Nominations for President

Nomination of Mark Green

Phillip A. Griffiths and Jill Pipher

Mark Green is an absolutely superb choice for president of the American Mathematical Society. His scientific contributions to mathematics are at the highest level and he has a remarkable record of service to our community. He has taken on leadership roles in the profession with great success, combining a skill for administration with a true spirit of generosity. His accomplishments in the profession have been recognized by the AMS, as a Fellow, by the IMU, as the Chern Medal Plenary Lecturer in Seoul and an ICM invited speaker in Berlin, by the American Academy of Arts and Sciences, as a Fellow, and through many awards and honors too numerous to list here. Mark's selection as the 2013 AMS Congressional Lecturer demonstrates the confidence of the mathematical community in his ability to speak eloquently on their behalf. Finally, he has shown extraordinary dedication to the mathematics profession through his leadership in establishing and directing the Institute for Pure and Applied Mathematics, through his membership in the AMS Strategic Planning Group and as an AMS Trustee, and over many years as author, editor, educator, organizer, speaker, and reviewer.

It seems natural to begin with an overview of Mark Green's mathematical contributions. Over more than four decades, Mark has done fundamental research in geometry, algebra, and some areas of applications. His work has answered outstanding basic questions, and, especially, has opened up new areas of research by establishing initial results and formulating conjectures that have given rise to entire streams of productive activity. Mark's work is characterized by great originality and an unsurpassed ability to originate and apply techniques from commutative algebra to geometric questions.

To cite some examples, the modern subject of "hyperbolicity" originated with Picard's theorem and now involves the study of holomorphic maps of $\mathbb{C}^n$ into a quasi-projective algebraic variety X. Early in his career, Mark solved a problem posed by Chern at the 1970 ICM about holomorphic maps from $\mathbb{C}^n$ into X when X is the complement of hyperplanes in $\mathbb{P}^n$ and later studied maps from $\mathbb{C}$ into X when X is a surface of general type. In the latter work, jet differentials were introduced, and in an original and suggestive step, their connection to the surface being of general type was established. This led to a conjecture that is the subject of considerable interest and much current work; among other things it was the topic of a recent Bourbaki seminar. The Green-Griffiths conjecture on holomorphic curves motivated conjectures of Lang on rational points on varieties of general type. Also in his early work, and in a completely different area, Mark formulated and established a Lie algebra theoretic classification of the differential invariants that determine curves in homogeneous spaces. As these examples suggest, Mark's mathematical research is remarkably broad: it covers much of geometry.

Beginning with the early work of Serre, Grothendieck and others, commutative and homological algebra have been absolutely fundamental tools in algebraic geometry. In the 1980s, Mark brought an extremely fruitful, geometrically motivated, perspective to the field. He was able to solve outstanding classical questions, including a question of Riemann on quadrics of rank four through a canonical curve. He went on to establish fundamental new results and formulate new and highly original questions. The Green conjecture on syzygies of a canonical curve is one of the deepest and most tantalizing questions about the geometry of algebraic curves. The commutative/homological techniques that he introduced provided effective methods for addressing geometric questions arising from Hodge theory and paved the way for important ongoing work applying Hodge theory to questions in algebraic geometry. Mark's work with Rob Lazarsfeld on deformation theory of cohomology groups continues to play a role in the classification theory of algebraic varieties.

Mark's research in the last twenty years has focused on a wide range of geometrically motivated questions in Hodge theory. He and his collaborators have results pertaining to algebraic cycles, general Neron models, and Mumford-Tate groups and domains. This work has led to various mathematical generalizations and is currently being applied to questions in physics. The Mumford-Tate groups are the natural symmetry groups in Hodge theory, and the corresponding Mumford-Tate domains are special homogeneous complex manifolds that have a very rich geometry, relating to representation theory in addition to Hodge theory and complex algebraic geometry. Using techniques from Lie theory, Mark and his collaborators completely classified the ways in which a simple algebraic group may be realized as a Mumford-Tate group. All of the results we describe here have opened new areas of currently active research.

Phillip A. Griffiths is Emeritus Professor of Mathematics and former director at the Institute for Advanced Study at Princeton. His email address is pg@ias.edu.

Jill Pipher is professor of mathematics at Brown University. Her email address is jill_pipher@brown.edu.

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We would now like to describe some of Mark's extraordinary service contributions to the profession. Over the years, these contributions range from specific service to AMS and SIAM to scientific and administrative leadership roles: the breadth of his service interests is striking. He has generously given time to support individuals and institutions that serve the mathematical community. His public presentations on policy and educational issues are noted for their focus and clarity.

He just completed a term (2010–2015) as a Trustee of the AMS and serves on its Strategic Planning Committee. Mark's perspective and advice on the profession has been widely sought: he has served as a consultant to many major scientific boards, societies and institutions, including the American Academy of Arts and Sciences Selection Panel, the American Council on Education, the National Research Council Board on Mathematical Sciences and Applications, (chair of) the 2013 NSF-DMS Committee of Visitors, the Simons Foundation, and numerous Canadian and US institute advisory boards.

One of the highlights in this extensive list is Mark's service as Vice-Chair of the National Research Council committee which produced the report, *The Mathematical Sciences in 2025*, known informally as *Math 2025*. Among the many excellent NRC reports in mathematics and other fields, *Math 2025* stands out in terms of its breadth and vision. Through concrete examples and cogent analysis, the mathematical sciences are portrayed in the report as having a role in the mathematical and scientific communities, and indeed in the larger society, that goes far beyond what could have been imagined even a few years ago. *Math 2025* is already receiving major attention from governmental, scientific and educational communities, and one might reasonably expect it to have a significant impact on the future of our field. Its companion volume, *Fueling Innovation and Discovery* has been distributed for uses that range from making the case for funding mathematics research to informing high school teachers about developments in mathematics. Mark's demonstrated leadership in communicating specific mathematical ideas, as well as the scope and impact of mathematics overall, is a tremendous resource for our community.

One milestone in Mark's service to the profession is his leadership at the Institute for Pure and Applied Mathematics (IPAM), an NSF Mathematics institute which he helped to found and served as co-director, and then director, for nearly a decade. Under his leadership, scientifically and administratively, IPAM went from the drawing board to its current unique place among math institutes, blending traditional areas of fundamental research in mathematics with synergistic opportunities for applications and impact in other scientific disciplines. Mark demonstrated a prescient vision for the impact of mathematical partnerships in science and technology, and it is his vision of this partnership to which IPAM owes its initial success.

Very recently, Mark's passion for communication and education has propelled him into a leadership role in finding funding for, and organizing meetings to promote the goals of, the broad initiative *Transforming Post-Secondary Education in Mathematics* or TPSE Math. Sponsored by the Carnegie Corporation of NY, the Sloan Foundation and four major mathematical societies, TPSE Math had its kickoff meeting in Austin in 2014. Mark and other members of the organizing committee led working groups exploring a variety of urgent issues in the undergraduate mathematics curriculum.

On the personal side, Mark is an engaging and warm colleague and teacher. His leadership style reflects his personality: considerate, informed, thoughtful and persuasive. We believe that the AMS would be well served by the unique blend of experiences, talent and passion that Mark would bring to the Presidency.

Nomination of Kenneth Ribet

Benedict Gross and Barry Mazur

It is an honor to nominate Kenneth Ribet for the Presidency of the AMS. We have both known Ken for over forty years. He has made fundamental contributions to number theory, and has served our profession in a variety of ways.

Ken's Background

Ken attended Brown as an undergraduate, receiving his AB and AM degrees in 1969. He came to Harvard as a graduate student in 1969—and promptly became an AMS member. His thesis advisor was John Tate. After receiving his PhD in 1973, Ken spent three years teaching at Princeton University and two years doing research in Paris before joining the UC Berkeley mathematics department in 1978. Ken has been a key member of his department since, teaching critical courses and winning several teaching awards. He has served in three different vice chairmanships as supervisor of the graduate program, the undergraduate program and the department's development efforts.

Ken has a deep and varied background in mathematics book and journal publishing. He began serving as journal editor almost thirty years ago and is currently an editor for a handful of number theory and general mathematics journals. After a brief stint as a book series editor for Cambridge University Press, he joined the New York-based Springer editorial board that looks after four book series, including the Graduate Texts in Mathematics series. Ken has served on the scientific advisor board of IPAM and is currently a member of the scientific board of the Simons Institute for the Theory of Computing.

Ken has been honored repeatedly over the course of his career. He won the Fermat Prize in 1989 and received an honorary doctorate from Brown University in 1998. Ken was elected to the American Academy of Arts and Sciences in 1997 and to the National Academy of Sciences in 2000. At the National Academy, he served on the US National Committee

Benedict Gross is George Vasmer Leverett Professor of Mathematics at Harvard University. His email address is gross@math.harvard.edu.

Barry Mazur is Gerhard Gade University Professor at Harvard University. His email address is mazur@math.harvard.edu.

for Mathematics, which represents the USA to the IMU. He also served three terms on the nominating committee, and chaired the mathematics section of the NAS for three years, beginning in 2009.

Ken is currently a member of the AMS Council. He serves on the Executive Committee of the Council, the Long Range Planning Committee, the Committee on Science Policy, and the committee that coordinates the collected works program. He is a much sought-after speaker, and with his varied professional experience would be an outstanding public face of the AMS.

Ken's Mathematics

Ken works in number theory and algebraic geometry. He is best known for his theorem in the 1980s that reduced the proof of Fermat's Last Theorem to the conjecture that all semi-stable elliptic curves over $\mathbb{Q}$ are modular (which was proved in the 1990s by Andrew Wiles and Richard Taylor). Ken was awarded the Fermat Prize for this contribution. But Ken's influence on number theory is more extensive than that single accomplishment: it spans four decades of important discoveries, during which Ken has been the inspiration for several generations of mathematicians. Many of his contributions are key to our understanding of the connections between the theory of modular forms and the ℓ-adic representations of the absolute Galois group of the field of rational numbers. We will briefly highlight three of them here. There are many other areas where Ken's work has been decisive, such as his construction with Deligne of p-adic L-functions for totally real fields [1].

To focus on a classical example in the theory of modular forms, consider the infinite product

$$\Delta(q) \;=\; q\prod_{n=1}^{\infty}(1-q^n)^{24} \;=\sum_{n=1}^{\infty}\tau(n)q^n.$$

The product $\Delta(q)$ can be thought of as a power series in the variable q; or putting $q = e^{2\pi i z}$ we may view it as an analytic function of the variable z in the upper half-plane, where it satisfies the additional symmetry $\Delta(-1/z) = z^{12}\Delta(z)$. As a consequence, Δ is a cuspidal modular form of level 1 and weight 12.

The Fourier coefficients, $\tau(n)$ of Δ have been studied by generations of mathematicians, starting with Ramanujan. Simple recurrence relations (first described by Mordell) allow one to retrieve the Ramanujan tau-function $n \mapsto \tau(n)$ from its values $\tau(p)$ for all prime numbers p. Serre conjectured, and Deligne proved, that the modular form $\Delta(q)$ has the following remarkable connection to Galois representations. Let $G_{\mathbb{Q}} := \mathrm{Gal}(\overline{\mathbf{Q}}/\mathbf{Q})$, the absolute Galois group of the field of rational numbers and let ℓ be a prime number. Then for every power ℓ^n there is a continuous representation

$$\rho_{\ell^n} : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{Z}/\ell^n\mathbb{Z})$$

which is unramified at all primes $p \neq \ell$. Moreover, the image of a Frobenius element at p has trace congruent to the pth Fourier coefficient $\tau(p)$ of Δ and determinant congruent to p^{11} modulo ℓ^n. The same result holds not only for Δ, but for the Fourier expansions of general Hecke eigenforms. Many of Ken's earliest articles involve a study of the images of these Galois representations.

On the Size of the Image of Galois Representations

Swinnerton-Dyer and Serre showed that for ℓ different from $2, 3, 5, 7, 23$, and 691, the image of the representation ρ_{ℓ^n} associated with Δ is as large as possible. Specifically:

$$\mathrm{image}(\rho_{\ell^n}) = \{g \in \mathrm{GL}_2(\mathbb{Z}/\ell^n\mathbb{Z}) \mid \det(g) \text{ is an eleventh power in} (\mathbb{Z}/\ell^n\mathbb{Z})^*\}.$$

In one of Ken's first published papers [2] he established an analogous result for the Galois representations mod ℓ^n (where $\ell \gg 0$) attached to general Hecke eigenforms of level 1. His later work amplifies and generalizes this result in various important directions; for example, [4] establishes the Tate conjecture for Jacobians of modular curves.

On the Theorem of Herbrand and Ribet

Returning to the cuspidal modular form Δ, consider the representation

$$\rho_\ell : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{Z}/\ell\mathbb{Z})$$

where $\ell = 691$, one of the primes for which the image of ρ_ℓ is *not* as large as possible; in fact it is contained in a Borel subgroup of $\mathrm{GL}_2(\mathbb{Z}/\ell\mathbb{Z})$. This is related to the Ramanujan congruence:

$$\tau(n) \;\equiv\; \sum_{d\,\mid\, n} d^{11} \mod 691$$

for every positive integer n. In particular,

$$\tau(p) \;\equiv\; 1 + p^{11} \mod 691$$

for every prime number $p \neq 691$. The number field fixed by the kernel of ρ_{691} is an *everywhere unramified* cyclic extension of degree 691 over the (cyclotomic) number field generated by 691th roots of unity. The existence of this unramified extension is related to the fact that 691 divides the numerator of the 12th Bernoulli number.

What one can take away from this example is that cuspidal modular forms such as Δ might be pressed into service to actually *construct* abelian everywhere unramified extensions of cyclotomic fields. That is precisely the approach that Ken took in his article [3], where he established the converse to a famous theorem of Herbrand. Specifically, Ken showed that for ℓ a prime number and k an integer with $2 < 2k < \ell - 1$, if the numerator of the $2k$th Bernoulli number is divisible by ℓ there is a cuspidal Hecke eigenform of weight $2k$ whose associated Galois representation mod ℓ has its image contained in a Borel subgroup, and the number field determined by the representation ρ_ℓ is an *everywhere unramified* cyclic extension of degree ℓ over the cyclotomic field generated by ℓth roots of unity. The extremely original viewpoint that Ken fashioned in his proof of 'Herbrand-Ribet', and the result itself, was seminal, and has been extraordinarily important for the later developments in the subject.

On Fermat's Last Theorem

The connection between automorphic forms and Galois representations can be run in either direction. An important conjecture of Serre (subsequently proved by Khare and Wintenberger) implies that any (irreducible) Galois representation

$$r : G_{\mathbb{Q}} \to \mathrm{GL}_2(\mathbb{Z}/\ell\mathbb{Z})$$

that has the property that complex conjugation, viewed as element of $G_{\mathbb{Q}}$, is not sent, under the representation r to $\pm 1 \in \mathrm{GL}_2(\mathbb{Z}/\ell\mathbb{Z})$, is associated to a cuspidal modular form modulo ℓ. Ken's remarkable contribution to Fermat's Last Theorem hinged on the (then conjectural) modularity theorem. (The modularity theorem is itself implied by Serre's conjecture.)

Here is a brief hint of how Ken's extraordinary contribution fits into the proof of Fermat's Last Theorem. A beautiful idea of Frey was to start with a putative non-trivial solution of Fermat's equation with exponent ℓ to produce an elliptic curve $\mathcal{E}$ over $\mathbf{Q}$ with very unusual properties. Assuming the modularity theorem, $\mathcal{E}$ would be parametrized by a cuspidal modular form ϕ of weight two with *correspondingly* unusual properties. Ken's ingenious idea [5] is to make use of those properties to construct a different modular form ϕ', which is also of weight two and whose Fourier expansion is congruent modulo ℓ to that of ϕ. The modular form ϕ' which Ken constructs has level 2. But there are no cusp forms of weight 2 and level 2, so ϕ' is constrained to be an Eisenstein series and therefore ϕ itself would have a Fourier expansion congruent modulo ℓ to an Eisenstein series. This would violate known results about rational torsion of elliptic curves; specifically about rational torsion in $\mathcal{E}$. So: the nontrivial solution of Fermat's equation cannot exist! Ken's argument is startling in its originality and makes use, among many other things, of the quaternionic description of the bad fibers of Shimura curves. The general technique Ken used for the construction of such a ϕ', as described above, might be called "level adjustment," Ken having initiated the important systematic study of the various possible levels of modular forms that are associated to the same mod ℓ Galois representation.

Ken as Teacher, Mentor, and Ambassador for Mathematics

Ken's marvelous talent for—and devotion to—teaching, lecturing, and generally guiding young mathematicians is recognized world-wide. At Berkeley he frequently gives large lecture classes in upper-level subjects, and takes special care to make genuine connections with each of his students. In 2014 there were over two hundred students in his linear algebra class, and he extended email invitations to each of them to join him for breakfasts and lunches at the Berkeley Faculty Club.

Ken won the department's distinguished teaching award on two occasions: soon after it was introduced in the 1980s and more recently in 2013. He has an impressively long list of students whose PhD's he supervised[1]. Many of his students have gone on to make notable contributions in teaching and research, both in academia and in industry.

Ken has engaged frequently in outreach in connection with Fermat's Last Theorem beginning with the Fermatfest in San Francisco in 1993. His AMS Invited Address at the 1994 annual meeting drew an overflow crowd, consisting essentially of all people who had registered for the Joint Math Meetings. Ken gave a public lecture on the history of Fermat's Last Theorem this fall at Bowdoin College and has given similar talks in the recent past at Humboldt State University and Southern Oregon University.

Conclusion

Ken Ribet has made outstanding contributions to research mathematics, and is a marvelous teacher and lecturer. With vision and immense energy he has already given tremendous service to his department, to the National Academy of Sciences, to the American Mathematical Society, and to the mathematical community in general. We feel he will do great things as President of the AMS.

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[1] *see* `genealogy.math.ndsu.nodak.edu/id.php?id=32910`.