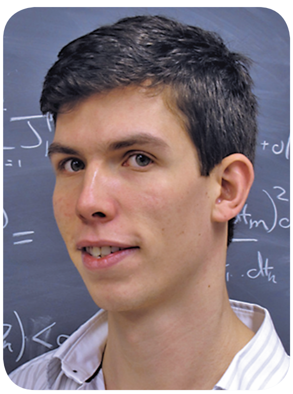
2020 Frank Nelson Cole Prize in Number Theory

James Maynard was awarded the 2020 Frank Nelson Cole Prize in Number Theory at the 126th Annual Meeting of the AMS in Denver, Colorado, in January 2020.


James Maynard

Citation

The Cole Prize in Number Theory is awarded to James Maynard for his many contributions to prime number theory. In particular, the prize recognizes the papers (1) "Small Gaps between Primes" (*Annals of Mathematics*, 2015), (2) "Large Gaps between Primes" (*Annals of Mathematics*, 2016), and (3) "Primes with Restricted Digits" (*Inventiones Mathematicae*, 2019).

Perhaps the central problem in prime number theory is the Hardy–Littlewood prime k-tuple conjecture which predicts (for a given k-tuple of distinct integers $h_1, \ldots, h_k$) a precise asymptotic formula for the number of integers $n \le N$ such that $n + h_1, \ldots, n + h_k$ are all prime. A special case is the famous twin prime problem, where it is still unproved whether there are infinitely many pairs of primes that differ by 2. Yitang Zhang, building on a breakthrough of Daniel Goldston, János Pintz, and Cem Yıldırım, famously established that there exists a natural number h (below 70 million) such that there are infinitely many prime pairs $n, n + h$. In the paper "Small Gaps between Primes," Maynard introduced new multidimensional sieve weights (discovered independently by Terence Tao) and used these weights to establish that for each k, there are k-tuples $h_1, \ldots, h_k$ such that $n + h_1, \ldots, n + h_k$ are all simultaneously prime for infinitely many n. In other words, Maynard established that infinitely often one can find k primes in intervals of bounded length $C(k)$. This result had seemed inaccessible to the methods of Goldston, Pintz, Yıldırım, and Zhang (which treated the case $k = 2$). Further, the new sieve weights are extremely flexible and have led to progress on many related questions.

Instead of making the gaps between prime numbers small, one can ask how large the gaps between consecutive primes can get. If p_n denotes the nth prime, then the prime number theorem shows that the gap $p_{n+1} - p_n$ is on average about $\log n$. Erik Westzynthius, in 1931, was the first to show that these gaps can be larger than an arbitrary constant times the average gap. Since then a number of mathematicians have worked on quantifying these maximal gaps, resulting in the Erdős–Rankin bound that $p_{n+1} - p_n$ can be larger than $C \log n \log_2 n \log_4 n/(\log_3 n)^2$ infinitely often. Here C is a positive constant, and $\log_j$ denotes the j-fold iterated logarithm. One of Erdős's favorite problems asked for an improvement over this result, replacing C by any function tending to infinity. After more than seventy years, this problem was resolved simultaneously by different methods: in Maynard's "Large Gaps between Primes," by an adaptation of his multidimensional sieve, and in Ford, Green, Konyagin, and Tao (*Annals of Mathematics* (2016)) by using the Green–Tao theorem on arithmetic progressions in the primes. Maynard's method allows for better quantitative control, and the state of the art is attained in joint work by both teams in *Journal of the American Mathematical Society* (2018).

Given a (naturally occurring) sparse subset of the natural numbers, one would like to count the primes in it. For example, do polynomial sequences (such as $n^2 + 1$) contain infinitely many primes? No example of such a univariate polynomial with degree larger than 1 is known, but breakthrough results of John Friedlander and Henryk Iwaniec and of David Heath-Brown establish such a result for polynomials in two variables, such as x^2+y^4 (Friedlander and Iwaniec) and $x^3 +2y^3$ (Heath-Brown). The sets of values in these examples are sparse in the sense that they contain only N^{1-c} natural numbers up to N for some $c > 0$. Maynard's "Primes with Restricted Digits" establishes that the Cantor-like set of natural numbers whose base b representations have no digit equal to a given $a \in \{0, \ldots, b-1\}$

contains infinitely many primes provided that the base b is at least 10. In particular, there are infinitely many primes with no 7 in their usual decimal representation. The number of such integers up to N is about $N^{\log 9/\log 10}$, so that this gives a further striking example of a sparse set that contains infinitely many primes.

Biographical Sketch

James Maynard studied at Cambridge for his undergraduate and master's degrees before doing a PhD at Oxford under Roger Heath-Brown. He was a CRM-ISM postdoctoral fellow for a year at the Université de Montréal and then returned to Oxford, first as a Junior Research Fellow of Magdalen College and then as a Fellow of the Clay Mathematics Institute. He also spent extended time as a research member at the Mathematical Sciences Research Institute, Berkeley; as a visitor at Aix-Marseille University, Marseilles; and as a member at the Institute for Advanced Study, Princeton, when a postdoc. Since 2018 he has been a research professor at Oxford.

His interests are in analytic number theory, particularly classical problems about prime numbers. He has previously been awarded the 2014 SASTRA Ramanujan Prize, the 2015 Whitehead Prize of the London Mathematical Society, the 2016 European Mathematical Society Prize, and the 2018 Compositio Prize. He was also a speaker at the 2018 International Congress of Mathematicians.

Response from James Maynard

It is a great honor to be awarded the 2020 Frank Nelson Cole Prize in Number Theory. My work builds on the development of a large number of ideas within analytic number theory, including the work of Brun, Selberg, Erdős, (A. I.) Vinogradov, Bombieri, (I. M.) Vinogradov, and, more recently, Pintz, Goldston, Yıldırım, and Zhang, as well as many others for whom there is no space to mention. Without these luminaries, my contributions would simply not have been possible, and so the final theorems owe rather more to them than they do to me.

I've been exceptionally fortunate to have been supported and encouraged to pursue my interest in mathematics throughout my career from a young age. My parents always encouraged me to follow my interests, and my schoolteachers allowed me to develop my independent tastes. More recently, many people have supported my career and mathematics in various ways, particularly my supervisor Roger Heath-Brown and my postdoc mentor Andrew Granville. Without this support, it is likely I would now be doing something rather less interesting than mathematical research!

I've always been attracted to the simplicity of the statements of many important open problems about primes—it is immensely satisfying to be personally involved in some of the partial progress on these questions. More generally, the field of analytic number theory feels revitalized and exciting at the moment, with new ideas coming from many different people, and hopefully this prize might inspire younger mathematicians to continue this momentum and make new discoveries about the primes.

About the Prize

The Cole Prize in Number Theory (and the Frank Nelson Cole Prize in Algebra) was founded in honor of Professor Frank Nelson Cole upon his retirement from the American Mathematical Society; he served as AMS Secretary for twenty-five years and as Editor-in-Chief of the *Bulletin* for twenty-one years. The original fund was donated by Professor Cole from moneys presented to him on his retirement, and was augmented by contributions from members of the Society. The fund was later doubled by his son, Charles A. Cole, and supported by family members. It has been further supplemented by George Lusztig and by an anonymous donor.

This prize recognizes a notable research work in number theory that has appeared in the last six years. The work must be published in a recognized, peer-reviewed venue. The current prize amount is US$5,000 and the prize is awarded every three years.

The Cole Prize is awarded by the AMS Council acting on the recommendation of a selection committee. The members of the selection committee for the 2020 prize were:

- Kannan Soundararajan (Chair)
- Andrew Wiles
- Melanie Matchett Wood

A list of the past recipients of the Cole Prize in Number Theory can be found at `https://www.ams.org/prizes-awards/pabrowse.cgi?parent_id=15`.

Credits

Photo of James Maynard is courtesy of James Maynard.