

2018 Frank Nelson Cole Prize in Algebra

ROBERT GURALNICK was awarded the 2018 Frank Nelson Cole Prize in Algebra at the 124th Annual Meeting of the AMS in San Diego, California, in January 2018.



Robert Guralnick

Citation

The 2018 Frank Nelson Cole Prize in Algebra is awarded to Robert Guralnick of the University of Southern California for his groundbreaking research on representation theory, cohomology, and subgroup structure of finite quasisimple groups and the wide-ranging applications of this work to other areas of mathematics.

Guralnick's paper "First Cohomology Groups of Chevalley Groups in Cross Characteristic" (with Pham Huu Tiep), published in the *Annals of Mathematics* in 2011, establishes an explicit upper bound for the dimension of the cohomology groups $H^1(G;V)$, where G is a finite Chevalley group defined over the finite field of characteristic p , and V is an irreducible representation of G in characteristic $\neq p$. This bound extends previous results of Cline, Parshall, and Scott (which apply to representations in characteristic p) and is of key importance for the Aschbacher-Scott program of understanding maximal subgroups of arbitrary finite groups.

The paper "Bounds on the Number and Sizes of Conjugacy Classes in Finite Chevalley Groups with Applications to Derangements" (with Jason Fulman), published in *Transactions of the American Mathematical Society* in 2012, gives a sharp bound for the total number of irreducible representations of finite Chevalley groups, resolving a long-standing question. This bound played a key role in recent advances on several old conjectures in group theory, including the Ore Conjecture, and the non-commutative Waring Problem.

The paper "Products of Conjugacy Classes and Fixed Point Spaces" (with Gunter Malle), published in the *Journal of the American Mathematical Society* in 2012, proves a strong generation result for finite simple groups. As consequences, the authors prove the 1966 conjecture of P. M. Neumann concerning fixed point subspaces in an irreducible representation of any finite group and a conjecture of Bauer, Catanese, and Grunewald concerning unmixed

Beauville structures (building on earlier work, including that of Garion, Larsen, and Lubotzky).

These papers represent only a small portion of Guralnick's overall contribution to group theory and adjacent areas, such as the inverse Galois problem, algebraic curves, arithmetic aspects of representation theory, expanders, etc. He is a prolific and dynamic problem solver, whose work has been getting more influential with the passage of time, and he has become the "go-to" person for a wide range of mathematicians in need of group-theoretic expertise.

Special note from the Selection Committee: Vladimir Voevodsky, our colleague on the Prize Committee, passed away just before the completion of the selection process. His input in our deliberations was invaluable, and we know he would have been very pleased with the final outcome.

Biographical Sketch

Robert Guralnick was born and raised in Los Angeles and received both his undergraduate and graduate degrees from the University of California Los Angeles. He then spent two years as a Bateman Research Instructor at the California Institute of Technology before moving to USC in 1979, where he is currently a professor. He served as department chair from 1990 to 1996 and has had visiting positions at Yale, Rutgers, MSRI, Caltech, Hebrew University, Cambridge, the Newton Institute, IAS, and Princeton. He is a Fellow of the AMS as well as the Association for the Advancement of Science. He was the G. C. Steward Fellow at Caius College in Cambridge (2009). He was a Simons Foundation Fellow in 2012. He has given plenary talks at the annual meeting of the AMS (2013) and the British Math Colloquium (2014). He gave a distinguished PIMS lecture (2014) and an invited talk at the 2014 International Congress in Seoul. He presented a distinguished lecture series at the Technion (2016). He served as the managing editor of the *Transactions of the AMS* (2004–2012) and is currently the managing editor of the *Forum of Math, Pi and Sigma*, an associate editor for the *Annals of Math*, and is also on the editorial board of the *Bulletin of the AMS*.

FROM THE AMS SECRETARY

Response from Robert Guralnick

I am tremendously honored and overwhelmed to receive the Frank Nelson Cole Prize of the American Mathematical Society. I would like to first thank Robert Steinberg, Michael Aschbacher, and John Thompson for their encouragement and help during my years as a graduate student and in my early career (and beyond). I would also like to thank my many fantastic coauthors—in particular, Jason Fulman, Gunter Malle, and Pham Huu Tiep, the coauthors on the papers mentioned in the citation for the prize—as well as Jan Saxl, Alex Lubotzky, Skip Garibaldi, Tim Burness, and many others.

The classification of finite simple groups is one of the most momentous results in mathematics. It is unique in its length and complication. John Thompson's revolutionary work, first on the odd order paper with Walter Feit and then even more significantly with the N -group papers, developed the tools that allowed just the possibility that a classification could be completed. Daniel Gorenstein then had the audacity to organize an attempt to classify finite simple groups, and as he had stated, the amazing achievements by Michael Aschbacher sped up the process immensely. The consequences of the classification have been ubiquitous and have had enormous consequences in group theory of course, but also in number theory, algebraic geometry, logic, and many other areas. The classification is much more than a list of the finite simple groups. It says that the typical finite simple group is a finite group of Lie type, i.e., a finite analog of a simple algebraic or Lie group. Then one can use the theory of algebraic groups to study these finite groups. This allows us to know about representations in the natural characteristic and about the subgroup structure of these groups. Moreover, the Deligne-Luzstig theory and its subsequent developments allow us to study the representation theory of these finite simple groups in characteristic zero and in cross characteristic.

Many problems in diverse areas can be translated to problems in group theory typically via representation theory or permutation group theory. There are many reduction theorems (perhaps most notably the Aschbacher-O'Nan-Scott theorem) which further reduce problems to properties of finite simple groups. I am especially enamored of problems where the translation of the problem is interesting as well as the group theoretic solution (and determining which group theoretic solutions give rise to solutions of the original problem).

About the Prize

The Cole Prize in Algebra is awarded by the AMS Council acting on the recommendation of a selection committee. For 2018 the members of the selection committee were:

- Robert K. Lazarsfeld (Chair)
- Zinovy Reichstein
- Vladimir Voevodsky

The Cole Prize in Algebra is awarded every three years for a notable research memoir in algebra that has appeared during the previous six years. The Cole Prize was established in 1928 to honor Frank Nelson Cole (1861–1926) on the occasion of his retirement as secretary of the AMS after twenty-five years of service. He also served as editor-in-chief of the *Bulletin* for twenty-one years. The Cole Prize carries a cash award of US\$5,000.

A list of previous recipients of the Cole Prize may be found on the AMS website at: www.ams.org/profession/prizes-awards/pabrowse?url=cole-prize-algebra.

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