

# 2019 Mary P. Dolciani Prize for Excellence in Research

STEPHAN RAMON GARCIA was awarded the inaugural Mary P. Dolciani Prize for Excellence in Research of the AMS at the 125th Annual Meeting of the AMS in Baltimore, Maryland, in January 2019.



Stephan Ramon Garcia

## Citation

The Mary P. Dolciani Prize for Excellence in Research is awarded to Stephan Ramon Garcia, W. M. Keck Distinguished Service Professor and Professor of Mathematics at Pomona College, for his outstanding record of research in operator theory, complex analysis, matrix theory, and number theory, for high-quality scholarship with a diverse set of undergraduates, and for

his service to the profession.

Garcia received his PhD in Mathematics in 2003 from the University of California at Berkeley. He is the author of eighty-nine research papers in several areas, including operator theory, linear algebra, complex analysis, mathematical physics, and number theory. His work has appeared in top research journals, as well as top expository journals, and he has been the Principal Investigator on four NSF research grants. He has co-authored four books and is currently writing two more. Garcia has also co-authored over twenty-nine articles with undergraduates, with papers appearing in the *American Mathematical Monthly*, the *Notices of the American Mathematical Society*, *Proceedings of the American Mathematical Society*, and the *Journal of Number Theory*, among others. His paper “G. H. Hardy: Mathematical Biologist,” written with a student, was included in the 2016 book series *The Best Writing on Mathematics*, published by Princeton University Press. Garcia currently serves as an editor of the *Notices of the American Mathematical Society*, the *American Mathematical Monthly*, *Proceedings of the Amer-*

*ican Mathematical Society*, *Annals of Functional Analysis*, and the undergraduate research journal *Involve*. He serves on the Human Resources Board of the American Institute of Mathematics (AIM), whose goal is to foster diversity in the activities of AIM. He is also a member of the advisory board of Research Experiences for Undergraduate Faculty (REUF), an NSF-funded program for faculty who are interested in conducting research with underrepresented minority students, students with disabilities, and first-generation college students.

Garcia’s research began with complex analysis and  $H^p$  spaces and now includes, among several other topics, operator theory on Hilbert spaces. One of his objectives is to develop models for various classes of operators. In a series of highly cited papers published in *Transactions of the AMS* and the *Journal of Functional Analysis*, he and his coauthors pioneered the study of complex symmetric operators. Specifically, the theory behind linear transformations  $T$  that are “almost” self-adjoint by means of a conjugate-linear, isometric involution  $C$ ; that is,  $T = CT^*C$ . Thus, the conjugation  $C$  works to express an operator in terms of its adjoint. These almost self-adjoint operators are called complex symmetric operators. Many unexpected and highly non-normal operators have been shown to be complex symmetric, as have several classes of familiar operators. Garcia and his colleagues have developed a structure theory for this important (and large) class of operators. They are currently developing the machinery to connect truncated Toeplitz operators and complex symmetric operators. They conjecture that every complex symmetric operator on a Hilbert space can be concretely represented in terms of truncated Toeplitz operators.

Garcia has also made significant contributions to number theory. His work in number theory has been primarily

in four areas: geometric lattice theory, exponential sums, arithmetic quotient sets, and the behavior of the Euler totient near prime arguments. Exponential sums, such as Gauss sums, Kloosterman sums, Ramanujan sums, and others, are classical objects of study in analytic number theory. Garcia's novel approach was to view these sums from the standpoint of supercharacter theory. From this perspective, classical exponential sums can be viewed as orthogonal functions on certain abelian groups. Garcia and his co-authors (many of whom were undergraduate students) used this approach to visualize exponential sums, exhibiting some rather remarkable and visually stunning graphical features of these objects. An arithmetic quotient set is a set of fractions  $a/b$ , where  $a$  and  $b$  are elements of an infinite arithmetically defined set. Garcia and his co-authors explored the relationship between the arithmetic properties of a set and the analytic properties of its corresponding quotient set, for example its density in the positive reals or in  $p$ -adic completions of the field of rational numbers. Concerning the Euler totient, one striking recent result of Garcia, his student Elvis Kahoro, and Florian Luca (subject to the Bateman–Horn conjecture) is that for an overwhelming majority of twin prime pairs  $(p, p + 2)$ , the first prime  $p$  has more primitive roots than the second,  $p + 2$ . Moreover, this is reversed for a small positive proportion of the twin primes.

Again, in these rich and deep subject areas, Garcia has been able to involve undergraduates in this work.

### Biographical Note

Stephan Ramon Garcia is W. M. Keck Distinguished Service Professor and Professor of Mathematics at Pomona College. He earned his BA and PhD in mathematics from UC Berkeley and was a postdoc at UC Santa Barbara. Since 2006 he has been on the faculty of Pomona College. He was recently elected a Fellow of the AMS (2019).

He is the author of over eighty-nine research articles in operator theory, complex analysis, matrix analysis, number theory, discrete geometry, and other fields. Several dozen of these papers were co-authored with students, many of whom are from underrepresented groups in the mathematical sciences. Garcia has also written four books: *Introduction to Model Spaces and Their Operators* (with W. T. Ross and J. Mashreghi, Cambridge, 2016), *A Second Course in Linear Algebra* (with R. A. Horn, Cambridge, 2017), *Finite Blaschke Products and Their Connections* (with W. T. Ross and J. Mashreghi, Springer, 2018), and *100 Years of Math Milestones: The Pi Mu Epsilon Centennial Collection* (with S. J. Miller, AMS, forthcoming).

Garcia has received four NSF research grants as principal investigator and five teaching awards. He serves on the editorial boards of the *Notices of the American Mathematical Society* (2019–), *Proceedings of the American Mathematical Society* (2016–), *Involve* (2011–), the *American Mathemat-*

*ical Monthly* (2017–), and *Annals of Functional Analysis* (2013–). He has served on the Human Resources Board of the American Institute of Mathematics since 2008.

### Response from Stephan Ramon Garcia

I am deeply honored to receive the inaugural Mary P. Dolciani Prize for Excellence in Research. Thanks go to the American Mathematical Society and the Mary P. Dolciani Halloran Foundation for initiating this award. Although I am the first recipient of this prize, there are many vibrant researchers at non-PhD-granting institutions who are also worthy. I look forward to celebrating the achievements of future prizewinners in the years to come.

This would not have been possible without the advice and support of my many colleagues in the profession and the members of my department. I owe a great deal of thanks to those mathematicians who mentored me during my formative years. My advisor, Donald Sarason, and my postdoctoral mentor, Mihai Putinar, are due special consideration. I also thank my innumerable co-authors, from whom I learned a great deal, and my many research students throughout the years. Finally, I wish to thank my wife, Gizem Karaali, and our children, Reyhan and Altay, for their constant support and affection.

### About the Prize

The Mary P. Dolciani Prize for Excellence in Research is awarded by the AMS Council acting on the recommendation of a selection committee. The members of the committee to select the inaugural winner of the Mary P. Dolciani Prize were:

- Linda Chen,
- Pamela Gorkin (Chair),
- Jeremy T. Teitelbaum.

The AMS Mary P. Dolciani Prize for Excellence in Research recognizes a mathematician from a department that does not grant a PhD who has an active research program in mathematics and a distinguished record of scholarship. It is funded by a grant from the Mary P. Dolciani Halloran Foundation. Mary P. Dolciani Halloran (1923–1985) was a gifted mathematician, educator, and author. She devoted her life to developing excellence in mathematics education and was a leading author in the field of mathematical textbooks at the college and secondary school levels.

### Credits

Photo of Stephan Ramon Garcia by Gizem Karaali.