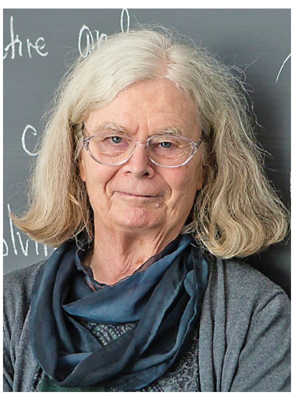
Karen Uhlenbeck Awarded Abel Prize

The Norwegian Academy of Science and Letters has awarded the Abel Prize for 2019 to **Karen Keskulla Uhlenbeck** of the University of Texas at Austin, "for her pioneering achievements in geometric partial differential equations, gauge theory and integrable systems, and for the fundamental impact of her work on analysis, geometry and mathematical physics." The Abel Prize recognizes contributions of extraordinary depth and influence in the mathematical sciences and has been awarded annually since 2003. It carries a cash award of six million Norwegian krone (approximately US$700,000).



Karen Keskulla Uhlenbeck

Citation

Karen Keskulla Uhlenbeck is a founder of modern geometric analysis. Her perspective has permeated the field and led to some of the most dramatic advances in mathematics in the last forty years.

Geometric analysis is a field of mathematics where techniques of analysis and differential equations are interwoven with the study of geometrical and topological problems. Specifically, one studies objects such as curves, surfaces, connections, and fields, which are critical points of functionals representing geometric quantities such as energy and volume. For example, minimal surfaces are critical points of the area and harmonic maps are critical points of the Dirichlet energy. Uhlenbeck's major contributions include foundational results on minimal surfaces and harmonic maps, Yang–Mills theory, and integrable systems.

An important tool in global analysis, preceding the work of Uhlenbeck, is the Palais–Smale compactness condition. This condition, inspired by earlier work of Morse, guarantees existence of minimizers of geometric functionals and is successful in the case of 1-dimensional domains, such as closed geodesics.

Uhlenbeck realized that the condition of Palais–Smale fails in the case of surfaces due to topological reasons. The papers of Uhlenbeck, coauthored with Sacks, on the energy functional for maps of surfaces into a Riemannian manifold, have been extremely influential and describe in detail what happens when the Palais–Smale condition is violated. A minimizing sequence of mappings converges outside a finite set of singular points, and, by using rescaling arguments, they describe the behavior near the singularities as bubbles or instantons, which are the standard solutions of the minimizing map from the 2-sphere to the target manifold.

In higher dimensions, Uhlenbeck in collaboration with Schoen wrote two foundational papers on minimizing harmonic maps. They gave a profound understanding of singularities of solutions of nonlinear elliptic partial differential equations. The singular set, which in the case of surfaces consists only of isolated points, is in higher dimensions replaced by a set of codimension 3.

The methods used in these revolutionary papers are now in the standard toolbox of every geometer and analyst. They have been applied with great success in many other partial differential equations and geometric contexts. In particular, the bubbling phenomenon appears in many works in partial differential equations, in the study of the Yamabe problem, in Gromov's work on pseudoholomorphic curves, and also in physical applications of instantons, especially in string theory.

After hearing a talk by Atiyah in Chicago, Uhlenbeck became interested in gauge theory. She pioneered the study of Yang–Mills equations from a rigorous analytical point of view. Her work formed a base of all subsequent research in the area of gauge theory.

Gauge theory involves an auxiliary vector bundle over a Riemannian manifold.

The basic objects of study are connections on this vector bundle. After a choice of a trivialization (gauge), a connection can be described by a matrix valued 1-form. Yang–Mills connections are critical points of gauge-invariant functionals. Uhlenbeck addressed and solved the fundamental question of expressing Yang–Mills equations as an elliptic system, using the so-called Coulomb gauge. This was the starting point for both Uhlenbeck's celebrated compactness theorem for connections with curvature bounded in L_p and for her later results on removable singularities for Yang–Mills equations defined on punctured 4-dimensional balls. The removable singularity theory for Yang–Mills equations in higher dimensions was carried out much later by Gang Tian and Terence Tao. Uhlenbeck's compactness theorem was crucial in non-Abelian Hodge theory and, in particular, in the proof of the properness of Hitchin's map and Corlette's important result on the existence of equivariant harmonic mappings.

Another major result of Uhlenbeck is her joint work with Yau on the existence of Hermitian Yang–Mills connections on stable holomorphic vector bundles over complex n-manifolds, generalizing an earlier result of Donaldson on complex surfaces. This result of Donaldson–Uhlenbeck–Yau links developments in differential geometry and algebraic geometry, and is a foundational result for applications of heterotic strings to particle physics.

Uhlenbeck's ideas laid the analytic foundations for the application of gauge theory to geometry and topology, to the important work of Taubes on the gluing of self-dual 4-manifolds, to the groundbreaking work of Donaldson on gauge theory and 4-dimensional topology, and many other works in this area. The book written by Uhlenbeck and Dan Freed on instantons and 4-manifold topology instructed and inspired a generation of differential geometers. She continued to work in this area, and in particular had an important result with Lesley Sibner and Robert Sibner on non-self-dual solutions to the Yang–Mills equations.

The study of integrable systems has its roots in nineteenth-century classical mechanics. Using the language of gauge theory, Uhlenbeck and Hitchin realized that harmonic mappings from surfaces to homogeneous spaces come in 1-dimensional parametrized families. Based on this observation, Uhlenbeck described algebraically harmonic mappings from spheres into Grassmannians relating them to an infinite-dimensional integrable system and Virasoro actions. This seminal work led to a series of further foundational papers by Uhlenbeck and Chuu-Lian Terng on the subject and the creation of an active and fruitful school.

The impact of Uhlenbeck's pivotal work goes beyond geometric analysis. A highly influential early article was devoted to the study of regularity theory of a system of nonlinear elliptic equations, relevant to the study of the critical map of higher order energy functionals between Riemannian manifolds. This work extends previous results by Nash, De Giorgi, and Moser on regularity of solutions of single nonlinear equations to solutions of systems.

Karen Uhlenbeck's pioneering results have had fundamental impact on contemporary analysis, geometry, and mathematical physics, and her ideas and leadership have transformed the mathematical landscape as a whole.

Biographical Sketch

The following is taken from a biography written by Jim Al-Khalili and published on the website `www.abelprize.no/c73996/binfil/download.php?tid=74122`.

"Karen Keskulla Uhlenbeck, the eldest of four children, was born in Cleveland, Ohio, in 1942. Her father, Arnold Keskulla, was an engineer, and her mother, Carolyn Windeler Keskulla, an artist and school teacher. The family moved to New Jersey when Karen was in third grade. As a young girl, she was curious about everything. Her parents instilled in her a love of art and music, and she developed a lifelong love of the outdoors, regularly roaming the local countryside near her home.

"Most of all, she loved reading, shutting herself away whenever she could to devour advanced science books, staying up late at night and even reading secretly in class. She dreamed of becoming a research scientist, particularly if it meant avoiding too much interaction with other people; not that she was a shy child, but rather because she enjoyed the peace and solitude of her own company. The last thing she wanted to do was to follow in her mother's footsteps and end up teaching—an attitude that would change dramatically later in life.

"Uhlenbeck's love affair with mathematics developed only after she had started at university. Having been inspired in high school by the writings of great physicists such as Fred Hoyle and George Gamow, she enrolled at the University of Michigan, initially planning to major in physics. However, she soon discovered that the intellectual challenge of pure mathematics was what really excited her. It also meant she didn't have to do any lab work, which she disliked.

"Graduating in 1964, she married her biophysicist boyfriend Olke Uhlenbeck a year later and decided to embark on postgraduate study. Already well aware of the predominantly male and often misogynistic culture in academia, she avoided applying to prestigious schools such as Harvard, where Olke was heading for his PhD and where competition to succeed was likely to be fierce. Instead, she enrolled at Brandeis University where she received a generous graduate fellowship from the National Science Foundation. There, she completed her PhD in mathematics [under Richard Palais], working on the calculus of variations; a technique that involves the study of how small changes in one quantity can help us find the maximum or minimum value of another quantity—like finding the shortest distance between two points. You might think this would be a straight line, but it is not always so straightforward. For example, if you have to drive through a busy city, the quickest route is not necessarily the shortest. Needless to say, Uhlenbeck's contribution to the field was somewhat more complicated than this!

"After a brief teaching period at MIT, she moved to Berkeley, California, where she studied general relativity and the geometry of space-time—topics that would shape her future research work. Although a pure mathematician, Uhlenbeck has drawn inspiration for her work from theoretical physics and, in return, she has had a major influence in shaping it by developing ideas with a wide range of different applications.

"For example, physicists had predicted the existence of mathematical objects called instantons, which describe the behavior of surfaces in four-dimensional space-time. Uhlenbeck became one of the world's leading experts in this field. The classic textbook *Instantons and 4-Manifolds*, which she cowrote in 1984 with Dan Freed, inspired a whole generation of mathematicians.

"In 1971, she became an assistant professor at the University of Illinois at Urbana-Champaign, where she felt isolated and undervalued. So, five years later she left for the University of Illinois at Chicago. Here, there were other female professors, who offered advice and support, as well as other mathematicians who took her work more seriously. In 1983, she took up a full professorship at the University of Chicago, establishing herself as one of the preeminent mathematicians of her generation. Her interests included nonlinear partial differential equations, differential geometry, gauge theory, topological quantum field theory, and integrable systems. In 1987, she moved to the University of Texas at Austin to take up the Sid W. Richardson Foundation Regents' Chair in mathematics. There, she broadened her understanding of physics by studying with Nobel Prize–winning physicist Steven Weinberg. She would remain at the University of Texas until the end of her working career.

"Uhlenbeck's most noted work focused on gauge theories. Her papers analyzed the Yang–Mills equations in four dimensions, laying some of the analytical groundwork for many of the most exciting ideas in modern physics, from the standard model of particle physics to the search for a theory of quantum gravity. Her papers also inspired mathematicians Cliff Taubes and Simon Donaldson, paving the way for the work that won Donaldson the Fields Medal in 1986.

"Uhlenbeck, now back in New Jersey, remains a staunch advocate for greater gender diversity in mathematics and in science. She has come a long way from the young girl who wished to be alone. For a while, she struggled to come to terms with her own success, but now says she appreciates it as a privilege. She has stated that she is aware of being a role model, for young female mathematicians in particular, but that 'it's hard, because what you really need to do is show students how imperfect people can be and still succeed. Everyone knows that if people are smart, funny, pretty, or well-dressed they will succeed. But it's also possible to succeed with all of your imperfections. I may be a wonderful mathematician and famous because of it, but I'm also very human.' Karen Uhlenbeck is certainly a remarkable human."

Karen Uhlenbeck received the National Medal of Science in 2000 and the AMS Steele Prize for Seminal Contribution to Research in 2007. She is a former MacArthur and Guggenheim Fellow and is a Fellow of the American Academy of Arts and Sciences and a member of the inaugural class of AMS Fellows. She gave the AWM Noether Lecture in 1988 and became the second woman after Emmy Noether to give a plenary lecture at the International Congress of Mathematicians in 1990. She is the first woman mathematician to be elected to the National Academy of Sciences (1986). She is currently a visiting senior research scholar at Princeton University and a visiting associate at the Institute for Advanced Study.

AMS President Jill C. Pipher said: "On behalf of the American Mathematical Society, it is my great pleasure to congratulate Professor Karen Uhlenbeck, recipient of the 2019 Abel Prize. Professor Uhlenbeck has made legendary advances in several fields of mathematics. Her early groundbreaking work on harmonic maps gave rise to a new field, geometric analysis. Her analysis via gauge theory of solutions of Yang–Mills equations had and will continue to have a profound influence on all future work in this field. She transformed the fields of geometry and analysis, crossing boundaries and making deep discoveries at the interfaces."

Read more about Uhlenbeck's life and work, including "A Glimpse of the Laureate's Work" by Arne B. Sletsjøe, as well as a list of previous recipients of the Abel Prize, at `www.abelprize.no/c73996/binfil/download.php?tid=74122`. See also the article "Karen Uhlenbeck and the Calculus of Variations," *Notices of the American Mathematical Society*, March 2019.

About the Prize

The Niels Henrik Abel Memorial Fund was established in 2002 to award the Abel Prize for outstanding scientific work in the field of mathematics. The prize is awarded by the Norwegian Academy of Science and Letters, and the choice of Abel Laureate is based on the recommendation of the Abel Committee, which consists of five internationally recognized research scientists in the field of mathematics. The Committee is appointed for a period of two years.

—From announcements of the Norwegian Academy of Science and Letters

Credits

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