



2020 Bôcher Memorial Prize

The 2020 Bôcher Memorial Prizes were presented at the 126th Annual Meeting of the AMS in Denver, Colorado, in January 2020. Three prizes were presented to **Camillo De Lellis**, **Lawrence Guth**, and **Laure Saint-Raymond**.



Camillo De Lellis

Citation for Camillo De Lellis

The 2020 Bôcher Memorial Prize is awarded to Camillo De Lellis for his innovative point of view on the construction of continuous dissipative solutions of the Euler equations, which ultimately led to Isett's full solution of the Onsager conjecture, and his spectacular work in the regularity theory of minimal surfaces, where he completed and improved

Almgren's program. Examples of two papers that represent his remarkable achievements are "Dissipative Continuous Euler Flows," *Inventiones Mathematicae* 193 (2013), in collaboration with L. Székelyhidi, and "Regularity of Area-Minimizing Currents III: Blow-Up", *Annals of Mathematics* 183 (2016), in collaboration with E. Spadaro.

Biographical Sketch: Camillo De Lellis

Camillo De Lellis was born in 1976 in San Benedetto del Tronto, Italy. After earning his undergraduate degree in mathematics at the University of Pisa in 1999, he wrote his doctoral dissertation in 2002 under the supervision of Luigi Ambrosio at the Scuola Normale Superiore di Pisa. He held a postdoctoral position at the Max Planck Institute for Mathematics in the Sciences in 2002–03, followed by a postdoctoral residency at the Eidgenössische Technische Hochschule in Zürich. De Lellis joined the faculty of the University of Zürich in 2004 as assistant professor of mathematics, and he was appointed full professor in 2005. In 2018 he moved to the Institute for Advanced Study in Princeton, where he holds the IBM von Neumann Professorship. He is active in the fields of calculus of variations, geometric

measure theory, hyperbolic systems of conservation laws, and fluid dynamics.

Response from Camillo De Lellis

I am grateful and deeply honored to be awarded the 2020 Bôcher Prize with Larry Guth and Laure Saint-Raymond, two colleagues whom I know personally and whose work I have admired for a long time. I am humbled by the list of previous recipients, but one name, Leon Simon, is particularly dear to me: during one sunny Italian summer of (slightly more than) twenty years ago his wonderful lecture notes infected me with many of the themes of my future research in geometric measure theory.

The results mentioned in the citation stem from long endeavors with two dear friends and collaborators, László Székelyhidi Jr. and Emanuele Spadaro: without their brilliance, their enthusiasm, and their constant will of progressing further, we would have not gotten there. But mostly I wish to thank them because working together has been really a lot of fun. Our research has been influenced by the work of several other mathematicians and takes advantage of important past and present developments in many topics in partial differential equations and differential geometry, but this is already explained in full detail in a few surveys and lecture notes.

Let me instead mention here that the seeds of both works were planted during a very important time of my life, while my beloved wife Lorenza was carrying the first of the three most beautiful prizes of my life. I take this opportunity to thank her for her constant support and immense patience.

Citation for Lawrence Guth

The 2020 Bôcher Memorial Prize is awarded to Lawrence Guth for his deep and influential development of algebraic and topological methods for partitioning the Euclidean space and multiscale organization of data and his powerful applications of these tools in harmonic analysis,



Lawrence Guth

incidence geometry, analytic number theory, and partial differential equations. These applications include the proof of the endpoint multilinear Kakeya conjecture, nearly sharp bounds for the distinct distances problem, best-known bounds on the Fourier restriction problem, further development of the Bourgain–Demeter decoupling theory which led to the resolution of the Vinogradov main conjecture, sharp estimates on oscillatory integrals, and a sharp almost-everywhere convergence theorem for solutions to the free Schrödinger equation. Examples of two papers that represent his remarkable achievements are “A Restriction Estimate Using Polynomial Partitioning,” *Journal of the American Mathematical Society* 29 (2016), no. 2, and “A Sharp Schrödinger Maximal Estimate in \mathbb{R}^2 ,” *Annals of Mathematics* (2) 186 (2017), no. 2, in collaboration with X. Du and X. Li.

year; I showed up with two pages of notes and started talking, and every ten minutes or so someone would ask a question and I would realize I hadn’t really understood what I was saying. The second time I spoke, I showed up with ten pages of notes.

As a graduate student I studied geometry, especially Misha Gromov’s work on metric geometry. A common theme in the work we studied in Tom’s seminar was using topological arguments to show that some geometric object had to exist and then using that object to prove new estimates in geometry and topology. For instance, we studied pseudoholomorphic curves in symplectic geometry and Yang–Mills connections in gauge theory. Another example, which was not as dramatic but more relevant to the work in this citation, was the way Gromov used the ham sandwich theorem to estimate eigenvalues of the Laplacian on a Riemannian manifold.

I started learning about restriction theory when I was a postdoc by reading Terence Tao’s notes from a graduate course he had taught. At that time, I had only worked in differential geometry, and I didn’t think I would be able to work in restriction theory. But I started reading the notes because I was curious about the Kakeya problem and about Fourier analysis, and I kept going because they are very readable and engaging. Now I give them to my graduate students to read, if they’re interested in Fourier analysis. One thing that impressed me about the field was how people looked to quite different parts of math to find new approaches to the problems, like when Bourgain brought in ideas from combinatorial number theory or when Wolff brought in the idea of partitioning from combinatorial geometry.

Just after I read those notes, Zeev Dvir’s paper solving the finite field Kakeya problem came out. This paper was a big surprise to people in the field. Until then it had looked plausible that the finite field Kakeya problem was as hard as the central problems of restriction theory, but Zeev’s proof was two pages long and accessible to a good undergraduate. The new idea of the proof had to do with high-degree polynomials, which was surprising because the problem was just about lines. The method used finite fields in a crucial way, however, and it wasn’t clear whether it could be adapted to say something about the original Kakeya problem or related problems in Fourier analysis, which take place over the real numbers. Zeev’s argument reminded me of geometry arguments that I had been studying and working on, and that led to my first paper in the area, using the ham sandwich theorem to prove an estimate related to the Kakeya problem.

The next year, I started my first professor job at the University of Toronto. Jim Colliander and Robert McCann invited me to help run a working group on analysis and geometry, and we made the Kakeya problem the theme

Biographical Sketch: Lawrence Guth

Lawrence Guth is a professor of mathematics at the Massachusetts Institute of Technology. He grew up in Brookline, Massachusetts, and went to Brookline High School. He was an undergraduate at Yale University, and he did his PhD at MIT in 2005 under the supervision of Tomasz Mrowka. Following a postdoctoral position at Stanford and a junior faculty appointment at the University of Toronto, he was appointed professor at the Courant Institute in 2011 and joined MIT in 2012.

Response from Lawrence Guth

I’m very honored to receive this award. Sitting down to write this response, and thinking back over the time I spent working on these projects, I realized just how many mentors and collaborators played an important role in them. I’ve been very lucky to have such mentors and collaborators.

Growing up, I learned math from my dad, and also from a lot of dedicated teachers at school, like Michael Narasny and Terry Leverich. In college, I learned from Serge Lang and Peter Jones. Peter first got me interested in analysis. It was from him that I first heard about the Kakeya problem about rotating a needle within a small area and that it had something to do with Fourier analysis. At the time I didn’t understand what the connection was, but it sounded intriguing.

In graduate school, I studied with Tom Mrowka. I picked Tom as an advisor because I liked how he worked with his students. I grew up mathematically in Tom’s weekly seminar. I remember the first time I spoke, early in my second

of the semester. At the end of the semester, we invited an expert, Nets Katz, to come and give some talks.

Over the next week, Nets and I started to collaborate. Nets taught me about combinatorics problems that were connected to the Kakeya problem in different ways. We explored using polynomials to study them, and eventually we worked out the idea of polynomial partitioning together. Ever since then, I've been trying to apply polynomial partitioning in different situations and seeing how much it can tell us about Fourier analysis.

A little bit later, I met Jean Bourgain, and I had a chance to work with him on restriction theory. I had spent a long time studying Bourgain's work, and so getting to actually work with him felt special, and a bit overwhelming. Our collaboration started when I showed him an idea about a problem he had worked on in the 1990s. My approach was based on polynomials, building on my first paper in Fourier analysis. But when Jean started to develop the approach, he saw that the polynomials weren't really needed.

Once we started to work together, he would send me notes of things he had worked out, and these notes piled up into a very large pile, which became a joint paper. It took me more than a year to read all those notes. Reading them was a real education in the field.

Since then, I've gotten to work on this circle of questions with more collaborators, including Josh Zahl, Ciprian Demeter, Jonathan Hickman, Marina Iliopoulou, Xiumin Du, Xiaochun Li, Hong Wang, Yumeng Ou, Bobby Wilson, Alex Iosevich, Noam Solomon, and Ruixiang Zhang. I want to thank all of them for their energy and ideas.

Finally I want to thank my family for all of their support and their love—my parents Susan and Alan, my sister Jenny, my wife Amy, and my children Elan and Bennett.



Laure Saint-Raymond

Citation for Laure Saint-Raymond

The 2020 Bôcher Memorial Prize is awarded to Laure Saint-Raymond for her transformative contributions to kinetic theory, fluid dynamics, and Hilbert's sixth problem on "developing mathematically the limiting processes...which lead from the atomistic view to the laws of motion of continua." With François Golse, she obtained solutions of incompressible Navier–Stokes and

Euler equations as limits of weak solutions of Boltzmann's equations. She also studied the Boltzmann–Grad limit in the hard sphere model, going beyond the kinetic time to derive Brownian motion by tracing a tagged particle in the limit of a Newtonian microscopic model. Examples of two

papers that represent her remarkable achievements are "The Brownian Motion as the Limit of a Deterministic System of Hard-Spheres," *Inventiones Mathematicae* 203 (2016), no. 2, in collaboration with T. Bodineau and I. Gallagher, and "Mathematical Study of Degenerate Boundary Layers: A Large Scale Ocean Circulation Problem," *Memoirs of the American Mathematical Society* 253 (2018), no. 1206, in collaboration with A.-L. Dalibard.

Biographical Sketch: Laure Saint-Raymond

Laure Saint-Raymond was torn between her taste for abstract theories and her wish to understand the world surrounding us a little bit more. She studied mathematics and physics, and she finally opted for applied mathematics and got her PhD under the supervision of François Golse in 2000.

An important part of her work is related to the sixth problem raised by Hilbert in 1900 on the occasion of the International Congress of Mathematicians, which addresses the question of the axiomatization of mechanics, and more precisely of describing the transition between atomistic and continuous models for gas dynamics by rigorous mathematical convergence results. Saint-Raymond has obtained major results concerning the asymptotic theory of the Boltzmann equation in kinetic theory of gases. She has also studied problems of scale separation in the context of geophysical flows, especially for the wind-driven oceanic dynamics.

Hired as a junior researcher at CNRS in 2000, she was promoted to professor at University Paris 6–Pierre et Marie Curie in 2002. She is currently professor at the École Normale Supérieure de Lyon and Fellow of the Institut Universitaire de France. She has been awarded many prizes, among which are the Prize of the European Mathematical Society in 2008, the Ruth Lyttle Satter Prize of the American Mathematical Society in 2009, and the Fermat Prize in 2015. She is a member of the French Academy of Sciences, the Academia Europae, and the European Academy of Sciences.

Response from Laure Saint-Raymond

I would like to begin these lines by congratulating my colleagues L. Guth and C. De Lellis, whose work is very inspiring to me, and with whom I am happy to share the Bôcher Memorial Prize.

I am actually very impressed and honored to receive this distinction, but I would like to associate my collaborators to this recognition: I always have a lot of pleasure working with them, and without them I would never have had the courage to embark on a program of this size.

Hilbert's sixth problem concerning the axiomatization of physics remains to this day a major challenge, especially since its statement is not very precise and its scope has expanded with the introduction of quantum and relativistic theories. Nevertheless, we have been able to open an

important gap in the setting of classical gas dynamics, showing the compatibility of atomic, statistical, and hydrodynamic descriptions (at least in some regimes).

A first significant advance, obtained in collaboration with F. Golse, was to understand the similarity of structure between the Boltzmann equation for rarefied gases and the Navier–Stokes equations of incompressible fluids, and in particular the hypoelliptic mechanism that explains viscous dissipation as a combination of microscopic transport and collision phenomena.

A second key advance concerns the derivation of the Boltzmann equation from the dynamics of a hard sphere billiard and has led to a series of joint works with T. Bodineau, I. Gallagher, and S. Simonella. The central question here is to understand the processes that are responsible for the decorrelation of particles and that make it possible to obtain a statistical description of the out-of-equilibrium system. Our most recent results, while they do not improve the (very short) time on which we can rigorously show that the Boltzmann equation predicts the most probable dynamics, give a much finer understanding of the limiting process by characterizing the (small or large) deviations with respect to this average dynamic.

In all these problems, entropy plays a major role, and one of the projects that is important to me now is to try to understand better its structure and physical meaning.

About the Prize

Established in 1923, the prize honors the memory of Maxime Bôcher (1867–1918), who was the Society's second Colloquium Lecturer in 1896 and who served as AMS president during 1909–10. Bôcher was also one of the founding editors of *Transactions of the AMS*. The original endowment was contributed by members of the Society. The prize is awarded for a notable paper in analysis published during the preceding six years. The work must be published in a recognized, peer-reviewed venue. The prize is given every three years and carries a cash award of US\$5,000.

The Bôcher Prize is awarded by the AMS Council acting on the recommendation of a selection committee. The members of the selection committee for the 2020 Bôcher Memorial Prize were:

- Assaf Naor
- Wilhelm Schlag
- Gigliola Staffilani (Chair)

A list of the past recipients of the Bôcher Prize can be found at https://www.ams.org/prizes-awards/pabrowse.cgi?parent_id=10.

Credits

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