Part one by C. Denson Hill and Paweł Nurowski

How the Green Light Was Given for Gravitational Wave Search

The recent detection of gravitational waves by the LIGO/Virgo team (B. P. Abbot et al. 2016) is an incredibly impressive achievement of experimental physics. It is also a tremendous success of the theory of general relativity. It confirms the existence of black holes, shows that binary black holes exist and that they may collide, and that during the merging process gravitational waves are produced. These are all predictions of general relativity theory in its fully nonlinear regime.

The existence of gravitational waves was predicted by Albert Einstein in 1916 within the framework of linearized Einstein theory. Contrary to common belief, even the very *definition* of a gravitational wave in the fully nonlinear Einstein theory was provided only after Einstein's death. Actually, Einstein advanced erroneous arguments against the existence of nonlinear gravitational waves, which stopped the development of the subject until the mid 1950s. This is what we refer to as the *red light* for gravitational wave research.

In this note we explain how the obstacles concerning gravitational wave existence were successfully overcome at the beginning of the 1960s, giving the *green light* for experimentalists to start designing detectors, which eventually produced the recent LIGO/Virgo discovery.

Gravitational Waves in Einstein's Linearized Theory

The idea of a gravitational wave comes directly from Albert Einstein. Immediately after formulating General Relativity Theory, still in 1916 Einstein [3] linearized his field equations

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \kappa T_{\mu\nu} \tag{0.1}$$

by assuming that the metric $g_{\mu\nu}$ representing the gravitational field has the form of a slightly perturbed Minkowski metric $\eta_{\mu\nu}$,

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon h_{\mu\nu}.$$

Here $0 < \epsilon \ll 1$, and his linearization simply means that he developed the left hand side of (0.1) in powers of ϵ and neglected all terms involving ϵ^k with $k > 1$. As a result of this linearization Einstein found the field equations of *linearized* general relativity, which can conveniently be written for an unknown

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h_{\alpha\beta}\eta^{\alpha\beta}$$

as

$$\Box\bar{h}_{\mu\nu} = 2\kappa T_{\mu\nu}, \qquad \Box = \eta_{\mu\nu}\partial^\mu\partial^\nu.$$

These equations, outside the sources where

$$T_{\mu\nu} = 0,$$

constitute a system of decoupled relativistic wave equations

$$\Box h_{\mu\nu} = 0 \tag{0.2}$$

for each component of $h_{\mu\nu}$. This enabled Einstein to conclude that *linearized* general relativity theory admits solutions in which the perturbations of Minkowski spacetime $h_{\mu\nu}$ are plane waves traveling with the speed of light. Because of the *linearity*, by superposing plane wave solutions with different propagation vectors k_μ, one can get waves having any desirable wave front. Einstein named these *gravitational waves*. He also showed that within the linearized theory these waves carry energy, and he found a formula for the energy loss in terms of the third time derivative of the quadrupole moment of the sources.

Since far from the sources the gravitational field is very weak, solutions from the linearized theory should coincide with solutions from the full theory. Actually the wave detected by the LIGO/Virgo team was so weak that it was treated as if it were a gravitational plane wave from the linearized theory. We also mention that essentially all visualizations of gravitational waves presented during popular lectures or in the news are obtained using linearized theory only.

The Red Light

We focus here on the fundamental problem posed by Einstein in 1916, which bothered him to the end of his life. The problem is: Do the fully nonlinear Einstein equations admit solutions that can be interpreted as gravitational waves?

If "yes," then far from the sources, it is entirely reasonable to use linearized theory. If "no," then it makes no sense to expend time, effort, and money to try to detect such waves: solutions from the linearized theory are not physical; they are artifacts of the linearization.

If the answer is "no" we refer to it as a "red light" for gravitational wave search. This red light can be switched to "green" only if the following subproblems are solved:

(1) What is a definition of a *plane* gravitational wave in the full theory?
(2) Does the so defined plane wave exist as a solution to the full Einstein system?
(3) Do such waves carry energy?
(4) What is a definition of a gravitational wave with *nonplanar front* in the full theory?
(5) What is the energy of such waves?
(6) Do there exist solutions to the full Einstein system satisfying this definition?

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(7) Does the full theory admit solutions corresponding to the gravitational waves emitted by bounded sources?

To give a green light here, one needs a satisfactory answer to all these subproblems. Let us explain: Suppose that only the questions (1)–(3) had been settled in a satisfactory manner. Could we have a green light? The answer is no, because, contrary to the linear theory, unless we are very lucky, there is no way of superposing plane waves to obtain waves with arbitrary fronts. Thus the existence of a plane wave does not mean the existence of waves that can be produced by bounded sources, such as for example binary black hole systems.

Search for Plane Waves in the Full Theory

Naive Approach

A naive answer to our question (1) could be: a gravitational plane wave is a spacetime described by a metric, which in some coordinates (t, x, y, z), with t being timelike, has metric functions depending on $u = t - x$ only; preferably these functions should be sin or cos. This is not a good approach as is seen in the following example:

Consider the metric

$$\begin{aligned} g =& (\eta_{\mu\nu} + h_{\mu\nu})\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2 \\ &+ \cos(t-x)\Big(2 + \cos(t-x)\Big)\mathrm{d}t^2 \\ &- 2\cos(t-x)\Big(1+\cos(t-x)\Big)\mathrm{d}t\mathrm{d}x \\ &+ \cos^2(t-x)\mathrm{d}x^2. \end{aligned}$$

We see here that the terms after the first row give the perturbation $h_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$ of the Minkowski metric $\eta = \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$. They are *oscillatory*, and one sees that the *ripples of the perturbation move with the speed of light*, $c = 1$, along the x-axis. A closer look shows also that the coefficients $h_{\mu\nu}$ of the perturbation satisfy the wave equation (0.2) (since they depend on a single null coordinate u only), and more importantly, that the full metric g has Ricci curvature 0 (is "*Ricci flat*").

Thus the above metric is not only an example of a "gravitational wave" in the linearized Einstein theory, but also it provides an example of a solution of the vacuum Einstein equations $R_{\mu\nu} = 0$ in the *fully nonlinear* Einstein theory. With all this information in mind, in particular having in mind the sinusoidal change of the metric with the speed of light in the x direction, we ask: is this an example of a plane gravitational wave?

The answer is *no*, as we created the metric g from the flat Minkowski metric $\eta = \mathrm{d}\bar{t}^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2$ by a *change of the time coordinate*: $\bar{t} = t + \sin(t - x)$. In view of this, the metric g is just the flat Minkowski metric, written in nonstandard coordinates. As such it does not correspond to any gravitational wave!

The moral from this example is that attaching the name of a "gravitational wave" to a spacetime that just satisfies an intuitive condition in some coordinate system is a wrong approach. As we see in this example we can always introduce a sinusoidal behaviour of the metric coefficients and their 'movement' with speed of light, by an appropriate change of coordinates.

Figure 1. Herman Bondi (left) here pictured with Peter G. Bergmann at the Jabłonna Relativity Conference, 1962, was one of the first to establish the possibility of planar gravitational waves.

We need a mathematically precise definition of even a plane wave.

Red Light Switched on: Einstein and Rosen

The first ever attempt to define a plane gravitational wave in the full theory is due to Albert Einstein and Nathan Rosen [4]. It happened in 1937, twenty years after the formulation of the concept of a plane wave in the linearized theory. They thought that they had found a solution of the vacuum Einstein equations representing a plane polarized gravitational wave. They observed that their solution had certain singularities and as such must be considered as *unphysical*. Their opinion is explicitly expressed in the subsequent paper of Rosen [7], which has the following abstract:

The system of equations is set up for the gravitational and electromagnetic fields in the general theory of relativity, corresponding to plane polarized waves. It is found that all nontrivial solutions of these equations contain singularities, so that one must conclude that strictly plane polarized waves of finite amplitude, in contrast to cylindrical waves, cannot exist in the general theory of relativity.

The Einstein-Rosen paper [4] was refereed by Howard P. Robertson, who recognized that the singularities encountered by Einstein and Rosen are merely due to the wrong choice of coordinates and that, if one uses correct coordinate patches, the solution may be interpreted as a cylindrical wave, which is nonsingular everywhere except on the symmetry axis corresponding to an infinite line source. This is echoed in Rosen's abstract quoted above in his phrase "in contrast to cylindrical waves," and is also mentioned in the abstract of the earlier Einstein-Rosen paper [4], whose first sentence is: *The rigorous solution for cylindrical gravitational waves is given.* Nevertheless, despite the clue given to them by Robertson, starting from 1937, neither Einstein nor Rosen believed that physically acceptable plane gravitational waves were admitted by the full Einstein theory. This belief of Einstein affected the views of his collaborators, such as Leopold Infeld, and more generally many other relativists. If a plane gravitational wave is not admitted by the theory, and if this statement comes from, and is fully supported by, the authority of Einstein, it was hard to believe at any fundamental level that the predictions of the linearized theory were valid.

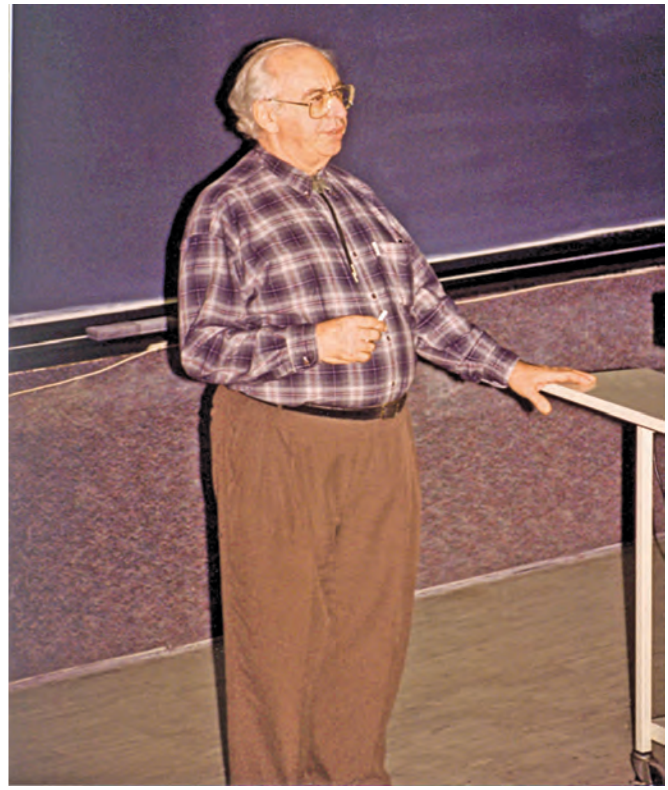


Figure 2. Ivor Robinson, shown here during Journées Relativistes in Dublin, 2001, was an independent discoverer of an exact solution describing planar gravitational waves.

Towards the Green Light: Bondi, Pirani, and Robinson

It is now fashionable to say that a new era of research on gravitational waves started at the International Conference on Gravitation held at Chapel Hill on 18–23 January 1957. To show that not everybody was sure about the existence of gravitational waves during this conference we quote Herman Bondi [1], one of the founding fathers of gravitational wave theory:

Polarized plane gravitational waves were first discovered by N. Rosen, who, however, came to the conclusion that such waves could not exist because the metric would have to contain certain physical singularities. More recent work by Taub and McVittie showed that there were no unpolarized plane waves, and this result has tended to confirm the view that true plane gravitational waves do not exist in empty space in general relativity. Partly owing to this, Scheidegger and I have both expressed the opinion that there might be no energy-carrying gravitational waves at all in the theory.

The last sentence in the quote refers to Bondi's opinion expressed during the Chapel Hill Conference. Interestingly, the quote is from Bondi's *Nature* paper announcing the discovery of a singularity-free solution of a plane gravitational wave that carries energy, received by the journal on March 24, 1957. A dramatic change of opinion between January and March of the same year!

Bondi in the *Nature* paper invokes the solution of Einstein's equations found in the context of gravitational waves by Ivor Robinson. This paper, and the subsequent paper written by Bondi, Felix Pirani, and Robinson [2], answers in positive our problems (1), (2), and (3).

In particular (1) is answered with the following definition of a *plane wave in the full theory*: The gravitational plane wave is a spacetime that (a) satisfies vacuum Einstein's equations $R_{\mu\nu} = 0$ and (b) has a 5-dimensional group of isometries. The motivation for this definition is the fact that a plane electromagnetic wave has a 5-dimensional group of symmetries. Bondi, Pirani, and Robinson do *not* assume that the 5-dimensional group of isometries is isomorphic to the symmetry of a plane electromagnetic wave. They inspect all Ricci flat metrics with symmetries of dimension greater than or equal to 4 given by A. Z. Petrov (1957), and find exactly one class of solutions with the same 5-dimensional group of isometries, which by a miracle *is* isomorphic to the symmetry group of the electromagnetic field.

It follows that the class of metrics obeying the Bondi-Pirani-Robinson definition of a plane gravitational wave depends on two *free functions of one variable* that can be interpreted as the wave amplitude and the direction of polarization. Using these free functions Bondi, Pirani, and Robinson obtained a *sandwich wave*, i.e. a gravitational wave that differs from the Minkowski spacetime only in a 4-dimensional strip moving in a given direction with the speed of light. They used this sandwich wave and analyzed what happens when it hits a system of test particles. It follows that the wave *affects* their motion, which leads to the conclusion that *gravitational plane waves in the full theory carry energy*.

In this way, the *Nature* paper of Bondi [1], together with the later paper of Bondi, Pirani, and Robinson [2], *solves our problems* (1), (2) *and* (3): the plane wave in the full theory is defined, it is realized as a class of solutions of Einstein field equations $R_{\mu\nu} = 0$, and it carries energy, since passing through the spacetime in a form of a sandwich it affects test particles.

As a last comment in this section we mention that the Bondi-Pirani-Robinson gravitational plane waves, sought

Figure 3. Felix Pirani, shown here in 1937, when Einstein and Rosen were writing their controversial paper, and in May 2015, a few months before his death, collaborated with Bondi and Robinson and gave an algebraic local criterion for gravitational waves.

with great effort by physicists for forty years, were actually discovered already in 1925 by a *mathematician*, H. W. Brinkmann. He discovered what are known as *pp-waves*, a class of Ricci flat metrics having radiative properties, which include Bondi-Pirani-Robinson plane waves as a special case. His discovery was published in English in *Mathematische Annalen* **94** (1925), 119–145. If only there had been better communication between mathematicians and physicists.

General Gravitational Waves

Closer to the Green: Pirani

The development of the theory of gravitational waves at the turn of the 1950s and 1960s was very rapid. The story, as we are presenting it here now, is more topical than chronological, so, breaking the chronology, we will now discuss an important paper of Felix Pirani [5], which appeared before Bondi's *Nature* announcement of the existence of a plane wave in Einstein's theory. It is also worthwhile to note that Pirani's paper [5] was submitted a few months *before* the Chapel Hill conference. For us, this paper is of fundamental importance, since, among other things, it gives the first attempt at a purely geometric *definition of a gravitational wave spacetime*.

Pirani argues that gravitational radiation should be detectable by analysis of the Riemann tensor. He suggests that a spacetime containing gravitational radiation should be *algebraically special*. This suggestion uses the so-called *Petrov classification* of gravitational fields. At every point it consists in the enumeration of the distinct *eigendirections* of the Weyl tensor (the traceless part of the Riemann tensor). These eigendirections are called *principal null directions* (PNDs). If at a point all four PNDs are distinct, the spacetime at this point is called *algebraically general*. If at least two of the PNDs coincide, the spacetime at this point is called *algebraically special*. At each point various coincidences of PNDs may occur, resulting in the stratification of the algebraically special spacetime points into four *Petrov types*: type II (two PNDs coincide, the other two are distinct), type III (three PNDs coincide), type N (four PNDs coincide), and type D (four PNDs are grouped in two different pairs of coinciding PNDs). Pirani's suggestion that spacetimes containing radiation should be algebraically special *everywhere* was not very precise, as all the Petrov types (II, III, D, N) had not yet been correctly spelled out (the fully correct Petrov classification was given later by Roger Penrose in 1960).

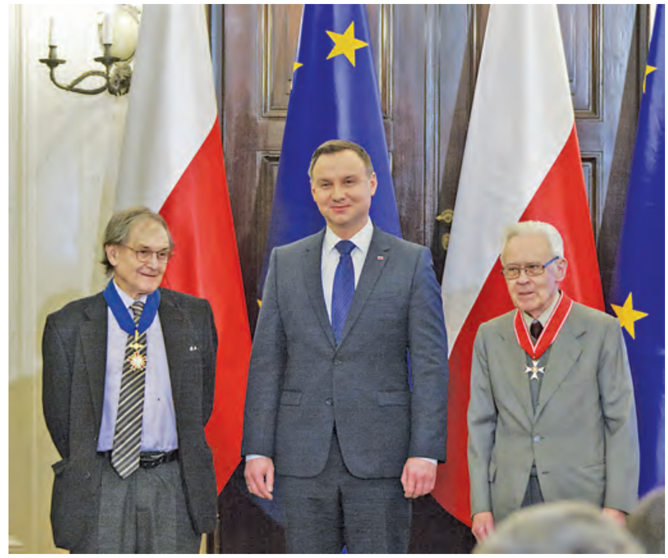


Figure 4. Roger Penrose (left), President of the Republic of Poland Andrzej Duda (center), and Andrzej Trautman at the ceremony at which Penrose got the highest Polish medal of merit for a foreigner and Trautman for a Pole, Warsaw 2016. Penrose and Trautman developed a nonlocal theory of radiation.

Pirani's intuition about the importance of algebraic speciality in the theory of gravitational waves was brilliant. However, he was wrong in insisting on algebraical speciality of radiative spacetimes everywhere. We know now ([9], p. 411, eq. (21)) that the Weyl tensor of a radiative spacetime must be of type N very *far from the sources*, or better said, *asymptotically*.

Switching on Green: Radiation is Nonlocal

Pirani's algebraic speciality condition for a gravitational wave spacetime refers to pointwise defined objects—the PNDs. As the Weyl tensor can change its algebraic type from point to point, the criterion is local. On the other hand, even in Maxwell theory, radiation is a nonlocal phenomenon. To illustrate this we recall a well-known conundrum:

Q: Does a unit charge hanging on a thread attached to the ceiling of Einstein's lift radiate or not?
A: Well... viewed by an observer in the lift—NO!, as it is at rest; but, on the other hand, viewed by an observer on the Earth—YES!, as it falls down with constant acceleration $\vec{g}$.

Here, the confusion in the answers is of course due to the fact that one tries to apply a *purely local* physical law—the equivalence principle[1]—to the very nonlocal phenomenon, which is radiation in electromagnetic theory.

This gives a hint as to how to define what radiation is in general relativity. One can not expect that in this nonlinear theory radiation can be defined in terms of local notions. This point is raised and consequently developed by Andrzej Trautman, in two papers [8, 9] submitted to *Bulletin de l'Academie Polonaise des Sciences*, behind the Iron Curtain, in April 1958. This led him to finally solve our problems (4)–(5), [9], and (6)–(7), [6], thereby switching the red light to green.

It is worthwhile to mention that although Trautman's two papers [8, 9] were published behind the Iron Curtain, their results were exposed to the Western audience. In the next two months after their submission to the Polish *Bulletin* (May–June, 1958) Trautman, on the invitation of Felix Pirani, gave a series of lectures at King's College London presenting their theses. The audience of his lectures included H. Bondi and F. Pirani, and the lectures were mimeographed and spread among Western relativists.

Another interesting thing is that Trautman's two papers were an abbreviated version of his PhD thesis. It had two supervisors: the official one—Leopold Infeld, the closest collaborator of Albert Einstein, who following Einstein did not believe in gravitational waves, and the unofficial one—Jerzy Plebański, for whom the existence of gravitational waves was obvious. It was Plebański who proposed gravitational waves as a subject of Trautman's PhD. Despite Infeld's disbelief in gravitational waves, Trautman obtained his PhD under Infeld.

Green Light: Trautman

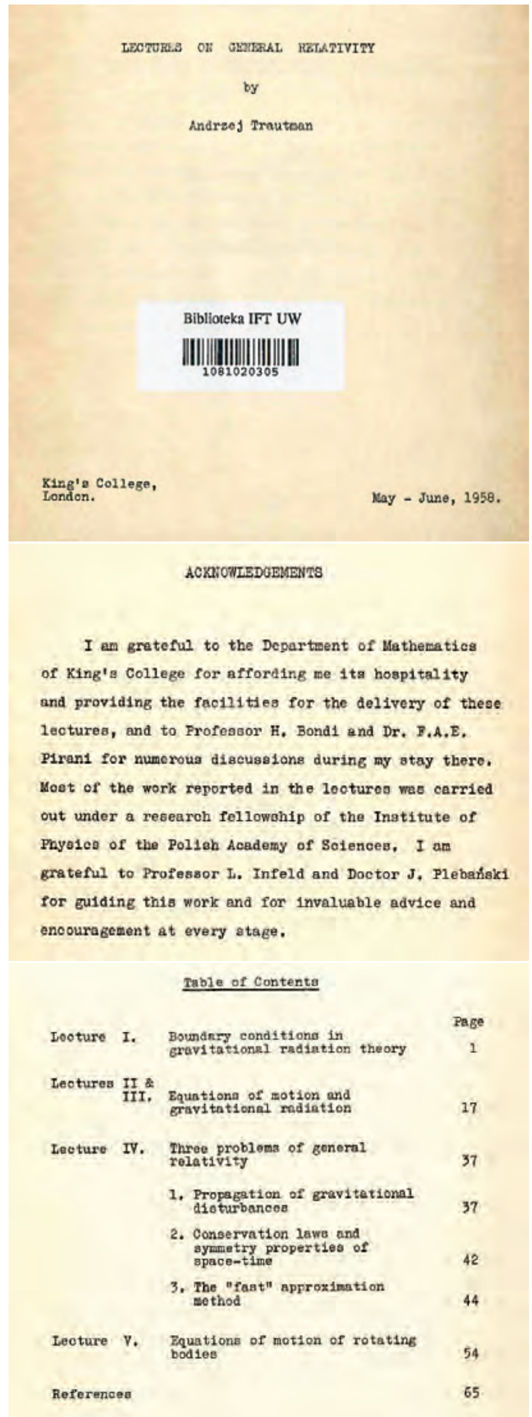
Trautman's general idea in defining what a gravitational wave is in the full Einstein theory was to say that it should satisfy certain boundary conditions at infinity. More precisely, from all spacetimes, i.e. solutions of Einstein's equations in the full theory, he proposed to select only those that satisfied boundary conditions at infinity, which were his *generalizations* of Sommerfeld's radiation conditions. These are known in the linear theory of a scalar field, and Trautman [8, 9] generalizes them to a number of *physical theories*. He reformulates Sommerfeld's radiation boundary conditions for the scalar inhomogeneous wave equation into a form that is then generalized to other field theories. As an example he shows how this generalization works in Maxwell's theory and that it indeed selects the outgoing radiative Maxwell fields from all solutions of Maxwell's equations.

In the next paper [9] Trautman does the same for Einstein's general relativity. Trautman defines the *boundary conditions to be imposed on gravitational fields due to isolated systems of matter*. This is the first step in solving our problems (4) and (5).

He then passes to the treatment of our problem (5). He uses the *Freud superpotential* 2-form $\mathcal{F}$ to split the Einstein tensor E into $E = \mathrm{d}\mathcal{F} - \kappa\mathfrak{T}$ so that the Einstein equations $E = \kappa T$ take the form

$$\mathrm{d}\mathcal{F} = \kappa(T + \mathfrak{T}).$$

[1] *The inability to locally distinguish between gravitational and inertial forces.*


LECTURES ON GENERAL RELATIVITY

by

Andrzej Trautman

King's College,
London.

May - June, 1958.

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Figure 5. The first three pages of Trautman's mimeographed King's College Lectures, which carried Trautman's work behind the Iron Curtain to Western relativists.

Figure 6. A blackboard discussion between Trautman's two advisors, Jerzy Plebański (left) and Leopold Infeld, at the Institute of Theoretical Physics of University of Warsaw. Ironically, Trautman got his PhD on gravitational waves as recommended by Plebański under Infeld, who didn't believe in them.

Here T is the energy-momentum 3-form, and κ is a constant related to the gravitational constant G and the speed of light c via $\kappa = \frac{8\pi G}{c^4}$ (in the following we work with physical units in which $c = 1$).

Since $\mathfrak{T}$ is a 3-form totally determined by the geometry, it is interpreted as the energy-momentum 3-form of *pure gravity*. The closed 3-form $T + \mathfrak{T}$ is then used to define the 4-momentum $P^\mu(\sigma)$ of a *gravitational field attributed to every space-like hypersurface* σ of a spacetime satisfying his radiative boundary conditions. He shows that $P^\mu(\sigma)$ is *finite* and *well defined*, i.e. that it does not depend on the coordinate systems adapted to the chosen boundary conditions. Using his boundary conditions he then calculates how much of the gravitational energy $p^\mu = P^\mu(\sigma_1) - P^\mu(\sigma_2)$ contained between the spacelike hypersurfaces σ_1 (initial one) and σ_2 (final one) *escapes to infinity*.

Finally, he shows that p^0 is *nonnegative*, saying that radiation is present when $p^0 > 0$.

Taken together, everything we have said so far about Trautman's results, *solves our problems* (4) and (5): What in popular terms is called a *gravitational wave in the full GR theory* is a *spacetime satisfying Trautman's boundary conditions with* $p^0 > 0$; the *energy of a gravitational wave* contained between hypersurfaces σ_1 and σ_2 is given by p^0.

Trautman proves only that $p^0 \geq 0$. If the inequality were sharp, $p^0 > 0$, this would give a proof of the statement that spacetimes satisfying Trautman's boundary conditions, or better said, the gravitational waves associated with them, *carry energy*. Trautman does not have such a proof. To handle this problem, one can try to find an example of an *exact solution* to the Einstein equations satisfying Trautman's boundary conditions, and to show that in this example p^0 is *strictly* greater than zero. This approach is taken by I. Robinson and Trautman [6], and we will comment on this later.

Figure 7. Andrzej Trautman established gravitational waves in the full Einstein theory.

As regards Trautman's paper [9], it is worthwhile to mention that Trautman shows there two other interesting things implied by his boundary conditions. The first of them is the fact that in the presence of electromagnetic radiation a spacetime satisfying his boundary conditions has far from the sources Ricci tensor in the form of a *null dust* $R_{\mu\nu} = \rho k_\mu k_\nu$, with k a *null vector*. This in particular means that the electromagnetic/gravitational radiation in his spacetimes travels with the speed of light. The second interesting feature he shows is that *far from the sources the Riemann tensor of a spacetime satisfying his radiative boundary conditions is of Petrov type N*. Since far from the sources $Riemann = Weyl$, this verifies the *intuition* of Pirani [5]: spacetimes satisfying radiative boundary conditions satisfy the algebraic speciality criterion, and from all the possibilities of algebraic speciality they choose a type N Weyl tensor as the leading term at infinity. This was later developed into the celebrated *peeling-off theorem* attributed to Ray Sachs.

The last two of our problems (6)–(7) were addressed by I. Robinson and Trautman [6]. There they *found a large class of exact solutions* of the full system of Einstein equations satisfying Trautman's boundary conditions.

Figure 8. Paul A. M. Dirac with Trautman and Infeld during the 1962 Jabłonna Conference.

The solutions describe waves with *closed fronts* so *they can be interpreted as coming from bounded sources.*

These solutions solve our last two problems (6) *and* (7). For some of them $p^0 > 0$, so they correspond to gravitational waves that *do carry energy*.

To conclude, we say that the Bondi-Pirani-Robinson papers [1, 2] and the Trautman-Robinson papers [9, 6] solve all our problems (1)–(7), giving the green light to further research on gravitational radiation. We will not comment on these further developments since they are well documented; see e.g. D. Kennefick's recent book *Traveling at the Speed of Thought: Einstein and the Quest for Gravitational Waves*.

Figure 9. Ivor Robinson and Andrzej Trautman, shown here in Trieste in the late 1980s, found a large class of exact solutions for gravitational waves with closed fronts in the full Einstein theory.

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