Professional Development in Teaching for Mathematics Graduate Students

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"My 'professional development' before going into a classroom as a TA was 'Here's the book, here's a syllabus, show up Monday. Turn in grades in December and don't sleep with any students in between.' Since then, I've gotten pretty good at handling teaching, but it has taken awhile."

"I was very lucky to get the mentoring in teaching I received when I started as a TA. I learned to teach well quickly, and didn't need much help after my first year."

"I am so thankful to have received the on-going professional development I had as a TA throughout my doctoral program. As I enter my first faculty position I'm confident in my abilities to adapt my teaching to a new environment and a new student population because of it."

Figure 1. Statements from mathematics graduate students describing TA experiences.

The purpose of this article is twofold: to give a sense of the current lay of the land in the preparation of mathematics graduate student teaching assistants (TAs) and to describe the collegiate mathematics education research base informing the next generation of college mathematics instructor preparation. We anchor discussion in three common types of TA preparation programs, each represented in one of the quotes above. Notably, the first quote represents a sink or swim experience that is becoming rare in US PhD-granting mathematics departments. Preparation of TAs for their instructional roles has blossomed in the last twenty years. The second quote is representative of current practices in many departments. The third quote illustrates the activities in innovative departments that are already implementing best practices suggested by research in collegiate mathematics education: sustained professional growth about teaching and learning.

To highlight the challenges and benefits of spending time paying attention to teaching, we provide information from postsecondary and related secondary-level educational research of several types. This includes basic and applied educational research that identifies good instructional practices, examines experiences TAs bring with them to teaching, provides frameworks for the structuring of TA preparation, and gives insight into the kinds of mathematics-specific and teaching-specific knowledge that needs to be developed among TAs. And, once a program for supporting TAs to learn and grow as instructors is put in place, evaluation research explores the implementation of efforts to improve TAs' teaching and the related impacts on undergraduate student learning. We close with promising practices and sketch anticipations for the future of the field of related research.

Though not common thirty years ago, today most doctorate- and master's-granting institutions provide some kind of TA preparation for teaching [4a]. The content for this professional development often comes from mathematicians offering their collective wisdom from practical

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DOI: http://dx.doi.org/10.1090/noti1260

experience. At its most basic, TA "training" these days consists of one to six hours of orientation to teaching during the week before classes begin and a one-semester weekly seminar on particular course topics (e.g., a course coordination meeting for those involved in teaching Calculus I). For a growing number of departments, a second kind of TA preparation is available as a semester or more of activities guided by materials such as Sarkisian's [15] book for international TAs or a college mathematics instruction-specific resource like DeLong and Winter's book [7] or Friedberg et al.'s [9] cases. A third level of TA instructional learning includes presemester and first-year support along with professional development during subsequent years of teaching. While TA development across time can occur in course-specific coordination meetings, some institutions have ongoing seminars attended by new and experienced TAs as well as faculty and others interested in college mathematics instruction. Currently, regardless of department-organized supports, about two-thirds of TAs (domestic and international) rely at least in part on "unofficial" types of professional learning that cannot be listed on a curriculum vitae and that depend on local department culture; these include regular unstructured conversations with faculty members who make themselves available to TAs to talk about teaching [4e].

Research to Inform Practice

Today's cutting-edge research on novice college mathematics instructor and TA development depends on frameworks that connect theories of learning and teaching to undergraduate student achievement. Like the much longer history of research on precollegiate school teaching, this new postsecondary work provides research-based guidance for instructor development. Rapid progress in the field has been possible largely because it has focused on extending results about good instructional practices and teacher knowledge development from the established precollegiate mathematics (and science) education literature.

While some issues of college teaching cut across disciplines, many are discipline specific. Researchers have documented features of the culture and community of mathematics and the influences in that community, particularly among TAs. For example, the research reported in a special issue of *Studies in Graduate and Professional Student Development* [4] represents some of the recent steps toward understanding how graduate students grow as instructors within the mathematics community. The articles in that volume include summative, statistical, and case study reports about TAs' teaching [4a], [4b], [4c], TA views about the teaching and learning of mathematics [4d], [4h], TA perceptions of mathematics departmental support [4e], what TAs do when they plan for class [4f], and enculturation experiences for international TAs in mathematics departments [4g]. The result of this and associated work is an understanding that mathematics TA development takes more than relying on a university center offering general professional support.

What Are Good Instructional Practices?

What instructors do in the classroom makes a difference in the learning opportunities students have. We know from collegiate and precollege education research that students learn more deeply with higher frequencies of particular instructional practices. These include working together on tasks, giving presentations, and engaging in whole-class and group discussions where explanation and justification are required [14]. For example, if we define a successful calculus program as Bressoud and others have in a recent study [5], then it turns out that instruction characteristic of highly successful programs has students work together to understand and explain their problem solving. In addition, as part of the same Mathematical Association of America (MAA) calculus study, researchers have identified structural components of successful programs. These include various course coordination policies and aspects of department culture (e.g., encouraging experimentation with student-active classroom strategies) and, notably, professional development for TAs. In particular, the study reports that TA professional preparation in successful programs includes both an intensive initial preparation experience plus ongoing learning opportunities through seminars, courses, and/or mentoring.

What Experience of Teaching and Learning Do TAs Bring to Instruction?

Like most mathematicians, novice college mathematics instructors' experiences as learners typically include a history of success in undergraduate mathematics, a notable absence of precalculus courses taken as an undergraduate, and great familiarity with lecture-based mathematics classes. So, not surprisingly, mathematics TAs report the greatest comfort with lecture and find some of the greatest challenge in working with students whose backgrounds or experiences are unlike their own (Figure 1). Yet, throughout graduate school and over a career, instructors most often teach students whose expertise and experience with mathematics is quite distinct from their own. Nearly two-thirds of instructional assignments in PhD-granting departments are in first-year courses like calculus and below [13].

While lecture is the most familiar teaching strategy, graduate students have a wealth of experience in learning mathematics in addition to attending lectures. They are familiar with sense-making strategies associated with the norms of seminars, particularly for proof writing, validation, and justification. Part of becoming an expert in advanced mathematics is developing skills to coordinate and choose flexibly among multiple representations and to critique one's reasoning (and that of others) using why, how, and why-not questions. When used to scaffold conversations in college classrooms, these questioning strategies also turn out to be the earmarks of successful instruction [5, 12]. Yet, as undergraduates, TAs may not have taken courses where instructors modeled these effective active-learning approaches. Current thinking in TA development includes how to leverage the fact that the approaches to learning we want graduate students to develop in their study of advanced mathematics have a great deal in common with the learning opportunities we would like them to create for their own undergraduate students.

> In all cases, good TAs are a benefit to a mathematics department, both in the actual teaching of mathematics to undergraduates and in relations with other departments and the administration. Bad TAs, as measured by student complaints, are a liability.
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> —*Friedberg* [8, p. 842]

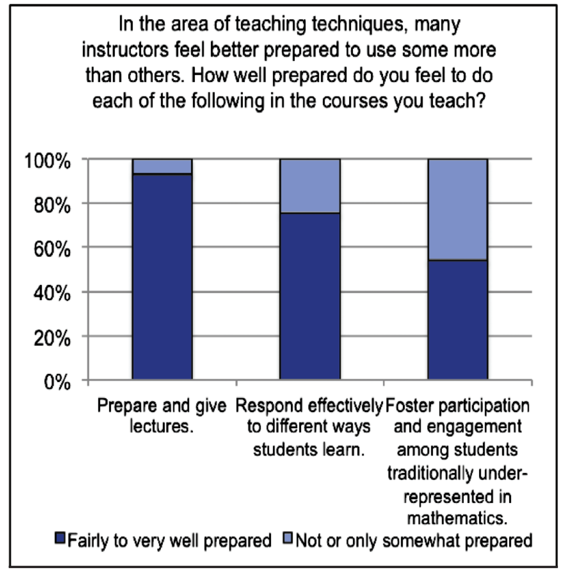

Figure 1. TAs views of their readiness to implement certain instructional techniques. Source: 203 responses by TAs at Group I universities; current work by Hauk.

What Are Good Models for Novice Instructor Development?

Suppose a department is moving into an expansion of TA development. What is reasonable to expect? Based on lessons learned in precollegiate education research, the greatest improvements in student learning come when teacher professional development has a format with [3]:

(1) An intensive initial experience, plus

(2) A spaced-across-time follow-up that includes sustained engagement with at least two components such as:

> (a) Examining student thinking and student work
>
> (b) Collaborating in teams to learn about teaching
>
> (c) Engaging in plan-implement-assess-reflect cycles
>
> (d) Analyzing and designing assessments
>
> (e) Working with a mentor through multiple classroom visits and conversations

Figure 2 illustrates this research-based constellation of effective professional learning components. Note that while public presentation to others about instructional practices (e.g., in a teaching seminar) is a support for better teaching, at least two other aspects appear to be necessary. We know from precollegiate education research that successful professional development programs focus on helping teachers understand how students learn and on building skill with pedagogical strategies that are effective for deepening students' subject-matter knowledge. Moreover, in the Blank and de las Alas [3] meta-analysis, effectiveness increased with collective participation of instructors in a department. Other development activities (e.g., by a university-wide office) may offer limited opportunities for such collaboration.

We know that when secondary teachers use active learning, reflective planning, student-responsive teaching, and make connections within and among these areas, teacher knowledge grows and desired changes in instructional practice occur. That similar results hold for college instruction has

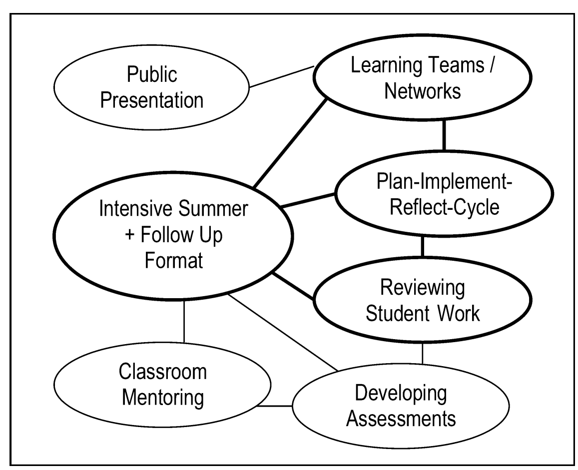

Figure 2. Constellations of effective professional development (based on [3]). Darkened set of connected ovals is one example of an effective constellation.

been confirmed in several studies, particularly as TAs attend to their students' thinking [4h]. Perhaps not surprisingly, when instructors observe, listen to, and talk with students they gain knowledge of how students think. What may be less obvious is that it is a cyclic process. That is, if instructors are armed with knowledge of student thinking, they are both more likely to provide students with learning opportunities in the present and to enrich their own opportunities to build knowledge of student thinking in the future. College-level research has illustrated that effective planning and instructional decisions take into consideration the ways students think [4f]. Such learning in and from teaching is quite context specific. As discussed below, it weaves together content knowledge about mathematics with knowledge about teaching.

What Is Pedagogical Content Knowledge (PCK)?

Those who have taught calculus recently have probably witnessed the following phenomenon. Students learn the chain rule to calculate derivatives. Some students then get carried away and either apply it to every function (whether or not it is appropriate) or iterate its application more times than necessary on "inner" functions. Similarly, experienced instructors are also likely to be familiar with a number of strategies that students might use successfully to factor second-degree polynomial expressions, even if they themselves were taught (and prefer) just one method. Evidence has accumulated that this kind of discipline-specific knowledge for teaching plays a substantial role in shaping instructional practices. This pedagogical content knowledge (PCK) is a kind of mathematical knowledge that supports effective mathematics instruction [1, 16]. The research on PCK in collegiate and precollegiate settings has provided evidence that the work of teaching demands more than just knowledge of the content being taught. In particular, knowledge of typical difficulties and various ways of thinking experienced by students along with knowledge of especially illuminating examples are major factors that shape instructors' practices and their students' learning. In addition, even with ongoing, sustained professional learning, it is possible to return to "novice instructor" status and be in need of additional professional development if faced with a new type of course or a variation of student population in a teaching assignment. Even experienced faculty struggle with the demands placed on their own PCK in new and highly interactive classroom contexts. The growing body of research in collegiate mathematics education points to several aspects of PCK development as key to effective instruction, like using student thinking, attending to computational and conceptual understanding, and orchestrating productive classroom discussions.

In part, mathematics TAs build pedagogical content knowledge through attention to student thinking when they grade or respond to student work. They also might have one-on-one conversations with students in office hours, a tutoring lab, or during class. TAs can learn to use all that student-based information productively as they plan for and implement interaction with students [11]. In addition to anticipation about student thinking, PCK includes awareness of what concepts are essential in a particular course and knowledge related to communicating about mathematics given the variety of thinking and preparation among students in the room [10]. TAs may gain some knowledge about prerequisite and subsequent concepts from their own experiences as students but need guidance to understand the experiences and needs of undergraduates who are not mathematics majors.

> Until recently, it was hardly acknowledged that teaching entails knowledge and skills that are more than academic subject-matter knowledge combined with formally lucid exposition and a sympathetic disposition toward students. In fact, it involves a kind of knowledge of mathematics itself distinct from what research mathematicians require for their research or typically know. Moreover, it is only recently being recognized that this knowledge and skill can be taught and learned.
>
> —*Bass* [2, p. 109]

What about Professional Development for Teaching after Graduate School?

In addition to classroom-specific skills, to flourish in their careers, graduate students need to develop awareness and knowledge of resources for professional learning beyond graduate school. There are many potential sources of professional development available for new college faculty that might be shared in a TA development program. For example, more than 10 percent of the people who complete PhDs in mathematics each year and are hired into tenure-track positions in US departments become MAA Project NExT Fellows (about eighty Fellows per year). Project NExT participants engage in professional learning and networking about teaching [17]. In addition, the AMS, MAA, and the American Mathematical Association of Two-Year Colleges (AMATYC) have programs, groups, and conference sessions that are valuable resources for new college faculty.

Determining the Effectiveness of TA Professional Development Programs

Given what is known about the structure, time, community supports, and pedagogical content knowledge development needed for effective TA preparation, we come to an obvious question. When we implement TA professional development, how do we know if it is working?

Let us start with the purpose of the endeavor: to help TAs develop as instructional professionals in both the short and long term. Research suggests that it is worthwhile to pay attention to knowledge and changes in knowledge (e.g., of mathematics, of pedagogy, of pedagogical content knowledge). It is also valuable to attend to shifts in TA beliefs about teaching, learning, and the doing of mathematics and to types and evolution in use of instructional practices while also considering increases in students' success.

Evaluation research focuses on examining how a program is working. First, there is a need to identify information to collect systematically. This will include triangulating among basic methods of capturing information about teaching: surveys, interviews, observations, and portfolios/artifacts (see [18] for an accessible primer on these methods). In some cases we can collect data before TAs participate in professional development so that it might be compared to another data set collected later. In other cases, it is more useful to identify departmental or development goals and gather data midstream and/or at strategic points in a program. For example, if the goal is that TAs are effective teachers, possible indicators could be that outcomes of interest (e.g., grade distribution, student evaluations, etc.) among TAs' students is within the 95 percent confidence interval of that among experienced faculty members' students. More than statistics, effective evaluation of a TA preparation program will include details that provide context for any quantitative results. This can include structured reporting in a teaching portfolio, TA comments on the departmental TA experience, analysis of undergraduate student written comments on midterm and end-of-term feedback forms, and TA response to and sense-making about these things.

Whatever the approach, it is essential to identify information to gather, get and analyze it, and to then determine what the lessons are to be learned from it. Equally critical is that the next iteration of TA development in the department be informed by those lessons learned.

Conclusion

Every future mathematician, scientist, and engineer takes many mathematics courses in college. Moreover, every future elementary, middle, and high school teacher takes mathematics courses as an undergraduate. For many, TAs provide some of this instruction. Thus, the quality of learning opportunities for a broad cross-section of the nation's college students is shaped by TAs' preparation to teach. The importance of teaching-related preparation and development increases even further as we recognize that subsequent generations of postsecondary faculty will come from current pools of graduate students. Many of these graduate students will go on to faculty positions at undergraduate institutions, dispersing among more than four thousand research and comprehensive universities, liberal arts colleges, and community colleges in the country. More than 70 percent of those who complete the PhD in mathematics and go into academic positions will spend their careers outside the kind of universities they attend [6]. Before TAs leave for jobs where the quality of their teaching is at least as important as their research production, graduate mathematics programs can serve TAs, the academy, and the nation with carefully designed and well-maintained programs of professional development in teaching. A graduate teaching assistantship may or may not prepare a newly minted doctoral graduate for the teaching challenges in a four-year or two-year college. However, a good TA development program produces instructors who can learn from their own practice and who know to seek collegial support. In a new faculty role, those skills are essential for continued professional growth.

We are only at the beginning of a complex process of understanding how the teaching and learning of college mathematics take place and how the experience of novice instructors, like TAs, plays out. Topics for research conducted in the college context that are likely to be important for work with TAs include: how novice college instructors learn to teach, how characteristics of instructors and their practices shape undergraduates' learning, how culture and context shape everything, and how college mathematics curriculum and teaching methods are understood, adopted, and enacted. Researchers in collegiate mathematics education are beginning to make progress with some of these topics. In particular, there is a special interest group of the MAA on research in undergraduate mathematics education and a working group within it that focuses on research around becoming an effective college mathematics instructor, including TA and faculty development (see `sigmaa.maa.org/rume`). In addition, a new, NSF-funded effort through the MAA (DUE-1432381) aims to create a Web portal for TA professional development materials and related research and evaluation (`www.maa.org/programs/faculty-and-departments/cominds`).

Finally, insights from experienced college mathematics instructors are crucial in the development of the next generation of college mathematics faculty. Say "Yes!" when collegiate mathematics education researchers ask if they can come visit your classroom or do an interview. Knowing what and how people have learned from their own teaching experiences will shape future research and development intended to help others learn to learn from practice.

Acknowledgments

This material is based upon work supported by the National Science Foundation (NSF) under Grant Number DUE-1432381 and the US Department of Education through Grant P116B060180. Any opinions, findings and conclusions or recommendations expressed are those of the authors and do not necessarily reflect the views of the NSF or the US Department of Education.

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