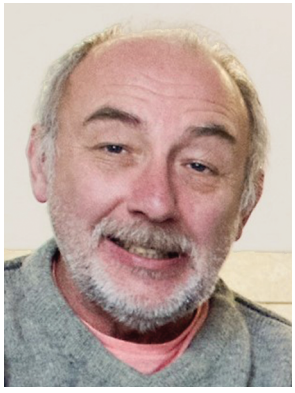
Beilinson and Kazhdan Awarded 2020 Shaw Prize

The Shaw Foundation has announced the awarding of the 2020 Shaw Prize to **Alexander Beilinson** of the University of Chicago and **David Kazhdan** of the Hebrew University of Jerusalem "for their huge influence on and profound contributions to representation theory, as well as many other areas of mathematics."


Alexander Beilinson


David Kazhdan

Citation for Beilinson and Kazhdan

Alexander Beilinson and David Kazhdan are two mathematicians who have made profound contributions to the branch of mathematics known as representation theory but who are also famous for the fundamental influence they have had on many other areas, such as arithmetic geometry, *K*-theory, conformal field theory, number theory, algebraic and complex geometry, group theory, and algebra more generally. As well as proving remarkable theorems themselves, they have created conceptual tools that have been essential to many breakthroughs of other mathematicians. Thanks to their work and its exceptionally broad reach, large areas of mathematics are significantly more advanced than they would otherwise have been.

Group theory is intimately related to the notion of symmetry, and one can think of a representation of a group as a "description" of it as a group of transformations, or symmetries, of some mathematical object, usually linear transformations of a vector space. Representations of groups are important, as they allow many group-theoretic problems to be reduced to problems in linear algebra, which is well understood. They are also important in physics because, for example, they describe how the symmetry group of a physical system affects the solutions of equations describing that system, and the representations also make the symmetry group better understood. In loose terms, representation theory is the study of the basic symmetries of mathematics and physics. Symmetry groups are of many different kinds: finite groups, Lie groups, algebraic groups, *p*-adic groups, loop groups, adelic groups. This may partly explain how Beilinson and Kazhdan have been able to contribute to so many different fields.

One of Kazhdan's most influential ideas has been the introduction of a property of groups that is known as Kazhdan's property (T). Among the representations of a group there is always the not very interesting "trivial representation," where we associate with each group element the "transformation" that does nothing at all to the object. While the trivial representation is not interesting on its own, much more interesting is the question of how close another representation can be to the trivial one. Property (T) gives a precise quantitative meaning to this question. Kazhdan used Property (T) to solve two outstanding questions about discrete subgroups of Lie groups. Since then it has had important applications to group representation theory, lattices in algebraic groups over local fields, ergodic theory, geometric group theory, expanders, operator algebras and the theory of networks, and has been recognized as a truly fundamental concept in representation theory.

After this first breakthrough, Kazhdan solved several other outstanding problems about lattices in Lie groups and representation theory, such as the Selberg conjecture about nonuniform lattices and the Springer conjecture on the classification of affine Hecke algebras.

While working with George Lusztig on this last problem, Kazhdan introduced an important family of polynomials, as well as formulating a very influential pair of (equivalent) conjectures. One of Alexander Beilinson's achievements was to prove these conjectures with Joseph Bernstein. (They

were also proved independently by Jean-Luc Brylinski and Masaki Kashiwara.) The methods introduced in this proof led to the area known as geometric representation theory, an area that Kazhdan also played an important part in developing, which aims to understand the deeper geometric and categorical structures that often underlie group representations. The resulting insights have been used to solve several open problems.

Another famous concept, this one established by Beilinson, Bernstein, and Pierre Deligne, is that of a perverse sheaf. It is not feasible to give a nontechnical explanation of what a perverse sheaf is—one well-known account begins by helpfully stating that it is neither perverse nor a sheaf—but it is another concept that can be described as a true discovery, in that it has a far from obvious definition, but it is now seen to be "one of the most natural and fundamental objects in topology" (to quote from the same account). One of the central goals of mathematics, the Langlands program, has been deeply influenced by this concept. For example, the whole work of Ngô on the "fundamental lemma" and the contributions of Laurent and Vincent Lafforgue (all three of them major prizewinners for this work) would have been unthinkable without it. Kazhdan too has brought extraordinary mathematical insight into this circle of ideas. By pointing out that orbital integrals could be interpreted as counting points on certain algebraic varieties over finite fields, he and Lusztig opened the way to the proof of the fundamental lemma, and since then Kazhdan has had and continues to have an enormous influence on the subject. Beilinson is also famous for formulating deep conjectures relating L-functions and motivic theory, which have completely changed the understanding of both topics and led to an explosion of related work.

Beilinson and Kazhdan are at the heart of many of the most exciting developments in mathematics over the last few decades, developments that continue to this day. It is for this that they are awarded the 2020 Shaw Prize in Mathematical Sciences.

Biographical Sketches

Alexander Beilinson was born in 1957 in Moscow and received his PhD in 1988 from the Landau Institute of Theoretical Physics, advised by Yuri Manin. He was a researcher at the Landau Institute from 1987 to 1993 and professor of mathematics at the Massachusetts Institute of Technology from 1988 to 1998, when he joined the University of Chicago. His research interests, along with representation theory, include algebraic geometry and mathematical physics. He was awarded the Moscow Mathematical Society Prize in 1985, the Ostrowski Prize (with Helmut Hofer) in 1999, and the Wolf Prize (with Vladimir Drinfeld) in 2018. He is a member of the US National Academy of Sciences and the American Academy of Arts and Sciences.

David Kazhdan was born in Moscow in 1946. He received a Kandidat Degree in 1969 from Moscow State University under the direction of Alexandre Kirillov. He left the Soviet Union in 1975 to take a position at Harvard University, where he is now professor emeritus. He moved to Israel in 2002. He received the Rothschild Prize in 2010; the Israel Prize, the country's highest honor, in 2012; and the EMET Prize in Exact Sciences (with Joseph Bernstein) in 2016. He was a MacArthur Fellow from 1990 to 1995. He was doctoral advisor to Vladimir Voevodsky, who won the Fields Medal in 2002. Kazhdan is a member of the US National Academy of Sciences, the American Academy of Arts and Sciences, and the Israel Academy of Sciences.

About the Prize

The Shaw Prize is an international award established to honor individuals who are currently active in their respective fields and who have achieved distinguished and significant advances, who have made outstanding contributions in culture and the arts, or who have achieved excellence in other domains. The award is dedicated to furthering societal progress, enhancing quality of life, and enriching humanity's spiritual civilization. Preference is given to individuals whose significant work was recently achieved.

The Shaw Prize consists of three annual awards: the Prize in Astronomy, the Prize in Science and Medicine, and the Prize in Mathematical Sciences. Established under the auspices of Run Run Shaw in November 2002, the prize is managed and administered by the Shaw Prize Foundation based in Hong Kong. The prize carries a cash award of US$1,200,000.

Previous recipients of the Shaw Prize in Mathematical Sciences are:

- Michel Talagrand (2019)
- Luis Caffarelli (2018)
- János Kollár and Claire Voisin (2017)
- Nigel J. Hitchin (2016)
- Gerd Faltings and Henryk Iwaniec (2015)
- George Lusztig (2014)
- David L. Donoho (2013)
- Maxim Kontsevich (2012)
- Demetrios Christodoulou and Richard S. Hamilton (2011)
- Jean Bourgain (2010)
- Simon K. Donaldson and Clifford H. Taubes (2009)
- Vladimir Arnold and Ludwig Faddeev (2008)
- Robert Langlands and Richard Taylor (2007)
- David Mumford and Wen-Tsun Wu (2006)
- Andrew Wiles (2005)
- Shiing-Shen Chern (2004)

—Shaw Foundation announcement

Credits

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