Communicating Mathematics to Children

Rich Schwartz
Communicated by Joel Hass

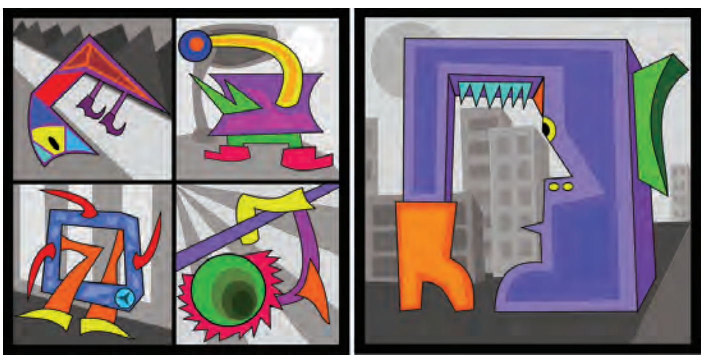

Figure 1. A few inkscape-drawn aliens from my unpublished comic book, ***Guardian of the Blue Metropolis*****.**

Introduction

I think of myself mostly as a research mathematician, but I also like to draw goofy pictures and explain things. Eventually these interests led me to write and illustrate some children's math books. In this article I will describe some aspects of my experience communicating mathematics to children through these books. I think that there is a great need for children's math books written by creative mathematicians, books which can show the vibrant, exciting, awe-inspiring nature of the subject. In case you are interested in doing this, I hope that my account will help you along.

In the first part of this article, I will describe how I got started in this business, as well as some obstacles I faced along the way. In the second part I will discuss my vision for what a children's book written by a creative mathematician might look like and why it doesn't fit into the conventional mold for a children's educational book.

How I Got Started

I have always enjoyed drawing comic books, and sometimes I take some time off from my research to do it. The books are usually about strange topics, like the collective intelligence of ants, or the Star Trek transporter problem, or alien life, or a monster made out of chicken lo mein. I sometimes draw the pictures by hand, but usually I use the computer programs **xfig** and **inkscape**. These are free drawing programs which I, like many mathematicians, use to illustrate my math papers. Figure 1 shows a few of the aliens I drew using inkscape.

I got interested in writing and illustrating comic books for children, naturally enough, when I had children of my own. In 2002, when my older daughter, Lucina, turned five, I drew a short booklet for her designed to teach her about prime numbers. Figure 2 shows roughly what the first version looked like.

The idea was for her to figure out the pattern from the pictures. Why are some of the numbers smiling?

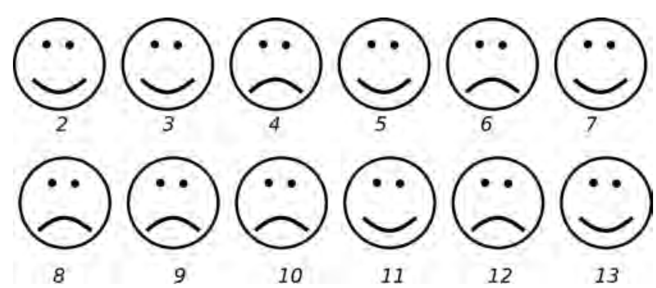

Figure 2. A drawing I made for my 5-year-old daughter to teach her about primes and composites.

Rich Schwartz is Chancellor's Professor of Mathematics at Brown University. His email address is res@math.brown.edu.

DOI: http://dx.doi.org/10.1090/noti1382

Figure 3. Some of my xfig pictures of the numbers.

Figure 4. A group photo involving the 2-monster, the 3-monster, and the 5-monster illustrates the number $30 = 2 \times 3 \times 5$**.**

Eventually I decided to jazz up the pictures a bit. Figure 3 shows the new pictures I drew using xfig.

I soon grew tired of drawing monsters, and it was then that I hit on the idea for my first children's math book, *You Can Count on Monsters*. Instead of having to create a new monster for every number, I decided to show a composite picture for composite numbers. The idea is to factor composite numbers into primes and then to arrange the corresponding prime monsters into a group photo. Figure 4 shows one example from the book.

It took me about five years to finish *You Can Count on Monsters* because I worked on it very sporadically. I don't mind saying that I did some of it while occupying Marcel Berger's office at IHES in the summer of 2002.

The Recursive Problem of Publishing

I tried occasionally to publish my comic books with big conventional publishers or smaller comic book publishers, but I never had a glimmer of success. The experience was always one of sending a manuscript off into a black hole. I had heard that it was a good idea to get an agent to help pitch your books to publishers, but I could never figure out how to get an agent. It seemed to me that you needed to know someone to get an agent. In short, you needed an agent to get an agent.

I went through similar struggles with the question of how to illustrate my books. Not having any formal training as an artist, I was very self-conscious about illustrating them myself. I had the persistent notion that my own illustrations would just be preliminary ones and that some great illustrator would go back through the book and re-do the pictures. So, sometimes I held off trying to publish something because I wanted to find an illustrator first. I could never find any illustrators, and so I started thinking about finding people who could introduce me to them.…

Eventually I decided to do the whole thing myself. With encouragement from my artist friends, my wife amongst them, I started thinking of my own illustrations as the final product. Also, I started using self-publishing companies like CreateSpace. With CreateSpace, you send in a PDF file containing your manuscript and then a few weeks later they send you a printed proof. You then tweak the manuscript and send it back. And so on. When you are happy with the manuscript, CreateSpace will sell it for you on amazon.com, print it on demand, and give you a small royalty. You may not get rich from this, but you can see a high-quality version of your book in print and for sale. A few of my comics ended up on amazon.com this way.

> *Write about what interests you, but think about the mind of the child.*

Shortly after making the original book for my daughter, I reimagined *You Can Count on Monsters* as a poster, where one could see all the pictures at once. Five years later, my wife convinced me to go back to the book format. I did this, and I was about to send it to CreateSpace, but on a whim I decided to ask Alice and Klaus Peters about it. At the time, Alice and Klaus owned A K Peters, a math publishing house which has published some wonderful and offbeat math books. They both liked the book and finally it was published.

Sadly, A K Peters no longer exists, but I think that there are still opportunities for publishing offbeat children's math books. I published my second book, *Really Big Numbers*, with the American Mathematical Society (AMS), and they plan to publish two more I wrote, *Gallery of the Infinite* and *Life on the Infinite Farm*. The AMS seems

Figure 5. Gracie, a cow with infinitely many feet, loves shoes.

interested in this sort of thing. I'd also like to mention that MSRI has a new program, called Mathical Books, which highlights and promotes mathematics in children's literature.

The Target Audience

Often when I tell someone that I have written a math book for children, they ask, *What is the age range for your book?* This is a very sensible question. When you start out on a commercial venture, you ought to know your target audience. Publishers, booksellers, and teachers will all want to know how to categorize, pitch, and place your book.

I hate this question. In my experience it has been the most discouraging thing anyone has asked me. Probably if I paid attention to the question I would never have written anything at all. Let me explain why I think that this question is not so appropriate for children's math books of the kind I imagine and how I dodged the question in one of my books.

I think that the kind of book a creative research mathematician might write for children would not appeal to the bulk of the children of any age. Rather, it will appeal to a few of the students at the top end of whatever grade they happen to be in, students who like the subject already but who are perhaps dissatisfied with what they are learning in school. I like the idea of writing books that have layers of meaning and detail, books whose depth unfolds before the reader as he or she spends more time with it.

Let me give an example from my book *Life on the Infinite Farm*. The book features a number of animals who are infinitely extended in one way or another. Figure 5 shows one of the characters, Gracie, an infinite cow who loves shoes. She has a shoe on every foot. Her dilemma is that she wants to wear the new shoes she gets as gifts from the other animals, but she doesn't want to take off her other shoes.

Anyone familiar with the famous Hilbert hotel will see the solution to her dilemma. She just places the shoes in front of her and steps out of each of the old shoes and into the new ones. All the shoes have moved back and she has the new shoes on her front feet. (The book has illustrations of this. If you are curious, you can find a link on my website.) It seems that a construction like this, which involves the nature of infinity, does not really have a target age; it ought to appeal to some people of all ages.

As a deeper example, I talk about Delores and Beena, two squids who have infinite branching trees of tentacles. The left half of Figure 6 shows Delores and the right half shows Delores and Beena together. Beena is asking Delores whether she can borrow some of Delores's jewelry.

Delores loves jewelry and wants to keep herself more or less completely covered in it. At the same time, Beena wants to be outfitted in a similar fashion. How does Delores keep herself covered in jewelry and outfit Beena as well? To solve this problem, Delores transfers some of her jewelry to Beena using local moves enabled by little fish. The fish move every piece of jewelry one unit towards Delores's head and thereby "double" the amount of jewelry she has. Then the fish transfer half the jewelry to Beena. Figure 7 shows two snapshots of the local move.

Some readers might recognize this as the heart of the proof that the free group on two generators is not an amenable group. I first saw this argument in a talk Shmuel Weinberger gave in Berkeley back in 1992, and it stayed with me all these years.

After presenting these kinds of problems and solutions, I raise the open-ended question as to how the animals

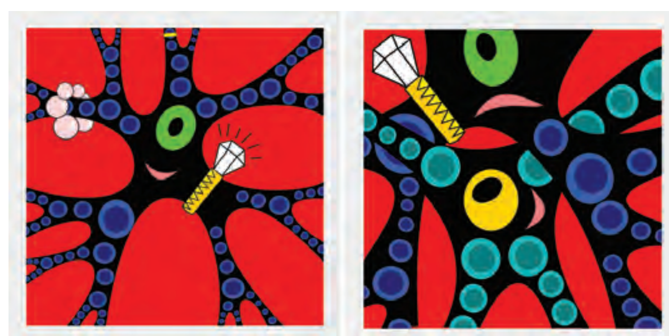

Figure 6. Delores and Beena are squids with infinite branching trees of tentacles. Beena asks Delores if she can borrow some of her jewelry.

Figure 7. Delores "doubles" her jewelry using local moves that transport all her jewelry closer to her head.

Figure 8. My book is like a bucking bronco, which you are supposed to ride until you fall off.

manage to move around on the farm, given their infinite sizes. How do they avoid crashing into each other? I suggest that the animals exist at many different scales, that they are quite acrobatic, and that some have features like trap doors and detachable parts. I also had in my mind that the infinite farm is a negatively curved space, so that the animals would not be stymied by the parallel postulate.

> *I imagine the proof of the Cantor-Bernstein Theorem as an analysis of the way cats and dogs are chasing each other.*

On the one hand, I can imagine that some very young children would be intrigued by the infinite and might like the pictures of the animals—I hope so, anyway. On the other hand, I can imagine older kids discussing how space might be designed to make something like this possible, or biologists wondering about an infinite periodic digestive tract, or physicists complaining about the nonrelativistic nature of the farm. It is hard to figure out an age range for *Life on the Infinite Farm*.

My book *Really Big Numbers* also presents material at many different levels of difficulty. It runs all the way from counting dots to recursive definitions akin to the Ackerman function. Figure 8 shows the method I used to dodge the problem of needing to write for a specific age or maturity level. At the beginning of the book, I explain that the book is a lot like the game of bucking bronco I used to play with my own children: The ride starts out slow and gradually gets faster until they fall off. The goal of the game is to stay on as long as possible, but it is no big deal if you fall off.

Once people get over the idea that they have to read everything in the book, the possibilities for exposition open up quite a bit.

Math in Slow Motion

In principle, all of mathematics reduces to a series of logical conclusions drawn from simple axioms. In practice, mathematicians take big strides through the system in an effort to reach deep and surprising results. One of the main issues involved in learning mathematics is the speed with which it piles up.

It might appear to take just a few pages to define, say, an integrable system on a smooth manifold, but if you wanted to explain it to a college student, you would have to slow down and explain what a manifold is and how calculus on manifolds works. If you wanted to reach a high school student, you would probably want to explain about the real numbers, continuity, linear algebra, and so on. Eventually the sheer length of time it would take you to explain the whole thing would make it impossible.

Given that children have very little math background, a children's math book which aims to impart real understanding has to be about a very small segment of mathematics. Fortunately, even very small pieces of mathematics—the Pythagorean Theorem, Heron's Formula, Pascal's triangle, the platonic solids, the infinitude of primes, the definition of Langton's ant, the hypercube,

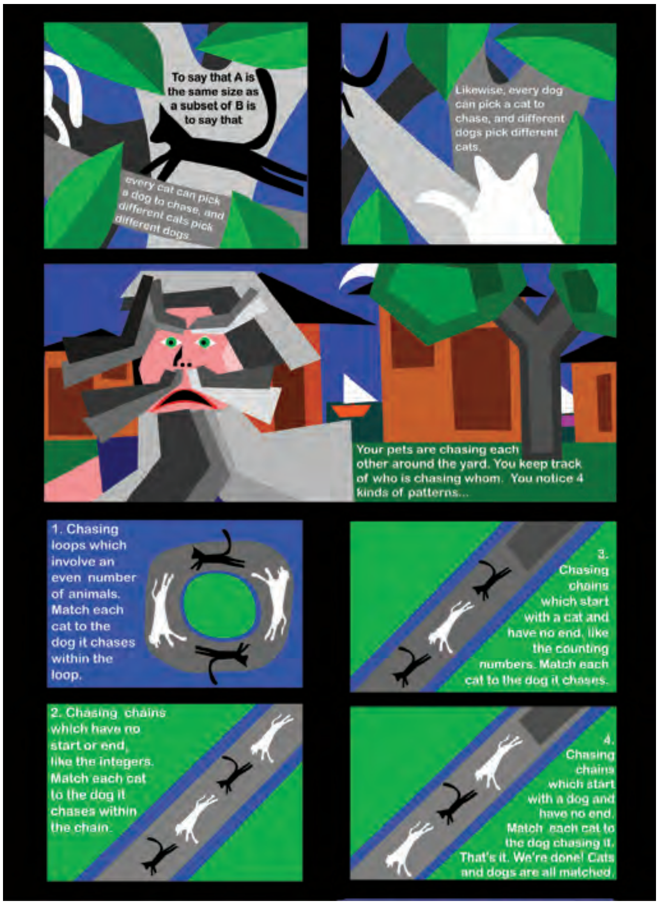

Figure 9. I imagine the proof of the Cantor-Bernstein Theorem as an analysis of the way cats and dogs are chasing each other.

the definition of continued fractions, scissors congruence of polygons, complex numbers, the Cantor diagonal argument, etc., etc., etc.—are beautiful and interesting. I think that a successful children's book about math should take a topic like this and present it vividly and in slow motion, so that a child could see every step.

In my book *Gallery of the Infinite* (which isn't quite for children) I tried to do this for some classic theorems in set theory, especially the famous Cantor diagonal argument. Figure 9 shows some of my exposition of the Cantor-Bernstein Theorem, which says that two sets A and B are bijective if there is an injection from A into B and an injection from B into A. I don't think that what I did is quite suitable for children, but I tried pretty hard to prove the result in a playful and engaging way.

> *Once people get over the idea that they have to read everything in the book, the possibilities for exposition open up quite a bit.*

It is hard to say exactly how to keep things slow and simple yet still present interesting mathematics, but it is easy to say what not to do. Like most mathematicians currently walking the planet, I have had the unpleasant experience (many times) of listening to a lecture in which the speaker assumes that the audience knows as much about the topic as the speaker does. Usually I am too embarrassed to stop the speaker, and I end up wasting an hour and coming out of the room with a headache. I can understand this happening when the speaker is addressing a large audience—perhaps the speaker has other listeners in mind—but sometimes this happens when it is just the two of us! Don't blow past your audience; write with empathy.

One positive suggestion I have is that you should illustrate your book lavishly. Children love catchy pictures. Also, the discipline of having a picture for every key concept in the book keeps the exposition going at a slow, measured pace. When I think about math, I often think in little cartoons, which later, in my papers, I have to transcribe into the written word. The reader then has to absorb all the words and formulas and (I hope) reassemble the cartoon pictures. It would be nice to be able to communicate the pictures directly. Maybe for simple topics this can be done.

The main idea is to understand the math all the way to the bottom, think about exactly how you understand it, and then put it all down on the page in a friendly and engaging way. Write about what interests you, but think about the mind of the child.

Courtesy of Brienne Brown.

Rich Schwartz and his daughters.