The Mathematics of Cathleen Synge Morawetz

Communicated by Christina Sormani

Morawetz, an avid sailor, invited us all to "sail with her, near the speed of sound."

Irene Gamba and Christina Sormani

Introduction

In this memorial we celebrate the mathematics of Cathleen Synge Morawetz (1923–2017). She was awarded the National Medal of Science in 1998 "for pioneering advances in partial differential equations and wave propagation resulting in applications to aerodynamics, acoustics and optics." In 2004 she won the Steele Prize for lifetime achievement and in 2006 she won the Birkhoff Prize "for her deep and influential work in partial differential equations, most notably in the study of shock waves, transonic flow, scattering theory, and conformally invariant estimates for the wave equation."

As it is impossible to review all her profound contributions to pure and applied mathematics, we have chosen instead to present some of her most influential work in depth. Terence Tao presents the Morawetz Energies and Morawetz Inequalities, which are ubiquitous in the analysis of nonlinear wave equations. Leslie Greengard and Tonatiuh Sánchez-Vizuet have written about her work on scattering theory. Kevin R. Payne describes the importance of her early work on transonic flows which both provided a new understanding of mixed-type partial differential equations and led to new methods of efficient aircraft design. In this introduction, we provide a little history about her career, and we close the article with a quote of hers thanking one of the mathematicians who supported her the most when she was young.

Morawetz was encouraged to study mathematics by her mother and a family friend, Cecilia Kreger, who was a mathematics professor at the University of Toronto. Her father, who was also a mathematician at Toronto did not encourage her pursuit of mathematics, but did encourage her to be "ambitious." Morawetz graduated with a bachelors in mathematics at Toronto in 1945 and completed her masters at MIT the following year.

In 1946 Morawetz was hired at New York University to edit the manuscript "Supersonic Flow and Shock Waves" by Richard Courant and Kurt Otto Friedrichs. She described this later in life as "an invaluable and immersive learning experience." Upon completing her doctorate in 1951 with Friedrichs, Morawetz first accepted a research associate position at MIT. However she quickly returned to New York University, where she stayed for the remainder of her career. Originally hired as research associate, she became assistant professor in 1957. At that time Courant also hired other NYU graduates to join the faculty, including Harold Grad, Anneli Cahn Lax, then Peter Lax and Louis Nirenberg. They remained close friends throughout her life. Morawetz was tenured in 1960 and earned a full professorship in 1965, the year before being awarded her first of two Guggenheim Fellowships. She was a Gibbs Lecturer in 1981, gave an invited address for SIAM in 1982, and was Noether Lecturer in 1983 and 1988. She served as the director of the Courant Institute at NYU from 1984 to 1988.

Morawetz was awarded the National Medal of Science in 1998 "for pioneering advances in partial differential equations and wave propagation."

Morawetz was an astounding mentor and a dedicated coauthor. Irene Gamba worked with her in 1992–1994 as an NSF postdoctoral fellow. She writes:

> Our discussions lasted for endless hours and were most illuminating and prolific. They culminated with two joint publications related to the approximation to transonic flow problems, and the life changing opportunity of joining the faculty as an assistant professor in the fall of 1994. She was an extraordinary role model for me.

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Cathleen Morawetz and fellow New York University PhD, Harold Grad, back at NYU on the faculty (1964).

Morawetz collaborated often with younger mathematicians, including Gregory Kriegsman, Walter Strauss, Alvin Bayliss, Kevin Payne, Susan Friedlander, Jane Gilman, and James Ralston. Among her doctoral students were Christian Klingenberg and Leslie Sibner.

Morawetz was elected president of the American Mathematical Society in 1993. At that time funding in core mathematics was under threat, the US government was shut down twice, the job market for new doctorates in mathematics was terrible, and universities were reconsidering the importance of having research mathematicians teaching their mathematics courses. We are facing these same difficulties today and can learn from her example.

Morawetz with Irene Gamba on the day Morawetz gave her Noether Lecture at the International Congress of Mathematicians in 1998. Their work was an extraordinary leap into an area that today remains quite unexplored.

As AMS president, Morawetz joined forces with the SIAM president, Margaret Wright, to defend the funding of both pure and applied mathematics. "Together, they formulated carefully worded statements for Congress and agency leaders, always stressing (equally) the remarkable track record of useful mathematics as well as the unexpected benefits that consistently emerge from undirected basic research." [1] Their work led eventually towards the creation of the NSF DMS Grants for Vertical Integration of Research and Education (VIGRE) "to increase the number of well-prepared US citizens, nationals, and permanent residents who pursue careers in the mathematical sciences." This program provided funding for postdocs, graduate students, and undergraduates engaged in research with one another and has directly influenced the careers of many young mathematicians.

Morawetz with Bella Manel (l) and Christina Sormani (r) at NYU in 1996. Manel received her doctorate at NYU in 1939.

Morawetz was a powerful leader, a wonderful mentor, and an amazing mathematician. All of us that were fortunate enough to be influenced by her aura through her ninety-four years of life can admire her relentless pursuit of excellence. Perhaps we too can strive to make a difference.

See Also

Morawetz's retiring AMS presidential address: https://www.ams.org/notices/199901/morawetz.pdf

Morawetz's work on the board of JSTOR: https://www.ams.org/notices/199806/comm-jstor.pdf

Happy 91st, Cathleen Synge Morawetz https://www.ams.org/notices/201405/rnoti-p510.pdf

[1] *Quote taken from a SIAM Memorial of Morawetz by Margaret Wright and John Ewing.*

Terence Tao

Morawetz Inequalities

Cast a stone into a still lake. There is a large splash, and waves begin radiating out from the splash point on the surface of the water. But, as time passes, the amplitude of the waves decays to zero.

This type of behavior is common in physical waves, and also in the partial differential equations used in mathematics to model these waves. Let us begin with the *classical wave equation*

$$-\partial_{tt}u + \Delta u = 0, \tag{1}$$

where $u : \mathbb{R} \times \mathbb{R}^3 \to \mathbb{R}$ is a function of both time $t \in \mathbb{R}$ and space $x \in \mathbb{R}^3$, which is a simple model for the amplitude of a wave propagating at unit speed in three dimensional space; here

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}$$

denotes the spatial Laplacian. One can verify that one has the family of explicit solutions

$$u(t,x) = \frac{F(t+|x|) - F(t-|x|)}{|x|} \tag{2}$$

to (1) for any smooth, compactly supported function $F : \mathbb{R} \to \mathbb{R}$, where

$$|x| = \sqrt{x_1^2 + x_2^2 + x_3^2}$$

denotes the Euclidean magnitude of a position $x \in \mathbb{R}^3$. The dispersive nature of this equation can be seen in the observation that the amplitude

$$\sup_{x \in \mathbb{R}^3} |u(t,x)|$$

of such solutions decays to zero as $t \to \pm\infty$, whilst other quantities such as the energy

$$\int_{\mathbb{R}^3} \frac{1}{2}|\partial_t u(t,x)|^2 + \frac{1}{2}|\nabla u(t,x)|^2 \, dx$$

stay constant in time (and in particular do not decay to zero).

The wave equation can be viewed as a special case of the more general *linear Klein-Gordon equation*

$$-\partial_{tt}u + \Delta u = m^2 u, \tag{3}$$

where $m \geq 0$ is a constant. Even more important is the *linear Schrödinger equation*, which we will normalize here as

$$i\partial_t u + \frac{1}{2}\Delta u = 0, \tag{4}$$

where the unknown field $u : \mathbb{R} \times \mathbb{R}^3 \to \mathbb{C}$ is now complex-valued. There are also nonlinear variants of these equations, such as the *nonlinear Klein-Gordon equation*

$$-\partial_{tt}u + \Delta u = m^2 u + \lambda |u|^{p-1} u, \tag{5}$$

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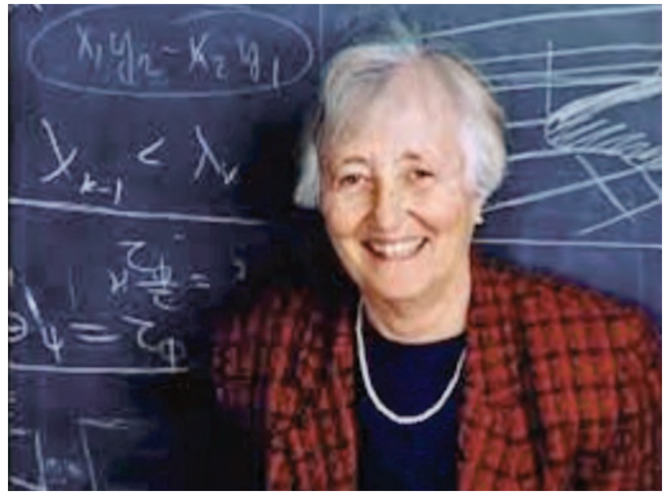

Morawetz worked to change the way we think about partial differential equations.

and the *nonlinear Schrödinger equation*

$$i\partial_t u + \frac{1}{2}\Delta u = \lambda |u|^{p-1} u, \tag{6}$$

where $\lambda = \pm 1$ and $p > 1$ are specified parameters. There are countless other further variations (both linear and nonlinear) of these dispersive equations, such as Einstein's equations of general relativity, or the Korteweg-de Vries equations for shallow water waves.

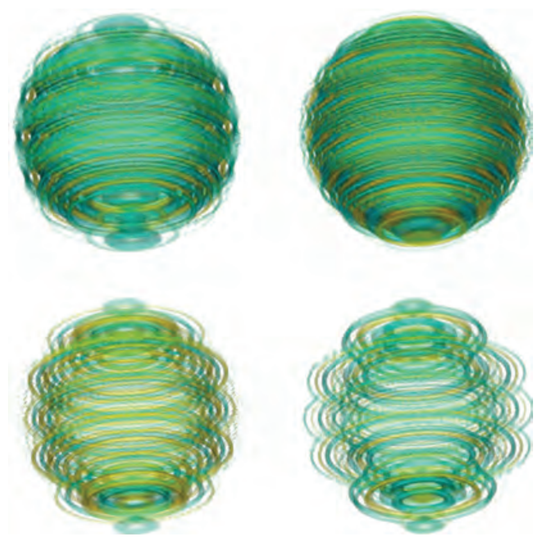

Figure 1. Dispersion is illustrated in this numerical simulation of the Klein-Gordon equation implemented by Brian Leu, Albert Liu, and Parth Sheth using XSEDE when they were undergrads at U Michigan in 2013. For a video see `www-personal.umich.edu/~brianleu`.

An important way to capture dispersion mathematically is through the establishment of *dispersive inequalities* that assert, roughly speaking, that if a solution u to one of these equations is sufficiently localized in space at an initial time, $t = 0$, then it will decay as $t \to \infty$. (If a solution u is not localized enough in space initially, it does not need to decay; consider for instance the traveling wave solution

$$u(t, x) = F(t - x_1)$$

to the wave equation (1).) This decay has to be measured in suitable function space norms, such as the $L^\infty_x(\mathbb{R}^3)$ norm.

One can represent any solution u to the linear Schrödinger equation explicitly in terms of the initial data $u(0)$ by the formula

$$u(t, x) = \frac{1}{(2\pi i t)^{3/2}} \int_{\mathbb{R}^3} e^{-i|x-y|^2/2t} u(0, y)\, dy$$

for all $t \neq 0$ and $x \in \mathbb{R}^3$, where the quantity $(2\pi i t)^{3/2}$ is defined using a suitable branch cut. From the triangle inequality, this immediately gives the dispersive inequality

$$\|u(t)\|_{L^\infty_x(\mathbb{R}^3)} \leq \frac{1}{(2\pi|t|)^{3/2}} \|u(0)\|_{L^1_x(\mathbb{R}^3)}. \tag{7}$$

If the solution is initially spatially localized in the sense that the L^1 norm

$$\|u(0)\|_{L^1_x(\mathbb{R}^3)}$$

is finite, then the solution $u(t)$ decays uniformly to zero as $t \to \pm\infty$. A similar (but slightly more complicated) dispersive inequality can also be obtained for solutions to the linear Klein-Gordon equation (3).

On the other hand, solutions to the linear Schrödinger Equation (4) satisfy the pointwise mass conservation law

$$\partial_t |u|^2 = \sum_{j=1}^{3} \partial_{x_j} \mathrm{Im}(\overline{u} \partial_{x_j} u). \tag{8}$$

From this, one can easily derive conservation of the spacial L^2 norm of the solution:

$$\|u(t)\|_{L^2_x(\mathbb{R}^3)} = \|u(0)\|_{L^2_x(\mathbb{R}^3)}.$$

In particular, the L^2 norm of the solution will stay constant in time, rather than decay to zero.

To reconcile this fact with the dispersive estimate, we observe that solutions to dispersive equations such as linear Schödinger equation spread out in space as time goes to infinity (much as the ripples on a pond do), allowing the L^∞ norm of such a solution to go to zero even while the L^2 norm stays bounded away from zero. As mentioned earlier, this effect can also be seen for the wave equation (1).

The above analysis of the linear Schrödinger equation relied crucially on having an explicit fundamental solution at hand. What happens if one works with nonlinear (and not completely integrable) equations, such as (5) or (6), in which no explicit and tractable formula for the solution is available? For linear equations (such as the wave or Schrödinger equation outside of an obstacle, or in the presence of potentials or magnetic fields) one can still hope to use methods from spectral theory to understand the long-time behavior (as is done for instance in the famous RAGE theorem of Ruelle (1969), Amrein-Georgescu (1973), and Enss (1977)). However, such methods are absent for nonlinear equations such as (5) or (6), particularly when dealing with solutions that are too large for perturbative theory to be of much use.

Cathleen Synge Morawetz in 1964.

Recall that in 1961, Morawetz proved the decay of solutions to the classical wave equation in the presence of a star-shaped obstacle. Morawetz used the "Friedrichs *abc* method," in which one multiplied both sides of a PDE such as (5) or (6) by a multiplier

$$a\partial_t u + b \cdot \nabla u + cu$$

for well chosen functions a, b, c, integrated over a space-time domain, and rearranging using integration by parts and omitting some terms of definite sign, obtained a useful integral inequality. The key discovery of Morawetz (a version of which first appeared in work of Ludwig) was that this method was particularly fruitful when the multiplier was equal to the radial derivative

$$\frac{x \cdot \nabla u}{|x|}$$

of the solution (in some cases one also adds a lower order term $\frac{u}{|x|}$).

In 1968, Morawetz applied this technique to study solutions u to the nonlinear Klein-Gordon equation (5), assuming one is in the nonfocusing case with $\lambda = m = +1$. (In the *focusing* case $\lambda = -1$, the equation (5) admits "soliton" solutions that are stationary in time and thus do not disperse.) By using a multiplier of the above form, Morawetz obtained an inequality of the form

$$\int_{\mathbb{R}} \int_{\mathbb{R}^3} U(t, x)\, dx\, dt \leq CE(u(0)), \tag{9}$$

where

$$U(t, x) = \frac{|u(t, x)|^2 + |u(t, x)|^{p+1}}{|x|}.$$

Here the constant C depends only on the exponent p and $E(u(0))$ is the energy:

$$E(u(0)) = \int_{\mathbb{R}^3} \frac{1}{2}|\nabla u(0,x)|^2 + \frac{1}{2}|\partial_t u(0,x)|^2 + \frac{1}{(p+1)}|u(0,x)|^{p+1}\,dx.$$

This type of estimate is now known as a *Morawetz inequality*. The key point here is that the left-hand side of the Morawetz inequality in (9) contains an integration over the entire time domain $\mathbb{R}$ (as opposed to a time integral over a bounded interval). It immediately rules out soliton-type solutions that move at bounded speed (as this would make the left-hand side of (9) infinite). It forces some time-averaged decay of the solution near the spatial origin $x = 0$. For instance, it is immediate from (9) that

$$\frac{1}{T}\int_0^T \int_K |u(t,x)|^2\,dx\,dt \to 0$$

as $T \to \infty$ for any compact spatial region $K \subset \mathbb{R}^3$.

Cathleen Morawetz and Walter Strauss in 2008.

Once one has some sort of decay estimate for a dispersive equation, it is often possible to "bootstrap" the estimate to obtain additional decay estimates. For instance one might use the decay estimate one already has to bound the right-hand side of a nonlinear PDE such as (5) or (6), and then solve the associated (inhomogeneous) linear PDE to obtain a new decay estimate for the solution.

An early result of this type was developed by Morawetz and Strauss in 1975. They showed that for any finite energy solution u to the nonlinear Klein-Gordon equation (5) with $\lambda = m = +1$ and $p = 3$, the solution decays like a solution to the linear Klein-Gordon equation (3). More precisely, there exist finite solutions u_+, u_- to (3) such that $u(t) - u_+(t)$ (resp. $u(t) - u_-(t)$) goes to zero in the energy norm as $t \to +\infty$ (resp. $t \to -\infty$). This can be developed further into a satisfactory *scattering theory* for such equations, which among other things gives a continuous *scattering map* from u_- to u_+ or vice versa. See Figure 2.

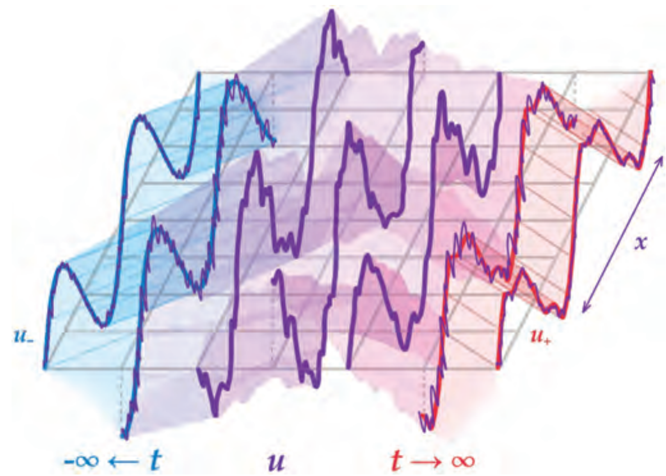

Figure 2. Here we see u_- on the left in blue and u_+ on the right in red, with u in purple approximating u_- as $t \to -\infty$ and approximating u_+ as $t \to \infty$.

In the decades since Morawetz's pioneering work, many additional Morawetz inequalities have been developed. For instance, in 1978, Lin and Strauss developed Morawetz inequalities for the nonlinear Schrödinger equation, and Morawetz herself discovered further such estimates for the wave equation outside of an obstacle. In more recent years, "interaction Morawetz inequalities" were introduced, which could control correlation quantities such as

$$\int_{\mathbb{R}}\int_{\mathbb{R}^3}\int_{\mathbb{R}^3} \frac{|u(t,x)|^2|u(t,y)|^p}{|x-y|}\,dxdydt \tag{10}$$

for solutions u to the nonlinear Schrödinger equation (6).

One way to view Morawetz inequalities is as an assertion of monotonicity of the *radial momentum*, which takes the form

$$\int_{\mathbb{R}^3} (\partial_t u)\left(\frac{x}{|x|}\cdot \nabla u\right)\,dx$$

for wave or Klein-Gordon equations, and

$$\int_{\mathbb{R}^3} \mathrm{Im}\left(\overline{u}\frac{x}{|x|}\cdot \nabla u\right)\,dx$$

for Schrödinger equations. Informally, this quantity is expected to be positive when waves propagate away from the origin, and negative when they propagate towards the origin. The intuition is that while waves can sometimes propagate towards the origin, eventually they will move past the origin and begin radiating away from the origin. However, in the absence of focusing mechanisms (such as a negative sign $\lambda = -1$ in the nonlinearity), the reverse phenomenon of outward net radial momentum being converted to inward net radial momentum cannot occur. Thus the radial momentum is always expected to be increasing in time.

On the other hand, under hypotheses such as finite energy, this radial momentum should be bounded. So by the fundamental theorem of calculus, the time derivative of the radial momentum should have a bounded integral in time. Intuitively, one expects this time derivative to be large when the solution has a strong presence near the origin, but not when the solution is far away from the origin. Far from the origin the radial vector field

The Morawetz inequalities are indispensible.

$$\frac{x}{|x|} \cdot \nabla$$

behaves like a constant, and the radial momentum approaches a fixed coordinate of the total momentum. This explains why Morawetz inequalities tend to involve factors such as $\frac{1}{|x|}$ that localize the estimate to near the origin.

The Morawetz inequalities are indispensible as an ingredient in controlling the long-time behavior of solutions to a wide array of dispersive defocusing equations, including a number of *energy-critical* or *mass-critical* equations in which the analysis is particularly delicate and interesting; see for instance the texts [1], [4], [3] for detailed coverage of these topics. They have also been successfully applied to many equations in general relativity (such as Einstein's equations for gravitational fields), for instance to analyze the asymptotic behaviour around a black hole. The fundamental tools that Morawetz has introduced to the field of dispersive equations will certainly underlie future progress in this field for decades to come.

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Leslie Greengard and Tonatiuh Sánchez-Vizuet

Cathleen Morawetz and the Scattering of Acoustic Waves

Cathleen Morawetz was a force at the Courant Institute when one of us (L.G.) arrived as a postdoctoral fellow. It was the last year of her directorship, but she made the time to welcome all newcomers. Her generosity of spirit was unmatched — she encouraged young people in every discipline, and her humour and enthusiasm were infectious.

When she began to study the decay properties of acoustic waves after impinging on an obstacle, essentially no general results were available. To understand the relevant issues, let us begin with the formulation of the problem in terms of the governing linear, scalar wave equation in $\mathbb{R}^3$, with a forcing term which is turned on for a finite time:

$$u_{tt}(\mathbf{x}, t) = \Delta u(\mathbf{x}, t) + f(\mathbf{x}, t). \tag{11}$$

Here, Δu is the Laplacian operator acting on the scalar function $u(\mathbf{x}, t)$ and $f(\mathbf{x}, t)$ is nonzero only in the finite time interval $0 \le t \le T$. We assume that we have zero initial (Cauchy) data at time $t = 0$:

$$u(\mathbf{x}, 0) = 0 \qquad \text{and} \qquad u_t(\mathbf{x}, 0) = 0.$$

We also assume that $f(\mathbf{x}, t)$ is a smooth, compactly supported and square integrable function in space-time, such as

$$f(\mathbf{x}, t) = W(\|\mathbf{x} - \mathbf{x}_0\|)\, W\left(\frac{2t - T}{T}\right), \tag{12}$$

where $\mathbf{x}_0 \in \mathbb{R}^3$ and $W(x)$ is a standard C^∞ bump function such as

$$W(x) = \begin{cases} e^{-\frac{1}{1-x^2}} & \text{for } |x| < 1; \\ 0 & \text{otherwise.} \end{cases}$$

Then, it is well known that

$$u(\mathbf{x}, t) = \frac{1}{4\pi} \int_{B_{\mathbf{x}_0}(1)} \frac{f(\mathbf{x}', t - \|\mathbf{x} - \mathbf{x}'\|)}{\|\mathbf{x} - \mathbf{x}'\|} d\mathbf{x}', \tag{13}$$

where $B_{\mathbf{x}_0}(1)$ denotes the unit ball centered at $\mathbf{x}_0$. From this formula it is clear that at any point $\mathbf{x}$ in space, the solution first becomes nonzero at $t = d_{min}$, where d_{min} is the distance from $\mathbf{x}$ to the closest point in $B_{\mathbf{x}_0}(1)$. It vanishes identically at $\mathbf{x}$ as soon as $t > T + d_{max}$, where d_{max} is the distance from $\mathbf{x}$ to the farthest point in $B_{\mathbf{x}_0}(1)$.

Suppose now that, rather than propagating in free-space, the outgoing spherical wavefront emanating from $\mathbf{x}_0$ hits an object as in Figure 3. That is, we assume there is a "sound-soft," smooth, bounded obstacle Ω with boundary $\partial\Omega$, which is at some distance from $B_{\mathbf{x}_0}(1)$, so that $\Omega \cap B_{\mathbf{x}_0}(1) = \emptyset$. Then, using the language of scattering theory, the total acoustic field is given by

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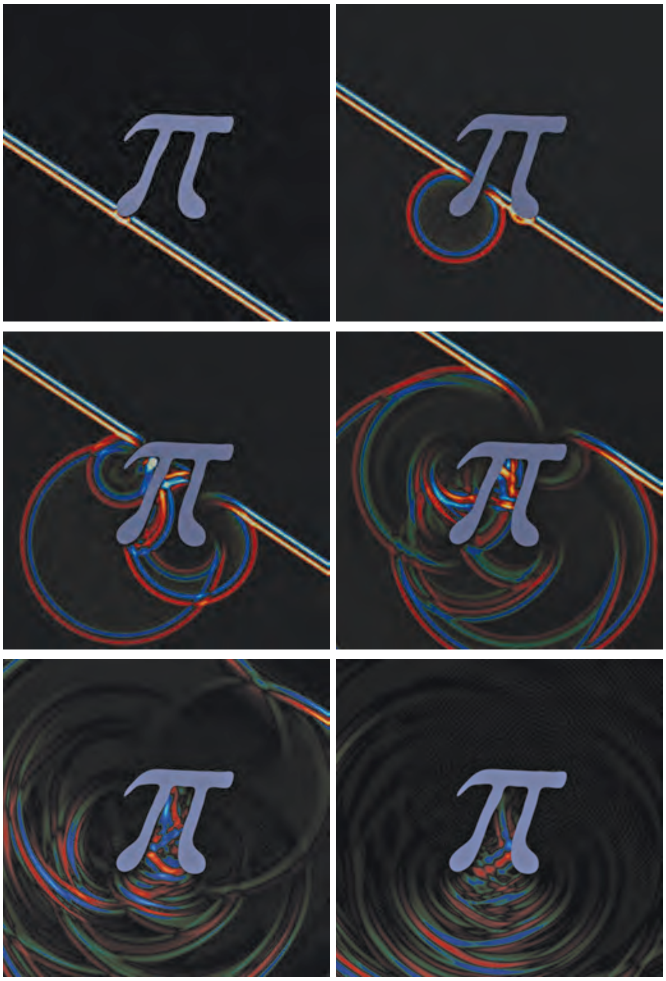

Figure 3. The evolution of an acoustic wave impinging upon a non-star shaped object, π, at times $t = 1, 4, 8, 13, 19, 26$ implemented using a high-order integral equation solver. A video of the simulation may be found at `https://cims.nyu.edu/~tonatiuh/morawetz.html`**.**

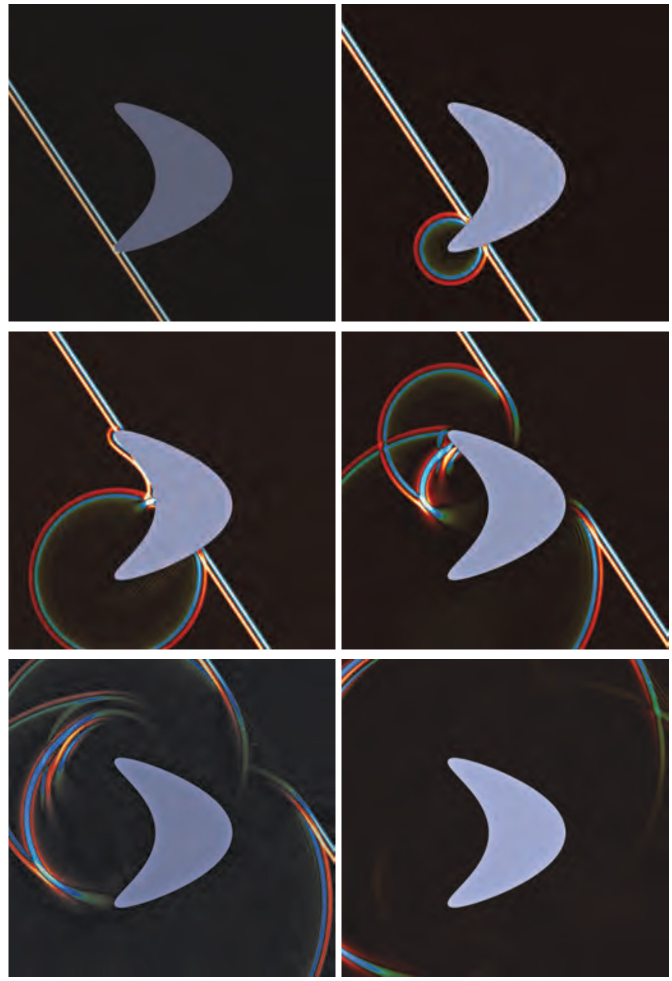

Figure 4. The evolution of an acoustic wave being scattered off a star shaped obstacle at the same sequence of times as in Figure 3. In the final panel, the scattered wave has almost completely left the simulation region, in contrast with the final panel of Figure 3.

$u(\mathbf{x}, t) + u^{scat}(\mathbf{x}, t)$, where the scattered field satisfies the homogeneous wave equation

$$u_{tt}^{scat}(\mathbf{x}, t) - \Delta u^{scat}(\mathbf{x}, t) = 0$$

for $t > 0$, with initial data

$$u^{scat}(\mathbf{x}, 0) = 0 \qquad \text{and} \qquad u_t^{scat}(\mathbf{x}, 0) = 0$$

and Dirichlet boundary conditions

$$u^{scat}(\mathbf{x}, t) = -u(\mathbf{x}, t)$$

for $\mathbf{x} \in \partial\Omega$.

Let $\mathbf{y}$ denote some fixed point away from both the ball $B_{\mathbf{x}_0}(1)$ and the obstacle Ω. The question is: can one prove that the scattered field decays at $\mathbf{y}$, and if so, at what rate? Very little progress had been made on this question until 1959, when Wilcox published a short note showing that in the case of a spherical obstacle, an exact solution could be expressed in terms of spherical harmonics. From this, Wilcox was able to conclude that the solution decays exponentially fast. While an important step, his result yielded no suggestion as to how to proceed in the general case.

In 1961, Morawetz [4] made a critical step forward. She showed that if the reflecting obstacle is star-shaped, then the solution to the wave equation decays like $t^{-1/2}$. A region Ω is said to be star-shaped if there exists a point $\mathbf{p} \in \Omega$, such that for all $\mathbf{x} \in \Omega$, the line segment from $\mathbf{p}$ to $\mathbf{x}$ is contained in Ω. The object in Figure 3, for example, is not star shaped, while the object in Figure 4 is. It is perhaps surprising that for non star-shaped obstacles, very little is understood to the present day.

The qualitative difference in the behavior of waves reflecting from obstacles that are not star-shaped and those that are is illustrated in Figures 3 and 4. In the first three panels of each figure, as the incoming wave hits the object, the scattered wave is clearly visible, with energy propagating outwards in all directions. In the next three panels, more of the energy is carried away. In Figure 3, some of the energy remains behind for quite some time, and in the last panel a significant amount of energy has focused in a small neighborhood. In Figure 4, the energy has propagated outward without significant concentration and appears to decay much more rapidly.

Remark. The simulations in these figures are actually for the two-dimensional wave equation, with an incoming plane wave of the form

$$u^{inc}(\mathbf{x}, t) = \chi(s/\alpha)\sin^3(s/\alpha), \quad s := \mathbf{x}\cdot\mathbf{d} - t.$$

The unit vector $\mathbf{d}$ points in the direction in which the wave propagates, $\chi(\cdot)$ is a smooth approximation to the characteristic function of the interval $[0, 2\pi]$, and α is a scaling factor that has the effect of shrinking (if $\alpha < 1$) or dilating (if $\alpha > 1$) the wave profile. In two dimensions, waves do *not* decay exponentially fast, even in the absence of a scatterer, but the focusing/trapping effect caused by nonstar shaped obstacles is similar.

Peter Lax with Cathleen Morawetz at the 2008 Conference on Nonlinear Phenomena in Mathematical Physics: Dedicated to Cathleen Synge Morawetz on her 85th Birthday.

Morawetz's writing style was very much that of a storyteller. To get a sense of that, here is the beginning of the proof of the main theorem in her 1961 paper [4]:

> The proof is based on energy identities, i.e. quadratic integral relations satisfied by all solutions. This is one of the most powerful tools for getting estimates for solutions of elliptic, hyperbolic or mixed equations. The most familiar identity of this kind for the wave equation is obtained by multiplying $u_{tt} = \Delta u$ by u_t and integrating in the slab $0 \le t \le t_1$; the resulting integral identity satisfies the conservation of energy. Here we use another multiplier in the place of u_t introduced by Protter for another purpose. The significance of using alternative multipliers has been frequently emphasized by Friedrichs and is often referred to as Friedrichs's a, b, c-method. The multiplier here is
>
> $$xu_x + yu_y + zu_z + tu_t + u \tag{14}$$
>
> and from the resulting identity we conclude that all the energy is carried outward.

In truth, Morawetz was being overly modest. It was her keen insight that allowed for the selection of a multiplier which would yield the desired result. The power and generality of this approach led to breakthroughs in many wave propagation problems, with the state of the art collected in Morawetz's 1966 monograph "Energy identities for the wave equation," originally released as a Courant Institute technical report.

A second major step forward in understanding the decay of waves scattered from star-shaped obstacles came in 1963, in joint work with Lax and Phillips. They showed that, in fact, such solutions decay exponentially (as they do for a sphere), not just as $t^{-1/2}$. The proof relies on an observation of Lax and Phillips that there is a function $Z(t)$ which satisfies the semigroup property

$$Z(t+s) = Z(t)Z(s), \tag{15}$$

and whose norm controls the decay of the solution. In this context, Morawetz's 1961 paper shows that for some time $t = \tau$, $|Z(\tau)|$ has decayed to less than one:

$$|Z(\tau)| < 1 = e^{-\alpha} \text{ for some } \alpha > 0.$$

That is enough to guarantee exponential decay! One simply writes

$$t = n\tau + t_1 \text{ where } t_1 < \tau,$$

from which

$$\begin{aligned} |Z(t)| &= |Z(t_1)||[Z(\tau)]^n| \\ &\le |Z(t_1)|e^{-\alpha n} = Ce^{-\alpha t/\tau}. \end{aligned}$$

Nontrapping Objects

One of the features of star-shaped objects is that rays impinging on them cannot be trapped. A ray here is the path taken by an infinitely thin beam of light which reflects from the surface according to geometrical optics. For a complicated scatterer, one can imagine that a ray could undergo successive bounces without escaping from the convolutions of the surface $\partial\Omega$ in any finite time interval (see Figure 5).

This situation was studied by Morawetz, Ralston, and Strauss in their 1977 article, where they proved a remarkable extension of Morawetz's earlier results; if the object Ω does not trap rays, then the scattered wave decays exponentially. The proof involves the introduction of an *escape function* (a generalization of the geometric intuition of an "escape path of finite length") and a different multiplier from that in Morawetz's 1961 paper. The use of such *Morawetz multipliers* is now ubiquitous in the analysis of PDEs.

Denoting by S a sphere which contains the smooth scatterer Ω, considering $\mathbf{x} \in S\backslash\Omega$, and letting ξ be a unit

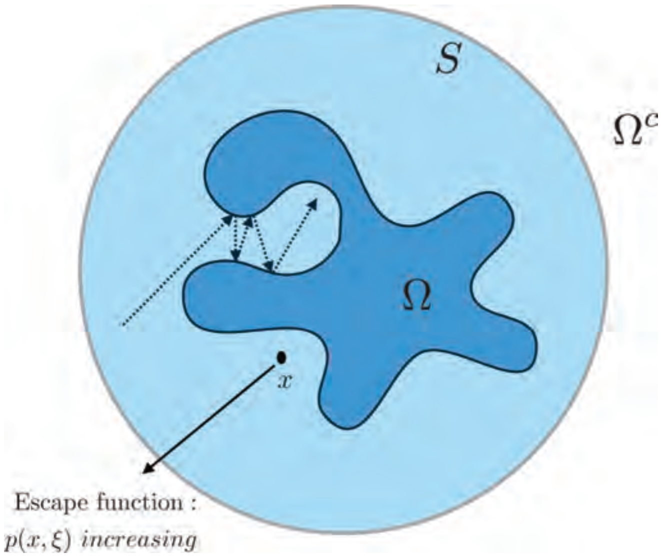

Figure 5. In 1977, Morawetz, Ralston, and Strauss generalized the class of scatterers for which decay results could be proven. They showed, in three dimensions, that if the object Ω does not trap rays, then the local energy of the wave must decay exponentially. Star-shaped objects are a subset of this much larger class. The path taken by a ray is depicted, reflecting from the surface each time according to geometrical optics.

vector in $\mathbb{R}^3$, $p(\mathbf{x},\xi)$ is said to be an escape function if it is real-valued, C^∞, and, informally speaking, "strictly increasing along rays, ξ being the ray direction at $\mathbf{x}$." Rays are said to be *not trapped* if the total path length in $S\backslash\Omega$ is bounded and waves are said to be *not trapped* if the local energy in $S\backslash\Omega$ decays to zero uniformly. Without entering into details, Morawetz, Ralston, and Strauss showed (1) that if rays are not trapped, then there exists an escape function and (2) that if there exists an escape function, then waves are not trapped, from which the result follows.

Geometric Optics and Frequency Domain Analysis

In the study of linear wave propagation, much of our understanding comes from the frequency domain — that is, analyzing the Fourier transform of the wave equation (11):

$$-k^2U(\mathbf{x},k) - \Delta U(\mathbf{x},k) = F(\mathbf{x},k). \tag{16}$$

Depending on the context, this is referred to as the Helmholtz or reduced wave equation. In 1968, Morawetz, together with Don Ludwig, began an investigation of exterior scattering from star-shaped surfaces in the frequency domain [5]. Two major results were presented there. First, they provided a key proof of the well-posedness of the scattering problem for sound-soft boundaries (homogeneous Dirichlet boundary conditions) with respect to the boundary data and forcing term $F(\mathbf{x},k)$ in (16). They also introduced what are now called Morawetz identities for the Helmholtz equation. Second, they showed that the formulas produced by the theory of geometrical optics are asymptotic to the exact solution. The relevant asymptotic regimes are illustrated in Figure 6.

Without entering into technical details, geometrical optics is based on expanding the incoming and scattered waves in terms of a series in inverse powers of the wavenumber k about the point $\mathbf{x}_0$ (see Figure 6). Ludwig had earlier proposed an expansion for the penumbra region as well. Morawetz and Ludwig showed that all of these expansions are truly asymptotic: to the solution in the illuminated region and penumbra, and asymptotically zero in the deep shadow.

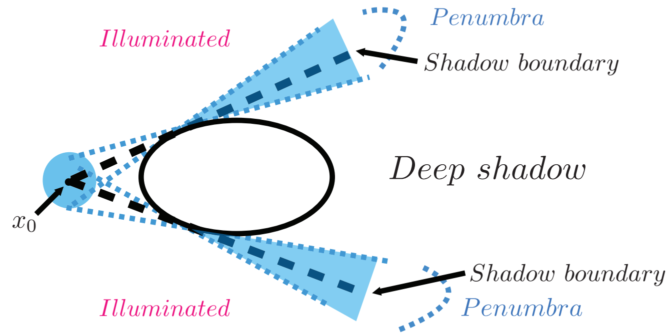

Figure 6. The asymptotic regimes for geometrical optics. For a fixed point $\mathbf{x}_0$, the two tangent lines to the scatterer define the *shadow boundary* (dashed black lines), which separates the illuminated region from the shadow region. The *penumbra* is a neighborhood of the shadow boundary, formed by the union of all shadow boundaries of spherical waves with centers in a neighborhood of $\mathbf{x}_0$. (Adapted from [5]).

Although Morawetz herself did little numerical computation, her analytic work (especially on multipliers) has played a major role in the design of numerical methods. We cannot do justice to the literature here, but refer the reader to three recent papers: one on eigenvalue computation, one on frequency domain scattering, and one on time-domain integral equations [1–3]. We have only been able to scratch the surface of her legacy in this note. Her contributions are profound and deep, and have changed the way we think about partial differential equations. She was a wonderful friend and colleague and is greatly missed.

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Kevin R. Payne

Transonic Flow and Mixed Type Partial Differential Equations

The work of Cathleen Morawetz on transonic fluid flow and the underlying PDEs of mixed elliptic-hyperbolic type spanned her career. Here we describe her earliest work. Beginning in the mid 1950s, Morawetz began working on transonic flow problems through her interactions with Kurt O. Friedrichs and Lipman Bers. This problem area was ripe for the unique blend of joyous ingenuity and practical tenacity which characterized her approach to happily doing mathematics in order to say something about a real world problem. Morawetz quickly made a name for herself by giving a mathematical answer to an important engineering question in transonic airfoil design.

Morawetz in 1958.

Morawetz periodically returned to this area with bursts of productivity that resulted in fundamental contributions over the next five decades. The photo of Morawetz in 1958 shows the happy face that Morawetz would display when discussing what interested her most. It was with the same gentle smile and glint in the eyes that she might also show her warm toughness and attachment to physical relevance when liquidating a night's calculations of a collaborator with a phrase like: "You know, the solutions should not really behave this way. Let's change the equation."

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What is Transonic Flow About?

In aerodynamics, a basic question is: *How does one fly at a relatively high speed, with relatively low cost and relatively low ecological damage?* In Morawetz's 1982 article in the *Bulletin of the AMS*, she described the problem as follows. The science of flight depends on the relative speed of the aircraft with respect to the speed of sound in the surrounding air. At relatively low speeds, the *subsonic range*, one can "sail" by designing wings to "get as much as possible of a free ride" from the wind. At very high speeds, the *supersonic range*, one needs "rocket propulsion" to overcome the *drag* produced by *shocks* that invariably form (the sonic boom). The goal of studying transonic flow is to find a compromise which allows for "sailing" efficiently "near the speed of sound." Shocks produce drag, which increases fuel consumption and hence increases cost. As seen in Figure 16, shocks (colored red) begin to appear on airfoils in wind tunnels when the upstream velocity is below, but near the speed of sound.

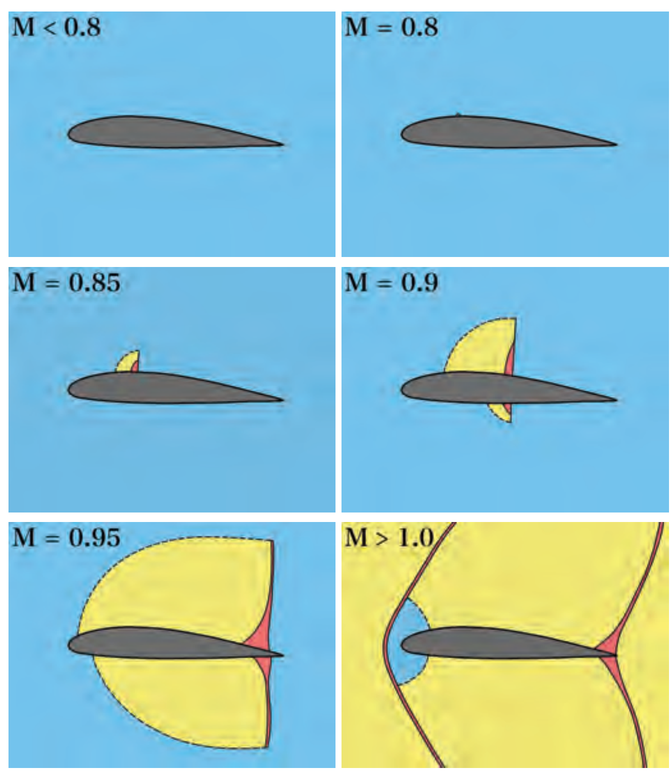

Figure 7. As wind tunnel speed increases from subsonic (blue regime, Mach $M < 1$) to supersonic (yellow regime, Mach $M > 1$), some supersonic shock (in red) appears over the wing already at Mach M=.85.

The 2-D irrotational, stationary, compressible and isentropic flow of air about a profile $\mathcal{P}$ is governed by an equation for the potential $\phi(x, y)$ whose gradient is the velocity field of the fluid with variable density ρ:

$$(c^2 - \varphi_x^2)\,\varphi_{xx} - 2\varphi_x\varphi_y\varphi_{xy} + (c^2 - \varphi_y^2)\,\varphi_{yy} = 0. \tag{17}$$

The natural boundary condition is to have normal derivative

$$\frac{\partial \varphi}{\partial n} = 0 \text{ on } \partial\mathcal{P}. \tag{18}$$

The nature of the flow is determined by the local Mach number $M = q/c$ where $q = |\nabla\varphi|$ is the flow speed and $c > 0$ is the local speed of sound defined by $c^2 = \partial p/\partial\rho$, where the adiabatic pressure density relation in air is $p = p(\rho) \sim \rho^\gamma$ with $\gamma \approx 1.4$. Observe that equation (17) is of the form

$$A\varphi_{xx} - 2B\varphi_{xy} + C\varphi_{xx}.$$

It is *elliptic* when

$$AC - B^2 > 0,$$

which occurs at points where the flow is subsonic ($q < c$). It is *hyperbolic* when

$$AC - B^2 < 0,$$

which occurs at points where the flow is supersonic ($q > c$) (see Figure 7). A transonic flow happens when there are both sub- and supersonic regions and the equation (17) is of mixed elliptic-hyperbolic type.

The presence of shocks in supersonic regions corresponds to drastic changes in air density and pressure coming from the compressibility, and these large pressure changes propagate at supersonic speeds, resulting in a shock wave which typically has a small but finite thickness. In Figure 7, the shock wave region is depicted in red. The velocity field $\nabla\varphi$ governed by (17) will experience jump discontinuities as one crosses the shock wave. One can use the presence of such discontinuities to detect the presence of shocks. The mathematical description of shocks requires a separate analysis of entropy effects, where equation (17) has broken down.

The Transonic Controversy

By the time of the Third International Congress for Applied Mechanics in 1930, a lively debate centered around the question: *Do transonic flows about a given airfoil always, never, or sometimes produce shocks?* In particular, *is it possible to design a viable airfoil capable of shock-free flight at a range of transonic speeds*? Contrasting evidence was presented at the congress which led many aerodynamicists to take opposing views. G.I. Taylor presented convergent Rayleigh series expansions for the velocity potential of some smooth transonic flows, while A. Busemann presented the results of wind tunnel experiments that indicated the presence of a lot of shocks. World War II moved attention to rocket propulsion. An answer would await the work of Morawetz in the 1950s. It was a case of "mathematics coming to the rescue."

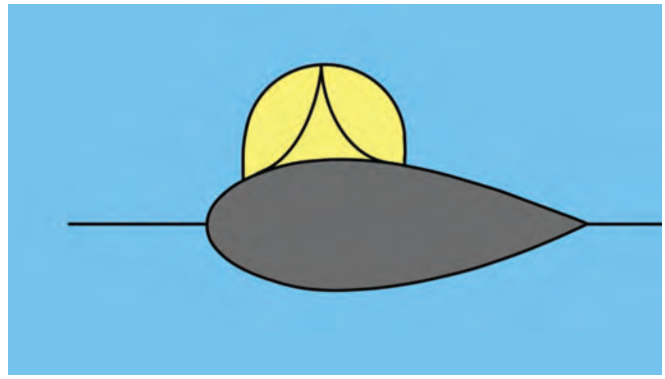

Figure 8. Morawetz's theorem proved that any perturbation of the wing inside the yellow supersonic regime creates shocks.

Morawetz's Answer to the Transonic Controversy

In a series of papers published in 1956–58 in *Comm. Pure Appl. Math.*, Morawetz gave a mathematical answer by proving that shock-free transonic flows are unstable with respect to arbitrarily small perturbations in the shape of the profile. Her theorem says that even if one can design a viable profile capable of a shock-free transonic flow, imperfection in its construction will result in the formation of shocks at the design speed.

Theorem. *Let φ be a transonic solution to* (17)-(18) *with continuous velocity field $\nabla\varphi$ and fixed speed q_∞ at infinity about a symmetric profile $\mathcal{P}$ as in Figure 8. For an arbitrary perturbation $\tilde{\mathcal{P}}$ of $\mathcal{P}$ along an arc inside the supersonic region attached to the profile which contains the point of maximum speed in the flow, there is NO continuous $\nabla\tilde{\varphi}$ solving the corresponding problem* (17)-(18) *with $\tilde{\mathcal{P}}$.*

Morawetz's proof involved two major steps. First, she determined the correct boundary value problem satisfied the perturbation of the velocity potential in the *hodograph plane* where a *hodograph transformation* linearizes the PDE (17) and sends the known profile exterior into an unknown domain. Then, using carefully tailored integral identities, she proved a uniqueness theorem for regular solutions of the transformed PDE with data prescribed on only a proper subset of the transformed boundary profile, which says that the transformed problem is *overdetermined* and no regular solutions exist. Morawetz extended this result to include fixed profiles but finite perturbations in q_∞, and the extension to non symmetric profiles was carried out by L. Pamela Cook (*Indiana Univ. Math. J.*, 1978).

Engineering Impact

While Morawetz's work left open the theoretical possibility of a *perfect transonic airfoil* capable of shock-free flight over a small range of transonic speeds, imperfection in its construction means the search for it is futile. Instead engineers must calibrate wing design to *minimize shock strength* over a useful range of transonic speeds.

Beginning in the early 1960s with the work of H.H. Pearcy and later R.R. Whitcomb on *supercritical airfoils*, transonic airfoil design paid close attention to the impact of Morawetz's findings. In the midst of the energy crisis of the 1970s, this direction of research exploded as part of the field of *computational fluid dynamics*. The *type-dependent difference scheme* of E.M. Murman and J.D. Cole (1971), the *complex characteristic method* of P. Garabedian and D. Korn (1971), and the *rotated difference scheme* of A. Jameson (1974) were some of the milestones in the economically viable calculation of steady transonic flows and codes for transonic airfoil design.

Morawetz with Paul Garabedian, whose complex characteristic method with D. Korn applied Morawetz's work to computational fluid dynamics.

Mathematical Impact

Cathleen Morawetz's early work on transonic flow both transformed the field of mixed type partial differential equations and served as excellent publicity for mathematics. Commenting on the transonic controversy in 1955, the celebrated aerodynamicist Theodore von Kármán observed: "… the mathematician may exactly prove existence and uniqueness of solutions in cases where the answer is evident to the physicist or engineer… On the other hand, if there is really serious doubt about the answer, the mathematician is of little help." Morawetz's surprising theorem on the nonexistence of smooth flows was a cheerful response to von Kármán's well-intentioned challenge.

Having settled the engineering question about the "exceptional nature" of shock-free transonic flows, Morawetz turned to related questions: *Can one prove robust existence theorems for weak shock solutions? Can one "contract" a weak shock to a sonic point on the profile?* The first question was supported by work of Garabedian-Korn in 1971, which demonstrated that small perturbations of continuous flows can have only weak shocks. The second question was inspired by the thinking of K.G. Guderley in the 1950s. Morawetz took two very different approaches to such questions.

Taking a singular perturbation with a hodograph transformation, the questions reduce to proving the existence of weak solutions to the Dirichlet problem for *linear* mixed type equations on domains Ω in the hodograph plane:

$$K(\sigma)\psi_{\theta\theta} + \psi_{\sigma\sigma} = f \quad \text{in} \quad \Omega \tag{19}$$

$$\text{and} \quad \psi = 0 \quad \text{on} \quad \partial\Omega, \tag{20}$$

where $K(\sigma) \sim \sigma$ as $\sigma \to 0$. Here ψ is the stream function of the flow, σ is a logarithmic rescaling of the flow speed which is sonic at $\sigma = 0$, and θ is the flow angle. For special domains, Morawetz [*Comm. Pure Appl. Math.* 1970] proved the surprising result of the existence of a unique weak solution to the problem.

Inspired by the differencing method of Jameson, Morawetz introduced an *artificial viscosity* parameter ν into the *nonlinear* potential equation by replacing the (inviscid) Bernoulli law

$$\rho = \rho_B(|\nabla\varphi|)$$

with a first order PDE which retards the density ρ. An ambitious program ensued in order to prove the existence of weak solutions to the inviscid problem as a weak limit of viscous solutions. Powerful but delicate tools in the application of the *compensated compactness method* of F. Murat, L. Tartar, and R. Di Perna were applied with success to complete parts of the program in Morawetz [*Comm. Pure Appl. Math.* 1985, 1991] and Gamba-Morawetz [*Comm. Pure Appl. Math.* 1996].

The Legacy of Cathleen Morawetz

Cathleen Morawetz with her family in 1958 when she solved the transonic controversy.

During the period 1952–2007, Morawetz produced 22 deep research papers and 10 survey papers on transonic flow and mixed type partial differential equations. She was an exemplary figure of the applied mathematician "who proves theorems to solve problems." Morawetz discovered and implemented a wide variety of tools to handle the complexity of mixed type PDEs. She developed energy methods and important identities by the skillful and ingenious use of multiplier methods championed by K. Friedrichs [*Comm. Pure Appl. Math.* 1958] and found surprising maximum principles which were calibrated to invariances in the equation.

The legacy of Cathleen Morawetz includes her dedication to the proposition that "there is no such thing as a distant relative," which she applied to every part of her well-lived life. Her grace, warmth and generosity to generations of mathematicians working in the area will be long remembered. She was a truly inspirational figure who invited us all to "sail with her, near the speed of sound."[2]

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Closing Thoughts

Cathleen Synge Morawetz with Richard Courant.

We close this article with a quote by Cathleen Synge Morawetz. Upon receiving the Birkhoff Prize in 2006, she said:

> There are many, many people whom I would have liked to thank for helping me over the years, but I would not have room for their names on this page. But one person stands out for supporting and encouraging me when I was between the crucial professional ages of twenty-three and thirty-five. I worked part-time on my PhD, part-time as a postdoc, and I had four children. That person was Richard Courant, the creator of the Courant Institute at New York University, where I have been a professor ever since.

It is truly rare for any department to support a woman's career in this way: with part-time research-associate positions and a long term commitment that does not require the woman to relocate every few years to eventually obtain a tenure track position. Many women leave academia after completing their doctorates, switching to jobs in industry, while others land in teaching positions and never have the opportunity to develop a research career. It is a great loss of talent. Imagine a world in which Morawetz had never developed her paramount results on transonic flow models, functional inequalities and scattering theory. Imagine a world in which more women's research were supported as well as hers was. It would be a better place.

ACKNOWLEDGMENTS. We would like to thank Cathleen Morawetz's son, John, and also the head of the Courant Institute Library, Carol Hutchins, for finding many wonderful photos and researching their dates and locations. We regret that we could only include a few here. We would also like to thank Penelope Chang for volunteering as an artist for the AMS *Notices*.

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Opening photo of Morawetz on boat courtesy of Morawetz family.

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Photo of Morawetz with Irene Gamba courtesy of Sylvia Wiegand and AWM.

Photo of Morawetz with Bella Manel and Christina Sormani courtesy of Roy Goodman.

Photo of Morawetz in front of chalkboard courtesy of James Hamilton.

Figure 1 by Brian Leu, Albert Liu, and Parth Sheth.

Photo of Morawetz in 1964 courtesy of NYU.

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Figures 5 and 6 by Leslie Greengard.

Figure 7 by Penelope Chang based on an image in `slideshare.net`.

Figure 8 adapted from Morawetz's diagram in her 1964 CPAM article by Penelope Chang.

Photos of Morawetz with Paul Garabedian and with Richard Courant courtesy of NYU.

Photo of Morawetz in 1958 courtesy of her family.

Photo of Irene M. Gamba courtesy of Irene M. Gamba.

[2] *Quotes in this paragraph are from Kevin Payne's talk "Cathleen's mathematics: transonic flow" and Nancy Morawetz's talk, respectively, at the Courant Institute "Celebration in Honor of Cathleen Synge Morawetz" on November 17, 2017.*

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Photo of Christina Sormani courtesy of Christina Sormani.
Photo of Terence Tao by Reed Hutchinson/UCLA.

Irene Martinez Gamba

ABOUT THE AUTHORS

Irene Martinez Gamba is an AMS Fellow and a SIAM Fellow. She completed her doctorate at the University of Chicago and completed groundbreaking work with Cathleen Morawetz as a post-doc at the Courant Institute. Her research areas include applied and computational analysis, mathematical and statistical physics, nonlinear kinetic and partial differential equations.

Leslie Greengard

Leslie Greengard is a SIAM Fellow, professor of mathematics and computer science at the Courant Institute, NYU, and director of the Center for Computational Biology at the Flatiron Institute, a division of the Simons Foundation. He served as director of the Courant Institute from 2006 to 2011. His research interests include the development of fast, high-order accurate integral equation based methods for problems in computational physics and inverse problems that arise in biological and biomedical imaging.

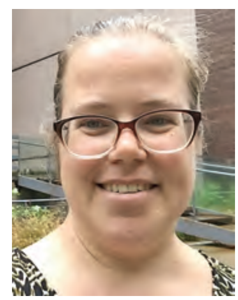
Kevin R. Payne

Kevin R. Payne is an AMS Fellow. His research centers on the use of energy methods and maximum principles for PDE of mixed elliptic-hyperbolic and degenerate types. He will be forever thankful to Cathleen Morawetz for getting him interested in such problems during a post-doc at the Courant Institute and for the honor of a happy collaboration over many years.

Tonatiuh Sánchez-Vizuet conducts research on the numerical analysis of time-domain integral equations arising in the study of wave propagation and in the application of continuous and discontinuous Galerkin methods to problems arising in computational physics.

Tonatiuh Sánchez-Vizuet

Christina Sormani is an AMS Fellow. She conducts research in geometric analysis and metric geometry using techniques from geometric measure theory with applications to Riemannian geometry and general relativity. She first met Professor Morawetz as an undergraduate at NYU, took courses with her as a doctoral student at the Courant Institute, and for years afterwards turned to Morawetz for suggestions as to how to best raise a family while pursing a career in research mathematics.



Christina Sormani

Terence Tao is a Fields Medalist and a MacArthur Fellow. He has completed groundbreaking research in harmonic analysis, partial differential equations, algebraic combinatorics, arithmetic combinatorics, geometric combinatorics, compressed sensing, and analytic number theory.

Terence Tao