2020 Levi L. Conant Prize

Amie Wilkinson was awarded the 2020 Levi L. Conant Prize at the 126th Annual Meeting of the AMS in Denver, Colorado, in January 2020.


Amie Wilkinson

Citation

The 2020 Levi L. Conant Prize is awarded to Amie Wilkinson for the article "What Are Lyapunov Exponents, and Why Are They Interesting?" published in the *Bulletin of the American Mathematical Society* in 2017.

The article provides a broad overview of the modern theory of Lyapunov exponents and their applications to diverse areas of dynamical systems and mathematical physics. It opens with an illuminating geometric example. The barycentric division of a triangle consists of the six triangles formed by joining each vertex to the midpoint of the opposite side. This process can be iterated, taking the barycentric subdivision of each of the six triangles, and continuing. How quickly do the triangles become skinnier as we iterate? A Lyapunov exponent gives the answer. This leads up to the introduction of Lyapunov exponents as the "magical numbers" that describe the expansion and contraction rates associated with a dynamical system. This concept is then placed in the mathematical framework of cocycles and hyperbolicity.

Starting with the foundational results of Oseledets and Anosov, Wilkinson then moves on to a wide range of modern developments, ranging from smooth dynamics and Pesin theory to translation surfaces and the spectral theory of ergodic Schrödinger operators. While these topics are quite diverse, the article emphasizes the connections between them that are made through the use of Lyapunov exponents and hyperbolicity. An additional common thread tying these subjects together is that they all feature the work of Artur Avila, who was awarded a Fields Medal in 2014. Wilkinson concludes with a short section on "metadynamics" and a summary of some of the major unifying themes running through this research area.

Wilkinson's exposition is original, elegant, passionate, and deep. Throughout the article, she maintains a very high standard of mathematical rigor. At the same time, she provides a great deal of geometric intuition through the use of well-chosen examples and striking visuals. Definitions of abstract concepts are followed by examples and special cases that are natural and relatively simple, but do not trivialize the subject and offer interesting phenomena for analysis. For instance, the discussion of random matrix cocycles, derivative cocycles, and the hyperbolic cocycle behind the barycentric subdivision clarifies the definition of a cocycle and provides valuable insights into how experts think about this concept and how they work with it in practice. The part related to mathematical physics is illustrated by the "Hofstadter butterfly," a fractal (discovered by Douglas Hofstadter of *Gödel, Escher, Bach* fame) whose each horizontal slice is the spectrum of a discrete magnetic Schrödinger operator. The same fractal also has a dynamical interpretation, and deep theorems relate its spectral properties to the dynamical ones. Wilkinson surveys these results. The explanations are clear and accessible to a wide audience. This is an impressive feat, given that this area of research has a reputation for being very technical and difficult to explain to nonexperts. The article could be skimmed for a quick introduction to a fascinating part of mathematics, but it also lends itself to careful and repeated study, rewarding the more invested reader with a deeper understanding of the subject. We expect that it will be a valuable resource for many years to come.

Biographical Sketch

Amie Wilkinson is a professor of mathematics at the University of Chicago working in ergodic theory and smooth dynamics. She received her undergraduate degree at Harvard and her PhD at Berkeley in 1995 with Charles Pugh.

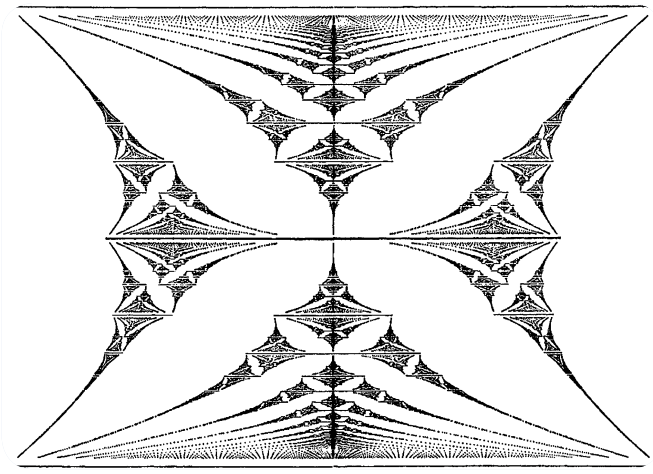

Figure 1. Hofstadter's butterfly.

She held postdoctoral positions at Harvard and Northwestern and rose to the level of full professor at Northwestern before moving to Chicago in 2011.

Wilkinson was the recipient of the 2011 AMS Satter Prize, and she gave an invited talk in the Dynamical Systems section at the 2010 ICM. In 2013 she became a Fellow of the AMS for "contributions to dynamical systems," and in 2019 she was elected to the Academia Europaea.

Wilkinson's research is concerned with the interplay between dynamics and other structures in pure mathematics—geometric, statistical, topological, and algebraic.

Response from Amie Wilkinson

I would like to thank the AMS for this great honor. The exponential growth rates measured by Lyapunov exponents are a powerful and yet elusive predictor of chaotic dynamics, and they aid in the fundamental task of organizing the long-term behavior of orbits of a system. Through the mechanism of renormalization, exponents of metadynamical systems direct the seemingly unrelated behavior of highly structured systems like rational billiard tables and barycentric subdivision. Lyapunov exponents can deliver delightful surprises as well, leading to crazy geometric structures such as the pathological foliations Mike Shub and I have studied. To summarize, I love the subject of this article and am delighted that I might have conveyed this affection to the reader.

I have many colleagues to thank for their input and support; three of them in particular I'd like to mention by name. Artur Avila, whose work was the guiding inspiration for this article, has in many ways shaped how I view Lyapunov exponents and has opened my eyes to their power and versatility. Curtis McMullen explained to me his beautiful analysis of the barycenter problem, which served as the perfect introduction to the subject. Svetlana Jitomirskaya was instrumental in helping me get the facts straight on ergodic Schrödinger operators. Finally, I would like to thank the AMS for first inviting me to talk on the work of Artur Avila at the AMS Current Events Bulletin in 2016, a lecture on which this article was based.

About the Prize

The Levi L. Conant Prize is awarded annually to recognize an outstanding expository paper published in either the *Notices of the AMS* or the *Bulletin of the AMS* in the preceding five years.

Established in 2001, the prize honors the memory of Levi L. Conant (1857–1916), who was a mathematician at Worcester Polytechnic Institute. The prize carries a cash award of US$1,000.

The Conant Prize is awarded by the AMS Council acting on the recommendation of a selection committee. The selection committee members for the 2020 Conant Prize were:

- Frank Calegari
- Thomas C. Hales
- Izabella J. Laba (Chair)

A list of the past recipients of the Levi L. Conant Prize can be found at `https://www.ams.org/prizes-awards/pabrowse.cgi?parent_id=29`.

Credits

Photo of Amie Wilkinson is by Jessica Wynne, used with permission.

Figure 1 reprinted with permission as follows: Douglas R. Hofstadter, "Energy Levels and Wave Functions of Block Electrons in Rational and Irrational Magnetic Fields," *Physical Review B* **14** (1976), 2239–2249. Copyright 1976, American Physical Society.