A Medieval Mystery: Nicole Oresme's Concept of *Curvitas*

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In a paper published in 1952, J. L. Coolidge (1873–1954) appreciates that the story of curvature is "unsatisfactory" [2], and he points out that "the first writer to give a hint of the definition of curvature was the fourteenth century writer Nicole Oresme, whose work was called to my attention by Carl Boyer." Then Coolidge comments: "Oresme conceived the curvature of a circle as inversely proportional to the radius; how did he find this out?" The scholarly conditions of the fourteenth century make this discovery phenomenal and the question as to how it was achieved worth researching. In the present article we describe how a fourteenth-century scholar (i) gave a correct definition for curvature of circles and attempted to extend it to general curves, (ii) tried to apply curvature to understand the behavior of real-life phenomena, and (iii) produced in his research a statement that anticipates the fundamental theorem of curves in the plane.

In various cases Oresme's work is not cited when the history of curvature is discussed (e.g., [5], [8]), while some authors (e.g., [1], p. 191) make note of his contribution to this concept. Several scholars have even concluded that the medieval sciences contributed very little to the modern scientific revolution. In addressing this perception, Edward

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The Rouen Cathedral where Nicole Oresme served first as canon, then as dean of the Cathedral, after 1362.

Grant [3] writes: "Even if the Middle Ages made few significant contributions to the advancement of the sciences themselves, or none at all, […] if no noteworthy medieval contributions were made to help shape specific scientific advances in the seventeenth century, in what ways did the Middle Ages contribute to the Scientific Revolution and, more to the point, lay the foundations for it?" We describe in the present article that curvature is one of the concepts that was first defined in the Middle Ages. The importance of the idea of curvature is described in many works (e.g., [1], [5], [8], [10]), and we don't feel we should elaborate on this point.

Nicole Oresme was born around 1320 in the village of Allemagne, near Caen, today Fleury-sur-Orne [6]. The first certain fact in his biography is that he was a "bursar" of the College of Navarre from 1348 to October 4, 1356, when he became a Master. The College of Navarre, established by Queen Joan I of Navarre in 1305, focused on teaching the arts, philosophy, and theology. Oresme's major was in theology. As a student, he had to observe the code at the College of Navarre, where the students were required to speak and write only in Latin; his ability to work in Latin would prove critical in his future work. Oresme studied, among others, with Jean Buridan and Albert of Saxony. It was at this institution where he wrote his most important works, e.g., *De proportionibus proportionum*, which is of particular importance for the history of mathematics, or *Ad pauca respicientes,* of interest for the history of ideas in celestial mechanics. Oresme remained Grand Master of the College until December 4, 1361, when he was forced to resign [6]. On November 23, 1362, he became a canon of the Rouen Cathedral (a place of major importance in the history of France), and on March 18, 1364, he was promoted to dean of the Cathedral. Oresme was in that period king's confessor and adviser, and some time before 1370 he became one of Charles V's (1364–80) chaplains; at the king's request he translated from Latin into French Aristotle's *Ethics* (1370) and *Politics*, as well as *Economics*.

As Marshall Clagett points out [7], it is very likely that *De configurationibus* was written in the interval 1351–55. To better depict this historical period, we recall here that during this period, Geoffrey Chaucer, later considered the father of English literature, was still a child in London. These are the same years when Giovanni Boccaccio wrote the *Decameron*, largely completed by 1352. In Florence, Francesco Petrarch, the first to coin the name of the "Dark Ages", was writing in Latin *De vita solitaria*. One of the main historical references for that historical period is Jean Froissart's *Chronicles*, describing the battles from the Hundred Years' War and the Black Death, impacting most of Europe in the interval 1346–53. The cathedral Notre-Dame de Paris was just completed a few years before, in 1345, and dominated the skyline of medieval Paris. In short, this period of time was infested with conflict and tragedy, greatly occupying civilian minds and making the main focus of life survival. The poets and the scientists worked in many cases in isolation for long intervals of time.

In the time frame in which Oresme wrote, the language of functions was not yet used in mathematics. It is impressive that Oresme reached the concept of curvature before the concept of function was established. He had to invent and express his thoughts without several fundamental mathematical concepts to refer to, thus making his explanations on curvature unique. These circumstances justify why Marshall Clagett is correct in discussing a "doctrine" [7] when he described Oresme's original contributions.

Clagett's critical edition including the treatise *De configurationibus* [7] was published in 1968, over a decade after J. L. Coolidge was hoping to see a more complete history of curvature. Clagett (1916–2005) notes (see [7], pp. 50–51) that the first instance of Oresme's comments on representation of quantity by either a line or surface of a body are his remarks to *Questions on the Geometry of Euclid*, more precisely when he discusses questions 10 and 11. Oresme refers to other authors, such as Witelo and Lincoln (i.e., Robert Grosseteste), "who in this manner imagine the intensity of light," and to "Aristotle, who in the fourth [book] of the *Physics* imagines time by means of a line." He also includes "the Commentator [Campanus] in the fifth [book] of this work [the *Elements*] where he holds, in expounding ratios, that everything having the nature of a continuum can be imagined as a line, surface, or body." Clagett points out ([7], p. 51) that the reference to Aristotle's work is "to the effect that every magnitude is continuous, and movement follows magnitude; therefore movement and hence time are continuous, for motion and time seem to be proportional." In short, Oresme's "doctrine" is actually a theory describing how quantities could be described by graphs. The concept was novel at that time, although it was based on Aristotle's earlier work.

In the first part (the first forty chapters) of *De configurationibus*, Oresme sets up the groundwork for the doctrine of configurations; then he applies the doctrine to qualities, focusing on "entities" which are permanent or enduring in time. While discussing these elements, he suggests that his theory could explain numerous physical and psychological phenomena. In the second part of *De configurationibus* (the next forty chapters), Oresme describes how graphical representation can be applied to "entities that are successive"; in particular, he applies the doctrine of "figurations" to motion. He concludes this part with several examples that could be extended to psychological effects, including the perceptions that are described as magic.

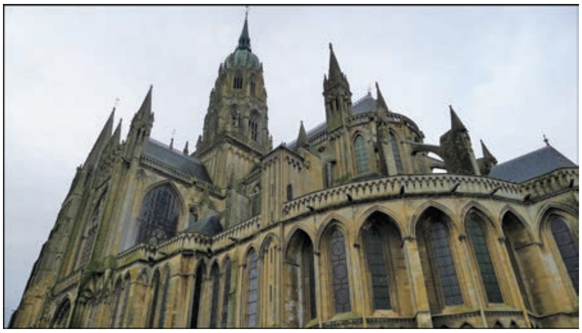
Finally, Oresme describes external geometrical figures used to represent qualities and motions. He compares the areas of such figures and concludes that by comparing the areas, one may have a basis for the comparison of different qualities and motions.

The donjon tower of the Château de Vincennes, in Paris, is 52 meters high and represents the tallest medieval fortified structure in Europe. King Philip VI of France started this work about 1337. The work was completed during Charles V's reign. When Nicole Oresme was a scholar in Paris, this donjon was in the process of being erected. Later on, during the reign of Charles V, this donjon served as a residence for the royal family. Its buildings are known to have once held the library and personal study of Charles V.

To fully describe his theory, Oresme begins his *De configurationibus* with the following clarification: "Every measurable thing except numbers is imagined in the manner of continuous quantity." Then he pursues a discussion of the latitude and longitude of qualities, followed with the presentation of their quantity. He leads into his argument that qualities can be "figured." He spends several chapters discussing suitability of figures and shape of various particular cases. This discussion suggests an early analysis of curves in general position, if we are to refer to the modern concept. One important distinction appears in chapter I.xi, where Oresme examines the differences between uniform and difform qualities. He continues his focus on this topic in I.xiv with a discussion of "simple difform difformity," which is of two kinds: simple and composite. In this chapter he uses "linea curva" for a curve, and "curvitas" to express its curvature. In I.xv he begins describing four kinds of simple difform difformity, which are explained by drawing graphs. There is little doubt that the author builds here an early approach to variable quantities and their corresponding graphical representation. After this extensive discussion, performed without any algebraic notation, Oresme approaches "surface quality." Finally, in chapters I.xix, I.xx, and I.xxi, he introduces curvature. Additionally, in chapters I.xv and I.xvi, Oresme describes graphs that are concave and convex. Due to the context of his analysis, Oresme actually performs the first exploration of the possible connections between curvature and convexity.

A particular case in this doctrine of qualities is represented by curvature (chapter I.xx), endowed with "both extension and intensity." Oresme writes (in M. Clagett's translation, [7], p. 215): "We do not know with what, or with regard to what, the intensity of curvature is measured. For now it appears to me that there are only two [possible] ways [to speak of the measure of curvature]. The first is that the increase in curvature is a function of its departure from straightness, i.e., of its distance from straightness. This is [to be measured] by the quantity of the angle constituted of a straight line and a curve, e.g., an angle of contingence or perhaps another angle also constructed from a straight line and a curve." This very intuitive description is very consistent with the modern study of signed curvature and its relationship with the change of turning angle with respect to arc length. Even further, Nicole Oresme reaches a more precise description. He writes specifically that the curvature of the circle is the inverse of its radius (in chapter I.xxi, where Oresme cites Aristotle's *On Curved Surfaces*). He delves more into this concept by covering more general curves: "Difform curvature is composed of an infinite number of parts of different nature and unrelatable [to each other]" (I.xx). Thus, his study of curvature is not limited to circles but is extended to more general cases. However, Oresme does not have any precise procedure to compute such a general curvature.

In classical differential geometry, the so-called fundamental theorem of curves states (e.g., [9], p. 29) that if two single-valued continuous functions $\kappa(s)$ and $\tau(s)$, for $s > 0$, are given, then there



The Southern Wall of the Bayeux Cathedral.
*[**NOTE:** This caption has been corrected and, so, differs from the print version.]*

exists one and only one space curve, determined by its position in space, for which s is the arc length, measured from an appropriate point on the curve, κ is the curvature, and τ is the torsion. This result was obtained in the nineteenth century. It is very surprising to read in Oresme the following reasoning (representative of Oresme's style and his intuition), which leads to a statement quite similar in conclusion to the fundamental theorem of curves ([7], p. 219):

> No intensity of difform curvature can be related to another dissimilar curvature in a ratio of 2 to 1 or [even] in a ratio of $\sqrt{2}$ to 1, i.e., either in a commensurable or incommensurable ratio—or, universally, in any ratio which could be found as existing between line and line. The conclusion is hence evident that intensity of curvature is not to be imagined by lines. Nor is there some curvature which is similar in intensity to some other quality of another species. Nor is curvature to be imagined by some figure. Nor is its intensity to be assimilated to the altitude of a figure, because the altitude of every figure is designated by lines. Finally, it is evident from this that no curvature is uniformly difform, for, by reason of accident, "uniformly difform" exists throughout a whole subject of the same nature and where the ratio of intensity to intensity, or excess of intensity, in the diverse parts is as the ratio of distance to distance, and consequently as the ratio of *lines*, as it is evident from the descriptions in chapter eleven, and this [reduction to ratios between lines] can not, as was just said, be suitable for difform curvature. And so it follows finally that every difform curvature is difform in a way different from that in which any other quality of another kind could be, and [so it is difform] with a strange, marvelous, diverse kind of difformity." [1]

[1] *The last sentence in the original is (*[7]*, p. 218): "Et inde sequitur ulterius quod omnis curvitas difformis est difformis aliter quam aliqua alia qualitas alterius generis possit esse et quadam extranea, mirabili, et diversa difformitate."*

Oresme does not work with the distinction between curvature and torsion for skew curves; all of his discussion is about planar curves, and the uniqueness part of the statement is suggested by "strange, marvelous, diverse" in the last sentence from the excerpt cited above. This shows that *curvitas difformis* is special in a unique way.

The generality of Oresme's doctrine resides in the attempt to model various phenomena by this approach. In *De configurationibus* we encounter his first attempt to apply his doctrine in chapter I.xxiv, where he discusses "On the variety of natural powers dependent on this figuration." He writes [7], p. 233: "It is manifest from natural philosophy and experience alike that all natural bodies determine in themselves their shapes, as, for example, animals, plants, some stones, and the parts of [all of] these. They also determine in themselves certain qualities which are natural to them. In addition to their shape that these qualities possess from their subject, it is necessary that they be figured with a figuration which they possess from their intensity—to employ the previously described imagery." To mention just one example, in chapter II.xl, titled "On the difformity of joys", Oresme discusses a subjective perception in the same terms as a physical quantity: "One ought to speak in the same way concerning a joy or a pleasure, which I suppose to be a certain quality extended in time and intended in degree."

The question asked by Julian Coolidge is where the idea of curvature comes from. There are many elements to suggest that the definition of curvature for curves is due to Nicole Oresme. One strong argument is that this definition was needed for his doctrine. Oresme developed it to serve his theoretical goals and to understand his configurations. Furthermore, Oresme builds upon Aristotle's conclusions and applies these ideas to a larger array of concepts where his graphs ("configurations") could be used. Some of the concepts he is interested in are today considered part of mathematics, some part of physics, while others approach the realm of psychology (e.g., the study of the question why certain perceptions lead to magic).

If Oresme clearly reached the first recorded definition of curvature for curves, then why do we see a certain hesitation to discuss and refer to his work? Perhaps because after the following generation his influence faded, his work was not continued, and his heritage was less understood. The historical reality of the Hundred Years' War limited the dissemination of Oresme's ideas. Later authors, such as Christiaan Huygens and Isaac Newton, discovered and developed fundamental concepts independently and did not build on Oresme's heritage. When mathematics benefited from the important revolution in sciences after 1600, Oresme's texts were perceived as inherited

Decorative architecture of the Rouen Cathedral.

from a different paradigm. However, by looking back at this work today, we should not imagine that *De configurationibus* is an obscure medieval text that could be described as “religious science.” Instead, it should be looked on as the initial approach to introduce curvature in the context of an early theory.

Addressing this type of understanding of the medieval books and types of arguments, Edward Grant writes in [3], p. 84: “Theologians had remarkable intellectual freedom and rarely permitted theology to hinder their inquiries into the physical world. If there was any temptation to produce a ‘Christian science,’ they successfully resisted it. Biblical texts were not employed to ‘demonstrate’ scientific truths by blind appeal to divine authority. When Nicole Oresme inserted some fifty citations to twenty-three different books of the Bible in his *On the Configurations of Qualities and Motions,* a major scientific treatise of the Middle Ages, he did so only as examples, or for additional support, but in no sense to demonstrate an argument.” There is no better answer to address the aforementioned concerns.

Our article does not aim more than to contribute to a long overdue discussion on the first recorded definition of curvature, pursuing J. L. Coolidge’s suggestion for a more complete history of this fundamental mathematical idea.

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