2020
Joseph L. Doob Prize

René Carmona and François Delarue were awarded the 2020 Joseph L. Doob Prize at the 126th Annual Meeting of the AMS in Denver, Colorado, in January 2020.

Citation

Stochastic game theory is a vastly complicated subject that attempts to model how rational individuals interact within a random environment, while each pursues her/his own, usually conflicting, goals. Understanding even simple-looking model problems requires designing strategies and counterstrategies of often great complexity. But “mean field games,” introduced a decade ago by J. M. Lasry and P. L. Lions and by M. Huang, R. P. Malhamé, and P. E. Caines, offer a resolution, looking for simpler strategic structures that sometimes appear in the limit as the number of identically motivated players goes to infinity. The analogy with statistical physics should be clear. The motivating insight is that each individual need not respond much to the precise decision of any other player, but rather to the empirical distribution of the cumulative effects of all the others’ actions.

These two magisterial volumes by René Carmona and François Delarue provide an accessible, fully detailed introduction to the exciting prospects for mean field games and to their considerable mathematical subtleties.

Volume I: Mean Field FBSDEs, Control, and Games introduces mean field games for which there are no random effects common to all players, so-called “games without common noise.” The reader will find here well-written chapters on finite-person stochastic games, optimality and forward-backward stochastic differential equations (FBSDEs), deriving mean field game limit PDEs, calculus on Wasserstein spaces, and much more.

More complicated stochastic effects lead to much more complex “games with common noise,” analyzed in Volume II: Mean Field Games with Common Noise and Master Equations. Most of the exposition here is devoted to building solutions of the master equation, a highly nonlinear second-order PDE defined on a space of probability measures.

With these two volumes, Carmona and Delarue contribute immensely to our understanding of mean field games (and they in turn generously acknowledge their debt to many other colleagues).

Biographical Sketch: René Carmona
René Carmona, PhD, is the Paul M. Wythes ’55 Professor of Engineering and Finance at Princeton University in the Department of Operations Research and Financial Engineering. He is an associate member of the Department of Mathematics, a member of the Program in Applied and Computational Mathematics, and director of Graduate Studies of the Bendheim Center for Finance, where he oversees the Master in Finance program. He obtained a PhD in probability from Marseille University, where he held his first academic job. After time spent at Cornell and a couple of stints at Princeton, he moved to the University of California at Irvine in 1981 and eventually to Princeton University in 1995.
FROM THE AMS SECRETARY

Dr. Carmona has been a Fellow of the Institute of Mathematical Statistics since 1984 and of the Society for Industrial and Applied Mathematics since 2009. He is the founding chair of the SIAM Activity Group on Financial Mathematics and Engineering and a founding editor of the journal Electronic Communications in Probability and the SIAM Journal on Financial Mathematics. He is on the editorial board of several peer-reviewed journals and book series.

His publications include over one hundred articles and eleven books in probability, statistics, mathematical physics, signal analysis, and financial mathematics. Over the last decade he has tried to understand from a probabilistic point of view a class of mathematical models introduced independently and simultaneously by a couple of French applied mathematicians (Jean Michel Lasry and Pierre Louis Lions) and a group of electrical engineers in Canada (Peter Caines, Minyi Huang, and Roland Malhamé), known under the name of Mean Field Games. Together with François Delarue, he developed a probabilistic approach to these models. This collaboration culminated in a two-volume book providing the state of the art on the subject.

More information can be found at www.princeton.edu/~rcarmona.

Biographical Sketch: François Delarue

François Delarue was born in 1976 in Normandy, France. He graduated from the École Normale Supérieure de Lyon (France), where he studied from 1996 to 1999. He received a PhD from the University of Marseille (France) in 2002, working on stochastic differential equations under the supervision of Etienne Pardoux. The same year, he was hired by University Paris 7–Diderot as an assistant professor (“maître de conférence” in the French system) at the research laboratory Laboratoire de Probabilités et Modèles Aléatoires. He stayed in Paris from 2002 to 2009 working on the team of Francis Comets, who was his advisor for the “habilitation à diriger des recherches” (French degree for supervising PhD theses) that he received in 2008. Since 2009, he has been a full professor at the research laboratory of mathematics Laboratoire Dieudonné of the University of Nice Sophia Antipolis. In 2014, he was appointed a junior fellow of the Institut Universitaire de France for five years. Since 2019, he has been supported by a chair allocation from the Institut 3IA (Institute for Artificial Intelligence in Nice).

François Delarue’s research is in stochastic analysis, including mean field particle systems and applications to partial differential equations. He met his coauthor René Carmona for the first time in 2009, and he invited him to spend one month in Nice in 2010 in order to initiate a collaboration. Since René’s visit, they have been working together on the probabilistic approach to mean field games. François Delarue’s research on the subject is supported by the French National Research Agency (ANR). Since 2011, François Delarue has been managing, at the Department of Mathematics of the University of Nice, the European MSc program in applied mathematics “Mathmods,” which runs in collaboration with other universities in Europe. Since 2018, he has also been managing the CNRS International Research Network of mathematics between southern Europe and northern Africa. Since 2019, he has been co-editor in chief, with Peter Friz, of the journal Annals of Applied Probability.

Response from René Carmona and François Delarue

We are deeply honored and grateful to receive the 2020 Joseph L. Doob Prize, and especially humbled by Doob’s towering figure in the history of probability and its foundations.

We began working together nearly a decade ago, driven by the common desire to develop stochastic analysis tools that could be used to solve applied problems in mathematical physics. A couple of published journal articles and two years of travel between Nice and Princeton later, we became excited by the potential of an otherwise nascent mathematical model for large anonymous games with mean field interactions. It quickly became clear to us that there was something really special and potentially far-reaching in these models that were introduced independently and simultaneously by a couple of French applied mathematicians (Jean Michel Lasry and Pierre Louis Lions) and a group of electrical engineers in Canada (Peter Caines, Minyi Huang, and Roland Malhamé). We are thankful for their groundbreaking innovation.

For us, the next step was to develop an approach that would facilitate the emergence of a subfield that was not limited to the analytic methods used in the subject’s earliest works. We worked to further academic understanding of the subject and to generate excitement among students, colleagues, and younger researchers by advising students, teaching short courses, organizing conferences, and generally proselytizing on the subject. In doing so, we came to the conclusion that there was a dire need for a comprehensive presentation of the subject if research was to thrive and ultimately become the basis of a subfield. We started compiling the many motivating examples of the theory’s applications, including those from finance, economics, population biology, logistics, and operations research. We streamlined results from all of the works that have appeared on the subject over the last decade, and presented them in a unified framework. As is often the case, in these early stages we did not have a clue just how extensive this project would be, what the final product would look like, and of the toll it would take on our lives. We also did not expect this project to prompt such profound honor and satisfaction, epitomized by this award, which has been granted to so
many prestigious and highly influential mathematicians before us.

Of particular note to us has been the wide range of scientific subfields beyond our immediate community that have already cited this book, and the extent to which new generations of researchers are both relying on our probabilistic approach and extending the scope of its applications. Witnessing this, and the many opportunities, conversations, and collaborations that this book has made possible, has been deeply heartening. We want to thank the committee for recognizing all of this, and above all else, for seeing this book’s potential to "provide an accessible, fully detailed introduction to the exciting prospects for mean field games and to their considerable mathematical subtleties" and to “contribute immensely to our understanding of mean field games.”

**About the Prize**

The Doob Prize was established by the AMS in 2003 and endowed in 2005 by Paul and Virginia Halmos in honor of Joseph L. Doob (1910–2004). Paul Halmos (1916–2006) was Doob’s first PhD student. Doob received his PhD from Harvard in 1932 and three years later joined the faculty at the University of Illinois, where he remained until his retirement in 1978. He worked in probability theory and measure theory, served as AMS president in 1963–64, and received the AMS Steele Prize in 1984 “for his fundamental work in establishing probability as a branch of mathematics and for his continuing profound influence on its development.” The Doob Prize recognizes a single, relatively recent, outstanding research book that makes a seminal contribution to the research literature, reflects the highest standards of research exposition, and promises to have a deep and long-term impact in its area. The book must have been published within the six calendar years preceding the year in which it is nominated. Books may be nominated by members of the Society, by members of the selection committee, by members of AMS editorial committees, or by publishers. The prize of US$5,000 is given every three years.

The Doob Prize is awarded by the AMS Council acting on the recommendation of a selection committee. The members of the selection committee for the 2020 prize were:

- Lawrence Craig Evans
- Mark Goresky
- Fan Chung Graham (Chair)
- H. W. Lenstra
- Terence Tao

A list of the past recipients of the Doob Prize can be found at [https://www.ams.org/prizes-awards/pabrowse.cgi?parent_id=21](https://www.ams.org/prizes-awards/pabrowse.cgi?parent_id=21).

**Credits**

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