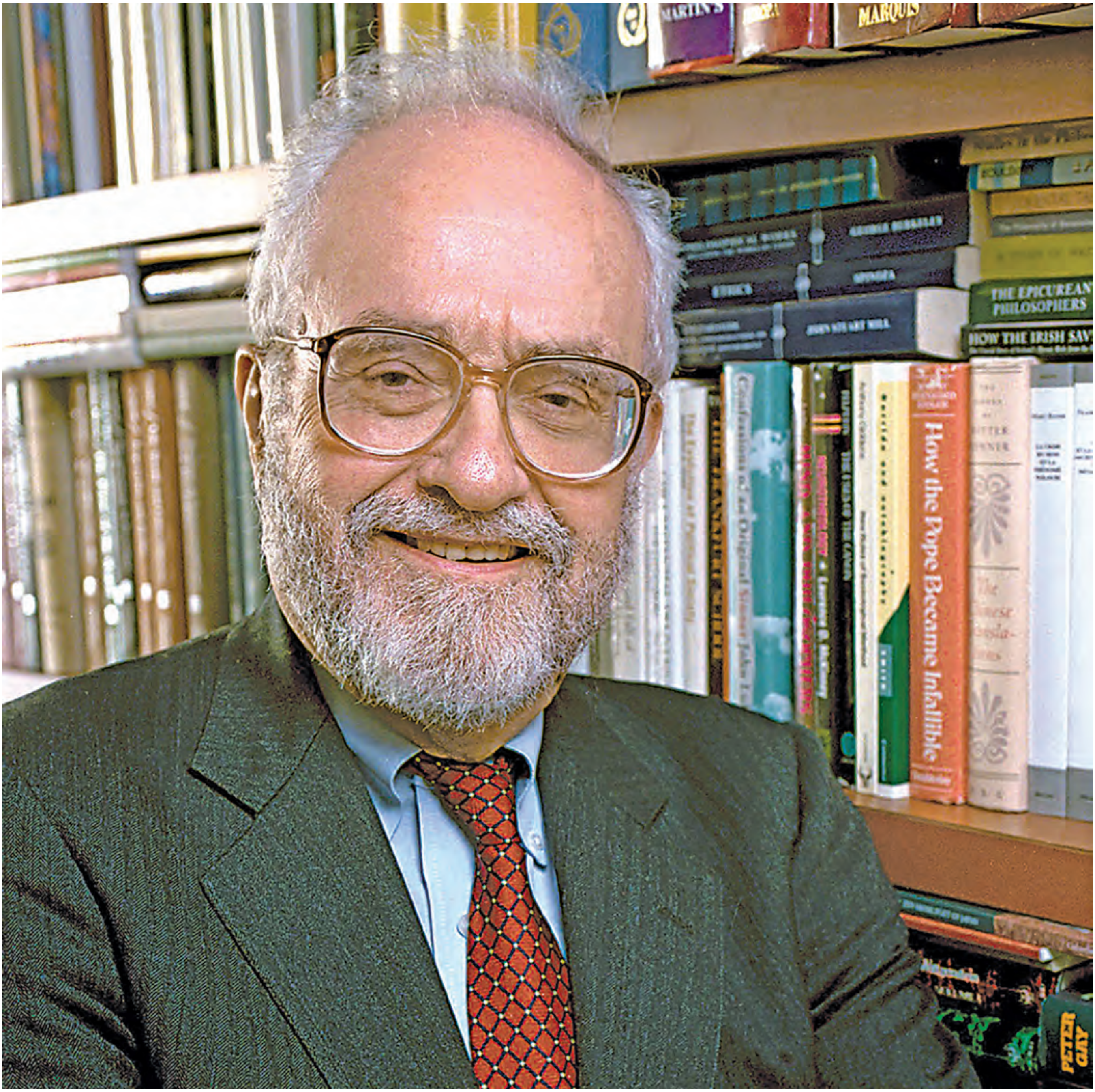
Felix Browder (1927–2016)



Felix Browder received the National Medal of Science and served as president of the American Mathematical Society.

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Haïm Brezis, Guest Editor

Felix Browder, who passed away exactly two years ago, was awarded the National Medal of Science by President Bill Clinton for "his pioneering work in nonlinear functional analysis and its applications to partial differential equations, and for leadership in the scientific community,...and in promoting science and math education for all." Indeed, there were two Felix Browders! There was Felix, the solver of nonlinear problems, who made major contributions to fixed point theory and was the leading architect of the theory of monotone operators and its generalizations. Some of his ideas have been successfully used in tackling equations arising in real-world models. See below the descriptions of his research by former student Roger D. Nussbaum and me. And there was Felix the visionary scientist who was able to foresee original directions and generously support emerging young talents. He was a great chairperson of the math department at the University of Chicago, and a very influential vice-president for research at Rutgers. Felix played an energizing role at the AMS throughout his career and served as its president for the years 1999–2000. In this capacity, and also as an extremely active member of the National Academy of Sciences, he lobbied Congress for additional funding for mathematical research and education. On another front, Felix developed close ties with a number of French mathematicians. His impact on the "post-Bourbaki" flourishing school of partial differential equations (PDEs) in France cannot be overestimated. All who knew Felix emphasize the friendly intellectual atmosphere radiating from him. No matter what topic, be it mathematics, philosophy, or history, Felix would communicate his joyful appetite and sheer pleasure of knowledge. In what follows, some of Felix's friends, colleagues, and former students share their memories and discuss the major contributions of this extraordinary mathematician.

See also:

Nomination for Felix Browder for AMS President Elect by Jerry Bona, September 1997 *Notices*, `https://www.ams.org/notices/199708/from.pdf`.

Reflections on the future of mathematics (retiring presidential address), June–July 2002 *Notices*, `https://www.ams.org/journals/notices/200206/fea-browder.pdf`.

Interview with AMS President Felix Browder, March 1999 *Notices*, `https://www.ams.org/notices/199903/comm-browder.pdf`.

Remembering Felix Browder by Thomas Lin, *The New Yorker*, December 20, 2016, `https://www.newyorker.com/tech/elements/remembering-felix-browder-a-nonlinear-genius-in-a-nonlinear-world`.

Felix Browder, mathematician shadowed by his father's life as a Communist, dies at eighty-nine, *The Washington Post*, December 15, 2016, `https://tinyurl.com/y9s7uhvh`.

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Photo Credit

Opening photo of Felix Browder courtesy of Rutgers University.

The Early Life of Felix Browder

Felix Earl Browder was born July 31, 1927 in Moscow, Russia, and died December 10, 2016 in Princeton, New Jersey. His father, Earl Browder, an American political activist, visited Russia in the 1920s as a representative of the Communist Trade Unions in the United States. There, he met and married Raissa Berkmann, born in a Jewish family, who had a law degree from the University of St. Petersburg. In the early 1930s the family settled in Yonkers, New York, where Felix attended high school. Felix was a child prodigy; he is said to have read at least a book a day from the time he was five years old. His only sport in high school was the debating team. He was a shark. His younger brother Bill vividly remembers the triumphal atmosphere, when their parents returned home with the conquering hero. At high school graduation, at the age of sixteen, he collected almost every academic prize and was awarded a New York State Regents scholarship. In the same year he entered MIT and, after only two years of study, graduated in 1946.

For his graduate studies, Felix attended Princeton University, where he worked on his PhD under the supervision of Solomon Lefschetz, a leading topologist who also made fundamental contributions to the theory of nonlinear ordinary differential equations. Felix submitted his doctoral thesis, *The Topological Fixed Point Theory and Its Applications in Functional Analysis*, and was awarded his PhD at the age of twenty.

The Difficult Years

The first positions held by Felix were instructorships at MIT (1948–1951) and at Boston University (1951–52), followed by a temporary position at the Institute for Advanced Study (IAS) in Princeton. During the McCarthy era it was perilous to be called "Browder": since the early 1930s his father Earl had been the leader of the American Communist Party and even its candidate for the presidential election of 1936. The renowned physicist Robert Oppenheimer was at the time the director of the Institute. Originally he pushed through the appointment of Felix, but later, when he was himself under investigation, he declined to sign a deferment request from military draft for Felix. Ironically, Earl had been expelled from the party six years earlier! In 1953, his case was brought to the Committee on Un-American Activities of the US House of Representatives. Norman Levinson, an MIT professor, made a very forceful deposition in support of Felix: "...He is the best student we had ever had in mathematics in the ninety years of existence of this institution...." To no avail—Felix was drafted to the army in 1953 and spent most of his two years of service pumping gas. It is fun to note that while at Fort Bragg, he published several papers with no institutional listing but only the laconic address, Fayetteville, North Carolina.

The Chicago Years

Felix was professor at the University of Chicago from 1963 to 1986. For eleven of those years he served as chair of the mathematics department.

His efforts as chair attest to his vision and dedication to excellence. He initiated many appointments of the highest quality—including those of Spencer Bloch, Luis Caffarelli, Charles Fefferman, Carlos Kenig, and Karen Uhlenbeck—which turned the Chicago math department into one of the most prestigious centers in the US. He was enormously supportive of young talented people and eager to promote them quickly to full professorship—maybe because of the difficulties he encountered at the beginning of his career? During these years he had an immense esteem for Alberto Calderón and much admiration for harmonic analysis; clearly he foresaw the impact this field would have on the future development of partial differential equations (PDEs). He had also established close contacts with leading Chicago physicists, such as the Nobel laureate S. Chandrasekhar and L. Kadanoff. Throughout his tenure in Chicago, there was a nonstop flow of distinguished international visitors. These were also the golden years of his mathematical research.

The Rutgers Years

Felix was appointed vice-president for research at Rutgers University in 1986 and occupied this position until 1991. Subsequently, he remained at Rutgers until his death. Felix was especially proud to have attracted to Rutgers—in his capacity as VP—the legendary Soviet mathematician Israel M. Gelfand. Felix encouraged and supported Gelfand's Correspondence Program for Mathematical Education. He was influential in other stellar appointments in Applied Mathematics, such as Bernard Coleman, Ingrid Daubechies, Martin Kruskal, and Norman Zabusky. Felix was also instrumental in getting a very strong group of string theorists, the so-called "String Quartet," to the Rutgers physics department. He prompted the creation of the Center for Nonlinear Analysis under the leadership of Haïm Brezis with the active participation of Abbas Bahri, Sagun Chanillo, and Yanyan Li. At the same time he initiated the successful NSF Science and Technology Center, which involved Rutgers, Bell Labs, Bellcore, and Princeton University, and which led to the formation of DIMACS, the Rutgers Center for Discrete Mathematics and Theoretical Computer Science. Felix clearly enjoyed and excelled in his responsibilities as VP for research; unfortunately his activities stopped due to the arrival of a new president who had different priorities for Rutgers.

Browder Family

Felix's wife, Eva nee Tislowitz, died in 2015. She had arrived in the US alone at age nine as a refugee from Austria just before the Holocaust. Eva met Felix at MIT, where she was a student and he was a young instructor.

Felix (center) with (from left to right) son Thomas, Ivar Ekeland, Haïm Brezis, wife Eva, and son Bill, at a reception after Felix received an honorary degree at the Sorbonne in 1990. Felix's impact on the post-Bourbaki flourishing school of PDEs in France cannot be overestimated.

While working as an administrator at Rutgers, Eva was also involved in the National Outreach Program in Mathematics inspired by I. M. Gelfand. Felix is survived by two younger brothers: Andrew Browder, who is professor emeritus at Brown University, and William Browder, a well-known topologist, who is professor emeritus at Princeton University. He is also survived by two sons: Tom Browder, a physics professor at the University of Hawaii, and Bill Browder, a bold international financier. Felix had five grandchildren. The oldest one, Joshua Browder, is the inventor of "DoNotPay," a program that allows motorists to appeal their parking tickets automatically.

The Mathematics of Felix Browder

Felix worked in three different directions:

i) Linear PDEs and functional analysis.
ii) Fixed point and degree theories.
iii) Monotone operators and beyond.

i) Linear PDEs and functional analysis. This topic preoccupied Felix from 1952 until 1962. He was concerned with questions of existence and regularity estimates (in terms of Sobolev and Schauder norms) for linear elliptic PDEs. These results overlap with the simultaneous ones by Agmon-Douglis-Nirenberg. He also made decisive contributions to the spectral analysis of nonselfadjoint elliptic differential operators. More importantly, Felix cemented the bridge connecting PDEs and functional analysis (at a time when functional analysis was becoming increasingly abstract and detached from PDEs). In his own words:

> …It is our purpose to present a general treatment in unified terms of these various PDEs and to exhibit explicitly the common methodological basis of the discussion of apparently diverse sorts of problems. In rough terms, this basis consists of the

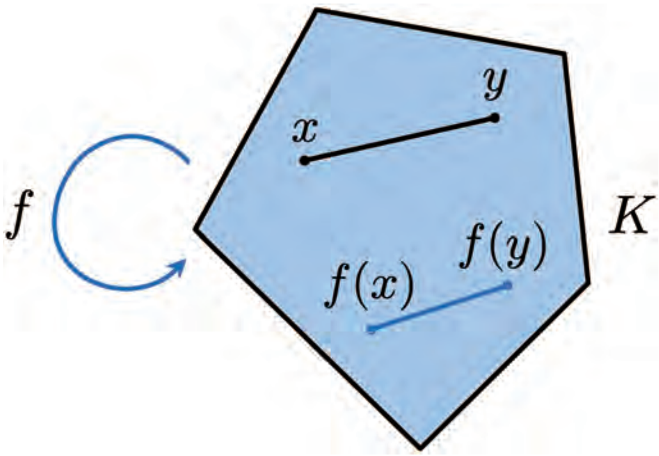

Figure 1. The 1909 Brouwer (and later the Schauder) fixed point theorems prove that a continuous function $f : K \to K$ on a compact convex set, K, in a Euclidean (or Banach space) has a fixed point. Browder's fixed point theorem does not require K to be compact. In his theorem, K is closed, bounded, and convex in a uniformly convex Banach space and $f : K \to K$ is a nonexpansive map: $\|f(x) - f(y)\| \leq \|x - y\|$ for all $x, y \in K$.

> combination of general principles from functional analysis with concrete analytical a priori estimates…(*Math. Ann.*, 1959)

This approach paved the way to a similar strategy Felix used in nonlinear problems.

One should also give credit to Felix for publicizing in the West important works of the Russian PDE community. In particular Felix translated the book by S. L. Sobolev, *Applications of Functional Analysis in Mathematical Physics* (Amer. Math. Soc., 1963), which received the following compliment in *Math. Reviews*: "the translation is exceedingly smooth." (At that time there was a massive effort to translate Russian math papers into English and this was often done by people with a limited math background.)

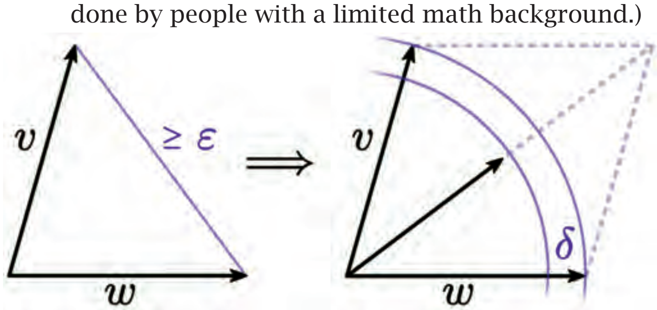

Figure 2. In Browder's fixed point theorem, the closed, bounded, convex set K is assumed to lie in a uniformly convex Banach space: for all $\varepsilon \in (0, 2]$ there exists a $\delta > 0$ such that for all unit vectors: $\|v - w\| \geq \varepsilon \Rightarrow \|(v + w)/2\| \leq 1 - \delta$.

ii) Fixed point and degree theories after the classic fixed point theorems of Brouwer and Schauder (Figure 1). Felix worked on this subject throughout his entire research life, from his PhD in 1948 until 1984. He made beautiful contributions in two directions. Firstly, he wrote a number of influential papers on nonexpansive mappings (maps with Lipschitz constant one). For example, he proved in 1965 that any nonexpansive self-mapping of a bounded, closed, convex subset of a uniformly convex Banach space has a fixed point (Figure 2). This result has interesting applications, e.g. in the study of periodic solutions of differential equations. Incidentally, it is still an open problem, more than fifty years later, whether a similar conclusion holds in all reflexive Banach spaces. Secondly, Felix achieved substantial progress in our understanding of topological fixed points and degree theory; see for example his 1983 expository paper in the *Bulletin of the American Mathematical Society*. A typical result is described by R. Nussbaum in his text "Felix Browder and a Useful Result in Fixed Point Theory" (page 1402 herein).

iii) Monotone operators and beyond. Felix has been the leading architect of the theory of monotone operators, its generalizations, and its applications to nonlinear PDEs. This theory is one of the most elegant and powerful tools in the study of nonlinear problems, and a cornerstone of nonlinear functional analysis. Felix has written many papers and two monographs ([B4] and [B6]) on this subject, spanning the period 1963–1997. Some of his early works were already very influential in the mid-1960s, e.g. in France, where J. Leray and J.-L. Lions published a paper under the title "Quelques Résultats de Višik sur les Problèmes Elliptiques Non Linéaires par les Méthodes de Minty-Browder" [LL]. Felix perceived immediately that this abstract theory could be successfully used in tackling nonlinear PDEs arising e.g. in differential geometry, physics, mechanics, biology, engineering, ecology, climate, finance, etc. Felix was not himself an "applied mathematician," but his ideas have had a lasting impact on real-world problems. On this subject see the contribution by H. Brezis "Felix Browder and Monotone Operators" (page 1403 herein).

Felix Browder and the AMS: A Life of Commitment

Felix Browder played an active role in the Society throughout his career. He served as president of the AMS for the years 1999–2000. In this capacity he lobbied Congress for additional funding for mathematical research and education. Throughout his life, Felix never missed an opportunity to be a spokesperson for the scientific community in the presence of politicians. In particular he was fond of his continued exchanges with Rush Holt, a US Representative for New Jersey. The list of AMS committees on which he served is too long to present here. His effective work as editor of the *Bulletin* and his service on the Science Policy Committee have left a permanent mark. He organized innumerable special sessions at regional and national meetings, as well as major international

Felix received the National Medal of Science in 1999. President Clinton had had a hard time placing the medal around the neck of the previous recipient, so Felix wisely removed his glasses beforehand, and Clinton remarked, "You see...these scientists they learn by experience," generating a big laugh.

conferences sponsored by the AMS. The proceedings that he edited are a very useful source of information. Let us mention in particular:

—Nonlinear Functional Analysis, Chicago, 1968; proceedings published by the AMS in 1970.

—Mathematical Developments Arising from Hilbert Problems, De Kalb, Illinois 1974; proceedings published by the AMS in 1976.

—The Mathematical Heritage of H. Poincaré, Bloomington, Indiana, 1980; proceedings published by the AMS in 1983.

— Nonlinear Functional Analysis and its Applications, Berkeley, 1983; proceedings published by the AMS in 1986.

— Mathematical Challenges of the 21st Century, UCLA, 2000; proceedings published in the AMS *Bulletin*.

Honors Received

In 1999, Felix was awarded the National Medal of Science by President Bill Clinton, for "his pioneering work in nonlinear functional analysis and its applications to partial differential equations, and for leadership in the scientific community." The citation added: "Through his accomplishments Browder laid the groundwork for the mathematics needed to study the array of complexities and intricacies we find in our biological and physical world. Throughout his career Browder has demonstrated unwavering commitment in broadening the interactions among the scientific disciplines and in promoting science and math education for all."

Other honors include his election to the National Academy of Science in 1973—he served as a member of the Council 1992–1995 and the Governing Board of the National Research Council 1994–1995. He was elected a fellow of the American Academy of Arts and Sciences in 1959 and awarded an honorary degree by the University of Paris-Sorbonne in 1990. An international conference honoring Felix was held at Rutgers in October 2001.

Browder, Knowledge, and the General Public

Felix was always eager to share with a wide general public his thoughts on the importance of mathematics, its history, its place in a changing world, and the major challenges it faces. During his tenure in Chicago Felix established a rule that colloquium speakers should include a historical perspective. To get a quick idea of his preoccupations it suffices to glance at the titles of some papers he wrote: "The Relevance of Mathematics" (1976), "Does Pure Mathematics Have a Relation to the Sciences?" (1976), "Mathematics and Society—a Historical View" (1977), "Mathematics and the Sciences" (1985), "Of Time, Intelligence and Institutions" (1992), "Reflections on the Future of Mathematics" (2002).

Particularly fascinating was Felix's unique breadth of interests: of course, mathematics, but also physics and science, history, religion, philosophy, political science, economics, literature, and you name it. He enjoyed discussing his views in long private conversations. He also taught philosophy classes at Rutgers for many years. His personal library had over 35,000 books.

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National Medal of Science photo courtesy of the National Science Technology Medals Foundation.

Roger D. Nussbaum

Felix Browder and a Useful Result in Fixed Point Theory

Fixed point theory was a life-long mathematical interest of Felix, beginning with his 1948 Princeton PhD dissertation. Here I would like to discuss a small, but very useful segment of Felix's work in this area—a sequence of three papers [1], [2], [3] rather unimaginatively titled "On a Generalization of the Schauder Fixed Point Theorem," "Another Generalization of the Schauder Fixed Point Theorem," and "A Further Generalization of the Schauder Fixed Point Theorem." All of these papers fit into the category of asymptotic fixed point theory, that is, results in which fixed point theorems for a map f are obtained with the aid of assumptions on the iterates f^n of f.

Specifically, I shall discuss without proofs results in [3] concerning the existence of "nonejective fixed points." As Felix mentions, his theorem is motivated by a question about [2] in a letter from G. Stephen Jones, who had used results from the first paper [1]. Jones's

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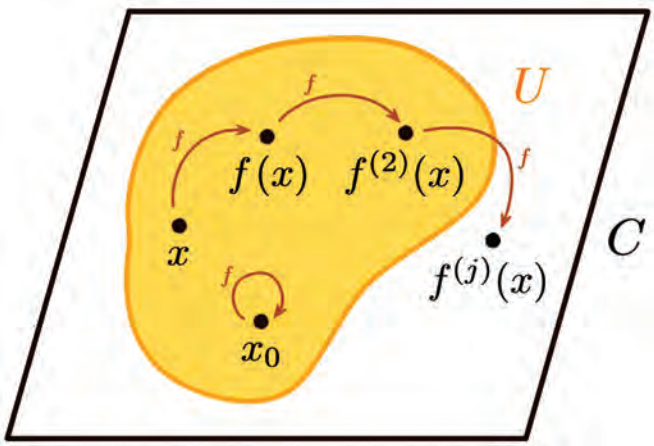

Figure 1. An ejective fixed point, x_0, of a map $f : C \to C$ has a neighborhood $U \subset C$, such that for all other $x \in U$, there exists $j = j(x)$ such that the j^{th} iteration, $f^{(j)}(x)$, is not in U.

original motivation came from the so-called Wright-Jones nonlinear differential-delay equation

$$x'(t) = -\alpha x(t-1)(1+x(t)). \tag{1}$$

Jones had used fixed point theory to prove that if $\alpha > \pi/2$, equation (1) has a nonconstant "slowly oscillating periodic solution" $x(\cdot)$.

E. M. Wright's name is associated with equation (1) because of his seminal paper [4], but it is interesting to note that Wright apparently had no idea that (1) has nonconstant slowly oscillating periodic solutions for $\alpha > \pi/2$.

Other applications of theorems from [1] were given by Halanay and Yoshizawa.

If C is a subset of a Banach space X and $f : C \to C$ is a continuous map with a fixed point x_0 (so $f(x_0) = x_0$), x_0 is called an "ejective fixed point" in [3] (see Figure 1) if there exists an open neighborhood U of x_0 in C (open in the relative topology on C) such that for every $x \in U \backslash \{x_0\}$ there exists a positive integer $j = j(x)$ such that $f^j(x) \notin U$.

If $C \subset X$ is convex, we shall call C infinite-dimensional if C is not contained in some finite-dimensional affine linear subspace of X.

Theorem 1 ([3]). *Let C be a compact, convex infinite-dimensional subset of a Banach space X and $f : C \to C$ a continuous map. Then f has a fixed point which is not ejective.*

If $C \subset X$ is compact and convex and contained in a finite-dimensional affine linear subspace of X, we let Y denote the smallest affine linear subspace that contains C and let $\partial(C)$ denote the boundary of C in Y and int (C) denote the interior of C in Y.

The following theorem is not stated by Browder [3] but follows easily by the same ideas.

Theorem 2. *Let C be a compact, convex finite-dimensional subset of a Banach space X and assume C is not a single point. Let $f : C \to C$ be a continuous map. Then either (i) f has a fixed point in int (C) or (ii) f has a fixed point in $\partial(C)$ which is not ejective.*

If $D \subset \mathbb{C} = \mathbb{R}^2$ and $D = \{z \in \mathbb{C} \,|\, |z| \le 1\}$, define $f : D \to D$ by $f(r\exp(i\theta)) = \sqrt{r}\exp(i\theta + i\pi/2), 0 \le r \le 1$. One can check that 0 is the only fixed point of f in D and 0 is an ejective fixed point of f; so, in the finite-dimensional case, f may fail to have a nonejective fixed point.

There have been numerous generalizations of the above results, and there have been applications, for example, to the study of nonlinear differential-delay equations.

References

[1] F. E. Browder, On a generalization of the Schauder fixed point theorem, *Duke Math. J.* **26** (1959), 291–304. MR0105629
[2] F. E. Browder, Another generalization of the Schauder fixed point theorem, *Duke Math. J.* **32** (1965), 399–406. MR0203718
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Image Credit

Figure 1 by Pen Chang.

Haïm Brezis

Felix Browder and Monotone Operators

The concept of monotone operator in Hilbert spaces was introduced by G. Minty [M] whose original motivation came from graph theory, electrical networks, and linear programming. Felix, who had a strong background in linear PDEs, foresaw immediately the enormous potential applicability of this abstract theory to nonlinear PDEs. This is already conspicuous in the first papers [B1], [B2] he wrote on this subject.

It is easy to explain some of the original ideas and far-reaching ramifications of this rich theory. Let us start with an elementary observation. Assume $\varphi : \mathbb{R} \to \mathbb{R}$ is a continuous nondecreasing function. Then the function $x \mapsto x + \varphi(x)$ is one-to-one and onto $\mathbb{R}$. This fact was extended by G. Minty to real Hilbert spaces H as follows. A mapping $A : H \to H$ is called *monotone* provided

$$(A(u_1) - A(u_2), u_1 - u_2) \ge 0 \text{ for all } u_1, u_2 \in H. \tag{2}$$

Assuming A is continuous and monotone, G. Minty [M] proved that $Id + A$ is one-to-one and onto H. Here is a typical extension due to F. Browder:

Theorem 1 ([B5]). *Assume $A : H \to H$ is continuous, monotone, and satisfies*

$$\lim_{|u|\to\infty} |A(u)| = \infty. \tag{3}$$

Then A is onto.

This theorem paved the way to a large collection of generalizations and applications to nonlinear PDEs. Firstly, Felix observed that the monotonicity condition (2) is natural in the framework of elliptic and parabolic boundary value problems. Secondly, he noticed that

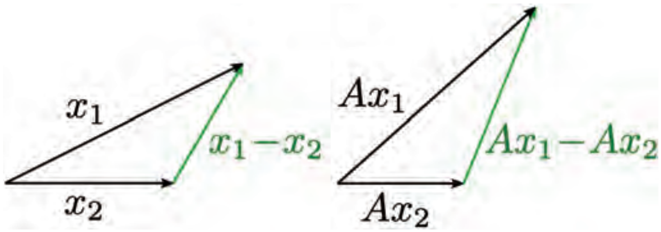

Figure 1. Browder made great advances in the study and applications of monotone maps, A, such that $(Ax_1 - Ax_2, x_1 - x_2) \geq 0$. Positive semidefinite linear operators on Hilbert spaces are easily seen to be monotone, but monotone maps need not be linear. More interesting examples are gradients of convex functions.

Work on compact perturbations of monotone operators by Browder led to further extensions by Leray, Lions, and Brezis, shown here with Browder many years later, in 2003.

condition (3) amounts to the fact that for every given $f \in H$ the equation $A(u) = f$ admits *a priori estimates*—a recurrent key phrase in PDEs. Thirdly, he popularized and made extensive use of a remarkable device (originally due to Minty [M]) that is both very elementary and extremely effective in a wide range of situations involving a passage to the limit (e.g. from finite to infinite-dimensional spaces). Here it is, used to derive the above theorem from Minty's result. Given $f \in H$ and $\varepsilon > 0$ there exists a unique u_ε solution of

$$\varepsilon u_\varepsilon + A(u_\varepsilon) = f. \tag{4}$$

The monotonicity of A yields $(A(u_\varepsilon) - A(0), u_\varepsilon) \geq 0$, so that $\varepsilon|u_\varepsilon| \leq |f - A(0)|$. Applying (4) and (3) we see that $|u_\varepsilon|$ remains bounded as $\varepsilon \to 0$, and thus a subsequence u_{ε_n} converges weakly to some u. Unfortunately nonlinear operators are badly behaved under weak convergence (a well-known source of difficulties in nonlinear problems); thus this fact alone does *not* guarantee that $A(u_{\varepsilon_n})$ converges, even weakly, to $A(u)$. Here enters monotonicity in its full glory. It allows one to produce solutions by a subtle mechanism that requires only bounds in some weak norms. By monotonicity, for all $v \in H$,

$$(A(v) - A(u_\varepsilon), v - u_\varepsilon) \geq 0,$$

so that by (4)

$$(A(v) - f + \varepsilon u_\varepsilon, v - u_\varepsilon) \geq 0.$$

Passing to the limit as $\varepsilon_n \to 0$ yields

$$(A(v) - f, v - u) \geq 0. \tag{5}$$

Choosing in (5) $v = u + tw$, with $t > 0$ and $w \in H$, gives $(A(u + tw) - f, w) \geq 0$. Letting $t \to 0$ we obtain $A(u) = f$, the desired conclusion.

The original setting of the above theorem has been generalized in many ways, thereby increasing tremendously its applicability to PDEs. Here are some typical directions:

i) The Hilbert space H is replaced by a Banach space V. Maps $A : V \to V^*$ are monotone provided they satisfy the condition

$$\langle A(u_1) - A(u_2), u_1 - u_2 \rangle \geq 0 \text{ for all } u_1, u_2 \in V.$$

ii) Nonlinear *compact* perturbations of monotone operators are admissible (see Browder [B2]). This has been pushed even further by J. Leray and J. L. Lions [LL] and H. Brezis [Bre], thus extending the classical Leray-Schauder theory, which relies heavily on compactness. Another standard tool in nonlinear problems is the "variational" approach, which consists of minimizing convex-type functionals. Since gradients of convex functionals are monotone operators, the current state of the art provides a unified "roof," which requires no compactness and no variational structure. The impact of Browder's ideas cannot be overestimated. In 1969 Lions published an influential book *Quelques Méthodes de Résolution des Problèmes aux Limites Non linéaires* [L], which is a collection of techniques used in solving nonlinear PDEs. Out of a total of 550 pages, about 150 pages are dedicated to monotonicity methods, and the bibliography includes twenty papers by F. Browder.

iii) The concept of *accretive maps* is another natural extension of monotone maps. Recall that in a Banach space X the semiscalar product is defined by

$$[x, y] = \lim_{t \downarrow 0} \frac{1}{2t}(\|x + ty\|^2 - \|x\|^2).$$

A map $A : X \to X$ is called *accretive* provided it satisfies the condition

$$[u_1 - u_2, A(u_1) - A(u_2)] \geq 0 \text{ for all } u_1, u_2 \in X.$$

Evolution equations associated with accretive maps play an important role because they generate semigroups of nonlinear contractions. Their systematic study was initiated by F. Browder in his pioneering paper [B3] (see also [B6]) and pursued by many people including T. Kato, J. L. Lions, M. Crandall, A. Pazy, Y. Komura, T. Liggett, V. Barbu, Ph. Bénilan, L. C. Evans, and myself. It has countless applications to problems coming from physics and mechanics. Recent uses include image processing, motion by mean curvature, etc.

iv) Maps A need not be defined on the whole space X, but just on a domain $D(A)$. This is especially relevant in applications to PDEs. In concrete examples $D(A)$ consists of smooth functions, while X may include rough functions.

v) One of the major achievements of the twentieth century in linear PDEs has been the systematic study of the concept of weak solutions in the sense of distributions. This notion requires an adjoint operator acting on a class of smooth testing functions. Such an approach has no analogue in nonlinear problems and the concept of weak solution becomes a very delicate issue even in simple models such as $u_t + uu_x = 0$ (Burgers equation) or $|\nabla u| = f(x)$ (Eikonal equation). Typically, such problems do not have classical (C^1) solutions and they admit too many weak solutions. It is necessary to select among these "fake" solutions the physical solution. Here again, monotone operators—and especially the weak formulation (5)—can be extremely useful. Property (5) suitably adapted, may provide a mechanism to detect the physically interesting weak solutions. For example, (5) adapted to the semiscalar product in $X = L^1$, picks up the "entropy solution" (in the sense of Lax–Oleinik theory of shock waves) for the Burgers equation.

To conclude, let's mention briefly an example of a problem that has raised considerable interest in recent years and confirms the lasting impact of Browder's ideas. Consider the *fully nonlinear* equation

$$F(x, u, Du, D^2u) = f(x) \text{ in } \Omega \subset \mathbb{R}^N, \tag{6}$$

where F satisfies the degenerate ellipticity condition

(7)
$$\begin{cases} F(x, u, p, X) \geq F(x, v, p, Y) \\ \text{whenever } u \geq v \text{ and } X \leq Y \\ \qquad\qquad \text{in the sense of symmetric matrices.} \end{cases}$$

It is easy to see (using the maximum principle) that the operator $A : D(A) \subset X \to X$, where $X = L^\infty(\Omega)$, $D(A) = \{u \in C^2(\overline{\Omega}); u = 0 \text{ on } \partial\Omega\}$, and $Au = F(x, u, Du, D^2u)$, is accretive. The solvability of the equation

$$A(u) = f, \text{ for a given (smooth) } f, \tag{8}$$

is a challenging task, especially in view of the fact that *weak solutions in the sense of distribution have no meaning whatsoever*. Returning to the above discussion (in particular (5)) it is natural to introduce a totally new concept of weak solution: a function $u \in L^\infty(\Omega)$ is a weak solution of (6) provided it satisfies

$$[v - u, A(v) - f] \geq 0 \text{ for all } v \in D(A), \tag{9}$$

where $[\,,\,]$ denotes the semiscalar product in $L^\infty(\Omega)$. Condition (9) corresponds basically to the notion of *viscosity solution* in the sense of M. Crandall and P. L. Lions (see e.g. the expository paper [CIL]). Of course, this is just the beginning of the story. Studying the existence, uniqueness, and regularity of solutions of (6) is a formidable task—still partially under investigation—rooted in the seminal plans of Felix Browder.

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Figure 1 by Pen Chang.

Photo of Haïm Brezis with Felix Browder courtesy of Rutgers University.

Amy Cohen

Remembering Felix Browder

My thesis advisor, Murray Protter, introduced me to Felix Browder one day before I left Berkeley for Cornell in the summer of 1971. I moved to Rutgers in 1972, never expecting that I'd meet Browder again. However, Browder came to Rutgers as vice president for research in 1986.

Browder was instrumental in bringing I. M. Gelfand to Rutgers in the late 1980s. While Gelfand is primarily known as a mathematician, Browder knew of Gelfand's work in applications of computer science. Browder introduced him to Rutgers faculty, who became co-workers in fields ranging from automated medical diagnostics to machine reading of handwriting.

At Rutgers, Browder encouraged and supported Gelfand's Mathematical Correspondence Program. This program continued the Gelfand School by Correspondence in the former Soviet Union, which connected grad students in Moscow with rural students whose mathematical education had progressed beyond what their school teachers could guide.

After leaving his vice presidency, Browder moved into an office in the math department. He worked on mathematics with colleagues. As his health failed, we saw less of him.

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I'll end with some personal anecdotes. As a member of the senior administration Browder learned that I had agreed to serve as acting dean of the Rutgers unit for returning adult bachelors degree candidates. When he next met me on campus he demanded, "What's a nice girl like you doing in a job like yours?" I must have answered correctly, since he later made me an AMS nominee to the Committee on the Undergraduate Program in Mathematics. Later I shamelessly asked him (and other senior faculty) to contribute financially to a math department project to enhance the success of first year students who placed into Calculus I but had "nonacademic indicators of risk." To my delight, Browder and others did contribute so the project could start before we received grant support.

Jean Mawhin

Besides his fundamental contributions to partial differential equations, Felix Browder is recognized as one of the founding fathers of nonlinear functional analysis, with numerous and fundamental contributions to its monotone, variational, and topological aspects. When he was still in Chicago, Felix invited me to talk at his seminar. After the lecture, Jerry Bona told me with a big smile: "Felix took notes during your lecture. You should expect to see some generalizations soon." It was indeed the case, and I was both happy and proud to have inspired him in a very small part of his outstanding work.

Roger Temam

I have always known Felix Browder, ever since I started to do research. Felix was a regular visitor to Paris and I always attended his seminars. After I graduated he invited me several times to visit him at the University of Chicago. The mathematics department was very diverse and inspiring, and it was a great pleasure to spend time there. Felix had a great influence on the theory of partial differential equations and functional analysis.

I would like to add a personal witness to the many witnesses that colleagues and friends will tell. When I was assistant professor working on my own thesis, a colleague, who was assistant professor in theoretical mechanics was stuck trying to write his thesis on shell theory. After some discussions where he told me the problem he was interested in, I gave him one day a paper of Felix. The paper contained exactly the theorem that he needed, and within a few months the colleague completed his thesis, and he moved on to become associate and then full professor. Although he probably never met Felix, he considered Felix as a sort of savior!

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Richard Beals

Advice from Felix

Everyone who knew Felix knew him not only as a brilliant intellect, but as a fount of stories, particularly stories about mathematicians. I will counter with a story about Felix. The spring of 1964 was a wonderful time to be on the job market. I had just completed a thesis under Felix's direction, and had three tempting instructorship offers, call them A, B, and C. Felix was at the Institute that year, so I went there to get his advice. Generous with both advice and time, he devoted three quarters of an hour to making a convincing case for choosing A rather than B or C. Felix's mind was not only brilliant, but supple. He went on for another forty-five minutes making the case for B, rather than A or C. As you have probably surmised, there followed yet another forty-five minutes on the case for C rather than A or B. I had much to think about on the train back to New Haven. A year later Felix made the whole A-B-C question moot by getting me to Chicago on a one-year visiting appointment that turned into tenure there. I have much to thank him for.

Louis Nirenberg

The death of Felix is a great loss for mathematics and for all who knew him. I first met Felix around the time that he received his PhD. We quickly became friends, and our friendship lasted all these years. Our mathematical interests had much in common. In addition to his research I admired many things about Felix. He read and seemed to remember everything. He was a superb chairman at the math department at the University of Chicago, and at Rutgers he was influential in bringing Gelfand and Brezis there. Talking with him was always a great pleasure, and always very informative.

Henri Berestycki

After sitting on my PhD committee in Paris, Felix brought me to the University of Chicago as a young Dickson instructor. This turned out to be a major experience for me that oriented my way of doing mathematics henceforth. When I arrived at the University of Chicago from France, he showed me to my apartment, explained how things worked around there with typical Felix warmth. When I asked him about safety as I had been alarmed by some reports (and after all this was Al Capone's city), he pointed to a park in the distance and said, "This park is so dangerous that even murderers do not venture there."

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Louis Nirenberg, pictured here with Felix and John Nash at Rutgers in 2007, said that Felix "seemed to remember everything."

When I arrived in Chicago, Felix asked me to teach an advanced graduate course on bifurcation theory. He grasped early on the importance of the subject as he understood the connections with other fields, not only in mechanics or physics but even in social sciences. And thanks to Felix on this occasion, I was able to meet economists.

A personal recollection of him has been a lesson for me on how to deal with apprentice mathematicians throughout my career. I once went along when Felix took a famous visiting speaker out to dinner at a fancy restaurant. This mathematician started to bully me a bit, poking at me simply I guess because I was young. I will always remember how Felix stopped this at once, nicely but quite firmly. Another quality of Felix was his immense trust. Once he had formed an opinion about you, he would go on to trust you in a generous manner.

Felix maintained an extraordinary lively and friendly intellectual atmosphere, creating a kind of Latin quarter by himself in his house in Hyde Park where he would often have people over. His library enchanted me. He constantly added volumes to it. I think that he viewed all his many trips as foraging expeditions for his unique collection. Particularly fascinating was his breadth of interest: his books covered nearly all fields of knowledge. This corresponded to an inclination I always felt myself and it brought us close. I felt elated by his example of a great mathematician who had such a large scope of interest and knowledge. Once, as I was teasing him that it was impossible to have read all these books, I picked one out of his shelves and opened it at a random page. He knew exactly what was on it! Mathematics, science, history, politics, philosophy—it seemed that there was no bound to his intellectual appetite and curiosity.

In his unique erudition, there was one thing that struck me particularly. It was the happiness of it all. It made him extraordinarily joyous to discuss an intellectual theme, and no matter what the topic, he would consider it with the same devoted attention and communicatively joyful appetite.

I vividly remember Felix grinning with joy on the many occasions of intellectual discussions. Be it when he could see some beautiful mathematical idea or because you brought in an interesting element or fact or argument in the conversation or because he could make some connection, you could always notice his smile. It was clear that knowledge and understanding gave him great joy.

Barbara Mastrian

I recall meeting Felix for the first time in 1991, in my office in the Rutgers mathematics department, which I shared with his secretary Sue. I was a new employee, and had heard about Felix but had not met him. I recall the feeling of Felix walking into the office and giving Sue a letter to type. I felt like I was meeting someone important and distinguished. Felix had a powerful presence, and was demanding of excellent work. He brought that letter back to Sue several times before she got it to his specifications.

A few years later I met Eva. She had come in the mathematics department to get some paperwork notarized, showed me pictures of her then infant grandson Josh, who was in Russia at the time. Periodically Eva would come in and show pictures and chat. Later after Eva and Felix were less mobile I would visit them at their home, update Felix on department functions and activities, and bring well wishes from faculty, along with piles of emails for Felix to go through.

My lasting impression of Felix was when I told him about Professor Gelfand's memorial, and he insisted on attending. He wanted to give a talk at the memorial, and even though it was a difficult task to get him there, as he was in a wheelchair, he made the event. He gave an impressive long talk about Gelfand.

Joel Lebowitz

One of the activities in which Felix played a central role was getting Gelfand to Rutgers. I remember Gelfand coming to see me at the Academy Hotel in Moscow where I was staying as a visitor to the Soviet Academy of Science. I transmitted to him the invitation from Felix to join Rutgers, and I wrote a letter to the American consul in Moscow for Gelfand to take to the consulate when applying for a visa.

Sagun Chanillo

Felix never shied away from bold decisions and action. This was a man with impeccable taste who took risks. A year after I arrived at Rutgers, Calderón was invited to a colloquium. At the dinner afterwards, Felix with a

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grand flourish pulled out an envelope from his jacket pocket, turned to Calderón, and said, “Alberto, this is for you.” Calderón opened the envelope and his eyebrows rose several inches. At that point Felix added, “Alberto you deserve it. I hope you can come.” Everyone at the table was stunned and taken by surprise, as nobody had been consulted, there was no committee, and no regard to “what if we fail and what will be the fallout?” Once Felix had a thought and a goal, it would seem he went after it in a bold and decisive way.

Felix was well known as a polymath and raconteur. Years ago, Felix and I found ourselves together at a Thanksgiving supper at the house of Abbas Bahri. The topic moved to something I deeply care about, ancient Indian astronomy. Felix had of course read the monumental treatise by Otto Neugebauer, *The Exact Sciences in Antiquity*, and quite remarkably also a translation of the *Surya Siddhanta*, which is Sanskrit for ‘the treatise of the sun.’ So he proceeded to tell me what Indian astronomy owed to Babylonian astronomy and vice versa. He also knew of the work of Frits Staal at Berkeley who was a pioneer in such studies. I have not met many since who could speak with such authority on so esoteric a subject.

Fred Roberts

Some Reflections on Felix Browder and the Formation of DIMACS

Not many people realize the critical role that Felix Browder played in the formation of DIMACS, the Center for Discrete Mathematics and Theoretical Computer Science, at Rutgers.

Felix came to Rutgers as vice president for research in 1986, and one of his early initiatives was to spearhead the Rutgers proposal for a National Science Foundation Science and Technology Center (STC). The STC program had three major components: cutting edge science in some field, education closely tied to research, and technology transfer. It required university, industry, and government partners.

When the STC program was announced, Rutgers, Princeton, and what were then AT&T Bell Labs and Bellcore (Bell Communications Research) had all developed strong programs in discrete mathematics and theoretical computer science, but they were only interacting sporadically and informally. Felix had the insight to see that a New Jersey Center for Discrete Mathematics and Theoretical Computer Science already existed in some sense. Felix managed to get Danny Gorenstein of Rutgers to agree to serve as director (a critical development), and he was joined by Ron Graham of Bell Labs and later Bob Tarjan of Princeton. Felix also played an important role in getting the New Jersey Commission on Science and Technology to join the team as the government partner.

The Rutgers-led proposal was a winner in the first-ever STC competition, one of 11 winners in all fields of science.

Browder played a critical role in the proposal that led to the formation of DIMACS, the host of the pictured $E + M = C^2$ workshop held in January 2017.

This led to the formation of DIMACS, the Center for Discrete Mathematics and Theoretical Computer Science, in early 1989. It is fair to say that we would not have gotten started on this without Felix. He played a pivotal role.

When Rutgers received the NSF award for DIMACS, it was at the time the largest award the university had ever gotten (some $US22 million) and a major center kickoff conference was held, with New Jersey Governor Tom Kean giving a keynote address.

Felix was there, but stayed in the background. He didn't need the public thanks that many of us who have benefited from DIMACS owed him.

Felix continued to play an important role as DIMACS grew and developed. He also found it useful to leverage the existence of DIMACS to recruit Israel Gelfand to Rutgers, arranging for DIMACS to provide both space and resources for Gelfand's many visitors. The scientific and educational collaboration between DIMACS and Gelfand continued until his death and indeed continues today through the work of Tanya Gelfand to get her late husband's educational books published. This too is part of the legacy of Felix Browder.

Felix remained interested in DIMACS long after he stepped down as vice president for research. He often asked how things were going. One of the last times I saw him was when he became aware of the Simons Foundation announcement for an Institute for the Theory of Computation and invited me to strategize about how DIMACS could apply. I visited him at home, and we had a wide-ranging discussion. It was still another reminder of how Felix stayed engaged with the center he played such a great role in founding.

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Jerry Bona

Browder the Professor

During my years as a young mathematician at the University of Chicago, I sat for three full courses from Felix. His lectures were fluid and his blackboard technique superb. He filled the room with his personality and kept the small audiences utterly focused. I still retain the notes from these courses. I came away from his lectures wondering how on earth he came up with the material, as most of it was his.

It was during the third course, which was on topological degree theory, when I was part of the Browder's social circle, that I had the temerity to ask Eva one evening if Felix prepared his lectures at home. Her response was immediate and forceful: no, he did not prepare lectures at home and she had no idea when he did. I found out a few weeks later. I was sitting in Felix's office discussing some mathematical point. Felix noticed that it was almost time for class. He immediately left off our conversation and concentrated for about a minute. He then rose and we walked together to class. All the while, he was clearly preparing the lecture.

I had the answer to my question: he prepared on the way to class, despite the complexity of what he was presenting! It was less than 15 meters from Felix's office to his favorite classroom. So once in a while, he would get stuck. You were then in for a treat. He would move to the side a little and start addressing the board and you got to see exactly how this great mind worked. It was a revelation to see the true inner workings of the beautiful, smoothly running mathematical theory that he was developing.

Probably this style did not go down so well in undergraduate, or even first-year graduate classes, but for someone a little more advanced and intensely interested, it was transformative.

Haïm Brezis

In 1964 I asked G. Choquet in Paris to give me a PhD thesis topic. He told me to learn fixed point theory. He was perhaps hoping that I would find connections between fixed points and extreme points—his main research interest at the time. A few months later, I went back to G. Choquet, told him that I had read many papers on fixed points, and asked him politely, "Monsieur le Professeur, what should I do next?" On his desk was a huge envelope he had just received from Felix Browder. Choquet knew Felix personally and held him in high esteem, but did not have much interest in these topics. So, instead of throwing away the envelope, he handed it to me and said, "This might have connections with fixed points."

These brief minutes turned out to be a defining moment in my career. I started reading the twenty or so reprints, all published in 1963–1964. Most of the papers had two parts. The first one was written in the language of abstract functional analysis, and fixed points were mentioned here and there; I was comfortable with their content. The second part was concerned with applications to PDEs, and I was totally lost. Under Bourbaki's influence, PDEs were simply not taught in Paris.

One day, in between two reading sessions at the library of the Institut Henri Poincaré, I noticed a flyer advertising a month-long summer school on PDEs in Montreal. 'F. Browder' was on the list of speakers; the other names, such as S. Agmon and G. Stampacchia—leading experts in PDEs—were totally unknown to me. The school was about to start the following week and the registration deadline had passed long ago. I mentioned it to my father, who immediately bought me a plane ticket. Luckily, I was admitted—even offered a room in the dorms—and started listening to Felix's classes. I introduced myself to Felix, who was pleased to hear that the package he had sent to Choquet had not been lost. We spoke for only a few minutes, but they were precious, particularly because Felix gave me ten more freshly published papers. At that time Felix was at the peak of his creativity, writing over twenty papers a year.

I greatly benefited from attending these classes. Felix taught me to enjoy PDEs through the eyes of a functional analyst. Originally most of functional analysis grew out of PDEs, but eventually it became increasingly abstract and detached from PDEs. In the mid-1960s, PDEs were hardly legitimate in Paris. J. Leray—who made celebrated contributions to PDEs in the 1930s—had shifted to other fields. J.-L. Lions was teaching some PDEs, disguised under the title "Numerical Analysis." His classes were held in "exile," in a building miles away from the "Holy of Holies," the legendary Institut Henri Poincaré.

Returning to Paris from Montreal, I realized that Felix's work had a rejuvenating impact on some French mathematicians, in particular, Leray and Lions. In fact, they had just published a PDE paper relying heavily on techniques developed by Felix. I studied it with great interest, and I understood it thanks to the background I had acquired from Felix's papers.

I also came across a new paper by Lions and G. Stampacchia concerning variational inequalities. The concept of weak solution that they had adopted was via integration by parts (not surprisingly, since Lions had been a student of L. Schwartz, the inventor of distributions); they could prove existence, but not uniqueness. Influenced by the ideas of Browder, I proposed a slightly different concept of weak solution, for which I could establish both existence and uniqueness (see page 1403 "Felix Browder and Monotone Operators"). Lions and Stampacchia were impressed and offered to collaborate with me on various projects using the same technique. Ironically, my first papers, at age twenty-three, were dealing with PDEs!

In 1967 Felix came to Paris for a conference organized by Lions. This time we had lengthy conversations! Felix invited me to give a talk at an AMS meeting to be held in Chicago the following year. My lecture was scheduled for the first day, and I came prepared to give a presentation on

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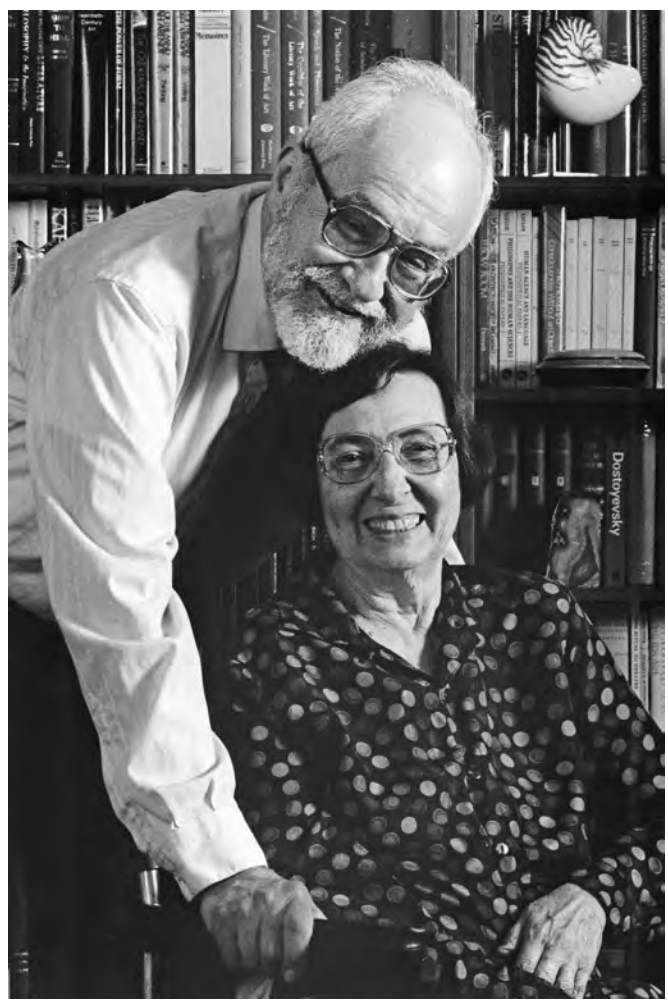
Felix and Yves Meyer in Paris in 1998 at the celebration for J.-L. Lions' seventieth birthday. Felix had developed close ties with a number of French mathematicians.

a blackboard. When I arrived at the Hilton Grand Ballroom, I searched frantically for the blackboards. Instead, there were strange gadgets that had not yet reached France: overhead projectors. My limited English added to the catastrophe. Nevertheless, Felix kindly invited me to spend the following week at the University of Chicago, where I gave a more relaxed talk on a blackboard. He was extremely receptive and made numerous comments. Subsequently, I spent a month every year with him in Chicago. Strictly speaking, I was not Felix's student, but I consider myself as an "adopted child" of Felix; very naturally he became my mentor.

When in Chicago, I enjoyed working with Felix for a few hours and then walking to one of his favorite used bookstores in Hyde Park: Powell's, O'Gara (with its dusty high shelves, and a cat), or 57th St Books (in a basement). Felix loved to buy books; he also spent hours at the copy machine in the library, copying papers, and then carefully cutting the black margins to make them look neater.

Felix had developed close ties with a number of French mathematicians. I remember the constant flow of visitors from France invited by Felix in the 1970s and early 1980s. Of course, senior professors (such as J. -L. Lions and R. Thom) were on his list. But also for many junior French mathematicians (such as I. Ekeland, R. Temam, H. Berestycki, and the late A. Bahri), the gate to the USA was Chicago. Felix was a superb host—even fully mobilized when a young instructor required special help in critical medical matters. In addition, Felix was the driving force in organizing two conferences celebrating two legendary French mathematicians: Henri Poincaré and Elie Cartan. On the other side of the Atlantic, Felix was awarded a well-deserved honorary degree at the Sorbonne in 1990. He often visited Paris, by himself or with his family. His favorite hotel was the Parisiana-Panthéon, run by two old ladies; they kept telling him fascinating stories about the famous Russian mathematician, Luzin, who had stayed there at the beginning of the 20th century. Felix read French fluently and was buying large numbers of books in the Latin Quarter. Needless to add that shipping them back home at the end of each visit was a major challenge!

When Felix moved to Rutgers in 1986, I followed him as a long-term distinguished visiting professor. Our mathematical collaboration became greatly reduced due to his time-consuming responsibilities as vice-president for research. Our last joint paper, "Partial Differential Equations in the 20th Century," written in 1997, had been commissioned by an encyclopedia on the history of science. We spent more time than expected working on this project; it was an unusual activity for both of us—especially for me. I watched with admiration as Felix proposed offhand a detailed table of contents and a list of topics to be discussed. He had a deep understanding of the evolution of concepts, and the main novelties each period produced. We also carefully read papers by Hilbert and Poincaré dating back to the beginning of the 20th century and discovered with great surprise that some preconceptions had to be reexamined. Felix was sharp-minded and had his personal views on almost every topic.



Felix and Eva at their home in New Jersey, circa 2002.

In our discussions, I found Felix always very respectful of my opinions, even if he did not share them. The same was true for religious beliefs. He regularly insisted that we go for lunch to a kosher restaurant to accommodate my dietary restrictions. If we worked on a Friday, he would send me back home early enough so that I could properly keep the Sabbath.

As years passed, our personal relationship grew deeper and sweeter. Occasionally, Felix would buy a book for me if he thought I might enjoy it. Eva and Felix had an open house for my whole family. Felix had a genuine interest for the work of my wife, the Israeli writer and poet, Michal Govrin. And Eva graciously agreed to be interviewed about her life by our daughter Rachel for a school project.

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Felix and Eva Browder photo courtesy of Haïm Brezis.