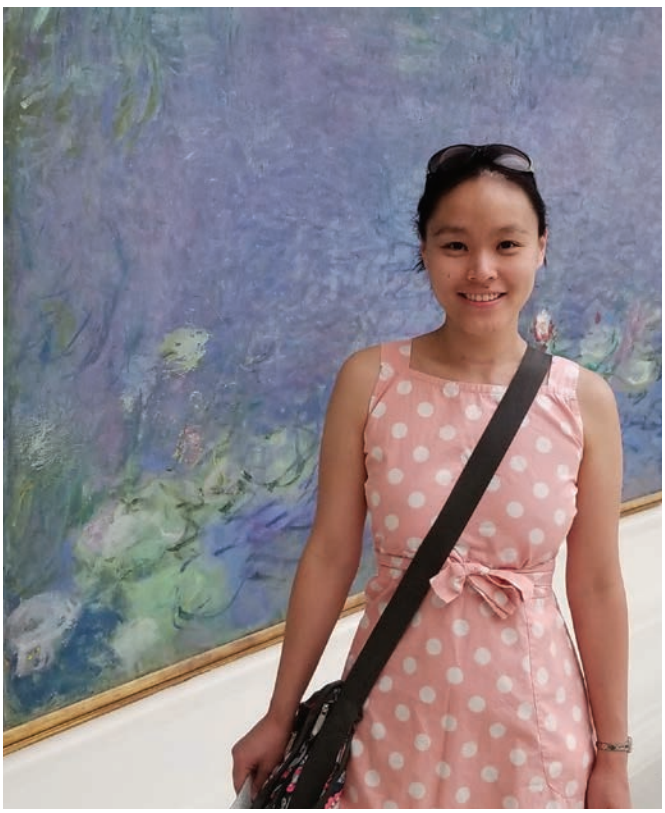
Wei Ho Interview

Conducted by Alexander Diaz-Lopez



Wei Ho is assistant professor at the University of Michigan. Recipient of several NSF grants, as well as a Sloan Research Fellowship, Wei works on algebraic geometry and its application to arithmetic problems.

Diaz-Lopez: *When did you know you wanted to be a mathematician?*

Ho: At the beginning of college, I was very undecided about what major to choose. I was so certain I would not major in math that in my first semester I took a distribution requirement from which math majors were exempt! By the end of my first year, however, I realized that all my favorite topics in my classes were the most mathematical: quantum mechanics, symmetry groups in inorganic chemistry, game theory. After working in an organic chemistry lab for the summer, I also found out that I was so clumsy that I might blow myself up if I continued in a lab science. So, during my sophomore year, I took (and enjoyed) some more math classes and switched by the end of the year. I spent the rest of my undergraduate years feeling like I was playing catch-up to the "real" math majors who had taken the hardest freshman math sequence, but I eventually realized that starting a year—or even many years—later does not matter and there is no "right" path.

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At every stage of my education and career, I have doubted whether to continue in academia, mostly due to my own insecurities (I'm told this is the infamous "imposter syndrome"). I seriously considered working as a quant at a hedge fund after college, dropping out of grad school to work for a tech company, and finding a consulting or tech job after grad school and again after my postdoc years. I am quite happy with my choice now, but I think there are many viable careers for someone with a background in math (and I would encourage graduate students to investigate all types of jobs as early as possible).

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DOI: http://dx.doi.org/10.1090/noti1762

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Diaz-Lopez: *Who else encouraged or inspired you?*

Ho: My parents encouraged my non-stop questions about math and everything else as a child, and they prioritized my brother's and my education over all else. They taught us to think analytically and precisely and question everything; while these are useful traits for a mathematician, I've learned these pedantic tendencies sometimes need to be suppressed in social situations!

I feel fortunate that many older mathematicians have been very kind and encouraging to me for no apparent reason, and there are so many people who had a profound influence on my mathematical path. I'll name just a few here. When I was deciding whether to switch to math, Daniel Allcock's number theory class was fun and accessible—and made me believe I could actually do math. During my year abroad after college, I loved Burt Totaro's class on geometric invariant theory, which strongly influenced my research directions in graduate school. My PhD advisor Manjul Bhargava and postdoc mentors Joe Harris and Johan de Jong have all been incredibly supportive throughout my career, and talking to any of them about a problem I'm mulling over is always fun and stimulating.

Someone who probably has no idea of her impact on me is Linda Chen, who was the first Asian-American woman mathematician I remember meeting. While Asians are not considered an underrepresented minority in math, there seem to be very few Asian-American mathematicians, and of course even fewer such women. The stereotypes gave me yet another reason for self-doubt: maybe the emphasis on obedience and duty ("聽話 and be 乖") in my Chinese cultural upbringing made me less able to think outside the box in my research, while my privileged and comfortable American childhood made me too lazy? These qualms have not gone away completely, but they have made me further appreciate how much representation matters, especially for people from groups that are underrepresented in whatever ways.

Diaz-Lopez: *How would you describe your work to a graduate student?*

Ho: I tend to think a lot about classical algebraic geometry constructions that can be adapted to arithmetic settings. Perhaps we understand some construction very well over the complex numbers, but more interesting phenomena appear when you change the base field to the rational numbers.

Most of the motivation for my work comes from simple questions in number theory and algebraic geometry. For example, given a multivariate polynomial with rational coefficients, how many rational solutions does it have? Maybe you can find the answer to be zero, or a finite number, or even "infinity"; but can you describe the set of solutions ("rational points") more precisely in any way? Sometimes it's easier to think about lots of polynomials all at once, in a big family, i.e., study a "moduli space" of the polynomials. Then you can try to study the structure of the relevant moduli space to answer your original question.

This strategy has proved immensely useful in a field now called arithmetic statistics, which studies the distribution of arithmetic invariants in families. For instance, we can associate a nonnegative integer, called the rank, to an "elliptic curve" defined over the rational numbers (the rank of the finitely generated abelian group made from the rational points of the curve). If we order all such elliptic curves (there are infinitely many) in a sensible way, can we predict what proportion of the curves have rank 0, or rank 21? Do any have rank 100? Mathematicians don't know the answer to these questions yet, but studying moduli spaces related to elliptic curves has given us some insight into this distribution. We also have various heuristic predictions and even some data to support them, thanks to much improved computational software over the last decades.

Diaz-Lopez: *What theorem are you most proud of and what was the most important idea that led to this breakthrough?*

Ho: This question is very hard to answer, since I tend to find all my theorems trivial once I know how to prove them!

I do remember being very excited about my recent work with Levent Alpoge, since the turning point was the trivial observation that $2^2=4$. The paper is about understanding the number of integer solutions (x,y) to equations of the form $y^2=x^3+Ax+B$, as A and B are varying over integers (with $4A^3+27B^2$ nonzero). Levent previously proved that when you order these so-called elliptic curves by the size of their coefficients, then the average number of integral points is bounded. To understand the distribution even better, you might study the higher moments of the number of integral points. We show that the second moment is also bounded, by combining a classical explicit construction of Mordell with some analytic techniques and a theorem of Bhargava-Shankar on the average size of Selmer groups of elliptic curves. Originally, we had hoped to prove this result with a rather complicated method, so we were both very happy when we realized there was a much simpler way that actually worked.

Random knowledge in one very specialized corner of math could be unexpectedly useful.

Another paper that has a fun backstory is a very short one with Johan de Jong. Johan and I had been meeting semi-regularly to chat about interesting math, and the paper came out of just a one-hour discussion. We realized that some constructions I knew from other projects could be adapted to solve a problem he liked. It was so simple that we tried very hard (but failed) to make the paper just one page long. But this was the first time where I concretely saw how random knowledge in one very specialized corner of math could be unexpectedly useful in a different area; it is helpful to pick up ideas from as many different fields as possible, even if they don't seem immediately relevant.

Diaz-Lopez: *What advice do you have for graduate students?*

Ho: Grad students are often stressed out about choosing a perfect advisor with a perfect problem. Neither exists. Choose an advisor based on compatibility, both mathematical and otherwise. And the problem you start with is almost definitely not going to be the problem you end with; it will evolve as you work on it.

Talk to people! Learn from your fellow grad students, as well as postdocs and faculty. Organize your own learning seminars if they aren't already happening.

Go to conferences. Read widely.

Go to seminars. Go to summer schools. Go to conferences. Read widely. Read survey papers, e.g., in the *Bulletin of the AMS*. You won't understand nearly anything at first, but it's useful to be exposed to lots of ideas—and to lots of people who have different ways of doing math.

If you aren't set on academia, explore your other options early. Do a summer internship if you can. Learn to code.

Diaz-Lopez: *All mathematicians feel discouraged occasionally. How do you deal with discouragement?*

Ho: I would say it is more than occasional! Research can be quite frustrating because much of your time is spent going down seemingly incorrect paths, but I try to remember that those supposed dead ends often are useful later.

I tend to have many projects at different stages. While it is sometimes hard to switch between topics (and most days I don't actively think hard about more than one project), I find it useful to switch to another when I am stuck on one. Having collaborators—who often are friends also—helps a lot with the discouragement. Finally, teaching and service responsibilities can make me feel productive even if I didn't get anywhere on my research on a given day!

Another strategy is to just do something completely non-mathematical. I go to ballet classes or play video games when my brain is not working well. It took me many years to not feel guilty whenever I was not working. As a research mathematician, I could in theory be working 24/7. But of course, I wouldn't be productive all of that time, and it was important to learn to make guilt-free time for friends, family, and outside interests. For example, I read novels and rarely work on airplanes.

Diaz-Lopez: *Any final comment or advice?*

Ho: I think grad students should learn early that their own mathematical adventure will be unique, so it is completely okay to not follow in other people's footsteps. You don't have to do things the way everyone else seems to, if something else works for you.

Most importantly, just have fun!

Photo Credit

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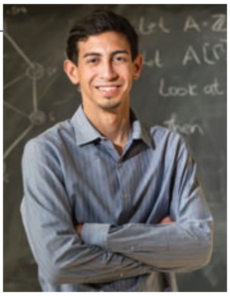
ABOUT THE INTERVIEWER



Alexander Diaz-Lopez

Alexander Diaz-Lopez, having earned his PhD at the University of Notre Dame, is now assistant professor at Villanova University. Diaz-Lopez was the first graduate student member of the *Notices* Editorial Board.