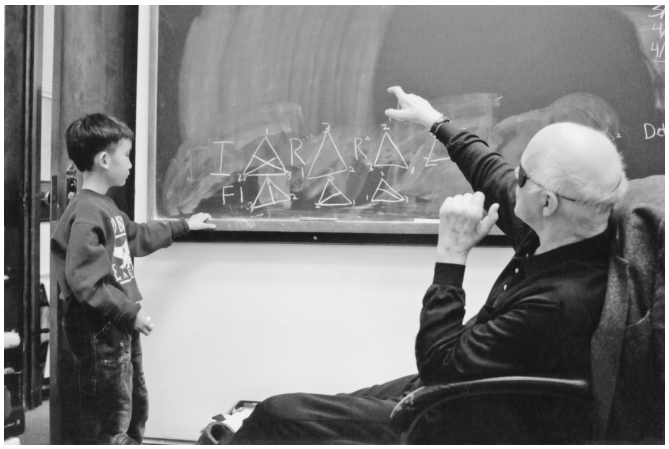
Paul J. Sally Jr. (1933–2013)

Jeffrey Adler, John Boller, Stephen DeBacker, and Loren Spice



Discussing symmetry in Sally's office.

Paul Sally, legendary leader and agitator on behalf of both mathematics and mathematics education, passed away on December 30, 2013. Paul was a one-of-a-kind character: boisterous yet humble, gruff yet charming, sophisticated yet coarse. Tough as nails and blessed with boundless energy, a phenomenal memory, and a heart of gold, Paul was relentless in the pursuit of his three great passions: mathematics, teaching, and basketball. Through a series of personal reminiscences and an afterword, we provide a glimpse of a remarkable man and his achievements.[1] We have ordered these reminiscences in an attempt roughly to parallel the course of his life, from beginning graduate student to the "powerhouse 'pirate'" [1] who was such an iconic part of the University of Chicago mathematics department.

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[1] *We encourage you to view the excellent 2010 interview* [4] *of Paul by Diane Herrmann and Hugo Rossi that is part of the Simons Foundation's Science Lives series. Additional biographical information may be found in the announcement of the 2014 AMS Award for Impact on the Teaching and Learning of Mathematics* [3].

DOI: http://dx.doi.org/10.1090/noti1676

One of Paul's great delights was the surprise of finding a problem unexpectedly easy or hard (or even unsolvable), and he insisted that *real* math problems do not come carefully posed to single out a unique answer (or even a unique interpretation). For him, figuring out a correct formal version of an informal problem was part of the fun, and he believed that the best problems lead to further problems rather than wrapping up a story. Throughout this article, we list Paul's favorite ten problems. Some hints and answers are included at the end of the article.

Kenneth I. Gross

Nowadays, the words "unique," "awesome," "legendary," and "larger than life" are thrown about casually, overused, and abused. However, when I write that these are the words that first come to mind when I think of Paul, I am using them in the strictest possible interpretation.

How does one write about a legend? I could focus on Paul's extensive research accomplishments; his extraordinary achievements as an educator; his commanding presence which characterized everything he did and separated him from mere mortals; or the immense courage he exhibited in the face of type I diabetes; which eventually robbed him of his vision and legs, but did not diminish his upbeat disposition or the passion with which he pursued his twin loves of mathematics and teaching. However, I am going to focus this encomium on another great passion of Paul's: basketball.

Paul and I first met at Brandeis University when I was an undergraduate and he was a graduate student there. Paul was in the inaugural graduate class, his wonderful wife, Judy, was in the second, and I was in the third.

The program was at a high level of rigor and abstraction, and it took Paul and me a while to find our stride. Fortunately, Ray Kunze [2] was on the faculty. "Find

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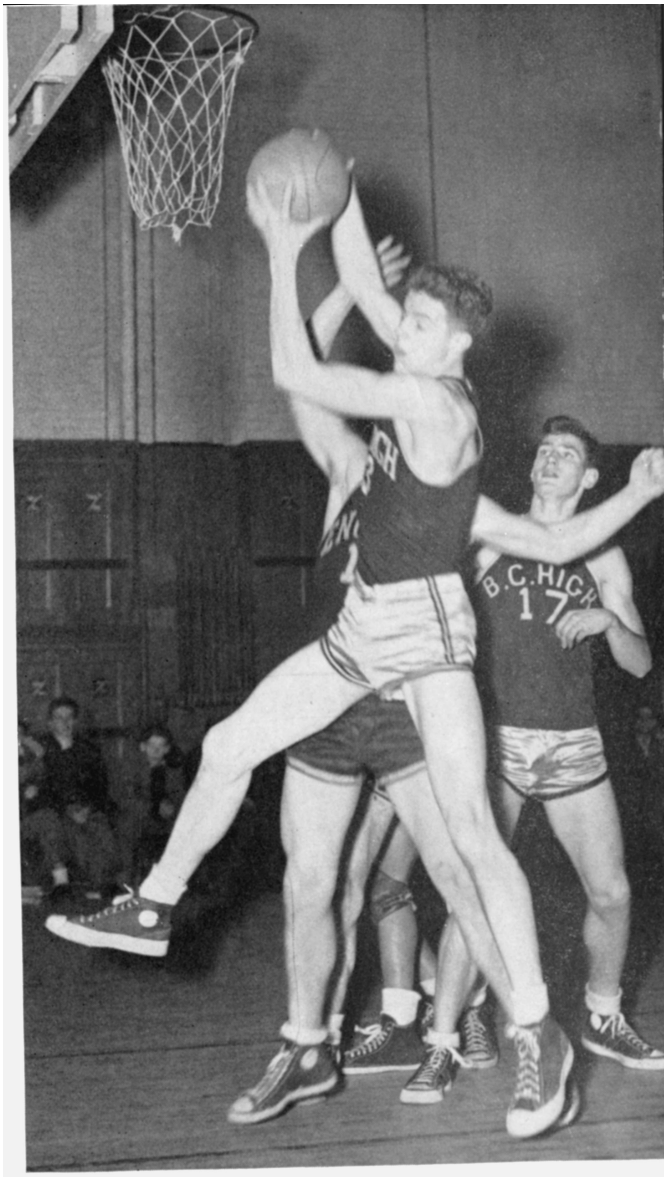

Sally grabs a rebound.

Judy and Paul Sally.

Sally leading his sons.

our stride" meant finding Ray, who was the ideal thesis advisor. Both of us owe our academic careers to Ray.

Ray left Brandeis in 1962 for Washington University in St. Louis, and Paul and I went with him. It was there that our life long friendship blossomed. Paul and Judy had three young children—David, Steven, and Paul III—for whom I served as baby sitter of last resort. Paul and I interacted mathematically on a regular basis, informally and in Ray's seminar, and we played basketball nearly every day.

Paul had been an All-Scholastic basketball player at Boston College High School and was on the Boston College team as an undergraduate. He was left-handed and had two signature shots: a hook shot that positioned his right elbow at precisely the level of one's jaw and a one-hand outside shot for which his left knee came up at the precise height to necessitate protecting one's most vulnerable organs. Like many competitive players, Paul had only one entry in his basketball rule book: do whatever you can get away with. One of many memorable vignettes will serve as an illustration.

As a backdrop, I will say something about Paul's career-long affinity for the representation theory of p-adic groups, which had its origin in Ray's seminar. The topic Ray chose was Gelfand and Graev's landmark paper on representations of SL(2) over a locally compact field. The seminar spawned a collaboration with Mitch Taibleson, and their joint paper launched Paul's p-adic career. Almost overnight, representation theory over p-adic fields was a thriving enterprise in which Paul was

River Problem. There are two boats docked on opposite sides of a river. The sides of the river are parallel and the current is negligible. The two boats set out at the same time to cross the river, and each maintains a constant speed. They first meet at a distance D_1 from side 1, then proceed to the opposite side and turn around. Next, they meet at a distance D_2 from side 2. How wide is the river? Does your answer make sense?

a pioneer. Paul proved the uniform boundedness of the analytic continuation of the principal series for SL(2) over local fields, and he and Joe Shalika proved the Plancherel formula for SL(2) over local fields.

Charles Gulizia, a doctoral student of mine at Dartmouth College, built on those papers to prove the Kunze–Stein L^p convolution theorem over a local field, and I invited Paul to be a member of the PhD examining committee. Paul made his acceptance contingent on my arranging a two-on-two basketball game. One of my undergraduate advisees joined us. He had been an all-state high school basketball player, was the Ivy League high-hurdles champion, and was Paul's height, but Paul manhandled him right from the start. Finally, having had enough of Paul's contemptible tactics, he overcame his reluctance to play physically with a distinguished professor from the University of Chicago who was twice his age, and he leveled Paul. Looking up from the floor, Paul said, "Hey, kid, where the H__ you been for the past ten minutes?"

That was the final time Paul and I played. Soon after, the ravages of diabetes started to set in, but you would never have known that from talking with him. He was always "great," and life was always "beautiful."

The last time I saw Paul, four months before his death, we were in the living room of his apartment, and he was unusually expansive about his life. We discussed mathematics and the book he and I were writing, and I told him I admired the way he could "see" the intricate details of mathematics though nearly blind. He replied that he had prepared for blindness for many years by doing mathematics without pencil and paper and with his eyes closed. We also talked about his education programs, and he revealed which of them he held most dear. If money ever got tight, he said he could reluctantly give up SESAME[2] and say that it had had a fantastic decades-long run. "But my Young Scholars Program, that's a different matter."

Now it is up to the ages to take care of Paul. If good deeds and a life well lived are the entrance requirements, then Paul is in a hallowed place. I hope for your sake, Paul, that the refs in heaven don't call a tight game.

[2] *Reflecting his enjoyment of word play, Paul was more attached to the name SESAME than to any particular interpretation of the acronym. He eventually settled on "Seminars for Endorsement of Science and Mathematics Educators."*

Three-Gap Problem. Let N be a positive integer, θ an angle in $(0, 2\pi)$, and S^1 the unit circle. Consider the map $f\colon \{0, 1, 2, \ldots, N-1, N\} \to S^1$, defined by $f(k) = k\theta \mod 2\pi$. Show that the image of f divides S^1 into arcs of exactly 1, 2, or 3 different lengths.

Norman Winarsky

I met Paul when I came to the College of the University of Chicago in 1966. I came as an eighteen-year-old boy, somewhat intimidated, alone, and concerned for my future.

I was remarkably fortunate to take classes from Paul and so to learn mathematics, from foundations to advanced analysis, from one of the best teachers in the world. The classes were small, perhaps fifteen people, and they were always exciting, challenging, and inspiring. He was the one who taught me how to think, from developing ideas from first principles, to proving theorems, to creating theories. Most of all, he taught me the beauty of mathematics.

I graduated in 1969. Paul advised me to go elsewhere for graduate school to have a broader mathematical and social experience. I was accepted at Princeton and decided to pursue my PhD in mathematics there.

The Vietnam War and the draft changed my plans. All graduate student deferments were discontinued, and the lottery determined who would be drafted. My number was low and I became 1A, which effectively guaranteed that I would go to Vietnam. Paul and Izaak Wirszup came up with a plan for which I am forever grateful. They worked to make me an instructor at Chicago, and so I petitioned my draft board for a deferment, not as a grad student, but as a teacher. The draft board agreed, and I received the deferment. That was great problem solving!

In graduate school Paul became my thesis advisor. Once again, he had a dramatic impact on my life. He decided to spend the 1971–72 academic year at the Institute for Advanced Study (IAS) at Princeton, where there was a seminar series on p-adic groups being led by Harish-Chandra. Paul arranged for me to be invited to join him as an IAS visitor. It was a remarkable time, learning from the greats in this field and developing my thesis topic on representations of semisimple p-adic groups.

After getting my PhD, I taught at the State University at Albany, SUNY, for two years, and then felt a deep need to do mathematics in the real world of industry. I think Paul was disappointed that I didn't continue in academia, but he never showed it. I joined RCA Labs' David Sarnoff Research Center and, over the next decades, led my division in helping to develop new technologies such as electron-beam systems, human vision analysis, HDTV, Siri, and more.

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The most important lesson I learned from Paul is not about mathematics, but about life in the face of what might seem to be overwhelming adversity. Paul lost two legs and one eye to diabetes, and he was nearly blind in the other eye. I never once heard him complain or even seem depressed. Every time I visited him, I heard the warm and friendly "Yo, Norman!" and then we caught up together like old times. His chalkboard was still full of mathematics, and his enthusiasm, energy, and happiness were infectious. He continued to do his research and had many plans for future books and papers. Whenever I visited, numerous visitors would be waiting in line, yet he always wanted to hear more about what I was doing.

Each of us is fortunate in our lives if we find a teacher who is interested in us, who gives us guidance and support, and who has passion for our chosen field. Paul was even more than that: he was a friend to me and a model for my life and for the lives of countless students and friends. His encouragement, values, enthusiasm, and love for mathematics were gifts that will go on forever.

David Vogan

I met Paul Sally sometime in the fall of 1972, when he returned to the University of Chicago after spending a year at the Institute for Advanced Study. I was an undergraduate, attached to math but with no idea what kind was most interesting. Fortunately, *Paul* knew very well what kind of math was interesting: harmonic analysis. His vita described his specialty as "harmonic analysis on semisimple groups."[3] None of this "representation theory" stuff: The goal was to understand spaces of functions with group actions. Representations were a tool for doing harmonic analysis.

I started taking courses from Paul, trying to understand a few of the seminar speakers he invited, and reading papers that he gave me. What I learned was that this mathematics was hard, but that Paul could always make it clearer and easier, or at least convince me that it was worth the effort to figure out.

I left Chicago in 1974, sent with Paul's guidance to the best place for studying group representations: MIT. From then on, I could absorb his wisdom (and his deflation of anything pretended) only in occasional short meetings, which still had long-term effects. I visited him in his fourth-floor office in Eckhart Hall about 1976, very anxious to tell him about all the great algebraic stuff I now knew about representations. He listened and asked, "So what's that tell you about *unitary* representations?" (In other words, how did all this algebra advance harmonic analysis?) The answer was that it didn't immediately tell you very much. That question—and all of his teaching before—had a serious effect on my work. Algebra is still almost all that I do, but at every opportunity it's algebra aimed at harmonic analysis.

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[3] *His business card described it slightly differently: "Representing Groups in All Fields." He enjoyed seeing how people interpreted this description.*

David Vogan and Sally discussing harmonic analysis.

> **House Problem.** Suppose your street has houses on only one side and the houses are numbered consecutively, from south to north, starting with 1 and ending with some number m. Suppose you live at a house where the sum of the numbers to the south of your house is equal to the sum of the numbers to the north of your house. What is the number of your house? What happens if the street has odd and even numbers on opposite sides?

Like many mathematicians, Paul understood well the enormous number of levels at which mathematics can be played. He sometimes compared it to basketball. Everyone understands the great difference between the kids on the school playground, high school teams, college teams, and the pros, and the further differences within each of those levels. (When Paul was playing basketball at Boston College, he sometimes practiced on the same court with the Boston Celtics. He said that sometimes, for a little while, he could feel that he was playing at their level. But then one of them would make a move, and he was just left in the dust.)

What made Paul unique was how deeply he cared about the people doing mathematics at all those levels, about Harish-Chandra, and about elementary school teachers. They were all doing math, and he wanted all of them to give it everything they had. As he did.

Rebecca Herb

I first got to know Paul and his work in the early 1970s while I was a graduate student at the University of Washington working with Garth Warner. Garth suggested that I look at his paper with Paul and attempt to generalize it to real rank two. It was slow work at the beginning but became easier for me after the Williamstown conference

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in the summer of 1972. I met Paul there, and his interest was instrumental in boosting my determination to write a thesis.

Motivated by efforts to generalize my thesis work to arbitrary real rank, Paul and I wrote a joint paper. We started working on it when I was a Dickson Instructor at the University of Chicago. Being there and working with Paul at the beginning of my research career was a wonderful opportunity because of his encouragement, enthusiasm, and knowledge of current work in the field. But it also had its frustrating side. On many occasions, I waited in the hall outside Paul's office while he listened patiently to an undergraduate talking about personal problems.

Over the years I kept coming back to the problem of the Fourier transform of orbital integrals and the Plancherel formula on semisimple Lie groups. Whenever I learned more about discrete series characters, I was able to refine the Fourier inversion formula. While I was working primarily on real Lie groups, Paul concentrated on p-adic groups, and he never gave up on his dream of gaining a good enough understanding of discrete series characters in the p-adic case to carry out the derivation of the Plancherel formula using the Fourier transform method.

As Paul's mobility and vision declined, he became even more focused on mathematics. As he remarked to David Goldberg, "After 80, you can really get things done." The title of an expository talk he gave a few years ago was "The Plancherel Theorem Done Right: Characters Tell All." For the Zuckerman conference in 2009, he planned an expository paper summarizing the real case and the program, goals, and current progress in the p-adic case. Paul asked me to join him in writing up the resulting paper summarizing my results in the real case. The paper emphasizes Paul's philosophy of representation theory as an extension of classical harmonic analysis and gives a brief history of this approach to the Plancherel formula, starting with the classical Plancherel theorem of 1910. Paul ends the paper with, "We expect to return to this subject in the near future."

Although he remained active as a research mathematician, Paul's biggest impact during the time that I knew him was on the research of others. I think that my experiences with him were typical. When I attended my first Joint Meetings as a graduate student, Paul was one of only two senior mathematicians who took the time to talk with me. Whenever we met, Paul asked what I was working on and reassured me that it was interesting and important. One afternoon at tea in Fuld Hall at IAS, when he was chatting with Harish-Chandra, Paul spotted me and called me over, saying, "So, what are you working on these days?" Of course, we had already had that conversation, but Paul wanted to give me the chance to tell Harish-Chandra about it and knew that I was too timid to approach him on my own. Paul also served as a clearinghouse for information about who was working on what in the days before email and online posting of preprints. Finally, he did his best to make sure that the work of junior mathematicians was appreciated, serving as a cheerleader for them at a time when others were less generous.

Harmonic Sum Problem. Let n be a positive integer. Define $S_n = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n}$. Show that for $n \geq 2$ the sum S_n is never an integer.

Paul was also a booster of the city of Chicago. When I was an instructor at Chicago in the mid-1970s, Paul had already initiated the Midwest Representation Theory Conferences[4] by inviting a few colleagues to Chicago during spring breaks. Besides talking math, we also sampled the attractions of Hyde Park and greater Chicago: beer and arm wrestling at Jimmy's; blood soup at an eastern European restaurant on the West Side; and driving tours of the Loop, including Lower Wacker Drive, a subterranean street used by delivery trucks and Paul. On my more recent visits to Chicago, Paul was proud to show off his Hyde Park penthouse apartment. The elevator only went as high as the floor below his apartment, and I was pretty nervous walking behind Paul as he wobbled up the flight and a half of stairs on his two prosthetic legs.[5] But, for Paul, it was worth the effort, since the apartment had a large roof terrace overlooking the city.

The past forty years wouldn't have been nearly as much fun without him.

Allen Moy

I became Paul Sally's student in the summer of 1979. At the time, I did not have a strong research direction, but I had learned some class-field theory and become acquainted with the Langlands conjectures. The fact that Paul was in representation theory, along with the general understanding among graduate students that Paul did not have any personality peculiarities, played important roles in my choice.

In the film *Kramer vs. Kramer*, Dustin Hoffman's character delivers truths about parenting that I have always felt applied to the way Paul mentored his students: "It has to do with constancy, it has to do with patience, it has to do with listening, it has to do with pretending to listen when you cannot listen anymore." The qualities of constancy, patience, and listening were, in my mind, Paul's best attributes as an advisor.

In spring 1982 I finished at Chicago and then went to Yale as a Gibbs instructor. At Yale I became acquainted with Paul's younger sons, who were undergraduates there. In May 1984 the family came to Yale for Steven's graduation, and they took me up on an offer to use my

[4] *Now named "The Paul J. Sally Jr. Midwest Representation Theory Conference."*

[5] *Later, Paul found it easier simply to remove his legs and scooch up backwards, lifting himself one by one up each of the tens of stairs between him and his penthouse—a move he called the "Sally-scooch." He never once considered moving.*

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The n-gon Problem. Let P_n be a regular n-gon. Place dots at the n vertices and the midpoints of the n edges. Can one place the integers from 1 to $2n$ in a one-to-one fashion at each of the dots in such a way that the sum of the three integers on any edge is the same?

apartment. I still vividly remember the Sally family walking down York Street, Judy beaming with happiness while arm-in-arm with her sons Steven and Paul.

My position at Michigan during 1990–2001 allowed me frequent contact with Paul. I spent May 1991 as Paul's academic visitor. One day, towards the end of my stay, I told Paul: "Look, I have $400. Let's go scalp two tickets to see the NBA playoff game between Michael Jordan's Chicago Bulls and the Detroit Pistons Bad Boys." We did. Our seats were in the rafters, but we had a joyous time.

In January 2000 my father passed away in my hometown of Chicago. We held the wake in Chicago's Chinatown. My mother and I were quite touched that Paul, by then assisted by a student helper because of his first leg amputation, came to my father's wake.

Roger Howe

Not many mathematicians evoke the phrase "larger than life," but it fits well with Paul Sally. For one thing, he *was* big—6 feet, 3 inches to the end of his days, even though his doctors recommended that after losing his lower legs to diabetes-related infections, he should use shorter prostheses. He refused, saying that 6 foot 3 is the perfect height from which to view the world.[6] He used the prostheses for more than walking. When he attended meetings, he sometimes rolled up in his wheelchair, took off his legs, and put them on the table in front of him, so that they functioned as a sort of nameplate.

Paul loved mathematics, he loved people, and he loved life. In my memory, the adjective he used the most was "beautiful." He loved telling stories about himself or friends. However, he was not reverent about mathematics. He was more likely to be reverent about people. His feelings about Harish-Chandra in particular showed tendencies toward awe. Paul's main area of research interest was the representation theory of p-adic groups, and he entered the field just as Harish was putting the finishing touches on his Plancherel theorem for real reductive groups and shifting his attention to the p-adic theory. Harish's theory was always Paul's model for describing p-adic representation theory.

I was about to start my first job when we met at a NATO Summer School in Namur, Belgium, in early September, 1969. Paul liked beer and at that time indulged freely, so being in Belgium was close to being in heaven. He and the conference organizer closed the bars every night. The organizer looked droopier with each succeeding day, but Paul continued to look fresh.

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[6] *Paul was also known to say, "Now that I have two prosthetic legs, I could be 7 feet tall if I wanted to be."*

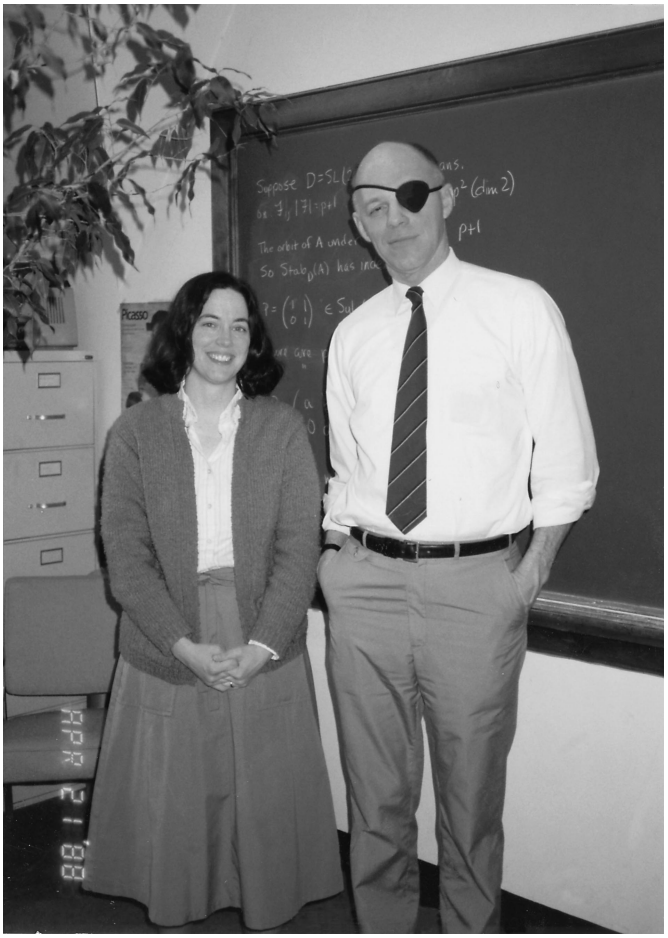
Sally at the annual Latke–Hamantash Debate.

Paul was generous with time and attention as well as with money to many people. My student Ju-Lee Kim was at the University of Illinois at Chicago from 2002 to 2007. During that time, she proved a major result about representations of p-adic groups. After complications from his diabetes necessitated a second leg amputation, Ju-Lee called him in the hospital to wish him a good recovery. The first thing he said was, "I heard the proof is right!"

Diane Herrmann

For more than thirty years I worked closely with Paul Sally. Maintaining respect for each other as colleagues and continuing to enjoy each other as friends was a remarkable gift. We worked together to sustain and improve the undergraduate program at Chicago, bringing it to a place where mathematics is the fourth most popular major on campus while upholding challenging and rigorous standards. For Paul, teaching at any level was a great joy. Whether his audience was the "pyrotechnically endowed" problem solvers in our Honors Analysis course (which he unashamedly called "Kick-Ass Mathematics"), hopeful but often mathematically less experienced middle-grade teachers, or our undergraduates, teaching mathematics was his passion. He often got sidetracked in class, going off on tangents that intrigued him, but he always pursued them with enthusiasm. I also watched and learned from him outside the classroom, as he used his knowledge of human nature to extract students' admissions of cheating (he seemed to have learned his fair share of techniques during his Jesuit high school years), to coax better spoken English out of an international graduate student trying to become a better teacher, or to say "no" to a student request, and mean it, and then have the student thank him on the way out of the office.

Diane Herrmann was associate director of undergraduate studies in mathematics at the University of Chicago from 1983 until her retirement in 2015.



Diane Herrmann and Sally

> **Packing Tetrahedra Problem.** How many congruent regular tetrahedra, with edge length 1, can be packed inside a sphere of radius 1 if each tetrahedron has a vertex at the center of the sphere?

Despite the fact that Paul and I had different styles, we shared things that mattered to both of us. He made the department, and my job, more human. He supported me as I balanced work and home life. He came from humble origins and respected my path as a first-generation college attendee from a small liberal arts college to a place like the University of Chicago. During some of our first conversations, he was almost as interested in my backstroke times as in my mathematics background. He let me know that life outside mathematics mattered and was important. I still try to follow his advice about professional meetings: take a day and see the place you're visiting. On a particularly icy evening during an AMS meeting in the 1980s, I enjoyed an Atlanta Hawks basketball game with Paul and his thesis advisor, Ray Kunze, where Paul sang along to the Bruce Springsteen tunes played at the breaks in the game. He loved music, and it was a particular joy to have him come to concerts I sang in, whether it was the Brahms Requiem in Rockefeller Chapel or the close harmony of my barbershop quartet. When I became more interested in art, especially mathematics and needlework, he was mystified and yet supportive at once. I'm not sure he understood how I could turn away from academic mathematics, but he encouraged my new direction. As long as you were working at something you were passionate about, that's what mattered to Paul.

Paul had a strong sense of fair play and often used his position and his knowledge to benefit those who did not have the advantages that come with privilege. He knew the effectiveness of Arnold Ross's summer program for high school students and wanted to offer that same kind of deep mathematical experience to Chicago youth. He and I founded the University of Chicago Young Scholars Program (YSP) in 1988 in the hope that by starting with the best young students from Chicago public school classrooms, we could "grow our own" mathematicians. Of the hundreds of YSP students over the years, there are many mathematical success stories, including those who have earned PhDs in mathematics and those who have become lead teachers in their own Chicago public school classrooms. Paul looked forward to each eager new crop of YSP students in July, and he enjoyed their enthusiasm and antics almost as much as he delighted in their mathematical discoveries.

We often talked about our students, our teaching, our successes, and our failures. No one else I have ever known had the capacity to inspire fear and admiration, disagreement and respect all at once. His passion for mathematics and his presence in the classroom have influenced countless mathematicians, as well as lawyers, teachers, and even basketball players. In the last few weeks of his life, as we were talking about our teaching, I read him a poem that he'd first shared with me. The first stanza of Billy Collins's "Schoolsville" reads:

> Glancing over my shoulder at the past,
> I realize the number of students I have taught
> is enough to populate a small town.

The students of Paul's who populate his small town are lucky to have been a part of his legacy.

Raja Malyala

My first encounter with Professor Sally was during summer 1982 at a lecture for high school students enrolled in an eight-week residential summer mathematics program. He had a tall, imposing frame and a pirate-like appearance, which made him at first appear somewhat less like a mathematician than a performer on a stage. However, as I had opportunity to observe on many occasions, Professor Sally was only a performer on the surface. As you stayed with him, you realized that there was great depth to this character.

Two years later, as a senior in high school, a classmate from the summer program and I were privileged to meet

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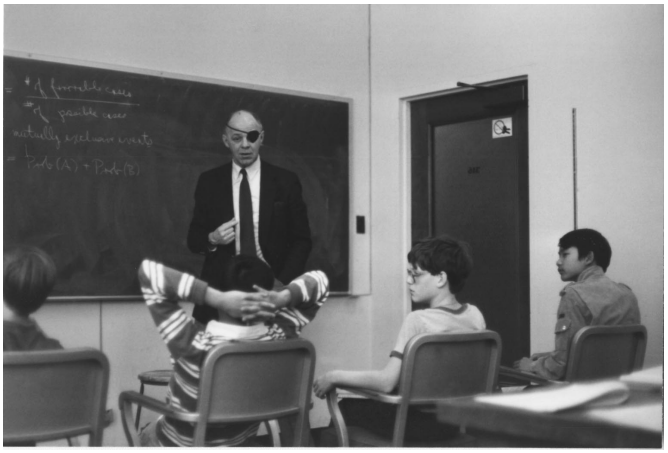

Sally instructing high school students.

> **Jim and John Problem.** John is 32. He is twice as old as Jim was when John was as old as Jim is now. How old is Jim?

for monthly two-hour problem-solving sessions at his home. Together we would solve problems from Herstein's *Topics in Algebra*, often spending an hour or more on one of the more difficult ones. He had a childlike curiosity; we would be scribbling something on paper, and he would be writing on the chalkboard, puffing his cigar and concentrating. There was absolutely no theater at all. This was the real Professor Sally, with deep curiosity and concentration. He was an extremely generous man, who tutored us for about a year without asking for any compensation, simply to attract young talent to mathematics. Professor Sally was a truly humble man who had no pretentiousness about him, and the theatrics were simply a way he was able to communicate with a wide variety of audiences, including high school students.

During my college years I did not have occasion to attend his courses, but I sat in on many of his lectures. He would start with the simplest concepts and, with a few pauses where he took some puffs from his famous pipe, would escalate slowly, leading to major theorems and explaining important concepts by unfolding them step by step, along the way challenging even the best students in the class. At the same time, he would laugh and make light of everything, which was not at all common among the faculty teaching the higher-level courses.

He was always a picture of strength and good humor and had the enthusiasm to struggle with the most difficult concepts through even the most difficult times. He realized that it is a gift not only to create great mathematics but even just to be able to understand and comprehend what the greatest mathematicians have left behind for us. His perspective undoubtedly also made him able to go on, despite his numerous health problems, as he could still find a way to feel positive. I am privileged to have known him.

Matthew Leingang

I entered the University of Chicago with the strong desire to become a mathematician but without much preparation beyond high school calculus and recreational reading. I placed into the most challenging calculus stream, Math 160s, which was the perfect course for me at the time. As director of undergraduate studies, Paul was adamant that each of the streams be serious. "We teach ε and δ in the 130s," he boasted, and to those who claimed the students in that sequence couldn't understand it he replied, "That's because you don't *expect* them to." Paul also declared that no calculus course at Chicago would use a textbook that did not prove the Mean Value Theorem. To him, the Mean Value Theorem was the most important theorem in calculus. I impart this very same wisdom to hundreds of my own students each year.

In my second year of college I took the Math 207–208–209 Honors Analysis sequence. This was my first experience with Paul in the classroom. Paul was the driving force behind 207–208–209 for decades, before and after I took it. He said that the true name of the course was "Kick-Ass Mathematics" but that the university would not let him put that in the catalog. Paul eventually wrote a textbook for the course, and the publisher likewise rejected this title, suggesting *Fundamentals of Mathematical Analysis* instead. At the beginning, it was not entirely clear whose posterior was to be kicked, ours or the mathematics'. I think he meant it both ways: in the beginning, ours were getting kicked, but, by the end, we were doing the kicking ourselves. On the first day of class, he gave the rules:

- Rule 1: Erase the boards before class begins and put them in the proper order.
- Rule 2: Minimum 25 hours per week of studying.
- Rule 3: Form study groups.
- Rule 4: Speak up in class. This is a participatory dictatorship.

He then gave us a 48-hour take-home diagnostic exam consisting of about half a dozen open-ended questions, many of which he and his wife, Judy, whom he described as "my first-class collaborator," later wrote about in one of their books.

Just as he expected the 130s students to handle εs and δs, Paul expected more from me than any other teacher ever has. He assigned homework on the fly during lecture by placing stars by various statements. In my notes, I dutifully transcribed his legend:

- One star: do it, don't turn it in.
- Two stars: do it, turn it in.
- Three stars: PhD thesis.

But he also loved us as no one else did. As most who spent any time with him soon discovered, his gruff mannerisms were an act and failed to hide his broad, warm smile. At one point in 207 I answered a question in a way he

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"Kick-Ass Mathematics" in Eckhart 206.

thought insightful. He invited me not to shake his hand nor to high-five, but just to touch his fingers.

In the spring of 1993 his diabetes claimed one of his legs, and during his recovery from the resulting surgery, class was moved temporarily to the hospital's lecture hall. (I doubt cancellation was ever considered.) The following summer I was working as a counselor in YSP when my grandfather died. My mother called the math department, and it fell upon Paul to break this news to me. I even recall seeking his advice about a relationship I was pursuing. He gave a much more colorful version of the adage that there are plenty of fish in the sea.

I was lucky enough to see Paul once or twice a year at conferences, and several years ago we were giving reports in the same session. Paul spoke after me and remarked, "By the way, Matt Leingang was my student at the University of Chicago. I taught him everything he knows." I felt he was telling me publicly, but without embarrassing me, that he was proud of me. It's a compliment that I will always remember.

Phil Kutzko

I know, I know. I am the guy who hit all the bars with him, who stayed up all night with him, telling jokes, swapping stories, smoking cigars. I am the guy who held the other foot.[7] But that is not who Paul Sally was to me. I could tell you stories, and they might make you laugh, but they would shed no light on the meaning of Paul's all-too-brief time with us. In fact, as he did for so very many other people, Paul changed the course of my life for the better, not once but many times.

I remember hearing my phone ring as I neared my office one day in my first semester at Iowa. I almost

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[7] *Contrary to the prize announcement* [3], *Paul's colleague was suspended, not from a hotel balcony, but near the railing, so the story that Paul told to the* Boston Globe [1] *is not completely apocryphal.*

Parallelepiped Problem. Find the volume of a parallelepiped in $\mathbf{R}^n$ by dissection.

didn't get to the phone in time. I heard Paul's voice, its Boston cadence not so different from the voices I grew up with in New York, and thought, "Maybe there is a place for me in this profession." He had no obligation to reach out to me, but he did, and almost immediately he set out—as we would say today—to mentor me. I was not an easy person to mentor. I had what is called these days an "attitude." Paul's approach to this was simple. When I would express angry sentiments, he would get quiet. When my expressions of these sentiments would get louder, he would get more quiet. No one had ever treated me that way before. Then, after thinking for some time, he would break his silence to tell me that he had found my opinions "very interesting." That was it; that was all he would say. It took me a while, but eventually I came to understand that Paul could see no point in these sorts of negative utterances, that he viewed them as a luxury that people from our background could not afford. He told me once that I didn't know what it meant to have power. It was a sentiment similar to those expressed by my minority colleagues, and I didn't understand. But then, one day, Paul came out to Iowa to give a math talk, and in the room were four African-American graduate students from other fields who, it turned out, were alumni of YSP. These students had come to thank Paul for changing their lives, and I began to understand what he had been trying to tell me.

Paul Sally was a giant among men—he towered over all of us. The programs that he started have touched tens of thousands of students, many from minority backgrounds, and provided them with a path to success. His programs for K–12 teachers indirectly affect the lives of even more students.

I visited Paul in the fall of 2013, and he asked me to tell him about the work I was doing in minority graduate education. I knew better than to try to tell him how much he had affected the course of my life, how much I was in his debt, but I could see that he knew. Instead, we talked about our families and our grandkids, and he told me about his plans for the years to come, full, as always, of enthusiasm and hope. And then we told a couple of jokes and made some amusing remarks about people we knew. Just like the old days. And that was the last time I saw him.

It is hard to think that we will never see you again, Paul, but if ever a person did what he was sent here to do, it was you.

Afterword

Nothing pleased Paul more than hammering away at a good math problem. If he could engage other people in the hammering, all the better. His enthusiasm attracted a wide spectrum of students, and he taught them all—from elementary school students, to graduate students, to teachers. Moreover, Paul would teach anywhere that

A Modest Differentiation Problem. Let f_α denote the function that takes the value $q^{-\alpha}$ at p/q in lowest terms and is zero otherwise. Show that when $\alpha > 1$ the function f_α is discontinuous at each rational and continuous at each irrational. Moreover, for $\alpha > 2$ it is both differentiable off a set of measure zero and differentiable at each irrational algebraic number.

Sally leading a discussion around 1960.

would have him—from Dedham Junior High School, to Chicago's Juvenile Detention Center, to the University of Chicago.

When he ran out of places to teach, he created his own teaching opportunities. During the 1970s more than a thousand Chicago public high school students per year participated in his annual math competitions, and many went on to attend his subsequent enrichment classes. In the mid-1970s Arnold Ross brought his Summer Science Training Program in mathematics to Chicago and operated it for several years. Paul eventually embraced the program, taught in it most years, and, following Ross's departure, sustained it through 1982. It was a precursor to YSP, which has operated from 1988 until the present. Reflecting his lifelong love of Paris, he created a study-abroad program for faculty and upper-level students at the University of Chicago's Center in Paris; while he fully intended to teach there, he never did.

Consistent with his adage that "to teach real mathematics, you had best know some," he worked with thousands of K–12 teachers throughout his career. From 1972 to 1982 he taught pedagogy courses for Chicago high school mathematics teachers, and in 1992 he founded SESAME, a staff development program for in-service teachers from the Chicago public schools which is still flourishing and was extended to the Boston area in the early 2000s. In addition, he helped to develop the Chicago-wide Algebra Initiative, and worked extensively with the Chicago Urban Teacher Education Program.

Isometries of ℓ^p space. Show that, if $p \neq 2$, then a linear isometry of $\mathbf{R}^n$ with the ℓ^p norm must be a generalized permutation matrix.

Nineteen graduate students completed doctorates under Paul's supervision:

Ernest Thieleker, 1968
Stephen Franklin, 1971
Norman Winarsky, 1974
Charles Asmuth, 1976
Wen-Min Chao, 1977
Charles David Keys, 1979
Courtney Moen, 1979
Walter Tuvell Jr., 1981
Allen Moy, 1982
Ronald Scott, 1984
Fiona Murnaghan, 1987
David Jabon, 1989
Daniel Goldstein, 1990
Chris Jantzen, 1990
Jeffrey Adler, 1996
Stephen DeBacker, 1997
Reid Huntsinger, 1997
John Boller, 1999
Loren Spice, 2004

Indeed, a significant number of the researchers studying the (complex) representation theory of reductive p-adic groups are Paul's students. It was not uncommon for his graduate students to show up for an appointment only to find it pushed back ("come back in half an hour") so that Paul could continue working with an undergraduate visitor or anyone else who needed his help. Occasionally, "in half an hour" would become the next day, or the next week. The meeting itself might be brief and punctuated by phone calls and knocks on the office door. In spite of this, Paul was always his students' best cheerleader. He was a perceptive and empathetic listener, attentive and genuinely interested in what they had to report, especially about mathematics they had learned or their progress on research projects, and he was remarkably effective at providing the kind of encouragement that inspired students to believe in themselves and to thrive and succeed.

Following the death of Joseph Shalika, Paul wrote a description of the work that led to their seminal papers. Nothing we write can convey Paul's love of mathematics better than his own words, so we close with them.

"I was at the Institute in Autumn 1967, lecturing on p-adic SL(2), following the works of Bruhat, Gelfand–Graev, and a few others, including Shalika. Joe was at Princeton. We finally got together in early 1968 and started working. It was an incredibly exciting adventure for two non-tenured, rambunctious rookies. We soon discovered the road map for our project (Harish-Chandra, *Plancherel formula for the* 2×2 *real unimodular group*). We thought we could do it all: Characters, Plancherel Theorem, and the Fourier Transform of Elliptic Orbital Integrals. We also had the Big Guy down the hall, for regular advice and direction.

"We worked mainly in the Seminar Room in Building C, computing, shouting, and wrangling for eight to ten hours at a time. It was spring, and the days were getting longer. So after we finished work, we would walk across the golf course to Andy's Bar on Alexander Street. There, we would drink four or five beers, eat two or three cheeseburgers, revel in the day's successes, and look forward to the same effort the next day. For those who have been in the chase, there is no need to talk further about the exhilaration that accompanies this."

Zappa-dappa.

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Selected Papers of Paul Sally

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[S9] P. J. Sally Jr. and M. H. Taibleson. (1966). Special functions on locally compact fields, *Acta Math.* **116**, 279–309. MR0206349

[S10] P. J. Sally Jr. and G. Warner. (1973). The Fourier transform on semisimple Lie groups of real rank one, *Acta Math.* **131**, 1–26. MR0450461

As promised, we provide some hints and answers to Paul's ten favorite problems. They are listed in no particular order.

- 24.
- Bertrand's postulate isn't necessary, but the Thue–Siegel–Roth theorem is.
- At least one problem remains unsolved at the time of writing.
- At least 20.

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