2018 Chevalley Prize in Lie Theory

Dennis Gaitsgory was awarded the 2018 Chevalley Prize in Lie Theory at the 124th Annual Meeting of the AMS in San Diego, California, in January 2018.

Dennis Gaitsgory

Citation

The 2018 Chevalley Prize is awarded to Dennis Gaitsgory for his work on the geometric Langlands program, especially his fundamental contributions to the categorical Langlands conjecture and its extension in his recent work with Dima Arinkin.

The original arithmetic Langlands program applies to number fields. What is now called the geometric Langlands program applies to function fields, in particular to fields $\mathbb{F}$ of meromorphic functions on complex nonsingular algebraic curves X. It arose from a series of ideas of Beilinson, Deligne, Drinfeld, and Laumon in the 1980s, following Langlands' far-reaching results and conjectures from the 1960s. The goal is to establish reciprocity laws between a certain type of geometric data attached to G-bundles on X (specifically, sheaves on the moduli space of G-bundles on X) and spectral data consisting of homomorphisms from the Galois group of $\mathbb{F}$ to the Langlands dual of G. Here G is a Chevalley (or, more generally, reductive) group over the ground field.

Dennis Gaitsgory is largely responsible for having created a systematic theory from what had been a collection of provocative ideas and insights. Gaitsgory's major results include: his proof of the "vanishing conjecture," which is a geometric analogue of regularity of Rankin-Selberg L-functions; his construction of the (geometric) Hecke eigensheaf corresponding to a (spectral) irreducible local system for $G = GL(n)$, joint with Frenkel and Vilonen and extending the work of Drinfeld for $n = 2$; his construction with Braverman of (geometric) Eisenstein series corresponding to (spectral) reducible local systems, following special cases established by Laumon; his remarkable application of the nearby-cycles functor from algebraic geometry to geometric Langlands; his work with Braverman and Finkelberg on the Uhlenbeck compactification—work which may also extend some of the theory of Eisenstein series from G to a Kac-Moody group; and his proof of a miraculous duality for the stack of G-bundles and its role in the functional equation for Eisenstein series.

The geometric Langlands program is most naturally formulated in terms of derived categories. The conjectural reciprocity law then becomes a statement about the existence of an equivalence between two categories and is known as the categorical Langlands conjecture. The derived category for the geometric side is the category of D-modules on the stack of G-bundles on X. On the spectral side, it is the category of quasi-coherent sheaves on the stack of local systems for the dual group of G. However, the category on the spectral side has suffered from a number of internal contradictions. In two recent, fundamental papers, Gaitsgory and Arinkin were able to correct this problem. The authors introduced a larger category for the spectral side, which they were then able to relate to a more familiar category based on parabolic subgroups of G. The revised categorical Langlands conjecture is very elegant and bears a closer resemblance to the original arithmetic Langlands program. In particular, it introduces objects that correspond in the arithmetic program to the expected automorphic representations that occur in the discrete spectrum but which do not satisfy the generalized Ramanujan conjecture.

The two papers of Gaitsgory with Arinkin represent the state of the art for the geometric Langlands program. Despite (or perhaps because of) their abstraction, they contain many beautiful ideas. They can be seen as marvelous examples of the unity of mathematics.

Gaitsgory's recent work on these topics appears in "Singular support of coherent sheaves and the geometric Langlands conjecture," *Selecta Math. (N.S.)* **21** (2015), 1–199 (with D. Arinkin), "Geometric constant term functor(s)," *Selecta Math. (N.S.)* **22** (2016), 1881–1951 (with V. Drinfeld), "A strange functional equation for Eisenstein series and miraculous duality on the moduli stack of bundles," arXiv:1404.6780, and "The category of singularities as a crystal and global Springer fibers", arXiv:1412.4394 (with D. Arinkin). Earlier notable publications include "On a vanishing conjecture appearing in the geometric Langlands correspondence," *Annals of Mathematics (2)* **160** (2004), no. 2, 617–682, and "Geometric Eisenstein series," *Inventiones Mathematicae* **150** (2002), no. 2, 287–384 (with A. Braverman).

Biographical Sketch

Dennis Gaitsgory received his PhD in 1998 from Tel Aviv University, where he studied with Joseph Bernstein. He was a Junior Fellow at Harvard (1997–2001) and a Clay Research Fellow (2001–2004). He held his first faculty position at the University of Chicago (2001–2005) and is currently a professor of mathematics at Harvard University.

His research focuses on the geometric Langlands theory in its various aspects (local and global, classical and quantum) and its relation to other areas of mathematics (geometry of moduli spaces of bundles on curves, the theory of *D*-modules, derived algebraic geometry, representations of Kac-Moody Lie algebras).

He was a recipient of the prize of the European Mathematical Society in 2000 and of a Simons Fellowship in 2015–2016.

Response from Dennis Gaitsgory

I am immensely honored to receive the Chevalley Prize.

I remember a phrase of my PhD advisor, Joseph Bernstein, that in addition to the three commonly known pillars of mathematics (algebra, analysis, and geometry), there is the fourth one—Lie theory, which describes the fundamental laws of symmetry. The mathematical objects produced by Lie theory are obtained by coupling a certain combinatorial data (Dynkin diagram, or, more generally, a root datum) to another type of mathematical structure. In its most basic incarnation, when this other piece of structure is a field, we obtain Chevalley groups.

As an aside, Claude Chevalley and my advisor's advisor, I. M. Gelfand, were at the origin of the above philosophy. A significant part of the work of the founder of this prize, G. Lusztig, can be seen in this light as well.

The Langlands correspondence is a striking property of Lie theory. It says that given a root datum, its coupling to a certain family of mathematical data (let us call it A-data) produces an object equivalent to one obtained by coupling the *dual root datum* with another type of data (call it B-data). The proximity of our terminology (i.e., A and B) to that appearing in quantum field theory is not a coincidence.

The geometric Langlands theory is a particular case of this phenomenon. Here, the A-coupling produces the category of *D*-modules on the moduli space of *G*-bundles on a given algebraic curve, and the B-coupling produces the category of quasi-coherent sheaves on the stack of *G*-local systems on the same curve.

My current perspective is that at its most fundamental, the geometric Langlands theory appears in its quantum version, where one can (hope to) trace the geometric Langlands phenomenon down to its source, i.e., directly relate the corresponding categories of *D*-modules to combinatorial data. Namely, one starts with a root datum and a quantum parameter and explicitly produces a certain geometric object, called a factorization algebra (technically, this is a family of perverse sheaves on configuration spaces of colored divisors). The categories on both the A and the B sides should be related by explicit procedures to the category of modules over this factorization algebra.

I am thrilled that my work has been recognized as a contribution to the development of Lie theory. On this occasion, I would like to thank the people who have mentored me throughout my career: Sasha Beilinson, Joseph Bernstein, Vladimir Drinfeld, and David Kazhdan.

I am grateful to Dima Arinkin for the very inspiring collaboration. Finally, I would like to thank Jacob Lurie for opening my eyes onto the world of higher categories, which became key technical tools in the geometric Langlands theory.

About the Prize

The Chevalley Prize is awarded by the AMS Council acting on the recommendation of a selection committee. The members of the selection committee for the 2018 Chevalley Prize were:

- James G. Arthur
- Jens Carsten Jantzen (Chair)
- Michele Vergne

The Chevalley Prize is for notable work in Lie theory published during the preceding six years; a recipient should be no more than twenty-five years past the PhD. The current prize amount is US$8,000, awarded in even-numbered years, without restriction on society membership, citizenship, or venue of publication. The Chevalley Prize was established in 2014 by George Lusztig to honor Claude Chevalley (1909–1984). Chevalley was a founding member of the Bourbaki group. He made fundamental contributions to class field theory, algebraic geometry, and group theory. His three-volume treatise on Lie groups served as a standard reference for many decades. His classification of semisimple groups over an arbitrary algebraically closed field provides a link between Lie's theory of continuous groups and the theory of finite groups, to the enormous enrichment of both subjects.

The inaugural prize in 2016 was awarded to Geordie Williamson.

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Photo of Dennis Gaitsgory courtesy of Dennis Gaitsgory.