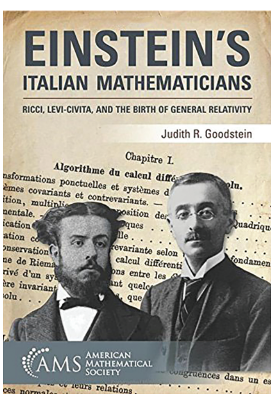
Einstein's Italian Mathematicians: Ricci, Levi-Civita, and the Birth of General Relativity

Reviewed by David E. Rowe



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Einstein's Italian Mathematicians:
Ricci, Levi-Civita, and the Birth of General Relativity
By Judith R. Goodstein

This delightful little book resulted from the author's longstanding enchantment with Tullio Levi-Civita (1873–1941), his mentor Gregorio Ricci Curbastro (1853–1925), and the special world that these and other Italian mathematicians occupied and helped to shape. The importance of their work for Einstein's general theory of relativity is one of the more celebrated topics in the history of modern mathematical physics; this is told, for example, in [Pais 1982], the standard biography of Einstein. Yet no one before Judith Goodstein has told this story with comparable empathy for the biographies of these two actors, whose lives and family connections here emerge against the background of turbulent political developments in modern Italy. Nor does the author shy away from topics like how Ricci developed his absolute differential calculus as a generalization of E. B. Christoffel's (1829–1900) work on quadratic differential forms or why it served as a key tool for Einstein in his efforts to generalize the special theory of relativity in order to incorporate gravitation. In like manner, she describes how Levi-Civita was able to give a clear geometric interpretation of curvature effects in Einstein's theory by appealing to his concept of parallel displacement of vectors (see below). For these and other topics, Goodstein draws on and cites a great deal of the vast secondary literature produced in recent decades by the "Einstein industry," in particular the ongoing project that has produced the first 15 volumes of *The Collected Papers of Albert Einstein* [CPAE 1–15, 1987–2018].

Her account proceeds in three parts spread out over twelve chapters, the first seven of which cover episodes in Ricci's career, culminating with his invention of the absolute differential calculus (*calcolo differenziale assoluto*). Italy was the land of great geometers, including two leading experts on differential geometry, Eugenio Beltrami (1835–1900) and Luigi Bianchi (1856–1928). The latter, like Ricci, had studied under the young Felix Klein (1849–1925) in Munich during the late 1870s, as the torch of higher geometry passed from Germany to the Italian peninsula. Beltrami and Bianchi both recognized the novelty of Ricci's new analytical tool, but they also took a wait-and-see attitude with regard to whether tensors and covariant derivatives could actually deliver significant new results. Recognition

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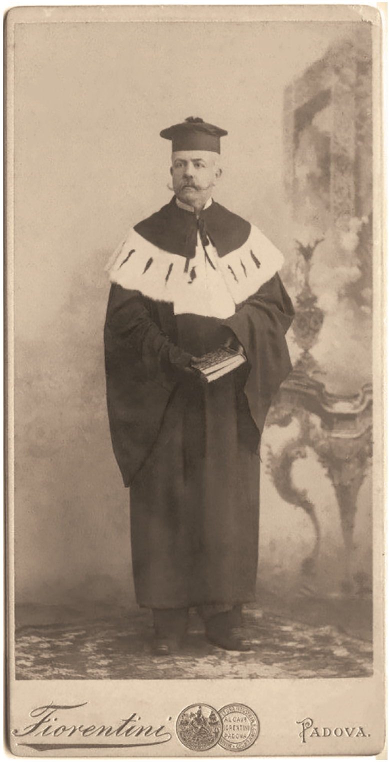

Figure 1. Gregoria Ricci Curbastro in formal academic attire.

thus proved hard to attain, and accolades came only late in Ricci's life (see Figure 1). Thus, he failed to win more than honorable mention for the three papers he submitted in 1887 to the Lincei Academy in hopes of winning its Royal Prize in Mathematics.

In Chapter 7 ("The Absolute Differential Calculus"), Goodstein describes the background ideas that motivated Ricci's invention. Modern differential geometry began with a famous work on the intrinsic geometry of surfaces, published by Carl Friedrich Gauss (1777–1855) in 1827. Long before Gauss, Leonhard Euler (1707–1783) had established several classical results on the curvature properties of surfaces, but Eulerian curvature depends on how a surface is embedded in 3-space. Gaussian curvature, on the other hand, does not; it is an intrinsic invariant and can be determined from measurements taken on the surface itself by means of the first fundamental form. In his famous Göttingen qualifying lecture (*Habilitationsvortrag*) from 1854, Bernhard Riemann (1826–1866) introduced a generalization of this surface invariant to manifolds of arbitrary finite dimension. Probably Gauss was the only one in the audience who understood this lecture, however, and so Riemann nearly took these daring ideas with him to his early grave [Klein 1927, p. 165]. Luckily, Richard Dedekind (1831–1916) later recovered the text and published it after his friend's death, thereby inspiring Christoffel to develop Riemann's ideas further. In his classic study [Christoffel 1869], he worked out what came to be called the Riemann-Christoffel curvature tensor. As Goodstein explains, Christoffel's analytical methods served as the starting point for Ricci's investigations. These centered on the problem of classifying Riemannian spaces by determining when two quadratic differential forms can be transformed into one another, a problem closely related to Beltrami's work. In short, this line of research, dating back to Gauss's classic investigation on the intrinsic geometry of surfaces, eventually spawned the modern theory of differential invariants. Ricci spent a good decade, from 1887 to 1896, elaborating on the methods first set forth in Christoffel's paper, a rather lonely intellectual journey until Levi-Civita came of age.

Goodstein's second hero makes his entrance in Chapter 8 ("The Alter Ego"), when as a brilliant young student Levi-Civita meets Ricci at the University of Padua. He enrolled there at age seventeen in 1890, the year in which Ricci was finally elevated to a full professorship with the title Professor of Algebra. Levi-Civita would later recall how Ricci's very first algebra course "remained for me a model of impeccable reasoning and fruitful mathematical enterprise" (p. 62). Ricci also continued to teach his upper-level course on mathematical physics, which would be of decisive significance for his star pupil. Their personal and mathematical relations come to the fore in Chapter 9 ("Intermezzo") in connection with the Bordin Prize of the Paris Academy, which again passed over Ricci's work in favor of closely related results concerning families of geodesics on surfaces obtained by Gabriel Koenigs (1858–1931). When Levi-Civita referred to Koenigs's work in the draft of a paper, his mentor reacted allergically, an interesting example of how delicate certain issues—such as when and how to cite a rival author—can sometimes be. In this case, Levi-Civita (see Figure 2) responded with his customary tact, leaving no room for Ricci to doubt his former pupil's loyalty and

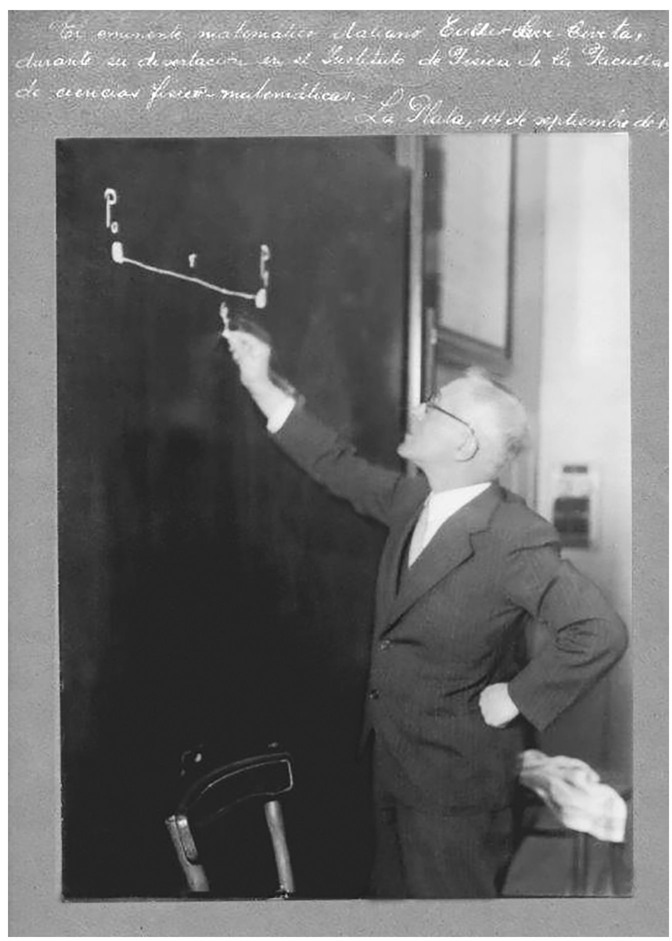

Figure 2. Levi-Civita lecturing in La Plata, Argentina, in 1923.

devotion. Recognition, prestige, loyalty, and fame: these are themes that resonate throughout Goodstein's narrative, making this book far more than just a good read.

When Ricci died in 1925, Levi-Civita eulogized him in a biographical memoir, republished in an English translation as Appendix B. The year 1928 saw the appearance of Levi-Civita's classical monograph *The Absolute Differential Calculus*, reviewed by George Rainich (1886–1968) for the *Bulletin of the American Mathematical Society*. On pages 118–119, Goodstein quotes a passage from this review, in which Rainich gave an intuitive description of parallel displacement in Riemannian spaces by way of surface theory. On a developable surface—such as a cylinder or a cone—the Gaussian curvature vanishes, which means there will be no effect when a vector is transported parallel to itself around a closed curve; the vector simply returns to its original position. This will not be the case, however, for surfaces with curvature. In the case of space-time geometries, the flat metric of Minkowski space reflects the fact that special relativity does not incorporate gravitational effects, whereas general relativity interprets these by means of space-times with curvature. Einstein's complicated journey from special to general relativity constitutes one of the most famous chapters in the history of science (for an excellent introduction to this theme, see [Gutfreund and Renn 2015]). As Goodstein rightly notes, however, these geometrical ideas had relatively little importance for Einstein; what he wanted from the mathematicians was an analytical tool. Thanks to his friend Marcel Grossmann (1878–1936), he found what he was looking for in a paper Ricci and Levi-Civita wrote for Klein's *Mathematische Annalen*.

Figure 3. Marcel Grossmann, Einstein's mathematical collaborator.

Unfortunately, Chapter 9 contains only a few brief remarks about this oft-cited survey article, "Méthodes de calcul différentiel absolu et leur applications" [Ricci and Levi-Civita 1901]. As Klein later emphasized [Klein 1927, p. 195], the methods presented therein were precisely those one finds in Albert Einstein's (1879–1955) classic presentation of general relativity from 1916 [Gutfreund and Renn 2015, pp. 183–226]. Goodstein cites the words of Robert Hermann, who called the paper [Ricci and Levi-Civita 1901] "one of the most influential and important in the history of *both* differential geometry and mathematical physics" (p. 77). Yet earlier she writes, "It is hard to reconstruct what Ricci saw in his work" before Grossmann stumbled upon it and pointed out its relevance for Einstein's approach to gravitational theory (p. 54). Clearly, [Einstein and Grossmann 1913] was the key text that set off the chain of events Hermann had in mind, though this paper, too, receives rather scant attention. In any event, I should remark that [Ricci and Levi-Civita 1901] contains a great wealth of applications and potential uses in geometry and physics, so there would seem to be little mystery about the research program Ricci had in mind.

Einstein's role in this story begins with Chapter 10 ("The Indispensable Mathematical Tool"). This is the one chapter where Goodstein's account flounders, due in part to an over-reliance on secondary sources rather than discussing the two aforementioned papers, which pertain most directly to this theme. For a clear discussion of the links between these key papers, readers should turn to Appendix A, an essay by Michele Vallisneri titled "From Ricci's Absolute Differential Calculus to Einstein's General Theory of Relativity." Vallisneri gives a somewhat more technical account of the transition from special to general relativity, a famously difficult topic; for detailed accounts, see the studies in [Renn 2007] and three papers by Michel Janssen [Janssen 1999, 2012, 2014].

Unlike the remainder of the book, parts of Chapter 10 contain some inaccurate information. This pertains to certain details, for example, about the seminar on electron theory offered by Minkowski and Hilbert in 1905 but also with regard to an important lecture that Minkowski presented in the Göttingen Mathematical Society in 1907 (published by Arnold Sommerfeld in 1915). The larger problem, however, stems from a widespread misconception about Einstein's special theory of relativity, as published in [Einstein 1905] and subsequently. In this justly famous paper, Einstein postulated that the laws of physics have the same form for all inertial observers, but furthermore that the velocity of light in a vacuum never varies no matter how these observers are moving, e.g., whether away from or toward a light source. From these assumptions, he quickly derived a law for adding the velocities of two frames moving in the same direction and then showed that the Maxwell-Lorentz electrodynamic equations remain unchanged under Lorentz transformations. This approach represented a clear break with that taken by ether theorists, including H. A. Lorentz (1853–1928), who postulated that this medium for the transmission of electromagnetic waves constituted an absolute rest frame in physical space. Einstein's principle of relativity made this luminiferous ether superfluous while undermining the notion that time measurements were independent for different observers. Einstein's 1905 paper thus recast fundamental ideas about time and space, thereby laying some of the cornerstones for what became the special theory of relativity. This theory, however, did not stand still, and only part of what transpired with relativity between 1905 and 1913 was due to Einstein. He alone, however, had the brilliant idea that noninertial, accelerated frames of reference would lead to gravitational fields, a notion that put him on the long intellectual journey that led to his general theory of relativity. But by the time he was prepared to tackle that problem, special relativity itself had greatly matured in the hands of several other investigators, including Henri Poincaré (1854–1912), Hermann Minkowski (1864–1909), Arnold Sommerfeld (1868–1951), and Max von Laue (1879–1960).

Goodstein's Chapter 10 implicitly raises the central question: What made the Ricci calculus such an "indispensable mathematical tool" for general relativity? In answering this, she largely skips over Einstein's early speculations concerning gravity and inertia, barely mentioning (page 89) the thought experiment from 1907 that led to his principle of equivalence. This principle postulates that an observer moving in a uniformly accelerated frame will measure precisely the same effects (locally) as if he or she were in a static frame within a homogeneous gravitational field. It is important to appreciate how Einstein's equivalence principle guided his whole approach to gravitation, stemming from this early insight that one should treat the inertial effects induced by accelerated reference frames as special types of gravitational fields. Initially, he tackled only the simplest cases, uniform linear acceleration and uniform rotation, trying to show how these induce a scalar field of gravitation. Yet even with these cases he encountered serious difficulties. Nevertheless, he clung to his inspiration, which eventually led him to realize that his whole approach, based on a generalized theory of relativity allowing for arbitrary motions, necessitated a mathematical formalism for handling expressions that remain invariant under arbitrary coordinate transformations. This, in a nutshell, is why the Ricci calculus proved to be just what Einstein was looking for.

Instead of focusing on this background, Goodstein begins Chapter 10 with remarks about Hermann Minkowski's (1864–1909) reconceptualization of special relativity. Clearly, she needed to deal with this aspect of the story in order to cross the bridge from the Ricci-calculus [Ricci and Levi-Civita 1901] to the first draft of general relativity in [Einstein and Grossmann 1913]. Indeed, Einstein himself wrote that without Minkowski's new framework, general relativity would "perhaps have remained stuck in its diapers" [CPAE 6, p. 463]. Still, she misses a crucially important part of what Minkowski achieved, namely, the invention of a 4-dimensional vector calculus adapted to the group of Lorentz transformations. In fact, Poincaré had already pointed out years earlier that the Lorentz transformations form a group, which he proceeded to exploit in his research on electrodynamics. Minkowski went further in reframing special relativity using invariants and covariants of the Lorentz group [Walter 1999]. Goodstein, in fact, hints at this by citing remarks made by Einstein's biographer, the physicist Abraham Pais, who noted how Minkowski was the first to write the "Maxwell-Lorentz equations [in electrodynamics] in their modern tensor form" (p. 88). A footnote then follows stating that these tensors have nothing to do with the tensors of the Ricci calculus. It would have been better to say that since covariant differentiation reduces to ordinary differentiation in Minkowski space, tensors corresponding to the Lorentz group are those proper to special relativity.

Goodstein faithfully recounts how Einstein was initially reluctant to accept Minkowski's packaging of special relativity as a 4-dimensional spacetime geometry, but she does not really explain when and why he changed his mind. She tells, though, how (with the help of the young physicist Jakob Laub (1884–1962)) he initially tried to dispense with it. After noting Minkowski's early and unexpected death—he was stricken by appendicitis in January 1909—Goodstein merely writes, "[i]n the end, Einstein's reluctant capitulation to complex mathematics came from a different source" (p. 88) but without really spelling out the source she has in mind. Reading further, we learn about his interactions in Prague with the mathematician Georg Pick (1859–1942), who probably told Einstein about the Ricci calculus. So perhaps he already had some inkling of what the Italians had long before cooked up when he returned to Zürich and spoke with his friend Grossmann about how he hoped to generalize relativity. But what about his "capitulation" to Minkowski's ideas?

When, at the very end of his life, Einstein recalled his conversations with Grossmann [Einstein 1956], the problem he raised was whether one could find generally covariant differential equations for the ten metric components of a 4-dimensional spacetime, i.e., equations that would retain their form under general coordinate transformations. If so, wrote Einstein, then solving those equations would yield the metric tensor needed to determine the trajectories of inertial point masses as geodesic curves in spacetime. If, on the other hand, a suitable transformation of the four spacetime coordinates happened to produce the Minkowski metric, then these trajectories would simply be time-like straight lines in Minkowski space, a flat pseudo-Euclidean 4-manifold. Even if Einstein's obviously condensed account might be somewhat historically inaccurate, it surely reveals that he had come to accept Minkowski's approach to special relativity well before he returned to Zürich. Contemporary evidence suggests, in fact, that this important intellectual turning point probably took place in 1910; moreover, the motivation behind this shift came from ongoing efforts to reconceptualize and polish special relativity. In that year, Einstein wrote to Sommerfeld expressing a favorable reaction to the latter's first paper on 4-vectors in *Annalen der Physik* [CPAE 5, p. 246]. The following year saw the publication of Laue's textbook on special relativity based on a similar formalism, a book that left a deep impression on Einstein. Indeed, the works of these three authors—Minkowski, Sommerfeld, and Laue—were prominently cited by Grossmann in [Einstein and Grossmann 1913, p. 328]. So Einstein, who by 1911 had to catch up with these developments, quickly saw the distinct advantages of a 4-dimensional formalism. (For a detailed account, see [Walter 2007]; on Laue's role, see [Rowe 2018].)

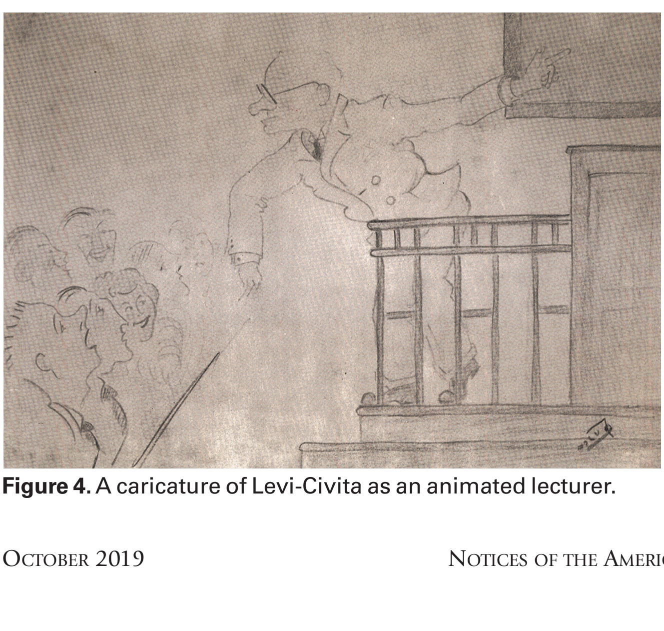

Figure 4. A caricature of Levi-Civita as an animated lecturer.

Einstein's general tendency, on the other hand, was often to downplay the importance of higher mathematics for theoretical physics. He saw himself—along with Paul Ehrenfest (1880–1933) and Niels Bohr (1885–1962)—as "mad advocates of principles," a type of theoretician he contrasted with another group, the "virtuosi," as exemplified by the Munich physicist Arnold Sommerfeld. To him, Einstein penned an oft-quoted letter from 29 October 1912, shortly after he had returned to Zürich from Prague (p. 95):

> I occupy myself exclusively with the problem of gravitation and now believe I will overcome all difficulties with the help of a friendly mathematician here. But this one thing is certain: that in all my life I have never labored at all as hard, and that I have become imbued with a great respect for mathematics, the subtle parts of which, in my innocence, I had till now regarded as pure luxury. Compared with this problem, the original theory of relativity is child's play.

The mathematics he was learning, with the help of his friend Marcel Grossmann (see Figure 3), was of course the Ricci calculus, soon to be dubbed tensor analysis. It is worth noting that from this time onward Einstein was rarely without a mathematical assistant or younger physicist of the "virtuoso" type (mini-biographies of his many assistants and collaborators can be found in [Pais 1982, pp. 483–501]).

Soon after he and Grossmann published the first version of general relativity in [Einstein and Grossmann 1913]—the so-called *Entwurf* theory—Einstein found himself stranded in Berlin without a suitable mathematical partner. It was during these early Berlin years—up until the spring of 1915 when Italy entered the war against the Central Powers—that he struck up a correspondence with Levi-Civita, as described in Chapter 11 ("Write to Me Next Time in Italian"); for details on this correspondence, see [Cattani and De Maria 1989]. In a letter to a friend in Zürich, Einstein had this to say about these exchanges:

> The theory of gravitation will not find its way into my colleagues' heads for a long time yet, no doubt. Only *one*, Levi-Civita in Padua, has probably grasped the main point completely, because he is familiar with the mathematics used. But he is seeking to tamper with one of the most important proofs in an incessant exchange of correspondence. Corresponding with him is unusually interesting; it is currently my favorite pastime. (p. 111)

Chapter 11 also deals with Levi-Civita's efforts to find a position in Italy for the Göttingen-trained physicist Max Abraham (1875–1922). An ether theorist famous for his sharp tongue, Abraham was perhaps the most formidable critic of the principle of relativity. Goodstein recounts some of his many clashes with Einstein, the most significant of which concerned electron theory. Abraham advanced a rigid model for the electron, as opposed to the Lorentzian electron, which underwent deformations at high velocities. Einstein's theory showed how these Lorentzian contractions could be understood as purely relativistic effects. For an amusing but very enlightening fictitious discussion between Lorentz, Poincaré, Einstein, et al. on the merits of Einstein's approach to the foundations of electrodynamics, see [Darrigol 2000, pp. 385–392].

In the closing twelfth chapter ("Parallel Displacements"), Goodstein recalls the reception of general relativity among differential geometers like Luther Eisenhart (1876–1965) in the US, as well as Jan Arnoldus Schouten (1883–1971) and his assistant Dirk Struik (1894–2000) in the Netherlands. Levi-Civita discovered his famous geometrical interpretation of the Riemann-Christoffel tensor during wartime, when scientific communication was both slow and difficult. As often happens in mathematics, Schouten had independently come up with a similar idea for interpreting covariant differentiation by making use of geodesic frames. Struik recalled how Schouten one day came running into his office waving an offprint of Levi-Civita's paper, alarmed to see that the Italian had already published the main idea behind parallel displacement. This notion turned out to be of great importance not only for differential geometry but also for spacetime physics due to the role played by affine connections in general affine spaces. In a Riemannian space, one has a special connection derived from the metric tensor, often called today the Levi-Civita connection. Soon afterward, Einstein, Hermann Weyl (1885–1955), and Arthur Eddington (1882–1944) would use general affine connections in their attempts to unify gravity and electromagnetism.

Goodstein also offers glimpses of what transpired in October 1921 when, following an invitation from Federigo Enriques (1871–1946), Einstein visited Bologna to deliver three lectures on relativity. Ricci was unable to attend, but the now elderly man still had the pleasure of meeting Einstein, who agreed to give a fourth lecture in Padua. A lighter episode took place during Levi-Civita's visit to Princeton in 1936. Leopold Infeld (1898–1968), who one day happened to be talking with Einstein in the latter's office, later recalled this scene.

> At this moment a knock at the door interrupted our conversation. A very small, thin man of about sixty entered, smiling and gesticulating, apologizing vividly with his hands, undecided in what language to speak. It was Levi-Civita, the famous Italian mathematician. …This time 'English' was chosen as the language of our conversation. …I watched the calm, impressive Einstein and the small, thin, broadly gesticulating Levi-Civita as they pointed out formulae on the

Figure 5. Tullio Levi-Civita with his wife, Libera Trevisani, New York, 1933.

> blackboard and then talked in a language which they thought to be English. The picture they made, and the sight of Einstein pulling up his baggy trousers every few seconds, was a scene, impressive and at the same time comic, which I shall never forget. [Infeld 1941/1980, p. 260]

Infeld described Einstein's English as very simple, but clear and distinct, whereas "Levi-Civita's English was much worse, and the sense of his words melted in the Italian pronunciation and vivid gestures" (see Figure 4).

The onset of fascism in Italy darkens subsequent events, particularly after 1938 when Mussolini's government enacted the racial laws that institutionalized discrimination against Jews. Here, as elsewhere, Judith Goodstein draws on interviews with Susanna Silberstein Ceccherini, the adopted daughter of Tullio and his much younger wife, Libera (see Figure 5). She recalled her mother's recollections of this sorrowful period in their family's life, leading up to Levi-Civita's death on 29 December 1941 at age sixty-eight. Goodstein's account also refers to the obituary of Levi-Civita written by W. V. D. Hodge (1903–1975), who summarized his life and noble character for the London Royal Society, a text reproduced as Appendix C in this volume.

This genuinely scholarly account of the lives of Ricci and Levi-Civita, who helped to shape a famous chapter in the history of modern mathematical physics, is a true joy to read from beginning to end. As an added bonus, the book contains thirty-three photos, many of which appear in print for the very first time. Perhaps a bit like Ricci, praise for Judith Goodstein, author of a noteworthy biography of Vito Volterra [Goodstein 2007], has been somewhat late in coming. In any event, for her efforts in producing this splendid book, high praise indeed is surely her just due.

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