entire class of chains, which are called *denumerable*.

In the important case in which all states communicate, they are either all recurrent or all transient. For the transient case, there is a potential theory and there is a boundary theory. The potential theory is modeled on the relationship between Brownian motion and the classical potential theory associated with static electrical charges (signed measures) in $\mathbf{R}^3$. The theories are linked (and are effectively equivalent) via the interpretation of the potential kernel in terms of the distributions for Brownian motion.

For a transient Markov chain with transition matrix P, the analog of the potential kernel is the matrix $\sum_{n=0}^{\infty} P^n$. Doob and G. A. Hunt developed a potential theory in this setting in the late 1950s. The theory does not work in the recurrent case. A modification is needed to have a recurrent potential theory, and Kemeny and Snell developed this theory in a collection of papers in 1961, at the same time redoing the transient case in matrix notation and showing the parallels of the recurrent and transient theories.

Creation of a boundary theory was also inspired by the classical case. Therein a boundary theory for a nice bounded region of $\mathbf{R}^n$ writes each positive harmonic function in the region as the integral over the boundary of the product $K(x,b)\,d\mu(b)$, where $d\mu$ is the boundary value measure for that harmonic function and where the Poisson kernel $K(x,b) = (\partial/\partial n)G(x,b)$ is the outward normal derivative of the potential kernel at the boundary point b. For general regions, possibly with a messy topological boundary, the classical theory has to be redone completely. The reworked theory is due to R. S. Martin and appeared in 1941. Analogs for Markov chains of Martin's boundary theory were produced independently by Doob and T. Watanabe, and by G. Hunt in the early 1960s. Kemeny and Snell, along with a precocious Dartmouth undergraduate Tony Knapp (later a professor of mathematics at Cornell and SUNY Stony Brook where he's now emeritus, as well as a former editor of the *Notices*), collected

[1] *Personal communication: "Memory of Kemeny" (2011), p. 10.*

[2] *Thanks to Anthony Knapp for providing much of the technical material in this section.*

these results, as well as a recurrent boundary theory of their own in a book *Denumerable Markov Chains* [3], which developed as an outgrowth of an undergraduate course on these matters for which Tony (as research assistant to Laurie) was a note taker. Knapp and Hans Föllmer (emeritus professor of mathematics at the Humboldt University of Berlin) recall those early Dartmouth days:

> *A. Knapp:* By 1959 when I was a freshman, except for three of the old guard who remained, all the other faculty were under forty. The place was a beehive of education, and the faculty and students functioned as an extended family. Faculty members made the mathematics majors a part of their lives—hanging out with them, hiring them as babysitters, inviting them to their homes as dinner guests, and so on, and generally caring about their lives beyond mathematics. I think of Snell as the chief architect of this atmosphere.
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> The department moved into its own new building in 1961, with Kemeny in a corner office and with Snell still at his right hand. Multiple books were in progress in the department. Snell would sing in the hall, and one could hear him approaching from a long way off. As he would enter the lounge, he would sometimes call out, "Who's for a game of Hearts?" He was *very* good at Hearts.
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> Kemeny and Snell had completely different personalities, yet they had great confidence in each other. Neither could have succeeded as well without the other. Snell was full of ideas, had a thorough intuitive grasp of probability, and had a command of the literature. Kemeny was a dynamo, able to work out long complex proofs and find just the right example to illustrate matters.
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> *H. Föllmer:* I first set eyes on Laurie Snell in Paris in the fall of 1965. Laurie was giving lectures on the potential theory of Markov chains at the Institut Henri Poincaré, and I had just arrived from Göttingen to study in Paris for a year. Laurie gave his lectures in French. Grammatically, his sentences were perfect; they had been carefully checked by his wife, Joan, who is fluent in French.
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> At that time I never talked to Laurie, but after his lectures I often saw him from afar in one of the nearby cafés. There he loved to play pinball with two of the French students, Jacques Azéma and Daniel Revuz, who went on to become major figures in French probability. Much later Laurie told me that he had found another source of great joy in Paris, namely, the singing lessons he was taking from a well-known French teacher.
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> The first time I actually spoke with Laurie Snell was in the spring of 1970, when I was an instructor at MIT. One day I received a phone call from Laurie, to my great surprise. He invited me to give a talk at Dartmouth College, and so I took a Greyhound bus up to New Hampshire. After the talk Laurie invited me for a beer on the porch of his house in Norwich. He was quite amused to hear that I had actually been a student in his class in Paris, and I was immediately taken in by his dry and self-deprecating New England sense of humor. So, when some weeks later I received an offer for an instructor position at Dartmouth, I accepted. The two years at Dartmouth turned out to be a great experience.
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> At that time the atmosphere in probability at Dartmouth was very stimulating, thanks to Laurie Snell, John Lamperti, and Itrel Monroe; to some very bright students that included David Griffeath and David Kreps; and to visitors such as Joe Doob and Frank Spitzer. In the late 1960s R. L. Dobrushin and Y. Sinai had made a major breakthrough in understanding the probabilistic structure of phase transitions in statistical mechanics. Frank Spitzer had started to work on closely related problems for interacting particle systems. Laurie immediately recognized the importance of this new development and reacted with great and highly contagious enthusiasm. Laurie's main concern, however, was to make this new field accessible to a wider audience, including students at an early stage. This focus is reflected in his book with Ross Kindermann [5].
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> I went back to Germany in 1972, but I always kept in touch with Laurie. The main reason was that Laurie was such a wonderful host and mentor, and in the end a close friend. For several years I returned to Dartmouth in the summer, and Laurie visited me in Bonn, Zürich, and Berlin. Over the years I was increasingly impressed to see how he maintained his youthful enthusiasm, especially for his ongoing project *Chance*. I will greatly miss these encounters, including his oft-repeated "Don't be silly."

Laurie's best-known book from that era (coauthored with Kemeny and Gerald Thompson) was *Introduction to Finite Mathematics* [4]. This was the first book to take seriously the applications of discrete mathematics in the social sciences. It eventually sold over 100,000 copies, and, in some sense, has never been matched as a textbook for mathematics in the social sciences. The review by D. Gale on MathSciNet says, "The book under review takes an entirely new tack. Instead of elementary analysis the mathematical subject matter is 'finite

mathematics' (actually certain branches of algebra). Instead of physical applications the book draws its illustrative material from the biological and social sciences…This book is more than original. It is unique."

The royalties from *Finite Mathematics* gave Laurie the ability to realize a lifelong dream of his: to create a performance space for amateur musicians and singers akin to Tanglewood, albeit on a smaller scale. "The Barn" on the property of the Snells' newly acquired house in Norwich, Vermont, was the center of local summer social life for several years. Faculty (including Laurie, singing), neighbors, visitors, and soon-to-be famous musicians all graced its stage. It was a visual and acoustic delight, the latter with the help of a team from New York's Metropolitan Opera that was at that time assisting in the construction of Dartmouth College's new Hopkins Center for the Arts. This was all for the pleasure of a local audience, seated in front of the stage in yellow captain's chairs, snacking on the wine and cheese that Laurie bought as part of his ritual preparations.

Chance

There are many people in the world who know Laurie only as the engine of *Chance News*, launched in 1992 and delivered at your electronic doorway approximately monthly until 2005, when Laurie finished the lengthy process of migrating the publication to Wiki form,[3] where it now lives, administered by his longtime friend and former student, Bill Peterson, professor of mathematics at Middlebury College. *Chance News* was in many ways a blog before there were blogs, at first providing extended abstracts of statistically interesting items from the news and evolving into a collection of more extended statistical and probabilistic investigations of news and other "real-life" stories, including many from Laurie's own life. The articles in *Chance News* provide terrific material for introductory probability and statistics courses. While friends and colleagues (most notably Bill Peterson, Jeanne Albert, Charles Grinstead, Peter Doyle, and Peter Kostelec) would from time to time share a story or help out on a problem, the vast majority of the work was done by Laurie. According to Tom Moore, during a Q&A period for one of the sessions at one of the Joint Statistical Meetings during the mid-1990s when the newsletter was just beginning to gain attention, a colleague from another institution got up to tell everyone about *Chance News* and what a wonderful resource it was. As detailed by Tom, the fellow described it clearly imagining a very large and impressive staff behind it, apparently unaware that it was being produced entirely by Laurie with a small support staff at Dartmouth and few regular contributors.

When it started, *Chance News* was a revolutionary use of the Web in terms of math pedagogy. Its continued relevance and importance are witnessed by a 2011 CAUSEweb Resource Award[4] presented to Bill Peterson, Jeanne Albert, and posthumously to Laurie. When *Chance News* started it was just Laurie's latest foray into using new technologies in teaching, a theme of his life that can be seen as far back as the significant use of BASIC in early editions of his *Introduction to Probability* (followed by his insistence that a second edition, published by the AMS [1], also be available for free on the Web). It continued in Laurie's pioneering work in integrating video and text for Web viewing (for the *Chance Lectures* that he and I organized in the late 1990s), and finally, in his last significant *Chance*-related work, when he migrated it into its current Wiki format. Had he been just a little younger or lived just a little longer, I'm sure that you'd be able to follow *Chance* on Twitter today.

Chance News was part of the larger *Chance* project that Laurie led [6], built around the idea of using real-world case studies to teach statistics. *Chance* courses were taught at Princeton, Dartmouth, and a host of other institutions and still live on today in many places. For these and other efforts, in 1996 Laurie was named a Fellow of the American Statistical Association.[5] Bill Peterson has this to say about the *Chance* years:

> After I began teaching at Middlebury College in 1989, I was reconnected with Laurie when he phoned our department to find out who was teaching statistics there. He was then nearing retirement and, as he described it, looking for a computing project in statistics that would parallel what he had done with probability and finite mathematics. We were intrigued by the success of a new Springer-Verlag magazine, *Chance* (now jointly published with the American Statistical Association), which had attracted leading statistics practitioners to describe their work in a form accessible to educated nonspecialists. This struck us as a promising model for a new course for liberal arts students that would introduce statistical topics in the context of real-world case studies, developing the necessary mathematical ideas only as needed.

[3] http://www.causeweb.org/wiki/chance/index.php/Main_Page

[4] *CAUSE is the Consortium for the Advancement of Undergraduate Statistics Education. The presentation was made at USCOTS 2011: United States Conference On Teaching Statistics in Raleigh-Durham, NC, on May 20, 2011.*

[5] *The official citation reads: "Laurie Snell, Benjamin Cheney Professor of Mathematics Emeritus, Dartmouth College: For an outstanding body of research in probability and its applications in the social sciences; for exemplary expository writing; and for curricular innovation and inspirational leadership as a teacher of probability and statistics extending over four decades."*

During the 1991–1992 academic year, the first *Chance* courses were offered. I taught a first-year seminar at Middlebury, following the case-study style we had originally planned. Meanwhile, Laurie was visiting Princeton University, where he cotaught a version of the course with Peter Doyle. Witnesses would later describe scenes of Peter and Laurie in the coffee room in the morning, poring over the day's news and selecting topics for discussion.

In fall 1992, Laurie produced the first issue of *Chance News* and also developed an NSF grant proposal to extend the *Chance* model to other institutions. With the additional funding, the project group expanded to include Joan Garfield from the University of Minnesota, Tom Moore from Grinnell College, and Nagambal Shah of Spelman College. Laurie became a traveling ambassador for the enterprise, coteaching *Chance* courses in a variety of formats at a variety of institutions.

Laurie officially retired from teaching in 1995, but his activities on the *Chance* project continued at full pace. He hosted a series of five-day summer workshops at Dartmouth for college faculty. Through a national application process, each of these drew about two dozen statistics educators interested in teaching their own versions of *Chance*. Laurie was a wonderfully generous host. He organized festive meals and after-workshop activities that included outdoor concerts and group hikes in the New Hampshire mountains. The participants appreciated these efforts, but above all they commented on how remarkable it was that such a well-known mathematician would take such a personal interest in their ideas and projects.

Final Thoughts

If you were to pick your mathematical specialization in order to maximize the number of people that you'd get to talk to about your work or the number of tales you'd be able to tell, probability and statistics would win. So, if Laurie was to be a mathematician, he had to be a probabilist, and *Chance* was the perfect capstone to his career. As his former colleague and student Claudia Henrion says, "Many people think of mathematics as something done alone, tucked away in an office with pen, paper, and now computer. But as Laurie Snell exemplified, it can also be a life filled with community, friendships, generosity and playfulness. These qualities all came together in his development of the *Chance* course, where he combined the theory of mathematics with people's everyday lives: events in the news, sports, gambling, and medicine."

As Laurie wound down his involvement with *Chance*, in his last few months he turned his attention to finally setting down as many of his stories as he could in one last volume, *Memory of Kemeny*. The last few weeks of his life were filled with visits by friends and several great dinners out and long stays with family. He played this last hand with his characteristic grace, humility, and good humor. To all of those whom he touched, Laurie was one in a million. Just by getting to know him, we all feel as though we've won the lottery.

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