2018 Levi L. Conant Prize

HENRY COHN was awarded the 2018 Levi L. Conant Prize at the 124th Annual Meeting of the AMS in San Diego, California, in January 2018.


Henry Cohn

Citation

The 2018 Levi L. Conant Prize is awarded to Henry Cohn for his article "A Conceptual Breakthrough in Sphere Packing," published in 2017 in the *Notices of the AMS*.

In 2016, Maryna Viazovska gave an astounding solution to the sphere packing problem in dimension 8. Just a week later, Cohn, Kumar, Miller, Radchenko, and Viazovska solved the sphere packing problem in dimension 24 by similar ideas.

Cohn's article unfolds the dramatic story behind these proofs. What is special about 8 and 24 that makes the proof work only in these dimensions? The answer is that there are truly extraordinary sphere packings in these dimensions, arising from the E_8 lattice in dimension 8 that appears in Lie theory, and the Leech lattice in dimension 24 that is so closely connected with finite simple sporadic groups.

In 2003, Cohn and Elkies showed that the solution to the sphere packing problem in dimensions $d \in \{8,24\}$ would follow from the existence of special functions on $\mathbb{R}_d$. They conjectured the existence of these functions, which have come to be known as *magic functions*. Calculations performed by Cohn, Elkies, Kumar, and Miller "left no doubt that the magic functions existed: one could compute them to fifty decimal places, plot them, approximate their roots and power series coefficients, etc. They were perfectly concrete and accessible functions, amenable to exploration and experimentation, which indeed uncovered various intriguing patterns. All that was missing was an existence proof." Viazovska not only provided the missing existence proof, she also gave a remarkable construction of the magic functions in terms of quasimodular forms, establishing a deep new connection between sphere packings and number theory.

Throughout the article, Cohn adds motivation and insight. What hints were there of the relevance of modular forms? How do magic functions relate to the density of sphere packings? Why is a strategy based on linear programming more sensible than it initially appears? Why is the Fourier transform a powerful tool for understanding periodic point configurations?

Viazovska's breakthrough was one of the mathematical highlights of the year 2016. However, nonexperts had no natural entry point to this exciting discovery. Cohn's beautiful exposition decisively addresses this lack, both illuminating the wide circle of ideas leading to the proof and drawing the contrast between the conclusive results in dimensions 8 and 24 and our almost complete lack of knowledge in other dimensions. This strange and striking tale will fascinate readers from every mathematical background.

Biographical Sketch

Henry Cohn received his PhD from Harvard University in 2000 and is now a principal researcher at Microsoft Research New England and an adjunct professor of mathematics at the Massachusetts Institute of Technology. His research interests include discrete mathematics, broadly interpreted, and he particularly enjoys applying abstract mathematics to concrete problems. His interest in concrete mathematical structures was kindled in 1990 at the PROMYS summer math program for high school students, and he now coteaches the number theory classes at PROMYS and PROMYS Europe with Glenn Stevens. He received an AIM five-year fellowship in 2000 and the Lester R. Ford award in 2005, spoke in the combinatorics section at the 2010 ICM, and has been a Fellow of the American Mathematical Society since 2015. Despite their isomorphic names, he and the French number theorist Henri Cohen are not in fact the same person.

Response from Henry Cohn

It's a pleasure and an honor to receive the 2018 Levi L. Conant Prize. The E_8 and Leech lattices are fascinating objects, and I hope readers will grow to love them as much as I do.

Of course this article would not exist if not for Maryna Viazovska's breakthrough. I am also grateful to Noam Elkies, Abhinav Kumar, Stephen D. Miller, and Danylo Radchenko for exploring this subject with me; to David Rohrlich and Glenn Stevens for the seminar that intro-

duced me to modular forms; to Donald Cohn, Noam Elkies, Steven Kleiman, James Propp, and Susan Ruff for their insights on mathematical writing; and to my wife, Rachel Miller, for affectionately indulging my preoccupation with higher dimensions.

About the Prize

The Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2018 prize, the selection committee consisted of the following individuals:

- Carolyn Gordon (Chair)
- Thomas C. Hales
- Serge L. Tabachnikov

The Levi L. Conant Prize is awarded annually to recognize an outstanding expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years.

Established in 2001, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic Institute. The prize carries a cash award of US$1,000.

A list of previous recipients of the Conant Prize may be found on the AMS website at: `www.ams.org/profession/prizes-awards/pabrowse?purl=-cole-prize-algebra#prize=Levi%20L.%20Conant%20Prize`.

Photo Credit

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