Luc Illusie

Michel Raynaud (1938–2018)

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Michel Raynaud's parents lived in Châtel-Guyon, a small spa town in the department of Puy-de-Dôme, located in the mountainous Auvergne region in the center of France. His father was a carpenter, and his mother, a cleaning lady. He was born on June 16, 1938, in Riom, a bigger town five miles to the southeast. He was their only child. He attended elementary school in Châtel-Guyon, then middle school in Riom, and, as a boarding student, high school in Clermont-Ferrand, the main city of the department. He entered the École Normale Supérieure (Paris) in 1958. I myself was admitted in 1959 and it is there that we met for the first time. I remember that he impressed me by his understanding of Lang's book on abelian varieties and Grothendieck's newly published first volumes of [EGA]. That was the beginning of a friendship that lasted until his death.

In 1961 he was number one on the "agrégation" exam, the national competitive examination selecting high school teachers in France. After one more year at the École Normale, he was admitted at the CNRS (Centre National de la Recherche Scientifique), where he stayed until 1967. Meanwhile he had met Michèle Chaumartin, who had entered the École Normale Supérieure de Jeunes Filles in 1958, and whom he married in 1962. They had a son, Alain, born in 1970, who now lives in California with his wife and two daughters.

In 1962, Michel and Michèle started attending Serre's course at the Collège de France and Grothendieck's algebraic geometry seminar [SGA] at the IHÉS. Grothendieck became their PhD advisor, and they contributed to his seminars ([SGA 3, 6] for Michel, and [SGA 1, 2, 7] for Michèle). After rejecting various suggestions of Grothendieck, Michel chose his own topic for his thesis, and defended it in 1968 [R70a]. In *Récoltes et semailles*, Grothendieck wrote: "Michel Raynaud prend une place à part, ayant trouvé par lui-même les questions et notions essentielles qui font l'objet de son travail de thèse, qu'il a de plus développé de façon entièrement indépendante; mon rôle de 'directeur de thèse' proprement dit s'est donc borné à lire la thèse terminée, à constituer le jury et à en faire partie."[1] He was hired as a professor at Orsay in 1967 and worked there until his retirement in 2001.

Raynaud was a modest person. He disliked honors, but several were nevertheless bestowed on him. He was an invited speaker at the Nice ICM in 1970 [R71]. In 1987 he was awarded the Ampère Prize of the French Academy of Sciences. He was elected corresponding member of the French Academy of Sciences in 1994. In 1995 he received the Frank Nelson Cole Prize, jointly with David Harbater, for the proof of the Abhyankar conjecture. On the occasion of his retirement a conference *Algebraic Geometry and Applications to Number Theory*, June 18–22, 2001, was held at Orsay in his honor.

Raynaud was the emblematic figure of algebraic geometry in our department at Orsay. In 1976 he founded the Équipe d'arithmétique et géométrie algébrique, a research unit associated with the CNRS, of which he was the head until 1983. He then let me succeed him. Together we founded the SAGA (Séminaire d'arithmétique et géométrie

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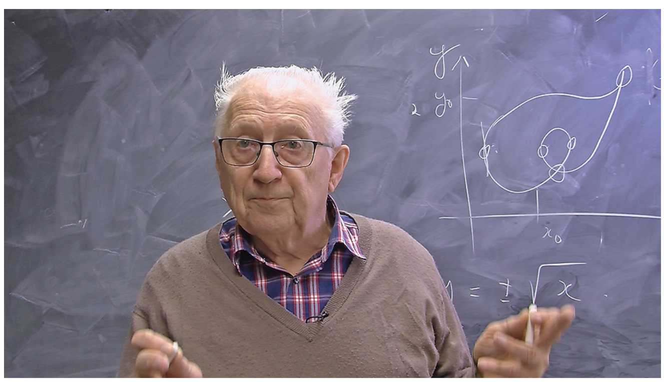
[1] *Michel Raynaud occupies a special place, as he found by himself the essential notions and questions that are the subject of his thesis, and which he developed in a totally independent way; my role of 'thesis advisor' was thus limited to reading the completed thesis, selecting the jury, and participating in it.*

algébrique) in 1985. It quickly became popular, and is still very active.

Every morning, he arrived at his office at 7:00 am. Many people came to consult him, sometimes from far away. Teaching mattered very much to him, at all levels. Models of clarity, his courses have inspired generations of students. He directed ten PhD theses. His students remember his demand for rigor, his generosity, and his kindness. For many years he chaired the committee in charge of the teaching assignments. Together with Guy Henniart he developed a successful teacher preparation (and pre-preparation) program for students planning to take the agrégation, in which his former students Lucile Bégueri and Renée Elkik, who had become his colleagues at Orsay, actively participated.

Raynaud was quite influential in the development of our ties with Japan and China. His Chinese student Xiao Gang, who died prematurely in 2014, created a school of complex algebraic geometry in Shanghai. In 1982, together with Tetsuji Shioda, he organized a Japan-France conference *Algebraic Geometry* [RS83] at Tokyo and Kyoto that was the starting point of a sustained and fruitful collaboration between French and Japanese algebraic geometers. He visited Japan several times. He also visited China once, in the spring of 2004: in the framework of a cooperative program supervised by Jean-Marc Fontaine, and kindly assisted by our Chinese colleague Yi Ouyang, Raynaud and I gave a course at Tsinghua University. At the end of it we selected students to come to Orsay to pursue a PhD (Raynaud's last student, Jilong Tong, was one of them). More came afterwards, and a good number are now university professors in China. This 2004 exchange with China led to many others in later years.



Michel Raynaud lecturing.

Raynaud's work is vast and touches several different topics. I will limit myself to brief comments on a few salient points.

(1) *The Manin–Mumford conjecture.* Raynaud proved the following generalization of this conjecture: let A be an abelian variety over an algebraically closed field k of characteristic zero, and let X be a closed integral subscheme of A. Denote by T the torsion subgroup of $A(k)$. If $T \cap X(k)$ is Zariski dense in X, then X is the translate of an abelian subvariety of A.[2] Raynaud first treated the case where X is a curve [R83a], by reduction modulo p^2 techniques, then the general case [R83b] using additional results from his approach (see (3) below) to rigid geometry.

[2] *The Manin–Mumford conjecture was for the case of a curve embedded in its Jacobian.*

(2) *The Abhyankar conjecture*. For the affine line X over an algebraically closed field k of characteristic $p>0$, this conjecture asserts that any finite group G which is generated by its p-Sylow subgroups is the Galois group of a connected étale Galois cover of X. Raynaud proved it in [R94a], again by rigid geometry techniques (based on the work described in (3) below). The problem had been proposed to him by Serre, who had treated the case where G is solvable. Shortly afterwards, Harbater [H94], elaborating on Raynaud's work, proved the general case of the Abhyankar conjecture (for affine smooth curves over k).

(3) *A new approach to rigid geometry.* Rigid analytic spaces over complete non-archimedean valued fields K were defined by Tate in 1961 and studied by him and, a few years later, by Kiehl. In the early 1970s Raynaud [R74] introduced a new way of thinking about them, by means of a category of formal models where certain blow-ups, called "admissible," are inverted: a typical example is his construction of the rigid *generic fiber* of a (flat, finitely presented) formal scheme over the ring of integers of K. The dictionary he built between formal and rigid geometry enabled him to recover Tate's and Kiehl's results by a simple application of techniques of [EGA]. He used it in [R71] to construct rigid uniformizations of abelian schemes over a complete discrete valuation field K having semi-abelian reduction (generalizing Tate's construction of the *Tate curve*), and, later, similar uniformizations for 1-motives over K in the sense of Deligne [R94b]. This viewpoint, which is crucial there and in (1) and (2), has been very influential, e. g. in crystalline cohomology (Berthelot's rigid cohomology) and p-adic Hodge theory (Faltings' almost purity theorem and applications).

(4) *Flatness.* Grothendieck showed the importance of flatness in algebraic geometry: the good families of algebraic varieties are those which correspond to a flat morphism of schemes f. Such morphisms, often with the extra condition that f be locally of finite presentation, were extensively studied in [EGA IV]. However, for the foundations of his new approach to rigid geometry, Raynaud needed more. In a seminal joint paper with Gruson [RG71] he proved that, given a morphism $f{:}X \rightarrow S$ satisfying mild assumptions (S quasi-compact and quasi-separated, and f of finite presentation), there exists a blow-up $g{:}S' \rightarrow S$ such that the strict transform $f'{:}X' \rightarrow S'$ of f is flat (and if f is flat over some open U in S, then g can be chosen to be an isomorphism over U). This flattening theorem was critical for

rigid geometry but has had many other applications (the above paper is Raynaud's most cited paper).

(5) *The Picard functor and Néron models.* The main part of this work was done in the 1960s. It reflects Raynaud's early interest for abelian varieties, and was partly influenced by Grothendieck, who, among other things, had proposed to him translating Néron's construction of Néron models into the language of schemes. In the late 1950s Grothendieck had proved the representability (by a scheme) of the relative Picard functor $\mathrm{Pic}_{X/S}$ for projective, flat morphisms $X \to S$ with integral geometric fibers. In [R70b] Raynaud studies the case where S is a trait (spectrum of a discrete valuation ring), but where the geometric fibers are no longer assumed to be integral. In particular, for X/S a proper, flat curve, with smooth, geometrically irreducible generic fiber, he gives practical conditions ensuring that $\mathrm{Pic}^0_{X/S}$ is the connected component of the Néron model A of the Jacobian of the generic fiber of X, and provides a combinatorial description of the group of connected components of the special fiber of A in terms of the irreducible components of the special fiber of X. These results had important applications: they were of critical use in Deligne-Mumford's article [DM69], and later in the analysis of the arithmetic of the modular curve $X_0(N)$ in work of Deligne-Mazur-Rapoport,[3] and Ribet (in his proof that the Shimura-Taniyama-Weil conjecture implies Fermat [R90]). An extended presentation of his results, together with useful pedagogical material, is given in a beautiful book coauthored with Bosch and Lütkebohmert [BLR90].

(6) *Group schemes of type $(p,\cdots,p)$.* Raynaud brought several contributions to the theory of group schemes: in [SGA 3] (Exp. XV, XVI, XVII), and in his thesis [R70a], but certainly the best known one is his paper [R74b] on group schemes of type $(p,\cdots,p)$, i.e., finite, flat, commutative group schemes annihilated by a prime number p. Let R be a strictly henselian discrete valuation ring of mixed characteristic $(0,p)$, absolute ramification index e, fraction field K, and let $\bar{K}$ be an algebraic closure of K and $I=\mathrm{Gal}(\bar{K}/K)$ the inertia group. Raynaud's paper [R74b] contains two independent main results:

(a) For $e \le p-1$, if G is a group scheme of type $(p,\cdots,p)$ over R, then any Jordan-Hölder quotient H of G is a group scheme in vector spaces over a field $\mathbf{F}_{p^r}$, the inertia I acts tamely on the $\mathbf{F}_{p^r}$-vector space $H(\bar{K})$ by homotheties through a character $\mathbf{F}^*_{p^r} \to \mathbf{F}^*_{p^r}$ which is a product of fundamental characters with exponents $\le e$. This proves a conjecture of Serre.

(b) If G is a p-divisible group over R, of height h and dimension d, with Tate module $T=T_p(G_K)$, then $\wedge^h T=\mathbf{Z}_p(d)$, where $\mathbf{Z}_p(d)=T_p((\mu_{p^\infty})_K)^{\otimes d}$.

These results have been very fruitful; for example, (b) was used by Faltings [F84] in his proof of the Mordell conjecture to bound the modular height in an isogeny class of abelian varieties (effective refinements were given by Raynaud in the Szpiro seminar [R85]).

(7) *Cartier-Dieudonné theory and crystalline cohomology.* In his proof of (6)(b) Raynaud used Cartier's theory of formal groups. He returned to it—and to the Picard functor—in [R79], where he made a thorough study of the proper R-scheme $\mathrm{Pic}^\tau_{X/R}$ for R as in (6)[4] and X/R proper and smooth. He then got interested in crystalline cohomology and the de Rham-Witt complex of proper, smooth schemes over a perfect field of characteristic $p>0$. It was known in the late 1970s that the initial term of the so-called slope spectral sequence associated with this complex gave, modulo p-torsion, formal p-divisible groups, but the structure of its p-torsion remained mysterious. We unraveled it in [IR83] in terms of certain graded modules over a graded ring, now called the *Raynaud ring*, which is an extension of the classical Cartier-Dieudonné ring. Raynaud, who often jokingly boasted of not understanding spectral sequences, in fact discovered the deepest property of the slope spectral sequence (and of its partner, the *conjugate spectral sequence*), that he suggested to call the *survival of the heart*. Despite its romantic name, this property is unfortunately too technical to be described here. This theory was extensively developed by Ekedahl [E84–86].[5] Other geometric and arithmetic applications were found later by Joshi and Milne-Ramachandran.

(8) *Geometry of curves and surfaces.* Raynaud [R78] gave the first example of a proper, smooth surface X over an algebraically closed field k of characteristic $p>0$ having an ample line bundle $\mathcal{L}$ such that $H^1(X,\mathcal{L}^{-1})\neq 0$, thus showing that the Kodaira vanishing theorem does not extend to positive characteristic.[6] His construction is based on certain curves studied by Tango. Relative variants were developed jointly with Szpiro and new examples were given. These curves are now called *Tango-Raynaud curves*.

In the mid 1970s Raynaud studied elliptic and quasi-elliptic fibrations of surfaces. His work remained unpublished but notes circulated. It was of essential use in joint papers with Liu and Lorenzini ([LLR04], [LLR05]) where they prove the following. Let X be a proper, smooth, and geometrically connected surface over a finite field k, $f{:}X \to V$ a proper and flat morphism to a proper, smooth,

[3] *In particular, for the modular curve $X_0(p)$, $p\ge 5$, the group of connected components of the special fiber at p of the Néron model of the Jacobian of $X_0(p)_{\mathbf{Q}}$ is, over an algebraic closure of $\mathbf{F}_p$, a cyclic group of order the numerator of $(p-1)/12$ [M77, Appendix].*

[4] *(supposed further to be complete, and have an algebraically closed residue field)*

[5] *See [RS83, 20–72] for a survey.*

[6] *Mumford had previously given an example with a singular, normal surface.*

geometrically connected k-curve with smooth, geometrically connected generic fiber X_K. Then: (a) (under a certain necessary hypothesis on local numerical invariants of f) the Artin-Tate and Birch and Swinnerton-Dyer conjectures are equivalent; (b) if, for some prime ℓ, the ℓ-part of the Brauer group $\mathrm{Br}(X)$ is finite, then the order of $\mathrm{Br}(X)$ is a square, which resolved a longstanding question.

Let now X be a proper, smooth, connected curve of genus $g\geq 1$ over an algebraically closed field k of characteristic $p>0$, with relative Frobenius $F:X\to X^{(p)}$. The sheaf $B\subset F_*\Omega^1_{X/k}$ of locally exact differentials, a vector bundle of rank $p-1$ and slope $g-1$ over $X^{(p)}$, appeared in the definition of Tango-Raynaud curves. Raynaud [R82] showed that B admits a *theta divisor* $D\subset J^{(p)}$ (where J is the Jacobian of X) parametrizing invertible sheaves L of degree zero such that $h^0\ (B\otimes L)\neq 0$. This divisor D turned out to be a key ingredient in the proof by Tamagawa [T04] of the fact that, over an algebraic closure of $\mathbf{F}_p$ there are only finitely many isomorphism classes of smooth, hyperbolic curves whose tame fundamental group is isomorphic to a given profinite group.

(9) *Bourbaki seminar.* Raynaud gave several talks in the Bourbaki seminar, on a wide range of topics. See the references at the end.

Let me finish on a more personal tone. Raynaud was fond of nature. His office at Orsay overlooked the woods. From time to time he would watch a red squirrel bouncing in the trees. He kept an amaryllis on his desk. In their youth he and Michèle climbed in the Alps, mostly in the Oisans massif. For almost forty years they went skiing each year in Val d'Isère. Starting in the late 1960s, together with Serre they regularly did rock climbing in Fontainebleau. Michel and Michèle also used to play tennis three times a week. He remained in rather good health until one day, in November 2017, he suddenly became disoriented near his house in Palaiseau. An MRI revealed a brain tumor. Most of it was successfully removed by surgery shortly before Christmas, and for a few weeks he seemed to have recovered. However, in February his condition deteriorated rapidly. He had to be hospitalized, he fell into coma, and he died on March 10.

Interview with Michel Raynaud.

Up to his last moments of consciousness he remained mentally alert and kept his sharp sense of humor. When I visited him at the hospital, we joked again about spectral sequences, of which he said that they had luckily disappeared in the surgery, and also about rigid annuli, which he loved, especially those of thickness zero, which had played a key role in his proof of the Abhyankar conjecture.

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