2020 Leonard Eisenbud Prize for Mathematics and Physics

Kevin Joseph Costello was awarded the 2020 Leonard Eisenbud Prize for Mathematics and Physics at the 126th Annual Meeting of the AMS in Denver, Colorado, in January 2020.

Kevin Joseph Costello

Citation

The Leonard Eisenbud Prize for Mathematics and Physics is awarded to Kevin Costello for his contributions to the mathematical foundations of quantum field theory and his gauge-theoretic explanation of solutions to the quantum Yang–Baxter equations. These have appeared in the works *Factorization Algebras in Quantum Field Theory I*, published by Cambridge University Press in 2017, *Renormalization and Effective Field Theory*, published by the American Mathematical Society in 2011, and the influential article posted on the arXiv: 1303.2632, "Supersymmetric Gauge Theory and the Yangian."

Quantum field theory is the physical framework underlying all modern understanding of elementary particles, fields, and their interactions. From its beginnings with quantum electrodynamics in the 1920s to the development of nonabelian gauge theories, mathematicians have struggled to formulate in precise terms the rigorous underpinnings of the subject, as well as to mathematically understand and justify the calculations done by physicists.

Included in this effort was the program of constructive quantum field theory, whose practitioners tried to give analytic constructions of field theories on Minkowski or Euclidean space satisfying the Wightman axioms starting from action principles that physicists used to describe the world. In spite of many beautiful results in infinite-dimensional analysis that came out of this effort, no one has yet succeeded in giving such a construction for realistic field theories in four dimensions.

The interface between mathematics and quantum field theory underwent a dramatic change in the 1980s under the influence of the rich structures underlying string theory, conformal field theory, and topological field theory, resulting in a new geometric synthesis via an axiomatization of the latter two by Graeme Segal and Michael Atiyah. This broad perspective ushered in an astounding panoply of applications, including the definition of Reshetikhin–Turaev–Witten invariants of three-manifolds, Gromov–Witten theory, Seiberg–Witten theory, various geometric ramifications of mirror symmetry, as well as a rapid influx of new geometric and algebraic tools and concepts into physics.

Amid the efforts of many mathematicians to explore the foundations of this new landscape, the recent work of Kevin Costello stands out for its depth, comprehensive view, and its relevance to physicists. In his book *Renormalization and Effective Field Theory*, Costello developed a complete mathematical foundation for perturbative renormalization as practiced by physicists, bringing it up to an unprecedented level of rigor and conceptual clarity. In particular, he gave a full mathematical exposition of the renormalization group flow on quantum field theories and a mathematical proof of the renormalizability of Yang–Mills theory, the physics counterpart of which earned Gerardus t'Hooft a Nobel Prize in 1999. The key mathematical ingredient in Costello's theory is *derived geometry*, the newest manifestation of the rich categorical generalizations of geometry that have grown out of the Grothendieck revolution of the 1960s. The application to physics lies in the insight that the zero locus of an action functional can exactly be enhanced to such a derived geometry in a way that naturally incorporates gauge symmetries via the Batalin–Vilkovisky formalism. The upshot is that the observables of a quantum field theory come together to form a remarkable mathematical structure called a *factorization algebra* on spacetime, an idea that Costello has been fleshing out systematically in collaboration with his student Owen Gwilliam in the books *Factorization Algebras in Quantum Field Theory I, II*. (The second volume is in the final stages of submission to Cambridge University Press.) These books set up a robust global analytic framework necessary to perform deformation quantization of a field theory expressed as an *elliptic moduli* problem and yield, in particular, rigorous perturbative quantizations of topological field theories such as Chern–Simons theory, the B-model, and Rozansky–Witten theory, as well as nontopological field theories such as Yang–Mills theory. This framework for understanding quantum field theory is developing rapidly and has already established itself as a new paradigm for mathematical approaches to quantum field theories that encompasses a substantial portion of the classical analytic tradition while interacting with modern geometry and topology in deep and novel ways.

One especially striking application of the theory has appeared in Costello's work on Yangians. The Yang–Baxter equations are consistency relations satisfied by matrices encoding the vertex interactions of lattice models and are understood to be the key to integrability. In the 1980s, Drinfeld interpreted solutions to these equations in terms of quantum groups, certain noncommutative deformations of classical and affine universal enveloping algebras, whereby concrete solutions to the Yang–Baxter equations arise from representations of special quantum groups called Yangians. Costello's remarkable insight is that such solutions can also be understood as arising from four-dimensional supersymmetric field theories that are topological in two directions and holomorphic in one complex direction. More precisely, Costello discovered the remarkable fact that *Yangian Hopf algebras are encoded in the factorization algebra of observables in "Chern–Simons deformations" of twisted* N = 1 *gauge theories in four dimensions*.

Even though a full explanation of this statement is a bit too elaborate to include here, it is worth emphasizing that Costello has used it to give a unified construction of all the standard rational, trigonometric, and elliptic solutions to the quantum Yang–Baxter equations. In the words of Edward Witten, Costello has explained why the Yang–Baxter equations have all those solutions, in a manner that goes substantially beyond Drinfeld's groundbreaking work, resolving a fundamental mystery that has shrouded the equations for many decades since their discovery.

Unlike many mathematicians working on topics originating in physics, Kevin Costello cares deeply about the foundations of physics itself as conceived of by physicists and is constantly generating physical insights that go beyond mathematical rigor. He is respected by mathematicians and physicists alike and plays a unique role in breaking new and fertile ground on which the two communities can jointly develop directions of research even while coming to a fuller understanding of important known phenomena. He is thus eminently deserving of the Eisenbud Prize for Mathematics and Physics.

Biographical Sketch

Kevin Joseph Costello was born in 1977 in Dublin, Ireland. He did his undergraduate degree at Cambridge University and received his PhD from Cambridge in 2003 under the supervision of Ian Grojnowski. He has worked at Imperial College London, the University of Chicago, and Northwestern University and is now a Krembil Foundation Chair at the Perimeter Institute in Waterloo, Ontario. He is a Fellow of the Royal Society, a former Sloan and Simons fellow, an invited speaker at the International Congress of Mathematicians in 2010, and an invited speaker at Strings 2016.

Response from Kevin Joseph Costello

It's a huge honor to receive this award, whose previous recipients include several personal heroes of mine. Some years ago I decided to leave the beaten path of algebra and topology and explore the foundations of perturbative quantum field theory. I've had a great deal of fun doing this, but I did not expect this to be a good career move. However, the mathematics community has been very supportive over the years, as evidenced by this wonderful prize. It's a real privilege to work in a community like ours which is open to interdisciplinary, curiosity-driven research. I'm very grateful to the AMS, the Eisenbud family, and the selection committee.

About the Prize

The Eisenbud Prize was established in 2006 in memory of the mathematical physicist Leonard Eisenbud (1913–2004) by his son and daughter-in-law, David and Monika Eisenbud. Leonard Eisenbud, who was a student of Eugene Wigner, was particularly known for the book *Nuclear Structure* (1958), which he coauthored with Wigner. A friend of Paul Erdős, he once threatened to write a dictionary of "English to Erdős and Erdős to English." He was one of the founders of the physics department at the State University of New York, Stony Brook, where he taught from 1957 until his retirement in 1983. His son David was president of the AMS during 2003–04. The Eisenbud Prize for Mathematics and Physics honors a work or group of works that brings the two fields closer together. Thus, for example, the prize might be given for a contribution to mathematics inspired by modern developments in physics or for the development of a physical theory exploiting modern mathematics in a novel way. The US$5,000 prize will be awarded every three years for a work published in the preceding six years.

FROM THE AMS SECRETARY

The Eisenbud Prize is awarded by the AMS Council acting on the recommendation of a selection committee. The selection committee members for the 2020 prize were:

- Jeffrey A. Harvey (Chair)
- Minhyong Kim
- Alexei Y. Kitaev

A list of the past recipients of the Eisenbud Prize can be found at `https://www.ams.org/prizes-awards/pabrowse.cgi?parent_id=23`.

Credits

Photo of Kevin Joseph Costello is courtesy of Gabriela Secara / Perimeter Institute.