Memorial Tribute: L. Gaunce Lewis Jr. (1949–2006) and Mark Steinberger (1950–2018)

J. Peter May

We have lost Gaunce Lewis and Mark Steinberger, two excellent algebraic topologists, to early deaths. Both were students, collaborators, and friends of mine, and Mark was also my nephew. Both were struck down by brain cancer, Gaunce dying on May 17, 2006, and Mark on September 15, 2018.

Gaunce and Mark, along with other students of mine from the early 1970s, especially Bob Bruner and Jim McClure, were in at the beginning of two major current directions in algebraic topology, equivariant stable homotopy theory and structured ring spectra. I will try to give something of the flavor of the work of Gaunce and Mark, focusing in part on the two books *Equivariant Stable Homotopy Theory* [11] and *H_∞ Ring Spectra and their Applications* [3] before going on to separate accounts of their later work. The first book, [11], was written by Gaunce, Mark, Jim, and me, and the second, [3], was written by Bob, Jim, Mark, and me. Both were published in 1986, based on a decade's worth of prior collaborative work. The first includes the results of Gaunce's 1978 thesis, and the two together include the results of Mark's 1977 thesis. Although Gaunce was older, Mark arrived at Chicago earlier, in 1972, so I will start with him back then.

I knew Mark as a child, although not well. His father's mother was my father's sister. That side of our family escaped from Nazi Germany in the 1930s. As teenagers, Mark's father Herbert and Herbert's brother Jack were sent to the United States on the first *Kindertransport* out of Germany in 1934. Their parents and younger brother Rudi followed in 1937. My father got out in 1936. They all started off in Chicago, strangely enough. Herbert died in 1994, Rudi in 2017, but Jack is still alive, age ninety-seven. Jack is a Nobel laureate in physics who befriended me and was my childhood role model, very much responsible for where I went to college and how I've spent my life, but that is another story.

Mark had already made up his mind to work with me before he entered graduate school, and I never knew whether that was more because of his mathematical interests or because of the family connection. He helped convince others to work in algebraic topology, spearheading a wonderful group of eight people who obtained their PhDs in just the three years 1977–79, including Gaunce, Bruner (PhD, 1977), and McClure (PhD, 1978).

Bruner wrote to me about Mark that "the first thing that comes to mind is his laugh and his ability to see things in a humorous light." Jeff Caruso (PhD, 1979) wrote, "I didn't know him very well but in our conversations he was

Figure 1. Mark Steinberger at the piano, 1982.

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Figure 2. Gaunce Lewis in his college years.

always helpful. He enjoyed explaining things, and helped me to learn about moduli spaces and other topics. I still remember vividly his witty portrayal of Prof. Rothenberg in the 1976 Beer Skit." Mark was charismatic and had a bubbling but caustic sense of humor. He was then still a teenager at heart. He had to be bailed out after one escapade, when he was caught for driving too slowly, but the details are hazy in my memory. Mathematically, he was quick, sharp, and incisive.

Gaunce, in contrast, was incredibly careful, precise, and methodical. McClure wrote to me, "Mark was a character, of course, and Gaunce was a man of great integrity." At a time when many young men were trying desperately to escape the draft, Gaunce volunteered for and served in the navy in the years 1972 to 1975 before entering graduate school. He made up for lost time by finishing his PhD in three years. In later life, Gaunce was long a teacher at the First United Methodist Church of Oswego, where he served as liturgy coordinator. He too had a great sense of humor.

To discuss their work, it seems best to start with [11] and Gaunce's contributions. It is no exaggeration to say that this book first consolidated the study of equivariant stable homotopy theory as a major branch of algebraic topology. Interest in it spiked early with Gunnar Carlsson's use of equivariant stable homotopy theory to prove the Segal conjecture [5] (also published in 1984), and it was resurrected by the extraordinary and unexpected role of equivariant stable homotopy theory in the remarkable solution by Mike Hill, Mike Hopkins, and Doug Ravenel of the Kervaire invariant problem [8] (published in 2016). To quote from Paul Goerss's Mathematics Reviews of [8], "This paper marks the renaissance and reinvigoration of equivariant stable homotopy theory. While this has been an important subfield since at least the 1970s, the unexpected application of equivariant techniques to such an important problem has brought the study of group actions in stable homotopy theory to the front of the stage. ... The foundation text remains [11]." His praise of [11] in his review of an earlier expository paper [9] was still more effusive.[1]

Gaunce's expertise, especially his remarkable application of Freyd's adjoint functor theorem to construct an adjunction between the prespectra and spectra of [11], is what made the original construction and analysis of the equivariant stable homotopy category possible. It is paradoxical that this abstract idea played this crucial role in building the approach to the stable homotopy category with the most precise point-set level description of the homotopically meaningful objects (technically, these are the strict Ω G-CW-spectra). I never had to take point-set topology questions seriously in our joint work, although there were serious issues to be sorted out, since I could just go to him for the answers. Several of his papers answer point-set questions of interest to algebraic topologists.

Gaunce's thesis was largely devoted to the development of highly structured Thom spectra, which are now very widely used; equivariant examples play a major role in [8]. That work and an early paper axiomatizing transfer operations were expanded and incorporated in [11]. With McClure and me, Gaunce proved that the equivariant and nonequivariant versions of the Segal conjecture are equivalent, which turned out to be a necessary step in Carlsson's proof. We also generalized that result to a result about classifying G-spaces that has since been used to study maps between classifying spaces. A short early paper by Gaunce has been particularly influential. It showed that a very natural set of axioms for a "convenient" category of spectra is inconsistent, meaning that to construct the best possible concrete category of spectra, one has to sacrifice having the best possible relationship to the category of spaces.

Gaunce's later work remained focused mostly on equivariant homotopy theory, both stable and unstable. He brought to it a powerful and unusual blend of categorical and computational thinking. Many fundamental features of nonequivariant algebraic topology require rethinking equivariantly. Gaunce wrote the definitive equivariant treatment of the Hurewicz theorem, the construction of Eilenberg–Mac Lane G-spaces associated to representations

[1] *It is unfortunate that, as the senior author in many collaborations, my name is often cited alone, giving me disproportionate credit for joint work.*

of G, the van Kampen theorem, and the Freudenthal suspension theorem in the papers [14, 13]. A comment by Bruner is relevant: "Gaunce found complications where people might not have expected them (or at least hoped there wouldn't be any), then found ways (often again surprising) to cope with them."

Figure 3. Gaunce Lewis in his mature years.

Gaunce pioneered the study of equivariant cohomology. People unfamiliar with modern algebraic topology think of equivariant cohomology with coefficients in an abelian group A as $H^*(EG\times_G X;A)$. That is Borel cohomology. While it is powerful and useful, it is only a very special case of Bredon cohomology, the equivariant cohomology theory that satisfies the dimension axiom. With McClure and myself, Gaunce introduced $RO(G)$-graded Bredon cohomology. That requires Mackey functor coefficients, and it is now understood to be central to equivariant algebraic topology. For example, for the obvious reason that one cannot embed a G-manifold equivariantly in any $\mathbb{R}^q$ with trivial G-action, one cannot even make sense of Poincaré duality without $RO(G)$-grading. However, $RO(G)$-graded cohomology is extraordinarily difficult to compute. It is a stroke of luck that the only actual equivariant calculation in the solution to the Kervaire invariant problems is flukishly easy; the genius is in the reduction to that calculation.

In the papers [12, 6], Gaunce and his student Kevin Ferland carried out what to this day are some of the most difficult and interesting calculations in equivariant algebraic topology. To the best of my memory, Gaunce was the first to have the idea that equivariant cohomology should not only be $RO(G)$-graded but should also be Mackey functor *valued*. That is, instead of an abelian group for each integer, as in classical algebraic topology, one has a Mackey functor for each element of $RO(G)$; these Mackey functors are interrelated by multiplicative structure. That rich structure raised foundational questions about the homological behavior of Mackey functors, which Gaunce addressed in the full generality of compact Lie groups rather than just finite groups and, still more generally, in an illuminating categorical framework for the relevant homological algebra. He shows that standard results, like projective implies flat for modules over a ring, can actually fail in such more general contexts.[2]

In two large-scale papers [15, 16], Gaunce made great progress in understanding equivariant stable homotopy theory for incomplete universes, which involves using parts, but not all of, $RO(G)$; that is closely, but mysteriously, related to recent work by Andrew Blumberg and Mike Hill [1] that grew out of the solution to the Kervaire invariant problem. With Halvard Fausk and me, he computed the Picard group, that is the group of invertible objects, of the equivariant stable homotopy category. With Mike Mandell, he made a systematic study of the equivariant universal coefficient and Künneth theorems, and they went on to give a valuable study of modules over a monoid in a general monoidal category.

Turning to Mark's work, we return to [11]. In preparing this tribute, I was startled to find that I had forgotten the contributions by Mark that are direct precursors to current work on equivariant infinite loop space theory. Operads of G-spaces are no more difficult to understand than operads of spaces, but they were first taken seriously in [11], where Mark was the first to consider actions of such G-operads on G-spectra. Even today some of the results obtained there seem surprising. Such operad actions are now understood to be fundamental to the study of equivariant stable homotopy theory. A plethora of examples were predicted to exist in [1] and were shown to exist in three 2017 preprints by different authors. Nonequivariantly, Mark studies operad actions on spectra homologically via chain complexes associated to extended powers of spectra.

This work established the foundations for Mark's study of Dyer Lashof operations on highly structured ring spectra, called E_∞-ring spectra or, in their weaker up-to-homotopy version, H_∞-ring spectra. This work appears in [3]. These operations are homology analogues of the classical Steenrod operations on the cohomology of spaces. In particular, he computed these operations on the homology of Eilenberg–Mac Lane spectra. I foolishly had expected such operations to be trivial, but Mark proved how very wrong I was. One highlight gives very general criteria for when a p-local, H_2-ring spectrum splits as a wedge of Eilenberg–Mac Lane spectra $H\mathbf{Z}_p$, or more generally EM spectra $H\mathbf{Z}_{(p^r)}$, or Brown-Peterson spectra BP.

Mark's calculations were quickly used in Bökstedt's celebrated calculation of the topological Hochshild homology of $\mathbb{Z}/p\mathbf{Z}$ [2]. To quote a referee who objected that this work deserves more credit than I gave it, these calculations "underlie all the research on THH and its cousins." They have also gained interest from their use in Tyler Lawson's remarkable proof [10] (published in 2018) that BP at p=2 is not an E_{12}-ring spectrum, answering a long-studied question that I asked in 1975. The book [3] as a whole (especially McClure's part) is the starting point of the study of power operations in stable homotopy theory, which now pervade that subject.

[2] *Unfortunately, much influential work of his in this direction remains unpublished.*

In contrast to Gaunce, Mark went in a different direction after his early work in algebraic topology. While his later work was still largely equivariant, it was now in geometric topology. It was done mostly in collaboration with James West and partly also with Sylvain Cappell, Julius Shaneson, and Shmuel Weinberger. I'll let Jim tell the story.

From Jim West:

Mark's most important body of work with me was, broadly speaking, in applications of topological simple homotopy theory. Simple homotopy theory is a fundamental ingredient in the study of the structure of manifolds. Equivariant versions of simple homotopy theory are essential to the study of the structure of group actions on manifolds. The classical formulations of simple homotopy theory are dependent on the combinatorial (or cellular) structure of simplicial (or CW) complexes. Homeomorphisms and group actions that are not differentiable or piecewise linear need not respect this combinatorial or cellular structure. It was important to find a formulation of simple homotopy theory that was invariant under topological homeomorphisms. Tom Chapman did this, using earlier work of mine, when he showed that the entire theory could be interpreted as the homeomorphism theory of compact Hilbert cube manifolds.

Mark arrived at Cornell excited by the idea of working with me to apply this extension in contexts where the classical formulation was difficult or impossible to apply. We discovered that the equivariant homeomorphism theory of locally linearizable actions of finite groups on (finite dimensional) manifolds was exactly such a situation. We had a very fruitful collaboration which involved, among other things, developing an equivariant surgery theory for locally linearizable actions. I think it would be fair to say that our work opened this subject for further research. The high points of our research were Mark's *Inventiones* paper [20] and the joint paper with Cappell and Shaneson in the *American Journal* titled "Non-linear similarity begins in dimension 6" [4]. I was congratulated very warmly in person on the latter result by Georges De Rham and by Ed Floyd.

In this collaboration, we were definitely equal partners. Mark was the strategist. He was usually the one who came up with the applications that might be accessible using our techniques. Technically, he made all the algebraic calculations, while I concentrated on the controlled infinite processes.

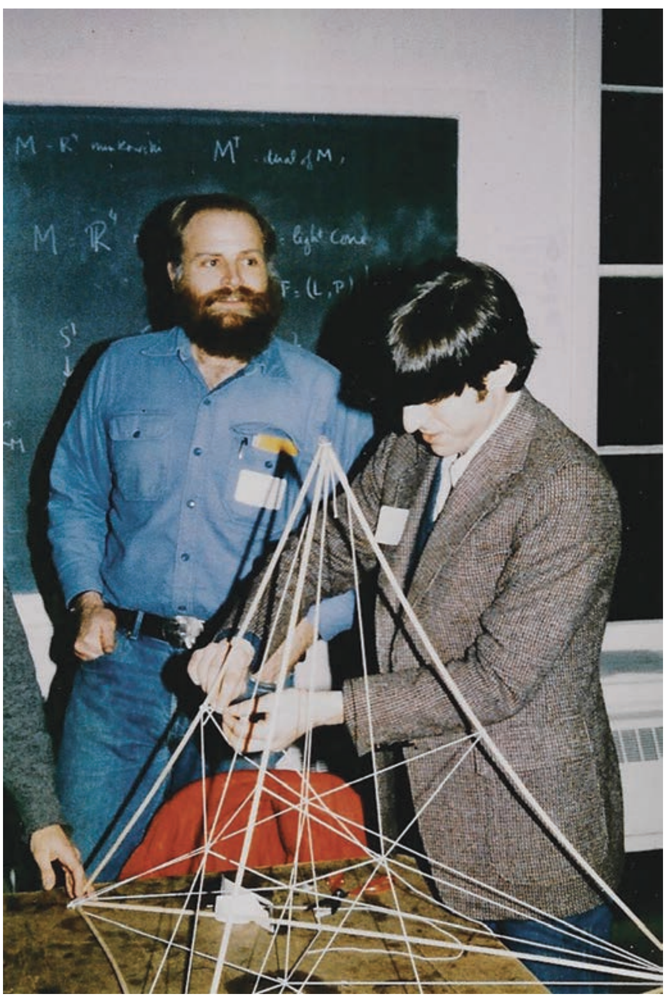

Figure 4. Mark Steinberger and Jim West, math and art; late 1970s.

To give a better idea of what this is all about, nonlinear similarity asks when linearly inequivalent representations of G can be G-homeomorphic. Analysis of the Picard group of the equivariant stable homotopy category is somewhat analogous, since in part it concerns the classification of representation spheres up to G-homotopy type.

Shmuel Weinberger wrote to me about the work of Mark and Jim:

My main mathematical interactions with Mark were about the work that he had done, partly with West, on topological simple homotopy theory for G-manifolds. Unlike the beautiful work of Chapman and Kirby–Siebenmann, which shows that topological manifolds behave just like smooth manifolds as far as their handlebody theory was concerned, this is not true with a group action. There were many examples and it looked like a complete mess before the work of Steinberger and West, and

> around the same time, Quinn came out and explained everything. Their points of view were very different, and it took a while for the community to understand why everything worked out consistently. I also took part in a collaboration with Cappell–Shaneson–Steinberger–West on nonlinear similarity that sought to develop constructive methods simultaneously with analyzing the obstructions coming out of topological equivariant simple homotopy theory. Ultimately, this led to the development of a theory of surgery on stratified spaces.

Jim does not allude to his first paper with Mark, in which they observe that a Serre fibration between CW complexes is a Hurewicz fibration. From the point of view of model categorical foundations for homotopy theory, in which these two kinds of fibrations play vastly different roles, this geometric result is quite curious. Another pair of papers by Mark alone clarifies the nature of PL fibrations.

Figure 5. Mark Steinberger speaking, 2015.

Turning to another kind of contribution, Mark deserves sole credit for the creation of an impressive new journal of mathematics, namely the *New York Journal of Mathematics* (*NYJM*). Mark founded that in 1994 as the first electronic general mathematics journal, and he was its editor-in-chief. Mark wrote an article about it that was published in the January 1996 issue of the *Notices* and is available online [19]. Mark was interested in both the economic and the technical benefits of electronic journals. For the former, he, along with many others at the time, was especially concerned about the sometimes exorbitant cost of many mathematics journals, and he saw it as a major advantage of electronic journals that they could be free.[3]

He also hoped that new journals like the *NYJM* might dent the hold of the top-ranked journals. He hated snobbishness in general and the rankings of journals in particular. He would have nodded in agreement with the article [7] about the tyranny of the top five journals in economics, which could just as well have been written about mathematics. In fact, as he knew, both [3] and [11] are in large part shotgun marriages of articles unpublishable in top journals at the time, hence their late appearance.

Mark was very much focused on the technical potential of electronic journals, and he wrote two informative (if perhaps technically dated) articles that focus on the creation of the *NYJM* [19] and on the existing and potential relevant technology [18]. One interesting technical innovation in the *NYJM* can be found at `nyjm.albany.edu/search/j/ghindex.html` where one can search all past publications of the *NYJM* at once for key strings of symbols or words. However, the focus of the journal is on quality and expertise, as its very strong editorial board attests.[4]

I wish I had words to do justice to the personalities of so many friends and colleagues now gone. I can hear both Mark and Gaunce laughing at me as I try.

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[3] *I can imagine his snort of laughter if he were told that pdf files of individual chapters of [11] are on sale by the publisher for $29.95 per chapter. (In fairness, the publisher sells the entire ebook version for "only" $59.99.)*

[4] *A blog by a satisfied author gives a readable description [17].*

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