2019 Levi L. Conant Prize

ALEX WRIGHT was awarded the 2019 Levi L. Conant Prize at the 125th Annual Meeting of the AMS in Baltimore, Maryland, in January 2019.

Alex Wright

Citation

The 2019 Levi L. Conant Prize is awarded to Alex Wright for his article "From rational billiards to dynamics on moduli spaces," published in 2016 in the *Bulletin (New Series) of the American Mathematical Society*; **53** (2016), no.1, 41–56.

In only sixteen pages, the article gives a panoramic view of the theory of translation surfaces and of the recent breakthrough by Alex Eskin, Maryam Mirzakhani, and Amir Mohammadi on the structure of the orbit closure of a translation surface. Wright's account combines brevity with clarity. It is a considerable feat: this active and highly technical research area comprises the work of many. The article gives nonspecialists a good entry point and a guide to further reading.

The article starts with motivation from billiards inside planar polygons. The billiard dynamical system describes the motion of a particle in a domain, subject to specular reflections off the boundary. Many mechanical systems with elastic collisions, that is, collisions in which the energy and momentum are preserved, are described as billiard systems. Little is known about billiards in general polygons (for example, we still do not know whether every obtuse triangle has a periodic billiard trajectory!); the situation is considerably better understood when the angles of the polygon are π-rational, because of their relation to translation surfaces. A translation surface is a surface that is presented as a finite collection of planar polygons, glued together along pairings of parallel edges. Reflected copies of rational polygons are special examples of translation surfaces.

Through ample figures and examples, Wright gives a simple definition of translation surfaces and their moduli space, clearly explains the relation to rational billiards, and describes an action of the general linear group GL(2, R) on the moduli space. He provides a brief survey of seminal work by Kerckhoff, Masur, Smillie, and Veech (in the 1980–1990s), including the surprising result by Veech that billiards in a regular polygon share a familiar property with billiards in a square: in countably many directions, every billiard trajectory is periodic, but in every other direction, trajectories are equidistributed.

The second half of the article is devoted to the recent breakthrough by Eskin, Mirzakhani, and Mohammadi: *the closure of the* GL(2, **R**) *orbit of a translation surface is always a manifold, defined locally by linear equations in* (*the standard*) *period coordinates*.

Wright outlines the proof and describes the relation of this theorem to other fundamental results, such as Ratner's orbit closure theorem and the high and low entropy methods of Einsiedler, Lindenstrauss, and Katok in homogeneous space dynamics. Wright also describes an intimate connection between moduli of translation surfaces and Teichmüller theory.

Several applications of the theorem are presented. For example, given a polygon and two points x and y inside it, the illumination problem asks whether there exists a billiard trajectory in the polygon from x to y. Recently, Lelièvre, Monteil, and Weiss proved that if the polygon is rational, then for every x there are at most finitely many y not illuminated by x; this work relies heavily on the theorem of Eskin, Mirzakhani, and Mohammadi.

Over the years, a number of surveys of the theory of translation surfaces and related topics have appeared, from lengthy and detailed ones to short overviews of the subject. Wright's article is based on his talk in the *Current Events Bulletin* at the Joint Mathematics Meetings in January of 2015. It is a tribute to the work of Maryam Mirzakhani, who passed away in 2017.

Biographical Note

Alex Wright received his BMath at the University of Waterloo in 2008 and his PhD at the University of Chicago in 2014. He was then awarded a five-year Clay Research Fellowship, which he held primarily at Stanford University. He is now at the University of Michigan. His research interests include Teichmüller theory, geometry, and dynamical systems, including special families of algebraic curves that arise in this context. In 2018 he received the Michael Brin Dynamical Systems Prize for Young Mathematicians.

Response from Alex Wright

I'm honored to receive this recognition for my expository article on the breakthrough work of Eskin, Mirzakhani, and Mohammadi. This work lies in Teichmüller dynamics, and yet it has remarkable connections to toy models in physics, other dynamical systems, ergodic theory on homogeneous spaces, and special families of algebraic curves. I am especially thankful to Alex Eskin and Maryam Mirzakhani for teaching me so much about the field. I'm also grateful to David Eisenbud for inviting me to speak on this topic at the *Current Events Bulletin*, and to Susan Friedlander for encouraging me to publish an article based on that talk.

About the Prize

The Levi L. Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. For the 2019 prize, the selection committee consisted of the following individuals:

- Thomas C. Hales (Chair),
- Izabella Joanna Laba,
- Serge L. Tabachnikov.

The Levi L. Conant Prize is awarded annually to recognize an outstanding expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years.

Established in 2001, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic Institute. The prize carries a cash award of US$1,000.

A list of previous recipients of the Levi L. Conant Prize may be found on the AMS website at `https://www.ams.org/profession/prizes-awards/pabrowse?purl=conant-prize`.

Credits

Photo of Alex Wright is courtesy of Alex Wright.