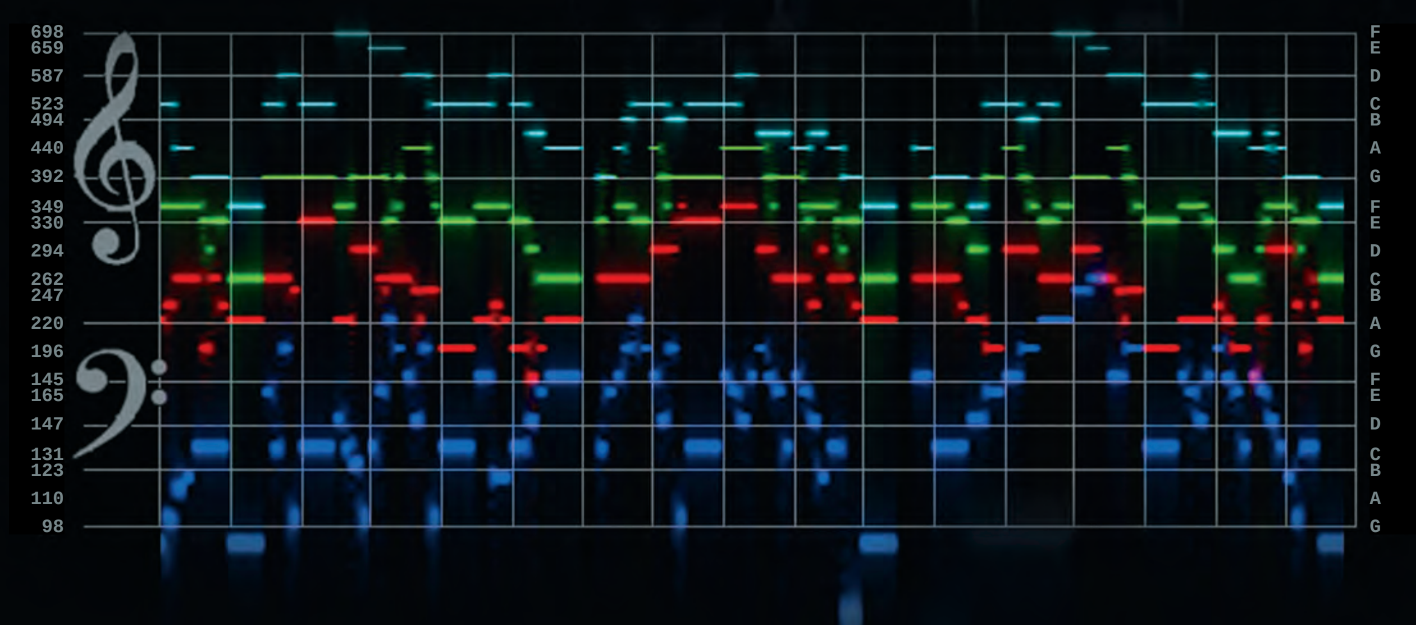
ISSN 0002-9920 (print)
ISSN 1088-9477 (online)

Notices

of the American Mathematical Society

June/July 2018 Volume 65, Number 6



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About the Cover, page 635.

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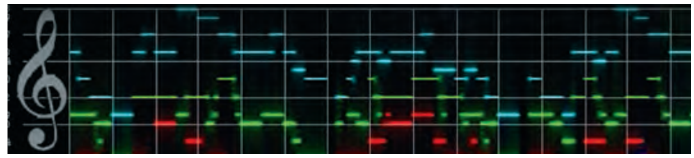


FEATURED

This summer we honor James G. Arthur, former president of the AMS, on the occasion of his 2017 AMS Steele Prize for Lifetime Achievement. Ron Solomon provides a progress report on the classification of all finite simple groups. Solomon Friedberg considers "WHAT IS...the Langlands Program?" And we've got a record number of Letters to the Editor. Relax with us and math.

—***Frank Morgan, Editor-in-Chief***

FROM THE AMS SECRETARY

ALSO IN THIS ISSUE

ALSO IN THIS ISSUE, CONT'D

Notices
of the American Mathematical Society

IN EVERY ISSUE

CURRENT INVITATIONS FROM THE AMS

About the Cover

This month we honor 2017 Steele Prize Lifetime Achievement winner James G. Arthur.

One of Arthur's main research interests is in harmonic analysis, a broad mathematical area with origins in the analysis of periodically recurring phenomena. Inspired by his research, this month's cover features the harmonic components of the sound waves produced by a piano playing an excerpt from Bach's *Gott sei uns gnädig und barmherzig* (BWV 323)–1725.[1]

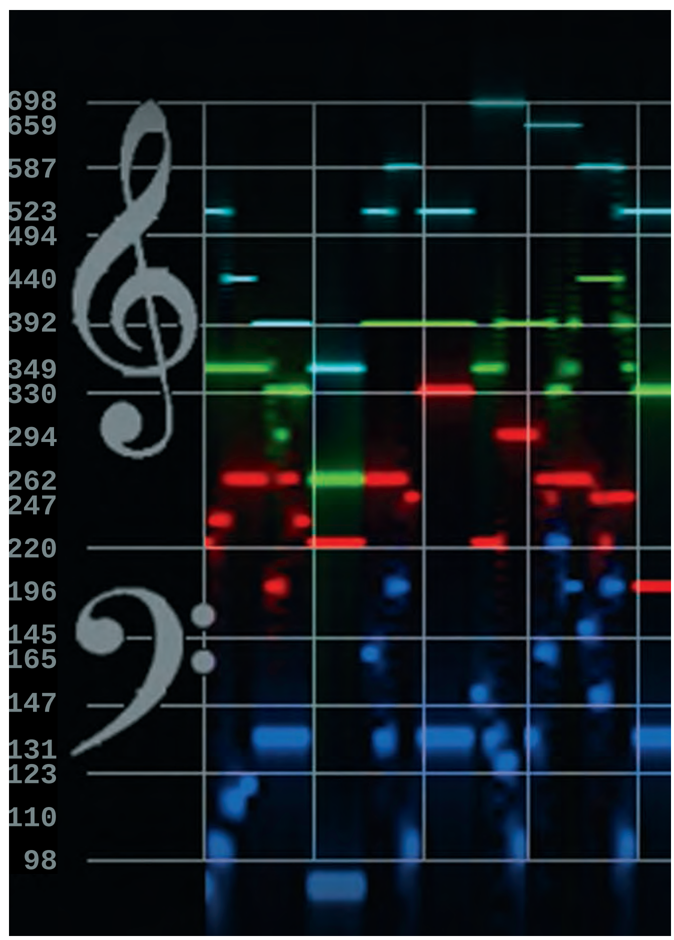
Of harmonic analysis in music and mathematics, Arthur once wrote:[2]

"Harmonic analysis in music is the study of chords, and of how they are used in combination to create musical effects. Harmonic analysis in mathematics takes on a somewhat different meaning. It too has roots in music, or at least in the mathematical analysis of sound. However, the term can also mean a kind of universal duality that runs throughout mathematics.

Different musical instruments make different kinds of sound, even when they play the same note. The analysis of this phenomenon can be very complicated, but to a first approximation, it is the shape of the instrument that determines the sound it creates. Shape is an obvious geometric property. Sound is an example of what is often called a spectral phenomenon… . We thus have an example of something geometric—the shape of a musical instrument—which corresponds to something spectral—the sound produced by the instrument. Harmonic analysis in the broadest sense refers to a general principle in mathematics that links geometric objects with spectral objects… ."

Photo Credit

Photo of James G. Arthur receiving the Steele Prize from Robert L. Bryant courtesy of Asif Ali Zaman.



[1] *This imagery is in the public domain.*

[2] www.claymath.org/library/cw/arthur/pdf/hainMath.pdf

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of the American Mathematical Society

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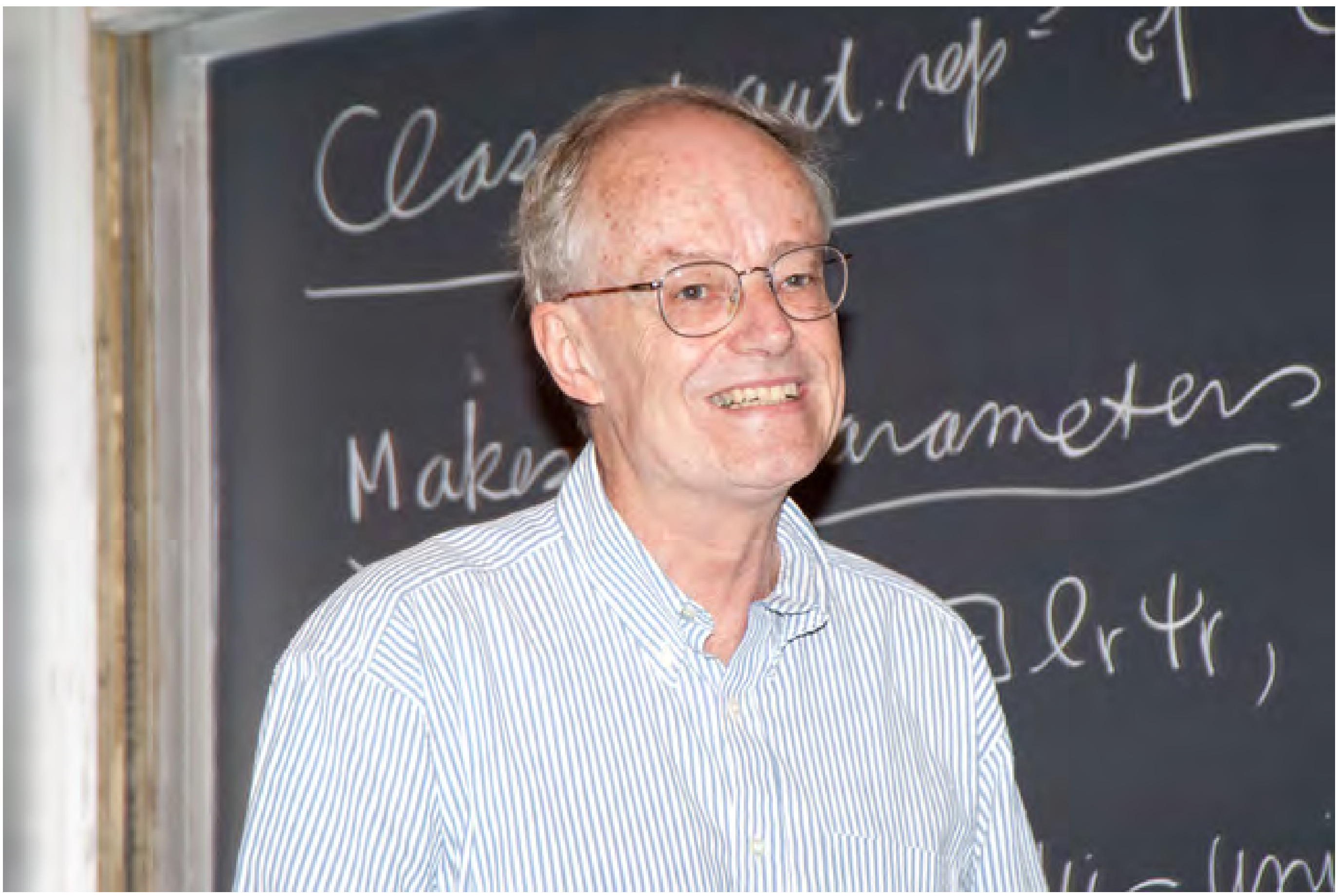
James G. Arthur: AMS 2017 Leroy P. Steele Prize for Lifetime Achievement

James Cogdell and Freydoon Shahidi, Guest Editors

Communicated by Steven J. Miller



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DOI: http://dx.doi.org/10.1090/noti1691

Jim Arthur's illustrious career, spread out over almost five decades, has contributed in a profound manner to our understanding of the theory of automorphic forms. He has single-handedly developed the trace formula, now known as the "Arthur–Selberg trace formula." This trace formula is a powerful tool in understanding the spectrum of reductive groups over number fields and Langlands's Program. In particular, it can be used to establish important cases of Langlands's Functoriality Principle, one of the central questions in the theory of automorphic forms and number theory. Some of its most general cases are proved by Arthur himself.

Throughout his career he has received many honors, including the Canada Gold Medal for Science and Engineering (1999), the Wolf Prize in Mathematics (2015), and the Leroy P. Steele Prize of the AMS for Lifetime Achievement (2016). He was elected as a Fellow of the Royal Society of Canada (1980), a Fellow of the Royal Society of London (1992), a Foreign Honorary Member of the American Academy of Arts and Sciences (2003), and a Foreign Associate of the National Academy of Sciences (US) (2014).

James Greig Arthur was born in Toronto, Canada, in May of 1944. He studied at the University of Toronto, receiving his BSc and MSc in 1966 and 1967, respectively. After that he attended Yale University, where he received his PhD under Robert Langlands in 1970. Upon graduation he served as an instructor at Princeton (1970–1972), an assistant professor at Yale (1972–1976), and a professor at Duke (1977–1979) before joining the University of Toronto as a professor in 1979. He became a University Professor at Toronto in 1987, where he currently holds a Mossman Chair. Arthur's career has a significant service component, including terms as president of the AMS (2005–2006) and a member of the Board of Trustees of the IAS (1997–2007). He has also served on various panels and committees of the IMU, as well as of scientific societies in Canada and the US. On a personal level, he married Dorothy Pendleton (Penny) Helm on June 10, 1972, and they have two sons, James and David.

For this tribute we have asked several of his colleagues and associates to contribute their remembrances and assessment of this Lifetime Achievement of Jim Arthur.

David Vogan received his PhD in 1976 at MIT. He has devoted his career to studying harmonic analysis on reductive groups, much like Jim Arthur has. Here he provides a gentle introduction to some of the ideas that figure into the work of Jim Arthur.

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Freydoon Shahidi is Distinguished Professor of Mathematics at Purdue University. His email address is fshahidi@purdue.edu.

Freydoon Shahidi, Steve Gelbart, and Penny and Jim Arthur in Montreal, August 1980.

David Vogan

Kernels of Beautiful Simplicity

I first met Jim Arthur around 1975 when he was delivering a colloquium lecture at MIT. I'm sorry to say that I don't know exactly what the subject was: although I have dozens of pages of notes from talks by Jim over the years, I wasn't able to find notes for that first one. What he was working on at that time was what he's been working on his whole career: harmonic analysis on reductive groups. A *reductive group* is a group of matrices defined by polynomial equations and admitting no normal subgroups of unipotent matrices. What's allowed by this definition are things like $\mathrm{GL}(n,\mathbb{R})$ and $O(n)$ (defined in the first case by no polynomial equations and in the second by the n^2 quadratic equations ${}^t gg = I_n$). What's ruled out are things like

$$\left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} \mid a \in \mathrm{GL}(p,\mathbb{R}), d \in \mathrm{GL}(q,\mathbb{R}) \right\}$$

(which has a normal subgroup

$$\left\{ \begin{pmatrix} I_p & b \\ 0 & I_q \end{pmatrix} \right\}$$

consisting of unipotent matrices).

Harmonic analysis is about understanding functions on G or on homogeneous spaces for G in terms of *representations* (π, V_π): vector spaces V_π carrying actions of G by linear transformations $\pi(g)$. Roughly speaking, harmonic analysis has two parts: a *Fourier transform* attaching to a function f a linear algebra object $\pi(f)$ for each representation V_π and *Fourier inversion* attaching a function to a collection of linear algebra objects T_π. The central example is the circle group $G = \{e^{i\theta}\}$. Irreducible representations π_n of G are indexed by integers. If f is a

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function on the circle, the Fourier transform is $\pi_n(f) = a_n$, the n^{th} Fourier coefficient of f (a complex number). One of the basic theorems is that the Fourier transform of a smooth function on the circle is a rapidly decreasing function on the integers, that all such rapidly decreasing functions on the integers occur, and that the original function f can be recovered by a convergent series

$$f(e^{i\theta}) = \sum_n a_n e^{-in\theta}.$$

We all learn many (more subtle and difficult) variations on this theme, beginning with the Fourier transform description of square-integrable functions.

In the world of reductive groups, some of these ideas can (through the amazing work of Harish-Chandra and others) be carried through very nicely. There are families of representations parametrized by things like $\mathbb{Z}^m \times \mathbb{C}^n$, so it makes sense to speak of a Fourier transform that is "rapidly decreasing" or "holomorphic" in the representation parameter. Jim's earliest work concerns such natural extensions of classical results about Fourier transforms to reductive groups. (Of course making *proofs* of these results is much more difficult than formulating them!) He is most famous for his many results about the "trace formula," which is an extension to reductive groups of the Poisson summation formula.

I'd like to tell you instead about ideas that Jim developed beginning in the early 1980s concerning phenomena in reductive groups that have no analogue in classical Fourier analysis. I said that some representations of a reductive group come in families $(\pi(\nu), V_{\pi(\nu)})$ with a parameter $\nu \in \mathbb{C}^n$. In the classical setting, all the spaces $V_{\pi(\nu)}$ are just $\mathbb{C}$, and so all the representations $\pi(\nu)$ are irreducible. For reductive groups, the spaces are infinite-dimensional. The representations $\pi(\nu)$ are irreducible for most ν, but for ν in certain affine hyperplanes in $\mathbb{C}^n$, $\pi(\nu)$ becomes *reducible*. This reducibility plays wonderful havoc with classical ideas. Functions that want to be holomorphic acquire poles along these hyperplanes, so interesting and complicated things happen when paths of integration are moved.

Langlands's description of harmonic analysis for square-integrable automorphic forms was phrased in terms of such behavior. He built the discrete part of the spectrum from the *cusp forms* (which are square-integrable for natural geometric reasons) and the *residual spectrum* (which arises from poles of an appropriate highest order in the meromorphic continuation of the continuous spectrum). The difficulty was that there was no general understanding of when these highest order poles might occur.

That is the problem addressed by Jim's work beginning around 1982. Here is a version of the central idea. The Langlands Program describes representations of a reductive group G in terms of semisimple elements in another complex reductive matrix group ${}^dG$. The complex parameters for representations mentioned above are eigenvalues of these matrices. What Jim understood was that the appearance of poles should be controlled by the existence of *nilpotent* matrices commuting with Langlands's matrices. That is, *poles should be related to elementary linear algebra questions about diagonalizing matrices with repeated eigenvalues.*

Jim Arthur, Ngô Bào Châu, Bill Casselman, Salim Ali Altuğ, and Jasmin Matz at the IAS conference in celebration of Langlands's eightieth birthday, October 2016.

Jim's work in the 1980s made this vague idea precise and drew powerful conclusions from it. He showed how to relate the idea to his work on the trace formula and to the Langlands–Shelstad ideas about stabilization and endoscopy. My friend Dan Barbasch and I have made our living since the 1980s on scraps from his table: Jim's ideas say where to look for interesting unitary representations of reductive groups over local fields, and so far he's been absolutely right.

The mathematics I love best addresses old and difficult problems using ideas with a kernel of beautiful simplicity. That's the mathematics that Jim does. He's an inspirational model, and I'm proud to count him as a friend. □

To turn the Arthur–Selberg trace formula into a workable tool, one needs the "fundamental lemma." Langlands expected this to indeed be a lemma, but it has required much work and ingenuity to establish it. A proof of the fundamental lemma was provided by a young Vietnamese mathematician, Ngô Bào Châu, and it garnered him a Fields Medal in 2010.

Ngô Bào Châu

The Laconic Professor Arthur

It was some time in 2000 when I met Jim Arthur for the first time. I was then a young mathematician working in an area not directly related to his. At that time, I worked with Tom Haines, who was a postdoc in Toronto, on questions related to bad reduction of Shimura varieties.

Ngô Bào Châu is Francis and Rose Yuen Distinguished Service Professor of Mathematics at the University of Chicago. Hie email address is ngo@uchicago.edu.

In a brief meeting I had with Jim, he handed me his recently completed papers on the stable trace formula. With his characteristic laconism, he said, "We need the fundamental lemma."

The fundamental lemma is a combinatorial identity discovered by Langlands in his study of points on Shimura varieties. In this context, the fundamental lemma is an ingredient in Langlands's strategy of expressing the Hasse–Weil zeta function of Shimura varieties in terms of automorphic L-functions. Slightly later, he realized similar combinatorial identities are needed in the stabilization of the trace formula for SL(2) that he worked out with Labesse. In the case of SL(2), the fundamental lemma can be proven by direct calculation of orbital integrals. In his lectures at École Normale Supérieure des Jeunes Filles, Langlands formulated the fundamental lemma in complete generality as a conjecture. Needless to say, nobody would care about these complicated combinatorial identities if they weren't important ingredients for establishing the stable trace formula. On the other hand, Jim's monumental work on the stable trace formula would remain conditional if the fundamental lemma wasn't proven. This little digression aims to provide some historical background to Jim's laconic sentence.

Jim's encouragement was important for me coming back to work full time on the fundamental lemma some years later. Some more years later, when I believed that I had the proof of that lemma, it was with a lot of emotion that I sent my preprint to Jim. His answer was again rather laconic: "You made my day." I still remember these words as a most treasured prize.

While working on the fundamental lemma, my understanding of Jim's work was rather superficial. My background is in algebraic geometry, which has very little to do with the arsenal Jim used in the construction of his trace formulas. Later, in learning Jim's work in more depth, I came closer to realizing all of its magnificence. There is little doubt that his construction of the trace formula for general reductive groups is one of the most important achievements of contemporary mathematics. □

Arthur was a student of another highly influential Canadian mathematician, Robert P. Langlands. Langlands has had a great influence on the theory of automorphic forms and anointed the trace formula, as subsequently developed by Arthur, as the preferred tool for establishing the endoscopic cases of his Functoriality Principle. This is precisely what Arthur did for the classical groups. It has been said that Arthur sprang fully formed from Langlands's head, much as Athena did from Zeus's (but less dramatically), but in fact he only began work on the trace formula after his PhD. Here Langlands puts Arthur's work in the broad context of twentieth- and twenty-first-century mathematics.

Robert Langlands and Jim Arthur at the conference in celebration of his sixtieth birthday, Fields Institute, October 2004.

Robert P. Langlands

James Arthur

The theory of automorphic forms as it evolved during the twentieth century contains one central theory, the theory over number fields, and two complementary theories of independent interest, the theory for function fields in one variable over finite fields and the geometric theory for function fields over complex algebraic curves. My views on the latter are explained in an article in preparation, "Об Аналитическом Виде Геометрической Теопии Автоморфных Форм" (An analytic view of the geometric theory of automorphic forms), and, so far as I can tell, for the former the decisive reference is the paper "*Chtoucas pour les groupes réductifs et paramétrisation de Langlands globale*" (Chtoucas for reductive groups and global Langlands parameterization) by Vincent Lafforgue. For analytic aspects of the theory over number fields, the principal contributor in recent decades has been Arthur.

He was preceded in the twentieth century by, above all, Hecke and Siegel, followed, with Hans Maass as intermediary, by Selberg. It is difficult to know just how much Selberg understood, but he was aware of the close relation between Eisenstein series and the spectral theory of differential equations on a half-line, a theory that appeared, for example, in a very early paper of Hermann Weyl on ordinary differential equations. A more striking, perhaps more imaginative, contribution of Selberg was the trace formula. This is a variant of the Frobenius reciprocity law for a subgroup H of a finite group G, but in a context where the spectrum is in part continuous, so that serious analytic problems appear. For GL(2) they are not too difficult. A typical application appears in Chapter 16 of the text *Automorphic Forms on* GL(2) by Jacquet–Langlands. For groups of larger rank, the difficulties are

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of a different degree. I began to think of the problem once I understood the theory of Eisenstein series, but with no success. Arthur began later, with no encouragement from me or, so far as I know, from anyone else, to study the problem. His thesis had been on quite a different matter. At the beginning of this century, thus almost thirty years after his initial efforts, I attempted to summarize, insofar as I had understood them, his contributions, which entail a great deal of technically and conceptually difficult analysis, so that they are, unfortunately, not easily accessible to those who most need to understand them: algebraic-number theorists and arithmetically oriented algebraic geometers. The reference is "The trace formula and its applications," *Can. Math. Bull.* (2001). That summary was a major undertaking; a summary of his more recent contribution would be even more difficult.

He has himself described in a book, *The Endoscopic Classification of Representations: Orthogonal and Symplectic Groups*, which exactly these mathematicians can be encouraged to read, the developments of the intervening years, again for the most part, but not solely, contributions of Arthur himself. What are the major goals? They are a theory of *functoriality* and a theory of *reciprocity*. This is not the place to define precisely these theories or the nature of their relation one to the other. They can be considered, in particular, as expressions of the nonabelian class field theory, whose existence Artin had begun to doubt in 1946, when he concluded that "… whatever can be said about non-abelian class field theory follows from what we know now. … " It is not a conviction, but a suspicion, not to be taken too seriously, that it was Chevalley and Artin who led mathematicians astray and that for these two theories—especially the first, about the second I have no clear idea—one has to return not to their expositions but to that of Helmut Hasse, *Bericht über neuere Untersuchungen und Probleme aus der Theorie der algebraischen Zahlkörper*, where the basic relation of class field theory appears as the equality of the number of solutions of two apparently different diophantine problems. I expect something similar to appear as a result of the trace formula and the related arithmetic in general. Arthur, in his book, does not reach this stage. He obtains, however, important and encouraging results for functoriality by an appeal to other arithmetic features of the problem, especially those related to the only partially developed and only partially arithmetic theory of endoscopy. Functoriality deals fully with the basic problem of analytic continuation of the Euler products of automorphic forms. It has, so far, nothing to offer to the Riemann hypothesis, although, on writing these lines, I recognize that I have never reflected on that question.

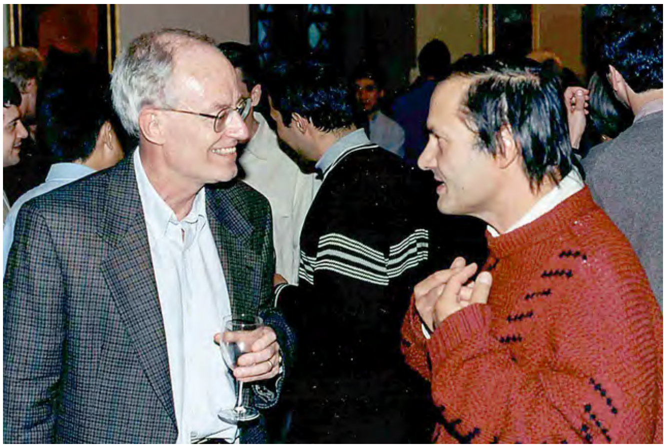
Moreover, in spite of the magnificent analytic achievements of Arthur, functoriality is far from a completely established concept. It can be considered as a part of harmonic analysis and as a part of analytic number theory. Its development in the second sense is only beginning. As a suggested introduction, I refer to the recent papers of Ali Altuğ, "Beyond endoscopy via the trace formula."

It is the issue of reciprocity that has the most attraction for number theorists. In brief, functoriality expresses the relation between automorphic forms on various groups according to principles that may be labelled *galoisian*, for the information is carried by groups and their relations. Knowing, as I wrote these lines, that unfortunately I had not found the time in recent years, even earlier, to study *reciprocity* and its several successes, among them Fermat's theorem, I spoke with my colleague Richard Taylor and, the conversation hardly begun, recognized that my grasp of the two theories, functoriality and reciprocity, and their relation was inadequate and misleading. Roughly speaking, it was explained to me that functoriality would aid in developing reciprocity. Namely, the first step would be to begin with a specific class of motives, those associated to the cohomology groups $H^i(X)$ of Shimura varieties, even to a specific class of Shimura varieties, and to show that they generated all the others. Functoriality established, the relevant operations, summands, sums, and products can be performed on both sides of the reciprocity—motives $\leftrightarrow$ automorphic forms—and thus lead to the desired reciprocity in general.

I explained my response to Taylor, who made it clear that although I had perhaps begun to understand the principles developed by him and his colleagues, I had exaggerated the possibilities offered by Shimura varieties and that there were many motives that were not accessible in this way, that like Thursday's child we had far to go. He did confirm, however, that "looking at the partial progress there has been on reciprocity in the last twenty-five years, functoriality, and in particular various instances of base change (and to a lesser extent endoscopy), have played a fundamental role. Without base change, and hence without Arthur's work, there would be many, many fewer theorems concerning reciprocity today."

Taylor had offered me reflections on a developing theory, but he did accompany them with references to specific work of him and Michael Harris, of Arthur–Clozel, and of Kottwitz. I did not have time to consult these before writing this essay, for which there was a short deadline, but it is clear that if L-functions are to play a role in the theory of diophantine equations, this will be in the context of functoriality, because the distribution of the Hecke–Frobenius parameters, thus the conjugacy classes in the group ${}^LG$ that define the Euler products attached to automorphic forms, is determined not by a single L-function but by all those associated to a given automorphic eigenform. □

Two of the mathematicians that probably have the greatest technical appreciation of Arthur's work and its influence are Colette Mœglin and Jean-Loup Waldspurger. Waldspurger did much early work on the fundamental lemma with an eye towards its eventual use by Arthur. Then together and separately, they have mined the consequences of Arthur's classification of local and global representations of the classical groups via the trace formula. Here they provide a more technical overview of the work of Arthur.



Jim Arthur and Jean-Loup Waldspurger at Arthur's sixtieth birthday conference, Fields Institute, October 2004.

Colette Mœglin and Jean-Loup Waldspurger

The Work of J. Arthur

Let G be an algebraic group, connected and semisimple, defined over $\mathbb{Q}$, for example, $G = \mathrm{SL}(n)$, $\mathrm{Sp}(2n)$, $\mathrm{SO}(n)$ for $n \geq 3$. Denote by $\mathbb{A}$ the ring of adèles of $\mathbb{Q}$. The group $G(\mathbb{A})$ acts by translation, on the right, in the vector space $L^2(G(\mathbb{Q})\backslash G(\mathbb{A}))$. Denote by L^2_{disc} the smallest closed subspace containing all the irreducible, closed, subrepresentations of $G(\mathbb{A})$. One of the goals of the theory of automorphic forms is to understand L^2_{disc} as a $G(\mathbb{A})$ module. Of course, the theory also holds for groups like $\mathrm{GL}(n)$ or $\mathrm{GSp}(2n)$; the only thing is to take care of the center in the definitions. The trace formula is one of the most powerful tools. This formula is an equality $J^G_{geom}(f) = J^G_{spec}(f)$. Here, $f \in C^\infty_c(G(\mathbb{A}))$ is any test function which takes value in the field of complex numbers. Both terms of this identity are complicated but contain a relatively simple principal term. For J^G_{spec} it is $\sum_\pi m(\pi)\,\mathrm{trace}\,\pi(g)$, where π describes all irreducible representations appearing in L^2_{disc} and $m(\pi)$ is precisely the multiplicity of π in L^2_{disc}. For the geometrical term, $J^G_{geom}(f)$, the principal term is $\sum_\gamma m(\gamma)I(\gamma, f)$, where γ runs over the set of elliptic semisimple elements in $G(\mathbb{Q})$, up to conjugacy by $G(\mathbb{Q})$, and $m(\gamma)$ is a certain volume. This is one of the major works of Arthur to have been able to obtain this formula in full generality. The first formula obtained had the disadvantage of containing, in the complementary terms, some terms which were not invariant by conjugacy. Arthur has given a second formulation of the trace formula that contains only invariant distributions. But even this second formulation is not suitable for all the consequences we want, as it does not allow one to compare the formulas for different groups as wanted by the principle of functoriality of Langlands.

Let us look at the geometrical part of the trace formula. In the case where we can hope to establish a correspondence between the conjugacy classes in $G(\mathbb{Q})$ and the conjugacy classes in $H(\mathbb{Q})$, for two groups G and H, this correspondence is in fact in general a correspondence for the conjugacy classes up to the action of $G(\overline{\mathbb{Q}})$ and $H(\overline{\mathbb{Q}})$ where $\overline{\mathbb{Q}}$ is an algebraic closure of $\mathbb{Q}$. This is called stable conjugacy. These two notions of conjugacy are different except for $\mathrm{GL}(n)$; an easy example can be found in $\mathrm{SL}(2, \mathbb{C})$. The idea of Langlands was that there should exist a stable version of the trace formula: $SI^G_{geom}(f) = SI^H_{spec}(f)$, at least if G is quasi-split (which is a technical condition). This formula has to be similar to the previous invariant trace formula. On the geometric side, the principal term is a sum over the stable conjugacy class of elements in $G(\mathbb{Q})$. In the spectral side the representations have to be grouped in linear combinations which are stable, i.e., which annihilate the test functions whose stable orbital integrals are zero. This stable formula is what Langlands called the theory of endoscopy. The notion of endoscopic elliptic data has been defined (Langlands, Shelstad, Kottwitz, Labesse) and is denoted here by $\underline{H}$. It is to each such datum that we try to associate a stable trace formula (independently of G of course) in such a way that we have an equality, for $* = geom$ or $spec$:

$$I^G_*(f) = \sum_{\underline{H}} i(G, H) SI^{\underline{H}}_*(f^{\underline{H}}), \qquad (*)$$

where $\underline{H}$ runs over the elliptic endoscopic data of G, $i(G, H)$ are explicit constants, and $f \mapsto f^{\underline{H}}$ is the transfer coming from the correspondence between stable orbital integrals. Among the endoscopic data, we have the quasi-split form of G, and if G is quasi-split, (*) is in fact the definition of SI^G_*, but one has to prove that the distribution so defined is stable. Arthur has established this theory in full generality. The proof of it not only occupies three papers of around one hundred pages but also a lot of preparatory papers. In fact, a lot of results had to be proved before in harmonic analysis and in the theory of representations. For example, the definition of weighted characters used the notion of normalized intertwining operators. But the theory, the actual state of knowledge, makes it impossible to compare the normalizing factors between a group and even one of its endoscopic data. Arthur has given a much better definition that we can control. But one of the most spectacular preparatory papers is the paper published in *Selecta*.

Arthur had previously made clear the importance of the elliptic representations (here the base field is a local one). These representations are the basis to obtain, via induction, all the tempered representations. Arthur proves that, for such representations, the stability and the endoscopic transfer can be read entirely on the subset of elliptic elements. A relatively direct consequence

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of such a result is the existence of the local spectral transfer. If $\underline{H}$ is an elliptic endoscopic datum of G, the Langlands–Shelstad transfer $f \mapsto f^{\underline{H}}$ gives a function $f^{\underline{H}}$ for which only the stable orbital integrals are well defined. Dually, to a stable distribution for the endoscopic datum, D^H, there is associated a distribution D for G by the equality $D(f) = D^H(f^{\underline{H}})$, and the local spectral transfer says that if D^H is a finite stable linear combination of representations of $\underline{H}$, then D is a finite linear combination of representations of G. The proof given by Arthur in *Selecta* is a local/global one and uses a simple form of the trace formula. This is a miniature of the general proof given in the three main papers, which we will now speak about. The difficulty of the stabilization is mainly on the geometric side. This comes from the way Arthur has made the trace formula invariant. The invariant form of the trace formula is no longer symmetric between a geometric and a spectral side; in fact both sides are now mixed. The weighted characters for tempered representation are now zero, but the information they have to carry has been put in the weighted invariant orbital integral.

Langlands and Kottwitz have handled the case of semisimple elliptic elements, and Arthur has to handle nonelliptic elements and especially those with a unipotent part. Already a semisimple nonelliptic element has complicated weighted orbital integrals, because this distribution contains information from the spectral side. Moreover, such an element is elliptic in a Levi subgroup, canonically attached to it, and the endoscopic data of this Levi subgroup also occur. The stabilization of the weighted orbital integrals attached to this kind of element occupies a great part of the proof. The proof is local/global using more than once the link between the geometric side and the spectral side. For the elements with a unipotent part, Arthur uses a descent method and a recurrence, and he is left with the purely unipotent elements. These elements are the only ones which can contradict the stability of the geometric side, but consideration of the spectral side proves that the way they will contradict the stability is not compatible with what is left as perhaps unstable on the spectral side. In fact, one can say that there are too few unipotent elements to block the process. Arthur has succeeded in walking around the unipotent part without computing it. This computation seems to be a very hard problem, and even though some progress has been made recently, this is perhaps the principal unsolved problem regarding the trace formula.

The stabilization of the trace formula has been a long-term job. It has been followed by another great deal of work, the explicit determination of L^2_{disc} when G is a classical group. This employs the stabilization of the trace formula and its twisted analogies. In this way, it is a whole part of the theory of automorphic forms which is due to Arthur. □

As we noted earlier, Arthur's career had a significant service component, which included a stint as president of the AMS. We have asked a subsequent president, and one of Jim's longest associates, to comment on his term as AMS president.

A lecture given at Arthur's sixtieth birthday conference at the Fields Institute, October 2004. On the stage is Steve Gelbart; in the first row are Bob Kottwitz, Jim Arthur, and Robert Langlands; in the next row are Bill Casselman, Freydoon Shahidi, and Lior Silberman; and in the back row, Tom Haines and Don Blasius.

Eric Friedlander

Leading the AMS with a Light Touch

The editors of this tribute to Jim Arthur have asked me to write something about Jim's role as president of the AMS. Tricky! Jim and I have been friends since we first arrived in Princeton in September 1970 as newly minted PhDs. In the almost half-century that has elapsed, we have had many amusing conversations and interesting mathematical discussions. The amusing conversations were often reflections upon our mathematical world and the delightful people who populate this world; those mathematical discussions typically involved my requests for enlightenment of Jim's very deep and difficult mathematics or Jim's requests for some targeted insight which I surely failed to satisfy. Looking back, I can't recall an unkind comment made by Jim (despite provocations), though there were some insightful comments about the idiosyncrasies of our mathematical community.

But what about Jim's AMS activities? First, Jim is a true gentleman and far more diplomatic than most mathematicians. Any critical remarks or complaints Jim would voice were shared in private, often with a big smile and a lowered voice. Perhaps he relied on me to some extent to rock the boat on difficult and thorny issues, such as the length of the Joint Prize Committee Ceremony or who would liven up AMS committees without causing mayhem. I presume that anyone with a strong opinion or a complaint who approached Jim during Jim's AMS presidency would have been fully satisfied with his

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response despite the fact that Jim held fast to his high standards and strong opinions.

My guess is that Jim might have suggested I write this "report" because of my role in assuring Jim that the "AMS green pages" given to members of the AMS Board of Trustees were not so impenetrable. Possibly, the color of these pages caused them to appear daunting; possibly, the size of sums considered was beyond our usual consideration (tens of millions of dollars); possibly, the detail and double bookkeeping of AMS finances at first threatened eyestrain and sleepless nights. But once Jim learned the tricks of asking a few innocent questions and appearing to be always on the correct page, he became a master.

AMS presidents are challenged by the need to appoint members to the uncountable number of AMS committees. Before this process began, Jim asked me to chair the AMS Committee on Committees. The dynamics of this committee proved somewhat challenging, but the entire process of selecting volunteers to advance the many positive efforts of the AMS was rewarding. Once again, humor leavened the burden, for we would discuss friends who were suitable as well as those who volunteered whom we did not know. We could proceed with some confidence, knowing that Bob Daverman's knowledge and wisdom would keep us from wandering too far astray.

Leading the Council is a delicate art, one which Jim mastered extremely well. Jim always kept the proceedings moving along with good humor, avoiding the pitfalls of lengthy personal anecdotes/remarks and curtailing discussions on topics not clearly relevant. Jim was invaluably guided by promptings from AMS Secretary Bob Daverman and the expertise of AMS Executive Director John Ewing. Flanked by Bob and John, exchanging handwritten notes and whispered comment, Jim had the AMS Council purring contentedly.

Indeed, I only recall one glitch. Once, after a meeting of the Executive Committee and Board of Trustees, we gathered to go out to enjoy a good dinner after all of our hard work. Having sorted out who had a car and who needed a ride, we arrived at the restaurant only belatedly to discover that we had left our leader (AMS President Jim Arthur) abandoned at the hotel. Of course, Jim rarely reminded us of the error of our ways.

Jim maintained a strong interest in preserving the important role that the AMS plays in advancing research mathematics through its publications, conferences, and prizes. A new distinguished lecture (the Einstein Lecture) is a consequence of Jim's efforts, and the evolution of AMS prizes and their selection committees owe much to Jim's interest.

So, what is my continuing impression of Jim Arthur and his AMS involvement? Reinforced by our continued "gossip sessions" and wry observations, my answer would be one of sense and sensibility, wit and good cheer. □

Jim Arthur receiving the Steele Prize for Lifetime Achievement from Robert Bryant, AMS Immediate Past President, at the Joint Mathematics Meetings in Atlanta, January 2017.

One thing we have not mentioned is Arthur's service as an advisor and mentor of graduate students. Arthur has been the advisor of eleven PhD students, one during his time at Yale (David DeGeorge, whom he shared with Bob Szczarba) and then ten while at the University of Toronto (Peter Mischenko, Jason Levy, Clifton Cunningham, Heng Sun, Cristina Ballantine, Paul Mezzo, Chao Li, Kam Fei Tam, Zhifeng Peng (whom he shared with Chung Pang Mok), and Bin Xu). One of his Toronto students was Cristina Ballantine, who received her PhD in 1998. Here she describes Arthur's talents as an advisor.

Cristina Ballantine

In Defense of the Semicolon

When mathematicians who are not number theorists ask me who my thesis advisor was, there might be a slight pause after I say Jim Arthur. Then, I just clarify: of the Arthur–Selberg trace formula. I don't need to add anything after that. Jim's name is by now somewhat interchangeable with the trace formula—the home of his mathematical life. I was a graduate student in Toronto for the better part of the 1990s. My first year there, he taught the graduate core course in real analysis. I loved analysis as an undergraduate and I felt comfortable with the subject. The way Jim taught it, however, had the effect of the right prescription glasses on a nearly blind person. I was in awe of his clarity.

At the end of the analysis course I knew I wanted to work with him. Soon, he became advisor to several students who had just started the PhD program. He was hoping to divide the work among us to establish the

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trace formula for the metaplectic group. We knew so little about the trace formula that, as Jim's fiftieth birthday was approaching, we decided to make him a T-shirt with the trace formula on it. Needless to say, we gave up once we realized that it wouldn't fit on a T-shirt.

At the time, there was no introductory book on automorphic forms, and Jim decided that we would learn the basics through a reading seminar in which we studied some articles from the Corvallis proceedings. It wasn't an easy way to learn. It was frustrating, even discouraging at times. There was a lot we didn't understand, and this had never happened to us in our undergraduate days. Yet Jim kept telling us to not worry and that one day it would all click. He called this *learning by osmosis*. It's one of the skills I learned from him that I appreciate most. It requires patience and overcoming being uncomfortable with your own inability to understand. And it works; eventually, it clicks. While the seminar was painful for us students, talking about mathematics we didn't fully understand, Jim sat in the back and observed. What he was observing was not clear to me until about a year later when it was time to start working on an actual problem.

For personal reasons I had to leave Toronto for a year, and it made sense that I would work on a problem that was not directly related to the metaplectic group. From the student seminar, Jim could tell that I really enjoyed working with spherical functions and he found just the right problem for me. Ramanujan graphs had just entered the mathematical scene a few years before, and I started working on generalizing the construction to hypergraphs and analyzing the spectrum via automorphic forms. The key to my work was Rogawski's classification of the representations of the unitary group in three variables, yet I would have never been able to use it successfully without translating it first into the language of the Arthur conjectures, which, of course, I did following Jim's advice.

Initially, Jim's advising style was hands off. He told me I could come to him with questions whenever I had some. For me, this didn't work well. I am most productive when under pressure. Adrenaline is my friend. I am not sure what made him change his strategy, but when I came back to Toronto and he had just returned from a sabbatical in France, all his students met with him individually for a half hour on Fridays. If I remember correctly, my time slot was 12:30. These are some of the most memorable times of my life as a graduate student. Probably not all his students had the same experience, but for me these office visits were often a sort of metamorphosis. I would feel frustrated with my work and then, after talking to Jim, I was excited again to be thinking about this particular problem and confident that I could solve it. Altogether I was a pretty happy graduate student, in no small part due to Jim's guidance. I still have the notes he wrote during office hours. They are as clear as his lecture notes, though much shorter. It was also during the weekly meetings that I had a glimpse into the way his memory worked. He always knew precisely what article would be relevant to a particular question. There were piles of papers on the window sill in his office and he knew exactly where to find the one he was looking for. Once, I asked him a question about Hecke algebras, and after answering it he said, "Didn't you ask this two years ago?" He was also generous with wisdom about life in general. When I was expecting my first child, he told me how I shouldn't expect a white page I can write on, how babies have their own personalities and that's a wonderful thing. When I told Jim that I wasn't going to take any time off once the baby is born, he just said, "Wait until you hold that baby in your arms. We'll talk about it then."

The day I finished proving the main result for my thesis his reaction was, "Great. Now keep the momentum." Then came the writing of the thesis. Jim himself is a wonderful writer. I'm pretty sure that if he hadn't been a mathematician he would have been a novelist. His general advice was that the thesis should read like prose but that I should also make it *short and snappy*. The day after I gave him the first draft of my thesis, he called me at home to ask what I meant by the definition on page 5. I explained, and ten minutes later he called back to ask another question about something on page 5. Another ten minutes later he called again and almost apologetically told me that I needed to rewrite the thesis. The next day I went into his office for more specific advice: no parentheses (if you need parentheses, it means you are not clear enough), do not start a sentence with the word *And* or a mathematical symbol, and no semicolons. I'm sure there was more, but I don't recall the details. Now, I can't help but repeat a story I told at Jim's sixtieth birthday conference. Years later, while searching for one of Jim's papers, among the many hits one read "'In Defense of the Semicolon' by James Arthur," a poem by his son of the same name.

I am extremely grateful that Jim Arthur took me as a PhD student. I learned a great deal of mathematics but also much about how to be part of the mathematical community, how to be gracious to my own students, and how to be happy and grateful for the chance to spend my life in mathematics. □

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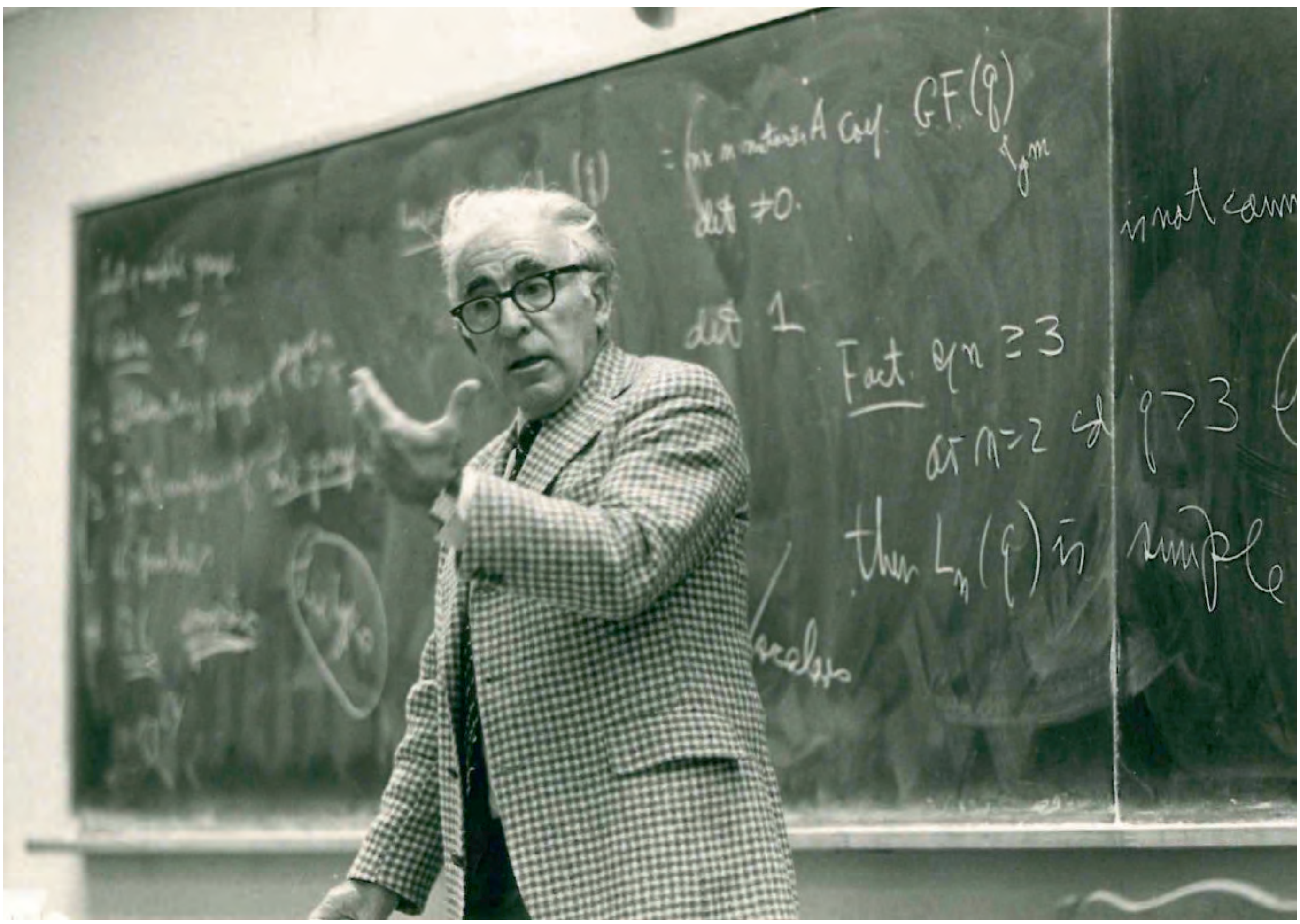
The Classification of Finite Simple Groups: A Progress Report

Ronald Solomon

Communicated by Alexander Diaz-Lopez and Harriet Pollatsek



Danny Gorenstein dubbed the classification of finite simple groups "The Thirty Years' War."

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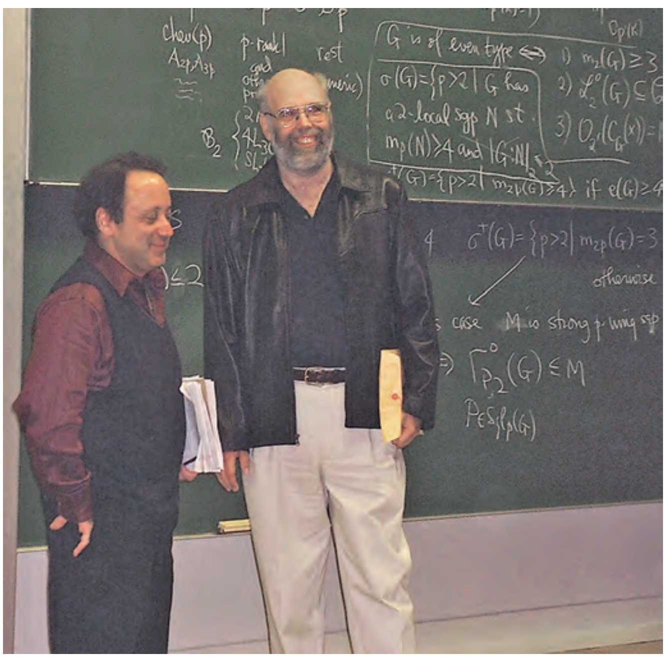
DOI: http://dx.doi.org/10.1090/noti1689

History of the Project

In 1981 the monumental project to classify all of the finite simple groups appeared to be nearing its conclusion. Danny Gorenstein had dubbed the project the "Thirty Years' War," dating its inception from an address by Richard Brauer at the International Congress of Mathematicians in 1954. He and Richard Lyons agreed that it would be desirable to write a series of volumes that would contain the complete proof of this Classification Theorem, modulo a short and clearly specified list of background results. As the existing proof was scattered over hundreds of journal articles, some of which cited other articles that were never published, there was a consensus that this was indeed a worthwhile project, and the American Mathematical Society (AMS) agreed to publish this series of volumes. In the spring of 1982, Danny and Richard recruited me to be a partner in this project. Richard Foote and Gernot Stroth were also recruited at an early stage to contribute specific portions of this work.



Richard Lyons (pictured here with the author) and Gorenstein agreed that it would be desirable to write a series of volumes [GLS] **that would contain the complete proof of this Classification Theorem.**

Considerable progress was made during the first decade of the project, and then, tragically, Danny Gorenstein died in August 1992. Nevertheless, the first six volumes of our series were published by the AMS during the decade 1994–2005. Then a hiatus ensued. I am happy to report that Volume 7 has just been published by the AMS, and Volume 8 is near completion and promised to the AMS by August 2018. The completion of Volume 8 will be a significant mathematical milestone in our work. It seems a good time to provide this progress report on the GLS project. Moreover, although the strategy outlined in Volume 1 [GLS] remains substantially unchanged, there is one significant change worthy of note. (A detailed overview of the original Classification Project may be found in [ALSS].)

We anticipate that there will be twelve volumes in the complete series [GLS], which we hope to complete by 2023. I will report on the current state of our project in this article.

Introduction to Classification

A common theme in mathematics is to study a particular mathematical structure and attempt to classify all instances of it, e.g., regular polyhedra, distance transitive graphs, and 3-manifolds, among others. Classifying infinite groups is hopeless, but already in the 1890s, inspired by the work of Killing and Cartan on finite-dimensional semisimple complex Lie algebras, some mathematicians began to contemplate the classification of all finite groups.

Can we classify all finite groups? Since we can combine any two finite groups, for instance via direct product, to create a new finite group, it is natural to begin by considering groups that are "building blocks" for all other finite groups. Similar to the factorization of integers into prime numbers, one can "break down" finite groups into smaller pieces called simple groups, which cannot be decomposed further. A group G is **simple** if it has no nontrivial proper normal subgroup; i.e., its only normal subgroups are the trivial group and G itself. The Jordan-Hölder Theorem then tells us that for any finite group G, there is an ordered sequence of subgroups, $1 = H_1 \lhd H_2 \lhd \cdots \lhd H_n = G$ called a composition series, such that each H_i is a maximal proper normal subgroup of H_{i+1} and all H_{i+1}/H_i, called composition factors, are simple. Moreover, any two composition series of a group G are equivalent in the sense that they have the same number of subgroups and the same composition factors, up to permutation and isomorphism.

Nonetheless, the problem of determining all ways to reassemble a set of composition factors into a finite group is daunting, perhaps infeasible. Although it is very easy to prove that the only abelian simple groups are $\mathbb{Z}/p\mathbb{Z}$, where p is a prime number, the problem of determining all finite p-groups, i.e., groups G all of whose composition factors are isomorphic to $\mathbb{Z}/p\mathbb{Z}$, is of frightening complexity. For example, there are billions of groups of size 2^{10}, and many of them are essentially indistinguishable.

Fortunately, in most applications of finite group theory, where the group arises as a group of permutations or symmetries or linear operators on some other structure, the problem can be reduced easily to the case where the group action is "primitive" in some sense. Using the classification of finite simple groups, Bob Guralnick, the latest Cole Prize recipient (see *Notices*, April 2018, p. 461), has demonstrated the efficacy of this strategy in a wide variety of contexts.

For the remainder of this article, I will discuss the classification of the finite simple groups. In contrast to the abelian case, the classification of nonabelian finite simple groups is quite complex and requires a full-scale classification strategy. The two smallest nonabelian simple groups are A_5, the alternating group on five symbols, with 60 elements, and $\mathrm{PSL}(2,7)$, a member of one of the families of groups of Lie type (see the next section), with 168 elements. In addition to the simple groups belonging to infinite families, there are also twenty-six so-called sporadic groups, five discovered by Mathieu in the nineteenth century, and the rest discovered between 1965 and 1975. The sporadic groups range in size from 7920 (the smallest Mathieu group) to approximately 8×10^{53} (the aptly named Monster).

In the next section, we discuss our classification strategy to prove the following theorem.

Theorem. *Every finite simple group is isomorphic to one of the following:*

(1) *a cyclic group of prime order,*
(2) *an alternating group of degree at least* 5*,*
(3) *a simple group of Lie type, or*
(4) *one of the* 26 *"sporadic" simple groups.*

The Classification Strategy

For about fifty years, the Classification Strategy has been schematically represented as a box subdivided into four smaller boxes:

small odd	small even
large odd	large even

Most of the finite simple groups are groups of Lie type defined over some finite field. If you are not familiar with Lie groups, you may consider the example of $\mathrm{PSL}(n,F)$, which is the quotient of the group $\mathrm{SL}(n,F)$ of $n \times n$ matrices of determinant 1 with entries from the finite field F by the normal subgroup of scalar matrices of determinant 1. In this context, odd and even refer to the parity of $|F|$, while small and large are measured by the size of n. When $n < 3$, the groups are definitely small, though $\mathrm{PSp}(4,F)$ is also small. If you know Tits's description of groups of Lie type as BN pairs, then it is more accurate to say that groups of BN rank 1 or 2 are small, while most groups of BN rank at least 3 are large.

Of course, there are also simple alternating groups and sporadic groups which must be fitted into this scheme. Much more serious is the fact that we must provide definitions for terms like "a group G of small odd type" that do not presuppose that G is a group of Lie type or indeed has any known property other than simplicity. However, if p is a prime divisor of $|G|$, then Sylow guarantees the existence of many p-subgroups of G, i.e., subgroups of order p^m for some $m \geq 1$. We call the centralizers and normalizers of such subgroups *p-local subgroups* of G. In his 1954 address, Richard Brauer made the case for attempting to characterize simple groups via their 2-local structures. A crucial validation for this strategy was provided in 1963 when Walter Feit and John G. Thompson [FT] published their proof that all nonabelian finite simple groups have even order.

Now, if $G = G(F)$ is a group of Lie type defined over a field F of even order, then the 2-local subgroups of G are contained in parabolic subgroups of G, and by a theorem of Borel and Tits they inherit significant structural properties from these parabolics. On the other hand, this is far from true in groups of Lie type when F has odd order, with a small number of interesting exceptions. This gave rise to the initial definitions of "even" and "odd": characteristic 2-type and "non"-characteristic 2-type. (We shall instead call the latter "odd type.")

Definition. A group H is of 2-**parabolic type** if H contains a normal 2-subgroup Q such that the centralizer $C_H(Q)$ is just $Z(Q)$. (This definition is nonstandard. It is used here for expository purposes.)

Definition. A group G is of **characteristic** 2-**type** if every 2-local subgroup of G is of 2-parabolic type.

A weakness of using this definition to define the line between even and odd is that, in the study of groups of odd type, it forces the consideration of all groups G having a 2-local, no matter how small, which is not of 2-parabolic type. A better definition is

Definition. A group G is of **even characteristic** (or parabolic characteristic 2) if every 2-local subgroup H of odd index in G is of 2-parabolic type.

In the GLS Project, we use another term, "even type," which is convenient for our approach but technically complicated to define. Recently, Magaard and Stroth have classified all groups that are of even type but not of even characteristic. The list is rather short: A_{12}, $\Omega_7(3)$, $\Omega_8^-(3)$, and the sporadic simple groups J_1, Co_3, and Fi_{23}. I shall speak henceforth of the odd/even dichotomy as the dichotomy between groups of even type and groups of odd (i.e., not even) type.

The small/large dichotomy was first formulated for groups of even type by John G. Thompson in his work on N-groups, i.e., nonsolvable finite groups all of whose local subgroups are solvable. He defined the parameter $e(G)$ to be the maximum rank of an abelian subgroup of odd order contained in some 2-local subgroup of G. A finite simple group G is of small even type if $e(G) \leq 2$, i.e., if no 2-local subgroup H of G contains a subgroup isomorphic to $C_p \times C_p \times C_p$ for any odd prime p. Such groups also came to be known as **quasithin groups**, and the classification of quasithin simple groups of even characteristic by Aschbacher and Smith [AS] was the culminating accomplishment of the original Classification Project. Notice that if G is a Chevalley group defined over a field of even order $q > 2$, then $e(G)$ typically measures the rank of a split torus of G and hence the BN-rank of G.

The small/large dichotomy for groups of odd type was originally formulated in terms of the 2-rank (or normal 2-rank or sectional 2-rank) of G. In the [GLS] volumes, the study of groups G of odd type focuses on the isomorphism types of components of involution centralizers, where an involution is an element of order 2 in G. The following definitions are of crucial importance.

Definitions.

1. A finite group L is **quasisimple** if $L/Z(L)$ is a nonabelian simple group and $L = [L,L]$.

Some of those who have worked on the Classification Project (l to r: David Goldschmidt, Ron Solomon, Stephen D. Smith, Richard Lyons, Michael Aschbacher, John G. Thompson, and Richard Foote), who gathered at Cambridge University in 2013 in celebration of Thompson's eightieth birthday.

2. A subgroup L of a group H is a **component** of H if L is a quasisimple subnormal subgroup of H. (Subnormality is the transitive extension of the normality relation.)

If, for example, $G = \mathrm{GL}(V)$, where V is an n-dimensional vector space over a finite field F of odd order, and if t is an involution in G, then the components of $C_G(t)$ typically are $\mathrm{SL}(V_-(t))$ and $\mathrm{SL}(V_+(t))$, where $V_-(t)$ and $V_+(t)$ are the -1 and $+1$ eigenspaces for t on V. Thus the components of $C_{\mathrm{GL}(V)}(t)$ are $[C_+, C_+] \cong \mathrm{SL}(V_+)$ and $[C_-, C_-] \cong \mathrm{SL}(V_-)$. See Figure 1. (Computations are similar in $\mathrm{SL}(V)$ and $\mathrm{PSL}(V)$, and the components are isomorphic to those in $\mathrm{GL}(V)$.)

$$t = \begin{pmatrix} I & O \\ O & -I \end{pmatrix}$$

$$\begin{aligned} C_{\mathrm{GL}(V)}(t) &= \left\{ \begin{pmatrix} A & O \\ O & B \end{pmatrix} : A \in \mathrm{GL}(V_+), B \in \mathrm{GL}(V_-) \right\} \\ &= C_+ \times C_- \cong \mathrm{GL}(V_+) \times \mathrm{GL}(V_-). \end{aligned}$$

Figure 1. The components of $C_{\mathrm{GL}(V)}(t)$ are $[C_+, C_+] \cong \mathrm{SL}(V_+)$ and $[C_-, C_-] \cong \mathrm{SL}(V_-)$.

Roughly speaking, groups of small odd type are defined to be simple groups G such that the only components occurring in any involution centralizer are isomorphic to $\mathrm{SL}(2, F)$ or $\mathrm{PSL}(2, F)$ for some field F of odd order. The classification of finite simple groups of small odd type appears in Volume 6 [GLS]. Since the [GLS] Project will quote [AS] for the classification of quasithin groups of even type, 2004 marked the publication of the volumes treating "small" groups of both odd and even type.

It should be noted here that in the outline of the [GLS] series in Volume 1, a slightly broader definition of small even type was formulated. Namely, it was stipulated that G was of small even type if G was of even type with $e(G) \le 3$ and no 2-local subgroup H of G contained a subgroup isomorphic to $C_p \times C_p \times C_p$, unless $p = 3$. It was anticipated that the Quasithin Problem would be enlarged to treat this wider problem. Given the monumental task accomplished by Aschbacher and Smith, it is quite understandable that they limited themselves to the original Quasithin Problem (though extended to groups of even type). It should also be noted that both Aschbacher–Smith [AS] and [GLS] treat only $\mathcal{K}$-proper simple groups, i.e., simple groups all of whose proper simple sections are known simple groups. This certainly suffices for an inductive proof of the Classification Theorem.

In the philosophy of the GLS Project, large simple groups of both even and odd type are to be treated as groups of component type, i.e., groups G that, for a suitable prime p, have p-rank at least 3 and have elements of order p whose centralizers have components of suitable type. Notice that if G is a simple group of Lie type defined over a field F of characteristic r, then centralizers of elements of order r have no components, while elements of prime order p different from r often do. For example, if $G = \mathrm{GL}(6, F)$, $|F| = 2^{2m}$, and x is an element of G of order 3 with three 2-dimensional eigenspaces, with eigenvalues 1, ω, and ω^2, then $C_{\mathrm{GL}(6,F)}(x)$ has three components, each isomorphic to $\mathrm{SL}(2, F)$ as in Figure 2. Again, the calculations are similar in $\mathrm{SL}(6, F)$ and in $\mathrm{PSL}(6, F)$.

$$x = \begin{pmatrix} I & O & O \\ O & \omega I & O \\ O & O & \omega^2 I \end{pmatrix}$$

and

$$\begin{aligned} C_{\mathrm{GL}(6,F)}(x) &= \left\{ \begin{pmatrix} A & O & O \\ O & B & O \\ O & O & C \end{pmatrix} : A, B, C \in \mathrm{GL}(2, F) \right\} \\ &\cong \mathrm{GL}(2, F) \times \mathrm{GL}(2, F) \times \mathrm{GL}(2, F). \end{aligned}$$

Figure 2. The centralizer $C_{\mathrm{GL}(6,F)}(x)$ of an order-3 element x has three components, each isomorphic to $\mathrm{SL}(2, F)$. Here $|F| = 2^{2m}$.

In general, in groups G of characteristic r, centralizers of elements of order p will typically have components that themselves are quasisimple groups of Lie type of characteristic r, indeed often smaller dimensional versions of G itself, as in the example in Figure 2. Thus, the principal criterion for "components of suitable type" is that they not be groups of Lie type of characteristic p. In this case, we shall call p a *semisimple* prime for G.

More accurately, we choose $p = 2$ if some involution centralizer $C_G(t)$ has a component which is not a group of Lie type in characteristic 2. We also exclude most sporadic components and, for the sake of "largeness," $\mathrm{SL}_2(q)$ and $\mathrm{L}_2(q)$. If we cannot choose $p = 2$, then we choose an odd prime p such that G has p-rank at least 3 and some p-element centralizer $C_G(x)$ has a component which has a noncyclic Sylow p-subgroup and is not a group of Lie type in characteristic p. (For $p = 3$, we also exclude several sporadic components.)

I have swept under the carpet a serious problem concerning "p-signalizers," and I shall continue to do so. But I will say that I should be saying "p-components" and not "components" in the previous paragraph, and the proof that we may reduce the problem from p-components to components when either $p = 2$ or p is odd and G has p-rank at least 4 is one of the principal results of Volume 5 [GLS]. For this reason, we use the term *generic even type* to refer to the subset of groups of large even type which have p-rank at least 4 for a suitable odd prime p. The signalizer problem for groups of characteristic 2-type and odd p-rank 3 was handled by Aschbacher in a pair of papers, which still need to be generalized to the case of groups of even type.

Building on the work in Volume 5 [GLS], Volumes 7 and 8 will complete the proof of the following two theorems.

Theorem O. *Let G be a finite $\mathcal{K}$-proper simple group of odd type. Then either G is an alternating group of degree $n \geq 5$ (but not 8 or 12) or G is a group of Lie type defined over a finite field of odd order or G is one of the following sporadic simple groups: M_{11}, M_{12}, J_1, Mc, $O'N$, or Ly.*

Theorem GE. *Let G be a finite $\mathcal{K}$-proper simple group of generic even type. Then either G is a group of Lie type defined over a finite field of even order or G has a proper p-uniqueness subgroup for some odd prime p or G has a "p-thin configuration" for some odd prime p.*

Volume 7 [GLS] has just been published by the AMS. It almost completes the identification of the alternating groups of degree $n \geq 13$ and the reduction of the Generic Case to the case where a suitable p-element centralizer has a component of Lie type in characteristic r, with either $p = 2$ or $r = 2$. Volume 8 will complete the identification of the alternating groups and the generic groups of Lie type. It is near completion, and we anticipate submission to the AMS for publication by August 2018.

Gernot Stroth's 600-page treatment of groups with a strongly p-embedded subgroup will probably appear as Volume 11 [GLS].

Inna Capdeboscq, coauthor of Volume 9 [GLS].

An important project stimulated by our work was the study of a class of amalgams of finite groups, known as Curtis–Tits–Phan amalgams, by a team of mathematicians including C. Bennett, R. Blok, R. Gramlich, C. Hoffman, M. Horn, B. Muhlherr, W. Nickel, and S. Shpectorov. Their results are added to our list of Background Results and used in our identification of many of the finite simple groups of Lie type. An extensive overview of this project, with a fuller list of authors and papers, may be found in Gramlich [Gr]. A very recent paper of Blok, Hoffman, and Shpectorov is also crucial to our work.

What will remain to be done after Volume 8 [GLS]? The remaining work falls primarily into three cases:

(1) the Bicharacteristic Case,
(2) the p-Uniqueness Case, and
(3) the $e(G) = 3$ Case.

The Bicharacteristic Case refers to the case when G is of even type with $e(G) \geq 4$ but there is no semisimple prime

p. Thus, G seems to have two different characteristics: 2 and p. Using an argument of Klinger and Mason, it is easy to see that $p = 3$. This is a phenomenon that occurs in many sporadic simple groups as well as a few groups defined over fields of order 2 or 3. Considerable work has been done on this problem, originally by Gorenstein and Lyons, and more recently by Inna Capdeboscq, Lyons, and me. We anticipate that this will be the principal content of Volume 9 [GLS], coauthored with Capdeboscq. This volume will also dispose of the dangling p-Thin case from Theorem GE.

The p-Uniqueness Case arises when the proof of Theorem GE leads to the construction of a proper subgroup M of G containing all (or almost all) of the p-local overgroups of a fixed Sylow p-subgroup of G. When $p = 2$, this problem was handled elegantly by Helmut Bender and Aschbacher, and their work is contained in Volume 4 [GLS]. (Similarly, the 2-Thin Problem was handled in Volume 6, which explains why the statement of Theorem O is so much cleaner than the statement of Theorem GE.) When p is odd, there is a major 600-page manuscript by Gernot Stroth treating groups with a strongly p-embedded subgroup, which will appear in the [GLS] series, probably in Volume 11. There are also substantial drafts by Richard Foote, Gorenstein, and Lyons for a companion volume (Volume 10?), which together with Stroth's volume will complete the p-Uniqueness Case. In particular, Volumes 9, 10, and 11 will strengthen Theorem GE to

Theorem GE+. *Let G be a finite $\mathcal{K}$-proper simple group of generic even type. Then G is a group of Lie type defined over a finite field of even order.*

There remains the gap between generic even type and large even type, i.e., the $e(G) = 3$ Case. If G is a $\mathcal{K}$-proper simple group of characteristic 2-type with $e(G) = 3$, then G was classified in a pair of papers by Michael Aschbacher. The first paper treats the case when a 2-local H contains an abelian p-subgroup A of rank 3 for some $p > 3$. Aschbacher's method in this case should extend fairly easily to our case, i.e., the case when G is of even type. In the second paper, 3 is the only choice for p. This is the case that was redefined as a subproblem of the Quasithin Problem in Volume 1 [GLS]. It has been given serious attention by several of our colleagues, notably Capdeboscq and Chris Parker, who have a manuscript overcoming one of the difficulties of extending Aschbacher's work to the even type situation. In this context, it might be very helpful to have a proof of the following result:

Statement. *Let G be a finite group with an abelian Sylow p-subgroup P. Suppose that P contains a strongly closed elementary abelian subgroup B. Then G contains a normal subgroup N such that $B = \langle x \in P \cap N : x^p = 1\rangle$.*

Using the Classification Theorem, this result is a special case of a theorem of Flores and Foote. However, we are not permitted to quote the Classification Theorem. For the purposes of our problem, it would suffice to assume that $p = 3$ and P has 3-rank 3.

It would be wonderful to complete our series by 2023, the sixtieth anniversary of the publication of the Odd Order Theorem. Given the state of Volumes 8, 9, 10, and 11, the achievement of this goal depends most heavily on the completion of the $e(G) = 3$ problem. It is a worthy goal.

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Photo of Gernot Stroth courtesy of Gernot Stroth.
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References

[ALSS] M. Aschbacher, R. Lyons, S. D. Smith, and R. Solomon, *The Classification of Finite Simple Groups*, Mathematical Surveys and Monographs Vol. 172, Amer. Math. Soc., 2011. MR2778190

[AS] M. Aschbacher and S. D. Smith, *The Classification of Quasithin Groups I, II*, Mathematical Surveys and Monographs **111, 112**, Amer. Math. Soc., 2004. MR2097623, MR2097624

[FT] W. Feit and J. G. Thompson, Solvability of groups of odd order, *Pacific J. Math.* **13** (1963), 775–1029. MR0166261

[GLS] D. Gorenstein, R. Lyons, and R. Solomon, *The Classification of the Finite Simple Groups*, Mathematical Surveys and Monographs **40**, 1–7, Amer. Math. Soc., 1994, 1996, 1998, 1999, 2002, 2005, 2018. MR1303592, MR1358135, MR1490581, MR1675976, MR1923000, MR2104668

[Gr] R. Gramlich, Developments in finite Phan theory, *Innov. Incidence Geom.* **9** (2009), 123–175. MR2658896

ABOUT THE AUTHOR

Ronald Solomon first learned to love words from his mother; to love math from his geometry teacher, Blossom Backal; and to love classical music from Mozart's Great G minor Symphony. He is the proud husband of Rose, father of Ari and Michael, and grandfather of Sofia.

Ronald Solomon

A Case Study in Noncommutative Topology

Claude L. Schochet

Communicated by Gerald B. Folland

ABSTRACT. This is an expository note focused upon one example, the irrational rotation C^*-algebra. We discuss how this algebra arises in nature—in quantum mechanics, group actions, and foliated spaces, and we explain how K-theory is used to get information out of it.

Introduction

This is the opposite of a survey paper. Here we are interested in one example, usually known as the *irrational rotation C^*-algebra* or *noncommutative torus* and written A_λ, where λ is some irrational number between 0 and 1. We will show that A_λ arises in at least three quite different contexts:

(1) quantum mechanics,
(2) action of a group on a compact Hausdorff space,
(3) foliated spaces.

Then we will use K-theory and traces to show that for λ irrational between 0 and 1/2, the A_λ are all nonisomorphic.

A word about the title. Commutative C^*-algebras with unit ("unital") and compact Hausdorff spaces are equivalent categories: given a compact Hausdorff space X you can form $C(X)$, the commutative unital C^*-algebra of all continuous complex-valued functions on X, and given a commutative unital C^*-algebra A, its maximal ideal space X is compact Hausdorff and $A \cong C(X)$. So studying commutative unital C^*-algebras is the same as studying compact Hausdorff spaces—a natural category for algebraic topology. Most C^*-algebras are noncommutative, and so studying them is doing noncommutative topology!

Mathematicians struggled to keep up, to keep the physics honest.

Our goal is to write as if we are sitting in a coffeehouse and explaining an idea to a good friend (on napkins, of course). So we are interested in getting an idea across but not at all interested in the technical details that, in any event, would be lost if the coffee spilled.[1]

Quantum Mechanics

In 1926–1927 the quantum-mechanical revolution in physics changed our understanding of the world. As has been the pattern since, the physicists knew what they wanted, and the mathematicians struggled to keep up, to keep the physics honest (as a mathematician would put it).

The simplest model of the hydrogen atom revolved about two self-adjoint operators P and Q that were to measure position and momentum of the electron. Heisenberg and Born showed that if Q is the position operator and P the momentum operator, then we have the *canonical commutation relation*

$$PQ - QP = -i\hbar I,$$

where $\hbar$ is Planck's constant.

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It is a pleasure to thank Marc Rieffel who, besides contributing the most important theorems to this note, was very helpful in its preparation, and to thank my old friend and co-author Jonathan Rosenberg.

DOI: http://dx.doi.org/10.1090/noti1684

[1] *I learned this technique from Dror Bar-Natan, who gave a great colloquium talk entitled "From Stonehenge to Witten, Skipping all the Details."*

It is easy to see that there are no $n \times n$ matrices P and Q such that
$$PQ - QP = -i\hbar I$$
with $\hbar \neq 0$: just observe that
$$tr(PQ - QP) = tr(PQ) - tr(QP) = 0,$$
but $tr(-\hbar I) = -in\hbar$. It is not much harder to see that there are no bounded self-adjoint operators on a Hilbert space with this property. There are unbounded ones, but to avoid technicalities with such operators it is best to pass to the corresponding one-parameter unitary groups
$$U_s = e^{isP}, \qquad V_t = e^{itQ},$$
for which the commutation relation $PQ - QP = -i\hbar$ becomes
$$U_s V_t = e^{-i\hbar st} V_t U_s.$$
This is called the Weyl form of the canonical commutation relation. Such pairs of unitary groups exist: for example, $U_s f(x) = f(x - \hbar s)$ and $V_t f(x) = e^{itx} f(x)$ on $L^2(\mathbb{R})$. Moreover, by the Stone-von Neumann theorem they are all unitarily equivalent, subject to an irreducibility condition. Taking $s = t = 1$ and $\lambda = -2\pi\hbar$, we obtain unitary operators U and V such that
$$UV = e^{2\pi i\lambda} VU.$$
These operators are bounded operators on the same Hilbert space, $U, V \in \mathcal{B}(\mathcal{H})$. So we may take the (non-commuting) polynomial algebra generated by U, V, and their adjoints. We then close up this algebra with respect to the operator norm and reach our goal, the C^*-algebra A_λ, constructed visibly as a norm-closed, $*$-closed subalgebra of $B(\mathcal{H})$.

This is the first construction of the A_λ. We may restrict attention to $\lambda \in [0, 1)$ and ask an elementary question: as λ changes, how is A_λ affected? It turns out that the case of greatest interest is when λ is irrational, and so we will restrict to that case as needed.

Homeomorphisms of the Circle

Let $\phi : S^1 \to S^1$ be rotation of the circle by $2\pi\lambda$ radians counterclockwise. Any rotation is a homeomorphism and thus determines an action of the integers on the circle by sending n to ϕ^n. This defines an action of the integers on $C(S^1)$ and from this one can construct a C^*-algebra
$$C(S^1) \rtimes \mathbb{Z}$$
as follows.[2]

For $\lambda = 0$ or 1, $C(S^1) \rtimes \mathbb{Z}$ is simply $C(T^2)$, continuous functions on the torus. For λ irrational we can realize this algebra as follows. Take $\mathcal{H}$ to be the Hilbert space $L^2(S^1)$ and let $T \in \mathcal{B}(\mathcal{H})$ be the bounded invertible operator $Tf = f \circ \phi$. Any $f \in C(S^1)$ gives a pointwise multiplication operator $M_f \in \mathcal{B}(\mathcal{H})$. Then $C(S^1) \rtimes \mathbb{Z}$ is the norm-closed $*$-algebra generated by T and by all of the M_f. Note that finite sums of the form
$$\Sigma_{n=-k}^{k} M_{f_n} T^n$$
are dense in $C(S^1) \rtimes \mathbb{Z}$. For λ irrational there is a unique normalized trace[3] τ on $C(S^1) \rtimes \mathbb{Z}$ given on finite sums by
$$\tau(\Sigma_n M_{f_n} T^n) = \int_{S^1} f_0(t)dt \ \in \mathbb{R}$$
where dt is normalized Lebesgue measure on the circle. It is not at all hard to prove that if λ is irrational, so that the action of $\mathbb{Z}$ on the circle is free, then
$$A_\lambda \cong C(S^1) \rtimes \mathbb{Z}.$$

Foliated Spaces

The local picture of a foliated space is $\mathbb{R}^p \times N$, where N is some topological space. A subset of the form $\mathbb{R}^p \times \{n\}$ is called a *plaque* and a measurable subset $T \subseteq \mathbb{R}^p \times N$ which meets each plaque at most countably often (the simplest being $\{x\} \times N$) is called a *transversal*. The global picture is more complicated. We say that a (typically compact Hausdorff) space X is a *foliated space* if each point in X has an open neighborhood homeomorphic to the local picture and locally the plaques fit together smoothly. A *leaf* is a maximal union of overlapping plaques; by construction it is a smooth p-dimensional manifold.

Here is an example. Start with the unit square $[0, 1] \times [0, 1]$ foliated by the vertical lines $\{x\} \times [0, 1]$. Glue the left and right sides of the square together—that is, identify $(0, t)$ with $(1, t)$—to make a cylinder, again foliated by vertical lines. If we now glue the top and bottom of the cylinder together, identifying $(x, 0)$ with $(x, 1)$ we get a torus foliated by circles that go around the central hole. (See Figure 1.)

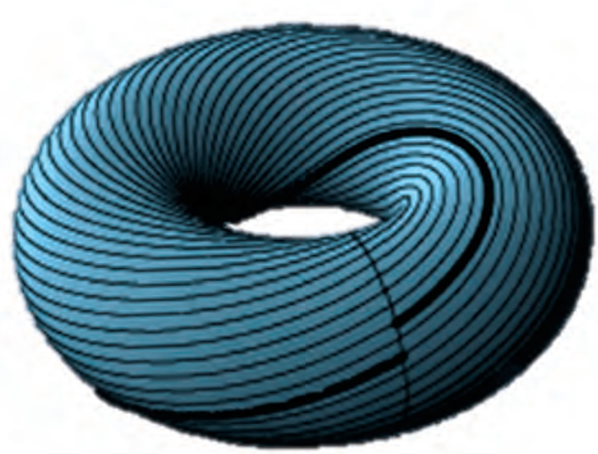

Figure 1. A torus foliated by lines of given irrational slope gives rise to a C^*-algebra.

To get something more interesting, we can give the cylinder a twist before gluing the ends together: we glue

[2] *This is actually a quite general construction. Starting with any C^*-algebra and locally compact group G acting on A you get a new crossed-product C^* algebra built by taking the algebraic crossed product and then completing. There are some topological choices for how the completion is done, but if the group is commutative then the completions coincide.*

[3] *A normalized trace on A is a linear functional on A such that $\tau(x^*x) = \tau(xx^*) \geq 0$ for all x and $\tau(1) = 1$.*

$(x, 0)$ to $(x \oplus \lambda, 1)$, where "$\oplus$" means *addition mod* 1. Now something quite unusual happens, and here we must specify whether λ is rational or not. If λ is rational, then each leaf of the foliated space is actually a circle wrapped around the torus several times. On the other hand, if λ is irrational, then you do not get circles: every leaf is a line $\mathbb{R}$. Furthermore, as in Figure 1, each line is wrapped about the torus infinitely often: each line is actually *dense* in the torus. This construction is called the *Kronecker flow on the torus.*

Every foliated space satisfying very minimal technical assumptions has a C^*-algebra associated to it. This is due to H. E. Winkelnkemper (1983) and to A. Connes (1979). In our context these always have the form $A \otimes \mathcal{K}$ where $\mathcal{K}$ is the C^*-algebra of the compact operators.[4] Here are some examples:

There is a natural trace that arises.

(1) If $X = F \times B$ or more generally, if X is the total space of a compact fibre bundle with fibre some smooth manifold F and base the compact Hausdorff space B, then X is foliated by the fibres and the foliation algebra is $C(B) \otimes \mathcal{K}$.

(2) If X is the torus foliated by circles, then it is a fibre bundle of the form $S^1 \to X \to S^1$ and by (1) the foliation algebra is $C(S^1) \otimes \mathcal{K}$ (where the S^1 in $C(S^1)$ is the base of the fibre bundle, not the generic fibre).

(3) (the punch line) If X is the Kronecker flow on the torus, then the foliation algebra is $A_\lambda \otimes \mathcal{K}$.

Note for $0 < \lambda < 1/2$ that using λ or $1 - \lambda$ gives the same foliated space, and so $A_\lambda \cong A_{1-\lambda}$. Thus, we restrict attention to irrational λ between 0 and $1/2$.

There is a natural trace that arises in this construction as well. What is needed is an invariant transverse measure. A *transverse measure* measures transversals, naturally enough. If it has enough nice properties then it is an *invariant transverse measure.* Not all foliated spaces have them, but the ones we are looking at do. In the case of the fibre bundle above, the foliation algebra is simply $C(B) \otimes \mathcal{K}$ and invariant transverse measures correspond to certain measures on B. Invariant transverse measures correspond to Ruelle-Sullivan currents in foliation theory (cf. [1] Ch. IV).

In the case of the Kronecker flow on the torus, the invariant transverse measure may be constructed from Lebesgue measure on a transverse circle to the foliated space. This passes to a trace on the foliation algebra which corresponds to the trace constructed above.

A *projection* p is an element of A that satisfies $p^2 = p = p^*$. Suppose that $p \in A$ is a projection. Then $0 \leq \tau(p) \leq 1$ by elementary considerations. But what is the range of the map τ ? In the case $A = M_n(\mathbb{C})$ the range of τ would be the set $\{0, \frac{1}{n}, \frac{2}{n}, \dots, 1\} \subseteq \mathbb{R}$. What happens for A_λ? Stay tuned.

To summarize, we have shown that the C^*-algebra A_λ arises in three disparate arenas of mathematics. (There are others as well, but this should be enough to convince you that it happens a lot!) At this point, though, it is not at all clear to what extent the algebra is dependent upon λ. Let's find out.

The World's Fastest Introduction to K-theory

Suppose first that A is a unital C^*-algebra. There are always projections, namely 0 and 1. If X is a connected space then these are the only projections in $C(X)$. On the other hand, $M_n(\mathbb{C})$ has lots of projections: for instance, take a diagonal matrix that has only ones and zeros on the diagonal. It turns out that $C(X) \otimes M_n(\mathbb{C})$ can have very interesting projections—these correspond to vector bundles over X.

Let $P_n(A)$ denote the set of projections in $A \otimes M_n(\mathbb{C})$, and define $P_\infty(A)$ to be the union of the $P_n(A)$ (where we put P_n inside of P_{n+1} by sticking it in the upper left corner and adding zeros to the right and below). Unitary equivalence and saying that p is equivalent to $p \oplus 0$ puts a natural equivalence relation $\sim$ on $P_\infty(A)$. Then $P_\infty(A)/\sim$ has a natural direct sum operation, and we can turn it into an abelian group by doing the so-called Grothendieck construction (taking formal differences of projections). If you don't like that, take the free abelian group on the equivalence classes and then divide out by the subgroup generated by all elements of the form $[P \oplus Q] - [P] - [Q]$. This gives an abelian group denoted $K_0(A)$.

For example, take $A = \mathbb{C}$. Then $P_n(A)$ consists of all of the projections in $M_n(\mathbb{C})$. We learned in the second semester of linear algebra that every projection is unitarily equivalent to a diagonal matrix of the form $diag(1, \dots, 1, 0, \dots, 0)$. Hence, the equivalence classes of $P_n(\mathbb{C})$ are classified (via rank) by the integers $\{0, 1, 2, \dots, n\}$, and the equivalence classes of $P_\infty(\mathbb{C})$ are classified by the natural numbers $\{0, 1, 2, \dots\}$. Taking formal inverses, we obtain $K_0(\mathbb{C}) \cong \mathbb{Z}$. Note that the same answer emerges if we take $A = M_j(\mathbb{C})$ for any j, since "matrices of matrices are matrices."

For commutative unital C^*-algebras $A = C(X)$ with compact Hausdorff maximal ideal space X, the Serre-Swan theorem tells us that $K_0(C(X))$ is given by

$$K_0(C(X)) \cong K^0(X)$$

where $K^0(X)$ is the Grothendieck group generated by complex vector bundles over X.

We may regard K_0 as a functor on unital C^*-algebras and maps, since if $f : A \to A'$ is unital, then f takes projections to projections, unitaries to unitaries, and preserves direct sums. If A is not unital, then we may form its unitization A^+ (for example, $C_o(X)^+ \cong C(X^+)$ where X is locally compact but not compact and X^+ is its one-point compactification), and then define $K_0(A)$ to be the kernel of the map

$$K_0(A^+) \longrightarrow K_0(A^+/A) \cong \mathbb{Z}.$$

[4] *If you are an analyst you can think of $\mathcal{K}$ as the smallest C^*-algebra inside the space of bounded operators $\mathcal{B}(\mathcal{H})$ on a separable Hilbert space $\mathcal{H}$ that contains all of the operators with finite-dimensional range. If you are an algebraist at heart then you will be pleased to hear that $\mathcal{K}$ is the unique maximal ideal of $\mathcal{B}(\mathcal{H})$.*

Since we are generally working with algebras tensored with the compact operators $\mathcal{K}$, it is good to know that $K_0(A) \cong K_0(A \otimes \mathcal{K})$.

Note that if A is separable then there are at most countably many equivalence classes of projections, and hence $K_0(A)$ is a countable abelian group. Every countable abelian group may be realized as $K_0(A)$ for some separable C^*-algebra A.

The K-theory of the Irrational Rotation C^*-algebra: The Bad News

Now, what is the K-theory of the irrational rotation C^*-algebra? A seemingly elementary question arises first: does A_λ have any non-trivial projections? This was open for several years, and it led to decisive work by Marc Rieffel whose results, together with those of Pimsner and Voiculescu, we now describe.[5]

We are altering the historical order a bit in what follows.

If λ is irrational then Pimsner and Voiculescu (1980) showed that

$$K_0(A_\lambda) \cong \mathbb{Z} \oplus \mathbb{Z}$$

independent of λ. This is just like for $A = C(T^2)$! So using K_0 by itself we cannot distinguish the various A_λ.

Traces to the Rescue

The key to distinguishing the family of C^*-algebras $\{A_\lambda\}$ is the trace defined at the end of the Homeomorphisms of the Circle section. If A is a C^*-algebra with a normalized trace τ and p and q are orthogonal projections in A (i.e. $pq = 0$) then

$$\tau(p \oplus q) = \tau(p) + \tau(q),$$

and so the trace gives us a homomorphism

$$\tau : K_0(A) \longrightarrow \mathbb{R}$$

of abelian groups. We have remarked previously that if A is separable then $K_0(A)$ is a countable abelian group, and hence $\tau(K_0(A_\lambda))$, the image of

$$\tau : K_0(A_\lambda) \longrightarrow \mathbb{R},$$

is a countable subgroup of $\mathbb{R}$. Pimsner and Voiculescu showed that the range of the trace

$$K_0(A_\lambda) \cong \mathbb{Z} \oplus \mathbb{Z} \xrightarrow{\tau} \mathbb{R}$$

lies inside $\mathbb{Z} + \lambda\mathbb{Z}$, the subgroup of $\mathbb{R}$ generated by 1 and by λ. Rieffel showed that every element of $(\mathbb{Z} + \lambda\mathbb{Z}) \cap [0, 1]$ is the image of a projection in A_λ. Combining these results gives us this omnibus isomorphism theorem:

Theorem 1 (Rieffel 1981 [2], Pimsner and Voiculescu 1980)**.**

(1) *If λ is irrational then the image of the trace*

$$\tau : K_0(A_\lambda) \longrightarrow \mathbb{R}$$

is exactly $\mathbb{Z} + \lambda\mathbb{Z}$.

(2) *If λ and μ are irrational numbers in the interval $[0, 1/2]$ with $A_\lambda \cong A_\mu$ then $\lambda = \mu$.*

(3) *More generally, if λ and μ are irrational numbers in the interval $[0, 1/2]$ and r and s are positive integers with*

$$A_\lambda \otimes M_r(\mathbb{C}) \cong A_\mu \otimes M_s(\mathbb{C})$$

then $\lambda = \mu$ and $r = s$.

(4) *Suppose that both λ and μ are irrational. Then $A_\lambda \otimes \mathcal{K} \cong A_\mu \otimes \mathcal{K}$ if and only if λ and μ are in the same orbit of the action of $GL(2, \mathbb{Z})$ on irrational numbers by linear fractional transformations.*

So we see that the A_λ retain all of the sensitive information about the angle λ. If we think back to the origins of A_λ this seems really astonishing: *The angle of the Kronecker flow deeply affects the geometry of the foliated space.*

[5] *It is easy to show that $A_\lambda \otimes \mathcal{K}$ has projections, and those projections determine the K-theory. It is a much deeper problem to deal with A_λ itself.*

References

[1] C. C. Moore and C. Schochet, *Global Analysis on Foliated Spaces*, Second edition, Mathematical Sciences Research Institute Publications, **9**, Cambridge University Press, New York, 2006. MR2202625

[2] M. A. Rieffel, C^*-algebras associated with irrational rotations, *Pacific J. Math.* **93** (1981), 415–429. MR623572

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Author photo courtesy of Rivka Schochet.

Claude L. Schochet

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Claude (Chaim) L. Schochet mostly works at home in the tiny village of Bar Yochai, Israel looking out at Mt. Meron when he is not visiting local wineries or playing folk music on his guitar. We welcome visitors!

Some Open Problems in Asymptotic Geometric Analysis

Bo'az Klartag and Elisabeth Werner

Communicated by Christina Sormani

ABSTRACT. We describe four related open problems in asymptotic geometric analysis: the hyperplane conjecture, the isotropic constant conjecture, Sylvester's problem, and the simplex conjecture.

High-dimensional systems are frequent in mathematics and applied sciences, and understanding high-dimensional phenomena has become increasingly important. Asymptotic geometric analysis emerged as a new area that deals with exactly such phenomena. It is at the crossroads of such disciplines as functional analysis, convex geometry, and probability theory and bears connection to mathematical physics and theoretical computer science as well. The last two decades have seen tremendous growth in this area.

A major impulse for the theory is the **hyperplane conjecture or slicing problem**. Motivated by questions arising in harmonic analysis, it was first formulated by J. Bourgain and made known through the work of many people, such as K. Ball and V. Milman and A. Pajor. It asks if every centered convex body of volume 1 has a hyperplane section through the origin whose volume is greater than an absolute constant $c > 0$:

Hyperplane Conjecture. *Every centered convex body K of volume $|K| = 1$ has a hyperplane section through the origin with volume greater than an absolute constant $c > 0$, independent of dimension.*

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DOI: http://dx.doi.org/10.1090/noti1685

Let us look at some easy examples for which the hyperplane conjecture holds. The first example is the Euclidean unit ball B_2^n, normalized to have volume 1:

$$K = \frac{B_2^n}{|B_2^n|^{\frac{1}{n}}}.$$

Every hyperplane section through the origin has $(n-1)$-dimensional volume

$$\frac{|B_2^{n-1}|}{|B_2^n|^{\frac{n-1}{n}}} = \frac{\Gamma(1+\frac{n}{2})^{\frac{n-1}{n}}}{\Gamma(1+\frac{n-1}{2})},$$

which, as n approaches infinity, approaches $\sqrt{e}$.

The second example is the cube of volume 1 centered at the origin. Now every coordinate hyperplane slice has hypervolume 1, a bit worse than the ball. The cube of course is the unit ball for the l^∞ norm, just as the Euclidean ball was the unit ball for the l^2 norm.

More generally, for $1 \le p < \infty$ consider the normalized unit ball in the l^p norm:

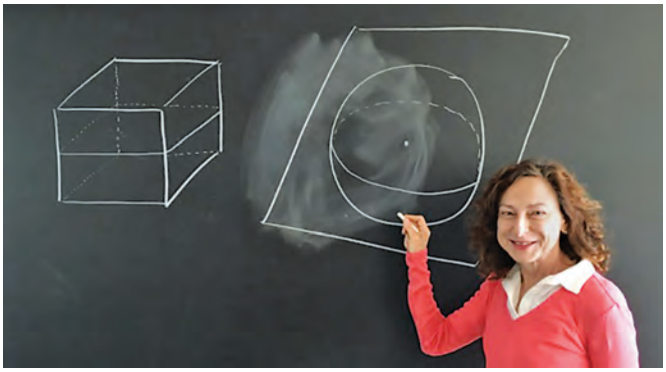
$$\|x\|_p = \left(\sum_{i=1}^n |x_i|^p\right)^{\frac{1}{p}}.$$

Then every coordinate hyperplane slice has $(n-1)$-dimensional volume

$$\frac{|B_p^{n-1}|}{|B_p^n|^{\frac{n-1}{n}}} = \frac{\Gamma(1+\frac{n}{p})^{\frac{n-1}{n}}}{\Gamma(1+\frac{n-1}{p})},$$

which approaches $e^{1/p}$. For the normalized l_p^n-unit balls, the maximal and minimal volume hyperplane sections are known. For instance, in the case of the normalized cube it was shown by K. Ball that $\sqrt{2}$ is the maximum, and by H. Hadwiger and D. Hensley that 1 is the minimum.

At the heart of the hyperplane conjecture lies the question of distribution of volume in a high-dimensional convex body. It is now understood that the convexity assumption forces most of the volume of a body to be



Elisabeth Werner shows hyperplane sections of the cube and the ball.

concentrated in some canonical way, and the main question is whether, under some natural normalization, the answer to many fundamental questions should be independent of the dimension. One such normalization, that in many cases facilitates the study of volume distribution, is the isotropic position, whose origin lies in classical mechanics of the nineteenth century. A convex body K in $\mathbb{R}^n$ is called isotropic if it has volume 1, is centered (has center of mass at the origin), and its inertia matrix is a multiple of the identity: there exists a constant $L_K > 0$ such that for all θ in the Euclidean unit sphere S^{n-1}

$$\int_K \langle x, \theta \rangle^2 dx = L_K^2.$$

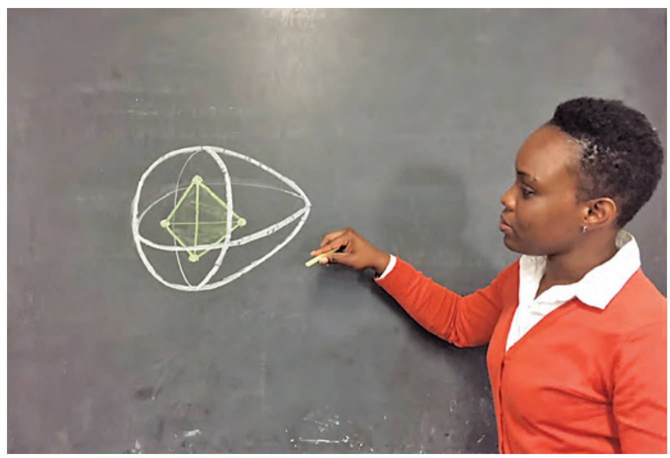
It is always possible to put a convex body in isotropic position via an non-degenerate affine transformation, and the isotropic position is unique up to orthogonal transformations. For instance, the aforementioned normalized l_p^n-unit balls are in isotopic position. The relevance of isotropic position for the hyperplane conjecture comes from the fact that if K is an isotropic convex body in $\mathbb{R}^n$, then all hyperplane sections through the origin have approximately the same volume and this volume is large enough if and only if L_K is small enough: for every θ in the Euclidean unit sphere S^{n-1} we have

$$\frac{c_1}{L_K} \leq |K \cap \theta^{\perp}| \leq \frac{c_2}{L_K}, \tag{1}$$

where c_1 and c_2 are absolute constants and $\theta^{\perp}$ is the hyperplane section through the origin orthogonal to θ. Of course, we can always put a convex body via an affine transformation in such a position that it has hyperplane sections that are arbitrarily large or arbitrarily small. The isotropic position prevents these very small or very large sections and puts investigation of the conjecture in the right setting. Thus, if we know a solution to the **isotropic constant conjecture**, which asks if there is an absolute constant C such that for all dimensions and all convex bodies K, $L_K \leq C$, then, from (1) we know the solution to the hyperplane conjecture for isotropic bodies and the general case follows by a standard argument. In fact, the two conjectures are equivalent: assume that the hyperplane conjecture has an affirmative answer. If K is isotropic, the above inequalities show that **all** sections $K \cap \theta^{\perp}$ have volume bounded by c_2/L_K from above and since $|K \cap \theta^{\perp}| \geq c$ for at least one θ, we get $L_K \leq c_2/c$.

It is known that $L_K \geq 1/\sqrt{e}$ for all convex bodies K in $\mathbb{R}^n$ and the minimum is attained for a Euclidean ball. There are recent connections to two classical problems. The first is that if among centrally-symmetric convex bodies L_K is maximized for the cube then a classical Minkowski conjecture on lattices follows, as was shown in a series of works, the most recent one by A. Magazinov. The second is that if among all convex bodies L_K is maximized for the simplex then an old conjecture by Mahler follows.

The best known upper bound to date for L_K is due to Klartag, who showed that $L_K \leq Cn^{1/4}$, removing the log factor in an earlier result by Bourgain that says $L_K \leq Cn^{1/4} \log n$. For numerous special classes of convex bodies one can do better. For instance, one knows that $L_{B_p^n}$ is bounded above for all $1 \leq p \leq \infty$ by a constant independent of dimension. More generally, the isotropy constant is bounded for unconditional convex bodies. Those are convex bodies that are symmetric about the coordinate hyperplanes. And bounds are known for many more classes of convex bodies.



Nadiesha Burgess (Lehman College master's student) illustrates Sylvester's problem, which seeks extreme cases.

Several problems are related or equivalent to the hyperplane conjecture. We only mention **Sylvester's problem** and the **simplex conjecture**. The setup is as follows: One chooses random points $x_1, \dots, x_{n+1}$ independently and uniformly distributed in an n-dimensional convex body K. Their convex hull $\text{conv}\{x_1, \dots, x_{n+1}\}$ is a random simplex contained in K.

For every $p > 0$, we consider

$$m_p(K) = \left(\frac{1}{|K|^{n+p+1}} \int_K \cdots \int_K |\text{conv}\{x_1, \dots, x_{n+1}\}|^p dx_{n+1} \dots dx_1 \right)^{1/p}.$$

Then $m_p(K)$ is invariant under nondegenerate affine transformations. Sylvester's problem asks to describe the affine classes of convex bodies for which $m_p(K)$ is minimized or maximized. It is known that for every

$p > 0$, $m_p(K) \geq m_p(B_2^n)$ with equality if and only if K is an ellipsoid. In the opposite direction the problem is open if $n \geq 3$. The quantity $m_1(K)$ is the expectation of the normalized volume of a random simplex in K. The simplex conjecture states that for every convex body K in $\mathbb{R}^n$, $m_1(K) \leq m_1(S_n)$, where S_n is a simplex in $\mathbb{R}^n$. This conjecture has been verified only in the case $n = 2$. If the simplex conjecture is correct, then the isotropic constant $L_K \leq C$ for every convex body K. So the simplex conjecture implies the hyperplane and isotropic constant conjectures.

Recent developments have led to investigations of properties of high-dimensional measures where the notion of independence has been replaced by geometric properties, such as convexity.

A good source for in-depth information, related problems, proofs, and references is the 2014 AMS Mathematical Surveys and Monographs *Geometry of Isotropic of Isotropic Convex Bodies* by S. Brazitikos, A. Giannopoulos, P. Valettas, and B. H. Vritsiou.

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Joe Harris Interview

Conducted by Laure Flapan

Communicated by Alexander Diaz-Lopez

Joe Harris is Higgins Professor of Mathematics at Harvard University. Harris' main research area is algebraic geometry. In addition to his more than 100 published research papers, Harris is a co-author of many popular textbooks, such as *Algebraic Geometry*, *3264 & All That*, and *Representation Theory*, among others.

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DOI: http://dx.doi.org/10.1090/noti1687

Flapan: *When did you know you wanted to be a mathematician?*

Harris: Around age 5 or 6—long before I had any real idea of what mathematicians do. Probably this was influenced by my parents—my father had wanted to be a mathematician, but because of quotas on the number of Jews in academia (yes, they really had those in the 30s) he pursued an MD instead.

Flapan: *Who encouraged or inspired you?*

Harris: My parents, of course, and also an older cousin, Dan Sankowski, who was a Berkeley math PhD.

Flapan: *How would you describe your research to a graduate student?*

Harris: I work on the classical side of algebraic geometry, dealing with questions about the geometry of varieties in projective space and their moduli or parameter spaces.

Flapan: *What theorem are you most proud of and what was the most important idea that led to this breakthrough?*

Harris: Probably that would be either the joint theorem with David Mumford that the moduli space of curves of large genus is of general type, or the theorem that the Severi varieties (parametrizing plane curves of given degree and genus) are irreducible, or the Brill-Noether theorem. Basically, I was just lucky to be in the right place at the right time: Grothendieck and others revolutionized the subject of algebraic geometry in the 1960s, introducing many new ideas and techniques. Those new developments made it possible to resolve a lot of outstanding open problems from the classical era, and I was fortunate to be in the first generation to grow up with these ideas.

It's important… to get a broad overview of the field you're working in.

Flapan: *According to the Math Genealogy Project, you have had 50 students and 176 total descendants. How has your involvement with so many students and young mathematicians shaped your own mathematical experience?*

Harris: It's kept me constantly aware of the need to explain what I do to other people, which has had the effect of keeping my work relatively concrete and example-driven.

Flapan: *How do you think the experience of graduate students today differs from when you first started advising students?*

I would urge students, if they feel they have a calling, to persevere.

Harris: I think students are better prepared now than when I was young—they start learning abstract math earlier, and have a pretty good idea of what mathematical research is about by the time they hit grad school.

Flapan: *Which of the books you've written is your favorite and why?*

Harris: That would probably be either *Representation Theory* with Bill Fulton or *3264 & All That* with David Eisenbud. Or maybe *Moduli of Curves* with Ian Morrison, or *Geometry of Schemes* with David Eisenbud, or *Principles of Algebraic Geometry* with Phil Griffiths… It's hard to stop.

Flapan: *All mathematicians feel discouraged occasionally. How do you deal with discouragement?*

Harris: Teach!

Flapan: *What advice do you have for current graduate students in math?*

Harris: One perennial piece of advice I give grad students is not to be in too much of a rush. In grad school, the push is always to focus on a narrow area, so you can write a thesis, but it's also important to take the time to get a broad overview of the field you're working in.

Flapan: *Any final comments or advice?*

Harris: Pursuing a career in academic mathematics is a pretty daunting prospect at present, but I would urge students, if they feel they have a calling, to persevere. We are members of a unique and wonderful community; they're unlikely to find one like it elsewhere. A good example of this is the Grothendieck revolution I mentioned earlier: Grothendieck, who was then in his 30s, came along and informed the algebraic geometers of the time that "they were doing it wrong." And, rather than burn him at the stake, they listened to what he had to say, saw that he was right, and set about rewriting the foundations of the subject—then almost 200 years old!—from scratch. I don't think that would happen in too many fields.

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Photo of Joe Harris by Anna Kreslavskaya.

Photo of Laure Flapan courtesy of Simons Center for Geometry and Physics.

Laure Flapan

ABOUT THE INTERVIEWER

Laure Flapan is a a postdoc at Northeastern University, working in algebraic geometry, particularly Hodge theory. Her email address is l.flapan@northeastern.edu.

WHAT IS...

the Langlands Program?

Solomon Friedberg

Communicated by Cesar E. Silva

The Langlands Program envisions deep links between arithmetic and analysis, and uses constructions in arithmetic to predict maps between spaces of functions on different groups. The conjectures of the Langlands Program have shaped research in number theory, representation theory, and other areas for many years, but they are very deep, and much still remains to be done.

For arithmetic, if K is a finite Galois extension of the rationals we have the Galois group $\mathrm{Gal}(K/\mathbb{Q})$ of all ring automorphisms of K fixing $\mathbb{Q}$. Frobenius suggested studying groups by embedding them into groups of matrices, so fix a homomorphism $\rho : \mathrm{Gal}(K/\mathbb{Q}) \to GL(V)$ where V is a finite dimensional complex vector space. For almost all primes p one can define a certain conjugacy class Frob_p in the Galois group. Artin introduced a function in a complex variable s built out of the values of the characteristic polynomials for $\rho(\mathrm{Frob}_p)$: $L(s,\rho) = \prod_p' \det(\mathrm{I}_V - \rho(\mathrm{Frob}_p)p^{-s})^{-1}$. Here the product is an Euler product, that is a product over the primes p; it converges for $\Re(s) > 1$. Also, there is a specific adjustment at a finite number of primes (indicated by the apostrophe in the product above) which we suppress. For example, if V is one dimensional and ρ is trivial, then $L(s,\rho)$ is exactly the Riemann zeta function. Artin made the deep conjecture that these "Artin L-functions" (which Brauer showed have meromorphic continuation to $s \in \mathbb{C}$) are entire if ρ is irreducible and nontrivial.

On the analysis side, let G be a (nice) algebraic group such as GL_n. Then one can study the space $L^2(\Gamma\backslash G(\mathbb{R}))$ consisting of complex-valued functions on $G(\mathbb{R})$ that are invariant under a large discrete subgroup Γ (such as a finite index subgroup of $GL_n(\mathbb{Z})$), transform under the center by a character, and that are square-integrable with respect to a natural group-invariant measure. The group $G(\mathbb{R})$ acts by the right regular representation, and it turns out that to each invariant subspace π (with some conditions) one can once again attach an Euler product $L(s,\pi)$ that converges for $\Re(s)$ sufficiently large, called the standard L-function of π. The space of functions π is called an automorphic representation. (One may also consider certain other functions on $\Gamma\backslash G(\mathbb{R})$ and certain quotient spaces.) The subject of automorphic forms studies the functions in π, and shows that for most π the "automorphic" L-series $L(s,\pi)$ is entire. A first example is $G = GL_1$, and there one recovers (from the adelic version) the theory of Dirichlet L-functions attached to a Dirichlet character $\chi : (\mathbb{Z}/N\mathbb{Z})^\times \to \mathbb{C}^\times$, and used by Dirichlet to prove his primes in progressions theorem.

Langlands's first insight is that Artin's conjecture should be true because Galois representations are connected to automorphic representations. More precisely, he conjectures that each Artin L-function $L(s,\rho)$ should in fact be an automorphic L-function $L(s,\pi)$, with π on $GL_{\dim V}$. If $\dim V = 1$ this assertion states that the Artin L-function is in fact a Dirichlet L-function; this is Artin's famous reciprocity law. Incidentally, the converse to this conjecture is false—there are far more analytic objects than algebraic ones.

This is already remarkable. But Langlands suggests much more. There are natural (and easy) algebraic constructions on the arithmetic side. Langlands conjectures that there should be matching (but, it seems, not easy) constructions on the analytic side.

For example, suppose V and W are two finite dimensional vector spaces and $\sigma : GL(V) \to GL(W)$ is a group homomorphism. (Concretely, W could be the symmetric nth power $\mathrm{Sym}^n(V)$ for some fixed n and σ the natural map.) Then there is a map on the Galois side given by composition: $\rho \mapsto \sigma \circ \rho$. If each of these Galois representations corresponds to an automorphic representation,

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DOI: http://dx.doi.org/10.1090/noti1686

$$\begin{tikzcd}
\mathrm{Gal}(K/\mathbb{Q}) \arrow[r, "\rho"] \arrow[dr, "\sigma\circ\rho"'] & GL(V) \arrow[r, squiggly] \arrow[d, "\sigma"] & L(s,\rho) & \text{a special case of} & L(s,\pi) & \pi \text{ on } GL_{\dim V} \arrow[l, squiggly] \arrow[d, dotted, "\text{Functoriality}"'] \\
& GL(W) \arrow[r, squiggly] & L(s,\sigma\circ\rho) & \text{a special case of} & L(s,\Pi) & \Pi \text{ on } GL_{\dim W} \arrow[l, squiggly]
\end{tikzcd}$$

Figure 1. Natural maps for Galois representations lead to conjectured functoriality maps.

$$\begin{tikzcd}
\mathrm{Gal}(K/\mathbb{Q}) \arrow[r, "\rho"] \arrow[dr, "\sigma\circ\rho"'] & H(\mathbb{C}) \arrow[r, squiggly] \arrow[d, hook, "\sigma"] & L(s,\rho) & \text{a special case of} & L(s,\pi) & \pi \text{ on dual group } G \arrow[l, squiggly] \arrow[d, dotted, "\text{Endoscopic Transfer}"'] \\
& GL(V) \arrow[r, squiggly] & {\scriptstyle L(s,\sigma\circ\rho) \atop =L(s,\rho)} & \text{a special case of} & {\scriptstyle L(s,\Pi) \atop =L(s,\pi)} & \Pi \text{ on } GL_{\dim V} \arrow[l, squiggly]
\end{tikzcd}$$

Figure 2. Endoscopic transfer: automorphic representations on different groups with the same L-function.

then there should be a map taking those spaces of functions attached to $GL_{\dim V}$ to certain spaces of functions attached to $GL_{\dim W}$. The Langlands Functoriality Conjecture asserts that there should be a full matching map on the analytic side, taking general automorphic representations π on quotients of $GL_{\dim V}(\mathbb{R})$ to automorphic representations Π on quotients of $GL_{\dim W}(\mathbb{R})$, such that the L-functions correspond. (More about this momentarily.) See Figure 1. There is no simple reason why the analysis on these two groups, each modulo a discrete subgroup, should be related; indeed, even if one can find a natural way to construct a function on $GL_{\dim W}(\mathbb{R})$ from one on $GL_{\dim V}(\mathbb{R})$, it is quite difficult to make one that is invariant under a large discrete subgroup of $GL_{\dim W}(\mathbb{R})$ and square-integrable on the quotient. The existence of this map in general would have important consequences including the generalized Ramanujan Conjecture. It is known in only a few cases.

The correspondence of L-functions is a key feature of the Langlands Functoriality Conjecture. For the conjectural map $\pi \mapsto \Pi$ described above, this means that if (for $\Re(s)$ sufficiently large)

$$L(s,\pi) = \prod_p{}' \det(\mathrm{I}_{\dim V} - A_p p^{-s})^{-1}$$

where A_p is an invertible diagonal matrix for all but finitely many primes p, then the new product

$$L(s,\pi,\sigma) := \prod_p{}' \det(\mathrm{I}_{\dim W} - \sigma(A_p) p^{-s})^{-1}$$

matches $L(s,\Pi)$ for almost all primes p. In fact, similarly to the Riemann zeta function, all automorphic L-functions should have analytic continuation to $\mathbb{C}$ and satisfy functional equations under $s \mapsto 1-s$. We know this property for $L(s,\pi)$ but not for $L(s,\pi,\sigma)$ in general (let alone that $L(s,\pi,\sigma)$ matches the standard L-function for an automorphic representation Π on $GL_{\dim W}$). The continuation of $L(s,\pi,\sigma)$ to $\mathbb{C}$ for a given symmetric power σ already carries important analytic information.

Just as Langlands predicts a map of spaces of functions that corresponds to the homomorphism σ, he also predicts maps of automorphic representations corresponding to other algebraic operations: a map that is a counterpart of the algebraic operation of tensor product, and maps corresponding to induction and restriction of Galois representations. These last maps require a tower of field extensions and base fields other than $\mathbb{Q}$, a topic that is best addressed by passing to the adeles. The automorphic induction and restriction maps for the general linear groups were constructed for cyclic extensions by Arthur and Clozel using the trace formula, following earlier work for GL_2 by Langlands and others.

So far we have only made use of general linear groups. Langlands also addresses how other algebraic groups enter the story. Suppose that the image of the Galois representation ρ is contained in a subgroup $H(\mathbb{C})$ of $GL(V)$, which is the complex points of a (nice) algebraic group H. Then Langlands conjectures that there should be an automorphic representation π on a group G, the dual group of H (the definition involves Lie theoretic data), whose L-function matches that of ρ. For example, suppose the image of ρ stabilizes a nondegenerate symplectic form on V, so after a choice of basis, ρ maps into the symplectic group $Sp_n(\mathbb{C}) \subset GL_n(\mathbb{C})$ for some even number n. Then Langlands conjectures that ρ should correspond (in the sense of matching L-functions) to an automorphic representation π on the special orthogonal group SO_{n+1}.

But wait a moment! If we simply forget that ρ maps into a subgroup $H(\mathbb{C})$ of $GL(V)$, then ρ should also correspond to an automorphic representation Π of $GL_{\dim V}$. Langlands then suggests that every automorphic representation π of G should correspond to an automorphic representation Π of $GL_{\dim V}$, with $L(s,\pi) = L(s,\Pi)$. See Figure 2. To summarize, given an automorphic representation on a symplectic or orthogonal group, there should be a corresponding automorphic representation on a general linear group with the same L-function, corresponding to the natural inclusion on the dual group side. Such maps are called endoscopic liftings (they 'see inside' the general linear group). They were established for certain classes of automorphic representations by Cogdell, Kim,

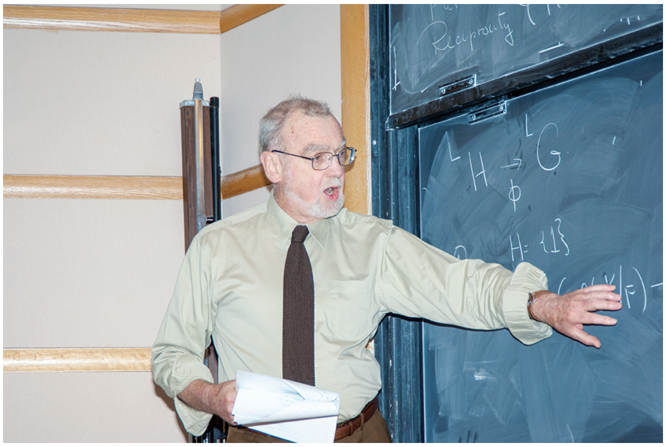

Figure 3. In the 1960s and 1970s Robert Langlands proposed a program relating arithmetic and certain spaces of functions on groups called automorphic representations.

Piatetski-Shapiro and Shahidi, and in full generality using the trace formula by Arthur.

More generally, whenever there is a homomorphism of dual groups the Langlands Functoriality Conjecture asserts that there should be a corresponding map of automorphic representations. That is, spaces of functions on the quotients of different real (or more generally, adelic) groups by discrete subgroups should be related. A further extension of functoriality replaces the dual group by the L-group, which involves the Galois group, so that the automorphicity of Artin L-functions is part of the same functoriality conjecture. Langlands is describing this in Figure 3.

The endoscopic lifting established by Arthur is a significant step in the Langlands Program. However, most of the program, including most cases of establishing functorial liftings of automorphic representations and most cases of the matching of Artin and automorphic L-functions, remains unproved. There is also a conjectured vast generalization of this matching, that every motivic L-function over a number field is an automorphic L-function (this includes Wiles's Theorem as one case), that is mostly unproved.

The Langlands Program also includes local questions, which allow one to say more about the L-functions at the finite number of primes p that were not discussed above (and where much progress has recently been announced by Scholze), and a version where a number field is replaced by the function field attached to a curve over a finite field, resolved for the general linear groups by L. Lafforgue. Another direction is the p-adic Langlands Program where one considers p-adic representations.

Langlands's vision connects arithmetic and analysis, and begins from Galois groups. We also know that Galois-like groups may be attached to coverings of Riemann surfaces. This substitution leads to the geometric Langlands Program, with connections to physics. The richness of Langlands's fundamental idea that simple natural maps for Galois representations are the shadows of maps for all automorphic representations continues to expand.

EDITOR'S NOTE. Robert Langlands in March was named winner of the 2018 Abel Prize for his program. More on Langlands is planned for the 2019 *Notices*. Meanwhile see the feature on James G. Arthur and his work on the Langlands Program in this issue.

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References

[1] J. Arthur, The principle of functoriality, *Bull. Amer. Math. Soc.* (N.S.) 40 (2003), no. 1, 39–53. MR1943132

[2] D. Bump, J. W. Cogdell, E. de Shalit, D. Gaitsgory, E. Kowalski, and S. S. Kudla, *An introduction to the Langlands program*, Birkhäuser Boston, Inc., Boston, MA, 2003. MR1990371

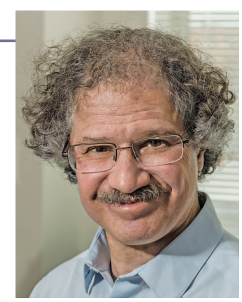
[3] E. Frenkel, *Elementary introduction to the Langlands program*, four video lectures (MSRI, 2015), available at `https://www.msri.org/workshops/804/schedules` or on YouTube.

[4] S. Gelbart, An elementary introduction to the Langlands program. *Bull. Amer. Math. Soc.* (N.S.) 10 (1984), no. 2, 177–219. MR733692

[5] The Work of Robert Langlands, papers, letters and commentary available from the Institute for Advanced Study website, `publications.ias.edu/rpl`.

ABOUT THE AUTHOR

In addition to his work in number theory and representation theory and involvement in K–12 math education, **Solomon Friedberg** enjoys outdoor activities, the arts, and foreign travel.



Solomon Friedberg

The AMS Graduate Student Blog, by and for graduate students, includes puzzles and a variety of interesting columns, such as the one from February 2018 excerpted here. `blogs.ams.org/mathgradblog`.

Introduction to Ideal Class Groups

by Tom Gannon, University of Texas at Austin

Algebraic number theory is a really interesting subject, but unlike some other subjects, it's not 100% clear what objects people study. This post provides an introduction to the class group of a finite dimensional field extension of $\mathbb{Q}$, an object often used in modern number theory....

One of the first cool facts about this is that the class group is always a finite group! This also develops the subject of class field theory, the study of Galois extensions of $\mathbb{Q}$ whose Galois groups are abelian over $\mathbb{Q}$. This can be used to prove the Kronecker-Weber theorem, which says that for any abelian extension $K/\mathbb{Q}$, there is a cyclotomic field containing K. In short – the class group of a number field is a rich object worth studying!

Photo Credit

Photo of Tom Gannon by Rachel Schlossman.

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Tom Gannon is a second-year graduate student at the University of Texas at Austin. When not doing mathematics, Tom can be found in the kitchen cooking new recipes, or can be found in the kitchen cooking old ones. His email address is `gannonth@math.utexas.edu`.

Tom Gannon

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Mathematics of Computation	**Susanne C. Brenner** (1 Feb 2012 – 31 Jan 2020)
Proceedings of the AMS	**Matthew A. Papanikolas** (1 Feb 2018 – 31 Jan 2022)
Transactions and Memoirs	**Alejandro Adem** (1 Feb 2013 – 31 Jan 2021)

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Society Governance

The American Mathematical Society has a bicameral governance structure consisting of the Council (created when the Society's constitution was ratified in December 1889) and the Board of Trustees (created when the Society was incorporated in May 1923). These bodies have the ultimate responsibility and authority for representing the AMS membership and the broader mathematical community, determining how the AMS can best serve their collective needs, and formulating and approving policies to address these needs. The governing bodies determine what the Society does and the general framework for how it utilizes its volunteer, staff, and financial resources.

The Governance Leadership consists of the Officers (President, President Elect or Immediate Past President, three Vice Presidents, Secretary, four Associate Secretaries, Treasurer, and Associate Treasurer), the Council, Executive Committee of the Council, and Board of Trustees.

The Council formulates and administers the scientific policies of the Society and acts in an advisory capacity to the Board of Trustees. Council Meetings are held twice a year (January and the spring).

The Board of Trustees receives and administers the funds of the Society, has full legal control of its investments and properties, and conducts all business affairs of the Society. The Trustees meet jointly with the Executive Committee of the Council twice a year (May and November) at ECBT Meetings.

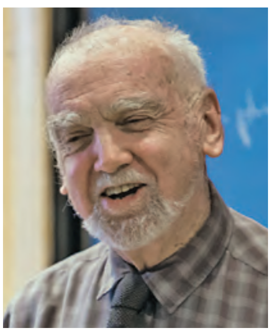
The Council and Board of Trustees are advised by nearly 100 Committees, including five Policy Committees (Education, Meetings and Conferences, Profession, Publications, and Science Policy) and over 20 Editorial Committees for the various journals and books it publishes.

The Council and Board of Trustees are also advised by the Executive Director and the Executive Staff, who are responsible for seeing that governance decisions are implemented by the Society's 210 staff members.

Learn more at `www.ams.org/about-us/governance`.

Robert Langlands Awarded Abel Prize

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2018 to Robert P. Langlands of the Institute for Advanced Study, Princeton, "for his visionary program connecting representation theory to number theory." The Abel Prize recognizes contributions of extraordinary depth and influence in the mathematical sciences and has been awarded annually since 2003. It carries a cash award of 6 million Norwegian krona (about US$715,000).



Robert Langlands

Citation

The Langlands Program predicts the existence of a tight web of connections between automorphic forms and Galois groups.

The great achievement of algebraic number theory in the first third of the twentieth century was class field theory. This theory is a vast generalization of Gauss's law of quadratic reciprocity. It provides an array of powerful tools for studying problems governed by abelian Galois groups. The non-abelian case turns out to be substantially deeper. Langlands, in a famous letter to André Weil in 1967, outlined a far-reaching program that revolutionized the understanding of this problem.

Langlands' recognition that one should relate representations of Galois groups to automorphic forms involves an unexpected and fundamental insight, now called Langlands functoriality. The key tenet of Langlands functoriality is that automorphic representations of a reductive group should be related, via L-functions, to Galois representations in a dual group.

Jacquet and Langlands were able to establish a first case of functoriality for GL(2), using the Selberg trace formula. Langlands' work on base change for GL(2) proved further cases of functoriality, which played a role in Wiles' proof of important cases of the Shimura-Taniyama-Weil conjecture.

The group GL(2) is the simplest example of a non-abelian reductive group. To proceed to the general case, Langlands saw the need for a stable trace formula, now established by Arthur. Together with Ngô's proof of the so-called Fundamental Lemma, conjectured by Langlands, this has led to the endoscopic classification of automorphic representations of classical groups, in terms of those of general linear groups.

Functoriality dramatically unifies a number of important results, including the modularity of elliptic curves and the proof of the Sato-Tate conjecture. It also lends weight to many outstanding conjectures, such as the Ramanujan-Peterson and Selberg conjectures and the Hasse-Weil conjecture for zeta functions.

Functoriality for reductive groups over number fields remains out of reach, but great progress has been achieved by the work of many experts, including the Fields medalists Drinfel'd, Lafforgue, and Ngô, all inspired by the guiding light of the Langlands Program. New facets of the theory have evolved, such as the Langlands conjectures over local fields and function fields and the geometric Langlands Program. Langlands' ideas have elevated automorphic representations to a profound role in other areas of mathematics, far beyond the wildest dreams of early pioneers such as Weyl and Harish-Chandra.

Biographical Sketch

Following is a biography written by Alexander Bellos and published on the Wolf Foundation site (`www.abelprize.no/c73016/binfil/download.php?tid=72984`).

In January 1967, Robert Langlands, a thirty-year-old associate professor at Princeton, wrote a letter to the great French mathematician André Weil, age sixty, outlining some of his new mathematical insights. "If you are willing to read it as pure speculation I would appreciate that," he wrote. "If not—I am sure you have a waste basket handy." Langlands' modesty now reads like an almost comic piece of understatement. His seventeen-page letter introduced a theory that created a whole new way of thinking about mathematics: it suggested deep links between two areas, number theory and harmonic analysis, that had previously been considered unrelated.

In fact, so radical were his insights, and so rich the mechanisms he suggested to bridge these mathematical fields, that his letter began a project, the Langlands Program, that has enlisted hundreds of the world's best mathematicians over the last fifty years. No other project

DOI: http://dx.doi.org/10.1090/noti1692

in modern mathematics has as wide a scope, has produced so many deep results, and has so many people working on it. As its depth and breadth have grown, the Langlands Program is frequently described as a grand unified theory of mathematics.

Robert Phelan Langlands was born in New Westminster, Greater Vancouver, Canada, in 1936. When he was nine, he moved to a small tourist town near the US border where his parents had a shop selling building supply materials. He had no intention of going to university until a teacher told him, in front of his classmates, that it would be a betrayal of his God-given talents.

Langlands enrolled at the University of British Columbia, aged sixteen. He completed his bachelor's degree in mathematics in 1957 and his master's degree a year later. He moved to Yale University for his doctorate, completing his PhD thesis, *Semi-Groups and Representations of Lie Groups*, in his first year there. In his second year he began to study the work of the Norwegian Atle Selberg, which later became central to his own research.

In 1960, Langlands joined Princeton University as an instructor, where he rubbed shoulders with Selberg, as well as André Weil and Harish-Chandra, all of whom were at the nearby Institute for Advanced Study. He was especially influenced by the work of Harish-Chandra on automorphic forms. Langlands was also learning other areas of mathematics, such as class field theory, an area he was nudged into by his colleague Salomon Bochner, who encouraged him to give a course in it. In 1962, Langlands was appointed a member in the Institute's School of Mathematics.

During the Christmas break of 1966, Langlands came up with the basic idea of "functoriality," a mechanism for linking ideas in number theory to those in automorphic forms. He bumped into Weil in a corridor in the beginning of January 1967 and began to explain his discovery. Weil suggested he write up his thoughts in a letter.

Langlands swiftly wrote the letter in longhand. Weil had the letter typed up, and it was widely circulated among mathematicians. Over the next few years, the letter provided many of them with a number of new, deep, and interesting problems, and, as more people joined the project to prove his conjectures, the enterprise became known as the Langlands Program. "There were some fine points that were right that rather surprise me to this day," Langlands later said about the letter. "There was evidence that these L-functions were good but that they would have these consequences for algebraic number theory was by no means certain."

Langlands spent the year 1967–1968 at the Middle East Technical University in Ankara. He speaks fluent Turkish. An enthusiastic learner of languages, he also speaks German and Russian.

Langlands returned to Yale, where he developed his twin ideas of functoriality and reciprocity and published them in "Problems in the Theory of Automorphic Forms" (1970). In 1972 he returned to Princeton as a professor at the Institute for Advanced Study, where he has been ever since.

Throughout the 1970s, Langlands continued to work on ideas within his program. In the mid-1980s, he turned his attention to percolation and conformal invariance, problems from theoretical physics. In recent years he has been looking back at ideas that he pioneered, such as one called "endoscopy".

Langlands' many honors include the first US National Academy of Sciences Award in Mathematics (1988); the Cole Prize in Number Theory (1982, with Barry Mazur); the Wolf Prize (1995–1996, with Andrew Wiles); the Steele Prize for Seminal Contribution to Research (2005); the Nemmers Prize (2006); and the Shaw Prize (2007, with Richard Taylor). He was a member of the inaugural class of AMS Fellows (2012).

AMS President Kenneth A. Ribet said, "It is my great pleasure to congratulate Professor Robert P. Langlands, winner of the 2018 Abel Prize. Robert Langlands is one of the most distinguished mathematicians alive today and a towering figure in the history of modern mathematics. His insights, which grew out of penetrating technical work early in his career, have transformed and enriched both number theory and representation theory. The deep relations between the two subjects that he predicted and probed have guided the work of countless mathematicians over the last 50 years."

Read more about Langlands' life and work, including "A Glimpse of the Laureate's Work" by Alex Bellos and "17 Handwritten Pages That Shaped a Whole Area of Mathematical Research" and "From Quadratic Reciprocity to Langlands' Program" by Arne B. Sletsjøe at `www.abelprize.no/c73016/seksjon/vis.html?tid=73017&strukt_tid=73016`.

About the Prize

The Niels Henrik Abel Memorial Fund was established in 2002 to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize is awarded by the Norwegian Academy of Science and Letters, and the choice of Abel Laureate is based on the recommendation of the Abel Committee, which consists of five internationally recognized research scientists in the field of mathematics. The Committee is appointed for a period of two years. The members of the current committee are:

- John Rognes, University of Oslo (Chair)
- Marie-France Vignéras, Institut de Mathématiques de Jussieu
- Ben J. Green, University of Oxford
- Irene Fonseca, Mellon College of Science
- Sun-Yung Alice Chang, Princeton University

Previous recipients of the Abel Prize are:

- Jean-Pierre Serre (2003)
- Michael Atiyah and I. M. Singer (2004)
- Peter Lax (2005)
- Lennart Carleson (2006)
- S. R. S. Varadhan (2007)
- John G. Thompson and Jacques Tits (2008)
- Mikhail L. Gromov (2009)
- John Tate (2010)
- John Milnor (2011)

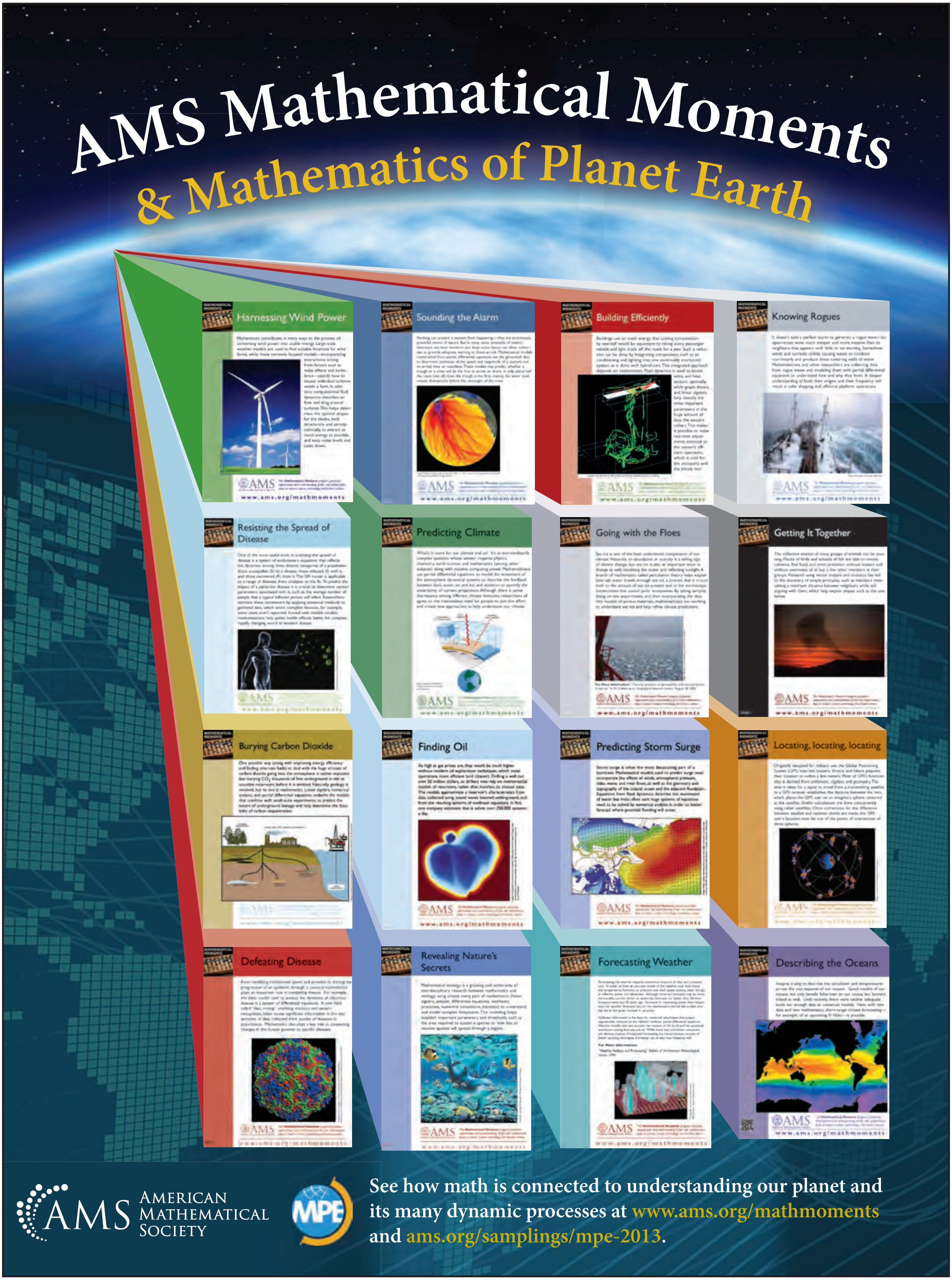
- Endre Szemerédi (2012)
- Pierre Deligne (2013)
- Yakov Sinai (2014)
- John F. Nash Jr. and Louis Nirenberg (2015)
- Andrew J. Wiles (2016)
- Yves Meyer (2017)

—From announcements of the Norwegian Academy of Science and Letters

Photo Credit

Photo of Robert Langlands courtesy of Dan Komoda/ Institute for Advanced Study.

See the WHAT IS? article on The Langlands Program in this issue, page 663.



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Finding Oil
Predicting Storm Surge
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Defeating Disease
Revealing Nature's Secrets
Forecasting Weather
Describing the Oceans
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See how math is connected to understanding our planet and its many dynamic processes at www.ams.org/mathmoments and ams.org/samplings/mpe-2013.

Call for Nominations

The selection committees for these prizes request nominations for consideration for the 2019 awards, which will be presented at the Joint Mathematics Meetings in Baltimore, MD, in January 2019. Information about past recipients of these prizes may be found at **www.ams.org/prizes-awards**.

LEVI L. CONANT PRIZE

The Levi L. Conant Prize, first awarded in January 2001, is presented annually for an outstanding expository paper published in either the *Notices* or the *Bulletin of the American Mathematical Society* during the preceding five years.

E. H. MOORE RESEARCH ARTICLE PRIZE

This prize was established in 2002 to honor E. H. Moore's extensive contributions to the discipline and to the Society. It is awarded every three years for an outstanding article published in one of the AMS primary research journals (namely, the *Journal of the AMS*, *Proceedings of the AMS*, *Transactions of the AMS*, *Memoirs of the AMS*, *Mathematics of Computation*, *Electronic Journal of Conformal Geometry and Dynamics*, and the *Electronic Journal of Representation Theory*) during the six calendar years ending a full year before the meeting at which the prize is awarded.

OSWALD VEBLEN PRIZE IN GEOMETRY

The Oswald Veblen Prize in Geometry, which was established in 1961 in honor of Professor Veblen, is awarded every three years for a notable research work in geometry or topology that has appeared in the last six years. The work must be published in a recognized, peer-reviewed venue.

AMS AMERICAN MATHEMATICAL SOCIETY

Nominations that reflect the diversity of our profession are encouraged.

NORBERT WIENER PRIZE IN APPLIED MATHEMATICS

The Norbert Wiener Prize was established in 1967 in honor of Professor Wiener and was endowed by a fund from the Department of Mathematics at the Massachusetts Institute of Technology. The prize is awarded every three years for an outstanding contribution to applied mathematics in the highest and broadest sense and is made jointly by the American Mathematical Society and the Society for Industrial and Applied Mathematics. The recipient must be a member of one of these societies.

RUTH LYTTLE SATTER PRIZE IN MATHEMATICS

The Ruth Lyttle Satter Prize is presented every two years in recognition of an outstanding contribution to mathematics research by a woman in the previous six years.

DAVID P. ROBBINS PRIZE

The David P. Robbins Prize, established in 2005 is awarded every three years for a paper with the following characteristics: it shall report on novel research in algebra, combinatorics, or discrete mathematics and shall have a significant experimental component; it shall be on a topic which is broadly accessible and shall provide both a clear statement of the problem and clear exposition of the work.

Further information about AMS prizes can be found at the Prizes and Awards website: **www.ams.org/prizes**.

Further information and instructions for submitting a nomination can be found at the prize nomination website: **www.ams.org/nominations**.

For questions contact the AMS Secretary at secretary@ams.org.

The nomination period is March 1 through June 30, 2018.

LETTERS TO THE EDITOR

Thanks for Black History Month and Women's History Month

Thank you so very much for the last two issues of the AMS *Notices* in honor of Black History Month and Women's History Month. These have been my two favorite *Notices* issues ever. These two issues not only recognized outstanding mathematicians, but they conveyed in a very powerful way that both mathematics and the AMS are for people from all backgrounds. I have shared these two editions with friends, particularly some who teach in high schools. Even personally, these two editions have made me feel even more a part of the AMS and made me very proud to be a member.

—Susan Kelly
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(Received March 9, 2018)

Admission Predictors for Success in a Mathematics Graduate Program

The process of pursuing/granting a PhD degree involves significant costs for both the individual students and the institutions; as a result, there is a need for universities to identify applicants who are most promising in their PhD program. In addition, understanding the predictors of success could potentially help institutions with the early identification of students who are encountering more challenges, and designing ways to better support them. Female and underrepresented minorities comprise an especially vulnerable population in graduate programs across the STEM fields, and in mathematics in particular. An effort is underway to understand the causes of underrepresentation and implement measures to mitigate them. In a recent study,[1] we gathered and analyzed data from about 200 students in a graduate program in mathematics at a public Tier I university in southern California. We were particularly interested in detecting any discrepancies between the success rates of different groups, which could not be explained by differences in their undergraduate record.

According to our analysis, GRE scores correlate with success, but interestingly, the verbal part of the GRE score has a higher predictive power for Math PhD, compared to the quantitative part. Further, we observe that undergraduate GPA does not correlate with success (there is even a slight negative slope in the relationship between GPA and the probability of success). This counterintuitive observation is explained once undergraduate institutions are separated by tiers: students from "higher tiers" have undergone a more rigorous training program; they on average have a slightly lower GPA but exhibit a slightly higher probability to succeed. Finally, a gender gap is observed in the probability to succeed with female students having a lower probability to finish with a PhD despite the same undergraduate performance, compared to males. This gap is reversed if we only consider foreign graduate students. It is our hope that our study will encourage other universities to perform similar analyses, in order to design better admission and retention strategies for math PhD programs.

—Timmy Ma, Karen E. Wood, Di Xu,
Patrick Guidotti, Alessandra Pantano,
and Natalia L. Komarova
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`komarova@uci.edu`

(Received March 5, 2018)

**We invite readers to submit letters to the editor at* `notices-letters@ams.org`*.*

[Ma] `arxiv.org/abs/1803.00595`

No Ancient Scottish Evidence of Fifth Platonic Solid

I read with dismay the inclusion of the oft-repeated claim in Mohammed Ghomi's article "Dürer's Unfolding Problem for Convex Polyhedra" (*Notices*, January 2018) that the Platonic solids "were already known to the ancient people of Scotland some 4,000 years ago." George Hart[1] and Lieven Le Bruyn[2] independently noted that the picture apparently showing the five Platonic solids (reproduced by Ghomi) in fact only gives four of them, with modern additions to give the impression that two near-identical objects are dual polyhedra. Thus either the dodecahedron or the icosahedron is missing from the photo, depending on one's conventions regarding what the 'knobs' correspond to. Bob Lloyd, in his study[3] on the matter, writes:

> No dodecahedral form with [20 knobs] has yet been found. There is therefore no evidence that the carvers were familiar with all five Platonic Solids. Even for the four which they did create, there is nothing to suggest that they would have thought these in any way different from the multitude of other shapes which they carved.

where the "other shapes" consist of over 400 carved stone objects with between 3 and 160 knobs. Lloyd's article was chosen to be part of the volume *The Best Writing on Mathematics 2013*, so deserves to be better-known.

—David Michael Roberts School of Mathematical Sciences, The University of Adelaide
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(Received January 31, 2018)

Roberts1 www.georgehart.com/virtual-polyhedra/neolithic.html

Roberts2 www.neverendingbooks.org/the-scottish-solids-hoax, *see also the follow-up* www.neverendingbooks.org/scottish-solids-final-comments

Roberts3 *DR Lloyd "How old are the Platonic Solids?"* BSHM Bulletin: Journal of the British Society for the History of Mathematics 27*(3) (2012)*, doi.org/10.1080/17498430.2012.670845

Native Script and Right-to-Left Languages

I enjoyed Allyn Jackson's article about Donald Knuth and native script in the January 2018 *Notices*. However, in the examples shown the right-to-left languages didn't come out correctly. At least in the ones using Hebrew and Arabic letters, the names are (almost) individually correct but are in the wrong order. For example, Elon Lindenstrauss' Hebrew name might be described as ssuartsnedniL nolE, using Hebrew characters instead of Latin ones. But it appeared as nolEssuartsnedniL with no space between the names, and Elon was misspelled in Hebrew. The correct rendering is אילון לינדנשטראוס. The same idea applies to Al-Khowārizmī's name.

—Alan Shuchat
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(Received February 23, 2018)

Women's History Month Advice Applies Also to Young Men

I'm not usually a fan of "special-interest-group in such-and-such-a-profession" promotions, but I came to appreciate your Women in Mathematics issue in March, 2018. The reason? Almost all of the "Advice to Young Women" sections could apply just as well to young men. A fact you might find it illuminating to reflect upon.

—W. F. Smyth
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(Received March 9, 2018)

Print and Electronic Versions of the *Notices*

The print version of *Notices* arrives near the end of the month, several weeks after the electronic version is available. This delay must be deliberate since it now arrives by mail much later than in the Olden Days when there were no e-versions. Is this an attempt to put the print version out of business? If the extra cost of printing and mailing is an issue then why not deliver a timely product but charge members more for the option of receiving the hard copy?.

—Erik Talvila
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(Received February 14, 2018)

EDITOR'S NOTE. We love both the print and electronic *Notices*. Each issue is posted and mailed out as soon as it is ready, about the middle of the previous month.

More References

The article "Ad Honorem Charles Fefferman" in the December 2017 issue of the *Notices* is an otherwise excellent piece, ruined by a lack of journal references for the many mentioned results and papers.

I am sure that experts in the field know precisely where to look for each of the mentioned results. But non-experts who wish to read further are stymied by the lack of full references and are reduced to guesswork to find relevant papers in the literature.

This is not the first occurrence of this problem in the *Notices*. It reflects a disturbing trend, which has been growing for at least a year. Other articles in the same issue display this problem to varying degrees.

Lack of space should not be any form of excuse. The *Notices* is and should remain fundamentally a scholarly journal, and not a disposable infotainment magazine.

—Gerard Jungman
Los Alamos, New Mexico

(Received February 1, 2018)

EDITOR'S NOTE. The current editorial guideline is for contributions to list at most five references of most interest to the casual reader. The Editor in Chief determines the content and form of *Notices* articles during their tenure.

ERRATUM. The following letter printed in the May 2018 *Notices* with two references accidentally omitted. We reprint the letter in full below and apologize to the letter writers.

Notices Reprint Omits Important Mathematical Background of 2016 Nobel Prize in Physics

We are writing about an article on the 2016 Nobel Prize in Physics, which appeared in the *Notices of the American Mathematical Society*, **64**, Number 6, 557–567, (2017). The work for which this prize was awarded has an important mathematical component. We thus applaud the decision by the editors of the *Notices* to publish an article about it. However, we are dismayed by some aspects of the presentation.

The article reprinted in the *Notices* was compiled by the "Class for Physics of the Royal Swedish Academy of Sciences"; (names of authors are not listed). It describes the groundbreaking work of F. Duncan Haldane, J. Michael Kosterlitz, and David J. Thouless on a "topological phase transition," the so-called Kosterlitz-Thouless transition, and on "topological states of matter." It was excerpted from a longer article authored by the Academy. Unfortunately, the *Notices* chose to selectively include only *nine* of *fifty-five* references in the original article. We would like to know why the editors of the *Notices* chose to eliminate so many references to important work, and why they did not include any references to mathematical results that had already strangely been missing in the original article released by the Royal Swedish Academy.

We wonder whether the article published in the *Notices* was refereed. It appears that knowledgeable mathematicians were not contacted in this matter. Although the article may be well suited to a physics audience, it neglects to mention a significant body of mathematical research closely related to the work of Haldane, Kosterlitz, and Thouless, some of which we have been involved in. We believe that the editors of the *Notices* should have consulted the mathematical physics community before publication of this article. Addition of a mathematical perspective would have enriched the article and made it more relevant for the readership of the *Notices*. It might also have inspired further mathematical research. Below, we include references to some of the important mathematical work related to the 2016 Nobel Prize that we feel are useful to a mathematical readership.

I. Papers on Spin Chains:

1) E. H. Lieb, T. D. Schultz, and D. C. Mattis, Two Soluble Models of an Antiferromagnetic Chain, *Ann. Phys.* **16**, 407 (1961).

The authors prove that the spin-1/2 chain is gapless.

2) I. Affleck, T. Kennedy, E. H. Lieb, and H. Tasaki, Rigorous Results on Valence-Bond Ground States in Antiferromagnets, *Phys. Rev. Lett.* **59**, 799 (1987).

This work is mentioned in the *Notices*, but no reference is provided.

3) T. Kennedy and H. Tasaki. Hidden $Z_2 \times Z_2$ symmetry breaking in Haldane-gap antiferromagnets, *Phys. Rev. B*, **45**, 304 (1992).

This paper is cited in the original article of the Academy.

Also, A. Polyakov's fundamental prediction of the gap in the two-dimensional classical Heisenberg model should have been mentioned in connection with Haldane's work on spin chains: A. Polyakov, *Phys. Lett.* **59B**, 79 (1975).

The results in this paper are closely related to the gap in the integer spin chain. It is cited in the original article of the Academy.

II. Papers on Phase Transitions:

4) J. Fröhlich, B. Simon, and T. Spencer, Phase Transitions and Continuous Symmetry Breaking, *Phys. Rev. Lett.* **36**, 804 (1976). Details appear in: Infrared Bounds, Phase Transitions and Continuous Symmetry Breaking, *Commun. Math. Phys.* **50**, 79 (1976).

This work contains the first proof of "infrared bounds" and applies them to prove the existence of phase transitions accompanied by continuous symmetry breaking in classical spin systems.

5) F. J. Dyson, E. H. Lieb, and B. Simon, Phase transitions in quantum spin systems with isotropic and nonisotropic interactions, *J. Stat. Phys.* **18**, 335 (1978). See also: T. Kennedy, E. H. Lieb, B. Shastry, Existence of Néel Order in Some Spin-1/2 Heisenberg Antiferromagnets, *J. Stat. Phys.* **53**, 1019 (1988).

In these papers, symmetry breaking and Néel order are established for anti-ferromagnetic quantum magnets.

6) J. Fröhlich and T. Spencer, The Kosterlitz-Thouless Transition in the Two-Dimensional Plane Rotator and Coulomb Gas, *Phys. Rev. Lett.* **46**, 1006 (1981). Details appear in: The Kosterlitz-Thouless-Transition in Two-Dimensional Abelian Spin Systems and the Coulomb Gas, *Commun. Math. Phys.* **81**, 527 (1981).

This work contains the first rigorous proof of existence of the Kosterlitz-Thouless transition; (it actually settled a controversy on this question).

A more detailed analysis of this transition appears in: P. Falco, Kosterlitz-Thouless Transition Line for the Two-Dimensional Coulomb Gas, *Commun. Math Phys.* **312**, 559–609 (2012).

III. Papers on topological states of matter:

7) D. J. Thouless, Mahito Kohmoto, MP Nightingale, and M Den Nijs, Quantized Hall conductance in a two-dimensional periodic potential, *Phys. Rev. Lett.* **49**, 405 (1982).

The relation of this work to homotopy groups of certain natural vector bundles is pointed out in: J. E. Avron, R. Seiler, and B. Simon, Homotopy and quantization in condensed matter physics, *Phys. Rev. Lett.* 51, 51 (1983).

8) B. Simon, Holonomy, the Quantum Adiabatic Theorem, and Berry's Phase, *Phys. Rev. Lett.* **51**, 2167 (1983).

This article establishes a connection between Berry's work and that of Thouless et al. quoted above. This connection allows the author to use Berry's ideas to interpret the integers of Thouless et al. in terms of eigenvalue degeneracies.

9) Jean Bellissard, Noncommutative Geometry and Quantum Hall Effect, in: *Proc. of ICM'94*, S. D. Chatterji (ed.), Basel, Boston, Berlin, Birkhäuser Verlag 1995.

J. E. Avron, R. Seiler, B. Simon, Charge deficiency, charge transport and comparison of dimensions, *Commun. Math. Phys.* **159**, 399 (1994).

Berry's phase for fermions (which has a quarternionic structure) was studied in: J. Avron, L. Sadun, J. Segert, and B. Simon, Chern numbers and Berry's phases in Fermi systems, *Commun. Math. Phys.* **124**, 595 (1989).

10) J. Fröhlich and U.M. Studer, Gauge Invariance and Current Algebra in Non-Relativistic Many-Body Theory, *Rev. Mod. Phys.* 65, 733 (1993).

To our knowledge, the "spin Hall effect" in time-reversal invariant topological insulators with chiral edge spin currents has been described in this paper for the first time.

11) J. Fröhlich et al., The Fractional Quantum Hall Effect, Chern-Simons Theory, and Integral Lattices, in: *Proc. of ICM'94,* S.D. Chatterji (ed.), Basel, Boston, Berlin: Birkhäuser Verlag 1995.

J. Fröhlich, U.M. Studer, and E. Thiran, Quantum Theory of Large Systems of Non-Relativistic Matter, in: *Proc. of Les Houches LXII, Fluctuating Geometries in Statistical Mechanics and Field Theory,* F. David, P. Ginsparg and J. Zinn-Justin (eds.), Amsterdam: Elsevier Science 1995.

In these papers (and refs. to original papers given therein), results on the Fractional Quantum Hall Effect and other phenomena related to topological states of matter are described. Topological Chern-Simons (field) theory and current algebra are applied to problems in condensed matter physics.

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(Received January 18, 2018)

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The Mechanization of Mathematics

Jeremy Avigad

Communicated by Daniel Velleman

Note: The opinions expressed here are not necessarily those of Notices.

ABSTRACT. In computer science, *formal methods* are used to specify, develop, and verify hardware and software systems. Such methods hold great promise for mathematical discovery and verification of mathematics as well.

Introduction

In 1998 Thomas Hales announced a proof of the Kepler conjecture, which states that no nonoverlapping arrangement of equal-sized spheres in space can attain a density greater than that achieved by the naive packing obtained by arranging them in nested hexagonal layers. The result relied on extensive computation to enumerate certain combinatorial configurations known as "tame graphs" and to establish hundreds of nonlinear inequalities.

He submitted the result to the *Annals of Mathematics*, which assigned a team of referees to review it. Hales found the process unsatisfying: it was more than four years before the referees began their work in earnest, and they cautioned that they did not have the resources to review the body of code and vouch for its correctness. In response, he launched an effort to develop a formal proof in which every calculation, and every inference, would be fully checked by a computer. To name the project, Hales searched for a word containing the initial letters of the words "formal," "proof," and "Kepler," and settled on "Flyspeck," which means "to scrutinize, or examine carefully." The project was completed in August of 2014.[1]

In May of 2016, three computer scientists, Marijn Heule, Oliver Kullmann, and Victor Marek, announced a solution to an open problem posed by Ronald Graham. Graham had asked whether it is possible to color the positive integers red and blue in such a way that there are no monochromatic Pythagorean triples, that is, no monochromatic triple a, b, c satisfying $a^2 + b^2 = c^2$. Heule, Kullmann, and Marek determined that it is possible to color the integers from 1 to 7,824 in such a way (see Figure 1), but that there is no coloring of the integers from 1 to 7,825 with this property. They obtained this result by designing, for each n, a propositional formula that describes a coloring of 1, . . . , n with no monochromatic triple. They then used a propositional satisfiability solver, together with heuristics tailored to the particular problem, to search for satisfying assignments for specific values of n.

For $n = 7{,}824$, the search was successful, yielding an explicit coloring of the corresponding range of integers. For the negative result, however, it is riskier to take the software's failure to find a coloring as an ironclad proof that there isn't one. Instead, Heule, Kullmann, and Marek developed an efficient format to encode a proof that the search was indeed exhaustive, providing a certificate that could be checked by independent means. The resulting

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DOI: http://dx.doi.org/10.1090/noti1688

[1] *Hales provided an engaging account of the refereeing process and the motivation behind the Flyspeck project in a talk presented to the Isaac Newton Institute in the summer of 2017,* `www.newton.ac.uk/seminar/20170710100011001`.

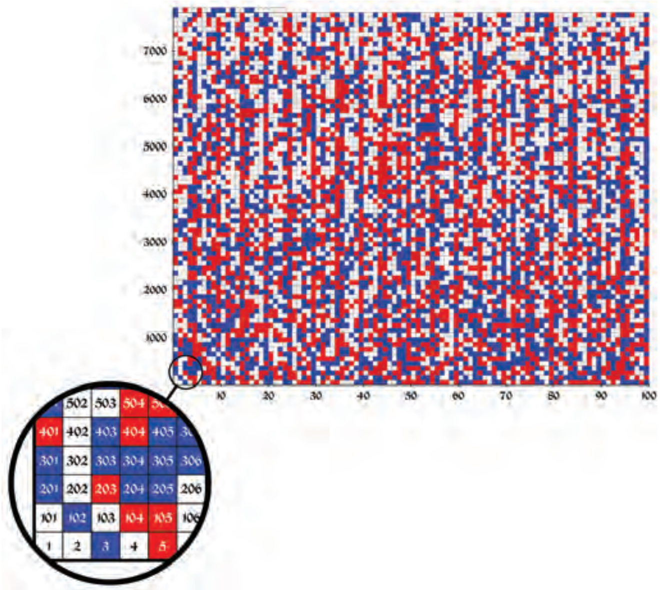

Figure 1. A family of colorings of the integers from 1 to 7,824 with no monochromatic Pythagorean triple. White squares can be colored either red or blue.

proof is 200 terabytes long, leading to popular reports in the international press of the longest proof ever found. They managed to produce a 68-gigabyte certificate with enough information for users to reproduce the proof on their own, and made it publicly available.

The use of computers in mathematics is by no means new. Numerical methods are routinely used to predict the weather, model the economy, and track climate change, as well as to make decisions and optimize outcomes in industry. Computer algebra systems like Mathematica, Maple, and Sage are widely used in applied mathematics and engineering.

By now we have even gotten used to the fact that computers can contribute to results in pure mathematics. The 1976 proof of the four color theorem by Kenneth Appel and Wolfgang Haken used the computer to check that each of a list of 1,936 maps had a required property, and to date there is no proof that can be checked by hand. In 2002, Warwick Tucker used careful calculation to show that the Lorenz attractor exists, that is, that Lorenz's original equations do indeed give rise to chaotic behavior in a precise sense. In doing so, he settled the fourteenth problem on a list of open problems prepared by Stephen Smale at the turn of the twenty-first century. In 2005 Manjul Bhargava and Jonathan Hanke used sophisticated computations to prove a conjecture by John Conway, now called the 290 theorem, which asserts that any positive definite quadratic form with integral coefficients that represents all positive integers up to 290 in fact represents all the positive integers. In 2013 Marc Lackenby and Rob Meyerhoff relied on computer assistance (as well as Perelman's proof of the geometrization conjecture) to provide a sharp bound on exceptional slopes in Thurston's Dehn surgery theorem. Other examples can be found under the Wikipedia entry for "computer-assisted proof," and in a survey by Hales [4].

But the uses of computation in the Flyspeck project and the solution to the Pythagorean triples problem have a different and less familiar character. Hales' 1998 result was a computer-assisted proof in the conventional sense, but the Flyspeck project was dedicated to *verification*, using the computer to check not only the calculations but also the pen-and-paper components of the proof, including all the background theories, down to constructions of the integers and real numbers. In the work on the Pythagorean triples problem, the computer was used to carry out a heuristic search rather than a directed computation. Moreover, in the negative case, the result of the computation was a formal proof that could be used to certify the correctness of the result.

What these two examples have in common is that they are mathematical instances of what computer scientists refer to as *formal methods*: computational methods that rely on formal logic to make mathematical assertions, specify and search for objects of interest, and verify results. In particular, both Flyspeck and the Pythagorean triples result rely crucially on formal representations of mathematical assertions and formal notions of mathematical inference and proof.

The thesis I will put forth in this article is that these two results are not isolated curiosities, but, rather, early signs of a fundamental expansion of our capacities for discovering, verifying, and communicating mathematical knowledge. The goal of this article is to provide some historical context, survey the incipient technologies, and assess their long-term prospects.

Both Flyspeck and the Pythagorean triples result rely crucially on… formal notions of mathematical inference and proof.

The Origins of Mechanized Reasoning

Computer scientists, especially those working in automated reasoning and related fields, find a patron saint in Ramon Llull, a thirteenth-century Franciscan monk from Mallorca. Llull is best known for his *Ars generalis ultima* ("ultimate general art"), a work that presents logical and visual aids designed to support reasoning that could win Muslims over to the Christian faith. For example, Llull listed sixteen of God's attributes—goodness, greatness, wisdom, perfection, eternity, and so on—and assigned a letter to each. He then designed three concentric paper circles, each of which had the corresponding letters inscribed around its border. By rotating the circles, one could form all combinations of the three letters, and thereby appreciate the multiplicity of God's attributes (see Figure 2). Other devices supported reasoning about the faculties and acts of the soul, the virtues and the vices, and so on.

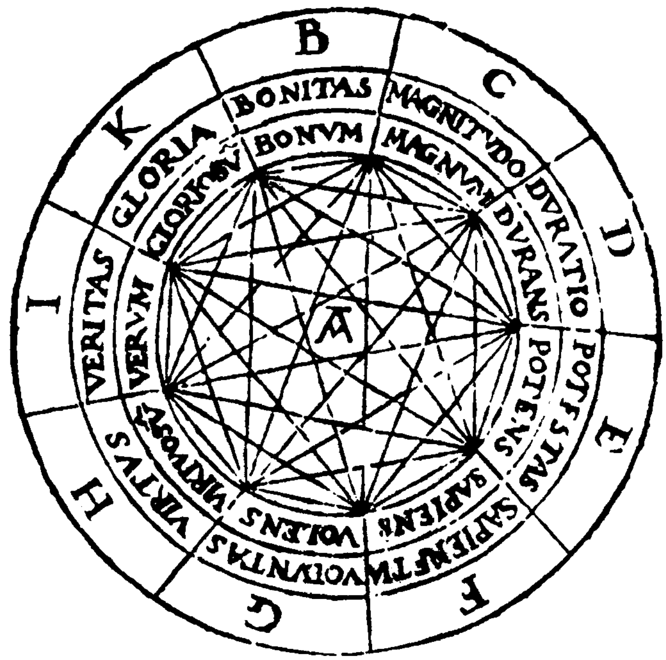

Figure 2. A thirteenth-century Franciscan monk, Ramon Llull, designed logical and visual aids to reason about the multipicity of God's attributes.

Although this work sounds quirky today, it is based on three fundamental assumptions that are now so ingrained in our thought that it is hard to appreciate their significance:

- We can represent concepts, assertions, or objects of thought with symbolic tokens.
- Compound concepts (or assertions or thoughts) can be obtained by forming combinations of more basic ones.
- Mechanical devices, even as simple as a series of concentric wheels, can be helpful in constructing and reasoning about such combinations.

Llull was influenced by an early Muslim thinker, al-Ghazali, and the first two assumptions can be found even earlier in the work of Aristotle. For example, the theory of the syllogism in *Prior Analytics* offers general arguments in which letters stand for arbitrary predicates, and Aristotle's other writings address the question of how predicates can combine to characterize or define a subject. But Llull's use of mechanical devices and procedures to support reasoning was new, and, in the eyes of many, this makes him the founder of mechanized reasoning.

> *Llull's ideas were an inspiration to Gottfried Leibniz.*

Almost 400 years later, Llull's ideas were an inspiration to Gottfried Leibniz, who, in his doctoral dissertation, dubbed the method *ars combinatoria* ("the art of combinations"). In 1666 he wrote a treatise, *Dissertatio de arte combinatoria*, which contained a mixture of logic and modern combinatorics. The unifying theme once again was a method for combining concepts and reasoning about these combinations. In this treatise, Leibniz famously proposed the development of a *characteristica universalis*, a symbolic language that could express any rational thought, and a *calculus ratiocinator*, a mechanical method for assessing its truth.

Although Leibniz made some initial progress towards this goal, his languages and calculi covered a very restricted fragment of logical inference. It is essentially the fragment we now call *propositional logic*, rediscovered by George Boole in the middle of the nineteenth century. But soon after Boole, others began to make good on Leibniz's promise of a universal language of thought, or, at least, languages that were sufficient to represent more complex assertions. Peirce, Schröder, Frege, Peano, and others expanded logical symbolism to include quantifiers and relations. In 1879 Gottlob Frege published his landmark work, *Begriffsschrift* ("concept writing"), which presented an expressive logical language together with axioms and rules of inference. In the introduction, he situated the project clearly in the Leibnizian tradition while carefully restricting its scope to scientific language and reasoning.

In the early twentieth century, the work of David Hilbert and his students and collaborators, Ernst Zermelo's axiomatization of set theory, and Bertrand Russell and Alfred North Whitehead's *Principia Mathematica* all furthered the project of using symbolic systems to provide a foundation for mathematical reasoning. The project was so successful that, in 1931, Kurt Gödel could motivate his incompleteness theorems with the following assessment:

> The development of mathematics toward greater precision has led, as is well known, to the formalization of large tracts of it, so that one can prove any theorem using nothing but a few mechanical rules. The most comprehensive formal systems that have been set up hitherto are the system of *Principia mathematica* (PM) on the one hand and the Zermelo-Fraenkel axiom system of set theory (further developed by J. von Neumann) on the other. These two systems are so comprehensive that in them all methods of proof used today in mathematics are formalized, that is, reduced to a few axioms and rules of inference. [3]

This brief historical overview will help situate the work I intend to present here. To properly bridge the gap from the beginning of the twentieth century to the present, I would have to survey not only the history of logic, foundations of mathematics, and computer science but also the history of automated reasoning and interactive theorem proving. Nothing I can do in the scope of this article would do these subjects justice, so I will now set them aside and jump abruptly to the present day.

Formal Methods in Computer Science

The phrase "formal methods" is used to describe a body of methods in computer science for specifying, developing, and verifying complex hardware and software systems. The word "formal" indicates the use of formal languages to write assertions, define objects, and specify constraints. It also indicates the use of formal semantics, that is, accounts of the meaning of a syntactic expression, which can be used to specify the desired behavior of a system or the properties of an object sought. For example, an algorithm may be expected to return a tuple of numbers satisfying a given constraint, C, expressed in some specified language, whereby the logical account spells out what it *means* for an object to satisfy the symbolically expressed constraint. Finally, the word "formal" suggests the use of formal rules of inference, which can be used to verify claims or guide a search.

Put briefly, formal methods are used in computer science to say things, find things, and check things. Using an approach known as *model checking*, an engineer describes a piece of hardware or software and specifies a property that it should satisfy. A tool like a satisfiability solver (SAT solver) or satisfiability-modulo-theories solver (SMT solver) then searches for a counterexample trace, that is, an execution path that violates the specification. The search is designed to be exhaustive so that failure to find such a trace guarantees that the specification holds. In a complementary approach known as *interactive theorem proving*, the engineer seeks to construct, with the help of the computer, a fully detailed formal proof that the artifact meets its specification.

It should not be surprising that such technologies bear on mathematical activity as well. Proving the correctness of a piece of hardware or software is an instance of proving a theorem, in this case, the theorem that states that the hardware or software, described in mathematical terms, meets its specification. Searching for bugs in hardware or software is simply an instance of searching for a mathematical object that satisfies given constraints. Moreover, claims about the behavior of hardware and software are made with respect to a body of mathematical background. For example, verifying software often depends on integer or floating point arithmetic and on properties of basic combinatorial structures. Verifying a hardware control system may invoke properties of dynamical systems, differential equations, and stochastic processes.

Of course, there is a difference in character between proving ordinary mathematical theorems and proving hardware and software correct. Verification problems in computer science are generally difficult because of the volume of detail, but they typically do not have the conceptual depth one finds in mathematical proofs. But although the focus here is on mathematics, you should keep in mind that there is no sharp line between mathematical and computational uses of formal methods, and many of the systems and tools I will describe can be used for both purposes.

Verified Proof

Interactive theorem proving involves the use of computational proof assistants to construct formal proofs of mathematical claims using the axioms and rules of a formal foundation that is implemented by the system. The user of such an assistant generally has a proof in mind and works interactively with the system to transform it into a formal derivation. Proofs are presented to the system using a specialized proof language, much like a programming language. The computational assistant processes the input, complains about the parts it cannot understand, keeps track of goals and proof obligations, and responds to queries, say, about definitions and theorems in the background libraries. Most importantly, every inference is checked for correctness using a small, trusted body of code, known as the *kernel* or *trusted computing base*. Some systems even retain, in memory, a complete description of the resulting axiomatic derivation, a complex piece of data that can be exported and verified by an independent reference checker.

The choice of axiomatic foundation varies. Some systems are based on set theory, in which every object denotes a set. Predicates are then used to pick out which sets represent objects like integers, real numbers, functions, triangles, and structures. Most systems, however, implement frameworks in which every object is assigned a *type* that indicates its intended use. For example, an object of type `int` is an integer, and an object of type `int → int` is a function from integers to integers. Such an approach often permits more convenient forms of input, since a system can use knowledge of data types to work out the meaning of a given expression. It also makes it possible for a system to catch straightforward errors, such as when a user applies a function to an object of the wrong type. The complexity of the typing system can vary, however. Some versions of type theory have a natural computational interpretation, so that the definition of a function like the factorial function on the nonnegative integers comes with a means of evaluating it.

Many core theorems of mathematics have been formalized in such systems, such as the prime number theorem, the four color theorem, the Jordan curve theorem, Gödel's first and second incompleteness theorems, Dirichlet's theorem on primes in an arithmetic progression, the Cartan fixed-point theorems, and the central limit theorem. Verifying a big name theorem is always satisfying, but a more important measure of progress lies in the mathematical libraries that support them. To date, a substantial body of definitions and theorems from undergraduate mathematics has been formalized, and there are good libraries for elementary number theory, real and complex analysis, point-set topology, measure-theoretic probability, abstract algebra, Galois theory, and so on. In November of 2008 the *Notices* devoted a special issue to the topic of interactive theorem proving, which provides an overview of the state of the field at the time (see also [1]). As a result, here I will discuss only a few landmarks that have been achieved since then.

In 2012 Georges Gonthier and thirteen co-authors announced the culmination of a six-year project that resulted in the verification of the Feit–Thompson odd order theorem. Feit and Thompson's journal publication in 1963 ran 255 pages, a length that is not shocking by today's standards but was practically unheard of at the time. The formalization was carried out in Coq, a theorem prover based on a constructive type theory using a proof language designed by Gonthier known as SSReflect. The formalization included substantial libraries for finite group theory, linear algebra, and representation theory. All told, the proof comprised roughly 150,000 lines of formal proof, including 4,000 definitions and 13,000 lemmas and theorems.

Another major landmark is the completion of the formal verification of the Kepler conjecture, described in the introduction. Most of the proof was carried out in a theorem prover known as HOL light, though one component, the enumeration of tame graphs, was carried out in Isabelle.

Yet another interesting development in the last few years stems from the realization, due to Steve Awodey and Michael Warren and, independently, Vladimir Voevodsky, that dependent type theory, the logical framework used by a number of interactive theorem provers, has a novel topological interpretation. In this interpretation, data types correspond to topological spaces or, more precisely, abstract representations of topological spaces up to homotopy. Expressions that would ordinarily be understood as functions between data types are interpreted instead as continuous maps. An expression of the form $x = y$ is interpreted as saying that there is a path between x and y, and the rules for reasoning about equality in dependent type theory correspond to a common pattern of reasoning in homotopy theory in which paths are contracted down to a base point. This opens up possibilities for using interactive theorem provers to reason about subtle topological constructions. Moreover, Voevodsky showed that one can consistently add an axiom that states, roughly, that isomorphic structures are equal, which is to say, the entire language of dependent type theory respects homotopic equivalence. The field has come to be known as *homotopy type theory*, a play on the homotopical intepretation of type theory and the theory of homotopy types.

At this stage, it may seem premature to predict that formally verified proof will become common practice. Even the most striking successes in formally verified mathematics so far have done little to alter the status quo. Hales' result was published in the *Annals of Mathematics* and widely celebrated long before the formal verification was complete, and even though the verification of the Feit–Thompson theorem turned up minor misstatements and gaps in the presentations they followed, the correctness of the theorem was not in doubt, and the repairs were routine.

But the mathematical literature is filled with errors, ranging from typographical errors, missing hypotheses, and overlooked cases to mistakes that invalidate a substantial result. In a talk delivered in 2014,[2] Vladimir Voevodsky surveyed a number of substantial errors in the literature in homotopy theory and higher category theory, including a counterexample, discovered by Carlos Simpson in 1998, to the main result of a paper he himself had published with Michal Kapronov in 1989. Voevodsky ultimately turned to formal verification because he felt that it was necessary for the level of rigor and precision the subject requires.

The situation will only get worse as proofs get longer and more complex. In a 2008 opinion piece in the *Notices*, "Desperately seeking mathematical truth" [5], Melvyn Nathanson lamented the difficulties in certifying mathematical results: "We mathematicians like to talk about the 'reliability' of our literature, but it is, in fact, unreliable." His essay was not meant to be an advertisement for formal verification, but it can easily be read that way.

The mathematical literature is filled with errors.

Checking the details of a mathematical proof is far less enjoyable than exploring new concepts and ideas, but it is important nonetheless. Rigor is essential to mathematics, and even minor errors are a nuisance to those trying to read, reconstruct, and use mathematical results. Even expository gaps are frustrating, and it would be nice if we could interactively query proofs for more detail, spelling out any inferences that are not obvious to us at first. It seems inevitable that, in the long run, formal methods will deliver such functionality.

[2] www.math.ias.edu/~vladimir/Site3/Univalent_Foundations_files/2014_IAS.pdf.

Verified Computation

When Hales submitted his proof of the Kepler conjecture to the *Annals*, a sticking point was that the mathematically trained referees were not equipped to vouch for the correctness of the code. Hales and his collaborators countered this concern by verifying these computations as well as the conventional mathematical arguments. This was not the first example of a formally verified proof that involved substantial computation: Gonthier's verification of the four color theorem in Coq was of a similar nature, relying on a simplified computational approach by Robertson, Sanders, Seymour, and Thomas.

This brings us to the subtle question as to what, exactly, it means to verify a computation. Researchers working in formal verification are very sensitive to the question as to what components of a system have to be trusted to ensure the correctness of a result. Ordinary pen-and-paper proofs are checked with respect to the axioms and rules of a foundational deductive system. In that case, the trust lies with the kernel, typically a small, carefully written body of code, as well as the soundness of the axiomatic system itself, the hardware that runs the kernel, and so on. To verify the nonlinear inequalities in the Flyspeck project, Hales and a student of his, Alexey Solovyev, reworked the algorithms so that they produce proofs as they go. Whenever a calculation depended on a fact like $12 \times 7 = 84$, the algorithm would produce a formal proof, which was then checked by the kernel. In other words, every computational claim was subjected to the same standard as a pen-and-paper proof. Checking the nonlinear inequalities involved verifying floating point calculations, and the full process required roughly 5,000 processor hours on the Microsoft Azure cloud.

Another approach to verifying computation involves describing a function in the formal foundational language of a theorem prover, proving that the description meets the desired specification, and then using an automated procedure to extract a program in a conventional programming language to compute its values. The target of the extraction procedure is often a functional programming language like ML or Haskell. This approach requires a higher degree of trust, since it requires that the extraction process preserve the semantics of the formal expression. Of course, one also has to trust the target programming language and its compiler or interpreter. Even so, the verification process imposes a much higher standard of correctness than unverified code. When writing ordinary mathematical code, it is easy to make mistakes like omitting corner cases or misjudging the properties that are maintained by an iterative loop. In the approach just described, every relevant property has to be specified, and every line of code has to be shown to meet the specifications. In the Flyspeck project, the combinatorial enumeration of tame graphs was verified in this way by Tobias Nipkow and Gertrud Bauer.

There is also a middle ground in which functions are defined algorithmically within the formal system and then executed using an evaluator that is designed for that purpose. There is then a tradeoff between the complexity of the evaluator and the reliability of the result. The verification of the four color theorem used such a strategy to evaluate the computational component of the proof.

One notable effort along these lines, by Frédéric Chyzak, Assia Mahboubi, Thomas Sibut-Pinote, and Enrico Tassi, yielded a verification of Apéry's celebrated 1973 proof of the irrationality of $\zeta(3)$. The starting point for the project was a Maple worksheet, designed by Bruno Salvy, that carried out the relevant symbolic computation. The group's strategy was to extract algebraic identities from the Maple computations and then construct formal axiomatic proofs of these identities in Coq. A fair amount of work was needed to isolate and manage side conditions that were ignored by Maple, such as showing that a symbolic expression in the denominator of a fraction is nonzero under the ambient hypotheses.

Yet another interesting project was associated with Tucker's solution to Smale's 14th problem. To demonstrate the existence of the Lorenz attractor, Tucker enclosed a Poincaré section of the flow defined by the Lorenz equations with small rectangles and showed that each rectangle (together with a cone enclosing the direction in which the attractor is expanding) is mapped by the flow inside another such rectangle (and cone). Tucker, a leading figure in the art of validated computation, relied on careful numeric computation for most of the region, coupled with a detailed analysis of the dynamics around the origin. Quite recently, Fabian Immler was able to verify the numeric computations in Isabelle. To do so, he not only formalized enough of the theory of dynamical systems to express all the relevant claims, but also defined the data structures and representations needed to carry out the computation efficiently and derived enough of their properties to show that the computation meets its specification.

Once again, on the basis of such examples, it may seem bold to predict that formally verified computation will become commonplace in mathematics. The need, however, is pressing. The increasing use of computation to establish mathematical results raises serious concerns as to their correctness, and it is interesting to see how mathematicians struggle to address this. In their 2003 paper, "New upper bounds on sphere packings. I," Cohn and Elkies provide a brief description of a search algorithm:

> To find a function g [with properties that guarantee an upper bound] … , we consider a linear combination of $g_1, g_3, \dots, g_{4m+3}$, and require it to have a root at 0 and m double roots at $z_1, \dots, z_m$. . . . We then choose the locations of $z_1, \dots, z_m$ to minimize the value r of the last sign change of g. To make this choice, we do a computer search. Specifically, we make an initial guess for the locations of $z_1, \dots, z_m$, and then see whether we can perturb them to decrease r. We repeat the perturbations until reaching a local optimum.

After presenting the bounds that constitute the main result of the paper, they write:

> These bounds were calculated using a computer. However, the mathematics behind the calculations is rigorous. In particular, we use exact rational arithmetic, and apply Sturm's theorem to count real roots and make sure we do not miss any sign changes.

The passage goes on to explain how they used approximations to real-valued calculations by rational calculations without compromising correctness of the results. In their 2013 paper "The maximal number of exceptional Dehn surgeries," Lackenby and Meyerhoff turn to the topic of computation:

> We now discuss computational issues and responses arising from our parameter space analysis. The computer code was written in C++.

They then proceed to sketch the algorithms they used to carry out the calculations described in the paper, as well as the methods for interval arithmetic, and some of the optimizations they used. They also discuss the use of Snap, a program for studying arithmetic invariants of hyperbolic 3-manifolds, which incorporates exact arithmetic based on algebraic numbers. In their preprint "Universal quadratic forms and the 290 theorem" Bhargava and Hanke are forthright in worrying about the reliability of their computations:

> As with any large computation, the possibility of error is a real issue. This is especially true when using a computer, whose operation can only be viewed intermittently and whose accuracy depends on the reliability of many layers of code beneath the view of all but the most proficient computer scientist. We have taken many steps to ensure the accuracy of our computations, the most important of which are described below.

These steps include checks for correctness, careful management of roundoff errors, and, perhaps most importantly, making the source code available on a web page maintained by the authors.

The paper by Cohn and Elkies appeared in the *Annals of Mathematics*, the one by Lackenby and Meyerhoff appeared in *Inventiones Mathematicae*, and the paper by Bhargava and Hanke will appear in *Inventiones* as well. This makes it clear that substantial uses of computation have begun to infiltrate the upper echelons of pure mathematics, and the trend is likely to continue. In the passages above, the authors are doing everything they can to address concerns about the reliability of the computations, but the mathematical community does not yet have clear standards for evaluating such results. Are referees expected to read the code and certify the behavior of each subroutine? Are they expected to run the code and, perhaps, subject it to empirical testing? Can they trust the reliability of the software libraries and packages that are invoked? Should authors be required to comment their code sufficiently well for a computer-savvy referee to review it?

Whatever means we develop to address these questions have to scale. Perhaps the bodies of code associated with the examples above are manageable, but what will happen when results rely on code that is even more complicated, and, say, ten times as long? With results like the four color theorem and Hales' theorem, we are gradually getting past the vain hope that every interesting mathematical theorem will have a humanly surveyable proof. But it seems equally futile to hope that every computational proof will make use of code that can easily be understood, and so the usual difficulties associated with understanding complicated proofs will be paired with similar difficulties in understanding complicated programs.

Formal Search

Formal verification does not have a visceral appeal to most mathematicians: the work can be painstakingly difficult, and the outcome is typically just the confirmation of a result that we had good reason to believe from the start. In that respect, the Pythagorean triple theorem of Heule, Kullmann, and Marek fares much better. Here the outcome of the effort was a new theorem of mathematics, a natural Ramsey-like result, and a very pretty one at that. The result relied on paradigmatic search techniques from the formal methods community, and it seems worthwhile to explore the extent to which such methods can be put to good mathematical work.

To date, such applications of formal methods to mathematics are few and far between. In 1996 William McCune proved the Robbins conjecture, settling a question that had been open since the 1930s as to whether a certain system of equations provided an equivalent axiomatization of Boolean algebras. The result was featured in an article by Gina Kolata in the *New York Times*. But the subject matter was squarely in the field of mathematical logic, and so it is not surprising that an automated theorem prover (in this case, one designed specifically for equational reasoning) could be used for such purposes.

Systems like McCune's can also be used to explore consequences of other first-order axioms. For example, McCune himself showed that the single equation $(w((x^{-1}w)^{-1}z))((yz)^{-1}y)=x$ axiomatizes groups in a language with a binary multiplication and a unary inverse, and Kenneth Kunen later showed that this is the shortest such axiom. Kunen went on to use interactive theorem provers to contribute notable results to the theory of nonassociative structures such as loops and quasigroups. (More examples of this sort are discussed in [2].)

Since the beginning of this century, propositional satisfiability solvers have been the killer app for formal methods, permitting algorithmic solutions to problems that were previously out of reach. On the heels of the Pythagorean triples problem, Heule has recently established that the Schur number $S(5)$ is equal to 160; in other words, there is a five-coloring of the integers from 1 to 160 with no monochromatic triple a, b, c with $a + b = c$, but no such coloring of the integers from 1 to 161.

A SAT solver had a role to play in work on the Erdös discrepancy problem. Consider a sequence $(x_i)_{i>0}$, where each x_i is $\pm$ 1, and consider sums of this sequence along multiples of a fixed positive integer, such as $x_1 + x_2 + x_3 + \ldots$ and $x_2 + x_4 + x_6 + \ldots$ and $x_3 + x_6 + x_9 + \ldots$. In the 1930s, Erdös asked whether it is possible to keep the absolute value of such sums—representing the discrepancy between the number of +1's and –1's along the sequence—uniformly bounded. In other words, he asked whether there are a sequence (x_i) and a value C such that for every n and d, $\left|\sum_{i=1}^{n} x_{id}\right| \le C$, and he conjectured that no such pair exists. In 2010 Tim Gowers launched the collaborative Polymath5 project on his blog to work on the problem. In 2014 Boris Konev and Alexei Lisitsa used a SAT solver to provide a partial result, namely, that there is no sequence satisfying the conclusion with $C = 2$. Specifically, they showed that there is a finite sequence $x_1, \ldots, x_{1,160}$ with discrepancy at most 2, but no such sequence of length 1,161. The following year, Terence Tao proved the full conjecture, with a conventional proof. This was a much more striking achievement, but we still have Konev and Lisitsa, and a SAT solver, to thank for exact bounds in the case $C = 2$. SAT solvers have been applied to other combinatorial problems as well.

The line between discovery and verification is not sharp. Anyone writing a search procedure does so with the intention that the results it produces are reliable, but, as with any piece of software, as the code becomes more complex, it becomes increasingly necessary to have mechanisms to ensure that the results are correct. This is especially true of powerful search tools, which rely on complicated tricks and heuristics to improve performance at the risk of compromising soundness. It is important that the solution to the Pythagorean triples problem produced a formal proof that could be verified independently, and, in fact, that proof has been checked by three proof checkers that themselves have been formally verified, one in Isabelle, one in Coq, and one in a theorem prover named ACL2. This provides a high degree of confidence in the correctness of the result.

Today, the use of formal methods in discovery is even less advanced than the use of formal methods in verification. The results described above depend, for the most part, on finding consequences of first-order axioms for algebraic structures, searching for finite objects satisfying combinatorial constraints, or ruling out the existence of such objects by exhaustive enumeration. It is not surprising that computers can be used to exhaust a large number of finite cases, but few mathematical problems are presented to us in that form. And spinning out consequences of algebraic axioms is a far cry from discovering consequences of rich mathematical assumptions involving heterogeneous structures and mappings between them.

But just as pure mathematicians have discovered uses for computation in number theory, algebraic topology, differential geometry, and discrete geometry, one would expect to find similarly diverse applications for formal search methods. The problem may simply be that researchers in these fields do not yet have a sense of what formal search methods can do, whereas the computer scientists who develop them do not have the expertise needed to identify the mathematical domains of application. If that is the case, it is only a matter of getting the communities to work more closely together. Combinatorics is a natural place to start, because the core concepts are easily accessible and familiar to computer scientists. But it will take real mathematical effort to understand how problems in other domains can be reduced to the task of finding finite pieces of data or ruling out the existence of such data by considering sufficiently many cases.

Indeed, for all we know, there may be lots of lovely theorems of mathematics that can *only* be proved that way. For the last two thousand years, we have been looking for proofs of a certain kind, because those are the proofs that we can survey and understand. In that respect, we may be like the drunkard looking for his keys under a streetlamp even though he lost them a block away, because that is where the light is. We should be open to the possibility that new technologies can open new mathematical vistas and afford new types of mathematical understanding. The prospect of ceding a substantial role in mathematical reasoning to the computer may be disconcerting, but it should also be exhilarating, and we should look forward to seeing where the technology takes us.

Digital Infrastructure

Contemporary digital technologies for storage, search, and communication of information provide another market for formal methods in mathematics. Mathematicians now routinely download papers, search the web for mathematical results, post questions on Math Overflow, typeset papers using LaTeX, and exchange mathematical content via email. Digital representations of mathematical knowledge are therefore central to the mathematical process. It stands to reason that mathematics can benefit from having better representations and better tools to manage them.

TeX and LaTeX have transformed mathematical dissemination and communication by providing precise means for specifying the appearance of mathematical expressions. MathML, building on XML, goes a step further, providing markup to specify the *meaning* of mathematical expressions as well. But MathML stops short of providing a foundational specification language, which is clearly desirable: imagine being able to find the statement of a theorem online, and then being able to look up the meaning of each defined term, all the way down to the primitives of an axiomatic system if necessary. That would provide clarity and uniformity, and help ensure that the results we find mean what we think they mean. The availability of such formal specifications would also support verification: we could have a shared public record of which results have been mechanically verified and how, and we could use theorems from a public repository to verify our own local results. Automated reasoning tools could make use of such background knowledge, and could, in turn, be used to support a more robust search. Contemporary *sledgehammer* tools for interactive theorem provers rely on heuristics to extract relevant theorems from a database and then use them to carry out a given inference. With such technology, one could ask whether a given statement is equivalent to, or an easy consequence of, something in a shared repository of known facts.

Figure 3. In the summer of 2017, the Isaac Newton Institute hosted a six-week workshop, *Big Proof*, dedicated to the technologies described here.

For all these purposes, formal specifications are essential. As a first step towards obtaining them, Hales has recently launched a Formal Abstracts Project, which is designed to encourage mathematicians to write formal abstracts of their papers. To process and check the definitions, he has chosen an interactive theorem prover called Lean, an open source project led by Leonardo de Moura at Microsoft Research (and to which I am a contributor). In the coming years, the Formal Abstracts project plans to seed the repository with core definitions from all branches of mathematics, and develop guidelines, tools, and infrastructure to support widespread use.

Conclusions

In the summer of 2017, the Isaac Newton Institute hosted a six-week workshop, *Big Proof*, dedicated to the technologies described here (see Figure 3).[3] As part of a panel discussion, Timothy Gowers gave a frank assessment of the new technology and the potential interest to mathematicians. He observed that the phrase "interactive proof assistant" is rather appealing until one learns that such assistants actually make proving a theorem a lot more difficult. The fact that a substantial body of undergraduate mathematics has been formalized is generally unexciting to the working mathematician, and existing tools currently offer little to improve our mathematical lives.

Gowers did enumerate three technologies that he felt would have widespread appeal. The first is a bona fide proof assistant that could work out small lemmas and results, at the level of a capable graduate student. The second is genuine search technology that can tell us whether a given fact is currently known, either because we would like to use it in a proof, or because we think we have a proof and are wondering whether it is worthwhile to work out the details. The third is a real proof checker, that is, something we can call when we think we have proved something and want confirmation that we have not made a mistake.

We are not there yet, but such technology seems to be within reach. There are no apparent conceptual hurdles that need to be overcome, though getting to that point will require a good deal of careful thought, clever engineering, experimentation, and hard work. And even before tools like these are ready for everyday use, we can hope to find pockets of mathematics where the methods provide a clear advantage: proofs that rely on nontrivial calculations, subtle arguments for which a proof assistant can provide significant validation, and problems that are more easily amenable to search techniques. Verification is not an all-or-nothing affair. Short of a fully formalized axiomatic proof, formalizing a particularly knotty or subtle lemma or verifying a key computation can lend confidence to the correctness of a result. Even just formalizing definitions and the statements of key theorems, as proposed by the Formal Abstracts project, adds helpful clarity and precision. Formal methods can also be used in education: if we teach students how to write formal proofs and informal

[3] *Talks delivered at the program are available online at* `www.newton.ac.uk/event/bpr`.

proofs at the same time, the two perspectives reinforce one another.[4]

The mathematics community needs to put some skin in the game, however. Proving theorems is not like verifying software, and computer scientists do not earn promotions or secure funding by making mathematicians happy. We need to buy into the technology if we want to reap the benefits.

To that end, institutional inertia needs to be overcome. Senior mathematicians generally do not have time to invest in developing a new technology, and it is hard enough to learn how to use the new tools, let alone contribute to their improvement. The younger generation of mathematicians has prodigious energy and computer savvy, but younger researchers would be ill-advised to invest time and effort in formal methods if it will only set back their careers. To allow them to explore the new methods, we need to give them credit for publications in journals and conferences in computer science, and recognize that the mathematical benefits will come only gradually. Ultimately, if we want to see useful technologies for mathematics, we need to hire mathematicians to develop them.

The history of mathematics is a history of doing whatever it takes to extend our cognitive reach, and designing concepts and methods that augment our capacities to understand. The computer is nothing more than a tool in that respect, but it is one that fundamentally expands the range of structures we can discover and the kinds of truths we can reliably come to know. This is as exciting a time as any in the history of mathematics, and even though we can only speculate as to what the future will bring, it should be clear that the technologies before us are well worth exploring.

References

[1] Jeremy Avigad and John Harrison, Formally verified mathematics, *Commun. ACM* **57**(4):66–75, 2014.

[2] Michael J. Beeson, The mechanization of mathematics. In *Alan Turing: Life and Legacy of a Great Thinker*, pages 77–134. Springer, Berlin, 2004. MR2172456

[3] Kurt Gödel, Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. *Monatsh. Math. Phys.*, **38**(1):173–198, 1931. Reprinted with English translation in *Kurt Gödel: Collected Works*, volume 1, Feferman et al, eds., Oxford University Press, New York, 1986, pages 144–195. MR1549910

[4] Thomas C. Hales, Mathematics in the age of the Turing machine. In *Turing's Legacy: Developments from Turing's Ideas in Logic*, volume 42 of Lecture Notes in Logic, pages 253–298. Assoc. Symbol. Logic, La Jolla, CA, 2014. MR3497663

[5] Melvyn B. Nathanson, Desperately seeking mathematical proof, *Math. Intelligencer*, **31**(2):8–10, 2009. MR2505014

ACKNOWLEDGMENT. I am grateful to Jasmin Blanchette and Robert Y. Lewis for corrections, suggestions, and improvements.

[4]*See the freely available textbook,* Logic and Proof, *by Robert Y. Lewis, Floris van Doorn, and me:* `leanprover.github.io/logic_and_proof/`.

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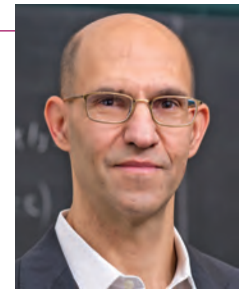
Jeremy Avigad's research interests include mathematical logic, formal verification, and the history and philosophy of mathematics.



Jeremy Avigad

A Full Fellowship?

I found that I could make deep contact only with the most serious students. Robin Forman was a mathematical whiz and in a band that did The Doors almost as well as The Doors themselves. His band played at my "Master Blasters," the master's open house I held periodically with loud music in my attempt to appear less geeky to my students. Robin bore an uncanny resemblance to my college roommate Wilfrid Schmid, Princeton valedictorian of 1964, now gone off to parts unknown."Harvard mathematics is my first choice, and I just got accepted, but with $5,000 minus tuition," Robin said, exuding disappointment, when he came to me in April for advice.

"That is an amazing coincidence," I replied. "My roommate, Wilfrid, also first in our class, applied to Harvard almost twenty years ago and was only given $5,000 minus tuition. He asked my advice, and I told him to phone the chairman of math at Harvard and tell him confidently, 'Perhaps, you don't know who I am.' Wilfrid did this and was promptly given a full fellowship at Harvard. Phone the chairman of math at Harvard and tell him, 'Perhaps you don't know who I am,'" I suggested. "I phoned the chairman of math at Harvard and said exactly those words," Robin reported back to me the next day. "Wilfrid Schmid is the chairman, and he gave me a full fellowship."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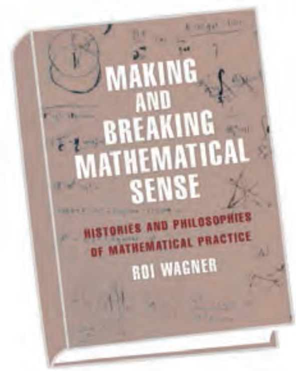
BOOK REVIEW

A Welcome Addition to the Philosophy of Mathematical Practice



by Brendan Larvor

Communicated by Daniel J. Velleman

Making and Breaking Mathematical Sense: Histories and Philosophies of Mathematical Practice
By Roi Wagner
Princeton University Press, 2017
Hardcover, 256 pages
ISBN: 9780691171715

In his celebrated *Proofs and Refutations*, Lakatos has one of his characters observe that "heuristic is concerned with language-dynamics, while logic is concerned with language-statics." The thought that as mathematics develops, its terms have their meanings stretched, distorted, and replaced, often unbeknownst to the mathematicians using them, is a main theme of Lakatos's essay. In spite of this, few philosophers in the field have picked up and elaborated Lakatos's suggestion that using mathematical terms to prove or disprove theorems changes the meanings of those terms. Most have been content with language-statics—at least until now.

Wagner's book is, among other things, a study in language-dynamics. In fact, Lakatos is not one of Wagner's principal sources, and he is mentioned only twice. Wagner's philosophical hinterland is nineteenth-century German idealism and the post-structural continental philosophy of Deleuze and Derrida.

Mention of post-structuralism may sound alarms in some readers, but one of the delightful features of this book is that it uses post-structuralist ideas without reproducing the jargon or prose-style associated with those traditions. In the first chapter, Wagner tells four different histories of the philosophy of mathematics in the twentieth century. In the second chapter he explains how terms in abbaco mathematical texts had a kind of fluidity about them and could stand for all sorts of things apart from their most obvious referents. Throughout the book, he uses accessible examples to show the dynamism and ambiguity of mathematical terms. The leading post-structuralist and post-modernist Derrida (see Figure 1) appears once, in a pair of quotations about how every meaningful remark can be quoted and can thereby lose its original context. Wagner makes this Derridean moment easy for us in two ways. First, he offers us a path to Derrida through Peirce, and second, he presents a simple example from matrix algebra to show how the same symbol can stand for either an object or an operation and on some occasions seems to stand for both. The tactical re-reading of a matrix

uses post-structuralist ideas without reproducing the jargon or prose-style

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DOI: http://dx.doi.org/10.1090/noti1690

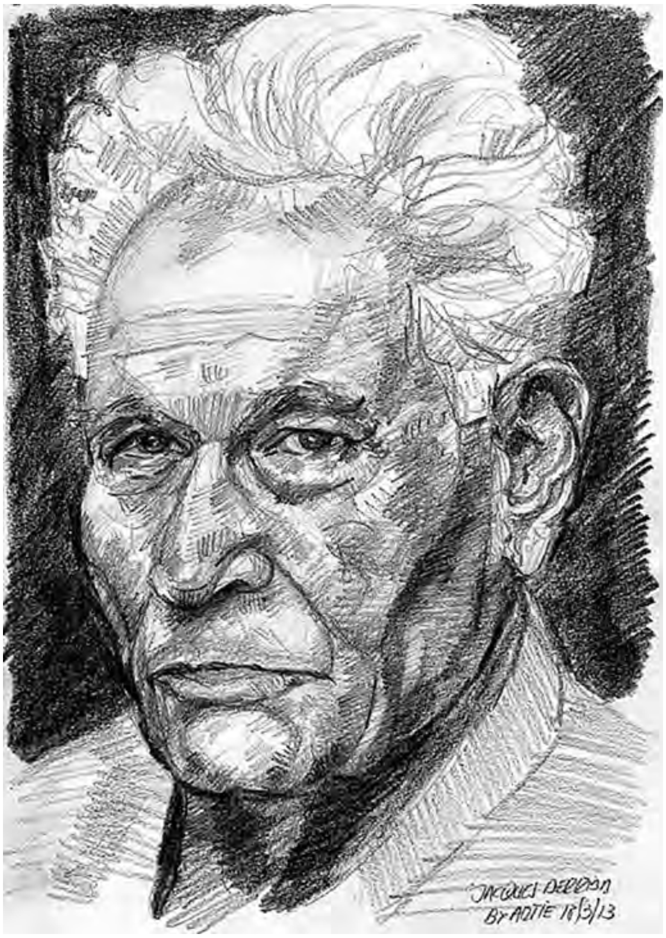

Figure 1. The leading post-structuralist and post-modernist Derrida claimed that every meaningful remark can be quoted and can thereby lose its original context.

as standing for an operation rather than for an object does not seem to be quite what either Peirce or Derrida had in mind, but this hardly matters. The story that Wagner tells about matrix algebra makes sense in its own terms, and it is interesting to see it juxtaposed with a little of the reading through which Wagner learned to think this way.

Similarly, the leading post-modernist Deleuze does not appear in person until a short section in Chapter 5, where he supplies some language and ideas for talking about the use of diagrams in mathematics from the point of view of the theory of embodied cognition. Here as elsewhere, Wagner offers readers the option of skipping the continental bits. In any case, the use of Deleuze is rather indirect—Wagner takes up a book that Deleuze wrote about the painter Francis Bacon called *The Logic of Sensation*. He picks out some of Deleuze's remarks about Bacon's working methods, translates them into a mathematical idiom, and then illustrates them with examples from Euclid. The core idea is quite simple: one of the important features of mathematical representations is that they are open to rule-governed manipulation and revision: algebraic terms can be gathered or multiplied out, lines can be added to Euclidean diagrams, axes can be changed, and so on. When these manual operations have become habitual, the body anticipates them without having to carry them out.

Overall, Derrida has four entries in the index, and Deleuze has five. Kant has twice as many, but the most often mentioned philosopher in this book is Wittgenstein. This is the Wittgenstein of the *Remarks on the Foundations of Mathematics*, who suggests that inconsistencies in mathematics need not be such a big deal so long as there is some practical way of making sure that no one uses an inconsistency to prove anything false. This is the Wittgenstein who notices that the same arithmetical statement might sometimes seem to function as an empirical statement and at other times seem more like a rule of grammar. Wagner attempts to make these two thoughts more plausible than they seem in Wittgenstein by attending more closely to real mathematical practice than Wittgenstein ever did. He cleverly includes quotations from Wittgenstein's discussions with Turing, which remind us that Turing took Wittgenstein seriously even when he was saying apparently crazy things. Turing's sceptical contributions also remind us that Wittgenstein's work was unfinished, that his observations are waiting to be explained by a larger, more perspicacious view of mathematical practice.

Semiosis (the process by which a symbol is connected to its meaning) is one large part of Wagner's argument. The other central idea is his proposal for a "constraints-based" philosophy of mathematics. Mathematics operates under various tensions: between empirical application and free creation, between unification and diversification, between isolation from and integration with other disciplines. The various options and dilemmas in mainstream philosophy of mathematics (Platonism, formalism, nominalism, structuralism, logicism, etc.) arise from trying to resolve these dilemmas once and for all by establishing a common foundation for all mathematics. This cannot work, according to Wagner, because mathematics is too diverse and above all because mathematical practice is too fluid, too semiotically slippery for any such foundational effort to do it justice. The point of a foundation is to fix things still, but living mathematics is mobile. The task of the philosopher is not to resolve the dilemmas but rather to understand the constraints that make mathematics what it is.

proposal for a "constraints-based" philosophy of mathematics

This makes it sound as if Wagner deliberately leaves all philosophical questions about mathematics unresolved. In fact, he does something surprising for an avowed post-structuralist. He says something so definite about the nature of mathematics that he comes close to positing an essence for it.

One of the characteristic features of mathematics is that mathematicians mostly agree on its results. There are disputes, but they almost always get resolved much more quickly than in other sciences, and once resolved are almost never re-opened. Why is this? Here, Wagner says something rather conventional:

> While the vast majority of mathematical everyday disputes are resolved by some sort of semi-formal shorthand,… this alone is not what allows mathematics to be much more consensual than other sciences. The consensus among mathematicians about the validity of proofs has a lot to do with formalization. By formalization I do not mean the translation of an entire proof into a strictly formal language… By formalization I mean a gradual process of piecemeal approximation of formality that is conducted only as far as required to resolve a given dispute. (p. 67)

This works, he explains, because mathematical arguments can be split up into sub-arguments, the disputed parts of which can be formalised. The formalised inferences are of a sort that can be checked by following rules that reliably give the same results regardless of which competent mathematician applies them. Partial formalisation of this sort is, according to Wagner, the final arbitrator of mathematical disputes. It is (as he observes) almost never carried out, but is nevertheless Wagner's explanation for the high level of consensus in mathematics. Indeed, it is one of two criteria that he offers for some intellectual practice to count as mathematics nowadays:

> Allowing myself to oversimplify, I might say that *the contemporary necessary and sufficient conditions of being mathematical is precisely the combination of dismotivation with respect to empirical application and potential formalization serving as highest arbitrator in disputes over the validity of arguments.* (p. 71)

When he identifies dismotivation as a defining characteristic of mathematics, what Wagner means is, roughly, that mathematicians are (as mathematicians) not terribly interested in the potential applications of their work, even if they are doing applied mathematics. More precisely, dismotivation is the process whereby a piece of mathematics floats free of the empirical enquiry that originally formed it. A partial differential equation developed to describe the flow of heat might serve to model something in economics, but the mathematician will not be interested unless this new connection brings with it new mathematics. Once the mathematics used to model some part of reality has developed to the point where it is mathematically interesting, mathematicians will work on it simply as mathematics and feel progressively less vulnerable to embarrassment if the model fails empirically. Whole numbers are useful for counting the sorts of things that can be counted, but number theorists are supremely uninterested in specifying those sorts of things.

Identifying the possibility of formalisation as his explanation of the highly consensual nature of mathematics puts Wagner in a surprisingly conservative position in the philosophy of mathematical practice. Recent work by others has started from the argument that since even partial formalisation almost never happens, something else must account for the consensus. To point to formalisation as the cause of consensus in those cases (the great majority) where it does not happen seems like magical thinking. One way Wagner might respond to this challenge is to broaden his notion of formalisation to include any procedure that can be broken down into rule-governed steps that give the same answer regardless of who carries them out. Before every phone had a calculator on it, people would settle calculation disputes by working slowly through the algorithms they learned in primary school. The rules in Euclid's plane geometry are sufficiently precisely specified that disputes could be settled in the same way, by breaking the argument into elementary steps and carefully checking that the process of proof was properly carried out. In these and similar cases, skilled practitioners can anticipate how the analysis into elementary steps would go (as Wagner notes). Perhaps what explains consensus in mathematics is not formalisation in the contemporary sense, but rather that there should be some procedures available to practitioners of mathematics that break down into elementary steps, that give consistent and reliable results and that are recognised as authoritative. Perhaps the formalised inferences of contemporary research mathematics have these features pre-eminently but not uniquely.

Be that as it may, this is the picture of mathematics that Wagner sets out in the first four chapters: mathematics as a collection of semiotically fluid practices that respond to external and internal constraints (commercial and military needs, capacities and limits of the human brain, inscription technologies, ideological and theoretical demands arising from mathematics itself and nearby disciplines, etc.) marked by dismotivation and high levels of consensus. Change and growth are explained by language-dynamics, as theorised by Derrida, while consensus and stability are explained by language-statics, made possible by formalisation.

The fifth and sixth chapters broach a new topic: the cognitive basis of mathematics. The chief business of Chapter 5 is to review the state of knowledge regarding the neural basis of elementary mathematics and to criticise the well-known theory of Lakoff and Núñez. Wagner pursues this theory in some detail, but the gist of his critique has three points. Lakoff and Núñez claim that mathematics arises in human thinking thanks to some basic metaphors. Wagner argues that this single model of a cognitive metaphor results in a very impoverished picture of the relations between various domains of mathematics. It explains how inferences can be mapped from one domain to another, but not how any other sort of transfer might take place. Second, because they have just one kind of relation (namely, cognitive metaphor), they end up constructing a rigidly hierarchical picture of mathematics that is structurally similar to the formal models of mathematics developed by foundationalist programmes in the philosophy of mathematics. No one who has absorbed the lessons of Wagner's first four chapters could find that satisfactory. Finally, Wagner suggests that the cognitive metaphor, as defined by Lakoff and Núñez, looks rather like a partial isomorphism as understood in modern, abstract mathematics. The explanation they offer for the human capacity for mathematical thinking therefore seems to run the risk of circularity.

The final chapter is interesting for philosophers curious about the history of their discipline and the genesis of Wagner's ideas. He leads us through a discussion of German idealism as found in Fichte, Schelling, and Hermann Cohen and into a brief meditation on the applicability of mathematics. The guiding thread here is the thought that the (allegedly) unreasonable applicability of mathematics arises from the fact that our world is increasingly formatted mathematically. Wagner follows this line as far as he can, but confesses in the end that it does not lead all the way to conviction.

This book is a welcome addition to the philosophy of mathematical practice, partly for the richness of its historical and contemporary mathematical examples, which this review has underplayed in favour of the more philosophical content. It is also valuable because it deploys rich stocks of philosophical theory in a sophisticated manner. This is needed in the philosophy of mathematical practice, much of which currently tends towards a kind of Baconian (the philosopher, not the painter) hope that case studies and borrowings from related empirical disciplines will spontaneously assemble themselves into a coherent account of mathematics. In Wagner's book, we find an impressive exhibition of the way in which philosophical theory can mediate between impossibly general questions (What is mathematics? How do proofs prove?) and detailed studies (How is algebra logically related to geometry in the third book by Rafael Bombelli?). It is not, of course, the last word on any of these subjects. No work of philosophy ever is.

Image Credits

Brendan Larvor

ABOUT THE REVIEWER

Brendan Larvor specialises in the history and philosophy of mathematics.

Beilinson and Drinfeld Awarded 2018 Wolf Prize in Mathematics

Alexander Beilinson and Vladimir Drinfeld of the University of Chicago have been awarded the Wolf Foundation Prize for Mathematics for 2018 by the Wolf Foundation.


Alexander Beilinson


Vladimir Drinfeld

The prize citation reads in part: "The Wolf Foundation Prize for Mathematics in 2018 will be awarded to Professors Alexander Beilinson and Vladimir Drinfeld, both of the University of Chicago, for their groundbreaking work in algebraic geometry (a field that integrates abstract algebra with geometry), in mathematical physics and in representation theory, a field which helps to understand complex algebraic structures. An 'algebraic structure' is a set of objects, including the actions that can be performed on those objects, that obey certain axioms. One of the roles of modern algebra is to research, in the most general and abstract way possible, the properties of various algebraic structures (including their objects), many of which are amazingly complicated."

Beilinson's "outstanding achievements include proofs of the Kashdan-Lustig and Jantzen conjectures, which play a key role in the representation theory, the development of important conjectures ('Beilinson's conjectures') in algebraic geometry, and a significant contribution to the interface between geometry and mathematical physics. The joint work of Beilinson and Vladimir Drinfeld on the Langlands Program—a woven fabric of theorems and conjectures designed to link key areas of mathematics—has led to impressive progress in implementing the program in important areas of physics, such as quantum field theory and string theory.

"Drinfeld has contributed greatly to various branches of pure mathematics, mainly algebraic geometry, arithmetic geometry, and the theory of representation—as well as mathematical physics. The mathematical objects named after him include the 'Drinfeld modules,' the 'Drinfeld chtoucas,' the 'Drinfeld upper half plane,' the 'Drinfeld associator'—and so many others that one of his endorsers jokingly said, 'one could think that "Drinfeld" was an adjective, not the name of a person.'

"In the seventies, Drinfeld began his work on the aforementioned 'Langlands Program,' the ambitious program that aimed at unifying the fields of mathematics. By means of a new geometrical object he developed, which is now called 'Drinfeld chtoucas,' Drinfeld succeeded in proving some of the connections that had been indicated by the Langlands Program. In the 1980s he invented the concept of algebraic 'quantum group,' which led to a profusion of developments and innovations not only in pure mathematics but also in mathematical physics (for example, in statistical mechanics).

"Drinfeld and Beilinson together created a geometric model of algebraic theory that plays a key role in both field theory and physical string theory, thereby further strengthening the connections between abstract modern mathematics and physics. In 2004 they jointly published their work in a book [*Chiral Algebras*, AMS], that describes important algebraic structures used in quantum field theory, which is the theoretical basis for the particle physics of today. This publication has since become the basic reference book on this complex subject."

Biographical Sketches

Alexander Beilinson was born in 1957 in Moscow and received his PhD in 1988 from the Landau Institute of Theoretical Physics. He was a researcher at the Landau Institute from 1987 to 1993 and professor of mathematics at the Massachusetts Institute of Technology from 1988 to 1998, when he moved to his present position at the

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DOI: http://dx.doi.org/10.1090/noti1693

University of Chicago. He was awarded the Moscow Mathematical Society Prize in 1985 and the Ostrowski Prize in 1999. He was elected to the National Academy of Sciences of the USA in 2017.

Vladimir Drinfeld, born in 1954 in Kharkiv, Ukraine, received his Candidate Sci. (Soviet equivalent of PhD) from Moscow University in 1978. From 1978 through 1980 he was assistant professor at Bashkir University, then held a number of research positions at the Institute for Low Temperature Physics in Kharkiv until accepting a professorship at the University of Chicago in 1999. He was awarded a Fields Medal in 1990. He is a member of the Ukrainian Academy of Sciences, the National Academy of Sciences of the USA, and the American Academy of Arts and Sciences.

About the Prize

The Wolf Prize carries a cash award of US$100,000. The science prizes are given annually in the areas of agriculture, chemistry, mathematics, medicine, and physics. Laureates receive their awards from the President of the State of Israel in a special ceremony at the Knesset Building (Israel's Parliament) in Jerusalem. The list of previous recipients of the Wolf Prize in Mathematics is available on the website of the Wolf Foundation:
`www.wolffund.org.il`.

—Elaine Kehoe

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Photo of Alexander Beilinson by Irene Ogievetskaya.
Photo of Vladimir Drinfeld courtesy of Vladimir Drinfeld.

Inside the AMS

AMS-AAAS Mass Media Fellow Chosen


Yen Duong

YEN DUONG, a recent graduate of the University of Illinois at Chicago, has been awarded the 2018 AMS-AAAS Mass Media Fellowship. Yen received her PhD in mathematics last year. She will work at the *Raleigh News and Observer* this summer.

The Mass Media Science and Engineering Fellows program is organized by the American Association for the Advancement of Science (AAAS). This competitive program is designed to improve public understanding of science and technology by placing advanced undergraduate, graduate, and postgraduate science, mathematics, and engineering students in media outlets nationwide. The fellows work for ten weeks over the summer as reporters, researchers, and production assistants alongside media professionals to sharpen their communication skills and increase their understanding of the editorial process by which events and ideas become news.

In its forty-fourth year, this fellowship program has placed more than 700 fellows in media organizations nationwide as they research, write, and report today's headlines. The program is designed to report science-related issues in the media in easy-to-understand ways so as to improve public understanding and appreciation for science and technology.

For more information on the AAAS Mass Media Science and Engineering Fellows program, visit the website:
`www.aaas.org/mmfellowship`.

—*Anita Benjamin*
AMS Office of Government Relations

Epsilon Awards Announced

The AMS has chosen eighteen summer mathematics programs to receive Epsilon grants for 2018. These summer programs give students a chance to see aspects of mathematics that they may not see in school and allow them to share their enthusiasm for mathematics with like-minded students.

The programs that received Epsilon grants for 2018 are:

All Girls/All Math Summer Camp, University of Nebraska, Mikil Foss, director

Baa Hózhó Math Camp, Diné College, David Auckly, director

Bridge to Enter Advanced Mathematics (BEAM), Bard College and Union College, Daniel Zaharopol, director

Canada/USA Mathcamp, Colorado School of Mines, Marisa Debowsky, director

GirlsGetMath@ICERM, Brown University, Brendan Edward Hassett, director

GirlsGetMath@Rochester, University of Rochester, Amanda M. Tucker, director

Hampshire College Summer Studies in Mathematics, Hampshire College, David C. Kelly, director

MathILy (serious Mathematics Infused with Levity), Bryn Mawr College, Sarah-Marie Belcastro, director

MathILy-Er, Bowdoin College, Jonah K. Ostroff, director

MathPath, Lewis & Clark College, Stephen B. Maurer, director

Mathworks Honors Summer Math Camp, Texas State University, Max Warshauer, director

New York Math Circle High School Summer Program, Courant Institute of Mathematical Sciences, New York University, Kovan Pillai, director

PROMYS (Program in Mathematics for Young Scientists), Boston University, Glenn Stevens, director

PROTaSM (Puerto Rico Opportunities for Talented Students in Mathematics), University of Puerto Rico, Mayaguez, Luis F. Caceres, director

Ross Mathematics Program, Ohio State University, Daniel B. Shapiro, director

Summer Institute for Math at UW (SIMUW), University of Washington, Seattle, Ron Irving, director

Summer Math Program for Young Scholars, Courant Institute of Mathematical Sciences, New York University, Selin Kalaycioglu, director

UPenn Summer Math Academy, University of Pennsylvania, Robert Strain, director

—*AMS announcement*

From the AMS Public Awareness Office

Who Wants to Be a Mathematician **winners Pravi Putalapattu (left) and Kyle Gatesman**

AMS at the USA Science and Engineering Festival (USASEF)

The AMS Public Awareness Office (PAO) organized both the AMS activity booth and the *Who Wants to Be a Mathematician* game at the USA Science and Engineering Festival in Washington, DC, in April 2018, which drew more than 350,000 people. The AMS activity was "Patterns: Parabolas and Polygons," line-drawing on patterns created by volunteer Susan Wildstrom, AMS member, donor, and high school teacher at Walt Whitman High School in Bethesda, Maryland. Children of all ages, parents, and teachers enjoyed the hands-on activity during the three-day event.

The AMS booth was staffed by Mike Breen, Annette Emerson, and Samantha Faria from the PAO; Susan Wildstrom and a rotating group of about eight of her students; Anita Benjamin, Paula Olugbemi, and interns Abby Quick and Eliot Melder from the AMS Office of Government Relations; and volunteer math enthusiast John Sadowsky from Johns Hopkins University. See a news item with video at `www.ams.org/news?news_id=4279` and download the pattern PDFs with instructions at "Making Patterns: Pushing the Envelope" at `www.ams.org/publicoutreach/curve-stitching`.

Who Wants to Be a Mathematician at USASEF

The game was held on "Sneak Peak" Friday, the day when local teachers brought their students to the festival. Kyle Gatesman, a senior at Thomas Jefferson High School for Science and Technology, won first place and US$1,000. An eighth grader, Pravi Putalapattu, finished in second place and won US$500. A math rapper, Professor Lyrical (Peter Michael Plourde from the University of the District of Columbia), entertained the audience during the game. See Kyle talk about why he likes math and more about the game at USASEF at `www.ams.org/programs/students/wwtbam/usasef2018`.

Evelyn Lamb, co-editor of the AMS Blog on Math Blogs

Lamb, the AMS–AAAS Media Fellow in 2012, has been writing for *AMS Math in the Media*, the JMM Blog, and the *AMS Blog on Math Blogs* since 2013. Some of her recent blog posts on topics found in the math "blogosphere" include "Math by the Book," "Genius Revisited," "Blind Review Review," and "Gold Medal Math." She is now moving on to be a full-time science writer, with pieces published in *Quanta Magazine*, *Scientific American*, and elsewhere. We wish her well! Find the vast archive of fascinating and thought-provoking writings by Lamb and co-editor Anna Haensch at `https://blogs.ams.org/blogonmathblogs`.

—Annette Emerson and Mike Breen
AMS Public Awareness Officers
paoffice@ams.org

An AMS Graduate Student Chapter Check-In

The AMS Graduate Student Chapter at Stony Brook University celebrates Pi Day 2018 with pies and AMS swag.

To celebrate this year's Pi Day, members of the American Mathematical Society Graduate Student Chapter at Stony Brook University celebrated with none other than a variety of pies! They tell us, "A respite from hard work was more than welcome as we all met in the mathematics [department] common room to catch up, win some AMS merchandise, and indulge our [collective] sweet tooth!"

—AMS Membership Department

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Who Wants to Be a Mathematician photo by Kate Awtrey, Atlanta Convention Photography.
Pi Day photos courtesy of AMS Graduate Student Chapter at Stony Brook University.

The CBMS Survey: A New Report

Thomas H. Barr

The latest report on the Conference Board of the Mathematical Sciences (CBMS) Survey was released in May, and it is brimming with both global and fine-grain information about undergraduate programs in the mathematical sciences throughout the US. This study has been conducted under the aegis of CBMS every five years since 1970 with support from the National Science Foundation. Written by Richelle Blair, Ellen E. Kirkman, and James W. Maxwell, the newest report *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the US* is, like its predecessors, published by the AMS. Among the many topical areas it covers are:

- Course enrollments by mathematical subject and instructional methodology
- Characteristics of bachelors programs and degree recipients
- Characteristics of course sections and instructional staff
- Faculty demographics

In addition to tracking these factors longitudinally, the new CBMS Survey examines a number of special situations, including curricula, distance learning, and dual enrollment.

A Sampling of the Findings

CBMS is one of the few national surveys that covers mathematical sciences in two-year colleges (2YCs). Data were gathered in the 2015–2016 academic year, through a survey instrument sent to a random sample of 222 2YCs selected from the 1,031 institutions in the US with two-year math programs. Approximately 54% of the sampled departments responded. Here is a small selection of findings:

- Between 2010 and 2015, 2YC enrollment in mathematics fell by 4% to 2.0 million.
- Between 2010 and 2015, enrollments in 2YC statistics courses increased by 104% to 445,000; calculus enrollments nudged up by 1% to 152,000.
- 58% of responding 2YCs implemented pathways course sequences, which include foundations, quantitative reasoning/literacy, and statistics as alternatives to many traditional pre-college courses.

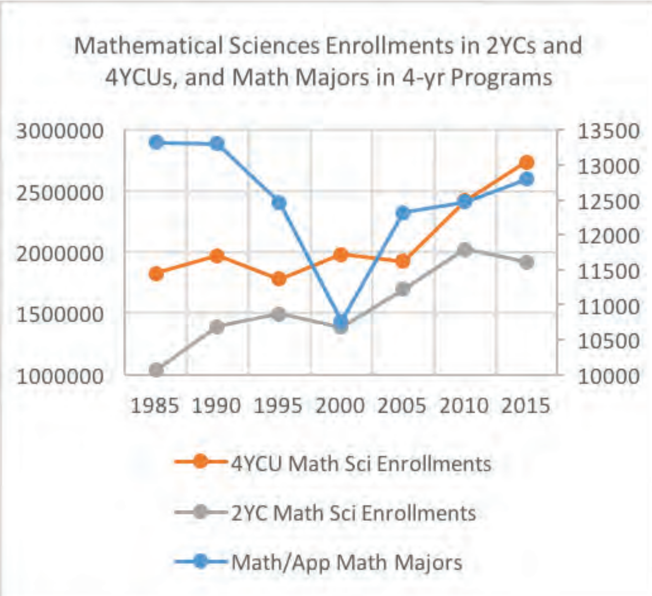
The CBMS Survey also covers mathematical sciences departments in four-year colleges and universities in the US (4YCUs). This sampling frame includes 1,470 bachelors-, masters-, and PhD-granting programs that offer degrees in mathematics, applied mathematics, statistics, mathematics education, actuarial science, computer science, and joint majors with other academic areas. These programs were stratified by degree offerings, and in academic year 2015–16, a total of 365 departments were sampled. Of these, a total of 269 responded.

Here are a few factoids regarding 4YCUs. In the study period:

- The total number of students earning bachelors degrees from all mathematical sciences departments increased by 18%.
- The total number of full-time and part-time mathematical sciences faculty increased by about 7%, much of that growth fueled by a 27% increase in the number of part-timers.
- Women comprised 31% of all full-time mathematical sciences faculty, 22% of all tenured faculty, and 36% of

Thomas H. Barr is AMS Special Projects Officer.

DOI: http://dx.doi.org/10.1090/noti1717



all tenure-eligible faculty—all figures up a percentage point or two from 2010.

A Glimpse of the Big Picture

The combination chart above illustrates one of the many perspectives that can be gleaned from the current report and the ones before it. This graphic focuses coarsely on two of the central elements of undergraduate education: students enrolled in mathematical sciences courses, and students graduating with degrees in mathematics. Over the span of thirty years, math enrollments in 2YCs have generally increased and more or less doubled. During the same period, mathematical sciences enrollments in 4YCUs increased by a factor of about 1.5. Along with these curves, the chart shows the numbers of "traditional" mathematics/applied mathematics majors graduating from 4YCUs. Except for a dip in 2000, these counts have hovered between 12,000 and 13,500 from 1985 to 2015. To set all of this in the wider higher-education context, in the same 30-year period, overall 4YCU enrollments grew by a factor of roughly 1.7 to about 20 million in 2015.

About the Survey and Looking to the Future

The full report is available in pdf form at `www.ams.org/profession/data/cbms-survey/cbms2015`, and print copies can be requested through this page. Earlier CBMS reports are also housed here. Discussions are already under way for a similar study in 2020, and suggestions or questions to the steering group can be made through this web page.

Reference

Richelle Blair, Ellen E. Kirkman, and James W. Maxwell, *Statistical Abstract of Undergraduate Programs in the Mathematical Sciences in the US*, American Mathematical Society, Providence, RI, 2018.


WORLD NEWS

Mathematics People

Tardos Named Kovalevsky Lecturer

Éva Tardos

Éva Tardos of Cornell University has been chosen as the 2018 AWM-SIAM Sonia Kovalevsky Lecturer by the Association for Women in Mathematics (AWM) and the Society for Industrial and Applied Mathematics (SIAM). She was honored for her "distinguished scientific contributions to the efficient methods for combinatorial optimization problems on graphs and networks, and her work on issues at the interface of computing and economics." According to the prize citation, she is considered "one of the leaders in defining the area of algorithmic game theory, in which algorithms are designed in the presence of self-interested agents governed by incentives and economic constraints." With Tim Roughgarden, she was awarded the 2012 Gödel Prize of the Association for Computing Machinery (ACM) for a paper "that shaped the field of algorithmic game theory." She received her PhD in 1984 from Eötvös Loránd University. She is also the recipient of a Fulkerson Prize (1988) and the George B. Dantzig Prize. She has received Packard, Sloan, and Guggenheim fellowships and is also a Fellow of the ACM and INFORMS. She was elected a Fellow of the AMS in 2013. She is a member of the National Academy of Engineering, the American Academy of Arts and Sciences, and the National Academy of Sciences and has served on the editorial boards of the *Journal of the ACM* and *Theory of Computing* and was editor in chief of the *SIAM Journal of Computing*. She will deliver the Kovalevsky Lecture at the 2018 SIAM Annual Meeting in Portland, Oregon, in July 2018.

The Sonia Kovalevsky Lectureship honors significant contributions by women to applied or computational mathematics.

—From an AWM announcement

Goldfarb and Nocedal Awarded 2017 von Neumann Theory Prize

Jorge Nocedal

Donald Goldfarb of Columbia University and Jorge Nocedal of Northwestern University have been awarded the 2017 John von Neumann Theory Prize by the Institute for Operations Research and the Management Sciences (INFORMS). According to the prize citation, "The award recognizes the seminal contributions that Donald Goldfarb and Jorge Nocedal have made to the theory and applications of nonlinear optimization over the past several decades. These contributions cover a full range of topics, going from modeling, to mathematical analysis, to breakthroughs in scientific computing. Their work on the variable metric methods (BFGS and L-BFGS, respectively) has been extremely influential."

About Goldfarb's work, the citation says, "Goldfarb's deep research contributions tie together the theoretical and the very practical in traditional linear and nonlinear programming, interior point methods, and the newly in vogue methods developed for signal processing and machine learning; and doing all that through a unique understanding of the fundamental issues in each and all of these areas. His contributions to the field are exceptionally broad, very influential and long-lasting, beginning with the famous Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm for nonlinear optimization in the 60s, then the revolutionary steepest edge simplex method for linear programming in the 80s and in the last decade first-order methods for large-scale convex optimization. The primal and dual steepest edge simplex algorithms, devised by Goldfarb with Reid and Forrest, respectively, are the most widely used variants of the simplex method. Goldfarb's work provides the theoretical foundation for many variants of this method implemented in most state-of-the-art commercial linear programming solvers. The Goldfarb-Idnani dual active set method for quadratic programming (QP) is one of the most widely used QP methods."

About Nocedal's work, the citation goes on to say, "Nocedal made seminal contributions to the area of unconstrained and constrained nonlinear optimization that have fundamentally reshaped this field. This includes the development of L-BFGS methods, extending interior point methods to non-convex constrained optimization, co-authoring a highly influential book in nonlinear optimization, and recently illuminating the interface between optimization and machine learning via efficient and effective second-order methods. In the 1980s, Nocedal invented the L-BFGS optimization algorithm, the limited memory version of the BFGS method. This opened the door to solving vastly larger unconstrained and box-constrained nonlinear optimization problems than previously possible: Nocedal's L-BFGS algorithm requires storage that is only a small multiple of the number of variables, whereas the original BFGS method required a quadratic amount of storage. The L-BFGS algorithm has had an immense practical impact, which is difficult to overstate. Nocedal was also instrumental in extending the interior-point revolution beyond convex optimization. In the late 1990s, he and his collaborators proposed the first theoretically sound algorithm for nonlinear and nonconvex optimization problems."

Donald Goldfarb received his PhD from Princeton University in 1966. He has held positions at the City College of New York, Cornell University, and the Courant Institute of Mathematical Sciences at New York University. His honors include the 1995 INFORMS Prize for Research Excellence in the Interface between Operations Research and Computer Science and the 2013 INFORMS Khachiyan Prize for Lifetime Accomplishments in Optimization. He was named a SIAM Fellow in 2012. Jorge Nocedal received his PhD from Rice University in 1978. He held positions at the National University of Mexico and the Courant Institute of Mathematical Sciences before joining Northwestern University in 1983. He was an invited speaker at the International Congress of Mathematicians in 1998, and his honors include the Charles Broyden Prize (2010) and the George B. Dantzig Prize (2012). He was named a SIAM Fellow in 2010. Nocedal tells the *Notices*: "As a teenager my brother and I built a telescope from scratch, and our desire to improve the optics led me to discover the field of computational optimization, the focus of my research since then."

—From an INFORMS announcement

Brendle and Masmoudi Awarded 2017 Fermat Prize

Simon Brendle

Nader Masmoudi

Simon Brendle of Columbia University and Nader Masmoudi of the Courant Institute of Mathematical Sciences, New York University, have been awarded the 2017 Fermat Prize. Brendle was honored "for his numerous and profound results in geometric analysis, involving partial differential equations of elliptic, parabolic and hyperbolic type; in particular for his elegant proof of Lawson's conjecture, for his characterization of soliton solutions of Ricci flows and mean curvature in dimension 3 as well as for his remarkable contributions, in collaboration with Gerhard Huisken, to the analysis of mean curvature flow of mean convex surfaces in manifolds of dimension 3." Masmoudi was recognized "for his remarkable work of depth and creativity in the analysis of nonlinear partial differential equations and in particular for his recent contributions to the rigorous and complete resolution of hydrodynamic stability problems raised at the end of the 19th century by the founding fathers of modern fluid mechanics." The prize is awarded every two years by the Institut de Mathématiques de Toulouse in fields in which Pierre de Fermat made decisive contributions.

—Jean-François Coulombel
Institut de Mathématiques de Toulouse

Oh Awarded Ho-Am Prize

Hee Oh

Hee Oh of Yale University has been awarded the 2018 Ho-Am Prize. According to the prize citation, she is "a world-leading expert in homogeneous dynamics, discrete subgroups of Lie groups, and its applications to geometry and number theory. A rare mathematician with expertise in all of ergodic, algebraic and analytic aspects of discrete subgroups of Lie groups, she combined them to develop innovative methods to solve many long-standing problems in geometry and number theory.

"One of her major achievements is the beautiful work on Apollonian circle packings, which originated from a theorem proven by the ancient Greek geometer Apollonius that, given any three tangent circles in the plane, there are exactly two more circles that are tangent to all three."

Among her other contributions are work on orbits of thin groups arising in Diophantine problems, on equidistribution of rational solutions of Diophantine equations, on geometric analogues of the prime number theorem, and on geodesic planes in hyperbolic 3-manifolds.

Oh received her PhD in 1997 from Yale University. She has held positions at Princeton University, the Institute for Advanced Study, the California Institute of Technology, and Brown University, and since 2013 she has been Abraham Robinson Professor at Yale. She is an inaugural Fellow of the AMS.

Oh says: "During my first math class in college, the professor made a rather surprising remark that 'mathematics is beautiful.' In retrospect my journey in mathematics was driven by curiosity about that statement and a desire to understand it. As it turned out, it was a journey of appreciation and confirmation of the beauty of mathematics."

The Ho-Am Prize is presented each year to individuals who have contributed to academics, the arts, and social development, or who have furthered the welfare of humanity through distinguished accomplishments in their respective professional fields.

—From a Ho-Am Foundation announcement

Logunov and Sawin Named Clay Research Fellows

Aleksandr Logunov

Will Sawin

Aleksandr Logunov of the Institute for Advanced Study and Will Sawin of ETH Zürich have been awarded Clay Research Fellowships by the Clay Mathematics Institute (CMI).

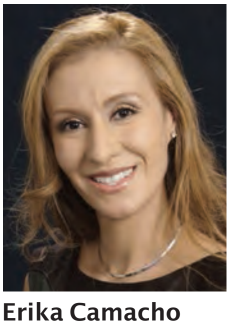
Aleksandr Logunov gained his PhD in 2015 under the supervision of Viktor Havin at the Chebyshev Laboratory, St. Petersburg State University. After two years as a postdoctoral fellow at Tel-Aviv University, he moved last year to the Institute for Advanced Study at Princeton. He will hold his Clay Research Fellowship at Princeton University. Together with Eugenia Malinnikova, he received a Clay Research Award in 2017. The award recognized Logunov's and Malinnikova's introduction of a novel geometric combinatorial method to study doubling properties of solutions to elliptic eigenvalue problems. This led to the solution of long-standing problems in spectral geometry, for instance, the optimal lower bound on the measure of the nodal set of an eigenfunction of the Laplace-Beltrami operator in a compact smooth manifold (Yau and Nadirashvili's conjectures). Logunov has been invited to speak on his work at the 2018 International Congress of Mathematicians in Rio. He has been appointed as a Clay Research Fellow for a term of two years beginning July 2018.

Will Sawin obtained his PhD in 2016 from Princeton University, under the supervision of Nicholas Katz. Since then he has worked with Emmanuel Kowalski as a Junior Fellow at ETH Zürich. Sawin's research is wide ranging but focused on the interactions of analytic number theory and algebraic geometry. Among the many areas in which he has made groundbreaking contributions are the application of étale cohomology to estimates of exponential sums over finite fields and, with Tim Browning, the adaptation of classical counting arguments in analytic number theory to explore compactly supported cohomology in spaces of interest in algebraic geometry. In a recent paper with Kowalski and Philippe Michel, he used l-adic cohomology to derive new bounds on certain bilinear forms that regularly arise in the study of automorphic forms. There are important applications, for example in the theory of twisted L-functions. He has also made many wider contributions to the mathematical community, not least through regular posts on diverse topics on the MathOverflow website. Sawin has been appointed as a Clay Research Fellow for a term of three years beginning July 2018.

Clay Research Fellowships are awarded on the basis of the exceptional quality of candidates' research and their promise to become mathematical leaders.

—From a CMI announcement

Camacho Receives Outstanding Latino/a Faculty Award



Erika Camacho

Erika Camacho of Arizona State University has been selected the recipient of the Outstanding Latino/a Faculty in Higher Education: Research/Teaching Award presented by the American Association of Hispanics in Higher Education (AAHHE). The selection criteria focused on demonstrated excellence in both research and teaching and significant contributions to the awardee's academic discipline.

—From an AAHHE announcement

Prizes of the Canadian Mathematical Society


Gordon Slade

Gordon Slade of the University of British Columbia has been awarded the Jeffery-Williams Prize for Research Excellence for his "outstanding work in rigorous statistical mechanics." He received his PhD from the University of British Columbia in 1984 and has been a member of the faculty there since 1999. He is a Fellow of the AMS, the Institute of Mathematical Statistics, the Fields Institute for Research in Mathematical Sciences, the Royal Society of Canada, and the Royal Society of London. He is the recipient of the Prize of the Institut Henri Poincaré, the CRM-Fields-PIMS Prize, and the 1995 CMS Coxeter-James Prize.

Slade tells the *Notices*: "I was raised in Toronto by parents who, due to the Great Depression, did not have the opportunity to finish high school. They understood the value of education, and did all they could to help me pursue mine. Sadly, they did not live long enough to see me graduate from university. Their encouragement is something I carry with me to this day."


Megumi Harada

Megumi Harada of McMaster University has been awarded the Krieger-Nelson Prize "for her research on Newton-Okounkov bodies, Hessenberg varieties, and their relationships to symplectic geometry, combinatorics, and equivariant topology, among others." She received her PhD from the University of California Berkeley in 2003. She was appointed a postdoc research fellow (academic) at the University of Toronto from 2003 to 2006, when she joined the faculty at McMaster University. She was awarded the Ruth I. Michler Memorial Prize of the Association for Women in Mathematics in 2013. She currently holds the Canada Research Chair in Equivariant Symplectic and Algebraic Geometry at McMaster. The prize recognizes outstanding research by a woman mathematician.

Harada tells the *Notices*: "I came to mathematics relatively late. I excelled in literature, history, and philosophy in high school, and was only passable in mathematics. When I entered college, I had serious intentions to become a cultural anthropologist. In fact, it was only a few weeks before my college graduation date that I finally, formally, declared mathematics to be my (sole) major."

Gary MacGillivray of the University of Victoria was awarded the CMS Excellence in Teaching Award. According to the prize citation, "MacGillivray's boundless energy, his love of teaching, his strong commitment and dedication to the success of his students have earned him the respect of his colleagues. His colleagues describe him as a great and effective teacher...truly involved in his community and by his students as master of explaining hard topics...a very approachable professor." He received his PhD in 1990 from Simon Fraser University. He held positions at Capilano College and the University of Regina before joining the faculty at the University of Victoria in 1992. Over the course of his career, he has supervised 48 undergraduate research projects and 39 graduate students or postdocs. He has written more than 100 papers, more than half of which are collaborations with students or postdocs.


Gary MacGillivray

MacGillivray tells *Notices*, "I've had a lifelong interest in sports and have been a volunteer coach off and on since 1976. Twenty years ago I ran marathons and did IRONMAN TRIATHALON®. Recently it has been less competitive things like cycling up Haleakala and hiking the Grand Canyon to the river and back in a day."

—From CMS announcements

Cirac Awarded Max Planck Medal


J. Ignacio Cirac

J. Ignacio Cirac of the Max Planck Institute of Quantum Optics has been awarded the 2018 Max Planck Medal "for his groundbreaking contributions to the field of quantum information and quantum optics." His research involves fundamental mathematical calculations in quantum information theory, modeling of quantum many-body systems, and concepts for the implementation of quantum optical systems. His group in the Theory Division has "developed new concepts for logical elements such as quantum gates that have already been implemented by experimental physicists. Furthermore, the group develops new algorithms for quantum communication, designs new quantum networks making use of the special properties of quantum particles, and creates new theoretical tools to characterize and quantify; e.g., entanglement of remote quantum systems." Cirac received his PhD in theoretical physics in 1991 from Universidad Complutense de Madrid. He was awarded the Wolf Prize in 2013.

—From a Max Planck Institute announcement

NEWS

Lu Awarded IMA Prize

Jianfeng Lu

JIANFENG LU of Duke University has been awarded the 2017 IMA Prize in Mathematics and Its Applications by the Institute for Mathematics and Its Applications (IMA). He was honored "for his many contributions in applied analysis, computational mathematics, and applied probability, in particular for problems from physics, chemistry, and material sciences. The unique strength of his research is to combine advanced mathematical analysis and algorithmic tools with a deep understanding of problems from science and engineering." According to the prize citation, "Some of Lu's major research achievements include groundbreaking contributions to electronic structure models, multiscale methods, rare events, and quantum molecular dynamics. His most recent contribution on the mathematical understanding of surface hopping algorithms has generated enormous excitement in the quantum chemistry community."

Lu received his PhD from Princeton University in 2009. He received an Alfred P. Sloan Foundation Research Fellowship in 2013 and an NSF CAREER Award in 2015.

—From an IMA announcement

Rolf Schock Prizes Awarded

Saharon Shelah

Ronald Coifman

SAHARON SHELAH of the Hebrew University of Jerusalem and Rutgers University and RONALD COIFMAN of Yale University have been awarded Rolf Schock Prizes for 2018 by the Royal Swedish Academy of Sciences. Shelah was awarded the prize in logic and philosophy "for his outstanding contributions to mathematical logic, in particular to model theory, in which his classification of theories in terms of so-called stability properties has fundamentally transformed the field of research of this discipline." Coifman was honored with the prize in mathematics "for his fundamental contributions to pure and applied harmonic analysis."

The prize citation for Shelah reads: "Saharon Shelah has made fundamental contributions to mathematical logic, particularly in model theory and set theory. In model theory, Shelah developed classification theory, concerning the classification of first-order theories in terms of properties of their classes of models. The classes of models of so-called stable theories have structural properties that can be characterized in geometrical terms, while the class of models of an 'unstable' theory lacks structure. Most of contemporary research in model theory builds on Shelah's work. Shelah has also made decisive contributions to set theory, including the development of a new variety of the forcing method and remarkable results in cardinal arithmetic, and he has solved deep problems in other areas, such as algebra, algebraic geometry, topology, combinatorics, computer science, and social choice theory. Shelah has had, and still has, an indisputable and exceptional position in mathematical logic, particularly in model theory. He is almost unbelievably productive, with seven books and more than 1,100 articles to date."

The citation for Coifman reads: "Ronald Coifman has made outstanding contributions to harmonic analysis. He has proven several important classical results and has recently dedicated his research to applied harmonic analysis and related areas. Along with Yves Meyer, he has played a crucial role in the development of the theory of wavelets, which has important applications in image compression, signal processing, and computer vision. He and his collaborators have recently initiated diffusion geometry, bringing the opportunity to create methods for finding structures in large data sets."

Shelah was born in Jerusalem in 1945. He received his PhD from Hebrew University under the direction of Michael O. Rabin. He held positions at Princeton University (1969–1970) and the University of California Los Angeles before joining the faculty at Hebrew University. He is also distinguished visiting professor at Rutgers University. His honors include the Erdős Prize (1977), the Rothschild Prize (1982), the Karp Prize (1983), the George Pólya Prize (1992), the Bolyai Prize (2000), the Wolf Prize (2001), the Steele Prize for Seminal Contribution to Research (2013), and the Hausdorff Medal (with Maryanthe Malliaris, 2017).

Coifman received his PhD from the University of Geneva in 1965 under the direction of Jovan Karamata. His honors include the DARPA Sustained Excellence Award and the Connecticut Science Medal (both in 1966), the 1999 Pioneer Award of the International Society for Industrial and Applied Science, and the 1999 National Medal of Science. He is a member of the American Academy of Arts and Sciences, the Connecticut Academy of Science and Engineering, and the National Academy of Sciences.

The Rolf Schock Prizes are awarded in logic and philosophy, mathematics, visual arts, and musical arts. Each prize carries a cash award of 400,000 Swedish krona (approximately US$47,000).

—From a Royal Swedish Academy announcement

Perkowski Awarded Rollo Davidson Prize

Nicolas Perkowski

Nicolas Perkowski of Humboldt-Universität zu Berlin and the Max Planck Institute for Mathematics in the Sciences has been awarded the 2018 Rollo Davidson Prize for his role in the development of the theory of paracontrolled distributions for singular stochastic partial differential equations and for advances in understanding of the Kardar-Parisi-Zhang equation. Perkowski received his PhD in 2013 from Humboldt-Universität zu Berlin and has held positions at Universität Wien and Université Paris Dauphine. In his free time he enjoys traveling and hiking.

—From a Davidson Trust announcement

Nisan Receives Rothschild Prize

Noam Nisan

Noam Nisan of The Hebrew University has been awarded the 2018 Rothschild Prize of Yad Hanadiv in mathematics/computer science. The prize was established to support, encourage, and advance the sciences and humanities in Israel and recognize original and outstanding published work in several disciplines. The prize in mathematics and computer science is awarded every two years.

—From a Yad Hanadiv announcement

ANZIAM Prizes Awarded

Australia and New Zealand Industrial and Applied Mathematics (ANZIAM), a division of the Australian Mathematical Society, has awarded medals for 2018 to three mathematical scientists. Philip G. Howlett of the University of South Australia was awarded the 2018 ANZIAM Medal for "sustained and outstanding contributions to both the theory and applications of mathematics, particularly in the development of control theoretic methods in the transport industry." The medal is given for outstanding merit in research achievements, activities enhancing applied or industrial mathematics or both, and contributions to ANZIAM. Yvonne Stokes of the University of Adelaide was awarded the E. O. Tuck Medal for fundamental contributions in viscous fluid mechanics and mathematical biology. The Tuck Medal is a midcareer award given for outstanding research and distinguished service to the field of applied mathematics. Claire Postlethwaite of the University of Auckland received the J. H. Michell Medal for research focusing on dynamical systems, in which she has made "important contributions to a wide range of areas including: heteroclinic cycles and networks; time-delayed feedback control; delay-differential equations; coupled cell dynamics; noise-induced dynamics; and bifurcations." The medal recognizes an outstanding young researcher in applied/industrial mathematics.

—From an ANZIAM announcement

Ruzhansky and Suragan Awarded Balaguer Prize

Michael Ruzhansky of Imperial College London and Durvudkhan Suragan of Nazarbayev University, Kazakhstan, have been awarded the 2018 Ferran Sunyer i Balaguer Prize for their monograph, "Hardy Inequalities on Homogeneous Groups (100 Years of Hardy Inequalities)." The prize is awarded for an unpublished mathematical monograph of an expository nature presenting the latest developments in an active area of research in mathematics in which the applicant has made important contributions. The prize carries a cash award of 15,000 euros (approximately US$18,600). The winning monograph will be published in Birkhäuser's series "Progress in Mathematics."

—From a Balaguer Foundation announcement

ACM Turing Award

John L. Hennessy of Stanford University and David Patterson of Google have been chosen recipients of the 2017 A. M. Turing Award of the Association for Computing Machinery (ACM) "for pioneering a systematic, quantitative approach to the design and evaluation of computer architectures with enduring impact on the microprocessor industry." The prize carries a cash award of US$1 million.

—From an ACM announcement

Putnam Prizes Awarded

The winners of the seventy-eighth William Lowell Putnam Mathematical Competition have been announced. The Putnam Competition is administered by the Mathematical Association of America (MAA) and consists of an examination containing mathematical problems that are designed to test both originality and technical competence. Prizes are awarded both to individuals and to teams.

The six highest ranking individuals each received a cash award of US$2,500. Listed in alphabetical order, they are:

- Omer Cerrahoglu, Massachusetts Institute of Technology
- Jiyang Gao, Massachusetts Institute of Technology
- Junyao Peng, Massachusetts Institute of Technology
- Ashwin Sah, Massachusetts Institute of Technology
- David Stoner, Harvard University
- Yunkun Zhou, Massachusetts Institute of Technology

Institutions with at least three registered participants obtain a team ranking in the competition based on the rankings of three designated individual participants. The five top-ranked teams (with members listed in alphabetical order) were:

- Massachusetts Institute of Technology (Allen Liu, Sammy Luo, Yunkun Zhou)
- Harvard University (Dong Ryul Kim, Stefan Spataru, David Stoner)
- Princeton University (Murilo Corato Zanarella, Zhuo Qun Song, Xiaoyu Xu)
- University of Toronto (Itai Bar-Natan, Michael Chow, Dmitry Paramonov)
- University of California Los Angeles (Xiaoyu Huang, Konstantin Miagkov, Ni Yan)

The first-place team receives an award of US$25,000, and each member of the team receives US$1,000. The awards for second place are US$20,000 and US$800; for third place, US$15,000 and US$600; for fourth place, US$10,000 and $400; and for fifth place, US$5,000 and US$200.

- Ni Yan of the University of California Los Angeles was awarded the Elizabeth Lowell Putnam Prize for outstanding performance by a woman in the competition. She received an award of US$1,000.

—From an MAA announcement

Simons Fellows in Mathematics

The Simons Foundation Mathematics and Physical Sciences (MPS) division supports research in mathematics, theoretical physics, and theoretical computer science. The MPS division provides funding for individuals, institutions, and science infrastructure. The Fellows Program provides funds to faculty for up to a semester-long research leave from classroom teaching and administrative obligations. The mathematical scientists who have been awarded Simons Fellowships for 2018 are:

- Marcelo Aguiar, Cornell University
- Anar Akhmedov, University of Minnesota
- Dmytro Arinkin, University of Wisconsin–Madison
- Aaron Bertram, University of Utah
- Lydia Bieri, University of Michigan
- Alexander Braverman, University of Toronto
- Ching-Li Chai, University of Pennsylvania
- Jingyi Chen, University of British Columbia
- Yingda Cheng, Michigan State University
- Jordan Ellenberg, University of Wisconsin–Madison
- Rui Loja Fernandes, University of Illinois at Urbana–Champaign
- Amanda Folsom, Amherst College
- Michael Goldstein, University of Toronto
- Alexander Goncharov, Yale University
- Anton Gorodetski, University of California, Irvine
- Antonella Grassi, University of Pennsylvania
- Lan-Hsuan Huang, University of Connecticut
- David Jerison, Massachusetts Institute of Technology
- Jeffrey Lagarias, University of Michigan
- Claude LeBrun, Stony Brook University
- Lionel Levine, Cornell University
- Marta Lewicka, University of Pittsburgh
- Max Lieblich, University of Washington
- Jacob Lurie, Harvard University
- Govind Menon, Brown University
- Antonio Montalban, University of California, Berkeley
- Mircea Mustata, University of Michigan
- Alexei Oblomkov, University of Massachusetts Amherst
- Sam Payne, Yale University
- Olga Plamenevskaya, Stony Brook University
- Kavita Ramanan, Brown University
- Sebastien Roch, University of Wisconsin–Madison
- Federico Rodríguez Hertz, Pennsylvania State University
- Sunder Sethuraman, University of Arizona
- Roman Shvydkoy, University of Illinois at Chicago
- Yannick Sire, Johns Hopkins University
- Christopher Sogge, Johns Hopkins University
- Frank Thorne, University of South Carolina
- Shankar Venkataramani, University of Arizona
- Alexander Vladimirsky, Cornell University

—From a Simons Foundation announcement

Guggenheim Fellowship Awards to Mathematical Scientists

The John Simon Guggenheim Memorial Foundation has announced the names of 173 scholars, artists, and scientists who were selected as Guggenheim Fellows for 2018. Selected as fellows in the mathematical sciences were:

- Moon Duchin, Tufts University
- Edriss S. Titi, Texas A&M University, Weizmann Institute of Science

Guggenheim Fellows are appointed on the basis of impressive achievement in the past and exceptional promise for future accomplishments.

—From a Guggenheim Foundation announcement

AAAS Fellows Elected

The American Academy of Arts and Sciences (AAAS) has elected its 2018 new fellows and foreign honorary members. Following are the new members in the section on Mathematics, Applied Mathematics, and Statistics:

- Alexei Borodin, Massachusetts Institute of Technology
- Sylvain Cappell, Courant Institute of Mathematical Sciences, New York University
- Larry D. Guth, Massachusetts Institute of Technology
- Svetlana Jitomirskaya, University of California Irvine
- Richard V. Kadison, University of Pennsylvania
- Guillermo R. Sapiro, Duke University

New fellows in other sections that involve the mathematical sciences are:

- James W. Demmel, University of California Berkeley (Computer Sciences)
- Leonidas J. Guibas, Stanford University (Computer Sciences)
- Arkadi Nemirovski, Georgia Institute of Technology (Class I Intersection)
- Marc S. Mangel, University of California Santa Cruz (Evolutionary and Population Biology and Ecology)
- Rosa L. Matzkin, University of California Los Angeles (Economics)
- Parag A. Pathak, Massachusetts Institute of Technology (Economics)
- H. Peyton Young, Johns Hopkins University, Oxford University (Economics)
- Itzhak Gilboa, Tel Aviv University, Ecole des Hautes Etudes Commerciales (International Honorary Member, Economics)

Cappell, Kadison, and Demmel are fellows of the AMS.

—From an AAAS announcement

Regeneron Science Talent Search

Two young scientists whose work involves the mathematical sciences are among the top winners in the 2018 Regeneron Science Talent Search.

Benjamin Firester, eighteen, of Hunter College High School received the first-place award of US$250,000 for developing a mathematical model to predict how disease data and weather patterns could spread a fungus that damages crops. David Wu, seventeen, of Montgomery Blair High School, Silver Spring, Maryland, received the fifth-place award of US$90,000 for his project, in which he studied the patterns of sequential prime numbers, improved the current methods for gathering data on prime number patterns by several orders of magnitude, and began connecting conjectures in number theory to irregularities in these patterns.

The Regeneron Science Talent Search is the United States' oldest and most prestigious science and mathematics competition for high school seniors. It is administered by the Society for Science and the Public.

—From a Society for Science and the Public announcement

SIAM Fellows Elected

The Society for Industrial and Applied Mathematics (SIAM) has elected its class of fellows for 2018. Their names and institutions follow.

- Todd J. Arbogast, University of Texas at Austin
- Liliana Borcea, University of Michigan
- Luis A. Caffarelli, University of Texas at Austin
- Ronald A. DeVore, Texas A&M University
- Stanley C. Eisenstat, Yale University
- Michael Elad, Technion-Israel Institute of Technology
- David A. Field, General Motors Corporation
- Margot Gerritsen, Stanford University
- Michael B. Giles, University of Oxford
- Alain Goriely, University of Oxford
- Peter Kuchment, Texas A&M University
- Madhav V. Marathe, Virginia Institute of Technology
- Alison L. Marsden, Stanford University
- Bojan Mohar, Simon Fraser University
- Helen Moore, AstraZeneca
- Pablo A. Parrilo, Massachusetts Institute of Technology
- Alex Pothen, Purdue University
- Helmut Pottmann, Technische Universität Wien
- Juan Restrepo, Oregon State University
- John N. Shadid, Sandia National Laboratories and University of New Mexico
- Arthur S. Sherman, National Institutes of Health
- Ralph C. Smith, North Carolina State University
- Tamas Terlaky, Lehigh University
- Robin Thomas, Georgia Institute of Technology
- Kim-Chuan Toh, National University of Singapore
- Panayot S. Vassilevski, Portland State University and Lawrence Livermore National Laboratory
- Homer F. Walker, Worcester Polytechnic Institute
- Karen E. Wilcox, Massachusetts Institute of Technology

—From a SIAM announcement

Hertz Foundation Fellowships Announced

The Fannie and John Hertz Foundation awards fellowships for graduate work in science and mathematics. Each fellow receives five full years of support toward his or her PhD degree. Four young scientists were awarded the fellowships in the mathematical sciences. They are:

- Colin Defant, Princeton University
- William Kuszmaul, Stanford University

- DAVID PALMER, Massachusetts Institute of Technology
- ETHAN SUSSMAN, Stanford University

—From a Hertz Foundation announcement

NSF Graduate Research Fellowships Awarded

The National Science Foundation (NSF) has awarded a number of Graduate Research Fellowships for fiscal year 2018. Further awards may be announced later in the year. This program supports students pursuing doctoral study in all areas of science and engineering and provides a stipend of US$30,000 per year for a maximum of three years of full-time graduate study. Information about the solicitation for the 2019 competition will be published in the "Mathematics Opportunities" section of an upcoming issue of the *Notices*.

Following are the names of the awardees in the mathematical sciences selected so far in 2018, followed by their undergraduate institutions (in parentheses) and the institutions at which they plan to pursue graduate work.

- IZABEL P. AGUIAR (Colorado School of Mines), University of Colorado at Boulder
- DYLAN ALTSCHULER (Princeton University), Princeton University
- RYAN ALWEISS (Massachusetts Institute of Technology), Massachusetts Institute of Technology
- OLIVIA ANGIULI (Harvard College), University of California Berkeley
- CHLOE I. AVERY (University of California Santa Barbara), University of California Santa Barbara
- ROBIN L. BELTON (Kenyon College), Montana State University
- SHAYNA BENNETT (Johnson State College), University of California Merced
- AARON BERGER (Yale University), Yale University
- ALEXI T. BLOCK GORMAN (Wellesley College), University of Illinois at Urbana–Champaign
- ELENA BONGIOVANNI (Michigan State University), Michigan State University
- CAITLYN BOOMS (University of Notre Dame), University of Notre Dame
- LUCAS C. BOUCK (George Mason University), George Mason University
- PAULA E. BURKHARDT (Pomona College), University of California Berkeley
- DYLAN M. CABLE (Stanford University), Stanford University
- ELIZABETH CARLSON (Carroll College), University of Nebraska-Lincoln
- AARON CHEN (Cornell University), University of Chicago
- ERIC CHEN (University of California Berkeley), University of California Berkeley
- EVAN CHEN (Massachusetts Institute of Technology), Massachusetts Institute of Technology
- KALEN P. CLIFTON (College of William and Mary), College of William and Mary
- SHELBY P. COX (University of Massachusetts Amherst)
- JONATHAN A. DEWITT (Haverford College), University of Chicago
- JUSTIN DONG (William Marsh Rice University), Brown University
- MARGARET DOUCETTE (University of Chicago), University of Chicago
- YONATAN DUKLER (University of California Los Angeles), University of California Los Angeles
- BRYAN FELIX (University of Texas at Austin), University of Minnesota–Twin Cities
- CHRISTIAN GAETZ (University of Minnesota–Twin Cities), Massachusetts Institute of Technology
- HAROLD X. GONZALEZ (Harvard University), Harvard University
- ISABELLA GRABSKI (Princeton University), Princeton University
- ANTHONY GRAVES-MCCLEARY (Vassar College)
- DEREK L. HANSEN (University of Oklahoma–Norman)
- ALI HASAN (University of North Carolina at Chapel Hill)
- SACHI HASHIMOTO (Victoria University of Chicago), Boston University
- XIAOYU HE (Harvard College), Stanford University
- HOWARD HEATON (Walla Walla College), University of California Los Angeles
- VINCENT C. HERR (Northwestern University), University of Colorado-Denver
- MELODY Y. HUANG (University of California Los Angeles)
- AARON HUDSON (University of Illinois at Urbana–Champaign), University of Washington
- IRIT HUQ-KURUVILLA (Columbia University), University of California Berkeley
- ALANA G. HUSZAR (The College of New Jersey), University of Michigan-Ann Arbor
- SUMUN IYER (Williams College), Williams College
- MARIA JAHJA (North Carolina State University), Carnegie–Mellon University
- ROBERT A. JEFFS (Harvey Mudd College), University of Washington
- LENA MIN JI (Columbia University), Princeton University
- ARUN KANNAN (University of Virginia), University of Virginia
- ALLISON KOENECKE (Massachusetts Institute of Technology), Stanford University
- STEPHANIE KU (University of California Los Angeles), Rensselaer Polytechnic Institute
- LUIS KUMANDURI (Stanford University), Stanford University
- BRIAN KWAN (University of California Irvine), University of California San Diego
- RAYLEIGH X. LEI (Columbia University), University of Michigan–Ann Arbor
- OSCAR F. LEONG (Swarthmore College), William Marsh Rice University
- GREYSON R. LEWIS (Brown University), University of California San Francisco

- Tamar Lichter (CUNY Queens College), Rutgers University
- Jared D. Lichtman (Dartmouth College), Dartmouth College
- Caitlin Lienkaemper (Harvey Mudd College), Pennsylvania State University
- Sharon Ling (The College of New Jersey), The College of New Jersey
- Amarise N. Little (California Institute of Technology), University of Washington
- Sammy Y. Luo (Massachusetts Institute of Technology), Massachusetts Institute of Technology
- Theresa Marlin (DeSales University)
- Ondrej Maxian (Case Western Reserve University), Case Western Reserve University
- Dayna Mercadante (Franklin Pierce College), Worcester Polytechnic Institute
- David W. Miller (Oberlin College), Northwestern University
- Alan T. Min (Purdue University), Purdue University
- Casandra D. Monroe (Princeton University), Princeton University
- Zachary B. Morrow (Wofford College), North Carolina State University
- Joshua P. Mundinger (Swarthmore College), Swarthmore College
- Riley Murray (University of California Berkeley), California Institute of Technology
- Isabella Nogues (Princeton University), Harvard University
- Ivan S. Paskov (Stanford University), Stanford University
- Rohil R. Prasad (Harvard University), Harvard University
- William M. Reese (North Carolina State University), North Carolina State University
- Sarah Reitzes (Tufts University), University of Chicago
- Colleen Robichaux (Louisiana State University and Agricultural and Mechanical College), University of Illinois at Urbana–Champaign
- Maximilian E. Ruth (University of Colorado at Boulder), University of Colorado at Boulder
- Andrew Senger (University of Minnesota–Twin Cities), Massachusetts Institute of Technology
- Jamshaid Shahir (University of Maryland Baltimore County), University of Maryland Baltimore County
- Melissa U. Sherman-Bennett (Simon's Rock of Bard College), University of California Berkeley
- Kevin Shu (California Institute of Technology), California Institute of Technology
- John V. Siratt (University of Arkansas at Little Rock), University of Arkansas at Little Rock
- Aaron A. G. Slipper (Harvard University), Harvard University
- Jacob V. Spertus (Bowdoin College)
- Shannon Sweitzer (University of California Riverside), University of California Riverside
- James H. Tao (Harvard University), Harvard University
- Libby Taylor (Georgia Institute of Technology), Georgia Institute of Technology
- Brian Tran (University of California San Diego), University of California San Diego
- Sang Truong (University of California Irvine), University of California Los Angeles
- Axel G. R. Turnquist (University of Washington), New Jersey Institute of Technology
- Roger Van Peski (Princeton University), Princeton University
- Sameera Vemulapalli (University of California Berkeley), University of California Berkeley
- Sarah M. Weinstein (Columbia University)
- Rebekah D. White (East Tennessee State University), North Carolina State University
- Zev Woodstock (James Madison University), North Carolina State University
- Lucy Z. Yang (University of Minnesota–Twin Cities), University of Minnesota–Twin Cities
- Christopher W. York (Lamar University Beaumont), Lamar University Beaumont
- Christopher T. Zhang (Princeton University), Prin-Princeton University
- Michael Zhao (University of Utah), Cambridge University
- Yuanyuan Zhao (Tongji University)

—NSF announcement

Marvin J. Greenberg (1935–2017)

Marvin Greenberg was famous for being the first mathematician to apply Grothendieck's theory of schemes to a problem in algebraic geometry. He also invented the so-called Greenberg functor in that subject.

Marvin Greenberg

Greenberg was a gifted expositor. His book *Lectures on Algebraic Topology* is widely used. He was awarded the Lester R. Ford Prize in 2010 for his book *Euclidean and Non-Euclidean Geometry* and his wonderful *American Mathematical Monthly* expository article, "Old and New Results in the Foundations of Elementary Plane and Non-Euclidean Geometries."

Greenberg was Serge Lang's first doctoral student, receiving his degree in 1959 from Princeton. In 1967, after stints at the University of California Berkeley and Northeastern University, he became a founding member of the University of California Santa Cruz mathematics department.

—Steven G. Krantz and Tony Tromba

NEWS

Edwin F. Beschler (1931–2018)

Edwin Beschler

Edwin Beschler, creator of Academic Press' Pure and Applied Mathematics series, died April 29 at the age of 86. He attended the American Academy of Dramatic Arts and pursued an acting career until he was drafted to serve in the US Army during the Korean War.

After his military service, Beschler resumed his acting career and just before his GI Bill expired, he enrolled at Columbia University where he became a student of mathematics. His career was not as a mathematician, but as a publisher of mathematical books and journals for Academic Press, Birkhäuser Boston, and Springer-Verlag.

Beschler was particularly active in the 1960s in the founding of a program of significant new research-level journals in mathematics, all of which continue to be published today. He retired in 1999, did language editing of mathematics for a number of international publishers, and returned to his first love, acting, accepting roles in student films, in commercials, and as an extra in movies. Beschler was an AMS member since 1962.

—Fern Beschler and Inna Mette

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Mathematics Opportunities

Listings for upcoming math opportunities to appear in Notices *may be submitted to* notices@ams.org *.*

AMS Mary P. Dolciani Prize for Excellence in Research

The AMS announces the competition for the inaugural Mary P. Dolciani Prize for Excellence in Research, recognizing a mathematician with a distinguished record of scholarship from a department that does not grant the PhD degree. The primary criterion for the prize is an active research program as evidenced by a strong record of peer-reviewed publications. The prize carries a cash award of US$5,000. The deadline for nominations is **June 30, 2018**. See www.ams.org/profession/prizes-awards/ams-prizes/dolciani-prize.

—AMS announcement

Call for Nominations for the SASTRA Ramanujan Prize

The Shanmugha Arts, Science, Technology, Research Academy (SASTRA) is seeking nominations for the 2018 SASTRA Ramanujan Prize, awarded to a mathematician not exceeding the age of thirty-two for outstanding contributions in an area of mathematics influenced by the late Indian mathematical genius Srinivasa Ramanujan. It carries a cash prize of US$10,000. The deadline for nominations is **July 31, 2018**. See the website qseries.org/sastra-prize/nominations-2018.html.

—Krishnaswami Alladi, University of Florida

Call for Nominations for the 2019 Hans Schneider Prize

The International Linear Algebra Society awards the Hans Schneider Prize once every three years to individuals who have made distinguished contributions to linear algebra. Nominations are sought for the 2019 prize. Nominations should be sent before **December 1, 2018**, to the Chair of the Prize Committee at rajendra.bhatia@ashoka.edu.in. See www.ilasic.org/misc/lecturers.html.

—Peter Semrl, University of Ljubljana

NSF Mathematical Sciences Postdoctoral Research Fellowships

The National Science Foundation (NSF) solicits proposals for the Mathematical Sciences Postdoctoral Research Fellowships. The deadline for full proposals is **October 17, 2018**. See www.nsf.gov/funding/pgm_summ.jsp?pims_id=5301.

—From an NSF announcement

NSF CAREER Awards

The National Science Foundation (NSF) solicits proposals for the Faculty Early Career Development Awards. The deadline for full proposals is **July 20, 2018**. See www.nsf.gov/funding/pgm_summ.jsp?pims_id=503214.

—From an NSF announcement

NSF Mathematical Sciences Innovation Incubator

The Mathematical Sciences Innovation Incubator (MSII) activity of the National Science Foundation (NSF) provides funding to support the involvement of mathematical scientists in research areas in which the mathematical sciences

**The most up-to-date listing of NSF funding opportunities from the Division of Mathematical Sciences can be found online at:* www.nsf.gov/dms *and for the Directorate of Education and Human Resources at* www.nsf.gov/dir/index.jsp?org=ehr. *To receive periodic updates, subscribe to the DMSNEWS listserv by following the directions at* www.nsf.gov/mps/dms/about.jsp.

are not yet playing large roles—for example, security and resilience of critical infrastructure, emerging technologies, innovative energy technology, and foundational biological and health research. For details, see www.nsf.gov/funding/pgm_summ.jsp?pims_id=505044.

—From an NSF announcement

IMA Prize in Mathematics and Its Applications

The Institute for Mathematics and Its Applications (IMA) awards the annual Prize in Mathematics and its Applications to an individual who has made a transformative impact on the mathematical sciences and their applications. The deadline for nominations is **July 20, 2018**. See www.ima.umn.edu/prize.

—From an IMA announcement

Fulbright Israel Postdoctoral Fellowships

The United States-Israel Educational Foundation (USIEF) plans to award eight fellowships to American postdoctoral scholars who seek to pursue research at Israeli institutions of higher education. The deadline for applications is **August 1, 2018**. See fulbright.org.il/content/fulbright-post-doctoral-fellowships.

—From a USIEF announcement

News from MSRI

With support from the National Science Foundation, the National Security Agency, academic departments, private foundations, and individuals, the Mathematical Sciences Research Institute (MSRI) will hold a number of workshops during the fall of 2018.

Established researchers, postdoctoral fellows, and graduate students are invited to apply for funding. MSRI actively seeks to achieve diversity in its workshops. Thus a strong effort is made to remove barriers that hinder equal opportunity, particularly for those groups that have been historically underrepresented in the mathematical sciences. MSRI has a resource to assist visitors with finding child care. Contact Sanjani Varkey at sanjani@msri.org. The workshops are as follows:

August 16–17, 2018: Connections for Women: Hamiltonian Systems, from Topology to Applications through Analysis.
www.msri.org/workshops/859

August 20–24, 2018: Introductory Workshop: Hamiltonian Systems, from Topology to Applications through Analysis.
www.msri.org/workshops/860

October 1–5, 2018: Hot Topics: Shape and Structure of Materials.
www.msri.org/workshops/900

October 8–12, 2018: Hamiltonian Systems, from Topology to Applications through Analysis I.
www.msri.org/workshops/871

November 26–30, 2018: Hamiltonian Systems, from Topology to Applications through Analysis II.
www.msri.org/workshops/872

—MSRI announcement

Frontiers for Young Minds: An Invitation

I would like to invite the mathematics research community to become involved with a new and rather unusual publishing outlet. *Frontiers for Young Minds* (See kids.frontiersin.org) is a free, open-access scientific journal with articles written by scientists for an audience of children and teens (ages 8-15). A distinctive feature of the journal is that submitted articles are reviewed not by the authors' peers, but by kids themselves, working under the guidance of experts.

Frontiers for Young Minds was founded in 2013 and has well-established sections in areas such as biodiversity, geoscience, and neuroscience. The newest section, of which I am the chief editor, is Understanding Mathematics. I believe that we need to do a better job of showing non-professionals the beauty and power of mathematics, and the earlier we start the better. I also believe that with enough care and craft, it is possible to communicate serious mathematics to a young audience.

All mathematicians are invited to contribute an article, whether on original research or on established topics.

— Jeremy Martin
jlmartin@ku.edu

Classified Advertisements

Positions available, items for sale, services available, and more

TENNESSEE

University of Memphis
Department of Mathematical Sciences

The Department of Mathematical Sciences at the University of Memphis is recruiting for a tenure- track Assistant Professor in Statistics to begin in August 2018. Qualifications include a PhD in Statistics or Biostatistics with research interests in Data Science and Bayesian Inference. Details are available at `www.memphis.edu/msci/news/positions.php`. Application should be completed on-line at `https://workforum.memphis.edu/postings/`. Review begins May 2018.

Email: `eogeorge@memphis.edu` for further questions. EOE.

011

CHINA

Southern University of Science and Technology (SUSTech)
Faculty Positions of Mathematics
The Department of Mathematics

The Department of Mathematics at Southern University of Science and Technology (SUSTech) is founded in 2015 with a dual mission of creating a first-class research and education organization for mathematics and providing service courses in support of other academic departments at SUSTech. We currently have 36 full-time faculty members, including 6 Chair Professors & 7 Full Professors, 3 Associate Professors, 12 Assistant Professors, and 8 teaching faculty members. Research interests of the faculty members cover a broad array of Mathematics including Pure Mathematics, Computational and Applied Mathematics, Probability and Statistics, and Financial Mathematics.

Call for Applications

We invite applications for full-time faculty positions at all ranks and in all areas of Mathematics, including Financial Mathematics and Statistics. SUSTech has a tenure system. Qualified candidates may apply for appointments with tenure.

Candidates should have demonstrated excellence in research and a strong commitment to teaching. A doctoral degree is required at the time of appointment. A candidate for a senior posi-

Suggested uses for classified advertising are positions available, books or lecture notes for sale, books being sought, exchange or rental of houses, and typing services. The publisher reserves the right to reject any advertising not in keeping with the publication's standards. Acceptance shall not be construed as approval of the accuracy or the legality of any advertising.

The 2018 rate is \$3.50 per word with a minimum two-line headline. No discounts for multiple ads or the same ad in consecutive issues. For an additional \$10 charge, announcements can be placed anonymously. Correspondence will be forwarded.

Advertisements in the "Positions Available" classified section will be set with a minimum one-line headline, consisting of the institution name above body copy, unless additional headline copy is specified by the advertiser. Headlines will be centered in boldface at no extra charge. Ads will appear in the language in which they are submitted.

There are no member discounts for classified ads. Dictation over the telephone will not be accepted for classified ads.

Upcoming deadlines for classified advertising are as follows: August 2018—June 6, 2018; September 2018—June 28, 2018; October 2018—July 27, 2018; November 2018—August 29, 2018; December 2018—September 21, 2018.

US laws prohibit discrimination in employment on the basis of color, age, sex, race, religion, or national origin. "Positions Available" advertisements from institutions outside the US cannot be published unless they are accompanied by a statement that the institution does not discriminate on these grounds whether or not it is subject to US laws. Details and specific wording may be found on page 1373 (vol. 44).

Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the US and Canada or 401-455-4084 worldwide for further information.

Submission: Promotions Department, AMS, P.O. Box 6248, Providence, Rhode Island 02904; or via fax: 401-331-3842; or send email to `classads@ams.org`. AMS location for express delivery packages is 201 Charles Street, Providence, Rhode Island 02904. Advertisers will be billed upon publication.

tion must have an established record of research and teaching, and a track-record in securing external funding.

To apply, please visit www.mathjobs.org and look up our job ad for instructions. For an informal discussion about applying to one of our positions, please contact Ms. Xianghui Yu, the Secretary of Department of Mathematics, by phone +86-755-88018703 or email: yuxh@sustc.edu.cn.

SUSTech offers competitive salaries, fringe benefits including medical insurance, retirement and housing subsidy, which are among the best in China. Salary and rank will be commensurate with qualifications and experiences of an appointee.

About the University

Established in 2012, SUSTech is a public institution funded by Shenzhen, a city with a designated special economic zone status in Southern China bordering Hong Kong. As one of China's key gateways to the world, Shenzhen is the country's fastest-growing city in the past three decades. From a small fishing village 30 years ago to a modern city with a population of over 10 million, the city has become the high-tech and manufacturing hub of southern China. It is home to the world's third-busiest container port and the fourth-busiest airport on the Chinese mainland. Being a picturesque coastal city, Shenzhen is also a popular tourist destination.

SUSTech is a pioneer in higher education reform in China. Its mission is to become a globally recognized institution that excels in research and promotes innovation, creativity and entrepreneurship. Ninety percent of SUSTech faculty members have overseas work experiences, and sixty percent studied or worked in top 100 universities in the world. The languages of instruction are English and Chinese. Sitting on five hundred acres of subtropical woodland with hills, rivers and a natural lake in Nanshan District of Shenzhen, the SUSTech campus is a beautiful place for learning and research.

The prosperity of Shenzhen is built on innovations and entrepreneurship of its citizens. The city has some of China's most successful high-tech companies such as Huawei and Tencent. SUSTech strongly supports innovations and entrepreneurship, and provides funding for promising initiatives. The university encourages candidates with intention and experience on entrepreneurship to apply.

010

AWM Submission Deadlines

Jan 31 **SIAM Poster Session**
Juried student poster session to be held in conjunction with AWM Workshops

Jan 31 **Student Essay Contest**
Interview-based biographies of contemporary women mathematicians and statisticians

Feb 1 **Mentoring Travel Grant**
Grant for untenured women PhD mathematicians to travel to a mentor's workplace to do research for one month

Feb 1 **Travel Grant**
Grant for women PhD mathematicians to attend meetings or conferences

Feb 15 **Birman Prize**$^{\pi}$
Highlights exceptional research in topology or geometry by a woman early in her career

Feb 15 **Microsoft Research Prize**$^{\int}$
Highlights exceptional research in algebra by a woman early in her career

Feb 15 **Sadosky Prize**$^{\int}$
Highlights exceptional research in analysis by a woman early in her career

Apr 15 **Student Chapter Awards**
Recognizes outstanding achievements in any of four categories: scientific excellence, outreach, professional development, and funding/sustainability

Apr 30 **Hay Award**
Recognizes outstanding achievement by a woman in any area of mathematics education

Apr 30 **Humphreys Award**
Recognizes outstanding mentorship from math teachers (female or male) who have encouraged female undergrad students to pursue mathematical careers and/or graduate mathematics study

May 1 **Travel Grant**
Grant for women PhD mathematicians to attend meetings or conferences

May 15 **AWM Fellows**
Recognizes individuals who have demonstrated a sustained commitment to the support and advancement of women in the mathematical sciences

Aug 15 **Service Award**
Recognizes individuals for promoting and supporting women in math through exceptional service to the AWM

Aug 15 **JMM Poster Session**
Juried student poster session to be held in conjunction with AWM Workshops

Sep 1 **Falconer Lecture**
Honors women who have made distinguished contributions to the mathematical science or mathematics education

Oct 1 **Schafer Prize**
Recognizes a woman undergraduate who has demonstrated excellence in mathematics

Oct 1 **Dissertation Prize**
Honors outstanding dissertations by women mathematical scientists (defended within the past two years)

Oct 1 **Travel Grant**
Grant for women PhD mathematicians to attend meetings or conferences

Oct 15 **Noether Lecture**
Honors women who have made fundamental and sustained contributions to the mathematical sciences

Nov 1 **Michler Prize**
Honors outstanding women who have recently been promoted to Associate Professor as Research Fellows in the Cornell Mathematics Department

Nov 1 **Kovalesky Lecture**
Recognizes those in the scientific or engineering community whose work highlights the achievement of women in applied or computational mathematics

π = Nominations in even years only
$\int$ = Nominations in odd years only

Nominations and applications should be submitted through MathPrograms at: https://www.mathprograms.org. Additional information about all of the AWM programs can be found on the AWM website: https://sites.google.com/site/awmmath/.

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Algebra and Algebraic Geometry

Szegő Kernel Asymptotics for High Power of CR Line Bundles and Kodaira Embedding Theorems on CR Manifolds

Chin-Yu Hsiao, *Academia Sinica, Taipei, Taiwan*

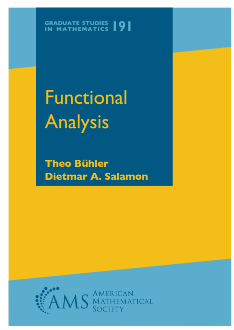
This item will also be of interest to those working in geometry and topology.

Contents: Introduction and statement of the main results; More properties of the phase $\varphi(x,y,s)$; Preliminaries; Semi-classical $\Box_{b,k}^{(q)}$ and the characteristic manifold for $\Box_{b,k}^{(q)}$; The heat equation for the local operatot $\Box_{s}^{(q)}$; Semi-classical Hodge decomposition theorems for $\Box_{s,k}^{(q)}$ in some non-degenerate part of Σ; Szegő kernel asymptotics for lower energy forms; Almost Kodaira embedding Theorems on CR manifolds; Asymptotic expansion of the Szegő kernel; Szegő kernel asymptotics and Kodairan embedding Theorems on CR manifolds with transversal CR S^1 actions; Szegő kernel asymptotics on some non-compact CR manifolds; The proof of Theorem 5.28; References.

Memoirs of the American Mathematical Society, Volume 254, Number 1217

June 2018, 140 pages, Softcover, ISBN: 978-1-4704-4101-2, 2010 *Mathematics Subject Classification:* 32V20, 32V30, **Individual member US$46.80**, List US$78, Institutional member US$62.40, Order code MEMO/254/1217

Analysis



Functional Analysis

Theo Bühler, and **Dietmar A. Salamon**, *ETH, Zurich, Switzerland*

Functional analysis is a central subject of mathematics with applications in many areas of geometry, analysis, and physics. This book provides a comprehensive introduction to the field for graduate students and researchers.

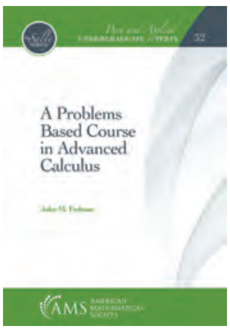
It begins in Chapter 1 with an introduction to the necessary foundations, including the Arzelà–Ascoli theorem, elementary Hilbert space theory, and the Baire Category Theorem. Chapter 2 develops the three fundamental principles of functional analysis (uniform boundedness, open mapping theorem, Hahn–Banach theorem) and discusses reflexive spaces and the James space. Chapter 3 introduces the weak and weak* topologies and includes the theorems of Banach–Alaoglu, Banach–Dieudonné, Eberlein–Šmulyan, Kreĭn–Milman, as well as an introduction to topological vector spaces and applications to ergodic theory. Chapter 4 is devoted to Fredholm theory. It includes an introduction to the dual operator and to compact operators, and it establishes the closed image theorem. Chapter 5 deals with the spectral theory of bounded linear operators. It introduces complex Banach and Hilbert spaces, the continuous functional calculus for self-adjoint and normal operators, the Gelfand spectrum, spectral measures, cyclic vectors, and the spectral theorem. Chapter 6 introduces unbounded operators and their duals. It establishes the closed image theorem in this setting and extends the functional calculus and spectral measure to unbounded self-adjoint operators on Hilbert spaces. Chapter 7 gives an introduction to strongly continuous semigroups and their infinitesimal generators. It includes foundational results about the dual semigroup and analytic semigroups, an exposition of measurable functions with values in a Banach space, and a discussion of solutions to the inhomogeneous equation and their regularity properties. The appendix establishes the equivalence of the Lemma of Zorn and the Axiom of Choice, and it contains a proof of Tychonoff's theorem.

With 10 to 20 elaborate exercises at the end of each chapter, this book can be used as a text for a one-or-two-semester course on functional analysis for beginning graduate students. Prerequisites are first-year analysis and linear algebra, as well as some foundational material from the second-year courses on point set topology, complex analysis in one variable, and measure and integration.

Contents: Foundations; Principles of functional analysis; The weak and weak* topologies; Fredholm theory; Spectral theory; Unbounded operators; Semigroups of operators; Zorn and Tychonoff; Bibliography; Notation; Index.

Graduate Studies in Mathematics, Volume 191

July 2018, 466 pages, Hardcover, ISBN: 978-1-4704-4190-6, LC 2017057259, 2010 *Mathematics Subject Classification:* 46-01, 47-01, **AMS members US$66.40**, List US$83, Order code GSM/191



A Problems Based Course in Advanced Calculus

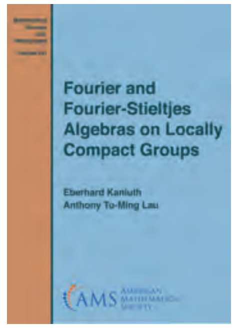
John M. Erdman, *Portland State University, OR*

This textbook is suitable for a course in advanced calculus that promotes active learning through problem solving. It can be used as a base for a Moore method or inquiry based class, or as a guide in a traditional classroom setting where lectures are organized around the presentation of problems and solutions. This book is appropriate for any student who has taken (or is concurrently taking) an introductory course in calculus. The book includes sixteen appendices that review some indispensable prerequisites on techniques of proof writing with special attention to the notation used the course.

Contents: Intervals; Toplogy of the real line; Continuous functions from ℝ to ℝ; Sequences of real numbers; Connectedness and the intermediate value theorem; Compactness and the extreme value theorem; Limits of real valued functions; Differentiation of real valued functions; Metric spaces; Interiors, closures, and boundaries; The topology of metric spaces; Sequences in metric spaces; Uniform convergence; More on continuity and limits; Compact metric spaces; Sequenctial characterization of compactness; Connectedness; Complete spaces; A fixed point theorem; Vector spaces; Linearity; Norms; Continuity and linearity; The Cauchy integral; Differential calculus; Partial derivatives and iterated integrals; Computations in $\mathbb{R}^n$; Infinite series; The implicit function theorem; Higher order derivatives; Quantifiers; Sets; Special subsets of ℝ; Logical connectives; Writing mathematics; Set operations; Arithmetic; Order propertiers of ℝ; Natural numbers and mathematical induction; Least upper bounds and greatest lower bounds; Products, relations, and functions; Properties of functions; Functions that have inverses; Products; Finite and infinite sets; Countable and uncountable sets; Bibliography; Index.

Pure and Applied Undergraduate Texts, Volume 32

July 2018, 360 pages, Hardcover, ISBN: 978-1-4704-4246-0, LC 2017051409, 2010 *Mathematics Subject Classification:* 00A07; 00-01, **AMS members US$63.20**, List US$79, Order code AMSTEXT/32



Fourier and Fourier-Stieltjes Algebras on Locally Compact Groups

Eberhard Kaniuth, and **Anthony To-Ming Lau**, *University of Alberta, Edmonton, AB, Canada*

The theory of the Fourier algebra lies at the crossroads of several areas of analysis. Its roots are in locally compact groups and group representations, but it requires a considerable amount of functional analysis, mainly Banach algebras. In recent years it has made a major connection to the subject of operator spaces, to the enrichment of both. In this book two leading experts provide a road map to roughly 50 years of research detailing the role that the Fourier and Fourier-Stieltjes algebras have played in not only helping to better understand the nature of locally compact groups, but also in building bridges between abstract harmonic analysis, Banach algebras, and operator algebras. All of the important topics have been included, which makes this book a comprehensive survey of the field as it currently exists.

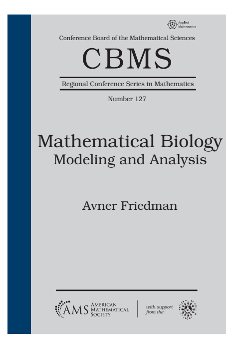
Since the book is, in part, aimed at graduate students, the authors offer complete and readable proofs of all results. The book will be well received by the community in abstract harmonic analysis and will be particularly useful for doctoral and postdoctoral mathematicians conducting research in this important and vibrant area.

Contents: Preliminaries; Basic theory of Fourier and Fourier-Stieltjes algebras; Miscellaneous further topics; Amenability properties of $A(G)$ and $B(G)$; Multiplier algebras of Fourier algebras; Spectral synthesis and ideal theory; Extension and separation properties of positive definite functions; Appendix; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 231

June 2018, 306 pages, Hardcover, ISBN: 978-0-8218-5365-8, LC 2017052436, 2010 *Mathematics Subject Classification:* 43-02, 43A10, 43A20, 43A30, 43A25, 46-02, 22-02, **AMS members US$97.60**, List US$122, Order code SURV/231

Applications



Mathematical Biology

Modeling and Analysis

Avner Friedman, *Ohio State University, Columbus, OH*

The fast growing field of mathematical biology addresses biological questions using mathematical models from areas such as dynamical systems, probability, statistics, and discrete mathematics.

This book considers models that are described by systems of partial differential equations, and it focuses on modeling, rather than on numerical methods and simulations. The models studied are concerned with population dynamics, cancer, risk of plaque growth associated with high cholesterol, and wound healing. A rich variety of open problems demonstrates the exciting challenges and opportunities for research at the interface of mathematics and biology.

This book primarily addresses students and researchers in mathematics who do not necessarily have any background in biology and who may have had little exposure to PDEs.

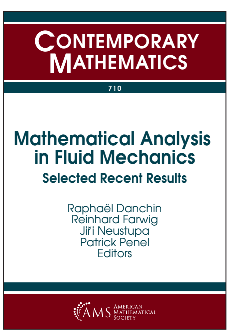
This item will also be of interest to those working in analysis.

Contents: Introductory biology; Introduction to modeling; Models of population dynamics; Cancer and the immune system; Parameters estimation; Mathematical analysis inspired by cancer models; Mathematical model of artherosclerosis: Risk of high cholesterol; Mathematical analysis inspired by the atherosclerosis model; Mathematical models of chronic wounds; Mathematical analysis inspired by the chronic wound model; Introduction to PDEs; Bibliography; Index.

CBMS Regional Conference Series in Mathematics, Number 127

August 2018, approximately 101 pages, Softcover, ISBN: 978-1-4704-4715-1, LC 2018015203, 2010 *Mathematics Subject Classification:* 35Q92, 35R35, 37N25, 49J10, 92C50, 92D25; 35B32, 35B35, 35B50, **AMS members US$41.60**, List US$52, Order code CBMS/127

Differential Equations



Mathematical Analysis in Fluid Mechanics

Selected Recent Results

Raphaël Danchin, *Université Paris-Est, Créteil, France*, **Reinhard Farwig**, *Technische Universität Darmstadt, Germany*, **Jiři Neustupa**, *Czech Academy of Sciences, Prague, Czech Republic*, and **Patrick Penel**, *Université du Sud-Toulon-Var, La Garde, France*, Editors

This volume contains the proceedings of the International Conference on Vorticity, Rotation and Symmetry (IV)—Complex Fluids and the Issue of Regularity, held from May 8–12, 2017, in Luminy, Marseille, France.

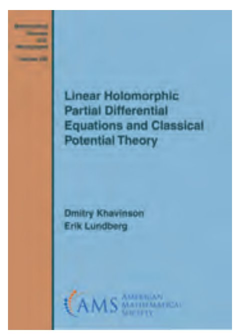
The papers cover topics in mathematical fluid mechanics ranging from the classical regularity issue for solutions of the 3D Navier-Stokes system to compressible and non-Newtonian fluids, MHD flows and mixtures of fluids. Topics of different kinds of solutions, boundary conditions, and interfaces are also discussed.

This item will also be of interest to those working in mathematical physics.

Contents: **H. Abels** and **M. Moser**, Well-posedness of a Navier-Stokes/mean curvature flow system; **M. Bulíček**, **J. Málek**, **V. Průša**, and **E. Süli**, PDE analysis of a class of thermodynamically compatible viscoelastic rate-type fluids with stress-diffusion; **D. Chamorro**, **P. G. Lemarié-Rieusset**, and **K. Mayoufi**, Local stability of energy estimates for the Navier-Stokes equations; **H. J. Choe** and **M. Yang**, Blow up criteria for the compressible Navier-Stokes equations; **W. Deng** and **P. Zhang**, Asymptotic stability of equilibrium to 3-D MHD system; **N. Filonov** and **T. Shilkin**, On some properties of weak solutions to elliptic equations with divergence-free drifts; **G. P. Galdi** and **M. Kyed**, Time-periodic solutions to the Navier-Stokes equations in the three-dimensional whole-space with a non-zero drift term: Asymptotic profile at spatial infinity; **T. Holding** and **E. Miot**, Uniqueness and stability for the Vlasov-Poisson system with spatial density in Orlicz spaces; **H. Kozono** and **S. Shimizu**, Strong solutions of the Navier-Stokes equations with singular data; **D. Maity** and **M. Tucsnak**, $L^p - L^q$-maximal regularity for some operators associated with linearized incompressible fluid-right body problems; **P. Maremonti**, On an interpolation inequality involving the Stokes operator; **K. Nakao** and **Y. Taniuchi**, Brezis-Gallouet-Wainger type inequality and its application to the Navier-Stokes equations; **T. Piasecki** and **M. Pokorný**, On steady solutions to a model of chemically reacting heat conducting compressible mixture with slip boundary conditions.

Contemporary Mathematics, Volume 710

July 2018, approximately 244 pages, Softcover, ISBN: 978-1-4704-3646-9, LC 2018003589, 2010 *Mathematics Subject Classification:* 35B40, 35B65, 35Q30, 35Q35, 76D05, 76D07, 76N10, 76W05, **AMS members US$93.60**, List US$117, Order code CONM/710



Linear Holomorphic Partial Differential Equations and Classical Potential Theory

Dmitry Khavinson, *University of South Florida, Tampa, FL*, and **Erik Lundberg**, *Florida Atlantic University, Boca Raton, FL*

Why do solutions of linear analytic PDE suddenly break down? What is the source of these mysterious singularities, and how do they propagate? Is there a mean value property for harmonic functions in ellipsoids similar to that for balls? Is there a reflection principle for harmonic functions in higher dimensions similar to the Schwarz reflection principle in the plane? How far outside of their natural domains can solutions of the Dirichlet problem be extended? Where do the continued solutions become singular and why?

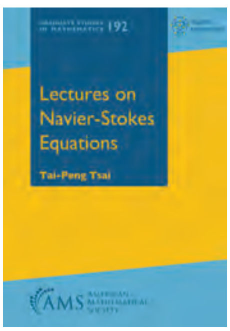
This book invites graduate students and young analysts to explore these and many other intriguing questions that lead to beautiful results illustrating a nice interplay between parts of modern analysis and themes in "physical" mathematics of the nineteenth century. To make the book accessible to a wide audience including students, the authors do not assume expertise in the theory of holomorphic PDE, and most of the book is accessible to anyone familiar with multivariable calculus and some basics in complex analysis and differential equations.

This item will also be of interest to those working in analysis.

Contents: Introduction: Some motivating questions; The Cauchy-Kovalevskaya theorem with estimates; Remarks on the Cauchy-Kovalevskaya theorem; Zerner's theorem; The method of globalizing families; Holmgren's uniqueness theorem; The continuity method of F. John; The Bony-Schapira theorem; Applications of the Bony-Schapira theorem: Part I - Vekua hulls; Applications of the Bony-Schapira theorem: Part II - Szegő's theorem revisited; The reflection principle; The reflection principle (continued); Cauchy problems and the Schwarz potential conjecture; The Schwarz potential conjecture for spheres; Potential theory on ellipsoids: Part I - The mean value property; Potential theory on ellipsoids: Part II - There is no gravity in the cavity; Potential theory on ellipsoids: Part III - The Dirichlet problem; Singularities encountered by the analytic continuation of solutions to the Dirichlet problem; An introduction to J. Leray's principle on propagation of singularities through $\mathbb{C}^n$; Global propagation of singularities in $\mathbb{C}^n$; Quadrature domains and Laplacian growth; Other varieties of quadrature domains; Bibliography; Index.

Mathematical Surveys and Monographs, Volume 232

July 2018, 214 pages, Hardcover, ISBN: 978-1-4704-3780-0, LC 2017055519, 2010 *Mathematics Subject Classification:* 35A20, 31B20, 32A05, 30B40, 14P05, **AMS members US$97.60**, List US$122, Order code SURV/232



Lectures on Navier-Stokes Equations

Tai-Peng Tsai, *University of British Columbia, Vancouver, BC, Canada*

The book is an excellent contribution to the literature concerning the mathematical analysis of the incompressible Navier-Stokes equations. It provides a very good introduction to the subject, covering several important directions, and also presents a number of recent results, with an emphasis on non-perturbative regimes. The book is well written and both beginners and experts will benefit from it. It can also provide great material for a graduate course.

—Vladimir Šverák, University of Minnesota

This book is a graduate text on the incompressible Navier-Stokes system, which is of fundamental importance in mathematical fluid mechanics as well as in engineering applications. The goal is to give a rapid exposition on the existence, uniqueness, and regularity of its solutions, with a focus on the regularity problem. To fit into a one-year course for students who have already mastered the basics of PDE theory, many auxiliary results have been described with references but without proofs, and several topics were omitted. Most chapters end with a selection of problems for the reader.

After an introduction and a careful study of weak, strong, and mild solutions, the reader is introduced to partial regularity. The coverage of boundary value problems, self-similar solutions, the uniform L^3 class including the celebrated Escauriaza-Seregin-Šverák Theorem, and axisymmetric flows in later chapters are unique features of this book that are less explored in other texts.

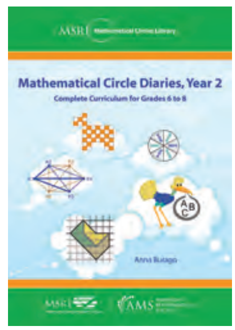
The book can serve as a textbook for a course, as a self-study source for people who already know some PDE theory and wish to learn more about Navier-Stokes equations, or as a reference for some of the important recent developments in the area.

Contents: Introduction; Steady states; Weak solutions; Strong solutions; Mild solutions; Partial regularity; Boundary value problem and bifurcation; Self-similar solutions; The uniform L^3 class; Axisymmetric flows; Bibliography; Index.

Graduate Studies in Mathematics, Volume 192

July 2018, 224 pages, Hardcover, ISBN: 978-1-4704-3096-2, LC 2017058342, 2010 *Mathematics Subject Classification:* 35Q30, 35Q35, 76D05, 76Dxx, **AMS members US$66.40**, List US$83, Order code GSM/192

General Interest



Mathematical Circle Diaries, Year 2

Complete Curriculum for Grades 6 to 8

Anna Burago, *Prime Factor Math Circle, Seattle, WA*

Mathematical circles, with their question-driven approach and emphasis on problem solving, expose students to the type of mathematics that stimulates the development of logical thinking, creativity, analytical abilities, and mathematical reasoning. These skills, while scarcely introduced at school, are in high demand in the modern world.

This book, a sequel to *Mathematical Circle Diaries, Year 1*, teaches how to think and solve problems in mathematics. The material, distributed among twenty-nine weekly lessons, includes detailed lectures and discussions, sets of problems with solutions, and contests and games. In addition, the book shares some of the know-how of running a mathematical circle. The book covers a broad range of problem-solving strategies and proofing techniques, as well as some more advanced topics that go beyond the limits of a school curriculum. The topics include invariants, proofs by contradiction, the Pigeonhole principle, proofs by coloring, double counting, combinatorics, binary numbers, graph theory, divisibility and remainders, logic, and many others. When students take science and computing classes in high school and college, they will be better prepared for both the foundations and advanced material. The book contains everything that is needed to run a successful mathematical circle for a full year.

This book, written by an author actively involved in teaching mathematical circles for fifteen years, is intended for teachers, math coaches, parents, and math enthusiasts who are interested in teaching math that promotes critical thinking. Motivated students can work through this book on their own.

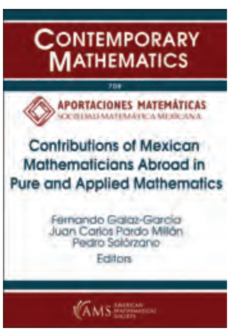
In the interest of fostering a greater awareness and appreciation of mathematics and its connections to other disciplines and everyday life, MSRI and the AMS are publishing books in the Mathematical Circles Library series as a service to young people, their parents and teachers, and the mathematics profession.

Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

Contents: Preliminaries; *Session plans:* Introduction; Checkerboard problems; Review: Math logic and other problem-solving strategies; Invariants; Proof by contradiction; Decimal number system and problems on digits; Binary numbers I; Binary numbers II; Mathematical dominoes tournament; Pigeonhole principle; Geometric pigeonhole principle; Mathematical Olympiad I; Combinatorics I. Review; Combinatorics II. Combinations; Mathematical auction; Combinatorics III. Complements. Snake pit game; Combinatorics IV. Combinatorial conundrum; Magic squares and related problems; Double counting, or there is more than one way to cut a cake; Mathematical Olympiad II; Divisibility I. Review; Divisibility II. Relatively prime numbers; GCF and LCM; Divisibility III. Mathematical race game; Mathematical auction; Divisibility IV. Divisibility by 3 and remainders; Divisibility V. Divisibility and remainders; Graph theory I. Graphs and their applications; Graph theory II. Handshaking theorem; Graph theory III. Solving problems with graphs; Mathematical Olympiad III; *Mathematical contests and competitions:* Mathematical contests; Mathematical auction; Mathematical dominoes; Mathematical snake pit; Mathematical race; Mathematical Olympiad; Short entertaining math games; *More teaching advice:* How to be a great math circle teacher; What comes next?; Solutions; Appendix to Session 6; Bibliography.

MSRI Mathematical Circles Library, Volume 20

August 2018, approximately 351 pages, Softcover, ISBN: 978-1-4704-3718-3, LC 2017058792, 2010 *Mathematics Subject Classification:* 97A20, 97A80, 00A07, 00A08, 00A09, 97D50, **AMS members US$44**, List US$55, Order code MCL/20



Contributions of Mexican Mathematicians Abroad in Pure and Applied Mathematics

Fernando Galaz-García, *Karlsruher Institut Für Technologie, Germany*, **Juan Carlos Pardo Millán**, *Centro de Investigación en Matemáticas, Guanajuato, Mexico*, and **Pedro Solórzano**, *Instituto de Matemáticas-Oaxaca, UNAM, Oaxaco, Mexico*, Editors

This volume contains the proceedings of the Second Workshop of Mexican Mathematicians Abroad (II Reunión de Matemáticos Mexicanos en el Mundo), held from December 15–19, 2014, at Centro de Investigación en Matemáticas (CIMAT) in Guanajuato, Mexico.

This meeting was the second in a series of ongoing biannual meetings aimed at showcasing the research of Mexican mathematicians based outside of Mexico.

The book features articles drawn from eight broad research areas: algebra, analysis, applied mathematics, combinatorics, dynamical systems, geometry, probability theory, and topology. Their topics range from novel applications of non-commutative probability to graph theory, to interactions between dynamical systems and geophysical flows.

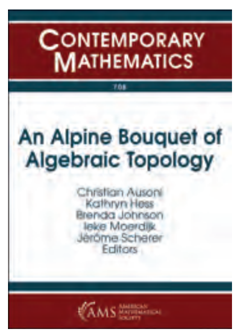
Several articles survey the fields and problems on which the authors work, highlighting research lines currently underrepresented in Mexico. The research-oriented articles provide either alternative approaches to well-known problems or new advances in active research fields. The wide selection of topics makes the book accessible to advanced graduate students and researchers in mathematics from different fields.

Contents: O. Arizmendi and **O. Juarez-Romero**, On bounds for the energy of graphs and digraphs; **P. Carrillo Rouse**, The Atiyah-Singer cobordism invariance and the tangent groupoid; **C. González-Tokman**, Multiplicative ergodic theorems for transfer operators: Towards the identification and analysis of coherent structures in non-autonomous dynamical systems; **R. Jiménez Rolland** and **J. Maya Duque**, Representation stability for the pure cactus group; **V. Kleptsyn** and **A. Rechtman**, Two proofs of Taubes' theorem on strictly ergodic flows; **H. Lange** and **A. Ortega**, The fibres of the Prym map of étale cyclic coverings of degree 7; **C. Lozano Huerta**, Extremal higher codimension cycles of the space of complete conics; **C. Meneses**, On Shimura's isomorphism and (Γ, G)-bundles on the upper-half plane; **M. Torres**, On the dual of BV; **C. Vargas**, A general solution to (free) deterministic equivalents.

Contemporary Mathematics, Volume 709

July 2018, approximately 163 pages, Softcover, ISBN: 978-1-4704-4286-6, LC 2017058161, 2010 *Mathematics Subject Classification:* 05C50, 58H05, 37H15, 20J06, 37C40, 14H40, 14N10, 30F35, 46E35, 46L54, **AMS members US$93.60**, List US$117, Order code CONM/709

Geometry and Topology



An Alpine Bouquet of Algebraic Topology

Christian Ausoni, *Université Paris 13, Villetaneuse, France*, **Kathryn Hess**, *École Polytechnique Fédérale de Lausanne, Switzerland*, **Brenda Johnson**, *Union College, Schenectady, NY*, **Ieke Moerdijk**, *Universiteit Utrecht, Netherlands*, and **Jérôme Scherer**, *École Polytechnique Fédérale de Lausanne, Switzerland*, Editors

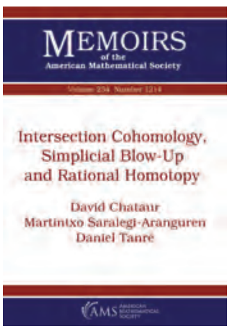
This volume contains the proceedings of the Alpine Algebraic and Applied Topology Conference, held from August 15–21, 2016, in Saas-Almagell, Switzerland.

The papers cover a broad range of topics in modern algebraic topology, including the theory of highly structured ring spectra, infinity-categories and Segal spaces, equivariant homotopy theory, algebraic K-theory and topological cyclic, periodic, or Hochschild homology, intersection cohomology, and symplectic topology.

Contents: A. Baker, Characteristics for $\mathscr{E}_\infty$ ring spectra; **P. Boavida de Brito**, Segal objects and the Grothendieck construction; **D. Chataur**, **M. Saralegi-Aranguren**, and **D. Tanré**, Blown-up intersection cohomology; **C. Costoya**, **D. Méndez**, and **A. Viruel**, Homotopically rigid Sullivan algebras and their applications; **E. Dotto**, A Dundas-Goodwillie-McCarthy theorem for split square-zero extensions of exact categories; **J. P. C. Greenlees**, Four approaches to cohomology theories with reality; **L. Hesselholt**, Topological Hochschild homology and the Hasse-Weil zeta function; **N. Kitchloo** and **J. Morava**, The stable symplectic category and a conjecture of Kontsevich; **M. Nakagawa** and **H. Naruse**, Universal Gysin formulas for the universal Hall-Littlewood functions; **B. Stonek**, Graded multiplications on iterated bar constructions; **K. Werndli**, Double homotopy (co)limits for relative categories.

Contemporary Mathematics, Volume 708

June 2018, 308 pages, Softcover, ISBN: 978-1-4704-2911-9, LC 2017055549, 2010 *Mathematics Subject Classification:* 11S40, 18Axx, 18Gxx, 19Dxx, 55Nxx, 55Pxx, 55Sxx, 55Uxx, 57Nxx, **AMS members US$93.60**, List US$117, Order code CONM/708



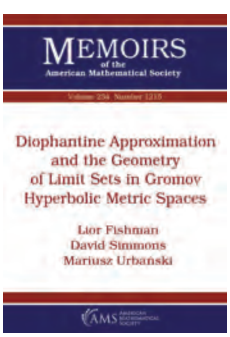
Intersection Cohomology, Simplicial Blow-Up and Rational Homotopy

David Chataur, *Université de Picardie Jules Verne, Amiens, France*, **Martintxo Saralegi-Aranguren**, *Université d'Artois, Lens, France*, and **Daniel Tanré**, *Université de Lille, Villeneuve d'Ascq, France*

Contents: Introduction; Simplicial blow-up; Rational algebraic models; Formality and examples; Appendix A. Topological setting; Bibliography; Index.

Memoirs of the American Mathematical Society, Volume 254, Number 1214

June 2018, 108 pages, Softcover, ISBN: 978-1-4704-2887-7, 2010 *Mathematics Subject Classification:* 55N33, 55P62, 57N80, **Individual member US$46.80**, List US$78, Institutional member US$62.40, Order code MEMO/254/1214



Diophantine Approximation and the Geometry of Limit Sets in Gromov Hyperbolic Metric Spaces

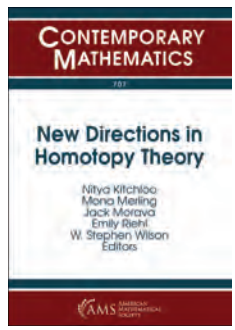
Lior Fishman, *University of North Texas, Denton, Texas*, **David Simmons**, *University of York, United Kingdom*, and **Mariusz Urbański**, *University of North Texas, Denton, Texas*

This item will also be of interest to those working in number theory.

Contents: Introduction; Gromov hyperbolic metric spaces; Basic facts about Diophantine approximation; Schmidt's game and McMullen's absolute game; Partition structures; Proof of Theorem 6.1 (Absolute winning of BA_ξ); Proof of Theorem 7.1 (Generalization of the Jarní–Besicovitch Theorem); Proof of Theorem 8.1 (Generalization of Khinchin's Theorem); Proof of Theorem 9.3 (BA_d has full dimension in $\Lambda_r(G)$); References.

Memoirs of the American Mathematical Society, Volume 254, Number 1215

June 2018, 137 pages, Softcover, ISBN: 978-1-4704-2886-0, 2010 *Mathematics Subject Classification:* 11J83, 20H10; 28A78, 37F35, **Individual member US$46.80**, List US$78, Institutional member US$62.40, Order code MEMO/254/1215



New Directions in Homotopy Theory

Nitya Kitchloo; **Mona Merling**, **Jack Morava**, **Emily Riehl**, and **W. Stephen Wilson**, *Johns Hopkins University, Baltimore, MD*, Editors

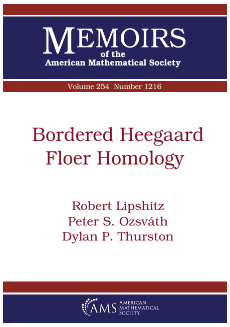
This volume contains the proceedings of the Second Mid-Atlantic Topology Conference, held from March 12–13, 2016, at Johns Hopkins University in Baltimore, Maryland.

The focus of the conference, and subsequent papers, was on applications of innovative methods from homotopy theory in category theory, algebraic geometry, and related areas, emphasizing the work of younger researchers in these fields.

Contents: **J. Heller** and **K. Ormsby**, The stable Galois correspondence for real closed fields; **J. L. Kass** and **K. Wickelgren**, An Étale realization which does NOT exist; **N. Kitchloo**, **V. Lorman**, and **W. S. Wilson**, Multiplicative structure on real Johnson-Wilson theory; **J. A. Lind** and **C. Malkiewich**, The Morita equivalence between parametrized spectra and module spectra; **C. McTague**, tmf is not a ring spectrum quotient of string bordism; **E. Peterson**, Cocycle schemes and $MU[2k,\infty)$-orientations; **K. Ponto** and **M. Shulman**, The linearity of fixed point invariants; **M. Szymik**, Homotopy coherent centers versus centers of homotopy categories; **G. Tabuada**, Recent developments on noncommutative motives; **I. Zakharevich**, The category of Waldhausen categories is a closed multicategory.

Contemporary Mathematics, Volume 707

June 2018, 194 pages, Softcover, ISBN: 978-1-4704-3774-9, LC 2017051421, 2010 *Mathematics Subject Classification:* 14Fxx, 18Dxx, 19Exx, 55Mxx, 55Nxx, 55Pxx, 55Rxx, **AMS members US$93.60**, List US$117, Order code CONM/707



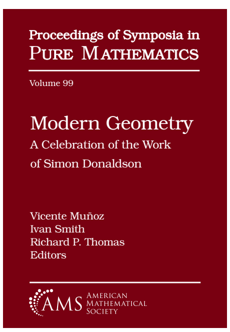
Bordered Heegaard Floer Homology

Robert Lipshitz, *University of Oregon, Eugene, Oregon*, **Peter S. Ozsváth**, *Princeton University, New Jersey*, and **Dylan P. Thurston**, *Indiana University, Bloomington, Indiana*

Contents: Introduction; $\mathcal{A}_\infty$ structures; The algebra associated to a pointed matched circle; Bordered Heegaard diagrams; Moduli spaces; Type D modules; Type A modules; Pairing theorem via nice diagrams; Pairing theorem via time dilation; Gradings; Bordered manifolds with torus boundary; Appendix A. Bimodules and change of framing; Index of definitions; Bibliography.

Memoirs of the American Mathematical Society, Volume 254, Number 1216

June 2018, 276 pages, Softcover, ISBN: 978-1-4704-2888-4, 2010 *Mathematics Subject Classification:* 57M25, 57R58, **Individual member US$46.80**, List US$78, Institutional member US$62.40, Order code MEMO/254/1216



Modern Geometry

A Celebration of the Work of Simon Donaldson

Vicente Muñoz, *Universidad Complutense de Madrid, Spain*, **Ivan Smith**, *University of Cambridge, United Kingdom*, and **Richard P. Thomas**, *Imperial College, London, United Kingdom*, Editors

This book contains a collection of survey articles of exciting new developments in geometry, written in tribute to Simon Donaldson to celebrate his 60th birthday. Reflecting the wide range of Donaldson's interests and influence, the papers range from algebraic geometry and topology through symplectic geometry and geometric analysis to mathematical physics. Their expository nature means the book acts as an invitation to the various topics described, while also giving a sense of the links between these different areas and the unity of modern geometry.

Contents: **G. Bèrczi**, **B. Doran**, and **F. Kirwan**, Graded linearisations; **A. Daemi** and **K. Fukaya**, Atiyah-Floer conjecture: A formulation, a strategy of proof and generalizations; **Y. Eliashberg**, Weinstein manifolds revisited; **N. Hitchin**, Remarks on Nahm's equations; **D. Joyce**, Conjectures on counting associative 3-folds in G_2-manifolds; **J. Li**, Toward an algebraic Donaldson-Floer theory; **H. Nakajima**, Introduction to a provisional mathematical definition of Coulomb branches of 3-dimesional $\mathcal{N} = 4$; **P. Ozsváth** and **Z. Szabó**, An overview of knot Floer homology; **R. Pandharipande**, Descendants for stable pairs on 3-folds; **J. Ross** and **D. Witt Nyström**, The Dirichlet problem for the complex homogeneous Monge-Ampère equation; **G. Székelyhidi**, Kähler-Einstein metrics; **A. Teleman**, Donaldson theory in non-Kählerian geometry; **E. Witten**, Two lectures on gauge theory and Khovanov homology.

Proceedings of Symposia in Pure Mathematics, Volume 99

July 2018, 416 pages, Hardcover, ISBN: 978-1-4704-4094-7, 2010 *Mathematics Subject Classification:* 32J25, 32L05, 53C07, 53C44, 53D35, 53D40, 53D50, 57R55, 57R57, 57R58, **AMS members US$106.40**, List US$133, Order code PSPUM/99

Math Education

Tactile Learning Activities in Mathematics

A Recipe Book for the Undergraduate Classroom

Julie Barnes, *Western Carolina University, Cullowhee, NC*, and **Jessica M. Libertini**, *Virginia Military Institute, Lexington, VA*, Editors

Q: What do feather boas, cookies, and paper shredders have in common?

A: They are all ingredients that have the potential to help your undergraduate students understand a variety of mathematical concepts. In this book, 43 faculty from a wide range of institutional settings share a total of 64 hands-on activities that allow students to physically engage with mathematical ideas ranging from the basics of precalculus to special topics appropriate for upper-level courses. Each learning activity is presented in an easy-to-read recipe format that includes a list of supplies; a narrative briefly describing the reasons, logistics, and helpful hints for running the activity; and a page that can be used as a handout in class. Purchase of the book also includes access to electronic printable versions of the handouts.

With so many activities, it might be hard to decide where to start. For that reason, there are four indices to help the reader navigate this book: a concept index, a course index, an author index, and a main ingredient index. In addition to providing activities for precalculus, calculus, commonly required mathematics courses for majors, and more specialized upper-level electives, there is also a section describing how to modify many of the activities to fit into a liberal arts mathematics class.

Whether you are new to using hands-on activities in class or are more experienced, the authors hope that this book will encourage and inspire you to explore the possibilities of using more hands-on activities in your classes.

Bon appétit!

Contents: *Appetizers (before calculus):* Precalculus; *Main courses (calculus):* Differential calculus; Integral calculus; Multivariable calculus; *Desserts (upper level courses):* Sophomore/junior courses; Junior/senior courses; Concept index; Author index; Main ingredient index; Course index.

Classroom Resource Materials, Volume 54

August 2018, approximately 314 pages, Hardcover, ISBN: 978-1-4704-4351-1, LC 2017057965, 2010 *Mathematics Subject Classification:* 97-01, 00-01, 26-01, 97-00, **AMS members US$36**, List US$45, Order code CLRM/54

The Alberta High School Math Competitions 1957–2006

A Canadian Problem Book

Andy Liu, Editor

Although there were some older contests in the Maritime region and in Lower and Upper Canada, the Alberta High School Mathematics Competition was the first and oldest in Canada to be run on a provincial scale. Started in 1957, the competition recently celebrated its fiftieth anniversary. These fifty years can be broken down to three periods, Ancient (1957–1966), Medieval (1967–1983) and Modern (1984–2006), with very distinctive flavors which reflect what was taught in the schools of the day. The first two periods are primarily of historical interest. During the Modern period, the talented problem committee was led by the world renowned problemist Murray Klamkin and composed many innovative and challenging problems.

In this book, you will find all the problems and answers for the first 50 years of the competition, up to 2005/2006, and full solutions are provided to those from the Modern period, often supplemented with multiple solutions or additional commentaries. Taken together, this unique collection of problems represent an interesting and valuable resource for students today preparing for these types of mathematics contests.

The Alberta High School Mathematics Competitions 1957–2006: A Canadian Problem Book is published by the American Mathematical Society (AMS) in collaboration with the Canadian Mathematical Society (CMS).

Problem Books, Volume 22

January 2008, 281 pages, Hardcover, ISBN: 978-0-88385-830-1, LC 2009933078, **Individual member US$30**, List US$40, Institutional member US$32, Order code PRB/22

Problems from Murray Klamkin

Andy Liu and **Bruce Shawyer**, Editors

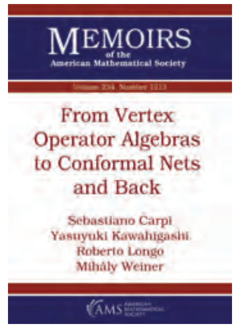
Murray Klamkin was a dedicated problem solver and problem proposer who left indelible marks on the problemist community. After working in industry and academe in the United States, he spent the last 30 of his 84 years in Canada. He was famous for his Quickies, problems that have quick and neat solutions. In this book, you will find all of the problems that he proposed for *Crux Mathematicorum*, including all of his Quickies. His problems covered a very wide range of topics and show a great deal of insight into what is possible in these areas. The problems are arranged into sets according to topic, and the lightly edited solutions are as published in *Crux Mathematicorum.*

This title is published by the American Mathematical Society (AMS) in collaboration with the Canadian Mathematical Society (CMS).

Problem Books, Volume 21

January 2009, 247 pages, Hardcover, ISBN: 978-0-88385-828-8, LC 2008939094, **Individual member US$48.75**, List US$65, Institutional member US$52, Order code PRB/21

Mathematical Physics



From Vertex Operator Algebras to Conformal Nets and Back

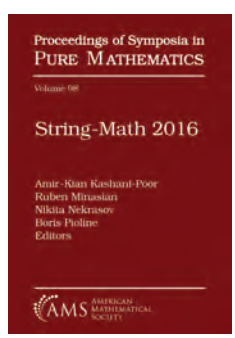
Sebastiano Carpi, *Università di Chieti-Pescara "G. d'Annunzio", Italy*, **Yasuyuki Kawahigashi**, *University of Tokyo, Japan*, **Roberto Longo**, *Università di Roma "Tor Vergata", Italy*, and **Mihály Weiner**, *Budapest University of Technology and Economics, Hungary*

This item will also be of interest to those working in algebra and algebraic geometry.

Contents: Introduction; Preliminaries on von Neumann algebras; Preliminaries on conformal nets; Preliminaries on vertex algebras; Unitary vertex operator algebras; Energy bounds and strongly local vertex operator algebras; Covariant subnets and unitary subalgebras; Criteria for strong locality and examples; Back to vertex operators; Appendix A. Vertex algebra locality and Wightman locality; Appendix B. On the Bisognano-Wichmann property for representations of the Möbius group; Bibliography.

Memoirs of the American Mathematical Society, Volume 254, Number 1213

June 2018, 85 pages, Softcover, ISBN: 978-1-4704-2858-7, 2010 *Mathematics Subject Classification:* 17B69, 46L60, 81T05, **Individual member US$46.80**, List US$78, Institutional member US$62.40, Order code MEMO/254/1213



String-Math 2016

Amir-Kian Kashani-Poor, *École Normale Supérieure, Paris, France*, **Ruben Minasian**, *Institut de Physique Théorique du CEA, Saclay, Gif-sur-Yvette, France*, **Nikita Nekrasov**, *Simons Center for Geometry and Physics, Stony Brook, NY*, and **Boris Pioline**, *Laboratoire de Physique Théorique et Hautes Energies, Paris, France*, Editors

This volume contains the proceedings of the conference String-Math 2016, which was held from June 27–July 2, 2016, at Collége de France, Paris, France.

String-Math is an annual conference covering the most significant progress at the interface of string theory and mathematics. The two fields have had a very fruitful dialogue over the last thirty years, with string theory contributing key ideas which have opened entirely new areas of mathematics and modern mathematics providing powerful concepts and tools to deal with the intricacies of string and quantum field theory.

The papers in this volume cover topics ranging from supersymmetric quantum field theories, topological strings, and conformal nets to moduli spaces of curves, representations, instantons, and harmonic maps, with applications to spectral theory and to the geometric Langlands program.

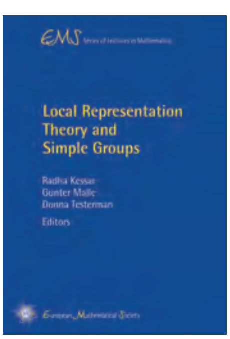
Contents: M. Bullimore, Three-dimensional $\mathcal{N} = 4$ gauge theories in omega background; **S. Cremonesi**, 3d supersymmetric gauge theories and Hilbert series; **R. Kodera** and **H. Nakajima**, Quantized Coulomb branches of Jordan quiver gauge theories and cyclotomic rational Cherednik algebras; **A. Balasubramanian** and **J. Teschner**, Supersymmetric field theories and geometric Langlands: The other side of the coin; **O. Dumitrescu**, A journey from the Hitchin section to the oper moduli; **D. Gaiotto**, S-duality of boundary conditions and the Geometric Langlands program; **P. Gavrylenko** and **O. Lisovyy**, Pure $SU(2)$ gauge theory partition function and generalized Bessel kernel; **L. Katzarkov**, **P. Pandit**, and **C. Simpson**, Reduction for $SL(3)$ pre-buildings; **A. Henriques**, Conformal nets are factorization algebras; **A. Polishchuk**, Contracting the Weierstrass locus to a point; **M. Mariño**, Spectral theory and mirror symmetry.

Proceedings of Symposia in Pure Mathematics, Volume 98

June 2018, 294 pages, Hardcover, ISBN: 978-1-4704-3515-8, LC 2017052181, 2010 *Mathematics Subject Classification:* 14D24, 14H60, 14D21, 14J33, 58E20, 81T60, 81T30, **AMS members US$106.40**, List US$133, Order code PSPUM/98

New AMS-Distributed Publications

Algebra and Algebraic Geometry



Local Representation Theory and Simple Groups

Radha Kessar, *City University of London, United Kingdom*, **Gunter Malle**, *University of Kaiserslautern, Germany*, and **Donna Testerman**, *EPF Lausanne, Switzerland*

The book contains extended versions of seven short lecture courses given during a semester programme on Local Representation Theory and Simple Groups, held at the Centre Interfacultaire Bernoulli of the EPF Lausanne. These courses focused on modular representation theory of finite groups, modern Clifford theoretic methods, the representation theory of finite reductive groups, as well as on various applications of character theory and representation theory, for example, to base sizes and to random walks.

These lectures are intended to form a good starting point for graduate students and researchers who wish to familiarize themselves with the foundations of the topics covered here. Furthermore, they give an introduction to current research directions, including the state of some open problems in the field.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Series of Lectures in Mathematics, Volume 29

May 2018, 369 pages, Softcover, ISBN: 978-3-03719-185-9, 2010 *Mathematics Subject Classification:* 20Bxx, 20Cxx, 20Gxx, **AMS members US$46.40**, List US$58, Order code EMSSERLEC/29

Brackets in the Pontryagin Algebras of Manifolds

Gwénaël Massuyeau, *Université de Strasbourg and CNRS, Dijon, France*, and **Vladimir Turaev**, *Indiana University, Bloomington*

A fundamental geometric object derived from an arbitrary topological space M with a marked point $\star$ is the space of loops in M based at $\star$. The Pontryagin algebra A of $(M, \star)$ is the singular homology of this loop space with the graded algebra structure induced by the standard multiplication of loops. When M is a smooth oriented manifold with boundary and $\star$ is chosen on ∂M, the authors define an "intersection" operation $A \otimes A \to A \otimes A$.

The authors prove that this operation is a double bracket in the sense of Michel Van den Bergh satisfying a version of the Jacobi identity. The authors show that their double bracket induces Gerstenhaber brackets in the representation algebras of A. These results extend the authors' previous work on surfaces, where A is the group algebra of the fundamental group of a surface and the Gerstenhaber brackets in question are the usual Poisson brackets on the moduli spaces of representations of such a group.

The present work is inspired by the results of William Goldman on surfaces and by the ideas of string topology due to Moira Chas and Dennis Sullivan.

This item will also be of interest to those working in geometry and topology.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Mémoires de la Société Mathématique de France, Number 154

April 2018, 138 pages, Softcover, ISBN: 978-2-85629-876-3, 2010 *Mathematics Subject Classification:* 17B63, 55N33, 55P50, 57R19, **AMS members US$41.60**, List US$52, Order code SMFMEM/154

Analysis

Factorization of Non-Symmetric Operators and Exponential *H*-Theorem

M. P. Gualdani, *University of Texas at Austin, Texas*, **S. Mischler**, *Université Paris IX-Dauphine, France*, and **C. Mouhot**, *University of Cambridge, UK*

The authors present an abstract method for deriving decay estimates on the resolvents and semigroups of non-symmetric operators in Banach spaces in terms of estimates in another smaller reference Banach space. The core of the method is a high-order quantitative factorization argument on the resolvents and semigroups, and it makes use of a semigroup commutator condition of regularization.

The authors then apply this approach to the Fokker-Planck equation, to the kinetic Fokker-Planck equation in the torus, and to the linearized Boltzmann equation in the torus. Thanks to the latter results and to a non-symmetric energy method, the authors obtain the first constructive proof of exponential decay, with sharp rate, towards global equilibrium for the full non-linear Boltzmann equation for hard spheres, conditionally to some smoothness and (polynomial) moment estimates; this solves a conjecture about the optimal decay rate of the relative entropy in the *H*-theorem.

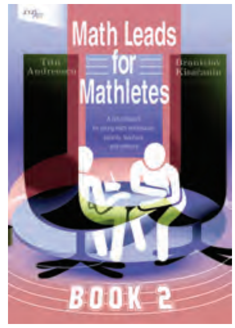
This item will also be of interest to those working in differential equations.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Mémoires de la Société Mathématique de France, Number 153

April 2018, 137 pages, Softcover, ISBN: 978-2-85629-874-9, 2010 *Mathematics Subject Classification:* 47D06, 34G10, 34K30, 35P15, 47H20, 37L05, 47J35, 54H15, 58D07, 35Q84, 76P05, 82B40, 82D05, **AMS members US$41.60**, List US$52, Order code SMFMEM/153

General Interest



Math Leads for Mathletes (Book 2)

A Rich Resource for Young Math Enthusiasts, Parents, Teachers, and Mentors

Titu Andreescu, *University of Texas at Dallas*, and **Branislav Kisačanin**, *NVIDIA Corporation and AwesomeMath*

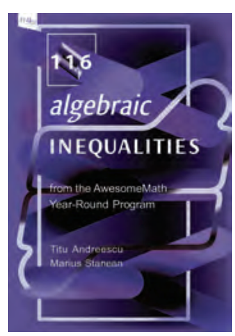
Math Leads for Mathletes (*Book 2*) is part of the Math Leads for Mathletes series, providing more challenging units for young math problem solvers and many others! The book draws on the authors' experience working with young mathletes and on the collective wisdom of mathematics educators around the world to help parents and mentors challenge and teach their aspiring math problem solvers. The topics contained in this book are best suited for middle schoolers, although students who discovered competitive mathematics in later grades will also benefit from the material. This book will help students advance in several directions important in competitive mathematics: algebra, combinatorics, geometry, and number theory. It presents a variety of problem solving strategies and challenges readers to explain their solutions, write proofs, and explore connections with other problems.

This item will also be of interest to those working in math education.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

XYZ Series, Volume 30

April 2018, 230 pages, Hardcover, ISBN: 978-0-9968745-5-7, 2010 *Mathematics Subject Classification:* 00A05, 00A07, 97U40, 97D50, **AMS members US$43.96**, List US$54.95, Order code XYZ/30



116 Algebraic Inequalities from the AwesomeMath Year-Round Program

Titu Andreescu, *University of Texas at Dallas*, and **Marius Stanean**, *Science Consultant with INDECO Software*

This book would certainly help Olympiad students who wish to prepare for the study of inequalities, a topic now of frequent use at various competitive levels. The inequalities from each section are ordered increasingly by the number of variables: one, two, three, four, and multivariables. Each problem has at least one complete solution and many problems have multiple solutions, useful in developing the necessary array of mathematical machinery for competitions.

This item will also be of interest to those working in math education.

A publication of XYZ Press. Distributed in North America by the American Mathematical Society.

XYZ Series, Volume 29

April 2018, 216 pages, Hardcover, ISBN: 978-0-9968745-8-8, 2010 *Mathematics Subject Classification:* 00A05, 00A07, 97U40, 97D50, **AMS members US$39.96**, List US$49.95, Order code XYZ/29

Geometry and Topology

Flag Varieties: An Interplay of Geometry, Combinatorics, and Representation Theory

Second Edition

V. Lakshmibai, *Northeastern University, Boston, MA*, and **Justin Brown**, *Northeastern University, Boston, MA*

Flag varieties are important geometric objects. Because of their richness in geometry, combinatorics, and representation theory, flag varieties may be described as an interplay of all three of these fields.

This book gives a detailed account of this interplay. In the area of representation theory, the book presents a discussion on the representation theory of complex semisimple Lie algebras as well as the representation theory of semisimple algebraic groups; in addition, the representation theory of symmetric groups is also discussed. In the area of algebraic geometry, the book gives a detailed account of the Grassmannian varieties, flag varieties, and their Schubert subvarieties. Because of the root system connections, many of the geometric results admit elegant combinatorial description, a typical example being the description of the singular locus of a Schubert variety. This discussion is carried out as a consequence of standard monomial theory (abbreviated SMT). Thus, the book includes SMT and some important applications—singular loci of Schubert varieties, toric degenerations of Schubert varieties, and the relationship between Schubert varieties and classical invariant theory.

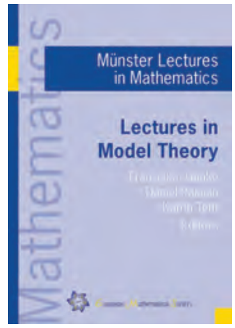
In the second edition, two recent results on Schubert varieties in the Grassmannian have been added. The first result gives a free resolution of certain Schubert singularities. The second result is about certain Levi subgroup actions on Schubert varieties in the Grassmannian and derives some interesting geometric and representation-theoretic consequences.

A publication of Hindustan Book Agency; distributed within the Americas by the American Mathematical Society. Maximum discount of 20% for all commercial channels.

Hindustan Book Agency

May 2018, 325 pages, Hardcover, ISBN: 978-93-86279-70-5, 2010 *Mathematics Subject Classification:* 14M15, **AMS members US$52**, List US$65, Order code HIN/76

Logic and Foundations



Lectures in Model Theory

Franziska Jahnke, *University of Münster, Germany*, **Daniel Palacín**, *Hebrew University of Jerusalem, Israel*, and **Katrin Tent**, *University of Münster, Germany*, Editors

Model theory is a thriving branch of mathematical logic with strong connections to other fields of mathematics. Its versatility has recently led to spectacular applications in areas ranging from diophantine geometry, algebraic number theory and group theory to combinatorics.

This volume presents lecture notes from a spring school in model theory which took place in Münster, Germany. The notes are aimed at Ph.D. students but should also be accessible to undergraduates with some basic knowledge in model theory. They contain the core of stability theory (Bays, Palacín), two chapters connecting generalized stability theory with group theory (Clausen and Tent, Simon), as well as introductions to the model theory of valued fields (Hils, Jahnke) and motivic integration (Halupczok).

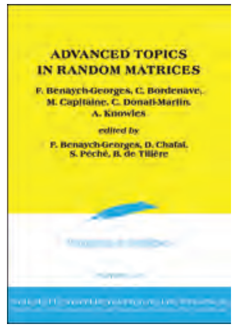
This item will also be of interest to those working in discrete mathematics and combinatorics.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Münster Lectures in Mathematics, Volume 2

May 2018, 222 pages, Softcover, ISBN: 978-3-03719-184-2, 2010 *Mathematics Subject Classification:* 03C45, 03C60, 03C98; 05E15, 12J20, 12L12, 14E18, 20E18, **AMS members US$38.40**, List US$48, Order code EMSMLM/2

Probability and Statistics



Advanced Topics in Random Matrices

F. Benaych-Georges, *Université Paris Descartes, France*, **C. Bordenave**, *Université de Toulouse, France*, **M. Capitaine**, *Institut de Mathématiques de Toulouse, France*, **C. Donati-Martin**, *Université Versailles St. Quentin, France*, and **A. Knowles**, *University of Geneva, Switzerland*

Edited by F. Benaych-Georges, D. Chafaï, S. Péché, and B. de Tilière

This book provides three accessible panoramas and syntheses on advanced topics in random matrix theory: (1) local semicircle law for Wigner matrices and applications to eigenvectors delocalization, rigidity of eigenvalues, and fourth moment theorem; (2) spectrum of random graphs, recent advances on eigenvalues and eigenvectors, and open problems; and (3) deformed random matrices and free probability, unified understanding of various asymptotic phenomena, such as spectral measure description, localization and fluctuations of extremal eigenvalues, and eigenvectors behavior.

This item will also be of interest to those working in discrete mathematics and combinatorics.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Panoramas et Synthèses, Number 53

April 2018, 190 pages, Softcover, ISBN: 978-2-85629-850-3, 2010 *Mathematics Subject Classification:* 60B20, 05C80, 46L54, **AMS members US$53.60**, List US$67, Order code PASY/53

MEETINGS & CONFERENCES OF THE AMS

JUNE/JULY TABLE OF CONTENTS

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated.

The most up-to-date meeting and conference information can be found online at: **www.ams.org/meetings**.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to page 88 in the January 2018 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit **www.ams.org/cgi-bin/abstracts/abstract.pl**. Questions about abstracts may be sent to `abs-info@ams.org`. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

MEETINGS IN THIS ISSUE

See `www.ams.org/meetings` *for the most up-to-date information on the meetings and conferences that we offer.*

ASSOCIATE SECRETARIES OF THE AMS

Central Section: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; email: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; email: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, email: brian@math.uga.edu; telephone: 706-542-2547.

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; email: lapidus@math.ucr.edu; telephone: 951-827-5910.

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See www.ams.org/meetings/.

Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL .

Shanghai, People's Republic of China

Fudan University

June 11–14, 2018

Monday – Thursday

Meeting #1140

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: April 2018
Program first available on AMS website: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

Yu-Hong Dai, Academy of Mathematics and System Sciences, *An overview of unconstrained optimization.*

Kenneth A. Ribet, University of California, Berkeley, *The Eisenstein ideal and the arithmetic of modular curves and their Jacobians.*

Richard M. Schoen, University of California, Irvine, *Geometry and general relativity.*

Sijue Wu, University of Michigan, *On the motion of water waves with angled crests.*

Chenyang Xu, Peking University, *Compact moduli spaces.*

Jiangong You, Nankai University, *Quasi-periodic Schrödinger operators.*

Special Sessions

Additive Combinatorics including its Interplay with Factorization Theory (SS 1), **Weidong Gao**, Nankai University, **Alfred Geroldinger**, University of Graz, and **David J. Grynkiewicz**, University of Memphis.

Algebraic Geometry (SS 3), **Davesh Maulik**, Massachusetts Institute of Technology, and **Chenyang Xu**, Peking University.

Algebraic and Geometric Topology (SS 2), **Michael Hill**, University of California at Los Angeles, **Zhi Lü** and **Jiming Ma**, Fudan University, and **Yifei Zhu**, Southern University of Science and Technology.

Asymptotically Hyperbolic Einstein Manifolds and Conformal Geometry (SS 4), **Jie Qing**, University of California Santa Cruz and Beijing International Center for Mathematical Research, **Mijia Lai** and **Fang Wang**, Shanghai Jiao Tong University, and **Meng Wang**, Zhejiang University.

Complex Geometry and Several Complex Variables (SS 5), **Qingchun Ji**, Fudan University, **Min Ru**, University of Houston, and **Xiangyu Zhou**, Chinese Academy of Sciences.

Computer Science (SS 6), **Erich Kaltofen**, North Carolina State University, and **Lihong Zhi**, Academy of Mathematics and Systems Science, Chinese Academy of Sciences.

Cybernetics (SS 7), **Alberto Bressan**, Pennsylvania State University, and **Xu Zhang**, Sichuan University.

Geometric Models and Methods in Quantum Gravity (SS 8), **Peng Wang**, Sichuan University, and **P. P. Yu**, Westminster College.

Geometric Representation Theory and the Langlands Program (SS 9), **Dihua Jiang**, University of Minnesota, **Yiqiang Li**, State University of New York at Buffalo, **Peng Shan**, Tsinghua University, and **Binyong Sun**, Academy of Mathematics and Systems Science, Chinese Academy of Sciences.

Geometry (SS 10), **Jiayu Li**, University of Science and Technology of China, and **Jie Qing**, University of California Santa Cruz and Beijing International Center for Mathematical Research.

Harmonic Analysis and Partial Differential Equations (SS 11), **Hong-Quan Li**, Fudan University, and **Xiaochun Li**, UIUC.

Harmonic Maps and Related Topics (SS 12), **Yuxin Dong**, Fudan University, **Ye-Lin Ou**, Texas A&M University-Commerce, **Mei-Chi Shaw**, University of Notre Dame, and **Shihshu Walter Wei**, University of Oklahoma.

Inverse Problems (SS 13), **Gang Bao**, Zhejiang University, and **Hong-Kai Zhao**, University of California at Irvine.

Mathematics of Planet Earth: Natural Systems and Models (SS 14), **Daniel Helman**, Ton Duc Thang University, and **Huaiping Zhu**, York University.

Noncommutative Algebra and Related Topics (SS 15), **Quanshui Wu**, Fudan University, and **Milen Yakimov**, Louisiana State University.

Nonlinear Analysis and Numerical Simulations (SS 16), **Jifeng Chu**, Shanghai Normal University, **Zhaosheng Feng**, University of Texas-Rio Grande Valley, and **Juntao Sun**, Shandong University of Technology.

Nonlinear Dispersive Equations (SS 17), **Marius Beceanu**, University at Albany SUNY, and **Chengbo Wang**, Zhejiang University.

Number Theory (SS 18), **Hourong Qin**, Nanjing University, and **Wei Zhang**, Columbia University.

Numerical Analysis (SS 19), **Jin Cheng**, Fudan University, and **Jie Shen**, Purdue University.

Operations Research (SS 20), **Yanqin Bai**, Shanghai University, **Yu-Hong Dai**, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, and **Jiming Peng**, University of Houston.

Ordinary Differential Equations and Dynamical Systems (SS 21), **Jiangong You**, Nankai University, and **Kening Lu**, Brigham Young University.

Partial Differential Equation–Elliptic and Parabolic (SS 22), **Xinan Ma**, University of Science and Technology of China, and **Lihe Wang**, University of Iowa.

Partial Differential Equations–Hyperbolic (SS 23), **Hongjie Dong**, Brown University, and **Zhen Lei**, Fudan University.

Probability (SS 24), **Zhenqing Chen**, University of Washington, and **Zenghu Li**, Beijing Normal University.

Quantum Algebras and Related Topics (SS 25), **Yun Gao**, York University, **Naihuan Jing**, North Carolina State University, and **Honglian Zhang**, Shanghai University.

Recent Advances in Numerical Methods in Partial Differential Equations (SS 26), **Ying Li**, Shanghai University, and **Jia Zhao**, Utah State University.

Recent Advances in Stochastic Dynamical Systems and their Applications (SS 27), **Xiaofan Li**, Illinois Institute of Technology, and **Yanjie Zhang**, Huazhong University of Science and Technology.

Singularities in Geometry, Topology, and Algebra (SS 28), **Rong Du**, East China Normal University, **Yongqiang Liu**, KU Leuven, and **Laurentiu Maxim** and **Botong Wang**, University of Wisconsin.

Statistics (SS 29), **Jianhua Guo**, Northeastern Normal University, and **Xumin He**, University of Michigan.

Symplectic Geometry (SS 30), **Qile Chen**, Boston College, **Huijun Fan**, Peking University, and **Yongbin Ruan**, University of Michigan.

Toplogical Thinking about Mathematics of Data and Complex Information (SS 31), **Amir Assadi**, University of Wisconsin and Beijing Institute of Technology, **Dan Burghelea**, Ohio State University, **Huafei Sun**, Beijing Institute of Technology, and **Yazhen Wang**, University of Wisconsin.

Newark, Delaware

University of Delaware

September 29–30, 2018

Saturday – Sunday

Meeting #1141

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: June 2018
Program first available on AMS website: August 9, 2018
Issue of *Abstracts*: Volume 39, Issue 3

Deadlines

For organizers: Expired
For abstracts: July 31, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Leslie Greengard, New York University, *Linear and nonlinear inverse problems in imaging.*

Elisenda Grigsby, Boston College, *Braids, surfaces, and homological invariants.*

Davesh Maulik, Massachusetts Institute of Technology, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Numerical Approximation of Partial Differential Equations (Code: SS 8A), **Constantin Bacuta** and **Jingmei Qiu**, University of Delaware.

Applied Algebraic Topology (Code: SS 2A), **Chad Giusti**, University of Delaware, and **Gregory Henselman**, Princeton University.

Billiard Dynamics: Standard and Alternative Collision Models (Code: SS 15A), **Tim Chumley**, Mount Holyoke College, **Chris Cox**, University of Delaware, and **Renato Feres**, Washington University in St. Louis.

Commutative Algebra (Code: SS 19A), **Ela Celikbas**, West Virginia University, **Sema Gunturkun**, University of Michigan, and **Oana Veliche**, Northeastern University.

Convex Geometry and Functional Inequalities (Code: SS 3A), **Mokshay Madiman**, University of Delaware, **Elisabeth Werner**, Case Western Reserve University, and **Artem Zvavitch**, Kent State University.

Fixed Point Theory with Application and Computation (Code: SS 7A), **Clement Boateng Ampadu**, Boston, MA, **Penumarthy Parvateesam Murthy**, Guru Ghasidas Vishwavidyalaya, Bilaspur, India, **Naeem Saleem**, University of Management and Technology, Lahore, Pakistan, **Yaé Ulrich Gaba**, Institut de Mathématiques et de Sciences Physiques (IMSP), Porto-Novo, Bénin, and **Xavier Udo-utun**, University of Uyo, Uyo, Nigeria.

Graph Theory (Code: SS 12A), **Sebastian M. Cioabă**, University of Delaware, **Brian Kronenthal**, Kutztown University of Pennsylvania, **Felix Lazebnik**, University of Delaware, and **Wing Hong Tony Wong**, Kutztown University of Pennsylvania.

Interplay between Analysis and Combinatorics (Code: SS 5A), **Mahya Ghandehari** and **Dominique Guillot**, University of Delaware.

Modern Quasiconformal Analysis and Geometric Function Theory (Code: SS 6A), **David Herron**, University of Cincinnati, and **Yuk-J Leung**, University of Delaware.

Nonlinear Water Waves and Related Problems (Code: SS 9A), **Philippe Guyenne**, University of Delaware.

Operator and Function Theory (Code: SS 4A), **Kelly Bickel**, Bucknell University, **Michael Hartz**, Washington University, St. Louis, **Constanze Liaw**, University of Delaware, and **Alan Sola**, Stockholm University.

Probability, Combinatorics, and Statistical Mechanics (Code: SS 18A), **Nayantara Bhatnagar** and **Douglas Rizzolo**, University of Delaware.

Quantum Correlation Sets in Quantum Information Theory (Code: SS 13A), **Elie Alhajjar** and **Travis B. Russell**, US Military Academy.

Recent Advances in Nonlinear Schrödinger Equations (Code: SS 1A), **Alexander Pankov**, Morgan State University, **Junping Shi**, College of William and Mary, and **Jun Wang**, Jiangsu University.

Recent Analytic and Numeric Results on Nonlinear Evolution Equations (Code: SS 10A), **Xiang Xu**, Old Dominion University, and **Wujun Zhang**, Rutgers University.

Representations of Infinite Dimensional Lie Algebras and Applications (Code: SS 16A), **Marco Aldi**, Virginia Commonwealth University, **Michael Penn**, Randolph College, and **Juan Villarreal**, Virginia Commonwealth University.

Stochastic Processes in Mathematical Biology (Code: SS 14A), **Yao Li**, University of Massachusetts Amherst, and **Abhyudai Singh**, University of Delaware.

The Mathematics of Swimmers and Active Particles (Code: SS 11A), **Louis Rossi**, University of Delaware.

Accommodations

Participants should make their own arrangements directly with the hotel of their choice. Special discounted rates were negotiated with the hotels listed below. Rates quoted do not include an 8% occupancy tax. Participants must state that they are with the **American Mathematical Society (AMS) Meeting at the University of Delaware/ AMS Fall Eastern Sectional Meeting** to receive the discounted rate. The AMS is not responsible for rate changes or for the quality of the accommodations. **Hotels have varying cancellation and early checkout penalties; be sure to ask for details.**

Candlewood Suites Newark South - University Area, 1101 South College Ave Newark, Delaware 19713; (302) 368-5500, `https://www.ihg.com/candlewood/hotels/us/en/newark/ilgcw/hoteldetai`. Rates are US$109 per night for a studio room with two queen beds, or a studio room with one queen bed, this rate is applicable for single or double occupancy. To reserve a room at these rates online please use this booking link: `https://www.candlewoodsuites.com/redirect?path=hd-amenities&brandCode=CW&localeCode=en&regionCode=1&hotelCode=ILGCW&_PMID=99801505&GPC=AMS&viewfullsite=true`. To reserve a room over the phone please contact the hotel directly and identify the block of rooms for the **American Mathematical Society**. This is an all suite property. All rooms include a fully equipped kitchen with appliances, including a full-sized fridge. Amenities include complimentary wired or wireless Internet access in sleeping rooms and public spaces, fitness center, 24-hour business center, on-site self-laundry services, and convenience store, *Candlewood Cupboard*. Complimentary parking is available on-site. Check-in is at 3:00 pm, check-out is at 11:00 am. This property is located less than 2 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **September 7, 2018.**

Comfort Inn and Suites, 3 Concord Lane, Newark, DE, 19713; (302) 737-3900, `https://www.choicehotels.com/delaware/newark/comfort-inn-hotels/de052`. Rates are US$102 per night for a guest room with two queen beds or one king bed, single or double occupancy. To reserve a room at these rates online please use this booking link: `https://www.choicehotels.com/reservations/groups/OJ83K5`. To reserve a room over the phone please contact the hotel directly and identify the block of rooms for the **AMS Sectional Meeting**. Amenities include complimentary wireless internet, fitness center, seasonal outdoor pool, in-room mini fridge and microwave, free hot breakfast (served between 6:00 am–10 am daily), and complimentary parking. Check-in is at 3:00 pm, check-out is at 11:00 am. This property is located approximately 5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. This rate is valid until the dates of the meeting or until there are no rooms available for booking at this rate**.**

Courtyard by Marriott Newark—University of Delaware, 400 David Hollowell Drive, Newark, Delaware, 19716; (302) 737-0900, https://www.marriott.com/hotels/travel/ilgud-courtyard-newark-university-of-delaware. Rates are US$159 per night for a deluxe single king bedded room or a deluxe double queen bedded room. To reserve a room at this rate online please use this booking link: https://protect-us.mimecast.com/s/fzbeCyPAvECKOpKfZ-sX6?domain=marriott.com. To reserve a room by phone please contact the hotel directly at the number listed above or use the Marriott Central Reservations line at (800) 321-2211 and identify the block of rooms for the **AMS Fall Eastern Sectional Meeting**. Amenities include complimentary high-speed internet access, fitness center, indoor swimming pool, *the Bistro* on-site restaurant offering room service and serving breakfast and dinner, and complimentary on-site parking. This property is pet-friendly with prior notice and a US$75 fee. Check-in is at 3:00 pm, check-out is at noon. This property is located less than one mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 31, 2018.**

Embassy Suites Newark-Wilmington/South, 654 South College Avenue, Newark, Delaware, 19713; (302) 368-8000; embassysuites3.hilton.com/en/hotels/delaware/embassy-suites-by-hilton-newark-wilmington-south-NEWDEES/index.html. Rates are US$140 per night for a guest room with double occupancy, with two queens or one king bed. Each additional adult added will be at a rate of US$15 per night. To reserve a room at this rate online please visit: protect-us.mimecast.com/s/TZtxCZ6G9JfAXrgTxJYoL. To reserve a room by phone please contact the hotel directly at (302) 368-8000 and identify the block of rooms for the **AMS Fall Eastern Sectional Meeting**. This is an all suite property. Amenities include complimentary wireless internet access, pull-out sleeper sofas, in-room microwaves and fridges, indoor pool, fitness center, business center, snack shop, free made-to-order hot breakfast and complimentary evening reception, on-site *T.G.I.Friday's* restaurant, and complimentary on-site parking. Check-in is at 3:00 pm, check-out is at noon. This property is located approximately 1.5 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 29, 2018.**

Red Roof Inn and Suites Newark - University, 1119 South College Avenue, Newark, DE 19713; (937) 328-1778, https://newark.redroof.com. Rates are US$60 per night for a guest room with one king bed or US$65 for a guest room with two queen beds, this rate is applicable for single or double occupancy. To reserve a room at this rate, please contact the hotel directly and identify the block of rooms for the **American Mathematical Society or block #094-358791**. Amenities include complimentary wireless internet access, in-room microwave and refrigerator, fitness center, business center, complimentary continental breakfast, on-property *Friendly's* restaurant, and complimentary parking. This property is pet-friendly. Check-in is at 4:00 pm, check-out is at 11:00 am. This property is located approximately 1.7 miles from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations in this block is **August 29, 2018,** however, if there is availability of rooms the rate will be honored beyond that date.

SpringHill Suites Newark Downtown, 402 Ogletown Road, Newark, Delaware 19711; (302) 273-1000, https://www.marriott.com/hotels/hotel-reviews/ilgss-springhill-suites-newark-downtown. Rates are US$129 per night for a king bedded suite, single or double occupancy; US$145 for a suite with two queen beds, single or double occupancy. To reserve a room please contact Marriott Reservations at (866) 877-7795 or contact the hotel directly at (302) 273-1000 and identify the block of rooms for the **AMS Fall Sectional Meeting at the University of Delaware**. Amenities include complimentary wired or wireless internet access, fitness center, indoor pool, in-room mini fridge and microwave, business center, complimentary made-to-order breakfast, and complimentary parking. This property is pet-friendly, with prior notice and a US$25 fee per pet, per night. Check-in is at 3:00 pm, check-out is at noon. This property is located approximately one mile from campus. Cancellation and early check-out policies vary and penalties exist at this property; be sure to check when you make your reservation. The deadline for reservations at this rate is **August 31, 2018.**

Housing Warning

Please beware of aggressive housing bureaus that target potential attendees of a meeting. They are sometimes called "room poachers" or "room-block pirates" and these companies generally position themselves as a meeting's housing bureau, convincing attendees to unknowingly book outside the official room block. They call people who they think will more likely than not attend a meeting and lure them with room rates that are significantly less than the published group rate – for a limited time only. And people who find this offer tempting may hand over their credit card data, believing they have scored a great rate and their housing is a done deal. Unfortunately, this often turns out to be the start of a long, costly nightmare.

These housing bureaus are not affiliated with the American Mathematical Society or any of its meetings, in any way. The AMS would never call anyone to solicit reservations for a meeting. The only way to book a room at a rate negotiated for an AMS Sectional Meeting is through AMS Sectional Meetings pages. The AMS cannot be responsible for any damages incurred as a result of hotel bookings made with unofficial housing bureaus.

Food Services

On Campus: There is one option nearby for participants looking to dine on campus. The closest dining facility to the meeting is a food court located in the Trabant Student Center. During weekends this facility is due to be open between the hours 8:00 am–10:00 pm. These hours are subject to change, please visit the AMS website closer to the dates of the meeting, for any changes.

Off Campus: Local organizers encourage participants to visit downtown Newark for options for lunch and dinner during the meeting. Main Street offers a variety of options and is within walking distance from Gore Hall and is adjacent to campus. For more information on dining throughout the downtown area please visit, `enjoydowntownnewark.com/businesses/`

Some dining options nearby to the Main Campus include:

Ali Baba's, 175 East Main Street, (302) 738-1111, `www.alibabade.com/`; Saturday and Sunday, 4:00 pm–11:00 pm; Authentic Middle Eastern cuisine; Moroccan, Lebanese, and Israeli.

Caffe Gelato, 90 East Main Street, (302) 738-5811, `www.caffegelato.net`; Saturday 10:00 am–10:00 pm and Sunday 10:00 am–8:00 pm; Award-winning restaurant and wine cellar serving brunch, lunch and dinner.

Catherine Rooney's Irish Pub, 102 East Main Street, (302) 369-7330, `catherinerooneys.com/cr-newark/`; Saturday and Sunday 10:00 am–1:00 am; serving pub fare.

Chef Tan, 108 E Main St, (302) 366-0900, `www.cheftan.com/`; Saturday 11:00 am–9:30 pm; Sun 11:00 am–9:00 pm; serving Chinese cuisine.

Home Grown Cafe, 126 East Main Street, (302) 266-6993, `www.homegrowncafe.com`; Saturday 9:30 am–1:00 am and Sunday 9:30 am–12:00 am; serving internationally inspired award-winning gourmet cuisine with a local flair; upscale dining with local art and music.

Iron Hill Brewery, 147 East Main Street, (302) 266-9000, `www.ironhillbrewery.com`, Saturday 11:00 am–1:00 am, Sunday 11:00 am–11:00 pm; upscale, casual brewery and restaurant.

Jake's Wayback Burgers, 250 South Main Street, Suite 110, (302) 861-6050, `https://waybackburgers.com`; Saturday 11:00 am–9:00 pm and Sunday 11:00 am - 7:00 pm; National burger chain started in Newark, DE.

Ramen Kumamoto, 165 E Main St, (302) 733-0888, `https://ramenkumamoto.eat24hour.com/menu`; Saturday 2:00 pm–10:30 pm and Sunday 2:00 pm–9:30 pm; Ramen and Japanese Cuisine.

The Perfect Blend, 249 E. Main Street (302) 276-5488, Saturday 7:00 am-4:00 pm and Sunday 8:00 am-3:00 pm; `www.theperfectblendinc.net`. Offering coffee, smoothies and Belgian Liege waffles.

Taverna, 121 East Main Street, (302) 444-4334, `www.tavernamainstreet.com/`; Saturday 11:30 am–1:00 am and Sunday 11:30 am–10:00 pm; rustic Italian eatery featuring coal oven pizza, hand crafted pastas & authentic Italian cuisine.

Registration and Meeting Information

Advance Registration: Advance registration for this meeting will open on **August 1st.** Advance registration fees are US$63 for AMS members, US$95 for nonmembers, and US$10 for students, unemployed mathematicians, and emeritus members. Participants may cancel registrations made in advance by emailing `mmsb@ams.org`. The deadline to cancel is the first day of the meeting.

On site Information and Registration: The registration desk, AMS book exhibit, AMS membership exhibit, and coffee service will be located in Gore Hall. The Invited Addresses will be held in Smith Hall, Room 120. Special Sessions and Contributed Paper Sessions will take place in classrooms in Gore Hall.

For additional information on building locations and a printable campus map, visit `https://www.udel.edu/content/dam/udelImages/main/pdfs/maps/CampusMap2015.pdf`

The registration desk will be open on Saturday, September 29, 7:30 am–4:00 pm and Sunday, September 30, 8:00 am–12:00 pm. The same fees apply for on-site registration, as for advance registration. Fees are payable on-site via cash, check, or credit card.

Other Activities

Book Sales: Stop by the on site AMS bookstore to review the newest publications and take advantage of exhibit discounts and free shipping on all on site orders! AMS and MAA members receive 40% off list price. Nonmembers receive a 25% discount. Not a member? Ask a representative about the benefits of AMS membership.

AMS Editorial Activity: An acquisitions editor from the AMS book program will be present to speak with prospective authors. If you have a book project that you wish to discuss with the AMS, please stop by the book exhibit.

Membership Activities: During the meeting, stop by the AMS Membership Exhibit to learn about the benefits of AMS Membership. Members receive free shipping on purchases all year long and additional discounts on books purchased at meetings, subscriptions to Notices and Bulletin, discounted registration for world-class meetings and conferences, and more!

Complimentary refreshments will be served courtesy, in part, by the AMS Membership Department.

Special Needs

It is the goal of the AMS to ensure that its conferences are accessible to all, regardless of disability. The AMS will strive, unless it is not practicable, to choose venues that are fully accessible to the physically handicapped.

If special needs accommodations are necessary in order for you to participate in an AMS Sectional Meeting, please communicate your needs in advance to the AMS Meetings Department by:

- Registering early for the meeting

-Checking the appropriate box on the registration form, and

- Sending an email request to the AMS Meetings Department at `mmsb@ams.org` or `meet@ams.org`.

AMS Policy on a Welcoming Environment

The AMS strives to ensure that participants in its activities enjoy a welcoming environment. In all its activities, the

AMS seeks to foster an atmosphere that encourages the free expression and exchange of ideas. The AMS supports equality of opportunity and treatment for all participants, regardless of gender, gender identity or expression, race, color, national or ethnic origin, religion or religious belief, age, marital status, sexual orientation, disabilities, or veteran status.

Harassment is a form of misconduct that undermines the integrity of AMS activities and mission.

The AMS will make every effort to maintain an environment that is free of harassment, even though it does not control the behavior of third parties. A commitment to a welcoming environment is expected of all attendees at AMS activities, including mathematicians, students, guests, staff, contractors and exhibitors, and participants in scientific sessions and social events. To this end, the AMS will include a statement concerning its expectations towards maintaining a welcoming environment in registration materials for all its meetings, and has put in place a mechanism for reporting violations. Violations may be reported confidentially and anonymously to 855-282-5703 or at www.mathsociety.ethicspoint.com. The reporting mechanism ensures the respect of privacy while alerting the AMS to the situation. Violations may also be brought to the attention of the coordinator for the meeting (who is usually at the meeting registration desk), and that person can provide advice about how to proceed.

For AMS policy statements concerning discrimination and harassment, see the AMS Anti-Harassment Policy at see the `www.ams.org/about-us/governance/policy-statements/anti-harassment-policy`.

Questions about this welcoming environment policy should be directed to the AMS Secretary at `www.ams.org/about-us/governance/sec-contact`.

Local Information and Maps

This sectional will take place on the Main Campus of the University of Delaware in Newark, Delaware. A campus map can be found at `css-rdms1.win.udel.edu/maps`. Information about the University of Delaware Department of Mathematical Sciences can be found at `https://www.mathsci.udel.edu`. For additional information about the University please visit the University of Delaware website at `www.udel.edu`.

The meeting will occupy space in Gore Hall and Smith Hall. For a printable version of the University map please visit `https://www.udel.edu/content/dam/udelImages/main/pdfs/maps/CampusMap2015.pdf`

Parking

Parking is available at the two UD parking garages located near Gore Hall. These are the Trabant Center Garage and the Center for the Arts Garage. Information about these locations can be found here: `https://sites.udel.edu/parking/permit-info`. Parking in these garages on the weekends costs US$6 per day, with no in and out privileges.

Travel

The University of Delaware is located in the heart of Newark, Delaware. Philadelphia International Airport is the closest airport to the University. Additionally Baltimore International Airport is an option for travel to the University, though it is located farther from campus. The most common types of transportation used from the airports, are rental cars, taxis, shuttles, and commuter trains. Shuttle service is available from both airports via Delaware Express. For more information please visit `https://delexpress.com/airport-shuttle -service`.

By Air:

Philadelphia International Airport (PHL) is the closest airport to the University of Delaware. Philadelphia Airport is located approximately 35 miles from the University.

Rental cars are available at PHL. Rental car companies may be contacted through the Ground Transportation Information desks located in Baggage Claims A-East, B/C and D/E. Agencies available at this airport include *Alamo*, *Avis*, *Budget*, *Dollar*, *Enterprise*, *Hertz*, and *National*.

Taxi, limousine, and shuttle services are available at Terminals A-East, B-C, D/E baggage claim areas. Terminal F ground transportation is accessible outside Terminal E baggage claim. For more information on these options, please visit the PHL website here; `www.phl.org/Pages/Passengerinfo/TransportationServices/taxi.aspx`.

To use SEPTA (Southeastern Pennsylvania Transportation Authority) trains to travel from PHL: Trains stop at Airport Terminals A, B, C & D, and E & F. For a terminal map of the Philadelphia International Airport, visit the airport's Web site at `www.phl.org/terminal_map.html`. Take SEPTA's Regional Rail Airport Line to the University City stop. At the University City stop take the Wilmington/Newark Regional Rail Line (M–F) to Newark Station. On the weekends, there is no SEPTA service to the Newark Train Station.

Baltimore International Airport (BWI) is located in Baltimore, Maryland, approximately 70 miles from the University.

Rental cars are available at BWI. BWI Marshall Airport has a rental car facility. The facility is located at Stoney Run Road and New Ridge Road. Free shuttle service carries customers to and from the airport approximately every ten minutes. Passengers arriving on flights should take the free shuttle from the lower level terminal for a ten-minute ride to the new facility. When returning a vehicle, look for highway directional signs to the facility. The Car Rental Facility is located at: 7432 New Ridge Rd. Hanover, MD 21076. Agencies available at this airport include *Alamo*, *Avis*, *Budget*, *Dollar*, *Enterprise*, *Hertz*, *National*, *Next Car*, *Payless*, and *Thrifty*.

Taxis are available at BWI Airport. Taxi stands are located just outside of the baggage claim area on the Lower Level of the terminal, near doors 5 and 13. Please note that this service is available from BWI Airport only. For cab service to BWI Airport, please consult your local cab company. BWI Airport taxicabs are prohibited from charging flat rates. For more information, call (410) 859-1100 or visit https://www.bwiairporttaxi.com

To use train to travel from BWI: AMTRAK service to/from BWI Airport is available from Newark Train Station (limited) and Wilmington Train Station. Please visit the AMTRAK website for more information on schedules, https://www.amtrak.com. AMTRAK tickets can be purchased online.

To use bus service to travel from BWI: Greyhound provides service between BWI airport and the University of Delaware. Greyhound service departs Newark (Park & Ride at 896 & Rt 4) at 2:40 p.m. and arrives at BWI at 6:30 p.m. (7 days a week) For more information on Greyhound Bus Service please visit their website at https://www.greyhound.com/

By Train: The Newark area is served by Amtrak and SEPTA (Southeastern Pennsylvania Transportation Authority) trains. Reservations can be made on Amtrak at www.amtrak.com. Information about regional trains on the SEPTA (Southeastern Pennsylvania Transportation Authority) lines can be found at www.septa.com. Please note that on the weekends, there is no SEPTA service to the Newark Train Station.

By Bus: The Newark area is served by bus lines including Greyhound and Megabus. Greyhound Buses use both the Delaware Welcome Center at 530 JFK Memorial Hwy, Newark; (800) 231-2222 and the Wilmington Bus Station, 101 N French St, Wilmington, DE; (302) 655-6111. For more information on Greyhound service please visit their website at https://www.greyhound.com. Megabus utilizes a bus stop on campus at Parking Lot #6 on Christiana Dr. For more information on Megabus service please visit their website at https://us.megabus.com.

By Car: All driving directions will end at the Visitors Center on the Newark Campus of the University of Delaware. Continue on campus on South College Ave to arrive at Gore and Smith Halls.

From the north: Take I-95 South to Delaware Exit 1-B — Route 896 North, which becomes South College Avenue at the intersection of Route 4. Continue straight on South College Avenue for about two miles, past such landmarks as the Bob Carpenter/Sports Convocation Center, Delaware Stadium, and the Delaware Field House. Continue going straight through the intersection of South College Avenue and Park Place until you reach the Visitors Center which will be on the corner of Winslow and South College Avenue on a campus map.

From the south: Take I-95 North to Delaware Exit 1 — Route 896 North, which becomes South College Avenue at the intersection of Route 4. Continue straight on South College Avenue for about two miles, past such landmarks as the Bob Carpenter/Sports Convocation Center, Delaware Stadium, and the Delaware Field House. The Visitors Center will be on the corner of Winslow and South College Avenue on a campus map.

From the west: Take the Pennsylvania Turnpike East to Route 283 South. Continue on Route 283 South, which becomes Route 30 East outside of Lancaster, to Route 896 South. As you reach campus and cross the railroad tracks, immediately turn left onto Delaware Avenue and right (at the second traffic light) onto South College Avenue. You will see the Visitors Center on the corner of Winslow and South College Avenue on a campus map.

Local Transportation

Car Rental: *Hertz* is the official car rental company for the meeting. To make a reservation accessing our special meeting rates online at www.hertz.com, click on the box "I have a discount", and type in our convention number (CV): 04N30008. You can also call Hertz directly at 800-654-2240 (US and Canada) or 405-749-4434 (other countries). At the time of your reservation, the meeting rates will be automatically compared to other Hertz rates and you will be quoted the best comparable rate available. Meeting rates include unlimited mileage and are subject to availability. Advance reservations are recommended, blackout dates may apply.

Taxi Service: Licensed, metered taxis are available in Newark. Some options in the Newark area include *Delaware Taxi*, (302) 357-1080, (delawaretaxi.weebly.com/); *Blue Hans UD Taxi*, (302) 442-0876 (blue-hans-ud-taxi-limousine-services.com/); and *Soso Taxi*, (302) 766-5615 (sosotaxi.com/).

Both *Lyft* and *Uber* also operate in the Newark area.

Bus Service: Delaware Transit Corporation (DTC) operates DART First State, offering transportation options in and around Newark. Services provided include buses, and commuter train service contracted through SEPTA. For additional information about bus fares and schedules please visit the DART website at https://www.dartfirststate.com.

Weather

The average high temperature for September is approximately 79 degrees Fahrenheit and the average low is approximately 57 degrees Fahrenheit. Visitors should be prepared for inclement weather and check weather forecasts in advance of their arrival.

Social Networking

Attendees and speakers are encouraged to tweet about the meeting using the hashtag #AMSmtg.

Information for International Participants

Visa regulations are continually changing for travel to the United States. Visa applications may take from three to four months to process and require a personal interview, as well as specific personal information. International participants should view the important information about traveling to the US found at https://travel.state.gov/content/travel/en.html. If you need a preliminary conference invitation in order to secure a visa, please send your request to mac@ams.org.

If you discover you do need a visa, the National Academies website (see above) provides these tips for successful visa applications:

* Visa applicants are expected to provide evidence that they are intending to return to their country of residence. Therefore, applicants should provide proof of “binding” or sufficient ties to their home country or permanent residence abroad. This may include documentation of the following:

- family ties in home country or country of legal permanent residence
- property ownership
- bank accounts
- employment contract or statement from employer stating that the position will continue when the employee returns;

* Visa applications are more likely to be successful if done in a visitor’s home country than in a third country;

* Applicants should present their entire trip itinerary, including travel to any countries other than the United States, at the time of their visa application;

* Include a letter of invitation from the meeting organizer or the US host, specifying the subject, location and dates of the activity, and how travel and local expenses will be covered;

* If travel plans will depend on early approval of the visa application, specify this at the time of the application;

* Provide proof of professional scientific and/or educational status (students should provide a university transcript).

This list is not to be considered complete. Please visit the websites above for the most up-to-date information.

Ann Arbor, Michigan

University of Michigan, Ann Arbor

October 20–21, 2018

Saturday – Sunday

Meeting #1143

Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: July 2018
Program first available on AMS website: August 30, 2018
Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: Expired
For abstracts: August 21, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Elena Fuchs, University of Illinois Urbana-Champaign, *Title to be announced.*

Andrew Putman, University of Notre Dame, *Title to be announced.*

Charles Smart, University of Chicago, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Commutative Algebra (Code: SS 15A), **Jack Jeffries**, University of Michigan, **Linquan Ma**, Purdue University, and **Karl Schwede**, University of Utah.

Advances on Analytical and Geometric Aspects of Differential Equations (Code: SS 24A), **Alessandro Arsie**, **Chunhua Shan**, and **Ekaterina Shemyakova**, University of Toledo.

Analytical and Numerical Aspects of Turbulent Transport (Code: SS 23A), **Michele Coti Zelati**, Imperial College London, and **Ian Tobasco** and **Karen Zaya**, University of Michigan.

Aspects of Geometric Mechanics and Dynamics (Code: SS 13A), **Anthony M Bloch** and **Marta Farre Puiggali**, University of Michigan.

Bio-inspired Mechanics and Propulsion (Code: SS 16A), **Silas Alben**, University of Michigan, and **Longhua Zhao**, Case Western Reserve University.

Canonical Operators in Several Complex Variables and Related Topics (Code: SS 21A), **David Barrett** and **Luke Edholm**, University of Michigan, and **Yunus Zeytuncu**, University of Michigan, Dearborn.

Cell Motility: Models and Applications (Code: SS 20A), **Magdalena Stolarska**, University of St. Thomas, and **Nicoleta Tarfulea**, Purdue University Northwest.

Cluster Algebra, Poisson Geometry, and Related Topics (Code: SS 9A), **Eric Bucher**, Michigan State University, and **Maitreyee Kulkarni** and **Bach Nguyen**, Louisiana State University.

Combinatorics in Algebra and Algebraic Geometry (Code: SS 14A), **Zachary Hamaker**, **Steven Karp**, and **Oliver Pechenik**, University of Michigan.

Commutative Algebra and Complexity (Code: SS 32A), **Harm Derksen**, **Francesca Gandini**, and **Visu Makam**, University of Michigan.

Commutative Ring Theory (Code: SS 22A), **Joe Stickles**, Millikin University, and **Darrin Weber**, University of Evansville.

Ergodic and Topological Quantum Systems (Code: SS 28A), **Matthew Cha**, **Ilya Kachkovskiy**, and **Shiwen Zhang**, Michigan State University.

Extensions-Interpolation-Shape Matching in R^d, Symmetry-Invariance, Algorithms and Related Topics (Code: SS 11A), **Steven Damelin**, American Mathematical Society, and **Nir Sharon**, Princeton University.

From Hyperelliptic to Superelliptic Curves (Code: SS 6A), **Tony Shaska**, Oakland University, **Nicola Tarasca**, Rutgers University, and **Yuri Zarhin**, Pennsylvania State University.

Geometry of Submanifolds, in Honor of Bang-Yen Chens 75th Birthday (Code: SS 1A), **Alfonso Carriazo**, University of Sevilla, **Ivko Dimitric**, Penn State Fayette, **Yun Myung Oh**, Andrews University, **Bogdan D. Suceava**, California State University, Fullerton, **Joeri Van der Veken**, University of Leuven, and **Luc Vrancken**, Universite de Valenciennes.

Interactions between Algebra, Machine Learning and Data Privacy (Code: SS 3A), **Jonathan Gryak**, University of Michigan, **Kelsey Horan**, CUNY Graduate Center, **Delaram Kahrobaei**, CUNY Graduate Center and New York University, **Kayvan Najarian** and **Reza Soroushmehr**, University of Michigan, and **Alexander Wood**, CUNY Graduate Center.

Large Cardinals and Combinatorial Set Theory (Code: SS 10A), **Andres E. Caicedo**, Mathematical Reviews, and **Paul B. Larson**, Miami University.

Mathematics of the Genome (Code: SS 30A), **Anthony Bloch**, **Daniel Burns**, and **Indika Rajapakse**, University of Michigan.

Modern Trends in Integrable Systems (Code: SS 12A), **Deniz Bilman**, **Peter Miller**, **Michael Music**, and **Guilherme Silva**, University of Michigan.

Multiplicities and Volumes: An Interplay Among Algebra, Combinatorics, and Geometry (Code: SS 19A), **Federico Castillo**, University of Kansas, and **Jonathan Montaño**, New Mexico State University.

New Trends in Numerical Methods for Partial Differential Equations: Theory and Applications (Code: SS 17A), **Fatih Celiker**, Wayne State University.

Nonlocality in Models for Kinetic, Chemical, and Population Dynamics (Code: SS 25A), **Christopher Henderson**, University of Chicago, **Stanley Snelson**, Florida Institute of Technology, and **Andrei Tarfulea**, University of Chicago.

Probabilistic Methods in Combinatorics (Code: SS 7A), **Patrick Bennett** and **Andrzej Dudek**, Western Michigan University, and **David Galvin**, University of Notre Dame.

Random Matrix Theory Beyond Wigner and Wishart (Code: SS 2A), **Elizabeth Meckes** and **Mark Meckes**, Case Western Reserve University, and **Mark Rudelson**, University of Michigan.

Recent Advances in Nonlinear PDE (Code: SS 31A), **Jessica Lin**, McGill University, and **Russell Schwab**, Michigan State University.

Recent Developments in Discontinuous Galerkin Methods for Differential Equations (Code: SS 34A), **Mahboub Baccouch**, University of Nebraska at Omaha.

Recent Developments in Mathematical Analysis of Some Nonlinear Partial Differential Equations (Code: SS 18A), **Mimi Dai**, University of Illinois at Chicago.

Recent Developments in the Mathematics of Tomography and Scattering (Code: SS 26A), **Shixu Meng**, University of Michigan, and **Yang Yang**, Michigan State University.

Recent Trends on Local, Nonlocal and Fractional Partial Differential Equations (Code: SS 27A), **Pablo Raúl Stinga**, Iowa State University, **Peiyong Wang**, Wayne State University, and **Jiuyi Zhu**, Louisiana State University.

Representations of Reductive Groups over Local Fields and Related Topics (Code: SS 8A), **Anne-Marie Aubert**, Institut Mathématiques de Jussieu, Paris Rive Gauche, **Jessica Fintzen**, IAS, University of Michigan, University of Cambridge, and **Camelia Karimianpour**, University of Michigan.

Self-similarity and Long-range Dependence in Stochastic Processes (Code: SS 4A), **Takashi Owada**, Purdue University, **Yi Shen**, University of Waterloo, and **Yizao Wang**, University of Cincinnati.

Structured Homotopy Theory (Code: SS 5A), **Thomas Fiore**, University of Michigan, Dearborn, **Po Hu** and **Dan Isaksen**, Wayne State University, and **Igor Kriz**, University of Michigan.

The Mathematics of Decisions, Elections, and Games (Code: SS 29A), **Michael A. Jones**, Mathematical Reviews, and **David McCune**, William Jewell College.

Topics in Graph Theory, Hypergraphs and Set Systems (Code: SS 33A), **John Engbers**, Marquette University, and **Cliff Smyth**, University of North Carolina, Greensboro.

San Francisco, California

San Francisco State University

October 27–28, 2018

Saturday – Sunday

Meeting #1144

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: July 2018
Program first available on AMS website: September 6, 2018
Issue of *Abstracts*: Volume 39, Issue 4

Deadlines

For organizers: Expired
For abstracts: August 28, 2018

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Srikanth B. Iyengar, University of Utah, *Title to be announced.*

Sarah Witherspoon, Texas A&M University, *Derivatives, derivations, and Hochschild cohomology.*

Abdul-Aziz Yakubu, Howard University, *Population cycles in discrete-time infectious disease models.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at `www.ams.org/cgi-bin/abstracts/abstract.pl`.

Advances in Operator Theory, Operator Algebras, and Operator Semigroups (Code: SS 14A), **Asuman G. Aksoy**, Claremont McKenna College, **Michael Hartglass**, Santa Clara University, **Zair Ibragimov**, California State University, Fullerton, and **Marat Markin**, California State University, Fresno.

Algebraic Geometry (Code: SS 21A), **Emily Clader** and **Dustin Ross**, San Francisco State University, and **Mark Shoemaker**, Colorado State University.

Analysis and Geometry of Fractals (Code: SS 7A), **Kyle Hambrook**, University of Rochester, **Chun-Kit Lai**, San Francisco State University, and **Sze-Man Ngai**, Georgia Southern University.

Applied Harmonic Analysis: Frame Theory and Applications (Code: SS 9A), **Chun-Kit Lai** and **Shidong Li**, San Francisco State University.

Big Data and Statistical Analytics (Code: SS 17A), **Tao He**, **Mohammad Kafai**, and **Alexandra Piryatinska**, San Francisco State University.

Combinatorial and Categorical Aspects of Representation Theory (Code: SS 10A), **Nicholas Davidson** and **Jonathan Kujawa**, University of Oklahoma, and **Robert Muth**, Tarleton State University.

Coupling in Probability and Related Fields (Code: SS 3A), **Sayan Banerjee**, University of North Carolina, Chapel Hill, and **Terry Soo**, University of Kansas.

Geometric Analysis (Code: SS 8A), **Ovidiu Munteanu**, University of Connecticut, and **David Bao**, San Francisco State University.

Geometric Methods in Hypercomplex Analysis (Code: SS 13A), **Paula Cerejeiras**, Universidade de Aveiro, **Matvei Libine**, Indiana University, Bloomington, and **Mihaela B. Vajiac**, Chapman University.

Geometric and Analytic Inequalities and their Applications (Code: SS 4A), **Nicholas Brubaker**, **Isabel M. Serrano**, and **Bogdan D. Suceavă**, California State University, Fullerton.

Homological Aspects in Commutative Algebra and Representation Theory (Code: SS 5A), **Srikanth B. Iyengar**, University of Utah, and **Julia Pevtsova**, University of Washington.

Homological Aspects of Noncommutative Algebra and Geometry (Code: SS 2A), **Dan Rogalski**, University of California San Diego, **Sarah Witherspoon**, Texas A&M University, and **James Zhang**, University of Washington, Seattle.

Markov Processes, Gaussian Processes and Applications (Code: SS 18A), **Alan Krinik** and **Randall J. Swift**, California State Polytechnic University.

Mathematical Biology with a focus on Modeling, Analysis, and Simulation (Code: SS 1A), **Jim Cushing**, The University of Arizona, **Saber Elaydi**, Trinity University, **Suzanne Sindi**, University of California, Merced, and **Abdul-Aziz Yakubu**, Howard University.

Mathematical Methods for the study of the Three Dimensional Structure of Biopolymers (Code: SS 22A), **Javier Arsuaga** and **Mariel Vazquez**, University of California Davis, Davis, and **Robin Wilson**, Cal Poly Pomona.

Noncommutative Geometry and Fundamental Applications (Code: SS 12A), **Konrad Aguilar**, Arizona State University, and **Frederic Latremoliere**, University of Denver.

Nonlocal PDEs via Harmonic Analysis (Code: SS 20A), **Tadele Mengesha**, University of Tennessee, Knoxville, and **Armin Schikorra**, University of Pittsburgh.

Probabilistic and Statistical Problems in Stochastic Dynamics (Code: SS 16A), **Tao He**, **Mohammad Kafai**, and **Alexandra Piryatinska**, San Francisco State University.

Research in Mathematics by Early Career Graduate Students (Code: SS 11A), **Michael Bishop**, **Marat Markin**, **Jenna Tague**, and **Khang Tran**, California State University, Fresno.

Social Change In and Through Mathematics and Education (Code: SS 19A), **Federico Ardila** and **Matthias Beck**, San Francisco State University, **Jamylle Carter**, Diablo Valley Community College, and **Kimberly Seashore**, San Francisco State University.

Statistical and Geometrical Properties of Dynamical Systems (Code: SS 6A), **Joanna Furno** and **Matthew Nicol**, University of Houston, and **Mariusz Urbanski**, University of North Texas.

Topics in Operator Theory: CANCELLED (Code: SS 15A), **Anna Skripka** and **Maxim Zinchenko**, University of New Mexico.

Fayetteville, Arkansas

University of Arkansas

November 3–4, 2018

Saturday – Sunday

Meeting #1142

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: July 2018
Program first available on AMS website: August 16, 2018
Issue of *Abstracts*: Volume 39, Issue 3

Deadlines

For organizers: Expired
For abstracts: September 4, 2018

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Mihalis Dafermos, Princeton University, *Title to be announced.*

Jonathan Hauenstein, University of Notre Dame, *Title to be announced.*

Kathryn Mann, Brown University, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Advances in Birational Geometry (Code: SS 11A), **Roi Docampo**, University of Oklahoma, and **Lance Edward Miller** and **Wenbo Niu**, University of Arkansas.

Commutative Algebra (Code: SS 3A), **Alessandro De Stefani**, University of Nebraska-Lincoln, **Paolo Mantero**, University of Arkansas, and **Thomas Polstra**, University of Utah.

Groups in Low-dimensional Topology and Dynamics (Code: SS 7A), **Matt Clay**, University of Arkansas, and **Kathryn Mann**, Brown University.

Harmonic Analysis and Partial Differential Equations (Code: SS 8A), **Ariel Barton**, University of Arkansas, and **Simon Bortz**, University of Minnesota.

Interactions Between Combinatorics and Commutative Algebra (Code: SS 10A), **Ashwini Bhat**, **Chris Francisco**, and **Jeffrey Mermin**, Oklahoma State University.

Interactions Between Contact and Symplectic Geometry and Low-dimensional Topology (Code: SS 5A), **Jeremy Van Horn-Morris**, University of Arkansas, and **David Shea Vela-Vick**, Louisiana State University.

Non-associative Algebraic Structures and their (Co)homology Theories (Code: SS 12A), **Michael Kinyon**, University of Denver, **Jozef H Przytycki**, The George Washington University, and **Petr Vojtechovsky** and **Seung Yeop Yang**, University of Denver.

Numerical Methods for Nonlinear Systems (Code: SS 9A), **Jonathan Hauenstein** and **Tingting Tang**, University of Notre Dame.

Operator Theory and Function Spaces of Analytic Functions (Code: SS 13A), **Daniel Luecking** and **Maria Tjani**, University of Arkansas.

Partial Differential Equations in Several Complex Variables (Code: SS 2A), **Phillip Harrington** and **Andrew Raich**, University of Arkansas.

Recent Advances in Mathematical Fluid Mechanics (Code: SS 1A), **Zachary Bradshaw**, University of Arkansas.

Recent Developments on Fluid Turbulence (Code: SS 6A), **Eleftherios Gkioulekas**, University of Texas Rio Grande Valley.

The Geometry of Curves and Applications (Code: SS 14A), **Jason Cantarella** and **Philipp Reiter**, University of Georgia.

Validation and Verification Strategies in Multiphysics Problems (Code: SS 4A), **Tulin Kaman**, University of Arkansas.

Baltimore, Maryland

Baltimore Convention Center, Hilton Baltimore, and Baltimore Marriott Inner Harbor Hotel

January 16–19, 2019

Wednesday – Saturday

Meeting #1145

Joint Mathematics Meetings, including the 125th Annual Meeting of the AMS, 102nd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM)and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2018
Program first available on AMS website: November 1, 2018
Issue of *Abstracts*: Volume 40, Issue 1

Deadlines

For organizers: Expired
For abstracts: September 25, 2018

Call for MAA Contributed Papers

The MAA Committee on Sessions of Contributed Papers solicits papers pertinent to the sessions listed below. Any paper that fits the subject of one of the themed sessions should be submitted directly to that session. All others should be submitted to the general sessions, which will accept abstracts in all areas of collegiate mathematics, mathematical pedagogy, and the undergraduate mathematics curriculum. Presentations in the themed sessions are normally 15 minutes in length while presentations in the general sessions are limited to 10 minutes each.

Each participant may make at most one presentation in an MAA Contributed Paper Session, either a presentation in one of the themed sessions or a presentation in one of the general sessions (exclusive or). If a paper cannot be accommodated in the themed session for which it was submitted, it will automatically be considered for the general contributed paper sessions. The session rooms are equipped with computer projectors and screens. Please note that the dates and times scheduled for these sessions remain tentative. Questions concerning the submission of abstracts should be addressed to abs-coord@ams.org. Abstracts may be submitted electronically at `jointmathematicsmeetings.org/meetings/abstracts/abstract.pl?type=jmm`.

The deadline for submission of abstracts is Tuesday, September 25, 2018.

Contributed Paper Sessions with Themes

Approaches to Mathematics Remediation in Baccalaureate-Granting Institutions, organized by **Michael Boardman**, Pacific University, **Helen E. Burn**, Highline College, and **Mary E. Pilgrim**, Colorado State University, Saturday morning. Mathematics programs in baccalaureate institutions increasingly offer developmental course work due to shifts in student demographics. Furthermore, a decade or more of research establishes the negative impact of such courses on student outcomes in the two-year context. As a result, mathematics programs at the baccalaureate level face increasing pressure to develop evidence-based models that effectively enable students to grow mathematically in developmental content areas while maintaining momentum towards degree attainment.
This session seeks scholarly presentations that address developmental mathematics at the baccalaureate level from a variety of perspectives that inform program leadership around developmental mathematics curriculum in the baccalaureate context. Presentations may focus on developmental models at a single institution or system-wide efforts (e.g., technology-enhanced, co-requisite, pathways); organizational factors leading to effective implementation of curriculum (e.g., redesigning the learning space, onboarding graduate teaching assistants, leading departmental change). The session also encourages historical or policy research on current external influences on developmental mathematics curriculum in the baccalaureate context. Sponsored by the Committee on the Undergraduate Program in Mathematics.

Discrete Mathematics in the Undergraduate Curriculum —Ideas and Innovations in Teaching, organized by **John Caughman**, Portland State University, **Oscar Levin**, University of Northern Colorado, and **Elise Lockwood**, Oregon State University, Wednesday morning and afternoon. Discrete mathematics offers many accessible points of entry for students to engage in deep mathematical thinking. Discrete mathematics is a fundamental aspect of computer science, and it is increasingly relevant in our digital world. The aim of this session is for researchers and teachers to share ideas for how to improve the teaching and learning of discrete mathematics at all undergraduate levels. We characterize discrete mathematics broadly to encompass topics of sets, logic, proof techniques, recurrences, combinatorics, graph theory, relations, and more. We hope to facilitate communication between mathematics education researchers and those who teach these topics. We welcome scholarly presentations that speak to pedagogical aspects of discrete mathematics, which may include, but are not limited to: research on student thinking about relevant concepts, research demonstrating effective instructional strategies, ideas for incorporating technology into the discrete mathematics classroom, innovative activities or pedagogical interventions, or philosophies toward teaching discrete mathematics.

The EDGE (Enhancing Diversity in Graduate Education) program: Pure and Applied talks by Women Math Warriors, organized by **Laurel Ohm**, University of Minnesota, and **Shanise Walker**, Iowa State University, Thursday afternoon. Since its beginning in 1998, over two hundred and forty women have participated in the EDGE program. Approximately seventy are currently working towards a PhD, over one hundred and twenty have earned Masters and more than eighty have gone on to successfully complete PhDs. This session will be comprised of research talks in a variety of different sub disciplines given by women involved with the EDGE program. For more information on the EDGE program, see `www.edgeforwomen.org`.

Ethnomathematics: Ideas and Innovations in the Classroom, organized by **Janet Beery**, University of Redlands, **Antonia Cardwell**, Millersville University of Pennsylvania, **Ximena Catepillan**, Millersville University of Pennsylvania, and **Amy Shell-Gellasch**, Eastern Michigan University, Wednesday afternoon. Ethnomathematics, the study of mathematical aspects of the cultures of indigenous peoples, has been an active subject area for many decades. As more institutions strive to present multicultural offerings to their students, courses dedicated to or incorporating ethnomathematics are becoming more popular. This session features talks that present ideas for incorporating ethnomathematics into mathematics courses, ethnomathematics focused courses, ideas for undergraduate research in ethnomathematics, as well as new ethnomathematical research that can be brought into the classroom. Well-tested ideas and innovations in ethnomathematics for its use in teaching are welcome. Sponsored by the History of Mathematics SIGMAA.

Formative and Summative Assessment of Mathematical Communication and Conceptual Understanding, organized by **Jessica OShaughnessy**, Shenandoah University, and **Jana Talley**, Jackson State University, Thursday afternoon. The MAA's Instructional Practices (IP) Guide was designed to inform effective teaching through evidence-based classroom, assessment, and course design practices. Relevant to the work of the MAA's Committee on Assessment, the document addresses several aspects of assessment practices that are critical to the future of mathematics education at the undergraduate level. In particular, the MAA's IP Guide provides insight on the use of formative and summative assessments in mathematics. Additionally, consideration is given to the need for proper assessment of each student's ability to communicate mathematics and demonstrate conceptual understanding. These assessment types allow for a wide variety of implementations and focus on students' general understanding of mathematics rather than only procedural knowledge. This session presents both preliminary and completed research reports on formative and summative assessment techniques that attend specifically to student communication skills and conceptual understanding. In addition to research reports, contributors are encouraged to submit anecdotal evidence of successful implementation of relevant assessment techniques in undergraduate classes. Sponsored by the Committee on Assessment.

Fostering Creativity in Undergraduate Mathematics Courses, organized by **Emily S. Cilli-Turner**, University of Washington Tacoma, **Houssein El Turkey**, University of New Haven, **Gulden Karakok**, University of Northern Colorado, **Milos Savic**, University of Oklahoma, and **Gail Tang**, University of La Verne, Saturday morning. Creativity is an integral part of practicing mathematicians' work, but it is seldom explicitly valued or fostered in undergraduate mathematics courses. While research into the promotion of mathematical creativity exists in the K–12 literature, studies at the undergraduate level are sparse. As such, theoretical frameworks, pedagogical techniques, tasks, and classroom environments that promote mathematical creativity for undergraduate students have not been extensively studied. For this session, we invite proposals that describe either a theoretical framework, an activity/assignment/project, or teaching practices that faculty believe can be successful in producing creative results from students in an undergraduate mathematics course. Talks in this session should describe outcomes and give evidence of success of the intervention.

Good math from bad: crackpots, cranks, and progress, organized by **Elizabeth T. Brown**, James Madison University, and **Samuel R. Kaplan**, University of North Carolina Asheville, Friday afternoon. There are many purveyors of bad mathematics; people who have become so obsessed with a flawed or crackpot idea that they ignore evidence to the contrary are called cranks. Squaring the circle, doubling a cube and trisecting angles are just a few well-known ill-conceived pursuits. Crank mathematicians are not always amateurs and sometimes good ideas are generated on bad problems. There are also people who, knowingly or not, abuse mathematics to advance arguments outside of the mathematics. Finally, there are straightforward mathematical errors that are nevertheless useful in advancing mathematics. The study of crackpot and other erroneous math exposes the interesting history of classical problems as well as contemporary issues that arise from the ease of communication and proliferation of unsound math on the internet. This unusual session offers the opportunity to explore good problems and some good mathematics with witty and sad stories of coincidence, pseudo-science, and eccentricity. All talks will be given in the context of learning without chiding or belittling those involved in these stories.

Humanistic Mathematics, organized by **Gizem Karaali**, Pomona College, and **Eric Marland**, Appalachian State University, Thursday morning. Humanistic mathematics is historical, going back about thirty years, and awakens many connotations in those who hear it. As a scholarly perspective, humanistic mathematics describes an approach to mathematics that views it as a human endeavor and focuses on the paths of inquiry that study it's aesthetic, cultural, historical, literary, pedagogical, philosophical, psychological, and sociological aspects. As a pedagogical stance, humanistic mathematics explores and builds on the relationship of mathematics with its nontraditional partners in the humanities, the fine arts, and social sciences, providing additional perspective for the role of mathematics in a liberal arts education. Submissions on all humanistic aspects of mathematics are invited. We are especially looking for submissions that will stimulate discussion and further inquiry related to collegiate mathematics in the first two years. Submissions should be aimed at a broad mathematical audience. Sponsored by the MAA Curriculum Renewal Across the First Two Years (CRAFTY) and the Journal of Humanistic Mathematics.

Inclusive Excellence—Attracting, Involving, and Retaining Women and Underrepresented Groups in Mathematics, organized by **Francesca Bernardi**, University of North Carolina at Chapel Hill, **Meghan DeWitt**, St Thomas Aquinas College, **Semra Kilic-Bahi**, Colby-Sawyer College, and **Minah Oh**, James Madison University, Saturday morning. The disparities in mathematics in terms of gender, race, background, and ethnicity continue to remain problematic for the sustained prosperity of the field. Focused and intentional efforts are required to close the gap. For this session, we invite presentations describing programs that have been specifically developed to attract, involve, and retain women and underrepresented groups via innovations in the curriculum, outreach, extracurricular activities, and STEM community-building efforts both inside and outside the classroom. Moreover, we invite presentations focused on effective social and academic support structures and scholarly efforts aimed to raise awareness on the issues surrounding these disparities. Sponsored by the MAA Committee on the Participation of Women in Mathematics.

Incorporating Programming and Computing in the Math Major Curriculum, organized by **Holly Peters Hirst**, Appalachian State University, and **Gregory S. Rhoads**, Appalachian State University, Saturday morning. Programming and computing play an important role in mathematics. Many undergraduate math majors encounter programming and computing concepts during their coursework. Some faculty require students to learn computer math systems, programming languages, specialized computing software, etc. Some departments have developed "mathematical computing" courses. Students also take computer science and statistical computing courses as part of their major or as electives. Papers in this session will address ways that faculty have incorporated programming or computing knowledge into the undergraduate mathematics major curriculum at their institutions. We hope to generate discussion of innovative approaches to exposing math majors to programming and computing concepts.

Inequalities and Their Applications, organized by **Titu Andreescu**, University of Texas at Dallas, and **Henry J. Ricardo**, Westchester Area Math Circle, Thursday morning. Since the 1934 publication of *Inequalities* by Hardy, Littlewood, and Pólya, the literature devoted to this subject and the importance of inequalities in various areas of mathematics and science have grown considerably. In addition, inequalities have become a staple of mathematical competitions on the secondary and college levels and in journal problem sections. This session solicits contributions related to mathematical inequalities (algebraic, analytic, geometric/trigonometric) and their applications. These include new inequalities, new proofs of old inequalities, expository presentations, and applications to various domains of science and mathematics. Talks in this session should be accessible to advanced undergraduate students, and descriptions of student research experiences in this area are particularly welcome.

Infusing Data Science and Big Data into the Statistics Classroom, organized by **Allen Harbaugh**, Boston University, Wednesday afternoon. Several schools around the world now have programs, both at the undergraduate as well as at the graduate levels, dedicated to Data Science education, and there have been many conference presentations about such programs. This session invites participants and attendees to examine how a statistics educator might introduce a subset of Data Science topics into a statistics course. We invite presentation proposals that showcase what Data Science topics a statistics instructor can incorporate into statistics courses of any level, how they accomplish this, and consequent assessments of impact on student learning. We encourage presentations on innovative classroom activities, curriculum plans, and external resources, that are accompanied by findings from having attempted to assess or evaluate the approaches that are advocated in the presentation. Sponsored by the SIGMAA on Statistics Education (SIGMAA STAT ED).

Innovative Curricular Strategies for Increasing Mathematics Majors, organized by **Stuart Boersma**, Central Washington University, **Eric Marland**, Appalachian State University, **Victor Piercey**, Ferris State University, and **Frank Savina**, University of Texas at Austin, Wednesday morning. Many colleges and universities are seeking information about new and strategic curricular efforts to increase the number of mathematics majors. Such curricular innovations may include alternate entry points to the mathematics major, alternate pathways to and through college level mathematics courses, first-year seminars aimed at STEM majors, and strategies to attract and retain specific populations (such as underrepresented groups or students with AP Calculus credit). This session seeks to identify such innovative practices where they exist and to share these successes with the MAA's Committee for the Undergraduate Program in Mathematics (CUPM) as well as the broader mathematical community. Ideally, papers should evaluate

Sponsored by the MAA Curriculum Renewal Across the First Two Years (CRAFTY).

Innovative and Effective Ways to Teach Linear Algebra, organized by **Sepideh Stewart**, University of Oklahoma, **Gil Strang**, Massachusetts Institute of Technology, **David Strong**, Pepperdine University, and **Megan Wawro**, Virginia Tech, Thursday morning. Linear algebra is one of the most interesting and useful areas of mathematics, because of its beautiful and multifaceted theory, as well as the enormous importance it plays in understanding and solving many real world problems. Consequently, many valuable and creative ways to teach its rich theory and its many applications are continually being developed and refined. This session will serve as a forum in which to share and discuss new or improved teaching ideas and approaches. These innovative and effective ways to teach linear algebra include, but are not necessarily limited to: (1) hands-on, in-class demos; (2) effective use of technology, such as Matlab, Maple, Mathematica, Java Applets or Flash; (3) interesting and enlightening connections between ideas that arise in linear algebra and ideas in other mathematical branches; (4) interesting and compelling examples and problems involving particular ideas being taught; (5) comparing and contrasting visual (geometric) and more abstract (algebraic) explanations of specific ideas; (6) other novel and useful approaches or pedagogical tools.

Innovative Pathways to Quantitative Literacy, organized by **Catherine Crockett**, Point Loma Nazarene University, **Keith Hubbard**, Stephen F. Austin State University, and **Jennifer Nordstrom**, Linfield College, Saturday morning. In recent years, numerous organizations have proposed pathways outside of traditional pre-calculus, college algebra, or intermediate algebra for entry-level college students. Much of the discussion has focused on the need for college students to improve their quantitative literacy. This session is dedicated to sharing examples of innovative curricula, innovative implementations of existing curricula, or research in the effectiveness of a particular curriculum in fostering quantitative literacy. Sponsored by the MAA Committee on Articulation and Placement MAA Subcommittee on Curriculum Renewal Across the First Two Year SIGMAA on Quantitative Literacy.

Inquiry-Based Learning and Teaching, organized by **Susan Crook**, Loras College, **Eric Kahn**, Bloomsburg University, **Brian Katz**, Augustana College, **Amy Ksir**, United States Naval Academy, **Victor Piercey**, Ferris State University, **Candice Price**, University of San Diego, and **Xiao Xiao**, Utica College, Friday morning and afternoon. The goal of Inquiry-Based Learning (IBL) is to transform students from consumers to producers of mathematics. Inquiry-based methods aim to help students develop a deep understanding of mathematical concepts and the processes of doing mathematics by putting those students in direct contact with mathematical phenomena, questions, and communities. Within this context, IBL methods exhibit great variety. Activities can take place in single class meetings and span entire curricula for students of any age; students can be guided to re-invent mathematical concepts, to explore definitions and observe patterns, to justify core results, and to take the lead in asking new questions. There is a growing body of evidence that IBL methods are effective and important for teaching mathematics and for fostering positive attitudes toward the subject. This session invites scholarly presentations on the use of inquiry-based methods for teaching and learning. We especially invite presentations that include successful IBL activities or assignments that support observations about student outcomes with evidence or that could help instructors who are new to IBL to try new methods. Among these and other topics, talks related to assessment are strongly encouraged. Sponsored by the SIGMAA on Inquiry-Based Learning (IBL SIGMAA).

Integrated STEM Instruction in Undergraduate Mathematics, organized by **Jeneva Clark**, University of Tennessee, Knoxville, and **Anant Godbole**, East Tennessee State University, Thursday afternoon. MAA's Common Vision project recommends that the mathematics community "articulate clear pathways between curricula driven by changes at the K–12 level and the first courses students take in college" (p.1), and one such significant curricular change is the integration of STEM disciplines. This session will focus on college courses that have injected multidisciplinary STEM content in a meaningful way into first and/or second year courses, including K-12 teacher preparation programs as well as preliminary mathematics coursework in STEM disciplines. For this session, we are soliciting talks on how faculty have injected integrated STEM content (that relies on an understanding of one or several fields of inquiry) into their first-year and/or second-year mathematics courses. Such augmented courses may be general education courses or not. We are thus looking for presenters to share pedagogical innovations from their implemented courses with evidence of impact. Artifacts might include relevant modules, units, learning objectives, and/or learning activities as well as assessment tools that lead to a broader understanding of both mathematics and the related and integrated science, technology, and/or engineering areas that are used to motivate the learning of the mathematics.

Integrating Research into the Undergraduate Classroom, organized by **Timothy B. Flowers**, Indiana University of Pennsylvania, and **Shannon R. Lockard**, Bridgewater State University, Wednesday afternoon. Undergraduate Research is a high-impact practice that inspires student learning, builds crucial skills, boosts retention and graduation rates, and particularly benefits underrepresented and at-risk students. While students often engage in undergraduate research outside of the classroom, incorporating research projects into the classroom can bring this impactful experience to even more students. This session will focus on incorporating research into the undergraduate classroom, from introductory to upper level mathematics courses. Presentations may describe a particular research project or activity, faculty experiences in mentoring undergraduate research in the classroom, or student experiences and feedback. All talks should emphasize why the project(s) being discussed is considered undergraduate research rather than a typical assignment. Participants are encouraged to share the impact on the students involved if possible.

Introducing Mathematical Modeling through Competitions, organized by **Chris Arney**, United States Military Academy, **William C. Bauldry**, Appalachian State University, and **Amanda Beecher**, Ramapo College, Thursday morning. COMAP's (`www.comap.org`) Mathematical Contest in Modeling (MCM), Interdisciplinary Contest in Modeling (ICM), and High School Mathematical Contest in Modeling (HiMCM), and SIMIODE's (`www.simiode.com`) Student Competition Using Differential Equations Modeling (SCUDEM) are team competitions in which students apply the mathematics they know to solve a real world problem. Students routinely report learning more in this 4-day period than any other during college, and find it one of the most rewarding experiences of their undergraduate careers. Students point to this experience in interviews as an example of working in a team environment, meeting a deadline, and as evidence of their problem-solving ability. The value of participating is worth much more than the four days of work, making this an impactful experience for faculty advisors as well. This session is aimed at faculty who wish to begin advising teams and for current advisors to share strategies for student success. We invite presentations from experienced advisors focused on building and supporting student teams, developing mentor relationships for faculty, and presentations elaborating the judging process in order to help advisors better prepare student teams. We especially encourage student teams who have achieved a "Meritorious" or higher rating to report on their contest experience. Sponsored by COMAP and SIMIODE.

It's Circular: Conjecture, Compute, Iterate, organized by **Thomas J. Clark**, Dordt College, and **James Taylor**, Math Circles Collaborative of New Mexico, Friday afternoon. Math Circles are a form of education outreach and enrichment through which mathematicians and mathematical scientists share their passion with K–12 teachers and students. Math Circles combine significant content with a setting that encourages a sense of discovery and excitement about mathematics through problem solving and interactive exploration. Great problems can often be solved by a variety of approaches working in concert. This session will focus on problems that are motivated, illuminated, or visualized through numerical computation or some form of computer modeling. For example, one might begin with a traditional math circle investigation—gaining insight into a problem, perhaps developing some conjectures—and follow it with computation or modeling, leading to greater insight and further analytical progress into the problem, or opening up new avenues for inquiry. Presentations in this session will generate new and interesting ideas providing a much-needed set of resources for circle organizers seeking problems unfolding the increasingly important topic of computation. Sponsored by SIGMAA-MCST.

MAA Session on Mathematical Experiences and Projects in Business, Industry, and Government (BIG), organized by **Robert Burks**, Naval Postgraduate School, and **Allen Butler**, Wagner Associates, Friday morning. The extraordinary growth of problems facing business, industry, and government seems overwhelming. It should not! As mathematicians, operations research analysts, and engineers, including those within academia, we experience and tackle these problems with experience, knowledge, and technological tools. We solve applied mathematics problems in business, industry, and government, including military applications, almost daily. We seek presenters to share examples of this type of problem solving. These may include problems where you have no clue how to proceed and are seeking ideas from our audience. Your talks will serve as inspiration to solve and tackle the real problems that we may face in the future. You do not have to be a BIG SIGMAA member to attend or present. This session is sponsored by the SIGMAA on Business, Industry, and Government. Sponsored by the Business Industry Government Special Interest Group of the Mathematical Association of America (BIG SIGMAA).

Mathematical Themes in a First-Year Seminar, organized by **Jennifer Bowen**, College of Wooster, **Mark Kozek,** Whittier College, **Pamela Pierce**, College of Wooster, and **Jennifer Schaefer**, Dickinson College, Friday afternoon. Does your college or university require students to take a first-year seminar? Often, these seminars include an introduction to college life and college-level academic culture through an emphasis on critical thinking, academic writing/research, information literacy, and collaborative learning. A mathematician tasked with teaching such a course may feel overwhelmed because the style of teaching and the assignments are typically different from those in our mathematics classes. At the same time, teaching such a seminar is a wonderful way to engage with students who may be open to learning more about the field. How can we, as mathematicians, engage students in mathematics or mathematically related themes? Speakers should share the theme/title of the seminar, the major learning goals of the course, the mathematical techniques and themes that were incorporated, and the degree to which these were successful.

Mathematics and the Arts, organized by **Karl Kattchee**, University of Wisconsin—LaCrosse, **Douglas Norton**, Villanova University, and **Anil Venkatesh**, Ferris State, Wednesday morning and afternoon. Learn, explore, and share interactions between mathematics and the arts. If you work in the arts with mathematical themes, if you use mathematics to create art, if you use mathematics and art together in your classrooms, if you explore interactions between mathematics and the arts in some of the usual or in some completely unexpected contexts, or if you want to hear reports of those who do any of the above, you are invited to this session. Just as "mathematics" includes any aspect of the mathematical sciences, "the arts" may be paint, sculpture, fiber arts, print, music, dance, architecture, poetry—from Escher to Oulipo, perspective to origami, fugues to fractals. Sponsored by SIGMAA-ARTS.

Mathematics and the Life Sciences: Initiatives, Programs, Curricula, organized by **Timothy D. Comar**, Benedictine University, **Carrie Diaz Eaton**, Unity College, and **Raina Robeva**, Sweet Briar College, Thursday morning. In the 2015 CUPM Curriculum Guide to Majors in the Mathematical Sciences, the life sciences were clearly identified as a key path through the mathematics major to graduate programs and the workforce. This account echoed many prior high-profile reports (e.g., Bio 2010 (2003), A New Biology for the 21st Century (2009), Vision and Change (2011), The Mathematical Sciences in 2025 (2013), and the SIAM white paper Mathematics: An Enabling Technology for the New Biology (2009)) that had previously discussed the changing landscape at the interface of mathematics and biology and had issued urgent calls for broadening students' exposure to mathematical methods for the life sciences. It appears that a wider array of curricular ideas, programs, and materials that can be scaled, modified, and assessed in a wide range of different institutions is still needed. Topics include scholarly contributions addressing initiatives, programs, curricula, and course materials at the interface of mathematics and the life sciences that have been implemented and tested successfully at institutions of higher education. Speakers will be invited to submit their work for consideration in the upcoming PRIMUS Special Issue: Mathematics and the Life Sciences: Initiatives, Programs, Curricula. Sponsored by the SIGMAA on Mathematical and Computational Biology (BIO SIGMAA).

Mathematics and Sports, organized by **John David**, Virginia Military Institute, and **Drew Pasteur**, College of Wooster, Wednesday afternoon. The expanding availability of play-by-play statistics and video-based spatial data have led to innovative research using techniques from across the mathematical sciences, with impacts on strategy and player evaluation. Other areas of interest include ranking methods, predictive models, physics-based analysis, etc. Research presentations, expository talks, and contributions related to curriculum or pedagogy are all welcome. With a broad audience in mind, talks should be accessible to undergraduate mathematics majors, and projects involving undergrads are particularly encouraged for submission. Sponsored by the Sports SIGMAA.

Meaningful Modeling in the First Two Years of College, organized by **William C. Bauldry**, Appalachian State University, and **Mary R. Parker**, Austin Community College, Saturday afternoon. Most major mathematical organizations, including the MAA in its 2015 CUPM Curriculum Guide, have encouraged programs to incorporate modeling at all levels of the mathematics curriculum, including the first two years of undergraduate coursework. There are good reasons to include modeling experiences in the first two years. These opportunities allow students majoring in mathematics to gain experience with an important (and often challenging) approach to problem-solving that will benefit them later in their coursework and careers, and all students—regardless of major—may find that they appreciate the role and value of mathematics more deeply by applying it to meaningful situations. This session welcomes papers that describe substantive mathematical modeling experiences for students who would typically be in their first two years as an undergraduate student, including (but not limited to) courses for non-majors and courses that do not have calculus as a prerequisite. Ideally, papers should also evaluate the effectiveness of the approach being taken. Sponsored by the MAA Mathematics Across the Disciplines (MAD) Subcommittee and the MAA Curriculum Renewal Across the First Two Years (CRAFTY) Subcommittee.

Open Educational Resources: Combining Technological Tools and Innovative Practices to Improve Student Learning, organized by **Benjamin Atchison**, Framingham State University, **Marianna Bonanome**, New York City College of Technology, **Margaret Dean**, Borough of Manhattan Community College, **Michael Gage**, University of Rochester, and **Annie Han**, Borough of Manhattan Community College, Friday morning. Experimentation in classroom methodologies is blooming. An increasing number of instructors are adapting more than one OER technological tool and combining them with active learning techniques to improve student learning in their classrooms. This session will provide a venue for exposition of these experiments thereby disseminating results (positive and negative), reducing duplication of effort, promoting collaboration between instructors and providing recognition for those on the front lines of experimental learning. Sponsored by the MAA Committee on Technologies in Mathematics Education (CTiME).

Philosophy of Mathematics, organized by **Jeffrey Buechner**, Rutgers University—Newark, and **Bonnie Gold**, Monmouth University (retired), Friday morning. This session invites talks on any topic in the philosophy of mathematics. Our special theme this year is "Do Choices of Mathematical Notation (and Similar Choices) Affect the Development of Mathematical Concepts?" Once mathematical concepts have gelled, they tend to feel "natural" to mathematicians. But in the process of exploring and developing new concepts, mathematicians make choices, including of notation and terminology that affect how the nascent concept solidifies. For example, to what extent does our decimal notation affect our understanding of numbers? Are there concepts and mathematical practices that can be understood in one notational framework and not in another? This session invites talks that look at this process, and the philosophical implications of the effect of our choice of mathematical notations on the development of mathematical concepts. Talks on the special theme will be given highest priority, but all talks on the philosophy of mathematics are welcome. Sponsored by POMSIGMAA.

Research in Undergraduate Mathematics Education (RUME), organized by **Stacy Brown**, California State Polytechnic University, **Megan Wawro**, Virginia Tech, and **Aaron Weinberg**, Ithaca College, Thursday morning and afternoon, and Friday morning and afternoon. The goals of this session are to promote high quality research in undergraduate mathematics education, to disseminate well-designed educational studies to the greater mathematics community, and to facilitate a productive impact of research findings on pedagogy in college mathematics. Presentations may be based on research in areas such as calculus, linear algebra, differential equations, abstract algebra, and mathematical proof. Examples include rigorous and scientific studies about students' mathematical cognition and reasoning, teaching practice in inquiry-oriented mathematics classrooms, design of research-based curricular materials, and professional development of instructors that supports college students' mathematical thinking. Presentations should report on completed research that builds on the existing literature in mathematics education and employs contemporary educational theories of the teaching and learning of mathematics. The research should use well-established or innovative methodologies as they pertain to the study of undergraduate mathematics education. Sponsored by SIGMAA RUME.

Revitalizing Complex Analysis, organized by **Michael Brilleslyper**, United States Air Force Academy, **Russell Howell**, Westmont College, and **Beth Schaubroeck**, United States Air Force Academy, Thursday afternoon. Complex Analysis, despite its beauty and power, seems to have lost some of the prominence it once enjoyed in undergraduate STEM fields. Growing out of an NSF grant, the Revitalizing Complex Analysis project seeks to remedy this situation. It has held successful contributed paper sessions at the past four Joint Mathematics Meetings. Proposals for the Baltimore JMM should be scholarly in nature, and collectively address a wide-range of questions: What are the essential components of an undergraduate complex analysis class from mathematical and scientific standpoints? What technologies seem to be promising? What pedagogical ideas have borne fruit? What interesting projects have worked well for student investigation? What novel connections have been made with other standard mathematics courses? What are some interesting applications to other disciplines? In general, what innovative approaches might be suggested in teaching the subject? Presentations that include evidence of success or failure in the classroom are especially welcomed.

The Scholarship of Teaching and Learning in Collegiate Mathematics, organized by **Tom Banchoff**, Brown University, **Curtis Bennett**, California State University, Long Beach, **Pam Crawford**, Jacksonville University, **Jacqueline Dewar**, Loyola Marymount University, **Edwin Herman**, University of Wisconsin-Stevens Point, and **Lew Ludwig**, Denison University, Wednesday morning & afternoon. In the scholarship of teaching and learning, faculty bring disciplinary knowledge to bear on questions of teaching and learning and systematically gather evidence to support their conclusions. Work in this area includes investigations of the effectiveness of pedagogical methods, assignments, or technology, as well as inquiries into student understanding. The session goals are to: (1) feature scholarly work on the teaching of postsecondary mathematics, (2) provide a venue for teaching mathematicians to make public their scholarly investigations into teaching/learning, and (3) highlight evidence-based arguments for the value of teaching innovations or in support of new insights into student learning. Appropriate for this session are preliminary or final reports of investigations of post-secondary teaching methods, student learning difficulties, curricular assessment, or insights into student (mis)understandings. Abstracts should: (1) have a clearly stated question that was or is under investigation and (2) indicate the type of evidence that has been or will be gathered and presented. For example, abstracts might refer to evidence such as student work, participation or retention data, pre/post tests, interviews, surveys, think-alouds.

The Teaching and Learning of Undergraduate Ordinary Differential Equations, organized by **Christopher S. Goodrich**, Creighton Preparatory School, and **Beverly H. West**, Cornell University, Friday afternoon. The teaching of undergraduate Ordinary Differential Equations (ODEs) provides a unique way to introduce students to the beauty and applicative power of the calculus. ODEs are also rich with aesthetically pleasing theory, which often can be successfully communicated visually and explored numerically. This session will feature talks that describe innovative teaching in the ODEs course as well as the description of either projects or pedagogy that can be used to engage students in their study of ODEs. Successful contributions could include but are not limited to: 1. Innovative ways of teaching standard topics in the ODEs course; 2. Strategies for teaching both differential equations and linear algebra simultaneously; 3. The inclusion of technology in the ODEs course; and 4. Descriptions of applications or nonstandard topics and how such topics can lead to student engagement and interest.
In addition, contributors should include some discussion of the success of their methods, such as in what ways the activity or method under discussion has improved student learning, retention, or interest in the differential equations course. Sponsored by CODEE (Consortium of Ordinary Differential Equations Educators).

Technology and Resources in Statistics Education, organized by **Stacey Hancock**, Montana State University, and **Karl RB Schmitt**, Valparaiso University, Friday afternoon. One of the five skill areas in the American Statistical Association's curriculum guidelines is "Data Manipulation and Computation" (pg. 9), embracing the need for students to be competent with programming languages, simulation techniques, algorithmic thinking, data management and manipulation, as well as visualization techniques. Additionally, the emphasis on using real data and problems and their inherent complexity means that technology is often necessary outside of specifically prescribed computational courses. This session invites instructors to contribute talks exploring the use of any software or technology in statistics education. Talks may include effective instructional or pedagogical techniques for linking programming to statistics, interesting classroom problems and the use of technology to solve them, or more. If you are unsure if your idea fits, please feel free to contact the organizers before submitting. Sponsored by the Committee on Technology in Mathematics Education SIGMAA-Statistics Education.

Touch it, Feel it, Learn it: Tactile learning activities in the undergraduate mathematics classroom, organized by **Chris Oehrlein**, Oklahoma City Community College, **Ann Trenk**, Wellesley College, and **Laura Watkins**, Glendale Community College, Thursday afternoon. This session invites presentations describing activities that use tactile teaching methods in any mathematics classes. Examples of tactile methods could include props or manipulatives that students can touch to understand concepts better, projects where students create physical models that represent a concept, and in-class activities in which students work together to create hands-on demonstrations of their understanding of a particular concept. This session seeks presentations that focus on engaging students through interaction with props, use of manipulative materials, or even inviting students to physically become a part of a function or concept; this does not include technology demonstrations such as computer visualizations. We seek innovative and creative ways for physically involving students in mathematics. Presentations detailing how to integrate a particular activity into class, student reactions, educational benefits, difficulties to avoid, and possible modifications of the activity are desired. Sponsored by the Professional Development committee and the Committee on Two-Year Colleges.

Undergraduate Student TAs in Mathematics, organized by **Aaron Peterson** and **Ursula Porod**, Northwestern University, Wednesday afternoon. Many mathematics departments around the country run undergraduate student teaching assistant (UGTA) programs. Roles of UGTAs vary from department to department. They range from behind-the-scenes graders to independent discussion section leaders, comparable in their TA duties to graduate student TAs. Proper training and mentoring of UGTAs are important parts of a successful program. However, while many departments run substantial professional training programs for their graduate students, UGTAs tend to receive far less teaching training. Reasons may include UGTAs' time limitations due to their own course work and limited departmental resources. We especially, but not exclusively, invite contributions of ideas for effective UGTA training and mentoring. More broadly, this session will be a forum for sharing current practices and critical evaluations of all aspects of existing UGTA programs.

General Contributed Paper Sessions

General Contributed Paper Sessions, organized by **Emelie Kenney**, Sienna College, and **Melvin Royer**, Indiana Wesleyan University, Wednesday, Thursday, Friday, and Saturday, mornings and afternoons. The MAA's General Contributed Paper Sessions accept contributions in all areas of mathematics, curriculum, and pedagogy. When you submit your abstract, you will be asked to classify it according to the following scheme: Assessment; History or Philosophy of Mathematics; Interdisciplinary Topics in Mathematics; Mathematics and Technology; Mentoring; Modeling and Applications; Outreach; Teaching and Learning Developmental Mathematics; Teaching and Learning Introductory Mathematics; Teaching and Learning Calculus; Teaching and Learning Advanced Mathematics; Algebra; Analysis; Applied Mathematics; Geometry; Graph Theory; Linear Algebra; Logic and Foundations; Number Theory; Probability and Statistics; Topology; and Other Topics.

Auburn, Alabama

Auburn University

March 15–17, 2019

Friday – Sunday

Meeting #1146

Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: January 2019
Program first available on AMS website: January 31, 2019
Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: August 15, 2018
For abstracts: January 29, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Grigoriy Blekherman, Georgia Institute of Technology, *Title to be announced.*

Carina Curto, Pennsylvania State University, *Title to be announced.*

Ming Liao, Auburn University, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Combinatorial Matrix Theory (Code: SS 2A), **Zhongshan Li**, Georgia State University, and **Xavier Martínez-Rivera**, Auburn University.

Commutative and Combinatorial Algebra (Code: SS 3A), **Selvi Kara Beyarslan**, University of South Alabama, and **Alessandra Costantini**, Purdue University.

Developments in Commutative Algebra (Code: SS 1A), **Eloísa Grifo**, University of Michigan, and **Patricia Klein**, University of Kentucky.

Honolulu, Hawaii

University of Hawaii at Manoa

March 22–24, 2019

Friday – Sunday

Meeting #1147

Central Section
Associate secretaries: Georgia Benkart and Michel L. Lapidus

Announcement issue of *Notices*: January 2019
Program first available on AMS website: February 7, 2019
Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: Expired
For abstracts: January 29, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Barry Mazur, Harvard University, *On the Arithmetic of Curves* (Einstein Public Lecture in Mathematics).

Aaron Naber, Northwestern University, *Analysis of Geometric Nonlinear Partial Differential Equations*.

Deanna Needell, University of California, Los Angeles, *Simple approaches to complicated data analysis*.

Katherine Stange, University of Colorado, Boulder, *Title to be announced*.

Andrew Suk, University of California, San Diego, *Title to be announced*.

Hartford, Connecticut

University of Connecticut Hartford (Hartford Regional Campus)

April 13–14, 2019

Saturday – Sunday

Meeting #1148

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: February 2019
Program first available on AMS website: February 21, 2019
Issue of *Abstracts*: Volume 40, Issue 2

Deadlines

For organizers: September 13, 2018
For abstracts: February 5, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Olivier Bernadi, Brandeis University, *Title to be announced*.

Brian Hall, University of Notre Dame, *Title to be announced*.

Christina Sormani, City University of New York, *Title to be announced*.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Analysis, Geometry, and PDEs in Non-smooth Metric Spaces (Code: SS 1A), **Vyron Vellis**, University of Connecticut, **Xiaodan Zhou**, Worcester Polytechnic Institute, and **Scott Zimmerman**, University of Connecticut.

Computability Theory (Code: SS 2A), **Damir Dzhafarov** and **Reed Solomon**, University of Connecticut, and **Linda Brown Westrick**, Pennsylvania State University.

Recent Development of Geometric Analysis and Nonlinear PDEs (Code: SS 3A), **Ovidiu Munteanu**, **Lihan Wang**, and **Ling Xiao**, University of Connecticut.

Stochastic Processes, Random Walks, and Heat Kernels (Code: SS 4A), **Patricia Alonso Ruiz**, University of Connecticut, and **Phanuel Mariano**, Purdue University.

Quy Nhon City, Vietnam

Quy Nhon University

June 10–13, 2019

Monday – Thursday

Meeting #1149

Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: April 2019

Deadlines

For organizers: November 30, 2018
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

Henry Cohn, Microsoft Research, *To be announced*.

Robert Guralnick, University of Southern California, *To be announced*.

Le Tuan Hoa, Hanoi Institute of Mathematics, *To be announced*.

Nguyen Dong Yen, Hanoi Institute of Mathematics, *To be announced*.

Zhiwei Yun, Massachusetts Institute of Technology, *To be announced*.

Nguyen Tien Zung, Toulouse Mathematics Institute, *To be announced*.

Madison, Wisconsin

University of Wisconsin-Madison

September 14–15, 2019
Saturday – Sunday

Meeting #1150
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: June/Julu 2019
Program first available on AMS website: July 23, 2019
Issue of *Abstracts*: Volume 40, Issue 3

Deadlines
For organizers: February 14, 2019
For abstracts: July 16, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Nathan Dunfeid, University of Illinois,Urbana-Champaign, *Title to be announced*

Teena Gerhardt, Michigan State University, *Title to be announced*

Lauren Williams, University of California, Berkeley, *Title to be announced* (Erdős Memorial Lecture).

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Homological and Characteristic $p > 0$ Methods in Commutative Algebra (Code: SS 1A), **Michael Brown**, University of Wisconsin-Madison, and **Eric Canton**, University of Michigan.

Binghamton, New York

Binghamton University

October 12–13, 2019
Saturday – Sunday

Meeting #1151
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: August, 2019
Program first available on AMS website: August 29, 2019
Issue of *Abstracts*: Volume 40, Issue 3

Deadlines
For organizers: March 12, 2019
For abstracts: August 20, 2019

Invited Addresses

Richard Kenyon, Brown University, *Title to be announced*

Tony Pantev, University of Pennsylvania, *Title to be announced*

Lai-Sang Young, New York University, Title to be announced

Gainesville, Florida

University of Florida

November 2–3, 2019
Saturday – Sunday

Meeting #1152
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: September 2019
Program first available on AMS website: September 19, 2019
Issue of *Abstracts*: Volume 40, Issue 4

Deadlines
For organizers: April 2, 2019
For abstracts: September 10, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Jonathan Mattingly, Duke University, *Title to be announced.*

Isabella Novik, University of Washington, *Title to be announced.*

Eduardo Teixeira, University of Central Florida, *Title to be announced.*

Riverside, California

University of California, Riverside

November 9–10, 2019
Saturday – Sunday

Meeting #1153
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: September 2019

Program first available on AMS website: September 12, 2019
Issue of *Abstracts*: Volume 40, Issue 4

Deadlines

For organizers: April 9, 2019
For abstracts: September 3, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Robert Boltje, University of California, Santa Cruz, *Title to be announced.*

Jonathan Novak, University of California, San Diego, *Title to be announced.*

Anna Skripka, University of New Mexico, Albuquerque, *Title to be announced.*

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Topics in Operator Theory (Code: SS 1A), **Anna Skripka** and **Maxim Zinchenko**, University of New Mexico.

Denver, Colorado

Colorado Convention Center

January 15–18, 2020

Wednesday – Saturday

Meeting #1154

Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2019
Program first available on AMS website: November 1, 2019
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2019
For abstracts: To be announced

Charlottesville, Virginia

University of Virginia

March 13–15, 2020

Friday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Fresno, California

California State University, Fresno

May 2–3, 2020

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Washington, District of Columbia

Walter E. Washington Convention Center

January 6–9, 2021

Wednesday – Saturday
Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Brian D. Boe
Announcement issue of *Notices*: October 2020
Program first available on AMS website: November 1, 2020
Issue of *Abstracts*: To be announced

Deadlines

For organizers: April 1, 2020
For abstracts: To be announced

Grenoble, France

Université Grenoble Alpes

July 5–9, 2021
Monday – Friday
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Buenos Aires, Argentina

The University of Buenos Aires

July 19–23, 2021
Monday – Friday
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Seattle, Washington

Washington State Convention Center and the Sheraton Seattle Hotel

January 5–8, 2022
Wednesday – Saturday
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2021
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2023
Wednesday – Saturday
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2022
Program first available on AMS website: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced

THE BACK PAGE

The April Contest Winner Is...

Randy K. Schwartz, who receives our book award.

Photographed and submitted by Annalisa M. Calini.

"Okay, who's the genius that gave these bagels the wrong genus?"

The June/July Caption Contest:

What's the Caption?

Drawn by James F. Bredt, © Frank Morgan

Submit your entry to `captions@ams.org` by July 25. The winning entry will be posted here in the October 2018 issue.

"In the broad light of day mathematicians check their equations and their proofs, leaving no stone unturned in their search for rigour. But, at night, under the full moon, they dream, they float among the stars and wonder at the miracle of the heavens. They are inspired. Without dreams there is no art, no mathematics, no life."

—Michael Atiyah, *Collected Works: 2002–2013*, p. 267

$\mathbf{P \neq NP}$

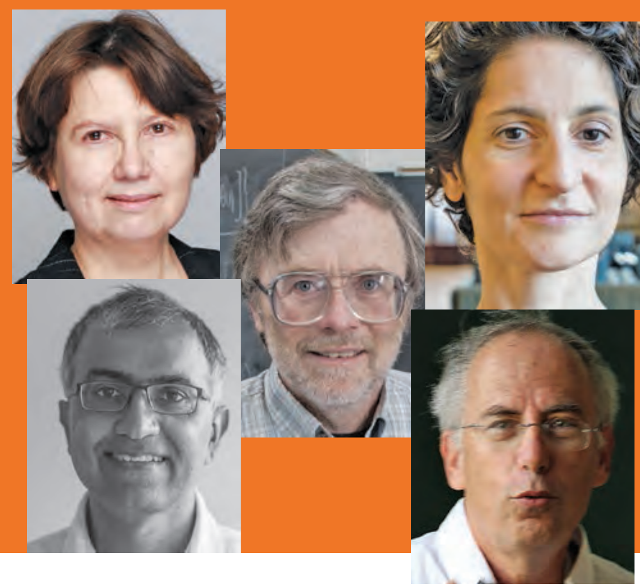
Submitted by Richard Pace.

Send in your favorite image of a math-related license plate. If *Notices* uses it, we'll send you a prize.

What crazy things happen to you? Readers are invited to submit original short amusing stories, math jokes, cartoons, and other material to: **`noti-backpage@ams.org`**.

IN THE NEXT ISSUE OF NOTICES

AUGUST 2018



The Mathematics of Cathleen Synge Morawetz

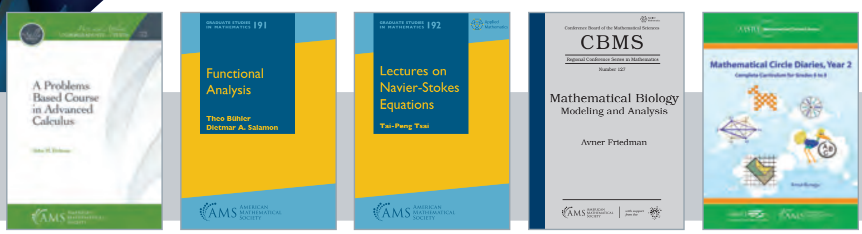
ICM Lecture Sampler

American Mathematical Society
Distribution Center

35 Monticello Place,
Pawtucket, RI 02861 USA

RECENT RELEASES

from the AMS



= Textbook
= Applied Mathematics
= Available in eBook Format

A Problems Based Course in Advanced Calculus

John M. Erdman, *Portland State University, OR*

This textbook for a course in advanced calculus promotes active learning through problem solving and can be used as a base for a an inquiry based class or as a guide in a traditional classroom setting where lectures are organized around the presentation of problems and solutions.

Pure and Applied Undergraduate Texts, Volume 32; 2018; 360 pages; Hardcover; ISBN: 978-1-4704-4246-0; List US$79; AMS members US$63.20; MAA members US$71.10; Order code AMSTEXT/32

Functional Analysis

Theo Bühler and **Dietmar A. Salamon**, *ETH, Zürich, Switzerland*

This book provides a comprehensive introduction to the field of functional analysis, a central subject of mathematics with applications in many areas of geometry, analysis, and physics.

Graduate Studies in Mathematics, Volume 191; 2018; 466 pages; Hardcover; ISBN: 978-1-4704-4190-6; List US$83; AMS members US$66.40; MAA members US$74.70; Order code GSM/191

Lectures on Navier-Stokes Equations

Tai-Peng Tsai, *University of British Columbia, Vancouver, Canada*

This graduate-level text on the incompressible Navier-Stokes system, which is of fundamental importance in mathematical fluid mechanics as well as in engineering applications, provides a rapid exposition on the existence, uniqueness, and regularity of its solutions, with a focus on the regularity problem.

Graduate Studies in Mathematics, Volume 192; 2018; 224 pages; Hardcover; ISBN: 978-1-4704-3096-2; List US$83; AMS members US$66.40; MAA members US$74.70; Order code GSM/192

Mathematical Biology

Modeling and Analysis

Avner Friedman, *Ohio State University, Columbus*

The fast growing field of mathematical biology addresses biological questions using mathematical models from areas such as dynamical systems, probability, statistics, and discrete mathematics. This book considers models that are described by systems of partial differential equations, and it focuses on modeling, rather than on numerical methods and simulations.

CBMS Regional Conference Series in Mathematics, Number 127; 2018; approximately 101 pages; Softcover; ISBN: 978-1-4704-4715-1; List US$52; AMS members $41.60; MAA members US$46.80; Order code CBMS/127

Mathematical Circle Diaries, Year 2

Complete Curriculum for Grades 6 to 8

Anna Burago, *Prime Factor Math Circle, Seattle, WA*

The material in this book, distributed among twenty-nine weekly lessons, teaches readers how to think and solve problems in mathematics via detailed lectures and discussions, sets of problems with solutions, and contests and games.

Titles in this series are co-published with the Mathematical Sciences Research Institute (MSRI).

MSRI Mathematical Circles Library, Volume 20; 2018; approximately 351 pages; Softcover; ISBN: 978-1-4704-3718-3; List US$55; AMS members $44; MAA members US$49.50; Order code MCL/20

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