OPINION

Don't Just Begin with "Let *A* be an algebra..."

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Note: The opinions expressed here are not necessarily those of Notices.

It is not unusual for a paper in our main research journals in mathematics to begin:

> *Let* A *be the algebra of formal Dirichlet series relative to the order relation of divisibility.*

Various definitions are introduced, and before long there are theorems and proofs, with perhaps a sentence or two along the way to guide the reader. If the paper is brief, that may be all the room one has. The *Satz-Beweis* style has long ago replaced the narrative style, where various observations are made and one is then shown the theorem that has just been proved. Presumably, for those who are in the center of a subfield who come fully prepared, the *Satz-Beweis* style is an efficient way of finding out about the latest advance in their sub-sub-field.

Gian-Carlo Rota[1] was one who believed that a paper should have an informative introduction that summarized the flow of the paper, pointed out the nature of the contribution, and placed the work in the context of the literature—perhaps even going back two centuries, much unlike our initial example above. So Rota would draft such an introduction for the papers of his former students, in effect educating them about what they had just done. In his "Ten Lessons I Wish I had Been Taught" [5], Rota wrote:

> Write informative introductions. If we wish our paper to be read, we had better provide our prospective readers with strong motivation to do so. A lengthy introduction, summarizing the history of the subject, giving everybody his due, and perhaps enticingly outlining the content of the paper in a discursive manner, will go some of the way towards getting us a couple of readers.

A paper should have an informative introduction.

Rota would urge you to give away the secret handshakes, the punchlines, the achievement and the contribution, the location within the research literature, all in the first page or so. And if the title is less cute but more informative, so much the better.

Rota's strategies, besides summarizing the contents of the paper, include informal and up-front introduction of notions that are novel in this context, connections with other branches of mathematics, and intellectual and historical motivations for the work. In many of these introductory passages there is what might be called a dramatic reversal: a stubborn problem is described; it would seem to be unavoidable; yet, here we have found a fruitful path.

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[1] *I knew Rota through a shared interest in phenomenology and mostly as a family friend. He would often talk about issues in his field and the better practices. I am trained as a physicist, so most of the time his remarks resonated with me.*

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Here are some brief examples:

Introducing novel notions and a dramatic presentation.

In Rota's paper [1] on the Möbius function, Rota first identifies the topic as a generalization of the familiar inclusion-exclusion principle:

> This work begins the study of a very general principle of enumeration, of which the inclusion-exclusion principle is the simplest, but also the typical case.

He then explains the nature of the generalization, from an order to a partial order:

> It often happens that a set of objects to be counted possesses a natural ordering, in general only a partial order. It may be unnatural to fit the enumeration of such a set into a linear order such as the integers; instead, it turns out in a great many cases that a more effective technique is to work with the natural order of the set.

Gradually he arrives at the specific concept of Möbius function as a generalization of the fundamental theorem of calculus:

> Looked at in this way, a variety of problems of enumeration reveal themselves to be instances of the general problem of inverting an "indefinite sum" ranging over a partially ordered set. The inversion can be carried out by defining an analog of the "difference operator" relative to a partial ordering. Such an operator is the Möbius function, and the analog of the "fundamental theorem of the calculus" thus obtained is the inversion formula on a partially ordered set. …In fact, the algebra of formal Dirichlet series turns out to be the simplest non-trivial instance of such a "difference calculus," relative to the order relation of divisibility.

Nominal authority [4].

> It was Hermann Weyl (or if it wasn't he, it should have been) who said a powerful technique for the study of an algebra is the classification of its endomorphisms and their infinitesimal analogues, namely, derivations.

Schematic history [2].

> Since the beginning of this century the development of group theory has been dominated by the notion of representation, and the seemingly more specialized theory of group actions (permutations) has been given short shrift. To be sure, every action of a group can be considered as a particular representation by matrices.…

Or, one might be setting the stage for a new approach while giving full credit to forbears, as in Rota's introduction to "On the foundations of combinatorial theory, I" [1]:

> One of the most useful principles of enumeration in discrete probability and combinatorial theory is the celebrated principle of inclusion-exclusion (cf. Feller, Fréchet, Riordan, Ryser). When skillfully applied, this principle has yielded to many a combinatorial problem.

Analogy with well-known mathematics [2].

> The idea for the solution of this problem bears a resemblance to Galois theory, an approach first fully presented by Rota and Smith.

Motivation

Motivation is only sometimes the actual motivation of the mathematician. More often, it is a hint for the reader, so that they can go from what they are acquainted with to what will follow [3]:

> The fact that joins and meets in the lattice of subspaces does not satisfy the distributive law that holds for union and intersection of sets was once seen as a fatal drawback.

But, surprisingly, there is no fatal drawback:

> …Thus, the failure of the distributive law of set theory has proved to be a purely psychological obstacle, at least as far as enumeration goes.

Following Rota's advice and writing a good introduction to a paper will make it possible for you to have more readers, more references to your work, and a chance for you yourself to see your work in a more general context. *Early on* introducing *novel notions*; mentioning *names* of other mathematicians, living or dead; providing a *schematic history*; pointing to *resemblances* with widely known mathematics; and claiming *motivation* for the work, whether it is the author's motivation or just meant for the reader—all will make your papers more effective. Hiding what you are doing, in the name of rigor or purity, is likely to reduce your readership.

References

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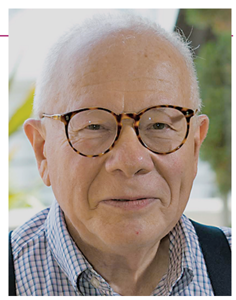
ABOUT THE AUTHOR

Martin H. Krieger does "urban tomography," systematic photographic and aural documentation of Los Angeles and New York City that aims to get at "an identity in a manifold presentation of profiles." He has written books on doing mathematics and mathematical physics. Currently, he is writing a book on uncertainty ("unk-unks").



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