The cover design is based on imagery in Hunting for Foxes with Sheaves, page 661.
Nominate a mathematics program that aims to bring more persons from underrepresented groups into some portion of the mathematics pipeline starting at the undergraduate level, or retains them once in the pipeline.

The AMS Committee on the Profession will select one award recipient for 2020, who will receive $1,000 provided by the Mark Green and Kathryn Kert Green Fund for Inclusion and Diversity.

For details on the award criteria and the nomination process, and to learn about past winners of the award, see www.ams.org/make-a-diff-award.

Email questions to aed-mps@ams.org.

Deadline: September 15, 2019
The American Mathematical Society (AMS) is dedicated to advancing research and connecting the diverse global mathematical community through our publications, meetings and conferences, MathSciNet®, professional services, advocacy, and awareness programs.

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This position will oversee the AMS education portfolio, with a focus on undergraduate and graduate education in the mathematical sciences (including the preparation of students to enter graduate programs, the mentoring of students for success in graduate school, the preparation for careers both inside and outside of academia, and the promotion of diversity and inclusiveness in all mathematics education).

RESPONSIBILITIES:

- Advance the Society’s involvement in student preparation for, and success in, graduate programs leading to an advanced degree in the mathematical sciences, with a focus on underrepresented groups, including women.
- Provide leadership for AMS efforts that support education in the mathematical sciences.
- Contribute to advocacy work focusing on education, engaging in discussions with policymakers and organizations, such as the National Academies and the US Department of Education.
- Interact with academic departments at the undergraduate and graduate levels.
- Work closely with the AMS Committee on Education.

EXPERIENCE AND QUALIFICATIONS:

- An earned doctorate in the mathematical sciences.
- Academic and administrative experience, including familiarity with PhD programs in the mathematical sciences.

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Submit your application on MathJobs.Org. Applications must include a letter describing your experience and interest in the position, a curriculum vitae, and contact information for three references. Applications received by March 31, 2019, will receive full consideration.

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Direct specific and confidential inquiries about this position to Karen Saxe, Associate Executive Director for Government Relations (kxs@ams.org), or Catherine Roberts, Executive Director (exdir@ams.org).
Dear Members of the American Mathematical Society,

It has been three years since my election as AMS Vice President, and my term is now ending. I was honored three years ago to win an election with a voting population as diverse as that of the AMS membership. I have been an AMS member since I was a graduate student in the early 1970s, and I have served on a number of AMS committees and attended many AMS meetings over the years. Being a VP has given me the opportunity to see the AMS in operation and to appreciate more of what it does. Since my term is now over I can pretend to be an expert and offer some reflections on the Society and the profession.

Perhaps the main thing that I came to appreciate about the AMS is the importance and efficiency of the full-time staff in Providence. They are the people who make sure things get done and that the budgets are balanced. As VP, I was a member of the Council and in a supporting role to the President who had primary responsibility to represent the Society.

Overall, being VP was a very enjoyable and informative job for me.

Like many mathematicians I spent the first 25 years of my career immersed in my research and teaching without much retrospection. Now that I am more senior I can look back over the profession and how it has evolved in the past 40 years. On the positive side, I believe that today's finishing PhDs who go into academia are far more prepared for their teaching duties than we were in the past. Also on the positive side there are now many more careers that are available to a mathematics PhD than there were in the early 1970s when I was a PhD student. In those days if you did not get an academic job you ended up doing something that did not use your mathematical training and skills. Now, in addition to academics, there are industries such as the financial industry and the computer and internet industries that employ highly trained mathematicians. This is a good thing for the discipline and for our students. It is also an indication that mathematics is a less isolated discipline than it was in the past. On the other hand, there is a difference between developing new mathematics and applying mathematical techniques in an application area. There is a danger that only the latter will be rewarded and that mathematics will become a service discipline. I think it is important that mathematics departments dictate the standard for the field and maintain high standards in both pure and applied fields.

These are a few of my thoughts and concerns at the end of my term as VP. It has been my pleasure to serve as an AMS Vice President for these past three years, and I am grateful for having had the opportunity to do it.

Sincerely Yours,

Richard Schoen
Excellence in Teaching Chair of Mathematics, UC, Irvine
Outgoing Vice President of the American Mathematical Society
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  Michael Robinson

- Topological Time Series Analysis
  
  Jose A. Perea

- WHAT IS...a Coleman Integral?
  
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Additions to Browder Biography in December 2018 Notices

To the Editors:

We write to correct the biographical sketch in "Felix Browder (1927–2016)" in Vol. 65, No. 11, of the Notices. The authors, in recounting "The Difficult Years," omit all mention of his career between his army service (ending 1955) and his appointment at the University of Chicago. The senior author of this letter was his colleague at Yale from 1956, and three of his first four graduate students, including the junior author, were his advisees there 1961–64.

Much credit for his (tenured) appointment at Yale in 1956 is due to G. A. Hedlund, Chair of the Department, and to A. Whitney Griswold, President of the University, who persisted in persuading members of the Yale Corporation of the merits of his appointment. While at Yale, Felix was very successful in research, in advising graduate students, and in promoting the status of the Department. He and the senior author convinced the Yale administration to introduce a prestigious postdoctoral program, the Gibbs Instructorships, that yielded immediate dividends in young mathematicians who have risen to distinction.

—George B. Seligman
—Richard Beals
Hamden, Connecticut

(Received December 20, 2018)

On “Looking at the Mathematical Literature” by Edward Dunne, Notices, February 2019

Dr. Dunne’s article gives a much needed explanation of the inner workings of MathSciNet. He makes it clear that the only articles that are reviewed are those published in Mathematical Journals or mathematical book chapters that are peer reviewed. The database that MathSciNet uses is gleaned from the included articles. Thus, in the information on citations, there is another reason why the numbers in, say Google Scholar and MathSciNet don’t match. A mathematics article that is cited in e.g. Physics Review Letters will not show up in the MathSciNet citations database but it will in that of Google Scholar. Similarly, mathematics published in a professional journal without mathematics in the title is not included in MathSciNet.

I am not advocating any changes in the policies of MathSciNet since it is an amazing, unique resource for mathematicians.

—Nolan R. Wallach
Professor Emeritus UCSD

(Received March 1, 2019)
Regarding the new eligibility criteria for Simons Collaboration Grants

I read with great interest Dr Randrianantoanina’s letter entitled “Regarding the new eligibility criteria for Simons Collaboration Grants” that appeared in the in the Notices of the AMS, Volume 66, Number 1. For the benefit of the readers of the Notices, I would like to share my own experience with the Simons Foundation, as it sheds some light on the current situation and raises some interesting questions.

On January 2018, just a few months after the new rules were implemented that exclude applicants from non-PhD granting departments, I applied for a Simons Collaboration Grant. As it turns out both the Office of Sponsored Programs here at East Carolina University and I missed this particular stipulation; my home department does not host a PhD program and therefore I was ineligible to apply. Nevertheless my application was processed by the Foundation and it was deemed successful as it was recommended for funding on May 24, 2018. (Award Number: 579144). It was a few months later that the mistake was caught when the contract letter arrived with all the stipulations clearly stated. (Actually the mistake was caught by a colleague of mine here at East Carolina University who was also awarded a grant.) Subsequently, my award was cancelled (and so was my colleague’s award).

In my subsequent communications with the Simons Foundation, I inquired about the nature of the exclusion. Dr. Elizabeth Roy, Senior Program Manager for the Division of Mathematics at Simons Foundation, offered the following explanation regarding the new rules:

“Our aim was not to be discriminatory, nor do we view primarily undergraduate-serving institutions as unworthy of consideration. Rather, it was an attempt to refine a program that has become increasingly popular and difficult to administer and review due to the large volume of applications we receive. Last year, we received 665 applications for a budget of 140 awards and this year, even with the eligibility change, we received almost 600.”

Like Dr Randrianantoanina, I am grateful for all the impactful support that the Simons Foundation provides to the mathematical community and I fully understand that the Foundation can do as it pleases when it comes distribution of funds. However I cannot help but wonder what happened to the funds that were supposed to support my award. Were the funds distributed to a less competitive proposal that originally did not make the cut? Have we reached the point where the ease of administering a program has become more important than the quality of the applications it is supposed to fund?

—Elias Katsoulis, Professor
Department of Mathematics, East Carolina University
(Received February 20, 2019)
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Hunting for Foxes with Sheaves

Michael Robinson

Michael Robinson is an Associate Professor in the Department of Mathematics and Statistics at American University. His email address is michaelr@american.edu.

All figures can be reproduced using The Jupyter Notebook at: https://github.com/kb1dds/foxsheaf.

Communicated by Notices Associate Editor Emilie Purvine.

Introduction

To a radio amateur (or “ham”), fox hunting has nothing to do with animals. It is a sport in which individuals race...
each other to locate a hidden radio transmitter on a known frequency. Since hams are encouraged to design and build their own equipment, the typical fox hunt involves a variety of different receivers and antennas with different capabilities. Some of these can display the received signal strength from the hidden transmitter (loosely measuring distance to the transmitter), while others estimate the compass bearing. Both of these estimates vary in accuracy and in precision depending on terrain, environmental conditions, equipment quality, and the skill of the operator.

Fox hunting also serves the purpose of preparing radio amateurs for emergency or disaster operations. Because disaster operations require the concerted efforts of multiple radio operators, it seems fitting to explore how the sport changes if fox hunting becomes cooperative. When participants combine their estimates of distance and bearing, how much faster can they find the transmitter? Part of the challenge of fox hunting is that measurements are taken infrequently, only once every few minutes. To win the hunt, every minute must count! In the most demanding scenario, each sensor only gets to take one measurement of the fox transmitter.

Locating the fox transmitter from a collection of different sensors is a model-based data fusion problem: combining disparate local observations into a global inference. Without a model that describes how signals from the transmitter arrive at each receiver, the signal reports are not helpful for locating the transmitter. Even with such a model, the effects of terrain, the transmitter’s antenna system, and the environment can cause substantial differences between the modeled signal and an actual received signal. Therefore, it is important that we remain even-handed about assumptions of the quality of the estimates and the quality of the model.

Though there are many techniques for solving data fusion problems, they broadly fall into two categories: (1) problem-specific deterministic methods, and (2) general statistical methods. Well-crafted problem-specific deterministic methods are very effective. Because of the physics of radio propagation, it is not too difficult to construct a deterministic method specifically for locating a fox transmitter. However, problem-specific methods often carry hidden assumptions that make it hard to transfer useful techniques to another problem. Worse, the bookkeeping associated with all combinations of sensors grows exponentially as more sensors are deployed. Regardless of their attraction, problem-specific methods for solving data fusion problems are costly and difficult to manage. Statistical methods automate the bookkeeping and tailoring needed for a problem-specific method, but they usually require many observations to produce accurate results. Since we are only using one measurement from each sensor, statistical methods are not the best option.

We are left with the need for a general deterministic method for finding the fox from a small number of measurements. This article explains how to meet this need using sheaves, mathematical objects that describe local consistency within data. We can perform data fusion for any sheaf, though the fox hunting problem will guide our selection of the specific sheaf we need and will be the context for its interpretation. The resulting fox hunting sheaf is modular; different sensors or models of their performance can be substituted easily without changing how their data are analyzed.

This article explains how to model a collection of sensors in the section called “Formalizing the Sensors in the Fox Hunt” so that they can be combined into a sheaf model in “Formalizing the Interactions between Sensors.” Once the sheaf is constructed, we show how to locate the fox transmitter in “Consistency Radius: Where is the Fox?” and determine if there are actually multiple fox transmitters in “Local Consistency Radius: Finding Multiple Foxes.” Finally, since applied sheaf theory is still in its infancy, “Frontiers” points the reader to some interesting directions for future study.

Formalizing the Sensors in the Fox Hunt

Each receiver (or sensor) \( A \) used in the fox hunt produces a signal report concerning its observation of the fox transmitter. Signal reports may be of different types, depending on the sensor. For instance, the strength of the received signal is typically reported as a single real number. In contrast, a compass bearing is reported as an angle, properly an element of the metric space \( S^1 \)—the unit circle. To handle both of these situations (and more), let us suppose that a signal report is an element of a pseudometric space \( D_A \), depending on the sensor \( A \). The sensor \( A \) produces reports through a continuous measurement function

\[
M_A : \mathbb{R}^2 \times C_F \times \mathbb{R}^2 \times C_A \rightarrow D_A,
\]

depending on fox transmitter location (in the plane), the fox transmitter equipment settings \( C_F \) (such as transmitter power and antenna orientation), the receiver location in the plane, and the receiver equipment settings \( C_A \) (such as antenna orientation).

Our sensor data will be drawn from a parameterized distribution \( S_A \), in which the noise level \( \sigma \) is taken as a parameter. To ensure consistency between the deterministic model and the stochastic one, our stochastic models satisfy

\[
S_A(x, y, c_f, x', y', c_a; \sigma) \rightarrow \delta_{M_A(x, y, c_f, x', y', c_a)} \text{ as } \sigma \rightarrow 0,
\]
in which \( \delta_{\alpha} \) is the unit impulse at \( \alpha \).

There are typically two kinds of sensors that are used in radio fox hunting: calibrated signal strength meters and directional antennas. Given knowledge of the fox transmitter’s power output, a calibrated signal strength meter
can help estimate the distance from the receiver to the fox. A directional antenna tells the operator the direction from which the fox’s signal appears to be the strongest.

Calibrated signal strength (RSSI) sensors. A received signal strength indication (RSSI) sensor measures the amount of power absorbed by its antenna from the fox transmitter. To model this accurately requires careful specification of the terrain and any obstacles between the transmitter and receiver. The Longley-Rice model is popular among engineers because it incorporates the effect of terrain and atmospheric losses on the received power, and its predictions are realistic [8]. Figure 1(a) shows the received signal power predicted by the Longley-Rice model for a fox transmitter placed on a mountain.

While the realism of the Longley-Rice model is a benefit, its precise specification is quite complicated. None of this complexity is necessary to demonstrate our approach, because all our analyses are modular. A different measurement function can be substituted later if desired without changing the analysis techniques. Therefore, we will simply model the power transferred from transmitter to receiver by an inverse-square law (Figure 1(b)). This corresponds to a measurement function in which there is only one transmitter power $p \in [0, \infty)$ and no receiver configuration,

$$M_{RSSI}(x, y, p, x', y') = \frac{p}{4\pi ((x-x')^2 + (y-y')^2)} \tag{1}$$

where $(x, y)$ is the transmitter location and $(x', y')$ is the receiver location.

From a statistical perspective, after the inverse square law, the next largest effect on the received signal is caused by self-interference as the signal travels along nearby paths with slightly different lengths. This effect is called Rician fading and is governed by the Rice distribution for signal amplitude (usually voltage) $V \in [0, \infty)$

$$Rice(V; \nu, \sigma) = \frac{V}{\sigma^2} \exp \left( -\frac{(V^2 + \nu^2)}{2\sigma^2} \right) I_0 \left( \frac{\nu V}{\sigma^2} \right)$$

where $I_0$ is the modified Bessel function of the first kind. The Rice distribution models the distance between the origin and a point in the plane drawn from a bivariate Gaussian with mean $\nu$ and standard deviation $\sigma$. Assuming the received signal is a combination of the inverse square law and Rician fading, the RSSI signal reports $P$ will follow the distribution

$$S_{RSSI}(P; x, y, p, x', y') = \frac{1}{2R} \left( Rice \left( P; \sqrt{2M_{RSSI}(x, y, p, x', y')}, \sigma \right) \right)^2 \tag{2}$$

in which antenna characteristic impedance $R$ (typically 50Ω for amateur equipment) and root mean square noise voltage $\sigma$ are assumed to be known in advance and constant.

Bearing sensors. A bearing sensor measures the angle between true north (or some other convenient, global direction) and the apparent direction of arrival of signals from the fox transmitter. Most amateur bearing sensors consist of an antenna (like the one shown in Figure 2) that is preferentially sensitive to signals arriving from a specific direction. The operator rotates the antenna until the signal strength is greatest, and then records its direction. For strong signals, the operator can also block incoming signals by holding a hand held radio against the chest. When the operator slowly turns around until the signal strength is minimized, the transmitter is then directly behind the operator!

A useful antenna for fox hunting produces a response like one of the two shown in Figure 3 (p.664). It is a continuous, unimodal function on the circle $S^1 \rightarrow \mathbb{R}$, since such a function has a single maximum (or minimum) in the direction of the fox transmitter (or in the exact opposite direction). A key performance criterion for an antenna is its beamwidth, the length of the interval for which the received signal exceeds half of the maximum value. In the presence of noise, an antenna with a small beamwidth will give more accurate bearing readings.
Table 1. Independent sensors for the fox hunting examples

<table>
<thead>
<tr>
<th>Sensor ID</th>
<th>Location</th>
<th>Type</th>
<th>Measurement</th>
<th>Noise model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1.0, 0.0)</td>
<td>Position</td>
<td>$\mathbb{R}^2$</td>
<td>(None)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bearing</td>
<td>$S^1$</td>
<td>von Mises</td>
</tr>
<tr>
<td>2</td>
<td>(1.0, 1.0)</td>
<td>Position</td>
<td>$\mathbb{R}^2$</td>
<td>(None)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Bearing</td>
<td>$S^1$</td>
<td>von Mises</td>
</tr>
<tr>
<td>3</td>
<td>(0.0, 0.0)</td>
<td>Position</td>
<td>$\mathbb{R}^2$</td>
<td>(None)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Signal strength</td>
<td>$\mathbb{R}$</td>
<td>Rician</td>
</tr>
<tr>
<td>4</td>
<td>(0.0, 0.5)</td>
<td>Position</td>
<td>$\mathbb{R}^2$</td>
<td>(None)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Signal strength</td>
<td>$\mathbb{R}$</td>
<td>Rician</td>
</tr>
</tbody>
</table>

We will model the observations recorded by a bearing sensor stochastically by way of the von Mises distribution, the probability distribution that is the closest analogue of the Gaussian distribution for the circle [9]. The von Mises distribution is given by

$$
VonMises(\theta; \mu, \kappa) = \frac{e^{\kappa \cos(\theta - \mu)}}{2\pi I_0(\kappa)}, \text{ for } \theta \in S^1
$$

where again $I_0$ is the modified Bessel function of the first kind. By analogy with the Gaussian distribution, $\mu$ in the von Mises distribution plays the role of the mean, while $1/\kappa$ is analogous to the variance. In order to model the measured bearing $\theta$ for a fox transmitter located at the true bearing $\mu$ from an antenna with beamwidth $B$, we set $\kappa = 4/B^2$.

Figure 3. Simulated received signal strength as a function of angle for typical antennas used in fox hunting. The solid red curve is typical for a directional antenna like the one shown in Figure 2, while the dashed blue curve is typical for a handheld receiver held against the body.

Given a fox transmitter located at $(x, y)$ and sensor located at $(x', y')$, the true bearing (in degrees) can be obtained by the measurement function

$$
M_{bearing}(x, y, x', y') = \begin{cases} 
180 & \text{if } y - y' > 0 \\
\frac{\pi}{180} \tan^{-1} \left( \frac{x-x'}{y-y'} \right) - 180 & \text{if } y - y' < 0 \text{ and } x - x' \geq 0 \\
\frac{\pi}{180} \tan^{-1} \left( \frac{x-x'}{y-y'} \right) + 180 & \text{if } y - y' < 0 \text{ and } x - x' < 0 \\
90 & \text{if } y - y' = 0 \text{ and } x - x' > 0 \\
-90 & \text{if } y - y' = 0 \text{ and } x - x' < 0 
\end{cases}
$$

where true north is oriented along the positive $y$-axis and angles are measured clockwise. This simple model ignores some important effects, such as the fact that the apparent bearing can be distorted by reflections, but it is a good model for relatively flat terrain. Then the appropriate stochastic model is

$$
S_{bearing}(\theta; x, y, x', y') = VonMises(\theta; M_{bearing}(x, y, x', y'), 4/B^2)
$$

for an antenna with a known, constant beamwidth $B$.

Coordinating multiple sensors. Consider a team of four radio operators each equipped with a position sensor (a GPS receiver) that records their own location and a sensor that measures either the received signal strength from or the bearing to the fox transmitter according to Table 1. Since GPS errors are small compared to those reported from amateur radio equipment, we will assume that GPS positions are known exactly. In all of the examples, we use the unit-less positions shown in the table rather than the latitude and longitude that would be reported by a GPS receiver.

Using this configuration of sensors, we will study five Cases (Table 2) that address two questions: (1) "Where is the fox?" and (2) "Are there multiple foxes?" We will start our analysis of the first question using Cases 1 and 2, before we demonstrate it with different realizations of stochastic noise in Case 3. Cases 4 and 5 will be used to address the second question.
Table 2. Fox hunting cases

<table>
<thead>
<tr>
<th>Case</th>
<th>Sensors</th>
<th>Number of foxes</th>
<th>Fox location known</th>
<th>Noise present</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>All</td>
<td>1</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>Sensors 1 &amp; 2</td>
<td>1</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>(bearings only)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>All</td>
<td>1</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>Sensors 1 &amp; 2</td>
<td>1</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>(bearings only)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>All</td>
<td>1</td>
<td>No</td>
<td>No</td>
</tr>
</tbody>
</table>

Formalizing the Interactions between Sensors

Neither of the measurement functions ($M_{RSSI}$ in (1) and $M_{bearing}$ in (3)) alone are sufficient to determine the location of the fox transmitter. It is only when multiple measurements from different locations or when different sensors’ measurements are taken together that the location of the fox may be determined. This joint interaction between sensors can be encoded as a sheaf of pseudometric spaces on a partial order, by taking account of deterministic, functional dependencies between sensors’ observations. Observations are of two types: (1) true observations: those actually made by sensors, and (2) virtual observations: those that could have been observed, but were not actually reported. The location and the transmitter power of the fox transmitter are in the latter class: the operator who hides the fox transmitter is legally required to observe its location and output power, but keeps these observations secret during the fox hunt!

It is reasonable to record position and a radio measurement simultaneously, so the typical measurement reported by Sensor 1 (a bearing sensor) will be of type $\mathbb{R}^2 \times S^1$.

This is a good strategy because there may be a statistical dependence between the position errors and the signal measurements. There are three observations that can be made: (1) the joint fox transmitter location and sensor location, (2) the fox transmitter’s location, and (3) the joint bearing sensor’s location and reported bearing. The first of these completely and functionally determines the other two, a situation that can be expressed as a partial order on the observations, which is shown in Figure 5(a) (p. 666). The joint fox transmitter location and sensor location (the minimal element in the partial order) is a virtual observation because we cannot make that joint observation without observing both simultaneously. The arrows in the diagram point from smaller to larger elements in the partial order, and express functional dependence. The spaces of observations and the actual functions themselves are shown in Figure 5(b), where $pr_k(x_1, x_2, \ldots) = x_k$ is the projection from a product onto its $k$-th factor. In much the same way, the relationship for an RSSI sensor is between a virtual observation (the fox transmitter location, power level, and sensor location, jointly) and two true observations is shown in Figure 6 (p. 666). Measurement functions play the role of transforming virtual observations into true observations; if a different measurement function is desired, it can be easily substituted without disrupting the partial order.

Combining all four sensors under the hypothesis that all of the fox transmitters are actually the same yields a somewhat larger collection of observations, shown in Table 3 along with their interpretations. They are named so that $B$ is for bearing sensors, $R$ is for RSSI sensors, and $F$ is for the fox.

These observations form a partial order, shown in Figure 7 (p. 666), obtained by “gluing together” several diagrams like those shown in Figures 5(a) and 6(a) along the common observations about the fox transmitter. The order relation $\leq$ is the transitive closure of the relation induced by the graph, so $R'_3 \leq F' \leq F$ for instance. This

Table 3. Observations used in the fox hunting examples

<table>
<thead>
<tr>
<th>Observation</th>
<th>Virtual</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F'$</td>
<td>Yes</td>
<td>Fox position, transmit power</td>
</tr>
<tr>
<td>$F$</td>
<td>Yes</td>
<td>Fox position</td>
</tr>
<tr>
<td>$B'_1$</td>
<td>Yes</td>
<td>Fox position, Sensor 1 position</td>
</tr>
<tr>
<td>$B_1$</td>
<td>No</td>
<td>Sensor 1 position and bearing</td>
</tr>
<tr>
<td>$B'_2$</td>
<td>Yes</td>
<td>Fox position, Sensor 2 position</td>
</tr>
<tr>
<td>$B_2$</td>
<td>No</td>
<td>Sensor 2 position and bearing</td>
</tr>
<tr>
<td>$R'_3$</td>
<td>Yes</td>
<td>Fox position, Sensor 3 position</td>
</tr>
<tr>
<td>$R_3$</td>
<td>No</td>
<td>Sensor 3 position and RSSI</td>
</tr>
<tr>
<td>$R'_4$</td>
<td>Yes</td>
<td>Fox position, Sensor 4 position</td>
</tr>
<tr>
<td>$R_4$</td>
<td>No</td>
<td>Sensor 4 position and RSSI</td>
</tr>
</tbody>
</table>

Figure 4. Spatial layout of the fox hunting examples
implies that the observation $R'_3$ functionally determines the observation $F$.

Figure 8 shows each observation’s space of values and specifies each of the measurement functions, which makes all of the functional dependencies between observations explicit. Figure 8 defines a bit more than a partial order; it determines a sheaf on the partial order.

Definition 1. [1] A sheaf $S$ on a partial order $(X, \leq)$ consists of the specification of:

1. A set $S(x)$ for each $x \in X$, called the stalk on $x$, and
2. A function $S(x \leq y) : S(x) \to S(y)$, called the restriction along $x \leq y$, for each $x \leq y \in X$, such that
3. Whenever $x \leq y \leq z \in X$, it follows that $S(x \leq z) = S(y \leq z) \circ S(x \leq y)$.

Notice that we need not specify the restriction along $R'_3 \leq F$ in Figure 7, for instance, since this is already completely determined by the composition of restrictions.
along $R_3' \leq F'$ and $F' \leq F$. If we have obtained the fox position and Sensor 3 position (observation $R_3'$), then we automatically know the fox position (observation $F$).

An important feature of a sheaf on a partial order is that the upward set $U_x$ for an element $x \in X$, given by

$$U_x = \{y \in X : x \leq y\}$$

is the set of observations $y \in U_x$ that are functionally determined by the observation $x$.

The collection of upward sets forms the base for a topology, called the Alexandrov topology $\text{Alex}(X, \leq)$. This suggests that we ought to assign measured quantities to open sections, not just individual observations.

Remark 1. The reader who is familiar with the usual definition of a sheaf on a topological space may notice that our definition of a sheaf $S$ on a partial order $(X, \leq)$ is merely a functor from the category generated by the partial order, and that the gluing axiom is apparently missing. The gluing axiom ensures that the value of $S$ on a typical open set $U$ is the space of sections for any open cover of $U$. Since we have only defined $S$ on the upward sets, not all open sets, we may use the gluing axiom to define sections and the rest of the sheaf accordingly.

Definition 2. Suppose that $U$ is an open set in $\text{Alex}(X, \leq)$. The set of sections on $U$ of a sheaf $S$ on $(X, \leq)$ is

$$S(U) = \left\{ s \in \prod_{x \in U} S(x) : s(y) = S(x \leq y) (s(x)) \right\},$$

for all $x \in U$ and $y \in U_x$,

namely the set of values from each stalk in $U$ that are consistent with all restrictions. A section on $X$ is called a global section.

It is immediate that $S(x)$ is in one-to-one correspondence with $S(U_x)$.

Case 1: All sensors, known fox, no noise. Suppose that the four sensors receive (with no noise) reports from a fox located at $(0.5, 0.5)$ transmitting with power level 1.0. This corresponds to the situation of a virtual observation $F'$ of $((0.5, 0.5), 1.0)$. Table 4 shows what all of the observations (both true and virtual) would be for this situation (positions of the sensors are from Figure 4). These observations form a global section $g$ of the sheaf $S$ shown in Figure 8. Although it is tedious to verify that $g$ really is a global section, it is enlightening to check a few restrictions. The virtual observation for the bearing Sensor 1 (observation $B_1'$) is consistent with the fox location (observation $F$) because

$$S\left(U_{B_1'} \subset U_F\right) g(B_1') = S\left(U_{B_1'} \subset U_F\right) ((0.5, 0.5), (1.0, 0.0)) = \text{pr}_1 ((0.5, 0.5), (1.0, 0.0)) = (0.5, 0.5) = g(F).$$

The observation at $B_1'$ is also consistent with the bearing Sensor’s true observation $B_1$ because

$$S\left(U_{B_1'} \subset U_{B_1}\right) g(B_1') = S\left(U_{B_1'} \subset U_{B_1}\right) ((0.5, 0.5), (1.0, 0.0)) = \left(\text{pr}_2 ((0.5, 0.5), (1.0, 0.0)), M_{\text{bearing}}(0.5, 0.5, 1.0, 0.0)\right) = ((1.0, 0.0), -45) = g(B_1').$$

The sheaf structure imposes constraints on sections over unions of overlapping upward sets. Observations $B_1'$ and $B_2'$ both stipulate a position for the fox transmitter in their first factor, but $F$ also stipulates a position for the fox transmitter. The model posited by a section over $U_{B_1'} \cup U_{B_2'}$ argues that there is only one transmitter, so each of these positions ought to be the same.

So while $S(B_1') = S(U_{B_1'}) = \mathbb{R}^4$ represents the fox position and Sensor 1 position, and $S(B_2') = S(U_{B_2'}) = \mathbb{R}^4$ represents the fox position and Sensor 2 position, their intersection $S(F) = S(U_F) = \mathbb{R}^2$ represents the fox position only. A section on the union

$$U_{B_1'} \cup U_{B_2'} = \{B_1', B_1, B_2, B_2, F\}$$

Table 4. Case 1: Known fox at location $(0.5, 0.5)$

<table>
<thead>
<tr>
<th>Observation</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^*$</td>
<td>Fox position, transmit power</td>
<td>$(0.5, 0.5), 1.0$</td>
</tr>
<tr>
<td>$F$</td>
<td>Fox position</td>
<td>$(0.5, 0.5)$</td>
</tr>
<tr>
<td>$B_1'$</td>
<td>Fox position, Sensor 1 position</td>
<td>$(0.5, 0.5), (1.0, 0.0)$</td>
</tr>
<tr>
<td>$B_1$</td>
<td>Sensor 1 position and bearing</td>
<td>$(1.0, 0.0), -45$</td>
</tr>
<tr>
<td>$B_2'$</td>
<td>Fox position, Sensor 2 position</td>
<td>$(0.5, 0.5), (1.0, 1.0)$</td>
</tr>
<tr>
<td>$B_2$</td>
<td>Sensor 2 position and bearing</td>
<td>$(1.0, 1.0), -135$</td>
</tr>
<tr>
<td>$R_3'$</td>
<td>Fox position and power, Sensor 3 position</td>
<td>$(0.5, 0.5), 1.0, (0.0, 0.0)$</td>
</tr>
<tr>
<td>$R_3$</td>
<td>Sensor 3 position and RSSI</td>
<td>$(0.0, 0.0), 0.16$</td>
</tr>
<tr>
<td>$R_4'$</td>
<td>Fox position and power, Sensor 4 position</td>
<td>$(0.5, 0.5), 1.0, (0.5, 1.0)$</td>
</tr>
<tr>
<td>$R_4$</td>
<td>Sensor 4 position and RSSI</td>
<td>$(0.5, 1.0), 0.32$</td>
</tr>
</tbody>
</table>
asserts that the fox positions between the two sensors are held in common, so that $S(U_{B_1} \cup U_{B_2}) = \mathbb{R}^6$ by the inclusion-exclusion principle.

Moreover, any global section is determined by its value on each of the minimal elements of the partial order in Figure 7. For this particular sheaf, specifying the fox position, fox transmitter power, and the positions of each of the four sensors completely determines a global section. Thus the set of global sections is $\mathbb{R}^{11} = \mathbb{R}^2 \times \mathbb{R} \times (\mathbb{R}^2)^4$.

**Consistency Radius: Where is the Fox?**

Sections on large open sets are rarely observed due to a variety of uncertainties about the model and the presence of stochastic noise. One cannot detect these uncertainties from a single observation; they are only apparent after comparing with other observations. This leads to the notion of an assignment, where the consistency described by restriction functions is not enforced.

**Definition 3.** [13]. For a sheaf $S$ on a partial order $(X, \leq)$ with the Alexandrov topology $\mathcal{T} = \text{Alex}(X, \leq)$, and a collection of open sets $\mathcal{U} \subseteq \mathcal{T}$, an assignment supported on $\mathcal{U}$ is an element of $\prod_{U \in \mathcal{U}} S(U)$.

If the space of sections $S(U)$ is a pseudometric space with pseudometric $d_U$ for each open $U \in \mathcal{U}$, the set of assignments supported on $\mathcal{U}$ has an assignment pseudometric for two assignments $a, b$ in $\prod_{U \in \mathcal{U}} S(U)$ given by

$$D(a, b) = \sqrt{\sum_{U \in \mathcal{U}} d_U ((a(U), b(U)))^2}.$$  

Suppose that $V \subseteq W$ are two open sets in $\mathcal{T} = \text{Alex}(X, \leq)$. This means that $\prod_{y \in V} S(y) \subseteq \prod_{x \in W} S(x)$, so that there is a natural projection $S(W) \to S(V)$ that merely discards the stalks that are in $W$ but not in $V$. In the case of upward sets $U_1 \subseteq U_2$, this projection is precisely the restriction $S(x \leq y)$. Without complicating our notation, we may unambiguously refer to the natural projection of sections $S(W) \to S(V)$ as a restriction $S(V \subseteq W)$.

**Definition 4.** For an assignment supported on all open sets, the consistency radius

$$c_S(a, \mathcal{T}) = \sqrt{\sum_{V \in \mathcal{T}} \sum_{U \in V} d_U ((S(U \subseteq V)) a(V), a(U))^2}$$

quantifies how far a given assignment is from being a global section.

Effective usage of sheaves in practice can hinge on careful weighting among the pseudometrics in Definition 4, but equal weights among the different pseudometrics in their natural units works well enough in our fox hunting examples.

Open question 1. How should one choose the weights for the assignment pseudometric and the consistency radius?

Implicit in both the assignment pseudometric and consistency radius is a choice of norm that aggregates multiple pseudometrics. Since different norms respond differently to outliers, what guides the selection of that norm?

The central relationship between global sections of $S$ and assignments is captured by the following bound, which interprets consistency radius as an obstruction to an assignment being a section.

**Proposition 1.** [13, Prop. 23] For an assignment $a$ to a sheaf $S$ of pseudometric spaces on $(X, \mathcal{T})$ in which each restriction map of $S$ is Lipschitz with constant $K$, then for every global section $s$ of $S(X)$,

$$c_S(a, \mathcal{T}) \leq (1 + K)D(a, s).$$

Strictly speaking, none of the restriction maps in this article are Lipschitz since their derivatives become unbounded when the fox and receiver approach each other. It is practical, however, to assume that the fox and the receivers are always separated by some minimum distance. This minimum distance establishes the Lipschitz constant $K$.

**Case 2:** Bearings only, unknown fox, no noise. To explore how consistency radius can help locate a fox transmitter, consider Sensor 1 and 2 in Figure 9(a) (p. 669), both of which are bearing sensors. Sensor 1 reports a bearing of $-45^\circ$ (corresponding to $B_1$ in Figure 9(b)) and Sensor 2 reports $-135^\circ$ (corresponding to $B_2$ in Figure 9(b)). Merely by intersecting sight lines from these sensors, we can infer that the fox is located at $(0.5, 0.5)$. The sheaf of observations also recovers this information and a little more besides. The two sensors correspond to five observations shown in Figure 9(b), and this collection of observations forms the sheaf shown in Figure 9(c) when associated to their measurement functions. As before, the fox position $F$ and both joint observations $B_1$ and $B_2$ are virtual observations, because we do not know their values.

We can encode the bearing reports as an assignment $a$ shown in Figure 10(a) (p. 669), where we note that the fox location can vary

$$a(U_F) = (x, y),$$

because we do not yet know its location. The true observations from the two sensors are given by

$$a(U_{B_1}) = ((1.0, 0.0), -45),$$

$$a(U_{B_2}) = ((1.0, 1.0), -135),$$

since we know both the sensor locations and the bearings. The two virtual observations at $B_1$ and $B_2$ are typically over constrained by these three facts because most points $(x, y)$ in the plane do not satisfy the simultaneous system

$$M_{\text{bearing}}(x, y, 1.0, 0.0) = -45,$$

$$M_{\text{bearing}}(x, y, 1.0, 1.0) = -135.$$
Figure 9. The setup for Case 2: (a) the spatial layout of the two bearing sensors and the fox whose location is unknown, (b) the partial order of the observations, and (c) the sheaf diagram.

Figure 10. The results of Case 2: (a) an assignment to the sheaf in Figure 9(c) with Sensor 1 receiving a bearing of $-45^\circ$ and Sensor 2 receiving a bearing of $-135^\circ$, (b) consistency radius as a function of fox transmitter position. Notice the prominent minimum where the bearing sight lines (marked with arrows) coincide.

Therefore, the consistency radius of the assignment $a$ is typically not zero. Figure 10(b) shows the consistency radius of $a$ as a function of the fox position $(x, y)$. Two conclusions can be drawn immediately from this plot: (1) the point of intersection $(0.5, 0.5)$ between the two sight lines minimizes the consistency radius (zero), (2) no other fox transmitter position will yield an assignment with consistency radius of zero. This strongly points to the conclusion that the fox transmitter is located at $(0.5, 0.5)$.

Optimally extending assignments. Case 2 above indicates that minimizing the consistency radius is a useful inference procedure. Instead of encoding the signal reports from the sensors as an assignment with values at both true and virtual observations, we really only need to consider values at the true observations. For instance, if we did not specify values at $B_1'$, $B_2'$, and $F$ in Figure 9(b), we could infer them from the values at $B_1$ and $B_2$. This warrants a more general definition of consistency radius.

Definition 5. The consistency radius of an assignment $a$ supported on $\mathcal{U}$—rather than all open sets—is the infimum of all consistency radii of assignments $b$ that restrict to $a$, namely

$$c_S(a, \mathcal{U}) = \inf \left\{ c_S(b, \mathcal{T}) : b \in \prod_{V \in \mathcal{T}} S(V) \right\}$$

such that $b(U) = a(U)$ whenever $U \in \mathcal{U}$. 

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We say that each such assignment $b$ extends $a$.

A nonzero consistency radius for an assignment that is supported on $\mathcal{U}$, rather than on all open sets, is still the obstruction to extending that assignment to a global section, as the next Proposition states.

Proposition 2. [14, Prop. 11] If $a$ is an assignment to a sheaf $S$ on a topological space $(X, \mathcal{T})$ supported on $\mathcal{T}$ (every open set) and $U \in \mathcal{T}$, then

$$c_S(a, \mathcal{T} \cap U) \geq d_{V_1}(S(V_1 \subseteq V_2)a(V_2), a(V_1))$$

for every $V_1 \subseteq V_2 \subseteq U$.

Case 3: All sensors, unknown fox, noise present. Consider the situation of trying to infer the location of a single unknown fox transmitter using all four sensors, in which the reports are contaminated with noise. Several simulations were run with different amounts of noise and different random draws to produce signal reports for each sensor. Each simulation produced four values, $(\theta_1, \theta_2, r_3, r_4)$ for the bearing from sensor 1, the bearing from sensor 2, RSSI from sensor 3, and RSSI from sensor 4, respectively.

These four sensors’ reports correspond to the open set $\{B_1, B_2, R_3, R_4\}$ in the partial order of the observations shown in Figure 11(a) (p. 671). The simulated reports were encoded as an assignment to the sheaf shown in Figure 11(b) supported on the open set $\{B_1, B_2, R_3, R_4\}$. Notice that the true observations—within the support of the assignment—are shown by an element of the stalk specifying the signal report, while the figure shows the entire stalk at each virtual observation.

Each simulation therefore corresponds to an assignment, for which we would like to compute consistency radius using Definition 5. This requires finding an extension of each assignment with minimal consistency radius. This extension was found using the “SLSQP” method of the popular SciPy optimizer scipy.optimize.minimize, which can solve general optimization problems. The default options for stopping conditions were sufficient in all our simulations, but more sophisticated optimizers or careful selection of parameters is sometimes required for other sheaves [15].

Figure 12 (p. 671) shows how consistency radius depends on stochastic noise present in simulated signal reports. In Figure 12(a), no noise is present in the reports from Sensor 3 and 4 (RSSI reports $r_3$ and $r_4$) but is present in the reports from Sensors 1 and 2 (bearing reports $\theta_1$ and $\theta_2$). The consistency radius generally increases steadily as the amount of noise in the bearing reports increases. Figure 12(b) shows the same situation but the bearings are noiseless, while noise is applied to the RSSI reports. The increase in consistency radius is somewhat more abrupt, but stabilizes for larger noise values.

The fox location appears in several different places in an assignment supported on all observations, namely $B_1'$, $B_2'$, $R_3'$, $R_4'$, and $F$. Since

$$U_F = \{F\} = U_{B_1'} \cap U_{B_2'} \cap U_{R_3'} \cap U_{R_4'} \cap U_F,$$

we deem that the predicted fox location is specified by the (virtual) observation at $F$.

Even though a given extension of an assignment may minimize consistency radius, it may not predict the correct location of the fox. Figure 13 (p. 672) shows the errors for the inferred location of the fox transmitter. The figure shows results from two ways to infer virtual observations when presented with a set of true observations as an assignment $a$ supported on $\mathcal{U}$: (1) extension: find the assignment extending $a$ to all open sets that minimizes consistency radius (green triangles), as given by Definition 5 or (2) fusion: find the nearest global section $s$ to $a$ in the assignment pseudometric (red dots) [13]. In both cases, the overall transmitter location error is largely independent of noise level, with angle errors being more detrimental. The green triangles in Figure 13 correspond to the same simulations shown in Figure 12, but the vertical scales are different: consistency radius is not the same as location error.

Open question 2. Solving the extension problem generally results in substantially better estimates of the fox transmitter location than solving the fusion problem, which is why we discuss it here. While this intriguing phenomenon has been observed in other settings [15], what are the precise conditions under which extension outperforms fusion?

Local Consistency Radius: Finding Multiple Foxes

The traditional amateur radio fox hunt involves only one fox transmitter. If there are actually multiple fox transmitters and the sensors are in general position, the minimum consistency radius of an assignment constructed from the signal reports will not be zero. Sheaves allow us to segment the observations into those of different foxes. This deduction can be fully justified if we can identify open sets of sensors whose reports are consistent, even when the collection of all sensors is not. The extent to which consistency is obtained on an open set is formalized by the following definition.

Definition 6. Let $a$ be an assignment to every open set in a sheaf $S$ of pseudometric spaces on $(X, \mathcal{T})$. For an open $U \in \mathcal{T}$, the local consistency radius on $U$ is

$$c_S(a, U) := \sqrt{\sum_{V_2 \subseteq U} \sum_{V_1 \subseteq V_2 \subseteq U} d_{V_1}((S(V_1 \subseteq V_2))a(V_2), a(V_1))^2}.$$

Using this, the $\epsilon$-consistent collection consists of every connected open set $U \in \mathcal{T}$, such that (1) $c_S(a, U) < \epsilon$ and
there is no other connected open set $V \in \mathcal{T}$ with $c_S(a, V) < \epsilon$ and $U \subset V$.

Usually, $\epsilon$-consistent collections only cover part of the space $X$. All observations $x \in X$ that are maximal in the partial order of observations will necessarily have $c_S(a, U_x) = 0$ for any assignment, so their upward sets are always included in any $\epsilon$-consistent collection for $\epsilon > 0$. Connectedness in Definition 6 is for ease of interpretation: two observations lying in different connected components of an open set $U$ are never tested for consistency when computing the local consistency radius on $U$.

**Lemma 1.** Local consistency radius is a monotonic function of the open set. Specifically, if $a$ is an assignment to a sheaf $S$ of pseudometric spaces and $U \subseteq V$ are open subsets of the base space, then

$$c_S(a, U) \leq c_S(a, V).$$

The Lemma follows from the fact that the sum in the expression for $c_S(a, V)$ is over a strictly larger set than the sum in the expression for $c_S(a, U)$.

A collection of open sets $\mathcal{V}$ is said to **refine** another collection of open sets $\mathcal{U}$ if every $V \in \mathcal{V}$ is contained in some $U \in \mathcal{U}$, which can be thought of—not uniquely—as a function $\mathcal{V} \to \mathcal{U}$. Therefore, if $\tau < \epsilon$, the $\tau$-consistent collection for an assignment refines its $\epsilon$-consistent collection.

**Definition 7.** For an assignment $a$ to every open set in a sheaf $S$ of pseudometric spaces on $(X, \mathcal{T})$, the **consistency filtration** $\mathcal{CF}_{(S,a)}$ is the set of all $\epsilon$-consistent collections for
all $\epsilon \in [0, \infty)$. We will use the notation $\text{CF}_{(S, a)}(\epsilon)$ to refer to a particular value of $\epsilon$.

Because of Lemma 1, the consistency filtration $\text{CF}_{(S, a)}$ is actually a sheaf on the partial order $(\mathbb{R}, \leq)$, in which the restrictions $\text{CF}_{(S, a)}(\tau < \epsilon)$ are refinement functions. As $\epsilon$ decreases, the open sets in $\text{CF}_{(S, a)}(\epsilon)$ become more refined.

Case 4: Bearings only, two unknown foxes, no noise. Let us consider the setting in which there are two bearing sensors, shown in Figure 14. The sheaf shown in Figure 14(b) assumes that there is one fox transmitter. The setup is thus far the same as in Case 2, but instead suppose that Sensor 1 reports a bearing of $-180^\circ$ and Sensor 2 reports a bearing of $0^\circ$. This can happen if there are two foxes, but cannot happen if there is only one fox. As before, we can encode the signal reports as an assignment $a$ shown in Figure 14(c), with

$$a(U_{B_1}) = ((1, 0), -180), \ a(U_{B_2}) = ((1, 1), 0).$$

Figure 14(d) shows the consistency radius of the assignment as a function of the fox location. The infimum consistency radius is strictly positive and is never attained, both of which indicate that there is a problem with the sheaf.
Both the maximality and connectedness requirements are important: \( \{F, B_1', B_1, B_2\} \) is consistent to a threshold of 100° but is not connected, and while \( \{B_1\} \) is sufficiently consistent and is connected, it is contained in a larger such set.

In Figure 15, \( \{B_2\} \) is farther from being absorbed into a larger consistent connected open set than any of \( \{F\}, \{B_1\} \), or \( \{F, B_1', B_1\} \), which suggests that its observation is incompatible with the others. The interpretation is that Sensor 2's report is harder to reconcile with the proposed fox transmitter location than Sensor 1’s report, because it is receiving a fox transmitter located somewhere else.

The consistency filtration is especially useful because it is robust to noise and other variations. If we weight the edges in the consistency filtration graph shown in Figure 15 with the difference in local consistency radius between their endpoints, we can interpret the differences between length of edges in the consistency filtration graph as the amount of noise (or other uncertainty) that we can accept and yet still detect the inconsistency between the signal reports. For instance, in order to reconcile \( B_1 \) with \( B_2 \) we must accept 135° of total error: the length from \( \{B_2\} \) to \( \{B_1', B_1, B_2, F\} \). The topology of the graph is unchanged when there is less than 45° of total error: the length from \( \{B_1', B_1, F\} \) to \( \{B_1', B_1, B_2, F\} \). These are guaranteed by the following theoretical result.

Theorem 1. [14, Thm. 3] For a sheaf of pseudometric spaces on a finite partial order, the consistency filtration is a continuous function of the assignment when the space of consistency filtrations is given the generalized interleaving distance [4, Def. 2.13] and the space of assignments is given the product topology.

The definition of generalized interleaving distance is rather technical, but the idea of the Theorem is straightforward. If we consider two different assignments \( a \) and \( b \) to a sheaf \( S \) such that their consistency filtrations \( CF_{(S,a)}(\epsilon) \) and \( CF_{(S,b)}(\phi(\epsilon)) \) are identical under an order preserving bijection \( \phi \), then the local consistency radius of any open set cannot change more than a constant factor times the distance between the two assignments. The size of \( \phi \) accounts for the difference in local consistency radii of the consistent collections, and also constrains the interleaving distance after some calculation.

Case 5: All sensors, two unknown foxes, no noise. As a final case, consider the situation in Table 5 where all four sensors are receiving noiseless signal reports, but where Sensor 3 (an RSSI sensor) receives signals from a fox located at \((0.5, 0.5)\) while the other Sensors receive signals from a fox located at \((0.5, 1.0)\). Both foxes use a transmit power of 1.0. If Sensor 3 had received the same fox as the others, it would report an RSSI observation of 1.0. If Sensor 3 had received the same fox as the others, it would report an RSSI observation of 1.0. If Sensor 3 had received the same fox as the others, it would report an RSSI observation of 1.0.

Let us use the same sheaf model as in Case 1 and Case 3 model. The problem is that there is no clearly “correct” location to be inferred for the single fox transmitter.

Without computing anything, observe that the local consistency radius of \( a \) on the open set \( \{B_1, B_2, F\} \) is always zero, because no comparisons between these observations are required. Each of the observations \( B_1, B_2, \) and \( F \) are perfectly self-consistent if one does not check for consistency between them! By choosing the fox location correctly at \( F \), one can easily ensure that \( c(a, U_{B_1'}) = 0 \) or \( c(a, U_{B_2'}) = 0 \), but one cannot make both zero at the same time. Given this information, Lemma 1 ensures that the global consistency radius, \( c(a, U_{B_1'} \cup U_{B_2'}) \) must be nonzero.

Definition 7 of the consistency filtration requires the assignment to be supported on every open set—since every open set is being tested for local consistency!—so let us suppose that the transmitter is located at the origin (marked with a star in Figure 14(d)). This can be encoded as the assignment shown in Figure 14(c) with \((x, y) = (0, 0)\). Figure 15 shows the consistency filtration of this assignment rendered as a directed graph, in which the vertices are labeled with open sets and their local consistency radius. The edges of this graph denote subset relations between open sets.

The consistency filtration begins at \( \epsilon = 0 \) with the three connected components \( \{B_1\}, \{B_2\}, \) and \( \{F\} \), since each of these are the largest connected open sets with consistency radius zero. \( B_1 \) and \( F \) become consistent above a threshold of 90° as both of these elements are subsumed into the open set \( \{B_1, B_1', F\} \). The \( \epsilon = 100\)-consistent collection, in which bearings are required to be closer than 100°, is

\[
\text{CF}_{(S,a)}(100) = \{\{B_1', B_1, F\}, \{B_2\}\}.
\]
Figure 16. Case 5: all sensors and two foxes (a) The data encoded as an assignment, constructed by extending an assignment on the shaded open set to the others by way of minimizing the overall consistency radius. (b) The resulting consistency filtration of this assignment.

Table 5. Case 5 signal reports

<table>
<thead>
<tr>
<th>Sensor ID</th>
<th>Type</th>
<th>Reported observation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Position</td>
<td>(1.0, 0.0)</td>
</tr>
<tr>
<td></td>
<td>Bearing</td>
<td>-26.5°</td>
</tr>
<tr>
<td>2</td>
<td>Position</td>
<td>(1.0, 1.0)</td>
</tr>
<tr>
<td></td>
<td>Bearing</td>
<td>-90°</td>
</tr>
<tr>
<td>3</td>
<td>Position</td>
<td>(0.0, 0.0)</td>
</tr>
<tr>
<td></td>
<td>RSSI</td>
<td>0.16</td>
</tr>
<tr>
<td>4</td>
<td>Position</td>
<td>(0.0, 0.5)</td>
</tr>
<tr>
<td></td>
<td>RSSI</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Theorem 1 is useful because it asserts that the consistency filtration changes continuously as the assignment changes, which itself varies continuously with the signal reports. The next largest open set to contain $R_3$ has a consistency radius of 0.08. All of the other true observations $B_1$, $B_2$, and $R_4$ are all contained in an open set with consistency radius 0.05. This means that Sensor 3’s report can be distorted by 0.03 before the topology of the consistency filtration graph changes. Tolerance to noise is typically expressed as a signal-to-noise ratio, with the typical fox hunting system providing the system maintains a signal-to-noise ratio of 0.03. Sensor 3’s report will be deemed inconsistent with the others provided the system maintains a signal-to-noise ratio of at least $0.08/0.03 = 5.3$, which will definitely be met by a typical fox hunting system. At such a signal-to-noise ratio, the value of $B_2$ will remain closest to the predicted fox location $F$ since the noise level would be not more than 0.02/5.3 = 0.004, which is smaller than the difference in local consistency radius for any of the other open sets.

Open question 3. How should consistency filtrations be analyzed in a systematic way? Certainly they can be used for segmentation of the observations, as is done above, but how can this be done most effectively?

Frontiers

Although sheaf theory got its start as an abstract tool, purposefully beyond any application, sheaves have a number of practical applications beyond fox hunting. The interested reader may enjoy reading how sheaves are useful in the theory of computation [3, 7], network coding [2], quantum graphs [11], systems of differential equations, graphical models [12], and signal processors [10]. One can also look to more general tools from category theory to combine observations, sensors, and decisions under very general conditions [5, 6].

The connections between statistics, topology, and sheaves have steadfastly resisted our attempts at complete understanding. While the simulations performed in this article used realistic noise models to produce signal reports, the sheaves that processed these reports were naïvely deterministic. Theoretical guarantees are available for small perturbations of the data, which provide a measure of robustness to statistical noise, but these appear to be of limited explanatory value.

The extension of an assignment supported on some open sets to the rest of the topology has the appearance of a statistical imputation problem. Should the consistency radius play the role of a decision statistic? If so, it seems that given the stochastic models, the sheaf should aid the specification of a maximum likelihood estimate of the fox transmitter. These connections are only just starting to be explored!

References


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Michael Robinson

Credits

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On Turbulence and Geometry
from Nash to Onsager

Camillo De Lellis, László Székelyhidi Jr.

Turbulence: a challenge for mathematicians. There is a huge literature on turbulent incompressible flows in applied mathematics, physics, and engineering. The outcome of such tremendous effort has been the derivation of several theories, which often allow quite accurate predictions of many phenomena. There is also a quite broad consensus on which fundamental partial differential equations (in short PDEs) describe with sufficient accuracy incompressible fluids, namely the Navier–Stokes and the Euler equations. Therefore, in a certain statistical or averaged sense, the predictions of the “theory of turbulence” should ultimately translate into mathematically verifiable claims about the behavior of solutions to the latter well-known PDEs. Indeed the working mathematician, even if not immediately confronted with clear-cut mathematical statements about turbulence (at least as we understand them in pure mathematics), will be nonetheless able to derive some precise mathematical consequences of the discussions right at the start of most textbooks about turbulent flows.

Yet, it seems very hard to prove any of these statements rigorously. Many valuable works in pure mathematics have shown the validity of some of the mechanisms identified
by applied mathematicians, physicists, and engineers. However these results are mostly confined either to models or to situations obeying some important (and as of yet unverifiable) a priori assumptions. In this note we want to report on a series of recent works which culminated in the complete verification of a famous statement in the theory of turbulence, not confined to some model or constrained by some special a priori assumption. These results have uncovered a surprising and interesting connection with a classical area of differential geometry and have also been used in other contexts.

K41 theory and the Onsager conjecture. Consider the incompressible Navier–Stokes equations

\[
\begin{cases}
    \partial_t \mathbf{v} + \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) + \nabla p - \mu \Delta \mathbf{v} = 0 \\
    \nabla \cdot \mathbf{v} = 0,
\end{cases}
\]

(1)

describing the motion of an incompressible homogeneous viscous fluid, where we set the density to be 1 for simplicity. The pair \((\mathbf{v}, p)\) consists, respectively, of a vector and a scalar function: \(\mathbf{v}(x, t)\) is the velocity of the fluid particle which occupies the point \(x\) at time \(t\), and \(p\) is the hydrodynamic pressure. Whilst acknowledging that most of the interesting hydrodynamic phenomena in reality happen in the presence of boundaries, in order to isolate a manageable mathematical situation we will assume in the rest of this note that the spatial domain is \(\mathbb{T}^3 = (\mathbb{R}/2\pi\mathbb{Z})^3\), the 3-dimensional flat torus with side-length \(2\pi\). In other words, we consider the problem (1) with periodic boundary conditions in the box \([0, 2\pi]^3\).

The coefficient \(\mu > 0\) is called the kinematic viscosity of the fluid. If the characteristic scale (which we have set to be unity) and the characteristic velocity of the flow are both fixed, then \(\mu\) is inversely proportional to the Reynolds number \(Re\), a quantity without physical dimensions, which is characteristic for the turbulent nature of the flow under consideration. A good intuition for the nature of turbulence is characteristic for the turbulent nature of the flow under consideration. A good intuition for the nature of turbulence is that in this situation the characteristic velocity is \(\mu\) and the Reynolds number is given by \(Re = \frac{\mu}{k_\mu}\), so that increasing \(\mu\) has the same effect as decreasing \(\mu\). This can be seen by a simple rescaling of time: given a solution \(\mathbf{v}(x, t)\) of (1) with the external force \(f\) above, set \(\mathbf{u}(x, t) = \mu^{-1}\mathbf{v}(x, \mu^{-1}t)\). Then \(\mathbf{u}\) is also a solution of the Navier–Stokes system on \(\mathbb{T}^3\), with kinematic viscosity equal to 1 and external force \(\mu^{-2}f(x) = Re \sin(x_1)e_2\). The simple stationary solution becomes \(\bar{u}(x) = \mu^{-1}\bar{v}(x) = Re \sin(x_1)e_2\), hence \(Re\) is the only parameter remaining.

Going back to the Navier–Stokes equations with no driving external force, elementary calculations show that for smooth solutions of (1) the total kinetic energy

\[ E(t) = \frac{1}{2} \int |\mathbf{v}(x, t)|^2 \, dx \]

satisfies the energy balance law

\[ \frac{d}{dt} E(t) = -\mu \int |\nabla \mathbf{v}|^2 \, dx. \]

Thus, at least formally, one would expect that as \(\mu \to 0\), the energy dissipation rate vanishes. This naive expectation is contradicted by observation, both physical and numerical: the dissipation rate remains finite and positive. This effect is called anomalous dissipation in the literature. Kolmogorov in the early 1940s pioneered the statistical theory of turbulent motions, assuming that generic flows can be seen as realizations of random fields. Kolmogorov’s theory postulates (cf. [36, Chapter 5]) that the energy dissipation is strictly positive and independent of the viscosity \(\mu\) when the latter goes to 0—in agreement with observation. The key insight is that anomalous dissipation arises from the fact that no matter how small the kinetic viscosity \(\mu\) is, there is a steady flow or cascade of energy from low to high frequencies, leading to large \(|\nabla \mathbf{v}|^2\). In the words of J. von Neumann [65], the decisive trait is that turbulence is not a matter of ergodic distribution of a fixed amount of energy, but the transport of a fixed flow of energy from sources in the low frequencies to sinks in the high frequencies in the Fourier-transform space. Assuming in addition local homogeneity and isotropy, Kolmogorov derived his famous \(k^{-5/3}\) law, expressing the mean distribution of kinetic energy density across an intermediate range of spatial frequencies \(k_0 < k < k_\mu\).

When \(\mu \downarrow 0\), (1) becomes formally the incompressible Euler equations

\[
\begin{cases}
    \partial_t \mathbf{v} + \nabla \cdot (\mathbf{v} \otimes \mathbf{v}) + \nabla p = 0, \\
    \nabla \cdot \mathbf{v} = 0.
\end{cases}
\]

(2)

The energy balance law implies that smooth solutions of (2) preserve the total kinetic energy.

Onsager suggested in his famous note [51] the possibility of anomalous dissipation for weak solutions of the Euler equations as a consequence of the energy cascade. Indeed, at least formally, as \(\mu \downarrow 0\), the inertial range of frequencies extends to infinity (namely \(k_\mu \to \infty\)), hence Kolmogorov’s \(k^{-5/3}\) law amounts to a certain regularity.
statement, when interpreted for single velocity fields rather than ensemble averages. This is exactly what Onsager proposed in 1949.\(^1\) It is important to emphasize that the theory of Kolmogorov is a statistical theory, dealing with random fields whose distribution laws need to satisfy several postulates, but even the mere existence of such random fields is not at all obvious in rigorous mathematical terms. In contrast, the suggestion of Onsager turned the problem into a “pure PDE” question that could be studied directly, and, after nearly 70 years, we can finally state the theorem:

**Theorem 1.** Let \((v, p)\) be a weak solution of \((2)\) with

\[
|v(x, t) - v(y, t)| \leq C|x - y|^\theta \quad \forall x, y, t
\]

(where \(C\) is a constant independent of \(x, y, t\)).

(a) If \(\theta > \frac{3}{2}\), then \(E(t)\) is necessarily constant;

(b) For \(\theta < \frac{3}{2}\) there are solutions for which \(E(t)\) is strictly decreasing.

In the historical context it is quite remarkable that Onsager was able to formulate a mathematically very precise statement. He gave, in particular, a rigorous definition of “weak solutions” expanding \((2)\) into an infinite system of ODEs for its Fourier coefficients,\(^2\) a point of view which coincides with the concept of “distributional solution” of the modern PDE literature. Nevertheless, it was only in the early 1990s that mathematicians took note of this statement as a mathematical conjecture, mostly as a result of Greg Eyink’s efforts in providing a modern account of Onsager’s unpublished work on the topic \([34]\) and in popularizing the subject in the mathematical fluid dynamics community \([33]\).

Part (a) of Theorem 1 was proved in \([18]\) using a regularization procedure and a clever and powerful, yet elementary, commutator estimate.

Part (b) of Theorem 1 took another 25 years, with a series of partial results and gradual improvements \([5, 6, 8, 22, 25–28, 40, 54, 56, 57]\), finally culminating in \([41]\) and the subsequent improvement \([10]\).

Well-formulated mathematical conjectures are not just about solving a problem. We associate with them the hope of learning something deeper about the context in which the problem was formulated, possibly revealing unexpected connections between different parts of mathematics. The story of Onsager’s conjecture fits very well in this ideal: the proof of Theorem 1 is quite possibly more interesting than the actual statement (which, in some sense, was already “clear” to physicists). It reveals a new mechanism by which energy cascades may appear in solutions of non-linear PDEs. This mechanism has already led to further breakthroughs, see below, and has uncovered an entirely unexpected connection between the mathematically elusive concept of turbulence and the “New Land” (cf. \([38]\)) discovered by John Nash in differential geometry.

The Scheffer–Shnirelman paradox. In \([54]\) V. Scheffer constructed a non-trivial weak solution to the Euler equations in \(\mathbb{R}^2\) with compact support in space and time. Subsequently A. Shnirelman in \([56]\) gave an entirely different proof in \(\mathbb{T}^2\). Such a result is hard to interpret physically, as it would correspond to a perfect incompressible fluid which can start and stop moving by itself, without the action of external forces. As such, for a long time Scheffer’s Theorem remained some sort of “paradox” in the PDE literature \([64]\), cited mostly as a warning example of unphysical behaviour if the notion of solution is too weak, with emphasis more on the non-uniqueness aspect rather than the violation of energy conservation. Indeed, since the weak solutions constructed here are merely square-integrable in space-time, there is no control on the regularity of the total kinetic energy \(E(t)\) (which is not even known to be bounded for every \(t\)), therefore in principle \(E\) need not be monotone on any time-interval. The first result in connection with Theorem 1(b) was obtained by Shnirelman in \([57]\)—he showed the existence of a solution on \(\mathbb{T}^3\) with finite and strictly monotone decreasing energy on some time interval. Nevertheless, this solution is not continuous, and, since it was obtained using a generalized flow model with sticky particles, it seemed to have no connection to the energy cascade picture postulated by Kolmogorov and Onsager.

**Differential inclusions, relaxation, and Reynolds stress.** Much less known than his works on the Euler and Navier–Stokes equations, the unpublished PhD thesis of Scheffer \([53]\) contains some remarkable precursors of fundamental results in the vectorial calculus of variations. One of them is akin to Evans’ \(\varepsilon\)-regularity theorem \([32]\) for minimizers of elliptic energies (the precise assumption is “uniform quasiconvexity”) and the other one is a precursor of a striking singularity theorem by Müller and Šverak \([49]\). The latter paper (which as we will see below has been an important source of inspiration for at least one other reason) proves the existence of a rather badly behaved Lipschitz critical point to elliptic energies which fall under the assumptions of Evans’ theorem: such a critical point

\(^1\)Towards the end of his note Onsager writes:

... It is of interest to note that in principle, turbulent dissipation as described could take place just as readily without the final assistance of viscosity \([A/N: \text{in the previous pages Onsager discusses the energy spectrum of turbulent solutions of (1) on the three-dimensional torus after rewriting the equations as an infinite-dimensional system of ODEs for its Fourier coefficients; the absence of viscosity refers to setting } \mu = 0 \text{ in (1)}]\). In the absence of viscosity, the standard proof of the energy conservation does not apply, because the velocity field does not remain differentiable! In fact it is possible to show that the velocity field in such “ideal” turbulence cannot obey any Lipschitz condition of the form

\[
|\nabla (F^2 + F) - \nabla (F^2)| < (\text{const.})^n
\]

for any \(n\) greater than \(\frac{1}{2}\), otherwise the energy is conserved.

\(^2\)Onsager’s continuation of the aforementioned paragraph reads:

Of course, under the circumstances, the ordinary formulation of the laws of motion in terms of differential equations becomes inadequate and must be replaced by a more general description; for example the formulation \((15)\) [A/N: the reference is to formula \((15)\) in Onsager’s paper] in terms of Fourier series will do.
is nowhere differentiable, in stark contrast with minimizers, which according to Evans must be smooth on a dense open subset.

The work [49] belongs, unlike the papers [54, 56, 57], to an established mathematical tradition, in the sense that during the 1990s and early 2000s several authors used analogous ideas to produce rather striking examples of irregular solutions to various systems of partial differential equations, all falling in the class of “differential inclusions” (the most notable examples are the works [21, 42, 59]). In fact an important functional analytic aspect of these ideas was pioneered in the context of ordinary differential equations was made possible by the seminal work [63].

The jump from this basic principle to partial differential equations was made possible by the seminal work [63] by L. Tartar (see also [31] for analogous ideas appearing at the same time), which examined the relations between weak convergence and differential constraints and uncovered new phenomena depending on the core PDE structure. As an example, consider the following $n$-dimensional generalization of (4):

$$X = \{ u : B \to \mathbb{R}^n | \nabla u \in \text{co}K \text{ a.e.} \}$$

$$S = \{ u : B \to \mathbb{R}^n | \nabla u \in K \text{ a.e.} \},$$

where $B \subset \mathbb{R}^n$ is the open unit ball, $K$ is a compact subset of $n \times n$ matrices, and $\text{co}K$ denotes the convex hull of $K$. If $K = O(n)$, the set of linear isometries, then $S$ is dense in $X$ with respect to the uniform topology [37]. However, if $K = SO(n)$, the set of orientation-preserving linear isometries, then $S$ consists only of affine functions [52]. The situation for a general $K$ lies somewhere in between these extreme cases, see [43, 48].

It turns out [26] that the Euler system (2), when interpreted as a differential inclusion similar to (5), falls in the same category as the $O(n)$-case above. In order to explain this, note that the density of $S$ in $X$ can be interpreted as a relaxation statement. Consider a sequence of Lipschitz maps $v_k : B \to \mathbb{R}^n$ with $\nabla v_k(x) \in O(n)$ for a.e. $x \in B$, i.e.

$$\partial_i u \cdot \partial_j u = \delta_{ij} \text{ a.e.},$$

which converge uniformly to a limit $u$. Without any additional information on the sequence, we can only say that the limit satisfies $\nabla u(x) \in \text{co}O(n)$ a.e., i.e.

$$\partial_i u \cdot \partial_j u \leq \delta_{ij} \text{ a.e.}$$

in the sense of quadratic forms. In more geometric language, the uniform limit of a sequence of isometries is a short map. The density of $S$ in $X$ is a kind of converse of this statement, that is, the system (7) is the relaxation of the system (6).

The analogous question for the Euler equations is as follows: Consider a sequence of uniformly bounded weak solutions $v_k$ to (2), which converge weakly in $L^2$ to some limit function $\bar{v}$. Then $\bar{v}$ satisfies the Euler equations with an error term:

$$\left\{ \begin{array}{ll}
\partial_i \bar{v} + \text{div}(\nabla \otimes \bar{v} + R) + \nabla p = 0 \\
\text{div}\bar{v} = 0,
\end{array} \right.$$  

where, denoting by $\nabla \otimes \bar{v}$ the weak limit of $v_k \otimes v_k$,  

$$R = \nabla \otimes \bar{v} - \nabla \otimes \bar{v}.$$  

Without any additional information on the sequence $v_k$, we have no reason to expect that the symmetric tensor $R$ vanishes, the only information we can gather is that $\bar{R}(x, t) \geq 0$ a.e. in the sense of quadratic forms (the passage from $\bar{R} = 0$ to $\bar{R} \geq 0$ is the precise analogue to the passage from $K$ to $\text{co}K$ in (5)). The tensor $R$ is, in a slightly different guise, a very well known object in the theory of turbulence, called Reynolds stress, which arises by a formal averaging of the equations (1) or (2). Thinking of a turbulent velocity field as the sum $v = \bar{v} + w$ of a mean flow $\bar{v}$ and a (random) fluctuation, the induced stress by the fluctuation on the mean flow is given by $R = w \otimes w$. This is the same formula as (9). Indeed, the process of taking weak limits is somewhat akin to averaging high-frequency oscillations, and weak convergence in place of averaging random fluctuations has been proposed by P. Lax [45] as a deterministic approach to turbulence.

The work in [25, 26] established the relationship between the system (8) and (2). More precisely, the main result in [25] states that (modulo some technical assumptions) any solution $\bar{v}$ of (8) can be weakly approximated by weak (bounded, but in general discontinuous) solutions $v$ of (2), so that, in this sense, the system (8) is the relaxation of the Euler system. In this way we were able to place the existence theorems of Scheffer and Shnirelman in a very general and flexible context, applicable not just to the incompressible Euler equations, but to several other PDEs, see e.g. [15, 20, 58] (note in contrast, that Shnirelman’s proofs heavily relied on generalized flows and thus on Arnold’s geodesic flow formulation of the incompressible Euler equations). Moreover, our work provided a surprising fil rouge between Scheffer’s PhD thesis and his work [54] on the incompressible Euler equations.
Note however that, in terms of regularity and the energy cascade picture, these results were no closer to Theorem 1(b) than [54, 56, 57].

Convex integration and the Nash–Kuiper paradox.

As mentioned above, there is a second reason why the paper [49] by S. Müller and V. Sverak was a major source of inspiration for us. The authors in [49] pointed out for the first time an important relation between the results in the theory of differential inclusions and Gromov’s h-principle in geometry. In particular the method of convex integration, introduced by M. Gromov [37] and extended in [49] to Lipschitz mappings, provides a very powerful tool to construct solutions to nonlinear PDEs.

The origin of Gromov’s convex integration lies in the famous Nash–Kuiper theorem on isometric embeddings of Riemannian manifolds. Let \( \Sigma \) be a smooth compact manifold of dimension \( n \geq 2 \), equipped with a Riemannian metric \( g \). A map \( u : \Sigma \to \mathbb{R}^N \) is isometric if it preserves the length of curves, i.e. if

\[
\ell_g(\gamma) = \ell_e(u \circ \gamma)
\]

for any \( C^1 \) curve \( \gamma \subset \Sigma \), (10)

where \( \ell_g(\gamma) \) denotes the length of \( \gamma \) with respect to the metric \( g \): 

\[
\ell_g(\gamma) = \int \sqrt{g(y(t))[\dot{y}(t), \dot{y}(t)]} \, dt. 
\]

If \( u \in C^1(\Sigma; \mathbb{R}^N) \) this means, using the language of Riemannian geometry, that the pull back of the Euclidean metric \( u^*e \) agrees with \( g \). In local coordinates such relation is the following system of partial differential equations

\[
\partial_i u \cdot \partial_j u = g_{ij},
\]

which can be seen as an (inhomogeneous) differential inclusion, compare with (6).

The existence of isometric immersions (and embeddings) of Riemannian manifolds into some Euclidean space is a classical problem, explicitly formulated for the first time by Schläfli, see [55], who conjectured that the system is solvable \emph{locally} if the dimension \( N \) of the target is at least \( \frac{n(n+1)}{2} \), matching the number of equations in (12). An isometric immersion in co-dimension 1 would seem a rare bird, since, with the exception of the case \( n = 2 \), the system (12) would be heavily overdetermined. Yet, in [50] J. Nash astonished the geometry world by proving essentially that the only obstruction to the existence of solutions of (12) is topological, at least in the class of \( C^1 \) maps.

In order to state Nash’s Theorem let us recall that an immersion \( u : \Sigma \to \mathbb{R}^N \) is called short if it “shrinks” the length of curves. For \( C^1 \) immersions and in local coordinates such condition is equivalent to the inequality

\[
\partial_i u \cdot \partial_j u \leq g_{ij}
\]

in the sense of quadratic forms, compare with (7).

\textbf{Theorem 2.} Let \( (\Sigma, g) \) be a smooth closed \( n \)-dimensional Riemannian manifold. Any \( C^\infty \) short immersion \( u : \Sigma \to \mathbb{R}^N \) with \( N \geq n + 1 \) can be uniformly approximated by \( C^1 \) isometric immersions. If it is, in addition, an embedding, then it can be approximated by \( C^1 \) isometric embeddings.

Nash proved Theorem 2 for \( N \geq n + 2 \) and suggested that his strategy could be suitably modified to work in the case \( N = n + 1 \); the details were then given in two subsequent works by N. Kuiper [44].

Theorem 2 can be seen as the \( C^1 \) analogue of the relaxation statement in (5). However, whilst it is rather easy to imagine Lipschitz isometric maps arising as “foldings” of the manifold \( \Sigma \), the \( C^1 \) case came as a complete surprise, in particular because the Nash–Kuiper theorem cannot hold for \( C^2 \) maps. For instance, a \( C^2 \) isometric immersion of a closed positively curved sphere in the three-dimensional space is necessarily convex, and in fact the shape is determined up to rigid motions by a classical result of Cohn–Vossen and Herglotz [16, 39]. In the 1950s Yu. Borisov showed that such result can be extended to \( C^{1,2+\varepsilon} \) isometries, cf. [1]. In fact, since for a \( C^1 \) surface the Gauss map is continuous, with a well-defined Brouwer degree, there was some hope that the rigidity statement of Cohn-Vossen and Herglotz can be extended to to the \( C^1 \) case. It was this hope that was shattered by Nash’s result.

Subsequently, Borisov announced that the Nash–Kuiper theorem can be extended to \( C^{1,\alpha} \) isometries provided \( \alpha \) is sufficiently small [2]. While a detailed proof of these announcements only appeared in one special case in [3], in the joint work [19] with Sergio Conti we revisited these extensions and provided a unified framework for all the results announced in [2]. In that paper we also noticed that the geometric considerations leading to Borisov’s rigidity statement [1] can be substituted by a short PDE argument which relies on the same commutator estimate as in Constantin, E, and Titi’s proof of Theorem 1(a).

Thus, a striking analogy between isometric immersions and solutions of the Euler equations arose: at sufficiently high regularity (\( C^2 \) for isometries, \( C^1 \) for Euler) solutions are well-behaved and with appropriate side-conditions uniquely determined, at sufficiently low regularity (Lipschitz/\( C^1 \) for isometries, bounded/\( C^0 \) for Euler) solutions with completely different behavior appear. It was thus natural to try to adapt the ideas of Nash in [50] to the Euler equations. It is important to emphasize, however, that this analogy concerns more the non-uniqueness aspect of weak solutions of Euler, i.e. the unphysical behavior already observed in connection with the Scheffer–Shnirelman paradox. On the other hand, in light of Gromov’s h-principle, one should perhaps view this aspect more as an expression of flexibility rather than non-uniqueness, the violation
of energy conservation is one aspect of this flexibility. Indeed, while part (a) of Theorem 1 shows that above the H"older exponent $\frac{1}{3}$ solutions cannot be too flexible, it is not at all clear what to expect about their uniqueness: one guess might be that the threshold for uniqueness is a small improvement of $C^1$ (in the Osgood sense).

Dissipative continuous solutions. Inspired by Nash’s proof in [50] we devised in [27] a “convex integration” scheme leading to continuous dissipative solutions of (2). Subsequently we showed in [28] that these solutions satisfy Theorem 1(b) with exponent $\theta < \frac{1}{10}$. The construction is based on an iteration, where at each step we add a highly oscillatory correction in order to decrease the defect to being a solution. More precisely, we construct inductively a sequence of solutions $(v_\theta, p_\theta, R_\theta)$, $\theta = 1, 2, \ldots$ to

$$
\begin{aligned}
\partial_t v_\theta + \text{div}(v_\theta \otimes v_\theta) + \nabla p_\theta &= -\text{div} R_\theta, \\
\text{div} v_\theta &= 0,
\end{aligned}
$$

such that $v_\theta \to v$ and $R_\theta \to 0$ uniformly. Observe that (14) is the same system as (8). Accordingly, $v_{\theta+1} = v_\theta + w_{\theta+1}$ where we think of $v_\theta$ as the “mean flow” on length-scales $\geq \lambda_{\theta}^{-1}$ and $w_{\theta+1}$ is the “fluctuation” on this scale. Thus, up to lower order corrections, $w_{\theta+1}$ should have the form

$$w_{\theta+1}(x, t) = W\left(v_\theta(x, t), R_\theta(x, t), \lambda_{\theta+1} x, \lambda_{\theta+1} t\right),
$$

where $W(v, R, \xi, \tau)$ is some “master function” and $\lambda_{\theta+1}$ a parameter which increases at least exponentially fast at each step. In comparison, in the proof of Nash [50] these “fluctuations” are spirals (and in [44] corrugations) aimed at increasing the metric—thereby reducing the metric error—in a single coordinate direction.

The basic idea for reducing the error with such an Ansatz is the following: assuming that $v_\theta$ is already the correct solution up to spatial frequencies of order $\lambda_\theta$, and $w_{\theta+1}$ is supported on spatial frequencies of order $\lambda_{\theta+1}$, it is easy to see that the only possibility for $w_{\theta+1}$ to correct the error $R_\theta$ is via the high-high to low interaction in the product $w_{\theta+1} \otimes w_{\theta+1}$. In other words, the master function $W$ should satisfy the properties that $\xi \mapsto W(v, R, \xi, \tau)$ is $2\pi$-periodic with average

$$\langle W \rangle := \frac{1}{(2\pi)^3} \int_{T^3} W(v, R, \xi, \tau) \, d\xi = 0;$$

and the average stress is given by $R$, i.e.

$$\langle W \otimes W \rangle = R.$$

Note how these requirements are consistent with (9). Now assume that $\xi \mapsto W(v, R, \xi, \tau)$ is a stationary solution of Euler for any $v, R, \tau$. Substituting this Ansatz for $v_{\theta+1}$ into (14) then yields as the main quadratic interaction term

$$\text{div}(w_{\theta+1} \otimes w_{\theta+1} - R_\theta) + \nabla p_{\theta+1},$$

leading to a new Reynolds stress $R_{\theta+1}$ with Fourier support on frequencies of order $\lambda_{\theta+1}$. In this way we can push the error to high frequencies by successively “undoing” the averaging process leading to Reynolds stresses in (8)-(9).

As explained in [29, 62], starting from the Ansatz above it is possible to write down a family of conditions that $W$ would have to satisfy, ideally, so to give a “clean” convex integration iteration leading to a proof of Theorem 1(b). Although this family of conditions is somewhat naive and unfortunately no such $W$ exists (indeed, the scaling of time in (15) is clearly “wrong”), approximations based on a special family of stationary solutions of Euler called Beltrami flows can be used. This lead to the results in [27, 28].

Climbing the Onsager ladder. The reason why the construction in [27, 28] was only able to produce weak solutions to the Euler system as in Theorem 1(b) with Hölder exponent $\theta < \frac{1}{10}$ is the rather poor control of the linear (i.e. transport) interaction term between “mean flow” $v_\theta$ and “fluctuation” $w_{\theta+1}$. Indeed, whilst it is quite clear from dimensional considerations that the scaling $(\lambda_{\theta+1} x, \lambda_{\theta+1} t)$ in (15) is unnatural, several modifications were introduced later in the precise implementation of the basic iteration scheme described above. P. Isett introduced in his PhD thesis [40] the correct space-time scaling, which eventually led to an improvement of Theorem 1(b) with Hölder exponent $\theta < \frac{1}{3}$, cf. [8]. Moreover, following an idea from the PhD thesis of T. Buckmaster to introduce temporal intermittency [5, 6], in the joint work [9] we reached for the first time the threshold $\theta < \frac{1}{3}$, although not in the desired scale of spaces: more precisely, for any $\theta < \frac{1}{3}$ we show the existence of nontrivial continuous solutions with compact temporal support (i.e. in the spirit of the Scheffer–Shnirelman paradox) which satisfy the condition

$$|v(x, t) - v(y, t)| \leq C(t)|x - y|^\theta \quad \text{for every } x, y, t,$$

where $C$ is an $L^1$ function of time.

The correct scale of spaces was finally achieved in [41], where P. Isett proved the existence of compactly supported nontrivial solutions in $C^\theta$ for every $\theta < \frac{1}{3}$. The proof in [41] contains two new ideas. Firstly, in [22] S. Daneri and the second author introduced a new class of “master functions” $W$ called Mikado flows, which are more stable under convection by a large-scale mean flow. However, it is not possible to use such $W$ directly in the scheme of [27] (and its subsequent refinements), not even to produce continuous solutions. The second key idea in [41] is a gluing technique, which combines the convex integration technique with the free (unforced) Euler dynamics in an alternating fashion. A shortcoming of [41] is the insufficient control of the kinetic energy $E(t)$. The final proof of Theorem 1(b) was then given in [10].
Further developments and open problems. Several interesting challenges remain in the area. One is to produce a sequence of Leray-Hopf solutions to the Navier-Stokes equations with vanishing viscosity which exhibit anomalous dissipation. This would be the case if, for instance, one were able to show that at least one of the solutions of Euler constructed by the convex integration method is a limit of Leray-Hopf solutions of the Navier-Stokes equations with vanishing viscosity. The latter problem might be linked to constructing dissipative solutions which are Onsager-critical, although it is not clear which scale of spaces is most natural.

A second important challenge in connection with the Kolmogorov-Onsager theory is to introduce intermittency and in this way to be able to recover the measured deviations from K41 self-similarity and possible multifractality. In their remarkable recent work [11] Buckmaster and Vicol have constructed a convex integration scheme which produces weak solutions of the Navier-Stokes equations, by introducing intermittency. In particular they were able to show that:

- For such weak solutions the Cauchy problem to the Navier-Stokes equations is ill-posed;
- Any "convex-integration solution" of Euler can be approximated strongly in $L^2$ by weak solutions of Navier-Stokes.

Furthermore, in the joint work [7] with M. Colombo they could produce such solutions with the additional property that they are smooth except for a small closed set of times (of Hausdorff dimension strictly smaller than 1). While the above results are striking, they seem for the moment far from producing solutions that belong to the energy space. The latter can be instead reached when the Laplacian in the viscosity is replaced by a fractional Laplacian with sufficiently low exponent (cf. [17, 30]; the latter works follow more closely the constructions in [8] and [10]). In a similar line of research, the second author in joint work with S. Modena [46, 47] constructed convex integration solutions to the transport and continuity equations with Sobolev vector fields which are not renormalized, thus showing optimality of the integrability assumptions in the DiPerna-Lions theory.

A further important issue is related to the closure problem and dynamical behavior of the Reynolds stress. While the results in [10, 22, 25] imply that in the absence of boundaries there is no constraint on the evolution of the Reynolds stress tensor, a series of results on initial value problems associated to classical hydrodynamic instabilities [12, 35, 60, 61] point at the possibility that in the presence of initial conditions or boundaries this is not the case anymore. The constrained evolution of the Reynolds stress in the general situation remains to be explored.

Regarding the analogy between Euler and isometric immersions, note that Theorem 1 provides a sharp threshold between two competing situations. A sort of analog of the Onsager’s conjecture has been recently proved for the isometric embedding problem, where the threshold exponent turns out to be $\frac{2}{3}$. In the recent work [23] D. Inauen and the first author have shown that, while isometric embeddings of class $C^{1,\frac{2}{3}+\varepsilon}$ satisfy a suitable generalization of a classical theorem in differential geometry (namely the Levi-Civita connection of the manifold coincides with the connection induced on the immersion by the ambient Euclidean space), for every $\varepsilon > 0$ it is possible to construct $C^{1,\frac{2}{3}-\varepsilon}$ isometric embeddings which violate it. Even though the latter result shows the criticality of the exponent $\frac{2}{3}$, the most compelling conjecture in the area is still unsolved:

**Conjecture 1.** Consider a positively curved sphere $(S^2, g)$ and isometric immersions $\nu \in C^{1,\alpha}(S^2, \mathbb{R}^3)$.

(a) If $\alpha > \frac{1}{2}$, then $\nu(S^2)$ is convex and it is unique up to ambient isometries.

(b) If $\alpha < \frac{1}{2}$ any short immersion (resp. embedding) $u$ can be uniformly approximated by a sequence of isometric immersions (resp. embedding) $\nu_k \in C^{1,\alpha}(S^2)$.

As already mentioned, part (a) of the Conjecture is known to hold for $\alpha > \frac{2}{3}$. Concerning part (b), the joint work of the authors with Inauen [24] prove the statement for positively curved disks when $\alpha < \frac{1}{3}$, while forthcoming work of the second author with W. Cao settles the case of general surfaces for $\alpha < \frac{1}{5}$.

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Topological Time Series Analysis

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Introduction

Time series are ubiquitous in our data rich world. In what follows I will describe how ideas from dynamical systems and topological data analysis can be combined to gain insights from time-varying data. We will see several applications to engineering and the life sciences, as well as some of the theoretical underpinnings.

\[ x'(t) = \sigma (y - x) \]
\[ y'(t) = x(r - z) - y \]
\[ z'(t) = xy - \beta z \]

Solving the Lorenz system yields a differentiable function
\[ \Phi : \mathbb{R} \times \mathbb{R}^3 \to \mathbb{R}^3 \]
where \( \Phi(t, v_0) = (x(t), y(t), z(t)) \) satisfies (1) for all \( t \in \mathbb{R} \) and \( \Phi(0, v_0) = v_0 \) for all \( v_0 \in \mathbb{R}^3 \). In fact, the orange curve from Figure 1 corresponds to \( \Phi(t, (5, 5, 5)) \) when \( (\sigma, \rho, \beta) = (10, 28, 8/3) \). The discrepancy between the orange and blue curves, as elucidated in [19], is a property inherent to the system. Lorenz realized that when manually entering \( v_j \) as input, he only used the first few significant digits instead of the full precision values. In other words, the system (1) can be extremely sensitive to initial conditions in that any errors are compounded exponentially with time.

\[ \Phi(0, v_0) = (x(0), y(0), z(0)) \]

\[ \Phi(200, v_0) = (x(200), y(200), z(200)) \]

\[ \Phi(400, v_0) = (x(400), y(400), z(400)) \]

Lorenz and the Butterfly

Imagine you have a project involving a crucial computer simulation. For an initial value \( v_0 = (x_0, y_0, z_0) \in \mathbb{R}^3 \), a sequence \( v_0, \ldots, v_n \in \mathbb{R}^3 \) is computed in such a way that \( v_{j+1} \) is determined from \( v_j \) for \( j = 0, \ldots, n-1 \). After the simulation is complete you realize that a rerun is needed for further analysis. Instead of initializing at \( v_0 \), which might take a while, you take a shortcut: you input a value \( v_j \) selected from the middle of the current results, and the simulation runs from there while you go for coffee. Figure 1 is displayed on the computer monitor upon your return; the orange curve is the sequence \( x_0, \ldots, x_n \) from the initial simulation, and the blue curve is the \( x \) coordinate for the rerun initialized at \( v_j \).

The results agree at first, but then they diverge widely; what is going on? Edward Norton Lorenz, a mathematician and meteorologist, asked himself the very same question while studying a simplified model for weather forecasting [19]. In the process of resolving the aforementioned discrepancy, which one could erroneously attribute to software error or a hardware malfunction, Lorenz laid out the foundations for what we know today as chaos theory. The relevant set of differential equations for the simplified model, called the Lorenz system, is shown in equation (1); \( x, y, \) and \( z \) are real-valued functions of time \( t \), and \( \sigma, \rho, \beta \in \mathbb{R} \) are physical constants.

\[ \begin{align*}
\Phi(t, v_0) &= \Phi(0, v_0) \\
\Phi(t_1, \Phi(t_0, v_0)) &= \Phi(t, v_0) \\
\Phi(t_2, \Phi(t_1, \Phi(t_0, v_0))) &= \Phi(t, v_0)
\end{align*} \]

Definition 0.1. A global continuous time dynamical system is a pair \((M, \Phi)\), where \( M \) is a topological space and \( \Phi : \mathbb{R} \times M \to M \) is a continuous map so that \( \Phi(0, p) = p \) and \( \Phi(s, \Phi(t, p)) = \Phi(s + t, p) \) for all \( p \in M \) and all \( t, s \in \mathbb{R} \).

The typical examples arising from differential equations (e.g., the Lorenz system) have as state space a smooth manifold \( M \) (e.g., \( \mathbb{R}^3 \)), and the dynamics are given by the integral curves (e.g., equation (2)) of a smooth vector field on \( M \) (e.g., (1)).

Some subsets of \( M \) are especially important since they attract the evolution of states in close proximity. Indeed, a set \( A \subset M \) is called an attractor (these are the kinds of sets we will focus on) if it satisfies three conditions: (1) it

\[ \begin{align*}
\Phi(t, v_0) &= \Phi(0, v_0) \\
\Phi(t_1, \Phi(t_0, v_0)) &= \Phi(t, v_0) \\
\Phi(t_2, \Phi(t_1, \Phi(t_0, v_0))) &= \Phi(t, v_0)
\end{align*} \]
is compact, (2) it is an invariant set — that is, if \( a \in A \) then \( \Phi(t, a) \in A \) for all \( t \geq 0 \) — and (3) it has an open basin of attraction. In other words, there is an invariant open neighborhood \( U \subset M \) of \( A \), so that
\[
\bigcap_{t \geq 0} \{ \Phi(t, p) : p \in U \} = A.
\]

An attractor \( A \) is called strange — and this is just a name — if (1) there are arbitrarily close points \( p, p' \) in a basin of attraction of \( A \) for which the distance between \( \Phi(t, p) \) and \( \Phi(t, p') \) grows exponentially quickly with \( t \), and (2) \( A \) has non-integral Hausdorff dimension. Lorenz’s butterfly \( Λ \subset \mathbb{R}^3 \) from Figure 2 is one of the most widely known examples of a strange attractor; its Hausdorff dimension is approximately 2.063 [31].

**Persistent Homology: Measuring Shape From Finite Samples**

The shape of attractors carries a great deal of information about the global structure of a dynamical system. Indeed, attractors with the shape of a circle \( S^1 = \{ z \in \mathbb{C} : |z| = 1 \} \) give rise to periodic processes; non-integral Hausdorff dimension is evidence of chaotic behavior; while high-dimensional tori \( \mathbb{T}^n = S^1 \times \cdots \times S^1 \) are linked to quasiperiodicity. The latter is a type of recurrence emerging from the superposition of periodic oscillators with incommensurate (i.e., \( \mathbb{Q} \)-linearly independent) frequencies. Quasiperiodicity appears, for example, in turbulent fluids [26], and the detection of biphonation in high-speed laryngeal videoseoscopy [30]. Similarly, the existence of chaos in brain activity is of considerable interest in neuroscience [17], as is the presence of periodic oscillators in biological systems [27].

A data analysis question that has received significant attention in recent years is how to measure the shape of a topological space \( X \) — e.g., an attractor — from a finite set \( X \) (the data) approximating it. This is the type of problem driving advances in Topological Data Analysis [7], and where tools like persistent homology [22] — which we will describe shortly — are relevant.

The space \( X \) around which the observed data \( X \) accumulates is unknown in practice, so the typical strategy in topological data analysis is to use \( X \) to construct another space whose shape approximates that of \( X \). One of the most widely used constructions is the Rips complex. If \( X \) (not necessarily finite) is equipped with a metric \( \rho \), then the Rips complex of \( X \) at scale \( \alpha \) is the set
\[
R_\alpha(X) := \{ \{x_0, \ldots, x_k\} \subset X : \rho(x_i, x_j) \leq \alpha \text{ for all } 0 \leq i, j \leq k \}.
\]

This is in fact a simplicial complex; points in \( X \) can be thought of as vertices, sets with two elements \( \{x_0, x_1\} \in R_\alpha(X) \) are edges, \( \{x_0, x_1, x_2\} \in R_\alpha(X) \) is a triangular face, and so on. A theorem of Janko Latschev [18] contends that if \( X \) is a closed Riemannian manifold, then under mild density hypotheses (which unfortunately cannot be checked in practice) the geometric realization of \( R_\alpha(X) \) is homotopy equivalent to \( X \) for small \( \alpha > 0 \).

The shape of a space (i.e., its homotopy type) refers to those properties that are invariant under continuous deformations; e.g., is it connected? are there holes? Said properties can be formalized and quantified using homology [13]. Given an integer \( n \geq 0 \), the \( n \)-th homology of a topological space \( B \) with coefficients in a field \( \mathbb{F} \), denoted \( H_n(B; \mathbb{F}) \), is a vector space over \( \mathbb{F} \). Its dimension \( \beta_n(B; \mathbb{F}) \) — the \( n \)-th Betti number of \( B \) with coefficients in \( \mathbb{F} \) — provides a count for the number of \( n \)-dimensional holes in \( B \). Indeed, \( \beta_0 \) counts the number of path-connected components, \( \beta_1 \) is the number of holes bounded by a closed loop in \( B \), \( \beta_2 \) is the number of voids bounded by a closed 2-dimensional region, and so on for \( n \geq 3 \). Here is an example: the 2-dimensional torus \( \mathbb{T}^2 = S^1 \times S^1 \) in Figure 3 (left) has Betti numbers \( \beta_0(\mathbb{T}^2; \mathbb{F}) = 1 \) since it is path-connected, \( \beta_1(\mathbb{T}^2; \mathbb{F}) = 2 \) since it has a horizontal and a vertical hole, \( \beta_2(\mathbb{T}^2; \mathbb{F}) = 1 \) since \( \mathbb{T}^2 \) itself encloses an empty volume, and \( \beta_n(\mathbb{T}^2; \mathbb{F}) = 0 \) for all \( n \geq 3 \). Similarly, the 2-sphere \( S^2 = \{ x \in \mathbb{R}^3 : ||x|| = 1 \} \) has Betti numbers \( \beta_0(S^2; \mathbb{F}) = \beta_2(S^2; \mathbb{F}) = 1 \) and \( \beta_n(S^2; \mathbb{F}) = 0 \) for \( n \geq 3 \), for the same reasons as the torus, but \( \beta_1(S^2; \mathbb{F}) = 0 \) since every closed loop on \( S^2 \) bounds a filled-in region.

We remark that \( \beta_n(B; \mathbb{F}) \) can change with the choice of field \( \mathbb{F} \). Indeed, if \( B \) is a closed connected \( n \)-dimensional manifold, \( p \geq 3 \) is a prime, and \( \mathbb{F}_p \) denotes the field with \( p \) elements, then \( B \) is orientable if and only if \( \beta_n(B; \mathbb{F}_p) = 1 \), and non-orientable if and only if \( \beta_n(B; \mathbb{F}_p) = 0 \) (see for example 3.26 and 3.28 in [13]).

For a realistic data set \( X \), the Betti numbers \( \alpha \mapsto \beta_n(R_\alpha(X); \mathbb{F}) \) are expected to be unstable as \( \alpha \) varies. Indeed, sampling artifacts or noise in \( X \) can produce holes that are present in \( R_\alpha(X) \) but not in \( R_{\alpha+\delta}(X) \) for small \( \delta > 0 \) (e.g., see Figure 4). This is where comparing the homology of spaces related via maps is useful. If \( \alpha \leq \alpha' \), then \( i^\alpha_{\alpha'} : R_\alpha(X) \rightarrow R_{\alpha'}(X) \) given by \( i^\alpha_{\alpha'}(\{x_0, \ldots, x_k\}) = \{x_0, \ldots, x_k\} \) induces a linear transformation
\[
i^\alpha_{\alpha'} : H_n(R_\alpha(X); \mathbb{F}) \rightarrow H_n(R_{\alpha'}(X); \mathbb{F}). \tag{4}
\]
Classes in $H_n(R_\alpha(X); \mathbb{F})$ that are not in the Kernel of $t_{\alpha}^{\alpha,\alpha'}$ for large $\alpha' - \alpha$ can thus be interpreted as being persistent in the data, and suggest true homological features of the underlying space $\mathcal{X}$. The collection of vector spaces and linear maps resulting from (4) is called the $n$-dimensional persistent homology, with coefficients in $\mathbb{F}$, of the Rips filtration $\mathcal{R}(X) = \{R_\alpha(X)\}_{\alpha \in \mathbb{R}}$. Thus far we have that the Betti numbers capture the homology of a space, yielding succinct shape descriptors for its topology. The persistent homology of $\mathcal{R}(X)$, on the other hand, describes the multiscale evolution of homological features underlying the data. More generally,

**Definition 0.2.** A persistence vector space $V$ is a collection of vector spaces $V_\alpha$, $\alpha \in \mathbb{R}$, and linear transformations $t^{\alpha,\alpha'} : V_\alpha \rightarrow V_{\alpha'}$, $\alpha \leq \alpha'$, so that:

1. $t^{\alpha,\alpha}$ is the identity of $V_\alpha$ for every $\alpha \in \mathbb{R}$.
2. $t^{\alpha',\alpha} \circ t^{\alpha,\alpha'} = t^{\alpha',\alpha'}$, whenever $\alpha \leq \alpha' \leq \alpha''$.

Two persistence vector spaces $V = \{V_\alpha, t^{\alpha,\alpha'}\}$ and $W = \{W_\alpha, \kappa^{\alpha,\alpha'}\}$ are isomorphic, denoted $V \cong W$, if there are linear isomorphisms $T_\alpha : V_\alpha \rightarrow W_\alpha$ for all $\alpha \in \mathbb{R}$, so that $\kappa^{\alpha,\alpha'} \circ T_\alpha = T_{\alpha'} \circ t^{\alpha,\alpha'}$ whenever $\alpha \leq \alpha'$.

We will concentrate on three quantities for a nonzero element $\gamma \in V_\alpha$:

$$
\text{birth}(\gamma) := \inf \left\{ \alpha \leq \alpha : \gamma \in \text{Image} \left( t^{\alpha,\alpha'} \right) \right\}
$$

$$
\text{death}(\gamma) := \sup \left\{ \alpha \geq \alpha : \gamma \in \text{Kernel} \left( t^{\alpha,\alpha'} \right) \right\}
$$

$$
\text{persistence}(\gamma) := \text{death}(\gamma) - \text{birth}(\gamma)
$$

When each $V_\alpha$ is finite dimensional, the isomorphism type of $V$ can be completely described via a simple invariant called the barcode $[9]$:

**Theorem 0.3.** Let $V = \{V_\alpha, t^{\alpha,\alpha'}\}$ be a persistence vector space so that $\dim(V_\alpha)$ is finite for all $\alpha \in \mathbb{R}$. Then, there exists a multiset (i.e., a set whose elements may appear with repetitions) of intervals $I \subset [-\infty, \infty]$ called the barcode of $V$, denoted $\text{bcd}(V)$, and so that:

1. $\text{bcd}(V)$ subsumes the Betti numbers: If $\alpha \in \mathbb{R}$, then $\dim(V_\alpha)$ is exactly the number of intervals $I \subset \text{bcd}(V)$, counted with repetitions, so that $\alpha \in I$.
2. $\text{bcd}(V)$ encodes persistence: For every $I \subset \text{bcd}(V)$ and every $\alpha \in I$, there exists $\gamma \in V_\alpha$ so that the left and right end-points of $I$ are birth($\gamma$) and death($\gamma$), respectively.
3. $\text{bcd}(V)$ is an invariant: If $W$ is a persistence vector space with $\dim(W_\alpha)$ finite for all $\alpha \in \mathbb{R}$, then $\text{bcd}(V) = \text{bcd}(W)$ if and only if $V \cong W$.

We will use $\text{bcd}^n(X; \mathbb{F})$ to denote the barcode for the $n$-dimensional persistent homology of $\mathcal{R}(X)$. Below in Figure 4 we show an example for $X \subset \mathbb{R}^2$ sampled with noise around the unit circle $S^1$, the Rips complex $R_\alpha(X)$ at scales $\alpha = 0, 0.36, 0.6, 1.21$, and the intervals (i.e., the horizontal blue lines) that comprise the barcode $\text{bcd}^1(X; \mathbb{F}_2)$. The computations were performed using the C++ library Ripser [4]. The single long interval is indicative of a persistent 1-dimensional hole in the data, which is consistent with $X$ being sampled around $S^1$; indeed, $\beta_1(S^1; \mathbb{F}_2) = 1$. The shorter intervals, on the other hand, are due to noise and sampling artifacts.

![Barcode for the Rips filtration on $X \subset \mathbb{R}^2$ near $S^1$.](image)

As we have seen thus far, persistent homology can be used to infer the topology of a space $\mathcal{X}$ given a finite sample $X$: the number of (comparatively) longer intervals in $\text{bcd}^n(X; \mathbb{F})$ suggests a value for $\beta_n(\mathcal{X}; \mathbb{F})$. The barcodes $\text{bcd}^n(X; \mathbb{F})$ can also be used to quantify/identify properties of dynamical systems: if $\mathcal{X}$ is an attractor, a barcode like the one in Figure 4 would be indicative of periodicity, while the barcodes in Figure 5 would point to quasiperiodic-odity. Indeed, as we alluded to at the beginning of this section, the superposition of incommensurate oscillators (i.e., quasiperiodicity) is tied to attractors with the topology of a high-dimensional torus [21]. In addition to periodicity and quasiperiodicity, measures of Hausdorff dimension can also be derived from persistent homology; see for instance [20, 25].

**Attractor Reconstruction: Time Series Data, Takens’ Theorem, and Sliding Window Embeddings**

In practice it is exceedingly rare to have an explicit mathematical description of a dynamical system of interest. Instead, one can often gather measurements of relevant quantities for each state $p \in M$ — e.g., in weather prediction one can estimate temperature, pressure, etc. A way of measuring can be thought of as a continuous map $F : M \rightarrow \mathbb{R}$ — called an observation function — and given an initial state $p \in M$, one obtains the time series

$$
\varphi_p : \mathbb{R} \rightarrow \mathbb{R}
$$

$$
t \mapsto F \circ \Phi(t, p)
$$

The blue and orange curves from Figure 1 are examples of time series from the Lorenz system. A single time series may appear to be a complete oversimplification of the
underlying dynamics. However, Takens’ embedding theorem, due to Floris Takens [28], implies that they can actually be very useful. For smooth manifolds $M$ and $N$, and for a nonnegative integer $k$, let $C^k(M, N)$ denote the set of functions from $M$ to $N$ whose derivatives up to degree $k$ exist and are continuous. $C^k(M, N)$ can be endowed with a topology, the (strong) Whitney topology, in which, roughly speaking, two functions are close if and only if the functions and all their derivatives up to degree $k$ are close on compact subsets of $M$. Here is Takens’ theorem:

**Theorem 0.4.** Let $M$ be a smooth, compact, Riemannian manifold; let $\tau > 0$ be a real number; and let $d \geq 2 \dim(M)$ be an integer. Then, for generic $\Phi \in C^2(\mathbb{R} \times M, M)$ and $F \in C^2(M, \mathbb{R})$, and for $\varphi_p(t)$ defined by (6), the delay map

$$\varphi : M \to \mathbb{R}^{d+1}$$

$$p \to (\varphi_p(0), \varphi_p(\tau), \varphi_p(2\tau), \ldots, \varphi_p(d\tau))$$

is an embedding (i.e., $\varphi$ is injective and its derivative has full-rank everywhere).

Generic means that the set of functions $\Phi, F$ for which (7) is an embedding is open and dense in the Whitney topology. In fact, if $A \subset M$ is a strange attractor, then $\varphi$ restricted to $A$ will be (generically) an embedding whenever $d$ is at least twice the Hausdorff dimension of $A$. Takens’ theorem motivates the following definition.

**Definition 0.5.** Let $f : \mathbb{R} \to \mathbb{R}$ be a function, $\tau > 0$ a real number, and $d > 0$ an integer. The **sliding window embedding** of $f$, with parameters $d$ and $\tau$, is the vector-valued function

$$SW_d,\tau f : \mathbb{R} \to \mathbb{R}^{d+1}$$

$$t \to (f(t), f(t + \tau), f(t + 2\tau), \ldots, f(t + d\tau))$$

(8)

The integer $d + 1$ is the dimension, $\tau$ is the delay, and the product $d\tau$ is the window size. For $T \subset \mathbb{R}$, the set

$$SW_d,\tau f = \{SW_d,\tau f(t) : t \in T\}$$

is the **sliding window point cloud** associated to the sampling set $T$.

Hence, given time series data $f(t) = \varphi_p(t)$ observed from a potentially unknown dynamical system $(M, \Phi)$, Takens’ theorem implies that (generically) the sliding window point cloud $SW_d,\tau f$ provides a topological copy of $\{\Phi(t, p) : t \in T\} \subset M$. In particular this will reconstruct attractors. The underlying shape of $SW_d,\tau f$ can be then quantified with persistent homology, and the associated barcodes $bcd_n(SW_d,\tau f ; F)$ can be used as features in inference, classification, and learning tasks [12, 25, 32]. We will see shortly several applications of these ideas to science and engineering, as well as a theoretical discussion on sliding window persistence and parameter choices. For now, here is an instantiation of the pipeline:

**Example.** Let $\omega \in \mathbb{R}$ be irrational; we will use $\omega = \sqrt{3}$ for computations but any other choice would do. Consider the dynamics $\Phi$ and the observation function $F$ on the torus $\mathbb{T}^2 = S^1 \times S^1 \subset \mathbb{C}^2$, given by

$$\Phi : \mathbb{R} \times \mathbb{T}^2 \to \mathbb{T}^2$$

$$(t, (z_1, z_2)) \mapsto (e^{it}z_1, e^{i\omega t}z_2)$$

$$F : \mathbb{T}^2 \to \mathbb{R}$$

$$(z_1, z_2) \mapsto \text{Re}(z_1 + z_2)$$

If $p \in \mathbb{T}^2$, then $\{\Phi(t, p) : t \in \mathbb{R}\}$ is dense in $\mathbb{T}^2$, and hence $\mathbb{T}^2$ is the only attractor; e.g., see [6], page 86, Example 6.15. For $p = (1, 1)$ we obtain the quasiperiodic time series $f(t) = F \circ \Phi(t, (1, 1)) = \cos(t) + \cos(\omega t)$, and we show in Figure 5 the dynamics $t \to \Phi(t, (1, 1))$ on the torus (left), the resulting time series $f(t)$ (center), and the barcodes $bcd_n = bcd_n(SW_d,\tau f ; F_2)$ for $d = 4, \tau = \frac{3}{4}\sqrt{3}\pi, n = 0, 1, 2$ (right).

**Figure 5.** Left: The dynamics $\Phi$ on the torus. The colors, blue through red, indicate the time $t$ for $t \to \Phi(t, (1, 1)) \in \mathbb{T}^2$. Center: The time series $f(t) = F \circ \Phi(t, (1, 1)) = \cos(t) + \cos(\sqrt{3}t)$. The colors indicate the time variable $t \in [0, 30\pi]$ and are coordinated with the planar torus on the left panel. Right: Barcodes for the Rips filtration $\mathcal{R}(SW_d,\tau f)$; the number of long intervals recovers the Betti numbers of the attractor: $\beta_0(\mathbb{T}^2 ; F_2) = \beta_2(\mathbb{T}^2 ; F_2) = 1$ and $\beta_1(\mathbb{T}^2 ; F_2) = 2$.

**Some Applications of Sliding Window Persistence**

**Wheeze detection.** A wheeze is an abnormal whistling sound produced while breathing. It is often associated with obstructed airways and lung diseases such as asthma, lung cancer, and congestive heart failure. In [11], Emran, Gentimis, and Krim show that the 1-dimensional barcode $bcd_n(SW_d,\tau f ; F)$, and particularly the length of its longest interval (i.e., its maximum persistence)

$$mp_1(SW_d,\tau f ; F) = \max \{\text{length}(I) : I \in bcd_1(SW_d,\tau f ; F)\}$$

is an effective feature for wheezing detection when $f$ is a recorded breathing sound. Indeed, the presence of wheezing in the sound signal $f$ leads to circular sliding window point clouds. When testing on a large database of sound...
recordings, Emrani et al. show that (10) leads to a higher detection accuracy than that of competing methods.

Periodicity quantification in gene expression data. Many biological processes, including the cell cycle, cell division, and the circadian clock, are periodic in nature. An important problem in systems biology is to describe these mechanisms at a genetic level [27], and biologists approach this by first collecting data. Specifically, how gene expression changes across time in a given model organism — e.g., yeast, mice, etc. In [24] databases of time series of gene expression data from the yeast cell cycle, the yeast metabolic cycle, and the mouse circadian clock are used to show that measures similar to (10) can outperform state-of-the-art methods for periodicity quantification, leading to the discovery of novel clock-regulated genes.

Segmentation of dynamic regimes. Complex high-dimensional systems can exhibit abrupt changes in qualitative behavior. For instance, the Earth’s climate has undergone several sudden transitions to and back from a “snowball Earth” [14]. Identifying markedly different regimes in a system’s evolution can thus be used for warning, modeling, and parameter estimation purposes [10]. Berwald et al. show in [5] that, given time series data, effective classifiers can be trained on features from the barcodes of sliding window point clouds, with the goal of automatically segmenting a system into different behavioral regimes. Some applications of their methodology include the detection of bifurcations on stochastic and chaotic systems, as well as the analysis of temperature and CO₂ levels in the Earth’s ice cores.

Chatter detection and classification in machining. Turning and milling are cutting processes used extensively in industrial manufacturing. Chatter, or machining vibrations, are wide oscillations of the cutting tool with respect to the metal workpiece; these undesired undulations leave surface flaws on the production piece during turning and milling. Khasawneh et al. show in [15, 16] that, given a time series $f$ describing the undulations of the cutting piece, $\text{bcd}_n^R(\mathcal{SW}_{d,T}f; \mathbb{F})$ can be used as an input feature in classification algorithms for chatter detection.

Periodicity and quasiperiodicity in video data. A video can be thought of as a multi-dimensional time series: the result of sampling a function $f: \mathbb{R} \rightarrow \mathbb{R}^k$. It follows that computing $\text{bcd}_n^R(\mathcal{SW}_{d,T}f; \mathbb{F})$ for the sliding window point cloud $\mathcal{SW}_{d,T}f \subset \mathbb{R}^{k(d+1)}$ can be used to detect recurrent behavior (e.g., periodicity and quasiperiodicity) in video data, without the need for tracking or surrogate signals. See for instance [30] for applications of these ideas to the problem of quantifying periodicity in video data, the detection of biphonation in high-speed laryngeal videolaryngoscopy, as well as the synthesis of slow-motion videos from recurrent movements [29].

Theoretical Investigations of Sliding Window Persistence

The barcodes $\text{bcd}_n^R(\mathcal{SW}_{d,T}f; \mathbb{F})$ have been used successfully in several applications as described above. In practice, there are two main challenges that must always be addressed. The first is parameter selection: appropriate values for $d \in \mathbb{N}$, $T > 0$, $\mathbb{F}$ and $T \subset \mathbb{R}$ need to be determined, given the application and computational resources at hand. The second challenge is validating the results. In tasks where the barcodes $\text{bcd}_n^R(\mathcal{SW}_{d,T}f; \mathbb{F})$ are used as features, one must quantify the likelihood that positive results are due to random fluctuations in the data. These challenges bring into stark focus the need for a theoretical understanding of $\text{bcd}_n^R(\mathcal{SW}_{d,T}f; \mathbb{F})$ as a function of all the parameters and the time series involved. The main difficulty here is that our knowledge of the homotopy type/homology of the Rips complex $R_\alpha(X)$, for arbitrary $\alpha$, is rather limited: planar circles [1] and gluings thereof [3] are essentially the only spaces where we have complete answers. I would like to highlight next some of what we do know.

Sliding window persistence of periodic functions. Let $\mathbb{T} = \mathbb{R}/2\pi\mathbb{Z}$. As a warm-up example for understanding the sliding window persistence of $f \in C^1(\mathbb{T}, \mathbb{R})$, let $L \in \mathbb{N}$, $\phi \in \mathbb{R}$, and $\zeta(t) = \sin(Lt + \phi)$. A bit of trigonometry shows that $\mathcal{SW}_{d,T}\zeta(t) = \sin(Lt + \phi)u + \cos(Lt + \phi)v$ where $u = \mathcal{SW}_{d,T}\cos(Lt)|_{t=0}$ and $v = \mathcal{SW}_{d,T}\sin(Lt)|_{t=0}$. It readily follows that if $d \geq 1$ and the set $\{u, v\}$ is linearly independent, then $\mathcal{SW}_{d,T}\zeta = \mathcal{SW}_{d,T}\zeta(\mathbb{T})$ is a planar ellipse. The semi-major and semi-minor axes can be computed explicitly as

$$a = \sqrt{(d + 1) + \frac{\sin(L(d+1)\tau)}{\sin(L\tau)}}$$

and

$$b = \sqrt{(d + 1) - \frac{\sin(L(d+1)\tau)}{\sin(L\tau)}}.$$  

The persistent homology of the Rips filtration on ellipses with small eccentricity, i.e. when $b < a < \sqrt{2}b$, has been recently studied by Adamszek et al. [2]. In particular, their work implies that if

$$\alpha_1 = \frac{4\sqrt{3}ab^2}{a^2 + 3b^2}$$

and

$$\alpha_2 = \frac{4\sqrt{3}a^2b}{3a^2 + b^2}$$

then the homotopy type of $R_\alpha(\mathcal{SW}_{d,T}\zeta)$ for $0 < \alpha \leq \alpha_2$ is either that of the circle, or that of a wedge of 2-dimensional
spheres as follows:

\[ R_\alpha(S\mathbb{W}_{d,\tau}) \approx \begin{cases} S^1 & \text{for } 0 < \alpha < \alpha_1 \\ S^2 & \text{for } \alpha = \alpha_1 \\ \sqrt{5} S^2 & \text{for } \alpha_1 < \alpha < \alpha_2 \\ \sqrt{3} S^2 & \text{for } \alpha = \alpha_2 \end{cases} \]

The range \( \alpha_1 < \alpha < \alpha_2 \) is especially interesting since only one of the five 2-dimensional classes persists. In other words, the linear transformation

\[ H_2(R_\alpha(S\mathbb{W}_{d,\tau})); \mathbb{F}) \rightarrow H_2(R_\nu(S\mathbb{W}_{d,\tau})); \mathbb{F} \]

has rank one for every \( \alpha_1 < \alpha < \alpha' < \alpha_2 \). We readily obtain the following theorem.

**Theorem 0.6.** Let \( \zeta(t) = \sin(Lt + \phi) \) for \( L \in \mathbb{N}, \phi \in \mathbb{R} \), and for \( d \in \mathbb{N} \), \( \tau > 0 \) let \( a, b \geq 0 \) be as in (11). If \( a < \sqrt{2}b \) and \( \mathbb{F} \) is a field, then the maximum persistence (10) in dimensions 1 and 2 satisfies

\[
\begin{align*}
\text{mp}^R_1(S\mathbb{W}_{d,\tau} ; \mathbb{F}) &= \frac{4\sqrt{3}ab^2}{a^2 + 3b^2} \\
\text{mp}^R_2(S\mathbb{W}_{d,\tau} ; \mathbb{F}) &\geq \frac{4\sqrt{3}ab^2}{a^2 + 3b^2} - \frac{4\sqrt{3}ab^2}{a^2 + 3b^2}.
\end{align*}
\]

The cases \( \alpha > \alpha_2 \) for \( a < \sqrt{2}b \) and \( \alpha > 0 \) for \( a \geq \sqrt{2}b \) are currently open. The case \( a = b \), i.e. when \( S\mathbb{W}_{d,\tau} \) is a circle, is much better understood. This happens when the window size \( dt \) is equal to (an integer multiple of) \( d(\frac{2\pi}{d + 1}) \), which is a little bit under \( L \) times the period length \( \text{Period}(\zeta) = \frac{2\pi}{L} \). Since the homotopy type of \( R_\alpha(S^1) \) is known for all \( \alpha > 0 \) [1], we get that

\[ R_\alpha(S\mathbb{W}_{d,\tau}) \approx \begin{cases} S^{2k+1} & \text{for } \sin\left(\frac{\pi k}{2k+1}\right) \leq \frac{\alpha}{\sqrt{2(d + 1)}} < \sin\left(\frac{\pi (k+1)}{2k+3}\right) \\ 2k^2 & \text{for } \alpha = \sqrt{2(d + 1)} \sin\left(\frac{\pi k}{2k+1}\right) \end{cases} \]

where the linear transformation

\[ H_{2k+1}(R_\alpha(S\mathbb{W}_{d,\tau})); \mathbb{F}) \rightarrow H_{2k+1}(R_\alpha(S\mathbb{W}_{d,\tau})); \mathbb{F}) \]

is an isomorphism (with rank one) for every

\[ \sqrt{2(d + 1)} \sin\left(\frac{\pi k}{2k+1}\right) < \alpha \leq \alpha' \]

\[ < \sqrt{2(d + 1)} \sin\left(\frac{\pi (k + 1)}{2k+3}\right). \]

Therefore,

**Theorem 0.7.** Let \( \zeta(t) = \sin(Lt + \phi) \), let \( \mathbb{F} \) be a field, and let \( \tau = \frac{2\pi}{a(d + 1)} \). Then, for every integer \( k \geq 1 \),

\[
\begin{align*}
\text{mp}^R_{2k}(S\mathbb{W}_{d,\tau} ; \mathbb{F}) &= 0 \\
\text{mp}^R_{2k-1}(S\mathbb{W}_{d,\tau} ; \mathbb{F}) &\geq \sqrt{2(d + 1)} \cdot \left( \sin\left(\frac{\pi k}{2k+1}\right) - \sin\left(\frac{\pi (k - 1)}{2k-1}\right) \right).
\end{align*}
\]

One strategy to understand \( \text{bcd}^R_n(S\mathbb{W}_{d,\tau}f; \mathbb{F}) \) for a function \( f \in C^1(\mathbb{T}, \mathbb{R}) \) is to first approximate \( f \) by its truncated Fourier series

\[
S_N f(t) = \sum_{|n| \leq N} \hat{f}(n) e^{int} , \quad \hat{f}(n) = \frac{1}{2\pi} \int_{\mathbb{T}} f(t) e^{-int} dt
\]

and then investigate the asymptotic behavior of the sequence of barcodes

\[ \text{bcd}^R_n(S\mathbb{W}_{d,\tau}S^n f; \mathbb{F}) , \quad N \in \mathbb{N}. \]

Indeed, the analysis of \( \zeta(t) = \sin(Lt + \phi) \) presented earlier can be bootstrapped to trigonometric polynomials, and the Stability Theorem [8] can be used to study \( \text{bcd}^R_n(S\mathbb{W}_{d,\tau}f; \mathbb{F}) \) via the behavior of (12) as \( N \rightarrow \infty \). This line of reasoning was explored in [23]. In particular, it yields insights for the choice of window size (it should be approximately \( \frac{d}{d+1} \) times the period length \( \text{Period}(f) \)), the embedding dimension (larger than twice the number of relevant harmonics), and the choice of field of coefficients (one whose characteristic does not divide \( 2\pi \)).

We end with a theorem [23, 6.8] relating the sliding window persistence of other families of functions. There are some results for quasiperiodic time series [21], but the rest of the landscape is essentially uncharted territory. There is also recent work in identifying families of functions whose sliding window point clouds recover other spaces [32] — e.g., Klein bottles, projective spaces, etc. It would be very interesting to see these models in naturally occurring phenomena, and perhaps in future applications of topology to the analysis of complex time varying data.
References


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Jose A. Perea
The AMS has not always been especially interested in education or outreach. In fact, in 1916 the *American Mathematical Monthly*, which had fallen on hard times, came to the Society for help. Not interested, replied the AMS leadership, education is not our thing. The Mathematical Association of America was created to save the *Monthly*. The AMS largely avoided education over the next half century, focusing mainly on mathematical research.

But this has changed in the past thirty years. The AMS Committee on Education (COE) was formed around 1990 and soon began to meet annually in Washington, DC. From 1995 onward those meetings included presentations, often from people based in Washington, and often from organizations outside mathematics. The presentations were of interest more broadly, and soon COE invited mathematicians from outside the committee to attend—to learn something about what other disciplines were doing as well as to hear from Washington-based organizations working on education at all levels. This not only gave research mathematicians a chance to find out what others were doing but also gave the others a chance to find out what was going on in mathematics.

In recent years, these COE presentations were less well attended, and in 2018 the Committee set out to rejuvenate them by organizing a one-day “mini-conference” as part of its annual meeting. Led by Katherine Stevenson, the conference took place on October 12 and offered seven presentations, augmented by two roundtables (micro-conferences?). Topics touched on everything from the push for *BIG* (Business, Industry, and Government) to recent attempts to establish Federal priorities for STEM.

One of the most thought-provoking talks was by Christopher Edley, president and co-founder of the Opportunity Institute (and former Dean of UC Berkeley Law School), who argued that current math requirements, both in high schools and undergraduate institutions, act as barriers rather than entry points to STEM. There is a growing movement for multiple pathways into STEM, and Edley made the argument for such efforts—that if mathematics requirements serve primarily as barriers, they are likely doing more harm than good, both to the students and to mathematics.

The seven presentations had a wide scope and engaged the audience:

- Supporting careers in business, industry and government (BIG), Rachel Levy
- TPSE: creation, evolution, and agenda, Karen Saxe and Uri Treisman
- The role of mathematics in the study of visual processing in the brain, Ellen Hildreth
- From the heat equation to financial security, Sonin Kwon
- Rethinking STEM, Jake Steel
- Math achievement: law, policy, and post-secondary opportunity, Christopher Edley
- What math do we want non-STEM college majors to know? Manil Suri

COE will have another mini-conference associated with its meeting in October of 2019. These small conferences help to connect mathematicians to the education world at large, which doesn’t seem to happen spontaneously. Twenty years ago, when the AMS Taskforce on Excellence interviewed deans about their mathematics departments, the deans were convinced that mathematicians are largely
focused inward on their own discipline. Six years ago, when the President’s Council of Advisors on Science and Technology produced a major report on undergraduate STEM education, they opined that mathematicians make little effort to improve math education. These are both misperceptions, but they reflect a reality: Mathematicians don’t always work hard to engage in dialogue with those outside the discipline. That’s especially true in education. The mini-conference is a way to remedy this.

—John Ewing, AMS Committee on Education

Modeling and Data Analysis
An Introduction with Environmental Applications

John B. Little, College of the Holy Cross, Worcester, MA

It is great that mathematics is finally taught as a tool to understand the challenges that the planet will be facing and to participate in the debate. The book aims at developing skills in mathematical modeling and data analysis, with a focus on the environment. The projects encourage active learning. A wonderful book!

—Christiane Rousseau, Initiator of Mathematics of Planet Earth (MPE2013) and Professor of Mathematics, Université de Montréal

This book on elementary topics in mathematical modeling and data analysis is intended for an undergraduate “liberal arts mathematics”-type course, but with a specific focus on environmental applications.

2019; 323 pages; Hardcover; ISBN: 978-1-4704-4869-1; List US$79; AMS members US$63.20; MAA members US$71.10; Order code MBK/120

Available at bookstore.ams.org/mbk-120.
The Early Career Section is a new community project, featured here in the Notices. This compilation of articles will provide information and suggestions for graduate students, job seekers, junior academics of all types, and those who mentor them. Angela Gibney serves as the editor of this section. This month’s theme is Plan for a productive summer. Next month’s theme will be Getting ready for the academic job market and applying for grants.

Summer Break: Go Explore the Opportunities

Natalie Hobson

In graduate school, with the stress of qualifying exams, advisor and personal research expectations, thesis writing, and job applications, it can be tempting to save your "break" to catch up on work that is piling up. But the most productive way to spend your time is at summer schools, conferences, workshops, and teaching opportunities that are available to graduate students in the summer. These activities can deepen your connection to the mathematics community, diversify your professional experiences, significantly strengthen your CV, and recharge you in ways you might not expect.

Summer Schools

Summer schools often take place at research institutes. For example, during the summer, the Mathematical Sciences Research Institute (MSRI) offers a variety of two-week programs focused on areas related to the current mathematical theme at MSRI. In some of these, participants attend lectures and participate in breakout sessions. This affords the opportunity to meet other graduate students, and to learn more about a particular research topic in an immersed environment designed for graduate students. At an MSRI summer school, student participants have lunch and snacks with visiting professors and stay in the dorms at Berkeley. This builds community between students and researchers.

The AMS also offers summer learning opportunities through its Mathematics Research Communities (MRCs). These include intensive week-long programs. The MRCs are typically hosted in a nice resort area (for Summer 2019, the MRC will be meeting in West Greenwich, Rhode Island). The structure of the MRCs typically begins with some initial introduction to the problems developed by the research leads, and then students break into groups. For the remainder of the week, these groups tackle research problems. Many MRCs result in journal publications.

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topic of MRCs varies from summer to summer. Check out the AMS website for what will be offered this summer.

Another program I attended as a graduate student was the Women in Mathematics program at the Institute of Advanced Study (IAS) in Princeton. This is an annual two-week school aimed at supporting women in math. The participants include women from undergraduate to emerita. Each year there is a different mathematical theme. Participants get to stay near Princeton and during the weekend, make trips to New York City, Philadelphia, and tour the Princeton area. In this program, I had the opportunity to engage in many informal conversations with others who have similar struggles in mathematics. Opportunities such as this can help normalize your experience and encourage persistence and success. The friends I made with the other researchers and students there continue to be my closest connections in the math community.

There are many other summer schools and programs available, and one can find out about them by talking to people and looking online. Talk to the graduate coordinator or department chair at your school and ask what programs they are aware of. Often these people get the first emails advertising summer opportunities. You can find postings for summer schools on the homepages of many of the professional organization websites (e.g., AMS, MAA, SACNAS, SIAM, etc.) and advertised on posters around your department.

Conferences and Workshops
Attending conferences and workshops can be a great opportunity to introduce yourself to people in your area and learn more about a particular field. Look for conferences that are happening in other states or countries. Visiting a new location can help you connect with researchers in a region you would otherwise not know. While attending conferences, always make an effort to meet new people. Mealtime and breaks at conferences are often great times to introduce yourself to people you don’t know. Having flexibility with your schedule in the summer months can give you the opportunity to start collaborations.

To find out about conferences, talk to your advisor about which ones she or he is attending (try to go along with them!). Some researchers have lists of conferences in their area on their websites (you can often Google and find these lists). Attend nearby AMS and MAA meetings, which are listed on the AMS and MAA websites; often departments will help students pay for their trips.

Teaching and Outreach
There are a number of outreach programs offered in the summer for younger students. Graduate students are often able to lead and assist in the activities and education of the participants. Some include weekly math circles for elementary school students, residential math camps for middle or high school students, or research experiences for undergraduates. Some programs are residential (such as REUs and summer math camps). Assistants typically live with the participants, sometimes acting as teacher or research assistant during the day and counselors in the evenings. Such an opportunity can provide you with the experience in mentoring younger students in academics and life. These can be very rewarding experiences.

Talk to people to find out about outreach opportunities! Ask your contacts if they know of research assistantships or teaching assistantships in their REUs. Write to the people you meet at conferences and workshops and see if they know of any openings.

Like anything else worth doing, summer programs can be competitive, so if you aren’t selected to participate the first time, keep the connections you made in the application process, be persistent, and try again another year. It took me three attempts to become the graduate assistant at MSRI-UP, and it was certainly worth the effort.

Conclusion
What if you are in your last summer? Which of these activities should you prioritize? It depends on your future career interests! If you are looking for a job at a teaching-focused school, then teaching and outreach activities are what you should look for. If your dream is a job at an R1, you may want to prioritize participating in an MRC or summer conference. Of course any of these opportunities will give you experiences to learn, grow, and network.

It is up to you to get the most out of your summer months. Take this time to explore summer schools, conferences, workshops, and teaching opportunities. This will allow you to gain new experiences, visit new places, meet new people, and learn new math. You can still take some time for yourself to relax and recharge. Conference in Hawaii? Why not stay a couple days to “network” on the beach?

Credits
Photo of Natalie Hobson is courtesy of the author.
When organizing the first camp, we used infrastructure and knowledge from the other outreach activities in the mathematics department. We decided against hosting an overnight camp. We had big dreams of what our camp could be, and hoped to expand in future summers. Since that initial summer, we have grown to include three different week-long camps, with a staff of nine graduate and undergraduate students.

In 2015, 23 rising ninth through twelfth grade students attended the inaugural SIM Camp. Students classified compact surfaces, developed strategies for tic-tac-toe on a torus and Klein bottle, learned the basics of proofs and modular arithmetic, and enjoyed encrypting and decrypting messages using ciphers. Each day included plenty of breaks to enjoy the lovely outdoors, a daily math puzzle challenge for prizes, and catered lunch.

In 2018, there were three one-week camps for 80 rising eighth through twelfth grade students. These students learned problem solving techniques, game theory and probability, set theory, and fractal geometry.
One thing we found helpful when starting SIM Camp was working from our own experiences and bringing in other people for their expertise. Melinda and Claire were inspired to create a camp based on summer math experiences at the Carleton Summer Math Program for Women (SMP) and Johns Hopkins Center for Talented Youth (CTY). Our first summer’s curriculum drew heavily on notes and course documents from SMP and CTY. Simone was brought in for her experience working with kids. Our business office helped with logistics, and our first year’s registration form was modified from a form used by another camp on our campus.

An important part of starting SIM Camp was knowing our department and university’s policies for having minors on campus. The University of Illinois Police Department has a form that we submit for every event we host. We provide plans for medical and other (i.e., weather, active shooter) emergencies and make sure to have emergency contact information for each participant. We make sure to review campus policies every year in case they change.

As SIM Camp grew, we encouraged new instructors to use existing mathematical resources when creating lesson plans. Between general audience math books, various math outreach programs, and math blogs, there are a lot of resources for teaching middle and high school students all kinds of college- and research-level mathematics. Pulling ideas from these resources means having some already-tested materials and saves time during course planning.

One ongoing challenge is funding, since we have tried to keep SIM Camp free to campers while paying the graduate and undergraduate student staff. While our first year was primarily a volunteer effort, in the past three years we’ve been lucky to be funded by the Department of Mathematics, National Science Foundation under Grant Number DMS-1449269, and the Mathematical Association of America’s Dolciani Mathematics Enrichment Grants. Another way to fund a camp would be to charge tuition (you could still apply for additional funding sources).

Another challenge is building connections with the community to recruit campers from groups who are traditionally underrepresented in math and on our university’s campus. We are always looking for better ways of getting the word out to students and parents, convincing students to apply, and getting them to come. One way to do this is to email schools in the area and local parent groups, which often have some kind of an online presence.

Every year, we collect feedback from staff and participants. As a result of this feedback, in preparation for the third year of camp we implemented a weeklong pedagogy training for SIM Camp staff, and a half day of community building activities for campers on the first day of camp. We developed three core values for campers to focus on: 1) noticing Aha! Moments and supporting others’ Aha! Moments, 2) viewing math as a collaborative process.
3) asking lots of questions (we say ‘the right answer is the next question’).

On the first day of camp, we organized activities to support discussion of each of these ideas, and continued those discussions throughout the week.

Over our four years of running this camp, we have learned how to take a dream, formulate it into a concrete idea, bring it to life, and grow an organization. SIM Camp is an example of the positive impact graduate students can have on their home institutions and community. As we move into the next phases of our professional careers, we encourage other graduate students to pursue opportunities for mathematical community involvement.

Think about starting math outreach activities for kids in your area! This is really all you need to start:

• a goal and target age group/demographic (i.e., driving-distance, girls, underrepresented minorities)
• dates and facilities (rooms on campus, etc.)
• a way to recruit campers, such as an email to teachers or parent groups
• lesson plans and someone to run activities, such as other graduate students or postdocs
• university policies for minors on campus
• registration forms and possibly application forms
• if using an application, a way to decide which students come

Good luck and have fun!

Credits

All photos are courtesy of the authors.
You Should Organize Conferences and Workshops

Izzet Coskun

Organizing conferences and workshops is a great opportunity for early career researchers to learn recent developments in an area of mathematics, meet senior researchers, and start new collaborations. Running such events can help hone the organizational skills that are crucial to many aspects of academic life, make their CVs more attractive to future employers, increase their broader impacts, and make their grant applications stronger.

At the University of Illinois at Chicago (UIC), we encourage our graduate students and postdocs in algebraic geometry to organize workshops as part of our regular training program. Our graduate students take turns organizing the Midwest Algebraic Geometry Graduate Conference, an annual event where a senior researcher and eight graduate students give talks about their research and another dozen students present posters. Similarly, our postdocs organize a weekend workshop in an area close to their research where they invite several senior mathematicians whose work they would like to master. Our students and postdocs have found these events very beneficial. They have formed new collaborations and made many valuable connections. Serving as organizers helped them integrate into the research community.

Organizing a successful workshop or conference does not have to be onerous and with practice can become streamlined. Typically an organizer has several duties including securing funding, arranging the venue, taking care of the participants’ travel and lodging, designing a webpage and advertising, providing coffee and refreshments, organizing social events, and dealing with reimbursements. Sharing these duties among several people can make the tasks more pleasant and manageable.

It is possible to run a successful workshop with relatively little money (even in an expensive city like Chicago). One can build the workshop around a researcher who is already visiting the department or invite local speakers. One can choose to fund participants’ hotel rooms, but not travel. By holding the workshop off-season, one can get cheaper accommodation rates. Many departments also have funding that can be used to organize small workshops. The department may have a Research Training Grant (RTG) or some colleagues may have Focused Research Grants (FRGs), or start up funds, which they may be willing to use to support a workshop. Departments may have visitor funds for inviting an outside speaker. It is also possible to secure conference funding from agencies such as the National Security Agency and the National Science Foundation.

Typically, a conference would take place at the organizer’s home institution or at one of the institutes dedicated to running mathematics conferences such as the Mathematical Sciences Research Institute (MSRI), the Fields Institute, the American Institute of Mathematics (AIM), the Mathematical Research Institute of Oberwolfach, Centre International de Rencontres Mathématiques (CIRM) in Luminy, the Banff International Research Station (BIRS) for Mathematical Innovation and Discovery, or the Institute for Computational and Experimental Research in Mathematics (ICERM). Conferences and workshops at such institutes have to be planned well in advance and have to follow the institute’s rules. There are competitive application processes. Early career researchers can team up with more senior colleagues to organize a workshop at such an institute. These institutes have experienced professionals who attend to the organizational details and make sure that the conferences and
workshops run smoothly. The organizers can concentrate more on the mathematical aspects. Running a workshop in one’s home institution has its advantages and disadvantages. One has more freedom in structuring the conference, but often the organizer has to be responsible for more of the organizational details. In this issue, Brendan Hassett has an article offering more details on organizing conferences at such institutes.

Each university has their own procedures and idiosyncrasies when it comes to reimbursements, booking travel, and reserving lecture or hotel rooms. Before organizing a conference or workshop, it is best to consult with a senior colleague who has organized successful conferences to learn these procedures. Communicating early and clearly with the academic professionals who handle hotel reservations and reimbursements and giving them accurate information in an organized and timely fashion is crucial both for making their jobs easier and the process run more smoothly.

To advertise the conference it is important to make a website that has all the crucial information such as the dates and venue, speakers, schedule, travel, accommodations, and local information. Advertising depends on the size of the event. If you would like a small workshop only for local participants, emailing the math departments of nearby universities may do the trick. For larger participation, emailing a wider variety of colleagues to direct their students and postdocs to the conference website is necessary. There are conference lists by field. You can reach a wide audience by posting the conference to these lists. If you are organizing a special session of an AMS meeting, you can also advertise in math publications like Notices of the AMS. Attracting a diverse group of participants can be challenging. Having a diverse group of speakers, and announcing the conference in lists reaching underrepresented groups can partially alleviate the problem.

A workshop or conference can take many different formats. One of my favorite formats is the weekend learning workshop. When we run these workshops at UIC, we choose a topic we would like to learn about. We invite two or three speakers to give mini-courses and allocate three hours to each speaker. We ask them to dedicate half an hour to a problem session. We often also invite several graduate students and postdocs working in the area to give hour-long talks. Over the years, I have found this format to be a good way to learn new developments in areas close to mine.

AIM style workshops, where speakers introduce problems and techniques in the morning and the participants work on an open problem in the afternoon can be rewarding and result in valuable publications. These workshops are harder to organize. Careful thought needs to go into choosing the problems. They have to be interesting, yet approachable. In my experience, these types of workshops succeed best when researchers from two different areas that do not usually interact come together to solve problems that are easier from the other perspective. Having too ambitious a research plan or too difficult a problem can make these workshops less successful.

Other typical formats include conferences where speakers are invited to present their work in hour-long talks. I find that having fewer talks and leaving time for mathematical discussions and interactions make such events more beneficial. For example, one can allocate time for people to discuss problems. A schedule too crammed with talks tends to be too tiring for participants and stifles interaction. Depending on the audience and the aims, one can add poster sessions, career development sessions, and problem sessions to the program. One can also mix and match and vary these formats.

I have found organizing Early Career Bootcamps and Math Research Communities to be especially rewarding. I believe participants really benefit from such events. I have made some of my life-long mathematical friends in such conferences. These events can benefit organizers immensely as well. They are a great way of meeting and learning about young mathematicians. They can be a gateway to hiring. Several of the participants have later become my postdocs or colleagues.

Whatever the format, it is crucial to choose the speakers carefully. Having a diverse set of speakers often helps the success of the conference. Having a large percentage of the speakers be good expositors in addition to being great mathematicians will help keep the audience engaged, increase participation, and promote the success of the conference.

When organizing a conference, it is important to be flexible. Something unexpected is bound to happen. I have had a storm wreak havoc on participants’ travel schedules, plenary speakers cancel at the last minute, and the department’s credit card fail. As long as you try your best, participants will understand. My final advice: always have a spare donut for the campus police when you find the conference building locked in the morning. A big smile and a donut can open many doors.

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Organizing Workshops and Programs at Math Institutes

Brendan Hassett

The only reliable way to be invited to good conferences is to organize them yourself!

I remember getting this advice from a distinguished mathematician—and prolific conference organizer—a few years after my PhD. This article will help you put this into practice.

What are the Institutes?
In the United States, the Division of Mathematical Sciences (DMS) of the National Science Foundation (NSF) currently supports eight Mathematical Sciences Research Institutes: the Institute for Advanced Study in Princeton (IAS), the Mathematical Sciences Research Institute in Berkeley (MSRI), the Institute for Mathematics and its Applications at the University of Minnesota (IMA), the American Institute of Mathematics in San Jose (AIM), the Institute for Pure and Applied Mathematics at UCLA (IPAM), the Mathematical Biosciences Institute at Ohio State University (MBI), the Statistical and Applied Mathematical Sciences Institute in the Research Triangle of North Carolina (SAMSI), and the Institute for Computational and Experimental Research in Mathematics at Brown University (ICERM). While these differ in mission and organizational structure, they all offer opportunities to early career mathematical scientists to participate in research programs, ranging from small collaboration groups through week-long workshops to year-long, large-scale thematic programs. Indeed, the long-term professional development of early career program participants is a crucial metric of the success of these institutes.

There are other US-based institutes sharing many attributes with the DMS-supported institutes, including the Center for Discrete Mathematics and Theoretical Computer Science at Rutgers (DIMACS), the Simons Center for Geometry and Physics at Stony Brook (SCGP), the National Institute for Mathematical and Biological Synthesis at the University of Tennessee (NIMBioS), etc. However they are funded and organized somewhat differently, so I will focus on the eight NSF Mathematical Sciences Research Institutes listed above.

What do they do?
Successful mathematicians have deep relationships with former mentors, friends from graduate school, past department colleagues, collaborators, etc. These connections form much of the human capital in the field. Institutes create connections among mathematical scientists based on communities of shared scholarly interests. Early career mathematicians often have the most to gain by cultivating such contacts, which can last a lifetime.

Institutes also help focus the attention of the mathematical community on important developments in the field. By helping organize a program, you can have a voice in deciding what these should be.

Who decides on Institute programs?
With the exception of the IAS, where the permanent faculty members play a central role in planning research programs, institutes choose programs through proposals originating from the mathematical sciences community. Typically, institute directors (leaders) help solicit ideas which are then evaluated by a scientific advisory board of outside experts. Criteria differ across institutes, reflecting their varying mis-
sions. Some prefer pure mathematics programs supporting well-defined disciplinary communities; others focus on applied topics; and still others promote connections between the mathematical sciences and other disciplines.

**Where do the proposals come from?**

It depends on the size and complexity of the activity. Small collaboration groups, like Collaborate@ICERM and SQuaREs at AIM, bring groups to the institutes for a short period of time to work on specific problems. These are probably the most accessible to early career mathematicians—some explicitly seek untenured participants.

Weeklong workshops can also include early career mathematicians as organizers, often in collaboration with more senior members. But there are successful meetings organized largely by untenured faculty and postdocs.

Graduate summer schools often include early career organizers. Many senior faculty cannot spare three weeks to give a series of lectures to a group of doctoral students. Summer schools often involve postdocs and untenured faculty as group leaders and TAs as well.

Semester and year-long programs are mostly organized by tenured faculty. Such a program may take years to develop. It is hard for untenured people to commit to participating in a program years in advance. However, within such programs there are organizational roles often performed by early career participants, e.g., graduate student and postdoc seminars are often coordinated by students and postdocs.

**Why organize a program at an Institute?**

Organizing conferences and programs, like writing grant proposals, focuses your own research program in a way that allows you to do more and better research. And it presents opportunities to influence the research directions of your mathematical community in a way that aligns with your mathematical values.

You might assume that it would be easier to run a workshop in your home department than at an institute far away. It saves the trouble of travelling. Also, writing a conference grant can be a great learning experience; they can be less competitive than individual grants and open a window on the grant-writing process.

Institutes do have advantages. Department staff have many regular duties and may not have the time to provide conference support. However, institute staff exist to support the organizers of their programs and are highly trained in the fine points of hosting conferences. The first time I organized a conference in my department (as a junior faculty member) I was personally responsible for creating the webpage, unlocking the lecture hall, delivering food, cleaning up afterward, etc. When I led programs at institutes, all these details were taken care of. Directors and scientific staff at institutes can also share their experience of what makes a successful meeting. A week away from the quotidian demands of students, colleagues, and family can also help one focus on research. It is easier to make mathematical connections over long coffee breaks and conference dinners. (However, institutes sometimes offer support for childcare and assistance to participants accompanied by young children.)

Another benefit of organizing conferences and programs at institutes is their visibility and prestige, which helps attract speakers and workshop participants and thus increases the impact of your program.

**How do I start?**

Talk to someone with a connection to an institute. This could be an organizer of a previous program, a member of the scientific or governing board, or an institute director. Sketch briefly what you have in mind and be prepared to get feedback. Most programs are the culmination of an iterative process starting with a conversation at tea leading to a phone call, then a program sketch, and finally a formal proposal. Make sure you review the solicitation instructions before submitting—scientific advisory boards do not want to turn back incomplete proposals.

**What are common challenges?**

**Scientific planning.** We have all been to events that bring together the same 25 people regularly to give the same talks to the same audience. Be prepared to involve people outside your immediate circle of friends and collaborators rather than just the ‘old boy network.’

Institutes exist to promote research and develop human capital, including individuals from underrepresented groups or schools with limited research resources. Keep this in mind as you choose speakers and review applications for funding.

Some institutes have specific guidelines for how workshops are structured. AIM emphasizes active collaboration rather than just a series of lectures. ICERM seeks a balance between mathematical theory and computational experiments and examples. There is no point pushing back against guidelines central to an institute’s missions.

Sometimes a workshop does not work out as planned due to factors outside your control and has to be re-imagined in mid-stream. The cause might be a research breakthrough, a competing event, the illness of a leader in the field, etc. Be flexible as new information comes in.

**Communication.** When an organizing committee maintains radio silence it multiplies the work of institute management. And if some of the group takes this approach it is a burden on the remaining conscientious organizers—the uncommunicative members are delegating their responsibilities.

Be cautious about making commitments to speakers and participants. Most institutes have standard policies and reimbursement rules reflected in their invitation language. Don’t create confusion by giving conflicting information.
Mathematicians tend to be self-reliant—we seldom have secretaries to manage schedules, webpages, refreshments, etc. At an institute, allow the staff to do their jobs and treat them with respect and courtesy. While they can make mistakes or misunderstand what organizers want from a program, institute staff often know more about running a workshop than most mathematicians!

**Professional conduct.** A challenging aspect of organizing a conference as an early career mathematician is confronting unprofessional behavior: a high status colleague badgering a speaker with repetitive and dismissive questions; heated arguments among participants; and even discrimination or sexual harassment. Bring these to the attention of the directors! They are responsible for maintaining a professional atmosphere and usually have experience with difficult conversations.

The popular view of mathematicians working alone in their offices is a myth. Mathematics is a social activity! Creating conditions where this can occur requires resources, planning, vision, and scientific direction. Institutes rely on members of the community—including early career researchers—for their vision and leadership. Organizing institute programs can have a profound impact on both your personal research and the future development of your scholarly community.

**Credits**

Author photo is courtesy of ICERM.

**ACKNOWLEDGMENTS.** Thanks to Ruth Crane, Erica Flapan, Angela Gibney, and Elisenda Grigsby for comments on this manuscript.

**POSTSCRIPT, 5-17-19:** It is now also possible to make a topic proposal for an IAS special year, see [www.math.ias.edu/special-years](http://www.math.ias.edu/special-years).
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Anatole Katok—A Half-Century of Dynamics

Boris Hasselblatt

Anatole Katok (August 9, 1944–April 30, 2018), a leading dynamicist, a Fellow of the American Mathematical Society, and a member of the American Academy of Arts and Sciences, began his career in Moscow, spent six years each at the University of Maryland and the California Institute of Technology, and worked at the Pennsylvania State University for almost three decades. He died in Danville, PA, from pneumonia and complications from an infection. Robert J. Zimmer of the University of Chicago called him “a whirlwind of mathematical activity” who expanded the boundaries of his field and “brought new connections and engaged all around him with an infectious and buoyant enthusiasm for mathematics and its mysteries” [32]. He “was an extraordinary man and a great mathematician, one of the giants in dynamical systems and ergodic theory” (Gregory Margulis, Yale), “a singular indomitable force of will” (Michael Boyle, Maryland), and “one of the most inspired, and inspiring, mathematicians of a generation” (Marcelo Viana, IMPA). Indeed, from early in his career Katok stood out for his ability to be involved in essentially all areas of dynamical systems. He always worked in dynamics in a broad sense, and his interests ranged widely, both scientifically and in terms of the character of his publications and activities. Beyond his own large body of work, at the breadth of which his Annals papers [1, 5, 7, 13, 15, 18] only hint, he fostered the creation of mathematics through mentorship; the sharing of ideas; and the organization of conference series, journals, and the Center for Dynamics and Geometry at the Pennsylvania State University, which was recently endowed and renamed in his honor.

Moscow

Anatole Katok, “Tolya” to his friends and family, was born a mere 200 miles from where he last lived, in Washington, DC. His father Boris (a metallurgical engineer) and his mother Dora (a chemist) belonged to a Soviet delegation working with the American lend-lease program. Begun even before the attack on Pearl Harbor, this program helped defeat the Axis powers by “lending” food, oil, and materiel to allied countries, principally the British Empire and the Soviet Union. The latter received some 11 billion USD worth of support (out of over 50 billion total), and this was crucial for the Soviet military effort. Even much later, Katok recalled ancient US vehicles from this program as a significant part of the Soviet automobile landscape. His

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Communicated by Notices Associate Editor Bryna Kra.

1Анатолий Борисович Каток in Russian usage.
parents were sent on this mission by the government, but not as diplomats. As a consequence, Katok was a US citizen by birth, a material point in his later biography. And he relished pointing out that he was therefore eligible to be elected president of the United States.

Katok’s father ranked high in the Soviet hierarchy, and he died in his early sixties (apparently due to heart problems). Katok was raised principally by his mother, from whose first marriage he had a half-brother, Alexander (Sasha) Gruz, 10 years his senior. In 1975 Sasha and Dora immigrated to the DC area, where she lived to the age of 93. She recognized and cherished Katok’s talents early, and he was an acknowledged wunderkind. Among his favorite childhood books was the Great Soviet Encyclopedia. He once told me that he had some regrets about the disdain for the practical that came with this admiration for the high feats of his intellect, which left him being “challenged” sometimes in practical or hands-on matters. Indeed, he did not take to technology naturally. In the initial years in Maryland he did not care to even acquire a driver’s license—but soon learned to drive cars with a manual transmission, in the ancient Beetle of Robbie Robinson, his first US doctoral student. He loved cars (his coolest was probably the yellow early 70s Mercedes-Benz convertible with hard and soft tops), though at times their basic functions would require some concentration. In the late 1980s there was also discernible surprise when colleagues first got an email from him. But he was no Luddite: he embraced email, and over time his hammering index fingers would type substantial texts. And he maintained a large and useful web page as well as a Google Scholar account.

In the ninth grade, he transferred to Moscow School N59, from where the likes of Arnold and Maslov had graduated, and he intended to enter the Lomonosov Moscow State University before his tenth and final grade.

In 1960, Katok’s first year at the university, with Kazhdan (first row, second from the left), Margulis (upper left corner), and Sergey Gelfand (right upper corner, third from the right). Katok liked this photo very much and kept it on his office wall.

With the help of Aleksei Markushevich, the vice-minister of education of the RSFSR, he obtained permission to take the external examination for high schoolers after the ninth grade and hence never needed to graduate from high school. Yulij Ilyashenko recalls a newspaper headline “Markushevich was struck by the mathematical talent of the youth.” Katok having been an alumnus of “mathematical circles” for high-school children (in his case supervised by A. Egorov and N. Vasiliev), Katok’s next move was to affect his life well beyond his career. Upon entering the university in 1960, he immediately began teaching such a circle himself, and among his students was Svetlana Rosenfeld, three years his junior (and also at Moscow School N59). They fell in love and married as early as legally possible. They were married for 52 years. Their daughter Elena was born in Moscow and is now the Ashbel Smith Professor at the Naveen Jindal School of Management of the University of Texas at Dallas. Their son Boris, also born in Moscow, is a Senior Software Developer and the owner of Coconut Tree Software in Sparks, Nevada. Danya Katok was born in Hollywood and is an acclaimed soprano in New York City. Katok was rightly proud of his wife and children and went out of his way to support them.

Katok started his third year under the supervision of Robert Minlos, but his enthusiasm waned by the end of the year. While Minlos was an inspiring teacher, the problems he assigned in his seminar required a lot of technical skill and bored Katok. Thus, Katok started attending Sinai’s special course

3Now archived at akatok.s3-website-us-east-1.amazonaws.com
on ergodic theory in 1963, just as Sinai began to receive wide recognition. They started meeting and discussing mathematical problems, and Katok worked with Sinai for his last two and a half years at the university as well as in graduate school.

The theory of dynamical systems (Katok was not fond of the term “chaos theory” used in connection with some applications) is a mathematical discipline that studies group actions—initially, \( \mathbb{Z}_p \), \( \mathbb{N}_p \), or \( \mathbb{R}_p \)-actions—with a view to describing the long-term behavior in probabilistic, geometric, smooth, and topological terms. With the seminal work of Kolmogorov, Smale, Anosov, and Sinai, dynamical systems emerged as an independent mathematical discipline in the 1960s. In this exciting time of transition young mathematicians found this rapidly developing theory attractive and began to work in it under the direction of Sinai, Alexeyev, and others.

Katok received his doctoral degree from Moscow State University in 1968, and his dissertation reflected much-cited work with Stepin in ergodic theory about periodic approximations of probability-preserving transformations. The work has numerous applications and helped solve some long-standing problems that went back to von Neumann and Kolmogorov. It earned Katok and Stepin the prestigious prize of the Moscow Mathematical Society for young mathematicians, and Katok presented it in a 15-minute communication at the 1966 International Congress of Mathematicians in Moscow. He went well past the allotted time until he was stopped by a question: “Is this a short communication or an invited talk?”

At that time, Katok took a reading course on algebraic topology with Dmitri Anosov, a dynamicist at the Steklov Institute (the Mathematical Institute of the USSR Academy of Sciences), who played a central role in the creation of the modern theory of hyperbolic dynamical systems. Soon after, Katok joined Anosov in the invention of the Anosov–Katok “approximation by conjugation” method or “method of fast periodic approximations” for the construction of dynamical systems with interesting, often exotic properties. Katok wrote a complete 50-page draft of the paper in three successive evenings and later said that the “best way to overcome a depression is to sit and write a mathematical paper.” This work was called “the main event of the year in ergodic theory” and is an ingenious tour de force in elliptic dynamics. Meanwhile, Anosov’s influence and style drew Katok’s interests toward hyperbolic dynamics, which made explosive progress in this period. The volume with Katok’s lectures from a 1971 summer school in Katsiveli (Ukraine) [17] became the main early Moscow text on differentiable dynamics. His lectures systematically present hyperbolic dynamics based on both some of the Western work and on the Anosov blueprint for studying the topological dynamics of hyperbolic sets using shadowing. This was much-needed and influential exposition but also cutting-edge mathematics, and notably, this summer school occurred a mere three years after Katok’s doctorate.

Searching the literature by authorship misses other essential contributions of his, however. As the mentor of Michael Brin and Yakov Pesin, Katok supported the development of the theories of both partially and nonuniformly hyperbolic dynamical systems. While in a (uniformly) hyperbolic dynamical system each tangent space splits into complementary subspaces in which the action expands or contracts, respectively, a partially hyperbolic dynamical system includes a third “center” subspace in which contraction and expansion are allowed but at lesser rates than in the former subspaces. Nonuniformly hyperbolic dynamical systems are just that: they possess expanding and contracting subspaces as well, but there is no uniform control of them nor of their contraction and expansion rates. While it is natural to expect that pushing the techniques from uniformly hyperbolic dynamics is the right strategy, the technical challenges are enormous. Pesin recalls struggling to explain his theory of nonuniform hyperbolicity in the publication, and Katok read drafts, discussed them generously, helped with the exposition and with putting this work in context—his vision and perspective were well ahead of Pesin’s at the time and crucial for this work. Yet, despite this deep involvement in the work, Katok refused to be named as a coauthor of what is now known as Pesin theory. This is why Brin and Pesin (and the Mathematics Genealogy Project) consider Katok their doctoral advisor even though he was not allowed to officially serve as such, and Anosov’s involvement was deep and crucial as well.

Not long after, Katok and Pesin discussed how to extend the theory of nonuniform hyperbolicity to systems with discontinuities. This had begun in the Anosov seminar and was motivated in great part by billiard systems. As they traveled to the seminar together one day, they used this lengthy commute to continue these discussions. Part
of this trip was a 30-minute bus ride, and it was on this bus that Pesin realized that the missing sufficient condition (beyond previously known assumptions on derivatives) was that the volume of a neighborhood of the discontinuity set be bounded by a power of its thickness. Just then, the passengers were asked to produce their tickets, and the pair realized that in their mathematical excitement they had forgotten to buy one. The fine was some 10 times as much as the bus fare, and Katok declared, “This is the price of a new mathematical discovery.” Their convulsions of laughter over having been caught without tickets because of this breakthrough caused some puzzlement among the other passengers. Because the impending departure of the Katoks made it infeasible to collaborate on writing this result up together, they agreed that Katok would publish it himself abroad—a tremendous effort. The resulting book with Strelcyn has since been the standard reference on hyperbolic systems with singularities.

Katok was at Moscow University during the golden age of Moscow mathematics, and at times he explained that his choice of mathematics as a vocation was influenced by the relative freedom mathematicians enjoyed because their discipline was least affected and controlled by ideological impositions. He thought that otherwise he might have chosen to become a historian or a diplomat. To those who knew him, diplomacy seems a less natural fit, but he had an inclination to statesmanship when it was for a good cause. The Katoks visited Kiev in 2014, and then again in June 2015 after the Russian attack. A planned conference had been canceled, then postponed, because participation had collapsed in the face of news coverage. He encouraged others to go, spoke at the conference, and made a point of staying longer to show support for Ukraine and Ukrainian mathematics. He understood that sometimes just being there goes a long way. He was also an admirer and a real friend of Poland and of Polish mathematicians, and he had a passion for organizing doctoral schools and conferences in Bedlewo near Poznan.

History remained a strong interest of his throughout his life, and stories abound about his prowess in that field. Many report on his command of the underlying facts, as demonstrated during a dinner conversation about nations that used to be part of the Soviet Union. When someone claimed that Armenia had “a population of 3 million,” Katok interjected “Not 3 million, I don’t think so, maybe 2.9 million at most,” whereupon a quick internet search revealed that the most recently recorded population was 2.89 million. Another example involves a challenge to name all the US presidents, which Katok dismissed as trivial, proceeding to name them all, plus all vice presidents and all Roman emperors in order without hesitation. He apologized for not knowing in every case the exact years of their reign. In his first conversation with Katok, Mariusz Lemanczyk from Poland had to answer the “exam” question of why the kingdom of Poland experienced exceptional prosperity during the second half of the 14th century (the answer being that the plague spared Poland). And when a conference in Bedlewo featured a bus excursion, Katok took the microphone at the front of the bus to bring everybody up to speed on the relevant pieces of Polish history. “The drive was not short, and neither was the history lesson.”

Of course, as in mathematics, recall of the facts is helpful, but understanding how to work with them is a higher skill—which Katok possessed in both mathematics and history. Around 1970 the well-known geometer Mikhail Postnikov gave a special lecture in Moscow on Morozov’s “new chronology,” a fringe theory in history, which had fallen into obscurity when Postnikov took it upon himself to bring mathematics to bear on this problem and rescue the theory. His lecture to this packed auditorium was compelling. After a few audience questions, Katok, a lowly junior mathematician and as such in the back of the auditorium, rose to say that he wanted to make some comments and that this might take some time. He then dismantled in a perfectly professional manner Postnikov’s position on almost every single fact from the lecture. The audience was persuaded and looked at Postnikov for a reply. After a pause Postnikov said, “A week ago I gave the same presentation at a meeting of the Moscow Historical Society to professional historians. None of them presented such serious professional comments, and I have to think about them.” Next, the well-known historian Lev Gumilyov came to the podium and said that he had been invited to present his view, but that his “young colleague” had already said everything he had to say.

Regrettably, mathematics was increasingly less sheltered by that time. From the late 1960s anti-Semitism and suppression of liberal thought grew at Moscow State University, and almost no Jews were accepted as students or faculty. So, Katok instead assumed an appointment at the Central Economics-Mathematics Institute (CEMI) of the USSR Academy of Science, which allowed him to combine work on mathematical problems in economics, if any, with research in pure mathematics. Katok had been attending the Alexeyev–Sinai seminar at Moscow State University regularly, and in 1969 he and Anosov started a seminar at the Steklov Institute (and moved to CEMI in 1975) with Katok as the driving force. At that time many mathematicians labored in day jobs unrelated to mathematics, and for them, these seminars were a vital connection to the research enterprise. In addition to talks about research by participants and guests, many seminar meetings were about works by other mathematicians, mostly foreigners, often based on preprints, which were rare and precious. The Alexeyev–Sinai and Anosov–Katok seminars were the main engine that put Moscow at the forefront of developing the modern theory of dynamical systems.
Katok’s active seminar and conference participation is well remembered for the way in which it enriched the experience and value of a talk for both the audience and the speaker. He would comment on the meaning of a result, the context that makes it interesting, prior work on which it builds, and why it is important and interesting. Likewise I remember from his classes those distinctive moments when he turned from the board to face the audience and explain what it all means. He did not prepare the technical points of his classes in advance, and the proofs were presented off the cuff, which meant that details could be missing or incorrect. But proofs serve to illuminate, and their ideas as well as the rich context he provided gave a unique picture of the subject.

In my Caltech days Katok ran a seminar as well as a working seminar; in the latter one students learned the communal practice of mathematics, and they would often be organized around a topic. I also do not ever remember him taking notes in a seminar or any other context. He confessed to this as a weakness, but to my mind it always reflected two major characteristics. Thanks to his prodigious memory he did not need notes in the first place, and furthermore, his encyclopedic knowledge and vision of mathematics made everything new that he learned about an organic part of an interconnected landscape he consummately inhabited and in which he could place and locate each item at will.

Returning to the post-doctoral decade in Moscow, it is striking how Katok’s work quickly spanned the breadth of dynamical systems well beyond his imprint on hyperbolic dynamics. He continued his work in ergodic theory with a substantial body of work on “monotone equivalence” (often called Kakutani equivalence and based on a far-reaching generalization of the concept of time-change in flows), and his student Satayev did “excellent thesis work on Kakutani equivalence theory that came earlier than a similar project by the giants of ergodic theory D. Ornstein, D. Rudolph and B. Weiss and was only marginally weaker than theirs.” A paper in elliptic dynamics astonishes experts to this day: He showed that without the required nondegeneracy assumption, the conclusions of the famed Kolmogorov–Arnold–Moser Theorem are completely reversed. His entry into parabolic dynamics proved lastingly influential as well: it involved the creation of the Katok–Zemlyakov “unfolding” method for the study of polygonal billiards; this idea was foundational for important parts of parabolic dynamics. Thus, within a decade from his doctorate, Katok was profoundly influential and internationally known not only in ergodic theory but in elliptic, parabolic, and hyperbolic dynamics.

Domestically, the situation started less ideally for the Katoks. In 1971 they had two children and lived in a single room of some 230 ft² in an apartment with two other families and a 4th room occupied until 1970 by Svetlana’s grandparents, quite a step down from Anatole Katok’s upbringing in a single-family apartment. Around that time, emigration became a realistic possibility, and some friends and colleagues started leaving. The Katoks considered the possibility but decided against it. Occasionally “cooperative apartments,” whose construction began in the 1960s, became available for purchase, and the family was eligible for an apartment of some 800 ft². These were difficult to get, but they found a rather large and expensive one—for which
they saw no way of producing the required down payment of 40%, worth some 18 months of their gross household income. They recalled with gratitude that Anosov lent them the entire sum without interest and with an indefinite term of repayment.

**Metropolitan Moves**

In 1972 the Banach Center in Warsaw was established by the Academies of Sciences of Bulgaria, Czechoslovakia, East Germany, Hungary, Poland, Romania, and the USSR, with the aim of promoting international cooperation in mathematics, especially between the East and West. Anatole Katok’s first trip abroad was for a 1975 conference there. By then his mother and brother had moved to the US, and the family’s tolerance for life in the Soviet Union was wearing thin. They finally decided to leave the country, and when Katok returned to Warsaw in 1977 for an international conference with many participants from the West, he made no secret of the plans to emigrate. Several people brought this to the attention of the mathematics department chair at Maryland, William Kirwan (later provost, president, and chancellor), suggesting that Katok was a rising star with the potential to become a superstar and that he would have many offers from prestigious universities. Kirwan called Katok in Moscow. Katok had no idea who was calling nor how much in demand he was, so Kirwan’s offer sounded very good to him. He accepted immediately.

On February 15, 1978 the Katoks emigrated from the Soviet Union. Vienna was the usual first stop on this exodus, and they still recall their joy at discovering a restaurant that offered a unique winning combination of cleanliness and price—and had the puzzlingly Scottish name of McDonald’s! (Twenty years later, we made a pilgrimage together to this very McDonald’s.) Next followed several weeks in Rome, and then a visit to IHES near Paris, where, for the first time in his life, Katok found himself with an appointment as a mathematician at a mathematics institute, and with an office of his own. Here he obtained some of his best-known and most widely quoted results on nonuniform hyperbolicity. Katok always recalled with gratitude the warmth with which the community of ergodic theorists in these cities received and supported them. That August, he turned 34, and the family arrived in Maryland, where Sasha and Dora, now residents of Rockville, picked them up at the airport. Anatole assumed his professorship, and Svetlana was finally able to pursue the doctorate in mathematics from which Soviet strictures had kept her.

A few remarks on some of the works from that period. Two *Annals* papers produced volume-preserving $C^\infty$ Bernoulli diffeomorphisms on any compact connected $C^\infty$ manifold, possibly with boundary.\footnote{Except in dimensions 3 and 4, where Dolgopyat and Pesin established this and more [6].} This is profoundly astonishing: these are volume-preserving diffeomorphisms that, up to a measurable change of “coordinates,” are isomorphic to a Bernoulli shift, i.e., a much more complex counterpart to the probabilistic model of a fair coin toss. The first was written by Katok at IHES and did this on the disk and on surfaces and provided two steps of four in the later joint proof on any manifold [1]. Also partially written at IHES, Katok’s by far most cited paper, “Lyapunov exponents, entropy and periodic orbits for diffeomorphisms,” combined the shadowing approach of Anosov and Bowen (to whom the paper is dedicated—he died the year the Katoks arrived in the US) with Pesin theory to great effect. Its best-known results are Katok’s Closing Lemma and that a diffeomorphism of a compact surface with positive topological entropy possesses a horseshoe. He was more proud of this work than anything else and lectured on it at the International Congress of Mathematicians in 1983 as well as the 1982 Rufus Bowen Memorial Lectures at Berkeley.
A 1982 paper gave effective lower bounds on the growth of the number of closed geodesics on a surface and proved that for a higher-genus Riemannian surface the topological and Liouville entropies agree only if the curvature is constant. With the help of Ralf Spatzier, Katok computed both entropies for all locally symmetric spaces and found that they agree. The resulting table is followed by the comment

It looks like a reasonable conjecture that those are the only cases of manifolds of negative curvature for which the Liouville measure has maximal entropy.

Before long, this understated aside came to be known as the Katok Entropy-Rigidity Conjecture. Rigidity theory has been active ever since, thanks on no small part to Katok himself. He not only worked on this conjecture but put much effort into popularizing it by giving talks, organizing or participating in a series of conferences over two decades, and directing the interest of mathematicians towards the conjecture. Rigidity theory was becoming an industry, and has been going strong since.

True to form, simultaneously with these papers on hyperbolic dynamics, Katok did notable work on both elliptic and parabolic dynamics as well as ergodic theory: the latter, in the spirit of his quest to understand which phenomena from ergodic theory are realized in smooth dynamics, shows that there are diffeomorphisms with the Kolmogorov property that do not have the aforementioned Bernoulli property.

Academic appointments in the US offered entirely new outlets for Katok's legendary organizational and collaborative energies. While he always maintained intense seminar activities, he immediately set about organizing special years such as 1979–1980 at Maryland and 1983–1984 at the Mathematical Sciences Research Institute. He was able to travel extensively, and rare was the year in which he did not visit multiple continents. Visits to Warwick in 1979 and 1980 led to the founding of the Journal Ergodic Theory and Dynamical Systems, which for some four decades now has been the global journal of record for the field. Twenty-five years after this founding, he did it again and started the Journal of Modern Dynamics with a view to being a highly selective dynamical systems journal with a broad focus and a democratic editorial board.

In the US, Katok was also in a position to build a research group. Maryland made significant hires during his time there (including Brin), and he began to have postdocs in addition to graduate students. Like others since, they recall that he always made time for talking about mathematics or career advice, even when he was obviously busy running a conference or in the midst of a collaboration with someone else. He would come in early or stay late, or find time during an excursion, whatever was needed. And he had a knack for cultivating talent and adjusting his mentoring to the students and their abilities.

Svetlana obtained her doctorate from the University of Maryland in 1983 (with Don Zagier), whereupon the Katoks departed for a year in Berkeley. Anatole was at MSRI, while Svetlana was a lecturer at UC Berkeley. For many years thereafter, Katok maintained close relations with MSRI, including service as a trustee. His time there during the 1992 special year on Lie Groups and Ergodic Theory with Applications to Number Theory and Geometry is worth mentioning for several reasons. One of these is that his facility with ideas is illustrated by an episode that started here and then in response to a question put to him by Keith Burns and Livio Flaminio. Without hesitation, he produced an example of a continuous foliation with smooth leaves and a set of full Lebesgue measure that intersects each leaf in a single point(!). Flaminio and Burns wrote this up, titled it “Fubini’s nightmare,” and did nothing with it until 1996 when Burns sent it to his colleague Amie Wilkinson, and via Charles Pugh and Michael Shub the example made its way to John Milnor—who immediately wrote it up for the Mathematical Intelligencer under the alliterative title “Fubini foiled.” This “pathological” phenomenon became instantly famous and has since been discovered to be somewhat typical of the center foliation (if defined) of a partially hyperbolic dynamical system.

In 1983–84 the need of the Katoks for two jobs in one region was no secret, and Barry Simon initiated an offer of

5The acknowledgments show that the ability to travel internationally and invite visitors to Maryland from the world over made a real difference in what Katok was now able to do.

6These are very close notions, and even in a purely measure-theoretic context it is not a simple exercise to find such an example.

7www.msri.org/programs/100.
a professorship for Anatole at Caltech, where they moved in 1984, Svetlana having a two-year adjunct position at UCLA. Now 40, Anatole settled into this position, while over time Svetlana was to hold faculty positions at most University of California campuses. From this time onward, Katok maintained a continuous stream, if not torrent, of doctoral students, with a grand total north of 40. For most of us this afforded a communal experience not entirely unlike what the Moscow seminars must have been like. This was not only due to the number of doctoral students but also to his ability to build a research group by hiring postdocs and faculty, inviting visitors, and drawing on interactions with related disciplines. A few salient items from my career may give an indication. Having come to the University of Maryland as a physics student on a Fulbright scholarship, I happened to hear that “someone called Katok” was going to give a course on classical mechanics in the spring of 1983. While taking it, I decided to become a mathematics doctoral student at Maryland, and in the spring of 1984 I got a letter from MSRI. Katok wrote that he was moving to Caltech and invited me to join him there. When I arrived, a former student of Simon’s began to work with Katok as well. Soon after, postdocs and faculty joined, and the number of doctoral students rose quickly. There was a steady stream of high-caliber seminar speakers and medium-term visitors from around the globe plus an active working seminar and almost annual courses on dynamics by Katok, never twice the same. He naturally interacted beyond dynamical systems. Simon was his office neighbor, and their animated conversations often rang through the hallways, the pitch ever increasing with the excitement of the interaction.

From the Caltech days, a few larger mathematics projects are worth singling out (in addition to the completion of [30]). With Gerhard Knieper, Mark Pollicott, and Howard Weiss, Katok carefully studied the dependence of entropy on perturbations of Anosov flows, and with Hurder he deeply connected dynamics with foliation theory [13,14]. Their study of the Godbillon–Vey class became seminal in an unintended direction: it necessitated a more careful study of the (transverse) regularity of the invariant foliations of an Anosov 3-flow, and in so doing led to another rigidity conjecture: that an Anosov flow is smoothly conjugate to an algebraic one if these invariant foliations are transversely $C^2$. Kanai almost immediately made a key construction (the Bott–Kanai connection) that enabled him to prove the first such result for geodesic flows on manifolds of dimension greater than 2. He came to Caltech as a Bateman Instructor, and Katok and Feres embarked on the chase of the comprehensive such result. And a chase it was: they were aware that Foulon and Labourie, soon joined by Benoist, were onto the same target. The Feres–Katok strategy was to buttress Kanai’s line of argument with ever more sophisticated dynamical refinements to progressively weaken the needed curvature-pinching hypothesis. Some of this work took place during a summer in Göttingen, and I remember the exhilaration of progress as well as the competitive spirit. In the end, Benoist–Foulon–Labourie got there first, and some of the Feres–Katok work remained unpublished. The consolation was that the French team had gotten there using a “big hammer,” the open-dense orbit theorem of Gromov. But they had also far exceeded the target by treating contact Anosov flows, i.e., they obtained rigidity without assuming the structure of a geodesic flow in the first place.\footnote{Contact Anosov flows whose invariant subbundles are $C^{\infty}$ or highly smooth are (essentially) smoothly conjugate to the geodesic of a locally symmetric space. Whether $C^2$ suffices remained open.}

The Göttingen memories also bring to mind the Oberwolfach conferences “Dynamische Systeme” then organized by Moser and Zehnder—I remember a reckoning that revealed Katok to have been second only to Zehnder in faithfulness of attendance. Helmut Hofer, who now co-organizes these, likes telling the emblematic story of the “Katok diagonal argument.” Katok’s usual seat was in the front right (sitting in front was his general custom, and the doors being on the right in the Oberwolfach meeting room makes this the natural side for late-comers). Michael
Herman, another prominent regular, might sit in the very left rear corner. It was not uncommon for one of Katok’s amendments to someone else’s talk to be countered by Herman from the opposite corner: “I can’t believe that I am hearing this! I have three counterexamples to that statement. No, I have infinitely many counterexamples to what you are saying...,” and so the argument across the diagonal of the room was on. This is memorable, of course, not only because it is amusing but because these interactions were illuminating for all those present and emblematic of the creative spark of those meetings.

Middle Pennsylvania

“Moscow, Vienna, Rome, Paris, Washington, Los Angeles—State College???” was a refrain briefly heard among dynamists in 1990. The intimacy of Caltech’s mathematics department made it challenging to build and maintain a large research group, and Svetlana’s position at UC Santa Cruz made for a longer commute than a family with a young child would want to keep up. Richard Herman from the Pennsylvania State University engineered a double-offer that drew the Katoks to State College, but their previous metropolitan biography raised doubts about how long they would last there. Though they spent nary a summer there, they never left. And they fully engaged from the start, helped by the simultaneous arrival of Pesin from Moscow via Chicago and H. Weiss from Caltech. They went on to build the department into the most prominent center of dynamical systems in the country. Mathematically as well, one might call this a new period.

In addition to the aforementioned seminal rigidity work and a continuing stream of publications in various parts of dynamical systems, Katok became engaged with the Zimmer program, the core conjectures of which were only just yielding at the end of his lifetime [2,3]. He heartily jumped in together with others by dedicating the first Dynamical Systems seminar to the “super-mathematics” of the day, Margulis–Zimmer superrigidity, following Robert Zimmer’s book. This continued for two years and then slowly morphed into a more general Dynamical Systems seminar with rigidity a cornerstone for years to come. The first meeting was packed with about 20 people, quite unusual for a topic of such depth, which required expertise in ergodic theory, functional analysis, Lie groups and algebras and their representations, unitary representations of groups, algebraic groups, and algebraic geometry. Katok kept the participants together, and encouraged everyone to participate in order to build a research school, even though he did not expect all the participants to write papers in the field. Remarkably, over the years most of the participants actually published in rigidity theory, and many defended PhD theses under his supervision.

Indeed, probably Katok’s most impressive contribution to dynamics in terms of research papers is the program on rigidity of actions of higher-rank abelian groups, i.e., to show that faithful actions of $\mathbb{R}^k$ or $\mathbb{Z}^k$ are standard. He carried this out in large part with former students, e.g., in more than twenty papers with Spatzier, Damjanović, Kalinin, Nîtićă, Török, and Kononenko. He also hired and collaborated prolifically with Manfred Einsiedler and Federico Rodriguez Hertz, mostly on rigidity questions, and Rodriguez Hertz has since succeeded him on the Raymond N. Shibley Chair. Compared to higher-rank lattice actions, he saw abelian actions as a more important (broader) subject because he wanted to understand dynamics from many different sources, and in abelian actions all types of dynamics appear and can be understood. It was icing on the cake rather than justification, that this could be applied to number theory when he, Einsiedler, and Elon Lindenstrauss addressed the Littlewood Conjecture. (Lindenstrauss went on to win a Fields Medal.) Icing as well, that this turned out to contribute to the recent breakthrough on the Zimmer program. Interestingly, Aaron Brown, who was central to proving the Zimmer conjecture, told me that all the tools he needed, he learned from Katok. Among these many works, the work with Spatzier on measure rigidity [28] and cocycle rigidity [29] started these developments, and his work with Kalinin and Rodriguez Hertz on nonuniform measure rigidity [15] then opened the door to the Littlewood conjecture and to the Zimmer program.

Throughout, Katok vigorously recruited students, and the vast majority of his doctoral students graduated from the Pennsylvania State University. Among Alena Erchenko’s early memories is that Katok told her at the beginning of the PhD program: “You will never know as much as I do. Therefore, the best route is to learn something that I do not know.” She and I think he liked when his students were able to prove something that he thought should be different. In her work with him, it first seemed that there might be restrictions on the pairs of Liouville and topological entropies, but then they discovered models that allowed them to vary Liouville entropy without changing topological
entropy much. Likewise, my dissertation result involved a geometric “threading” mechanism rather than one of a cohomological nature as in any previous work. Another memorable piece of advice was given at home, when he entertained Jana Rodriguez Hertz and Raúl Ures: “If you want to succeed in mathematics, then one possibility is to be Margulis. But if, like in my case, you are not Margulis, then you have to work like crazy. There is no other way.”

His care for others in the community went well beyond mathematical needs. H. Weiss called Katok the most gregarious mathematician he has known, and the frequent social events hosted by the Katoks were an important center of a community. He helped his students with moves and medical appointments, and he made a point to include young mathematicians in conferences, over time he wrote letters of recommendation for most dynamicists, and at times his active intervention rescued careers.

My largest project with Katok mainly took place during the Penn State years. In our Caltech days we had vaguely entertained the idea that my polished notes from his first dynamical systems course there might become a book, but we did not make concrete plans until 1989. Helped by notes from his dynamics courses since, we had a strong base for modest plans. It surely helped that we had no idea what we were getting ourselves into. Mostly in the course of mutual visits over five years, Introduction to the Modern Theory of Dynamical Systems [24] acquired its final form and 800-page heft. By now, it is The Book on dynamics, and most of those who earned PhDs in dynamical systems in the present millennium grew up in no small part with this book. On a 1996 visit in Montevideo we dug into a follow-on project, A first course in dynamics | with a panorama of recent developments [12], which had half the size—but took twice as long. Other book(-size) projects of his include the 200-page Principal Structures [11], which provides a common background for two Handbook volumes [8,9], the 130+ page current-research survey The theory of dynamical systems and general transformation groups with invariant measure [27] from Moscow days, and the monographs Invariant Manifolds, Entropy and Billiards. Smooth Maps with Singularities [30], Rigidity in higher rank abelian group actions. Volume I. Introduction and cocycle problem [26], and Combinatorial constructions in ergodic theory and dynamics [19].

Two more books are worth mentioning in the context of the Katoks’ imprint on undergraduate mathematics education at the Pennsylvania State University. They created the Mathematics Advanced Study Semesters program (MASS), which provides a springboard to graduate education for many students from small colleges who might otherwise not be prepared or inspired for graduate education, and two courses Katok taught in it resulted in books [4,23] with Vaughn Climenhaga, then a graduate teaching assistant for the course.

At the Pennsylvania State University Katok was able to build lasting structures to support dynamical systems worldwide. One is the aforementioned Center for Dynamical Systems and Geometry. The other is a major annual workshop. Upon arrival at the Pennsylvania State University, Katok set upon organizing a large rigidity conference for March 1991, and it was promptly followed by a Penn State–University of Maryland Workshop in Dynamical Systems and Related Topics in October 1991, which has taken place annually ever since and is always followed by a companion meeting at the University of Maryland in the Spring. Participants travel from one of these institutions to the other, and they are joined by participants from across the US and the globe. This along puts Penn State on the mental map of every dynamicist in the world. To raise the visibility of dynamical systems, Katok and Pesin persuaded Brin to create an international prize for outstanding work in the theory of dynamical systems and related areas. The first Michael Brin Prize was presented in 2008 at the Maryland session of the semi-annual workshop, which was dedicated to Brin’s 60th birthday, and this (now annual) prize has since been complemented by the annual Michael Brin Dynamical Systems Prize for Young Mathematicians.

Together with the countless doctoral students and postdocs who cycled through the department in these
three decades, plus those hired during his time, whether still at Penn State or not, Katok made Penn State a center of dynamics, and dynamics central to Penn State—with a broad perspective. In the estimation of George Andrews, a former department chair, “The rise of the reputation of our department owes much to his leadership both in his own research and teaching and in bringing outstanding mathematicians to our department.”

He had a zest for life that had many dimensions. He loved conversations (about mathematics, history, and almost any other topic), art, music, wine, books, and food—locally, abroad, and at home. The aforementioned Jana Rodriguez Hertz, an Argentinian mathematician from Uruguay, recalls dinner at his house: “He cooked steaks, and he really knew how to do it. And let me tell you that it is not easy to impress an Argentinian or Uruguayan at that.”

While an inveterate urban traveler, Anatole also enjoyed the great outdoors. Many remember the obligatory vigorous conference hikes, such as the clamber up the San Gabriel Mountains or the workshop Sundays in the hills of Pennsylvania. Alas, in the mid 1980s this had to pause when Katok underwent aggressive cancer treatment. Yet, he came to every seminar and continued to work as usual, so the gallows humor in the research group had it that this crisis had reduced Katok’s energies to those of a mere earthly being. He was before long pronounced fully cured, but resulting heart and lung problems started appearing a decade later and increased over time. Nevertheless, he persistently threw himself into life and mathematics. Aside from our second book, we completed several other large projects and started yet others now left in limbo. All this alongside his extensive research productivity on higher-rank rigidity and many other topics, plus an unabated flow of books and expository writing, including historic/sociological works with unique and thoughtful perspectives that will likely remain a useful part of our historical record [10, 16, 20–22, 25]. On my last visit we reworked a perennial book project into a different form that would make it more feasible in reasonable time, but he warned me that because he was doing well enough to work on such projects, he was going to have a few others on his front burner. More recently, the Swedish Research Council appointed him to the Tage Erlander Guest Professorship 2018 in recognition of his excellent services to science, and he was scheduled to consummate in the second half of the calendar year. His enthusiasm and engagement never stopped, but this was not to be, and numerous projects are now suspended.

Anatole Katok is gone, but he created enough momentum, infrastructure, and leadership talent that we can expect dynamical systems to do him proud in years to come.

References

14. Hurder S, Katok A, Differentiability, rigidity and Godbil-


Credits

Photo of Katok in Leningrad is by Konrad Jacobs, Archives of the Mathematisches Forschungsinstitut Oberwolfach. Photo of Katok with Dmitry Anosov is from the archives of the Mathematisches Forschungsinstitut Oberwolfach, ©Dirk Ferus. Photo of Helmut Hofer is by Boris Hasselblatt. Photo of Katok in his office is courtesy of the Pennsylvania State University. Photo at the Pennsylvania State University during the 2015 Fall Workshop is by Jana Rodriguez Hertz. All other article photos are courtesy of the Katok family.
Anatole Katok Center for Dynamical Systems and Geometry

Svetlana Katok and Yakov Pesin

Creation of the Center
The Penn State research group in dynamical systems was formed in 1990 when Anatole and Svetlana Katok, Yakov Pesin, and Howard Weiss moved to Penn State to join Eugene Wayne who already was there. By that time Anatole had already established himself as a leader in dynamical systems who had co-organized several major events (such as a year program in dynamics at MSRI in 1983-84) and had trained a number of graduate students and postdocs at University of Maryland and Caltech. He came to Penn State with the plan to build a strong group in dynamics and to attract talented young mathematicians to the subject. Full of energy and ideas, he immediately started a weekly seminar in dynamical systems and was thinking of organizing some major scientific meetings.

At that time the mathematics department of the University of Maryland had an active group of researchers working in dynamical systems with whom Anatole had had close scientific relations. Anatole came up with the idea of organizing biannual workshops in “Dynamical Systems and Related Topics” as a joint enterprise of the two mathematics departments with fall workshops held at Penn State and spring workshops held at Maryland. The first such meeting happened in the fall of 1991.

Other major events were the AMS Summer Workshop on Dynamical Systems organized by Anatole jointly with Yakov Pesin and Howard Weiss in Seattle in 1999 and the International Conference “Ergodic Theory, Geometric Rigidity, and Number Theory,” which was co-organized by Anatole at Newton Institute (Cambridge, UK) in July, 2000.

Within a few years after its creation, the dynamics group at Penn State had grown and so had its activities. This included a visitor program, intensive collaborations, a weekly seminar, etc. The group also had a large number of graduate students interested in dynamics. In early 1990, due to the fall of the Soviet Union, many undergraduates from Eastern European countries had the opportunity to pursue a PhD in the West. Quite a few came to Penn State, which was already known for its extensive program in dynamics.

In 1997, after being named Raymond S. Shibley Professor of Mathematics, Anatole Katok initiated the creation of the Center for Dynamical Systems (the Center) at Penn State and became its director. A few years later the Center was expanded to become the Center for Dynamical Systems and Geometry, which has now become one of the major centers in dynamics in the US.

At the time the Center was created Anatole Katok wrote a memo in which he stated the Center’s goals as follows:

• To stimulate research in the theory of dynamical systems and related areas at Penn State.
• To serve as a forum for the exchange of ideas and discussion of achievements within the worldwide dynamical systems community.
• To foster interaction, exchange of ideas, and joint projects between mathematicians and researchers in other areas interested in non-linear dynamics.
• To disseminate the knowledge of the achievements in dynamical systems theory within the scientific community.
• To contribute to the training of the next generation of researchers in dynamical systems and related areas.
Financial Support and Endowments

Initially, the financial support for the Center’s activities came primarily from Katok’s departmental fund and his fund as Raymond S. Shibley Professor, while the financial support for the fall workshop came primarily from the NSF. After Anatole Katok’s untimely death, three donors have stepped forward to make generous gifts in Katok’s memory totaling $2.7 million.

- A lead gift of $2 million came from Michael Brin, a colleague and former student of Katok, who endowed the Anatole Katok Chair in Mathematics in the Eberly College of Science with the goal of supporting outstanding faculty members who are leaders in the study of dynamical systems, as Katok was throughout his career.
- Additional contributions totaling $700,000 came from Brin and two of Katok’s other former students, Sergey Ferleger and Alexey Kononenko, to provide permanent program support for the Center for Dynamical Systems and Geometry, which now bears Anatole Katok’s name.
- Additional matching funds came from the Eberly College of Science.

Penn State University and, in particular, its Eberly College of Science were instrumental in helping to build the Center with the goal of preserving Anatole Katok’s legacy and his outstanding contributions to mathematics.

“The endowment from Michael, Serge, and Alexey will honor Anatole’s memory and extend his legacy in perpetuity,” said Douglas Cavener, dean of the Eberly College of Science.

“These meaningful gifts to honor the late Dr. Anatole Katok will propel cutting-edge mathematics research for generations to come,” said O. Richard Bundy III, vice president for development and alumni relations. “This is a fitting way, indeed, to honor a professor and mentor who was profoundly respected by so many not only at Penn State, but throughout the mathematics world.”

“Anatole was an extraordinary academic and contributed great value to his profession,” said Brin. “Through these gifts, Alexey, Serge, and I hope to honor his outstanding legacy by supporting faculty members who will continue the research and teaching to which he devoted his life.”

The Center Structure and Programs

The Anatole Katok Center for Dynamical Systems and Geometry at Penn State has a strong and active group of faculty working in a broad spectrum of disciplines related to dynamical systems and geometry. At present, the Center consists of 14 faculty members, 7 associate members, 14 graduate students, and a postdoc. Yakov Pesin is its current director. The Center has a broad research program that includes:

- A visitor program (about 10 short- and long-term visitors a year)
- A weekly Dynamical Systems Seminar
- Center for Dynamics and Geometry Colloquium at which talks are given about once a month by distinguished invited speakers
- Working Seminar: “Dynamics and its Working Tools,” which operates “in old Russian style” where each lecture lasts about two hours and includes all relevant details
- Dynamics Student Seminar organized by students interested in dynamical systems
- Basic courses in dynamical systems and related topics at graduate and undergraduate levels, special topic courses and special lecture series (on an annual basis)
- An annual workshop in “Dynamical Systems and Related Topics.” Starting as regional meetings, these workshops have now grown into major national conferences on dynamical systems with many participants coming from all over the world.

In the last several years during these workshops two major prizes in dynamical systems have been awarded: Michael Brin Prize in Dynamical Systems (for researchers no more than 14 years from their PhD) and Michael Brin Dynamical Systems Prize for Young Mathematicians (no more than 4 years from their PhD). The monetary fund for these two awards came from the endowments established by Michael Brin, and these are the two major and highly prestigious awards for young mathematicians working in dynamical systems.

For more information on Anatole Katok Center for Dynamical Systems and Geometry see https://math.psu.edu/dynsys

AUTHOR NOTE: We are happy to report that Federico Rodriguez Hertz has accepted the position of the Anatole Katok Chair in Mathematics. His appointment started April 1, 2019.

Credits

Photos of Svetlana Katok and Yakov Pesin are courtesy of the authors.
MATHEMATICAL MOMENTS

Scoring with New Thinking

When did you last pay a late fee for a video you watched at home? Whaaa? In the 1980s, the biggest challenge for a video store was how to keep customers from walking out with a title. But when videos became available via streaming, the problem changed. As of 2018, the average US households had 3.3 streaming devices, and Netflix alone has 147 million members. How do you keep that many people satisfied? The answer: the computer. When you sign up for Netflix, it asks you to complete a 5-minute survey—a task that no less than 68% of users complete. The answers are used to build a model, which then generates recommendations that suit your viewing habits. With more than 100,000 titles in its catalog, Netflix has millions of recommendations to choose from. The company allocates resources (computing power) based on the number of times the recommendation is used. For example, if a title is recommended and then used 10 times, it will receive 10 times the computing power to help generate new recommendations. And early buses encounter fewer passengers, thus creating shorter boarding times. But when the gap between buses is too small, passengers are forced to stand on the platform waiting for another bus—a frustrating problem known as bus bunching. Researchers have tested solutions in simulations using mathematical models and on the road with real bus routes. The most successful approaches are those that focus on directions that pull them closer together.

Winning the Race

The book and film Hidden Figures tells the story of African-American women mathematicians who worked on the early days of the space program. Among the film’s many memorable moments is when future Presidential Medal of Freedom recipient Katherine Johnson, who worked as “computers” for NASA in the early days of the space program, showed that at the time videos could be delivered by mail cheaply, with no need to visit a store and no need for late fees, so long as you didn’t keep too many videos. But the astronaut John Glenn, whose life depended on the calculations being correct, insisted that the human computer, Katherine Johnson, verify the machine’s work. Only then did he feel confident that he could rocket into space and return home.

Hidden Figures

Four mathematicians introduced to the public by the movie Hidden Figures: Katherine Johnson, Dorothy Vaughan, Mary Jackson, and Christine Darden. The team also discovered an explanation for why some shells have spikes. The spikes form perpendicular to the shell opening during mantle growth spurts, when the shell is twisting. Expanding and rotating form a shell like the one pictured, with an increasing number of spikes. Mollusks are not known for their math skills, but there is mathematics behind the going into a shell. New research suggests that the length of a wind turbulence event determines the number of spikes that form. The model can be applied to the growth of all shell lines, including those of the sea cucumber, crustaceans, and jellyfish. For More Information:

Hidden Figures

The Mathematical Moments program promotes appreciation and understanding of the role mathematics plays in science, nature, technology, and human culture. The Mathematical Moments program promotes appreciation and understanding of the role mathematics plays in science, nature, technology, and human culture.
The Controversy over the AMS Publishing Original Investigations

Steve Batterson

The publication of mathematical research is presently at the core of the AMS mission. This role for the Society seems so natural that the drama and contention behind the founding of its first research journal, the Transactions, is surprising. In 1898, however, the most prominent American mathematicians in the organization stood in opposition to the AMS sponsoring a research journal. The dispute began when some AMS members expressed dissatisfaction with the existing opportunities for publishing their research in the United States. The issue centered on whether the AMS should step in to address the problem. This time the division fell along generational, rather than geographic, lines.

Since 1884, when astronomer Ormond Stone started the Annals of Mathematics, two American journals had existed for the dissemination of mathematical research. The American Journal of Mathematics (AJM) continued to publish scholarship at a high level. The Annals, with a mission to serve a wider audience, printed papers of a more intermediate quality.

For Stone the Annals was a labor of love. He self-financed the journal after coming to the University of Virginia to direct its observatory. Stone often published articles for which, though the originality of the results was marginal, the presentation made mathematical topics more accessible. In the 1890s the journal continued to fill this niche as it added stronger material from the rapidly developing American mathematical community. At the same time, resentment was building among young Americans over the editorial management of the AJM. An analysis of their grievances requires some background on the AJM and its administration at Johns Hopkins University.

Prior to the founding of the AJM in 1878, American mathematicians had no domestic periodical for conveying important discoveries to an international audience. Since major results were then exceedingly rare in the United States, the deficiency had infrequent effect. Nevertheless, Benjamin Peirce’s 1870 paper, on linear associative algebras, and G. W. Hill’s 1877 paper, with the differential equation that now carries his name, were necessarily printed privately.
HISTORY

Johns Hopkins President Daniel Coit Gilman\(^2\) recognized the need for an American mathematics journal to support scholarship in the country. He pushed his mathematics professor J. J. Sylvester to begin a mathematics journal attached to Hopkins. Sylvester at first demurred, pointing to the dearth of material originating in the United States.

With the assurance of financial support from the University and administrative assistance from his junior colleague William Story, Sylvester finally agreed to undertake the journal project. The venture was highly speculative, depending as it did on an increase in American research. To fill its early issues, and maintain the journal’s distinction, Sylvester solicited papers from prominent European associates. Along with Hill’s seminal work on the lunar theory and articles by Sylvester, Simon Newcomb, and Henry Rowland, the first issues included papers by the British mathematicians Arthur Cayley and W. K. Clifford. The work of Sylvester’s students was soon supplementing the articles by Hopkins faculty, other Americans, and European contributors. The international combination was an overwhelming success, bringing credit to Johns Hopkins. Over Sylvester’s six-year tenure the authorship breakdown was 25% European, 45% Hopkins, and 30% other American.

During the 1880s the editorial roles of the departing Sylvester and Story were assumed by Newcomb and Sylvester’s former student, Thomas Craig. Over time Craig’s duties expanded. In 1894 he was promoted from associate editor to editor, with Newcomb taking on advisory status. The practice of soliciting papers from Europeans continued. Meanwhile, since the departure of Sylvester, Hopkins was a declining source of mathematical research.

Talented young American faculty recently returned from study in Europe, such as Maxime Bôcher (Harvard), W. F. Osgood (Harvard), E. H. Moore (Chicago), Henry Seely White (Northwestern), and James Pierpont (Yale), naturally looked to place their papers in the still prestigious AJM. They had mixed results. Bôcher recorded the following comment about an article he published in the 1892 Annals:

“A similar paper was sent to Craig for the American Journal in October 1891 and ignored by him.”\(^3\)

Whether Craig was disrespecting Bôcher or had a reasonable explanation, the AJM did come to recognize the contributions by the new generation, to some extent. Bôcher, Osgood, Moore, and White each had one paper, and Pierpont had two in the

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\(^2\)The remainder of this paragraph and the following one are drawn largely from [5], where more is available on the AJM.

\(^3\)From the notebook, with annotations on his publications, among the Papers of Maxime Bôcher. HUG 1224.5. Harvard University Archives.
volumes over the years 1895–1897. During this three-year period the percentage of Hopkins papers dropped, from the Sylvester days, to 19% as the other American institutions' share rose to 40%. European contributions, at 40%, had also increased.

With contact facilitated by the meetings of the AMS, the young cohort became aware of worthy domestic results that were not appearing in the AJM. They resented the AJM practice of committing in advance to publish papers of Europeans, leaving inadequate space for Americans. Moreover, they shared stories of sloppy editing and poor refereeing for which they blamed Craig. By 1898 the notion had arisen of the AMS sponsoring a research journal.

For mathematicians in their thirties such as Bôcher, Moore, Pierpont, White, and Osgood, the AJM had existed throughout their careers, operating, in their opinion, with a European bias. The developing state of American mathematics in 1898 simply called for another journal. Bulletin editor Thomas Fiske appreciated the delicacy of the situation. Although he was from the younger generation and shared their views, he, alone of them, had participated in the discussions earlier in the decade. Any proposal would involve the current AMS president, Simon Newcomb, who would be especially sensitive to the treatment of the AJM with which he had been associated from its conception.

Fiske sought a course to accommodate all parties. Journals were financial burdens for universities. The University of Virginia had taken over support of the Annals from Stone, only recently finding itself unable to sustain the funding. The AJM was a continuing expense for Johns Hopkins. Perhaps Hopkins would relinquish control to the AMS if the Society assumed financial responsibility while perpetuating the AJM’s Baltimore heritage. On March 29, 1898 Fiske wrote Newcomb:

> It has on several occasions been suggested that the American Mathematical Society undertake the publication of a journal of original investigation. The supply of original material suitable for publication has been increasing rapidly in the past two years, and it seems desirable either that the American Journal should be enlarged or that a new journal should be established. How does the following proposition impress you?

> That, the American Journal be transferred to the American Mathematical Society, it being agreed that the title-page and cover shall always bear in a conspicuous position the inscription “Founded by the Johns Hopkins University”

Although turning over the AJM proved to be a nonstarter with Hopkins, Newcomb seemed open to some sort of cooperative relationship with the AMS. At the 1898 AMS Summer Meeting a special committee was constituted with the broad charge "to consider the question of securing improved facilities for the publication of original mathematical articles in this country." The committee members were Bôcher, Moore, Newcomb, and Pierpont, with Fiske

It was not the first time that members of the Society had discussed the possibility of publishing original investigation. In 1891, when Emory McClintock was the Society president, the New York Mathematical Society had rejected this course for its Bulletin. McClintock and his presidential successor, G. W. Hill, were from an older generation. Their professional lives were markedly changed by the creation of the AJM. McClintock and Hill felt it impudent for the Society to infringe on what they regarded as Johns Hopkins' franchise. A committee, which included McClintock and Fiske, decided on a policy for the Bulletin to carry primarily articles of a “critical and historical nature.”

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4 From the Simon Newcomb Papers. Manuscript Division, Library of Congress.

5 P. 2 in the October 1898 Bulletin.
as chair. Its first efforts were in pursuit of an arrangement with Hopkins for joint custody of the AJM.\textsuperscript{6}

Fiske found himself functioning as an intermediary, between Bôcher, Moore, and Pierpont on one side and Newcomb on the other. The agenda of the four younger men was simple. The AMS needed to gain editorial control of the AJM. Over the short run they would be satisfied with substantially reducing the authority of Craig while publishing more papers by Americans.

Guided by a report of the special committee and reaction from the AMS Council, Fiske and Moore drew up a formal proposal. Its business aspects were straightforward. The number of AJM issues per year would increase from four to six, with the AMS footing the added expense. Seemingly more controversial was their plan for editorial reorganization. Rather than a single editor selected by Hopkins, “the Journal shall have a board of seven editors, of whom several (two) shall be selected by Johns Hopkins University, and the others (five) by the Council of the American Mathematical Society.” The board was to elect, “from among themselves,” an editor in chief under the stipulation that Newcomb would serve as the first editor in chief.\textsuperscript{7}

The parenthetical entries in the proposal suggest that Fiske and Moore anticipated some negotiation over the number of editors to be selected by the two entities. Ironically, the integers two and five were rare aspects of the proposal over which there is no record of disagreement. The negotiations between Fiske and Newcomb were tense, eliciting frustration on the part of the otherwise diplomatic Fiske. From the outset they haggled over the AMS subvention. To offset the cost of the additional issues Newcomb proposed the AMS contribute $500 per year and assume responsibility for 100 new subscribers. A skeptical Fiske. From the outset they haggled over the AMS subvention. To offset the cost of the additional issues Newcomb proposed the AMS contribute $500 per year and assume responsibility for 100 new subscribers. A skeptical Fiske began asking for, and not receiving, specifics on the current AJM operating expenses.

The biggest dispute was over the powers and titles, rather than the number, of the Hopkins editors. The nomination of Newcomb as the first editor in chief, suggested by Moore, was a short-term gesture intended to ease Craig out of authority and acknowledge Newcomb’s eminence. The younger committee members were adamant that Newcomb’s successors be elected from, and by, the entire board. Newcomb maintained that fundamental editorial responsibilities could only be performed by someone on the ground in Baltimore where the journal was printed. To handle these duties required an editor in chief and an associate editor in chief, both from Johns Hopkins. Under Newcomb’s scheme the AMS designees would carry the title of associate editor.

As the Bulletin was printed in Pennsylvania and edited in New York, Newcomb’s reasoning was received with considerable skepticism. Moreover, his counter-proposal was completely unacceptable to the other committee members. Their nightmare scenario was having Craig continue his management function under the title of associate editor in chief, leaving the AMS representatives merely as referees. Bôcher, Moore, Pierpont, and Fiske were unanimous that all board members, other than the editor in chief, have equal standing.

The bottom line was that both sides were determined to hold editorial control of the AJM. Newcomb invited the committee to Hopkins for a discussion with Gilman. Fiske was willing to make the trip himself, but was reluctant to urge his colleagues on the committee to join him. He wrote Newcomb: “Before we can ask our representatives to attend at their own expense a conference to be held in Baltimore, we must assure ourselves that there is some probability of an agreement being reached—I must confess that I am becoming quite doubtful as to whether our negotiations can lead to a mutually satisfactory understanding—”\textsuperscript{8}

Other committee members were more pessimistic. Even so, Fiske wanted to make every effort on behalf of the AMS. He wrote to Bôcher, Moore, and Pierpont asking them to accompany him to Baltimore. Only Pierpont agreed to join Fiske for the meeting. At about the same time, Pierpont made arrangements to attend the Chicago section meeting at the end of the year to, among other things, fill in Moore and White on developments.

On Saturday, December 17, 1898, Fiske and Pierpont met with Newcomb and Gilman in Baltimore to discuss a cooperative arrangement between Hopkins and the AMS on the AJM. The following day Pierpont sent this postmortem to White from Washington.

…the Amer Math Soc met its Waterloo. Gilman, contrary to the notion the Newcomb letters to Fiske awoke, insisted that the J.H.U. retain absolute control of the journal. Gilman would not listen to a board of editors appointed by J.H.U. & the Amer Math Soc which should guide the policy of the journal. Fiske tried by diplomatic word twisting to get Gilman to recede in some respects. But Gilman would not move an inch. On the other hand a plan was evolved that the JHU under leadership of Newcomb should try in every possible way to meet the wishes of the Society. Increase number of pages, ask certain members of the Am M. S. to become advisory editors & &c. All this shall not cost the Soc a

\textsuperscript{6}The following discussion of the committee’s work is drawn from two archival collections and from p. 56–61 in [2]. The archival collections are the Henry Seely White Correspondence in the Columbia University Rare Book and Manuscript Library and the Simon Newcomb Papers in the Manuscript Division of the Library of Congress.

\textsuperscript{7}The proposal, dated 10/29/1898, is among the Simon Newcomb Papers. It is also printed, without the parentheses, on p. 56 in [2].

Pierpont was left to ponder what steps Newcomb would take, and whether they would be satisfactory in any way. After some time as a tourist in Washington, Pierpont stopped in Baltimore on his return to New Haven. A meeting with Craig ended any hope of meaningful accom-
modation. Craig told Pierpont that, without additional funds from the AMS, there was no prospect of enlarging the AJM. The practice of soliciting papers from Europeans would continue.

Upon the failure of the AJM negotiations, contemporary mathematicians might ask why the AMS did not pursue an arrangement to upgrade the Annals. After all, the Annals has been one of the world’s most prestigious journals throughout the careers of all living mathematicians. At the end of 1898, however, the rise in stature of the Annals was a few decades and two venue changes in the future. As strange as it may now be to think of the Annals typecast in the second class, the ordering of the two American journals was taken for granted. Discussion about the Annals concerned surviving its financial difficulties, rather than competing with the AJM.

Many of the younger Americans were disheartened. Johns Hopkins stood as the effective arbiter of significant mathematics in the United States. The only alternative course, starting a new AMS journal, faced three formidable obstructions. The first was financial. In 1898 total AMS receipts were $1831 and disbursements were $1614 with $1363 going to the Bulletin. While a $500 subvention to the AJM seemed doable, launching a journal would necessitate obtaining on the order of $1500 in additional annual revenue. Then there was the question of whether sufficient quality research was being produced to fill the pages of the new journal. That the AJM was shunning good material did not imply Americans were writing enough to sustain an entire new periodical. Finally came the political barri-
ers. Former presidents McClintock and Hill and outgoing president Newcomb were firmly against the AMS publishing original research. Overcoming their opposition risked provoking the biggest crisis in the history of the Society.

One week after his unsettling meeting with Craig, Pier-
pont arrived in Chicago and began talks with Moore and White. As depressing as the Baltimore developments had been, another effect was to focus attention on the feasibility of a new journal. Moore, Pierpont, and White confronted the second barrier directly. They made a careful analysis of the output of mathematical research in the United States. As Fiske later wrote Newcomb: “After much hand calculation and discussion of statistics, they reached a conclusion which surprised them, but which they believed trustworthy. It was that the amount of available good material produced at home, together with the supply the AJM regularly receives from abroad, was more than double the present capacity of the AJM.”

From this point on Pierpont campaigned for the new journal with an evangelical zeal. On his return to Yale, Pierpont stopped in Philadelphia, Princeton, and New York to lay out the case for other Society members. He corresponded with Osgood. The supply issue was resolved. Money and politics remained.

Paralleling Pierpont’s lobbying was an effort by Newcomb to placate the AMS by signing up select members, including Fiske and Moore, as associate editors for the AJM. Fiske initially agreed to serve, but continued to communicate with others, such as Moore, who was stalling for time to wait on developments with the AMS journal. In mid-January Moore and Fiske sent Newcomb their regrets. With his response to Fiske, Newcomb portrayed himself as an aggrieved party:

The Mathematical Society desired improved facilities, etc. I suggested that we could get these

Thomas Scott Fiske, AMS President, 1903–1904.

9Henry Seely White Correspondence. Letter from Pierpont, 12/18/1898. Columbia Rare Book and Manuscript Library.

McClintock felt caught in the middle. He agreed to speak, unofficially, on Newcomb’s behalf at the dinner. On February 24, 1899, Fiske was at the head of a table in the restaurant of the Grand Union Hotel. Bôcher and McClintock sat on each side of Fiske. About twenty AMS members were present. When the discussion began, a number of the younger men expressed support for a new research journal. Bôcher then asked McClintock to share his view. McClintock spoke from the heart, strongly opposing any action that would bring the Society into competition with Johns Hopkins and the journal that had meant so much to American mathematicians of his generation. The pleasant mood of the gathering abruptly ended. A confrontation loomed. Then Bôcher asked McClintock whether it would be improper for the Society to publish its Transactions. McClintock’s response, that any Society was free to publish its Transactions, ended the impasse. A second barrier was overcome with a name for the new journal.

The Society wished a body of associate-editors. This also was granted, and after consultation with yourself and others, the desired members were asked to act as my associates. Thus, with a great deal of labor, past and future, I have succeeded, I believe, in getting for the Society everything it could consider really necessary to the end in view. Moreover, this was done without any call on our funds. And now, after I have done all this, the Society runs away from me, as it were, and refuses to furnish the associate-editors, or to have anything to do with the arrangement.\textsuperscript{11}

Newcomb had glossed over crucial areas of contention. Fiske prefaced his rebuttal by stipulating that, while others were enthusiastically advocating a new journal, he, himself, remained neutral. He went on to point out the unresolved grievances over the roles of associate editors and Craig, and the two-track handling of European and domestic submissions. In a postscript, Fiske presented the position of Pierpont, Moore, and White, first citing their study and closing with “[o]n the basis of this calculation they demand increased facilities for publication. On the ground that the A.M.J. [sic] cannot provide these facilities without our financial assistance, and that we cannot render such assistance while the J.H.U. maintains its present attitude, they ask for the establishment of a new journal.”\textsuperscript{12}

A showdown over the controversial proposal for a new journal was shaping up for the February AMS Council meeting in New York. To clear the air, Fiske scheduled a dinner get-together for interested parties on the prior evening. Pierpont marshalled all the eastern supporters to attend. Newcomb decided to remain at home. He had just handed over the AMS presidency to Robert Woodward, a Columbia mechanics professor. Approaching his 50\textsuperscript{th} birthday, Woodward was born between the two generations involved in the dispute. Newcomb conveyed his views in a letter to McClintock. Fully aware of the arguments on both sides,

\textsuperscript{11}P. 57 in \cite{2}  

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The special committee recommended to the Council that the Society begin publication of the Transactions, if possible, on January 1, 1900. The Council then appointed a committee consisting of Fiske (chair), Woodward, Bôcher, Pierpont, and Moore (i.e., the special committee with Woodward in place of Newcomb) to work on the financial arrangements.

\textsuperscript{13}Our discussion of the dinner is drawn largely from Osgood’s recollections on p. 58 in \cite{2}.
Pierpont had already studied the matter. The approach became to have Society members solicit their universities to pledge $100 a year for each of the next five years. Fortified with talking points from Pierpont, mathematicians became fundraisers.

The campaign was a success. Chicago, Columbia, Cornell, Northwestern, and Yale followed through with subventions for ten years. Other institutions provided support for shorter periods. The scheme supplied the Transactions with $8100 over its first ten years. Sufficient funds were assured by its April 1899 meeting for the Society to authorize the journal. The Council appointed Moore, Ernest Brown, and Fiske as the first editorial board, which in turn elected Moore as the editor in chief.

Although a member of the Council, Moore did not travel to New York for the February or April meetings. He did take at least one trip during this period. On March 22 Moore visited Baltimore to explore an offer of a Johns Hopkins professorship. The recruitment began in January while Moore was considering whether to serve as an associate editor of the AJM. To a query from Newcomb, Moore responded that, while happy in Chicago, he was open to considering better circumstances. Over the following two months Newcomb and Gilman pressed for a campus visit while Moore sought details on remuneration and opportunities for building a department. Despite receiving neither, Moore was sufficiently interested to make the trip to Baltimore. Shortly after returning to Chicago, he declined the offer. If Moore had accepted, he likely would have become associated with the AJM, rather than the Transactions.

That Moore was the top choice of both Johns Hopkins and his AMS colleagues attested to his standing in the American mathematical community. A major international recognition came just after being named editor in chief. Moore was the only American among seven scientists awarded honorary degrees in a ceremony at Göttingen. The occasion was the unveiling of statues of Gauss and Wilhelm Weber.14

The timing of the 37-year-old Moore’s acceptance of the demanding editorial position, with his career so rapidly ascending, was indicative of his commitment to elevating American scholarship. It should be said that he was not alone in giving of himself to advance the Transactions. Making the journal a reality had drawn together a group in common cause. Mathematicians outside the editorial board, such as Heinrich Maschke, White, Pierpont, Osgood, and Bôcher, were passionately devoted to bringing distinction to the journal.

A number of qualities made Moore an ideal editor in chief. His mathematical connoisseurship and his willingness to work with authors to improve exposition ensured that the published papers were of high quality. But the Transactions was intended to showcase American mathematics, and Moore possessed a knack for subtle promotion that had already served the University of Chicago. The first issue of the Transactions included articles by White, Leonard Dickson (then at Texas), F. R. Moulton (Chicago), Maschke, Bôcher, Oskar Bolza (Chicago), G. A. Miller (Cornell), and Moore, all from American universities. As with the AJM, the Transactions carried papers by authors from European institutions, specifically Paul Gordan of Erlangen and Édouard Goursat of the University of Paris in the first issue. What was different with the Transactions was that, rather than being solicitations of the authors’ choosing, the papers of Gordan and Goursat were inspired by earlier work of Americans. In planning the issue layout to lead off with White, Gordan, and then Goursat, Moore was deftly bringing attention to the acknowledgements by Gordan and Goursat that their work drew on papers by White and Osgood, respectively.

The creation of the Transactions coincided with the 1899 transfer of the Annals to Harvard. Bôcher, especially, worked diligently to make the Annals a valuable resource for faculty and graduate students. Nonetheless he operated with a clear understanding that the best material belonged in the Transactions. The timing of the Annals move to Harvard probably explains the absence of Böcher from the original Transactions board. Osgood was abroad for the 1899–1900 academic year, residing with his family in Göttingen for most of the period. Throughout the year he followed the progress of both journals with great interest.

Osgood’s own research began slowly in Europe. For several months he sought relief from insomnia and a neurological disorder. The symptoms abated, at least temporarily, toward the end of the fall. In the first days of 1900 Osgood’s spirits were lifted by a letter from Newcomb with the passage

Sylvester’s chair at the Johns Hopkins has never been filled. The university is now desirous of filling it, if only one of the leading men of the time can be found for the place. I have taken the liberty of suggesting you. Please tell me whether you are willing to allow the use of your name in this connection.15

14Klein selected Moore while Hilbert chose Hadamard for the prestigious international awards. See P. 34 in October 1899 Bulletin and Henry Seely White Correspondence. Letter from Osgood, 10/30/1899. Columbia University Rare Book and Manuscript Library.

15Henry Seely White Correspondence. In a 1/6/1900 letter from Osgood he quotes a 12/21/1899 letter from Newcomb to Osgood. Columbia Rare Book and Manuscript Library.
Nine months after Moore turned Hopkins down, Newcomb was recruiting another mathematician from the younger generation. Osgood appeared more movable. At Harvard he was slated to serve three additional years at the rank of assistant professor before becoming eligible for promotion to professor. The most famous chair in the history of American mathematics would have seemed tempting. Yet Osgood was certain that, whatever the emoluments, they would not include the authority to hire “a body of men of my selection to man the department with and to guarantee keeping the department up to a first class standard.” He saw better prospects with Böcher at Harvard. Osgood declined further consideration.

Meanwhile, at Göttingen Osgood found mathematical stimulation in his interactions with Felix Klein, Arthur Schoenflies, and particularly David Hilbert. Osgood wrote to White with excitement over Hilbert’s progress on the Dirichlet problem. His last extant letter to White is dated February 28, 1900, expressing the intention of auditing Hilbert’s upcoming course. On June 2 Osgood boarded a ship for his return to New York. On the voyage he completed a manuscript that, soon after his return, was submitted to the Transactions. The paper, entitled “On the existence of the Green’s function for the most general simply connected plane region,” appeared in the journal’s third (October) issue. Building on the work of Riemann, Poincaré, and others, Osgood resolved the crucial element of what is now known as the Riemann mapping theorem. Osgood’s work on the Riemann mapping theorem placed him at the cutting edge of mathematical scholarship.

The younger American mathematicians were elated with the early volumes of the Transactions. Finally, the AMS provided an opportunity for meritorious papers to receive competent review and prompt publication. The editors sprinkled articles by distinguished Europeans, such as Hilbert and Jacques Hadamard, around the domestic material to confer an international prestige.

As with any American journal in 1900, the Transactions faced the challenges of traveling across the Atlantic Ocean and circulating through Europe. Consider the receptions, such as they were, to Osgood’s article and to G. D. Birkhoff’s seminal proof of Poincaré’s Last Geometric Problem a dozen years later. In the first edition of his classic text Complex Analysis [1], analytic function authority Lars Ahlfors states that many proofs have been given for the Riemann Mapping Theorem, without naming any names. For the second and third editions Ahlfors credits Paul Koebe with the first correct proof of the theorem, accompanied by the footnote: “A related theorem from which the mapping theorem can be derived had been proved earlier by W. F. Osgood, but did not attract the attention it deserves.”

Richard Courant was at Göttingen in 1913 when Birkhoff’s result was published in the Transactions. Many years later, Courant recalled that the paper appeared in “some obscure American journal.” Courant learned of the work from a review in Jahrbuch über die Fortschritte der Mathematik. Europeans were better able to access the proof when the Bulletin de la Société Mathématique de France took the unusual step of publishing a French translation of Birkhoff’s Transactions paper. Unfortunately, the experiences of Osgood and Birkhoff demonstrate that the success in conveying the early twentieth century rise in American mathematics was mixed. As a consequence, the young Americans did not receive the respect abroad for their community that they desired and deserved.

While the Transactions became more a record than a showcase for American mathematics, its significance went well beyond its actual pages. The struggle to establish the journal drew Böcher, Fiske, Moore, Osgood, Pierpont, White, and others of their generation together in solidarity. Pierpont’s letters to White, after their crucial collaboration with Moore in Chicago, were written as if to a family member. Implementation of the Transactions was a team effort under the direction of Moore. The success was widely felt, empowering a generation in their thirties to take control and change the direction of the AMS. In 1900 Moore became the first 1860s-born president of the Society. He was followed in two-year terms by Fiske, Osgood, Pierpont, White, and others of their generation together in solidarity. The leaders for the new century carried aspirations conveying the early twentieth century rise in American mathematics on to flourish. In [3], I advance the view that by 1913 the significance of mathematical scholarship in the United States warranted comparable ranking with that of the leading European nations. The Transactions remains in operation today, the second sibling of a robust and evolving AMS periodical program in which the Journal of the AMS serves as the organization’s flagship for papers of the highest quality.

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16Henry Seely White Correspondence. Letter from Osgood, 2/28/1900s. Columbia Rare Book and Manuscript Library.

17For more on Osgood’s work see [7] and the discussion by R. Narasimhan in [3].
References


Credits

Photos of Thomas Fiske, Simon Newcomb, E. H. Moore, and Emory McClintock are courtesy of the AMS. Photo of Pierpont, Osgood, and Bôcher is courtesy of Theodore Osgood with electronic enhancements by Beverly Ruedi. Photo of Steve Batterson is by Ellen Neidle.
The Award for Impact on the Teaching and Learning of Mathematics is given annually to a mathematician or group of mathematicians who have made significant contributions of lasting value to mathematics education. Priorities of the award include recognition of (a) accomplished mathematicians who have worked directly with precollege teachers to enhance teacher impact on mathematics achievement for all students or (b) sustainable and replicable contributions by mathematicians to improving the mathematics education of students in the first two years of college.

The $1,000 award is provided through an endowment fund established by a contribution from Kenneth I. and Mary Lou Gross in honor of their daughters Laura and Karen. The AMS Committee on Education selects the recipient.

Nominations with supporting information should be submitted online to www.ams.org/impact. Letters of nomination may be submitted by one or more individuals. The letter of nomination should describe the significant contributions made by the nominee(s) and provide evidence of the impact these contributions have made on the teaching and learning of mathematics. The letter of nomination should not exceed two pages and may include supporting documentation not to exceed three additional pages. A brief curriculum vitae for each nominee should also be included.

Deadline for nominations is September 15, 2019.
mentoring 80–100 middle and high school students every summer from 1996 to 2004, he developed the curriculum related to mathematical logic and computer science for the national program. Dr. Siadat is perhaps best known for his Keystone Project, developed in close partnership with his two-time dissertation advisor, Professor Sagher, in which he worked on improving freshman and sophomore performance through frequent student assessment and feedback, adjusting teaching practices, and building study skills to achieve mastery of the topics. This research showed that the proper use of mathematics can actually improve one’s working memory toward enhancing fluid intelligence.

Built on his belief that mathematics can be a keystone to student learning, the Keystone program for teaching mathematics has helped students at different college levels. For example, in a multiyear study involving hundreds of community college students, the method led to an increase in student success in Elements of Algebra to 59%, as compared to 18% in the control group. Interestingly, without teaching any reading, it also led to a 12.3 percentile-point increase in reading comprehension scores of students, which could be attributed to their improved concentration skills. The control group showed no increase in the reading scores. The cumulative data also confirmed that the retention rate for the project classes was 74% versus 67% in traditional classes.

Dr. Siadat has made significant contributions toward improving mathematics education at the precollege and two-year college levels through regional and national projects, research, and presentations. His influence is not just limited to the students at his own institution, as is evident from the many honors he has received from various regional and national organizations, including the Carnegie Foundation for the Advancement of Teaching and Learning.
Council for Advancement and Support of Education, the Illinois Council of Teachers of Mathematics, the National Council of Instructional Administrators, the University of Illinois at Chicago, the Illinois Community College Board, the National Institute for Staff and Organizational Development, Northeastern Illinois University, and the Mathematical Association of America. Dr. Siadat’s work is a gift both to his many generations of students and to mathematics education.

For his many sustainable and replicable contributions to mathematics and mathematics education at both the precollege and college levels, the AMS Committee on Education is delighted to award Dr. M. Vali Siadat the AMS Award for Impact on the Teaching and Learning of Mathematics.

Biographical Note

M. Vali Siadat earned his BSEE from the University of California, Berkeley, and, later on, an MSEE from San Jose State University, while working as a professional engineer in the Silicon Valley in California. Throughout his academic and professional work, however, mathematics remained the subject of his utmost passion. With the desire to get rigorous training in mathematics and inspired by teaching in this subject, he pursued his intensive graduate studies in mathematics, resulting in his being awarded an MS in applied mathematics, a PhD in pure mathematics (harmonic analysis), and a DA (Doctor of Arts) degree in mathematics, majoring in mathematics education with a minor in applied statistics, from the University of Illinois at Chicago (UIC). He earned his doctoral degrees under the supervision of his two-time dissertation advisor and mentor, Professor Yoram Sagher.

Dr. Siadat has previously taught at several institutions of higher learning, including California State University at Dominguez Hills, the University of Southern California, Chicago State University, Loyola University Chicago, and the City Colleges of Chicago, where he is currently distinguished professor of mathematics at its Richard J. Daley campus. He was the Director/Co-Principal Investigator of a grant of over three quarters of a million dollars from NASA to conduct the Project Access/Chicago PREP program. He was also the Director/Co-Principal Investigator of a grant of nearly US$100,000 from Gabriella and Paul Rosenbaum Foundation intended to expand the Keystone Project. Dr. Siadat has extensive publications in mathematics and mathematics education journals and has had numerous presentations at local, statewide, and national mathematics meetings.

Dr. Siadat won the Distinguished Professor Award of Richard J. Daley College in 1999–2000. He won the 1999 Exemplary Initiatives in the Classroom Award from the National Council of Instructional Administrators, the 2001 Award for Excellence in Teaching from the Illinois Council of Teachers of Mathematics, and the 2001 Excellence in Learning–Centered Instruction Award from the Illinois Community College Board. He received the MAA Illinois Section’s Outstanding Teaching Award in 2002 and the Carnegie Foundation for the Advancement of Teaching and Council for Advancement and Support of Education Illinois Professor of the Year Award in 2005, followed by the 2009 NISOD Excellence Award from the University of Texas at Austin. He was also awarded the 2009 Mathematical Association of America’s Deborah and Franklin Tepper Haimo Award for distinguished teaching of mathematics.

Response

I am deeply honored to have received this prestigious award. Let me first highly commend the American Mathematical Society for creating this award in recognition of the importance of the mathematics education of the young minds attending colleges and universities in the United States. This is another testimonial that AMS not only promotes pure mathematics research and scholarship but also supports and promotes the efforts to upgrade the mathematics education of our precollege and college students. Let me also thank the AMS Committee on Education and the Awards Committee for bestowing upon me this great honor. I am immensely grateful.

About the Award

The Award for Impact on the Teaching and Learning of Mathematics was established by the AMS Committee on Education (COE) in 2013. The US$1,000 award is given annually to a mathematician (or group of mathematicians) who has made significant contributions of lasting value to mathematics education. Priorities of the award include recognition of (a) accomplished mathematicians who have worked directly with precollege teachers to enhance teachers’ impact on mathematics achievement for all students, or (b) sustainable and replicable contributions by mathematicians to improve the mathematics education of students in the first two years of college. The endowment fund that supports the award was established in 2012 by a contribution from Kenneth I. and Mary Lou Gross in honor of their daughters Laura and Karen. The award is presented by the AMS COE acting on the recommendation of a selection subcommittee. For the 2019 award, the members of the subcommittee were:

- Douglas Ensley
- Manmohan Kaur (chair)
- Jon Wilkening

A list of previous recipients of the Impact Award may be found on the AMS website at: [www.ams.org/profession/prizes-awards/ams-awards/impact](http://www.ams.org/profession/prizes-awards/ams-awards/impact).

Credits

Photo of M. Vali Siadat is by Neil Stern.
Many of us who believe passionately that math belongs to everyone used to think of math outreach as puzzles and games; media coverage such as books, articles, or films highlighting some math or a math personality in an accessible way; or art and design, showcasing beauty in two or three dimensions.

Those things are wonderful. But as we at MSRI have developed the programming for the National Math Festival (NMF)—appearing in its third incarnation on Saturday, May 4, 2019 at the Washington DC convention center—we have been amazed at the breadth of the ways math can be appreciated by the public. Hoping that others will see how to realize even more variety, we list some of what will happen at the NMF this year (all different from the events in past versions):

Among twenty-one distinct stage presentations and performances will be:

- A world-class New York dance ensemble with Dance of the Diagram, commissioned by Jim Simons to highlight the beauty of differential cohomology;
- A recent winner of the “Dance Your PhD” contest (AAAS and Science magazine), Nancy Scherich, to share her insights on braiding and anchoring a math Maypole;
- A whirlwind tour of 300 years of mathematical and musical history with Lillian Pierce—complete with a violin solo;
- Marcus du Sautoy’s reflections on creativity and artificial intelligence;
- Francis Su’s exploration of The Mathematics of Human Flourishing;
- Holly Krieger’s peek into prime numbers, by way of the YouTube channel Numberphile (2.7 million subscribers and counting…);
- Former Baltimore Ravens player John Urschel’s sharing of his enthusiasms for the math of chess and the math of football; and
- An opportunity to touch a prehistoric ice core, accompanied by a talk for ages 8+ about

The Amazing Variety of Public Math

David Eisenbud and Kirsten Bohl

the mathematics of sustainability, with Mary Lou Zeeman and the Mathematics of Planet Earth team.

The National Science Foundation will screen the winners of its first-ever “We Are Mathematics” short film contest; Joseph Teran will draw a young audience with clips from his team’s animation work in Frozen and Moana; and MSRI will screen a trailer of the forthcoming public television film about the inspiring, joyous life of Maryam Mirzakhani. All of this takes place in the Alfred P. Sloan Foundation Film Room.

Many organizations besides MSRI work hard and successfully to encourage math appreciation, and the NMF will give them a platform, too. New to the Festival, the Make or Take Spiral is not your regular exhibit setup (10’ x 10’ booths with skirted 6’ tables). Instead, it is a large Fibonacci-spiral-shaped structure with interior rooms housing about 20 different math organizations who individually and collectively will mobilize Festival-goers to “take math home with them” in a wide variety of ways. For example:

- The National Council of Teachers of Mathematics (NCTM) will bring tips and resources for classroom teachers and homeschool parents.
- The Erikson Institute Early Math Collaborative and DREME Network out of Stanford University will bring ideas, energy, and resources for preschool families.
- The National Science Foundation will bring two scavenger hunts, one for children and one for adults, both designed to showcase just how much math research they support.
- Four local Math Circles will come ready to sign up families for the next academic year.

Along with the novelties mentioned above, games and puzzles are a staple of public interest in math, and they will be well represented. New to the 2019 Festival is a newly released collection of board games for family-style play called South of the Sahara, created by the MIND Research Institute. Drawing on ancient math games from Ghana, Mozambique, and Madagascar, these games are available in English and Spanish, best for ages 7 and up. Many more games will make an appearance at the Festival including a chance to bring along and share your own favorite math-themed board games.

The National Math Festival has also catalyzed associated events around the country. Bubbles and soap films are always a source of wonder, and dozens of science museums around the US will host events on the same day as the Festival. With the support of the construction-set maker Zometool®, member museums affiliated with the Association of Science-Technology Centers (ASTC) will offer hands-on bubble blowing fun for all ages. Participants will dip Zometool wands of three-dimensional shapes such as cubes or tetrahedra into buckets of a homemade “bubble goop” made with Dawn® dish soap, and explore what happens when the wand is later shaken, or an additional bubble is deposited into the center of the frame with a straw, and so on. For starters, the geometric shape of the bubble inside the structure is often quite beautiful, and also not what one might expect. These events will serve as an amplification of the NMF on Saturday, May 4.

The world of insightful online recreational math content continues to amaze as well, and we salute its expansion with an ever-growing but carefully chosen set of books, videos, puzzles, games, and toys sortable by age appropriateness (ages 2-18+), in the More Math! section of the NMF web site NationalMathFestival.org.

The 2019 National Math Festival is organized by the Mathematical Sciences Research Institute (MSRI) in cooperation with the Institute for Advanced Study (IAS) and...
the National Museum of Mathematics (MoMath). We are grateful to the many dedicated mathematicians and math organizations—such as the AMS, AWM, Bridges, Julia Robinson Mathematics Festival, NAM, NOVA, The Young People’s Project, and many, many others—who will bring this large-scale event to life.

We can only look forward to yet more surprises in this wonderful landscape of public engagement!

Credits
Photo of David Eisenbud is courtesy of the Simons Foundation.
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Notices of the American Mathematical Society
WHAT IS...

a Coleman integral?

Jennifer S. Balakrishnan

Introduction

Robert Coleman wrote a series of papers in the 1980s [Col82, Col85b, CdS88] where he developed a theory of $p$-adic line integration on curves and higher-dimensional varieties with good reduction at $p$. He gave a number of spectacular applications for these integrals, thereby demonstrating that they are a powerful tool in arithmetic geometry. These integrals are now known as Coleman integrals. The theory has since been extended in a number of different directions by Berkovich, Besser, Colmez, Vologodsky, and Zarhin, among others. Moreover, these integrals are amenable to computation, and implementations in various contexts are available in the computer algebra systems SageMath and Magma.

We discuss how to construct and compute these integrals and conclude by mentioning a few applications. For ease of exposition, we will assume that $X$ is a smooth projective curve of genus $g$ defined over $\mathbb{Q}$. We will also assume that $p$ is a prime of good reduction for $X$, which means that locally, the equations defining $X$ can be written with coefficients in $\mathbb{Z}_p$, so that when we reduce these modulo $p$, we obtain a smooth curve over $\mathbb{F}_p$.

Suppose $\omega$ is a meromorphic 1-form of the second kind on $X_{\mathbb{Q}_p}$, i.e., a 1-form with residue zero at all poles. What does it mean to compute the line integral $\int_P^Q \omega$ for points $P, Q \in X(\mathbb{Q}_p)$? The crux of the matter is understanding a $p$-adic “path” between the points $P$ and $Q$, in particular if $P$ and $Q$ are $p$-adically far away from each other.

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First we introduce a bit of terminology. Let $X^{an}$ denote the rigid analytic space over $\mathbb{Q}_p$ associated to $X$ (this is an analogue of a complex analytic space over a non-archimedean field). There is a natural specialization map from $X^{an}$ to its special fibre $\overline{X}$ given by reduction mod $p$. The fibres of this map are open unit disks, called residue disks.

Now fix a residue disk $D$ and consider $P \in D$. We can compute a uniformizing parameter $t$ at $P$, and if $P' \equiv P$ (mod $p$), the “tiny integral” $\int_P^{P'} \omega = \int_0^{t(P')} \omega(t)dt$, computed by a change of variables, converges. In particular, this is the Coleman integral of $\omega$ between $P$ and $P'$.

Example 1. Let $X : y^2 = x^5 + 2x + 1$ over $\mathbb{Q}_5$, and consider the points $P = (0, 1), P' = (5, 56)$ and the differential form $dx/2y$. We have that $t = x$ is a uniformizing parameter at $P$, and we compute $y = 1 + t - \frac{1}{2} t^2 + \frac{1}{2} t^3 - \frac{5}{8} t^4 + \frac{11}{16} t^5 - \frac{29}{16} t^6 + \frac{47}{16} t^7 - \frac{589}{256} t^8 + O(t^9)$. We have

$$\int_P^{P'} \frac{dx}{2y} = \int_0^{t(P')} \frac{dx(t)}{2y(t)}dt$$

$$= 3 \cdot 5 + 3 \cdot 5^2 + 2 \cdot 5^3 + 2 \cdot 5^4 + 3 \cdot 5^5 + 3 \cdot 5^6 + O(5^7).$$

However, if $Q \not\equiv P$ (mod $p$), the strategy above does not work. Indeed, as open unit disks in the $p$-adic topology are either disjoint or identical, we seem to have a serious problem: we can compute tiny integrals within any
given residue disk, but how do we carry out analytic continuation to ensure compatibility between different residue disks?

Coleman solved this problem using Dwork’s principle of analytic continuation along Frobenius, that is to say, using the fact that these integrals are Frobenius equivariant. By computing the action of Frobenius on differentials, Coleman showed that there is a canonical path between two points, which allowed him to compute integrals between different disks. One works over a wide open subspace $V$ of $X^{an}$ (obtained by removing from $X^{an}$ a finite number of closed disks of radius less than 1) and uses Frobenius (a rigid analytic map) to write down a linear system. The key insight is that by (proofs of) the Weil conjectures, we know the possible eigenvalues of Frobenius on $p$-adic (Monsky-Washnitzer or rigid) cohomology. We make this more precise below.

**Integrals Between Different Residue Disks**

First we record a number of useful properties of the Coleman integral:

**Theorem 2** (Coleman). Let $\eta, \xi$ be 1-forms of the second kind on a wide open subspace $V$ of $X^{an}$ and $P, Q, R \in V$. The definite Coleman integral has the following properties:

1. **Linearity:** $\int_P^Q (a\eta + b\xi) = a\int_P^Q \eta + b\int_P^Q \xi$.
2. **Additivity in endpoints:** $\int_P^Q \xi = \int_P^R \xi + \int_R^Q \xi$.
3. **Change of variables:** If $V' \subset X$ is a wide open subspace of a rigid analytic space $X$ and $\Phi: V \to V'$ a rigid analytic map then $\int_P^Q \Phi^* \xi = \int_{\Phi(P)}^{\Phi(Q)} \xi$.
4. **Fundamental theorem of calculus:** $\int_P^P df = f(Q) - f(P)$ for $f$ a rigid analytic function on $V$.

The rigid cohomology group $H^1_{rig}(X/\mathbb{Q}_p)$ is a 2-dimensional vector space that is equipped with an action of $p$-power Frobenius $F_p$ that is lifted from the special fibre $\overline{X}$. By the work of Baldassarri and Chiarellotto, there is an isomorphism between rigid and algebraic de Rham cohomology. Let $\{\omega_0, \ldots, \omega_{2g-1}\}$ be a basis of $H^1_{rig}(X/\mathbb{Q}_p)$.

One first computes the action of $p$-power Frobenius $F_p$ on each differential in the basis and reduces using relations in cohomology to obtain

$$F_p^* (\omega_i) = df_i + \sum_{j=0}^{2g-1} \Phi_{ij} \omega_j,$$

where each $f_i$ is an element of a ring of overconvergent functions associated to $X_{\mathbb{Q}_p}$, and $\Phi \in M_{2g \times 2g}(\mathbb{Q}_p)$. In the case of hyperelliptic curves, the computation (1) is Kedlaya’s algorithm, and in the case of smooth curves, this was recently made into a practical algorithm by Tuitman. Then one uses properties of the Coleman integral to compute the values of integrals on basis differentials between points $P, Q$ where the $f_i$ converge, starting from

$$\int_{F_p(P)}^Q \omega_i = \int_P^Q F_p^* \omega_i,$$

and using (1) to deduce the following:

$$\sum_{j=1}^{2g} (\Phi - I)_{ij} \left( \int_P^Q \omega_j \right) = f_i(P) - f_i(Q) - \int_P^Q \omega_i - \int_P^{F_p(Q)} \omega_i.$$

In particular, since the eigenvalues of the matrix $\Phi$ are algebraic numbers of complex absolute value $p^{1/2}$, the matrix $\Phi - I$ is invertible, and we obtain the integrals of basis differentials between $P$ and $Q$.

**Example 3.** As in our previous example, let $X : y^2 = x^5 + 2x + 1$ over $\mathbb{Q}_5$. Now consider the points $P = (0, 1), Q = (1, 2)$. Using (2), we compute in SageMath that

$$\int_P^Q \frac{dx}{2y} = 2.5 + 4.5^2 + 3.5^3 + 4.5^4 + 4.5^5 + 2.5^6 + O(5^7).$$

**Remark 4.** One can instead compute the Coleman integrals of regular 1-forms by passing to the Jacobian of the curve and rescaling so that the endpoints of integration are in the residue disk of the identity. Pulling back to the curve, this allows one to rewrite any given Coleman integral as a sum of tiny integrals. The main advantage of the approach with Frobenius is that it generalizes easily to iterated Coleman integrals.

Coleman and de Shalit, as well as Besser [Bes02], defined iterated Coleman integrals

$$\int_P^Q \xi_n \cdots \xi_1,$$

which behave formally like iterated path integrals

$$\int_0^1 \cdots \int_0^1 f_n(t_n) \cdots f_1(t_1) \, dt_n \cdots dt_1.$$

Besser and de Jeu were the first to give an algorithm to compute iterated Coleman integrals, in the case of $X = \mathbb{P}^1 \setminus \{0, 1, \infty\}$. These integrals are defined by the $p$-adic differential equations

$$\begin{cases} 
L_0(z) = \frac{z}{1-z} \\
L_{n+1}(z) = L_n(z) \frac{dz}{z}, \ n \geq 0
\end{cases}$$

and are $p$-adic polylogarithms, which via a $p$-adic analogue of Beilinson’s conjecture, are conjecturally related to special values of $p$-adic $L$-functions. In the case of higher genus curves, one can compute $n$-fold iterated Coleman integrals by applying (1) to each differential in the integrand, then relating $n$-fold integrals to $(n-1)$-fold integrals and making an observation about the eigenvalues of the matrix $\Phi^n$, to produce the appropriate analogue of (2).
Applications

Coleman and Gross described how to use Coleman integrals to compute $p$-adic heights on Jacobians of curves, which allows one to compute interesting arithmetic invariants, such as $p$-adic regulators. Indeed, there is a (global) $p$-adic height pairing on the Jacobian of a curve that, in many ways, parallels the story of the canonical Néron–Tate height: the Coleman–Gross $p$-adic height is a bilinear form $h$ that can be computed via a decomposition

$$h(P_1, P_2) = \sum_v h_v(P_1, P_2)$$

into local heights $h_v$. While the local heights at primes $v \neq p$ are computed via arithmetic intersection theory (just as in the Néron–Tate height, though the intersection multiplicities are weighted with $p$-adic logarithms), the local height pairing at the prime $v = p$ is given in terms of a Coleman integral. By computing global $p$-adic heights on a basis of the Mordell–Weil group of the Jacobian, one can calculate a $p$-adic regulator, which is one of the invariants appearing in a $p$-adic analogue of the Birch and Swinnerton-Dyer conjecture.

Coleman also used these integrals to compute torsion points on Jacobians. He further gave a beautiful reinterpretation [Col85a], via his eponymous integrals, of the classical method of Chabauty for showing finiteness of the number of rational points on curves $X/\mathbb{Q}$ whose Jacobians have rank less than their genus. On each residue disk, the Coleman integrals are locally analytic and can be written as convergent $p$-adic power series, with finitely many zeros. By bounding the number of zeros of these integrals (where this zero locus, which we denote as $X(\mathbb{Q}_p)_1$, is a finite set of $p$-adic points containing the set of rational points), he gave an upper bound on the number of rational points on such curves. The Chabauty–Coleman method has been generalized in a number of ways, for instance, by removing the hypothesis that $p$ must be a prime of good reduction. This variant was used by Stoll and Katz–Rabinoff–Zureick-Brown to give a uniform bound on the number of rational points on curves whose Jacobians have rank at most $g - 3$.

Iterated Coleman integrals are conjectured to satisfy striking relationships when evaluated on rational points on curves, with no restriction on the rank of the Jacobian of the curve. Kim’s construction of Selmer varieties [Kim09] gives a sequence of sets

$$X(\mathbb{Q}_p)_1 \supset X(\mathbb{Q}_p)_2 \supset \cdots \supset X(\mathbb{Q}_p)_n,$$

where each set $X(\mathbb{Q}_p)_k$ is described by $k$-fold iterated Coleman integrals and contains the set of rational points $X(\mathbb{Q})$. One would like to show that, for a given curve $X/\mathbb{Q}$, there is a computable depth $\ell$ at which the set $X(\mathbb{Q}_p)_\ell$ is finite. Moreover, one would like to compute the set $X(\mathbb{Q}_p)_\ell$. Some recent progress has been made on understanding quadratic Chabauty—i.e., studying the set $X(\mathbb{Q}_p)_2$—by constructing integrals that vanish on the set of points $X(\mathbb{Q}_p)_2$ for curves with extra structure and Mordell–Weil rank equal to $g$. More generally, by carrying out Kim’s nonabelian Chabauty program, one hopes to discover new relationships among iterated Coleman integrals, thereby leading to a new, constructive proof of finiteness of rational points on higher genus curves.

References

Treasurer
of the American Mathematical Society

The American Mathematical Society is seeking applications and nominations of candidates for the position of Treasurer. The Treasurer is an officer of the Society and is appointed by the Council for a two-year term. The first term of the new Treasurer will begin February 1, 2021, with initial appointment expected in January 2020 in order that the Treasurer-designate may observe the conduct of Society business for a full year before taking office.

All necessary expenses incurred by the Treasurer in performance of duties for the Society are reimbursed.

QUALIFICATIONS

The Treasurer should be a research mathematician and must have substantial knowledge of Society activities. Although the Treasurer is appointed by the Council for a term of two years, candidates should be willing to make a long-term commitment, as it is expected that the new Treasurer will be reappointed for subsequent terms pending successful performance reviews.

DUTIES

- Administer or supervise the administration of fiscal policies in the interest of the mathematical community, as laid down by the Trustees.
- Monitor the receipt and expenditure of funds and the care of investments.
- Monitor budgets and trends of finance over periods of years.
- Review salary policy for AMS employees and its applications to individuals.
- Serve ex officio as a member of the Board of Trustees, Council, and several other committees; the Treasurer chairs the Audit and Risk, the Investment, and the Salary committees.

APPLICATIONS & NOMINATIONS

A Search Committee, with Alejandro Adem as Chair, has been formed to seek and review applications. Persons wishing to apply should do so through MathPrograms.Org. Nominations and questions should be directed to the Chair of the Search Committee: tsc-chair@ams.org.

For full consideration, applications, nominations, and supporting documentation should be received by May 15, 2019.
2019 Mathematics Programs that Make a Difference Award

The Women and Mathematics Program at the Institute for Advanced Study is the recipient of the 2019 AMS Mathematics Programs that Make a Difference Award.

WAM Program Committee Members
Back, left to right: Margaret Readdy, Antonella Grassi, Nancy Hingston, Dusa McDuff, Karen Uhlenbeck.
Front, left to right: Alice Chang, Lisa Traynor (former program committee member), Chuu-Lian Terng, Christine Taylor, Michelle Huguenin.

Citation
The American Mathematical Society, through its Committee on the Profession, is pleased to announce that the 2019 Award for Mathematics Programs that Make a Difference recognizes the Women and Mathematics Program (WAM) at the Institute for Advanced Study for its outstanding program of encouraging women to pursue advanced study and careers in mathematics. Created in 1994 with support from the Institute, Princeton University, and the National Science Foundation, WAM brings together around 60 female participants yearly, including undergraduate students, graduate students, and postdocs, for a week-long summer program of engaging lectures by leading female mathematicians and problem sessions run by teaching assistants. Panels and informal discussions with academic and industrial mathematicians round out the mentoring and networking experience. Approximately half the undergraduate participants come from schools without PhD programs.
Program participants collaborating on bench behind Fuld Hall.

and approximately a third of the recent participants are members of ethnically underrepresented groups. During its existence, WAM alumnae have won numerous accolades, including 55 out of 311 female NSF graduate fellows in mathematics, 26 out of 99 female NSF postdoctoral fellows in mathematics, and 22 out of 123 female ICM invited speakers. Recently, WAM alumnae have been encouraged to become WAM ambassadors, who, with small grants generously supported by Lisa Simonyi, initiate similar programs at other institutions. The Ambassador Program will help sustain and grow this successful paradigm which has made a substantial impact on encouraging women to pursue careers in the mathematical sciences.

About the Program
Following an initial workshop led by Antonella Grassi in 1993, the WAM Program was founded in 1994 at the IAS by Karen Uhlenbeck and Chuu-Lian Terng. The program seeks both to inspire talented women at the undergraduate level to pursue and complete their educational goals at the highest academic levels and to address the challenges encountered by female graduate students and postdoctoral researchers. It encourages female mathematicians to form collaborative relationships and to be part of a large network that provides support and reduces the sense of isolation experienced by many women in mathematics.

WAM is organized by a committee of local mathematicians headed by Alice Chang, Dusa McDuff, and Margaret Readdy (Academic Manager) and advised by Peter Sarnak in the IAS School of Mathematics and Administrative Manager Michelle Huguenin. Lisa Carbone, Maria Chudnovsky, Nancy Hingston, Elizabeth Milicević, Linda Ness, Lillian Pierce, and Karen Uhlenbeck complete the committee.

The annual May program centers around a mathematical topic of current interest. The core of the program consists of two week-long invited lecture series, named in honor of Uhlenbeck and Terng, supplemented by TA-run problem sessions, a postdoc research seminar, a computer workshop on a related topic, a distinguished colloquium named in honor of Grassi, and panel discussions that include industry and academic mathematicians. Midweek the program spends a day at Princeton University, where the participants hear talks and interact with Princeton faculty and students.

The program has been experimenting with new ways to extend its outreach beyond the local community. In 2017 former WAM manager Christine Taylor initiated the WAM Ambassador Program to provide small grants to WAM alumni to act as ambassadors to run outreach programs at their home institutions or regions. Recent activities include a series of fifteen research talks, panels, workshops, and social events for the University of California Santa Barbara student chapter of the Association for Women in Mathematics (www.facebook.com/UCSBWAM); University of Michigan Women in Mathematics (WIM) and AWM events, including graduate school panels and linear algebra study nights; and a Harvard/Massachusetts Institute of Technology Graduate Workshop in Algebraic Geometry for Women and Mathematicians of Minority Genders (sites.google.com/view/GWAG).

About the Award
In 2005, the American Mathematical Society Committee on the Profession (CoProf) conferred the Mathematics Programs that Make a Difference award in order to profile those programs that are succeeding and could serve as a model for others. Specifically, the Committee seeks to honor programs that:

1) aim to bring more persons from underrepresented backgrounds into some portion of the pipeline beginning at the undergraduate level and leading to advanced degrees in mathematics and professional success, or retain them once in the pipeline;

2) have achieved documentable success in doing so; and

3) are replicable models.

Preference is given to programs with significant participation by underrepresented minorities.

Beginning with the 2018 award, this recognition includes an award of US$1,000 provided by the Mark Green and Kathryn Kert Green Fund for Inclusion and Diversity.

A list of previous recipients of the Mathematics Programs that Make a Difference Award can be found on the AMS website at: https://www.ams.org/make-a-diff-award.

Credits
Article photos are by Andrea Kane, courtesy of IAS.
Mathematical Approaches to Secure In-Vehicle Networks

Robert A. Bridges

Modern vehicles rely on scores of small computers called electronic control units (ECUs) that continually communicate life-critical messages, e.g., wheel speeds and steering angle, through a few controller area networks (CANs). Originally designed as a closed network for the life of the vehicle, CAN protocol developed bereft of security features (e.g., authentication or encryption). For emission testing, updates, and other diagnostics, vehicle CANs are now physically accessible, but this comes with vulnerability to cyber attack. Mandatory on-board diagnostic ports (OBD-II) provide adversaries with direct access to the vehicle. Remote attacks, e.g., via Bluetooth or cellular connections, are possible but require deep expertise. In short, the critical networks for vehicle functionality are exposed to remote and local cyber attacks with little to no built-in defenses. Most notably, the infamous 2015 research of Miller and Valasek [Miller, Valasek] exhibited a remote takeover of a Jeep's brakes resulting in a massive recall and ample media coverage.

While the CAN protocol allows anyone to craft and send or passively monitor packets, original equipment manufacturers (OEMs e.g., Ford, GM) employ proprietary (secret) encodings, packing multiple signals or messages with potentially different formats into each CAN packet. In short, one can see or form packets, but how the constituent bits are translated to functional values is not public and varies per make, model, year, and even trim! Hence, pioneering after-market solutions for vehicle-security is a problem of personal and national security ripe for contributions of mathematicians and data scientists.

Oak Ridge National Laboratory (ORNL) has entered this research space as a collaboration of ongoing cyber security research and ORNL’s National Transportation Research Center. Our initial efforts along with others [Moore, et al. 2017; Gmiden, et al. 2016; Song, et al. 2016] use stationary stochastic processes to model timing properties of a given vehicle’s messages and identify injected signals. We exhibited an after-market, plug-in detector prototype that can self-tune in a matter of minutes learning timing char-
characteristics of the packets, then accurately identify packet injections to the CAN. Because various signals in the CAN data are communicating properties of related subsystems, we expect many covariates in these signals (e.g., pedal angle and vehicle speed). Attacks that seek to change vehicle states abruptly may manipulate the messages in an unexpected and discontinuous way. To test this hypothesis, we employed manifold learning techniques to learn a lower dimensional representation of CAN data and found that testing on emulated attacks shows a discontinuous jump in the lower dimensional representation providing a novel avenue for detection [Tyree, et al. 2018]. The above method is a “black box” method (knowing nothing of the data’s meaning, but identifying correlations mathematically), which detracts from interpretability. We seek to expand the method by developing a pre-processing algorithm to tokenize (partition) and translate the CAN packets into their encoded messages. Our process involves regression and optimal packing techniques to accurately decode many of the tested vehicles’ CAN data.

This ongoing research is a collaboration of mathematicians, data scientists, computer scientists, and automotive engineers and hopefully provides an example of the exciting interplay a mathematician can find in an applied research setting seeking real-world impact. Contributing researchers include Miki Verma (ORNL), Zachariah Tyree (GM), Samuel Hollifield (ORNL), Michael Iannacone (ORNL), Michael Moore (ORNL), Frank Combs (ORNL), and Michael Starr (USAF).

Robert A. Bridges

The Effect of Climate Change on Hurricanes

Michael F. Wehner and Christina M. Patricola

The past two Atlantic hurricane seasons have seen a number of particularly devastating storms. Hurricanes Harvey, Maria, Irma, Florence, and Michael affected millions of people in both the United States and the Caribbean island nations. As theoretical and climate modelling evidence provides solid support that the most intense hurricanes will become even more intense in a warmer world [Walsh et al., 2014], it is natural to ask whether such changes have already occurred. Although trends in observed Atlantic hurricane statistics are difficult to detect due to natural variability and a limited period of consistent observations, it is now possible to use extreme weather event attribution techniques borrowed from epidemiology to answer such questions. The CAlibrated and Systematic Characterization, Attribution, and Detection of Extremes (CASCADE) group, a DOE BER Scientific Focus Area at the Lawrence Berkeley National Laboratory has been refining these techniques to apply to individual intense storms.

Recently, we published a paper in *Nature* [Patricola & Wehner, 2018] examining 15 different intense historical tropical cyclone events using a publically available weather forecast model configured to simulate these storms in two ways. The first, a hindcast of the storms under actual observed environmental conditions, was compared to a second, counterfactual simulation under conditions such that humans had not warmed the planet. In order to most accurately simulate these storms, we used the Weather Research and Forecasting (WRF) model configured at resolutions down to 3 km, and 18 million processor hours on Cori KNL, a Cray XC40 at the National Energy Research Supercomputer Center (NERSC), also located at the Berkeley Lab. There are multiple ways to construct a simulated counterfactual world. We used previously performed global climate model simulations configured both with and

This work was supported by the US Department of Energy, Office of Science, Office of Biological and Environmental Research, Climate and Environmental Sciences Division, Regional & Global Climate Modeling Program, under Award Number DE-AC02-05CH11231. This research used resources of the National Energy Research Scientific Computing Center (NERSC), a DOE Office of Science User Facility supported by the Office of Science of the US Department of Energy under Contract No. DE-AC02-05CH11231. Michael Wehner is a Senior Scientist in the Computational Research Division at LBNL. His email address is mfwehner@lbl.gov. Christina Patricola is a Research Scientist in the Climate and Ecosystems Sciences Division at LBNL. Her email address is cmpatricola@lbl.gov.
without human changes to the composition of the atmosphere to estimate anthropogenic changes to atmospheric temperatures and humidity and ocean-surface temperatures and humidity. These were applied to the observed quantities used as initial and boundary conditions of the actual hindcast to generate a “hindcast that might have been” in the absence of climate change. This enables the attribution of the human influence on these storms interpreted in a Pearl causal framework [Pearl, 2009].

We found that although the expected increase in the wind speed of intense hurricanes due to human induced global warming has not yet emerged, it will by the end of the twenty-first century. However an increase in extreme hurricane precipitation has already emerged in our simulations and in some cases is greater than expected from thermodynamic considerations alone. These simulations indicate that global warming can cause a structural change in the storm dynamics that causes precipitation in the most intensely raining part of the storm to increase the most. This plausible mechanism of dynamical changes in intense hurricanes in a warmer world explains previously unexplained results from four independent studies of Hurricane Harvey (including our own). These found extreme precipitation increases in excess of that expected from the 19th century Clausius-Clapeyron relationship that establishes a relationship between temperature and saturation specific humidity [Risser & Wehner 2017; van Oldenborgh et al. 2017; Wang et al. 2018], and an increase in the probability of extreme rainfall from hurricanes like Harvey due to climate change [Emanuel 2017].

References


Song HM, Kim HR, Kim HK, 2016. Intrusion detection system based on the analysis of time intervals of CAN messages for in-vehicle network, in: ICOIN.


Credits

Photo of Robert A. Bridges is courtesy of ORNL.
Photo of Michael F. Wehner is by Rachel Lance, LBNL.
Photo of Christina M. Patricola is by Berkeley Lab photographer Marilyn Chung.
2019 Award for an Exemplary Program or Achievement in a Mathematics Department

The Department of Mathematics and Computing at Franklin College is the recipient of the 2019 AMS Award for an Exemplary Program or Achievement in a Mathematics Department.

At MathFest in 2016 and 2017. The Franklin Curriculum is designed around the following principles:

1) Research-based, all-inclusive goals and objectives
2) Scaffolded developmental strands
3) Engaging department culture
4) Innovative curriculum
5) Deliberate integration of co-curricular experiences
6) Comprehensive assessment

In addition to having well-defined programmatic goals, the goals and objectives for students are clear and comprehensive; the department is in the process of implementing online individual experiential portfolios to assist students in collecting, organizing, and reflecting on artifacts documenting their achievements. This integrated and engaging learning experience for mathematics students results in large numbers of majors highly involved in math-related activities during college and extremely successful professionally after graduation.

Their approach to developing career-readiness skills is well thought out and effective. All department instructors ensure that their students can work effectively in a team, solve complex problems, and communicate effectively both orally and in writing with a variety of audiences within and outside of mathematics. The department also brilliantly leverages the knowledge and experience of the older students to teach younger students. Older students with internship or workplace experience meet with younger students so that they can ask questions and learn about these opportunities. The older students learn to articulate how these experiences influenced them, a helpful skill to have for future job interviews, something the department is very aware of. This intermingling of students from different years is replicated in the department’s extracurricular and social activities. Engagement with alumni is another im-

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FROM THE AMS SECRETARY

Students engaged in SOMA activity with faculty member at departmental Homecoming tent.

portant pillar of the department’s strategy. The department asks alumni what they learned during their four years that is actually helpful in the workplace. This allows instructors to further tailor their approach. Informal events permit current students to mingle with alumni. Students participate in “Shadow Day” each semester, to get firsthand experience in a workplace.

Franklin College’s Department of Mathematics and Computing has made a sustained and imaginative effort over several years to create an inspiring culture of collaboration and excellence for their students. That culture has fostered an admirable level of academic and professional success for its students and alumni.

Thus we are delighted to present the 2019 AMS Award for an Exemplary Program to the Department of Mathematics and Computing at Franklin College.

About the Program

In 2019, Franklin College will launch a year-long celebration of “A Legacy of Mathematics.” The triggering achievement is the fact that three mathematics faculty members—Dwight Heath (1919–1961), Richard Park (1958–2003), and Daniel Callon (1987–today)—collectively have spanned a century at the college. But the larger focus is the culture of creative, integrated, experiential curricular and co-curricular learning that has been carefully nurtured by the combined efforts of the faculty, students, alumni, community partners, and donors. This program has been cited by the Association of College and University Educators as a national model of twenty-first-century career skill preparation.

A carefully scaffolded curriculum features innovative courses at all levels that incorporate technology and the development of professional skills and dispositions, including an emphasis on written and oral communication and collaborating effectively in formal and informal team environments. Annual co-curricular programming averages 200 distinct participants on a campus of 1,000 students, with each student involved in an average of four individual activities. Capstone experiences include internships, independent research projects, team statistical consulting projects for local nonprofit organizations, and alumni-coached team projects to analyze big data sets.

The foundations of this current program have been under development for over thirty years, building on funding from the National Science Foundation and Lilly Endowment Inc. to acquire technological infrastructure and to write curricular materials, resulting in selection as one of
Students work collaboratively on a project in student resource room.

EDUCOM’s 101 Success Stories of Information Technology in Teaching and Learning in Higher Education in 1993. A student learning plan based on principles of active learning was developed in 1995; a major component of the associated comprehensive assessment program was featured in Assessment Practices in Undergraduate Mathematics (MAA Notes 49) in 1999. Departmental hiring procedures have emphasized the department’s philosophy of experiential learning and involvement in curricular and co-curricular programming for over two decades. The institution of weekly lunches including all mathematics faculty in 2009 has fostered communication and solidified departmental unity, resulting in a program graduating an average of twelve mathematics majors and six minors per year.

**About the Award**

The Award for an Exemplary Program or Achievement in a Mathematics Department was established by the AMS Council in 2004 and was given for the first time in 2006.

This award recognizes a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Departments of mathematical sciences in North America that offer at least a bachelor’s degree in mathematical sciences are eligible. Through the generous support of an anonymous donor, the award carries a cash prize of US$5,000. The award is presented by the AMS Council acting on the recommendation of a selection committee. The members of the 2019 selection committee were Craig Evans, Rhonda Hughes, Brea Ratliff, Cristoph Thiele, and Sylvia Wiegand (chair).

**Credits**

Article photos by Renee Kean.
Maker of Patterns

Steven J. Miller

Through voluminous correspondence with his family in England, in this book the mathematical physicist Freeman Dyson gives us an entertaining story of physics and physicists from the perspectives of someone whose work was at the forefront of so many subjects. It is perspectives and not perspective, as we are treated to his thoughts from when the events happened, as well as commentary written for this book where he has the benefit of the passage of time. Very few technical details are given on his professional work, but for the purposes of the book very little detail is needed. While those with expertise in his areas will know the backstory behind the theories hinted at in his letters, the purpose of this book is not to expound on those. Rather, it is to discuss the life lessons and observations from someone who has significantly contributed in a variety of fields over sixty years. The book is filled with rich, insightful commentary on many issues; as this is a review for a mathematical audience, below I will mostly concentrate on some valuable professional lessons one can gain from his experiences.

To set the stage: Freeman Dyson is one of the giants of 20th century science, famous for his theories and advocacy on many social issues. Born in England in 1923, he studied with many of the greats at Cambridge in the early 1940s before joining the British war effort in the Operational Research Section of the RAF Bomber Command. After the war, in 1947 he travelled to America under a Commonwealth Scholarship to study with Bethe at Cornell, meeting, among others, Richard Feynman, whose perspective on quantum electrodynamics he helped explain and develop. After a year at the Institute for Advanced Study, he returned to England, working at the University of Birmingham until 1951 when he joined the faculty at Cornell. He moved back to IAS permanently in 1953, where he is now an emeritus professor.

In over six decades of scientific study he has made contributions in varied fields such as quantum electrodynamics, Diophantine approximation, and random matrix theory. Though numerous concepts bear his name, from the Dyson crank (which emerges in the study of modular forms and partition problems) to the Dyson sphere (a staggering engineering feat where one builds a shell completely enveloping a star, allowing one to harness its energy and provide almost limitless area for habitation), this book on the Maker of Patterns is not about the technical details of his contributions. Instead, it is a collection of letters written by Dyson, mostly to his parents, with commentary from him looking back at those earlier moments in his life. We are thus treated to a personal account of some of the most pivotal moments of the last century, from the horrors of World War II and the reconciliations afterwards to the development and acceptance of new theories of physics to efforts by scientists concerned about nuclear war to preserve the peace and the ethical issues of biological research.

The letters cover his life from the start of his student days at Cambridge to the late 1970s. We read his views on everything from family to politics, both with the immediacy of someone living it as well as the perspective in the commentaries that comes from years of experience. As the book is almost 400 pages, for the purposes of this review most of the passages chosen below highlight not just big moments, but more importantly good advice on how to
be a successful and happy scientist. Though such a focus omits the details of his science and peace advocacy, as well as his personal life, doing so keeps this review to a readable length, and these observations deserve a wide audience.

Below are several gems grouped by content, and not chronology. Many of the excerpts are deliberately taken from 1948 as he was reconciling the works of Schwinger and Feynman, and this coherent collection gives the reader a sense of what is in this book. Feynman and Schwinger shared the 1965 Nobel Prize in Physics with Sin-Itiro Tomonaga for their work building theories of quantum electrodynamics. The last is from an exciting time in mathematical physics, when random matrix theory was being recognized as a useful tool for nuclear physics; sadly there are no letters describing his famous ‘chance encounter’ with Hugh Montgomery, where the connections to number theory (through the Riemann zeta function) were first noticed.

The first set of excerpts concerns the new theories by Schwinger and Feynman on quantum electrodynamics. We are transported back to this exciting time as new ideas are being proposed, tested, adapted, and then adopted. Dyson becomes the person in the right place at the right time with the right training. These letters provide a wonderful commentary on the emergence of a new perspective, and how the establishment reacts.

January 24, 1948: A new period in physics started with the Columbia University experiments last summer which for the first time contradicted the existing quantum theory outside the nucleus. The first step was taken by Bethe when he showed how the theory could be extended to explain the Columbia results. My calculations of last term were part of the detailed carrying out of this extension. Then there was another big step in November when Julian Schwinger at Cambridge, Mass., produced a formally unified theory including Bethe’s work and covering the whole of nonnuclear physics.

September 14, 1948: On the third day of the journey a remarkable thing happened; going into a sort of semistupor as one does after forty-eight hours of bus riding, I began to think very hard about physics, and particularly about the rival radiation theories of Schwinger and Feynman. Gradually my thoughts grew more coherent, and before I knew where I was, I had solved the problem that had been in the back of my mind all this year, which was to prove the equivalence of the two theories. Moreover, since each of the two theories is superior in certain features, the proof of equivalence furnished a new form of the Schwinger theory which combines the advantages of both.

September 30, 1948: To arrive at the frontiers of physics is like breaking through a crust, after which one finds plenty of room to move in a lot of directions. …. One thing which I must always keep in mind to prevent me from getting too conceited is that I was extraordinarily lucky over the piece of work I have just finished. The work consisted of a unification of radiation theory, combining the advantageous features of the two theories put forward by Schwinger and Feynman. It happened that I was the only young person in the world who had worked with the Schwinger theory from the beginning and had also had long personal contact with Feynman at Cornell, so I had a unique opportunity to put the two together. …. It is for the sake of opportunities like this that I want to spend five more years poor and free rather than as a well-paid civil servant.

The final passage of this section is, for the expert, painfully brief. Freeman Dyson played a major role in the development of random matrix theory, which is able to accurately model phenomena ranging from the energy levels of heavy nuclei to zeros of L-functions. There are a few brief mentions to his collaboration with Mehta and others, but the purpose of this book is not to go into the details of what he studied and proved, and letters like this are but markers to what he has done.

May 25, 1961: You ask what I have been calculating so industriously for the last three months. I will try to explain what it is about. The idea is to work out a new kind of statistical theory which will apply to the dynamics of heavy nuclei.

The next set of excerpts give a wonderful view of what it is like to be a scientist. These range from a frank description of interactions with senior colleagues, to the joys of being absorbed in a problem and the challenges of finding time to work, to the creation of institutes. There are also personal recollections of how nervous one is when giving talks to audiences of experts, and the stresses and mental health issues people encounter in their careers. These latter are being recognized more and more, with institutions devoting significant resources to these problems. As a mathematician who has trained and mentored many postdocs, graduate students, and undergraduates, I found these passages especially impactful.

Commentary on the letter of October 17, 1948: The following letter to Oppenheimer contains some technical language which nonexpert readers should skip. Translated into plain language, the letter tells Oppenheimer to listen to what Feynman has to say and stop raising silly ob-
November 14, 1948: Oppenheimer is in California this weekend, .... I have been observing his behavior rather carefully during seminars. If one is saying, for the benefit of the audience, things he knows already, he cannot resist hurrying one on to something else; when one says things that he doesn’t know or immediately agree with, he breaks in before the point is fully established with acute and sometimes devastating criticisms, .... On Tuesday we had our fiercest public battle so far, .... He came down on me like a ton of bricks and conclusively won the argument as far as the public was concerned. However, afterwards he was very friendly to me and even apologized to me. When life is like this the great thing is to keep a sense of proportion, and avoid becoming a nervous wreck like Oppy. So far I think I’m succeeding, but you should not be surprised when I write melancholy letters occasionally.

May 1, 1949, Princeton: The Washington meeting lasted three days and was on the whole very successful. .... Being in the most fashionable branch of physics, I was put into the largest auditorium, a grandiose monstrosity with enormous gold-painted columns stretching up to a domed and bright blue roof. .... About half an hour before I was due to start, I came in and had a look around this place, and the sight of it made me so nervous that none of my previous agony at Chicago and elsewhere could faintly compare with it. For that last half-hour I was in a terrible state, sitting in a chair and sweating all over and feeling I could not even stand up.

October 25, 1952: Two days after the phone call from Oppy, I had a conversation with Bethe, and out of it I got a new idea for a major piece of research in physics. This has absorbed me completely during the last ten days. .... These last ten days have been great fun, but they make it even more clear how necessary it is to go to Princeton. I simply cannot go on at this pace. All my routine jobs are left undone and are piling up ahead of me.

November 1, 1952: My activities in the department are growing by leaps and bounds. I now am directing an empire of eight people who are working hard on the meson calculations which I started six weeks ago. It is amazing how things are humming. Everyone is happy, and they are getting interesting results. It is easy to run such a group once you have a suitable job for them to do. I am happy about it all. When I leave here, they will say “Look how he built the department up in two years” instead of “He didn’t like it so he quit after two years.” This makes a great difference.

Chapter 20 Introductory Commentary: Adventures of a Psychiatric Nurse. When I was appointed a professor at the Institute for Advanced Study, I used to say that my real job was to be a psychiatric nurse, giving consolation and comfort to the young members when they suffered from loneliness or depression. The visiting members were in a highly stressful situation, facing a year or two of complete freedom, with the expectation that they should do something brilliant. If they failed to perform, given this unique opportunity, there was a real danger of psychological collapse. In my time as a professor I lost three young people whom I had invited as members, one by suicide and two who ended up in mental institutions. I do not know how many I saved. I only know that the institute is a dangerous place for young people, and as a professor, I bore a heavy responsibility for their mental health. The letters are as usual arranged chronologically, beginning with family affairs and then telling stories of psychological disasters.

As much of the book concerns his personal life, it is worth including a few of these items. The next set of quotes is representative of his impressions of America and Americans.

January 2, 1948: The fact that they [Americans] are more alone in the world than average English people probably accounts for their great spontaneous friendliness. I had heard this friendliness attributed to the size of the country and to people’s loneliness in space, but I think the loneliness in time is more important.

November 14, 1948: After I wrote to you from Boston, I had an unequalled opportunity of seeing the real Boston, .... Boston is the most European of American cities, superficially reminiscent of London. Even the slums are old and built of brick, in contrast to the typical American slums which are built of wood and corrugated iron.

April 4, 1948: The revolutionary thing about the atomic bomb is not that it is so lethal but that it is so cheap. .... The upshot of this is that
when two powers both have even moderate quantities of plutonium at their disposal, to have the greater quantity is not a decisive advantage. The decisive factor in military strength is vulnerability. And the United States is likely to remain enormously more vulnerable to this sort of attack than Russia (let alone Western Europe). Hence in the course of time, and especially if Russia starts building a navy, there will be increasing pressure upon the United States to strike first before it is too late. As you say, it is very like 1914. I am an optimist too, but only in the very long run.

This last passage is from one of the final sections of the book. It and the letters that follow, which describe his interactions with others on the committee, give some sense of the challenges and importance of such work.

January 29, 1977: Today I went to the first meeting of a citizens’ committee which is supposed to decide for the town of Princeton whether the biologists at the university are to be permitted to work with recombinant DNA. I was asked to serve on the committee and agreed to do so because this is an important question and I should not stand aside. The committee will involve a great deal of work, and is supposed to produce a final report by May 1. We were told to expect to put into it about ten hours of work per week for ten weeks. It will probably add up to more than that.

For those looking for a technical description of some of the many areas where Dyson contributed, this is not the book to read. The most we get are the faintest outlines of subjects, with the barest mention of the issues. What we do get, and get well, are the thoughts and reflections of a major player from those times. We relive these times through letters written to non-experts, and thus the emphasis cannot be on the details, but rather on the people involved. This ranges from their personalities to how he interacted with them. There are many lessons here that transcend field and time, and are as relevant to the researchers of today as they were to those from 60 years ago. Thus, the lack of detail in the end gives the book more universality; the messages here can (and should) be read and understood by a wide audience.
New and Noteworthy Titles on our Bookshelf
May 2019

Closing the Gap: The Quest to Understand Prime Numbers
by Vicky Neale (Oxford University Press, 2017, 176 pages)

This short book chronicles some of the recent spectacular developments in the study of prime numbers. It revolves around the explosive events of 2013–2014, which were initiated by Yitang Zhang’s unexpected proof that there are infinitely many pairs of primes that differ by at most 70,000,000. Subsequent refinements and generalizations evolved at a rapid pace, with dozens of authors (individually and collectively) contributing in a short amount of time.

Although this is a “popular science” book about prime numbers, a basic level of familiarity with calculus and infinite series is assumed. Later on, the Hardy–Littlewood circle method approach to Waring’s Problem is discussed, and there is even a short section devoted to unpacking the meaning of the corresponding singular series—some non-trivial mathematics! However, Neale always tries to explain things in a down-to-earth and friendly manner.

Neale interweaves recent events with historical background and related results. The book features a creative structure that lends itself well to the subject matter. Apart from the introduction, the odd-numbered chapters have titles such as “June 2013” and chronicle the relevant number-theoretic events that occurred in a given month. The even-numbered chapters discuss related number-theoretic topics at a leisurely pace. Results ranging from Euclid’s Theorem and the Prime Number Theorem to Szemerédi’s Theorem and Lagrange’s Four-Square Theorem are explored in a conversational tone and with many illustrations.

The reporting begins in earnest in Chapter 3 (“May 2013”), which introduces one of the main characters in the drama, Yitang Zhang. Later chapters discuss the contributions of other mathematicians, in particular James Maynard, Terence Tao, and the Polymath8 group. A great deal of attention is paid to the role played by Polymath projects, in which groups of mathematicians collaborate on difficult problems online and in the open via editable wikis. For instance, a good four pages of this slender volume are devoted to the question “Is Polymath the future?”

A curious undergraduate mathematics major should enjoy this book and learn a great deal. For mathematicians who do not specialize in number theory but who are curious about the flurry of recent activity in the field, this book provides an excellent entry point.

Heretics! The Wondrous (and Dangerous) Beginnings of Modern Philosophy
by Steven Nadler and Ben Nadler (Princeton University Press, 2017, 192 pages)

It might seem unusual to discuss a graphic narrative on seventeenth-century philosophy in the Notices. However, this is not as much of a stretch as it might at first appear. Many of the key players in the story are familiar to us because of their seminal contributions to mathematics and science. For example, Pascal, Descartes, Leibniz, and Galileo are all central characters, along with other roughly contemporary philosophers, such as Bacon, Hobbes, Locke, Spinoza, and Bruno, who are not as intimately associated with the development of modern mathematics. The authors (Steven Nadler is a professor of philosophy and Ben Nadler is an illustrator) combine their disparate backgrounds and skills to present an appealing tale that is well suited for mathematicians who are familiar with the famous names, but unfamiliar with the philosophy behind them. Although very little mathematics is discussed, this book successfully depicts how some of the seminal characters of modern mathematics fit into the larger intellectual framework of seventeenth-century intellectual discourse.
CALL FOR APPLICATIONS & NOMINATIONS

Secretary
of the American Mathematical Society

The American Mathematical Society is seeking candidates for the position of Secretary, one of the most important and influential positions within the Society. The Secretary participates in formulating policy for the Society, participates actively in governance activities, plays a key role in managing committee structures, oversees the scientific program of meetings, and helps to maintain institutional memory.

The first term of the new AMS Secretary will begin February 1, 2021, with initial appointment expected in January 2020 in order that the Secretary-designate may observe the conduct of Society business for a full year before taking office.

All necessary expenses incurred by the Secretary in performance of duties for the Society are reimbursed, including staff support. The Society is prepared to negotiate a financial arrangement with the successful candidate and the candidate’s employer in order that the new Secretary be granted sufficient release time and provided staff support to carry out the many functions of the office.

QUALIFICATIONS
The Secretary should be a research mathematician and must have substantial knowledge of Society activities. Although the AMS Secretary is appointed by the Council for a term of two years, candidates should be willing to make a long-term commitment, as it is expected that the new Secretary will be reappointed for subsequent terms pending successful performance reviews.

DUTIES
• Organize and coordinate the Council and its committees.
• Serve as a member of the Council, Executive Committee, and several other committees, including the five policy committees, the Development Committee, and the Editorial Boards Committee.
• Work closely with the President to coordinate and administer the activities of committees.
• Oversee, together with the Associate Secretaries, the scientific programs of all AMS meetings.

APPLICATIONS & NOMINATIONS
A Search Committee, with Tara Holm as Chair, has been formed to seek and review applications. Persons wishing to apply should do so through MathPrograms.Org. Nominations and questions should be directed to the Chair of the Search Committee: ssc-chair@ams.org.

For full consideration, applications, nominations, and supporting documentation should be received by May 15, 2019.

The American Mathematical Society is committed to diversity. All qualified applicants will receive consideration without regard to race, color, religion, gender, gender identity or expression, sexual orientation, national origin, genetics, disability, age, or veteran status.
Mathematical Reviews is not just a bibliographic database of the mathematical literature; it also contains reviews of many of the items in the database. The reviews have been a hallmark of the operation from the very beginning. While the full-time staff of Mathematical Reviews puts in a lot of work to produce what is now MathSciNet®, the expertise of the many reviewers is an essential part of what makes Mathematical Reviews exceptional. As of this writing, we have 22,652 active reviewers.

The list of the 285 reviewers in the first volume (covering all of 1940, the first year of publication) is quite impressive. It reads like a who’s who of mathematics from the time. The reviewers include at least one Nobel Laureate, one Fields Medalist (the first two medals were awarded in 1936, then not again until 1950), and several future Wolf Prize winners. There are 15 AMS presidents in the list, plus the father of an AMS President. You can find it here: https://www.ams.org/publications/math-reviews/reviewersvolumel.

The list of all reviewers continues to include remarkable mathematicians. Here are some award-winning mathematicians who are reviewers:

- **Fields Medals**: 30 of 62 medalists have written at least one review. The most prolific are Michael Atiyah (229 reviews), Pierre-Louis Lions (218 reviews), and Lars Ahlfors (170 reviews). Of the most recent medalists (2018), Alessio Figalli has written the most reviews (28). Of the medalists in this century, Cédric Villani is the most prolific reviewer, having written 98 reviews.
- **Abel Prize**: 16 of the 19 laureates have written reviews. The most prolific are Michael Atiyah, Srinivasa Varadhan (106 reviews), and Mikhail Gromov (101 reviews).
- **Steele Prize for Lifetime Achievement**: 22 of the 29 prizewinners have written at least one review. The most prolific are Henry P. McKean (315 reviews), Harry Kesten (262 reviews), Frederick Gehring (159 reviews), and Ralph S. Phillips (128 reviews).
- **Steele Prize for Seminal Contribution**: 19 of the 38 prizewinners have written at least one review. The most prolific are Louis de Branges (257 reviews), Srinivasa Varadhan, and Mikhael Gromov.

### Counting Reviews

A frequent question we receive from new reviewers or prospective reviewers is “How many reviews do people normally write in a year?” Well, as a database, we have the data. From 2000 to 2017 (the last year for which we have complete data at this time), the median number of reviews written per year (per reviewer) has held steady at 3. The average number of reviews has varied from 3.17 to 4.45, with the low coming in 2016 and the high in 2005. From this you might deduce (correctly) that we have some reviewers who write a lot of reviews. There are 20 reviewers who are still active and who have written 500 or more reviews:

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Edward Dunne
Historically, we also had some rather prolific reviewers, some of whom you may recognize:

Some Prolific Past Reviewers:
- Joel, Jeffrey S. ................................................................. 3152
- Boas, Ralph P. .............................................................. 2400
- Lehmer, D. H. ................................................................. 990
- Coxeter, H. S. M. .......................................................... 891
- Carlitz, Leonard ............................................................ 866
- Struik, Dirk J. ................................................................. 811
- Erdős, Paul ...................................................................... 638
- Hewitt, Edwin .............................................................. 602
- Dieudonné, Jean ............................................................ 591
- Dyson, Freeman J. ........................................................ 393
- Niven, Ivan .................................................................. 361
- De Branges, Louis .......................................................... 256
- Fleming, Wendell H. ....................................................... 192

The name Jeffrey Joel may not be familiar to many. He was an Associate Editor at Mathematical Reviews from 1973 to 1991. Ralph Boas, who has several claims to fame, was the Executive Editor of Mathematical Reviews from 1945 to 1950. His son, Harold, is keeping up the family tradition of writing reviews, appearing on the list of current prolific reviewers.

The reviewers come from all over the world. We have active reviewers from 141 different countries. (For comparison, the United Nations has 193 member states.) About one-sixth come from the United States. Half of the reviewers come from six countries: United States, China, Italy, India, France, and Germany. Three quarters of the reviewers come from 16 countries: the above six plus Spain, Japan, Brazil, England, Iran, Turkey, Poland, Russia, Canada, and Romania.

Reviewers and AMS Members

Mathematical Reviews is a service of the American Mathematical Society. So, what is the correlation between membership and reviewing? Overall, about 10% of AMS members are active reviewers. This number is misleading, however, since the count of “all AMS members” used to compute this includes all the undergraduate and graduate student members, who are unlikely to be reviewers. About 17% of regular AMS members are reviewers. By category, the affiliate members (those living in a country classified by the World Bank as a low-income, lower middle-income, or upper middle-income economy) have the greatest percentage who are reviewers: 53%. The next is reciprocity members (those who also belong to one of the foreign societies with which the AMS has established a reciprocity agreement), of whom a bit over 27% are reviewers.

Reviewers by Subject Area

Reviewers are not evenly distributed among the subject areas. In particular, we tend to have more reviewers in traditional areas of pure mathematics than in applied areas. Most reviewers are willing to review in multiple classes, so it is hard to pigeonhole someone as a “calculus of variations” reviewer, since they might also write reviews of papers in functional analysis or PDEs. So, there is a thinning out factor that might be necessary. Also, some reviewers are willing (and able) to write more reviews per year than others. Finally, it is not always clear what is pure mathematics or applied mathematics just by a 2-digit class. For instance, I’m about to count MSC 35 (PDEs) as “pure” and MSC 60 (probability) as “applied,” but neither characterization seems fair. Nevertheless, counting the number of reviewers who have listed a particular 2-digit class gives an idea of how hard or easy it is to match a reviewer to a paper in that class. By this rough estimate, we have about four times the density of reviewers in the classes from 03 to 58 as in the classes from 60 to 86. Mathematical Reviews is always looking for reviewers in all areas, but you can see why we would be particularly happy to have more reviewers in these “applied areas,” which leads us to the next section.

Becoming a Reviewer

How do you become a reviewer? Statistically, most reviewers are suggested by our editors. We contact them to ask if they are interested. Normally, we look for someone who has already published some papers, which gives us an idea of the person’s research interests. The rule of thumb (a rough guideline) is to have three papers in MathSciNet. If you haven’t been contacted by us, or if you were, said “No,” and have now changed your mind, then you can volunteer to become a reviewer by writing to math-
rev@ams.org. If you are going to volunteer, it is best to include your MR Author ID. Find it by searching from the URL: https://mathscinet.ams.org/mathscinet/MRAuthorID/search. This link works whether your institution has a subscription to MathSciNet or not. In the block at the top of the page, you will see your MR Author ID. Mine is 239650. Note: if you don’t have any publications in the Mathematical Reviews database, then you won’t have an MR Author ID.

Why should you become a reviewer? One reason is a sense of duty: helping out your fellow mathematicians. Another reason is to keep up on mathematics. As a reviewer, you would be sent papers on topics related to your own interests. Often, these might be papers you already know about. However, they may also be papers you haven’t heard about. As a reviewer myself, I found this second reason the more compelling. Reviewing was a great way to keep up on the literature. Now that I am Executive Editor, I don’t write reviews any more, but I do miss it.

Edward Dunne

Credits
Author photo is courtesy of Edward Dunne.
Carmichael’s totient conjecture lacks the name recognition of such media darlings as the twin prime conjecture or the Riemann hypothesis, but in 2018 this open question in number theory blipped briefly into the public consciousness. It cropped up in, of all places, a DriveTime® commercial.

Besides bearing a name beginning with the syllable *ca*, Carmichael’s totient conjecture (CTC) has nothing to do with DriveTime’s stock—in-trade, which is the sale and financing of used automobiles. CTC concerns the multiplicity of values of Euler’s totient function. Euler’s totient function \( \phi(n) \) returns for a positive integer \( n \) the number of positive integers at most \( n \) that are relatively prime to \( n \) (where 1 is counted as being relatively prime to all numbers). CTC posits that for every \( n \in \mathbb{Z}^+ \) there is at least one \( m \in \mathbb{Z}^+, m \neq n \) such that \( \phi(m) = \phi(n) \).

The DriveTime spot risks leaving credulous viewers under the same misapprehension the conjecture’s eponym harbored for over a decade: namely, that the CTC has been proven (or “solved” as the ad’s copywriters might inaptly put it). Robert Daniel Carmichael (1879–1967) published a paper [1] purporting to establish the CTC in 1907. Deeming the proof trivial enough even for students, Carmichael included it as an exercise—Chapter 2, #8—in his 1914 textbook *The Theory of Numbers*. By 1922, however, several readers had pointed out a gap in Carmichael’s argument, a gap he could not fix.

“So far I have been unable to supply a proof of the theorem,” Carmichael conceded in a note [2] in the *Bulletin*, “though it seems probable that it is correct.” He felt “compelled to allow it to stand in the status of a conjectured or empirical theorem.”

Now if any conjecture warrants being designated an “empirical theorem,” CTC is it. The statement remains unproven, but mathematicians have over the decades increased by impressive orders of magnitude the lower bound on a counterexample (see Table 1). Carmichael himself began the push, establishing in his 1922 note that if there exists an \( n \) such that the value \( \phi(n) \) is attained only by that unique \( n \), that \( n \) exceeds \( 10^{37} \). In the 1994 paper [10] in which they raised the bar to \( 10^{10,000,000} \), Aaron Schlafly and Stan Wagon remarked, “We do not know of another unsolved problem..."
Math Outside the Bubble

“But did he buy his car at DriveTime, using the industry’s smartest online tools?” continues the voiceover as a man in coveralls enters stage left and Zoolof’s satisfied smile transforms into a wide-eyed look of alarm. “No, no he did not,” narrates Lyman as the janitor’s wet cloth begins to obliterate the mathematician’s chalkings. “Dr. Gunter Zoolof was almost a genius.”

That last line underscores what Carmen Latterell, who has written about pop-culture’s portrayal of those who do mathematics, sees as the DriveTime spot’s most deleterious implication. Yes it’s unfortunate in Latterell’s eyes that the ad depicts a mathematician stereotypical in his demographics—male, white—and appearance. But she finds it even worse that the commercial perpetuates the myth that a mathematician is brilliant in mathematics but stupid in all other areas.

“This is the harmful stereotype that makes students not want to study math,” she says.

Not all ads featuring mathematicians reinforce all stereotypes, however. A 2013 commercial for Beautyrest’s ComforPedic mattress, for instance, shows graph theorist Maria Chudnovsky exercising sound judgment in choosing a sleep surface. And in a 2016 TurboTax spot a smartly clad Chudnovsky (one mathematician, two commercials? surely a record!) deftly deploys an on-screen help function to demonstrate that “it doesn’t take a genius to do your taxes.”

Many mathematics educators find the equation of “mathematician” and “genius” problematic, but were someone to resolve CTC at last, would s/he have claim to the “genius” mantle?

Stan Wagon thinks so.

“Whenever a big conjecture, around for over 100 years, that is ‘obviously true’ by various heuristic considerations is actually proved, that is a big deal,” he says. “Any such

<table>
<thead>
<tr>
<th>Lower bound</th>
<th>Reference</th>
<th>Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n &gt; 10^{37} )</td>
<td>R. D. Carmichael [2]</td>
<td>1922</td>
</tr>
<tr>
<td>( n &gt; 10^{400} )</td>
<td>V. L. Klee, Jr. [6]</td>
<td>1947</td>
</tr>
<tr>
<td>( n &gt; 10^{10,000} )</td>
<td>P. Masai &amp; A. Valette [8]</td>
<td>1982</td>
</tr>
<tr>
<td>( n &gt; 10^{10,000,000} )</td>
<td>A. Schlafly &amp; S. Wagon [10]</td>
<td>1994</td>
</tr>
<tr>
<td>( n &gt; 10^{10,000,000,000} )</td>
<td>K. Ford [3]</td>
<td>1998</td>
</tr>
</tbody>
</table>

Table 1. The lower bound on a counterexample to CTC

Kevin Ford, who in 1998 [3] bumped that lower bound up to a staggering 10^{10,000,000,000}, calls CTC his “favorite problem” and has puzzled over it on and off since his grad school days.

In 1999, Ford proved a result related to an alternate formulation of CTC. If \( A(f) \) denotes the number of positive integers \( n \) for which \( \phi(n)=f \), CTC says that \( A(f) \) can never equal 1. Ford showed [4] that every positive integer other than 1 occurs as a value of \( A(f) \).

In 2014, Ford shed some light on the difficulty of a particular strategy for resolving the CTC. One approach to the problem is based on showing that any counterexample must be divisible by many primes; if the set of such primes were infinite, CTC would follow. Ford showed [5] that the set \( S \) of such primes is very “thin” (more precisely, if \( P(x)=\#\{p \in S: p \leq x\} \), then \( P(x)=O(x^{1-c}) \) for some \( c>0 \)). Proving the infinitude of this set, Ford says, “seems to be very hard.”

In 2018, Kannan Soundararajan sent Ford a link to the DriveTime commercial.

“I was pleasantly surprised,” Ford recalls, “as this conjecture is not that widely known.”

The DriveTime commercial (https://bit.ly/2sNH6B7), unfortunately, does little more than name-drop CTC and bolster well-worn stereotypes.

“This is Dr. Gunter Zoolof,” the spot opens, panning upward from a pocket protector to a drooping bowtie to a bespectacled face framed in an Einstein-esque mane.

“Did he solve Carmichael’s totient conjecture?” intones Will Lyman, the narrator of PBS’s Frontline, as the camera cuts to Zoolof atop a ladder at a multi-story blackboard crammed with the integrals, derivatives, and summations that to the lay public pass for cutting-edge mathematics. 2 “Yes, yes he did.”

2“‘The board work seems to have nothing to do with any coherent thoughts,’” says Carl Pomerance, whose first published paper [9] was on CTC. “I saw the quadratic discriminant formula, some calculus, some other unrecognizable stuff.”

Kevin Ford (https://faculty.math.illinois.edu/~ford) practices for a lecture on CTC.
person would have gained a measure of immortality, and yes, ‘genius’ would be a reasonable description.”

“Of course,” Wagon hedges, “it depends a bit on how the proof goes.”

References


Credits

Photo of Kevin Ford is courtesy of Kevin Ford. Author photo is by David Gabel.
Mathematics People

2019 AWM Awards

The Association for Women in Mathematics presented several awards at the Joint Mathematics Meetings held in Baltimore, Maryland, in January 2019.

Jacqueline Dewar of Loyola Marymount University in Los Angeles has been named the recipient of the 2019 Louise Hay Award for Contributions to Mathematics Education “in recognition of her many achievements as a professor, a leader in outreach, and a contributor to the scholarship of teaching and learning.” She has been an advocate for active learning, initiated a biomathematics program, and developed courses in computer literacy, the history of women in mathematics, and mathematics in civic engagement. She was a cofounder of the Math Science Interchange in Los Angeles, which still provides an annual career day, “Expanding Your Horizons—LA”, for K–12 students and teachers. Thousands of girls and their teachers have attended these events. She continues to lead workshops and train other leaders. Dewar tells the Notices: “My interest in mathematics goes back to my wonderful freshman algebra teacher, Mr. Kramer, followed by being selected as one of forty students (thirty-six boys and four girls) to attend a four-week NSF summer program for talented high school students at St. Louis University. That hooked me on mathematics! As a university faculty member for forty years, some of the most eye-opening experiences I have had in mathematics education occurred inside K–12 schools doing things like talking about math careers, coaching junior high students for math competitions, leading a ‘math for girls’ after-school program, and visiting the classrooms of my former students who became K–12 teachers, and conversing with them, professional to professional. I heard from the teachers about their successes and the challenges they faced. My wish is that many more of us in the higher education mathematics community could find ways to have K–12 mathematics education experiences.” Outside of her professional work, Dewar loves gardening and swing dancing.

Suzanne Weekes of Worcester Polytechnic Institute has been honored with the 2019 M. Gweneth Humphreys Award for Mentorship of Undergraduate Women in Mathematics “for her exceptional track record of support, guidance, unvarnished feedback, and inspiration.” The prize citation notes that “Weekes is a founding director and has offered a strong shaping hand in the deeply impactful MSRI-UP [Mathematical Sciences Research Institute Undergraduate Program] program, devoted to ‘cultivating heretofore untapped mathematical talent’ with a focus on communities traditionally underrepresented in mathematics. Over her tenure at MSRI-UP, over eighty women, including more than fifty women of color, have passed through the program, with the majority continuing to graduate programs after college.” She received her PhD in mathematics and scientific computing from the University of Michigan in 1995. Her research involves dynamic materials, numerical methods, and computational fluid dynamics. She chairs the Education Committee of the Society for Industrial and Applied Mathematics (SIAM).

Kathryn Mann of Brown University has been awarded the 2019 Joan and Joseph Birman Research Prize in Topology and Geometry “for breakthrough work in the theory of dynamics of group actions on manifolds.” The prize citation reads: “Mann uses a broad array of mathematical tools to obtain results at the juncture of topology, group theory, geometry, and dynamics, and she finds new connections between them. She has discovered new phenomena, built general theory, and has solved long-open problems. As an example, in a solo paper she introduced a new method to study the topology of the space of surface group representations in the space of orientation-preserving circle homeomorphisms and to prove a rigidity result about geometric such representations. Building on this paper, jointly with M. Wolff, Mann proved that conversely this rigidity property characterizes the geometric surface group actions on the circle. A leading expert describes this as one of the best results obtained in
the area in the last couple of decades and another mathematician describes Mann as ‘that once-in-a-generation thinker who opens significant new directions for research.’” Mann received her PhD from the University of Chicago in 2014, working under Benson Farb. She has received a Sloan Research Fellowship for 2019. Mann tells the Notices: “I’ve always enjoyed the outdoors, and like to spend as much of my non-mathematical time outside as I can, hiking, biking, and with the recent move to Providence I’ve even taken up rowing.”

—from AWM announcements

Daubechies and Voisin Receive International Awards for Women in Science

Ingrid Daubechies of Duke University and Claire Voisin of the Collège de France have been honored for Women in Science. Daubechies, representing North America, was recognized “for her exceptional contribution to the numerical treatment of images and signal processing, providing standard and flexible algorithms for data compression. Her innovative research on wavelet theory has led to the development of treatment and image filtration methods used in technologies from medical imaging equipment to wireless communication.” Voisin, representing Europe, was honored “for her outstanding work in algebraic geometry. Her pioneering discoveries have allowed [mathematicians and scientists] to resolve fundamental questions on topology and Hodge structures of complex algebraic varieties.” Each award is worth 100,000 euros (about US$113,000). The L’Oréal-UNESCO Women in Science program annually honors five outstanding women scientists from five regions—the Arab and African States, Europe, Latin America, and North America—for their contributions to the sciences, including mathematics, computer science, chemistry, physics, and materials science.

—from a L’Oréal-UNESCO announcement

2019 MAA Awards

The Mathematical Association of America (MAA) awarded several prizes at the Joint Mathematics Meetings in Baltimore, Maryland, in January 2019.

Tom Leinster of the University of Edinburgh was awarded the Chauvenet Prize for his article “Rethinking Set Theory,” *American Mathematical Monthly* 121 (2014), no. 5. The prize citation reads in part: “Every mathematician knows that modern mathematics is an axiomatic system based on a theory of sets defined by the Zermelo–Fraenkel axioms plus the Axiom of Choice (ZFC). But how many of us can recite these axioms? Even after looking them up, are they in accord with our working understanding of sets? Or is the ZFC conception of sets necessarily nonintuitive as a result of having to rectify the difficulties of naive set theory discovered by Russell? In this paper, Tom Leinster tackles this issue with clarity and finesse.” Leinster studied in Oxford and Cambridge, doing a PhD on higher category theory with Martin Hyland, followed by postdoctoral positions in Cambridge and Paris and a stint at the University of Glasgow before joining the faculty at Edinburgh. His interests lie mainly in applications of category theory, recently focusing on applications to geometry, analysis, and the quantification of biological diversity. He is the author of three books: *Higher Operads, Higher Categories* (Cambridge University Press, 2004), *Basic Category Theory* (Cambridge University Press, 2014), and *Entropy and Diversity: The Axiomatic Approach* (in press). He has also written about the role played by mathematicians in the mass surveillance of citizens by governments and is a contributor to the research blog The n-Category Café. Leinster tells the Notices: “I spend much of my free time campaigning for democratic rights in Catalonia, and am donating the Chauvenet prize money to the legal fund of the Catalan prisoners on trial for holding a referendum.”

Cathy O’Neil of ORCAA was awarded the Euler Book Prize for *Weapons of Math Destruction* (Crown, 2016). According to the prize citation, this is “a singularly important book especially at this current historical juncture. It is well-written, engaging, and tackles an important issue, ‘the dark side of data science,’ in a thoughtful way. O’Neil convincingly and passionately argues that math is not just for solving the world’s problems; it is responsible also for fueling some of them. Her discussion of ethical issues and how mathematical models, data, and algorithms are used to manipulate society is important both socially and politically.” O’Neil received her PhD in mathematics from Harvard University and taught at Barnard College before...
entering the private sector with the hedge fund D. E. Shaw and for the software company RiskMetrics. In 2011 she began working as a data scientist. She is the founder of ORCAA, an algorithmic auditing company.

Philip Uri Treisman of the University of Texas at Austin was honored with the 2019 Gung and Hu Award for Distinguished Service to Mathematics “for his extraordinary leadership in strengthening mathematics and science education throughout the K–20 spectrum, supporting mathematics achievement and equity for historically disenfranchised groups, and promoting innovation, productive partnerships, and community service.” He “may be best known for his seminal research on factors that support high achievement for students historically disenfranchised in mathematics,” having created, with collaborators, Emerging Scholars Programs to help eliminate barriers to success, particularly for ethnic minority students. Throughout his career he has worked to improve mathematics education in the United States and has received numerous awards and recognition for his leadership and research. He received his PhD from the University of California Los Angeles under the direction of Leon Henkin. He has been on the faculty at UT Austin since 1991 and is director of the Charles A. Dana Center for Mathematics and Science Education.

The Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching of Mathematics were awarded to Suzanne Dorée of Augsburg University, Minneapolis; Carl Lee of Central Michigan University; and Jennifer Switkes of California State Polytechnic University Pomona.

Dorée was recognized “for her exemplary teaching innovation and leadership, not only at Augsburg University, but also nationally through her work with the MAA, the Charles A. Dana Center, and numerous presentations and workshops on campuses throughout the United States.” She “takes great care in the design of classes, authentic assessments, and highly interactive classrooms, creating an environment where the students build routines that support a high-level of effort, time on task, and success.” She developed the university’s developmental algebra course, which focuses on learning in applied contexts, and also a discrete mathematics course, which includes transitioning to the ideas of mathematical logic, axioms, and proof writing. She chaired the national Curriculum Renewal Across the First Two Years (CRAFTY) committee and proposed, organized, coordinated, and assisted in rewrites of a series of articles in MAA Focus highlighting mathematics curriculum renewal projects throughout the United States. Dorée received her PhD from the University of Wisconsin–Madison. Her research interests include curriculum and materials development and directing undergraduate research in combinatorics.

Lee was recognized “for his outstanding contributions to teaching and learning in the mathematical sciences and particularly in statistics. He is an innovative and engaging teacher and an inspired mentor” who “has successfully worked to develop and promote statistics programs at both undergraduate and graduate levels.” With NSF support, he developed the Real-Time Online Hands-on Activities Database, through which students can mimic statistical practices. He is one of the founding members of the Consortium for the Advancement of Undergraduate Statistics Education (CAUSE), founded in 2003. He founded the Undergraduate Statistics Project Competition (USPROC) in 2006 and chaired the competition committee from 2006 to 2013. Lee was raised in a small mountain village in southern Taiwan and received his PhD in statistics from Iowa State University in 1984. He is the cofounder and co-chief editor of the Journal of Statistical Distributions and Applications. He is a true believer that “teaching is to give people hope and learning is by doing.”

Switkes “is recognized for bringing her educational core values of excellence, honor, integrity, love, and purpose to all students, and specifically to traditionally underserved students.” She is an officially designated teacher-scholar who has directed thirteen master’s theses and twelve undergraduate research projects in the past thirteen years. She is also a dedicated volunteer in working with underserved populations. She teaches prison inmates through the Prison Education Project and spent a sabbatical teaching in Uganda, both at the university level and with prison inmates. She created a Math/Science Forum at the California Rehabilitation Center, and she has invited STEM faculty from Cal Poly Pomona to give guest lectures there during several terms. She received her PhD from Claremont Graduate University in 2000 and has been on the faculty at Cal Poly Pomona since 2001. Her primary mathematical interests are in mathematical modeling and operations research. She serves as a volunteer pastor at a church focusing on community service and mentoring of leaders. In her spare time, she loves hiking and camping. She tells the Notices: “I love visiting prisons, because some inmates never receive a visitor. In addition to doing mathematics with the inmates, I want the inmates to know that there is hope for their lives.”

—From MAA announcements
Lillian Pierce of Duke University has been awarded the AMS Joan and Joseph Birman Fellowship for Women Scholars for the academic year 2019–2020. Pierce’s research is in analytic number theory and harmonic analysis. Her work in number theory involves counting integral points on varieties and studying properties of class groups of number fields, for which problems she has developed new methods involving the circle method, sieves, and character sums. Her work in analysis focuses on oscillatory integral operators, Radon transforms, and Carleson operators, as well as their discrete analogues, which have deep ties to number theoretic questions.

Pierce grew up in a small town in California and was primarily home-schooled as a child. She began playing the violin at age four and was performing professionally by age eleven. She entered Princeton University as a mathematics major but also completed a premed curriculum. Under the mentorship of Elias Stein and others, her interest turned to pure mathematics. She was valedictorian of the 2002 class of Princeton and a Rhodes Scholar. After two years studying at Oxford University with Roger Heath-Brown, she returned to Princeton for her PhD, which she received in 2009 under the direction of Stein. She did postdoctoral work at Oxford, the Institute for Advanced Study, and the Hausdorff Center for Mathematics as a Bonn Junior Fellow. She joined the faculty at Duke in 2014, where she is currently the Nicholas J. and Theresa M. Leonardy Associate Professor of Mathematics.

Pierce has received a Marie Curie Fellowship, an NSF Mathematical Sciences Postdoctoral Research Fellowship, an NSF CAREER award, a von Neumann Fellowship at the Institute for Advanced Study, and a Sloan Research Fellowship. She was awarded the AWM Sadosky Research Prize in 2018 and gave an AMS Invited Address at the 2019 Joint Mathematics Meetings in Baltimore, Maryland, a Bourbaki Seminar in 2017, and an MAA Invited Address at the 2017 JMM in Atlanta, Georgia. Pierce plans to use the Fellowship funding to buy out teaching and to bring one or more collaborators to her home institution, thus reducing the effect of travel on her three young children. Pierce is particularly grateful to the Joan and Joseph Birman Fellowship for the purposeful flexibility of the funding it provides.

The Joan and Joseph Birman Fellowship for Women Scholars, established in 2017 with a generous gift from Joan and Joseph Birman, seeks to give exceptionally talented women extra research support during their mid-career years. The first three Fellowships are also being supported by the Stephen and Margaret Gill Family Foundation, in memory of Hilda Geiringer von Mises. The primary selection criterion for the Birman Fellowship, which carries a stipend of US$50,000, is the excellence of the candidate’s research.

Read an interview ([www.ams.org/giving/honoring/the-line-newsletter-fall2017-PDF.pdf](http://www.ams.org/giving/honoring/the-line-newsletter-fall2017-PDF.pdf)) with Joan Birman about her decision to create the Fellowship with the goal of “helping more women mathematicians to develop their creative voices.”

The recipient of the inaugural Birman Fellowship in 2018 was Margaret Beck. For more information about the Fellowship see: [www.ams.org/profession/prizes-awards/Birman-fellowship]

—Elaine Kehoe

The AMS has awarded its Centennial Fellowship for the academic year 2019–2020 to Piotr Przytycki of McGill University. He will use the Fellowship for full support for the academic year and to travel to visit collaborators in the United Kingdom, France, Poland, and the United States.

Przytycki told Notices: “My research interests are geometric group theory and low dimensional topology. In one of my research directions, I am studying arcs systems on surfaces. With Hensel and Webb we found a simple proof of the uniform hyperbolicity of the arc graph, using arcs that are unicorns, and unicorn paths. With Hensel and Osajda we discovered that dismantlable graphs are omnipresent in low dimensional topology and in particular give a uniform understanding of arc graphs, sphere graphs, and hyperbolic groups. In another direction, we studied with Wise separability in 3-dimensional manifolds. We proved in particular that the fundamental groups of all knot complements in the 3-sphere have faithful representations in $SL(n,Z)$.

“I received my PhD at the Polish Academy of Sciences in 2008 under the supervision of Jacek Świątkowski in Wroclaw. I stayed at the Academy until I joined McGill University in 2014. I also spent a year at University of Illinois at Urbana–Champaign UIUC in 2011 and at Paris Sud Orsay in 2013–2014.”
“I come from a family of mathematicians. My mother Jolanta Słomińska is an algebraic topologist, and my father Feliks Przytycki does dynamical systems. My husband, Marcin Sabok, is also a mathematician, a logician.

“My great passion is theatre. In particular, in 2010 I directed David Auburn’s Proof in amateur theatre, ‘Kontrapunkt,’ in Warsaw—created and directed by a professional actor and director Zbigniew Bogdański.”

The Centennial Fellowship carries a stipend of US$93,000, a travel expense allowance of US$9,300, and a complimentary Society membership for one year. The award was made at the recommendation of the Centennial Fellows Selection Committee. The primary selection criterion is the excellence of the candidate’s research.

Please note: Information about the competition for the 2020–2021 AMS Centennial Fellowship will be published in the Mathematics Opportunities section of an upcoming issue of the Notices.

—Elaine Kehoe

Gwynne and Song Awarded Clay Research Fellowships

Ewain Gwynne of the University of Cambridge and Antoine Song of Princeton University have been awarded Clay Research Fellowships by the Clay Mathematics Institute (CMI).

Ewain Gwynne obtained his PhD in 2018 from the Massachusetts Institute of Technology under the supervision of Scott Sheffield. Since then he has held the Herschel Smith Fellowship at the University of Cambridge and is a Junior Research Fellow at Trinity College Cambridge. Gwynne is a remarkably productive and ingenious researcher with broad interests across probability, especially conformal probability, Schramm-Loewner evolution, Liouville quantum gravity, and random geometry in dimension 2. He has already made landmark contributions to these areas, often developing collaborations with other leading talents in the field. For example, he made breakthroughs with Ding on the fractal dimension of Liouville quantum gravity, with Miller on self-avoiding walks and percolation interfaces on random planar maps, and with Sun on the Fortuin-Kasteleyn model on random planar maps. Ewain has been appointed as a Clay Research Fellow for a term of four years beginning July 1, 2019. He will be based at the University of Cambridge.

Antoine Song will receive his PhD in 2019 from Princeton University, where he has been working under the guidance of Fernando Codá Marques. Song has already established himself as an expert in geometric analysis, solving long-standing problems of fundamental importance concerning the nature of minimal hypersurfaces in compact Riemannian manifolds. First he proved that in dimensions 3 to 7 the closed minimal hypersurface of least area in such a manifold is always embedded. Then, in joint work with Codá Marques and Neves, he showed that for generic metrics on closed manifolds in these dimensions, one can always find a sequence of minimal embedded hypersurfaces that become equidistributed in the sense that the average of the induced measures on the first n hypersurfaces in the sequence converges to the normalized volume measure on the ambient manifold as n tends to infinity. This was a dramatic improvement in the state of the art concerning a circle of problems inspired by Yau’s 1982 conjecture that every closed 3-dimensional Riemannian manifold contains infinitely many closed minimal surfaces. Building on work of Codá Marques and Neves, in 2018 Song proved Yau’s conjecture in complete generality. Song has been appointed as a Clay Research Fellow for a term of five years beginning July 1, 2019. He will be based at the University of California at Berkeley.

Clay Research Fellowships are awarded on the basis of the exceptional quality of candidates’ research and their promise to become mathematical leaders.

—CMI announcement

Aslanyan Awarded Emil Artin Junior Prize

Vahagn Aslanyan of Carnegie Mellon University has been awarded the 2019 Emil Artin Junior Prize in Mathematics. Aslanyan was chosen for his paper “Definability of Derivations in the Reducts of Differentially Closed Fields,” Journal of Symbolic Logic 82 (2017).

Established in 2001, the Emil Artin Junior Prize in Mathematics, now under the auspices of the Armenian Mathematical Union, carries a cash award of US$1,000 and is presented usually every year to a student or former student of an Armenian educational institution under the age of thirty-five for outstanding contributions to algebra, geometry, topology, and number theory—the fields in which Emil Artin made major contributions. The prize committee consisted of A. Basmajian, Y. Movsisyan, and V. Pambuccian.

—Victor Pambuccian

New College, Arizona State University
Winternitz Awarded 2018 Wigner Medal

Pavel Winternitz of the University of Montreal has been named the recipient of the 2018 Wigner Medal “for his fundamental contribution to the determination and application of symmetries in the resolution of differential equations and (super-) integrable systems.” He works in mathematical physics, symmetries, and nonlinear phenomena, particularly Lie groups and Lie algebras. The medal is administered by the Group Theory and Fundamental Physics Foundation.

—Elaine Kehoe

2019 NAS Awards Announced

Two researchers whose work involves the mathematical sciences have been honored with the National Academy of Sciences awards for 2019.

Ola Svensson of École Polytechnique Fédérale de Lausanne received the Michael and Sheila Held Prize for "a series of groundbreaking new algorithms for the traveling salesman problem, one of the most heavily studied and important questions in theoretical computer science." The prize honors outstanding, innovative, creative, and influential research in the areas of combinatorial and discrete optimization, or related parts of computer science, such as the design and analysis of algorithms and complexity theory. It carries a cash award of US$100,000.

Tom Griffiths of Princeton University received a 2019 Troland Research Award for his “pioneering work bringing the methods of Bayesian inference to bear on understanding a broad range of cognitive functions, from perception to language, decision making, reasoning, and cognitive control, and for bringing formal rigor to the notion of bounded rationality, explaining apparent irrationalities of behavior in rational terms.” Griffiths is the author of the book Algorithms to Live By: The Computer Science of Human Decisions. The award is given annually to recognize unusual achievement by young investigators (defined as no older than forty) and to further empirical research within the broad spectrum of experimental psychology. It carries a cash award of US$75,000.

—From an NAS announcement

Compositio Mathematica Prize Awarded

James Maynard of Oxford University has been awarded the Compositio Mathematica Prize for the best paper for the period 2014–2016 for "Dense Clusters of Primes in Subsets," 152 (2016). The prize is awarded every third year by the Foundation Compositio Mathematica for an outstanding piece of research published in the journal during that period.

—From a Foundation Compositio Mathematica announcement

2019 Sloan Fellows Announced

The Alfred P. Sloan Foundation has announced the names of 126 recipients of the 2019 Sloan Research Fellowships. Each year the foundation awards fellowships in the fields of mathematics, chemistry, computational and evolutionary molecular biology, computer science, economics, neuroscience, physics, and ocean sciences. Grants of US$70,000 for a two-year period are administered by each Fellow’s institution. Once chosen, Fellows are free to pursue whatever lines of inquiry most interest them, and they are permitted to employ fellowship funds in a wide variety of ways to further their research aims.

Following are the names and institutions of the 2019 awardees in the mathematical sciences.

- Xiuyuan Cheng, Duke University
- Florian Frick, Carnegie Mellon University
- Shirshendu Ganguly, University of California Berkeley
- Kristen Hendricks, Michigan State University
- Mihaela Ifrim, University of Wisconsin, Madison
- Philip Isett, California Institute of Technology
- Junehyuk Jung, Texas A&M University
- Andrew W. Lawrie, Massachusetts Institute of Technology
- Bao Le Hung, Northwestern University
- John Lesieutre, Pennsylvania State University
- Francesco Lin, Princeton University
- Kathryn Mann, Brown University
- Davi Maximo, University of Pennsylvania
- Barna Saha, University of Massachusetts, Amherst
- Mahdi Soltanolkotabi, University of Southern California
- Konstantin Tikhomirov, Georgia Institute of Technology
- Botong Wang, University of Wisconsin, Madison
- Yufei Zhao, Massachusetts Institute of Technology
- Tianyi Zheng, University of California San Diego
- Xin Zhou, University of California Santa Barbara

—From a Sloan Foundation announcement
Credits
Photo of Piotr Przytycki is by Kinga Osajda.
Photo of Lillian Pierce is courtesy of Duke Photography.
Photo of Ewain Gwynne is courtesy of Ewain Gwynne.
Photo of Antoine Song is courtesy of Antoine Song.
Photo of Suzanne Doré is courtesy of Augsburg University.
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THE NEXT GENERATION FUND

The Next Generation Fund is a new endowment at the AMS that exclusively supports programs for doctoral and postdoctoral scholars. It will assist rising mathematicians each year at modest but impactful levels, with funding for travel grants, collaboration support, mentoring, and more.

To learn more or make a gift, go to www.ams.org/nextgen

Want to learn more?
Visit www.ams.org/giving
Or contact development@ams.org
401.455.4111
Karen Uhlenbeck, professor emerita at the University of Texas at Austin, can remember the first time she ran into another woman in a math department. It was in the 1980s, more than a decade into her career, at her fourth academic position since graduating. When she earned her PhD in 1968, she was one of very few women who were entering the tenure track at a research-oriented math department. She kept her head down and focused on research and, despite barriers such as anti-nepotism rules that made it difficult for her to get a position at the same institution as her husband at the time, forged a successful research career. “The assumption was that things would get better,” she says. But in the early 1990s, she and others who had been part of the influx of women into math departments in the 1970s realized they were still the youngest women in their departments. “A lot of us who were not very involved in women’s issues looked around and said, ‘Something needs to be done’.”

In 1993 Antonella Grassi, then a postdoc at Duke University, organized a mentoring program for women in mathematics at the Mathematical Sciences Research Institute (MSRI). The next year, the program moved to the Institute for Advanced Study (IAS) campus in Princeton, New Jersey. Uhlenbeck and Chuu-Lian Terng, then a mathematician at Northeastern University in Boston, organized the program. Twenty-five years and over 1000 alumnae later, the Women and Mathematics program (WAM) at the Institute for Advanced Study is being recognized with the 2019 AMS Mathematics Programs that Make a Difference Award. “It’s a wonderful recognition,” IAS director Robbert Dijkgraaf says. “We’re very happy about it.”

WAM began under the auspices of the Park City Mathematics Institute (PCMI), a summer program run by the IAS that brings mathematicians and mathematics teachers from various backgrounds and geographical areas together to study and exchange ideas. PCMI was founded in 1991, and Uhlenbeck was one of the founders. Because of that connection, it made sense for the women’s program to be part of PCMI. In the early years, women students who had been accepted into PCMI would be invited to a summer school at the Institute in May, before PCMI itself, to meet each other and get an introduction to the mathematics that would be the focus of the program.

WAM has changed and evolved over the decades. After a few years, WAM separated from PCMI in order to remove the restriction that participants must attend PCMI as well. For many years, WAM was a two-week program. Today, due
to both budgetary and scheduling considerations, it is one week long, hosting about 60 participants at various stages in their mathematical education and career paths, from advanced undergraduate students to postdocs. Instructors and guest lecturers participate as well. The two lecture series and related problem sessions form the core of the program, with other programming such as outreach events and career panels as an important focus. “We have a good mix of academic and social interactions,” says Michelle Huguenin, the administrative program manager of WAM.

Although there was some resistance among mathematicians to the idea that a program for women in mathematics was necessary, the IAS under then-director Phillip Griffiths was supportive, providing funding and generous administrative assistance. “When you’re offered [something] like that, you don’t turn it down,” Uhlenbeck says. “They were terrifically supportive all along.” Currently, funding for WAM comes from both Princeton University and the National Science Foundation.

Dijkgraaf notes that WAM is an important part of the IAS mission to foster excellence in mathematics research. Mathematics is a field that “belongs to all of us,” he says. Or at least it should. When, due to race- or gender-based discrimination, entire groups of people are forced to overcome barriers not present for white men or are excluded from mathematics entirely, the field as a whole suffers. “It is extremely important, I think, that we bring a maximum of diversity to math,” Dijkgraaf says. From the point of view of mathematicians, it is a self-serving goal—a larger talent pool means better mathematics. But of course, access to a quality mathematics education is a question of equity as well. Mathematics is one of the most widely taught subjects across the world, and, for better or worse, it is a gatekeeper for many other classes. Students with strong math backgrounds who can demonstrate facility in learning new mathematics have a wide variety of career paths open to them.

Much of the focus at the IAS is on giving outstanding individual researchers the freedom to work on the questions that most fascinate them without teaching obligations or pressures from commercial interests or funding agencies. But WAM and PCMI make their marks differently. They are part of broadening the reach of the IAS beyond the lucky few who get to spend years strolling through the campus’s serene woods and settling in for lunches at its legendary dining hall. Those programs “maximize our impact on the mathematical community,” Dijkgraaf says. Though IAS is a relatively small institution, its historical importance in the mathematics community allows it to, as Dijkgraaf puts it, “punch a little bit above our weight.” The Institute’s investment in a program designed to address the issue of women’s underrepresentation in mathematics makes the statement that the problems women face in mathematics education and career development are worth understanding and combating.

**Continuing Importance**

From the beginning, the need for a program like WAM was obvious to Uhlenbeck and other women mathematicians of her generation. “Everybody my age knew there was a problem. People reminded you of it every day,” she says. “But it wasn’t clear that hanging around with other women would solve the problem.” She and other women mathematicians wanted to show that women consciously grouping and working together could help students get ahead and find a place in the mathematics research world.

Dusa McDuff, professor of mathematics at Columbia University and Barnard College and current chair of the WAM steering committee, is a few years younger than Uhlenbeck but also graduated at a time when very few women went into math research careers. When she was starting her career, “there were some women mathematicians, but there were none I knew,” she says. She was aware of some historical women in mathematics but never had female mathematician role models to look up to. “Having an existence proof is something, but it would be nice to actually know somebody.”

Open prejudice against women in mathematics is much less common than it used to be. It is no longer condoned by university policies, and mathematicians, whether male, female, or nonbinary, are increasingly likely to speak up when they encounter it. But there are still lingering prejudices in attitudes about who can be taken seriously as a mathematician. Dijkgraaf notes that WAM is an important part of the IAS mission to foster excellence in mathematics research. Mathematics is a field that “belongs to all of us,” he says. Or at least it should. When, due to race- or gender-based discrimination, entire groups of people are forced to overcome barriers not present for white men or are excluded from mathematics entirely, the field as a whole suffers. “It is extremely important, I think, that we bring a maximum of diversity to math,” Dijkgraaf says. From the point of view of mathematicians, it is a self-serving goal—a larger talent pool means better mathematics. But of course, access to a quality mathematics education is a question of equity as well. Mathematics is one of the most widely taught subjects across the world, and, for better or worse, it is a gatekeeper for many other classes. Students with strong math backgrounds who can demonstrate facility in learning new mathematics have a wide variety of career paths open to them.

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Uhlenbeck offers, “Women don’t do different math than men.” But male-dominated environments can make it more difficult for women to participate fully. The supportive atmosphere of WAM and the participation of women in many different career stages can make younger students feel less vulnerable as they wade into research mathematics. “For many people who attend, they have not met so many female mathematicians at once,” University of Kentucky mathematician and WAM Academic Program Manager Margaret Readdy says. Seeing so many successful women ahead of them can help them see a path forward.

“There is something different about a room full of women,” University of Michigan mathematician Anna Gilbert says. She participated in WAM in one of the first years of its existence and has returned since as a lecturer. “I think it’s palpable to everybody.” She says programs like WAM can be especially valuable when they help students build their confidence and comfort talking about mathematics to a variety of people. “It’s nice to spend a while talking about exciting research and feeling comfortable doing it,” she says. “If meetings like this can facilitate that, they’re important. It’s also important to get to the point where you feel comfortable talking excitedly about your research no matter the setting.”

Doing the Math

The core of WAM has always been the math content courses. There are two lecture courses each summer. They used to be referred to as the beginning and the advanced courses, but they were recently renamed for WAM founders Terng and Uhlenbeck. In the early years, it was especially important to the organizers that the program be seen as a rigorous mathematical experience so it could gain credibility. Uhlenbeck remembers a student whose (male) advisor was skeptical about the need for women-only math programs. “My thesis advisor did not believe in this,” she recalled the student saying, “but, he said, ‘it’s a program run by Karen Uhlenbeck and the mathematics is good, so you should go.’”

The program themes are tied to the yearly themes of the IAS seminar School of Mathematics and have covered applied topics such as mathematical biology and cryptography to theoretical mathematics fields like geometric group theory and the Langlands program. For many of the undergraduates, this is their first exposure to research-level mathematics.

After lectures, daily TA-led problem sessions get students working together to digest the mathematics from their courses. “It’s much more hands-on and significantly more challenging than TA-ing any other course I’ve done,” says freelance math and science writer Yen Duong, who was a TA during the 2017 program. “It’s more similar to a postdoc or professor’s work of doing a topics class in more current research.” The atmosphere was different for her as a TA than it had been when she was participating as a grad student, but, “I was still able to have those deep, insightful conversations that come about when you throw these women together who have had similar experiences,” she says.

Advanced graduate students and postdoctoral participants organize a Women in Science Seminar in the afternoons. The seminar enables students to give talks about their work, often for the first time outside of their home institutions. “When I think of when I first started to give talks—they were awful!” says freelance math and science writer Yen Duong, who was a TA during the 2017 program. “It’s more similar to a postdoc or professor’s work of doing a topics class in more current research.” The atmosphere was different for her as a TA than it had been when she was participating as a grad student, but, “I was still able to have those deep, insightful conversations that come about when you throw these women together who have had similar experiences,” she says.

For anyone who has finished a talk feeling like they needed to read tea leaves to tell whether their audience was engaged and following them, the prospect of honest, kind feedback from mathematicians who really know what they are talking about is tantalizing. “That’s something you never get” at other conferences, Readdy says. She is working on putting together guidelines for new speakers that she hopes will help raise the caliber of their first talks and allow them to mature even more quickly.

“As a mathematical community, we should always think, ‘Are there new ways in which we could structure ourselves?’”
—Robbert Dijkgraaf
Becoming Mathematical Citizens

In addition to course lectures, problem sessions, and research talks, WAM programming includes other educational, social, and outreach opportunities. The programming depends largely on the interests of the participants and changes from year to year. In the early years, the Women in Science Seminar was organized by Uhlenbeck and had a diverse set of offerings on topics outside of the course content. She would invite science historians to share histories and biographies of women in math and organize discussions of issues facing women mathematicians.

Participants can organize evening programs about many hot-button issues. Questions of equity and justice in mathematics classrooms and in research spaces are of increasing interest to young participants, who initiate discussions about these topics. For example, as the broader society’s understanding of gender identity has evolved more toward seeing it as a spectrum (or even a higher-dimensional mathematical object) rather than a binary variable, some participants have organized discussions on the topic of gender in mathematics. Being willing to tackle these topics with provocative speakers “incites more, and better, and deeper conversations and questions,” Duong says. “The undergraduate and beginning graduate students almost feel like kindling, and they’re catching the flame for more justice in the mathematics world. That gives me great hope.”

Interested participants also have the opportunity to perform local outreach while they attend WAM. Aside from visits to local schools, participants have attended local 5K races and set up tables demonstrating fun extracurricular math like a non-transitive dice game and Eugenia Cheng’s method for cutting a bagel into two linked halves.

Participants are housed together, either on campus in Institute-owned apartments or at a hotel in downtown Princeton. They walk to classes together, share meals on campus, and socialize in the evenings. “For this group of female mathematicians from various backgrounds and various stages of their scientific career, spending time together is very important,” Dijkgraaf says. “It creates bonds that last a lifetime.”

Focus on Careers

Career and work-life balance panels are common programming at WAM. Although many graduate students still see it as the one true career path, becoming a research mathematician is an alternate career in the sense that most people who enter graduate school to study mathematics will not eventually have tenure at a university. WAM—like many organizations that serve mathematics students—is increasingly focusing on giving career advice and guidance that acknowledges that particular reality of mathematical careers.

“As a community, I personally feel we could spend more time on thinking about what it means to be a mathematician,” Dijkgraaf says. What does it mean to have a career in math? What sacrifices are involved in the academic career path, or industry, or government? How will a mathematical career align with your broader values? “This is something quite general,” he says. He feels that as a student, these questions were often neglected in favor of a more myopic focus on courses and content. “Spending a week or two together and having the opportunity to have these exchanges might be as valuable as the math you learn and the contacts you make,” he says.

Organizers have found that students are more and more interested in hearing from nonacademic career panelists. They want to know about the range of options available and how to prepare for both applying to and working in nonacademic jobs. “One of the values is that younger women come here and get to see such a broad variety of older women around them,” Uhlenbeck says. “I’d hate to think that everybody with a PhD has to become a research mathematician.” They can see role models who have a variety of mathematical careers and get a better understanding of the skills and background that would be most helpful for each one.

One recent addition to WAM that is useful for both academic and nonacademic careers is a focus on computer skills. “As a community, I personally feel we could spend more time on thinking about what it means to be a mathematician,” Dijkgraaf says. What does it mean to have a career in math? What sacrifices are involved in the academic career path, or industry, or government? How will a mathematical career align with your broader values? “This is something quite general,” he says. He feels that as a student, these questions were often neglected in favor of a more myopic focus on courses and content. “Spending a week or two together and having the opportunity to have these exchanges might be as valuable as the math you learn and the contacts you make,” he says.

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One recent addition to WAM that is useful for both academic and nonacademic careers is a focus on computer skills. “We feel strongly that every young mathematician these days should know how to do something with computers,” McDuff says. For example, during the 2018 WAM program, which had a theme of modern cryptography, participants learned to use Sage and LMFDB, an online database of $L$-functions, modular forms, and related objects.

In recent years, New Jersey-based manufacturer Johnson & Johnson has hired one WAM undergraduate or beginning graduate student as a summer intern, starting after the end of WAM that summer. The intern works in several different roles in the company and learns about the ways math modeling can be used in pharmaceuticals and medical devices as well as business marketing and sales. Johnson & Johnson has been consistently impressed with the quality
of interns they have gotten, and students have enjoyed the hands-on experience. “It is a symbiotic relationship where both parties—the intern and the company—are happy,” Readdy says.

**Impact**

In its 25 years, many prominent and successful women in mathematics have participated in the WAM program. About a third of the recipients of the Alice T. Schafer Mathematics Prize for excellence in mathematics by an undergraduate women have participated in the program, along with about a sixth of women recipients of NSF graduate fellowships, and a third of women recipients of NSF and Sloan postdoctoral fellowships. A sixth of women ICM invited speakers and more than half of women ICM plenary speakers have gone through the program. To be clear, correlation is not causation. Women with strong interests and backgrounds in math are more likely to be interested in and participate in WAM and all of the aforementioned programs. But many alumnae of the program who continue to progress in mathematics research careers cite the rigorous, collegial atmosphere as a place of growth and encouragement.

One of the most ringing endorsements of the program is the fact that so many people involved come back. “We have many participants who return in various forms,” Huguenin says. Women who participate as undergraduates often come back as graduate students or postdocs; former participants return as lecturers; some alumnae end up returning as members of the IAS, spending one or two years working on their research at the Institute.

After participating in WAM as a graduate student in 2016, Duong knew she wanted to be involved in the future. “I brought it up before I even left,” she says. She asked the organizers whether there was a way she could come back to the program. By a lucky coincidence, the next year’s mathematical theme was also related to her research, so she came back as a TA.

Professional collaborations begin at WAM as well. “I’ve talked with people who say they’ve changed their research area as a result of attending WAM,” Readdy says. Spending so much time talking with other young women who are interested in the same area of math can ignite interest in proving theorems together. Huguenin recalls a pair that started working together at WAM several years ago and recently went back to the Institute as part of the Summer Collaborators program.

One of the criteria for the Mathematics Programs that Make a Difference Award is that the program has a replicable model. While the tranquil setting of the IAS and its history and prestige in the mathematics community are unique, women-only programs that focus on current mathematics research have proliferated in the decades since WAM was established. WAM has shown the value of bringing women at many different stages of their career together for intensive workshops and conferences. Today, women mathematicians can find a host of conferences and informal networks of others in their specific research fields: Women in Numbers, Women in Topology, Women in Probability, Women in Mathematical Biology, Women in Sage. The list goes on. MSRI semester programs begin with short Connections for Women conferences to help build a community of women who will be participating either in the weeklong conferences or the full semester. The EDGE (Enhancing Diversity in Graduate Education) program has helped women from underrepresented racial and ethnic groups and first-generation college students start graduate school on the right foot. The WAM Ambassador Program

**WAM Ambassadors**

Two years ago through a donation from Lisa Simonyi, the WAM Ambassador Program began. Through small grants, this initiative allows participants in the summer program at IAS to take their ideas to the broader community. Each year, up to six graduate student alumnae will receive funding to help support undergraduate math majors at their universities, and up to three postdoctoral or advanced graduate student alumnae receive grants to fund small conferences or outreach programs. “We want them to take their experience from here, bring it home and put it to use in whatever way works best for them,” Huguenin says.

In 2017, the first year of the program, Yen Duong organized the Carolinas Women in Mathematics Symposium at the University of North Carolina at Charlotte. Similar conferences took place at several other schools, including Carnegie Mellon University and the University of Wisconsin. Undergraduate grants have funded Association for Women in Mathematics student chapter activities, reading groups about both mathematics and gender equity, and outreach activities geared toward children in the community. This coming year, one undergraduate grant will be used to fund a Wikipedia edit-a-thon where students will research women mathematicians and add or expand on biographies of women in math on Wikipedia.

The year after the program is funded, each Ambassador comes back for the summer program to discuss their grant-funded award and its outcome. “I think the point is to harness the creativity of these young women in mathematics,” Duong says. She recalls talking with other WAM participants about how nice it would be to be able to take some of the energy and camaraderie from the summer program to institutions around the country. “It took off from there,” she says. “They have the ideas from the program, and then with the grants they have the tools and resources to actually carry those ideas into fruition.”
has helped create small regional conferences and other activities to support the work of women mathematicians around the country (see sidebar).

“One lesson that I have learned [from WAM] is that we should be experimental in always thinking of new ways to bring mathematicians together,” Dijkgraaf says. He hopes WAM will also provide a model for people looking to increase participation by members of other groups that are underrepresented in mathematics. “As a mathematical community, we should always think, ‘Are there new ways in which we could structure ourselves?’ particularly, I think, if our aim is to make math the most inclusive subject,” he says. “Math has always benefited from having new points of view.”

**Increasing Access**

WAM has, on average, 150 applicants each year, but the size of the venues and the desire to keep the atmosphere intimate keeps the cap on attendees around 60. Organizers are continuously assessing their recruitment and admission procedures to try to encourage diversity along many axes.

In 2016 the program established childcare grants to make it more accessible to participants with children. Moms, dads, and nonbinary parents all have childcare responsibilities, but these responsibilities tend to fall disproportionately on women, especially when children are very young, and the cost of childcare can be a barrier to women’s participation in conferences and other professional activities. Duong was able to come back to WAM as a TA because of a childcare grant that allowed her to bring her mother to care for her infant daughter. Shortly after implementing the childcare grant program at WAM, the IAS expanded the scope and now offers childcare grants to participants at all of their programs.

Other recent efforts at broadening access have focused on the types of schools participants attend. Many R1 universities have strong ties to the IAS, and as a result their students have had easier access to a broader range of upper-level math courses and programs like WAM. “The goal of the program is broader than just encouraging the women who already have everything going for them when they arrive at the point of applying to graduate school,” Uhlenbeck says. To help draw in students from other backgrounds, the WAM committee actively recruits students from historically black colleges and universities and from smaller and less research-oriented universities. About a third of the undergraduate participants have been members of racial or ethnic groups that are underrepresented in mathematics and about half have been students at universities that do not confer doctorates in mathematics.

WAM is flexible. Organizers and participants are adapting the program to changes in math department offerings and the job market. The organizers are optimistic about its continued relevance and influence in the broader mathematical community. Uhlenbeck says, “The program is in good hands and...thriving.”

**Credits**

All WAM program photos are by Andrea Kane, courtesy of IAS. Photo of Evelyn Lamb is by Jon Chaika.
Mathematics Opportunities

Listings for upcoming mathematics opportunities to appear in Notices may be submitted to notices@ams.org.

Early Career Opportunity

Call for Nominations for the SASTRA Ramanujan Prize
The Shanmuga Arts, Science, Technology, Research Academy (SASTRA) is seeking nominations for the 2019 SASTRA Ramanujan Prize, awarded to a mathematician not exceeding the age of thirty-two for outstanding contributions in an area of mathematics influenced by the late Indian mathematical genius Srinivasa Ramanujan. It carries a cash prize of US$10,000. The deadline for nominations is July 31, 2019. See the website qseries.org/sastra-prize/nominations-2019.html.

—Krishnaswami Alladi, University of Florida

IPAM Call for Proposals

The Institute for Pure and Applied Mathematics (IPAM) seeks program proposals from the mathematical, statistical, and scientific communities for long programs and workshops, to be reviewed at IPAM’s Science Advisory Board meeting in November. For more information, go to www.ipam.ucla.edu/propose-a-program or contact the IPAM Director at director@ipam.ucla.edu. Proposals should also address the inclusion of women and members of underrepresented minorities as speakers, organizers, and participants.

—Stacey Beggs
Assistant Director, IPAM

Early Career Opportunity

NSF Postdoctoral Research Fellowships
The National Science Foundation (NSF) awards Mathematical Sciences Postdoctoral Research Fellowships in all areas of the mathematical sciences, including applications to other disciplines. Awards are either Research Fellowships or Instructorships. The Research Fellowship provides full-time support for any eighteen academic-year months in a three-year period. The Research Instructorship provides either two academic years of full-time support or one academic year of full-time and two academic years of half-time support. The deadline for proposals is October 16, 2019. See www.nsf.gov/funding/pgm_summ.jsp?pims_id=5301&org=NSF.

—NSF announcement

Early Career Opportunity

Research Training Groups in the Mathematical Sciences
The National Science Foundation (NSF) Research Training Groups in the Mathematical Sciences program provides funds for the training of US students and postdoctoral associates through structured research groups that include vertically integrated activities spanning the entire spectrum of educational levels from undergraduate through postdoctoral. The deadline for full proposals is June 4, 2019. See www.nsf.gov/funding/pgm_summ.jsp?pims_id=5732.

—NSF announcement

Early Career Opportunity

International Mathematics Competition for University Students
The Twenty-Sixth International Mathematics Competition for University Students will be held July 28–August 3, 2019, at American University in Blagoevgrad, Bulgaria. Students completing their first, second, third, or fourth years of university education are eligible. See www.imc-math.org.uk.

—John Jayne, University College London
Call for Papers for Haifa Workshop

The Nineteenth Haifa Workshop on Interdisciplinary Applications of Graph Theory, Combinatorics, and Algorithms will be held June 11–12, 2019, at the University of Haifa, Caesarea Rothschild Institute, and June 13, 2019, at the Israel Mathematics Union Meeting. Contributed talks are invited. The workshop emphasizes the diversity of the use of combinatorial algorithms and graph theory in application areas. Abstracts of one to two pages should be sent by April 30, 2019 to HaifaGraph2019@gmail.com. See the website www.cri.haifa.ac.il.

—Martin Charles Golumbic, General Chair
University of Haifa

2019 National Math Festival

The 2019 National Math Festival will be held in Washington, DC, on May 4, 2019, from 10 a.m. to 4 p.m. It will feature a day of hands-on activities, speakers, and performances to show that math is for everyone. The event is free and open to the public. See https://www.nationalmathfestival.org/2019-festival.

—From an MSRI announcement

Early Career Opportunity

NRC Research Associateship Programs

The National Academy of Sciences, Engineering, and Medicine offers postdoctoral and senior research awards on behalf of twenty-three US federal research agencies and affiliated institutions with facilities at over 100 locations throughout the United States and abroad. Applications are sought from highly qualified candidates, including recent doctoral recipients and senior researchers. The deadline for the current review cycle is May 1, 2019. See sites.nationalacademies.org/pga/rap.

—NRC announcement
OHIO

University of Central Missouri
School of Computer Science and Mathematics
Non-Tenure Track Position in Mathematics

The School of Computer Science and Mathematics at the University of Central Missouri is accepting applications for a non-tenure-track position in Mathematics at the rank of Assistant Professor. The appointment will begin August 2019.

A PhD in mathematics is required and preference will be given to candidates with expertise in abstract algebra. Applicants should be able to teach a wide variety of undergraduate and graduate courses including general education, calculus, linear algebra and abstract algebra courses. The successful candidate will have an ability and enthusiasm for directing undergraduate research and master’s theses. Other duties include service activities and advising of majors. The typical teaching load is 12 credit hours per semester. Contract is renewable, contingent upon performance and budget.

The Application Process: To apply online, go to https://jobs.ucmo.edu and apply to position #997165. The following items should be attached:

- A letter of interest
- A curriculum vita
- Copies of transcripts
- Teaching and research statements

- A list of at least three professional references including telephone numbers and email addresses. Official transcripts and three letters of recommendation will be requested for candidates invited for an on-campus interview.

For more information, contact:
Rhonda McKee, Search Committee Chair
School of Computer Science and Mathematics
University of Central Missouri
Warrensburg, MO 64093
(660) 543-4930
mckee@ucmo.edu

Initial screening of applications begins immediately, and continues until position is filled.

AA/EEO/ADA. Women and minorities are encouraged to apply.

Founded in 1871, UCM is a comprehensive state university that offers approximately 150 different degree programs within four academic colleges. The University is located in Warrensburg, MO, approximately 35 miles southeast of the Kansas City metropolitan area. The School of Computer Science and Mathematics offers BS and MS degrees in Mathematics.
Korea Institute for Advanced Study (KIAS)
Assistant Professor & Research Fellow in Pure and Applied Mathematics

The School of Mathematics at the Korea Institute for Advanced Study (KIAS) invites applicants for the positions at the level of KIAS Assistant Professor and Postdoctoral Research Fellow in pure and applied mathematics. KIAS, founded in 1996, is committed to the excellence of research in basic sciences (mathematics, theoretical physics, and computational sciences) through high-quality research programs and a strong faculty body consisting of distinguished scientists and visiting scholars.

Applicants are expected to have demonstrated exceptional research potential, including major contributions beyond or through the doctoral dissertation. The annual salary starts from 50,500,000 Korean Won (approximately US$45,500 at current exchange rate) for Research Fellows, and 57,500,000 Korean Won for KIAS Assistant Professors, respectively. In addition, individual research funds of 10,000,000 ~ 13,000,000 Korean Won are available per year. The initial appointment for the position is for two years and is renewable for up to two additional years, depending on research performance and the needs of the research program at KIAS.

Applications will be reviewed twice a year, May 20 and November 20, and selected applicants will be notified in a month after the review. In exceptional cases, applications can be reviewed other times based on the availability of positions. The starting date of the appointment is negotiable. Application materials must include a standard cover sheet which is posted on the website (www.kias.re.kr), a cover letter, a curriculum vitae including a list of publications, a research plan, and three letters of recommendation. All documents, prepared in English, should be submitted by post or e-mail to:

Ms. Sojung Bae (mathkias@kias.re.kr)
School of Mathematics
Korea Institute for Advanced Study (KIAS)
85 Hoegiro (Cheongnyangni-dong 207-43), Dongdaemun-gu,
Seoul 02455, Republic of Korea

Dozens of positions at all levels are available at the recently founded Center for Applied Mathematics, Tianjin University, China. We welcome applicants with backgrounds in pure mathematics, applied mathematics, statistics, computer science, bioinformatics, and other related fields. We also welcome applicants who are interested in practical projects with industries. Despite its name attached with an accent of applied mathematics, we also aim to create a strong presence of pure mathematics. Chinese citizenship is not required.

Light or no teaching load, adequate facilities, spacious office environment and strong research support. We are prepared to make quick and competitive offers to self-motivated hard workers, and to potential stars, rising stars, as well as shining stars.

The Center for Applied Mathematics, also known as the Tianjin Center for Applied Mathematics (TCAM), located by a lake in the central campus in a building protected as historical architecture, is jointly sponsored by the Tianjin municipal government and the university. The initiative to establish this center was taken by Professor S. S. Chern. Professor Molin Ge is the Honorary Director, Professor Zhiming Ma is the Director of the Advisory Board. Professor William Y. C. Chen serves as the Director.

TCAM plans to fill in fifty or more permanent faculty positions in the next few years. In addition, there are a number of temporary and visiting positions. We look forward to receiving your application or inquiry at any time. There are no deadlines.

Please send your resume to mathjobs@tju.edu.cn. For more information, please visit cam.tju.edu.cn or contact Ms. Erica Liu at mathjobs@tju.edu.cn, telephone: 86-22-2740-6039.
New Books Offered by the AMS

Applications

**Mathematical Modeling in Economics and Finance**

Probability, Stochastic Processes, and Differential Equations

Steven R. Dunbar, University of Nebraska–Lincoln

Designed for an upper-division course on modeling in the economic sciences, this textbook combines a study of mathematical modeling with exposure to the tools of probability theory, difference and differential equations, numerical simulation, data analysis, and mathematical analysis. Readers will gain a deep appreciation for the modeling process and learn methods of testing and evaluation driven by data.

AMS/MAA Textbooks, Volume 49

bookstore.ams.org/text-49

**Modeling and Data Analysis**

An Introduction with Environmental Applications

John B. Little, College of the Holy Cross, Worcester, MA

Can we coexist with the other life forms that have evolved on this planet? Are there realistic alternatives to fossil fuels that would sustainably provide for human society’s energy needs and have fewer harmful effects? Mathematical models are fundamental for understanding the potential consequences of choices we make. This book on elementary topics in mathematical modeling and data analysis is intended for an undergraduate “liberal arts mathematics”-type course, but with a specific focus on environmental applications.

This item will also be of interest to those working in applications


bookstore.ams.org/mbk-120
strategy to study the fluctuations of strongly interacting random variables.

**Virtual Fundamental Cycles in Symplectic Topology**

*John W. Morgan*, Simons Center for Geometry and Physics, Stony Brook, NY, Editor  
*Dusa McDuff*, Columbia University, New York, NY, *Mohammad Tehrani*, University of Iowa, Iowa City, *Kenji Fukaya*, Simons Center for Geometry and Physics, Stony Brook, NY, and *Dominic Joyce*, Mathematical Institute, Oxford, United Kingdom

The method of using the moduli space of pseudo-holomorphic curves on a symplectic manifold was introduced by Mikhail Gromov in 1985. From the appearance of Gromov’s original paper until today this approach has been the most important tool in global symplectic geometry. This volume brings together three approaches to constructing the “virtual” fundamental cycle for the moduli space of pseudo-holomorphic curves.

Mathematical Surveys and Monographs, Volume 237  
[bookstore.ams.org/surv-237](bookstore.ams.org/surv-237)

**Asymptotics of Random Matrices and Related Models**

*Alice Guionnet*, Université de Lyon, CNRS, ENS de Lyon, France

The celebrated law of large numbers and the central limit theorem describe the asymptotics of the sum of independent variables. However, there are many models of strongly correlated random variables: for instance, the eigenvalues of random matrices or the tiles in random tilings. These lecture notes describe a general

**Mathematical Physics**
Number Theory and Geometry
An Introduction to Arithmetic Geometry
Álvaro Lozano-Robledo, University of Connecticut, Storrs

This introduction to number theory and arithmetic geometry uses geometry as the motivation to prove the main theorems in the book. This text describes many applications including modern applications in cryptography and also presents some recent results in arithmetic geometry. With many exercises, this book can be used as a text for a first course in number theory or for a subsequent course on arithmetic (or diophantine) geometry at the junior-senior level.

Pure and Applied Undergraduate Texts, Volume 35
May 2019, 488 pages, Hardcover, ISBN: 978-1-4704-5016-8,
LC 2018056079, 2010 Mathematics Subject Classification: 11–XX, 11Gxx, 14Gxx, List US$109, AMS members US$87.20,
MAA members US$98.10, Order code AMSTEXT/35
bookstore.ams.org/amstext-35

New in Contemporary Mathematics

Algebra and Algebraic Geometry

Groups, Algebras and Identities
Eugene Plotkin, Bar-Ilan University, Ramat-Gan, Israel, Editor

This volume contains the proceedings of the Research Workshop of the Israel Science Foundation on Groups, Algebras and Identities, held from March 20–24, 2016, at Bar-Ilan University and the Hebrew University of Jerusalem, Israel, in honor of Boris Plotkin’s 90th birthday. The papers in this volume cover various topics of universal algebra, universal algebraic geometry, logic geometry, and algebraic logic, as well as applications of universal algebra to computer science, geometric ring theory, small cancellation theory, and Boolean algebras.

This book is co-published with Bar-Ilan University (Ramat-Gan, Israel).

Contemporary Mathematics, Volume 726
April 2019, 224 pages, Softcover, ISBN: 978-1-4704-3713-8,
LC 2018040270, 2010 Mathematics Subject Classification: 03Gxx, 08Axx, 08Bxx, 20Axx, 20Fxx, 16Sxx, 17Axx, 18Fxx,
List US$117, AMS members US$93.60, MAA members US$105.30,
Order code CONM/726

bookstore.ams.org/conm-726

Analysis

Nonlinear Dispersive Waves and Fluids
Shijun Zheng, Georgia Southern University, Statesboro, Marius Beceanu, University at Albany, NY, Jerry Bona, University of Illinois at Chicago, Geng Chen, University of Kansas, Lawrence, Tuoc Van Phan, University of Tennessee, Knoxville, and Avy Soffer, Rutgers University, Piscataway, NJ, Editors

This volume contains the proceedings of the AMS Special Session on Spectral Calculus and Quasilinear Partial Differential Equations and the AMS Special Session on PDE Analysis on Fluid Flows, which were held in January 2017 in Atlanta, Georgia. The articles address the latest trends and perspectives in the area of nonlinear dispersive equations and fluid flows. The topics mainly focus on using state-of-the-art methods and techniques to investigate problems of depth and richness arising in quantum mechanics, general relativity, and fluid dynamics.

This item will also be of interest to those working in mathematical physics

Contemporary Mathematics, Volume 725
April 2019, 275 pages, Softcover, ISBN: 978-1-4704-4109-8,
LC 2018040209, 2010 Mathematics Subject Classification: 35–XX, 37–XX, 47–XX, 58–XX, 76–XX, 93–XX, List US$117,
AMS members US$93.60, MAA members US$105.30,
Order code CONM/725

bookstore.ams.org/conm-725
New AMS-Distributed Publications

Algebra and Algebraic Geometry

**Courbes et Fibrés Vectoriels en Théorie de Hodge $p$-Adique**

Laurent Fargues, Institut de Mathématiques de Jussieu, Paris, France, Jean-Marc Fontaine, Université de Paris-Sud, Orsay, France, and Pierre Colmez, CNRS, IMJ-PRG, Sorbonne Université, Paris, France

This work is dedicated to the discovery, the definition, and the study of the fundamental curve in $p$-adic Hodge theory. The authors define and study the $p$-adic period rings as rings of holomorphic functions of the variable $p$. Then they classify the vector bundles on the curve, a theorem that generalizes in some sense the classification theorem of vector bundles on the projective line. As an application, they give geometric proofs of the two main theorems in $p$-adic Hodge theory: weakly admissible implies admissible and de Rham implies potentially semi-stable.

This item will also be of interest to those working in number theory

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the US, Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Astérisque, Number 406


[bookstore.ams.org/ast-406](bookstore.ams.org/ast-406)

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Analysis

**Measure and Integration**

S. Kesavan

This book deals with topics usually studied in a master’s or graduate level course on the theory of measure and integration. It starts with the Riemann integral and points out some of its shortcomings which motivate the theory of measure and the Lebesgue integral.

Starting with abstract measures and outer-measures, the Lebesgue measure is constructed and its important properties are highlighted. Measurable functions, different notions of convergence, the Lebesgue integral, the fundamental theorem of calculus, product spaces, and signed measures are studied. There is a separate chapter on the change of variable formula and one on $L^p$-spaces.

Most of the material in this book can be covered in a one-semester course. The prerequisite for following this book is familiarity with basic real analysis and elementary topological notions, with special emphasis on the topology of the $N$-dimensional euclidean space. Each chapter is provided with a variety of exercises.

A publication of Hindustan Book Agency; distributed within the Americas by the American Mathematical Society. Maximum discount of 20% for all commercial channels.

Hindustan Book Agency


[bookstore.ams.org/hin-77](bookstore.ams.org/hin-77)
Differential Equations

Estimates for Differential Operators in Half-space
Igor W. Gel’man, Akko, Israel, and Vladimir G. Maz’ya, Linköping University, Sweden, University of Liverpool, UK, and Peoples’ Friendship University of Russia, Moscow
Translated from the German by Darya Apushkinskaya.
Originally published in 1981 by Akademie-Verlag as Abschätzungen für Differentialoperatoren im Halbraum.

Inequalities for differential operators play a fundamental role in the modern theory of partial differential equations. Among the numerous applications of such inequalities are existence and uniqueness theorems, error estimates for numerical approximations of solutions and for residual terms in asymptotic formulas, as well as results on the structure of the spectrum. The inequalities cover a wide range of differential operators, boundary conditions and norms of the corresponding function spaces.

The book focuses on estimates up to the boundary of a domain. It contains a great variety of inequalities for differential and pseudodifferential operators with constant coefficients. Results of final character are obtained, without any restrictions on the type of differential operators. Algebraic necessary and sufficient conditions for the validity of the corresponding a priori estimates are presented. General criteria are systematically applied to particular types of operators found in classical equations and systems of mathematical physics (such as Lamé’s system of static elasticity theory or the linearized Navier–Stokes system), Cauchy–Riemann’s operators, Schrödinger operators, among others. The well-known results of Aronszajn, Agmon–Douglis–Nirenberg and Schechter fall into the general scheme and sometimes are strengthened.

The book will be interesting and useful to a wide audience, including graduate students and specialists interested in the theory of differential equations.

A publication of the European Mathematical Society (EMS). Distributed within the Americas by the American Mathematical Society.

EMS Tracts in Mathematics, Volume 31

bookstore.ams.org/emstm-31

General Interest

Mathematics of Takebe Katahiro and History of Mathematics in East Asia
Tsuikane Ogawa, Yokkaichi University, Yokkaichi Mie, Japan, and Mitsuo Morimoto, Seki Kowa Institute of Mathematics, Yokkaichi University, Yokkaichi Mie, Japan, Editors

This volume is a collection of papers contributed by participants at the International Conference on Traditional Mathematics in East Asia and Related Topics, held on August 25–30, 2014, at Ochanomizu University, Tokyo, Japan. This conference was one of the satellite conferences of the Seoul ICM 2014. The year 2014 also coincided with the 350th anniversary of the birth of Takebe Katahiro (1664–1739), one of the great mathematicians of the Edo period. In his honor, the conference was called the Takebe Conference 2014.

This volume is divided into four parts and an appendix. Part I is concerned with Takebe Katahiro, his mathematics and his times. The editors believe that wasan represents one phase of the mathematics of East Asia, especially of China, Japan, and Korea, for which ancient Chinese mathematics served as a basis, and Chinese characters as a lingua franca.

Part II concerns the old mathematics of Korea in the Joseon Dynasty. Part III treats the mathematics of Ancient China. Part IV follows the subsequent development of Japanese mathematics and mathematical education after the Meiji Restoration (1868).

The very substantial appendix contains English translations of three volumes (12, 17 and 19) of the Taisei Sankei (twenty volumes, 1711), a monograph written by the master mathematician Seki Takakazu and the two Takebe brothers, Kata’akira and Katahiro.

Published for the Mathematical Society of Japan by Kinokuniya, Tokyo, and distributed worldwide, except in Japan, by the AMS.

Advanced Studies in Pure Mathematics, Volume 79
December 2018, 561 pages, Hardcover, ISBN: 978-4-86497-057-0, 2010 Mathematics Subject Classification: 01A27; 01A13, 01A25, List US$97, AMS members US$77.60, Order code ASPM/79

bookstore.ams.org/aspm-79
The selection committees for these prizes request nominations for consideration for the 2020 awards, which will be presented at the Joint Mathematics Meetings in Denver, CO, in January 2020. Information about past recipients of these prizes may be found at www.ams.org/prizes-awards.

**BÔCHER MEMORIAL PRIZE**

The Bôcher Prize is awarded for a notable paper in analysis published during the preceding six years. The work must be published in a recognized, peer-reviewed venue.

**CHEVALLEY PRIZE IN LIE THEORY**

The Chevalley Prize is awarded for notable work in Lie Theory published during the preceding six years; a recipient should be at most twenty-five years past the PhD.

**LEONARD EISENBUD PRIZE FOR MATHEMATICS AND PHYSICS**

The Eisenbud Prize honors a work or group of works, published in the preceding six years, that brings mathematics and physics closer together.

**FRANK NELSON COLE PRIZE IN NUMBER THEORY**

This Prize recognizes a notable research work in number theory that has appeared in the last six years. The work must be published in a recognized, peer-reviewed venue.
LEVI L. CONANT PRIZE

The Levi L. Conant Prize, first awarded in January 2001, is presented annually for an outstanding expository paper published in either the Notices of the AMS or the Bulletin of the AMS during the preceding five years.

JOSEPH L. DOOB PRIZE

The Doob Prize recognizes a single, relatively recent, outstanding research book that makes a seminal contribution to the research literature, reflects the highest standards of research exposition, and promises to have a deep and long-term impact in its area. The book must have been published within the six calendar years preceding the year in which it is nominated. Books may be nominated by members of the Society, by members of the selection committee, by members of AMS editorial committees, or by publishers. The prize is awarded every three years.

AWARD FOR DISTINGUISHED PUBLIC SERVICE

The Award for Distinguished Public Service recognizes a research mathematician who has made recent or sustained distinguished contributions to the mathematics profession through public service. The award is given every other year.

Further information about AMS prizes can be found at the Prizes and Awards website: www.ams.org/prizes.
Further information and instructions for submitting a nomination can be found at the prize nomination website: www.ams.org/nominations.
For questions contact the AMS Secretary at secretary@ams.org.
The nomination period is March 1 through June 30, 2019.
Meetings & Conferences of the AMS
May Table of Contents

The Meetings and Conferences section of the Notices gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited on this page for more detailed information on each event.

Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. Information in this issue may be dated. The most up-to-date meeting and conference information can be found online at: www.ams.org/meetings.

Important Information About AMS Meetings: Potential organizers, speakers, and hosts should refer to page 127 in the January 2019 issue of the Notices for general information regarding participation in AMS meetings and conferences.

Abstracts: Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of \LaTeX is necessary to submit an electronic form, although those who use \LaTeX may submit abstracts with such coding, and all math displays and similarly coded material (such as accent marks in text) must be typeset in \LaTeX. Visit www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

### Meetings in this Issue

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See https://www.ams.org/meetings for the most up-to-date information on the meetings and conferences that we offer.

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Associate Secretaries of the AMS

**Central Section:** Georgia Benkart, University of Wisconsin–Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; email: benkart@math.wisc.edu; telephone: 608-263-4283.

**Eastern Section:** Steven H. Weintraub, Department of Mathematics, Lehigh University, Bethlehem, PA 18015-3174; email: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

**Southeastern Section:** Brian D. Boe, Department of Mathematics, University of Georgia, 220 D W Brooks Drive, Athens, GA 30602-7403, email: brian@math.uga.edu; telephone: 706-542-2547.

**Western Section:** Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; email: lapidus@math.ucr.edu; telephone: 951-827-5910.
Meetings & Conferences of the AMS

Hartford, Connecticut
University of Connecticut Hartford (Hartford Regional Campus)

Announcement issue of Notices: February 2019
Program first available on AMS website: February 21, 2019
Issue of Abstracts: Volume 40, Issue 2

Deadlines
For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses
- Brian C Hall, University of Notre Dame, Eigenvalues of random matrices in the general linear group in the large-N limit.
- Christina Sormani, Lehman College and CUNYGC, Compactness theorems for sequences of Riemannian Manifolds.
- Olivier Bernardi, Brandeis University, Percolation on triangulations, and a bijective path to Liouville quantum gravity.

Special Sessions
- Algebraic Number Theory, Harris Daniels, Amherst College, and Alvaro Lozano-Robledo and Erik Wallace, University of Connecticut.
- Banach Space Theory and Metric Embeddings, Mikhail Ostrovskii, St. John's University, and Beata Randrianantoanina, Miami University.
- Chip-firing and Divisor Theory, Caroline Klivans, Brown University, and David Perkinson, Reed College.
- Cluster Algebras and Related Topics, Emily Gunawan and Ralf Schiffler, University of Connecticut.
- Combinatorial Commutative Algebra and Polyhedral Geometry, Elie Alhajjar, UIS Military Academy, and McCabe Olsen, Ohio State University.
- Computability Theory, Damir Dzafarov and Reed Solomon, University of Connecticut, and Linda Brown Westrick, Pennsylvania State University.
- Convergence of Riemannian Manifolds, Lan-Hsuan Huang and Maree Jaramillo, University of Connecticut, and Christina Sormani, City University of New York Graduate Center and Lehman College.
Discrete Dynamical Systems and Applications, Elliott J. Bertrand, Sacred Heart University, and David McArdle, University of Connecticut.

Invariants of Knots, Links, and Low-dimensional Manifolds, Patricia Cahn, Smith College, and Moshe Cohen and Adam Lowrance, Vassar College.

Knot Theory, the Colored Jones Polynomial, and Khovanov Homology, Adam Giambrone, Elmira College, and Katherine Hall, University of Connecticut.

Mathematical Cryptology, Lubjana Beshaj, United States Military Academy, and Jaime Gutierrez, University of Cantabria, Santander, Spain.

Mathematical Finance, Oleksii Mostovyi, University of Connecticut, Gu Wang, Worcester Polytechnic Institute, and Bin Zhou, University of Connecticut.

Modeling and Qualitative Study of PDEs from Materials Science and Geometry, Yung-Sze Choi, Changfeng Gui, and Xiaodong Yan, University of Connecticut.

Recent Advances in Structured Matrices and Their Applications, Maxim Derevyagin, University of Connecticut, Olga Holz, University of California, Berkeley, and Vadim Olshevsky, University of Connecticut.

Recent Development of Geometric Analysis and Nonlinear PDEs, Ovidiu Munteanu, Lihan Wang, and Ling Xiao, University of Connecticut.

Representation Theory of Quantum Algebras and Related Topics, Drew Jaramillo, University of Connecticut, Garrett Johnson, North Carolina Central University, and Margaret Rahmoeller, Roanoke College.

Special Session on Regularity Theory of PDEs and Calculus of Variations on Domains with Rough Boundaries, Murat Akman, University of Connecticut, and Zihui Zhao, University of Washington.

Special Values of L-functions and Arithmetic Invariants in Families, Ellen Eischen, University of Oregon, Yifeng Liu, Yale University, Liang Xiao, University of Connecticut, and Wei Zhang, Massachusetts Institute of Technology.


Stochastic Processes, Random Walks, and Heat Kernels, Patricia Alonso Ruiz, University of Connecticut, and Phanuel Mariano, Purdue University.


Quy Nhon, Vietnam

International Centre for Interdisciplinary Science and Education

June 10–13, 2019
Monday – Thursday

Meeting #1149
Associate secretary: Brian D. Boe
Announcement issue of Notices: April 2019

Program first available on AMS website: To be announced
Issue of Abstracts: N/A

Deadlines
For organizers: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/internmtgs.html.

Invited Addresses

Henry Cohn, Microsoft Research, Sphere packing, Fourier interpolation, and ground states in 8 and 24 dimensions.

Robert Guralnick, University of Southern California, Fixed point free permutations and applications.

Le Tuan Hoa, Institute of Mathematics, Vietnam Academy of Science and Technology, On the complexity of polynomial systems.


Zhiwei Yun, Massachusetts Institute of Technology, Endoscopy for Hecke categories and character sheaves.

Nguyen Tien Zung, Toulouse Mathematics Institute, On a universal conservation law in dynamics.
**Special Sessions**

Algebraic Topology, **Nguyen Huu Viet Hung**, Vietnam National University, Hanoi (Vietnam), **Le Minh Ha**, Vietnam Institute for Advanced Study in Mathematics (Vietnam), and **Dev Sinha**, University of Oregon (USA).

Applied and Industrial Mathematics, **John Birge**, University of Chicago (USA), and **Vu Hoang Linh**, Vietnam National University, Hanoi (Vietnam).

Arithmetic Algebraic Geometry and related topics, **Phung Ho Hai**, Institute of Mathematics, Vietnam Academy of Science and Technology (Vietnam), **Le Hung Viet Bao**, Northwestern University (USA), and **Ngo Dac Tuan**, CNRS and University of Caen Normandy (France).

Commutative Algebra and Its Interactions to Combinatorics, **Le Tuan Hoa**, Vietnam Academy of Science & Technology (Vietnam), and **Ha Huy Tai**, Tulane University (USA).

Complex Geometry and Dynamical Systems, **Pham Hoang Hiep**, Vietnam Academy of Science and Technology (Vietnam), **Daniel Burns**, University of Michigan (USA), **Tien-Cuong Dinh**, National University of Singapore (Singapore), and **Thai Thuan Quang**, Quy Nhon University (Vietnam).

Discrete Mathematics, **Phan Thi Ha Duong**, Vietnam Academy of Science & Technology (Vietnam), and **Vu Ha Van**, Yale University (USA).

Formal Mathematics, **Thomas C. Hales**, University of Pittsburgh (USA), and **Tran Nam Trung**, Vietnam Academy of Science & Technology (Vietnam).

Geometry and Physics, **Nguyen Tien Zung**, Toulouse Mathematics Institute (France), and **Tudor Ratiu**, Shanghai Jiao Tong University (China).

Groups, Representations and Applications, **Pham Huu Tiep**, Rutgers University (USA/Vietnam), and **Robert M. Guralnick**, University of Southern California (USA).

Homological Methods in the Representation Theory of Groups and Algebras, **Daniel K. Nakano** and **Jon Carlson**, University of Georgia (USA), and **Ngo Vo Nham**, University of North Georgia (USA).

Optimization and Variational Analysis, **Nguyen Dong Yen**, Vietnam Academy of Science & Technology (Vietnam), **James Burke**, University of Washington (USA), **Phan Quoc Khanh**, Vietnam National University, Ho Chi Minh City (Vietnam), **Huynh Van Ngai**, Quy Nhon University (Vietnam), and **Hung M. Phan**, University of Massachusetts (USA).

Singularities and Algebraic Geometry, **Le Quy Thuong**, Vietnam National University, Hanoi (Vietnam), **Nero Budur**, KU Leuven (Belgium), and **Pho Duc Tai**, Vietnam National University, Hanoi (Vietnam).

Value Distribution Theory, Complex Geometry, Diophantine Approximation, and Related Topics, **William Cherry**, University of North Texas (USA), **Ta Thi Hoai An**, Vietnam Academy of Science & Technology (Vietnam), and **Min Ru**, University of Houston (USA).

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**Madison, Wisconsin**

*University of Wisconsin–Madison*

**September 14–15, 2019**

Saturday – Sunday

**Meeting #1150**

Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: June 2019

Program first available on AMS website: July 23, 2019

Issue of *Abstracts*: Volume 40, Issue 3

**Deadlines**

For organizers: Expired

For abstracts: July 16, 2019

The scientific information listed below may be dated. For the latest information, see [www.ams.org/amsmtgs/sectional.html](http://www.ams.org/amsmtgs/sectional.html).

**Invited Addresses**

- **Nathan Dunfield**, University of Illinois, Urbana-Champaign, *Title to be announced*.
- **Teena Gerhardt**, Michigan State University, *Title to be announced*.
- **Lauren Williams**, University of California, Berkeley, *Title to be announced* (Erdős Memorial Lecture).

**Special Sessions**

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at [www.ams.org/cgi-bin/abstracts/abstract.pl](http://www.ams.org/cgi-bin/abstracts/abstract.pl).
MEETINGS & CONFERENCES

Algebraic and Geometric Combinatorics (Code: SS 12A), Benjamin Braun, University of Kentucky, Marie Meyer, Lewis University, and McCabe Olsen, Ohio State University.

Analysis and Probability on Metric Spaces and Fractals (Code: SS 10A), Guy C. David, Ball State University, and John Dever, Bowling Green State University.

Applications of Algebra and Geometry (Code: SS 38A), Shamgar Gurevich and Jose Israel Rodriguez, University of Wisconsin-Madison.

Arithmetic of Shimura Varieties (Code: SS 26A), Chao Li, Columbia University, and Solly Parenti and Tonghai Yang, University of Wisconsin-Madison.

Association Schemes and Related Topics – in Celebration of J.D.H. Smith’s 70th Birthday (Code: SS 8A), Kenneth W. Johnson, Penn State University, Abington, and Sung Y. Song, Iowa State University.

Automorphic Forms and L-Functions (Code: SS 16A), Simon Marshall and Ruixiang Zhang, University of Wisconsin-Madison.

Classical and Geophysical Fluid Dynamics: Modeling, Reduction and Simulation (Code: SS 17A), Nan Chen, University of Wisconsin-Madison, and Honghu Liu, Virginia Tech University.

Combinatorial Algebraic Geometry (Code: SS 21A), Juliette Bruce and Daniel Erman, University of Wisconsin-Madison, Chris Eur, University of California Berkeley, and Lily Silverstein, University of California Davis.


Computability Theory in honor of Steffen Lempp’s 60th birthday (Code: SS 6A), Joseph S. Miller, Noah D. Schweber, and Mariya I. Soskova, University of Wisconsin-Madison.


Connections between Noncommutative Algebra and Algebraic Geometry (Code: SS 15A), Jason Gaddis and Dennis Keeler, Miami University.

Extremal Graph Theory (Code: SS 14A), Józef Balogh, University of Illinois, and Bernard Lidicky, Iowa State University.

Floer Homology in Dimensions 3 and 4 (Code: SS 29A), Jianfeng Lin, UC San Diego, and Christopher Scaduto, University of Miami.

Fully Nonlinear Elliptic and Parabolic Partial Differential Equations, Local and Nonlocal (Code: SS 25A), Fernando Charro, Wayne State University, Stefania Patrizi, The University of Texas at Austin, and Peiyong Wang, Wayne State University.


Geometry and Topology in Arithmetic (Code: SS 41A), Rachel Davis, University of Wisconsin-Madison.

Geometry and Topology of Singularities (Code: SS 13A), Laurentiu Maxim, University of Wisconsin-Madison.

Hall Algebras, Cluster Algebras and Representation Theory (Code: SS 27A), Xueqing Chen, UIW-Whitewater, and Yiqiang Li, SUNY at Buffalo.

Hodge Theory in Honor of Donu Arapura’s 60th Birthday (Code: SS 11A), Ajneet Dhillon, University of Western Ontario, Kenji Matsuki and Deepam Patel, Purdue University, and Botong Wang, University of Wisconsin-Madison.

Homological and Characteristic p > 0 Methods in Commutative Algebra (Code: SS 1A), Michael Brown, University of Wisconsin-Madison, and Eric Canton, University of Michigan.

Homotopy Theory (Code: SS 34A), Gabe Angelini-Knoll and Teena Gerhardt, Michigan State University, and Bertrand Guillou, University of Kentucky.


Lie Representation Theory (Code: SS 19A), Mark Colarusso, University of South Alabama, Michael Lau, Université Laval, and Matt Ondrus, Weber State University.

Model Theory (Code: SS 5A), Uri Andrews and Omer Mermelstein, University of Wisconsin-Madison.

Nonlinear Dispersive Equations and Water Waves (Code: SS 37A), Mihaela Ifrim, University of Wisconsin-Madison, and Daniel Tataru, University of California, Berkeley.

Number Theory and Cryptography (Code: SS 40A), Eric Bach, University of Wisconsin-Madison, and Jon Sorenson, Butler University.

Quasigroups and Loops – in honor of J.D.H. Smith’s 70th birthday (Code: SS 35A), J.D. Phillips, Northern Michigan University, and Petr Vojtechovsky, University of Denver.

Recent Developments in Harmonic Analysis (Code: SS 3A), Theresa Anderson, Purdue University, and Joris Roos, University of Wisconsin-Madison.
Recent Trends in the Mathematics of Data (Code: SS 39A), Sebastien Roch, University of Wisconsin-Madison, David Sivakoff, Ohio State University, and Joseph Watkins, University of Arizona.

Recent Work in the Philosophy of Mathematics (Code: SS 4A), Thomas Drucker, University of Wisconsin-Whitewater, and Dan Sloughter, Furman University.

Relations Between the History and Pedagogy of Mathematics (Code: SS 32A), Emily Redman, University of Massachusetts, Amherst, Brit Shields, University of Pennsylvania, and Rebecca Vinsonhaler, University of Texas, Austin.

Several Complex Variables (Code: SS 7A), Hanlong Fang and Xianghong Gong, University of Wisconsin-Madison.

Special Functions and Orthogonal Polynomials (Code: SS 2A), Sarah Post, University of Hawai‘i at Manoa, and Paul Terwilliger, University of Wisconsin-Madison.


Supergeometry, Poisson Brackets, and Homotopy Structures (Code: SS 36A), Ekaterina Shemyakova, University of Toledo, and Theodore Voronov, University of Manchester.

Topics in Graph Theory and Combinatorics (Code: SS 20A), Songling Shan and Papa Sissoko, Illinois State University.

Topology and Descriptive Set Theory (Code: SS 18A), Tetsuya Ishiu and Paul B. Larson, Miami University.

Uncertainty Quantification Strategies for Physics Applications (Code: SS 9A), Qin Li, University of Wisconsin-Madison, and Tulin Kaman, University of Arkansas.

Wave Phenomena in Fluids and Relativity (Code: SS 24A), Sohrab Shahshahani, University of Massachusetts, and Willie W.Y. Wong, Michigan State University.

Zero Forcing, Propagation, and Throttling (Code: SS 23A), Josh Carlson, Iowa State University, and Nathan Warnberg, University of Wisconsin-La Crosse.

Binghamton, New York

Binghamton University

October 12–13, 2019

Saturday – Sunday

Meeting #1151

Eastern Section

Announcement issue of Notices: August 2019

Program first available on AMS website: August 29, 2019

Issue of Abstracts: Volume 40, Issue 3

Deadlines

For organizers: Expired

For abstracts: August 20, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Richard Kenyon, Brown University, Title to be announced.

Tony Pantev, University of Pennsylvania, Title to be announced.

Lai-Sang Young, New York University, Title to be announced.

Special Sessions

If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Groups and Their Representations (Code: SS 1A), Jamison Barsotti and Rob Carman, College of William and Mary, and Daniel Rossi and Hung P. Tong-Viet, Binghamton University.

Homotopy Theory and Algebraic K-theory (Code: SS 5A), Cary Malkiewich, Binghamton University, Marco Varisco, University at Albany, and Inna Zakharevich, Cornell University.

Low-Dimensional Topology (Code: SS 3A), Jenya Sapir and Jonathan Williams, Binghamton University.

Representations of Lie algebras, Vertex Operators, and Related Topics (Code: SS 2A), Alex Feingold, Binghamton University, and Christopher Sadowski, Ursinus College.

What’s New in Group Theory? (Code: SS 4A), Luise-Charlotte Kappe, Binghamton University, and Justin Lynd and Arturo Magidin, University of Louisiana at Lafayette.
MEETINGS & CONFERENCES

Gainesville, Florida

University of Florida

November 2–3, 2019
Saturday – Sunday

Meeting #1152
Southeastern Section
Associate secretary: Brian D. Boe

Announcement issue of Notices: September 2019
Program first available on AMS website: September 19, 2019
Issue of Abstracts: Volume 40, Issue 4

Deadlines
For organizers: Expired
For abstracts: September 10, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses
Jonathan Mattingly, Duke University, Title to be announced.
Isabella Novik, University of Washington, Title to be announced.
Eduardo Teixeira, University of Central Florida, Title to be announced.

Special Sessions
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Algebras, Analysis and Physics (Code: SS 6A), Craig A. Nolder, Florida State University, Carmen Judith Vanegas Espinoza, Technical University of Manabi (Ecuador), and Soren Krauss, Universitat Erfurt (Germany).

Combinatorial Lie Theory (Code: SS 3A), Erik Insko, Florida Gulf Coast University, Martha Precup, Washington University in St. Louis, and Edward Richmond, Oklahoma State University.

Fractal Geometry and Dynamical Systems (Code: SS 2A), Mrinal Kanti Roychowdhury, University of Texas Rio Grande Valley.

Geometric and Topological Combinatorics (Code: SS 1A), Bruno Benedetti, University of Miami, Steve Klee, Seattle University, and Isabella Novik, University of Washington.

Probabilistic and Geometric Tools in High-Dimension (Code: SS 4A), Arnaud Marsiglietti, University of Florida, and Artem Zvavitch, Kent State University.

Topological Complexity and Related Topics (Code: SS 5A), Daniel C. Cohen, Louisiana State University, and Alexander Dranishnikov and Yuli B. Rudyak, University of Florida.

Riverside, California

University of California, Riverside

November 9–10, 2019
Saturday – Sunday

Meeting #1153
Western Section
Associate secretary: Michel L. Lapidus

Announcement issue of Notices: September 2019
Program first available on AMS website: September 12, 2019
Issue of Abstracts: Volume 40, Issue 4

Deadlines
For organizers: Expired
For abstracts: September 3, 2019

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses
Robert Boltje, University of California, Santa Cruz, Title to be announced.
Jonathan Novak, University of California, San Diego, Title to be announced.
Anna Skripka, University of New Mexico, Albuquerque, Title to be announced.
Special Sessions
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Geometric Partial Differential Equations and Variational Methods (Code: SS 4A), Longzhai Lin, University of California, Santa Cruz, Xiangwen Zhang, University of California, Irvine, and Xin Zhou, University of California, Santa Barbara.
Inverse Problems (Code: SS 3A), Hanna Makaruk, Los Alamos National Laboratory, and Robert Owczarek, University of New Mexico, Albuquerque and University of New Mexico, Los Alamos.
Random Matrices and Related Structures (Code: SS 2A), Jonathan Novak, University of California, San Diego, and Karl Liechty, De Paul University.
Topics in Operator Theory (Code: SS 1A), Anna Skripka and Maxim Zinchenko, University of New Mexico.

Denver, Colorado
Colorado Convention Center

January 15–18, 2020
Wednesday – Saturday

Meeting #1154
Joint Mathematics Meetings, including the 126th Annual Meeting of the AMS, 103rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM)

Charlottesville, Virginia
University of Virginia

March 13–15, 2020
Friday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of Notices: Title to be announced

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Special Sessions
If you are volunteering to speak in a Special Session, you should send your abstract as early as possible via the abstract submission form found at www.ams.org/cgi-bin/abstracts/abstract.pl.

Einstein Public Lecture in Mathematics, Moon Duchin, Tufts University, Title to be announced.
Curves, Jacobians, and Abelian Varieties (Code: SS 1A), Andrew Obus, Baruch College (CUNY),, Tony Shaska, Oakland University, and Padmavathi Srinivasan, Georgia Institute of Technology.
MEETINGS & CONFERENCES

Medford, Massachusetts
Tufts University

March 21–22, 2020
Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: To be announced

Program first available on AMS website: February 6, 2020
Issue of Abstracts: Volume 41, Issue 2

Deadlines
For organizers: August 22, 2019
For abstracts: January 28, 2020

West Lafayette, Indiana
Purdue University

April 4–5, 2020
Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: To be announced

Program first available on AMS website: February 13, 2020
Issue of Abstracts: Volume 41, Issue 2

Deadlines
For organizers: September 5, 2019
For abstracts: February 4, 2020

Fresno, California
California State University, Fresno

May 2–3, 2020
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced

Program first available on AMS website: March 19, 2020
Issue of Abstracts: Volume 41, Issue 2

Deadlines
For organizers: October 3, 2019
For abstracts: March 3, 2020

El Paso, Texas
University of Texas at El Paso

September 12–13, 2020
Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: To be announced

Program first available on AMS website: July 23, 2020
Issue of Abstracts: Volume 41, Issue 3

Deadlines
For organizers: February 2, 2020
For abstracts: July 14, 2020

State College, Pennsylvania
Pennsylvania State University, University Park Campus

October 3–4, 2020
Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: To be announced
Program first available on AMS website: August 11, 2020

Issue of Abstracts: Volume 41, Issue 3

Deadlines
For organizers: March 3, 2020
For abstracts: August 11, 2020
MEETINGS & CONFERENCES

Chattanooga, Tennessee
University of Tennessee at Chattanooga

October 10–11, 2020
Saturday – Sunday
Southeastern Section
Associate secretary: Brian D. Boe
Announcement issue of Notices: To be announced

Salt Lake City, Utah
University of Utah

October 24–25, 2020
Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced

Washington, District of Columbia
Walter E. Washington Convention Center

January 6–9, 2021
Wednesday – Saturday
Joint Mathematics Meetings, including the 127th Annual Meeting of the AMS, 104th Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the Winter Meeting of the Association of Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Grenoble, France
Université de Grenoble-Alpes

July 5–9, 2021
Monday – Friday
Associate secretary: Michel L. Lapidus
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced
Issue of Abstracts: To be announced

Deadlines

Program first available on AMS website: August 27, 2020
Issue of Abstracts: Volume 41, Issue 4

Deadlines

For organizers: March 10, 2020
For abstracts: August 18, 2020

Deadlines

For organizers: March 24, 2020
For abstracts: September 1, 2020

Deadlines

For organizers: April 1, 2020
For abstracts: To be announced

Deadlines

For organizers: To be announced
For abstracts: To be announced
MEETINGS & CONFERENCES

Buenos Aires, Argentina
The University of Buenos Aires
July 19–23, 2021
Monday – Friday
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: To be announced
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Omaha, Nebraska
Creighton University
October 9–10, 2021
Saturday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of Notices: To be announced

Program first available on AMS website: To be announced
Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Seattle, Washington
Washington State Convention Center and the Sheraton Seattle Hotel
January 5–8, 2022
Wednesday – Saturday
Associate secretary: Georgia Benkart
Announcement issue of Notices: October 2021
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced

Boston, Massachusetts
John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel
January 4–7, 2023
Wednesday – Saturday
Associate secretary: Steven H. Weintraub
Announcement issue of Notices: October 2022
Program first available on AMS website: To be announced

Issue of Abstracts: To be announced

Deadlines
For organizers: To be announced
For abstracts: To be announced
Frank and Brennie Morgan
AMS-MAA-SIAM Prize
for Outstanding Research in Mathematics
by an Undergraduate Student

The prize is awarded each year to an undergraduate student (or students for joint work) for outstanding research in mathematics. Any student who is an undergraduate in a college or university in the United States or its possessions, Canada, or Mexico is eligible to be considered for this prize.

The prize recipient’s research need not be confined to a single paper; it may be contained in several papers. However, the paper (or papers) to be considered for the prize must be completed while the student is an undergraduate; they cannot be written after the student’s graduation. The research paper (or papers) may be submitted for the committee’s consideration by the student or a nominator. Each submission for the prize must include at least one letter of support from a person, usually a faculty member, familiar with the student’s research. Publication of research is not required.

The recipients of the prize are to be selected by a standing joint committee of the AMS, MAA, and SIAM. The decisions of this committee are final. Nominations for the 2020 Morgan Prize are due no later than June 30, 2019. Those eligible for the 2020 prize must have been undergraduates in December 2018.

Questions may be directed to:
James Sellers, Secretary
Mathematical Association of America
Penn State University
University Park, PA 16802
Telephone: 814-865-7528
Email: sellersj@psu.edu

Nominations and submissions should be sent to:
Carla Savage, Secretary
American Mathematical Society
Computer Science Department
North Carolina State University
Raleigh, NC 27695-8206
or uploaded via the form available at: www.ams.org/nominations
American Mathematical Society Distribution Center
35 Monticello Place, Pawtucket, RI 02861 USA

RECENT RELEASES
from the AMS

AMS / MAA Press
Mathematical Modeling in Economics and Finance
Probability, Stochastic Processes, and Differential Equations
Steven R. Dunbar, University of Nebraska–Lincoln
Designed for an upper-division course on modeling in the economic sciences, this textbook combines a study of mathematical modeling with exposure to the tools of probability theory, difference and differential equations, numerical simulation, data analysis, and mathematical analysis.

Modeling and Data Analysis
An Introduction with Environmental Applications
John B. Little, College of the Holy Cross, Worcester, MA
Using diverse examples with environmental science data, this text provides a lively review of high school math. Ideal for a quantitative literacy course, it offers an excellent alternative to the typical math rehash.
—Louis J. Gross, Chancellor’s Professor of Ecology and Evolutionary Biology and Mathematics, University of Tennessee, Knoxville
This book on elementary topics in mathematical modeling and data analysis is intended for an undergraduate “liberal arts mathematics”-type course, but with a specific focus on environmental applications.
2019; 323 pages; Hardcover; ISBN: 978-1-4704-4869-1; List US$79; AMS members US$63.20; MAA members US$71.10; Order code MBK/120

Number Theory and Geometry
An Introduction to Arithmetic Geometry
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