Introduction to Ideal Class Groups

by Tom Gannon, University of Texas at Austin

Algebraic number theory is a really interesting subject, but unlike some other subjects, it’s not 100% clear what objects people study. This post provides an introduction to the class group of a finite dimensional field extension of \( \mathbb{Q} \), an object often used in modern number theory.

One of the first cool facts about this is that the class group is always a finite group! This also develops the subject of class field theory, the study of Galois extensions of \( \mathbb{Q} \) whose Galois groups are abelian over \( \mathbb{Q} \). This can be used to prove the Kronecker-Weber theorem, which says that for any abelian extension \( K/\mathbb{Q} \), there is a cyclotomic field containing \( K \). In short – the class group of a number field is a rich object worth studying!

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Photo of Tom Gannon by Rachel Schlossman.

ABOUT THE AUTHOR

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