Jean-Pierre Wintenberger

Pierre Colmez and Chandrashekhar Khare

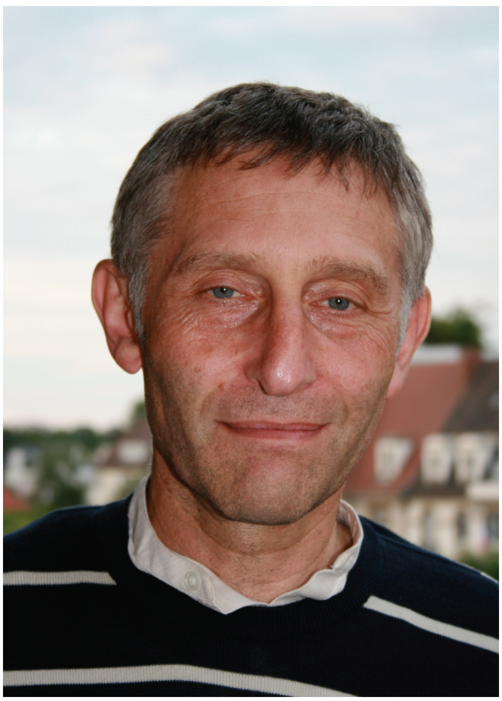

Figure 1. Strasbourg, France, 2008.

A Brief Bio

Jean-Pierre Wintenberger was born in Neuilly-sur-Seine, near Paris, in 1954. His parents were scientists who transmitted to him their curiosity, interest, and passion for science and research. He entered the École Normale Supérieure de Paris in 1973, along with his friend Jean-Yves Mérindol, who later became an algebraic geometer and the president of Strasbourg University. Some of the other students who entered the ENS at the same time were Jean-Francois Mestre, Joseph Oesterlé, Guy Henniart (and Hélène Esnault in ENS Sèvres—for women: the two ENSs merged in 1985). Just the previous year Henri Carayol, Laurent Clozel, Étienne Fouvry, Gérard Laumon, Jean-Loup Waldspurger (and Colette Moeglin in ENS Sèvres) had entered the ENS, to be followed by Bernadette Perrin-Riou at ENS Sèvres in 1974. This was a spectacularly rich crop of students, forming a core group who went on to play a big role in the growth of the French school of number theory, particularly arithmetic geometry and automorphic forms.

Jean-Pierre got his first thesis in 1978 and his Thèse d'État (Habilitation) in 1984 in Grenoble, under the supervision of Jean-Marc Fontaine. He held the position of researcher in CNRS from 1978 to 1991, first in Grenoble, then in Orsay. He was a professor at the Université de Strasbourg from 1991 till he retired in 2017. He was a member of the Institut Universitaire de France, received the Prix Thérèse Gautier from the French Academy of Science in 2008, and was an invited speaker in the Number Theory Section at the International Congress of Mathematicians in 2010. He received the 2011 AMS Frank Nelson Cole Prize in Number Theory (jointly with Khare) for their proof of Serre's modularity conjecture.

Jean-Pierre passed away on January 23, 2019. He is survived by his son, Olivier Wintenberger, who is an applied mathematics professor at Sorbonne Université, and his daughter, Claire Guillet, who is a doctor in Grenoble.

Grenoble

Pierre Colmez

Jean-Pierre Wintenberger did his thèse de 3-ième cycle and his thèse d'État in Grenoble, under the supervision of Jean-Marc Fontaine. This is where I met him for the first time. I spent two years (1985–87) in Grenoble for my PhD: my

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thesis problem had been given to me by John Coates, who was then a professor at Orsay, but I had ended up in Grenoble, with Fontaine as an official adviser, by some bizarre twist. My first year there was rather miserable, as my thesis (about complex L-functions) had a big gap. The second year was much more fun: Fontaine had just returned from his year at Minneapolis, where he was collaborating with William Messing on their proof [6] of Fontaine's C_{cris} conjecture [3] on periods of p-adic algebraic varieties with good reduction, and everybody was speaking of p-adic periods (including me: I was fantasizing about a product formula for these numbers, analogous to the product formula for rational numbers, and most of what was being discussed found its way in the ouput [1] of my fantasies). Roland Gillard [8] had just proved, in the case of ordinary reduction, a p-adic analog of Shimura's multiplicative relations between periods of CM abelian varieties (a vast generalization of the celebrated Chowla–Selberg formula expressing periods of elliptic curves with complex multiplication in terms of values of the Γ-function at rational arguments, the simplest formula of this type being $\int_1^\infty \frac{dx}{\sqrt{x^3-x}} = \frac{\Gamma(1/4)\Gamma(1/2)}{2\,\Gamma(3/4)}$) using methods introduced by Gross [9] in his geometric proof of the Chowla–Selberg formula. Wintenberger had started to attack the general case, which is more difficult, as the periods do not live in $\mathbb{C}_p$ (the completion of the algebraic closure of $\mathbb{Q}_p$) anymore but in the bigger (and much scarier) ring $\mathbb{B}_{\mathrm{dR}}$ of p-adic periods, constructed by Fontaine [3]. I remember him telling me one day that $\mathbb{B}_{\mathrm{dR}}$ lives in families (i.e., you have relative $\mathbb{B}_{\mathrm{dR}}$'s over bases), and p-adic periods of a family satisfy the same kind of differential equations as complex periods. His remark played a big role in my work with Fontaine [2] some twelve years later. He completed half of this project, namely, the definition of the p-adic periods of a family of abelian varieties [18], but part II announced in the introduction of that paper never materialized, which is a pity.

At the time, Wintenberger already had a solid reputation in the field. He had developed, with Fontaine [7, 15], the Field of Norms theory, which attaches to any "reasonable" infinite extension L of the field $\mathbb{Q}_p$ of p-adic numbers a characteristic p field $X(L)$ isomorphic to $\mathbb{F}_q((T))$ for some $q = p^f$. The association $L \mapsto X(L)$ looks very strange at first (see the formula for the addition law below), but it is functorial and provides a bridge between the absolute Galois groups of finite extensions of $\mathbb{Q}_p$ and those of finite extensions of $\mathbb{F}_p((T))$. The Field of Norms theory is the foundation upon which rests the powerful theory of (φ, Γ)-modules of Fontaine [4], which gives a description of all $\mathbb{Q}_p$-representations of these absolute Galois groups. It is also the 0-dimensional case of Scholze's tilting equivalence [13] between characteristics 0 and p.

Figure 2. Monte Verità, Switzerland, 2009.

Important examples of reasonable infinite extensions of $\mathbb{Q}_p$ are the cyclotomic extension $\mathbb{Q}_p(\mu_{p^\infty})$, the Kummer extension $\mathbb{Q}_p(p^{1/p^\infty})$, or extensions fixed by the kernel of representations $\rho : G_{\mathbb{Q}_p} \to \mathrm{GL}_d(\mathbb{Q}_p)$ "coming from geometry" (i.e., from the étale cohomology of algebraic varieties defined over $\mathbb{Q}_p$ or its finite extensions). The case of the cyclotomic extension gives a dévissage of the absolute Galois group $G_{\mathbb{Q}_p}$ of $\mathbb{Q}_p$ with the following shape: one has a natural exact sequence

$$1 \to G_{\mathbb{F}_p((T))} \to G_{\mathbb{Q}_p} \to \mathbb{Z}_p^* \to 1,$$

where $G_{\mathbb{F}_p((T))}$ is the absolute Galois group of $\mathbb{F}_p((T))$. A reasonable infinite extension of $\mathbb{Q}_p$ can be written as an increasing union of finite extensions L_n of $\mathbb{Q}_p$, and $X(L)$ is the set of sequences $(x_n)_{n\in\mathbb{N}}$, with $x_n \in L_n$ and $\mathrm{N}_{L_{n+1}/L_n}(x_{n+1}) = x_n$ for all $n \in \mathbb{N}$. The set $X(L)$ is turned into a field of characteristic p by setting $(x_n) + (y_n) = (s_n)$ and $(x_n)(y_n) = (t_n)$, with

$$t_n = x_n y_n \quad \text{and} \quad s_n = \lim_{k\to\infty} \mathrm{N}_{L_{n+k}/L_n}(x_{n+k} + y_{n+k})$$

(that the limit exists is the nontrivial part of this construction and uses crucially the fact that the extension is reasonable).

Another striking contribution was his construction [16] of a natural splitting of the Hodge filtration for varieties over a p-adic field. If X is a smooth projective algebraic variety of dimension d defined over a characteristic 0 field

K, Grothendieck has defined its algebraic de Rham cohomology $H^{\bullet}_{\mathrm{dR}}(X/K)$ by means of algebraic differential forms. The $H^{i}_{\mathrm{dR}}(X/K)$'s are finite-dimensional K-vector spaces that vanish for $i > 2d$ and are endowed with a decreasing filtration, the Hodge filtration, by sub-K-vector spaces. If K is a subfield of $\mathbb{C}$, then $\mathbb{C} \otimes_K H^{\bullet}_{\mathrm{dR}}(X/K)$ is isomorphic to the de Rham cohomology of the $2d$-dimensional differentiable manifold $X(\mathbb{C})$, and Hodge theory provides a description of $\mathbb{C} \otimes_K H^{\bullet}_{\mathrm{dR}}(X/K)$ in terms of harmonic forms, which, in turn, induces a canonical splitting of the Hodge filtration on $\mathbb{C} \otimes_K H^{\bullet}_{\mathrm{dR}}(X/K)$ (but not on $H^{\bullet}_{\mathrm{dR}}(X/K)$ itself, as this splitting usually involves complex numbers that are transcendental over K).

Now, if K is a finite extension of $\mathbb{Q}_p$, there is nothing like harmonic forms at our disposal (at least, up to now). But Wintenberger managed to define a natural splitting of the Hodge filtration in the case where X has "good reduction modulo p" and $K/\mathbb{Q}_p$ is unramified. In that case the cohomology of X is controlled by that of its reduction, and the morphism $x \mapsto x^p$ that exists in characteristic p, the Frobenius morphism, induces a morphism φ on the $H^{i}_{\mathrm{dR}}(X/K)$'s. Hence $H^{i}_{\mathrm{dR}}(X/K)$ is what Fontaine calls a filtered φ-module (i.e., a K vector space with a φ and a filtration). Now, p-adic Hodge theory (nothing to do with harmonic forms) implies that this filtered φ-module has special properties: there exists an $\mathcal{O}_K$-lattice M (with $\mathcal{O}_K$ the ring of integers of K) such that φ is divisible by p^i on $M \cap \mathrm{Fil}^i$ and $M = \sum_i p^{-i}\varphi(M \cap \mathrm{Fil}^i)$ (such a lattice is said to be strongly divisible). Wintenberger's result is a linear algebra result concerning these filtered φ-modules admitting a strongly divisible lattice, and there is no geometry involved. This result has remained a mystery: is there a theory of p-adic harmonic forms that would explain the existence of this natural splitting? Does this splitting exist without assuming $K/\mathbb{Q}_p$ to be unramified or X to have good reduction?

Wintenberger was interested in this splitting for the construction [17] of special representations $\rho : G_K \to \mathrm{GL}_d(\mathbb{Q}_p)$ of the absolute Galois group G_K of K with $\rho(G_K)$ open in a given algebraic subgroup of $\mathrm{GL}_d(\mathbb{Q}_p)$ (some kind of inverse Galois problem for finite extensions of $\mathbb{Q}_p$): Fontaine-Laffaille theory [5] allows us to translate the problem in terms of φ-modules admitting a strongly divisible lattice. This was not the last time that Wintenberger used this splitting for questions related to representations of Galois groups (see e.g. [19]).

I did not really follow very closely what he was doing later on after he took a position in Strasbourg, and I was amazed to discover at a conference that he organized in Strasbourg about Serre's conjecture [14] on the modularity of mod p representations of the absolute Galois group of $\mathbb{Q}$ that he was actually proving, in collaboration with Chandrashekhar Khare [10–12], this very conjecture (a dream of quite a few number theorists at the time)! He had been thinking about a strategy to attack it for a long time.

Strasbourg

Chandrashekhar Khare

I got to know Jean-Pierre well through our work together on Serre's modularity conjecture. We arrived at working on the conjecture by different paths. Jean-Pierre was interested in proving cases of the Mumford–Tate conjecture for abelian varieties defined over number fields and in particular in the first case of it that was still open: that of a 4-dimensional abelian variety defined over a number field. I came to it more directly, and since my PhD thesis in 1995 I had been interested in Serre's conjecture. I studied congruences between modular forms in my thesis and then got interested in questions of lifting Galois representations. Thus we came from different mathematical backgrounds to Serre's conjecture, and I think our collaboration benefited from this diversity of interest and training we brought to it.

Jean-Pierre invited me to visit him for a month in Strasbourg, and I visited him in the fall of 2004. Little did I expect that we would spend the month proving the first cases of Serre's conjecture, which made no superfluous hypotheses on the residue characteristic and image of the representation! Combining observations we had each made independently, we had a result on Serre's conjecture almost the day I arrived in Strasbourg, and then we spent my month's stay making sure of the details. Jean-Pierre explained to me his beautiful idea of "killing ramification" in the first days of my visit. The idea is used to reduce the general case of Serre's modularity conjecture to the level one case. It struck me as an idea that could perhaps naturally occur only to someone who thought very p-adically, and Jean-Pierre since his thesis with Jean-Marc Fontaine in 1979 had thought about the then emerging field of p-adic Hodge theory, proving foundational results and finding new applications of it.

We were happy to part at the end of my visit, when I returned to Mumbai, having convinced ourselves that we could show that there were no irreducible odd Serre-type

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Figure 3. The Vosges, France, 2012.

Galois representations of certain low weights and levels as predicted by Serre and most satisfyingly that the only odd irreducible Serre-type representation of level 1 and Serre weight 12 was the one that arose from Ramanujan's Δ-function. We also had a strategy to prove all of Serre's conjecture assuming broad generalizations of the modularity lifting results pioneered by Wiles. At the end of the productive visit we celebrated by going for dinner, along with the number theorists at Strasbourg, to an elegant restaurant, La Casserole, in one of the small twisting alleys near the Cathédrale.

We wrote our first paper on Serre's conjecture expecting that to prove the full conjecture using our strategy would require very elaborate developments of the modularity lifting machinery by specialists in the area. But within a few months we found a plausible path using a modification of our original strategy that made extensive and novel use of congruences between Galois representations and was consequently less demanding in terms of the modularity lifting theorems we would need. The topography of this path was reminiscent of the winding, intersecting paths surrounding the Cathédrale in Strasbourg that led to different views of its lone Gothic spire, its motif orienting a visitor's meanderings around it.

It still took us almost five years (2004–09) to complete all the details and have the proof of the full conjecture published. We communicated throughout this period mainly via email, interspersed with meeting at conferences and short visits to Salt Lake City, Paris, Montréal, Strasbourg, Monte Verità, ... Our work benefited enormously from the rigor and technical deftness Jean-Pierre brought to sorting through the niceties of proving modularity lifting theorems in delicate cases, like the 2-adic lifting theorems we needed for our strategy. I admired Jean-Pierre's focus on what was central to mathematics and his wide mathematical culture, part of which seemed to be due to his education as a Normalien and then being trained in the formidable French school of arithmetic geometry. I also admired the sense of adventure in his mathematical work that led him to work on Serre's modularity conjecture, a subject that was a little distant from the mathematics that had occupied him prior to our joint work on it.

Jean-Pierre passed away before he reached sixty-five. I had last seen him in the hospital Pitié-Salpêtrière in Paris in July 2018. He seemed mentally alert but physically worn out. I had hoped to see him again this year, but that was not to be. In his last years he suffered from Parkinson's disease. Within a week of Jean-Pierre's death, his advisor, Jean-Marc Fontaine, passed away, without whose foundational work on p-adic Hodge theory our proof of Serre's conjecture would not have been possible.

I will miss Jean-Pierre's friendship, unassuming nature, and keen intellect.

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Credits

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