2019 David P. Robbins Prize

Roger Behrend, Ilse Fischer, and Matjaž Konvalinka were awarded the 2019 David P. Robbins Prize at the 125th Annual Meeting of the AMS in Baltimore, Maryland, in January 2019.


Roger Behrend


Ilse Fischer


Matjaž Konvalinka

Citation

The David P. Robbins Prize is awarded to Roger Behrend, Ilse Fischer, and Matjaž Konvalinka for the paper "Diagonally and antidiagonally symmetric alternating sign matrices of odd order," published in 2017 in *Advances in Mathematics*.

In this work, Behrend, Fischer, and Konvalinka prove, after more than thirty years, the conjectured formula for the number of odd-order diagonally and antidiagonally symmetric alternating sign matrices, the last remaining of David Robbins's conjectures on alternating sign matrices.

An alternating sign matrix (ASM) is a square matrix in which every entry is 0, 1, or –1, and along each row and column the nonzero entries alternate in sign and have a sum of 1. They were introduced by David Robbins and Howard Rumsey in work on a certain generalization of the determinant where these matrices surfaced naturally. Robbins, in the mid-1980s, initiated a program of counting symmetry classes of ASMs of a given size and conjectured remarkably simple product formulae for most of these symmetry classes. The quote from his 1991 survey paper reads: "These conjectures are of such compelling simplicity that it is hard to understand how any mathematician can bear the pain of living without understanding why they are true."

All had been proven by 2006, with the exception of the conjecture for diagonally and antidiagonally symmetric ASMs, which had resisted proof until the present paper.

The Robbins conjectures have led to the development of new methods of enumeration, as well as to the discovery of deep connections to statistical physics. The first breakthrough came in 1996, with the proof by Doron Zeilberger that $n \times n$ ASMs are equinumerous with totally symmetric, self-complementary plane partitions in a $2n \times 2n \times 2n$ box, for which George Andrews had derived a simple product formula. In the same year, Greg Kuperberg made the connection to statistical physics by deriving the same ASM enumeration from the Izergin–Korepin determinant for a partition function for the six-vertex model on a square grid with domain wall boundary conditions. Kuperberg subsequently used this approach to enumerate three other symmetry classes of ASMs, and the enumeration by Roger Behrend, Ilse Fischer, and Matjaž Konvalinka builds on his work.

The main technical tool introduced by Kuperberg is a set of determinants and Pfaffian formulae for ASM partition functions, and it is these formulae that explain why the enumeration formulae are products of small factors. Behrend, Fischer, and Konvalinka arrive at a partition function with a compact formula by introducing vertex weights, depending on many parameters, into the model. Through computational experiments, they were able to guess the form of the partition function, which of course depends fundamentally on the choice of weights. To arrive at the compact formula, they took advantage of the observation by Soichi Okada, and by Alexander Razumov and Yuri Stroganov, that parti-

tion functions associated with ASM enumeration can often be written in terms of determinants which are associated with characters of irreducible representations of classical groups. In this case the partition function turns out to be a sum of two determinants, and each determinant reduces at some point to a Schur function.

The David P. Robbins Prize is awarded to a novel research paper in algebra, combinatorics, or discrete mathematics with a significant experimental component. This proof of the last remaining of David Robbins's conjectures on alternating sign matrices is a shining example. Not only is it a crowning achievement that makes use of deep methods developed by a community of researchers over more than twenty-five years, it is also a paper that makes new problems accessible.

Biographical Sketch: Roger Behrend

Roger Behrend was born in Melbourne, Australia. He studied mathematics and physics at the University of Melbourne and Imperial College London, receiving a PhD in mathematical physics from the University of Melbourne in 1997. Between 1997 and 2000, he held postdoctoral positions at the Physics Institute of the University of Bonn and the C. N. Yang Institute for Theoretical Physics at Stony Brook University. He has worked in the School of Mathematics at Cardiff University since 2001 and held a visiting position in the Faculty of Mathematics at the University of Vienna during 2017–2018. His research throughout the past decade has been in combinatorics. Much of his spare time is spent listening to classical music.

Response from Roger Behrend

I feel deeply honored to receive the David P. Robbins Prize together with my collaborators Ilse Fischer and Matjaž Konvalinka. It is fitting that in the research recognized by this award, we proved a conjecture of Robbins himself, and that this conjecture involved alternating sign matrices, which were first encountered by David Robbins and Howard Rumsey.

I believe that our construction of a proof of Robbins's conjecture for the number of odd-order diagonally and antidiagonally symmetric alternating sign matrices lies some distance from both the beginning and the end of the overall story of alternating sign matrices. Looking back, the proof depended on a significant body of earlier work, including that of Mills, Robbins, Rumsey, Izergin, Korepin, Zeilberger, Kuperberg, Okada, Razumov, and Stroganov. Looking forward, there remain many intriguing mysteries still to be resolved. As an important example, bijective proofs are currently lacking for known equalities between numbers of alternating sign matrices and numbers of certain plane partitions.

I am thankful to my wife Rachael and to my colleagues, family, and friends for their support throughout my exploration of the fascinating world of alternating sign matrices.

Biographical Sketch: Ilse Fischer

Ilse Fischer received her doctoral degree in 2000 from the University of Vienna under the direction of Christian Krattenthaler. After some years as a postdoctoral researcher at the University of Klagenfurt, she returned to a faculty position at the University of Vienna in 2004. In 2009 she was awarded the START prize of the Austrian Federal Ministry for Science, the most prestigious award for young researchers in Austria, and a 1.1 million € research grant endowment. In 2017 she was promoted to full professor. Her research is devoted to enumerative and algebraic combinatorics, and its connections to statistical physics and other fields.

Response from Ilse Fischer

The idea of working on Robbins's last open conjecture on alternating sign matrices slowly manifested in my mind as I was writing a grant proposal about ten years ago, when I identified it as an ultimate, albeit unrealistic, goal. In the beginning I hardly dared spend much time on it, but every now and then I discussed it with other combinatorialists. Roger Behrend and Matjaž Konvalinka were obviously among them, but I also had a particularly fruitful exchange with Arvind Ayyer back in 2012, which led us to several conjectures on the enumeration of extreme diagonally and antidiagonally symmetric alternating sign matrices of odd order. About three years later, Arvind, Roger, and I were able to prove these conjectures, and to some extent also this work paved the way for the eventual proof of Robbins's conjecture. I feel deeply honored and moved to now receive, together with Matjaž and Roger, the David P. Robbins Prize.

I would like to express my appreciation for the initiative to support mathematical research with an experimental component. Results discovered through experiment rather than intuition have the potential to be particularly surprising, and proving them can present a challenge because initially one may have no clue as to the reason why they are true. The area of enumerative combinatorics Robbins and several others originated serves as a good example: They introduced objects such as alternating sign matrices, plane partitions, and lozenge tilings, and while for most enumerations no explicit formula exists containing, say, only the basis arithmetic operations, certain enumerations of those objects are expressible by simple product formulas, which were usually discovered through computer experiments. Although all of Robbins's conjectures have now been proven, the proofs are complicated and we still lack thorough understanding just in what situations to expect a simple enumeration formula, nor are we able to explain phenomena such as the same enumeration formula ap-

pearing in the context of two very different combinatorial objects. Much of my past and current research has been driven by these questions.

Biographical Sketch: Matjaž Konvalinka

Matjaž Konvalinka was born in Ljubljana, Slovenia. He obtained his bachelor's and master's degrees at the University of Ljubljana, and his PhD at the Massachusetts Institute of Technology in 2008 under Igor Pak. He held a postdoctoral position at Vanderbilt University until 2010, and has been a professor at the Faculty of Mathematics and Physics, University of Ljubljana, since then. In 2012, he received a University award for excellent teaching and research. He mostly works in enumerative and algebraic combinatorics, and particularly enjoys bijective proofs, Schur functions, and tableaux combinatorics.

Response from Matjaž Konvalinka

I am deeply honored to be one of the recipients of the AMS David P. Robbins Prize. One of the reasons I love combinatorics is that many of its problems can be explained to a child, even when they are fiendishly hard to solve, and they inspire deep new tools and theorems. Problems involving alternating sign matrices are a prime example of this. Combinatorialists will forever be grateful to David Robbins and his coauthors for introducing them to the community and for the conjectures related to their enumeration.

I owe a debt of gratitude to many people. First and foremost I have to thank Ilse and Roger, my coauthors, both amazing mathematicians and people. They are truly worthy recipients of this prize. I am also deeply grateful to Marko Petkovšek for my first combinatorics courses; to my PhD advisor Igor Pak for everything he taught me and for always knowing what problems I will like; to Richard Stanley for his wonderful lectures, papers, and books; and to Sara Billey for being the best collaborator and friend one could imagine. My colleagues and students at the University of Ljubljana are a big part of why I enjoy my job. Many thanks also go to my husband Danijel and our daughter Ana, to the rest of my family, and to my friends, not least for seeming less surprised by this prize than I am.

About the Prize

The David P. Robbins Prize was established in 2005 in memory of David P. Robbins by members of his family. Robbins, who died in 2003, received his PhD in 1970 from the Massachusetts Institute of Technology. He was a longtime member of the Institute for Defense Analysis Center for Communications Research and a prolific mathematician whose work (much of it classified) was in discrete mathematics. The prize is given for a paper published during the preceding six calendar years that (1) reports on novel research in algebra, combinatorics, or discrete mathematics, (2) has a significant experimental component, (3) is on a topic broadly accessible, and (4) provides a simple statement of the problem and clear exposition of the work. The US$5,000 prize is awarded every three years.

The David P. Robbins Prize is awarded by the AMS Council acting on the recommendation of a selection committee. The members of the 2016 David P. Robbins Prize Committee were:

- Nola Alon
- Robert Calderbank (Chair)
- Timothy Chow
- Sylve Corteel
- Avi Wigderson

A list of previous recipients of the David P. Robbins Prize can be found on the AMS website at: `http://www.ams.org/profession/prizes-awards/ams-prizes/robbins-prize`.

Credits

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